



Lecture 2 : Number System

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Decimal Number System

Base (also called radix) = 10

- 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Digit Position

- Integer & fraction

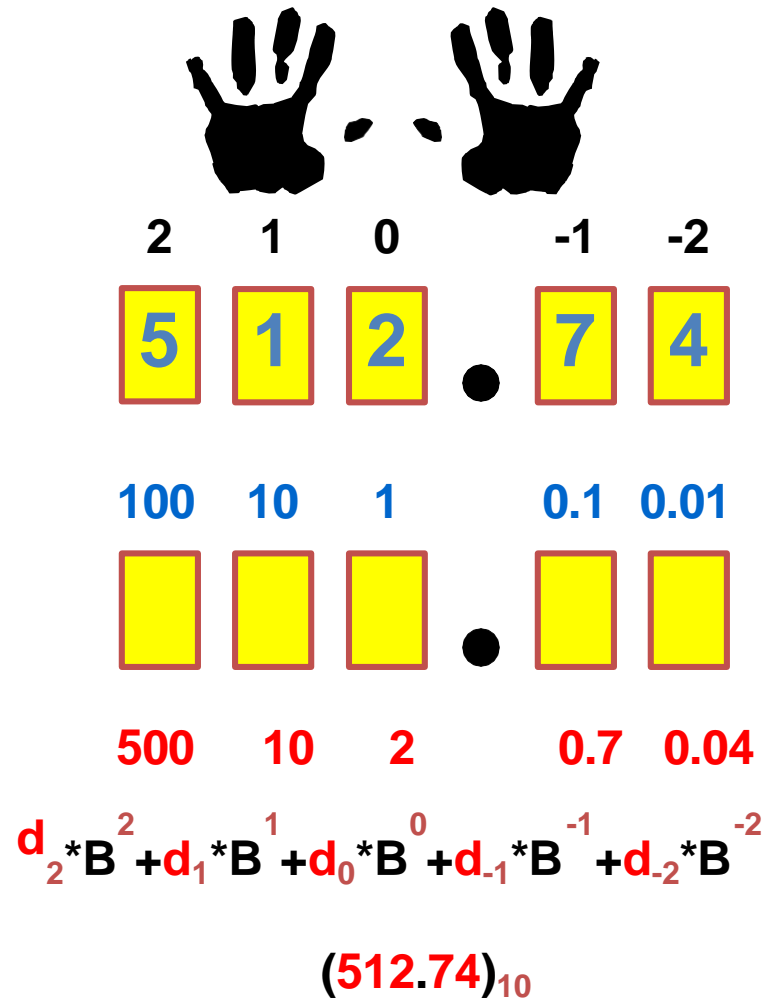
Digit Weight

- Weight = $(Base)^{Position}$

Magnitude

- Sum of “*Digit x Weight*”

Formal Notation



Octal Number System

Base = 8

- 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }

Weights

- Weight = $(Base)^{Position}$

Magnitude

- Sum of “*Digit x Weight*”

Formal Notation

64	8	1		1/8	1/64
5	1	2	.	7	4
2	1	0		-1	-2

$$5 * 8^2 + 1 * 8^1 + 2 * 8^0 + 7 * 8^{-1} + 4 * 8^{-2}$$
$$= 320 + 8 + 2 + 0.875 + .0625$$
$$= (330.9375)_{10}$$
$$(512.74)_8$$

Binary Number System

Base = 2

- 2 digits { 0, 1 }, called *binary digits* or “*bits*”

Weights

- Weight = $(Base)^{Position}$

Magnitude

- Sum of “*Bit x Weight*”

Formal Notation

Groups of bits 4 bits = *Nibble*
 8 bits = *Byte*

1 0 1 1
1 1 0 0 0 1 0 1

4	2	1		1/2	1/4
1	0	1	.	0	1
2	1	0		-1	-2

$$1 * 2^2 + 0 * 2^1 + 1 * 2^0 + 0 * 2^{-1} + 1 * 2^{-2}$$
$$=(5.25)_{10}$$
$$(101.01)_2$$

Hexadecimal Number System

Base = 16

- 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }

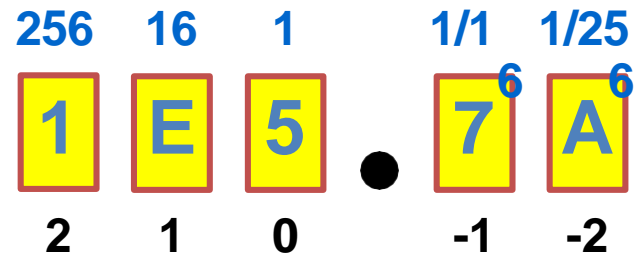
Weights

- Weight = $(Base)^{Position}$

Magnitude

- Sum of “*Digit x Weight*”

Formal Notation



$$\begin{aligned} & 1 * 16^2 + 14 * 16^1 + 5 * 16^0 + 7 * 16^{-1} + 10 * 16^{-2} \\ & = 256 + 224 + 5 + 0.4375 + 0.0390625 \\ & = (485.4765625)_{10} \end{aligned}$$

$$(1E5.7A)_{16}$$

Base-r Number System

For Base - r system $(a_n a_{n-1} \dots a_1 a_0 . a_{-1} a_{-2} \dots a_{-m})_r$

$$a_n \times r^n + a_{n-1} \times r^{n-1} + \dots + a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + \dots + a_{-m} \times r^{-m}$$

