Chapter 1

Theory

1.1 Ionized Gas

A plasma is considered to be a form or subset of ionized gas. That is, a volume of gas in which some fraction of the neutral atoms and/or molecules have been separated into electron and ion pairs. For a sufficiently large number of particles, the behavior of each species in the ionized gas can be described by a continuous distribution function.

This probability distribution expresses the likelihood of finding a particle with a specific range of velocities in a specific volume, as a function of time. This function is denoted as $f_\alpha(\vec{r}, \vec{v}, t)$, where the subscript α denotes the species, f is the probability distribution, $\vec{r}$ is the position, $\vec{v}$ is the velocity, and t is the time.

The behavior of f_α can be shown to be governed by the Boltzmann equation,

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + q_\alpha \left(\vec{E} + \vec{v} \times \vec{B} \right) \cdot \nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t} \right)_{\text{coll}}. \tag{1.1}$$

Here, q is the charge of the species, $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, and $\partial f_\alpha/(\partial t)_{\text{coll}}$ is a term which represents changes to the distribution function as a result of collisions. Coupled with Maxwell's equations, equation 1.1 provides a complete description of the behavior of the fields and particles in a plasma.

For a species in equilibrium in the absence of external forces and $(\partial f_\alpha/\partial t)_{\text{coll}} = 0$, it can be shown that the distribution of energies is

$$f_\alpha(\varepsilon) = \frac{3\sqrt{3}}{\sqrt{2\pi}}(k_{\text{B}}T_\alpha)^{-3/2}\exp\left(-\frac{3\varepsilon}{2k_{\text{B}}T_\alpha}\right) \tag{1.2}$$

where ε is the energy, k_{B} is Boltzmann's constant, and T_α is the temperature of the species. This is referred to as the Maxwell-Boltzmann distribution. It should be emphasized that this solution only applies when the species can be considered to be in equilibrium. Gradients and electromagnetic fields can both significantly alter the distribution function of a species. This can be of particular importance in the calculation or reaction rates, or the measurement of temperatures.

Additionally, the Boltzmann equation may be solved for electrons, assuming they only interact through elastic collisions, and that they are in equilibrium with a small, constant electric field. This result was originally presented by Druyvesteyn and Penning [1] and has come to be known as the Druyvesteyn distribution. It is defined as,

$$f_\alpha = \frac{3\sqrt{3}}{4\Gamma(3/4)}(k_{\text{B}}T_\alpha)^{-3/2}\exp\left[-\frac{9}{16}\left(\frac{\varepsilon}{k_{\text{B}}T_\alpha}\right)\right] \tag{1.3}$$

where Γ is the gamma function. This solution tends to suppress the probability of higher and lower-energy electrons in favor of more intermediate values. Figure 1.1 compares the probably distributions from equations 1.2 and 1.3 for the

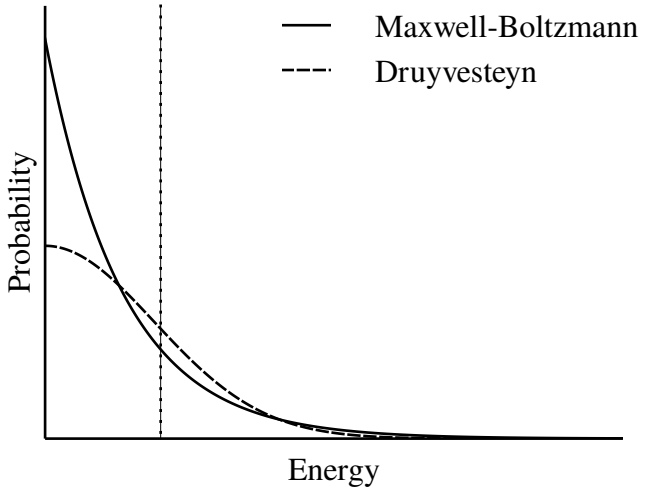

Figure 1.1: Comparison of the Maxwell-Boltzmann energy distribution and the Druyvesteyn distribution for the same average energy (illustrated by the dotted line).

same temperature α. The dotted line illustrates the average energy for the two distributions, which is not the same as the most probably energy.

Additional solutions of equation 1.1 in anything but these simple cases can be very challenging. In most situations, it is reduced to more tenable equations by integrating over velocity-space (leaving f as a function of space and time). The first so-called moment is often called the conservation equation or continuity equation,

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{u_\alpha}) = G_\alpha - L_\alpha. \tag{1.4}$$

In this case, $\vec{v}$ has been replaced by a mean velocity $\vec{u}$, and the collision operator has been replace by gain (G) and loss (L) terms. The gain and loss terms are

generally calculated by integrating a cross section over a known velocity distribution.

