第三章 动态电路频域特性分析

3.2 动态电路频域特性求解

动态电路频域特性求解

- 频域也属于变换域

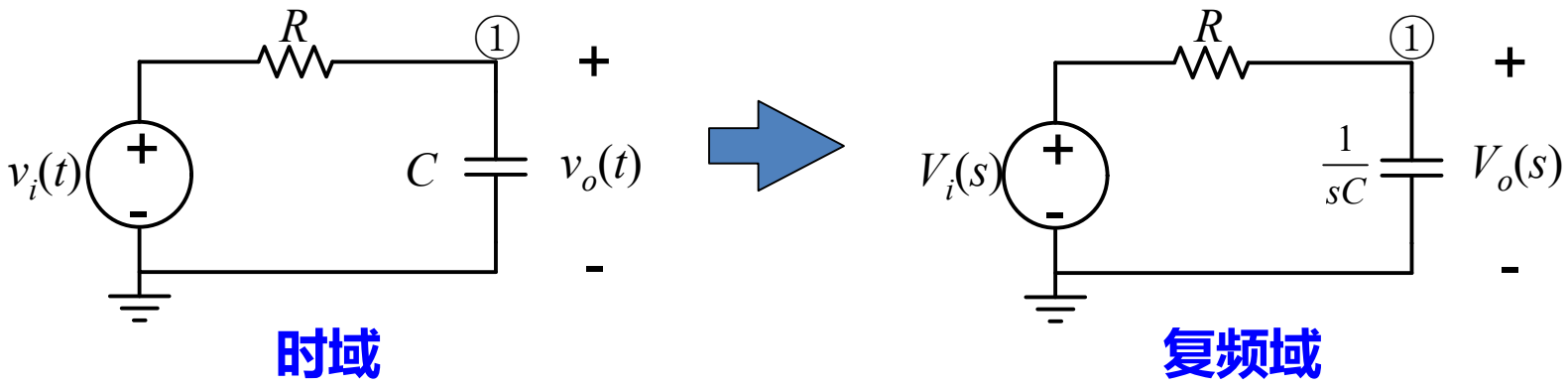
复频域 → **频域**

$s = \sigma + j\omega$ → $\sigma = 0,\ s = j\omega$

- 系统函数 $H(s) = \dfrac{V_o(s)}{V_i(s)}$
  - 也称为传递函数(或转移函数)
- 频域特性 $H(j\omega) = H(s)\big|_{s=j\omega}$

1阶RC电路系统函数

- 假设满足零状态初始条件



$$\frac{V_o(s)-V_i(s)}{R}+sCV_o(s)=0$$

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{1}{sCR+1}$$

1阶RC电路频域特性

$$H(j\omega) = H(s)\Big|_{s=j\omega} = \frac{1}{1+j\omega CR}$$

- 特征角频率 $\omega_n = \frac{1}{RC}$

$$H(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_n}}$$

$$H(j\omega) = \left|H(j\omega)\right| e^{j\varphi(j\omega)}$$

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1+(\omega^2/\omega_n^2)}}$$

幅频特性

$$\varphi(j\omega) = -\arctan\left(\omega/\omega_n\right)$$

相频特性

1阶RC电路频域特性

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega^2/\omega_n^2)}}$$

$$\varphi(j\omega) = -\arctan(\omega/\omega_n)$$

- 当频率很低

$$\omega \ll \omega_n, \quad \frac{\omega}{\omega_n} \ll 1$$

$$|H(j\omega)| \approx 1, \quad \varphi(j\omega) \approx 0$$

- 信号可以“通”过电路

1阶RC电路频域特性

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega^2/\omega_n^2)}}$$

$$\varphi(j\omega) = -\arctan(\omega/\omega_n)$$

- 当频率很高

$$\omega \gg \omega_n, \quad \frac{\omega}{\omega_n} \gg 1$$

$$|H(j\omega)| \approx \frac{\omega_n}{\omega} \to 0, \quad \varphi(j\omega) \to -\frac{\pi}{2}$$

