Elastic wave propagation in a solid layer with laser-induced point defects

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The results of the studies of the propagation of plane harmonic waves and vibrations in an isotropic, elastic solid layer with atomic point defects (interstitial atoms, vacancies, and electron-hole pairs) generated by a pulsed laser beam are presented. The study is based on coupled evolution equations for the elastic displacement of the medium and atomic defect density fields. The defect dynamics are governed by the strain-stimulated generation, diffusion, and annihilation processes. The frequency equations corresponding to the symmetric and antisymmetric elastic-concentration modes of vibration of the layer are obtained. Some limiting cases and special cases of the frequency equations are considered and a procedure for determining the phase velocity and the attenuation (or amplification) constants is discussed. © *2011 American Institute of Physics*. [doi:10.1063/1.3633524]

I. INTRODUCTION

Excitation of elastic-concentration waves by a pulsed laser beam in solids is of great interest because of extensive applications of pulsed laser technologies in material processing and nondestructive detecting and characterization, including the detection of defect concentration regions in them and the coating quality testing. When a solid is illuminated with a laser pulse, absorption of the laser pulse results in a localized temperature increase, which in turn causes lattice defects and generates an elastic-concentration wave in the solid. Various structural imperfections in the crystal lattice, i.e., atomic point defects, which are produced from the lattice site atoms because of pulsed laser beam, introduce a significant strain of the medium as a result of the difference between the radii of lattice atoms and defects,[1] and play an important role in surface modification of solids exposed to laser radiation.[2–5] The formation of defects may occur also as a result of mechanical, thermal, and electric treatments of materials. Examples of such defects are interstitial atoms, vacancies, color centers and their clusters, electron-hole pairs, and impurities atoms. Strains in an elastic wave cause the defects to move within a crystal cell (a strain-induced drift), whereas the strains and a variation in the temperature in the wave modulate the probabilities of generation and recombination of defects of the thermal-fluctuation origin (via variations in the energy parameters of the defect subsystem, i.e., the energies of the defect formation and migration).[2]

Under certain conditions, the nonlinearities related to these interactions may become essential for the propagation of elastic perturbations in condensed matter and result in a renormalization of lattice parameters (elastic modulus). The presence of point lattice defects with a finite recombination rate in a medium may induce the appearance of dissipative terms, which are absent in ordinary equations for elastic waves.[3]

The study of the behavior of elastic waves in view of their interaction with structural defects is of certain theoretical and practical interest, in particular, when analyzing the mechanisms of anomalous mass transport observed in the cases of the laser–matter interaction and ion implantation into metallic materials and in the studies of mechanical activation of components in the case of solid-phase chemical reactions.[6]

From the point of view of laser semiconductor technology, the phenomenon of elastic strain-wave generation is of interest in connection with the radiant energy transfer by acoustic waves through distances far exceeding the size of the energy absorption region. If the transferred energy density is sufficiently high, the acoustic waves can be one of the sources of the so-called long-range effect observed in semiconductor structures exposed to laser radiation.[7] The strain-wave generation is also one of the factors explaining the physical mechanisms of ion-beam gettering, which is widely used in modern microelectronics for improving the electrophysical characteristics of layered semiconductor structures.[8]

The elastic waves propagating in a condensed medium carries information about distortions of their form and energy and about the energy losses related to the defect structure; this information is needed for optical–acoustical diagnostics of various parameters and the structure of solids.

A large amount of work has been devoted to solving elastic wave propagation with the consideration of the coupling effect between defect density and strain fields. A model of the propagation of bending elastic waves in phononic crystal thin plates with a point defect has been considered in Ref. 9. They show that the frequency and number of the defect modes are strongly dependent on the filling fraction of the system and the size of the point defect. The theory of elasticity concerning the solid elastic material consisting of a distribution of atomic defects,[3] is receiving greater attention because of its theoretical and practical relevance. In the above theory, the atomic defects have been included as an additional independent kinematic variable. This theory reduces to the classical theory of elasticity in the limiting case when the defect density field vanishes. This new theory can play an important role in practical problems of laser material treatment where the classical theory is inadequate.

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In Refs. 10–12, nonlinear dynamics of self-consistent longitudinal strain waves in laser-irradiated solids, without taking into account the interaction with lattice defects, has been theoretically investigated. In these studies, attention was mostly focused on the study of an influence of the strain-induced diffusion, generation, and recombination of defects on the propagation of one-dimensional (1D) elastic strain disturbances and their dispersion and dissipation properties. Self-organization of stationary elastic-concentration periodic structures on the surface of the laser-irradiated solids has been considered in Ref. 13.

Research has been proposed to solve 1D dispersive strain-wave propagation in an isotropic solid with quadratic nonlinearity of elastic continuum by taking into account the interaction of the longitudinal displacements with the temperature field and the field of concentration of recombining atomic defects.[14] The governing nonlinear equation describing the evolution of the self-consistent strain fields has been derived and discussed. The influence of stress-induced decay of defect complexes and clusters on the evolution of strain waves has been also considered.[15]

Our aim in the present investigation is to study the propagation of elastic-concentration waves in an isotropic solid elastic layer containing a distribution of atomic point defects. The secular equation that governs the propagation of elastic-concentration waves has been derived by solving a system of coupled partial differential equations. We obtain the frequency equations corresponding to the symmetric and antisymmetric modes of vibration of the plate. Some limiting cases of the frequency equations are considered and a procedure for determining the phase velocity and the amplification factor is discussed. Relevant results of previous investigations are deduced as special cases

II. BASIC EQUATIONS

The basic equations governing the defect density and elastic displacement fields are given in this section. Let us assume that an external energy flux (e.g., laser radiation) creates point atomic defects in a surface layer. Let $n^{(j)}(x_i,t)$ $(i=1,2,3)$ be the density of these defects of the jth type [$j=v$ for vacancies (v-defects) and $j=i$ for interstitials (i-defects)]. In the context of thermal-fluctuation model of the point-defect production, the rate of defect generation from the lattice sites is governed by temperature (or intensity of laser beam) and stresses. Therefore, this rate may vary under the effect of elastic deformation fields, i.e., thermal-fluctuation-related defects may be generated and annihilated. Deformation fields affect the characteristics of the defects. Thus, when the strain waves propagate, the formation energy of atomic defects changes in the compression and dilatation zones. If $e=u_{ll}=\nabla\cdot\vec{u}$ is the dilatation ($\vec{u}$ is the displacement vector of the medium) and $\vartheta_g^{(j)}$ is the deformation potential characterizing the variation of the formation activation energy of defects under the lattice deformation, the renormalized formation energy of atomic defects can be represented as $w_g^{(j)}=w_{g0}^{(j)}-\vartheta_g^{(j)}e$ ($w_{g0}^{(j)}$ is the formation energy for the jth-type defect in an unstrained crystal). If there is a deformation-related perturbation of the lattice, not only does the formation energy of defects decrease, but so does the activation energy for the defect migration $w_m^{(j)}=w_{m0}^{(j)}-\vartheta_m^{(j)}e$ ($w_{m0}^{(j)}$ is the migration energy of the defects in the absence of deformation and $\vartheta_m^{(j)}$ is the deformation potential characterizing the variation of the migration activation energy of defects under the lattice deformation). This results in an increase in the diffusion coefficient of defects (D). Modulation of the formation brings about the corresponding modulations of the source function (g) and recombination rate (γ) of atomic defects:

$$g_j=g\exp\left(\vartheta_g^{(j)}e/k_BT\right),$$

$$\gamma_j=\tau_j^{-1}\exp\left(\vartheta_m^{(j)}e/k_BT\right).$$

Here $g=g_0\exp\left(-w_{g0}/k_BT\right)$ is the rate of generation of atomic defects by an external source in the absence of the strain field, T is the absolute temperature, k_B is the Boltzmann constant, g_0 is the constant of defect generation, and τ_j is the relaxation time of defects of the jth type in the absence of the strain field.

