A model for the propagation of strain solitary waves in solids with relaxing atomic defects

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A model for the propagation of nonlinear dispersive one-dimensional longitudinal strain waves in an isotropic solid with quadratic nonlinearity of elastic continuum is proposed by taking into account the interaction of the longitudinal displacements with the temperature field and the field of concentration of nonequilibrium (recombining) atomic point defects (vacancies and interstitial atoms). The governing nonlinear equation describing the evolution of the self-consistent strain fields is derived. It is shown that the thermoelastic effect on the strain waves manifests itself in the appearance of dissipative terms, which describe the heat transfer and the thermoelastic interaction caused by the strain-induced heat release due to the recombination of atomic defects. The equation that describes the evolution of the amplitude of nonlinear traveling localized waves with time in the single-wave approximation is derived, and on the basis of this equation, the damping increments of these waves are determined with allowance the dissipative losses. The influence of stress-induced decay of defect complexes on the evolution of nonlinear strain waves is considered. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2838478]

I. INTRODUCTION

The studies of the generation and propagation of nonlinear dispersive strain waves in solids under intense external actions (in particular, under laser or electron-beam radiation or under high-speed loading) represent one of the rapidly progressing lines of research in nonlinear wave dynamics.[1–10] The presence of such waves is usually attributed to the balance of two competing processes: the spreading due to the dispersion of the medium and the wave-front breaking due to the nonlinearity of the elastic system. The dispersion may be caused by the geometric dimensions of the system (the finiteness of the crystal lattice period[11] or the thickness of the sample[3]), as well as by its molecular structure[1] or regular inhomogeneities of the medium. As a rule, the physical origin of nonlinearity is assumed to be the nonlinear dependence of strain on the displacement gradient.[1] Nonlinear waves are described by nonlinear equations. In contrast to the linear waves, an amplitude and a velocity of nonlinear waves are connected to one another.

The study of the generation and propagation of nonlinear localized strain waves is important for the development of both the general theory of nonlinear wave processes and the modern methods of nondestructive testing of materials, including the detection of defect concentration regions in them and the coating quality testing. From the point of view of laser semiconductor technology, the phenomenon of strain wave generation is of interest in connection with the radiant energy transfer by acoustic waves through distances far exceeding the size of the energy absorption region. If the transferred energy density is sufficiently high, the acoustic waves can be one of the sources of the so-called long-range effect observed in semiconductor structures exposed to laser radiation.[12] The strain wave generation is also one of the factors explaining the physical mechanisms of ion-beam gettering, which is widely used in modern microelectronics for improving the electrophysical characteristics of layered semiconductor structures.[13] Possible application of localized strain waves for the nondestructive control and for the problems of durability of constructional materials is discussed in Ref. 3.

Of special interest are the single nonlinear traveling strain waves that keep their shapes on propagation. This is a result of the balances between factors (dispersion, nonlinearity, and dissipation) affecting the wave behavior.[3] Among the sources of dissipation, one can mention a microstructure, nonequilibrium point defects, and thermal effects.

The generation of atomic point defects of the crystal structure (vacancies and interstitials) may occur as a result of the action of intense external energy fluxes (laser and corpuscular radiations) on condensed media or as a result of mechanical, thermal, and electric treatments of materials. A high concentration of nonequilibrium atomic defects is a source of internal mechanical stresses. These stresses are caused by the distortion (strain) of the crystal lattice near the defects arising as a result of the atomic bond breakage. The defects generated in a crystal may diffuse through the crystal and recombine either at various internal inhomogeneities in the bulk of the crystal (or emerge at the surface) or with each other (a mutual recombination). The recombination of defects may be accompanied by a heat release, which leads to a local increase in the temperature of the matrix lattice. The rate of the heat release due to recombination obeys the activation law and is determined by the height of the defect motion activation barrier (E_m) and by the temperature of the medium (T). The propagation of perturbations of the elastic strain field leads to a decrease in the activation barrier (E_m)

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owing to the deformation potential, which gives rise to a strain-induced recombination and, hence, to an increase in the temperature of the matrix.[14] The resulting inhomogeneous distributions of temperature and defect concentration give rise to forces proportional to their gradients, which additionally distort the lattice.

The presence of a high concentration of nonequilibrium atomic defects $[n=(10^{19}-10^{20})\ \mathrm{cm}^{-3}]$ in the medium and its relation to the elastic strain and temperature fields may affect the propagation of nonlinear elastic disturbances in a condensed medium and produce qualitatively new physical effects. For example, physical nonlinearities caused by atomic defects may lead to the appearance of relaxation components in the lattice parameters (in both linear and nonlinear elastic moduli). The presence of defects with a finite relaxation rate may give rise to dissipative terms, which are absent in the conventional equations describing nonlinear elastic waves in a medium.

The propagation of solitary strain waves in elastic plates without taking into account the interaction with structural defects was investigated theoretically in Refs. 6–9. In Refs. 15–20, the models of the evolution of nonlinear longitudinal strain waves in media were considered with allowance for the interaction with laser-induced nonequilibrium atomic defects. The influence of the strain-induced diffusion, generation, and recombination of defects on the propagation of elastic strain disturbances and their dispersion and dissipation properties were analyzed. However, the temperature variations in the medium and the thermoelastic stresses associated with them were ignored.

In recent years, considerable attention has been given to studying the possibility of using the thermoelastic effect for diagnosing mechanical stresses and various defects of the crystal structure.[21,22] Despite the relatively large number of publications, the possibilities offered by the thermoacoustic diagnostics remain poorly investigated. Considerable interest has been attracted to studying the characteristic features of thermoelastic wave propagation with allowance for the interaction with structural inhomogeneities of the matrix and to determining the mechanism of this interaction.

