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A model for the propagation of nonlinear dispersive strain waves in a solid with defect clusters

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ABSTRACT

A model for the propagation of nonlinear dispersive one-dimensional longitudinal strain waves in an isotropic solid with quadratic nonlinearity of elastic continuum is developed with taking into account the interaction with atomic defect clusters. The governing nonlinear dispersive–dissipative equation describing the evolution of longitudinal strain waves is derived. An approximate solution of the model equation was derived which describes asymmetrical distortion of geometry of the solitary strain wave due to the interaction between the strain field and the field of clusters. The contributions of the finiteness of the relaxation times of cluster-induced atomic defects to the linear elastic modulus and the lattice dissipation and dispersion parameters are determined. The amplitudes and width of the nonlinear waves depend on the elastic constants and on the properties of the defect subsystem (atomic defects, clusters) in the medium. The explicit expression is received for the sound velocity dependence upon the fractional cluster volume, which is in good agreement with experiment. The critical value of cluster volume fraction for the influence on the strain wave propagation is determined.

1. Introduction

It is known that the nonlinear dispersive strain waves may propagate in an elastic solid under an intense impulse exposure (in particular, under impulse laser-beam exposure) [1–7]. Existence of these waves is caused due to the balance between nonlinearity and dispersion. Dispersion of the medium can be caused by the finiteness of the crystal-lattice period or by the sample thickness. Nonlinearity is provided by both the elastic features of a condensed medium (physical nonlinearity) and a nonlinear relationship between displacements and strains (geometrical nonlinearity) [1]. Various structural imperfections in the crystal lattice, i.e., atomic point defects (vacancies and interstitial atoms), which are produced from the lattice-site atoms due to external effects (for example, pulsed laser beam), introduce a significant strain of the medium as a result of the difference between the radii of lattice atoms and defects, and play an important role in solids exposed to radiation [8].

Strains in an elastic wave cause the defects to move within a crystal cell (a strain-induced drift), whereas the strains and a variation in the temperature in the wave modulate the probabilities of generation and recombination of defects of the thermal-fluctuation origin (via variations in the energy parameters of the defect subsystem, i.e., the energies of the defect formation and migration) [8].

Under certain conditions, the nonlinearities related to these interactions may become essential for the propagation of nonlinear elastic perturbations in condensed matter and result in a renormalization of lattice parameters (of both linear and nonlinear elastic modulus) [9–12]. It was shown in Ref. [12] that the presence of point lattice defects with a finite recombination rate in a medium may induce the appearance of dissipative terms, which are absent in ordinary equations for nonlinear elastic waves.

The study of the behavior of strain waves in view of their interaction with structural defects is of certain theoretical and practical interest, in particular, when analyzing the mechanisms of anomalous mass transport observed in the cases of the laser–matter interaction and ion implantation into metallic materials [13] and in the studies of mechanical activation of components in the case of solid-phase chemical reactions. The strain solitary waves may propagate and transfer energy over long distances along elastic medium. The wave of elastic strains propagating in a condensed medium carries information about distortions of their form and energy and about the energy losses related to the defect structure; this information is needed for optical–acoustical diagnostics of various parameters and the structure of solids.

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The surface of various nanoclusters (voids, dislocation loops, etc.) may also acts as effective sources of atomic defects. The clusters can be present in a medium or can be generated in the case of the laser irradiation. Clusters may also be developed under certain conditions in rapidly quenched metals. The rate of atomic defect generation by a "nanocluster" obeys the activation law and increases with increasing temperature (T) and decreasing activation-barrier height (Q). When the perturbation of the elastic strain field is propagated, the activation-barrier height is reduced, and results in the strain-stimulated "evaporation" of clusters; finally, the atomic defect density in the host increases. The efficiency of nanoclusters as the sources of atomic defects is especially important if the initial volumes and concentrations of clusters are large.

In our previous works [9–12] the influence of the generation–recombination processes and the strain-induced drift in a subsystem of atomic defects on propagation of nonlinear longitudinal strain waves was investigated. In the present paper, we develop a model of propagation of the longitudinal strain nonlinear wave in a plate [9–12]; the model accounts for the direct interaction between the strain fields and the atomic defect cluster. We derive model equation describing the propagation of nonlinear longitudinal strain waves. We specify the renormalization of the linear elastic modulus and the lattice dispersion and dissipation parameters; this renormalization is related to interaction between clusters and strains. We estimate the critical value of cluster volume fraction when they start to influence the propagation of strain waves.

