RF/Microwave Circuit and System

Lecture 5

Contents

- Basic Concepts in RF Design
  - Nonlinearity and Time Variance
  - Intersymbol Interference
  - Random Processes and Noise
  - Sensitivity and Dynamic Range
  - Passive Impedance Transformation

Nonlinearity and Time Variance

If input $x1(t)$ and $x2(t)$

$x1(t) \to y1(t)$, $x2(t) \to y2(t)$,

$ax1(t) + bx2(t) \to ay1(t) + by2(t)$

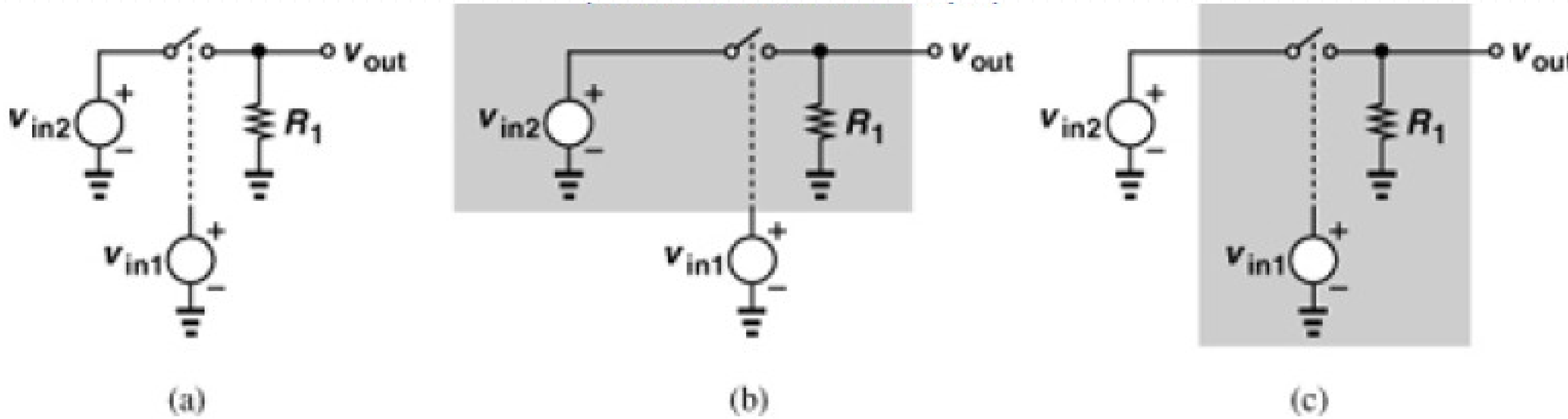
Not satisfy -> Nonlinear

$x(t) \to y(t)$,

$x(t-\tau) \to y(t-\tau)$

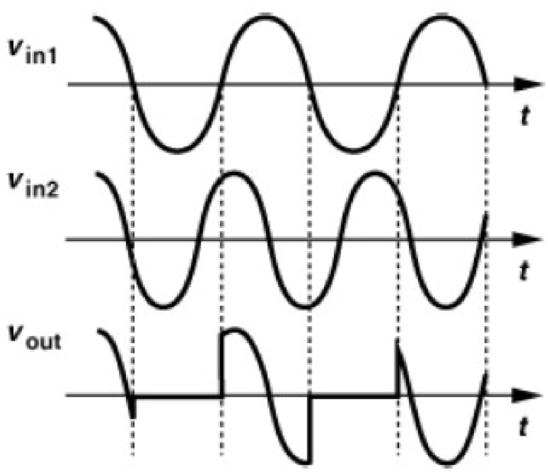
Not satisfy -> Time variant

Nonlinearity and Time Variance



(a) Simple switching circuit, (b) system with V_{in1} as the input, (c) system with V_{in2} as the input

$$v_{out}(t) = v_{in2}(t) \cdot S(t).$$



$$V_{out}(f) = V_{in2}(f) * \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi}\delta\left(f - \frac{n}{T_1}\right)$$
$$= \sum_{n=-\infty}^{+\infty} \frac{\sin(n\pi/2)}{n\pi} V_{in2}\left(f - \frac{n}{T_1}\right).$$

Nonlinearity and Time Variance

- 1. Effects of Nonlinearity

$$y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

- 1) Harmonics

$$
\begin{aligned}
&If\ x(t) = Acoswt,\\
&y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)\\
&\quad = \alpha_1 Acoswt + \frac{\alpha_2 A^2}{2}(1+\cos 2wt) + \frac{\alpha_3 A^3}{4}(3\cos wt + \cos 3wt)\\
&\quad = \frac{\alpha_2 A^2}{2} + (\alpha_1 A + \frac{3\alpha_3 A^3}{4})\cos wt + \frac{\alpha_2 A^2}{2} cos2wt + \frac{\alpha_3 A^3}{4} cos3wt
\end{aligned}
$$

Nonlinearity and Time Variance

- 2) Gain Compression
  - 1dB compression point
    - Input signal level that causes the small-signal gain to drop by 1dB
    - If $\alpha_3 < 0$, gain is decreasing function of A

$$\left(\alpha_1 + \frac{3\alpha_3 A^2}{4}\right) A \cos wt$$

$$20\log\left|\alpha_1 + \frac{3}{4}\alpha_3 A_{1-dB}^2\right| = 20\log|\alpha_1| - 1\text{dB}$$

$$A_{1-dB} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

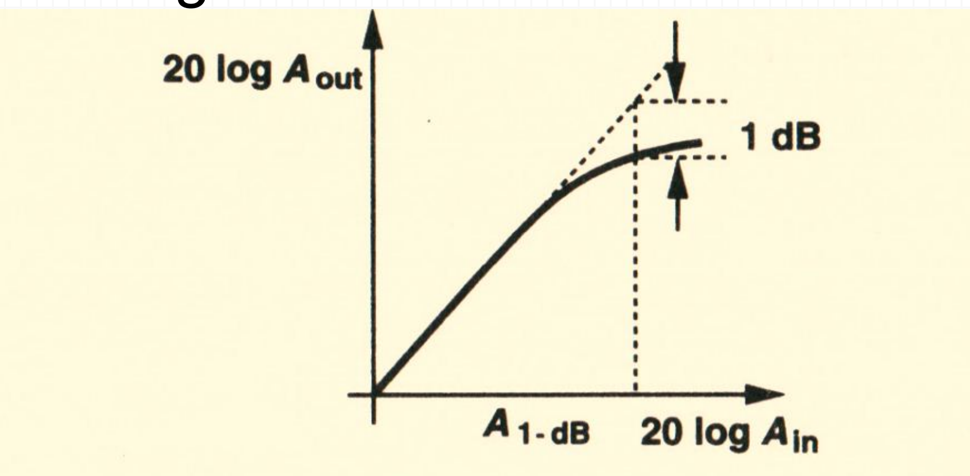


Figure 2.4 Definition of the 1-dB compression point.

Nonlinearity and Time Variance

- 3) Desensitization and Blocking
  - Desensitization
    - Since a large signal tends to reduce the average gain of the circuit, the weak signal may experience a vanishingly small gain.
  - Blocking
    - Gain drop to Zero (α3 < 0)

$$\text{If } x(t) = A_1 \cos w_1 t + A_2 \cos w_2 t,$$

$$y(t) = (\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2)\cos w_1 t + \ldots$$

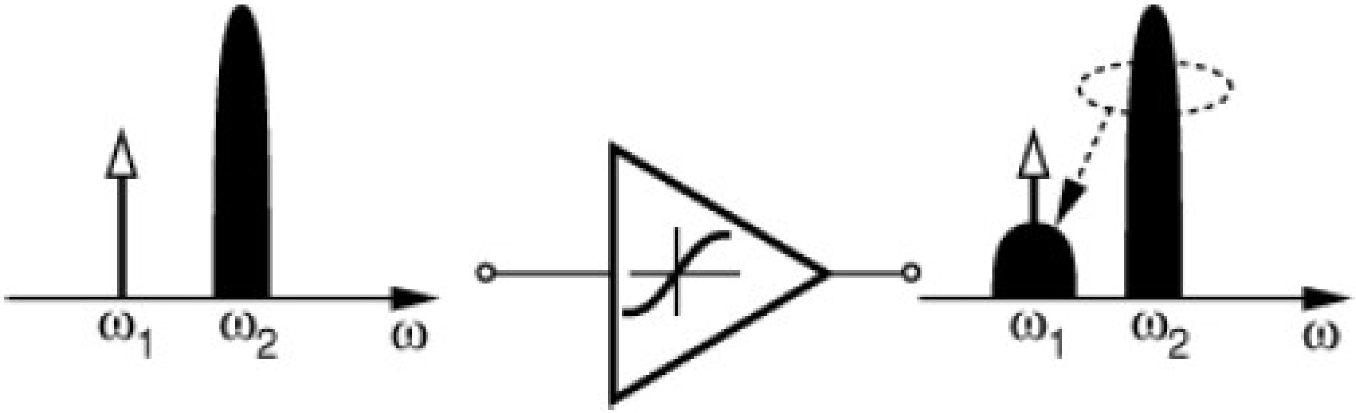
A1 << A2

$$y(t) = (\alpha_1 + \frac{3}{2}\alpha_3 A_2^2) A_1 \cos w_1 t + \ldots$$

Nonlinearity and Time Variance

- 4) Cross Modulation

$$y(t) = \left[\alpha_1 + \frac{3}{2}\alpha_3 A_2^2\left(1 + \frac{m^2}{2} + \frac{m^2}{2}\cos 2\omega_m t + 2m\cos\omega_m t\right)\right]A_1\cos\omega_1 t + \cdots$$



Nonlinearity and Time Variance

- 5) Intermodulation
  - When two signals with different frequencies are applied to a nonlinear system, the output in general exhibits some components that are not harmonics of the input frequencies.

$$If\ x(t) = A_1 \cos w_1 t + A_2 \cos w_2 t,$$

$$y(t) = \alpha_1 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2$$
$$+ \alpha_3 (A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Nonlinearity and Time Variance

