

# Learning-based Attributed Graph Matching

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# Outline

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  - Overview
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  - Matching
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# Background

- Attributed graphs

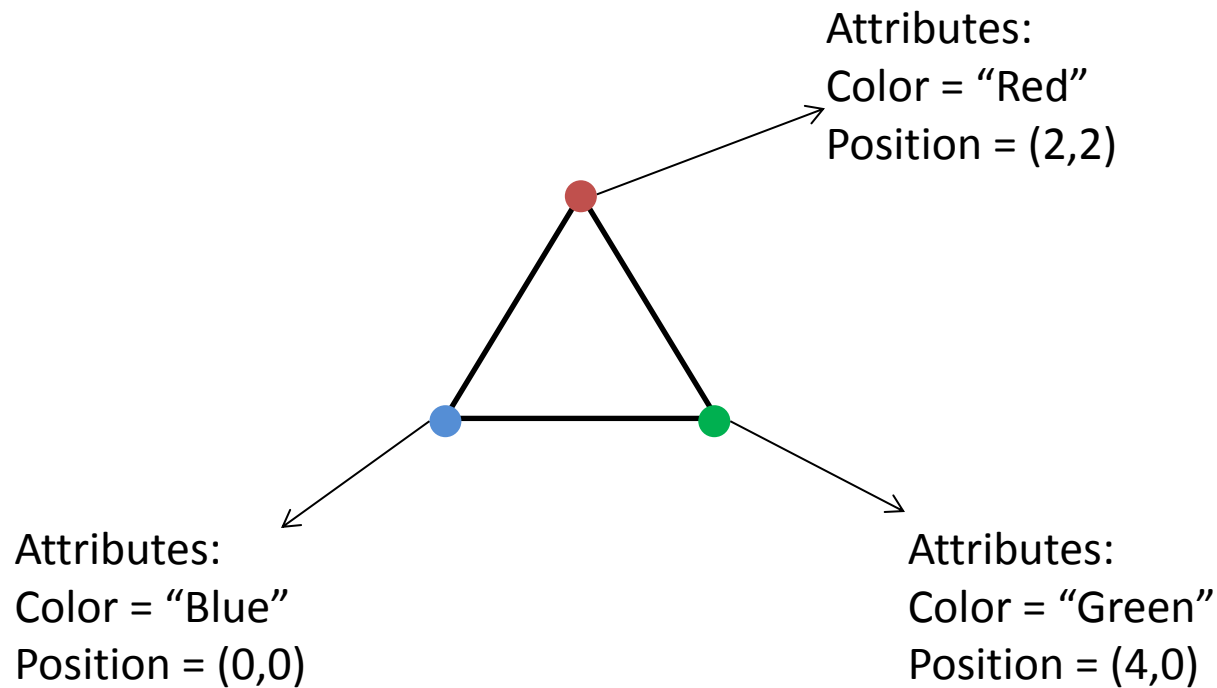


Figure 1: Example of Attributed Graph

# Background

- Attributed graph Matching

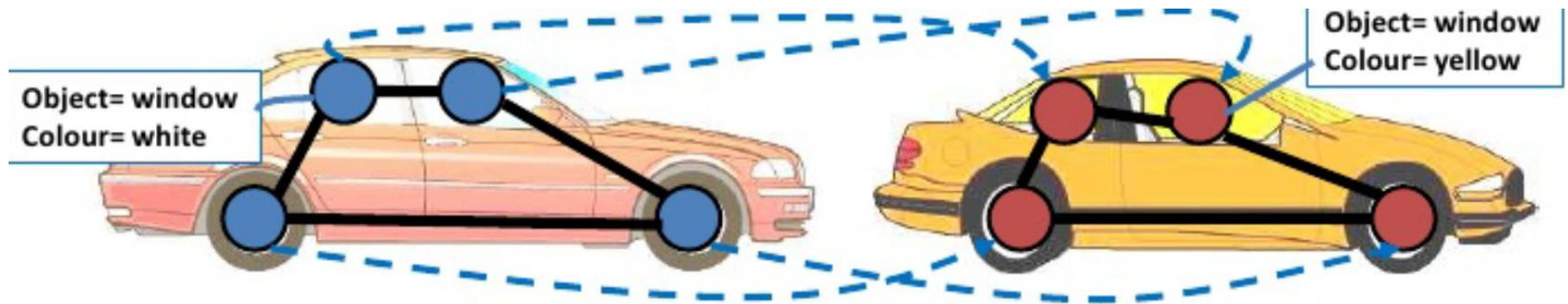


Figure 2: Example of graph matching<sup>[1]</sup>. Both the structure of the car(denoted by the structure of nodes) and the attributes are matched.