The definition of the mean velocity, $\vec{u}$ can be obtained by multiplying equation 1.1 by v and integrating over velocity-space, to obtain the second moment,

$$m_\alpha n_\alpha \left[\frac{\partial \vec{u_\alpha}}{\partial t} + (\vec{u_\alpha} \cdot \nabla)\vec{u_\alpha} \right] = q_\alpha n_\alpha (\vec{E} + \vec{u_\alpha} \times \vec{B}) - \nabla \cdot \vec{\Pi} + \vec{f}|_{\text{coll}}. \qquad (1.5)$$

This expresses the conservation of momentum by the plasma. It provides a means by which to solve for the mean velocity of the system, however it also introduces two additional terms. $\vec{f}|_{\text{coll}}$ deals with the forces transferred to α via collisions. Though complex, it only depends on known quantities. The second term, $\vec{\Pi}$, is the pressure tensor and can only be defined by the third moment of the Boltzmann equation. In fact, each additional moment introduces a new term requiring a higher order moment, *ad infinitum*. In most situations, this chain of equations is terminated after the first two or three moments by the use of an additional assumption such as an equation of state.

For our purposes, one more moment will suffice. Assuming that the pressure is isotropic, one can multiply equation 1.1 by $mv^2/2$, and integrate over velocity-space. This yields the energy conservation equation,

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p_\alpha\right) + \nabla \cdot \frac{3}{2}(p_\alpha \vec{u}_\alpha) + p_\alpha \nabla \cdot \vec{u}_\alpha + \nabla \cdot \vec{q}_\alpha = \frac{\partial}{\partial t}\left(\frac{3}{2}p_\alpha\right)\Big|_{\text{coll}}. \qquad (1.6)$$

In this case, p represents the isotropic pressure, and $\vec{q}$ is the heat flow. The first term on the LHS represents the total energy contained by the species, the second term is the energy flux in and out of the volume, and the third term accounts for

changes due to compression or expansion. The RHS is the collision operator which describes energy added or removed from the system as a result of collisions.

Equations 1.4 and 1.6 are particularly important for this study. As will be detailed in chapter **??**, the two can be used to create a global model of the plasma. While such a model only considers globally averaged quantities, it can be extremely useful in characterizing the complex evolution of multi-component plasmas.

1.2 Plasma Criteria

Though the Boltzmann equation can describe both an ionized gas and a plasma, the two are conceptually distinct. A plasma is unique in that its dynamics are governed by long range electromagnetic forces unlike gases in which short-range collisions dominate. As a result, plasmas frequently feature large scale structure and organization. Examples of these structures are ubiquitous in astronomy where phenomena such as the aurora borealis, coronal mass ejections, and even interstellar media all qualify as forms of plasma. How then, does one define a plasma? There are three criteria required for an ionized gas to be considered a plasma.

Debye Length

As mentioned, an ionized gas features a number of electron and ion pairs. If an electrical perturbation is introduced into the ionized gas, the charged particles

will tend to rearrange themselves to shield it out. A plasma is an ionized gas which is large enough for this shielding effect to occur. The characteristic length scale for this shielding effect to take place is referred to as the Debye length, denoted λ_D. It can be shown to be equal to $\sqrt{\varepsilon_0 T_e/(en_0)}$, where ε_0 is the vacuum permittivity, T_e is the electron temperature, and n_0 is the plasma density. If the characteristic length scale of the ionized gas is L, then $\lambda_D < L$ for it to be considered a plasma.

Debye Sphere

However, the previous condition is necessary, but not sufficient for shielding to occur. It is possible that an ionized gas may have a relatively small Debye length, but also lack enough charged particles for shield to occur. More simply put, it would be impossible for a single electron to shield out even the smallest of perturbations. For that reason, the number of particles in a Debye sphere must be greater than unity in a plasma, or $n_0(4\pi\lambda_D^3/3) >> 1$.

Plasma Oscillations

Finally, a plasma may exhibit Debye shielding, but lack the long-range interactions of a plasma. This can occur when the collision frequency with neutral particles is too high. In this case, the behavior of the ionized gas would be determined more by the random collisions. Therefore, the characteristic response frequency of a plasma, commonly called the plasma frequency, must be greater

than the neutral collision frequency, or $\omega_p > \nu$. The plasma frequency can be shown to be $\omega_p = \sqrt{e^2 n_0/(\varepsilon_0 m_e)}$.

There are many natural and man-made ionized gases which can be considered plasmas. Figure 1.2 shows several categories of plasma, plotted as a function of their electron density and temperature. As can be seen, the electron densities span seven decades, and the densities cover in excess of 20. This broad range of conditions presents a particularly challenging problem for both simulations and experimental measurements.

1.3 Discharge Initiation

The Boltzmann equation is a continuous, statistical description of a plasma. By comparison, the initial breakdown of a plasma is a highly discontinuous process marked by its stochasticity. The initiation of a discharge is typically the result of electron avalanches which occur randomly throughout a volume of gas. Often, the seed electrons for a plasma are the products of ionizing cosmic rays which generate a few electrons per cubic-centimeter. As a result, it is necessary to consider the initiation of a discharge separately from a pre-existing plasma.

