

Melting front of ice: What do the positrons can say about it?

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Abstract. The thickness of the near-surface melting layer of ice is estimated by measuring the melting kinetics of the ice spherical balls, temperature readings of the ice surface, and the bulk ice temperature. The temperature monitoring system is programmed on an Arduino-based controller with thermocouple sensors and non-linear semiconducting thermistors. By the melting front, we consider the near-surface layer where multiple types of structural defects accumulate (owing to the latent heat absorption during melting). The melting front contains “point” defects (e.g. vacancies, interstitials) and macroscopic defects, such as dislocations, cracks, stress and strain fields, and vacancy pores. We also measured positron annihilation lifetime spectra in ice and water at temperatures above -8 °C and up to room temperature. We demonstrate that efficient accumulation of the structural defects in melting ice begins only when some temperature overheating above the melting temperature (+0.1...+0.4 °C) is applied and does not occur at negative temperatures.

Introduction

The traditional point of view on melting (as a first kind phase transition) is the following. As the solid heats up at temperatures below the melting point, the molecules increase the amplitude of their oscillations relative to their equilibrium positions. So each new portion of heat increases the temperature of the body. However, when T approaches the melting point, the incoming heat flux is spent not on increasing the amplitude of molecular vibrations, but mostly on breaking intermolecular bonds, and on an accumulation of structural defects of the crystal lattice. These defects can be not only “point” defects (like vacancies and interstitials), but the larger ones like microcracks, dislocations, local stresses, and deformations of the lattice. For example, when vacancies meet, they can form vacancy-type clusters or pores [1]. Then the question arises about the layer’s characteristic thickness where these processes occur. Since heat supply is necessary for defect formation, a temperature gradient (“overheating”) must be inside this layer, extending deep into the solid phase to a certain depth d_i . We will call this value as the thickness of the melting front, and this layer itself as the melting front [2].

The starting point in considering the melting mechanism can be attributed to the experiments carried out by M. Faraday back in 1850 [3]. However, the question of how the destruction of the molecular order on the ice surface occurs, how the resulting disorder penetrates the solid phase, and how ice finally melts remains unsolved till today.

A large number of works performed by the molecular dynamics method [4, 5] show that melting (transformation of an ordered solid phase into a disordered liquid) occurs sequentially layer by layer. Thus, it is concluded that the thickness of the melting front is very small – of a molecular scale. However, it should be noted that the number of molecules representing an ordered lattice in the calculation

cell, where the melting process is played out on a computer, is only a few thousand. Periodic boundary conditions are also used at the cell edge. This is not enough to reproduce the real picture of melting and take into account the appearance of macroscopic defects such as stress and strain fields, dislocations, microcracks, and vacancy pores. It turns out impossible to include grain boundary melting, which is responsible for the premelting phenomenon. Therefore, it is necessary to consider conclusions obtained with the help of molecular dynamics calculations with much caution.

Note that in the classical Stefan problem of phase transition (the phase change problem or the problem with free boundary condition) [6] it is also implied that the thickness of the melting front (where the phase transition is currently occurring, molecular bonds are broken and latent heat of melting is absorbed) is infinitely thin (of a molecular scale).

In part, discussions about the structure of defrosted (thawed) water shed light on the mechanism of crystallization and melting. Some researchers insist on the existence of ice-like structures in it. In particular, small-angle X-ray scattering experiments [7] indicate that the size of these structures is of the order of tens of angstroms. Similar results were obtained in the positron annihilation measurements in freshly defrosted water [8]. It was shown that the lifetime of these small ice structures is about several hours after ice melting and keeping thawed water at a temperature slightly above the melting point (1-2 °C). Such a manifestation of structural memory of thawed water is explained as follows. When a solidification occurs, a dendritic skeleton is formed therein first. Then “amorphous” water freezes in between the branches of the dendrites. The melting process proceeds in reverse order: first the inter-dendritic regions are melting, and then the branches of the dendritic skeleton are “breaking off”, forming those tiny ice cubes. So one obtains a “broth with ice cubes”, which are melting later on in thawed water [9].

