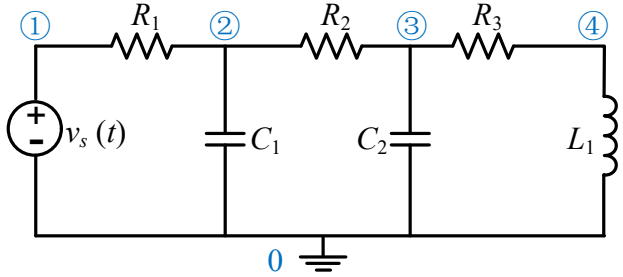
第二章 动态电路瞬态特性分析

2.2 动态电路微分方程复频域求解

动态电路微分方程复频域求解

电路

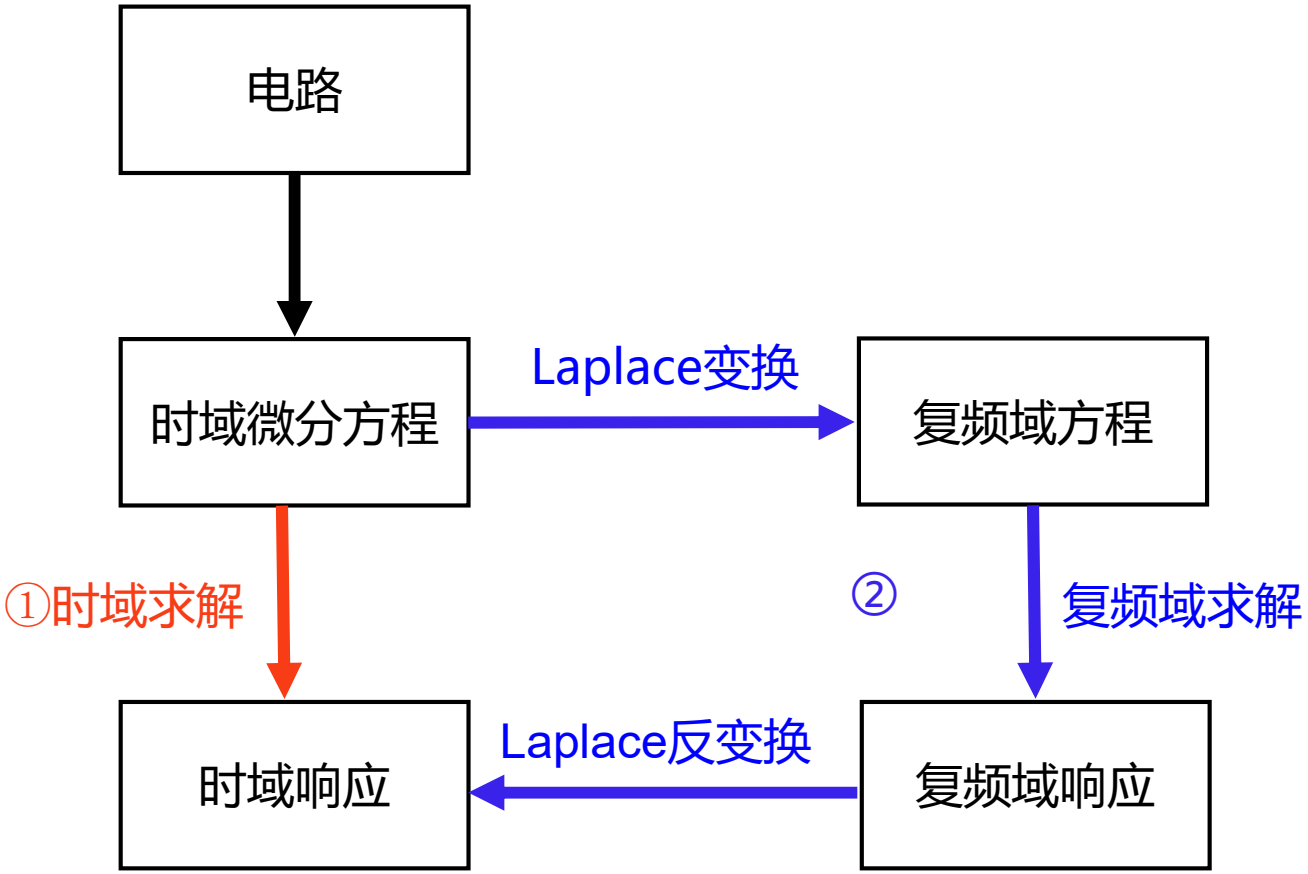


$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 & 0 & -1 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 0 & 0 & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} & 0 & 0 \\ 0 & 0 & -\frac{1}{R_3} & \frac{1}{R_3} & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_s \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & L_3 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ i_s \\ i_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ v_s \\ 0 \end{bmatrix}$$

