**Hamming Codes**

All Hamming codes have a minimum distance of 3 which means they can either correct a single error or detect two errors.

If is the number of parity bits for a Hamming code then:

The following two Wikipedia articles are very good treatments of the Hamming codes. The first is specific to the Hamming(7,4) code while the latter is more general.

<https://en.wikipedia.org/wiki/Hamming(7,4)>

<https://en.wikipedia.org/wiki/Hamming_code>

When reading the first article note the matrix equations use column vectors for the code words whereas Sklar’s textbook and my class notes use row vectors. This means the terms and equations in the various sources are related by matrix transpose operations. For example in the textbook and class notes encoding and syndrome calculations are accomplished by:

In the Wikipedia article these operations are accomplished by:

where the subscript *W* denotes the generator and parity-check matrices in the Wikipedia article.

where signifies a row-wise flipping of the elements of . The syndrome calculations in my class notes are *big-endian* meaning the most significant error locator bit is encountered first in the leftmost position of the syndrome row vector. The syndrome calculated in the Wikipedia article is also big-endian with the most significant error locator bit at the top of the syndrome column vector.

**Systematic Hamming Codes**

If the information symbols are grouped together the code is systematic. The MATLAB function [H, G] = hammgen(m) creates systematic parity-check and generator matrices for a Hamming code with parity symbols. The MATLAB generated Hamming codewords have the parity symbols first followed by the information symbols.

For example with :

>> [H, G] = hammgen(3)

H =

1 0 0 1 0 1 1

0 1 0 1 1 1 0

0 0 1 0 1 1 1

G =

1 1 0 1 0 0 0

0 1 1 0 1 0 0

1 1 1 0 0 1 0

1 0 1 0 0 0 1

For systematic Hamming codes there is no easy rule for determining the error pattern associated with each syndrome. A standard array must be constructed for decoding. For the MATLAB generated Hamming code in this example the standard array is:

**i si ei**

0 000 0000000

1 101 0000001

2 111 0000010

3 011 0000100

4 110 0001000

5 001 0010000

6 010 0100000

7 100 1000000