

# Learning Attributed Subgraph Matching

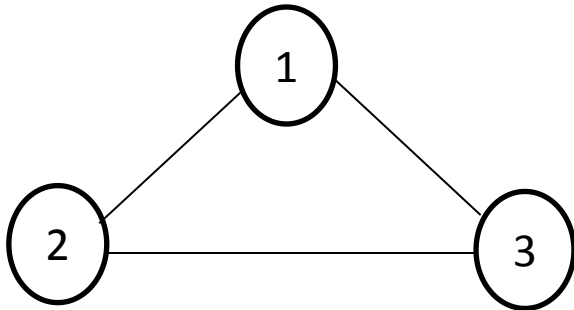
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# Outline

- Introduction and problem definition
- Existing methods and Motivation
- Methodology
- Experiments and Results
- Future Work

# Graph



**Nodes {1, 2, 3}**

**Edges {(1, 2),  
(2, 3),  
(1, 3)}**

Enough information for real tasks?

# Attributed graphs

Richer graph representations

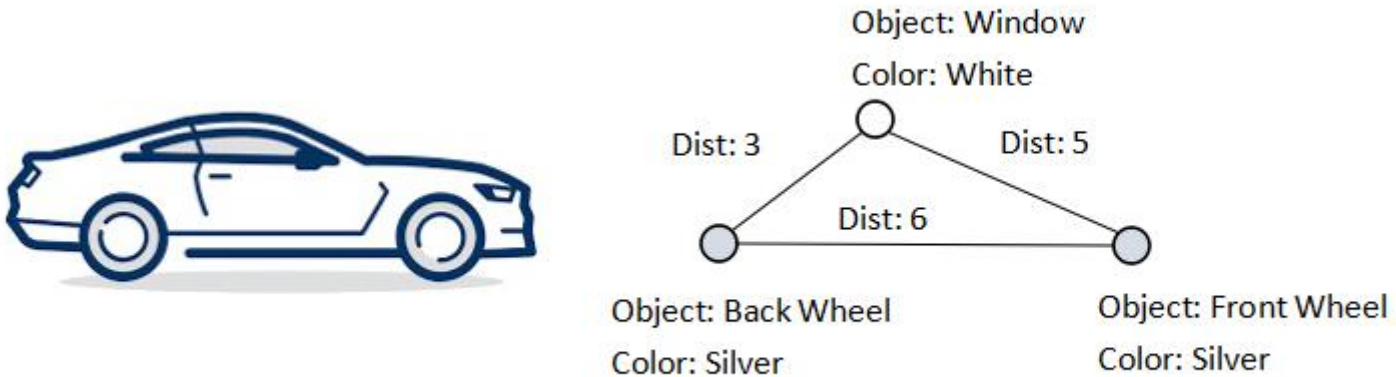


Figure 1: Example Attributed Graph

# Application of Attributed Graph

- Pattern representation in images

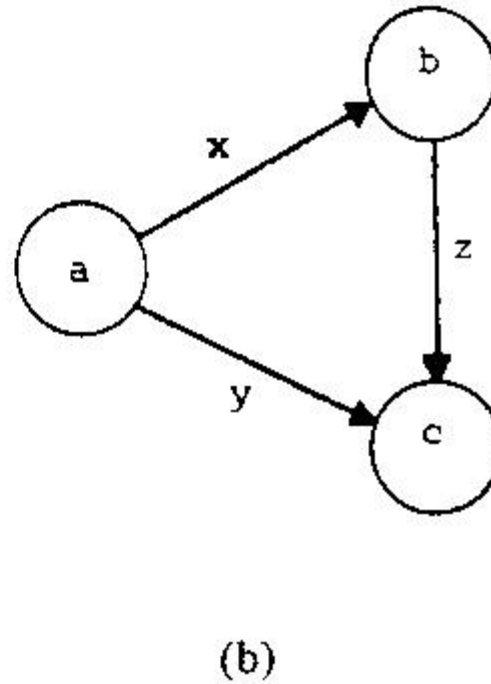
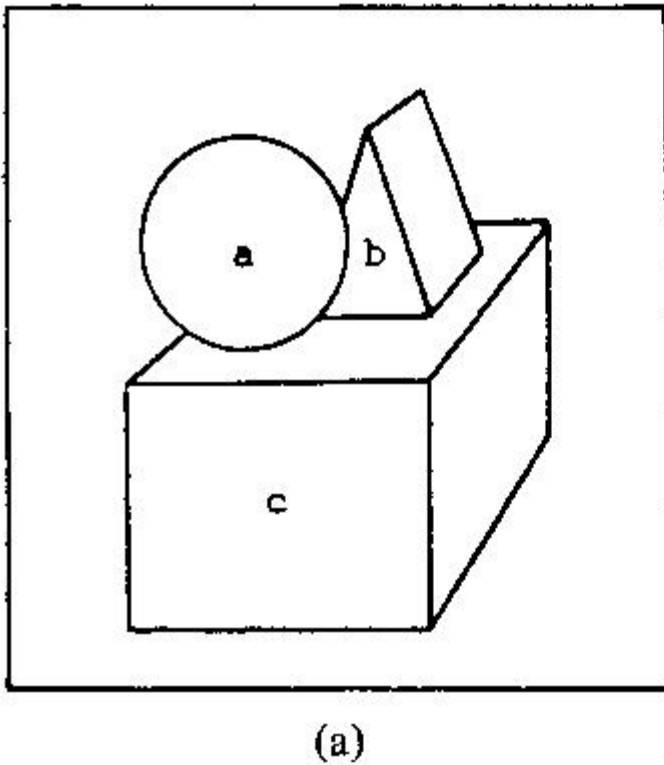


Figure 2: Attributed graph representation of image features<sup>[2]</sup>

# Application of Attributed Graph

Social system

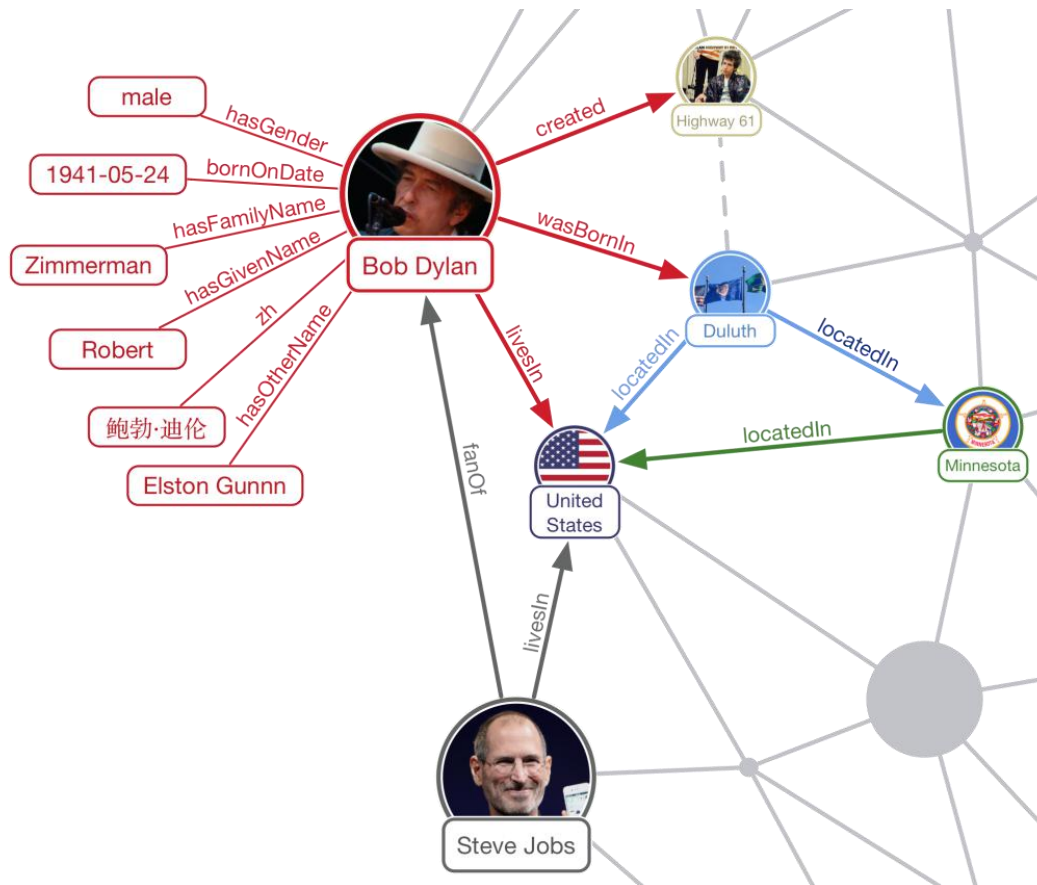
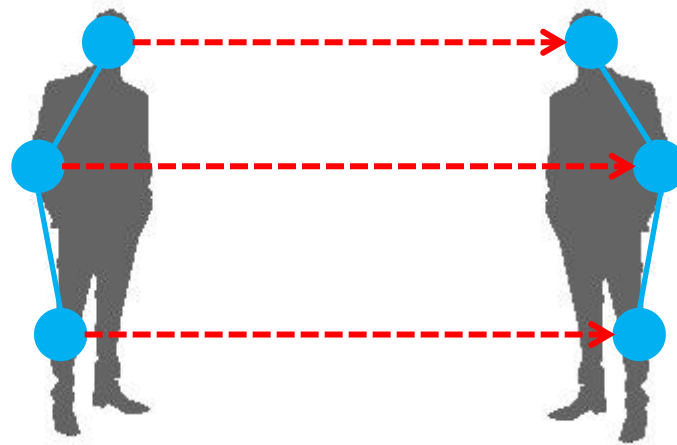


