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Nonlinear longitudinal strain wave interacting with point defect in metal plates

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A model of nonlinear longitudinal strain wave propagation in metal plates with quadratic nonlinearity of elastic continuum, exposed to laser impulses, is developed in view of the interaction with the fields of point defects (vacancies and interstitials). The influence of the recombination-generation processes in a defect subsystem on elastic strain wave propagation is analyzed. The existence of a nonlinear elastic shock wave of low intensity is revealed in the system and its structure is studied. The shock waves can have either an oscillatory or a monotonic profile. The estimations of the wave front depth and velocity are performed. The contributions to the linear elastic modulus, as well as to the lattice dispersion parameters are found, which are due to the interaction of the elastic strain field and the field of defects. *© 2005 American Institute of Physics*. [DOI: 10.1063/1.1861516]

I. INTRODUCTION

The prominent features of the nonlinear behavior of a solid under an intense impulse exposure (in particular, under impulse laser-beam exposure) are the appearance and propagation of nonlinear strain waves (in particular, solitons) of various nature. This phenomenon is of considerable interest, and numerous theoretical and experimental papers[1–7] are devoted to its study. In considering the evolution of nonlinear elastic waves in a crystal, the deviation of elastic lattice properties from Hooke's law is commonly considered as nonlinearity.[1] In the exposed solids, various structural imperfections of the lattice, namely, the point defects generated under external influences and resulting in a considerable strain of the medium, may be of essential importance. The strain of the medium is caused by the difference in the covalent radii of the lattice and defect atoms. The defect-strain interaction may occur both through changing the energy parameters of the subsystem of defects (the energy of defect formation and the energy of their migration) and via the appearance of diffusion current (strain-induced drift). Under some certain conditions, nonlinearities related to these interactions may become important for the propagation of nonlinear elastic perturbations in solids, and they may lead to the renormalization of lattice parameters (both linear and nonlinear elastic moduli). The presence of the point defects with a finite recombination velocity in the medium may induce the appearance of dissipative terms, which are absent in ordinary equations for nonlinear elastic waves. The wave dynamics may depend on the dispersion that is due to the finiteness of the lattice spacing[8] or to the specimen width,[9] as well as on the dispersion related to nonequilibrium defects.

The propagation of an elastic strain wave in such systems may occur in the form of shock waves. In this case, the influence of generation and recombination processes turns out to be similar to the dissipation of elastic oscillation energy in a viscoelastic medium with aftereffect and relaxation. The formation of the shock front of the acoustic wave in insulators exposed to 0.15-μs laser pulses with energy up to 5 J was experimentally observed in Ref. 10. The study of wave dynamics in view of the interaction of waves with structure defects is of certain theoretical and practical interest, in particular, in analyzing the mechanisms of anomalous mass transfer detected in laser and ion implantation of metal materials[11] and in studying the mechanical activation of components in solid-phase chemical reactions. The elastic waves propagating in a condensed medium carry the information about the distortion of their shape and velocity, energy loss, defect structure, etc. This information is necessary to determine the various parameters and structure of solids.

In this paper, the possibility of the existence of shock waves in the propagation of a nonlinear longitudinal strain wave in a crystal plate with quadratic nonlinearity of elastic continuum, exposed to laser pulses, is studied in view of the generation of point defects (vacancies and interstitials). It is shown that generation and recombination processes in the defect subsystem result in dissipative effects and, hence, in the appearance of elastic shock waves of low intensity. The renormalization of linear elastic moduli and dispersion parameters is found, which is due to the defect-strain interaction.