2. Model

Let us consider an isotropic elastic plate of thickness h with atomic defect clusters. Let us introduce a nonstationary function $f(a,t)$ describing the distribution function of spherically shaped clusters in their radii a at a moment t, so that $dN(a,t)=f(a,t)da$ is the number of clusters per unit volume with the radii belonging to the interval $(a,a+da)$. Then the total number of clusters per unit volume $N(t)$ and the total cluster volume per unit volume $V(t)$ (cluster volume fraction) remaining after strain-stimulated "evaporation" for time t are given by the following integrals:

$$\int_0^\infty f(a,t)\mathrm{d}a=\int_0^\infty \mathrm{d}N(a,t)=N(t)$$

and

$$V(t)=\int_0^\infty v_{cl}\mathrm{d}N=\frac{4\pi}{3}\int_0^\infty a^3 f(a,t)\mathrm{d}a.$$

Let $n_j(x,t)$ the volume concentration of cluster-induced point defects of the j type (a subscript for vacancies $j=v$, for interstitial atoms $j=i$). For simplicity, we will further restrict our consideration to only one type of point defect generation, for example, vacancies, and set $n_j(x,t)\equiv n(x,t)$.

The clusters are basically unstable and, given sufficient thermal and deformation activation, will shrink so as to reduce the total interfacial energy of the system. Clusters shrinking under the action of strain field due to deformation potential emit single atomic defects that migrate by diffusion to internal sinks (e.g., dislocations, boundaries, and coherent inclusions) in the matrix where they are annihilated. It is assumed that the temperature field in a plate is homogeneous.

We consider a special form of the wave motions, specifically, the nonlinear plane longitudinal waves of small but finite amplitude, propagating along the x-axis ($-\infty<x<\infty$) in a plate with free lateral surfaces, that is, one-dimensional (1D) waves in which the deformation field depends upon the x-coordinate only. We study the long-wavelength approximation, when the characteristic length of a wave exceeds the thickness of a plate. Then the 2D governing equation of motion for an elastic plate is simplified and 1D equation can be used for describing the behavior of nonlinear longitudinal strain waves. Dispersion in the strain wave, such as rods, arises due to the finite transverse size of the plate and in the presence of transverse component in the displacement vector. The matrix, that is, the basic medium is considered to be homogeneous. We assume also that the average intercluster distance is much larger than the linear dimension (radius) of clusters, and is smaller than the wavelength.

The fractional cluster volume $p(t)$ in a plate is given by

$$p(t)=\frac{V(t)}{V_0}=\frac{4\pi}{3V_0}\int_0^\infty a^3 f(a,t)\mathrm{d}a,$$

where V_0 is the initial cluster volume fraction.

The distribution function $f(a,t)$ obeys the continuity equation [14]

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial a}\left(\frac{\mathrm{d}a}{\mathrm{d}t}f\right)=0,$$

where $\mathrm{d}a/\mathrm{d}t$ is the decay rate for a single cluster of radius a.

If the change of cluster sizes is limited by diffusion of atomic defects, the decay kinetics of a single cluster in solids is described by [14]

$$a\frac{\mathrm{d}a}{\mathrm{d}t}=-n_{th}\Omega D\left[\exp\left(\frac{2\sigma_{cr}\Omega}{ak_\mathrm{B}T}\right)-1\right],$$

where D is the point defect diffusion coefficient, n_{th} is the thermodynamic equilibrium concentration of atomic defects at a flat cluster/matrix interface, σ_{crys} is the interfacial tension between the cluster and matrix, k_B is the Boltzmann constant; $\Omega\approx d_0^3$ is the atomic volume (d_0 is the interatomic spacing).

Then, according to Ref. [15], the behavior of a variable $p(t)$ can be described by the activation equation, which, in view of influence of the strain field, can be written as

$$\frac{\partial p}{\partial t}=-\frac{p}{\tau_0}\exp\left(\frac{Q_0-\vartheta_d e}{k_BT}\right), \quad (1)$$

where τ_0^{-1} is the constant of the relaxation rate of clusters, $\tau_0^{-1}=4\pi R_p N_0 D n_{th}\Omega/V_0$ (N_0 is the initial density of clusters, $R_p=2\sigma_{crys}\Omega/k_BT$), Q_0 is the self-diffusion activation energy for cluster relaxation, and ϑ_d is the deformation potential. The strain of the medium $e=\partial u/\partial x$ ($u(x,\ t)$ is the x-component of the displacement vector ($\vec{u}$)). The exponential in (1) takes into account the decrease of the self-diffusion activation energy Q due to the strain field according to the formula: $Q=Q_0-\vartheta_d e$. Since the change in the self-diffusion activation energy is small compared to the unstrained value, let $\tau_{p0}(T)=\tau_0\exp(Q_0/k_BT)$ (τ_{p0} is the cluster lifetime without taking into account the strain influence), and express the exponential in (1) as

$$\tau_{p0}^{-1}(T)\exp(\vartheta_d e/k_BT)\approx\tau_{p0}^{-1}(T)(1+\vartheta_d e/k_BT).$$

We assume that clusters are immobilized in the plate so that there is no transport term in (1).

