

6.630 Solution to Problem Set 10

Solution P10.1

- (a) $f_{c10} = \frac{\omega_{c10}}{2\pi} = \frac{c}{2\pi} \left(\frac{\pi}{a} \right) = \frac{3 \times 10^8}{2\pi} \times \frac{\pi}{6.55} = 22.9 \text{ (MHz)} < f < f_{c01} = \frac{c}{2\pi} \left(\frac{\pi}{b} \right) = \frac{3 \times 10^8}{2\pi} \times \frac{\pi}{4.19} = 35.8 \text{ (MHz)}$
- (b) An AM radio operates in the range of 500 to 1600 (KHz) is below the cutoff frequency of the fundamental mode TE_{10} . Therefore AM signals can not be received in the tunnel.
- (c) FM singals operate in the range of 88 to 108 (MHz) can be received in the tunnel.

Solution P10.2

In the region of $z > 0$,

$$H_y = \sum_{m=1}^{\infty} H_m^{(1)} \cos(m\pi x/d) e^{ik_z z},$$

$$E_x = \sum_{m=1}^{\infty} \frac{k_z H_m^{(1)}}{\omega \epsilon} \cos(m\pi x/d) e^{ik_z z}.$$

In the region of $z < 0$,

$$H_y = \sum_{m=1}^{\infty} H_m^{(2)} \cos(m\pi x/d) e^{-ik_z z},$$

$$E_x = \sum_{m=1}^{\infty} \frac{-k_z H_m^{(2)}}{\omega \epsilon} \cos(m\pi x/d) e^{-ik_z z}.$$

At the boundary $z = 0$,

$$\sum_{m=1}^{\infty} (H_m^{(2)} - H_m^{(1)}) \cos \frac{m\pi x}{d} = J_s \cos \frac{3\pi x}{d},$$

$$\sum_{m=1}^{\infty} \frac{k_z H_m^{(1)}}{\omega \epsilon} \cos \frac{m\pi x}{d} = \sum_{m=1}^{\infty} \frac{-k_z H_m^{(2)}}{\omega \epsilon} \cos \frac{m\pi x}{d}.$$

So $m = 3$ and $H_m^{(2)} = -H_m^{(1)} = J_s/2$.

In the regin of $z > 0$, $H_y = -\frac{J_s}{2} \cos \frac{3\pi x}{d} e^{ik_z z}$, $E_x = -\frac{k_z J_s}{2\omega \epsilon} \cos \frac{3\pi x}{d} e^{ik_z z}$

and $E_z = \frac{iJ_s}{2\omega \epsilon} \left(\frac{3\pi}{d} \right) \sin \frac{3\pi x}{d} e^{ik_z z}$.

In the regin of $z < 0$, $H_y = \frac{J_s}{2} \cos \frac{3\pi x}{d} e^{-ik_z z}$, $E_x = -\frac{k_z J_s}{2\omega \epsilon} \cos \frac{3\pi x}{d} e^{-ik_z z}$

and $E_z = \frac{-iJ_s}{2\omega \epsilon} \left(\frac{3\pi}{d} \right) \sin \frac{3\pi x}{d} e^{-ik_z z}$.

Solution P10.3

- (a) Only TE_{10} mode is propagating. $t = \ell/v_g = 1.8 \text{ } \mu\text{s}$.
- (b) Three modes, TE_{10} , TE_{01} and TE_{20} , whose cutoff frequencies are 5 GHz, 10GHz, and 10GHz, respectively.
For TE_{10} , $t = 1.14 \text{ } \mu\text{s}$. For TE_{01} and TE_{20} , $t = 3.28 \text{ } \mu\text{s}$.