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Question 1		Mark Allocation

$$(a) \text{ Transfer function (TF)} = \frac{C(s)}{R(s)}$$

where  $C(s) = \underline{\{ \text{system output} \}}$

$R(s) = \underline{\{ \text{system input} \}}$

$\underline{\{ \quad \}} = \text{Laplace transform}$

For system output to be an impulse response,  
 $R(s) = 1$

$$\therefore \text{TF} = C(s)$$

$$= \underline{\{ h(t) \}}$$

$$= \frac{2}{(s+2)^2 + 2^2} \quad (\text{from Laplace Transform table})$$

$$= \frac{2}{s^2 + 4s + 8}$$

$\therefore$  The poles (roots of the denominator of TF) are  
at  $s = -2 \pm j2$

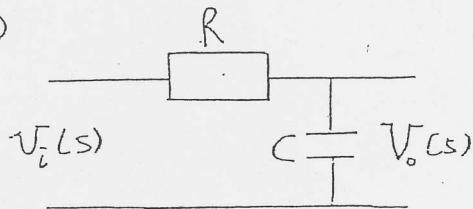
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2

Since the complex conjugate pair of poles  
are on left-half of s-plane, the system  
is stable.

2

(b)



$$V_o(s) = \frac{I(s)}{sC}$$

where  $I(s)$  is Laplace transform of current in the circuit

$$V_i(s) = R I(s) + \frac{I(s)}{sC}$$

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$$\begin{aligned}
 H(s) &= \frac{V_o(s)}{V_i(s)} = \frac{\frac{I(s)}{sC}}{R\frac{I(s)}{sC} + \frac{I(s)}{sC}} \\
 &= \frac{1}{sRC + 1} \\
 &= \frac{1}{1 + sT} \quad \text{where } T = RC
 \end{aligned}$$

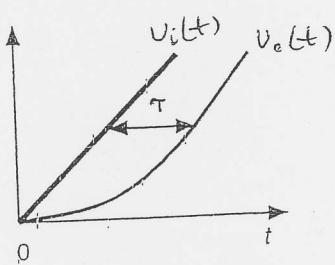
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For a unit ramp input  $V_i(s) = \frac{1}{s^2}$

$$\begin{aligned}
 V_o(s) &= \frac{1}{1 + sT} \times \frac{1}{s^2} \\
 &= \frac{T}{s + \frac{1}{T}} - \frac{1}{s} + \frac{1}{s^2} \\
 \therefore v_o(t) &= Te^{-\frac{t}{T}} - T + t \quad (\text{using Laplace Transform table}) \\
 &= t - T(1 - e^{-\frac{t}{T}})
 \end{aligned}$$

5

The dynamic response of the servo to a unit ramp input is as shown below, where the output  $v_o(t)$  lags the input  $v_i(t)$ . As  $t \rightarrow \infty$ ,



$v_o(t) \rightarrow v_i(t-T)$ ,  
i.e., value of the output at time t  
is equal to value of the input at  
time  $t-T$ . In other words,

$v_o(t)$  always lags  $v_i(t)$  by a time equal to  $T$   
for large  $t$ . The smaller  $T$  is, the shorter  
is the time lag.

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Total = 25 marks

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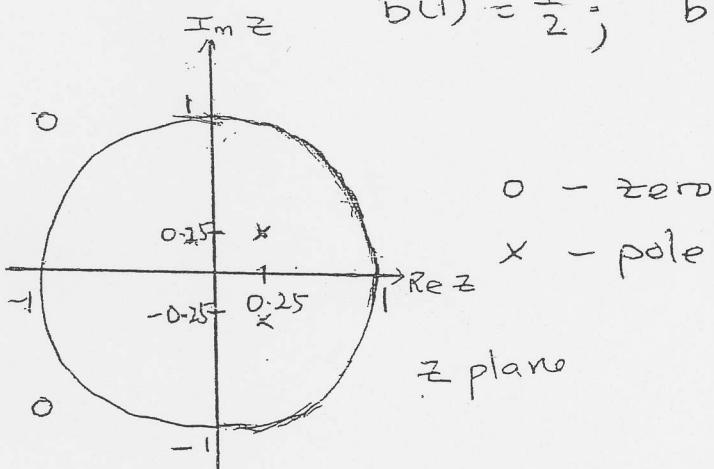
Given poles at  $z = \frac{1}{4} \pm j\frac{1}{4}$  and zeros at  $z = -1 \pm j$ ,

