ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examinations : Summer 2008

SIGNAL PROCESSING

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed : 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. Show that the Laplace transfer function of the second-order filter shown in Figure 1 is:

$$\frac{V_o(s)}{V_i(s)} = \frac{(1+R_1C_1s)(1+R_2C_2s)}{1+(R_1C_1+R_2C_2+R_1C_2)s+R_1C_1R_2C_2s^2} \ .$$

(12 marks)

Determine the poles of the filter for $R_1 = 20\,\mathrm{k\Omega}$, $C_1 = 2\,\mu\mathrm{F}$, $R_2 = 110\,\mathrm{k\Omega}$ and $C_2 = 100\,\mu\mathrm{F}$. Hence determine whether the system is stable, unstable or marginally stable.

(7 marks)

Sketch the expected filter impulse response. (6 marks)

(25 marks total)

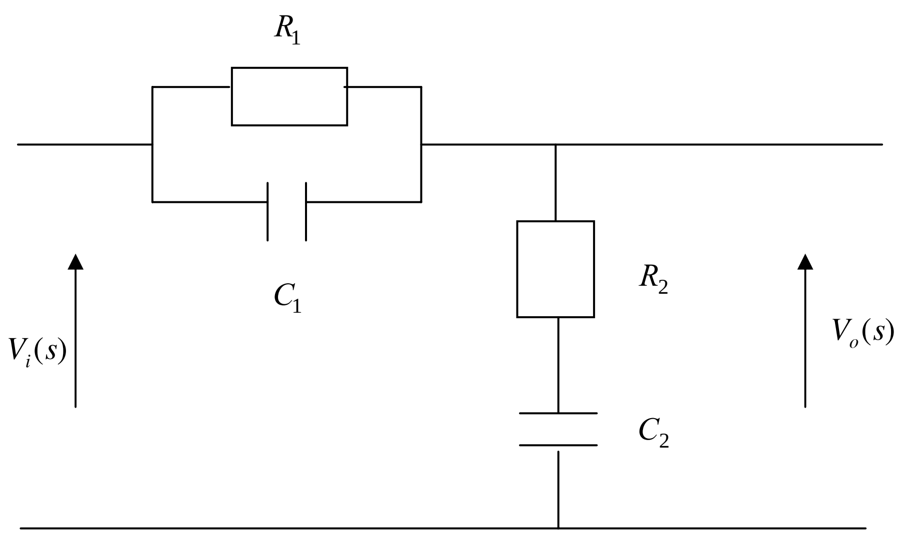

Figure 1. A second-order filter.

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2. Derive the state equations of the second-order infinite impulse response (IIR) filter shown in Figure 2. Hence show that the Z-transform transfer function of the filter is

$$H(z) = \frac{\sum_{m=0}^{2} a(m) z^{-m}}{1 - \sum_{m=1}^{2} b(m) z^{-m}}$$

where *a(m)* and *b(m)* are filter coefficients. (15 marks)

Given that the filter has zeros at $z = 1 \pm j$ and poles at $z = -\frac{1}{4} \pm j\frac{1}{4}$, sketch the Z-plane pole-zero plot. Hence state whether the filter is lowpass, highpass or bandpass. Determine also the value of the filter coefficients. (10 marks)

(25 marks total)

