Problem 1

a. $L_{2,0}(x) = \frac{(x-0.3)(x-0.6)}{-0.3\times(-0.6)} \cdot e^{0}\cos 0$

$L_{2,1}(x) = \frac{x\cdot(x-0.6)}{0.3\times(-0.3)} \cdot e^{2\times 0.3}\cos 0.9$

$L_{3,1}(x) = \frac{x(x-0.3)}{0.6\times 0.3} \cdot e^{2\times 0.6}\cos(3\times 0.6)$

$\therefore P(x) = 3.75x^2 + 1.62x + 1$

$\Delta = \left|\frac{f^{(3)}(\xi)}{3!}(x-0.6)(x-0.3)x\right| \leq 0.25$

b.

$L_{2,0}(x) = \frac{(x-2.4)(x-2.6)}{-0.4\times(-0.6)}$

$L_{2,1}(x) = \frac{(x-2)(x-2.6)}{0.4\times(-0.2)}$

$L_{2,2}(x) = \frac{(x-2)(x-2.4)}{0.6\times 0.2}$

$P(x) = -1.6\times 10^{-3}x^2 + 0.01x - 0.01$.

$\Delta = \left|\frac{f^{(3)}(\xi)}{3!}\right| \cdot \left|(x-2)(x-2.4)(x-2.6)\right| \leq 3.2\times 10^{-4}$.

Problem 2.

$P(x) = 6x^3 + bx^2 + cx$

$\therefore \; 6 + b + c = 3 \qquad \therefore \begin{cases} b = -20 \\ c = 17 \end{cases}$

$48 + 4b + 2c = 2$

$\therefore \; \cancel{y = 6 - 20 + 17 = 3}$

$y = 6 \times 0.5^3 - 20 \times 0.5^2 + 17 \times 0.5 = 4.25$

Problem 3

$$P_{0,1,2} = \frac{(x-x_0)P_{1,2} - (x-x_2)P_{0,1}}{x_2 - x_0}$$

$$P_{0,2} = \frac{(x-x_0)P_2 - (x-x_2)P_0}{x_2 - x_0} = \frac{f(0.5)\cdot x - (x-0.5)}{0.5} = x+1$$

$\therefore \; P_2 = f(0.5) = 1.5$

$$P_{1,2} = \frac{(x-x_1)P_2 - (x-x_2)P_1}{x_2 - x_1} = \frac{(x-0.25)\times 1.5 - (x-0.5)\times 2}{0.25}$$

$$= -2x + 2.5$$

$$\therefore \; P_{0,1,2} = \frac{(x-0)(-2x+2.5) - (x-0.5)(2x+1)}{0.5}$$

$$= -8x^2 + 8x + 1$$

$$\therefore \; P_{0,1,2,3}(2.5) = \frac{(2.5-0)\cdot P_{1,2,3}(2.5) - (2.5-0.75)\cdot P_{0,1,2}(2.5)}{x_3 - x_0}$$

$$= 77.7$$

Problem 4:

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1}$$

$\therefore f[x_1] = 3$

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$\therefore f[x_0, x_1] = 5$

$$\because f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0}$$

$\therefore f[x_0] = 1$.

Problem 5:

① $h_0 = 1-0 = 1$, $h_1 = 2-1 = 1$

$S_j(x) = a_j + b_j(x-x_j) + c_j(x-x_j)^2 + d_j(x-x_j)^3$

$a_0 = 0$, $a_1 = 1$, $a_2 = 2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore [c_0 \; c_1 \; c_2]^T = [0 \quad 0 \quad 0]^T$

$\therefore [b_0 \; b_1 \; b_2]^T = [1 \quad 1 \quad 1]$

$[d_0 \; d_1 \; d_2]^T = [0 \; 0 \; 0]$

$$\therefore S(x) = \begin{cases} x \\ 1 + (x-1) = x \\ 2 + (x-2) = x \end{cases}$$

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②

$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$\therefore [C_0 \quad C_1 \quad C_2]^T = [0, 0, 0]^T$

$\therefore S(x) = x$