Investigations of ice with the help of positron annihilation spectroscopy have been carried out for many years. First of all, we must mention the contribution of the Danish group (Riso National Laboratory, Danmark; www.risoe.dtu.dk) headed by Ole Mogensen and then by Morten Eldrup [10, 11]. Extensive studies of the melting process in different media were done in the positron laboratory at Lublin University (T. Goworek et al.). They particularly studied thermal bursts in ice in the e^+ blobs (i.e. in the terminal part of the e^+ track) [12].

Unlike previous research works, where ice and water were studied under thermodynamically equilibrium conditions, in this paper we investigate an essentially nonequilibrium object – the melting front. It does not exist at equilibrium. For its existence, it is necessary to have a heat flow entering the solid phase and absorbing therein in the form of the latent heat of melting. The energy supplied by this flow from the outside is spent on destroying the crystal lattice and formation of the structural defects. In [2] the thickness of the ice melting front in the case of the ice ball melting in the air was obtained. In this paper, we repeat the same experiment using the additional strain sensor, better thermistors (with lower intrinsic heat release), and an improved system for the suspension of the ice ball.

Another aim of our work was to observe the defect formation in the ice lattice at melting, analyzing the temperature dependence of the positron annihilation lifetime (LT) spectra at different temperatures close to the melting point. We also wanted to understand at which temperature range the latent heat of melting is absorbed by the crystal lattice – below the melting point, or if this process requires some overheating above the melting point.

Experiment

Bidistilled water with a conductivity of 2-4 $\mu\text{S}/\text{cm}$ was frozen in spherical and cylindrical plastic and aluminum molds in the freezing chamber at $T = -11 \dots -10^\circ\text{C}$. The produced ice balls were from 10 to 15 cm in diameter, and cylinders were 24-25 mm in diameter and 35 mm in height. Thermistors ($\approx 10 \text{ k}\Omega$) were frozen inside the balls and in the bulk of one of the cylinders. All temperature sensors were calibrated using the Lauda Eco Gold thermostat in the temperature range of $-7 \dots +20^\circ\text{C}$.

The ice balls were melted in the air at room temperature ($21 \dots 22^\circ\text{C}$). Using a plastic ring, mesh, and thin wires, Fig. 1, the ball was suspended above the glass into which the thawed water dropped down. The glass was placed on an electronic scale assembled on a strain gauge (CZL611; 3 kg). The strain gauge readings, i.e. the current mass of thawed water, $m(t)$, were continuously transmitted via the HX711 ADC to the Arduino Uno and then via the USB port to the computer memory. The temperature sensor readings were recorded similarly. The procedure for measuring the kinetics of accumulated thawed water is described in detail in [2].

Ice ball melting in air

In 1889 Austrian physicist Josef Stefan considered the one-dimensional problem of thermal conductivity in a two-phase system (ice in liquid water) when the opposite ends of the system are maintained at negative/positive temperatures. The position of the interphase boundary was not fixed in space, and it could move with time. This problem became classical and now is included in many textbooks on equations of mathematical physics [6]. It is assumed that the so-called “Stefan condition” is fulfilled at the ice-water interphase boundary. This condition is just the energy conservation law at the interphase boundary (heat flux balance condition that accounts for the absorption of latent heat of melting). As we have mentioned above, the temperature of the ice-water interphase boundary was assumed to be 0°C and the thickness of this boundary was considered as infinitely small. J. Stefan obtained an analytical solution to this problem expressed in terms of the error function. For our purposes, it will be enough to use an even simpler approach based just on the Stefan condition.

