International Journal on Artificial Intelligence Tools
Vol. 12, No. 3 (2003) 375–394
© World Scientific Publishing Company

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

COMPUTING WITH BEES: ATTACKING COMPLEX TRANSPORTATION ENGINEERING PROBLEMS

PANTA LUČIĆ* and DUŠAN TEODOROVIĆ†

The Charles E. Via Jr. Department of Civil and Environmental Engineering
Virginia Polytechnic Institute and State University
Northern Virginia Center, 7054 Haycock Road
Falls Church, VA 22043-2311, U.S.A.
**plucic@vt.edu*
†duteodor@vt.edu

Received 1 February 2003
Accepted 1 April 2003

The Bee System (an artificial bee swarm) is introduced in this paper. The proposed approach is applied to the Traveling Salesman Problem. The obtained results are very promising. The potential applications of the developed Bee System in the field of transportation engineering are discussed. The Bee System represents the new, successful application of emergent techniques based on natural metaphors, such as simulated annealing, genetic algorithms, and neural networks, to the complex engineering and management problems.

Keywords: Bee System; artificial life; traveling salesman problem; evolutionary computation; transportation engineering.

1. Introduction

Natural systems teach us that very simple individual organisms can form systems capable of performing highly complex tasks by dynamically interacting with each other. Bee swarm behavior in nature is primarily characterized by autonomy and distributed functioning and self-organizing. In the last couple of years, the researchers started studying the behavior of social insects in an attempt to use the Swarm Intelligence concept in order to develop various Artificial Systems. The development of Artificial Systems does not entail the complete imitation of natural systems, but explores them in the search for ideas and models.

It is of course of great importance to investigate both advantages and disadvantages of autonomy, distributed functioning and self-organizing in relation to traditional engineering methods that rely on control and centralization. The first goal of this paper is to acquaint the reader with the applications of swarm intelligence to date in engineering, management and control, and to indicate the directions for future research in this area. The main goal of this paper is to introduce the Bee System (an artificial bee swarm) that represents a new approach in the field of Swarm Intelligence. The third goal of the paper is to show how we can use proposed approach in development of Artificial Systems aimed at solving complex problems in transportation engineering. The organization of the paper is described below. Solving complex engineering problems by emergent techniques based on natural metaphors is discussed in Section 2. Basic characteristics of the Swarm Intelligence are explained in Section 3. The Bee System (an artificial bee swarm) is introduced in Section 4. Section 5 is devoted to the experimental study of the Bee System. Potential applications of the Bee System are discussed in Section 6. Conclusions and recommendations for further research are given in section 7.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

2. Solving Complex Engineering Problems by Emergent Techniques Based on Natural Metaphors

Combinatorial Programming is utilized for solving optimization problem that have combinatorial and/or discrete structure. Integer Programming, Dynamic Programming and Graph Theory techniques are traditional approaches for solving combinatorial optimization problems. A great number of practical real-world problems was formulated and solved using these techniques during the last four decades. It is important to note however that the majority of real-world problems solved by some of the optimization techniques were of small dimensionality. Many engineering problems are combinatorial by their nature. Most of the combinatorial optimization problems are difficult to solve either because of the large dimensionality or because it is very difficult to decompose then into smaller sub-problems. These are most commonly found to be NP-complete problems that cannot be solved exactly in polynomial time. Typical representatives of this type of problems are the vehicle fleet planning and static and dynamic routing and scheduling of vehicles and crews for airlines, railroads, truck operations and public transportation services, designing utility and transportation networks and optimizing alignments for highways and public transportation routes through complex geographic spaces, different locations problems, etc.

Many times in real life we only need a "good" solution. In other words, very often the decision makers are satisfied with sub-optimal solution obtained by some "good" heuristics ("Good" heuristics assumes, first of all, polynomial computer time and small computer memory requirement).

In recent years, the so-called metaheuristic algorithms have increasingly been used in solving difficult combinatorial optimization problems. These include the simulated annealing [1 2 3], genetic algorithms [4 5], and tabu search [6 7 8]. Simulated annealing and genetic algorithms are based on the principles of respective analogies with the processes occurring in nature.

The simulated annealing technique is one of the methods increasingly used in the last few years in solving complex combinatorial problems. Kirkpatrick et al. [2] and separately Cerny [3] were the first to suggest the simulated annealing technique in

solving combinatorial optimization problems. This method is based on the analogy with certain problems in the field of statistical mechanics found in the studies of Metropolis et al. [1]. The simulated annealing technique has already been used in solving complex transportation engineering problems [9 10 11].
Genetic algorithms represent search techniques based on the mechanics of nature selection used in solving complex combinatorial optimization problems. These algorithms were developed by analogy with Darwin's theory of evolution and the basic principle of the "survival of the fittest." The most significant results in the field of genetic algorithms were achieved by Holland [4] and Goldberg [5]. Genetic algorithms have been extensively used in the last few years in solving complex transportation engineering problems [12 13 14 15 16 17 18 19 20 21 22 23 24]. Recent successful applications of genetic algorithms are optimizing 3-dimensional highway alignments [25 26] and scheduling interdependent waterway transportation projects [27].
Artificial neural networks [4 5] are also very successful engineering tools inspired by biology. Artificial neural networks are composed of the elements that function similarly to a biological neuron. These elements are organized in a way that is reminiscent of the anatomy of a brain. In addition to this superficial similarity, artificial neural networks display a striking number of the brain's properties. For example, they are able to learn from experience, to apply to new cases generalizations derived from previous instances, and to abstract essential characteristics of input data that often contain irrelevant information. In the 1990s, hundreds of papers were published applying neural network models to different engineering problems [23].
Often times in some of the decision-making problems that are combinatorial by nature there exist multiple conflicting criteria. Development of the hybrid models (models based on the combination of the Multiple Objective Decision-Making techniques and the Metaheuristics Techniques) for solving certain class of difficult combinatorial optimization problems in engineering has already been a significant contribution to the current research in the field.
The successful applications of emergent techniques based on natural metaphors, such as simulated annealing, genetic algorithms, and neural networks, to the complex engineering and management problems are certainly encouraging; they most definitely point out to the natural systems as a source of ideas and models for development of various artificial systems.

