ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examinations : Summer 2010

SIGNAL PROCESSING

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed: 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. ANSWER BOTH PARTS

(a) The Laplace transfer function of a system is

$$H(s) = \frac{5s+14}{s^2+5s+4} \ .$$

Determine the poles of the system and hence state whether the system is stable, unstable or marginally stable. Determine also the impulse response, $h(t)$, of the system.

(11 marks)

(b) Show that the transfer function of the system shown in Figure 1 is

$$G(s) = \frac{sCR_2+1}{sC(R_1+R_2)+1}.$$

Determine the dynamic response of the system to a unit step input.

(14 marks)

(25 marks total)

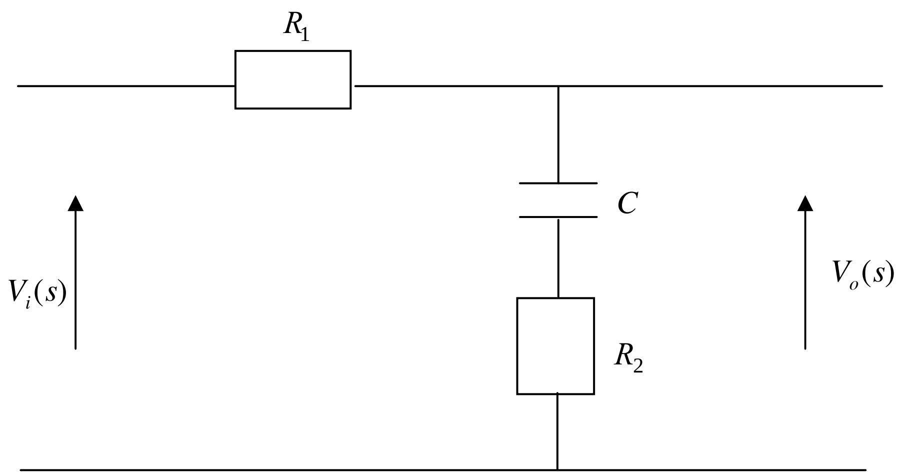

Figure 1. A second-order system.

Continued ....

2. Write the Z-transform transfer function, $H(z)$, of the second-order infinite impulse response (IIR) filter shown in Figure 2.

(7 marks)

Using the transfer function and noting that $e^{j\pi} = -1$, compute $|H(0)|$ and $|H(\pi)|$, i.e., the gain of the filter at $\omega = 0, \pi$. Hence explain whether the IIR filter is lowpass or highpass.

(10 marks)

It is required to modify the filter such that its gain at $\omega = \pi$ is 1. Draw the block diagram of the modified filter.

(8 marks)

(25 marks total)

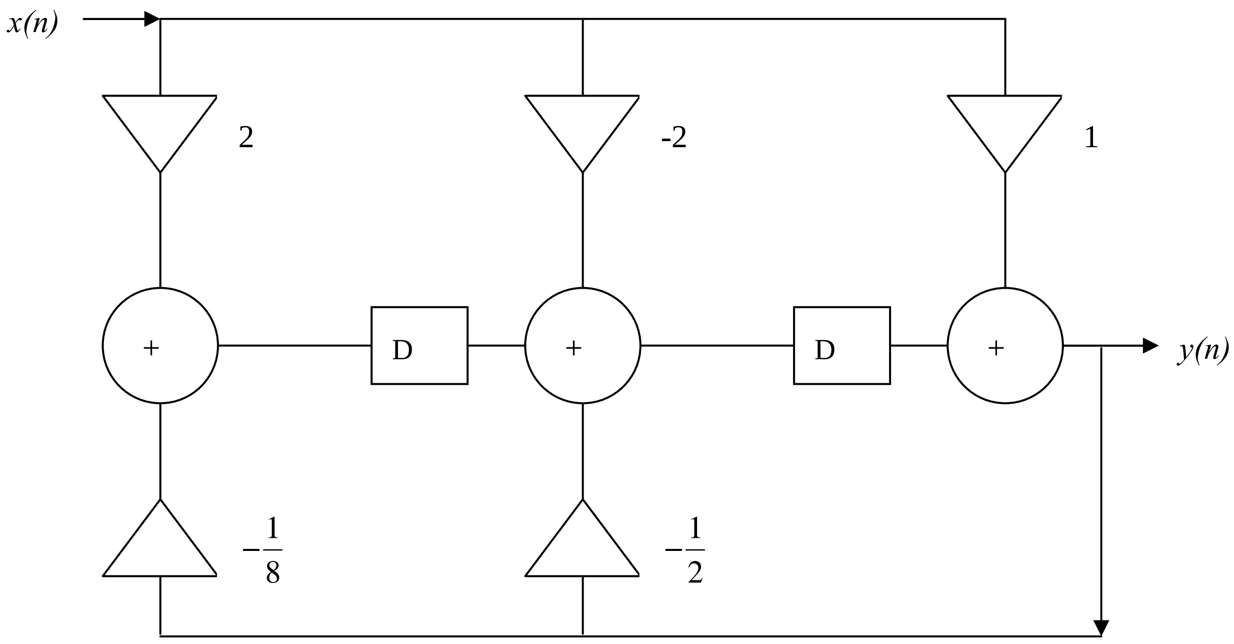

Figure 2 A second-order IIR filter.

