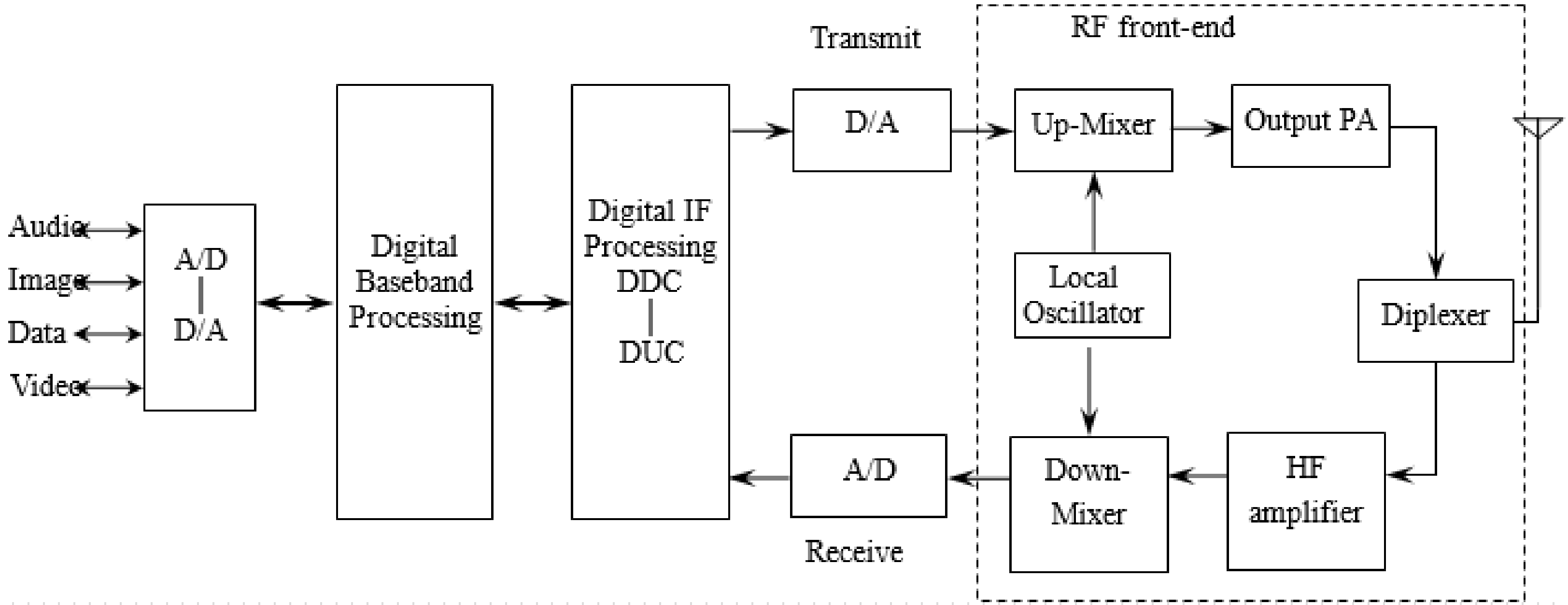
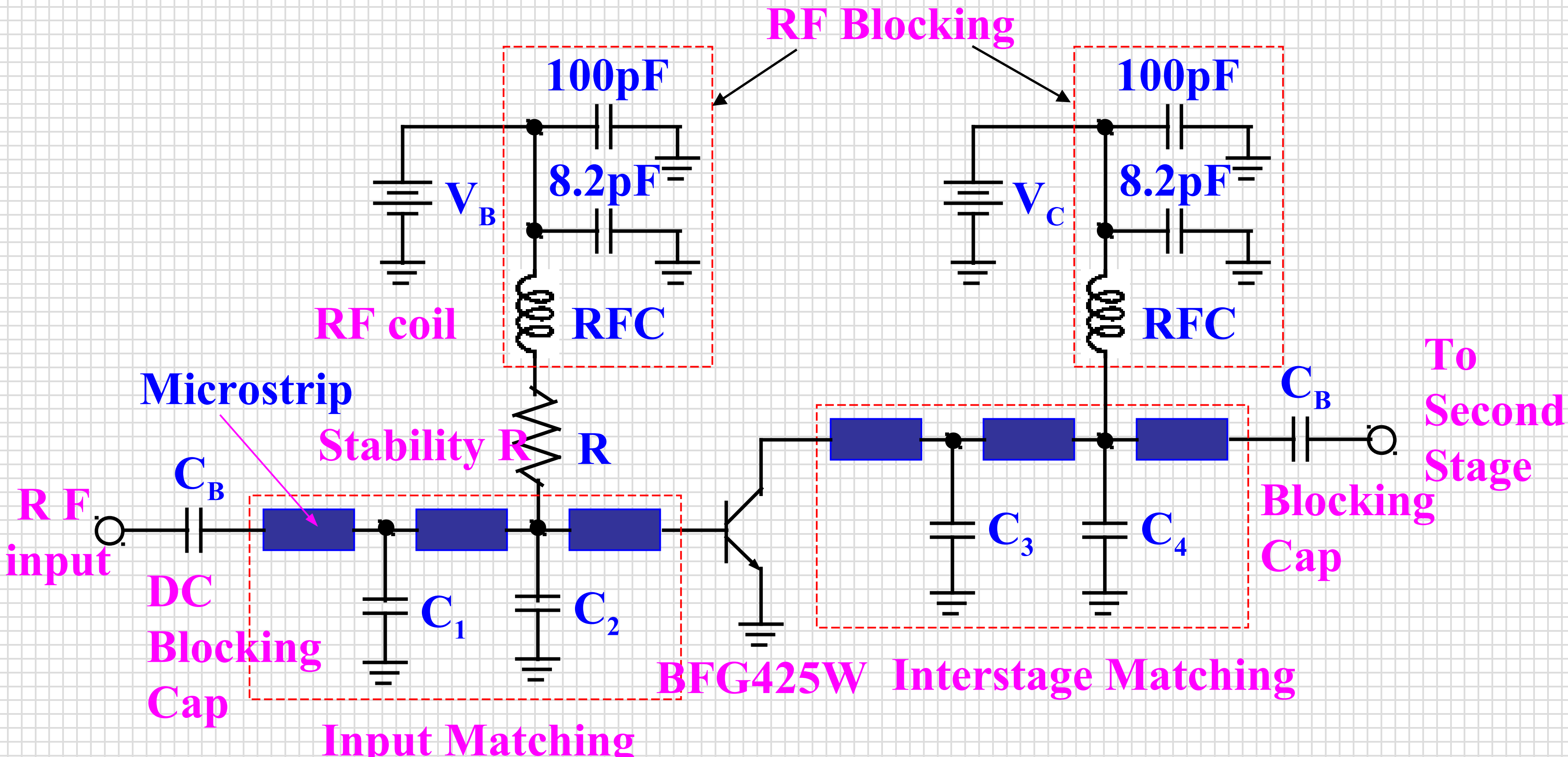
RF/Microwave Circuit and System

Lecture 2

System structure

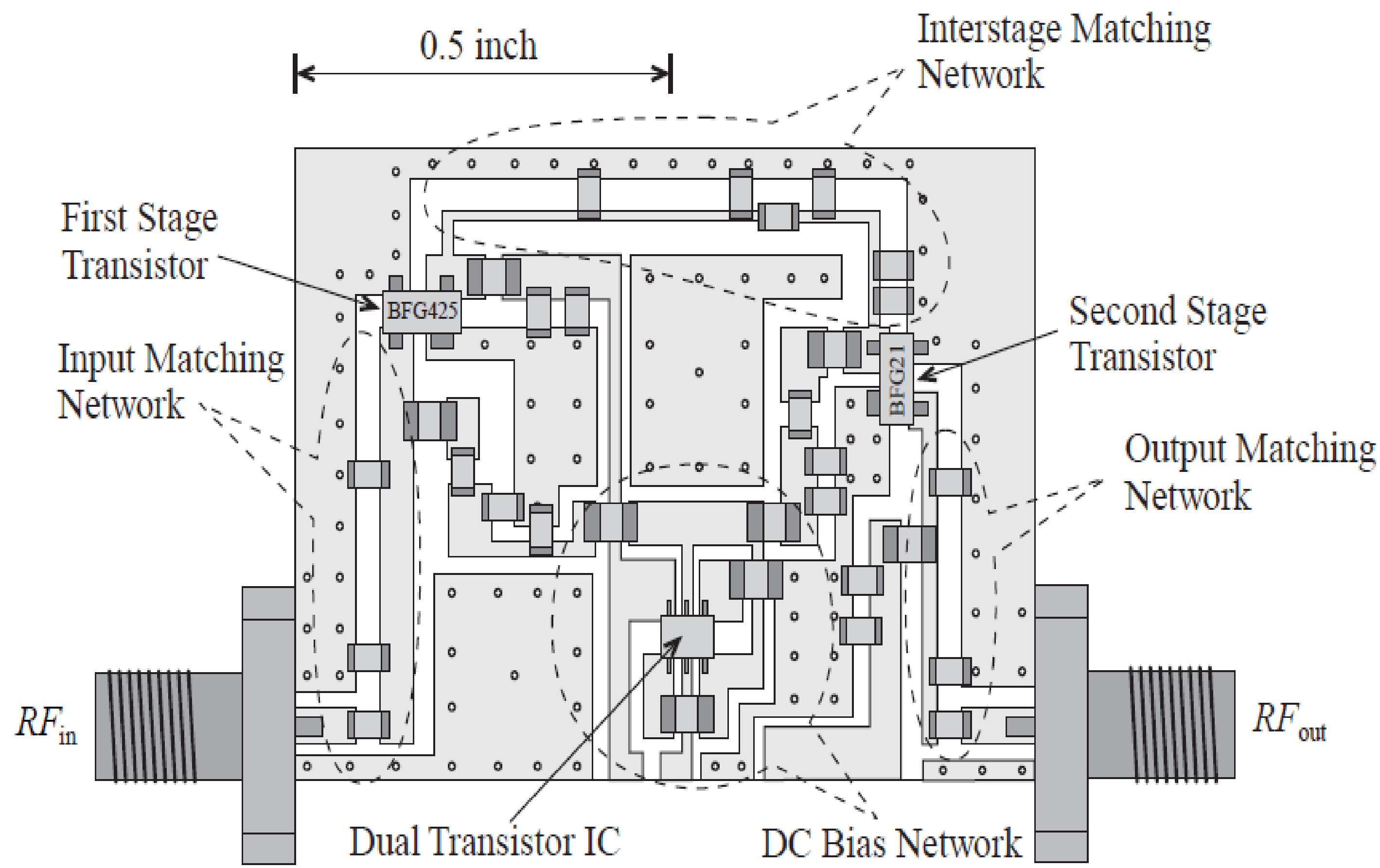


2GHz first stage power amplifier for Mobile phone



The matching is needed to ensure optimal power transfer as well as to eliminate performance degrading reflections;The separation of high-Frequency signals from the DC bias conditions is achieved through two RF blocking networks (RFC)

Printed circuit board layout of the power amplifier.



