Lecture 2 : Number System

By
A. L. Jannat Tariq

Decimal Number System

Base (also called radix) = 10

- 10 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

Digit Position

- Integer & fraction

Digit Weight

- Weight = (*Base*) ^{Position}

Magnitude

- Sum of "*Digit* x *Weight*"

Formal Notation

2	1	0		-1	-2
5	1	2	.	7	4
100	10	1		0.1	0.01
			.		
500	10	2		0.7	0.04

$$d_2 * B^2 + d_1 * B^1 + d_0 * B^0 + d_{-1} * B^{-1} + d_{-2} * B^{-2}$$

$$(512.74)_{10}$$

Octal Number System

Base = 8

- 8 digits { 0, 1, 2, 3, 4, 5, 6, 7 }

Weights

- Weight = (*Base*) ^{Position}

Magnitude

- Sum of "*Digit* x *Weight*"

Formal Notation

64	8	1		1/8	1/64
5	1	2	●	7	4
2	1	0		-1	-2

$5 *8^{2}+1 *8^{1}+2 *8^{0}+7 *8^{-1}+4 *8^{-2}$

$=320+8+ 2+0.875+.0625$

$=(330.9375)_{10}$

$(512.74)_{8}$

Binary Number System

Base = 2

- 2 digits { 0, 1 }, called *b*inary dig*its* or "*bits*"

Weights

- Weight = $(Base)^{Position}$

Magnitude

- Sum of "*Bit* x *Weight*"

Formal Notation

Groups of bits 4 bits = *Nibble*

8 bits = *Byte*

4	2	1		1/2	1/4
1	0	1	.	0	1
2	1	0		-1	-2

$$1 * 2^{2} + 0 * 2^{1} + 1 * 2^{0} + 0 * 2^{-1} + 1 * 2^{-2}$$

$$= (5.25)_{10}$$

$$(101.01)_2$$

1 0 1 1

1 1 0 0 0 1 0 1

Hexadecimal Number System

Base = 16

- 16 digits { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F }

Weights

- Weight = (*Base*) ^{Position}

Magnitude

- Sum of "*Digit* x *Weight*"

Formal Notation

256	16	1		1/16	1/256
1	E	5	.	7	A
2	1	0		-1	-2

$1 * 16^{2} + 14 * 16^{1} + 5 * 16^{0} + 7 * 16^{-1} + 10 * 16^{-2}$

$= 256 + 224 + 5 + 0.4375 + 0.0390625$

$= (485.4765625)_{10}$

$(1E5.7A)_{16}$

Base-r Number System

For Base - r system $(a_n a_{n-1} .... a_1 a_0 . a_{-1} a_{-2} .. a_{-m})_r$

$a_n \times r^n + a_{n-1} \times r^{n-1} ...... a_1 \times r^1 + a_0 \times r^0 + a_{-1} \times r^{-1} + ... a_{-m} \times r^{-m}$

Find the decimal equivalent of

$(123.4)_8$ [Octal] $= 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0 + 4 \times 8^{-1} = 83.5$

$(B2.4)_{16}$ [Hexa decimal] $= 11 \times 16^1 + 2 \times 16^0 + 4 \times 16^{-1} = 178.25$

$(110101)_2$ [Binary]

$= 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 53$

Decimal to Binary Conversion

Convert $(49)_{10}$ to $()_2$

		Remainder	
2	49		
2	24	1	LSB
2	12	0	
2	6	0	
2	3	0	
2	1	1	
	0	1	MSB

$(110001)_2$

Decimal to Binary Conversion

Convert $(50)_{10}$ to $()_2$

			Remainder	
2	50			
2	25	$\rightarrow$	0	LSB
2	12	$\rightarrow$	1	
2	6	$\rightarrow$	0	
2	3	$\rightarrow$	0	
2	1	$\rightarrow$	1	
	0	$\rightarrow$	1	MSB

