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Geometric properties of the single-diode photovoltaic model and a new very simple method for parameters extraction

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ABSTRACT

One of the most important models to predict the electrical behavior of a photovoltaic (PV) module is the so-called single-diode model. This model is derived from the electrical equivalent circuit formed by a current source in parallel with one diode, a shunt resistor and a series resistor. The model equation depends on five parameters, if these parameters are obtained, it has been tested that this model fits accurately the real behavior of the PV module under a minimum of illumination. Nevertheless, the extraction of the parameters is quite difficult since none of the variables in the model equation can be expressed in explicit form. This fact also implies the difficulty of knowing the real properties of the current as an implicit function of the voltage in this model. Knowing these properties deeply will involve a more suitable use of the model and better understanding of the behavior of the photovoltaic module. The first goal of this paper is the rigorous mathematical study of the model. In particular, it is demonstrated that the model equation actually defines the current as an implicit function of the voltage which is indefinitely differentiable along the real line. We will provide the most significant geometric properties of the current function by means of the study of the first and the second derivatives functions which are also implicitly given. The second goal of the paper is, given real data of voltage and current measured from a PV module, to show how the parameters of the model equation can be extracted in a very simple way, giving rise to an estimated curve which fits accurately the real one. The proposed new analytical method is as good, for instance, as the well-known analytical five point method but significantly simpler.

1. Introduction

The single-diode equivalent electrical circuit of a photovoltaic (PV) module with N_p cells in parallel and N_s cells in series is depicted in Fig. 1.

At a given illumination, the relationship between the current i (measured in Amperes [A]) and the voltage v (measured in Volts [V]) obtained from the previous circuit is given by the model equation

$$i = N_p I_{ph} - N_p I_{sat}\left(\exp\left(\frac{1}{nV_T}\left(\frac{v}{N_s}+\frac{iR_s}{N_p}\right)\right)-1\right) - N_p\frac{1}{R_{sh}}\left(\frac{v}{N_s}+\frac{iR_s}{N_p}\right) \tag{1}$$

where I_{ph} is the photocurrent measured in A; I_{sat} is the diode saturation current measured in A; R_s is the series resistance measured in Ohms [Ω]; R_{sh} is the shunt resistance measured in Ω; n is the diode ideality factor; $V_T = (k/q)T$, where T is the temperature of the cells measured in Kelvin degrees [K], $k = 1.3806488 \times 10^{-23}$ is the Boltzmann's constant measured in Joules per Kelvin [J]/[K] and, $q = 1.60217653 \times 10^{-19}$ is the electronic charge measured in Coulombs [C].

The solutions (v, i) of the previous equation generate a curve in the Euclidean plane. We will refer to this theoretical curve as i–v curve to distinguish it from the real I–V curve (or I–V characteristic) generated by the PV module.

The theoretical i–v curve fits the real I–V characteristic of most of the PV modules with very good accuracy (see, for instance, Refs. [1,2]) under a minimum of illumination (about half AM1 according to [2]). If the five parameters I_{ph}, I_{sat}, n, R_s, and R_{sh} are extracted for certain conditions of irradiance and temperature, the electrical behavior of the PV module can be precisely predicted for any other conditions. A large amount of papers (see, among others, Refs. [3–28]) have dealt with the problem of

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extracting the previous parameters using different methodologies. Such a quantity of papers dealing with this model demonstrate its importance and its acceptance for modeling a PV module.

Some of the previous reported methodologies consist on solving a system of five non-linear equations obtained after substituting real data, measured on the I–V curve, on the model equation and on the equation obtained derivating the previous one (see, Refs. [4–6,18]). Another kind of methodology consists on obtaining the best theoretical i–v curve fitting the real one by solving an optimization problem [2,7–10,13,24]. There are also methods which use non-elementary functions as the Lambert W function to extract the parameters in a different way [19,23,25].

Some papers dealing with simplified models of three and four parameters [26–28], have been able to obtain exactly and explicitly the corresponding parameters just by using information of the I–V curve near the operating point of the PV module.

We would like to point out that to properly use a model, it is important to know its mathematical properties such as differentiability, boundedness, sign of the function and its derivatives, asymptotic behavior, etc. which provide its geometric properties. The knowledge of these properties help researchers to better understand the electrical behavior of the PV modules, as well as, to study new techniques to extract the parameters. This paper aims to study deeply some of these properties and, taking the advantage of the theoretical knowledge, to provide a new method to obtain easily, quickly and accurately the parameters of the single-diode model taking real measurements of a PV module.

Before starting with the mentioned results, let us rewrite Eq. (1) in a more simplified way which depends on new five parameters gathering the original ones in a reduced form.

