6.630 Solution to Problem Set 8

Solution P8.1

$\theta_c = \arcsin^{-1}(n_2/n_1) = 89.7°$

Solution P8.2

(a) The Brewster angle for $\epsilon_t = 9$ is $\theta_B = \tan^{-1}\sqrt{\epsilon_t} = \tan^{-1}\sqrt{9} = 71.57°$.

(b) The dominant portion of the sun glares is TE polarized wave. The polaroid glasses absorb the TE component of the incident light, thus the TM component reaches the eyes after passing through the polaroid glasses.

Solution P8.3

The critical angle $\theta_c = \sin^{-1}\sqrt{\frac{\epsilon_t}{\epsilon_o}}$. The Brewster angle $\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_t}{\epsilon_o}}$. The critical angle θ_c is always larger than the Brewster angle θ_B. It is impossible to have totally transmission and total reflection at the same time.

Solution P8.4

$\theta_B = \tan^{-1} n = \tan^{-1} 1.46 = 55.59°$, $\theta = 90° - \theta_B = 34.41°$.

Part of TE's power and none of TM's power is reflected at the boundary each time. Eventually, only TM is left in the last output. Hence, $\overline{E}$ is parallel to the paper.

Solution P8.5

(a) $\theta_B = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = \tan^{-1}\frac{1}{\sqrt{3}} = 30°$

(b.i) $\overline{k}$ is on the $x - z$ plane, and $\overline{E}\cdot\overline{k} = 0$. We need to find a $\overline{E}$ vector that is on the $x - z$ plane, which is given by the $\overline{E}_t$ and is in the direction of $\hat{z} - \sqrt{3}\hat{x}$. Thus $\overline{k}_t$ is in the direction of $\sqrt{3}\hat{z} + \hat{x}$. From that calculation, we have $\theta_t = \tan^{-1}(\sqrt{3})$, as the angle is measured from the normal of the surface. Thus, $\theta_i = 30°$, $\theta_t = 60°$

(b.ii) To find the polarization, we look at $\overline{E}_t|_{x,z=0}$ and find the magnitude and phase of the polarization vectors.

$$\overline{E}_t|_{x=z=0} = \hat{y}\frac{E_0}{\sqrt{2}}\cos(-\omega t) + E_0\frac{\hat{z} - \hat{x}\sqrt{3}}{2\sqrt{2}}\sin(-\omega t) = \hat{e_1}E_1\cos(-\omega t) + \hat{e_2}E_2\sin(-\omega t)$$

It is seen that E_1 and E_2 has the same magnitude and $\hat{e_1}\times\hat{e_2}$ is in the $\hat{k}$ direction.

At $\omega t = 0$, $\overline{E}_t|_{x,z=0}(\omega t = 0)$ is in the $\hat{e}_1$ direction.

At $\omega t = \pi/2$, $\overline{E}_t|_{x,z=0}(\omega t = \pi/2)$ is in the $-\hat{e}_2$ direction.

So, transmitted field is LHCP.

(b.iii) Reflected field is linearly polarized, because the TM component is totally transmitted already.

(b.iv)

$$E_i^{TE} = E_t^{TE}/T^{TE}$$

$$H_i^{TM} = H_t^{TM}, E_i^{TM} = \frac{\eta_1}{\eta_2}E_t^{TM}$$

$$\overline{E}_i = \hat{y}\frac{2E_0}{3\sqrt{2}}e^{ik_xx+ik_zz} + E_0\left[\frac{\hat{z}}{2\sqrt{2}} - \frac{\hat{x}}{2\sqrt{6}}\right]e^{ik_xx+ik_zz-i\pi/2}$$

$$\overline{E}_r = \hat{y}\frac{E_0}{3\sqrt{2}}e^{-ik_xx+ik_zz}$$

(c.i) $\phi = 0$
(c.ii) constant, maximum value $2E_0$