$$\begin{aligned} H(z) &= \frac{(z+1-j)(z+1+j)}{(z-\frac{1}{4}-j\frac{1}{4})(z-\frac{1}{4}+j\frac{1}{4})} \\ &= \frac{z^2 + 2z + 2}{z^2 - \frac{1}{2}z + \frac{1}{8}} \\ &= \frac{1 + 2z^{-1} + 2z^{-2}}{1 - (\frac{1}{2}z^{-1} - \frac{1}{8}z^{-2})} \end{aligned}$$

cf.  $H(z) = \frac{\sum_{m=0}^2 a(m) z^{-m}}{1 - \sum_{m=1}^2 b(m) z^{-m}}$  where  $a(m)$  and  $b(m)$  are filter coefficients

$$\Rightarrow a(0) = 1 ; a(1) = 2 ; a(2) = 2$$

$$b(1) = \frac{1}{2} ; b(2) = -\frac{1}{8}$$



pole-zero plot

$$|H(0)| = \frac{\sqrt{2^2+1^2} \times \sqrt{2^2+1^2}}{\sqrt{0.75^2+0.25^2} \times \sqrt{0.75^2+0.25^2}} = 8$$

$$|H(\frac{\pi}{2})| = \frac{1 \times \sqrt{2^2+1^2}}{\sqrt{0.25^2+0.75^2} \times \sqrt{0.25^2+1.25^2}} = 2.2188$$

$$|H(\pi)| = \frac{1 \times 1}{\sqrt{1.25^2+0.25^2} \times \sqrt{1.25^2+0.25^2}} = 0.6154$$

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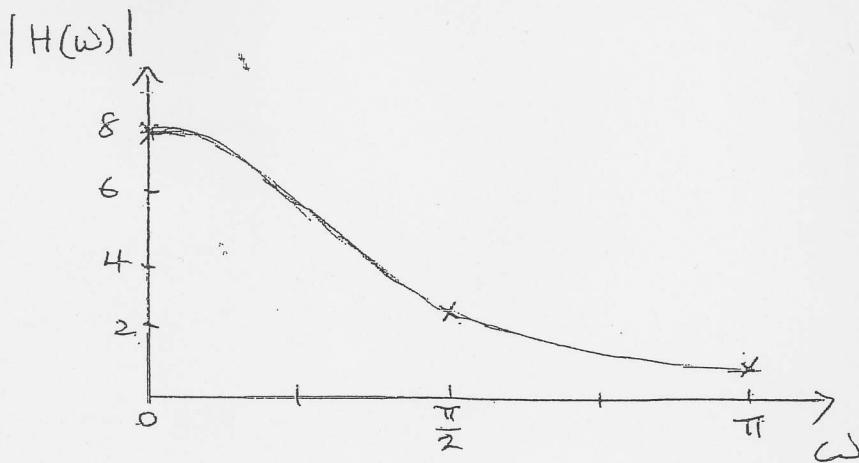
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2

2

The filter is lowpass because it passes (amplifies) low frequencies and attenuates high frequencies.

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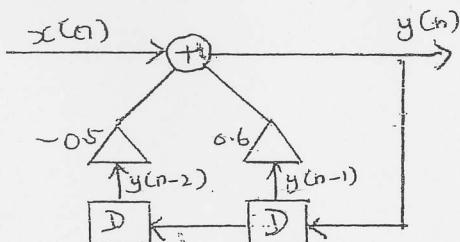
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Question 3

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Difference equation

$$\{y(n)\} = \{x(n)\} + 0.6 \{y(n-1)\} - 0.5 \{y(n-2)\} \quad -① \quad 5$$

For impulse response

$$\{x(n)\} = \{1, 0, 0, 0, 0, 0, 0, 0, 0\}$$

2

Assuming quiescent condition:

$\{x(n)\}$	$\{y(n-2)\}$	$\{y(n-1)\}$	$\{y(n)\}$
1	0	0	1
0	0	1	0.6
0	1	0.6	-0.1
0	0.6	-0.1	-0.4
0	-0.1	-0.4	-0.2
0	-0.4	-0.2	0.1
0	-0.2	0.1	0.2
0	0.1	0.2	0.1
0	0.2	0.1	0.0

Impulse response

$$\{y(n)\} = \{1, 0.6, -0.1, -0.4, -0.2, 0.1, 0.2, 0.1, 0.0\}$$

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From ①

$$Y(z) = X(z) + 0.6 z^{-1} Y(z) - 0.5 z^{-2} Y(z)$$

$$\begin{aligned}\frac{Y(z)}{X(z)} &= \frac{1}{1 - 0.6z^{-1} + 0.5z^{-2}} \\ &= \frac{z^2}{z^2 - 0.6z + 0.5}\end{aligned}$$

4

Zeros are roots of numerator of transfer function  
 $\therefore$  double zeros at  $z=0$

2

Poles are roots of denominator of transfer function

$\therefore$  poles are at  $z = -0.3 \pm j0.64$

2

Since the complex conjugate pair of poles are within the unit circle of the  $z$ -plane

pole-zero plot, i.e.  $|z| = 0.71 < 1$ ,

the system is a stable system.

2

Since the impulse response tends to 0 as  $n$  tends to infinity, the system is stable.

2

Total = 25  
marks

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(a)  $R_{xx}(\gamma) = E(x(t), x(t+\gamma))$

At  $\gamma=0$   $P(1,1) = 0.6$

$P(0,0) = 0.4$

$P(0,1) = P(1,0) = 0$

But only  $x(t) = x(t+0) = 1$  will contribute to

the ACF so  $R_{xx}(0) = P(1,1) = 0.6$

4

After switching points  $t = \lambda, 2\lambda, 3\lambda, \dots, n\lambda$

$P(1,1) = 0.6 \times 0.6 = 0.36$

$P(0,0) = 0.4 \times 0.4 = 0.16$

$P(1,0) = 0.6 \times 0.4 = 0.24$

$P(0,1) = 0.4 \times 0.6 = \frac{0.24}{1.00} \checkmark$

But again only  $x(t) = x(t+\gamma) = 1$  contributes to

$R_{xx}(\gamma)$ , so  $\gamma \geq \lambda$ ,  $R_{xx}(\gamma) = E[x(t), x(t+\gamma)]$

$= P(1,1) = 0.36$

For  $\gamma \leq \lambda$  the probability of an event in the

interval  $t \rightarrow t + \lambda = \frac{\gamma}{\lambda}$

(ie passing a switching point)

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4 cont.

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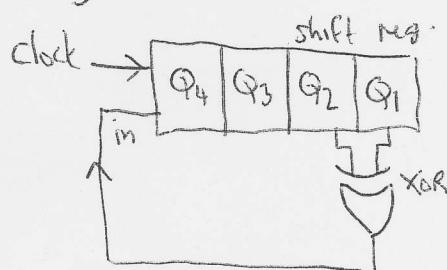
In which case  $E[x(t) \cdot x(t+\tau)] = 0.36$ , or  
 the probability of no event is  $(1 - \frac{\tau}{\lambda})$ , in which case  
 $E[x(t) \cdot x(t+\tau)] = P(1,1) \Big|_{\tau=0} = 0.6$

$$\text{Thus } R_{xx}(\tau < \lambda) = (1 - \frac{\tau}{\lambda}) 0.6 + (\frac{\tau}{\lambda}) 0.36$$

$$\text{ACF is an even function so } R_{xx}(\tau < \lambda) = 0.6 - 0.24 \left( \frac{|\tau|}{\lambda} \right)$$