The evolution equation for the concentration of atomic defects in the "source–sinks" approximation has the form

$$\frac{\partial n}{\partial t}=-\frac{V_0}{\Omega}\frac{\partial p}{\partial t}-\frac{n}{\tau_d(e)}. \quad (2)$$

The source-related term in Eq. (2) depends [according to Eq. (1)] on the strain in the continuous medium, on the initial density and volume of clusters, on the temperature of the medium, and on physical properties. The second term accounts for the strain-induced recombination of atomic defects [8]:

$$\tau_d^{-1}(e) = \tau_{d0}^{-1}\,\exp(\vartheta_m e/k_B T) \approx \tau_{d0}^{-1}\left(1+\frac{\vartheta_m e}{k_B T}\right),$$

where $\vartheta_m = K\Omega_m$ [16] (K is the bulk modulus Ω_m is the dilatation parameter describing the variation of the crystal volume as the result of vacancy formation $\Omega_m<0$, $|\Omega_m| = (0.2\text{–}0.4)\Omega$); $\tau_{d0}^{-1} = v_d/l_d$ is the recombination rate in the absence of strain (τ_{d0} is the defect-relaxation time, v_d is the average velocity of defects, and l_d is the average length of free run of defects).

The nonlinear dynamic equation, which describes the propagation of a 1D longitudinal strain wave in a plate with quadratic nonlinearity of an elastic continuum, is given by

$$\frac{\partial^2 e}{\partial t^2} - c_s^2\frac{\partial^2 e}{\partial x^2} - \frac{\beta}{2\rho}\frac{\partial^2}{\partial x^2}(e^2) - \frac{\partial^2}{\partial x^2}\left(g_1\frac{\partial^2 e}{\partial t^2} - g_2\frac{\partial^2 e}{\partial x^2}\right) = -\frac{\vartheta_m}{\rho}\frac{\partial^2 n}{\partial x^2}. \quad (3)$$

Here $c_s = (E/\rho(1-\sigma^2))^{1/2}$ is the longitudinal sound velocity in a plate, ρ is the density of the medium, E is Young's modulus, and σ is Poisson's ratio. The coefficient of nonlinearity β and the dispersion parameters g_1 and g_2 are given by [3,5]

$$\beta = \frac{3E}{1-\sigma^2} + v_1\left[1-\left(\frac{\sigma}{1-\sigma}\right)^3\right] + 3v_2\left[\frac{1-4\sigma+6\sigma^2}{(1-\sigma)^3}\right] + v_3\left(\frac{1-2\sigma}{1-\sigma}\right)^3,$$

$$g_1 = \frac{\sigma(1-2\sigma)h^2}{12(1-\sigma)^2}, \quad g_2 = \frac{\sigma h^2 c_t^2}{2(1-\sigma)^2},$$

where v_1, v_2, and v_3 are the elastic modulus of the third order, $c_t = \sqrt{\mu/\rho}$ is the velocity of the bulk shear waves, and μ the second-order elastic coefficient. For most solids (metals, many of the polymers), $\beta<0$. There are also metals in which the deviation of the elastic lattice properties from Hooke's law is insignificant. In this case, $\beta>0$. In (3), we restricted our consideration to smooth strain disturbance and took into account the contributions of the elastic modulus to the spatial dispersion in the first nonvanishing approximation.

The second term in the left-hand part of Eq. (3) accounts for the linear elasticity theory, the third term describes the nonlinearity of elastic medium due to anharmonic lattice vibrations, the next two terms reflect the dispersions (temporal and spatial) caused by the finite thickness of a solid. The right-hand side of this equation takes into concentration the concentration stresses generated by cluster-induced point defects.

Eq. (3) represents the generalization of the well-known equation (called the refined equation with two dispersions (g_1, g_2)) for a nonlinear longitudinal elastic strain wave in a plate [3–6] to the case of a system with concentration stresses [8], which are caused by the generation–recombination processes in the nonequilibrium atomic defect subsystem. Such a generalization can be realized by adding the terms

$$\sum_{j=i,v} n_j \vartheta_m^{(j)} e$$

to the free energy of the system [8]. These terms take into account the interaction of the elastic strain field of the plate with the atomic defect field.

