第三章 动态电路频域特性分析

3.4 RLC电路频率响应

RLC电路的频率响应

- 基本RLC电路是理解滤波电路频率特性的基础
- 假设所有电路均满足零状态初始条件

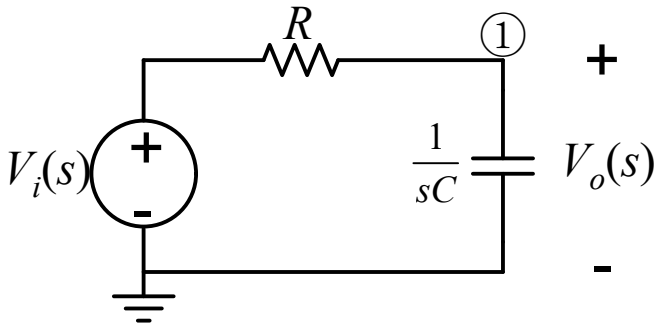
电感/电容的阻抗

- 电感和电容的阻抗都与频率有关
- 电感阻抗 $j\omega L$
  - 大小随频率升高而增大
  - 通直流阻交流
- 电容阻抗 $\frac{1}{j\omega C}$
  - 大小随频率升高而减小
  - 隔直流通交流

RC电路频率响应

- 系统函数

$$H(s)=\frac{1}{1+sRC}$$

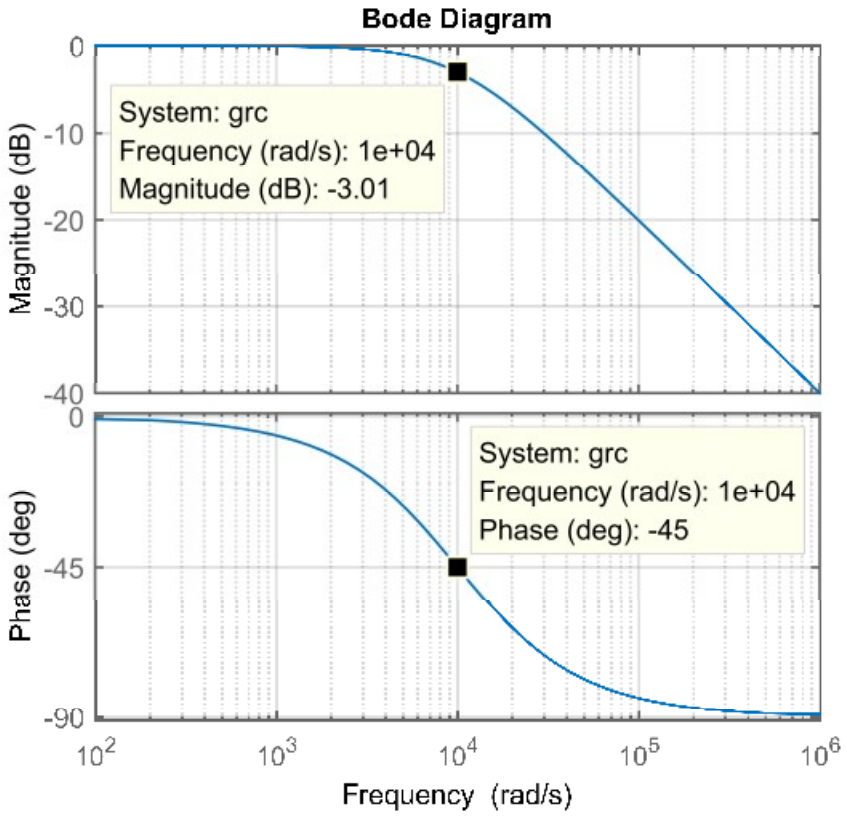


- 频率响应

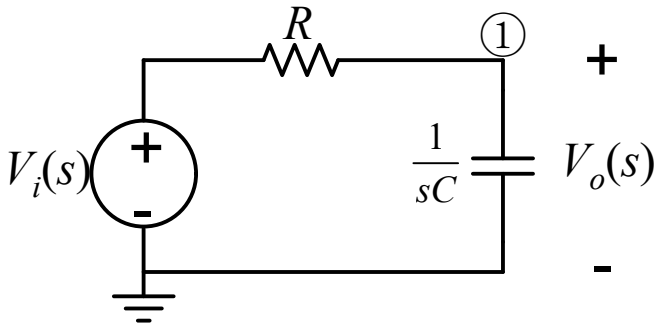
$$\omega_n=\frac{1}{RC} \qquad H(j\omega)=\frac{1}{1+j\frac{\omega}{\omega_n}}$$