Following [2], measuring the ice ball melting kinetics, we also recorded the surface temperature of the melting ice by attaching to it a miniature thermistor, and the bulk temperature of the ice ball, Fig. 1. It was obtained that the difference between these two temperatures is $0.4 \pm 0.1^\circ\text{C}$. Then, equating the heat flux coming from air into the ball (through a thin film of thawed water formed on the surface of the ball) to the rate of ice melting multiplied by the latent heat of fusion, we obtain the following equation:

$$-h \frac{dm_i(t)}{dt} = \frac{\kappa_i S(t) \Delta T_i}{d_i}. \quad (1)$$

Here $m_i(t) = \frac{4\pi}{3} R^3(t) \rho_i$ is the mass of an ice ball of radius $R(t)$ at time t , ρ_i is the density of ice, $\kappa_i = 2.25 \text{ W}/(\text{m}\cdot\text{K})$ is the specific thermal conductivity of ice, $h = 332.4 \text{ J/g}$ is its latent heat of melting, d_i is the thickness of the melting front, $\Delta T_i \approx 0.4^\circ\text{C}$ is the difference between surface and bulk ice temperatures, $S(t) = 4\pi R^2(t)$ is the surface area of the ball. This equation is easily solved if we go to the variable $R(t)$, which has the meaning of the radius of the ball:

$$\frac{dR(t)}{dt} = -\frac{\kappa_i \Delta T_i}{h \rho_i d_i} \quad \rightarrow \quad R(t) = R_0 \left(1 - \frac{t}{t_0} \right), \quad t_0^{-1} = \frac{\kappa_i \Delta T_i}{h \rho_i R_0 d_i}. \quad (2)$$

Here R_0 is the radius of the original ice ball (before melting), and t_0 is the melting time of the ball. It is seen from this that the melting rate, R_0/t_0 , defined as a time derivative of the ball radius, does not

depend on the size of the ball and the time elapsed from the beginning of melting. It depends on the temperature gradient on the ice surface. Based on the above equation, it is possible to write down the final expression for the kinetics of thawed water accumulation:

$$m(t < t_0) = m_0 - m_i(t) - (\lambda_e - \lambda_c) \int_0^t S(t) dt - \lambda_e S_g t,$$

$$m(t > t_0) = m(t_0) - \lambda_e S_g \cdot (t - t_0), \quad (3)$$

where $m_0 = \frac{4\pi}{3} R_0^3 \rho_i$ is the initial mass of the ice ball. The last terms in Eq. 3 take into account the effect of evaporation of thawed water from the glass where it is accumulated, $S_g = 78 \text{ cm}^2$ is the surface area of water in the glass, $\lambda_e \approx 0.0087 \text{ g}/(\text{cm}^2 \cdot \text{h})$ is the specific evaporation rate (this value was found by fitting the dependence of the mass of thawed water vs. time at $t > t_0$, when ice melting ended; in this case the mass of water in the glass decreases linearly with time), Fig. 1.

The term $-(\lambda_e - \lambda_c) \int_0^t S(t) dt$ takes into account the process of evaporation/condensation of water from/to the surface of the ice ball during its melting. Condensation of moisture from the air is possible since the temperature of the air adjacent to the ball is below the dew point. Fitting the kinetics of thawed water accumulation at $t < t_0$, Fig. 1, using Eq. 3, we found that $t_0 = 19.4 \text{ h}$, and the difference of the evaporation and condensation rates on the surface of the melting ball is $\lambda_e - \lambda_c = 0.0014 \text{ g}/\text{cm}^2/\text{s}$, that is, these two processes almost completely compensate each other. The uncertainty of t_0 determination is 3-4%.

Using Eq. 2 it is possible to recalculate the obtained value for t_0 to the corresponding thickness d_i of the ice melting front. For the kinetics shown in Fig. 1, it results in $d_i = 3.3 \text{ mm}$. It agrees well with measurements of the melting kinetics of the ice balls of different masses and sizes. Despite large variations in melting times and initial radii of the balls, the obtained values of the melting front thickness are close to each other: $d_i = 3.2 \dots 3.6 \text{ mm}$ [2, 13].

Measurement of the lifetime positron annihilation spectra in ice and water

Since the thickness of the melting front in the ice turned out to be quite a macroscopic value, comparable to the ionization slowing down distance of fast positrons born at the Ti-44 nuclear decay, and the propagation rate of the melting front under given conditions is rather slow, about $R_0/t_0 \approx 3 \text{ mm/h}$, one can try to observe an approach of the melting front to the e^+ source through variations of the parameters of the LT spectra.

