第三次作业

3.3

$$\frac{d^2A_{z1}}{dr^2}=\frac{d}{dr}\left[\frac{d}{dr}\left(C_1\frac{e^{-jkr}}{r}\right)\right]=C_1\frac{d}{dr}\left[-jk\frac{e^{-jkr}}{r}-\frac{e^{-jkr}}{r^2}\right]$$

$$\frac{2}{r}\frac{dA_{z1}}{dr}=\frac{2}{r}C_1\left(-jk\frac{e^{-jkr}}{r}-\frac{e^{-jkr}}{r^2}\right)$$

$$k^2A_{z1}=k^2C_1\frac{e^{-jkr}}{r}$$

$\therefore \frac{d^2A_{z1}}{dr^2}+\frac{2}{r}\frac{dA_{z1}}{dr}+k^2A_{z1}=0$

同理可得：$\frac{d^2A_{z2}}{dr^2}+\frac{2}{r}\frac{dA_{z2}}{dr}+k^2A_{z2}=0$

$\therefore A_{z1}, A_{z2}$ 是方程的解.

3.6

$\nabla\times E_1=-\hat{z}H_1-M_1$ ，$\nabla\times H_1=\hat{y}E_1+J_1$

$\nabla\times E_2=-\hat{z}H_2-M_2$ ，$\nabla\times H_2=\hat{y}E_2+J_2$

$H_2\cdot\nabla\times E_1=-\hat{z}H_2\cdot H_1-H_2\cdot M_1$

$E_1\cdot\nabla\times H_2=\hat{y}E_1\cdot E_2+E_1\cdot J_2$

$\therefore E_1\cdot\nabla\times H_2-H_2\cdot\nabla\times E_1=\hat{y}E_1\cdot E_2+\hat{z}H_2\cdot H_1+E_1\cdot J_2+H_2\cdot M_1$

$\because \nabla\cdot(H_2\times E_1)=-\nabla\cdot(E_1\times H_2)=\hat{y}E_1\cdot E_2+\hat{z}H_2\cdot H_1+E_1\cdot J_2+H_2\cdot M_1$

$E_2\cdot\nabla\times H_1=\hat{y}E_2\cdot E_1+E_2\cdot J_1$

$H_1\cdot\nabla\times E_2=-\hat{z}H_1\cdot H_2-M_1\cdot M_2$

$E_2\cdot\nabla\times H_1-H_1\cdot\nabla\times E_2=\hat{y}E_2\cdot E_1+\hat{z}H_1 H_2+E_2\cdot J_1+H_1 M_2$

$\therefore -\nabla\cdot(E_1\times H_2-E_2\times H_1)=E_1\cdot J_2+H_2\cdot M_1-E_2\cdot J_1-H_1\cdot M_2$

4.1

a. $\sin\psi = \sqrt{1-(\sin\theta \cdot \cos\varphi)^2}$

$E_\varphi = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \cdot \sin\psi = j\eta \frac{kI_0 \cdot l e^{-jkr}}{4\pi r} \sqrt{1-(\sin\theta \cos\varphi)^2}$

$H_x = j \frac{kI_0 l e^{-jkr}}{4\pi r} \cdot \sin\psi = \frac{E_\psi}{\eta}$

b. $U = U_0 (1-\sin^2\theta\cos^2\varphi)$

$P_{rad} = U_0 \int_0^{2\pi}\int_0^{\pi} (1-\sin^2\theta \cdot \cos^2\varphi) \cdot \sin\theta \, d\theta \, d\varphi = U_0 \cdot \frac{8}{3}\pi$

$D_0 = \frac{4\pi U_0}{P_{rad}} = 1.5$

4.5

a. $\varphi = 0°$, $E_\varphi = j\eta \frac{kI_0 l e^{-jkr}}{4\pi r} \sqrt{1-\sin^2\theta}$

∴ E_φ只有$\hat{\theta}$方向

4.29

(a) $P_{rad} = P_{in} = 1$

$U_0 = \frac{P_{rad}}{4\pi} = \frac{1}{4\pi}$

$U_{dip} = U_0 D_0 \approx 0.131$

(b)

$W_{dip} = \frac{U_{dip}}{r^2} \approx 5.23 \times 10^{-4}\ w/m^2$

$W_0 = \frac{U_0}{r^2} \approx 3.183 \times 10^{-9}\ w/m^2$