ACTSC 845 - Term Project

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December 17, 2016

1 Introduction

In the world of finance, rational risk-averse investors are seeking to maximize their returns on investments while also minimizing the risks they take. To be able to minimize the latter, investors must be able to measure risk in a quantifiable manner. There are several measures one can use to quantify risks; one of the most common measures is Value at Risk (VaR). The focus of our paper is to assess the adequacy of methodologies used in calculating the VaR of various assets classes. Namely, we will review the results of Johansen and Sowa's paper which analyzes three conditional volatility models' performances in regards to forecasting the VaR of equities, commodities and exchange rates.

1.1 Value at Risk

VaR as a concept emerged in the late 1980s, largely because of the 1987 stock market crash. The crash was so unlikely according to the models in use at the time that it called for the creation of new ones. VaR was developed to provide the ability to segregate large events from quantitative analysis such that they did not dominate the results but also were not ignored. This was mainly achieved through conditioning on the recent past. VaR was further popularized by its usage in the Basel II Accord in 1999 which is still in place today.

The mathematical definition of VaR can be formulated as:

$$\begin{aligned} \mathrm{VaR}_\alpha(X) &= \inf\{x \in \mathbb{R} : \Pr(X + x < 0) \leq 1 - \alpha\} \\ &= \inf\{x \in \mathbb{R} : 1 - F_X(-x) \geq \alpha\} \end{aligned}$$

In words this describes the VaR at a given confidence level α as the smallest threshold x such that the probability of losing at least x is at least α. VaR as a risk measure is monotonic, positive homogeneous, and translation invariant. However, one should keep in mind that VaR is not sub-additive and thus does not satisfy the conditions of being a proper risk measure. The key shortcoming of VaR is that it does not describe the behaviour within the tail of a distribution.

The definition of VaR is non-constructive; it defines the property that VaR must satisfy but not how to obtain VaR itself. There are several ways to estimate VaR: analyzing historical data, historical simulation, Monte Carlo simulation, guessing, to name a few. A good estimate of VaR should be wrong α % of the time and the violations should be independent of time, each other, and the level of VaR. The level $1-\alpha$ is said to be the nominal size of a VaR forecast, it is often compared to the empirical size of the forecast which is the observed percentage predictions which result in violations in either a real-world or backtest setting.

1.2 GARCH Models

Over years of observation, financial practitioners and scholars alike have noted that financial time series tend to exhibit heteroskedasticity. That is to say, volatility is observed to vary over time. Thus a good estimate for the VaR of a financial series should incorporate this stylized fact exhibited by most financial series. By far the most widespread technique to model this behaviour is to use generalized autoregressive conditional heteroskedasticity (GARCH) models first developed by Bollerslev(1986). Our paper examines work done to assess the suitability of various adaptations of GARCH models in computing the VaR for various different assets.

In the Johansson and Sowa paper, they used the ARCH(1) , GARCH(1,1) and EGARCH(1,1) models. We plan to examine the quality of the VaR forecasts for each of these models. In addition we will evaluate the quality of the ARMA(1,1)-GARCH(1,1) model's VaR predictions.

The log return of a security is defined as $x_t = \ln(\frac{X_t}{X_{t-1}})$ where X_t is the adjusted closing price of the security at time t. The GARCH(p,q) model is defined as follows

$$x_t = \sigma_t z_t$$

where z_t is a strong white noise process and the series σ_t^2 is modeled by

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i x_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

where ω, α_i, and β_i are constants. For GARCH(1,1) model, stationary solution requires $\alpha + \beta < 1$.

An ARCH(q) model is simply a GARCH(0,q) model.

The Exponential GARCH (EGARCH) model was developed by D.B. Nelson in 1991 to extend the GARCH model. EGARCH is an asymetric model that attempts to model the stylized fact of asset returns that negative returns tend to have larger impacts on volatility than do positive returns of the same magnitude. The EGARCH model is defined as follows

$$x_t = \sigma_t z_t$$

where z_t is a strong white noise process and the series $\ln(\sigma_t^2)$ is modeled by

$$\ln(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i g(z_{t-i} + \sum_{i=1}^{p} \beta_i \ln(\sigma_{t-i}^2)$$

and $g(z_t)$ is defined by

$$g(z_t) = \theta(z_t) + \lambda(|z_t| - \mathbb{E}(|z_t|))$$

1.3 ARMA-GARCH model

The ARMA(P,Q)-GARCH(p,q) model allows for more flexibility in forecasting the mean.The ARMA model specifies the conditional mean of the process and the GARCH model specifies the conditional variance of the process. These two models are perfectly compatible with each other and this leads to the complete specification of ARMA(P,Q)-GARCH(p,q) model. We decided to include it to extend the research of Johansson and Sowa to see if more flexibility in forecasting the conditional mean would translate into better performing VaR forecasts. To implement ARMA(P,Q)-GARCH(p,q) model, We can consider it as an ARMA(p,q) model with GARCH innovations.

$$X_t - \sum_{i=1}^{p} a_i X_{t-i} = z_t - \sum_{i=1}^{p} b_i z_{t-i}$$

where p is the lag parameter of the observed variables X_t, q is the lag parameter of white noise z_t which is the GARCH white noise following the form

$$z_t = \sigma_t \omega_t$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i z_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

where ω, α_i, and β_i are constants, ω_t follows a specific distribution.

2 Methodology

2.1 Forecasting procedure

Given a fitted GARCH model, one can obtain the VaR at a given confidence level α by evaluating the quantile function of the forecasted distribution at the α level. In the case of a single step ahead this simplifies to the following:

$$\hat{X}_{t+1} = \hat{\sigma}_{t+1} z_{t+1}$$

$$\hat{\sigma}_{t+1}^2 = \omega + \sum_{i=1}^{q} \omega_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2$$

where $\hat{\sigma}_{t+1}$ is estimated from GARCH(1,1) model. ω_{t+1} follows a specific distribution. All other parameters are known by time t.

$$\mathrm{VaR}_{\alpha,t+1} = \hat{\sigma}_{t+1} F_{\alpha}^{-1}$$

where $\hat{\sigma}_{t+1}$ comes from the GARCH(1,1) fit, and F is the distribution of GARCH residuals.

One can see that at a one period horizon the distribution of the error term is still either student's t or normal, which only has been shifted and/or scaled. As in Johansson and Sowa's we perform the analysis twice, one on models with normally distributed errors and the other on models with student's t distributed errors. This allows us to compare the resulting forecasts to determine if one of the distributions describes the data better.

To evaluate and compare the different models, a common technique is to perform either a rolling or expanding window n-step ahead forecast. For a given data set, the basic steps for rolling or expanding window forecasting are as follow:

- choose a rolling window size, m, which is the number of consecutive observation per rolling window. The size of the rolling window depends on the sample size, N and periodicity of the data;
- choose a forecast horizon, h. The forecast horizon depends on the application and periodicity of the data;
- if the number of increments between successive rolling windows is 1 ($h = 1$), then partition the entire data set into $n = N - m + 1$ subsamples. The first rolling window contains observations for period 1 through m, the second rolling window contains observations for period 2 through $m + 1$, and so on. The first expending window contains observations for period 1 through m. The second expending window contains observations for period 1 through $m + 1$, and so on.

In our paper we have chosen a window size of 250 observations to roughly correspond with the number of trading days in a year.

For each series, the parameters for the four different GARCH models are estimated. Subsequently, these fitted models are used to make four one period ahead out of sample VaR predictions at the 90, 95, 99, and 99.5 % confidence levels. This entire process is repeated a second time but with student's t distributed errors in place of normally distributed errors. To summarize, every series is fit with a total of 8 models for which, at each time step and each model, the VaR is predicted at 4 confidence levels.

