Spectroscopic Investigation of a Repetitively-Pulsed Nanosecond Discharge

by

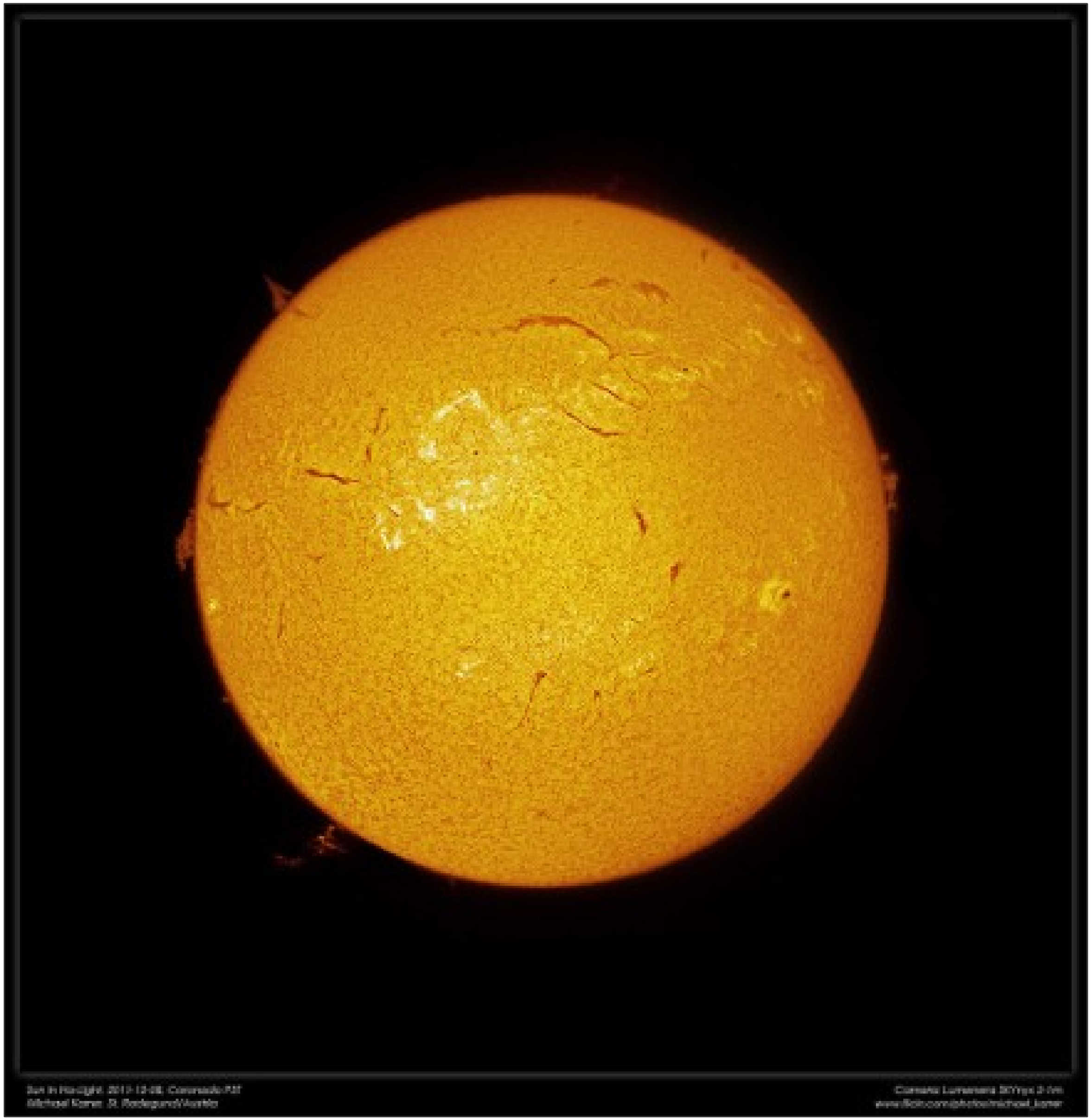
Benjamin T. Yee

Chair: John E. Foster

UNIVERSITY OF MICHIGAN

- Introduction
- Pulsed plasmas and outstanding questions
- Metastable measurements
- Global model
- Emission measurements
- Conclusions

Introduction



Karrer, M. "Sun 2011-12-08." URL: http://www.flickr.com/photos/michael_karrer/6487199145/, Accessed: August 26, 2013.

What Is a Plasma? (Sun)

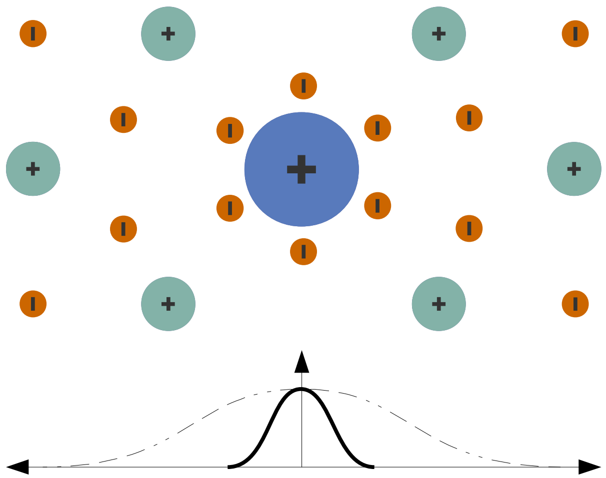
Hunt, P. "Lightning." URL: http://www.flickr.com/photos/michael_karrer/6487199145/, Accessed: Aug. 26, 2013.

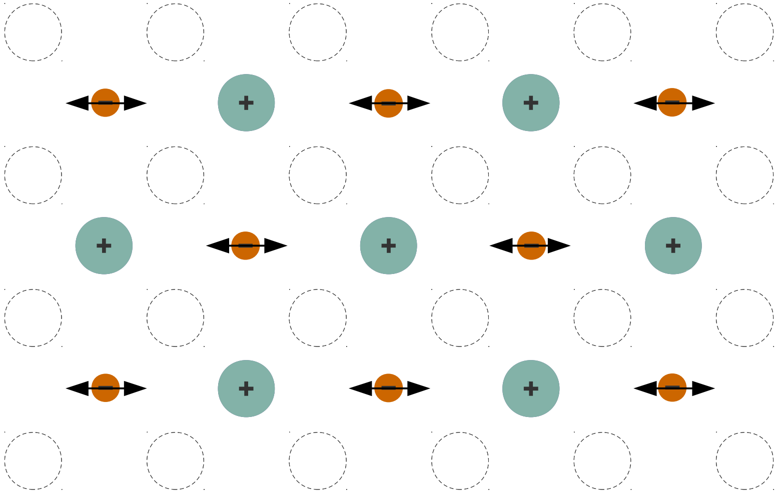
What Is a Plasma? (Lighting)

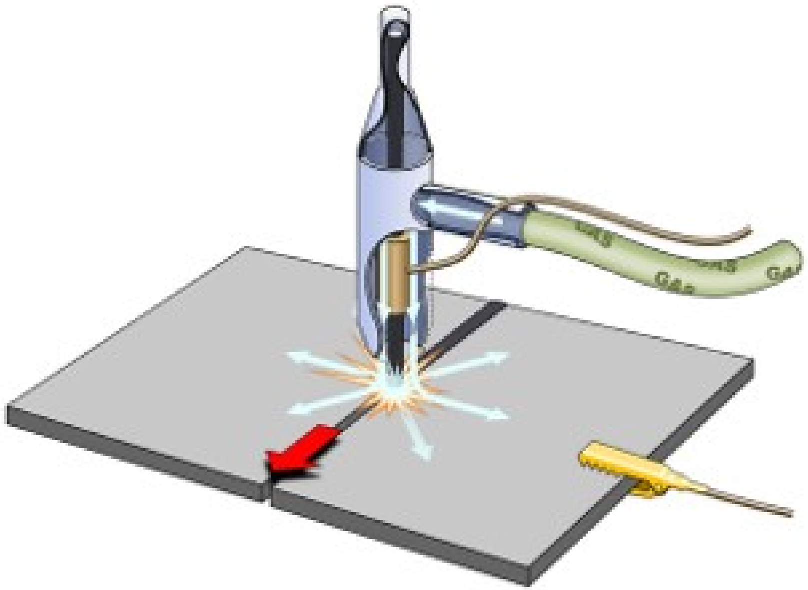
NASA, ESA, and the Hubble Heritage Team. "Star-Forming Region LH 95 in the Large Magellanic Cloud." URL: http://hubblesite.org/gallery/album/entire/pr2006055a/, Accessed: August 27, 2013.

What Is a Plasma? (Interstellar Gases)

- Ionized gas
  - Neutral particles
  - Positive (and negative) ions
  - Electrons
- Exhibits large-scale electrostatic effects
  - Debye shielding
  - Electron oscillations

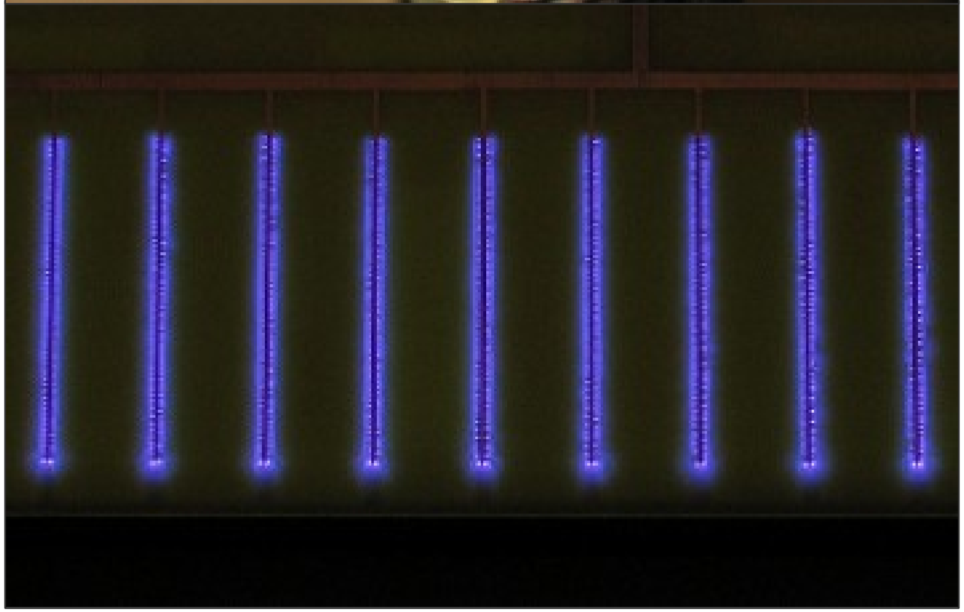






Bowden, M. “A fluorescent light bulb.” URL: http://www.sxc.hu/photo/203835, Accessed: August 26, 21013.
Shigeru23. “Gas arc welding (TIG & MIG).” URL: https://commons.wikimedia.org/wiki/File:Gas_arc_welding_%28TIG_%26_MIG%29.PNG, Accessed: August 27, 2013.
Schmid, M. “Wasabi-Chips (Migros).” URL: https://commons.wikimedia.org/wiki/File:Wasabi-Chips_%28Migros%29.JPG, Accessed: August 27, 2013.
Volant, L. “LG LV3550 15 R.” URL: http://www.flickr.com/photos/27048731@N03/5589223946/, Accessed: August 27, 2013.

Applications



- Modification or enhancement of fuel combustion properties
- Modification of airflow over surfaces without mechanical actuators

Wong, N. “Tip of a spark plug.” URL: http://www.flickr.com/photos/14029705@N00/375024452/, Accessed: August 27, 2013.
Xunger, “Plasma glow discharge.” URL: https://commons.wikimedia.org/wiki/File:Streamwise.JPG, Accessed: August 27, 2013.

- Ionization of gas flow for MHD generators and energy bypasses

- Sterilization of bacterial, viral, and chemical contaminants (e.g. MRSA or anthrax)

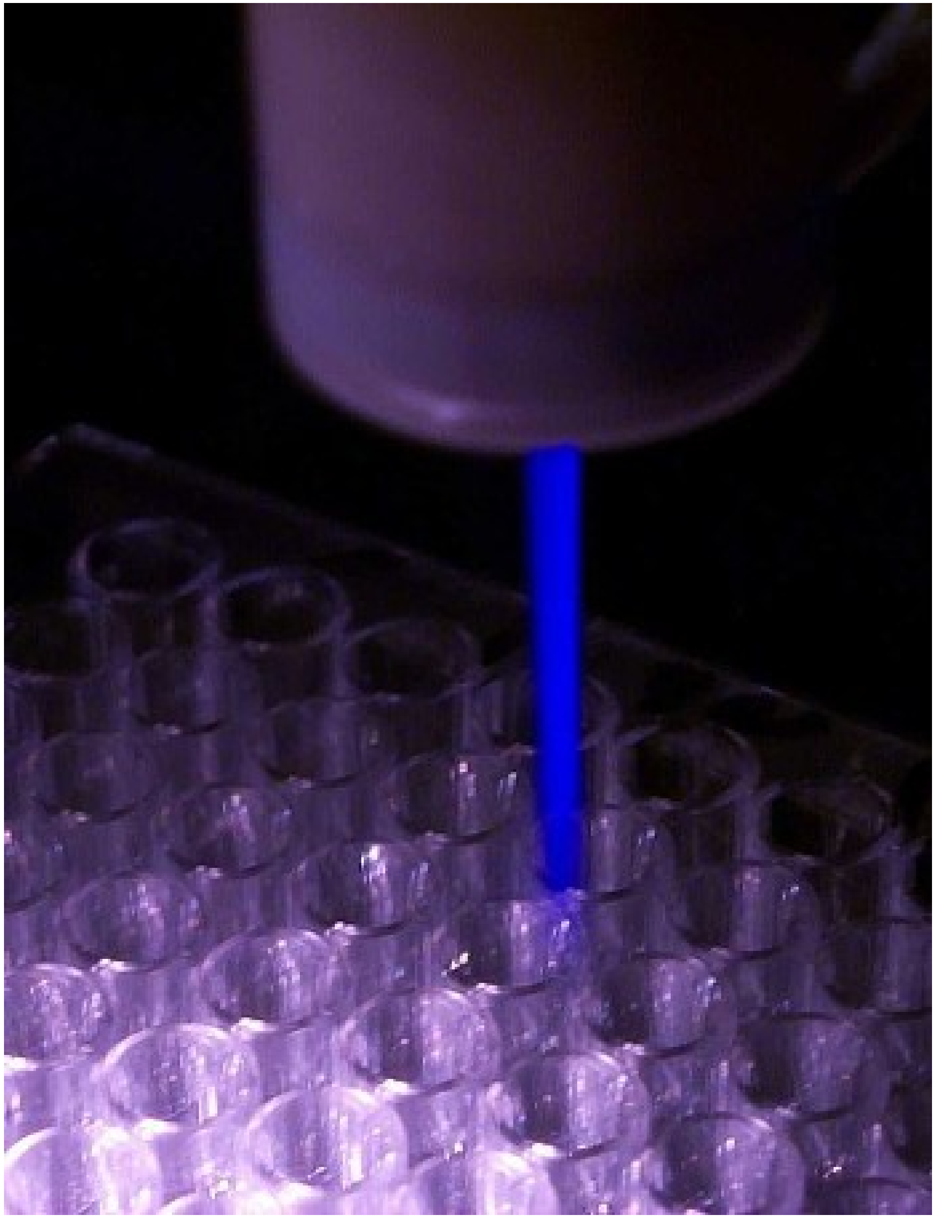
Bellay, C. “X-51A Waverider on B-52 2009.” URL: https://commons.wikimedia.org/wiki/File:X-51A_Waverider_on_B-52_2009.jpg, Accessed: August 27, 2013.
parfe_alp, “Operation.” URL: http://www.flickr.com/photos/parfe/2239523465/, Accessed: August 27, 2013.

- Common in welding, cutting, and arc lighting
- All particle species at approximately the same temperature
  - Electrons
  - Ions
  - Neutrals
- Often several thousand degrees Celsius

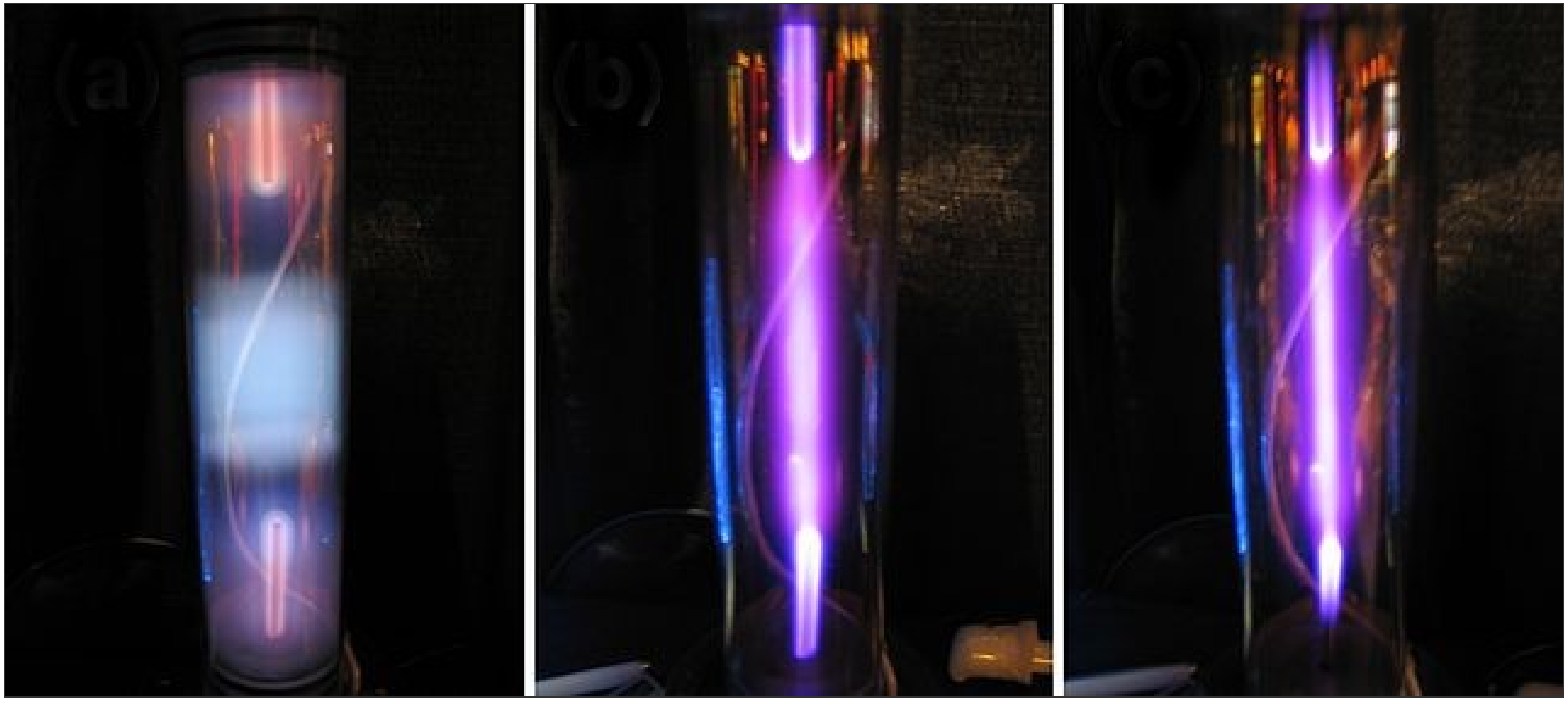
D-Kuru/Wikimedia Commons. "Afterglowing electrodes of an arc lamp." URL: https://commons.wikimedia.org/wiki/File:Arc_lamp-afterglow_2_PNr%C2%B00038.jpg, Accessed: August 28, 2013.

Equilibrium Plasmas

- Common in fluorescent lighting and material processing.
- Different temperatures for each species
  - Often, $T_{neutrals} \sim T_{ions} < T_{electrons}$
- Desirable for many high-pressure applications; high gas temperatures can cause damage



Entropi5. "A low temperature plasma jet: Plasma Pencil." URL: https://commons.wikimedia.org/wiki/File:Plasma_Pencil.jpg, Accessed: August 28, 2013.

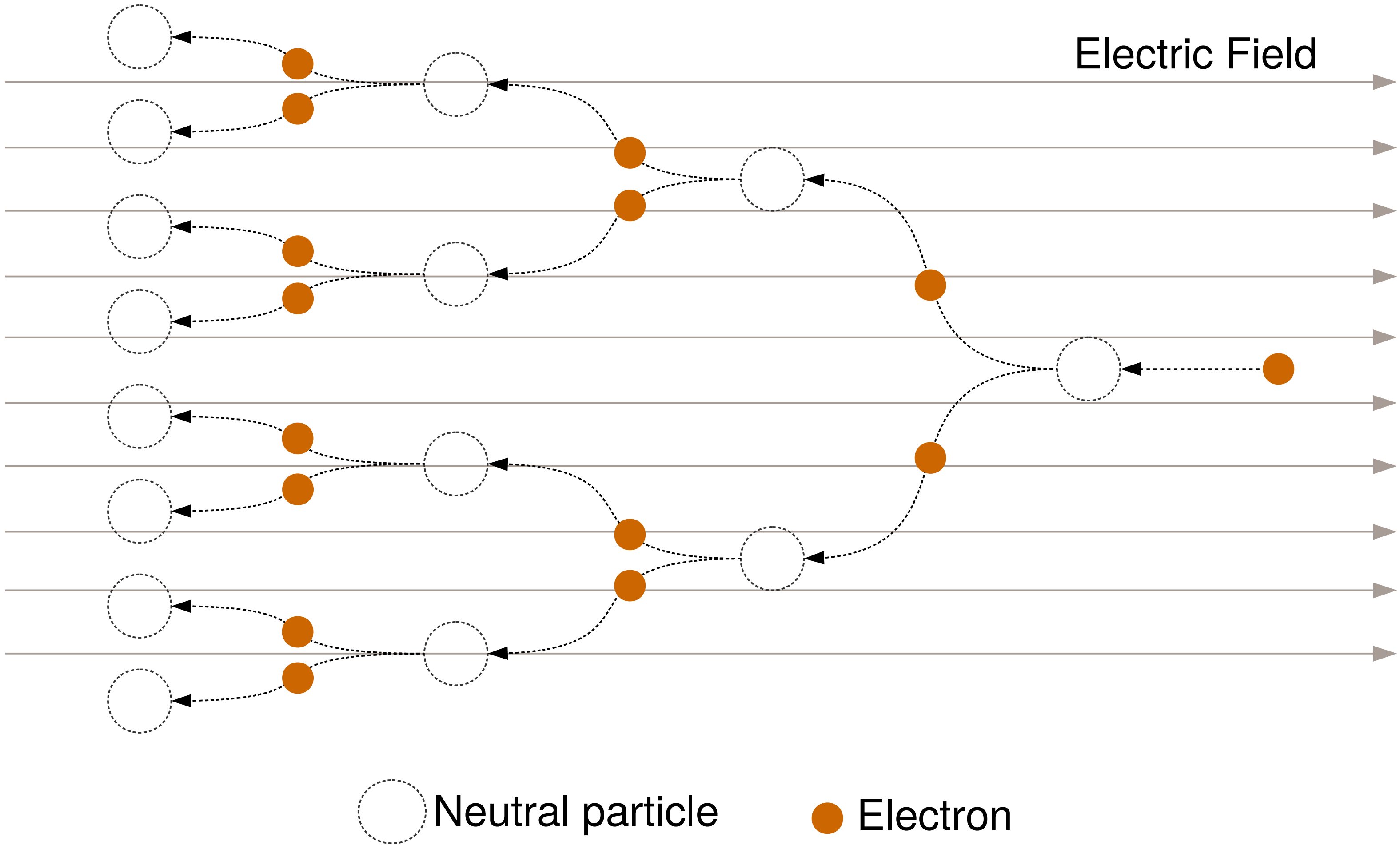


- Increase in pressure $\rightarrow$ tendency to equilibrate
- Related to ionization instability
- Prevented by careful control of power input
  - Charging of dielectric barriers
  - Pulsed discharges

Wikigan, "Transition from a glow discharge in Argon, to an arc." URL: https://commons.wikimedia.org/wiki/File:Glow2arc.jpg, Accessed: August 28, 2013.

Glow-to-Arc Transition

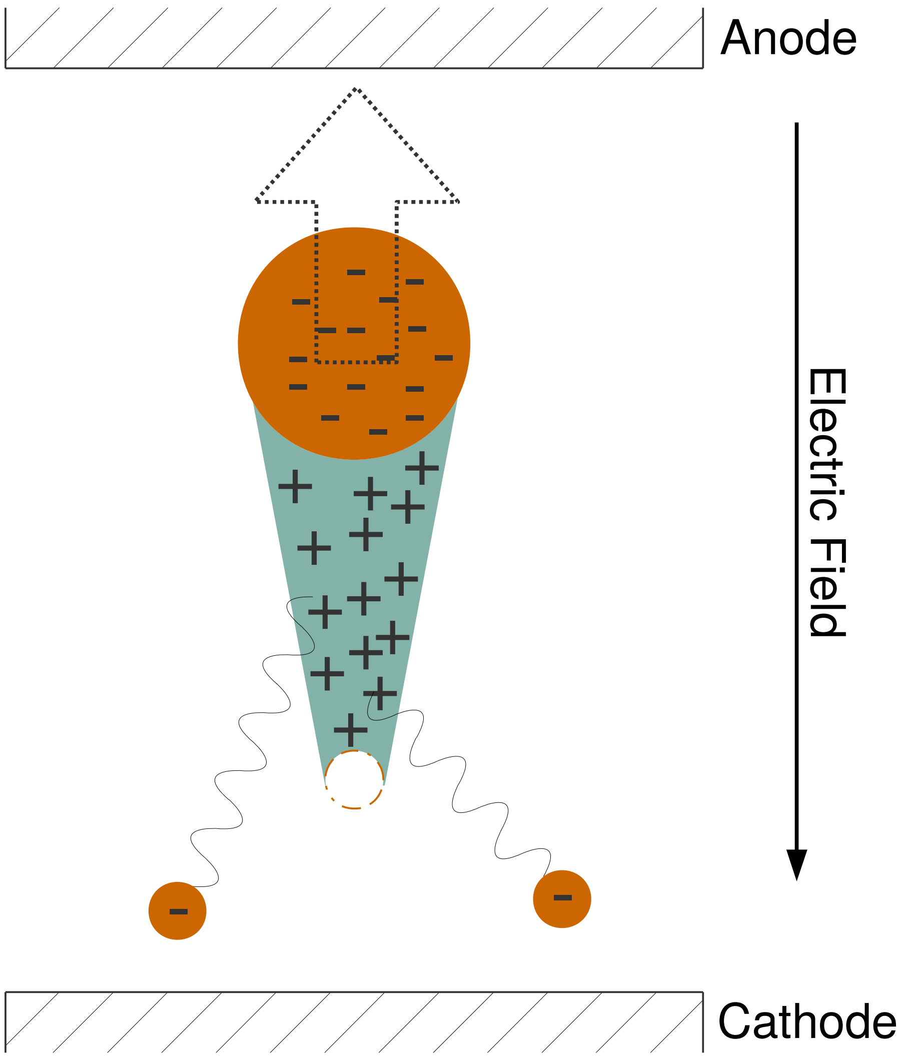
Pulsed Plasmas and Outstanding Questions

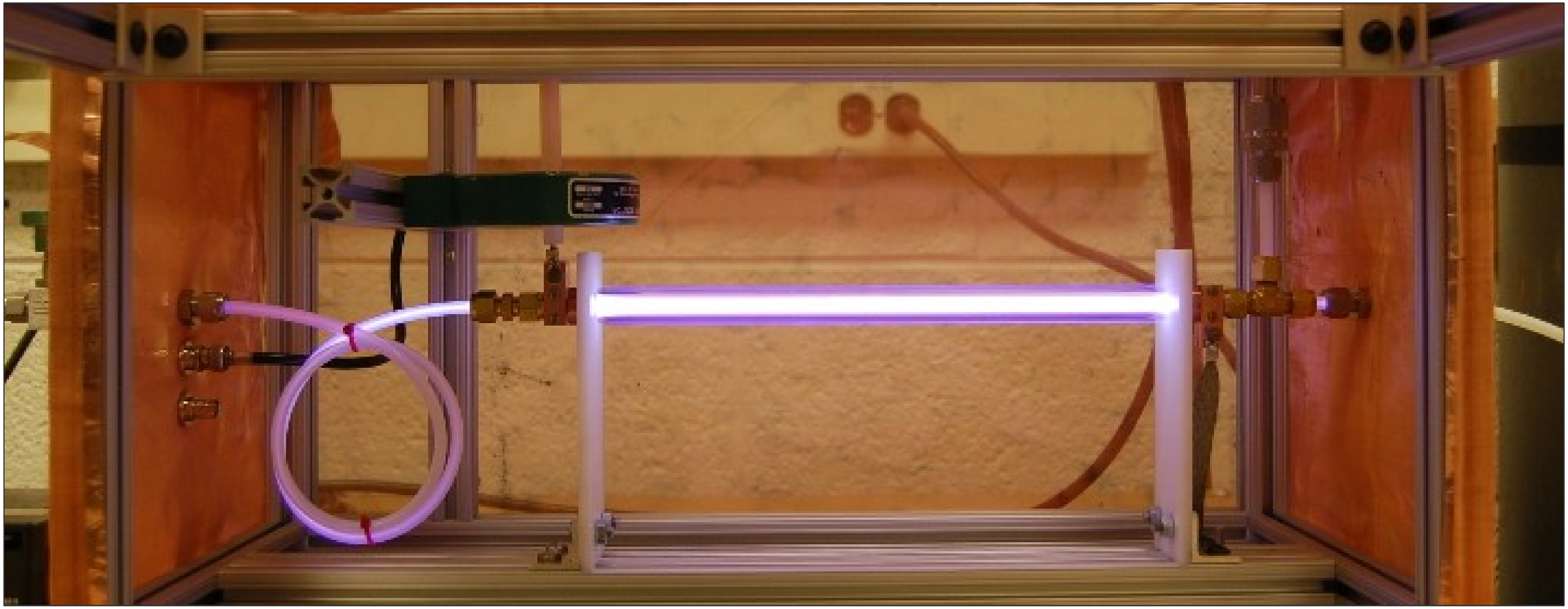


- Most pulsed plasmas initiated by electron avalanche
- Initial electron from cosmic rays, UV light, previous pulse, ...
- Sufficient electric field → exponential growth