The present paper is devoted to the study of the propagation of one-dimensional (1D) longitudinal waves in an isotropic plate with a quadratic elastic nonlinearity with allowance made for the interaction between the strain, temperature, and nonequilibrium atomic defect concentration fields. A nonlinear evolution equation is derived for describing the propagation of longitudinal thermoelastic waves. It has the form of a dissipation-perturbed Korteweg–de Vries (KdV) equation. The influence of both the relaxation properties of atomic defects and the heat transfer through the plate surface on the characteristics of waves and on the dissipative parameters of the medium is considered. The influence of deformation-induced decay of defect complexes on the evolution of nonlinear strain waves is discussed.

II. MODEL

Let $n^{(j)}(x,t)$ be the concentration of nonequilibrium atomic point defects of the jth type [$j=v$ for vacancies (v-defects) and $j=i$ for interstitials (i-defects)], which are generated by an external energy flux (e.g., laser radiation) in a plate with a thickness h. We consider a plane nonlinear longitudinal thermoelastic wave of small but finite amplitude that propagates along the x axis ($-\infty<x<\infty$) in the plate with free lateral surfaces. We will assume that the thickness of the plate is much less than the typical size of the wave Λ, $h\ll\Lambda$ (long-wave approximation). In such 1D waves, the strain field depends on the x-coordinate, but not the y-coordinate. Dispersion in the strain wave, such as rods, arises due to the finite transverse size of the plate and in the presence of transverse component in the displacement vector.[3]

If thermal equilibrium in the plate is established (the thermal relaxation time is $\tau_\chi=h^2/\chi$, where χ is the thermal diffusivity) much faster than the equilibrium of the plate with a thermostat (the characteristic time of heat transfer is $\tau_T=h\rho C_p/2b$, where ρ is the density of the medium, C_p is the thermal capacity, and b is the heat transfer coefficient), i.e., $\tau_\chi\ll\tau_i$ or $h\ll\rho C_p\chi/2b$, we can assume that the temperature distribution in the plate is homogeneous throughout its thickness while in the plane of the plate, it is determined by the strain-stimulated recombination of defects and the heat transfer.

The main processes that govern the behavior of the atomic defect subsystem in time are the recombination at neutral centers and the mutual annihilation in the volume. The interaction of thermal, strain, and concentration fields occurs through the direct mechanism due to the modulation of the rate of recombination heat release owing to the deformation potential.

With the assumptions listed above, the nonlinear dynamic equation describing the propagation of 1D waves of longitudinal displacements in a plate (in the continual approximation) with allowance for the effect of concentration (caused by nonequilibrium atomic defects) and temperature stresses can be represented in the form

$$\frac{\partial^2 u}{\partial t^2}-c_p^2\left(1+\beta\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2}-\frac{\partial^2}{\partial x^2}\left(g_1\frac{\partial^2 u}{\partial t^2}-g_2\frac{\partial^2 u}{\partial x^2}\right)=-\frac{1}{\rho}\sum_{j=v,i}\vartheta_d^{(j)}\frac{\partial n^{(j)}}{\partial x}-\frac{\vartheta_T}{\rho}\frac{\partial T}{\partial x}. \tag{1}$$

Here, $u(x,t)$ is the longitudinal displacement, $c_p=[E/\rho(1-\sigma^2)]^{1/2}$ is the velocity of longitudinal waves in the plate in the absence of atomic defects, where E is Young's modulus and σ is Poisson's ratio, and $\vartheta_d^{(j)}=K\Omega_d^{(j)}$ is the defect deformation potential, where K is the bulk modulus and $\Omega_d^{(j)}$ is the volume elastic strain caused by the relaxation of the jth-type defect volume. For v-defects, $\Omega_d^{(v)}=-\delta^{(v)}\Omega<0$ (here, the coefficient is $\delta^{(v)}=0.2-0.4$ and Ω is the atomic volume), whereas for i-defects, $\Omega_d^{(i)}=\delta^{(i)}\Omega>0$ (the coefficient is $\delta^{(i)}=1.7-2.2$). A vacancy and an interstitial are represented as a substitutional atom whose volume is smaller or greater than the volume of the matrix atoms, respectively. Finally, $\vartheta_T=K\alpha_T$, where α_T is the thermal expansion coefficient.

In Eq. (1), the third term on the left-hand side characterizes the elastic nonlinearity of the medium (β is the nonlinearity coefficient). For most solids (metal and a lot of poly-

mers), $\beta<0$. There are also metals in which the deviation of elastic lattice properties from Hooke's law is insignificant. In this case, $\beta>0$. The fourth and fifth terms describe the dispersion [temporal (g_1) and spatial (g_2)] due to the thickness of the plate.[7,23] The right-hand side of Eq. (1) takes into account the forces applied to the lattice because of the thermal-deformation interaction.

The coefficient of nonlinearity β and the dispersion parameters g_1 and g_2 are given[7,9]

$$\beta = \frac{3E}{1-\sigma^2} + 3\nu_2\left[\frac{1-4\sigma+6\sigma^2}{(1-\sigma)^3}\right] + \nu_1\left[1-\left(\frac{\sigma}{1-\sigma}\right)^3\right] + \nu_3\left(\frac{1-2\sigma}{1-\sigma}\right)^3,$$

$$g_1 = \frac{\sigma(1-2\sigma)h^2}{12(1-\sigma)^2}, \quad g_2 = \frac{\sigma h^2 c_t^2}{2(1-\sigma)^2},$$

where ν_1, ν_2, and ν_3 are the Landau moduli of the third order, $c_t=\sqrt{\mu/\rho}$ is the velocity of the bulk shear waves, and μ is the second-order Lame coefficient.

