RF/Microwave Circuit and System

Lecture 11

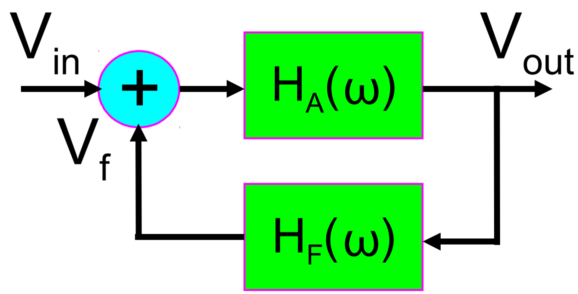
Huangfu Jiangtao

浙江大学 信息与电子工程学院
College of Information Science & Electronic Engineering, Zhejiang University

Chapter10 Oscillators and Mixers

10.1 Basic Oscillator Model

The core of any oscillator circuit is a loop that causes a positive feedback at a selected frequency.



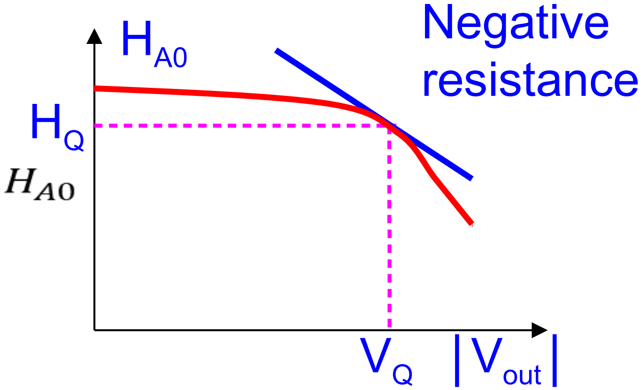
closed-loop transfer function:

$$H_{CL}(\omega)=\frac{V_{out}}{V_{in}}=\frac{H_A(\omega)}{1-H_A(\omega)H_F(\omega)}$$

Since $V_{in}=0$, loop gain:

$$H_A(\omega)H_F(\omega)=1$$

Written $H_F(\omega)=H_{Fr}(\omega)+H_{Fi}(\omega)$ and $H_A(\omega)=H_{A0}$



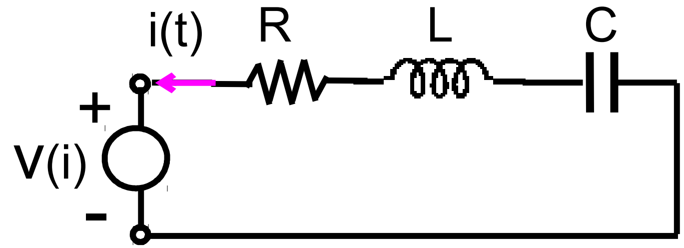
steady-state situation:

$$H_{A0}=\frac{1}{H_{Fr}(\omega)} \qquad H_{Fi}(\omega)=0$$

Initially, must ensure that $H_{A0}(\omega)H_{Fr}(\omega)>1$

At point $|V_{out}|=V_Q$, a negative slope of the curve is needed to ensure a decrease in gain for increasing voltage.

10.1.1 Negative Resistance Oscillator



Using a current-controlled voltage source, the Governing equation in terms of the current is Written as:

$$L\frac{d^2 i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=-\frac{dv(i)}{dt}$$

Set the right-hand side to zero (i.e., reach steady state and the voltage amplitude is stable), obtain the standard solution:

$$i(t)=e^{\alpha t}\left(I_1 e^{j\omega_Q t}+I_2 e^{-j\omega_Q t}\right)$$

Where $\alpha=-\frac{R}{2L}$ $and\ \omega_Q=\sqrt{\frac{1}{LC}-(\frac{R}{2L})^2}$, because $\alpha<0$, the harmonic response of the resonance circuit will reduce to zero as time progresses.

If we succeed in selecting a nonlinear device whose voltage-current response is $v(i)=v_0+R_1 i+R_2 i^2+\cdots$

$$L\frac{d^2 i}{dt^2}+R\frac{di}{dt}+\frac{1}{C}i=-\frac{dv(i)}{dt}=-R_1\frac{di}{dt}$$

ISEE

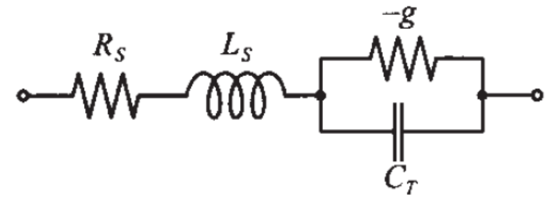
$$L\frac{d^2 i(t)}{dt^2} + (R + R_1)\frac{di(t)}{dt} + \frac{1}{C}i(t) = 0$$

To set the attenuation coefficient to zero:

$$R_1 = -R$$

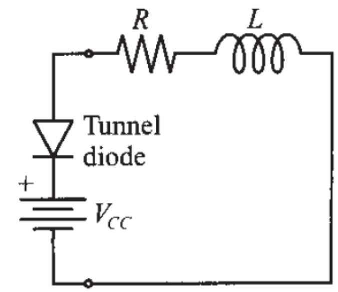
Moreover, to get the oscillations started, we require a positive attenuation coefficient, which implies R_1 to be less than $-R$

A direct way to implement such a negative resistance condition is via a tunnel diode, whose electric representation is :

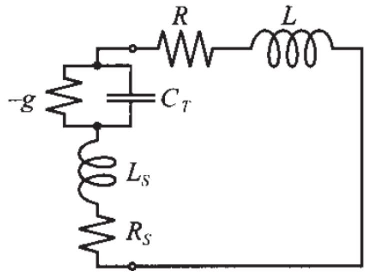


Electric circuit representation of a tunnel diode.

Since the tunnel diode already possesses an inherent capacitance, an extra capacitor in the external circuit is not required.



Tunnel diode oscillator circuit

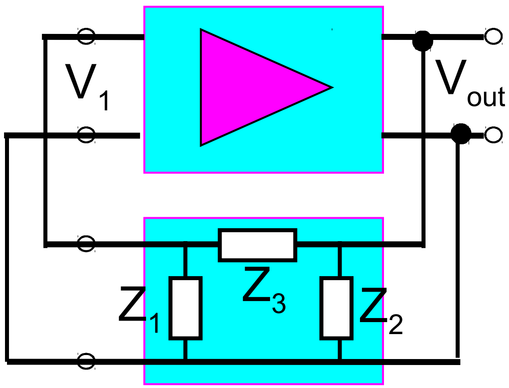


Small-signal equivalent circuit

10.1.2 Feedback Oscillator Design

For Pi-network, under high-impedance input and Output assumption:

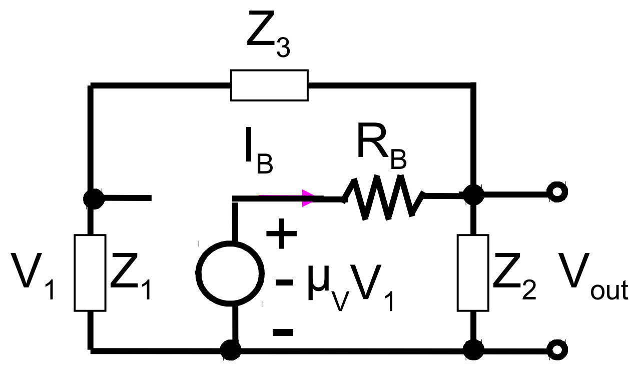
$$H_F(\omega) = \frac{V_1}{V_{out}} = \frac{Z_1}{Z_1 + Z_3}$$



Using a simple, low-frequency FET model with voltage gain μ_V and output resistance R_B

The corresponding loop equation:

$$\mu_V V_1 + I_B R_B + I_B Z_C = 0$$



Where: $1/Z_C = Y_C = 1/Z_2 + 1/(Z_1 + Z_3)$

Thus: $$H_A(\omega) = \frac{V_{out}}{V_1} = \frac{-\mu_V}{Y_C R_B + 1}$$

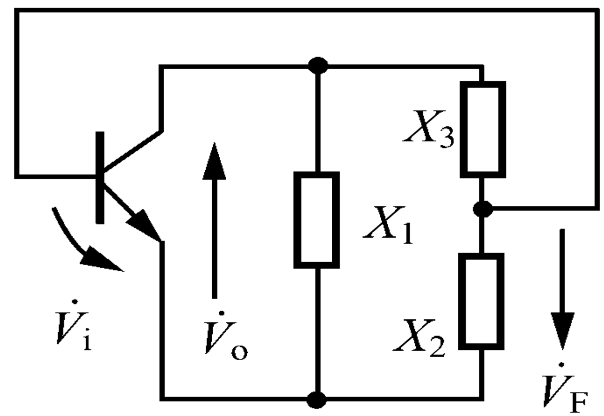
Closed-loop transfer function:

$$H_F(\omega) H_A(\omega) = \frac{-\mu_V Z_1 Z_2}{Z_1 Z_2 + Z_2 Z_3 + R_B(Z_1 + Z_2 + Z_3)} \equiv 1$$

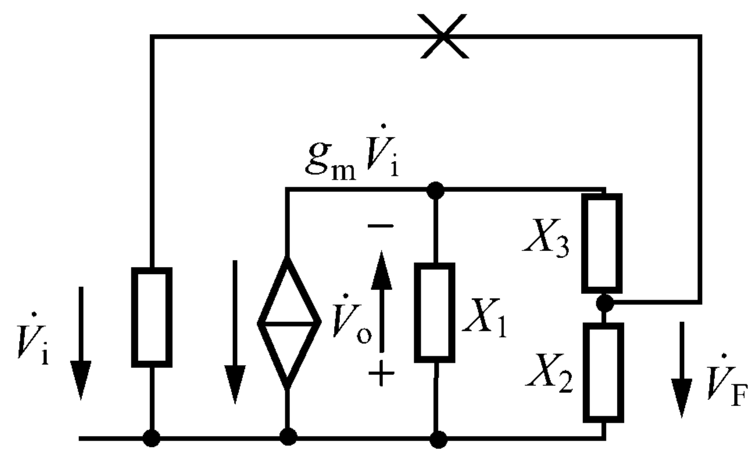
This equation allows us to design various oscillator types depending on the choice of the three impedances in the feed-back loop

10.1.2 Feedback Oscillator Design

$$V_o = g_m Z_C V_i$$



$$X_1 + X_2 + X_3 = 0$$

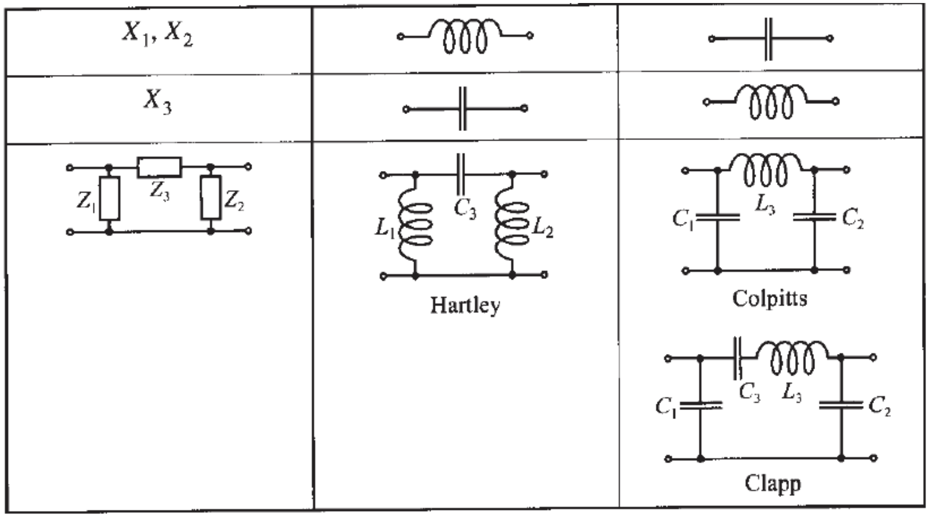


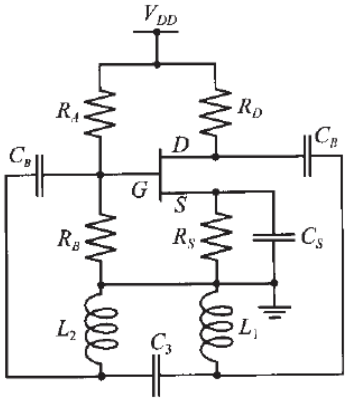
$$V_F = \frac{-V_o}{X_2 + X_3} \times X_2 = g_m Z_C \times \frac{X_2}{X_1} \times V_i$$

V_F and V_i are inphase, so X_1 and X_2 **must have the same properties**

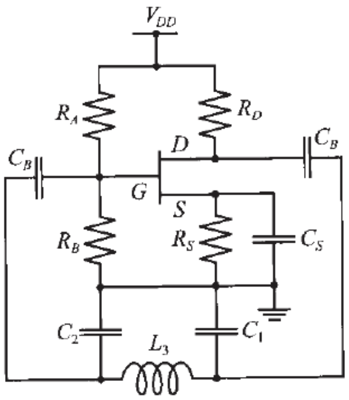
10.1.2 Feedback Oscillator Design

$$Z_1 + Z_2 + Z_3 = 0$$

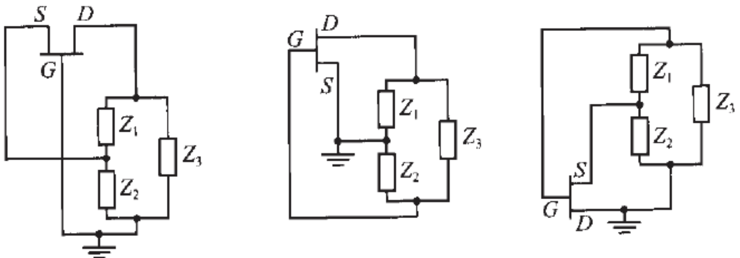




(a) Hartley oscillator

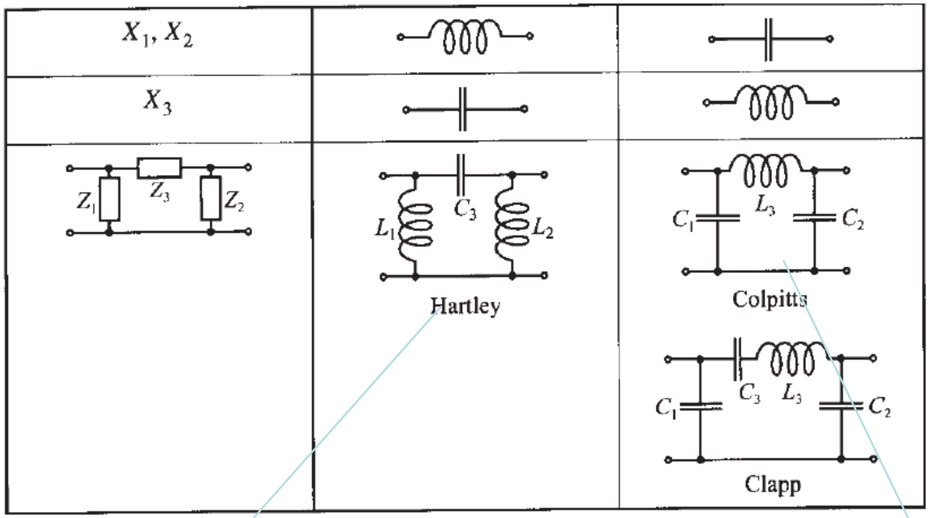


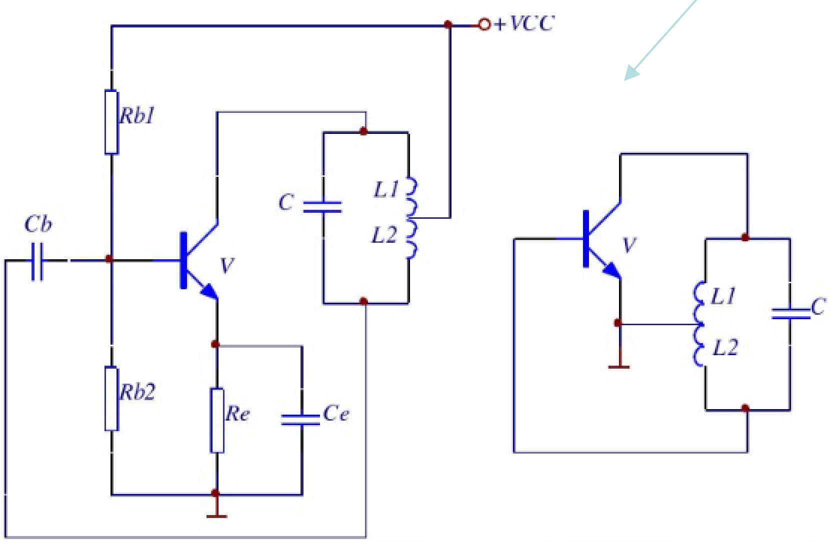
(b) Colpitts oscillator

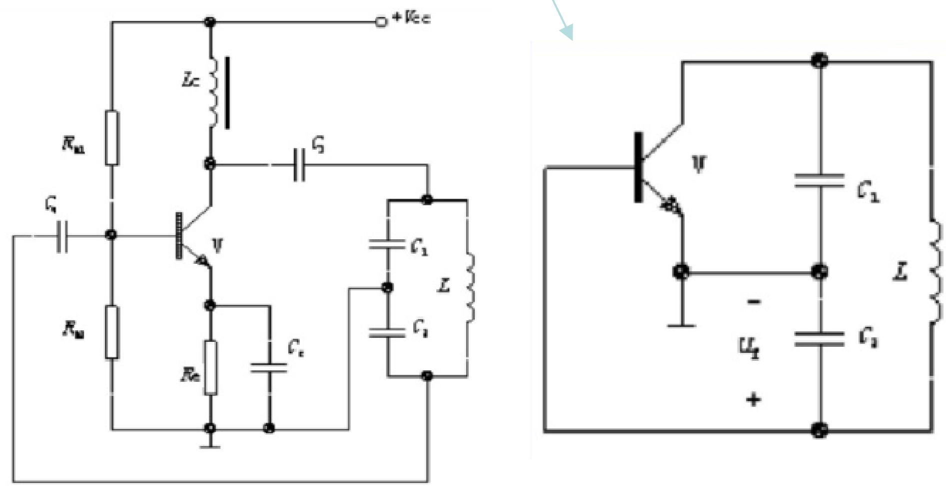


(a) Common gate (b) Common source (c) Common drain

10.1.2 Feedback Oscillator Design

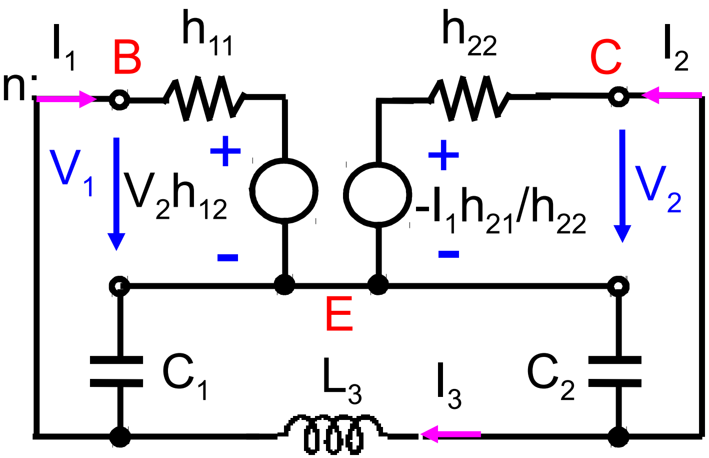






10.1.3 Design Steps

The corresponding Kirchhoff voltage mesh equation:



$$\left(h_{11}-jX_{C1}-\frac{h_{12}h_{21}}{h_{22}}\right)I_1+\frac{h_{12}}{h_{22}}I_2+jX_{C1}I_3=0$$

$$-\frac{h_{21}}{h_{22}}I_1+\left(\frac{1}{h_{22}}-jX_{C2}\right)I_2-jX_{C2}I_3=0$$

$$jX_{C1}I_1-jX_{C2}I_2+j\left(X_{L3}-X_{C1}-X_{C2}\right)I_3=0$$

Computing the determinant and setting its Imaginary portion to zero:

$$f=\frac{1}{2\pi}\sqrt{\frac{h_{22}/h_{11}+\left(C_1+C_2\right)/L_3}{C_1C_2}}$$

Setting the real part of the determinant to zero, and assuming that $h_{12} \ll 1$, yields:

$$\frac{C_1^2}{C_2^2}\left(h_{11}h_{22}-h_{12}h_{21}\right)-\frac{C_1}{C_2}h_{21}+1=0$$

Under the assumption that $h_{21}^2 \gg 4(h_{11}h_{22}-h_{12}h_{21})$

$$C_1 \approx h_{21}C_2/\left(h_{11}h_{22}-h_{12}h_{21}\right)$$

The preceding treatment deals with the h-parameters as real quantities may not be generally applicable.

