Someone, Signals and Systems Have Fun!

1 Modulation

Sinusoidal Amplitude Modulation

Synchronous Demodulation:

$y(t) = cos(\omega_c t)x(t)$

$c'(t) = cos(\omega_c t + \Delta\omega t + \phi)$ (local carrier)

$x_r(t) = x(t)cos(\Delta\omega t + \phi)$

Asynchronous Demodulation:

$y(t) = cos(\omega_c t)[x(t) + A]$, $A + x(t) > 0$

no DC component $\rightarrow \eta = \frac{E(x^2(t))}{E(x^2(t))+A^2} < \frac{1}{1+PAPR_x}$

Quadrature Modulation

$y(t) = x_I(t)cos(\omega_c t) - x_Q(t)sin(\omega_c t)$

$y(t)cos(\omega_c t + \phi) \rightarrow LPF(\text{Gain} = 2) \rightarrow x_I(t)cos(\phi) + x_Q(t)sin(\phi)$

$-y(t)sin(\omega_c t) \rightarrow LPF \rightarrow x_Q(t)$

Single-Sideband Modulation

$x(t) = x_I(t)$

$H_h(j\omega) = \begin{cases} -j, & \omega > 0 \\ j, & \omega < 0 \end{cases}$

$y_{USB}(t) \rightarrow X_Q(j\omega) = X(j\omega)H_h(j\omega)$, note the ω_{c0} of LPF

$y_{SSB}(t) \rightarrow X_Q(j\omega) = -X(j\omega)H_h(j\omega)$

spectrum efficiency: SSB = QM = $2\times$AM

Pulse-Amplitude Modulated Signals

$y(t) = \Sigma x(nT)p(t-nT)$, $p(t) = Sa(\frac{\pi t}{T}) + p_1(t)$

$P_1(j\omega)$ is real, $\omega_M = \pm\frac{2\pi}{T}$, even, odd around $\frac{\pi}{T}$ (ISI-free)

Frequency Modulation

$y(t) = cos(\omega_c t + \theta_c(t))$

$\frac{d\theta_c(t)}{dt} = k_f x(t)$, $x(t) = Acos(\omega_m t)$

$m_f = \frac{k_f A}{\omega_m}$, $y(t) = cos(\omega_c t + m_f sin(\omega_m t))$

NB: $m_f << \frac{\pi}{2}$

$y(t) \approx cos(\omega_c t) - m_f(sin(\omega_m t))(sin(\omega_c t))$

$= cos(\omega_c t) + 0.5m_f cos(\omega_c t + \omega_m t) - 0.5m_f cos(\omega_c t - \omega_m t)$

WM:

Multiplexing and Multiple Access

2 Laplace Transform

$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

Unilateral: $\int_{0-}^{\infty}$

Properties

Reduced rational fraction $X(s) = \frac{N(s)}{D(s)}$ (When $s = \infty$ is zero or pole)

ROC

Right sided:($\sigma \in$ ROC) $\rightarrow$ ($\Re e s > \sigma \subseteq$ ROC)

Left sided:($\sigma \in$ ROC) $\rightarrow$ ($\Re e s < \sigma \subseteq$ ROC)

Two sided: $\sigma_1 < \Re e s < \sigma_2$

Bounded by poles or ∞, no poles contained in ROC

ILT

$X(s) = \mathcal{F}\{x(t)e^{-\sigma t}\} \rightarrow x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}dt$

$= \frac{e^{\sigma t}}{2\pi}\int_{-\infty}^{+\infty} X(\sigma + j\omega)e^{j\omega t}d\omega$ (when $\Re e s = \sigma$ in ROC)

$\mathcal{L}^{-1}\{s^n\} = u_n(t), all\ s$

$\mathcal{L}^{-1}\{\frac{1}{(s+a)^n}\} = \begin{cases} \frac{-t^{n-1}}{(n-1)!}e^{-at}u(-t), & \Re e s < -a \\ \frac{t^{n-1}}{(n-1)!}e^{-at}u(t), & \Re e s > -a \end{cases}$

Properties(ROC)

$X_1(s) = \mathcal{L}\{x_1(t)\}$, ROC $= R_1$

$X_2(s) = \mathcal{L}\{x_2(t)\}$, ROC $= R_2$

$aX_1(s) + bX_2(s) = \mathcal{L}\{ax_1(t) + bx_2(t)\}$, ROC $\supseteq R_1 \cap R_2$ ($R_1 \cap R_2 = \emptyset \rightarrow$ ROC $= \emptyset$)

