Problem 1

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = -0.999948 \quad \cancel{\approx -1}$$

$$p_2 = p_1 - \frac{f(p_1)}{f'(p_1)} = -0.999949$$

$f'(x) = -3x^2 + \sin x \quad \therefore f'(p_0 = 0) = 0 \quad \therefore p_0 = 0$ can't be used

Problem 2

(1) $$p_{k+1} = p_k - \frac{b - \frac{1}{p_k}}{\frac{1}{p_k^2}} = 2p_k - bp_k^2.$$

$$|\varepsilon_{k+1}| = \left| \frac{\frac{1}{b} - (2p_k - bp_k^2)}{\frac{1}{b}} \right| = |1 - 2bp_k + b^2p_k^2| = (bp_k - 1)^2 = \left(\frac{\frac{1}{b} - p_k}{\frac{1}{b}} \right)^2.$$

$\because \varepsilon_k = \dfrac{\frac{1}{b} - x_k}{\frac{1}{b}} \quad \therefore |\varepsilon_{k+1}| = \varepsilon_k^2.$

(2)

$f'(x) = \dfrac{1}{x^2}, \; f''(x) = -\dfrac{2}{x^3} \quad \therefore f(x) \in C^2(0, \frac{2}{b}].$

$f'(x) \neq 0, \; x \in (0, \frac{2}{b})$

$\therefore \exists\, \delta = \frac{1}{b}$, st sequence $\{p_n\}_{n=1}^{\infty}$ converge

for $p_0 \in (0, \frac{2}{b})$.

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Problem 3:

(1) $J = \begin{bmatrix} 3 & x_3 \sin(x_2 x_3) & x_2 \sin(x_2 x_3) \\ 8x_1 & -1250x_2 + 2 & 0 \\ -x_2 e^{-x_1 x_2} & -x_1 e^{-x_1 x_2} & 20 \end{bmatrix}$

$$X^{(1)} = X^{(0)} - J(X^{(0)}) F(X^{(0)}) = \begin{pmatrix} \frac{9}{2} \\ 2 \\ -\frac{200\pi}{3} \end{pmatrix}$$

$$X^{(2)} = X^{(1)} - J(X^{(1)}) \cdot F(X^{(1)}) = \begin{pmatrix} -0.009 \\ -4.84 \\ 3.97 \end{pmatrix} \times 10^3$$

(2) $J = \begin{bmatrix} 2x_1 & 1 & 0 \\ 1 & -2x_2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$$X^{(1)} = X^{(0)} - J(X^{(0)}) \cdot F(X^{(0)}) = \begin{bmatrix} 5 \\ 37 \\ 45 \end{bmatrix}$$

$$X^{(2)} = \begin{bmatrix} 1124 \\ -101294 \\ 1305 \end{bmatrix}$$

Problem 4:

(1) $g(x_1, x_2, x_3) = (15x_1 + x_2^2 - 4x_3 - 13)^2 + (x_1^2 + 10x_2 - x_3 - 11)^2 + (x_2^3 - 25x_3 + 22)^2$

$$\nabla g = \begin{bmatrix} 30(15x_1 + x_2^2 - 4x_3 - 13) + 4x_1(x_1^2 + 10x_2 - x_3 - 11) \\ 4x_2(15x_1 + x_2^2 - 4x_3 - 13) + 20(x_1^2 + 10x_2 - x_3 - 11) + 6x_2^2(x_2^3 - 25x_3 + 22) \\ -8(15x_1 + x_2^2 - 4x_3 - 13) - 2(x_1^2 + 10x_2 - x_3 - 11) - 50(x_2^3 - 25x_3 + 22) \end{bmatrix}$$

取 ~~$x^{(0)} = [0;0;0]$~~ $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\alpha = 0.1$

解得: $X = \begin{bmatrix} 1.0353 \\ 1.0855 \\ 0.9271 \end{bmatrix}$

(2) $g(x_1, x_2, x_3) = (10x_1 - 2x_2^2 + x_2 - 2x_3 - 5)^2 + (8x_2^2 + 4x_3^2 - 9)^2 + (8x_2x_3 + 4)^2$

$$\nabla g = \begin{bmatrix} 20(10x_1 - 2x_2^2 + x_2 - 2x_3 - 5) \\ 2(-4x_2 + 1)(10x_1 - 2x_2^2 + x_2 - 2x_3 - 5) + 36x_2(8x_2^2 + 4x_3^2 - 9) + 16x_3(8x_2x_3 + 4) \\ -4(10x_1 - 2x_2^2 + x_2 - 2x_3 - 5) + 16x_3(8x_2^2 + 4x_3^2 - 9) + 16x_2(8x_2x_3 + 4) \end{bmatrix}$$

取 $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\alpha = 0.1$

解得: $X = \begin{bmatrix} 0.5 \\ 1 \\ -0.5 \end{bmatrix}$.

Problem 5

(1) $\frac{\partial g_1}{\partial x_1} = \frac{x_1}{5} < \frac{1.5}{5} = \frac{3}{10} < \frac{1}{2}$, $\frac{\partial g_2}{\partial x_1} = \frac{x_2^2+1}{10} < \frac{13}{40} < \frac{1}{2}$,

$\frac{\partial g_1}{\partial x_2} = \frac{x}{1005} < \frac{1.5}{5} = \frac{3}{10} < \frac{1}{2}$, $\frac{\partial g_2}{\partial x_2} = \frac{2x_1x_2}{10} < \frac{9}{20} < \frac{1}{2}$.

$\therefore$ G has only fixed-point in D.

(2).

$$x^{(1)} = \cancel{x^{(0)}} \ G(x^{(0)}) = \begin{pmatrix} \frac{9}{10} \\ \frac{4}{5} \end{pmatrix}$$

$$x^{(2)} = G(x^{(1)}) = \begin{pmatrix} 0.945 \\ 0.9476 \end{pmatrix}$$