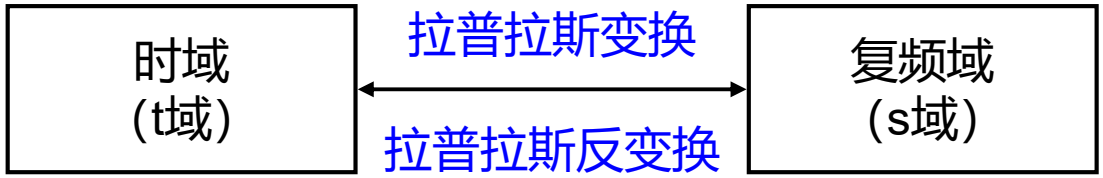
微分形式电路方程

怎么求解？

动态电路微分方程复频域求解



复频域



$f(t)$ $F(s)$

拉普拉斯变换
（Laplace transform）

$$F(s)=\int_{0^-}^{\infty} f(t)e^{-st}dt$$

复频率 $s=\sigma+j\omega$

拉普拉斯反变换
（Inverse Laplace transform ）

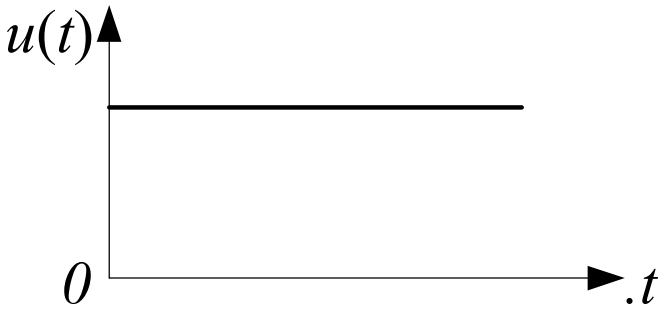
$$f(t)=\frac{1}{2\pi j}\int_{\sigma-j\omega}^{\sigma+j\omega} F(s)e^{st}ds$$

$$F(s)\big|_{\sigma=0}=F(j\omega)$$ **对应频域**

常用激励函数的复频域形式

单位阶跃函数$u(t)$

$$u(t)=\begin{cases}0 & t<0\\ 1 & t>0\end{cases}$$



$$F(s)=\int_{0^-}^{\infty}u(t)e^{-st}dt=\frac{1}{s}$$

Matlab代码:

```
>> syms t;
>> ft=heaviside(t)

ft =

heaviside(t)

>> Fs=laplace(ft)

Fs =

1/s
```

