ES3350

THE UNIVERSITY OF WARWICK

Third Year Examinations: Summer 2009

COMMUNICATIONS SYSTEMS

SECTION A

SECTION B

Candidates should answer FOUR QUESTIONS.

Time Allowed: 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided. Data sheets for Bessel functions are attached to the paper.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

PLEASE USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. (a) A full AM wave is represented by the expression:

$$e(t) = \left[5 + 2.5\cos\left(3140t\right)\right]\cos(2\pi 10^5 t) \quad \text{Volts}$$

(i) Determine the modulation index.

(ii) Determine the modulating frequency, in Hz.

(iii) Determine the carrier frequency, in Hz.

(iv) Determine the peak amplitude of the modulated wave.

(v) Determine the frequency components, in Hz, present in the AM waveform and the amplitude of each component.

(vi) Sketch the spectrum of *e*(t).

(vii) Determine the power efficiency.

(16 marks)

(b) A full AM transmitter has an output of 24 kW when modulated to a depth of 100%.

Determine from first principles:

(i) The power output when the carrier is unmodulated;

(ii) The power output when after modulation to a depth of 60% one sideband is suppressed and the carrier component is reduced by 26 dB.

(9 marks)

2. An FM modulator output is given by

$$e(t) = 100\cos\left[2000\pi t + \phi(t)\right] \quad \text{Volts}$$

The modulator operates with frequency sensitivity k_f = 8 Hz/V and has the input message signal

$$m(t) = 5\cos(16\pi t) \quad \text{Volts}$$

Note: To aid you in your answers, a table of Bessel functions and a Bessel function plot are attached.

(a) Determine:

(i) The carrier frequency, in Hz.

(ii) The modulating signal frequency, in Hz.

(iii) The modulation index.

(iv) The bandwidth using the 1% significant sideband frequency approach.

(v) The bandwidth using Carson's rule.

(13 marks)

(b) Assuming a 1 Ω load, determine the fraction of the total power contained in the frequency band 972 Hz to 1028 Hz.

(6 marks)

(c) The frequency sensitivity k_f is gradually reduced until the first sideband amplitude in the output is zero. Assuming a 50 Ω load, determine:

(i) The average power at the carrier frequency.

(ii) The average power in all the remaining sidebands.

(6 marks)

3. (a) A system consists of a cascade of 3 networks as shown in Figure 1. All networks are matched. The three networks have power gains G_1, G_2 and G_3 respectively, and noise figures F_1, F_2 and F_3. The noise figures are all measured under similar input conditions of noise power N_{in} e.g. a matched resistor at ambient temperature (290 K). The cascaded arrangement is preceded by a matched noise source N_{in}. Starting from the definition of the noise figure i.e.

$$F = \frac{\text{signal to noise ratio at the input}}{\text{signal to noise ratio at the output}}$$

derive, showing in detail all steps of the derivation, the following expression for the overall noise figure of the system

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}$$

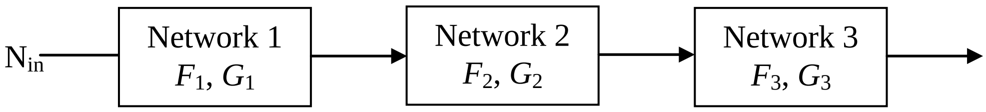

Figure 1

(7 marks)

(b) Three matched amplifiers, with the characteristics shown below, are available to amplify a signal.

Amplifier	Power Gain	Noise figure
A	6.02 dB	3.01 dB
B	3.01 dB	6.02 dB
C	9.03 dB	3.01 dB

Question 3 continued overleaf

Question 3 continued

The amplifiers are to be connected in a cascade.

(i) Determine the order in which the amplifiers must be connected to achieve the lowest overall noise figure for the cascaded system.

(ii) Determine the value of the lowest overall noise figure for the cascaded system.

(8 marks)

(c) The receiver system of a satellite communication Earth station is shown in Figure 2. The various stages may be considered matched for maximum power transfer. Calculate the equivalent noise temperature, T_e, of the system at the waveguide input.