For the measurements, we used the positron annihilation lifetime spectrometer assembled on two fast PMT (XP2020q and H3378-50) with BaF_2 scintillators. Signals from the gamma detectors accompanying the positron birth and its annihilation (the start-stop coincidences) were registered by standard fast-timing ORTEC electronic units. The width of the time resolution function was FWHM=290 ps, and the spectra acquisition rate was about 30 cps. Such a low count rate was due to the ice cylinders being placed inside the thermostat's copper coil tube (the inner diameter of the coil tube is 28 mm, and the external one is 44 mm). Outside the coil was thermal insulation (a 3 mm layer of polyurethane foam), preventing direct contact with air. All these increased the distance from the e^+ source to the detectors, which reduced the count rate.

For measuring the LT spectra, the ice cylinders were loaded vertically one on another into the thermostat coil tube. A positron source was made of a Ti-44 isotope of 7 MBq activity welded between

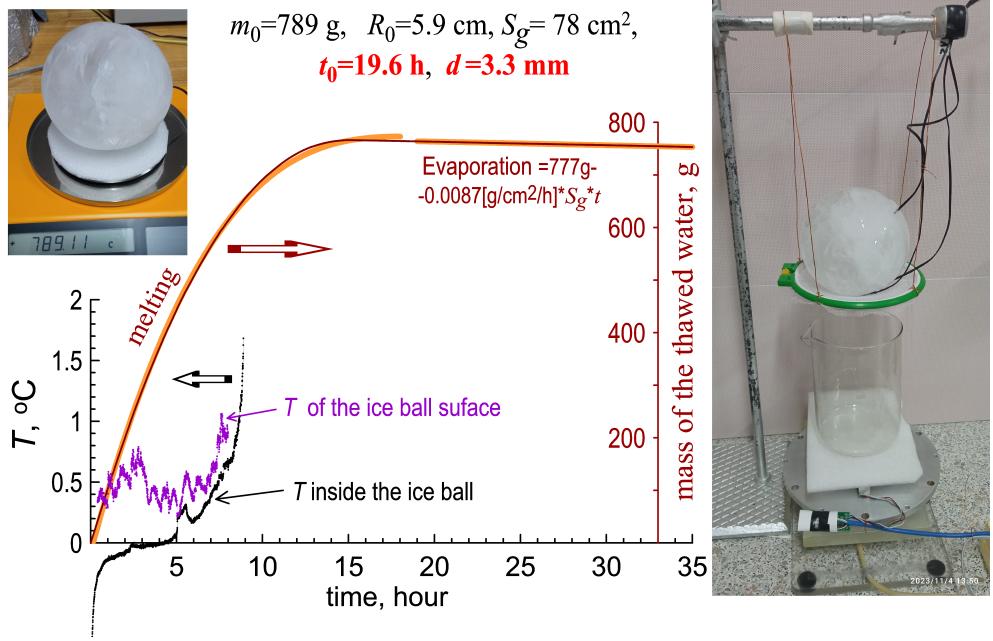


Fig. 1: Accumulation of thawed water during the melting of the ice ball (its initial mass was 789 g, and the initial radius was 5.9 cm). The thin brown line shows raw experimental data on the accumulation of thawed water, $m(t)$, in a glass mounted on an electronic scale. The orange curve shows the fitting of experimental data using Eq. 3. There was also a small time shift in $m(t)$, $t \rightarrow t - t_1$, taking into account that at $t < t_1 \approx 10$ min, the ice ball warmed up from the freezer temperature to the melting point, so the formation of thawed water has not yet begun. The thermistor readings are shown below, one sensor was frozen inside the ice ball (black curve), and the second was pressed against the ball surface with a thin plastic disk (violet curve). It can be seen that during melting, the temperature difference between the ice surface and ice bulk is about $T_i \approx 0.4\ldots0.5$ °C. The increase in T readings is associated with the loss of thermal contacts of the sensors with the ice.

two 10 μm Ti foils, 10 mm in diameter with the help of a 0.2 mm Ti washer. The Ti-44 isotope was present in the form of titanium dioxide and hydroxide. e^+ source was placed between the ice cylinders.