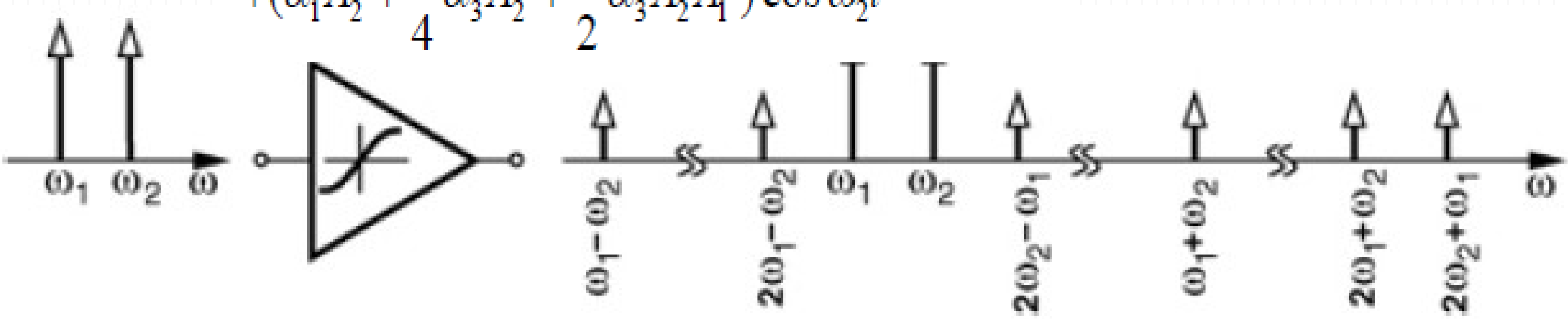
$$\omega = \omega_1 \pm \omega_2 : \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t$$

$$= 2\omega_1 \pm \omega_2 : \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4}\cos(2\omega_1 - \omega_2)t$$

$$= 2\omega_2 \pm \omega_1 : \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_2^2 A_1}{4}\cos(2\omega_2 - \omega_1)t$$

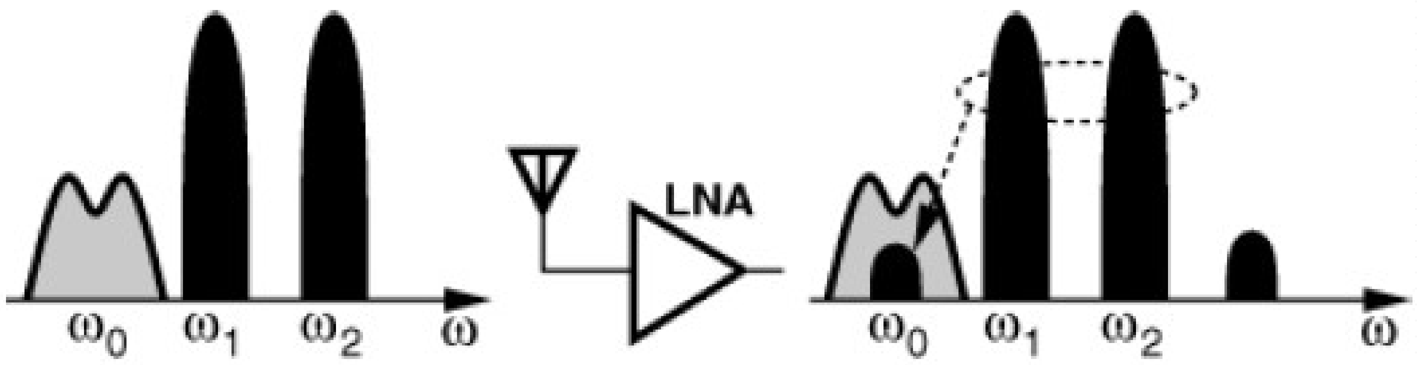
$$\omega = \omega_1, \omega_2 : (\alpha_1 A_1 + \frac{3}{4}\alpha_3 A_1^3 + \frac{3}{2}\alpha_3 A_1 A_2^2)\cos\omega_1 t$$

$$+(\alpha_1 A_2 + \frac{3}{4}\alpha_3 A_2^3 + \frac{3}{2}\alpha_3 A_2 A_1^2)\cos\omega_2 t$$



Nonlinearity and Time Variance

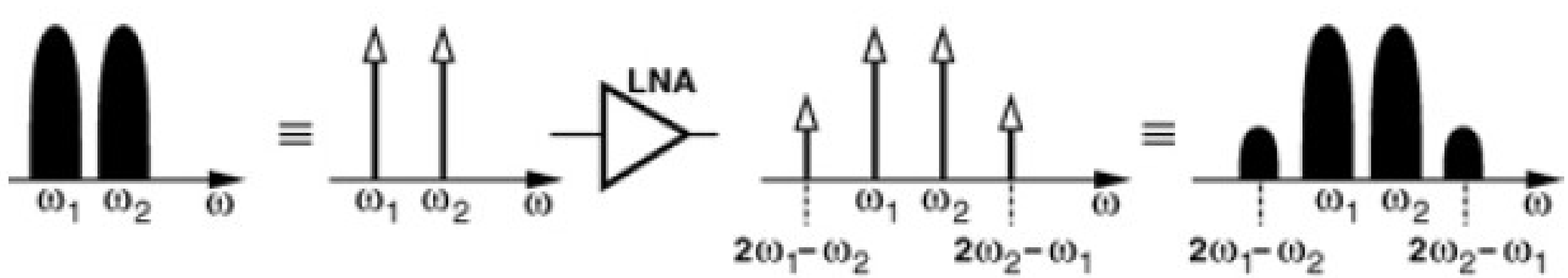
- Third Intercept Point(IP3)
  - If the difference between w1 and w2 is small, the components at 2w1-w2 and 2w2-w1 appear in the vicinity of w1 and w2

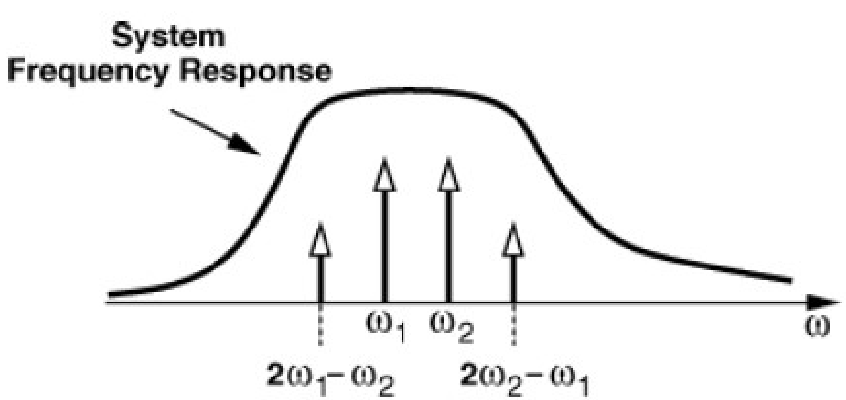


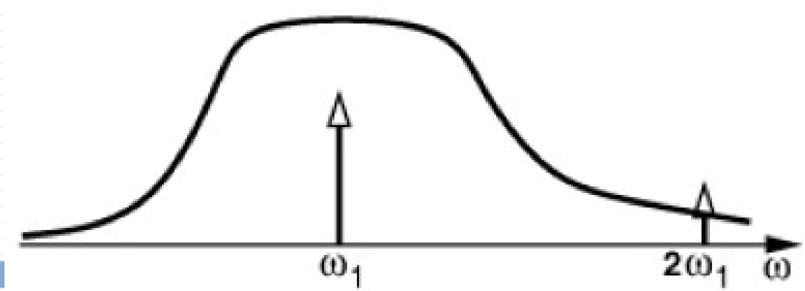
Nonlinearity and Time Variance

- Third Intercept Point(IP3)

(a)







Nonlinearity and Time Variance

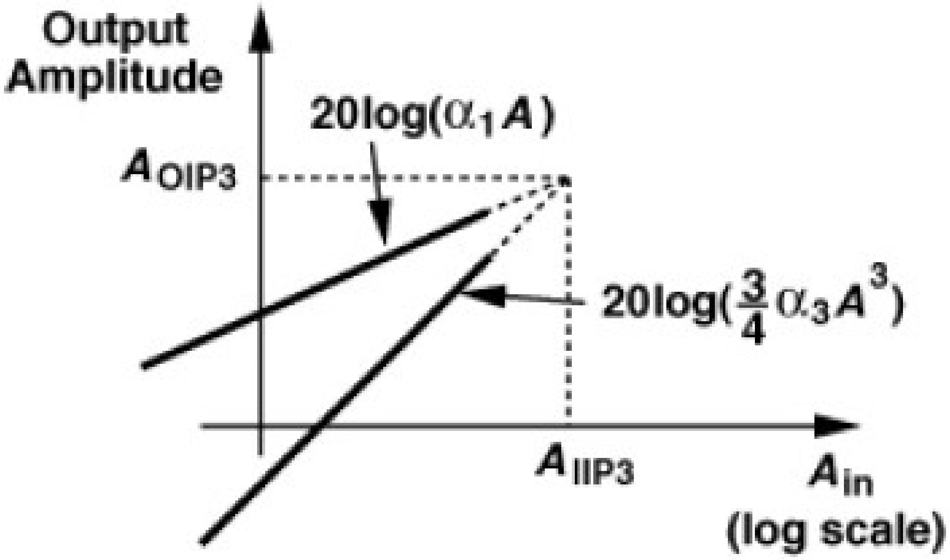
- Third Intercept Point(IP3)

$$|\alpha_1 A_{IIP3}| = \left|\frac{3}{4}\alpha_3 A_{IIP3}^3\right|$$

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

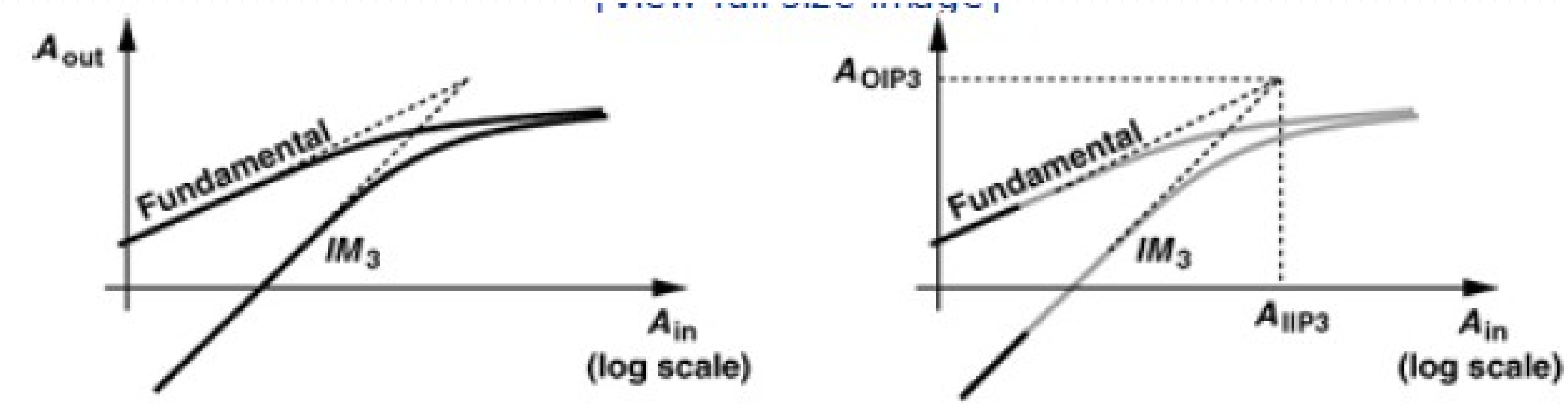
$$\frac{A_{IIP3}}{A_{1dB}} = \sqrt{\frac{4}{0.435}}$$

$$\approx 9.6\,\mathrm{dB}.$$



Nonlinearity and Time Variance

- Third Intercept Point(IP3)



$$\frac{A_{\omega 1,\omega 2}}{A_{IM3}} \approx \frac{|\alpha_1| A_{in}}{3|\alpha_3| A_{in}^3/4} = \frac{4|\alpha_1|}{3|\alpha_3| A_{in}^2} \qquad \frac{A_{\omega 1,\omega 2}}{A_{IM3}} = \frac{A_{IP3}^2}{A_{in}^2}$$

$$20\log A_{\omega 1,\omega 2} - 20\log A_{IM3} = 20\log A_{IP3}^2 - 20\log A_{in}^2$$

$$20\log A_{IP3} = \frac{1}{2}(20\log A_{\omega 1,\omega 2} - 20\log A_{IM3}) + 20\log A_{in}$$

