



hole concentrations when  $E_t$  coincides with the Fermi level, and  $n_i$  is the intrinsic carrier concentration.

For the plot of Fig. 1,  $U(x)$  was determined by imposing several conditions that simplify the analysis and lead to a maximum value of  $m_{DR}$ . (As described in Section 3, some of the theoretical models are restricted to these “special-case” conditions). The conditions are that the junction voltage is sufficiently large (0.3 V) to ensure that  $n(x)p(x) \gg n_i^2$ ;  $E_t$  is equal to the intrinsic Fermi level  $E_i$  throughout the depletion region, giving  $n_1(x) = p_1(x) = 0$ ; and the minority carrier lifetime coefficients are equal and uniform throughout the depletion region, giving  $\tau_{n0}(x) = \tau_{p0}(x) = \tau_0$ . With these restrictions, Eq. 1 simplifies to

$$U(x) = \frac{n(x)p(x)}{\tau_0[n(x) + p(x)]}, \quad (2)$$

from which it is evident that  $U(x)$  is strongly peaked and centered at the point where  $n(x) = p(x)$ , defined here to be  $x = 0$  (Fig. 1).

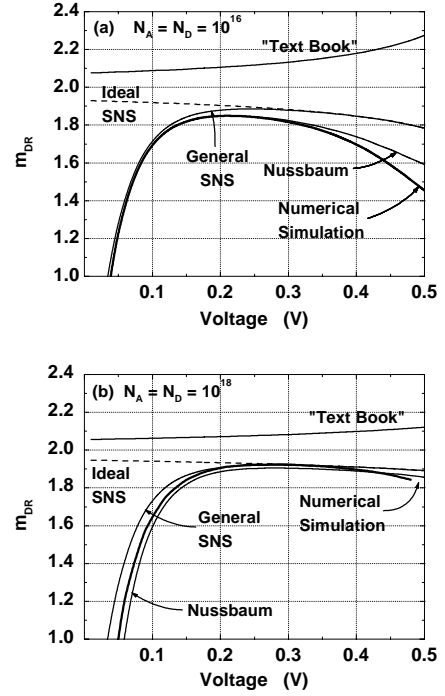
An estimate for  $m_{DR}$  can be attained from Eq. 2 if it is assumed that the quasi-Fermi levels are constant across the depletion region and separated by  $V_j$ , that is,  $pn = n_i^2 \exp(qV_j/kT)$  [4]. Hence, at  $x = 0$ ,  $n(0) = p(0) = n_i \exp(qV_j/2kT)$ , and Eq. 2 simplifies further to  $U(0) = n_i \exp(qV_j/2kT)/\tau_0$ . Thus, with the special-case conditions,  $U(x)$  is a maximum at  $x = 0$  and is proportional to  $\exp(qV_j/2kT)$ . If the depletion-region recombination current  $J_{DR}$  were due solely to the recombination that occurs at  $x = 0$ ,  $m_{DR}$  would equal 2.

But recombination from other locations in the depletion region also contributes to  $J_{DR}$ . Far from the centre of the depletion region,  $U(x)$  becomes like bulk SRH recombination and is proportional to  $\exp(qV_j/kT)$ . (For example, for  $p(x) \gg n(x)$ , Eq. 2 is simplified to  $U(x) = n(x)/\tau_0$ , and since as a majority carrier,  $p(x)$  depends weakly on  $V_j$ ,  $n(x)$  and  $U(x)$  are proportional to  $\exp(qV_j/kT)$ .) Thus, if  $J_{DR}$  were due solely to the recombination that occurs at large  $|x|$ ,  $m_{DR}$  would equal 1.

From the above discussion, it is evident that as  $|x|$  increases,  $U(x)$  changes from being proportional to  $\exp(qV_j/2kT)$  to being proportional to  $\exp(qV_j/kT)$ . This change occurs due to the increasing difference between  $n(x)$  and  $p(x)$ . Since  $J_{DR}$  is the integration of  $U(x)$  across the width of the depletion region,  $m_{DR}$  takes a value between 1 and 2.

Further to this discussion on  $m_{DR}$ , it is significant that the above discussion is based on the special-case conditions, which cause  $U(x)$  to be strongly peaked at  $x = 0$ . This gives rise to a larger value of  $m_{DR}$  (but that is still less than 2). In almost all cases, variations from these conditions lead to a reduction in the value of  $m_{DR}$ . Such variation include when  $E_t \neq E_i$  [1][2], and when there is more than one trap level [2][3]. The value of  $m_{DR}$  is increased if  $\tau_{n0}(x)$  and  $\tau_{p0}(x)$  are smaller near  $x = 0$  than elsewhere in the depletion region, since  $U(x)$  would become more strongly peaked about  $x = 0$ . This latter situation is discussed further in Section 4.

