ES3C50

THE UNIVERSITY OF WARWICK

Third Year Examinations : Summer 2007

SIGNAL PROCESSING

SECTION A

SECTION B

Candidates should answer FOUR QUESTIONS, TWO from SECTION A and TWO from SECTION B.

Time Allowed : 3 hours.

Only calculators that conform to the list of models approved by the School of Engineering may be used in this examination. The Engineering Databook and standard graph paper will be provided.

Read carefully the instructions on the answer book and make sure that the particulars required are entered on each answer book.

USE A SEPARATE ANSWER BOOK FOR EACH SECTION

SECTION A

1. **ANSWER ALL PARTS**

(a) The impulse response of a system is $h(t) = e^{-2t} \sin 2t$. Determine the Laplace transfer function of the system and hence its poles. Is the system stable? (8 marks)

(b) Show that the transfer function of the filter in Figure 1 is:

$$\frac{V_o(s)}{V_i(s)} = \frac{K}{1 + s\tau}$$

where $K = -\frac{R_2}{R_1}$, $\tau = CR_2$ and assume a perfect operational amplifier. (8 marks)

Determine the pole of the transfer function and sketch the pole-zero plot. Use the pole-zero plot to sketch the magnitude frequency response of the filter. Hence state whether the response corresponds to a low pass filter or a high pass filter. (9 marks)

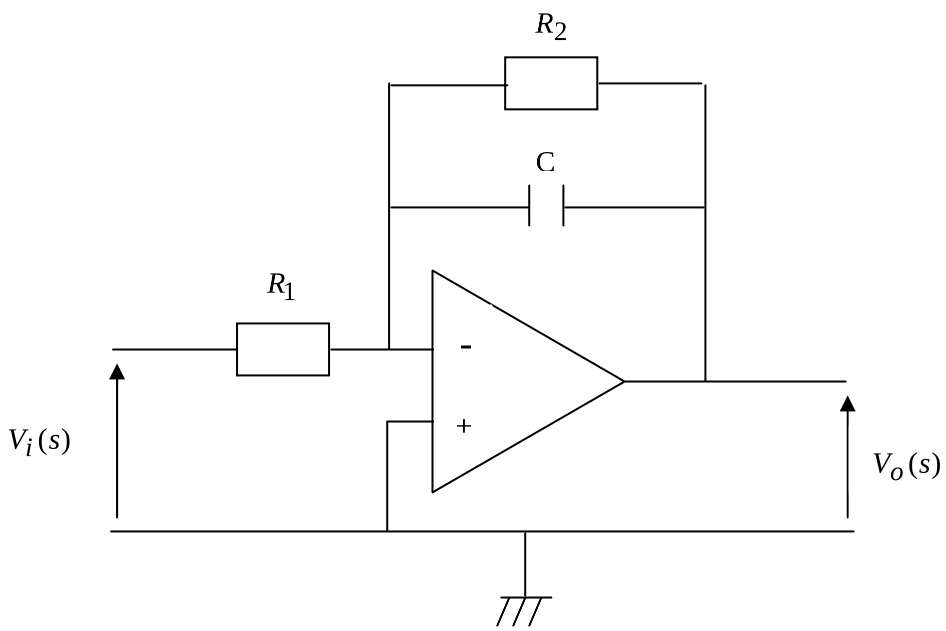

Figure 1. A first-order filter

(25 marks total)

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2. Write down the ZT transfer function of a general second-order recursive filter in terms of the filter coefficients a(0), a(1), a(2), b(1) and b(2). The ZT transfer function of an Infinite Impulse Response (IIR) filter is:

$$H(z) = \frac{\frac{1}{8} - \frac{1}{4}z^{-1} + \frac{1}{4}z^{-2}}{1 + \frac{1}{2}z^{-1} + \frac{1}{8}z^{-2}}$$

Hence state the filter coefficients of the given IIR filter. (7 marks)

Sketch the Z-plane pole-zero plot of the IIR filter. (8 marks)

Use the plot to sketch the magnitude frequency response of the filter, indicating the response at angular frequency ω of 0, $\frac{\pi}{2}$ and π. Hence state whether the IIR filter is lowpass, highpass or bandpass. (10 marks)

(25 marks total)

Continued ….