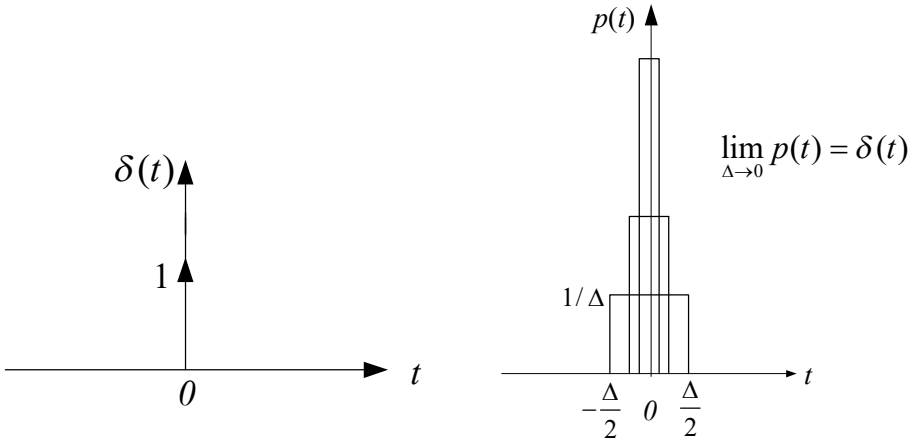
laplace: 拉普拉斯变换

常用激励函数的复频域形式

冲击函数 $\delta(t)$

$$\delta(t)=\begin{cases}0 & t\neq 0\\ \infty & t=0\end{cases}$$

$$\int_{-\infty}^{\infty}\delta(t)dt=1$$



$$F(s)=\int_{0^-}^{\infty}\delta(t)e^{-st}dt=1$$

Matlab代码:

```
>> syms t;
>> ft=dirac(t)

ft =

dirac(t)

>> Fs=laplace(ft)

Fs =

1
```

