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RESEARCH ARTICLE

Fennec Fox Optimization: A New Nature-Inspired Optimization Algorithm

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ABSTRACT This paper proposes a new nature-based metaheuristic algorithm called Fennec Fox Optimization (FFA), mimicking two natural behaviors of the animal Fennec Fox in nature. Concretely, Fennec's digging ability and escape strategy from wild predators were the fundamental inspiration for the proposed FFA. The mathematical model of FFA is presented in two phases based on imitating these two behaviors. First, the efficiency of FFA was evaluated in the optimization of sixty-eight standard benchmark functions and four engineering design problems. Second, FFA performance is challenged against eight well-known optimization algorithms. The optimization results show that FFA perfectly balances exploration and exploitation in searching for the global optimum. Hence, FFA can provide suitable solutions to optimization problems. The comparison of results indicates the superiority of FFA in most objective functions over competitor algorithms in providing the optimal solution.

INDEX TERMS Fennec fox, optimization, nature-inspired, optimization algorithm, metaheuristic, exploration, exploitation.

I. INTRODUCTION

Optimization is a fundamental challenge in various sciences that has become more prominent with the advancement of science and technology [1]. In general, any problem with more than one solution is an optimization problem, and the main purpose of optimization is to find the best solution among these existing solutions. The first step of any optimization strategy is to find all the necessary decision variables, constraint conditions, and the optimized objective function [2]. After identifying and modeling the problem, the optimal solution can be obtained using problem-solving methods. Optimization problem-solving strategies can be divided into two groups: deterministic methods and stochastic methods.

Deterministic methods are generally divided into two distinct groups. The first group includes gradient-based methods that use derivatives of the objective function and constraints along with the values of these functions to find the optimal solution. The second group comprises non-gradient-based methods that are not based on gradients and need to evaluate the values of the objective function and constraints for further searches [3].

Deterministic methods have effective performance in solving linear or convex non-linear problems. However, these methods fail in solving more complex types of problems, especially those with a discontinuous, non-derivative, and non-convex objective function. Real-world problems usually include a high number of decision variables, stochastic behavior of the objective functions, unknown search spaces, discrete search spaces, non-convex behaviors, and many local solutions. The weaknesses and inadequacies of deterministic methods led to the introduction of random methods to solve optimization problems [4].

Stochastic methods, while not requiring knowledge of a derivative of an optimized function, or even the existence of its derivative, can solve the problem through random scanning of the search space and using random operators. Metaheuristic algorithms are stochastic methods with a high speed of convergence, when solving optimization problems with swarm intelligence [5]. The solution-finding process in these methods is such that a certain number of candidate solution alternatives are initially randomly generated for the problem.

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Then, in an iterative process, these candidate solutions are successively enhanced based on repeating algorithm steps. Finally, the best candidate solution to the considered problem will be found after the complete execution of the algorithm.

Due to the nature of the random search, metaheuristic algorithms do not guarantee that they will converge to the optimal global solution. Therefore, the found solution to optimization problems by these types of algorithms are called quasi-optimal [6].

Exploration and exploitation are the two fundamental concepts that enable metaheuristics to provide suitable quasi-optimal solutions to optimization problems. Exploration refers to the algorithm's ability to search comprehensively and globally the problem-solving space, which allows the algorithm to avoid getting caught in optimal local areas. Exploitation is the ability of an algorithm to search locally in the neighborhood of the previously obtained solution to find quasi-optimal solutions much closer to the exact global optimal. What is significant about these two capabilities is that an algorithm can converge to suitable solutions by providing the appropriate balance between exploration and exploitation [7].

Metaheuristic algorithms, based on the main idea employed in their design, belong to nine main groups: swarm-based, biology-based, physics-based, social-based, sport-based, chemistry-based, music-based, mathematical-based, and hybrid algorithms [8].

Swarm-based algorithms are based on modeling natural phenomena, behaviors, and strategies of animals, birds, insects, and other living things in nature. Ant Colony Optimization (ACO) [9], the Particle Swarm Optimization (PSO) [10], the Artificial Bee Colony (ABC) [11], and the Firefly Algorithm (FA) [12] are popular methods that belong to this group. The ants' natural behavior and ability to find the shortest path between the nest and food has been the main inspiration for the introduction of ACO. PSO design simulates the movement of a flock of birds or a group of fish in search of food. Modeling the intelligent behavior of honey bees in search of food has led to the creation of the ABC. Simulating the optical relationship between fireflies is a crucial idea in the development of the FA. The behavior and strategy of animals hunting in nature have been the main idea in designing algorithms such as the Gray Wolf Optimization (GWO) [13] inspired by the gray wolves, the Whale Optimization Algorithm (WOA) [14] inspired by the humpback whale, the Marine Predator Algorithm (MPA) [15] inspired by ocean predators. Some of the other swarm-based metaheuristics are, e.g., the Reptile Search Algorithm (RSA) [16], the Raccoon Optimization Algorithm (ROA) [17], the African Vultures Optimization Algorithm (AVOA) [18], and the Golden Eagle Optimizer (GEO) [19].

Biology-based algorithms have been developed by simulating the concepts of biological sciences, genetics, and evolutionary laws. The Genetic Algorithm (GA) [20] and the Differential Evolution (DE) [21] are the two most widely used metaheuristics in this group. The main idea of GA is derived from natural selection and the reproduction process using operators of selection, crossover, and mutation. The DE algorithm is developed to overcome the main drawback of GA, namely the lack of local search in this algorithm. The main difference between GA and DE is in the selection operator. Some other biology-based metaheuristics are the Evolution Strategy (ES), the Cultural Algorithm (CA) [22], and Genetic programming (GP) [23].

Physics-based algorithms are introduced based on simulations of laws and phenomena in physics. The Simulated Annealing (SA) [24] and the Gravitational Search Algorithm (GSA) [24] and the Gravitational Search Algorithm (GSA) [25] are two widely used physics-based algorithms. The idea used in SA design is modeling the annealing process in which the metal is heated, kept at a certain temperature, and finally, gradually cooled. The use of gravitational force in a system consisting of masses at different distances from each other has been a fundamental inspiration in introducing the GSA. Some of other physics-based metaheuristics are: the Water Cycle Algorithm (WCA) [26], Optics Inspired Optimization (OIO) [27], [28], the Central Force Optimization (CFO) [29], the Nuclear Reaction Optimization (NRO) [30], the Multi-Verse Optimizer (MVO) [31], and the Atom Search Optimization (ASO) [32].

Social-based algorithms are developed based on human behaviors and interactions. War Strategy Optimization (WSO) [33], Teaching-Learning Based Optimization (TLBO) [34], Doctor and Patient Optimization (DPO) [35], and Driving Training-Based Optimization (DTBO) [36] are social-based algorithms.

Sport-based algorithms are designed based on the simulation of game rules and players' behavior. For example, volleyball Premier League (VPL) [37], Football Game Based Optimization (FGBO) [38], Puzzle Optimization Algorithm (POA) [39], and Ring Toss Game Based Optimizer (RTGBO) [40] are sport-based algorithms.

Chemistry-based algorithms are developed based on chemistry concepts, e.g., the artificial Chemical Reaction Optimization Algorithm (ACROA) [41] belongs to this group.

Music-based algorithms have been introduced based on the simulation of music concepts, e.g., the Musical Composition Algorithm (MCA) [42] and Harmony Search (HS) [43] are the most usable in practice.

Mathematical-based algorithms are designed based on modeling theories and concepts in mathematics, e.g., the Golden Sine Algorithm (GSA) [44] and the Base optimization algorithm (BOA) [45] are methods of this type.

Hybrid algorithms are developed based on combining two or more algorithms in order to improve their performance. For example, Sine–cosine and Spotted Hyena-based Chimp Optimization Algorithm (SSC) [46] and the Hybrid firefly algorithm with grouping attraction (HFA-GA) [47] are hybrid algorithms.

Plant intelligence has been the main inspiration source in designing plant-based algorithms [48]. Water-based algorithms have been introduced based on simulation the process in hydrology [49].

Whether new algorithms are required despite the numerous metaheuristics designed so far is the main research question in optimization algorithm analysis. The answer to this question lies in the No Free Lunch (NFL) theorem [50]. According to the NFL, there is no guarantee that an algorithm with excellent performance in optimizing a specific group of problems will have the same performance in solving all other optimization problems. The NFL theorem encourages researchers to develop new metaheuristic algorithms for solving optimization problems in different fields of science more effectively. The NFL theorem also motivated the authors of this paper to introduce a new metaheuristic algorithm to provide better optimal solutions to optimization problems.

The fennec fox is the smallest member of the fox species, with its large ears being the most prominent feature of this animal. Large ears and high hearing power enable the fennec fox to hear the sound of underground prey. It is a powerful digger that digs to catch these prey underground. The fennec fox has a strategy for confrontation with predator attacks that, by suddenly changing movement directions, mislead the predator and escape from it.

Based on the best of our knowledge from the literature review, the behavior characteristics of the fennec fox have not been used in designing any optimization algorithm so far, which motivated the authors of this article to propose a new optimization method based on the mathematical modeling of the behavior of fennec foxes in nature.

Many bio-inspired metaheuristic algorithms have been proposed that simulate the natural behavior of animals. In order to confirm the originality of the proposed FFA approach, we decided to compare it with one of the most successful algorithms in this category GWO, which mimics the natural behavior of gray wolves and their hierarchical hunting leadership structure. The main idea of GWO is the strategy of gray wolves in hunting and attacking prey. GWO models hunt in three steps: search, surround, and attack prey.

The first significant difference between FFA and GWO is their primary sources of inspiration (fennec fox vs. gray wolf behavior). The fennec fox usually hunts in limited areas, and therefore the hunting process in FFA is simulated as a local search and covers only a small area of the search space. On the contrary, the hunting process in GWO is simulated in three steps based on hierarchical management with the goal of global and local search. In FFA design, the behavior of fennec foxes while escaping from predators is simulated to provide a global search in the problem-solving space. Meanwhile, in the design of GWO, there is no such step that simulates the escape behavior of gray wolves from their predators.

The novelty and contribution of this paper are to introduce a new nature-inspired optimization algorithm called Fennec Fox Optimization (FFA) that mimics the biological activities of the fennec fox. The fundamental inspiration of FFA is the description of two types of fennec fox behaviors, including digging ability and escape predator attack strategy. The various steps of the FFA theory are described and mathematically modeled. The FFA's performance in optimizing sixty-eight objective functions of a variety of unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017 types has been evaluated. The optimization results obtained from FFA are compared with the performance of eight well-known algorithms. In addition, the efficiency of FFA in real-world applications has been evaluated to optimize four engineering design problems.

The rest of this paper is as follows. First, the proposed algorithm is introduced in Section 2. Then in Section 3, simulation studies are presented. Next, the application of the proposed algorithm in solving real-world problems is studied in Section 4. Finally, conclusions and suggestions for future studies are provided in Section 5.

II. FENNEC FOX OPTIMIZATION

This section presents the inspiration concepts and mathematical modeling of the proposed Fennec Fox Optimization (FFA) algorithm.

A. INSPIRATION AND FENNEC FOX BEHAVIOR

The fennec fox is a member of the Vulpes family of foxes living in the Sinai Peninsula in Egypt, the sandy Sahara Desert, and elsewhere in North Africa. The fennec fox is the smallest canid species and is easily identifiable due to its enormous ears [31]. The fennec fox has straw-colored fur. It has a black tip on its tapered tail. Its large ears contain longitudinal reddish stripes on the back and are so thickly hairy inside that the external auditory meatus is hidden. The ear margins are whitish but darker on the rear. Its paw pads are densely furred, making walking easier in sandy and hot soil. Figure 1 shows an image of a fennec fox. Female species weigh 1 to 1.9 kg and have head-to-body size of 34.5 to 39.5 cm, a 23 to 25 cm long tail, and 9 to 9.5 cm long ears. Males are slightly larger than females, with head-to-body measurements ranging from 39 to 39.5 cm, a 23 to 25 cm long tail and 10 cm long ears, and weighing at least 1.3 kg. The fennec fox is an omnivorous animal whose diet includes

FIGURE 1. Fennec fox (take from Wikimedia).

lizards, small rodents, skinks, geckos, small birds and their eggs, fruits, and some tubers [51].

The two behaviors of fennec fox are more significant than its other features. These features include a robust digging ability and escape strategy from predators.

The two features of the behavior of the fennec fox are more significant than the others. These features include strong digging abilities and an escape strategy from predators. Fennec foxes dig their nests in the sand. They feed on small vertebrates and insects that they quickly find underground, as they are sensitive to prey movements below ground and subsequently dig them out of the sand [52].

Pharaoh eagle-owls and other African horned owls feed on fennec fox pups. There have been anecdotal accounts of striped hyenas, jackals, and caracals preying on fennec foxes. However, nomads claim that the fennec fox is so swift and changes directions that even their Salukis are hardly capable of catching it [51].

Two types of intelligent strategies of the fennec fox, namely digging to find its prey in the sand and its escape strategy from an attacking predator, are the essential inspiration of the proposed FFA optimization method. Moreover, these two types of intelligent fennec fox behavior are the basis of the FFA mathematical model.

B. MATHEMATICAL MODELING

This subsection presents mathematical simulations of the natural behavior of a fennec fox to model the proposed FFA.

1) INITIALIZATION

FFA is a population-based metaheuristic algorithm in which fennec foxes make up search members. In FFA, each fennec fox represents a candidate solution to the problem whose position in the search space determines the values of the decision variables. So, the mathematical expression of any fennec fox is a vector, and its population is a matrix called the population matrix. At startup, fennec foxes are randomly initialized in the search space using (1).

$$X_i : x_{i,j} = lb_j + r \cdot \left(ub_j - lb_j\right), \quad i = 1, 2, \ldots, N, j = 1, 2, \ldots, m \tag{1}$$

where X_i is the ith fennec fox, $x_{i,j}$ is the its jth dimension (i.e., decision variable), N is the total number of fennec foxes, m is the number of decision variables, r is a random number in the interval [0, 1], lb_j is the lower bound, and ub_j is the upper bound of the jth decision variable.

In (2), a population matrix for the FFA, consisting of N fennec foxes, is specified.