The concentration of atomic defects is dependent on temperature of the medium. One thus needs to know how the laser irradiation affects the local temperature field of the surface at the laser spot. We will consider here situations where the laser irradiation only heats the crystal (the light energy absorbed by the medium is transformed into heat), and where an equilibrium between laser radiation and the temperature field (T) is reached on time scales much shorter than the characteristic time scale of defect density evolution. Typically, the time scale for equilibration between photon absorption and defect generation is on the order of picoseconds, while that for defect diffusion is of the order of microseconds. We also assume that the contribution of thermal strains to deformation fields is negligible compared to lattice dilatation because of atomic defects and the phase changes and chemical reactions in the medium are absent.

In this paper, we will analyze the problem of the wave propagation in an elastic solid irradiated over a large area by CW or pulsed lasers. Furthermore, we will assume that the temperature profile has reached its equilibrium value. Its evolution is sufficiently slow compared to atomic defect generation, and can be considered as quasi-stationary. The solution of the heat conduction equation for this case is given by Duley.[16]

The governing field equations for the displacement vector and defect density in the absence of body forces, are given by:[3]

$$\rho\frac{d^2u_i}{dt^2}=\nabla_j\sigma_{ij}^{el}+\nabla_j\sigma_{ij}^{d}, \tag{1}$$

$$\frac{dn_j}{dt}=-\nabla_iQ_i+g_j-\gamma_jn_j-\beta n_in_v. \tag{2}$$

Here ρ is the density of the medium, $\sigma_{ik}=\sigma_{ik}^{el}+\sigma_{ik}^{d}$ is the symmetric stress tensor, and Q_i is the defect flux.

The second term in the right-hand side of Eq. (1) takes into account the forces applied to the lattice because of the defect–deformation interaction. Eq. (1) represents the

generalization of the known equation for the elastic waves in a solid[17] to the case of a system with concentration-related stresses,[3] which are caused by the generation–recombination processes in the non-equilibrium atomic defect subsystem. In Eq. (2), the first term represents diffusion of the defects, the second term characterizes generation of atomic defects stimulated because of the presence of the elastic field, and the last two terms describe the strain-stimulated recombination of defects.[2]

The constitutive equations for the elastic solids with defect generation relate the stress tensor (σ_{ik}) and defect flux ($\vec{Q}$) in the media to the strain and the change in defect density.

Thus,

$$\sigma_{ik} = c_{iklm}e_{lm} - \sum_j \vartheta_{ik}^{(j)} n_j = \sigma_{ik}^{el} + \sigma_{ik}^{d},$$

$$Q_i = -D_{ik}\nabla_k n_j - \vec{v}_j n_j.$$

Here $e_{ij} = \left(\nabla_j u_i + \nabla_i u_j\right)/2$, c_{iklm} is the elastic modulus, $\vartheta_{ik}^{(j)}$ is the deformational potential, and D_{ik} the diffusion coefficients of the defects.

The defect flux Q_i, where the first term represents diffusion with a coefficient D_j, and the second term corresponds to the drift of defects at the velocity $\vec{v}_j = \left(D_j/k_BT\right)\vec{F}_j$ under the influence of the force $F_j = -\nabla U_{\text{int}}$ resulting from the interaction of defects with an inhomogeneous strain field ($U_{\text{int}} = -K\Omega_d^{(j)}\nabla \cdot u$ is the energy of the interaction between one defect and the strain field).

If, the material is isotropic, then c_{iklm}, D_{ij}, and ϑ_{ik} are given by

$$c_{iklm} = \lambda\delta_{ik}\delta_{lm} + \mu(\delta_{il}\delta_{km} + \delta_{im}\delta_{lk}),$$

$$D_{ik} = D\delta_{ik},\ \vartheta_{ik}^{(j)} = \vartheta_d^{(j)}\delta_{ik},$$

and the constitutive equations becomes

$$\sigma_{ik} = \delta_{ik}\lambda e_{ll} + 2\mu e_{ik} - \delta_{ik}\sum_j \vartheta_d^{(j)} n_j, \tag{3}$$

$$Q_j = -D\nabla n_j - \vec{v}n_j, \tag{4}$$

where λ and μ are the Lame coefficients;[17] and $\vartheta_d^{(j)} = K\Omega_d^{(j)}$, where K is the bulk modulus and $\Omega_d^{(j)}$ is the volume elastic strain caused by the relaxation of the jth-type defect volume. For v-defects, $\Omega_d^{(v)} = -\delta^{(v)}\Omega < 0$ (here, the coefficient is $\delta^{(v)} = 0.2$–0.4 and Ω is the atomic volume), whereas, for i-defects, $\Omega_d^{(i)} = \delta^{(i)}\Omega > 0$ (the coefficient is $\delta^{(i)} = 1.7$–2.2). v- and i-defects are represented as substitutional atoms whose volumes are smaller or greater than the volumes of the matrix atoms, respectively.

The field equations governing the displacement field and the defect density fields are obtained by substituting the constitutive relations (3) and (4) into the equations of motion (1) and (2) as

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} - \mu\nabla^2\vec{u} - (\lambda+\mu)\nabla\left(\nabla\cdot\vec{u}\right) = -\sum_{j=i,v}\vartheta_d^{(j)}\nabla n_j, \tag{5}$$

$$\frac{\partial n_j}{\partial t} = g_j - D\nabla Q_j - \beta n_i n_v - \tau_j^{-1}n_j. \tag{6}$$

Here $\nabla^2 = \partial^2/\partial x_\alpha \partial x_\alpha$, where $\alpha = 1, 2$. In the absence of defect generation, we have $n_j = 0$; and Eq. (1) reduces then to the Navier's equation of classical elasticity. From this point on, we limit our consideration to the case of only one type of atomic defect and drop the superscript j in Eqs. (5) and (6), i.e., we assume that $\vartheta_d^{(j)} = \vartheta_d$, $\Omega_d^{(j)} = \Omega_d$, $n_j \equiv n$, $\tau_j \equiv \tau$, $D_j = D$, etc.

III. FORMULATION OF THE PROBLEM

We consider an isotropic elastic solid, which occupies the Cartesian space $x_3 \in [-h, h]$, $x_1, x_2 \in [-\infty, \infty]$. The boundary $x_3 = \pm h$ is supposed to be free of stresses and strains. We choose the x_1-axis along the direction of wave propagation in such a way so that all the particles on a line parallel to the x_2-axis are equally displaced and, hence, all the field quantities are independent of x_2-coordinates. Thus, the motion of the plate is supposed to take place in the x_1x_3 plane and, for the assumed motion of the plate, the displacement vector $\vec{u}$ has the component $(u, 0, w)$ and all the other variables depend on x_1, x_3, and t only.

We seek solutions of (5) and (6) subject to the boundary conditions

$$\sigma_{33} = \sigma_{31} = 0 \text{ on } x_3 = \pm h. \tag{7}$$

Also, the boundary condition for n is given by

$$\frac{\partial n}{\partial x_3} = 0, \quad \text{for} \quad x_3 = \pm h. \tag{8}$$

We can express the defect density field as $n = n_0 + n_1$ ($n_0 = g\tau$ is a spatially homogeneous solution and n_1 is a small non-homogeneous perturbation). Inserting in (6) and neglecting the nonlinear terms, we get the linearized equation as

$$\frac{\partial n_1}{\partial t} = ge - g_d\Delta e + D\nabla^2 n_1 - \gamma n_1, \tag{9}$$

where $g = g_0\left(\vartheta_g - \vartheta_m\right)/k_BT$, $g_d = Dn_0\vartheta_d/k_BT$.

