P2.1

(1) $k = \sqrt{\omega^2 \mu \varepsilon_c} = \sqrt{\omega^2 \mu \varepsilon_0} \cdot \sqrt{40(1+0.3i)} \approx \sqrt{\omega^2 \mu \varepsilon_0} \cdot \sqrt{40 \times \frac{\sqrt{109}}{10} \times e^{i0.3}}$

$\approx \overset{0.339}{\cancel{0.054}} e^{i0.15} = \overset{0.339}{\cancel{0.054}}(\cos 0.15 + i \sin 0.15)$

$\therefore$ penetration depth: $\Delta = \frac{1}{\underset{0.339}{\cancel{0.054}} \times \sin 0.15} \approx \cancel{123.9m}$ $19.7m$

(2)

$\tan\delta_1 = \frac{6}{\omega \varepsilon} = 7.49 \times 10^{13}$

$\tan\delta_2 = 4.49 \times 10^{8}$

(3)

~~$\because \varepsilon = 80\varepsilon_0 \quad \therefore E|_{x=100} = 1V/m$~~

$\varepsilon_c = \varepsilon + i\frac{6}{\omega} = 80\varepsilon_0 + 0.04i$

$\therefore k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon_0} \cdot \sqrt{80 + \frac{0.04}{\varepsilon_0} i}$

$\because \frac{0.04}{\varepsilon_0} >> 80$

$\therefore k \approx k_R + 0.0224i$

~~$\therefore E|_{x=100} = e^{-0.0224 \times 100} \cdot 1 V/m \approx 0.11 V/m$~~

$\therefore E|_{r=100} = e^{-0.0224 \times 100} \cdot 1 \cdot e^{ik_R r} \approx 0.11 V/m$

~~$H|_{r=100} = \int \frac{i}{\mu_0} E\, dt = \frac{1}{\mu_0} e^{-2.24} \cdot \frac{1}{k} e$~~

~~$\therefore \langle\vec{S}\rangle = \frac{1}{2\pi}\int_0^{2\pi} \vec{E} \times \vec{H}\, d(\omega t) =$~~

~~$\therefore \langle\vec{S}\rangle = \vec{E}(r) \times \vec{H}(r) =$~~

$\vec{H} = \frac{\mu_0}{i\omega} e^{-k_B} e^{ik_R r}$

$\therefore \langle\vec{S}\rangle = \vec{E}(r) \times \vec{H}(r) = \frac{\mu_0}{\omega} \cdot 0.11^2 = 2.42 \times 10^{-11}\ W/m^2$

Scanned with CamScanner

P2.2

(1) $\chi = -\frac{\omega_p^2}{\omega^2}$, $\omega_p = \frac{Ne^2}{m_e\varepsilon_0}$

$\varepsilon_m = \varepsilon_0(1+\chi) = \varepsilon_0(1-\frac{\omega_p^2}{\omega^2})$

∵ N 极大 ∴ $\omega_p >> \omega^2$

∴ $k = \omega\sqrt{\mu_0\varepsilon_m} = \omega\sqrt{\mu_0\varepsilon_0}\cdot\sqrt{1-\frac{\omega_p^2}{\omega^2}} = \sqrt{\mu_0\varepsilon_0}\cdot\sqrt{\omega^2-\omega_p^2}$

$\approx \omega_p\sqrt{\mu_0\varepsilon_0}\,i = \sqrt{\frac{Ne^2\mu_0}{m}}\,i$

∴ $d_p = \sqrt{\frac{m}{Ne^2\mu_0}}$

(2) $d_p = \sqrt{\frac{m_e}{Ne^2\mu_0}} \approx 2\times10^{-8}m$

(3)

∵ $d_p = \sqrt{\frac{m_e}{Ne^2\mu_0}}$

∴ 当 $N_1 > N_2$ 时，$d_{p1} < d_{p2}$

P2.4

assume: $\vec{E_1} = \hat{x}E_0\cos(\omega t-kx+\frac{\pi}{4}) + \hat{y}E_0\cos(\omega t-kx-\frac{\pi}{4})$

$\vec{E_2} = \hat{x}E_0\cos(\omega t-kx-\frac{\pi}{4}) + \hat{y}\cos(\omega t-kx+\frac{\pi}{4})$

E_1 和 E_2 都是圆偏振波

$\vec{E} = \vec{E_1}+\vec{E_2} = \hat{x}\sqrt{2}E_0\cos(\omega t-kx) + \hat{y}E_0\sin(\omega t-kx)$

$\vec{E}$ 是线偏波

∴ 题目得证.