Figure 3: Attributed graph representation of social system<sup>[1]</sup>

# Attributed Graph Matching

**Given:** 2 attributed graphs

**Find:** pairwise matching of the nodes across graphs



**NP-complete**

Figure 4: attributed graph matching example<sup>[3]</sup>

# Attributed Graph Matching

## Applications

- Computer Vision
  - Object recognition
- Medicine
  - Diagnostics
- Biology
  - Biometric identification
- NLP
  - Document matching
- Recommender System

...



# Inexact Attributed graph Matching

Observation: exact matches don't always exist, find the best match

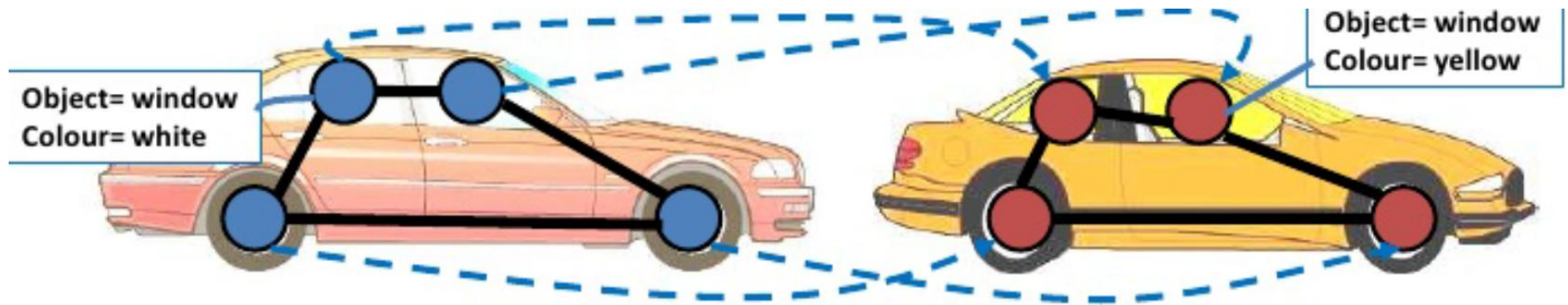


Figure 5: Inexact attributed graph matching example<sup>[4]</sup>.

# Attributed subgraph Matching

Find best match among in the graph(both structural and attribute)



Figure 6: Crowd<sup>[5]</sup>.