The relation between the strain-induced generation and drift terms is given by $4\pi^2\left(\vartheta_d/\vartheta_g\right)\left(D\tau/d_{latt}^2\right)$, where d_{latt} is the characteristic scale of the waves. Thus, the generation term dominates at formation of "large-scale" structures ($d_{latt} \geq 2\pi\left(\vartheta_d/\vartheta_g\right)^{1/2}\sqrt{D\tau}$), and the drift term dominates at generation of "small-scale" structures [$d_{latt} < 2\pi\left(\vartheta_d/\vartheta_g\right)^{1/2}\sqrt{D\tau}$]. As $\sqrt{D\tau} = l$ (l is the average distance between sinks), and the scale of structures is always supposed to be greater than l, the choice of drift or generation terms depends on a ratio ϑ_d/ϑ_g. In this paper, we believe that $\vartheta_d/\vartheta_g < 1$.

IV. BASIC SOLUTIONS

To solve the problem, we use Helmholtz decomposition theorem to express the displacement vector as $\vec{u} = \nabla\varphi + \nabla\times\vec{\psi}$, $\nabla\cdot\varphi = 0$, so that the displacement components are written as

$$u = \frac{\partial\varphi}{\partial x_1} + \frac{\partial\psi}{\partial x_3},\ w = \frac{\partial\varphi}{\partial x_3} - \frac{\partial\psi}{\partial x_1}, \tag{10}$$

where the vector point potential function is defined as where $\vec{\psi} = (0, -\psi, 0)$ and φ and ψ depend only on x_1, x_3, and time t.

Upon introducing Eqs. (10) in (5), we obtain

$$\frac{\partial^2 \varphi}{\partial t^2} = c_L^2 \nabla^2 \varphi - \frac{\vartheta}{\rho} n_1, \tag{11}$$

$$\frac{\partial^2 \psi}{\partial t^2} = c_T^2 \nabla^2 \psi, \tag{12}$$

where $c_L (= \sqrt{(\lambda + 2\mu)/\rho})$ is the sound velocity for longitudinal acoustic waves and $c_T (= \sqrt{\mu/\rho})$ is the sound velocity for transverse acoustic waves.[17] Equation (12) corresponds to purely transverse waves in the solid that get decoupled from rest of the motion and are not affected by the defect density fields.

Eq. (9) becomes

$$\frac{\partial n_1}{\partial t} = g \nabla^2 \varphi + D \nabla^2 n_1 - n_1 \tau^{-1}. \tag{13}$$

Now using (10) and (13), the boundary conditions (7) may be expressed in terms of φ and ψ as follows:

$$2 \frac{\partial^2 \varphi}{\partial x_1 \partial x_3} - \frac{\partial^2 \psi}{\partial x_3^2} + \frac{\partial^2 \psi}{\partial x_1^2} = 0, \tag{14}$$

$$\frac{\partial^2 \varphi}{\partial x_1 \partial x_3} - \frac{\partial^2 \psi}{\partial x_1^2} + \frac{1}{2c_T^2} \frac{\partial^2 \psi}{\partial t^2} = 0. \tag{15}$$

We consider the case of harmonic waves so that the solutions φ, ψ, and n, of Eqs. (11) and (12) take the forms

$$\begin{aligned} \varphi(x_1, x_3, t) &= \tilde{\varphi}(x_3) \exp i(qx_1 + \omega t), \\ \psi(x_1, x_3, t) &= \tilde{\psi}(x_3) \exp i(qx_1 + \omega t), \\ n_1 &= \left(\rho c_L^2 / \vartheta_d\right) \tilde{n} \exp i(qx_1 + \omega t), \end{aligned} \tag{16}$$

where q and ω are the wave number and complex angular frequency of the waves, respectively; and $\tilde{\phi}(x_3)$ and $\tilde{\psi}(x_3)$ are functions of x_3. Evidently, expressions (16) correspond to plane surface waves propagating along the positive x_1-direction with wavelength $2\pi/q$. Here, the phase velocity is given by $c = \mathrm{Re}(\omega)/q$ and attenuation constant by $R = \mathrm{Im}(\omega)$, where $\mathrm{Re}(\omega)$ and $\mathrm{Im}(\omega)$ denote the real and imaginary part of ω, respectively.

Substitution of (16) into (11) and (12) led to a coupled system of three equations in terms of $(\tilde{\varphi}, \tilde{\psi}, \tilde{n})$:

$$\frac{d^2 \tilde{\varphi}}{dx_3^2} - \left(q^2 - \frac{\omega^2}{c_L^2}\right) \tilde{\varphi} - \tilde{n} = 0, \tag{17}$$

$$\frac{d^2 \tilde{\psi}}{dx_3^2} - \left(q^2 - \frac{\omega^2}{c_T^2}\right) \tilde{\psi} = 0, \tag{18}$$

$$\frac{d^2 \tilde{n}}{dx_3^2} - \left(q^2 + \frac{i\,\omega\tau + 1}{l^2}\right) \tilde{n} + \frac{\delta}{l^2} \left(\frac{d^2 \tilde{\varphi}}{dx_3^2} - q^2 \tilde{\varphi}\right), \tag{19}$$

where $\delta = g_0 \vartheta_d (\vartheta_g - \vartheta_m) \tau / \rho\, c_L^2 kT$ is the defect-strain coupling constant. The parameter δ may be of either sign. As signs of the deformational potentials ϑ_d, ϑ_g, and ϑ_m are the same, we have for v–defects: $\delta > 0$, if $|\vartheta_g| > |\vartheta_m|$; $\delta < 0$, if $|\vartheta_g| < |\vartheta_m|$.

Eliminating $\tilde{n}$ from (17) and (19), we obtain

$$\left\{ \left[\frac{d^2}{dx_3^2} - q^2 \left(1 - \frac{\omega^2}{c_L^2 q^2}\right)\right] \left[\frac{d^2}{dx_3^2} - q^2 - \frac{1}{l^2}(1 + i\omega\tau) + \frac{\delta}{l^2} \left(\frac{d^2}{dx_3^2} - q^2\right)\right] \right\} \tilde{\varphi} = 0. \tag{20}$$

The requirement of the existence of the non-trivial solution of Eqs. (17)–(19) provides us with a quadratic polynomial characteristic equation in α^2, which gives us three pairs of the characteristic roots $\pm\alpha_j$ $(j = 1, 2, 3)$. In general, the roots $\pm\alpha_j$ are complex and, therefore, the solution is a superposition of the plane waves attenuating with depth. As we are considering surface waves only, we choose only that form of α_i, which satisfies the radiation condition, viz., $\mathrm{Re}(\alpha_j) > 0$. Thus, we obtain the following formal solution for the functions $(\tilde{\varphi}, \tilde{\psi})$:

$$\begin{aligned} \tilde{\phi} &= \sum_{j=1}^{2} a_j \cosh \alpha_j x_3 + b_j \sinh \alpha_j x_3, \\ \tilde{\psi} &= a_3 \sinh \alpha_3 x_3 + b_3 \cosh \alpha_3 x_3, \end{aligned} \tag{21}$$

where a_j and b_j, with $j = 1, 2, 3$, are the arbitrary constants,

$$\alpha_1^2 = q^2 - \omega^2 / c_T^2. \tag{22}$$

α_2^2 and α_3^2 are the roots of the equation

$$\alpha^4 - \alpha^2 \left[2q^2 - \omega^2 c_L^{-2} + (1 + i\omega\tau - \delta) l^{-2}\right] + \left(q^2 - \omega^2 c_L^{-2}\right)\left((i\omega\tau + 1) l^{-2} + q^2\right) - \delta q^2\, l^{-2} = 0\,. \tag{23}$$