Example 10-1: For a 200MHz oscillation frequency, a Colpitts BJT oscillator in common-emitter configuration has to be designed. For the bias point of $V_{CE} = 3V$ and $I_C = 3mA$, the following circuit parameters are given at room temperature of 20°C: $C_{BC} = 0.1fF$, $r_{BE} = 2k\Omega$, $r_{CE} = 10k\Omega$, $C_{BE} = 100fF$. If the inductance should not exceed $I_C = 3mA$, find values for the capacitances in the feedback loop.

Solution: compute the h-parameters for DC

200MHz

$$h_{11} = \frac{r_{BE}}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}} = 2000\Omega$$

1881-j473

$$h_{12} = \frac{j\omega C_{BC} r_{BE}}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}} = 0$$

(59+j2.4)10^{-4}

$$h_{21} = \frac{r_{BE}\left(g_m - j\omega C_{BC}\right)}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}} = 233.32$$

219-j55

$$h_{22} = \frac{1}{r_{CE}} + \frac{j\omega C_{BC}\left(1 + g_m r_{BE} + j\omega C_{BE} r_{BE}\right)}{1 + j\omega\left(C_{BE} + C_{BC}\right)r_{BE}} = 0.1\mathrm{mS}$$

0.11+j0.03

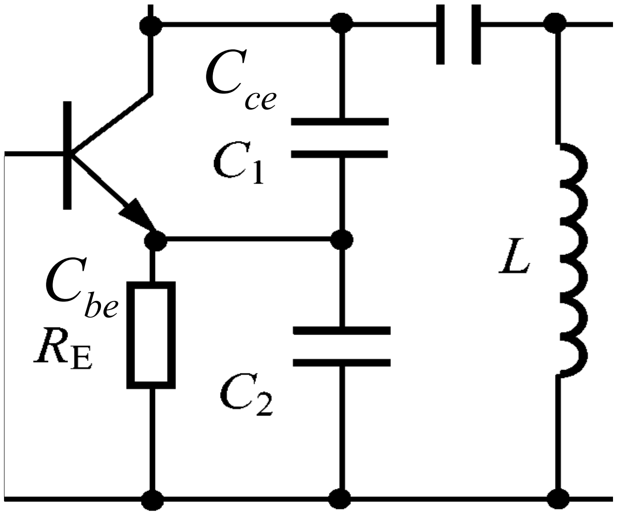
Obtain: $C_1 \approx h_{21}C_2 / \left(h_{11}h_{22} - h_{12}h_{21}\right) = 1166.6C_2 = KC_2$ A minimal amount of tuning

L=50 *nH* $f = \frac{1}{2\pi}\sqrt{\frac{h_{22}/h_{11} + \left(C_1 + C_2\right)/L_3}{C_1C_2}}$ Thus: $C_2 = 12.638pF$

Considering Interelectrode Capacitance

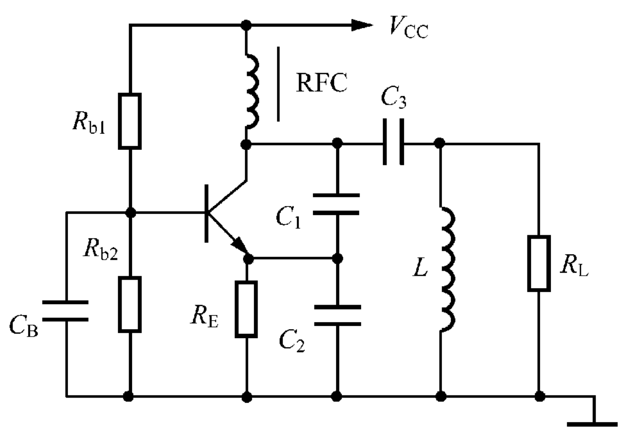
$$C_1' = C_1 + C_{ce}$$

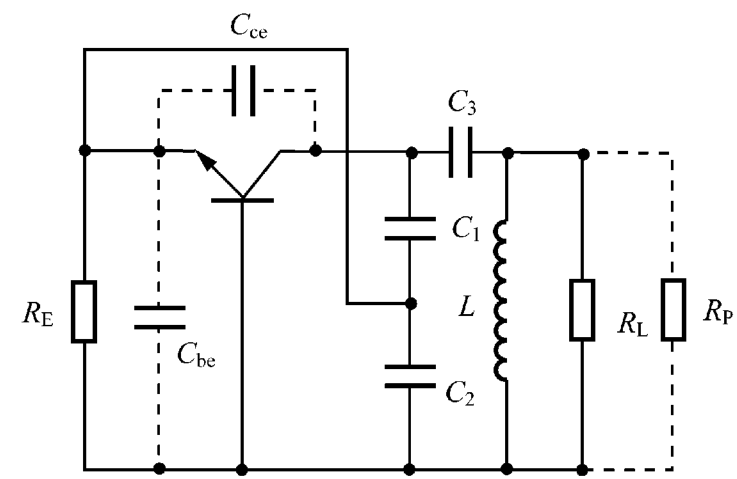
$$C_2' = C_2 + C_{be}$$



$$\omega_{osc}' = \frac{1}{\sqrt{LC_\Sigma}} = \frac{1}{\sqrt{L\dfrac{C_1'C_2'}{C_1'+C_2'}}} < \omega_{osc} = \frac{1}{\sqrt{LC_\Sigma}} = \frac{1}{\sqrt{L\dfrac{C_1C_2}{C_1+C_2}}}$$

Clapp Oscilloator





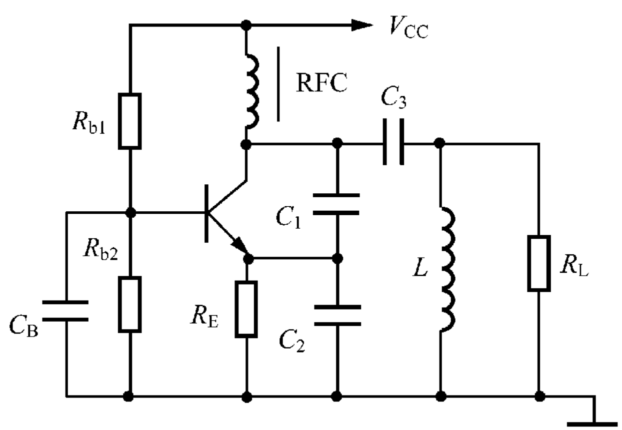
Add a small C_3 (C_3 $C_1 \ll C_3$ 、 C_2

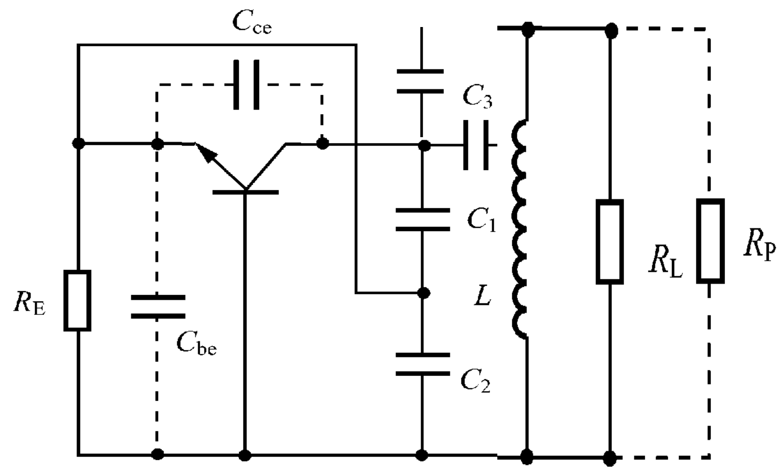
$\ll$)

$$C_\Sigma = C_1 // C_2 // C_3 \approx C_3$$

$$\omega_{osc} = \frac{1}{\sqrt{LC_\Sigma}} \approx \frac{1}{\sqrt{LC_3}}$$

Clapp Oscilloator





Add a small C_3 (C_3 $C_1 \ll C_3$ 、 C_2

$\ll$)

$$R_P' = R_P \,//\, R_L$$

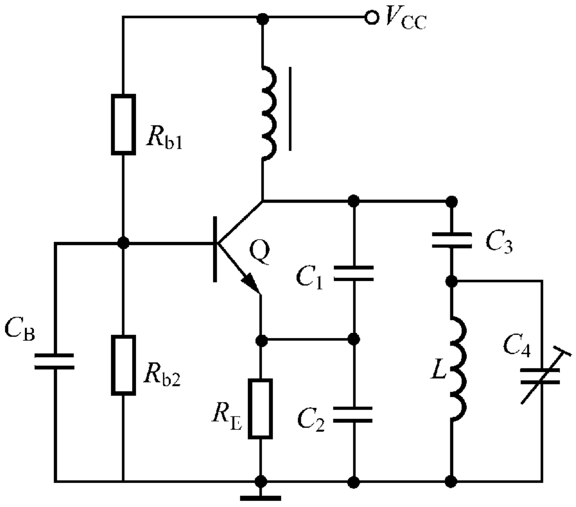
$$P_{cb} = \frac{C_3}{C_{12}+C_3} \qquad C_{12} = \frac{C_1C_2}{C_1+C_2} \qquad R_P'' = \left(\frac{C_3}{C_{12}+C_3}\right)^2 R_P'$$

$C_3 \Downarrow$ $P_{cb} \Downarrow$ $R_P'' \Downarrow$ $A = g_m R_P'' \Downarrow$ Gain $\Downarrow$ Stop work

Sillery Oscillator

The capacitors C4 are connected in parallel at both ends of the inductor L.

$$C_\Sigma \approx C_3 + C_4$$

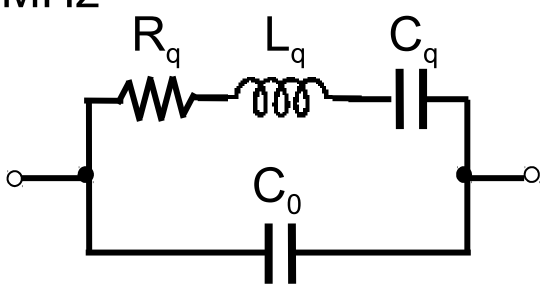


OSC frequency：

$$\omega_{osc} = \frac{1}{\sqrt{LC_\Sigma}} = \frac{1}{\sqrt{L(C_3 + C_4)}}$$

10.1.4 Quartz Oscillators

Quartz resonators can offer a number of advantages: a much higher quality factor, improved frequency stability, and immunity to temperature fluctuations. Unfortunately, they cannot be constructed to exceed approximately 250MHz



The admittance of this model:

$$Y = j\omega C_0 + \frac{1}{R_q + j\left(\omega L_q - 1/\omega C_q\right)} = G + jB$$

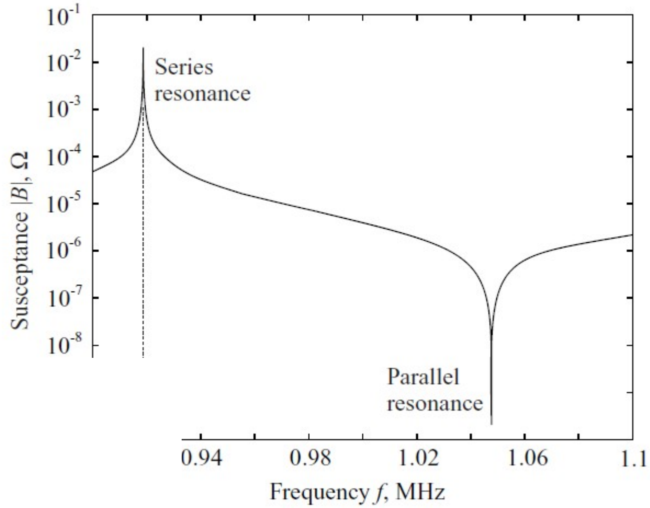
Setting the imaginary component to zero:

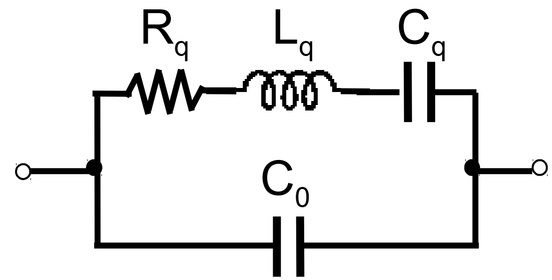
$$\omega_0 C_0 = \frac{\omega_0 L_q - 1/\omega_0 C_q}{R_q^2 + \left(\omega_0 L_q - 1/\omega C_q\right)^2}$$

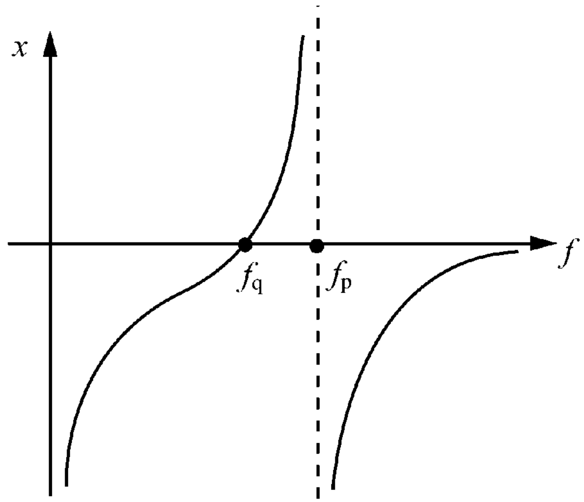
Series resonance frequencies: $f_S \approx \dfrac{1 + \dfrac{R_q^2 C_q}{2L_q}}{2\pi\sqrt{L_q C_q}}$

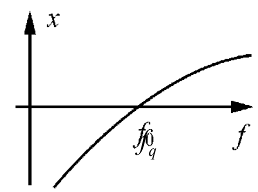
Parallel resonance frequencies: $f_P \approx \dfrac{1}{2\pi}\sqrt{\dfrac{C_q + C_0}{L_q C_q C_0}\left(1 - \dfrac{R_q^2 C_0}{2L_q}\right)}$

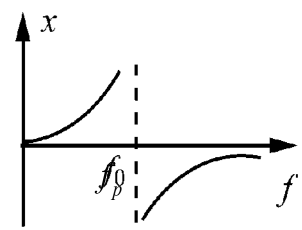
10.1.4 Quartz Oscillators







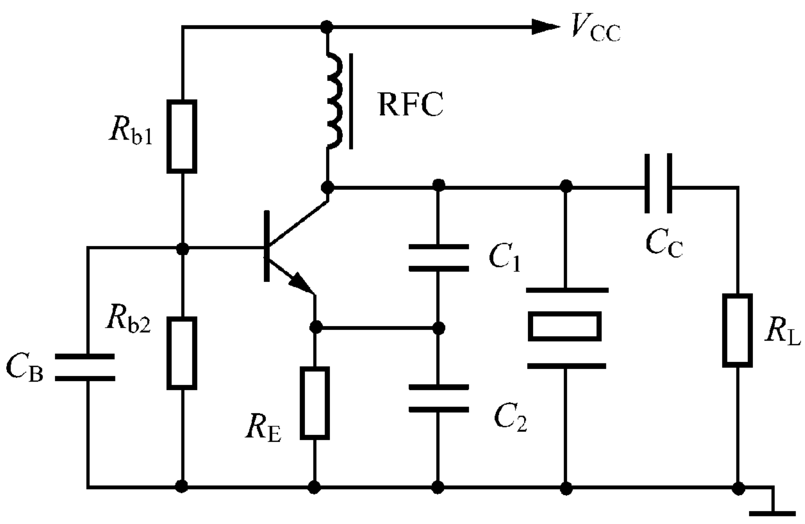


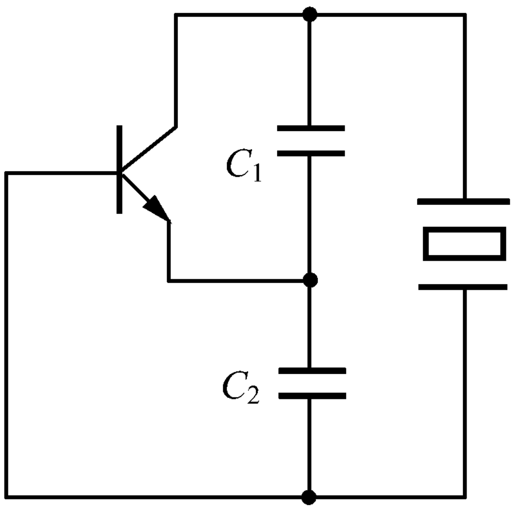


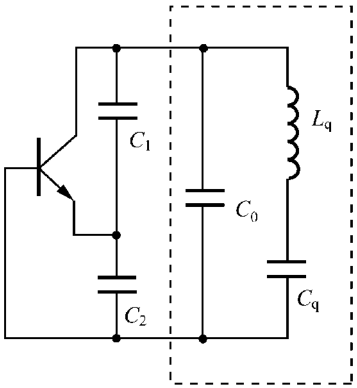
Series resonance frequencies:

Parallel resonance frequencies:

