Amplitude of sine-waves generated by an oscillating IIR-filter

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Problem: Calculate the amplitude of a sine-wave, knowing two samples of it at phase distance $\Delta\phi$.

$y_1 = A\cos(\phi)$

$y_2 = A\cos(\phi - \Delta\phi)$

Known: y_1, y_2, $\Delta\phi$

Unknown: A, ϕ

Solution:

$$y_2 = A\cos(\varphi - \Delta\varphi) = A\cos(\varphi)\cos(\Delta\varphi) + A\sin(\varphi)\sin(\Delta\varphi)$$

$$y_2 = y_1\cos(\Delta\varphi) + A\sqrt{1 - \frac{y_1^2}{A^2}}\sin(\Delta\varphi)$$

$$y_2^2 + y_1^2\cos^2(\Delta\varphi) - 2y_1\,y_2\cos\Delta\varphi = A^2\sin^2(\Delta\varphi) - y_1^2\sin^2(\Delta\varphi)$$

$$A = \frac{\sqrt{y_2^2 + y_1^2 - 2y_1\,y_2\cos(\Delta\varphi)}}{\sin(\Delta\varphi)}$$

For the task of generating a sine-wave using an IIR-filter with poles on the unit circle, this is an issue when changing the frequency of the sinusoid (parameter $2\cos\Omega$) and the filter is not energy-free, i.e. it has in its memory initial values from previously generated samples.

Substitute in the above formula of A the value y_1 by the output of the filter at time instant (k-1) (i.e. the value in the memory z^{-1}) and the value y_2 by the output of the filter at time instant (k-2) (i.e. the value in the memory z^{-2}). For $\Delta\phi$ use:

$$\Delta\varphi = \frac{2\pi f_{new}}{f_s}$$

where f_{new} is the changed frequency of the sinusoid, i.e. the new one which shall be in effect starting from time instant k, and f_s is the sampling frequency.

Knowing the amplitude A of the sinusoid, normalize the output of the filter to A, so that the final output has amplitude 1. Take care to normalize only the output value, not the feedback-values y (i.e. the normalization has to be outside the feedback-loop).

Remember: for k=0 you have chosen $y_1=1$ and $y_2=0$, so the initial amplitude will be $A=1/\sin(\Delta\phi)$.