Journal of Research on Educational Effectiveness

ISSN: 1934-5747 (Print) 1934-5739 (Online) Journal homepage: www.tandfonline.com/journals/uree20

Causal Connections Between Mathematical Language and Mathematical Knowledge: A Dialogic Reading Intervention

David J. Purpura, Amy R. Napoli, Elizabeth A. Wehrspann & Zachary S. Gold

To cite this article: David J. Purpura, Amy R. Napoli, Elizabeth A. Wehrspann & Zachary S. Gold (2017) Causal Connections Between Mathematical Language and Mathematical Knowledge: A Dialogic Reading Intervention, Journal of Research on Educational Effectiveness, 10:1, 116-137, DOI: 10.1080/19345747.2016.1204639

To link to this article: https://doi.org/10.1080/19345747.2016.1204639

Published online: 30 Sep 2016.

JOURNAL OF RESEARCH ON EDUCATIONAL EFFECTIVENESS
2017, VOL. 10, NO. 1, 116–137
http://dx.doi.org/10.1080/19345747.2016.1204639

Causal Connections Between Mathematical Language and Mathematical Knowledge: A Dialogic Reading Intervention

David J. Purpura[a], Amy R. Napoli[a], Elizabeth A. Wehrspann[a], and Zachary S. Gold[a]

ABSTRACT
The acquisition of early mathematical knowledge is critical for successful long-term academic development. Mathematical language is one of the strongest predictors of children's early mathematical success. Findings from previous studies have provided correlational evidence supporting the importance of mathematical language to the development of children's mathematics skills, but there is limited causal evidence supporting this link. To address this research gap, 47 Head Start children were randomly assigned to a mathematical language intervention group or a business-as-usual group. Over the course of eight weeks, interventionists implemented a dialogic reading intervention focused on quantitative and spatial mathematical language. At posttest, students in the intervention group significantly outperformed the students in the comparison group not only on a mathematical language assessment, but on a mathematical knowledge assessment as well. These findings indicate that increasing children's exposure to mathematical language can positively affect their general mathematics skills. This study is an important first step in providing causal evidence of the importance of early mathematical language for children's general mathematical knowledge and the potential for mathematical language interventions to increase children's overall mathematics abilities.

KEYWORDS
mathematical language
mathematics
vocabulary
preschool
dialogic reading

The development of early mathematical knowledge, which begins before school entry, is critical to the development of later mathematics skills (Dowker, 2005; Sarama & Clements, 2009; Starkey, Klein, & Wakeley, 2004). One of the strongest predictors of early mathematical success is language ability (LeFevre et al., 2010; Purpura, Hume, Sims, & Lonigan, 2011). Notably, the language associated with mathematics—or mathematical language—is highly content specific (Harmon, Hedrick, & Wood, 2005) and parent and teacher usage of this specific language is predictive of growth in children's mathematical knowledge (Boonen, Kolkman, & Kroesbergen, 2011; Gunderson & Levine, 2011; Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Levine, Suriyakham, Rowe, Huttenlocher, & Gunderson, 2010). Despite significant evidence that mathematical language (e.g., knowledge of terms such as more, less, near, and far) is related to young children's mathematical knowledge (e.g., skills such as counting, relations, and operations), there is limited causal evidence supporting this

CONTACT David J. Purpura purpura@purdue.edu Purdue University, Human Development and Family Studies, 1202 W. State Street, Rm. 231, West Lafayette, IN 47907-2055, USA.
[a]Purdue University, West Lafayette, Indiana, USA

relation. Thus, the central purpose of this study was to examine the impact of a mathematical language intervention on the mathematical knowledge of preschool children.

The Relations of Language and Mathematics Development

A strong relationship between mathematics and language is apparent early in children's academic development (Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010; Purpura et al., 2011; Romano, Babchishin, Pagani, & Kohen, 2010). Language skills are presumed to support mathematical development (Miura & Okamoto, 2003) as children utilize language skills to refine their quantitative understanding (Spelke, 2003). Further, difficulties in mathematics and difficulties in literacy and language often co-occur (Lewis, Hitch, & Walker, 1994; Mann Koepke & Miller, 2013). Children who experience difficulties in both mathematics and language tend to have poorer mathematical knowledge than children who have difficulties in mathematics alone (Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000). Unfortunately, children from low socioeconomic status (SES) families often have deficits in both mathematics (Jordan, Huttenlocher, & Levine, 1994; Jordan & Levine, 2009; Starkey et al., 2004) and language skills (Pungello, Iruka, Dotterer, Mills-Koonce, & Reznick, 2009). Language deficits are thought to be one explanation for SES-based differences in mathematics performance (Jordan et al., 1994; Jordan & Levine, 2009; Starkey et al., 2004). Despite the relation between language and mathematics, improving children's general language skills does not directly benefit their mathematics knowledge (Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). A likely explanation for this lack of improvement is that the language targeted through a general language intervention includes different words, or words that are used differently in mathematical contexts, than those words utilized in mathematical instruction (Harmon et al., 2005).

Mathematics-Specific Language

In contrast to mathematical knowledge (children's ability to work with exact numbers such as counting, comparing, and adding), mathematical language is a child's understanding of the key words used in early mathematics. Two specific aspects of mathematical language have been identified as potentially important for early mathematical learning: quantitative language (Barner, Chow, & Yang, 2009) and spatial language (Pruden, Levine, & Huttenlocher, 2011; Ramani, Zippert, Schweitzer, & Pan, 2014). Quantitative language includes terms such as "more than," "less than," "many," and "fewer." Understanding these terms allows children to make and describe comparisons between groups or numbers. For example, knowing that the term "more" can mean an increase in quantity ("give me more") or can be used in comparative statements ("five is more than two") may allow children to refine their understanding of quantity more precisely. Spatial language includes such terms as "before," "above," and "near." Understanding spatial language is related to children's spatial thinking (Pruden et al., 2011), and it may allow children to talk about relations between physical objects and between numbers on a number line, as well as develop spatial skills that have been found to be important for mathematical development (Mix & Cheng, 2012; Verdine, Irwin, Golinkoff, & Hirsh-Pasek, 2014). For example, many spatial words such as "before" and "after" are often connected to the number sequence and indicate increases and decreases in quantity ("five comes after four").

Assessments of mathematical language that broadly include quantitative and spatial language are some of the strongest predictors of the acquisition of early mathematical knowledge (Purpura & Logan, 2015; Toll & Van Luit, 2014a), and mathematical language mediates the relation between general language and early mathematics skills (Toll & Van Luit, 2014b). Unfortunately, young children are not always able to use mathematical language appropriately (Hurewitz, Papafragou, Gleitman, & Gelman, 2006), and preschool children from families of low SES have significantly lower mathematical language than their peers from families of middle and higher SES (Purpura & Reid, 2016). Children who do not understand specific aspects of mathematical language also struggle to acquire certain early mathematics skills (Barner et al., 2009).

Researchers have postulated that interactions rich in mathematical language are important in developing children's early mathematical knowledge (Clements, Baroody, & Sarama, 2013). Studies have shown that interactions that include mathematical language, both with caregivers and teachers, are predictive of children's mathematics outcomes (Gunderson & Levine, 2011; Levine et al., 2010; Ramani, Rowe, Eason, & Leech, 2015). Notably, a strong emphasis on mathematical language is believed to be a critical component of the success of early mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2006). Despite evidence that there are strong concurrent and predictive relations between children's mathematical language knowledge and their mathematics performance, the causal relation between mathematical language and mathematics knowledge has not been investigated.

Interventions to Improve Mathematical Language

Researchers have emphasized the importance of including a mathematical language component in general mathematics instruction (Chard et al., 2008; Clements & Sarama, 2011). However, to date only a few studies have focused on directly improving children's mathematical language skills (Hassinger-Das, Jordan, & Dyson, 2015; Hojnoski, Columba, & Polignano, 2014; Jennings, Jennings, Richey, & Dixon-Krauss, 1992; Powell & Driver, 2014). For different reasons, none of these interventions addressed the potential causal link between children's mathematical language and mathematical knowledge, particularly in preschool children.