$(110010)_2$

Decimal to Binary Conversion

Convert $(0.125)_{10}$ to $()_2$

	Integer
$0.125 \times 2 = 0.25$	0
$0.25 \times 2 = 0.5$	0
$0.5 \times 2 = 1.0$	1

$(0.125)_{10} = (0.001)_2$

$(0.125)_{10} = (0.0010)_2$ Expand to required number of digits Required

Decimal to Binary Conversion

Convert $(0.49)_{10}$ to $()_2$

	Integer
$0.49 \times 2 = 0.98$	0
$0.98 \times 2 = 1.96$	1
$0.96 \times 2 = 1.92$	1
$0.92 \times 2 = 1.84$	1

$(0.49)_{10} = (0.011111\ldots.)_2$ Limited to required number of digits

Binary to Decimal Conversion

Convert $(110110)_2$ to $(\ )_{10}$

$=1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$=1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$

$=1 \times 32 + 1 \times 16 + 0 \times 8 + 1 \times 4 + 1 \times 2 + 0 \times 2^0$

$=54$

Binary to Octal Conversion

1110001010101

For Octal- 2^3, 8bit: 2^3

001	110	001	010	101
1	6	1	2	5

$(1110001010101)_2 \longrightarrow (16125)_8$

Octal to Binary Conversion

octal to binary   Octal: base 8 digits used 0-7

6373

$$\begin{array}{cccc} 6 & 3 & 7 & 3 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 110 & 011 & 111 & 011 \end{array}$$