II. GOVERNING EQUATIONS

Consider an isotropic crystal plate with point defects induced by laser radiation. Let $n_j(x,t)$ be the volume density of point defects ($j=v$ for vacancies and $j=i$ for interstitials). In the propagation of longitudinal strain waves, the activation energy for defect formation varies in the regions of compression and tension. The renormalized energy of point defect formation may be represented as $E=E_0-\varphi\, div\vec{u}$, where $\varepsilon=div\vec{u}$ is the strain of the medium, $\vec{u}$ is the vector of medium displacement, φ is the strain potential, and E_0 is the energy of defect formation in the strain-free crystal. The variation of

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the formation energy results in a corresponding variation of the defect source function and, consequently, in the spatial redistribution of defects.[12] In addition, defects may migrate over the crystal and recombine at various centers, e.g., dislocations, interstitial impurities, etc. In the framework of the aforementioned assumptions, the nonlinear dynamic equation describing the propagation of the longitudinal displacement wave in a crystal in view of defect generation (in the one-dimensional case) may be written as

$$\frac{\partial^2 u}{\partial t^2} - c_s^2\frac{\partial^2 u}{\partial x^2} - \frac{\beta_N}{\rho}\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial x} - l^2\left(\frac{\partial^4 u}{\partial t^2\,\partial x^2} - c_\tau^2\frac{\partial^4 u}{\partial x^4}\right) = -\frac{K\Omega_j}{\rho}\frac{\partial n_j}{\partial x}. \quad (1)$$

Here, $u(x,t)$ is the displacement of the medium, $c_s=[E/\rho(1-\sigma^2)]^{1/2}$ and $c_\tau=[\mu/\rho(1-\sigma^2)]^{1/2}$ are the velocities of the longitudinal and transverse waves, respectively, ρ is the density of the medium, K is the modulus of uniform compression, μ is the rigidity modulus, and Ω_j is the dilation parameter describing the variation of the crystal volume as the result of point defect formation ($\Omega_j<0$ for $j=v$ and $\Omega_j>0$ for $j=i$). The coefficient of nonlinearity β_N and the dispersion parameter l are given by[13]

$$\beta_N = \frac{3E}{1-\sigma^2} + 3B\left[\frac{1-4\sigma+6\sigma^2}{(1-\sigma)^3}\right] + A\left[1-\left(\frac{\sigma}{1-\sigma}\right)^3\right] + C\left(\frac{1-2\sigma}{1-\sigma}\right)^3,$$

$$l^2 = \frac{h^2\sigma^2}{12(1-\sigma)^2},$$

where A, B, and C are the Landau moduli of the third order, E is the Young modulus, σ is the Poisson ratio, and h is the thickness of the plate. For most solids (metals and many of the polymers), $\beta_N<0$. There are also metals in which the deviation of the elastic lattice properties from Hooke's law is insignificant. In this case, $\beta_N>0$. In (1), we restricted our consideration to smooth strain disturbance and took into account the contributions of the elastic moduli to the spatial dispersion in the first nonvanishing approximation.

Equation (1) in the absence of elastic concentration stresses, which is called the double-dispersion equation, was studied in detail in Refs. 2–5. The generalization of this equation to the case of the presence of elastic concentration stresses in the system is performed in Ref. 12 in the framework of the Hamiltonian approach.

The distribution of point defects determining the right side of Eq. (1) depends on the strains, stresses, and, therefore, displacements in the medium. Thus, in order to describe adequately the propagation of the elastic deformation wave, we have to supplement Eq. (1) with an equation for the density of defects. If the processes of defect generation from the lattice sites and their recombination at the centers of various nature are basic to the determination of the defect behavior in time, the kinetic equation

$$\frac{\partial n_{1j}}{\partial t} = q_\varepsilon\frac{\partial u}{\partial x} - \beta_j n_{1j} \quad (2)$$

takes place for weak nonuniform density perturbations $n_{1j}=n_j-n_{j0}$ and $n_{1j}\ll n_{j0}$, where $n_{j0}=q_0\tau_j$ is the uniform stationary defect distribution. Here, q_0 is the rate of point defect generation in the absence of strain, the first term in the right-hand side of (2) corresponds to strain-induced generation ($\varepsilon=\partial u/\partial x$ is the strain of the medium), the second term allows for the defect decrease due to the recombination ($\beta_j=1/\tau_j=\rho_j D_j$ is the recombination rate at sinks, ρ_j is the sink density, D_j is the diffusion coefficient for a defect of j type, and τ_j is the relaxation time of the defects), the bulk mutual recombination of dissimilar defects is neglected, and $q_\varepsilon=q_0(K\Omega_j/kT)$ for the thermal mechanism of point defect generation.

