P60

33.

(1) $\int_{-1}^{2} f(x)dx = 1$

$\therefore c = \frac{1}{9}$

(2) 当 $y \leq -3$ 时, $F_Y(y) = 0$

当 $y \geq 6$ 时, $F_Y(y) = 1$

当 $-3 < y < 6$ 时,

$P\{Y \leq y\} = P\{3X \leq y\} = P\{X \leq \frac{y}{3}\} = \int_{-1}^{\frac{y}{3}} \frac{1}{9}(4-x^2)dx$

$\therefore f_Y(y) = \frac{1}{3}\left[\frac{4}{9} - \frac{1}{9}\left(\frac{y}{3}\right)^2\right] = \frac{4}{27} - \frac{1}{243}y^2$

$$\therefore f_Y(y) = \begin{cases} \frac{4}{27} - \frac{1}{243}y^2, & -3 < y < 6 \\ 0, & \text{other} \end{cases}$$

(3) 当 $z \leq 0$ 时, $F_Z(z) = 0$

当 $z \geq 2$ 时, $F_Z(z) = 1$

当 $0 < z < 2$ 时,

~~$F_Z(z)$ $P\{z <$~~

$P\{Z < z\} = P\{|X| < z\} = P\{-z < X < z\}$

① 当 $0 < z < 1$ 时,

$F_Z(z) = \int_{-z}^{0} \frac{1}{9}(4-x^2)dx + \int_{0}^{z} \frac{1}{9}(4-x^2)dx$

$= \frac{8}{9}z - \frac{2}{27}z^3$

② 当 $1 < z < 2$ 时,

$F_Z(z) = \int_{-1}^{0} \frac{1}{9}(4-x^2)dx + \int_{0}^{z} \frac{1}{9}(4-x^2)dx$

$= \frac{11}{27} + \frac{4}{9}z - \frac{1}{27}z^3$

$$\therefore F_Z(z) = \begin{cases} 0, & z \leq 0 \\ \frac{8}{9}z - \frac{2}{27}z^3, & 0 < z < 1 \\ \frac{11}{27} + \frac{4}{9}z - \frac{1}{27}z^3, & 1 < z < 2 \\ 1, & z \geq 2 \end{cases}$$

$$f_Z(z)=\begin{cases}\frac{8}{9}-\frac{2}{9}z^2, & 0<z<1\\ \frac{4}{9}-\frac{1}{9}z^3, & 1<z<2\\ 0, & \text{others}.\end{cases}$$

35.

由 $f_X(x)=\begin{cases}1, & 0<x<1\\ 0, & \text{others}.\end{cases}$

当 $n=2k+1$ 时.

$$P\{Y<y\}=P\{X^n<y\}=P\{X<\sqrt[n]{y}\}=\int_0^{\sqrt[n]{y}}dx=\sqrt[n]{y}$$

$$\therefore F_Y(y)=\begin{cases}0, & y<0\\ \sqrt[n]{y}, & 0<y<1\\ 1, & y>1.\end{cases}\qquad \therefore f_Y(y)=\begin{cases}\frac{1}{n}\frac{\sqrt[n]{y}}{y}, & 0<y<1\\ 0, & \text{others}.\end{cases}$$

当 $n=2k$ 时

$$P\{Y<y\}=P\{X^n<y\}=P\{-\sqrt[n]{y}<X<\sqrt[n]{y}\}=\int_0^{\sqrt[n]{y}}dx=\sqrt[n]{y}$$

$$\therefore F_Y(y)=\begin{cases}0, & y<0\\ \sqrt[n]{y}, & 0<y<1\\ 1, & y>1\end{cases}\qquad \therefore f_Y(y)=\begin{cases}\frac{1}{n}\cdot\frac{\sqrt[n]{y}}{y}, & 0<y<1\\ 0, & \text{others}\end{cases}$$

36. $f_X(x) = \begin{cases} \frac{2}{3\pi}, & 0 < x < \frac{3}{2}\pi \\ 0, & \text{others} \end{cases}$

