1 Lecture 1 - Modulation

$$f(t) = \underbrace{A_c}_{\text{Amplitude}} \times \cos(\overbrace{\omega}^{\text{Angular frequency}} t + \underbrace{\phi_c}_{\text{Phase term}}) \tag{1}$$

Modulation is the process of applying information to a carrier, in this case, the carrier frequency is ω_c (as an angular frequency) or f_c as a frequency directly where $\omega_c = 2\pi f_c$.

If we modify A in the above in sympathy with the information we want to send, then this is amplitude modulation.

If however we change ω_c or ϕ_c in sympathy with the information, these are forms of angle modulation - varying ω_c is frequency modulation, varying phi_c is phase modulation.

If the amplitude etc, varies linearly with the information then this is analogue, alternatively varying these quantities in an on/off manner or between two state is digital.

1.1 Amplitude Modulation

The function $f(t)$ becomes

$$f(t) = \underbrace{A_0(1 - m\cos\omega_m t)}_{A} \times \underbrace{cos\omega_0 t}_{\text{No } \phi_c \text{ term}} \tag{2}$$

f_m or ($\omega_m = 2\pi f_m$) is a simple sine/cosine wave modulation function

Expanding $f(t)$:-

$$\begin{aligned} f(t) &= A_0\cos\omega_0 t + A_0 m \cos\omega_m t x \cos\omega_0 t \\ &= A_0\cos\omega_0 t + \frac{A_0 m}{2}\cos(\omega_m + \omega_0)t + \cos(\omega_0 - \omega_m)t \end{aligned} \tag{3}$$

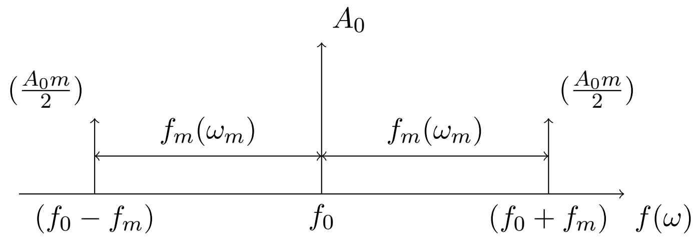


Figure 1: Spectrum of an AM signal, including sidebands

The simple process of amplitude modulation produces an upper sideband (USB) at $f_0 + f_m$ and a lower sideband at $f_0 - f_m$. Note that only the sidebands contain any useful information about the amplitude m of the modulation signal.

This modulation has a power penalty because the carrier conveys no useful amplitude information itself.

$$\begin{aligned} \left(\frac{\text{Power in sidebands}}{\text{Total power transmitted}}\right) &= \frac{(0.5A_0 m)^2 + (0.5A_0 m)^2}{(0.5A_0 m)^2 + (0.5A_0 m)^2 + A_0^2} \\ &= \frac{0.5A_0^2 m^2}{0.5A_0^2 m^2 + A_0^2} \\ &= \frac{0.5m^2}{0.5m^2 + 1} \end{aligned} \tag{4}$$

$m =$ modulation depth $= \left(\frac{A_m}{A_0}\right)$ which is a ratio of modulation voltage to the carrier voltage (or current).

It is essential not to let $|m| \geq 1$ because this is overmodulation and it distorts the AM waveform.

Going back to equation 4 in the limit $m = 1$ and so the ratio is:-

$$\frac{0.5 \times 1^2}{(0.5 \times 1^2) + 1} = \frac{0.5}{1.5} = \frac{1}{3} = 33.3\% \tag{5}$$