The contribution of positron annihilation within the e^+ source was about 13%. It was estimated in independent experiments using the reference samples of monocrystalline Si and annealed polycrystalline Al. It was found that the contribution of the e^+ annihilation within the source is reasonably approximated by two exponentials decaying in time: 93%, 0.27 ns, and 7%, 1.1 ns.

To keep the ice cylinders at the required temperatures, we used the thermostat which runs coolant through the coil tube. One thermistor measured the temperature in the bulk of the upper ice cylinder, and the second one (sealed in a thin Kapton film) was placed in the gap between the coil tube and ice cylinders.

The measurement of the LT spectra began after reaching the required temperature by ice and lasted more than a day at a given T , Fig. 3. Total number of the start-stop coincidences at each temperature was at least 3-4 million. Seven measurements were carried out in the temperature range -7.6...-0.35 C under thermodynamically equilibrium conditions. At $T = 0$ °C, the spectra were recorded one by one every 2 hours for three days. Then the LT spectra were measured when the surface of the ice cylinders was overheated by +0.1 and +0.4 °C. After that when most of the ice around the e^+ source was melted, the source sank to the bottom of the Al container.

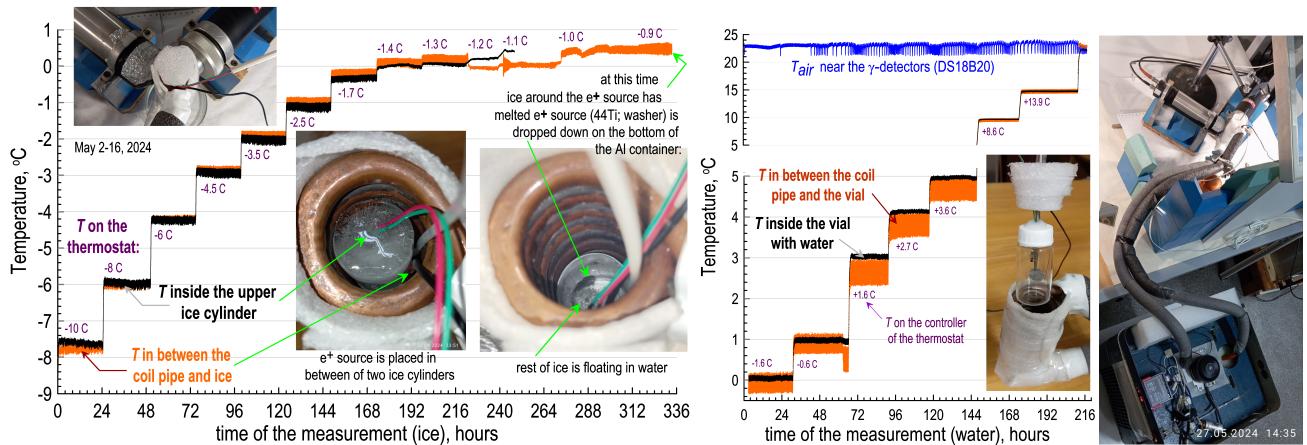


Fig. 2: On the left: the temperature inside the ice cylinder (black), and the temperature between the cylinder and the thermostat coil tube (orange). Both cylinders are placed inside the thermostat coil. At the end of the experiment, the ice surrounding the positron source melted, and the e^+ source fell to the bottom of the Al container. On the right: the temperature of the water inside the vial, where the positron source is placed (black) and the temperature between the vial and the coil tube (orange) during the measurement of the positron annihilation lifetime spectra.

After that, the measurement of the LT spectra in pure water was started at temperatures from 0.1 °C up to room temperature, Figs. 2 and 3 (on the left). A vial with water was arranged at the same place in the thermostat coil tube, where the melting of ice cylinders was studied. The water temperature was controlled by a thermistor submerged in the vial, and the second thermistor was placed between the coil tube and the vial. The measurements of the LT spectra began after both temperatures stopped to change, and lasted for more than a day.

