Problem P7.1

A plane wave in free space is incident at an angle θ on a conducting half-space. For large $\sigma/\omega\epsilon_o$, show that the transmitted wave is almost perpendicular to the boundary by finding $\theta_t = \tan^{-1}(k_x/k_{zR})$.

Problem P7.2

When the incident k vector is normal to a plane boundary, a TE wave becomes a TEM wave; a TM wave also becomes a TEM wave. Compare the reflection and transmission coefficients for TE and TM waves at normal incidence. Do both TE and TM results reduce to the same number? If not, why? Do the reflectivities and transmissivities for TE and TM waves at normal incidence reduce to the same result?

Problem P7.3

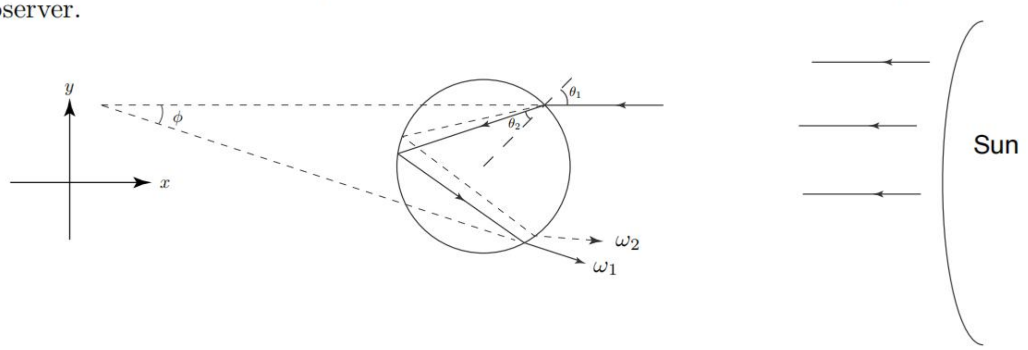
A plane wave is incident from free space ($z > 0$) on a perfect electric conductor ($z < 0$) with the incident electric field

$$\overline{E}_i = (\frac{1}{\sqrt{2}}\hat{x} + \frac{1}{\sqrt{2}}\hat{z} + i\hat{y})e^{i\frac{1}{\sqrt{2}}k_o x - i\frac{1}{\sqrt{2}}k_o z}.$$

What is the reflected electric field $\overline{E}_r$? Compare the handedness of polarizations of $\overline{E}_i$ and $\overline{E}_r$.

Problem P7.4

Rainbow arc often appears when sunlight shines on water droplets after a brief shower late in the afternoon. When a sun ray is refracted as it enters the raindrop, total internally reflected from inside the drop, and refracted again as it leaves the drop and passes to the observer.



(a) Consider the ray path with only one internal reflection. Show that the scattering angle ϕ between the incident ray and the exit ray is $2(2\theta_2 - \theta_1)$, where θ_1 is the incident angle and θ_2 is the refracted angle.

(b) For a sphere with a radius a where $ka \gg 1$, the direction where the scattering angle is stationary ($\frac{d\phi}{d\theta_1} = 0$) corresponds to the least cancellation between different rays and hence a large scattering amplitude. Show that the maximum scattering angle ($\phi_{\max}$) occurs at $\theta_1 = \sin^{-1}\sqrt{(4-n^2)/3}$ and $\phi_{max} \approx 42°$ for $n = 4/3$, with the scattering angle between the incident ray and scattered ray $\theta_s = 138°$.

(c) The refractive index for a raindrop is $n = 1.330$ for red light ($\lambda = 0.7\mu$m), $n = 4/3 = 1.333$ for orange light, and $n = 1.342$ for violet light ($\lambda = 0.4\mu$m). Determine the scattering angles for the red and violet light rays. What are the relative positions of the different color bands in a rainbow?

