

P.S.1

$$\frac{B_1}{A_1} = R_{12} e^{i2k_1 x d}, \quad \frac{A_1}{B_1} = R_{10}$$

$$\therefore R_{10} R_{12} e^{i2k_1 x d} = 1 = e^{i2m\pi}$$

$$\vec{H}_2 = \hat{y} A_1 (1 + R_{12}) e^{ik_1 x d} e^{ik_2 x (x-d)} e^{ik_2 z z}$$

$$\vec{H}_1 = \hat{y} A_1 \left\{ e^{ik_1 x X} + \frac{1}{R_{10}} e^{-ik_1 x X} \right\} e^{ik_2 z z}$$

$$\vec{H}_0 = \hat{y} A_1 \left(1 + \frac{1}{R_{10}} \right) e^{-ik_1 x X} e^{ik_2 z z}$$

$$\therefore k_{0x} = i k_{1x} I, \quad k_{0x} = i k_{1x} I, \quad R_{10} = e^{i2\varphi_{10}}, \quad R_{12} = e^{i2\varphi_{12}}$$

$$\therefore 2\varphi_{10} + 2\varphi_{12} + 2k_1 x d = 2m\pi, \quad k_z^2 + k_{1x}^2 = k_1^2, \quad k_z^2 - k_{1x}^2 = k^2$$

$$\therefore m\pi - \varphi_{10} - \varphi_{12} = k_{1x} d = \sqrt{k_1^2 - k^2} d = k_1 \sqrt{1 - \frac{\mu_0 \epsilon_2}{\mu_0 \epsilon_1}} d$$

$$\therefore k_1 = \frac{m\pi + \arctan \frac{\epsilon_1 \sqrt{\epsilon_2 - \epsilon_0}}{\epsilon_0 \sqrt{\epsilon_1 - \epsilon_2}}}{d \sqrt{1 - \frac{\epsilon_2}{\epsilon_1}}}$$

Pg.1

$$\vec{E} = \hat{y} (Ae^{ik_x x} + Be^{-ik_x x}) e^{ik_z z}$$

$$x=0: A+B=0$$

$$x=d: Ae^{ik_x d} + Be^{-ik_x d} = 0$$

$$\therefore 2i \sin(k_x d) = 0$$

$$\therefore \text{导波条件: } k_x d = m\pi, m=1, 2, \dots$$

$$\therefore \text{当 } m=1 \text{ 时,}$$

$$k_{cm} = \frac{\pi}{d}$$

$$\therefore f_{cm} = c \cdot k_{cm} / 2\pi = 1875 \text{ Hz}$$

Pg.2

(1) 当 $\frac{\omega}{c} > \frac{m\pi}{d}$ 时, 波才能传导

$$\therefore m < \frac{\omega d}{c\pi} = 3.464$$

对于 TE, $m=1, 2, 3$ 对于 TM, $m=0, 1, 2, 3$

$$(2) k_x d = 2\pi, k_x = \frac{2\pi}{d}$$

$$\text{假设, } \vec{H} = \hat{y} H_0 \cos \frac{2\pi}{d} x e^{ik_z z}$$

$$\vec{E} = \frac{H_0}{\omega \epsilon_0} \left[\hat{x} k_z \cos \frac{2\pi}{d} x - \hat{z} i \frac{2\pi}{d} \sin \frac{2\pi}{d} x \right] e^{ik_z z}$$

$$(3) v_p = \frac{\omega}{k_z} = 3.674 \times 10^8 \text{ m/s},$$

$$v_g = \frac{d\omega}{dk_z}, k_z^2 + k_x^2 = k^2 = \omega^2 \mu_0 \epsilon_0$$

$$\therefore v_g = \frac{k_z}{\omega \mu_0 \epsilon_0} = 2.45 \times 10^8 \text{ m/s}$$

(4) 当发生全反射时,

$$n_2 \sin \theta = n_1 \therefore \sin \theta = \sqrt{\frac{\epsilon_0}{\epsilon_1}} = \sqrt{3}, \text{ 不成立}$$

\therefore 不可能发生全反射.

(5)

$$\theta_B = \arctan \sqrt{\frac{\epsilon_1}{\epsilon_0}} = \frac{\pi}{3}$$

$$\because \omega = 30 \text{ GHz} \quad \therefore \lambda = \frac{2\pi \cdot c}{\omega} = 0.01 \cdot 2\pi \text{ m}$$

$$\therefore \frac{k_x}{\sin \theta_B} = \lambda$$

$$\therefore \frac{m\pi}{d} = \lambda \sin \theta_B \quad \therefore m = 3$$

P9.3

$$\cancel{k_x d = m\pi} \quad 2k_x d + 4\varphi_{10} = 2m\pi$$

$$k_z^2 + k_x^2 = \omega^2 \mu \epsilon = k^2$$

$$k_z^2 - k_{xI}^2 = \omega^2 \mu_0 \epsilon_0 = k_0^2$$

当 cutoff 时, $k_z = k$, $k_{xI} = 0$, $\varphi_{10} = 0$

$$\therefore m\pi = k_x d = \sqrt{k^2 - k_0^2} d = k \sqrt{1 - \frac{\mu_0 \epsilon_0}{\mu \epsilon}} d$$