Electron Avalanche

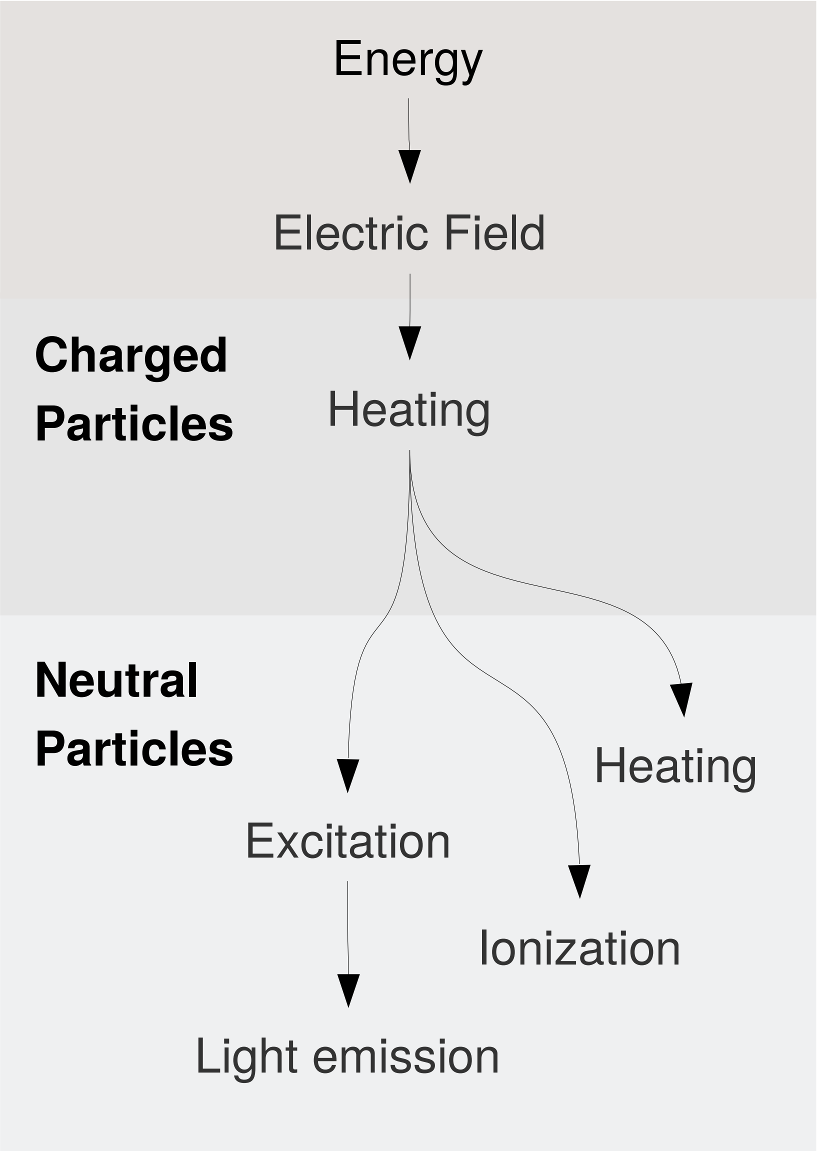
- In certain cases (Raether-Meek criterion), avalanche becomes streamer discharge
- Internal field of avalanche comparable to applied field
- Reduces energy transfer to streamer
- “Injected” electrons and photoionization can become important





- Repetitively pulsed nanosecond discharge (RPND)
  - Development involves significant space charge, similar to streamer
- Attractive for high-pressure applications
  - Uniform over large volumes (on the order of liters)
  - Little gas heating
- Pulse characteristics
  - Pulse-widths: 1 – 100 ns
  - Voltages: 1 – 100+ kV
  - Repetition rates: 1 – 100 kHz

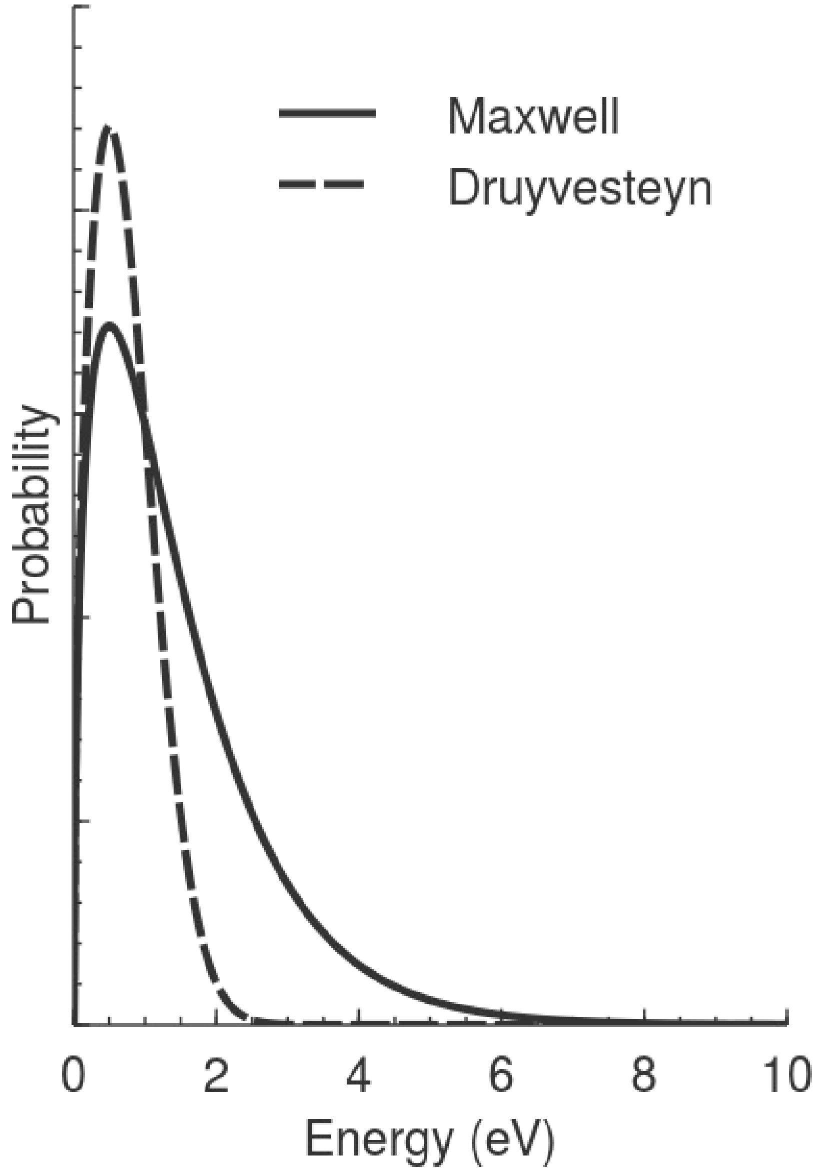
Repetitively-Pulsed Nanosecond Discharge

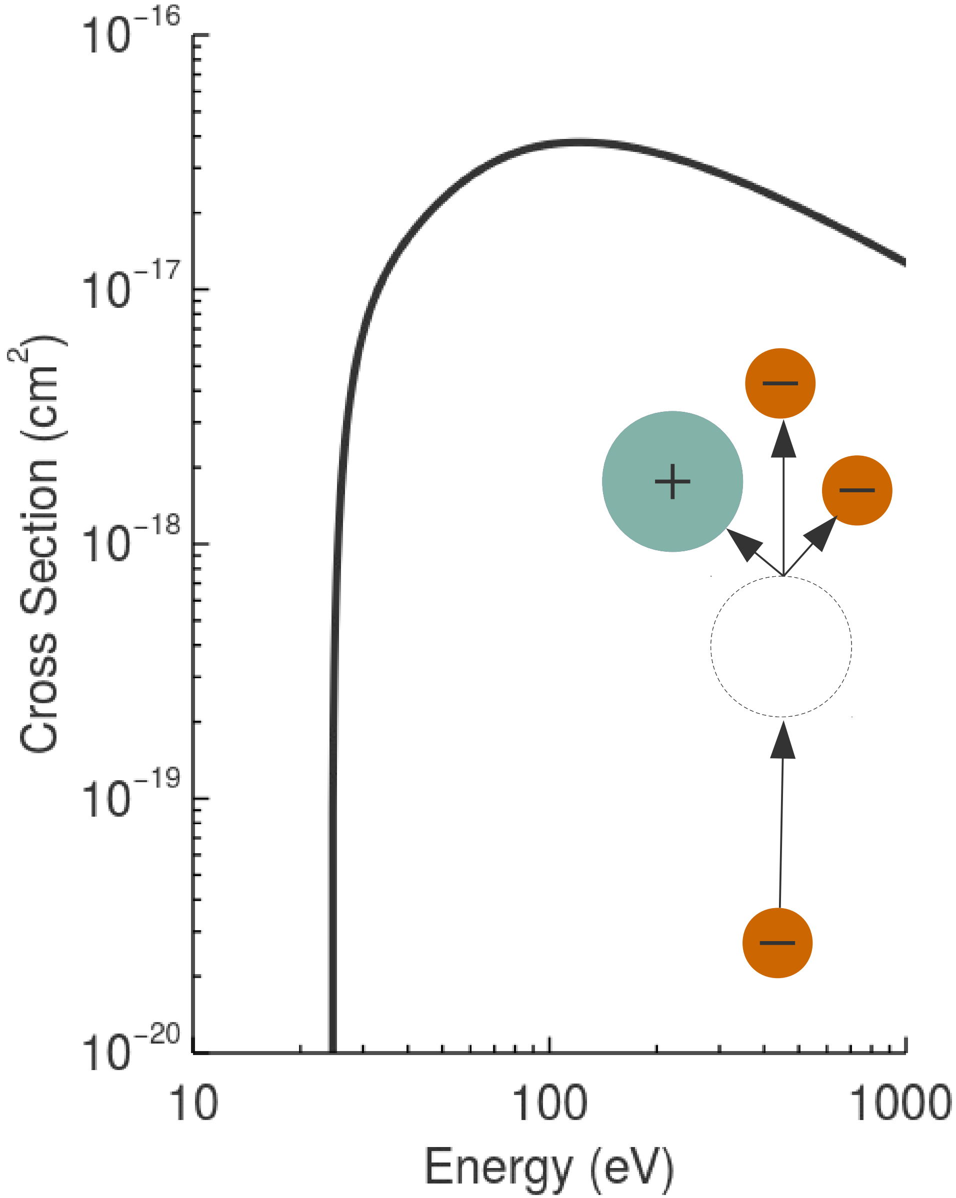


- Energy added to charged particles via electric field
- Some energy heats electrons
- Remainder enters neutral particles through collisions
- How is energy divided?
  - Important as neutral particle products are often desirable for applications

Questions: Energy Coupling

- An EEDF is a continuous probability distribution for electron energies
- With assumptions can be obtained
  - Analytically
  - Numerically
- However, assumptions may not be valid for RPND

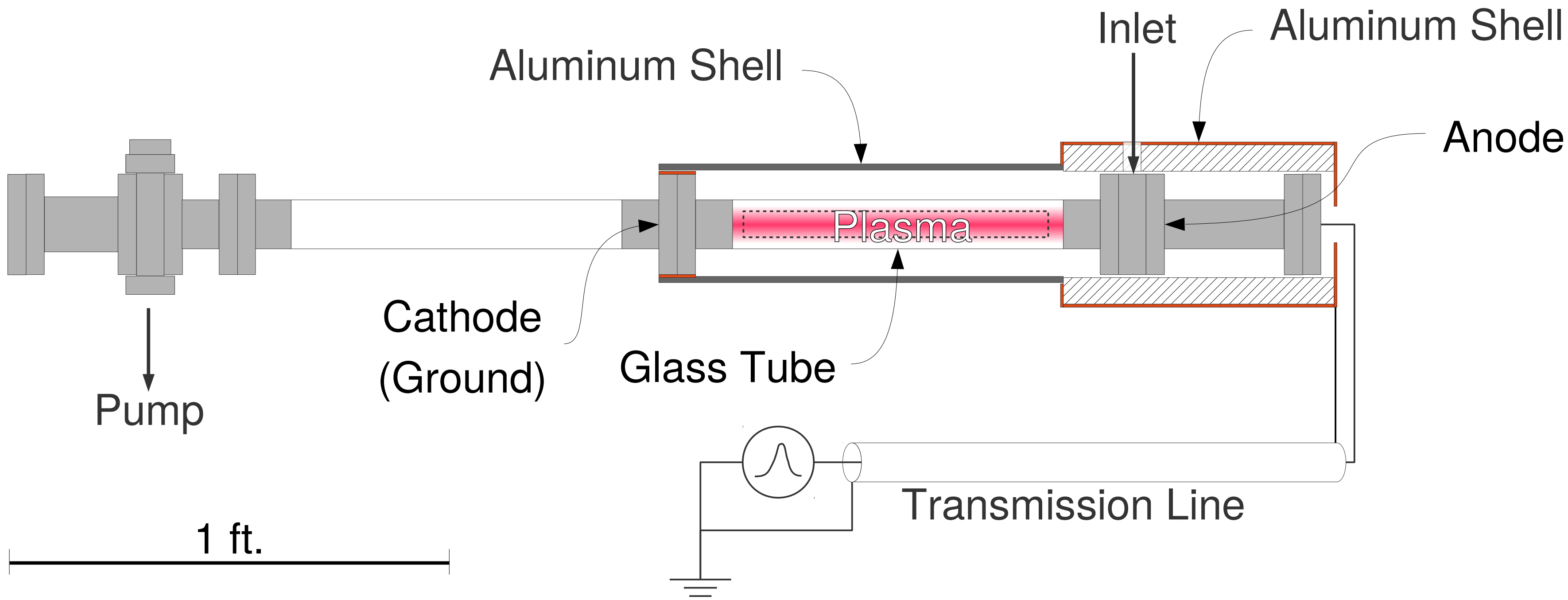




- Reactions in plasma determined by cross section and EEDF
- Cross section expresses probability of specific reactions
  - Excitation
  - Ionization
  - Dissociation
  - ...
- Must know EEDF to determine reaction rates

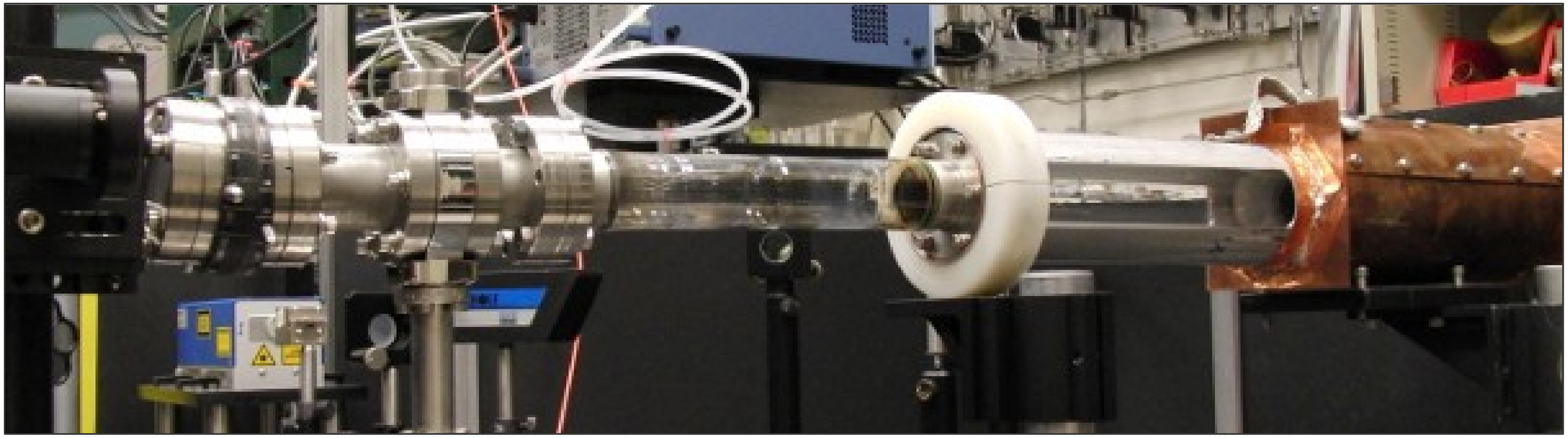
Questions: Electron Energy Distribution

Experimental Setup

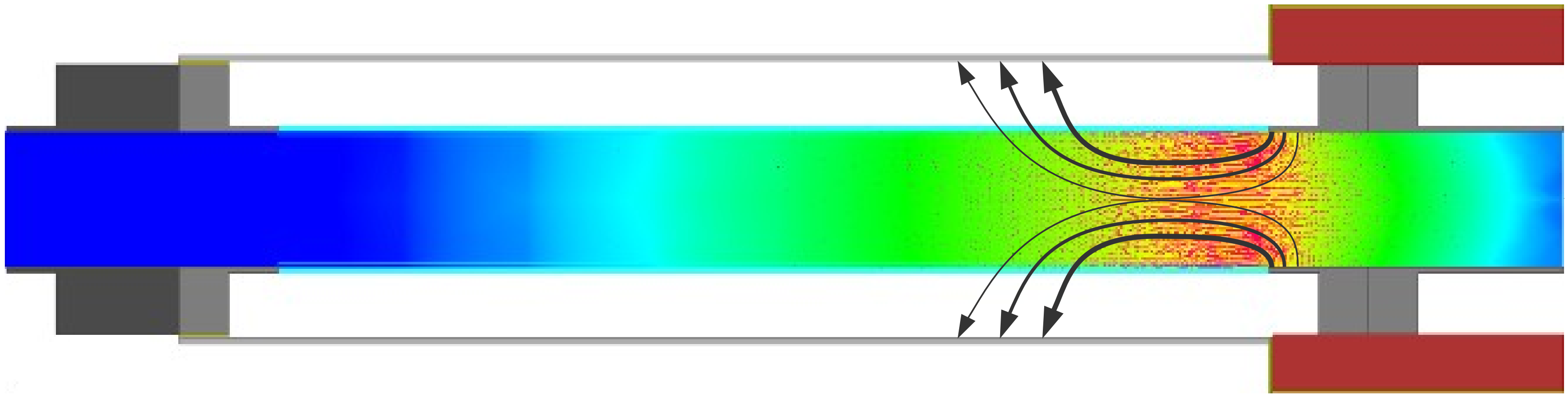


Coaxial-type geometry

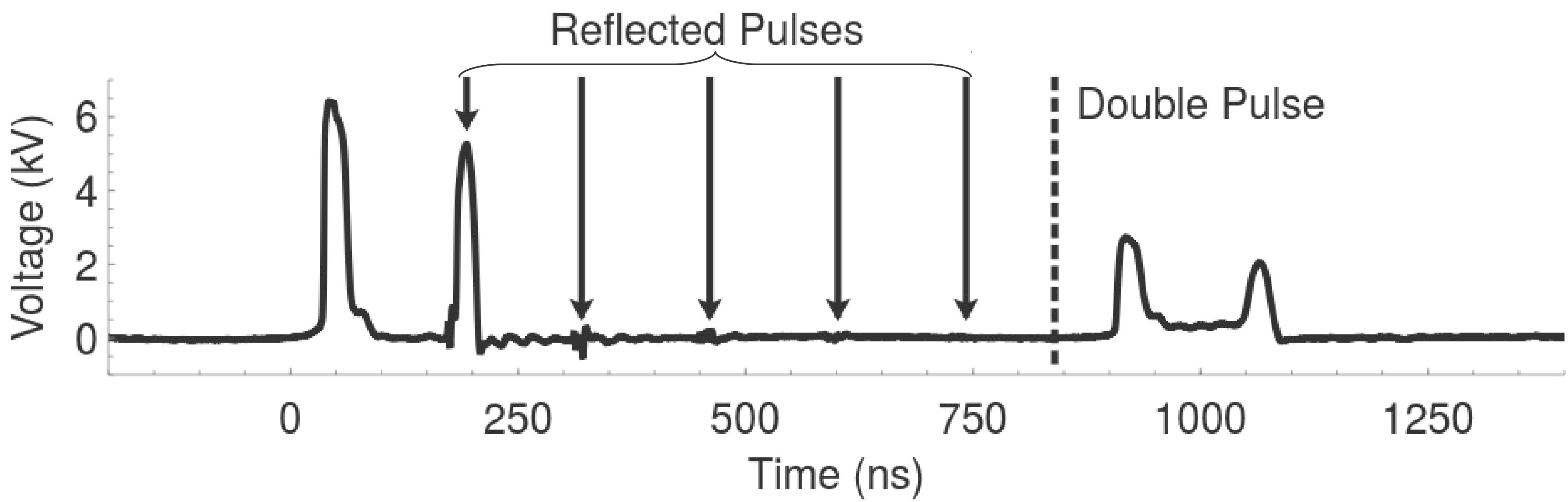
- Inner conductor: plasma
- Dielectric: borosilicate vacuum tube, air, and teflon
- Outer conductor: copper and aluminum shells



- Ultra-high purity helium: 0.3 – 16.0 Torr
- Rare gas discharges → interesting properties
  - Minimal gas heating
  - UV emissions
  - Data available to create detailed population kinetics model
- Voltage pulses: 25 ns FWHM, +6.4 kV, 1.0 kHz



- Peak field: 3.8 kV / cm
- Non-uniform field, concentrated near anode-glass interface
- Notable radial component a result of outer conductor



- Impedance mismatch causes numerous reflections between power supply and anode
  - Additional opportunity for energy absorption
- 13 m transmission line used to isolate incident and reflected pulses in time
- Double-pulsing by power supply observed after 800 ns

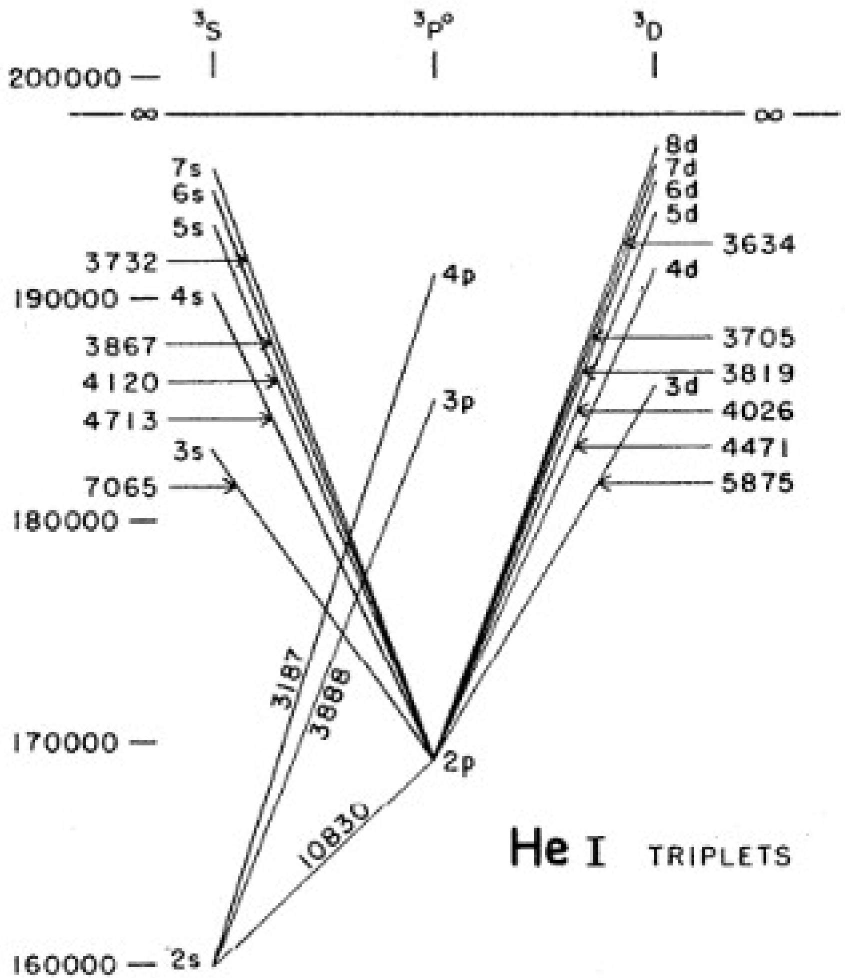
Input Waveform

Metastable Measurements

- Physical probes
  - Perturb plasma
  - Not fast enough (fastest Langmuir probes: 11 MHz)
  - Suggests an <u>optical</u> approach
- Electrons
  - No detectable emissions (i.e. bremsstrahlung or synchrotron)
  - Insufficient density for laser scattering
  - Suggests <u>atomic</u> measurements

→ <u>Optical measurements of atomic states</u>

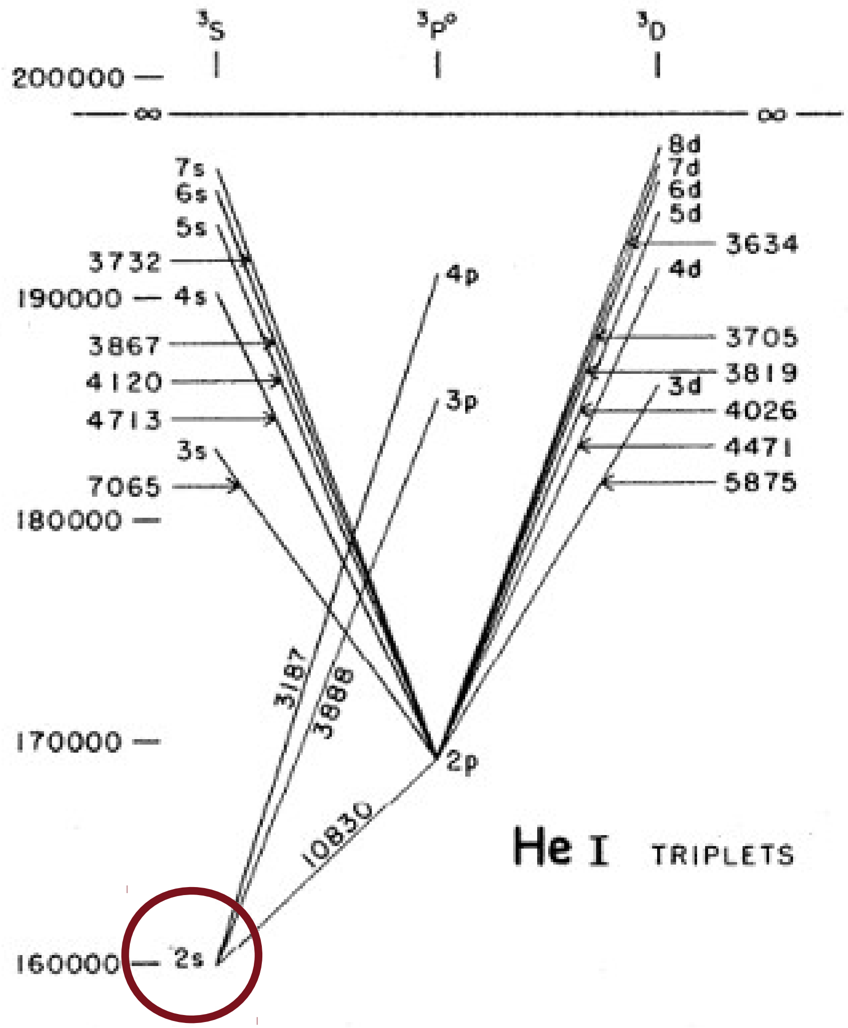
What Can Be Measured?



- Atomic states can absorb light with specific wavelengths
  - Light must be close to transition energy with another state
- Consider volume of gas with a mixture of states
  - Light incident on volume and matching specific transition would be absorbed
  - Absorption proportional to density of initial atomic state
- Higher temporal resolution than Langmuir prove, photodiodes available with 1+ GHz bandwidth

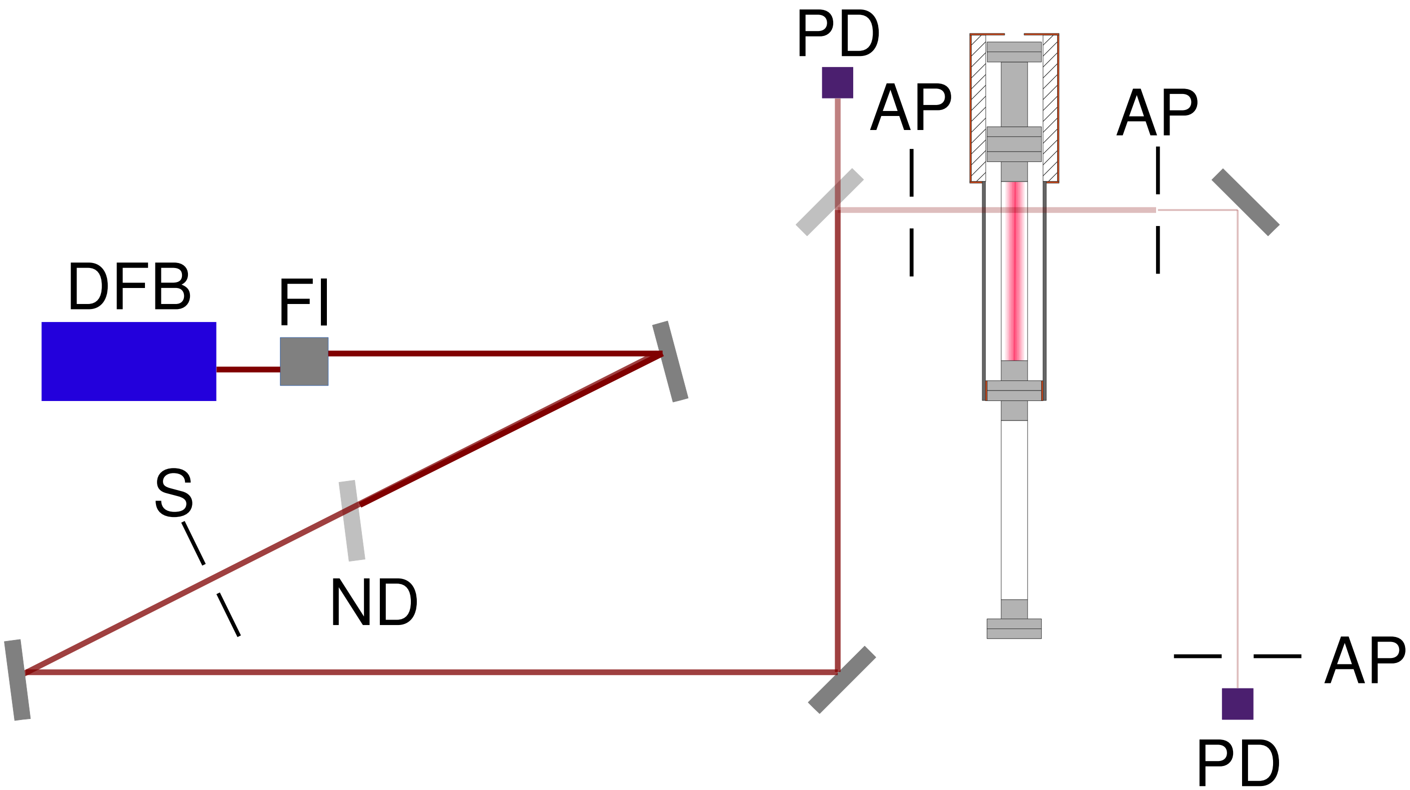
Moore, C. and Merrill, P. "Partial Grotrian Diagrams of Astrophysical Interest." National Bureau of Standards. Washington, D.C. (1968).