RC电路频率响应

$R = 100\Omega, \quad C = 1\mu F, \quad \omega_n = 10^4 rad/s$



低通滤波特性



分压公式：

$$V_o(s) = \frac{1/sC}{R + 1/sC} V_i(s)$$

频率越低，分压越大

频率越高，分压越小

RC电路频率响应

- 分压公式

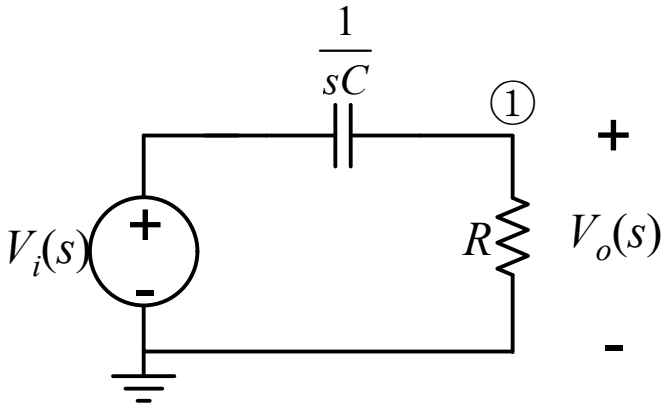
$$V_o(s)=\frac{R}{R+1/sC}V_i(s)$$

- 系统函数

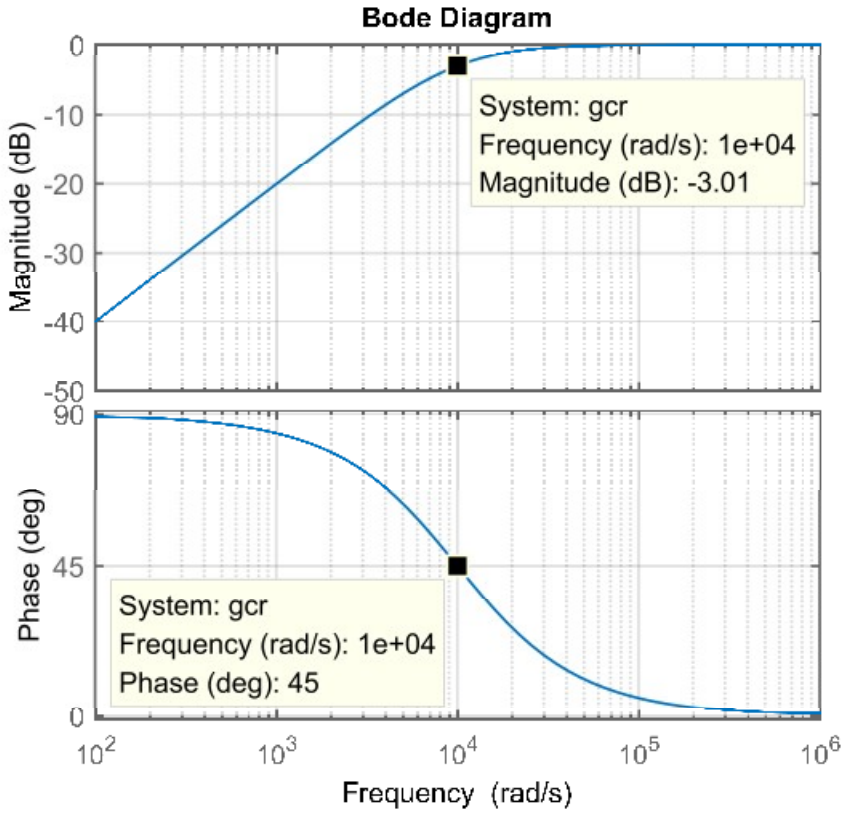
$$H(s)=\frac{sRC}{1+sRC}$$

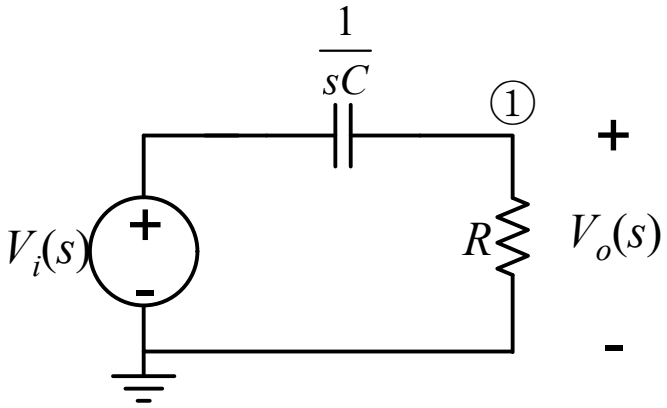
- 频率响应

$$\omega_n=\frac{1}{RC}\qquad H(j\omega)=\frac{j\dfrac{\omega}{\omega_n}}{1+j\dfrac{\omega}{\omega_n}}$$



RC电路频率响应





$$R = 100\Omega, \quad C = 1\mu F,$$