# Problem Definition

- Given two attributed graphs  $G$  and  $G'$ , for any connected query subgraph  $g$  of  $G$ , find top- $k$  matching **connected** subgraphs in  $G'$ .
- Real-life Applications
  - Computer Vision
    - 2D & 3D Image analysis
    - Object Recognition
  - Graph Database Indexing and Retrieval
    - Biometric Identification
    - Recommender System
  - NLP
    - Document matching
  - Many others

# Why learning?

- NP problem - Computationally expensive, requires approximation
- Common Mathematical approach<sup>[2]</sup>
  - Step 1: Compute pairwise distance(similarity) between nodes in both graph to give a distance matrix. The distance is computed using a predefined measure.
  - Step 2: Match nodes using approximation algorithms
- Shortcoming of the approach above:
  - time & space complexity
  - handcrafted distance measure

# Existing Learning Approaches

- Graph Neural Network(GNN)

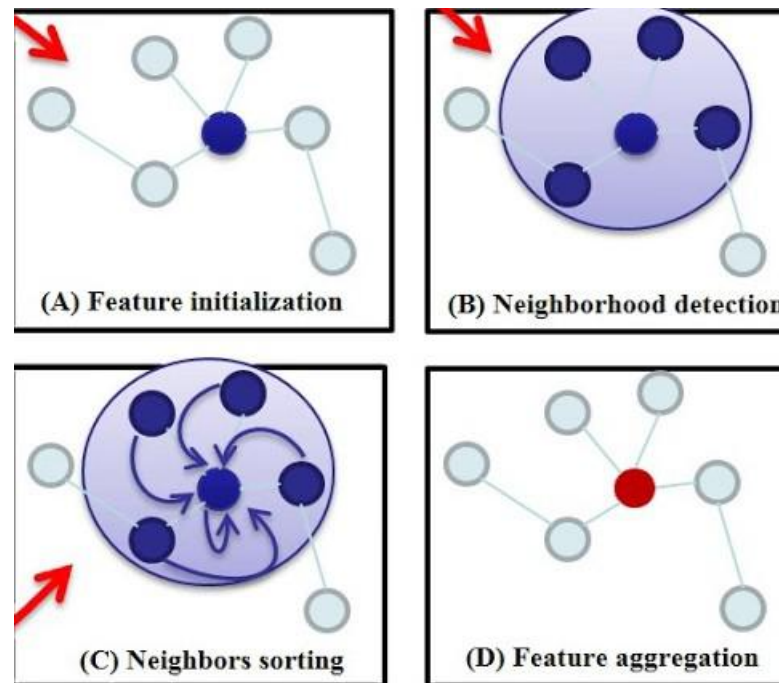


Figure 3: An illustration of 1-degree (hop) graph neural network<sup>[3]</sup>. The network works based on the principle of message passing.

# Existing Learning Approaches

- GNN for graph matching

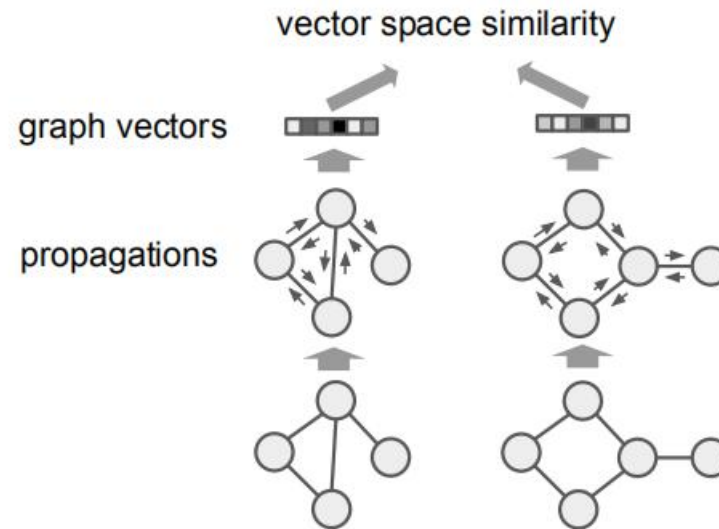


Figure 4: Graph matching using GNN<sup>[4]</sup>. GNN is used to process each graph to generate a graph level embedding, then the two embeddings are compared to compute the similarity.



