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Parameters Extraction of the Au/SnO_2-Si(n)/Al p-n Junction Solar Cell Using Lambert W Function

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Abstract The paper present a simple method for the extraction of solar cells parameters with a single diode circuit model from its dark characteristic and considering the series and shunt resistances. In order to extract the four parameters of the solar cell, an optimized technique is presented by using a combination of Lambert W function in its Padé-type approximation and the *FindFit function* in the Mathematica software package. By using the exact explicit analytical solutions, the current-voltage curves of the Au/SnO_2-Si(n)/Al p-n junction solar cell are calculated and simulated. The results have the good agreement between the fitted current-voltage curves and the experimental data of the corresponding solar cell. A closed form expression for the Lambert W function based on a Padé-type approximation is also provided.

Keywords Parameters extraction · Solar cell · Lambert W function

1 Introduction

Knowledge of solar cell parameters from experimental data is the basic prerequisite for technology modification and improvements. Many methods have been developed to extract the solar cell parameters (the ideality factor (n), the saturation currents (I_s), the series resistance (R_s) and the shunt conductance (G_p)), both direct and indirect.

The standard method widely used to extract the parameters requires the presence of a linear region in the ln(I) versus V plot [1]. Only two parameters n and I_s can be obtained from the slope and intercept of the linear region. This relatively simple analysis fails when a large series resistance R_s is present.

Numerous alternative methods have been proposed over the last few years to circumvent the problem introduced by R_s when extracting the desired parameters [2–7]. Practically all of the published methods use the construction of some auxiliary functions that allows the separation of the effect of R_s. In other suggested analytical methods, the R_s problem is solved using the small-signal conductance, differentiation or integration of the current with respect to voltage and addition of an external resistance in series with the diode in the measured I-V data [8–13].

Numerical methods have also been used to calculate the parameters; one of these techniques uses polynomials to demonstrate the current-voltage relationship and implements iterative method such as the recursive least-squares method [14] and Newton-Raphson method (NRM) [15] to obtain all the model parameters. But, the disadvantage of this approach is its dependency on the initial values used in the suggested iterative technique. Furthermore, NRM does not always converge. Recently, several high precision algorithms techniques have been reported, such as differential evolution (DE) [16], genetic algorithm (GA) [17] particle swarm optimization (PSO) [18] and pattern search (PS) [19, 20]. While the calculation process is more complicated, this is not conducive to master.

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Given that many advantages of these methods, in this paper, we report the development of a combination method of the Lambert W function and the FindFit function in the Mathematica software package for determining the I-V characteristics of solar cell. The parameters are extracted from dark current-voltage characteristics of an efficient p-n junction Au/SnO_2-Si(n)/Al solar cell at different temperatures (298 K, 323 K and 363 K). The samples were prepared by a vacuum evaporation technique using Balzer unit (BA- 510). The silicon n- type wafers are subjected to a rigorous cleaning cycle in three steps, in order to reduce the pin holes formation phosphorus doped Si wafer with the thickness of 300 μm and resistivity of 4.5 Ω.cm orientated in (100) plane were used. A thin insulator layer was formed on the Si surface by thermal treatment.

The aluminum thin film was deposited as back contacts for the fabricated samples. After the deposition of the aluminum film on the rough face of the wafer the samples were annealed under vacuum to a temperature of 350 °C for half an hour. This heat treatment is necessary to obtain an ohmic contact (back contact) between the aluminum and the wafers. After the deposition of SnO_2 with the required thickness on the silicon wafer, the wafer is placed inside an evacuated chamber for annealing purpose up to 400 °C. Au thin film was deposited as front contact by vacuum evaporation technique. The I-V measurements of the device were achieved using a conventional d. c. technique and a high-impedance Keithley 610 electrometer in air and at dark conditions.

2 Parameter Extraction Method

The dark forward current of a typical p-n junction device can be expressed by the relation

$$I = I_s\left\{\exp\left(\frac{\beta}{n}(V - R_s I)\right) - 1\right\} + G_p(V - R_s I) \quad (1)$$

Where $\beta = \frac{q}{kT}$ is the usual inverse thermal voltage, n is the ideal factor, I_s is the saturation current, R_s is series resistance and G_p the shunt conductance.