In IP3 point

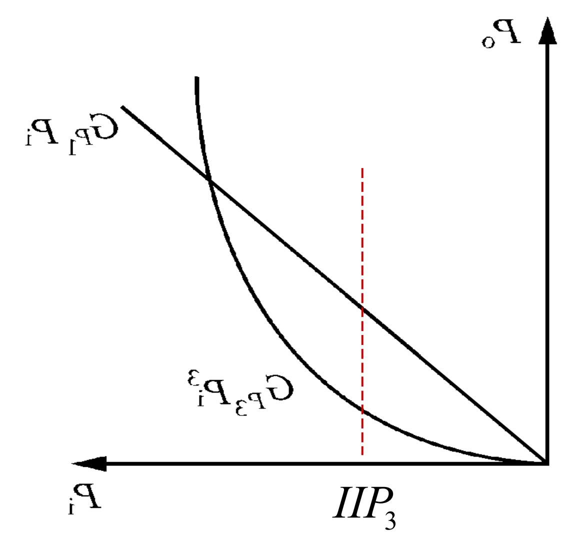
$$P_{o1} = P_{o3}$$

$$P_i = IIP_3$$

$$P_{o1} = G_{p1}(IIP_3)$$

$$P_{o3} = G_{p3}(IIP_3)^3$$

$$\frac{G_{p1}}{G_{p3}} = (IIP_3)^2$$

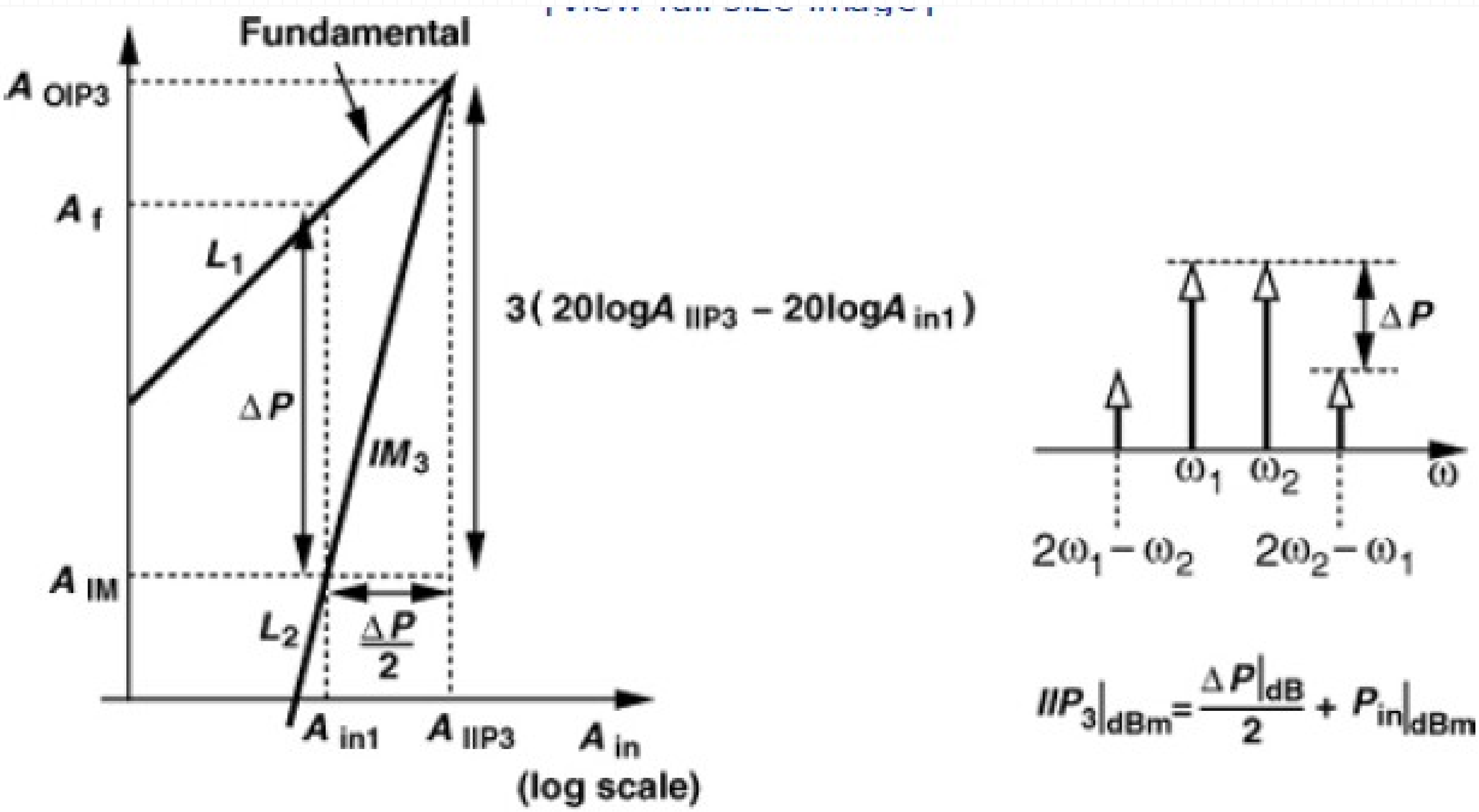


$$P_{IMR} = \frac{P_{o3}}{P_{o1}} = \frac{G_{p3}P_i^3}{G_{p1}P_i} = \frac{P_i^2}{(IIP_3)^2}$$

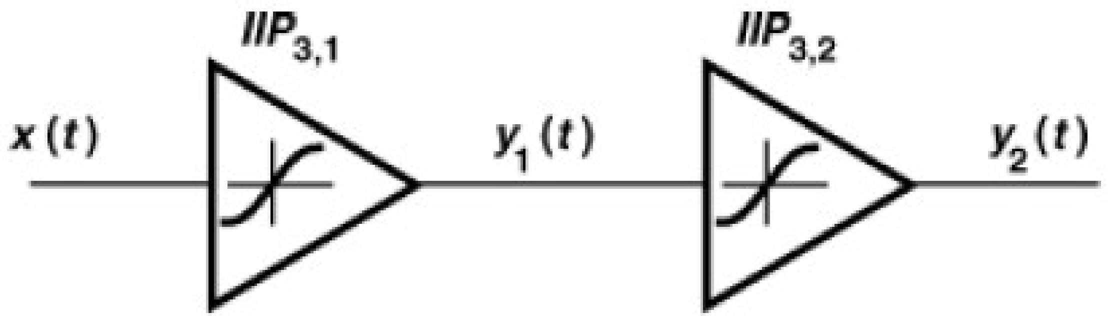
$$P_{IMR}(dB) = 2P_i(dBm) - 2(IIP_3)(dBm)$$

Nonlinearity and Time Variance

- Third Intercept Point(IP3)



Cascaded Nonlinear Stages



$$y_1(t) = \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t) \quad y_2(t) = \beta_1 y_1(t) + \beta_2 y_1^2(t) + \beta_3 y_1^3(t)$$

$$y_2(t) = \beta_1[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)] + \beta_2[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^2 + \beta_3[\alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)]^3.$$

$$y_2(t) = \alpha_1\beta_1 x(t) + (\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3)x^3(t) + \cdots.$$

$$A_{IP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}.$$

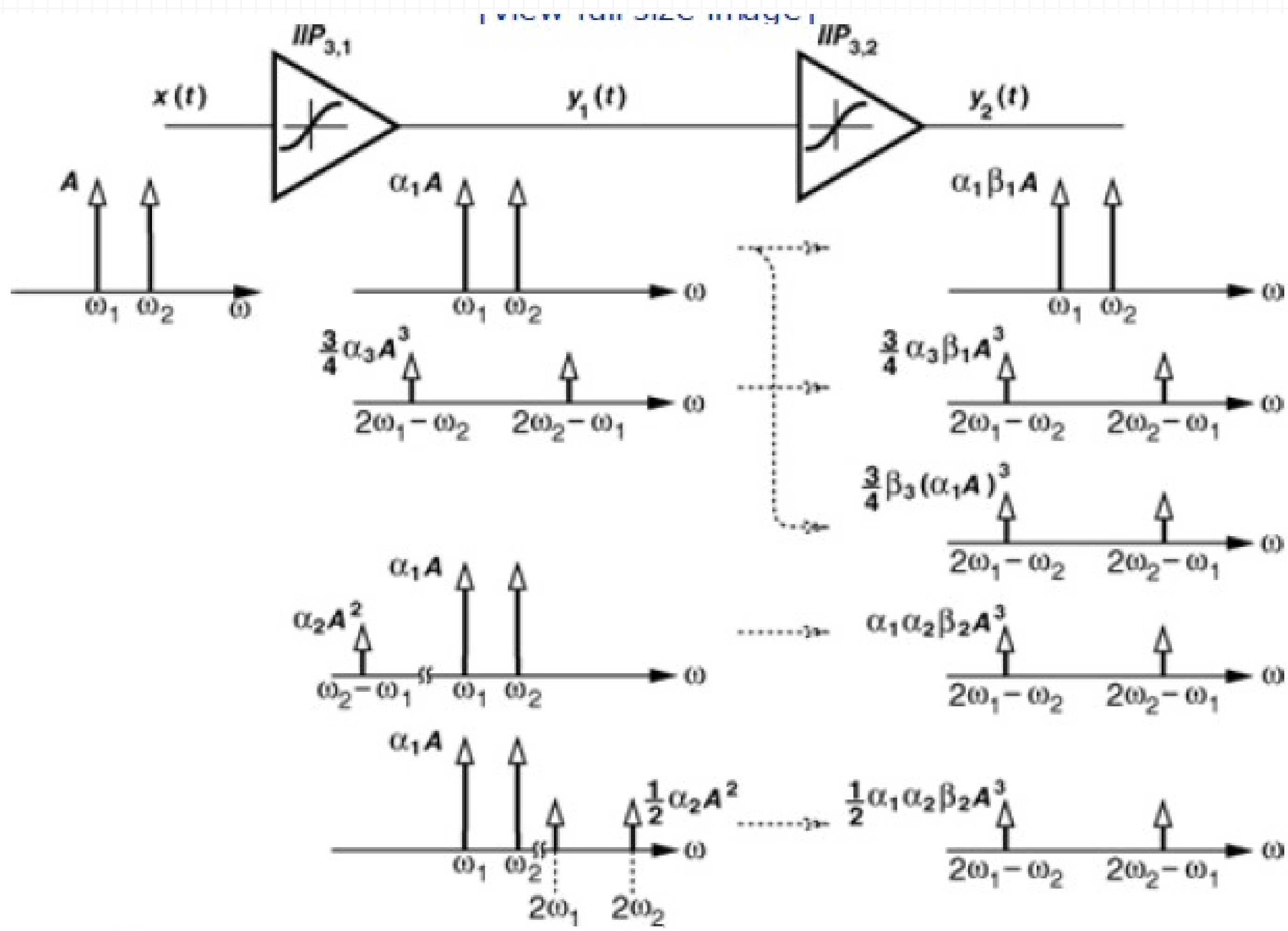
Cascaded Nonlinear Stages

$$\frac{1}{A_{IP3}^2} = \frac{3}{4}\left|\frac{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}{\alpha_1\beta_1}\right|$$

$$= \frac{3}{4}\left|\frac{\alpha_3}{\alpha_1} + \frac{2\alpha_2\beta_2}{\beta_1} + \frac{\alpha_1^2\beta_3}{\beta_1}\right|$$