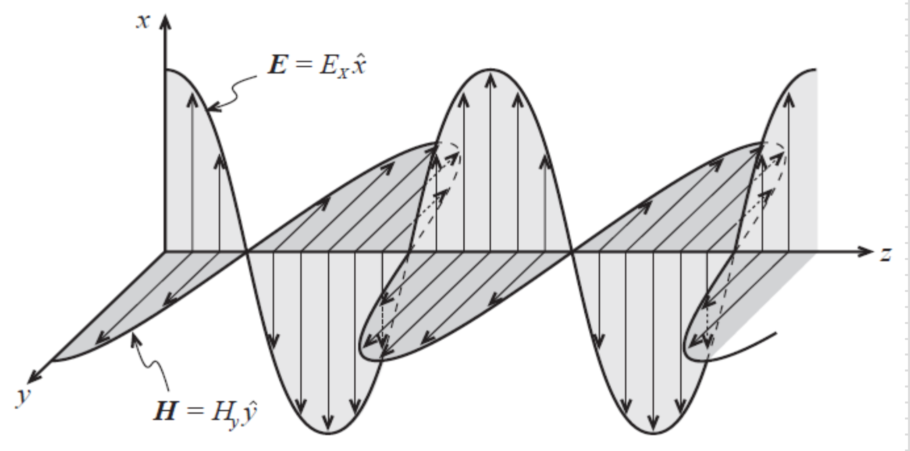
To understand,analyze and ultimately build such a PA requires Knowledge of a number of RF topics

1.2 Dimensions and units

In free space, plane electromagnetic wave progapation in the positive z-direction

$$E_x = E_{0x}\cos(\omega t - \beta z) \ \text{V/m}$$ x-directed electric field vector

$$H_y = H_{0y}\cos(\omega t - \beta z) \ \text{A/m}$$ y-directed magnetic field vector



eature of plane wave：

Transverse electromagnetic mode，

and H are orthogonal to the direction of propagation

E and H are orthogonal to each other and same phase

Based on Maxwell， intrinsic impedance Z_0

$$Z_0 = E_x / H_y = \sqrt{\mu / \varepsilon} = \sqrt{(\mu_0 \mu_r)/(\varepsilon_0 \varepsilon)} = 377\Omega \sqrt{\mu_r / \varepsilon_r}$$

Permeability μ and permittivity ε based on material，

$\mu_0 = 4\pi \times 10^{-7}$(H/m)， $\varepsilon_0 = 8.85 \times 10^{-12}$ (F/m)， μ_r and ε_r are relative values.

In the direction of wave propagation, the change of the phase Kz of the unit distance is called the phase constant (propagation constant).

$$\beta = k = \omega \sqrt{\mu\varepsilon}$$

Wavelength is the distance when kz changes 2πphase $\lambda = 2\pi / \beta$

Phase velocity is the speed of phase plane of sine wave 。

$$\because \quad \omega t - \beta z = consant, so\ \omega dt - \beta dz = 0$$

∴phase velocity of TEM：

$$v_p = \frac{dz}{dt} = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}} \text{m/s} \qquad (1.3)$$

Transverse electromagnetic mode

Example 1.1 compute the intrinsic impedance ,phase velocity, and Wavelengths of an electromagnetic wave in free space for

$f = 30\text{MHz}$， 300MHz， 30GHz

Solution： **Relative permeability and permittivity of free space are equal to unity**

Intrinsic impedance： $$Z_0 = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi\times10^{-7}}{8.85\times10^{-12}}} = 377\Omega$$

Phase velocity： $$v_p = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\times10^8\ m/s$$

wavelength： $$\lambda = \frac{2\pi}{\beta} = \frac{2\pi v_p}{\omega} = \frac{v_p}{f} = \begin{cases} \textbf{10 m} \\ \textbf{1 m} \\ \textbf{1 cm} \end{cases}$$

1.4 RF Behavior of Passive Components

Conventional circuit， *R* is *f* independent， $X_C = \frac{1}{\omega C}$ ， $X_L = \omega L$ 。

resistances,inductances,capacitances are not only created by wires,coils and plates, even a single straight wire or a copper segment of a PCB prossesses frequency dependent R and L 。 **DC resistance of a conductor：**

$$R_{DC} = l/\left(\pi a^2 \sigma_{cond}\right)$$

DC signal,the entire conductor cross-sectional area is utilized. At AC, the alternating charge carrier flow establishes a magnetic field that induces an electric field whose associated current density opposes the initial current flow.The effect is strongest at the center, the result is a current flow that tends to reside at the outer perimeter with increasing frequency ——Skin effect

Z-directed current density：

$$J_z = \left[pIJ_0(pr)\right]/\left[2\pi a J_1(pa)\right]$$

where $p^2 = -j\omega\mu\sigma_{cond}$, J_0, J_1 **are bessel functions of zeroth and first order, and I is the total curren flow**

in high freq ($f \geq$500MHz)，

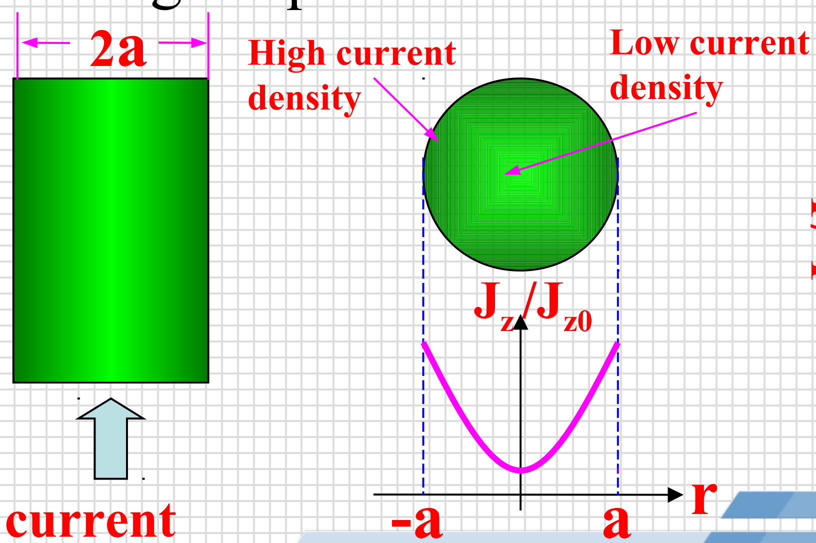
Normalized R： $R/R_{DC} \cong a/2\delta$

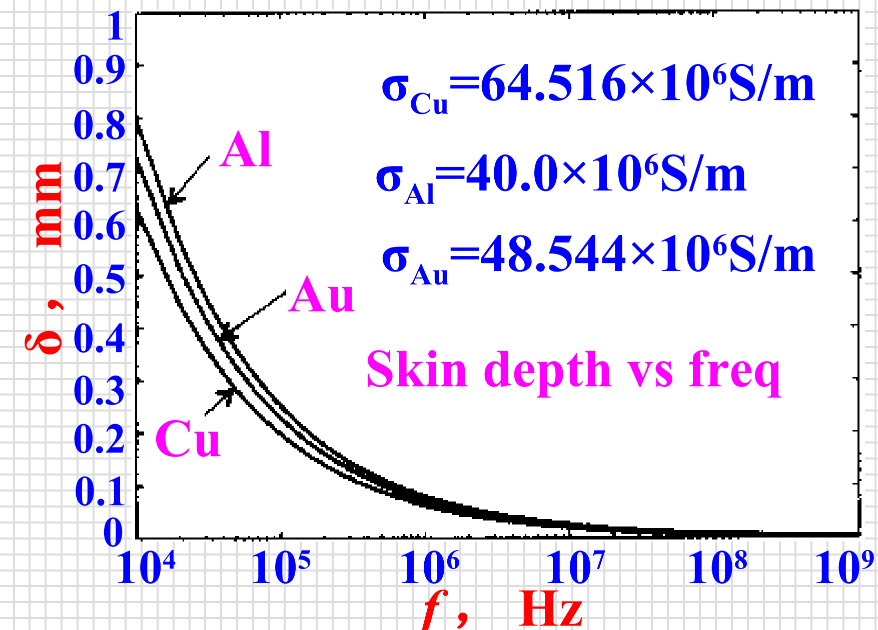
Normalized L： $\omega L/R_{DC} \cong a/2\delta$

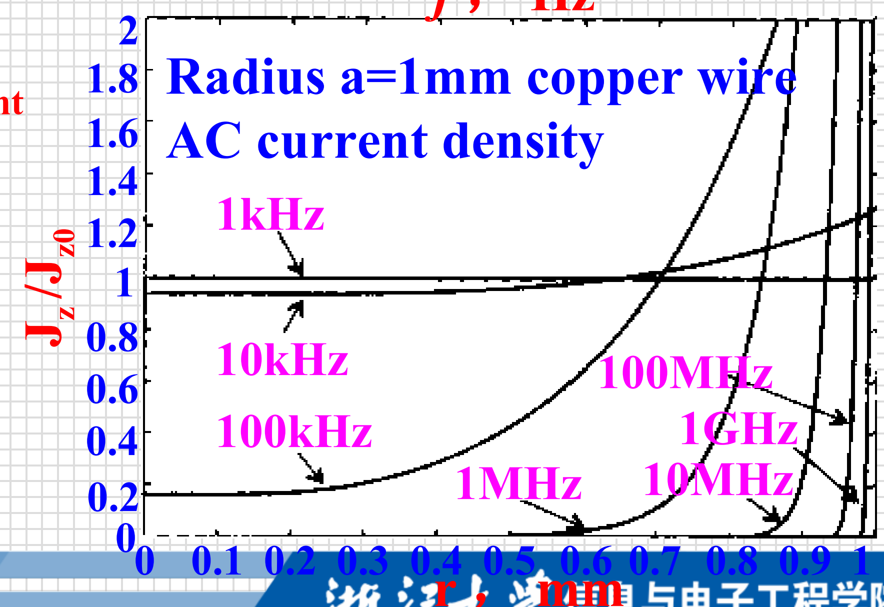
Skin depth： $\delta=(\pi f\mu\sigma_{cond})^{-1/2}$

Most case, for conductor $\mu_r=1$，

Skin depth rapidly decreases for Increasing frequencies







American wire gauge， diameter doubles every six wire gauges

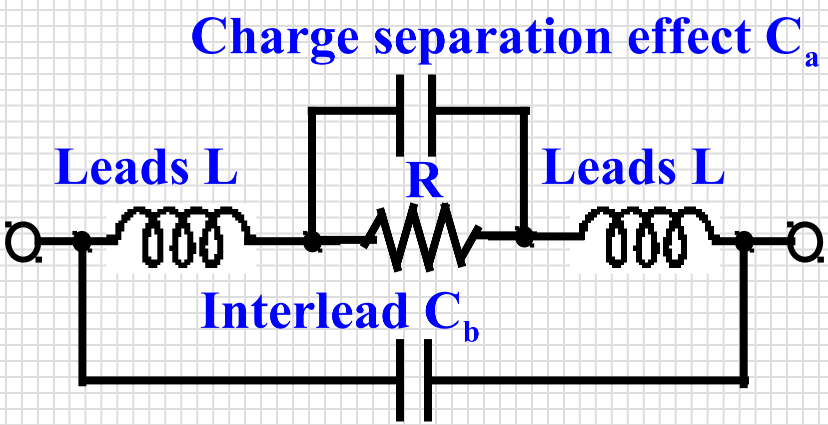
AWG50： d=1mil， AWG44： d=2mil， AWG38： d=4mil， ··