Equation (1) represents the generalization of the known equation [called the refined equation with two dispersions (g_1, g_2) for a nonlinear longitudinal elastic strain wave in a plate][3,7,23] to the case of a system with concentration and thermoelastic stresses,[11] which are caused by the generation-recombination processes in the nonequilibrium atomic defect subsystem. Such a generalization can be realized by adding the terms

$$\sum_{j=i,v} n^{(j)}\vartheta_d^{(j)}\,\mathrm{div}\,\mathbf{u} \quad \text{and} \quad T\vartheta_T\,\mathrm{div}\,\mathbf{u}$$

to the free energy of the system.[14] These terms take into account the interaction of the elastic strain field of the plate with the defect field and the temperature field, respectively.

The concentration of the nonequilibrium (relaxing) atomic defects $n^{(j)}(x,t)$ obeys the activation-type kinetic equation. With the allowance for the effect of the elastic strain field, this equation can be represented in the form

$$\frac{\partial n^{(j)}}{\partial t} = -\frac{n^{(j)}}{\tau_d^{(j)}}\exp\left(\frac{\vartheta_m^{(j)}s}{k_BT}\right) - \gamma^{(iv)}n^{(v)}n^{(i)}\exp\left(\frac{\vartheta_m^{(i)}s}{k_BT}\right). \quad (2)$$

Here, $s=\partial u/\partial x$ is the strain of the medium, $\tau_d^{(j)}=\tau_0^{(j)}\exp(E_{m0}^{(j)}/k_BT)$ is the relaxation time of defects of the jth type in the absence of the strain field, $E_{m0}^{(j)}$ is the diffusion activation energy in the absence of strain, $\tau_0^{(j)}$ is the relaxation rate constant, k_B is the Boltzmann constant, and $\vartheta_m^{(j)}$ is the deformation potential characterizing the variation of the diffusion activation energy of defects under the lattice deformation. In Eq. (2), the first term determines the strain-induced recombination of the defects of type j at internal inhomogeneities (e.g., grain and phase boundaries, dislocations, interstitial impurities, etc.) playing the role of neutral sinks, and the second term determines the mutual recombination of the defects of different types ($\gamma^{(iv)}=4\pi\bar{R}D_0^{(i)}$ is the mutual recombination rate in the absence of deformation and $\bar{R}$ is the radius of the recombination zone).

The corresponding equation for the temperature distribution $T(x,t)$ in the plate has the form[14]

$$\frac{\partial T}{\partial t} - \chi\frac{\partial^2 T}{\partial x^2} + \frac{T}{\tau_T} = \frac{E_k^{(j)}}{\rho C_p}\frac{n^{(j)}}{\tau_d^{(j)}}\exp\left(\frac{\vartheta_m^{(j)}s}{k_BT}\right) + \frac{E_k^{(iv)}}{\rho C_p}\gamma^{(iv)}n^{(i)}n^{(v)}\exp\left(\frac{\vartheta_m^{(i)}s}{k_BT}\right). \quad (3)$$

Here, $E_k^{(j)}$ is the energy released in a unit volume as a result of the recombination of a single defect of type j at the centers (in order of magnitude, $E_k^{(j)}\propto E_f^{(j)}$, where $E_f^{(j)}$ is the energy of the defect formation), $E_k^{(iv)}$ is the energy released as a result of the mutual recombination of defects, and $E_k^{(i)}+E_k^{(v)}<E_k^{(iv)}$.

The terms on the right-hand side of Eq. (3) determine the thermal powers released as a result of the recombination of atomic defects at the centers and as a result of the mutual annihilation of defects, respectively. The effects of self-heating due to volume deformations (terms of the type of $3K\alpha_T T\partial^2 u/\partial x\partial t$) are small compared to the relaxation-caused heat release and the heat transfer through the surface and, hence, are ignored.

Equations (1)–(3) form a closed set of equations for describing the propagation of 1D weakly nonlinear longitudinal waves in a plate with a quadratic nonlinearity of the elastic continuum with allowance for the deformation, temperature, and atomic defect concentration fields.

III. THE EVOLUTION EQUATION FOR A NONLINEAR WAVE

Let ε, n_0, and T_0 be the characteristic scales of variation for the strain, the concentration of atomic defects, and the temperature, respectively, and c_p and Λ be the characteristic velocity and wavelength of a wave. From this point on, we limit our consideration to the case of only one type of defects and drop the superscript j in Eqs. (1)–(3); i.e., we assume that $n^{(j)}(x,t)\equiv n(x,t)$, $\tau_d^{(j)}\equiv\tau_d$, $D^{(j)}=D$, $\vartheta_m^{(j)}=\vartheta_m$, $\Omega_d^{(j)}=\Omega_d$, etc.

We introduce the following dimensionless variables and parameters:

$$u' = \frac{u}{\Lambda\varepsilon}, \quad s' = \frac{s}{\varepsilon}, \quad x' = \frac{x}{\Lambda}, \quad t' = \frac{c_p t}{\Lambda},$$

$$T' = \frac{T}{T_0}, \quad n' = \frac{n}{n_0},$$

$$\beta' = \beta\varepsilon, \quad \chi' = \frac{\chi}{c_p\Lambda}, \quad g_1' = \frac{g_1}{\Lambda^2}, \quad g_2' = \frac{g_2}{c_p^2\Lambda^2},$$

$$\vartheta_d' = \frac{\vartheta_d n_0}{\rho c_p^2\varepsilon}, \quad \vartheta_T' = \frac{\vartheta_T T_0}{\rho c_p^2\varepsilon}, \quad \vartheta_m' = \frac{\vartheta_m\varepsilon}{k_BT_0},$$

$$E_k' = \frac{E_k n_0}{\rho C_p T_0}, \quad \tau_{T,d}' = \frac{\tau_{T,d}c_p}{\Lambda}.$$