2.2 VaR evaluation method

2.2.1 Kupiec test

Once we have calculated the predicted VaR values, we form a new series which consists of indicator variables describing whether or not a violation of the predicted VaR occurred. For a given series we define this variable as

$$I_{\alpha,t} = \begin{cases} 1 & \text{if } x_t < \mathrm{VaR}_{\alpha,t} \\ 0 & \text{Otherwise} \end{cases}$$

We can then define V_α to be the total number of violations of the α level predicted VaR (henceforth denoted VaR_α) over the time period. As a result $\frac{V_\alpha}{N}$ is the empirical size by definition.

In Johansson and Sowa's paper, the likelihood ratio test developed by Paul Kupiec (1995) is used. The test compares the empirical size to the nominal size using the following likelihood ratio

$$K = 2\ln\left((1-\frac{V}{N})^{N-V}(\frac{V}{N})^V\right) - 2\ln\left((1-\alpha)^{N-V}(\alpha)^V\right)$$

We will also use the standard 5 percent significance level for our analysis. The test statistic K is distributed according to a chi-square distribution with 1 degree of freedom (Orhan and Köksal). That means that Kupiec test statistic values of above 3.84 will result in rejection of the null hypothesis of the predicted VaR_α's empirical size matching the nominal size in the long run.

2.2.2 Christoffersen's Test for independence and correct exceedances

As addressed in Johansson and Sowa's paper, the Kupiec test does not test for independence of violations. Further the test is indifferent to the magnitude of the violation; a violation of 1.32 is equivalent to a violation of 1337 with regards to the Kupiec test. To account for the first flaw, we use Christoffersen's 1998 exceedance independence test(Holton 2016).

the Christoffersen test focuses on the frequency of consecutive violations and operates under the null that the empirical size will equal the nominal size in the longrun and that the violations are independent of one another.

For the Christoffersen test, we first define

$$q_0^* = \Pr(I^t = 0 | i^{t-1} = 0)$$
$$q_1^* = \Pr(I^t = 0 | i^{t-1} = 1)$$

These were the VaR measure's conditional coverages which was its actual probabilities of not experiencing an exceedance given that it did not (in the case of $\hat{q}_0$ or did in the case of $\hat{q}_1$) experience an exceedance in the previous period. The null hypothesis H_0 was that $\hat{q}_0 = \hat{q}_1 = \hat{q^*}$ If a VaR measure is observed for $\alpha + 1$ periods, there will be α pairs of consecutive observations(i^{t-1}, i^t). Dis-aggregate these as

$$\alpha_{00} + \alpha_{01} + \alpha_{10} + \alpha_{11} = \alpha$$

where α_{00} is the number of pairs$(i^{t-1}, i^t)$of the form$(0, 0)$; α_{01} is the number of the form $(0, 1)$; etc. We want to test if

$$\frac{\alpha_{0,0}}{\alpha_{00} + \alpha_{01}} \approx \frac{\alpha_{10}}{\alpha_{10} + \alpha_{11}}$$

which would support our null hypothesis. We apply a likelihood ratio test as follows. Assuming H_0 didn't hold, we estimate $\widehat{q}_0$ and $\widehat{q}_1$ with

$$\widehat{q}_0^* = \frac{\alpha_{00}}{\alpha_{00} + \alpha_{01}} \qquad \widehat{q}_1^* = \frac{\alpha_{10}}{\alpha_{10} + \alpha_{11}}$$

Assuming H_0 did hold,we estimated q^* with

$$\widehat{q}^* = \frac{\alpha_{00} + \alpha_{10}}{\alpha_{00} + \alpha_{10} + \alpha_{01} + \alpha_{11}}$$

Our likelihood ratio is

$$\Lambda = \frac{(1 - \hat{q}^*)^{\alpha_{01}+\alpha_{11}} (\hat{q}^*)^{\alpha_{00}+\alpha_{10}}}{(1 - \hat{q}_0^*)^{\alpha_{01}} (\hat{q}_0^*)^{\alpha_{00}} (1 - \hat{q}_1^*)^{\alpha_{11}} (\hat{q}_1^*)^{\alpha_{10}}}$$

$$= \left(\frac{\hat{q}^*}{\hat{q}_0^*}\right)^{\alpha_{00}} \left(\frac{1 - \hat{q}^*}{1 - \hat{q}_0^*}\right)^{\alpha_{01}} \left(\frac{\hat{q}^*}{\hat{q}_1^*}\right)^{\alpha_{10}} \left(\frac{1 - \hat{q}^*}{1 - \hat{q}_1^*}\right)^{\alpha_{11}}$$

and $-2\ln(\Lambda)$ is approximately centrally chi-squared with one degree of freedom, that is, $-2\ln(\Lambda) \sim \chi^2(1, 0)$ assuming H_0. We will use the standard 5% significance level and reject the null for values of Λ greater than 5.91.

One weakness of the Christoffersen test is that the expected number of subsequent violations can be very small at higher confidence levels of the VaR. The test statistic is not defined for the case where no subsequent observations are observed.

3 Implementation

3.1 Data

All calculations were performed using R with the rugarch library.

The equities and exchange rate data were downloaded from Yahoo finance while the commodities data was downloaded from Quandl. As in Johansson and Sowa's paper, the series are categorized into three groups; equities, commodities and currencies. All series selected are based in the United States.

Each model was estimated using a 250 day rolling window from the beginning of the series until December 1st 2016. Due to reasons of availability, equity data begins in 2000, commodities in 2008, and currencies in 2004. All series begin on January 1st.

3.2 Descriptive Statistics

Unsurprisingly, the unconditional means tended to 0. All of the returns series were leptokurtotic implying that they have thicker tails than the normal distribution. This leads us to believe that the student's t distribution may model the

Category	Security	Description
	Microsoft	Large technology company
Equities	Apple	Large technology company
	Bank of America	Large financial institution
	Gold	Cash commodity price for Gold, Engelhard fabricated products
Commodities	Cheddar	Cash commodity price for Cheddar Cheese, Barrels, Chicago. Units: $ per lb.
	Soybean meal	Cash commodity price for Soybean Meal, Central Illinois. Units: $ per ton.
	CAD to USD	Exchange rate between CAD to USD
Currencies	JPY to USD	Exchange rate between JPY to USD
	GBP to USD	Exchange rate between GBP to USD

Table 1: Data Series

security	n	mean	std	Min	Max	Excess Kurtosis	Skewness
Microsoft	4257	9.46E-05	0.0198	-0.1696	0.1787	9.4861	-0.1176
Apple	4257	7.995E-04	0.0278	-0.7312	0.1302	112.9497	-4.3365
Bank of America	4257	8.10E-05	0.0305	-0.3421	0.3021	25.4080	-0.3352
Gold	2244	1.481E-04	0.0125	-0.09592	0.0651	5.0035	-0.3165
Cheddar	2217	-8.09E-05	0.0171	-0.1567	0.0870	11.0693	-1.1203
Soybean meal	2218	-4.64E-06	0.0203	-0.1307	0.1412	4.4261	-0.2182
CAD to USD	3359	1.05E-05	0.0061	-0.0369	0.0360	2.7076	0.2058
JPY to USD	3344	1.99E-05	0.0083	-0.1685	0.1629	106.1095	-0.5701
GBP to USD	3357	1.057531E-04	0.0061	-0.0348	0.0791	10.4910	0.9196

Table 2: Descriptive Statistics

data bettre as it has thicker tails than the normal distribution. All of the series display skewness as opposed to those featured in Johnasson and Sowa's paper. Two of the currencies have positive skews, the rest have negative skews, which agrees with empirical observations of financial series.

3.3 Analysis of the Unconditional Distributions

We chose three financial assets from three markets and analyzed the tails in comparison with the normal and student's t distributions.

