CSCI-GA-2271 Fall 2025 Assignment 0

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Question 1

P (1 wins) = 1/5, P (1 fail) = 4/5 P (2 wins) = 1/4, P (2 fail) = 3/4

P (1 fail, 2 fail) = (1-p1) (1-p2) = (4/5) (3/4) = 3/5

P(1 wins) = P(1 wins) + P(1 fail, 2 fail, 1 wins) + P(1 fail, 2 fail , 1 fail, 2 fail, 1 wins) ..

P(1 wins) = 1/5 + P(1 fail, 2 fail) * (1/5) + P(1 fail, 2 fail)$^2 * $(1/5).. = (1/5)$* \frac{1}{1-P(1fail,2fail)}$

$$P(1wins) = \frac{1}{5} + \frac{1}{1-3/5} = 0.5$$

because of geometric series, to simplify 1 + P(1 fail, 2 fail) + P(1 fail, 2 fail)^2 + $P(1fail,2fail)^3$..

Question 2

P(COVID) = 0.01, P(no COVID) = 0.99, P(+—COVID) =0.9, P(-—COVID)=0.1

$$P(COVID|+) = \frac{P(COVID \cap +)}{P(+)}$$

Expand numerator,

$$P(COVID \cap +) = P(COVID) * P(+|COVID) = 0.01 * 0.9 = 0.009$$

Denominator 0.009+(0.10)(0.99) = 0.009+0.099 = 0.108

$$\frac{0.009}{0.108} \approx 0.0833$$

Question 3

Valid iff it maintains non-negativity and the $\int_{-\infty}^{\infty} \frac{1}{1+x} f(x)dx = 1$

so

i) f(x)=0 for $x < 0$ and $f(x) = \frac{1}{1+x} > 0$ for $x \geq 0$

ii) $\int_{-\infty}^{\infty} \frac{1}{1+x} f(x)dx = \int_{0}^{\infty} \frac{1}{1+x} dx = [ln(1+x)]_0^{\infty}$

$ln(1+0) = 0$
$ln(1+100,000) \approx 11.513$
$ln(1+1,000,000) \approx 13.8155$

$f(x)$ is not a valid PDF since $\int_{-\infty}^{\infty} \frac{1}{1+x} f(x)dx$ diverges as x grows larger.

Question 4

$P(X+Y \leq C) = \int\int_{x+y \leq c} f_x(x) f_y(y) dxdy, X \perp Y$

$P(X+Y \leq C) = \int_0^1 \int_0^{1-x} f_X(x) f_Y(y) dxdy$
$= \int_0^1 \int_0^{1-x} (2x)(2y) dxdy$
$= \int_0^1 (4x)[\frac{y^2}{2}]_0^{1-x} dx$
$= \int_0^1 (2x(1-x)^2 dx$
$= \int_0^1 (2x - 4x^2 + 2x^3) dx$
$= [x^3 - \frac{4}{3}x^3 + \frac{1}{2}x^4]_0^1 = 1 - \frac{4}{3} + \frac{1}{2} = \frac{1}{6}$

$P(X+Y \leq C) = \frac{1}{6}$

Question 5

X $\sim$ Unif$(0, 1)$, $\qquad Y = e^X$.

The expectation of Y is

$\mathbb{E}[Y] = \mathbb{E}[e^X] = \int_0^1 e^x \cdot f_X(x)\, dx$

Since $X \sim$ Unif$(0, 1)$, the pdf is

$f_X(x) = 1 \quad$ for $0 < x < 1$

continued

$\mathbb{E}[Y] = \int_0^1 e^x dx$

$\int_0^1 e^x dx = \mathrm{e}^x$

$\mathrm{e}^1 - e^0 = e - 1$

Question 6

We know that $X_i \sim \text{Poisson}(\lambda = 5), ; i = 1, \dots, 125$.

So the sum $S = \sum_{i=1}^{125} X_i \sim \text{Poisson}(125 \cdot 5) = \text{Poisson}(625)$.

$\bar{X} = \frac{S}{125}$, and the question is about $\mathbb{P}(\bar{X} < 5.5)$.

Since $125 \cdot 5.5 = 687.5$, this means $\mathbb{P}(\bar{X} < 5.5) = \mathbb{P}(S \leq 687)$.

$\mathbb{P}(S \leq 687) = e^{-625} \sum_{k=0}^{687} \frac{625^k}{k!}$, but that is not computable by hand.

CLT

For S, $\mu = 625, \quad \sigma^2 = 625, \quad \sigma = \sqrt{625} = 25$.

So we can do a normal approx: $Z = \frac{687.5 - 625}{25} = \frac{62.5}{25} = 2.5$.

Therefore $\mathbb{P}(S \leq 687) \approx \Phi(2.5) \approx 0.9938$.

So the answer is $\mathbb{P}(\bar{X} < 5.5) \approx 0.994$.