Absorption Spectroscopy



- Triplet metastable chosen for study
- Lowest-energy excited state
- Cannot spontaneously transition to ground state (spin-forbidden) → long-lived
- Reservoir for energy
- Can cause excitation and ionization long after applied voltage pulse

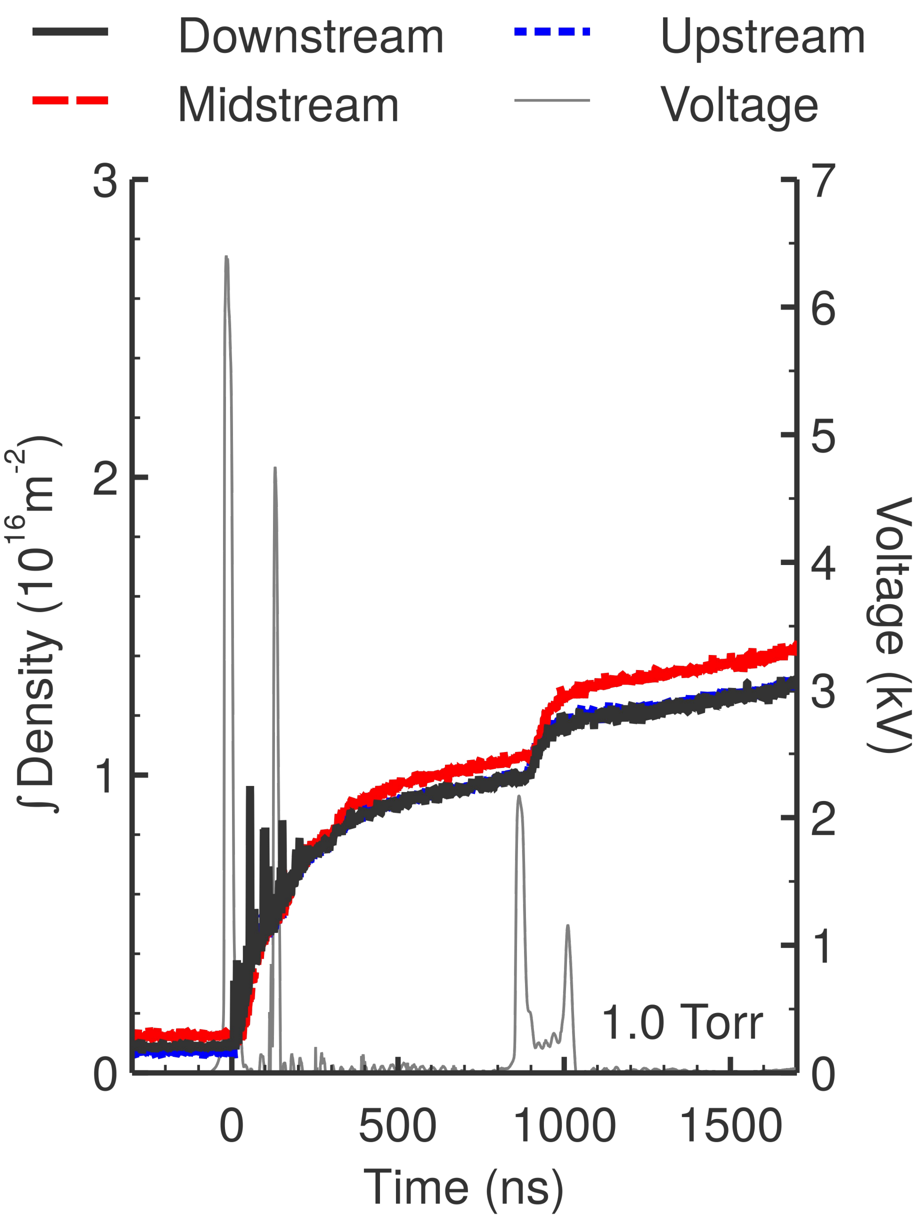
Moore, C. and Merrill, P. "Partial Grotrian Diagrams of Astrophysical Interest." National Bureau of Standards. Washington, D.C. (1968).



- $2^3S \rightarrow 2^3P_0$ transition (1083 nm), across diameter of discharge
- Pressures: 0.3, 0.5, 1.0, 2.0, 3.0, 4.0, 8.0, and 16.0 Torr
  - Covered transition between low and high pressure behavior for metastable losses
- Locations: 5.1 cm (Upstream), 12.7 cm (Midstream), and 20.3 cm (Downstream)
  - Allowed observation of axial variations in plasma formation

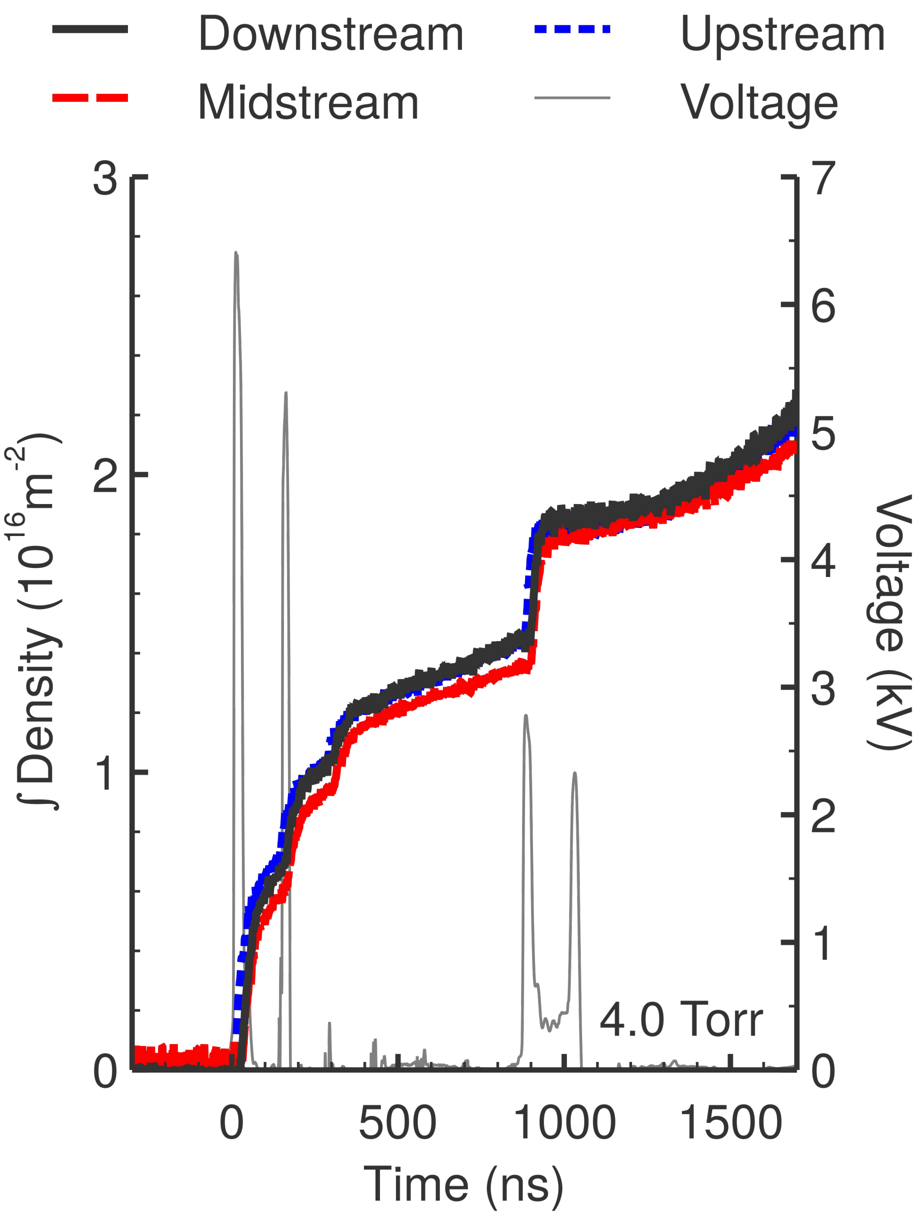
Laser-Absorption Spectroscopy Setup

- Line-integrated densities (will not assume radial distribution)
- Observed persistent metastable population between pulses
- Electrical noise present in measurements near anode
- No significant changes with location relative to anode



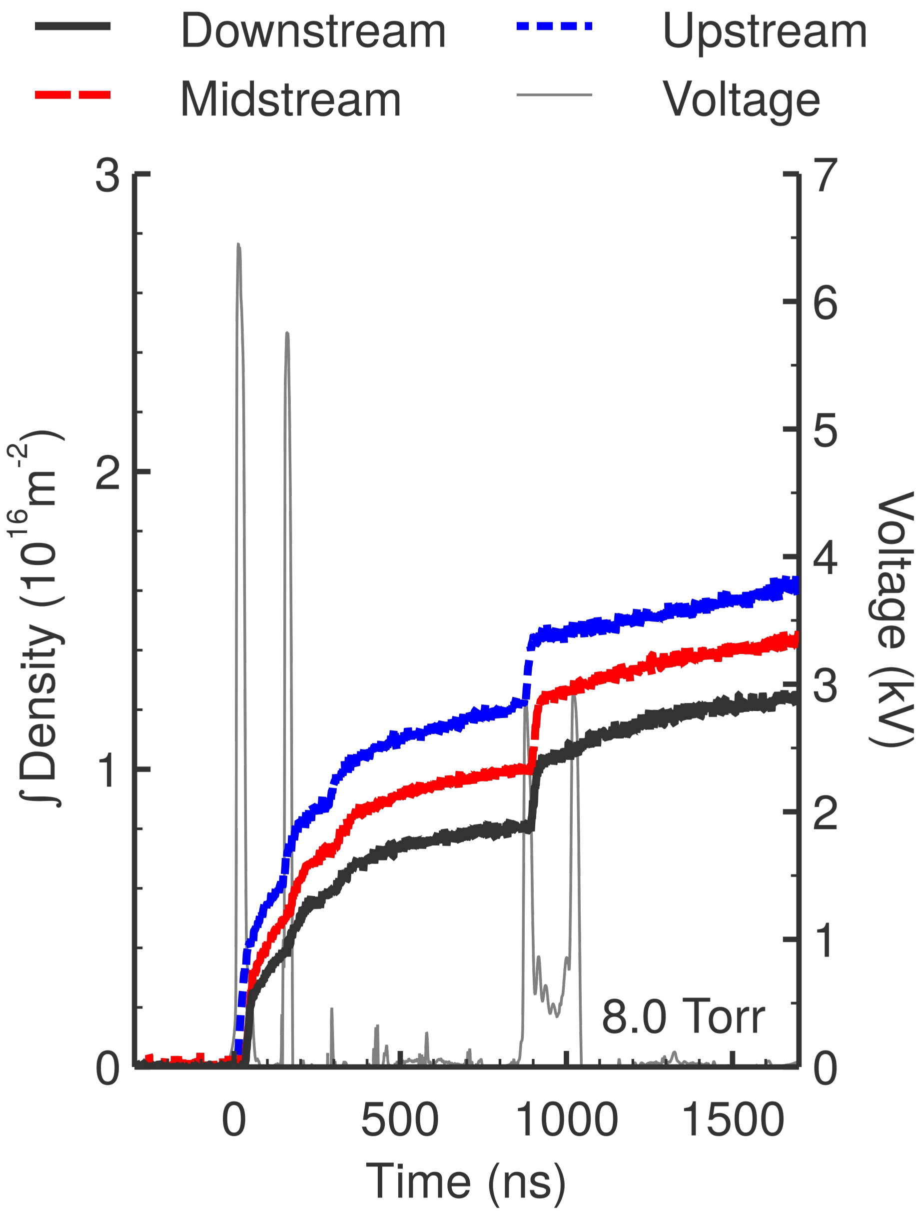
Metastable Densities (Low Pressure)

- Decrease in pre-pulse metastable population
- Peak metastable population for examined range
- Reflections from power supply deposit additional energy, boost metastable generation

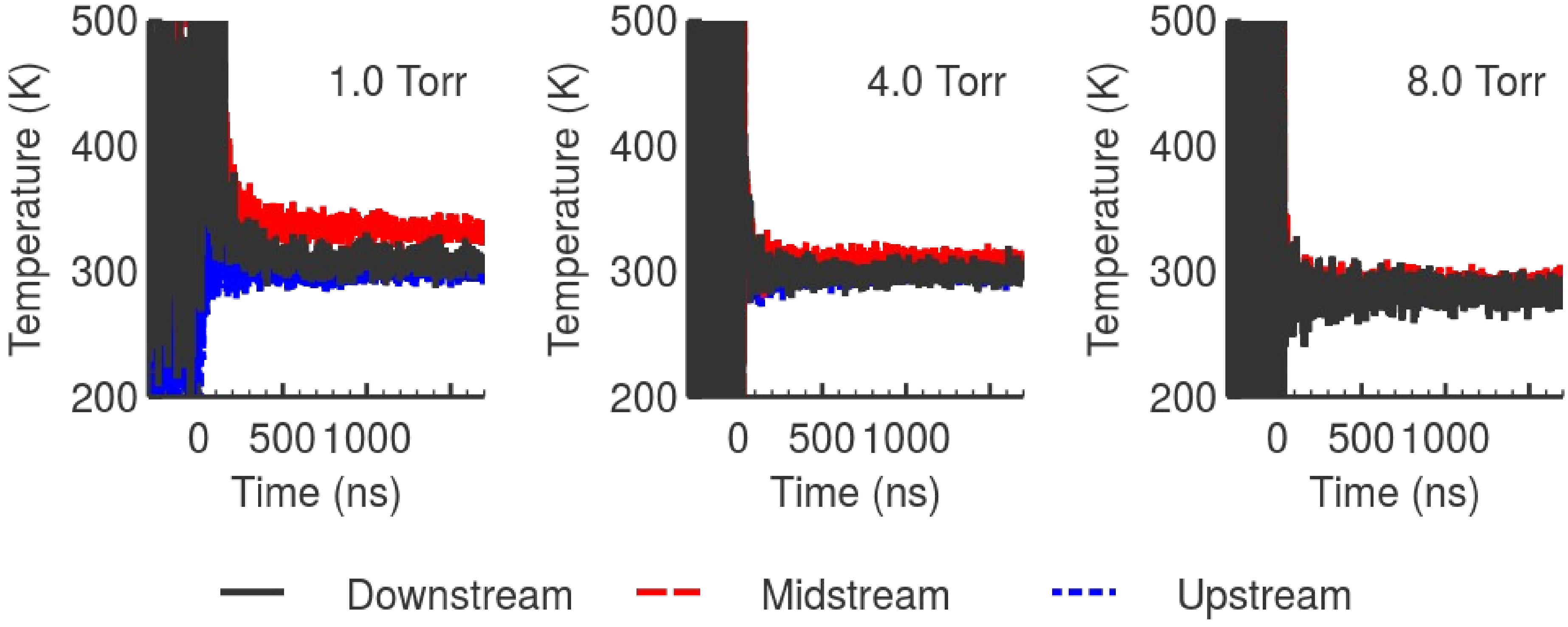


Metastable Densities (Moderate Pressure)

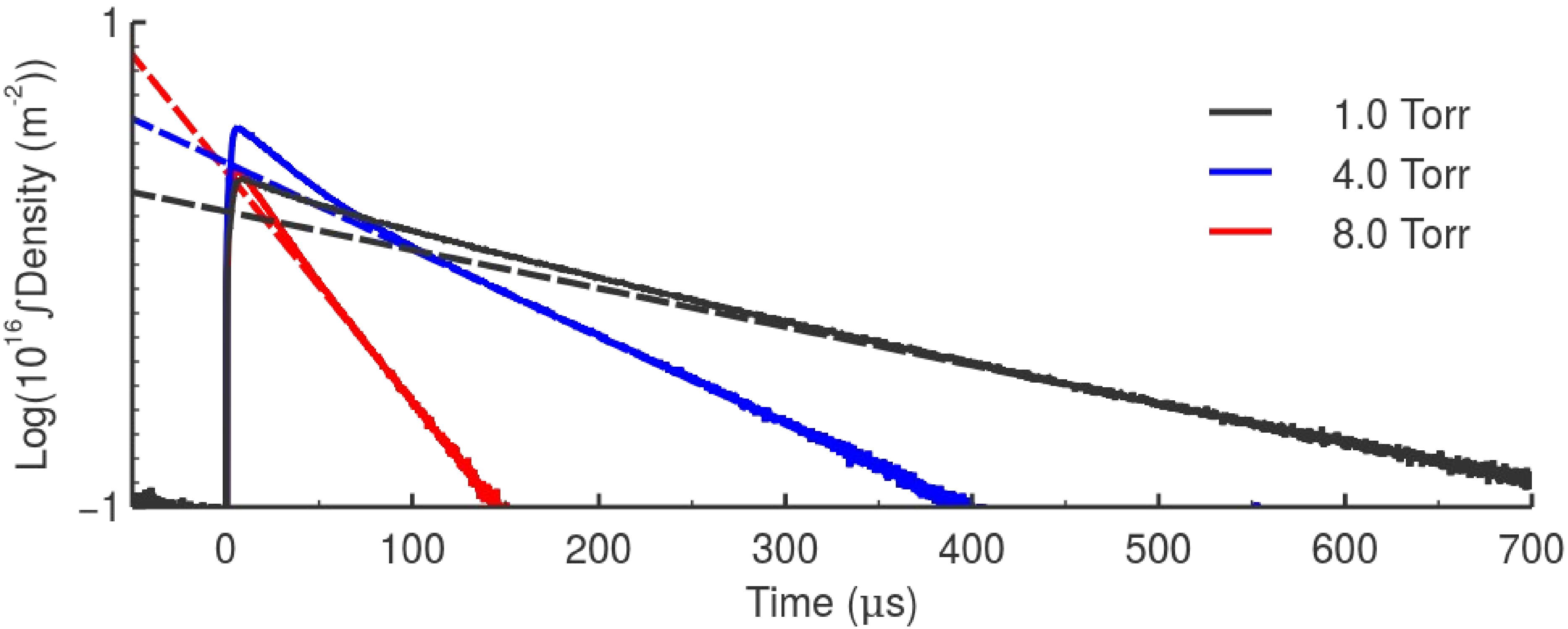
- No detectable metastables before pulse (< 3.0×10^{14} m^{-2})
- Overall decline in metastable density
- Reflection effects still observable
- Metastables decrease with distance from anode
  - Suggests plasma attenuation



Metastable Densities (High Pressure)

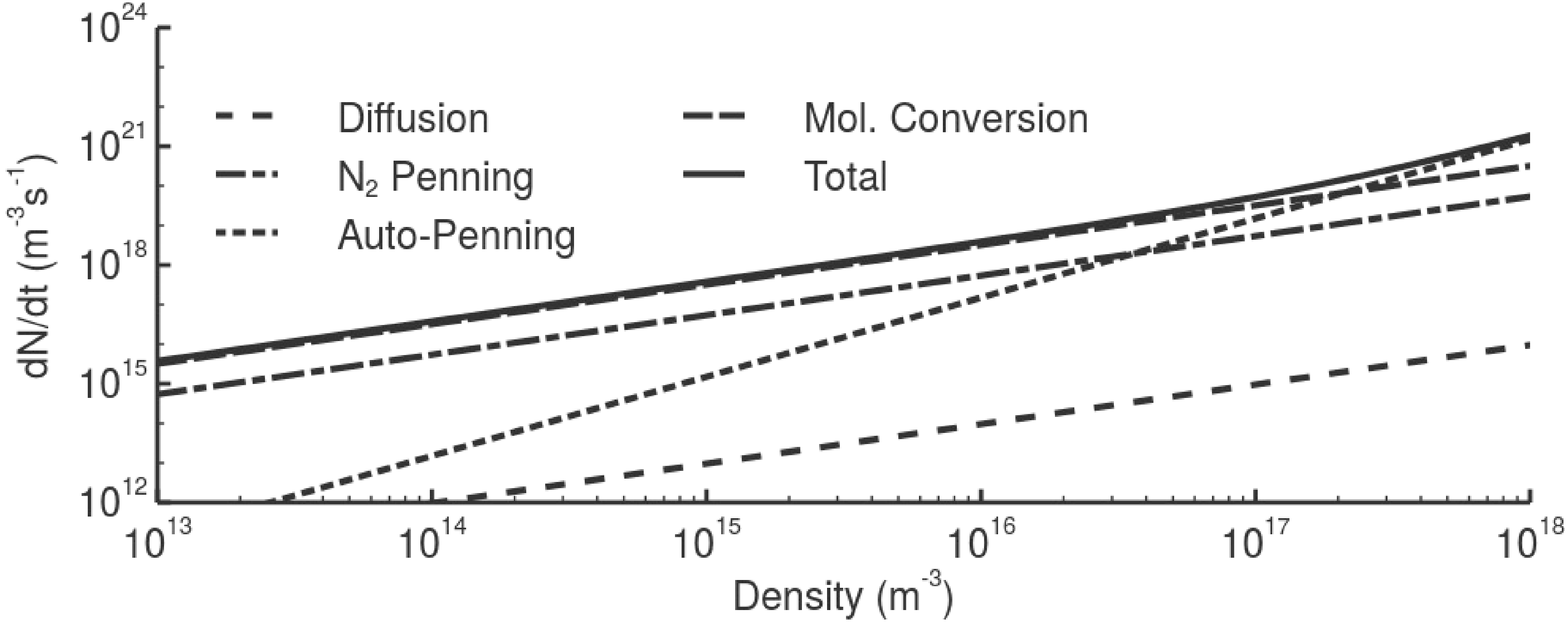


- Change in absorption resulting from small changes in laser wavelength yields gas temperatures (from Doppler broadening)
- Results indicate minimal gas heating
- No significant difference from 300 K for all conditions



- Long-duration metastable measurements can reveal loss mechanisms
- Straight line indicates exponential process (e.g. diffusion)
- Deviation from straight line implies a non-exponential process (e.g. Penning ionization between metastables)

Metastable Destruction



- Initially dominated by non-exponential processes
  - Superelastic electron collisions (previous measurements show electron density comparable to metastable)
  - Penning ionization between metastables
- Molecular conversion dominates at lower metastable densities
- Pre-pulse metastables only detectable for 4.0 Torr and below

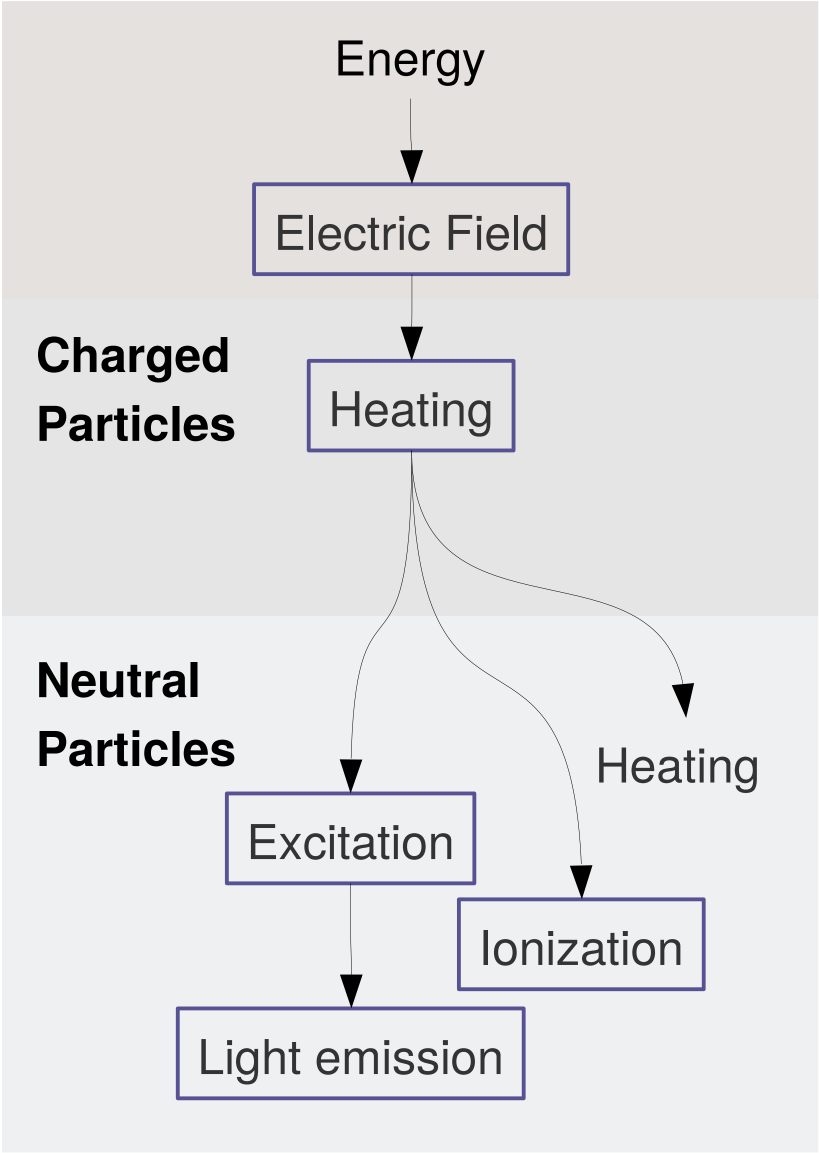
Destruction Processes

- Metastable states only persist between pulses at lower pressures
- There is an optimal pressure for metastable generation
- Additional energy deposited by pulses reflected from power supply
- Plasma appears to attenuate at higher pressures
- No detectable gas heating
- Data suggest losses initially dominated by superelastic electron collisions and Penning ionization between metastables

Metastable Summary

Global Model

- Metastables only one of many excited states
- Use metastable measurements to infer other properties
  - Electric fields
  - Electron densities and temperatures
  - Excited states
- Can be accomplished with a numerical model of plasma

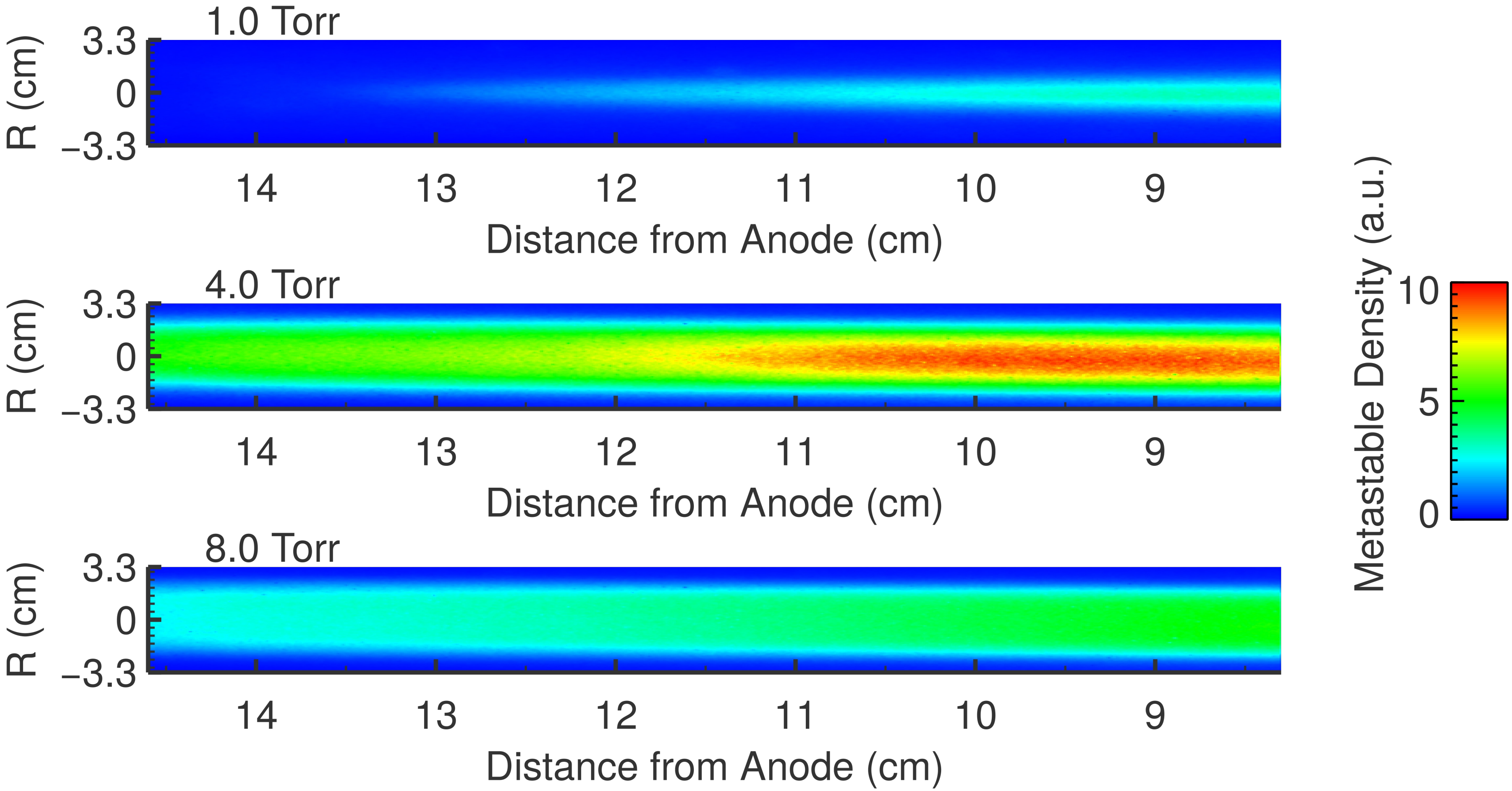


Applying the Metastable Measurements

- Several approaches and degrees of complexity
  - Fluid, kinetic, or particle
  - 0,1,2, or 3 dimensions
  - Particles and reactions to include
- Used global model based on first and third moments of Boltzmann equation
  - Simple starting place
  - Allows inclusion of large number of reactions
  - Does not solve for spatial variations (homogeneous)

- Species:
  - Helium, all states $n < 5$ (multiplet averaged)
  - Helium ions
  - Electrons
- Reactions:
  - Elastic scattering (Pack)
  - Electron (de-)excitation (380 transitions, Ralchenko 2008)
  - Optical transitions (126 transitions, NIST ASD)
  - Excitation transfer (35 transitions, Dubreuil and Catherinot)
- Assume Maxwell-Boltzmann EEDF based on comparisons with particle-in-cell and two-term expansion results.

Model Development



- Previous LCIF measurements showed radial variations in metastable and electron densities
- Cause and pressure-dependence not well-understood
- Assume uniform radial density distribution, results can be modified for different profiles

Weatherford, B.R., Barnat, E.V., Xiong, Z., Kushner, M., 65th Gaseous Elec. Conf., October 22-26, 2012, Austin, TX.