3. Basic Characteristics of the Swarm Intelligence

Social insects (bees, wasps, ants, termites) have lived on Earth for millions of years, building nests and more complex dwellings, organizing production and procuring food. It has also been noted that they care about order and cleanliness, occasionally move around, have a communication and warning system, maintain an army, wage wars, and divide labor. In addition, the colonies of social insects are very flexible and can adapt well to the changing environment. This flexibility allows the colony of social insects to be robust and maintain its life in spite of considerable disturbances.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

The dynamics of the social insect population is a result of the different actions and interactions of individual insects with each other, as well as with their environment. The interactions are executed via multitude of various chemical and/or physical signals. The final product of different actions and interactions represents social insect colony behavior. Interaction between individual insects in the colony of social insects has been well documented. The examples of such interactive behavior are bee dancing during the food procurement, ants' pheromone secretion, and performance of specific acts which signal the other insects to start performing the same actions. These communication systems between individual insects contribute to the formation of the "collective intelligence" of the social insect colonies. Recently, the term "Swarm Intelligence", denoting this "collective intelligence", has come into use [28 29 30 31].

The self-organization of the ants is based on relatively simple rules of individual insect's behavior [32 33 34 35 36]. In the majority of ant species a number of "scouts" leave the nest foraging for food [37]. The ants successful at finding food leave behind them the pheromone trail that the other ants follow in order to reach the food. The appearance of the new ants at the pheromone trail reinforces the pheromone signal. This comprises typical autocatalytic behavior, i.e., the process that reinforces itself and thus converges fast. The "explosion" in such processes is regulated by a certain limitation mechanism. In the ant case, the pheromone trail evaporates with time. In this behavioral pattern the decision of an ant to follow a certain path to the food depends on the behavior of his nestmates. At the same time, the ant in question will also increase the chance that the nestmates leaving the nest after him follow the same path. In other words, one ant's movement is highly determined by the movement of previous ants.

The important result of the Artificial System development that was based on the Swarm Intelligence is the creation of the Ant System. Ant System [38 39] is a relatively new metaheuristic for hard combinatorial optimization problems. Dorigo et al. [39] applied Ant System to the classical Traveling Salesman Problem. They also tested the approach proposed on asymmetric Traveling Salesman Problem, the quadratic assignment and the job-shop scheduling problem. Bullnheimer et al. [40 41] used the Ant System to solve the Vehicle Routing Problem in its basic form (homogenous fleet, capacity restriction, distance restriction, one central depot) and obtained very good results.

4. Bee System

4.1. *Bees in the nature*

By examining the bees' behavior, the researchers tried is to describe how bee colonies function in nature. Self-organization of the bees is based on a few relatively simple rules of individual insect's behavior [42 43 44 45 46 47 48 49 50 51 52 53 54 55]. In spite

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

of the existence of a large number of different social insect species, and variation in their behavioral patterns, it is possible to describe individual insects' as capable of performing a variety of complex tasks [56]. The best example is the collection and processing of nectar, the practice of which is highly organized. Each bee decides to reach the nectar source by following a nestmate who has already discovered a patch of flowers. Each hive has a so-called dance floor area in which the bees that have discovered nectar sources dance, in that way trying to convince their nestmates to follow them. If a bee decides to leave the hive to get nectar, she follows one of the bee dancers to one of the nectar areas. Upon arrival, the foraging bee takes a load of nectar and returns to the hive relinquishing the nectar to a food storer bee. After she relinquishes the food, the bee can (a) abandon the food source and become again uncommitted follower, (b) continue to forage at the food source without recruiting the nestmates, or (c) dance and thus recruit the nestmates before the return to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area the bee dancers "advertise" different food areas. The mechanisms by which the bee decides to follow a specific dancer are not well understood, but it is considered that "the recruitment among bees is always a function of the quality of the food source" [56]. It is also noted that not all bees start foraging simultaneously. The experiments confirmed that "new bees begin foraging at a rate proportional to the difference between the eventual total and the number presently foraging".

4.2. *Artificial bees*

Bees' behavior in nature has been a great inspiration for the authors of this paper and a source of ideas and models for development of various artificial systems capable to solve complex engineering and management problems. Our artificial bee colony behaves partially alike, and partially differently from bee colonies in nature. The bee system described in this paper represents a significant modification and improvement of our previous work [57]. Our artificial system is the system composed of a number of precisely defined elements (individuals). We define the behavior rules of our artificial bees (agents) and simulate the interaction between them. In this way, different scenarios with different rules can be simulated and explored.

A large portion of social insects' activities is tied to the food foraging. It is known that honeybees "normally spend the last part of their life collecting food" [58]. Also, they "spend a considerable portion of their life span learning and improving their foraging skills" [49]. Every bee colony has scouts that are colony's explorers [59]. The explorers do not have any guidance while looking for food. They are primarily concerned with finding any kind of food source. As a result of such behavior, the scouts are characterized by low search costs and a low average in food source quality. Occasionally, the scouts can accidentally discover rich, entirely unknown food sources. In the case of artificial bees, the artificial scouts attempting to solve difficult combinatorial optimization problems could have as a task the fast discovery of the group of feasible solutions. Some of those feasible solutions to the difficult combinatorial optimization problems could then prove to be solutions of very good quality.

In the case of honeybees, the recruitment rate represents a "measure" of how quickly the bee colony finds and exploits a newly discovered food source. Artificial recruiting could similarly represent the "measurement" of the speed with which the feasible solutions or the "good quality" solutions are found.

The cooperation between the insects decreases foragers' costs in finding new food sources. This suggests that cooperation between artificial bees would also allow for the fast discovery of the feasible solution.

It is also known that cooperation increases the quality of the food sources located by foragers. This implies that cooperation could also help us find the best solutions of the difficult combinatorial optimization problems.

The survival and progress of the bee colony is dependent upon rapid discovery and efficient utilization of the best food resources. In other words, the successful solution of difficult engineering problems (especially those that need to be solved in real time) is connected to the relatively fast discovery of "good solutions".

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

4.3. *Solving the traveling salesman problem by the bee system*

The primary goal of this paper is to explore the possible applications of swarm intelligence (and especially collective bee intelligence) in solving complex transportation engineering problems. The development of the new heuristic algorithm for the Traveling Salesman Problem will serve as an illustrative example for such applications and will show the characteristics of the proposed concept. The well known Traveling Salesman problem is defined in the following way: Given n nodes, find the shortest itinerary that starts in a specific node, goes through all other nodes exactly once and finishes in the starting node.