# Problem Definition

**Given:** 2 attributed graphs  $G$  and  $G'$ , as well as a (query) subgraph  $g$  from  $G$

**Find:** The best matching subgraph *in*  $G'$

**NP-hard**

**Gap in existing methods**

# Existing Approaches

- Possible to enumerate all the candidate subgraphs, but impractical.
- Existing approaches are all approximations

# Existing Approaches

## Index-based

1. Develop index functions for nodes to capture node information.
2. Apply approximation algorithms to find the optimal matching.

pros: index usually intuitive, e.g. node neighbourhood

cons: Require handcraft index function  
Do not generalize well

# Existing Approaches

## Graph kernels

- ML kernel methods applied on graphs.
- Measures graph similarity.
- stackable
- Subgraph matching kernel, Optimal Assignment(OA) kernel

pros: many available kernels

cons: handcraft kernel function

# Existing Approaches

## Substructure-similarity based

1. Compute similarity for every pairs of substructures across graphs
2. Matching based on substructure similarity

pros: Can use ML to learn similarity measure

cons: Prone to error

Hard to reason about matching process

Space complexity

# Our model

Purpose: To mitigate the problems in current approaches

- Use neural network to learn node representations and similarity
- considering the query subgraph as a whole



# Our model

## An overview

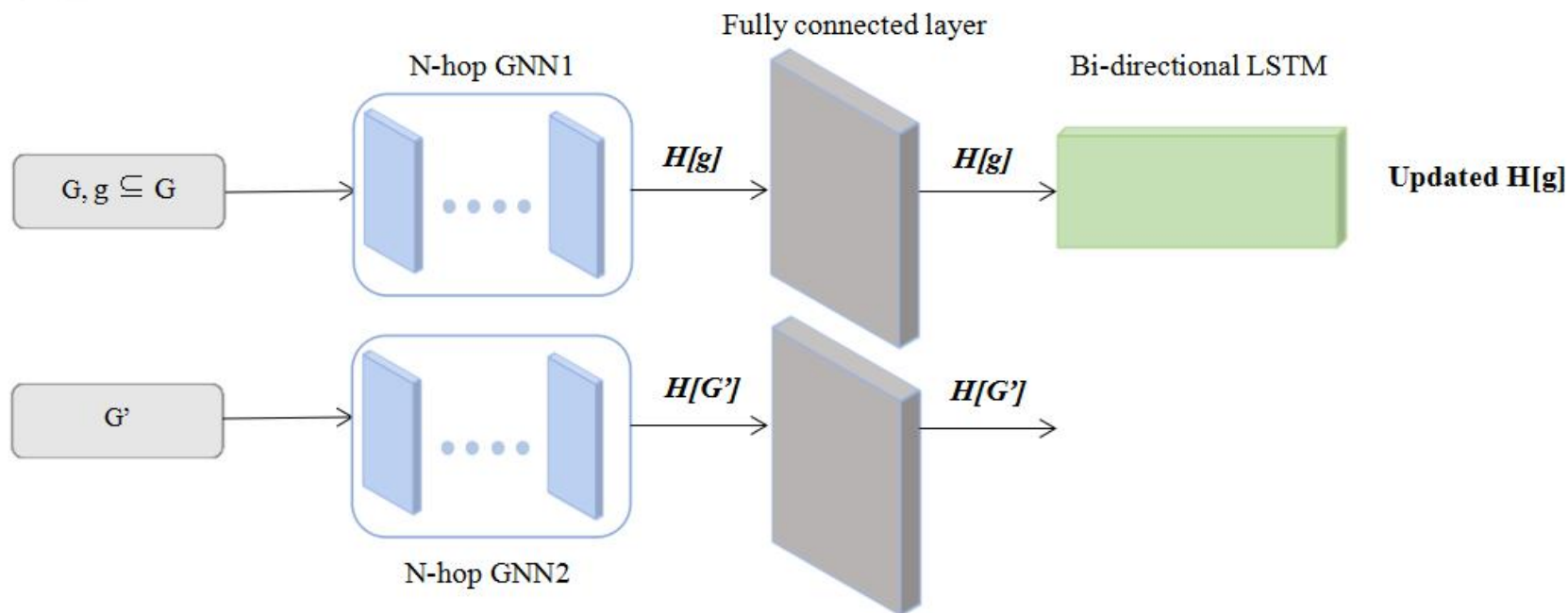


Figure 7: Model Overview.  $H(g)$  is the set of node embedding for nodes in  $g$

