# Vision and Image Processing: Shading, Photometric Stereo

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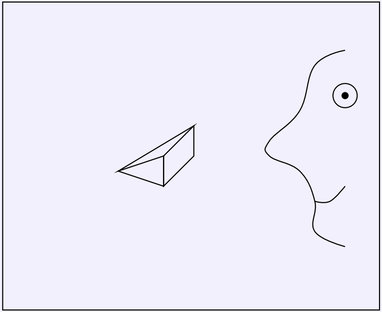
## Plan for today and January 3rd

- Image Formation and reflectance.
- Lighting Models.
- The Photometric Stereo Problem.

# Outline

# The Photometric Stereo (PS) Problem [Woodham, 1980]



- 1 fixed camera + 1 “fixed” scene
- $m$ lightings

Slide by Y. Quéau

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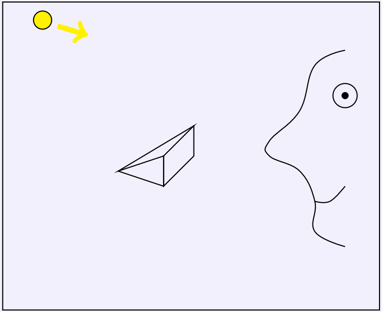

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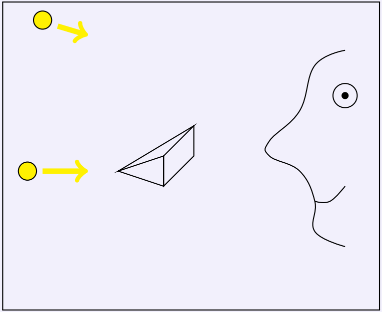

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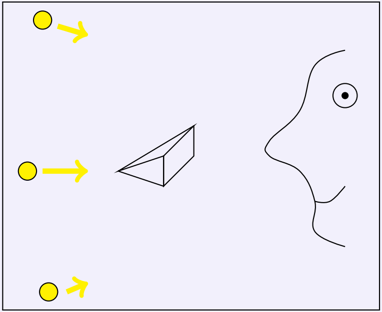

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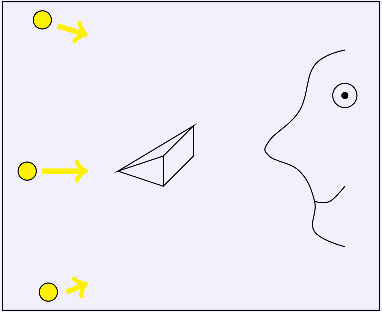

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- $m$ lightings

**Goal:**

3D-reconstruction of the scene from the 2D images

Slide by Y. Quéau

## Ingredients for Photometric Stereo

Photo metric Stereo

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Suggest. Volume recovery from measured light. Needed:

- Understand reflectance: how is light reflected from an object.
- How can we measure it.
- How object geometry is linked to light.

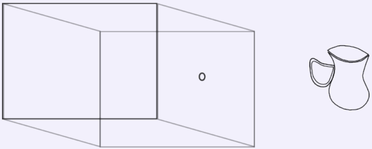
# Outline

# Light and Image Formation

**Ingredients**

- Object

## Light and Image Formation



**Ingredients**

- Object
- Camera

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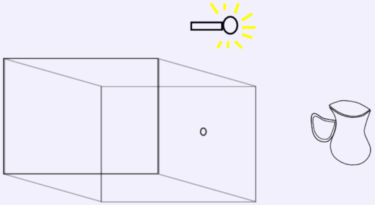

**Ingredients**

- Object
- Camera
- Light source

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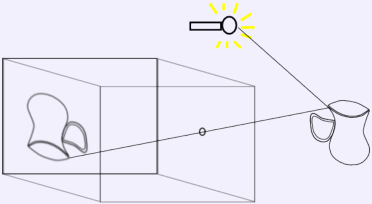

**Ingredients**

- Object
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- Light source
- Light reflection by object surface.