Eqs. (1)–(3) are the closed system of equations. This system completely describes the distribution of 1D longitudinal strain waves of small but finite amplitude in a plate due to nonstationary and nonuniform distributions of the atomic defect subsystem. The inverse effect, i.e., variation in the concentration field of defects in a plate as a result of perturbations of elastic strains, is also accounted for.

3. Nonlinear model equation for longitudinal strain wave and its solution

For the convenience analysis we introduce the following dimensionless variables

$$e^* = e/\varepsilon, \quad x^* = x/\Lambda, \quad t^* = tc_s/\Lambda, \quad n^* = n/C,$$

where ε and C are the characteristic scales of elastic strain and concentration of atomic defects, respectively, Λ is the wavelength of perturbations; we represent Eqs. (1)–(3) in the following form (the respective stars are dropped for simplicity)

$$\frac{\partial p}{\partial t} = -\frac{p}{\tau_p}(1+\bar\vartheta_d e), \quad (4)$$

$$\frac{\partial n}{\partial t} = -\frac{V_0}{\Omega C}\frac{\partial p}{\partial t} - \frac{n}{\bar\tau_d}(1+\bar\vartheta_m e), \quad (5)$$

$$\frac{\partial^2 e}{\partial t^2} - \frac{\partial^2 e}{\partial x^2} - \frac{\bar\beta}{2}\frac{\partial^2}{\partial x^2}(e^2) - \frac{\partial^2}{\partial x^2}\left(\bar g_1\frac{\partial^2 e}{\partial t^2} - \bar g_2\frac{\partial^2 e}{\partial x^2}\right) + \theta\frac{\partial^2 n}{\partial x^2} = 0. \quad (6)$$

Here the following dimensionless parameters are entered:

$$\bar\beta = \beta\varepsilon/\rho c_s^2, \quad \bar g_1 = g_1/\Lambda^2, \quad \bar g_1 = g_2/c_s\Lambda^2,$$
$$\bar\tau_d = \tau_{d0}c_s/\Lambda, \quad \bar\tau_p = \tau_{p0}c_s/\Lambda,$$
$$\theta = \vartheta_m C/\rho c_s^2\varepsilon, \quad \bar\vartheta_{m,d} = \vartheta_{m,d}\varepsilon/k_B T.$$

We now represent the solution to Eqs. (4) and (5) as the sum of the spatially homogeneous and inhomogeneous solutions

$$p = p_0 + \delta p, \quad n = n_0 + \delta n,$$

where p_0 and n_0 are the spatially uniform (along the x-axis) parts and δp and δn are small spatially nonuniform parts ($|\delta p| \ll p_0$, $|\delta n| \ll n_0$), which appear due to propagation of weakly nonlinear perturbations (e) of longitudinal strains.

Then from Eqs. (4) and (5), we have the following set of equations for the leading order terms p_0 and n_0:

$$\frac{\partial p_0}{\partial t} = -\frac{p_0}{\bar\tau_p},$$
$$\frac{\partial n_0}{\partial t} = -\frac{V_0}{\Omega C}\frac{\partial p_0}{\partial t} - \frac{n_0}{\bar\tau_d}.$$

These equations are similar to the equations used in the theories of combustion and chemical reactions. Their solutions were analyzed in the book of Zeldovich et al. [17]. Similar equations are used also in the theory of laser-induced recrystallization process of amorphous semiconductor thin films [18].

Although because of generation–recombination processes the solutions p_0 and n_0 can vary with time, this variation can be ignored for strain perturbations characterized by time scales ($\tau_s \propto \Lambda/c_s$) shorter than the minimum time ($\min\{\bar\tau_d, \bar\tau_p\}$) of cluster ($\bar\tau_p$) and atomic defect ($\bar\tau_d$) relaxation. Below we will consider this case, namely. The stationary values of p_0 and n_0 are connected by

$$n_0 = p_0(V_0/\Omega C)(\bar\tau_d/\bar\tau_p).$$

Neglecting nonlinear terms in the perturbations, for δp and δn, we have the following equations:

$$\frac{\partial}{\partial t}(\delta p) + \frac{\delta p}{\bar\tau_p} = -\frac{p_0}{\bar\tau_p}\bar\vartheta_d e, \quad (7)$$

$$\frac{\partial}{\partial t}(\delta n) + \frac{\delta n}{\bar\tau_d} = -\frac{V_0}{\Omega C}\frac{\partial}{\partial t}(\delta p) - \frac{n_0\bar\vartheta_m}{\bar\tau_d}e. \quad (8)$$

In derivation of (7) and (8) we took into account

$\partial p_0/\partial t \ll \partial(\delta p)/\partial t$ and $\partial n_0/\partial t \ll \partial(\delta n)/\partial t$.