We define an artificial bee environment. This environment is an artificial representation of the space. Our artificial bees (agents) perform only activities defined by our model and a corresponding computer program. We define the way in which artificial bees communicate with each other. In other words, we perform Multi-agent simulation.

Let us denote by $G = (N, A)$ the network in which the bees are collecting nectar (the graph in which the traveling salesman route should be discovered). Let us also randomly locate the hive in one of the nodes. When foraging, the bees are trying to collect as much nectar as possible. Let us also assume that the nectar quantity that is possible to collect flying along a certain link is inversely proportional to the link length. In other words, the shorter the link, the higher the nectar quantity collected along that link. This means that the greatest possible nectar quantity could be collected when flying along the shortest traveling salesman route. Our artificial bees will collect the nectar during the certain prescribed time interval. After that, we will randomly change the hive position. The bees will start to collect the nectar

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

from the new location. We will then again randomly change the hive location etc. The iteration in searching process represents one change of the hive position. We assume that our artificial bees live in an environment characterized by the discrete time. Each iteration is composed of a certain number of stages. The stage is an elementary time unit in bees' environment. During one stage the bee will visit s nodes, create partial traveling salesman tour, and after that return to the hive (the number of nodes s to be visited within one stage is prescribed by the analyst at the beginning of the search process). In the hive the bee will participate in a decision making process. The bee will decide whether to abandon the food source and become again uncommitted follower, continue to forage at the food source without recruiting the nestmates, or dance and thus recruit the nestmates before returning to the food source. Let us denote by B the total number of bees in the hive and by $B(u,\ z)$ the total number of bees collecting nectar during stage u ($u = 0,\ 1,\ 2,\ ...,$ $\left\lceil \frac{|N|-1}{s} \right\rceil$) in iteration z.

We will assume that at the beginning of every iteration z all bees are in the hive, i.e.:

$$B(0, z) = 0 \tag{1}$$

It is noted in the nature that not all bees start foraging simultaneously. In the case of artificial bees we have been increasing the number of foraging bees in every subsequent stage in the following way. Let us introduce the binary variables $b_k(u,$ $z)$, defined as:

$$b_k(u,z) = \begin{cases} 1, & \text{if } k\text{-th bees participates in foraging during stage } u \\ & \text{in iteration } z \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$\begin{aligned} k &= 1, 2, ..., B \\ u &= 0, 1, 2, ..., \left\lceil \tfrac{|N|-1}{s} \right\rceil \\ z &= 1, 2, ..., M \end{aligned}$$

where:

M - maximum number of iterations.

Some bees will start foraging in the first stage. The remaining bees will join the nestmates in the second stage, or in the third stage, or in the fourth stage, etc. Once bee starts foraging, it will remain "active" until the end of the considered iteration. We describe the foraging activity of every bee by the array composed of 0's and 1's. The array [1,1,1, ...,1] describes foraging activity of the bee that has been foraging from the first stage; the array [0,0,0,1,1,1, ...,1] describes foraging activity of the bee that has been foraging from the fourth stage, and so forth.

Let us also introduce the binary variables $h_k(u,\ z)$, defined in the following way:

$$h_k(u,z) = \begin{cases} 1, & \text{if } w > r_k(u,z) \text{ and } b_k(u-1,z) = 0 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where:

w - the parameter given by the analyst $(0 \leq w \leq 1)$,

$\mathrm{r}_k(u,z)$ - the random number taken form the unit interval [0,1].

The binary variables $h_k(u,\ z)$ indicate the stage in which particular bee starts with foraging. For every bee k that has not been participating in the foraging process in the stage (u-1), we have chosen the random number $r_k(u,\ z)$. The k-th bee will join her nestmates in foraging during stage u, if the following relation has been satisfied:

$$w > r_k(u,z) \tag{4}$$

The higher the value of the parameter w given by the analyst, the higher the chance for any bee to become foraging. That is, practically all bees from the hive will start foraging process rather quickly. The smaller value of the parameter w corresponds with the slow increase of the number of foraging bees.
The binary variable $b_k(u,\ z)$ equals:

$$b_k(u,z) = b_k(u-1,z) + h_k(u,z) \tag{5}$$

The total number of foraging bees during u-th stage in the z-th iteration equals:

$$B(u,z) = \sum_{k=1}^{B} b_k(u,z) \tag{6}$$

During any stage, bees are choosing nodes to be visited in a random manner. Logit model is one of the most successful and widely accepted discrete choice model. We have assumed that the probability of choosing node j by the k-th bee, located in the node i (during stage $u+1$ and iteration z) equals:

$$p_{ij}^k(u+1,z) = \begin{cases} \dfrac{e^{-ad_{ij}\frac{z}{\sum_{r=\max(z-b,1)}^{z-1} n_{ij}(r)}}}{\sum_{l\in N_k(u,z)} e^{-ad_{il}\frac{z}{\sum_{r=\max(z-b,1)}^{z-1} n_{il}(r)}}}, & \begin{array}{l} i = g_k(u,z), \\ j \in N_k(u,z), \\ \forall k,u,z \end{array} \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

where:

$i, j-$	node indexes (i, j = 1, 2, ..., $\|N\|$),
$d_{ij}-$	length of link (i, j),
$k-$	bee index (k = 1,2, ..., B),
$z-$	iteration index (z = 1, 2, ..., M),
$n_{il}(r)-$	total number of bees that visited link (i, l) in r-th iteration,
$b-$	"memory length"[a] ,
$g_k(u, z)-$	last node that bee k visits at the end of stage u in iteration z,
$N_k(u, z)-$	set of unvisited nodes for bee k at stage u in iteration z (in one stage bee will visits s nodes; we have $\|N_k(u, z)\| = \|N\| - us$),
$a-$	input parameter.

Let us discuss relation (7) that we propose in a more details. The greater the distance between node i and node j, the lower the probability that the k-th bee located in the node i will choose node j during stage u and iteration z. The distance d_{ij} is obviously a very important factor influencing the bee's choice of the next node. The influence of the distance is lower at the beginning of the search process. The greater the number of iterations z, the higher the influence of the distance. This is expressed by the term z in the nominator of the exponent (relation (7)). In other words, at the beginning of the search process bees have more freedom of flight. They have more freedom to search the solution space. The more iterations we make, the bees have less freedom to explore the solution space. The more we are approaching the end of the search process, the more focused the bees are on the flowers (nodes) in the neighborhood. Our artificial bees have memory and they can remember how many bees visited a certain link during last b iterations. The greater the total number of bees that visited a certain link in the past, the higher the probability of choosing that link in the future. This represents the interaction between individual bees in the colony.