Townsend Mechanism

Classically, plasmas are created by two different mechanisms, the choice of which depends on the strength of the electric field. At lower electric fields, the Townsend mechanism is responsible for the formation of a plasma. Consider a two elec-

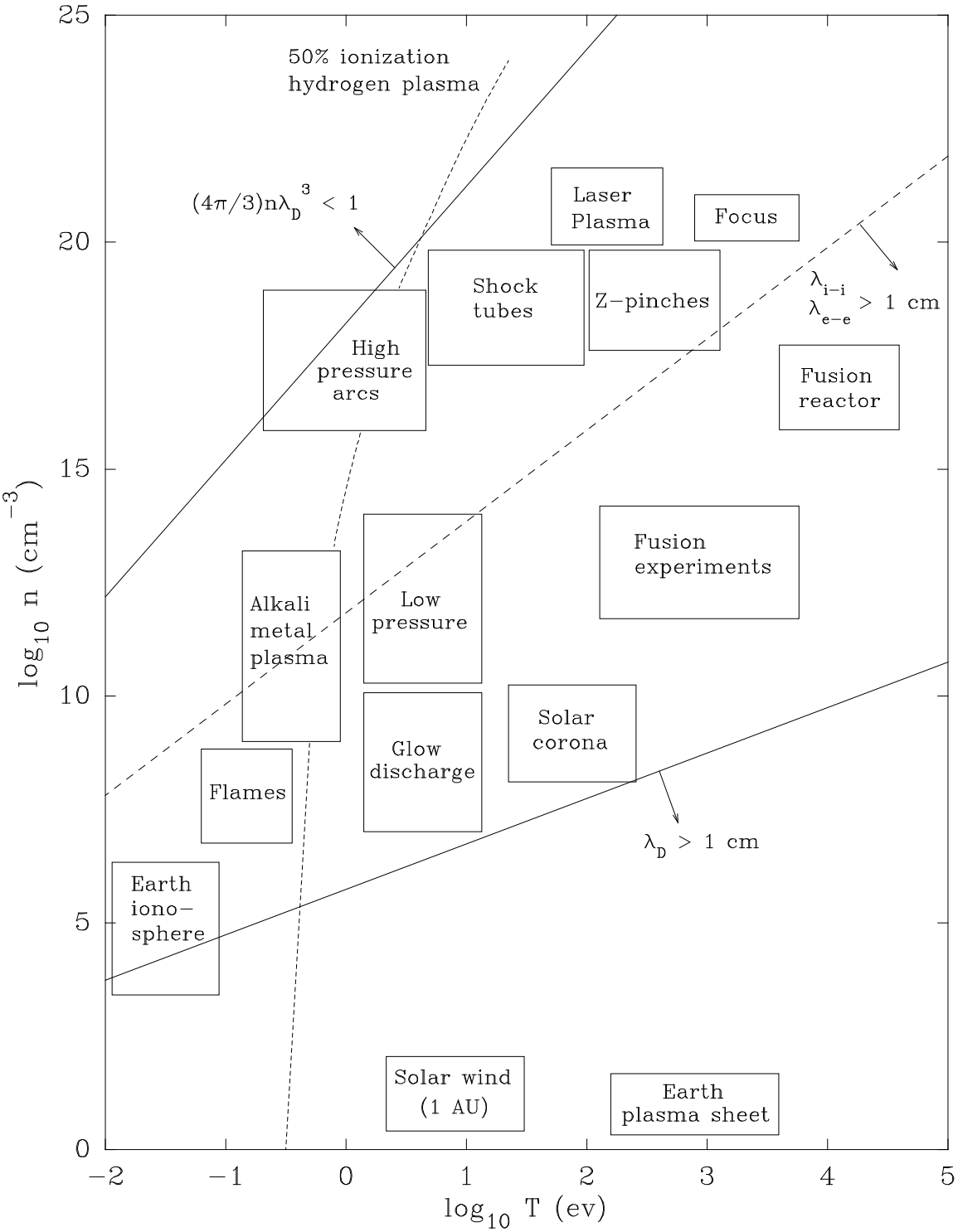

Figure 1.2: Illustration of the various regimes of plasma in terms of electron temperature and density [2].

trodes separated by a gap filled with some gas. An electron starting near the cathode will drift toward the anode. For a large enough electric field the electron will gain enough kinetic energy to ionize a neutral atom, producing a second electron. The two electrons are now accelerated by the field, instigating further ionization of the background gas. The population of electrons quickly grows, thus the process is referred to as an electron avalanche. Eventually, the avalanche electrons are collected at the anode.

In their wake are ions which slowly drift toward the cathode. As the ions impact the surface of the cathode, they occasionally cause a secondary electron to be emitted. This secondary electron initiates a new avalanche and helps to sustain the discharge. Townsend breakdown occurs when the electron multiplication in the avalanche compensates for the finite probability that an ion striking the cathode releases a secondary electron. Therefore, the time required for the ions to drift across the gap is the characteristic time it takes for a discharge to develop via the Townsend mechanism (on the order of $1\,\mu$s).

