

## Spectroscopic Investigation of a Repetitively-Pulsed Nanosecond Discharge

by

Benjamin T. Yee

Chair: John E. Foster



Karrer, M. "Sun 2011-12-08." URL: [http://www.flickr.com/photos/michael\\_karrer/6487199145/](http://www.flickr.com/photos/michael_karrer/6487199145/). Accessed: August 26, 2013.

### What Is a Plasma? (Sun)

4/105  
Sep 3, 2013

- Introduction
- Outstanding questions in RPNDs
- Metastable measurements
- Global model
- Emission measurements
- Conclusions

### Outline

2/105  
Sep 3, 2013



Hunt, P. "Lightning." URL: [http://www.flickr.com/photos/michael\\_karrer/6487199145/](http://www.flickr.com/photos/michael_karrer/6487199145/). Accessed: Aug. 26, 2013.

### What Is a Plasma? (Lightning)

5/105  
Sep 3, 2013

# Introduction

3/105  
Sep 3, 2013

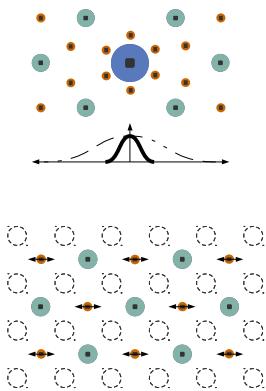


NASA, ESA, and the Hubble Heritage Team. "Star-Forming Region LH 95 in the Large Magellanic Cloud." URL: <http://hubblesite.org/gallery/album/entire/p2006055a/>. Accessed: August 27, 2013.

### What Is a Plasma? (Interstellar Gases)

6/105  
Sep 3, 2013

- Ionized gas
  - Neutral particles
  - Positive (and negative) ions
  - Electrons
- Exhibits large-scale electrostatic effects
  - Debye shielding
  - Electron oscillations



## What Is a Plasma?

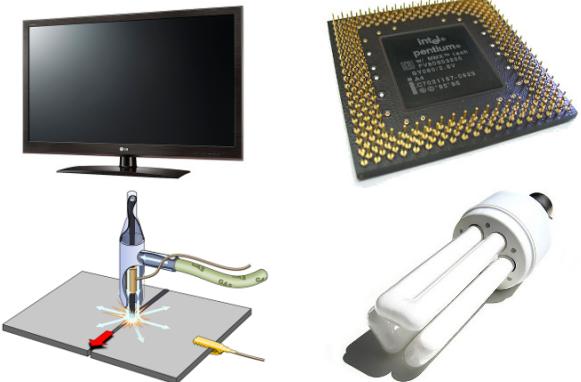
7/105  
Sep 3, 2013

$$T_{\text{neutrals}} \sim T_{\text{ions}} \sim T_{\text{electrons}}$$

D-Kuru/Wikimedia Commons. "Afterglowing electrodes of an arc lamp." URL: [https://commons.wikimedia.org/wiki/File:Arc\\_lamp-afterglow\\_2\\_PN%C2%B0B00038.jpg](https://commons.wikimedia.org/wiki/File:Arc_lamp-afterglow_2_PN%C2%B0B00038.jpg). Accessed: August 28, 2013.

## Equilibrium Plasmas

10/105  
Sep 3, 2013



Bowden, M. "A fluorescent light bulb." URL: <http://www.sxc.hu/photo/203835>. Accessed: August 26, 21013.  
Shigenz23. "Gas arc welding (TIG & MIG)." URL: [https://commons.wikimedia.org/wiki/File:Gas\\_arc\\_welding\\_%28TIG,%26,MIG%29.PNG](https://commons.wikimedia.org/wiki/File:Gas_arc_welding_%28TIG,%26,MIG%29.PNG). Accessed: August 27, 2013.  
Schmid, M. "Wasabi-Chips (Micro)." URL: [https://commons.wikimedia.org/wiki/File:Wasabi-Chips\\_%282Migros%29.JPG](https://commons.wikimedia.org/wiki/File:Wasabi-Chips_%282Migros%29.JPG). Accessed: August 27, 2013.  
Volant, L. "LG LV3550 15 R." URL: <http://www.flickr.com/photos/27048731@N03/5589223946>. Accessed: August 27, 2013.

## Applications

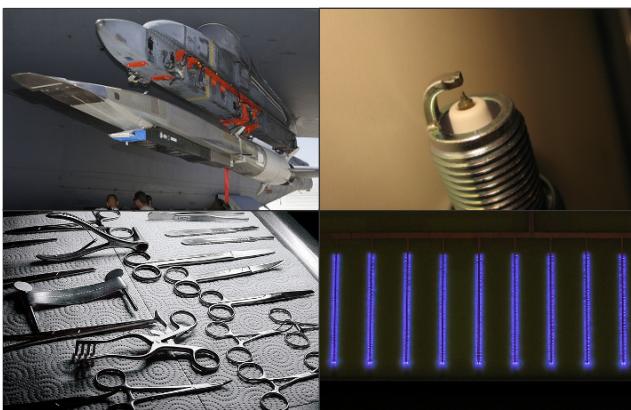
8/105  
Sep 3, 2013

$$T_{\text{neutrals}} < T_{\text{ions}} < T_{\text{electrons}}$$

Entrop5. "A low temperature plasma jet: Plasma Pencil." URL: [https://commons.wikimedia.org/wiki/File:Plasma\\_Pencil.jpg](https://commons.wikimedia.org/wiki/File:Plasma_Pencil.jpg). Accessed: August 28, 2013.

## Non-Equilibrium Plasmas

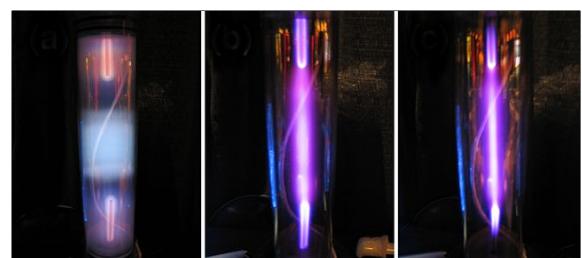
11/105  
Sep 3, 2013



Belyay, C. "X-51A Waverider on B-52 2009." URL: [https://commons.wikimedia.org/wiki/File:X-51A\\_Waverider\\_on\\_B-52\\_2009.jpg](https://commons.wikimedia.org/wiki/File:X-51A_Waverider_on_B-52_2009.jpg). Accessed: August 27, 2013.  
Wong, N. "Tip of a spark plug." URL: <http://www.flickr.com/photos/14297050@N00/375024452>. Accessed: August 27, 2013.  
pafef\_atp. "Operation." URL: <http://www.flickr.com/photos/pafef/2299324465>. Accessed: August 27, 2013.  
Xunger. "Plasma glow discharge." URL: <https://commons.wikimedia.org/wiki/File:Streamwise.JPG>. Accessed: August 27, 2013.

## Atmospheric-Pressure Plasma Applications

9/105  
Sep 3, 2013



- Increase in pressure → tendency to thermalize
- Caused by ionization instability
- Prevented by careful control of power input

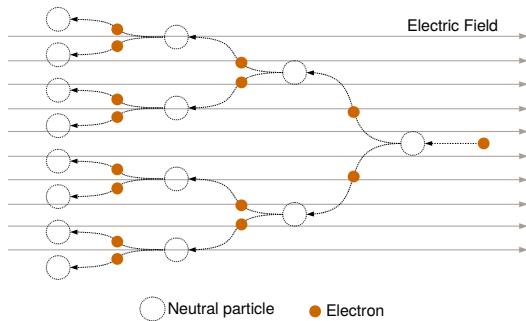
Wikigan. "Transition from a glow discharge in Argon, to an arc." URL: <https://commons.wikimedia.org/wiki/File:Glow2arc.jpg>. Accessed: August 28, 2013.

