KU1102 – Pengenalan Komputasi

Tugas pekan ke-7

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1. Kerjakan Latihan 1 s.d 6 pada modul ke-1 pekan ke-8 “Array dan Pemrosesannya (Python)”! Tuliskan data yang akan digunakan sebagai komentar pada bagian kamus. Contoh:
   ```
   # KAMUS
   # ...
   # arrayNilai = [85, 90, 95, 90, 85]
   ```
   Tuliskan hasil yang diharapkan dalam bentuk komentar pada akhir program. Contoh:
   ```
   # ALGORITMA
   # ...
   # avgNilai = 89
   ```
2. Buatlah program untuk melakukan inversi matrix dengan menggunakan Eliminasi Gauss-Jordan!
   (contoh perhitungan pada halaman-halaman berikutnya)

Matrix Inversion using Gauss-Jordan Elimination

Inverse Matrix

A square matrix A has an inverse matrix A^{-1} in which:

- $\det(A) \neq 0$
- $AA^{-1} = I$, where I is an identity matrix

Given $A \begin{bmatrix} a_{11} & \cdots & a_{11} \\ a_{21} & \cdots & a_{11} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix}$

Reference: https://mathworld.wolfram.com/MatrixInverse.html

Inverse Matrix

We can use Gauss-Jordan Elimination to find the inverse of A by adding an augmented matrix of I

$$[A \quad I] \equiv \begin{bmatrix} a_{11} & \cdots & a_{11} & 1 & 0 & \cdots & 0 \\ a_{21} & \cdots & a_{11} & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} & 0 & 0 & \cdots & 1 \end{bmatrix}$$

and modifying the matrix (row-wise) so that

$$[I \quad B] \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 & b_{11} & \cdots & b_{11} \\ 0 & 1 & \cdots & 0 & b_{21} & \cdots & b_{11} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & b_{n1} & \cdots & b_{nn} \end{bmatrix}, where\ B = A^{-1}$$

Reference: https://mathworld.wolfram.com/Gauss-JordanElimination.html

Inverse Matrix

Example

Given square matrix A

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 3 & 7 \\ 2 & 4 & 7 \end{bmatrix}$$

Determine A^{-1}!

Inverse Matrix

Example

Solution:

- Check $\det(A)$
  Determinant of A is non-zero, A invertible.
- Write augmented matrix

$$[A \quad I] \equiv \begin{bmatrix} 3 & 4 & 5 & 1 & 0 & 0 \\ 1 & 3 & 7 & 0 & 1 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Matrix

Example

- Find the pivot in the 1st column, swap the 2nd and the 1st rows

$$[A \quad I] \equiv \begin{bmatrix} 1 & 3 & 7 & 0 & 1 & 0 \\ 3 & 4 & 5 & 1 & 0 & 0 \\ 2 & 4 & 7 & 0 & 0 & 1 \end{bmatrix}$$

- Eliminate the 1st column using the pivot

$$[A \quad I] \equiv \begin{bmatrix} 1 & 3 & 7 & 0 & 1 & 0 \\ 0 & -5 & -16 & 1 & -3 & 0 \\ 0 & -2 & -7 & 0 & -2 & 1 \end{bmatrix}$$

Inverse Matrix

Example

$$[A \quad I] \equiv \begin{bmatrix} 1 & 3 & 7 & 0 & 1 & 0 \\ 0 & -5 & -16 & 1 & -3 & 0 \\ 0 & -2 & -7 & 0 & -2 & 1 \end{bmatrix}$$

- Normalize the 2nd row (divide by -5) to make it the pivot of 2nd column

$$[A \quad I] \equiv \begin{bmatrix} 1 & 3 & 7 & 0 & 1 & 0 \\ 0 & 1 & 16/5 & -1/5 & 3/5 & 0 \\ 0 & -2 & -7 & 0 & -2 & 1 \end{bmatrix}$$

- Eliminate the 2nd column using the pivot

$$[A \quad I] \equiv \begin{bmatrix} 1 & 0 & -13/5 & 3/5 & -4/5 & 0 \\ 0 & 1 & 16/5 & -1/5 & 3/5 & 0 \\ 0 & 0 & -3/5 & -2/5 & -4/5 & 1 \end{bmatrix}$$

Inverse Matrix

Example

$$[A \quad I] \equiv \begin{bmatrix} 1 & 0 & -13/5 & 3/5 & -4/5 & 0 \\ 0 & 1 & 16/5 & -1/5 & 3/5 & 0 \\ 0 & 0 & -3/5 & -2/5 & -4/5 & 1 \end{bmatrix}$$

- Normalize the 3rd row (divide by -3/5)

$$[A \quad I] \equiv \begin{bmatrix} 1 & 0 & -13/5 & 3/5 & -4/5 & 0 \\ 0 & 1 & 16/5 & -1/5 & 3/5 & 0 \\ 0 & 0 & 1 & 2/3 & 4/3 & -5/3 \end{bmatrix}$$

- Eliminate the 3rd column using the pivot

$$[A \quad I] \equiv \begin{bmatrix} 1 & 0 & 0 & 7/3 & 8/3 & -13/3 \\ 0 & 1 & 0 & -7/3 & -11/3 & 16/3 \\ 0 & 0 & 1 & -2/3 & -4/3 & -5/3 \end{bmatrix}$$

A^{-1}

Inverse Matrix

Example

Given square matrix A

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 3 & 7 \\ 2 & 4 & 7 \end{bmatrix}$$

Determine A^{-1}!

$$A^{-1} = \begin{bmatrix} 7/3 & 8/3 & -13/3 \\ -7/3 & -11/3 & 16/3 \\ -2/3 & -4/3 & -5/3 \end{bmatrix}$$