P1.4.1

The magnetic field $\overline{H}$ and electric field $\overline{E}$ of a Hertzian dipole at very large distances $(kr \gg 1)$ are

$$\overline{H} = -\hat{\phi}\frac{\omega k q \ell}{4\pi r}\sin\theta\cos(kr-\omega t)$$
$$\overline{E} = -\hat{\theta}\frac{k^2 q \ell}{4\pi\epsilon_o r}\sin\theta\cos(kr-\omega t)$$

(a) Find the Poynting's power density vector $\overline{S}$ as a function of time. What is the time-averaged power density vector $\langle\overline{S}\rangle$?

(b) By integrating the Poynting vector over the surface of a sphere of radius r, find the time-averaged power P radiated by the Hertzian dipole.

(c) The amplitude of the current in the Hertzian dipole is $I_o = \omega q$. By using $P = \frac{1}{2}I_o^2 R_{rad}$, find the radiation resistance R_{rad} of the Hertzian dipole.

(d) A radio station is 15 km away from a city. The transmitting antenna tower may be modeled as a Hertzian dipole antenna of dipole moment $q\ell$. To maintain the FCC standard of 25 mV/m field strength in the city, how much radiation power P must be provided?

P1.6.1

Derive boundary conditions for $\overline{E}$ and $\overline{H}$ by applying Stokes' theorem to [P1.6.1.1].

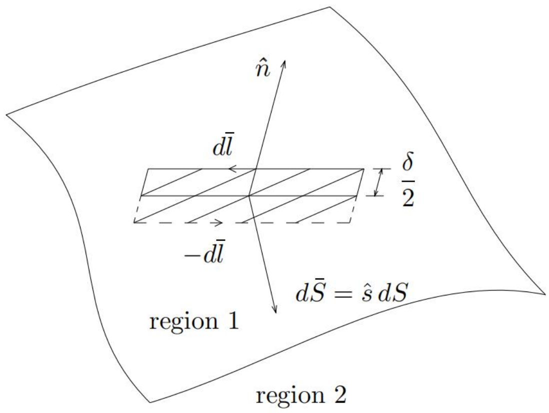


Figure P1.6.1.1 Derivation of boundary condition with Stokes' theorem.

P1.6.2

Derive the boundary conditions for $\overline{H}$ by applying the curl theorem to a small pill-box volume on the x-y plane which has an area A and an infinitesimal thickness Δz.

P1.6.3

Applying the divergence theorem (1.1.19) and integrating over the pillbox volume in Fig. E1.6.1.1 with area a and circumferential length l to find boundary condition for $\overline{D}$.

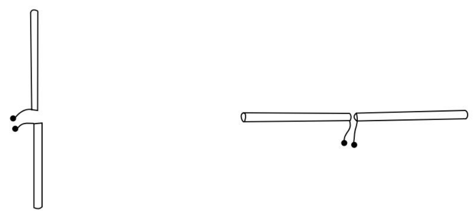
Problem P2.1

In the United States, broadcasting to the general public is regulated by the Federal Communications Commission (FCC), which allocates frequencies and establishes technical standards. Three general classes of broadcast stations have been established. *Standard broadcast stations* (amplitude modulation-AM) are licensed for operation on channels spaced by 10 kHz and occupying the band from 535 to 1605 kHz. *Frequency modulation* (FM) *broadcast stations* are authorized for operation on 100 allocated channels, each 200 kHz wide, extending consecutively from 201 on 88.1 MHz to channel 300 on 107.9 MHz. *Television broadcast stations* (operating with vestigial-sideband amplitude modulation of the visual carrier and frequency modulation of the aural carrier) are authorized for commercial and educational operation on designated channels 2–83, each 6 MHz wide, extending from 54–806 MHz. (See reference: *Reference Data for Radio Engineers*, Howard W. Sams & Co., ITT, 1983.)

The electromagnetic waves radiated by AM stations have the $\overline{E}$ field perpendicular to the ground and parallel to the antenna towers (vertical polarization). Most FM stations broadcast with circular polarization. For television broadcasting, the $\overline{E}$ field is parallel to the ground (horizontal polarization).

The induced current on a receiving antenna is largest when the antenna is aligned with the electric field. Given are two wire antenna configurations. For each type of broadcast, AM, FM, and TV, specify which configuration(s) gives maximum reception

(i) (ii)



Problem P2.2

Obtain the phasor notation of the following time-harmonic functions (if possible):

(a) $V(t) = 6\cos(\omega t + \pi/4)$
(b) $I(t) = -8\sin(\omega t)$
(c) $A(t) = 3\sin(\omega t) - 2\cos(\omega t)$
(d) $C(t) = 6\cos(120\pi t - \pi/2)$
(e) $D(t) = 1 - \cos(\omega t)$
(f) $U(t) = \sin(\omega t + \pi/3)\sin(\omega t + \pi/6)$

Problem P2.4

Wave polarization can be viewed by either taking a series of still pictures at several fixed times, called the spatial view point or by making observations at a fixed point in space, called the temporal view point. We define polarization from the temporal view point. Let us now look at polarization from the spatial view point.

Consider an electromagnetic wave with $k = 100\mathrm{K_o}$ propagating in the $\hat{z}$ direction.

$$\overline{E}(\overline{r}, t) = E_0[\hat{x}\cos(kz - \omega t) + \hat{y}\sin(kz - \omega t)]$$

What is the wavelength and the polarization of this wave?

From the spatial point of view, by taking a picture at $t = 0$, the tips of the electric field vectors form a helix. Is the helix right-handed or left-handed? What is the pitch(wavelength) of this helix?

Problem P2.5

The Earth receives over all frequency bands about $1.5\,\mathrm{kW/m^2}$ of power from the Sun.

(a) The Earth-Sun distance is 150×10^9 m. How long does it take the sunlight to reach the Earth?

(b) The Earth radius is 6400 km. What is the total power received by the Earth?

(c) Assume the Sun's mass is $2 \times 10^{30}\,\mathrm{kg}$ which converts to radiated energy according to mc^2 at 1 percent efficiency. How long can the Sun radiate at the present level?

(c) The Sun radiates $10^{-20}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$ at $3\,\mathrm{GHz}$. Assuming constant power level over $1\,\mathrm{GHz}$ bandwidth, what is the Poynting power density and the corresponding electric field amplitude?