Find the decimal equivalent of

$$(123.4)_8 \text{ [Octal]} = 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 4 \times 8^{-1} = 83.5$$

$$(B2.4)_{16} \text{ [Hexa decimal]} = 11 \times 16^1 + 2 \times 16^0 + 4 \times 16^{-1} = 178.25$$

$$(110101)_2 \text{ [Binary]}$$

$$= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 53$$

Decimal to Binary Conversion

Convert $(49)_{10}$ to $()_2$

		Remainder	
	2 49		
	2 24	→ 1	↑ LSB
	2 12	→ 0	
	2 6	→ 0	
	2 3	→ 0	
	2 1	→ 1	
$(110001)_2$	0	→ 1	MSB

Decimal to Binary Conversion

Convert $(50)_{10}$ to $(\)_2$

			Remainder	
	2	50		
	2	25	→ 0	↑ LSB
	2	12	→ 1	
	2	6	→ 0	
	2	3	→ 0	
	2	1	→ 1	
$(110010)_2$		0	→ 1	MSB

Decimal to Binary Conversion

Convert $(0.125)_{10}$ to $()_2$

Integer

$$0.125 \times 2 = 0.25$$

0

$$0.25 \times 2 = 0.5$$

0

$$0.5 \times 2 = 1.0$$

1



$$(0.125)_{10} = (0.001)_2$$


$$(0.125)_{10} = (0.0010)_2$$

Expand to required number of digits Required

Decimal to Binary Conversion

Convert $(0.49)_{10}$ to $()_2$

	Integer
$0.49 \times 2 = 0.98$	0
$0.98 \times 2 = 1.96$	1
$0.96 \times 2 = 1.92$	1
$0.92 \times 2 = 1.84$	1



$(0.49)_{10} = (0.011111\dots)_2$ Limited to required number of digits

Binary to Decimal Conversion

Convert $(110110)_2$ to $()_{10}$

$$= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$$

$$= 1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$$

$$= 1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$$

$$= 54$$

Binary to Octal Conversion

1110001010101

For Octal- 2^3 , 8bit: 2^3

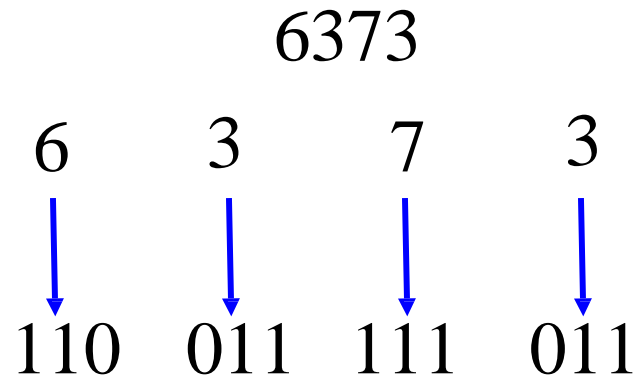
001	110	001	010	101
↓	↓	↓	↓	↓
1	6	1	2	5

$(1110001010101)_2 \longrightarrow (16125)_8$

Octal to Binary Conversion

octal to binary

Octal: base 8 digits used 0-7



$(6373)_8 \longrightarrow (11001111011)_2$

Binary to Hexadecimal Conversion

1110001010101 4 bit, then Hexa decimal-2⁴

0001 1100 0101 0101



1



C



5



5

$(1110001010101)_2 \longrightarrow (1C55)_{16}$

Binary Addition

Rules for Binary Addition

bits to be added {

1	1	1	1	0	0	0	0
1	0	1	0	1	0	1	0
1	1	0	0	1	1	0	0
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
11	10	10	01	10	01	01	00

the carry into the column

the carry out of the column

Binary Addition

Binary Addition Example

carry into the second column

	0	10	110	0110
0110	0110	0110	0110	0110
0111	0111	0111	0111	0111
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
	1	01	101	1101

result for first column

final result

Binary Subtraction

Rules of Binary Subtraction

$$\Rightarrow 1 - 0 = 1$$

$$\Rightarrow 1 - 1 = 0$$

$$\Rightarrow 0 - 0 = 0$$

$$\Rightarrow 0 - 1 = 1$$

(This can not be done directly, hence we borrow one digit from the digit to the left or the next higher order digit.)

Binary Subtraction

$$\begin{array}{r} \\ \\ (-) \\ \hline \\ \hline \end{array}$$

Diagram illustrating binary subtraction. The top row shows the minuend (1010) and the subtrahend (1010). The result is 0010. A borrow is indicated by an arrow pointing from the 10s place to the 1s place.

Binary Multiplication

$$\begin{array}{r} 110 \\ \times 11 \\ \hline \textcircled{1} 110 \\ 110x \\ \hline 10010 \end{array}$$

Binary Division

Diagram illustrating Binary Division:

Divisor: 11

Dividend: 001100

Quotient: 1001

Remainder: 1

The diagram shows the step-by-step process of binary division, including the subtraction of the divisor from the dividend and the shifting of the divisor to the right.

Step 1: 11 divides 100100. The quotient is 1, and the remainder is 100.

Step 2: 11 divides 10010. The quotient is 1, and the remainder is 10.

Step 3: 11 divides 1001. The quotient is 1, and the remainder is 0.

Step 4: 11 divides 00100. The quotient is 0, and the remainder is 0100.

Step 5: 11 divides 01001. The quotient is 1, and the remainder is 001.

Step 6: 11 divides 00100. The quotient is 0, and the remainder is 00100.

Step 7: 11 divides 00101. The quotient is 1, and the remainder is 0001.

Step 8: 11 divides 00010. The quotient is 0, and the remainder is 00010.

Step 9: 11 divides 00011. The quotient is 1, and the remainder is 0000.

Step 10: 11 divides 00001. The quotient is 0, and the remainder is 00001.

Step 11: 11 divides 00001. The quotient is 0, and the remainder is 00001.



Thank you