拉普拉斯变换特性

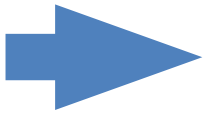
- 零状态条件下

$$f\left(0^{-}\right)=0$$

$$f(t)\leftrightarrow F(s)$$

$$\frac{d}{dt}f(t)\leftrightarrow sF(s)$$

时域：微分
微分方程



复频域：乘s
代数方程

二阶RC电路微分方程列写

节点①　　时域：自变量t

$$\frac{v_1 - v_2}{R_1} - i_s = 0, \quad v_1 = v_i$$

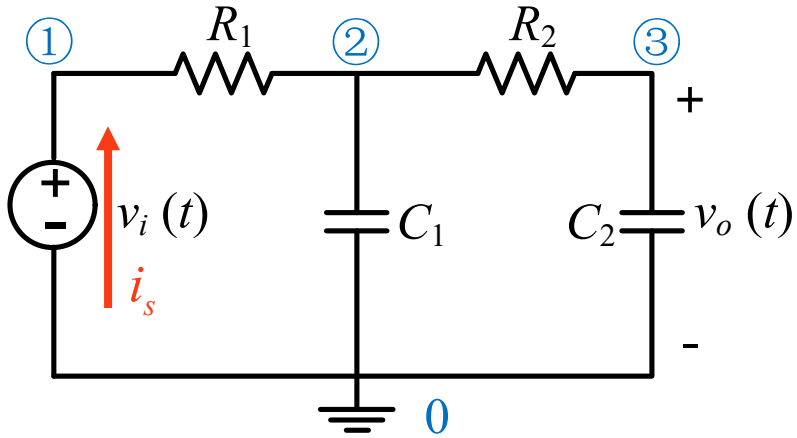
增加了变量i_s

节点②

$$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1 \frac{dv_2}{dt} = 0$$

节点③

$$-\frac{v_2 - v_3}{R_2} + C_2 \frac{dv_3}{dt} = 0$$



R_1=100 Ω　R_2=100 Ω　$v_i(t)=u(t)$

C_1=1 μF　C_2=0.01 μF

二阶RC电路复频域方程

拉普拉斯变换 $v(t) \leftrightarrow V(s), \quad \frac{d}{dt}v(t) \leftrightarrow sV(s)$

复频域：自变量s

节点①

$$\frac{v_1 - v_2}{R_1} - i_s = 0, \quad v_1 = v_i$$

$$\frac{V_1 - V_2}{R_1} - I_s = 0, \quad V_1 = V_i$$

节点②

$$-\frac{v_1 - v_2}{R_1} + \frac{v_2 - v_3}{R_2} + C_1\frac{dv_2}{dt} = 0$$

$$-\frac{V_1 - V_2}{R_1} + \frac{V_2 - V_3}{R_2} + C_1 sV_2 = 0$$

节点③

$$-\frac{v_2 - v_3}{R_2} + C_2\frac{dv_3}{dt} = 0$$

$$-\frac{V_2 - V_o}{R_2} + C_2 sV_3 = 0$$

二阶RC电路复频域方程

$$v_i(t) = u(t) \leftrightarrow V_i(s) = \frac{1}{s}$$

复频域：自变量s

$$\begin{bmatrix} \frac{1}{R_1} & -\frac{1}{R_1} & 0 & -1 \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+sC_1 & -\frac{1}{R_2} & 0 \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2}+sC_2 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_o \\ I_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{s} \end{bmatrix}$$

复频域方程求解

```
syms R1 R2 C1 C2;

A = ...
    [1/R1, -1/R1, 0, -1; ...
     -1/R1, 1/R1+1/R2+C1*s, -1/R2, 0; ...
     0, -1/R2, 1/R2+C2*s, 0; ...
     1, 0, 0, 0];

B = [0; 0; 0; 1/s];

V = A \ B;  V(3)
```

$$V_3(s)=\frac{1}{s}\times\frac{1}{R_1C_1R_2C_2s^2+s(R_1C_1+R_1C_2+R_2C_2)+1}$$

R_1=100 Ω　R_2=100 Ω　C_1=1 μF　C_2=0.01 μF

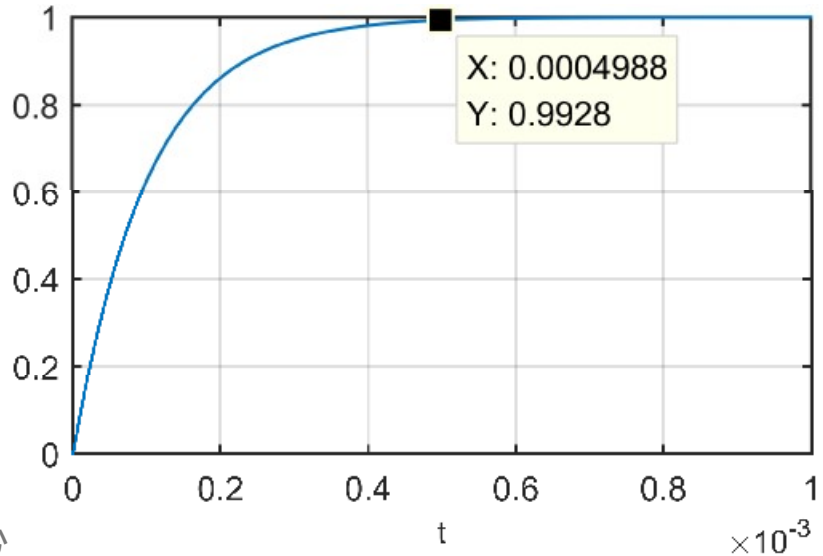
$$\rightarrow V_3(s)=\frac{10^{10}}{s\left(s^2+1.02\times10^6+10^{10}\right)}$$