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_i \\ \vdots \\ X_N \end{bmatrix}_{N\times m} = \begin{bmatrix} x_{1,1} & \cdots & x_{1,j} & \cdots & x_{1,m} \\ \vdots & \ddots & \vdots & ⋰ & \vdots \\ x_{i,1} & \cdots & x_{i,j} & \cdots & x_{i,m} \\ \vdots & ⋰ & \vdots & \ddots & \vdots \\ x_{N,1} & \cdots & x_{N,j} & \cdots & x_{N,m} \end{bmatrix}_{N\times m}, \tag{2}$$

where $X_i = (x_{i1}, x_{i2}, \ldots, x_{im})$ (the ith row of the matrix X) represents the ith fennec fox in the population of the range N and each column $\left(x_{1j}, x_{2j}, \ldots, x_{Nj}\right)^T$ represents the candidate values for the jth decision variable.

The candidate values proposed by each fennec fox are placed in the variables of the objective function, and then the objective function is evaluated. As a result, the estimated values of the objective function are mathematically modeled using the vector specified in (3).

$$F = \begin{bmatrix} F_1 \\ \vdots \\ F_i \\ \vdots \\ F_N \end{bmatrix}_{N\times 1} = \begin{bmatrix} F(X_1) \\ \vdots \\ F(X_i) \\ \vdots \\ F(X_N) \end{bmatrix}_{N\times 1}, \tag{3}$$

where F is the vector of the values of the objective function and F_i is the value of the objective function obtained for the ith fennec fox.

The criterion for deciding which of the candidate solutions is the best solution to the problem is the value of the objective function. The candidate solution with the objective function's best value is selected as the best solution. Because candidate solutions are updated at each iteration, the best candidate solution is updated in each iteration too.

Two natural fennec fox behaviors have been used to update the position of FFA members in the search space. These behaviors include (i) digging to eat prey beneath the sand and (ii) escaping from predators.

2) PHASE 1: THE DIGGING TO LOOK FOR PREY UNDER THE SAND (EXPLOITATION)

The fennec fox hunts at night and alone. It uses the power of hearing its large ears to detect prey beneath the sand and, after finding its location, digs with its feet to expose and hunt its prey. This fennec fox behavior is a local search, and simulating this behavior increases the FFA's exploitation power in achieving a solution closer to the global optima. To model the behavior of a fennec fox during digging, we consider a neighborhood with a radius R around its actual position. The fennec fox with a local search in this area can converge to a better solution. This phase of updating FFA members is mathematically simulated using (4) to (6).

$$x_{i,j}^{P1} = x_{i,j} + (2 \cdot r - 1) \cdot R_{i,j}, \tag{4}$$

$$R_{i,j} = \alpha \cdot \left(1 - \frac{t}{T}\right) \cdot x_{i,j}, \tag{5}$$

$$X_i = \begin{cases} X_i^{P1}, & F_i^{P1} < F_i; \\ X_i, & else, \end{cases} \tag{6}$$

where X_i^{P1} is the new suggested status of the ith fennec fox based on first phase, $x_{i,j}^{P1}$ is the its jth dimension, F_i^{P1} is the its objective function value, $R_{i,j}$ is the neighborhood radius for $x_{i,j}$, t is the iteration counter, T is the total number of iterations, and α is a constant set to 0.2.

3) PHASE 2: ESCAPE STRATEGY FROM THE PREDATORS' ATTACK (EXPLORATION)

The fennec fox is exposed to attacks by wild predators such as Pharaoh eagle-owl, striped hyenas, and caracals. However, it escapes predators thanks to its incredible speed and sudden change of direction of movement. In our mathematical model, this escape strategy of the fennec fox is the basis of global scanning of the search space. Simulation of this escape strategy enhances the exploration power of the proposed FFA. It helps to avoid getting stuck in the optimally local areas and thus identifying the optimal global one. Hence, the random position of each candidate solution in the search space can be considered a model of the behavior of the fennec fox during its escape. The second phase of the FFA population update is mathematically simulated using (7) to (9).

$$X_i^{rand} : x_{i,j}^{rand} = x_{k,j}, k \in \{1, 2, \ldots, N\}, i = 1, 2, \ldots, N, \tag{7}$$

$$x_{i,j}^{P2} = \begin{cases} x_{i,j} + r \cdot (x_{i,j}^{rand} - I \cdot x_{i,j}), & F_i^{rand} < F_i; \\ x_{i,j} + r \cdot \left(x_{i,j} - x_{i,j}^{rand}\right), & else, \end{cases} \tag{8}$$

$$X_i = \begin{cases} X_i^{P2}, & F_i^{P2} < F_i; \\ X_i, & else, \end{cases} \tag{9}$$

where X_i^{rand} is the target position intended for the escape of the ith fennec fox, $x_{i,j}^{rand}$ is the its jth dimension, F_i^{rand} is the its objective function value, X_i^{P2} is the new suggested status of the ith fennec fox based on second phase, $x_{i,j}^{P2}$ is its jth dimension, F_i^{P2} is the value of its objective function, and I is a random number from the set $\{1, 2\}$.

C. REPETITIONS PROCESS, FLOWCHART, AND PSEUDO-CODE OF FFA

The first FFA iteration is completed after updating the position of all fennec foxes based on the first and second phases. This update process continues until the end of the total number of iterations of the algorithm based on (4) to (9). Finally, the FFA provides the candidate solution to the given problem after full implementation. The FFA implementation steps are presented as a flowchart in Figure 2, and its pseudo-code is in Algorithm 1.

D. COMPUTATIONAL COMPLEXITY

In this section, the computational complexity of FFA is analyzed. The FFA preparation and initialization process has the complexity $O(Nm)$, where N is the number of fennec foxes and m is the number of problem variables. In each iteration, each fennec fox is updated in two different phases whose total complexity is equal to $O(2NmT)$. Furthermore, the computational complexity of the random members to simulate the escaping target of the population members is equal to $O(NmT)$. Consequently, the total computational complexity of the FFA is equal to $O(Nm(1 + 3T))$.

Algorithm 1 Pseudo-Code of the Proposed FFA.

Start FFA.
1. Input the optimization problem information.
2. Set the number of iterations (T) and the number of fennec foxes' population (N).
3. Initialization of the position of fennec foxes and evaluation of the objective function.
4. For $t = 1 : T$
6. For $i = 1 : N$
7. **Phase 1: The digging to look for prey under the sand (exploitation).**
8. Calculate new status of the ith fennec fox using (4) and (5).
9. Update the ith fennec fox using (6).
10. **Phase 2: Escape strategy from the predators' attack (exploration).**
11. Generate the target position intended for the escape of the ith fennec fox and evaluate its objective function using (7).
12. Calculate new status of the ith fennec fox using (8).
13. Update the ith fennec fox using (9).
14. end for $i = 1 : N$
15. Save the best candidate solution so far.
16. end for $t = 1 : T$
17. Output the best solution of the given optimization problem.

End FFA.

III. SIMULATION STUDIES AND DISCUSSION

In this section, the proposed algorithm is employed to optimize 68 standard benchmark functions to evaluate its performance in providing the optimal solution. These benchmark functions include unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, and CEC2017. In addition, the quality of the proposed algorithm is compared with eight well-known algorithms, namely GA, PSO, GSA, TLBO, GWO, WOA, TSA, and MPA. The values of the control parameters of these algorithms are specified in Table 1.

The initial population for FFA and competitor algorithms is randomly generated using (1). In order to provide a fair comparison, each of the algorithms, in twenty independent executions, each containing 1000 iterations, is employed to optimize each of the benchmark functions. The simulations are performed in Matlab®version 2020a using 64-bit Core i7 processor with 3.20 GHz and 16 GB main memory in Windows 10.

The simulation results are reported using four indicators (i) the best candidate solution "best," (ii) the average of the candidate solutions "mean," (iii) the median of the candidate solutions "median," and (iv) the standard deviation of the candidate solutions "std."

Suppose that during individual iterations of FFA, the variable's value goes outside the specified search space. In that case, we use the usual procedure that the value of this variable

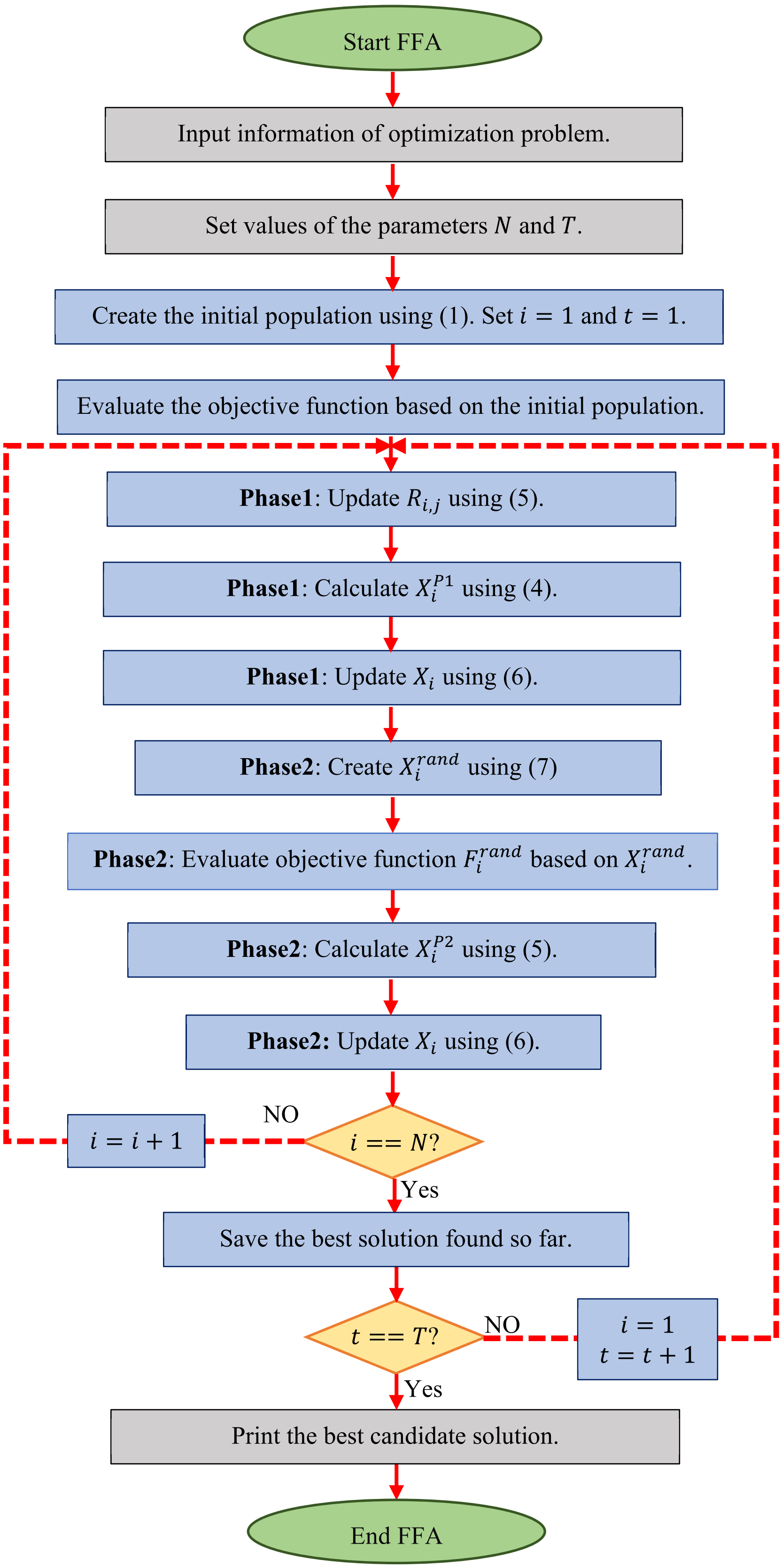

FIGURE 2. Flowchart of FFA.

TABLE 1. Parameter values for the competitor algorithms.

Algorithm	Parameter	Value
MPA		
	Constant number	$P = 0.5$
	Random vector	R is a vector of uniform random numbers from $[0, 1]$.
	Fish aggregating devices (FADs)	FADs $= 0.2$
	Binary vector	$U = 0$ or 1
TSA		
	$P_{\min}$ and $P_{\max}$	1, 4
	c_1, c_2, c_3	random numbers from the interval $[0, 1]$.
WOA		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
	r is a vector of random numbers from $[0, 1]$.	
	l is a random number in $[-1,1]$.	
GWO		
	Convergence parameter (a)	a: Linear reduction from 2 to 0.
TLBO		
	T_F: teaching factor	T_F = round $[(1 + \text{rand})]$
	random number	rand is a random number from the interval $[0, 1]$.
GSA		
	Alpha, G_0, R_{norm}, R_{power}	20, 100, 2, 1
PSO		
	Topology	Fully connected
	Cognitive and social constant	$(c_1, c_2) = (2, 2)$
	Inertia weight	Linear reduction from 0.9 to 0.1
	Velocity limit	10% of dimension range
GA		
	Type	Real coded
	Selection	Roulette wheel (Proportionate)
	Crossover	Whole arithmetic (Probability $= 0.8$, $\alpha \in [-0.5, 1.5]$)
	Mutation	Gaussian (Probability $= 0.05$)

is placed into the closest endpoint of the interval that determines this variable.

A. EVALUATION OF UNIMODAL BENCHMARKS

The characteristic of unimodal functions is that they have one extremum. The purpose of optimizing these functions is to evaluate the exploitation ability of algorithms in the local search of problem-solving space and convergence towards the optimal global solution. Table 2 reports the optimizing results of the unimodal functions using the proposed FFA and eight competitor algorithms. FFA is the best optimizer compared to the eight competitor algorithms in F1, F2, F3, F4, F5, F6, and F7. Analysis of the simulation results shows the high exploitation ability of the FFA in convergence to the global optimal and its competitive superiority over eight compared algorithms.