P3.2

(1) $\varepsilon_c = 40\varepsilon_0 + i\frac{\sigma}{\omega}$

$\therefore k = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu_0\varepsilon_0}\cdot\sqrt{40}\cdot\sqrt{1+i\frac{\sigma}{40\varepsilon_0\omega}}$

$\approx \frac{2\pi\times2.5\times10^9}{3\times10^8}\cdot\sqrt{40}\times\left(1+i\cdot\frac{1}{2}\cdot\frac{\sigma}{40\varepsilon_0\omega}\right)$

$\therefore d_p = \frac{1}{k_z} = \frac{3\times10^8}{2\pi\times2.5\times10^9}\times\frac{1}{\sqrt{40}}\cdot2\times\frac{40\varepsilon_0\omega}{\sigma} = 0.0168m$

(2)

$\frac{\omega\varepsilon}{\sigma} << \sigma \longrightarrow \frac{\omega\varepsilon}{\sigma} < 0.1$

$\therefore \omega < \frac{0.1\sigma}{\varepsilon}$

$\therefore f < \frac{0.1\sigma}{2\pi\varepsilon} = 0.899\times10^6 Hz$

(3)

$\varepsilon = \varepsilon_0 + i\frac{\sigma}{\omega} \quad \because \frac{\sigma}{\varepsilon_0\omega} >> 1$

$\therefore d_p \approx \sqrt{\frac{2}{2\pi\times10^8\times4\pi\times10^{-7}\times3.54\times10^7}} \approx 8.46\times10^{-6}$

$5d_p = 4.24\times10^{-5}m$

$\therefore$ 不够厚

(4) $f = 100Hz$, $d_p = \sqrt{\frac{2}{\omega\mu_0\sigma}} = 25.2m$

$f = 5MHz$, $d_p = 0.11m$

(5) $d_p = \sqrt{\frac{2}{\omega\mu_0\sigma}} = 7.96m$

$\because \langle\vec{S}\rangle = \vec{E}\times\vec{H} = e^{-k_z r}\cdot e^{-k_z r} = e^{-2k_z r}$

$\therefore \langle\vec{S}\rangle\big|_{r=7.96} = 1.22\times10^{-11}$

5.1

(1) $R = \frac{1-p}{1+p}$, $T = \frac{2}{1+p}$

$p = \frac{\mu_i k_{tx}}{\mu_t k_{ix}} = \frac{\mu_0 \omega\sqrt{\mu_0 \varepsilon_{t}}}{\mu_0 \omega\sqrt{\mu_0 \varepsilon_{ix}}} = 2$

$\therefore R_{12} = -\frac{1}{3}$, $T_{12} = \frac{2}{3}$

(2)

$S_{ix} = S_{rx} + S_{tx}$

$\vec{S}_i = \vec{E} \times \vec{H} = \hat{y} \times (-\frac{\hat{x} k_{iz} - \hat{z} k_{ix}}{\omega\mu_0}) = \frac{k_{ix}}{\omega\mu_0}\hat{x}$

$\vec{S}_r = \hat{y} \cdot R \times \frac{R}{\omega\mu_0}(-\hat{x} k_{rz} + \hat{z} k_{zx}) = \frac{R^2 k_{rx}}{\omega\mu_0}\hat{x}$

$\vec{S}_t = \frac{T^2 k_{tx}}{\omega\mu_0}\hat{x}$

PS.2

(1) $\nabla \times \vec{H}(\vec{r}) = -i\omega\vec{D}(\vec{r}) + \vec{J}(\vec{r})$

当无外加的源时,

$\nabla \times \vec{H}(\vec{r}) = -i\omega\vec{D_0}(\vec{r}) = -i\omega\varepsilon_0\vec{E}(\vec{r})$

$\because \vec{P} = \vec{D} - \varepsilon_0\vec{E}$

$\therefore \nabla \times \vec{H}(\vec{r}) = -i\omega\varepsilon_0\vec{E} - i\omega\vec{P}$

$\therefore \vec{J}(\vec{r}) = -i\omega\vec{P}$

(2)

$\vec{J}_b = \vec{J}_e - \vec{J}_c = -i\omega\vec{P} - 6\vec{E}$

$= -i\omega\chi\varepsilon_0\vec{E} - 6\vec{E}$

$= -(i\omega\chi\varepsilon_0 + 6)\vec{E}$

$= -i\omega(\chi\varepsilon_0 + i\frac{6}{\omega})\vec{E}$

$= -i\omega(\varepsilon_0\varepsilon_r + i\frac{6}{\omega} - \varepsilon_0)\vec{E} = -i\omega(\varepsilon - \varepsilon_0)\vec{E}$