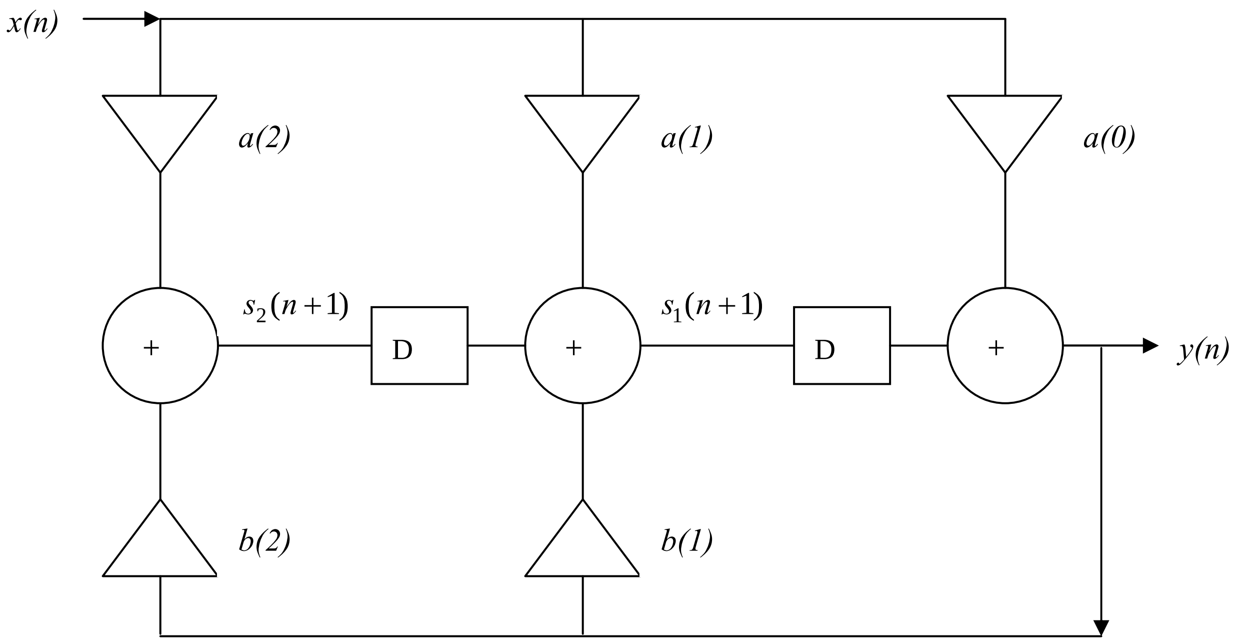

Figure 2 A second-order IIR filter.

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3. Derive the difference equation of the digital integrator shown in Figure 3 and show that its Z-transform transfer function is

$$H(z) = \frac{1}{1 - z^{-1}} .$$

Hence determine the impulse response and use it to determine the stability of the integrator.

(11 marks)

Determine also the pole of the integrator and use it to verify the stability of the integrator.

(6 marks)

Show that the state equations for the integrator can be written in the form

$$s(n+1) = a\, s(n) + b\, x(n)$$

$$y(n) = c\, s(n) + d\, x(n)$$

and determine the constants a, b, c, d. (8 marks)

(25 marks total)

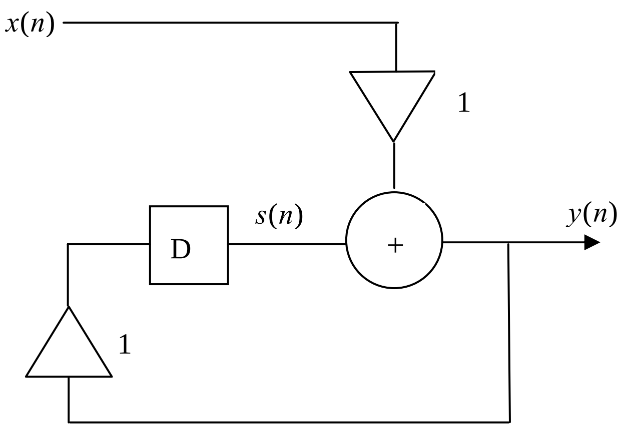

Figure 3 A digital integrator.

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SECTION B

4. Sketch the block diagram of a second-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights. (3 marks)

It is required to design a FIR filter to approximate the ideal low-pass characteristic with cut-off frequency f_c. The folding frequency of the digital filter is to be 4 f_c. By Fourier series show that the ideal filter weights are given by

$$g_n = 0.25 \quad , \qquad n = 0$$
$$g_n = \frac{1}{n\pi}\sin\frac{n\pi}{4}, \qquad n = \pm 1, \pm 2, \pm 3, \ \ldots$$

(7 marks)

A Blackman window with $N = 4$ is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for n = 0, ±1, ±2, ± 3, and ± 4. For the Blackman window, weight

$$w_n = 0.42 + 0.50\cos\left(\frac{2n\pi}{2N+1}\right) + 0.08\cos\left(\frac{4n\pi}{2N+1}\right), \quad -N \le n \le N$$
$$w_n = 0 \qquad , \quad \text{all other values of } n$$

(4 marks)

Find the magnitude of the gain of the filter at D.C., and at the folding frequency. (3 marks)

Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

Show how a matched filter can be formed from an FIR filter, and how it can perform correlation between a known and a received signal. (6 marks)

(25 marks total)

Continued ….

