We will start in 3-4 mins

Problem Solving Using Recursion

Recursion

Special class

$\boxed{1 - 5}$

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Agenda

↳ 2-3 mins Intro

↳ Problem Solving

String based Recursion

few traditional Recursion problem

Lay foundation for backtracking

Q => You are given a number (n) print all possible numbers from 1 → n in lexicographical order.

↳ Try to solve without sorting

<n

01 → 1

→ Dictionary order

2 2 2 2
0, 1, 2, 3
A B C D

→ A
→ B
→ BA

0 ✓
1 ✓
10 ✓

n = 13 all no's from 1 — 13

1 — n

1 — n

1, 10, 11, 12, 13, 2, 3, 4, 5, 6, 7, 8, 9,

1 — n

lexicographic

No sorting

Recursion

Recursion

$$5$$

$$f(5) = 5 \times f(4)$$

extra space

$$4 \times f(3)$$

call stack

f^n that returns factorial of 5

$$f(n) = n \times f(n-1)$$

recurrence

code is very easy

n 1 — n lexicographic 1 — n

0 — A 1 — B 2 — C

n = 13

1 — 1000

n = 1000

= 1

1
10
100
1000
10000

11 lengths

B
BA
BAA
BAAA

BAB

BB → 11

BB BAB

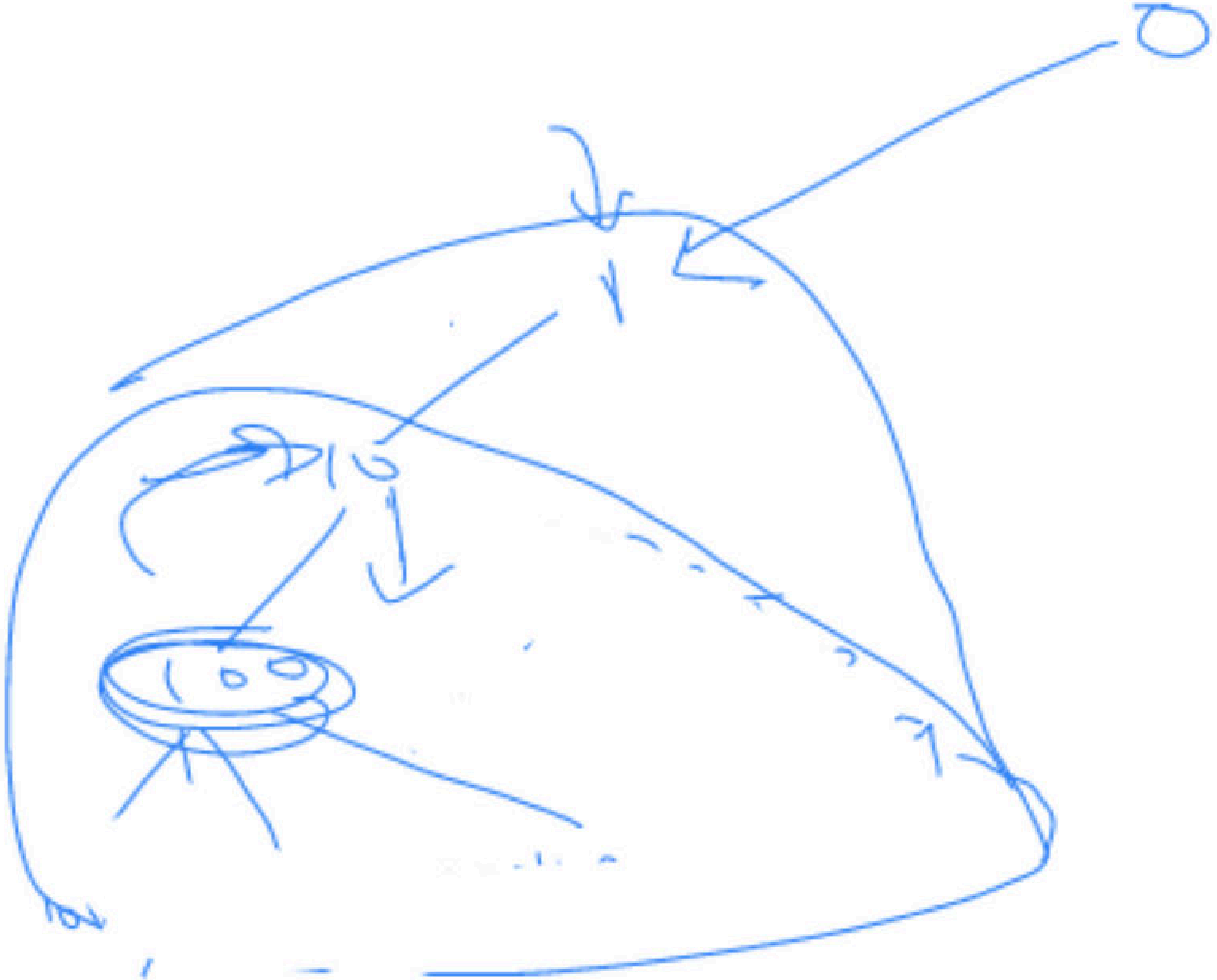
(1, 10, 100, 1000, 101, 102 ----- 109, 11, 110, 111 ---- 119)

1 — 3

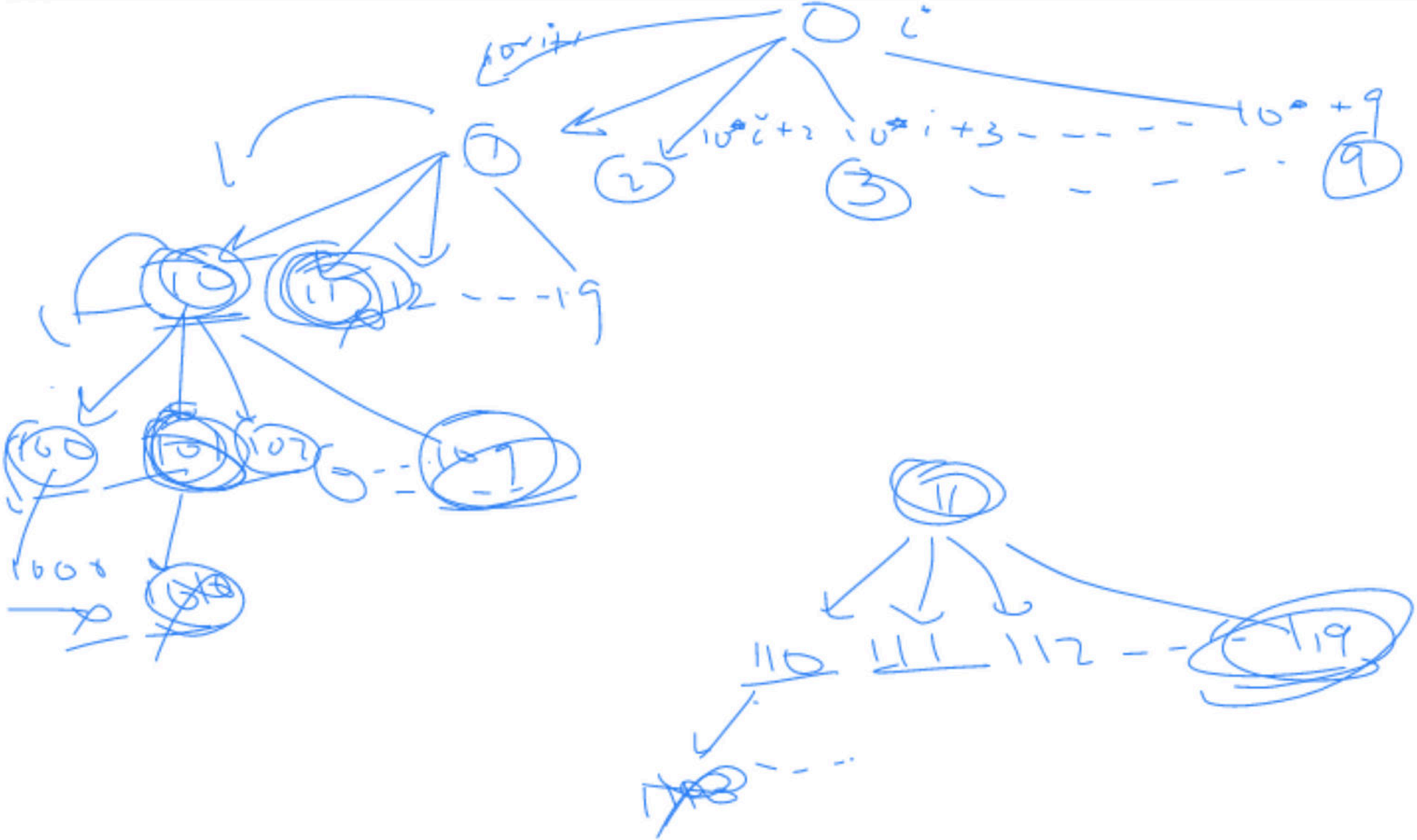
3 → Recurrence Intuition ⇄ for K it is true

→ Base case → K = 1

→ Self work → K + 1



0
1
10
100


0
i
10*i+1
1
2
10*i+2
10*i+3
3
10*i+9
9
10
11
12
19
100
101
102
109
1008
11
110
111
112
119

Clear??