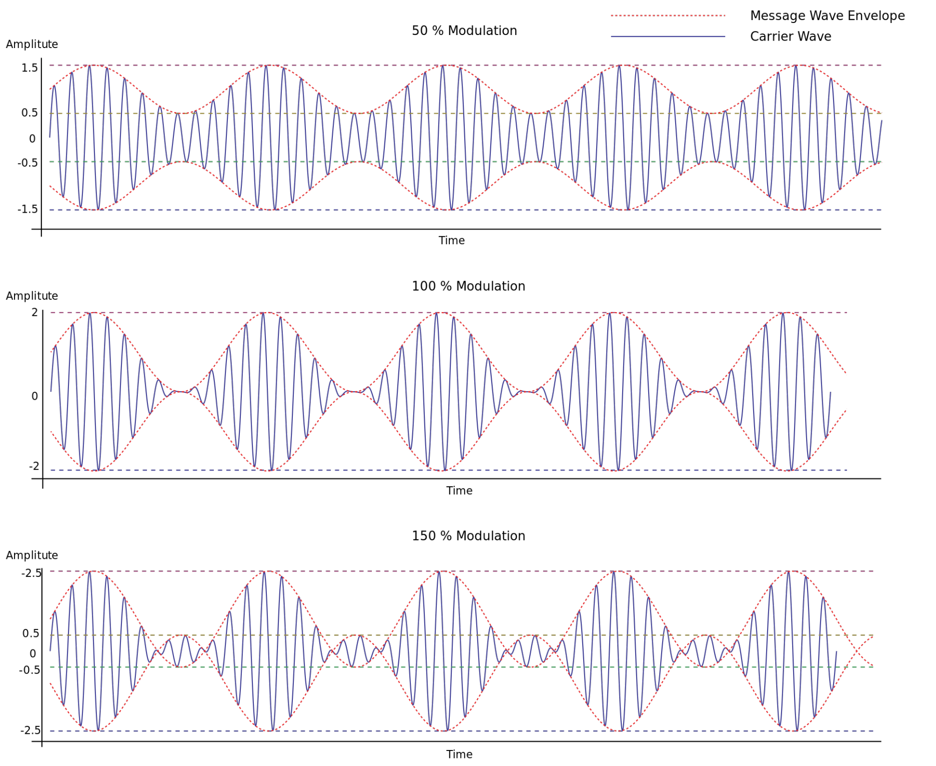
$\frac{2}{3}$ of the transmitted power is wasted.

Typically, $m \approx 0.4$ from which we have:-

$$\frac{0.5 \times 0.4^2}{(0.5 \times 0.4) + 1} = \frac{0.08}{0.08 + 1} \tag{6}$$

so this ratio is around 0.08 ie. only 8% of the transmitted power is used, or 92% is wasted!

1.1.1 Waveforms



It is essential, when considering the above waveform, to have $|m| \leq 1$ as mentioned before.

Detecting AM requires an envelope detector in its simplest form:-

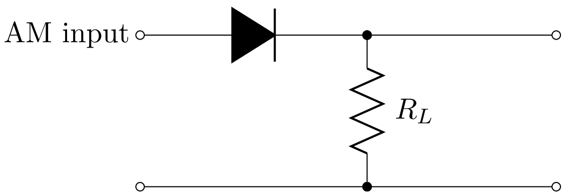


Figure 2: Simple envelope detector

A more advanced envelope detector adds a low pass filter to remove high frequency components from the carrier wave.

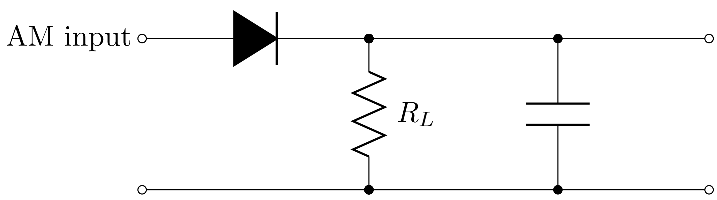


Figure 3: More advanced envelope detector using smoothing

If the carrier frequency is high compared to the modulation frequency, the recovered envelope is a "quite good" approximation to the original given a certain combination of R_L and C.

2 Lecture 2 - Double Sideband and Single Sideband AM

Recall AM signal (Equation 2. Power is wasted in the carrier, so a better form of A.M. could be:-

$$f(t) = A_0 \cdot \cos \omega_0 \cdot m \cos \omega_m t \tag{7}$$

Where $m = \frac{A_m}{A_0}$ = modulating depth and A_m = amplitide of modulating signal

$$f(t) = \left(\frac{A_0 m}{2}\right) (\cos{(\omega_0 + \omega_m)t} + \cos{(\omega_0 - \omega_m)t}) \tag{8}$$

This is known as double sided sideband, supressed carrier (DSB).