- 信号无法“通”过电路

1阶RC电路频域特性

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega^2/\omega_n^2)}}$$

$$\varphi(j\omega) = -\arctan(\omega/\omega_n)$$

- 当 $\omega = \omega_n$

$$|H(j\omega)| = \frac{1}{\sqrt{2}} = 0.7, \quad \varphi(j\omega) = -\frac{\pi}{4}$$

- 信号幅度只有输入的70%

- 电路具有**低通**的特点

波特图

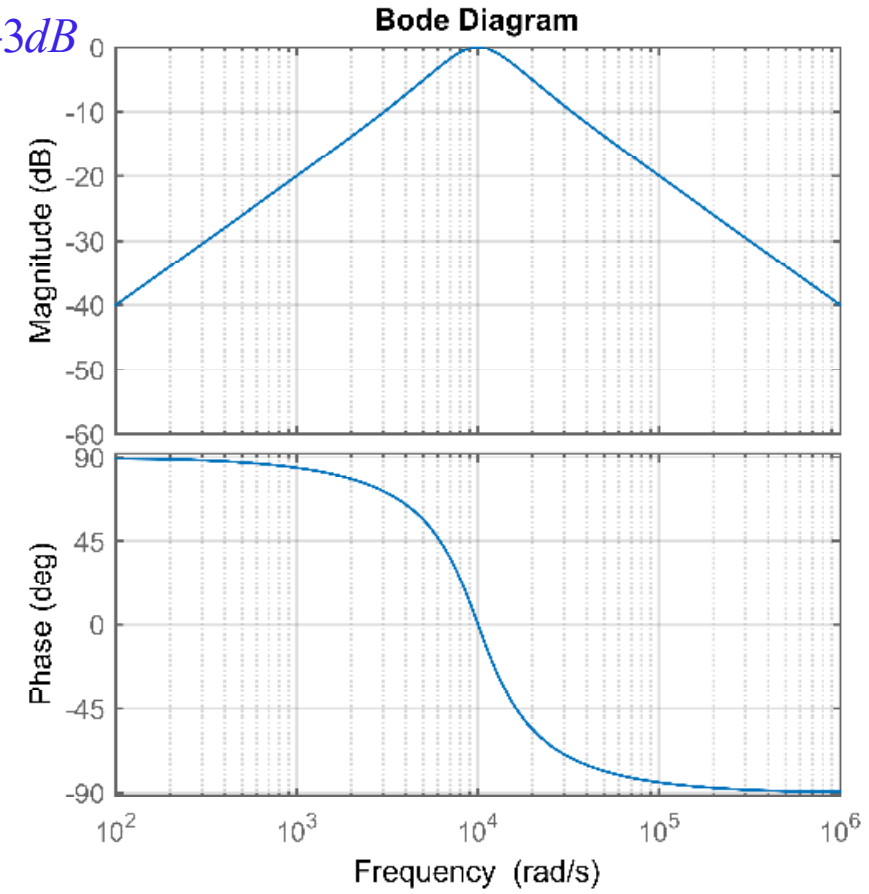
$$|H(j\omega)|(dB) = 20\log 10|H(j\omega)|$$

$$H(j\omega) = |H(j\omega)|e^{j\varphi(j\omega)}$$

$$20\log 10(0.7) = -3dB$$

$|H(j\omega)|(dB)$

$\varphi(j\omega)$



幅频特性

相频特性

1阶RC电路波特图

$$H(s)=\frac{Q(s)}{P(s)}=\frac{a_m s^m+a_{m-1}s^{m-1}+\cdots+a_1 s+a_0}{b_n s^n+b_{n-1}s^{n-1}+\cdots+b_1 s+b_0}$$