The LT spectra in ice are well decomposed into two components. On the left in Fig. 3 the corresponding intensities and lifetimes are shown. In the middle of the figure, the intensities and lifetimes of the spectra are given at 0 °C (three days' measurement) and two overheating of the surface of the ice cylinders (+0.1 and +0.4 °C).

On the right in Fig. 3, the variations of the parameters of LT spectra in liquid water are presented. The LT spectra in liquid water were deconvoluted in a standard way into three exponents. The lifetime of the short-lived (parapositronium) component was assumed to be 160 ps following the results of variation of the R-parameter vs. an external magnetic field [14]. The long-lived component is associated with orthopositronium atoms and the short-lived one with the annihilation of “free” positrons and the internal annihilation of parapositronium.

Discussion and Summary

By definition, there are no fluxes at thermodynamic equilibrium. Therefore, at equilibrium melting does not proceed and the melting front does not exist. A two-phase system (ice-water mixture at 0 °C) will survive for an infinite time maintaining the volume fractions of the solid and liquid phases. To start melting and to observe the melting front, it is necessary to create overheating on the surface of the solid, to disturb the system from equilibrium.

A suitable object for this is ice melting. By the melting front, we mean the near-surface layer of the interphase boundary, where the heat flux enters the lattice, heats it, and becomes absorbed therein. This

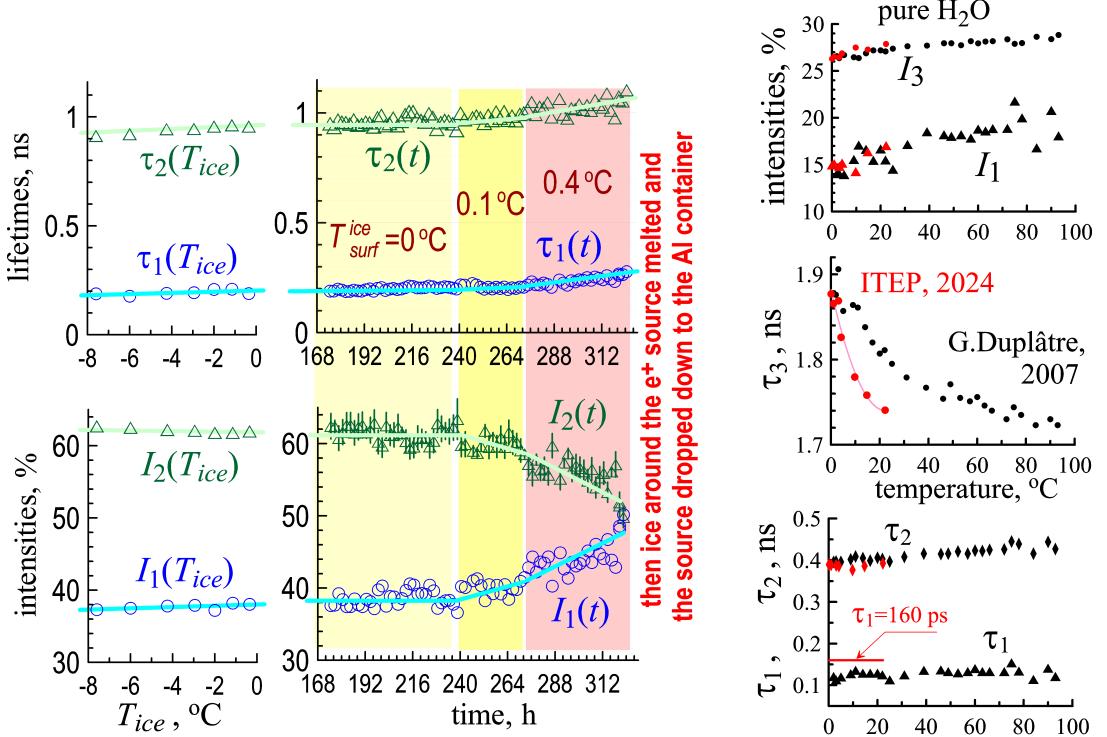


Fig. 3: Parameters of the LT spectra in ice (on the left and in the middle; 2-exponential deconvolution) and in water (on the right; 3-exponential deconvolution; red dots are intensities and lifetimes obtained in this work, data shown in black are taken from [14]).

energy is spent on the rupture of intermolecular bonds, and accumulation of structural defects in the lattice, including “macroscopic” ones such as dislocations, microcracks, stresses and strain fields, and vacancy pores. This means that the thickness of the melting front cannot be too small (on a molecular scale), but should be noticeably larger. Within this layer, the latent heat is absorbed during melting.