# Training stage

Novel idea: Train with pairs of matching subgraphs

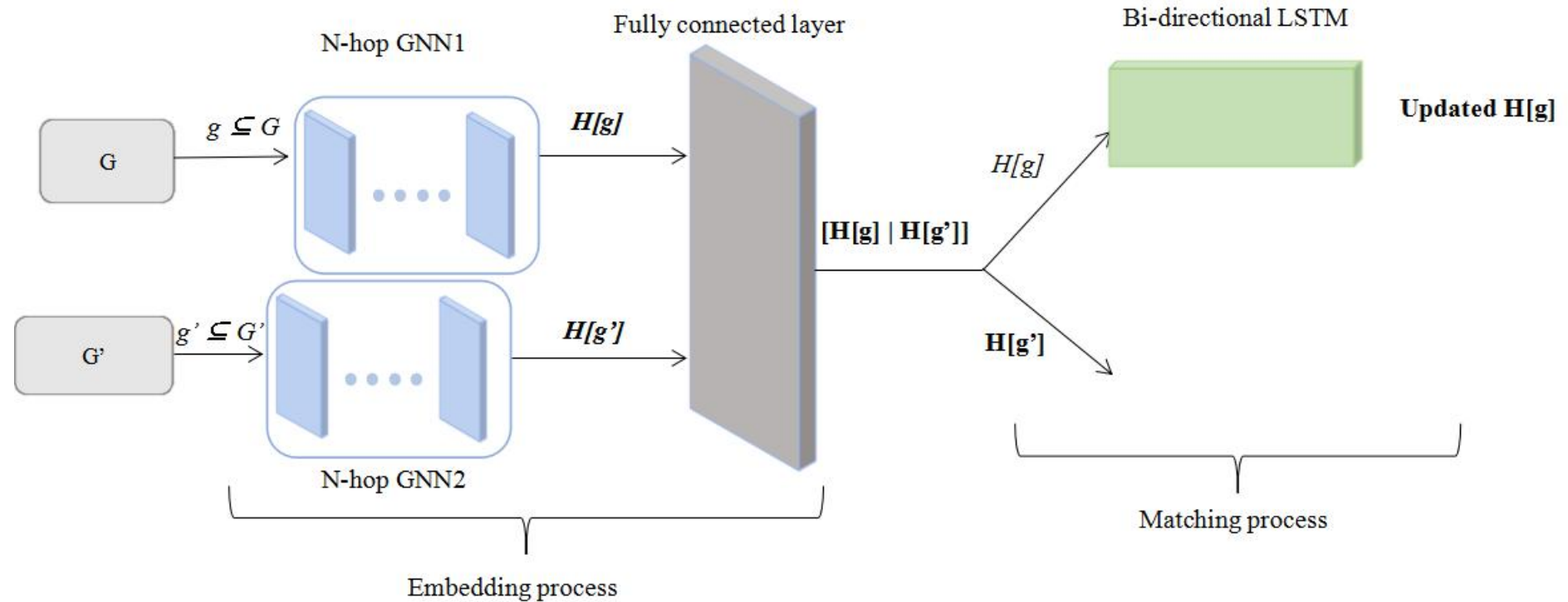


Figure 8: Model for the training stage.

# Embedding process

- Graph Neural Network(GNN)