From the plots shown below, we reach one conclusion: the student's t distribution captures the heavy tails of the original data in three different markets. The normal distribution seems to overestimate the risk when $1-\alpha$ is small and underestimate the risk when $1-\alpha$ is large. In this paper, we compare the most popular distributions used to model the error terms in GARCH models: the normal and student's t distributions. From the definition of GARCH models, it is obvious that the conditional distribution of future returns one period ahead has the same distribution as the innovations. Heavy tails in financial time series refer to the fact that the downside tail of the distribution decays in a power-law speed as opposed to an exponential speed as in the normal distribution. The most often applied GARCH(1,1) model assumes that the innovation term follows a standard normal distribution. Recent literature in risk management shows that the conditional normality assumption does not perform well in evaluating the downside risk at higher confidence levels. It is also shown that the GARCH process with normal innovation generates much thinner tail than that obtained from empirical observations which results in the underestimation of potential downside risk. The results above can be shown from the graphs

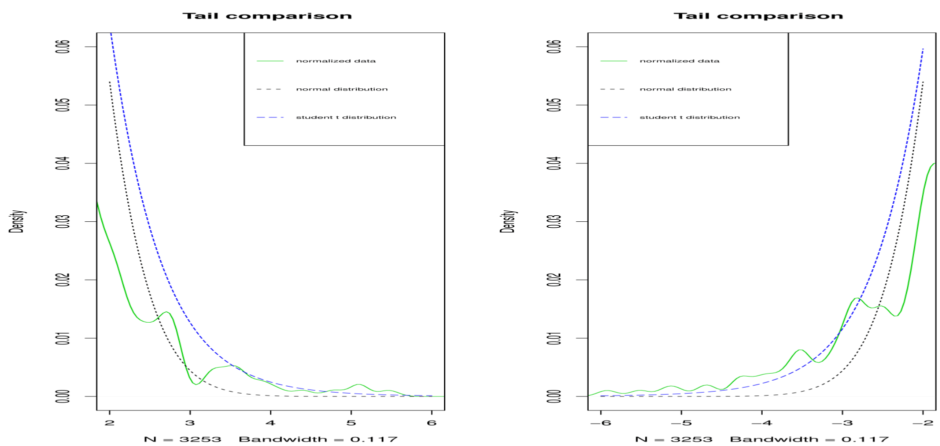


Figure 1: Tail comparison of normalized Microsoft stock price with normal and student's t distribution

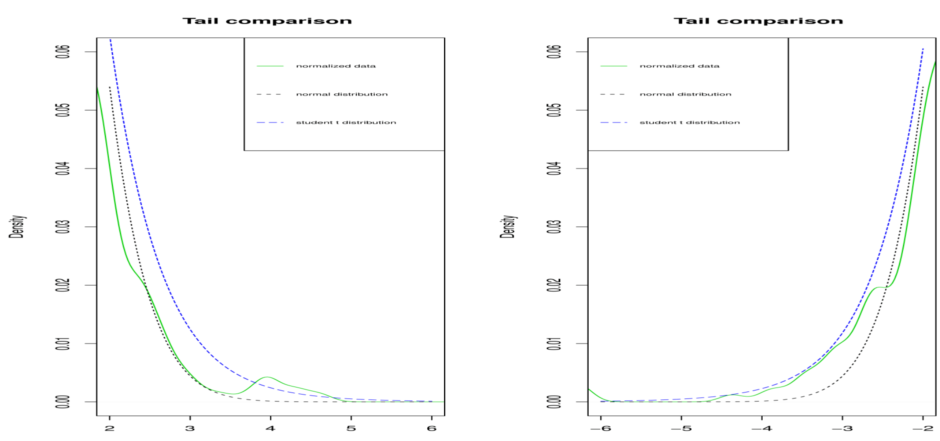


Figure 2: Tail comparison of normalized Commodity soybean meal price with normal and student's t distribution

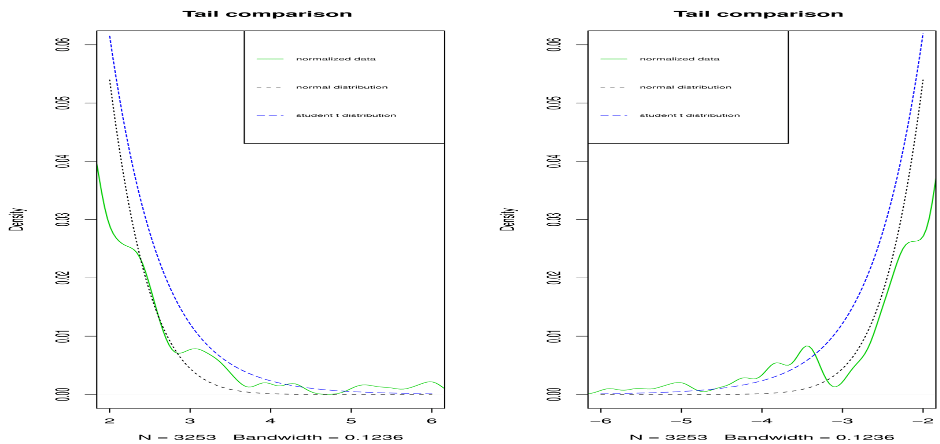


Figure 3: Tail comparison of normalized JYP to USD currency with normal and student's t distribution

provided in section 4.

3.4 Challenges

During the process of computing the forecasts and fitting the models we ran into several hurdles. The first such hurdle was the amount of computation we needed to perform. With each series having a few thousand observations, refitting on each observation was progressing rather slowly. To overcome this we used the parallel package in R which allowed us to triple the speed of our forecasts with just a few lines of code. Another notable road block was that some of the windows were not converging and were thus preventing us from performing our tests and observing the empirical size. Because we used log returns, our values tended to be rather small. This created an unnecessary burden of keeping leading 0's, multiplying the values by 100 allowed many non-converging windows to converge. Another approach to get all of the windows to converge was to use multiple different optimization algorithms (The R routines used were: nlminb,solnp,gosolnp,lbfgs, and nloptr). The combination of these two approaches reduced the number of non convergent windows from thousands down to only handful. The last few were coerced into convergence by increasing the relative tolerance of the algorithms by a factor of 10.

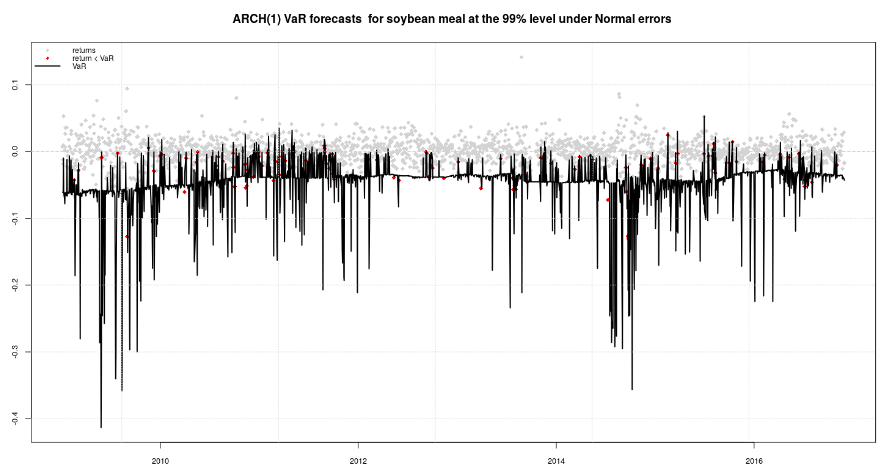
4 Results

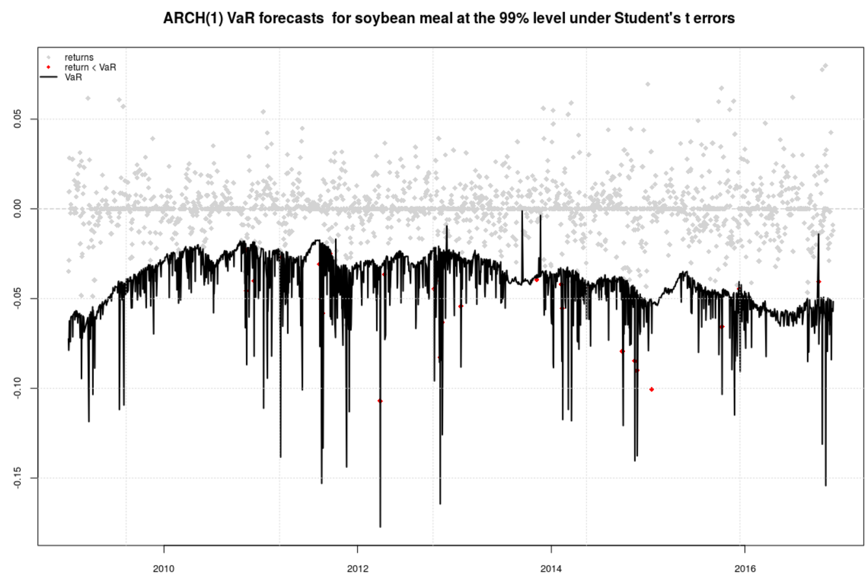
The corresponding tables containing the backtest results for each model can be found at the end of the paper. The results are presented individually for each model. It should be noted that values of NaN for the Christoffersen test statistic imply that no consecutive violations occurred since the statistic is not defined for such cases.