Yes/No

$i \rightarrow 10*i+x \quad x \in [0-9]$

Recurrence Intuition

i

$10*i+x < n \quad 10*i+1 < n$

$f(x, n)$ → Print all the no's from x–n in

$1 < x < i$ order

$f(0, 1000)$ → Bigger problem

→ Recursion Intuition
→ Self work
→ Base case

f(0, 1000)
→ 0

f(1,1000) f(2,1000) f(3,1000) - - - f(9,1000)

print

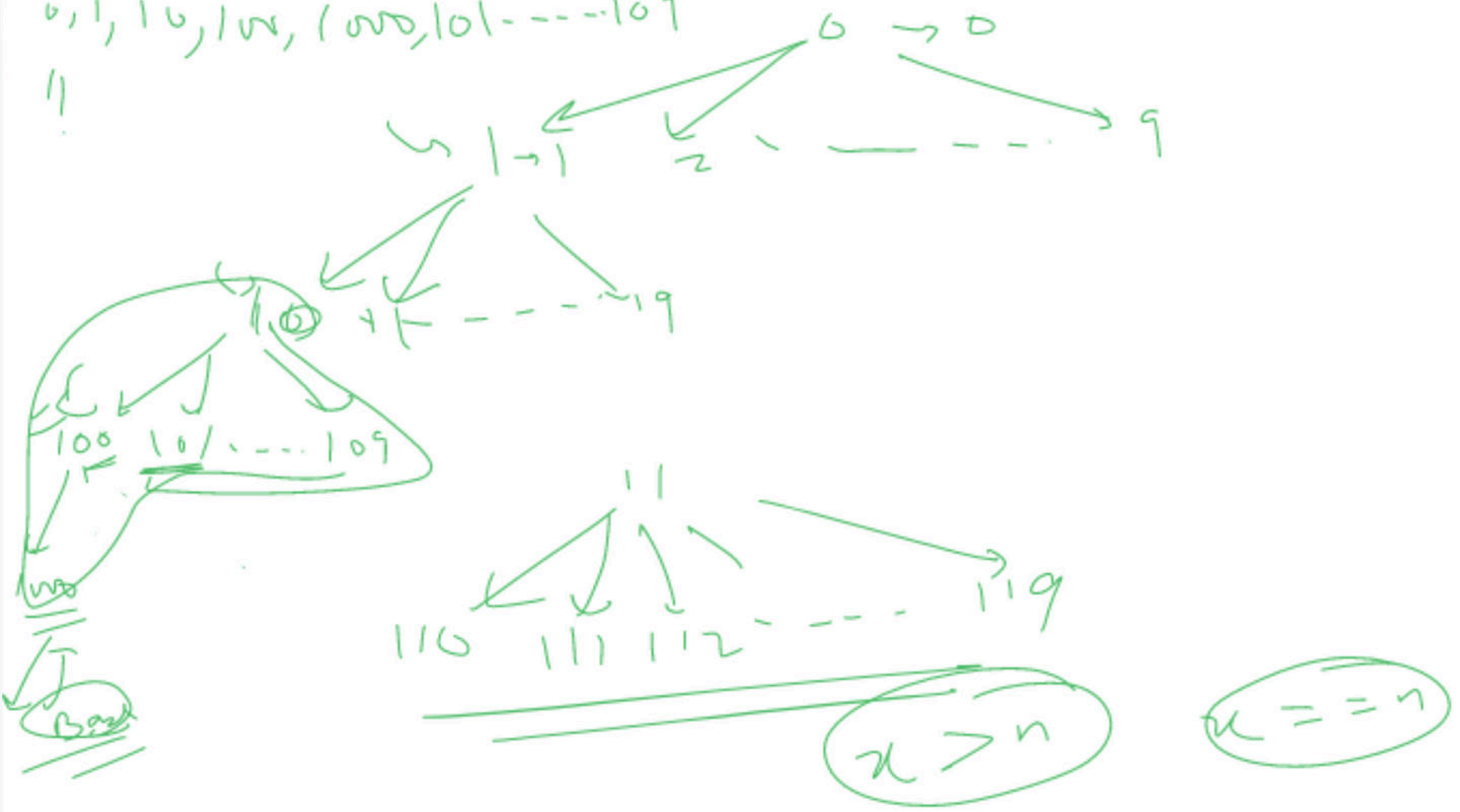
f(10,1000) f(11,1000) - - - f(19,1000)

$10 * x + 1$ — $10 * x + 9$

10

f(100,1000) f(101,1000) - - - f(109, 1000)

