ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examination: Summer 2012

SIGNAL PROCESSING

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed: 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. a) Show that the transfer function of the filter in Figure 1 is

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{1+sT}$$

where $K = -\frac{R_2}{R_1}$ and $T = CR_2$, assuming perfect operational amplifier, i.e., the voltage difference in the amplifier inputs is zero and the inputs draw no current.

(10 marks)

b) Determine the pole of the transfer function and sketch the pole-zero plot.

(6 marks)

c) Use the pole-zero plot to sketch the magnitude frequency response of the filter. Hence state whether the response corresponds to a low pass filter or a high pass filter. (9 marks)

(25 marks total)

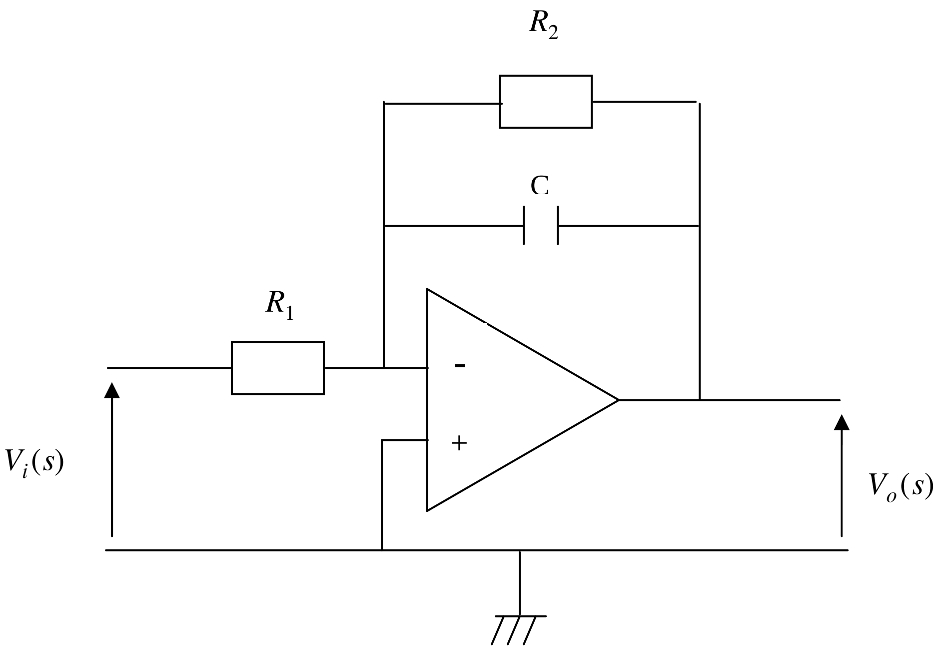

Figure 1. A first-order filter.

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2. a) The Z-transform transfer function of an Nth-order filter is

$$H(z) = \frac{\sum_{m=0}^{N} a(m) z^{-m}}{1 - \sum_{m=1}^{N} b(m) z^{-m}}$$

where $a(m)$ and $b(m)$ are the filter coefficients. If the transfer function has poles at $z = \frac{3}{4} \pm j\frac{\sqrt{3}}{4}$ and zeros at $z = -\frac{\sqrt{2}}{4} \pm j\frac{\sqrt{6}}{4}$, determine the filter coefficients.

(12 marks)

b) Sketch the Z-plane pole-zero plot of the filter. Determine the radial frequencies where the magnitude frequency response of the filter is maximum. Use the pole-zero plot to determine the magnitude frequency response at radial frequency ω of 0 and π. Hence state whether the filter is lowpass, highpass or bandpass.

(13 marks)

(25 marks total)

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3. a) Derive the difference equation of the filter shown in Figure 2. Write down the input sequence $\{x(n)\}$ for $n = 0,1,2,\ldots,4$ that will give the impulse response of the filter. Use the difference equation and a tabular method to determine the filter impulse response $\{y(n)\}$, assuming quiescent conditions.

(11 marks)

b) Derive the Z-transform transfer function of the filter and use it to determine the first five elements of the filter impulse response. State the filter type and hence state whether the filter has any poles and zeros. Determine, if any, the poles and zeros of the filter. Hence state whether the filter is stable or unstable. How is this confirmed by the impulse response?

(14 marks)

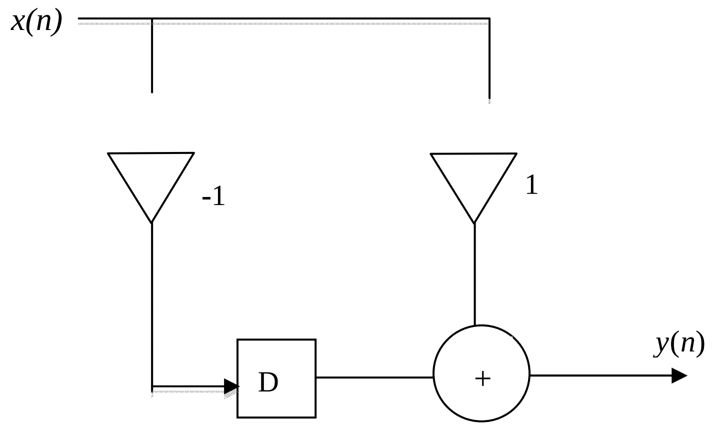

Figure 2 A filter.