Let us consider self-similar solutions of the forms

$$e = e(\xi), \quad \delta n = \delta n(\xi), \quad \delta p = \delta p(\xi),$$

where $\xi = x - vt$ (v is the velocity of the nonlinear wave). These solutions describe nonlinear longitudinal waves of strain and atomic defect concentration and waves of annealing of clusters; these waves propagate with a constant velocity (v) in the positive direction of the x-axis. The system of partial differential equations (6)–(8) is then transformed into the following system of ordinary differential equations (ODEs):

$$(v^2-1)\frac{d^2e}{d\xi^2} - \frac{\bar{\beta}}{2}\frac{d^2}{d\xi^2}(e^2) - (\bar{g}_1 v^2 - \bar{g}_2)\frac{d^4e}{d\xi^4} + \theta\frac{d^2}{d\xi^2}(\delta n) = 0, \tag{9}$$

$$-v\frac{d}{d\xi}(\delta p) + \frac{\delta p}{\bar{\tau}_p} = -\frac{p_0}{\bar{\tau}_p}\bar{\vartheta}_d e, \tag{10}$$

$$-v\frac{d}{d\xi}(\delta n) + \frac{\delta n}{\bar{\tau}_d} = \frac{V_0 v}{\Omega C}\frac{d}{d\xi}(\delta p) - \frac{v_0\bar{\vartheta}_m}{\bar{\tau}_d}e. \tag{11}$$

We choose the boundary conditions to Eqs. (9)–(11) in the following form

$$e(-\infty) = e_-, \quad e(+\infty) = e_+, \quad \delta n(\pm\infty) = 0, \quad \delta p(\pm\infty) = 0. \tag{12}$$

The boundary conditions given by (12) mean that propagation of wave perturbations of strain in a medium with clusters transfers the system under consideration from a state with a strain $e = e_-$ to the state with a strain $e = e_+$.

Eqs. (10) and (11) may be combined to obtain

$$-v\frac{d}{d\xi}(\delta n) + \frac{\delta n}{\bar{\tau}_d} = \frac{V_0}{\Omega C}\left[\frac{\delta p}{\bar{\tau}_p} + (\bar{\vartheta}_d - \bar{\vartheta}_m)\frac{p_0}{\bar{\tau}_p}e\right]. \tag{13}$$

We then take into account that the second term in the parentheses in Eq. (13) is much larger than the first term; therefore, the latter may be ignored. The solution of the resultant inhomogeneous differential equation has the following form with allowance for the boundary condition (12)

$$\delta n(\xi) = \int_{-\infty}^{+\infty} d\zeta z_p(\zeta)G(\xi-\zeta), \tag{14}$$

where

$$G(\xi-\zeta) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk\frac{\exp\{ik(\xi-\zeta)\}}{\bar{\tau}_d^{-1} - ikv},$$

$$z_p(\xi) = z_{p1}e(\xi), \quad z_{p1} = \frac{V_0 p_0(\bar{\vartheta}_d - \bar{\vartheta}_m)}{\Omega C\bar{\tau}_p}.$$

Eliminating the variable δn and using formula (14), we obtain the following equation that describes the propagation of nonlinear strain wave in a medium with point defect clusters:

$$(v^2-1)\frac{d^2e}{d\xi^2} - \frac{\bar{\beta}}{2}\frac{d^2}{d\xi^2}(e^2) - (\bar{g}_1 v^2 - \bar{g}_2)\frac{d^4e}{d\xi^4} = -\theta\frac{d^2}{d\xi^2}\int_{-\infty}^{+\infty} d\zeta z_p(\zeta)G(\xi-\zeta). \tag{15}$$

Eq. (15) represents the integral–differential equation in which the occurrence of an integrated term is caused by cluster–strain interaction. Equations similar to (15) are characteristic of dissipative media with deformational memory (or relaxation) [1]. Similar equation appears also in the theory of evolution of a longitudinal strain wave in an elastic plate with nonequilibrium laser-generated point defects [9–12]. At $\theta = 0$ (without cluster-induced atomic defects), this equation has the same form as in the case of a nonlinear longitudinal wave in the free solid with nonlocal interatomic interaction [1].