0, 1, 10, 100, 1000, 101 ---- 109



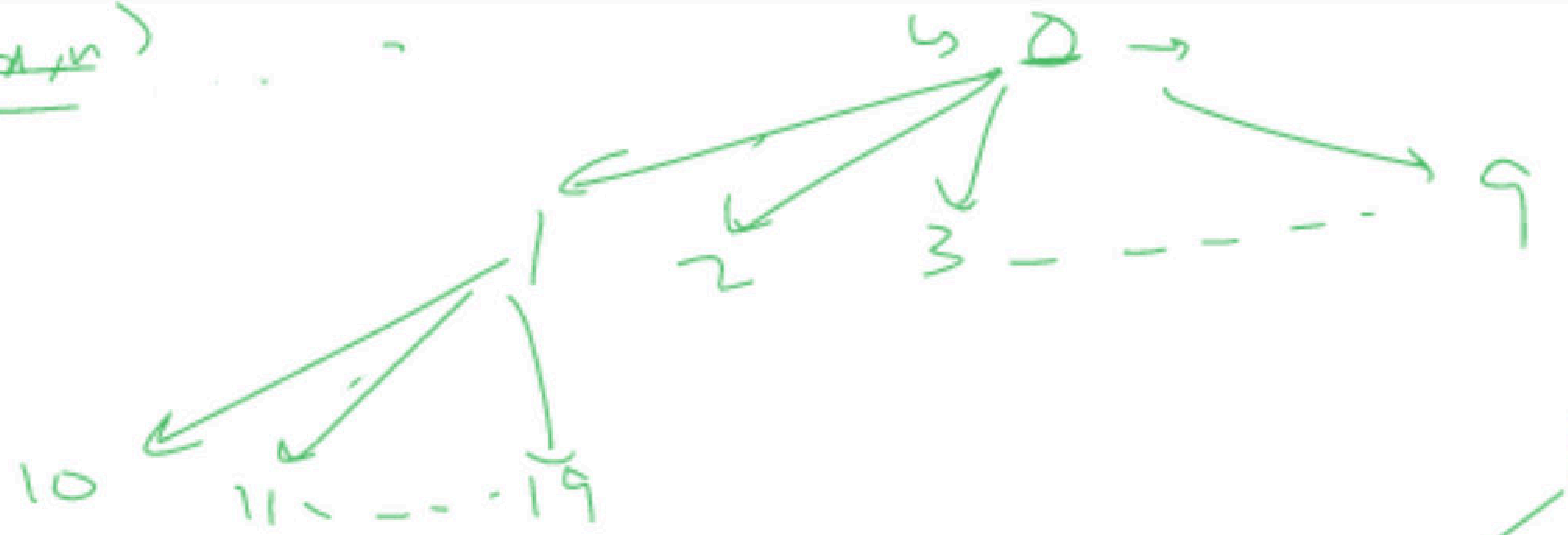
$f(x, n) \rightarrow$ print x

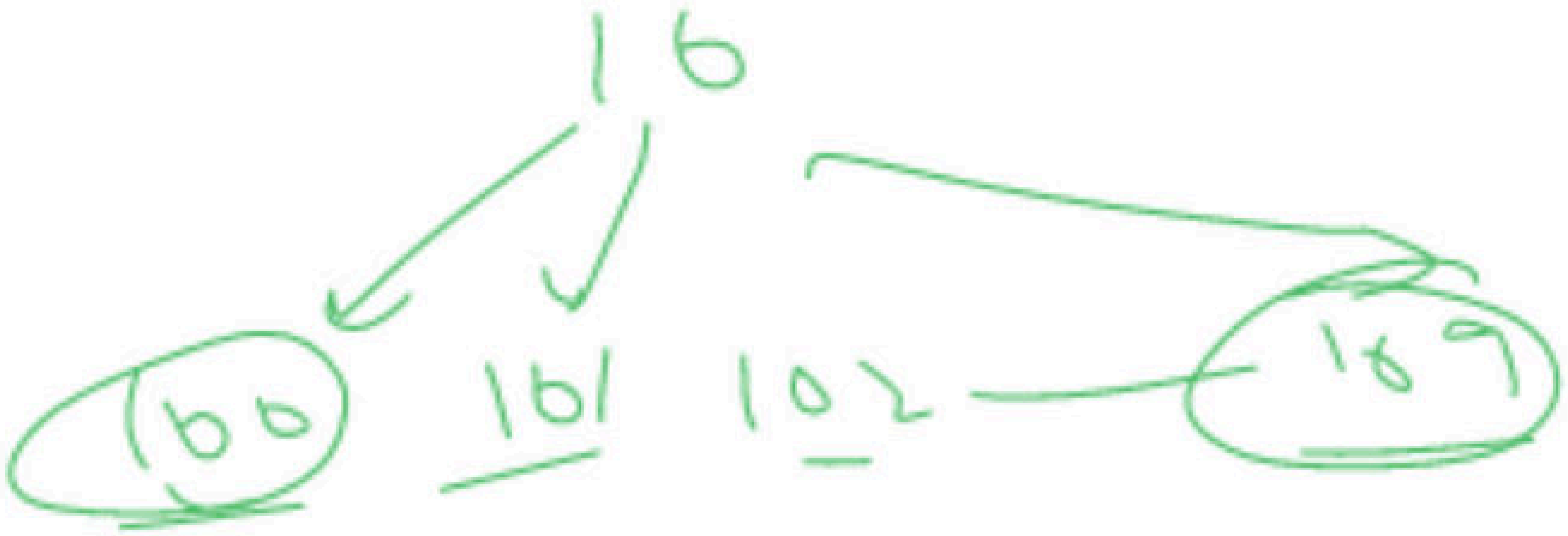
$f(10x+1, n), f(10*x+7, n), \ldots$

Tree

$f(x,n)$

$(n==0)$





$6*10+0$ 1.0

10

$10*10+0$

$10*10+1$

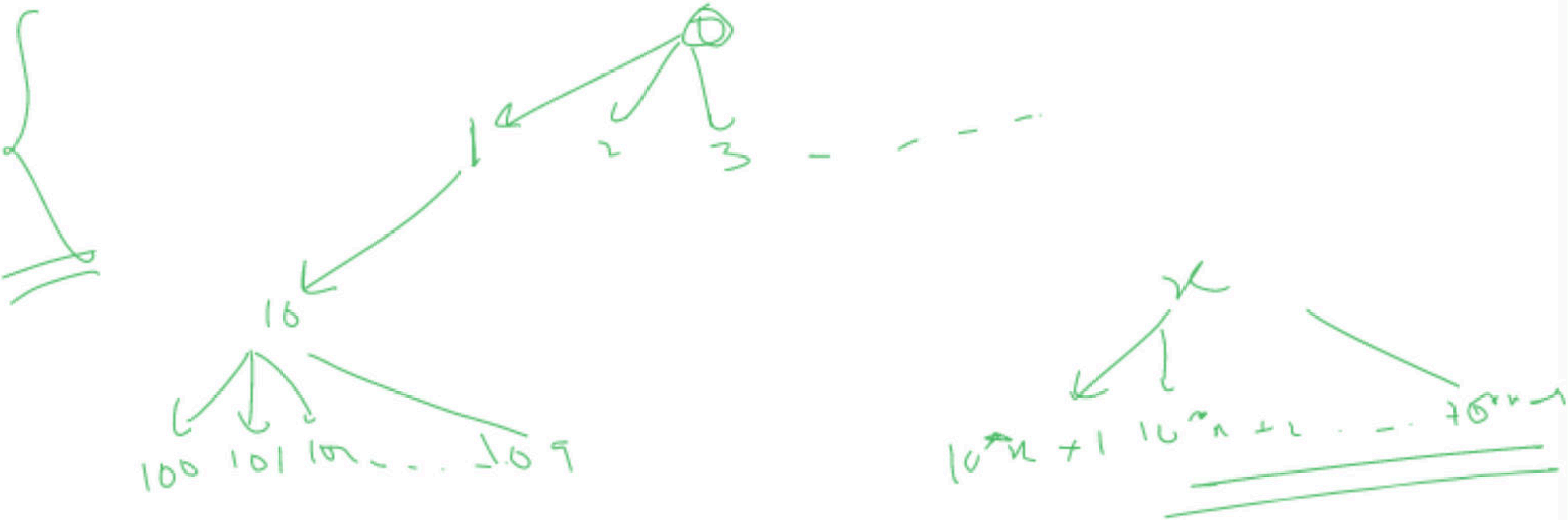
$\vdots$

9

$n = 13$

1, 10, 11, 12, 13, 2, 3, 4, 5, 6, 7, 8, 9

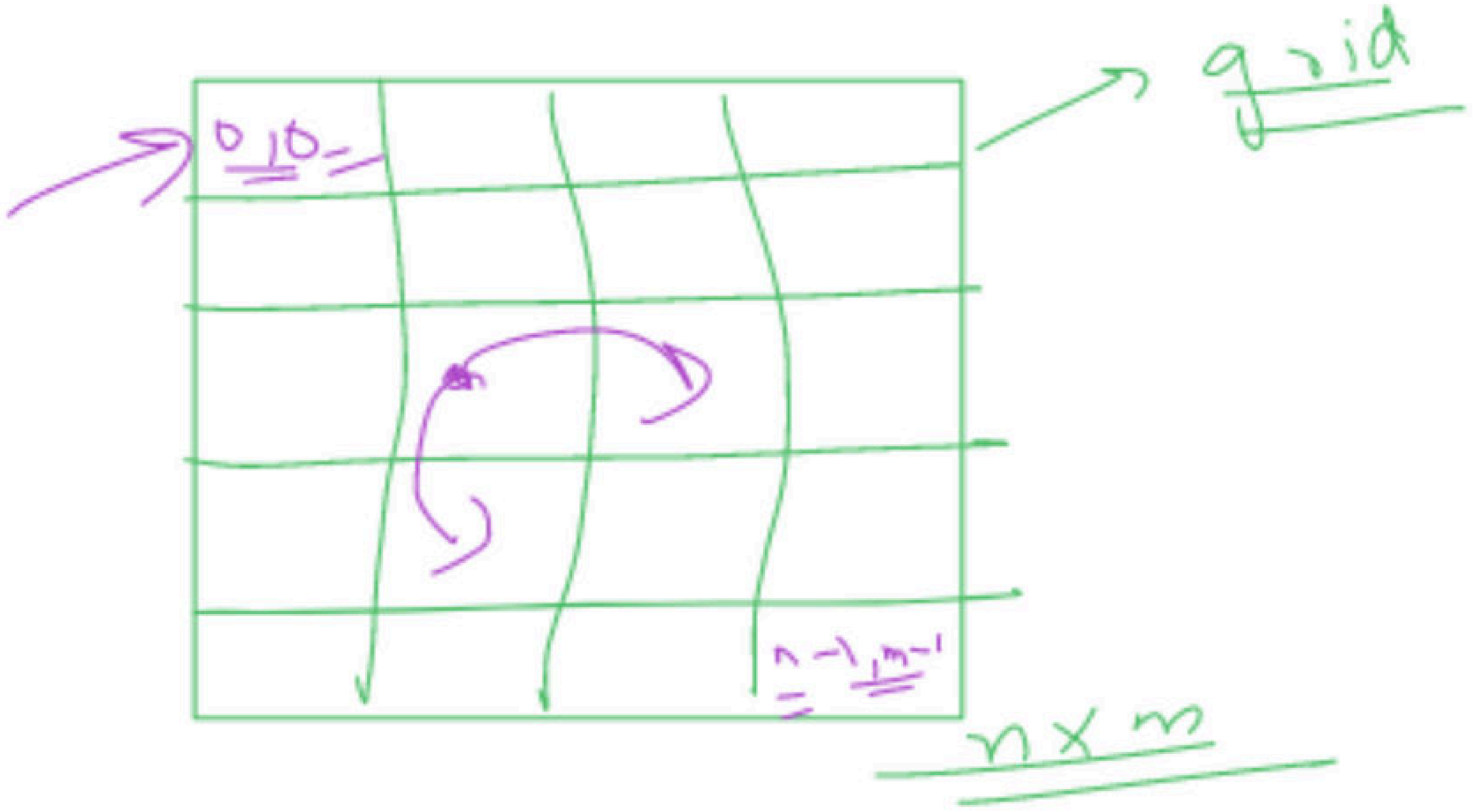