where： $1\text{mil}=2.54\times10^{-5}\text{m}=2.54\times10^{-2}\text{mm}$

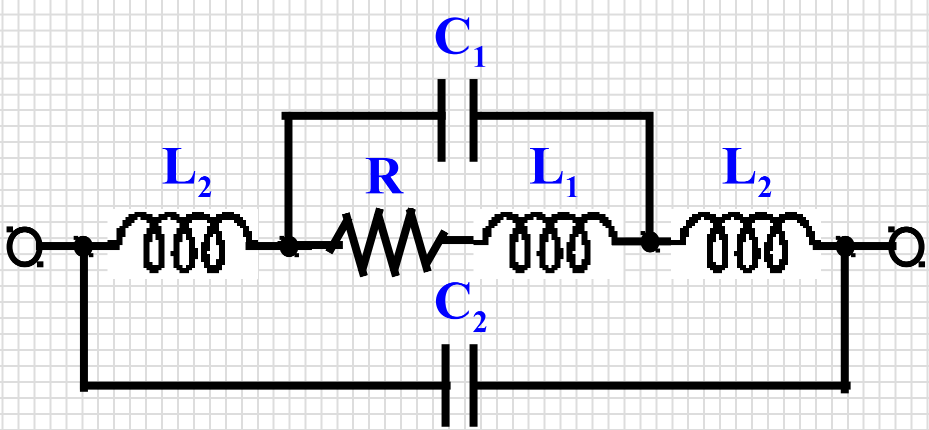
1.4.1 High-frequency Resistors

In RF and MW circuits, thin-film chip resistors，

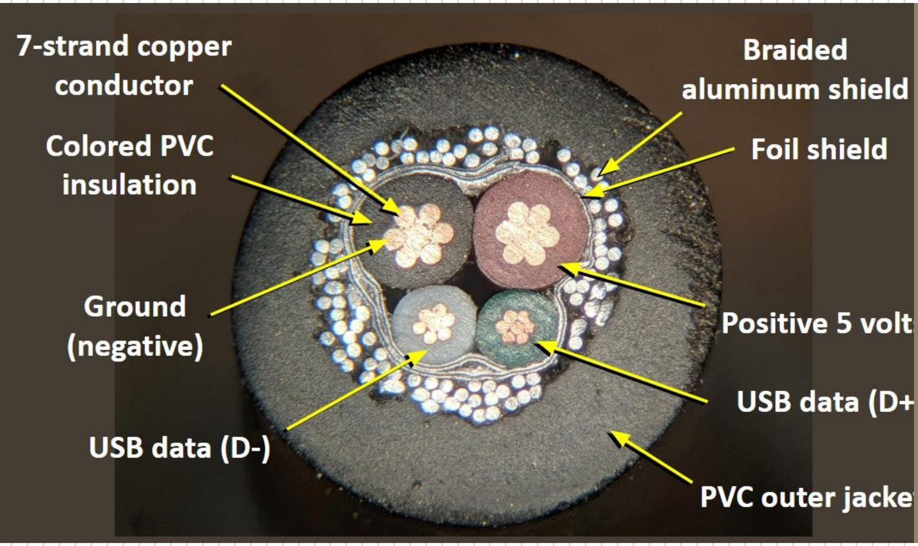
Equivalent circuit：

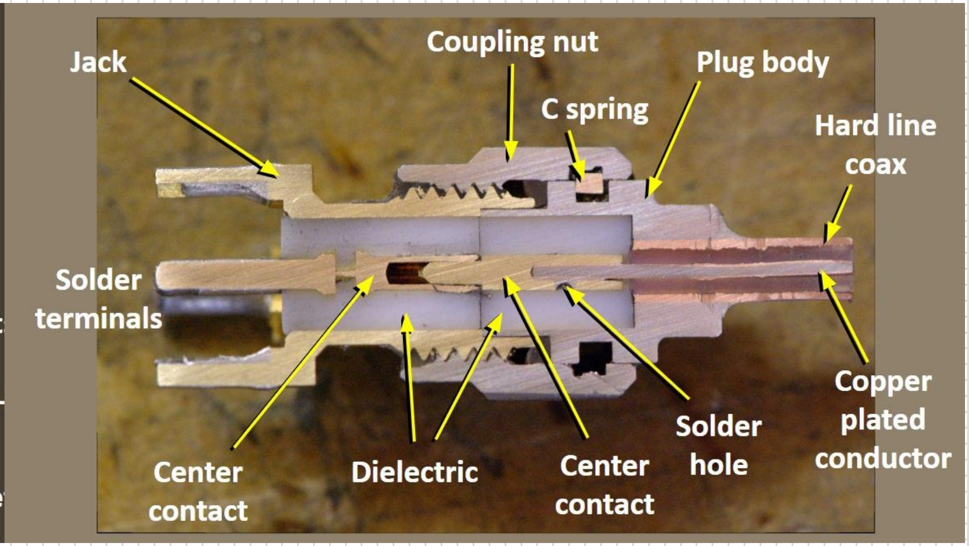


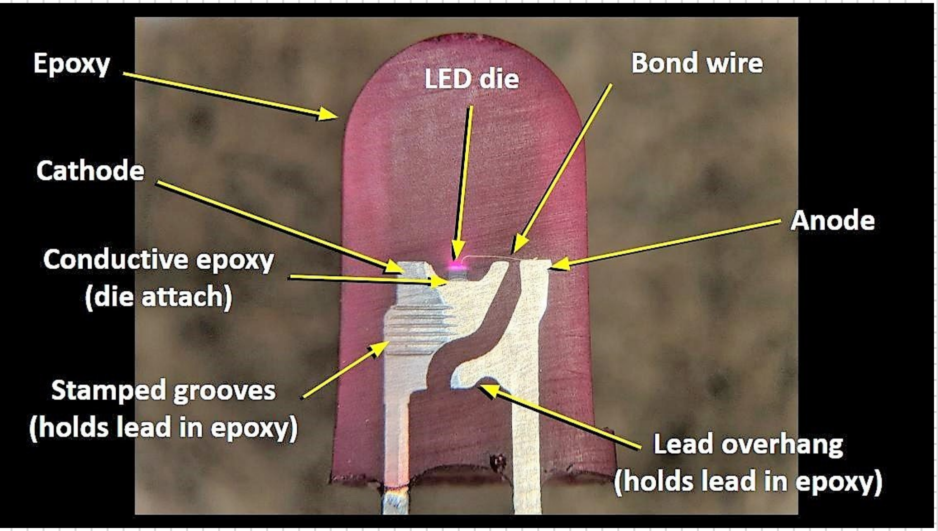
Equivalent circuit of the resistor

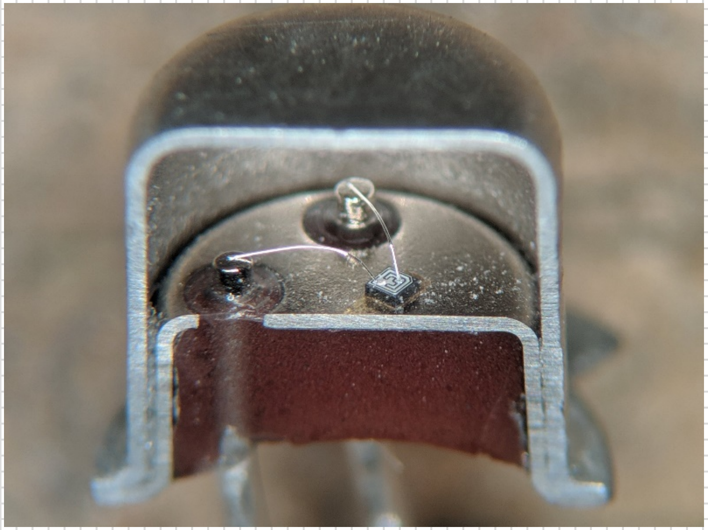


Equivalent circuit of the wire-wound resistor


7-strand copper conductor
Braided aluminum shield
Colored PVC insulation
Foil shield
Ground (negative)
Positive 5 volt
USB data (D+
USB data (D-)
PVC outer jacke


Jack
Coupling nut
Plug body
C spring
Hard line coax
Solder terminals
Copper plated conductor
Solder hole
Center contact
Dielectric
Center contact


Epoxy
LED die
Bond wire
Cathode
Anode
Conductive epoxy (die attach)
Stamped grooves (holds lead in epoxy)
Lead overhang (holds lead in epoxy)



Example 1.3 find the high frequency impedance behavior of a 500Ω Metal film resistor with 2.5cm copper wire connections of AWG 26 And a stray capacitance C_a=5pF 。

Solution： AWG26 d=16mil， $a = 8\times2.54\times10^{-5}m = 0.2032mm$

from 1.10 and 1.11，

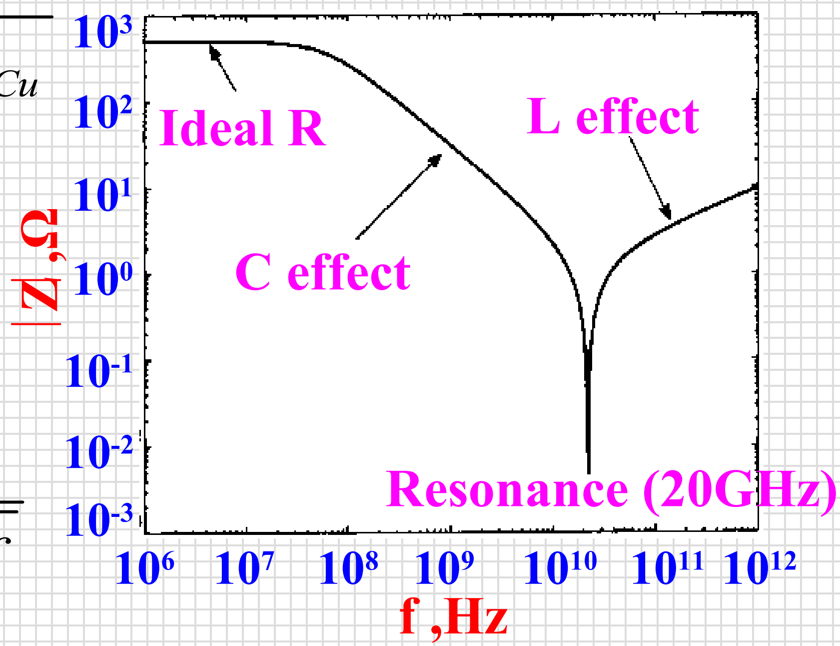
$$L = \frac{aR_{DC}}{2\omega\delta_{Cu}} = \frac{a}{4\pi f}\frac{2l}{\pi a^2\sigma_{Cu}}\sqrt{\pi f\mu_0\sigma_{Cu}}$$

$$= \frac{0.25\sqrt{\mu_0}}{2a\pi\sqrt{\pi f\sigma_{Cu}}}$$

$$= \frac{0.125\sqrt{4\pi\times10^{-7}}}{2.032\times10^{-4}\pi\sqrt{64.516\times10^{6}\pi f}}$$

$$= \frac{1.54}{\sqrt{f}}\mu H$$

$$Z = j\omega L + \frac{1}{j\omega C + 1/R}$$



1.4.2 High-frequency Capacitors

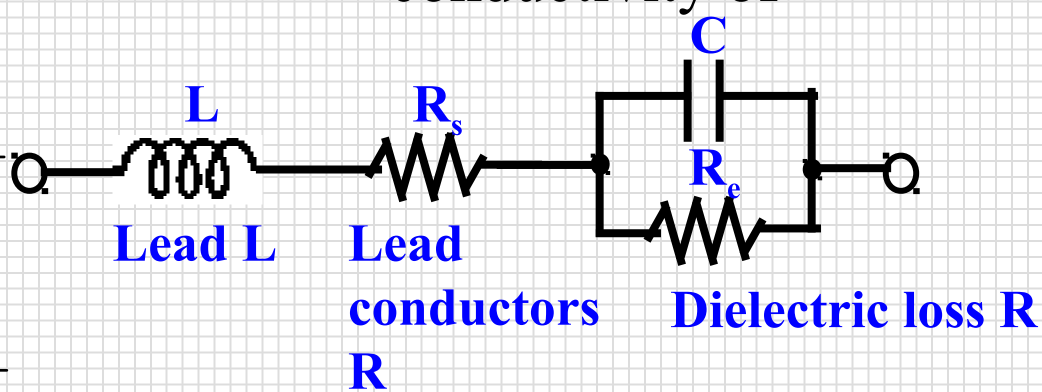
In circuit analysis defines capacitance for a parallel plate with plate surface by the plate separation : $C = \dfrac{\varepsilon A}{d}$