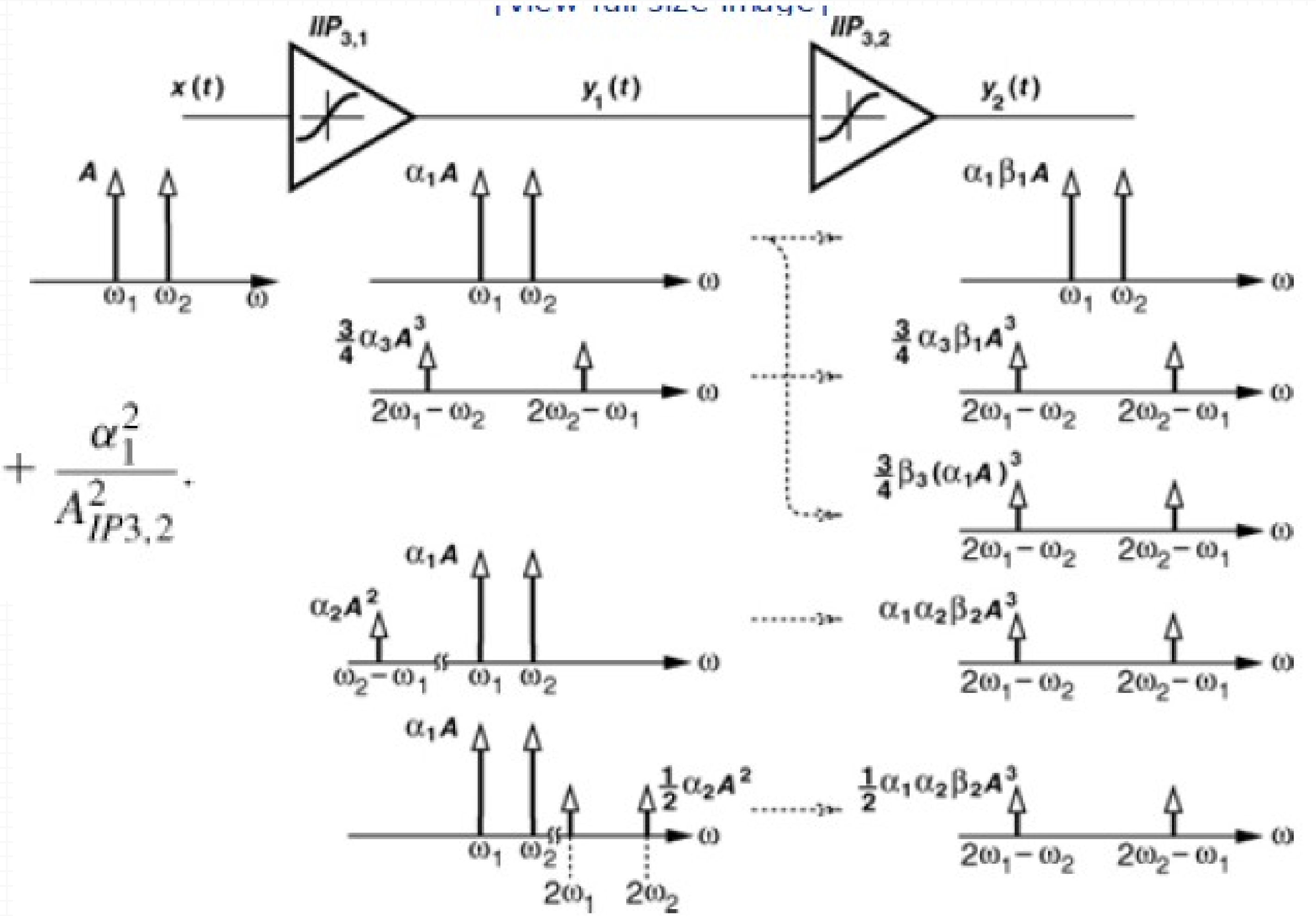
$$= \left|\frac{1}{A_{IP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IP3,2}^2}\right|,$$

Cascaded Nonlinear Stages



$$
\begin{aligned}
y_2(t) &= \alpha_1\beta_1 A(\cos\omega_1 t + \cos\omega_2 t) \\
&\quad + \left(\frac{3\alpha_3\beta_1}{4} + \frac{3\alpha_1^3\beta_3}{4} + \frac{3\alpha_1\alpha_2\beta_2}{2}\right) A^3[\cos(\omega_1 - 2\omega_2)t \\
&\quad + \cos(2\omega_2 - \omega_1)t] + \cdots ,
\end{aligned}
$$

Cascaded Nonlinear Stages



$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2}.$$

$$\frac{1}{A_{IP3}^2} \approx \frac{1}{A_{IP3,1}^2} + \frac{\alpha_1^2}{A_{IP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IP3,3}^2} + \cdots.$$

Intersymbol Interference

- Linear time-invariant systems can also distort a signal if they do not have sufficient bandwidth
- Each bit level is corrupted by decaying tails created by previous bits
- Solution
  - Pulse shaping(Nyquist signaling)
  - Raised cosine

Intersymbol Interference-ISI

- Pulse Shaping

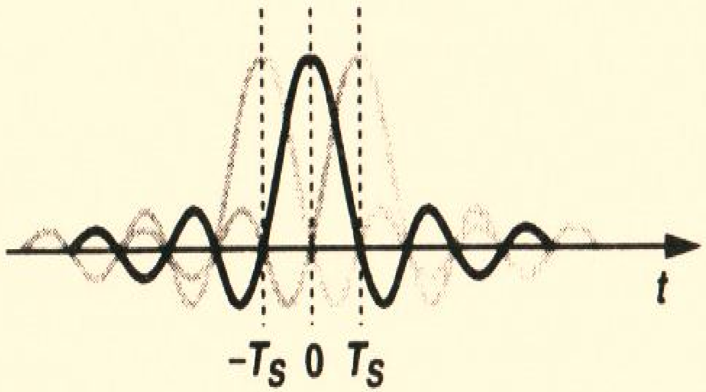


Figure 2.14 Pulse shape with no ISI.

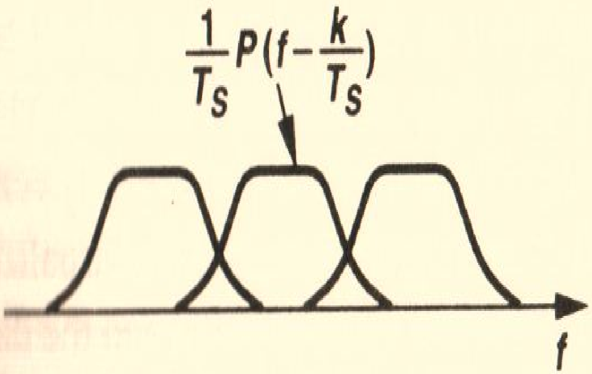


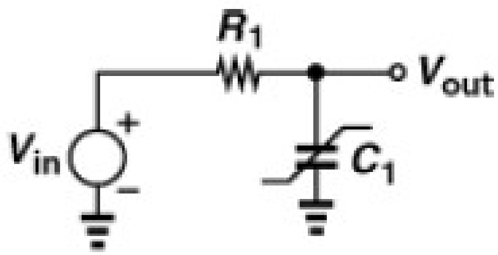
Figure 2.15 Nyquist's condition for the spectrum of a pulse shape that gives no ISI.

The shape is selected such that ISI is zero at certain points in time

AM/PM Conversion

In some RF circuits, e.g., power amplifiers, amplitude modulation (AM) may be converted to phase modulation (PM), thus producing undesirable effects.

$$V_{out}(t) = V_2 \cos[\omega_1 t + \phi(V_1)].$$

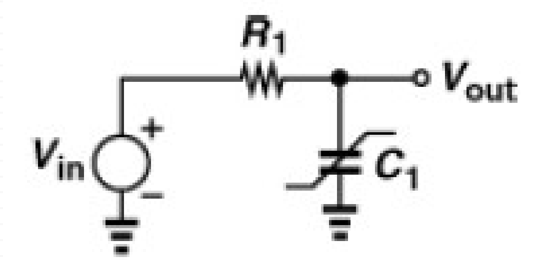


$$C_1 = (1 + \alpha V_{out})C_0$$

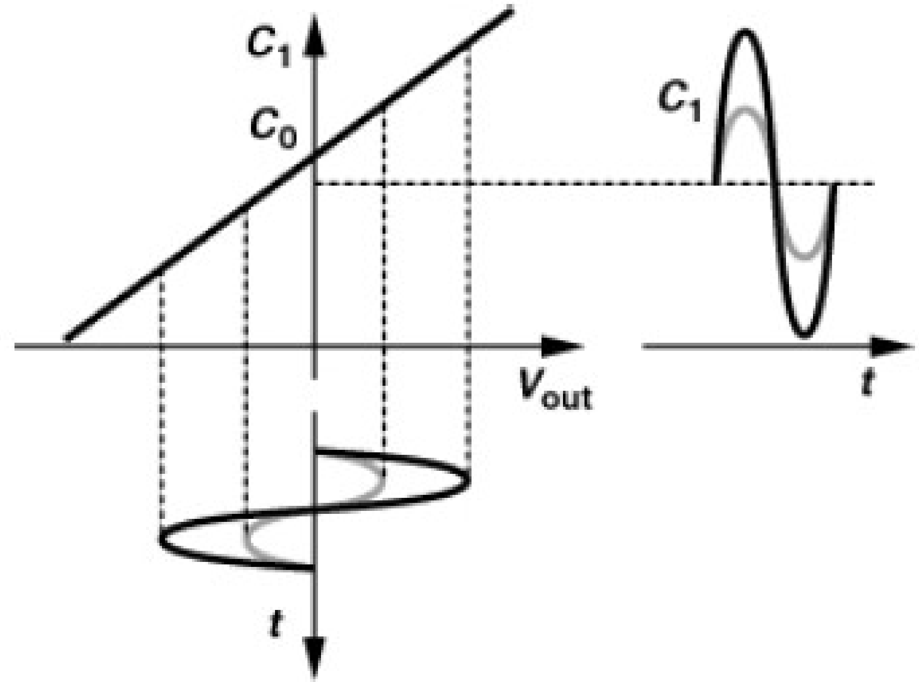
$$V_1 \cos \omega_1 t = R_1(1 + \alpha V_{out})C_0 \frac{dV_{out}}{dt} + V_{out}$$

$$V_{out}(t) \approx V_1 \cos(\omega_1 t - R_1 C_0 \omega_1 - \alpha R_1 C_0 \omega_1 V_1 \cos \omega_1 t).$$