Pitch crystal oscillator circuit







External load capacitance of crystal C_L

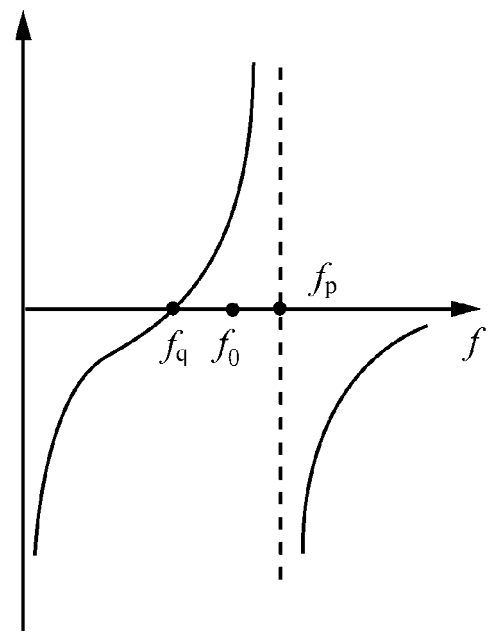
$$C_L = \frac{C_1 C_2}{C_1 + C_2}$$

C_1 **and** C_2 **in series**

$$f_o = \frac{1}{2\pi\sqrt{L_q \frac{C_q(C_0 + C_L)}{C_q + (C_0 + C_L)}}} \approx f_q\left(1 + \underline{\frac{1}{2} \times \frac{C_q}{C_0 + C_L}}\right)$$

$$f_P \approx f_q\left(1 + \underline{\frac{C_q}{2C_0}}\right)$$

$$f_q < f_o < f_p$$



External load capacitance of crystal C_L

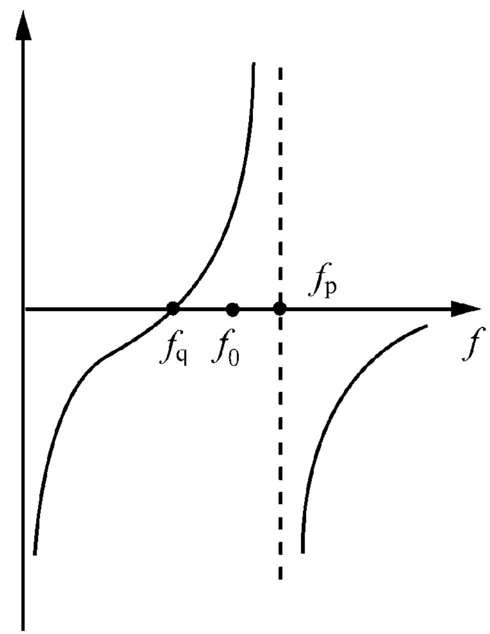
$$C_L = \frac{C_1 C_2}{C_1 + C_2}$$

C_1 **and** C_2 **in series**

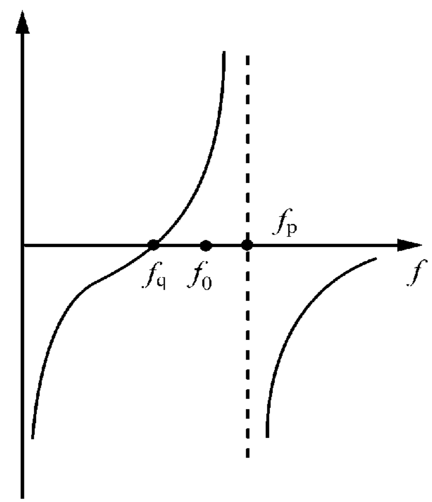
$$f_o = \frac{1}{2\pi\sqrt{L_q \frac{C_q(C_0 + C_L)}{C_q + (C_0 + C_L)}}} \approx f_q\left(1 + \underline{\frac{1}{2} \times \frac{C_q}{C_0 + C_L}}\right)$$

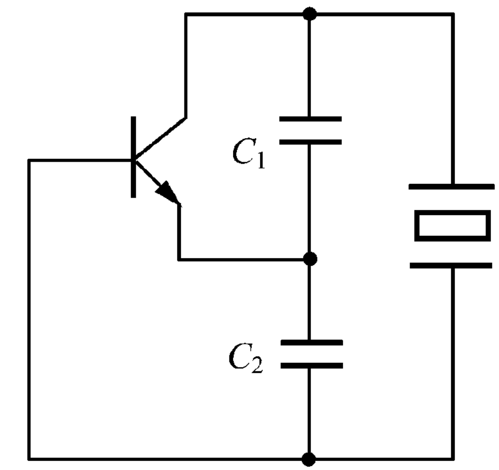
$$f_P \approx f_q\left(1 + \underline{\frac{C_q}{2C_0}}\right)$$

$$f_q < f_o < f_p$$



Crystals are electrically inductive

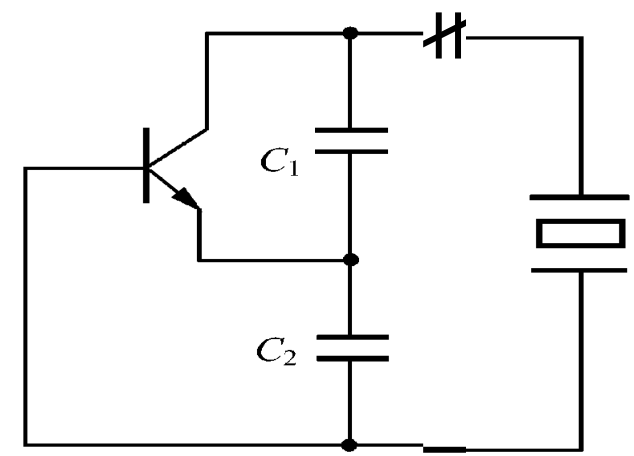




To satisfy the three-point constitution rule

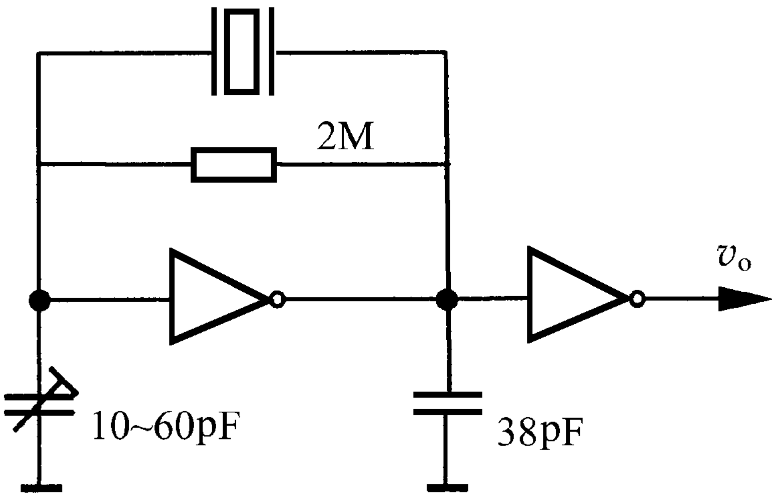
$$f_{osc} = f_o$$

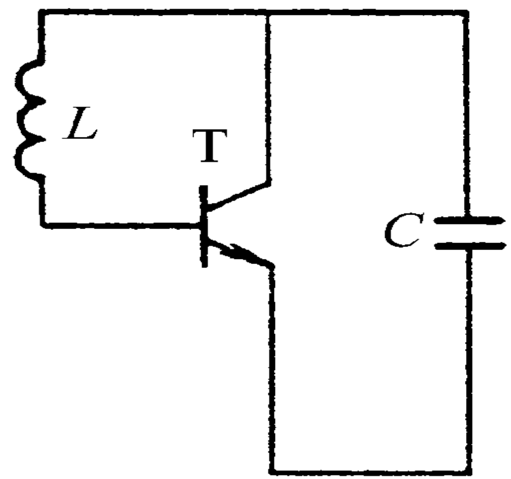
Oscillation frequency depends on the crystal and load capacitance.



Oscillation frequency can be fine tuned between f_q and f_p

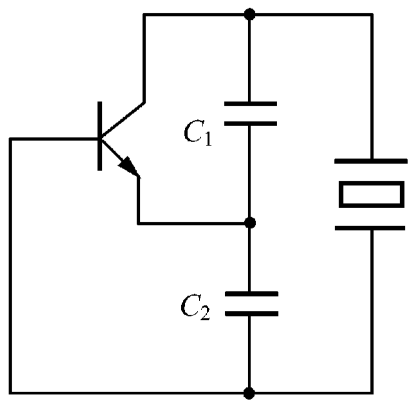
Parallel Pitch Crystal Oscillator Circuit

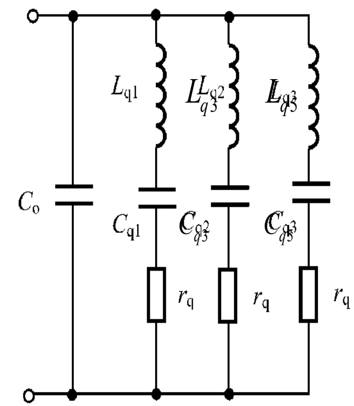


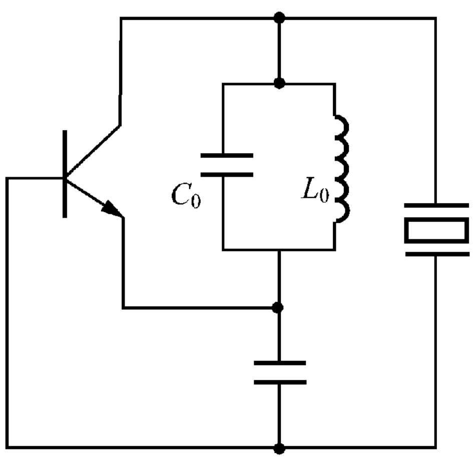


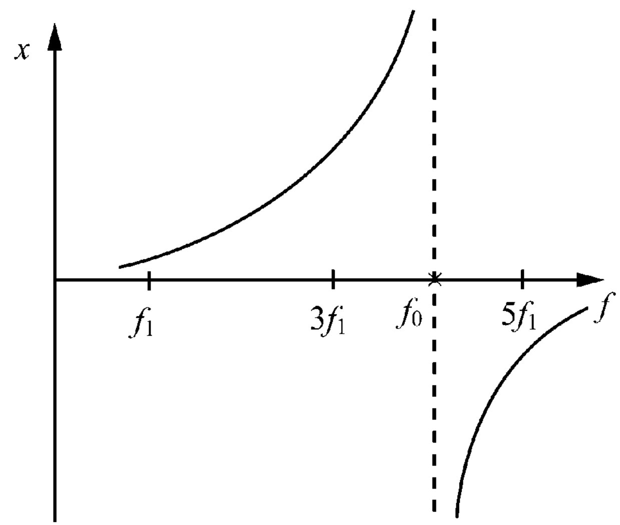
Inverter is equivalent to common reflection transistor

Overtone Crystal Oscillator





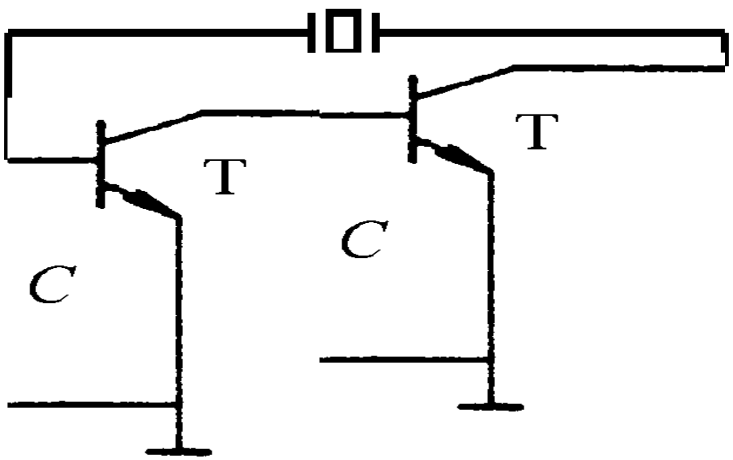


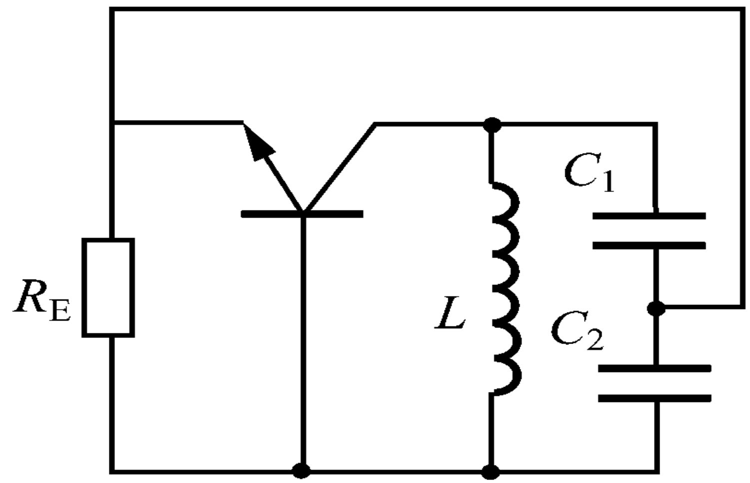


The three-point condition is satisfied at the fifth overtone, the pitch and the third overtone are not satisfied

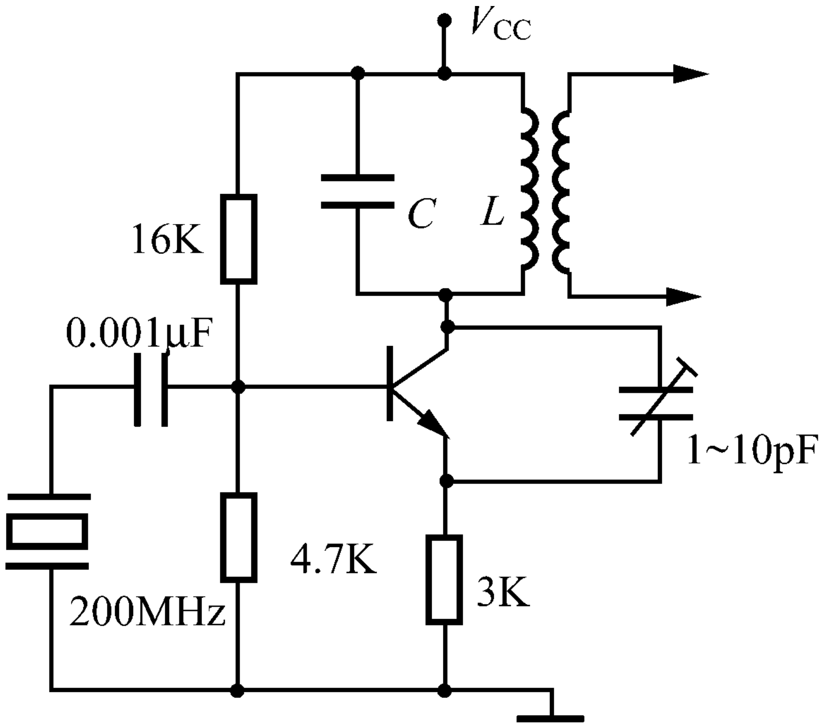
Series crystal oscillator

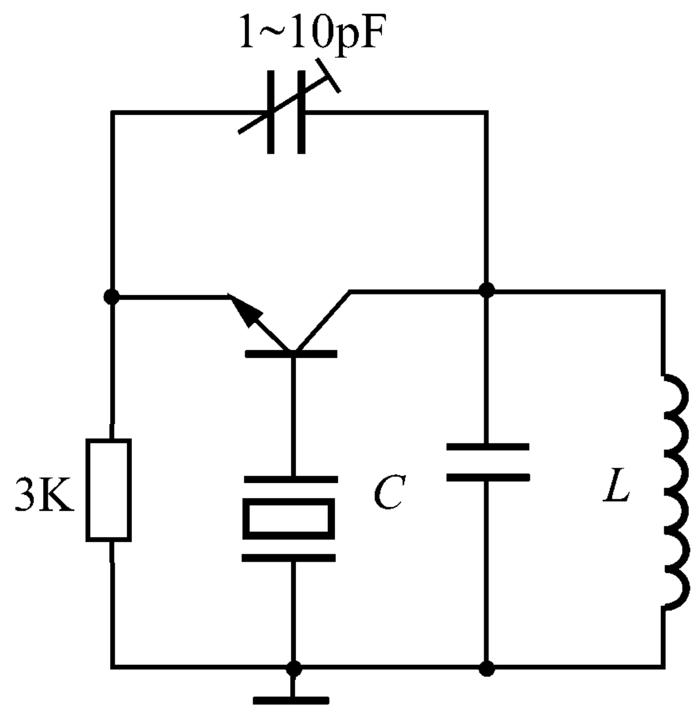
- Near the oscillation frequency, the crystal is equivalent to short circuit





Crystal on Feedback Branch



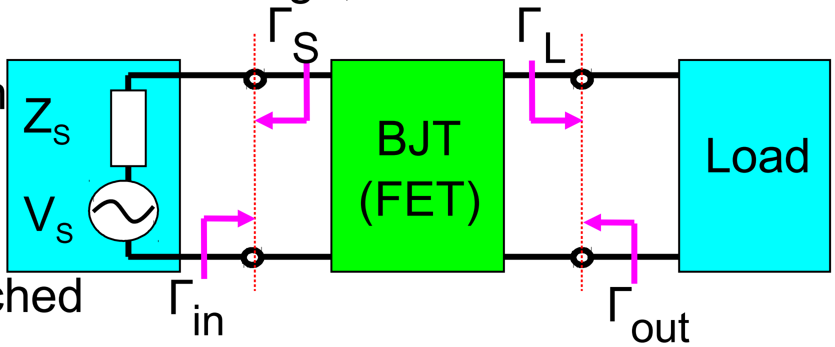


The gain of common base triode is the largest when the crystal is short-circuited.

ISEE

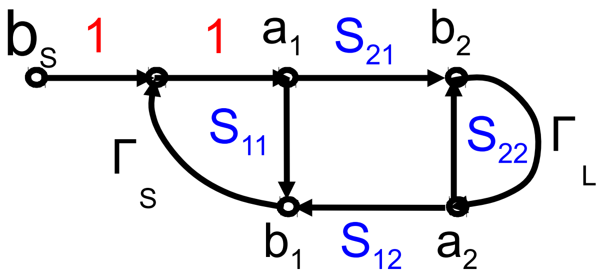
10.2 High-Frequency Oscillator Configuration

As the operating frequency approaches the GHz range, the wave nature of voltages and currents cannot be neglected the associated S-parameter representation Are required to represent the circuit's functionality.



The input reflection coefficient for matched Source impedance is:

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$



Thus: $b_S = \sqrt{Z_0}V_G/(Z_G + Z_0)$

Define the loop gain: $$\frac{b_1}{b_S} = \frac{\Gamma_{in}}{1-\Gamma_S\Gamma_{in}}$$

At a particular frequency: $\Gamma_{in}\Gamma_S=1$, the circuit is unstable and begins to oscillate

For the output side: $\Gamma_{out}\Gamma_L=1$

Ensure: $$k = \frac{1-|S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}||S_{21}|} < 1$$

10.2 High-Frequency Oscillator Configuration

It is interesting to note that if the oscillation condition is met either at the input or output port, the circuit is oscillating at both ports. This is directly seen by comparing the reflection coefficients at the input and output ports. We know that

$$\frac{1}{\Gamma_{\text{in}}} = \frac{1 - S_{22}\Gamma_L}{S_{11} - \Delta\Gamma_L} \equiv \Gamma_S \tag{10.27}$$

and solving for Γ_L yields

$$\Gamma_L = \frac{1 - S_{11}\Gamma_S}{S_{22} - \Delta\Gamma_S} \tag{10.28}$$

However, Γ_{out} can also be written as

$$\Gamma_{\text{out}} = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S} \tag{10.29}$$

Therefore, we conclude that (10.28) is the inverse of (10.29), and thus

$$\Gamma_L = 1/\Gamma_{\text{out}} \tag{10.30}$$

ISEE

10.2.1 Fixed-Frequency Oscillators

Example 10-4: Design a series feedback oscillator at $f = 1.5GHz$. And $V_{CE} = 3V$ and $V_{BE} = 3V$, $S_{11} = 1.47\angle 125°$, $S_{12} = 0.327\angle 130°$, $S_{21} = 2.2\angle -63°$ and $S_{22} = 1.23\angle -45°$.

