1 Part 1

1.1 Motivation

How long does it take to double anything

$$\left(1+\frac{x}{100}\right)^n = 2 \tag{1}$$

Where x is the growth rate and n is the number of years.

$$n \approx \frac{70}{x} \tag{2}$$

1.2 Fundamental Concepts

For a probability distribution

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{3}$$

Expectation $E[X]$ and variance σ^2 are given in the databook.

1.2.1 Fourier Transforms

Fourier Transforms (F) changes between time domain and frequency domain

$$X(f) = F\left\{x(t)\right\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \tag{4}$$

$$x(t) = F^{-1}\left\{X(f)\right\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}dt \tag{5}$$

1.2.2 Delta function

Infinitesimally brief

$$\delta(x-a) = a \qquad x \neq a \tag{6}$$

Normalised

$$\int_{-\infty}^{\infty} \delta(x-a)dx = 1 \tag{7}$$

Sifting

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a) \tag{8}$$

Fourier transform of delta function

$$\int_{-\infty}^{\infty} \delta(f \pm f_0)e^{\pm j2\pi f\tau}df = e^{\pm j2\pi f_0\tau} \tag{9}$$

1.2.3 Transfer function

The FT of an impulse response is the transfer function

For an RC circuit the impulse is given by Equation (10).

$$h(t) = \frac{1}{RC}e^{-\frac{t}{RC}} \qquad t \geq 0 \tag{10}$$

The transfer function is given by Equation (11)

$$H(t) = \frac{1}{1+j2\pi fRC} \tag{11}$$

1.2.4 Autocorrelation

Measure of signal similarity at different times

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t+\tau)f^*(t)d\tau \tag{12}$$

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t)f^*(t-\tau)d\tau \tag{13}$$

Where f^* represents the complex conjugate (real function, $f* = f$).

Also,

$$R_{xx} = E\left\{x(t)x(t-\tau)\right\} = E\left\{x(t+\tau)x(t)\right\} \tag{14}$$

Can be written as $R_x(\tau)$

1.2.5 Signal Energey and Power

The energy of a signal is given by

$$E = \int_{-\infty}^{\infty} \left|f(t)\right|^2 dt\text{Joules} \tag{15}$$

which is a slight problem for infinitely long signals, so practaically,

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left|f(t)\right|^2 dt\text{Watts} \tag{16}$$

The DC power is given from the autocorrelation function

$$\lim_{\tau\to\pm\infty} \left\{R_x(\tau)\right\} \tag{17}$$

and the mean power is given as

$$R_x(0)$$

1.2.6 Parsevcal's Theorem

$$E = \int_{-\infty}^{\infty} \left|f(t)\right|^2 dt = \int_{-\infty}^{\infty} \underbrace{\left|F(t)\right|^2}_{\text{Energy Density}} df\text{Joules} \tag{18}$$

or

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left|f(t)\right|^2 dt = \int_{-\infty}^{\infty} \underbrace{\left|S(f)\right|}_{\text{Power Spectral Density}} df\text{Watts} \tag{19}$$

1.2.7 Wiener-Kinchine Theorem

For random signals, the autocorelation function and PSD form a Fourier transform pair

$$P(f) = F(R(\tau))$$

1.3 Digitisation

2 Part 2

2.1 Baseband Pulse Transmission

3 Part 3

3.1 Modulated Data Transmission

4 Part 4

4.1 ECC

5 Part 5

5.1 Data Compression and Cryptography