4.1 ARCH(1) results

The ARCH(1) model performed better under student's t distributed errors than under normally distributed errors for all series and confidence levels except for the $VaR_{0.90}$ of the Cheddar series. Under normally distributed errors, the ARCH(1) model vastly underestimates the risk at all confidence levels. Across all series, the ARCH model under normal errors performs better the lower the confidence level is. The ARCH(1) under normal errors did not pass a single Kupiec or Christoffersen test.

The ARCH(1) model under student's t errors produces much more accurate VaR forecasts. In regards to equities, the ARCH(1) model produces VaR forecasts that have the proper empirical size and the Kupiec Null of correct exceedances cannot be rejected for any of the three securities at any of the confidence levels. However, the Christoffersen test statistic exceeded the critical value of 5.91 at multiple levels which means that the violations are not independent of one another. This indicates that the ARCH(1) model is not capturing equities' volatility clustering behaviour. With regards to the cheddar series, there were many days without any price movement which adversely impacted the fit of all the models unfortunately. Though the ARCH(1) model with student's t errors performed exceedingly well at forecasting the VaR's of gold and soybean meal. The ARCH(1) model under student's t errors performed very well at higher confidence levels.





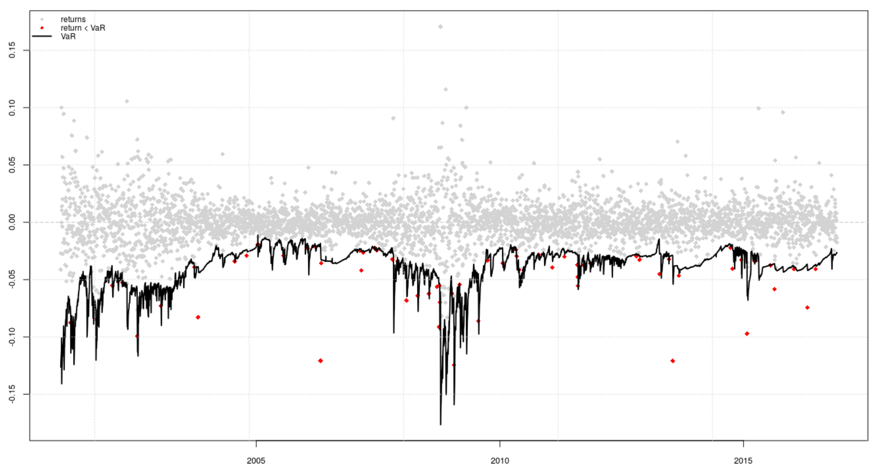
When comparing the two different error schemes, it appears that as the rolling window progresses, the model under student's t errors is much more sensitive to changes in behaviour of the series than the model under normal errors. Where the model with normal errors produces a noisier forecast in the short-run, it's mean does not change very much over time. This is in direct contrast to the model with student's t distributed errors. This is significant as it implies that the size of the window used to fit the model may play a considerable role in the quality of the forecasts. This could be a topic to explore in a future paper.

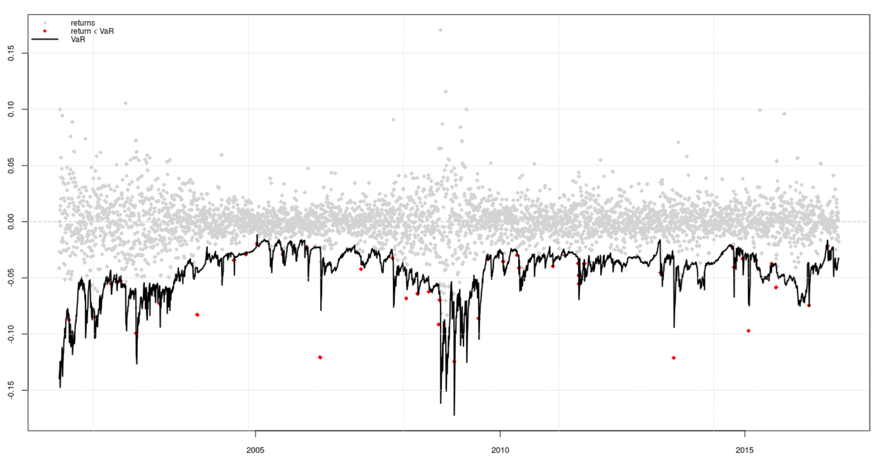
4.2 GARCH(1,1) results

The GARCH(1,1) with normal errors consistently under estimated the risk at higher confidence levels, though not nearly as much as the ARCH(1) with normal errors. On the other hand, at lower confidence levels, the GARCH(1,1) under normal errors tended to over estimate the level of risk. In the middle, at the 95% confidence level, the GARCH(1,1) performed strongly. At the 95% level the GARCH(1,1) under normal errors passed the Kupiec test for frequency of violations in most series but did fail the Christoffersen test for independence for a few series. Though still poor, the GARCH(1,1) with normal errors performs much stronger than the ARCH(1) with normal errors.

Overall, the GARCH(1,1) produced much better VaR forecasts using student't distributed errors when compared to normally distributed errors. The model generally outperformed its ARCH counterpart but fell short in forecasting the VaR of commodities series as well as for Bank Of America. Given that BAC is a financial institution, it is interesting that the ARCH model was better able to forecast the VaR. For the remaining series, the forecasts from the GARCH(1,1) model with student's t errors appeared have some slight independence issues at some confidence levels but generally performed well.

GARCH(1,1) VaR forecasts for Microsoft at the 90% level under Normal errors





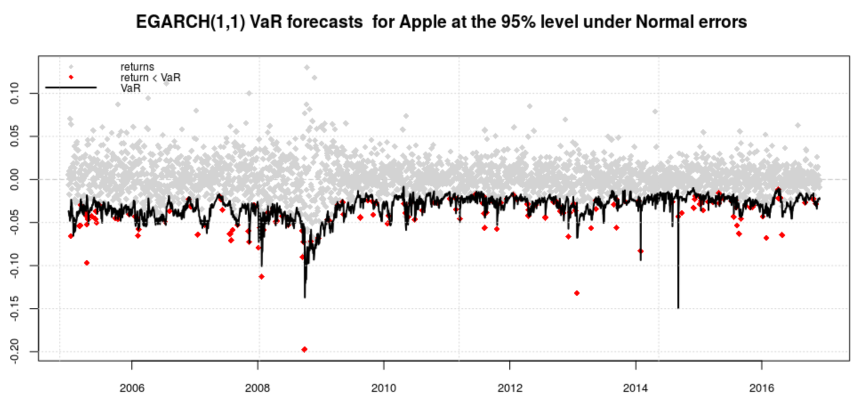
The two models under the different errors appear to behave in a similar manner with the normal forecasts consistently over estimating the amount of risk. Overall the GARCH(1,1) model appears to be more reactive than the ARCH(1) model and displays less noise in its forecasts. This is to be expected due to the GARCH(1,1) model's long term memory.

