# 6.632 Solution to Problem Set 5

## Solution P5.1

At the cutoff, the Goos-Hanchen shift at $x = d$ is zero. Combing guidance condition and dispersion relations, we can get

$$k_{cm} = \frac{m\pi + \tan^{-1}\frac{\epsilon_1\sqrt{\epsilon_2-\epsilon_0}}{\epsilon_0\sqrt{\epsilon_1-\epsilon_2}}}{d\sqrt{1-\epsilon_2/\epsilon_1}}$$

## Solution P5.2

(a) $\overline{J}_s = \hat{z}\frac{2k_0}{\omega\mu}e^{i\sqrt{3}k_0 z/2}$

(b) The waves radiated by the surface current sheet are

$$\overline{E}_+ = (\hat{x}\sqrt{3}/2 - \hat{z}1/2)e^{ik_0x/2}e^{ik_0z\sqrt{3}/2}$$

$$\overline{E}_- = (-\hat{x}\sqrt{3}/2 - \hat{z}1/2)e^{-ik_0x/2}e^{ik_0z\sqrt{3}/2}$$

which cancel the incident wave in region $x > 0$ and form the reflected wave in region $x < 0$.

## Solution P5.3

(a)

$$\frac{d^2S_\rho}{d\ell d\lambda} = \frac{d^2S_\rho}{d\ell d\omega}\frac{d\omega}{d\lambda} = \frac{\mu q^2}{4\pi}\omega(1-\frac{1}{n^2\beta^2})\frac{2\pi c}{\lambda^2}$$

$$\frac{d^2N}{d\ell d\lambda} = \frac{q^2c}{2\lambda^2\hbar}\mu(1-\frac{1}{n^2\beta^2})$$

Since $1-\frac{1}{n^2\beta^2} = \sin^2\theta$, $\frac{dN}{d\ell} \propto \frac{d\lambda}{\lambda^2}\sin^2\theta$.

(b) $\theta = 3.6°$

(c) $\ell = 0.528$ m