P1.4.1

The magnetic field $\overline{H}$ and electric field $\overline{E}$ of a Hertzian dipole at very large distances $(kr \gg 1)$ are

$$\overline{H} = -\hat{\phi}\frac{\omega k q\ell}{4\pi r}\sin\theta\cos(kr-\omega t)$$
$$\overline{E} = -\hat{\theta}\frac{k^2 q\ell}{4\pi\epsilon_o r}\sin\theta\cos(kr-\omega t)$$

(a) Find the Poynting's power density vector $\overline{S}$ as a function of time. What is the time-averaged power density vector $\langle\overline{S}\rangle$?
(b) By integrating the Poynting vector over the surface of a sphere of radius r , find the time-averaged power P radiated by the Hertzian dipole.
(c) The amplitude of the current in the Hertzian dipole is $I_o = \omega q$. By using $P = \frac{1}{2}I_o^2 R_{rad}$, find the radiation resistance R_{rad} of the Hertzian dipole.
(d) A radio station is 15 km away from a city. The transmitting antenna tower may be modeled as a Hertzian dipole antenna of dipole moment $q\ell$. To maintain the FCC standard of 25 mV/m field strength in the city, how much radiation power P must be provided?

P5.4.1

(a) Consider an array of two out-of-phase but equal amplitude $\hat{z}$-directed Hertzian dipoles as shown in Fig. P5.4.1.1.

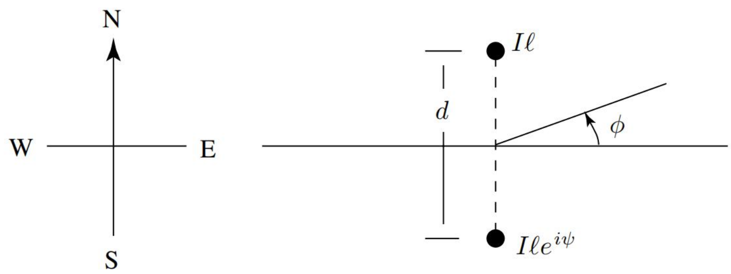

Figure P5.4.1.1

Show that the array factor $|F(\phi)|$ may be expressed as

$$|F(\phi)| = \left|2\cos\left[\frac{kd}{2}\sin\phi - \frac{\psi}{2}\right]\right|$$

(b) A broadcast array of two vertical towers with equal current amplitude is to have a horizontal plane pattern such that
  (i) maximum field intensity is to the north $(\phi = 90°)$
  (ii) the only nulls are at $\phi = 225°$ and $\phi = 315°$.

Specify the arrangement of the towers, their spacing and phasing.