Then α_2^2 and α_3^2 are defined from (23) as follows:

$$\begin{aligned} \alpha_2^2 + \alpha_3^2 &= 2q^2 - \omega^2 c_L^{-2} + (1 + i\omega\tau - \delta) l^{-2}, \\ \alpha_2^2 \alpha_3^2 &= \left(q^2 - \omega^2 c_L^{-2}\right)\left(q^2 + (i\omega\tau + 1) l^{-2}\right) - \delta\, l^{-2} q^2. \end{aligned} \tag{24}$$

Upon inserting solutions for φ and ψ from (21) in Eq. (10), the displacement components are obtained as

$$\begin{aligned} u &= \left(iq\tilde{\phi}(x_3) + \alpha_3 (a_3 \cosh \alpha_3 x_3 + b_3 \sinh \alpha_3 x_3)\right) \times \exp i(qx_1 + \omega t), \\ w &= \left(-iq a_3 \tilde{\psi}(x_3) + \sum_{j=1}^{2} \alpha_j \left(a_j \cosh \alpha_j x_3 + b_j \sinh \alpha_j x_3\right)\right) \times \exp i(qx_1 + \omega t). \end{aligned}$$

Clearly, the displacements get modified as a result of the characteristic roots corresponding to diffusion field equations, because of coupling among interacting fields. Similarly, we can obtain expressions for the normal stresses.

V. DISPERSION EQUATIONS OF THE WAVE IN AN INFINITIVE MEDIUM

In this section, as a special case, we consider the generation of plane harmonic structures in unbounded mediums. Setting $\alpha_r = 0$ and $\gamma = 1,\ 2,\ 3$ in (22) and (23) we obtain

$$\omega^2 = c_T^2 q^2, \tag{25a}$$

$$\left(q^2 - \omega^2 c_L^{-2}\right)\left[q^2 + (i\omega\tau + 1)l^{-2}\right] - \delta l^{-2}\, q^2 = 0. \tag{25b}$$

Eq. (25a) corresponds to transverse waves, which are independent of δ, having speed of propagation c_T. Eq. (25b) pertains to the coupled strain and concentration waves.

To explore and delineate the strain and defect generation effects, we shall seek solutions of (25b) for small values of δ. For $\delta = 0$, Eq. (25b) admits the following solutions $\omega_{1,2}^{(0)} = \pm c_L q$ (acoustical mode) and $\omega_3^{(0)} = i(l^2q^2 + 1)\tau^{-1}$ (diffusion mode). Now, for small $\delta \ll 1$, we may write

$$\omega_{1,2} = \omega_{1,2}^{(0)} + \delta\omega_{1,2}^{(1)} + \delta^2\omega_{1,2}^{(2)} + ...,$$

$$\omega_3 = \omega_3^{(0)} + \delta\omega_3^{(1)} + \delta^2\omega_3^{(2)} + ...,$$

substituting into (25b), and equating the coefficients of like powers of δ, we finally arrive at

$$\omega_{1,2} = \omega_{1,2}^{(0)} - \frac{\delta}{2}\frac{\omega_{1,2}^{(0)}\left(q^2l^2 + 1 - i\omega_{1,2}^{(0)}\tau\right)}{(q^2l^2+1)^2 + \left(\omega_{1,2}^{(0)}\tau\right)^2} + \mathrm{O}(\delta^2), \tag{26}$$

$$\omega_3 = \omega_3^{(0)} - \frac{i\delta c_L^2 q^2 \tau}{c_L^2 q^2\tau^2 + (q^2l^2+1)^2} + \mathrm{O}(\delta^2). \tag{27}$$

From Eq. (26) we obtain

$$\mathrm{Re}\left(\omega_{1,2}\right) = \omega_{1,2}^{(0)}\left[1 - \frac{\delta}{2}\frac{q^2l^2+1}{(q^2l^2+1)^2 + (c_Lq\tau)^2}\right], \tag{28a}$$

$$\mathrm{Im}\left(\omega_{1,2}\right) = \frac{\delta\tau}{2}\frac{c_L^2q^2}{(q^2l^2+1)^2 + (c_Lq\tau)^2}. \tag{28b}$$

If $c_Lq \ll \tau^{-1}(l^2q^2+1)$ and the viscosity is taken into account [by adding in Eq. (5) the terms $\eta_T\Delta\dot{\vec{u}}$ and $\eta_L\nabla(div\dot{\vec{u}})$, where $\eta_T = \eta$ and $\eta_L = 4\eta/3 + \zeta$; η and ζ are the first and second viscosity coefficients], the dispersion Eq. (28b) describes an instability of the amplitude of acoustic waves (i.e., laser generation of acoustic waves)

$$\mathrm{Im}\left(\omega_{1,2}\right) = \frac{1}{2\rho}\eta q^2 + \frac{\delta}{2\tau}. \tag{29}$$

From Eq. (29), it follows that excitation of acoustic waves as a result of the instability occurs, if $|\delta| > \eta q^2\tau/\rho$.

It follows that the frequency spectrum of acoustic wave is hardly changed ($\mathrm{Re}(\omega_{1,2}) \approx \omega_{1,2}^{(0)}$), but the increment is renormalized.

If $c_Lq \ll \tau^{-1}(l^2q^2+1)$,

$$\mathrm{Re}\left(\omega_{1,2}\right) = \omega_{1,2}^{(0)}\left(1 - \frac{\delta}{2(q^2l^2+1)}\right), \tag{30a}$$

$$\mathrm{Im}\left(\omega_{1,2}\right) = \frac{1}{2\rho}\eta q^2 + \frac{\delta\tau}{2}\frac{c_L^2q^2}{(q^2l^2+1)^2}. \tag{30b}$$

In this case, there is a softening of frequencies of acoustic waves [instability of frequencies $\mathrm{Re}\left(\omega_{1,2}\right) \to 0$, $\mathrm{Im}\left(\omega_{1,2}\right)0$] and this is related to taking into account the generation of atomic defects. It is necessary to note that reduction of frequency does not occur up to zero, and up to value $\omega_\eta = \eta q^2/2\rho \ll c_Lq$. The Eq. (30b) describes attenuation of the amplitudes of acoustic waves.

At $\omega \ll c_Lq$ from (25b) we obtain

$$\omega = -i\left(\delta - l^2q^2 - 1\right)\tau^{-1}. \tag{31}$$

The dependences of the growth increment $\Gamma = -\mathrm{Im}(\omega)$ on the wave number are plotted in Fig. 1 for different values of defect-strain coupling constant (δ). It can be seen from Fig. 1 that, if

$$\delta \geq \delta_{cr} = 1, \text{i.e.}, \; g \geq g_{cr} = \rho\, c_L^2 k_B T/\tau\vartheta_d(\vartheta_g - \vartheta_m), \tag{32}$$

there is a range of wave numbers $0 < q < q_0 = [(\delta - 1)/l^2]^{1/2} = 1.7 \times 10^4\mathrm{cm}^{-1}$ (at $l^2 = 10^{-8}\mathrm{cm}^2$), such that $\mathrm{Im}\omega < 0$, i.e., the amplitudes of the Fourier harmonics of the small perturbations increase ($\Gamma > 0$) with time without limit. The result is an instability leading to the formation of coupled defect-strain structures. The period of these structures is

$$d_{lat} = \frac{2\pi}{q_0} = 2\pi l\sqrt{\frac{g_{cr}}{g - g_{cr}}}.$$

As the temperature is increased, the period grows and this is related to an increase in the lifetime of defects.

