

# HW 1: Quantum Mechanics as a 1 + 0 Dimensional Quantum Field Theory

March 8, 2019

*will be officially posted on **March 7** and due on March 24. Before March 7, any modifications to the problems are possible.*

## 1 Problem 1: $q^4$ theory at $\mathcal{O}(\lambda^3)$

In this problem, we continue our study of the system governed by the Hamiltonian

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2 + \frac{\lambda}{4!}q^4, \quad (1)$$

where we have set  $m = 1$  and  $\hbar = 1$ . We want to calculate the ground energy  $E_0$  of the system by evaluating the transition amplitude

$$\int dx \langle x | e^{-i\hat{H}T} | x \rangle, \quad (2)$$

perturbatively in  $\lambda$ . *All calculations below should be performed by assuming the analytic continuation  $T \rightarrow -iT$  and we only consider the large  $T$  limit, just like what we did in the class.*

- **Draw** all possible connected Feynman diagrams contribute at  $\mathcal{O}(\lambda^3)$ . (4 distinct diagrams)
- Based on the diagrams you draw above, **construct** the integrands using Feynman rules. Note that you should multiply each diagram by its symmetry factor.
- **Integrate over** the “time”  $\tau_1, \tau_2$  and  $\tau_3$  to find the  $\lambda^3$  contributions to the transition amplitude.
- **Find** the  $\lambda^3$  correction to the ground Energy  $E_0$ , i.e.,

$$E_0 = \frac{1}{2}\omega + \frac{\lambda}{4!} \left(\frac{1}{2\omega}\right)^2 \times 3 - \left(\frac{\lambda}{4!}\right)^2 \left(\frac{1}{2\omega}\right)^4 \frac{21}{\omega} + \lambda^3 \times ? + \mathcal{O}(\lambda^4). \quad (3)$$

## 2 Problem 2: Instanton

*(not finished yet!)*

Now we consider

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2 - \frac{\lambda}{4!}q^4, \quad \text{or} \quad L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega^2 q^2 + \frac{\lambda}{4!}q^4. \quad (4)$$

The “−” sign before the  $\frac{\lambda}{4!}q^4$  term makes the ground energy  $E_0$  in-accessible by a perturbative calculation in  $\lambda$ . Again we work with the imaginary time  $T \rightarrow -iT$ .

- **Find** the  $E_0$  using Feynman path integral. From the result that you obtain, explain why the perturbative calculation in  $\lambda$  will not work.