If we denote

$$
\begin{aligned}
A &:= \frac{N_p I_{ph} R_{sh}}{R_{sh}+R_s} \\
B &:= \frac{N_p I_s R_{sh}}{R_{sh}+R_s} \\
C &:= \exp\left(\frac{1}{N_s n V_T}\right) \\
D &:= \exp\left(\frac{R_s}{N_p n V_T}\right) \\
E &:= \frac{N_p}{N_s}\,\frac{1}{R_{sh}+R_s}
\end{aligned}
\tag{2}
$$

Eq. (1) becomes

$$i = A - B\left(C^v D^i - 1\right) - Ev \tag{3}$$

Since the parameters I_{ph}, I_{sat}, n, R_s, and R_{sh} are strictly positive, the constants in the previous equation satisfy

$$A>0, B>0, C>1, D>1, E>0 \tag{4}$$

If the new parameters are extracted, the original ones can be retrieved as:

$$
\begin{aligned}
I_{ph} &= \frac{1}{N_p}\,\frac{A\ln C}{\ln C - E\ln D} \\
I_s &= \frac{1}{N_p}\,\frac{B\ln C}{\ln C - E\ln D} \\
nV_T &= \frac{1}{N_s}\,\frac{1}{\ln C} \\
R_s &= \frac{N_p}{N_s}\,\frac{\ln D}{\ln C} \\
R_{sh} &= \frac{N_p}{N_s}\left(\frac{1}{E} - \frac{\ln D}{\ln C}\right)
\end{aligned}
\tag{5}
$$

2. Properties of the intensity function defined by the single-diode model

In this section, let us prove that for a voltage v, the model equation (3) gives a unique intensity value i providing then a real function $i(v)$ which can be derivated as many times as wanted at any voltage v. We will prove that $i(v)$ is strictly decreasing and strictly concave. Moreover, we will see that the second derivative function has a local (indeed global) minimum. The behavior of the functions i, i' and i'' when v tends to ∞ will also be studied showing that function i' has horizontal asymptotes at right and left and, function i has an oblique asymptote at left which is often perceptible near the short circuit point. This asymptotic behavior will give the key tool to develop a new analytical method to extract the parameters of the single-diode model.

2.1. Single-diode model equation defines i as a function of v

First, let us prove that variable i in Eq. (3) can be seen as a function of v along the real line R which is, moreover, indefinitely differentiable at any point of R.

Given an open subset Λ of the n-dimensional Euclidean space R^n, a real function defined in Λ is said to be of class $C^\infty(\Lambda)$ if it has continuous partial derivatives of any order at any point of Λ.

Theorem 1. *Eq. (3) defines i as an implicit function of v of class $C^\infty(R)$.*

Proof. Consider the function $F(v,i) := i - A + B(C^v D^i - 1) + Ev$. Observe that $F(v,i) = 0$ if and only if (v,i) is a solution of (3).

Let us divide the proof into two steps.

Step 1: Eq. (3) defines i as a function of v.

Given $v_0 \in R$, define the function $F_{v_0}(i) := F(v_0, i)$ which is differentiable with $F'_{v_0}(i) = 1 + BC^{v_0}D^i \ln D + E > 0$.

For $v_0 \geq 0$, taking $i_1 := -Ev_0 - v_0 \log_D C$ and $i_2 := A + B$ one has $F_{v_0}(i_1) < 0$ *and* $F_{v_0}(i_2) > 0$.

For $v_0 < 0$, taking $i_3 := v_0 \log_D C$ and $i_4 := A + B - Ev_0$ one has $F_{v_0}(i_3) < 0$ and $F_{v_0}(i_4) > 0$.

From Bolzano's theorem and the fact that F_{v_0} is strictly increasing, one has for any $v_0 \in R$ a unique $i_0 \in R$ such that $F(v_0, i_0) = F_{v_0}(i_0) = 0$. So, we can define the function $i : R \to R$ which assigns to each v the unique i satisfying $F(v,i) = 0$.

