

## Question 1

1 i) number of comparisons:

- pass 1: compare positions 1-2, 2-3, 3-4 ... (n-1)-n  
→ n-1 comparisons
- pass 2: compare up to position n-1  
→ n-2 comparisons
- ...
- pass n-1: 1 comparison

total number of comparisons:

$$(n-1) + (n-2) \dots + 2 + 1$$

$$\Rightarrow \sum_{k=1}^{n-1} k = \boxed{\frac{n(n-1)}{2}}$$

1 ii) average case number of swaps (random order)

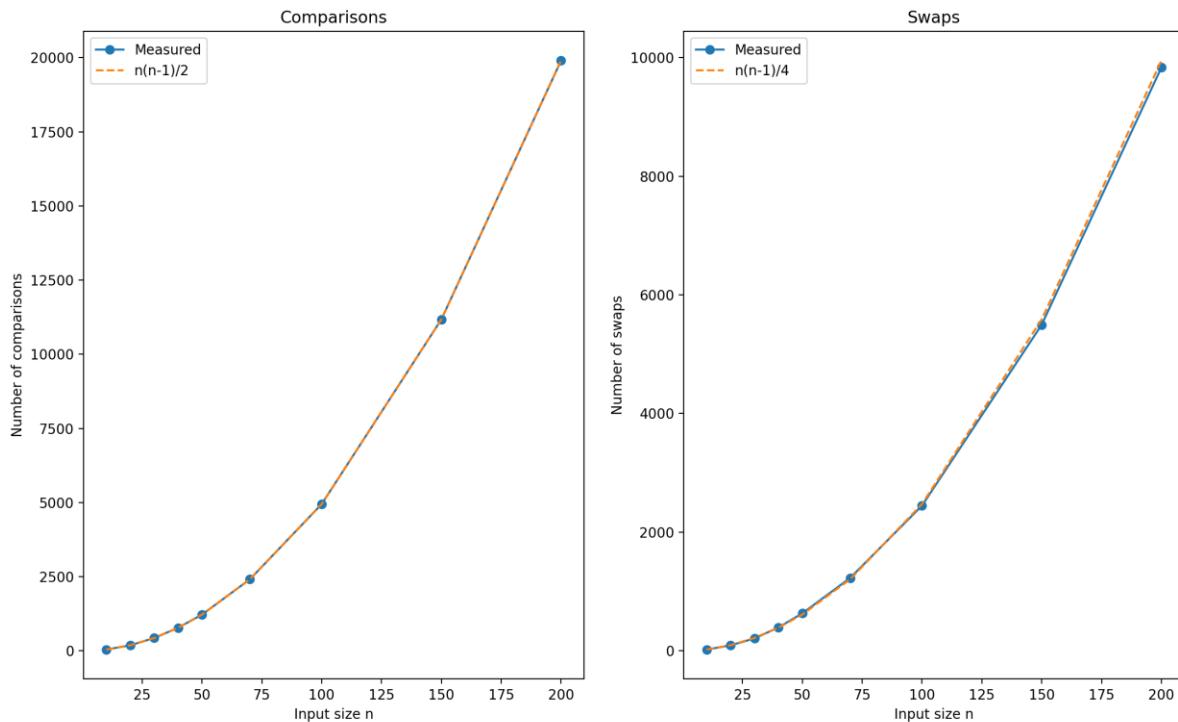
1 swap for every pair out of order

for a random permutation of n elements, total possible pairs:  $\binom{n}{2} = \frac{n(n-1)}{2}$

any pair has a  $\frac{1}{2}$  probability of being out of order

$$\text{so } \frac{1}{2} \times \frac{n(n-1)}{2} = \boxed{\frac{n(n-1)}{4}}$$

## Question 4



The experimental results of the comparison graph match the expected values (follows  $n(n-1)/2$ ) as bubble sort performs the same number of comparisons for a given  $n$  regardless of its input values, while the experimental results of the swaps graph closely follow  $n(n-1)/4$  with some variation due to the randomly generated inputs (appropriate for average-case complexity analysis). Bubble sort is  $O(n^2)$  for both comparisons and swaps, which matches the experimental results as both graphs show quadratic growth.