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3. The Z-transform transfer function of a second-order system is

$$H(z) = \frac{z^2}{z^2 - z + 0.5}.$$

Draw the block diagram of the direct implementation of $H(z)$.

(9 marks)

Using the transfer function, determine the difference equation of the system. Use the difference equation and a tabular method to obtain the system impulse response to 2 decimal places, assuming quiescent conditions. Does the response represent a stable system?

(10 marks)

Determine the poles of the system and use them to verify the stability.

(6 marks)

(25 marks total)

Continued ....

SECTION B

4. ANSWER BOTH PARTS

(a) A random, discrete-interval ternary signal, *x(t),* can take the level +1 with a probability of 0.4, -1 with a probability of 0.4, and 0 with a probability of 0.2. Show that its autocorrelation function is

$$R_{xx}(\tau) \quad = \quad 0.8\left(1-\frac{|\tau|}{\lambda}\right) \qquad\qquad 0 \le |\tau| \le \lambda$$

$$R_{xx}(\tau) \quad = \quad 0 \qquad\qquad |\tau| \ge \lambda,$$

where λ is the clock pulse interval of the signal. (15 marks)

(b) An anti-aliasing filter is to be designed for an analogue to digital converter. The continuous time signal will be sampled at a frequency ω_s rad.s^{-1}, and each sample will be converted to a 10-bit binary coded digital word. The filter cut-off frequency is to be $\omega_s/7$ rad.s^{-1}, and any aliased component must not cause a change in any sampled value greater than plus or minus one least significant bit. The anti-aliasing filter will be an analogue Butterworth filter with a normalised frequency response:

$$\left|G(j\omega)\right| = \frac{1}{\left(1+\omega^{2n}\right)^{1/2}} \quad ,$$

where ω is angular frequency in rad.s^{-1}, and n is the order of the filter. Sketch the gain of the filter against frequency, labelling the cut-off frequency, and the sampling and folding frequencies of the discrete system. Calculate the minimum order of the filter, n.

(10 marks)

(25 marks total)

Continued ….

5. Sketch the block diagram of a third-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights. (3 marks)

It is required to design a FIR filter to approximate the ideal low-pass characteristic with cut-off frequency f_c . The folding frequency of the digital filter is to be 3 f_c . By Fourier series show that the ideal filter weights are given by

$$g_n = 0.33 \quad , \qquad n = 0$$

$$g_n = \frac{1}{n\pi} \sin \frac{n\pi}{3}, \qquad n = \pm 1, \pm 2, \pm 3, \; ...$$

(7 marks)

A Hamming window with $N = 5$ is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for n = 0, ±1, ±2, ± 3, ± 4, and ± 5. For the Hamming window, weight

$$w_n = 0.54 + 0.46 \cos \frac{2\pi n}{2N+1} \quad , \quad -N \le n \le N$$

$$w_n = 0 \qquad , \quad \text{all other values of } n$$

(4 marks)

Find the magnitude of the gain of the filter at D.C., and at the folding frequency. (3 marks)

Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

Show how a matched filter can be formed from an FIR filter, and how it can perform correlation between a known and a received signal. (6 marks)

(25 marks total)

Continued ….

6. ANSWER ALL PARTS

The system of Figure 3 has time delays, T, of one clock period. The input sequence applied is

$$x[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$

(a) Calculate values for y[0], y[1], y[2], y[3] and y[4]. Find an equation for y[k] in terms of present and past inputs of x[k]. (5 marks)

(b) The Discrete Time Fourier Transform (DTFT) is defined as:

$$Y(\omega) = \sum_{k=-\infty}^{\infty} y[k] \exp\left(-jk\omega T\right).$$

Find the DTFT of y[k], as calculated in (a), and sketch its magnitude response against ω. You can assume that T=1. Comment on the characteristics of the filter from observations of your sketch. Justify your reasoning. (10 marks)

(c) The Discrete Fourier transform is defined as:

$$Y[m] = \sum_{n=0}^{N-1} y[n] \exp\left(-j2\pi mn / N\right).$$

For an input data window width of 12, calculate the Discrete Fourier transform of the first 12 samples of the output of the system in Figure 3. Sketch $|Y[m]|$ and compare with $|Y(\omega)|$ as calculated in part (b). (10 marks)

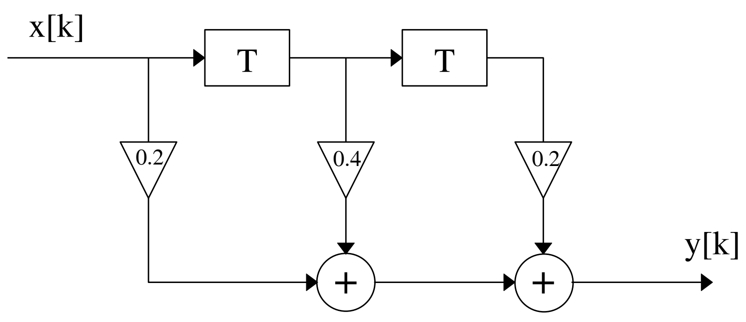

Figure 3. Discrete System

(25 marks total)

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