(3)

$$\nabla\times\vec{E}(\vec{r}) = i\omega\vec{B}(\vec{r})$$

$$\therefore \vec{B}(\vec{r}) = \hat{y}E_0 k_I e^{-k_I z} e^{ik_R z} - \hat{y}E_0 e^{-k_I z}\cdot ik_R e^{ik_R z}$$

$$k = k_R + ik_I = \omega\sqrt{\mu_0\varepsilon} = \omega\sqrt{\mu_0(\varepsilon_R + i\varepsilon_I)}$$

$$\therefore \vec{H}(\vec{r}) = \cancel{\hat{y}E_0(k_I e^{-k_I z}}$$

$$= \hat{y}\mu E_0(k_I - ik_R)e^{-(k_I - ik_R)z}$$

记 $\bar{k} = k_R - ik_I = \overline{k_R + ik_I}$

$$\therefore \vec{H}(\vec{r}) = \hat{y}\mu_0 E_0 \bar{k} e^{-kz}$$

P5.3

(1) $\varepsilon = \varepsilon_0(1 - \frac{\omega_p^2}{\omega^2}) = \varepsilon_0(1 - \frac{\omega_p^2}{4\omega_p^2}) = \frac{3}{4}\varepsilon_0$

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{\sqrt{\mu_1\varepsilon_1}}{\sqrt{\mu_2\varepsilon_2}} = \frac{\cancel{\sqrt{\mu_0\varepsilon_0}}}{\sqrt{\mu_0\frac{3}{4}\varepsilon_0}} = \frac{\sqrt{\mu_0\cdot\frac{3}{4}\varepsilon_0}}{\sqrt{\mu_0\cdot\varepsilon_0}} = \frac{\sqrt{3}}{2}$$

$$\therefore \theta_c = \frac{\pi}{3}$$

(2) $\theta_t + \theta_i = \frac{\pi}{2}$

$$2\sin\theta_i = \sqrt{3}\sin\theta_t$$

$\sin\theta_i \approx 0.65$ $\quad\theta_c = \theta_i \approx 0.71$

(3) 能

~~tan~~ $\frac{n_1}{n_2}$ $\quad\tan\theta_b = \frac{n_1}{n_2}$

$$\sin\theta_c = \frac{n_1}{n_2}$$

假设 $n_1 = 1,\ n_2 = 2$

则 $\theta_c = 30°$

$$\theta_b = \arctan(\frac{n_2}{n_1}\tan\theta_c) \approx 67°$$

P5.4

(1) $\omega_{pE} = \sqrt{\frac{N_E e^2}{m_e \varepsilon_0}} \approx 1.78\times10^7$

$f_{pE} = \frac{\omega_{pE}}{2\pi} = \cancel{1.12\times10^8 Hz}\ 2.83\times10^6 Hz$

$f_{pF} = \frac{1}{2\pi}\sqrt{\frac{N_F e^2}{m_e \varepsilon_0}} = \cancel{2.75\times10^8 Hz}\ 6.95\times10^6 Hz$

(2)

$\varepsilon_E = \varepsilon_0 (1 - \frac{\omega_{pE}^2}{\omega^2}) = \cancel{0.968\varepsilon_0}\ 0.999\varepsilon_0$

$\varepsilon_F = \varepsilon_0 (1 - \frac{\omega_{pF}^2}{\omega^2}) = \cancel{0.922\varepsilon_0}\ 0.995\varepsilon_0$

$\therefore \frac{\sin\theta}{\sin\theta_t} = \frac{0.995\varepsilon_0}{0.999\varepsilon_0}$

$\therefore \sin\theta_t = \cancel{1.05}\ 1.004 \sin\theta$

$\therefore \theta_t = \arcsin(\cancel{1.05}\ 1.004 \sin\theta)$

(3) $\sin 30^\circ = \frac{\sqrt{\mu_0 \varepsilon_F}}{\sqrt{\mu_0 \varepsilon_E}}$

$\therefore \frac{\varepsilon_F}{\varepsilon_E} = \frac{1}{4}$

$\therefore \frac{1 - \frac{\omega_{pF}^2}{\omega^2}}{1 - \frac{\omega_{pE}^2}{\omega^2}} = \frac{1}{4}$ $\quad \therefore f_1 \approx 7.86\times10^6 Hz$

$\sin 30^\circ = \frac{\sqrt{\mu_0 \varepsilon_F}}{\sqrt{\mu_0 \varepsilon_0}}$

$\therefore \frac{\varepsilon_F}{\varepsilon_0} = \frac{1}{4}$

$\therefore f_2 = 8.02\times10^6 Hz$