AM/PM Conversion



$$C_1 = C_0(1 + \alpha_1 V_{out} + \alpha_2 V_{out}^2).$$



$$V_{out}(t) \approx V_1 \cos[\omega_1 t - R_1 C_0 \omega_1 (1 + \alpha_1 V_1 \cos\omega_1 t + \alpha_2 V_1^2 \cos^2\omega_1 t)]$$

$$\approx V_1 \cos(\omega_1 t - R_1 C_0 \omega_1 - \frac{\alpha_2 R_1 C_0 \omega_1 V_1^2}{2} - \cdots).$$

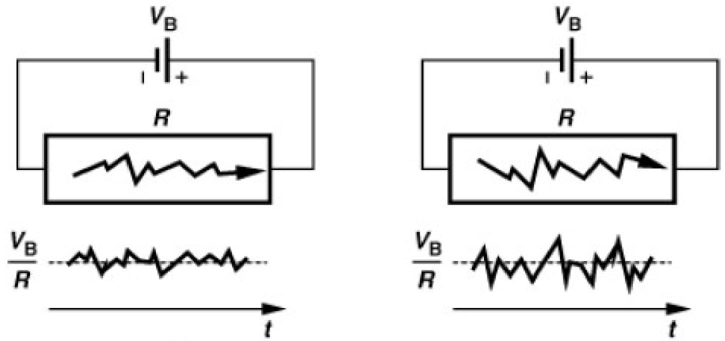
Random Processes and Noise

- 1. Random Processes
  - A family of time functions
- 1) Statiscal Ensembles
  - Doubly infinite(infinite measurements X infinite time)
  - Time average (n(t) : noise voltage)

$$< n(t) > = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} n(t) dt$$

  - Ensemble average(Pn(n) : PDF)

$$\overline{n(t)} = \int_{-\infty}^{+\infty} n(t) P_n(n) dn$$

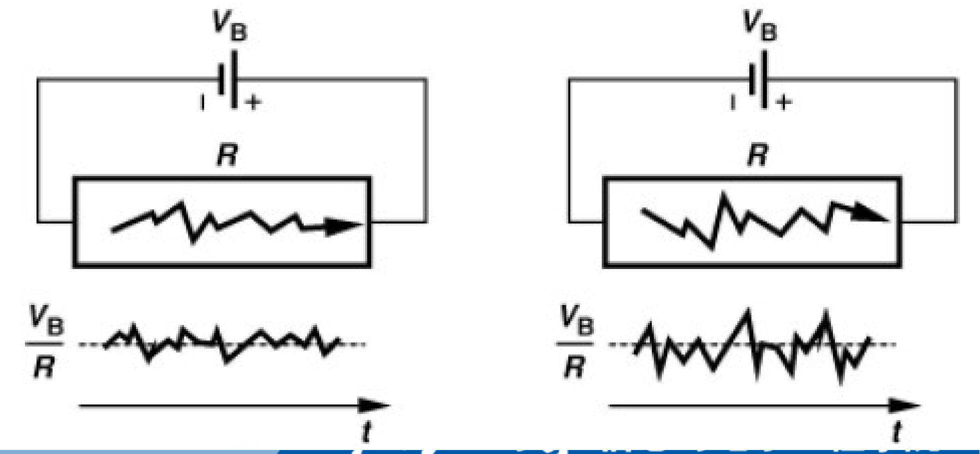


Random Processes and Noise

– Second-order average(mean square)

$$< n^2(t) > = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} n^2(t)dt$$

$$\overline{n^2(t)} = \int_{-\infty}^{+\infty} n^2(t)P_n(n)dn$$



Random Processes and Noise

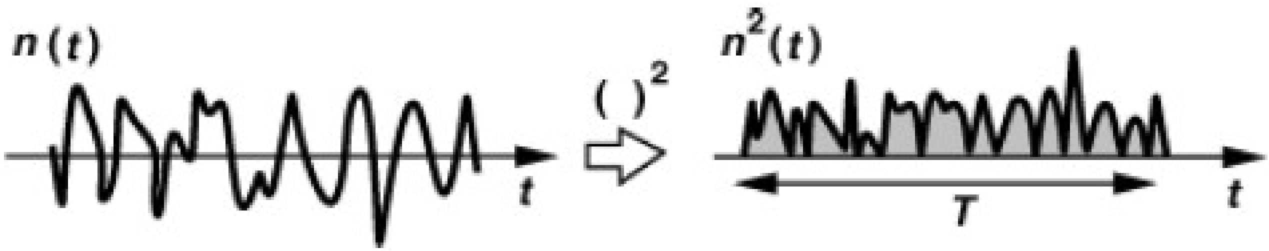
- 2) PDF(Probability Density Function)
  - $P_x(x)dx$ = probability of $x < X < x+dx$
  - X is the measured value of x(t) at some point in time
  - Gaussian distribution
    - PDF of the sum approaches a Gaussian distribution

$$P_x(x) = \frac{1}{\sigma\sqrt{2\pi}} exp \frac{-(x-m)^2}{2\sigma^2}$$

Random Processes and Noise

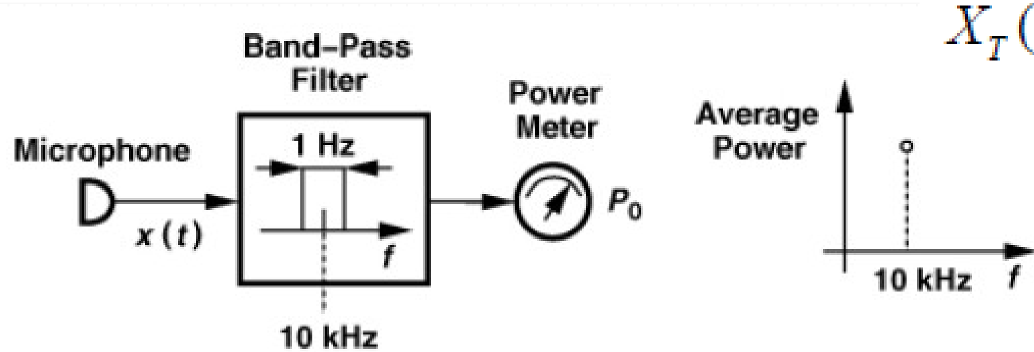
- 3) PSD(Power Spectral Density)

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 dt < \infty$$



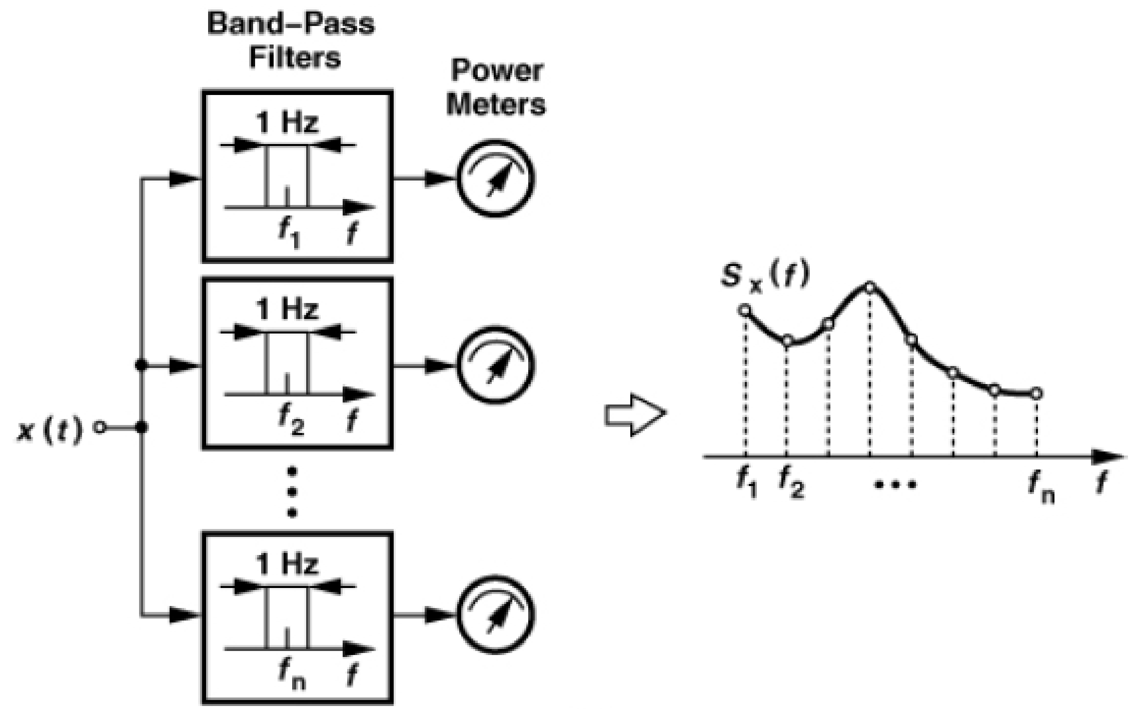
$$S_x(f) = \lim_{T\to\infty} \frac{\overline{|X_T(f)|^2}}{T}$$

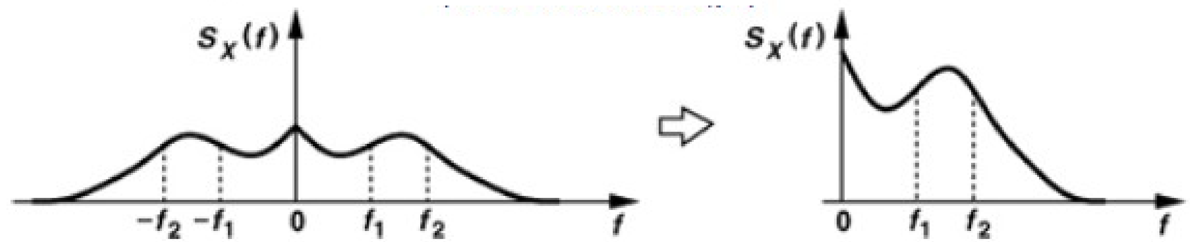
$$X_T(f) = \int_0^T x(t)\exp(-j2\pi ft)dt$$