3. Derive the difference equation of the second-order system shown in Figure 2. Assuming quiescent conditions, use the difference equation and a tabular method to determine the output sequence $\{y(n)\}$ of the system, to 1 decimal place, when it is subjected to the following input sequence:

$$\{x(n)\} = \{1,0,0,0,0,0,0,0\}.$$

What can you say about the system stability from the output $\{y(n)\}$? (13 marks)

Derive the Z-transform transfer function of the system and hence determine its poles and zeros. State how the poles and/or zeros confirm the system stability that you have determined from the output $\{y(n)\}$. (12 marks)

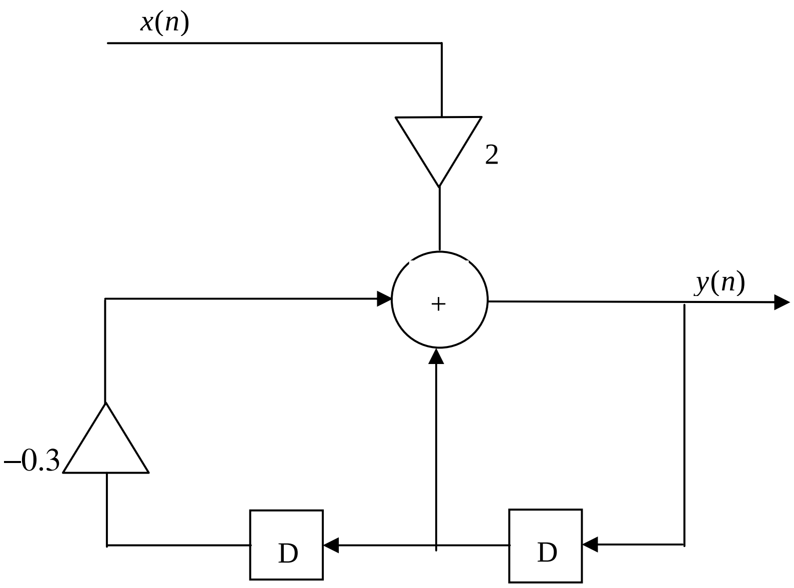

Figure 2. A second-order system

(25 marks total)

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SECTION B

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4. Sketch the block diagram of a second-order Finite Impulse Response (FIR) filter and derive an expression for the output in terms of the input sequence and filter weights. (3 marks)

It is required to design a FIR filter to approximate the ideal high-pass characteristic with a cut-off frequency f_c. The folding frequency of the digital filter is to be $4f_c$. By Fourier series show that the ideal filter weights are given by

$$g_n = 0.75 \quad , \quad n = 0$$

$$g_n = -\frac{1}{n\pi}\sin\frac{n\pi}{4}, \quad n = \pm 1, \pm 2, \pm 3, \ldots$$

(7 marks)

A Hamming window with $N = 5$ is applied to the filter weights. Determine the resulting filter weights, accurate to three decimal places, for $n = 0, \pm 1, \pm 2, \pm 3, \pm 4$, and ± 5. For the Hamming window, weight

$$w_n = 0.54 + 0.46 \cos\frac{2\pi n}{2N+1} \quad , \quad -N \leq n \leq N$$

$$w_n = 0 \quad , \quad \text{all other values of } n.$$

(4 marks)

Find the magnitude of the gain of the filter at D.C. and at the folding frequency. (3 marks)

Write out the causal form of the filter in difference equation form, expressing the present value of the output in terms of past and present values of the input. (2 marks)

Show how a matched filter can be formed from an FIR filter, and how it can perform correlation between a known and a received signal. (6 marks)

(25 marks total)

Continued ....

