P206

7-9.

(1) $EX = \int_{0}^{+\infty} x \frac{2x}{\theta^2} dx = \frac{2}{3}\theta$

$\therefore \hat{\theta}_1 = \frac{3}{2}\overline{X}$

$L(\theta) = \prod f(x_i,\theta) = \frac{2^n}{\theta^{2n}} \prod x_i$, $0 \le x < \theta$

$\therefore$ 当 θ 取 min, $L(\theta)$ 取 max

$\therefore \hat{\theta}_2 = \max\{x_1 \cdots x_n\}$

(2) $EX_n = \int_0^{\theta} \frac{2n}{\theta^{2n}} x^{2n} dx = \frac{2n}{2n+1}\theta < \theta$

$E\hat{\theta}_2 = EX_n$

$\therefore \hat{\theta}_2$ 的估计更优

(3) $\frac{2}{3}\overline{X}_1 \to \theta$

$E(\hat{\theta}_2) = \frac{2n}{2n+1}\theta$

$\therefore \frac{2n}{2n+1} X_2 \longrightarrow \theta_2$

$\therefore$ 是 θ 的相合估计

7-10

(1) $EX = \int_0^{\infty} x \cdot \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = \theta$

$\therefore \hat{\theta}_1 = \overline{X}$

$L(\theta) = \prod f(x_i,\theta) = \theta^{-n} e^{-\sum \frac{x_i}{\theta}}$

$\therefore \frac{d \ln L(\theta)}{d\theta} = -\frac{n}{\theta} + \frac{\sum x_i}{\theta^2} = 0$ $\therefore \hat{\theta}_2 = \overline{X}$

(2) $E(c\sum X_i) = cE(\sum X_i) = cn\overline{X} = cn\theta$

$D(c\sum X_i) = c^2 D(\sum X_i) = c^2 n\theta^2$

$\therefore E[(c\sum x_i - \theta)^2] = \theta^2 E[c^2n^2 - 2cn + 1]$

$\therefore$ 当 $c = \frac{1}{n+1}$ 时, 在均方误差下最优

Scanned with CamScanner

(3) 是：

$DX = EX^2 - (EX)^2 = \int_0^{\infty} x^2 \cdot \frac{1}{\theta} e^{-\frac{x}{\theta}} dx - \theta^2 = \theta^2$

$D\hat{\theta} = D\bar{X} = \frac{\theta^2}{n}$

$P\{|\hat{\theta} - \mu| < \varepsilon\} \geq 1 - \frac{D\hat{\theta}}{\varepsilon^2} = 1 - \frac{\theta^2}{n} \to 1$

$\therefore \hat{\theta}$ 是 θ 的相合估计.

11.

(1) $L(\theta) = \prod f(x_i, \theta) = n e^{-\sum(x_i - \theta)} = ne^{-(x-\theta)}$

$\therefore \theta$ 取 min 时，$L(\theta)$ 取 min

$\therefore \hat{\theta} = \min\{x_1 \cdots x_n\}$

(2) $f(x-\theta, \theta) = \begin{cases} ne^{-x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$

$\therefore \hat{\theta} - \theta$ 密度函数 $h(t) = \begin{cases} ne^{-nt}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

(3)

$\because h(t)$ 与 θ 无关

$\therefore \hat{\theta} - \theta$ 可以为枢轴量.

(4)

$\because \theta$ 的置信水平为 $1-\alpha$ 的单侧置信下限

$\therefore$ 可取 $\min\{x_1 \cdots x_n\} + \frac{1-\alpha}{n}$

13.

(1) $\bar{X}=3400.9$

$1-\alpha=0.95 \rightarrow \alpha=0.05$

$\bar{X}-\frac{S}{\sqrt{n}}t_{n-\frac{\alpha}{2}}(n-1)=3172.3$

$\bar{X}+\frac{S}{\sqrt{n}}t_{n-\frac{\alpha}{2}}(n-1)=3629.5$

∴置信区间为(3172.3, 3629.5)

(2)

单侧置信下限为 $3172.3+2\times21.4=3215.1$

18.

$\bar{X}=803$, $S_1^2=75$

$\bar{Y}=938$, $S_2^2=102$

$\mu_1-\mu_2=\bar{X}-\bar{Y}=-135$

$\cancel{Z_{0.025}}\ Z_{\frac{\alpha}{2}}=1.96$

∴ ~~$\bar{X}-\frac{S}{\sqrt{n}}t_{0.025}(n-1)=-159.8$~~ $\bar{X}-\bar{Y}-Z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}=-159.8$

$\bar{X}-\bar{Y}+$ ～ $=-110.2$

∴置信区间为(-159.8, -110.2)

单侧置信上限为 -114.2

19.

(1) $\mu_1 - \mu_2 = 5.3$

$z_{\frac{\alpha}{2}} = 1.96$

$\therefore \mu_1 - \mu_2 - z_{\frac{\alpha}{2}}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = -4.01$

$\mu_1 - \mu_2 + \sim = 14.61$

$\therefore$ 置信区间为 $(-4.01, 14.61)$

(2)

$\mu_1 - \mu_2 + t_{\frac{\alpha}{2}}(k)\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = 14.867$

$\mu_1 - \mu_2 - t_{\frac{\alpha}{2}}(k)\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = -4.269$

$\therefore$ 置信区间为 $(-4.269, 14.867)$

(3)

$$\frac{S_1^2/S_2^2}{F_{\frac{\alpha}{2}}(n_1-1, n_2-1)} = 0.385$$

$$\frac{S_1^2/S_2^2}{F_{1-\alpha/2}(n_1-1, n_2-1)} = 3.859$$

$\therefore$ 置信区间为 $(0.385, 3.859)$