Random Processes and Noise

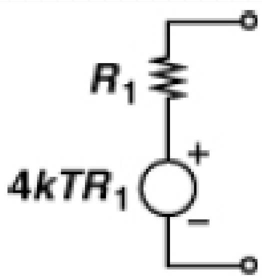
- 3) PSD(Power Spectral Density)

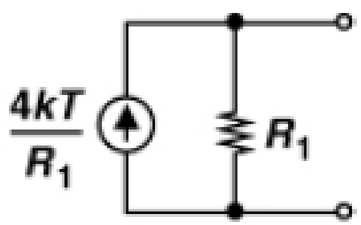




Random Processes and Noise

- 2. Noise
  - 1) Thermal Noise
    - resistor
    - base and emitter resistance of bipolar devices
    - channel resistance of MOSFETs





$$\overline{V_n^2} = 4kTR_1$$

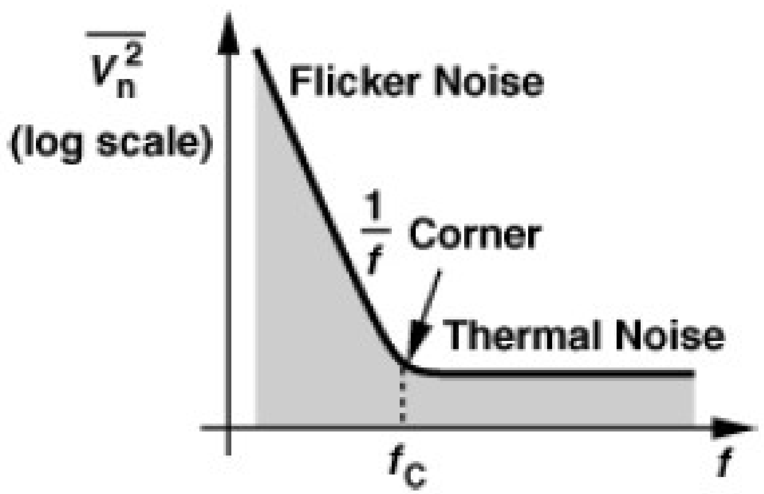
$$\overline{I_n^2} = \overline{V_n^2}/R_1 = 4kT/R_1$$

Noise power=(Vn/2R)^2*R=kTB

Random Processes and Noise

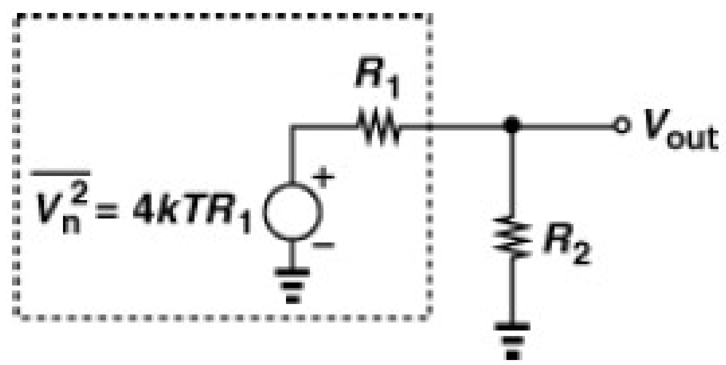
- 2. Noise
  - Flicker Noise

$$\overline{V_n^2} = \frac{K}{WLC_{ox}}\frac{1}{f},$$



Random Processes and Noise

– 2) Input-Referred Noise



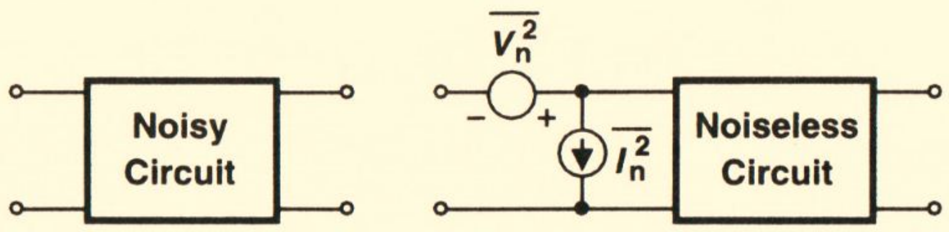


Figure 2.27 Representation of noise by input noise generators.

Random Processes and Noise

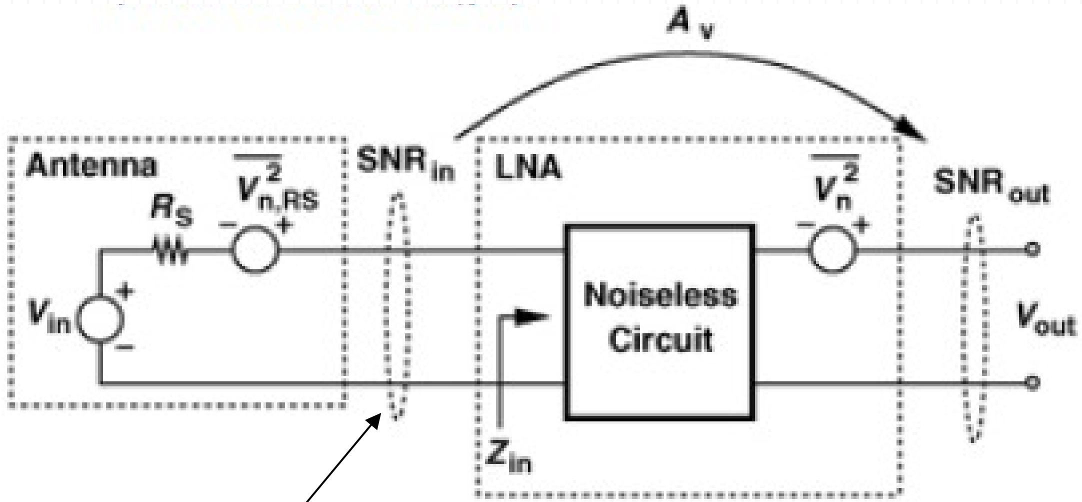
- 3) Noise Figure
  - SNR - Analog circuits
  - NF – RF circuits
  - Consider noise of the circuit & SNR of the pre-stage

$$\text{noise figure} = \frac{SNR_{\text{in}}}{SNR_{\text{out}}},$$

$$NF|_{dB} = 10\log\frac{SNR_{in}}{SNR_{out}}.$$

Random Processes and Noise

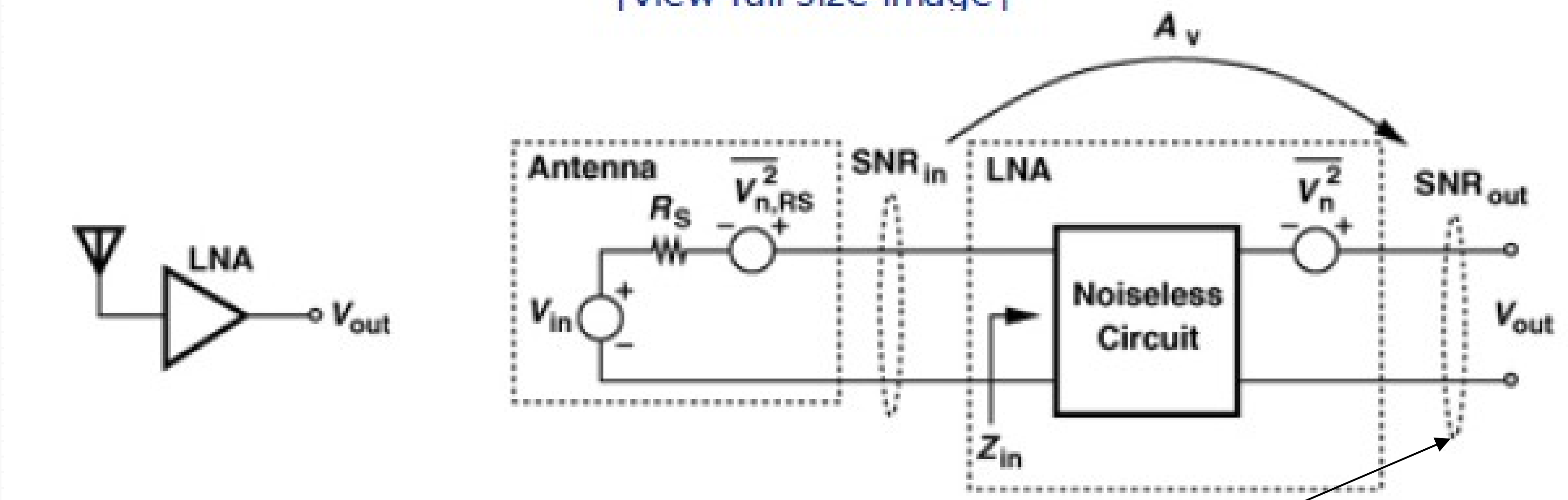
- 3) Noise Figure



$$SNR_{in} = \frac{|\alpha|^2 V_{in}^2}{|\alpha|^2 \overline{V_{RS}^2}}$$

Random Processes and Noise

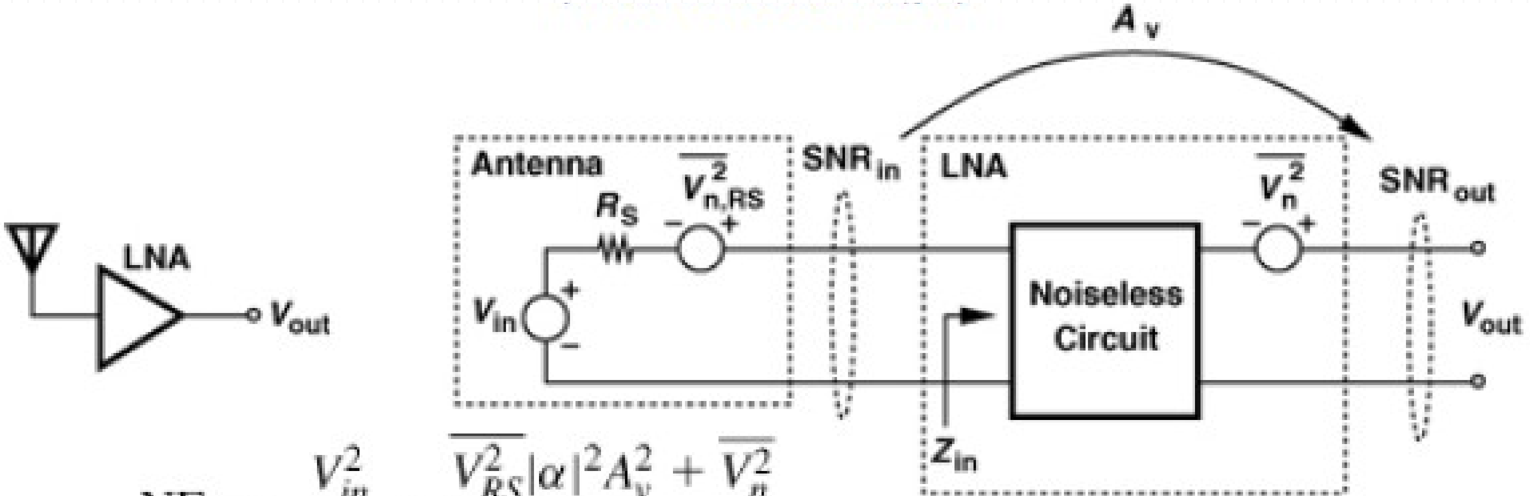
- 3) Noise Figure



$$SNR_{out} = \frac{V_{in}^2 |\alpha|^2 A_v^2}{\overline{V_{RS}^2} |\alpha|^2 A_v^2 + \overline{V_n^2}}.$$