B. EVALUATION OF HIGH-DIMENSIONAL MULTIMODAL BENCHMARKS

The characteristic of high-dimensional multimodal functions is that their problem-solving space has several optimal local areas. The purpose of optimizing these functions is to evaluate the exploration ability of algorithms in the global search for problem-solving space to pass through local areas and identify the main optimal area. Table 3 releases the optimization results of the F8 to F13 benchmark functions. This table shows that FFA (similar to TBLO and MPA) always reached the global optimum for F9 and F11. Furthermore, we can see from the analysis of the results in Table 3 that FFA is also the best optimizer for optimizing the functions F8, F10, F12, and F13. Analysis of the performance of optimization algorithms in solving the benchmark functions of F8 to F13 indicates the high exploration power of FFA in identifying the main optimal region of search space and not catching in the optimal local area compared to the eight competitor algorithms.

C. EVALUATION OF FIXED- DIMENSIONAL MULTIMODAL BENCHMARKS

The characteristic of fixed-dimensional multimodal functions is that their problem-solving space has a low number of optimal local areas. This feature makes these functions useful for evaluating algorithms' exploration and exploitation capabilities simultaneously and analyzing the balance between the two indicators. Table 4 presents the results of the FFA implementation of eight competing algorithms in optimizing the objective functions F14 to F23. Based on this table, it is clear that FFA is the best optimizer for optimizing the function F18. The simulation results also show that in other functions of this group, FFA has provided optimal solutions with better values for statistical indicators. Comparing optimization algorithms shows that FFA has provided more efficient performance in optimizing F14 to F23 functions by creating a proper balance between exploration and exploitation compared to eight competitor algorithms. Figure 3 shows the convergence curves of the performance of FFA and eight competitor algorithms in optimizing F1 to F23.

TABLE 2. Optimization results of the proposed FFA and competitor algorithms on unimodal test functions.

F		GA	PSO	GSA	TLBO	GWO	WOA	MPA	TSA	FFA
F1	mean	30.50201	0.100957	9.26E-17	4.99E-74	1.55E-58	3.5E-153	1.55E-49	7.37E-47	1.4E-212
	best	17.92696	0.000487	4.1E-17	2.98E-77	2.2E-61	4.8E-165	1.66E-53	7.12E-51	1.6E-218
	std	10.46286	0.31078	3.41E-17	1.07E-73	2.82E-58	1.4E-152	2.31E-49	2.35E-46	0
	median	28.19897	0.00972	8.86E-17	2.32E-75	1.94E-59	1.5E-159	4.47E-50	1.36E-48	1.3E-214
	rank	9	8	7	3	4	2	5	6	1
F2	mean	2.788395	0.895505	5.26E-08	5.74E-39	8.33E-35	4.6E-102	9.33E-28	1.2E-28	3.5E-112
	best	1.745356	0.045282	4.12E-08	1.71E-40	4.07E-36	4E-114	4.41E-31	9.3E-31	5.5E-114
	std	0.544788	0.722723	1.05E-08	7.46E-39	7.76E-35	1.5E-101	1.72E-27	1.59E-28	9.6E-112
	median	2.741555	0.584164	4.93E-08	3.23E-39	6.08E-35	1.2E-108	1.37E-28	6.42E-29	6.3E-113
	rank	9	8	7	3	4	2	6	5	1
F3	mean	2168.983	388.1315	376.9763	3.47E-24	1.81E-15	21190.41	4.21E-12	6.52E-12	2.51E-31
	best	1424.187	21.76826	159.109	5.42E-29	2.2E-18	2450.06	8.64E-20	5.91E-22	1.85E-35
	std	639.6914	288.4306	109.2613	9.66E-24	3.84E-15	11633.63	8.47E-12	2.8E-11	7.32E-31
	median	2100.7	293.0444	400.2444	9.11E-26	7.92E-17	21499.71	1.51E-13	1.99E-14	4.3E-33
	rank	8	7	6	2	3	9	4	5	1
F4	mean	2.829395	6.279883	1.243185	1.97E-30	2.23E-14	39.47074	3.71E-19	0.005269	2.03E-79
	best	2.216469	2.290268	7.91E-09	1.34E-31	5.25E-16	1.25418	5.63E-20	5.51E-05	3.75E-81
	std	0.466936	2.502379	1.281418	2.73E-30	5.81E-14	30.21706	3.58E-19	0.012634	2.8E-79
	median	2.783478	5.882471	0.973078	9.06E-31	5.26E-15	36.66701	1.97E-19	0.000853	6.8E-80
	rank	7	8	6	2	4	9	3	5	1
F5	mean	595.3854	4611.934	40.10492	26.4769	26.50784	27.21939	25.90411	28.21019	25.44749
	best	228.808	26.28099	25.22295	25.31057	25.22167	26.4883	24.43897	26.28923	25.2627
	std	424.9867	20116.61	33.2699	0.584548	0.6401	0.452078	0.658538	0.868321	0.112592
	median	475.573	86.09804	26.2573	26.35559	26.45352	27.05858	25.91693	28.64535	25.45164
	rank	8	9	7	3	4	5	2	6	1
F6	mean	33.9	49.6	1.27E-09	1.105033	0.640456	0.091444	1.81E-06	3.737059	1.14E-10
	best	13	10	6.01E-10	0.268374	1.38E-05	0.005876	7.69E-07	2.838003	5.85E-12
	std	15.07141	30.48624	4.23E-10	0.380047	0.371018	0.128087	6.24E-07	0.558395	1.28E-10
	median	32.5	42	1.3E-09	1.11594	0.748136	0.036256	1.77E-06	3.687136	7.79E-11
	rank	8	9	2	6	5	4	3	7	1
F7	mean	0.010589	0.184141	0.057003	0.001619	0.000807	0.001583	0.000672	0.005196	0.000383
	best	0.003032	0.069017	0.022195	0.000541	0.000142	0.00017	0.000153	0.001939	5.49E-05
	std	0.004819	0.079015	0.024145	0.000726	0.000532	0.001484	0.000335	0.002585	0.000348
	median	0.010178	0.177731	0.05208	0.001658	0.000761	0.001098	0.000568	0.005071	0.000275
	rank	7	9	8	5	3	4	2	6	1
Mean rank		8	8.285714	6.142857	3.428571	3.857143	5	3.571429	5.714286	1
Total ranking		8	9	7	3	4	5	2	6	1

D. STATISTICAL ANALYSIS

A statistical analysis of the FFA performance and eight competing algorithms is performed to analyze the simulation results further to optimize functions F1 to F23. Wilcoxon sign-rank test [53] is a non-parametric statistical analysis used to detect significant differences between two data samples. In this regard, the Wilcoxon sign rank test has been employed to determine whether the superiority of FFA over competitor algorithms is significant or not.

The results of the Wilcoxon sign-rank test of implementation of FFA performance against competitor algorithms are reported in Table 5. This test is performed for a significant level of 0.05. Accordingly, in cases where the p-value is smaller than the specified significance level, FFA has a considerable superiority and a substantial difference over the corresponding competitor algorithm. The results of the Wilcoxon sign-rank test indicate that the FFA, in most cases, has a p-value of less than 0.05, indicating its statistically significant superiority over competitor algorithms.

E. SENSITIVITY ANALYSIS

The FFA provides a solution to the problem in an iterative-based process by using the search for a problem-solving space by its population members. For this reason, it can be assumed that the choice of T and N parameter values should affect the effectiveness of FFA to achieve an optimal solution to

TABLE 3. Optimization results of the FFA and competitor algorithms on high dimensional multimodal test functions.

F		GA	PSO	GSA	TLBO	GWO	WOA	MPA	TSA	FFA
F8	mean	-8421.5	-6547.41	-2650.03	-5436.39	-6040.54	-11320.1	-9772.51	-6098.27	-11343.5
	best	-9681.18	-8244.17	-3207.63	-6509.71	-7276.86	-12569.1	-10256.9	-7083.91	-11722.2
	std	641.2242	748.5216	428.173	478.6985	850.4632	1704.092	419.3788	633.2125	367.767
	median	-8399.11	-6693.09	-2772.35	-5358.52	-6066.5	-12454.4	-9912.21	-6156.24	-11468.1
	rank	4	5	9	8	7	2	3	6	1
F9	mean	54.68123	67.7144	26.71463	0	0.406254	0	0	159.6377	0
	best	23.23239	39.79836	14.92438	0	0	0	0	70.97103	0
	std	13.80758	18.84112	7.735754	0	1.397792	0	0	49.16026	0
	median	52.61443	65.06856	24.87397	0	0	0	0	160.6867	0
	rank	4	5	3	1	2	1	1	6	1
F10	mean	3.5751	2.727233	7.69E-09	4.26E-15	1.74E-14	4.26E-15	3.73E-15	1.689996	8.88E-16
	best	2.881962	1.693449	5.66E-09	8.88E-16	1.15E-14	8.88E-16	8.88E-16	1.51E-14	8.88E-16
	std	0.396644	0.857798	1.13E-09	7.94E-16	4.35E-15	2.44E-15	1.46E-15	1.584771	0
	median	3.62958	2.733921	7.64E-09	4.44E-15	1.51E-14	4.44E-15	4.44E-15	2.588698	8.88E-16
	rank	8	7	5	3	4	3	2	6	1
F11	mean	1.473471	0.185266	7.618567	0	0.001175	0.003539	0	0.004467	0
	best	1.288095	0.002367	3.379227	0	0	0	0	0	0
	std	0.123868	0.228487	2.950602	0	0.005253	0.015825	0	0.005724	0
	median	1.447709	0.122356	7.791353	0	0	0	0	0	0
	rank	6	5	7	1	2	3	1	4	1
F12	mean	0.274894	1.501058	0.182176	0.084035	0.0355	0.00661	2E-08	4.783887	1.76E-08
	best	0.060841	0.000107	3.27E-19	0.051343	0.00706	0.001018	8.67E-09	0.17976	4.01E-09
	std	0.138648	1.285627	0.38514	0.023314	0.016389	0.011423	7.44E-09	3.3571	1.3E-08
	median	0.264424	1.285267	9.41E-19	0.07631	0.033928	0.00343	1.92E-08	4.058888	1.46E-08
	rank	7	8	6	5	4	3	2	9	1
F13	mean	2.707835	3.607621	0.30556	1.042083	0.54941	0.189519	0.232311	2.814447	0.009141
	best	1.291959	0.009572	5.4E-18	0.448692	0.293445	0.043184	1.08E-07	1.707989	8.13E-08
	std	0.754476	3.031014	1.185068	0.291014	0.169957	0.107823	0.486847	0.673751	0.014988
	median	2.867222	3.305798	1.15E-17	1.042979	0.553487	0.180446	2.75E-07	2.825118	5.1E-06
	rank	7	9	4	6	5	2	3	8	1
Mean rank		6	6.5	5.666667	4	4	2.333333	2	6.5	1
Total ranking		6	7	5	4	4	3	2	7	1

the problem. Therefore, this section provides FFA sensitivity analysis to parameters T and N. First, the FFA's sensitivity to the parameter T is analyzed by optimization of the functions F1 to F23 for $T = 100, 500, 800,$ and 1000 (Table 6 presents these simulation results). Figure 4 shows the convergence curves of the proposed FFA in solving the functions F1 to F23 under the influence of changes in the value of the parameter T. In the second study, the FFA's sensitivity to the parameter N is analyzed based on optimizing the functions F1 to F23 for $N = 20, 30, 50,$ and 100 (Table 7 and Figure 5 present these simulation results). Based on these results, we can see that for increasing values of N or T, the searching power of the algorithm increases, and the FFA provides better solutions closer to the global solution.

F. EVALUATION OF CEC2015 BENCHMARK

Table 8 shows the results of FFA and eight competitor algorithms in optimizing the CEC2015 functions. Analysis of the simulation results indicates that FFA has performed better in optimizing CEC1, CEC3, CEC5, CEC6, CEC7, CEC8, CEC9, CEC11, CEC12, CEC13, CEC14, and CEC15 functions. On the other hand, TSA in optimizing the functions CEC2 and CEC4 and GA in optimizing the function CEC10 have produced better results, while FFA was the second-best optimizer for these functions.

G. EVALUATION OF CEC2017 BENCHMARK

Table 9 provides the performance results of FFA and eight competitor algorithms in solving the CEC2017 functions. Based on the comparison of the results obtained, it is clear that FFA has provided better results compared to the eight competitor algorithms in solving C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, C12, C13, C15, C16, C17, C18, C21, C22, C23, C24, C25, C26, C27, C28, C29, and C30.

TABLE 4. Optimization results of the proposed FFA and competitor algorithms on fixed dimensional multimodal test functions.

