ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examination: Summer 2011

SIGNAL PROCESSING

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed: 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. ANSWER BOTH PARTS

(a) The impulse response of a system is

$$h(t) = e^{-2t} \sin 2t.$$

Determine the Laplace transfer function of the system and hence its poles. Is the system stable?

(10 marks)

(b) Show that the Laplace transfer function of the servo shown in Figure 1 is

$$H(s) = \frac{1}{1 + s\tau}$$

where $\tau = RC$. Determine the dynamic response $v_o(t)$ of the system to a unit ramp input $v_i(t)$. Sketch and comment on the response.

(15 marks)

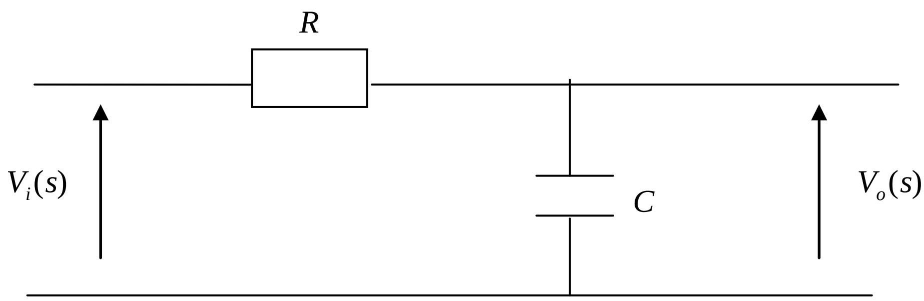

Figure 1 A servo.

(25 marks total)

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2. The Z transform transfer function of a filter has poles at $z = \frac{1}{4} \pm j\frac{1}{4}$ and zeros at $z = -1 \pm j$. Determine the transfer function and hence the filter coefficients.

(12 marks)

Sketch the Z-plane pole-zero plot of the filter. Use the plot to sketch the magnitude frequency response of the filter, indicating the value of the response at radial frequency ω of 0, $\frac{\pi}{2}$ and π. Hence state whether the filter is lowpass, highpass or bandpass.

(13 marks)

(25 marks total)

Continued ….

3. Derive the difference equation of the system shown in Figure 2. Write down the input sequence $\{x(n)\}$ for $n = 0,1,2,\ldots,8$ that will give the impulse response of the system. Use the difference equation and a tabular method to determine the impulse response $\{y(n)\}$ to 1 decimal place, assuming quiescent conditions.

(13 marks)

Derive the Z transform transfer function of the system and hence determine its poles and zeros. Hence state whether the system is stable or unstable. How is this confirmed by the impulse response?

(12 marks)

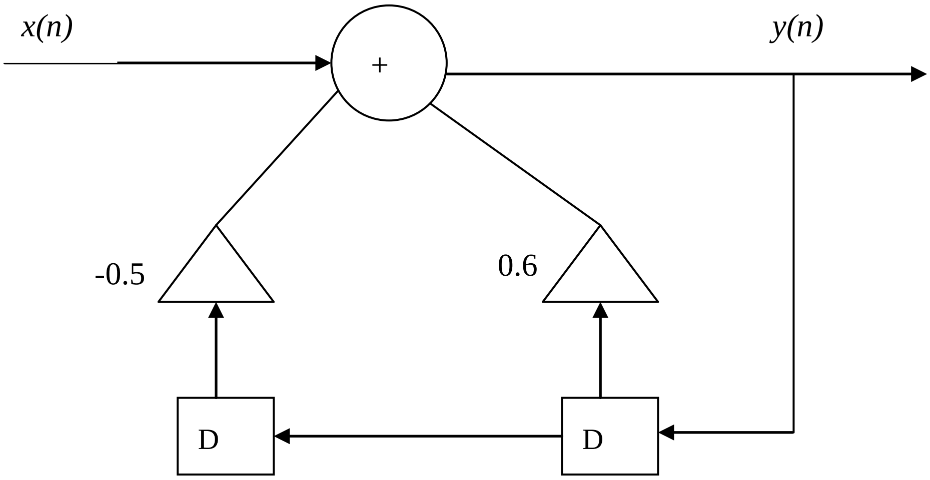

Figure 2 A second-order system.