5

(b) Generator



Sequence

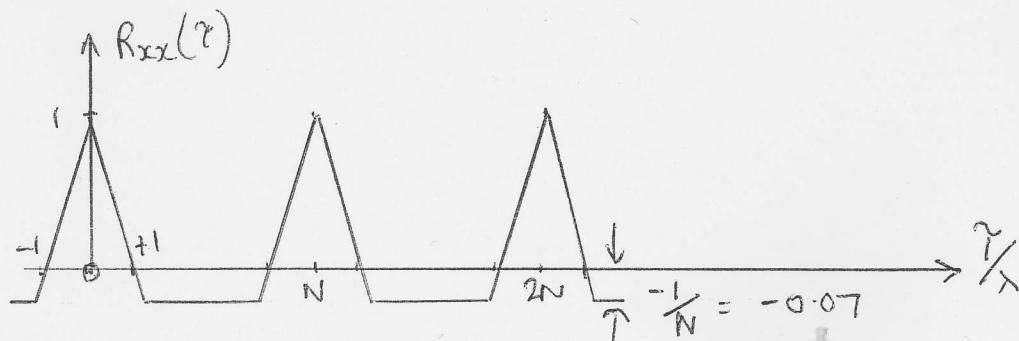
Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Decimal
0	0	0	1	1
0	0	1	0	2
0	1	0	0	4
1	0	0	1	9
etc ...				

Take one bit, say Q<sub>1</sub>, as binary output. Start with any number except 0000 to get maximum length.

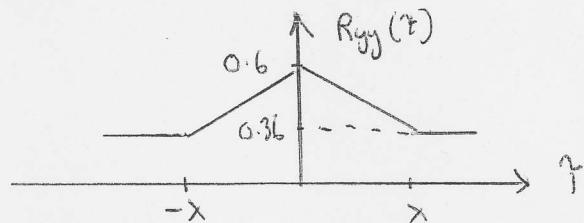
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ACF of PRBS is



ACF of discrete interval BS is



- ACF of PRBS repeats periodically whereas for Discrete interval it does not.
- ACF of PRBS has max = 1 , DIBS max. = 0.6
- ACF of PRBS has min =  $-0.07$  , DIBS min =  $+0.36$
- ACF of PRBS crosses horizontal axis at  $\pm\lambda$ , DIBS, bottom of slope at  $\pm\lambda$

Examples of Use - one from say :-

Monte Carlo

Injection into fluid flow to measure speed

Seismology

Determine noise transition path in vehicles

System identification

One paragraph sufficient

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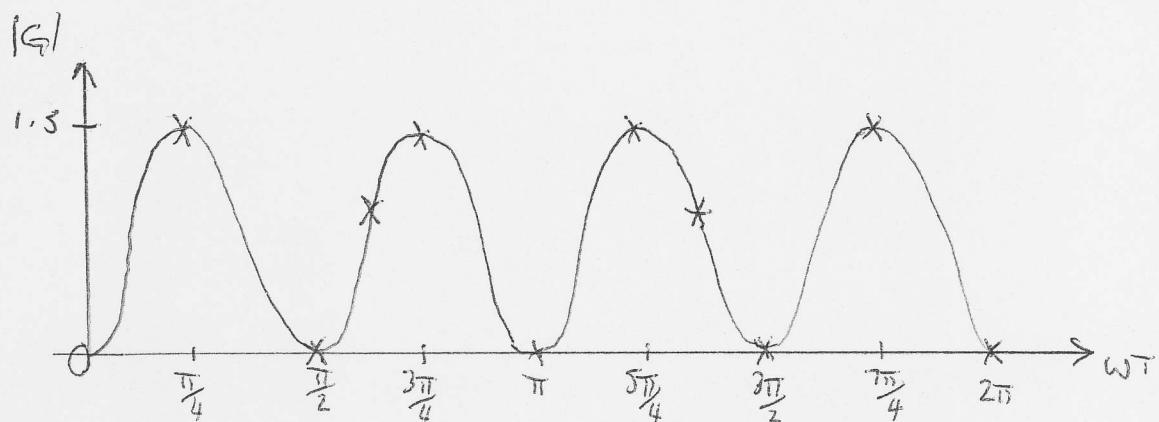
$$(a) \text{ Using } G(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} g[k] e^{-jk\omega T}$$

$$\begin{aligned} G(e^{j\omega T}) &= -0.5 e^{+j\omega 3T} - 0.5 e^{+j\omega T} + 0.5 e^{-j\omega T} + 0.5 e^{-j\omega 3T} \\ &= -0.5 \left\{ (e^{j\omega 3T} - e^{-j\omega 3T}) + (e^{j\omega T} - e^{-j\omega T}) \right\} \\ &= -j (\sin 3\omega T + \sin \omega T) \\ &= \end{aligned}$$