On the basis of Eq. (15), we can analyze the propagation of a nonlinear stationary strain wave taking into account the effects of (i) dispersive properties of the medium and (ii) elastic properties of both the lattice and a subsystem of point defect clusters. An exact analysis of this equation with arbitrary values of the parameters involved seems impossible. Further, we will consider it assuming that $l_d\Delta \ll 1$, where Δ is the characteristic scale of wave perturbations. We replace the integral term in Eq. (15) with a differential one. For this purpose we represent the function $G(\xi-\zeta)$ in the form of following expansion:

$$G(\xi-\zeta) = \bar{\tau}_d\delta(\xi-\zeta) + \bar{\tau}_d^2 v\delta'(\xi-\zeta) + \bar{\tau}_d^3 v^2\delta''(\xi-\zeta) + \ldots,$$

where $\delta(x)$ is the delta function; a prime denotes differentiation with respect to the argument.

Then retaining the first three terms in this expansion, we find

$$\frac{d^2}{d\xi^2}\int_{-\infty}^{+\infty} d\zeta z_p(\zeta)G(\xi-\zeta) \approx z_{p1}\left(\bar{\tau}_d\frac{d^2e}{d\xi^2} + \bar{\tau}_d^2 v\frac{d^3e}{d\xi^3} + \bar{\tau}_d^3 v^2\frac{d^4e}{d\xi^4}\right).$$

Substituting this expression into Eq. (15), we arrive at the following equation:

$$(v^2 - c_{sp}^2)\frac{d^2e}{d\xi^2} - \frac{\beta}{2}\frac{d^2}{d\xi^2}(e^2) + \gamma\frac{d^3e}{d\xi^3} - g_p\frac{d^4e}{d\xi^4} = 0, \tag{16}$$

where

$$c_{sp} = c_s\left(1 - p_0\frac{V_0}{\Omega C}\frac{\tau_{d0}}{\tau_{p0}}\frac{\vartheta_m|\vartheta_d - \vartheta_m|}{\rho c_s^2 k_B T}\right)^{1/2} \tag{17}$$

is the velocity of sound renormalized owing to the interaction between the strain field and clusters. The dissipation (γ) and dispersion (g_p) coefficients are expressed in the following form in terms of the generation–relaxation parameters of the atomic defects and their clusters

$$\gamma = z_{p1}\bar{\tau}_d^2 v\theta, \quad g_p = v^2(\bar{g}_1 - \bar{\tau}_d^3 z_{p1}\theta) - \bar{g}_2. \tag{18}$$

Integrating twice (16), we have the following ODE for the strain waves

$$g_p\frac{d^2e}{d\xi^2} - \gamma\frac{de}{d\xi} + \frac{\beta}{2}e^2 - (v^2 - c_{sp}^2)e + l_1\xi + l_2 = 0, \tag{19}$$

where l_1 and l_2 are arbitrary constants. For localized waves, the arbitrary constants are equal to zero due to the asymptotical conditions $e(\xi)|_{\xi\to\pm\infty} = 0$.

Therefore, (16) is equivalent to the equation of the second order

$$\frac{d^2e}{d\xi^2} - \frac{\gamma}{g_p}\frac{de}{d\xi} + \frac{\beta}{2g_p}e^2 - \frac{v^2 - c_{sp}^2}{g_p}e = 0. \tag{20}$$

It is seen that Eq. (20) coincides with the ODE reduction of the Burgers and Korteweg-de Vries (KdV) equations [19].

In the weakly dissipative limit the solution of this equation can be presented in the quasistationary form

$$e = e_0(\xi)e_1(\xi), \tag{21}$$

where $e_1(\xi)$ is slowly varying amplitude, and $e_0(\xi)$ is the solution of the Eq. (20) without a dissipative term:

$$\frac{d^2e_0}{d\xi^2} - \frac{v^2 - c_{sp}^2}{g_p}e_0 + \frac{\beta}{2g_p}e_0^2 = 0. \tag{22}$$

Substituting (21) in (20), and taking into account Eq. (22), we receive

$$e_1 = 1 + \frac{2\gamma}{\beta e_0^2}\frac{de_0}{d\xi}.$$

The solitary wave solutions (at $v^2 > c_{sp}{}^2$) of Eq. (22) has the form

$$e_0 = a\cosh^{-2}(k\xi), \tag{23}$$

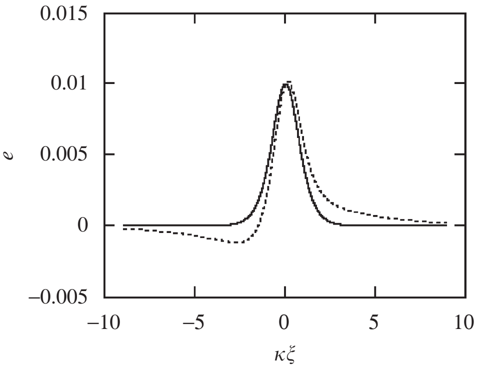

Fig. 1. Asymmetrical distortion of geometry of a solitary strain wave due to the interaction with clusters (continuous line—$e_0(\xi)$ and dashed—$e(\xi)$).

