Silicon Solar Energy Converters

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(Received August 12, 1954)

Theory is given for the design of silicon solar energy converters commonly known as the Bell Solar Battery. Values are given for the various parameters in the design theory. Experimental data are presented and compared with the theoretical relations based on a simple model.

It is found that with present techniques, units can be made with up to 6 percent efficiency in the conversion of solar radiant energy to electrical energy. An important factor in obtaining such high efficiencies is the reduction of the series resistance of the cell to as low a value as possible.

INTRODUCTION

With the development of the technique of solid diffusion in producing *p-n* junctions in silicon,[1] new silicon devices[2,3] are now possible for the first time. Among these is a large area silicon photodiode or solar energy converter[3] commonly known as the Bell Solar Battery which has a higher conversion efficiency of solar radiant energy to electrical energy than any other device.

This paper will present the theory for the design of solar energy converters. General relations will be derived from which it will be shown that with the present status of semiconductors, silicon is the best material available from which efficient solar energy converters can be made for use at temperatures near 300°K.

With the use of reasonable anticipated values of various parameters, it will be shown that silicon solar energy converters should be able to be made with efficiencies as high as 10 percent. At present, the best units are 6 percent efficient.[3] Reasons for this result will be given.

GENERAL THEORY

The problem of the photoelectric power converter may best be defined by considering the case of an ideal *p-n* junction with a constant current source in parallel with the junction. The constant current source results from the excitation of an excess over the thermal value of electrons into the conduction band and the diffusion and drift of these carriers across the barrier. The nature of the source need not concern us; it may be a photon source (solar radiation, γ radiation, incandescent lamp, x-rays, etc.) a high-energy particle source (electron gun, β-radioactive elements,[4] α particles, protons, neutrons, etc.), or any other means for creating electron-hole pairs without changing the properties of the ideal junction appreciably (that is, without creating a large electric field or high temperatures in the vicinity of the junction).

The $I-V$ characteristic of such a device is given as

$$I = I_0(e^{qV/kT}-1)-I_L, \tag{1}$$

where I_0 = reverse saturation current of ideal junction; q = charge of electron, k = Boltzmann's constant, T = absolute temperature, and I_L = strength of constant current source.

A plot of Eq. (1) is given in Fig. 1 for selected values of the parameters. It is seen that the curve passes through the fourth quadrant and therefore that power can be extracted from the device. By properly choosing a load, it is possible to extract close to 80 percent of the product $I_{sc} \times V_{oc}$ where I_{sc} is the short circuit current and V_{oc} is the open circuit voltage of the device.

Let us calculate the maximum power that can be obtained from a solar energy converter exclusive of losses by recombination and series and shunt resistances. In bright sunlight at sea level, if every photon falling on a unit created one hole-electron pair that caused

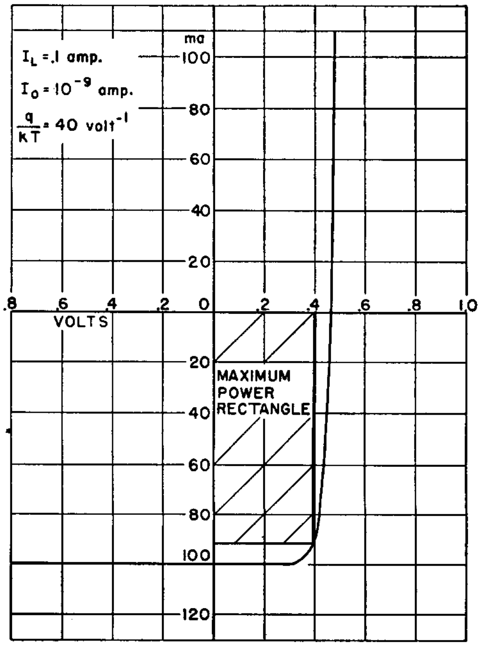

FIG. 1. Ideal $I-V$ characteristic of a photocell or solar energy converter.