The solution of Eq. 1 can be expressed by using the Lambert W function [21]:

$$I = \frac{V\,G_p - I_s}{1 + R_s\,G_p} + \frac{n}{\beta\,R_s} W\left\{\frac{\beta\,I_s\,R_s}{n(1 + G_p\,R_s)}\exp\left[\frac{\beta\,V}{n} + \frac{\beta\,R_s\,(I_s - G_p V)}{n(1 + G_p R_s)}\right]\right\} \quad (2)$$

Here W denotes the short-hand notation for the Lambert W function.

Parameters extraction n, I_s, R_s and G_p of p-n junction solar cell was done using the FindFit function in the Mathematica software package. Function FindFit gives the best fit parameters for a given model function and a set of data. This function can be used for linear and nonlinear problems using last square curve fitting. In the case of nonlinear problems the function FindFit can use internally several methods like: Newton, quasi Newton, conjugate gradient and principal axis method of Brent [22], which can be chosen depending on solved problem. The function FindFit work most efficiently that means finds globally optimal fit if the given input parameters values are not much different from the optimal and if symbolic derivatives can be computed. Both of these conditions are performed due to the analytical form of Eq. 2.

3 Results and Discussion

In Fig. 1 fitting results obtained with Eq. 2 and with relative experiment results obtained at 298 K are compared. As shown, the agreement between the experimental curve and this theoretical one is not complete and the deviation becomes evident. The four extracted parameters are presented in Fig. 1.

To go farther, a key point that necessities to be discussed in association with the Lambert W function is what kind of approximation should be used to compute it. In this regard, let us point out that Eq. 2 is considered an exact result in the sense that the Lambert W function can be numerically determined within the required precision.

Efficient solutions to this problem can be delivered by many commercial software packages like Mathematica. But, a most significant question with the analytic treatment of this function is that its Taylor series fluctuates between ever larger negative and positive values becoming insufficient for practical resolutions [23]. An alternative technique other than a series expansion is therefore imperatively

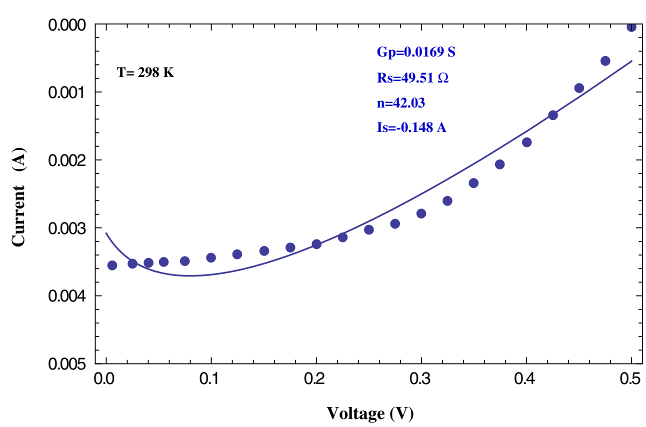

Fig. 1 Current–voltage characteristics of the Au/SnO_2-Si(n)/Al p-n junction solar cell at 298 K, experimental (·) data and the fitted curve (-) with the Lambert W function

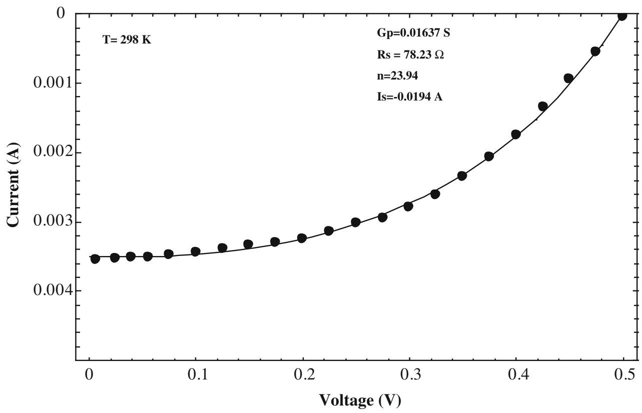

Fig. 2 Current–voltage characteristics of the Au/SnO_2-Si(n)/Al p-n junction solar cell at 298 K, experimental (·) data and the fitted curve (-) with the Padé -type approximation of the Lambert W function