时域响应求解

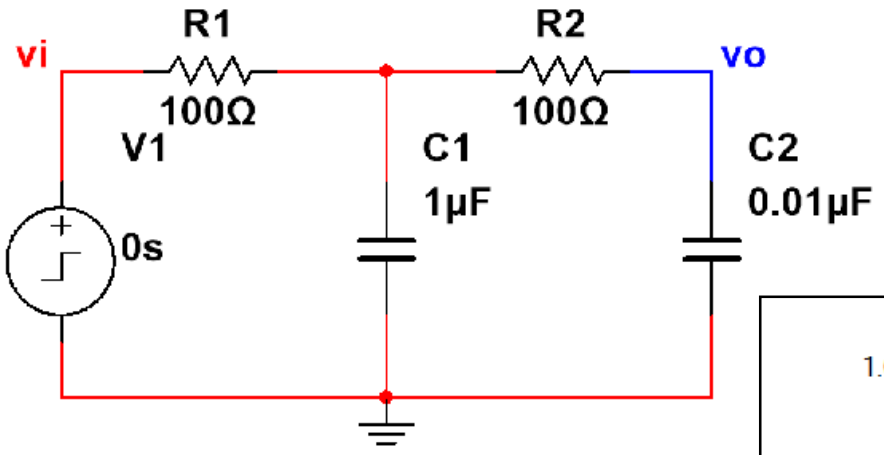
```
>> V(3)

ans =

10000000000/(s*(s^2 + 1020000*s + 10000000000))

>> vt=ilaplace(V(3))

vt =

1 - exp(-510000*t)*(cosh(10000*2501^(1/2)*t) + (51*2501^(1/2)*sinh(10000*2501^(1/2)*t))/2501)
```

ilaplace: 拉普拉斯反变换

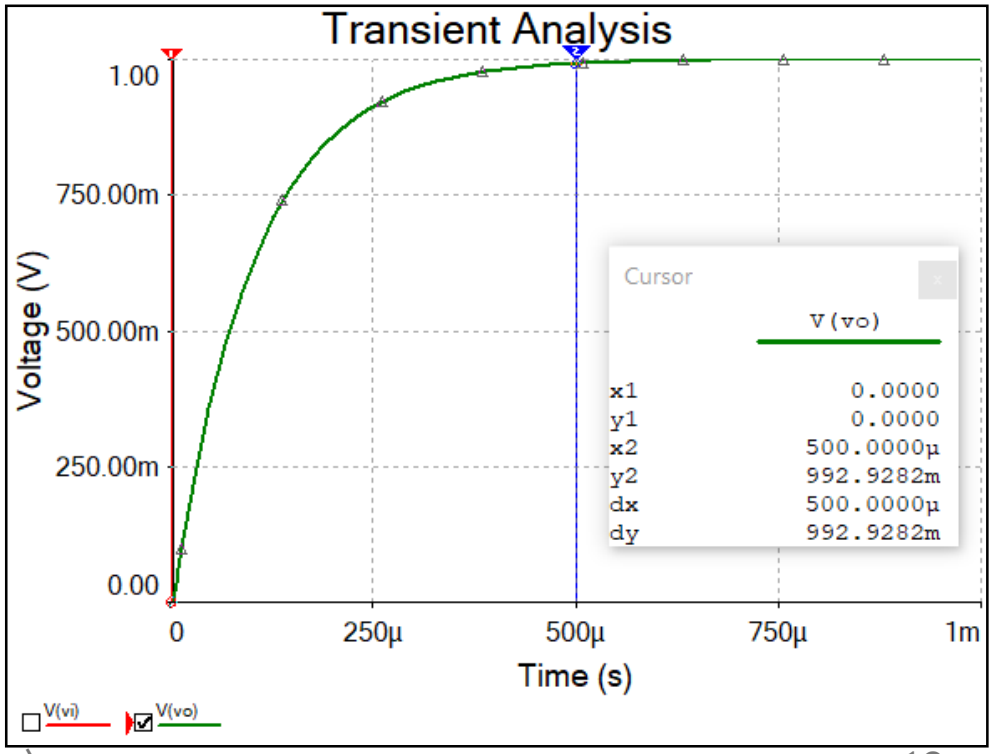
```
ezplot(vt,[0,1e-3]); grid on; ylim([0,1]);
```