Problem P8.1

That the exhausted travelers in desert often see an unreal “lake” in the distance is called the phenomenon of mirage. The hot sand in desert heats the air close to the ground and thus forms a layer of air with less density as well as less refractive index. The refractive index of air in the visible region is about 1.000271374 at the temperature of $20°C$, while about 1.000257559 at $35°C$. Consider the two air layers with different temperatures as shown in Fig. 1, what is the critical angle θ_c of the wave in this case? Try to explain the mirage using this model.

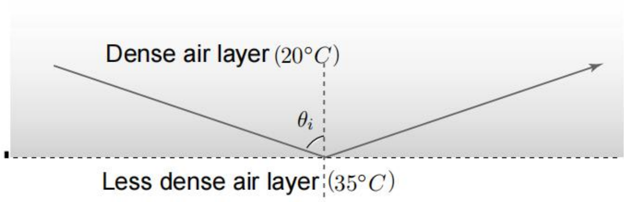


Fig.1

Problem P8.2

Sun light glares caused by reflections from plane surfaces are partially linearly polarized.

(a) Determine the Brewster angle for $\epsilon_t = 9\epsilon_0$ The Brewster angle, θ_B, is also called the polarization angle because at θ_B the reflected wave is entirely perpendicularly polarized (TE polarized).

(b) Your polaroid glasses absorb one linear component of incident light. To minimize sun glare, what component, perpendicularly or parallelly polarized (TM polarized), reaches your eyes after passing through the glasses? Explain why.

Problem P8.4

The gas laser depicted in Fig. 2 uses “Brewster angle” quartz windows on the gas discharge tube in order to minimize reflection losses. Determine the angle θ if the index of refraction for quartz at the wavelength of interest is $n = 1.46$. Because of these windows, the laser output is almost completely linearly polarized. What is the direction of polarization, i.e., is $\overline{E}$ parallel or perpendicular to the paper? Why?

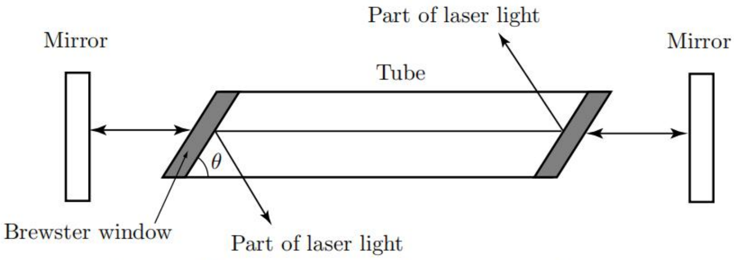


Fig. 2. A gas laser with Brewster windows.

Problem P8.5

Consider a plane wave incident from a dielectric region with permittivity $\epsilon = 3\epsilon_o$ upon a halfspace with $\epsilon = \epsilon_o$ as shown in Fig. 3.

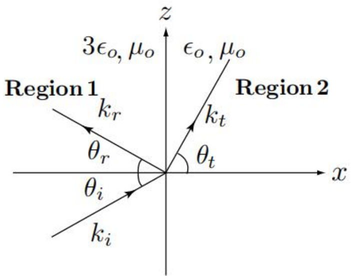


Fig. 3.

(a) Find the Brewster angle for Region 1.

(b) Suppose the transmitted electric field is given by

$$\overline{E}_t = \hat{y}\frac{E_0}{\sqrt{2}}e^{ik_{tx}x+ik_{kz}z} + E_0\frac{\hat{z}-\hat{x}\sqrt{3}}{2\sqrt{2}}e^{ik_{tx}x+ik_{tz}z-i\frac{\pi}{2}}$$

(i) Determine the incident and transmitted angles, θ_i and θ_t.

(ii) What is the polarization of the transmitted field? Be sure to specify the handedness (left or right) if necessary.

(iii) What is the polarization of the reflected wave? Be sure to specify the handedness (left or right) if necessary.

(iv) Give an expression for the incident electric field, $\overline{E}_i$, and the reflected electric field, $\overline{E}_r$.