Spatial Dependence

$$\frac{dN_i}{dt} = n_e \left[\sum_{j \neq i} N_j K^e_{j,i}(T_e) - N_i \sum_{j \neq i} K^e_{i,j}(T_e) \right]$$

Electron (de)excitation

$$+ \left[\sum_{j > i} N_j K^o_{j,i} - N_i \sum_{j < i} K^o_{i,j} \right]$$

Radiative transitions

$$+ N_g \left[\sum_{j \neq i} N_j K^a_{j,i} - N_i \sum_{j \neq i} K^a_{i,j} \right]$$

Atomic excitation transfer

Particle Density Equation (First Moment)

$$\frac{d}{dt}\left(\frac{3}{2}n_e T_e\right) = \frac{e^2 n_e E(t)^2}{m_e k_m(T_e) N_g}$$

Electric field heating

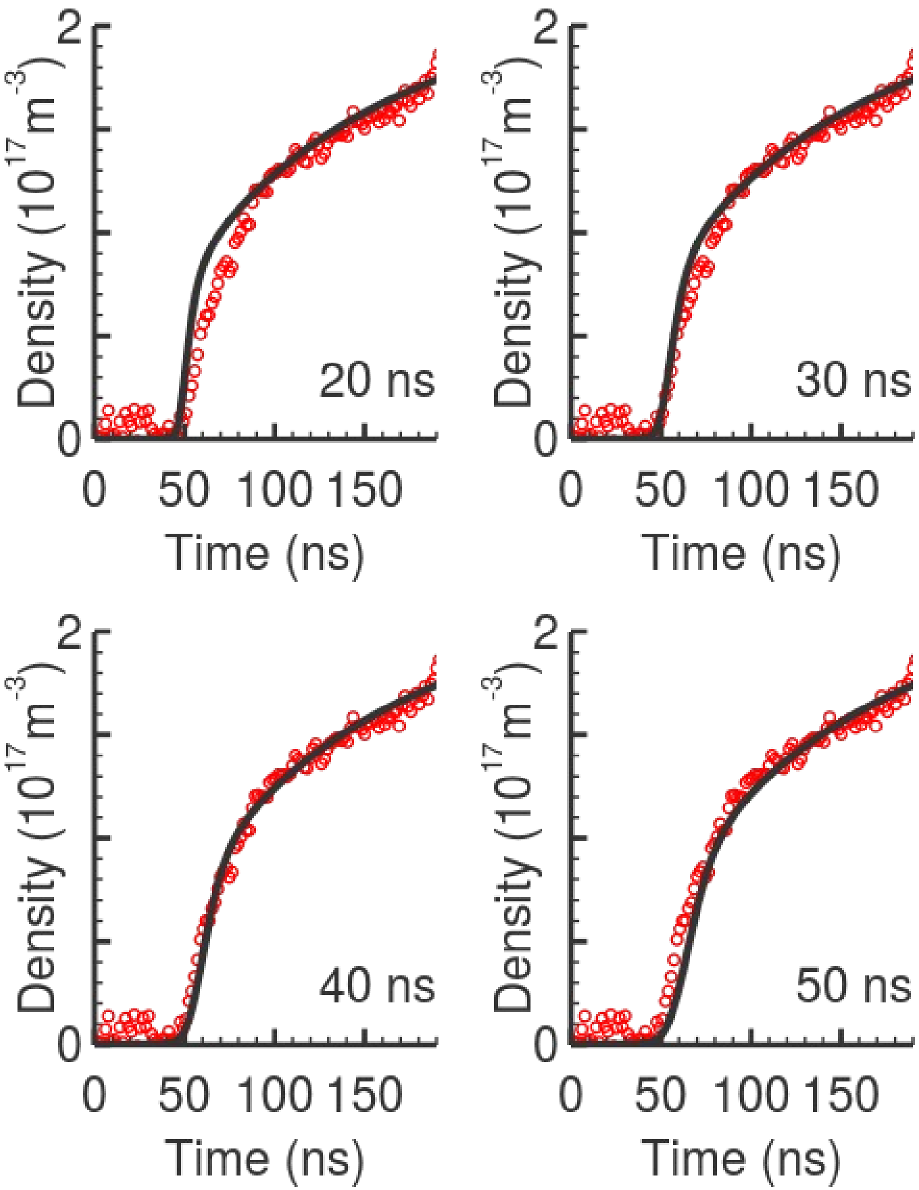
$$- n_e k_m(T_e) N_g \left(\frac{3m_e}{M}\right) \frac{3}{2}(T_e - T_g)$$

Elastic collisions

$$-n_e \sum_i \sum_{j \neq i} K_{ij}^e N_i \Delta\epsilon_{ij}$$

Inelastic collisions

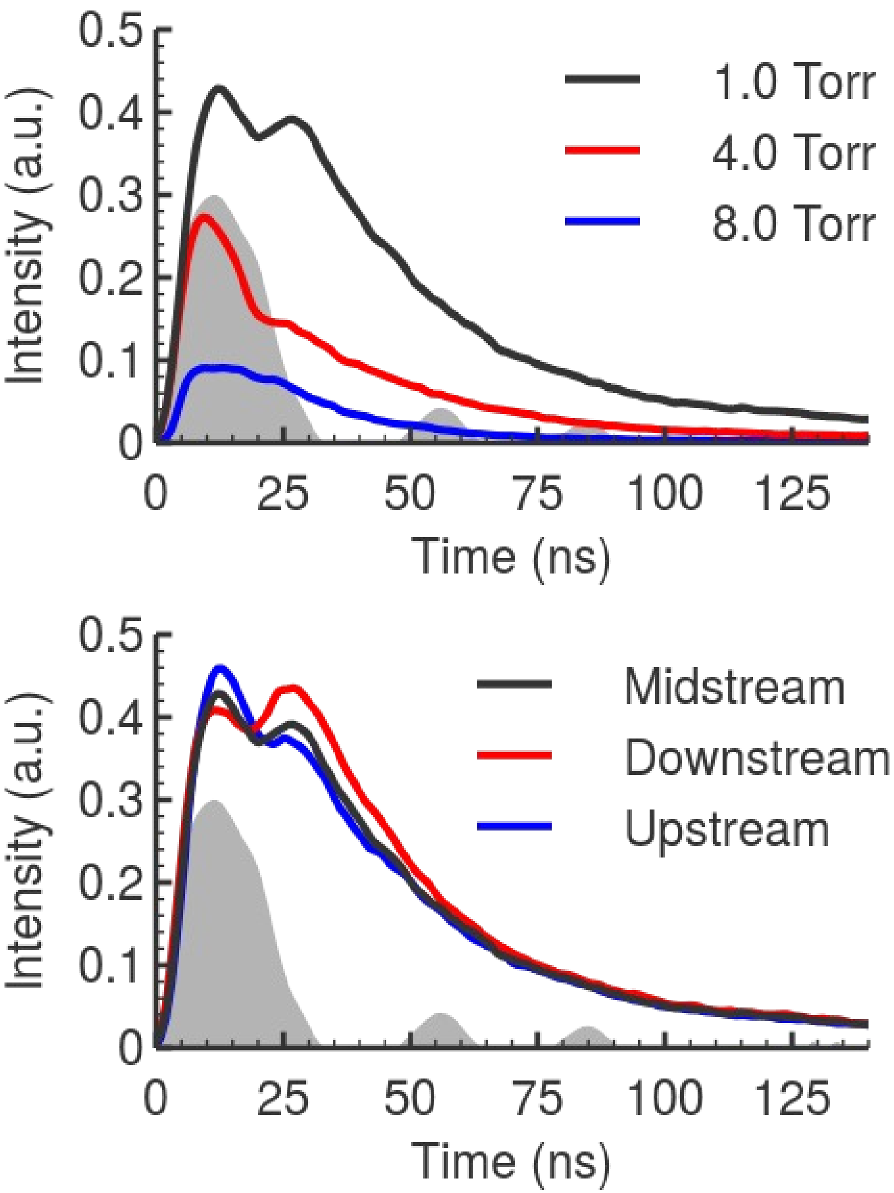
Electron Energy Equation (Third Moment)



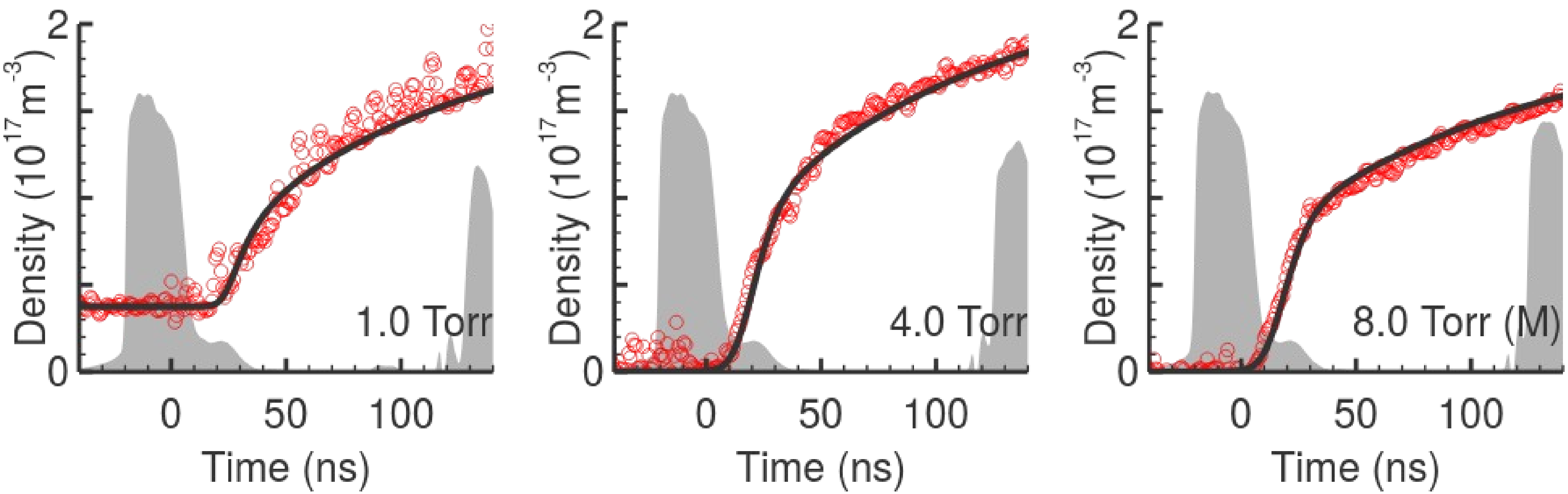
- Assumed electric field applied as a Gaussian pulse
- Free variables: peak electric field and pulse width
- Best fit obtained with 40 ns pulse-width.
- Why is this longer than the applied pulse (25 ns)?

Pulse-width

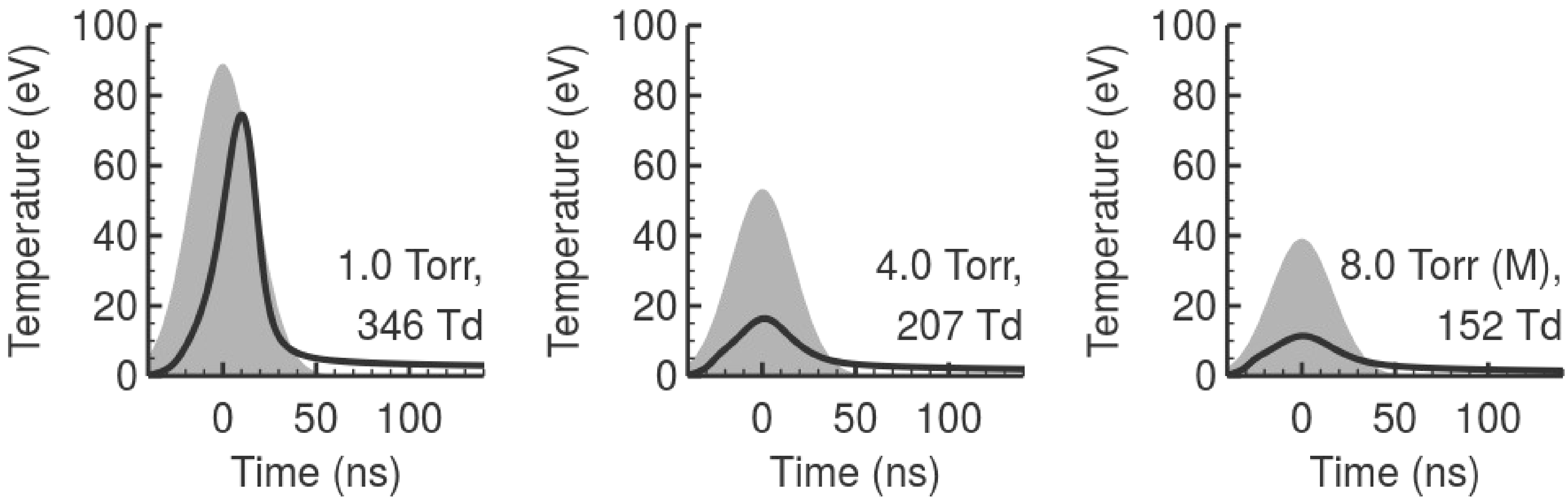
- Optical emissions showed evidence of return stroke
  - Counter-propagating excitation from cathode to anode
- Double peak at several pressures
  - Different from power supply reflections
  - Second peak most intense closest to cathode
- *Effective* excitation period longer than 25 ns



Evidence for Return Stroke



- Good agreement for most cases—trends and magnitudes were consistent
- Largest discrepancies at 1.0 Torr
  - Initial rise too fast
  - Radiative cascade too slow
  - Extended excitation?

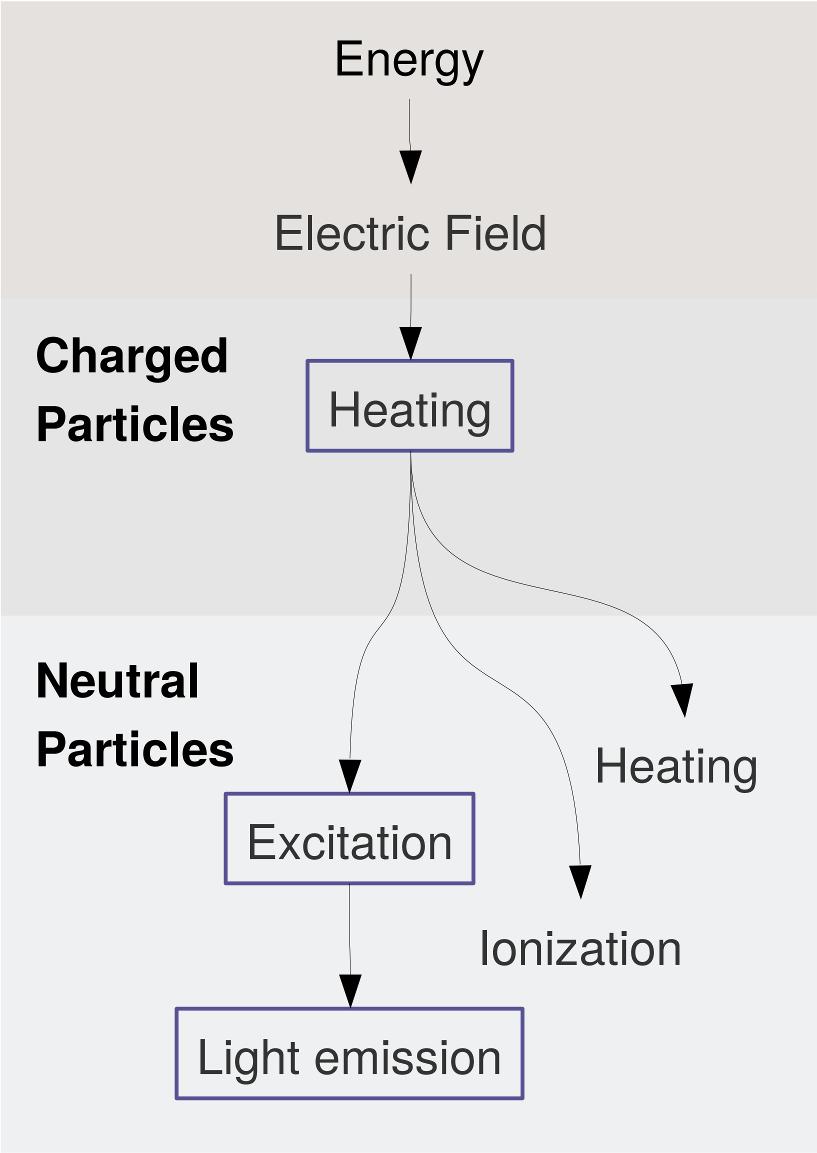


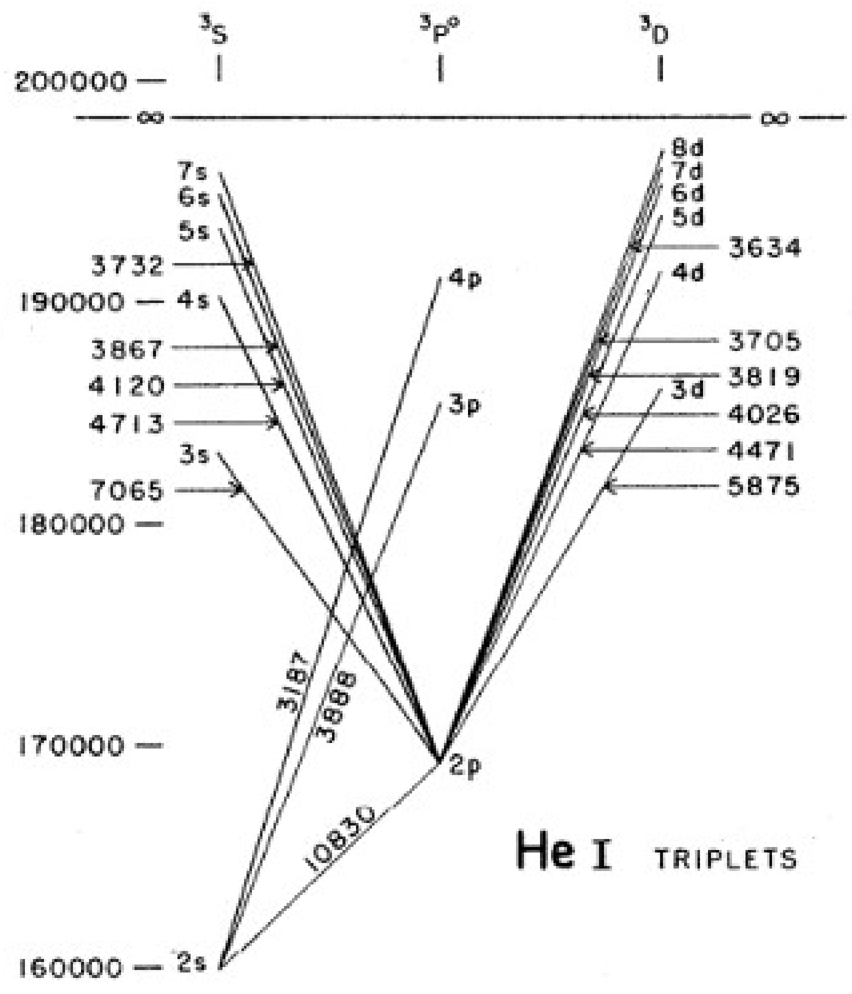
- Peak fields consistent with other studies, but in range where EEDF is not well-matched by Maxwell-Boltzmann
- Electron temperatures (particularly 1.0 Torr) are excessive
- Suggests potential violation of assumptions
- Different distribution or model necessary?

- Created and applied global model of helium plasma and applied to RPND
- Observed evidence of extended excitation period in population of metastables
- Simulations reproduced metastable density dynamics and magnitudes
- Temperatures and fields at, especially at low pressures, suggest violation of underlying assumptions (e.g. Maxwell-Boltzmann distribution)

Emission Measurements

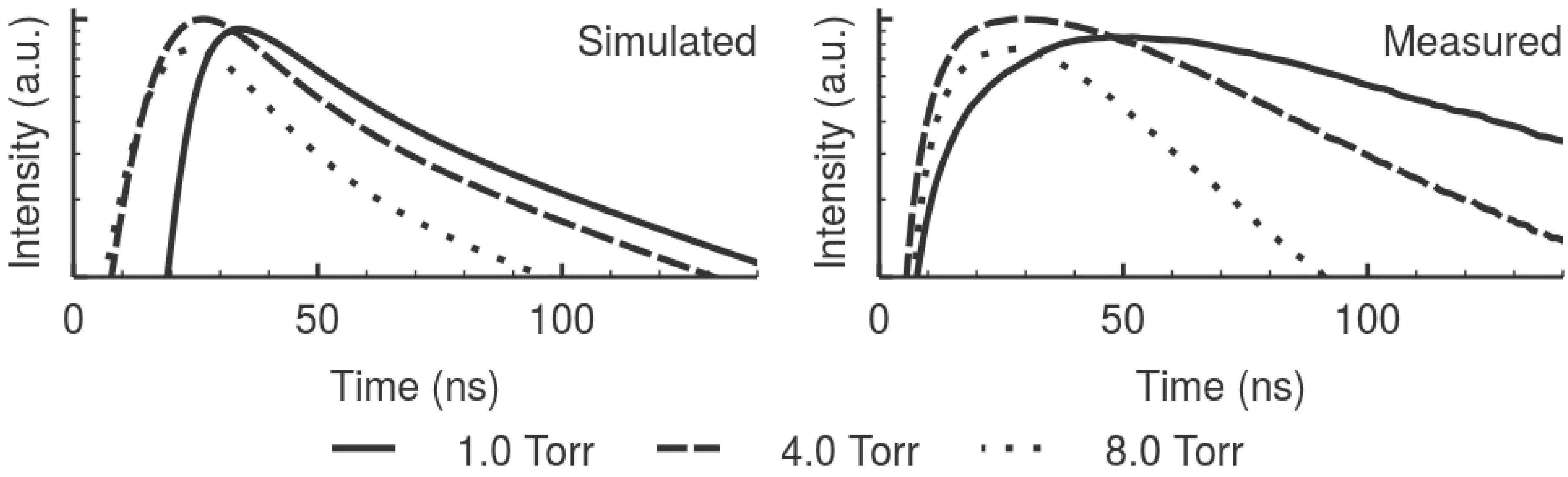
- Simulations based on metastable densities
  - Generated predictions of other excited state densities and emissions
- Optical emission spectroscopy
  - Experimental check of simulated emissions
  - May reveal other phenomena
  - Potential estimate electron temperature





- Observed optical emissions in visible range (triplet and singlet)
- Calibrated relative intensities with tungsten lamp
- Same conditions as absorption spectroscopy
  - 0.3 – 16.0 Torr
  - Three axial locations

Helium Emissions



- Relative peak heights and timing similar, distinctly different in shape
- Measurements suggest pressure-dependent de-excitation process
- Decay rate increases with pressure
  - Penning ionization of impurities?
  - Missing excitation transfer rates?
- Long emission lifetime suggest an extended excitation process
  - Beam-like electrons
  - Persistent electric field (seen in other experiments)

Extended Excitation ($3^1D \rightarrow 2^1P^o$, 668 nm)

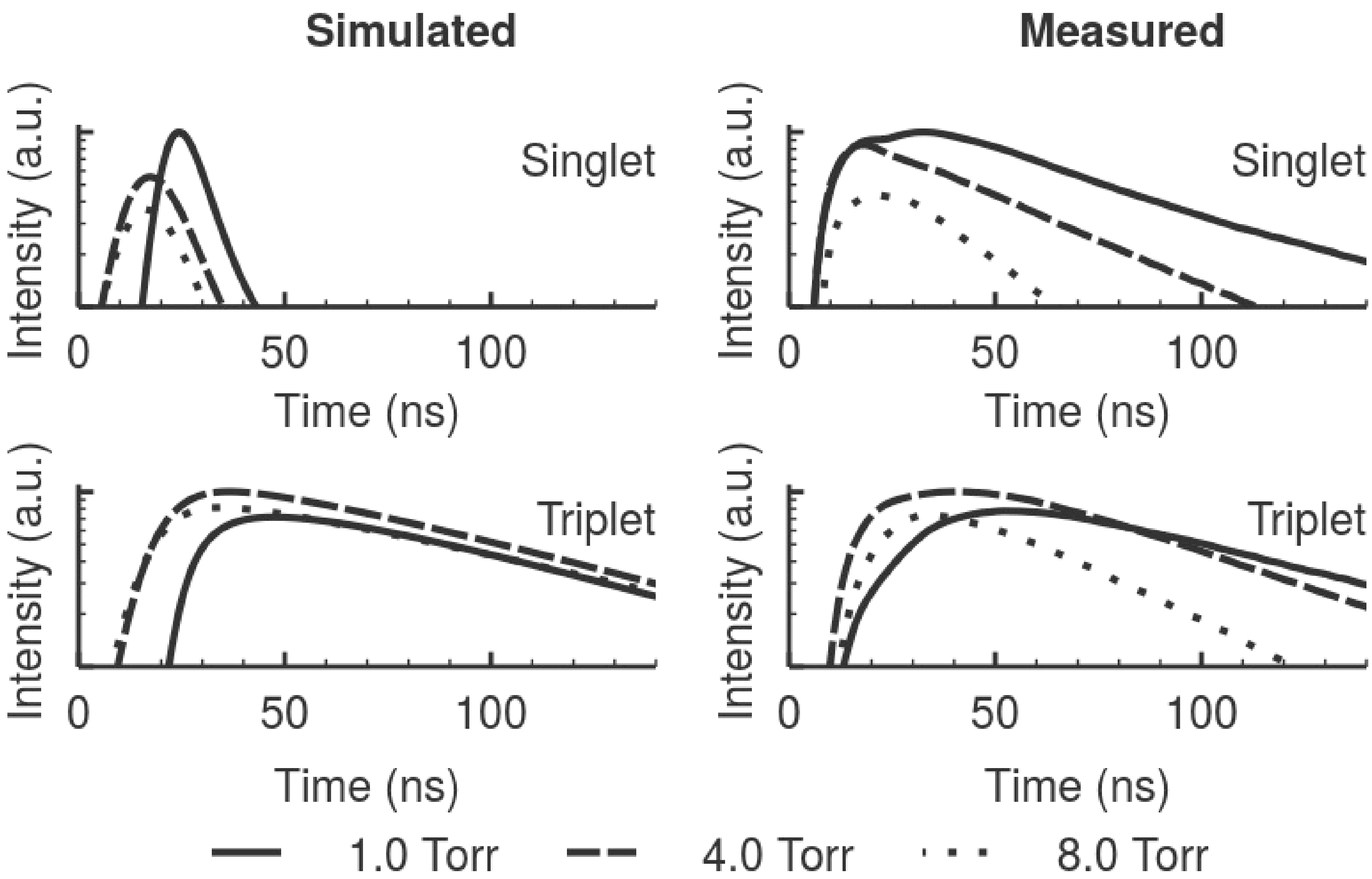
- Resonance radiation (transition from excited state to ground) can be “trapped” in plasmas
- Absorbed and re-emitted many times before exiting
- Increases energy residence time
- Described by “trapping factor”, multiplier of normal lifetime
  - Geometry and lineshape dependent
- Could contribute to extended excitation

Pressure (Torr)	Trapping Factor	Effective Lifetime (s)
1.0	8,773	1.549×10^{-5}
4.0	38,031	6.715×10^{-5}
8.0	78,837	1.392×10^{-4}

Lifetime increase for $3^1P \rightarrow 1^1S$ transition

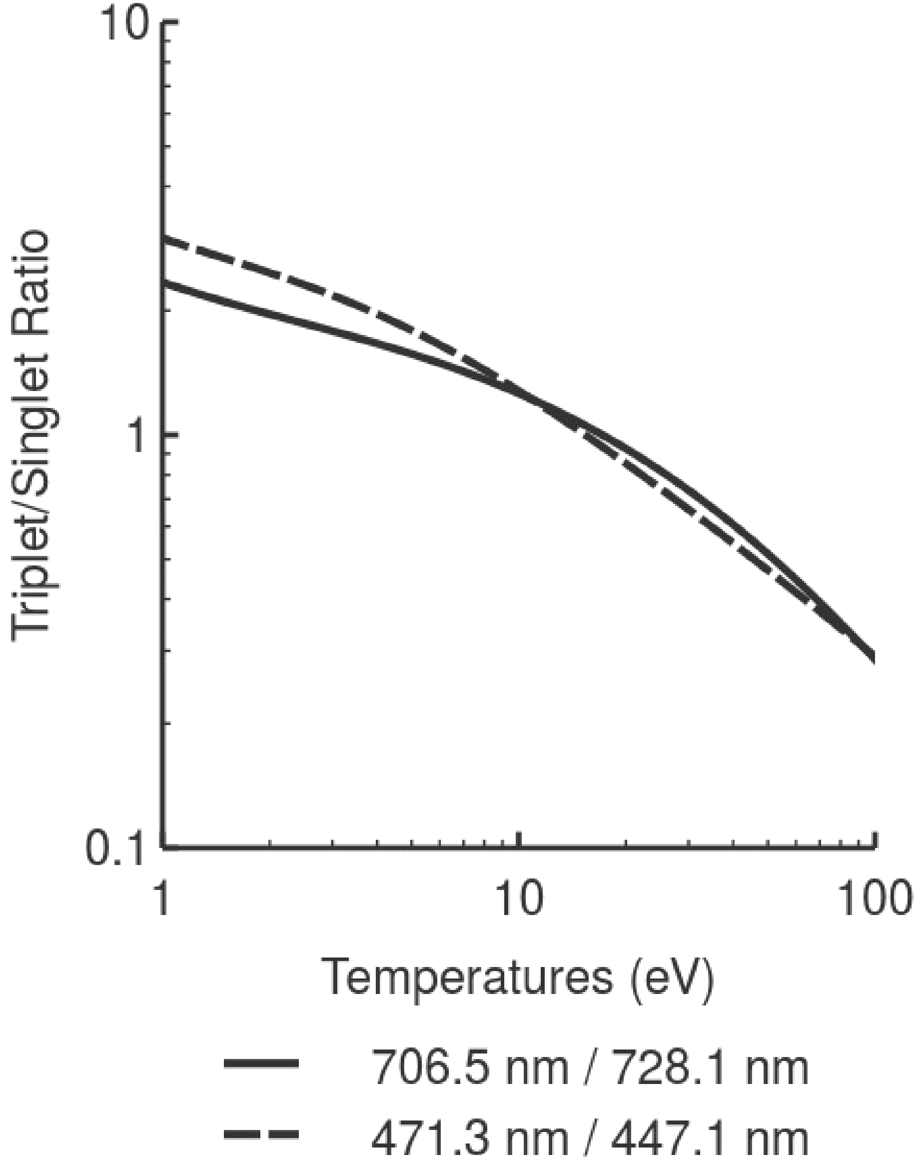
Radiation Trapping

- Compare same transition within triplet and singlet manifolds at 501.5 nm and 388.8 nm ($3P \rightarrow 2S$)
  - Only 3^1P has a resonance transition, 3^3P does not
- If radiation trapping is absent: comparison of simulated and measured singlet emissions should be the same as the triplets
- If radiation trapping is present: simulated singlet emissions will decay faster than measured emissions, but triplets will be unaffected



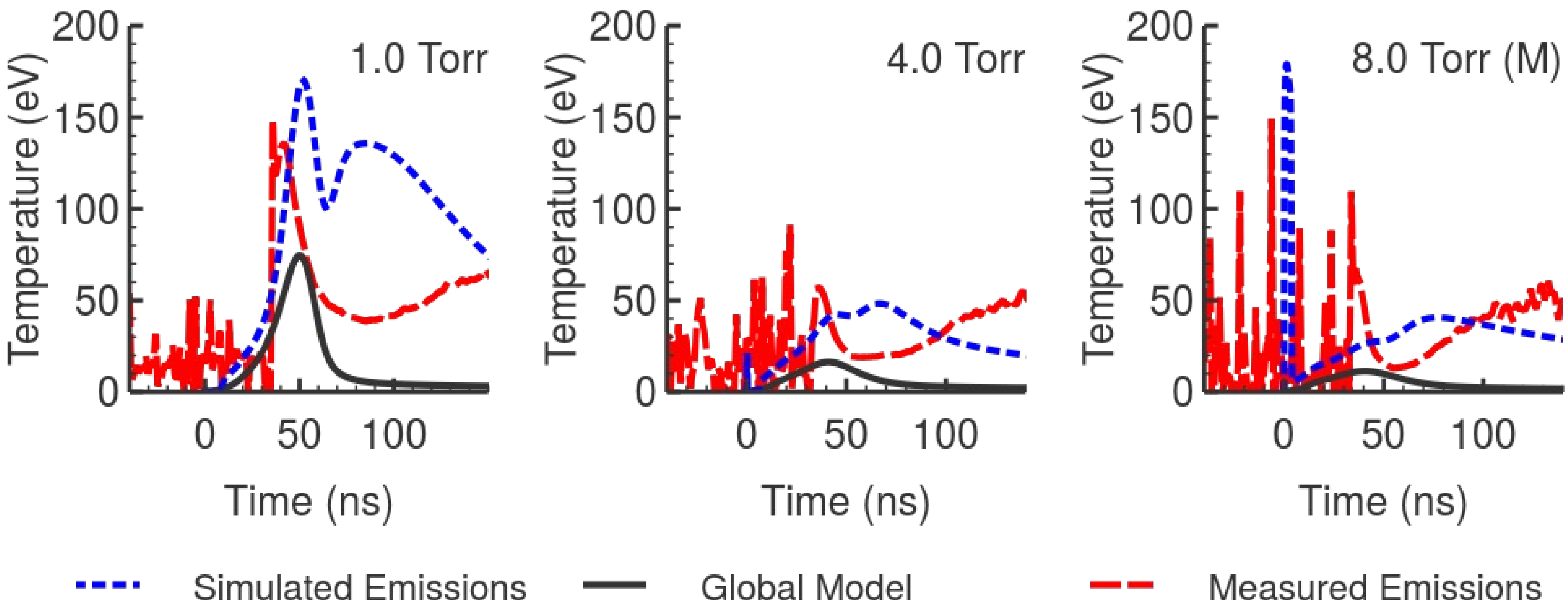
- Simulated singlet emissions fall off more rapidly than measured
- Comparison of triplet emissions show a lack of the same behavior
- Suggests importance of radiation trapping

