ES3350

THE UNIVERSITY OF WARWICK

Third Year Examinations: Summer 2010

COMMUNICATIONS SYSTEMS

SECTION A

SECTION B

Candidates should answer FOUR QUESTIONS.

Time Allowed: 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided. Data sheets for Bessel functions are attached to the paper.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

PLEASE USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. (a) An oscilloscope display of a full AM wave $e(t)$ is shown in Figure 1. The modulating signal $m(t)$ is a sinusoidal signal of the form $E_m \cos(\omega_m t)$. The carrier signal is of the form $E_c \cos(\omega_c t)$. To assist you, the full AM wave envelope is also plotted.

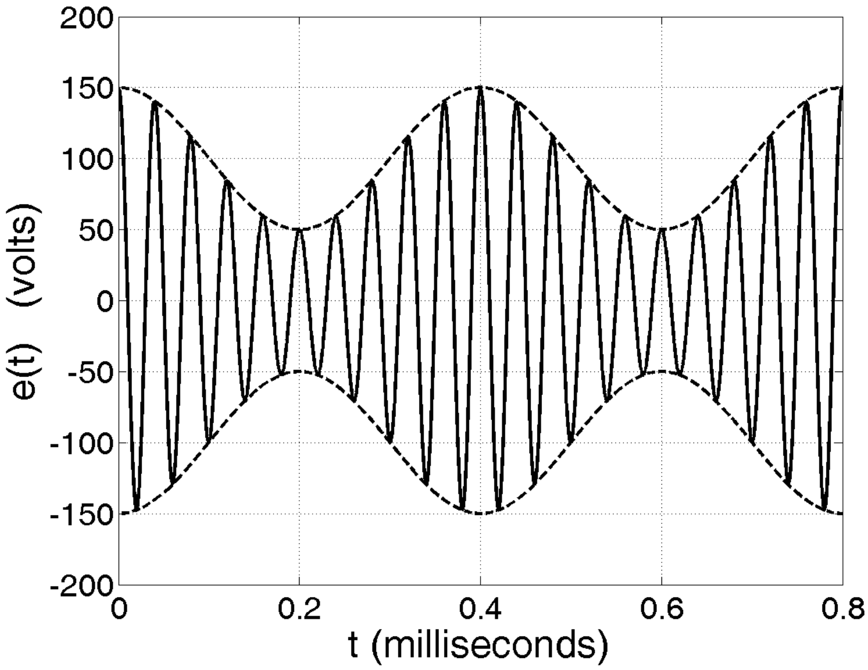

Figure 1

(i) Determine the modulating signal frequency, in Hz.

(ii) Determine the carrier signal frequency, in Hz.

(iii) Determine the amplitudes of the modulating signal and the carrier signal.

(iv) Determine the modulation index.

(v) Determine the mathematical expression of $e(t)$.

(vi) Sketch the spectrum of $e(t)$.

(vii) Determine the power efficiency.

(16 marks)

Question 1 continued
Question 1 continued overleaf

(b) A 75% modulated full AM wave has 1 kW of power in the lower sideband. The modulating signal is a single sine wave. The carrier component is attenuated by 4 dB before the transmission but the sideband components are unchanged. Calculate the total transmitted power.

(5 marks)

(c) The instantaneous amplitude of a full AM signal is given by the expression

$$e(t) = 1000\left[1 + \sum_{n=1}^{3} \frac{1}{2n}\cos(2\pi n \times 10^3 t)\right]\cos(2\pi \times 10^3 t) \qquad \text{volts}$$

Calculate the mean power that the signal would dissipate in a 50 Ω resistive load.

(4 marks)

Continued

2. An FM modulator output is given by the expression

$$e(t)\big|_{FM} = E_c \cos[\omega_c t + \beta \sin(\omega_m t)] \qquad (1)$$

where β is the modulation index. The modulating signal $m(t)$ is a single modulating sinusoid of the form $m(t) = E_m \cos(\omega_m t)$. The unmodulated carrier signal is $e_c(t) = E_c \cos(\omega_c t)$.

