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7.

(1) $E\left(\frac{1}{n}\sum_{n=1}^{n} X_n^2\right) = E(X_{n1}^2) = \int x^2 \lambda e^{-\lambda x} dx = \frac{2}{\lambda^2}$

(2) $\frac{1}{50}\sum_{i=1}^{100} X_i = \frac{1}{100}\sum_{i=1}^{100} 2X_i \sim N\left(\frac{2}{\lambda}, \frac{1}{25\lambda^2}\right)$.

(3) $E(X_i^2) = \frac{2}{\lambda^2}$, $D(X_i^2) = \frac{24}{\lambda^4}$

$\frac{1}{100}\sum X_i^2 \sim N\left(\frac{2}{\lambda^2}, \frac{24}{100\lambda^4}\right)$.

$\therefore P\left(\frac{1}{100}\sum X_i^2 \leq \frac{2}{\lambda^2}\right) = 0.5$

10.

$X = \begin{cases} 1 & 0.3 \\ 2 & 0.5 \\ 0 & 0.2 \end{cases}$ $\therefore EX = 1.3$

$DX = 0.61$

$\therefore \frac{1}{800}\sum X_i \sim N\left(1.3, \frac{0.61}{800}\right)$.

$\therefore \frac{\frac{1}{800}\sum X_i - 1.3}{\sqrt{\frac{0.61}{800}}} \sim N(0,1)$.

$\therefore P(\sum X_i > 1000) = \Phi(1.81) = 96.48\%$.

11.

(1) $X_i = \begin{cases} 1 & 第i论得0 \\ 0 & 第i论不得0 \end{cases}$ $P(X_i=1)=0.3$

$$P(\sum X_i \le 35) = \Phi\left(\frac{0.35-0.3}{\sqrt{\frac{0.3\times 0.7}{100}}}\right) = 86.21\%.$$

(2)

$P(X_i=0)=0.2$ $P(X_i=1)=0.16$

$P(X_i=2)=0.128$ $P(X_i=4)=0.512$

故 $E(X_i)=2.464$ $D(X_i)=2.793$

$\therefore P(\sum X_i \geq 220) = \cancel{1-} \Phi(1.58) = 94.3\%.$

4.

(1) $\bar{X} \sim (0, \frac{1}{16})$.

(5) $\bar{X} - X_1 \sim (0, \frac{1}{16})$.