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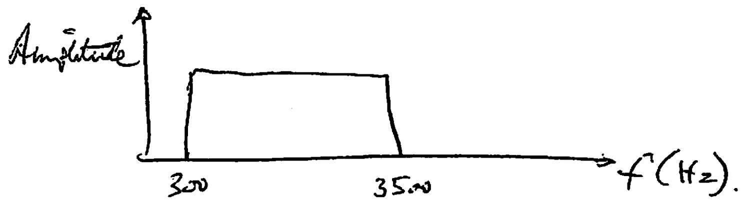
Question 1		Mark Allocation

The signal occupies from 300Hz to 3.5KHz :–



The carrier is at 10MHz and its amplitude at peak is 15 volts. So it can be represented by: $V_c(t) = 15 \cos \omega_c t$ where $\omega_c = 2\pi f_c$ with $f_c = 10^7$. The modulation frequency within the band is of amplitude 10 volts peak, and so may be represented by: $V_m(t) = 10 \cos \omega_m t$. where $(2\pi \times 300) \leq \omega_m \leq (2\pi \times 3500)$.

(a) For full amplitude AM :–

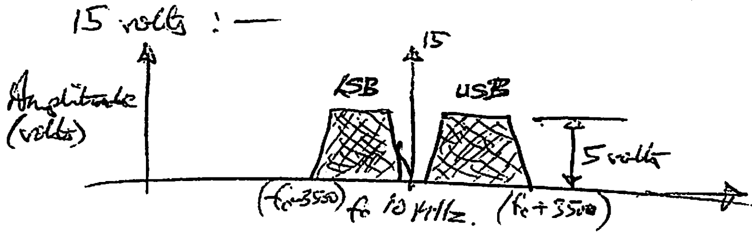
$$f(t) = V_{co}(1 + m \cos \omega_m t) . \cos \omega_c t .$$

where V_{co} = 15 volts = peak carrier amplitude.

Hence $f(t) = V_{co} \cos \omega_c t + \left(\frac{V_{co} . m}{2}\right) . \cos(\omega_m + \omega_c)t + \left(\frac{V_{co} . m}{2}\right) . \cos(\omega_c - \omega_m)t .$

where $m = \left|\frac{V_m(t)}{V_c(t)}\right|$ or $\left(\frac{V_{mo}}{V_{co}}\right)$ where V_{mo} = 10 volts, V_{co} = 15 volts.

– The sidebands at $(\omega_c \pm \omega_m)$ are of amplitude : $\left(\frac{10}{15}\right)\left(\frac{15}{2}\right) = 5$ and the carrier is 15 volts :—



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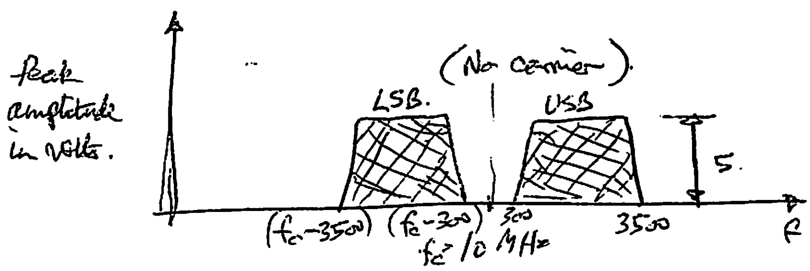
(b) For double sideband, suppressed carrier :–

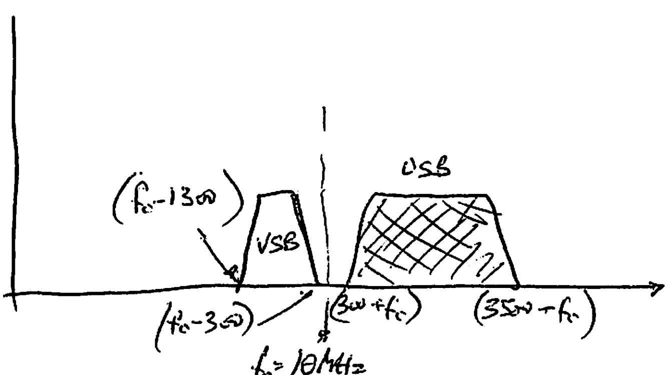
$$f(t) = (V_{co} \cdot m) \cdot \cos \omega_m t \cdot \cos \omega_c t.$$

$$= \left(\frac{V_{co} \cdot m}{2}\right) \left(\cos(\omega_m + \omega_c)t + \cos(\omega_c - \omega_m)t\right)$$

$$= 5\left(\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t\right)$$



(c) For vestigial sideband → like DSB but with only 1KHz of bandwidth for LSB :–



(A formula for VSB may be given : $f(t) = \left[5\left(\cos(\omega_c + \omega_m)t + \alpha \cos(\omega_c - \omega_m)t\right)\right]$ where $0 < \alpha \leq 1$ depending on f_m.