Verification of Radiation Trapping



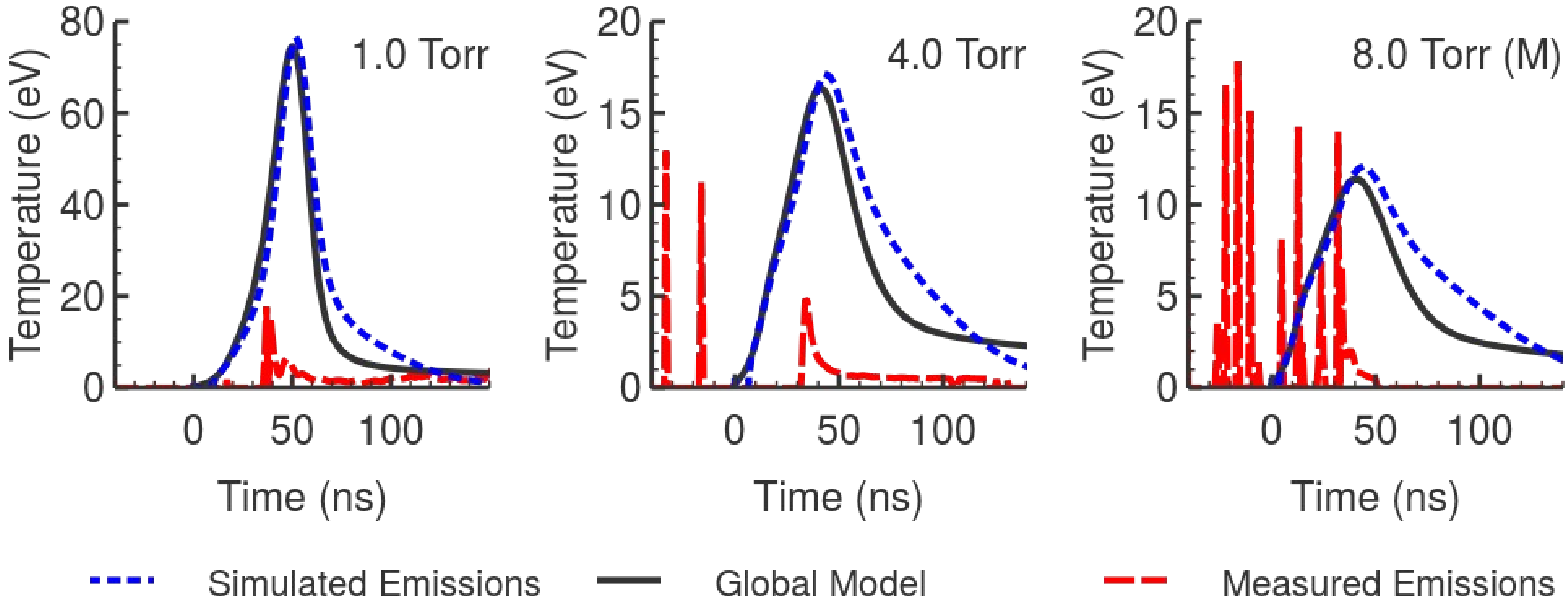
- Plasma emissions can be used to estimate electron temperature
- Several approaches, line ratios based on coronal model possibly suitable
- Assumes
  - Excited states generated by electron collisions with unexcited atoms
  - Excited states decay by radiative emission only
- Maxwell-Boltzmann distribution used for consistency, others possible
- Two candidate ratios selected for evaluation
  - 706.5 nm / 728.1 nm ($3^3S \rightarrow 2^3P^o$ / $3^1S \rightarrow 2^1P^o$)
  - 471.3 nm / 447.1 nm ($4^3S \rightarrow 2^3P^o$ / $4^1D \rightarrow 2^1P^o$)

Coronal Model



- Compared temperatures from electron energy equation with those predicted by simulated line ratios and measured line ratios
- Temperatures from simulated line ratios did not match solutions of electron energy equation
  - Suggests coronal model does not apply for these transitions
- Possible cause is collisional redistribution of n = 4 states
- Agreement may be improved by inclusion of n = 5 states

Ratio 1: 471.3 nm / 447.1 nm



- Simulated emissions consistent with solutions of electron energy equation
  - Coronal model may be applicable
- Application to measured emissions was limited by signal-to-noise ratio
- Distribution likely not Maxwell-Boltzmann (concept of temperature not applicable)
  - May still be useful as a measure of mean electron energy

Ratio 2: 706.5 nm / 728.1 nm

- Global model captures some features of RPND population kinetics
- Missing physics
  - Extended excitation (distribution or field related?)
  - Radiation trapping
- $3^3S \rightarrow 2^3P^o$ / $3^1S \rightarrow 2^1P^o$ ratio may be useful indicator of electron energy in helium RPDN

Conclusions

- Developed laser-absorption spectroscopy system and analysis software
- Obtained detailed measurements of metastable dynamics in a helium RPND
- Created a global model code for helium plasmas
- Used code to infer plasma parameters from metastable density information
- Measured optical emissions of the RPND
- Found several additional phenomena that are potentially important in the description of the RPND

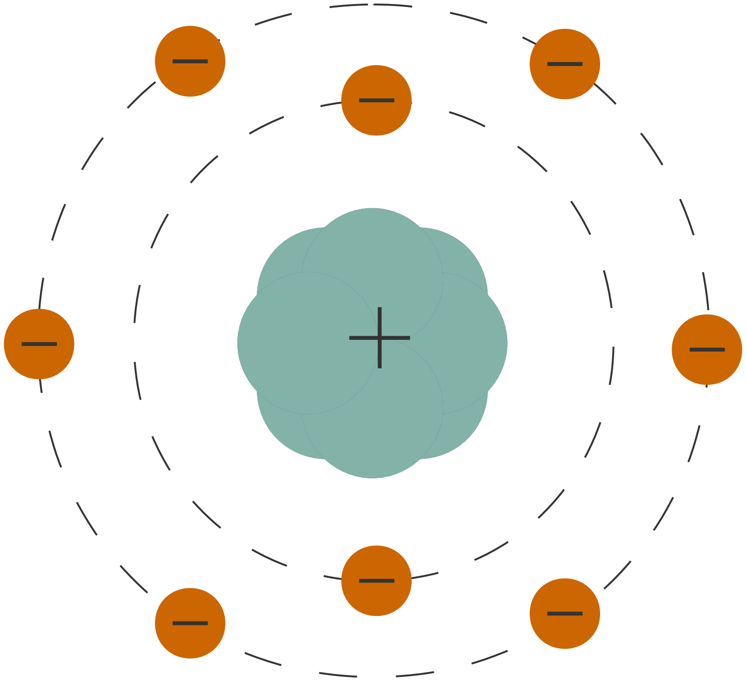
- Pulse reflections can deposit significant amounts of additional energy
- Metastable states only persist between pulses at lower pressures
- Excitation period lasts significantly longer than expected
  - Partially a result of a return stroke
  - Additional physics necessary to completely account for behavior
- May be possible to employ simple line ratio diagnostics for estimates of mean electron energy

- Address temporal evolution of metastable density profile (Abel inversion)
  - Provides insight on the relation between wall effects and radial profiles
- Incorporate additional physics (Penning ionization of impurities, radiation trapping, etc.) in present global model
  - Would address whether post-pulse behavior can be completely described by mechanisms unrelated to EEDF
- Employ different EEDF models to better match excited state dynamics
  - May provide alternative explanation for extended excitation in RPND
- Develop a 0D Monte-Carlo simulation of RPND
  - Could provide better insight on EEDF evolution under high electric fields

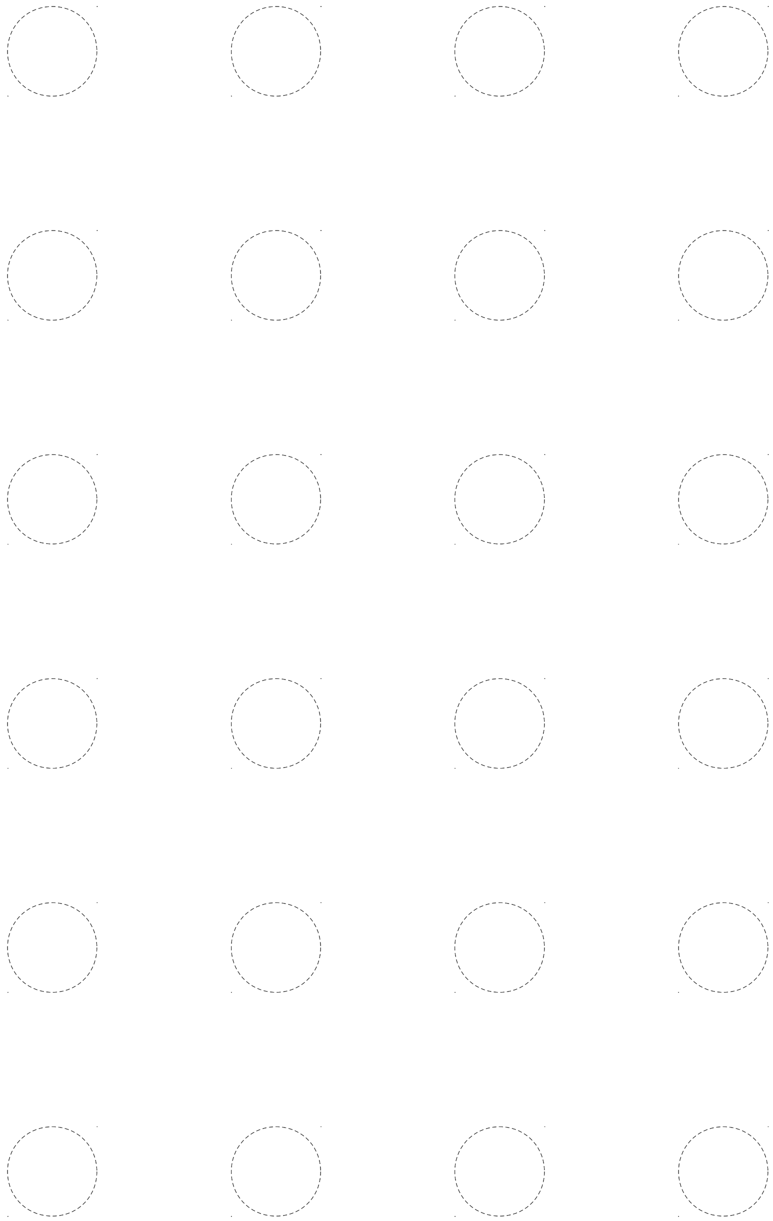
Acknowledgements

Questions

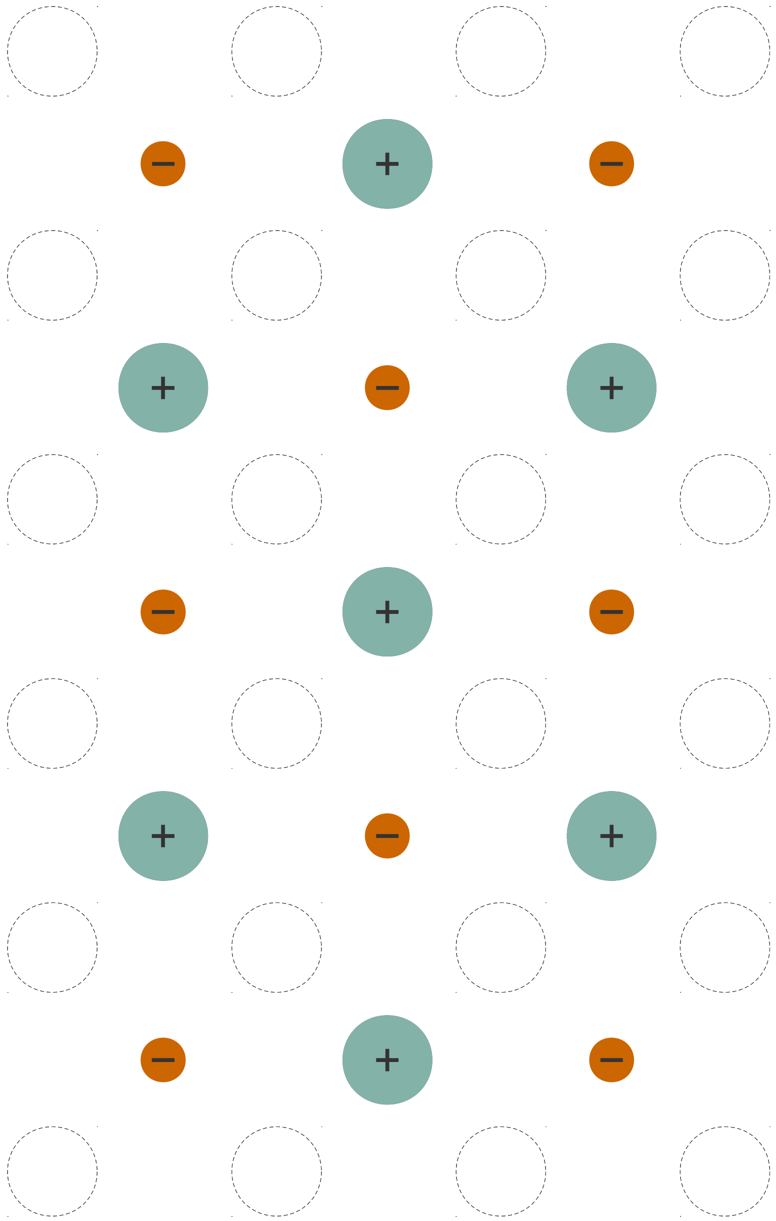
Backup Slides



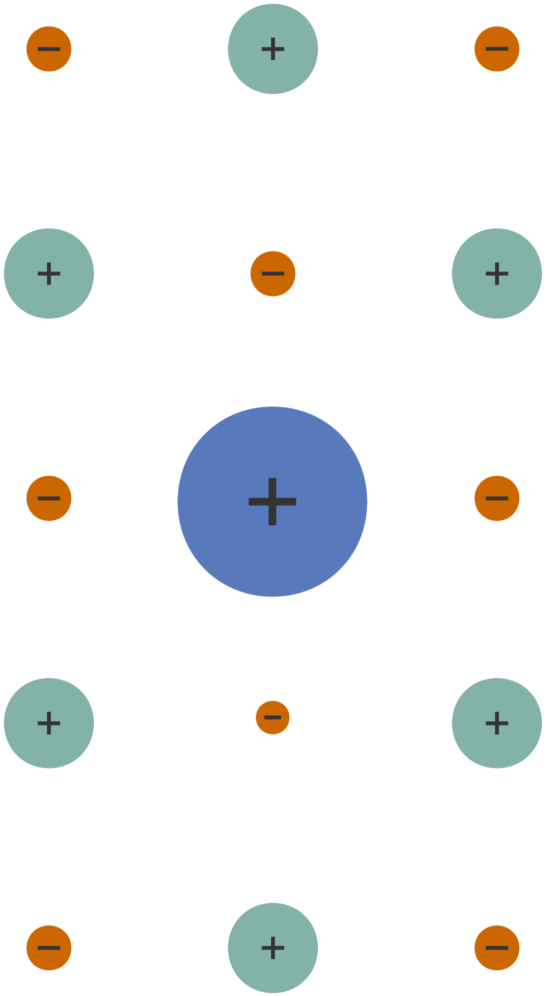
- Positively-charged nucleus
- Orbited by negatively-charged electrons
- Overall, electrically neutral



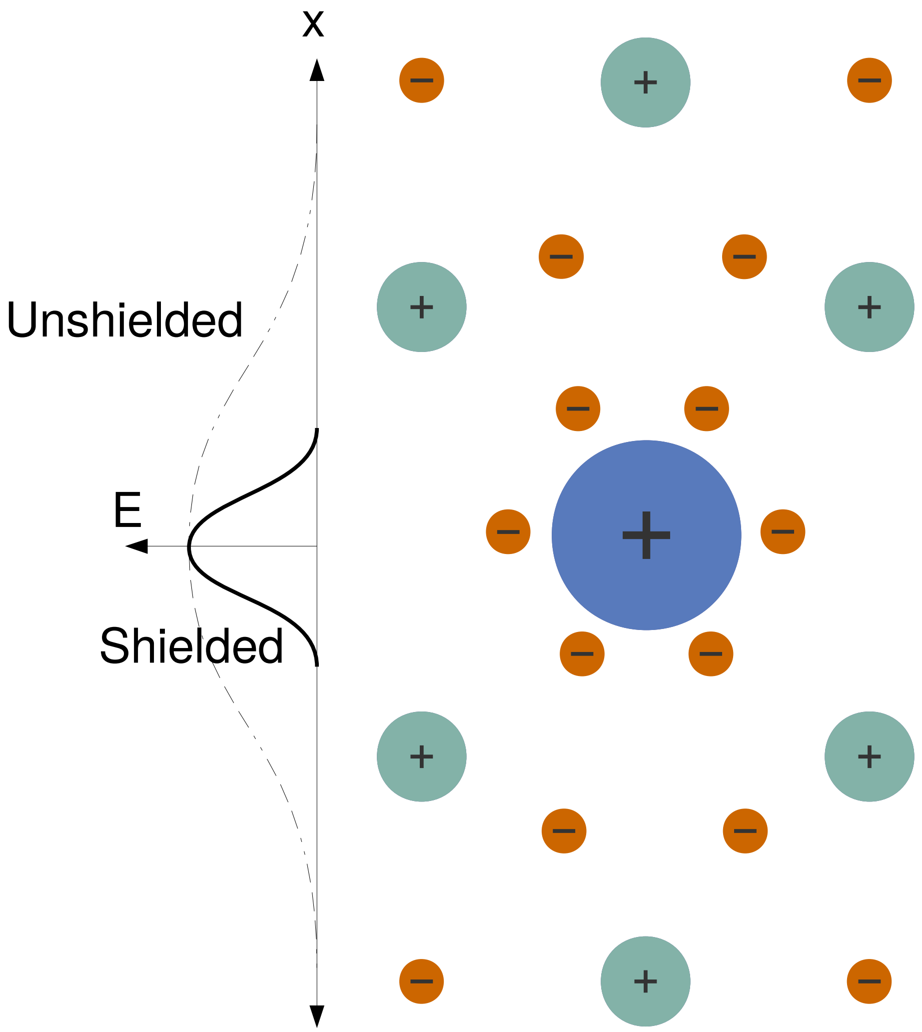
- Consider an atomic gas
- Many neutral atoms with random motion
- No electrical interaction



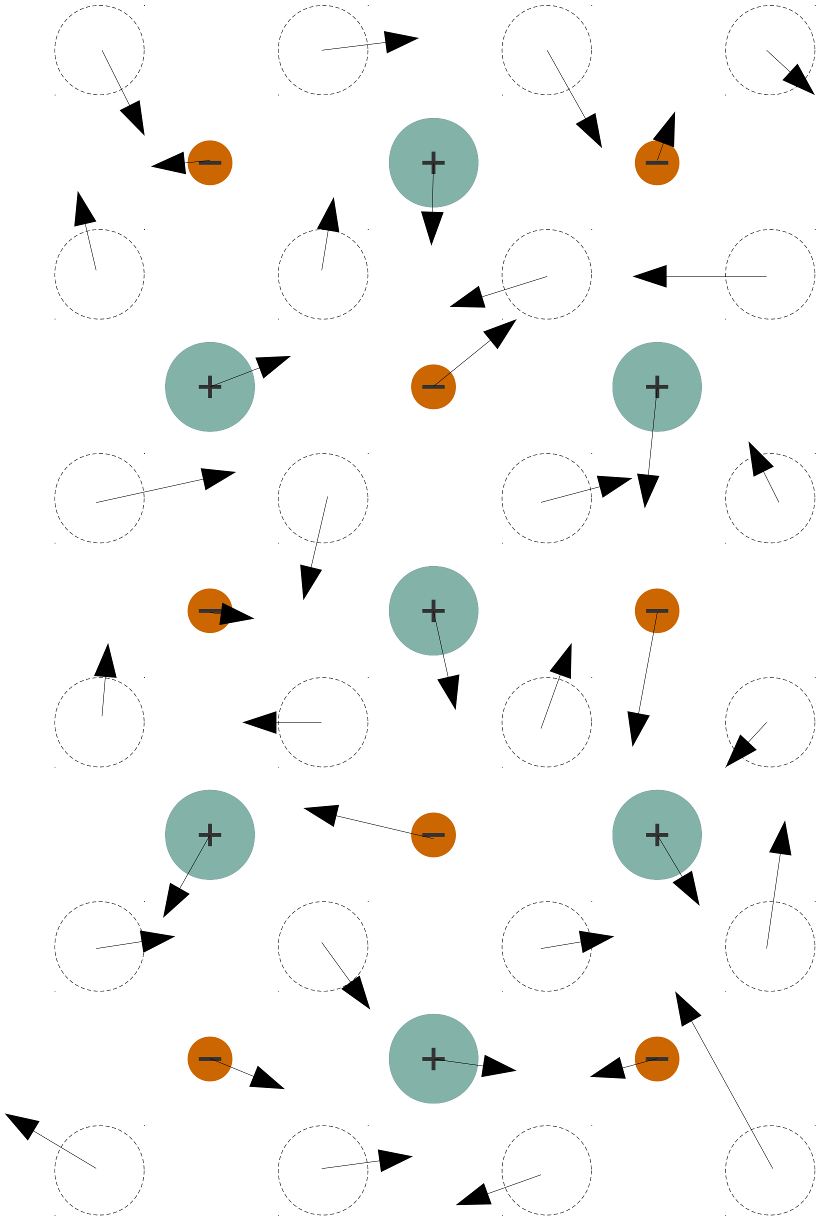
- Add energy to gas
  - Electricity
  - Shock
  - Light
- Electrons separate from atoms
- Collection of neutral atoms, (positive) ions, and (negative) electrons
- What makes a plasma?



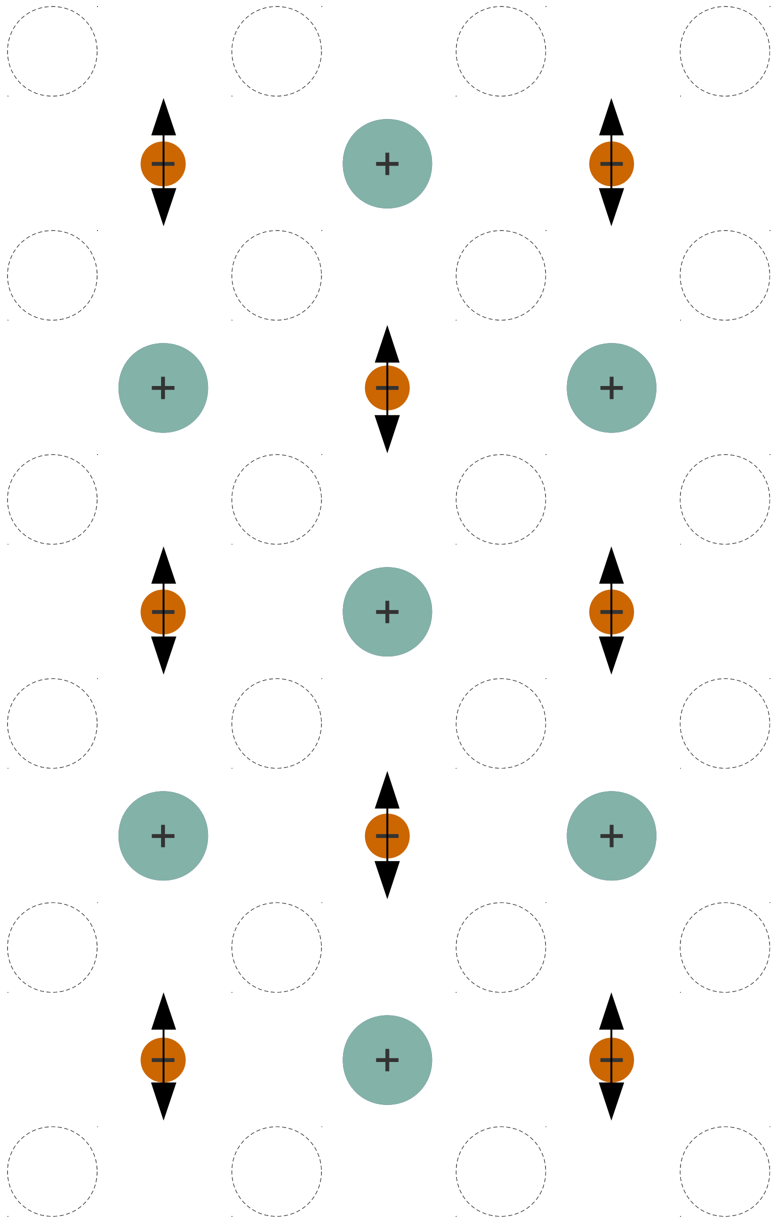
Consider what happens with an electrical perturbation.



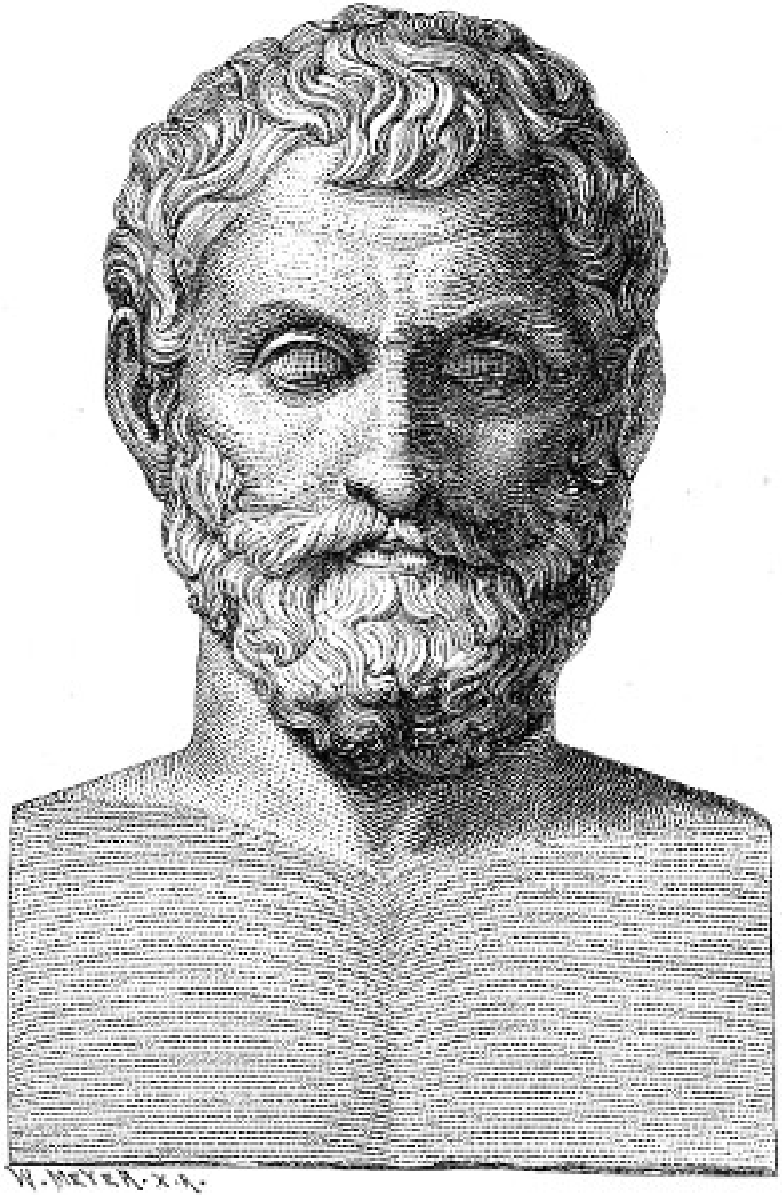
- Electrons move to shield charge
- Similar for positive and negative perturbation
- Electric field falls off quickly
- $\lambda < L,\ n\lambda^3 >> 1$



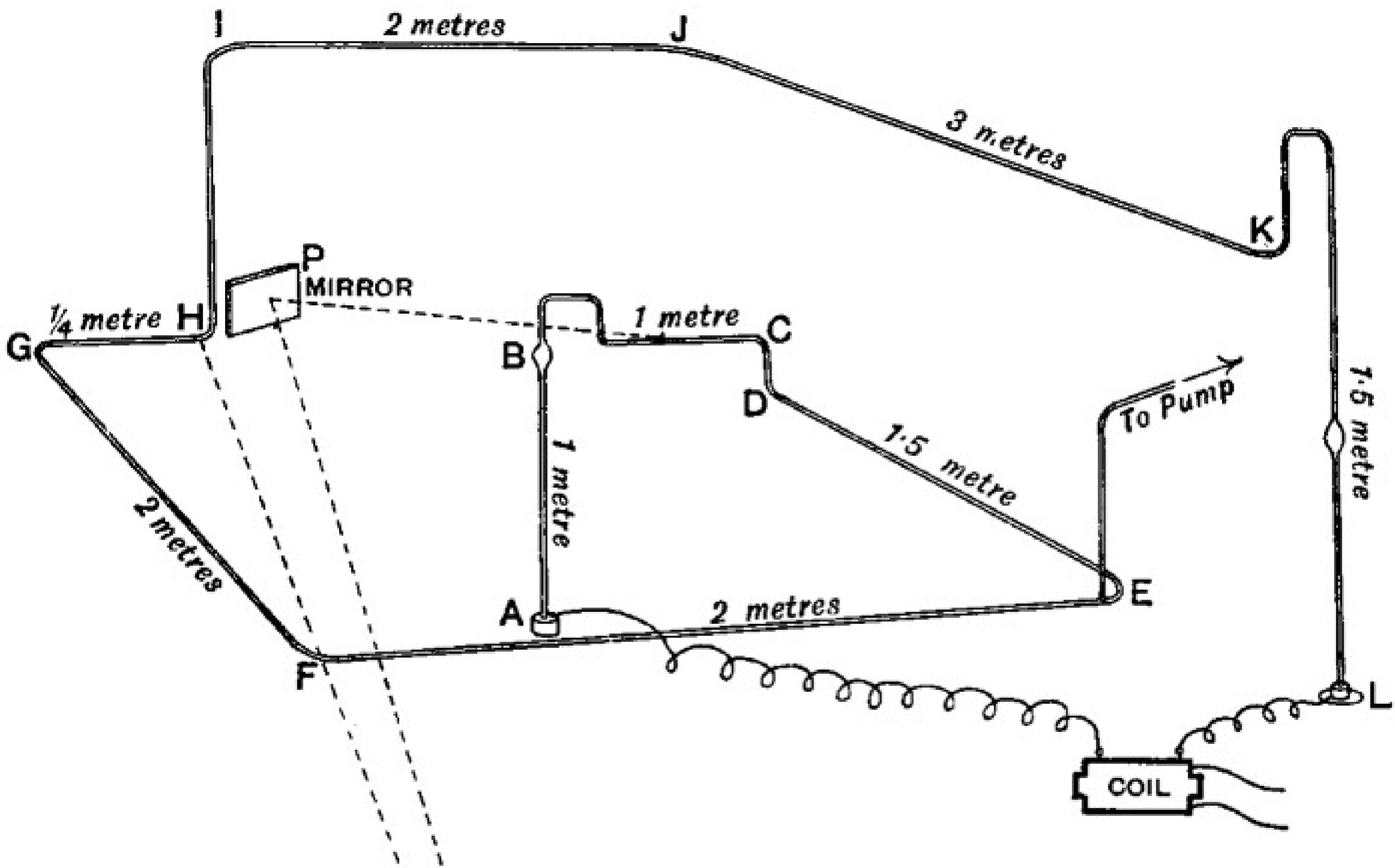
- Random collisions with neutral particles
- Neutral collisions can still determine properties of ionized gas
- Electric interactions should be dominant in plasma



- Electrons possess natural oscillation frequency
- Characteristic of electrical interaction
- $\omega_p > \nu$



Wallis, E. et al. "Illustrerad verldshistoria utgifven av E. Wallis. Volume I." (1875).
Stotesbury, H. "J. J. Thomson." Popular Science, Vol. 56 (1899).



