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6.

(1) $E(X) = \int_0^{+\infty} x \cdot \frac{2}{x} e^{-2x} dx \int_0^{x} dy = \int_0^{+\infty} 2x e^{-2x} dx = \frac{1}{2}$

$\therefore E$

(2) $E(3X-1) = 3E(X) - 1 = \frac{1}{2}$

(3) $E(XY) = \int_0^{+\infty} \int_0^{x} xy \cdot \frac{2}{x} e^{-2x} dx dy$

$= \frac{1}{4}$

7.

(1) $\begin{cases} 1-x & 0<x<l \\ x & 1>x>l \\ x & x=l,\ l \le \frac{1}{2} \\ 1-x & x=l,\ l>\frac{1}{2} \end{cases}$

$\therefore E(l) = \int_0^{l} (1-x) dx + \int_l^{1} x dx = \frac{1}{2} + q - q^2$

(2) $\therefore$ 当 $l=\frac{1}{2}$ 时

$E(l)_{max} = \frac{1}{4}$

8.

$f(T_1) = x, \quad f(T_2) = y$

$\therefore E(T) = \int_0^1 \int_0^1 |x-y| \cdot xy \, dx dy = \frac{1}{3}$

10.

$\because \int_0^{+\infty} t k e^{-\lambda t} dt = \frac{1}{\lambda}$

$\therefore k = \lambda$

$\therefore f(t) = \lambda e^{-\lambda t}$

$\therefore E(X) = \int_0^8 t \lambda e^{-\lambda t} dt = \frac{1}{\lambda} + 7\lambda e^{-8\lambda}$

11.

(1) $f(x,y) = \frac{1}{\pi r^2}$

$\therefore E(X) = E(Y) = \iint_{x^2+y^2 \le r^2} x f(x,y) dx dy = 0$

(2) $E(\sqrt{X^2+Y^2}) = \int_0^{2\pi} \int_0^r \frac{\rho^2}{\pi r^2} d\rho d\theta = \frac{2}{3} r$

12.

(1) $\frac{a}{15}\cdot\frac{a-1}{14}\cdot 2+\frac{a}{15}\cdot\frac{14-(a-1)}{14}+\frac{15-a}{15}\cdot\frac{a}{14}=\frac{4}{3}$

$\therefore a=10$

(2)

$E(\xi_9)=4\cdot\frac{C_{10}^4\cdot C_5^5}{C_{15}^9}+5\cdot\frac{C_{10}^5 C_5^4}{C_{15}^9}+6\cdot\frac{C_{10}^6\cdot C_5^3}{C_{15}^9}+7\cdot\frac{C_{10}^7\cdot C_5^2}{C_{15}^9}+8\cdot\frac{C_{10}^8\cdot C_5^1}{C_{15}^9}$

$+9\cdot\frac{C_{10}^9\cdot C_5^0}{C_{15}^9}=6$

13.

$M=\max\{x_1\cdots x_n\}$

$K=\min\{x_1\cdots x_n\}$

$\therefore F_M(m)=m^n$, $F_K(k)=(1-k)^n$

$\therefore f_M(m)=n\cdot m^{n-1}$ $f_K(k)=\cancel{nk^{n-1}}\,n(1-k)^{n-1}$

$\therefore E(M-K)=E(M)-E(K)=\int_0^1 mf_M(m)dm-\int_0^1 kf_K(k)dk$

$=\frac{n-1}{n+1}$

14.

~~$f(x)=$~~ $P\{X=k\}=\frac{e^{-\lambda}\lambda^k}{k!}$

$P\{Y=n\}=\Sigma P\{X=k\}\cdot C_k^n p^n(1-p)^{k-n}$

$=\frac{(\lambda p)^n e^{-\lambda p}}{n!}$

$\therefore E(Y)=\lambda p$

17.

$Var(X) = E(X^2) - E(X)^2$

$E(X) = \int_{-\infty}^{+\infty} x \frac{1}{2} e^{-|x|} dx = \int_{-\infty}^{0} x \frac{1}{2} e^{x} dx + \int_{0}^{+\infty} x \cdot \frac{1}{2} e^{-x} dx = 0$

$E(X^2) = \frac{1}{2}\int_{-\infty}^{+\infty} x^2 e^{-|x|} dx = 2$

$\therefore Var(X) = 2$

$E|X| = \frac{1}{2}\int_{-\infty}^{+\infty} |x| e^{-|x|} dx = 1$

$D[|X|] = E(|X|^2) - (E|X|)^2 = 1$.

18.

(1) $E(X) = 0.7 \times 2\% \times 100 + 0.2 \times 10\% \times 100 + 0.1 \times 24\% \times 100 = 6$

$p = 2\% \times 0.7 + 0.2 \times 10\% + 0.1 \times 24\% = 0.06$

$\therefore D(X) = 100p(1-p) = 5.64$

(2)

$p' = \frac{2\% \times 0.7}{0.7} = 2\%$

$\therefore E(Y) = 100 \times (1 - 2\%) = 98$

$D(Y) = 100 \times 98\% \times 2\% = 1.96$

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19.

(1) $P\{X+Y \geq 1\} = 1 - P\{X+Y<1\} = 1 - P\{X=0, Y=0\}$
$= \frac{3}{4}$

(2)

$E(X\cdot(-1)^Y) = \cancel{0\cdot(-1)^0 P(X=0)P(Y=0)} +$
$1\cdot(-1)^0 P(X=1)P(Y=0) + 1\cdot(-1)^1 P(X=1)P(Y=1) = 0$

$D(X\cdot(-1)^Y) = \frac{1}{2}$