For every bee we now know the nectar quantity collected by the bee (the length of the partial traveling salesman tour). After returning to the hive bees relinquish the nectar to a food storer bee. After relinquishing the food, the bee is making the decision about abandoning the food source or continuing the foraging at the food source. It is assumed in this paper that every artificial bee (agent) can obtain the information about nectar quantity collected by every other artificial bee. The probability that the bee k will at beginning of the stage $u + 1$ use the same partial tour that is defined in stage u in iteration z equals:

$$p_k(u+1, z) = e^{-\frac{L_k(u,z) - \min_{r \in w(u,z)}(L_r(u,z))}{uz}} . \tag{8}$$

[a]While foraging in stage u, every artificial bee has the ability to notice the total number of bees in every link. Every artificial bee has the same ability for previous stages as well. That is, artificial bees have the capacity to remember former bee assignments in the network. However, bee recollection is limited. The maximum number of stages that bee can recall represents memory length.

where $L_k(u, z)$ is the length of partial route that is discovered by bee k in stage u in iteration z.

We can see from relation (8) that if a bee has discovered the shortest partial traveling salesman tour in stage u in iteration z, the bee will fly along the same partial tour with the probability equal to one. The longer the tour that the bee has discovered, the smaller the probability that the bee will fly again along the same tour. When bee decides not to abandon the food source she can: (a) continue of foraging at the food source without recruiting the nestmates; (b) fly to the dance floor area and start dancing, thus recruiting the nestmates before the return to the food source. The bee opts for one of the above alternatives with a certain probability. Within the dance area the bee dancers "advertise" different food areas. The mechanisms by which the bee decides to follow a specific dancer are not well understood, but it is considered that "the recruitment among bees is always a function of the quality of the food source" [56]. Since the bees are, before all, social insects (the interaction between individual bees in the colony has been well documented), it is assumed in this paper that the probability p^* of an event that the bee will continue foraging at the food source without recruiting the nestmates is very low:

$$p^* \ll 1 \tag{9}$$

After relinquishing the food, and after making the decision to continue foraging at the food source, the bee flies to the dance floor and starts dancing with the probability equal to (1 - p^*). Bee dancing represents the interaction between individual bees in the colony. This kind of communication between individual bees contributes to the formation of the collective intelligence of the bee colony.

In the case when at the beginning of stage $u + 1$, bee does not use the same partial traveling salesman tour, the bee will go to the dancing area and will follow another bee(s). We assume that every partial traveling salesman tour ξ that is being advertised in the dance area has two main attributes: (a) the total length, and (b) the number of bees that are advertising the partial route. We introduce the normalized value of the total length of the partial traveling salesman tour and the normalized value of the number of bees advertising the partial tour. Both normalized values are defined in the following way: (a) Both normalized values can take any value between 0 and 1; (b) The smaller the total length normalized value the better the partial tour; (c) The bigger the number of bees normalized value the better the partial tour.

Let us denoted by $Y(u, z)$ - the set of partial tours that were visited by at least one bee and by $B_\xi(u, z)$ - the number of bees that discovered partial route ξ. The normalized value of the partial route length equals:

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

$$\alpha_\xi(u,z) = \begin{cases} \frac{L_\xi(u,z)-\min_{r\in Y(u,z)}(L_r(u,z))}{\max_{r\in Y(u,z)}(L_r(u,z))-\min_{r\in Y(u,z)}(L_r(u,z))}, & \begin{array}{l}\max\limits_{r\in Y(u,z)}(L_r(u,z)) \neq \\ \min\limits_{r\in Y(u,z)}(L_r(u,z)), \\ j \in N_k(u,z), \\ \forall k,u,z\end{array} \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

The normalized value of the number of bees advertising the partial tour equals:

$$\beta_\xi(u,z) = \begin{cases} \frac{B_\xi(u,z)-\min_{r\in Y(u,z)}(B_r(u,z))}{\max_{r\in Y(u,z)}(B_r(u,z))-\min_{r\in Y(u,z)}(B_r(u,z))}, & \begin{array}{l}\max\limits_{r\in Y(u,z)}(B_r(u,z)) \neq \\ \min\limits_{r\in Y(u,z)}(B_r(u,z)), \\ j \in N_k(u,z), \\ \forall k,u,z\end{array} \\ 0, & \text{otherwise} \end{cases} \tag{11}$$

We have assumed in this paper that the probability that the partial route ξ will be chosen by any bee that decided to choose the new route equals:

$$p_\xi(u,z) = \frac{e^{\rho\beta_\xi(u,z)-\theta\alpha_\xi(u,z)}}{\sum_{r\in Y(u,z)} e^{\rho\beta_r(u,z)-\theta\alpha_r(u,z)}}, \quad \xi \in Y(u,z),\ \forall u,z \tag{12}$$

where:

ρ, θ - parameters given by the analyst.

Before relocating the hive to the next location we tried to improve the solution obtained by the bees in current iteration by applying different tour improvement algorithms. The most frequently used tour improvement algorithms are based on k-opt procedure. The basic idea is to replace subset of k arcs in previously defined tour by the subset of new arcs with same cardinality and smaller total length such that new shorter tour will result. Complexity of k-opt algorithm is $O(n^k)$ where n is number of nodes considered in TSP instance. In the literature researchers have used 2-opt and 3-opt algorithms most frequently [60]. In this paper we used 2-opt algorithm, 3-opt algorithm and modified 3-opt algorithm (this modified algorithm will be explained later in the paper). Creating of the partial Traveling Salesman Tours is shown in Fig. 1.

Four bees chose their first nodes and created four different Traveling Salesman tours during the first stage. The second stage shows three bees continuing to further develop their tours. The fourth bee did not use the same partial traveling salesman tour in the second stage. This bee started to follow another bee. All four bees continued to further develop partial Traveling salesman tours during the third stage.

