P1.1.2

Prove $|\overline{A}\times\overline{B}|^2 = A^2B^2 - (\overline{A}\cdot\overline{B})^2$ by using $\overline{C}\times(\overline{A}\times\overline{B}) = \overline{A}(\overline{C}\cdot\overline{B}) - (\overline{C}\cdot\overline{A})\overline{B}$.

P1.1.10

The function $\Phi = x^2+2y^2$ describes a family of ellipses. Find its gradient and show that $\nabla\Phi$ is normal to the ellipse and pointing in the directions of an expanding ellipse.

P1.1.12

Prove the following identities:

$$\nabla\cdot(\overline{E}\times\overline{H}) = \overline{H}\cdot(\nabla\times\overline{E}) - \overline{E}\cdot(\nabla\times\overline{H}) \tag{1.1.9}$$

$$\nabla\cdot(\nabla\times\overline{A}) = 0 \tag{1.1.10}$$

$$\nabla\times(\nabla\Phi) = 0 \tag{1.1.11}$$

$$\nabla\times(\nabla\times\overline{E}) = \nabla(\nabla\cdot\overline{E}) - \nabla^2\overline{E} \tag{1.1.12}$$

P1.1.17

Prove that $\nabla(\overline{A}\cdot\overline{B}) = (\overline{A}\cdot\nabla)\overline{B} + (\overline{B}\cdot\nabla)\overline{A} + \overline{A}\times(\nabla\times\overline{B}) + \overline{B}\times(\nabla\times\overline{A})$.

P1.2.3

An electromagnetic wave has spatial frequency $k_o = 100\,\mathrm{K}_o$. Determine the wavelength in meters and the temporal frequency in GHz.

Determine the spatial frequency in unit of K_o for a laser light at wavelength $\lambda = 0.6328\,\mu\mathrm{m}$.

Determine the spatial frequency in unit of K_o for a microwave oven at frequency 2.4 GHz.

P1.2.6

Consider an electromagnetic wave propagating in the $\hat{z}$-direction with

$$\overline{E} = \hat{x}e_x\cos(kz-\omega t+\psi_x) + \hat{y}e_y\cos(kz-\omega t+\psi_y)$$

where e_x, e_y, ψ_x, and ψ_y are all real numbers.

(a) Let $e_x = 2$, $e_y = 1$, $\psi_x = \pi/2$, $\psi_y = \pi/4$. What is the polarization?

(b) Let $e_x = 1$, $e_y = \psi_x = 0$. This is a linearly polarized wave. Prove that it can be expressed as the superposition of a right-hand circularly polarized wave and a left-hand circularly polarized wave.

(c) Let $e_x = 1$, $\psi_x = \pi/4$, $\psi_y = -\pi/4$, $e_y = 1$. This is a circularly polarized wave. Prove that it can be decomposed into two linearly polarized waves.