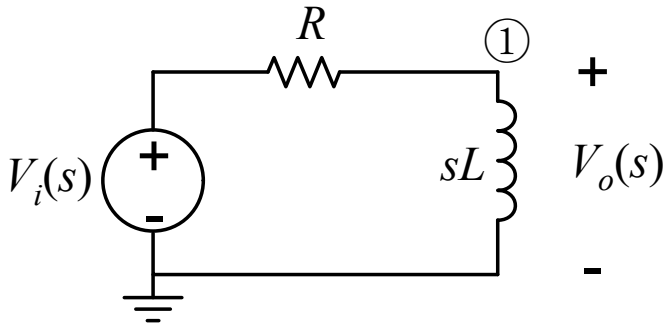
$$\omega_n = 10^4 rad/s$$

高通滤波特性

RL电路频率响应

- 分压公式

$$V_o(s) = \frac{sL}{R + sL} V_i(s)$$



- 系统函数

$$H(s) = \frac{sL/R}{1 + sL/R}$$

- 频率响应

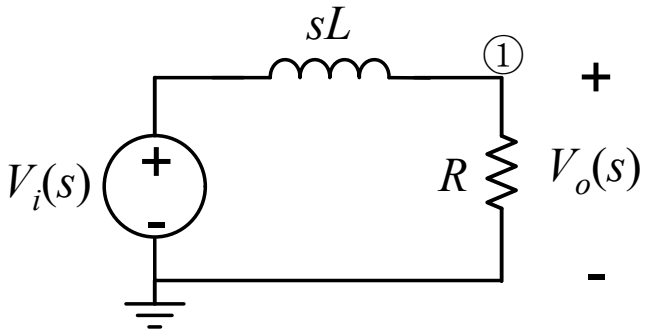
$$\omega_n = \frac{R}{L} \qquad H(j\omega) = \frac{j\frac{\omega}{\omega_n}}{1 + j\frac{\omega}{\omega_n}}$$

高通滤波特性

RL电路频率响应

- 分压公式

$$V_o(s)=\frac{R}{R+sL}V_i(s)$$



- 系统函数

$$H(s)=\frac{1}{1+sL/R}$$

- 频率响应

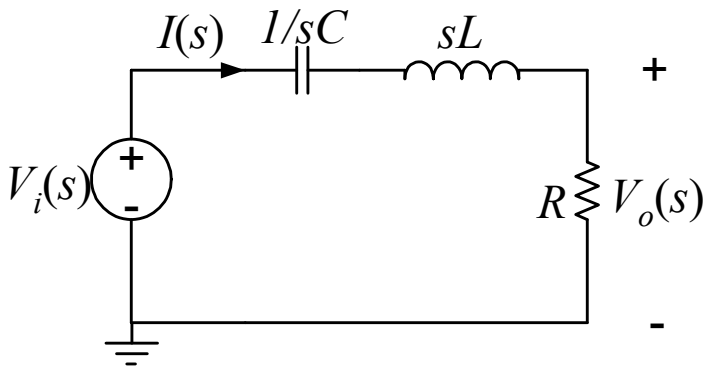
$$\omega_n=\frac{R}{L}\qquad H(j\omega)=\frac{1}{1+j\dfrac{\omega}{\omega_n}}$$