In summary,  $m_{DR}$  takes a value between 1 and 2 but  $m_{DR} \sim 2$  only under very specific conditions that lead to the recombination being very strongly peaked about the point where  $n(x) = p(x)$ .



**Figure 2:**  $m_{DR}$  vs  $V_j$  as predicted by the theoretical models and by numerical simulation for the special-case conditions—see text.

### 3. COMPARISON OF MODELS FOR DEPLETION-REGION RECOMBINATION

There are several theoretical models of depletion-region recombination that can be used to determine a specific value of  $m_{DR}$ . Four of these models are compared in this section; they are, a simple model found in some text books (e.g., [4][5]), the ideal model of Sah, Noyce and Shockley (SNS) [1]; the general model of SNS [1]; and the model of Nussbaum [6].

Fig. 2 plots the value of  $m_{DR}$  that is predicted by each of the theoretical models, for the same special-case conditions used in Fig. 1. The numerical solution is also plotted for comparison. The figure indicates that there is some discrepancy between the models. Following a summary of the assumptions entailed in the models, the reasons for the discrepancy are described. (These and other models have also been compared with regards to  $J_{DR}$  [9]).

#### 3.1 Assumptions common to all theoretical models

To determine  $J_{DR}$ , each of the theoretical models follow a similar derivation. First it is assumed that the current flow across the junction is one-dimensional, so that  $J_{DR}$  is given by the integration of  $U(x)$  across the width of the depletion region  $W$ ,  $J_{DR} = q \int_W U(x) dx$ .

To determine  $U(x)$ , it is assumed the recombination rate follows the SRH formula, which requires that there be a single trap at an energy  $E_t$ , and that the semiconductor is non-degenerate. It is further assumed that the traps are uniformly distributed and the lifetime coefficients are constant throughout the depletion region; thus from Eq. 1,  $U(x)$  varies only with  $n(x)$  and  $p(x)$ .

$n(x)$  and  $p(x)$  are determined by assuming that

they are related to  $\psi(x)$  following Boltzmann statistics and that the electron and hole quasi-Fermi levels are constant and separated  $V_j$  [1].

Finally, by determining an expression for  $\psi(x)$ ,  $J_{DR}$  can be found. To simplify the expression for  $\psi(x)$ , all of the models assume that the junction is a step junction.

Once  $J_{DR}$  is known,  $m_{DR}$  can be determined from  $m_{DR} = \frac{q}{KT} \left( \frac{dV_j}{d \ln J_{DR}} \right)$  [3].

### 3.2 “Text book” model

A “text-book” model provides the simplest derivation of  $J_{DR}$  (e.g., [4][5]). It is restricted to the special-case conditions that lead to Eq. 2 (Section 2). The model assumes that  $U(x)$  is constant across the depletion region and equal to  $U(0)$ . The integration of  $U(x)$  across  $W$  is then trivial and given by  $J_{DR} = qWU(0)$ . It can be seen in Fig. 1, that the assumption,  $U(x) = U(0)$  leads to an overestimation of  $J_{DR}$ .

At first glance, the “text book” model appears to show that  $m_{DR} = 2$ , since  $U(0) \propto \exp(qV_j/2kT)$ . However,  $W$  is not constant with  $V_j$ ; from the depletion approximation,  $W \propto \sqrt{V_{bi} - V_j}$  [4]. It follows that  $m_{DR} = 2$  only when  $V_{bi} \gg V_j$ , but this situation does not arise for practical silicon solar cells. Thus the “text-book” model predicts  $m_{DR} > 2$ , contradicting the qualitative discussion of the preceding section. Fig. 2 shows the value of  $m_{DR}$  as a function of  $V_j$  for (a)  $V_{bi} = 0.71$  V and (b)  $V_{bi} = 0.95$  V, and indicates that the model overestimates  $m_{DR}$ .

### 3.3 Ideal SNS model

With the first paper on the subject, Sah, Noyce and Shockley (SNS) presented an idealised model that provides a good intuitive description of depletion-region recombination [1]. (Nussbaum provides an alternative derivation of the same model [6]). Like the “text book” model, the ideal SNS model is restricted to the special-case conditions that lead to Eq. 2. But rather than setting  $U(x) = U(0)$ , this model determines  $U(x)$  by assuming that the potential gradient  $\psi'(x)$  is constant throughout the depletion region and equal to  $(V_{bi} - V_j)/W$ , where  $W$  is again defined by the depletion approximation. Fig. 1 indicates that this assumption becomes increasingly less valid with increasing  $|x|$ , as  $\psi'(x)$  actually decreases and approaches zero. Consequently, the model overestimates  $J_{DR}$ , though to a lesser extent than the “text book” model.

