1 Lecture 1 - Modulation

$$f(t) = \underbrace{A_c}_{\text{Amplitude}} \times \cos(\overbrace{\omega}^{\text{Angular frequency}} t + \underbrace{\phi_c}_{\text{Phase term}}) \tag{1}$$

Modulation is the process of applying information to a carrier, in this case, the carrier frequency is ω_c (as an angular frequency) or f_c as a frequency directly where $\omega_c = 2\pi f_c$.

If we modify A in the above in sympathy with the information we want to send, then this is amplitude modulation.

If however we change ω_c or ϕ_c in sympathy with the information, these are forms of angle modulation - varying ω_c is frequency modulation, varying phi_c is phase modulation.

If the amplitude etc, varies linearly with the information then this is analogue, alternatively varying these quantities in an on/off manner or between two state is digital.

1.1 Amplitude Modulation

The function $f(t)$ becomes

$$f(t) = \underbrace{A_0(1 - m\cos\omega_m t)}_{A} \times \underbrace{cos\omega_0 t}_{\text{No } \phi_c \text{ term}} \tag{2}$$

f_m or ($\omega_m = 2\pi f_m$) is a simple sine/cosine wave modulation function

Expanding $f(t)$:-

$$\begin{aligned} f(t) &= A_0\cos\omega_0 t + A_0 m\cos\omega_m t x\cos\omega_0 t \\ &= A_0\cos\omega_0 t + \frac{A_0 m}{2}\cos(\omega_m + \omega_0)t + \cos(\omega_0 - \omega_m)t \end{aligned} \tag{3}$$

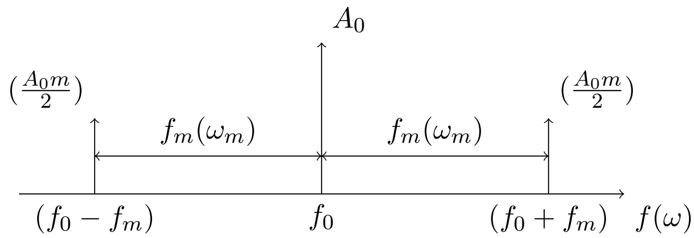


Figure 1: Spectrum of an AM signal, including sidebands

The simple process of amplitude modulation produces an upper sideband (USB) at $f_0 + f_m$ and a lower sideband at $f_0 - f_m$. Note that only the sidebands contain any useful information about the amplitude m of the modulation signal.

This modulation has a power penalty because the carrier conveys no useful amplitude information itself.

$$\begin{aligned} \left(\frac{\text{Power in sidebands}}{\text{Total power transmitted}}\right) &= \frac{(0.5A_0 m)^2 + (0.5A_0 m)^2}{(0.5A_0 m)^2 + (0.5A_0 m)^2 + A_0^2} \\ &= \frac{0.5A_0^2 m^2}{0.5A_0^2 m^2 + A_0^2} \\ &= \frac{0.5m^2}{0.5m^2 + 1} \end{aligned} \tag{4}$$

m = modulation depth = $\left(\frac{A_m}{A_0}\right)$ which is a ratio of modulation voltage to the carrier voltage (or current).

It is essential not to let $|m| \geq 1$ because this is overmodulation and it distors the AM waveform.

Going back to equation 4 in the limit $m = 1$ and so the ratio is:-

$$\frac{0.5 \times 1^2}{(0.5 \times 1^2) + 1} = \frac{0.5}{1.5} = \frac{1}{3} = 33.3\% \tag{5}$$

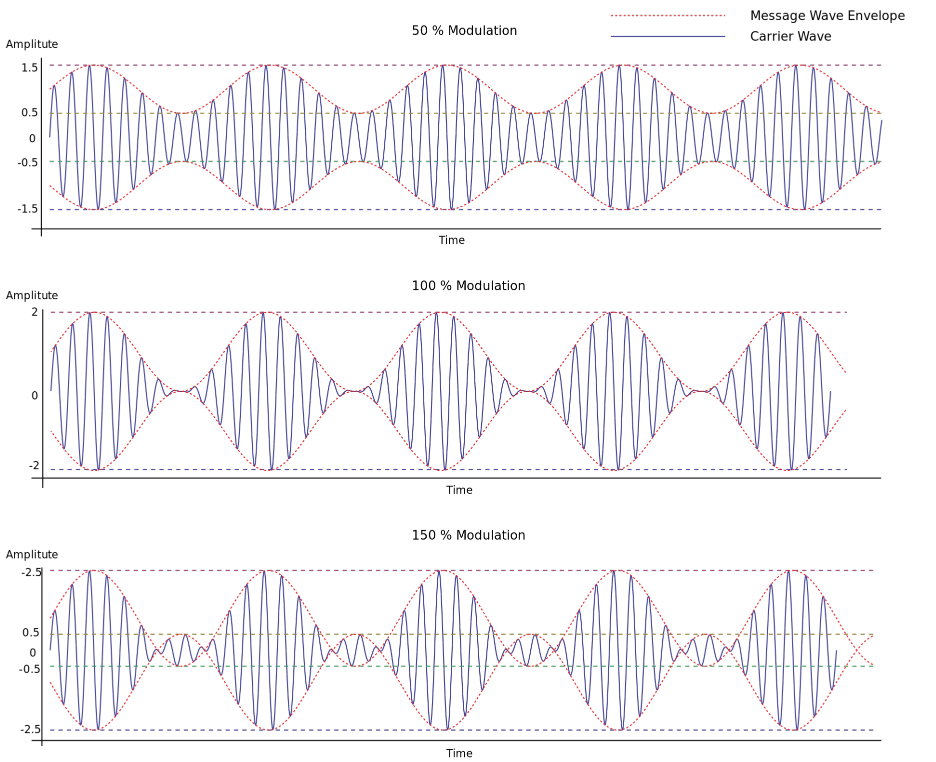
$\frac{2}{3}$ of the transmitted power is wasted.