There is an important property of a DSB $\rightarrow$ for no modulation, no signal is sent, so that it is very power effiicient, but not bandwidth efficient.

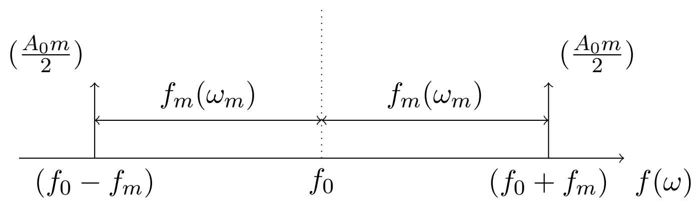


Figure 4: Spectrum of a DSB signal

The envelope of a DSB signal for a single frequency of modulation is:-

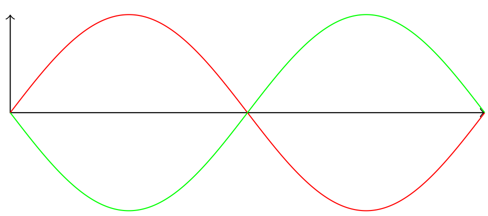


Figure 5: DSB Signal envelope

Because there is no carrier present, the envelope of the DSB waveform no longer represents the modulating function, therefore an envelope detector cannot be used to detect it - a product detector is used instead.

To illustrate the operation, instead of a DSB being the input, simply use a carrier frequency with no modulation

The output after filtering is $(\frac{A_0 E}{2})$ which is proportinal to A_0, the signal amplitude.

Replace the simple input by:-

$$A_0 \cos \omega_0 t \times \cos \omega_m t \tag{9}$$

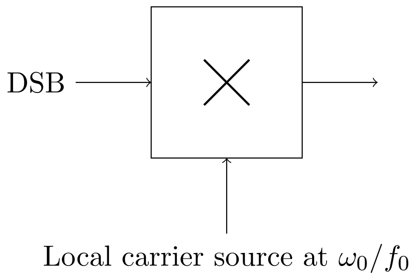


Figure 6: Product detector/mixer/multiplexer with simple input

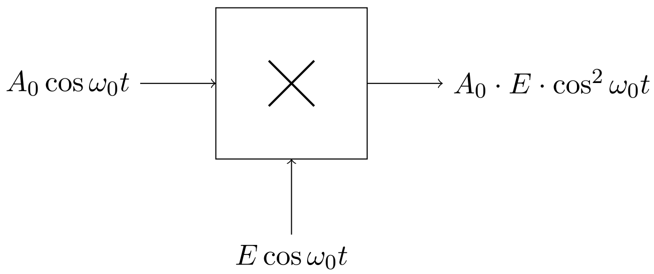


Figure 7: Product detector/mixer/multiplexer with real input

output is

$$\begin{aligned}(A_0 \times \cos \omega_0 t \times \cos \omega_m t)(E \times \cos \omega_0 t) &= A_0 E \cos^2 \omega_0 t \times \cos \omega_m t && (10)\\ &= \left(\frac{A_0 E}{2}\right)(1 + \cos 2\omega_0 t) \times \cos \omega_m t && (11)\end{aligned}$$

After low pass filtering to remove all frequencies above ω_m:-

$$OP = \left(\frac{A_0 E}{2}\right) \times \cos \omega_m t \tag{12}$$

which represent the modulation function directly.

This system can be improved even further because bandwidth is wasted. Bandwidth is important because it is a scarce resource.

If we consider the expanded form of that for DSB:-

$$f(t) = \left(\frac{A_0 m}{2}\right) \cos{(\omega_0 + \omega_m)t} + \left(\frac{A_0 m}{2}\right) \cos{(\omega_0 - \omega_m)t} \tag{13}$$

If we filter carfully, then we need only send either the USB or LSB.