Magnitude  $|G(e^{j\omega T})| = |\sin 3\omega T + \sin \omega T|$

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$\omega T$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\pi$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	$2\pi$
$\sin 3\omega T$	0	0.767	-1	0	0.767	0	-0.767	0	1	-0.767	0
$\sin \omega T$	0	0.767	1	0.866	0.707	0	-0.707	-0.866	-1	-0.767	0
$ G(e^{j\omega T}) $	0	1.414	0	0.866	1.414	0	+1.414	+0.866	0	1.414	0



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$$(b) \quad y = [1.000 \quad 0.760 \quad 0.156 \quad -0.522 \quad -0.951 \quad -0.924 \\ \quad \quad \quad \quad \quad \quad -0.454 \quad +0.233]$$

$$N = 8$$

$$Y[m] = \sum_{n=0}^7 y[n] \exp\left(-j\frac{mn\pi}{4}\right)$$

$$Y[0] = \sum_{n=0}^7 y[n] = -0.762 \quad \text{note } m=0 \quad 2$$

$$Y[1] = y[0]e^{-j\frac{\pi}{4}} + y[1]e^{-j\frac{\pi}{2}} + y[2]e^{-j\frac{3\pi}{2}} + y[3]e^{-j\frac{3\pi}{4}} + y[4]e^{-j\pi} \\ + y[5]e^{-j\frac{5\pi}{4}} + y[6]e^{-j\frac{3\pi}{2}} + y[7]e^{-j\frac{7\pi}{4}}$$

$$y[0] = 1.000$$

$$y[1]e^{-j\frac{\pi}{4}} \rightarrow 0.760 (\cos \frac{\pi}{4} - j \sin \frac{\pi}{4}) = 0.760(0.707 - j 0.707) = 0.537 - j 0.537$$

$$y[2]e^{-j\frac{\pi}{2}} \rightarrow 0.156 (\cos \frac{\pi}{2} - j \sin \frac{\pi}{2}) = 0.156(0 - j) = -j 0.156$$

$$y[3]e^{-j\frac{3\pi}{4}} \rightarrow -0.522 (\cos \frac{3\pi}{4} - j \sin \frac{3\pi}{4}) = 0.369 + 0.369j$$

$$y[4]e^{-j\pi} \rightarrow -0.951 (\cos \pi - j \sin \pi) = 0.951 + 0j$$

$$y[5]e^{-j\frac{5\pi}{4}} \rightarrow -0.924 (\cos \frac{5\pi}{4} - j \sin \frac{5\pi}{4}) = 0.653 - 0.653j$$

$$y[6]e^{-j\frac{3\pi}{2}} \rightarrow -0.454 (\cos \frac{3\pi}{2} - j \sin \frac{3\pi}{2}) = 0 - 0.454j$$

$$y[7]e^{-j\frac{7\pi}{4}} \rightarrow 0.233 (\cos \frac{7\pi}{4} - j \sin \frac{7\pi}{4}) = 0.165 + 0.165j$$

$$\text{Sum to get } Y[1] = \underline{\underline{3.675 - 1.268j}} \quad 3$$

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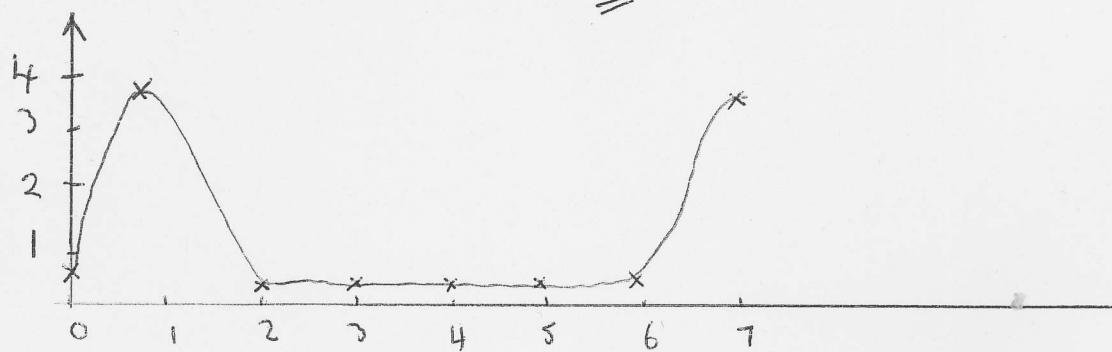
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$$|Y[0]| = 0.702, |Y[1]| = |3.675 - 1.267j| = 3.89,$$