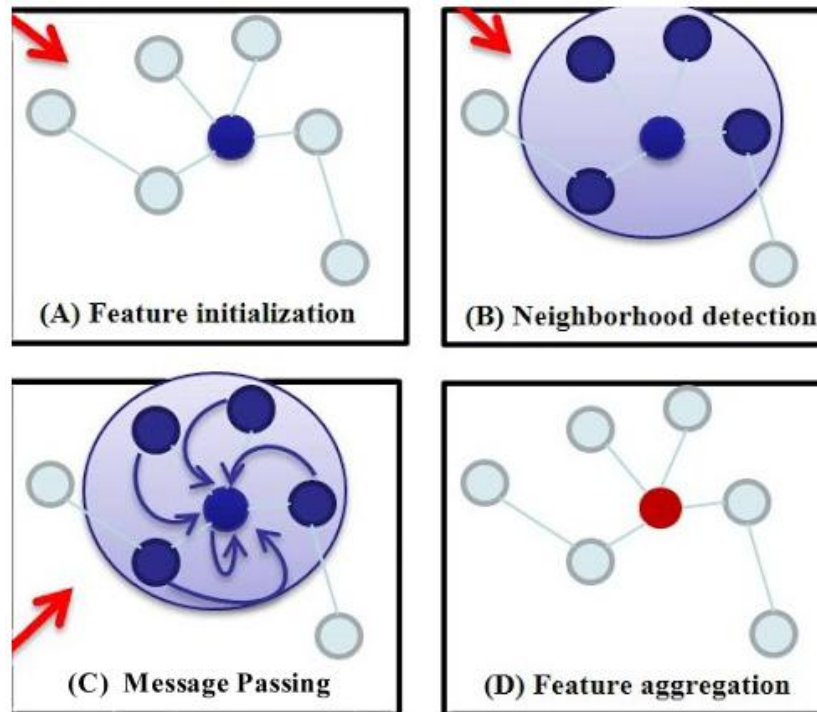


Figure 9: An illustration of 1-degree (hop) graph neural network<sup>[6]</sup>. The network works based on the principle of message passing.

# Embedding process

- Fully connected layer
  - Combine the separate node embeddings, learn two graphs jointly
- Embedding Loss
  - $L_e$  = difference in embeddings between graphs in the training pair

# Matching process

- Inspired by NLP
- In sentences, words are closely correlated.
- Same for nodes in the same graph
- Novel idea: Treat graphs as sequence of nodes
- Learn the sequence
- LSTM: capture long-short term dependencies

# Matching process

- How to sequence the nodes?
  - random sequence



Bi-directional LSTM learns dependencies in both directions

# Matching process

- Bi-directional LSTM(BiLSTM)

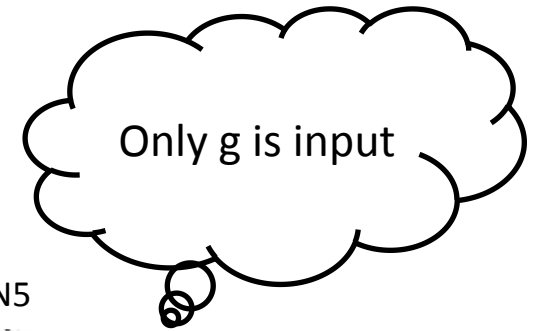
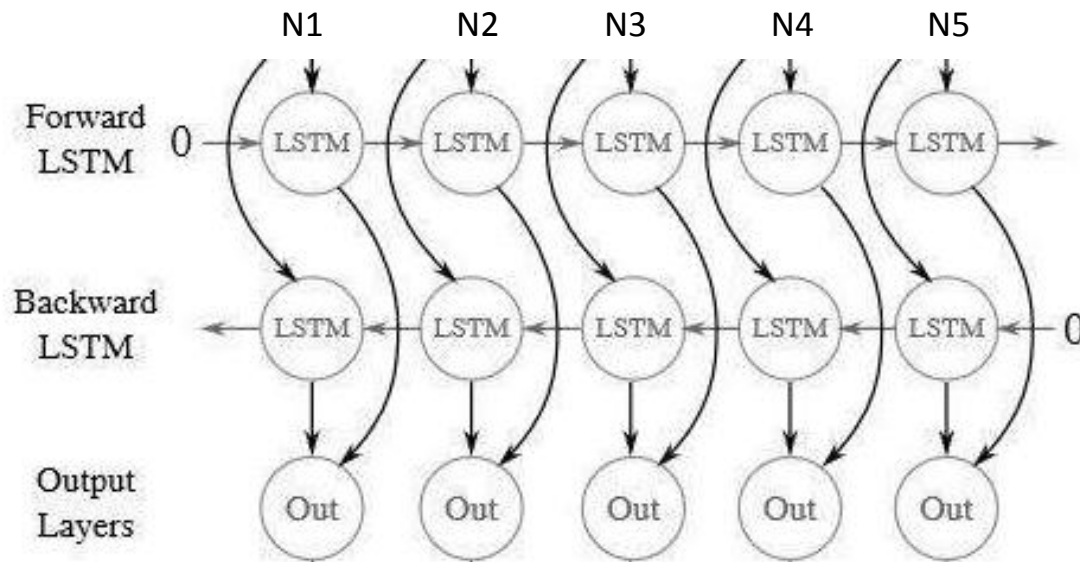