Ideally no current, at high freq, the dielectric material become lossy

Impedance of C : $Z = \dfrac{1}{G_e + j\omega C}$

where : $G_e = \dfrac{\sigma_{diel} A}{d}$, σ_{diel} conductivity of dielectric, Series connection

series loss tangent $\tan\Delta_s = \dfrac{\omega\varepsilon}{\sigma_{diel}}$



Then : $G_e = \dfrac{\omega\varepsilon A}{d\tan\Delta_s} = \dfrac{\omega C}{\tan\Delta_s}$

Equivalent circuit for capacitor

Examples 1.4 Compute the high frequency impedance of a 47pF Capacitor whose dielectric medium consists of an aluminum oxide possessing a series loss tangent of 10^{-4} and whose leads are 1.25cm AWG26 copper wires

Solution： similar example 1.3， lead inductance：

$$L=\frac{aR_{DC}}{2\delta\omega}=\frac{2l}{4\pi a}\sqrt{\frac{\mu_0}{\pi\sigma_{Cu}f}}=\frac{771}{\sqrt{f}}nH$$

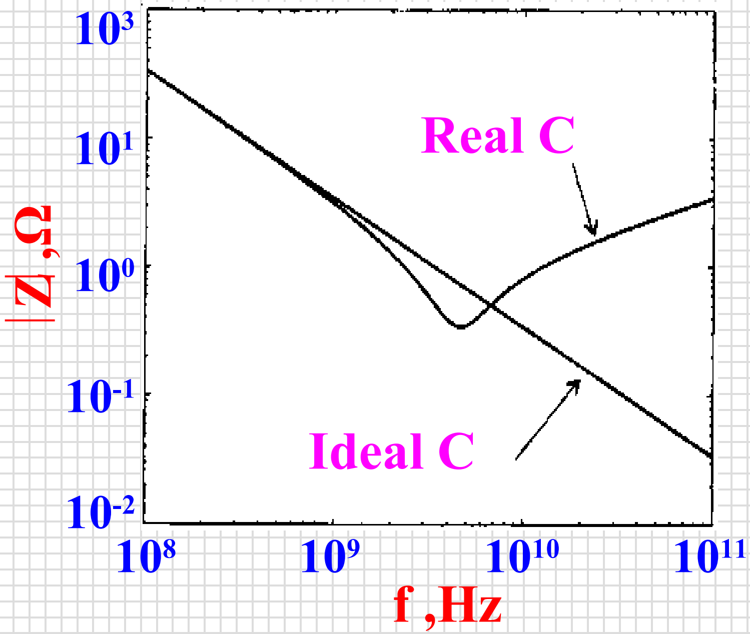
From 1.13 式， leads R：

$$R_s=\frac{aR_{DC}}{2\delta}=\frac{l}{a}\sqrt{\frac{\mu_0 f}{\pi\sigma_{Cu}}}=4.8\sqrt{f}\,\mu\Omega$$

From 1.16 式， leakage R：

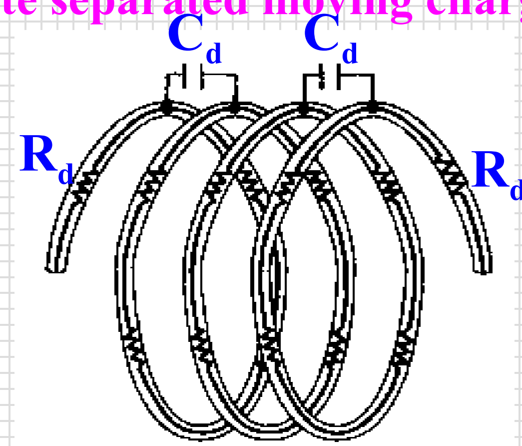
$$R_e=\frac{1}{G_e}=\frac{\tan\Delta_s}{2\pi fC}=\frac{33.9\times10^6}{f}M\Omega$$

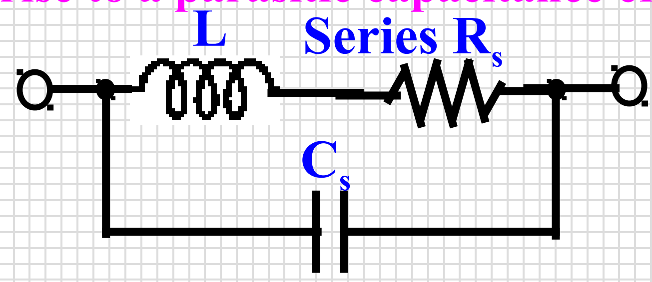
$$Z=j\omega L+R_s+\frac{1}{j\omega C+1/R_e}$$



1.4.3 High-frequency Inductors

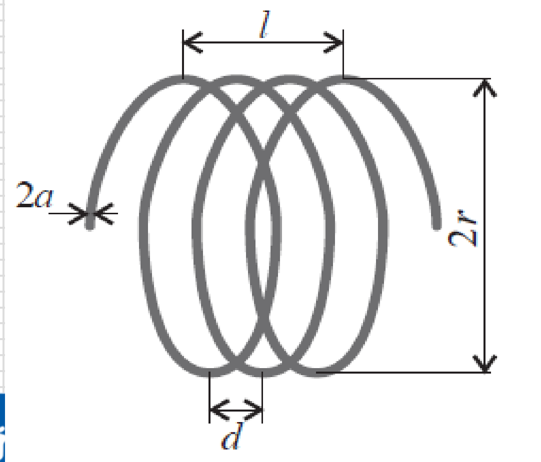
Coil is formed by winding a wire on a cylindrical former， represent an inductance in addition to the frequency-dependent wire resistance. Adjacently positioned wires constitute separated moving charges, giving rise to a parasitic capacitance effect.





Equivalent circuit inductor

Example 1.5 estimate the frequency response of an RFC formed by 3.5 turns of AWG36 copper wire on a 0.1inch Air core. assume that the length of the coil is 0.05 inch



Solution： Table A.4： AWG36 a = 2.5mil=63.5μm

radius： r = 50mil=1.27mm (1inch=1000 mil)

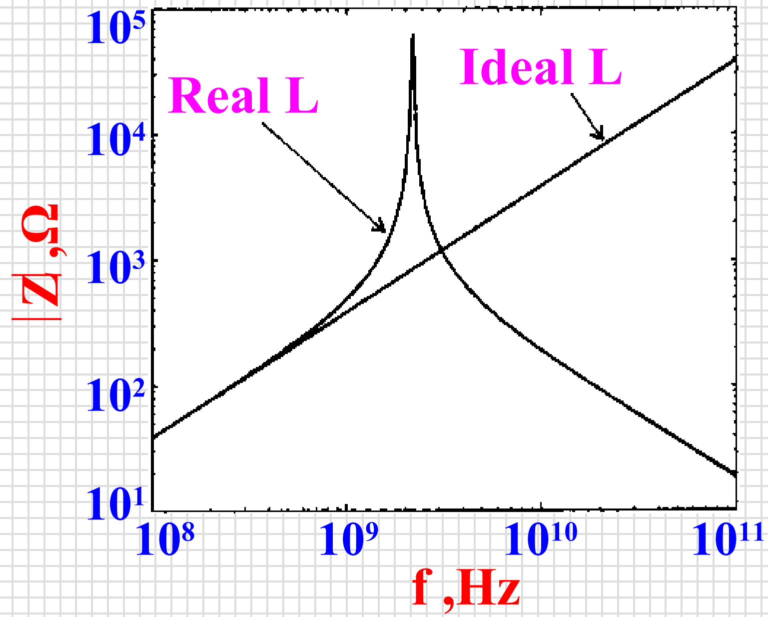
Coil length： l =50mil=1.27mm

Distance between two turns：

d= l/N≈3.6×10^{-4}m

Air core solenoid inductance：

$$L = \frac{\pi r^2 \mu_0 N^2}{l} = 61.4nH$$



from parallel-plate capacitor 1.14，

area A=2a l_{wire}(=2πrN length of wire)，

So equivalent C：$C_s = \frac{\varepsilon A}{d} = \frac{\varepsilon_0 \cdot 2\pi r N \cdot 2a}{l / N} = 4\pi\varepsilon_0 \frac{raN^2}{l} = 0.087\,pF$

Neglect skin effect, series resistance：$R_s = \frac{l_{wire}}{\sigma_{Cu}\pi a^2} = \frac{2\pi r N}{\sigma_{Cu}\pi a^2} = 0.034\Omega$

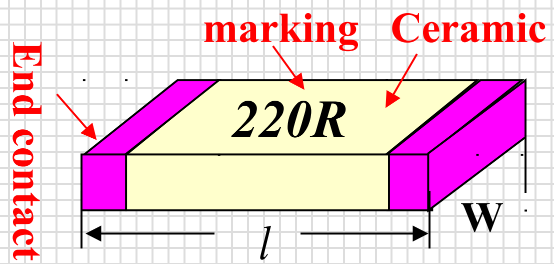
RFC find widespread use for biasing RF circuits， as tuning elements，

Quality factor： $Q = X / R_s$

Easy welding

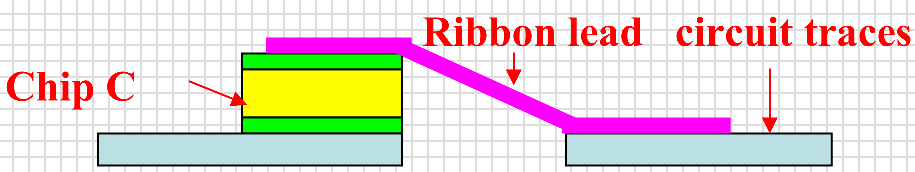
1.5 Chip components and circuit board considerations

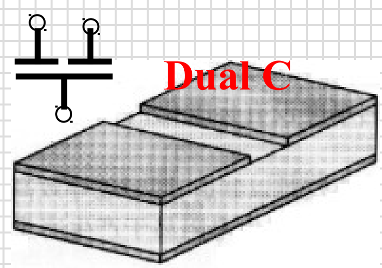
1.5.1 chip resistors

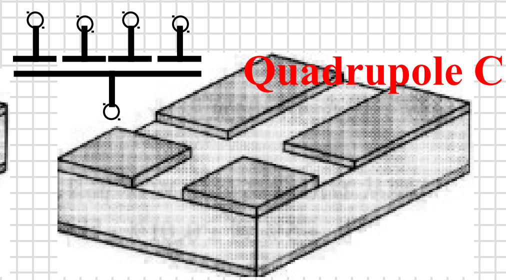


geometry	Size code	Length(l)， mil	width(w)， mil
	0402	40	20
	0603	60	30
	0805	80	50
	1206	120	60
	1218	120	180

1.5.2 chip capacitors



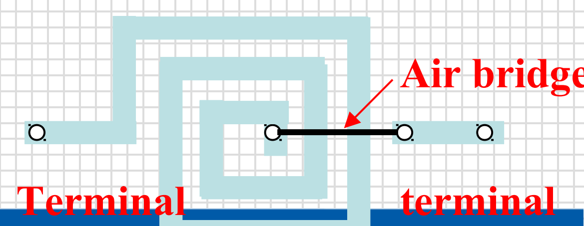




1.5.3 surface-mounted Inductors

Still the wire-wound coil

Flat inductors can be integrated with microstrip



Summary

Electromagnetic wave nature begins to dominate over Kirch-hoff's current and voltage laws

Issues constant： $\beta = 2\pi/\lambda, \quad v_p = \omega/\beta = 1/\sqrt{\varepsilon\mu}$

Consequence of the electromagnetic wave nature is skin effect： $\delta = 1/\sqrt{\pi f \mu \sigma}$

Cylindrical lead wire： $R \approx R_{DC}\dfrac{a}{2\delta}, \quad X = \omega L \approx R_{DC}\dfrac{a}{2\delta}$

These wires,in conjunction with the respective R C and L elements, form eletric equivalent circuits whose performance deviate from the ideal element behavior

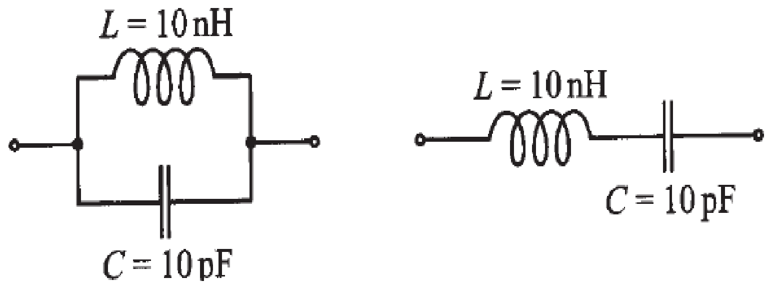
The physical dimensions of R C L are small ASAP， when the wavelength is comparable in size with the discrete electronic components, basic circuit analysis no longer applies

Exercise 1

1.1 Compute the phase velocity and wavelength in an FR4 printed circuit board whose relative dielectric constant is 4.6 and where the operational frequency is 1.92 GHz.