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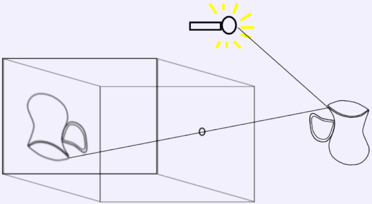

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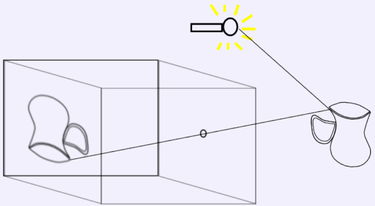

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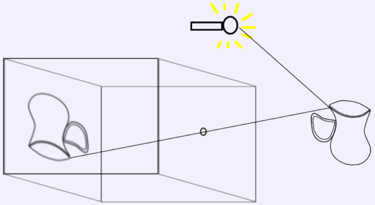

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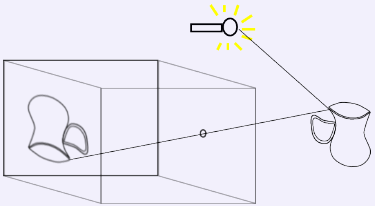

**Ingredients**

- Object
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- Image formation inside camera: when light, scene and camera parameters known: reflectance function.
- BTW: Camera detectors react almost truly linearly to received luminance.
- Can image formation model give enough information about the object surface to reconstruct it?

# Materials and Light

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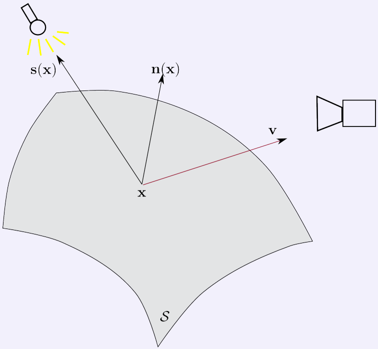
From now, only opaque objects.

# Matte vs Brilliant

# Bidirectional Reflectance Distribution Function – BRDF



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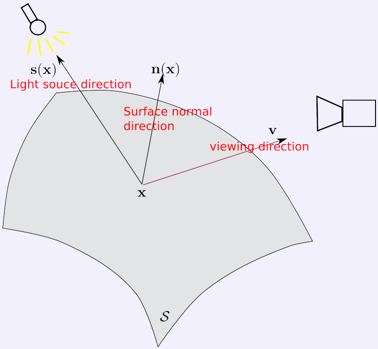


- Luminance emitted by punctual object **x** on a surface $\mathcal{S}$ with normal direction $\mathbf{n}(\mathbf{x})$ at **x**, in emission direction **v** characterized by spherical angles $(\theta_e, \varphi_e)$ w.r.t $\mathbf{n}(\mathbf{x})$:

$$L(\mathbf{x}, \theta_e, \varphi_e) = \int_{\theta_i=0}^{\frac{\pi}{2}} \int_{\varphi_i=0}^{2\pi} \kappa(\mathbf{x}, \theta_i, \varphi_i, \theta_e, \varphi_e) \bar{L}(\theta_i, \varphi_i) \sin\theta_i \cos\theta_i \, d\theta_i d\varphi.$$

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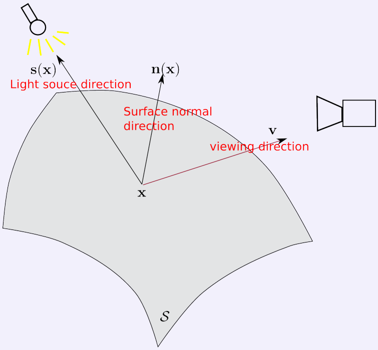


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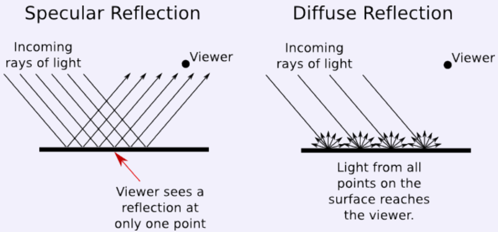
- Nice formula but I won't explain what it means!

## Specular Vs. Matte Objects




Two standard reflection models: specular: mirror like surface, diffuse: rough surface (at very small scale): Lambertian model. Others, especially useful in Computer Graphics.