Solution:

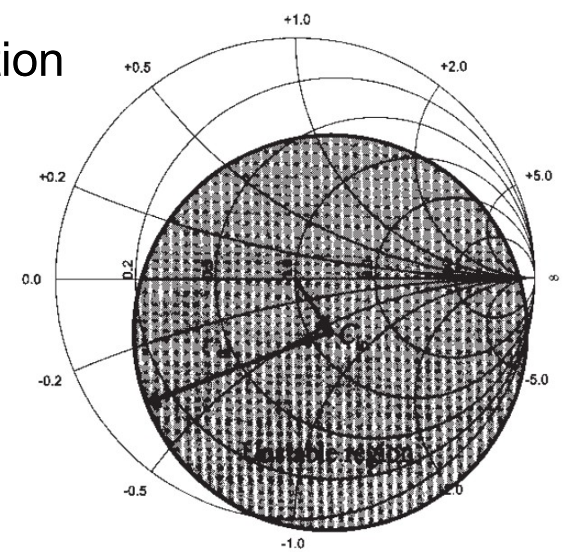
Ensure that the transistor is at least potentially unstable

$$k = \left(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2\right) / \left(2|S_{12}||S_{21}|\right) = -0.975$$

Plot the input stability circle to choose a reflection coefficient for the input matching network

$$r_{in} = \frac{|S_{12}S_{21}|}{\left||S_{11}|^2 - |\Delta|^2\right|} = 0.82$$

$$C_{in} = \frac{\left(S_{11} - \Delta S_{22}^*\right)^*}{|S_{11}|^2 - |\Delta|^2} = 0.27\angle -57^\circ$$



Since $|C_{in}| < r_{in}$ *and* $|S_{22}| > 1$, the stable region is outside of the shaded circle.

Theoretically, any Γ_s residing inside of the stability circle would satisfy the requirements. In practice, however, we would like to choose Γ_s such that it maximizes the output reflection coefficient:

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

When $\Gamma_S = S_{11}^{-1}$, we obtain an infinite output Γ_S , which results in $\Gamma_L = 0$. The Slightest deviation from the 50Ω value results in ceasing all oscillations, so we choose Γ_S somewhat close, but not exactly equal to S_{11}^{-1}.

Select: $\Gamma_S = 0.65\angle - 125°$

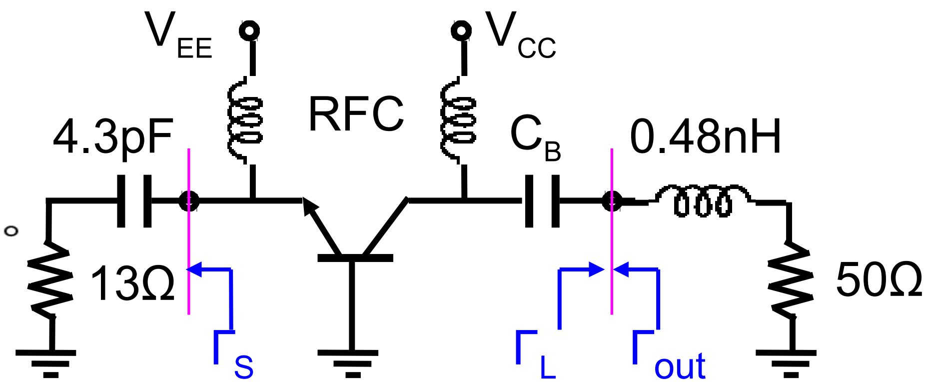
$$Z_S = (13 - j25)\Omega$$

Result in:

$$\Gamma_{out} = 14.67\angle - 36.85°$$

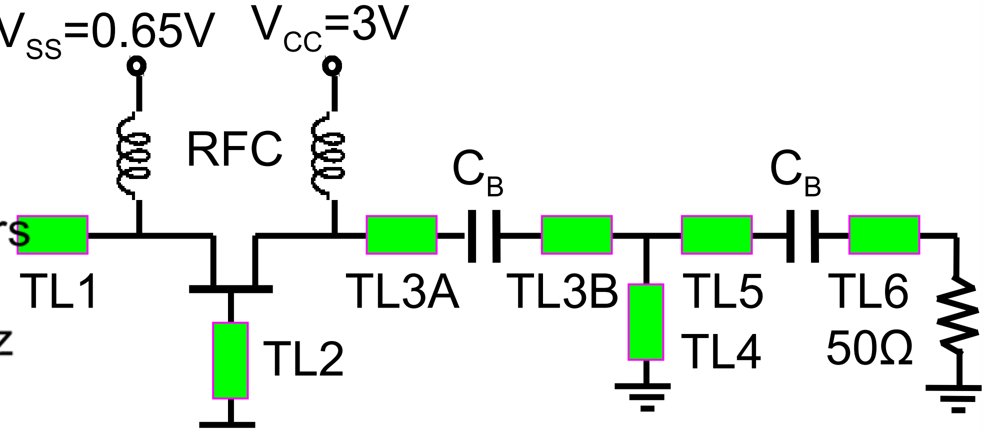
Obtain: $\Gamma_L = \Gamma_{out}^{-1} = 0.068\angle 36.85°$

$$Z_L = (55.6 + j4.57)\Omega = -Z_{out}$$



As the output power of the oscillator begins to build up, the power dependence of The transistor's S-parameters results in a less negative $R_{out} = Re\{Z_{out}\}$ for increasing output power. Thus, it is necessary to choose $R_L = Re\{Z_L\}$ such that $R_L + R_{out} < 0$. In practice, a value of $R_L = -\frac{R_{out}}{3}$ is often used.

For high-frequency applications, a more realistic design requires the use of distributed elements.

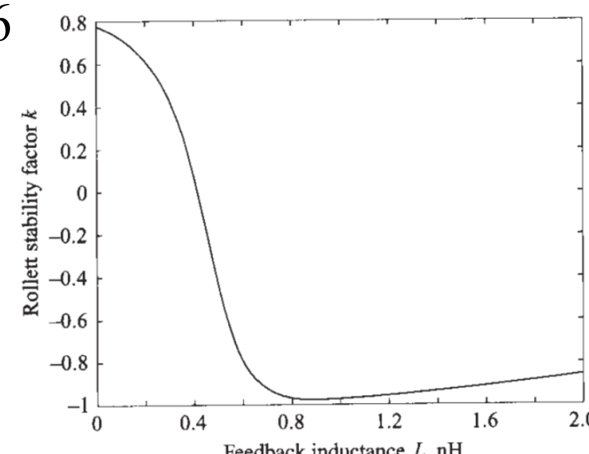


Example 10-5: The S-parameters of the GaAs FET in common-gate configuration are measured at 10GHz and have the following values: $S_{11} = 0.37\angle -176°$, $S_{11} = 0.37\angle -176°$, $S_{11} = 0.37\angle -176°$, and $S_{11} = 0.37\angle -176°$. Design an oscillator with 10GHz fundamental frequency. Furthermore, match the oscillator to a 50Ω load impedance.

Solution:

1. Check the stability of the transistor by computing the Rollett stability factor:

$$k = \left(1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2\right) / \left(2|S_{12}||S_{21}|\right) = 0.776$$



Attempt to increase the instability by connecting a feedback inductor to the gate of the transistor. It is Seen that maximum instability is achieved for $L = 0.9nH$. One of the ways to realize an inductance is to replace it with a short-circuit transmission line stub. The electrical length of the transmission line assuming 50Ω characteristic line impedance:

$$\beta l = \arctan\left(\frac{\omega L}{Z_0}\right) = 48.5^\circ$$

The resulting S-parameters for the FET with a short-circuited stub connected to the gate contact are:

$$[S] = \begin{bmatrix} 1.01\angle 169^\circ & 0.29\angle 148^\circ \\ 2.04\angle -33^\circ & 1.36\angle -34^\circ \end{bmatrix}$$

2. Develop an input matching network.

Select $\Gamma_S = 1\angle -160°$, which corresponds to a source impedance of $Z_S = -j8.8\Omega$ and which can be realized as an open-circuit stub with a 50Ω characteristic impedance and $80°$ electrical length.

The output reflection coefficient:

$$\Gamma_{out} = S_{22} + S_{12}S_{21}\Gamma_S / (1 - S_{11}\Gamma_S) = 4.18\angle 26.7^\circ$$

$$Z_{out} = (-74.8 + j17.1\Omega)$$

Due to the power dependence of the transistor's S-parameters, choose the real Portion of the load impedance to be slightly smaller than $-R_{out}$:

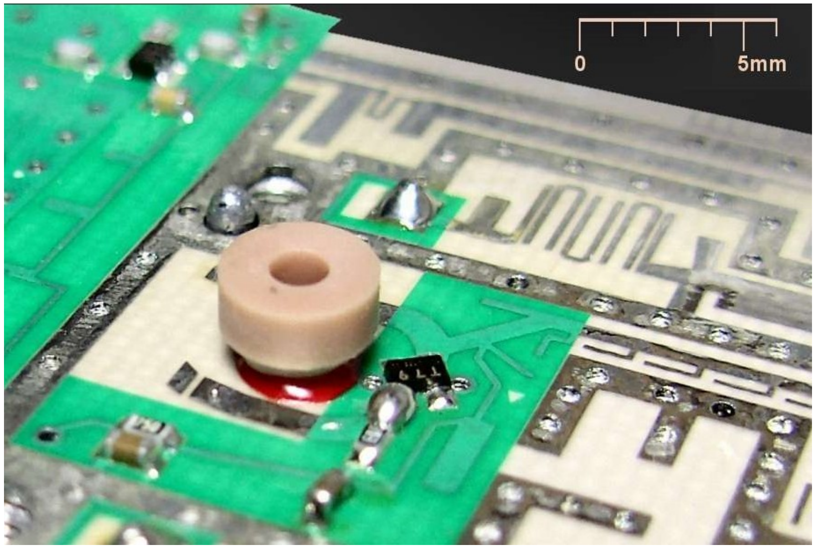
$$Z_L = (70 - j17.1\Omega)$$

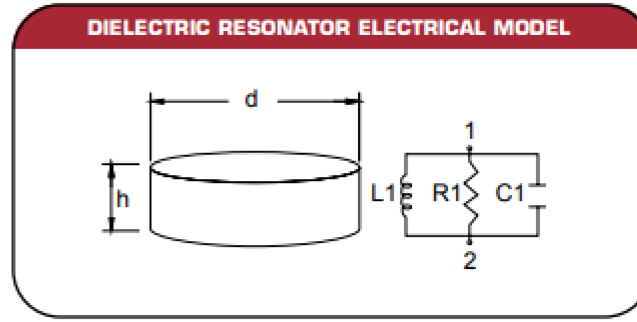
Transmission line	Electrical length, deg.	Width, mil	Length, mil
TL1	80	74	141
TL2	48.5	74	86
TL3	67	74	118
TL4	66	74	116

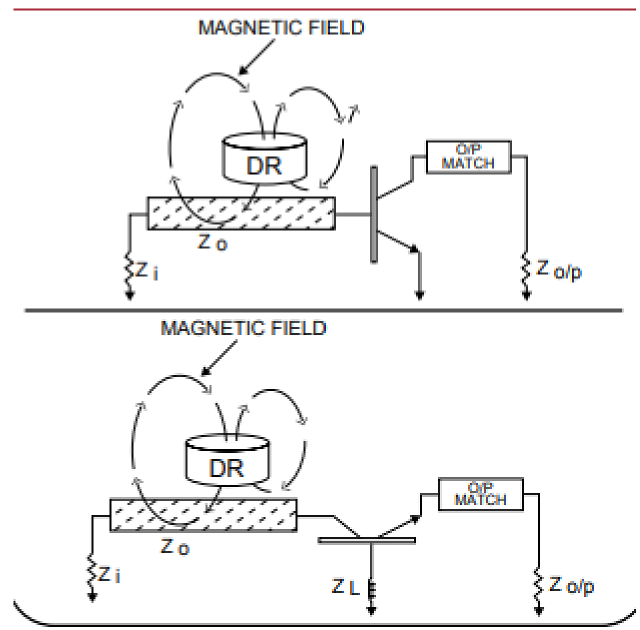
The transformation of the 50Ω load impedance to Z_L is done through a matching network consisting of a 50Ω transmission line with an electrical length of $67°$ and a short-circuit stub of $66°$ length.

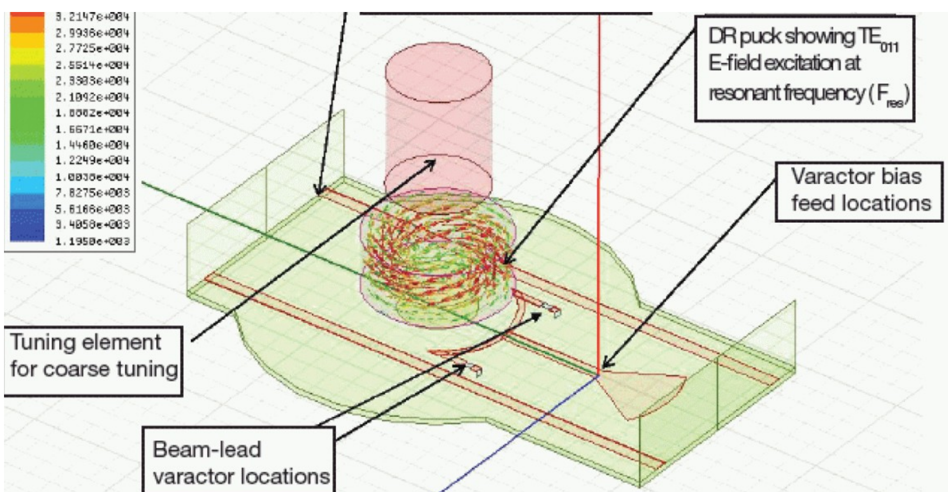
The TL3 line is cut into two halves, to accommodate the blocking capacitor. The lines TL5 and TL6 can have any length since they are connected to a 50Ω load.

10.2.2 Dielectric Resonator Oscillators



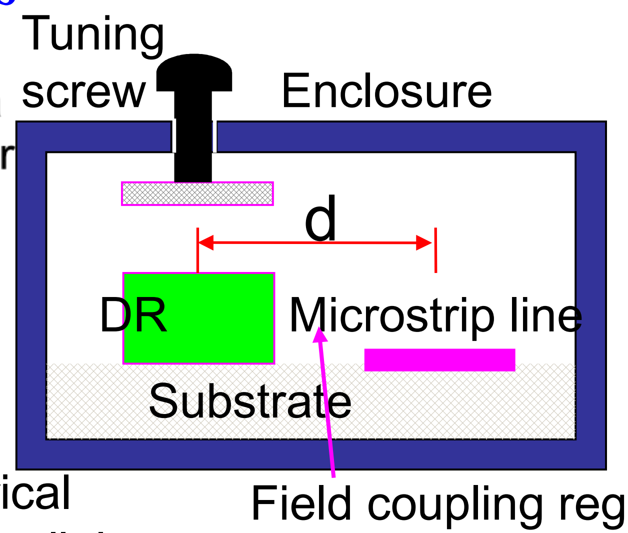






10.2.2 Dielectric Resonator Oscillators

Where dealing with microstrip line realizations, a DR can be added to provide a very high-Q oscillator design(up to 10^5) with extraordinary temperature stability of better than $\pm 10\ ppm/°C$.

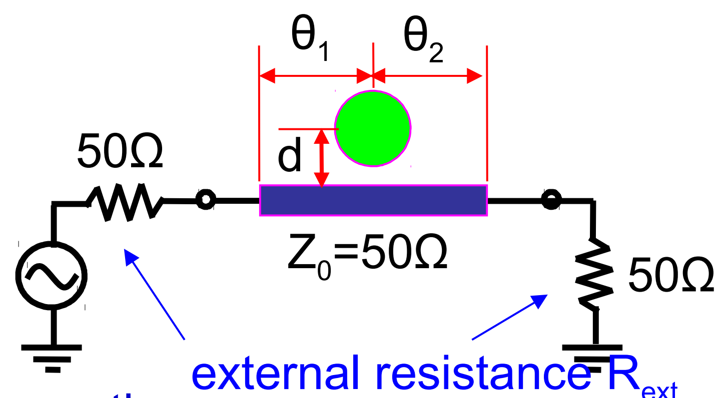


DR can either be placed on top or next to the Microstrip line in a metallic enclosure. The electric Field coupling between the strip line and the cylindrical Resonator can be modeled near resonance as a parallel RLC circuit. The tuning screw permits a geometric adjustment which translates into a change of the resonance frequency.

Resonance frequency: $\omega_0 = 1/\sqrt{LC}$

Unloaded Q: $Q_u = \dfrac{R}{\omega_0 L} = \omega_0 RC$

Coupling coefficient: $\beta = \dfrac{R}{R_{ext}} = \dfrac{R}{2Z_0} = \dfrac{\omega_0 Q_u L}{2Z_0}$

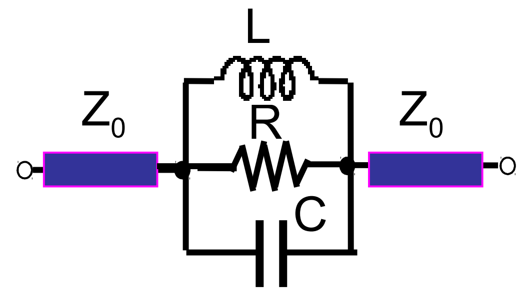


Similar to a transformer, β quantifies the electromagnetic linkage between the resonator and the microstrip line, with typical values in the range of 2 to 20.

β is also employed to describe the relationship between the unloaded(Q_u), loaded (Q_L), and external(Q_E) quality factors: $Q_u = \beta Q_E = (1+\beta) Q_L$

$$Z_{DR} = \frac{R}{1 + j\omega RC - jR/\omega L} = \frac{R}{1 + jQ_u\left(\omega/\omega_0 - \omega_0/\omega\right)}$$

$$= \frac{R}{1 + jQ_u\left(\omega^2 - \omega_0^2\right)/\omega\omega_0} \approx \frac{R}{1 + j2Q_u \Delta f / f_0}$$



Transmission line model

where $\Delta f = f - f_0$, $\omega + \omega_0 \approx 2\omega_0$

Normalized with respect to Z_0 near resonance: $z_{DR} \approx \dfrac{R/Z_0}{1 + j2Q_u \Delta f / f_0} = 2\beta$

S parameter of the transmission line segments:

$$[S] = \begin{bmatrix} 0 & e^{-j\theta_1} \\ e^{-j\theta_1} & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\beta}{\beta+1} & \dfrac{1}{\beta+1} \\ \dfrac{1}{\beta+1} & \dfrac{\beta}{\beta+1} \end{bmatrix} \begin{bmatrix} 0 & e^{-j\theta_2} \\ e^{-j\theta_2} & 0 \end{bmatrix} = \begin{bmatrix} \dfrac{\beta e^{-j(\theta_1+\theta_2)}}{\beta+1} & \dfrac{e^{-j(\theta_1+\theta_2)}}{\beta+1} \\ \dfrac{e^{-j(\theta_1+\theta_2)}}{\beta+1} & \dfrac{\beta e^{-j(\theta_1+\theta_2)}}{\beta+1} \end{bmatrix}$$

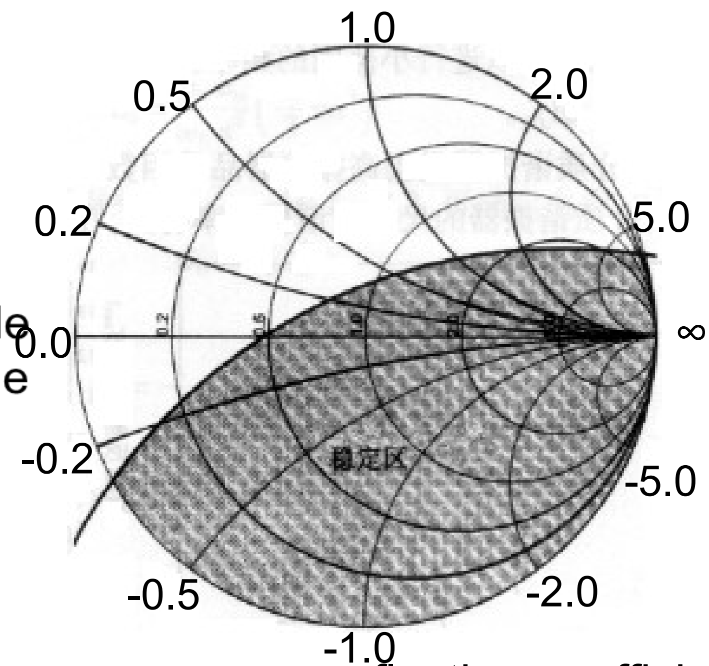
If the electric line length is equal on both sides of the DR, we obtain:

$$\theta_1 = \theta_2 = \theta = 2\pi\lambda/(l/2)$$

Thus: $$\Gamma_{in}(\omega_0) = S_{11} = \beta e^{-j2\theta}/(\beta+1) = S_{22} = \Gamma_{out}(\omega_0)$$

The design engineer specifies a particular resonance frequency and board material, and the manufacturer will provide DR in terms of diameter, length, tuning screw extension, distance d from the microstrip line, cavity material, the coupling parameter and the unloaded Q.