10 marks

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$$X = \left(\frac{\text{Power in sidebands}}{\text{Total transmitted power}}\right)$$

, In each case, power is in $50\,\Omega$ and $P = \frac{(V_{RMS})^2}{R_L = 50}$,

In case (a): Power in sidebands $= \frac{(5/\sqrt{2})^2}{50} + \frac{(5/\sqrt{2})^2}{50}$

Power in total $= \frac{(5/\sqrt{2})^2}{50} + \frac{(5/\sqrt{2})^2}{50} + \frac{(15/\sqrt{2})^2}{50}$

$$\therefore \text{Ratio } X = \frac{\left[\left(\frac{25}{2}\right) + \left(\frac{25}{2}\right)\right] / 50}{\left[\left(25/2\right) + \left(\frac{25}{2}\right) + \left(\frac{225}{2}\right)\right] / 50}$$

$$= \left(\frac{50}{275}\right) = \underline{0.18}$$

In case (b), similarly :–

Power in sidebands $= \left[\left(50/2\right) / 50\right]$

Power in total $= \left[\left(50/2\right) / 50\right]$

$\therefore \underline{X = 1}$

In case (C), and for frequencies less than (1KHz + 300)

$\Rightarrow \underline{X = 1}$

For frequencies $f_m > 1KHz$. $\therefore$ Power in sidebands $= (25/2)/50$ (6 marks)

Total power $= (25/2)/50$

<u>So X = 1 again</u>

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(d) Where power is not a problem, (mains electricity), a number of options exist (1) full AM with carrier & both sidebands, (2) DSB which saves power – which is not an issue (3) VSB, for which a similar argument is relevant, (4) FM, for which full power is always transmitted, as also for (5) PM.

The key issue here is not so much power consumed – there is plenty on tap, but more how communication around the world is achieved. Usually this would be lower HF frequencies at reasonable power, for which SSB is usually employed. Typical frequencies would be 1.5 MHz to 30 MHz (the lower the better). DSB would reduce range (because of the extra bandwidth required, thus increasing noise), as would AM (full) which wastes power on the carrier.

(e) When mobile telephony is considered, with the factors of cost, power consumption, and the given range, a form of modulation is suggested which saves power but is easy (and therefore cheap) to implement. AM (DSB/c) is easy and cheap to implement but much power is wasted. The same goes for FM and PM. Therefore the remaining options are DSB/sc, SSB and VSB. VSB is easy to implement (from Full AM = AM (DSB/c)) but wasteful of power. DSB is very power efficient and comparatively easy to generate, and SSB is better still but not as easy as DSB to generate and detect (an extra filter is needed). Therefore, DSB would / or could be the preferred option.

(f) Where cost is crucial and power consumption etc, much less so, AM (DSB/c) is the preferred (and much used) option — 9 marks

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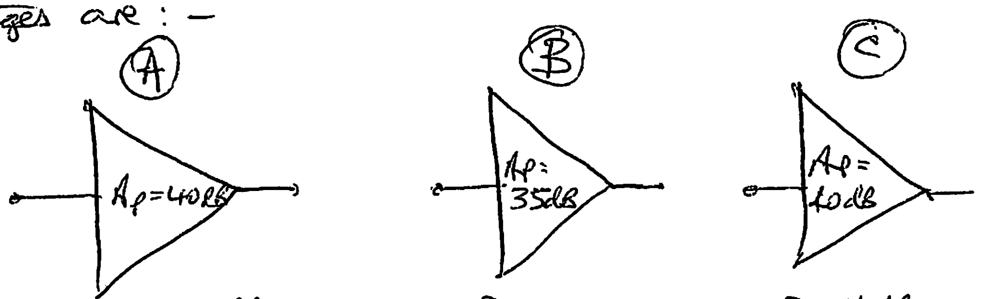
Question 2		Mark Allocation

a) The stages are :-



F = 10dB   F: 7dB.   F = 11dB.

The first step is to convert these power ratios in dB to numeric quantities :-

	A_p (dB)	A_p	F(dB)	F
A	40	10^4	10	10
B	35	$10^{3.5}$	7	5.01
C	40	10^4	11	12.6

According to the Friis formula, the noise figure as a numeric ratio is given by:

$$F = F_1 + \frac{(F_2 - 1)}{G_1} + \frac{(F_3 - 1)}{G_1 G_2}$$

Where F_1 is the noise figure of the first stage, G_1 the power gain of the first stage, etc. It is very clear that the highest gain / lowest noise figure should be first, because of the powerful effect of G_1, and subsequently for G_2 and so on. In this case, there are two stages with the highest gain — A and C. However, A has the lower noise figure. Stage ~~C~~ ~~should~~ could come second because it has higher gain than stage B, but this needs a little more examination, to be sure.