Then, setting $E_k^{(iv)}=\gamma^{(iv)}=0$ in Eqs. (1)–(3), and changing to the dimensionless variables (the primes indicating the dimen-

sionless variables will be omitted), we arrive at the set of equations,

$$\frac{\partial n}{\partial t} = -\frac{n}{\tau_d}\exp\left(\frac{\vartheta_m}{T}\frac{\partial u}{\partial x}\right), \tag{4}$$

$$\frac{\partial T}{\partial t} + \frac{T}{\tau_T} - \chi\frac{\partial^2 T}{\partial x^2} = -E_k\frac{\partial n}{\partial t}, \tag{5}$$

$$\frac{\partial^2 u}{\partial t^2} - \left(1+\beta\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2}{\partial x^2}\left(g_1\frac{\partial^2 u}{\partial t^2} - g_2\frac{\partial^2 u}{\partial x^2}\right) = -\vartheta_d\frac{\partial n}{\partial x} - \vartheta_T\frac{\partial T}{\partial x}. \tag{6}$$

We present the plate temperature in the form $T=T^{(0)}+T^{(1)}$, where $T^{(0)}$ is the spatially uniform part and $T^{(1)}=T^{(1)}\times(x)$ is the spatially nonuniform part, which appears due to propagation of longitudinal strain waves. Correspondingly, the defect concentration $n=n^{(0)}+n^{(1)}$, where $n^{(1)}=n^{(1)}(x)$.

Then, from Eqs. (4) and (5), we have the following set of equations for $(T^{(0)},n^{(0)})$:

$$\frac{\partial n^{(0)}}{\partial t} = -\frac{n^{(0)}}{\tau_0}\exp\left(\frac{E_m}{T^{(0)}}\right),$$

$$\frac{\partial T^{(0)}}{\partial t} + \frac{T^{(0)}}{\tau_T} = -E_k\frac{\partial n^{(0)}}{\partial t}.$$

This equations are similar to equations used in the theories of combustion and chemical reactions. Their solutions was analyzed in the book of Zeldovich *et al.*[24] Similar equations are used also in the theory of laser-induced recrystallization process of amorphous semiconductor thin films.[25]

For $(T^{(1)},n^{(1)})$, we have

$$\frac{\partial n^{(1)}}{\partial t} + \frac{n^{(1)}}{\tau_{d0}} = -\frac{n^{(0)}\vartheta_m}{\tau_{d0}}\frac{\partial u}{\partial x}, \tag{7}$$

$$\frac{\partial T^{(1)}}{\partial t} + \frac{T^{(1)}}{\tau_T} - \chi\frac{\partial^2 T^{(1)}}{\partial x^2} = -E_k\frac{\partial n^{(1)}}{\partial t} \tag{8}$$

$[\tau_{d0}=\tau_d(T=T^{(0)})]$.

Under the condition $\omega\tau_{d0}\gg 1$ (ω is the frequency), we neglect the term proportional to $n^{(1)}$ on the left-hand side of Eq. (7). Combining the resulting equation with Eq. (8), we obtain

$$\frac{\partial T^{(1)}}{\partial t} + \frac{T^{(1)}}{\tau_T} - \chi\frac{\partial^2 T^{(1)}}{\partial x^2} = E_k\frac{n^{(0)}\vartheta_m}{\tau_{d0}}\frac{\partial u}{\partial x}. \tag{9}$$

From Eq. (8), neglecting the heat transfer ($\chi=\tau_T^{-1}=0$), we obtain the following relation reflecting the energy conservation law:

$$n^{(1)} \approx -E_k^{-1}T^{(1)}. \tag{10}$$

Then, after the preliminary differentiation with respect to x and the substitution of $\partial T/\partial x$ taken from Eq. (6), with allowance for Eq. (10), we reduce Eq. (9) in the form

$$\frac{\partial}{\partial t}\left[\frac{\partial^2 u}{\partial t^2} - \left(1+\beta\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2}{\partial x^2}\left(g_1\frac{\partial^2 u}{\partial t^2} - g_2\frac{\partial^2 u}{\partial x^2}\right)\right] = -\eta_0\frac{\partial^2 u}{\partial x^2} + \left(\chi\frac{\partial^2}{\partial x^2} - \frac{1}{\tau_T}\right)\left[\frac{\partial^2 u}{\partial t^2} - \left(1+\beta\frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2}{\partial x^2}\left(g_1\frac{\partial^2 u}{\partial t^2} - g_2\frac{\partial^2 u}{\partial x^2}\right)\right], \tag{11}$$

where $\eta_0=n^{(0)}\vartheta_m(E_k\vartheta_T-\vartheta_d)\tau_{d0}^{-1}$.

Equation (11) is a differential analog of equations characteristic of dissipative media with a strain memory (or with relaxation).[1] In the absence of defect generation and temperature variation ($\vartheta_m=\chi=\tau_T^{-1}=0$), it coincides with the equation of a longitudinal elastic wave in a medium with local interactions.[1]

Neglecting the nonlinear terms in Eq. (11) and representing the solution to the equation obtained above in the form of plane waves

$$u = u_k\exp[-i(wt-kx)]$$

(w and k are the frequency and the wavelength, respectively), we arrive at the dispersion equation for linear thermoelastic waves in a medium with relaxing defects,

$$(w^2 - k^2 + g_1w^2k^2 - g_2k^4)[w + i(\chi k^2 + \tau_T^{-1})] = -i\eta_0k^2. \tag{12}$$

The effects of energy dissipation due to the heat transfer and the role of lattice dispersions in the elastic wave dynamics are adequately described in the literature.[1,3] Therefore, in analyzing the effect of generation-relaxation processes on the propagation of linear harmonic waves, for simplification we assume that $g_1=g_2=\chi=0$. Then, assuming that the right-hand side of the dispersion equation is small (for large relaxation times τ_d), we obtain

$$w = k - \eta_0\frac{k\tau_T}{1+ik\tau_T} + O(\eta_0^2),$$

$$\mathrm{Re}(w) = k - \eta_0\frac{k}{1+k^2\tau_T^2}, \quad \mathrm{Im}(w) = \eta_0\frac{k^2\tau_T}{1+k^2\tau_T^2}.$$

The propagation of harmonic strain disturbances will be stable when $\mathrm{Im}(w)<0$ and unstable when $\mathrm{Im}(w)>0$. Note that for $\eta_0>0$, we have $\mathrm{Im}(w)>0$ for all the wave numbers (k). Hence, the presence of recombination processes in the defect subsystem in a thermoelastic medium may lead to the instability of wave-type disturbances.