VI. DISPERSION EQUATION OF THE PLANE WAVE IN AN ELASTIC LAYER

Applying the boundary conditions (14) and (15), one obtains a set algebraic equation for the determination of the arbitrary constants d_j. Elimination of this constant, gives

$$|S_{ik}| \times |A_{ik}| = 0, i, k = 1, 2, 3, \tag{33}$$

where

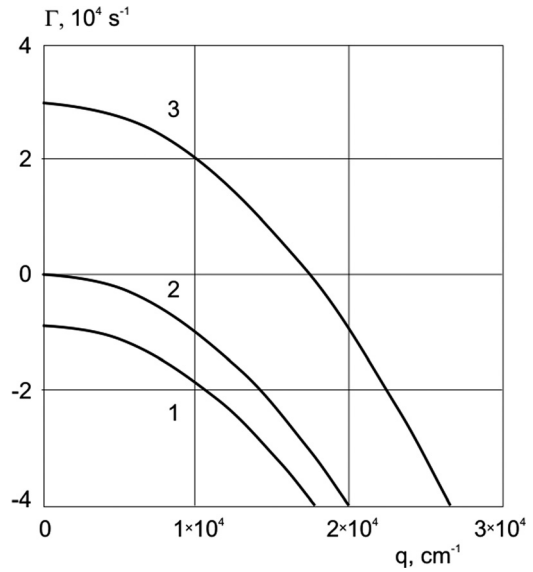

FIG. 1. Dependence of the growth increment (Γ) on the wave number for different values of defect-strain coupling constant: $\delta = 0.1$ (line 1), $\delta = 1$ (line 2), and $\delta = 4$ (line 3).

$$S_{1l} = \left(2 - \omega^2/c_T^2 q^2\right)\coth\alpha_l h, \quad S_{13} = 2i\alpha_3 q^{-1}\coth\alpha_3 h,$$
$$S_{2l} = 2i\alpha_l q^{-1}, \quad S_{23} = -(2 - \omega^2/q^2 c_T^2),$$
$$S_{3l} = \alpha_l q^{-1}\left(\alpha_l^2 + \omega^2 c_T^{-2} - q^2\right), \quad S_{33} = 0, \quad l = 1, 2.$$

Expressions for A_{ik}'s can be obtained from S_{ik} by replacing $\coth(\alpha_j h)$ with $\tanh(\alpha_j h)$. Equations $|S_{ik}| = 0$ and $|A_{ik}| = 0$ are associated with the symmetric and antisymmetric vibrations of the plate relative to the middle surface, respectively. The frequency equations $|S_{ik}| = 0$ and $|A_{ik}| = 0$ can be combined and represented a

$$\left(2q^2 - \frac{\omega^2}{c_T^2}\right)^2 = \frac{4q^2\alpha_1\alpha_2\alpha_3\left(\alpha_2^2 - \alpha_1^2\right)}{\frac{\alpha_2\left(\alpha_2^2 + \omega^2 c_L^{-2} - 1\right)}{\gamma_1^{\pm 1}} - \frac{\alpha_1\left(\alpha_1^2 + \omega^2 c_L^{-2} - 1\right)}{\gamma_2^{\pm 1}}}, \tag{34}$$

where $\gamma_i = \tanh\alpha_3 h/\tanh\alpha_i h$, $i = 1, 2$.

In Eq. (34), the positive power index is associated with the symmetrical motion and negative index with the antisymmetrical motion.

Eq. (34) contains $c(= \mathrm{Re}(\omega)/q)$ as the unknown quantities and, hence, this equation may be used to determine c in terms of q. This equation is, therefore, the phase velocity equation for the waves considered. Because c depends on q, in general, the waves are generally dispersive. This dispersive nature of the general waveform arises because of the presence of defect generation in the medium. The discussion of transcendental equations, in general, is difficult; we, therefore, consider the results for some limiting cases.

VII. LIMITING FORMS OF THE FREQUENCY EQUATIONS AND THEIR SOLUTIONS

We now discuss the limiting forms of the frequency equation that are defined by the inequalities: (i) $\alpha h \ll 1$, very long waves; and (ii) $\alpha h \gg 1$, very short waves.

In the first case, the hyperbolic tangent in (34) approaches unity and, hence, (34) for symmetrical motion becomes

$$\left(2q^2 - \omega^2 c_T^{-2}\right)^2\left(\alpha_1^2 + \alpha_2^2 + \alpha_1\alpha_2 - \omega^2 c_L^{-2} + q^2\right) = 4q^2\alpha_1\alpha_2\alpha_3(\alpha_1 + \alpha_2). \tag{35}$$

Eq. (35) determines the velocity of Rayleigh surface waves in a half-space solid with atomic defects. We shall now consider this in the following special cases.

(a) In the particular case $\delta = 0$, which means that the defect-strain interaction is absent, we obtain

$$Ra(c^2)\left(i\omega\tau + l^2 q^2 + 1\right) = 0,$$
$$Ra(c^2) = \left(1 - \frac{c^2}{2c_T^2}\right)^4 - \left(1 - \frac{c^2}{c_L^2}\right)\left(1 - \frac{c^2}{c_T^2}\right),$$

which yields

$$Ra(c^2) = 0, \tag{36a}$$
$$w = i\tau^{-1}\left(l^2 q^2 + 1\right). \tag{36b}$$

Equation (36a) is equivalent to the classical Rayleigh equation, while (36b) defines the attenuation constant for the concentration waves. With $c_R^2 = \omega^2/k^2$, Eq. (36a) reduces to

$$\left(2 - c_R^2 c_T^{-2}\right)^4 = 16\left(1 - c_R^2 c_T^{-2}\right)\left(1 - c_R^2 c_L^{-2}\right). \tag{37}$$

(b) $\delta \neq 0$. In this case, there are solutions describing qualitatively different types of instability: (1) if the viscosity is taken into account, the dispersion Eq. (35) describes laser generation of surface acoustical waves (SAWs); (2) generation of ordered static structures.

At $\delta \ll 1$, roots of Eqs. (24) may by rewritten as

$$\alpha_2^2 = q^2 - \omega^2 c_L^{-2} + \eta_L i\omega^3\left(\rho\, c_L^4\right)^{-1} - \delta\frac{\omega^2 c_L^{-2}}{i\omega\tau + 1 + \omega^2 c_L^{-2} l^2} + \mathrm{O}(\delta^2),$$
$$\alpha_3^2 = q^2 + l^{-2}(i\omega\tau + 1) - \frac{\delta}{l^2}\frac{i\omega\tau + 1}{i\omega\tau + 1 + \omega^2 c_L^{-2} l^2} + \mathrm{O}(\delta^2).$$

Then we have

$$\frac{\left(1 - \omega^2/2q^2 c_T^2 + i\omega^3\eta_T/2q^2\rho\, c_T^4\right)^2}{\left(1 - \omega^2/q^2 c_T^2 + i\omega^3\eta_T/q^2\rho\, c_T^4\right)^{1/2}} = W^{1/2}\frac{\left[2\sqrt{W} + 2q^2 - \omega^2 c_L^{-2} + (i\omega\tau + 1 - \delta)l^{-2} + \eta_L i\omega^3/\rho\, c_L^4\right]^{1/2}}{q\left[q^2 + (i\omega\tau + 1 - \delta)l^{-2} + \eta_L i\omega^3/\rho\, c_L^4 + \sqrt{W}\right]}, \tag{38}$$

where

$$W = \left(q^2 - \omega^2 c_L^{-2} + \eta_L i\omega^3\left(\rho c_L^4\right)^{-1}\right)\left(q^2 + (i\omega\tau + 1)l^{-2}\right) - q^2\delta\, l^{-2}.$$

Assuming that $\delta \ll 1$ is a sufficiently small set,

$$\omega^2/k^2 = c_R^2\left(1 + \delta\xi_1 + \delta^2\xi_2 + \ldots\right), \tag{39}$$

where $c_R = \omega_R/q$ is the classical Rayleigh wave speed, which is a root of (37).

