Problem 1:

a. $h=0.1$

$f'(1.1) = \frac{1}{2h}\cdot[-3f(1.1)+4f(1.2)-f(1.3)] \approx 17.769705$

$f'(1.2) = \frac{1}{2h}[f(1.3)-f(1.1)] = 22.193635$

$f'(1.3) = \frac{1}{2h}[f(1.4)-f(1.2)] = 27.10735$

$f'(1.4) = \frac{1}{2h}[f(1.2)-4f(1.3)+3f(1.4)] = 32.51085$

b. $h=0.2$

$f'(8.1) = \frac{1}{2h}\cdot[-3f(8.1)+4f(8.3)-f(8.5)] = 3.09205$

$f'(8.3) = \frac{1}{2h}[f(8.5)-f(8.1)] = 3.11615$

$f'(8.5) = \frac{1}{2h}[f(8.7)-f(8.3)] = 3.139975$

$f'(8.7) = \frac{1}{2h}[f(8.3)-4f(8.5)+3f(8.7)] = 3.163525$

Problem 2:

$$M = N(h) + k_1 h^2 + k_2 h^4 + k_3 h^6 + \cdots \quad ①$$

$$M = N\left(\frac{h}{3}\right) + k_1 \frac{h^2}{9} + k_2 \frac{h^4}{81} + k_3 \frac{h^6}{3^6} + \cdots \quad ②$$

$9 \times ② - ①$:

$$\therefore 8M = 9N\left(\frac{h}{3}\right) - N(h) - \frac{8}{9}h^4 - \frac{80}{81}h^6 + \cdots \quad ③$$

$$\therefore M = N\left(\frac{h}{9}\right) + k_1 \frac{h^2}{9^2} + k_2 \frac{h^4}{9^4} + k_3 \frac{h^6}{9^6} + \cdots \quad ④$$

$$M = \frac{9}{8}N\left(\frac{h}{3}\right) - \frac{1}{8}N(h) - \frac{10}{9}h^4 - \frac{10}{81}h^6 + \cdots \quad ⑤$$

$9 \times ④ - ②$:

$$\therefore 8M = 9N\left(\frac{h}{9}\right) - N\left(\frac{h}{3}\right) - \frac{8}{729}h^4 + \left(\frac{1}{9^8} - \frac{1}{3^6}\right)h^6 + \cdots \quad ⑥$$

$$\therefore M = \frac{9}{8}N\left(\frac{h}{9}\right) - \frac{1}{8}N\left(\frac{h}{3}\right) - \frac{1}{729}h^4 + \frac{1}{8}\left(\frac{1}{9^8} - \frac{1}{3^6}\right)h^6 + \cdots \quad ⑦$$

$\therefore 81 \times ⑦ - ⑤$:

$$81M = \frac{9^3}{8}N\left(\frac{h}{9}\right) - \frac{9^6}{8}N\left(\frac{h}{3}\right) + \frac{1}{8}N(h) + O(h^6)$$

$$\therefore M = \frac{9}{8}N\left(\frac{h}{9}\right) - \frac{8^9}{9}N\left(\frac{h}{3}\right) + \frac{1}{8648}N(h) + O(h^6)$$

Problem 3:

a.

T: $\int_{-0.25}^{0.25} (\cos x)^2 dx = \frac{0.25+0.25}{2}\left[(\cos\frac{1}{4})^2 + (\cos -\frac{1}{4})^2\right] = 0.5$

S: 原式 $= \frac{0.25}{3}\left[(\cos -0.25)^2 + 4((\cos 0)^2 + (\cos 0.25)^2\right] = 0.5$

b.

T: 原式 $= \frac{0.5}{2}\left[0 + (-0.5)\ln 0.5\right] = 0.0866$

S: 原式 $= \frac{0.25}{3}\left[-0.5\ln 0.5 - 4\times 0.25\ln(0.75) + 0\right] = 0.0528$

c.

T: 原式 $= \frac{1.3-0.75}{2} \times \left[f(x_0) + f(x_1)\right] = 0.528$

S: 原式 $= \frac{0.275}{2} \times \left[f(x_0) + 4f(x_1) + f(x_2)\right] = 0.529$

d.

