2.3

$$\hat{z}\times\vec{H}_1 = \begin{bmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 0 & 1\\ H_{1x} & H_{1y} & H_{1z}\end{bmatrix} = \hat{y}H_{1x} - \hat{x}H_{1y}$$

∵ $\bar{\bar{z}}\cdot\vec{H}_1 = \hat{y}H_{1x} - \hat{x}H_{1y} = \bar{\bar{z}}\cdot[H_{1x}, H_{1y}, H_{1z}]$

∴ $$\bar{\bar{z}} = \begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

(2) $g\mu_0\vec{M}\times\vec{H} = g\mu_0(\hat{z}M_0+\vec{M}_1)\times(\hat{z}H_0+\vec{H}_1)$

∵ $\vec{M}_1\times\vec{H}_1\approx 0$, $\hat{z}\times\hat{z}=0$

∴ 原式 $= \hat{z}M_0\times\vec{H}_1 + \vec{M}_1\times\hat{z}H_0 = M_0\hat{z}\times\vec{H}_1 - H_0\hat{z}\times\vec{M}_1$

根据上题结论: $g\mu_0\vec{M}\times\vec{H} \approx (M_0\bar{\bar{z}}\cdot\vec{H}_1 - H_0\bar{\bar{z}}\cdot\vec{M}_1)g\mu_0$

∴ $\frac{d\vec{M}}{dt} = \frac{d(\hat{z}M_0+\vec{M}_1)}{dt} = \frac{d\vec{M}_1}{dt} = -i\omega\vec{M}_1 = (M_0\bar{\bar{z}}\cdot\vec{H}_1 - H_0\bar{\bar{z}}\cdot\vec{M}_1)g\mu_0$

~~∴ $\vec{M}_1 = -\frac{g\mu_0}{i\omega}(M_0\bar{\bar{z}}\cdot\vec{H}_1 - H_0\bar{\bar{z}}\cdot\vec{M}_1)$~~ $\vec{M}_1 = \frac{g\mu_0 M_0\bar{\bar{z}}}{g\mu_0 H_0\bar{\bar{z}} - i\omega}\vec{H}_1$

∵ $\vec{B}_1 = \mu_0(\vec{H}_1+\vec{M}_1)$

∴ $\vec{B}_1 = \mu_0\left(1+\frac{g\mu_0 M_0\bar{\bar{z}}}{g\mu_0 H_0\bar{\bar{z}} - i\omega}\right)\vec{H}_1$

∴ $\bar{\bar{\mu}} = \mu_0\left(1+\frac{g\mu_0 M_0\bar{\bar{z}}}{g\mu_0 H_0\bar{\bar{z}} - i\omega}\right)$

∴ $$\bar{\bar{\mu}} = \begin{bmatrix}\mu & i\mu_g & 0\\ -i\mu_g & \mu & 0\\ 0 & 0 & \mu_z\end{bmatrix}$$

3)

$$\bar{\bar{k}}^2 = \omega^2 \bar{\bar{\mu}} \cdot \bar{\bar{\varepsilon}}, \quad \bar{\bar{\mu}} = \begin{bmatrix} \mu & i\mu_g & 0 \\ -i\mu_g & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}, \quad \bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon & & \\ & \varepsilon & \\ & & \varepsilon \end{bmatrix}$$

$$\therefore \bar{\bar{k}}^2 = \omega^2 \begin{bmatrix} \varepsilon\mu & i\varepsilon\mu_g & 0 \\ -i\varepsilon\mu_g & \varepsilon\mu & 0 \\ 0 & 0 & \varepsilon\mu_z \end{bmatrix}$$

