MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

Problem Set No. 5	6.632 Electromagnetic Wave Theory	Issued: 20090305 R
Spring Term 2009		Due: 20090312 R

Reading assignment: Section 4.2, 5.1, 6.1C ; J. A. Kong, "*Electromagnetic Wave Theory*," EMW Publishing, 2008.

Problem P5.1

Consider an asymmetric slab waveguide shown in Figure 1. It is known that $\epsilon_1 > \epsilon_2 > \epsilon_0$. Derive the guidance condition for TM modes. What is the cutoff spatial frequency for TM$_m$ mode?

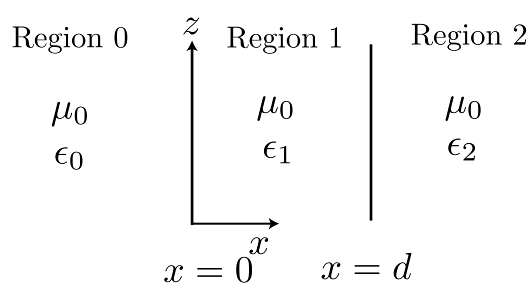

Fig. 1

Problem P5.2

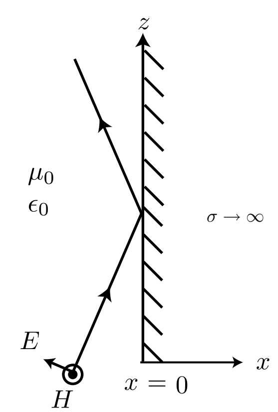

Fig. 2

A plane wave with expression $\overline{E} = (-\hat{x}\frac{\sqrt{3}}{2} + \hat{z}\frac{1}{2})e^{ik_0x/2+ik_0z\sqrt{3}/2}$ which is incident onto a PEC (perfect electric conductor) boundary at $x = 0$ will induce surface current on the interface.

(a) What is the expression of the surface current $\overline{J}_s$?

(b) From another point of view, we can treat the surface current on the interface as a source which generates two waves in $+x$ and $-x$ directions, respectively. The $-x$ travelling wave corresponds to the reflected wave while the $+x$ travelling wave cancels the incident wave in the region of $x > 0$ such that there is no field inside PEC. Show that the radiated waves from the surface current obeys the above statement.

Problem P5.3

In the Čerenkov radiation, the total energy radiated out of a cylinder of path ℓ and radius ρ is given by

$$S_\rho = \frac{\mu q^2 \ell}{4\pi} \int_0^\infty d\omega\, \omega (1 - \frac{1}{n^2 \beta^2}).$$

So the energy lost per unit length per unit frequency band is

$$\frac{d^2 S_\rho}{d\ell d\omega} = \frac{\mu q^2}{4\pi} \omega (1 - \frac{1}{n^2 \beta^2}).$$

(a) By $E_{\text{photon}} = \hbar\omega$ and $d\omega/d\lambda = 2\pi c/\lambda^2$, show that the number of photons radiated on unit path at wavelength λ is

$$\frac{d^2 N}{d\ell d\lambda} = \frac{q^2 c}{2\lambda^2 \hbar} \mu (1 - \frac{1}{n^2 \beta^2})$$

and show the frequently used formula $\frac{dN}{d\ell} \propto \frac{d\lambda}{\lambda^2} \sin^2\theta$, which gives the dependence of N on λ and θ.

(b) Gas Čerenkov detector is widely used in high energy particle experiments. The refractive index of the gas n is typically 1.002. What will be the angle for the Čerenkov radiation in case of $\beta = 1$?

(c) Most energy is radiated by the waves in the band 350 nm $\sim$ 550 nm. How many photons can you get on unit path? In order to get 100 photons for the detector, how long is the path (ℓ)? This is the size the detector should be. Note that the parameters are as follows: $\hbar = 6.63 \times 10^{-34}/2\pi\ \mathrm{J \cdot s/rad}$, $q = 1.6 \times 10^{-19}\ \mathrm{C}$, $\beta = 1$.