## Glow-to-Arc Transition

12/105  
Sep 3, 2013

# Pulsed Plasmas and Outstanding Questions

13/105  
Sep 3, 2013

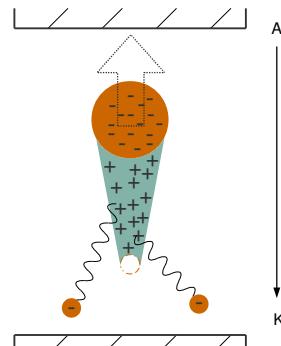


- Initial electron from cosmic rays, UV light, previous pulse, ...
- Sufficient electric field → exponential growth

## Electron Avalanche

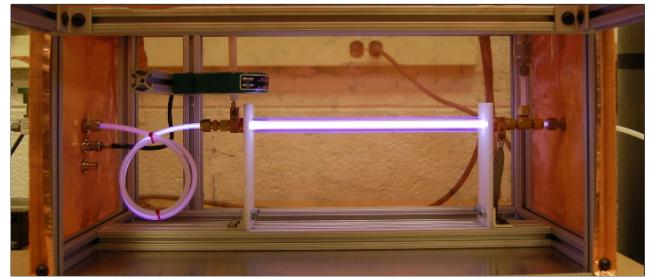
14/105  
Sep 3, 2013

- Internal field of avalanche comparable to applied field
- Reduces energy transfer to streamer
- “Injected” electrons and photoionization become important



## Streamer Discharge

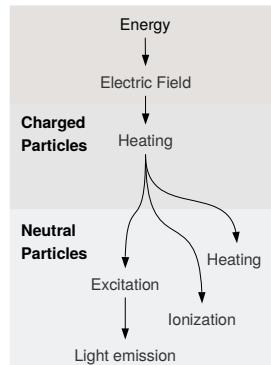
15/105  
Sep 3, 2013



- Similar processes to streamer discharge
- Short pulse-widths: 0 – 100 ns
- High voltages: 1 – 100+ kV
- Moderate repetition rates: 1 – 100 kHz

## Repetitively-Pulsed Nanosecond Discharge

16/105  
Sep 3, 2013

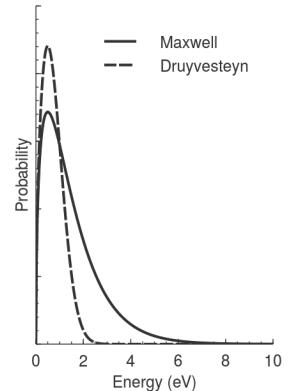


- Energy added to charged particles via electric field
- Fraction of energy heats electrons
- Remainder enters neutral particles through collisions
- How is energy divided?

## Questions: Energy Coupling

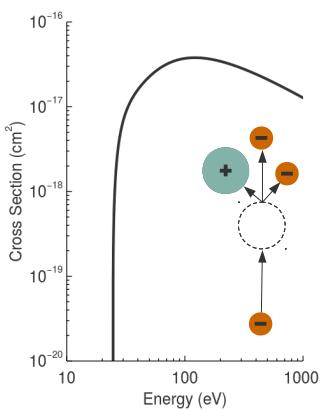
17/105  
Sep 3, 2013

- Continuous probability distribution for electron energies
- Analytic expressions and approximate calculations possible
- Assumptions may limit use for RPND
- 



## Questions: Electron Energy Distribution

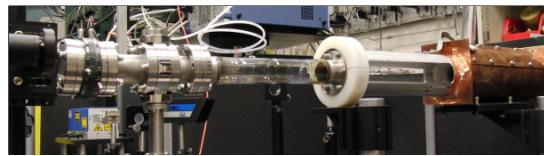
18/105  
Sep 3, 2013



- Plasma products determined by cross sections
- Probability to form a product for a given electron energy

## Questions: Electron Energy Distribution

19/105  
Sep 3, 2013



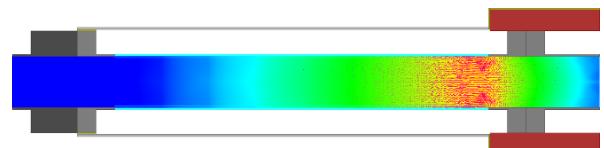
- Ultra-high purity helium: 0.3 – 16.0 Torr
- Rare gas discharges → interesting properties
  - Minimal gas heating
  - UV emissions
- Data available to create detailed population kinetics model
- Voltage pulses: 25 ns FWHM, +6.4 kV, 1.0 kHz

## Experimental Setup

22/105  
Sep 3, 2013

# Experimental Setup

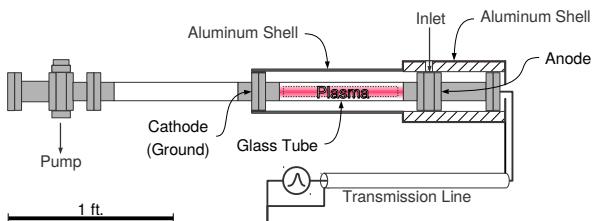
20/105  
Sep 3, 2013



- Peak field: 3.8 kV / cm
- Non-uniform field, concentrated near anode-glass interface
- Notable radial component a result of outer conductor

## Vacuum Electric Fields

23/105  
Sep 3, 2013

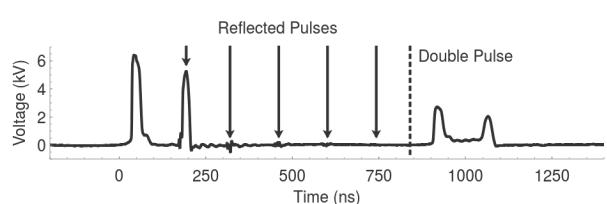


Coaxial-type geometry

- Inner conductor: plasma
- Dielectric: borosilicate vacuum tube, air, and teflon
- Outer conductor: copper and aluminum shells

## Discharge Geometry

21/105  
Sep 3, 2013



- Impedance mismatch at anode should cause doubling of voltage
- Long (13 m) transmission line used to isolate incident and reflected pulse

## Input Waveform

24/105  
Sep 3, 2013

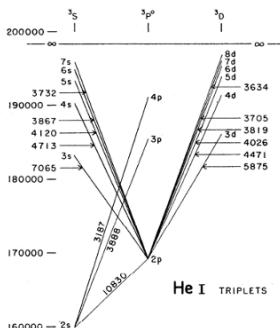
# Metastable Measurements

25/105  
Sep 3, 2013

- Physical probes
  - Perturb plasma
  - Not fast enough
  - Suggests an optical approach
- Electrons
  - No detectable emissions
  - Insufficient density for laser scattering
- Atoms and atomic states

## What Can Be Measured?

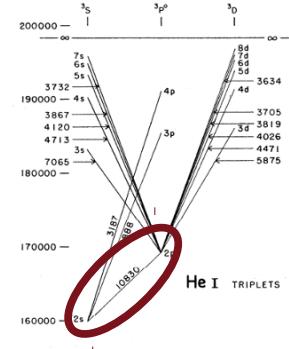
26/105  
Sep 3, 2013



## Excited States of Helium

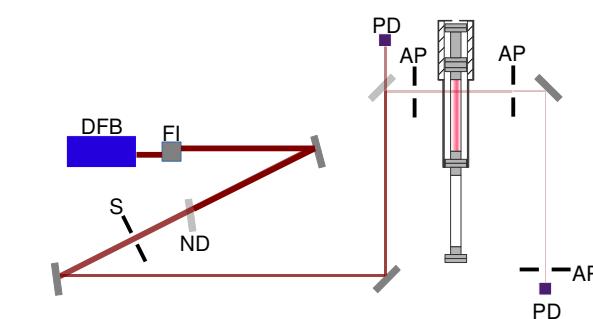
27/105  
Sep 3, 2013

- Absorption of light
  - Photon energy must match energy difference between states
- Time resolution determined by detector
- Fraction of light absorbed proportional to density of lower state