$$|Y[2]| = 0.37, |Y[3]| = 0.23, |Y[4]| = 0.20, |Y[5]| = 0.23,$$

$$|Y[6]| = 0.37, |Y[7]| = 3.89.$$



2

This is not a line spectrum because we have broadening of the peaks. Values away from  $|Y[1]|$  and  $|Y[7]|$  not zero. Spectral leakage because start and end samples not identical. Could use a window function to reduce end samples to zero.

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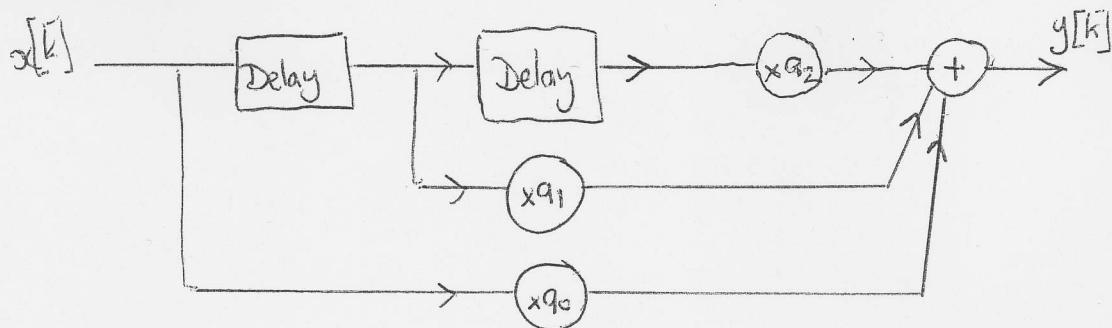
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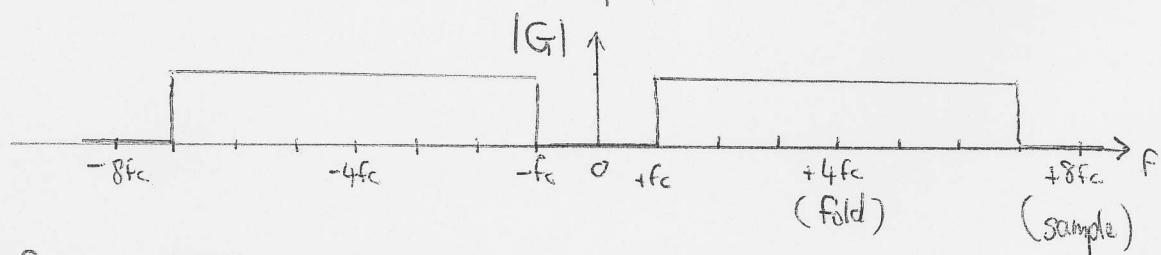
Mark Allocation



$$y[k] = x[k]q_0 + x[k-1]q_1 + x[k-2]q_2$$

3

Periodically extended filter response



$$\text{Period} = 8f_c = 2L, \quad L = 4f_c$$

Fourier series

$$\begin{aligned} a_n &= \frac{1}{4f_c} \int_{-4f_c}^{4f_c} G(f) \cos \frac{n\pi f}{4f_c} df = \frac{2}{4f_c} \int_{f_c}^{4f_c} \cos \frac{n\pi f}{4f_c} df \\ &= \frac{1}{2f_c} \left[ \frac{4f_c}{n\pi} \sin \frac{n\pi f}{4f_c} \right]_{f_c}^{4f_c} \\ &= \frac{2}{n\pi} \left( \sin \frac{n\pi}{4} - \sin \frac{n\pi}{4} \right) = -\frac{2}{n\pi} \sin \frac{n\pi}{4} \end{aligned}$$

