V2-Linear Programming Assignment

SC42055 Optimization in Systems and Control

E_1, E_2 and E_3 are parameters changing from 0 to 18 for each group according to the sum of the last three numbers of the student IDs:

$$E_1 = D_{a,1} + D_{b,1}, \quad E_2 = D_{a,2} + D_{b,2}, \quad E_3 = D_{a,3} + D_{b,3},$$

where $D_{a,3}$ is the right-most digit of one student and $D_{b,3}$ is the right-most digit of the other student.

Edison Automotive is a disruptive car manufacturer that produces electric cars. Due to its infancy in the segment, they only produce two cars: the model R and the model W. While all the cars they are able to produce are sold in the market after one month, they have a technological limiting factor: Edison Automotive can only produce $(5+E_1)\cdot 10^6$ battery cells per month and each model R and each model W respectively require $4\cdot 10^3$ and $6\cdot 10^3$ battery cells.

In addition to the batteries, a second limiting factor is number of hours available: building a model R requires 10 hours and building a model W requires 15 hours. At the moment, Edison has $100 + E_2$ employees available; each of them can work 160 hours per month and have a monthly salary of $3000 + 50E_3$ €. In addition, because of the unions, the company cannot fire any employee and have to pay their full monthly salary independently of the number of hours worked.

A third limiting factor is the storage space available for stock: Edison Automotive only has available $(15+E_3)\cdot 10^3\,\mathrm{m}^2$ and each model R and W respectively occupy $10\,\mathrm{m}^2$ and $12\,\mathrm{m}^2$. As the produced cars need to be stored for one month, this is a very important limiting factor.

The model R and model W are respectively being sold for 55000 € and 75000 €. Moreover, the cost of manufacturing the cars without considering the worker salaries is 30000 € for the model R and 45000 € for the model W.

Given the restrictions above, Edison Automotive wants to optimize their benefits, but as nobody in their team have expertise in numerical optimization, they have decided to hire you. These are your tasks:

1. Formulate the optimization problem and transform it into a standard form of the linear programming problem considering the number of model R and model W as continuous variables.

2. Find the optimal solution to the LP problem using MATLAB (use the continuous formaluation and `linprog`. Do not use `intlinprog`). What is optimal benefit and the optimal number of model R and model W to be manufactured? (Take into account that the actual number of model R and model W are integers). Which of the constraints is limiting the production?

3. While you worked on the previous optimization problem, something has happened on the market: a batch of model R had an issue in their battery system and as a result customers are now not willing to buy as many model R as before. From the marketing department, they tell you that the maximum number of model R that the company can

manufacture/sell is now limited to 1000 per month. Formulate the new LP problem. What is the optimal benefit and the optimal number of model R and model W to be manufactured?

4. After you optimize the manufacturing process, they ask you to propose ideas on how to further increase the profits of the company. You start reading some of the company internal reports and find that, for each additional worker that the company hires, the time required to manufacture each car is reduced by 5 minutes. In the reports, you also notice that this time reduction is limited to 6 hours, i.e. the maximum number of extra workers is limited to 72. In addition, after further reading, you realize that the manufacturing constraints can be improved:

   - Changing the way the battery cells are produced, the number of battery cells produced per month can be increased from $(5 + E_1) \cdot 10^6$ to $(8 + E_1) \cdot 10^6$.
   - Edison Automotive has an abandoned factory that can be used for storage at no additional cost. Therefore, the new storage capacity is increased from $(15 + E_3) \cdot 10^3\,\mathrm{m}^2$ to $(22 + E_3) \cdot 10^3\,\mathrm{m}^2$.

   Formulate the new optimization problem in order to maximize the profit. Is it possible to formulate it as a single LP problem? Keep in mind that the number of model R cars that can be manufactured is still limited to 1000 per month.

5. Find in MATLAB the optimal number of workers, models R, and models W. (Advice: You can solve as many LP problems as necessary).

6. Edison Automotive has hired the number of optimal employees obtained in part 5 and significantly increased their benefit. Because of the success, the 1000 limit on the model R is no longer in place and they have signed contracts to produce at least 1250 model R and 1000 model W per month. Now they want to start manufacturing a new car: the model V, which requires $2 \cdot 10^3$ battery cells, occupies $8\,\mathrm{m}^2$, and uses 8 hours independently of the number of workers, i.e. the production time of this model cannot be reduced increasing the number of workers. In addition, the cost of production for the new model is 15000 €, the car is being sold for 45000 €, and creating the new manufacturing process is only economically beneficial if they can guarantee a minimum production of 1500 models V per month. Considering that there is a lack of skilled workers in the sector and that the number of workers cannot be increased w.r.t. the optimal number obtained in step 4, is it economically beneficial to build the new model V? If not, can Edison Automotive at least satisfy the new two contracts on the model W and model R?

The written report on the practical exercise should be uploaded to Brightspace before Wednesday, October 3, 2018 at 17.00 p.m. as one pdf file. The MATLAB code used should also be uploaded to Brightspace as one .m file; please make sure that the code is error free. Please also note that you will lose 0.5 point from your grade on the report for each (started) day of delay in case you exceed the deadline.