Equations (1) and (2) make up a closed system. This system completely describes the distribution of one-dimensional elastic displacement perturbations in the solid due to nonstationary and nonuniform distributions of the point defect subsystem. The inverse effect, i.e., variation in the concentration field of defects in a solid as a result of perturbations of elastic strains, is also accounted for.

III. NONLINEAR STATIONARY WAVES

Consider self-similar solutions of the forms of $u=u(\xi)$ and $n_{1j}=n_{1j}(\xi)$, where $\xi=x-vt$, describing the longitudinal waves of elastic displacement and defect density, propagating along the x axis with a velocity of $v=$const. In this case, the system of partial differential equations [Eqs. (1) and (2)] passes into the following system of ordinary differential equations:

$$(v^2-c_s^2)\frac{d^2u}{d\xi^2} - \frac{\beta_N}{\rho}\frac{d^2u}{d\xi^2}\frac{du}{d\xi} - l^2(v^2-c_\tau^2)\frac{d^4u}{d\xi^4} = -\frac{K\Omega_j}{\rho}\frac{dn_{1j}}{d\xi}, \quad (3)$$

$$-v\frac{dn_{1j}}{d\xi} + \frac{n_{1j}}{\tau_j} = q_\varepsilon\frac{du}{d\xi}. \quad (4)$$

We use the conditions

$$u_\xi(-\infty)=\varepsilon_0, \quad u_\xi(+\infty)=0, \quad n_{1j}(\pm\infty)=0 \quad (5)$$

as boundary conditions for Eqs. (3) and (4).

The boundary conditions in (5) imply that the waves propagating in a medium take the system under consideration from a deformationless state to that with a constant deformation (ε_0). We will further restrict our consideration to a system with only one type of defect and take $n_{1j}(\xi)\equiv n_1(\xi)$, $\tau_j\equiv\tau$, and $\Omega_j=\Omega$ in (3)–(5).

The solution of the nonuniform differential equation [Eq. (4)], in view of the boundary conditions in (5), has the form

$$n_1(\xi) = \int_\xi^{+\infty} d\xi' q(\xi')\exp\left(\frac{\xi-\xi'}{\tau v}\right), \quad (6)$$

where $q(\xi)=q_\varepsilon v^{-1}du/d\xi$.

Having eliminated the defect density, using (6), we arrive at

$$(v^2-c_s^2)\frac{d^2u}{d\xi^2}-\frac{\beta_N}{\rho}\frac{d^2u}{d\xi^2}\frac{du}{d\xi}-l^2(v^2-c_\tau^2)\frac{d^4u}{d\xi^4}+\frac{K\Omega}{\rho}\frac{d}{d\xi}\int_\xi^{+\infty}\exp\left(\frac{\xi-\xi'}{\tau v}\right)q(\xi')d\xi'=0, \qquad (7)$$

describing the propagation of a nonlinear elastic displacement wave.

The equations similar to (7) are typical for the systems with strain memory (or relaxation).[1] At $l=0$ (without dispersion) and $q_\varepsilon=\beta=0$ (without defect generation), this equation has the same form as in the case of a longitudinal wave in the free space. On the basis of Eq. (7), a thorough analysis of the displacement wave propagation is possible in view of both the dispersion properties of the medium and elastic properties of the lattice and the defect subsystem. An exact analysis of this equation with arbitrary values of the parameters involved seems impossible. Further, we will consider it at small (compared to the period of the wave t_0) defect relaxation times ($\tau \ll t_0$). In this case, the integral term in (7) can be replaced by the differential one. Let us expand the function $q(\xi - z)$ by its Taylor series expansion at ξ. Retaining the first three terms in this expansion, we find

$$\frac{d}{d\xi}\int_\xi^{+\infty}d\xi' q(\xi')\exp\left(\frac{\xi-\xi'}{\tau\, v}\right)\approx q_\varepsilon\left(\tau\frac{d^2u}{d\xi^2}+\tau^2\nu\frac{d^3u}{d\xi^3}+\tau^3v^2\frac{d^4u}{d\xi^4}\right).$$