(a) Show, through derivation starting from equation (1), that the FM signal expression can be rewritten as follows

$$e(t)\big|_{FM} = E_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[(\omega_c + n\omega_m)t]$$

where $J_n(\cdot)$ is the nth-order Bessel function of the first kind.

Note: To aid you in your derivation the following formulae are given

$$\cos[\beta \sin(\omega_m t)] = J_0(\beta) + 2\sum_{n=1}^{\infty} J_{2n}(\beta)\cos(2n\omega_m t),$$

$$\sin[\beta \sin(\omega_m t)] = 2\sum_{n=0}^{\infty} J_{2n+1}(\beta)\sin[(2n+1)\omega_m t)]$$

(6 marks)

(b) When the carrier signal $e_C(t) = 10\cos(10^6\pi t)$ volts is frequency modulated by the message signal $m(t) = 2\cos(10^4\pi t)$ volts, the carrier frequency varies within ±4% of its unmodulated value.

Note: To aid you in your answers, a table of Bessel functions and a Bessel function plot are attached.

Question 2 continued overleaf

Determine:

(i) The frequency sensitivity of the modulator.

Question 2 continued

(ii) The modulation index.

(iii) The bandwidth using the 1% significant sideband frequency approach.

(iv) The bandwidth using Carson's rule.

(v) Determine the percentage of the total power contained in the frequency band 473 kHz to 526 kHz.

(vi) The frequency deviation is carefully increased from its current value until the second sideband amplitude in the output is zero. Under these conditions and assuming a 50 Ω resistive load, determine the average power in the first-sidebands.

(19 marks)

Question 2 continued overleaf

3. (a) Consider a 75 Ω resistor maintained at ambient temperature. Assuming a bandwidth of 1 MHz, calculate the following:

   (i) The root-mean-square value of the voltage appearing across the terminals of this resistor due to thermal noise.

   (ii) The available noise power delivered to a matched load.

(6 marks)

(b) A system consists of a cascade of 2 networks as shown in Figure 2. Network 1 has a gain G_1 = 20 dB and a noise figure F_1 = 5 dB. Network 2 has a gain G_2 = –2 dB and a noise figure of F_2 = 12 dB.

   (i) Calculate the noise figure of the system in dB.

   (ii) Calculate the signal to noise ratio in dB at the output of Network 2 if the signal to noise ratio at the input of Network 1 is 30 dB.

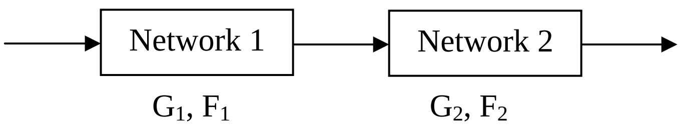

Figure 2

(7 marks)

(c) Consider the receiver system of Figure 3, which consists of a lossy waveguide, a low-noise amplifier, a frequency down converter and an intermediate frequency amplifier. Figure 3 includes the noise figures (F) and power gains (G) of these four networks. The various stages are matched for maximum power transfer. The antenna temperature is 50 K.

Question 3 continued overleaf

Question 3 continued

(i) Calculate the equivalent noise temperature for each of the four networks in Figure 3 assuming that the noise figure of each network was measured under matched input conditions with the noise source at an ambient temperature.

(ii) Using the values obtained in (i), calculate the equivalent noise temperature, T_e, at the waveguide input.

(iii) Calculate the total effective noise temperature at the waveguide input.

(iv) If the available signal power from the antenna is 10^{-13} W and the equivalent noise bandwidth of the receiver system is 10 MHz calculate the signal to noise ratio at the output of the system.