The Townsend mechanism is characterized by two parameters: α and γ, the first and second Townsend coefficients. α is the number of ionization events that occur per unit path-length. The value depends on the applied electric field, and is typically expressed in terms of the reduced electric field; the electric field divided by the neutral gas density. The second Townsend coefficient is the probability that an ion impinging on the cathode produces a secondary electron. The values for γ can vary widely and depend on the type of ion, its energy, the cathode material, contamination of the surface, and many other factors. That said, typical values

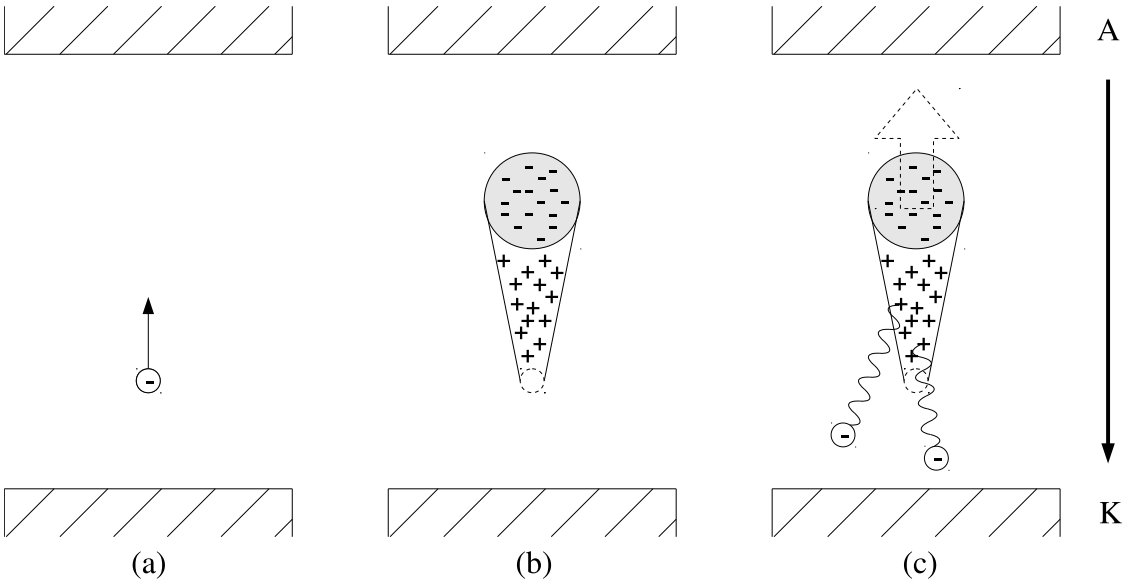

Figure 1.3: An illustration of the development of a single streamer. (a) A seed electron is accelerated by the applied electric field. (b) The initial electron develops into an avalanche which leaves a large region of positive space charge, halting further advance. (c) The streamer propagates toward the cathode via photoionization and the anode via nonlocal electrons and photoionization.

are around 0.01-0.1.

Streamer Mechanism

In contrast, the streamer discharge does not require secondary emission processes. Additionally, streamer discharges can develop in time periods as short as 1 ns, much less than the time required for Townsend breakdown. In order to describe the streamer mechanism, again consider an electron between two electrodes, as seen in (a) of figure 1.3. The electron acquires energy from the applied electric field, and begins to create an avalanche of electrons. The electrons in the

avalanche travel with a velocity characterized by the electron mobility, μ. This term expresses the frictional force exerted by the gas on the electrons. Like the first Townsend coefficient, it is a function of the reduced electric field. Consequently, the mean velocity of electrons drifting in a time-varying field $E(t)$ can be expressed as

$$u(t) = \mu(E/N_g)E(t), \tag{1.7}$$

where N_g is the neutral gas density. Given this definition of the electron drift velocity, the total length of the avalanche may be written as

$$\xi = \int_{t_0}^{t} u(t)dt. \tag{1.8}$$

Here, t_0 is the time at which $E(t)$ becomes high enough that the first Townsend coefficient, α, exceeds 0. This is considered the beginning of the avalanche as there is no multiplication before this time.