The subsequent integration of  $U(x)$  across  $W$  yields  $J_{DR} \propto U_{\max}/\sqrt{V_{bi} - V_j}$  [1][6]. Similar to the “text-book” model, it follows that  $m_{DR} = 2$  only when  $V_{bi} \gg V_j$ ; otherwise,  $m_{DR} < 2$ . This result indicates that by taking into account the variation of  $W$  with respect to  $V_j$ , and the recombination that occurs where  $n(x)$  and  $p(x)$  are not equal,  $m_{DR}$  must take a value that is less than 2. As seen in Fig. 2, the ideal SNS model slightly overestimates  $m_{DR}$ .

### 3.4 General SNS model

SNS also provide a more general model of depletion-region recombination for symmetrical step junctions, that is not restricted to the special-case conditions described above [1]. (The model is extended by Choo to include asymmetrical step junctions [8]). Like their ideal model, their general model also makes the as-

sumption that  $\psi'(x)$  is constant, and subsequently overestimates both  $J_{DR}$  and  $m_{DR}$ .

As seen in Fig. 2, the general SNS model differs from the ideal SNS model only at low voltages. This difference arises from the simplification in the ideal SNS theory that  $p(x)n(x) \gg n_i^2$ .

### 3.5 Nussbaum model

The most complex and the most accurate of the theoretical models is that presented by Nussbaum [6][7]. Like the general SNS theory, Nussbaum’s model is not limited to the special-case conditions, but it differs from the SNS theory in two ways: firstly,  $\psi(x)$  is determined with Poisson’s equation; and secondly, the depletion approximation is not used to determine  $W$ , but rather, the limits of the integral are defined as where  $\psi'(x) = 0$ . (These integral limits can be redefined to conform with  $W$  as calculated by the depletion approximation [9].) Fig. 2 demonstrates that of the four models, the Nussbaum model most closely matches the numerical simulation.

### 3.6 Numerical simulation

The semiconductor device simulator, DESSIS [10], was also used to determine  $m_{DR}$ . DESSIS solves the fully coupled set of semiconductor differential equations, and does not require any of the afore-mentioned assumptions. Consequently, when appropriately applied, DESSIS can be used to determine the most accurate value of  $m_{DR}$ .

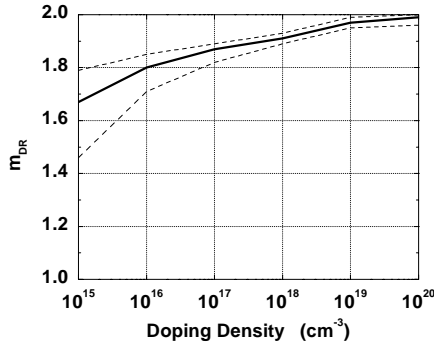
To provide a meaningful comparison with the theoretical models, the DESSIS model was constructed so that the carrier flow was essentially one-dimensional, and so that  $U(x)$  follows the SRH formula (Eq. 1), where  $n_1(x) = p_1(x) = 0$ , and  $\tau_{n0}(x) = \tau_{p0}(x) = \tau_0$ . The numerical solutions are shown in Fig. 2.

Note that the numerical solution gives the total recombination current and not  $J_{DR}$ . To determine  $J_{DR}$  (and hence  $m_{DR}$ ), the diffusion recombination current, which is proportional to  $\exp(qV_j/kT)$ , was subtracted from the total recombination current. In most cases, however, this subtraction was insignificant in the voltage range of interest (0.2–0.4 V), since at these voltages,  $J_{DR}$  was the dominant source of the recombination current.

## 4. MAXIMUM $m_{DR}$ for SILICON SOLAR CELLS

DESSIS was used to determine the maximum value of  $m_{DR}$  for silicon solar cells, for the case where  $\tau_{n0}(x)$  and  $\tau_{p0}(x)$  are uniformly distributed across the depletion region; this value occurs when the special-case conditions are applied (Section 2). As well as providing the most accurate estimate of  $m_{DR}$ , the numerical solutions can be applied to diffused junctions.