In the general form, Eq. (11) is likely to be solvable only numerically. However, if the dissipation effects associated with the thermal-deformation interaction are small, the right-hand side of Eq. (11) can be considered as a small deviation of the wave processes from the "unperturbed" state. Then, assuming that, in the zero-order approximation, we have $u_{tt}\approx u_{xx}$, from Eq. (11) we obtain the following equation for the self-consistent field of elastic displacements:

$$\frac{\partial^2 u}{\partial t^2} - \left(1 + \beta \frac{\partial u}{\partial x}\right)\frac{\partial^2 u}{\partial x^2} - g\frac{\partial^4 u}{\partial x^4} = -\eta_0 \frac{\partial u}{\partial t} + \eta_1 \frac{\partial^3 u}{\partial x^2 \partial t} - \eta_2 \frac{\partial^5 u}{\partial x^4 \partial t}, \tag{13}$$

where

$$g = g_1 - g_2, \quad \eta_1 = g/\tau_T, \quad \eta_2 = \chi g.$$

IV. DISSIPATION MODIFIED NONLINEAR DISPERSIVE EQUATION

To simplify the analysis, we follow the approach used in Ref. 9 and pass from Eq. (13) to the equations of coupled normal waves. Introducing new (wave-related) variables s_1 and s_2 for long-wave perturbations (small dispersion) according to the formulas

$$\frac{\partial u}{\partial t} = s_1 - s_2, \quad \frac{\partial u}{\partial x} = -(s_1 + s_2) + \frac{g}{2}\frac{\partial^2}{\partial x^2}(s_1 + s_2),$$

we arrive at the set of equations,

$$\pm\frac{\partial s_{1,2}}{\partial t} + \frac{\partial s_{1,2}}{\partial x} + \frac{g}{2}\frac{\partial^3 s_{1,2}}{\partial x^3} + \frac{\beta}{4}\frac{\partial}{\partial x}(s_1 + s_2)^2 \pm \eta_0 s_{1,2} - \eta_1 \frac{\partial^2 s_{1,2}}{\partial x^2} + \eta_2 \frac{\partial^4 s_{1,2}}{\partial x^4} = 0, \tag{14}$$

where the upper sign corresponds to the equation for s_1, and the lower sign to the equation for s_2.

Equation (14) is evolution equations with a nonlinearity of the Burgers–KdV type. For such equations, the quantity e is conventionally interpreted as the momentum flux density, and the quantity $s^2/2$ as the energy flux density. Equation (14) leads to the following momentum and energy conservation laws for localized perturbations:

$$\frac{\partial}{\partial t}\left(\int_{-\infty}^{+\infty} s dx\right) = 0,$$

$$\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}(s^2/2)dx = \mp\eta_0\int_{-\infty}^{\infty}(s^2/2)dx - \eta_1\int_{-\infty}^{+\infty}s_x^2 dx - \eta_2\int_{-\infty}^{+\infty}s_{xx}^2 dx$$

(in the case of periodic waves, the integration in these expressions is performed over the wave period Λ). Since the coefficients are $\eta_0, \eta_1, \eta_2 > 0$, the latter three terms on the left-hand side of Eq. (14) are responsible for the wave energy dissipation. The coefficient η_0 characterizes the wave energy dissipation at low frequencies, and the coefficients η_1 and η_2 at high frequencies.

Thus, the influence of the thermoelastic effect on the strain waves ($s_{1,2}$) manifests itself in the appearance of the dissipative terms caused by the heat transfer processes and the thermoelastic interaction due to the strain-induced recombination heat release.

As one can see from the set of Eq. (14), the functions s_1 and s_2 represent counterpropagating waves that interact owing to the quadratic nonlinearity. The change from Eq. (13) to the equations of coupled waves simplifies the investigation of the problem and allows us to separate it into two stages.[9] At the first stage, we ignore the interaction of counterpropagating waves and take into account only the effects that are responsible for the formation of nonlinear waves. In this case, the set of Eq. (14) falls into two independent equations of the single-wave approximation. These equations allow analytical solutions in the form of traveling steady-state nonlinear (localized or periodic) waves, whose shape depends on the relative values of the nonlinearity parameter (β) and the dispersion (g) of the medium. At the second stage, the effects of interaction of counterpropagating waves are taken into account. Here, we use the method of averaging over steady-state solutions obtained at the first stage.

Below, we limit our consideration to the evolution of a single strain wave $s_1 = s(x,t)$ propagating in the positive direction of the x axis. Note that, already in this approximation, the problem is of interest and can be used for studying the dynamics of nonlinear deformation-concentration structures in solids.

From Eq. (14) for $s(x,t)$, we obtain the following nonlinear equation:

$$\frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} + g\frac{\partial^3 s}{\partial x^3} + \frac{\beta}{2}\frac{\partial}{\partial x}(s^2) = -\eta_0 s + \eta_1 \frac{\partial^2 s}{\partial x^2} - \eta_2 \frac{\partial^4 s}{\partial x^4}. \tag{15}$$

V. ASYMPTOTIC SOLUTION IF THE GOVERNING EQUATION

Assuming that the dissipative terms involved in this equation are small compared to the nonlinear and dispersion terms, we introduce the notations $\eta_0 = \delta\tilde{\eta}_0$, $\eta_1 = \delta\tilde{\eta}_1$, and $\eta_2 = \delta\tilde{\eta}_2$, where $\delta \gg 1$.