低通滤波特性

RLC串联电路频率响应

- 分压公式

$$V_o(s) = \frac{R}{R + sL + 1/sC} V_i(s)$$



- 系统函数

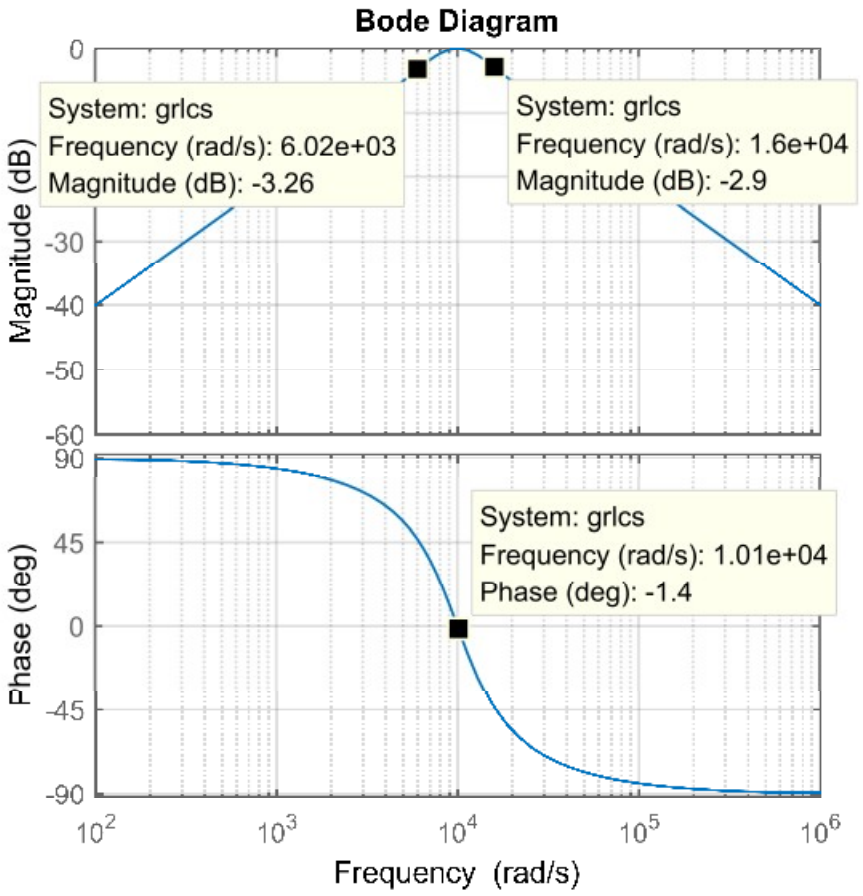
$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

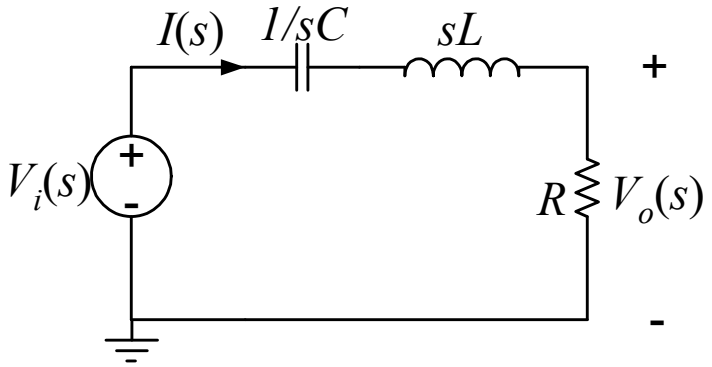
- 频率响应

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q_s = \frac{\omega_0 L}{R}$$

$$H(j\omega)\Big|_{\text{串联}RLC} = \frac{1}{1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