5. **ANSWER BOTH PARTS**

(a) A random, discrete-interval binary signal, *x(t)*, can take the level 1 with a probability of 0.6, and 0 with a probability of 0.4. Show that its autocorrelation function is:

$$R_{xx}(\tau) = 0.6 - 0.24\left(\frac{|\tau|}{\lambda}\right) \qquad 0 \le |\tau| \le \lambda$$

$$= 0.36 \qquad |\tau| \ge \lambda,$$

where λ is the clock pulse interval of the signal. (12 marks)

(b) Analogue to digital converters (A/D) introduce quantisation noise, e_n , to the input signal. Assuming e_n depends on the number of bits, *b*, in the A/D output, find expressions for:

(i) The quantisation interval *q* in terms of *b*.

(ii) The range of the quantisation error e_n in terms of *b*.

(iii) The quantised signal $U_q(k)$ in terms of a noise-free signal $U(k)$ and e_n .

(iv) The variance of e_n in terms of *q*.

(v) The signal to noise ratio *SNR* of the A/D in terms of the noise-free signal power, P_u , and the noisy signal power, P_n. (6 marks)

Assuming a sinusoidal input to the A/D with a peak voltage that maps onto the full range of the A/D's output, show that

$$SNR = 6.02b + 1.76 \text{ dB}.$$

If the quantisation noise power is to be at 75dB below the signal power level, calculate the required number of bits for the A/D converter. (7 marks)

(25 marks total)

Continued **….**

6. **ANSWER ALL PARTS**

The non-recursive filter (FIR) of Figure 3 has time delays, T, of one clock period. The input sequence applied is

$$x[k] = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}.$$

(a) Calculate values for y[0], y[1], y[2], y[3] and y[4]. Find an equation for y[k] in terms of present and past inputs of x[k]. (5 marks)

(b) Find the Discrete Time Fourier Transform (DTFT) of y[k] and sketch its magnitude response against ω. You can assume that T=1. The DTFT is defined as follows:

$$Y(\omega) = \sum_{k=-\infty}^{\infty} y[k]\exp(-jk\omega T).$$

Comment on the characteristics of the filter from observations of your sketch. Justify your reasoning. (10 marks)

(c) For an input data window width of 16, calculate the Discrete Fourier transform of the first 16 samples of the output. Sketch $|Y[m]|$ and compare with $|Y(\omega)|$ in part (b). The Discrete Fourier transform is defined as follows: $Y[m] = \sum_{n=0}^{N-1} y[n]\exp(-j2\pi mn/N)$. (10 marks)

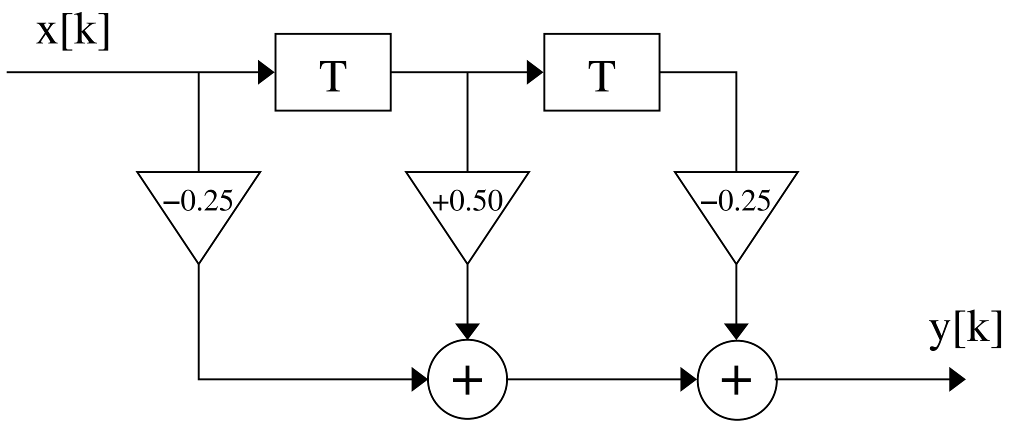

Figure 3. A FIR Filter

(25 marks total)

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