# Diffuse Reflection

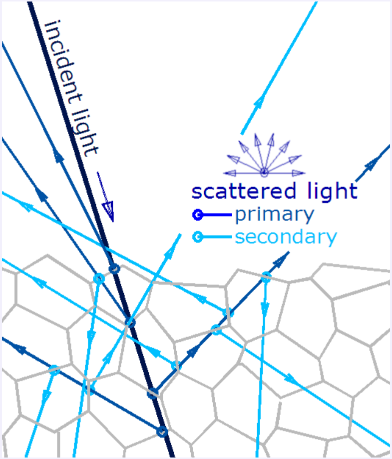



Image Source GianniG46, Wikipedia

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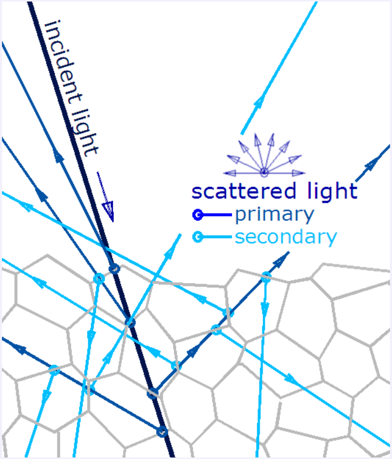


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Rough surface at micro-scale.

- Light bounces.
- Reflections in all directions
- Some light is absorbed.
- Only a percentage of light energy is reemitted.

## Lambert's Cosine Law

### Reflectance

- Linearised Lambertian model: $I(\mathbf{p}) = \rho(\mathbf{x})\mathbf{s}(\mathbf{x}) \cdot \mathbf{n}(\mathbf{x})$
- $\rho(\mathbf{x})$ is the *albedo* at $\mathbf{x}$ – material light absorption property, $\rho \in [0, 1]$. Assumes matte material such as chalk...

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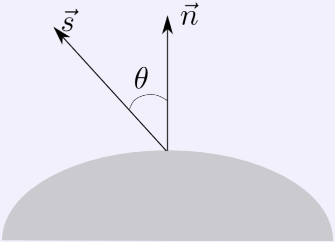

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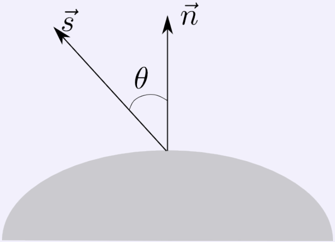


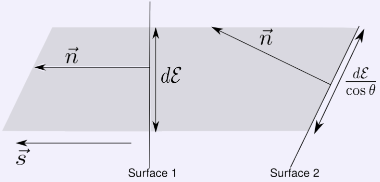

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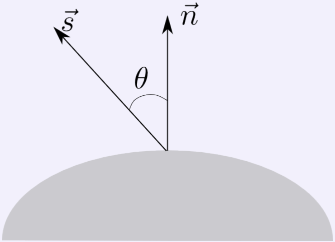
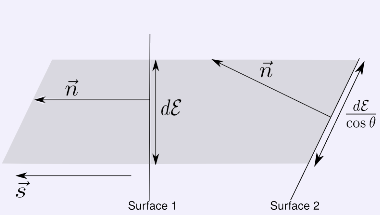
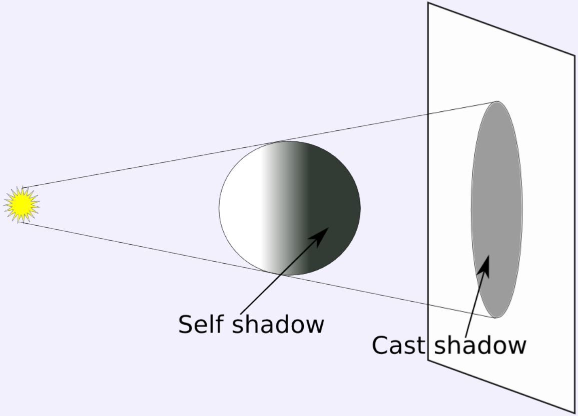



### In words

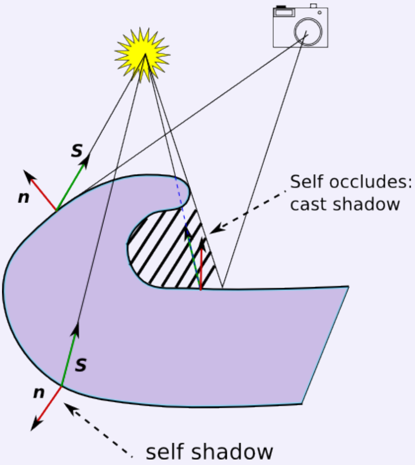
- Local orientation of object w.r.t. light: In surface 1: surface area matches ray "section". In surface 2: surface area larger than ray section, but receive same amount of light.