Substituting expression (39) into (38), and retaining only the terms of $\mathrm{O}(\delta)$, we receive the following dispersion relation:

$$\omega^2 = q^2 c_R^2\left(1 - \delta\frac{(1 - \beta\zeta)f(c_R, q)}{l^2 c_R^2 Ra'(c_R)}\right), \tag{40}$$

where

$$f(c_R,q)=\frac{1}{b}\left(1-\frac{a}{q^2}\frac{b+2\sqrt{ab}+q^2}{a+b+2\sqrt{ab}}\right),$$

$$a=q^2\left(1-c_R^2c_L^{-2}\right),$$

$$b=q^2+(ic_Rq\tau+1)l^{-2}+\eta_L iq^3c_R^3\left(\rho\, c_L^4\right)^{-1},$$

$$\beta=c_R^2/c_L^2,\quad \zeta=c_L^2/c_T^2,\quad Ra'(c_R)=\left.\frac{dRa}{d(\omega^2/q^2)}\right|_{\omega^2/k^2=c_R^2}.$$

If $\omega_R^2\eta D\ll\rho c_T^4$ and $c_Rq\gg Dq^2+\tau^{-1}$, from Eq. (40) we have

$$\omega^2=q^2c_R^2-iq|\delta|\,\frac{c_R(1-\beta\zeta)}{\tau c_L^2Ra'(c_R)}+iq^3\eta\frac{c_R\beta\zeta}{\rho}. \tag{41}$$

After decomposition of Eq. (41) into real and imaginary parts, for the frequency of SAWs, we have the familiar expression $\omega_R=c_Rq$. The increment $\Gamma=\mathrm{Im}(\omega)$ of SAWs obtained from Eq. (41) is

$$\Gamma=\eta q^2\frac{\beta\zeta}{2\rho}-|\delta|\frac{(1-\beta\zeta)}{2\tau c_L^2Ra'(c_R)}. \tag{42}$$

The dispersion Eq. (42) describes an instability of the amplitude of acoustic waves.

In the absence of defect-strain interaction ($\delta=0$), waves decay because of viscosity of the medium. The excitation of SAWs as a result of the instability occurs if

$$|\delta|>\delta_{cr}=\eta\tau q^2\frac{\beta\,\zeta\,c_L^2Ra'(c_R)}{\rho(1-\beta\,\zeta)}.$$

It follows that the SAW frequency is hardly changed, but the increment is renormalized.

If $c_Lq\ll\tau^{-1}(l^2q^2+1)$, from Eq. (40) we have

$$\mathrm{Re}\left(\omega_{1,2}\right)=qc_R\left(1-\delta\frac{1-(1-\beta)/\varphi(\beta)}{(q^2l^2+1)c_R^2R'(c_R)}\right)^{1/2},$$

$$\varphi(\beta)=\frac{2\left(1+\sqrt{1-\beta}\right)-\beta}{2+\sqrt{1-\beta}}.$$

It is seen that, in this case, there is a softening (as with the infinite volume) of frequencies of acoustic waves [instability of frequencies $\mathrm{Re}\left(\omega_{1,2}\right)\to 0$, $\mathrm{Im}\left(\omega_{1,2}\right)>0$], and this is related to taking into account the defect-strain interaction. This reduction of frequency does not occur up to zero, and up to value $\omega_\eta=\sigma\eta q^2/2\rho c_L^2\ll c_Rq$, where $0.87<\sigma<0.95$.[17]

Numerical values of $s=c/c_T$ (non-dimensional Rayleigh wave velocity) have been calculated from Eq. (35) for $\omega=10^6s^{-1}$ and typical values (for *Si*) of the constants: $l^2=10^{-8}\mathrm{cm}^2$, $\tau=10^{-4}s$ (at $T=1.6\times10^3\mathrm{K}$). The results are presented in Fig. 2 for different values of δ and ζ. Figure 2 shows that the velocity of Rayleigh waves in the presence of atomic defect generation in an elastic layer decreases when the value of δ increases for a particular value of ζ. Also, for a particular value of δ, the wave velocity increases with the increase of ζ.

Equation (38) leads to the dispersion equation for static structures [on the condition that $\Gamma^2/c_{T,L}^2\ll q^2$, $(\Gamma+\tau^{-1})/D$, $\delta>0$, $\mathrm{Re}(\omega)=0$] in the form

$$\Gamma=\left(\frac{\delta}{1-\zeta^{-1}}-l^2q^2-1\right)\tau^{-1}. \tag{43}$$

From expression (43), it follows that, if $\delta\geq\delta_{cr}=1-\zeta^{-1}$, i.e.,

$$g\geq g_{cr}^{(s)}=\rho\,c_L^2\left(1-\zeta^{-1}\right)k_BT/\tau\vartheta_d\left(\vartheta_g-\vartheta_m\right),$$

there is a range of wave numbers,

$$0<q<q_0=l^{-1}\sqrt{\delta\left(1-\zeta^{-1}\right)^{-1}-1},$$

in which $\Gamma>0$, i.e., the amplitudes of the Fourier harmonics of the perturbations increase with time without limit. This gives rise to an instability and its growth results in the formation of coupled periodic structures of defect density and strain on surface. Dependences of an increment Γ of a defect-strain surface structure on the wave number are plotted in Fig. 3 for different values of defect-strain coupling constant (δ) in accordance with formula (43). Comparing Figs. 1 and 3, we see that the maximum growth increment for the case of surface structures is greater, and the critical pumping parameter is less.

The period of surface static structures is

$$d_{latt}^{(s)}=2\pi l\sqrt{\frac{g_{cr}^{(s)}}{g-g_{cr}^{(s)}}}. \tag{44}$$

The value of the maximum increment is

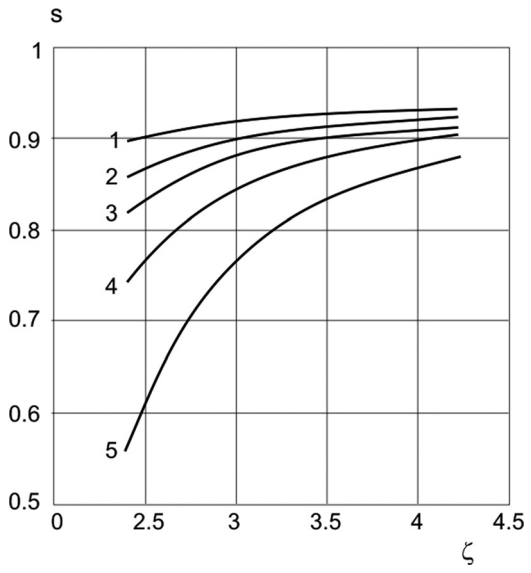

FIG. 2. Rayleigh wave velocity as a function of ζ for different δ values: 0 (line 1), 0.2 (line 2), 0.3 (line 3), 0.4 (line 4), and 0.5 (line 5).

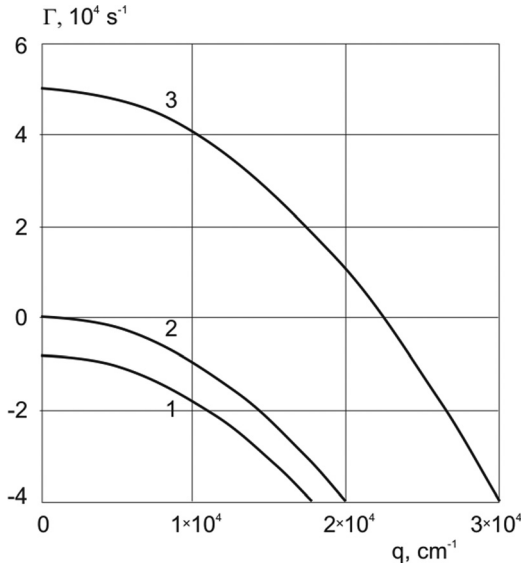

FIG. 3. Dependence of the growth increment (Γ) of surface structures for different values of defect-strain coupling constant: $\delta = 0.1$ (line 1), $\delta = 0.65$ (line 2), and $\delta = 4$ (line 3).

$$\Gamma_{\max} = \delta\left(1 - \zeta^{-1}\right)^{-1} - \tau^{-1}.$$

Then the critical value of the pump parameter is found from the condition $\Gamma_{\max} = 0$:

$$\delta_{cr} = (\zeta - 1)/\zeta.$$

The formation time of a surface elastic-concentration structure is given by $\tau_s \approx \Gamma_{\max}^{-1} = \tau(\delta/\delta_{cr} - 1)^{-1}$, which tends to infinity near an instability threshold ($\delta \to \delta_{cr}$) (critical slowing down occurs at a phase transition). When the threshold is exceeded significantly ($\delta \gg \delta_{cr}$), we find that $\tau_s = \tau\delta_{cr}\delta^{-1}$.