Three well-conducted randomized controlled trial intervention studies with primary school children were designed to improve both mathematical knowledge and mathematical language (Hassinger-Das et al., 2015; Jennings et al., 1992; Powell & Driver, 2014). Hassinger-Das and colleagues randomly assigned kindergartners to a mathematical language storybook intervention, a number sense intervention, or a business-as-usual comparison group. The storybook and number sense interventions both included a game and activity component, but differed in that the storybook intervention also included books as a tool to teach mathematical language. Jennings and colleagues also used storybook reading to target kindergarten children's mathematical language abilities. Similar to the intervention design in Hassinger-Das et al., the Jennings et al. intervention was not limited to mathematical language. The intervention utilized books as well as a free-play station with a variety of props to target both mathematical language and mathematical knowledge. Additionally, instruction was delivered at the classroom level with only two teachers in each condition, making it difficult to discern if the effects were due to the intervention or to classroom-level factors. Finally, Powell and Driver randomly assigned first graders to one of three groups: an addition group, an addition-plus-mathematical-language group, or a comparison group. In the two intervention groups, children engaged in explicit instruction of addition that involved a flashcard activity, a lesson, and a review. The addition-plus-mathematical-language group also engaged in an introduction to mathematical language and was exposed to targeted words during the instructional sessions. As all three of these studies showed promising results, it is quite clear that mathematical language skills are malleable through intervention. However, it is not possible to differentiate the effects of improving mathematical language from the effects of the number-training content of the interventions because these interventions engaged children in both mathematical language and mathematical performance activities within the same intervention conditions. Further, all of these studies were conducted with elementary school children, whereas the breadth of research on mathematical language talk by teachers has targeted younger children. To effectively test the causal relation between mathematical language and mathematical knowledge, an intervention study focused on only mathematical language instruction in preschool is needed.

A novel study conducted by Hojnoski and colleagues (2014) is the only one to focus directly on improving mathematical language in preschool children without also targeting mathematical knowledge. The researchers used a single-subject design to determine whether parent training on the use of mathematical concepts during shared reading increased the frequency of mathematics-specific utterances during parent–child book reading. Positive intervention effects were found on parents' and children's general language and mathematical language use during shared reading. Although these findings are important in indicating the potential for parent–child storybook reading to increase mathematical language usage, they do not indicate whether such increased usage translates to increases in the depth of children's mathematical language or to their mathematical knowledge outcomes.

Using Dialogic Reading to Improve Mathematical Language

To conduct a study evaluating the causal relation between mathematical language and mathematical knowledge, a randomized controlled trial focusing solely on mathematical language instruction is needed. One potential mechanism for teaching children mathematical language is through a dialogic reading storybook intervention, similar to a component of the broader intervention used by Hassinger-Das et al. (2015). Dialogic reading involves a role shift in typical adult–child book sharing (Lonigan & Whitehurst, 1998). In traditional book sharing, the adult is the storyteller and the child is the listener; dialogic reading calls for the child to become the storyteller while the adult uses questions and prompts and adds information in order to scaffold the child's language development. Dialogic reading has been found to be effective in both home (Mol, Bus, de Jong, & Smeets, 2008) and school (Hargrave & Sénéchal, 2000; Lever & Sénéchal, 2011; Lonigan, Anthony, Bloomfield, Dyer, & Samwel, 1999; Lonigan, Purpura, Wilson, Walker, & Clancy-Menchetti, 2013) settings to improve children's general language and vocabulary. The utility of dialogic reading for improving children's language skills is likely derived from the emphasis on eliciting verbal responses and descriptions from children (Lever & Sénéchal, 2011). This framework may be effective for improving mathematical language because mathematical curricula rich in interaction have been shown to be effective for improving both mathematics and language skills (Sarama, Lange, Clements, & Wolfe, 2012).

Current Study

A growing body of evidence has indicated that mathematical language is malleable (Hassinger-Das et al., 2015; Hojnoski et al., 2014; Jennings et al., 1992; Powell & Driver, 2014) and is a strong predictor of mathematical knowledge (Purpura & Logan, 2015; Toll & Van Luit, 2014a, 2014b), and that exposure to mathematical language is highly predictive of mathematical knowledge growth across the early years (Boonen et al., 2011; Gunderson & Levine, 2011; Klibanoff et al., 2006; Levine et al., 2010). However, there is limited evidence supporting the causal connection between mathematical language and mathematical knowledge. The few studies in which interventions have targeted mathematical language have not been designed to assess this causal relation. As such, there is a critical need to investigate whether an intervention targeting mathematical language alone (i.e., without the additional focus on mathematical knowledge and content such as counting and number words) would result in improvement in mathematical performance. The current study was designed to meet this need by assessing the impact of improving children's mathematical language on their mathematical knowledge via a randomized controlled trial. It was hypothesized that children in the intervention group would significantly outperform their peers in the business-as-usual comparison condition on both mathematical language skills and on mathematical knowledge. As mathematical language is a targeted subgroup of more general language skills and is more proximal to mathematical knowledge than is general language, it was not expected that the intervention group would outperform the comparison group on general vocabulary skills.

Method

Participants

Children from 10 classrooms in four Head Start centers participated in the intervention. Parents of 64 children consented for their children to participate in the study. Of these children, 14 did not complete the pretest for reasons including: they left the school before testing could begin ($n = 6$), had significant language barriers ($n = 5$), or refused to assent ($n = 3$). Of the 50 children who completed pretesting, three were not assigned to condition for one of two reasons: they left their school after pretesting, but prior to assignment ($n = 1$), or their availability during the school day did not meet project requirements ($n = 2$). Thus, 47 children were randomly assigned within school to either the treatment ($n = 24$) or comparison condition ($n = 23$). Two children in the intervention condition (8%) and six children in the comparison condition (26%) left their schools before the end of the posttesting resulting in 17% total attrition and 18% differential attrition. Noncompleters did not differ from completers on age, $t(45) = 1.50$, $p = .141$, mathematical language, $t(45) = 1.07$, $p = .292$, or expressive vocabulary, $t(45) = 0.75$, $p = .456$. However, noncompleters did perform significantly worse on mathematical knowledge at pretest than completers, $t(45) = 2.14$, $p = .038$.

Children who completed the study ranged in age from 3.33 to 5.24 years old ($M = 4.67$, $SD = 0.41$). Just over half (56.4%) of the participants were male, 30.8% were Caucasian, 30.8% Hispanic, 12.8% African American, and 25.6% Other/Not-Specified. Fourteen participants (seven per condition) were English Language Learners. Median parental education was a high school diploma (51.3% of children's parents had no higher than a high school diploma, 23.1% of children had a parent with some college education, but no degree, and

25.6% of children had a parent with at least an associate's degree). The participating schools all used Creative Curriculum (Dodge et al., 2010) as their standard curriculum.

Measures

Preschool Early Numeracy Skills Screener—Brief Version (PENS-B)

The PENS-B (Purpura, Reid, Eiland, & Baroody, 2015) is a 24-item test of mathematical knowledge that focuses on numeracy skills. The items are representative of the broad numeracy skills children are expected to attain in preschool and kindergarten. Areas of assessment include the following: set comparison, numeral comparison, one-to-one correspondence, number order, numeral identification, ordinality, story problems, and number combinations. Both receptive and expressive items are included on this measure. Children were asked questions by the assessor and asked to respond orally (e.g., "How many dogs are there?") or by pointing to a picture (e.g., "Point to the eighth duck in the line"). Twenty-one questions were presented with a visual and verbal stimulus and three were verbal story problems. One point was awarded for each correct response. The test had a high internal consistency in the current sample ($\alpha = .93$) and previous evidence has indicated it is highly correlated with the Test of Early Mathematics Ability—Third Edition (TEMA-3; $r = .73$; Purpura et al., 2015).