当 $Y<-1$ 时，$F_Y(y)=0$

当 $Y>1$ 时，$F_Y(y)=0$

当 $-1<Y<1$ 时，

$$P\{Y<y\} = P\{\cos X<y\} = \cancel{P\{\arccos y < X\}}\ P\{X<\arccos y\}$$

① 当 $-1<Y<0$ 时，

$$F_Y(y) = \int_{\pi}^{\arccos y} \frac{2}{3\pi}dx + \int_{2\pi-\arccos y}^{\pi} \frac{2}{3\pi}dx$$

$$= \frac{2\arccos y - 2\pi}{3\pi}$$

② 当 $0<Y<1$ 时

$$F_Y(y) = \int_{\arccos y}^{\frac{\pi}{2}} \frac{2}{3\pi}dx + \int_{\pi}^{\frac{3}{2}\pi} \frac{2}{3\pi}dx + \int_{\frac{\pi}{2}}^{\pi} \frac{2}{3\pi}dx = \frac{2}{3}$$

$$\cancel{= \frac{1}{3} - \frac{2\arccos y}{3\pi}} = 1 - \frac{2\arccos y}{3\pi}$$

$$\therefore F_Y(y) = \begin{cases} 0, & y<-1 \\ \frac{2\arccos y - 2\pi}{3\pi}, & -1<y<0 \\ 1-\frac{2\arccos y}{3\pi}, & 0<y<1 \\ 1, & y>1 \end{cases}$$

37. $f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, $-\infty < x < +\infty$

~~$\because g(x) = x^2$, g~~

$P\{Y<y\} = P\{X^2<y\} = P\{-\sqrt{y}<X<\sqrt{y}\}$

$\therefore F_Y(y) = \int_{-\sqrt{y}}^{\sqrt{y}} \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

$\therefore f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\sqrt{y}-\mu)^2}{2\sigma^2}} \cdot \frac{1}{2\sqrt{y}} + \frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{(\sqrt{y}+\mu)^2}{2\sigma^2}} \cdot \frac{1}{2\sqrt{y}}$

38.

(1) $\int_0^2 (ax+b)dx = 1$, $\int_0^1 (ax+b)dx = \frac{1}{3}$

$\therefore a = \frac{1}{3}$, $b = \frac{1}{6}$

(2) $Y = g(X)$, $\because g'(x) = \frac{1}{2\sqrt{x}} > 0$

$\therefore f_Y(y) = \begin{cases} (ay^2+b)\cdot 2y = \frac{2}{3}y^3 + \frac{1}{3}y , & 0<y<\sqrt{2} \\ 0 , & \text{others} \end{cases}$

39.

(1) $Y = g(X) = e^X$, $g'(x) = e^x > 0$

$\therefore f_Y(y) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(\ln y)^2}{2}} \cdot \frac{1}{y}$

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(2)

$P\{Z<z\} = P\{\ln|X| < z\} = P\{|X|<e^z\} = P\{-e^z<X<e^z\}$

$\therefore F_Y(y) = \int_{-e^z}^{e^z} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

$\therefore f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-\frac{e^{2z}}{2}} \cdot e^z - \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{e^{-2z}}{2}} e^z$

P92:

1. $P\{X=3, Y=3\} = \frac{3}{5}\times\frac{3}{5}+\frac{2}{5}\times\frac{2}{5} = \frac{13}{25}$

$P\{X=4, Y=2\} = \frac{2}{5}\times\frac{3}{5} = \frac{6}{25}$

$P\{X=2, Y=4\} = \frac{3}{5}\times\frac{2}{5} = \frac{6}{25}$

$\therefore$

X \ Y	2	3	4	P(X=i)
2	0	0	$\frac{6}{25}$	$\frac{6}{25}$
3	0	$\frac{13}{25}$	0	$\frac{13}{25}$
4	$\frac{6}{25}$	0	0	$\frac{6}{25}$

2. $\begin{cases} a+b+0.5=1 \\ P\{X=0\} = 0.3+a \\ P\{X+Y=1\} = \cancel{0.3+a}\ a+b \end{cases}$

$\because \{X=0\}$ 与 $\{X+Y=1\}$ 独立

$\therefore P\{X=0 \cap X+Y=1\} = P\{X=0\}\cdot P\{X+Y=1\}$

$\therefore a = (0.3+a)(a+b)$

$\therefore a=0.3,\ b=0.2$

3.

$$P\{Y\le 0 \mid X<2\} = \frac{P\{Y\le 0, X<2\}}{P\{X<2\}} = \frac{a+0.1}{a+b+0.1} = 0.5$$

$$P\{Y=1\} = b+c = 0.5$$

$$a+b+c = 0.7$$

$$\therefore \begin{cases} a=0.2 \\ b=0.3 \\ c=0.2 \end{cases}$$

5.