1.3 A coaxial cable that is assumed lossless has a wavelength of the electric and magnetic fields of $\lambda = 20$ cm at 960 MHz. Find the relative dielectric constant of the insulation

1.5 Find the frequency response of the impedance magnitude of the following series and parallel *LC* circuits:



1.10 The leads of a resistor in an RF circuit are treated as straight aluminum wires ($\sigma_{Al} = 4.0 \times 10^7$ S/m) of AWG size 14 and of total length of 5 cm. (a) Compute the DC resistance. (b) Find the AC resistance and inductance at 100 MHz, 1 GHz, and 10 GHz operating frequencies.

Chapter2 Transmission Line

Higher frequencies imply decreasing wavelengths， Voltages and currents no longer remain spatially uniform when compared to the geometric size of the discrete circuit elements， they have to betreated as propagating waves

2.1 why trasmission line theory

assume the wave is confined to a conducting medium,

aligned with z-axis， E-field has a longitudinal component E_z

， give a voltage drop : $V = -\int E_z dl_z$

$E_x = E_{0x}\cos(\omega t - \beta z)$ in free space

Space characterized λ=2πβ/

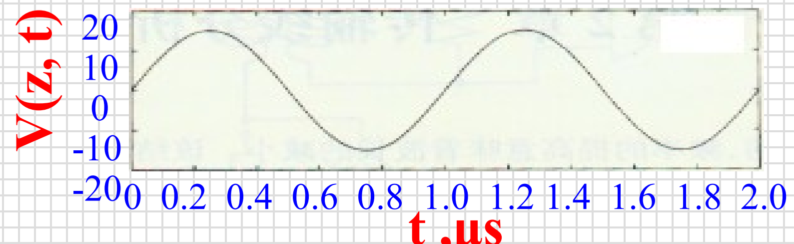
Time period T=1/f

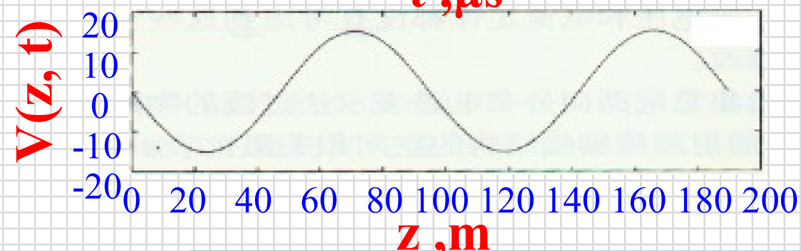
assume $f = 1MHz, \varepsilon_r = 10, \mu_r = 1$

From 2.1， $v_p = \frac{\omega}{\beta} = \lambda f = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$

λ=94.86m， for voltage wave :

$$V = -\int \cos(\omega t - \beta z)dz = \sin(\omega t - \beta z)/\beta$$





Spatially and temporally depicted

wire along z-zxis, length 1.5cm

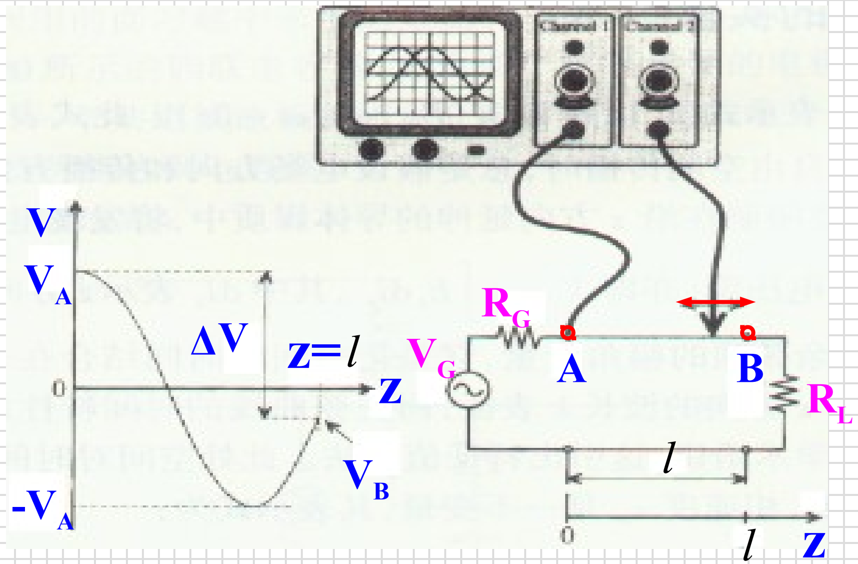
resistance is negligible，

f=1MHz spatial voltage variations

insignificant。

f =10GHz ,λ =0.949cm， same

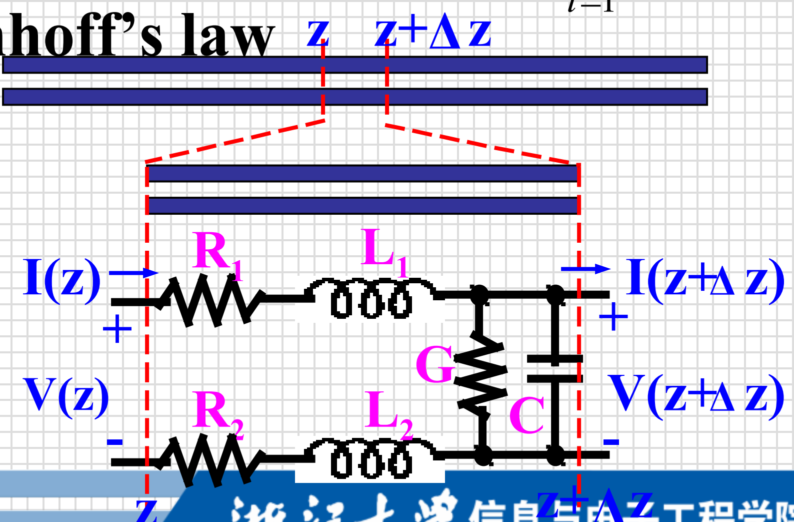
length， result as figure



Low-frequency,resistance negligible,

no patial voltage variation, use Kirchhoff's law

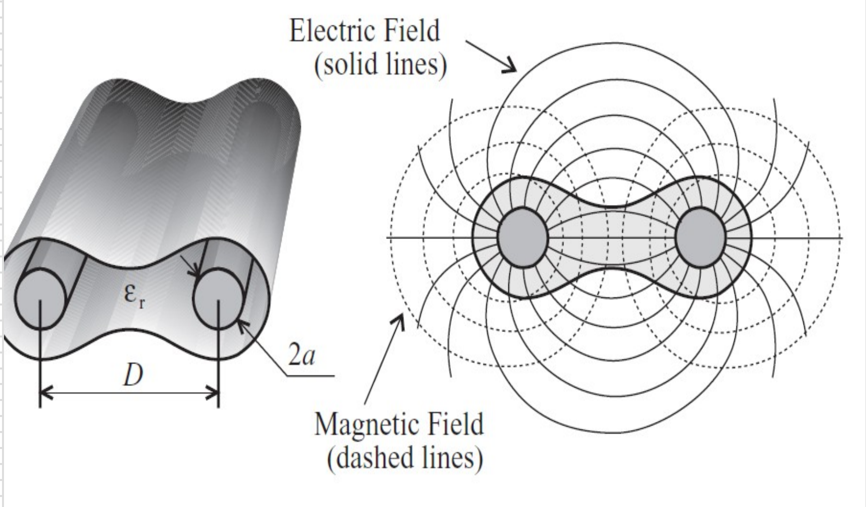
$$\sum_{i=1}^{N} V_i = 0$$



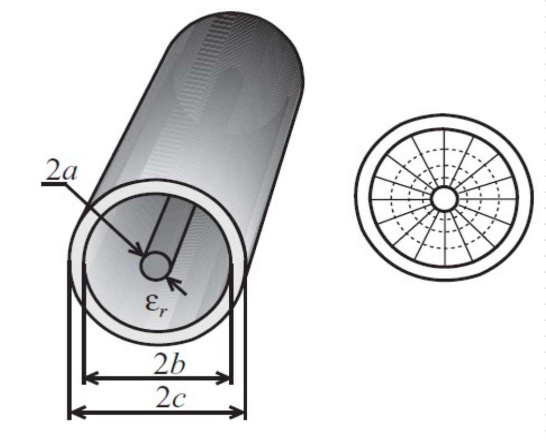
High frequency, spatial behavior of Voltage and current， distributed Parameters R L C G (size more than a tenth of wavelength)。

2.2 Transmission line

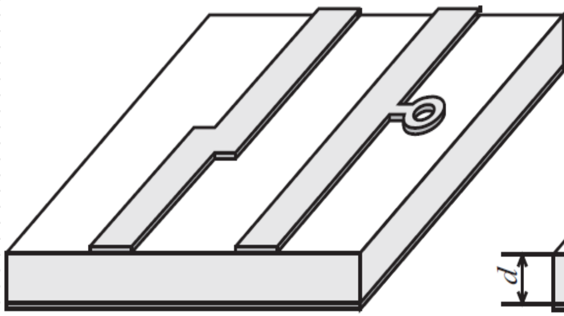
2.2.1 two-wire lines

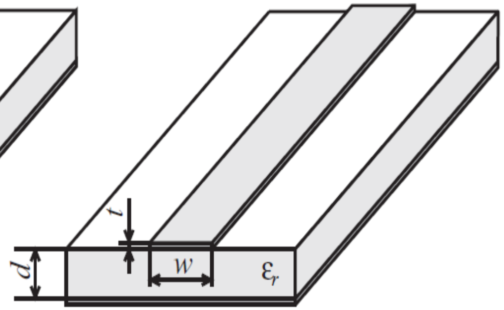


2.2.2 coaxial line

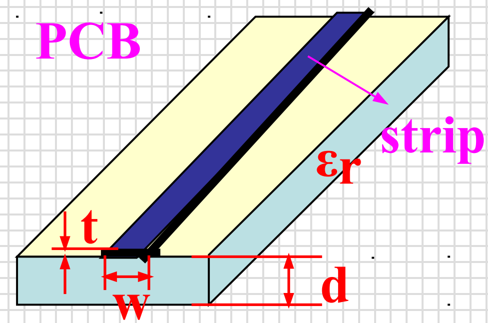


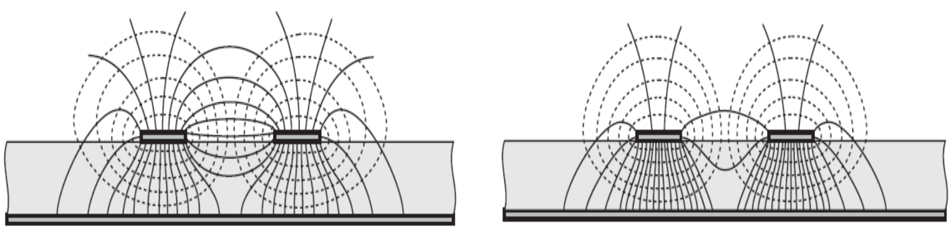
2.2.3 Microstrip

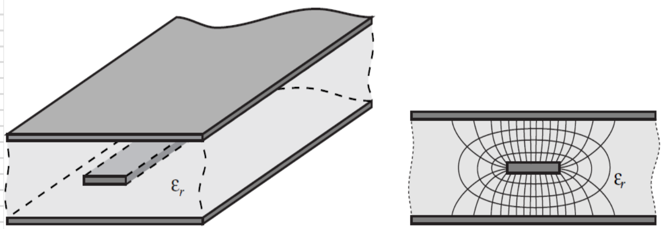




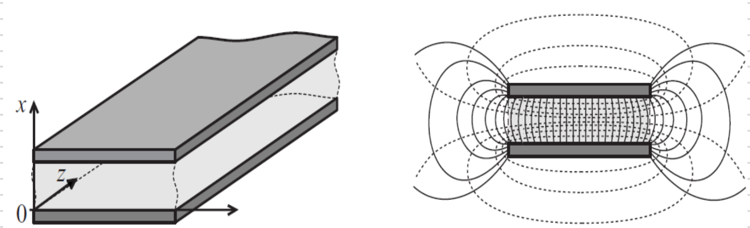
(a) Printed circuit board section (b) Microstrip line