Typically, $m \approx 0.4$ from which we have:-

$$\frac{0.5 \times 0.4^2}{(0.5 \times 0.4) + 1} = \frac{0.08}{0.08 + 1} \tag{6}$$

so this ratio is around 0.08 ie. only 8% of the transmitted power is used, or 92% is wasted!

1.1.1 Waveforms



It is essential, when considering the above waveform, to have $|m| \leq 1$ as mentioned before.

Detecting AM requires an envelope detector in its simplest form:-

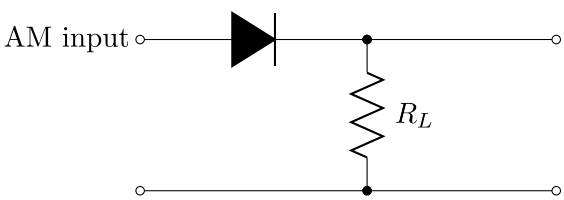


Figure 2: Simple envelope detector

A more advanced envelope detector adds a low pass filter to remove high frequency components from the carrier wave.

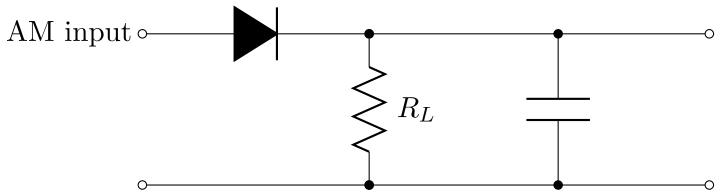


Figure 3: More advanced envelope detector using smoothing

If the carrier frequency is high compared to the modulation frequency, the recovered envelope is a "quite good" approximation to the original given a certain combination of R_L and C.

2 Lecture 2 - Double Sideband and Single Sideband AM

Recall AM signal (Equation 2. Power is wasted in the carrier, so a better form of A.M. could be:-

$$f(t) = A_0 \cdot \cos\omega_0 \cdot m\cos\omega_m t \tag{7}$$

Where $m = \frac{A_m}{A_0}$ = modulating depth and A_m = amplitide of modulating signal

$$f(t) = \left(\frac{A_0 m}{2}\right)(\cos(\omega_0 + \omega_m)t + \cos(\omega_0 - \omega_m)t) \tag{8}$$

This is known as double sided sideband, supressed carrier (DSB).

There is an important property of a DSB $\rightarrow$ for no modulation, no signal is sent, so that it is very power effiicient, but not bandwidth efficient.

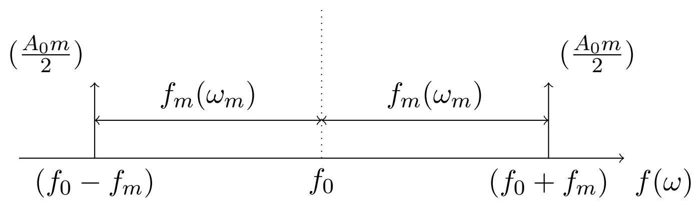


Figure 4: Spectrum of a DSB signal

The envelope of a DSB signal for a single frequency of modulation is:-

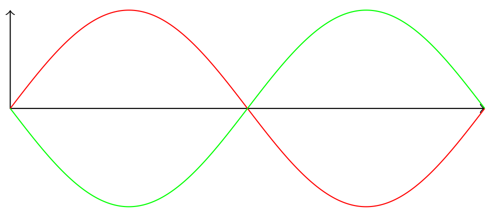


Figure 5: DSB Signal envelope

Because there is no carrier present, the envelope of the DSB waveform no longer represents the modulating function, therefore an envelope detector cannot be used to detect it - a product detector is used instead.