F		GA	PSO	GSA	TLBO	GWO	WOA	MPA	TSA	FFA
F14	mean	1.048667	3.595793	3.321838	1.196415	5.68611	2.816294	0.998004	9.860703	0.998004
	best	0.998004	0.998004	1.002443	0.998004	0.998004	0.998004	0.998004	1.992031	0.998004
	std	0.222066	3.787922	3.019179	0.610693	4.959962	3.013364	0	4.508939	0
	median	0.998004	1.992031	2.490722	0.998004	2.982105	0.998004	0.998004	10.76318	0.998004
	rank	2	6	5	3	7	4	1	8	1
F15	mean	0.015388	0.002499	0.002094	0.004587	0.003453	0.000695	0.000307	0.010218	0.000307
	best	0.000782	0.000307	0.001425	0.000311	0.000307	0.000309	0.000307	0.000308	0.000307
	std	0.016221	0.006126	0.000501	0.008102	0.007296	0.000407	4.18E-10	0.014552	3.65E-11
	median	0.014273	0.000307	0.00201	0.00062	0.000308	0.000619	0.000307	0.000518	0.000307
	rank	8	4	3	6	5	2	1	7	1
F16	mean	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.02214	-1.03163
	best	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	std	4.78E-06	1.14E-16	1.02E-16	1.68E-06	7.28E-09	2.16E-11	2.22E-16	0.01487	1.25E-16
	median	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163	-1.03163
	rank	4	1	1	5	3	2	1	6	1
F17	mean	0.466023	0.744637	0.397887	0.398002	0.397888	0.397888	0.397887	0.397911	0.397887
	best	0.397887	0.397887	0.397887	0.397888	0.397887	0.397887	0.397887	0.397888	0.397887
	std	0.302731	0.709302	0	0.000129	6.53E-07	1.07E-06	0	4.26E-05	0
	median	0.397905	0.397887	0.397887	0.397942	0.397888	0.397888	0.397887	0.397891	0.397887
	rank	6	7	1	5	2	3	1	4	1
F18	mean	7.302903	3	3	3.000001	3.000009	3.000017	3.0000001	17.85004	3
	best	3	3	3	3	3	3	3	3.000001	3
	std	10.54375	3E-15	2.33E-15	8.55E-07	8.88E-06	3.66E-05	1.2E-15	29.6742	3.67E-16
	median	3.00117	3	3	3	3.000006	3.000004	3	3.00001	3
	rank	8	3	4	5	6	7	2	9	1
F19	mean	-3.86262	-3.86278	-3.86278	-3.86013	-3.8621	-3.85926	-3.86278	-3.86199	-3.86278
	best	-3.86278	-3.86278	-3.86278	-3.86271	-3.86278	-3.86278	-3.86278	-3.86278	-3.86278
	std	0.000295	2.06E-15	1.97E-15	0.003598	0.001705	0.00391	2.28E-15	0.002273	2.09E-15
	median	-3.86278	-3.86278	-3.86278	-3.86235	-3.86276	-3.85976	-3.86278	-3.86272	-3.86278
	rank	2	1	1	5	3	6	1	4	1
F20	mean	-3.2283	-3.26462	-3.322	-3.26952	-3.23514	-3.12218	-3.322	-3.18397	-3.322
	best	-3.32163	-3.322	-3.322	-3.31846	-3.32199	-3.32197	-3.322	-3.32164	-3.322
	std	0.078203	0.074972	3.67E-16	0.066903	0.068243	0.42223	3.95E-16	0.322711	4.56E-16
	median	-3.23661	-3.322	-3.322	-3.30352	-3.20278	-3.32126	-3.322	-3.20248	-3.322
	rank	5	3	1	2	4	7	1	6	1
F21	mean	-6.26023	-5.62381	-6.54245	-7.41815	-9.27181	-8.36691	-10.1532	-7.5422	-10.1532
	best	-9.73855	-10.1532	-10.1532	-9.93926	-10.153	-10.153	-10.1532	-10.1329	-10.1532
	std	2.711083	2.883857	3.709254	1.721422	2.197948	2.493281	2.31E-05	2.97703	8.18E-06
	median	-7.06069	-5.10077	-6.71492	-7.9886	-10.1529	-10.1487	-10.1532	-9.36034	-10.1532
	rank	7	8	6	5	2	3	1	4	1
F22	mean	-7.37187	-6.38293	-10.0204	-7.01912	-10.1368	-9.37097	-10.4029	-7.73607	-10.4029
	best	-9.9828	-10.4029	-10.4029	-10.019	-10.4028	-10.4026	-10.4029	-10.3823	-10.4029
	std	1.916626	3.469587	1.710817	2.001537	1.188443	2.56418	3.51E-15	3.550652	3.65E-15
	median	-7.86313	-5.10825	-10.4029	-7.72738	-10.4026	-10.4005	-10.4029	-10.1476	-10.4029
	rank	6	8	3	7	2	4	1	5	1
F23	mean	-6.36016	-6.42082	-10.5364	-7.74116	-10.1302	-7.0399	-10.5364	-5.45276	-10.5364
	best	-10.1845	-10.5364	-10.5364	-10.0566	-10.5363	-10.5363	-10.5364	-10.4744	-10.5364
	std	2.608634	3.847923	1.68E-15	1.969913	1.814381	3.021576	2.41E-15	3.740792	2.31E-15
	median	-6.88826	-3.83543	-10.5364	-8.30556	-10.5359	-5.12846	-10.5364	-2.84909	-10.5364
	rank	6	5	1	3	2	4	1	7	1
Mean rank		5.4	4.6	2.6	4.6	3.6	4.2	1.1	6	1
Total ranking		7	6	3	6	4	5	2	8	1

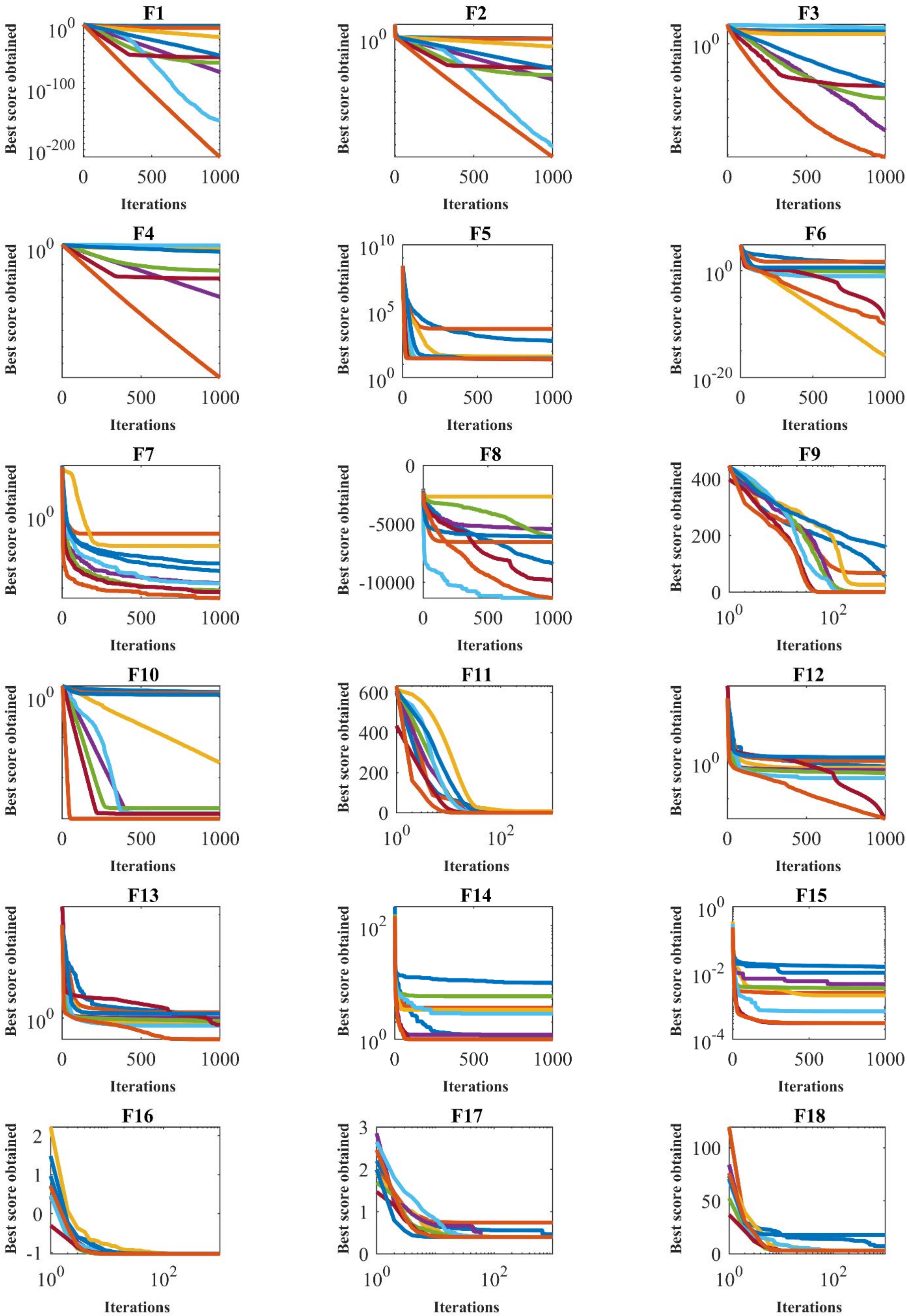

FIGURE 3. Convergence curve of FFA and eight competitor algorithms in solving test functions.

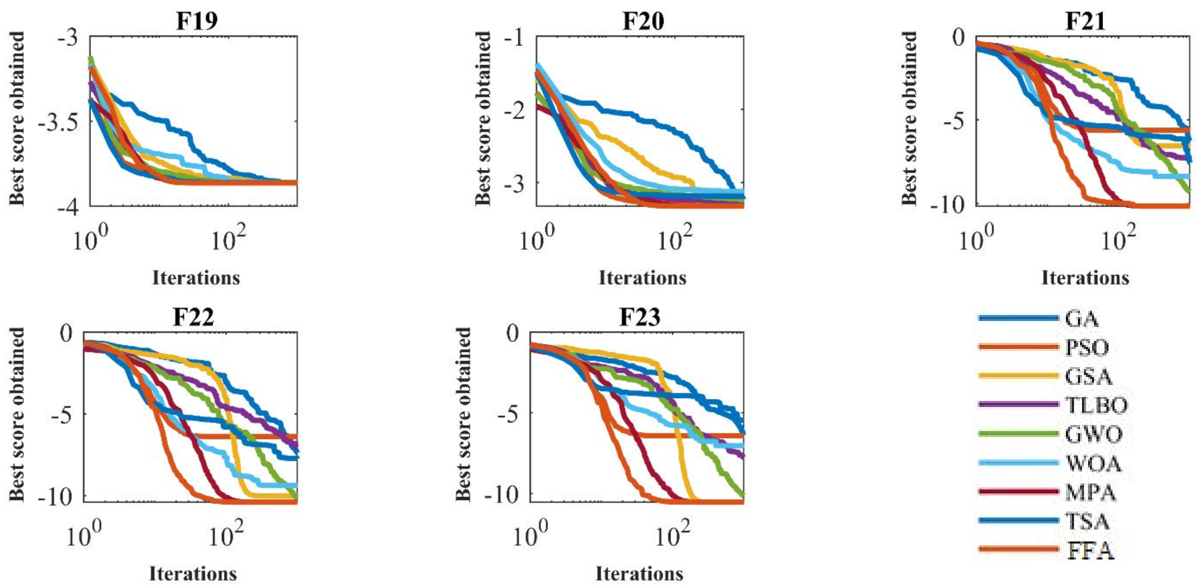

FIGURE 3. *(Continued.)* Convergence curve of FFA and eight competitor algorithms in solving test functions.

TABLE 5. *p*-values obtained using the Wilcoxon rank sum test.

Compared Algorithms	Functions type		
	Unimodal	High-Multimodal	Fixed-Multimodal
FFA vs. MPA	1.01E-24	1.97E-21	1.44E-34
FFA vs. TSA	1.01E-24	1.97E-21	2.64E-14
FFA vs. WOA	6.77E-24	4.04E-13	4.98E-13
FFA vs. GWO	5.28E-23	1.04E-14	1.44E-34
FFA vs. TLBO	6.33E-19	5.3E-16	1.44E-34
FFA vs. GSA	2.14E-22	0.001395	2.09E-34
FFA vs. PSO	8.04E-17	0.000309	0.000798
FFA vs. GA	1.01E-24	1.87E-19	1.44E-34

H. DISCUSSION

Exploration and exploitation are the most important for the success of metaheuristic algorithms in handling optimization problems.

The exploitation ability of FFA in local search is evaluated on seven unimodal functions. The proposed approach has performed better than competitor algorithms in handling functions F1 to 7. The values obtained for the unimodal objective functions using FFA are much better than those obtained by the competitor algorithms. Simulating the digging ability of the Fennec fox has led to FFA's high ability in local search and exploitation. Comparing the optimization results and Wilcoxon statistical analysis in handling unimodal functions shows the superior exploitation capability of the FFA compared to the competitor algorithms.

The exploration ability of FFA in global search is tested on six high-dimensional multimodal objective functions. The proposed approach has provided superior performance in solving functions F8 to F13 compared to competitor algorithms. In the competition with the compared algorithms, FFA ranked as the best optimizer in handling all six functions F8 to F13. The values obtained for the objective functions in most cases of high-dimensional multimodal functions with FFA are much better than those obtained from competitor algorithms. Simulating the strategy of Fennec foxes when they escape and change their position suddenly has led to the high ability of FFA in global search and exploration. Results of optimization and statistical analysis clearly show the superior exploration capability of the FFA compared to competitor algorithms.

In addition to the high ability of the optimization algorithm in exploration and exploitation, it is also necessary that these two search abilities are very well balanced. The ability of FFA to establish a balance between exploration and exploitation has been evaluated on ten fixed-dimensional multimodal functions. Compared to competitor algorithms, the proposed method has performed better in optimizing functions F14 to F23. Furthermore, FFA is the best optimizer compared to competitor algorithms to handle all functions F14 to F23. Based on the optimization and statistical analysis results, it is evident that the proposed approach is highly able to balance exploration and exploitation compared to competitor algorithms.

The power of FFA in handling complex optimization problems is tested on functions from CEC2015 and CEC2017. In most cases, the values obtained for the objective function using FFA are better than those obtained using competitor algorithms. Furthermore, the analysis of the optimization results shows that the proposed FFA approach has ranked as the best optimizer to solve the test results of CEC2015 and CEC2017 compared with competitor algorithms.

Among the reasons for the superiority of the proposed FFA approach compared to competing algorithms, we can point to the updating strategies of the FFA members by considering

TABLE 6. FFA sensitivity analysis on different numbers of population members.

Objective Function	Number of Population Members			
	20	30	50	100
F1	1.8E-209	1.44E-212	1.4E-215	1.1E-215
F2	2.8E-102	3.51E-112	3.5E-113	1.6E-113
F3	2.93E-27	2.51E-31	2.09E-32	1.04E-32
F4	3.81E-75	2.03E-79	5.46E-80	7.78E-81
F5	26.16941	25.44749	25.31288	25.30576
F6	0.014485	1.14E-10	1.33E-11	1.99E-12
F7	0.000846	0.000383	0.000274	0.000147
F8	-9363.71	-11343.5	-11817.70	-12015
F9	0	0	0	0
F10	1.07E-15	8.88E-16	8.88E-16	8.88E-16
F11	0	0	0	0
F12	2.93E-05	1.76E-08	2.08E-09	7.23E-10
F13	0.683425	0.009141	0.004642	0.003296
F14	1.097209	0.998004	0.998004	0.998004
F15	0.000324	0.000307	0.000307	0.000307
F16	-1.03163	-1.03163	-1.03163	-1.03163
F17	0.397887	0.397887	0.397887	0.397887
F18	3	3	3	3
F19	-3.86278	-3.86278	-3.86278	-3.86278
F20	-3.29822	-3.322	-3.322	-3.322
F21	-9.34023	-10.1532	-10.1532	-10.1532
F22	-10.0123	-10.4029	-10.4029	-10.4029
F23	-9.28954	-10.5364	-10.5364	-10.5364

TABLE 7. Sensitivity analysis of the FFA for the maximum number of iterations.