RLC串联电路频率响应





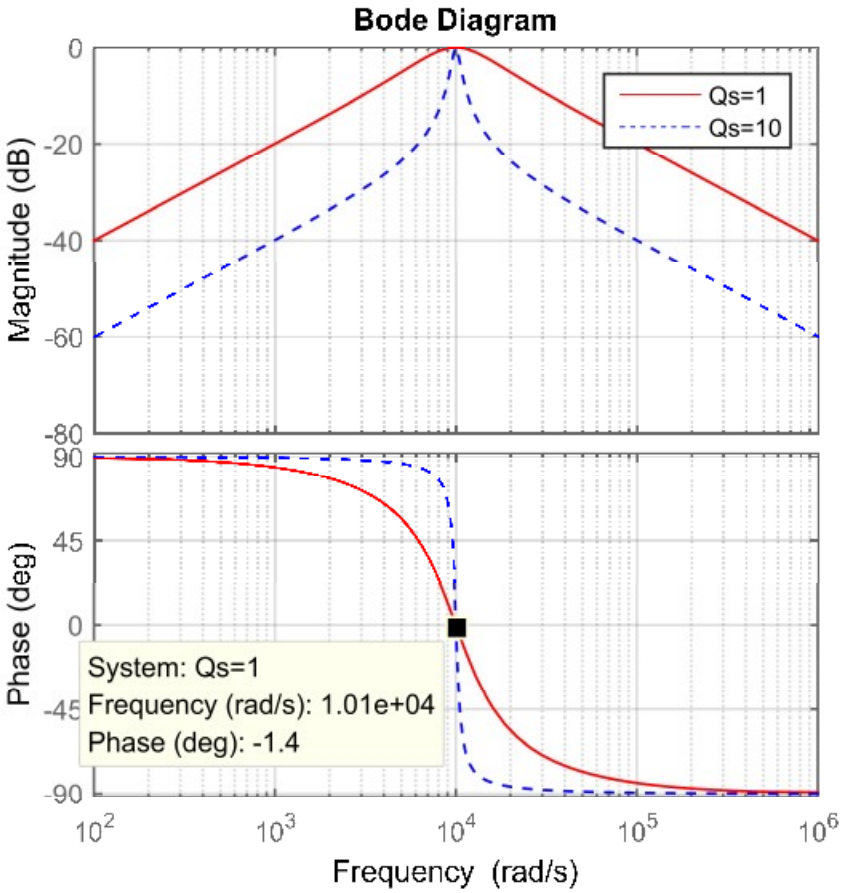
$$R = 100\Omega, \quad L = 10mH, \quad C = 1\mu F$$