$$\therefore k = k_{cm} = \frac{m\pi}{d \sqrt{1 - \frac{\mu_0 \epsilon_0}{\mu \epsilon}}}$$

$$k_z = k_0 = k \sqrt{\frac{\mu_0 \epsilon_0}{\mu \epsilon}}$$

$\therefore m$ 可以为 0, 1, 2

对应截止频率为 0, 12 GHz, 24 GHz

P10.1

$$1) k_x = \frac{m\pi}{a}, k_y = \frac{n\pi}{b}, k_x^2 + k_y^2 + k_z^2 = \frac{\omega^2}{c^2}$$

$$\therefore \frac{\pi}{a} < k_x < \frac{2\pi}{a}, 0 < k_y < \frac{\pi}{b}$$

$$a \leftarrow \infty$$

$$\therefore \frac{c\pi}{2\pi a} < f < \frac{c\pi}{2\pi b}$$

$$\therefore 22.9\text{MHz} < f < 35.8\text{MHz}$$

(2)

从第(1)问知, AM的频率小于能通过的频率, 所以AM不能在该波导中传播。

(3)

FM可以在该波导中传播, 比当 $m=3$ 时, $f=68.7\text{MHz}$ 高, 故能以至少 TE_{32} 的模式传播。

P10.2

由于电流沿 x 正方向, 则在 z 方向产生的是TM波。

$$\text{在 } z > 0 \text{ 时, } \vec{H}_1 = \hat{y} \sum_{m=1}^{\infty} H_m \cos\left(\frac{m\pi}{a}x\right) e^{ik_z z}$$

$$\vec{E}_1 = \hat{x} \sum_{m=1}^{\infty} \frac{k_z H_m}{\omega \epsilon} \cos\left(\frac{m\pi}{a}x\right) e^{ik_z z}$$

$$\text{在 } z < 0 \text{ 时, } \vec{H}_2 = \hat{y} \sum_{m=1}^{\infty} -H_m \cos\left(\frac{m\pi}{a}x\right) e^{-ik_z z}$$

$$\vec{E}_2 = \hat{x} \sum_{m=1}^{\infty} -\frac{k_z H_m}{\omega \epsilon} \cos\left(\frac{m\pi}{a}x\right) e^{-ik_z z}$$

在 $z=0$ 处, 利用边界条件 $\hat{n} \times (\vec{H}_1|_{z=0} - \vec{H}_2|_{z=0}) = \vec{J}_s$

$$\therefore \sum_{m=1}^{\infty} 2H_m \cos\left(\frac{m\pi}{a}x\right) = J_s \cos\left(\frac{3\pi}{a}x\right)$$

$$\therefore H_m = -\frac{J_s}{2}, m=3$$

$$\therefore \vec{H}_1 = \frac{J_s}{2} \cos\left(\frac{3\pi}{a}x\right) e^{ik_z z}, \vec{H}_2 = -\frac{J_s}{2} \cos\left(\frac{3\pi}{a}x\right) e^{-ik_z z}$$

\therefore 该模式下的幅度为 $\pm \frac{J_s}{2}$

10.3

(1) 在基波模式下, 波以 TE_{m0} 或 TE_{0n} 的模式传播.

$$k_x a = m\pi, \quad k_y b = n\pi, \quad \frac{\omega^2}{c^2} = k_x^2 + k_z^2 + k_y^2$$

当 $n=0$ 时, $k_x = \frac{m\pi}{a}$, $\frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + k_z^2$

取 $m=1$, $k_z = 69.46 \text{ rad/m}$

$$V_g = \frac{d\omega}{dk_z} = \frac{k_z}{\omega \mu_0 \epsilon_0} = 1.66 \times 10^8 \text{ m/s}$$

$$\therefore t = \frac{l}{V_g} = 1.8 \times 10^{-6} \text{ s}$$

当 $m>1$ 或 $n>0$ 时, $k_z^2 < 0$, 无法传播

\therefore 只有 TE_{10} 可以传播

(2)

$$\frac{\omega^2}{c^2} = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + k_z^2$$

TE_{10} 对应 cutoff: 5 GHz

TE_{20} ~ : 10 GHz

TE_{01} ~ : 10 GHz

\therefore 此3种模式可以传播

$$TE_{10}: t = \frac{l}{V_g} = \frac{l \omega \mu_0 \epsilon_0}{k_z} = 1.14 \times 10^{-6} \text{ s}$$

$$TE_{20}: t = \frac{l}{V_g} = 3.28 \times 10^{-6} \text{ s}$$

$$TE_{01}: t = \frac{l}{V_g} = 3.28 \times 10^{-6} \text{ s}$$