Mathematical Language

The mathematical language subtest is an author-developed measure of mathematics-specific language. The measure has 16 items assessing comparative language (e.g., combine, more, less, take away) and spatial language (e.g., near, far, before). Children were awarded one point for each correct response. In prior work (Purpura & Logan, 2015), these items were selected from a larger battery (i.e., broader range) of items using an item response theory framework. The selected items had a range of difficulty parameters and strong discrimination parameters. The specific words included on this measure were intended to be broadly representative of the quantitative and spatial language associated with mathematics. Quantitative words included: take away, a little bit, most, more, fewest, and less. Spatial words included: nearest, under, first, far, below, front, middle, end, last, and before. All items were designed to be completed without exact quantitative skills and in a nonnumeracy context. For example, the quantitative questions were asked in different ways: (a) comparing dots with such a gross difference that children would be able to respond correctly regardless of numeracy ability as long as they knew the meaning of the language terms (e.g., 10 vs. 2) and (b) using a picture of mostly full and mostly empty glasses when asking "Which glass has the most water?" or "Which glass has less water?" This mathematical language task had an internal consistency of .78 for this sample. In previous work, it was found that the correlation between mathematical language and mathematical knowledge was not significantly different from the correlation between mathematical language and general vocabulary, suggesting it accounts for the common aspects across domains (Purpura & Reid, 2016).

Expressive One-Word Picture Vocabulary Test—Fourth Edition (EOWPVT)

The EOWPVT (Martin & Brownell, 2011) was used to measure children's expressive vocabulary ability. In this task, children were shown a colored picture of an object(s) and asked "What is this?" "What is this for?" or "What are these?" The EOWPVT has excellent reliability ($\alpha = .95$–$.96$ for 3- to 5-year-old children; Martin & Brownell, 2011). Children received one point for each correct response. Basal and ceiling rules were administered per the instruction manual.

Covariates

Covariates were child age and highest parental education level (scored on an 8-point scale ranging from eighth grade or less to doctoral/postgraduate degree). Parental education was included as a covariate because although all children met criteria to be enrolled in Head Start centers, there was a range of educational backgrounds of parents which may contribute to children's acquisition of mathematical knowledge over time.

Procedures

Overview

Consent forms were sent to the parents of children from participating schools in September. Approximately 225 total consent forms were delivered to schools, and parents of 64 children returned the consent forms. Pretesting began in late October and ran through the first week in December. In early January, the schools were contacted to ensure that pretested children were still attending before random assignment was conducted. A total of 47 children from four Head Start centers were randomly assigned to either the treatment or comparison condition. Children were randomly assigned within schools to ensure that school-level variance was randomly distributed across groups.

Beginning in late January, children in the treatment condition participated in a mathematical language storybook reading intervention in small groups for approximately 15 to 20 minutes per day, two to three days per week, for eight weeks. In addition to randomly assigning children to condition, attempts were made to randomly assign children to the small groups for each session to ensure that the same students were not always in sessions together and to minimize group-level effects. However, because children could not always be taken outside of their classrooms and there were not always enough students in each class to create multiple groups, some groups comprised the same children together throughout the duration of the intervention. There were a total of 10 small groups, six of which always comprised the same children and four of which children were randomly assigned to groups for each day. One of the small-group intervention sessions took place outside of the classroom, and sessions for the nine other groups were held in corners, nooks, or reading areas of each classroom. When the intervention was conducted in the classrooms, interventionists made efforts to limit distractions, such as turning children's chairs away from other classroom activities and asking teachers to keep other children from wandering into reading areas. Because children from both conditions were in each classroom, interventionists prevented diffusion of treatment by ensuring that only intervention children were included in each reading session. Posttesting started two weeks after completion of the intervention (the last week in March/first week in April).

Pretest and Posttest

Trained undergraduate students completed all pretest assessments in the Head Start centers at times identified by the school directors. Assessments were completed in a room or area designated by the school director or teachers. All testers completed two 2–3 hour training sessions, followed by individual practice and a testing-out session to ensure they were fluent with the assessment measures and general principles of working with children. During testing-out sessions, undergraduate students administered the tests to one of the lead project staff who ensured that all assessments were administered correctly.

Posttesting followed an identical process to pretesting except that both undergraduate and graduate students administered the assessments. The two graduate students who completed posttest assessments were also interventionists; however, they did not complete assessments at schools in which they conducted the intervention. All testers were blind to children's condition assignments.

Intervention Design

The framework of the storybook reading intervention in the current study was modeled after dialogic reading. The current intervention used this dialogic reading framework but focused on terms, concepts, and pictures that involved mathematical language. A key aspect of this intervention was that it was designed to include only mathematical language terms (e.g., more, less, near, far) and not mathematical knowledge content (e.g., number names, counting). Interventionists read with children using strategies identified in prior research; specifically, the PEER (Prompt, Evaluate, Expand, and Repeat) strategy and CROWD (Completion, Recall, Open-ended, Wh- [what, where, why], and Distancing) prompts were employed during readings (Morgan & Meier, 2008).

Before the intervention began, the interventionists worked together and developed questions that were written onto note cards and placed at specific points in each book. These questions incorporated the use of mathematical language and were asked by each interventionist. Questions were designed to expand upon the existing mathematical language of the book (e.g., "How do we know the girl has more than the boys?") or to incorporate mathematical language in places where the book did not (e.g., "Is the bird above or below the tree?"). The prompts for each book continually became more complex. Initially, interventionists introduced the book, asked basic prompting questions, and made mathematical language comments that were not always framed as questions. These prompts allowed children to become familiar with each book's content and its mathematical language before advancing to more in-depth prompts and discussions including distancing strategies (i.e., children were asked to describe scenarios in which something happened in their lives similar to events occurring in each book). Table 1 includes examples of this question process, which allowed children to become familiar with each book as they advanced to more in-depth dialogic processes throughout the intervention. Thus, familiarity with each book was established as part of the intervention process over each day.

Interventionists asked these questions at the same points in each book. Spontaneous feedback emerged between interventionists and children each day as well. Interventionists

Table 1. Examples of dialogic reading prompts and questions.

	Basic prompts/explanations	More complex prompts	Most complex prompts/discussions
Quantitative language	"There are just a few people in the stands."	"How do we know there were a lot and not just a few?	"Have you ever seen a lot of people together?"
Spatial language	"Is this rollercoaster near the merry-go-round?"	"Tell me about the ride. Did it go up and down? Was it tall?"	"Can you think of something that you know that goes around like a merry-go-round?"

expanded upon children's various responses to questions in an effort to reinforce children's understanding of the mathematical language used. Importantly, each interventionist employed standard dialogic reading strategies and made efforts to respond in ways that did not reinforce children's numeracy skills or counting (see section on fidelity).

Intervention Books. Six books were chosen for the eight-week intervention (see Table 2). Books were chosen based on content, length, and age-appropriateness. To ensure that mathematical language was the sole focus of the intervention, books were not selected if they included counting, and books that included numbers/numerals were modified and read without using number words (e.g., "Three boys were near the trees" was read as "A few boys

Table 2. Intervention schedule: Books and questions used each week.

Week	Book	Number of questions	Mathematical language terms	Total instances[a]	Language usage rate[b]
1	*Many is How Many?* by Illa Podendorf	19	a lot, behind, far, big, few, fewer, fewest, in front of, longer, more, most, near, over, smaller, under	28	QL = 64% SL = 36%
2	*Albert Is Not Scared* by Eleanor May	24	a lot, away, around, back, down, more, farther, few, fewer, fewest, first, front, in, inside, last, near, out of, outside, short, tall, top, toward, under, up	35	QL = 26% SL = 74%
3	*Just Enough Carrots* by Stuart J. Murphy	27	a lot, any, enough, far, few, fewer, fewest, in, inside, many, most, more, near, out, outside, same, too many	38	QL = 76% SL = 24%
4	*Rosie's Walk* by Pat Hutchins	30	a lot, above, around, behind, below, beneath, between, bottom, down, few, higher, highest, in, in front, inside, lowest, many, near, on, over, through, toward, under	41	QL = 10% SL = 90%
5	*Albert's Bigger Than Big Idea* by Eleanor May	29	a lot, away, below, bigger, biggest, few, high, higher, inside, middle, more, most, near, on, on top, outside, smallest, too big, up	36	QL = 56% SL = 44%
6	*Little White Rabbit* by Kevin Henkes	33	a lot, above, away, below, between, big, bigger, bottom, closer, far, farther, few, many, more, near, over, same, small, smaller, through, top, under	51	QL = 43% SL = 57%
7	Review sessions—Children's choice				
8	Make-up sessions				