(4)

$$\bar{\bar{\nu}} = \begin{bmatrix} \nu & i\nu_g & 0 \\ -i\nu_g & \nu & 0 \\ 0 & 0 & \nu_z \end{bmatrix}, \quad \nu = \frac{\mu}{\mu^2-\mu_g^2}, \quad \nu_g = \frac{-\mu_g}{\mu^2-\mu_g^2}, \quad \nu_z = \frac{1}{\mu_z}$$

$$K = \frac{1}{\varepsilon}$$

在kDB中，

$$\begin{bmatrix} u^2-\nu K & -iK\nu_g\cos\theta \\ iK\nu_g\cos\theta & u^2-K(\nu\cos^2\theta+\nu_z\sin^2\theta) \end{bmatrix} \cdot \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \frac{B_1}{B_2} = -\frac{u^2-K(\nu\cos^2\theta+\nu_z\sin^2\theta)}{iK\nu_g\cos\theta}$$

$$\therefore \frac{B_2}{B_1} = -i\frac{u^2-K(\nu\cos^2\theta+\nu_z\sin^2\theta)}{K\nu_g\cos\theta} = i\tan\alpha \neq 0$$

∴存在法拉第旋转.

扫描全能王 创建

P3.1

记 $K=\frac{1}{\varepsilon}$, $\nu=\frac{1}{\mu}$; $\tau=\frac{1}{\xi}$

$\therefore \vec{E}=K\vec{D}+\tau\vec{B}$

$\quad \vec{H}=\tau\vec{D}+\nu\vec{B}$

$\because \omega\vec{B}=\vec{k}\times\vec{E}$ $\qquad \therefore \omega B_1=-k(KD_2+\tau B_2)$

$\quad -\omega\vec{D}=\vec{k}\times\vec{H}$ $\qquad \omega B_2=k(KD_1+\tau B_1)$

$$-\omega D_1=-k(\nu B_2+\tau D_2)$$

$$-\omega D_2=k(\nu B_1+\tau D_1)$$

$$\therefore \begin{bmatrix} -u & -\tau \\ -\tau & u \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}=\begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix},\quad u=\frac{\omega}{k}$$

$$\begin{bmatrix} u & -\tau \\ -\tau & -u \end{bmatrix}\begin{bmatrix} D_1 \\ D_2 \end{bmatrix}=\begin{bmatrix} 0 & \nu \\ \nu & 0 \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0 & \frac{1}{K} \\ \frac{1}{K} & 0 \end{bmatrix}\begin{bmatrix} -u & -\tau \\ -\tau & u \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}=\begin{bmatrix} \frac{u}{u^2+\tau^2} & \frac{-\tau}{u^2+\tau^2} \\ \frac{-\tau}{u^2+\tau^2} & \frac{-u}{u^2+\tau^2} \end{bmatrix}\begin{bmatrix} 0 & \nu \\ \nu & 0 \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$\therefore \quad K\nu=u^2+\tau^2$

$\therefore u^2=\frac{\omega^2}{k^2}=K\nu+\tau^2$

$\therefore \omega^2=k^2\left(\frac{1}{\varepsilon\mu}+\frac{1}{\xi^2}\right)$

3.3

$$\frac{d\vec{M}}{dt} = -i\omega\vec{M_1} = g\mu_0(M_0\hat{z}\times\vec{H_1} - H_0\hat{z}\times\vec{M_1})$$

$\therefore \vec{B_1} = \mu_0(\vec{H_1}+\vec{M_1}) = \bar{\bar{\mu}}\cdot\vec{H_1}$

$$\therefore \bar{\bar{\mu}} = \begin{bmatrix} \mu & i\mu_g & 0 \\ -i\mu_g & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix}$$

$$\therefore \bar{\bar{\nu}} = \begin{bmatrix} \nu & i\nu_g & 0 \\ -i\nu_g & \nu & 0 \\ 0 & 0 & \nu_z \end{bmatrix}$$

$$\therefore \begin{bmatrix} u^2-\nu k & -ik\nu_g\cos\theta \\ ik\nu_g\cos\theta & u^2-k(\nu\cos^2\theta+\nu_z\sin^2\theta) \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$

$$\frac{D_2}{D_1} = -\frac{B_1}{B_2} = i\,\frac{u^2-k(\nu\cos^2\theta+\nu_z\sin^2\theta)}{k\nu_g\cos\theta} = i\tan\alpha \neq 0$$

$\therefore$ 存在法拉第旋转.