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7.

(1) $\frac{\bar{X}-\mu}{\sqrt{\sigma^2/n}} \sim N(0,1)$

(2) $\frac{\bar{X}-\mu}{\sqrt{S^2/n}} \sim t(8)$

(3) $\frac{\sum(X_i-\bar{X})^2}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2} \sim \cancel{t(9)}\ \chi^2(8)$

(4) $\chi^2(8)$

(5) $\chi^2(1)$

(6) $F(1,8)$

(7) $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ ∴ 原式 $\sim F(1,2)$

(8) $Y_1 = \bar{X_1}$, $Y_2 = \bar{X_2}$

∴原式 $\sim F(2,2)$

8.

(1) $E(X) = \int_{-\infty}^{+\infty} \frac{1}{2} e^{-|x|} = 0$

$D(X) = EX^2 - 0 = \frac{1}{5}$

(2)

$\cancel{D(S^2)}$

$E(S^2) = E(X^2) - (EX)^2 = 2.$

10.

(1) $X \sim E(\lambda)$

$\therefore E(\bar{X}) = \frac{1}{\lambda}$, $S^2(X) = \frac{1}{\lambda^2}$

$S^2(\bar{X}) = \frac{1}{10} S^2(X) = \frac{1}{10\lambda^2}$

(2)

$E(X_{(1)}) = \frac{1}{10} E(\bar{X}) = \frac{1}{10\lambda}$.

$S^2(X_{(1)}) = \frac{1}{10} S^2(\bar{X}) = \frac{1}{100\lambda^2}$

11.

$X_{n+1} \sim N(0,1)$

$\bar{X} \sim N(\mu, \frac{6^2}{n}) = N(0, \frac{1}{8})$

$\therefore Y = \sqrt{\frac{8}{9}} \cdot \frac{X_{n+1} - \bar{X}}{S}$

$= \frac{(X_{n+1} - \bar{X}) - (0-0)}{S\sqrt{1+\frac{1}{8}}} \sim t(1+8-2) = t(7)$

12. 12题在最后

~~(1) $\bar{X} \sim N(\mu, \frac{6^2}{5})$~~

~~$\bar{Y} \sim N(\mu, \frac{6^2}{9})$~~

~~$\frac{\bar{X}-\mu}{6/\sqrt{5}} \sim N(0,1)$~~

~~$\frac{\bar{Y}-\mu}{6/3} \sim N(0,1)$~~

~~$\frac{\bar{X}-\mu}{6/\sqrt{5}} - \frac{\bar{Y}-\mu}{6/3} \sim N(0,2)$~~

~~$\therefore (\sqrt{5}-3) \frac{\bar{X}-\bar{Y}}{6} \sim N(0,2)$~~

~~$\therefore a = \frac{\sqrt{5}-3}{12}$~~

~~(2) $E\left(\frac{(n-1)S^2}{6^2}\right) = n-1$~~

13.

$\frac{S_1^2}{S_2^2} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(6,6)$. $\frac{S_2^2}{S_1^2} \sim F(6,6)$

$\therefore P\{\max(\frac{S_1^2}{S_2^2}, \frac{S_2^2}{S_1^2}) > c\} = 2P(\frac{S_1^2}{S_2^2} > c) = 0.05$

$\therefore$ c 为上α分位点

$c = F_{0.025}(6,6) = 5.82$

14.

$\sum_{i=1}^{8} X_i = (n_1-1)S_1^2$

$\sum_{i=9}^{16} X_i = (n_2-1)S_2^2$

$\sigma_1^2 = \sigma_2^2$

$\therefore \frac{\sum_{i=1}^{8} X_i}{\sum_{i=9}^{16} X_i} = \frac{(n_1-1)S_1^2/\sigma_1^2}{(n_2-1)S_2^2/\sigma_1^2} = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \sim F(8,8)$

$\therefore P_1 = F_{(8,8)}(1) = 0.5$

$P_2 = 0$

12.

(1)

$\bar{X} \sim N(\mu, \frac{\sigma^2}{5})$, $\bar{Y} \sim N(\mu, \frac{\sigma^2}{9})$

$\bar{X} - \bar{Y} \sim N(0, (\frac{1}{5}+\frac{1}{9})\sigma^2)$

$\therefore \frac{\bar{X}-\bar{Y}}{\sqrt{(\frac{1}{5}+\frac{1}{9})\sigma^2}} \sim N(0,1)$

$\therefore a = \sqrt{\frac{45}{14}}$

(2) $\frac{4S_1^2}{\sigma_1^2} \sim \chi^2(4)$. $\frac{8S_2^2}{\sigma_2^2} \sim \chi^2(8)$

$\therefore \frac{4S_1^2}{\sigma_1^2} + \frac{8S_2^2}{\sigma_2^2} \sim \chi^2(12)$

$\therefore \frac{\frac{\bar{X}-\bar{Y}}{\sqrt{\frac{14}{45}\sigma^2}}}{\sqrt{(\frac{4S_1^2}{\sigma_1^2}+\frac{8S_2^2}{\sigma_2^2})/12}} \sim t(12)$

$\therefore b = \sqrt{\frac{135}{14}}$