APMA 3100, **Group Work** - Section 5.7 - 5.8

NAME (Print last name first): Wise Madeline SCORE:_______/14

[4 pts] 1. The joint PMF of X and Y is below, along with the marginal PMFs of X and Y.

X / Y	0	1	2	$P_Y(y)$
~~0~~	~~0.1~~	~~0.2~~	~~0.2~~	0.5
2	0.1	0.25	0.15	0.5
$P_X(x)$	0.2	0.45	0.35	

$E[X] = x \cdot P_X(x)$

$= (0)(0.2) + (1)(0.45) + 2(0.35)$

$= 0.45 + 0.7$

$= 1.15$

$E[Y] = 0(0.5) + 2(0.5)$

$= 1.0$

(a) Find $Cov(X,Y)$.

$cov(x,y) = E[xy] - E[x] \cdot E[y]$

$E[xy] = 2(0.25) + 4(0.15) = 0.5 + 0.6 = 1.1$

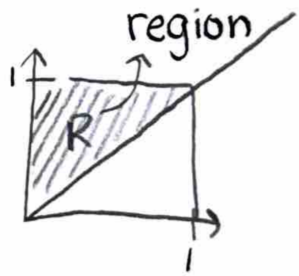
$cov(x,y) = 1.1 - 1.15(1.0)$

$= -0.05$

(b) Determine whether X and Y are independent. Justify your answer.

no, because $cov(x,y) \neq 0$, and $P_{X,Y}(x,y) \neq P_X(x)\,P_Y(y)$ for all (x,y) pairs

[10 pts] 2. Let X and Y be continuous random variables with joint PDF:



$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find marginal PDFs of X and Y.

$f_X(x) = \int_x^1 2\,dy = 2y\Big]_x^1 = 2 - 2x$

$f_Y(y) = \int_0^y 2\,dx = 2x\Big]_0^y = 2y$

$$\boxed{\begin{aligned} f_X(x) &= \begin{cases} 2-2x & 0<x<1 \\ 0 & \text{otherwise} \end{cases} \\ f_Y(y) &= \begin{cases} 2y & 0<y<1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}}$$

(b) Determine whether X and Y are independent.

$f_{x,y}(x,y) = 2$

$2 \neq 2y(2-2x)$

not independent

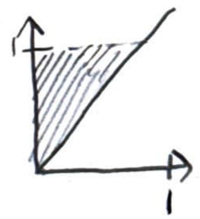
(c) $E(X)$

$\int_0^1 x(2-2x)dx = \int_0^1 2x-2x^2 dx = x^2 - \frac{x^3 2}{3}\Big]_0^1 = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$

(d) $E(Y)$

$\int_0^1 y(2y)dy = \int_0^1 2y^2 dy = \frac{2y^3}{3}\Big]_0^1 = \boxed{\frac{2}{3}}$

(e) $E(XY)$



$E(xy) = \int_0^1 \int_x^1 xy2\, dy\, dx$

$= \boxed{\frac{1}{4}}$

(f) Cov(X, Y)

$\text{cov}(x,y) = E(xy) - E(x)E(y)$

$\text{cov}(x,y) = \frac{1}{4} - \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)$

$= \frac{1}{4} - \frac{2}{9}$

$= \frac{9}{36} - \frac{8}{36} = \boxed{\frac{1}{36}}$