if ($x > n$) → base case


0
1
2
3
10
100 101 102 ... 109
x

$f(x, n)$

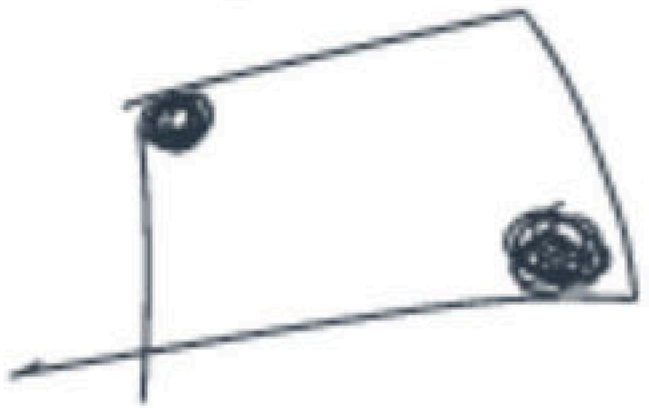
Tra

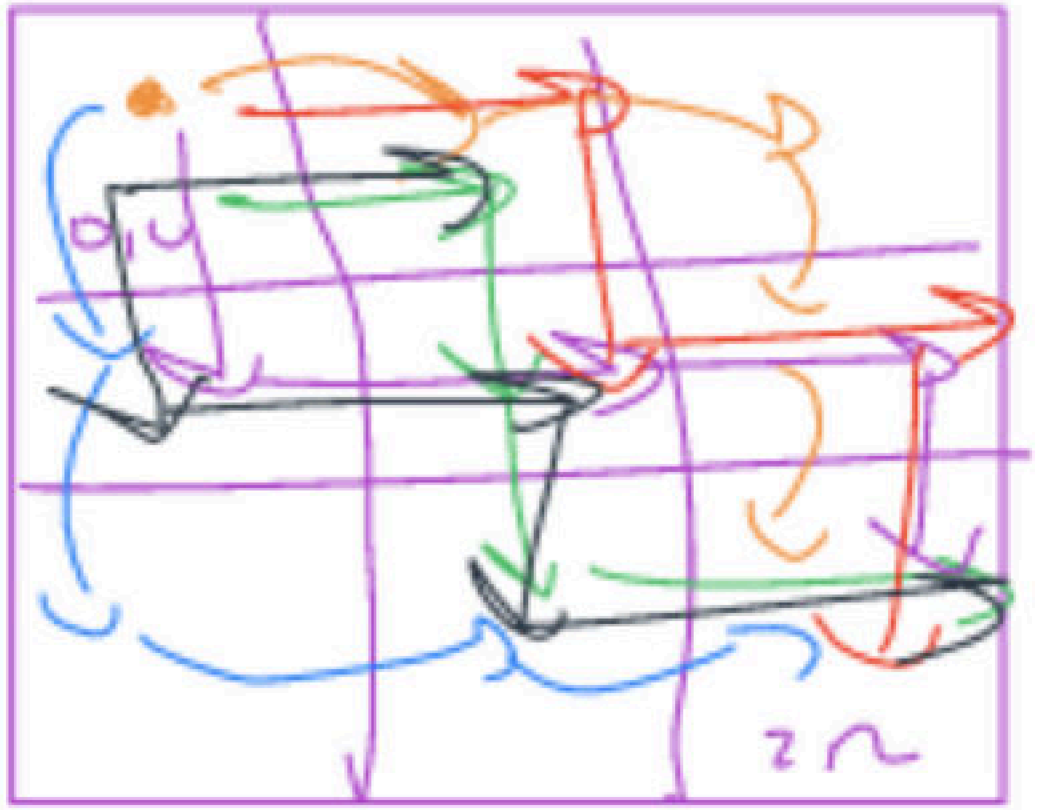
$f(10x+1)$ $f(10x+2)$ $- - - f(1m-1)$



right
or
down

n, m → no. of ways to reach from 0,0 to n-1, m-1
→ print the ways





1 1 1 1

1 6

6

on Em grid

Permutation

RRDD
DDRR
RDDR
RDRD
DRRD
DRDR

$\frac{4!}{2!\,2!}$