For antisymmetic modes, the ratio of hyperbolic tangents in Eq. (34) may by approximated to unity and Eq. (34) reduces to Eq. (35).

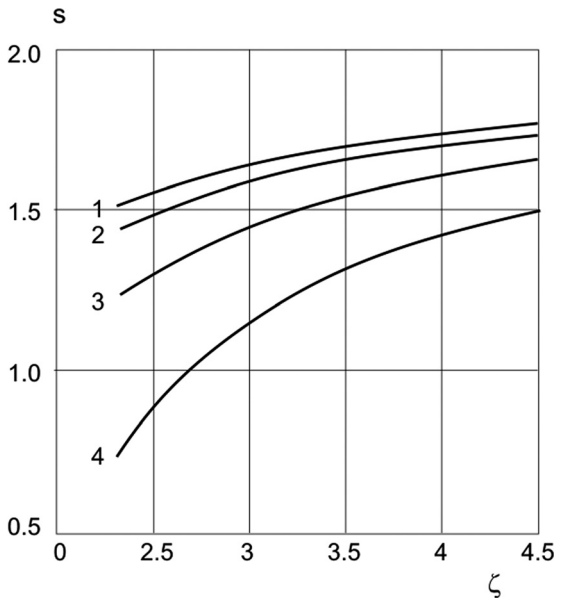

FIG. 4. Wave velocity in a thin plate as a function of ζ for different δ values: 0 (line 1), 0.1 (line 2), 0.3 (line 3), and 0.5 (line 4).

From the results presented, we observe that the influence of coupling between the strain and defect fields affects the phase velocity of the wave motion and the amplification factor as well.

Consider now the case of very long waves, by making use of the approximation $\tanh x \approx x - x^3/3$.

So, (28) for symmetrical motion becomes

$$4\alpha_1^2\alpha_2^2 - \left(2 - c^2c_T^{-2}\right)^2\left[\alpha_1^2 + \alpha_2^2 - q^2(1 - c^2c_L^{-2})\right]q^2 = 0. \tag{45}$$

Eq. (45) determines the vibration of thin plates with defect generation. When the medium is free from defects we have $\alpha_1^2 = q^2\left(1 - c^2/c_L^2\right)$ and we get the classical results of Rayleigh:

$$\left(2 - c^2c_T^{-2}\right)^2 - 4\left(1 - c^2c_L^{-2}\right) = 0.$$

For small frequency waves, we ignore higher degree terms. In view of this approximation, Eq. (45) becomes

$$\left(2 - \frac{c^2}{c_T^2}\right)^2 = 4\left(1 - \frac{c^2}{\tilde{c}_L^2}\right)^2, \tag{46}$$

where $\tilde{c}_L^2 = c_L^2(1 - \delta)$. From (46) we obtain

$$c = 2c_T\{1 - 1/\zeta(1 - \delta)\}^{1/2}.$$

It is observed that the wave velocity in thin plates decreases monotonically with increasing values of δ for a particular value of ζ (Fig. 4). We note that for a particular value of δ, the wave velocity increases with the increases of values of ζ.

For antisymmetic modes, Eq. (34) reduces to

$$\begin{aligned}&\left(2 - c^2c_T^{-2}\right)^2\left[\alpha_2\left(\alpha_2^2 + q^2\left(c^2c_L^{-2} - 1\right)\right)\left(1 - \frac{1}{3} - h^2\alpha_1^2\right)\right.\\&\left.-\alpha_1\left(\alpha_1^2 + q^2\left(c^2c_L^{-2} - 1\right)\right)\left(1 - \frac{1}{3}h^2\alpha_2^2\right)\right]\\&= 4\,q\left(1 - c^2c_T^{-2}\right)\left(\alpha_2^2 - \alpha_1^2\right)\left(1 - \frac{1}{3}h^2\alpha_3^2\right).\end{aligned} \tag{47}$$

Eq. (47) can be considered a modified version of the classical result obtained by Rayleigh in an elastic plate with atomic defect generation. If the plate is free from defects, (47) transforms to

$$c^2/c_T^2 = \left(\frac{4}{3}\right)q^2h^2\left(1 - \varsigma^{-2}\right),$$

which is the classical result of Rayleigh.[17]

VIII. CONCLUSIONS

In the present work, the problem of elastic wave propagation in an isotropic solid layer containing a distribution of non-equilibrium atomic defects (interstitial atoms, vacancies, and electron-hole pairs) has been investigated. The formation of atomic defects occurs as a result of the action of intense external energy fluxes (laser radiations) on the solid layers.

The analysis is based on coupled evolution equations for the elastic displacement and defect density fields. The defect dynamics is governed by the strain-stimulated generation, transport, and annihilation processes. We have provided an exact formal solution for the displacements and defect density fields in an infinite layer of finite thickness. We have obtained dispersion equations corresponding to the symmetric and antisymmetric modes of vibration of the layer assuming that the boundaries of the layer are stress-free. We used Lame's potentials to derive the dispersion equations. A procedure for determining the phase velocity and the amplification factor has been discussed. The proposed analysis is applied for the special cases of very short and very long waves. The phase velocity and attenuation constants of the waves get modified because of the generation–recombination effects in a defect subsystem. Relevant results of previous investigations are deduced as special cases.

We also showed that at certain conditions elastic-concentration instabilities in a system of atomic point defects on the surface of the plate can be developed. The underlying idea of the model of an instability considered in this paper is related to a reduction in the activation (formation and migration) energy of atomic defects under the influence of elastic fields. This influence modulates the rates of generation (or recombination) processes. At small concentrations of atomic defects, their return influence on strain fields is insignificant and can be ignored. However, under strongly non-equilibrium conditions, when the pump parameter is $g > g_{cr}$ (or the density of atomic defects $n > n_{cr}$), it is necessary to take into account that the processes of recombination or generation react on the stress fields. The equation for the kinetics of point defects is then supplemented by an equation from the theory of elasticity.

We have observed that if the pump parameter is above the critical value, because of strain-defect instability, a coupled elastic-defect periodic structure on the surfaces of plates arises. A mechanism on the development of the instability is because of the coupling between defect dynamics and the elastic field of the solids. Laser radiation (or, in general, a flux of particles) generates high concentrations of atomic defects in the surface layer of the irradiated material. When a fluctuation harmonic of the elastic deformation field appears in a medium because of the generation of atomic defects, the activation energies of formation and migration of the defects are modulated and a strain-induced drift of atomic defects occurs. This is a consequence of defect-strain interaction. The associated modulation of the rates of defect generation (recombination) and strain-induced flux of defects gives rise to periodic spatial–temporal fields of the defect concentration. The redistribution of defects creates forces proportional to their gradients. These forces lead in turn to additional growth of strain fluctuations. When the defect density or a critical rate of defect generation exceeds the critical value, elastic-concentration instabilities develop as a result of positive feedback, which result in the formation of ordered concentration-strain structures.