[a]Total number of instances of all mathematical language terms used in the questions throughout the book.
[b]Percent of instances of mathematical language usage for quantitative (QL) and spatial (SL) language in each book.

were near the trees"). Interventionists were instructed not to elaborate on children's mathematics usage (e.g., if a child began counting when prompted for a mathematical language answer, the interventionists did not encourage or expand on the counting). Three of the six books emphasized quantitative language (e.g., more, fewer, a lot, a little) and three emphasized spatial language (e.g., higher, lower, above, below, before, after); efforts were made to integrate either type of mathematical language into the questions when possible. Interventionists explicitly defined or explained mathematical language terms during the intervention if a child demonstrated a lack of understanding of the term (e.g., incorrectly responded to a question or prompt). The interventionists did not explicitly define each word for all participants because they worked to teach the terms through the context of the pictures, text, and questions. For the first six weeks, interventionists and children read one book per week and the foci of the books alternated so that quantitative language was emphasized one week and spatial language was emphasized the next week (see Table 2)—though both quantitative and spatial language were included in questions each week when possible. During the seventh week, children were allowed to choose a book for each session. This was done both to review content and to ensure prolonged engagement. In the final week, interventionists made efforts to make up sessions with students who had missed sessions during the first six weeks.

Mathematical Language Questions and Terms. On average, 27 mathematical language questions or prompts were identified per book, ranging from 19–33. Mathematical language terms that were used in the questions were selected based on terms frequently utilized in state and national mathematical knowledge standards (e.g., NCTM, 2006) and on other language tests or assessments that captured similar constructs (Bracken, 2006; Cannon, Levine, & Huttenlocher, 2007). Specific words were selected for each book based on the ease with which words could be effectively elicited from the text and pictures of the book. A total of 56 mathematical language words were included across all six books, and there were 229 total instances of mathematical language terms (45% quantitative, 55% spatial) built into the questions, with the number of instances of each word ranging from 1–22 ($M = 4.02$, $SD = 4.54$). The words "a lot," "more," "inside," and "near" were more commonly included in questions than other words. On average, each book contained approximately 38 instances of mathematical language.

Interventionists. Three graduate students implemented the intervention. Each interventionist had experiences working with children prior to this study. One interventionist was a former elementary school teacher with additional previous experience reading to kindergarteners as a community volunteer. The second interventionist had experience as an early childhood tutor and had previously managed other research projects observing and assessing young children. The third interventionist had experience working with preschoolers in child care settings and observing young children.

Interventionist Training. The interventionists were trained in dialogic reading using a training manual comprised of detailed descriptions of dialogic reading components, examples of each component, and details regarding how to ask questions to appropriately address each component during implementation. These manuals were developed by the research team and used in tandem with training videos created by experts in the field of dialogic reading. Interventionists were also trained on strategies for providing feedback to children in ways that reinforced their understanding of mathematical language, as well as avoided reinforcement of numeracy and counting. Training on feedback strategies that reinforced mathematical language was accomplished by practicing dialogic reading with children not in the current study, including using the specific prompts and questions used during the intervention. The principal investigator observed interventionists reading to several children and provided structured feedback on dialogic reading methods, as well as effective delivery, use of mathematical language, avoiding use of numbers and counting, and general behavior management. The interventionists also discussed individual experiences during the practice week to revise strategies as needed. Throughout the duration of the intervention, interventionists held weekly meetings to discuss progress, ensure that scheduling and child absences were documented, and verify that each book was correctly annotated with questions.

Fidelity

Interventionists audio-recorded each of the intervention sessions. Fidelity of implementation was assessed by randomly selecting approximately 33% of each interventionist's sessions (62 of 185 total sessions) and evaluating whether or not they utilized each of the preidentified questions and whether or not they used any exact numbers or counting words/procedures. The intent of the latter aspect was to ensure that the intervention was solely focused on mathematical language and not mathematical content, and thus, no use of exact numbers or counting words/procedures was desired. On average, the three interventionists maintained high levels of fidelity (90%), used only an average of 1.4 number words per session, and never used counting. Fidelity for each interventionist was: 97.4% (Range = 90.9% to 100%, for 26 of 78 total sessions), 89.1% (Range = 72.9% to 100%, for 22 of 65 total sessions), and 78.1% (Range = 56.3% to 100%, for 14 of 42 total sessions). The average amount of number-related utterances by children was also calculated from the selected sessions. On average, there were 2.4 total uses of number words or counting sequences spoken by children in each session, though not all number or counting responses by children were correct. Note that the average number of words used was by all children in a session, not by individual children. It was not possible to determine which child made each utterance given that we used audio recordings and not video recordings.

Dosage

Most children in the intervention group participated in 13 to 18 sessions ($M = 14.5$ sessions; $SD = 3.7$). One child in the intervention condition did not participate in any sessions, but was included in the analyses because we used an intent-to-treat model. When the child was removed from the analyses, the results were slightly more robust.

Analytic Procedures

Among completers, there were only two instances of missing data. Parents of two children did not provide educational levels. These two missing values were replaced by the average parent education in their schools. Three separate ANCOVAs were conducted—one for each key outcome variable (i.e., mathematical language, mathematical knowledge [numeracy], and expressive vocabulary)—to evaluate the effects of the intervention. Covariates included age, parental education, and pretest score. Because children were randomly assigned to condition within school, condition was not expected to be confounded with school. However, all analyses were also conducted with school as a random-effect covariate. School was not statistically significant for any of the analyses and was not included in the analyses reported below. Further, given the importance of reporting effect size (Lipsey et al., 2012; Wilkinson & APA Task Force on Statistical Inference, 1999), the magnitude of the effects was evaluated by using Hedges's g; both statistical significance (p values) and effect size are presented in the results section. Effect sizes were calculated using posttest means that were adjusted to account for pretest and all covariates. According to the Institute of Education Sciences (IES) What Works Clearinghouse guidelines (IES, 2014) Hedges's g effect sizes of "0.25 standard deviations or larger are considered to be substantively important."

Results

Preliminary Analyses

Descriptive statistics for children's pretest and posttest scores in both of the conditions are presented in Table 3. Raw scores were used in the analyses. All pretest scores were normally distributed. Preliminary analyses indicated that children in the mathematical language intervention condition and the business-as-usual condition did not differ on parental education, $t(37) = -1.21$, $p = .235$ or age, $t(37) = -1.07$, $p = .290$. There also were no significant pretest differences between groups on mathematical language, $t(37) = -0.08$, $p = .936$, Hedges's $g = -0.02$, mathematical knowledge, $t(37) = -0.57$, $p = .570$, Hedges's $g = -0.18$, or on expressive vocabulary, $t(37) = -0.18$, $p = .861$, Hedges's $g = -0.06$. Correlations between all pretest and posttest variables are presented in Table 4.

Primary Analyses

The intervention group significantly outperformed the business-as-usual comparison group on the mathematical language measure, $F(1, 34) = 4.26$, $p = .047$, Hedges's $g = 0.42$ and on the mathematical knowledge measure, $F(1, 34) = 4.18$, $p = .049$, Hedges's $g = 0.32$. The intervention group did not significantly outperform the comparison group on the expressive vocabulary measure, $F(1, 34) = 0.18$, $p = .677$, Hedges's $g = -0.08$. Among the covariates,

Table 3. Pretest and adjusted posttest mean scores, standard deviations, and effect sizes for all key measures.

	Mathematical language storybook intervention ($n = 22$)				Business-as-usual comparison condition ($n = 17$)				
	Pretest		Adjusted posttest		Pretest		Adjusted posttest		
Variable	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	Hedges's *g*
Mathematical language	9.32	3.70	12.25	3.65	9.41	3.37	10.85	2.66	.42*
Early numeracy skills	7.59	5.11	10.70	5.25	8.59	5.72	8.92	5.85	.32*
Expressive vocabulary	48.09	18.04	50.66	23.68	49.06	15.40	52.26	14.04	$-.08^{ns}$

Note. Posttest means are adjusted for age, parent education, and pretest.
$^*p < .05$; ns = not significant.