$$\text{and } g_c = q_0 = \text{mean of } |G| = \frac{6}{8} = 0.75;$$

$$g_{\pm n} = \frac{a_n}{2} = -\frac{1}{n\pi} \sin \frac{n\pi}{4}$$

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Truncating at  $n = \pm 4$

<u>n</u>	<u><math>g_n</math></u>	<u>Blackman weight</u>	<u>Windowed <math>g_n w_n</math></u>
0	0.75	$0.42 + 0.5 + 0.08 = 1.000$	0.750
$\pm 1$	-0.225	$0.42 + 0.5 \cos \frac{2\pi}{9} + 0.08 \cos \frac{4\pi}{9} = 0.817$	-0.184
$\pm 2$	-0.159	$0.42 + 0.5 \cos \frac{4\pi}{9} + 0.08 \cos \frac{8\pi}{9} = 0.432$	-0.069
$\pm 3$	-0.075	$0.42 + 0.5 \cos \frac{6\pi}{9} + 0.08 \cos \frac{12\pi}{9} = 0.130$	-0.010
$\pm 4$	0	-	0.000

$$\text{So } G(z) = -0.010z^3 - 0.069z^2 - 0.184z + 0.75 - 0.184z^{-1} \\ - 0.069z^{-2} - 0.010z^{-3}$$

$$G(e^{j\omega T}) = -2(0.01 \cos 3\omega T + 0.069 \cos 2\omega T + 0.184 \cos \omega T) + 0.75$$

$$\text{At dc } (\omega=0), G = 0.75 - 2(0.01 + 0.069 + 0.184) = 0.224$$

$$\text{At folding frequency } (\omega = \frac{\pi}{T}) \quad G = 0.75 - 2(-0.01 + 0.069 - 0.184) = 1.000$$

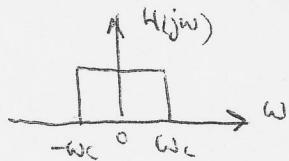
Causal Filter

$$G(z) = -0.01z^0 - 0.069z^{-1} - 0.184z^{-2} + 0.75z^{-3} - 0.184z^{-4} \\ - 0.069z^{-5} - 0.01z^{-6}$$

$$g[k] = -0.01x[k] - 0.069x[k-1] - 0.184x[k-2] + 0.75x[k-3] \\ - 0.184x[k-4] - 0.069x[k-5] - 0.01x[k-6]$$

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Ideal low-pass



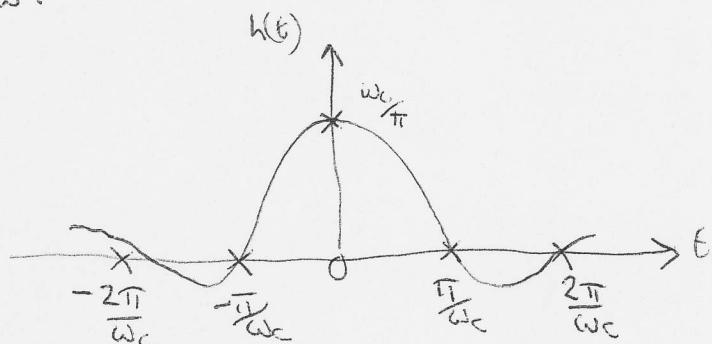
$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-w_c}^{w_c} (\cos \omega t + j \sin \omega t) d\omega$$

$$= \frac{1}{2\pi t} \left[ \sin \omega_c t - j \cos \omega_c t \right]_{-\omega_c}^{\omega_c} = \frac{1}{2\pi t} \cdot 2 \sin \omega_c t$$

$$= \frac{\sin \omega_c t}{\pi t} \quad \text{and} \quad h(0) = \lim_{t \rightarrow 0} \frac{\sin \omega_c t}{\pi t} = \frac{\omega_c}{\pi}$$

 $h(t)$  crosses time axis when  $\sin \omega_c t = 0$ ie  $\omega_c t = n\pi$  where  $n$  = integer ( $|n| > 0$ )

$$t = \frac{n\pi}{\omega_c}$$



Causality requires  $h(t)$  to be zero for  $t < 0$  so the filter is not realisable

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