ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examinations : Summer 2009

SIGNAL PROCESSING

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed : 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook (Edition 5) and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. ANSWER BOTH PARTS

(a) The impulse response of a system is

$$e^{-3t}\cosh 2t.$$

Determine the Laplace transfer function and hence its poles. Is the system stable?

(12 marks)

(b) Show that the Laplace transfer function of the R-C circuit in Figure 1 is

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{sRC+1}.$$

Determine the dynamic response of the circuit to a unit step input.

(13 marks)

(25 marks total)

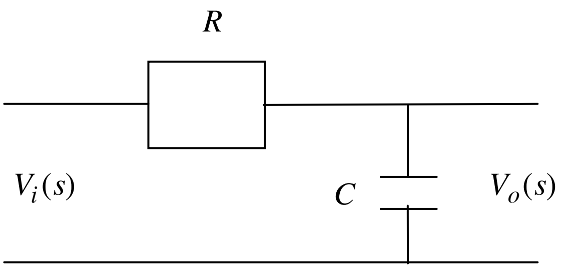

Figure 1. An R-C circuit.

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2. Figure 2 shows the direct implementation of an infinite impulse response (IIR) filter. State the filter coefficients $a(0)$, $a(1)$, $a(2)$, $b(1)$ and $b(2)$. Hence determine the Z-transform transfer function of the filter

(10 marks)

Determine the poles and zeros of the transfer function and sketch the Z-plane pole-zero plot of the filter. Use the plot to sketch the filter magnitude frequency response and state whether the filter is lowpass, highpass or bandpass.

(15 marks)

(25 marks total)

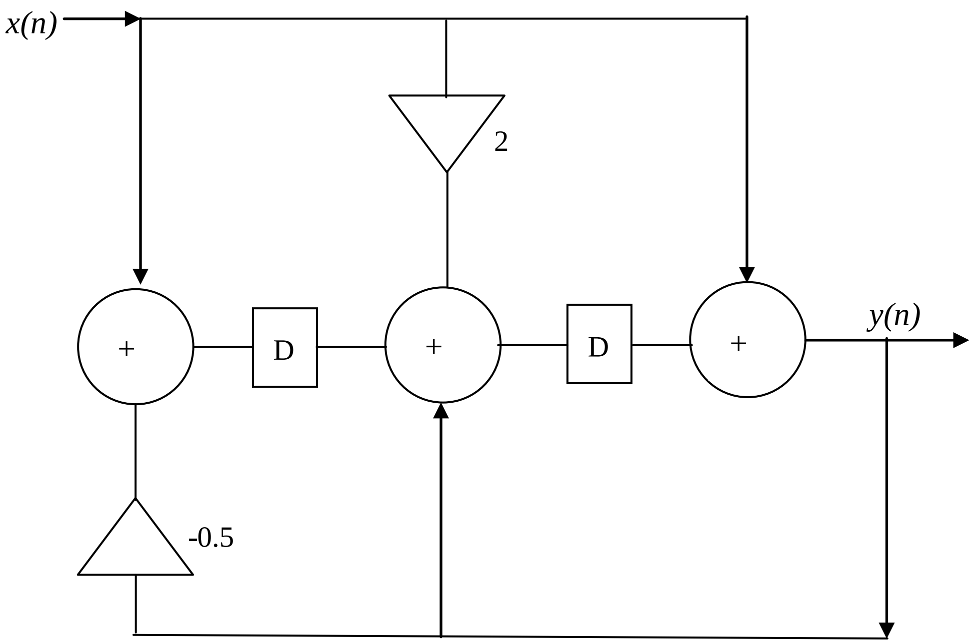

Figure 2 A second order IIR filter.

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3. Derive the difference equation of the second order system shown in Figure 3. Use the difference equation and a tabular method to derive the impulse response $\{y(n)\}$ to 2 decimal places, assuming quiescent conditions.

(13 mark)

Derive the Z-transform transfer function of the system and hence determine the poles and zeros of the system. State whether the system is stable or unstable. How is this confirmed by the impulse response?

(12 marks)

(25 marks total)

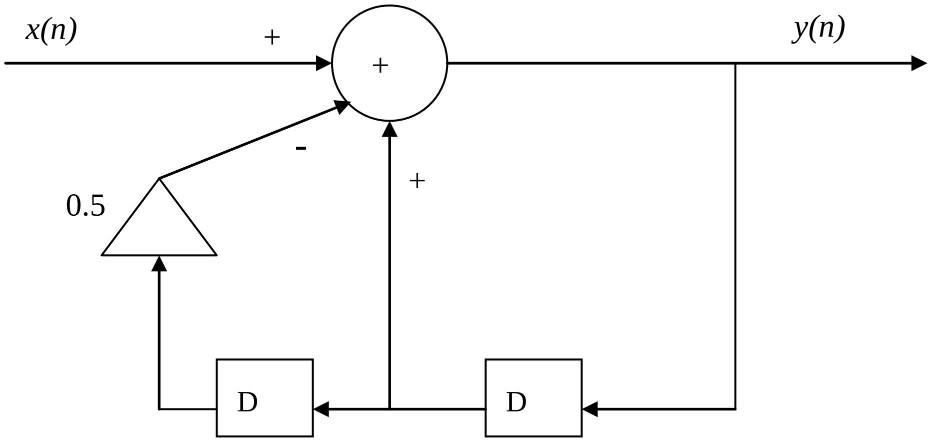

Figure 3. A second order system.