$$(6373)_8 \longrightarrow (110011111011)_2$$

Binary to Hexadecimal Conversion

1110001010101 4 bit, then Hexa decimal-2^4

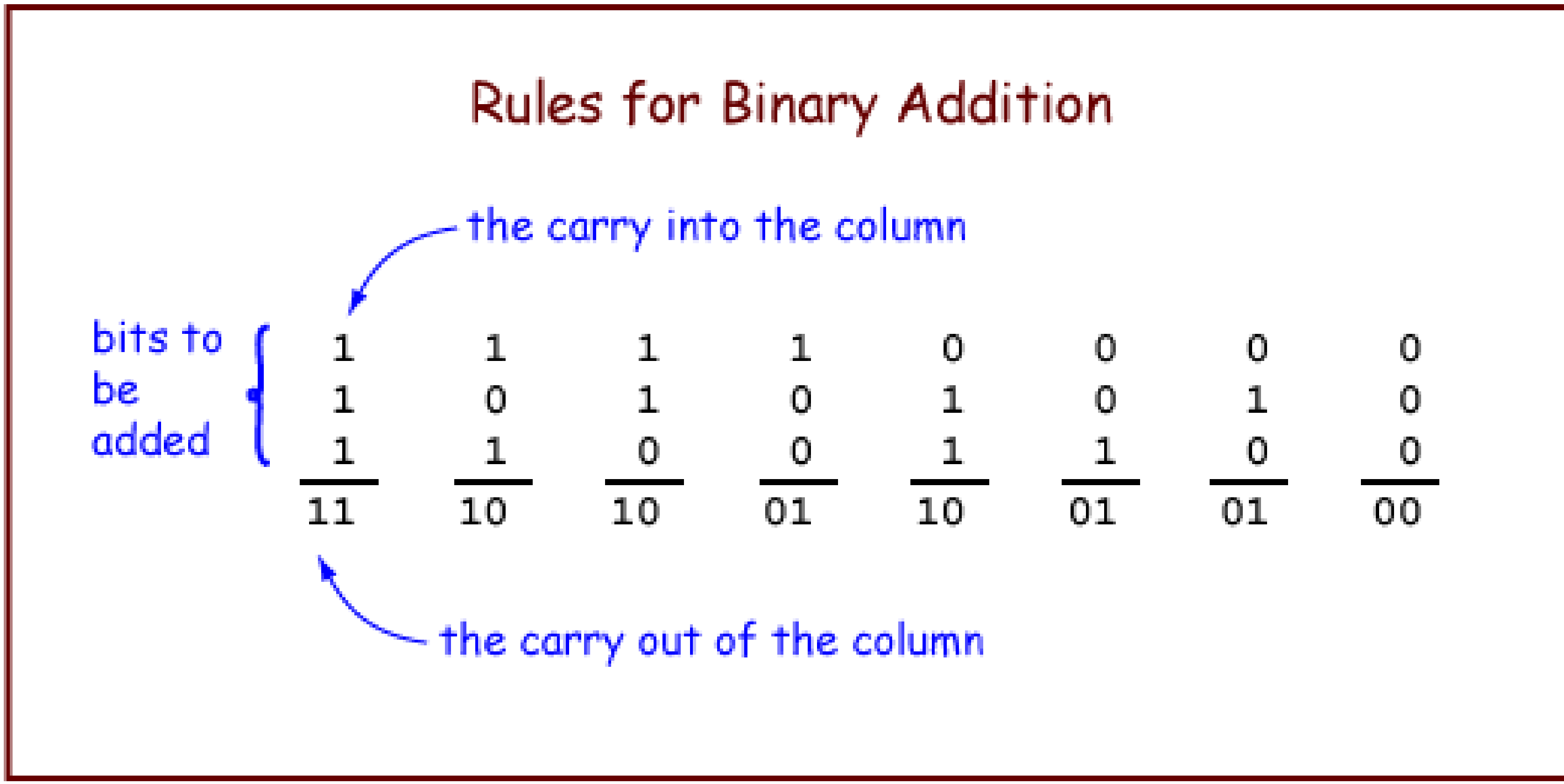
0001 1100 0101 0101

$\downarrow$ $\downarrow$ $\downarrow$ $\downarrow$

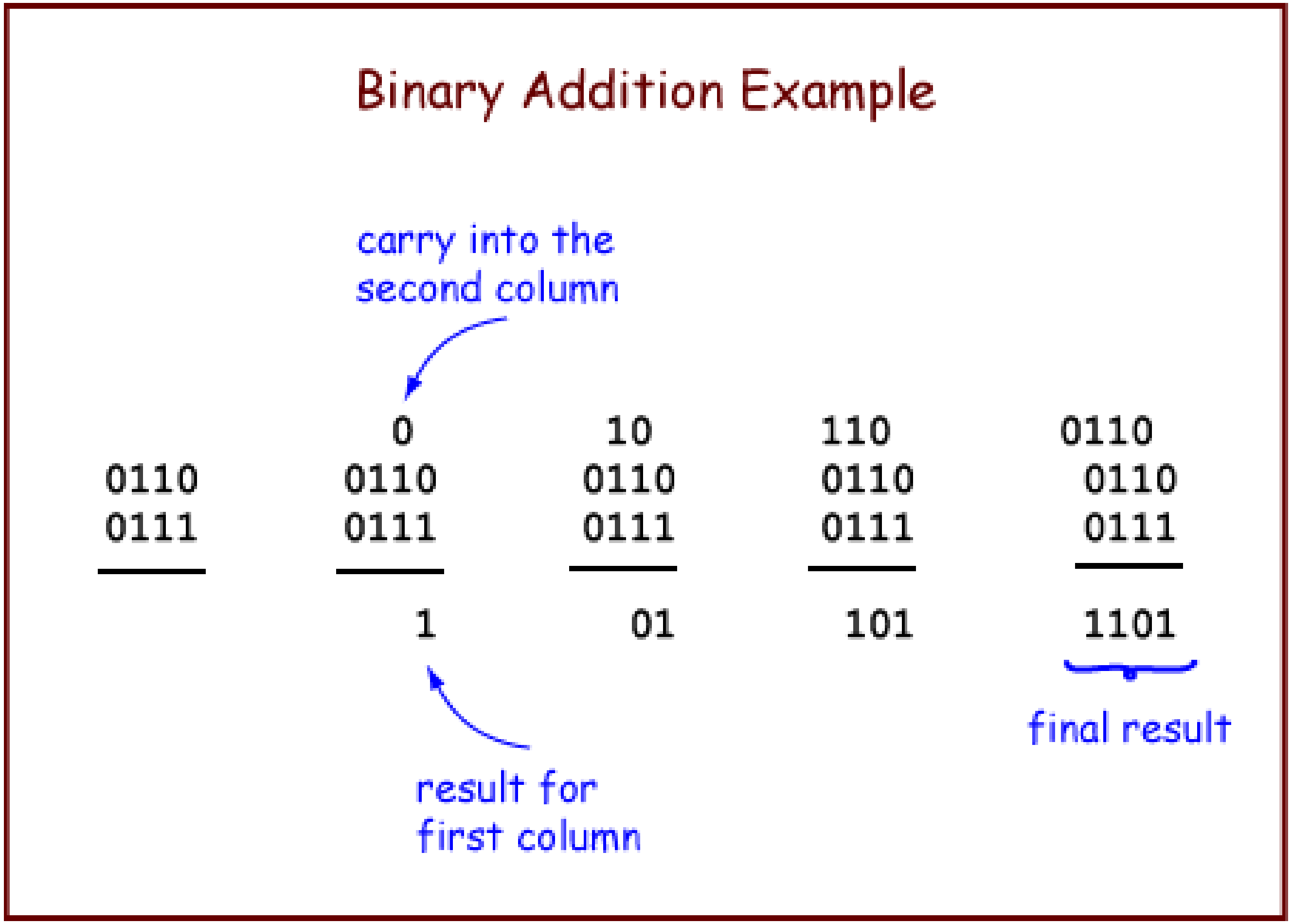
1 C 5 5

$(1110001010101)_2 \longrightarrow (1C55)_{16}$

Binary Addition



Binary Addition



Binary Subtraction

Rules of Binary Subtraction

$\Rightarrow$ 1 - 0 = 1

$\Rightarrow$ 1 - 1 = 0

$\Rightarrow$ 0 - 0 = 0

$\Rightarrow$ 0 - 1 = 1