## Triplet Metastables

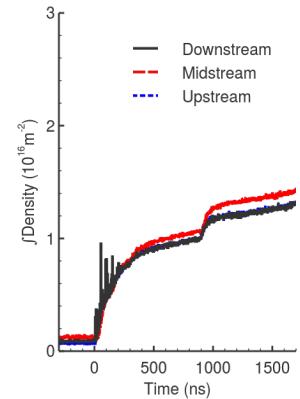
28/105  
Sep 3, 2013



## Laser-Absorption Spectroscopy Setup

29/105  
Sep 3, 2013

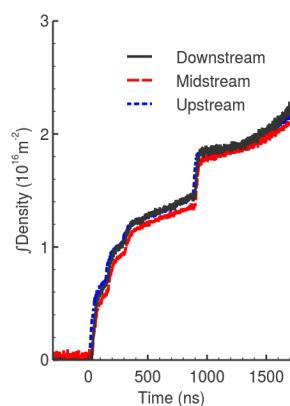
- Observable metastable density before pulse
- Noise present in downstream measurements from pulse
- No significant variation with location



## Metastable Densities (Low Pressure)

30/105  
Sep 3, 2013

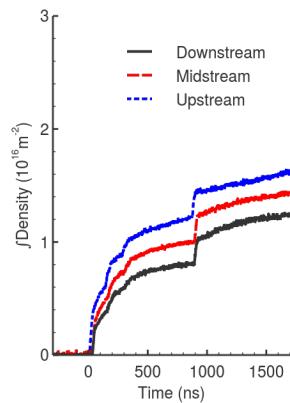
- Reduced pre-pulse density
- Increase in final density
- Reflections cause boost in metastable generation rate



### Metastable Densities (Moderate Pressure)

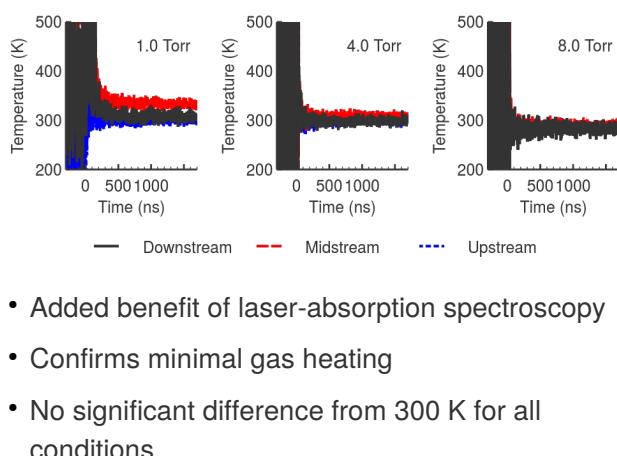
31/105  
Sep 3, 2013

- No metastables before pulse
- Overall decline in metastable density
- Reflection effects still observable
- Plasma attenuation with distance from anode



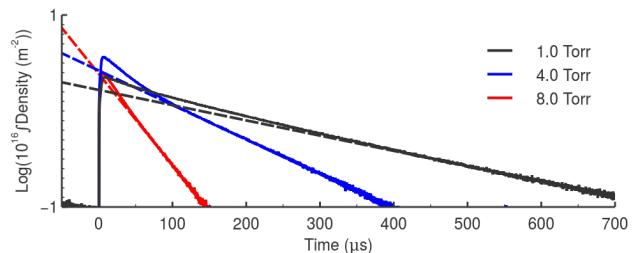
### Metastable Densities (High Pressure)

32/105  
Sep 3, 2013



### Temperatures

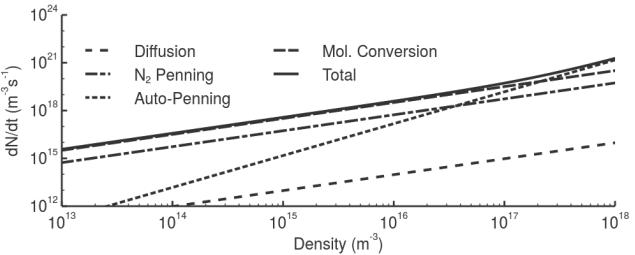
33/105  
Sep 3, 2013



- Long-duration measurements reveal loss mechanisms
- Deviation from straight line  $\rightarrow$  non-exponential process

### Metastable Destruction

34/105  
Sep 3, 2013



- Initially dominated by non-exponential processes
  - Superelastic electron collisions
  - Penning ionization between metastables
- Molecular conversion dominates at lower metastable densities
- Pre-pulse metastables only detectable for 4.0 Torr and below

### Destruction Processes

35/105  
Sep 3, 2013

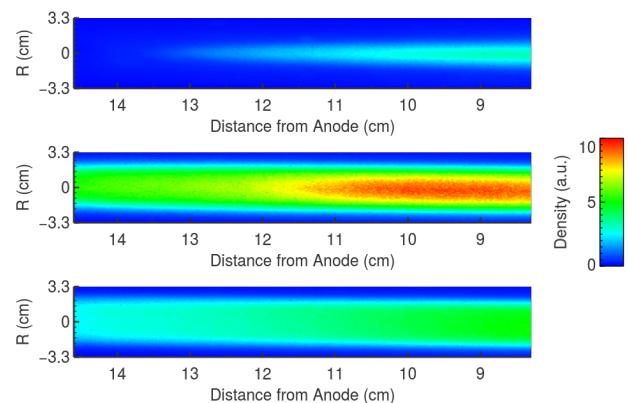
- Metastable states only persist between pulses at lower pressures
- There is an optimal pressure for metastable generation
- Significant attenuation of plasma occurs at higher pressures
- No detectable gas heating
- Losses initially dominated by superelastic electron collisions and Penning ionization between metastables

### Metastable Summary

36/105  
Sep 3, 2013

# Global Model

37/105  
Sep 3, 2013

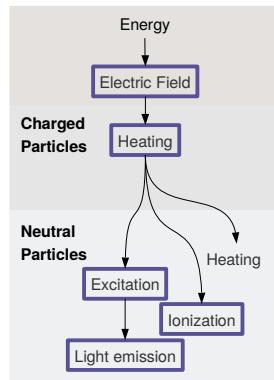


Weatherford, B.R., Barnat, E.V., Xiong, Z., Kushner, M., 65th Gaseous Elec. Conf., October 22-26, 2012, Austin, TX.

## Spatial Dependence

40/105  
Sep 3, 2013

- Metastables only one of many excited states
- Use metastable measurements to infer other properties
  - Electric fields
  - Electron densities and temperatures
  - Excited states



## Applying the Metastable Measurements

38/105  
Sep 3, 2013

- Assumed spatial dependence (global model)
- Species:
  - Helium, all states  $n < 5$
  - Helium ions
  - Electrons
- Reactions:
  - Elastic scattering (Pack)
  - Electron (de-)excitation (380 transitions, Ralchenko 2008)
  - Optical transitions (126 transitions, NIST ASD)
  - Excitation transfer (35 transitions, Dubreuil and Catherinot)

## Model Parameters

39/105  
Sep 3, 2013

$$\frac{dN_i}{dt} = n_e \left[ \sum_{j \neq i} N_j K_{j,i}^e(T_e) - N_i \sum_{j \neq i} K_{i,j}^e(T_e) \right] \text{Electron (de)excitation}$$

$$+ \left[ \sum_{j > i} N_j K_{j,i}^o - N_i \sum_{j < i} K_{i,j}^o \right] \text{Radiative transitions}$$

$$+ N_g \left[ \sum_{j \neq i} N_j K_{j,i}^a - N_i \sum_{j \neq i} K_{i,j}^a \right] \text{Atomic excitation transfer}$$

## Particle Density Equation

41/105  
Sep 3, 2013

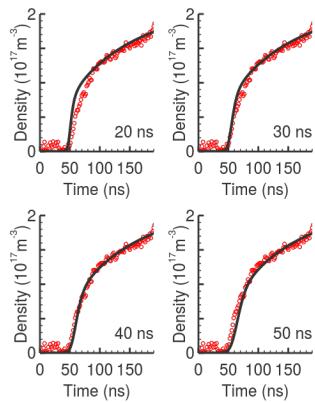
$$\frac{d}{dt} \left( \frac{3}{2} n_e T_e \right) = \frac{e^2 n_e E(t)^2}{m_e k_m(T_e) N_g} \text{Electric field heating}$$

$$- n_e k_m(T_e) N_g \left( \frac{3 m_e}{M} \right) \frac{3}{2} (T_e - T_g) \text{Elastic collision losses}$$

$$- n_e \sum_i \sum_{j \neq i} K_{ij}^e N_i \Delta \epsilon_{ij} \text{Inelastic collisions losses}$$

## Electron Energy Equation

42/105  
Sep 3, 2013

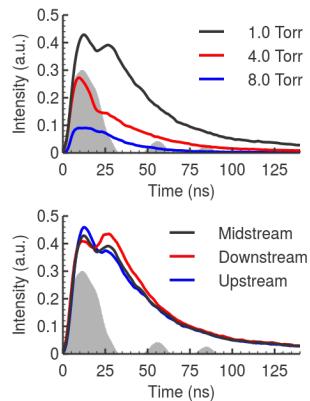


- Assumed electric field applied as a Gaussian pulse
- Free variables: electric field and pulse width
- Best fit obtained with 40 ns pulse-width, but this is longer than applied pulse?