or

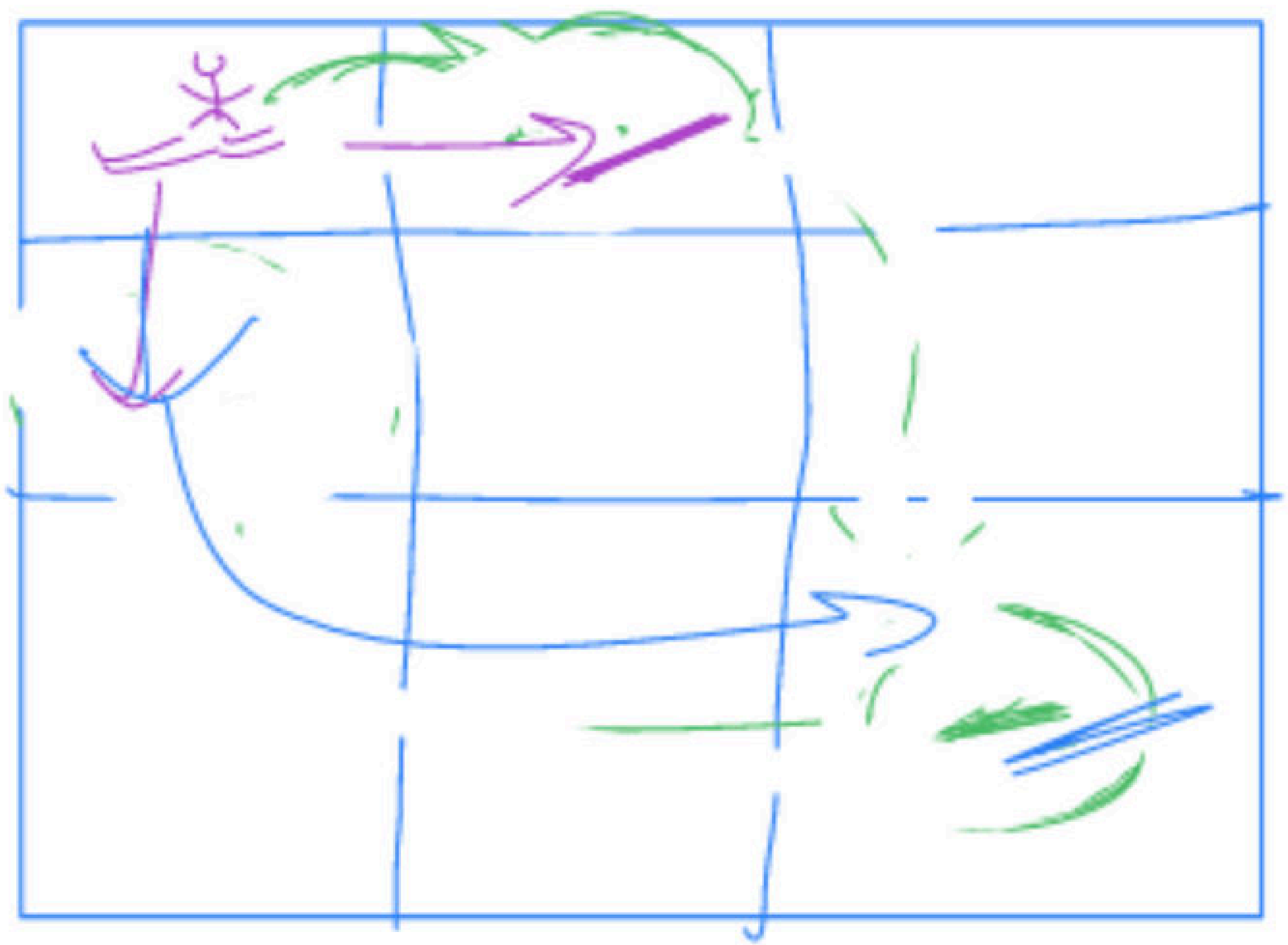
$C_1 + C_2$

Recursion Intuit

2 2

TL

OR

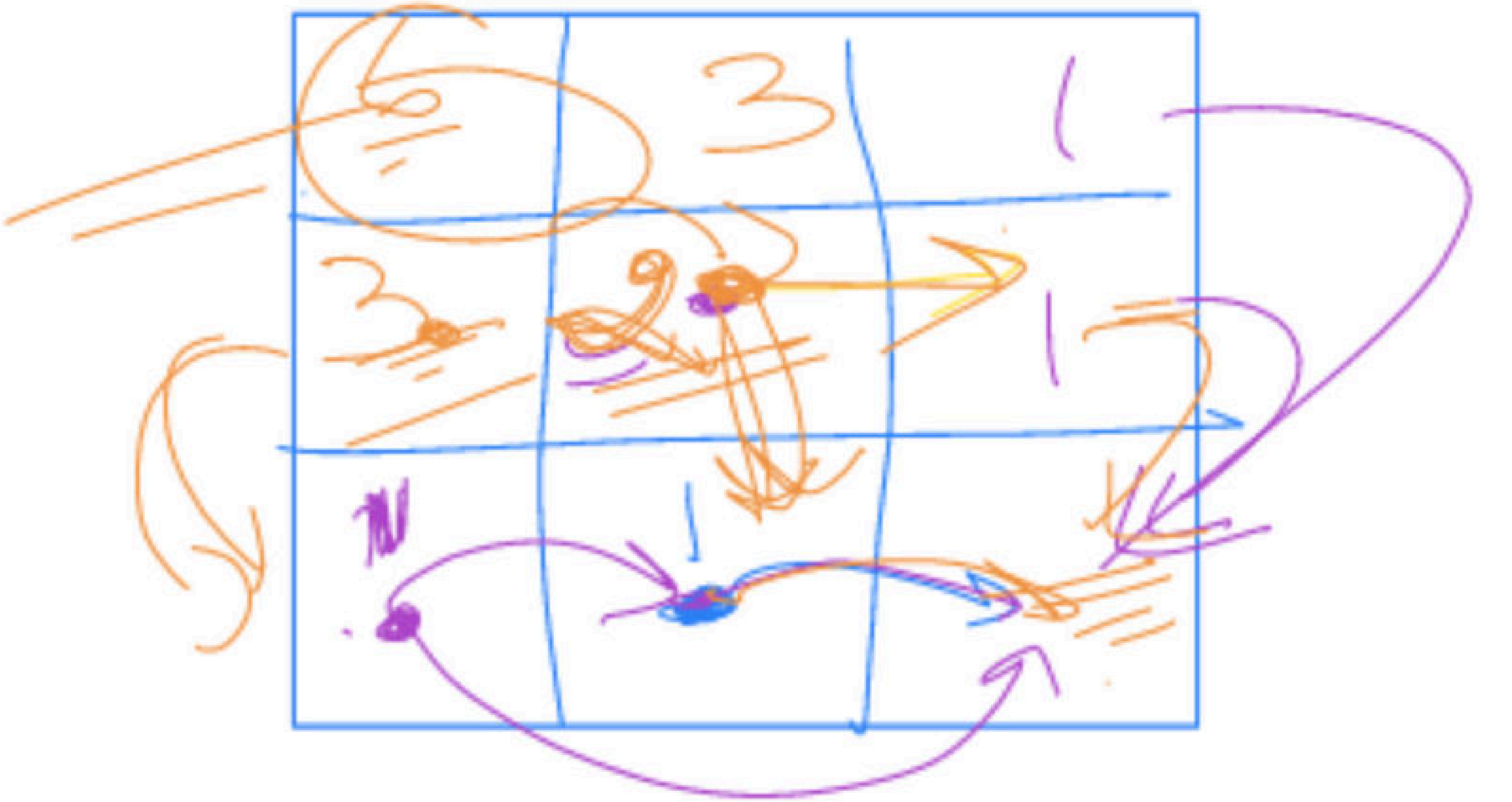


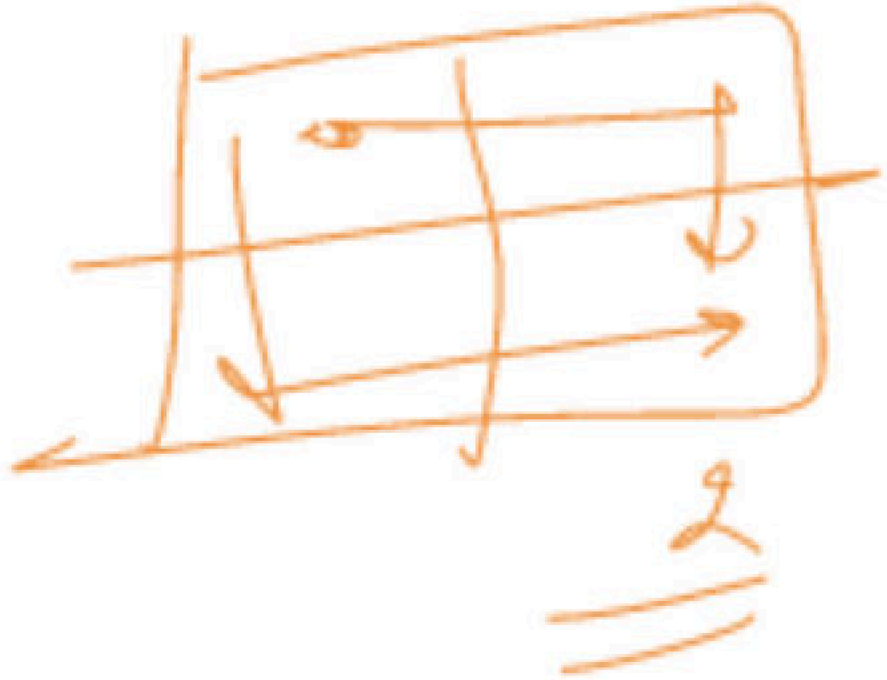
x, y

6 1

$n \times m$

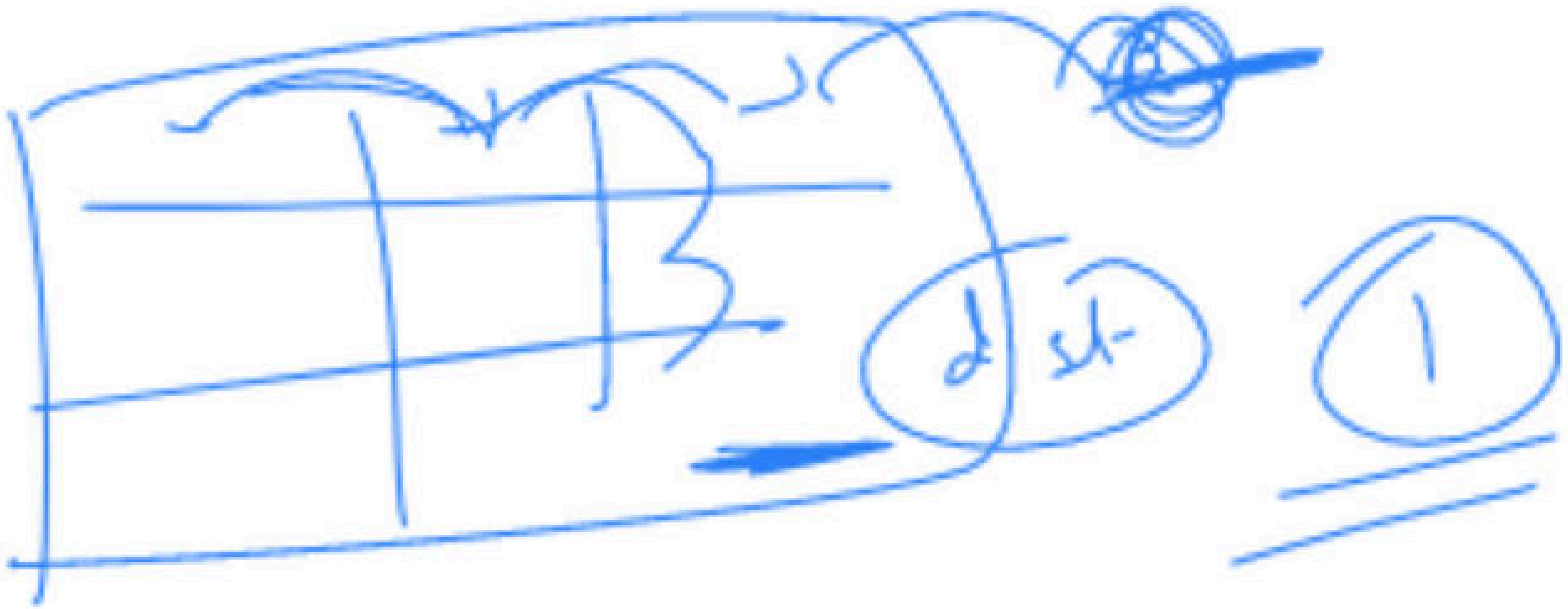
no of ways

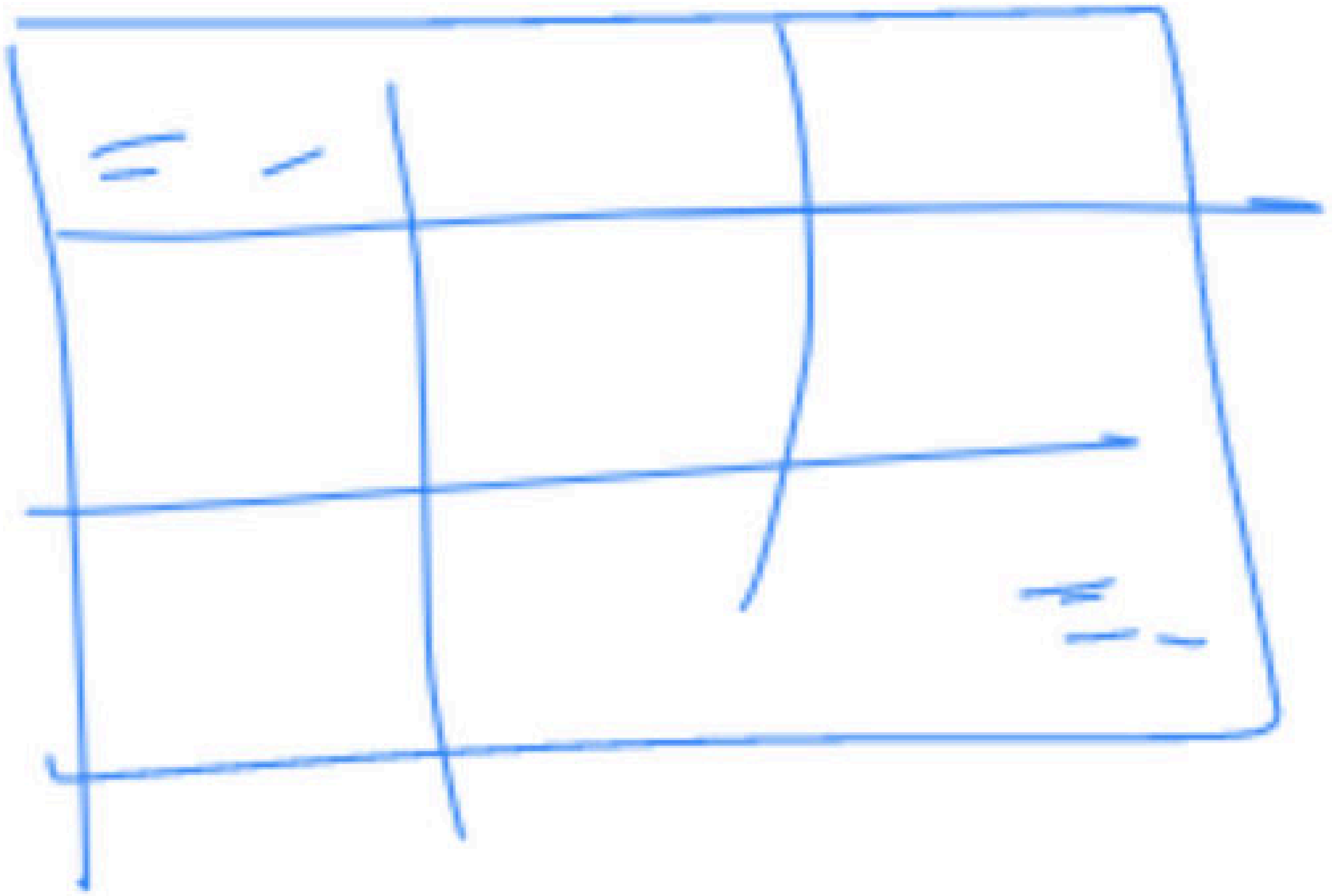




1 + 1

$\Rightarrow$ 2



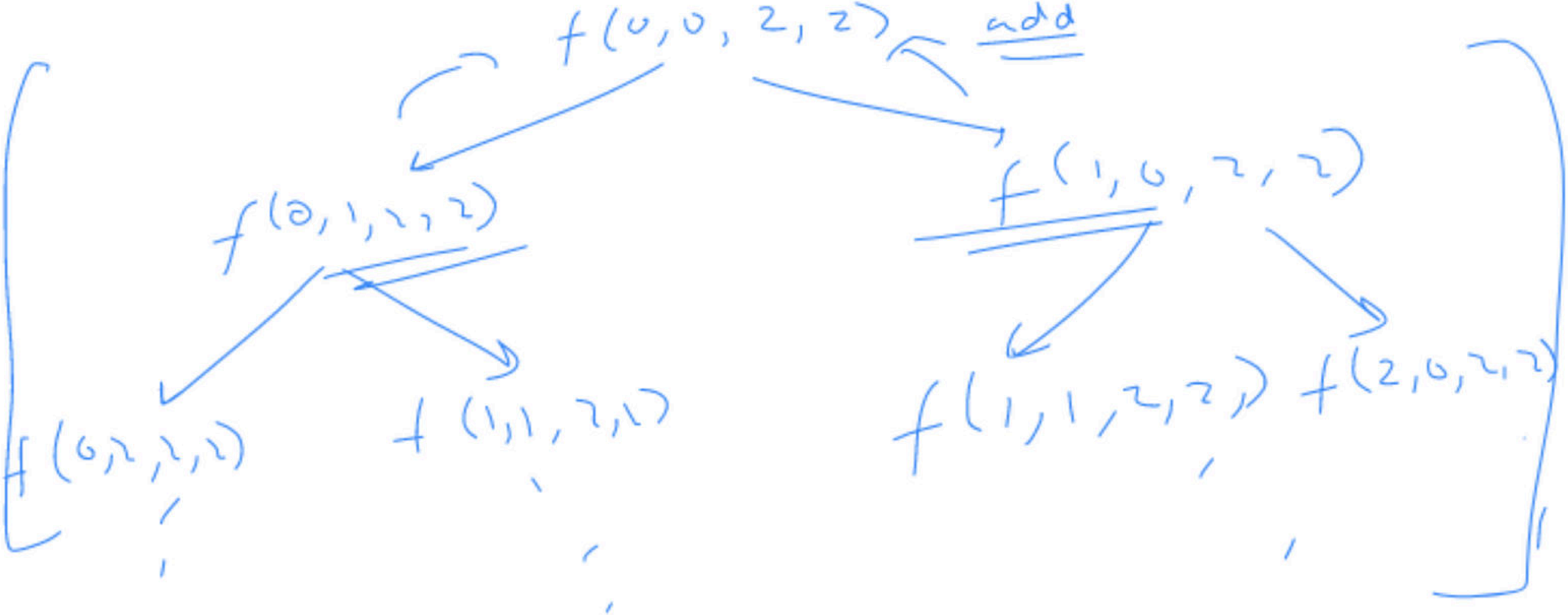