Figure 10: 1 layer Bi-directional LSTM used in the model.<sup>[7]</sup> The input is one subgraph in each pair.

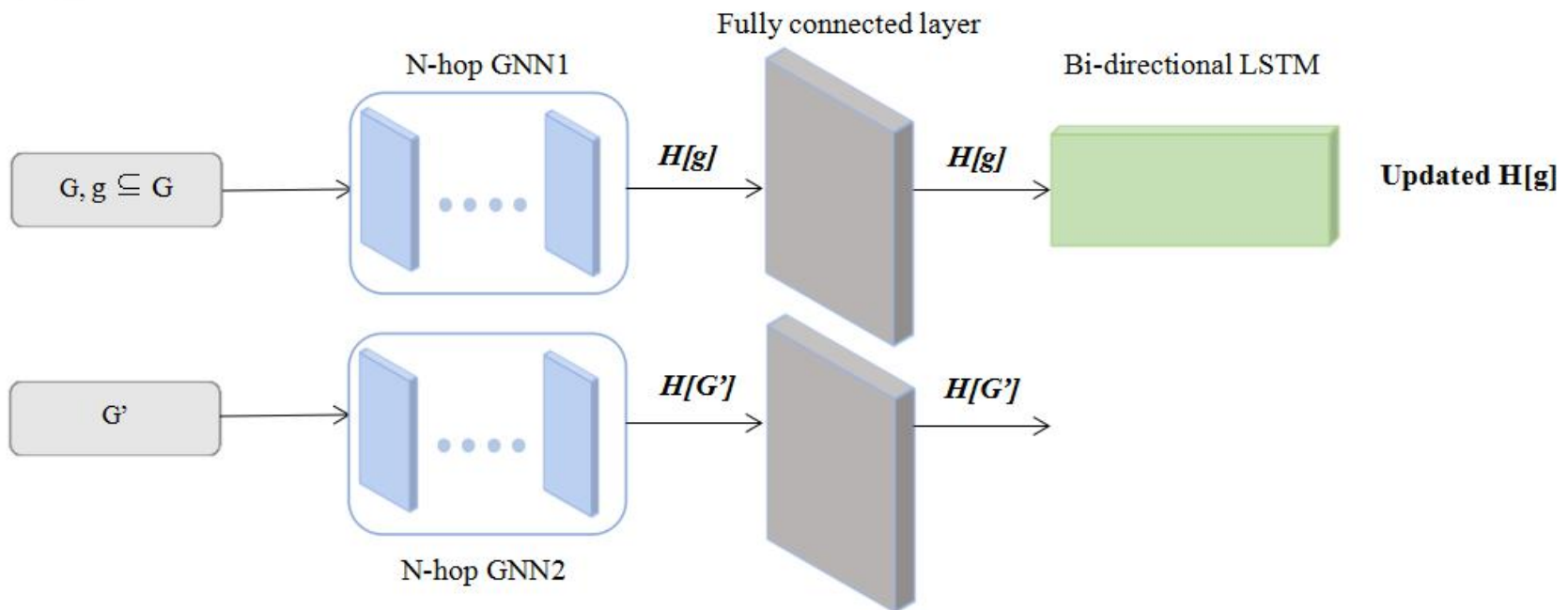
# Matching process

- Finding the best match
  - look for closest neighbour for each node using its embedding
- Matching Loss:
  - Let  $g''$  be the best matching found,  
 $L_M$  = difference in distance matrix between query graph  $g$  and  $g''$ .
- Total loss:  
 $L_E + \gamma L_M$  where  $\gamma$  is a hyperparameter, default is 0.5.



