Problem P4.1

Consider a conductive uniaxial medium with

$$\overline{\overline{\epsilon}} = \begin{pmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

and

$$\overline{\overline{\sigma}} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma_z \end{pmatrix}$$

Find dispersion relations for this medium when a plane wave is propagating in $\hat{x}$ direction. Explain the operation of a polaroid with this model by assuming $\sigma_z/\sigma \gg 1$. Show that a piece of polaroid turns any wave into a linearly polarized wave.

Problem P4.2

Consider a circularly polarized electromagnetic wave normally incident upon a slab as shown in the figure. The incident electric field is expressed by

$$\overline{E}_i = E_o\hat{x}e^{ik_y y} + \alpha E_o\hat{z}e^{i(k_y y+\beta)}.$$

In this problem, neglect the reflection of the slab.

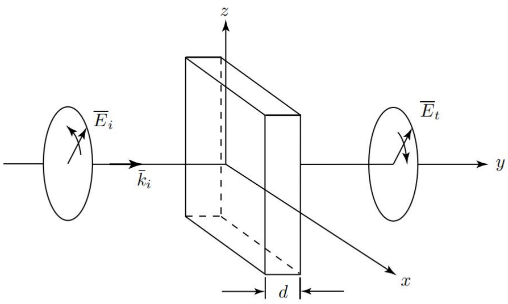

Fig. 1

(a) Let the incident wave be left-hand circularly polarized and assume that both α and β are positive, what is α and what is β?

(b) Let the slab be a uniaxial medium with the permittivity tensor

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}$$

where $\epsilon_x = \epsilon_y = 4\epsilon_o$, $\epsilon_z = 9\epsilon_o$ and the permeability $\mu = \mu_o$. Inside the uniaxial slab, what is the wave number k_y for the $\hat{x}$-polarized electric wave in terms of the wave number in free space k_o, where $k_o = \omega\sqrt{\mu_o\epsilon_o}$?

(c) For the uniaxial slab as in Part (b), let the incident wave be left-hand circularly polarized, what is the minimum thickness d in terms of the wavelength in free space λ_o, where $\lambda_o = 2\pi/k_o = 2\pi/\omega\sqrt{\mu_o\epsilon_o}$, such that the output electric field is right-hand circularly polarized?

Problem P2.1

In a negative uniaxial medium ($\epsilon_z < \epsilon$), a wave vector $\overline{k}$ makes an angle $\theta_1 < \pi/2$ with the optic axis. The Poynting's vector $< \overline{S} >$ makes an angle θ_2 with the optic axis. Determine whether θ_2 is larger or smaller than θ_1. Sketch the curve of θ_2 vs θ_1 for $0 < \theta_1 < \pi/2$.

Problem P2.2

Consider a slab of thickness d, as shown in Fig. 1, with the following permittivity and conductivity tensors:

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \qquad \overline{\overline{\sigma}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

where $\epsilon_x = 12\epsilon_\circ$, $\epsilon_y = \epsilon_\circ$, $\epsilon_z = 4\epsilon_\circ$, and $\mu = \mu_\circ$. The conductivity for polarized wave in z-direction is $\sigma_z = 0.2\epsilon_\circ\omega$ mho/meter.

Let the incident electric field propagate in the $\hat{w}$-direction. Neglect reflections.

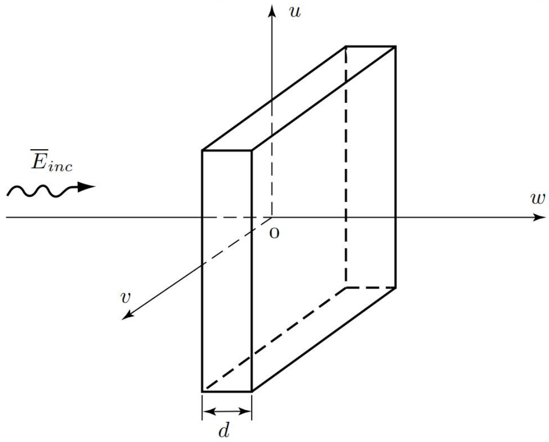

Fig. 1

Polarizer:

(a) Assign x, y, z to u, v, w (not necessarily in that order) so that, for any arbitrarily-polarized incident electric field and sufficiently thick slab, the transmitted field is linearly polarized.

(b) Determine the minimum thickness d in terms of free space wavelength such that the undesirable component of the incident field is attenuated by $1/e$.

Wave plate:

(c) Assign x, y, z to u, v, w (not necessarily in that order) so that, for a given linearly polarized incident electric field, the transmitted field is circularly polarized. Specify the axes so that there is no power absorption. Give an expression for an incident electric field such that, given the correct thickness d, the transmitted electric field is circularly polarized. What is the minimum correct d? Sketch the rotation of E field on the right surface (i. e. $w = d$) of the slab with this minimum d.

Problem P3.2

A Nicol prism made of calcite is cut diagonally and then joined together with a film of Canada balsam (refractive index n=1.53). Calcite is a negative uniaxial crystal with $\sqrt{\epsilon_z/\epsilon} = 1.49/1.66$. An incident light from the left will become a linearly polarized light when it leaves the crystal from the right. Show that with the arrangement shown in Fig. 1 an incident light from the left becomes a linearly polarized light when it leaves the crystal from the right. ($\alpha \approx 30.5°$)

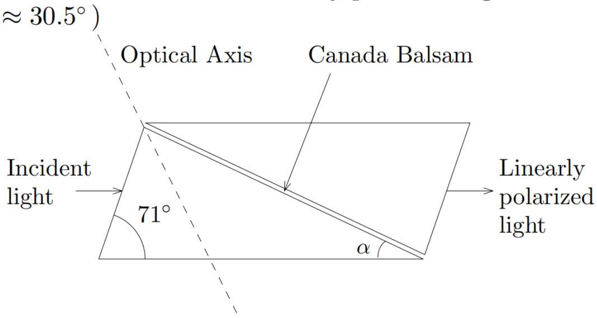

Fig. 1.