J. J. Thomson. "Notes on Recent Researches in Electricity and Magnetism." Clarendon Press, Oxford, UK, 1893.

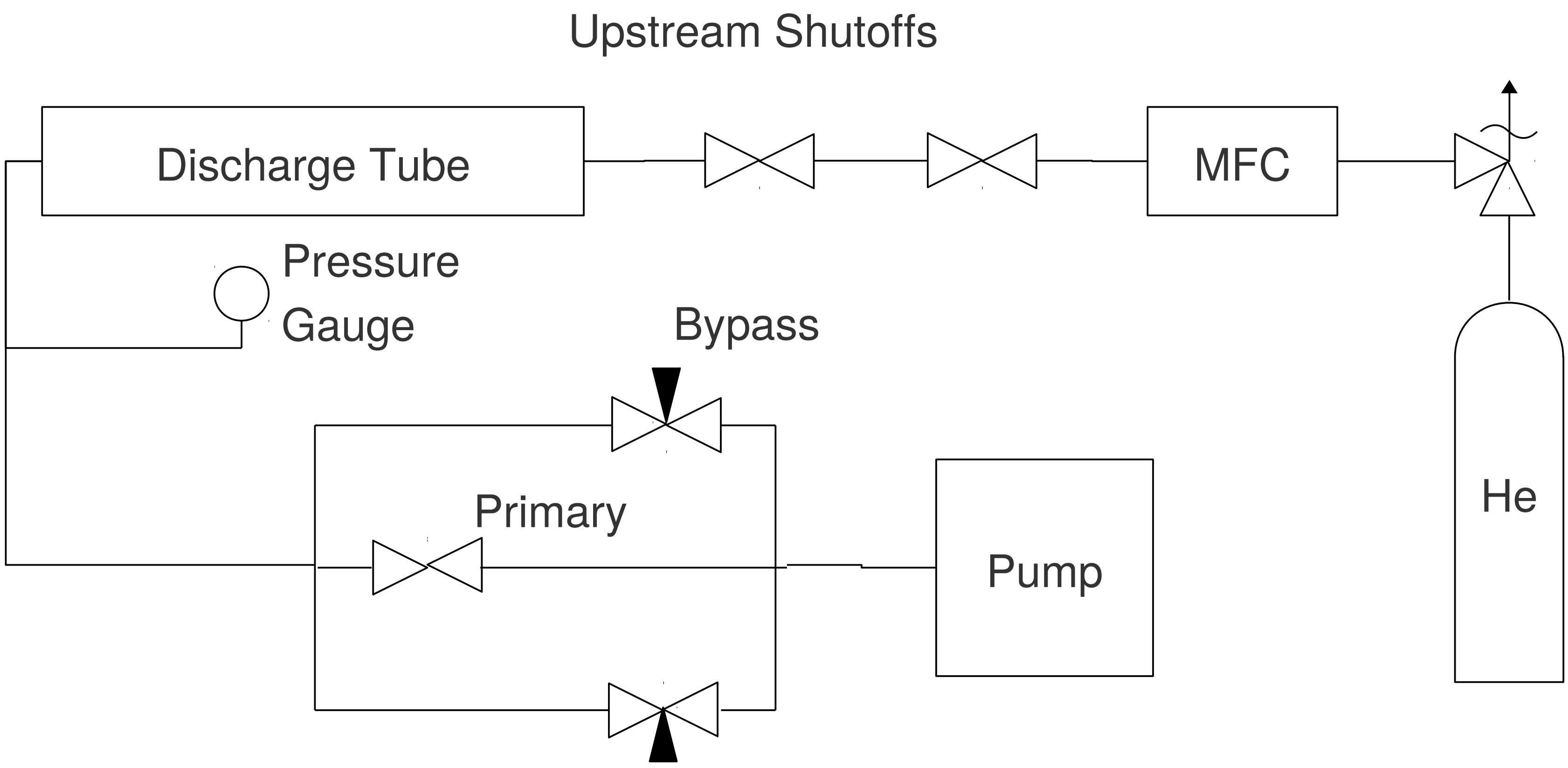
First Pulsed Plasma Measurements

- Normally controlled by boundaries which limit gas flow out of plasma

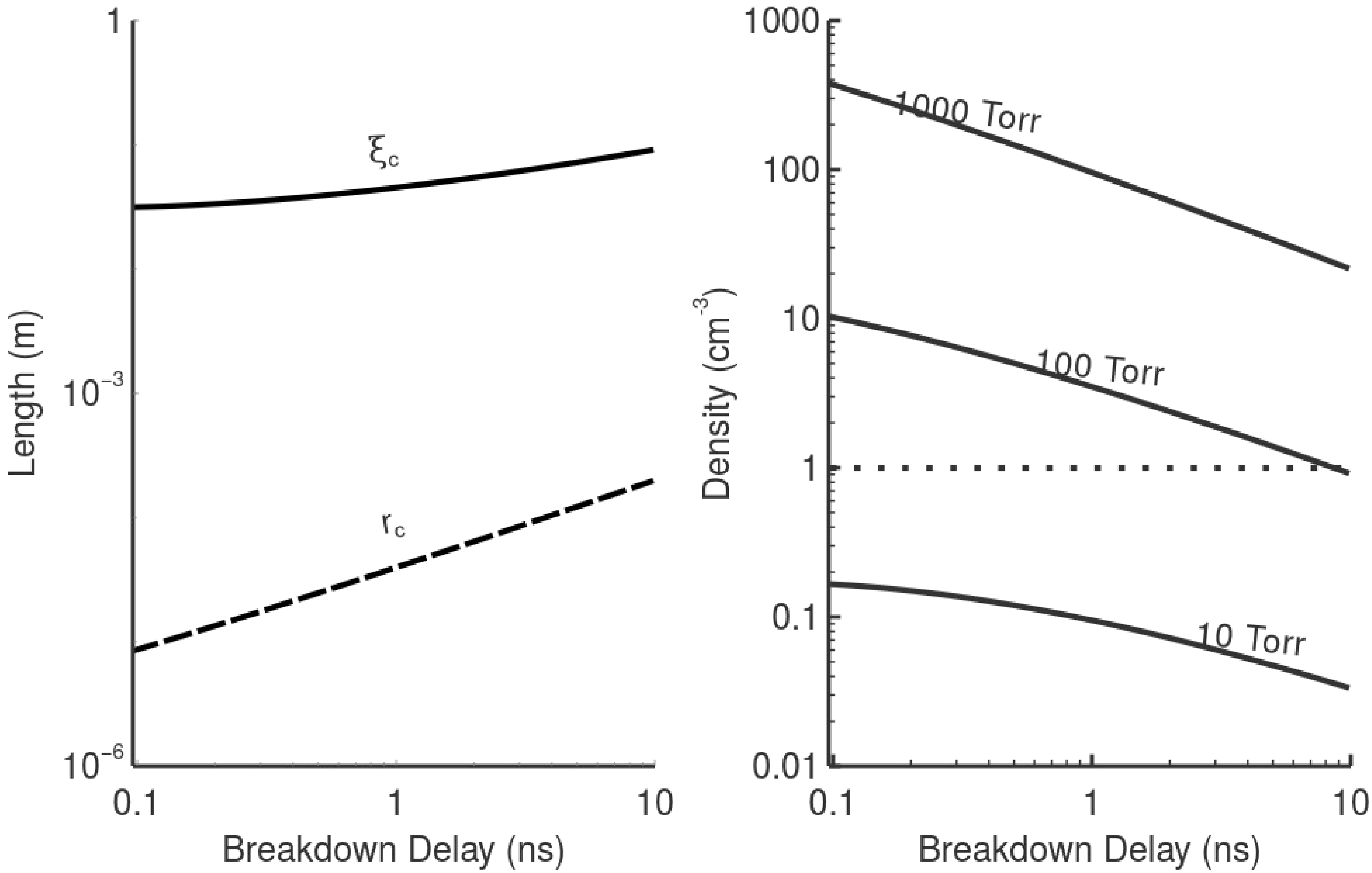
1. Discharge causes gas heating
2. Results in local rarefaction
3. Increases reduced electric field
4. Increases ionization rate and power deposition
5. Results in increased gas heating

$$\exp(\alpha d) \approx 10^8 \qquad \rightarrow \qquad \alpha d \approx 18 - 21$$

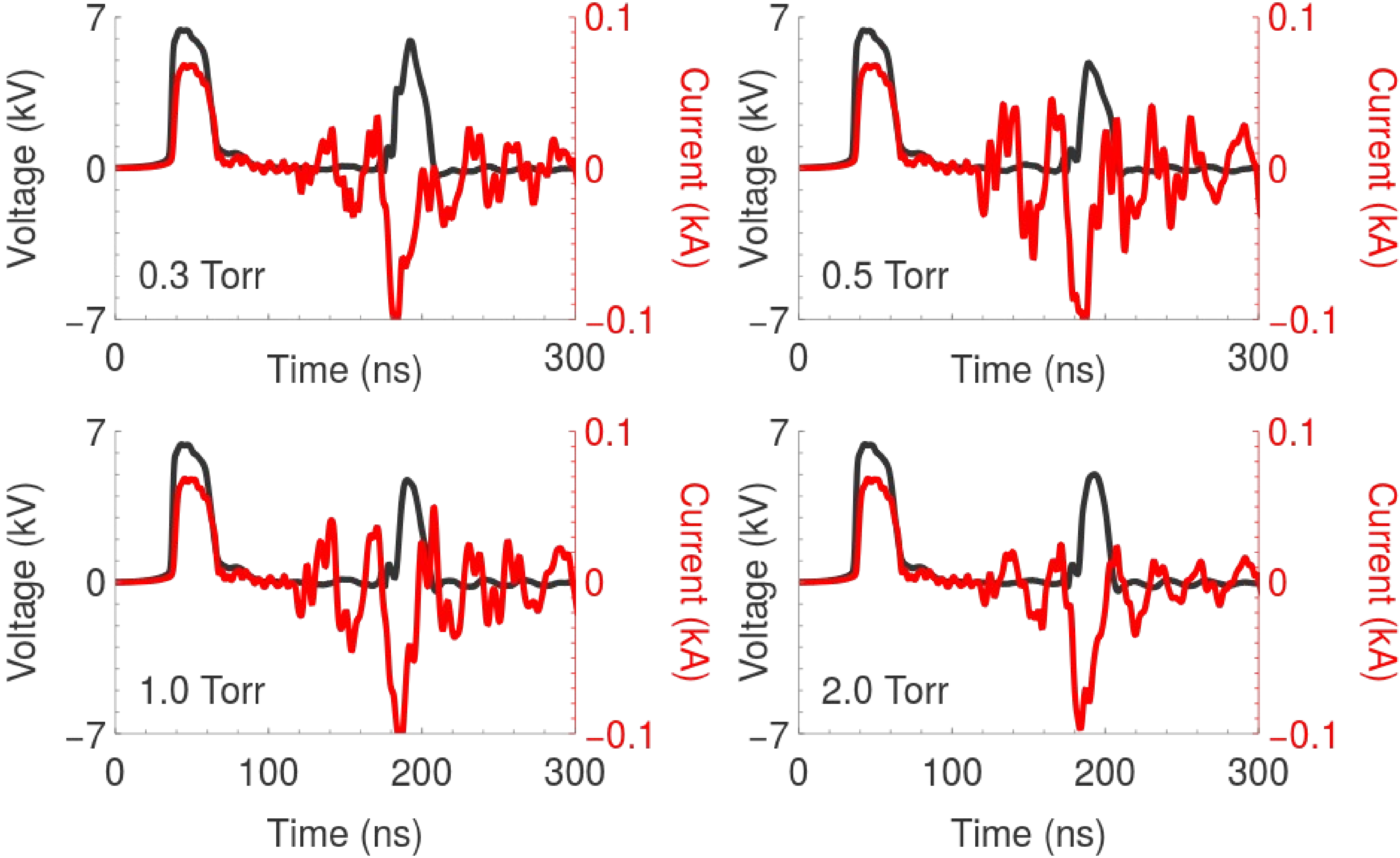
- Condition at which space charge ~ applied electric field
- States that total number of electrons ~ 10^8
- α(3.8 kV/cm / (16 Torr)) / 18 $\rightarrow$ d = 6 mm



Pumping System

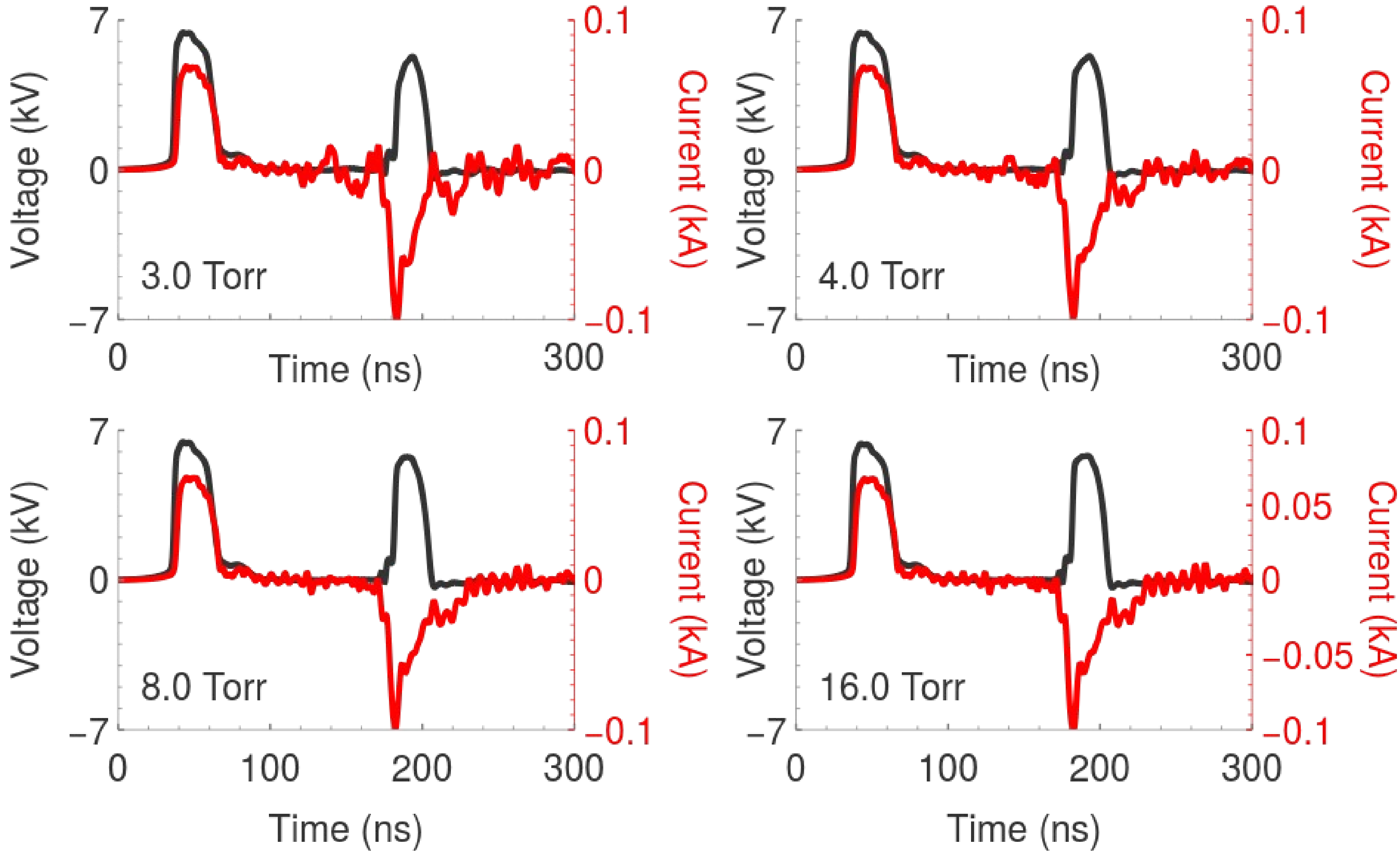


Homogeneous Streamers

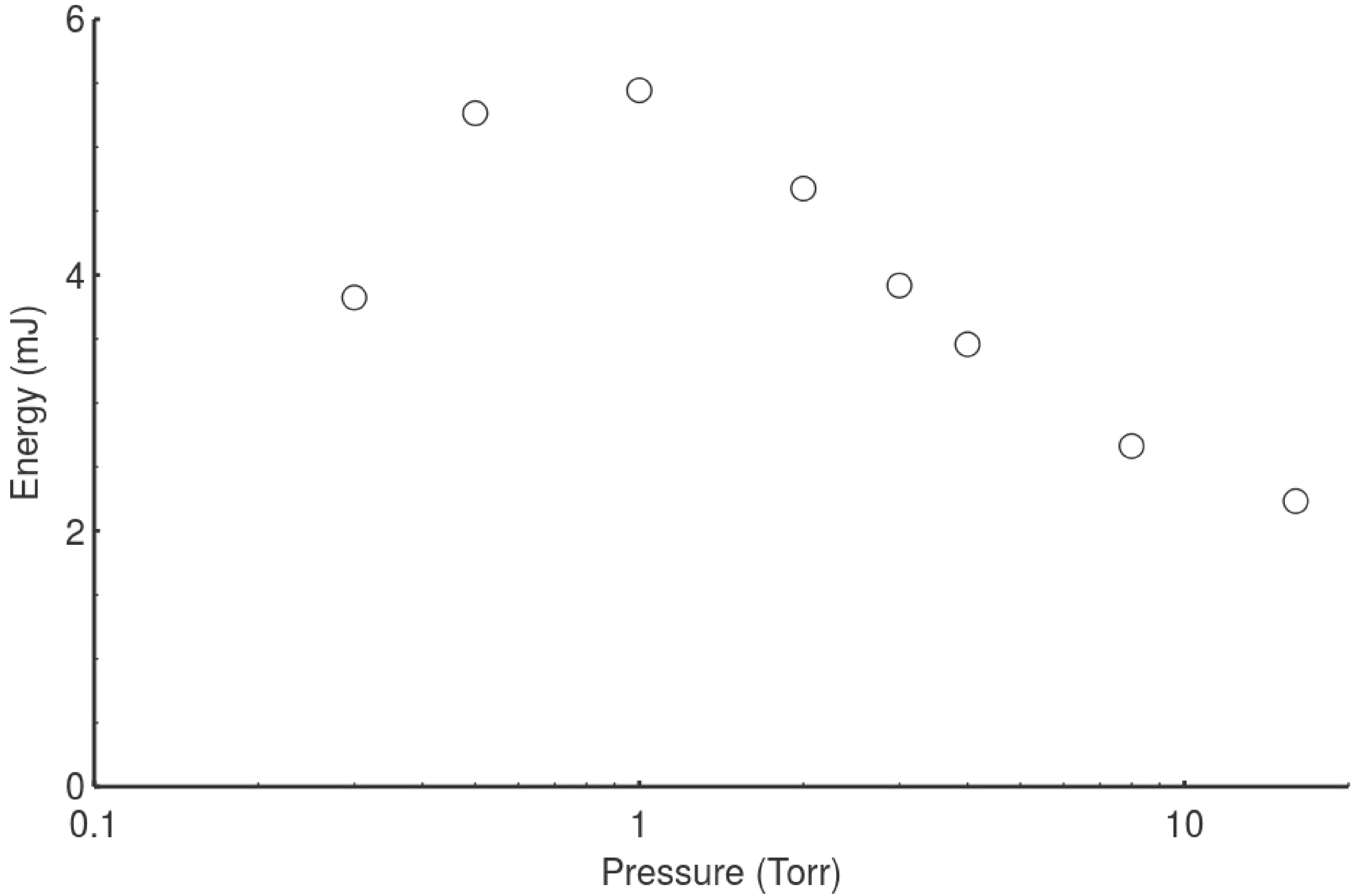


7
0
−7
0.1
0
−0.1
Voltage (kV)
Current (kA)
0.3 Torr
0.5 Torr
1.0 Torr
2.0 Torr
0
100
200
300
Time (ns)

Waveforms 1


3.0 Torr
4.0 Torr
8.0 Torr
16.0 Torr
Voltage (kV)
Current (kA)
Time (ns)
7
0
−7
0.1
0.05
0
−0.05
−0.1
0
100
200
300

Waveforms 2



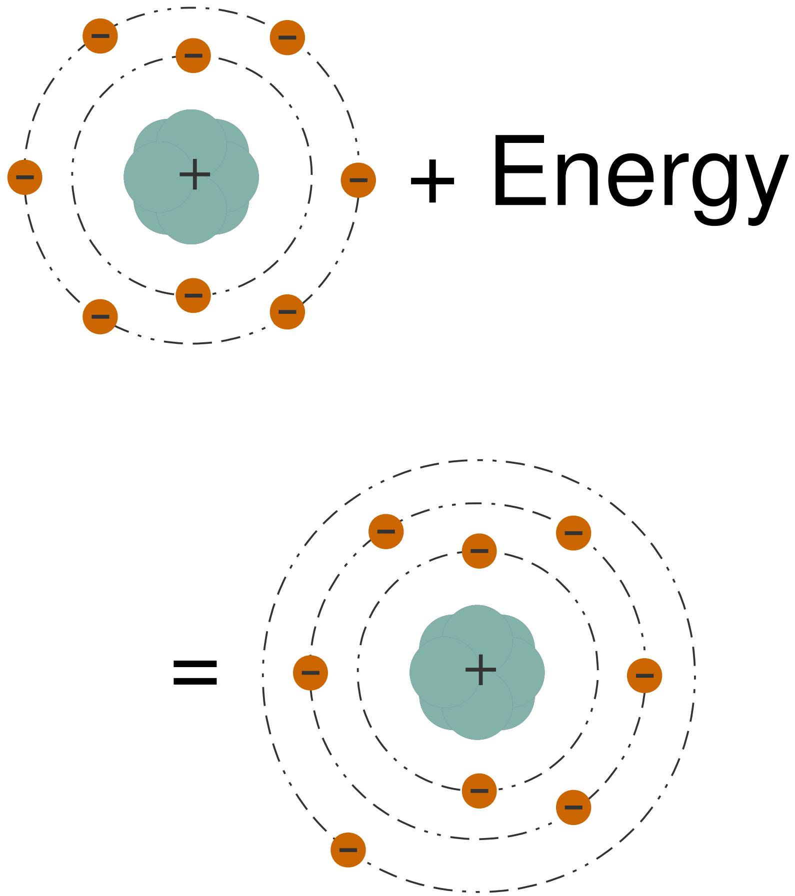
6
4
2
0
Energy (mJ)
0.1
1
10
Pressure (Torr)

$$\Delta N = \Delta N_0 \frac{1}{1 + W\tau_{\text{eff}}} = \Delta N_0 \frac{1}{1 + I/I_{\text{sat}}}$$

$$W = \frac{\sigma I}{h\nu} \qquad \rightarrow \qquad I_{\text{sat}} = \frac{h\nu}{\sigma \tau_{\text{eff}}}$$

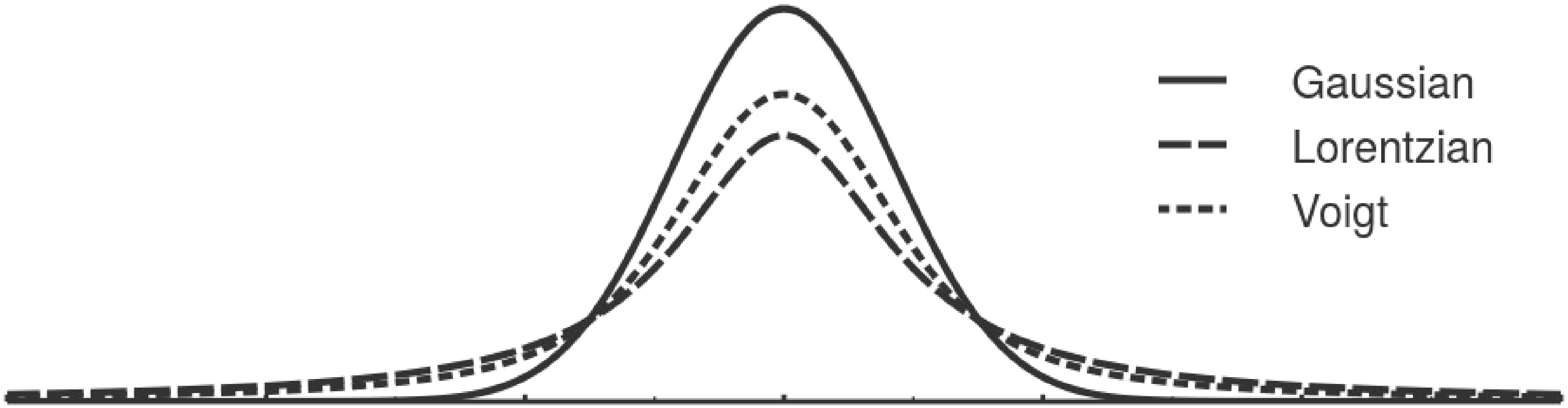
$$I_{\text{sat}} = \frac{2\sqrt{2} h_{21}}{\sigma\tau} = 0.45 \quad \text{mW} \cdot \text{cm}^{-2}$$

- Energy added to atom
  - Collisions
  - Absorption of light
- One or more electrons to higher orbit → excited atom
- Excited state can emit photon to reach lower energy state
  - Occurs with a specific rate



- $\Delta S = 0$
- $\Delta L = +1$ or 0
- $\Delta J = +1$ or 0
- $L = 0$ cannot transition to $L = 0$
- $J = 0$ cannot transition to $J = 0$

Electric-Dipole Selection Rules



Lorentzian $$g(\omega) = -\frac{1}{4\pi^2}\frac{A\lambda^3}{\Delta\omega_a}\frac{1}{1+[2(\omega-\omega_a)/\Delta\omega_a]^2}$$

Voigt $$g(\omega) = \sqrt{\frac{2\ln 2}{\pi^3}}\frac{\Delta\omega_d}{\Delta\omega_d}\int_{-\infty}^{\infty}\frac{1}{[(\omega-\omega_a)-\omega']^2+4^2} \times \exp\left[4\ln 2\left(\frac{\omega'}{\Delta\omega_d}\right)^2\right]d\omega'$$

Gaussian $$g(\omega) = \sqrt{\frac{4\log 2}{\pi\Delta\omega_d^2}}\exp\left[-(4\log 2)\left(\frac{\omega-\omega_a}{\Delta\omega_d}\right)^2\right]$$

$$\Delta\omega_a = A + BP$$

$$\Delta\omega_d = \omega_0\sqrt{\frac{8k_\mathrm{B}T_\mathrm{g}\ln 2}{Mc^2}}$$

- Pressure and natural broadening
- Lorentzian lineshape
- *A* from NIST ASD
- *B* from Urabe et al. (empirical measurement)
- *P* is measured pressure

- Doppler broadening (thermal motion)
- Gaussian lineshape

Broadening mechanisms

$$\sigma(\omega) = A\frac{\lambda^2}{8\pi}\frac{g_1}{g_2}g(\omega)$$

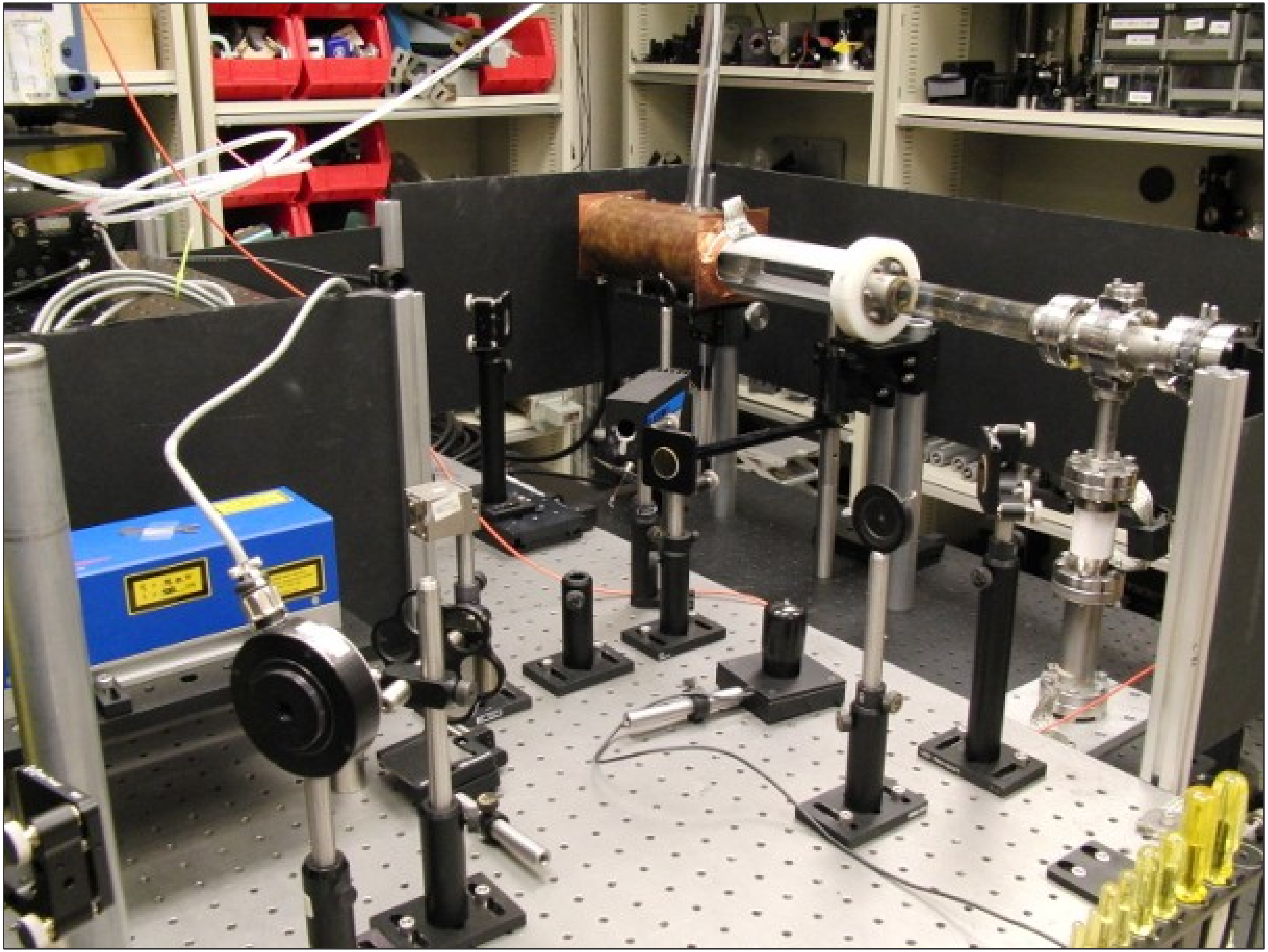
$$\frac{dI(x,\omega)}{dx} = -\sigma(\omega)N(x)I(x,\omega)$$

$$T(\omega) = \frac{I(x,\omega)}{I_0(\omega)} = \exp\left[-\sigma(\omega)\int_0^x N(x')dx'\right]$$

