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24.

$$f_{X_i}(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$F_{X_i}(x) = \begin{cases} 1-e^{-\lambda x}, & x > 0 \\ 0, & x < 0 \end{cases}$$

$$\therefore F_T(t) = P(X_2 < t, X_3 < t \cup X_3 < t, X_1 < t)$$
$$= 2(1-e^{-\lambda x})^2$$
$$= 2 - 4e^{-\lambda x} + e^{-2\lambda x}$$

~~25.~~

$$\cancel{f_{X_i}(x) = \begin{cases} p & x=1 \\ 1-p & x=0 \end{cases}}$$

$$\therefore \cancel{F_{X_i}(x) =}$$

25.

(1) $P\{X_i = k\} = p^k (1-p)^{1-k}$

$$P\{Z=k\} = P\{\sum_{i=1}^{n} X_i = k\} = C_n^k p^k (1-p)^{n-k}.$$

(2)

$$P\{X=k\} = C_m^k p^k (1-p)^{m-k}$$
$$P\{Y=k\} = C_n^k p^k (1-p)^{n-k}.$$
$$\therefore P\{W=k\} = P\{X+Y=k\} = \sum_{w=0}^{\min(m,n)} P\{X=w, Y=k-w\}$$
$$= \sum P\{X=w\} \cdot P\{Y=k-w\} = C_m^w p^w (1-p)^{m-w} \cdot C_n^{k-w} p^{k-w} \cdot (1-p)^{n-k+w}$$

26.

$$f_X(x)=\begin{cases}\frac{1}{2a}, & -a<x<a\\ 0 & \text{other}\end{cases} \qquad F_X(x)=\begin{cases}0 & x<-a\\ \frac{x+a}{2a} & -a<x<a\\ 1 & x>a\end{cases}$$

$$f_Y(y)=\frac{1}{\sqrt{2\pi}\sigma}\cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$\cancel{F_Z(z)=}$

$\cancel{f_Z(z)=\iint_{x+y\le z} f_X(z-y)\, f_Y(y)\, dy}$

$$F_Z(z)=\iint_{x+y\le z} f_X(\cancel{z-y}\; x)\, f_Y(y)\, dx\, dy$$

$$=\int_{-\infty}^{+\infty} f_X(\cancel{z-y}\; x)\, dx \int_{-\infty}^{z-x} f_Y(y)\, dy$$

$$=N\left(\frac{z-x-\mu}{\sigma}\right).$$

27.

$$f_Z(z)=\int_{-\infty}^{+\infty} f(x, z-\cancel{y}\; x)\, dx$$

$$\begin{cases}0<x<1\\ 0<z-x<2\end{cases} \longrightarrow \begin{cases}0<x<1\\ z-2<x<z\end{cases}$$

$z>3,\ f_Z(z)=0$

$\cancel{3<z}\ 2<z<3 \quad f_Z(z)=\int_{z-2}^{1}\frac{3-\cancel{x}-z+\cancel{x}}{3}\,dx=\frac{(3-z)^2}{3}$

$1<z<2 \quad f_Z(z)=\int_0^{1}\frac{3-z}{3}\,dx=\frac{(3-z)}{3}$

$0<z<1 \quad f_Z(z)=\int_0^{z}\frac{3-z}{3}\,dx=\frac{(3-z)z}{3}$

$z<0 \quad f_Z(z)=0$

28. $Z = X + Y$

$P\{Z<z\} = P\{X+Y<z\} = P\{Y<z-m, X=m\}$

当 $X=0$ 时,

$P\{Z<z\} = 0.5 \cdot P\{Y < z \cancel{-0.5}\}$

$= 0.5 \cdot \int_0^{z} f(y)\,dy$

$\cancel{= 0.5 \int}$

$\cancel{\therefore f_Z(z) = 0.5 f(z-0.5)}$

当 $X=1000$ 时

$P\{Z<z\} = 0.3 \int_0^{z-1000} f(y)\,dy$

当 $X=5000$ 时

$P\{Z<z\} = 0.2 \int_0^{z-5000} f(y)\,dy$

$\therefore f_Z(z) = 0.5 f(z) + 0.3 f(z-1000) + 0.2 f(z-5000)$

30.

