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16.

(1) $\int_0^2 c(4-x^2)dx = 1$

$\therefore c = \frac{3}{16}$

(2) $F(x) = \int_{-\infty}^{x} f(x) = \frac{3}{16}(4x - \frac{1}{3}x^3)$

$$\therefore F(x) = \begin{cases} 0, & x \le 0 \\ \frac{3}{16}(4x - \frac{1}{3}x^3), & 0 < x < 2 \\ 1, & x \ge 2 \end{cases}$$

(3)

$P\{-1 < x \le 1\} = \int_0^1 f(x) = \frac{3}{16} \times (4 - \frac{1}{3}) = \frac{11}{16}$

(4) $P(Y=2) = C_5^2 (\frac{11}{16})^2 \cdot (\frac{5}{16})^3 \approx 0.144$.

17.

(1) $\because F(1-) = F(1+)$ $\quad \because F(2-) = F(2)$

$\therefore a = b$ $\quad \therefore 2b = 1 \quad \therefore b = \frac{1}{2}$

$\therefore a = b = \frac{1}{2}$

(2) $f(x) = F'(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ \frac{1}{2}, & 1 \le x < 2 \\ 0, & x \ge 2. \end{cases}$

(3)

$P\{0.5<X<1.5\} = F(1.5) - F(0.5) = \frac{1}{2}\times 1.5 - \frac{1}{2}\times 0.5^2 = \frac{5}{8}$

18

(1) $P_1 = \frac{10+10}{45-20} = \frac{4}{5}$.

(2) $P_2 = \frac{5+10}{45-20} = \frac{3}{5}$

(3) $P_3 = \frac{\frac{5}{25}}{\frac{5+15}{25}} = \frac{1}{4}$

20.

(1) $P\{X>2.5\} = 1 - P\{X\le 2.5\} = 1 - \Phi_0\left(\frac{2.5-5}{1}\right) = \Phi_0(2.5) = 0.9938$

(2) $P\{X<3.52\} = \Phi_0(-1.48) = 1 - \Phi_0(1.48) = 0.0694$.

(3) $P\{4<X<6\} = \Phi(6) - \Phi(4) = \Phi_0(1) - \Phi_0(-1) = 2\Phi_0(1) - 1$

$= 0.6826$

(4) $P\{|X-5|>2\} = P\{X>7 \cup X<3\} = 1 - \Phi(7) + \Phi(3)$

$= 1 - \Phi(2) + \Phi(-2)$

$= (1 - \Phi(2))\times 2 = 0.0456$

22.

(1) $P\{X>170\} = 1-\Phi(170) = 1-\Phi_0\left(\frac{170-170}{5}\right) = 0.5$

(2) $P\{165<X<175\} = \Phi(175)-\Phi(165) = \Phi_0(1)-\Phi(-1) = 2\Phi_0(1)-1$

$= 0.6826$

(3) $P\{X<172\} = \Phi(172) = \Phi_0(0.4) = 0.6552.$

24.

(1) $P(X<200) = \Phi(200) = \Phi_0(-0.8) = 1-\Phi_0(0.8) = 0.2119.$

$P(200\le X\le 400) = \Phi(240)-\Phi(200) = 2\Phi_0(0.8)-1 = 0.5762$

$P(X>240) = 1-\Phi(240) = 1-\Phi_0(0.8) = 0.2119$

$\therefore \alpha = 0.2119\times 0.1 + 0.5762\times 0.001 + 0.2\times 0.2119$

≈ 0.064

(2) $\beta = \frac{0.2119\times 0.2}{0.064} \approx 0.66$

(3)

$\theta = C_3^2\alpha(1-\alpha)^2 + (1-\alpha)^3 = 0.988$

25.

$\Phi(12800) = \Phi_0\left(\frac{12800-\mu}{\sigma}\right) = 0.3$

$1-\Phi(10000) = 1-\Phi_0\left(\frac{10000-\mu}{\sigma}\right) = 0.95$

$\therefore \Phi_0\left(\frac{12800-\mu}{\sigma}\right) = 0.3 \quad \therefore \frac{12800-\mu}{\sigma} = -0.52$

$\Phi_0\left(\frac{10000-\mu}{\sigma}\right) = 0.05 \quad \frac{10000-\mu}{\sigma} = -1.65$

$\therefore \begin{cases} \mu = 12129 \\ \sigma = 1290 \end{cases}$

26.

$\Phi(x_1) : \Phi(x_2)-\Phi(x_1) : 1-\Phi(x_2) = 50:34:16$

$\frac{\Phi(x_1)}{\Phi(x_2)} = \frac{25}{8}$, $\frac{\Phi(x_1)}{\Phi(x_2)} = \frac{25}{42}$, $\Phi(x_2) = \frac{21}{25}$

$\therefore \Phi(x_1) = \frac{21}{42} = \frac{1}{2}$

$\therefore x_1 = 15$

$\Phi(x_2) = \Phi_0\left(\frac{x_2-15}{2}\right) = \frac{21}{25}$ $\therefore \frac{x_2-15}{2} = 1$

$\therefore x_2 = 17$

27.

(1) $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$\therefore 2\sigma^2 = 1$ $\therefore \sigma = \frac{\sqrt{2}}{2}$

$\mu = 0$

$\therefore a = \sqrt{\pi}$

(2) $P\{X > \frac{1}{2}\} = 1-\Phi(\frac{1}{2}) = 1-\Phi_0\left(\frac{0.5-0}{\frac{\sqrt{2}}{2}}\right) = 0.2389$.

28.

~~(1) $F(x) = e^{-\frac{1}{8}t}$, $f(x) = \lambda e^{-\lambda x} = \frac{1}{8}e^{-\frac{1}{8}t}$~~

~~(2) $F(10) = e^{-\frac{5}{4}}$~~

~~(3) $P(8<X<16) = e^{-\frac{1}{8}\times 16} - e^{-}$~~

28.

(1) $f(x) = \lambda e^{-\lambda x} = \frac{1}{8}e^{-\frac{1}{8}t}$, $F(x) = 1-e^{-\lambda x} = 1-e^{-\frac{1}{8}x}$

(2) $F(10) = 1-e^{-\frac{5}{4}}$

(3) $F(8<X<16) = 1-e^{-\frac{1}{8}\times 16} - (1-e^{-\frac{1}{8}\times 8}) = \frac{1}{e} - \frac{1}{e^2} = \frac{e-1}{e^2}$

29.

$f_1(x) = \frac{1}{3}e^{-\frac{1}{3}x}$, $f_2(x) = \frac{1}{6}e^{-\frac{1}{6}x}$

(1) $F(X>6) = 0.4F_1(X>6) + 0.6F_2(X>6)$

$= \frac{0.4}{e^2} + \frac{0.6}{e}$

(2)

$F(X\geq 1 \mid X\geq 0.4) = \frac{F(X\geq 1)}{F(X\geq 0.4)} = e^{-\frac{1}{5}}$

30

(1) $f(x) = F(x) = 0.2e^{-0.2x}$

(2) $P\{5<X<10\} = F(10) - F(5) = \frac{1}{e} - \frac{1}{e^2}$

31. $F(x) = 1 - e^{-0.01x}$

(1) $P_1 = C_3^2 \cdot (1-F(150))^2 \cdot F(150) = 3e^{-3}(1-e^{-1.5})$

(2) $P_2 = P_1 + (1-F(150))^3 = 3e^{-3}(1-e^{-1.5}) + e^{-4.5}$

$= 3e^{-3} - 2e^{-4.5}$