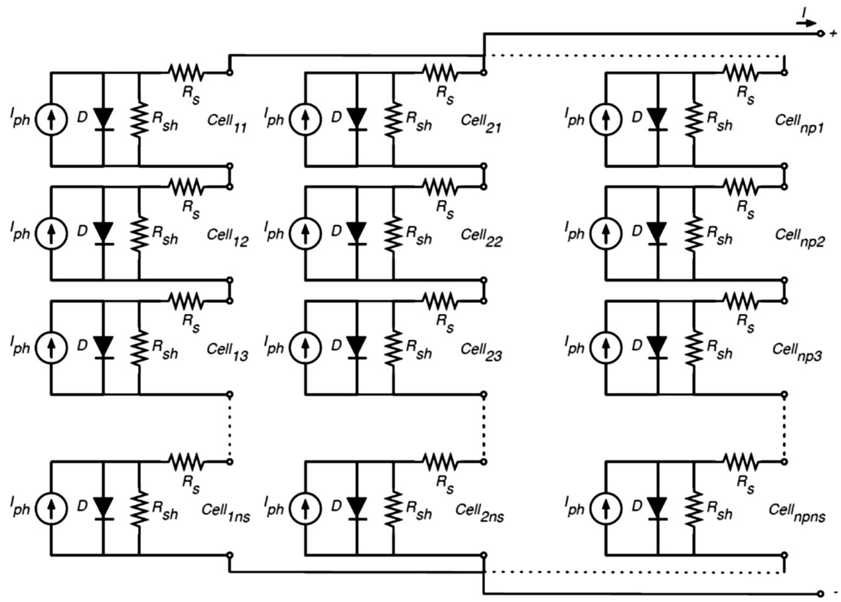

Fig. 1. PV module single-diode model equivalent electrical circuit.

Step 2: the function i is of class $C^\infty(R)$

Given an arbitrary $v \in R$, we have that

$F(v, i(v)) = 0$

$\frac{\partial F}{\partial i}(v, i(v)) = 1 + BC^v D^{i(v)} \ln D + E > 0$

F is of class $C^\infty(R^2)$

Implicit function theorem ensures the existence of a neighborhood $M \times N$ of $(v, i(v))$ and a unique function $\phi : M \to N$ of class $C^\infty(M)$ such that $\varphi(v) = i(v)$ and $F(v, \varphi(v)) = 0$ for all $v \in M$. The arbitrariness of $v \in R$ implies that i is of class $C^\infty(R)$.

2.2. *The sign of the derivatives of the function i*

Let us determine the sign of the derivative functions of i. Let us prove that the first derivative is lower and upper bounded. We will see in the next subsection that the obtained bounds are the tightest ones.

Theorem 2. *One has the following properties:*

(1) $i' < 0$ *and, moreover,*

$$-\frac{\ln C}{\ln D} < i'(v) < -E \tag{6}$$

(2) $i'' < 0$

(3) $i'''(v_{0\text{jerk}}) = 0$, $i'''(v) < 0$ *for* $v < v_{0\text{jerk}}$, *and* $i'''(v) > 0$ *for* $v > v_{0\text{jerk}}$, *where*

$$v_{0\text{jerk}} = \frac{\frac{1}{2} - (A+B)\ln D - \ln(2B \ln D)}{\ln C - E \ln D}$$

Proof of (1). The first assertion of (1) is obtained just derivating with respect to v in Eq. (3) and clearing i'

$$i' = -\frac{E + BC^v D^i \ln C}{1 + BC^v D^i \ln D} < 0 \tag{7}$$

Now, from the previous expression and the fact that $E < \ln C / \ln D$, one obtains the bounds of (6).

Proof of (2). Derivating with respect to v in (7) and clearing i'' we obtain

$$i'' = -\frac{BC^v D^i (\ln C + i' \ln D)^2}{1 + BC^v D^i \ln D} < 0 \tag{8}$$

Proof of (3). From (7), we have

$$-BC^v D^i (\ln C + i' \ln D) = i' + E \tag{9}$$

Substituting this expression in (8) and clearing i'' one obtains

$$i'' = \frac{(\ln C + i' \ln D)^2}{\ln C - E \ln D}(i' + E) \tag{10}$$

and, derivating this expression with respect to v one has

$$i''' = i'' \frac{\ln C + i' \ln D}{\ln C - E \ln D}(\ln C + i' \ln D + 2(i' + E)\ln D)$$

From (8) we know that $i'' < 0$, and from (6) we know that

$$\ln C - E \ln D > \ln C + i' \ln D > 0$$

so

$$i''' = 0 \Leftrightarrow \ln C + i' \ln D + 2(i' + E)\ln D = 0 \Leftrightarrow i' = \frac{-1}{3}\frac{\ln C}{\ln D} - \frac{2}{3}E$$

Denote

$$i'_{0\text{jerk}} := \frac{-1}{3}\frac{\ln C}{\ln D} - 2E$$

and call $v_{0\text{jerk}}$ and $i_{0\text{jerk}}$ the voltage and the current associated to $i'_{0\text{jerk}}$, respectively.

From (3) we can write

$$-BC^v D^i = i - A - B + Ev$$

and, substituting this expression in (9) one obtains

$$i = \frac{i' + E}{\ln C + i' \ln D} + A + B - Ev$$

In particular,

$$i_{0\text{jerk}} = \frac{-1}{2\ln D} + A + B - Ev_{0\text{jerk}}$$

Substituting $v_{0\text{jerk}}$ and the previous value in (3) we obtain

$$v_{0\text{jerk}} = \frac{\frac{1}{2} - (A+B)\ln D - \ln(2B\ln D)}{\ln C - E\ln D}$$

Now, let us study the sign of i'''.