For the special-case conditions,  $m_{DR}$  is approximately constant with  $V_j$  in the range 0.2–0.4 V (Fig. 2). For silicon solar cells, this voltage range is the most appropriate for an investigation into depletion-region recombination, since at lower voltages,  $n(x)p(x) \not\gg n_i^2$ , and at higher voltages, other sources of recombination contribute significantly to the total recombination current. In the results that follow, the average value of  $m_{DR}$  in this voltage range and is shown as the thick



**Figure 3:** Numerical simulation for the maximum  $m_{DR}$  for a silicon step junction, where  $\tau_{n0}$  and  $\tau_{p0}$  are uniform across the depletion region. Bold line shows the average value and the dashed lines show the limits of  $m_{DR}$  between 0.2–0.4 V.

line in Figs. 3 and 4. The dashed lines indicate the highest and lowest value of  $m_{DR}$  in the voltage range.

Fig. 3 plots the maximum value of  $m_{DR}$  against the doping density for a symmetrical step junction, when  $\tau_{n0}$  and  $\tau_{p0}$  are constant with  $x$ . It indicates that for very heavy doping,  $m_{DR}$  approaches 2. This result arises from  $U(x)$  being extremely peaked about the  $p$ - $n$  junction. For lighter doping,  $m_{DR}$  decreases to  $\sim 1.7$ . To a good approximation, it was found that  $m_{DR}$  takes the value of the more lightly doped side of an asymmetrical step junction, and that  $m_{DR}$  is independent of  $\tau_{n0}$  and  $\tau_{p0}$ .

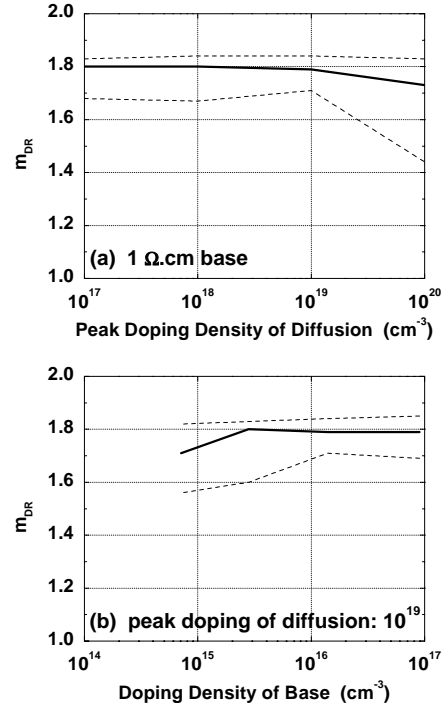
Fig. 4 plots  $m_{DR}$  against the doping density for a diffused  $n^+-p$  junction, when  $\tau_{n0}$  and  $\tau_{p0}$  are constant with  $x$ . It indicates that for all practical doping levels, the maximum value of  $m_{DR}$  is  $\sim 1.8$ . To a good approximation, it was found that  $m_{DR}$  is independent of  $\tau_{n0}$  and  $\tau_{p0}$ .

Note that the values plotted in Figs. 3 and 4 could be exceeded if  $\tau_{n0}(x)$  and  $\tau_{p0}(x)$  were smaller at  $x = 0$  than elsewhere in the depletion region (since this would cause  $U(x)$  to be more strongly peaked where  $n(x) = p(x)$ ). Such a situation might arise for a grown step junction where there is a greater number of defects at the boundary between  $n$ -type and  $p$ -type silicon. The situation is less likely to occur in diffused silicon junctions.

In summary, if a solar cell exhibits an  $m_{DR}$  that is greater than that plotted in Fig. 3 (for a given doping level), it can be concluded that either the trap density near the location where  $n(x) = p(x)$  is greater than elsewhere, or the recombination mechanism does not follow the SRH equation.

## 5. CONCLUSION

This paper examined the value of  $m_{DR}$ , with a particular emphasis on its maximum value. It was demonstrated that there is a discrepancy between several theoretical models of depletion-region recombination; the models that contain more assumptions tend to overestimate  $m_{DR}$ . Numerical simulations, which make fewer assumptions than the theoretical models, were then used to determine the maximum value of  $m_{DR}$  for both step-junction and diffused-junction silicon solar



**Figure 4:** Numerical simulation for the maximum  $m_{DR}$  for a silicon diffused junction, where  $\tau_{n0}$  and  $\tau_{p0}$  are uniform across the depletion region. Bold line shows the average value and the dashed lines show the limits of  $m_{DR}$  between 0.2–0.4 V.

cells, for the case when the trap density is uniformly distributed across the depletion region. The maximum value of  $m_{DR}$  was found to increase with doping from 1.7 to 2 for step junctions; and to be approximately 1.8 for all practical doping levels of diffused junctions.

It is concluded from this study that for silicon solar cells that exhibit a strong  $\exp(qV_j/2kT)$  recombination current, this recombination current can be explained as a consequence of depletion-region recombination only if the solar cell contains a very heavily doped step junction, or if it contains a greater density of traps at the centre of the depletion region than elsewhere in the depletion region.

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