As seen in figure 1.3, the ions generated by the ionization events remain in place as the electron avalanche passes. This follows from the much larger mass of the ions, and the relatively short time scale on which the avalanche occurs. The cone-like structure of the ions results from the collision-induced, transverse diffusion of the electrons. The free diffusion coefficient is $D = \lambda v_m athrmth(E/N_g)/3$, where λ is the mean free path of the electrons, and v_{th} is their thermal velocity. Following Levatter and Lin [3], if the electrons are assumed to diffuse equally in all directions, the electron density in the head of the avalanche is

$$n_e(r) = \frac{N_e}{\pi^{3/2}R(t)^2} \exp\left(-\frac{r^2}{R(t)^2}\right), \tag{1.9}$$

where r is the radial coordinate (relative to the center of the avalanche), N_e is the absolute electron population, $\Delta t = t - t_0$, and R is the diffusion radius. The diffusion radius can be calculated from

$$R(t) = \int_0^{\xi} \lambda v_{\mathrm{th}}(\xi')d\xi', \tag{1.10}$$

as the thermal velocity is expected to change as more energy is deposited into the electrons over the length of the avalanche. The dependence of R on time is implied from its dependence on ξ.

In a Townsend discharge, the avalanche would continue across the entire width of the gap. On the other hand, the avalanche of a streamer grows at such a rate that its space charge shields out the applied field. This stalls the head of the avalanche mid-gap as seen in part (c) of figure 1.3. It is commonly assumed that this occurs when the peak field within the head of the avalanche becomes equal to the applied field. Again, from Levatter and Lin, electric field can be found to equal,

$$E_a(r) = \frac{eN_e}{4\pi\varepsilon_0 R^2} F(r/R), \qquad \text{where} \tag{1.11}$$

$$F(r/R) = \frac{1}{R^2}\left[\mathrm{erf}(r/R) - \frac{2}{\pi^{1/2}}(r/R)\exp(-r^2/R^2)\right], \tag{1.12}$$

and erf is the error function. F is a dimenionless function which has a peak value of 0.428. The number of electrons in the avalanche, is equal to

$$N_e = \int_0^{\xi} \alpha(\xi')d\xi'. \tag{1.13}$$

Here, Levatter and Lin make a number of assumptions in order to develop an analytic and dimensionless solution for $E_{a,\max}(t) = E(t)$. However, it is possible

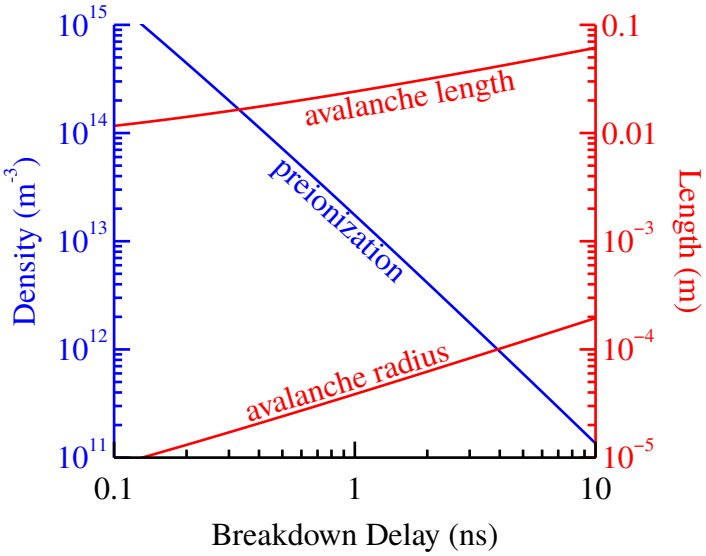

Figure 1.4: Numerical calculations of the preionization density for homogeneous excitation, avalanche length, and avalanche radius in helium at a pressure of 1.0 Torr as a function of the slope of the electric field.

to numerically integrate equations 1.8 and 1.10 over time until $E_{a,\max}$ is equal to the applied field. This should provide a more accurate, but less general result for the final radius and length of the avalanche. Assuming a linearly increasing electric field, figure 1.4 shows the results of such calculations for an avalanche in 1.0 Torr of helium, as a function of various breakdown delays. The breakdown delay is defined as the time it takes for $\alpha > 0$. The mobilities, mean velocities, and Townsend coefficients were interpolated from solutions of the Boltzmann equation provided by the BOLSIG+ code with Phelps' cross sections [4]. For this range of breakdown delays, the avalanche was able to develop up to 6 cm in length before it stalled. The times required for the avalanche to stall ranged from around

13 ns for the shortest breakdown delay, and 330 ns for the longest.

Once stalled, the avalanche can no longer continue to advance toward either electrode. At this point the avalanche can be considered a streamer as it begins to increase its extent by several other processes. The large internal fields of the avalanche can accelerate individual electrons and "inject" them in the direction of the anode [5]. In addition, as the excited atoms in the wake of the avalanche begin to radiate, they can cause photoionization throughout the volume. Photoelectrons generated close enough to the negative head, or positive tail of the streamer will initiate secondary avalanches which eventually connect to the primary one. Both of these methods take place primarily along the axis of the original avalanche, thus the streamer remains relatively constricted in the direction transverse to the electric field.

However, these processes are not critical in the formation of a large-volume discharge by an RPND. The previous description of a streamer only considers an avalanche generated by a single electron. In reality, many can form simultaneously assuming that there is more than one seed electron in the volume. With sufficient pre-ionization of the volume, the strong fields of the individual avalanches can begin to overlap. This smoothes out the field gradients which result in constricted streamers and leads to a relatively homogeneous breakdown.