(a) Teflon epoxy (ε_r = 2.55) (b) Alumina (ε_r = 10.0)



(a) Sandwich structure (ε_r = 2.55) (b) Cross-sectional field distribution



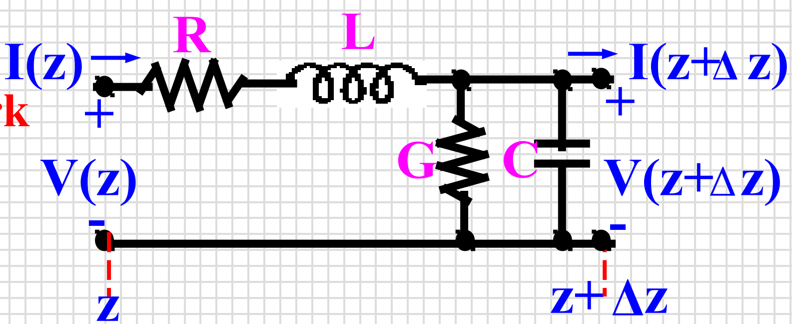
(a) Geometric representation (b) Field distribution (ε_r = 2.55)

2.3 equivalent circuit representation

On the geometric scale of RF circuit， voltage and currents are no longer spatially constant，Kirchhoff's laws cannot be applied over the macroscopic line Dimension. However, transmission line can be broken down into smaller segments， They are still large enough to contain all relevant electric characteristics, loss, inductive, capacitive effect. general equivalent circuit is shown in figure:

Advantage：

- **Porvides a clear intuitive physical picture**
- **lends itself to a standardized tow-port network representation**
- **permits the analysis with Kirchhoff's laws**
- **provied building blocks that allow the expansion from microscopic to macroscopic forms**



Disadvantage：

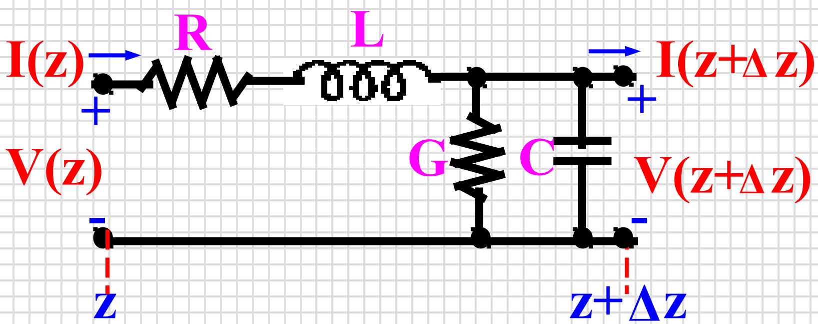
- **It is basically a one-dimensional analysis that does not take into account filed fringing in the plane orthogonal to the direction of propagation and therefore can not predict interference with other components of the circuit；**
- **Material-related nonlinearities due to hysteresis effects are neglected**

2.6 Different line configurations

Parameter	Two-Wire Line	Coaxial Line	Parallel-Plate Line	Unit
R	$\dfrac{1}{\pi a \sigma_{\text{cond}} \delta}$	$\dfrac{1}{2\pi \sigma_{\text{cond}} \delta}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$	$\dfrac{2}{w \sigma_{\text{cond}} \delta}$	Ω/m
L	$\dfrac{\mu}{\pi} \operatorname{acosh}\left(\dfrac{D}{2a}\right)$	$\dfrac{\mu}{2\pi} \ln\left(\dfrac{b}{a}\right)$	$\mu \dfrac{d}{w}$	H/m
G	$\dfrac{\pi \sigma_{\text{diel}}}{\operatorname{acosh}(D/(2a))}$	$\dfrac{2\pi \sigma_{\text{diel}}}{\ln(b/a)}$	$\sigma_{\text{diel}} \dfrac{w}{d}$	S/m
C	$\dfrac{\pi \varepsilon}{\operatorname{acosh}(D/(2a))}$	$\dfrac{2\pi \varepsilon}{\ln(b/a)}$	$\varepsilon \dfrac{w}{d}$	F/m

2.7 Transmission line equation

2.7.1 Kirchhoff voltage and current law representations



Use KVL : $(R+j\omega L)I(z)\Delta z+V(z+\Delta z)=V(z)$ （2.26）

Differential : $\lim\limits_{\Delta z\to 0}\left(-\dfrac{V(z+\Delta z)-V(z)}{\Delta z}\right)=-\dfrac{dV(z)}{dz}=(R+j\omega L)I(z)$ （2.28）

From KCL : $I(z)-V(z+\Delta z)(G+j\omega C)\Delta z=I(z+\Delta z)$ （2.29）

Differential : $\lim\limits_{\Delta z\to 0}\dfrac{I(z+\Delta z)-I(z)}{\Delta z}=\dfrac{dI(z)}{dz}=-(G+j\omega C)V(z)$ （2.30）

2.7.2 Traveling voltage and current waves

$$-\frac{dV(z)}{dz}=(R+j\omega L)I(z) \qquad \frac{dI(z)}{dz}=-(G+j\omega C)V(z)$$ differentiating

Obtain: $\frac{d^2V(z)}{dz^2}-k^2V(z)=0 \qquad \frac{d^2I(z)}{dz^2}-k^2I(z)=0$

where $k=k_r+jk_i=\sqrt{(R+j\omega L)(G+j\omega C)}$ complex propagation constant （2.32）

Solution $V(z)=V^+e^{-kz}+V^-e^{kz}, \quad I(z)=I^+e^{-kz}+I^-e^{kz}$

+ +z-direction

- -z-direction

2.7.3 General Impedance definition

substituted $V(z)=V^+e^{-kz}+V^-e^{kz}$ into $-\frac{dV(z)}{dz}=(R+j\omega L)I(z)$ differentiationg

: $I(z)=k\left(V^+e^{-kz}-V^-e^{kz}\right)/(R+j\omega L)$

Characteristic impedance :

$$\boxed{Z_0=\frac{R+j\omega L}{k}=\sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$ （2.37）

$$I(z) = \frac{k}{R + j\omega L}\left(V^{+}e^{-kz} - V^{-}e^{kz}\right) = I^{+}e^{-kz} + I^{-}e^{kz}$$ **so :** $$Z_0 = \frac{V^{+}}{I^{+}} = \frac{V^{-}}{I^{-}}$$

2.7.4 Lossless transmission line model

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{d}{w} \qquad (2.41)$$

lossless： R=G=0

L and C given in Table 2.1

2.8 Microstrip transmission lines

Substrate thickness increases or conductor width decreases, fringing fields become more prominent and cannot be ignored in the mathematical models. Aliens have developed approximate expressions for calculation of the characteristic line impedance,taking into account conductor width and thickness. (when： conductor thickness/substrate height=t/h ＜ 0.005).

when $w/h<1$ ： $Z_0=\dfrac{Z_f}{2\pi\sqrt{\varepsilon_{eff}}}\ln\left(\dfrac{8h}{w}+\dfrac{w}{4h}\right)$ **Empirical formulas**

Wave impedance in free space $Z_f=\sqrt{\mu_0\varepsilon_0}=376.8\Omega$

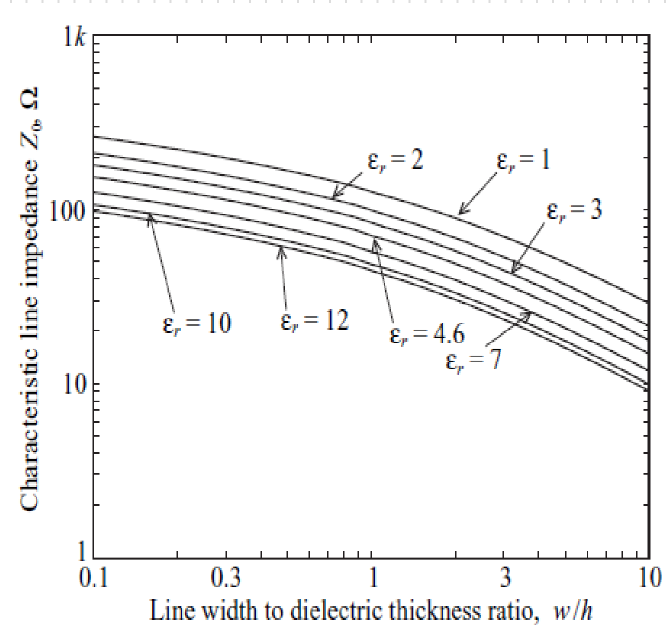
Effective dielectric constant

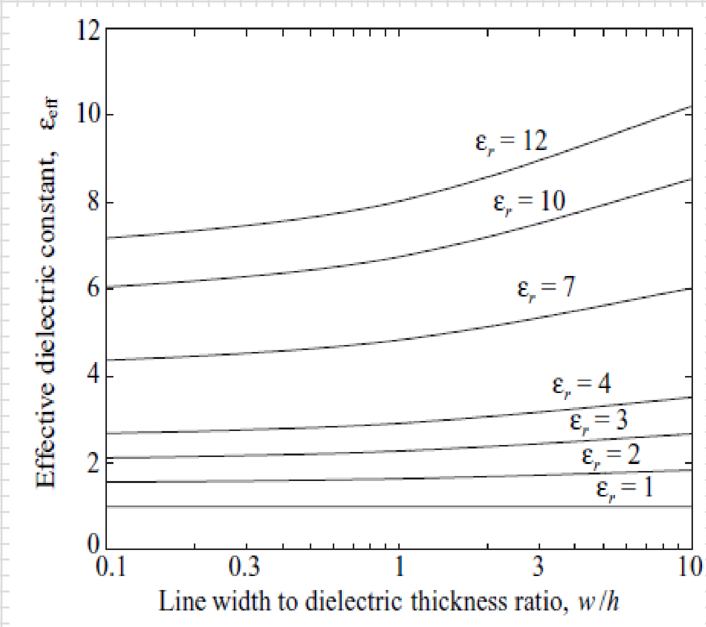
$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left[\left(1+12\frac{h}{w}\right)^{-1/2}+0.04\left(1-\frac{w}{h}\right)^2\right]$$

when $w/h>1$

$$Z_0=\frac{Z_f}{\sqrt{\varepsilon_{eff}}\left(1.393+\frac{w}{h}+\frac{2}{3}\ln\left(\frac{w}{h}+1.444\right)\right)}$$

$$\varepsilon_{eff}=\frac{\varepsilon_r+1}{2}+\frac{\varepsilon_r-1}{2}\left(1+12\frac{h}{w}\right)^{-1/2}$$