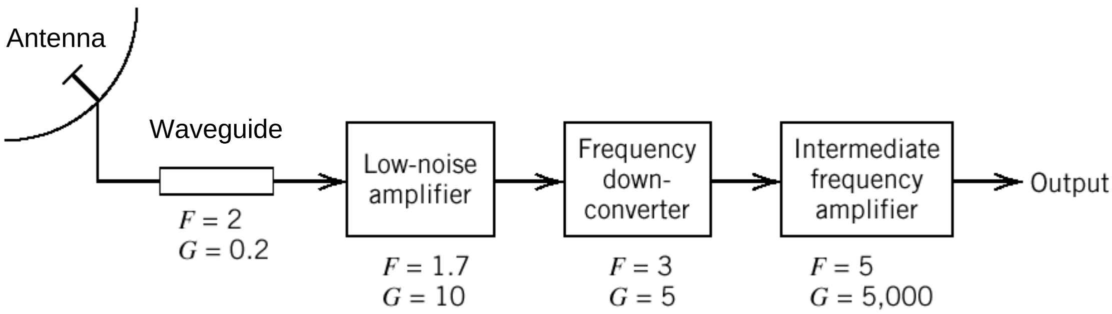

Figure 3

(12 marks)

Continued

SECTION B

4. (a) Explain the signal to noise ratio benefit obtained through the use of frequency shift keying (FSK) or phase shift keying (PSK) rather than amplitude shift keying (ASK). Indicate which method is likely to be the optimum choice when bandwidth is also a design consideration.

(5 marks)

(b) The bit error rate of a coherent carrier borne digital transmission system is :

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left[\sqrt{\frac{\int_{-\infty}^{\infty} p^2(t)dt}{4N_0}}\right]$$

where $p^2(t)$ is the difference signal and N_0 is the single-sided noise power spectral density. Starting from this equation, obtain an expression for incoherent FSK in terms of the average energy per bit and N_0, indicating any assumptions made.

(10 marks)

(c) Calculate the probability of error for an FSK system operating at 250 kHz using a carrier of peak amplitude 0.5 V when the *two-sided* noise power spectral density is $4\times10^{-8}\ \mathrm{V}^2\,\mathrm{Hz}^{-1}$. You may use the approximation $\mathrm{erf}(x) \approx \exp(-x^2)/(x\sqrt{\pi})$.

(5 marks)

(d) Explain the advantage of M-ary transmission for modulated data using a limited bandwidth channel. Sketch the constellation diagrams for binary PSK, QPSK and 16-QAM.

(5 marks)

5. (a) Explain how the phenomenon of intersymbol interference (ISI) arises in telecommunication channels indicating how it may cause transmission errors.

(4 marks)

(b) A pulse arrives after transmission through a non-ideal telecommunication channel, with bandwidth B, possessing a shape given by:

$$P(t) = e^{-\pi B|t|/2} \cos(-\pi Bt)$$

Sketch the shape of the pulse when B is 100 kHz.

(4 marks)

(c) Describe, using appropriate diagrams and mathematical expressions, the technique of zero-forcing equalisation that may be used to improve the shape of received pulses that have undergone channel distortion.

(7 marks)

(d) Design a three-tap zero-forcing equaliser for the pulse in part (b), assuming that the bit rate is 100 kbps. Sketch the equalised pulse shape.

(10 marks)

6. (a) What is *Perfect Secrecy*? Describe a system which achieves it.

(2 marks)

(b) Explain what is meant by *one-way functions* and *trapdoor one-way functions*, illustrating your answer with reference to factorisation.

(4 marks)

(c) Briefly outline the principles of Public Key Cryptosystems.

(5 marks)

(d) Describe the operation of the RSA Public Key Cryptosystem, and how suitable public and private keys p and s are derived from two large primes q and r.

(10 marks)

(e) Find the secret key for a simple RSA system employing the values $q = 11$, $r = 7$ and $p = 7$.