This field overlap is roughly equivalent to a spatial overlap of the heads of the avalanches. In this case, the condition for homogeneous breakdown is simply $n_{e,c} > r_c^{-3}$, where $n_{e,c}$ is the preionization density, and r_c is the radius of the stalled avalanche. A plot of this condition with the breakdown time can also be

observed in figure 1.4.

It is apparent that as the avalanche radius increases, the necessary pre-ionization decreases. It is less obvious as to why the avalanche radius decreases as the breakdown delay increases. This can be explained by the lower slope, implied by the long breakdown delays. As the slope decreases, the avalanche has a longer period of time to diffuse in the system, thus the increased radius.

1.4 Atomic Spectroscopy & Notation

Given a particular plasma, it is often desirable to measure various critical parameters. Such measurements can be broadly split into perturbing and non-perturbing measurements. Perturbing measurements, such as those made with physical probes inserted into the plasma, can alter the local or even global qualities of a plasma. In contrast, non-perturbing measurements, are passive and do not influence the plasma in any way.

A number of non-perturbing diagnostics involve the practice of plasma spectroscopy, or the interaction of light and plasma. Careful measurements of the light emitted from excited atomic states can yield electron densities and temperatures, excited state densities and temperatures, electric fields, and magnetic fields. The topic of spectroscopy is extensive and it is neither necessary nor desirable to cover it in full. Instead we will only consider what is necessary to understand the emissions from a singly-excited, multi-electron atom.

An atom is composed of a small, positively charged nucleus, orbited by nega-

tively charged electrons. The actual position of any single electron is probabilistic and described by a wavefunction; solutions of the Schrödinger equation for the atom in question. Many different wavefunctions or orbitals exist, each described by four quantum numbers:

- $n = 1, 2, \ldots$: the principal quantum number,
- $l = 0, 1, \ldots, n-1$: the orbital angular momentum,
- $m_l = -l, \ldots, l$: the magnetic quantum number, and the
- $s = \pm 1/2$: spin quantum number.

The quantum numbers are hierarchical such that each n, or shell, possesses a series of subshells, l, while each subshell possesses a number of individual states, m_l, and each state possess one of two spins. The Pauli exclusion principle forbids the electrons of an atom from possessing the same combination of quantum numbers. As a result, each subsequent shell of an atom can only contain $2(2l+1)$ electrons, after which it is considered full. The subshells are often referred to using the nomenclature $0, 1, 2, 3, \ldots = s, p, d, f, \ldots$.

As a result of their separation from the nucleus, the electrons in an atom have some degree of potential energy. As the n and l of an electron increase, so does its potential energy. In most cases, m_l and s do not affect the potential energy of an electron. As an example, an electron in the 1s ($n = 1$ and $l = 0$) subshell has the lowest possible potential energy.

Absent from external influences, the subshells are populated with electrons so as to minimize the total potential energy of the system. This natural arrangement is referred to as the ground state configuration. Often, but not always, the subshells are filled sequentially and in order from lowest to highest l. Provided some input energy, one or more of the electrons surrounding the atom may transition to another orbital, increasing the potential energy of the system. In low-temperature plasmas it usually one of the electrons from the outermost or unfilled subshell to be excited.

In hydrogen and alkali metals, all the subshells are filled with the exception of a lone electron in the outer (or valence) subshell. For these atoms, the potential energy of any singly-excited configuration is uniquely determined by this electron. As a result, the initial and final states of the atom can be uniquely identified based on the initial and final n and l of the aforementioned electron. In contrast, the potential energies of configurations in other atoms are determined by the collective effects of all outer electrons.

This necessitates an additional means of classifying the electron configurations in these atoms. It turns out that the states of these types of atoms can be specified based on their total orbital angular momentum $L = \sum l_i$, the total spin, $S = \sum s_i$, and the total angular momentum, $J = L + S$, where i are all the electrons of the valence shell. In addition, each state can be said to have either even or odd parity, defined as $(-1)^{\sum l_i}$, where -1 is odd, and 1 is even.

Together, these values are sufficient to identify the electron configuration in these atoms. They are often written out in a form called the term symbol which

lists the filled subshells and appends a symbol representing the outer subshell configuration. For example, the so called triplet metastable state of helium can be written as $1s2p^3P^o_{0,1,2}$. This describes a helium atom with one electron in the 1s subshell and a second atom in the 2p subshell. This configuration has a total orbital angular moment of 1 (denoted by the 'P'), odd parity (denoted by the superscript 'o'), a total spin of $1/2$ (the superscript 3 is equal to $2S+1$) and three possible values for the total angular momentum: 0, 1, or 2 which varies depending on the spin-orbit interaction.