- tf函数语法

$$\mathrm{tf}\left(\left[a_m\ a_{m-1}\cdots a_0\right],\left[b_n\ b_{n-1}\cdots b_0\right]\right)$$

$$H(s)=\frac{1}{sCR+1}$$

$$\left[a_m\ a_{m-1}\cdots a_0\right]=\left[1\right]$$

$$\left[b_n\ b_{n-1}\cdots b_0\right]=\left[RC\ 1\right]$$

```
R=100; C=1e-6;
H=tf([1],[R*C 1])
bode(H);
```

```
>> H=tf([1],[R*C 1])

H =

      1
  ------------
  0.0001 s + 1

Continuous-time transfer function.

>>
```

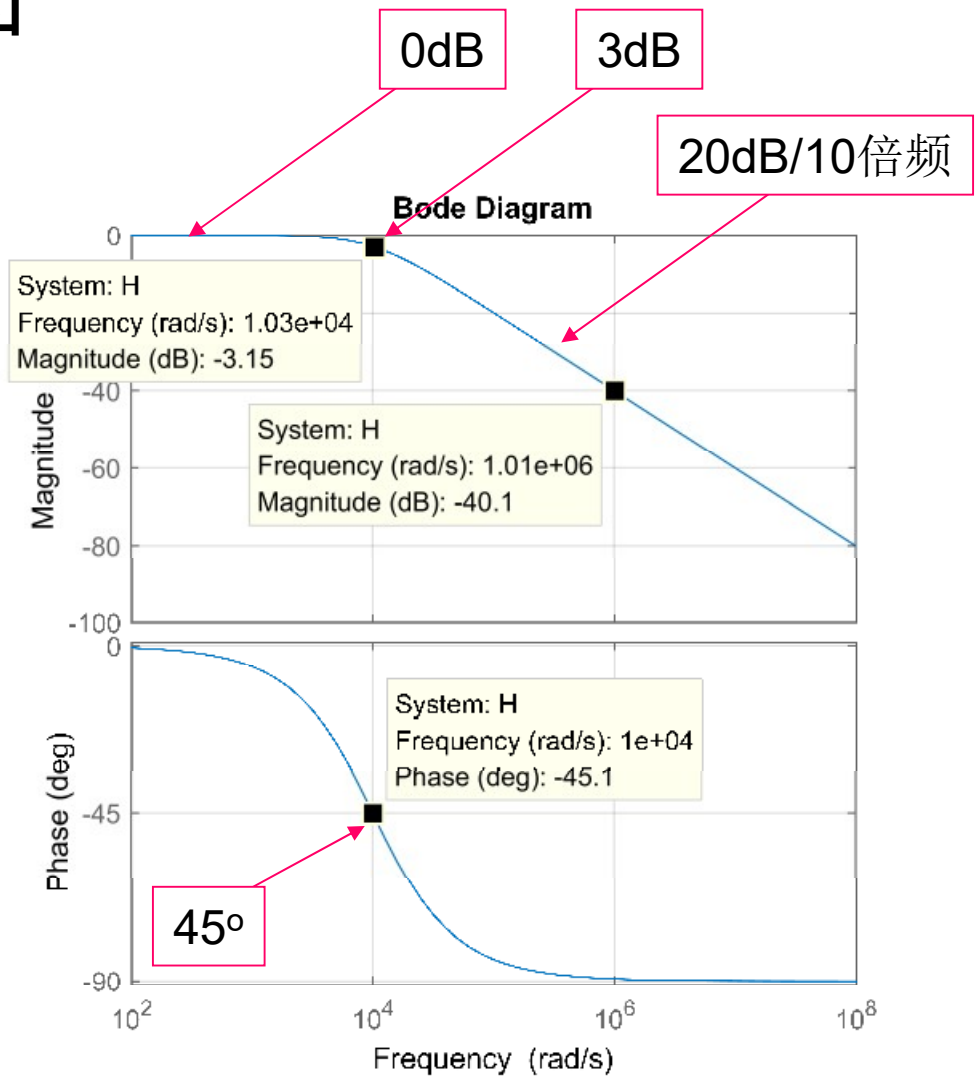
1阶RC电路波特图

$$R = 100\Omega \quad C = 1\mu F$$

$$\omega_n = 1000 rad/s$$

$$|H(j\omega)|_{\omega \gg \omega_n} \approx \frac{\omega_n}{\omega}$$

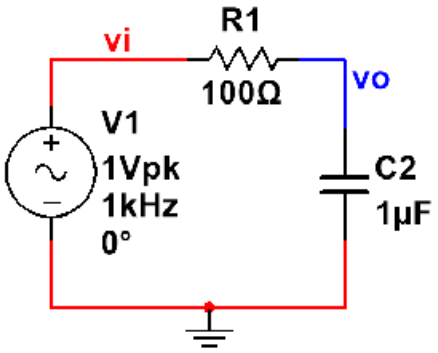
$$20\log 10\left(\frac{\omega_n}{10\omega_x}\right) - 20\log 10\left(\frac{\omega_n}{\omega_x}\right) = -20$$