## Pulse-width

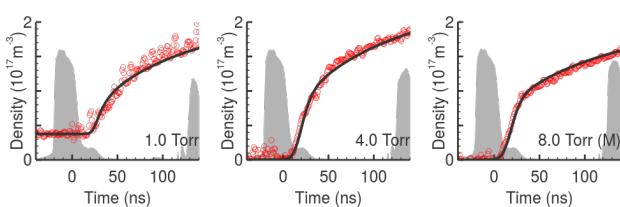
43/105  
Sep 3, 2013

- Optical emissions showed evidence of return stroke
- Double peak at several pressures
- Second peak most intense closest to ground electrode
- Excitation period effectively longer than 25 ns



## Evidence for Return Stroke

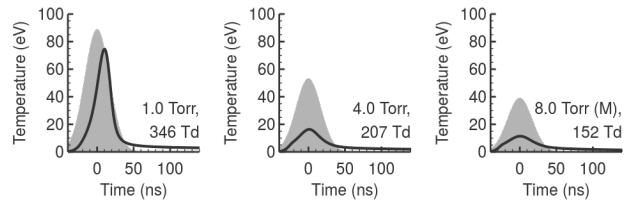
44/105  
Sep 3, 2013



- Good agreement for most cases—trends and magnitudes were consistent
- Largest discrepancies at 1.0 Torr
  - Initial rise too fast
  - Radiative cascade too slow
  - Extended excitation?

## Metastable Matching

45/105  
Sep 3, 2013



- Peak fields consistent with other studies
- 1.0 Torr fields well into range where two-term expansion solutions performed better
- 1.0 Torr temperatures are not reasonable

## Peak Fields and Electron Temperatures

46/105  
Sep 3, 2013

- Created and applied global model of RPND
- Observed evidence of extended excitation period
- Simulations able to reproduce metastable density dynamics
- Temperatures and fields at 1.0 Torr suggest potential violation of underlying assumptions

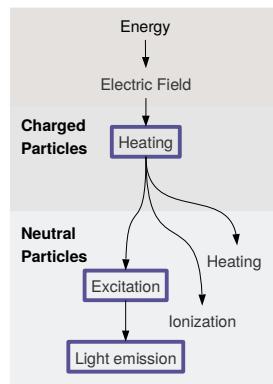
## Summary of Simulation Results

47/105  
Sep 3, 2013

# Emission Measurements

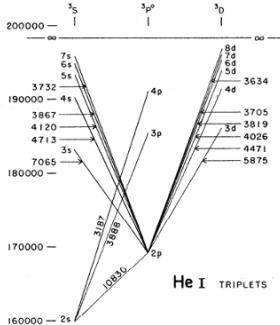
48/105  
Sep 3, 2013

- Simulations based on metastable densities
- Optical emissions spectroscopy can check other excited states
- Can reveal other phenomena
- May be able to estimate electron temperature



## Corroborating Simulations

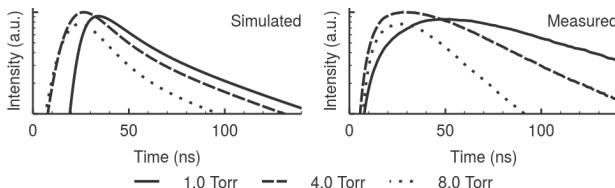
49/105  
Sep 3, 2013



- Observed transitions in visible (triplet and singlet)
- Calibrated relative intensities with tungsten lamp
- Same conditions as absorption spectroscopy

## Helium Emissions

50/105  
Sep 3, 2013



- Relative peak heights and timing similar, distinctly different in shape
- Measurements suggest pressure-dependent de-excitation process (Penning ionization of impurities?)
- Long emission lifetime suggest an extended excitation process
  - Beam-like electrons
  - Persistent electric field

## Extended Excitation ( $3^1D \rightarrow 2^1P^o$ , 668 nm)

51/105  
Sep 3, 2013

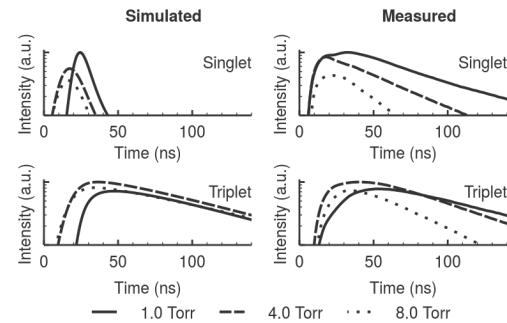
- Resonance radiation can be “trapped” in plasmas
- Increases energy residence time
- Could contribute to extended excitation

Pressure (Torr)	Trapping Factor	Effective Lifetime (s)
1.0	8,773	$1.549 \times 10^{-5}$
4.0	38,031	$6.715 \times 10^{-5}$
8.0	78,837	$1.392 \times 10^{-4}$

Lifetime increase for  $3^1P \rightarrow 1^1S$  transition

## Radiation Trapping

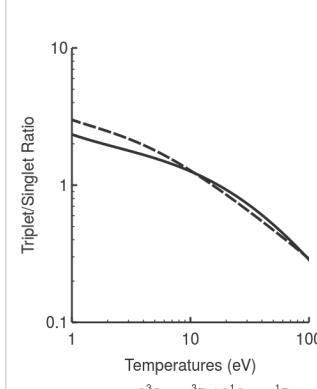
52/105  
Sep 3, 2013



- Verify by comparison of 389 nm and 501 nm
- If radiation trapping affects system, measured lifetime of 501 line should be greater than simulation

## Radiation Trapping

53/105  
Sep 3, 2013

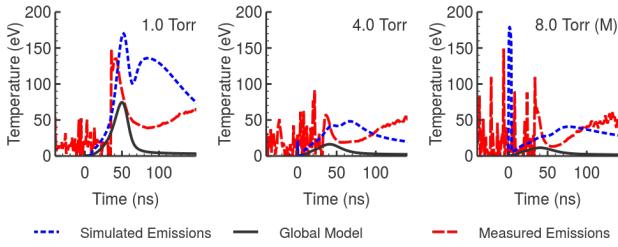


- Can be used to estimate electron temperature
- Assumes
  - Electron-ground excitation only
  - Radiative losses
- Maxwell-Boltzmann distribution

$$\frac{I_{i,j}}{I_{i',j'}} = \frac{\lambda_{i',j'} A_{i,j} \sum A_{i'} K_{0,i}(T_e)}{\lambda_{i,j} A_{i',j'} \sum A_i K_{0,i'}(T_e)}$$

## Coronal Model

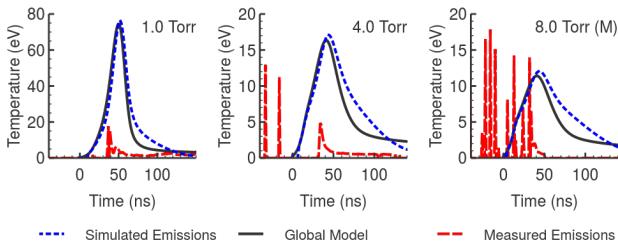
54/105  
Sep 3, 2013



- Poor match with simulated and measured emissions
- Possible need to include n = 5 states
- Collisional redistribution of n = 4 states potentially a factor

