

Uniformization procedure.

Denote $h = \max_{i=0, \bar{W}} (-D_0)_{ii}$.

Then the matrix $P(n, t)$, $n \geq 1$, can be represented in the following form:

$$P(n, t) = \sum_{j=0}^{\infty} e^{-ht} \frac{(ht)^j}{j!} K_n^{(j)}, \quad n \geq 0, \quad (1)$$

where the matrices $K_n^{(j)}$, $n \geq 1, j \geq 0$, are calculated by recursion

$$\begin{aligned} K_0^{(0)} &= I, \quad K_n^{(0)} = O, \quad n \geq 1, \\ K_0^{(j+1)} &= K_0^{(j)}(I + h^{-1}D_0), \\ K_n^{(j+1)} &= h^{-1}K_{n-1}^{(j)}D_1 + K_n^{(j)}(I + h^{-1}D_0), \quad n \geq 1, \quad j \geq 0. \end{aligned}$$

Computation of transition probabilities

We consider the constant service time. In this case, the distribution function $B(t)$ has the form

$$B(t) = \begin{cases} 0, & t \leq b_1, \\ 1, & t > b_1. \end{cases}$$

- $\varphi_k(t) = \frac{(\gamma t)^k}{k!} e^{-\gamma t}, \quad k \geq 0.$
- $\hat{\varphi}_k(t) = \sum_{i=k}^{\infty} \varphi_i(t), \quad k \geq 0.$
- $\varphi_k = \varphi_k(b_1), \quad k \geq 0.$
- $\hat{\varphi}_k = \sum_{i=k}^{\infty} \varphi_i, \quad k \geq 0.$
- $\sum_{i=0}^{\infty} P(i, t) z^i = e^{(D_0 + D_1 z)t}, \quad |z| < 1.$
- $\Phi(i, k) = \int_0^{\infty} P(i, t) \varphi_k(t) dB(t) = P(i, b_1) \varphi_k(b_1) = e^{-(h+\gamma)b_1} \frac{(\gamma b_1)^k}{k!} \sum_{j=0}^{\infty} \frac{(hb_1)^j}{j!} K_i^{(j)}, \quad i \geq 0, \quad k \geq 0.$
- $\hat{\Phi}(i, k) = \int_0^{\infty} P(i, t) \hat{\varphi}_k(t) dB(t) = P(i, b_1) \hat{\varphi}_k(b_1) = e^{-(h+\gamma)b_1} \sum_{j=0}^{\infty} \frac{(hb_1)^j}{j!} K_i^{(j)} \sum_{l=k}^{\infty} \frac{(\gamma b_1)^l}{l!}, \quad i \geq 0, \quad k \geq 0.$
- $N(m) = \int_0^{\infty} P(m, t) \gamma e^{-\gamma t} dt = \gamma \sum_{j=0}^{\infty} \frac{h^j}{j!} K_m^{(j)} \int_0^{\infty} e^{-(h+\gamma)t} t^j dt = \frac{\gamma}{h+\gamma} \sum_{j=0}^{\infty} \left(\frac{h}{h+\gamma}\right)^j K_m^{(j)}, \quad m \geq 0.$
- $M(r) = \int_0^{\infty} e^{D_0 t} \varphi_r(t) D_1 dt = \int_0^{\infty} e^{D_0 t} \frac{(\gamma t)^r}{r!} e^{-\gamma t} D_1 dt = \gamma^r (-D_0 + \gamma I)^{-(r+1)} D_1.$
- $\hat{M}(r) = \sum_{l=r}^{\infty} M(l) = \gamma^r (-D_0 + \gamma I)^{-r} (-D_0)^{-1} D_1, \quad r \geq 0.$

Подскажите, нужно ли расписывать переходные вероятности следующим образом? Если да, то что еще можно упростить?

$$\begin{aligned}
P\{(0,0) \rightarrow (j,k')\} &= M(0) \sum_{n=0}^j N(n) \Phi(j-n, k') + N(0) \sum_{m=0}^{k'} M(m) \Phi(j, k'-m) = \\
&(-D_0 + \gamma I)^{-1} D_1 \sum_{n=0}^j \gamma \sum_{l=0}^{\infty} \left(\frac{h}{h+\gamma}\right)^l \frac{1}{h+\gamma} K_n^{(l)} e^{-(h+\gamma)b_1} \frac{(\gamma b_1)^{k'}}{k'!} \sum_{m=0}^{\infty} \frac{(hb_1)^m}{m} K_{j-n}^{(m)} + \\
&\gamma \sum_{j=0}^{\infty} \left(\frac{h}{h+\gamma}\right)^j \frac{1}{h+\gamma} K_0^{(j)} \sum_{m=0}^{k'} \gamma^m (-D_0 + \gamma I)^{-(m+1)} D_1 e^{-(h+\gamma)b_1} \frac{(\gamma b_1)^{k'-m}}{(k'-m)!} \sum_{l=0}^{\infty} \frac{(hb_1)^l}{l!} K_j^{(l)} = \\
&(-D_0 + \gamma I)^{-1} D_1 \frac{\gamma}{h+\gamma} e^{-(h+\gamma)b_1} \frac{(\gamma b_1)^{k'}}{k'!} \sum_{n=0}^j \sum_{l=0}^{\infty} \left(\frac{h}{h+\gamma}\right)^l K_n^{(l)} \sum_{m=0}^{\infty} \frac{(hb_1)^m}{m} K_{j-n}^{(m)} + \\
&\gamma \frac{1}{h+\gamma} e^{-(h+\gamma)b_1} \sum_{j=0}^{\infty} \left(\frac{h}{h+\gamma}\right)^j K_0^{(j)} \sum_{m=0}^{k'} \gamma^m (-D_0 + \gamma I)^{-(m+1)} D_1 \frac{(\gamma b_1)^{k'-m}}{(k'-m)!} \sum_{l=0}^{\infty} \frac{(hb_1)^l}{l!} K_j^{(l)}, \\
&k' = \overline{0, K-2};
\end{aligned}$$