[1] C. S. Fuller (private communication).
[2] G. L. Pearson and C. S. Fuller, Proc. Inst. Radio Engrs. 42, 760 (1954).
[3] Chapin, Fuller, and Pearson, J. Appl. Phys. 25, 676 (1954).
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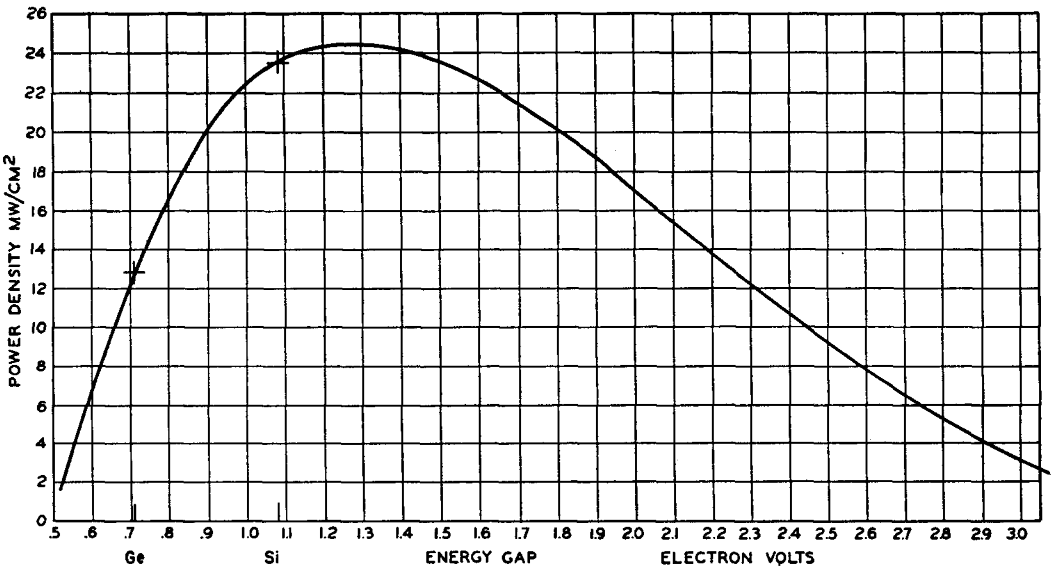

FIG. 2. Maximum converted power density in bright sunlight as a function of energy gap of semiconductor.

current to flow across the junction, there would be a short-circuit current of 0.080 ampere per square centimeter of effective device area. However, the long wavelength limit for the creation of hole-electron pairs by photons depends on the energy gap of the semiconductor involved and this reduces the magnitude of the highest possible short-circuit current to 0.044 ampere/cm² in silicon and 0.068 ampere/cm² in germanium. Further reduction of these values occurs due to reflection losses. Even with "nonreflective" coatings the maximum short-circuit currents will be near 0.035 ampere/cm² in silicon and 0.055 ampere/cm² in germanium. More will be said about this possibility of improving present efficiencies later in the paper. One might think that by choosing semiconductors with lower energy gaps, one would obtain larger short-circuit currents and thus higher efficiencies. However, as will be shown in the next paragraph, decreasing the energy gap reduces the open-circuit voltage at a much faster rate than the short circuit current increases.

The open-circuit voltage V_{oc} at room temperature is given by

$$V_{oc}=0.0575\log_{10}(I_L/I_0)\text{ volts.}\tag{2}$$

Therefore V_{oc} can be maximized by minimizing I_0. I_0 is approximately given by

$$I_0=\left\{qp_n\left(\frac{D_P}{\tau_P}\right)^{\frac{1}{2}}+qn_P\left(\frac{D_n}{\tau_n}\right)^{\frac{1}{2}}\right\}\times\text{area}\tag{3}$$

where p_n = equilibrium density of holes in n-region; n_p = equilibrium density of electrons in p-region; D_p, τ_p = diffusion constant and lifetime of holes in n-region; and D_n, τ_n = diffusion constant and lifetime of electrons in p-region. Consider a heavily doped p-region in contact with a moderately doped n-region, a favorable condition for low I_0 using the diffusion technique. Then

$$J_0=\frac{I_0}{\text{area}}\simeq qp_n\left(\frac{D_p}{\tau_p}\right)^{\frac{1}{2}},\tag{3a}$$

using the relationships

$$p_n=\frac{n_i^2}{n_n}=\frac{2.23\times10^{31}T^3e^{-E_g/kT}}{n_n}\tag{4}$$

and

$$n_n=\frac{1}{\rho_nq\mu_n}\tag{5}$$

where n_i = equilibrium density of electrons in intrinsic semiconductor, n_n = equilibrium density of electrons in n-region, ρ_n = resistivity of n-region, μ_n = mobility of electrons in n-region, and E_g = energy gap of semiconductor (volts). The open-circuit voltage can be expressed at 300°K as

$$V_{oc}=0.0575\log_{10}J_L\left\{\frac{0.062e^{39E_g}}{\rho_n\mu_n}\left(\frac{\tau_P}{D_p}\right)^{\frac{1}{2}}\right\},\tag{6}$$

where J_L is the light current density.