The ~~second~~ third term is : $\dfrac{(F_3-1)}{G_1 G_2}$.

If the second stage is B, then $G_2 = 10^{3.5} = 3162.3$ and $F_3 = 12.6$, so in this case (A, B, C) the ~~third~~ ~~term~~ is $\dfrac{(12.6-1)}{(10^4 \times 3162.3)} = \left(\dfrac{11.6}{3.1623\times10^7}\right)$

$= \underline{\underline{3.67\times10^{-7}}}$

If the second stage is C, then $G_2 = 10^4$ and $F_3 = 5.01$, so in this case (A, C, B), the third term is :–

$\dfrac{(5.01-1)}{(10^4\times10^4)} = \underline{\underline{4.01\times10^{-8}}}$ which is nearly 10 times smaller than for the previous option.

Therefore, the order which gives the best noise performance is: $(A \rightarrow C \rightarrow B)$. | 12 marks

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If B becomes more noisy, then its contribution at the last stage is changed :–

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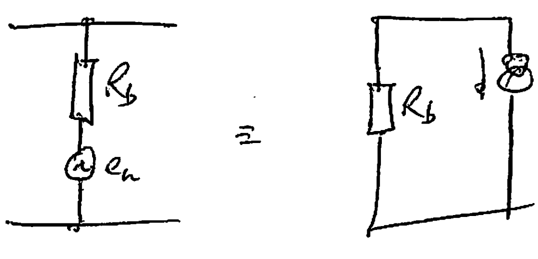
Question 2		Mark Allocation

b) The gain stays the same (not used anyway), but its noise figure ($\equiv f_3$) increases from 7dB ($\equiv 5.01$) to 10dB ($\equiv 10$), so the last term doubles (approximately). However, it will have a very small effect in practice as the overall system noise figure is greatly dominated by that of the first stage i.e. f_1 or 10. (3 marks)

c) The expressions for shot noise are: $\overline{i_n^2} = 2qI_p \cdot \Delta f$ and for thermal | Johnson noise :- $\overline{e_n^2} = 4KT.R_b.\Delta f$.

It is convenient to redraw the noise sources in (b) so that they are all current- or voltage sources, and so may be directly compared. Converting the Johnson noise to a current source :-



$$\bar{i}_J = \left(\frac{e_n}{R_b}\right) = \frac{2\sqrt{KTR_b\Delta f}}{R_b}$$

$$= 2\sqrt{\frac{KT.\cancel{R_b}.\Delta f}{R_b^{\cancel{2}}}}$$

$$= 2\sqrt{\frac{KT.\Delta f}{R_b}}$$

The issue is then of comparing i_J with i_n, the shot noise for the photodiode :-

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$\overline{i_n^2} = 2q \cdot I_p \cdot \Delta f$. and $\overline{i_J^2} = \left(\frac{4KT \cdot \Delta f}{R_b}\right)$.

$$\therefore \left(\frac{\overline{i_n^2}}{\overline{i_J^2}}\right) = \frac{(\cancel{2}q \cdot I_p \cdot \cancel{\Delta f}) \cdot R_b}{2\cancel{4}KT \cdot \cancel{\Delta f}}$$

$$= \left(\frac{q \cdot I_p \cdot R_b}{2KT}\right) = \frac{(1.6\times\cancel{10^{-19}})(50\times10^{-6})\times10^{7}}{2\times(1.38\times\cancel{10^{-23}}_{-4})\times300}$$

$$= \left(\frac{1.6\times50\times10^{-6}\times10^{7}}{600\times1.38\times10^{-4}}\right) = \underline{9662 \text{ (approx)}}$$

∴ The noise due to the photodiode (shot noise), is nearly 10,000 times the Johnson or thermal noise in the resistance. (10 marks)

(10/25)

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Question 3		Mark Allocation

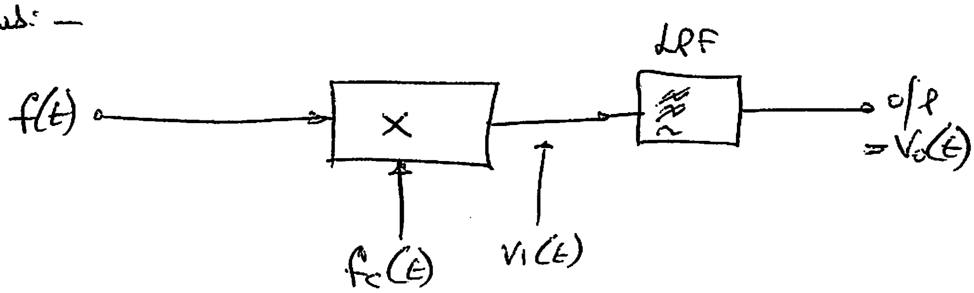
a) An AM DSB/SC wave can be represented by: –

$f(t) = A . \cos\omega_c t . \cos\omega_m t$. where ω_c = angular carrier frequency.