Filtering therefore reduces the bandwidth, as each sideband carries the same information (each has an "m" term). Difficulty arises in obtaining sharp cutoff filters. The PHASING METHOD can be used to generate SSB (single side band).

$$E_0 = E_1 + E_2 \tag{14}$$

$$\begin{aligned}E_1 &= [A_0 \cos \omega_0 t] \times \left[\cos \omega_m t + \frac{\pi}{2}\right] \\ &= A_0 \cos \omega_0 t \times \sin \omega_m t && (15)\end{aligned}$$

therefore

$$E_2 = A_0 \sin \omega_0 t \times \cos \omega_m t \tag{16}$$

therefore

$$
\begin{aligned}
E_0 &= A_0 \times [\cos \omega_0 t \times \sin \omega_m t + \sin \omega_0 t \times \cos \omega_m t] \\
&= A_0 \sin (\omega_0 + \omega_m) t
\end{aligned}
\tag{17}
$$

USB, no filtering needed.

Similarly, the LSB can be generated by changing the sign of the second term above (on the first term).

The method works very well only when a single frequency of modulation is used. In practice, a range of frequencies is used and it is very difficult to obtain a 90°phase shift over a range of frequencies.

2.1 Vestigial Sideband

Remember all the abbreviations, "in case they get slipped into a question" This is an approximation to SSB which has been used extensively in analogue T.V. ver many decades. Basically, a DSB/C (Double Sideband with Carrier)

The main benefits are: simplicity and cheapness compared to SSB, when using valve/vacuum tube technologies.

The disadvantage is carrier still there, wasting power, but allowing an envelope detector (cheap, easy to make) to be used.

3 Lecture 3 - Angle Modulation

Starting with **??**

$$
f(t) = A_0 \cdot \cos (\omega_0 t + \phi) \tag{18}
$$

In the expression for $f(t)$ if we make $(\omega_c t + \phi) = \phi(t)$ then ϕ_c may be made instantaneously proportional to the modulating signa $f_m(t)$:-

$$
\begin{aligned}
\phi(t) &= \omega_c t + k_p f_m(t) \\
\Longrightarrow f(t) &= A_0 \cos \theta(t)
\end{aligned}
\tag{19}
$$

where $k_p =$ constant of modulation. This is Phase Modulation.

Alternatively, we can have

$$
\begin{aligned}
\omega(t) &= \left[\frac{d\phi(t)}{dt} \right] \\
&= \omega_c + k_f \cdot f_m(t)
\end{aligned}
\tag{20}
$$

where, as before k_f is a constant.

In this case, the instantaneous frequency has the value ω_c in the absence of a modulating signal. By integrating the above expression $\frac{d\theta(t)}{dt}$:-

$$
\begin{aligned}
\omega(t) &= \frac{d\theta(t)}{dt} \Longrightarrow \int \omega(\theta) dt = \theta(t) \\
\Longrightarrow \theta(t) &= \omega_c t + k_f \int f_m(t) dt + \phi_0
\end{aligned}
\tag{21}
$$

This is frequency modulation.

Frequency modulation is thus an indirect form of pulse modulation and is mathematically related.

Phase and frequency modulation are thus similar in form and are both angle modulation.

Frequency modulation can be generated easily using a voltage controlled oscillator.

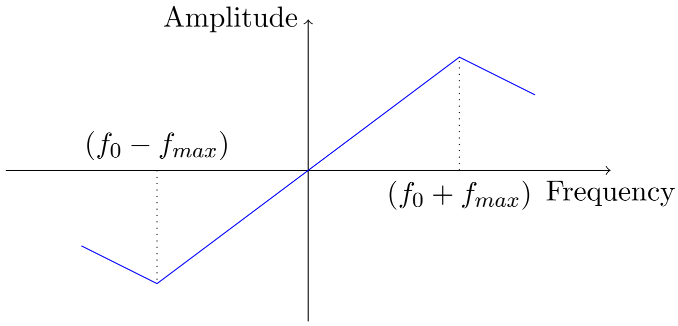


Figure 8: FM spectrum usage

3.1 Detection of FM

An FM discriminator (or Phase Locked Loop) is used to convert frequency changes to amplitude changes:- At frequency f_0 the output of the FM demodulator is zero. For frequencies above f_0 (but less than $\pm f_{max}$ then the output is greater than zero (or inverted and greater than 0 for $f < f_0$. The output of such a discriminator is then fed to a circuit or system which detects amplitude changes.