## Shadows




- Self shadow: surface is behind the light source. $\mathbf{s} \cdot \mathbf{n} \leq 0$.
- Cast shadow: part of the scene occludes another part.

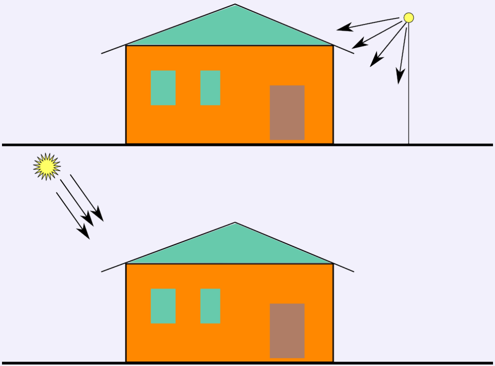
# Shadows again




- Lambert's law and self-shadows: $I = \rho \max(\mathbf{s} \cdot \mathbf{n}, 0)$.
- Cast shadows: Non local phenomenon, Lambert's law is local...

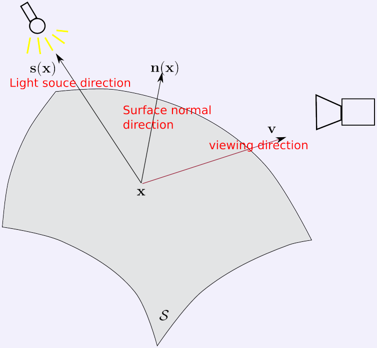
# Outline

## Type of Light Sources



- Top: near light source, radial.
- Bottom: far light source: parallel. Our choice in these lectures.
- Other types?

# Shape From Shading (SfS) – B. Horn 1970




- Use Lambert's Law to gain information on visible surface via normal vector $\mathbf{n}(\mathbf{x})$.
- Need link between surface equations and normal vector.

# Settings

- Representation of the surface. Assume surface parameterized by $(u, v) \mapsto \mathcal{S}(u, v) \in \mathbb{R}^3$. Even better: depth map:

$$S(u, v) = [u, v, z(u, v)]^T : \text{ Monge Patch.}$$

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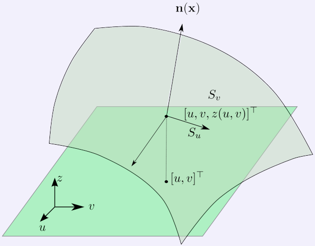


Tangent vectors at $\mathbf{x}(u, v) = [u, v, z(u, v)]^\top$

$$\frac{\partial \mathcal{S}}{\partial u} = S_u = \begin{bmatrix} 1 \\ 0 \\ \frac{\partial z}{\partial u} \end{bmatrix}, \quad \frac{\partial \mathcal{S}}{\partial v} = \mathcal{S}_v \begin{bmatrix} 0 \\ 1 \\ \frac{\partial z}{\partial v} \end{bmatrix},$$

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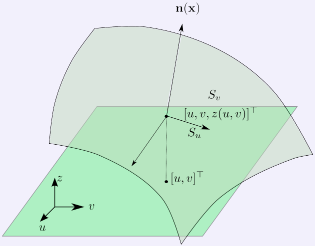


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- Normal vector $\mathbf{n}(u, v) = \mathbf{n}(\mathbf{x}(u, v))$

$$\mathbf{n}(\mathbf{x}) = \frac{\mathcal{S}_u \times \mathcal{S}_v}{|\mathcal{S}_u \times \mathcal{S}_v|} = \frac{1}{\sqrt{|\nabla z|^2 + 1}} \begin{bmatrix} -z_u \\ -z_v \\ 1 \end{bmatrix}$$

## Camera Model

- Two Choices: pinhole and orthographic.
- Orthographic Camera Model.
  - $[x = u, y = v, z] \mapsto [u, v]$: orthographic projection.
  - Formula from previous slide: Coordinates of normal vector in orthographic projection:

$$\mathbf{n} = \frac{1}{\sqrt{|\nabla z|^2 + 1}} \begin{bmatrix} -z_u \\ -z_v \\ 1 \end{bmatrix}$$