5. Experimental Study of the Bee System

The proposed Bee System was tested on a large number of numerical examples. In all the cases one of tour improving algorithms has been employed. There are three

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

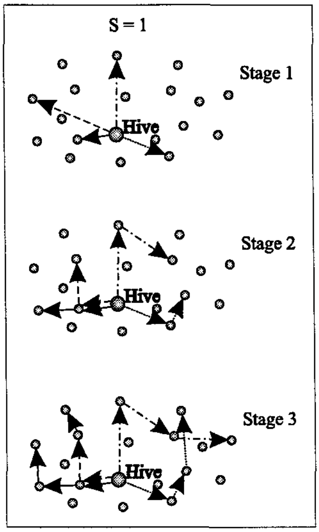

Fig. 1. Creation of the partial Traveling Salesmen Tours.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

sets of results that correspond to 2opt, 3-opt and modified 3-opt ("short version") tour improving algorithm respectively. The benchmark problems were taken from the following Internet address:
http://www.iwr.uni-heidelberg.de/iwr/comopt/software/TSPLIB95/tsp/. The following 10 problem instances were considered: Eil51.tsp, Berlin52.tsp, St70.tsp, Pr76.tsp, Kroa100.tsp, Eil101.tsp, Tsp225.tsp, A280.tsp, Pcb442.tsp and Pr1002.tsp. Problem instances Pcb442.tsp and Pr1002.tsp were considered only with modified 3opt ("short version") tour improving algorithm. All tests were run on an IBM compatible PC with PIII processor (533MHz). Tables 1, 2 and 3 present the results obtained by Bee System when search is limited on 100 cycles.
From the table 1 it could be seen that Bee System reinforced with 2-opt tour improvement heuristic provides excellent results for small size of TSP instances. However, if the size of instances is increased the solution quality will start to fall down (last column in table 1). In order to increase the solution quality we also coupled the Bee System with the 3-opt heuristic algorithm (Table 2).
The Bee System enriched with 3-opt heuristic algorithm produced significantly better results than the Bee System enriched with 2-opt heuristic algorithm. On the other hand, 3-opt algorithm is time consuming because the algorithm explores all possible combinations of three arcs replacement. There is a possibility to run the algorithm much faster with a small chance of diminishing solution quality. One simple way to do it (that was applied in this paper) is to modify 3-opt algorithm and to reduce the size of the searching space. In this paper, the reduction of the search space was done in the following way: three arcs were considered removed if

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Table 1. The results obtained by the Bee System enriched with 2-opt heuristic

Problem (Number of nodes)	Optimal Value (O)	The best value obtained by the Bee System (B)	(B-O)/O (%)	Time required to find the best solution (sec)	Average value obtained by the Bee System over 20 runs (A)	(A-O)/O (%)
Eil51 (51)	428.87	431.12	0.53	44	433.76	1.14
Berlin52 (52)	7544.37	7544.37	0.00	18	7634.37	1.19
St70 (70)	677.11	678.62	0.22	238	684.28	1.06
Pr76 (76)	108159.00	108790.00	0.58	127	109444.60	1.19
Kroa100 (100)	21285.40	21441.50	0.73	58	21575.70	1.36
Eil101 (101)	640.21	642.45	0.35	146	665.62	3.97
Tsp225 (225)	3859.00	4065.56	5.35	2076	4113.71	6.60
A280 (280)	2586.77	2740.63	5.95	1855	2784.81	7.66

their corresponding starting nodes were "close enough" (if the distances between the starting nodes were greater than the threshold value that we prescribed, the corresponding arcs were not removed). The results obtained by the Bee System enriched with modified 3-opt algorithm are given in the table 3.

Table 2. The results obtained by the Bee System enriched with 3-opt heuristic

Problem (Number of nodes)	Optimal Value (O)	The best value obtained by the Bee System (B)	(B-O)/O (%)	Time required to find the best solution (sec)	Average value obtained by the Bee System over 20 runs (A)	(A-O)/O (%)
Eil51 (51)	428.87	428.87	0.00	37	428.87	0.00
Berlin52 (52)	7544.37	7544.37	0.00	1	7544.37	0.00
St70 (70)	677.11	677.11	0.00	22	677.11	0.00
Pr76 (76)	108159.00	108159.00	0.00	11	108159.00	0.00
Kroa100 (100)	21285.40	21285.40	0.00	10	21285.40	0.00
Eil101 (101)	640.21	640.21	0.00	1741	643.05	0.44
Tsp225 (225)	3859.00	3876.05	0.44	5153	3905.32	1.20
A280 (280)	2586.77	2600.34	0.53	13465	2627.45	1.57

We can see from the tables 1, 2 and 3 that the proposed Bee System produced results of a very high quality. The Bee System was able to obtain the objective function values that are very close to the optimal values of the objective function. In all instances with less than 100 nodes the Bee System has produced the optimal solution (tables 2 and 3). The times required to find the best solutions by the Bee System are very low. In other words, the Bee System was able to produce "very good" solutions in a "reasonable amount" of computer time. The best solutions of the two studied benchmark problems discovered by the Bee System are presented in the figures 2 and 3.

Table 3. The results obtained by the Bee System enriched with 3-opt heuristic

Problem (Number of nodes)	Optimal Value (O)	The best value obtained by the Bee System (B)	(B-O)/O (%)	Time required to find the best solution (sec)	Average value obtained by the Bee System over 20 runs (A)	(A-O)/O (%)
Eil51 (51)	428.87	428.87	0.00	29	428.87	0.00
Berlin52 (52)	7544.37	7544.37	0.00	0	7544.37	0.00
St70 (70)	677.11	677.11	0.00	7	677.11	0.00
Pr76 (76)	108159.00	108159.00	0.00	2	108159.00	0.00
Kroa100 (100)	21285.40	21285.40	0.00	10	21285.40	0.00
Eil101 (101)	640.21	640.21	0.00	61	643.07	0.45
Tsp225 (225)	3859.00	3899.90	1.06	11651	3909.69	1.31
A280 (280)	2586.77	2608.33	0.83	6270	2632.42	1.76
Pcb442 (442)	50783.55	51366.04	1.15	4384	51756.89	1.92
Pr1002 (1002)	259066.77	267340.70	3.19	28101	268965.60	3.82

6. Some Potential Applications of the Bee System in Transportation Engineering

In this section we will shortly describe some of the potential applications of the Bee System in development of Artificial Systems aimed at solving complex problems in transportation engineering. The following examples were chosen among a great number of potential applications of the developed Bee System.