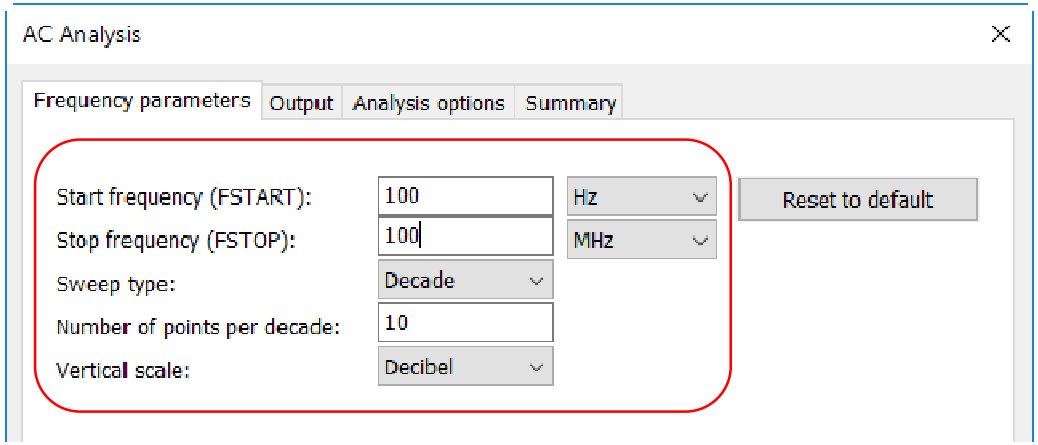


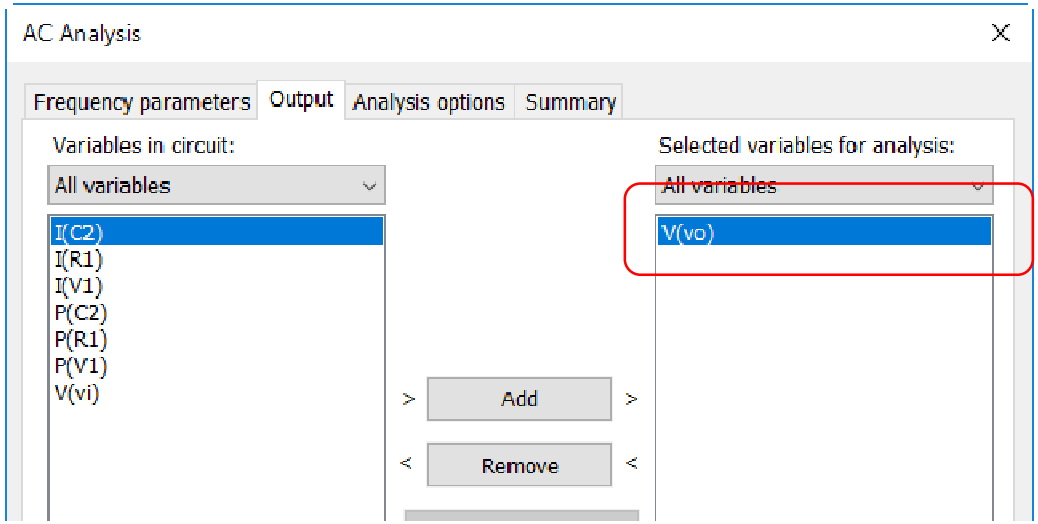
1阶RC电路波特图仿真

$R = 100\Omega \quad C = 1\mu F$

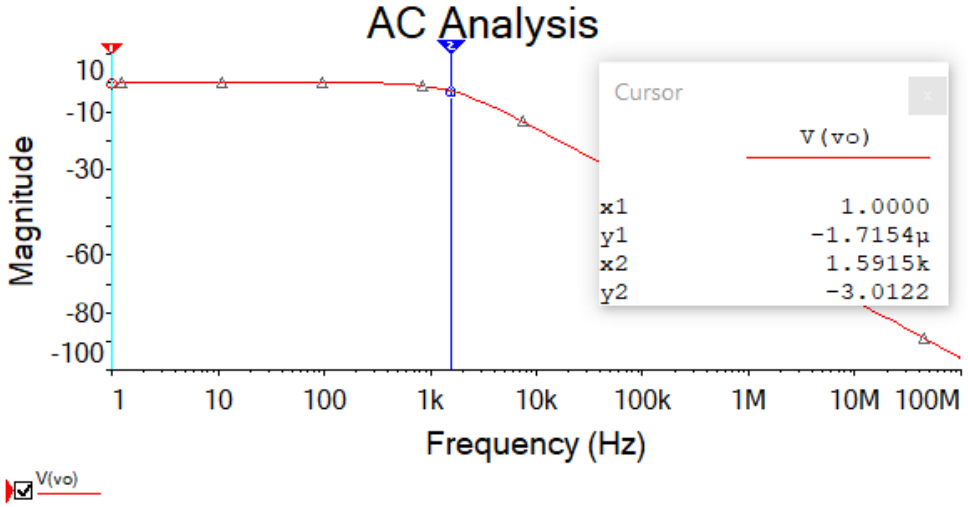