### Ratio 1: $4^3S \rightarrow 2^3P / 4^1D \rightarrow 2^1P$

55/105  
Sep 3, 2013



- Coronal model and global model: consistent
- Application limited by signal-to-noise ratio
- Likely not a Maxwellian temperature
- May still be useful as a measure of mean electron energy

### Ratio 2: $4^3S \rightarrow 2^3P / 3^1S \rightarrow 2^1P$

56/105  
Sep 3, 2013

- Global model captures some features of RPND population kinetics
- Missing physics
  - Extended excitation (distribution or field related?)
  - Radiation trapping
- $4^3S \rightarrow 2^3P_0 / 3^1S \rightarrow 2^1P_0$  ratio may be useful indicator of electron energy in helium RPND

### Summary of Emission Measurements

57/105  
Sep 3, 2013

## Conclusions

- Developed laser-absorption spectroscopy system and analysis software
- Obtained detailed measurements of metastable dynamics in a helium RPND
- Created a global model code for helium plasmas
- Used code to infer plasma parameters from metastable density information
- Measured optical emissions of the RPND
- Found several additional phenomena that are potentially important in the description of the RPND

### Summary of Work

58/105  
Sep 3, 2013

- Excited states only persist at lower pressures, what ensures consistent breakdown?
- Unexpectedly long excitation period, even accounting for return stroke
  - Slowing of beam-like electrons
  - Radiation trapping
  - Persistent electric fields
- May be possible to employ simple line ratio diagnostics for estimates of mean electron energy

### Synthesis of Results

59/105  
Sep 3, 2013

- Address temporal evolution of metastable density profile (Abel inversion) → wall effects
- Incorporate additional physics (Penning ionization of impurities, radiation trapping, etc.) in present global model.
- Employ different EEDF models to better match excited state dynamics
- Move to a 0D Monte-Carlo simulation of RPND to avoid EEDF issues
- 

## Future Work

61/105  
Sep 3, 2013

## Acknowledgements

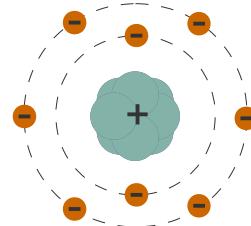
62/105  
Sep 3, 2013

## Questions

63/105  
Sep 3, 2013

# Backup Slides

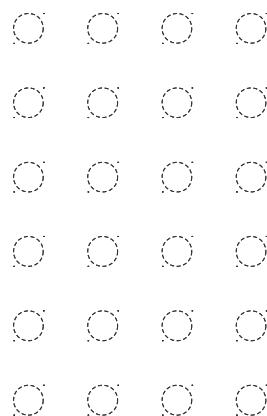
64/105  
Sep 3, 2013



- Positively-charged nucleus
- Orbited by negatively-charged electrons
- Overall, electrically neutral

## Bohr Atom

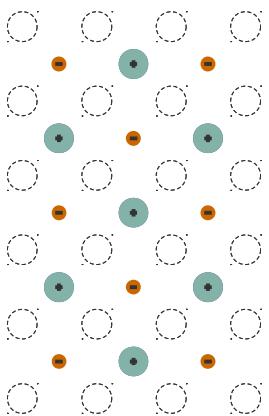
65/105  
Sep 3, 2013



- Consider an atomic gas
- Many neutral atoms with random motion
- No electrical interaction

## Gas

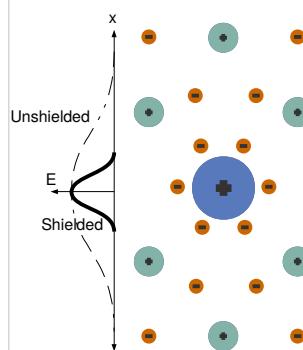
66/105  
Sep 3, 2013



- Add energy to gas
  - Electricity
  - Shock
  - Light
- Electrons separate from atoms
- Collection of neutral atoms, (positive) ions, and (negative) electrons
- What makes a plasma?

## Ionized Gas

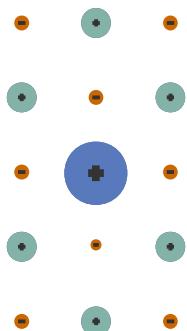
67/105  
Sep 3, 2013



- Electrons move to shield charge
- Similar for positive and negative perturbation
- Electric field falls off quickly
- $\lambda < L, n\lambda^3 \gg 1$

## Debye Length

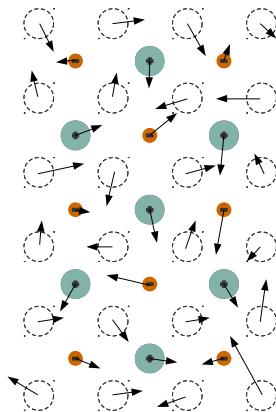
70/105  
Sep 3, 2013



Consider what happens with an electrical perturbation.

## Perturbation

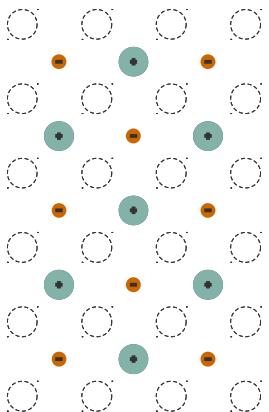
68/105  
Sep 3, 2013



- Random collisions with neutral particles
- Neutral collisions can still determine properties of ionized gas
- Electric interactions should be dominant in plasma

## Neutral Particle Interactions

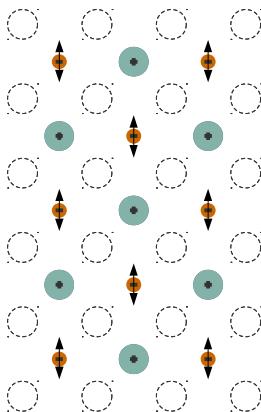
71/105  
Sep 3, 2013



- Add energy to gas
  - Electricity
  - Shock
  - Light
- Electrons separate from atoms
- Collection of neutral atoms, (positive) ions, and (negative) electrons
- What makes a plasma?

## Ionized Gas

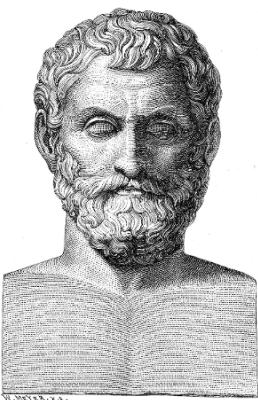
69/105  
Sep 3, 2013



- Electrons possess natural oscillation frequency
- Characteristic of electrical interaction
- $\omega_p > v$

## Charged-Particle Oscillations

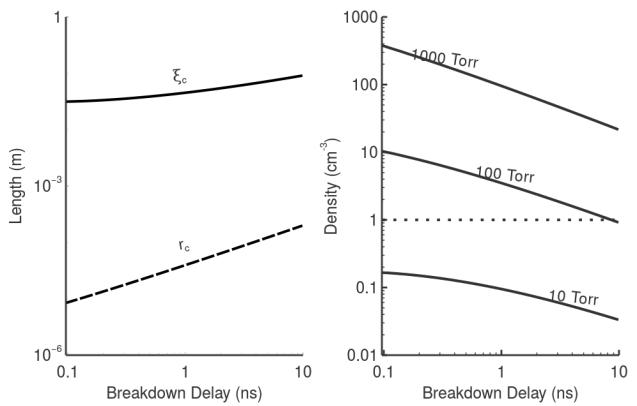
72/105  
Sep 3, 2013



Wallis, E. et al. "Illustrerad verldshistoria utgiven av E. Wallis. Volume I." (1875).  
Statesbury, H. "J. J. Thomson." Popular Science, Vol. 56 (1895).