A = Peak amplitude

ω_m = Modulation angular frequency.

Demodulation uses a balanced demodulator/product ~~detector~~, as follows: –



$f_c(t)$ is a local carrier at the same frequency as was used to generate the DSB signal. Substituting the terms given in the question: –

$f_c(t) \equiv \varepsilon_c = 0.1 \cos\omega_1 t$.

$f(t) \equiv A \cos\omega_c t . \cos\omega_m t . = \varepsilon_c . \varepsilon_m$

$= (100)(50) \cos\omega_1 t . \cos\omega_2 t$.

where $A = 5000$ and $\omega_c \equiv \omega_1$, $\omega_m \equiv \omega_2$.

Now $V_1(t)$ [as in the above diagram] $= f(t) \times f_c(t)$

$= (5000)(0.1) . \cos^2\omega_1 t . \cos\omega_2 t$.

$= \left(\frac{500}{2}\right).(1 + \cos 2\omega_1 t).\cos\omega_2 t$.

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Hence $V_1(t) = 250.\cos w_2 t + 250.\cos 2\omega_1 t.\cos\omega_2 t.$

$$= 250.\cos w_2 t + 125\cos(2\omega_1+\omega_2)t + 125\cos(2\omega_1-\omega_2)t.$$

Thus $Vo(t)$ [after the L.P.F] removes all frequencies above the maximum modulation frequency ie:

$$Vo(t) = 250.\cos w_2 t$$
$$= 250.\cos(2\pi\times 3\times 10^3)t.$$

(A sinewave of 3KHz).

10 marks

b) The noise analysis can be treated in parallel with the signal path: $v_n(t)$ is written as:–

$v_n(t) = x(t).\cos\omega_1 t + y(t).\sin\omega_1 t$ where $x(t)$ and $y(t)$ are in phase and quadrature components of noise respectively. Their combination produces noise of ~~the~~ the same mean value as $v_n(t)$, and they have the same ~~statistics~~. They are equal in mean value.

When this noise passes through the demodulation process, the output noise is (before the low pass filter):–

$$\hat{f}(t) = \left(E_{ct} \times E_{cr}\right) + \left(v_n(t) \times E_{cr}\right)$$

$$= \left(100\cos\omega_1 t \times 0.1\cos\omega_1 t\right) + \left(0.1\cos\omega_1 t \times v_n(t)\right)$$

$$= \frac{10}{2}\left(1+\cos 2\omega_1 t\right) + \left(0.1\cos\omega_1 t \times \left[x(t).\cos\omega_1 t + y(t).\sin\omega_1 t\right]\right)$$

$$= \frac{10}{2}\left(1+\cos 2\omega_1 t\right) + \frac{0.1}{2}x(t)\left(1+\cos 2\omega_1 t\right) + \left(\frac{0.1}{2}\right)y(t)\left(\sin 2\omega_1 t\right)$$

After the low-pass filter, which allows only modulation frequencies through, all the $2\omega_1$ terms are removed, leaving:–

$$f_o(t) = \frac{1}{2}\left(10 + 0.1\,x(t)\right)$$

The output SNR, using $(\text{voltages})^2$ to represent power, is:–

$$SNR_o = \left(\frac{\text{Signal power out}}{\text{Noise power out}}\right) = \frac{\cancel{(0.5)^2}(10)^2}{\cancel{(0.5)^2}(0.1)^2 x(t)^2}$$

$$= \left[10^4 / x(t)^2\right].$$

(also cancelling the $\frac{1}{2}$ factor for converting $\hat{V}^2$ to V_{RMS}^2 in both numerator & denominator)

At the input to the demodulator, SNR_i

$$= \frac{(100)^2}{v_n(t)^2} = \left[10^4 / v_n(t)^2\right]$$

Now, at the input the noise power, which is proportional to $v_n(t)^2$, is equally split between $x(t)^2$ and $y(t)^2$, so $v_n(t)^2 = 2x(t)^2$

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$$\therefore \left(\frac{SNR_o}{SNR_i}\right) = \frac{\left(10^4 / x(t)^2\right)}{\left(10^4 / 2(x(t))^2\right)} = \underline{\underline{2}}.$$

Therefore, there is an improvement of 2 times in the SNR because of coherent detection. (11 marks)

c) An AM detector responds to amplitude aspects only, so any noise affecting the phase or frequency is ignored by such a detector.

In contrast, an FM detector responds only to frequency (or phase) variations, so noise which affects the signal amplitude (assuming above signal threshold condition) is ignored by such a detector. (4 marks)

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