P1.4.1

(1) $\vec{S} = \vec{E}\times\vec{H} = \hat{r}\,\frac{\omega k q L}{4\pi r}\cdot\frac{k^2 q L}{4\pi\varepsilon_0 r}\sin^2\theta\cos^2(kr-\omega t)$

$= \hat{r}\,\eta\left(\frac{\omega q k L}{4\pi r}\right)^2\sin^2\theta\cos^2(kr-\omega t)$, $\eta=\sqrt{\frac{\mu_0}{\varepsilon_0}}$

∴ $\langle\vec{S}\rangle = \frac{1}{2\pi}\int_0^{2\pi} d(\omega t)\,\vec{E}\times\vec{H} = \hat{r}\,\frac{\eta}{2}\left(\frac{\omega q k L}{4\pi r}\right)^2\sin^2\theta$

(2)

$P = \int_0^{\pi} d\theta\, 2\pi r^2\sin\theta\,\eta\left(\frac{\omega q k L}{4\pi r}\right)^2\sin^2\theta = \cancel{\frac{4\pi}{3}(\omega k)^2}\,\eta\,\frac{4\pi}{3}\left(\frac{\omega k q L}{4\pi}\right)^2$

(3)

$R_{rad} = \frac{2P}{I_0^2} = \frac{8\pi}{3}\left(\frac{kL}{4\pi}\right)^2\eta$

(4)

当θ取$\frac{\pi}{2}$时，$\langle\vec{S}\rangle$最大

∴此时，$E_0 = \frac{k^2 q L}{4\pi\varepsilon_0 r}$

∴ $P=\frac{2\pi}{3\eta_0}(E_0 r)^2 = 781.25W$

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Ps. 4.1

(1) $\vec{J}(\vec{r'}) = \hat{z} I L + \hat{z} I L \, \delta(x' - d)$

$\therefore \vec{f}(\theta, \varphi) = \hat{z} I L \sum_{n=0}^{1} e^{in(kd \sin\theta \cos\varphi - \psi)}$

$= \hat{z} I L \left(e^{-i(kd \sin\theta \cos\varphi - \psi)} + 1 \right)$

在平面内，$\theta = 0$

$\therefore \vec{f}(\varphi) = \hat{z} I L \left(e^{-ikd \sin\varphi - \psi} + 1 \right)$

$\therefore \vec{F}(\varphi) = e^{-ikd \sin\varphi + \psi} + 1$

$\therefore |\vec{F}(\varphi)| = \left| e^{-ikd\cos\varphi + \psi} + 1 \right|$

$= \sqrt{[\cos(kd \sin\varphi - \psi) + 1]^2 + \sin^2(kd \sin\varphi - \psi)}$

$= \sqrt{2 + 2\cos(kd \sin\varphi - \psi)} = \sqrt{2 \cdot 2\cos^2\left(\frac{kd}{2} \sin\varphi - \frac{\psi}{2}\right)}$

$= \left| 2\cos\left(\frac{kd}{2} \sin\varphi - \frac{\psi}{2}\right) \right|$

(2)

当 $\varphi = \frac{\pi}{2}$ 且 $|\vec{F}(\varphi)|$ 取最大时，$\frac{kd}{2} \sin\varphi - \frac{\psi}{2} = m\pi$

$\therefore \frac{kd}{2} - \frac{\psi}{2} = m\pi \quad \therefore kd = 2m\pi + \psi$

$\because$ 当 $\varphi = 225°$ 和 $\varphi = 315°$ 时，$|\vec{F}(\varphi)| = 0$

$\therefore \frac{kd}{2} \sin 225° - \frac{\psi}{2} = \frac{\pi}{2} + n\pi$，$\frac{kd}{2} \sin 315° - \frac{\psi}{2} = \frac{\pi}{2} + n\pi$

$\therefore -\frac{\sqrt{2}}{2} \cdot \frac{kd}{2} - \frac{\psi}{2} = \frac{\pi}{2} + n\pi$

可以取 $kd = 2\pi + \psi$，$\frac{kd}{\sqrt{2}} + \psi = -\frac{\pi}{2}$

$\because k = \frac{2\pi}{\lambda}$

$\therefore$ 解得：$d = \frac{\lambda}{2 + \sqrt{2}}$，$\psi = \pi\left(\frac{\sqrt{2}}{\sqrt{2}+1} - 2\right)$

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