Someone, Signals and Systems
Have Fun!

1 Laplace Transform

$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\mathrm{d}t$
Unilateral: $\int_{0-}^{\infty}$

Properties

Reduced rational fraction $X(s) = \frac{N(s)}{D(s)}$ (When $s = \infty$ is zero or pole)

ROC

Right sided:($\sigma \in$ ROC) $\rightarrow$ ($\Re\mathfrak{e}\, s > \sigma \subseteq$ ROC)
Left sided:($\sigma \in$ ROC) $\rightarrow$ ($\Re\mathfrak{e}\, s < \sigma \subseteq$ ROC)
Two sided: $\sigma_1 < \Re\mathfrak{e}\, s < \sigma_2$
Bounded by poles or ∞, no poles contained in ROC

ILT

$X(s) = \mathcal{F}\{x(t)e^{-\sigma t}\} \rightarrow x(t) = \frac{1}{2\pi j} X(s)e^{st}\mathrm{d}t$
$= \frac{e^{\sigma t}}{2\pi} \int_{-\infty}^{+\infty} X(\sigma + j\omega)e^{j\omega t}\mathrm{d}\omega$ (when $\Re\mathfrak{e}\, s = \sigma$ in ROC)
$\mathcal{L}^{-1}\{s^n\} = u_n(t), all\ s$

$$\mathcal{L}^{-1}\{\frac{1}{(s+a)^n}\} = \begin{cases} \frac{-t^{n-1}}{(n-1)!}e^{-at}u(-t), & \Re\mathfrak{e}\, s < -a \\ \frac{t^{n-1}}{(n-1)!}e^{-at}u(t), & \Re\mathfrak{e}\, s > -a \end{cases}$$

Properties(ROC)

$X_1(s) = \mathcal{L}\{x_1(t)\}, \mathrm{ROC} = R_1$
$X_2(s) = \mathcal{L}\{x_2(t)\}, \mathrm{ROC} = R_2$
$aX_1(s) + bX_2(s) = \mathcal{L}\{ax_1(t) + bx_2(t)\}, \mathrm{ROC} \supseteq R_1 \cap R_2$ ($R_1 \cap R_2 = \emptyset \rightarrow \mathrm{ROC} = \emptyset$) $X_1^*(s^*) = \mathcal{L}\{x_1^*(t)\}, \mathrm{ROC} = R_1$
$e^{-s\tau}X_1(s) = \mathcal{L}\{x_1(t-\tau)\}, \mathrm{ROC} = R_1$
$X_1(s-\tau) = \mathcal{L}\{x_1(t)e^{\tau t}\}, \mathrm{ROC} = R_1 + \Re\mathfrak{e}\,\tau$
$\frac{1}{|a|}X_1(\frac{s}{a}) = \mathcal{L}\{x_1(at)\}, \mathrm{ROC} = aR_1 = \{s|\frac{s}{a} \in R_1\}$
$X_1(s)X_2(s) = \mathcal{L}\{x_1(t) * x_2(t)\}, \mathrm{ROC} \supseteq R_1 \cap R_2$ (This equality may not hold if $R_1 \cap R_2 = \emptyset$)
$sX(s) = \mathcal{L}x'(t), \mathrm{ROC} \supseteq R_1, X'(s) = \mathcal{L}\{-tx(t)\}, \mathrm{ROC} = R_1$
$\frac{1}{s}X(s) = \mathcal{L}\{\int_{-\infty}^{t} x(\tau)\mathrm{d}\tau\}, \mathrm{ROC} \supseteq (R \cap (\Re\mathfrak{e}\, s > 0))$

Examples

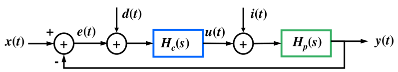
$\mathcal{L}\{cos(\omega_0 t)u(t)\} = \frac{s}{s^2+\omega_0^2}, \mathcal{L}\{cos(\omega_0 t)u(t)\} = \frac{\omega}{s^2+\omega_0^2}$