# Existing Learning Approaches

- Graph Matching network

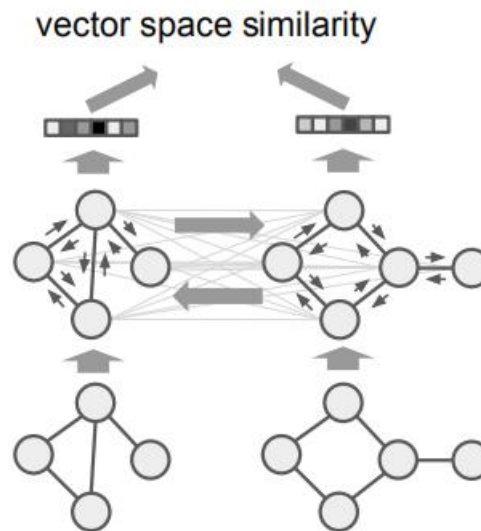


Figure 5: Graph matching network<sup>[4]</sup>. Uses attention mechanism to learn cross-graph correlation when training.

# Existing Approaches - Problems

- Similarity based on graph-level representations
- Does not specify which part matches and on what basis do the two graphs match
- Generally focused on matching the attributes
- Capture more structural information with deeper GNNs, however, exponentially more computationally expensive
- Does not actively search for matching subgraphs, only compute the similarity of given graphs
- Searching for nodes based on closest embeddings often gives unconnected graphs

# My Approach

- Two parts in the model
  - Embedding part
  - Matching part
- General idea: first generate node embeddings, then learn to matching the subgraphs

# Synthetic Dataset

- Randomly generate 150 pairs of matching subgraphs of size 5 with attributes  $\text{Color} \in \{R, G, B\}$
- Matching based on attributes/structure or both
- Aggregate the first subgraph of each pair to form  $G$
- Aggregate the second subgraph of each pair to form  $G'$
- 2:1:1 for training data, validation data and testing data

# Embedding part

- Model

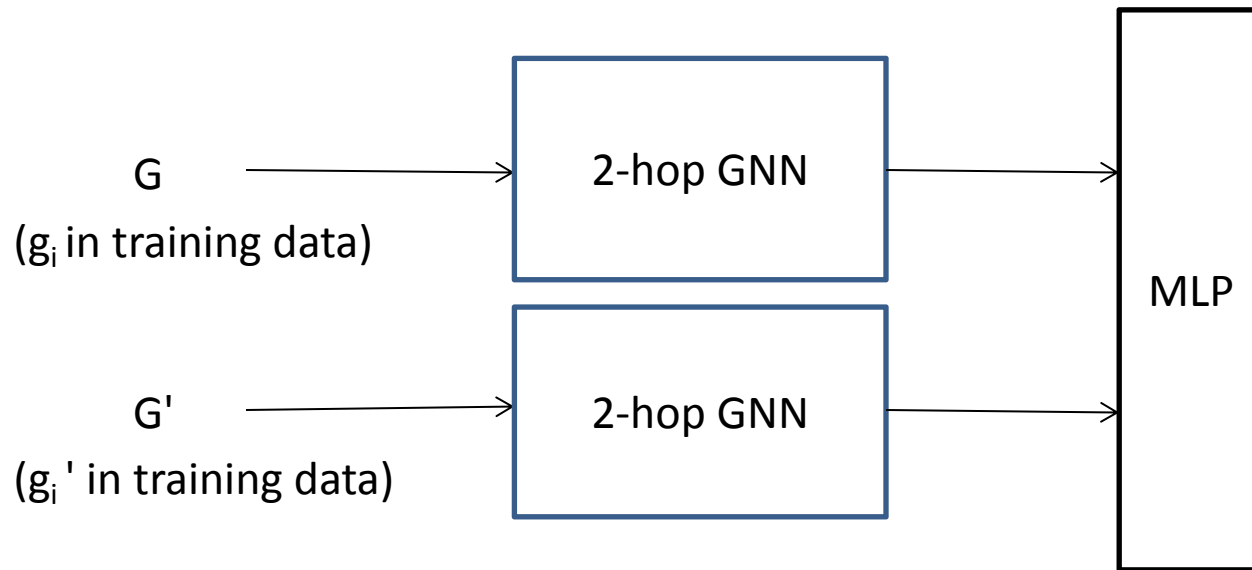


Figure 6: I use separate 2-hop GNNs for each graph. Matching subgraphs are passed into the model one pair at one time.