Since i' is strictly decreasing and continuous, one has

$$i'(v) < i_{0\text{jerk}} \Leftrightarrow v > v_{0\text{jerk}} \quad \text{and} \quad i'(v) > i_{0\text{jerk}} \Leftrightarrow v < v_{0\text{jerk}}$$

Moreover, i''' is also continuous and, so, Bolzano's theorem ensures that i''' has constant sign below and above $v_{0\text{jerk}}$.

For

$$i'_1 := \frac{-1}{3}\frac{\ln C}{\ln D} - \frac{1}{3}E > i_{0\text{jerk}}$$

one obtains

$$i'''_1 = i''_1 \frac{\ln C + i'_1 \ln D}{\ln C - E\ln D}\left(\ln C + i'_1 \ln D + 2(i'_1 + E)\ln D\right) < 0$$

hence, $i'''(v) < 0$ for $v < v_{0\text{jerk}}$.

For

$$i'_2 := \frac{-2}{3}\frac{\ln C}{\ln D} - \frac{2}{3}E < i_{0\text{jerk}}$$

one obtains

$$i'''_2 = i''_2 \frac{\ln C + i'_2 \ln D}{\ln C - E\ln D}\left(\ln C + i'_2 \ln D + 2(i'_2 + E)\ln D\right) > 0$$

hence, $i'''(v) > 0$ *for* $v > v_{0\text{jerk}}$.

Remark 1. The following properties are consequence of the previous result.

- i is strictly decreasing and strictly concave
- i' is strictly decreasing and it is concave below $v_{0\text{jerk}}$ and convex above $v_{0\text{jerk}}$.
- i'' is strictly decreasing below $v_{0\text{jerk}}$ and strictly increasing above $v_{0\text{jerk}}$, so i'' attains a global minimum at $v_{0\text{jerk}}$ with value

$$i''_{0\text{jerk}} = \frac{-4}{27}\frac{1}{\ln D}(\ln C - E\ln D)^2$$

2.3. Behavior of the function i and its derivatives when v tends to infinite

Now let us study the asymptotic behavior of the functions i, i', and i''. Although in practice the interval of interest is $[0, v_{oc}]$, we will see how this study is applied to obtain a new analytical method to extract the parameters of the single-diode model.

Theorem 3. *One has the following asymptotic properties:*

(1) $\lim_{v\to-\infty} i(v) = +\infty$ *and* $\lim_{v\to+\infty} i(v) = -\infty$
(2) $\lim_{v\to-\infty} C^v D^i = 0$ *and* $\lim_{v\to+\infty} C^v D^i = +\infty$
(3) $\lim_{v\to-\infty} i'(v) = -E$ *and* $\lim_{v\to+\infty} i'(v) = -(\ln C/\ln D)$
(4) $\lim_{v\to\pm\infty} i''(v) = 0$

Proof of (1). Let us see that i is not upper bounded. Reasoning by contradiction, in other case there would exist a constant M with $i < M$ and, so $0 < C^v D^i < C^v D^M$. Since $\lim_{v\to-\infty} C^v = 0$, by the squeeze theorem we would have $\lim_{v\to-\infty} C^v D^i = 0$ and

$$\lim_{v\to-\infty} i(v) = \lim_{v\to-\infty} A + B - BC^v D^i - Ev = +\infty$$

leading to a contradiction. Therefore, i is not upper bounded and, since i is strictly decreasing

$$\lim_{v\to-\infty} i(v) = +\infty.$$

By other hand, we have $A + B - BC^v D^i - Ev < A + B - Ev$ and $\lim_{v\to+\infty} A + B - Ev = -\infty$, so

$$\lim_{v\to+\infty} i(v) = \lim_{v\to+\infty} A + B - BC^v D^i - Ev = -\infty$$

Proof of (2). Since $i < A + B - Ev$ then

$$0 < C^v D^i < e^{v(\ln C - E\ln D) + (A+B)\ln D}$$

From (6) we have $\ln C - E\ln D > 0$, then $\lim_{v\to-\infty} e^{v(\ln C - E\ln D) + (A+B)\ln D} = 0$ and, by the squeeze theorem

$$\lim_{v\to-\infty} C^v D^i = 0$$

By other hand, from (6) we have $\ln C + i' \ln D > 0$, then function $f(v) := C^v D^i$ satisfies

$$f'(v) = C^v D^i (\ln C + i' \ln D) > 0$$

then f is strictly increasing and, therefore, $\lim_{v\to+\infty} C^v D^i$ is finite or $+\infty$.