Random Processes and Noise

- 3) Noise Figure



$$NF = \frac{V_{in}^2}{4kTR_S} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{V_{in}^2|\alpha|^2 A_v^2}$$

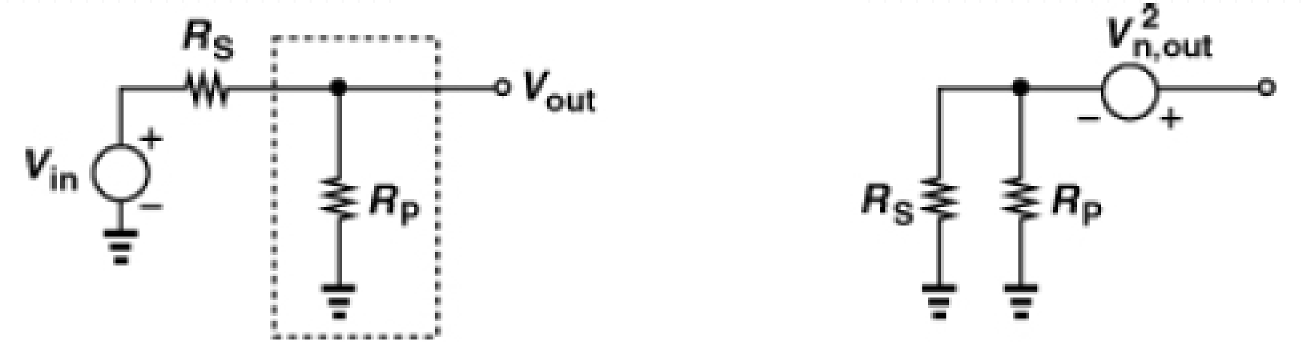
$$= \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2 A_v^2 + \overline{V_n^2}}{|\alpha|^2 A_v^2}$$

$$= 1 + \frac{\overline{V_n^2}}{|\alpha|^2 A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}.$$

Random Processes and Noise

- 3) Noise Figure

$$\mathrm{NF} = \frac{1}{4kTR_S} \cdot \frac{\overline{V_{n,out}^2}}{A_0^2},$$



(a) (b)

$$\overline{V_{n,out}^2} = 4kT(R_S||R_P).$$

$$A_0 = \frac{R_P}{R_P + R_S}$$

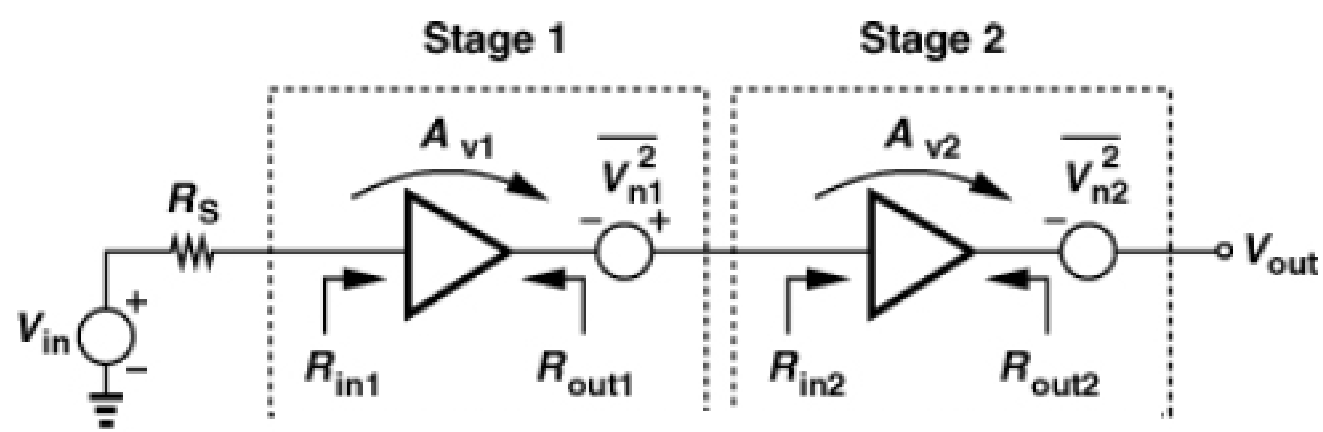
$$\mathrm{NF} = 4kT(R_S||R_P)\frac{(R_S + R_P)^2}{R_P^2}\frac{1}{4kTR_S}$$

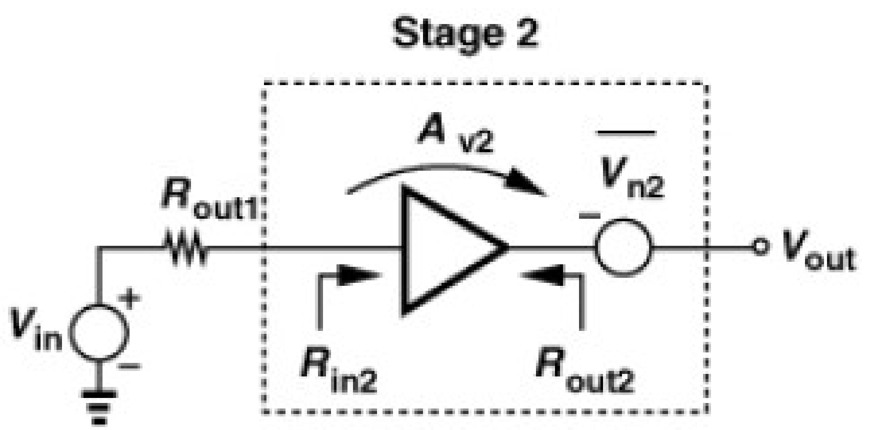
$$= 1 + \frac{R_S}{R_P}.$$

Random Processes and Noise

- 3) Noise Figure

Noise in a cascade of stages





Random Processes and Noise

- 3) Noise Figure

Noise in a cascade of stages

$$A_0 = \frac{V_{out}}{V_{in}} = \frac{R_{in1}}{R_{in1} + R_S} A_{v1} \frac{R_{in2}}{R_{in2} + R_{out1}} A_{v2}.$$

$$\overline{V_{n,out}^2} = \overline{V_{n2}^2} + \overline{V_{n1}^2} \frac{R_{in2}^2}{(R_{in2} + R_{out1})^2} A_{v2}^2$$

$$\begin{aligned}
\mathrm{NF}_{tot} &= 1 + \frac{\overline{V_{n,out}^2}}{A_0^2} \cdot \frac{1}{4kTR_S} \\
&= 1 + \frac{\overline{V_{n1}^2}}{\left(\frac{R_{in1}}{R_{in1} + R_S}\right)^2 A_{v1}^2} \cdot \frac{1}{4kTR_S} \\
&\quad + \frac{\overline{V_{n2}^2}}{\left(\frac{R_{in1}}{R_{in1} + R_S}\right)^2 A_{v1}^2 \left(\frac{R_{in2}}{R_{in2} + R_{out1}}\right)^2 A_{v2}^2} \cdot \frac{1}{4kTR_S}
\end{aligned}$$

Random Processes and Noise

- 3) Noise Figure

Noise in a cascade of stages

$$NF_2 = 1 + \frac{\overline{V_{n2}^2}}{\frac{R_{in2}^2}{(R_{in2}+R_{out1})^2}A_{v2}^2}\frac{1}{4kTR_{out1}}.$$

$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{\frac{R_{in1}^2}{(R_{in1}+R_S)^2}A_{v1}^2\frac{R_S}{R_{out1}}}.$$

Random Processes and Noise

- 3) Noise Figure

Noise in a cascade of stages

$$P_{out,av} = V_{in}^2 \frac{R_{in1}^2}{(R_S + R_{in1})^2} A_{v1}^2 \cdot \frac{1}{4R_{out1}}. \qquad P_{S,av} = \frac{V_{in}^2}{4R_S}$$

$$NF_{tot} = NF_1 + \frac{NF_2 - 1}{A_{P1}},$$

Friis equation

$$NF_{tot} = 1 + (NF_1 - 1) + \frac{NF_2 - 1}{A_{P1}} + \cdots + \frac{NF_m - 1}{A_{P1} \cdots A_{P(m-1)}},$$

Equivalent noise temperature

$$N_i = kT_0B$$

$$N_{i\text{total}} = k(T_0 + T_e)B$$

$$N_o = G_P k(T_0 + T_e)B$$

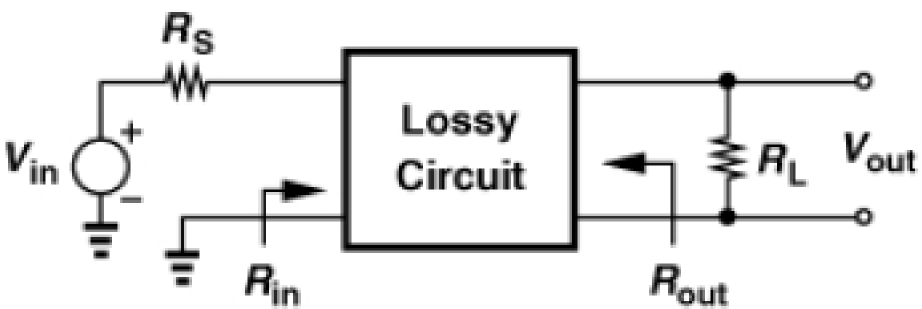
$$F = \frac{P_i / N_i}{P_o / N_o} = \frac{P_i / kT_0B}{G_P P_i / G_P k(T_0 + T_e)B} = 1 + \frac{T_e}{T_0}$$

