

Polynomial Multiplication

3/3 points (100%)

Practice Quiz, 3 questions

1 / 1
points

1.

For $n = 1024$, compute how many operations will the faster divide and conquer algorithm from the lectures perform, using the formula $3^{\log_2 n}$ for the number of operations.



1024



1048576



59049

Correct

$\log_2 n = \log_2 1024 = 10$, so $3^{\log_2 n} = 3^{10} = 59049$.

1 / 1
points

2.

What is the key formula used in the faster divide and conquer algorithm to decrease the number of multiplications needed from 4 to 3?



$$a_1b_0 + a_0b_1 = (a_0 + a_1)(b_0 + b_1) - a_0b_0 - a_1b_1$$

Correct

Correct! This means that we only need to do 3 multiplications a_0b_0 , a_1b_1 and $(a_0 + a_1)(b_0 + b_1)$ instead of 4 multiplications a_0b_0 , a_1b_1 , a_0b_1 and a_1b_0 .



$$a_1(b_0 + b_1) = a_1b_0 + a_1b_1$$



$$a_0 + b_0 = a_1 + b_1$$



$$(a_0 + a_1)(b_0 + b_1) = a_0b_0 + a_0b_1 + a_1b_0 + a_1b_1$$

1 / 1
points

3.

(This is an advanced question.)

Polynomial Multiplication

How to apply Karatsuba polynomial multiplication algorithm to multiply very big integer numbers (containing hundreds of thousands of digits) faster?

Practice Quiz, 3 questions

3/3 points (100%)



For a number $A = \overline{a_1 a_2 \dots a_n}$ with n digits create a corresponding polynomial $a(x) = a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_n$. Then $a(10) = A$. Do the same with number $B = \overline{b_1 b_2 \dots b_n}$ and create polynomial $b(x)$. Multiply polynomials $a(x)$ and $b(x)$, get polynomial $c(x) = \overline{c_1 c_2 \dots c_n}$. If we create a number $C = \overline{c_1 c_2 \dots c_n}$, it is almost the same as product of A and B, but some of its "digits" may be 10 or bigger. If the last "digit" is 52, for example, make the last digit just 2 and add 5 to the previous digit. Go on until all the digits are from 0 to 9.

Suppose we need to multiply numbers 13 and 24. The correct result is 312. To get this result, we first create polynomials $a(x) = x + 3$ and $b(x) = 2x + 4$ corresponding to numbers 13 and 24 respectively. We then use Karatsuba's algorithm to multiply those polynomials and get polynomial $c(x) = 2x^2 + 10x + 12$. To get the answer, we need to compute $c(10) = 2 \times 10^2 + 10 \times 10 + 12$. You see that some of the coefficients of polynomial c are not digits, because they are bigger than 9. To fix that, for each such coefficient from right to left we subtract 10 from it and add 1 to the previous coefficient:

$$c(10) = 2 \times 10^2 + 10 \times 10 + 12 = 2 \times 10^2 + 11 \times 10 + 2 = 3 \times 10^2 + 1 \times 10 + 2 = 312$$


Correct

First we need to convert number with n digits to polynomial with n coefficients in $O(n)$ time. Then we need to multiply two polynomials of degree n in $O(3^{\log_2 n})$ time. After that, we need to convert the polynomial back to number and "fix" it in $O(n)$. The total time for multiplication of the numbers will be $O(n) + O(3^{\log_2 n}) + O(n) = O(3^{\log_2 n})$ as opposed to $O(n^2)$ time for the grade school number multiplication algorithm.



For number A , create a polynomial $a(x) = A$, for number B create a polynomial $b(x) = B$, multiply those polynomials and get the answer.

Suppose we need to multiply numbers 13 and 24. The correct result is 312. To get this result, we first create polynomials $a(x) = 13$ and $b(x) = 24$ corresponding to numbers 13 and 24 respectively. We then use Karatsuba's algorithm to multiply those polynomials and get polynomial $c(x) = 312$. Now we know that $13 \times 24 = 312$.