It is easy to see from (3) that

$$\ln\left(C^v D^i\right) = \frac{-i}{E}(\ln C - E\ln D) + \frac{\ln C}{E}\left(-BC^v D^i + A + B\right)$$

Then, reasoning by contradiction, if we would have $\lim_{v\to+\infty} C^v D^i = k > 0$, taking into account that $\lim_{v\to+\infty} i = -\infty$, we would obtain from the previous expression that

$$\ln(k) = +\infty + \left(\frac{\ln C}{E}\right)(-Bk + A + B)$$

which is a contradiction, therefore it must be

$$\lim_{v\to+\infty} C^v D^i = +\infty$$

Proof of (3). It is immediate from (7) and statement (2).

Proof of (4). It is immediate from (10) and statement (3).

The previous theorem suggests the following result.

Theorem 4. *The function*

$$L(v) = -Ev + A + B$$

is an oblique asymptote of i when v tends to $-\infty$.

Proof. From (3) and statement (2) of the previous theorem we have that

$$\lim_{v\to-\infty} \frac{i}{v} = \lim_{v\to-\infty} \frac{A+B}{v} - B\frac{C^v D^i}{v} - E = -E$$

and

$$\lim_{v\to-\infty} i - (-Ev) = A + B - BC^v D^i = A + B$$

Remark 2. It can be checked that $\lim_{v\to+\infty} i/v = -(\ln C/\ln D)$, nevertheless i has not an oblique asymptote when v tends to $+\infty$ because $\lim_{v\to+\infty} i - (-(\ln C/\ln D))v = +\infty$.

3. The oblique asymptote method: a new and simple method for the extraction of the five parameters of the single-diode model

In this section, let us provide a new analytical method, we will call the oblique asymptote method, to obtain the five parameters of the single-diode model with real measurements of a PV module. This method is based on Theorem 4, presented in the previous section. The method is very simple and just need to know:

- The short circuit point of the I–V curve: $(0, I_{sc})$.
- The slope of the I–V curve at the short circuit point: I'_{sc}.
- Three points (different of the previous one) of the I–V curve $(V_1, I_1), (V_2, I_2)$ and (V_3, I_3).

In theoretical conditions, these three points could be three arbitrary points in the I–V curve. Nevertheless, since real curves are not ideal, it is better to select these three points with a voltage greater than the maximum power point (at the right of this point in the curve) and preferably uniformly distributed.

The method is based on considering the approximation $A + B = I_{sc}$ and $E = -I'_{sc}$, in other words, the line $I_{sc} + I'_{sc}v$ is assumed to be the oblique asymptote of the single-diode model. The other parameters are obtained explicitly as follows:

Taking logarithms in Eq. (3) and denoting $f(v,i) := \ln(A + B - Ev - i)$, one has

$$\ln(B) + v\ln(C) + i\ln(D) = f(v,i)$$

Since the theoretical curve must pass through the points $(V_k, I_k),\ k = 1,2,3$, the following system must be satisfied

$$\ln(B) + V_k\ln(C) + I_k\ln(D) = f(V_k, I_k),\quad k = 1,2,3$$

This system is very easy to solve and provides the following solutions

$$D = \exp\left(\frac{(f(V_1,I_1) - f(V_2,I_2))(V_2 - V_3) - (f(V_2,I_2) - f(V_3,I_3))(V_1 - V_2)}{(I_1 - I_2)(V_2 - V_3) - (I_2 - I_3)(V_1 - V_2)}\right)$$

$$C = \exp\left(\frac{f(V_2,I_2) - f(V_3,I_3) - (I_2 - I_3)\ln(D)}{V_2 - V_3}\right)$$

$$B = \exp(f(V_1,I_1) - V_1\ln(C) - I_1\ln(D))$$

Finally, $A = I_{sc} - B$ and recall that $E = -I'_{sc}$.

The original parameters could now be extracted as in (5).

Remark 3. Assume a PV module with $N_s = N_p = 1$ and denote $R_{sh0} = -1/I'_{sc}$. The analytical five-point method [5] uses as a key assumption $R_{sh} = R_{sh0}$. We would point out that the assumption $E = -I'_{sc}$ of our method is similar to the previous one, just observing that $E = -I'_{sc}$ becomes $R_{sh} = R_{sh0} - R_s$. Exactly this approximation was already used in [6].

The main difference between our method and the analytical five-point method or the one proposed in [6], is that we do not need to know the slope of the I–V curve at the open circuit point $(V_{oc}, 0)$.

4. Experimental results

In this section, the oblique asymptote method (OAM) is applied to estimate the single-diode model parameters from a real PV module. The estimation will be compared with the one obtained from the well-known analytical five point method (5pM), in order to evaluate the goodness of the new method. This experimental test has been carried out with an I–V curve provided by the PV module manufacturer EURENER.