(1)

$Y \backslash X$	1	2	3	4	5	6	y_{ij}
1	$\frac{1}{36}$	0	0	0	0	0	$\frac{1}{36}$
2	$\frac{1}{36}$	$\frac{1}{18}$	0	0	0	0	$\frac{1}{12}$
3	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{12}$	0	0	0	$\frac{5}{36}$
4	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{9}$	0	0	$\frac{7}{36}$
5	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{5}{36}$	0	$\frac{1}{4}$
6	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{36}$	$\frac{1}{6}$	$\frac{11}{36}$
x_{ij}	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	

(2) $P\{X=i \mid Y=1\} = \frac{P\{X=i, Y=1\}}{P(Y=6)} = \frac{36}{11} P\{X=i, Y=6\}$

$\therefore$

X	1	2	3	4	5	6
P	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{1}{11}$	$\frac{6}{11}$

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6.

(1)

X \ Y	1	2	3
1	$\frac{7}{18}$	$\frac{1}{18}$	$\frac{1}{18}$
0	$\frac{11}{30}$	$\frac{1}{15}$	$\frac{1}{15}$

(2)

Y	1	2	3
P	$\frac{34}{45}$	$\frac{11}{90}$	$\frac{11}{90}$

(3) $P\{X=i \mid Y=1\} = \frac{P\{X=i, Y=1\}}{P\{Y=1\}}$

X	1	0
P	$\frac{35}{68}$	$\frac{33}{68}$

7. ~~$f_X(x) = \frac{\lambda^x e^{-\lambda}}{x!}$~~

$P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$

~~$P(Y=m) = P(X=k) \cdot P(Y=m \mid X=k) \cdot P(X=k) = 0.1 \frac{\lambda^k e^{-\lambda}}{k!}$~~

(1) $P(X=k, Y=m) = \frac{\lambda^k e^{-\lambda}}{k!} C_k^m \, 0.1^m \, 0.9^{k-m}$

(2) $P(Y=m) = \sum_{k=0}^{+\infty} P(Y=m \mid X=k) \cdot P(X=k)$

$= \sum_{k=0}^{+\infty} P(X=k, Y=m)$

$= \sum_{k=0}^{+\infty} C_k^m \, 0.1^m \, 0.9^{k-m} \cdot \frac{\lambda^k e^{-\lambda}}{k!}$

$= \sum_{k=m}^{+\infty} \frac{1}{(k-m)!\,m!} 0.1^m \, 0.9^{k-m} \lambda^k e^{-\lambda}$

$= \frac{0.1^m \lambda^m}{m!} e^{-\lambda} \sum_{k=m}^{+\infty} \frac{(0.9\lambda)^{k-m}}{(k-m)!} = \frac{(0.1\lambda)^m}{m!} e^{-\lambda} \cdot e^{0.9\lambda}$

$= \frac{(0.1\lambda)^m}{m!} \cdot e^{-0.1\lambda}$

9.

(1) $P\{X=1\}=0.3 \quad P\{X=2\}=0.7$

$P\{Y=0\}=0.4 \quad P\{Y=1\}=0.6$

$\cancel{P\{X=1, Y=0\} = P\{X=1 \mid Y=}$

X \ Y	0	1
1	0.1	0.2
2	0.3	0.4

(2)

$$P\{X=i \mid Y=0\} = \frac{P\{X=i, Y=0\}}{P\{Y=0\}}$$

$$\therefore P\{X=1 \mid Y=0\} = \frac{0.1}{0.4} = \frac{1}{4},$$

$$P\{X=2 \mid Y=0\} = \frac{0.3}{0.4} = \frac{3}{4}$$

$\therefore \cancel{P\{X\ P\{X\le i \mid Y=0\}}$

$$\therefore F_{X|Y}(x_i \mid 0) = \begin{cases} 0 & x<1 \\ \frac{1}{4} & 1\le x<2 \\ \frac{3}{4} & x\geq 2 \end{cases}$$