Objective Function	Maximum Number of Iterations			
	100	500	800	1000
F1	1.15E-17	4.8E-105	1.5E-169	1.44E-212
F2	3.95E-10	1.74E-55	8.11E-90	3.51E-112
F3	1.547557	3.11E-14	4.5E-25	2.51E-31
F4	1.02E-06	5.88E-39	6.4E-63	2.03E-79
F5	28.57374	26.52898	25.79317	25.44749
F6	2.049845	1.3E-05	1.1E-08	1.14E-10
F7	0.002465	0.00049	0.000443	0.000383
F8	-5298.1	-9518.75	-11042.8	-11343.5
F9	0.011748	0	0	0
F10	8.88E-16	8.88E-16	8.88E-16	8.88E-16
F11	3.33E-17	0	0	0
F12	0.051921	0.000385	3.68E-07	1.76E-08
F13	0.790266	0.068216	0.056731	0.009141
F14	0.998004	0.998004	0.998004	0.998004
F15	0.00056	0.00035	0.000309	0.000307
F16	-1.03163	-1.03163	-1.03163	-1.03163
F17	0.397888	0.397887	0.397887	0.397887
F18	3	3	3	3
F19	-3.86278	-3.86278	-3.86278	-3.86278
F20	-3.32158	-3.322	-3.322	-3.322
F21	-10.1143	-10.1532	-10.1183	-10.1532
F22	-10.1502	-10.3375	-10.4029	-10.4029
F23	-10.1704	-10.5319	-10.4699	-10.5364

exploration in achieving the main optimal region in the search space, exploitation in converging to possible better solutions and creating a suitable balance between exploration and exploitation in the search process. The exploration phase of FFA, with the goal of global search in the problem-solving space, avoids directing the FFA population towards the best

TABLE 8. Optimization results of the FFA and competitor algorithms on CEC2015 test functions.

		FFA	MPA	TSA	WOA	GWO	TLBO	GSA	GA	PSO
CEC1	Ave	1.97E+05	2.02E+06	4.37E+05	1.47E+06	6.06E+05	7.65E+06	1.50E+06	1.50E+06	3.20E+07
	std	1.42E+05	2.29E+06	5.20E+05	2.89E+06	5.52E+05	3.38E+06	1.33E+06	1.33E+06	9.21E+06
CEC2	Ave	1.03E+04	5.65E+06	9.41E+03	1.97E+04	1.43E+04	7.33E+08	6.70E+06	6.70E+06	4.58E+04
	std	5.06+03	6.63E+06	1.19E+04	1.61E+04	1.13E+04	2.56E+08	1.47E+08	1.47E+08	1.20E+04
CEC3	Ave	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02	3.20E+02
	std	6.37E-03	7.79E-02	9.47E-02	1.01E-01	3.51E-02	8.28E-02	1.28E-02	1.28E-02	1.22E-02
CEC4	Ave	4.12E+02	4.16E+02	4.08E+02	4.26E+02	4.18E+02	4.42E+02	4.40E+02	4.19E+02	4.39E+02
	std	2.68E+01	1.13E+01	4.36E+00	1.29E+01	1.13E+01	8.49E+00	6.17E+01	6.17E+01	7.98E+00
CEC5	Ave	8.30E+02	9.20E+02	8.65E+02	1.33E+03	1.09E+03	1.76E+03	9.81E+02	9.81E+02	1.75E+03
	std	7.09E+01	1.96E+02	2.38E+02	3.80E+02	3.09E+02	2.53E+02	2.27E+02	2.27E+02	3.07E+02
CEC6	Ave	1.20E+03	2.26E+04	1.86E+03	7.35E+03	3.82E+03	2.30E+04	4.05E+03	4.05E+03	3.91E+06
	std	1.49E+02	2.70E+04	2.12E+03	4.20E+03	2.68E+03	2.65E+04	1.16E+04	1.16E+04	2.97E+06
CEC7	Ave	7.01E+02	7.02E+02	7.02E+02	7.02E+02	7.02E+02	7.06E+02	7.02E+02	7.02E+02	7.08E+02
	std	1.73E-02	7.78E-01	8.53E-01	1.21E+00	1.03E+00	9.98E-01	6.05E-01	6.05E-01	1.45E+00
CEC8	Ave	1.41E+03	3.49E+03	3.43E+03	9.93E+03	2.58E+03	6.73E+03	1.47E+03	1.47E+03	6.07E+05
	std	8.96E+02	2.24E+03	3.05E+03	9.61E+03	1.77E+03	3.70E+03	2.57E+03	2.57E+03	5.29E+05
CEC9	Ave	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03	1.00E+03
	std	7.28E-03	1.41E-01	7.95E-02	2.42E-01	5.82E-02	1.08E+00	1.66E+01	1.66E+01	5.86E+00
CEC10	Ave	1.37E+03	4.00E+03	3.27E+03	8.39E+03	2.62E+03	9.91E+03	1.24E+03	1.23E+03	3.42E+05
	std	7.03E-02	3.10E+03	2.02E+03	1.23E+04	1.96E+03	9.71E+03	2.76E+04	2.76E+04	1.91E+05
CEC11	Ave	1.29E+03	1.40E+03	1.35E+03	1.37E+03	1.39E+03	1.35E+03	1.35E+03	1.35E+03	1.41E+03
	std	5.61E+00	6.39E+01	1.23E+02	9.87E+01	5.96E+01	1.22E+02	1.55E+01	1.55E+01	8.50E+01
CEC12	Ave	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.31E+03	1.30E+03	1.30E+03	1.31E+03
	std	9.01E-03	7.36E-01	7.63E-01	1.01E+00	8.88E-01	1.69E+00	8.25E+00	8.25E+00	2.26E+00
CEC13	Ave	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.30E+03	1.35E+03
	std	6.52E-05	2.11E-03	5.98E-03	1.14E-03	2.67E-03	4.16E-03	7.07E-03	7.07E-03	5.17E+01
CEC14	Ave	6.12E+03	7.29E+03	7.10E+03	7.60E+03	7.34E+03	7.51E+03	6.22E+03	6.22E+03	9.30E+03
	std	5.37E+01	2.70E+03	3.43E+03	1.42E+03	2.72E+03	1.67E+03	2.33E+03	2.33E+03	4.44E+02
CEC15	Ave	1.60E+03	1.61E+03	1.60E+03	1.61E+03	1.60E+03	1.62E+03	1.60E+03	1.60E+03	1.64E+03
	std	6.39E-01	5.43E+00	2.93E+07	1.24E+01	1.98E+02	4.00E+00	6.26E+01	6.26E+01	1.23E+01

member, which leads to local convergence. Furthermore, in the design of FFA, global search is based on randomly generated positions for the predators, which leads to accurate and efficient scanning of the search space. On the other hand, the exploitation phase of FFA, with the aim of local search, as shown in (4) and (5), makes the search area smaller during the algorithm progress and leads to an increase in accuracy in local search and convergence to possible better solutions near discovered solutions.

IV. FFA APPLICATION FOR ENGINEERING DESIGN PROBLEMS

In this section, the efficiency of the FFA in solving four engineering design problems is evaluated, including the design of a pressure vessel, a speed reducer, a welded beam, and a tension/compression spring.

In dealing with constrained optimization problems, if the values of the variables go out of the allowed range, they are set to the closest endpoint of the interval that determines this variable. Also, to satisfy all constraints in the form of equalities and inequalities, we used the penalty coefficient, which added to the objective function's value. In this case, the solution that does not comply with the constraints is identified as an inappropriate solution.

A. TENSION/COMPRESSION SPRING DESING OPTIMIZATION PROBLEM

The tension/compression spring design is an engineering optimization problem whose main objective is reducing the weight of the tension/compression spring. Figure 6 shows the schematic of the tension/compression spring design problem. This issue has three design variables as follows:

d: the wire diameter,
D: the mean coil diameter,
P: the number of active coils.

The mathematical model of tension/compression spring design problem is as follows.

$$\text{Consider}: X = [x_1, x_2, x_3] = [d, D, P].$$
$$\text{Minimize}: f(x) = (x_3 + 2)\, x_2 x_1^2.$$

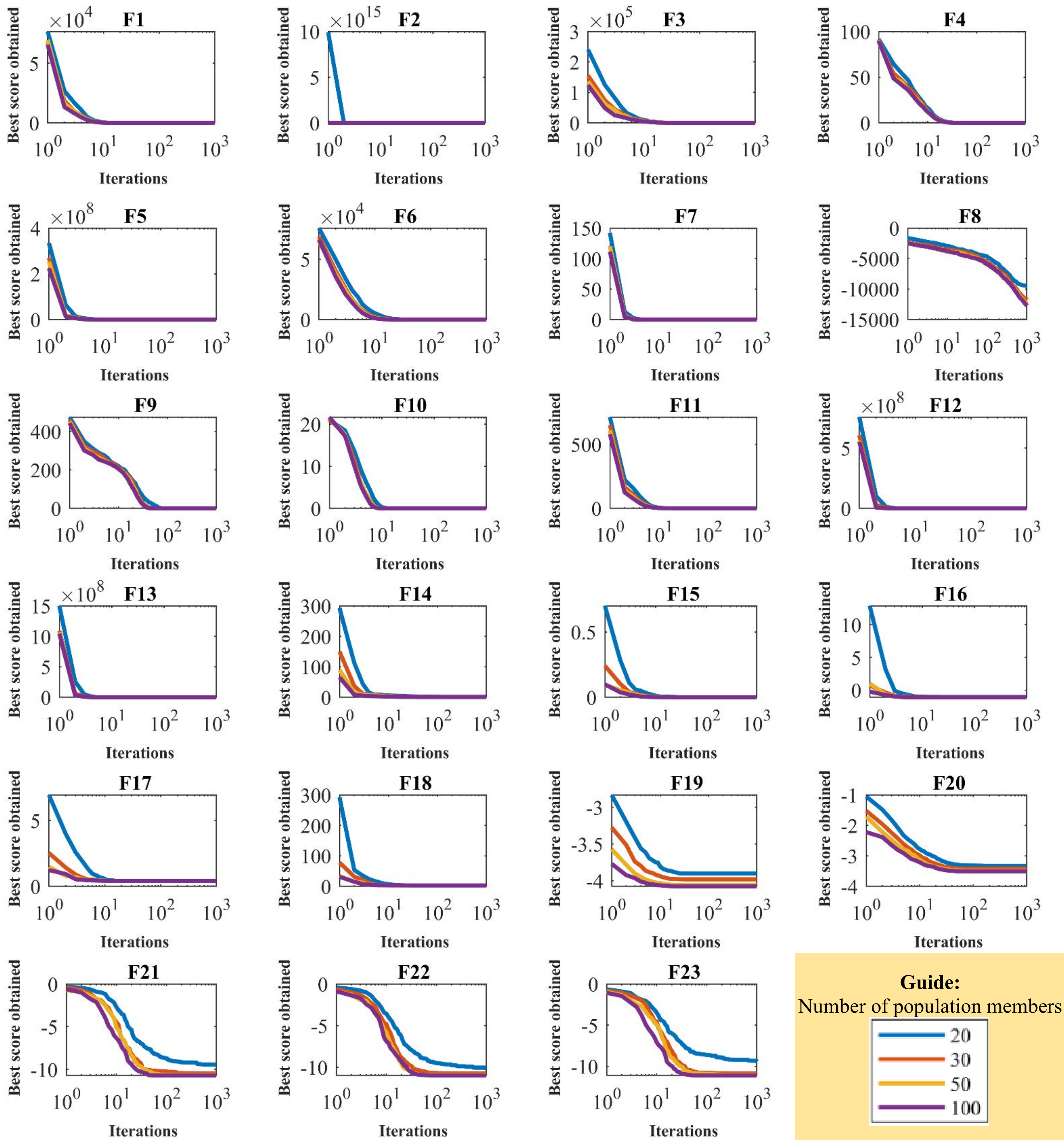

FIGURE 4. Sensitivity analysis of the FFA for the number of population members.

