7-1

$E(X)=\int_0^\theta x\cdot\frac{6x(\theta-x)}{\theta^3}dx=\frac{\theta}{2}$

$\therefore \hat{\theta}=2\bar{X}$

$E(\hat{\theta})=E(2\bar{X})=2E(\bar{X})=2E(X)=\theta$

$Var(\hat{\theta})=D(2\bar{X})=4D(\bar{X})=\frac{4}{n}D(X)$

$D(X)=E(X^2)-E(X)^2$

$\because E(X^2)=\int_0^\theta x^2\cdot\frac{6x(\theta-x)}{\theta^3}dx=\frac{3}{10}\theta^2$

$\therefore Var(X)=\frac{\theta^2}{20}$

$\therefore Var(\hat{\theta})=\frac{\theta^2}{5n}$

7-3

(1) $\bar{X}=\frac{1}{10}\Sigma x_i=\frac{9}{10}$

$E(X)=0\times\theta+1\times\frac{1-\theta}{2}+2\times\frac{1-\theta}{2}=\frac{3}{2}(1-\theta)$

$\therefore E(X)=\bar{X}$

$\therefore \frac{9}{10}=\frac{3}{2}(1-\theta)$ $\therefore \theta=0.4$

$L(\theta)=(\frac{1-\theta}{2})^3$

$\therefore \ln L(\theta)=3\ln\frac{1-\theta}{2}$

$\therefore \frac{d\ln L(\theta)}{d\theta}=0$

$\therefore$ 解得: $\hat{\theta}=\frac{1}{3}$

(2) $\overline{X}=\frac{9}{10}$，$EX=2\theta(1-\theta)+2(1-\theta)^2$

$\therefore \hat{\theta}=0.55$

$L(\theta)=2\theta(1-\theta)^5$

$\therefore \ln L(\theta)=\ln 2+\ln\theta+5\ln(1-\theta)$

$\therefore \frac{d\ln L(\theta)}{d\theta}=0$

$\therefore \theta=\frac{1}{4}$

(3) $EX=\lambda+2-2\theta-2\lambda=2-2\theta-\lambda$

$\therefore \theta=1-\frac{3}{2}\times\frac{9}{10}+\frac{\lambda}{2}$

$L(\theta)=n_1\cdot n_2^2$ $\lambda=2\overline{X}-A_2$

$\frac{d\ln L(\theta)}{d\theta}=0$

$\therefore \hat{\theta}=\frac{n_0}{n}=\frac{4}{10}=\frac{2}{5}$，$\lambda=\frac{1}{3}$

7-4.

(1) $EX=\int_0^2 2^{-\theta}\theta x^{\theta}=\overline{X}$

$\therefore \hat{\theta}=\frac{\overline{X}}{2-\overline{X}}$

$L(\theta)=\prod f(x_i,\theta)$

$\therefore \frac{d\ln L(\theta)}{d\theta}=0$

$\therefore$ 解得：~~$\theta=\ln 2\cdot x$ 0.336或0.577~~ $\hat{\theta}=0.336$或0.577

(2) 同理，$\hat{\theta}=2\overline{X}-2$

$\frac{d\ln L(\theta)}{d\theta}=0$

$\therefore \hat{\theta}=0.186/0.35$

(3) $EX = \int_{-\infty}^{+\infty} \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}$

$\therefore \hat{\theta} = \sqrt{\frac{\bar{x}}{2}}$

$\frac{d\ln L(\theta)}{d\theta} = 0$

$\therefore \hat{\theta} = 0.485/0.468$

7-5

(1) $L(\theta) = \pi \frac{x_i}{\theta} e^{-\frac{x_i^2}{2\theta}}$

$\therefore \ln L(\theta) = -n\ln\theta + \sum \ln x_i - \frac{1}{2\theta}\sum x_i^2$

$\therefore \frac{d\ln L(\theta)}{d\theta} = 0$

取得: $\hat{\theta} = \frac{1}{2n}\sum x_i^2$

(2)

$E(x^2) = \int_0^{+\infty} \frac{x^3}{\theta} e^{-\frac{x^2}{2\theta}} dx = 2\theta \int_0^{+\infty} \frac{x^2}{2\theta} e^{-\frac{x^2}{2\theta}} d\left(\frac{x^2}{2\theta}\right)$
$= 2\hat{\theta}$

$\therefore E(x^2) = \frac{\sum x_i^2}{n}$

(3)

$P\{x>1\} = \int_1^{+\infty} \frac{x}{\theta} e^{-\frac{x^2}{2\theta}} dx$

$\therefore$ 同理可得:

$P\{x>1\} = e^{-\frac{n}{\sum x_i^2}}$

7-6

(1) $E(\hat{\sigma}^2) = a\sum(X_{i+1}-X_i)^2$

$= a\sum[D(X_i)+D(X_{i+1})+\mu-\mu]$

$= a\cdot 18\sigma^2 = \sigma^2$

$\therefore a=\frac{1}{18}$

(2) $E(\hat{\sigma}^2) = b\sum[D(X_i)+D(X_{i+5})+\mu-\mu]$

$=b\cdot 10\sigma^2=\sigma^2$

$\therefore b=\frac{1}{10}$

7-7

$E(X_1)=E(X_2)=E(X_3)=\mu$

$E(\hat{\mu}_1)=\frac{1}{2}\mu+\frac{1}{4}\mu+\frac{1}{4}\mu=\mu$

$E(\hat{\mu}_2)=2\mu-2\mu+\mu=\mu$

$E(\hat{\mu}_3)=\frac{1}{3}\mu+\frac{1}{3}\mu+\frac{1}{3}\mu=\mu$

$\therefore$都是无偏估计

$D(\hat{\mu}_1)=\frac{3}{8}\sigma^2$，$D(\hat{\mu}_2)=\sigma^2$，$D(\hat{\mu}_3)=\frac{1}{3}\sigma^2$

$\therefore \mu_3$最有效

7-8

1) $ET=a\sigma^2+b\sigma^2+c\sigma^2=(a+b+c)\sigma^2=\sigma^2$

$\therefore a+b+c=1$

2) $\because \frac{(n-1)S^2}{\sigma^2}\sim\chi^2(n-1)$

$\therefore D(S^2)=\frac{2\sigma^4}{n-1}$

$\therefore D(S_1^2)=2\sigma^4,\ D(S_2^2)=\sigma^4,\ D(S_3^2)=\frac{2}{3}\sigma^4$

$\therefore DT=\cancel{(2+1+\frac{2}{3})\sigma^4}\ (2a^2+b^2+\frac{2}{3}c^2)\sigma^4$

令 $L=a^2+\frac{b^2}{2}+\frac{c^2}{3}-\lambda(a+b+c-1)$

依 $\begin{cases}\frac{\partial L}{\partial a}=0\\ \frac{\partial L}{\partial b}=0\\ \frac{\partial L}{\partial c}=0\end{cases}$ 解得：$\begin{cases}a=\frac{1}{6}\\ b=\frac{1}{3}\\ c=\frac{1}{2}\end{cases}$