Having substituted this expression into (7), we arrive at the following equation:

$$(v^2-\tilde{c}_s^2)\frac{d^2u}{d\xi^2}-\frac{\beta_N}{\rho}\frac{d^2u}{d\xi^2}\frac{du}{d\xi}+\frac{K\Omega q_\varepsilon\tau^2 v}{\rho}\frac{d^3u}{d\xi^3}-\left[l^2(v^2-c_\tau^2)-q_\varepsilon\tau^3v^2\frac{K\Omega}{\rho}\right]\frac{d^4u}{d\xi^4}=0, \qquad (8)$$

where $\tilde{c}_s=c_s(1-K\Omega q_\varepsilon\tau/\rho c_s^2)^{1/2}$ is the velocity of sound renormalized due to the defect-strain interaction.

In Eq. (8), the term with the third derivative of the displacement (the Burgers term) describes the wave energy dissipation. The appearance of this term is clearly related to the generation and recombination processes in the defect subsystem. In addition, the finiteness of the defect recombination rate (τ^{-1}) results in an additional contribution to the nonlinear equation [Eq. (8)]. This may influence the nonlinear wave properties.

Having performed a single integration with respect to ξ and introduced the notation $\varepsilon=du/d\xi$, we find an equation coinciding with the first integral of the well-known stationary Korteweg-de Vries–Burgers equation,[14]

$$g\frac{d^2\varepsilon}{d\xi^2}-\delta\frac{d\varepsilon}{d\xi}+(\beta_N/\rho)\varepsilon^2-\alpha\varepsilon=0,$$

$$\alpha=v^2-\tilde{c}_s^2,\quad g=l^2(v^2-c_\tau^2)-K\Omega q_\varepsilon\tau^3v^2\rho^{-1},$$

$$\delta=K\Omega q_\varepsilon\tau^2 v\rho^{-1}. \qquad (9)$$

Here, the integration constant is equal to zero in view of the boundary conditions $du/d\xi|_{\xi\to\pm\infty}=0$.

Equation (9) has been repeatedly considered in the literature. In particular, the characteristic features of solitary-wave (solitons) attenuation at low dissipation were studied. Shock-wave attenuation in elastic media without dissipation was considered in the framework of the Burgers equation.[1,14,15] Equation (9) also describes the propagation of nonlinear ion-sound waves in plasma in view of the Landau damping.[16] Equation (9), with the boundary conditions

$$\varepsilon(-\infty)=\varepsilon_0,\quad \varepsilon(+\infty)=0 \qquad (10)$$

admits a solution in the form of shock waves of low intensity (step-function solution).[14]

If the dispersion is negligible, Eq. (9) yields the stationary Burgers equation

$$\delta\frac{d\varepsilon}{dx}-(\beta_N/\rho)\varepsilon^2+\alpha\varepsilon=0,$$

having a solution in the form of shock waves with a monotonic profile,

$$\varepsilon=\frac{\varepsilon_0}{2}\left(1-th\frac{x-vt}{b}\right),\quad b=\frac{2\rho\delta}{\beta_N\varepsilon_0}.$$

.

In the general case, we will qualitatively determine the pattern of the solution to Eq. (9) in view of the influence of dispersion, using the analysis performed in Ref. 14. Note that an explicit analytical solution of Eq. (9) with additional restrictions on its coefficients can be found by the method described in Ref. 5.