Subject to :

$$g_1(x) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0,$$

$$g_2(x) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3)} + \frac{1}{5108 x_1^2} - 1 \le 0,$$

$$g_3(x) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0,$$

$$g_4(x) = \frac{x_1 + x_2}{1.5} - 1 \le 0.$$

With

$0.05 \le x_1 \le 2, 0.25 \le x_2 \le 1.3,$ and$2 \le x_3 \le 15.$

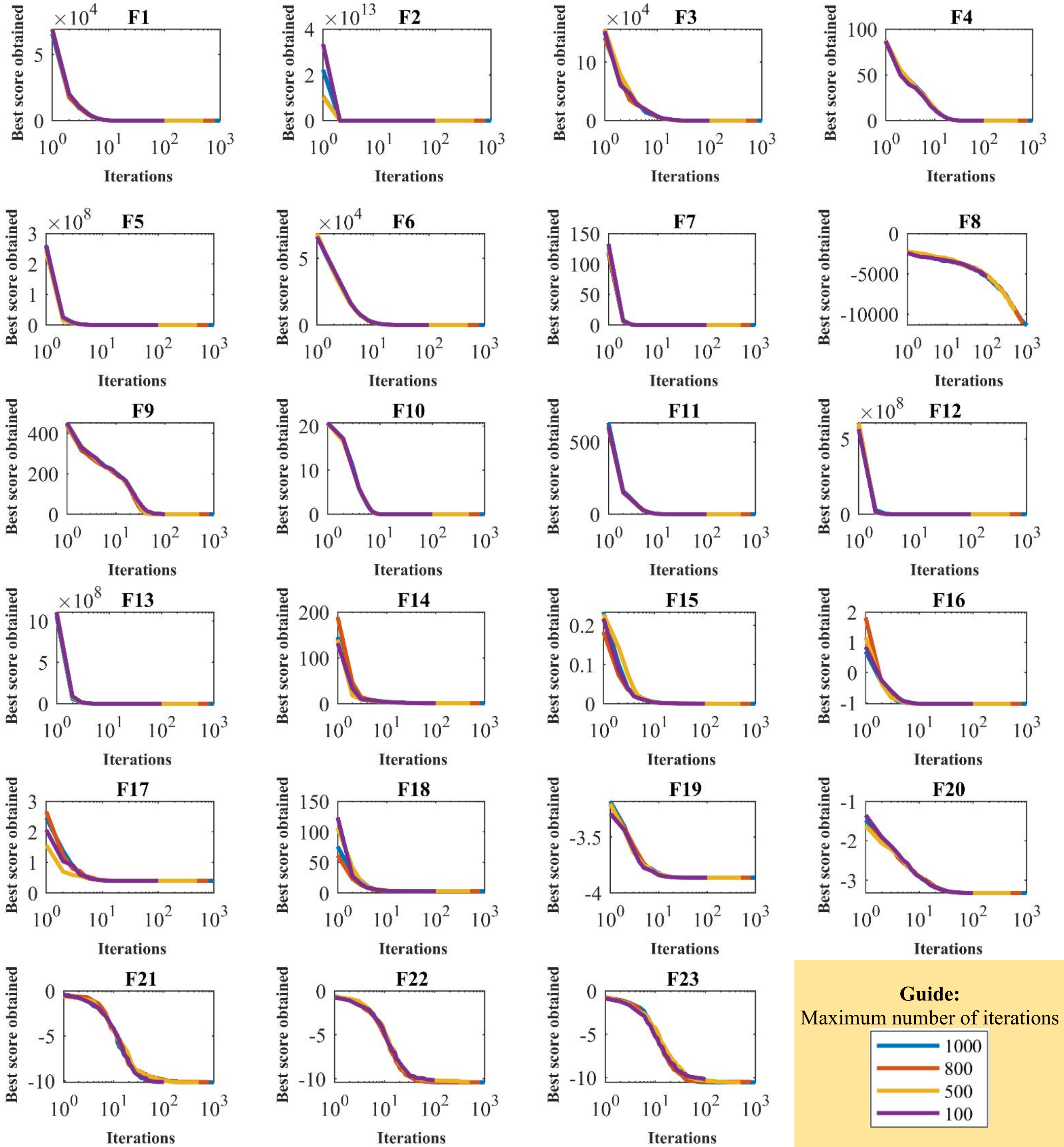

FIGURE 5. Sensitivity analysis of the FFA on different total number of iterations.

The results of FFA implementation and competitor algorithms in assigning the values of the tension /compression spring design variables are reported in Table 10. FFA provides the optimal solution to this problem with the values of the design variables equal to (0.0512889, 0.347136, 11.8781) and the corresponding values of the objective function equal to 0.012673. The statistical results obtained from the performance of FFA and competitor algorithms are presented in Table 11. The simulation results imply that FFA with better statistical indicators generates the optimal solution to the tension/compression spring design problem compared to competitor algorithms. The convergence curve to achieve the optimal design in this problem, obtained from FFA, is shown in Figure 7.

TABLE 9. Optimization results of the FFA and competitor algorithms on CEC2017 test functions.

		FFA	MPA	TSA	WOA	GWO	TLBO	GSA	GA	PSO
C1	Ave	8.25E+02	2.38E+05	4.47E+04	2.12E+05	1.57E+05	6.16E+04	3.30E+06	7.75E+05	8.05E+07
	std	7.91E+02	2.51E+07	5.31E+06	2.40E+07	3.00E+07	5.63E+06	9.32E+07	3.49E+07	3.87E+06
C2	Ave	1.64E+02	3.23E+04	9.51E+03	5.75E+05	1.07E+03	1.53E+04	4.68E+03	7.43E+07	6.91E+06
	std	5.68E+00	4.72E+06	1.30E+05	6.74E+07	1.72E+05	1.24E+05	1.31E+04	2.67E+09	3.45E+05
C3	Ave	3.00E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02	3.30E+02
	std	1.77E-03	4.25E-03	9.58E-03	7.90E-03	1.02E-02	3.62E-03	1.33E-02	8.39E-03	7.90E-01
C4	Ave	4.00E+02	4.21E+02	4.19E+02	4.26E+02	4.36E+02	4.28E+02	4.49E+02	4.52E+02	5.62E+02
	std	1.72E+01	1.99E+02	3.37E+01	1.24E+02	1.40E+02	1.24E+02	8.09E+02	8.60E+02	5.37E+03
C5	Ave	6.31E+02	9.23E+02	8.75E+02	9.30E+02	1.43E+04	1.19E+04	1.85E+03	1.86E+03	1.64E+04
	std	1.03E+02	2.15E+03	2.49E+03	2.07E+03	3.91E+04	3.20E+03	3.18E+03	2.64E+03	3.81E+02
C6	Ave	9.38E+02	1.39E+04	1.96E+03	2.36E+03	7.45E+03	3.92E+04	3.01E+05	2.40E+04	1.09E+06
	std	7.14E+02	1.38E+05	1.13E+04	2.81E+05	4.31E+04	2.79E+04	3.08E+07	2.76E+05	2.88E+06
C7	Ave	7.12E+02	7.12E+02	7.12E+02	7.12E+02	7.12E+02	7.12E+03	7.18E+02	7.16E+02	7.65E+02
	std	5.20E-03	7.55E-02	8.64E-02	7.89E-02	1.32E+01	1.05E-01	1.56E+01	1.01E-01	3.82E+00
C8	Ave	9.61E+02	1.96E+03	3.53E+03	3.59E+04	9.03E+03	2.68E+04	6.17E+04	6.83E+03	1.70E+03
	std	8.76E+02	1.19E+04	3.16E+04	2.35E+04	9.72E+05	1.88E+04	5.40E+08	3.81E+04	3.45E+03
C9	Ave	1.07E+03	1.10E+03	1.10E+03	1.10E+04	1.10E+03	1.10E+03	1.10E+04	1.10E+03	1.12E+03
	std	1.25E-03	1.68E-02	8.06E-02	1.52E-02	2.53E-01	5.93E-03	5.97E+01	1.09E-01	0.00E+00
C10	Ave	1.37E+03	2.10E+03	3.37E+03	4.10E+04	8.49E+04	2.72E+03	3.52E+04	9.01E+04	3.14E+03
	std	1.91E+02	3.11E+04	2.13E+04	3.21E+04	1.34E+05	2.07E+04	2.02E+06	9.82E+04	4.04E+02
C11	Ave	1.12E+03	1.48E+03	1.45E+03	1.50E+04	1.47E+03	1.49E+03	1.51E+03	1.45E+03	2.79E+03
	std	1.53E+01	2.77E+02	1.34E+03	6.50E+02	8.88E+02	6.07E+02	8.61E+02	1.33E+03	3.66E+03
C12	Ave	1.40E+03	1.40E+03	1.40E+03	1.40E+03	1.40E+03	1.40E+05	1.41E+06	1.41E+03	1.68E+04
	std	1.25E-03	8.79E-02	6.64E-02	7.47E-02	1.02E-01	8.99E-02	2.37E+01	1.80E+01	5.75E+01
C13	Ave	1.40E+02	1.40E+02	1.40E+03	1.40E+06	1.40E+04	1.40E+03	1.45E+02	1.40E+04	3.06E+04
	std	2.62E-06	3.15E-05	6.09E-04	1.12E-05	1.25E-04	2.78E-05	5.28E+02	4.27E-04	2.33E+04
C14	Ave	1.45E+03	4.35E+04	1.40E+03	7.39E+03	7.70E+04	7.44E+04	9.40E+03	7.61E+03	6.50E+03
	std	7.10E+04	2.01E+04	3.54E+04	2.81E+04	1.53E+04	2.83E+04	4.55E+03	1.78E+04	2.06E+02
C15	Ave	1.52E+03	1.70E+03	1.70E+03	1.71E+04	1.71E+06	1.70E+03	1.74E+06	1.72E+06	2.39E+06
	std	4.24E+01	4.25E+01	3.04E+02	4.44E+01	1.35E+02	2.09E+02	1.34E+01	4.11E+01	2.74E+05
C16	Ave	1.60E+03	3.28E+05	5.37E+05	3.02E+06	2.47E+05	7.06E+05	4.20E+06	8.65E+05	4.51E+05
	std	3.96E-01	3.50E+09	6.30E+08	3.39E+09	3.99E+09	6.62E+09	1.03E+10	4.48E+09	1.52E+04
C17	Ave	3.68E+03	4.13E+04	8.41E+04	6.65E+06	2.97E+04	2.43E+05	5.58E+03	8.33E+06	2.83E+04
	std	3.40E+00	5.71E+04	2.29E+04	7.73E+05	2.71E+04	2.23E+03	2.30E+03	3.66E+07	9.47E+04
C18	Ave	4.20E+02	4.20E+02	4.20E+02	4.20E+02	4.20E+02	4.20E+03	4.20E+03	4.20E+02	2.43E+03
	std	3.52E-05	5.24E-04	1.06E-03	8.89E-04	8.95E-04	4.61E-04	2.32E-07	9.38E-04	2.22E+03
C19	Ave	3.73E+01	5.11E+03	1.09E+01	5.16E+02	5.26E+02	5.18E+03	5.39E+03	5.42E+02	1.41E+03
	std	3.21E+00	2.98E+02	5.39E-01	2.23E+02	2.39E+02	2.23E+02	9.08E+02	9.59E+02	2.49E+03
C20	Ave	8.01E+02	8.13E+03	7.65E+02	8.20E+02	2.33E+03	2.09E+03	2.75E+03	2.76E+04	1.40E+04
	std	1.64E+01	3.14E+01	3.48E+01	3.06E+00	4.90E+02	4.19E+01	4.17E+01	3.63E+02	8.51E+03
C21	Ave	2.25E+03	2.29E+03	2.86E+03	3.26E+04	8.35E+04	4.82E+04	4.91E+06	3.30E+04	2.19E+04
	std	5.01E-13	2.37E+04	3.22E+03	3.80E+05	5.30E+03	3.78E+04	4.07E+07	3.75E+04	4.15E+03
C22	Ave	8.01E+02	8.02E+02	8.02E+02	8.02E+03	8.02E+02	8.02E+03	8.08E+02	8.06E+03	8.49E+03
	std	3.04E-01	8.54E-01	9.63E-01	8.88E-01	2.31E+01	9.24E-01	2.55E+00	8.88E-02	1.93E+03
C23	Ave	2.61E+03	2.86E+03	4.43E+04	4.49E+04	8.93E+04	3.58E+03	7.07E+04	7.73E+03	4.30E+04
	std	1.32E+00	3.28E+04	4.15E+04	3.34E+04	1.07E+05	2.87E+04	6.39E+06	4.80E+04	6.86E+03
C24	Ave	2.50E+03	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	2.00E+04	5.07E+04
	std	0.00E+00	2.67E-02	7.95E-03	2.51E-02	3.52E-02	6.92E-03	6.96E+01	9.67E-02	4.54E+01
C25	Ave	2.92E+03	3.00E+04	4.27E+04	5.00E+04	9.39E+04	3.62E+04	4.42E+06	8.91E+04	4.81E+03
	std	1.35E+00	4.10E+04	3.12E+04	4.20E+04	2.33E+05	3.06E+04	3.01E+06	7.51E+04	3.08E+03
C26	Ave	1.90E+03	2.38E+04	2.35E+05	2.40E+04	2.37E+04	2.39E+04	2.41E+04	2.35E+04	1.13E+04
	std	7.07E-13	3.76E+02	2.33E+03	7.49E+02	1.10E+03	7.06E+02	9.60E+02	2.32E+03	6.18E+02

TABLE 9. *(Continued.)* **Optimization results of the FFA and competitor algorithms on CEC2017 test functions.**

C27	Ave	4.08E+03	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.30E+04	2.31E+04	2.31E+04	5.11E+03
	std	1.12E-01	9.78E-02	8.73E-02	8.46E-02	8.95E-02	9.98E-02	3.36E+00	3.89E+01	1.22E+02
C28	Ave	9.17E+03	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.30E+04	5.35E+04	5.30E+04	4.60E+04
	std	5.01E-13	4.14E-05	7.08E-04	4.31E-05	2.24E+04	3.77E-05	5.17E+02	5.26E-04	7.52E+00
C29	Ave	3.16E+03	5.25E+04	8.10E+04	8.29E+03	8.60E+03	8.34E+04	8.30E+04	8.51E+03	5.63E+03
	std	1.40E+01	3.00E+04	4.53E+04	3.80E+04	2.52E+05	3.82E+05	5.54E+03	2.77E+04	1.45E+01
C30	Ave	1.17E+04	2.60E+04	2.60E+04	2.61E+04	2.61E+04	2.60E+04	2.64E+04	2.62E+04	6.05E+05
	std	5.04E+02	5.24E+01	4.03E-04	6.53E+01	2.34E+02	3.08E-03	2.33E+02	5.10E+01	3.27E+04

FIGURE 6. Schematic view of tension/compression spring problem.

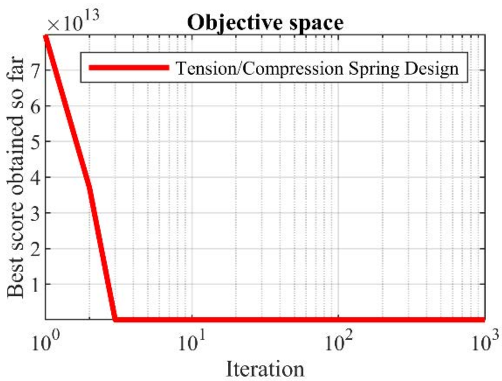

FIGURE 7. Convergence analysis of the FFA for the tension/compression spring design optimization problem.