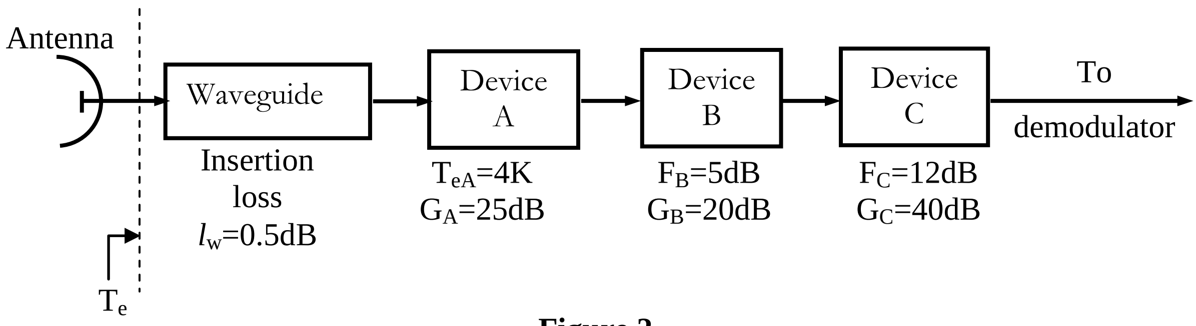

Figure 2

(10 marks)

SECTION B

4. A discrete random interference source, *n(t)*, can take one of four voltage levels, which have the values and probabilities shown in the table below.

$n_1 = 0.1V$	$n_2 = 0.15V$	$n_3 = 0.2V$	$n_4 = 0.25V$
$p_1 = 0.2$	$p_2 = 0.3$	$p_3 = 0.4$	$p_4 = \alpha$

(a) Determine the value of α. (2 marks)

(b) Calculate the expected values of n(t) and n^2(t). (8 marks)

(c) Determine the average powered delivered by n(t) into a 50 Ω load. (2 marks)

A continuous random signal, s(t), is uniformly distributed over a voltage range from –3V to +3V.

(d) Determine the probability density function of s(t). (2 marks)

(e) Calculate the average powered delivered by s(t) into a 50Ω load. (6 marks)

(f) Estimate the bit error rate when s(t) is the signal in a system where n(t) is the only noise source. (5 marks)

5. (a) Discuss the technique of algebraic coding and explain how the inclusion of redundant bits enables errors to be corrected.

(3 marks)

(b) Explain what is meant by a 3-bit repetition code and discuss why such an approach to error correction is inefficient. (3 marks)

(c) Outline how the addition of parity check bits to a block of data produces an error correction method that is more efficient than the code in part (b). (3 marks)

(d) Considering the block code generated by the matrix.

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Determine:

(i) The number of code words in the code and the rate of the code. (2 marks)

(ii) Find the parity check matrix and hence the syndrome for the received code word.

(7 marks)

$$\mathbf{R}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(iii) Indicate from the syndrome why $\mathbf{R}^{\mathrm{T}}$ is corrupted and, assuming that it is unlikely that more than one bit is in error, find and correct the error. (7 marks)

6. (a) Describe what is meant by the terms *prefix code* and *uniquely decodable (UD).*

(2 marks)

(b) Draw the code tree for the code $\{1, 10, 110, 011, 00\}$ and hence show that it is not a prefix code.

(4 marks)

(c) A discrete memoryless source (DMS) has the output alphabet $\{A, B, C, D, E\}$ and this is encoded by the mapping

$$\begin{array}{lcl} A & \mapsto & 00 \\ B & \mapsto & 01 \\ C & \mapsto & 10 \\ D & \mapsto & 110 \\ E & \mapsto & 111 \end{array}.$$

Confirm that this is a prefix code and determine its length when the source symbols are equiprobable.

(4 marks)

(d) Now consider the case when the probabilities of the appearance of the symbols from the source in part (c) are such that

$$P(A) = 2P(B); \quad P(B) = 2P(C); \quad P(C) = 2P(D) = 2P(E)$$

(i) Determine the DMS symbol probabilities and hence the entropy of the DMS.

(8 marks)

(ii) Write down a UD code for the DMS with lengths equal to the minus the logarithm to the base 2 of the symbol probabilities.

(3 marks)

(iii) Calculate the length of the code in part (ii) and comment on it in relation to the entropy of the source.