# Testing stage

- Only have the query subgraph



# Experiments

- Three datasets:
  1. Synthetic dataset
  2. Cora
  3. MovieLens
- Three early (~2010) baselines:
  1. TALE(index-based)
  2. Optimal Assignment(OA) kernel
  3. LG method(substructure-similarity based).
- Measure:
  1. Accuracy
  2. Precision

# Synthetic Dataset

- Randomly generate 200 pairs of matching subgraphs of size 3, 4, 5 with attributes  $\text{Color} \in \{\text{R}, \text{G}, \text{B}\}$
- Each pair obtained by randomly generating one and get the other with small modifications
- Aggregate the pairs to form the large graphs

# Cora Dataset

- Classification dataset
- Entries: 2708 machine learning papers
- Each has 1433 features denoting the presence of 1433 keywords
- Class: type of paper
- Randomly extract 200 pairs of matching subgraphs of size 3, 4, 5 using a greedy approach.

# MovieLens Dataset

- Dataset recording movie rating
- Entries: movies
- Each has 22 features denoting its information and rating
- Randomly extract 200 pairs of matching subgraphs of size 3, 4, 5.

# Experimental Result

- Result for our model on different datasets, taking average of 10 runs

Dataset	Subgraph size	Accuracy	Precision
Synthetic	3	0.61	0.70
Synthetic	4	0.57	0.70
Synthetic	5	0.50	0.62
Cora	3	0.62	0.72
Cora	4	0.52	0.68
Cora	5	0.48	0.66
MovieLens	3	0.59	0.71
MovieLens	4	0.52	0.64
MovieLens	5	0.40	0.59

Results: Ok but not exceptional

For different datasets, scalability changes

# Comparison with baselines

- Synthetic dataset: Simple attributes, Little number of nodes

Table 4.1: Accuracy comparison on the four models on the 40 instances in the synthetic dataset

Subgraph size	Our model	LG	OA kernel	TALE
3	0.61	0.52	0.49	<b>0.75</b>
4	0.57	0.43	0.45	<b>0.70</b>
5	0.50	0.29	0.38	<b>0.63</b>

Second among the four methods

# Comparison with baselines

- Cora dataset: Very complex attributes, Large number of nodes

Table 4.2: Accuracy comparison on the four models on the 40 instances in the Cora dataset

Subgraph size	Our model	LG	OA kernel	TALE
3	<b>0.62</b>	0.39	0.28	0.56
4	<b>0.52</b>	0.34	0.19	0.49
5	<b>0.48</b>	0.32	0.15	0.44

The best model in this set of experiment.



# Result analysis

- For a model which introduces novel ideas, achieved moderate performance
- Performance problem-dependent, probably attributed to the loss function
- Some error may be resulted by the dataset processing step(the matching pairs found not accurate)
- Naive graph sequencing process

# Future Work

- Compare with more recent baselines
- Try other models for processing sequential data
- Try embedding into other spaces
- Include edge attributes
- Involve heterogeneous graphs



# Thank you!

# References

- [1]: Image from: <https://medium.com/@brkyataman/knowledge-graph-and-youtube-29d259fd3dc1>
- [2]: Image from: A. Wong and M. You, Entropy and Distance of Random Graphs with Applications to Structural Pattern Recognition
- [3]: Image from: <https://www.kissclipart.com/silhouette-person-thinking-clipart-silhouette-clip-10cl2p/>
- [4]: Image from: [https://www.slideshare.net/mobile/raul\\_A/attributed-graph-matching-of-planar-graphs](https://www.slideshare.net/mobile/raul_A/attributed-graph-matching-of-planar-graphs)
- [5]: Image from: <https://huaban.com/pins/1112150236/>
- [6]. Image from: Jun Wu, Jingrui He, Jiejun Xu, <https://www.kdd.org/kdd2019/>
- [7]. Image from: [https://www.sohu.com/a/283978749\\_717210](https://www.sohu.com/a/283978749_717210)