- Pinhole Camera model.
  - Projection

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} u = \frac{fx}{z} \\ v = \frac{fy}{z} \end{bmatrix}$$

  - Normal vector in camera coordinates: complicated!

$$\mathbf{n} = \frac{1}{\sqrt{|\nabla z|^2 + \left(\frac{z + [u,v]\nabla z}{f}\right)^2}} \begin{bmatrix} -z_u \\ -z_v \\ \frac{z + [u,v]\nabla z}{f} \end{bmatrix}$$

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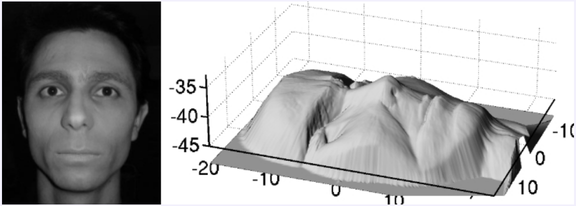
Complicated Math and Research Topics

# Examples, Problems



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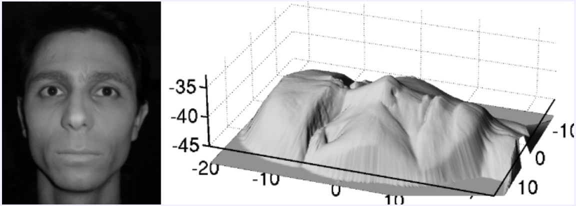


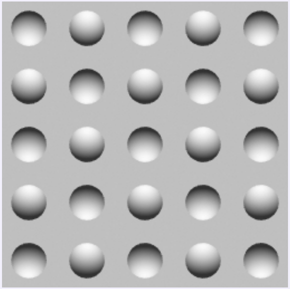

# SfS (Counter)Example

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## Why Does It Go Wrong

Assume Light **s** known and constant (far light source with known intensity). Per Pixel:

- Number of unknowns:
  - Normal vector $\mathbf{n}(u, v)$, 3 components $[\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3]$, but
    $$\|\mathbf{n}\| = 1 : \mathbf{n}_1^2 + \mathbf{n}_2^2 + \mathbf{n}_3^2 = 1.$$
    2 degrees of freedom (DoF).
  - Albedo $\rho(u, v)$: 1 value, 1 DoF.
  - Total: 3 DoF.
- Known information per pixel:
  - Reflectance $I(u, v) = \rho(u, v)\mathbf{s} \cdot \mathbf{n}(u, v)$: 1 equation linking 3 unknowns.
- Remaining DoFs: 2.
- For unambiguous solution, need remaining DoF = 0.
- In counterexample, 1DoF removed by assuming $\rho(u, v) \equiv 1$: Wrong!

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- Answer is geometric!

## Woodham Original PS – 1980

- How to remove Degrees of Freedom?
- Woodham Solution: Use more images $I^1, I^2, \dots, I^k$.
- Don't change Camera Position – keep same depth function, normals, change lights.
- $k$ far (parallel) light sources $\mathbf{s}^1, \dots, \mathbf{s}^k$: $k$ equations

$$\begin{cases} I^1(u,v) & = \rho(u,v)\mathbf{s}^1 \cdot \mathbf{n}(u,v) \\ I^2(u,v) & = \rho(u,v)\mathbf{s}^2 \cdot \mathbf{n}(u,v) \\ \dots & \dots \\ I^k(u,v) & = \rho(u,v)\mathbf{s}^k \cdot \mathbf{n}(u,v) \end{cases}$$

$$(\mathbf{s}^i \cdot \mathbf{n} = \mathbf{s}^i_1 \mathbf{n}^1 + \mathbf{s}^i_2 \mathbf{n}_2 + \mathbf{s}^i_3 \mathbf{n}_3)$$

- Which $k$ to choose? 3 DoF: $k \geq 3$. Exactly 3, more?
- Answer is geometric!
- From $(u,v) \mapsto \mathbf{n}(u,v)$ to surface? Integration of normals: out of scope; Matlab / Python functions will be provided.