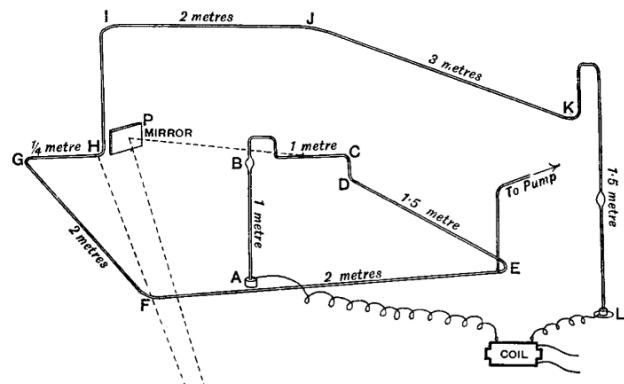
## Early History

73/105  
Sep 3, 2013



## Homogeneous Streamers

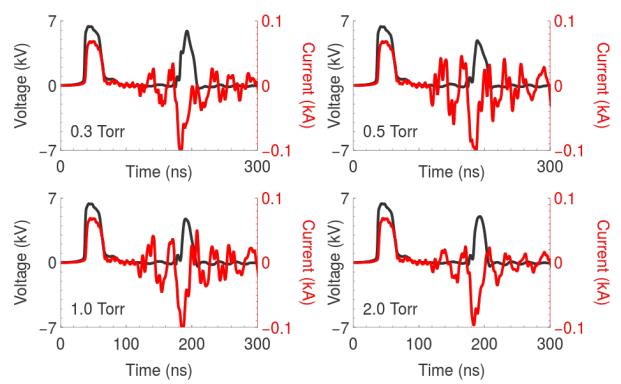
76/105  
Sep 3, 2013



J. J. Thomson. "Notes on Recent Researches in Electricity and Magnetism." Clarendon Press, Oxford, UK, 1893.

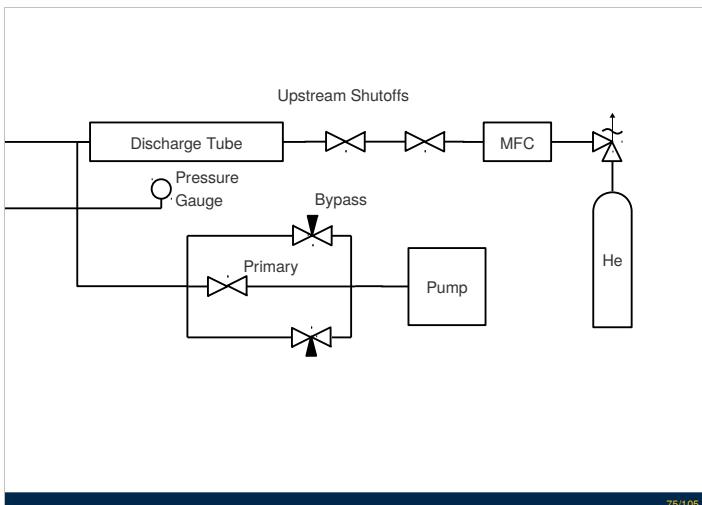
## First Pulsed Plasma Measurements

74/105  
Sep 3, 2013



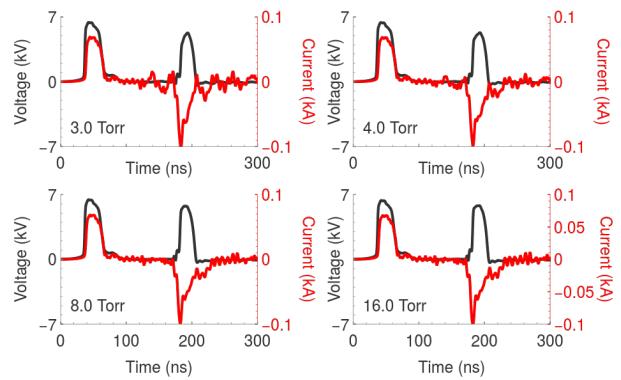
## Waveforms 1

77/105  
Sep 3, 2013



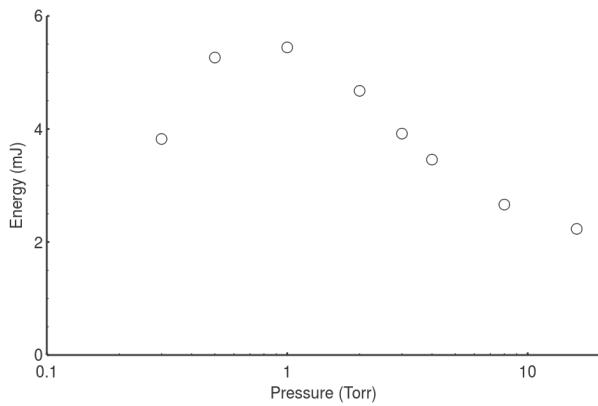
## Pumping System

75/105  
Sep 3, 2013



## Waveforms 2

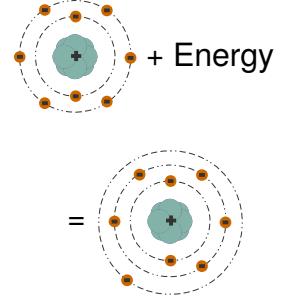
78/105  
Sep 3, 2013



## Energy Coupling

79/105  
Sep 3, 2013

- Energy added to atom
  - Collisions
  - Absorption of light
- One or more electrons to higher orbit  $\rightarrow$  excited atom
- Excited state can emit photon to reach lower energy state
  - Occurs with a specific rate



## Excited States of Helium

82/105  
Sep 3, 2013

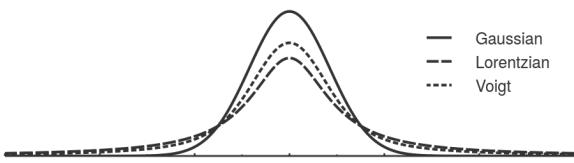
$$\Delta N = \Delta N_0 \frac{1}{1 + W\tau_{\text{eff}}} = \Delta N_0 \frac{1}{1 + I/I_{\text{sat}}}$$

$$W = \frac{\sigma I}{h\nu} \quad \rightarrow \quad I_{\text{sat}} = \frac{h\nu}{\sigma\tau_{\text{eff}}}$$

$$I_{\text{sat}} = \frac{2\sqrt{2}h_{21}}{\sigma\tau} = 0.45 \text{ mW} \cdot \text{cm}^{-2}$$

## Two-Level Saturation

80/105  
Sep 3, 2013



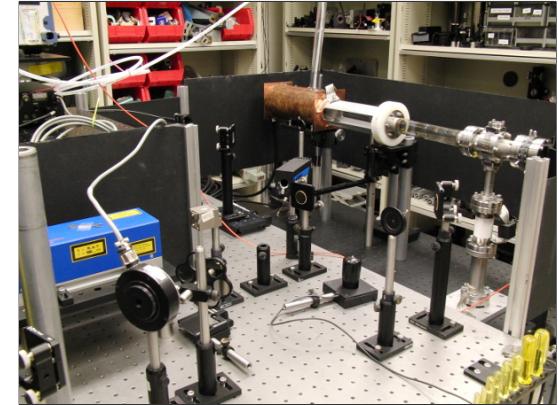
$$\text{Lorentzian} \quad g(\omega) = -\frac{1}{4\pi^2} \frac{A\lambda^3}{\Delta\omega_a} \frac{1}{1 + [2(\omega - \omega_a)/\Delta\omega_a]^2}$$

$$\text{Voigt} \quad g(\omega) = \sqrt{\frac{2 \ln 2}{\pi^3}} \frac{\Delta\omega_d}{\Delta\omega_d} \int_{-\infty}^{\infty} \frac{1}{[(\omega - \omega_a) - \omega']^2 + 4^2} \times \exp \left[ 4 \ln 2 \left( \frac{\omega'}{\Delta\omega_d} \right)^2 \right] d\omega'$$

$$\text{Gaussian} \quad g(\omega) = \sqrt{\frac{4 \log 2}{\pi \Delta\omega_d^2}} \exp \left[ -(4 \log 2) \left( \frac{\omega - \omega_a}{\Delta\omega_d} \right)^2 \right]$$