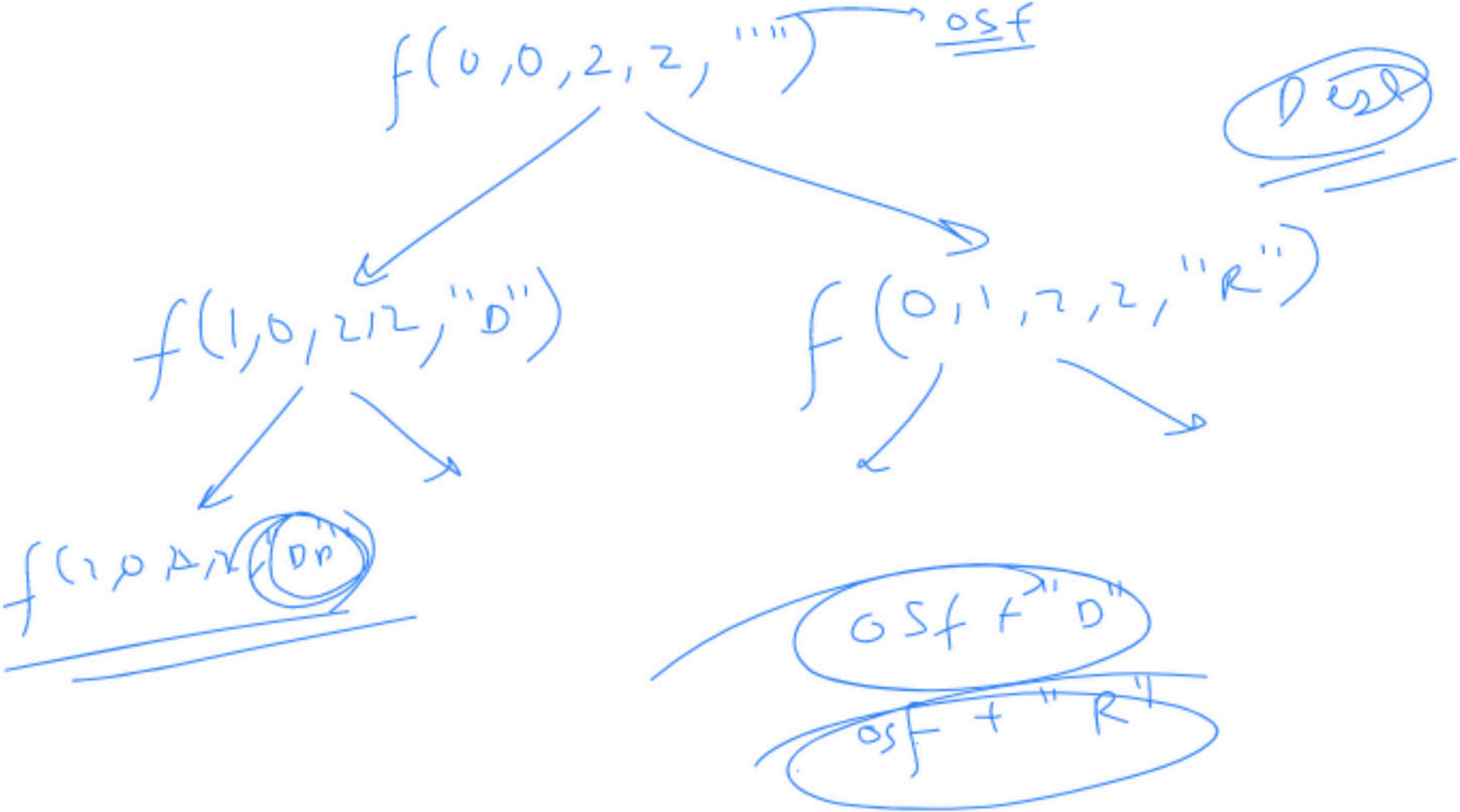
Right

or

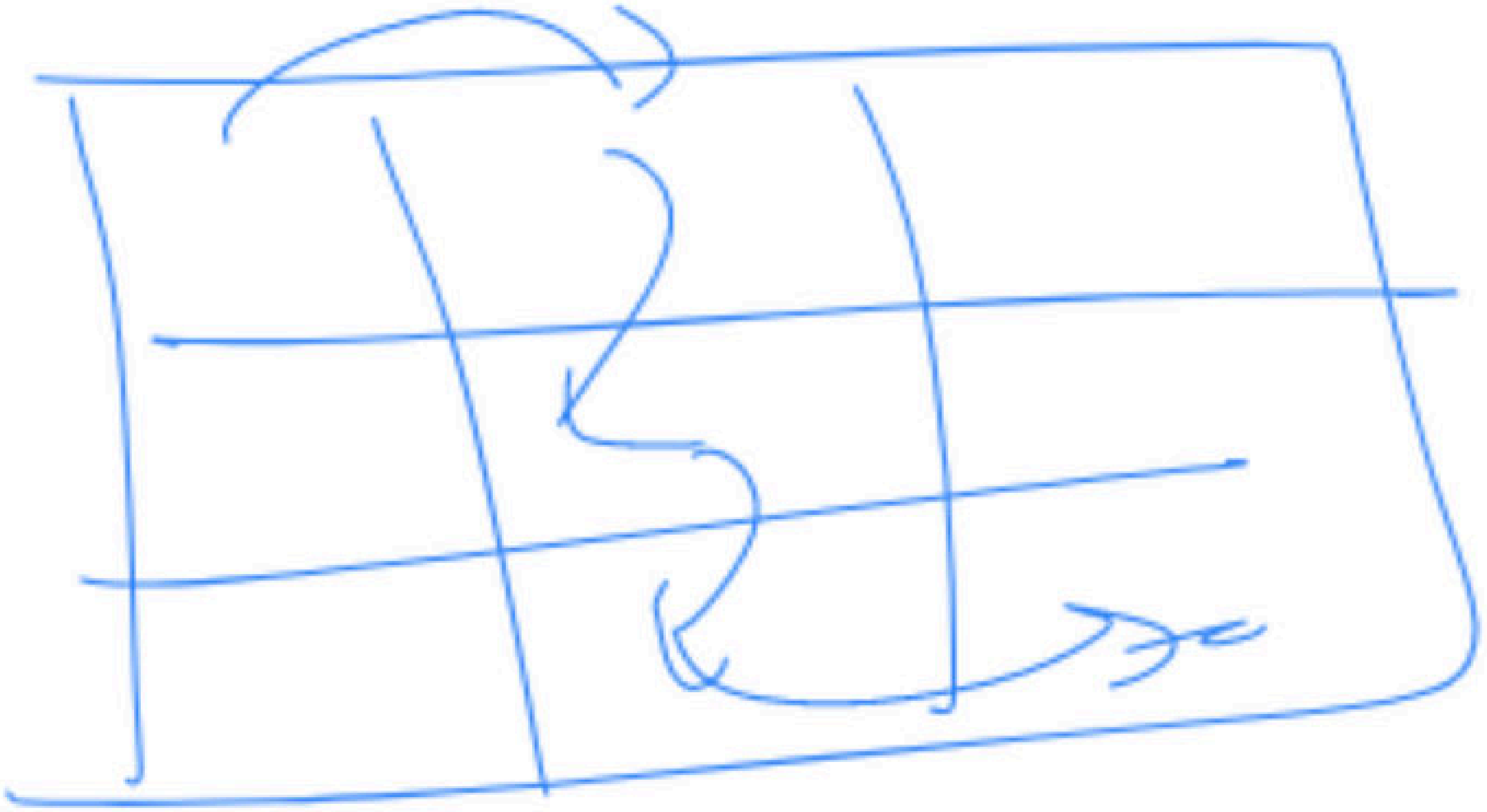
down

Print


f(0,0,2,2)
add
f(0,1,2,2)
f(1,0,2,2)
f(0,2,2,2)
f(1,1,2,2)
f(1,1,2,2)
f(2,0,2,2)


f(0,0,2,2,"")
osf
Dest
f(1,0,2,2,"D")
f(0,1,2,2,"R")
osf + "D"
osf + "R"

RDD



↳ array (N)

↳ Sorted? Recursively

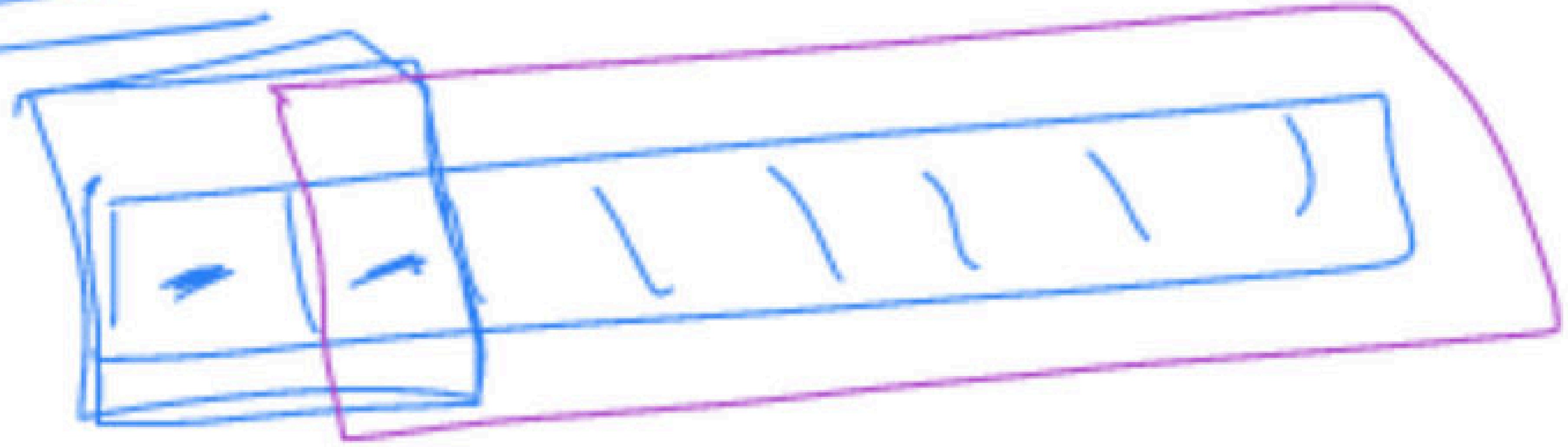
1, 2, 3, 4 → true

3, 4, 1, 2 → false

arr

arr[0] > arr[1]