We seek the solution to Eq. (15) in the form of traveling waves,[2]

$$s = s(\xi, \theta), \quad \xi_x = 1, \quad \xi_t = -V(\theta), \quad \theta = \delta t.$$

Substituting them into Eq. (15), we obtain

$$(1 - V)\frac{\partial s}{\partial \xi} + g\frac{\partial^3 s}{\partial \xi^3} + \frac{\beta}{2}\frac{\partial}{\partial \xi}(s^2) = \delta\left(\frac{\partial s}{\partial \theta} - \tilde{\eta}_0 s + \tilde{\eta}_1 \frac{\partial^2 s}{\partial x^2} - \tilde{\eta}_2 \frac{\partial^4 s}{\partial x^4}\right). \tag{16}$$

We represent the solution to Eq. (15) in the form

$$s = s^{(0)} + \delta s^{(1)} + \delta^2 s^{(2)} + \cdots.$$

Then, in the zero-order approximation, for $s^{(0)}$ we obtain the stationary KdV equation,

$$(1 - V)\frac{\partial s^{(0)}}{\partial x} + g\frac{\partial^3 s^{(0)}}{\partial \xi^3} + \frac{\beta}{2}\frac{\partial}{\partial \xi}(s^{(0)})^2 = 0. \tag{17}$$

It allows solutions in the form of solitary waves (solitons) or cnoidal waves. The solution to Eq. (17) in the form of solitary waves vanishing at $\xi \to \pm\infty$ is

$$s^{(0)}(\xi) = A\cosh^{-2}(k_m \xi),$$

$$A = 3(V-1)/\beta, \quad k_m = \sqrt{-\beta/6g}A^{1/2} = k_0 A^{1/2}.$$

In the next approximation, for $s^{(1)}$ we obtain the equation

$$\hat{L}s^{(1)} = (1-V)\frac{\partial s^{(1)}}{\partial \xi} + g\frac{\partial^3 s^{(1)}}{\partial \xi^3} + \frac{\beta}{2}\frac{\partial}{\partial \xi}(s^{(0)}s^{(1)}) = -\frac{\partial s^{(0)}}{\partial \theta} - \tilde{\eta}_0 s^{(0)} + \tilde{\eta}_1 \frac{\partial^2 s^{(0)}}{\partial \xi^2} - \tilde{\eta}_2 \frac{\partial^3 s^{(0)}}{\partial \xi^3} = \hat{\psi}^{(1)}(s^{(0)}). \quad (18)$$

The operator conjugate to $\hat{L}$ has the form

$$\hat{L}^A = V\frac{\partial}{\partial \xi} - \beta s^{(0)}\frac{\partial}{2\partial \xi} - g\frac{\partial^3}{\partial \xi^3}.$$

It is evident that $\hat{L}^A s^{(0)} = 0$.

Using the condition of orthogonality of the first-order and zero-order approximations

$$\int_{-\infty}^{+\infty} s^{(0)}\hat{\psi}^{(1)}(s^{(0)})d\xi = 0,$$

we obtain the equation for the amplitude of the nonlinear wave,

$$\frac{dA}{dt} = -\frac{4}{3}\eta_0 A + 14k_0^2\eta_1 A^2 - 26\eta_2 k_0^3 A^3. \quad (19)$$

The first term on the right-hand side of Eq. (19) describes the time variation of the amplitude of thermoelastic waves due to the defect-deformation interaction, and other two terms (with η_1, η_2) due to a heat transfer. In the absence of temperature variation ($\vartheta_T = \chi = \tau_T^{-1} = 0$), it coincides with the equation for the amplitude of the nonlinear wave in a plate with defect generation obtained in (Ref. 19).

For a steel plate with the parameters $\chi = 0.1$ cm/s, $C_p = 1$ J/(K), $b = 10^{-2}$ W/(cm^2 K), $\beta = 10^{12}$ dyn/cm^2, $\sigma = 0.29$, $\vartheta_m = 5$ eV, $\vartheta_d = 10$ eV, $E = 2\times 10^{11}$ Pa, and $\alpha_T = 2\times 10^{-5}$ K^{-1} with the characteristic values of $n^{(0)} = 5\times 10^{18}$ cm^{-3} and $\tau_{d0} = 10^{-3}$ s, the estimates of the coefficients involved in Eq. (19) yield

$$\eta_0 \propto 10^{-12}(\Lambda/h), \quad 14k_0^2\eta_1 \propto 8\times 10^{-14}(\Lambda/h), \quad 26\eta_2 k_0^3 \propto 10^{-14}(\Lambda/h).$$

Hence, for the characteristic amplitude values $A \leqslant 1 (s \leqslant 10^{-4})$, the energy dissipation at high frequencies is small and, therefore, the second and third terms on the right-hand side of Eq. (19) can be ignored. Then, we have

$$dA/dt = -4\eta_0 A/3.$$

From this expression, for the amplitude of the nonlinear solitary wave (or for its velocity), we obtain the exponential attenuation

$$A = A_0 \exp(-\Gamma t),$$

with the increment

$$\Gamma = 4n^{(0)}\vartheta_m(|\vartheta_d| + E_k\vartheta_T)/3\tau_{d0} = \Gamma_d + \Gamma_h.$$

It should be noted that the characteristic time of attenuation of the solitary wave is proportional to the defect recombination time: $\Gamma^{-1} \sim \tau_{d0}$. The damping increment contains two contributions: the contribution (Γ_d) caused by a relaxation of atomic defects and the contribution (Γ_h) caused by the strain-induced heat release due to the recombination of defects. At $E_k = 1.4$ eV and $|\Omega_d| = 10^{-23}$ cm^3, we have

$$\Gamma_d/\Gamma_h = |\Omega_d|C_p/\alpha_T E_k \approx 2.$$

From Eq. (18) with allowance for Eq. (19), we obtain the solution

$$s^{(1)} = \frac{\eta_0}{6k_m}\{\tanh\phi - 1 + [3(1-\phi\tanh\phi) + \phi(2 - \phi\tanh\phi)]\cosh^{-2}\phi\}$$

$$|\phi| \ll O(\delta^{-1/2}), \quad (20)$$

where $\phi = k_m\xi$.

