Problem Set 8: Maximum Score Estimation

ECON833

Jaewoo Lee

1 Part A: Problem Set 7

1.1 Context

It is easy to find products around us that have a limited selling period, so having no value after a certain time point. For example, sellers of event tickets (e.g., concert, sports, and movies) and travel business (hotels and airlines) owners (or managers) have a limited time window to sell their product with zero salvage value after the time window. They should be careful to price their products to maximize their profit. I also have a relevant experience like them. I worked at a retail company (like Walmart) in Korea and spent the first two years as an assistant manager at the deli session of one of the company's branches. Some of my products had to be carefully managed in terms of their selling time period. In particular, fresh sushi products should be sold within five hours because of their perishability and customers' sensitivity to freshness. Otherwise, they had to be discarded (and I sampled and tasted part of them quality check). To increase the probability of selling those products, I discounted products that had not been sold within the first two hours since they were displayed in summer (three hours in the other seasons). Although the original price were officially fixed in a relatively short term (e.g., one year), the discount rate were adjusted depending on my demand forecasting based on the number of unsold items and different factors influencing customer demands like weather, the number of customers visiting the store until making discount decisions, the time of the day, and the day of the week. One interesting belief that many Koreans have is that eating fresh seafood on a rainy day is not good for health, leading people to buy fresh seafood products on a rainy day less than on a sunny day. I model this context for dynamic programming.

1.2 Model description

I assume that a single seller (she) produces and sells multiple units of a particular product (e.g., 10 packs of 6 sushi pieces) in a time window of two points (i.e., t = 1 and 2).I assume that the period length of t = 1 is longer than the one of t=2, which implies that t=1 is more likely to have more selling opportunities. She has to find the optimal initial price at t = 0, which impacts her production plan of the product along with the expected weather condition of the selling period. To simplify the model, I assume that the system is in one of two states: good weather (G) and bad weather(B). While those states is time-invariant, I introduce changes in the weather condition in a way that two types of weather information are given to her: Expected weather condition and actual weather condition. As I mention above, the former affects her production plan at t = 0 while the latter does the realized revenue at t = 1 and 2). To simplify the model, I assume that if the weather at t = 1 is expected to be bad, the actual weather is always bad while it is forecast to be good, it can be either good or bad. The relationship between expected and actual weather works based on a one-step memory, discrete time and discrete state Markov chain process with a transition probability matrix as shown in Equation (1).

$$\Pi = \begin{vmatrix} \pi_{GG} & \pi_{GB} \\ \pi_{BG} & \pi_{BB} \end{vmatrix} \tag{1}$$

If the system is expected to be or is actually in state G, the probability of the item being sold is α_G. If the system is in state B, the probability of the item being sold is α_B. Assuming consumers' more favorable behavior for fresh products in nice weather, thus, $\alpha_G > \alpha_B$.

If she hopes her inventory gets empty without discounting the initial price, she may want to absolutely reduce her production quantity regardless of the expected weather condition or the initial price. But then, it is more likely to a reduced profit due to a waste of her production capacity (e.g., labor cost). Thus, I assume that her production quantity for the selling period is a function of the expected weather condition (i.e., either α_G or α_B) and the initial price, p_1 as shown in Equation (2). I assume that the price variable and the state of the system have an impact on the production quantity and actual demands independently of each other.

$$Q_0(p_1, w_1) = (E - Fp_1) \times \alpha_{w_1},\ w \in \{G, B\} \tag{2}$$

, where E and F are constants, w_1 is the expected weather condition she is informed of at t = 0.

The realized sales quantity at t= T for the time period are determined by Equation (3),

$$Q_T(p_T, w_2) = (E - Fp_T) \times \alpha_{w_2},\ T \in \{1, 2\}, w \in \{G, B\} \tag{3}$$

, where w_2 is the actual weather condition during the sales period. $Q_T(p_T, w_2) \leq Q_0(p_1, w_1)$. Since the actual weather condition for the selling period is realized based on Equation (1), α_{w_2} is a function of α_{w_1} and the elements of Equation (1) as shown in Equation (4)

$$\alpha_{w_2=G} = \alpha_{w_1=G}\pi_{GG} + \alpha_{w_1=B}\pi_{BG} \tag{4}$$

$$\alpha_{w_2=B} = \alpha_{w_1=G}\pi_{GB} + \alpha_{w_1=B}\pi_{BB} \tag{5}$$

Since unsold products must be discarded without salvage value at the end of t=2, which gives her nothing but zero value, her objective at t = 2 is to sell out her inventory (or cooling display stand) to avoid loss. To reach this goal, at the beginning of t=2 she should discount the original price to increase the probability of the product being sold during the remaining time (because t=2 is shorter). I assume that the discounted price at t=2 is a function of the initial price, the remaining quantity from t=1, and the actual weather condition as shown in Equation (4),

$$p_2 = p_1 - \mathbb{K}\{Q_0(p_1, w_1) - Q_1(p_1, w_2)\}(1 - \alpha_{w_2}) \tag{6}$$

, where $\mathbb{K}$ is a constant penalizing the inventory and $p_2 < p_1$. $p_2 < p_1$ is possible (i.e., there is no inventory) but she doesn't have to care about selling the product at t = 2. Intuitively, the discount rate increases in the remaining quantity of the product at the end of t = 1 and decreases in the actual weather condition.

The total sales revenue that she collects from a batch of the item during the sales period is the sum of the products of the price and product quantity at two time points (i.e., t=1 and t=2).The Bellman equation is

$$V(w) = \max_{p_1} p_1 Q_1(p_1, w_2) + p_2 Q_2(p_2, w_2) \tag{7}$$

If she still has unsold products at the end of t=2, they have zero value. Thus, equation (5) doesn't include the sale revenue at t=3. Since p_2 and $Q_2(p_2, w_2)$ are both a function of p_1, the first order condition is

$$Q_1(p_1, w_2) + p_1 \frac{\partial Q_1(p_1, w_2)}{\partial p_1} + Q_2(p_2, w_2) \frac{\partial p_2}{\partial p_1} + p_2 \frac{\partial Q_2(p_2, w_2)}{\partial p_1} = 0 \tag{8}$$

2 Part B:Problem Set 8