Numerical Analysis – Fall 2022

Assignment #2

Issued: Sept. 27, 2022 **Due: Oct. 17, 2022**

Problem 1:

Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find p_2. Could $p_0 = 0$ be used?

Problem 2:

Assume that we wish to use the Newton-Raphson method to approximate the root $\frac{1}{b}$ of the nonlinear equation

$$f(x) = b - \frac{1}{x} = 0,$$

where we assume $b > 0$.

(i) Show that $|\epsilon_{k+1}| = \epsilon_k^2$ where ϵ_k is the relative error in x_k at the k-th iteration, given by

$$\epsilon_k = \frac{\frac{1}{b} - x_k}{\frac{1}{b}}.$$

(ii) Show that the Newton-Raphson iterations will converge to $\frac{1}{b}$ for any starting value x_0 provided that

$$0 < x_0 < \frac{2}{b}.$$

Problem 3:

Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(2)}$ for each of the following nonlinear systems.

a.

$$\begin{aligned} 3x_1 - \cos(x_2x_3) - \frac{1}{2} &= 0, \\ 4x_1^2 - 625x_2^2 + 2x_2 - 1 &= 0, \\ e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0. \end{aligned}$$

b.

$$\begin{aligned} x_1^2 + x_2 - 37 &= 0, \\ x_1 - x_2^2 - 5 &= 0, \\ x_1 + x_2 + x_3 - 3 &= 0. \end{aligned}$$

Problem 4:

Use the method of Steepest Descent with $TOL = 0.05$ to approximate the solutions of the following nonlinear systems.

a. $$\begin{aligned} 15x_1 + x_2^2 - 4x_3 &= 13, \\ x_1^2 + 10x_2 - x_3 &= 11, \\ x_2^3 - 25x_3 &= -22. \end{aligned}$$

b. $$\begin{aligned} 10x_1 - 2x_2^2 + x_2 - 2x_3 - 5 &= 0, \\ 8x_2^2 + 4x_3^2 - 9 &= 0, \\ 8x_2x_3 + 4 &= 0. \end{aligned}$$

Problem 5: The nonlinear system

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0, \qquad x_1x_2^2 + x_1 - 10x_2 + 8 = 0$$

can be transformed into the fixed-point problem

$$x_1 = g_1(x_1, x_2) = \frac{x_1^2 + x_2^2 + 8}{10}, \qquad x_2 = g_2(x_1, x_2) = \frac{x_1x_2^2 + x_1 + 8}{10}$$

(a) Show that $\mathbf{G} = (g_1, g_2)^t$ mapping $D \subset R^2$ into R^2 has a unique fixed point in

$$D = \{(x_1, x_2)^t \mid 0 \le x_1, x_2 \le 1.5\}$$

(b) Let $\mathbf{x}^{(0)} = [0, 1]^t$, and perform two steps of the fixed point iteration to find $\mathbf{x}^{(2)}$