For the asymptotic cases, formula (20) yields the expressions

$$s^{(1)} \to -(\eta_0/3k_m)[1 - 2\phi^2\exp(2\phi)], \quad 1 \ll -\phi < O(\delta^{-1/2}),$$

$$s^{(1)} \to -(2\eta_0/3k_m)\exp(-2\phi), \quad 1 \ll \phi < O(\delta^{-1/2}),$$

which show the presence of a "shelf" behind the perturbed soliton. The appearance of such a shelf is presumably related to the dissipative term caused by the recombination heat release in the atomic defect subsystem.

VI. PROPAGATION OF STRAIN WAVES IN PLATES WITH DEFECT COMPLEXES

Let us consider a role of the traps, capable to grasp atomic defects, forming the bonded states with v-defects (va-complexes) or i-defects (ia-complexes). Such traps always are present in real solids (or can be entered into him as a result of various intense external actions). A role of traps, for example, can play impurity atoms. For example, in silicon, v-defects form complexes with oxygen, phosphorus, and arsenic. In such crystals, the energy reserved by nonequilibrium defects much more that is caused by an opportunity of accumulation of defects due to small mobility of those from them which are seized by impurities. Complexes create significant distortions of a lattice and can interact with a field of elastic deformations. Various effects and the phenomena can be connected to the presence of complexes (including, strain wave propagation) in solids and consequently studying of their role is rather actual.

The longitudinal strain wave interacting with complexes (for definiteness, with ia-complexes) may locally (in the interaction region) reduce the activation barrier (the activation energy of the diffusion of defects) of complex decay owing to the deformation potential and, hence, cause an increase in the rate of recombination processes. The latter, in their turn, are accompanied by a local heat release and a deformation of the medium.

In modeling the propagation of nonlinear waves in such systems, Eq. (2) for the concentration of atomic defects should be replaced by the following set of equations:

$$\frac{\partial n^{(i)}}{\partial t} = -\frac{1}{\tau^{(i)}}n^{(i)} - \gamma^{(ia)}n^{(i)}n^{(a)} + \frac{1}{\tau^{(ia)}}n^{(ia)}, \quad (21)$$

$$\frac{\partial n^{(ia)}}{\partial t} = \gamma^{(ia)} n^{(i)} n^{(a)} - \frac{1}{\tau^{(ia)}} n^{(ia)} - D^{(ia)} k_{ia}^2 n^{(ia)}, \tag{22}$$

$$\frac{\partial n^{(a)}}{\partial t} = -\gamma^{(ia)} n^{(i)} n^{(a)} + \frac{1}{\tau^{(ia)}} n^{(ia)}, \tag{23}$$

where $n^{(ia)}$ and $\tau^{(ia)} = \tau_0^{(ia)} \exp(E_b^{(ia)}/k_B T)$ are the concentration and lifetime of *ia*-complexes, respectively. $\gamma^{(ia)} = r_{iai} D^{(i)}$ is the generation rate of *ia*-complexes, $D^{(ia)} = D_0^{(ia)} \exp(-E_m^{(ia)}/k_B T)$ is the diffusion coefficient for the *ia*-complexes, $E_m^{(ia)}$ and $E_b^{(ia)}$ are the migration and band energies of *ia*-complexes, respectively, and $\tau_0^{(ia)}$, r_{iai}, and $D_0^{(ia)}$, are constants.

The second term on the right-hand side of Eq. (21) describes the time variation of the *i*-defect concentration due to the formation of complexes. Equation (22) describes the kinetic of *ia*-complexes. Here, the first term on the right-hand side characterizes the generation of *ia*-complexes, and the second and third terms reflect their losses connected with the dissociation and an output on a surface. Equation (23) describes the rate of the impurity concentration variation.

At low temperatures, the complex dissociation is small in comparison to the recombination on a surface, that is,

$$1/\tau^{(ia)} < D^{(ia)} k_{ia}^2.$$

At high temperatures, on the contrary,

$$1/\tau^{(ia)} > D^{(ia)} k_{ia}^2.$$

Hence, at the temperature $T = T_{\max}$, appropriate to a maximum of dependence of *ia*-complex concentration, leaving on a surface to the moment of time $t = t_{\max}$, the following condition[14] should be satisfied:

$$1/\tau^{(ia)}(T_{\max}) \approx D^{(ia)}(T_{\max}) k_{ia}^2.$$

Let us consider a case of higher temperatures, when the rate of complex dissociation is great and the processes of complex formation are complicated, that is,

$$\gamma^{(ia)} n^{(i)} n^{(a)} \tau^{(ia)} < n^{(ia)}.$$

Then, neglecting the term $\gamma^{(ia)} n^{(i)} n^{(a)}$ in Eqs. (21)–(23), we come to system of the equations,

$$\frac{\partial n^{(i)}}{\partial t} = -\frac{n^{(i)}}{\tau_d^{(i)}} \exp\left(\frac{\vartheta_m^{(i)} s}{k_B T}\right) + \frac{n^{(it)}}{\tau^{(it)}} \exp\left(\frac{\vartheta_m^{(it)} s}{k_B T}\right), \tag{24}$$

$$\frac{\partial n^{(it)}}{\partial t} = -\frac{n^{(it)}}{\tau^{(it)}} \exp\left(\frac{\vartheta_m^{(it)} s}{k_B T}\right). \tag{25}$$