2 PID

Linear Feedback

Forward $H(s)$, Feedback $G(s)$ (z when discrete).
$Q(s) = \frac{Y(s)}{X(s)} = \frac{H(s)}{1+H(s)G(s)}$
Open loop transfer: $\frac{R(s)}{E(s)} = H(s)G(s)$
Error transfer: $\frac{E(s)}{X(s)} = \frac{1}{1+H(s)G(s)}$
Inverse system: $Q(s) = \frac{1}{\frac{1}{H(s)}+G(s)} \approx \frac{1}{G(s)}$ (when $|H(s)| \gg 1$)
Compensation: (forward $H(j\omega)$, backward K), $Q(s) \approx \frac{1}{K}$, (approx irrelevant to H)
Gain $\times$ BW $\equiv$ Constant

PID Control



$H_o(s) \triangleq H_c(s)H_p(s)$
$E(s) = \frac{X(s)}{1+H_o(s)} - \frac{H_p(s)}{1+H_o(s)}I(s) - \frac{H_o(s)}{1+H_o(s)}D(s)$
$Y(s) = \frac{H_oX(s)}{1+H_o(s)} + \frac{H_p(s)}{1+H_o(s)}I(s) + \frac{H_o(s)}{1+H_o(s)}D(s)$
Error transfer:$H_e(s) = \frac{1}{1+H_o(s)}$, close loop: $H(s) = \frac{H_o(s)}{1+H_o(s)}$, $H(s) + H_e(s) = 1$

Steady State Error

Assume the system to be steady system.
$\lim_{s\rightarrow 0} \int \frac{\mathrm{d}f(t)}{\mathrm{d}t} e^{-st}\mathrm{d}t = \lim_{s\rightarrow 0} sF(s) - f(0)$
$f(+\infty) - f(0) = \lim_{s\rightarrow 0} sf(s) - f(0)$
$x(t) = \frac{t^k}{k!}u(t), X(s) = s^{-(k+1)}$
$e_{ss} = \lim_{s\rightarrow 0} sE(s) = \lim_{s\rightarrow 0} \frac{s^{-k}}{(1+H_c(s)H_p(s))}$
type-k system: limited non-zero e_{ss} for k
$H_p(s) = s^{-m}\hat{H}_p(s), H_c(s) = s^{-n}\hat{H}_c(s), 0 < |\hat{H}_p(s)|, |\hat{H}_c(s)| < \infty$
$e_s s = \frac{1}{s^{n+m}+\hat{H}_p(s)\hat{H}_c(s)} s^{k-(n+m)}$, type- $m+n$ system for tracking x
Similar analysis for interference suppression $I(s)$, type- n

$H_c(s) = k_P$

$x(t) = u(t)$
$H_p(s) = \frac{A}{s+a}, H(s) = \frac{Ak_p}{s+a+Ak_p}, e_{ss} = \frac{a}{a+k_pA}$
k_P increase $\Rightarrow$ faster response, less error
$H_p(s) = \frac{A}{s^2+a_1s+a_2}, H(s) = \frac{Ak_P}{s^2+a_1s+a_2} = \frac{Ak_P}{Ak_P+a_2} \frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}$, damped oscillation
$e_{ss} = \frac{a_2}{a_2+k_PA}$ k_P increase $\Rightarrow$ damp decrease, enhance oscillation, less error

$H_c(s) = \frac{k_I}{s}$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{k_IA}{s^2+as+k_IA}, H_e(s) = \frac{s(s+a)}{s^2+as+k_IA}$, damping factor decrease to zero
$x(t) = u(t) \rightarrow e_{ss} = 0, x(t) = tu(t) \rightarrow e_{ss} = \frac{a}{k_IA}$

$H_c(s) = k_Ds$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{k_DAs}{(k_DA+1)s+a}, H(s) = \frac{s+a}{(k_DA+1)s+a}$
$x(t) = u(t), y(\infty) = 0$ (can't track constant error)

$H_c(s) = k_P + k_Ds$

$H_p(s) = \frac{A}{s+a}, H(s) = \frac{(k_P+k_Ds)A}{(Ak_D+1)s+Ak_P+a}$
$x(t) = u(t), y(\infty) = H(0) = \frac{k_PA}{a+k_PA}, y(0+) = \frac{k_DA}{1+k_DA}$

$H_c(s) = k_P + \frac{k_I}{s}$ (lower ss error)

$x(t) = u(t) \rightarrow e(\infty) = 0; x(t) = tu(t) \rightarrow e(\infty) = \frac{a}{K_IA}$

PLL