Example 10-6: Design an 8GHz DRO using a GaAs FET whose S-parameter at $f_0 = 8GHz$ are $S_{11} = 1.1\angle 170°$, $S_{12} = 0.4\angle -98°$, $S_{21} = 1.5\angle 163°$, and $S_{22} = 0.9\angle -170°$, $\beta = 7$, $Q_u = 5000$. Find the length of the 50Ω microstrip line at the input port side of the FET, if the DR is located in the middle. Assume the DR is terminated with a 50Ω resistor.



Solution:

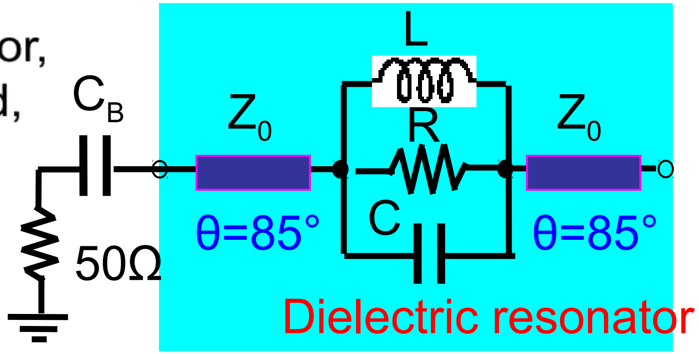
The input stability circle of the FET at $f_0 = 8GHz$

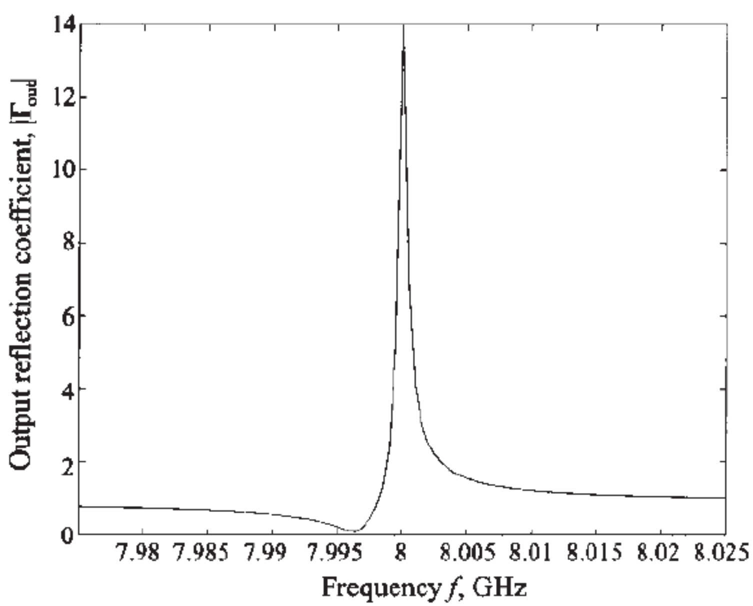
To satisfy the oscillation conditions we have to choose a source reflection coefficient Somewhere in the non-shaded area.

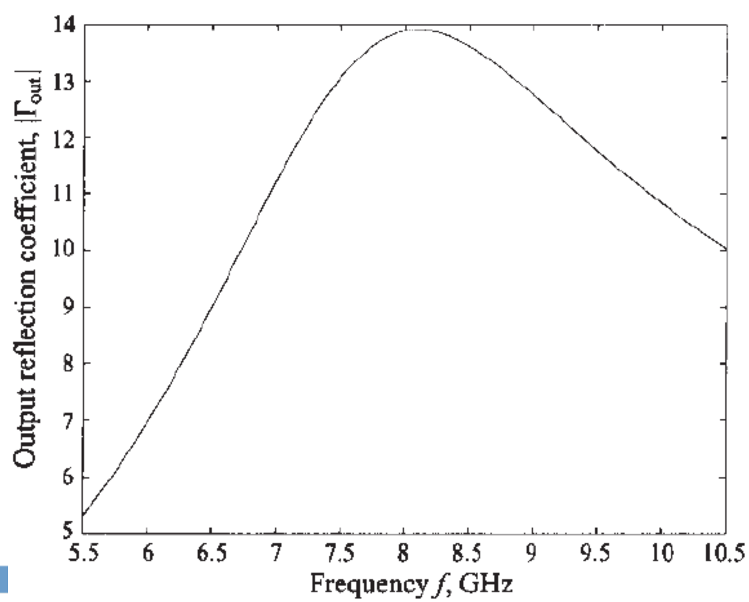
Since the termination resistance for the dielectric resonator is equal to the characteristic line impedance, the output reflection coefficient of the DR is:

$$\Gamma_S = \frac{\beta}{1+\beta} e^{-j2\Theta} = 0.875 e^{-j2\Theta}$$

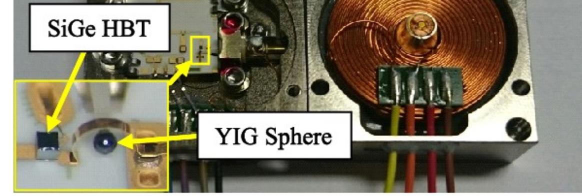
To maximize the output reflection of the transistor, we have to chose Γ_S close to S_{11}^{-1}. Since $|\Gamma_S|$ is fixed, the best we can do is to select Θ such that the phase angle of Γ_S is equal to the phase of S_{11}^{-1}, or $-2\Theta = \angle S_{11}^{-1} = -\angle S_{11}$, leading to $\Theta = 85°$







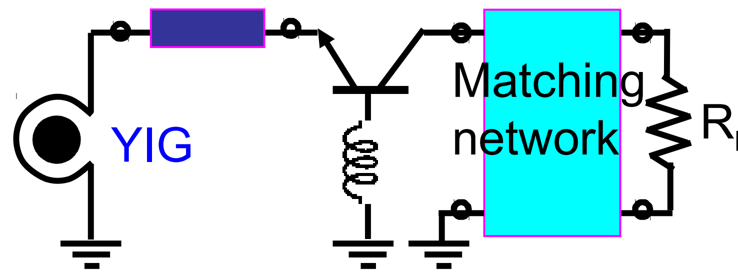
10.2.3 YIG-Tuned Oscillator



The dielectric resonator allows tuning over a very narrow band around the Resonance frequency. A magnetic element named YIG whose effective permeability can be externally controlled through a static magnetic bias field H_0 offers a wideband tunable oscillator design with a tuning range of more than a decade of bandwidth.

Unloaded quality factor: $$Q_u = \frac{-4\pi M_S/3 + H_0}{H_L}$$

Where M_S is the saturation magnetization and H_L is the resonance line width



Precessional motion angular frequency: $$\omega_m = 2\pi\gamma\left(4\pi M_S\right) = 8\pi\gamma M_S$$

Resonance frequency: $$\omega_0 = 2\pi\gamma H_0$$

Thus: $$L_0 = \frac{\mu_0\omega_m}{\omega_0 d^2}\left(\frac{4}{3}\pi a^3\right), \quad C_0 = L_0\omega_0^2, \quad G_0 = \frac{d^2}{\mu_0\omega_m Q_u\left(\frac{4}{3}\pi a^2\right)}$$

With a being the radius of the YIG sphere, d is the diameter of the coupling loop.

ISEE

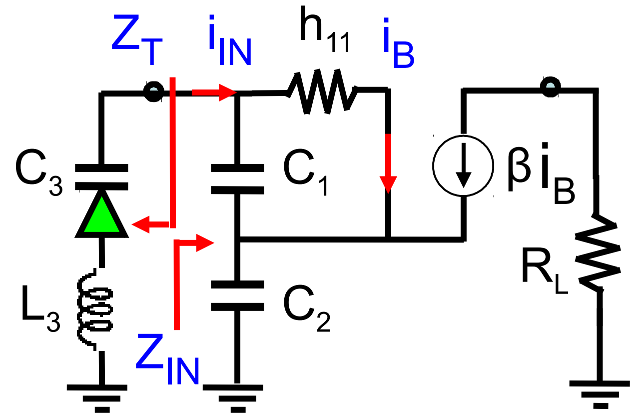
10.2.4 Voltage-Controlled Oscillator

For a varactor diode, the capacitance: $C_V = \dfrac{C_{V0}}{\sqrt{1 - V_Q / V_{diff}}}$ that can be affected by the reverse bias V_Q

The feedback loop euqtion:

$$\left(Z_{C1} + Z_{C2}\right) i_{IN} - Z_{C1} i_B + Z_{C2} \beta i_B = v_{IN}$$

$$\left(h_{11} + Z_{C1}\right) i_B - Z_{C2} i_{IN} = 0$$



Thus: $$Z_{IN} = \frac{h_{11}\left(Z_{C1} + Z_{C2}\right) + Z_{C1} Z_{C2}\left(1 + \beta\right)}{h_{11} + Z_{C1}}$$

$$\approx \frac{1}{j\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) - \frac{\beta}{h_{11}}\left(\frac{1}{\omega^2 C_1 C_2}\right)$$

$h_{11} >> Z_{C1}$， $1+\beta \approx \beta$, $g_m = \dfrac{\beta}{h_{11}}$

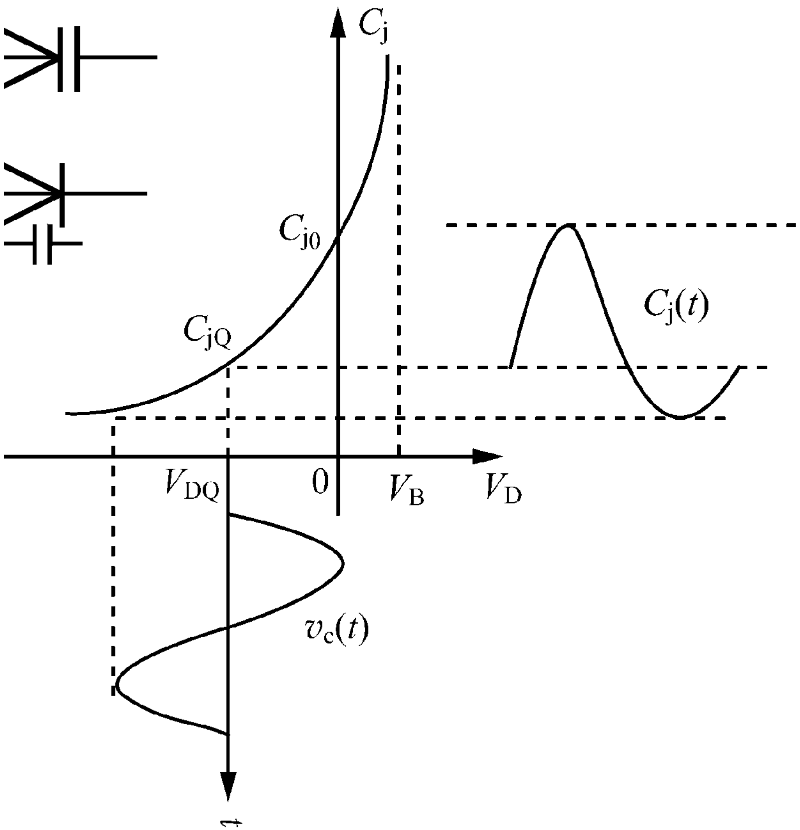
$$X_1 + X_2 + X_3 = j\left(\omega_0 L_3 - \frac{1}{\omega_0 C_3}\right) + \frac{1}{j\omega_0}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = 0$$

Resonance frequency: $$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}$$

The combined resistance of the varactor diode must be equal to or less than $|R_{IN}|$ in order to create sustained oscillations

Diode reverse bias

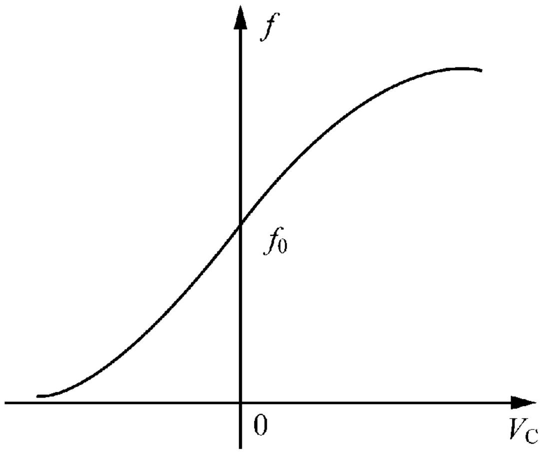
$$C_j = \frac{C_{j0}}{\left(1 - \frac{V_D}{V_B}\right)^n} \qquad (V_D < 0)$$



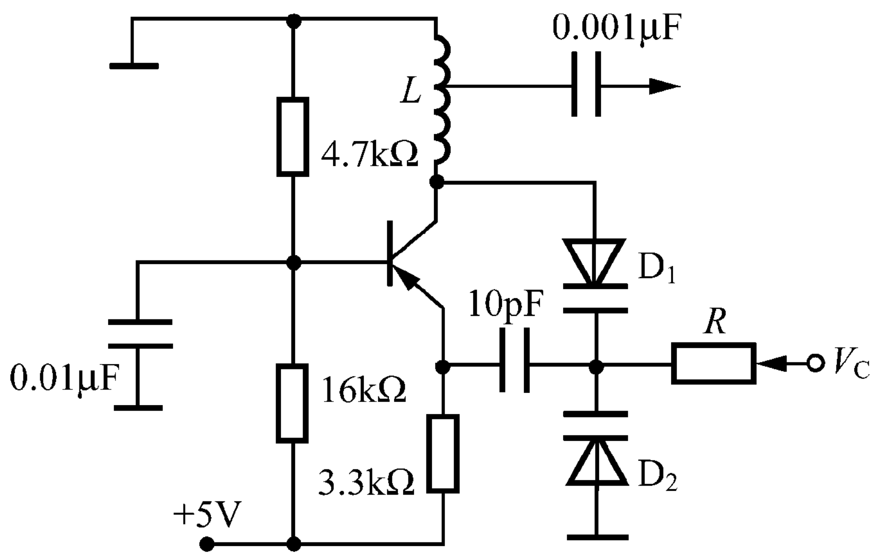
Unit $(rad/s)/V \quad Hz/V$

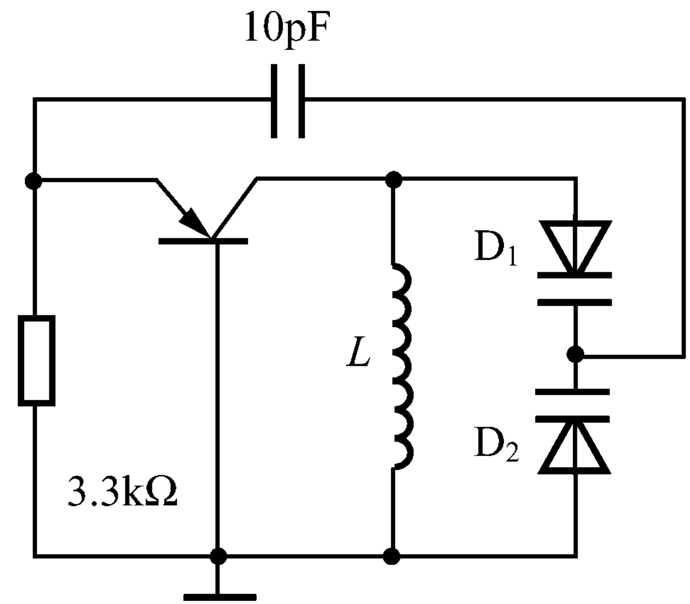
frequency range $\boldsymbol{f}_{\mathbf{max}} \sim \boldsymbol{f}_{\mathbf{min}}$