To illustrate the operation, instead of a DSB being the input, simply use a carrier frequency with no modulation

The output after filtering is $(\frac{A_0 E}{2})$ which is proportinal to A_0, the signal amplitude.

Replace the simple input by:-

$$A_0 \cos\omega_0 t \times \cos\omega_m t \tag{9}$$

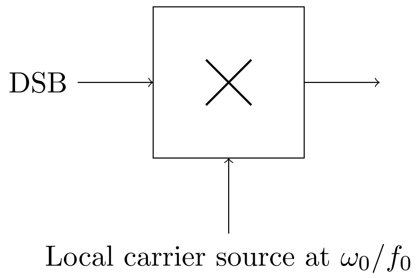


Figure 6: Product detector/mixer/multiplexer with simple input

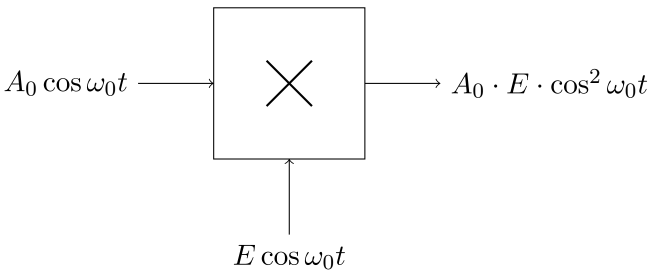


Figure 7: Product detector/mixer/multiplexer with real input

output is

$$\begin{aligned}
(A_0 \times \cos \omega_0 t \times \cos \omega_m t)(E \times \cos \omega_0 t) &= A_0 E \cos^2 \omega_0 t \times \cos \omega_m t && (10)\\
&= \left(\frac{A_0 E}{2}\right)(1 + \cos 2\omega_0 t) \times \cos \omega_m t && (11)
\end{aligned}$$

After low pass filtering to remove all frequences above ω_m:-

$$OP = \left(\frac{A_0 E}{2}\right) \times \cos \omega_m t \tag{12}$$

which represent the modulation function directly.

This system can be improved even further because bandwidth is wasted. Bandwith is important because it is a scarce resource.

If we consider the expanded form of that for DSB:-

$$f(t) = \left(\frac{A_0 m}{2}\right) \cos(\omega_0 + \omega_m)t + \left(\frac{A_0 m}{2}\right) \cos(\omega_0 - \omega_m)t \tag{13}$$

If we filter carfully, then we need only send either the USB or LSB.

Filtering therefore reduces the badnwidth, as each sideband carries the same information (each has an "m" term). Difficulty arrises in obtaining sharp cuttof filters. The PHASING METHOD can be used to generate SSB (single side band).

$$E_0 = E_1 + E_2 \tag{14}$$

$$\begin{aligned}
E_1 &= [A_0 \cos \omega_0 t] \times \left[\cos \omega_m t + \frac{\pi}{2}\right] \\
&= A_0 \cos \omega_0 t \times \sin \omega_m t && (15)
\end{aligned}$$

$\therefore$

$$E_2 = A_0 \sin \omega_0 t \times \cos \omega_m t \tag{16}$$

$\therefore$

$$\begin{aligned} E_0 &= A_0 \times [\cos \omega_0 t \times \sin \omega_m t + \sin \omega_0 t \times \cos \omega_m t] \\ &= A_0 \sin (\omega_0 + \omega_m) t \end{aligned} \tag{17}$$

USB, no filtering needed.

Similarly, the LSB can be generated by changing the sign of the second term above (on the first term).

The method works very well only when a single frequency of modulation is used. In practise, a range of frequences is used and it is very difficult to obtain a 90°phase shift over a range of frequencies.

2.1 Vestigial Sideband

Remember all the abbreviations, "in case they get slipped into a question" This is an approximation to SSB which has been used extensively in analoge T.V. ver many decades. Basically, a DSB/C (Double Sideban with Carrier)

The main benefits are: simplicty and cheapness compared to SSB, when using valve/vacuum tube technologies.

The disadvantage is carrier still there, wasting power, but alowing an envelope detector (cheap, easy to make) to be used.

3 Lecture 3 - Angle Modulation

Starting with **??**

$$f(t) = A_0 \cdot \cos (\omega_0 t + \phi) \tag{18}$$

In the expression for $f(t)$ if we make $(\omega_c t + \phi) = \phi(t)$ then ϕ_c may be made instantaneously proportional to the modulating signa $f_m(t)$:-

$$\begin{aligned} \phi\,(t) &= \omega_c t + k_p f_m(t) \\ \Longrightarrow f(t) &= A_0 \cos \theta(t) \end{aligned} \tag{19}$$

where k_p = constant of modulation. This is Phase Modulation.

