As a first-order modeling approach, the composite beam is considered to be isotropic with homogeneous properties, where the issues of anisotropic and heterogeneous composite material properties will be addressed in future iterations of the current model. Although physical composite beams are generally anisotropic in nature, the primary goal of the current modeling is to explore the effect of CNT damping, thus the study is conducted using an isotropic material approximation to elucidate the principal effects and trends produced by CNT damping in the rotating structures. The governing physics of the solid rotating beam are described by the conservation of momentum equation [31], written as

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$$\rho\frac{\partial^2 \boldsymbol{u}}{\partial t^2} - \nabla\cdot\left(\frac{EI}{A}\nabla\boldsymbol{u}\right) - (\boldsymbol{F_v} + \boldsymbol{F_c} + \boldsymbol{F_r}) = 0 \tag{1}$$

where ρ, E, I, and A are the density, modulus, moment of inertia, and cross-sectional area of the specific geometry of study, respectively, $\boldsymbol{u}$ is the displacement vector of the structure, and $\boldsymbol{F_v}$, $\boldsymbol{F_c}$, and $\boldsymbol{F_r}$ are the viscous, structural coulomb, and rotational body force vectors, respectively. The viscous damping model is applied to describe the composite material damping without CNTs, and the effect of the matrix-embedded CNTs is described by the structural coulomb friction model based on the assumption of stick-slip action between the CNT-resin interface [22,32]. Using a first mode assumption, the viscous and coulomb friction damping models can be expressed as body forces [33,34], written, respectively, as:

$$\boldsymbol{F_v} = \delta_v\rho\omega_1\frac{\partial\boldsymbol{u}}{\partial t} \tag{2}$$

$$\boldsymbol{F_c} = \delta_c\rho\omega_1^2|\boldsymbol{u}|sgn\left(\frac{\partial\boldsymbol{u}}{\partial t}\right) \tag{3}$$

where the subscripts v and c represent viscous and coulomb damping, respectively, δ is the material loss factor, ω_1 is the first natural frequency of the structure, and $sgn()$ is the signum function which returns the sign of its argument. The values for the material loss factors in Eqs. 2 and 3 are obtained via experimental measurements of CNT-infused, carbon-fiber reinforced composite beams with a fiber volume fraction of 0.58 using dynamic mechanical analysis (DMA) in the dual cantilever mode at an excitation frequency of 1 Hz as presented in [23], expressed as

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$$\delta_v = 0.0359\varepsilon^{0.08} \tag{4}$$

$$\delta_c = 0.0359\left[(\alpha^{0.172}+1)\varepsilon^{(0.0492+0.0308e^{-1.11\alpha})} - \varepsilon^{(0.08)}\right] \tag{5}$$

where ε is the material strain and α is the CNT weight percentage loading within the composite matrix. The expressions in Eqs. 4 and 5 are empirical equations based on the experimentally measured damping data, and the viscous loss factor expression, δ_v, is found using the damping measurements of the fiber-reinforced composite beams without CNTs. As CNTs are incorporated into the composite material, the additional damping is attributed to the stick-slip

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phenomena at the CNT-matrix interface discussed above and in [17,20,22], and therefore the coulomb material loss factor expression, δ_c, is derived as the difference between the measured damping in the composite using matrix-embedded CNTs and the baseline damping expression without CNTs. These two equations are stated in [23] to be valid for material strain values ranging from 0 to 0.05 percent and CNT weight percentage loadings from zero to two weight percent, which fall within the range of parameters used in the current numerical simulations. It is evident from inspection of Eqs. 4 and 5 that each damping function increases monotonically with increasing strain, and that the predicted damping in Eq. 5 is zero when $\alpha = 0$ and increases at higher CNT weight percentage values. Together, Eqs. 2–5 describe the strain-dependent damping loads which affect the beam during the transient simulation cases and allow the effect of various CNT weight percentage loadings to be explored on the resulting beam dynamics.