(4 marks)

Plot of Bessel Functions

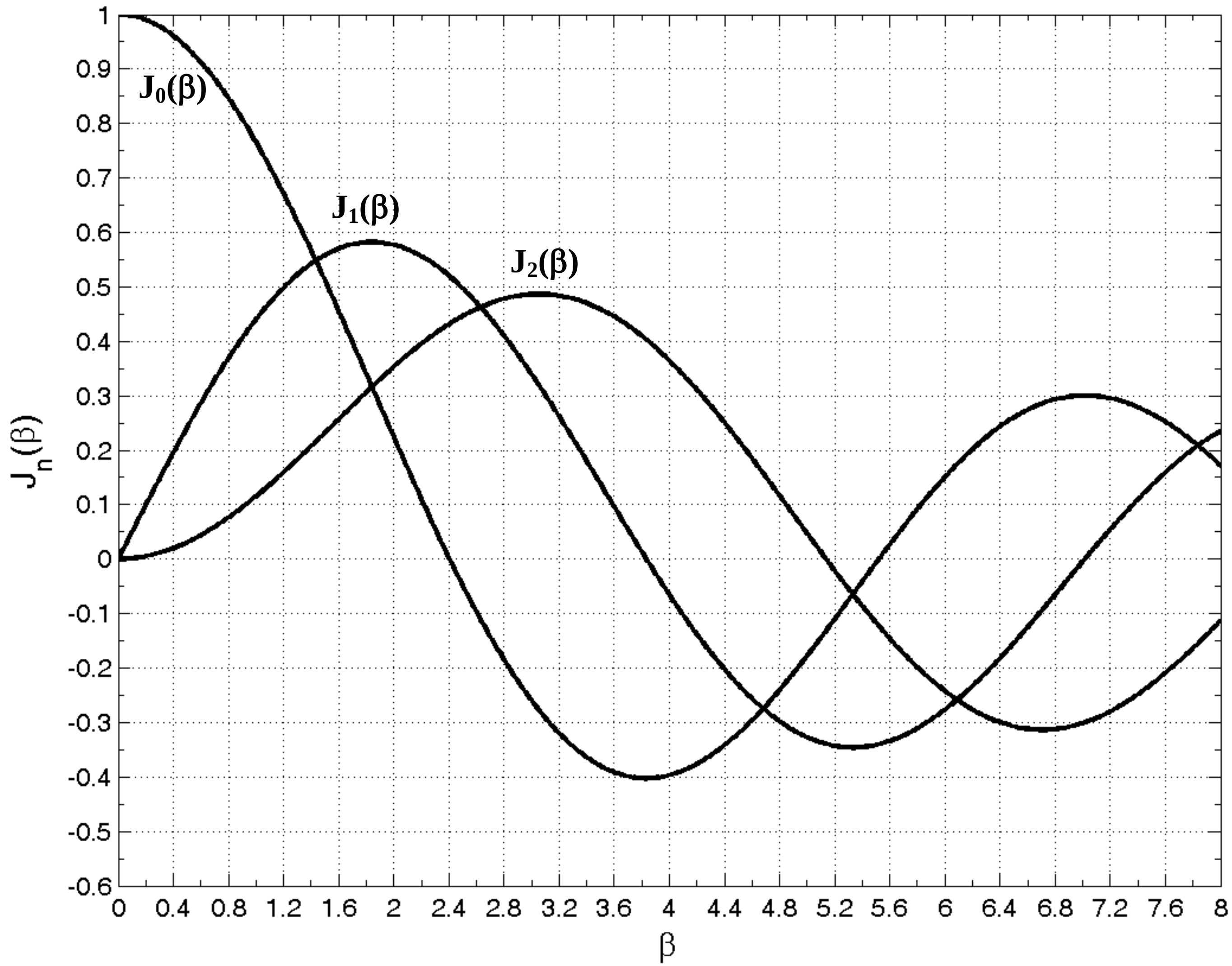

Table of Bessel Functions to four decimal places

β	$J_0(\beta)$	$J_1(\beta)$	$J_2(\beta)$	$J_3(\beta)$	$J_4(\beta)$	$J_5(\beta)$	$J_6(\beta)$	$J_7(\beta)$	$J_8(\beta)$	$J_9(\beta)$	$J_{10}(\beta)$	$J_{11}(\beta)$	$J_{12}(\beta)$	$J_{13}(\beta)$	$J_{14}(\beta)$	$J_{15}(\beta)$	$J_{16}(\beta)$
0	1.0000	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.10	0.9975	0.0499	0.0012	–	–	–	–	–	–	–	–	–	–	–	–	–	–
0.20	0.9900	0.0995	0.0050	0.0002	–	–	–	–	–	–	–	–	–	–	–	–	–
0.25	0.9844	0.1240	0.0078	0.0003	–	–	–	–	–	–	–	–	–	–	–	–	–
0.50	0.9385	0.2423	0.0306	0.0026	0.0002	–	–	–	–	–	–	–	–	–	–	–	–
1.0	0.7652	0.4401	0.1149	0.0196	0.0025	0.0002	–	–	–	–	–	–	–	–	–	–	–
1.5	0.5118	0.5579	0.2321	0.0610	0.0118	0.0018	0.0002	–	–	–	–	–	–	–	–	–	–
2.0	0.2239	0.5767	0.3528	0.1289	0.0340	0.0070	0.0012	0.0002	–	–	–	–	–	–	–	–	–
2.4048	–	0.5192	0.4318	0.1990	0.0647	0.0164	0.0034	0.0006	0.0001	–	–	–	–	–	–	–	–
3.0	−0.2601	0.3391	0.4861	0.3091	0.1320	0.0430	0.0114	0.0025	0.0005	0.0001	–	–	–	–	–	–	–
4.0	−0.3971	−0.0660	0.3641	0.4302	0.2811	0.1321	0.0491	0.0152	0.0040	0.0009	0.0002	–	–	–	–	–	–
5.0	−0.1776	−0.3276	0.0466	0.3648	0.3912	0.2611	0.1310	0.0534	0.0184	0.0055	0.0015	0.0004	0.0001	–	–	–	–