$X_1^*(s^*) = \mathcal{L}\{x_1^*(t)\}$, ROC $= R_1$

$e^{-s\tau}X_1(s) = \mathcal{L}\{x_1(t-\tau)\}$, ROC $= R_1$

$X_1(s-\tau) = \mathcal{L}\{x_1(t)e^{\tau t}\}$, ROC $= R_1 + \Re e\tau$

$\frac{1}{|a|}X_1(\frac{s}{a}) = \mathcal{L}\{x_1(at)\}$, ROC $= aR_1 = \{s|\frac{s}{a} \in R_1\}$

$X_1(s)X_2(s) = \mathcal{L}\{x_1(t) * x_2(t)\}$, ROC $\supseteq R_1 \cap R_2$ (This equality may not hold if $R_1 \cap R_2 = \emptyset$)

$sX(s) = \mathcal{L}\{x'(t)\}$, ROC $\supseteq R_1$, $X'(s) = \mathcal{L}\{-tx(t)\}$, ROC $= R_1$

$\frac{1}{s}X(s) = \mathcal{L}\{\int_{-\infty}^{t} x(\tau)d\tau\}$, ROC $\supseteq (R \cap (\Re e s > 0))$

Examples

$\mathcal{L}\{\cos(\omega_0 t)u(t)\} = \frac{s}{s^2+\omega_0^2}$, $\mathcal{L}\{\sin(\omega_0 t)u(t)\} = \frac{\omega_0}{s^2+\omega_0^2}$

Analysis

Casual $\rightarrow$ ROC = right-half plane

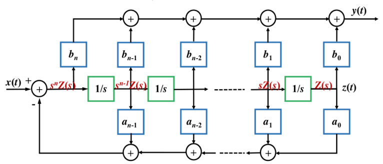
Casual(Rational) $\leftrightarrow$ ROC = right-half plane

LTI Stability $\leftrightarrow$ $j\omega$ − axis $\subseteq$ ROC

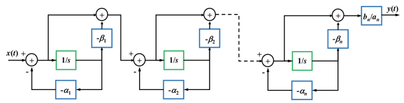
Casual Stability $\leftrightarrow$ all poles have negative real part

Block Diagram

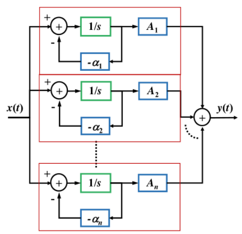
Direct: $H(s) = \frac{\sum b_i s^i}{\sum a_i s^i}$,



Parrallel: $H(s) = A\frac{\prod s-\alpha_i}{\prod s-\beta_i}$



Cascade: $H(s) = \sum\frac{A_i}{s-a_i}$



Unilateral Laplace

3 z Transform

$X(z) = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$, $z = e^s$: $\Re e s = \sigma \rightarrow |z| = e^{\sigma}$

Examples

$\mathcal{Z}\{\cos(\omega_0 n)u[n]\} = \frac{1-z^{-1}cos\omega_0}{1-2z^{-1}cos\omega_0+z^{-2}}$

$\mathcal{Z}\{\sin(\omega_0 n)u[n]\} = \frac{z^{-1}sin\omega_0}{1-2z^{-1}cos\omega_0+z^{-2}}$

inverse z-Transform

$x[n] = \frac{1}{2\pi j}\oint X(z)z^{n-1}dz$

4 PID

Linear Feedback

Forward $H(s)$, Feedback $G(s)$ (z when discrete).

$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1+H(s)G(s)}$

Open loop transfer: $\frac{R(s)}{E(s)} = H(s)G(s)$

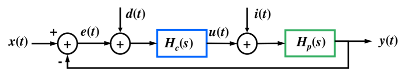
Error transfer: $\frac{E(s)}{X(s)} = \frac{1}{1+H(s)G(s)}$

Inverse system: $Q(s) = \frac{1}{\frac{1}{H(s)}+G(s)} \approx \frac{1}{G(s)}$ (when $|H(s)| \gg 1$)

Compensation: (forward $H(j\omega)$, backward K), $Q(s) \approx \frac{1}{K}$, (approx irrelevant to H)

Gain $\times$ BW $\equiv$ Constant

PID Control



$H_o(s) \triangleq H_c(s)H_p(s)$

$E(s) = \frac{X(s)}{1+H_o(s)} - \frac{H_p(s)}{1+H_o(s)}I(s) - \frac{H_o(s)}{1+H_o(s)}D(s)$

$Y(s) = \frac{H_o X(s)}{1+H_o(s)} + \frac{H_p(s)}{1+H_o(s)}I(s) + \frac{H_o(s)}{1+H_o(s)}D(s)$

Error transfer: $H_e(s) = \frac{1}{1+H_o(s)}$, close loop: $H(s) = \frac{H_o(s)}{1+H_o(s)}$, $H(s) + H_e(s) = 1$

Steady State Error

Assume the system to be stable system.

