HW 1: Quantum Mechanics as a $1+0$ Dimensional Quantum Field Theory

March 17, 2019

will be officially posted on March 7 and due on March 24. Before March 7, any modifications to the problems are possible.

1 Problem 1: q^4 theory at $\mathcal{O}(\lambda^3)$

In this problem, we continue our study of the system governed by the Hamiltonian

$$H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2 + \frac{\lambda}{4!}q^4\,, \tag{1}$$

where we have set $m = 1$ and $\hbar = 1$. We want to calculate the ground energy E_0 of the system by evaluating the transition amplitude

$$\int \mathrm{d}x \, \langle x|e^{-i\hat{H}T}|x\rangle\,, \tag{2}$$

perturbatively in λ. *All calculations below should be performed by assuming the analytic continuation $T \to -iT$ and we only consider the large T limit, just like what we did in the class.*

- **Draw** all possible connected Feynman diagrams contribute at $\mathcal{O}(\lambda^3)$. (4 distinct diagrams)
- Based on the diagrams you draw above, **construct** the integrands using Feynman rules. Note that you should multiply each diagram by its symmetry factor.
- **Integrate over** the "time" τ_1, τ_2 and τ_3 to find their contributions to the $\mathrm{O}(\lambda^3)$ corrections to the transition amplitude.
- **Find** the λ^3 correction to the ground Energy E_0, i.e.,

$$E_0 = \frac{1}{2}\omega + \frac{\lambda}{4!}\left(\frac{1}{2\omega}\right)^2 \times 3 - \left(\frac{\lambda}{4!}\right)^2 \left(\frac{1}{2\omega}\right)^4 \frac{21}{\omega} + \lambda^3 \times ? + \mathcal{O}(\lambda^4). \tag{3}$$

2 Problem 2: Instanton

Follow what we discussed in the class to derive yourselves to strengthen your understanding. Please show sufficient steps!!

Now we consider

$$H = \frac{1}{2}p^2 - \frac{1}{2}\omega^2 q^2 + \frac{\lambda}{4!}q^4\,, \qquad \text{or} \;\; L = \frac{1}{2}\dot{q}^2 + \frac{1}{2}\omega^2 q^2 - \frac{\lambda}{4!}q^4\,. \tag{4}$$

The "$-$" sign before the $\frac{1}{2}\omega^2 q^2$ term makes the ground energy E_0 in-accessible by a perturbaive calculation in λ. Again we work with the imaginary time $T \to -iT$.

- **Find** the E_0 using Feynman path integral. From the result that you obtain, explain why the perturbative calculation in λ will not work.