5. **ANSWER BOTH PARTS**

(a) A random, discrete-interval ternary signal, *x(t),* can take the level +1 with a probability of 0.375, -1 with a probability of 0.375, and 0 with a probability of 0.25. Show that its autocorrelation function is

$$R_{xx}(\tau) = 0.75\left(1 - \frac{|\tau|}{\lambda}\right) \qquad 0 \le |\tau| \le \lambda$$

$$R_{xx}(\tau) = 0 \qquad |\tau| \ge \lambda,$$

where λ is the clock pulse interval of the signal. (15 marks)

(b) An anti-aliasing filter is to be designed for an analogue to digital converter. The continuous time signal will be sampled at a frequency ω_s rad.s^{-1}, and each sample will be converted to a 9-bit binary coded digital word. The filter cut-off frequency is to be $\omega_s/8$ rad.s^{-1}, and any aliased component must not cause a change in any sampled value greater than plus or minus one least significant bit. The anti-aliasing filter will be an analogue Butterworth filter with a normalised frequency response:

$$|G(j\omega)| = \frac{1}{\left(1+\omega^{2n}\right)^{1/2}} \quad ,$$

where ω is angular frequency in rad.s^{-1}, and n is the order of the filter. Sketch the gain of the filter against frequency, labelling the cut-off frequency, and the sampling and folding frequencies of the discrete system. Calculate the minimum order of the filter, n.

(10 marks)

(25 marks total)

Continued ….

6. **ANSWER BOTH PARTS**

(a) The Discrete Time Fourier Transform (DTFT) of a signal $g[k]$ is defined as

$$G(e^{j\omega T}) = \sum_{k=-\infty}^{\infty} g[k]\exp(-jk\omega T).$$

Find the DTFT of the signal shown in Figure 4. Assume all other samples are zero.

(7 marks)

Sketch the magnitude and phase of $G(e^{j\omega T})$ against ω. You can assume that T = 1.

(8 marks)

(b) The Discrete Fourier Transform (DFT) of the sequence $x[n]$ is defined as

$$X[m] = \sum_{n=0}^{N-1} x[n]\exp\left(-j2\pi mn/N\right).$$

Find the DFT of the sequence $x[n]$ of length 12 samples, where $x[0] = 1/4$, $x[1] = 1/2$, $x[2] = 1/4$, and all other $x[k]$ are zero. Sketch the magnitude of $X[m]$ for $m = 0,1,\ldots,11$.

(10 marks)

(25 marks total)

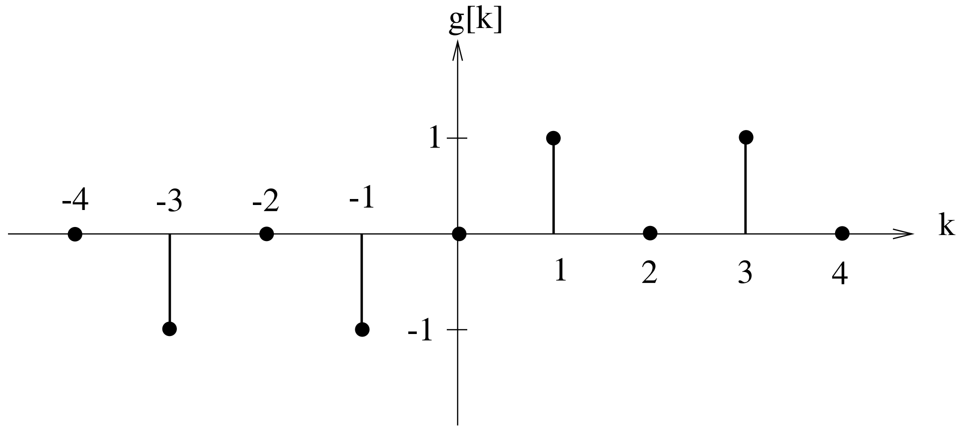

Figure 4 Signal $g[k]$.

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