The limiting case of these instabilities, when the spatial inhomogeneity of the defect distribution is the result solely of the strain-induced fluxes, can be called the diffusion-elastic instability and the case in which only modulation of the defect generation rate is important can be considered as the generation–elastic instability. It follows that the system of atomic defects with the elastic interaction is internally unstable against a transition to a spatially inhomogeneous state.

We have shown that, the increment of the elastic-concentration structures is proportional to pump parameter. The criterion of self - organization of atomic defects and the dependence of a lattice period on temperature of medium are determined analytically. The growth in period with increase in temperature is related to a decrease in the life-time of atomic defects. For a fixed value of the temperature field the period of the lattice decreases with increase in pump parameter. The critical value of the pump parameter for self-organization of periodic structures is governed by the dilatation volume of defects, by the potential energy of elastic interaction, and the elastic parameters of the medium.

As an example let us consider the formation of concentration-strain structures in laser-irradiated semiconductors (in particular, *Si*). To evaluate the concentration of generated lattice defects (n_0), we consider here conditions when the duration of a laser pulse (τ_{Las}) exceeds the defect-relaxation time (τ). In this case the density of defects on the surface of the solid reaches a steady-state value $n_0 = g_0\tau = g^{(0)}\tau^{(0)} \exp\left(-w_{f0}/k_BT_0\right)$, where $w_{f0} = w_{f0} - w_{m0}$, $g_0 = g(T_0)$ is the defect generation rate, T_0 is the steady-state value of the temperature field on the surface, $g^{(0)}$ and $\tau^{(0)}$ are constants.

If $I_{Las} = \text{const}$ (uniform irradiation) and the optical absorption length (α_{Las}^{-1}) sufficiently less than heat diffusion length $l_T = (\chi\tau_{Las})^{1/2}$, the maximum temperature rise cat the substrate surface owing to the laser pulse action may be evaluated[16] as

$$T_0 = \frac{2(1-R)I_0}{\lambda_T}\sqrt{\frac{\chi\tau_{Las}}{\pi}}, \tag{48}$$

where R is the reflectivity coefficient, λ_T the thermal conductivity coefficient; χ the thermal diffusion coefficient. So putting values into (48) for *Si* ($\lambda_T = 0.8\,W/(cm^0K)$, $\chi = 0.4\ cm^2/s$, $I_0 = 10^5\text{W/cm}^2$, pulse duration $2\times10^{-3}s$ and $R = 0.4$ one can get $T_0 = 1.6\times10^3\ ^0\text{K}$. Then, taking $\Omega = 2\times10^{-22}\text{cm}^3$, $w_{d0} = 1\text{eV}$, a value of $10^{19}1/\text{cm}^3$ may be estimated for the critical density of defect concentration ($n_{cr} = g_{cr}\tau$), which is several orders of magnitude less than the concentration of the host atoms and shows that this mechanism of the formation of ordered structures may be realized on practice. For the period (d_{latt}) of the resultant surface structure we have an estimate 0.9 μm, which follows from expression (44) (for typical values of parameters $\rho = 2.3\ \text{g/cm}^3$, $\lambda = 6.4\times10^{10}$ Pa, $\mu = 7.9\times10^{10}$ Pa, $|\vartheta_g| = 15$ eV, $|\vartheta_m| = 10$ eV, $|\vartheta_d| = 10^2$ eV, $\zeta = 3$, $g = 1.1g_{cr}^s$, $D = 10^{-5}\ \text{cm}^2/\text{s}$ and $\tau = 3\times10^{-4}$ s). The maximum growth increment of the instability is $\Gamma_{\max} = 2\times10^4\ \text{s}^{-1}$; the instability increment exceeds the reciprocal of the duration of a laser pulse acting on the surface of a solid ($\Gamma_{\max}\tau_{Las} \gg 1$).

In addition to one-dimensional (or two-dimensional) lattices, the growth of the elastic-concentration instability on the surface can also create structures of different types. If the laser field is radially symmetric relative to the z axis, then surface structures in the form of radial rays and rings are

formed. Then, the substitution $\exp(iqx_1) \rightarrow J_m(qr)\cos(m\varphi)$ is made in solutions (16); here, J_m is a Bessel function of the first kind and of order m (m is an integer). The specific case of $m = 0$ corresponds to structures which are concentric rings; r and φ are the polar coordinates in the x_1x_2 -plane. An analysis similar to that given in Sec. IV leads to a dispersion equation which is identical with Eq. (34), and the increment is independent of the number of a harmonic.

The degeneracy with respect to m is lifted if the Gaussian dependence of the laser radiation intensity on the radial coordinate, $I = I_0 \exp\left(-r^2/r_0^2\right)$, is taken into account. This leads to the appearance of structures in the form of radial rays, and it is then found that $\exp(ikx_1) \rightarrow (r/r_0)^m \cos(m\varphi)$ $\exp\left(-r^2/r_0^2\right)$.

The dispersion dependences for these structures are again described by Eq. (34), but with q^2 replaced by $\tilde{q}^2 = 4m/r_0^2$. The maximum number of rays in a structure is deduced from the condition $q_m^2 = \tilde{q}^2$ is $m \approx \pi^2 r_0^2/d_{latt}^2$.

The appearance of ordered surface-relief structures in the form of radial and ring rays was detected experimentally[18] when the surface of nickel was irradiated in air by pulses from a solid-state laser ($\lambda_{Las} = 1.06\,\mu$m, duration $\tau_{Las} = 1.6 \times 10^{-3}$ s) with a Gaussian distribution and with the maximum intensity $I = 5 \times 10^5$ W/cm^2. For $r_0 = 3 \times 10^{-2}$ cm and $d_{latt} = 30\ \mu$m, the maximum number of rays is 10^3, roughly in agreement with the experimental results.[18] Radial ring periodic structures of the precipitate have been also observed after laser precipitation of ions from the liquid phase on the surfaces of metals in Ref. 19.

We considered the self-organization of atomic defects of same jth type because of the interaction of these defects through the self-consistent strain field. In an irradiated medium, defects are generated as pairs of interstitial atoms ($j = i$) and vacancies ($j = v$). Usually, the inequality $\vartheta_i \gg \vartheta_v$ is met for such processes. Therefore, we kept only the term with $\vartheta_d^{(i)} \equiv \vartheta_d > 0$ in Eqs. (1) and (2) for self-consistent displacements. Interstitial atoms form self-consistent periodic structures with defect-concentration maxima within the regions where strain field $divu_{ll} = \zeta > 0$. Vacancy defects are accumulated within the regions where $\zeta < 0$. Thus, double interstitial atom-vacancy periodic structures are formed.

Clearly, the linear theory considered here describes the early stage of the development of an instability only. However, the nature of generated ordered structures (because of an instability) and the amplitudes of these structures as functions on material and irradiation conditions can only be determined by considering the influence of nonlinear effects in the model. In this connection, computer simulation of fluctuation development is needed.

For the elastic-concentration instabilities, ordered structures of different types may appear depending on the parameters of the radiation and the sample itself. Formation of one-dimensional lattices, and of concentric-ring and radial structures, is governed by the spatial characteristics of the laser radiation (uniform irradiation of the whole sample, a laser spot with a uniform intensity distribution, a circular spot with a radial intensity distribution). More complex structures can also be obtained. Variation of the spatial characteristics of the laser beam provides an effective means for the control of the formation of specific (specified in advance) surface structures.

It is noted that the model developed in this paper can also be used to study the generation of thermal-elastic waves in irradiated insulating and semiconducting wafers. In this case, the roles of the formation energy and the defect-relaxation time can be played by the bandgap of semiconductor (or insulator) and by the characteristic time of heat exchange with the surrounding medium (the thermal-relaxation time), respectively.

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