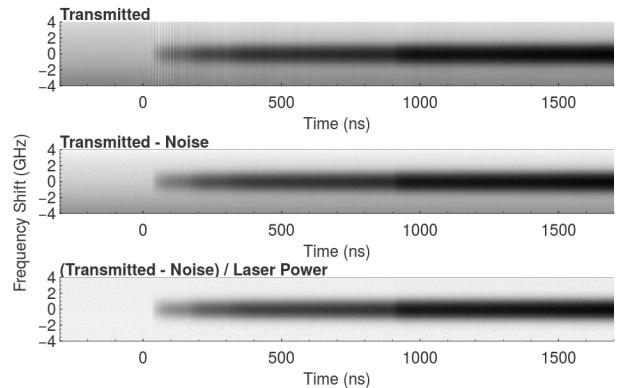
## Lineshapes

81/105  
Sep 3, 2013



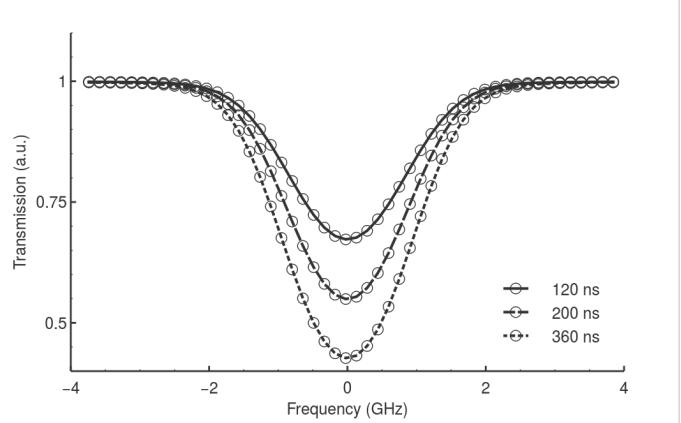
## Laser-Absorption Spectroscopy Setup

83/105  
Sep 3, 2013



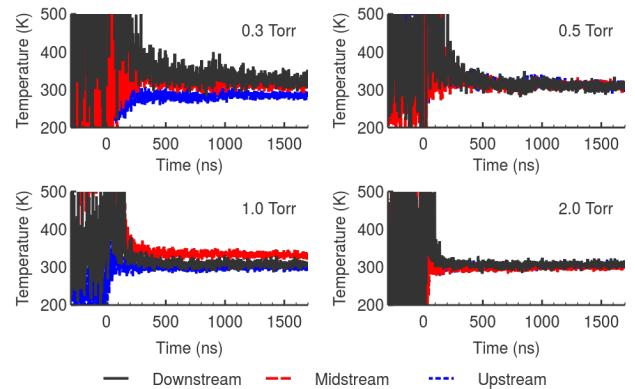
## Spectral Post-Processing

84/105  
Sep 3, 2013



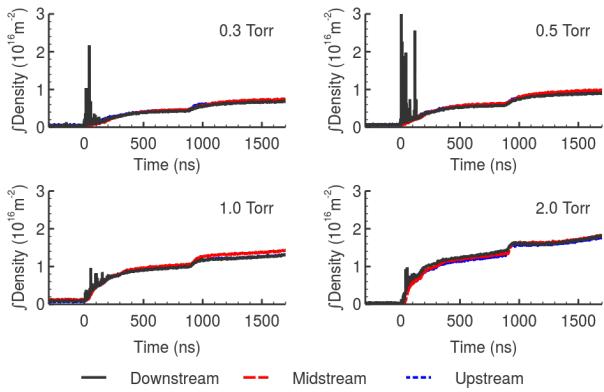
### Absorption Spectra Matching

85/105  
Sep 3, 2013



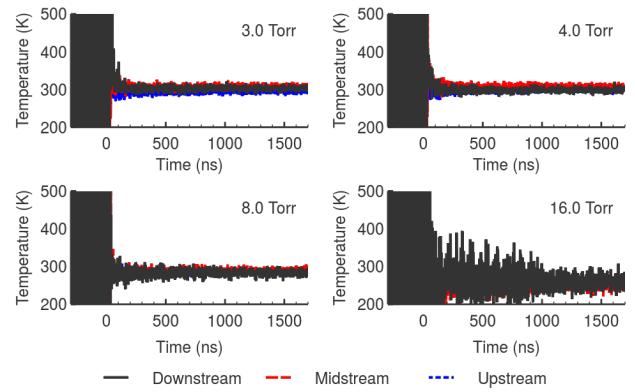
### Temperatures 1

88/105  
Sep 3, 2013



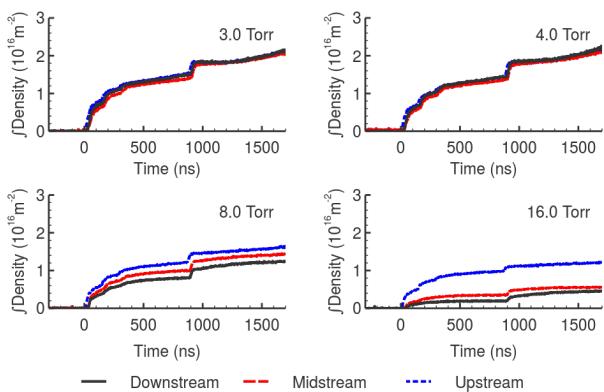
### Metastables 1

86/105  
Sep 3, 2013



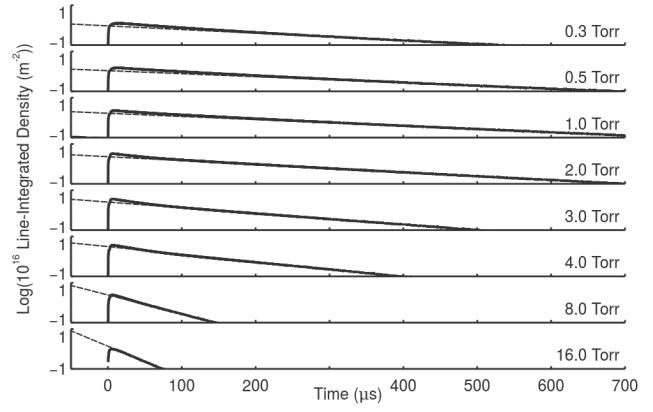
### Temperatures 2

89/105  
Sep 3, 2013



### Metastables 2

87/105  
Sep 3, 2013



### Metastable Decay

90/105  
Sep 3, 2013

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} (\vec{E} + \vec{v} \times \vec{B}) \cdot \nabla_v f_\alpha = \left( \frac{\partial f_\alpha}{\partial t} \right)_{\text{coll}}$$

- Describes the evolution of the distribution function for a particle  $\alpha$
- 7 independent variables
- Simple equilibrium solutions possible
  - Maxwell-Boltzmann
  - Druyvesteyn
- Generally difficult to solve otherwise

## Boltzmann Equation

91/105  
Sep 3, 2013

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{u}_\alpha) = G_\alpha - L_\alpha.$$

- Boltzmann equation can be converted into moments by integrating over velocity space
- Introduces particle density  $n$  and mean fluid velocity,  $u$
- $G$  – source terms
- $L$  – loss terms

## First Moment: Continuity Equation

92/105  
Sep 3, 2013

$$m_\alpha n_\alpha \left[ \frac{\partial \vec{u}_\alpha}{\partial t} + (\vec{u}_\alpha \cdot \nabla) \vec{u}_\alpha \right] = q_\alpha n_\alpha (\vec{E} + \vec{u}_\alpha \times \vec{B}) - \nabla \cdot \vec{\Pi} + \vec{f}_{\text{coll}}$$

- Second momentum obtained by multiplying by velocity and integrating over velocity space
- Describes changes in fluid velocity over space
- Introduces pressure tensor  $\Pi$ .
- $\vec{f}_{\text{coll}}$  expresses change in momentum due to collisions