$$\omega_n = 10^4 rad/s, \quad Q_s = 1$$

带通滤波特性

RLC串联电路品质因数

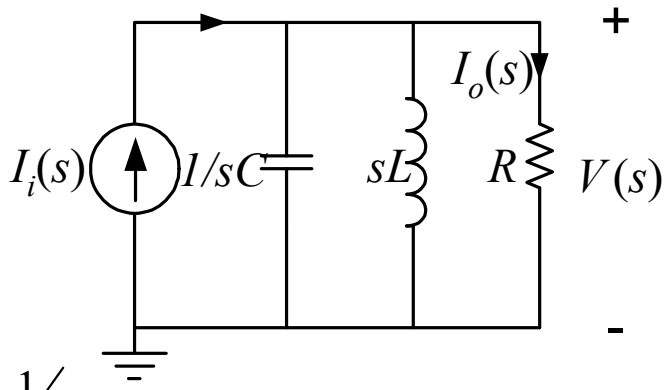
- 品质因数高，曲线更尖锐，带宽更窄
- 滤波电路的频率选择性更好



RLC并联电路频率响应

- 分流公式

$$I_o(s) = \frac{1/R}{1/R + 1/sL + sC} I_i(s)$$



- 系统函数

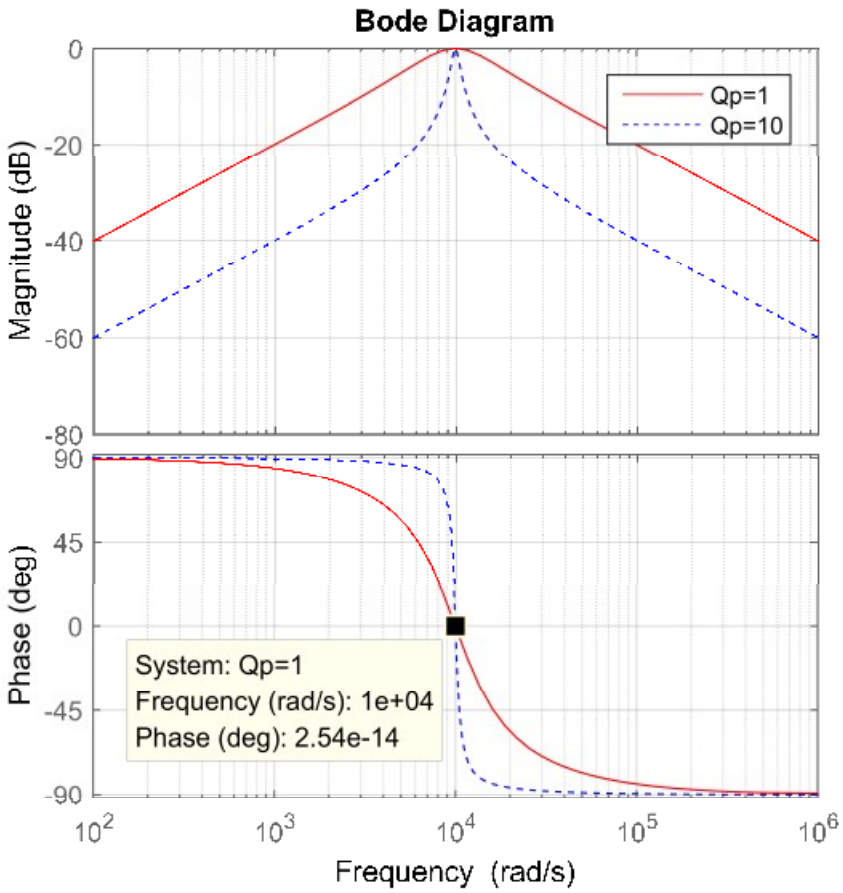
$$H(j\omega) = H(s)\big|_{s=j\omega} = \frac{G}{G + j\omega L + \frac{1}{j\omega C}}, \quad G = 1/R$$

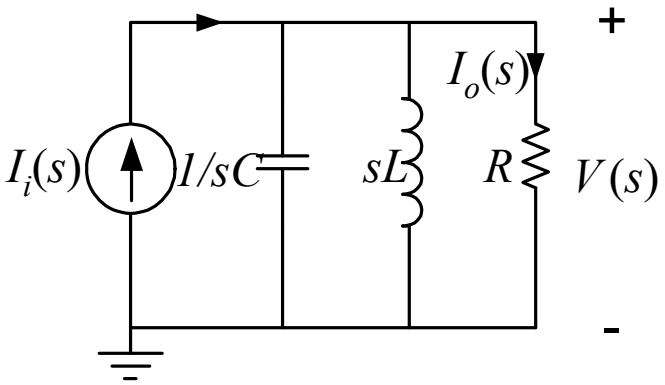
- 频率响应

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q_p = \frac{\omega_0 C}{G}$$

$$H(j\omega)\big|_{\text{并联}RLC} = \frac{1}{1 + jQ_p\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

RLC并联电路频率响应





$$R = 100\Omega, \quad L = 10mH, \quad C = 1\mu F$$

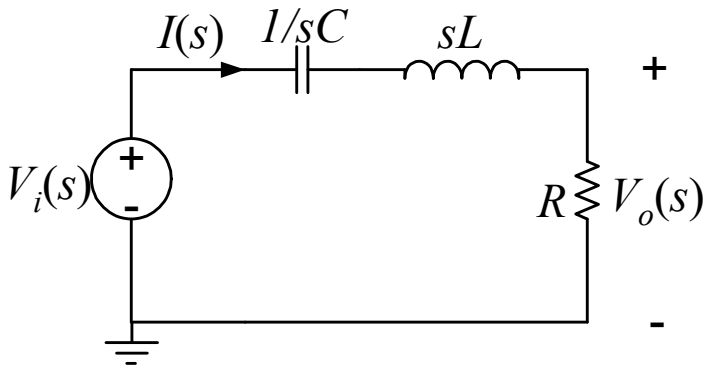
$$\omega_n = 10^4 rad/s, \quad Q_p = 1$$