Tables 1 and 2 summarize the data used for calculation purposes, concretely, $P1$, $P2$ and $P3$ are three voltage–current points, and the other values in the tables have been described previously. In this test, the slope at short circuit point I'_{sc} and the slope at open circuit point I'_{oc} have been estimated applying linear regression analysis to the measured points near the corresponding points $(0, I_{sc})$ and $(V_{oc}, 0)$.

Table 1
Data for OAM.

I_{sc} [A]	P1 ([V], [A])	P2 ([V], [A])	P3 ([V], [A])	I'_{sc} [V]/[A]
8.479	(21.936, 7.850)	(24,780, 5.825)	(28.175, 0.016)	-4.986×10^{-3}

Table 2
Data for 5pM.

I_{sc} [A]	MPP ([V], [A])	V_{oc} [V]	I'_{sc} [V], [A]	I'_{oc} [V]/ [A]
8.479	(21.936, 7.850)	28.207	-4.986×10^{-3}	-2.085

Fig. 2 shows the real data and the estimated i–v curves applying OAM and 5pM. One can observe that both methods provide an accuracy estimation of the real curve.

In Tables 3 and 4 a detailed comparison of the estimated parameters is provided. It can be seen that all the parameters are very similar except the parameters I_s and B which present a slightly difference.

To compute the area between the real I–V curve and an estimated one, we use the following approximative technique, based on the trapezoidal rule, which takes into account if the segment between two consecutive points (V_k, I_k), (V_{k+1}, I_{k+1}) of the real I–V curve intersect the segment between the corresponding points (V_k, i_k), (V_{k+1}, i_{k+1}) of the estimated one inside the interval $[V_k, V_{k+1}]$. The voltage of the intersection point, which exists if $i_k - i_{k+1} - (I_k - I_{k+1}) \neq 0$, is given by

$$W_k = \frac{i_k V_{k+1} - i_{k+1} V_k - (I_k V_{k+1} - I_{k+1} V_k)}{i_k - i_{k+1} - (I_k - I_{k+1})}$$

To control if the previous voltage falls into the interval $[V_k, V_{k+1}]$, we define for $j = 1, 2$

$$V_{kj} = \begin{cases} W_k & \text{if } i_k - i_{k+1} - (I_k - I_{k+1}) \neq 0 \text{ and } V_k < w_k < V_{k+1} \\ (2-j)V_k + (j-1)V_{k+1} & \text{otherwise} \end{cases}$$

Table 3
Original parameters.

	I_{ph} [A]	I_s [A]	nVt [J C]	R_s [Ω]	R_{sh} [Ω]
OAM	8.492	2.147×10^{-8}	1.425	2.916×10^{-1}	2.003×10^{2}
5pM	8.493	9.186×10^{-9}	1.368	3.156×10^{-1}	2.006×10^{2}

Table 4
Simplified parameters.

	A	B	C	D	E
OAM	8.479	2.144×10^{-8}	2.018	1.227	4.986×10^{-3}
5pM	8.479	9.172×10^{-9}	2.078	1.260	4.979×10^{-3}

Table 5
Area between real and estimated curves.

A_{OAM} : 4.876×10^{-1}	A_{OPM}/A : 2.265×10^{-3}
A_{5pM} : 4.999×10^{-1}	A_{5pM}/A : 2.322×10^{-3}

The area between the segment connecting the real points (V_k, I_k), (V_{k+1}, I_{k+1}) and the segment connecting the estimated points (V_k, i_k), (V_{k+1}, i_{k+1}) is given by

$$A_k = \frac{|i_k - I_k|(V_{k2} - V_k) + |i_{k+1} - I_{k+1}|(V_{k+1} - V_{k1})}{2}$$

Assuming that we have n points of the PV module, an approximation of the area between the real I–V curve and an estimated one is given by

$$\sum_{k=1}^{n-1} A_k$$

Let us denote by A_{5pM} and A_{OAM} the previous area when the considered methods are the analytical five point method and the oblique asymptote one, and let us denote by A the area in the first