5.5201	–	−0.3403	−0.1233	0.2509	0.3960	0.3230	0.1891	0.0881	0.0344	0.0116	0.0035	0.0009	0.0002	–	–	–	–
6.0	0.1506	−0.2767	−0.2429	0.1148	0.3576	0.3621	0.2458	0.1296	0.0565	0.0212	0.0070	0.0020	0.0005	0.0001	–	–	–
7.0	0.3001	−0.0047	−0.3014	−0.1676	0.1578	0.3479	0.3392	0.2336	0.1280	0.0589	0.0235	0.0083	0.0027	0.0008	0.0002	0.0001	–
8.0	0.1717	0.2346	−0.1130	−0.2911	−0.1054	0.1858	0.3376	0.3206	0.2235	0.1263	0.0608	0.0256	0.0096	0.0033	0.0010	0.0003	0.0001
9.0	−0.0903	0.2453	0.1448	−0.1809	−0.2655	−0.0550	0.2043	0.3275	0.3051	0.2149	0.1247	0.0622	0.0274	0.0108	0.0039	0.0013	0.0004
10	−0.2459	0.0435	0.2546	0.0584	−0.2196	−0.2341	−0.0145	0.2167	0.3179	0.2919	0.2075	0.1231	0.0634	0.0290	0.0120	0.0045	0.0016
11	−0.1712	−0.1768	0.1390	0.2273	−0.0150	−0.2383	−0.2016	0.0184	0.2250	0.3089	0.2804	0.2010	0.1216	0.0643	0.0304	0.0130	0.0051
12	0.0477	−0.2234	−0.0849	0.1951	0.1825	−0.0735	−0.2437	−0.1703	0.0451	0.2304	0.3005	0.2704	0.1953	0.1201	0.0650	0.0316	0.0140
13	0.2069	−0.0703	−0.2177	0.0033	0.2193	0.1316	−0.1180	−0.2406	−0.1410	0.0670	0.2338	0.2927	0.2615	0.1901	0.1188	0.0656	0.0327
14	0.1711	0.1334	−0.1520	−0.1768	0.0762	0.2204	0.0812	−0.1508	−0.2320	−0.1143	0.0850	0.2357	0.2855	0.2536	0.1855	0.1174	0.0661
15	−0.0142	0.2051	0.0416	−0.1940	−0.1192	0.1305	0.2061	0.0345	−0.1740	−0.2200	−0.0901	0.1000	0.2367	0.2787	0.2464	0.1813	0.1162