Get w/h ratios based on a given characteristic impedance

when $w/h \le 2$ $$\frac{w}{h}=\frac{8e^{A}}{e^{2A}-2}$$

when $w/h \ge 2$ $$\frac{w}{h}=\frac{2}{\pi}\left\{B-1-\ln(2B-1)+\frac{\varepsilon_r-1}{2\varepsilon_r}\left[\ln(B-1)+0.39-\frac{0.61}{\varepsilon_r}\right]\right\}$$

where $A=2\pi\frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r+1}{2}}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right)$, $B=\frac{\pi Z_f}{2Z_0\sqrt{\varepsilon_r}}$

Example 2.5 line impedance Z_0=50Ω， selected FR-4 PCB, with ε_r=4.6， h=40 mil， What are the width of the trace, phase velocity and wavelength at 2GHz

Solution： Use Figure 2.20 ε_r=4.6， Z_0=50Ω determine w/h=1.9 。

$$A=2\pi\frac{Z_0}{Z_f}\sqrt{\frac{\varepsilon_r+1}{2}}+\frac{\varepsilon_r-1}{\varepsilon_r+1}\left(0.23+\frac{0.11}{\varepsilon_r}\right)=1.5583$$

$$w/h = 8e^{A}/\left(e^{2A}-2\right) = 1.8477$$

$$w = 1.8477 \times 40 = 73.9\ mil$$

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{w}\right)^{-1/2} = 3.4575$$

$$v_p = c/\sqrt{\varepsilon_{eff}} = 1.61\times 10^{8}\ m/s, \qquad \lambda = v_p / f = 80.67\ mm$$

The assumption of zero thickness of the strip line may not be valid and corrections to be preceding equations are needed.

Effective width：

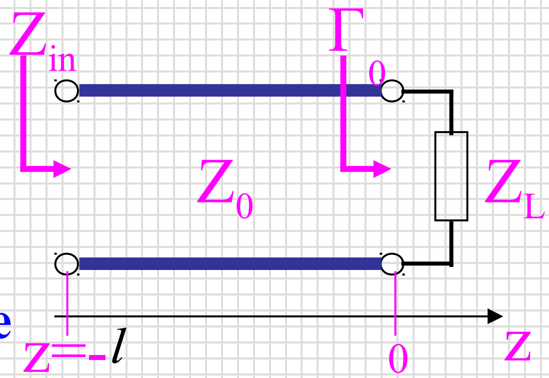
$$w_{eff} = w + \frac{t}{\pi}\left(1 + \ln\frac{2x}{t}\right)$$

when $w > h/2\pi > 2t, \qquad x = h$

when $h/2\pi > w > 2t, \qquad x = 2\pi w$

2.9 Terminated lossless transmission line

2.9.1 Voltage reflection coefficient



The load is located at z=0， the voltage wave is coupled into the line at z=-l .

So the voltage anywhere along the line: $V(z) = V^+ e^{-kz} + V^- e^{kz}$

Reflection coefficient: $\boxed{\Gamma_0 = \dfrac{V^-}{V^+}}$ **Reflected voltage wave** / **Incident voltage wave**

At z=0： $V(z) = V^+\left(e^{-kz} + \Gamma_0 e^{kz}\right)$, $I(z) = \dfrac{V^+}{Z_0}\left(e^{-kz} - \Gamma_0 e^{kz}\right)$

then：$Z(0) = Z_L = \dfrac{V(0)}{I(0)} = Z_0 \dfrac{1+\Gamma_0}{1-\Gamma_0}$ So： $\boxed{\Gamma_0 = \dfrac{Z_L - Z_0}{Z_L + Z_0}}$ （2.52）

when $Z_L \to \infty$ **open line** $\Gamma_0 = 1$， **reflected and incident wave the same ploarity**

when $Z_L = 0$ **short** $\Gamma_0 = -1$， **reflected wave inverted amplitude**

When $Z_L = Z_0$ **match** $\Gamma_0 = 0$， **no reflection， abosorbed by the load**

2.9.2 Propagation Constant and Phase Velocity

Complex propagation constant : $k = k_r + jk_i = \sqrt{(R + j\omega L)(G + j\omega C)}$

For lossless line : $k = j\omega\sqrt{LC}$

Engineering notation : $\alpha \equiv k_r = 0$ **Attenuation coefficient**

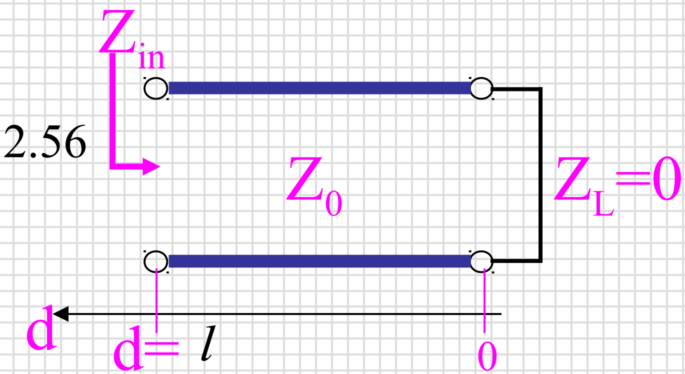
$\beta \equiv k_i = \omega\sqrt{LC}$ **Wave number or propagation constant**

Then : $V(z) = V^+\left(e^{-j\beta z} + \Gamma_0 e^{j\beta z}\right)$, $I(z) = \dfrac{V^+}{Z_0}\left(e^{-j\beta z} - \Gamma_0 e^{j\beta z}\right)$ （2.56） （2.57）

n1.3 : $v_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}}$ independent with frequency

Dispersion-free transmission 。 In reality we have to take into account a certain Degree of frequency dependence or dispersion of the phase velocity that causes Signal distortion

2.9.3 standing waves

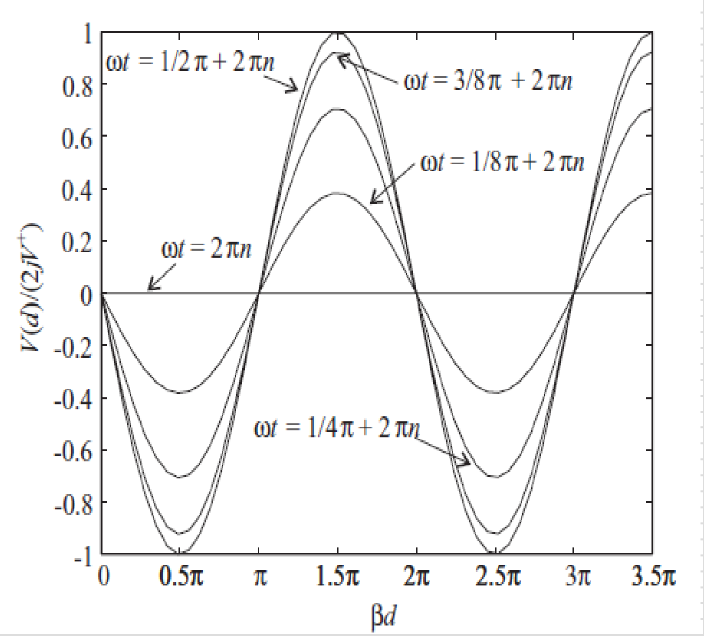


Insert the reflection coefficient for a short-circuit line into 2.56 change to a new coordinate d：

$$V(d) = V^+\left(e^{j\beta d} - e^{-j\beta d}\right)$$

Phase expression to time domain： $e^{j\beta d} - e^{-j\beta d} = 2j\sin(\beta d)$

$$\begin{aligned} v(d,t) &= \mathrm{Re}\{Ve^{j\omega t}\} \\ &= \mathrm{Re}\{2jV^+\sin(\beta d)e^{j\omega t}\} \\ &= -2V^+\sin(\beta d)\sin\omega t \end{aligned}$$



sin(β) for d=0 voltage maintains the Short condition at all time instance t.

The incident wave is 180°out of phase

Fixed zero crossing at 0, λ/2, λ, 3λ/2……

Voltage： $V(d) = V^{+}e^{j\beta d}\left(1+\Gamma_0 e^{-j2\beta d}\right) = A(d)\left[1+\Gamma(d)\right]$

Current： $I(d) = \dfrac{V^{+}}{Z_0}e^{j\beta d}\left(1-\Gamma_0 e^{-j2\beta d}\right) = \dfrac{A(d)}{Z_0}\left[1-\Gamma(d)\right]$

Reflection coefficient： $\boxed{\Gamma(d) = \Gamma_0 e^{-j2\beta d}}$ （2.64）

Matched condition， Γ_0= 0， Γ(d)= 0， maintaining propagation wave

Standing wave ratio： $SWR = \dfrac{|V_{\max}|}{|V_{\min}|} = \dfrac{|I_{\max}|}{|I_{\min}|} = \dfrac{1+|\Gamma_0|}{1-|\Gamma_0|}$

Extreme values of (2.64) can only be +1 and －1

match termination SWR=1, open or short SWR→∞

Strictly speaking, SWR can only be applied to lossless lines. The magnitude of The voltage or current waves diminishes as a function of distance due to attenuation。 Because most RF systems possess very low losses, SWR can be Safely applied.

2.10 Special termination Condition

2.10.1 Input impedance of terminated lossless line

$$V(d) = A(d)\left[1+\Gamma(d)\right]$$

$$I(d) = \frac{A(d)}{Z_0}\left[1-\Gamma(d)\right]$$

$$\Gamma(d) = \Gamma_0 e^{-j2\beta d}$$

$$\Gamma_0 = \frac{Z_L - Z_0}{Z_L + Z_0}$$

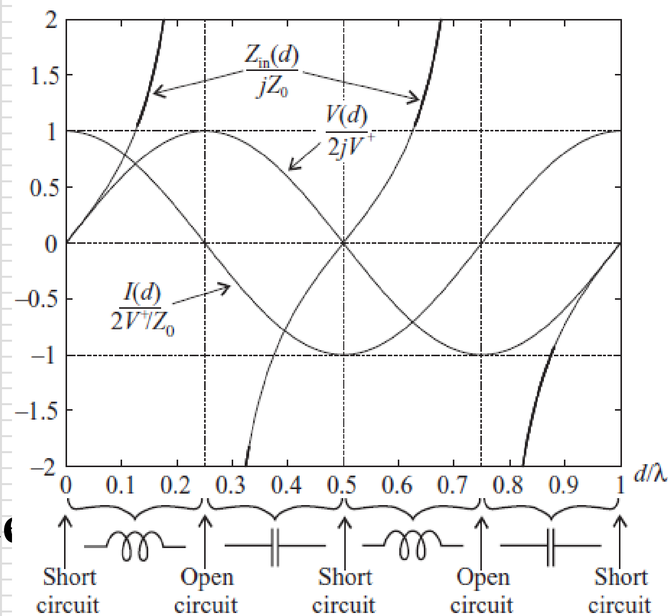
Distance d from the load, the impedance：

$$Z_{in}(d) = \frac{V(d)}{I(d)} = Z_0\frac{1+\Gamma(d)}{1-\Gamma(d)} = Z_0\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}$$

Predict how the load impedance Z_L is transformed along a transmission line of Characteristic impedance Z_0 and length d.

