In[5]:=
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(* Run the script in the directory of the present notebook *)
Get[NotebookDirectory[] <> "prin_symb.m"]
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Use the following Pythagorean quadruples as incoming directions

{{3, 2, 2, -1}, {6, 2, 4, -4}, {6, 4, 2, -4}, {9, 4, 4, -7}, {11, 6, 2, -9}, {14, 6, 4, -12}}

Trying 15 choices of 4-tuples of incoming directions

Choice 1

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 6 & 9 \\ 2 & 2 & 4 & 4 \\ 2 & 4 & 2 & 4 \\ -1 & -4 & -4 & -7 \end{pmatrix}$$

SKIPPING: the vectors b_j do not form a basis

Choice 2

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 6 & 11 \\ 2 & 2 & 4 & 6 \\ 2 & 4 & 2 & 2 \\ -1 & -4 & -4 & -9 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} \frac{192}{5} & \frac{232}{5} & 80 & 32 \\ \frac{232}{5} & \frac{272}{5} & 80 & 32 \\ 80 & 80 & \frac{192}{5} & -\frac{144}{5} \\ 32 & 32 & -\frac{144}{5} & -\frac{192}{5} \end{pmatrix}$$

Choice 3

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 6 & 14 \\ 2 & 2 & 4 & 6 \\ 2 & 4 & 2 & 4 \\ -1 & -4 & -4 & -12 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -24 & 0 & 104 & 88 \\ 0 & 24 & 104 & 88 \\ 104 & 104 & 64 & -32 \\ 88 & 88 & -32 & -64 \end{pmatrix}$$

Choice 4

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 9 & 11 \\ 2 & 2 & 4 & 6 \\ 2 & 4 & 4 & 2 \\ -1 & -4 & -7 & -9 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} \frac{72}{5} & \frac{12}{5} & 16 & 4 \\ \frac{12}{5} & -\frac{48}{5} & 16 & 4 \\ 16 & 16 & \frac{32}{5} & \frac{16}{5} \\ 4 & 4 & \frac{16}{5} & -\frac{32}{5} \end{pmatrix}$$

Choice 5

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 9 & 14 \\ 2 & 2 & 4 & 6 \\ 2 & 4 & 4 & 4 \\ -1 & -4 & -7 & -12 \end{pmatrix}$$

SKIPPING: one of the vectors xi_j is zero

Choice 6

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 11 & 14 \\ 2 & 2 & 6 & 6 \\ 2 & 4 & 2 & 4 \\ -1 & -4 & -9 & -12 \end{pmatrix}$$

SKIPPING: the vectors b_j do not form a basis

Choice 7

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 9 & 11 \\ 2 & 4 & 4 & 6 \\ 2 & 2 & 4 & 2 \\ -1 & -4 & -7 & -9 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -\frac{78}{5} & -\frac{73}{5} & -10 & 1 \\ -\frac{73}{5} & -\frac{68}{5} & -10 & 1 \\ -10 & -10 & -\frac{16}{5} & \frac{16}{5} \\ 1 & 1 & \frac{16}{5} & \frac{16}{5} \end{pmatrix}$$

Choice 8

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 9 & 14 \\ 2 & 4 & 4 & 6 \\ 2 & 2 & 4 & 4 \\ -1 & -4 & -7 & -12 \end{pmatrix}$$

SKIPPING: one of the vectors xi_j is zero

Choice 9

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 6 & 11 & 14 \\ 2 & 4 & 6 & 6 \\ 2 & 2 & 2 & 4 \\ -1 & -4 & -9 & -12 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -152 & -140 & -96 & 12 \\ -140 & -128 & -96 & 12 \\ -96 & -96 & -32 & 32 \\ 12 & 12 & 32 & 32 \end{pmatrix}$$

Choice 10

The incoming directions b_j as column vectors

$$\begin{pmatrix} 3 & 9 & 11 & 14 \\ 2 & 4 & 6 & 6 \\ 2 & 4 & 2 & 4 \\ -1 & -7 & -9 & -12 \end{pmatrix}$$

SKIPPING: one of the vectors xi_j is zero

Choice 11

The incoming directions b_j as column vectors

$$\begin{pmatrix} 6 & 6 & 9 & 11 \\ 2 & 4 & 4 & 6 \\ 4 & 2 & 4 & 2 \\ -4 & -4 & -7 & -9 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -\frac{279}{10} & -\frac{117}{5} & \frac{3}{2} & \frac{27}{2} \\ -\frac{117}{5} & -\frac{189}{10} & \frac{3}{2} & \frac{27}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{18}{5} & -\frac{6}{5} \\ \frac{27}{2} & \frac{27}{2} & -\frac{6}{5} & -\frac{18}{5} \end{pmatrix}$$

Choice 12

The incoming directions b_j as column vectors

$$\begin{pmatrix} 6 & 6 & 9 & 14 \\ 2 & 4 & 4 & 6 \\ 4 & 2 & 4 & 4 \\ -4 & -4 & -7 & -12 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -\frac{168}{5} & -\frac{128}{5} & \frac{88}{5} & \frac{136}{5} \\ -\frac{128}{5} & -\frac{88}{5} & \frac{88}{5} & \frac{136}{5} \\ \frac{88}{5} & \frac{88}{5} & \frac{64}{5} & -\frac{32}{5} \\ \frac{136}{5} & \frac{136}{5} & -\frac{32}{5} & -\frac{64}{5} \end{pmatrix}$$

Choice 13

The incoming directions b_j as column vectors

$$\begin{pmatrix} 6 & 6 & 11 & 14 \\ 2 & 4 & 6 & 6 \\ 4 & 2 & 2 & 4 \\ -4 & -4 & -9 & -12 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} -\frac{1688}{5} & -\frac{1408}{5} & -\frac{1064}{5} & \frac{168}{5} \\ -\frac{1408}{5} & -\frac{1128}{5} & -\frac{1064}{5} & \frac{168}{5} \\ -\frac{1064}{5} & -\frac{1064}{5} & -\frac{384}{5} & \frac{288}{5} \\ \frac{168}{5} & \frac{168}{5} & \frac{288}{5} & \frac{384}{5} \end{pmatrix}$$

Choice 14

The incoming directions b_j as column vectors

$$\begin{pmatrix} 6 & 9 & 11 & 14 \\ 2 & 4 & 6 & 6 \\ 4 & 4 & 2 & 4 \\ -4 & -7 & -9 & -12 \end{pmatrix}$$

The full 4th order interaction

$$\begin{pmatrix} 352 & 256 & \frac{408}{5} & -\frac{696}{5} \\ 256 & 160 & \frac{408}{5} & -\frac{696}{5} \\ \frac{408}{5} & \frac{408}{5} & \frac{32}{5} & -\frac{64}{5} \\ -\frac{696}{5} & -\frac{696}{5} & -\frac{64}{5} & -\frac{32}{5} \end{pmatrix}$$

Choice 15

The incoming directions b_j as column vectors

$$\begin{pmatrix} 6 & 9 & 11 & 14 \\ 4 & 4 & 6 & 6 \\ 2 & 4 & 2 & 4 \\ -4 & -7 & -9 & -12 \end{pmatrix}$$

SKIPPING: the vectors b_j do not form a basis

Number of successful choices

9

The dimension of the subspace spanned by the principal symbols

6