return false



else

→ Recursion

→ Self

→ Base

$f(i,n) = arr[i] > arr[i+1]$? return false : ret $f(i+1, n)$

$\downarrow$

$\underbrace{arr[i] - arr[n-1]}$

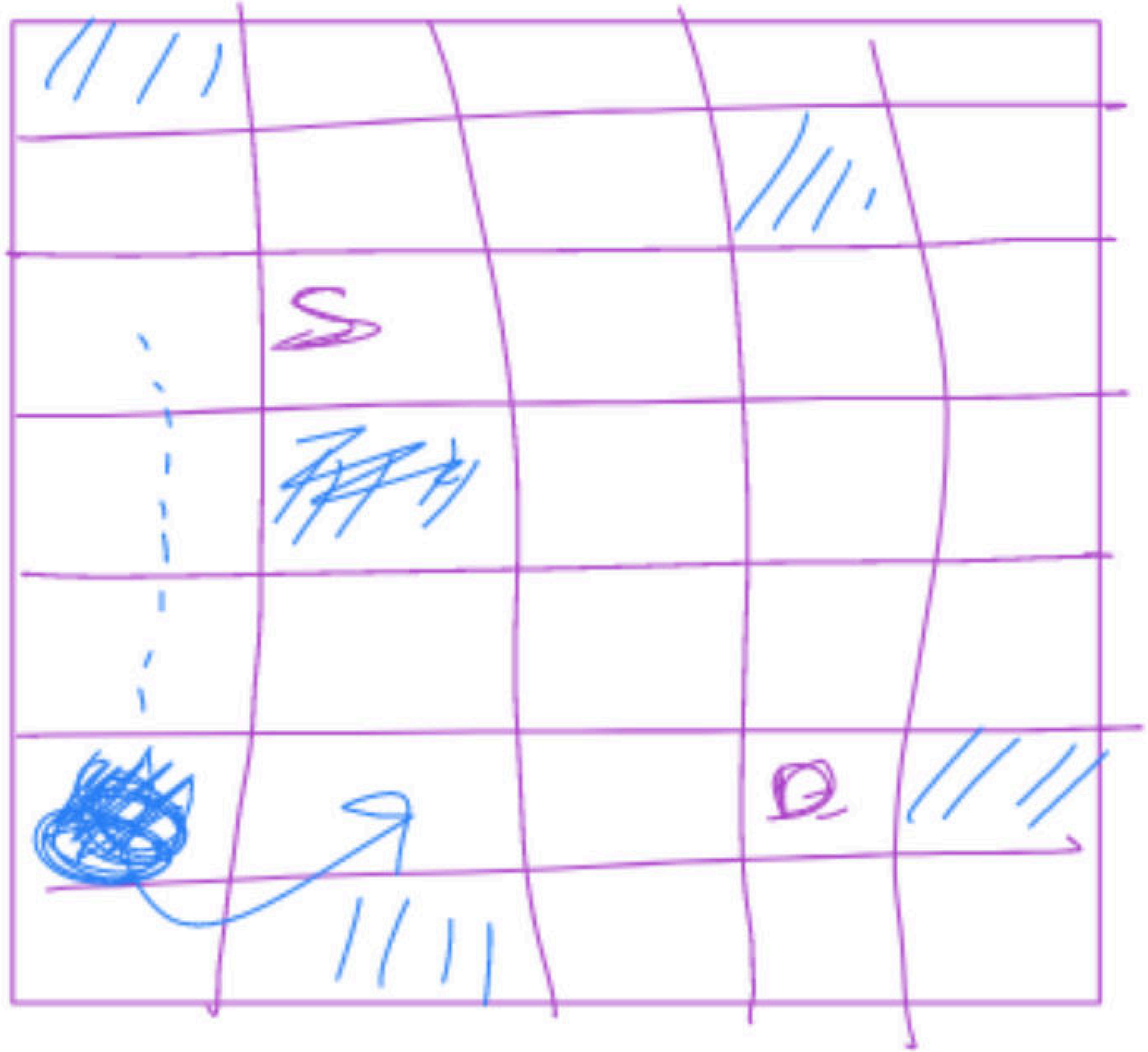
sorted

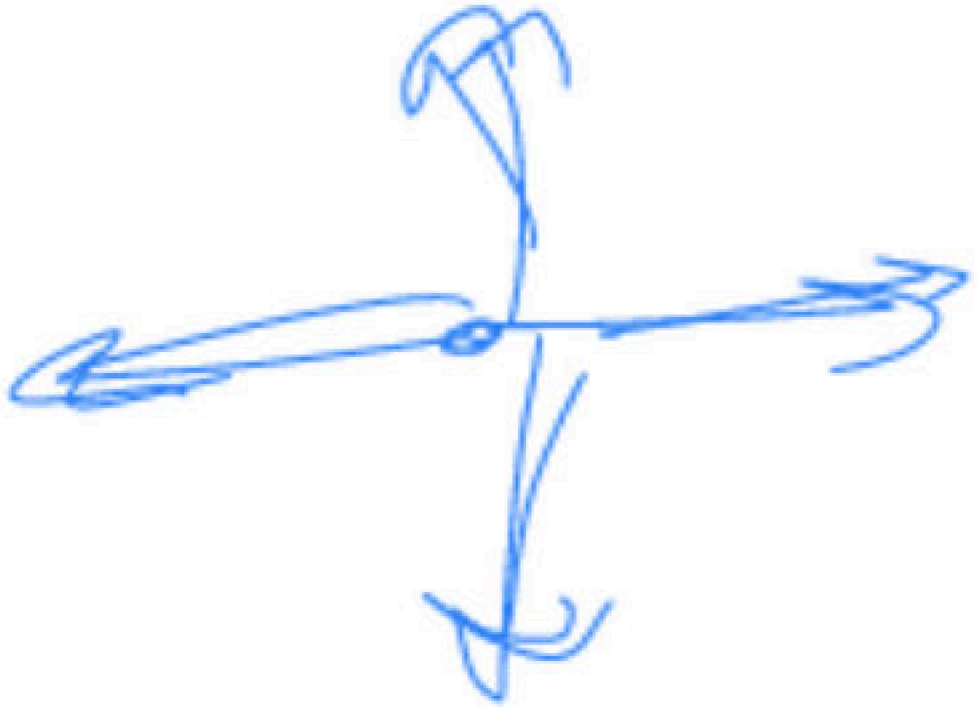
$i == n-1$

if ($i == n-1$) return true ;