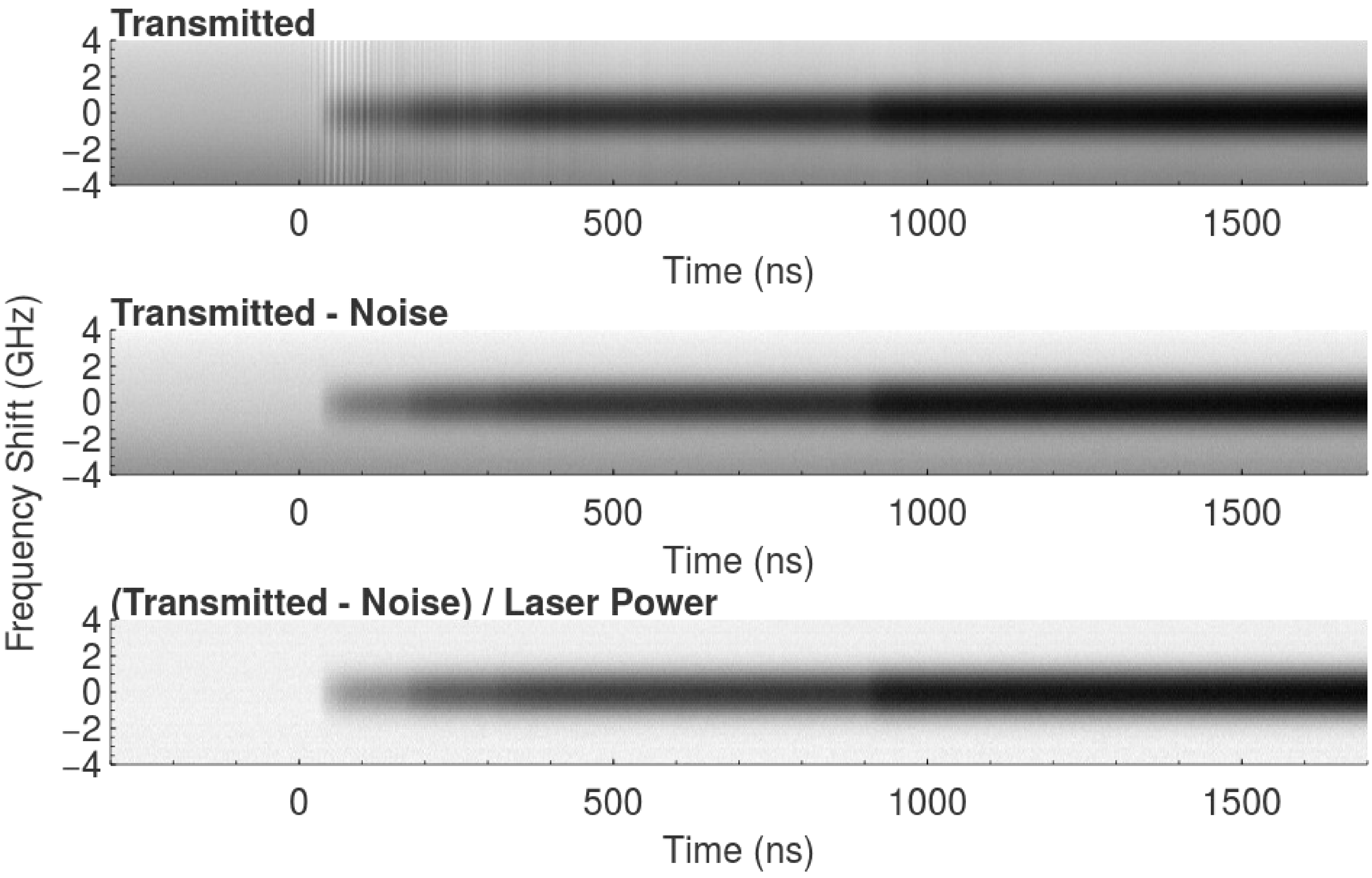
$$\langle N\rangle = \int_0^x N(x')dx'$$

- Absorption cross section is dependent on wavelength and lineshape
  - Voigt lineshape used: Doppler and pressure broadening
- Determine best-fit between spectrum and model with Levenburg-Marquardt algorithm, free variables:
  - Temperature
  - Line-integrated density

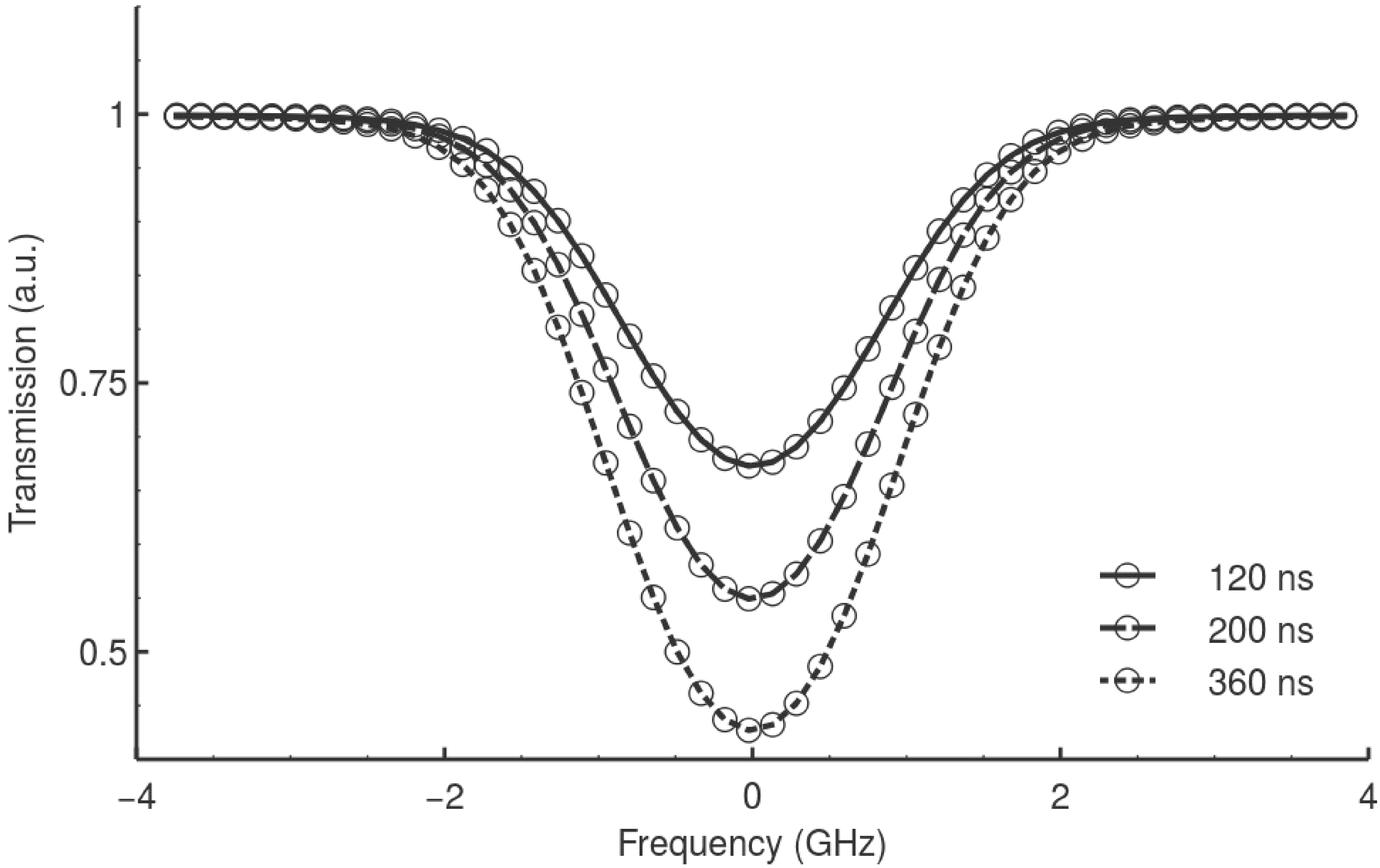
Absorption Model



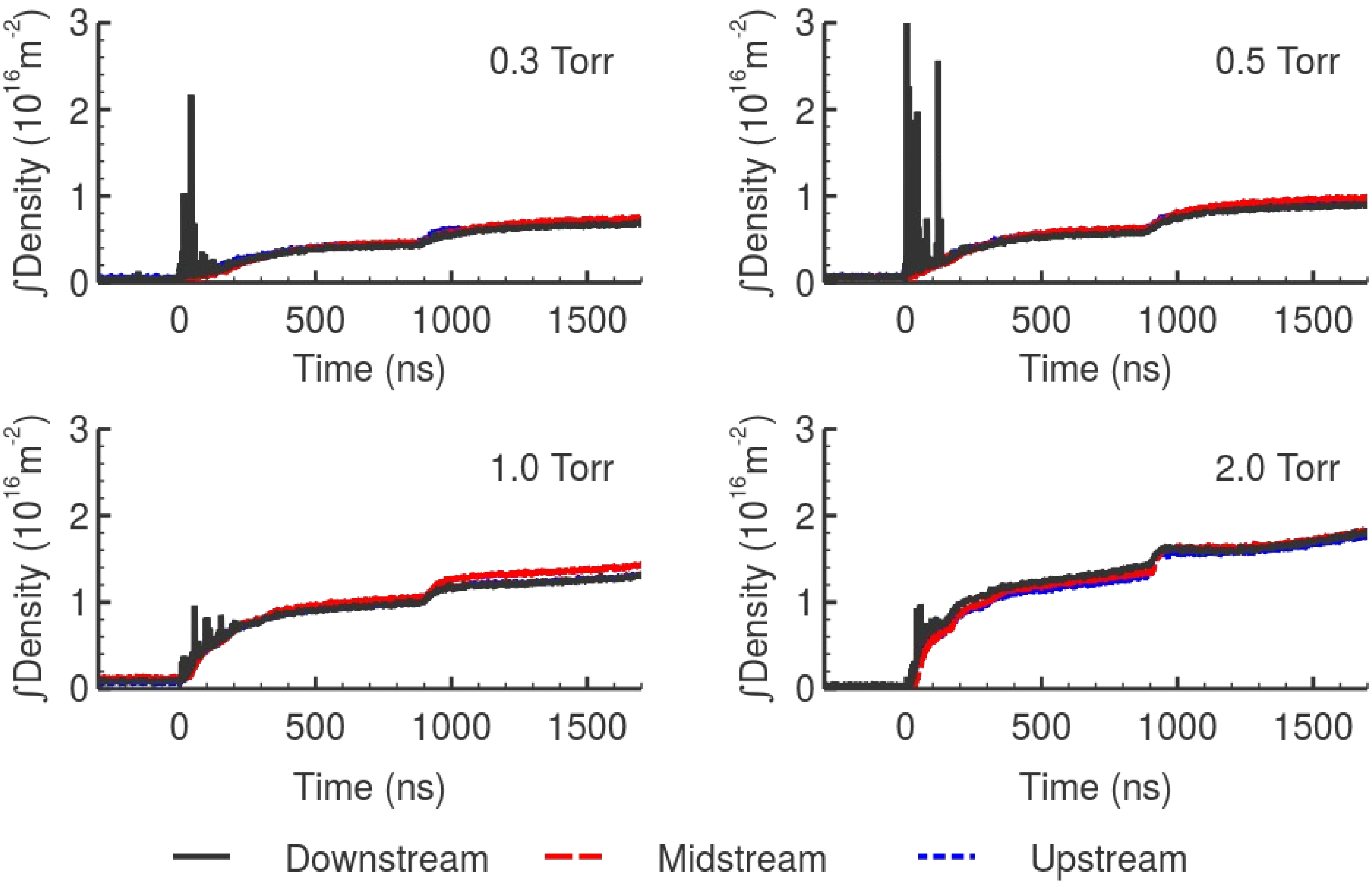
Laser-Absorption Spectroscopy Setup



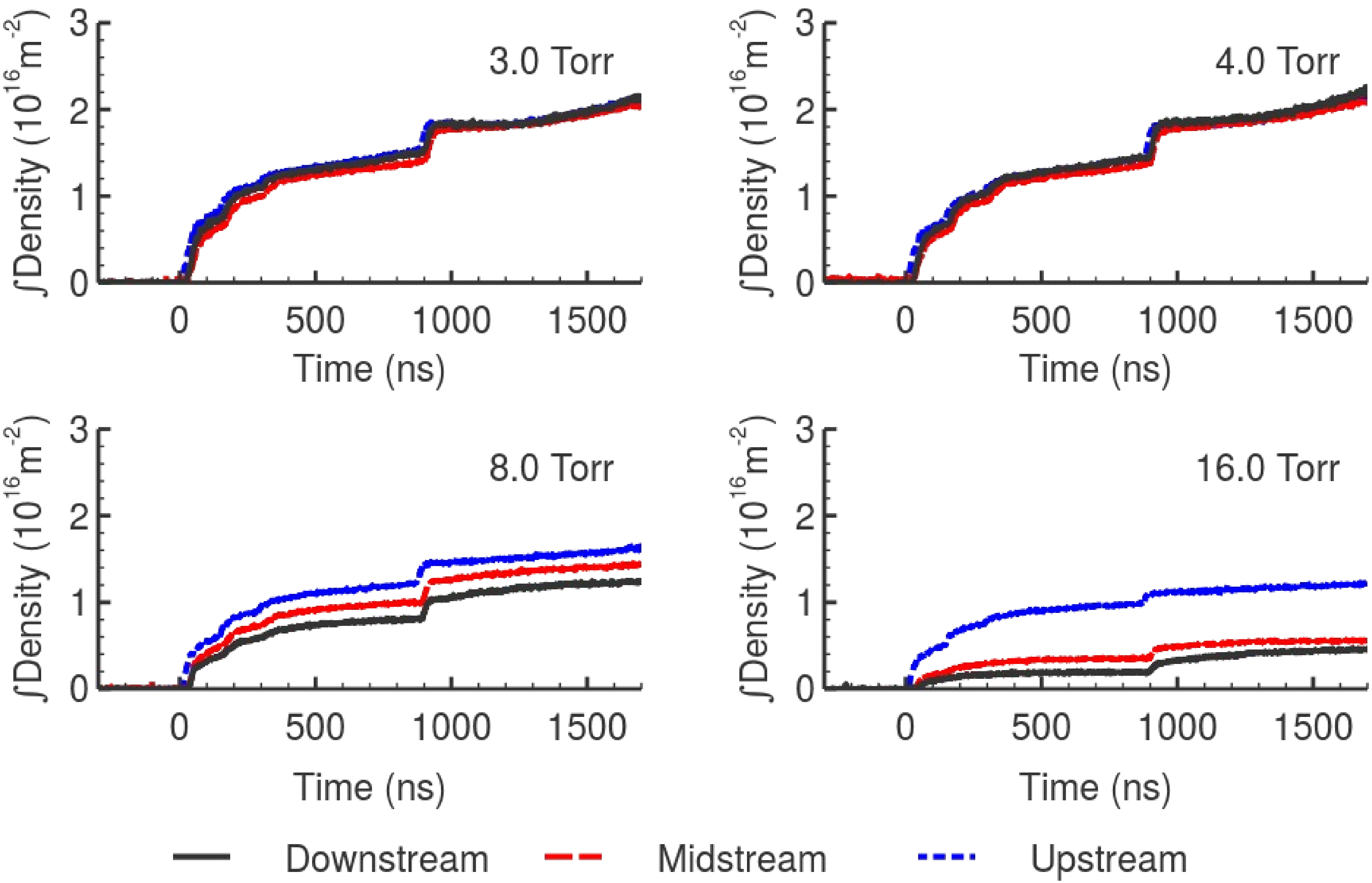
Spectral Post-Processing



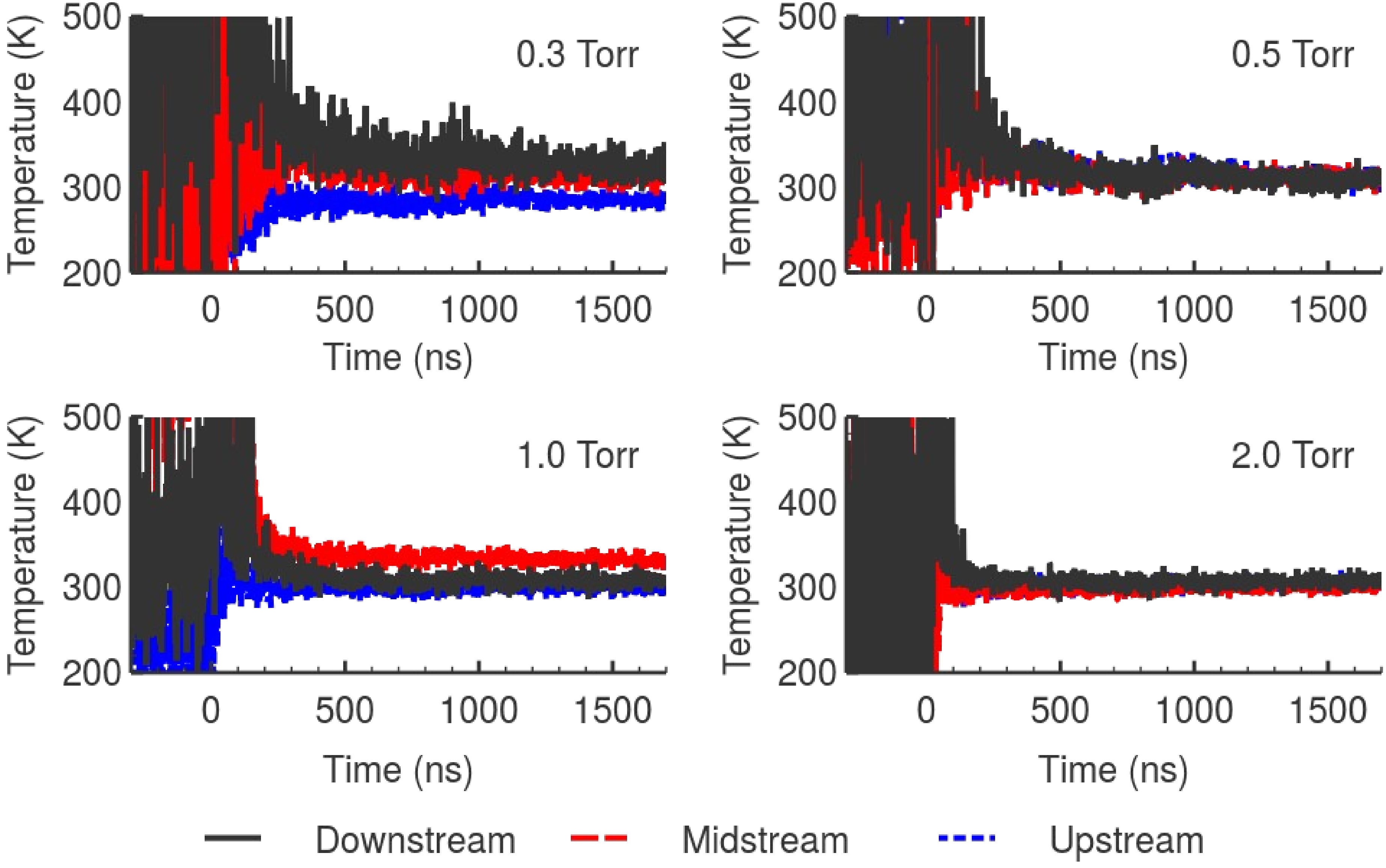
Absorption Spectra Matching

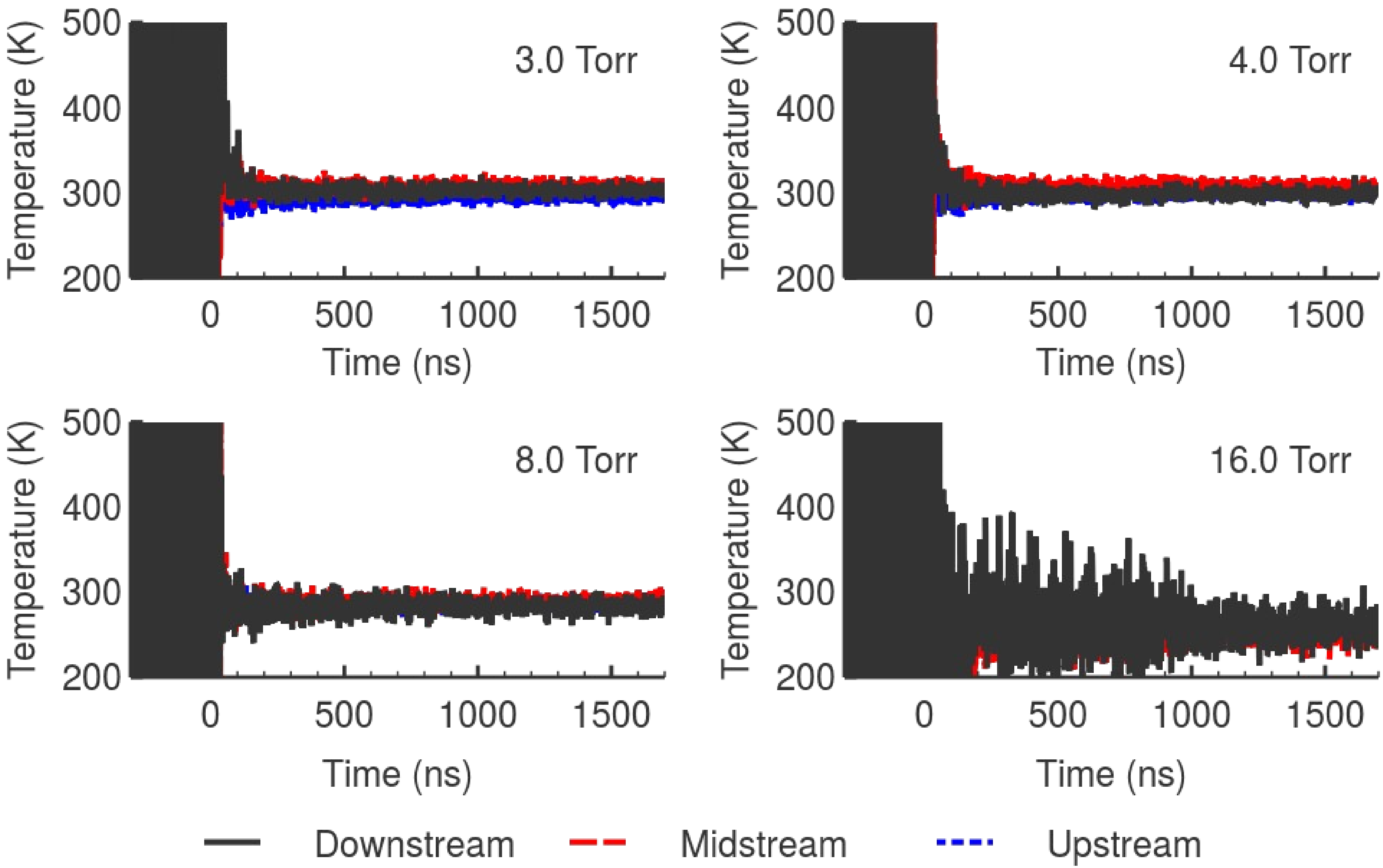


Metastables 1



3.0 Torr
4.0 Torr
8.0 Torr
16.0 Torr
∫Density (10^{16}m^{-2})
Time (ns)
Downstream
Midstream
Upstream


0.3 Torr
0.5 Torr
1.0 Torr
2.0 Torr
Temperature (K)
Time (ns)
500
400
300
200
0
500
1000
1500
Downstream
Midstream
Upstream

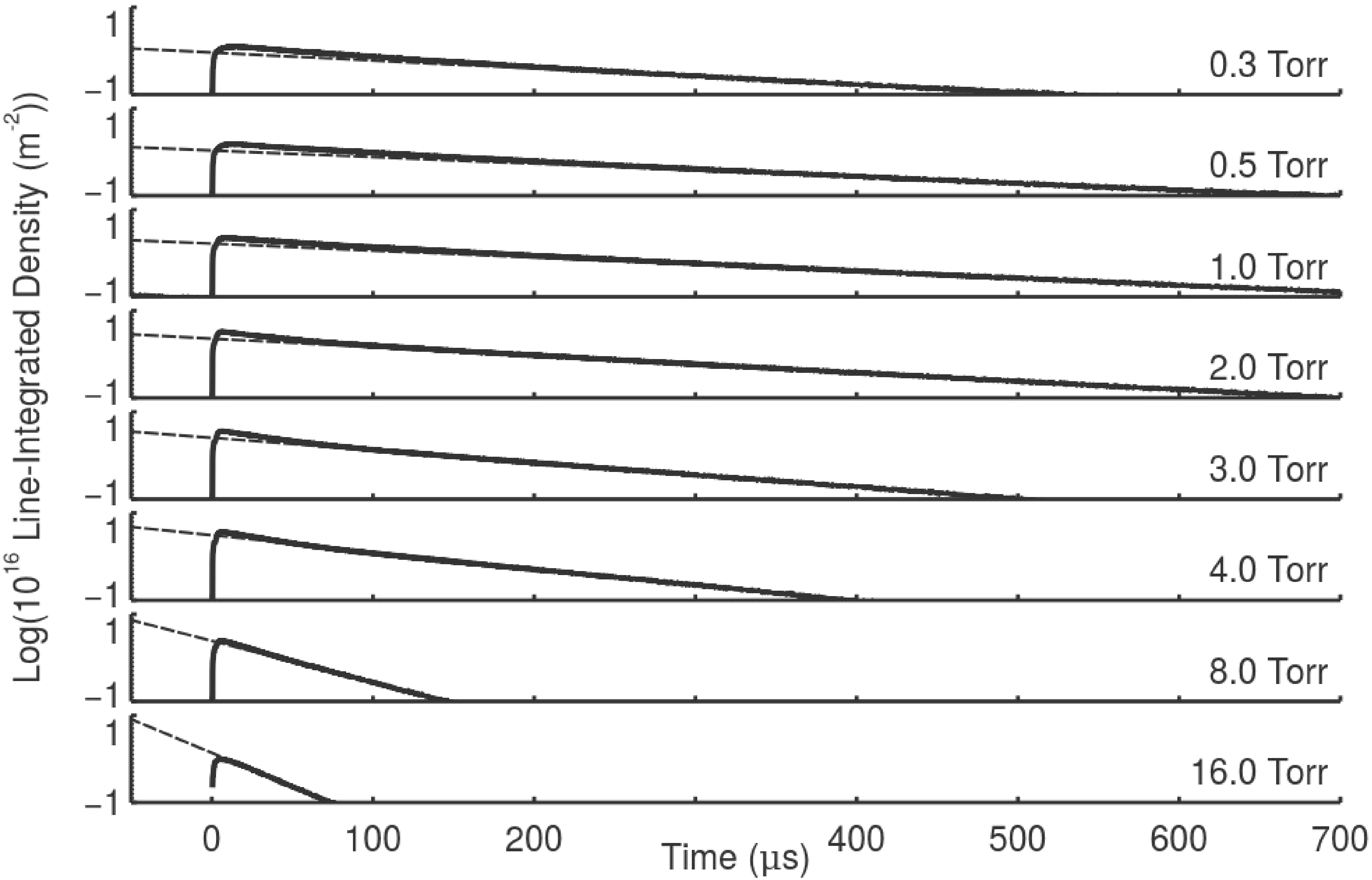


Temperatures 2

$$\frac{dN}{dt} = \frac{D_0 N}{L^2} + \delta P^2 N + \beta_{11} N^2 + C N_{imp} N$$

- From Deloche et al., *C* from Pouvesle et al.
- D0 = 410 cm^2 Torr/s
- δ = 0.2 $Torr^{-2}$/s
- β11 = 1.5 x 10^{-9} cm^3/s
- C = 7.0 x 10^{-11} cm^3/s

Metastable Destruction Coefficients



Metastable Decay

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\vec{E} + \vec{v} \times \vec{B}\right) \cdot \nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{coll}}$$

- Describes the evolution of the distribution function for a particle α
- 7 independent variables
- Simple equilibrium solutions possible
  - Maxwell-Boltzmann
  - Druyvesteyn
- Generally difficult to solve otherwise

Boltzmann Equation

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \vec{u_\alpha}) = G_\alpha - L_\alpha .$$

- Boltzmann equation can be converted into moments by integrating over velocity space
- Introduces particle density *n* and mean fluid velocity, *u*
- *G* – source terms
- *L* – loss terms

$$m_\alpha n_\alpha\left[\frac{\partial\vec{u_\alpha}}{\partial t}+(\vec{u_\alpha}\cdot\nabla)\vec{u_\alpha}\right]=q_\alpha n_\alpha(\vec{E}+\vec{u_\alpha}\times\vec{B})-\nabla\cdot\vec{\Pi}+\vec{f}|_{\text{coll}}$$

- Second momentum obtained by multiplying by velocity and integrating over velocity space
- Describes changes in fluid velocity over space
- Introduces pressure tensor Π.
- $f|_{coll}$ expresses change in momentum due to collisions