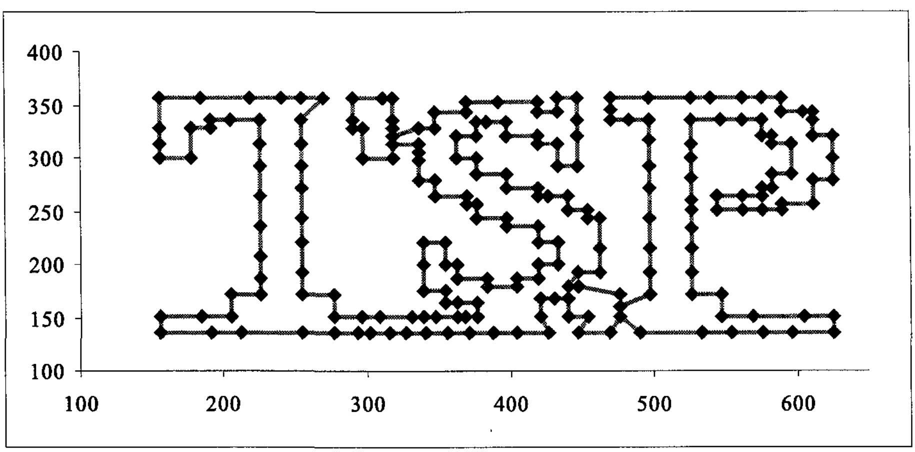

Fig. 2. Bee System, 100 cycles solution to the TSP problem instance Tsp225.tsp.

Planning and designing the transit, airline, or utility network networks is an ex-

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

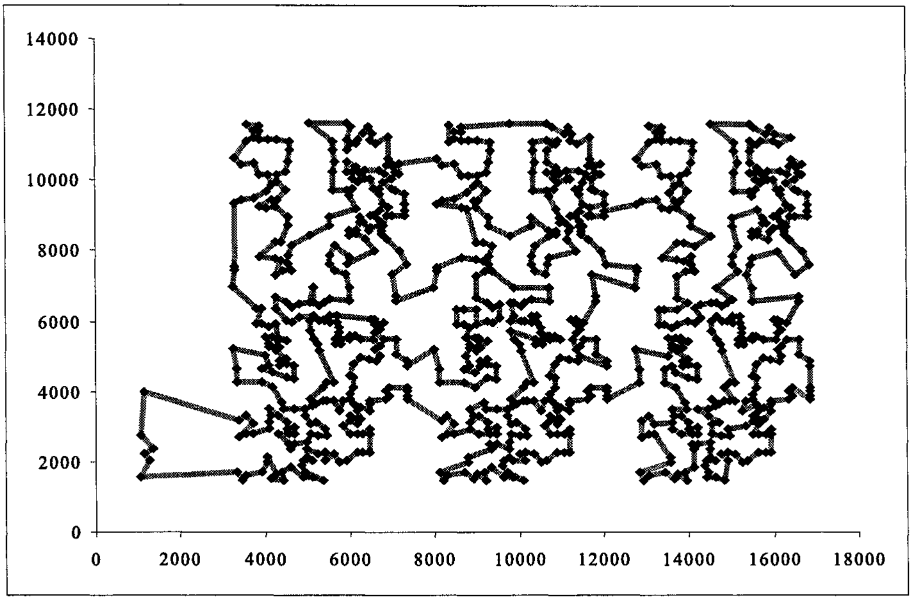

Fig. 3. Bee System, 100 cycles solution to the TSP problem instance Pr1002.tsp.

tremely complex planning task combinatorial by its nature. The chosen network shape and the vehicle frequency on individual links directly affect the business results of the operator and the level of service provided to passengers. Passengers in intercity transport and packages are very frequently routed from the origins to the destinations through one or more hubs. Instead of routing the passengers from each origin directly to their destination, the hub and spoke system transports passengers and packages through the hubs. Highly competitive environment in air transportation, logistics and communication assumes the best possible hub and spoke architecture. It should also be underlined that when transportation is to be established among a large number of nodes, the dimensions of the problem become very large. The transportation network planning and design problem is the ideal problem for the application of the developed Bee System.

The classical vehicle routing problem consists of finding the set of routes that minimizes transport costs. Further variations of the classical vehicle routing problem include existing few depots in the network, doing service with a few types of different vehicles, uncertain demand at nodes, or existing time windows for doing service at certain nodes. Developing hybrid models (Bee System, Fuzzy Logic and Visualization) for solving complex vehicle routing and scheduling problems would be of a great benefit for both the distribution companies, and the wider public.

The highway alignment problem assumes selection of the "best" path to connect

two points in the space. The "best" alignment minimizes total costs and satisfies the engineering design, operational constraints. Developing hybrid model (Bee System and Fuzzy Logic) for solving highway alignment problem would be of a great benefit for transportation agencies, as well as to the wider public.
Different versions of the Dial-A-Ride problem are found in every day practice: transportation of people in low-density areas, transportation of the disabled and elderly persons, and parcel pick-up and delivery service in urban areas are some of the examples. The dynamic Dial-A-Ride problem could be described as follows: all customers demanding fast service define pick-up and delivery location, as well as preferred beginning of the service. The problem is to assign every new passenger request to one of the vehicles already on the network, and to design a new route and schedule for this vehicle. This assignment must be done in real time.
The Gate Assignment Problem represents the assignment of arriving aircraft to available gates. During last decade, with the increase of number of flights and the number of passengers in air transportation, the airport gate assignment problem became much more complex. This problem is a difficult combinatorial optimization problem. By its nature this problem is also dynamic. Originating passengers walk from the check-in to the departure gate. Transfer passengers walk from the arriving gate to the new departing gate, while terminating passengers walk from arriving gate to the baggage claim area. The total passenger walking distance depends on the passenger transfer volume between every pair of aircrafts, as well as the distance between every pair of gates. It is very logical to try to minimize the total passenger walking distance. While solving this problem it is necessary to take into account different operational constraints. Developing Decision Support Systems based on the Bee System for the aircraft gate assignments would be also of great importance for the mitigation of airline schedule disturbances.