To our opinion, studies of melting kinetics using molecular dynamics methods at current computer capacities cannot adequately take into account the appearance of large-scale defects due to the insufficient size of the simulation cell. Therefore, we consider as incorrect the conclusion that the melting front thickness is of a molecular scale. Note that the melting front (as a finite-size object) is also skipped in the classical one-dimensional Stefan’s problem on heat propagation in a two-phase system (ice-water). But there it was done for the sake of obtaining a solution in an analytical form.

We have studied the propagation of the melting front in two cases: 1) melting of ice balls in the air at room temperature and 2) melting of ice balls submerged in cold water (at $T \lesssim 1$ °C) [13]. In the last case, the thickness of the melting front was estimated by variation of the Archimedean force during melting. However, this also required temperature measurements of the surface of the melting ice and its bulk temperature. In both these cases, the overheating turned out to be the same, namely, ≈ 0.4 °C (in the case of melting in cold water its temperature was specially selected). So the thickness of the melting front d_i in these cases is the same, about 3 mm. The rate of the melting front propagation, $-dR/dt = R_0/t_0$, depends only on the magnitude of overheating, and under these conditions, it is ≈ 3 mm/h.

Since d_i coincides in order of magnitude with the e^+ projected range, and the propagation velocity of the melting front is not too high, we tried to observe the approaching of the melting front to the

e^+ source. For this purpose, a special coil tube was made, into which two ice cylinders were placed with the e^+ source between them. An external thermostat was used to control the temperature of the ice cylinders. First, we measured the LT spectra of ice at negative temperatures, then reached exactly the melting point and continuously measured the spectra for three days. Then small overheating (+0.1; +0.4 °C) was applied and ice cylinders were melted.

At low temperatures (from -7.6 to -0.35 °C), a slight increase in the lifetime, τ_2 , of the long-lived component was observed. This is probably due to the loosening of amorphous regions of the ice, i.e. the areas between the branches of its dendritic skeleton. Just within these regions with larger free volume the positronium atom is localized and annihilated.

These areas were formed during the freezing of water. A dendritic skeleton is formed first, and then water freezes between the branches of the dendrites. During melting, everything occurs oppositely: amorphous “inter-dendritic” water melts first, and then branches of the dendritic skeleton break off. These lead to the formation of under-melted ice pieces in freshly defrosted (thawed) water, which exist there for some time (hours). It can be recorded by variations of the LT positron annihilation spectra [8].

When 170 h had passed since the beginning of the LT measurements of ice cylinders, the temperature was set to 0°C, and the spectra were measured for three days. No changes in the parameters of the spectra were observed. This looks natural since the entire system stays in equilibrium at a constant temperature, there are no heat fluxes, and the ice melting has not yet begun. Then the temperature of the ice surface was increased by +0.1 °C, melting began, but slowly enough. Nevertheless, the defect formation process started and the melting front began to propagate to the location of the e^+ source. The accumulation of defects reduces the mobility of both the thermalized positron and the track electrons. So the Ps formation probability, I_2 , decreases, and the fraction of positron annihilation in the “free” state increases. When the overheating of the ice surface was increased to 0.4 °C, the defect formation process (and melting as well) went faster. It is seen from the parameters of the LT spectra, Fig. 3.

Hence we draw the following conclusion: the intensive defect formation process (and melting) is associated with the absorption of latent heat of fusion. It begins only when the temperature of the ice surface exceeds the point of phase transition (0 °C). This can be seen by the variation of the parameters of the LT annihilation spectra before (no variation) and after 0°C (noticeable changes).