The dependence of wave velocity v on its amplitude ε_0 is determined by the formula

$$v^2=\tilde{c}_s^2+\varepsilon_0\beta_N\rho^{-1}. \qquad (11)$$

From the analysis of the asymptotic wave behavior, it follows that a solution satisfying the boundary conditions in (10) exists if the wave velocity $v>\tilde{c}_s$. Then, according to (11), the wave excited is a tension wave ($\varepsilon_0>0$) for $\beta_N>0$, and a compression wave ($\varepsilon_0<0$) for $\beta_N<0$.

IV. WAVE STRUCTURE

The structure of a nonlinear wave can be determined by studying the asymptotic behavior of the solutions of Eq. (9) subject to the boundary conditions in (10). Hereafter, we follow the qualitative analysis conducted in Ref. 14.

The structure of the shock wave depends on the ratio between the dispersion and dissipation parameters (g and δ) in Eq. (9). At sufficiently small values of δ, the shock wave has an oscillating structure. If the dissipation parameter exceeds a certain critical value, $\delta>\delta_*$, the shock wave has a monotonous profile. The critical values of the dissipation parameter δ_* corresponding to monotonous and oscillating wave profiles are determined by the formula $\delta_*=\sqrt{4|\alpha g|}$. The structure of the shock wave is schematically represented in Figs. 1(a) and 1(b).

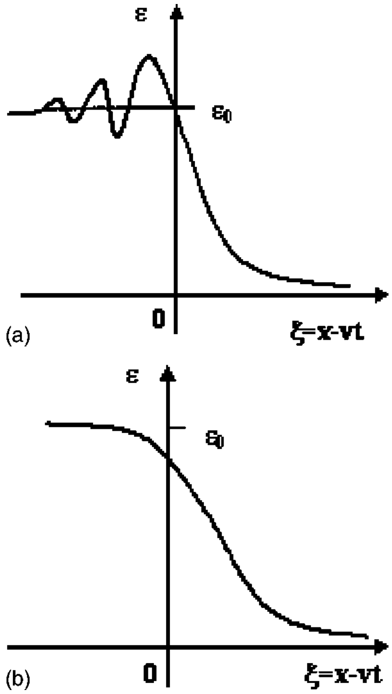

FIG. 1. Schematic representation of a shock wave structure. At small values of the dissipation parameter (δ), the shock wave has an oscillating structure (case a). If the dissipation parameter exceeds a critical value ($\delta > \delta_*$), the shock wave has a monotonous profile (case b).

Using (11), this equation can be represented in the form $|\varepsilon_0| = \varepsilon_*$, where the critical amplitude value is

$$\varepsilon_* = \left(\frac{K\Omega}{kT}\right)(q_0 \tau \Omega).$$

Thus, the shock wave has an oscillating structure at $|\varepsilon_0| > \varepsilon_*$ and a monotonic structure at $|\varepsilon_0| < \varepsilon_*$. The critical amplitude value ε_* dividing the shock waves with an oscillating structure from the monotonic shock waves is determined by the elastic modulus, temperature of the medium, intensity of defect generation, their relaxation time, and dilation volume of defects. At the characteristic values of parameters (for vacancies) $K = 5 \times 10^{11}$ dyn/cm, $q_0 \tau = 10^{19}$ cm^{-3}, $\rho = 8$ g/cm^3, $\beta_N = 10^{12}$ dyn/cm, and $|\Omega| = 10^{-23}$ cm^3, we find $\varepsilon^* = 4 \times 10^{-2}$.

The spatial scale of the variation of the solution to Eq. (9) evidently determines the width of the shock wave (L), i.e., the distance at which the oscillations die out.[14] An estimation yields

$$L = \frac{(4|g|/\delta)\sqrt{\varepsilon_* + |\varepsilon_0|}}{\sqrt{\varepsilon_* + |\varepsilon_0|} - \sqrt{\varepsilon_*}}. \tag{12}$$

The oscillation period (d) is found from the solution linearized in the neighborhood of the homogeneous solution $\varepsilon_0 = \alpha\rho/\beta_N$ of Eq. (9),

$$g\frac{d^2\varepsilon}{d\xi^2} - \delta\frac{d\varepsilon}{d\xi} + \alpha\varepsilon = 0.$$

Finally, we find

$$d = (4\pi|g|/\delta)\sqrt{\frac{\varepsilon_*}{|\varepsilon_0| - \varepsilon_*}}. \tag{13}$$

.