F.M. and P.M. generate a different spectrum to let $f_m(t) = \cos(\omega_m t)$ for F.M.

$$\begin{aligned}\therefore f(t) &= A_0 \cdot \cos\left[\omega_0 t + k_f \int_0^t \cos(\omega_m t) dt\right] \\ &= A_0 \cdot \cos\left[\omega_0 t + \left(\frac{k_f}{\omega_m}\right) \sin \omega_m t\right] \qquad (22)\end{aligned}$$

The instantaneous frequency of this value is $f_i = \frac{(\omega_0 + k_f \cos \omega_m t)}{2\pi}$ which oscillates between two extreme values:-

$$\left[\frac{\omega_0}{2\pi}\right] \pm \left[\frac{k_f}{2\pi}\right] \text{ for } cos(\omega_m t)_m ax = \pm 1 \qquad (23)$$

The peak value of the difference between f_i and the carrier frequency $f_0 = \left(\frac{\omega_0}{2\pi}\right)$ is the frequency deviation, which is $\left[\frac{k_f}{2\pi}\right]$ in the above example.

Since k_f is proportional to the real amplitude of the modulating signal, so is the frequency deviation.

If f_d is the maximum frequency deviation and f_a is the actual frequency deviation then the deviation ratio, m is defined as:-

$$m = \frac{f_a}{f_d} \qquad (24)$$

Now $f_a = \frac{k_f}{2\pi}$ so $m = \left[\frac{k_f}{2\pi \cdot f_d}\right]$, therefore

$$\left(\frac{k_f}{\omega_m}\right) = \frac{2\pi f_a}{2\pi f_m} = \frac{f_a}{f_m} = m\frac{f_d}{f_m} \qquad (25)$$

(Comparing actual frequency deviations to the modulating frequency f_m).

$$\begin{aligned}\Longrightarrow \frac{m f_d}{f_m} &= m_p = \text{modulation index} \\ \therefore f(t) &= A_0 \cdot \cos[\omega_0 t + m_p \cdot \sin \omega_m t] \qquad (26)\end{aligned}$$

Using Bessel functions,

$$f(t) = A_0 \sum_{m=\infty}^{\infty} I_n(m_p) \cdot \cos(\omega_c + n\omega_m)t \qquad (27)$$

therefore firstly, FM thus generates a range of frequencies from $-\infty$ to ∞ in practice, the bandwidth is restricted and also the Bessel coefficient ($= I$) have finite values which tend to zero as n tends to $\pm\infty$

Modulation Index m_p	No. of sidebands n	Bandwidth (a) as multiples of f_m	Bandwidth (b) as multiples of f_a
0.1	2	2	20
0.5	4	4	8
1	6	6	6
5	16	16	3.2
30	70	70	2.3

Table 1: Modulation index to bandwidths

4 Lecture 4

$$B = n \cdot f_m = N \cdot \left(\frac{f_a}{mp}\right) \tag{28}$$

Where f_a = frequency deviation and f_M = modulating frequency.

E.g: For VHF/FM mono transmissions, the maximum value of f_a = 75kHz and the maximum value of f_m = 15kHz (speech and music quality). therefore $m = \frac{f_{a(max}}{f_{m(max)}} = 5$. Looking at table **??** above, we can see that, for $m = 5$ we need a bandwidth of $(3.2 \times 75\text{kHz}) = 240\text{kHz}$. The reason why such a large bandwidth is needed is because F.M. causes the frequency changes to occur at a high rate, resulting in a much larger change in spectrum.

4.1 Zenith-GE FM Stereo System

The FM stereo signal has to have backwards compatibility with mono, and uses independent left (L) and right (R) signals. When a mono source is used, the stereo signal has to revert to a mono representation for backward compatibility. Conversely, when a stereo signal is received by a mono receiver, then the latter should be able to interpret the stereo as a mono source (forward compatibility).

The L and R signals are formatted at the transmit end into $(\underbrace{L+R}_{\text{mono}})$ and $(\underbrace{L-R}_{\text{“stereo”}})$.