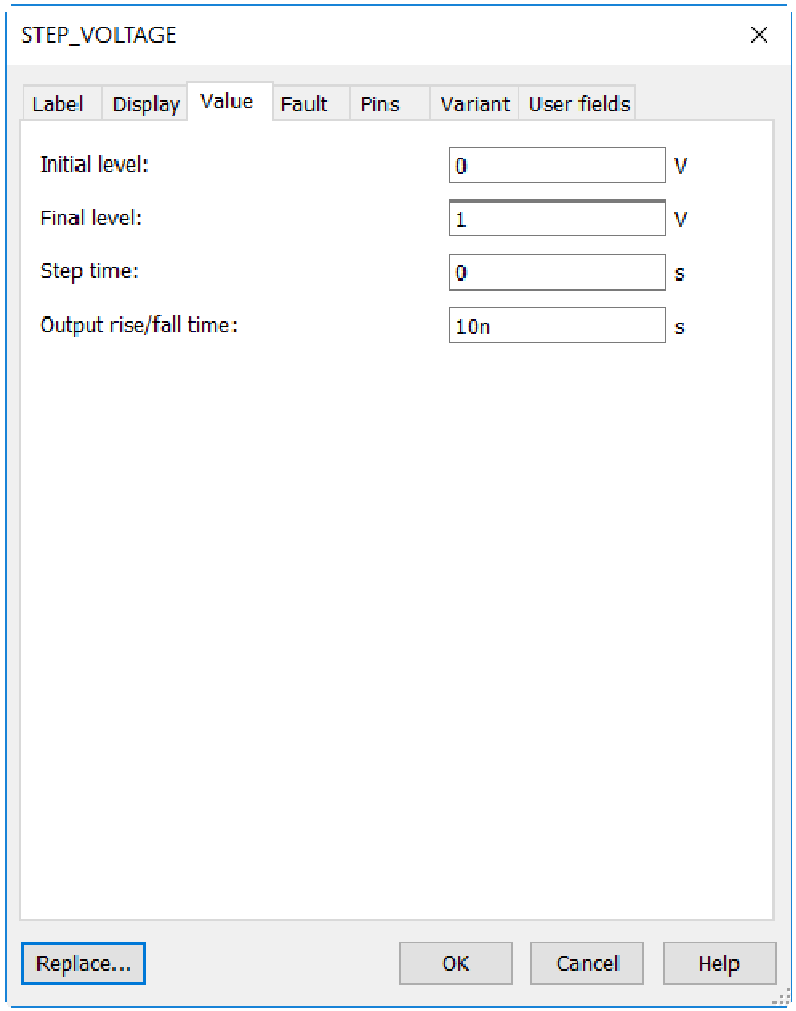
电路仿真结果参照



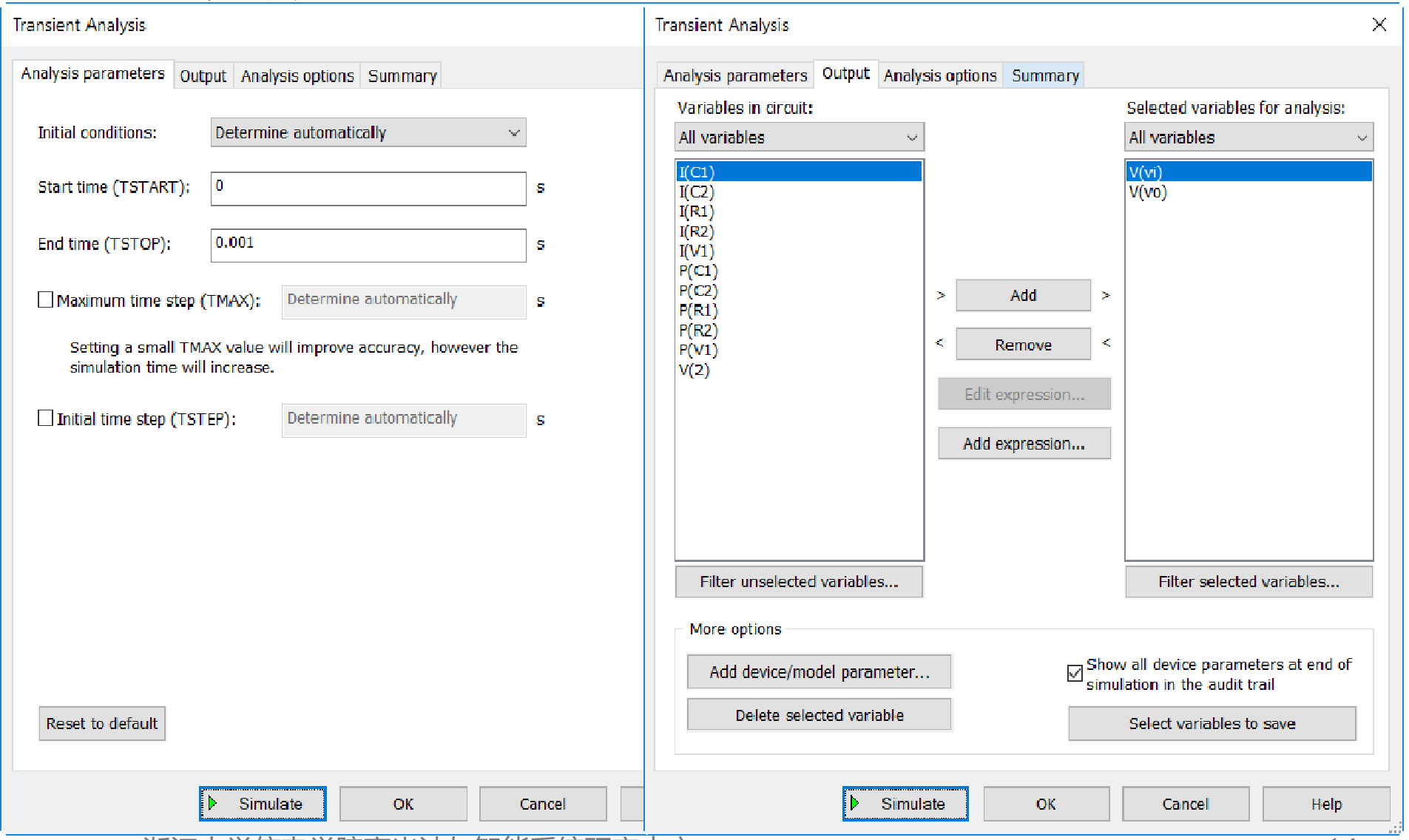
仿真时间：1ms



信号源设置


STEP_VOLTAGE
Label
Display
Value
Fault
Pins
Variant
User fields
Initial level:
0
V
Final level:
1
V
Step time:
0
s
Output rise/fall time:
10n
s
Replace...
OK
Cancel
Help

仿真设置


Transient Analysis
Analysis parameters
Output
Analysis options
Summary
Initial conditions:
Determine automatically
Start time (TSTART):
0
s
End time (TSTOP):
0.001
s
Maximum time step (TMAX):
Determine automatically
s
Setting a small TMAX value will improve accuracy, however the simulation time will increase.
Initial time step (TSTEP):
Determine automatically
s
Reset to default
Simulate
OK
Cancel
Transient Analysis
Analysis parameters
Output
Analysis options
Summary
Variables in circuit:
All variables
I(C1)
I(C2)
I(R1)
I(R2)
I(V1)
P(C1)
P(C2)
P(R1)
P(R2)
P(V1)
V(2)
Add
Remove
Edit expression...
Add expression...
Selected variables for analysis:
All variables
V(vi)
V(vo)
Filter unselected variables...
Filter selected variables...
More options
Add device/model parameter...
Delete selected variable
Show all device parameters at end of simulation in the audit trail
Select variables to save
Simulate
OK
Cancel
Help

小结

- 动态电路微分方程求解
  - 时域微分方程->复频域代数方程
  - 复频域求解
  - 拉普拉斯反变换->时域响应