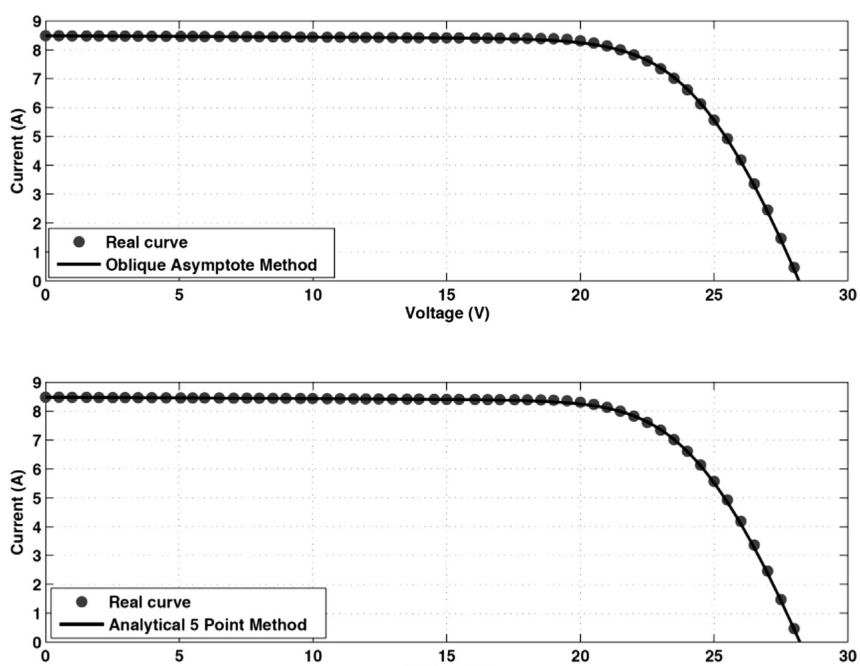

Fig. 2. Estimated i–v curves.

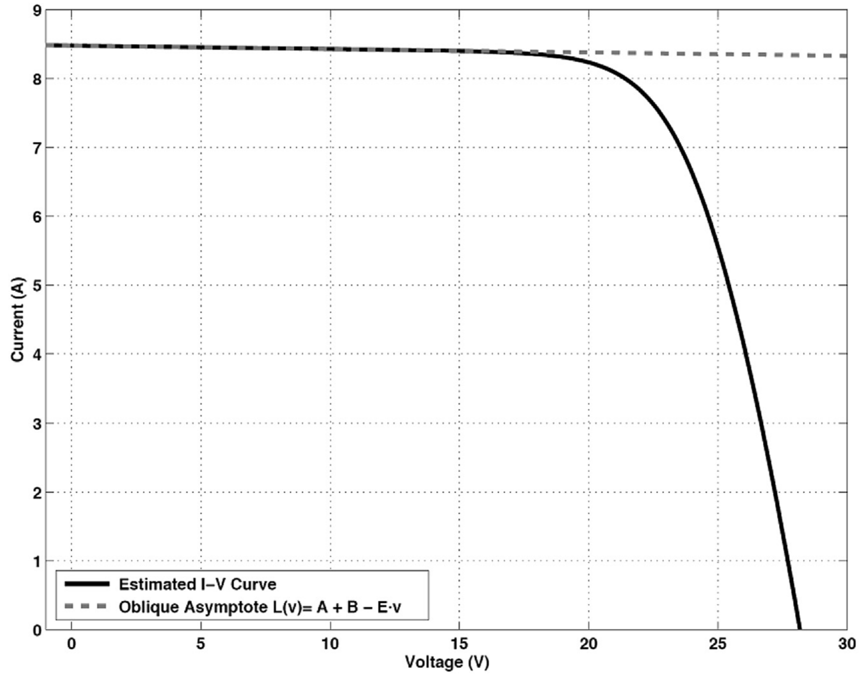

Fig. 3. Oblique asymptote.

quadrant below the real I–V curve computed approximately by the trapezoidal rule. Table 5 presents the values of the areas A_{5pM} and A_{OAM} as well as the relative ones with respect to A.

It is evident from the results that both methods fit very well the real I–V curve in terms of the area between the curves, in this case OAM is a little bit better than 5pM.

To finish this section, let us show graphically some of the theoretical results obtained in this paper. We provide three pictures based on the previous curve estimated by the oblique asymptote method given by the equation

$$i = 8.48 - 2.14 \times 10^{-8}\left(2.02^{v} \times 1.23^{i} - 1\right) - 4.99 \times 10^{-3} v$$