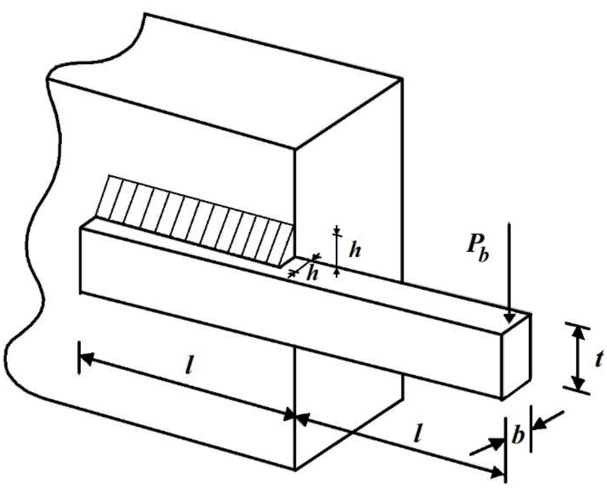

FIGURE 8. Schematic view of the welded beam design problem.

B. WELDED BEAM DESING OPTIMIZATION PROBLEM

The design of welded beams is a minimization issue with the primary aim of reducing the fabrication cost of the welded beam. Figure 8 shows the schematic of the problem of welded beam design. This problem has four design variables as follows:

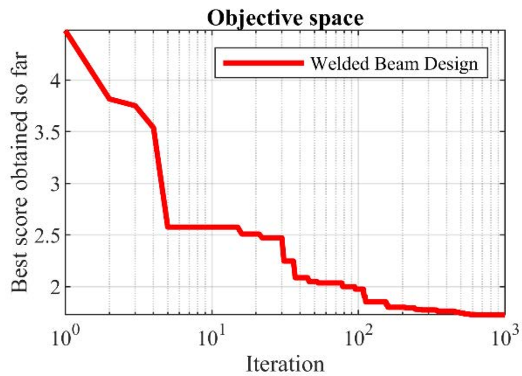

FIGURE 9. Convergence analysis of the FFA for the welded beam design optimization problem.

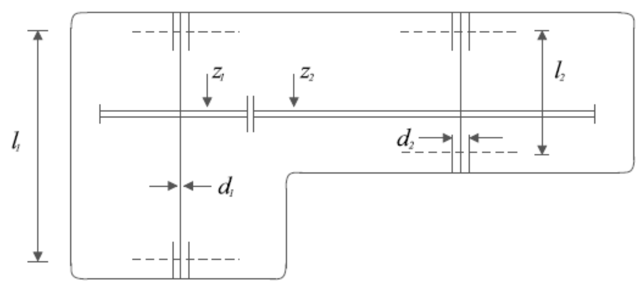

FIGURE 10. Schematic view of speed reducer design problem.

h : the thickness of the weld,
l: the length of the clamped bar,
t: the height of the bar,
b: the thickness of the bar.

The mathematical model of this engineering problem is as follows.

$$\text{Consider}: X = [x_1, x_2, x_3, x_4] = [h, l, t, b].$$

$$\text{Minimize}: f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2).$$

$$\text{Subject to}:$$

$$g_1(x) = \tau(x) - 13600 \leq 0,$$

$$g_2(x) = \sigma(x) - 30000 \leq 0,$$

TABLE 10. Comparison of optimal solutions of the algorithms for the tension/compression spring design problem.

Algorithm	Optimum Variables			Optimum Cost
	d	D	P	
FFA	0.0512889	0.3471360	11.8781	0.012673000
TSA	0.0511440	0.3437510	12.0955	0.012674000
MPA	0.050178	0.341541	12.07349	0.012678321
WOA	0.050000	0.310414	15.00000	0.013192580
GWO	0.050000	0.315956	14.22623	0.012816930
TLBO	0.050780	0.334779	12.72269	0.012709667
GSA	0.050000	0.317312	14.22867	0.012873881
PSO	0.050100	0.310111	14.00000	0.013036251
GA	0.050250	0.316351	15.23960	0.012776352

TABLE 11. Statistical results for the tension/compression spring design problem.

Algorithm	Best	Mean	Worst	SD	Median
FFA	0.012673000	0.012681272	0.012710292	0.000021	0.012681796
TSA	0.012674000	0.012684106	0.012715185	0.000027	0.012687293
MPA	0.012678321	0.012697116	0.012720757	0.000041	0.012699686
WOA	0.013192580	0.014817181	0.017862507	0.002272	0.013192580
GWO	0.012816930	0.014464372	0.017839737	0.001622	0.014021237
TLBO	0.012709667	0.012839637	0.012998448	0.000078	0.012844664
GSA	0.012873881	0.013438871	0.014211731	0.000287	0.013367888
PSO	0.013036251	0.014036254	0.016251423	0.002073	0.013002365
GA	0.012776352	0.013069872	0.015214230	0.000375	0.012952142

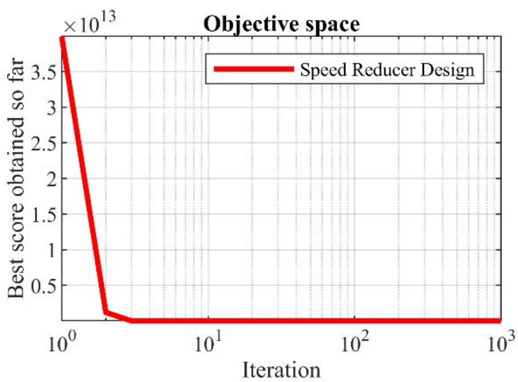

FIGURE 11. Convergence analysis of the FFA for the speed reducer design optimization problem.

$$
\begin{aligned}
g_3(x) &= x_1 - x_4 \le 0,\\
g_4(x) &= 0.10471x_1^2\\
&\quad +0.04811x_3x_4(14 + x_2) - 5.0 \le 0,\\
g_5(x) &= 0.125 - x_1 \le 0,\\
g_6(x) &= \delta(x) - 0.25 \le 0,\\
g_7(x) &= 6000 - p_c(x) \le 0,
\end{aligned}
$$

where

$$
\tau(x) = \sqrt{(\tau')^2 + (2\tau'\tau'')\frac{x_2}{2R} + (\tau'')^2},
$$

$$
\tau' = \frac{6000}{\sqrt{2}\,x_1x_2},\ \tau'' = \frac{MR}{J},
$$

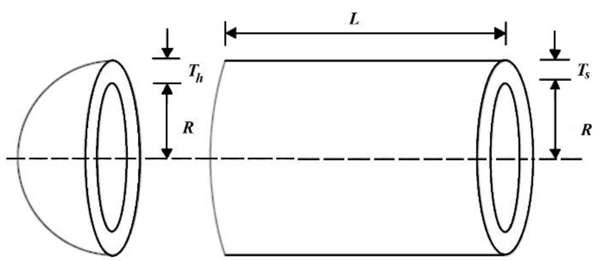

FIGURE 12. Schematic view of the pressure vessel design problem.

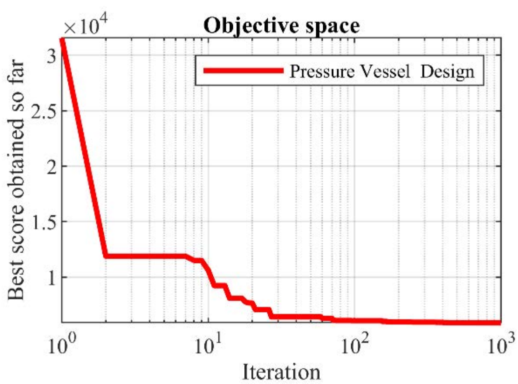

FIGURE 13. Convergence analysis of the FFA for the pressure vessel design optimization problem.

$$
M = 6000\left(14 + \frac{x_2}{2}\right), R = \frac{1}{2}\sqrt{x_2^2 + (x_1 + x_3)^2},
$$

$$
J = \frac{\sqrt{2}}{6}x_1x_2\left(x_2^2 + 3(x_1 + x_3)^2\right),
$$

TABLE 12. Comparison of optimal solutions of the algorithms for the welded beam design problem.

Algorithm	Optimum Variables				Optimum Cost
	h	l	t	b	
FFA	0.205780	3.469900	9.036800	0.205790	1.725301
TSA	0.205563	3.474846	9.035799	0.205811	1.725661
MPA	0.205678	3.475403	9.036964	0.206229	1.726995
WOA	0.197411	3.315061	10.00000	0.201395	1.820395
GWO	0.205611	3.472103	9.040931	0.205709	1.725472
TLBO	0.204695	3.536291	9.004290	0.210025	1.759173
GSA	0.147098	5.490744	10.00000	0.217725	2.172858
PSO	0.164171	4.032541	10.00000	0.223647	1.873971
GA	0.206487	3.635872	10.00000	0.203249	1.836250

TABLE 13. Statistical results for the welded beam design problem.

Algorithm	Best	Mean	Worst	SD	Median
FFA	1.725301	1.725332	1.725414	0.000012	1.725227
TSA	1.725661	1.725828	1.726064	0.000287	1.725787
MPA	1.726995	1.727128	1.727564	0.001157	1.727087
WOA	1.820395	2.230310	3.048231	0.324525	2.244663
GWO	1.725472	1.729680	1.7261651	0.048662	1.727420
TLBO	1.759173	1.817657	1.873408	0.027543	1.820128
GSA	2.172858	2.544239	3.003657	0.255859	2.495114
PSO	1.873971	2.119240	2.320125	0.034820	2.097048
GA	1.836250	1.363527	2.035247	0.139485	1.9357485

TABLE 14. Comparison of optimal solutions of the algorithms for the speed reducer design problem.

Algorithm	Optimum Variables							Optimum Cost
	b	m	p	l_1	l_2	d_1	d_2	
FFA	3.500000	0.7	17	7.3	7.8	3.350210	5.286680	2996.3482
TSA	3.501590	0.7	17	7.3	7.8	3.351270	5.288740	2998.5507
MPA	3.506690	0.7	17	7.380933	7.815726	3.357847	5.286768	3001.2880
WOA	3.500019	0.7	17	8.3	7.8	3.352412	5.286715	3005.7630
GWO	3.508502	0.7	17	7.392843	7.816034	3.358073	5.286777	3002.9280
TLBO	3.508755	0.7	17	7.3	7.8	3.461020	5.289213	3030.5630
GSA	3.600000	0.7	17	8.3	7.8	3.369658	5.289224	3051.1200
PSO	3.510253	0.7	17	8.35	7.8	3.362201	5.287723	3067.561
GA	3.520124	0.7	17	8.37	7.8	3.366970	5.288719	3029.002

TABLE 15. Statistical results for the speed reducer design problem.

Algorithm	Best	Mean	Worst	SD	Median
FFA	2996.3496	2997.221	3000.652	1.16428	2997.027
TSA	2998.5507	2999.640	3003.889	1.93193	2999.187
MPA	3001.288	3005.845	3008.752	5.83794	3004.519
WOA	3005.763	3105.252	3211.174	79.6381	3105.252
GWO	3002.928	3028.841	3060.958	13.0186	3027.031
TLBO	3030.563	3065.917	3104.779	18.0742	3065.609
GSA	3051.120	3170.334	3363.873	92.5726	3156.752
PSO	3067.561	3186.523	3313.199	17.1186	3198.187
GA	3029.002	3295.329	3619.465	57.0235	3288.657

$$\sigma(x) = \frac{504000}{x_4 x_3^2}, \delta(x) = \frac{2.1952}{x_4 x_3^3},$$
$$p_c(x) = 64746.022\,(1 - 0.0282346 x_3)\, x_3 x_4^3.$$

With

$$0.1 \le x_1, x_4 \le 2 \text{ and } 0.1 \le x_2, x_3 \le 10.$$

TABLE 16. Comparison of optimal solutions of the algorithms for the pressure vessel design problem.

Algorithm	Optimum Variables				Optimum Cost
	T_s	T_h	R	L	
FFA	0.7780598	0.3847198	40.31254	200	5883.5865
TSA	0.8303737	0.4162057	42.75127	169.3454	6048.7844
MPA	0.779035	0.384660	40.327793	199.65029	5889.3689
WOA	0.778961	0.384683	40.320913	200.00000	5891.3879
GWO	0.845719	0.418564	43.816270	156.38164	6011.5148
TLBO	0.817577	0.417932	41.74939	183.57270	6137.3724
GSA	1.085800	0.949614	49.345231	169.48741	11550.2976
PSO	0.752362	0.399540	40.452514	198.00268	5890.3279
GA	1.099523	0.906579	44.456397	179.65887	6550.0230

TABLE 17. Statistical results for the pressure vessel design problem.

Algorithm	Best	Mean	Worst	SD	Median
FFA	5883.5865	5888.1192	5890.2625	1.1605	5885.0206
TSA	6048.7844	6052.6241	6071.2496	2.893	6050.2282
MPA	5889.3689	5891.5247	5894.6238	13.910	5890.6497
WOA	5891.3879	6531.5032	7394.5879	534.119	6416.1138
GWO	6011.5148	6477.3050	7250.9170	327.007	6397.4805
TLBO	6137.3724	6326.7606	6512.3541	126.609	6318.3179
GSA	11550.2976	23342.2909	33226.2526	5790.625	24010.0415
PSO	5890.3279	6264.0053	7005.7500	496.128	6112.6899
GA	6550.0230	6643.9870	8005.4397	657.523	7586.0085

TABLE 18. Unimodal objective functions.

Objective Function	Range	*Dim*	F_{min}
$F_1(X)=\sum_{i=1}^{m} x_i^2$	$[-100,100]$	30	0
$F_2(X)=\sum_{i=1}^{m} \lvert x_i\rvert+\prod_{i=1}^{m} \lvert x_i\rvert$	$[-10,10]$	30	0
$F_3(X)=\sum_{i=1}^{m}\left(\sum_{j=1}^{i} x_i\right)^2$	$[-100,100]$	30	0
$F_4(X)=max\{\lvert x_i\rvert\},\quad 1\le i\le m$	$[-100,100]$	30	0
$F_5(X)=\sum_{i=1}^{m-1}\left[100\left(x_{i+1}-x_i^2\right)^2+(x_i-1)^2)\right]$	$[-30,30]$	30	0
$F_6(X)=\sum_{i=1}^{m}([x_i+0.5])^2$	$[-100,100]$	30	0
$F_7(X)=\sum_{i=1}^{m} ix_i^4+r$, where r is a random real number from the interval $[0,1]$.	$[-1.28,1.28]$	30	0

The proposed values for the welded beam design variables using FFA and eight competitor algorithms are reported in Table 12. FFA provides the optimal solution for this design with values of variables equal to (0.2057, 3.4699, 9.0368, 0.20579) and the corresponding value of the objective function equal to 1.725301. Statistical results from implementing FFA and competitor algorithms are presented in Table 13. FFA reveals performance superiority over

TABLE 19. High-dimensional multimodal objective functions.