2.10.2 Short Circuit transmission line

$Z_L=0$： $Z_{in}(d) = jZ_0\tan(\beta d)$



d=0， Z_{in}(d)= Z_L =0； increasing d,

the impedance is increase and inductive behavior

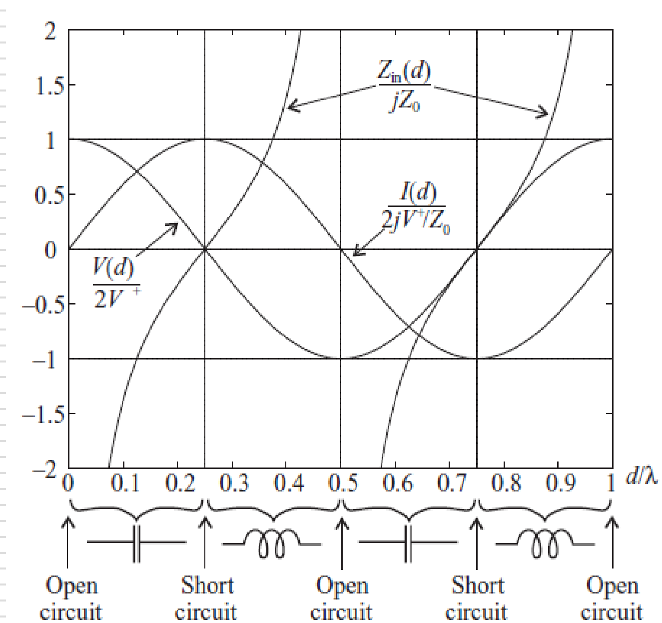
d=λ/4， Z_{in}(d)=∞， open-circuit dondition；

Further increase， negative imaginary impedance Capacitive behavior

d=λ/2， Z_{in}(d)=0， entire periodic repeated

2.10.3 Open-circuit transmission line

If $Z_L \to \infty$: $$Z_{in}(d) = \frac{-jZ_0}{\tan(\beta d)}$$

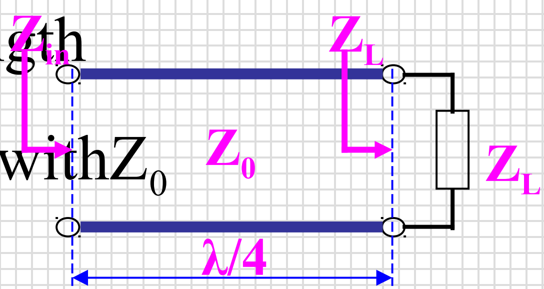


Keep the length d fixed, sweep the frequency over a specified range

2.10.4 Quarter-wave transmission line

If $Z_L = Z_0$: $$Z_{in}(d) = Z_0$$ regardless length

If $d=\lambda/2$: $$Z_{in}(d) = Z_L$$ independent withZ_0

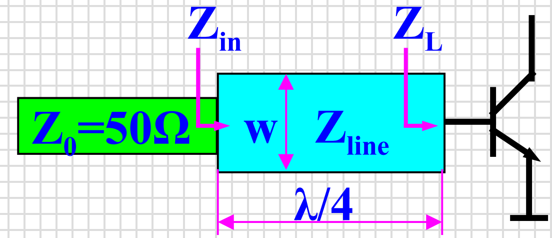


And : $$Z_{in}(d=\lambda/4) = Z_0^2 / Z_L$$

Lambda-quarter transformer， matching of a real load impedance to a desired Real input impedance by choosing a segment： $$Z_0 = \sqrt{Z_L Z_{in}} \quad (2.82)$$

Example 2.8 A transistor input impedance 25Ω， at 500MHz matched to a 50Ωmicrostrip line 。 Thickness of dielectric 1mm, ε= 4， loss is neglected，Find the length, width and characteristic impedance of λ/4 parallel-plate line Transformer for which matching is achieved 。

Solution： $Z_0 = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{\mu}{\varepsilon}}\dfrac{d}{w}$ $\qquad Z_0 = \sqrt{Z_L Z_{in}}$



Line impedance：

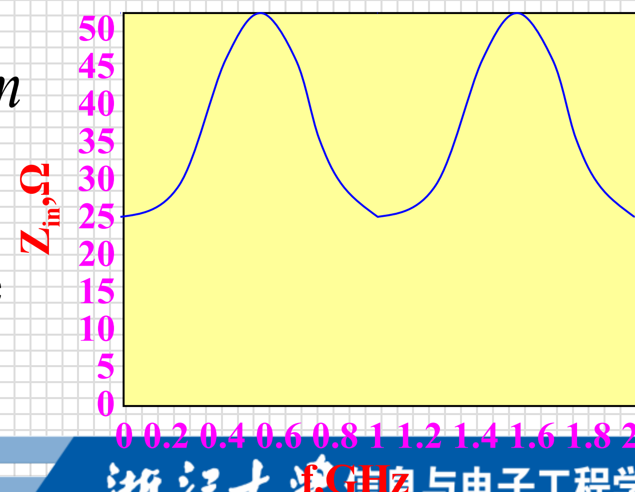
$$Z_{line} = \sqrt{Z_{in} Z_L} = 35.355\Omega = \sqrt{\mu / \varepsilon}\, d / w$$

Then： $w = \sqrt{\mu_0 / \varepsilon_0 \varepsilon_r}\, d / Z_{line} = 5.239 \ \ mm$

$$L = \mu d / w = 235.8 \ \ nH/m, \qquad C = \varepsilon w / d = 188.6 \ \ pF/m$$

$$l = \frac{\lambda}{4} = \frac{v_p}{4f} = \frac{1}{4f\sqrt{LC}} = 74.967 \ \ mm$$

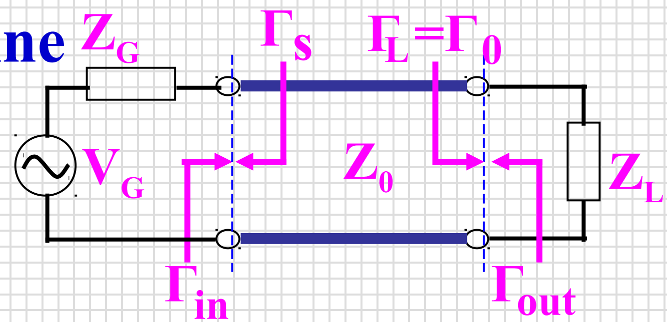
From： $Z_{in}(d) = Z_{line}\dfrac{1+\Gamma_0 e^{-j2\beta d}}{1-\Gamma_0 e^{-j2\beta d}}$ Get figure



matched not only at 500MHz, but also 1.5 GHz

Cannot work over a wide frequency band

2.11 Sourced and loaded transmission line



2.11.1 Phasor representation of source

Input voltage at the beginning of the trasmission line：

$$V_{in}=V_{in}^{+}+V_{in}^{-}=V_{in}^{+}\left(1+\Gamma_{in}\right)=V_G\frac{Z_{in}}{Z_{in}+Z_G}$$

仿照 $\Gamma_0=\dfrac{Z_L-Z_0}{Z_L+Z_0}$ 得出

from $\Gamma(d)=\Gamma_0 e^{-j2\beta d}$, input reflection： $\Gamma_{in}=\Gamma(d=l)=\Gamma_0 e^{-j2\beta l}=\dfrac{Z_{in}-Z_0}{Z_{in}+Z_0}$

Transmission coefficient：

$$T_{in}=1+\Gamma_{in}=\frac{2Z_{in}}{Z_{in}+Z_0},\quad T_0=1+\Gamma_0=\frac{2Z_L}{Z_L+Z_0}$$

example2.9

The connected source introduces an additional difficulty.Since the voltage reflected from the Load is traveling toward the source, we need to consider a mismatch between the transmission Line and the source impedance 。

The source reflection coefficient： $\Gamma_S=\dfrac{Z_G-Z_0}{Z_G+Z_0}$ And： $\Gamma_{out}=\Gamma_S e^{-j2\beta l}$

2.11.2 Power considerations for a transmission line

tage : $V_{in}=V_{in}^{+}\left(1+\Gamma_{in}\right)$ current : $I_{in}=V_{in}^{+}\left(1-\Gamma_{in}\right)/Z_0$

Time-averaged power :

$$P_{in}=P_{in}^{+}+p_{in}^{-}=\frac{1}{2}\mathrm{Re}\left\{VI^{*}\right\}=\frac{1}{2}\frac{\left|V_{in}^{+}\right|^{2}}{Z_0}\left(1-\left|\Gamma_{in}\right|^{2}\right)$$

n2.69 : $Z_{in}=Z_0\dfrac{1+\Gamma_{in}}{1-\Gamma_{in}}$ 2.87 : $Z_G=Z_0\dfrac{1+\Gamma_S}{1-\Gamma_S}$

Then : $V_{in}^{+}=\dfrac{V_{in}}{1+\Gamma_{in}}=\dfrac{V_G}{1+\Gamma_{in}}\dfrac{Z_{in}}{Z_{in}+Z_G}=\dfrac{V_G}{2}\dfrac{1-\Gamma_S}{1-\Gamma_S\Gamma_{in}}$

$$P_{in}=\frac{1}{8}\frac{\left|V_G\right|^{2}}{Z_0}\frac{\left|1-\Gamma_S\right|^{2}}{\left|1-\Gamma_S\Gamma_0e^{-j2\beta l}\right|^{2}}\left(1-\left|\Gamma_0e^{-j2\beta l}\right|^{2}\right)$$

$$\left|V_L^{+}\right|=\left|V_{in}^{+}\right|e^{-\alpha l}$$

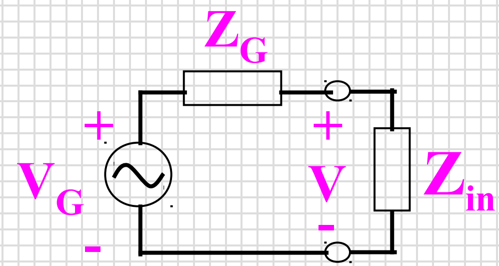
a attenuation coefficient

Lossless line and matched : $P_{in}=\dfrac{1}{8}\dfrac{\left|V_G\right|^{2}}{Z_0}=\dfrac{1}{8}\dfrac{\left|V_G\right|}{Z_G}$

Lossy line : $P_L=\dfrac{\left|V_L^{+}\right|^{2}}{2Z_0}\left(1-\left|\Gamma_L\right|^{2}\right)=\dfrac{1}{8}\dfrac{\left|V_G\right|^{2}}{Z_0}\dfrac{\left|1-\Gamma_S\right|^{2}}{\left|1-\Gamma_S\Gamma_{in}\right|^{2}}\left(1-\left|\Gamma_L\right|^{2}\right)e^{-2\alpha l}$

2.11.3 Input impedance matching

$$P_{in}=\frac{1}{2}\operatorname{Re}\left\{V_{in}\frac{V_{in}^*}{Z_{in}^*}\right\}=\frac{1}{2}\frac{|V_G|^2}{\operatorname{Re}\{Z_{in}^*\}}\left|\frac{Z_{in}}{Z_G+Z_{in}}\right|^2$$



Maximum pwer value : $\dfrac{\partial P_{in}}{\partial R_{in}}=\dfrac{\partial P_{in}}{\partial X_{in}}=0$

$Z_{in}=Z_G^*$

$Z_{out}=Z_L^*$

2.11.4 Return loss and insertion loss

Practical circuit suffer a degree of mismatch Γ_{in} is not zero

Return Loss : $RL=-10\log\dfrac{P_r\left(=P_{in}^-\right)}{P_i\left(=P_{in}^+\right)}=-10\log|\Gamma_{in}|^2=-20\log|\Gamma_{in}|=-\ln|\Gamma_{in}|$

Reflected power

Incident power

Decibel(dB) Nepers(N)

In addition to the return loss,also introduce insertion loss

Insertion Loss :

Transmitted power

$$IL=-10\log\frac{P_t}{P_i}=-10\log\frac{P_i-P_r}{P_i}=-10\log\left(1-|\Gamma_{in}|^2\right)$$

Incident power