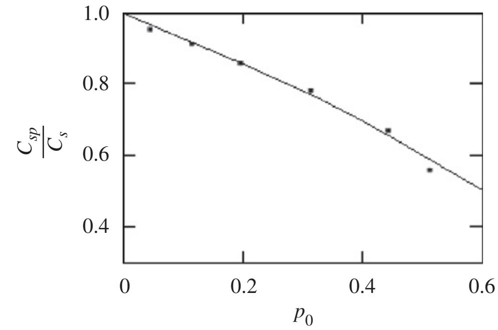

Fig. 2. Sound velocity c_{sp} as a function of the volume fraction of clusters (p_0). The solid line represents the theoretical results. The points represent the experimental data for iron from Ref. [22].

with

$$a = \frac{3(v^2 - c_{sp}^2)}{\beta}, \quad k = \frac{v^2 - c_{sp}^2}{4g_p}.$$

The solution of Eq. (20) then can be written as

$$e = a\cosh^{-2}(k\xi)\left(1 + \frac{\gamma}{6g_p}\sinh(2k\xi)\right). \tag{24}$$

This is a well-known solution for the KdV–B equation in the weakly dissipative limit. It can be seen from (24) that the influence of the generation–recombination processes in a defect subsystem results in asymmetrical distortion of the profile of a solitary strain wave. The forward wave front becomes less abrupt, and the back front becomes sloper. The structure of the solitary wave has the characteristic asymmetric form of "the spark fin". The shape of the nonlinear wave is shown in Fig. 1, where $\gamma = 0.11$, $g_p = 0.12$, $a = 0.01$. The continuous line in Fig. 1 accounts for the unperturbed solution (23), which corresponds to nondissipative approximation ($\gamma = 0$), while solution (24) is shown by the dashed line. This solution differs from the soliton-like waves by the presence of a splitting of asymptotes: $e(\xi \to \infty) = e(\xi \to -\infty) - \Delta$.

4. Numerical estimates and discussion

In Eq. (20), the appearance of the term with the first derivative with respect to strains is evidently related to the strain-stimulated generation of atomic defects at the surfaces of nanoclusters and to recombination. In addition, the finiteness of the defect-recombination rate ($\bar{\tau}_d^{-1}$) brings about a variation in the dispersion contributions to the nonlinear equation, which may affect the characteristics of propagating nonlinear strain waves.

Equations similar to Eq. (20) are used in many physical problems, such as nonlinear longitudinal waves in a viscoelastic rods and plates [4–6], nonlinear gravitational waves in a shallow liquid [19], and propagation of nonlinear ion-acoustic waves in plasma with Landau damping [20].

As follows from Eq. (18), the cluster-related contribution to the dispersion parameter is important if the relative volume of clusters exceeds a critical value, i.e.,

$$p_0 > p_{0^*} = \frac{\bar{\tau}_p C \Omega \bar{g}}{\bar{\tau}_d^3 \theta V_0 \bar{\vartheta}_m(\bar{\vartheta}_m - \bar{\vartheta}_d)},$$

or in dimensional values

$$p_0 > p_{0^*} = \frac{\rho \tau_{p0} k_B T \Omega (g_1 v^2 - g_2)}{\tau_{d0}^3 V_0 \vartheta_m(\vartheta_m - \vartheta_d)}. \tag{25}$$

Critical value p_{0^*} is determined by deformation potentials, initial cluster volume fraction and cluster concentration, atomic defect-relaxation times, and also temperature and material constants of the medium. Since the dispersion parameters g_1 and g_2 are proportional to h^2, we have the dependence of critical value p_{0^*} on thickness of the plate: $p_{0^*} \propto h^2$. The temperature behavior of $p_{0^*}(T)$, dissipation coefficient $\gamma(T)$, and sound velocity c_{sp} are determined by the Arrhenius-type relation.