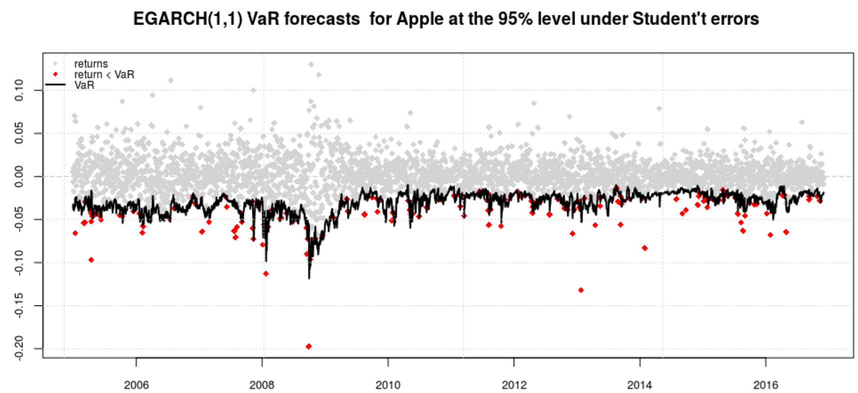
4.3 EGARCH(1,1) results

Due to the large amount of days with zero returns in the cheddar series, our forecasts using the EGARCH(1,1) model with student's t errors did not yield relevant results.

The EGARCH(1,1) with normally distributed errors underestimates the risk at high confidence levels and overestimates it at lower confidence levels in a similar fashion the the GARCH(1,1) with normal errors. The EGARCH(1,1) with normal errors performs very well at the 95% confidence level across all series in terms of the Kupiec test but fails the Christoffersen test for Cheddar and Gold. As opposed to the GARCH(1,1) with normal errors, the EGARCH model performs well at the 90% confidence level when forecasting the VaR of currencies.

The EGARCH(1,1) with student's t errors performs worse than the ARCH(1) and GARCH(1,1) at the 99.5% confidence level. The EGARCH(1,1) with student's t distributed errors struggled with the Bank of America, Gold, and Japanese Yen series. However the EGARCH(1,1) with student's t errors significantly outperformed it's ARCH and GARCH counterparts in forecasting the VaR at the 90% and 95% confidence levels.





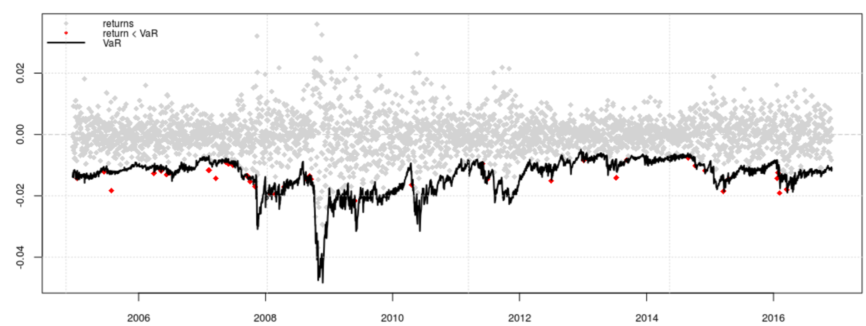
Student's t errors appears to create forecasts with less variance than the forecasts from normally distributed errors. Both models performed well but the student's t errors appear to create a tighter more consistent forecast. Though our paper did not analyze this, it appears that the correlation between the magnitude of the forecasts and the probability of a violation is smaller under student's t errors.

4.4 ARMA(1,1)-GARCH(1,1) results

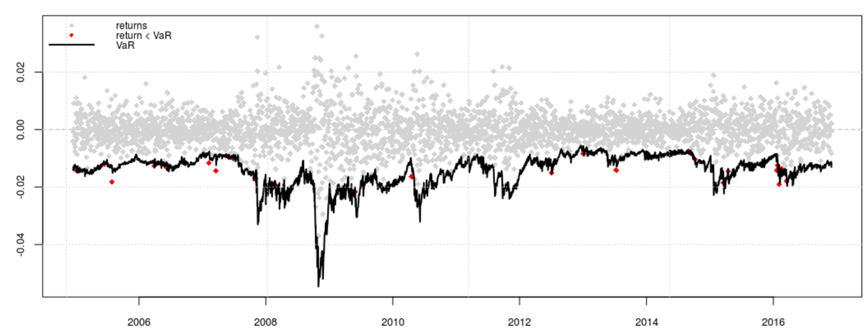
The ARMA(1,1)-GARCH(1,1) with normally distributed errors offered slight improvements over the EGARCH(1,1) in all areas except for forecast the VaR of Bank of America at the 95% level. Passing the Kupiec and Christoffersen tests at the 99% confidence level for the CAD/USD exchange rate, made ARMA(1,1)-GARCH(1,1) the only model with normal errors to pass at the 99% level for a series.

The ARMA(1,1)-GARCH(1,1) with student's t distributed errors performed poorly in forecasting the VaR of Bank of America and of cheddar. ARMA(1,1)-GARCH(1,1) with student's t errors performed better than the EGARCH(1,1) at the 99.5% level but was still outperformed by the ARCH(1) model in that respect. In general the ARMA(1,1)-GARCH(1,1) with student's t errors performed well for currencies as well as both the Microsoft and soybean meal series.

ARMA(1,1)-GARCH(1,1) VaR forecasts for CAD/USD at the 99% level under Normal errors



ARMA(1,1)-GARCH(1,1) VaR forecasts for CAD/USD at the 99% level under Student's t errors



The ARMA(1,1)-GARCH(1,1) model appears to be the most reactive without introducing large amounts of noise into its forecasts. In the ARMA(1,1)-GARCH(1,1) model, the difference between student's t errors and normal errors is the least noticeable.

5 Conclusion

The EGARCH(1,1) and ARMA(1,1)-GARCH(1,1) models performed the best using normally distributed errors and were very similar.

The model that performed the best at the 99% and 99.5% levels was the ARCH(1) with student's t distributed errors. At lower confidence levels, EGARCH(1,1) with student's t distributed errors performed the best, though the EGARCH(1,1) with normally distributed errors matched it at the 95% level.

Overall the student's t distribution yielded better VaR forecast at the 90%,99%, and 99.5% confidence level while the normal distribution provided slightly better results at the 95% level. In Johansson and Sowa's paper they found that the VaR was better forecasted by the normal distribution in most cases. In line with earlier results, our results show that this can be attributed to their use of 95% as their sole confidence level and that at other confidence levels the student's t distribution produces better forecasts in all of the models.