# Embedding part

- Result: vector representation of each node in  $G$  and  $G'$
- Loss:
  - Embedding of a subgraph = concatenation of constituent node embeddings
  - $L_E$  = Euclidean distance between the embedding of each training pair

# Matching part

- High-degree GNN can capture more structural information, but
  - computationally expensive
  - structural information not clearly shown in the result
- My approach:
  - inspired by NLP methods
  - Nodes in the same graph are related
  - Treat graphs as sequence of nodes
  - Use LSTM to learn about the correlation in the sequence

# Matching part

- Bi-directional LSTM(BiLSTM)

input: one subgraph  $g$  of  $G$

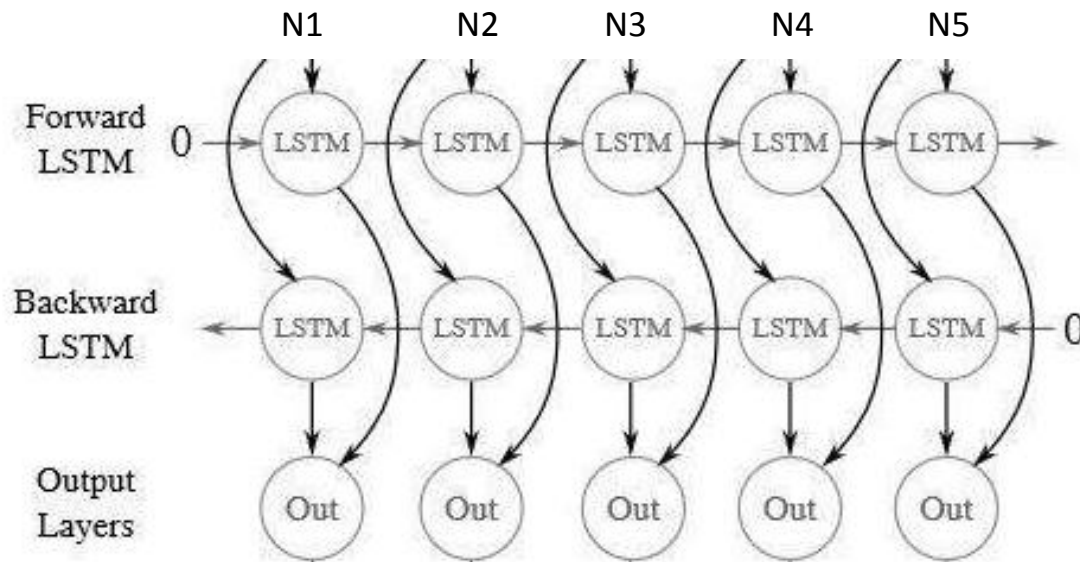


Figure 7: 1 layer Bi-directional LSTM used in the model.<sup>[5]</sup> The input is one subgraph in each pair.



# Matching part

- LSTM: capable of catching long-short term dependency between nodes, may be capable to capture all levels of relations between nodes.
- How to sequence the nodes?
  - random sequence



Nodes before may also have dependencies on the nodes after. Bi-directional LSTM allows learning the connections in both directions

# Matching part

- Result: vector representation of nodes, with structure information of the subgraphs considered. To get the matching subgraph  $g''$  for  $g$ , find the closest node in  $G'$  for each node in  $g$  with the shortest euclidean distance.
- Matching Loss:  
For each training pair  $g$  in  $G$  and  $g'$  in  $G'$ ,  
 $L_M = \text{euclidean distance between } g'' \text{ and } g'$
- Total loss:  
 $L_E + \gamma L_M$  where  $\gamma$  is a parameter to be optimized via validation

# Experimental Result

- Based on the synthetic dataset with subgraphs of 5 nodes
- $\sim 20\%$  accuracy rate( $g'' = g'$ )
- $\sim 20\%$   $g''$  found connected
- $> 40\%$  all nodes in  $g'$  within a 2 hop neighbourhoods of  $g''$
- Numbers are not good apparently
- relatively unresearched field, hard to find baselines

# Future Work

- Try using GMN for the embedding part
- Try further imbedding the nodes into Wasserstein space
- Fine-tune the model to improve the result
- Test on complex real-world datasets
- Find baselines to compare with



# Thank you!

# References

[1]: Image from: [https://www.slideshare.net/mobile/raul\\_A/attributed-graph-matching-of-planar-graphs](https://www.slideshare.net/mobile/raul_A/attributed-graph-matching-of-planar-graphs)

[2]: D.H.Kim,I.D.Yun and S.U.Lee,A New Attributed Relational Graph Matching Algorithm Using the Nested Structure of Earth Mover's Distance, 17th International Conference on Pattern Recognition, p. 48-51, 2004.

[3]. Image from: Jun Wu, Jingrui He, Jiejun Xu, <https://www.kdd.org/kdd2019/>

[4]. Image from: Y. Li et.al. Graph Matching Networks for Learning the Similarity of Graph Structured Objects. ICLR 2019.

[5]. Image from: [https://www.sohu.com/a/283978749\\_717210](https://www.sohu.com/a/283978749_717210)