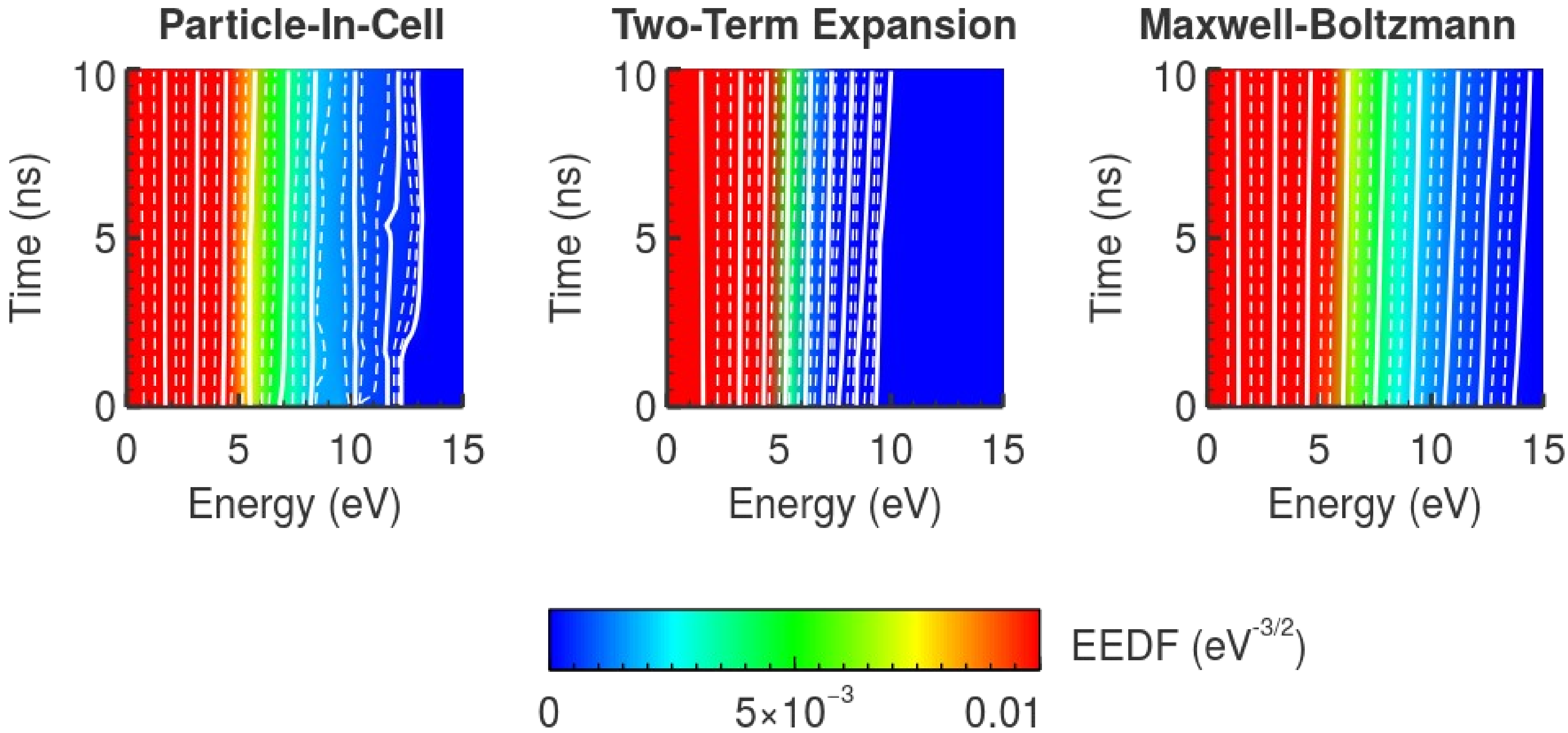
Second Moment: Conservation of Momentum

$$\frac{\partial}{\partial t}\left(\frac{3}{2}p_\alpha\right) + \nabla \cdot \frac{3}{2}(p_\alpha \vec{u}_\alpha) + p_\alpha \nabla \cdot \vec{u}_\alpha + \nabla \cdot \vec{q}_\alpha = \frac{\partial}{\partial t}\left(\frac{3}{2}p_\alpha\right)\Big|_{\text{coll}}$$

- Third momentum obtained by multiplying by energy and integrating of velocity space
- Energy expressed in terms of pressure, *p*
- RHS includes energy losses and gains resulting from collisions

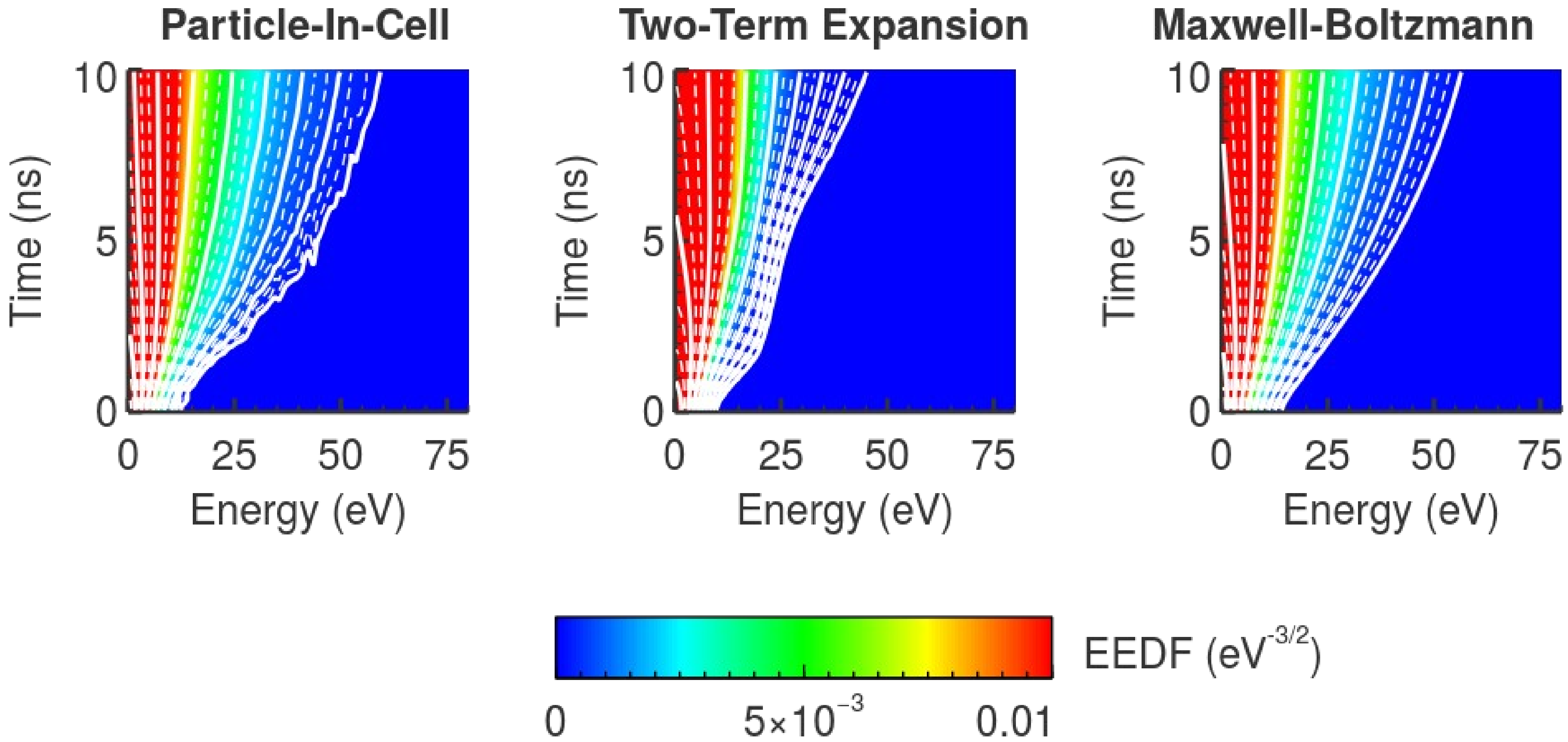
Third Moment: Conservation of Energy

- Must assume an EEDF for reaction rates
- Baseline: 0D particle-in-cell (XPDP1)
- Two approaches considered:
  - Maxwell-Boltzmann distribution
  - Solutions of two-term expansion of Boltzmann equation (BOLSIG+)



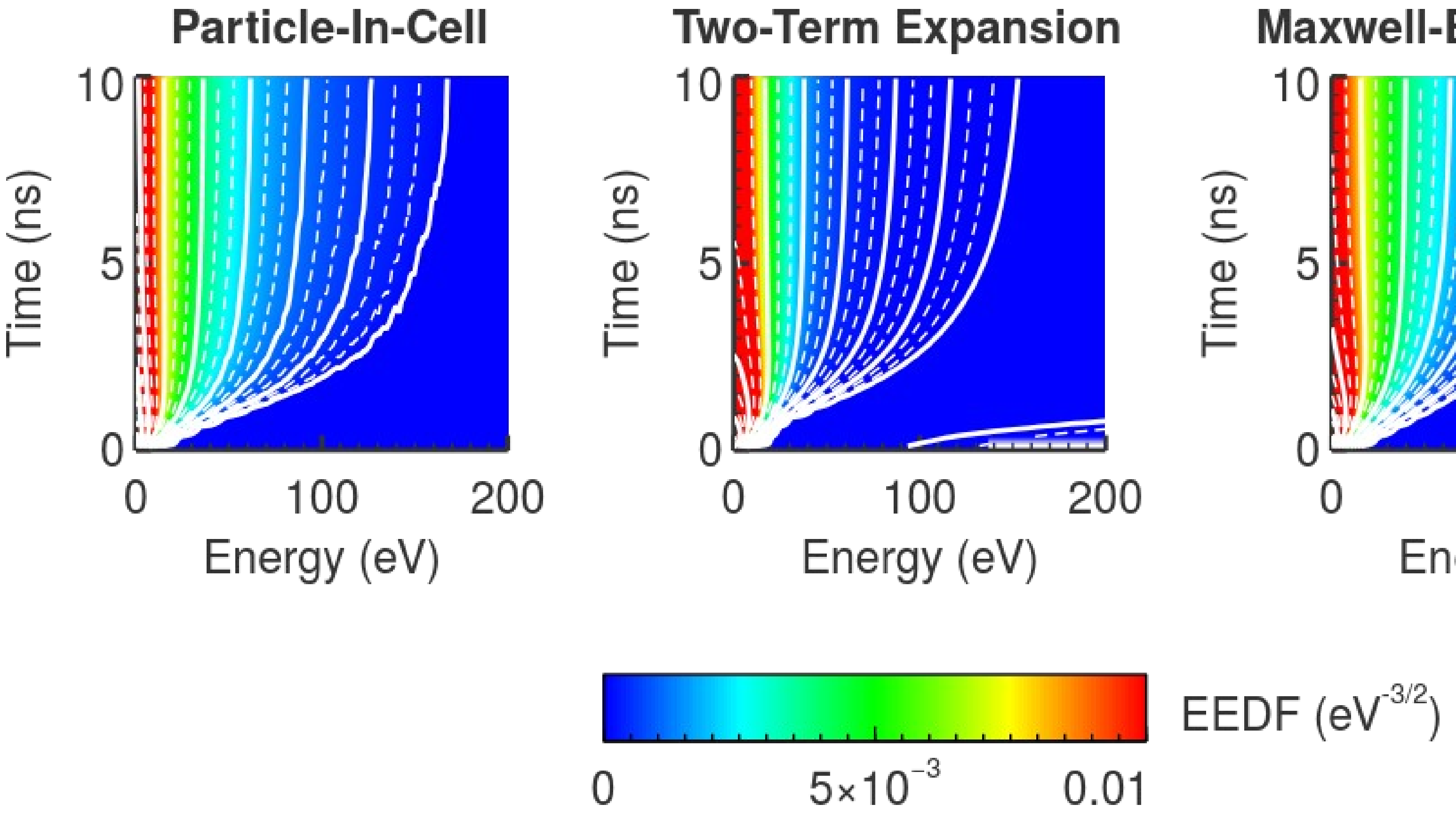
- Both approaches show reasonable agreement with particle-in-cell
- Two-term: less high-energy electrons
- Maxwell-Boltzmann: more high-energy electrons

EEDF Comparisons (10 Td)



- Unexpected contour variations in two-term expansion solutions
- Better agreement between Maxwell-Boltzmann and particle-in-cell

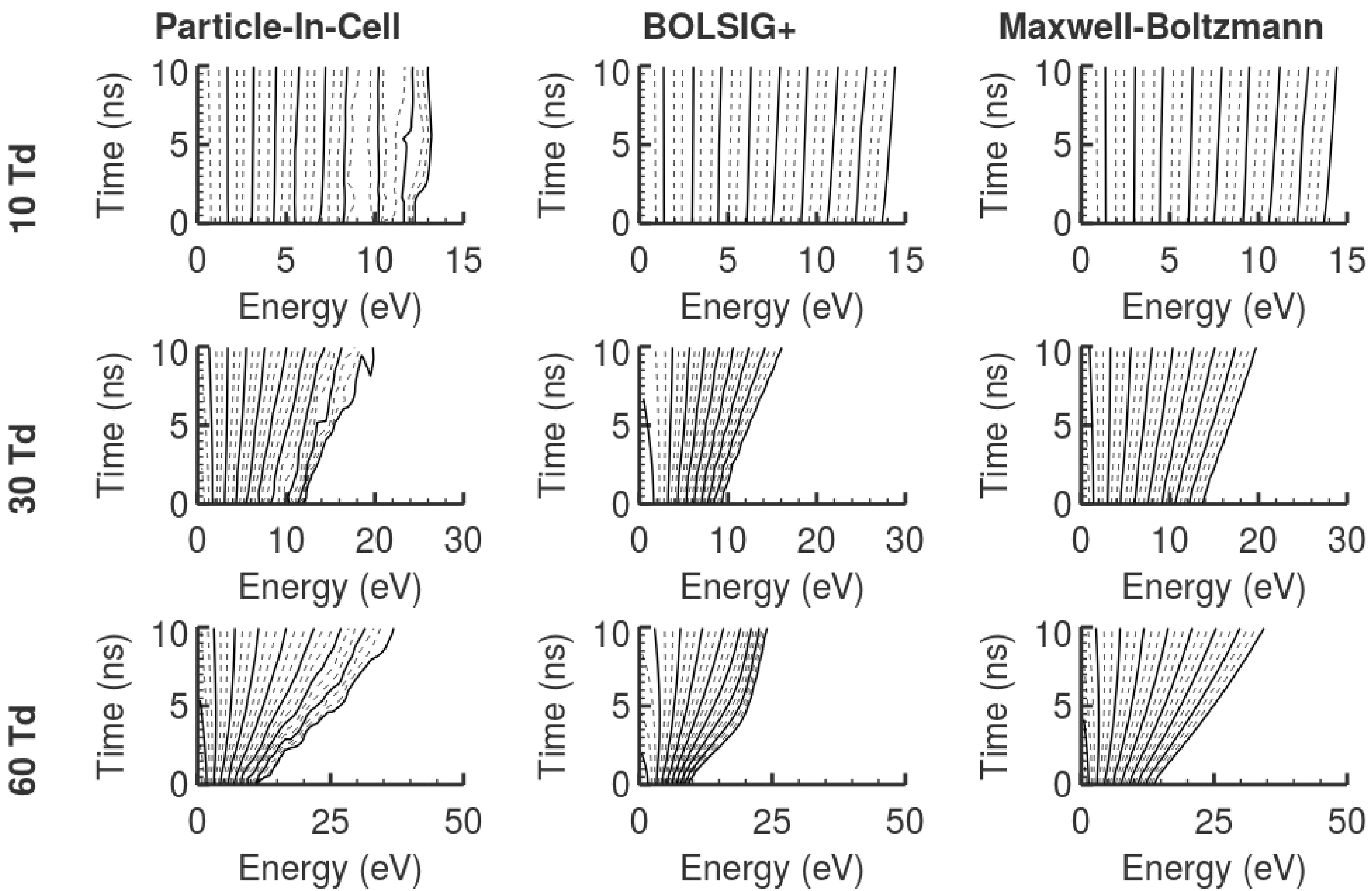
EEDF Comparisons (100 Td)



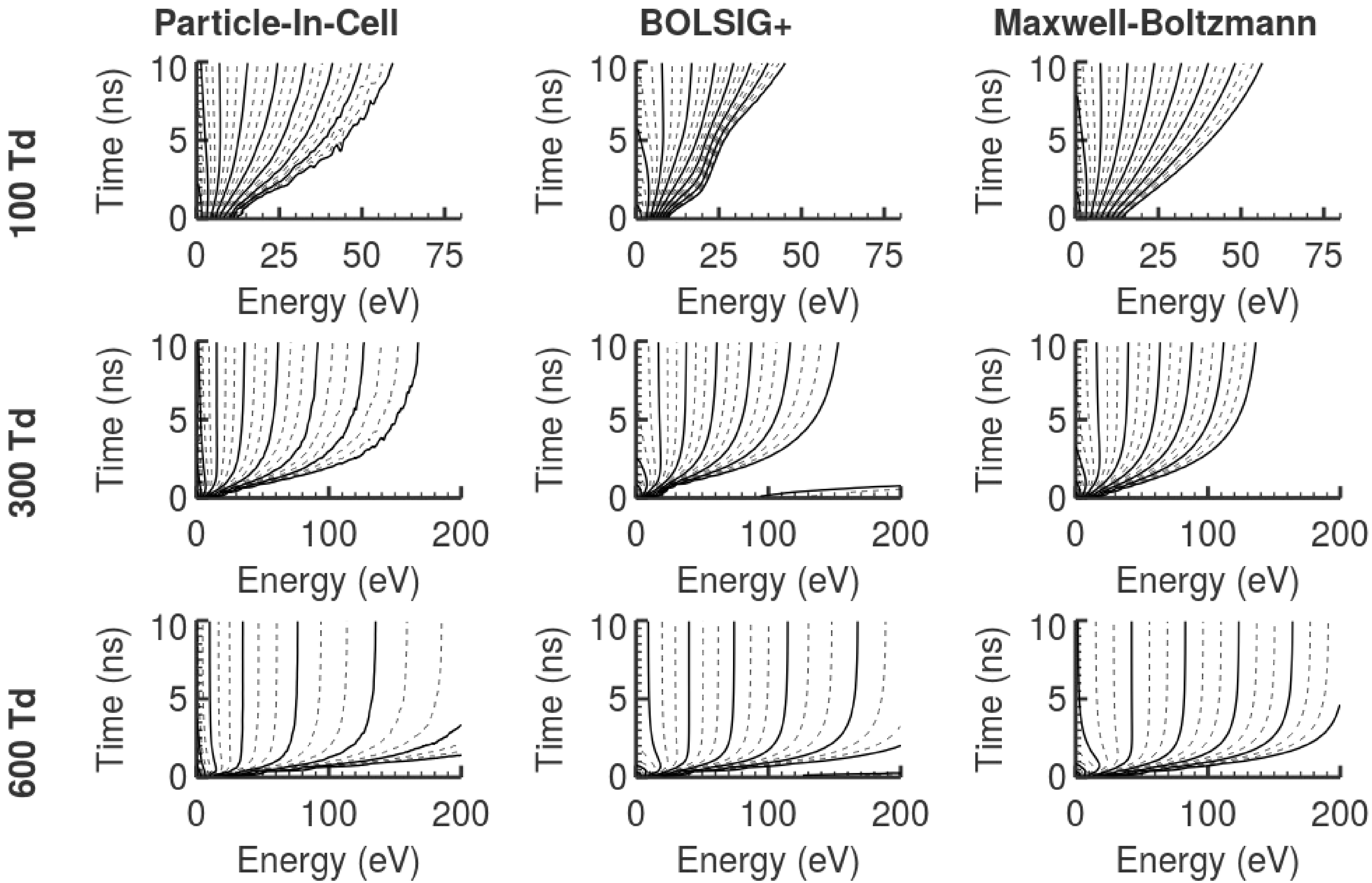
- Particle-in-cell: more 100 eV+ electrons than other results
- Two-term expansion solution best match this behavior
- Beginning of beam-like behavior?

EEDF Comparisons (300 Td)

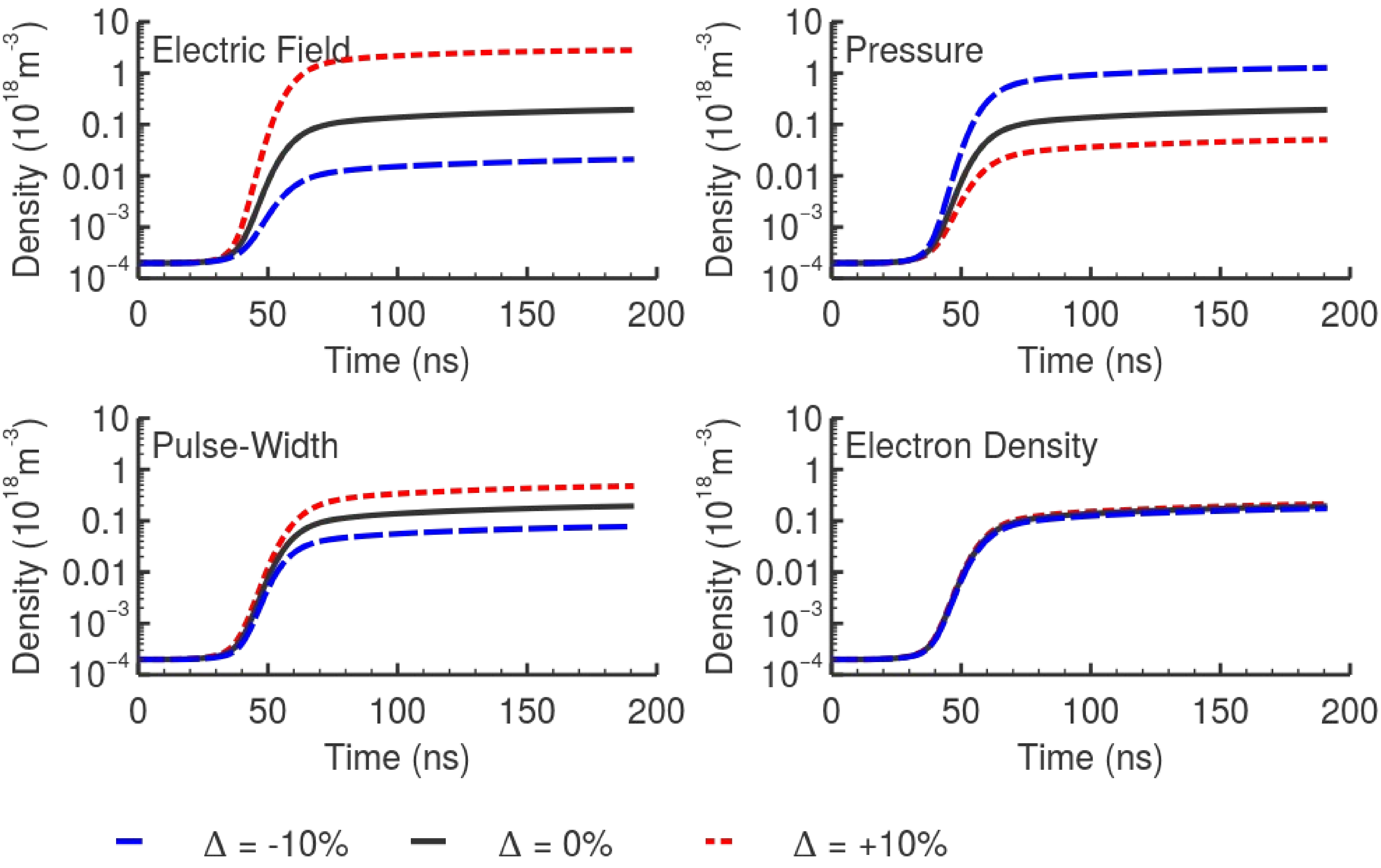
- $<$ 100 Td: Maxwell-Boltzmann gives best agreement with particle-in-cell
- $>$ 100 Td: BOLSIG+ gives best agreement with particle-in-cell
- Primary discrepancy in the number of high energy electrons: Particle-in-cell $>$ BOLSIG+ $>$ Maxwell-Boltzmann
- Better overall agreement led to use of Maxwell-Boltzmann distribution



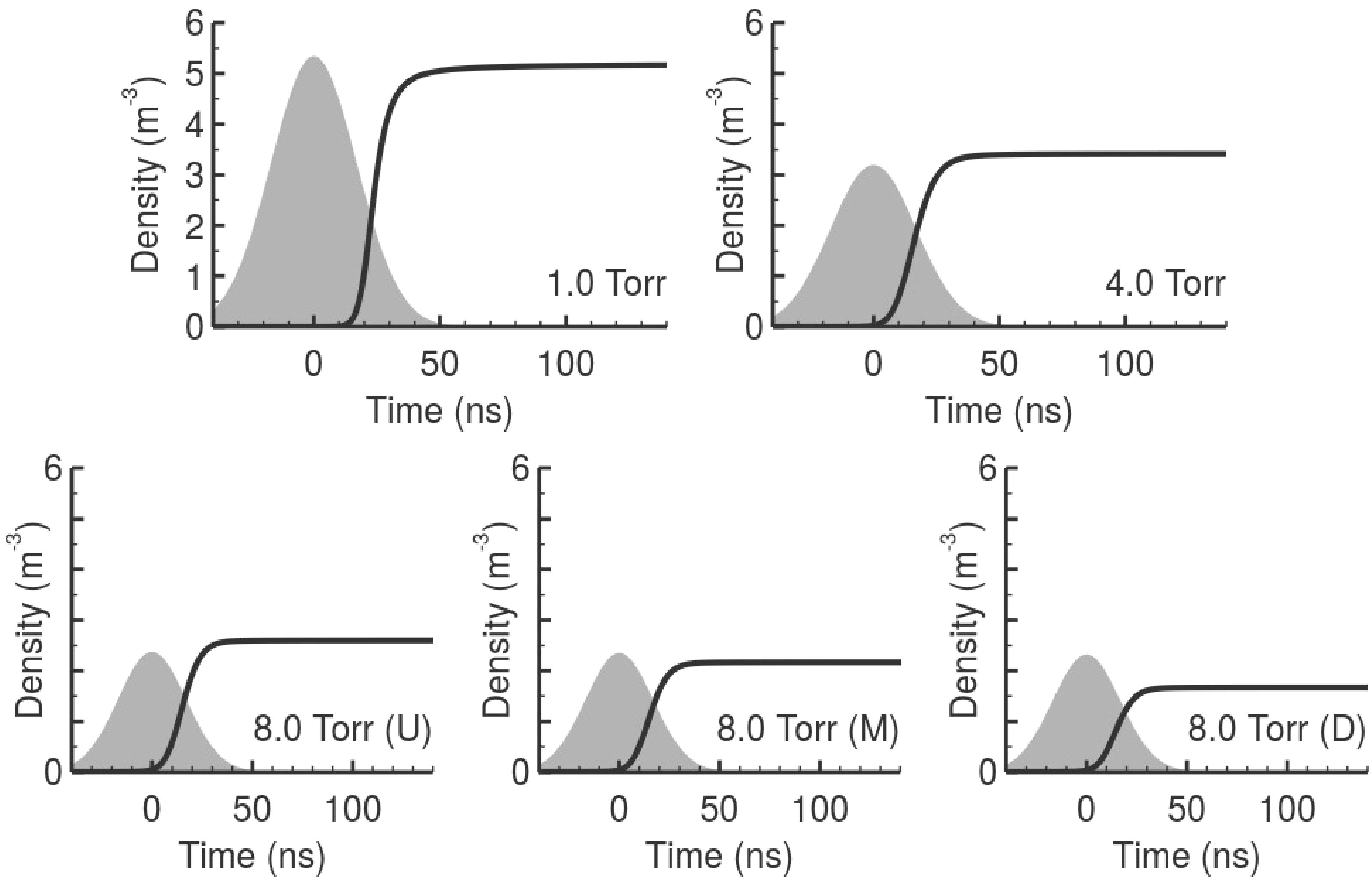
EEDF Contours 1



EEDF Contours 2



Perturbation Results



Density (m^-3)
Time (ns)
1.0 Torr
4.0 Torr
8.0 Torr (U)
8.0 Torr (M)
8.0 Torr (D)

- Pressures: 0.3 – 16.0 Torr
- Spex HR460 monochromator, 0.88 nm bandpass
- 1,200 grooves/mm
- Photomultiplier tube, maximum 3.0 ns rise time
- Relative intensities calibrated with tungsten blackbody lamp

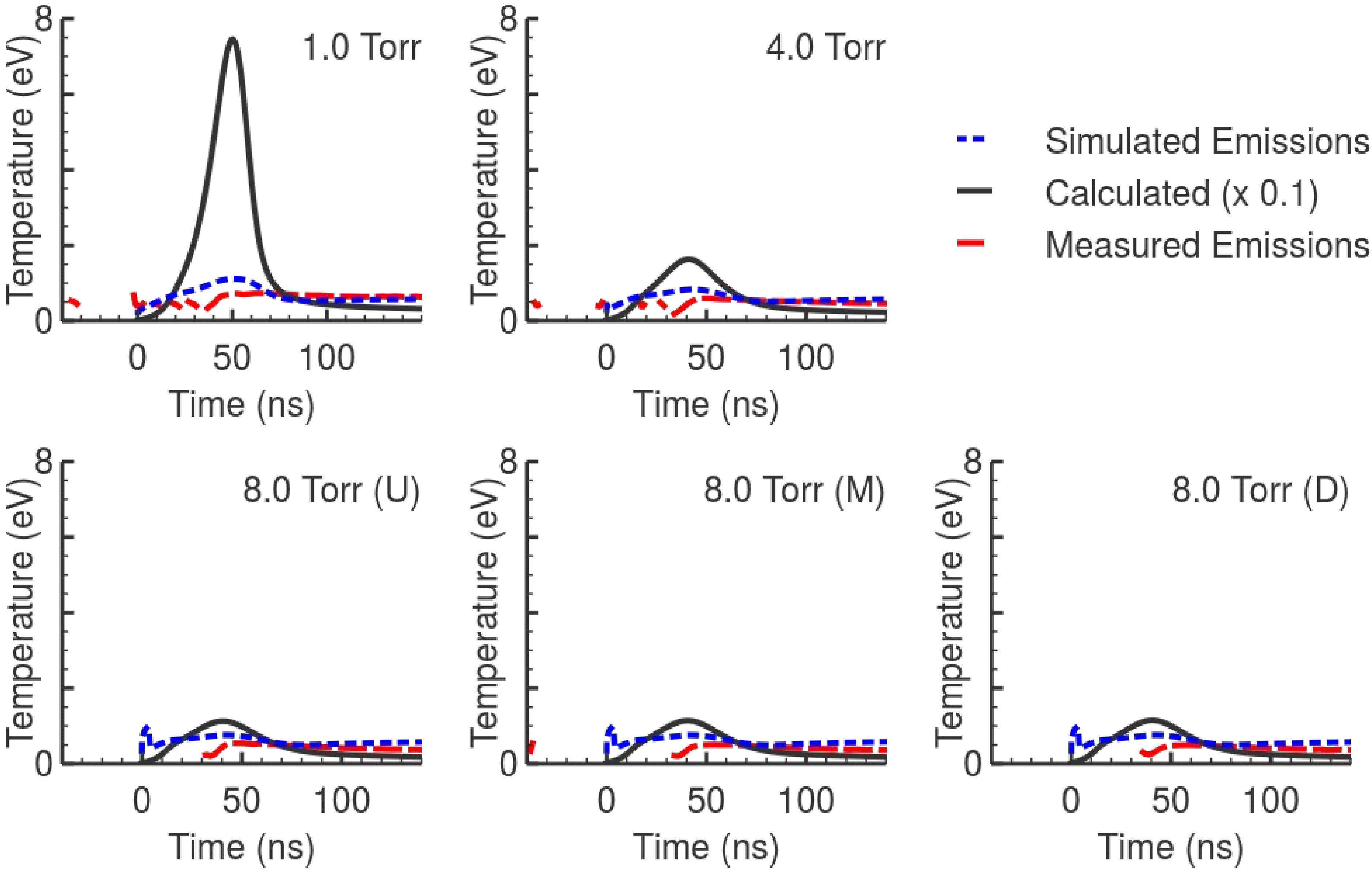
$$T = \frac{k_0 R \left[\pi \log\left(k_0 R\right)\right]^{1/2}}{1.6}$$

$$k_0 = \frac{\lambda_0^3 N_1}{8\pi} \frac{g_2}{g_1} \frac{A_{21}}{\pi^{1/2} v_{\mathrm{th}}}$$

- Used Holstein's trapping factor calculations
- Assumed Doppler broadening
- Infinite cylinder

$$\frac{I_{i,j}}{I_{i',j'}} = \frac{\lambda_{i',j'} A_{i,j} \sum A_{i'} K_{0,i}(T_e)}{\lambda_{i,j} A_{i',j'} \sum A_i K_{0,i'}(T_e)}$$

- From Kunze
- Dependent on radiative lifetime for transition and overall lifetime for state
- Excitation rates calculated from Ralchenko cross sections



Temperature (eV)
8
0
1.0 Torr
0
50
100
Time (ns)
4.0 Torr
Simulated Emissions
Calculated (x 0.1)
Measured Emissions
8.0 Torr (U)
8.0 Torr (M)
8.0 Torr (D)