6 References

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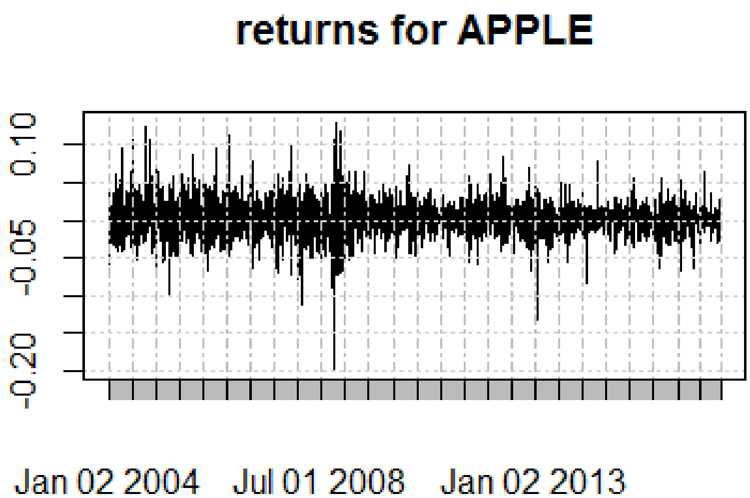
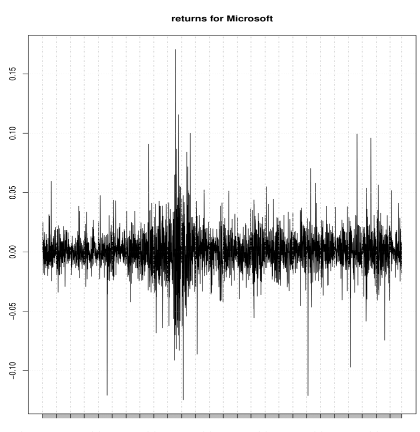
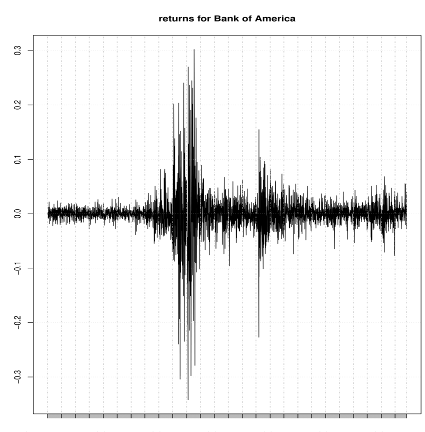
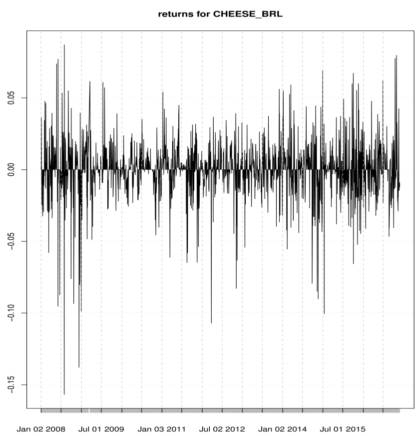
7 Appendix

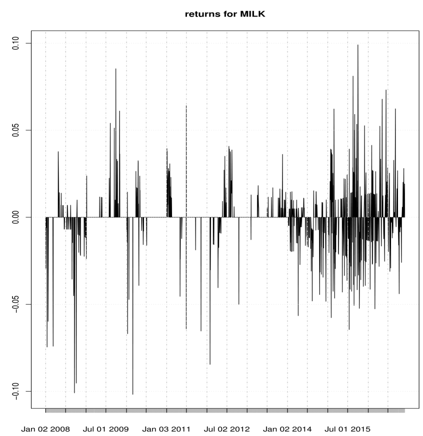
7.1 Return Series and Parameter Estimation

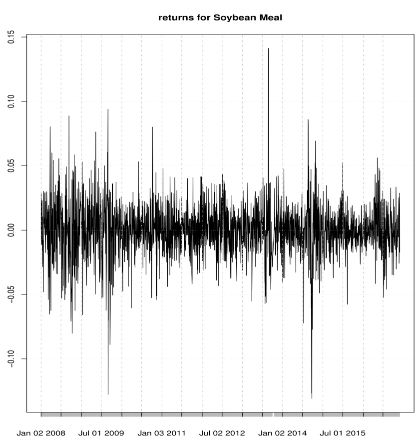


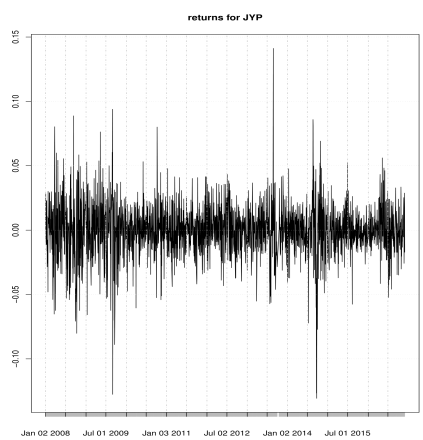


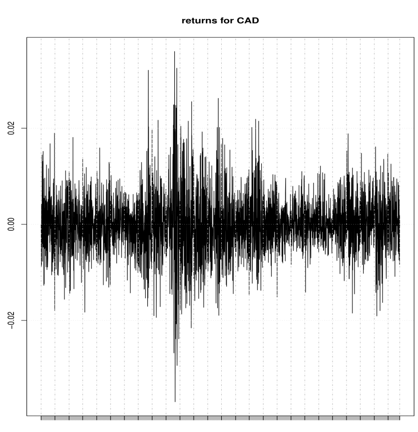


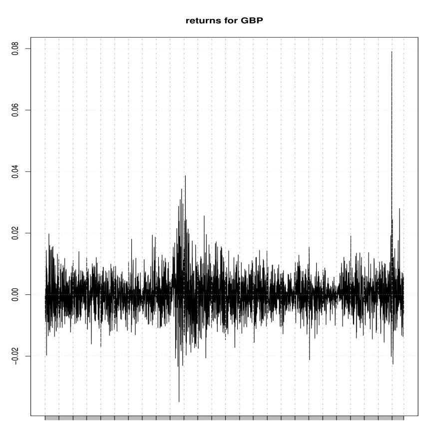

returns for CHEESE_BRL
0.05
0.00
-0.05
-0.10
-0.15
Jan 02 2008
Jul 01 2009
Jan 03 2011
Jul 02 2012
Jan 02 2014
Jul 01 2015


returns for MILK
0.10
0.05
0.00
-0.05
-0.10
Jan 02 2008
Jul 01 2009
Jan 03 2011
Jul 02 2012
Jan 02 2014
Jul 01 2015


returns for Soybean Meal
0.15
0.10
0.05
0.00
-0.05
-0.10
Jan 02 2008
Jul 01 2009
Jan 03 2011
Jul 02 2012
Jan 02 2014
Jul 01 2015


returns for JYP
0.15
0.10
0.05
0.00
-0.05
-0.10
Jan 02 2008
Jul 01 2009
Jan 03 2011
Jul 02 2012
Jan 02 2014
Jul 01 2015


returns for CAD
0.02
0.00
-0.02
Jan 01 2004
Jul 03 2006
Jul 02 2008
Jul 01 2010
Jul 02 2012
Jul 01 2014
Jul 01 2016


returns for GBP
0.08
0.06
0.04
0.02
0.00
-0.02
Jan 01 2004
Jul 03 2006
Jul 02 2008
Jul 01 2010
Jul 02 2012
Jul 01 2014
Jul 01 2016

ARCH(1) with normal errors

security	mu	omega	alpha1
Microsoft	-0.00042	0.00023	0.28088
Apple	0.00339	0.00036	0.20363
Bank of America	0.00001	0.00046	0.28046
Gold	0.00106	0.0001	0.20753
Cheddar	-0.00356	0.00016	0.26573
Soybean meal	0.00321	0.00025	0.25381
CAD to USD	0.00004	0.00002	0.18373
JPY to USD	0.00027	0.00004	0.22598
GBP to USD	0.00033	0.00003	0.18633

ARCH(1) with student's t errors

security	mu	omega	alpha1	shape
Microsoft	0.00007	0.00032	0.14718	6.14165
Apple	0.00138	0.00042	0.1308	7.07937
Bank of America	-0.00004	0.00074	0.26973	12.20337
Gold	0.00033	0.00016	0.04643	8.70409
Cheddar	0.00041	0.00059	0.81668	2.18691
Soybean meal	0.00043	0.00034	0.17601	17.19794
CAD to USD	-0.00007	0.00003	0.10685	15.56564
JPY to USD	0.00007	0.00005	0.19315	7.70366
GBP to USD	0.00001	0.00003	0.08605	22.27503

GARCH(1,1) with normal errors

security	mu	omega	alpha1	beta1
Microsoft	0.00029	0.00003	0.07018	0.85681
Apple	0.0018	0.00002	0.06381	0.88218
Bank of America	0.00012	0.00002	0.07955	0.8927
Gold	0.00025	0	0.05321	0.91893
Cheddar	0.0003	0.00001	0.08291	0.87479
Soybean meal	0.00019	0.00003	0.0888	0.84695
CAD to USD	-0.00005	0	0.04136	0.9377
JPY to USD	0.00008	0.00001	0.07074	0.86612
GBP to USD	0.00002	0	0.04897	0.92477