7. Conclusion

The Bee System that was developed in this paper has been successfully applied to the classical Traveling Salesman Problem. The results obtained are considered to be very good. This successful application of the Bee System to the difficult combinatorial optimization problem is very encouraging as it confirms the value of natural systems as a source of ideas and models for development of various useful artificial systems.
In future research, one of the main goals should be the exploration of the different types of artificial bee organizations and interactions and their influence on the dynamics of the population. There are questions relating to the above mentioned characteristics of the social insects that need to be further answered in future research: (a) Should artificial bees (agents) be equal, or should there sometimes be several types of agents? (b) Are there further possibilities for hybrid combination of swarm intelligence and other artificial intelligence approaches and different heuristic algorithms? (c) Could swarm intelligence be combined with other techniques (e.g.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Fuzzy Sets Theory techniques) for solving complex engineering problems characterized by uncertainty? (d) What are the costs and benefits of the development of Artificial Systems based on natural Swarm Intelligence in engineering, computer science and management science?
Swarm Intelligence (Ant System) has already been applied in some other engineering areas such as robotics. The results obtained are also very good.
In further research, models inspired by the developed Bee System could be created for different transportation engineering problems. Preliminary results with the Bee System that was developed in this paper are very promising. These results indicate that the development of new models based on swarm intelligence principles could significantly contribute to the solution of complex transportation engineering problems.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Acknowledgements

We wish to thank the National Science Foundation for this research was supported by a grant #CMS-0085430.

References

[1] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, E. Teller, Equation of State Calculations by Fast Computing Machines. Journal of Chemical Physics **21**, (1953) 1087–1092.
[2] S. Kirkpatrick, C.D. Gelatt, M.P. Vecchi, Optimization by Simulated Annealing. Science **220**, (1983) 671–680.
[3] V. Cerny, Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm. Journal of Optimization, Theory and Applications **45**, (1985) 41–55.
[4] J. Holland, Adaptation in Natural and Artificial Systems. University of Michigan Press, Ann Arbor (1975).
[5] D. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning. Addison-Wesley, Reading (1989).
[6] F. Glover, Tabu Search, Part I. ORSA Journal on Computing **2**, (1989) 4-32.
[7] F. Glover, Tabu Search, Part II. ORSA Journal on Computing **2**, (1990) 4-32.
[8] F. Glover, M. Laguna, Tabu Search, In Modern Heuristic Techniques for Combinatorial Problems, 70-150, Reeves, C.R., (ed.), Blackwell Scientific, Oxford (1993).
[9] B. Golden, C. Skicsim, Using Simulated Annealing to Solve Routing and Locations Problems. Working paper MS/S 85-010, University of Maryland at College Park, College Park (1985).
[10] D. Teodorović, G. Pavković, A Simulated Annealing Technique Approach to the Vehicle Routing Problem in the Case of Stochastic Demand. Transportation Planning and Technology **16**, (1992) 261–273.
[11] D. Teodorović, E. Krčmar-Nožić, G. Stojković, Airline Seat Inventory Control by Application of the Simulated Annealing. Transportation Planning and Technology **17**, (1993) 219–233.
[12] H. Kopfer, G. Pankratz, E. Erkens, Development Of A Hybrid Genetic Algorithm For Vehicle-Routing. OR Spektrum **16**, (1994) 21–31.

[13] A. Wren, D. Wren, A Genetic Algorithm For Public Transport Driver Scheduling. Computers Operations Research **22**, (1995) 101–110.

[14] T.F. Fwa, W.T. Chan, C.Y. Tan, Genetic-algorithm Programming of Road Maintenance and Rehabilitation. Journal of Transportation Engineering-ASCE **122**, (1996) 246–253.

[15] D. Levine, Application of a Hybrid Genetic Algorithm to Airline Crew Scheduling. Computers Operations Research **23**, (1996) 547–558.

[16] C.J. Malmborg, A Genetic Algorithm for Service Level Based Vehicle Scheduling. Eurpoean Journal of Operational Research **93**, (1996) 121–134.

[17] K. Nachtigall, S. Voget, A Genetic Algorithm Approach to Periodic Railway Synchronization. Computers and Operations Research **23**, (1996) 453–46.

[18] J.Y. Potvin, C. Duhamel, F. Guertin A Genetic Algorithm for Vehicle Routing with Backhauling. Applied Intelligence **6**, (1996) 345–355.

[19] N. Sannomiya, K. Tatemura, Application of Genetic Algorithm to a Parallel Path Selection Problem. International Journal of Systems Science **27**, (1996) 269–274.

[20] G. Ulusoy, S. Sivrikaya, F. Serifoglu, U. Bilge, A Genetic Algorithm Approach to the Simultaneous Scheduling of Machines and Automated Guided Vehicles. Computers Operations Research **24**, (1997) 335–351.

[21] M. Qiu, Prioritising and Scheduling Road Projects by Genetic Algorithm. Mathematics And Computers In Simulation **43**, (1997) 569–574.

[22] S.B. Pattnaik, S. Mohan, V.M. Tom, Urban Bus Transit Route Network Design Using Genetic Algorithm. Journal of Transportation Engineering-ASCE **124**, (1998) 368–375.

[23] D. Teodorović, K. Vukadinović, Traffic Control and Transport Planning: A Fuzzy Sets and Neural Networks Approach, Kluwer Academic Publishers, Boston/Dordrecht/London (1998).

[24] Y. Gu, C.A. Chung, Genetic Algorithm Approach to Aircraft Gate Reassignment Problem. Journal of Transportation Engineering-ASCE **125**, (1999) 384–389.

[25] J.C. Jong, P. Schonfeld, Genetic Algorithms for Optimizing 3-Dimensional Highway Alignments, accepted for Transportation Research Part B (2002).

[26] M. Jha, P. Schonfeld, Integrating Genetic Algorithms and GIS to Optimize Highway Alignment. Transportation Research Record, available on CD ROM of January 2000 TRB Annual Meeting (2000).

[27] J.C. Jong, P. Schonfeld, A Genetic Algorithm for Selecting and Scheduling Interdependent Projects, forthcoming in the Journal of Waterway, Port, Coastal and Ocean Engineering ASCE (2002).

[28] G. Beni, The concept of Cellular Robotic System. In Proceedings 1988 IEEE International Symposium on Intelligent Control, 57–62, Los Alamitos, CA, IEEE Computer Society Press (1988).

[29] G. Beni, J. Wang, Swarm Intelligence. In Proceedings Seventh Annual Meeting of the Robotics Society of Japan, (1989) 425–428, Tokyo, RSJ Press.

[30] G. Beni, S. Hackwood, Stationary Waves in Cyclic Swarms. In Proceedings 1992 International Symposium on Intelligent Control, (1992) 234–242, Los Alamitos, CA, IEEE Computer Society Press.

[31] E. Bonabeau, M. Dorigo, G. Theraulaz, Swarm Intelligence, Oxford University Press, Oxford (1999).