Linearity $f = f_0 + A_0 v$



Voltage control sensitivity——A_0（VCO gain）

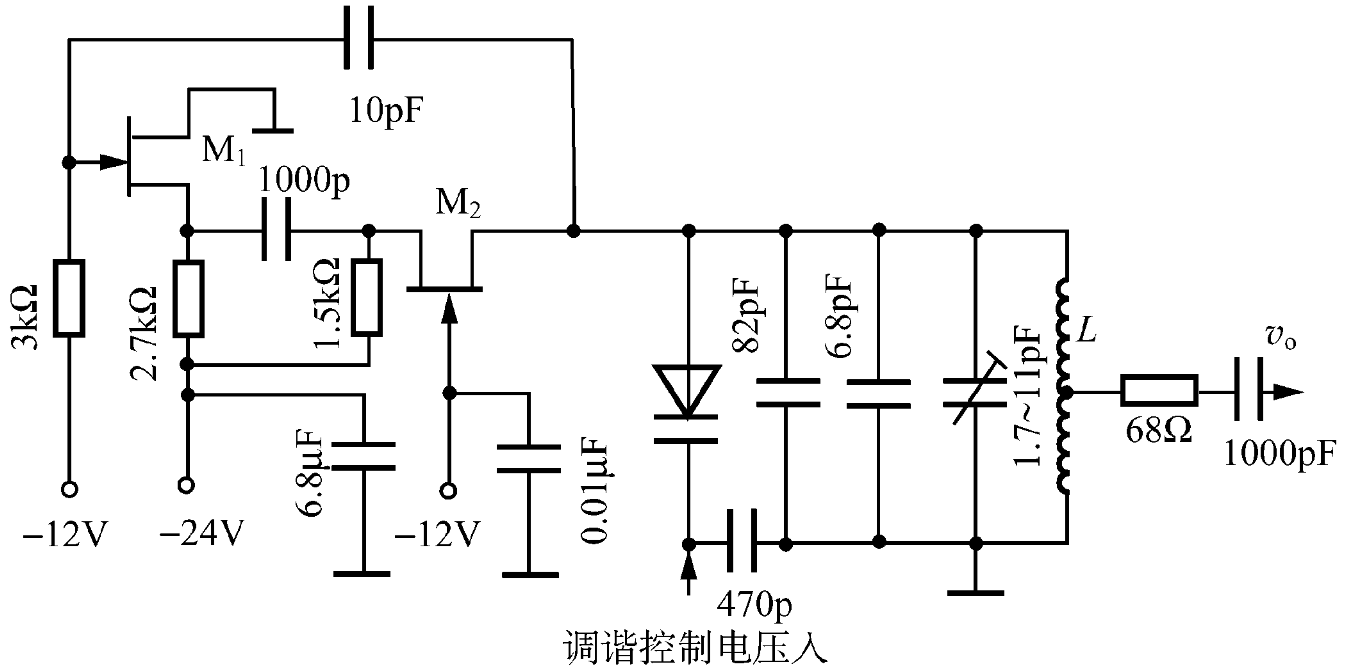




1. Common Base Transistor Configuration

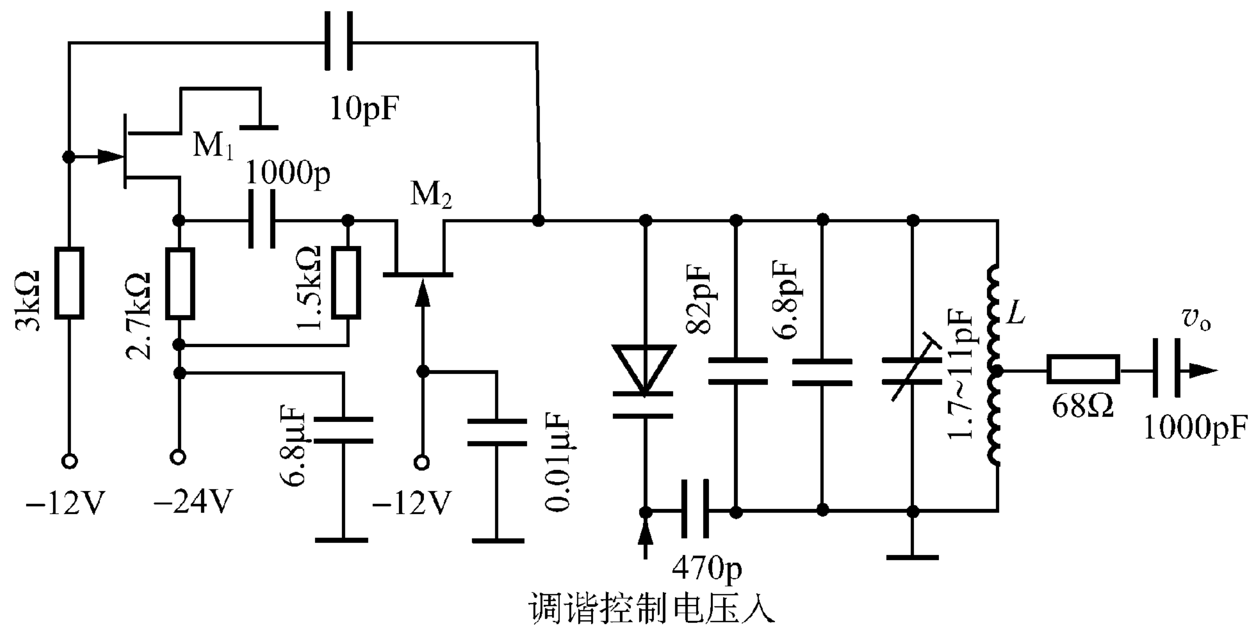
2. Two diodes in series

3. Oscillation frequency is determined by four reactor elements.

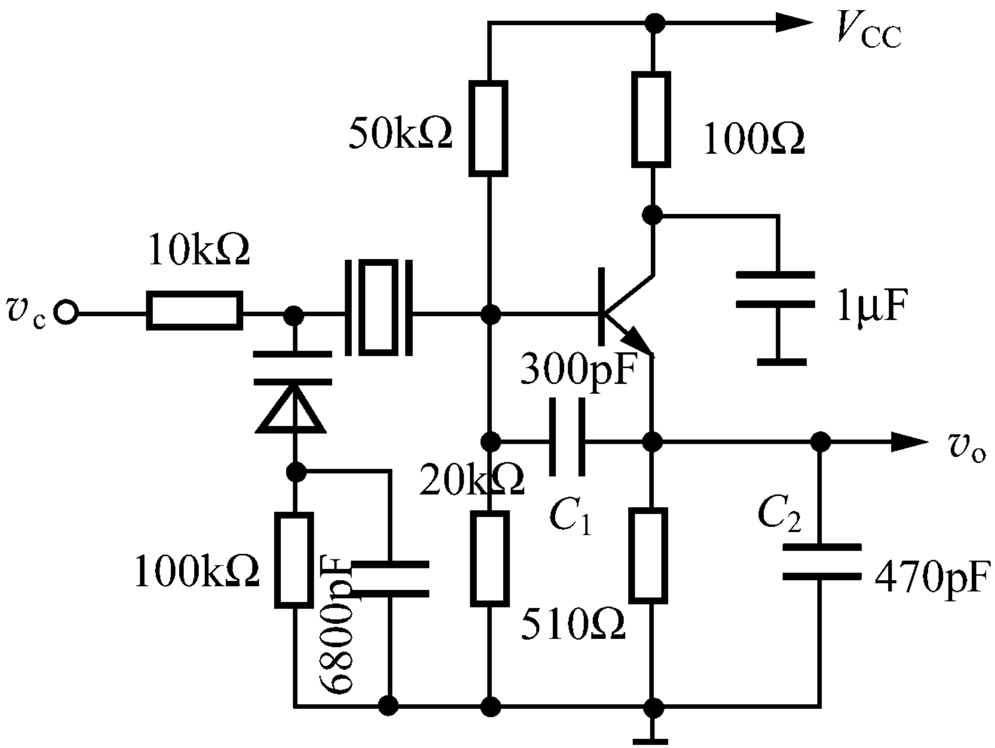


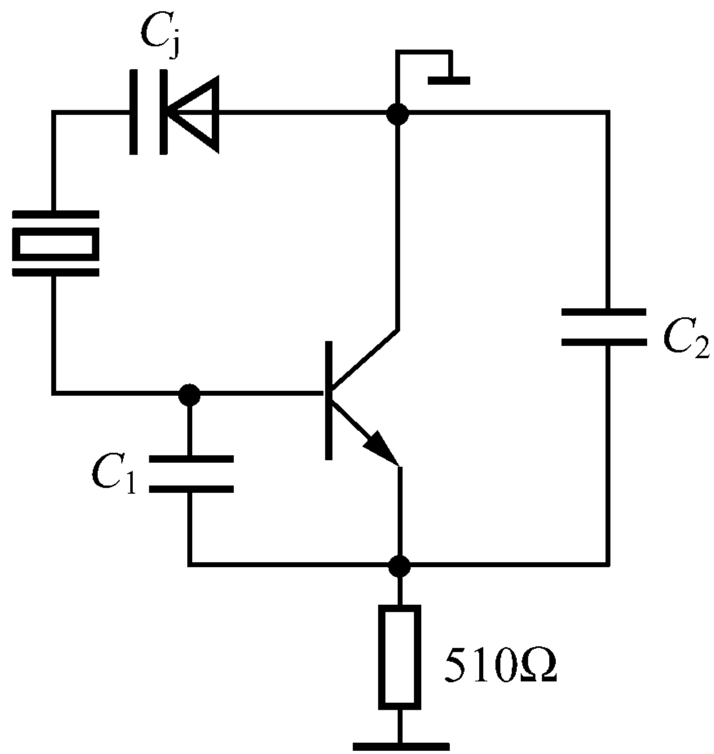
M_1 **Follower**

M_2 **Common gat triode**



Quartz Crystal Voltage Controlled Oscillator



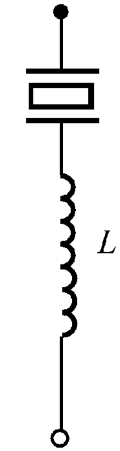


Parallel Crystal Oscillator-Crystal is an Inductive Element

Load capacitance $C_1 // C_2 // C_j$

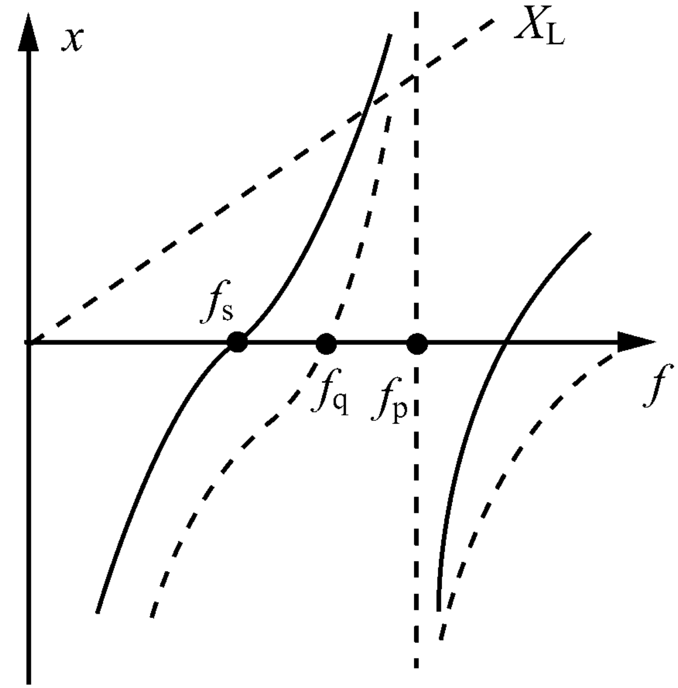
Control voltage change C_j to change the frequency of crystal oscillator

A Method of Expanding Frequency Range-Series Inductance



The range of inductance of crystal is from

$$f_q \sim f_p \longrightarrow f_S \sim f_p$$

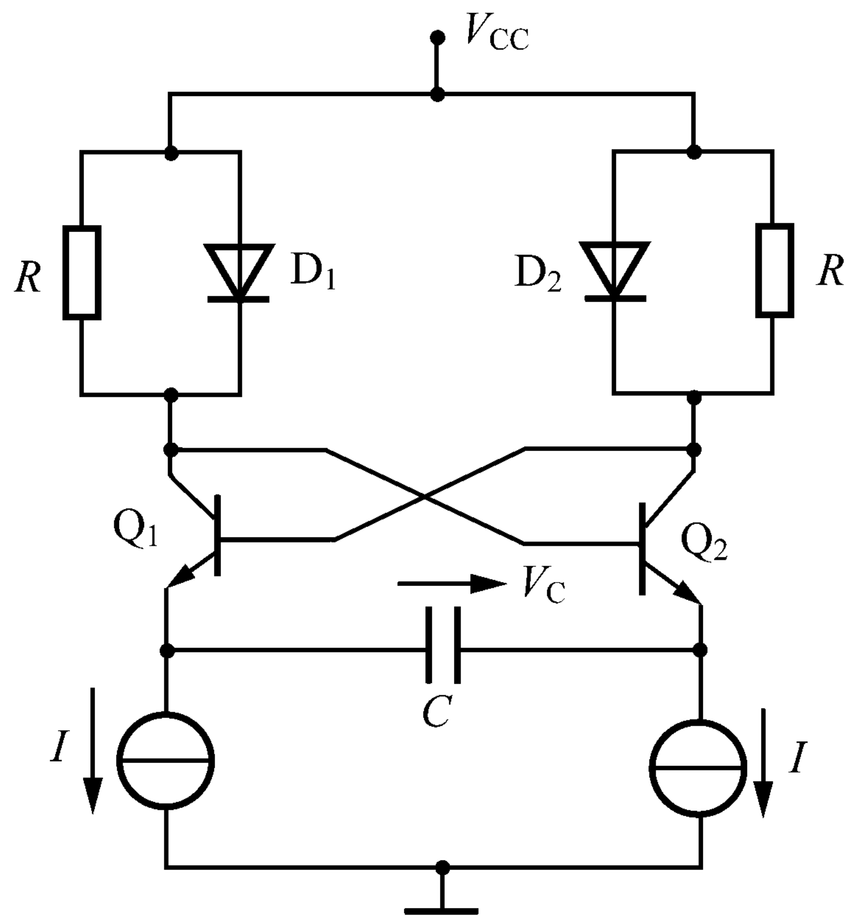


Frequency range of the oscillator is also expanded accordingly

Emitter coupled multivibrator

1.Two cross-coupled transistors const
positive feedback

2.The capacitor is charged and discha
alternately between two threshold leve
form oscillation.

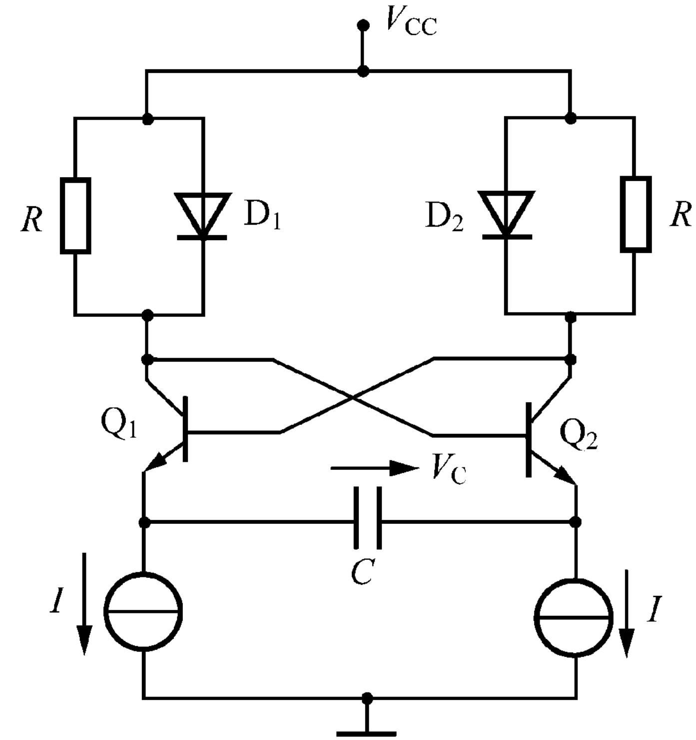


Initial state Q_1 on Q_2 off $V_C = 0$

Turn-on voltage V_{on}

$$Q_1 \begin{cases} V_{c1} = V_{cc} - V_{on} \\ V_{b1} = V_{c2} = V_{cc} \\ \boxed{\begin{aligned} V_{e1} &= V_{b1} - V_{on} \\ &= V_{cc} - V_{on} \end{aligned}} \end{cases}$$

$$Q_2 \begin{cases} V_{b2} = V_{cc} - V_{on} \\ V_{b2} - V_{e2} < V_{on} \end{cases}$$

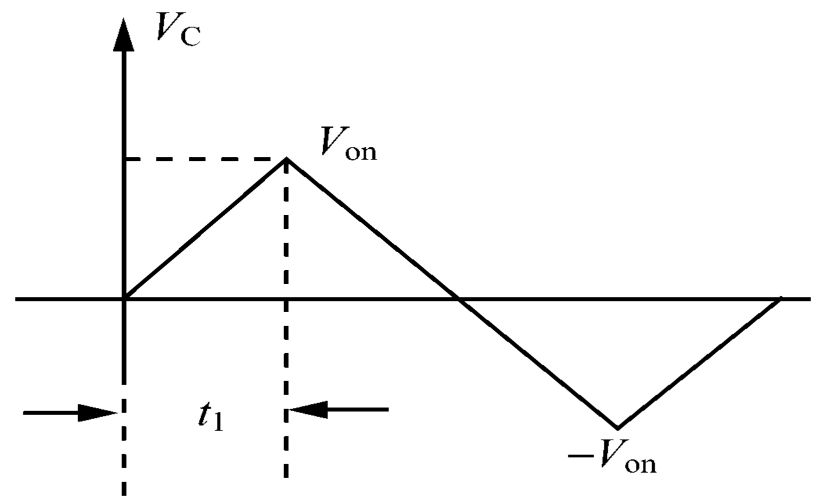


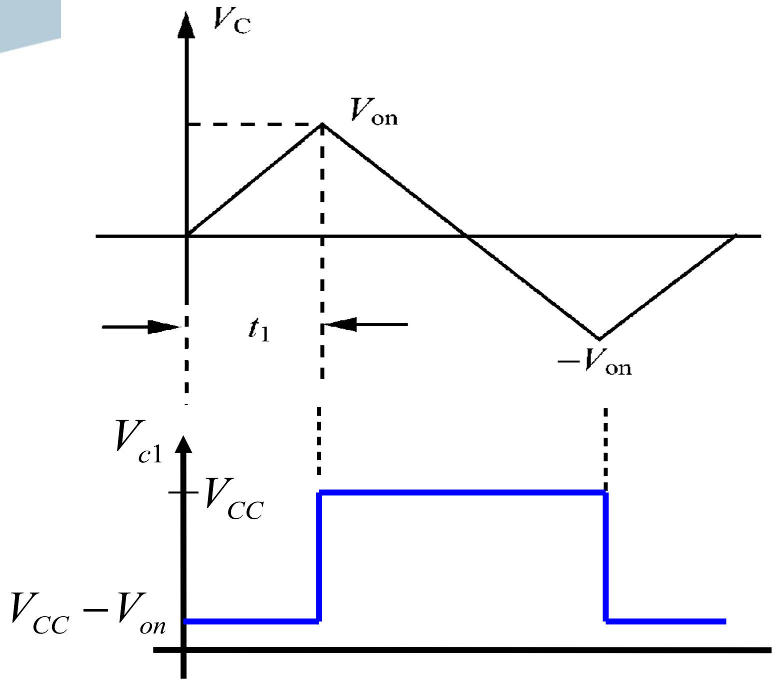
Capacitor C Constant Current I Charging ，

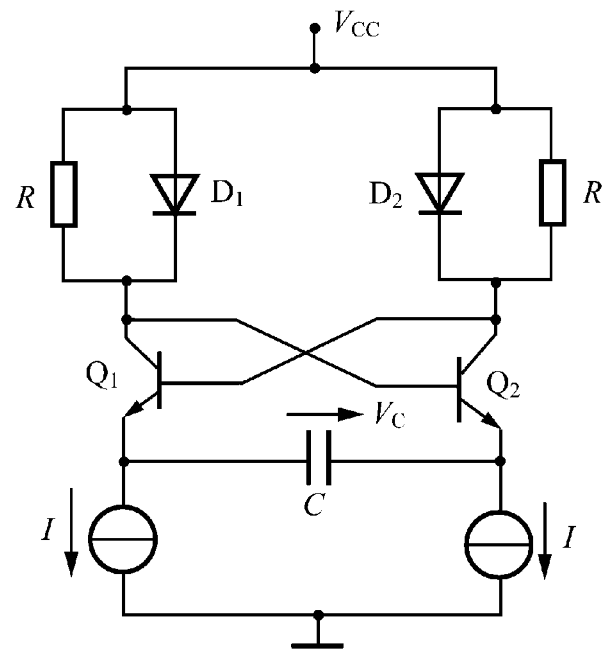
when $V_C = V_{on} \longrightarrow \begin{cases} V_{e2} = (V_{cc} - V_{on}) - V_{on} \\ V_{b2} - V_{e2} = V_{on} \end{cases}$

Q_2 **On ，** Q_1 **off** $\longrightarrow$

Capacitor C Constant Current I Discharge







There is no frequency selection circuit in the circuit, so the output is not sinusoidal wave.

$$V_{on} = \frac{1}{C}\int_0^{t_1} I\,dt = \frac{I}{C}t_1 \longrightarrow f = \frac{1}{T} = \frac{1}{4t_1} = \frac{I}{4CV_{on}}$$

The frequency of the multivibrator can be controlled by controlling the charging current I or changing the capacitor C.