This conclusion contradicts the statements of several works that the absorption of the heat of melting by the crystal lattice occurs **below** the melting point and that overheating in the case of melting is impossible. Now we see that this is not the case. It should also be mentioned that in the case of melting, we always deal with a thermodynamically nonequilibrium situation. Overheating is necessary to pump the hidden heat into a solid phase, create structural defects therein, and destroy the crystal lattice.

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References

- [1] A.R. Ubbelohde: *Melting and crystal structure*. London: Oxford University Press (1965).
- [2] I.S. Stepanov, L.I. Budaeva, S.V. Stepanov: “Estimation of the thickness of the ice melting front based on the study of kinetics of the ice ball melting in air” Colloid Journal. V.86(3) (2024), p.448-455 <https://doi.org/10.1134/S1061933X24600179>

- [3] N. Maeno: *The Science of Ice*. Sapporo: Hokkaido University Press (1981).
- [4] W. Wei, S. Xiao and J. Ni: “Studies of ice melting using molecular dynamics”. Molecular Simulation. V.36(11) (2010), p.823-830 DOI: 10.1080/08927021003774287
- [5] M. Fitzner, G.C. Sosso, S.J. Cox and A. Michaelides: “Ice is born in low-mobility regions of supercooled liquid water”. PNAS. V.116(6) (2019), p.2009-2014 www.pnas.org/cgi/doi/10.1073/pnas.1817135116
- [6] A.N. Tichonov and A.A. Samarskii: *Equations of mathematical physics*. New York et al., USA: Pergamon Press Ltd. (1963).
- [7] C. Huang, K.T. Wikfeldt, T. Tokushima, D. Nordlund, Y. Harada, U. Bergmann, M. Niebuhr, T.M. Weiss, Y. Horikawa, M. Leetmaa, M.P. Ljungberg, O. Takahashi, A. Lenz, L. Ojamae, A.P. Lyubartsev, S. Shin, L.G.M. Pettersson and A. Nilsson: “The inhomogeneous structure of water at ambient conditions” PNAS V.106(36) (2009), p.15214-15218 www.pnas.org/cgi/doi/10.1073/pnas.0904743106
- [8] L.I. Zemskaya, O.V. Ilyukhina, M.V. Karpov and S.V. Stepanov: “Positronium annihilation in freshly defrosted thawed water”, Physics of Wave Phenomena, V.31 (2023), p.105-109 <https://doi.org/10.3103/s1541308x23020115>
- [9] B. Monserrat, J.G. Brandenburg, E.A. Engel and B. Cheng: “Liquid water contains the building blocks of diverse ice phases” Nature Communications, V.11 (2020), p.5757 <https://doi.org/10.1038/s41467-020-19606-y>
- [10] O.E. Mogensen: *Positron annihilation in chemistry*, Springer-Verlag, Berlin (1995).
- [11] Y. Nagai, M. Kakimoto, T. Hyodo, K. Fujiwara, H. Ikari, M. Eldrup and A.T. Stewart: “Temperature dependence of the momentum distribution of positronium in MgF₂, SiO₂, and H₂O”. Physical Review B., V.62(9) (2000), p.5531-5535 <https://doi.org/10.1103/PhysRevB.62.5531>
- [12] B. Zgardzinska and T. Goworek: “Search for premelting at the end of positron track in ice”, Physics Letters A V.378 (2014), p.915-917 <http://dx.doi.org/10.1016/j.physleta.2014.01.029>
- [13] I.S. Stepanov, L.I. Budaeva, O.A. Raznitsyn, M.V. Karpov and S.V. Stepanov: “What is the thickness of the ice melting front?” Nuclear Physics and Engineering, V.15(6) (2024), p.1-11 DOI: 10.56304/S2079562924060411
- [14] S.V. Stepanov, D.S. Zvezhinski, G. Duplatre, V.M. Byakov, Yu.Yu. Batskikh and P.S. Stepanov: “Incorporation of the Magnetic Quenching Effect into the Blob Model of Ps Formation. Finite Sized Ps in a Potential Well”. Materials Science Forum, Vol.666 (2011), p.109-114 doi:10.4028/www.scientific.net/MSF.666.109