Table 4. Correlations among pretest and posttest measures.

	1	2	3	4	5
Fall measures					
1. Mathematical language	—				
2. Early numeracy skills	.71**	—			
3. Expressive vocabulary	.64**	.58**	—		
Spring measures					
4. Mathematical language	.78**	.61**	.54**	—	
5. Early numeracy skills	.76**	.86**	.62**	.73**	—
6. Expressive vocabulary	.52*	.51*	.83**	.48*	.50*

**$p < .001$, *$p < .01$.

the pretest variable was significant in each analysis, age was significant only for the mathematical knowledge analysis, and parental education was not significant in any of the analyses.

Discussion

An abundance of evidence indicates that mathematical language predicts growth in mathematical knowledge for young children (Adams, 2003; Knight & Hargis, 1977; Lansdell, 1999; Schleppegrell, 2007; Toll & Van Luit, 2014a, 2014b). The need to include mathematical language in curricula and instructional activities has been emphasized both in research and policy documents (Clements et al., 2013; NCTM, 2006). However, given that no prior study investigated the causal relation between mathematical language and mathematical knowledge, these assertions were based on correlational evidence. Though several prior studies did examine the impacts of mathematical language training (Hassinger-Das et al., 2015; Hojnoski et al., 2014; Jennings et al., 1992; Powell & Driver, 2014), these studies did not allow for the examination of the causal relation between mathematical language and mathematical knowledge. The findings from the current study address this critical gap in the research literature because the intervention was focused solely on improving mathematical language without direct emphasis on mathematical knowledge. Notably, the mathematical language intervention resulted in statistically significant and substantively important improvements not only on mathematical language outcomes, but on mathematical knowledge outcomes as well. Because only mathematical language, and not mathematical knowledge, was intentionally emphasized in the intervention condition, this study provided a well-controlled test of the causal relation between mathematical language and the numeracy aspects of mathematical knowledge. The findings indicate that improving mathematical language also appears to have an impact on early mathematics development in this sample.

As hypothesized, the intervention led to significant improvements in both mathematical language skills and mathematical knowledge; however, the reasons for these improvements likely differ for each construct because mathematical language skills were explicitly emphasized and mathematical knowledge was intentionally not taught. Across all six stories, children were exposed to 56 different mathematical language terms between 1 and 22 times each. This high level of exposure to words in an interactive context of dialogic reading provided children with an opportunity to both understand and apply the mathematical language terms.

In contrast to mathematical language, the children in the intervention group were not intentionally exposed to any counting or number/quantity naming concepts (though an incidental 1.4 number words, on average, were utilized by the interventionists in each session). The improvements in mathematical knowledge are potentially due to four plausible explanations. First, as can be seen with the comparison group, children effectively did not demonstrate any growth across the duration of the study in their mathematical knowledge skills as a result of the standard classroom instruction. It is possible that learning targeted mathematical language enables children to better understand the mathematical instruction in which they were engaged during their regular classroom time. Such an explanation would suggest that early mathematical language skills are necessary for the acquisition of the mathematical knowledge typically taught during classroom instruction.

Second, because young preschool children often do not have a strong understanding of basic number-line principles (Reid, Baroody, & Purpura, 2015), the acquisition of mathematical language concepts may have enabled children to generate a basic understanding of the ordering of quantities which, in turn, may have provided a foundation for better grasping exact quantity concepts. For example, terms such as "few," "more," and "a lot" may have enabled children to form benchmarks on a number line of quantitative magnitude. Terms such as "before," "last," and "after" may have enabled children to better grasp spatial aspects of the number line. Future research involving number-line tasks (similar to those used by Siegler & Ramani, 2008) as pretest and posttest measures should be conducted to examine this hypothesis.

Third, the use of mathematical language may have prompted children to utilize their existing mathematical knowledge when discussing mathematical language terms which, in turn, may have enabled subsequent development of these skills. On average, 2.4 number words or instances of counting (though not always correct) per intervention session were used by children. In prior correlational research demonstrating a link between parent and teacher math talk (Boonen et al., 2011; Levine et al., 2010) and children's mathematical knowledge growth, the use of mathematical language may have provided children with rich opportunities to use and apply their mathematical knowledge.

Fourth, it is possible that due to the highly language-based nature of most early mathematics skills (Purpura & Ganley, 2014) and, therefore, the high language base of measures assessing those skills, the improvements generated on the mathematical knowledge task may have resulted from children better understanding what was being asked on the assessment, rather than actual growth in mathematics skills. Such an explanation would indicate that children with deficits in the mathematical language skills may actually have underlying mathematical capabilities that they are not able to express in standard assessment settings because they cannot understand what is being asked of them. Ultimately, these potential explanations need to be examined intentionally through future research in a systematic manner.

It is evident that children begin to acquire mathematical language early on in development (Barner et al., 2009; Durkin, Shire, Riem, Crowther, & Rutter, 1986; Fuson, 1988; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007) and the acquisition of these skills is typically supported through their interactions with both teachers and parents (Boonen et al., 2011; Levine et al., 2010). Not only is caregiver use of basic and advanced mathematical language related to children's usage of similar language, but more frequent use of such language is related to growth in mathematical knowledge even after controlling for the frequency of engaging in mathematical activities (Ramani et al., 2015). It has been argued that improving mathematical language enhances children's mathematical knowledge. Specifically, it is believed that understanding mathematical vocabulary allows children to understand the meaning of mathematical discussions with parents and teachers as well as within instructional learning activities. For example, for a child to engage in a quantity comparison activity (e.g., finding the bigger collection), the child must understand the meaning of the word "bigger" to understand the task. Similarly, to fully engage in a number-order activity (e.g., the teacher tells children that the number 4 comes after the number 3), children must have an understanding of the words "after" and "before." Thus, when children understand these terms, they are more likely to be able to engage in mathematical activities in a constructive and meaningful way.

Children must have access to a variety of mathematical language terms in order to understand mathematical content. It is not just quantity, but rather quality, of language interactions that support children's typical language development (Hirsh-Pasek et al., 2015). Many children from lower SES families engage in fewer, and less complex, mathematical interactions with their caregivers compared to their peers from middle and higher SES families (Saxe, Guberman, & Gearhart, 1987; Vandermaas-Peeler, Nelson, Bumpass, & Sassine, 2009). The dialogic reading framework used in the current study provided children with an opportunity to engage in mathematical language interactions in an increasingly complex manner; as the stories progressed and as the books were read multiple times, the questions increased in complexity. Essentially, the specific questions, and the reading process in general, were designed to engage children with in-depth usage of mathematical language and this process was shown to be a potential method for providing low SES children with mathematical language instruction.

In contrast to the findings for mathematical language and mathematical knowledge, no significant effects were found on general vocabulary. This lack of effect was found despite the use of dialogic reading as the instructional framework—a reading strategy that has previously shown reliable effects on general vocabulary skills (Hargrave & Sénéchal, 2000; Lever & Sénéchal, 2011; Lonigan et al., 1999, 2013). One plausible explanation for not finding a benefit on expressive vocabulary in the current study is that the words and concepts on which the dialogic reading prompts were focused were targeted in nature and are not ones that typically appear on measures of general vocabulary. As such, children may have benefited in their acquisition of language skills, but this improvement was targeted to domain-specific words, such as those directly assessed on the mathematical language measure. It is possible, however, that as children were engaged in dialogue related to the books and storylines, effects on other oral language measures such as narrative retelling could be found as was done in previous work. For example, Sarama and colleagues (2012) demonstrated that an engaging and interactive mathematics curriculum resulted in improvement in targeted aspects of oral language. Future work with a broader set of general language measures should be used to investigate this possibility.