Cross-Coupled Positive Feedback Transistor

$Q_{19}Q_{20}$、$Q_{21}Q_{22}$

（Q_{19}、Q_{22} fowller）

Bias current source

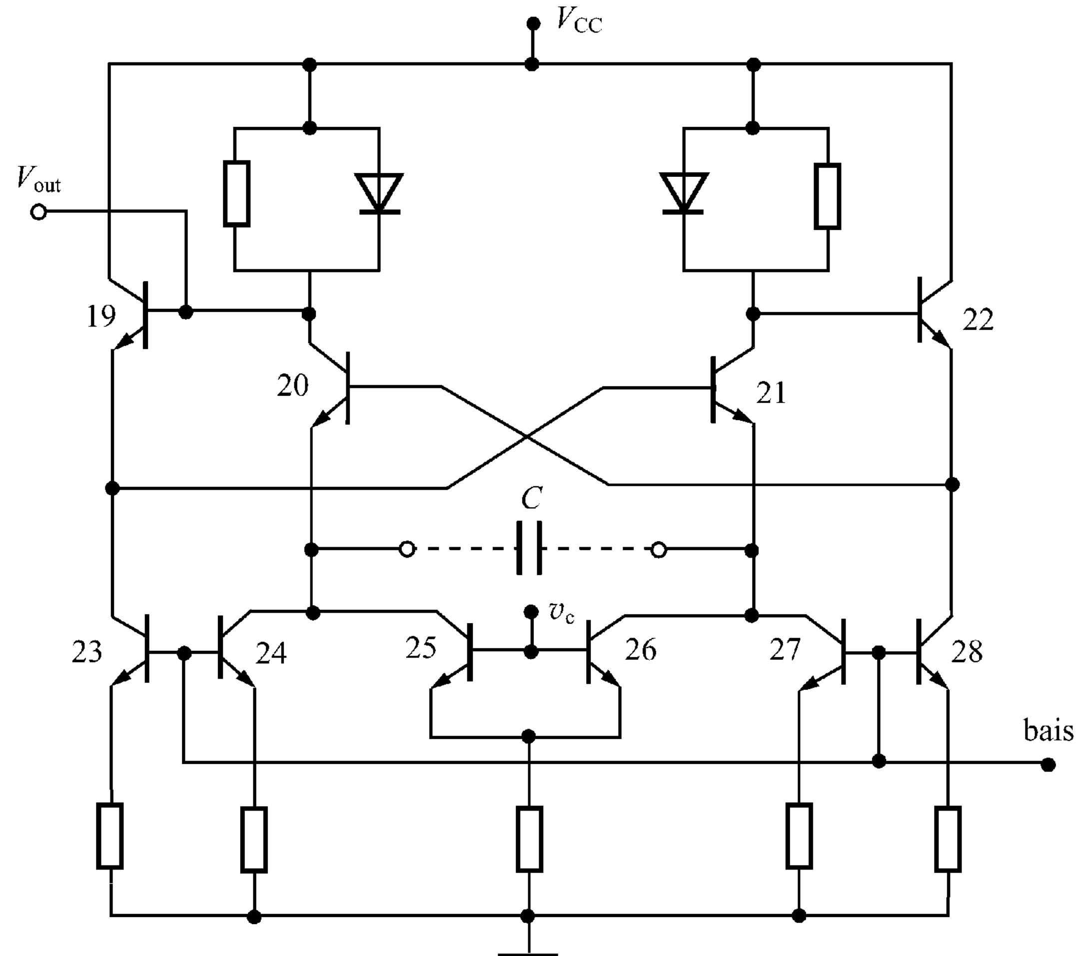
$Q_{23}Q_{24}Q_{25}$、$Q_{26}Q_{27}Q_{28}$

External capacitance C

Ccontrol voltage V_c

Changing the current of

Q_{25}、Q_{26} $\longrightarrow$ **Changing oscillation frequency**



Oscillator Comparison (typical performance)

Parameter /Device	Quartz OCXO	Rubidium	Cesium	GPSDO
Frequency accuracy after 30 minute warm-up	No guaranteed accuracy, must be set on frequency	5×10^{-10}	1×10^{-12}	1×10^{-12}
Stability at 1 s	1×10^{-12}	1×10^{-11}	1×10^{-11}	Parts in 10^{10}
Stability at 1 day	1×10^{-10}	2×10^{-12}	1×10^{-13}	2×10^{-13}
Stability at 1 month	Parts in 10^{9}	5×10^{-11}	Parts in 10^{14}	Parts in 10^{15}
Aging	1×10^{-10} / day	5×10^{-11}/month	None, by definition	None, frequency is corrected by satellites
Cost (USD)	$200 to $2000	$1000 to $8000	$25000 to $50000	$2000 to $15000

Frequency Accuracy (Offset)

The degree of conformity of a measured value to its definition at a given point in time. Accuracy tells us how closely an oscillator produces its nominal or nameplate frequency.

Frequency Offset, Frequency Error, Frequency Bias

Frequency Difference, Relative Frequency, Fractional Frequency, Accuracy

Stability

Stability indicates how well an oscillator can produce the same time and frequency offset over a given period of time. **Stability doesn't indicate whether the time or frequency is "right" or "wrong", but only whether it stays the same.** In contrast, accuracy indicates how well an oscillator has been set on time or set on frequency.

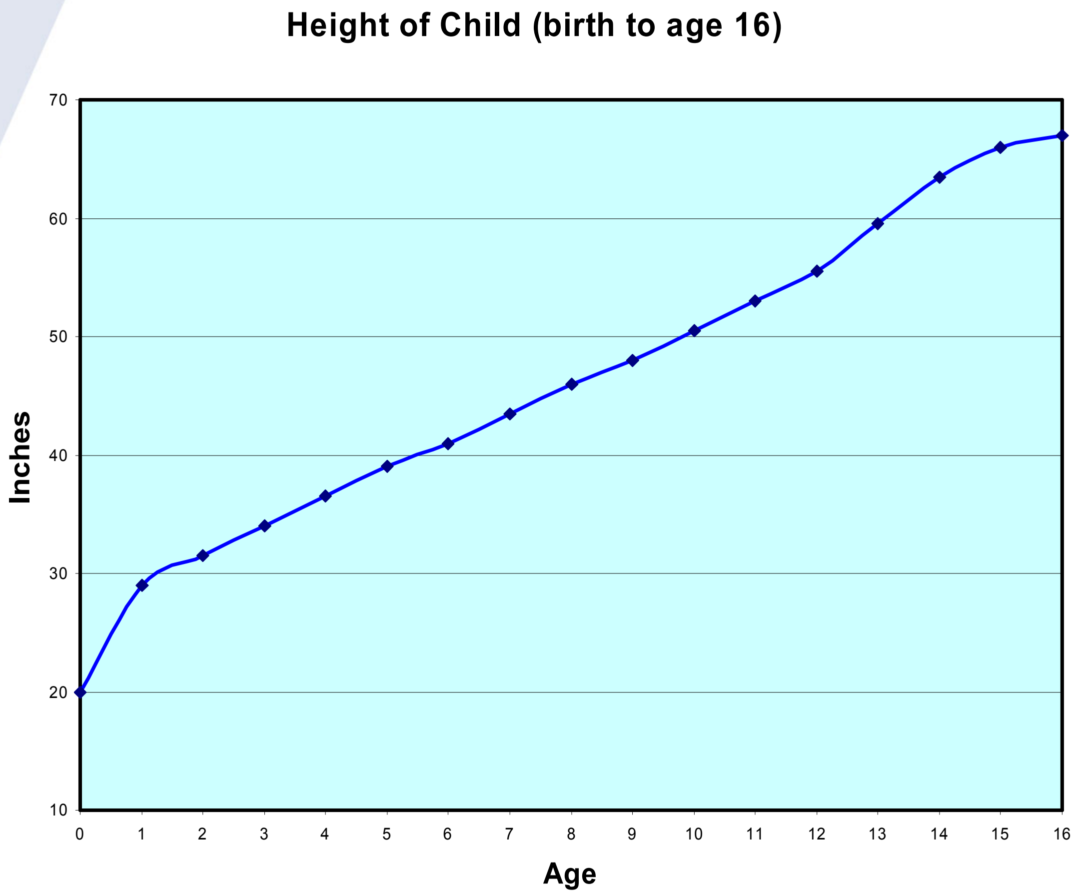
The relationship between accuracy and stability

Estimating Stability

Stability is estimated with statistics that evaluate the frequency fluctuations of an oscillator that occur over time. The most common statistic used to estimate stability is the Allan deviation.

Short-term stability usually refers to intervals off less than 100 seconds, longer-term stability can refer to intervals greater than 100 seconds, but usually refers to periods longer than 1 day.

Why standard deviation doesn't work with oscillators



Consider this graph of the growth of a child who was 20 inches tall at birth, and 67 inches tall at age 16.

At age 8, the child's average height was about 36 inches, and the standard deviation was about 8 inches. By age 16, the average height had increased to 46 inches, and the standard deviation was about 14 inches.

These numbers are meaningless! Taking the mean and the standard deviation from the mean is pointless if the data has a trend, or is "non-stationary". That's why standard deviation is seldom used in time and frequency.

Using the Allan Deviation with time domain data

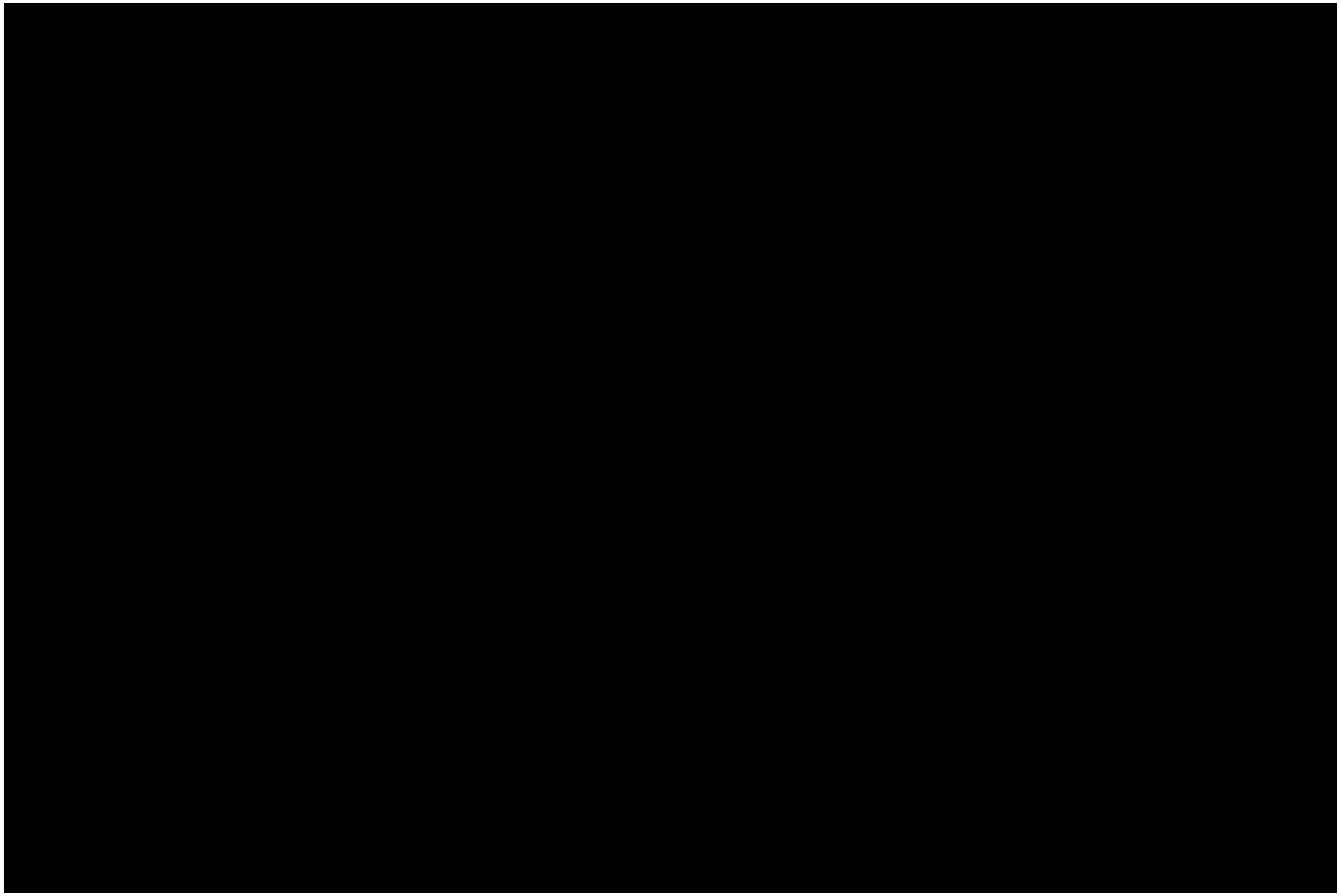
$$\sigma_a{}^2(\tau) = \lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m}\left[\left(f_{j1}-\frac{f_{j1}+f_{j2}}{2}\right)^2+\left(f_{j2}-\frac{f_{j1}+f_{j2}}{2}\right)^2\right]$$
$$= \lim_{m\to\infty}\frac{1}{m}\sum_{i=1}^{m}\frac{(f_{j2}-f_{j1})^2}{2}$$

f_j is a set of time equally spaced frequency measurements within a time period τ (tau)

m is the number of average factor

ADEV is computed using an iterative method (multiple passes through a loop). Normally, ADEV is computed using the octave method, so m is doubled on each pass. For example, stability would be estimated at 1, 2, 4, 8, 16, 32 s, etc. But it is possible to increment m by 1 each time, and calculate stability for every possible averaging time. That takes longer, but reveals more information.

A graph of frequency stability



Noise Floor

Allan deviation graphs show stability estimates at different averaging times. These estimates improve (the stability gets better and better), until the devices reaches its noise floor. The noise floor is the point where more averaging doesn't help; you'll get the same answer or a worse answer if you continue to average. When the noise floor is reached, the remaining noise is **non-white**, and cannot be removed by averaging.

Specification sheets for frequency sources usually only provide stability estimates out to the point where the noise floor is reached. For example, if an oscillator's specification sheet only shows stability estimates out to an average time of 10 seconds, you'll know that the stability at longer intervals (like 1 hour or 1 day) is worse than the stability at 10 seconds.

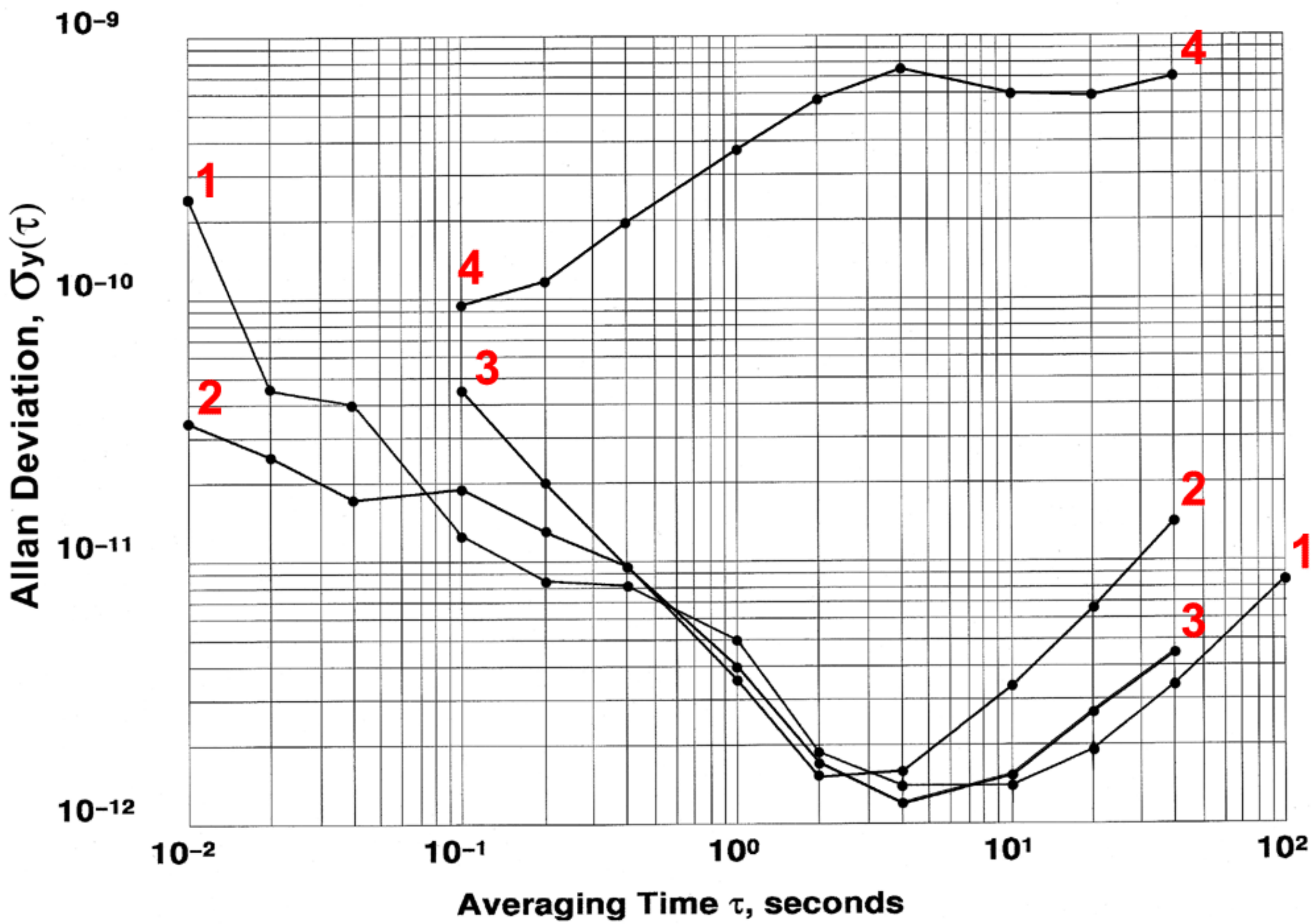
How long does it take for a oscillator to reac h its noise floor?