(This can not be done directly, hence we borrow one digit from the digit to the left or the next higher order digit.)

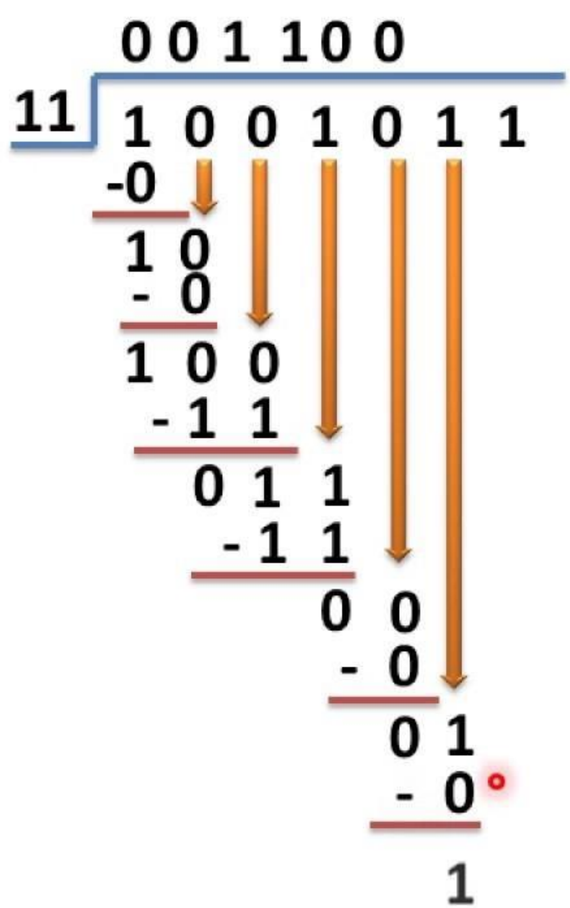
Binary Subtraction

$$\begin{array}{rccccl} & & 0 & 10 & & \longleftarrow \text{borrow} \\ & 1 & \cancel{1} & \cancel{0} & 0 & \\ (-) & 1 & 0 & 1 & 0 & \\ \hline & 0 & 0 & 1 & 0 & \\ \hline \end{array}$$

Binary Multiplication

$$
\begin{array}{ccccc}
 & & 1 & 1 & 0 \\
 & \times & & 1 & 1 \\
\hline
 & 1 & 1 & 1 & 0 \\
 & 1 & 1 & 0 & \mathrm{x} \\
\hline
1 & 0 & 0 & 1 & 0 \\
\hline
\end{array}
$$

Binary Division



Thank you