It is seen from Eq. (6) that in order to obtain a large V_{oc} one should use a semiconductor with a large energy gap, low resistivity consistent with the diffusion process, material with low mobility, and high lifetime. Assuming a lifetime of ten microseconds, a resistivity of one-tenth ohm centimeters, a mobility variation of $\mu_nE_g^{2.5}$ = constant, and a diffusion constant variation of $D_PE_g^{2.5}$ = constant, one can calculate the maximum power density as a function of the energy gap. The last two assumptions are not important in that almost any other assumption would give results with less than a few percent difference from the results given in Fig. 2. It is seen that for optimum power with respect to the solar spectrum the energy gap in the semiconductor

TABLE I.

Item	Best value	Limiting factor
$J_{sc}=J_L$	25 ma/cm²	reflection
τ_p	10^{-7} sec	diffusion technique
ρ	0.1 ohm-cm	surface solubility of B in Si is about 10^{18} cm^{-3}
D_p	10 cm²/sec	consistency with high τ_p and ρ[a]
μ_n	$700\frac{\text{cm}^2}{\text{volt-sec}}$	consistency with high τ_p and ρ[a]

[a] M. B. Prince, Phys. Rev. 93, 1204 (1954).

should lie between 1.0 and 1.6 electron volts. Thus silicon with a room temperature energy gap of 1.08 electron volts is ideally suitable as a material for solar energy converters. It is interesting to note that similar devices made from germanium would have about half the conversion efficiency of silicon devices. Since the solar radiant power density is 108 milliwatts per square centimeter at sea level on a bright clear day with the sun at the zenith and maximum power density that can be obtained from a silicon unit is 23.5 milliwatts per square centimeter, the maximum possible efficiency for a silicon energy converter would be 21.7 percent.

It is the purpose of this paper to explain why this efficiency is unattainable and to predict maximum designable efficiencies.

THEORY OF SILICON SOLAR ENERGY CONVERTER

As has been mentioned in the last section, the maximum I_{sc} in silicon solar energy converters even with "nonreflective" coatings is about 35 milliamperes/cm². This fact immediately reduces the maximum efficiency to 17.2 percent or 18.6 milliwatts/cm². However these figures are based on anticipated values

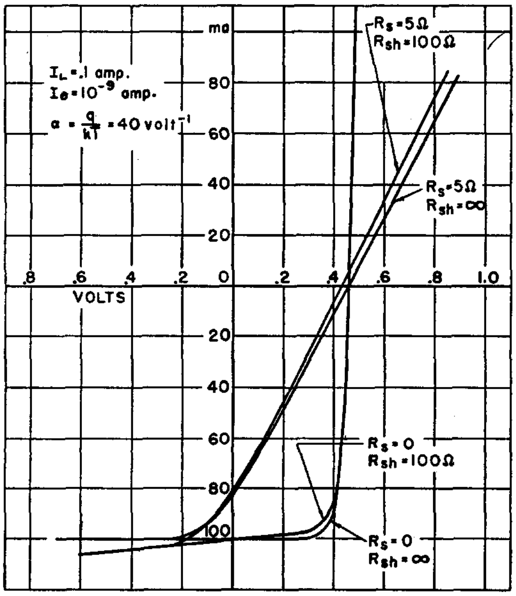

FIG. 3. Theoretical $I-V$ characteristic for various converters that include series and shunt resistances.

of various parameters; that is, improvement in the quality of the present silicon. The best values at present using the solid diffusion technique of introducing boron into n-type silicon are given in Table I. Using these values, the maximum expected power density P_A is

$$P_A=0.8J_{sc}V_{oc}=0.8\times 0.025\times 0.58=0.0116 \text{ watts/cm}^2$$

or the maximum expected efficiency is 10.7 percent.

