1 Part 1

1.1 Motivation

How long does it take to double anything

$$\left(1+\frac{x}{100}\right)^n = 2 \tag{1}$$

Where x is the growth rate and n is the number of years.

$$n \approx \frac{70}{x} \tag{2}$$

1.2 Fundamental Concepts

For a probability distribution

$$\int_{-\infty}^{\infty} f(x)dx = 1 \tag{3}$$

Expectation $E[X]$ and variance σ^2 are given in the databook.

1.2.1 Fourier Transforms

Fourier Transforms (F) changes between time domain and frequency domain

$$X(f) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \tag{4}$$

$$x(t) = F^{-1}\{X(f)\} = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}dt \tag{5}$$

1.2.2 Delta function

Infinitesimally brief

$$\delta(x-a) = a \qquad x \neq a \tag{6}$$

Normalised

$$\int_{-\infty}^{\infty} \delta(x-a)dx = 1 \tag{7}$$

Sifting

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a) \tag{8}$$

Fourier transform of delta function

$$\int_{-\infty}^{\infty} \delta(f \pm f_0)e^{\pm j2\pi f\tau}df = e^{\pm j2\pi f_0\tau} \tag{9}$$

1.2.3 Transfer function

The FT of an impulse response is the transfer function. For an RC circuit the impulse is given by Equation (10).

$$h(t) = \frac{1}{RC}e^{-\frac{t}{RC}} \qquad t \geq 0 \tag{10}$$

The transfer function is given by Equation (11)

$$H(t) = \frac{1}{1+j2\pi fRC} \tag{11}$$

1.2.4 Autocorrelation

Measure of signal similarity at different times

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t+\tau)f^*(t)d\tau \tag{12}$$

$$R_{ff}(\tau) = \int_{-\infty}^{\infty} f(t)f^*(t-\tau)d\tau \tag{13}$$

Where f^* represents the complex conjugate (real function, $f* = f$). Also,

$$R_{xx} = E\{x(t)x(t-\tau)\} = E\{x(t+\tau)x(t)\} \tag{14}$$

Can be written as $R_x(\tau)$

1.2.5 Signal Energy and Power

The energy of a signal is given by

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt \qquad \text{Joules} \tag{15}$$

which is a slight problem for infinitely long signals, so practically,

$$P = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt \qquad \text{Watts} \tag{16}$$

The DC power is given from the autocorrelation function

$$\lim_{\tau\to\pm\infty}\{R_x(\tau)\} \tag{17}$$

and the mean power is given as

$$R_x(0)$$

1.2.6 Parseval's Theorem

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} \underbrace{|F(t)|^2}_{\text{Energy Density}} df \qquad \text{Joules} \tag{18}$$

or

$$P = \lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |f(t)|^2 dt = \int_{-\infty}^{\infty} \underbrace{|S(f)|}_{\text{PSD}} df \qquad \text{Watts} \tag{19}$$

1.2.7 Wiener-Kinchine Theorem

For random signals, the autocorelation function and PSD form a Fourier transform pair

$$P(f) = F(R(\tau)) \tag{20}$$

$$R(\tau) = F^{-1}(P(f)) \tag{21}$$

1.3 Digitisation

Sample is quantised into multiple steps

1.3.1 Sampling

SQER is given by

$$sqer = \frac{3N^2}{2} \tag{22}$$

Where N is the number of steps between limits.

Highest frequency measurable is signal rate divided by 2 word length

2 Part 2

2.1 Baseband Pulse Transmission

2.1.1 Gaussian Noise Model

$$p(v)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(v-\overline{v})^2}{2\sigma^2}\right) \tag{23}$$

For thermal noise, zero mean, so $\overline{v} = 0$. PSD is constant, usually $\frac{N_0}{2}$

3 Part 3

3.1 Modulated Data Transmission

3.1.1 Amplitude Shift Keying

ASK, also known as OOK is very simple, but susceptable to noise. If the signal amplitude is small, SNR is poor. Tends to be avoided.