For the numerical estimates we use the following values for the parameters: $h = 10^{-2}$ cm, $\rho = 8\,\mathrm{g \times cm^{-3}}$, $R_p = 3 \times 10^{-7}$ cm (at $\Omega = 10^{-23}\,\mathrm{cm^3}$, $\sigma_{\mathrm{crys}} = 10^3\,\mathrm{erg \times cm^{-2}}$, $T = 700$ K), $D = 3 \times 10^{-6}\,\mathrm{cm^2 \times s^{-1}}$ [19], $c_s = 3 \times 10^5\,\mathrm{cm \times s^{-1}}$, $\rho_s = 10^{12}\,\mathrm{cm^{-2}}$, $V_0 = 6 \times 10^{-3}$, $N_0 = 10^{14}\,\mathrm{cm^{-3}}$ [21], $Q_0 = 1.2$ eV, $\sigma = 3 \times 10^{-1}$, $\vartheta_m = 10$ eV [14], $\vartheta_d = 5$ eV. For the τ_{d0} and τ_{p0} we obtain the following $\tau_{p0} = 3 \times 10^{-7}$ s, $\tau_{d0} = 6 \times 10^{-9}$ s. Then from Eq. (25) for a critical value p_{0^*} we have the estimation: $p_{0^*} = 3 \times 10^{-2}$.

Now let us compare the theoretical results obtained with the experimental results. Consider formula (17) for the sound velocity. Fig. 2 represents the sound velocity (c_{sp}) as a function of the volume fraction of clusters for iron with above given values of the parameters. The solid line represents the theoretical results. The points represent the experimental data for iron from [22]. As follows from Fig. 2 with increase of a volume fraction of clusters the sound velocity monotonously decreases. We note that formula (17) reproduces well the experimental dependency [22]. Similar dependence takes place also for porous silicon (pore-Si).

Propagation of ultrasonic signals excited in pore-Si by nanosecond laser pulses was experimentally investigated in Ref. [23]. The samples in the shape of layers of $h = (0.5-4) \times 10^{-3}$ cm thickness and $p = 0.5-0.7$ porosity were prepared. At $h = 2.4 \times 10^{-3}$ cm, $p = 0.5$, $c_s(Si) = 8.43 \times 10^5\,\mathrm{cm \times s^{-1}}$ [23], we have from Eq. (17) the estimation $c_{sp} = 3.5 \times 10^5\,\mathrm{cm \times s^{-1}}$, which also agree satisfactorily with experimental data ($c_{sp} = 3.1 \times 10^5\,\mathrm{cm \times s^{-1}}$) [23].

5. Conclusion

The problem of propagation of a dispersive longitudinal strain wave in an elastic plate with quadratic nonlinearities is considered in the context of a model that accounted for the self-consistent behavior of the strain field in the medium and the concentration of the cluster-induced atomic point defects. We derived the governing equation that describes the nonlinear longitudinal waves of self-consistent strain and depends on the parameters of a defect subsystem; this equation is, in fact, the combination of the KdV and Burgers equations.

We analyzed the effect of the relaxation–generation parameters of the cluster-induced atomic defects on the characteristics of propagation of these waves. We assess quantitatively the contributions related to finiteness of the atomic defect recombination rate to the linear elastic modulus, the lattice dissipation, and dispersion parameters. We received an approximate solution of the governing equation, which describes asymmetrical distortion of the nonlinear wave profile due to the interaction between the strain and clusters.

We determined the critical value of cluster volume fraction for the case where their influence on the strain wave propagation was appreciable. A satisfactory agreement between theory and experiments was realized for the sound velocity (c_{sp}) dependence upon the volume fraction of clusters in iron and silicon.

The physical and geometrical properties of solitary waves, e.g. amplitude, velocity, asymmetry, contain information about the elastic constants and the properties of the defect subsystem (atomic defects, clusters) of the medium. Consequently, analytical predictions of this study make it possible to measure independently the elastic moduli and the parameters of a subsystem of defects (for example, the recombination rate, the migration and formation energies of atomic defects, the self-diffusion activation energy for the cluster decay rate and so on) in solids on the basis of nonlinear distortions of the strain longitudinal waves.

Note, in conclusion, that it is also of significant interest to study the thermal effects at strain wave propagation in a medium with defect clusters. A strain wave interacting with clusters may cause a local temperature increase and, hence, strengthen the recombination processes. The latter, in turn, are accompanied by local heat generation and deformation of the medium. The study of the nonlinear interaction of deformation fields and temperature with structure defects (both atomic defects and clusters) is of interest from scientific and practical points of view. This will be studied in the near future.

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