2.1

(a) $\Omega_1 = \int_{\pi/4}^{\pi/3}\int_{\pi/6}^{\pi/3} d\Omega = \int_{\pi/4}^{\pi/3} d\varphi \int_{\pi/6}^{\pi/3} \sin\theta\, d\theta = \frac{\sqrt{3}-1}{24}\pi$

$\Omega_A = \Delta\theta \cdot \Delta\varphi = (\theta_2-\theta_1)(\varphi_2-\varphi_1) = \frac{\pi^2}{72}$

Ω_1 与 Ω_A 还是明显有区别的，$\Delta = \frac{|\Omega_1-\Omega_A|}{\Omega_1} \approx 30.1\%$.

(b)

$D_0 = \frac{4\pi}{\Omega_A} \approx 91.7$

$\therefore D_{0dB} = 10\log_{10}(91.7) = 19.6\ dB$

2.2

$\because \vec{W} = \vec{e}\times\vec{h}$

$e(x,y,z;t) = Re[E(x,y,z)e^{j\omega t}]$.

$h(x,y,z;t) = Re[H(x,y,z)e^{j\omega t}]$

~~$\vec{W} = Re[\vec{E}(x,y,z)\times\vec{H}(x,y,z)\cdot e^{j2\omega t}]$~~

$\therefore Re[\vec{E}e^{j\omega t}] = \frac{1}{2}[\vec{E}e^{j\omega t} + \vec{E}^* e^{-j\omega t}]$

$Re[\vec{H}e^{j\omega t}] = \frac{1}{2}[\vec{H}e^{j\omega t} + \vec{H}^* e^{-j\omega t}]$

$\therefore \vec{W} = \frac{1}{2}Re[\vec{E}\times\vec{H}^*] + \frac{1}{2}Re[\vec{E}\times\vec{H}\,e^{j2\omega t}]$.

2.3

(a) $W_{rad} = \frac{1}{2}Re(\vec{E}\times\vec{H}^*) = \frac{|E|^2}{2\eta} = \frac{1}{2}|E|^2 = 12.5\ V/m^2$.

(b) $P_{rad} = \frac{1}{2}\oint_S Re[\vec{E}\times\vec{H}^*]\cdot ds = \frac{|E|^2}{2\eta}\oint_S ds = 5\times10^5\pi$

2.8

HPBW = 10°

(a) ∵ 当 $\theta_h = 5^\circ$ 时, $V(\theta_h) = 0.5$

∴ $(\cos 5^\circ)^n = 0.5$

∴ $n \log(\cos 5^\circ) = \log 0.5$

∴ $n = \frac{\log 0.5}{\log(\cos 5^\circ)} \approx 181.81$

(b)

$P_{rad} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} U \sin\theta \, d\theta \, d\varphi = 0.03446$

∴ $D_0 = \frac{4\pi}{\Omega_A} = 364.68$

∴ $D_A(dB) = 10 \log 2 \approx 25.62 dB$

(c)

$D_2 \simeq \frac{4\pi}{\theta_{1r}\theta_{2r}} = \frac{41253}{\theta_{1d}\theta_{2d}} = 41253$

∴ $D_2(dB) = 26.15 dB$

(d)

$D_3 \simeq \frac{32 \ln 2}{\theta_{1r}^2 + \theta_{2r}^2} = \frac{72815}{\theta_{1d}^2 + \theta_{2d}^2} = 364.075$

$D_3(dB) = 25.61 dB$

2.29

$G = 16 dB = 10^{1.6}$

∵ 无损耗

∴ $P_{out} = G \cdot P_{in} = 8 \times 10^{1.6} W$

∴ $W_{rad} = \frac{P_{out}}{S} = 2.53 \times 10^{-7} W/cm^2$