The Zenith stereo signal, overall, uses both of the above. The $(L + R)$ signal is baseband, and the $(L - R)$ signal is used with DSB/SC (AM) at $38kHz$:-

The “pilot tone” is used a the receiver to generate $38kHz$ at the correct phase so as to be able to demodulate the $(L - R)$, DSB/SC signal optimally. The $(L - R)$ and pilot tone signals have to be constrained in amplitude to prevent “over modulation” of the FM carrier and this prevent distortions and consequent unwanted frequencies being generated. Typically, $(L - R)$ is limited to 45% of its maximum value, in which case the overall signal (i.e. $(L+R)$, pilot tone and $(L-R)$) never exceeds 90% of the overall possible maximum amplitude. Note that $(L+R)$ can be recovered directly by using a $15kHz$ low pass filter (LPF) at the receiver – which is the mono component, and can be easily interpreted by a mono receiver.

The $(L - R)$ signal must be recovered using a $38kHz$ signal derived from the $19kHz$ $\left(= \frac{38}{2}\right)$ pilot tone as mentioned. The base bandwidth of the stereo signal is $53kHz$ (from figure **??**)

Using Carson's Rule (an approximation), we can calculate the required overall FM bandwidth as

$$B = 2 \cdot \left[f_{a(max)} + f_{m(max)}\right] \tag{29}$$

where $f_{a(max)} = 75kHz$ (to be compatible with mono) $f_m(max) = 53kHz$, therefore $B = 256kHz$.

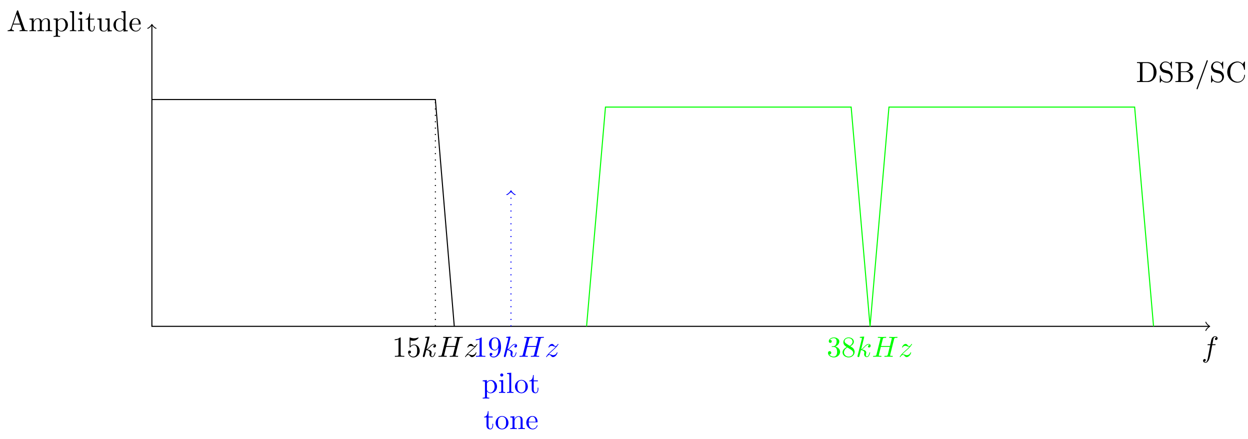


Figure 9: Zenith-GE spectrum

FM has a SNR (signal to noise ratio) advantage over AM above a crtical threshold. Below the threshold the FM signal becomes unusable very quickly. FM as above is a simple form of "spread spectrum".

Figure 10: Zenith-GE reciever diagram

5 Superheterodyne receiver

SUPERHETERODYNE or "Superhet"

Figure 11: Superheterodyne reciever

A mixer is (typically) a device (or system) of the following characteristic:-

$$v_0 = k_1 v_i + k_2 v_i^2 + k_3 v_i^3 \cdots \tag{30}$$

Wehre v_o is the output voltage, v_i is the input voltage and $k_1 \cdots k_n$ are coefficients.