Even 10 percent efficient units have not been made and reasons for this fact must be given. Up to the present discussion only ideal junctions have been considered. Now let us consider a practical unit. It may have some shunt resistance R_{sh} and certainly has some series resistance R_s due to the body material and the contact to the body. It can be readily shown

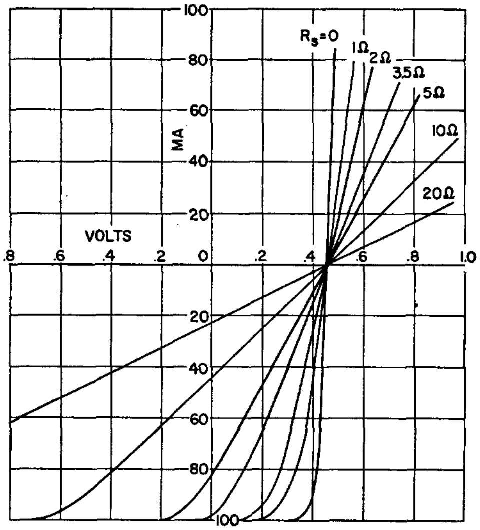

FIG. 4. Theoretical $I-V$ characteristic for various converters with different series resistance.

that for a model containing a R_s and a R_{sh}, the $I-V$ characteristic is given by

$$\ln\left[\frac{I+I_L}{I_0}-\frac{V-IR_s}{I_0R_{sh}}+1\right]=\frac{q}{kT}(V-IR_s). \qquad (7)$$

Plots of this equation with all combinations of $R_s=0$, 5 and $R_{sh}=\infty$, 100 ohms are given in Fig. 3 with the same parameters I_0, I_L, and kT/q as in Fig. 1. It can be seen that a shunt resistance even as low as 100 ohms does not appreciably change the power output of a unit whereas a series resistance of only 5 ohms reduces the available power to less than 30 percent of the optimum power with $R_s=0$. Since the R_{sh} does not affect our results, let us neglect it in future calculations. Figure 4 shows the theoretical $I-V$ characteristics of units having $R_s=0$, 1, 2, 3.5, 5, 10, and 20 ohms. Table II gives the ratio of the relative maximum available power from such units. These data are plotted in Fig. 5 which shows graphically how extremely important it is to reduce the series resistance to as low a value as possible.

Now that it has been shown that the series resistance is a controlling parameter in reducing the available power from a silicon solar energy converter, let us consider a possible design of a unit and place some limits on the controlling parameters.

Consider Fig. 6 which shows a possible configuration for a solar energy converter. Radiation is incident on the top surface. Contacts are made to the n island and p ring on the bottom surface. The geometrical parameters are the length L, width W, thickness H, and the depth of the p-type layer t. The length and width of the unit are limited by the size of crystal from which the unit is cut. At the present time L is limited to about 6 cm and W to 1.5 cm. The thickness of the wafer H should be made small such that the resistance in the n-type region of the device is a minimum. However this parameter is not critical since it is easy

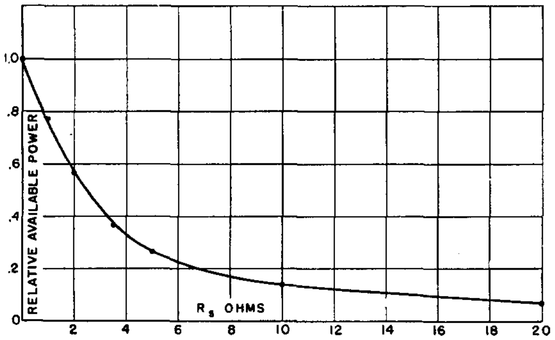

FIG. 5. Relative maximum available power as a function of the series resistance.

to keep the resistance of this part of device below $\frac{1}{4}$ ohm with thicknesses between 0.1 and 0.5 cm. Therefore in the interest of conservation of material, it is desirable to make $H \simeq 0.1$ cm. A more critical dimension is the thickness of the p-layer t. Practically all of the electron-hole pairs produced by sunlight are within 10^{-4} cm of the surface. Thus two conflicting demands are made on t; thick t to reduce the resistance of the layer and thin t to prevent loss of minority carriers by recombination before diffusing to the junction. It can be shown that the p-layer resistance is given by

$$R = W\rho/4tL. \tag{8}$$

The power that can be obtained from such a device is given by

$$P = -IV = -I\left[\frac{kT}{q}\ln\left(\frac{I+I_L}{I_0}+1\right)+IR_s\right]. \tag{9}$$

For the purposes of calculations let us assume the following values of some of the parameters:

$$p_n = 10^5\ \mathrm{cm}^{-3} \qquad kT/q = 0.025\ \mathrm{volt}$$
$$D_p = 10\ \mathrm{cm}^2/\mathrm{sec}$$
$$\tau_p = 10^{-7}\ \mathrm{sec}$$

TABLE II.