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SECTION B

4. **ANSWER BOTH PARTS**

(a) A random, discrete-interval binary signal, *x(t)*, can take the level 1 with a probability of 0.3, and 0 with a probability of 0.7. Sketch a particular realization of the signal. Show that its autocorrelation function is

$$R_{xx}(\tau) = 0.3 - 0.21\left(\frac{|\tau|}{\lambda}\right) \qquad 0 \le |\tau| \le \lambda$$

$$R_{xx}(\tau) = 0.09 \qquad |\tau| \ge \lambda,$$

where λ is the clock pulse interval of the signal. (15 marks)

(b) Sketch the autocorrelation function of a pseudo-random binary signal based on a maximum-length sequence of period 15 digits. Compare this with a sketch of the autocorrelation function described in part (a). (4 marks)

Describe how such a pseudo-random signal could be generated using a circuit based on a shift register. How can a maximum-length sequence be ensured? (3 marks)

Give an example of a practical use of a pseudo-random binary signal. (3 marks)

(25 marks total)

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5. Sketch the block diagram of a second-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights. (3 marks)

It is required to design a FIR filter to approximate the ideal high-pass characteristic with cut-off frequency f_c. The folding frequency of the digital filter is to be 5 f_c. By Fourier series show that the ideal filter weights are given by

$$g_n = 0.8, \qquad n = 0$$

$$g_n = -\frac{1}{n\pi}\sin\frac{n\pi}{5}, \qquad n = \pm 1, \pm 2, \pm 3, \ldots$$

(7 marks)

A Hanning window with N = 5 is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for n = 0, ±1, ±2, ± 3, ± 4 and ± 5. For the Hanning window, weight

$$w_n = 0.5 + 0.50\cos\left(\frac{2n\pi}{2N+1}\right), \qquad -N \le n \le N$$

$$w_n = 0, \qquad \text{all other values of } n$$

(4 marks)

Find the gain of the filter at D.C., and at the folding frequency. (3 marks)

Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

The inverse Fourier transform of a signal $H(j\omega)$ is defined as $h(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} H(j\omega)exp(j\omega t)\, d\omega$.

By inverse transformation into the time domain, show that the ideal low-pass filter characteristic is not physically realisable. (6 marks)

(25 marks total)

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6. **ANSWER BOTH PARTS**

(a) The Discrete Time Fourier Transform (DTFT) of a signal $g[k]$ is defined as

$$G(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} g[k]\exp(-jk\omega T).$$

Find the DTFT of the signal shown in Figure 4. Assume all other samples are zero. Sketch the magnitude of $G(e^{j\omega T})$ against ω. You can assume that $T = 1$. Comment on the phase of $G(e^{j\omega T})$ (13 marks)

(b) The Discrete Fourier Transform (DFT) of the sequence $x[n]$ is defined as

$$X[m] = \sum_{n=0}^{N-1} x[n]\exp\left(-j2\pi mn/N\right).$$

A complete cycle of a sinusoid has been sampled by eight equally spaced samples. After processing by the DFT a line spectrum was obtained. In a second experiment, the waveform was truncated, and eight such samples were obtained from just 90% of the cycle. The samples were y = [0.000 0.649 0.988 0.853 0.309, -0.383 -0.891 -0.972].

Calculate the values Y[0] and Y[1] of the DFT of the sequence $y[n]$. Given the remaining values $|Y[2]| = |Y[6]| = 0.44$, $|Y[3]| = |Y[5]| = 0.29$, and $|Y[4]| = 0.26$, find a value for $|Y[7]|$ and sketch $|Y[m]|$ for $m = 0,1,\ldots,7$. Why is this not a line spectrum? How could it be made to appear more line like? (12 marks)

(25 marks total)

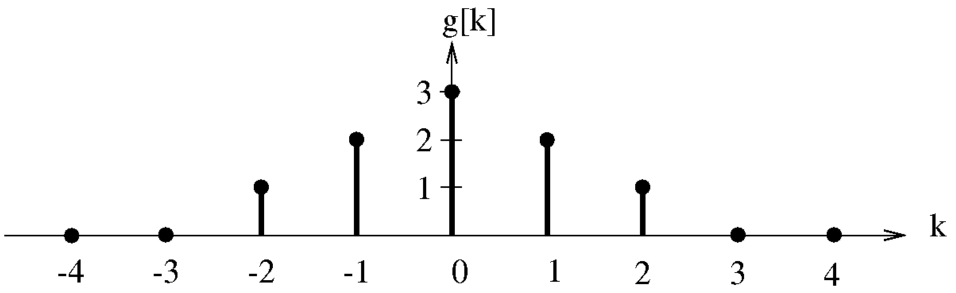

Figure 4 Signal $g[k]$.

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