This curve has the following oblique asymptote at left

$$L(v) = 8.48 + 2.14 \times 10^{-8} - 4.99 \times 10^{-3} v$$

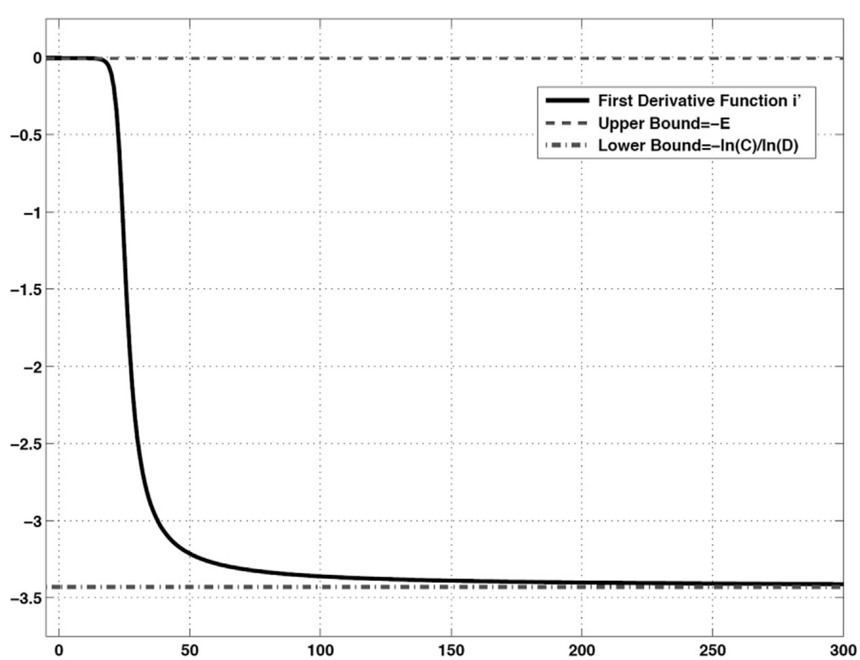

Fig. 4. First derivative and bounds.

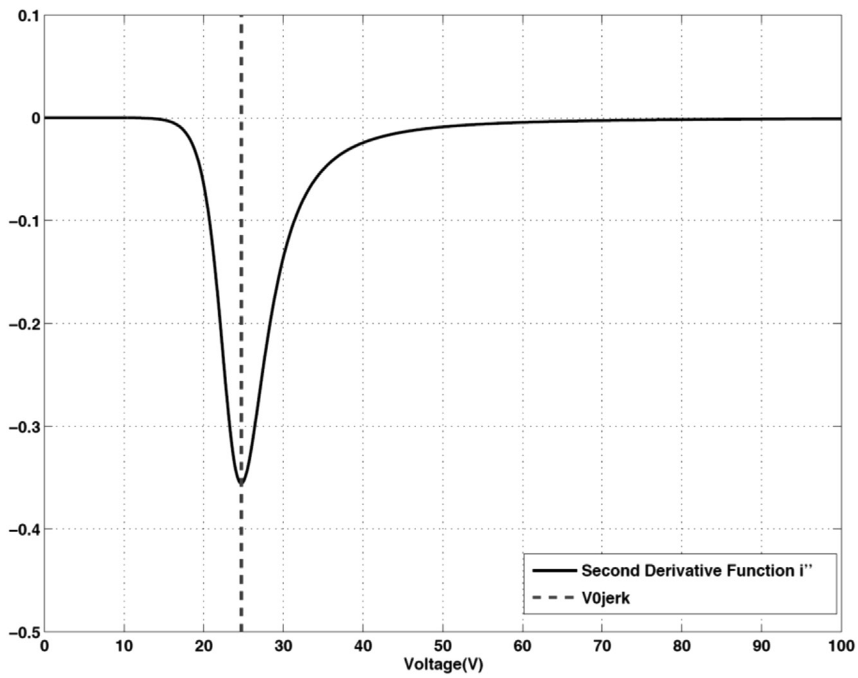

Fig. 5. Second derivative and $v_{0\text{jerk}}$.

and Fig. 3 shows the graph of the estimated i–v curve together with this asymptote.

In Fig. 4 it is drawn the first derivative function of the estimated curve as well as their horizontal asymptotes.

Fig. 5 shows the graph of the second derivative function attaining its minimum at $v_{0\text{jerk}} \approx 24.7$ V. It can also be observed the horizontal asymptote of this function.

5. Conclusions

The geometric properties of the single-diode model have been determined through a rigorous mathematical study of the model equation. Concretely, we have proved that the model equation gives for any voltage v a unique current i determining a function $i(v)$ which is indefinitely differentiable along the real line. So, we are enabled to study the derivatives of the function $i(v)$ and deduce the geometry of the i–v curve obtained with the model. We have studied the sign of the first, the second and the third derivatives of $i(v)$ obtaining different bounds which have allowed us to demonstrate that the i–v curve generated by the model is strictly decreasing and strictly concave, moreover, we have seen that the second derivative attains a global minimum at a certain voltage $v_{0\text{jerk}}$. Some interesting bounds on the first derivative function have allowed us to predict and further prove a usual phenomena observed in the real behavior of several PV modules, namely, the points near the short circuit point take the shape of a line. Indeed, we have proved that the i–v curve has an oblique asymptote at left-hand that is actually perceptible close to the short circuit point. The expression of this asymptote has been provided and has allowed us to describe a new analytical method based in this fact. It has been tested that this method is comparable, for instance, with the well-known analytical five point method but with a clear advantage, the new method does not need to know the slope of the real I–V curve near the open circuit point, and the calculi involved in its resolution are significantly simpler than those used in the analytical five point method. We conclude that the new method “the oblique asymptote method” we are proposing can be a useful tool to characterize PV modules and to analyze their behavior. Further investigations should be focused on analyzing the parameter sensitivity of this method in presence of variation in the environmental conditions.

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