## Linear System for Normal + Albedo

- Woodham idea: Normal + Albedo Simultaneously

$$\mathbf{m} := \rho\mathbf{n}, \quad \rho = \|\mathbf{m}\|, \mathbf{n} = \frac{\mathbf{m}}{\|\mathbf{m}\|}$$

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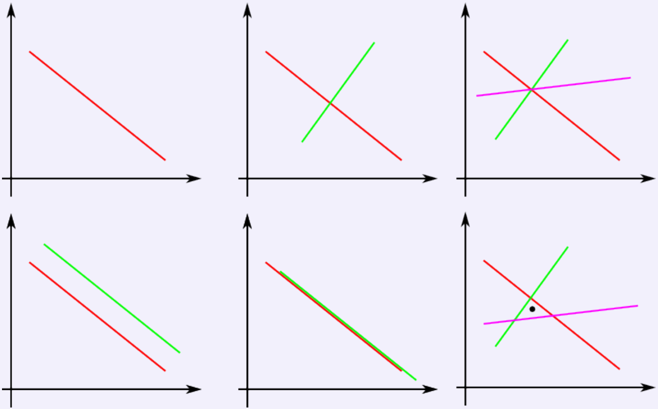
- Number of solutions?
- Can vary from none to a lot!

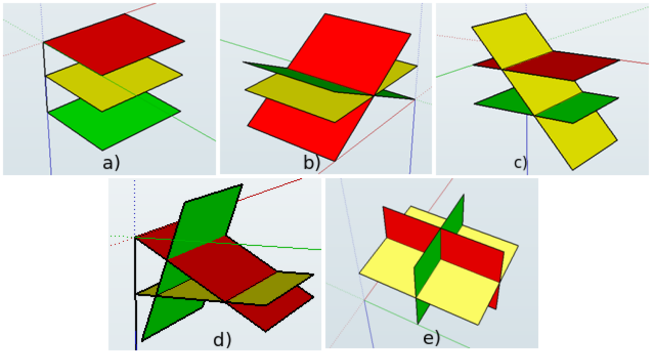
## Linear Algebra Again

- System of equations in 2 unknown.



- Need at least 2 independent equations! ...but not 3!
- For 3 or more: concept of “closest solution in least squares”.

# In 3D




Same light source, incompatible measurements, b) coplanar light sources, compatible measurements, c) 2 light source, 3 incompatible measurements, d) coplanar light sources, incompatible measurements, e) 3 non coplanar light sources

Picture from Guillermo Bautista, mathandmultimedia.com

# What Can Go Wrong?

- Parameters / devices:
  - Measurement errors: Camera?
  - Coplanar light sources: example?
- Reflectance
  - Lambert's law only valid for matte materials: specularities.
  - Shadows / penumbra: non black cast shadow areas.
- More?

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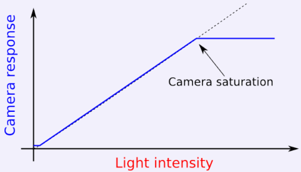

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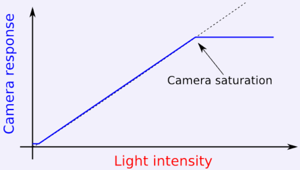


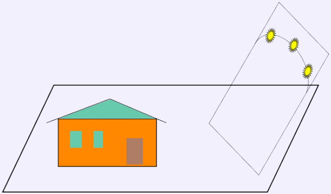

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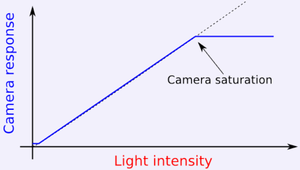


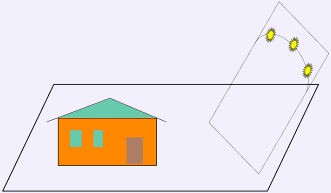

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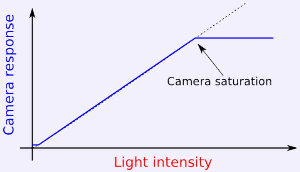
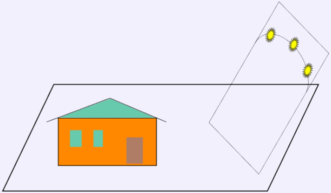
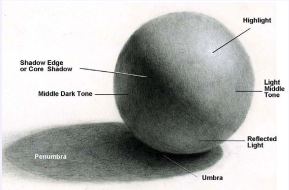




## Solving For Normals: Pseudo-Inverse Approach

Assume 3 or more non coplanar light sources.