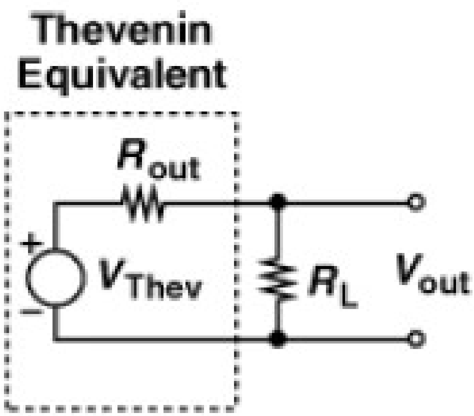
$$T_e = (F - 1)T_0$$

Random Processes and Noise

- 3) Noise Figure

Noise Figure of Lossy Circuits





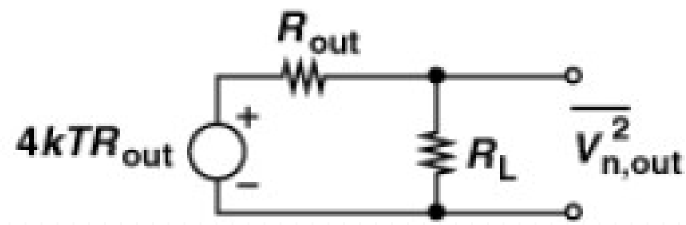
$$L = \frac{V_{in}^2}{V_{Thev}^2} \frac{R_{out}}{R_S}$$

Random Processes and Noise

- 3) Noise Figure

Noise Figure of Lossy Circuits

$$\overline{V^2_{n,out}} = 4kTR_{out}\frac{R_L^2}{(R_L + R_{out})^2}$$



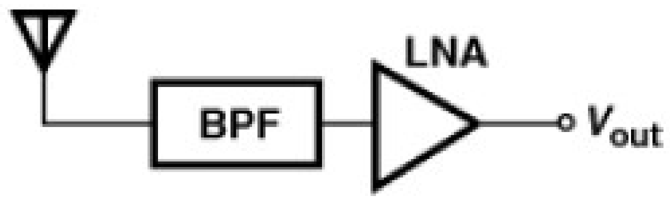
$$A_0 = \frac{V_{Thev}}{V_{in}}\frac{R_L}{R_L + R_{out}}$$

$$NF = 4kTR_{out}\frac{V_{in}^2}{V_{Thev}^2}\frac{1}{4kTR_S}$$
$$= L.$$

Random Processes and Noise

- 3) Noise Figure

Noise Figure of Lossy Circuits



$$NF_{tot} = NF_{filt} + \frac{NF_{LNA} - 1}{L^{-1}}$$
$$= L + (NF_{LNA} - 1)L$$
$$= L \cdot NF_{LNA},$$

if L = 1.5 dB and $NFLNA$ = 2 dB, then $NFtot$ = 3.5 dB.

Sensitivity and Dynamic Range

- 1. Sensitivity
  - Minimum signal level that the system can detect with acceptable SNR.

$$NF = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/P_{RS}}{SNR_{out}}$$

$$P_{sig} = P_{RS} \cdot NF \cdot SNR_{out}.$$

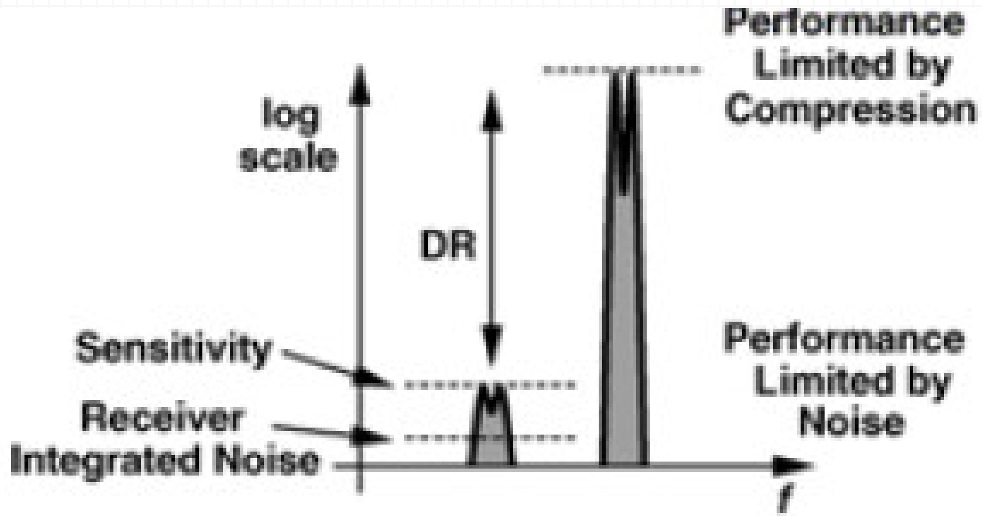
$$P_{sig,tot} = P_{RS} \cdot NF \cdot SNR_{out} \cdot B.$$

$$P_{sen}|_{dBm} = P_{RS}|_{dBm/Hz} + NF|_{dB} + SNR_{min}|_{dB} + 10\log B,$$

  - Pin.min = -174dBm/Hz + NF + 10logB + SNRmin
  - Noise floor : -174dBm/Hz + NF + 10logB

Sensitivity and Dynamic Range

- 2. Dynamic Range
  - Ratio of the maximum input level that the circuit can tolerate to the minimum input level at which the circuit provides a resonable signal quality.



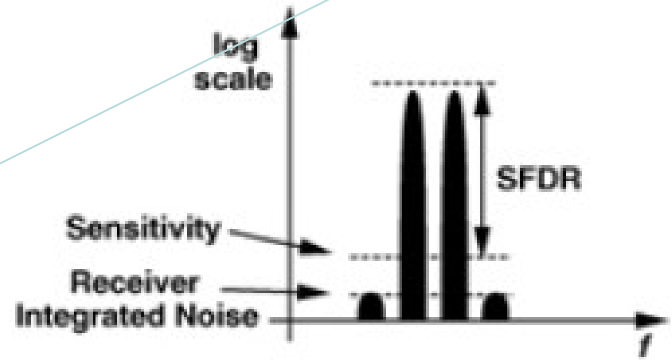
Sensitivity and Dynamic Range

- 2. Dynamic Range
  - SFDR(Spurious-Free Dynamic Range)
    - Upper end of the dynamic range on the intermodulation behavior
    - Lower end on the sensitivity

$$P_{IIP3} = P_{in} + \frac{P_{in} - P_{IM,in}}{2}$$

$$P_{in} = \frac{2P_{IIP3} + P_{IM,in}}{3}$$

$$SFDR = P_{in,max} - (-174\,\text{dBm} + NF + 10\log B + SNR_{min})$$
$$= \frac{2(P_{IIP3} + 174\,\text{dBm} - NF - 10\log B)}{3} - SNR_{min}.$$



- F : Noise floor

$$SFDR = \frac{2(P_{IIP3} + F)}{3} - SNR_{\min}$$

Passive Impedance Transformation

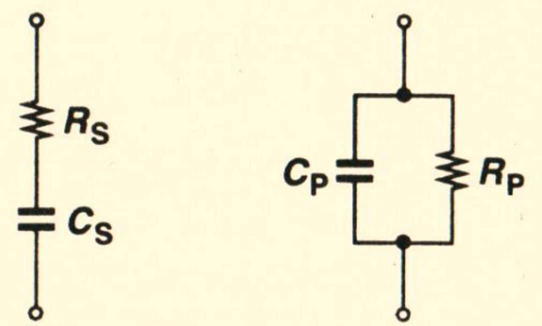


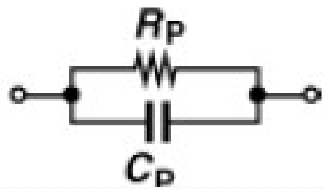
Figure 2.37 Equivalent series and parallel RC circuits.

Q of the series combination : 1/RsCsw
Q of the parallel combination : RpCpw

If Q is relatively high and the band of interest relatively narrow, then one network can be converted to the other

Passive Impedance Transformation

$$\frac{R_P}{R_P C_P s + 1} = \frac{R_S C_S s + 1}{C_S s}$$



$$R_P C_S j\omega = 1 - R_P C_P R_S C_S \omega^2 + (R_P C_P + R_S C_S) j\omega.$$

$Q_S = Q_P.$ $\longrightarrow$

$$R_P C_P R_S C_S \omega^2 = 1$$

$$R_P C_P + R_S C_S - R_P C_S = 0.$$

$$R_P = \frac{1}{R_S C_S^2 \omega^2} + R_S.$$

$$R_P = (Q_S^2 + 1) R_S.$$

$$C_P = \frac{Q_S^2}{Q_S^2 + 1} C_S.$$

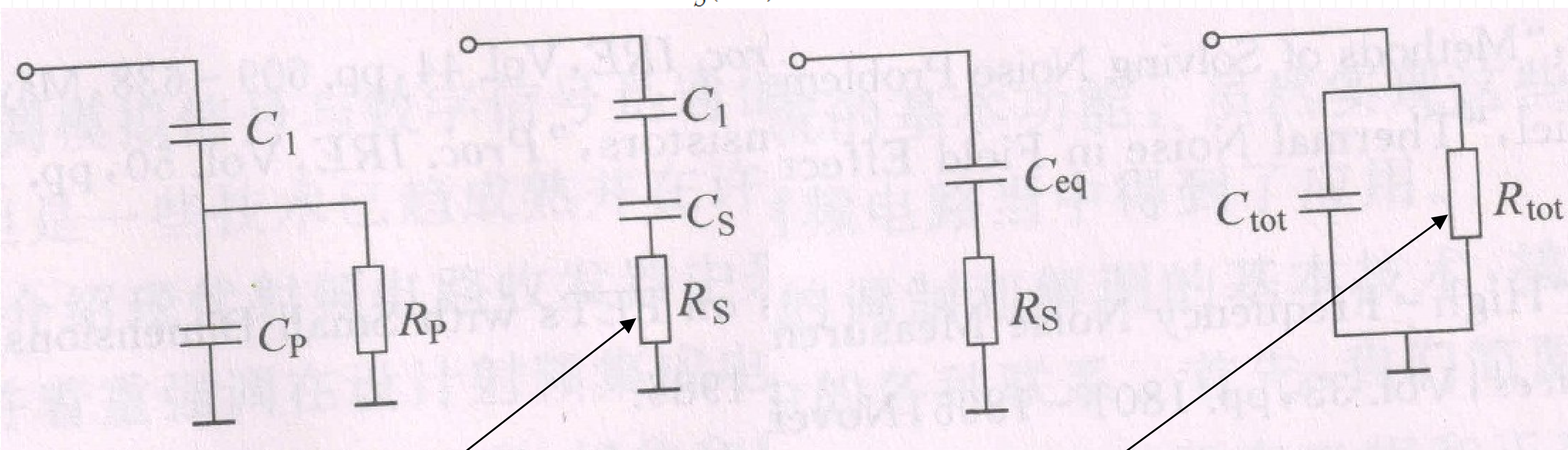
When $Q_S^2 \gg 1$

$$R_P \approx Q_S^2 R_S \qquad C_P \approx C_S.$$

Passive Impedance Transformation

$$Assuming\, R_P \gg Rs,\, C_P \approx C_S$$

$$R_P \approx \frac{1}{R_S(Cw)^2}$$



$$R_S \approx 1/[R_P(C_P\omega)^2]$$

$$R_{tot} \approx 1/[R_S(C_{eq}\omega)^2] = (1 + C_P / C_1)^2 R_P$$

Resistor increase $(1 + C_P / C_1)^2$