Implications

The intervention methods used in this study provide a relatively straightforward and useful tool for delivering mathematical language instructional opportunities in a developmentally appropriate manner. Recent evidence indicates that, although preschool teachers now spend more time engaging their children in mathematics activities than they historically have, the total instructional time remains limited—particularly for teachers with lower levels of education and those who teach children from families of low SES (Piasta, Pelatti, & Miller, 2014). Storybook reading is an accessible means of incorporating mathematical language and mathematical content into daily classroom and home activities for teachers and parents (Anderson, Anderson, & Shapiro, 2005; Casey, Kersh, & Young, 2004; Hojnoski et al., 2014; Hong, 1994; Jennings et al., 1992; Young-Loveridge, 2004). For example, Hojnoski et al. were able to successfully train parents to increase the frequency of children's mathematics talk through storybook reading. It is possible that interventions such as these can meaningfully affect children's mathematical skills in an engaging manner early in their educational development. Future work to intentionally develop children's books that convey key mathematical language and skill content may be a novel approach to providing mathematical instruction across the early years in an engaging manner.

Limitations

Despite the many strengths of this study, there are several limitations to note. First, the external validity of the study is limited. Although many efforts were made to recruit participants, only a small number of parents gave consent for their child to participate. Further, several children who were enrolled in the study moved away, refused to participate, or did not meet English language requirements. The initial participation rates and the attrition limited the sample size. There may have been unmeasurable differences between the children who participated in the study and those who did not, limiting the generalizability (external validity) of the findings. Therefore, generalizing the benefits of the intervention beyond the current sample may not be possible. Similarly, it was not possible to reliably evaluate age-related differences in improvement with this sample size. Further work is needed to understand if younger and older preschool children differentially benefit from the intervention. Second, the comparison condition was a business-as-usual comparison group so it is possible that the effects of the intervention may be due to children in the intervention group simply receiving additional instructional time and attention. However, this limitation is unlikely to have affected the findings because the effects were circumscribed to mathematical language and mathematical knowledge, but not to general vocabulary.

Third, some children were not allowed to leave their classrooms due to school policies, and therefore, even though children were fully randomized to condition, randomization of the small groups each day was not always possible. Thus, there may have been some unmeasured effects of the individual groups that could not be accounted for in this study. Future work in which all groups are fully randomized each day to reduce grouping effects or a study with enough power to statistically control for these effects is needed. Fourth, though the effect sizes on mathematical language and mathematical knowledge are classified as "substantively important" (IES, 2014), they were still in the small range. The intervention took place during wintertime (February to March), and weather cancellations and delays limited the number of days for the intervention and resulted in some children missing intervention sessions. Had all intervention children participated in the full number of sessions, the findings may have been more robust.

Fifth, because audio recordings, not video recordings, were utilized to assess the fidelity of the sessions, it was not possible to determine the mathematical language usage by individual children. Children's usage of mathematical language and mathematical knowledge in response to the instructors' use of mathematical language needs to be explored as a potential mechanism by which the domains are related. Sixth, from the current study, it is not possible to determine if the impact of mathematical language on mathematical knowledge resulted from changes in (a) mathematical language as a whole, (b) specific components of mathematical language such as quantitative or spatial, or (c) specific individual words more prevalent in mathematical instruction and assessment. Because the mathematical language terms utilized during the intervention ranged from 1 to 22 instances across the books, it is not possible to determine the exact mechanism behind this change. Future research training children on specific words should be considered to better evaluate this question.

Though not direct limitations of the study itself, there were aspects of the intervention that may limit the feasibility for scaling the intervention to the classroom level. The reading groups were small (typically two or three children), and teachers may not have the time or resources available to engage in small-group book reading. Although dialogic reading has been successfully used in large groups to improve expressive and receptive language (Wasik, Bond, & Hindman, 2006), at the present time, it is not clear whether the large-group approach would also work for improving children's mathematical language. Additionally, the books chosen for the intervention were selected for their richness in opportunities for promoting mathematical language. Teachers may not have access to these or other books that can be used to easily facilitate conversations promoting mathematical language. Although teachers may find opportunities in typical storybooks to encourage children's use of mathematical language, effectively incorporating these strategies requires time and training that may be limited for many teachers. Scale-up work is needed to ensure that teachers and parents can effectively promote mathematical language without extensive training.

Future Directions

The findings from this study are a first step in understanding how increasing children's mathematical language can potentially improve their mathematical knowledge. In the present study, mathematical language was the sole focus of instruction, whereas other interventions have also incorporated mathematical knowledge content (e.g., Hassinger-Das et al., 2015; Powell & Driver, 2014). In major policy and research documents (Clements et al., 2013; NCTM, 2006), an emphasis has been placed on using mathematical language instruction, not to replace, but to enhance mathematical knowledge instruction. It is critical to examine whether or not mathematical language instruction may enhance general mathematical knowledge instruction compared to mathematical knowledge instruction alone and how (and when) mathematical language instruction can be effectively integrated into existing classroom curricula. Given that Powell and Driver (2014) found that their mathematical knowledge intervention alone increased both general mathematical knowledge and mathematical language, it is also possible that mathematical language is a byproduct of effective mathematical knowledge instruction. Thus, a future study that includes a mathematical language intervention group, a mathematical knowledge intervention group, and a group that includes both mathematical language and mathematical knowledge may provide insight as to which individual facet best promotes mathematical knowledge, or if an integrated model is ideal.

A second focus for future research is identifying the components of mathematical language most related to improvement in mathematical knowledge and whether specific components of mathematical language are related to specific components of mathematical knowledge. In the current study, both quantitative and spatial language were incorporated throughout the intervention, making it impossible to identify the unique contributions of each component. In future studies, it is necessary to separate the effects of these two components to see if the findings were primarily due to one component of mathematical language or a combination of the two. Further, as different aspects of mathematical knowledge may rely on the acquisition of different aspects of mathematical language, the effects of targeted language instruction (e.g., specific mathematical language terms) on targeted mathematical knowledge components needs to be examined. For example, spatial terms such as "before" and "after" may be related to mathematical knowledge such as number order whereas terms such as "more" and "fewer" may be related to basic comparative skills.

Finally, application of this intervention in less controlled settings and with other at-risk groups is needed. The current study was implemented by trained interventionists who delivered the intervention with high levels of fidelity and to children who had adequately developed English language skills. Future studies should explore the effectiveness of the intervention in more typical school and home settings (e.g., when delivered by teachers to larger groups or by parents without intense training). Similarly, it is also important to investigate the effectiveness of the intervention with nonnative English speakers who may ultimately benefit the most from focused practice in using mathematical language.

Conclusion

Overall, the findings from the current study provide a foundational step in understanding the role of mathematical language in general mathematical knowledge development. Notably, it appears that improving mathematical language may have causal impacts on children's acquisition of mathematical knowledge. Further research is needed to examine how mathematical language instruction can be combined with mathematical knowledge instruction to enhance children's early learning opportunities, as well as to identify which components, or combination of components, are most important for building early mathematical knowledge. Ultimately, the support of a causal relation between mathematical language and general mathematics knowledge begins to fill a critical gap in the research literature and complements and enhances prior correlational evidence.

ARTICLE HISTORY

Received 31 August 2015
Revised 31 May 2016
Accepted 19 June 2016

References

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher*, *56*, 786–795.

Anderson, A., Anderson, J., & Shapiro, J. (2005). Supporting multiple literacies: Parents' and children's mathematical talk within storybook reading. *Mathematics Education Research Journal*, *16*, 5–26. doi:10.1007/BF03217399

Barner, D., Chow, K., & Yang, S.-J. (2009). Finding one's meaning: A test of the relation between quantifiers and integers in language development. *Cognitive Psychology*, *58*, 195–219. doi:10.1016/j.cogpsych.2008.07.001

Boonen, A. J. H., Kolkman, M. E., & Kroesbergen, E. H. (2011). The relation between teachers' math talk and the acquisition of number sense within kindergarten classrooms. *Journal of School Psychology*, *49*, 281–299. doi:10.1016/j.jsp.2011.03.002

Bracken, B. A. (2006). *Bracken Basic Concept Scale—Third Edition: Receptive*. San Antonio, TX: Psychological Corp.

