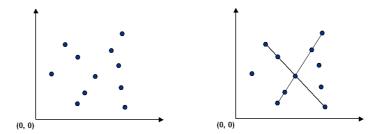


Write a program to recognize line patterns in a given set of points.

Computer vision involves analyzing patterns in visual images and reconstructing the real-world objects that produced them. The process is often broken up into two phases: *feature detection* and *pattern recognition*. Feature detection involves selecting important features of the image; pattern recognition involves discovering patterns in the features. We will investigate a particularly clean pattern recognition problem involving points and line segments. This kind of pattern recognition arises in many other applications such as statistical data analysis.

The problem. Given a set of n distinct points in the plane, find every (maximal) line segment that connects a subset of 4 or more of the points.



Point data type. Create an immutable data type Point that represents a point in the plane by implementing the following API:

```
public class Point implements Comparable<Point> {
  public Point(int x, int y)
                                                       // constructs the point (x, y)
                                                       // draws this point
            void draw()
            void drawTo(Point that)
                                                         draws the line segment from this point to that point
   public String toString()
                                                       // string representation
   public
                        int compareTo(Point that)
                                                          compare two points by y-coordinates, breaking ties by x-coordinates
                                                          the slope between this point and that point
   public
                     double slopeTo(Point that)
                                                       // compare two points by slopes they make with this point
  public Comparator<Point> slopeOrder()
}
```

To get started, use the data type Point.java , which implements the constructor and the draw(), drawTo(), and toString() methods. Your job is to add the following components.

- The compareTo() method should compare points by their y-coordinates, breaking ties by their x-coordinates. Formally, the invoking point (x_0, y_0) is less than the argument point (x_1, y_1) if and only if either $y_0 < y_1$ or if $y_0 = y_1$ and $x_0 < x_1$.
- The slopeTo() method should return the slope between the invoking point (x_0, y_0) and the argument point (x_1, y_1) , which is given by the formula $(y_1 y_0) / (x_1 x_0)$. Treat the slope of a horizontal line segment as positive zero; treat the slope of a vertical line segment as positive infinity; treat the slope of a degenerate line segment (between a point and itself) as negative infinity.
- The slopeOrder() method should return a comparator that compares its two argument points by the slopes they make with the invoking point (x_0, y_0) . Formally, the point (x_1, y_1) is *less than* the point (x_2, y_2) if and only if the slope $(y_1 y_0) / (x_1 x_0)$ is less than the slope $(y_2 y_0) / (x_2 x_0)$. Treat horizontal, vertical, and degenerate line segments as in the slopeTo() method.
- Do *not* override the equals() or hashCode() methods.

Corner cases. To avoid potential complications with integer overflow or floating-point precision, you may assume that the constructor arguments x and y are each between 0 and 32,767.

Line segment data type. To represent line segments in the plane, use the data type LineSegment.java , which has the following API:

Brute force. Write a program BruteCollinearPoints.java that examines 4 points at a time and checks whether they all lie on the same line segment, returning all such line segments. To check whether the 4 points p, q, r, and s are collinear, check whether the three slopes between p and q, between p and q, and between p and q are all equal.

The method segments() should include each line segment containing 4 points exactly once. If 4 points appear on a line segment in the order $p \rightarrow q \rightarrow r \rightarrow s$, then you should include either the line segment $p \rightarrow s$ or $s \rightarrow p$ (but not both) and you should not include *subsegments* such as $p \rightarrow r$ or $q \rightarrow r$. For simplicity, we will not supply any input to BruteCollinearPoints that has 5 or more collinear points.

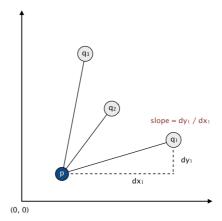
Corner cases. Throw an IllegalArgumentException if the argument to the constructor is null, if any point in the array is null, or if the argument to the constructor contains a repeated point.

Performance requirement. The order of growth of the running time of your program should be n^4 in the worst case and it should use space proportional to n plus the number of line segments returned.

A faster, sorting-based solution. Remarkably, it is possible to solve the problem much faster than the brute-force solution described above. Given a point p, the following method determines whether p participates in a set of 4 or more collinear points.

- Think of p as the origin.
- For each other point q, determine the slope it makes with p.
- Sort the points according to the slopes they makes with p.
- Check if any 3 (or more) adjacent points in the sorted order have equal slopes with respect to p. If so, these points, together with p, are collinear.

Applying this method for each of the n points in turn yields an efficient algorithm to the problem. The algorithm solves the problem because points that have equal slopes with respect to p are collinear, and sorting brings such points together. The algorithm is fast because the bottleneck operation is sorting.



Write a program FastCollinearPoints.java that implements this algorithm.

The method segments() should include each *maximal* line segment containing 4 (or more) points exactly once. For example, if 5 points appear on a line segment in the order $p \rightarrow q \rightarrow r \rightarrow s \rightarrow t$, then do not include the subsegments $p \rightarrow s$ or $q \rightarrow t$.

Corner cases. Throw an IllegalArgumentException if the argument to the constructor is null, if any point in the array is null, or if the argument to the constructor contains a repeated point.

Performance requirement. The order of growth of the running time of your program should be $n^2 \log n$ in the worst case and it should use space proportional to n plus the number of line segments returned. FastCollinearPoints should work properly even if the input has 5 or more collinear points.

Sample client. This client program takes the name of an input file as a command-line argument; read the input file (in the format specified below); prints to standard output the line segments that your program discovers, one per line; and draws to standard draw the line segments.

```
public static void main(String[] args) {
    // read the n points from a file
   In in = new In(args[0]);
   int n = in.readInt();
   Point[] points = new Point[n];
   for (int i = 0; i < n; i++) {
        int x = in.readInt();
        int y = in.readInt();
       points[i] = new Point(x, y);
    // draw the points
   StdDraw.enableDoubleBuffering();
   StdDraw.setXscale(0, 32768);
    StdDraw.setYscale(0, 32768);
    for (Point p : points) {
       p.draw();
   StdDraw.show();
    // print and draw the line segments
    FastCollinearPoints collinear = new FastCollinearPoints(points);
   for (LineSegment segment : collinear.segments()) {
        StdOut.println(segment);
        segment.draw();
    StdDraw.show();
}
```

Input format. We supply several sample input files (suitable for use with the test client above) in the following format: An integer n, followed by n pairs of integers (x, y), each between 0 and 32,767. Below are two examples.

```
% cat input6.txt
                         % cat input8.txt
6
19000
       10000
                          10000
                                  10000
18000
       10000
                           3000
32000
       10000
                                   7000
21000
       10000
                           7000
                                   3000
                          20000
 1234
        5678
                                  21000
14000
                          14000
                                  15000
                           6000
```

```
% java-algs4 BruteCollinearPoints input8.txt
(10000, 0) -> (0, 10000)
(3000, 4000) -> (20000, 21000)

% java-algs4 FastCollinearPoints input8.txt
(3000, 4000) -> (20000, 21000)
(0, 10000) -> (10000, 0)

% java-algs4 FastCollinearPoints input6.txt
(14000, 10000) -> (32000, 10000)
```

Web submission. Submit a .zip file containing only BruteCollinearPoints.java, FastCollinearPoints.java, and Point.java. We will supply LineSegment.java and algs4.jar. You may not call any library functions except those in java.lang, java.util, and algs4.jar. You may use library functions in java.util only if they have already been introduced in the course. For example, you may use Arrays.sort(), but not java.util.HashSet.

This assignment was developed by Bob Sedgewick and Kevin Wayne. Copyright © 2008.