These excited atomic states usually have finite lifetimes as the electrons in the excited states will often transition to lower states. This can occur spontaneously, through the emission of a photon, or via a superelastic collision with another particle. In the case of spontaneous transitions, only certain ones are allowed, as defined by a set of selection rules:

- $\Delta S = 0$
- $\Delta L = \pm 1$ or 0
- $\Delta J = \pm 1$ or 0
- $L = 0$ cannot transition to $L = 0$
- $j = 0$ cannot transition to $J = 0$

These rules are determined using the electric dipole approximation. As a result, some transitions forbidden by these rules can occur, though they occur at a much lower rate.

Figure 1.5 is a diagram of the energy levels in neutral helium and the allowed transitions. In this case, the atomic states are separated into the singlet ($S = 0$) and triplet ($S = 1$) manifolds. The singlet manifold represents excited states where the electron spins are anti-parallel, and the triplet manifold represents excited states where the electron spins are parallel. As indicated by the first selection rule, transitions between these two manifolds is forbidden, thus each is something of a self-contained system.

Spectral Lineshapes

It is the transitions between these various excited states which concern spectroscopy. Electrons which transition to lower energy states emit photons which can be detected. Conversely, if an atom is exposed to a photon with an energy matching a transition, the atom may absorb the photon. Both processes are useful in determining the prevalence and dynamics of the excited states. This, in turn, can be used to infer various plasma properties.

Conservation of energy requires that the energy of the absorbed or emitted photon match the energy difference between the two states. However, the finite lifetime of excited atomic states implies, via the time-energy formulation of the uncertainty principle, some uncertainty in the actual energy difference between the states. As a result, the emitted photon will possess an energy selected from a distribution of energies.

This distribution is referred to as the spectral lineshape. The natural lineshape

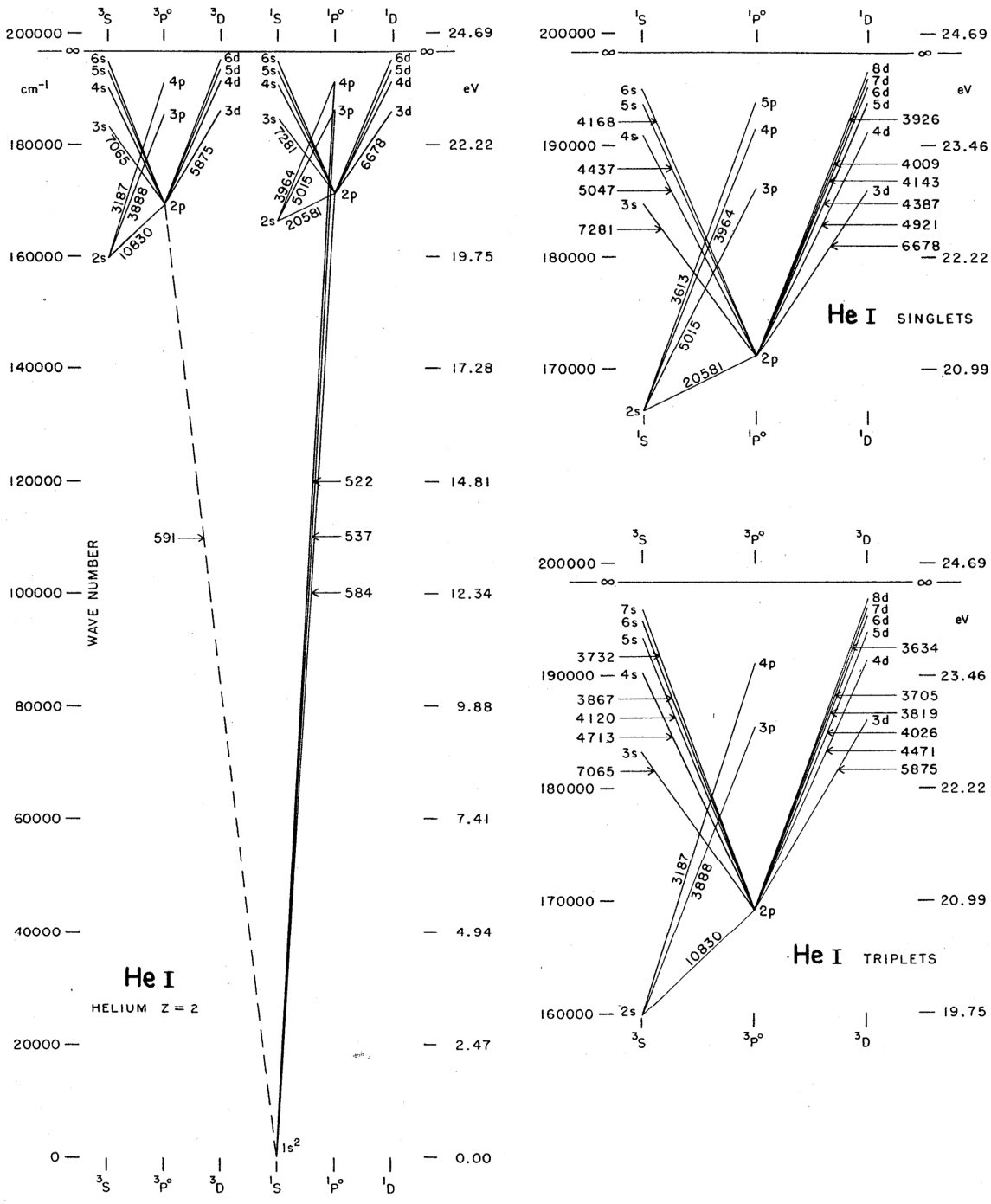

Figure 1.5: A partial Grotrian diagram of neutral helium [6].