(4 marks)

END

Plot of Bessel Functions

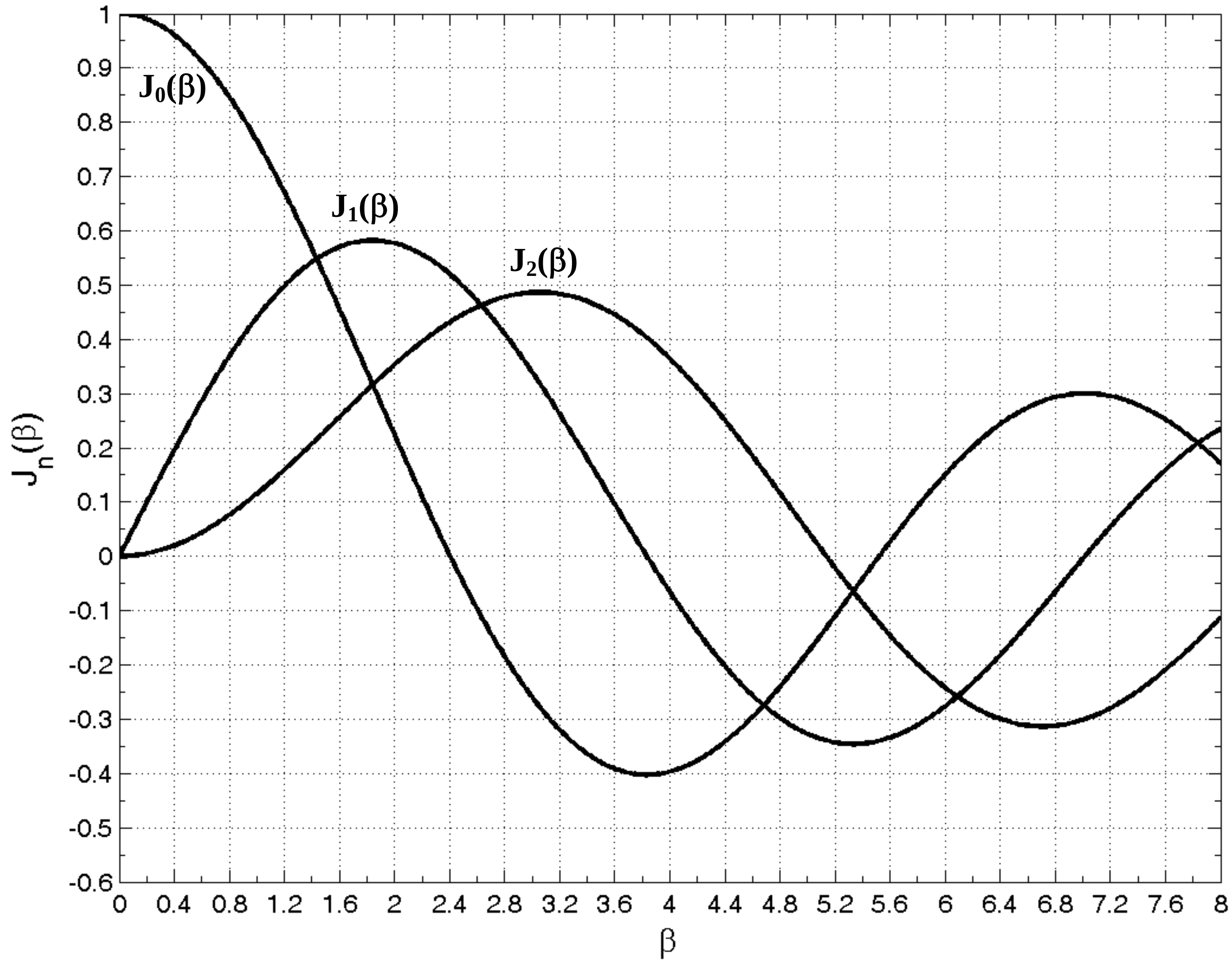

Table of Bessel Functions to four decimal places

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$	$J_{10}(\beta)$	$J_{11}(\beta)$	$J_{12}(\beta)$	$J_{13}(\beta)$	$J_{14}(\beta)$	$J_{15}(\beta)$	$J_{16}(\beta)$
0	1.0000	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.10	0.9975	0.0499	0.0012	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.20	0.9900	0.0995	0.0050	0.0002	–	–	–	–	–	–	–	–	–	–	–	–	–
0.25	0.9844	0.1240	0.0078	0.0003	–	–	–	–	–	–	–	–	–	–	–	–	–
0.50	0.9385	0.2423	0.0306	0.0026	0.0002	–	–	–	–	–	–	–	–	–	–	–	–
1.0	0.7652	0.4401	0.1149	0.0196	0.0025	0.0002	–	–	–	–	–	–	–	–	–	–	–
1.5	0.5118	0.5579	0.2321	0.0610	0.0118	0.0018	0.0002	–	–	–	–	–	–	–	–	–	–
2.0	0.2239	0.5767	0.3528	0.1289	0.0340	0.0070	0.0012	0.0002	–	–	–	–	–	–	–	–	–
2.4048	–	0.5192	0.4318	0.1990	0.0647	0.0164	0.0034	0.0006	0.0001	–	–	–	–	–	–	–	–
3.0	−0.2601	0.3391	0.4861	0.3091	0.1320	0.0430	0.0114	0.0025	0.0005	0.0001	–	–	–	–	–	–	–
4.0	−0.3971	−0.0660	0.3641	0.4302	0.2811	0.1321	0.0491	0.0152	0.0040	0.0009	0.0002	–	–	–	–	–	–
5.0	−0.1776	−0.3276	0.0466	0.3648	0.3912	0.2611	0.1310	0.0534	0.0184	0.0055	0.0015	0.0004	0.0001	–	–	–	–
5.5201	–	−0.3403	−0.1233	0.2509	0.3960	0.3230	0.1891	0.0881	0.0344	0.0116	0.0035	0.0009	0.0002	–	–	–	–