Cannon, J., Levine, S., & Huttenlocher, J. (2007). *A system for analyzing children and caregivers' language about space in structured and unstructured contexts* (Spatial Intelligence and Learning Center (SILC) Technical Report). Retrieved from http://www.silccenter.org/media/silc_pdfs/resources/testsandinstruments/Spatial%20Language%20Coding%20Manual%201-10-12.pdf

Casey, B., Kersh, J. E., & Young, J. M. (2004). Storytelling sagas: An effective medium for teaching early childhood mathematics. *Early Childhood Research Quarterly*, *19*, 167–172. doi:10.1016/j.ecresq.2004.01.011

Chard, D. J., Baker, S. K., Clarke, B., Jungjohann, K., Davis, K., & Smolkowski, K. (2008). Preventing early mathematics difficulties: The feasibility of a rigorous kindergarten mathematics curriculum. *Learning Disability Quarterly*, *31*, 11–20. doi:10.2307/30035522

Clements, D. H., Baroody, A. J., & Sarama, J. (2013). *Background research on early mathematics* (Background research for the National Governors Association [NGA] Center Project on Early Mathematics). Retrieved from http://www.nga.org/cms/home/nga-center-for-best-practices/meeting–webcast-materials/page-edu-meetings-webcasts/col2-content/main-content-list/strengthening-early-mathematics.html

Clements, D. H., & Sarama, J. (2011). Early childhood mathematics intervention. *Science*, *333*, 968–970. doi:10.1126/science.1204537

Dodge, D. T., Heroman, C., Berke, K., Bickart, T., Colker, L., Jones, C., & Dighe, J. (2010). *The creative curriculum for preschool* (5th ed.). Bethesda, MD: Teaching Strategies.

Dowker, A. (2005). Early identification and intervention for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*, 324–332. doi:10.1177/00222194050380040801

Durkin, K., Shire, B., Riem, R., Crowther, R. D., & Rutter, D. R. (1986). The social and linguistic context of early number word use. *British Journal of Developmental Psychology*, *4*, 269–288. doi:10.1111/j.2044-835X.1986.tb01018.x

Fuson, K. C. (1988). *Children's counting and concepts of number*. New York, NY: Springer-Verlag.

Gunderson, E. A., & Levine, S. C. (2011). Some types of parent number talk count more than others: Relations between parents' input and children's cardinal-number knowledge. *Developmental Science*, *14*, 1021–1032. doi:10.1111/j.1467-7687.2011.01050.x

Hanich, L. B., Jordan, N. C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, *93*, 615–626. doi:10.1037/0022-0663.93.3.615

Hargrave, A. C., & Sénéchal, M. (2000). A book reading intervention with preschool children who have limited vocabularies: The benefits of regular reading and dialogic reading. *Early Childhood Research Quarterly*, *15*, 75–90. doi:10.1016/S0885-2006(99)00038-1

Harmon, J. M., Hedrick, W. B., & Wood, K. D. (2005). Research on vocabulary instruction in the content areas: Implications for struggling readers. *Reading and Writing Quarterly*, *21*, 261–280. doi:10.1080/10573560590949377

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *Elementary School Journal*, *116*(2), 242–264. doi:10.1086/683986

Hirsh-Pasek, K., Adamson, L. B., Bakeman, R., Owen, M. T., Golinkoff, R. M., Pace, A., Suma, K. (2015). The contribution of early communication quality to low-income children's language success. *Psychological Science*. doi:10.1177/0956797615581493

Hojnoski, R. L., Columba, H. L., & Polignano, J. (2014). Embedding mathematical dialogue in parent–child shared book reading: A preliminary investigation. *Early Education and Development*, *25*, 469–492. doi:10.1080/10409289.2013.810481
Hong, H. (1996). Effects of mathematics learning through children's literature on math achievement and dispositional outcomes. *Early Childhood Research Quarterly*, *11*, 477–494. doi:10.1016/S0885-2006(96)90018-6
Hooper, S. R., Roberts, J., Sideris, J., Burchinal, M., & Zeisel, S. (2010). Longitudinal predictors of reading and math trajectories through middle school for African American versus Caucasian students across two samples. *Developmental Psychology*, *46*, 1018–1029. doi:10.1037/a0018877
Hurewitz, F., Papafragou, A., Gleitman, L., & Gelman, R. (2006). Asymmetries in the acquisition of numbers and quantifiers. *Language Learning and Development*, *2*, 77–96. doi:10.1207/s15473341lld0202_1
Institute of Education Sciences. (2014). *What Works Clearinghouse: Procedures and standards handbook* (Version 3). Retrieved from http://ies.ed.gov/ncee/wwc/pdf/reference_resources/wwc_procedures_v3_0_standards_handbook.pdf
Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly*, *7*, 263–276. doi:10.1016/0885-2006(92)90008-M
Jordan, N. C., Glutting, J., Dyson, N., Hassinger-Das, B., & Irwin, C. (2012). Building kindergartners' number sense: A randomized controlled study. *Journal of Educational Psychology*, *104*, 647–660. doi:10.1037/a0029018
Jordan, N. C., & Hanich, L. B. (2000). Mathematical thinking in second-grade children with different forms of LD. *Journal of Learning Disabilities*, *33*, 567–578. doi:10.1177/002221940003300605
Jordan, N. C., Huttenlocher, J., & Levine, S. C. (1994). Assessing early arithmetic abilities: Effects of verbal and nonverbal response types on the calculation performance of middle- and low-income children. *Learning and Individual Differences*, *6*, 413–432. doi:10.1016/1041-6080(94)90003-5
Jordan, N. C., & Levine, S. C. (2009). Socioeconomic variation, number competence, and mathematics learning difficulties in young children. *Developmental Disabilities*, *15*, 60–68. doi:10.1002/ddrr.46
Klibanoff, R. S., Levine, S. C., Huttenlocher, J., Vasilyeva, M., & Hedges, L. V. (2006). Preschool children's mathematical knowledge: The effect of teacher "math talk". *Developmental Psychology*, *42*, 59–69. doi:10.1037/0012-1649.42.1.59
Knight, L. N., & Hargis, C. H. (1977). Math language ability: Its relationship to reading in math. *Language Arts*, *54*, 423–428.
Lansdell, J. M. (1999). Introducing young children to mathematical concepts: Problems with "new" terminology. *Educational Studies*, *25*, 327–333. doi:10.1080/03055699997837
LeFevre, J.-A., Fast, L., Skwarchuk, S.-L., Smith-Chant, B. L., Bisanz, J., Kamawar, D., & Penner-Wilger, M. (2010). Pathways to mathematics: Longitudinal predictors of performance. *Child Development*, *81*, 1753–1767. doi:10.1111/j.1467-8624.2010.01508.x
Lever, R., & Sénéchal, M. (2011). Discussing stories: On how a dialogic reading intervention improves kindergartners' oral narrative construction. *Journal of Experimental Child Psychology*, *108*, 1–24. doi:10.1016/j.jecp.2010.07.002
Levine, S. C., Suriyakham, L. W., Rowe, M. L., Huttenlocher, J., & Gunderson, E. A. (2010). What counts in the development of young children's number knowledge? *Developmental Psychology*, *46*, 1309–1319. doi:10.1037/a0019671
Lewis, C., Hitch, G. J., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9- to 10-year-old boys and girls. *Journal of Child Psychology and Psychiatry*, *35*, 283–292. doi:10.1111/j.1469-7610.1994.tb01162.x
Lipsey, M. W., Puzio, K., Yun, C., Hebert, M. A., Steinka-Fry, K., Cole, M. W., Busick, M. D. (2012). *Translating the statistical representation of the effects of education interventions into more readily interpretable forms*. Washington, DC: IES National Center for Special Education Research, Institute of Education Sciences.
Lonigan, C. J., Anthony, J. L., Bloomfield, B. G., Dyer, S. M., & Samwel, C. S. (1999). Effects of two shared-reading interventions on emergent literacy skills of at-risk preschoolers. *Journal of Early Intervention*, *22*, 306–322. doi:10.1177/105381519902200406

Lonigan, C. J., Purpura, D. J., Wilson, S. B., Walker, P. M., & Clancy-Menchetti, J. (2013). Evaluating the components of an emergent literacy intervention for preschool children at risk for reading difficulties. *Journal of Experimental Child Psychology*, *114*, 111–130. doi:10.1016/j.jecp.2012.08.010

Lonigan, C. J., & Whitehurst, G. J. (1998). Relative efficacy of parent and teacher involvement in a shared-reading intervention for preschool children from low-income backgrounds. *Early Childhood Research Quarterly*, *13*, 263–290. doi:10.1016/S0885-2006(99)80038-6

Mann Koepke, K. M., & Miller, B. (2013). At the intersection of math and reading disabilities: Introduction to the special issue. *Journal of Learning Disabilities*, *46*, 483–489. doi:10.1177/0022219413498200

Martin, N. A., & Brownell, R. (2011). *Expressive One-Word Picture Vocabulary Test* (4th ed.). Novato, CA: Academic Therapy Publications.