T: d. 原式 $= \frac{2e-e}{2} \times \left[\frac{1}{e\ln e} + \frac{1}{2e\ln 2e}\right] = 0.648$

S: 原式 $= \frac{e}{6} \times \left[\frac{1}{e\ln e} + \frac{1}{2e\ln 2e} + 4\cdot\frac{1}{\frac{3e}{2}\ln\frac{3e}{2}}\right] = 0.532$

Problem 4:

a.

$R_{11} = \frac{x_2-x_1}{2}(f(x_1)+f(x_2)) = 0.58$

$R_{21} = \frac{x_2-x_1}{4}(f(x_1)+2f(x_1+h)+f(x_2)) = 1.29$

$R_{31} = \frac{x_2-x_1}{8}[f(x_1)+2f(x_1+h)+2f(x_1+2h)+2f(x_1+3h)]+f(x_2)] = 1.42$

$R_{22} = R_{21} + \frac{1}{3}(R_{21}-R_{11}) = 1.53$

$R_{32} = R_{31} + \frac{1}{3}(R_{31}-R_{21}) = 1.46$

$R_{33} = R_{32} + \frac{1}{15}(R_{32}-R_{22}) = 1.45$

b.

$R_{11} = \frac{1.5}{2}[x_2\ln(x_2+1) + x_1\ln(x_1+1)] = 1.09$

$R_{21} = \frac{1.5}{4}[x_1\ln(x_1+1)+2(x_1+h)\ln(x_1+h+1)+x_2\ln(x_2+1)] = 0.547$

$R_{31} = 0.268$

$R_{22} = R_{21} + \frac{1}{3}(R_{21}-R_{11}) = 0.366$

$R_{32} = R_{31} + \frac{1}{3}(R_{31}-R_{21}) = 0.175$

$R_{33} = 0.111$

c.

$R_{11} = 11.48$ $R_{21} = 3.29$

$R_{31} = 1.82$ $R_{22} = 0.56$

$R_{32} = 1.34$ $R_{33} = 1.59$

d.

$R_{11} = 0.648$ $R_{21} = 0.56$

$R_{31} = 0.536$ $R_{22} = 0.53$

$R_{32} = 0.528$ $R_{33} = 0.527$

Problem 5:

a.

$w_0 = 1$

$w_1 = w_0 + 0.1 \cdot f(1, 1) = 1$

$w_2 = w_1 + 0.1 \cdot f(1.1, 1) = \frac{122}{121} \approx 1$

$w_3 = w_2 + 0.1 \cdot f(1.2, 1) = 1.014$

$w_4 = w_3 + 0.1 \cdot f(1.3, 1.014) = 1.03116$

$w_5 = w_4 + 0.1 \cdot f(1.4, w_5) = 1.051$

$w_6 = w_5 + 0.1 \cdot f(1.5, w_6) = 1.072$

$w_7 = w_6 + 0.1 \cdot f(1.6, w_6) = 1.09411$

$w_8 = w_7 + 0.1 \cdot f(1.7, w_7) = 1.117$

$w_9 = w_7 + 0.1 \cdot f(1.8, w_8) = 1.14$

$w_{10} = w_9 + 0.1 \cdot f(1.9, w_9) = 1.164$

b. $w_0 = 0$, $w_1 = w_0 + 0.2 f(1, w_0) = 0.2$

$w_2 = w_1 + 0.2 f(1.2, w_1) = 0.439$

$w_3 = w_2 + 0.2 f(1.4, w_2) = 0.721$

$w_4 = w_3 + 0.2 f(1.6, w_3) = 1.05$

$w_5 = w_4 + 0.2 f(1.8, w_4) = 1.43$

$w_6 = w_5 + 0.2 f(2, w_5) = 1.875$

$w_7 = w_6 + 0.2 f(2.2, w_6) = 2.39$

$w_8 = w_7 + 0.2 f(2.4, w_7) = 2.99$

$w_9 = w_8 + 0.2 f(2.6, w_8) = 3.6845$

$w_{10} = w_9 + 0.2 f(2.8, w_9) = 4.494$

Problem 6:

$$E = \sum_{i=1}^{4} [y(x_i) - (ax_i + b)]^2$$

$$\therefore \frac{\partial E}{\partial a} = 0 \quad \frac{\partial E}{\partial b} = 0$$

$$\therefore a = 2.0338 \quad b = 6.4067$$

$$\therefore E = \sum [y(x_i) - (ax_i + b)^2]^2 = 18.3$$