(25 marks total)

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SECTION B

4. ANSWER BOTH PARTS

(a) A random, discrete-interval binary signal, *x(t)*, can take the level 1 with a probability of 0.6, and 0 with a probability of 0.4. Show that its autocorrelation function is

$$R_{xx}(\tau) = 0.6 - 0.24\left(\frac{|\tau|}{\lambda}\right) \qquad 0 \le |\tau| \le \lambda$$

$$= 0.36 \qquad |\tau| \ge \lambda,$$

where λ is the clock pulse interval of the signal. (14 marks)

(b) Describe how a pseudo-random binary signal based on a maximum length sequence of period 15 digits could be generated using a circuit based on a shift register. How can a maximum-length sequence be ensured? (4 marks)

Sketch the autocorrelation function of such a signal and compare this with a sketch of the autocorrelation function described in part (a). (4 marks)

Give an example of a practical use of a pseudo-random binary signal. (3 marks)

(25 marks total)

Continued **….**

5. ANSWER BOTH PARTS

(a) The Discrete Time Fourier Transform (DTFT) of a signal $g[k]$ is defined as

$$G(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} g[k]\exp(-jk\omega T).$$

Find the DTFT of the signal shown in Figure 3. Assume all other samples are zero.

Sketch the magnitude of $G(e^{j\omega T})$ against ω. You can assume that T = 1.

(13 marks)

(b) The Discrete Fourier Transform (DFT) of the sequence $x[n]$ is defined as

$$X[m] = \sum_{n=0}^{N-1} x[n]\exp\left(-j2\pi mn/N\right).$$

A complete cycle of a sinusoid has been sampled by eight equally spaced samples. After processing by the DFT a line spectrum was obtained. In a second experiment, the waveform was truncated, and eight such samples were obtained from just 90% of the cycle. The samples were y = [1.000 0.760 0.156 -0.522 -0.951, -0.924 -0.454 0.233].

Calculate the values Y[0] and Y[1] of the DFT of the sequence $y[n]$. Given the remaining values $|Y[2]| = |Y[6]| = 0.37$, $|Y[3]| = |Y[5]| = 0.23$, and $|Y[4]| = 0.20$, find a value for $|Y[7]|$ and sketch $|Y[m]|$ for $m = 0,1,\ldots,7$. Why is this not a line spectrum? How could it be made to appear more line like? (12 marks)

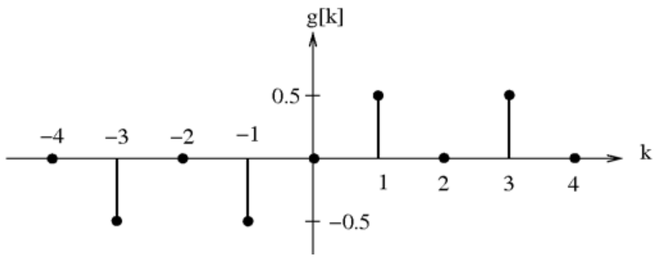

Figure 3 Signal $g[k]$.

(25 marks total)

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6. Sketch the block diagram of a second-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights. (3 marks)

It is required to design a FIR filter to approximate the ideal high-pass characteristic with cut-off frequency f_c . The folding frequency of the digital filter is to be 4 f_c . By Fourier series show that the ideal filter weights are given by

$$g_n = 0.75 \quad , \qquad n = 0$$

$$g_n = -\frac{1}{n\pi}\sin\frac{n\pi}{4}, \qquad n = \pm 1, \pm 2, \pm 3, \; ...$$ (7 marks)

A Blackman window with $N = 4$ is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for $n = 0, \pm 1, \pm 2, \pm 3$, and ± 4. For the Blackman window, weight

$$w_n = 0.42 + 0.50\cos\left(\frac{2n\pi}{2N+1}\right) + 0.08\cos\left(\frac{4n\pi}{2N+1}\right), \quad -N \le n \le N$$

$$w_n = 0 \qquad , \quad \text{all other values of } n$$ (4 marks)

Find the magnitude of the gain of the filter at D.C., and at the folding frequency. (3 marks)

Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

The inverse Fourier transform of a signal $H(j\omega)$ is defined as $h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(j\omega) exp(j\omega t)\, d\omega$.

By inverse transformation into the time domain, show that the ideal low-pass filter characteristic is not physically realisable. (6 marks)

(25 marks total)

END