The corresponding equation for the temperature distribution $T(x,t)$ has the form

$$\frac{\partial T}{\partial t} - \chi \frac{\partial^2 T}{\partial x^2} + \frac{T}{\tau_T} = \frac{E_k^{(i)}}{\rho C_p} \frac{n^{(i)}}{\tau_d^{(i)}} \exp(\vartheta_m^{(i)} s/k_B T) - \frac{E_k^{(it)}}{\rho C_p} \frac{n^{(it)}}{\tau^{(it)}} \exp(\vartheta_m^{(it)} s/k_B T). \tag{26}$$

The last term on the right-hand side of Eq. (26) takes into account the temperature change due to the allocation of energy at capture of *i*-defects on traps ($E_k^{(it)}$ is the energy released in a unit volume). Other terms have the former sense.

Equations (24)–(26) together with the Eq. (1) completely describes the nonlinear dynamics of longitudinal strain waves in thermoelastic plates with *ia*-complexes.

We represent the solution to the set of Eqs. (24) and (25) as a sum of spatially uniform part $(n_0^{(i)}, n_0^{(it)})$ and nonuniform part $(n_1^{(i)}, n_1^{(it)})$,

$$n^{(i)} = n_0^{(i)} + n_1^{(i)}, \quad n^{(it)} = n_0^{(it)} + n_1^{(it)}.$$

In the linear approximation, we have

$$\frac{\partial n_1^{(i)}}{\partial t} + \frac{n_1^{(i)}}{\tau_d^{(i)}} = \frac{n_1^{(it)}}{\tau^{(it)}} + \left(\frac{n_0^{(it)} \vartheta_m^{(it)}}{\tau^{(it)}} - \frac{n_0^{(i)} \vartheta_m^{(i)}}{\tau_d^{(i)}}\right) \frac{\partial u}{\partial x}, \tag{27}$$

$$\frac{\partial n_1^{(it)}}{\partial t} + \frac{n_1^{(it)}}{\tau^{(it)}} = -\frac{n_0^{(it)} \vartheta_m^{(it)}}{\tau^{(it)}} \frac{\partial u}{\partial x}. \tag{28}$$

The appropriate linear equation for nonuniform distribution of a temperature field looks like

$$\frac{\partial T_1}{\partial t} + \frac{T_1}{\tau_T} - \chi \frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\rho C_p} \left(E_k^{(i)} \frac{n_0^{(i)} \vartheta_m^{(i)}}{\tau_d^{(i)}} - E_k^{(it)} \frac{n_0^{(it)} \vartheta_m^{(it)}}{\tau^{(it)}}\right) \frac{\partial u}{\partial x}. \tag{29}$$

Operating similarly to Sec. III, from Eqs. (1) and (27)–(29), we receive the following evolution equation for a nonlinear wave of displacement:

$$\frac{\partial^2 u}{\partial t^2} - \left(1 + \beta \frac{\partial u}{\partial x}\right) \frac{\partial^2 u}{\partial x^2} - g \frac{\partial^4 u}{\partial x^4} = -\eta_0^{(it)} \frac{\partial u}{\partial t} + \eta_1 \frac{\partial^3 u}{\partial x^2 \partial t} - \eta_2 \frac{\partial^5 u}{\partial x^4 \partial t}, \tag{30}$$

where

$$\eta_0^{(it)} = \left(E_k^{(i)} \frac{n_0^{(i)} \vartheta_m^{(i)}}{\tau_d^{(i)}} - E_k^{(it)} \frac{n_0^{(it)} \vartheta_m^{(it)}}{\tau^{(it)}}\right) \left(\vartheta_T - \frac{\vartheta_d}{E_k^{(i)}}\right).$$

This equation coincides with Eq. (13) if we replace $\eta_0^{(it)} \to \eta_0$ (after transition to dimensional variables). Note that under certain conditions (the certain values of average concentration of complexes), the coefficient $\eta_0^{(it)}$ is negative. In this case, the nonlinear elastic perturbations and also the fields of concentration of atomic defects and temperatures can propagate as solitary pulses (solitons) to amplitude increasing in time (growth increment $\propto 4\eta_0^{(it)}/3$). The asymptotic traveling wave solutions of Eq. (30) can be found by the method described in Sec. V.

VII. CONCLUSIONS

We proposed a model of the propagation of 1D longitudinal thermoelastic waves in nonlinearly elastic isotropic plates with nonequilibrium (relaxing) atomic point defects. The model is based on the equations that uniquely describe the combined dynamics of the fields of longitudinal displacements, concentration of point defects, and temperature of the medium. The nonequilibrium concentrations of defects in the plate are caused by the absorption of external energy fluxes (electromagnetic laser radiation). It is shown that in a plate where the interaction of the strain field with the nonequilib-

rium atomic defect subsystem and the temperature field of the medium occurs because of the modulation of the rate of recombination heat release due to the deformation in an elastic wave, the evolution of weakly nonlinear thermoelastic strain disturbances is described by the dissipative perturbed KdV equation characteristic of media with dissipation and dispersion. The equation that describes the evolution of the amplitude of nonlinear traveling localized strain waves with time in the single-wave approximation is derived, and on the basis of this equation, the damping increments of these waves are determined with allowance for the low- and high-frequency dissipative losses. The influence of stress-induced decay of defect complexes on evolution of nonlinear dispersive strain waves was analyzed.

Strain solitary waves propagating in laser-irradiated plates carries information about distortions of their form and energy and about the energy losses related to the defect structure, which may eventually be used for optical-acoustical diagnostics of various parameters and the structure of solids.

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