Question 7

We have the recurrence

$X_n = f(W_n, X_{n-1}), ; n = 1, \ldots, p,$

and the error

$E = |c - X_p|^2 = (c - X_p)^\top (c - X_p).$

First,

$\frac{\partial E}{\partial X_p} = 2(X_p - c).$

Then by chain rule (back through the recurrence):

$\frac{\partial E}{\partial X_{p-1}} = \frac{\partial E}{\partial X_p}, \frac{\partial X_p}{\partial X_{p-1}},$

$\frac{\partial E}{\partial X_{p-2}} = \frac{\partial E}{\partial X_{p-1}}, \frac{\partial X_{p-1}}{\partial X_{p-2}},$

and so on, until we reach

$\frac{\partial E}{\partial X_0} = \frac{\partial E}{\partial X_p} \cdot \frac{\partial X_p}{\partial X_{p-1}} \cdot \frac{\partial X_{p-1}}{\partial X_{p-2}} \cdots \frac{\partial X_1}{\partial X_0}.$

Finally, substituting the form of each X_n:

$\frac{\partial E}{\partial X_0} = 2(X_p - c), \frac{\partial f(W_p, X_{p-1})}{\partial X_{p-1}}, \frac{\partial f(W_{p-1}, X_{p-2})}{\partial X_{p-2}} \cdots \frac{\partial f(W_1, X_0)}{\partial X_0}.$

Question 8

$Ax, A^T x, x^T A$

Ax
2(2) + 6(3) + 7(4) = 4+18+28 = 50
3(2) + 1(3) + 2(4) = 6+3+8 = 17
5(2) + 3(3) + 4(4) = 10+9+16 = 35
$Ax = \begin{bmatrix} 50 \\ 17 \\ 35 \end{bmatrix}$
A$^T x$
$A^T = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 1 & 3 \\ 7 & 2 & 4 \end{bmatrix}$

$A^T x$
2(2) + 3(3) + 5(4) = 4+9+20 = 33
6(2) + 1(3) + 3(4) = 12+3+12 = 27
2(7) + 2(3) + 4(4) = 14 + 6 + 16 = 36
$\begin{bmatrix} 33 \\ 27 \\ 36 \end{bmatrix}$

$x^T A$
$$\begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$
2(2) + 3(3) + 4(5) = 4+9+20 = 33
2(6) + 3(1) + 4(3) = 12+3+12 = 27
2(7) + 3(2) + 4(4) = 14+6+16 = 36
$x^T A = \begin{bmatrix} 33 & 27 & 36 \end{bmatrix}$

Question 9

$a = 6, b = 2, c = 3, d = 3, e = 1, f = 1, g = 10, h = 3, i = 4$

$det A = 6(1*4-1*3)-2(3*4-1*10)+3(3*3-1*10) = 6(1)-2(2)+3(-1) = -1! = 0$

A is invertible

I gave an LLM a snapshot of my work prompting it with the following: "Can you make this in latex"

$$C_{11} = +\det\begin{bmatrix}1 & 1\\3 & 4\end{bmatrix} = 4-3 = 1,$$

$$C_{12} = -\det\begin{bmatrix}3 & 1\\10 & 4\end{bmatrix} = -(12-10) = -2,$$

$$C_{13} = +\det\begin{bmatrix}3 & 1\\10 & 3\end{bmatrix} = 9-10 = -1,$$

$$C_{21} = -\det\begin{bmatrix}2 & 3\\3 & 4\end{bmatrix} = -(8-9) = +1,$$

$$C_{22} = +\det\begin{bmatrix}6 & 3\\10 & 4\end{bmatrix} = 24-30 = -6,$$

$$C_{23} = -\det\begin{bmatrix}6 & 2\\10 & 3\end{bmatrix} = -(18-20) = +2,$$

$$C_{31} = +\det\begin{bmatrix}2 & 3\\1 & 1\end{bmatrix} = 2-3 = -1,$$

$$C_{32} = -\det\begin{bmatrix}6 & 3\\3 & 1\end{bmatrix} = -(6-9) = +3,$$

$$C_{33} = +\det\begin{bmatrix}6 & 2\\3 & 1\end{bmatrix} = 6-6 = 0.$$

$$C = \begin{bmatrix}1 & -2 & -1\\1 & -6 & 2\\-1 & 3 & 0\end{bmatrix}, \qquad \mathrm{adj}(A) = C^{\top} = \begin{bmatrix}1 & 1 & -1\\-2 & -6 & 3\\-1 & 2 & 0\end{bmatrix}.$$

Divide by the adj A/det A:

$$\begin{bmatrix}-1 & -1 & 1\\2 & 6 & -3\\1 & -2 & 0\end{bmatrix}.$$

b)

$$a = 1,\ b = 2,\ c = 3,\ d = 0,\ e = 2,\ f = 2,\ g = 1,\ h = 4,\ i = 5$$

$\det(B) = 1(2\cdot5-2\cdot4) - 2(0\cdot5-2\cdot1) + 3(0\cdot4-2\cdot1) = (10-8)-2(0-2)+3(0-2) = 2+4-6 = 0$

Therefore, B is not inversible

Question 10

Eigenvectors are vectors, which represent the direction and can be scaled by a scalar value λ, where the eigenvector's direction remains unchanged. Eigenvalues are features of a matrix, which can also prove if a matrix is unequal or not.

$$\det\left(\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}1 & 0 & -1\\ 1 & 0 & 0\\ -2 & 2 & 1\end{bmatrix}\right)$$

$$\begin{bmatrix}\lambda - 1 & 0 & 1\\ -1 & \lambda - 0 & 0\\ 2 & -2 & \lambda - 1\end{bmatrix}$$

1st term$(\lambda-1)(\lambda(\lambda-1)) - (0_(- 2)) = (\lambda-1)(\lambda(\lambda-1)) = (\lambda-1)(\lambda^2-\lambda)$

2nd term b=0

3rd term $c(dh - eg) = 1((-1)(-2) - (\lambda)(2)) = 2 - 2\lambda$

$\lambda^3 - 2\lambda^2 + \lambda + (2 - 2\lambda)$

$=\lambda^3 - 2\lambda^2 - \lambda + 2$

$=(\lambda-2)(\lambda-1)(\lambda+1)$

eigenvalues $\lambda \in 2, 1, -1$