[32] J.L.Deneubourg, S. Aron, S. Goss, J.M. Pasteels, The Self-Organizing Exploratory Pattern of the Argentine Ant. Journal of Insect Behavior **3**, (1990) 159–168.

[33] N.R. Franks, A. Wilby, B.W. Silverman, C. Tofts, Self-organizing Nest Construction in Ants: Sophisticated Building by Blind Buldozing. Animal Behaviour **44**, (1992) 357–375.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.

[34] R. Beckers, J.L. Deneubourg, S. Goss, Trails and U-turns in the Selection of a Path by the Ant Lasius niger. Journal of Theoretical Biology **159**, (1992) 397–415.

[35] P. Nonacs, J.L. Soriano, Patch Sampling Behaviour and Future Foraging Expectations in Argentine Ants, Linepithema humile. Animal Behaviour **55**, (1998) 519–527.

[36] K. Vienne, C. Erard, A. Lenoir, Influence of the Queen on Worker Behaviour and Queen Recognition Behaviour in Ants. Ethology **104**, (1998) 431–446.

[37] J.L. Deneubourg, S. Goss, N. Franks, J.M. Pasteels, The Blind Leading the Blind: Modeling Chemically Mediated Army Ant Raid Patterns. Journal of Insect Behavior **2**, (1989) 719–725.

[38] A. Colorni, M. Dorigo, V. Maniezzo, Distributed Optimization by Ant Colonies. In Proceedings of the First European Conference on Artificial Life, (1991) 134–142, Varela, F., and Bourgine, P., (eds.), Paris, Elsevier.

[39] M. Dorigo, V. Maniezzo, A. Colorni, Ant System: Optimization by a Colony of Cooperating Agents. IEEE Transactions on Systems, Man and Cybernetics, Part B **26**, (1996) 29–41.

[40] B. Bullnheimer, R.F. Hartl, C. Strauss, A New Rank Based Version of the Ant System - a Computational Study. Central European Journal of Operations Research **7**, (1999) 25–38.

[41] B. Bullnheimer, R.F. Hartl, C. Strauss, An Improved Ant System Algorithm for the Vehicle Routing Problem. Annals of Operations Research **89**, (1999) 319–328.

[42] J.L. Gould, Landmark Learning by Honey Bees. Animal Behaviour **35**, (1987) 26–34.

[43] P.S. Hill, P.H. Wells, H. Wells, Spontaneous Flower Constancy and Learning in Honey Bees as a Function of Colour. Animal Behaviour **54**, (1997) 615–627.

[44] V.S. Baschbach, K.D. Waddington, Risk-sensitive Foraging in Honey Bees: No Consensus Among Individuals and No Effect of Colony Honey Stores. Animal Behavior **47**, (1994) 933–941.

[45] R. Dukas, L.A. Real, Learning Foraging by Bees: a Comparison Between Social and Solitary Species. Animal Behaviour **42**,(1991) 269–276.

[46] R. Kadmoon, A. Shmida, Departure Rules Used by Bees Foraging for Nectar: a Field Test. Evolutionary Ecology **6**, (1992) 142–151.

[47] B. Peleg, A. Shmida, S. Ellner, Foraging Graphs: Constraint Rules on Matching Between Bees and Flowers in a Two-sided Pollination Market. Journal of Theoretical Biology **157**, (1992) 191–201.

[48] T.D. Seeley, The Tremble Dance of the Honey Bee: Message and Meanings. Behavior Ecology Sociobiology **31**, (1992) 375–383.

[49] R. Dukas, P.K. Visscher, Lifetime Learning by Foraging Honey Bees. Animal Behavior **48**, (1994) 1007–1012.

[50] T. Keasar, A. Shmida, U. Motro, Innate Movement Rules in Foraging Bees: Flight Distances are Affected by Recent Rewards and are Correlated with Choice of Flower Type. Behaviour Ecology Sociobiology **39**, (1996) 381–388.

[51] L. Chittka, A. Gumbert, J. Kunze, Foraging Dynamics of Bumble Bees: Correlates of Movements Within and Between Plant Species. Behavioral Ecology **8**, (1997) 239–249.

[52] L. Chittka, J.D. Thompson, Sensori-motor Learning and its Relevance for Task Specialization in Bumble Bees. Behaviour Ecology Sociobiology **41**, (1997) 385–398.

[53] R.G. Collevatti, L.A.O. Campos, J.H. Schoereder, Foraging Behaviour of Bee Pollinators on the Tropical Weed Triumfetta semitriloba: Departure Rules from Flower Patches. Insectes Sociaux **44**, (1997) 345–352.

[54] K.D. Waddington, C.M. Nelson, R.E.Jr. Page, Effects of Pollen Quality and Genotype on the Dance of Foraging Honey Bees. Animal Behaviour **56**, (1998) 35–39.

[55] N.M. Williams, J.D. Thompson, Trapline Foraging by Bumble Bees: III. Temporal

Patterns of Visitation and Foraging Success at Single Plants. Behavioral Ecology **9**, (1998) 612–621.

[56] S. Camazine, J. Sneyd, A Model of Collective Nectar Source by Honey Bees: Self-organization Through Simple Rules. Journal of Theoretical Biology **149**, (1991) 547–571.

[57] P. Lučić, D. Teodorović, Bee System: Modeling Combinatorial Optimization Transportation Engineering Problems by Swarm Intelligence. In Preprints of the TRISTAN IV Triennial Symposium on Transportation Analysis, (2001) 441–445, Sao Miguel, Azores Islands.

[58] J.C. Biesmeijer, M.G.L. van Nieuwstadt, S. Lukacs, M.J. Sommeijer, The Role of Internal and External Information in Foraging Decisions of Melipona Workers (Hymenoptera: Meliponinae). Behavior Ecology Sociobiology **42**, (1998) 107–116.

[59] T.D. Seeley, P.K. Visscher, Assesing the Benefits of Cooperation in Honeybee Foraging: Search Costs, Forage Quality, and Competitive Ability. Behavior Ecology Sociobiology **22**, (1988) 229–237.

[60] S. Lin, B.W. Kernighan, An Effective Heuristic for the Traveling Salesman Problem. Operations Research **21**, (1973) 498–516.

Int. J. Artif. Intell. Tools 2003.12:375-394. Downloaded from www.worldscientific.com
by NORTHEASTERN UNIVERSITY on 02/13/15. For personal use only.