带通滤波特性

品质因数越高，电路频率选择性越好

RLC串联电路的阻抗频率特性

$$Z(s) = R + sL + 1/(sC)$$
$$= \frac{s^2LC + sRC + 1}{sC}$$



$$Z(\omega) = Z(s)\Big|_{s=j\omega}$$
$$= R + j\omega L + 1/(j\omega C)$$
$$= R\left(1 + jQ_s\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q_s = \frac{\omega_0 L}{R}$$

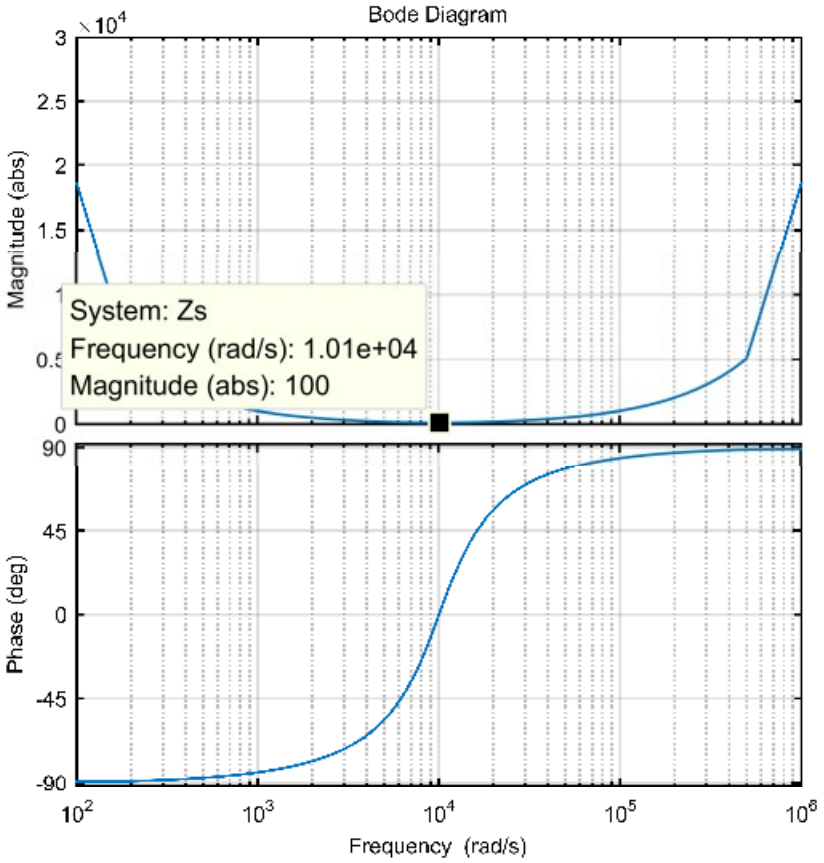
RLC串联电路的阻抗频率特性

$$Z(s) = R + sL + 1/(sC)$$
$$= \frac{s^2LC + sRC + 1}{sC}$$

$$R = 100\Omega, \quad L = 10mH, \quad C = 1\mu F$$

```
R=100; L=10e-3; C=1e-6; G=1/R;
Zs=tf([L*C R*C 1],[C 0])
```

```
>> P = bodeoptions;
>> P.MagUnits='abs';
>> bodeplot(Zs, P)
>>
```