- Equation $\mathbf{I} = M_{\mathbf{s}}\,\mathbf{m}$ may not have solution if $\mathbf{M_s}$ is $k \times 3$, $k > 3$.
- Via Moore-Penrose Pseudo-Inverse $M_{\mathbf{s}}^{\dagger}$, solution of

$$\mathbf{m} \text{ such that } \|\mathbf{I} - M_{\mathbf{s}}\mathbf{m}\|^2 = \min : \quad \mathbf{m} = M_{\mathbf{s}}^{\dagger}\mathbf{I}.$$

Pros.

- Good news: Matlab `pinv` function, Python `pinv` function in package `numpy.linalg`.

Cons.

- Does not separate wrong and accurate measurements.

# Solving For Normals: Equations Selection - I

In a nutshell.

- Per pixel: find best constraints: 3 "good measurements" $I^a$, $I^b$, $I^c$ and corresponding "good lights" $\mathbf{s}^a$, $\mathbf{s}^b$, $\mathbf{s}^c$.

$$\underbrace{\begin{bmatrix} I^a \\ I^b \\ I^c \end{bmatrix}}_{\mathbf{I}_{abc}} = \underbrace{\begin{bmatrix} \mathbf{s}_1^a & \mathbf{s}_2^a & \mathbf{s}_3^a \\ \mathbf{s}_1^b & \mathbf{s}_2^b & \mathbf{s}_3^b \\ \mathbf{s}_1^c & \mathbf{s}_2^c & \mathbf{s}_3^c \end{bmatrix}}_{M_{abc}} \begin{bmatrix} \mathbf{m}^1 \\ \mathbf{m}^2 \\ \mathbf{m}^3 \end{bmatrix}$$

Good measurements.

- $I_i$ good measurement: not too small (shadow, penumbra) and not too large (saturation, potential specularity).
- Good light sources: $\det M_{abc}$ as large as possible.

# Solving For Normals: Equations Selection - II

Pros.

- Best system of equations per pixel.

Cons.

- Lack of spatial coherence: different lights can be chosen for neighbor pixels.
- Need thresholds for intensities: parameters of the algorithm.
- More complicated to code.

Size Matters.

- For relatively small $k$: equation selection can be a good idea.
- For very large $k$ (1000, more...) pseudo-inverse very good: statistical reason.

# Algorithm in a Nutshell

Input.

- $k$ known parallel light sources $\mathbf{s}_1, \ldots \mathbf{s}_k$. $k$ recorded images $I_1, \ldots, I_k$.

Normals and Albedo recovery.

- For each (valid) pixel $[u, v]$ in image domain:

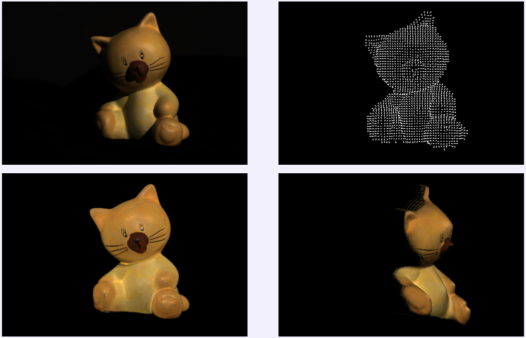
  1. Solve $\mathbf{m}(u, v)$ either via pseudo-inverse or equation selection.
  2. Get albedo and normal:

$$\rho(u,v) = \|\mathbf{m}(u,v)\|, \quad \mathbf{n}(u,v) = \frac{1}{\rho(u,v)}\mathbf{m}(u,v).$$

Surface Recovery.

- Get surface from normals via surface integration.
- May "paint surface" with albedo.

# Process



Top left: 1 of the input images, right: normal field. Bottom left: albedo, right: a 3D reconstruction.

# Mezigue

# Mezigue

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## Literature, available in AbasIon

- Horn, Shape from shading 1970
- Woodham, Photometric Stereo, 1980

## Summary

- φῶς/φωτός
- μέτρον
- στερεός

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