Objective Function	Range	Dim	F_{min}
$F_8(X)=\sum_{i=1}^{m} -x_i \sin\left(\sqrt{\lvert x_i\rvert}\right)$	$[-500,500]$	30	−12569
$F_9(X)=\sum_{i=1}^{m}\left[x_i^2-10\cos(2\pi x_i)+10\right]$	$[-5.12,5.12]$	30	0
$F_{10}(X)=-20\exp\left(-0.2\sqrt{\frac{1}{m}\sum_{i=1}^{m}x_i^2}\right)-\exp\left(\frac{1}{m}\sum_{i=1}^{m}\cos(2\pi x_i)\right)+20+e$	$[-32,32]$	30	0
$F_{11}(X)=\frac{1}{4000}\sum_{i=1}^{m}x_i^2-\prod_{i=1}^{m}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$	$[-600,600]$	30	0
$F_{12}(X)=\frac{\pi}{m}\{10\sin(\pi y_1)+\sum_{i=1}^{m}(y_i-1)^2[1+10\sin^2(\pi y_{i+1})]+(y_n-1)^2\}+\sum_{i=1}^{m}u(x_i,10,100,4)$, where $y_i=1+\frac{x_i+1}{4}$, $u(x_i,a,i,n)=\begin{cases}k(x_i-a)^n, & x_i>a;\\ 0, & -a\le x_i\le a;\\ k(-x_i-a)^n, & x_i<-a,\end{cases}$	$[-50,50]$	30	0
$F_{13}(X)=0.1\{\sin^2(3\pi x_1)+\sum_{i=1}^{m}(x_i-1)^2[1+\sin^2(3\pi x_i+1)]+(x_n-1)^2[1+\sin^2(2\pi x_m)]\}+\sum_{i=1}^{m}u(x_i,5,100,4)$, where $u(x_i,a,i,n)=\begin{cases}k(x_i-a)^n, & x_i>a;\\ 0, & -a\le x_i\le a;\\ k(-x_i-a)^n, & x_i<-a.\end{cases}$	$[-50,50]$	30	0

competitor algorithms in terms of statistical indicators. The FFA convergence process to provide the optimal design for this problem is shown in Figure 9.

C. SPEED REDUCER DESING OPTIMIZATION PROBLEM

The design of a speed reducer is a minimization problem in engineering challenges whose main objective in its design implementation is to reduce the weight of the speed reducer. Figure 10 shows the schematic of the speed reducer design problem. This problem has seven design variables as follows:

- b: the face width,
- m: the module of teeth,
- p: the number of teeth in the pinion,
- l_1 : the length of the first shaft between bearings,
- l_2 : the length of the second shaft between bearings,
- d_1 : the diameter of first shafts,
- d_2 : the diameter of second shafts.

The mathematical model of this engineering problem is as follows.

$$\begin{aligned}\text{Consider}: X &= [x_1,x_2,x_3,x_4,x_5,x_6,x_7]\\ &= [b,m,p,l_1,l_2,d_1,d_2].\\ \text{Minimize}: f(x) &= 0.7854x_1x_2^2\left(3.3333x_3^2+14.9334x_3\right.\\ &\quad\left.-43.0934\right)-1.508x_1\left(x_6^2+x_7^2\right)\\ &\quad+7.4777\left(x_6^3+x_7^3\right)+0.7854(x_4x_6^2+x_5x_7^2).\end{aligned}$$

Subject to :

$$\begin{aligned}g_1(x) &= \frac{27}{x_1x_2^2x_3}-1\le 0,\ [[space]]g_2(x)\\ &= \frac{397.5}{x_1x_2^2x_3}-1\le 0,\\ g_3(x) &= \frac{1.93x_4^3}{x_2x_3x_6^4}-1\le 0,\ g_4(x)=\frac{1.93x_5^3}{x_2x_3x_7^4}-1\le 0,\\ g_5(x) &= \frac{1}{110x_6^3}\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2+16.9\cdot 10^6}-1\le 0,\\ g_6(x) &= \frac{1}{85x_7^3}\sqrt{\left(\frac{745x_5}{x_2x_3}\right)^2+157.5\cdot 10^6}-1\le 0,\\ g_7(x) &= \frac{x_2x_3}{40}-1\le 0,\ g_8(x)=\frac{5x_2}{x_1}-1\le 0,\\ g_9(x) &= \frac{x_1}{12x_2}-1\le 0,\\ g_{10}(x) &= \frac{1.5x_6+1.9}{x_4}-1\le 0,\\ g_{11}(x) &= \frac{1.1x_7+1.9}{x_5}-1\le 0.\end{aligned}$$

TABLE 20. Fixed-dimensional multimodal objective functions.

Objective Function	Range	Dim	F_{min}
$F_{14}(X)=\left(\frac{1}{500}+\sum_{j=1}^{25}\frac{1}{j+\sum_{i=1}^{2}(x_i-a_{ij})^6}\right)^{-1}$	$[-65.53, 65.53]$	2	0.998
$F_{15}(X)=\sum_{i=1}^{11}\left[a_i-\frac{x_1(b_i^2+b_ix_2)}{b_i^2+b_ix_3+x_4}\right]^2$	$[-5, 5]$	4	0.00030
$F_{16}(X)=4x_1^2-2.1\cdot x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	$[-5, 5]$	2	−1.0316
$F_{17}(X)=\left(x_2-\frac{5.1}{4\pi^2}x_1^2+\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos x_1+10$	[-5, 10] × [0, 15]	2	0.398
$F_{18}(X)=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)]$ $[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	$[-5, 5]$	2	3
$F_{19}(X)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2\right)$	$[0, 1]$	3	−3.86
$F_{20}(X)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2\right)$	$[0, 1]$	6	−3.22
$F_{21}(X)=-\sum_{i=1}^{5}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.1532
$F_{22}(X)=-\sum_{i=1}^{7}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.4029
$F_{23}(X)=-\sum_{i=1}^{10}[(X-a_i)\cdot(X-a_i)^T+6c_i]^{-1}$	$[0, 10]$	4	−10.5364

With

$$2.6 \le x_1 \le 3.6,\ 0.7 \le x_2 \le 0.8,\ 17 \le x_3 \le 28,$$
$$7.3 \le x_4 \le 8.3,\ 7.8 \le x_5 \le 8.3,\ 2.9 \le x_6 \le 3.9,\ \text{and } 5 \le x_7 \le 5.5.$$

The performance of FFA and eight competitor algorithms in optimizing the speed reducer design problem are presented in Table 14. FFA provides the optimal solution with the vector of variables values equal to (3.5, 0.7, 17, 7.3, 7.8, 3.35021, 5.28668) and the objective function value equal to 2996.3482. Statistical results of the performance of FFA against competitor algorithms are reported in Table 15. These results indicate that FFA is a more effective optimizer in solving the speed reducer design problem by providing better statistical indicators. The FFA convergence behavior to achieve the optimal design for this problem is shown in Figure 11.

D. PRESSURE VESSEL DESING OPTIMIZATION PROBLEM

The main goal of pressure vessel design is to minimize the total cost of material, forming, and welding of a cylindrical vessel. Figure 12 shows the schematic of the pressure vessel design problem. This problem has four design variables as follows:

T_s : the thickness of the shell,
T_h : the thickness of the head,
R : the inner radius,
L : the length of the cylindrical section without considering the head.

TABLE 21. IEEE CEC2015 benchmark test functions.

	Functions	Related basic functions	Dim	F_{min}
CEC1	Rotated Bent Cigar Function	Bent Cigar Function	30	100
CEC2	Rotated Discus Function	Discus Function	30	200
CEC3	Shifted and Rotated Weierstrass Function	Weierstrass Function	30	300
CEC4	Shifted and Rotated Schwefel's Function	Schwefel's Function	30	400
CEC5	Rotated Katsuura	Katsuura Function	30	500
CEC6	Shifted and Rotated HappyCat Function	HappyCat Function	30	600
CEC7	Shifted and Rotated HGBat Function	HGBat Function	30	700
CEC8	Shifted and Rotated Expanded Griewank's plus Rosenbrock's Function	Griewank's Function Rosenbrock's Function	30	800
CEC9	Shifted and Rotated Expanded Scaffer's F6 Function	Expanded Scaffer's F6 Function	30	900
CEC10	Hybrid Function 1 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1000
CEC11	Hybrid Function 2 ($N = 4$)	Griewank's Function Weierstrass Function Rosenbrock's Function Scaffer's F6 Function	30	1100
CEC12	Hybrid Function 3 ($N = 5$)	Katsuura Function HappyCat Function Expanded Griewank's plus Rosenbrock's Function Schwefel's Function Ackley's Function	30	1200
CEC13	Composition Function 1 ($N = 5$)	Rosenbrock's Function High Conditioned Elliptic Function Bent Cigar Function Discus Function High Conditioned Elliptic Function	30	1300
CEC14	Composition Function 2 ($N = 3$)	Schwefel's Function Rastrigin's Function High Conditioned Elliptic Function	30	1400
CEC15	Composition Function 3 ($N = 5$)	HGBat Function Rastrigin's Function Schwefel's Function Weierstrass Function High Conditioned Elliptic Function	30	1500

The mathematical model of this engineering problem is as follows.

$$\begin{aligned}
\text{Consider} : X &= [x_1, x_2, x_3, x_4] = [T_s, T_h, R, L]. \\
\text{Minimize} : f(x) &= 0.6224x_1x_3x_4 + 1.778x_2x_3^2 \\
&\quad + 3.1661x_1^2x_4 + 19.84x_1^2x_3. \\
\text{Subject to} :& \\
g_1(x) &= -x_1 + 0.0193x_3 \le 0, \\
g_2(x) &= -x_2 + 0.00954x_3 \le 0, \\
g_3(x) &= -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \le 0, \\
g_4(x) &= x_4 - 240 \le 0.
\end{aligned}$$

With

$$0 \le x_1, x_2 \le 100 \text{ and } 10 \le x_3, x_4 \le 200.$$

The results of FFA implementation and eight competitor algorithms to determine the values of the variables of the design of pressure vessels are reported in Table 16. FFA proposes the optimal design of this problem with the values

TABLE 22. IEEE CEC2017 benchmark test functions.

	Functions	f_{min}
C1	Shifted and Rotated Bent Cigar Function	100
C2	Shifted and Rotated Sum of Different Power Function	200
C3	Shifted and Rotated Zakharov Function	300
C4	Shifted and Rotated Rosenbrock's Function	400
C5	Shifted and Rotated Rastrigin's Function	500
C6	Shifted and Rotated Expanded Scaffer's Function	600
C7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
C8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
C9	Shifted and Rotated Levy Function	900
C10	Shifted and Rotated Schwefel's Function	1000
C11	Hybrid Function 1 ($N = 3$)	1100
C12	Hybrid Function 2 ($N = 3$)	1200
C13	Hybrid Function 3 ($N = 3$)	1300
C14	Hybrid Function 4 ($N = 4$)	1400
C15	Hybrid Function 5 ($N = 4$)	1500
C16	Hybrid Function 6 ($N = 4$)	1600
C17	Hybrid Function 6 ($N = 5$)	1700
C18	Hybrid Function 6 ($N = 5$)	1800
C19	Hybrid Function 6 ($N = 5$)	1900
C20	Hybrid Function 6 ($N = 6$)	2000
C21	Composition Function 1 ($N = 3$)	2100
C22	Composition Function 2 ($N = 3$)	2200
C23	Composition Function 3 ($N = 4$)	2300
C24	Composition Function 4 ($N = 4$)	2400
C25	Composition Function 5 ($N = 5$)	2500
C26	Composition Function 6 ($N = 5$)	2600
C27	Composition Function 7 ($N = 6$)	2700
C28	Composition Function 8 ($N = 6$)	2800
C29	Composition Function 9 ($N = 3$)	2900
C30	Composition Function 10 ($N = 3$)	3000

of the variables equal to (0.7780598, 0.3847198, 40.31254, 200), and the corresponding value of the objective function is equal to 5883.5865. The statistical results obtained from implementing FFA and competitor algorithms are presented in Table 17. Based on the analysis of the results of this table, it is concluded that FFA has a more effective performance in optimizing the pressure vessel design problem by providing better values in statistical indicators. The FFA convergence curve for this problem to achieve the optimal design is shown in Figure 13.

V. CONCLUSION AND FUTURE RESERCH

This paper developed a new nature-based optimization algorithm called Fennec Fox Optimization (FFA). The essential inspiration in FFA design is modeling the behavior of fennec foxes in nature when digging to hunt prey and the strategy used against predator attacks.

FFA efficiency was tested based on optimizing sixty-eight standard benchmark functions of unimodal, high-dimensional multimodal, fixed-dimensional multimodal, CEC2015, CEC2017, and four real-world optimization applications.

In addition, the FFA's performance in achieving the optimal solution was compared to eight well-known optimization methods. Experimental results showed that FFA, thanks to its proper balance between exploration and exploitation, is significantly better than eight competitor algorithms for most test benchmark functions.

It can be expected that future research following the results of this study could focus on developing multi-objective, discrete, and binary versions of the proposed FFA approach.

A multi-objective version can be used to deal with multi-objective optimization problems. A discrete version can be considered for issues that require variables from a discrete set (often a subset of integers). Finally, a binary version can be applied for problems that need to be addressed using binary algorithms, primarily feature selection problems.

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APPENDIX

See Tables 18–22.

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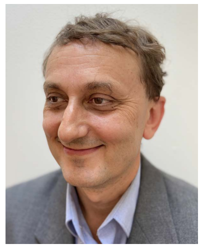

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