P94

11. $f(x,y)=\begin{cases} 0.1 & x=0,\ y=0 \\ kxy & 0<x<1,\ 0<y<1 \\ 0.1 & x=1,\ y=1. \end{cases}$

$\therefore \iint_D f(x,y)\,dx\,dy = \frac{1}{4}k = 0.8 \quad \therefore k = 3.2$

$\therefore F(x,y)=\begin{cases} 0, & x<0 \text{ or } y<0 \\ 0.1, & x=0,\ y=0 \\ 0.1x^2y^2+0.1, & 0<x<1,\ 0<y<1 \\ 1, & x\geq 1,\ y\geq 1. \end{cases}$

12.

(1) $\int_0^1\int_0^x c(y-x)\,dx\,dy = 1$

$\therefore c = 6$

(2) $P\{X+Y\leq 1\} = \iint_{\Omega} f(x,y)\,dx\,dy = 6\int_0^{\frac{1}{2}}dx\int_x^{1-x}(y-x)\,dy = \frac{1}{2}$

(3) $P\{X<0.5\} = 6\int_0^{\frac{1}{2}}dx\int_x^1(y-x)\,dy = \frac{7}{8}$

13.

(1) $f_X(x) = \int_0^2 x\,dy = 2x$

$f_Y(y) = \int_0^1 x\,dx = \frac{1}{2}$

(2) $P\{Y\leq 2X\} = \int_0^1 x\,dx\int_0^{2x}dy = \frac{2}{3}$

扫描全能王 创建

14.

(1) $c\int_{1}^{2}(x-1)dx\int_{x}^{4-x}dy=1$

$\therefore c=3.$

(2) $f_X(x)=\int_{x}^{4-x}3(x-1)dy=3(x-1)(4-2x)\quad 1<x<2$

~~$f_Y(y)=\int_{1}^{2}3(x-1)dx=$~~

$$f_Y(y)=\begin{cases}3\int_{1}^{y}(x-1)dx=\dfrac{3(y-1)^2}{2}, & 1<y<2\\[2ex] 3\int_{1}^{4-y}(x-1)dx=\dfrac{3(y-3)^2}{2}, & 2<y<3.\end{cases}$$

16.

(1) $\because f_{Y|X}(y|x)=\dfrac{f(x,y)}{f_X(x)}$

$\therefore f(x,y)=f_{Y|X}(y|x)\cdot f_X(x)=\lambda^2e^{-\frac{y}{x}-\lambda x},\quad x>0,y>0$

(2) $$F_{Y|X}(y|x)=\begin{cases}\int_0^y\lambda^2e^{-\frac{y}{x}-\lambda x}dy=1-e^{-\frac{y}{x}}, & y>0\\ 0, & ~~\text{other}~~\ y\le 0\end{cases}$$

(3) $P\{Y>1|X=1\}=$ ~~$F_{Y|X}$~~ $1-P\{Y\le 1|X=1\}$

$=1-F_{Y|X}(1|1)=1-(1-\frac{1}{e})=\frac{1}{e}$

扫描全能王 创建

17.

(1) $f_Y(y) = \begin{cases} 0, & |y| \geq 1 \\ \int_{y^2}^{1} \frac{5}{4} x\,dx = \frac{5}{8} - \frac{5}{8} y^4, & |y| < 1 \end{cases}$

(2) $f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{2x}{1-y^4}, \quad y^2 < x < 1$

$\therefore f_{X|Y}(x|y) = \begin{cases} \frac{2x}{1-y^4}, & y^2 < x < 1 \\ 0, & \text{other} \end{cases}$

(3)

$P\{X > \frac{1}{2} | Y = \frac{1}{2}\} = \int_{\frac{1}{2}}^{1} \frac{2x}{1-(\frac{1}{2})^4} dx = 0.8$

19.

(1) $f_X(x) = \begin{cases} 1, & 1 < x < 2 \\ 0, & \text{other} \end{cases}$

$f(x,y) = f_{Y|X}(y|x) \cdot f_X(x) = \begin{cases} \frac{2(4-y)}{(3-x)^2}, & 1 < x < 2, x+1 < y < 4 \\ 0, & \text{other} \end{cases}$

$P\{Y < 3\} = \int_1^2 \frac{1}{(3-x)^2} dx \int_{x+1}^{3} 2(4-y) dy = \frac{1}{2}$

(2) $\cancel{f_Y(y) = \int_{x+1}^{4} \frac{2(4-y)}{(3-x)^2} d}$

$f_Y(y) = \begin{cases} \int_1^{y-1} \frac{2(4-y)}{(3-x)^2} dx = y-2, & 2 < y < 3 \\ \int_1^{2} \frac{2(4-y)}{(3-x)^2} dx = 4-y, & 3 < y < 4 \\ 0, & \text{other} \end{cases}$

扫描全能王 创建

(3)

$$P\{X\le 1.5 \mid X+Y=3\} = \frac{P\{X\le 1.5,\ X+Y=3\}}{P\{X+Y=3\}} = 0 \quad ?$$

21. $f(x,y) = \begin{cases} \frac{2}{\pi}, & (x,y)\in D \\ 0, & \text{other} \end{cases}$

(1) $f_X(x) = \begin{cases} \int_{\cancel{0}\,-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{\pi}\, dy = \frac{4}{\pi}\sqrt{1-x^2}, & \cancel{0}\,< x < 1 \\ 0, & \text{other} \end{cases}$

(2) $P\{X<\frac{1}{2}\} = \int_{0}^{\frac{1}{2}} f_X(x)\, dx = \frac{1}{\frac{\pi}{6}+\frac{\sqrt{3}}{8}} \quad \frac{1}{\frac{\pi}{6}+\frac{\sqrt{3}}{4}}$

~~(3) $f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \frac{1}{\sqrt{1-x^2}},\ -1<x<1$.~~

~~$F\{Y\le 1 \mid X=0\} =$~~

(3) $\cancel{f(x,y)}\ f_Y(y) = \int_{0}^{\sqrt{1-y^2}} \frac{2}{\pi}\, dx = \frac{2}{\pi}\sqrt{1-y^2}$

$\therefore f_Y(y)\cdot f_X(x) \ne f(x,y)$

$\therefore$ 不独立.

扫描全能王 创建