If we let $v_i = a_1 \cos \omega_1 t + \_2 \cos \omega_2 t$ $(\omega_1 = 2\pi f_1 \ldots)$

It may be shown that the effect f the polynomial expression is to produce a spectrum of signals at the output involving:-

$$f_1, f_2, (f_1 \pm f_2), *2f_1 \pm f_2), (2f_2 \pm f_1) \cdots \tag{31}$$

Where the optimal polynomial expression for a mixer is

$$\begin{aligned}
v_0 &= k_2 v_i^2 \text{ so using equation } \textbf{??} \\
&= k_2(a_1 \cos \omega_1 t + _2 \cos \omega_2 t)^2 \\
&= k_2(a_1^2 \cos^2 \omega_1 t + 2a_1 a_2 \cos \omega_1 t \cos \omega_2 t + a_2^2 \cos^2 \omega_2 t \\
&= k_2(\frac{a_1^2}{2}(1 + \cos 2\omega_1 t) + \frac{2a_1 a_2}{2}(\cos(\omega_1 + \omega 2)t + \cos(\omega_1 - \omega_2)t) + \frac{a_2^2}{2}(1 + cos 2\omega_2 t) \qquad (32)
\end{aligned}$$

DC term

$$k_2 \left(\frac{a_1^2}{2} + \frac{a_2^2}{2} \right) \tag{33}$$

$2\omega_1$ term

$$k_2 \frac{a_1^2}{2} \cos 2\omega_1 t \tag{34}$$

$2\omega_2$ term

$$k_2 \frac{a_1^2}{2} \cos 2\omega_2 t \tag{35}$$

Sum $(\omega_1 + \omega_2)$ term

$$a_1 a_2 (\cos(\omega_1 + \omega 2)t \tag{36}$$

Difference term $(\omega_1 - \omega 2)$

$$a_1 a_2 (\cos(\omega_1 - \omega 2)t \tag{37}$$

The useful frequency in this case is $(\omega_2 - \omega_1)$ (or $(\omega_1 - \omega_2)$ depending on which is the larger), which is the Intermediate Frequency (IF). To extract this in a superhet receiver, a bandpass filter centered at $)\omega_2 - \omega_1)$ is used. All other frequencies are filtered out.

E.G. Suppose we are receiving 540kHz in the "Medium wave" band. Suppose also we use 440kHz as the I.F. We can now choose two different local oscillator frequencies (i.e. for f_2) which give the same I.F. It can be either 0.1MHz (100kHz) or 980kHz.

Using another example, suppose we need to receive 1.6MHz, then the local oscillator could be either 1.16MHz or 2.04MHz. Overall, therefore, the local oscillator could tune from 0.1MHz to 1.16MHz or from 0.98MHz to 2.04MHz

Conventionally, we always choose the solution requiring the smaller percentage frequency changes. In ths example, if we calculate $\frac{f_{max} - f_{min}}{f_{max}}$ for the L.O. (Local Oscillator) in each case, then for the range 0.1MHz to 1.16MHz we have

$$\frac{f_{max} - f_{min}}{f_{max}} = \frac{1.16 - 0.1}{1.16} = 0.91 \tag{38}$$

For 1.6MHz to 2.04MHz

$$\frac{f_{max} - f_{min}}{f_{max}} = \frac{2.04 - 0.98}{2.04} = 0.52 \tag{39}$$

The second option is the one chosen.

Alternatively, we have the following dilemma

Figure 12: Superheterodyne receiver

The second problem with the superhet approach is that it can "double tune" i.e. in the above example either 1.5MHz or 0.5MHz produces the same IF at 0.5MHz in this case. Conventionally we would receive the 0.5MHz signal in order for the LO to be greater than f_1, the input frequency.

To ensure this, we need to filter out the "wrong" input frequency $(f_2 + I.F.)$:-

Figure 13: Superheterodyne receiver

The "wrong" frequency is called the Image Frequency. Normally, "tracking" is used:-

Figure 14: Superheterodyne receiver

The L.O. frequency and BPF centre frequency change at the same time. This is called TRACKING. The B.P.F is arranged to have the minimum bandwidth necessary, but if the L.O. frequency is increased, so the centre frequency at the B.P.F is increased by the same amount.