GARCH(1,1) with student's t errors

security	mu	omega	alpha1	beta1	shape
Microsoft	0.00011	0.00002	0.05769	0.8964	5.99055
Apple	0.00155	0.00003	0.06007	0.88575	6.87539
Bank of America	0.00013	0.00002	0.0842	0.88631	10.76796
Gold	0.00033	0	0.03199	0.95325	7.55002
Cheddar	0.0004	0.00009	0.22227	0.76751	2.40566
Soybean meal	0.00025	0.00003	0.06568	0.87427	11.80798
CAD to USD	-0.00008	0	0.04426	0.92643	13.39331
JPY to USD	0.00007	0.00001	0.08783	0.85139	8.00566
GBP to USD	0.00002	0	0.03746	0.94221	13.59791

EGARCH(1,1) with normal errors

security	mu	omega	alpha1	beta1	gamma1
Microsoft	-0.00014	-0.98465	-0.04128	0.88331	0.09928
Apple	0.00122	-1.23641	-0.15709	0.84692	0.06889
Bank of America	-0.00064	-0.75064	-0.06687	0.90856	0.14302
Gold	0.00024	-1.2744	-0.08207	0.86113	0.03519
Cheddar	0.00055	-1.45361	-0.06756	0.82012	0.18952
Soybean meal	0.00023	-0.62453	-0.01235	0.92125	0.10638
CAD to USD	0.00003	-0.78073	0.0646	0.92559	0.03351
JPY to USD	0	-2.71478	-0.07336	0.72233	0.16691
GBP to USD	0.0001	-1.66415	0.06088	0.84204	0.02671

EGARCH(1,1) with student's t errors

security	mu	omega	alpha1	beta1	gamma1	shape
Microsoft	-0.0002	-0.79851	-0.06623	0.90563	0.08327	7.10544
Apple	0.00123	-1.01104	-0.14154	0.87302	0.04952	10.04667
Bank of America	-0.00041	-0.98984	-0.1185	0.88186	0.09022	18.13254
Gold	0.0003	-0.84556	-0.05372	0.90731	-0.0358	10.24023
Cheddar	0.00032	-1.52361	-0.02509	0.79557	0.55441	2.14182
Soybean meal	0.00033	-0.99113	-0.02245	0.87558	0.11882	19.39285
CAD to USD	0	-1.74622	0.06705	0.83253	0.03303	31.39975
JPY to USD	0.00005	-2.74031	-0.05252	0.72061	0.11532	12.5127
GBP to USD	0.00007	-2.52744	0.05953	0.75999	0.01156	35.75916

ARMA(1,1)-GARCH(1,1) with normal errors

security	mu	ar1	ma1	omega	alpha1	beta1
Microsoft	0.00029	-0.01972	-0.01785	0.00003	0.06873	0.86751
Apple	0.00181	-0.05563	0.06971	0.00003	0.06673	0.87642
Bank of America	0.00017	-0.08594	0.06705	0.00002	0.08105	0.8898
Gold	0.00023	0.15444	-0.17075	0	0.05385	0.91685
Cheddar	0.00027	0.45668	-0.25572	0.00001	0.08739	0.85728
Soybean meal	0.00013	0.12753	-0.09745	0.00003	0.07929	0.86252
CAD to USD	-0.00004	-0.11262	0.0776	0	0.03936	0.94205
JPY to USD	0.00007	0.02369	-0.09388	0	0.07221	0.90021
GBP to USD	0.00003	-0.0357	0.01553	0	0.04376	0.9344

7.2 Backtesting results

	model:	ARCH(1)							
	alpha:	0.9		0.95		0.99		0.995	
		Normal	student's t	Normal	student's t	Normal	student's t	Normal	student's t
	V/N	14.0%	9.2%	10.4%	4.6%	7.3%	1.0%	6.4%	0.6%
	Kupiec	64.195	2.855	187.454	1.107	668.287	0.029	846.74	0.741
Microsoft	Christofferson	64.309	4.615	187.582	6.747	669.068	NaN	847.611	NaN
	V/N	10.6%	9.6%	6.6%	4.9%	3.4%	1.0%	2.8%	0.7%
	Kupiec	1.48	0.606	18.855	0.1	145.371	0.022	210.73	2.193
Apple	Christofferson	4.732	8.065	30.74	4.118	147.472	NaN	211.601	NaN
	V/N	14.1%	10.7%	9.6%	5.5%	5.9%	1.3%	5.3%	0.7%
	Kupiec	65.731	1.882	142.752	2.383	462.246	3.827	630.547	2.193
Bank of Americ	Christofferson	67.695	3.364	143.043	2.635	462.513	8.157	630.591	8.399
	V/N	15.1%	10.0%	11.0%	5.5%	6.3%	0.9%	5.8%	0.5%
Gold	Kupiec	50.566	0.002	113.741	0.887	258.259	0.197	358.032	0
	Christofferson	56.096	0.512	116.899	1.6	264.193	NaN	363.703	NaN
	V/N	12.3%	13.5%	9.2%	8.3%	5.8%	1.5%	5.3%	0.8%
	Kupiec	10.42	24.036	57.914	37.664	216.605	3.9	306.831	3.262
Cheddar	Christofferson	21.053	48.532	62.658	40.147	217.49	4.478	307.956	NaN
	V/N	14.7%	9.8%	10.9%	4.8%	6.7%	1.2%	5.7%	0.7%
	Kupiec	43.454	0.082	108.628	0.21	284.408	0.537	350.863	0.926
Soybean meal	Christofferson	45.083	1.642	111.614	4.176	284.571	NaN	350.907	NaN
	V/N	14.4%	9.7%	9.9%	4.7%	6.1%	1.0%	5.3%	0.5%
	Kupiec	59.958	0.225	125.601	0.756	382.114	3.841	492.788	0.16
CAD to USD	Christofferson	60.366	1.119	126.673	0.854	383.713	NaN	496.793	NaN
	V/N	14.5%	10.5%	10.3%	5.9%	6.4%	1.2%	5.7%	0.9%
	Kupiec	63.173	0.861	142.483	4.811	414.054	1.517	548.353	8.216
JPY to USD	Christofferson	64.086	1.233	148.51	4.981	415.546	1.99	549.239	NaN
	V/N	14.6%	10.2%	10.1%	5.2%	5.4%	1.0%	4.4%	0.6%
	Kupiec	64.388	0.189	131.801	0.214	295.942	0.028	353.946	1.182
GBP to USD	Christofferson	64.388	2.517	132.06	1.932	296.465	NaN	354.123	NaN

