Problem P2.1

(a) The complex permittivity for bottom round steak is about $\epsilon = 40(1 + i0.3)\epsilon_o$ at the operating frequency (2.5 GHz) of a microwave oven. What is the penetration depth?

(b) Calculate loss tangents and skin depths for sea water at frequencies 60 Hz and 10 MHz. Sea water can be characterized by conductivity $\sigma = 4\,\mathrm{mho/m}$, permittivity $\epsilon = 80\epsilon_0$, and permeability $\mu = \mu_o$ at those frequencies.

(c) A 100-Hz electromagnetic wave is propagating down into the sea water with an electric field intensity E of 1 V/m just beneath the sea surface. What is the intensity of E at a depth of 100 m? What are the time-average Poynting's power densities just beneath the surface and at a depth of 100 m?

Problem P2.2

Superconductivity was first observed by Kamerlingh Onnes in 1911. In 1933 Meissner and Ochsenfeld discovered that superconducting metals cannot be penetrated by magnetic fields. Magnetic fields are expelled from a normal metal when it is cooled to the superconducting state. The macroscopic theory of superconductivity was developed by London and London in 1935 followed by the microscopic theory of Bardeen, Cooper and Schrieffer in 1957.

A simple model of superconductivity calls for an electron plasma with a very high electron density N.

(a) Show that the penetration depth of a plasma with very large N takes the form

$$d_p = \sqrt{\frac{m}{Ne^2\mu_0}}$$

(b) Letting $N = 7 \times 10^{28}\,\mathrm{m}^{-3}$, calculate d_p.

(c) Compare the above result with the skin depth of a good conductor. Explain why a very slowly varying magnetic field can penetrate a good conductor but not a superconductor.

Problem P2.4

Show that a linearly polarized wave can be decomposed into a right-hand circularly polarized wave and a left-hand circularly polarized wave.

Problem P3.2

(a) At the operating frequency (2.5 GHz) of a microwave oven, the permittivity for bottom round steak is about $\epsilon = 40\epsilon_0$ and the conductivity $\sigma = 2$ mho/meter. What is the penetration depth? Compare this penetration depth to that of polystyrene foam which has the permittivity $\epsilon = 1.03\epsilon_0$ and conductivity $\sigma = 4 \times 10^{-6}$ mho/meter.

(b) Earth is considered to be a good conductor when $\omega\epsilon/\sigma \ll 1$. Determine the highest frequency for which earth can be considered a good conductor if $\ll 1$ means less than 0.1. Assume $\sigma = 5 \times 10^{-3}$ mho/meter and $\epsilon = 10\epsilon_0$.

(c) Aluminum has $\epsilon = \epsilon_0$, $\mu = \mu_0$ and $\sigma = 3.54 \times 10^7$ mho/m. If an antenna for VHF reception is made of wood coated with a layer of aluminum and if its thickness ought to be five times greater than the skin depth of the aluminum at that frequency, determine the thickness of the aluminum layer. Is ordinary aluminum foil thick enough for that purpose? Use $f = 100$ MHz. Ordinary aluminum is approximately 1/1000 inch thick.

(d) Calculate skin depths for sea water at frequencies 100 Hz and 5 MHz. Sea water can be characterized by conductivity $\sigma = 4$ mho/m, permittivity $\epsilon = 80\epsilon_0$, and permeability $\mu = \mu_0$ at those frequencies.

(e) A ship at the ocean surface wishes to communicate electromagnetically with a deeply submerged vehicle 100 meters below the surface. Consider a ULF signal at 1 KHz propagating down into the sea water. What fraction of the incident power density reaches the submerged vehicle?

Problem P5.1

Consider an electromagnetic wave impinging normally upon a dielectric half space (Region 2) with permittivity ϵ_2 from a medium (Region 1) with permittivity ϵ_1.

(a) Let $\epsilon_1 = \epsilon_o$ and $\epsilon_2 = 4\epsilon_o$. What are the reflection coefficient R_{12} and the transmission coefficient T_{12}?
(b) What is the sum of Poynting power of the wave on either side of the interface? Do they conserve?

Problem P5.2

The constitutive relation for a lossy dielectric with $\epsilon = \epsilon_R + i\epsilon_I$ and $\mu = \mu_0$ can be written as a material contribution in excess of the "free space" part $\epsilon_0\bar{E}$ by defining the polarization $\bar{P} = \bar{D} - \epsilon_0\bar{E}$.
(a) Show that the material contribution in maxwell equations can be contained within an effective electric current density $\bar{J}_e = -i\omega\bar{P}$.
(b) In a conductor, a conduction current $\bar{J}_c = \sigma\bar{E}$ is responsible for the loss of electromagnetic energy. More generally, we can write $\bar{J}_c = \omega\epsilon_I\bar{E}$. If we define the free current density by $\bar{J}_c$ and a bound current density $\bar{J}_b$ such that $\bar{J}_e \equiv \bar{J}_c + \bar{J}_b$, what is $\bar{J}_b$?

Now consider an electromagnetic wave $\bar{E} = \hat{x}E_0 e^{-k_I z} e^{ik_R z}$ propagating in a lossy dielectric with $k = k_R + ik_I = \omega\sqrt{\mu_0\epsilon}$.
(c) What is the magnetic field of the electromagnetic wave?

Problem P5.3

A plane wave of angular frequency ω is incident on a plasma medium with permeability μ_o and permittivity $\epsilon = \epsilon_o \left(1 - \omega_p^2/\omega^2\right)$, where ω_p is the plasma frequency and $\omega = 2\omega_p$.

(a) Calculate the critical angle θ_C such that the incident wave is totally reflected.

(b) Calculate the Brewster angle θ_B such that TM waves are totally transmitted.

(c) In general for any two isotropic media, can you find an incident angle θ such that $\theta = \theta_B > \theta_C$? If you can, give an example. If you cannot, explain why not.

Problem P5.4

The ionosphere extends from approximately 50 km above the earth to several earth radii (mean earth radius is about 6371 km) with the maximum in ionization density at about 300 km. For simplicity, assume that the ionosphere consists of a 40 km thick E layer with electron density $N = 10^{11}\,\mathrm{m}^{-3}$ below a 200 km thick F layer with $N = 6 \times 10^{11}\,\mathrm{m}^{-3}$.

(a) What are the plasma frequencies of the E and F layers ?

(b) Consider a plane wave of 10 MHz incident at an angle θ upon the ionosphere from below the E layer, what is the angle θ_t of the wave in the ionospheric E layer ?

(c) Let $\theta = 30°$, below what frequency will the wave be totally reflected by the E layer and below what frequency will it be totally reflected by the F layer ?