R_s	Relative maximum available power
0 ohms	1
1	0.77
2	0.57
3.5	0.37
5	0.27
10	0.14
20	0.07

thus

$J_0 = 10^{-10}$ amp/cm^2 = reverse saturation current density,

$J_{L0} = 0.03$ amp/cm^2 = light current density at surface of unit,

$L_D = 10^{-3}$ cm = diffusion length in p-layer.

Therefore $I_0 = J_0 WL = 10^{-10} WL$ amp and $I_L \cong J_{L0} \times WLe^{-t/L_D} = 0.03 WL\exp(-10^3 t)$ amp. The last relation is a good approximation for $t \geq L_D$. For $t \ll L_D$, the approximation fails but in the following calculations, the error introduced by the approximation is negligible. Substituting these values and (8) into (9), we find

$$P = -0.025 I \ln\left[\frac{I + 0.03 WL\exp(-10^3 t)}{10^{-10} WL} + 1\right] - I^2 \frac{W}{4}\frac{\rho}{tL}. \tag{10}$$

The first term represents the actual power developed and the second term represents the power lost in the p-type layer. Equation (10) indicates that one should make L as large as possible and W small. Let $L = 5$ cm as this is a convenient limit for this parameter and $W = 1$ cm as anything smaller will make the fabrication unnecessarily difficult. Since the surface resistivity of

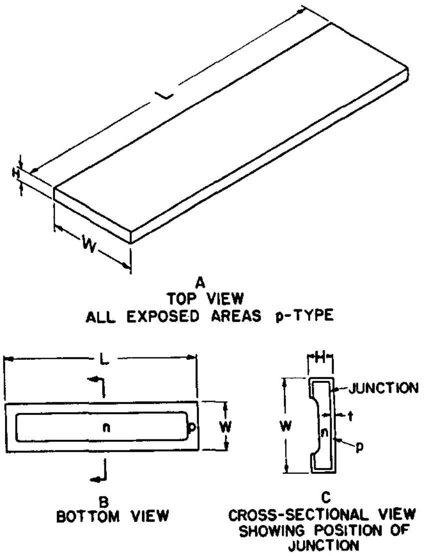

FIG. 6. Geometry of solar energy converter.

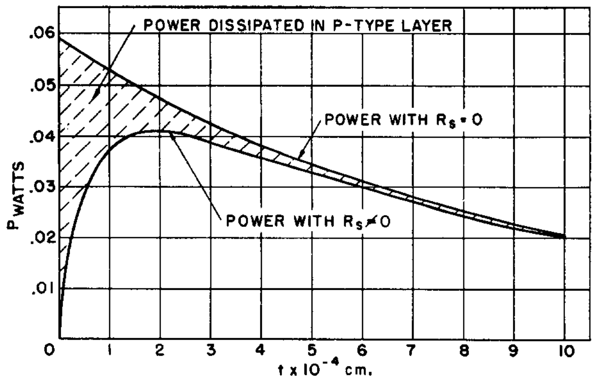

FIG. 7. Maximum available power as a function of active p-layer thickness.

boron diffused silicon is about 10^{-3} ohm-cm,[1] let ρ be 2×10^{-3} ohm-cm as an average resistivity of the p-type layer. Then (10) can be rewritten as

$$P=-0.0575I\log[2\times10^9 I+3\times10^8\exp(-10^3 t)]-10^{-4}\frac{I^2}{t}. \qquad (11)$$

Equation (11) has been maximized with respect to I for various values of t and plotted in Fig. 7. The upper curve gives the maximum value of the power for the first term in the above expression and the lower curve represents the maximum value for the entire expression. It is observed that the maximum power occurs for our chosen geometry with $t=2\times10^{-4}$ cm and that the maximum is quite broad. For this unit (5 cm²) there is an efficiency of over 8 percent and a p-layer resistance of 0.5 ohm.