Figure 4: Backtesting results for ARCH(1) model

	model:	GARCH(1,1)							
	alpha:	0.9		0.95		0.99		0.995	
		Normal	student's t	Normal	student's t	Normal	student's t	Normal	student's t
	V/N	8.0%	9.9%	4.3%	5.1%	1.6%	1.1%	1.1%	0.6%
	Kupiec	19.759	0.02	4.112	0.166	13.186	0.589	21.445	0.421
Microsoft	Christofferson	19.888	2.712	10.676	2.859	13.189	NaN	NaN	NaN
	V/N	8.6%	10.3%	4.6%	5.2%	1.4%	1.2%	1.1%	0.7%
	Kupiec	9.657	0.292	1.269	0.482	6.389	1.147	19.882	3.539
Apple	Christofferson	15.524	2.984	5.732	10.234	6.431	NaN	NaN	NaN
	V/N	9.1%	10.6%	5.2%	5.6%	1.9%	1.4%	1.2%	0.8%
	Kupiec	3.632	1.357	0.388	2.835	27.074	5.034	26.403	6.074
Bank of Americ	Christofferson	10.768	3.676	5.987	6.132	30.16	9.011	31.934	11.01
	V/N	9.1%	10.6%	5.4%	6.0%	1.8%	1.2%	1.3%	0.7%
Gold	Kupiec	1.941	0.869	0.709	3.713	10.548	0.784	16.019	0.844
	Christofferson	2.405	0.983	1.509	4.915	10.716	NaN	17.004	NaN
	V/N	7.7%	13.6%	4.8%	8.0%	2.3%	2.0%	1.6%	1.3%
	Kupiec	12.148	25.403	0.205	32.428	24.152	16.336	29.078	16.434
Cheddar	Christofferson	28.064	59.55	15.154	39.421	26.808	16.377	31.845	NaN
	V/N	8.8%	10.2%	4.8%	5.3%	1.9%	1.5%	1.2%	0.8%
	Kupiec	3.042	0.058	0.21	0.456	13.54	3.891	14.579	2.341
Soybean meal	Christofferson	11.998	7.378	7.891	6.801	17.738	7.095	19.104	10.517
	V/N	8.7%	9.7%	4.8%	5.0%	1.4%	1.0%	0.8%	0.4%
	Kupiec	6.543	0.225	0.382	0.014	4.797	0.039	5.872	1.488
CAD to USD	Christofferson	7.121	0.234	2.08	0.273	NaN	NaN	NaN	NaN
	V/N	8.8%	10.8%	4.9%	5.5%	1.7%	1.2%	1.3%	0.8%
	Kupiec	4.935	1.957	0.05	1.353	14.203	0.794	27.134	4.043
JPY to USD	Christofferson	4.975	1.957	0.091	2.208	15.14	1.391	27.499	5.833
	V/N	9.3%	10.4%	5.1%	5.4%	1.4%	1.1%	1.0%	0.7%
	Kupiec	1.72	0.625	0.018	1.229	4.133	0.119	13.407	1.74
GBP to USD	Christofferson	2.433	3.021	1.389	3.752	NaN	NaN	NaN	NaN

Figure 5: Backtesting results for GARCH(1,1) model

	model:	EGARCH(1,1)							
	alpha:	0.9		0.95		0.99		0.995	
		Normal	student's t	Normal	student's t	Normal	student's t	Normal	student's t
	V/N	8.6%	10.5%	4.9%	5.2%	1.9%	1.3%	1.3%	0.7%
	Kupiec	9.318	0.916	0.059	0.23	27.074	3.827	37.461	4.318
Microsoft	Christofferson	18.008	3.766	3.931	5.005	27.246	3.942	37.577	NaN
	V/N	8.3%	9.6%	5.0%	5.0%	1.7%	1.3%	1.3%	0.9%
	Kupiec	14.234	0.606	0.01	0.001	15.208	3.827	35.518	9.177
Apple	Christofferson	20.464	3.576	4.77	5.889	17.467	3.942	35.659	10.186
	V/N	9.1%	10.3%	5.3%	5.5%	1.9%	1.4%	1.2%	0.7%
	Kupiec	3.841	0.416	0.7	1.969	25.762	5.693	29.926	3.539
Bank of Americ	Christofferson	7.873	1.957	1.999	2.045	27.108	9.501	35.032	9.204
	V/N	9.5%	10.5%	5.6%	6.2%	1.9%	1.4%	1.3%	0.7%
Gold	Kupiec	0.612	0.616	1.539	5.806	11.774	2.273	17.913	0.844
	Christofferson	4.617	1.888	6.26	8.168	16.297	3.047	NaN	NaN
	V/N	7.8%		5.0%		2.6%		1.9%	
	Kupiec	11.046		0.001		35.025		46.802	
Cheddar	Christofferson	22.343		10.714		36.7		48.366	
	V/N	9.1%	10.4%	5.0%	5.5%	1.7%	1.3%	1.3%	1.0%
	Kupiec	1.839	0.375	0.002	1.163	8.645	1.338	18.339	6.726
Soybean meal	Christofferson	2.806	2.115	3.154	3.644	10.841	NaN	NaN	NaN
	V/N	9.7%	10.3%	4.8%	4.8%	1.7%	1.4%	1.0%	0.7%
	Kupiec	0.354	0.233	0.285	0.285	13.978	3.485	10.605	3.129
CAD to USD	Christofferson	0.384	0.349	0.289	0.498	13.982	NaN	NaN	NaN
	V/N	9.0%	10.4%	5.0%	5.1%	1.8%	1.5%	1.6%	0.8%
	Kupiec	3.196	0.656	0.011	0.125	16.535	6.441	46.292	5.974
JPY to USD	Christofferson	5.174	0.715	0.014	0.215	16.536	6.573	46.354	7.501
	V/N	9.7%	10.4%	5.4%	5.2%	1.7%	1.1%	1.2%	0.6%
	Kupiec	0.34	0.452	0.899	0.214	11.843	0.271	19.716	1.182
GBP to USD	Christofferson	0.775	0.457	1.024	0.27	11.862	NaN	20.318	NaN

Figure 6: Backtesting results for EGARCH(1,1) model

	model:	ARMA(1,1)-GARCH(1,1)							
	alpha:	0.9		0.95		0.99		0.995	
		Normal	student's t	Normal	student's t	Normal	student's t	Normal	student's t
	V/N	8.2%	10.2%	4.6%	5.2%	1.6%	1.2%	1.1%	0.6%
	Kupiec	15.088	0.238	1.269	0.482	12.221	1.147	23.053	1.631
Microsoft	Christofferson	16.163	4.993	3.557	2.778	12.222	1.449	NaN	NaN
	V/N	8.9%	10.3%	4.7%	5.2%	1.5%	1.1%	1.1%	0.7%
	Kupiec	5.999	0.486	0.572	0.388	10.388	0.846	19.882	4.318
Apple	Christofferson	14.244	3.351	4.292	7.332	10.39	NaN	NaN	NaN
	V/N	9.7%	11.0%	5.6%	5.9%	1.8%	1.4%	1.2%	0.9%
	Kupiec	0.452	4.376	3.076	6.683	21.99	7.12	28.144	9.177
Bank of Americ	Christofferson	8.409	12.509	7.245	9.266	22.294	13.645	30.366	13.469
	V/N	9.3%	10.9%	5.5%	6.0%	1.7%	1.3%	1.3%	0.7%
Gold	Kupiec	1.021	1.877	1.085	4.097	8.267	1.2	17.913	1.454
	Christofferson	2.435	2.117	1.238	5.189	8.528	NaN	18.788	NaN
	V/N	7.6%	12.8%	5.0%	8.2%	2.6%	2.0%	1.8%	1.2%
	Kupiec	13.909	15.457	0.001	35.529	35.025	14.921	41.447	12.838
Cheddar	Christofferson	15.983	22.904	6.472	38.267	35.116	NaN	NaN	NaN
	V/N	9.0%	10.3%	4.9%	5.1%	1.8%	1.3%	1.3%	0.8%
	Kupiec	2.283	0.151	0.062	0.072	10.979	1.862	18.339	3.256
Soybean meal	Christofferson	3.463	3.878	2.243	1.551	15.712	2.72	19.197	5.692
	V/N	9.1%	9.7%	4.5%	4.6%	1.3%	0.9%	0.8%	0.4%
	Kupiec	3.071	0.354	1.897	1.077	2.365	0.145	4.876	0.882
CAD to USD	Christofferson	3.164	0.687	2.878	1.135	NaN	NaN	NaN	NaN
	V/N	9.1%	11.0%	5.2%	5.7%	1.8%	1.3%	1.3%	0.8%
	Kupiec	2.77	3.061	0.267	3.24	17.757	2.453	29.075	4.968
JPY to USD	Christofferson	3.6	3.085	2.733	5.573	22.874	5.01	31.481	10.81
	V/N	9.5%	10.4%	5.1%	5.5%	1.4%	1.2%	1.1%	0.6%
	Kupiec	0.895	0.452	0.018	1.818	4.814	1.077	14.894	0.725
GBP to USD	Christofferson	3.207	2.616	1.389	4.68	NaN	NaN	NaN	NaN

Figure 7: Backtesting results for ARMA(1,1)-GARCH(1,1) model