$f(+\infty) = \lim_{s\to 0} sF(s)$

$x(t) = \frac{t^k}{k!}u(t)$, $X(s) = s^{-(k+1)}$

$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{s^{-k}}{(1+H_c(s)H_p(s))}$

type-k system: limited non-zero e_{ss} for k

$H_p(s) = s^{-m}\hat{H}_p(s)$, $H_c(s) = s^{-n}\hat{H}_c(s)$, $0 < |\hat{H}_p(s)|, |\hat{H}_c(s)| < \infty$

$e_{ss} = \frac{1}{s^{n+m}+\hat{H}_p(s)\hat{H}_c(s)s^{k-(n+m)}}$, type- $m+n$ system for tracking x

Similar analysis for interference suppression $I(s)$, type- n

1. $H_c(s) = k_P$

$x(t) = u(t)$

$H_p(s) = \frac{A}{s+a}$, $H(s) = \frac{Ak_P}{s+a+Ak_P}$, $e_{ss} = \frac{a}{a+k_P A}$

k_P increase $\Rightarrow$ faster response, less error

$H_p(s) = \frac{A}{s^2+a_1 s+a_2}$, $H(s) = \frac{Ak_P}{s^2+a_1 s+a_2} = \frac{Ak_P}{Ak_P+a_2}\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$, damped oscillation

$e_{ss} = \frac{a_2}{a_2+k_P A}$ k_P increase $\Rightarrow$ damp decrease, enhance oscillation, less error

2. $H_c(s) = \frac{k_I}{s}$

$H_p(s) = \frac{A}{s+a}$, $H(s) = \frac{k_I A}{s^2+as+k_I A}$, $H_e(s) = \frac{s(s+a)}{s^2+as+k_I A}$ k_I increase $\Rightarrow$ damping factor decrease to zero

$x(t) = u(t) \rightarrow e_{ss} = 0$, $x(t) = tu(t) \rightarrow e_{ss} = \frac{a}{k_I A}$

3. $H_c(s) = k_D s$

$H_p(s) = \frac{A}{s+a}$, $H(s) = \frac{k_D As}{(k_D A+1)s+a}$, $H(s) = \frac{s+a}{(k_D A+1)s+a}$

$x(t) = u(t)$, $y(\infty) = 0$ (can't track constant error)

4. $H_c(s) = k_P + k_D s$

$H_p(s) = \frac{A}{s+a}$, $H(s) = \frac{(k_P+k_D s)A}{(Ak_D+1)s+Ak_P+a}$

$x(t) = u(t)$, $y(\infty) = H(0) = \frac{k_P A}{a+k_P A}$, $y(0+) = \frac{k_D A}{1+k_D A}$

5. $H_c(s) = k_P + \frac{k_I}{s}$ **(lower ss error)**

$x(t) = u(t) \rightarrow e(\infty) = 0$; $x(t) = tu(t) \rightarrow e(\infty) = \frac{a}{K_I A}$

PLL

5 State-Space Analysis

CT

$[q_1(t), ..., q_L(t)]^T = \boldsymbol{q}(t)$

$\frac{d\boldsymbol{q}(t)}{dt} = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{b}x(t)$

$y(t) = \boldsymbol{c}^{\boldsymbol{T}}\boldsymbol{q}(t) + dx(t)$

Stability

for LTI: Asymptotic Stable(about λ_i)=BIBO

Modal Coordinates(CT for example)

$[\boldsymbol{v}_1, ..., \boldsymbol{v}_L] = \boldsymbol{V}$, $\boldsymbol{A}$'s eigenvectors

$\boldsymbol{q}(t) = \boldsymbol{V}\boldsymbol{r}(t)$

evolution:

$\boldsymbol{b} = \boldsymbol{V}\boldsymbol{\beta}$

$\frac{dr_i(t)}{dt} = \lambda_i r_i(t) + \beta_i x(t)$

output:

$\boldsymbol{\xi} = \boldsymbol{V}^{\boldsymbol{T}}\boldsymbol{c}$

$y(t) = \boldsymbol{\xi}^{\boldsymbol{T}}\boldsymbol{r}(t) + dx(t)$

Reachability

i − th mode is reachable: $\beta_i \neq 0$

Matrix: $\boldsymbol{R}_{\boldsymbol{L}} = [\boldsymbol{A}^{\boldsymbol{L-1}}\boldsymbol{b}, ..., \boldsymbol{b}]$ is full rank

Observability

$i - th$ mode is observable: $\xi_i \neq 0$

Matrix: $\boldsymbol{O}_{\boldsymbol{L}} = \begin{bmatrix} c^T \\ c^T A \\ \vdots \end{bmatrix}$ L rows