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21.

由17题知：

$Var(X)=2$, $E(X)=0$

$Var(|X|)=1$, $E(|X|)=1$

$Cov(X,|X|) = E(X|X|) - E(X)E(|X|) = 0 - 0$

$\therefore \rho_{X|X|} = 0$

$\therefore X, |X|$ 不相关

$\because E(X|X|) = E(X)E(|X|)$

$\therefore X$与$|X|$独立

22.

(1) $E(XY) = \int_{-1}^{1}\int_{-1}^{1} xy \cdot \frac{1}{4}(1+xy)\,dx\,dy = \frac{1}{9}$

$E(X) = \int_{-1}^{1}\int_{-1}^{1} x \cdot \frac{1}{4}(1+xy) = 0$

$E(Y) = 0$

$\therefore cov(X,Y) = E(XY) - E(X)\cdot E(Y) = \frac{1}{9}$,

$DX = E(X^2) = \frac{1}{2}\int_{-1}^{1}\int_{-1}^{1} x^2 \cdot \frac{1}{4}(1+xy)\,dx\,dy = \frac{1}{3}$

$DY = E(Y^2) = \frac{1}{3}$

$\therefore \rho_{XY} = \frac{cov(X,Y)}{\sqrt{DX}\cdot\sqrt{DY}} = \frac{1}{3} > 0$

$\therefore X, Y$正相关

$\because f(x,y) \neq f_X(x)\cdot f_Y(y)$

$\therefore X, Y$不独立.

(2)

~~E(X²)~~ 且

$E(X^2) = \iint x^2 f(x,y)\,dxdy = \frac{1}{3}$

$E(Y^2) = \frac{1}{3}$

$E(X^2Y^2) = \int_{-1}^{1}\int_{-1}^{1} x^2y^2 f(x,y)\,dxdy = \frac{1}{9}$

$\therefore \cancel{Cov}\ cov(x^2, y^2) = 0$

$\therefore \rho_{x^2, Y^2} = 0$

$\therefore X^2$与Y^2独立且不相关.

23.

~~$P\{X=k\} = C_n^k \cdot (\frac{1}{6})^k \cdot (\frac{5}{6})^{n-k}$.~~

~~$P\{Y=m\} = C_n^m \cdot (\frac{1}{6})^m \cdot (\frac{5}{6})^{n-m}$~~

$E(X) = \frac{1}{6}n$, $E(Y) = \frac{1}{6}n$

$P\{X_i = 1, \cancel{X} Y_j = 1\} = \begin{cases} \frac{1}{36}, & i \neq j. \\ 0 & i = j \end{cases}$

$\therefore E(X,Y) = \sum_{i\neq j} \cancel{\theta} P = \frac{n(n-1)}{36}$

$\therefore Cov(X,Y) = -\frac{n}{36}$

$\therefore \rho_{X,Y} = -\frac{1}{5}$

$\therefore X, Y$负相关.

26.

(1) $E(\xi)=E(XY)=E(X)\cdot E(Y)=0$

~~$D(\xi)=D(XY)$~~ 记ξ为Z

$$F_Z(z)=P\{XY\le z\}$$
$$=P\{Y=1\}P\{XY\le z|Y=1\}+P\{Y=-1\}P\{XY\le z|Y=-1\}$$
$$=P\{Y=1\}P\{X\le z\}+P\{Y=-1\}P\{X\ge -z\}$$
$$=\Phi(z)$$

$\therefore \xi\sim N(0,1)$

(2) $Cov(X,\xi)=E(X^2Y)-E(X)E(XY)=0$

$\therefore X,\xi$不相关

$P\{X\le 1, XY\le 1\}$

$=P\{Y=1\}P\{X\le 1\}+P\{Y=-1\}P\{X\ge -1\}$

$=0.5\Phi(1)$

$\because \xi\sim N(0,1)$

$\therefore P\{X\le 1, XY\le 1\}\ne P\{X\le 1\}\cdot P\{XY\le 1\}$

$\therefore X$、ξ不独立

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(1)

X	0	1
P	$\frac{3}{5}$	$\frac{2}{5}$

Y	0	1

(1) $P\{Y=1 \mid X=1\} = \frac{1}{2}$

$\therefore P\{Y=1, X=1\} = P\{Y=1 \mid X=1\} \cdot P\{X=1\} = \frac{1}{5}$

$P\{Y=0 \mid X=1\} = \frac{1}{2}$

$\therefore P\{Y=0, X=1\} = \frac{1}{5}$

$P\{Y=1 \mid X=0\} = \frac{1}{3}$

$\therefore P\{Y=1, X=0\} = \frac{1}{5}$

$P\{Y=0, X=0\} = \frac{2}{5}$

$\therefore$

$X \backslash Y$	0	1	
0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{3}{5}$
1	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{2}{5}$
	$\frac{3}{5}$	$\frac{2}{5}$	

$\therefore P\{X=k, Y=j\} \neq P\{X=k\} \cdot P\{Y=j\}$

$\therefore X, Y$ 不独立

(2) $E(XY) = \frac{1}{5}$

$Cov(X, Y) = E(XY) - E(X)E(Y) = \frac{1}{25}$

$\therefore X, Y$ 正相关

31.

$$X_1+X_2+X_3+X_4 \sim N(4\times170, \cancel{4}\ 144)$$

$\therefore \ \bar{X} \sim N(170, 36)$

$\therefore \ P\{\bar{X} \geq 176\} = 1 - P\{\bar{X} < 176\} = 1 - \Phi_0\left(\frac{176-170}{6}\right) = 0.2578$

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2.

$$P\left\{\left|\frac{1}{500}\sum X_i - 10\%\right| < 5\%\right\} \geq 1 - \frac{D\left(\frac{1}{500}\sum X_i\right)}{(5\%)^2} = 92.8\%$$

3.

$P\{\xi = k\} = \left(\frac{1}{2}\right)^{k-1}$

$E(\xi) = \sum_{n=2}^{\infty} n \cdot P\{\xi = n\} = 3$

$D(\xi) = E(\xi^2) - E^2(\xi) = 2$

$\therefore P\{|\xi - E(\xi)| \geq \varepsilon\} \geq 1 - \frac{D(\xi)}{\varepsilon^2} = 75\%$

$\therefore \varepsilon = 2\sqrt{2}$

$\therefore$ 当 $a = 3-2\sqrt{2}$, $b = 3+2\sqrt{2}$ 时成立.

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5.

$\because X_i$ 独立同分布

$\therefore E(X_i)=E(X_j)$

$\therefore E\left(\sum_{i=1}^{n} iX_i\right)=\sum_{i=1}^{n} iE(X_i)=E(X_1)\cdot\sum_{i=1}^{n} i=\frac{n(n+1)}{2}\cdot E(X_1)$

$\therefore E\left(\frac{2}{n(n+1)}\sum_{i=1}^{n} i\cdot X_i\right)=E(X_1)$

$\therefore \frac{2}{n(n+1)}\sum_{i=1}^{n} i\cdot X_i \xrightarrow{P} E(X_1)$

6.

(1) 收敛 $\sigma^2+\mu^2$

(2) 收敛 σ^2

(3) 收敛 $\frac{\mu}{\sigma^2+\mu^2}$

(4) 不收敛

$\lim_{n\to\infty} E\left(\sqrt{n\sum_{i=1}^{n}(X_i-\mu)^2}\right)=\infty$