Alternatively, we can have

$$\begin{aligned} \omega(t) &= \left[\frac{d\phi(t)}{dt}\right] \\ &= \omega_c + k_f \cdot f_m(t) \end{aligned} \tag{20}$$

where, as before k_f is a constant.

In this case, the instantaneous frequency has the value ω_c in the absence of a modulating signal. By integrating the above expression $\frac{d\theta(t)}{dt}$:-

$$\begin{aligned} \omega(t) &= \frac{d\theta(t)}{dt} \Longrightarrow \int \omega(\theta) dt = \theta(t) \\ \Longrightarrow \theta(t) &= \omega_c t + k_f \int f_m(t) dt + \phi_0 \end{aligned} \tag{21}$$

This is frequency modulation.

Frequency modulation is thus an indirect form of pulse modulation and is mathematically related.

Phase and frequency modulation are thus similar in form and are both angle modulation.

Frequency modulation can be generated easily using a voltage controlled oscillator.

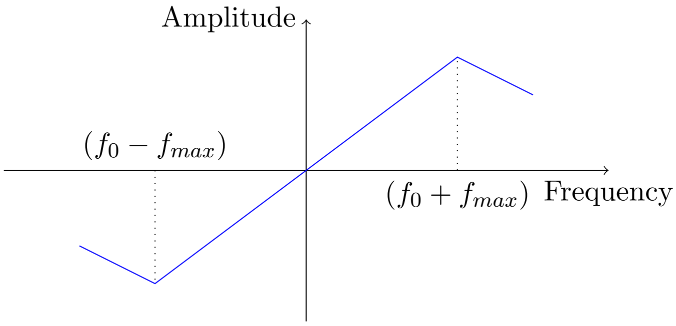


Figure 8:

3.1 Detection of FM

An FM discriminator (or Phase Locked Loop) is used to convert frequenct changes to amlitude changes:- At frequency f_0 the output of the FM demodulator is zero. For frequencies above f_0 (but less than $\pm f_{max}$ then the output is greater than zero (or inverted and greater than 0 for $f < f_0$. The output of such a discriminator is then fed to a circuit or system which detects amplitude changes.

F.M. and P.M. generate a different spectrum to let $f_m(t) = \cos(\omega_m t)$ for F.M.

$$\begin{aligned}
\therefore f(t) &= A_0 \cdot \cos\left[\omega_0 t + k_f \int_0^t \cos(\omega_m t) dt\right] \\
&= A_0 \cdot \cos\left[\omega_0 t + \left(\frac{k_f}{\omega_m}\right)\sin\omega_m t\right] \qquad (22)
\end{aligned}$$

The instantaneous frequency of this value is $f_i = \frac{(\omega_0 + k_f \cos\omega_m t)}{2\pi}$ which oscillates between two extreme values:-

$$\left[\frac{\omega_0}{2\pi}\right] \pm \left[\frac{k_f}{2\pi}\right] \text{ for } cos(\omega_m t)_m ax = \pm 1 \qquad (23)$$

The peak value of the difference between f_i and the carrier frequency $f_0 = \left(\frac{\omega_0}{2\pi}\right)$ is the frequency deviation, which is $\left[\frac{k_f}{2\pi}\right]$ in the above example.

Since k_f is proportional to the real amplitide of the modulating signal, so is the frequency deviation.

If f_d is the maximum frequency deviation and f_a is the actual frequency deviation then the deviation ratio, m is defined as:-

$$m = \frac{f_a}{f_d} \qquad (24)$$

Now $f_a = \frac{k_f}{2\pi}$ so $m = \left[\frac{k_f}{2\pi \cdot f_d}\right] \therefore$

$$\left(\frac{k_f}{\omega_m}\right) = \frac{2\pi f_a}{2\pi f_m} = \frac{f_a}{f_m} = m\frac{f_d}{f_m} \qquad (25)$$

(Comparing actual frequency deviations to the mdoulating frequency f_m).

$$\begin{aligned}
\Longrightarrow \frac{m f_d}{f_m} &= m_p = \text{modulation index} \\
\therefore f(t) &= A_0 \cdot [\omega_0 t + m \cdot \sin\omega_m t] \qquad (26)
\end{aligned}$$

Usign Bessel functions,

$$f(t) = A_0 \sum_{m=\infty}^{\infty} I_n(m_p) \cdot \cos(\omega_c + n\omega_m)t \qquad (27)$$

$\therefore$ firstly, FM thus generates a range of frequencies from $-\infty$to∞ in practice, the bandwidth is restrictred and also the Bessel coefficient ($= I$) have finite values which tend to zero as n tends to $\pm\infty$

Modulation Index m_p	No. of sidebands n	Bandwidth (a) as multiples of f_m	Bandwidth (b) as multiples of f_a
0.1	2	2	20
0.5	4	4	8
1	6	6	6
5	16	16	3.2
30	70	70	2.3