

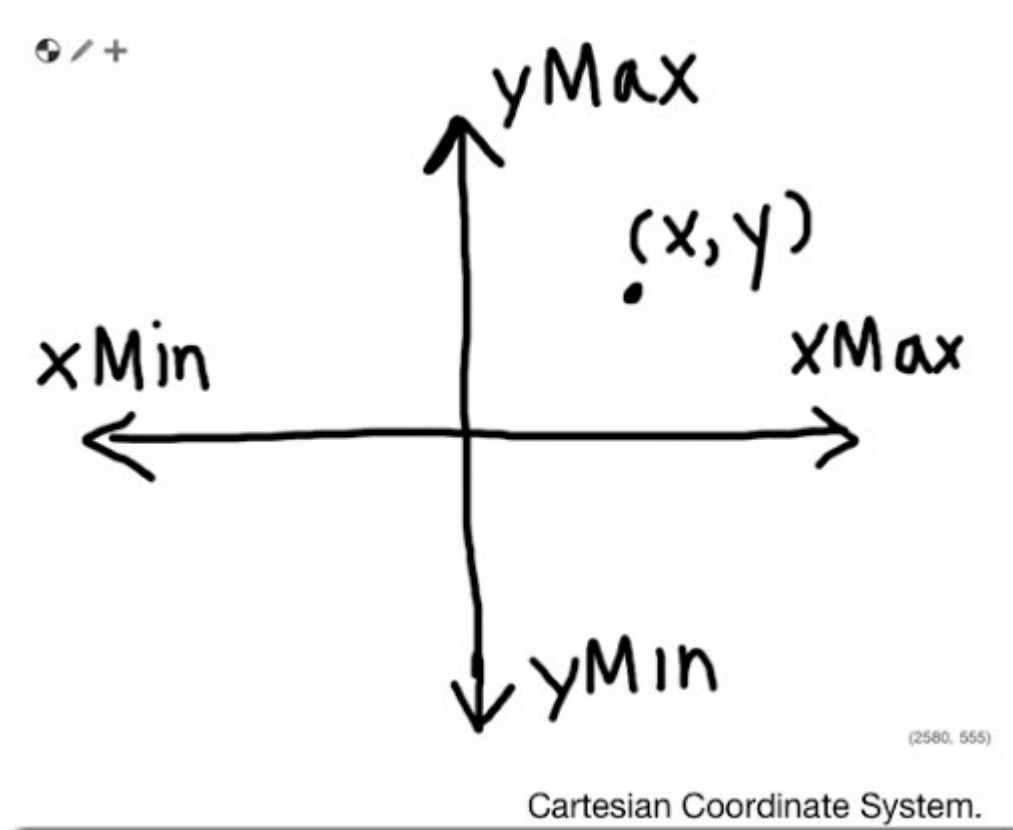
**Thursday, September 13, 2012**

## **Cartesian Coordinates to Pixel (Screen) Coordinates**

### **Converting Cartesian Coordinates to Screen Coordinates**

When working with computer or calculator graphics, sometimes we have to work with screen coordinates. The screen coordinate system has the following:

1. The upper corner pixel (point) is (0,0).
2. The lower corner pixel is (A, B).
3. The x axis increases in the right direction.
4. The y axis increases in the down direction, the opposite direction of the Cartesian plane.





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representing the bottom edge of the screen you are working with.

In order to convert a point  $(x, y)$  in the Cartesian coordinates to point  $(x_p, y_p)$  in Screen coordinates, first observe the following:

1. Picture the plane you are working with as a screen. If you are working with a calculator or computer, this is fairly easy.
2. Let  $X_{\min}$  be the least-valued  $x$  of the screen (left edge) and  $X_{\max}$  be the most-valued  $x$  (right edge). Similarly, let  $Y_{\min}$  be the least-valued  $y$  of the screen (bottom) and  $Y_{\max}$  be the most-valued  $y$  of the screen (top).

You can check with the window settings on a graphing calculator to verify  $X_{\max}$ ,  $X_{\min}$ ,  $Y_{\max}$ , and  $Y_{\min}$ .

For the Hewlett Packard 28 series, 48 series and 50g, the variables  $XRNG$  and  $YRNG$  list the screen's dimensions,  $\{X_{\min}, X_{\max}\}$  for  $XRNG$  and  $\{Y_{\min}, Y_{\max}\}$  for  $YRNG$ .

To transform Cartesian coordinates to screen coordinates, we can use transformation matrices for scaling and translation. The general form

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x + x_s \\ y + y_s \end{bmatrix}$$

show the transformation from coordinates  $(x, y)$  to  $(x', y')$  where:

$s_x$  = scale of the new  $x$  axis. If  $s_x < 0$ , the  $x$  point is reflected with respect to the  $x$  axis.

$s_y$  = scale of the new y axis. If  $s_y < 0$ , the y point is reflected with respect to the y axis. In going from Cartesian coordinates to screen coordinates,  $s_y < 0$ .

$x_s$  = Is the translation (shift to the new center) in the x-direction. If  $x_s > 0$ , the new center is to the right of the original center. Similarly, if  $x_s < 0$ , the new center is to the left of the original center.

$y_s$  = Is the translation (shift to the new center) in the y-direction. If  $y_s > 0$ , the new center is above the original center. Similarly, if  $y_s < 0$ , the new center is below the original center.

For the transformation from Cartesian coordinates to screen coordinates:

$$s_x = A / (X_{\max} - X_{\min})$$

$$s_y = -B / (Y_{\max} - Y_{\min})$$

Since the new center will be at  $(X_{\min}, Y_{\max})$ :

$$x_s = -X_{\min}$$

$$y_s = -Y_{\max}$$

Therefore by matrix multiplication, with  $x' = x_p$  and  $y' = y_p$ :

$$x_p = (x - X_{\min}) * A / (X_{\max} - X_{\min})$$

$$y_p = (y - Y_{\max}) * -B / (Y_{\max} - Y_{\min})$$

On the HP 48 series and 50g, the translation from Cartesian to screen coordinates can be accomplished with the  $C \rightarrow PX$  function.

Example:

Using the following window with settings  $X_{\min} = -5$ ,  $X_{\max} = 5$ ,  $Y_{\min} = -4$ , and  $Y_{\max} = 4$ , transform the Cartesian coordinate  $(1, 0)$  to screen coordinates. The screen has pixel size  $130 \times 79$ .

In this case,  $A = 130$  and  $B = 79$ .

Then:

$$x_p = 130 / (5 - (-5)) * (1 - (-5)) = 78$$

$$y_p = -79 / (4 - (-4)) * (0 - 4) = 39.5$$

The screen coordinate of  $(1, 0)$  is  $\{78, 39.5\}$ .