According to (13), the oscillation period decreases with increasing the amplitude of a nonlinear wave ε_0 and takes on the value $d \approx 4\pi\sqrt{\rho\, g/|\beta_N \varepsilon_0|}$ in the limit $|\varepsilon_0| \gg \varepsilon_*$.

V. DISCUSSION

From the conditions of the existence of an oscillating shock wave ($\varepsilon_* < |\varepsilon_0|$) and smallness of its amplitude $|\varepsilon_0| \ll 1$, we obtain the restriction on its velocity,

$$\tilde{c}_s^{\,2}[1 + \varepsilon_*(|\beta_N|/\rho\, \tilde{c}_s^{\,2})] < v^2 < \tilde{c}_s^{\,2}(1 + |\beta_N|/\rho\tilde{c}_s^{\,2}).$$

Since monotonic shock waves can appear in the media with rather large values of ε_* ($\varepsilon_* > |\varepsilon_0|$), we obtain the following constraints on the velocities of these waves:

$$v^2 < \tilde{c}_s^{\,2}(1 + |\beta_N|/4\rho\tilde{c}_s^{\,2}).$$

In the solids with negative dispersion ($g > 0$), an oscillating structure takes place before the wave front. On the contrary, in the media with positive dispersion ($g < 0$), the oscillating "tail" is behind the front.

According to formulas (12) and (13), the parameters L and d of oscillatory shock waves depend on the dispersion parameter g. Therefore, for these waves to originate, the medium should possess a dispersion. In contrast to oscillatory waves, monotonic shock waves can also exist in the systems without dispersion.

Let us discuss now the contribution of defects to the dispersion properties of a medium. The correction to the dispersion parameters due to nonequilibrium defects is of importance if $K\Omega q_\varepsilon \tau^3 v^2 \rho^{-1} > l^2(v^2 - c_\tau^2)$.

This implies the following restriction on the defect density:

$$q_0 \tau > \frac{\rho l^2 (v^2 - c_\tau^2) kT}{(v\tau K\Omega)^2}.$$

This condition can be satisfied for rather high defect densities ($q_0 \tau \geq 10^{19}$ cm^{-3}), which are typical for powerful impulse laser effect on most solids. However, if the linear elastic moduli for a lattice, the dilation volume of defects, and their relaxation time are large, the corrections due to defects become substantial even at lower densities.

Thus, in a solid exposed to external energy fluxes bringing about the generation of nonequilibrium point defects, the strain wave may propagate as a shock wave of low intensity. The shock waves may have an oscillating profile as well as a monotonic one. The existence of such waves is determined by the dissipation processes of generation (recombination) of defects (rate of defect generation and recombination time), by dispersion of the medium, and by elastic properties of the lattice and the defect subsystem.

The model equation describing the propagation of the nonlinear wave of elastic deformation in an elastic medium is derived in view of generation and recombination processes. Its structure presents a combination of the stationary Korteweg-de Vries and Burgers equations. The quantitative estimations of the contributions to linear elastic moduli and bulk dispersion due to finite defect recombination rate are presented. The critical defect densities are found in the case when their influence on the strain wave propagation is significant.

Note, in conclusion, that the strain wave propagation is also of significant interest in a medium with clusters of point defects (vacancies, interstitial clusters, etc.). A strain wave interacting with such defects may cause a local temperature increase and, hence, strengthen the recombination processes. The latter, in turn, are accompanied by local heat generation and deformation of the medium. Further study of the nonlinear interaction of deformation fields and temperature with structure defects (both point defects and clusters) is of interest from scientific and practical standpoints. A propagating strain wave carries information about various defects of the condensed medium that is necessary for diagnosing the defect structure of solids.

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