Someone, Signals and Systems
Have Fun!

1 PID

Linear Feedback

Forward $H(s)$, Feedback $G(s)$ (z when discrete).

$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1+H(s)G(s)}$

Open loop transfer: $\frac{R(s)}{E(s)} = H(s)G(s)$

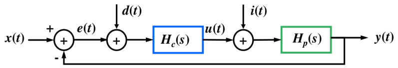
Error transfer: $\frac{E(s)}{X(s)} = \frac{1}{1+H(s)G(s)}$

Inverse system: $Q(s) = \frac{1}{\frac{1}{H(s)}+G(s)} \approx \frac{1}{G(s)}$ (when $|H(s)| \gg 1$)

Compensation: (forward $H(j\omega)$, backward K), $Q(s) \approx \frac{1}{K}$, (approx irrelevant to H)

Gain $\times$ BW $\equiv$ Constant

PID Control



$H_o(s) \triangleq H_c(s)H_p(s)$

$E(s) = \frac{X(s)}{1+H_o(s)} - \frac{H_p(s)}{1+H_o(s)}I(s) - \frac{H_o(s)}{1+H_o(s)}D(s)$

$Y(s) = \frac{H_o X(s)}{1+H_o(s)} + \frac{H_p(s)}{1+H_o(s)}I(s) + \frac{H_o(s)}{1+H_o(s)}D(s)$

Error transfer:$H_e(s) = \frac{1}{1+H_o(s)}$, close loop: $H(s) = \frac{H_o(s)}{1+H_o(s)}$, $H(s) + H_e(s) = 1$

Steady State Error

Assume the system to be steady system.

$\lim_{s\to 0} \int \frac{\mathrm{d}f(t)}{\mathrm{d}t} e^{-st}\mathrm{d}t = \lim_{s\to 0} sF(s) - f(0)$

$f(+\infty) - f(0) = \lim_{s\to 0} sf(s) - f(0)$

$x(t) = \frac{t^k}{k!}u(t)$, $X(s) = s^{-(k+1)}$

$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s^{-k}}{(1+H_c(s)H_p(s))}$

type-k system: limited non-zero e_{ss} for k

$H_p(s) = s^{-m}\hat{H}_p(s), H_c(s) = s^{-n}\hat{H}_c(s), 0 < |\hat{H}_p(s)|, |\hat{H}_c(s)| < \infty$

$e_s s = \frac{1}{s^{n+m}+\hat{H}_p(s)\hat{H}_c(s)} s^{k-(n+m)}$, type- $m+n$ system for tracking x

Similar analysis for interference suppression $I(s)$, type- n

$H_c(s) = k_P$

$x(t) = u(t)$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{Ak_P}{s+a+Ak_P}, e_{ss} = \frac{a}{a+k_P A}$

k_P increase $\Rightarrow$ faster response, less error

$H_p(s) = \frac{A}{s^2+a_1 s+a_2}, H(s) = \frac{Ak_P}{s^2+a_1 s+a_2} = \frac{Ak_P}{Ak_P+a_2}\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$, damped oscillation

$e_{ss} = \frac{a_2}{a_2+k_P A}$ k_P increase $\Rightarrow$ damp decrease, enhance oscillation, less error

$H_c(s) = \frac{k_I}{s}$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{k_I A}{s^2+as+k_I A}, H_e(s) = \frac{s(s+a)}{s^2+as+k_I A}$, damping factor decrease to zero

$x(t) = u(t) \to e_{ss} = 0, x(t) = tu(t) \to e_{ss} = \frac{a}{k_I A}$

$H_c(s) = k_D s$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{k_D As}{(k_D A+1)s+a}, H(s) = \frac{s+a}{(k_D A+1)s+a}$

$x(t) = u(t), y(\infty) = 0$ (can't track constant error)

$H_c(s) = k_P + k_D s$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{(k_P+k_D s)A}{(Ak_D+1)s+Ak_P+a}$

$x(t) = u(t), y(\infty) = H(0) = \frac{k_P A}{a+k_P A}, y(0+) = \frac{k_D A}{1+k_D A}$

$H_c(s) = k_P + \frac{k_I}{s}$ (lower ss error)

$x(t) = u(t) \to e(\infty) = 0; x(t) = tu(t) \to e(\infty) = \frac{a}{K_I A}$

PLL