Quartz
- < 10 s, sometimes < 1 s

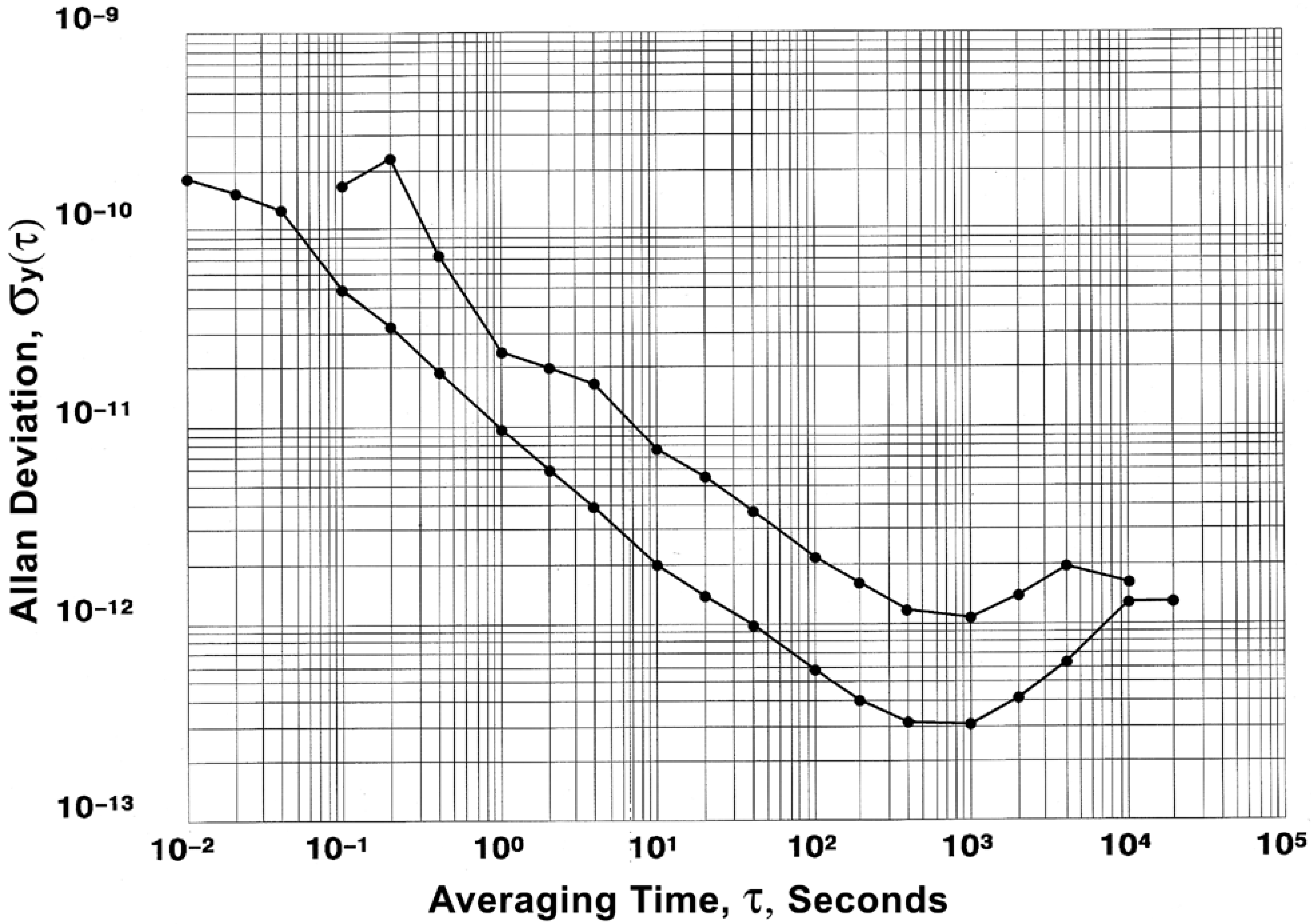
Rubidium
- 1000 s

Cesium
- Several days to 30 days

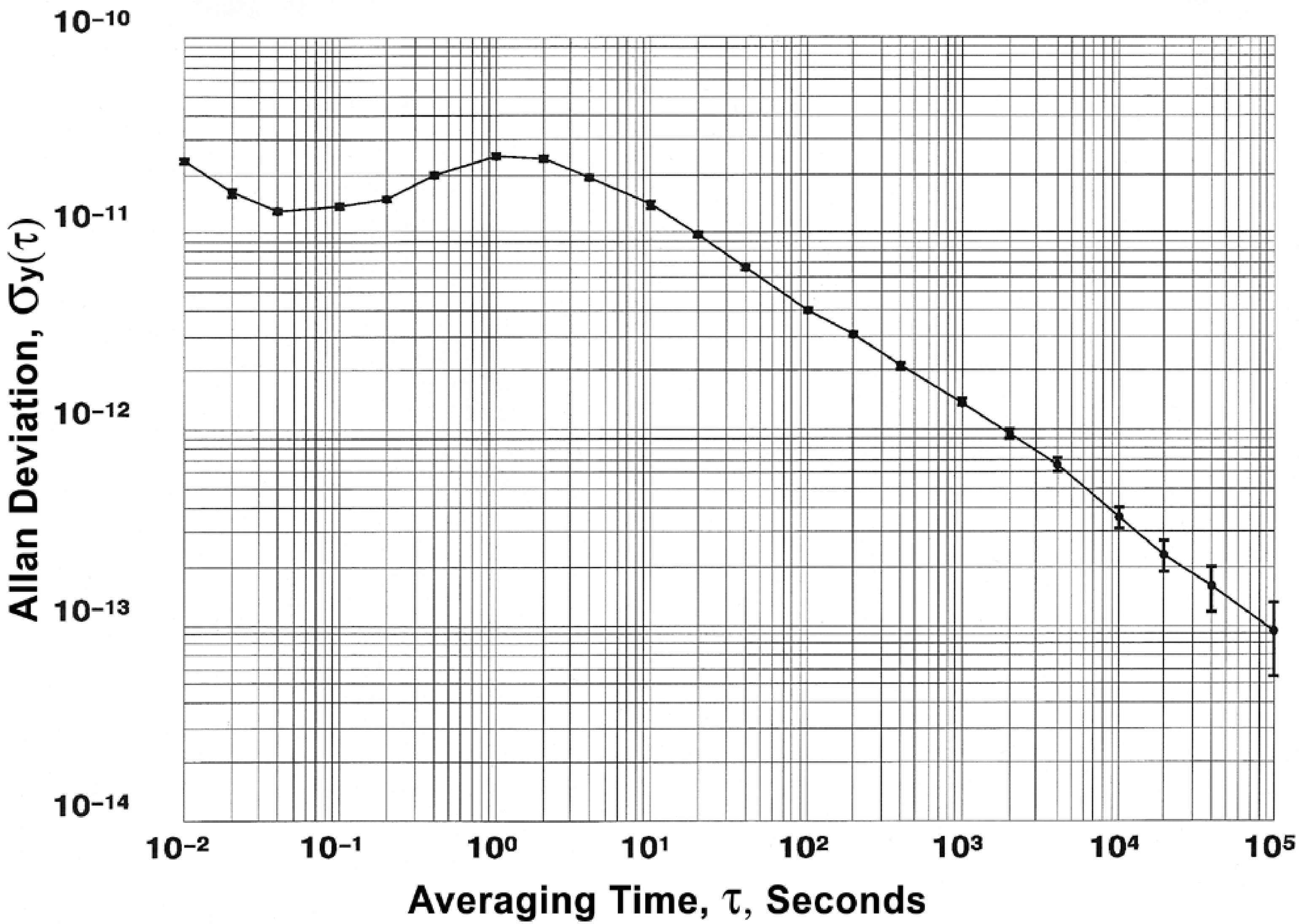
Comparison of Quartz Oscillators



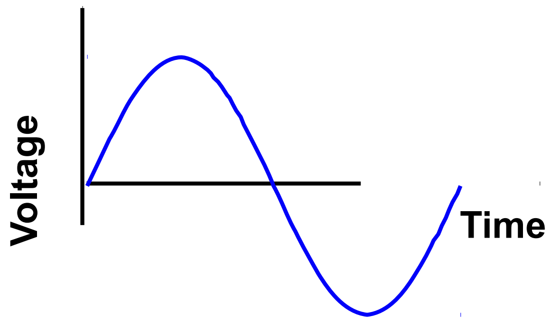
Comparison of Rubidium Oscillators



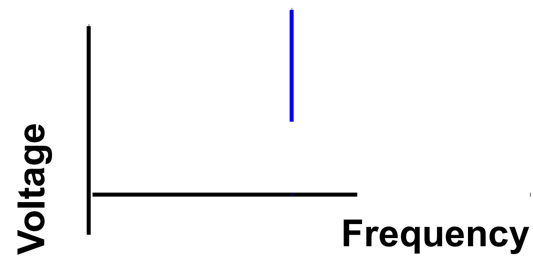
Cesium Oscillator



Sources Generate Sine Waves

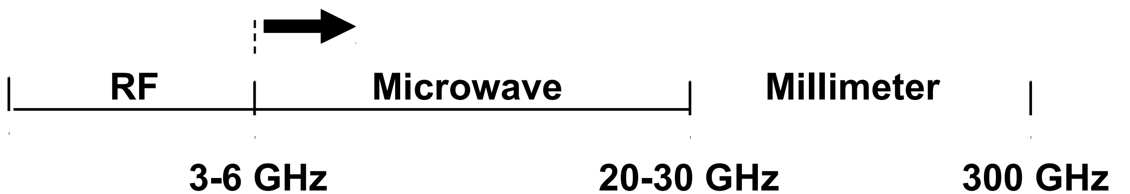


Oscilloscope



Spectrum Analyzer

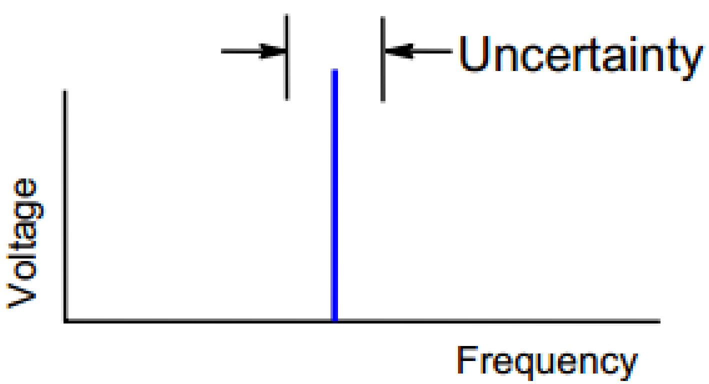
This is the ideal output: most specs deal with deviations from the ideal and adding modulation to a sine wave



CW Source Specifications

…Frequency

- **Range: Range of frequencies covered by the source**
- **Resolution: Smallest frequency increment.**
- **Accuracy: How accurate is the displayed frequency?**



EXAMPLE

Accuracy = $\pm f_{CW} * \tau_{aging} * \tau_{cal}$

f_{CW} = CW frequency = 1 GHz

τ_{aging} = aging rate = 0.152ppm/year

τ_{cal} = time since last calibrated = 1 year

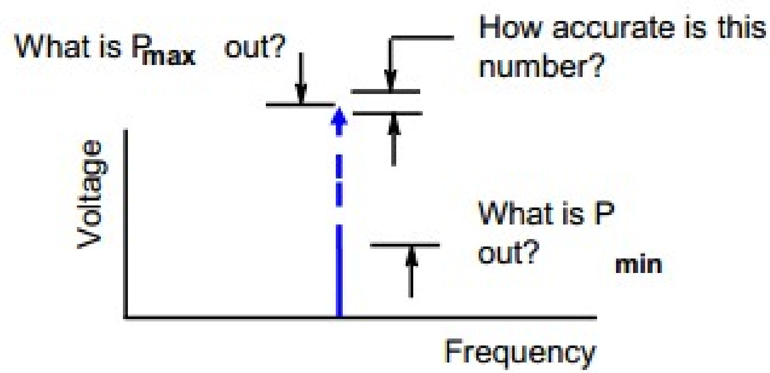
Accuracy = $\pm$152 Hz

CW Source Specifications

…Amplitude

- Range (-136dBm to +13dBm)
- Accuracy (+/- 0.5dB)
- Resolution (0.02dB)
- Switching Speed (25ms)
- Reverse Power Protection

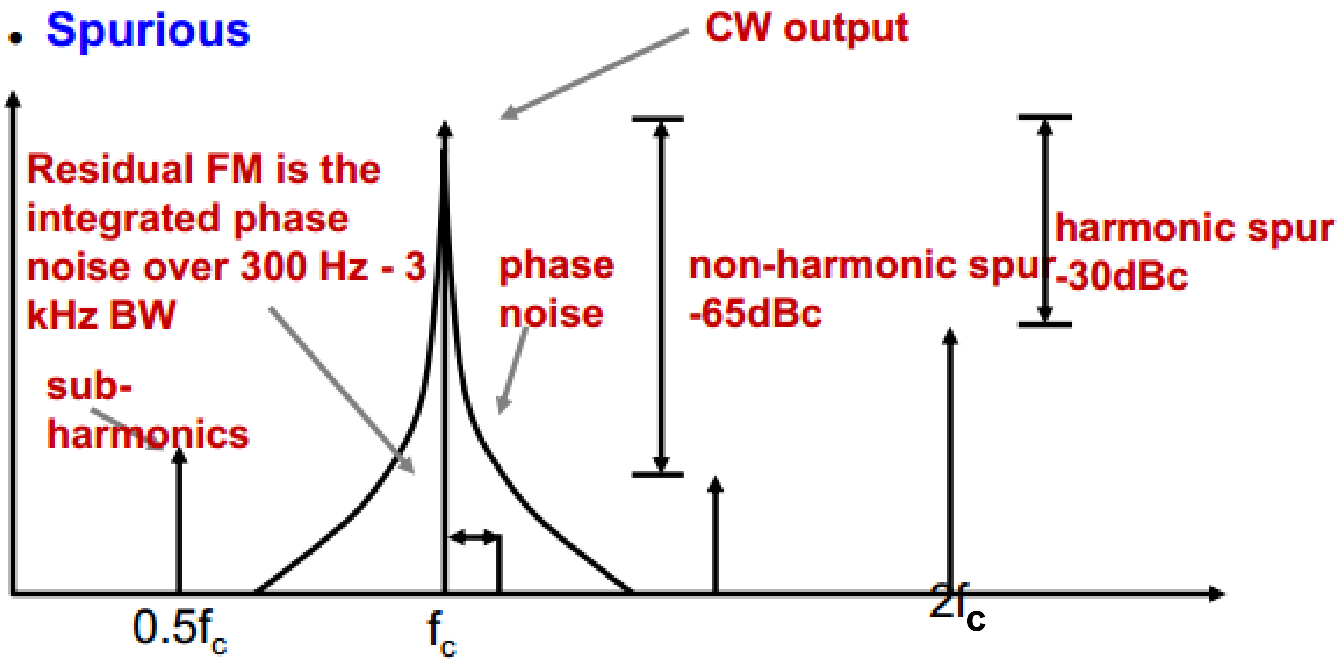
Source protected from accidental transmission from DUT



CW Source Specifications

…Spectral Purity

- Phase Noise
- Residual FM
- Spurious



Phase noise

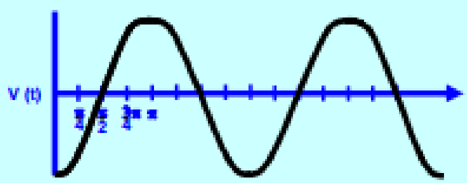
理想CW 信号

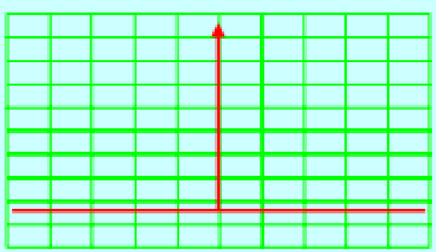
$$V(t) = A_o \sin 2\pi f_o t$$

Where

A_o = nominal amplitude

f_o = nominal frequency





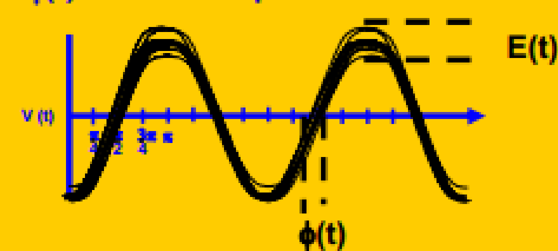
真实CW 信号

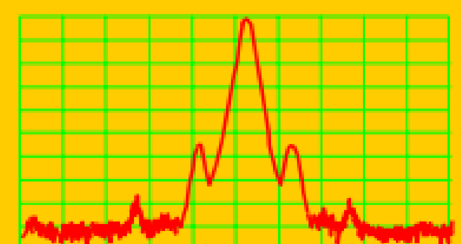
$$V(t) = [A_o + E(t)] \sin [2\pi f_o t + \phi(t)]$$

Where

E(t) = random amplitude fluctuations

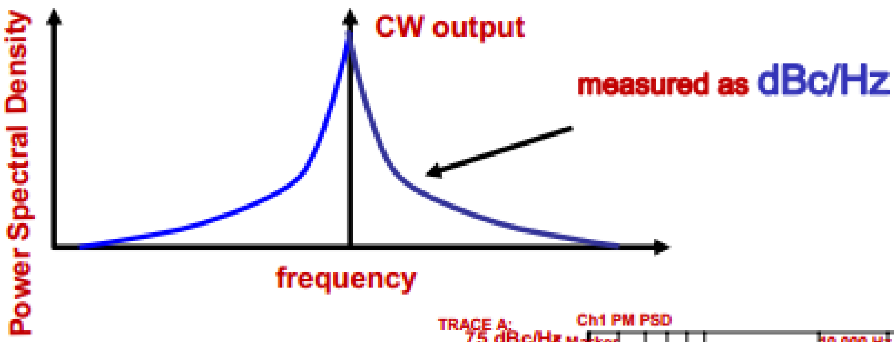
$\phi(t)$ = random phase fluctuations

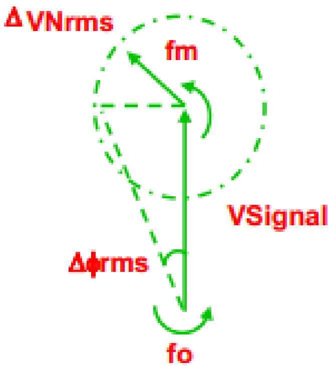




CW Source Specifications

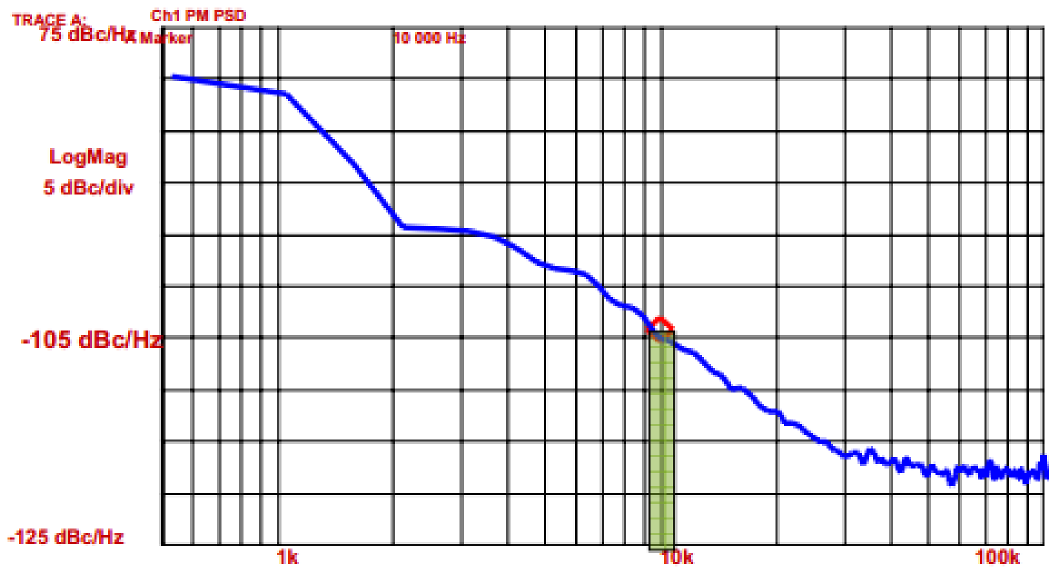
…Spectral Purity： Phase Noise





-80dBc/Hz @10Hz offset
-90dBc/Hz @ 100Hz offset
-105dBc/Hz @10kHz offset

……………………………………



Choosing a Source

- Frequency
  - range
  - resolution
  - accuracy
- Amplitude
  - range
  - accuracy
- Spectral Purity
  - harmonics, sub-harmonics
  - non-harmonics
  - SSB phase noise
  - residual FM
- Modulation type
  - am
  - fm
  - phase
  - pulse
  - digital
- Sweep capabilities
  - CW
  - ramp
  - step
  - list

-Capability

-Deviation from ideal