(1) $P\{\sum_{i=1}^{10} X_i \geq 2\} = 1 - P\{\sum_{i=1}^{10} X_i < 2\}$

$= 1 - P\{\sum_{i=1}^{10} X_i = 0\} - P\{\sum_{i=0}^{10} X_i = 1\}$

$= 1 - e^{-10\lambda} - C_{10}^{1} \lambda e^{-10\lambda}$

$= 1 - e^{-10\lambda} - 10\lambda e^{-10\lambda}$

(2) $P\{\max X_i \geq 2\} = 1 - P\{\max X_i < 2\}$

$= 1 - P(X_i<2)^{10}$

$= 1 - (1+\lambda)^{10} e^{-10\lambda}$

(3)

$P\{\min X_i = 0\} = 1 - [P\{X_i > 0\}]^{10} = 1-(1-e^{-\lambda})^{10}$

$P\{\max X_i \geq 2, \min X_i = 0\} = P\{\min X_i = 0\} - P\{\max X_i > 2\} + [P\{0 < X_i < 2\}]^{10}$

$= 1-(1-e^{-\lambda})^{10} - (1+\lambda)^{10} e^{-10\lambda} + \lambda^{10} e^{-10\lambda}$

$\therefore P\{\max X_i \geq 2 \mid \min X_i = 0\} = \dfrac{P\{\max X_i \geq 2, \min X_i = 0\}}{P\{\min X_i = 0\}}$

$= \dfrac{1-(1-e^{-\lambda})^{10} - (1+\lambda)^{10} e^{-10\lambda} + \lambda^{10} e^{-10\lambda}}{1-(1-e^{-\lambda})^{10}}$

31.

$P(X+Y=Z) = P(X=k, Y=Z-k)$

$\therefore$

Z	1	2	3	4	5
P	~~0.4~~ 0.04	0.14	0.3	0.32	0.2

$M = \max(X, Y)$

M	0	1	2	3
P	0	0.1	0.5	0.4

$N = \min(X, Y)$

N	0	1	2	3
P	0.2	0.4	0.4	0

33.

~~P{2X=Y}~~

~~P{Z=z} f_Z(z)=~~

$$F_Z(z)=\iint\limits_{2x-y\le z} f(x,y)\,dx\,dy$$

当 $z>0$ 时，$F_Z(z)=1$

当 $-4<z<0$ 时

$$F_Z(z)=\int_{\frac{z}{2}}^{2}dx\cdot\int_{0}^{2x-z}\frac{1}{4}dy$$

$$=1+\frac{z^2}{8}-\frac{9}{16}z$$

当 $z<-14$ 时，$F_Z(z)=0$

$$\therefore f_Z(z)=\begin{cases}\frac{1}{4}z-\frac{9}{16} & -4<z<0\\ 0, & \text{other}\end{cases}$$

~~35.~~

$$\cancel{F_M(m)=P\,F_X(m)\cdot F_Y(m)=\begin{cases}2y, & 0<x<1,\ 0<y<1\\ 0, & \text{other}\end{cases}}$$

$$\cancel{F_N(n)=1-(1-}$$

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35.

$$F_X(x) = \begin{cases} 0, & x<0 \\ x, & 0<x<1 \\ 1, & x>1 \end{cases}$$

$$F_Y(y) = \begin{cases} 0, & y<0 \\ y^2, & 0<y<1 \\ 1, & y>1 \end{cases}$$

$$\therefore F_M(m) = F_X(x)\cdot F_Y(y) = \begin{cases} 0, & \cancel{x<0,\ y<0}\ t<0 \\ m^3 & 0<m<1 \\ 1, & t>1 \end{cases}$$

$$\therefore f_M(m) = \begin{cases} 3m^2, & 0<m<1 \\ 0, & \text{other} \end{cases}$$

$$F_N(n) = 1-(1-F_X(n))(1-F_Y(n)) = \begin{cases} 0, & n<0 \\ n^2+n-n^3, & 0<n<1 \\ 1 & n>1 \end{cases}$$

$$\therefore f_N(n) = \begin{cases} 2n-3n^2+1 & , 0<n<1 \\ 0 & , \text{other} \end{cases}$$

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1.

次数 $k = n\cdot\frac{N}{n}$

2.

$1.2+3\times0.8=3.6>3$

∴选方案二

~~3.~~

~~X~~	~~1~~	~~2~~	~~3~~	~~4~~	~~5~~	~~6~~	~~7~~	~~8~~
~~P~~	~~0~~	~~$\frac{1}{64}$~~	~~$\frac{1}{32}$~~	~~$\frac{3}{64}$~~	~~$\frac{1}{16}$~~	~~$\frac{5}{64}$~~	~~$\frac{3}{32}$~~	~~$\frac{1}{8}$~~

3.

X	1	2	3	4	5	6	7	8
P	0	$\frac{1}{28}$	$\frac{1}{14}$	$\frac{3}{28}$	$\frac{1}{7}$	$\frac{5}{28}$	$\frac{3}{14}$	$\frac{1}{4}$

∴ $E(x)=6$

4.

$\eta_n = np$

$S_n = np - n(1-p) = 2np - 1$

5.

$$P\{X=k\} = \begin{cases} \left(\frac{1}{2}\right)^{k-1}\cdot\frac{1}{2} = \left(\frac{1}{2}\right)^{k}, & k>1 \\ 0 & k=1 \end{cases}$$

∴ $E(X) = \lim_{n\to+\infty} 2\times\frac{1}{4}+3\times\frac{1}{8}+\cdots+n\left(\frac{1}{2}\right)^n = 1.5$