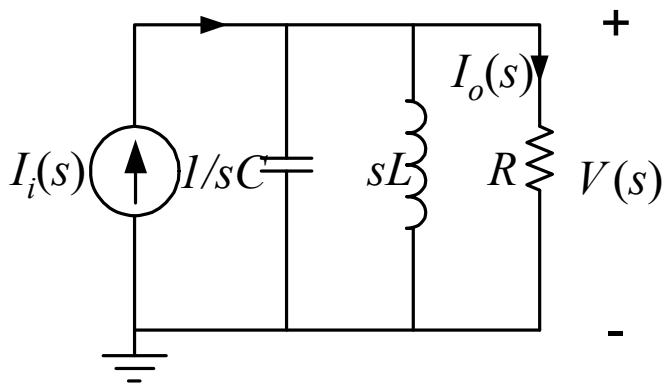


电路谐振时，阻抗只有实部，且达到最小值R

RLC并联电路的阻抗频率特性

$$Z(s)=\frac{1}{Y(s)}=\frac{1}{G+sC+1/(sL)}$$

$$=\frac{sL}{s^2LC+sLG+1}$$



$$Y(\omega)=Y(s)\big|_{s=j\omega}$$

$$=G+j\omega C+1/(j\omega L)$$

$$=G\left(1+jQ_p\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)\right)$$

$$\omega_0=\frac{1}{\sqrt{LC}}\qquad Q_p=\frac{\omega_0 C}{G}$$

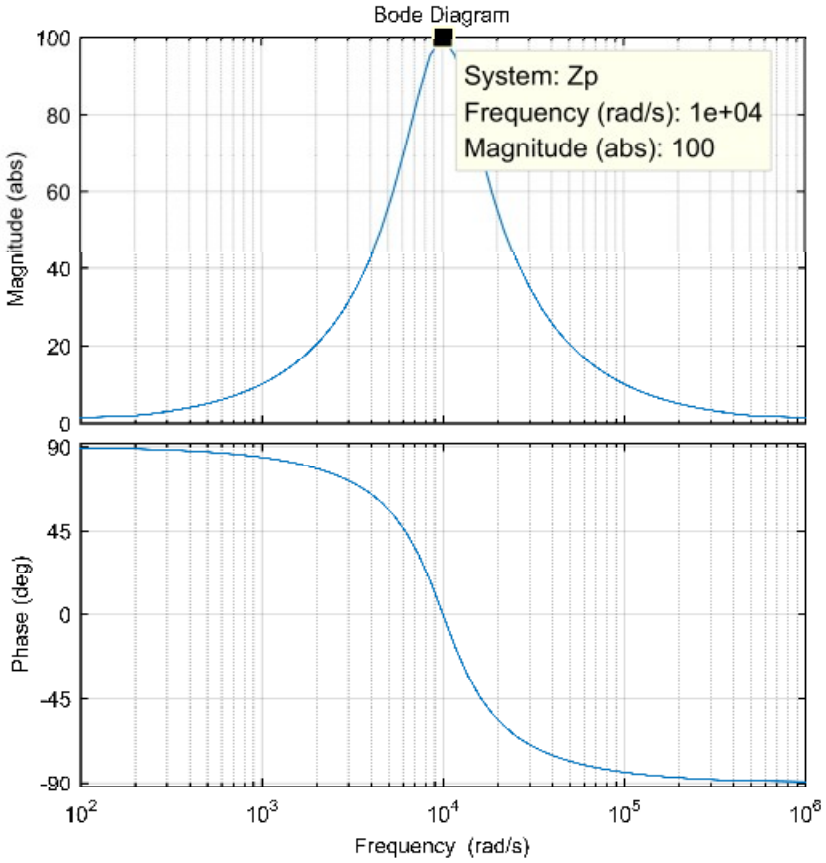
RLC并联电路的阻抗频率特性

$$Z(s)=\frac{1}{Y(s)}=\frac{1}{G+sC+1/(sL)}$$

$$=\frac{sL}{s^2LC+sLG+1}$$

$$R=100\Omega,\quad L=10mH,\quad C=1\mu F$$

```
R=100; L=10e-3; C=1e-6; G=1/R;
Zp=tf([L 0],[L*C G*L 1])
```



电路谐振时，导纳只有实部，且达到最小值G

对应阻抗达到最大值R

小结

- 频率特性
  - RC电路
  - RL电路
  - RLC串联电路
  - RLC并联电路