6.0	0.1506	−0.2767	−0.2429	0.1148	0.3576	0.3621	0.2458	0.1296	0.0565	0.0212	0.0070	0.0020	0.0005	0.0001	–	–	–
7.0	0.3001	−0.0047	−0.3014	−0.1676	0.1578	0.3479	0.3392	0.2336	0.1280	0.0589	0.0235	0.0083	0.0027	0.0008	0.0002	0.0001	–
8.0	0.1717	0.2346	−0.1130	−0.2911	−0.1054	0.1858	0.3376	0.3206	0.2235	0.1263	0.0608	0.0256	0.0096	0.0033	0.0010	0.0003	0.0001
9.0	−0.0903	0.2453	0.1448	−0.1809	−0.2655	−0.0550	0.2043	0.3275	0.3051	0.2149	0.1247	0.0622	0.0274	0.0108	0.0039	0.0013	0.0004
10	−0.2459	0.0435	0.2546	0.0584	−0.2196	−0.2341	−0.0145	0.2167	0.3179	0.2919	0.2075	0.1231	0.0634	0.0290	0.0120	0.0045	0.0016
11	−0.1712	−0.1768	0.1390	0.2273	−0.0150	−0.2383	−0.2016	0.0184	0.2250	0.3089	0.2804	0.2010	0.1216	0.0643	0.0304	0.0130	0.0051
12	0.0477	−0.2234	−0.0849	0.1951	0.1825	−0.0735	−0.2437	−0.1703	0.0451	0.2304	0.3005	0.2704	0.1953	0.1201	0.0650	0.0316	0.0140
13	0.2069	−0.0703	−0.2177	0.0033	0.2193	0.1316	−0.1180	−0.2406	−0.1410	0.0670	0.2338	0.2927	0.2615	0.1901	0.1188	0.0656	0.0327
14	0.1711	0.1334	−0.1520	−0.1768	0.0762	0.2204	0.0812	−0.1508	−0.2320	−0.1143	0.0850	0.2357	0.2855	0.2536	0.1855	0.1174	0.0661
15	−0.0142	0.2051	0.0416	−0.1940	−0.1192	0.1305	0.2061	0.0345	−0.1740	−0.2200	−0.0901	0.1000	0.2367	0.2787	0.2464	0.1813	0.1162