

RHODES UNIVERSITY

Department of Mathematics (Pure & Applied)

CLASS TEST NO. 1: 15 August, 2019

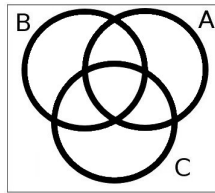
Algebra and Analysis (in MAM 2)

Time: 90 minutes
Marks Available: 40
Maximum Marks: 40

All questions may be answered. Marks are shown in brackets.

Question 1: [3 Marks]

Reproduce the diagram below and shade the area corresponding to $(A \cup B) \triangle C$.



Question 2: [6 Marks]

Show that for any sets A, B we have

$$\overline{(A \triangle B)} \cap A = A \cap B.$$

Question 3: [3+6 Marks]

- State the definition of an invertible mapping.
- Suppose that the map $f : A \longrightarrow B$ is invertible. Prove that f is injective.

Question 4: [6 Marks]

Let

$$\begin{aligned} f : \mathbb{Z} &\longrightarrow \mathbb{Z} \\ n &\longmapsto 10n - 1. \end{aligned}$$

Determine if f is surjective or not. Justify your answer.

Question 5: [9+7 Marks]

Let \diamond and \star be two operations on \mathbb{R} define for any $a, b \in \mathbb{R}$ by

$$a \diamond b = \frac{3a + 3b - 1}{3} \quad \text{and} \quad a \star b = \frac{ab}{3} - 1.$$

- Show that \diamond has an identity in \mathbb{R} . Find the inverse of 3 relative to \diamond , if it exists.
- Determine if \star has an identity or not. Justify your answer.

—————**END**—————