## Second Moment: Conservation of Momentum

93/105  
Sep 3, 2013

$$\frac{\partial}{\partial t} \left( \frac{3}{2} p_\alpha \right) + \nabla \cdot \frac{3}{2} (p_\alpha \vec{u}_\alpha) + p_\alpha \nabla \cdot \vec{u}_\alpha + \nabla \cdot \vec{q}_\alpha = \frac{\partial}{\partial t} \left( \frac{3}{2} p_\alpha \right) \Big|_{\text{coll}}$$

- Third momentum obtained by multiplying by energy and integrating of velocity space
- Energy expressed in terms of pressure,  $p$
- RHS includes energy losses and gains resulting from collisions

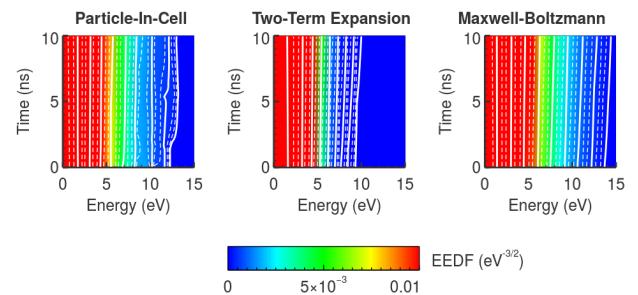
## Third Moment: Conservation of Energy

94/105  
Sep 3, 2013

- Must assume an EEDF for reaction rates
- Baseline: 0D particle-in-cell (XPDP1)
- Two approaches considered:
  - Maxwell-Boltzmann distribution
  - Solutions of two-term expansion of Boltzmann equation (BOLSIG+)

## EEDF Assumption

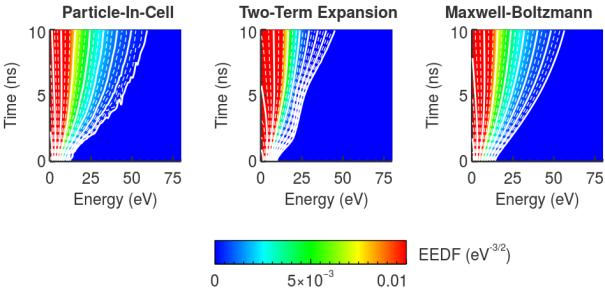
95/105  
Sep 3, 2013



- Both approaches show reasonable agreement with particle-in-cell
- Two-term: less high-energy electrons
- Maxwell-Boltzmann: more high-energy electrons

## EEDF Comparisons (10 Td)

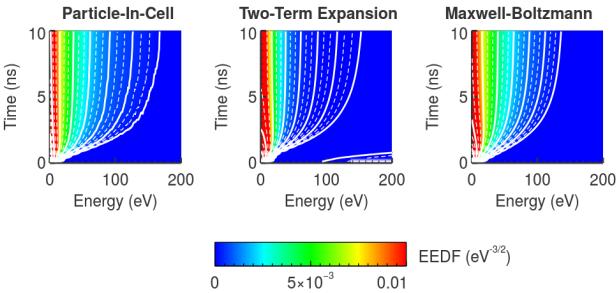
96/105  
Sep 3, 2013



- Unexpected contour variations in two-term expansion solutions
- Better agreement between Maxwell-Boltzmann and particle-in-cell

### EEDF Comparisons (100 Td)

97/105  
Sep 3, 2013



- Particle-in-cell: more 100 eV+ electrons than other results
- Two-term expansion solution best match this behavior
- Beginning of beam-like behavior?

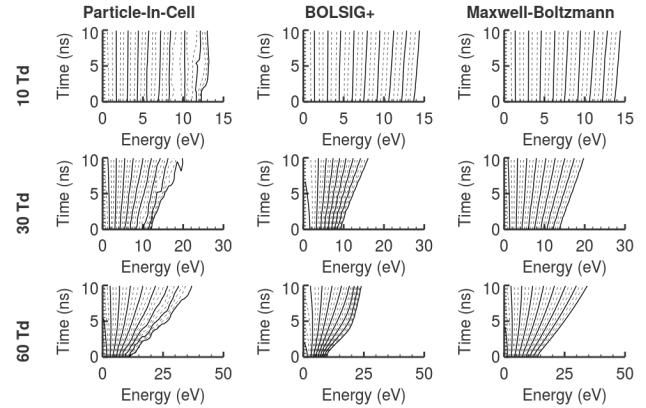
### EEDF Comparisons (300 Td)

98/105  
Sep 3, 2013

- < 100 Td: Maxwell-Boltzmann gives best agreement with particle-in-cell
- > 100 Td: BOLSIG+ gives best agreement with particle-in-cell
- Primary discrepancy in the number of high energy electrons: Particle-in-cell > BOLSIG+ > Maxwell-Boltzmann
- Better overall agreement led to use of Maxwell-Boltzmann distribution

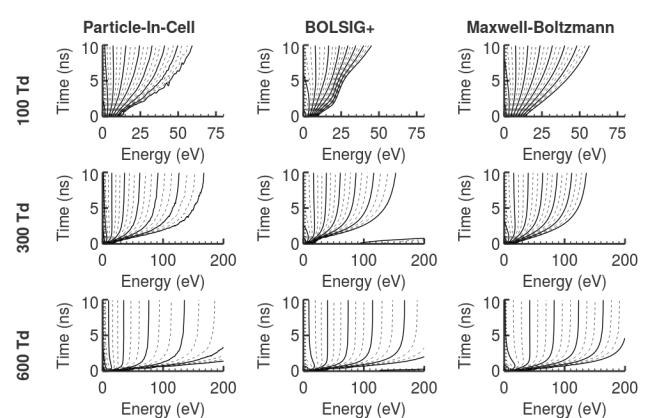
### EEDF Comparison Summary

99/105  
Sep 3, 2013



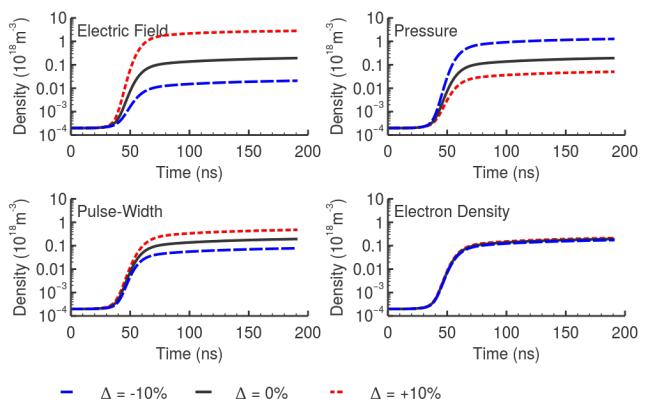
### EEDF Contours 1

100/105  
Sep 3, 2013



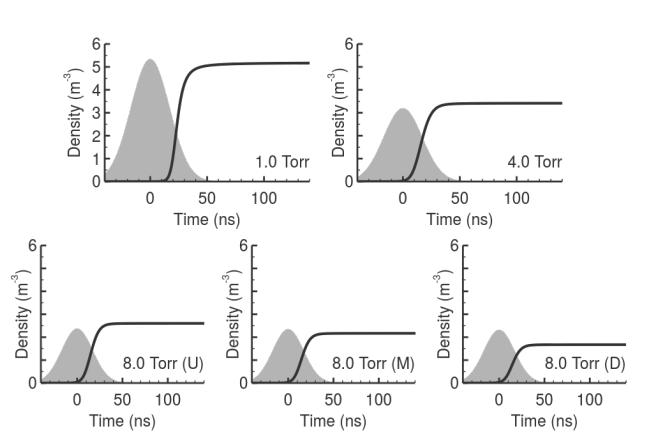
### EEDF Contours 2

101/105  
Sep 3, 2013



### Perturbation Results

102/105  
Sep 3, 2013



- Pressures: 0.3 – 16.0 Torr
- Spex HR460 monochromator, 0.88 nm bandpass
- 1,200 grooves/mm
- Photomultiplier tube, maximum 3.0 ns rise time
- Relative intensities calibrated with tungsten blackbody lamp