Miura, I. T., & Okamoto, Y. (2003). Language supports for mathematics understanding and performance. In A. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills: Constructing adaptive expertise* (pp. 229–242). Mahwah, NJ: Lawrence Erlbaum Associates.

Mix, K. S., & Cheng, Y. L. (2012). The relation between space and math: Developmental and educational implications. *Advances in Child Development and Behavior*, *42*, 197–243.

Mol, S. E., Bus, A. G., de Jong, M. T., & Smeets, D. J. (2008). Added value of dialogic parent–child book readings: A meta-analysis. *Early Education and Development*, *19*, 7–26. doi:10.1080/10409280701838603

Morgan, P. L., & Meier, C. R. (2008). Dialogic readings' potential to improve children's emergent literacy skills and behavior. *Preventing School Failure*, *52*, 11–16. doi:10.3200/PSFL.52.4.11-16

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through Grade 8 mathematics*. Reston, VA: Author.

Piasta, S. B., Pelatti, C. Y., & Miller, H. L. (2014). Mathematics and science learning opportunities in the preschool classrooms. *Early Education and Development*, *25*, 445–468. doi:10.1080/10409289.2013.817753

Powell, S. R., & Driver, M. K. (2014). The influence of mathematics vocabulary instruction embedded within addition tutoring for first-grade students with mathematics difficulty. *Learning Disability Quarterly*, 1–13. doi:10.1177/0731948714564574

Pruden, S. M., Levine, S. C., & Huttenlocher, J. (2011). Children's spatial thinking: Does talk about the spatial world matter? *Developmental Science*, *14*, 1417–1430. doi:10.1111/j.1467-7687.2011.01088.x

Pungello, E. P., Iruka, I. U., Dotterer, A. M., Mills-Koonce, R., & Reznick, J. S. (2009). The effects of socioeconomic status, race, and parenting on language development in early childhood. *Developmental Psychology*, *45*, 544–557. doi:10.1037/a0013917

Purpura, D. J., & Ganley, C. M. (2014). Working memory and language: Skill-specific or domain-general relations to mathematics? *Journal of Experimental Child Psychology*, *122*, 104–121. doi:10.1016/j.jecp.2013.12.009

Purpura, D. J., Hume, L. E., Sims, D. M., & Lonigan, C. J. (2011). Early literacy and early numeracy: The value of including early literacy skills in the prediction of numeracy development. *Journal of Experimental Child Psychology*, *110*, 647–658. doi:10.1016/j.jecp.2011.07.004

Purpura, D. J., & Logan, J. A. (2015). The nonlinear relations of the approximate number system and mathematical language to early mathematics development. *Developmental Psychology*, *51*, 1717–1724. doi:10.1037/dev0000055

Purpura, D. J., & Reid, E. E. (2016). Mathematics and language: Individual and group differences in mathematical language skills in young children. *Early Childhood Research Quarterly*, *36*, 259–268. doi:10.1016/j.ecresq.2015.12.020

Purpura, D. J., Reid, E. E., Eiland, M. D., & Baroody, A. J. (2015). Using a brief preschool early numeracy skills screener to identify young children with mathematics difficulties. *School Psychology Review*, *44*, 41–59. doi:10.17105/SPR44-1.41-59

Ramani, G. B., Rowe, M. L., Eason, S. H., & Leech, K. A. (2015). Math talk during informal learning activities in Head Start families. *Cognitive Development*, *35*, 15–33. doi:10.1016/j.cogdev.2014.11.002

Ramani, G. B., Zippert, E., Schweitzer, S., & Pan, S. (2014). Preschool children's joint block building during a guided play activity. *Journal of Applied Developmental Psychology*, *35*, 326–336. doi:10.1016/j.appdev.2014.05.005

Reid, E. E., Baroody, A. J., & Purpura, D. J. (2015). Assessing young children's number magnitude representation: A comparison between novel and conventional tasks. *Journal of Cognition and Development*, *16*, 759–779. doi:10.1080/15248372.2014.920844

Romano, E., Babchishin, L., Pagani, L. S., & Kohen, D. (2010). School readiness and later achievement: Replication and extension using a nationwide Canadian survey. *Developmental Psychology*, *46*, 995–1007. doi:10.1037/a0018880

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.

Sarama, J., Lange, A. A., Clements, D. H., & Wolfe, C. B. (2012). The impacts of an early mathematics curriculum on oral language and literacy. *Early Childhood Research Quarterly*, *27*, 489–502. doi:10.1016/j.ecresq.2011.12.002

Sarnecka, B. W., Kamenskaya, V. G., Yamana, Y., Ogura, T., & Yudovina, Y. B. (2007). From grammatical number to exact numbers: Early meanings of "one," "two," and "three" in English, Russian, and Japanese. *Cognitive Psychology*, *55*, 136–168. doi:10.1016/j.cogpsych.2006.09.001

Saxe, G. B., Guberman, S. R., & Gearhart, M. (1987). Social processes in early number development. *Monographs of the Society for Research in Child Development*, *52*, i–162. doi:10.2307/1166071

Schleppegrell, M. J. (2007). The linguistic challenges of mathematics teaching and learning: A research review. *Reading & Writing Quarterly*, *23*, 139–159. doi:10.1080/10573560601158461

Siegler, R. S., & Ramani, G. B. (2008). Playing linear numerical board games promotes low-income children's numerical development. *Developmental Science*, *11*, 655–661. doi:10.1111/j.1467-7687.2008.00714.x

Spelke, E. (2003). What makes us smart? Core knowledge and natural language. In D. Genter & S. Goldin-Meadow (Eds.), *Language in mind* (pp. 277–311). Cambridge, MA: MIT Press.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly*, *19*, 99–120. doi:10.1016/j.ecresq.2004.01.002

Toll, S. W. M., & Van Luit, J. E. H. (2014a). The developmental relationship between language and low early numeracy skills throughout kindergarten. *Exceptional Children*, *81*, 64–78. doi:10.1177/0014402914532233

Toll, S. W. M., & Van Luit, J. E. H. (2014b). Explaining numeracy development in weak performing kindergartners. *Journal of Experimental Child Psychology*, *124*, 97–111. doi:10.1016/j.jecp.2014.02.001

Vandermaas-Peeler, M., Nelson, J., Bumpass, C., & Sassine, B. (2009). Numeracy-related exchanges in joint storybook reading and play. *International Journal of Early Years Education*, *17*, 67–84. doi:10.1080/09669760802699910

Verdine, B. N., Irwin, C. M., Golinkoff, R. M., & Hirsh-Pasek, K. (2014). Contributions of executive function and spatial skills to preschool mathematics achievement. *Journal of Experimental Child Psychology*, *126*, 37–51. doi:10.1016/j.jecp.2014.02.012

Wasik, B. A., Bond, M. A., & Hindman, A. (2006). The effects of a language and literacy intervention on Head Start children and teachers. *Journal of Educational Psychology*, *98*, 63–74. doi:10.1037/0022-0663.98.1.63

Wilkinson, L., & APA Task Force on Statistical Inference. (1999). Statistical methods in psychology journals: Guidelines and explanations. *American Psychologist*, *54*, 594–604. doi: 10.1037/0003-066X.54.8.594

Young-Loveridge, J. (2004). Effects on early numeracy of a program using number books and games. *Early Childhood Research Quarterly*, *19*, 82–98. doi:10.1016/j.ecresq.2004.01.001