$\omega_n = 1000 rad / s$



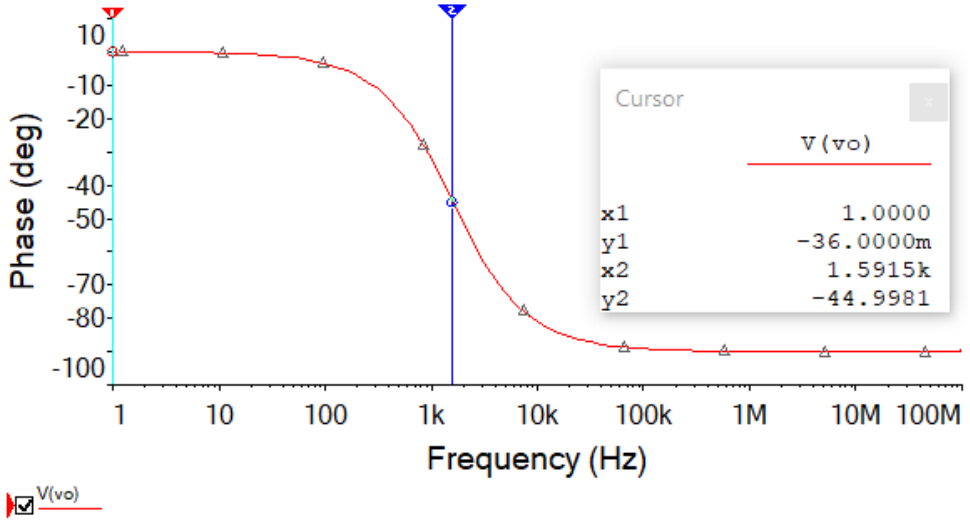




1阶RC电路波特图仿真



$$f_n = \frac{10^4}{2\pi} \approx 1.6 \times 10^3$$



小结

- 1阶RC电路
  - 系统函数
  - 频域特性
  - 波特图及其仿真