

Chapter 1

Library phi

Require Import Relations.Relation_Definitions.

Inductive Process :=

- | Nil : Process
- | Input : nat → (nat → Process) → Process
- | Output : nat → Process → Process
- | Sum : Process → Process → Process
- | Res : nat → Process → Process.

Definition small_step : relation Process :=

```
fun p1 p2 ⇒  
  match p1 with  
  | Input x f ⇒  
    ∃ v, p2 = f v  
  | Output x p ⇒  
    p2 = p  
  | Sum p1' p2' ⇒  
    p2 = Sum p1' p2' ∨ p2 = Sum p1 p2'  
  | Res x p ⇒  
    ∃ y, p2 = Res y p ∧ x ≠ y  
  | _ ⇒ False  
end.
```

Inductive multi_step : relation Process :=

- | refl : ∀ p, multi_step p p
- | step : ∀ p1 p2 p3, small_step p1 p2 → multi_step p2 p3 → multi_step p1 p3.

Inductive bisim : relation Process :=

- | bisim_refl : ∀ p, bisim p p
- | bisim_trans : ∀ p1 p2 p3, bisim p1 p2 → small_step p2 p3 → bisim p1 p3
- | bisim_symm : ∀ p1 p2, bisim p1 p2 → bisim p2 p1.

Definition bisimulation (R : relation Process) :=

∀ p q, R p q → (∀ p', small_step p p' → ∃ q', R p' q' ∧ small_step q q') ∧

$$(\forall q', \text{small_step } q \ q' \rightarrow \exists p', R \ p' \ q' \wedge \text{small_step } p \ p').$$

Definition bisimilarity ($R : \text{relation Process}$) :=

$$\forall p \ q, R \ p \ q \rightarrow \exists q', \text{bisim } q \ q' \wedge \text{multi_step } p \ q'.$$