$$f(t) = \begin{cases} A_c os(2\pi f_c t) & 0 \leq t < T \quad \text{``1'} \\ 0 & 0 \leq t < T \quad \text{``0'} \end{cases} \tag{24}$$

with T the bit time, and $\frac{1}{T}$ the signalling rate. The bandwidth is $\frac{2}{T}$.

$$P_e = \frac{1}{2}erfc\left[\frac{A_c}{2\sqrt{2}\sigma}\right] \tag{25}$$

The energy per bit E_b is half E_d, given by

$$P_e = \frac{1}{2}erfc\left[\frac{E-b}{2N_0}\right] \tag{26}$$

3.1.2 Frequency Shift Keying

Easy to implement, but bandwidth heavy. Used in low data rate situations

$$f(t) = \begin{cases} A_c\cos(2\pi f_1 t) & 0 \leq t < T \quad \text{``1'} \\ A_c\cos(2\pi f_0 t) & 0 \leq t < T \quad \text{``0'} \end{cases} \tag{27}$$

PSD is

$$\frac{1}{T} + \frac{1}{T} + f_1 - f_0 = 2B_t\left(1 + \frac{\delta f}{2B_T}\right) \tag{28}$$

Can be detected with two BPFs, one at f_0 and one at f_1.

$$P_e = \frac{1}{2}erfc\left[\frac{A_c}{2\sigma}\right] \tag{29}$$

The energy per bit E_b is half E_d, given by

$$P_e = \frac{1}{2}erfc\left[\frac{E-b}{2N_0}\right] \tag{30}$$

3.1.3 Phase Shift Keying

Same bandwidth as ASK, but best error performance

$$f(t) = \begin{cases} A_c\cos(2\pi f_c t) & 0 \leq t < T \quad \text{``1'} \\ -A_c\cos(2\pi f_c t) & 0 \leq t < T \quad \text{``0'} \end{cases} \tag{31}$$

$$P_e = \frac{1}{2}erfc\left[\frac{A_c}{\sqrt{2}\sigma}\right] \tag{32}$$

Bandwidth is $2B_T$ like ASK.

The energy per bit E_b is given as

$$P_e = \frac{1}{2}erfc\left[\frac{E-b}{N_0}\right] \tag{33}$$

To increase throughput with a limited bandwidth, use multiple levels. Can implement encoding schemes.

Constellation diagrams are used.

PSK can be extended to multiple phase shifts, such as 45° rather than 180°.

QAM uses phase and amplitude to achieve multiple levels per transmission. Uses phase and amplitude.

4 Part 4

4.1 ECC

Achieve detection and correction of errors by adding extra data and redundancy.

To achieve increased throughput (data rate must be trebled for 3x redundancy), BER goes up gives a disadvantage.

4.1.1 Parity

Using a block check byte or in conjunction with per-word parity, can detect and correct errors

4.1.2 Linear Block Codes

Uses addition (XOR, so no carry) and multiplcation (AND).

For a souce of M messages of length k ($M = 2^k$), then with r redundant bits, the message length is 2^{k+r}. Bit rate is increaesed to $\frac{k+r}{k}$ times the uncoded rate - original message is still present. $n = k + r$

Hamming distance d_{min} is the minimum difference between any pair of codes in a given set.

Given D errors in recieved codeword, if we can detect with certainty that the received codeword is not valid when

$$D \leq d_{min} - 1 \tag{34}$$

t erors can be detected when

$$2t + 1 \leq d_{min} \leq 2t + 2 \tag{35}$$

and

$$d_{min} \leq n - k + 1 \tag{36}$$

Generators

$$\mathbf{A} = \begin{bmatrix} a_1 & a_2 & \dots & a_k \end{bmatrix} \tag{37}$$

encode via a generator matrix

$$\mathbf{T} = \mathbf{A} \cdot \mathbf{G} \tag{38}$$

For (7,4),

$$\mathbf{T} = \begin{bmatrix} a_1 & a_2 & a_3 & a_4 & c_1 & c_2 & c_3 \end{bmatrix} \tag{39}$$

5 Part 5

5.1 Data Compression and Cryptography