2022 年电磁场与电磁波英文班回忆卷

Problem 1 (20%)

Consider a rectangular waveguide as shown in Fig.1(A) and a rectangular cavity resonator as shown in Fig.1(B).

Suppose a=20mm, b=15mm and d=10mm.

The velocity of light in the air is $c = 3 \times 10^8 m/s$.

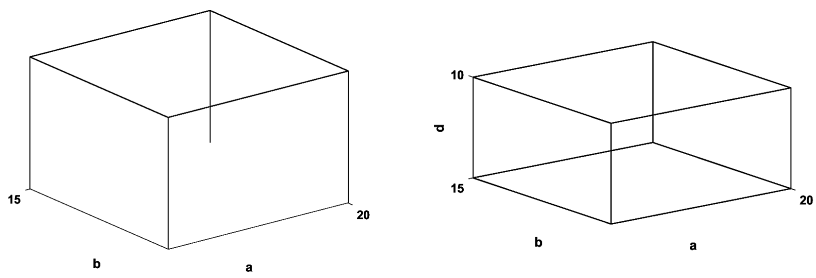

Figure1

(1) List the three lowest distinct cutoff frequencies (in the unit of Hz) of the rectangular waveguide and their corresponding mode designations (TE_{mn}, TM_{mn}).

(2) In the rectangular waveguide,what is the frequency range (in the unit of Hz) in which only the fundamental mode can propagate?

(3) List the three lowest distinct resonator frequencies (in the unit of Hz) of the cavity resonator and their corresponding mode designations (TE_{mnp}, TM_{mnp}).

Problem 2 (20%)

Consider a plane wave incident on a planar boundary at $x = 0$ as shown in Fig.2, where ϵ_0 is the permittivity in the air and μ_0 is the permeability in the air. Suppose the incident electric field is:

$$\vec{E}_i = (-\hat{x} + \sqrt{2}i\hat{y} + \hat{z})E_0 e^{i(k_x x + k_z z)}$$

where E_0 is a real constant. The incident wave vector $\vec{k}_i$ is on the x-z plane.

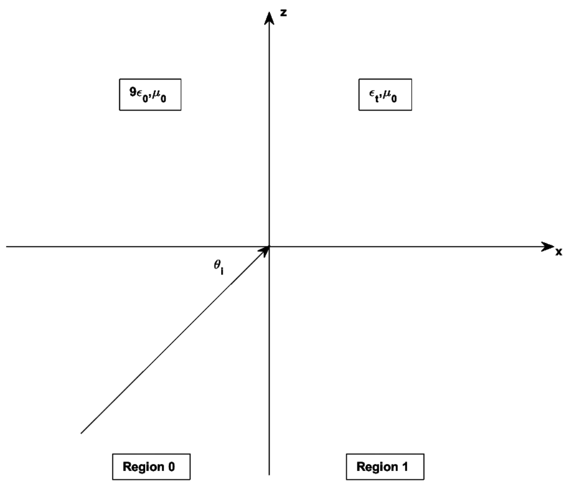

Figure2

(1) Prove that the incident angle is 45°.

(2) Suppose $k_z = \pi m^{-1}$, the velocity of light in the air $c = 3 \times 10^8 m/s$. Find the wavelength (in the unit of m) and the frequency (in the unit of Hz) in region 0.

(3) When the incident angle is equal to the critical angle, calculate the value of ϵ_t $(0 < \epsilon_t/\epsilon_0 < \infty)$, find the reflected electric field $\vec{E}_r$ and compare the handedness of polarization of $\vec{E}_i$ and $\vec{E}_r$. (Hint: Decompose $\vec{E}_i$ into the TM and TE components)

Problem 3 (20%)

Consider a three-layer structure as shown in Fig.3.

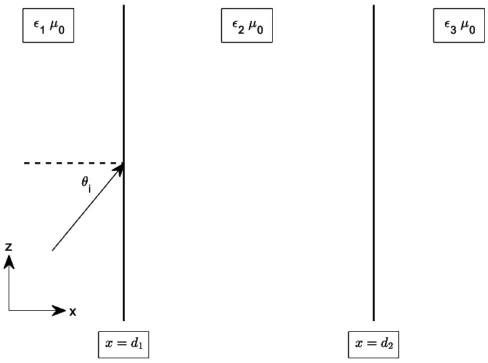

Figure3

The permittivity in each region is given as follows: $\epsilon_1 = 3\epsilon_0$,$\epsilon_2 = 9\epsilon_0$, and $\epsilon_3 = \epsilon_0$, where ϵ_0 is the permittivity in the air and μ_0 is the permeability in the air.

In region 1, the incident magnetic field vector is

$$\vec{H}_i = \hat{y} H_1 e^{i(k_{1x}x + k_{1z}z)}$$

where H_1 is real constant.$R_{l(l+1)}$ denotes the reflection coefficient in region l, caused by the boundary separating region l and region $l+1$. $R_{(l+1)l}$ is equal to the negative of the reflection coefficient $R_{l(l+1)}$.

Suppose $R_{12} = 0$.

(1) What is the value of incident angle θ_i? (Hint: The Brewster angle)

(2) Calculate the reflection coefficient R_{23}.

Problem 4 (20%)

(1) Name at least 5 types of media with different constitutive relations.

(2) Consider a medium with the permittivity and the permeability

$$\bar{\bar{\epsilon}} = \begin{bmatrix} \epsilon_x & & \\ & \epsilon_y & \\ & & \epsilon_z \end{bmatrix} \quad \bar{\bar{\mu}} = \begin{bmatrix} \mu_x & & \\ & \mu_0 & \\ & & \mu_0 \end{bmatrix}$$

where μ_0 is the permeability in the air.

In the kDB system, suppose the wave vector $\vec{k}$ is on the y-z plane. (Hint: $\phi = \pi/2$ in the kDB system)

(2.1) Let $\vec{E} = \bar{\bar{\kappa}} \cdot \vec{D}$ and $\vec{H} = \bar{\bar{\nu}} \cdot \vec{B}$, find $\bar{\bar{\kappa}}$ and $\bar{\bar{\nu}}$.

(2.2) Suppose $\bar{\bar{T}}$ is the transformation matrix, calculate $\bar{\bar{\kappa}}_k = \bar{\bar{T}}\bar{\bar{\kappa}}\bar{\bar{T}}^{-1}$.

(2.3) Calculate the dispersion relation for type I wave with $\vec{D} = \hat{e}_1 D_1$ and type II wave with $\vec{D}_2 = \hat{e}_2 D_2$ respectively. Among these two types of waves, which one corresponds to the ordinary wave and which one corresponds to the extraordinary wave?

Problem 5 (20%)

A Hertzian dipole is located at the origin of corrdinates with the dipole momentum along the $\hat{z}$ direction. In the far field, the electric field is

$$\vec{E}(\vec{r}) = -\hat{\theta} i\omega\mu Il \frac{e^{ikr}}{4\pi r} sin\theta$$

where r is the distance between the source and observation point.

(1) Sketch the radiation field pattern on the x-z plane.

(2) Find the magnetic field $\vec{H}(\vec{r})$, the time-average Poynting's power density $\langle \vec{S} \rangle$, and the total radiated power P_r.

(3) Consider an antenna array of 4 elements, pointing in the $\hat{z}$ direction and placed along the x axis with equal spacing d. Each element has a progressive phase shift α relative to its adjacent element. Suppose $d = 5\lambda/6$ and $\alpha = 5\pi/3$. Calculate the array factor $F(u)$ and draw the radiation pattern on the x-y plane.