Base

Rat in a maze



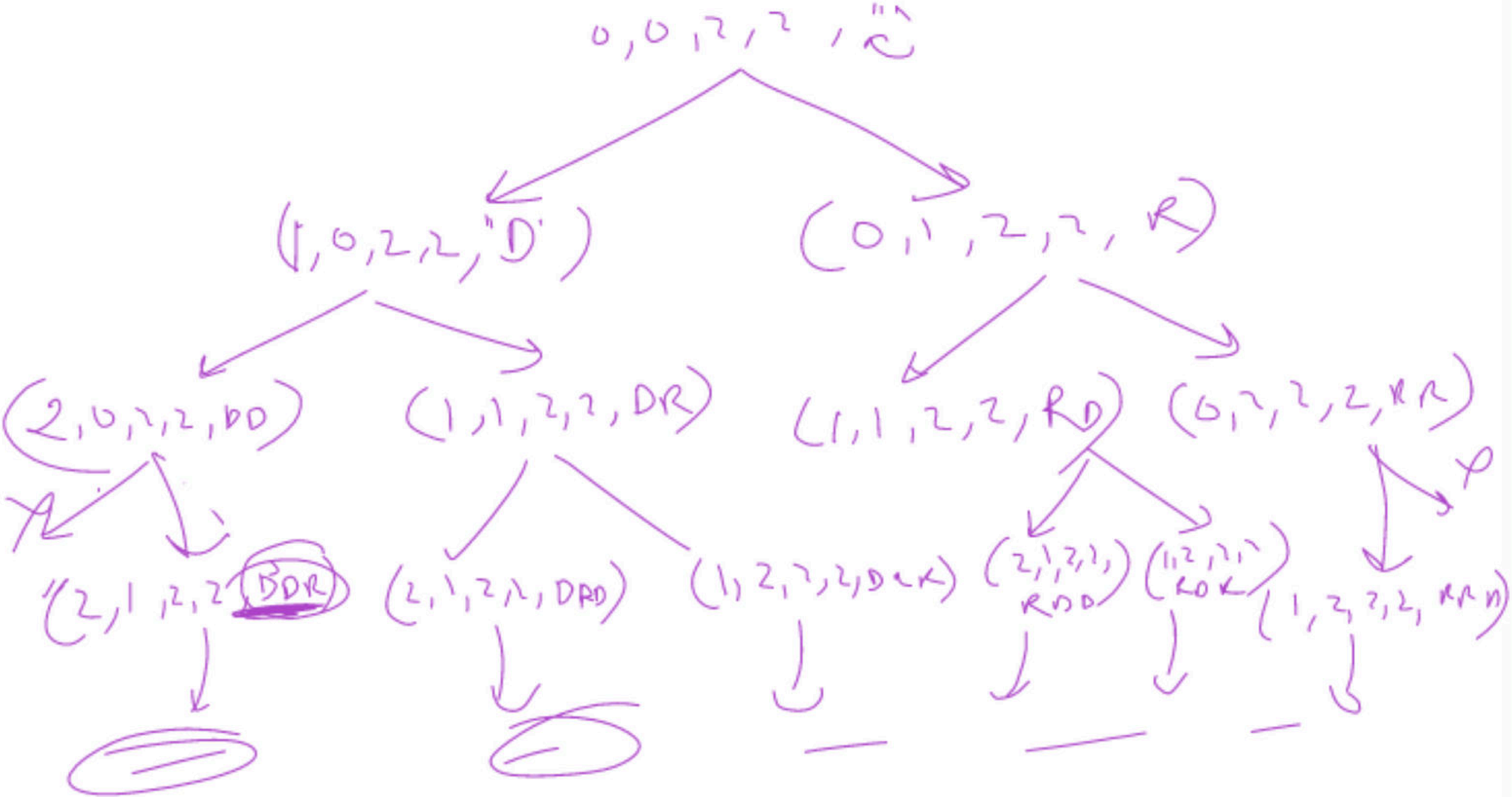


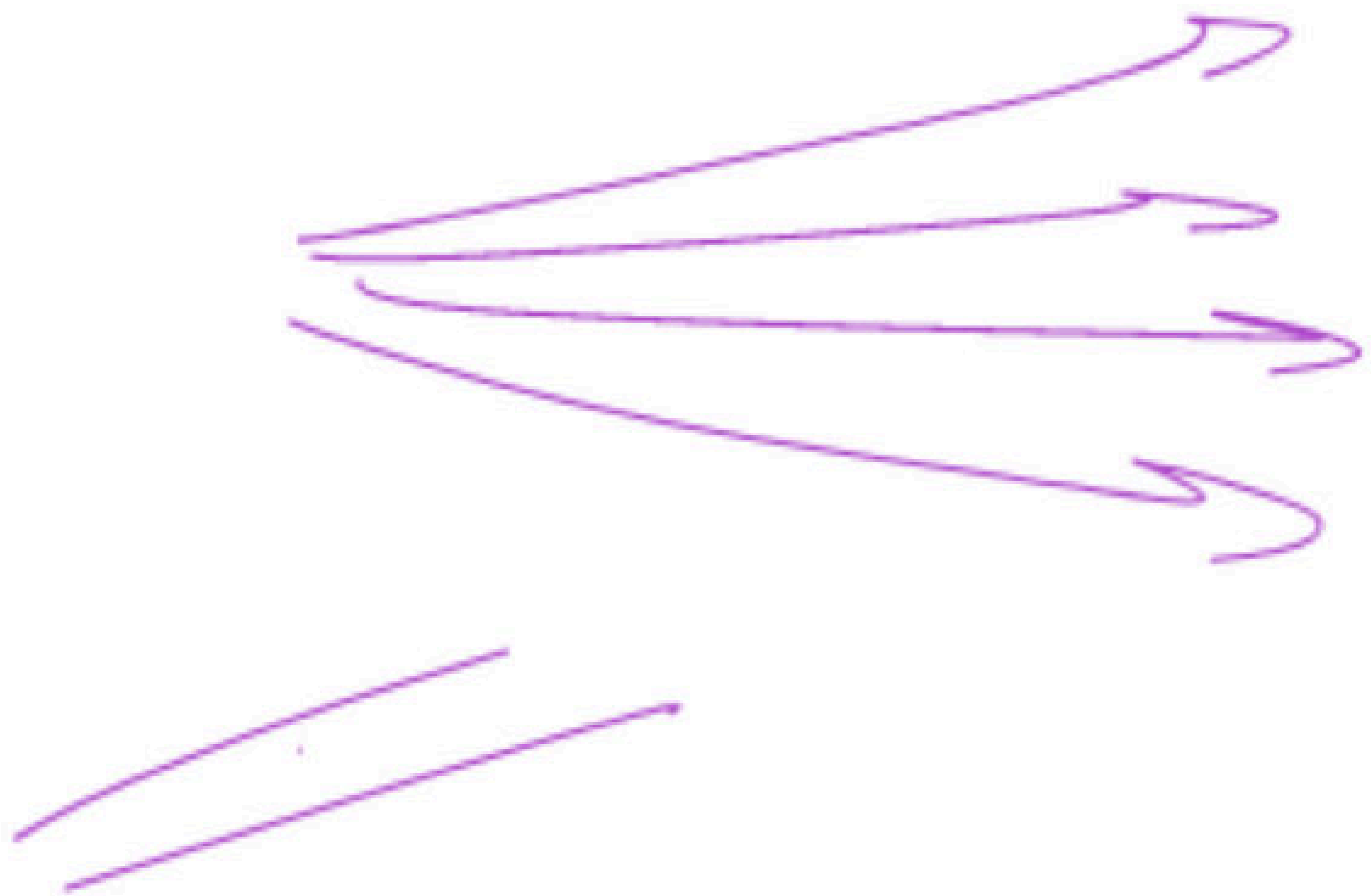
How many way to run from

S — D

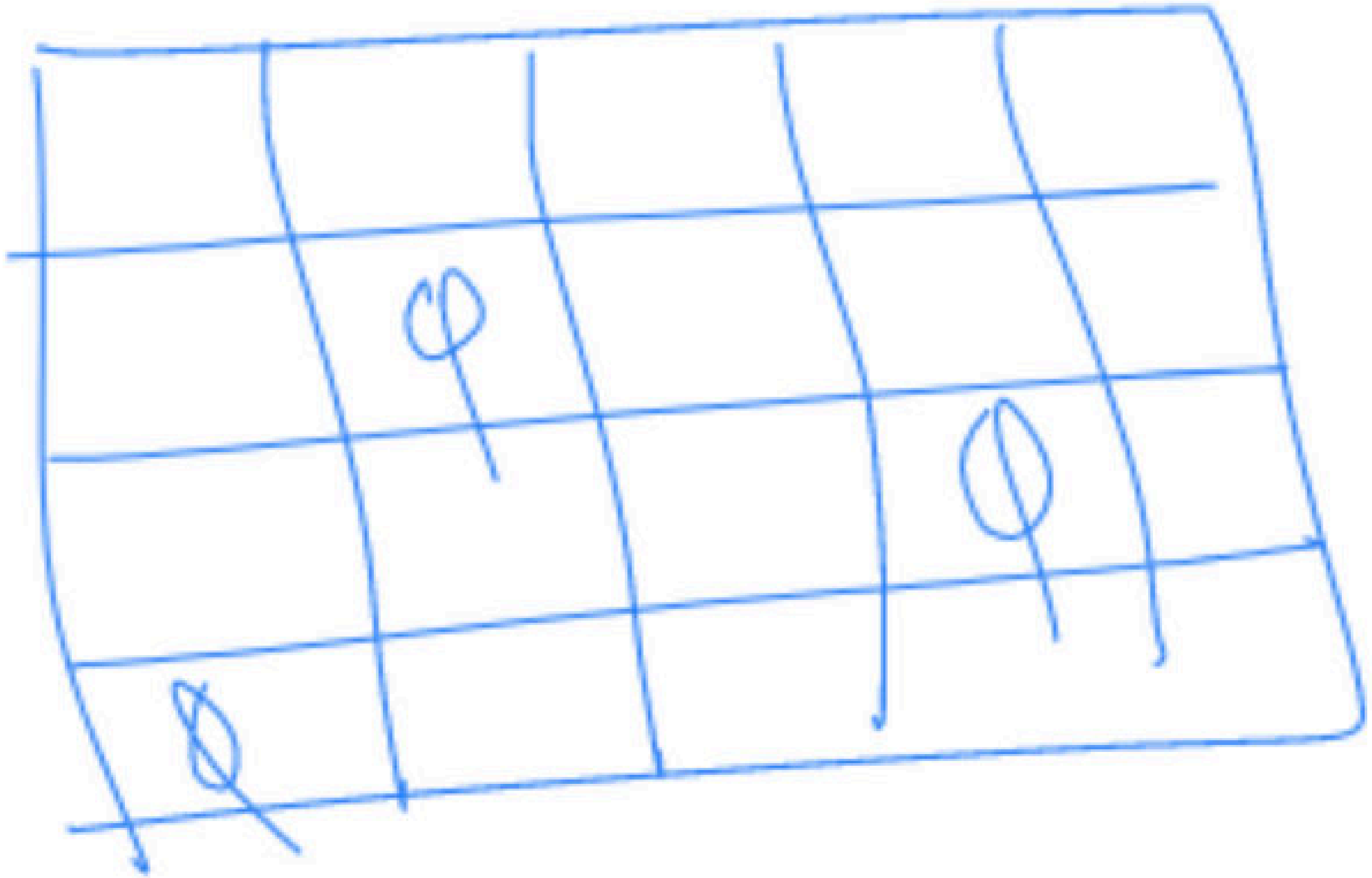
cand1 + cand2

$$f(i, j, n, m) = f(i+1, j, n, m) + f(i, j+1, n, m)$$


0,0,2,2,""
(1,0,2,2,"D")
(0,1,2,2,R)
(2,0,2,2,DD)
(1,1,2,2,DR)
(1,1,2,2,RD)
(0,2,2,2,RR)
(2,1,2,2,DDR)
(2,1,2,2,DRD)
(1,2,2,2,DRR)
(2,1,2,2,RDD)
(1,2,2,2,RDR)
(1,2,2,2,RRD)

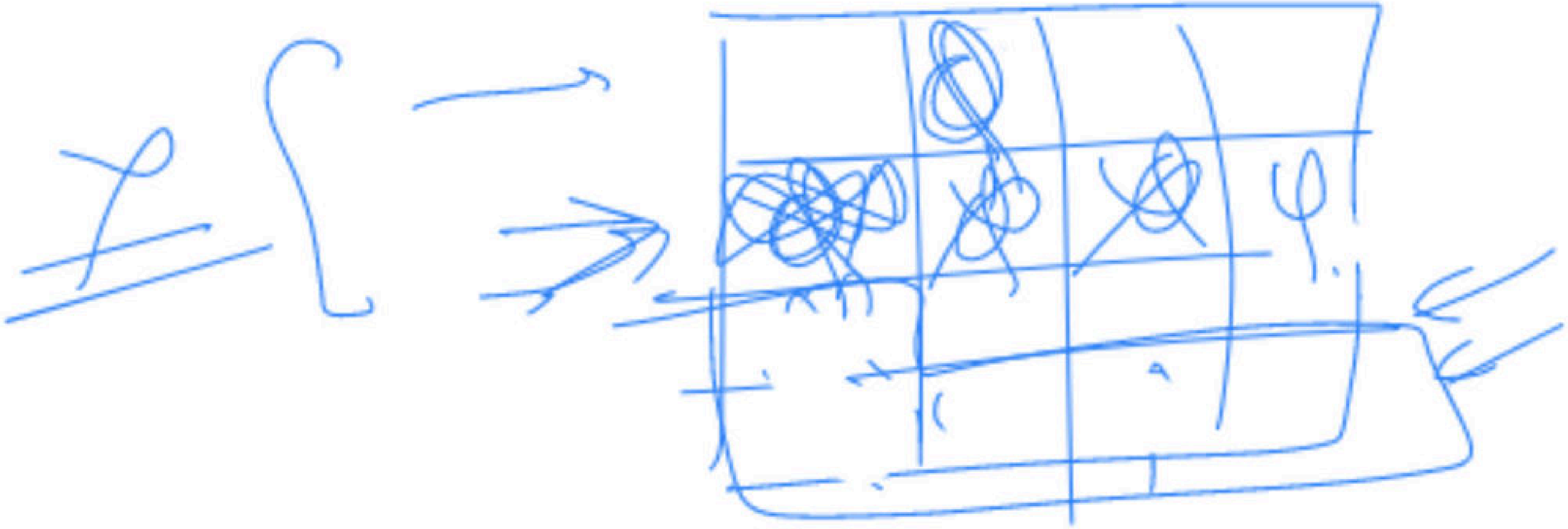


N queen



N queen

N×N



N Queen → N Knights → N Kings

$$f(0,0,n,m) = f(1,0,n,m) + f(0,1,n,m)$$

add

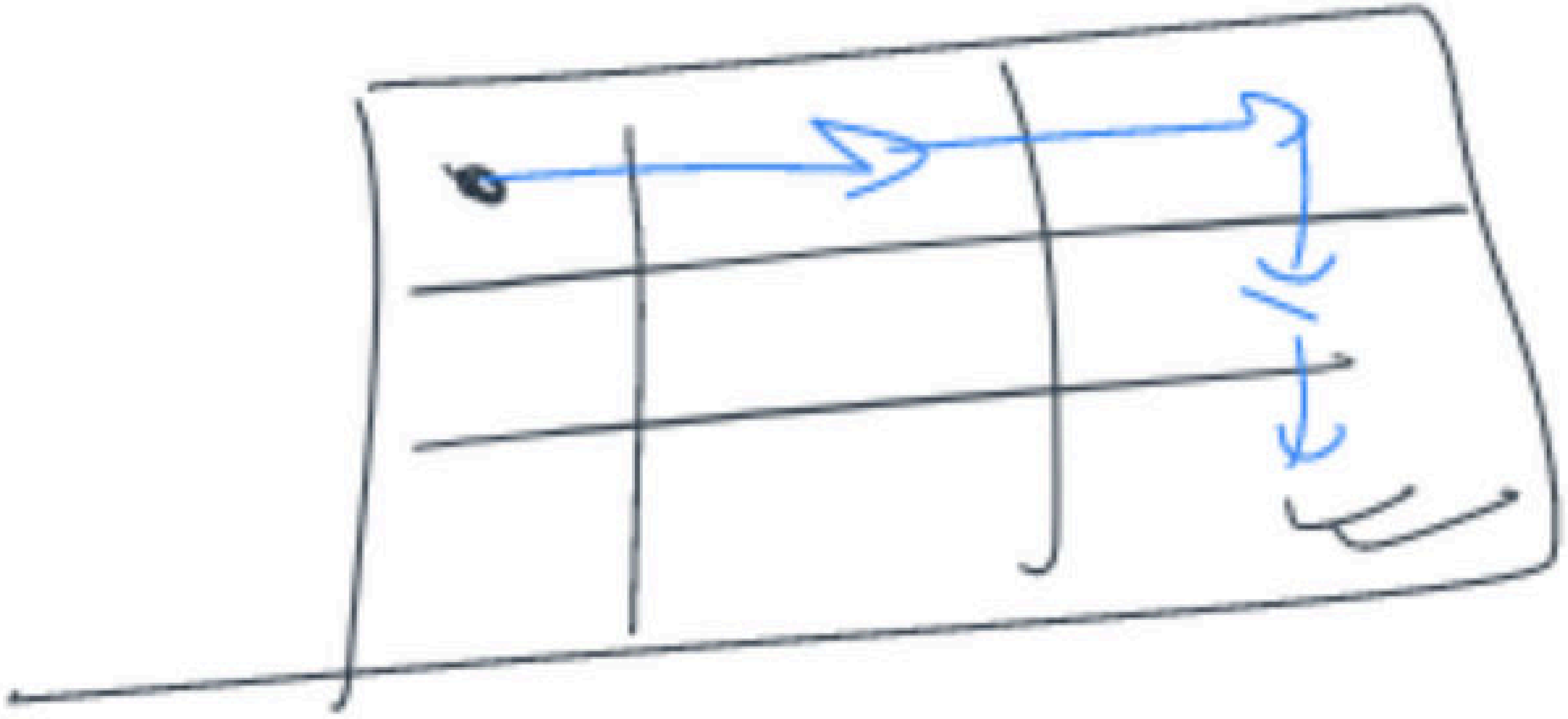
no. of ways to reach from

0,0 → n, m

→ right

or

↓ down



1

count all

no ways