**Golay Codes**

The generator matrix for a systematic extended Golay(24,12) code is:

where is the identity matrix and is the parity array matrix:

This parity array matrix has the following two useful properties:

The Golay(24,12) code is self-dual meaning that the generator matrix and the parity-check matrix are the same:

The decoding algorithm determines the error pattern such that:

The syndrome is calculated as usual:

We split the error pattern into two equal length 12-tuples . Then:

Post-multiply both sides of the above equation by and rearrange to find:

The Golay(24,12) code corrects up to three errors. For correctable error patterns:

There are four possibilities:

These four possibilities define four different types of correctable error patterns:

* If then where is the all-zero 12-tuple. From this:

.

For this case and the correctable error pattern is:

* If then where is the unit 12-tuple with a one in the th position.

where denotes the th row of the matrix . Rearranging,

Now . The correctable error pattern is:

* If or and then . It follows that:

Here and the correctable error pattern becomes:

* Finally if and then . This gives:

For this possibility . The correctable error pattern is:

A decoding algorithm can be devised for the Golay(24,12) code based on this analysis:

**Step 1:** Compute the syndrome .

**Step 2:** If then set .

Go to Step 8.

**Step 3:** If for some row of then set .

Go to Step 8.

**Step 4:** Compute the second syndrome .

**Step 5:** If then set .

Go to Step 8.

**Step 6:** If for some row of then set .

Go to Step 8.

**Step 7:** The received sequence cannot be decoded with this algorithm.

Possible actions include:

* Declare a decoding failure
* Request retransmission
* Report the detection of a 4-bit error pattern
* Use the syndrome and a lookup table to decode a select subset of all the possible 4-bit error patterns

**Step 8:** The estimated code word is .

The Golay(23,12) code can be generated using the generator matrix for the extended Golay(24,12) code after deleting the last column. The decoding algorithm for the Golay(24,12) code can be used to decode the Golay(23,12) code if an overall parity bit is appended to the end of the 23-tuple using *odd* parity. This appended bit is discarded after decoding.