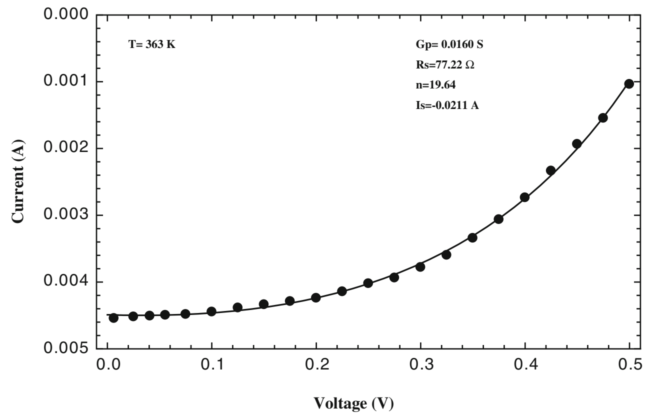

Fig. 4 Current–voltage characteristics of the Au/SnO_2-Si(n)/Al p-n junction solar cell at 363 K, experimental (·) data and the fitted curve (-) with the Padé-type approximation of the Lambert W function

necessary. In [24], it was exposed that a practical formula to compute Lambert W function is given by the expression:

$$W(z) \approx Log(1+z) \quad \left\{1 - \frac{Log\,[1 + Log(1+z)]}{2 + Log(1+z)}\right\} \quad . \quad (3)$$

This is based on a Padé-type approximation. Other more complicated expressions can also be found in literature [13–18].

With this approximation of the Lambert W function, Eq. 2 can be approximate as follow:

$$I \approx \frac{V\,G_p - I_s}{1 + R_s\,G_p} + \frac{n}{\beta\,R_s}\left[Log(1+X) \quad \left\{1 - \frac{Log\,[1 + Log(1+X)]}{2 + Log(1+X)}\right\}\right] \quad (4)$$

where

$$X = \frac{\beta\,I_s\,R_s}{n(1 + G_p\,R_s)}\ \exp\left[\frac{\beta\,V}{n} + \frac{\beta\,R_s\,(I_s - G_p V)}{n(1 + G_p R_s)}\right] \quad (5)$$

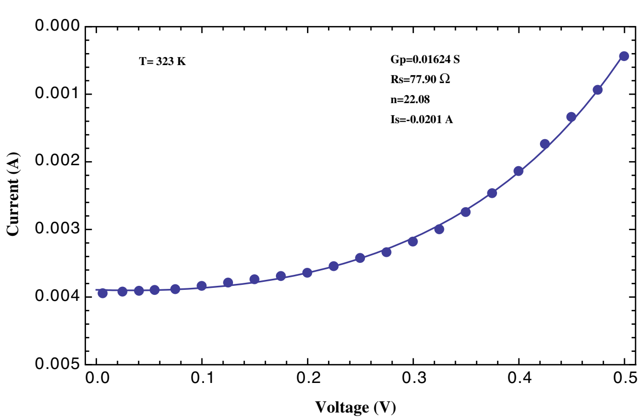

Fig. 3 Current–voltage characteristics of the Au/SnO_2-Si(n)/Al p-n junction solar cell at 323 K, experimental (·) data and the fitted curve (-) with the Padé -type approximation of Lambert W function

The solar cell parameters were extracted by using Eq. 4. In order to show that Eq. 4 is accurately appropriate, Fig. 2 illustrates the fitting curves given by Eq. 2 calculated by Eq. 4. The agreement is excellent within the experimental I-V. The four extracted parameters are presented in Fig. 2.

To provide a comprehensive evaluation, the presented procedure has also been applied in parameter extraction from current–voltage characteristics measured as a function of temperature. Current-voltage characteristics of Au/SnO_2-Si(n)/Al p-n junction solar cell measured at temperatures 325 K, 363 K and calculated for extracted parameters are presented in Figs. 3 and 4. In these last, a good agreement is shown between experimental and fitting curves, the four extracted parameters are also presented.

4 Conclusion

This paper presents a simple method to extract the solar cell parameters from the dark I-V experimental data. By using the exact analytical solutions, the I-V curves of Au/SnO_2-Si(n)/Al solar cell are calculated and simulated. A simple method is presented by using a combination of the Lambert *W* function and Findfit function in the Mathematica software package to extract the parameters. The results obtained are in good agreement with the experimental data. The proposed method is easy, simple, does not involve prior knowledge of any of the parameters of interest. An approximate expression based on a Padé-type approximation was also provided for the Lambert W function.

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