(25 marks total)

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SECTION B

4.

a) A random, discrete-interval ternary signal, *x(t)*, can take the level +V with a probability of 0.3, -V with a probability of 0.2, and 0 with a probability of 0.5. Show that its autocorrelation function is given by:

$$R_{xx}(\tau) = 0.5V^2 - 0.49V^2\left(\frac{|\tau|}{\lambda}\right), \qquad 0 \le |\tau| < \lambda$$
$$= 0.01V^2, \qquad |\tau| \ge \lambda$$

where λ is the clock pulse interval of the signal. (12 marks)

b) Analogue to digital converters (A/D) introduce quantisation noise, e_n , to the input signal. Assuming e_n depends on the number of bits, *b*, in the A/D output, find expressions for:

(i) The quantisation interval *q* in terms of *b*.

(ii) The range of the quantisation error e_n in terms of *b*.

(iii) The quantised signal $U_q(k)$ in terms of a noise-free signal *U(k)* and e_n .

(iv) The variance of e_n in terms of *q*.

(v) The signal to noise ratio (*SNR*) of the A/D in terms of the noise-free signal power, P_u , and the noisy signal power, P_n. (7 marks)

c) Assuming a sinusoidal input to the A/D with a peak voltage that maps onto the full range of the A/D's output, show that:

$$SNR = 6.02b + 1.76 \text{ dB}.$$

If the quantisation noise power is to be at 90dB below the signal power level, calculate the required number of bits for the A/D converter. (6 marks)

(25 marks total)

Continued ….

5.

a) The Discrete Time Fourier Transform (DTFT) of a signal $g[k]$ is defined as

$$G(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} g[k]\exp(-jk\omega T).$$

Find the DTFT of the signal shown in Figure 3. Assume all other samples are zero. Sketch the magnitude of $G(e^{j\omega T})$ against ω. You can assume that T = 1.

(13 marks)

b) The Discrete Fourier Transform (DFT) of the sequence $x[n]$ is defined as

$$X[m] = \sum_{n=0}^{N-1} x[n]\exp\left(-j2\pi mn/N\right).$$

Find the DFT of the sequence $g[k]$ of length 16 samples, where $g[k]$ is defined as in Figure 3, and all other $g[k]$ are zero. Sketch the magnitude of $G[m]$ for m = 0,1,…,15. Compare the graphs that you sketched in parts (a) and (b).

(12 marks)

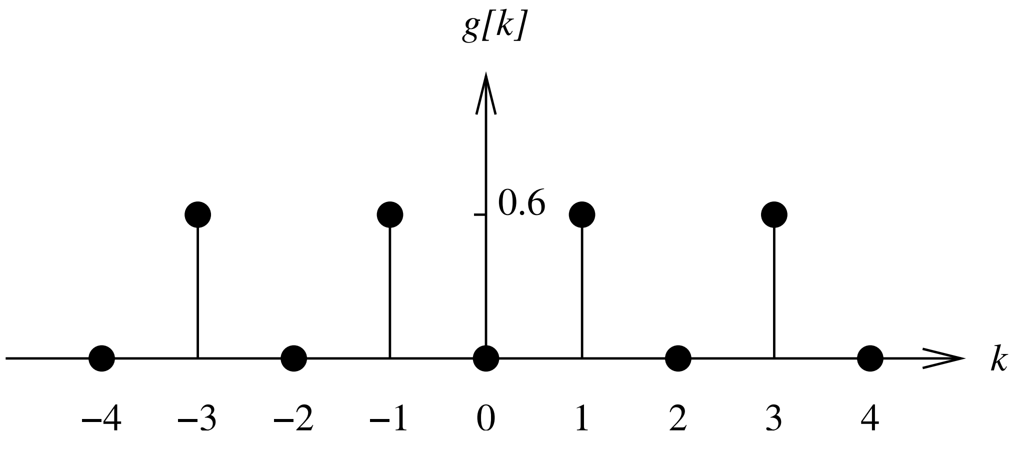

Figure 3 Signal $g[k]$.

(25 marks total)

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6. a) Sketch the block diagram of a fourth-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights.

(3 marks)

b) It is required to design a FIR filter to approximate the ideal high-pass characteristic with cut-off frequency f_c. The folding frequency of the digital filter is to be 5 f_c. By Fourier series show that the ideal filter weights are given by

$$g_n = 0.8, \qquad n = 0$$

$$g_n = -\frac{1}{n\pi}\sin\frac{n\pi}{5}, \qquad n = \pm 1, \pm 2, \pm 3, \ \ldots$$

(7 marks)

c) A Hanning window with $N = 5$ is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for $n = 0, \pm 1, \pm 2, \pm 3, \pm 4$ and ± 5. For the Hanning window, weight

$$w_n = 0.5 + 0.50\cos\left(\frac{2n\pi}{2N+1}\right), \qquad -N \le n \le N$$

$$w_n = 0, \qquad \text{all other values of } n$$

(4 marks)

d) Find the gain of the filter at D.C., and at the folding frequency. (3 marks)

e) Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

f) Show how a matched filter can be formed from an FIR filter, and how it can perform correlation between a known and a received signal. If the known sequence is 1, 3, 8, 2, what would be a suitable set of filter weights? (6 marks)

(25 marks total)

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