To account for the effects of rotation on the composite structure, the geometry is modeled in a rotating frame of reference and results in a body load vector expressed in general as

$$\boldsymbol{F_r} = \rho\left(\Omega\boldsymbol{e}\times(\Omega\boldsymbol{e}\times\boldsymbol{u}) + \frac{\partial\Omega}{\partial t}\boldsymbol{e}\times\boldsymbol{u} + 2\Omega\boldsymbol{e}\times\frac{\partial\boldsymbol{u}}{\partial t}\right) \tag{6}$$

where the subscript r denotes rotation and $\boldsymbol{e}$ is a unit-length vector describing the axis of rotation. In the current simulation, each geometry is considered to rotate around the z-axis in a counter-clockwise fashion when looking in the negative z-direction from above the system, and therefore $\boldsymbol{e} = (0,0,1)$. The first term in Eq. 6 represents the centrifugal force, the second term is the Euler force arising from the angular acceleration of the system, and the last term is referred to as the Coriolis force. To complete the necessary modeling requirements, the rotating structure is considered to be cantilevered at its base at a fixed distance from the axis of rotation, R, representing the radius of a centrally rotating hub, as illustrated in Fig. 1a. These boundary conditions are given by

$$\boldsymbol{u}(t, x = R, y, z) = 0 \tag{7}$$

$$\frac{\partial\boldsymbol{u}}{\partial x}(t, x = R, y, z) = \frac{\partial\boldsymbol{u}}{\partial y}(t, x = R, y, z) = \frac{\partial\boldsymbol{u}}{\partial z}(t, x = R, y, z) = 0 \tag{8}$$

and the remaining surfaces of the geometry are specified as free boundaries. In addition, each structure is considered to be initially at rest, written as:

$$\boldsymbol{u}(t = 0, x, y, z) = 0 \tag{9}$$

$$\frac{\partial\boldsymbol{u}}{\partial t}(t = 0, x, y, z) = 0 \tag{10}$$

where Eqs. 1–10 provide a full mathematical description of the physics of the rotating composite beam.

Three different rotating composite structures are studied separately according to the model developed above, and the various geometries and associated meshes are depicted in Fig. 2: (a) a

10b). In Fig. 10c, the contours for the wind turbine geometry show that the decrease in u^* is greatest at 40% (solid line) for a geometric thickness ratio greater than approximately 0.8 and high values of Ω_0. In addition, the settling times are improved by up to 60% (dashed line) at high values of a^* and moderate to high angular speeds. For example, the design Point D in Fig. 10c at $\Omega_0 \approx 70\ RPM$ and $a^* \approx 1.5$ corresponds to a decrease in the maximum amplitude of 40% (solid line) and a decrease in the settling time of 60% (dashed line) by the addition of 2% CNT.

The trends shown in Fig. 10 demonstrate that the addition of two weight percent CNTs to the composite matrix of each geometry has a significant impact on the structure's performance, decreasing the maximum tip displacement by 10–50% and the settling time by 40–72% across all simulation cases. In general, the t_s^* values show a minor variation across the parameter sweep for each different structure, changing by about 4%, 8%, and 20% for the simple beam (Fig. 10a), helicopter rotor (Fig. 10b), and wind turbine blade (Fig. 10c), respectively, between the different simulation cases. In contrast, the performance improvement in u^* varies from 50 to 10 percent for the simple beam and helicopter rotor and from 40 to 10 percent for the wind turbine blade across the same parameter space, indicating that the maximum tip displacement has a greater dependence on the CNT weight percentage loading over the studied ranges of Ω_0 and a^*. Fig. 10 may be applied as a first-order design tool for developing operating ranges and geometric parameters of rotating composite structures with material damping augmentation from matrix-embedded CNTs.

Rev. 1 Com. 3 Although the current study has demonstrated the benefits of increased damping in rotating composite structures using matrix-embedded CNTs, an increase in internal damping may also cause rotor instability above certain critical speeds, as discussed in [44–46]. These studies have demonstrated that rotor instability typically occurs as the structure is rotated at angular speeds greater than the first natural frequency of the rotor. In the current work, each of the various structures have been rotated at angular speeds which do not exceed the first natural frequency of the rotor when accounting for spin-stiffening effects, and therefore the problem of rotor instability should not affect the current results. However, this issue should be carefully considered in the design of physical composite rotors with matrix-embedded CNTs to avoid instability problems at high angular speeds. In future studies, the current model may be extended by including the effects of anisotropic and heterogeneous material properties to better simulate the composite material effects on the structural dynamics of each geometry and add further fidelity to the current model. (Rev. 1 Com. 2) In addition, higher modes of vibration and the associated modal damping parameters may be included in the governing equations to improve the accuracy of the current numerical model, and the effects of higher CNT weight percentage loadings within the composite material may also be considered.