of an atomic transition can be shown [7] to be a Lorentzian of the form,

$$g(\omega) = -\frac{1}{4\pi^2}\frac{A\lambda^3}{\Delta\omega_a}\frac{1}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2}, \tag{1.14}$$

where ω is the photon frequency, A is the Einstein coefficient for the transition, λ is the wavelength of the transition, ω_a is central frequency of the transition, and $\Delta\omega_a$ the full-width half maximum (FWHM) of the transition. In the ideal case, where the atoms motionless and unaffected by external perturbations, $\Delta\omega_a = A$. This is known as the natural linewidth.

Other processes can affect the spectral lineshape. For example, inter-atomic collisions can reduce the lifetimes of excited states. This results in additional broadening of the line, though it retains its Loretnzian nature. As the frequency of inter-atomic collisions increases linearly with pressure, this phenomena is referred to as pressure broadening. It can be included in equation 1.14 by using $\Delta\omega_a = A + BP$, where B is a measured or calculated broadening coefficient, and P is the pressure.

Atomic motion can also play a role in the spectral lineshape. If an atom is moving toward or away an observer as it emits a photon, the emitted photon will be blue or red shifted. If this effect is averaged over the random motion of atoms in a gas, the result is an additional broadening of the lineshape, called Doppler broadening. Unlike pressure broadening, Doppler broadening introduces a Gaus-

sian component to the lineshape such that,

$$g(\omega) = \sqrt{\frac{2\ln 2}{\pi^3}}\frac{\Delta\omega_a}{\Delta\omega_d}\int_{-\infty}^{\infty}\frac{1}{[(\omega-\omega_a)-\omega']^2+4\Delta\omega_a^2} \times \exp\left[4\ln 2\left(\frac{\omega'}{\Delta\omega_d}\right)^2\right]d\omega'. \quad (1.15)$$

Here, $\Delta\omega_d = \omega_a\sqrt{\frac{8k_B T_g \ln 2}{Mc^2}}$, is the width of the Doppler broadening. This form of the spectral lineshape is known as the Voigt profile, and it must be numerically integrated. In the case that $\Delta\omega_d >> \Delta\omega_a$, equation 1.15 can be simplified to a standard Gaussian distribution,

$$g(\omega) = \sqrt{\frac{4\log 2}{\pi\Delta\omega_d^2}}\exp\left[-(4\log 2)\left(\frac{\omega-\omega_a}{\Delta\omega_d}\right)^2\right]. \quad (1.16)$$

The effect of the various broadening mechanisms is most apparent in the wings of the lineshape, far from the peak. Figure 1.6 illustrates the three major lineshapes with equivalent full widths. The Voigt profile is composed of equally broad Lorentzian and Gaussian distributions. As can be seen, the wings of the Gaussian distribution fall off very quickly. In comparison, the Lorentzian component is observable well out to the edges of the figure.

The spectral lineshape can be altered by a number of other processes. Electric fields can influence the emissions via the Stark effect, while magnetic fields can split up degenerate states via the Zeeman effect. The fields of electrons and nearby molecules can also alter the lineshape of a transition. While not used in this study, such effects can be used as effective diagnostic tools for plasmas.

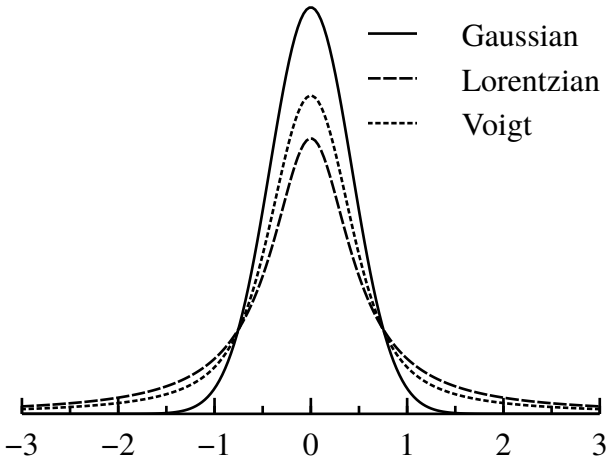

Figure 1.6: A comparison of the three primary spectral lineshapes, each with the same full width.

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