On a unit area basis, Eq. (10) can be written as

$$P_A=\frac{P}{WL}=-0.025J\times\ln\left[\frac{J+0.03\exp(-10^3 t)}{10^{-10}}+1\right]-J^2\frac{W^2\rho}{4t}, \qquad (12)$$

$$=-0.0575J\log[10^{10}J+3\times10^8\exp(-10^3 t)]-J^2\frac{W^2\rho}{4t}. \qquad (12a)$$

Equation (12) indicates that the power per unit area developed is independent of the length of the specimen but depends on the square of the width. A plot of maximum power per unit area *versus* the width of specimen is given in Fig. 8 along with a plot of the product as a function of the width. It is seen in Fig. 8 that with our assumptions and from any standpoint, one would not want to make any units with widths greater than about 1.7 cm. If one wants a unit with high photosensitivity (high efficiency), the width should be made as small as possible whereas if one wants as much power as possible from a single unit then the width should be made about 1.5 cm. A more accurate calculation indicates that the product of $P_A\times W$ approaches an asymptotic value and does not fall off with W as is shown in Fig. 8.

Another serious source of series resistance is in the actual contact of the conductors to the n- and p-type regions of the silicon.

The temperature dependence of the operating characteristics is always of interest in the description of semiconductor devices. Since the short circuit current depends only on the light current intensity, it will have no temperature dependence (except for minor corrections due to lifetime changes and series resistance changes). The most important temperature effect is through the change in open circuit voltage. It can be shown that the temperature variation of the open circuit voltage can be represented in silicon by

$$dV_{oc}/dT=-0.00288\ \text{volt/°C}. \qquad (13)$$

Since the output power of the device varies linearly

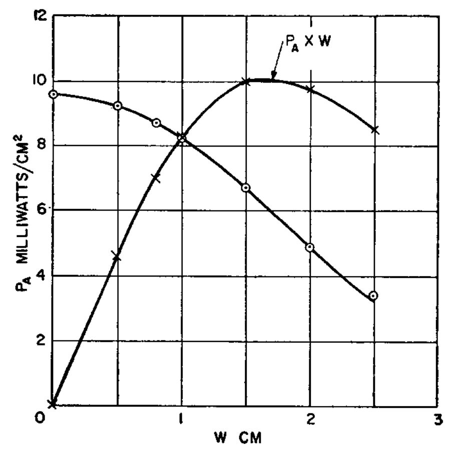

FIG. 8. Maximum available power density as a function of width of solar energy converter.

with the open-circuit voltage, the power decreases with temperature at a rate greater than ½ percent per degree centigrade. It will be shown in the next section that Eq. (13) is satisfied by experiment.

RESULTS OF EXPERIMENTAL MEASUREMENTS

The results to be presented in this section consist of the following:

1. The $I-V$ characteristic of a unit before and after reducing the series resistance of the device. 2. The $I-V$ characteristic of a unit as a function of the light current. 3. The $I-V$ characteristic of 10 units connected in series. 4. The open circuit voltage as a function of temperature.

The units whose properties are given below were made by diffusing boron into n-type silicon. The dimensions of these units are L=5.7 cm, W=1.2 cm, H=0.1 cm, and t=0.0002 cm.

Figure 9 shows the results of decreasing the series resistance of a cell on the $I-V$ characteristic of the converter. The sample originally had a series resistance of 6.1 ohms. After the contact was improved the series resistance dropped to about 2.7 ohms. This decrease in series resistance allowed one to obtain from the unit at optimum load 2.2 times as much power after the improvement compared to that obtained before the improvement. Further reduction of the series resistance of this converter would lead to further increase in the maximum available power. This unit has a shunt resistance of the order of 1000 ohms.

The $I-V$ characteristics of a converter as a function of the light intensity are given in Fig. 10. This unit has a series resistance of 1.8 ohms and on comparing the 102 ma short-circuit current curve with those of Fig. 4, it is seen that the experimental curve fits the theoretical curve extremely well. It has been observed on this and on other units that the maximum available power under different intensities of illumination occurs at a constant voltage for a particular converter. In the case of the unit whose characteristics are illustrated in Fig. 10, this voltage is 0.30 volt or $\frac{2}{3}$ the open circuit voltage. As a consequence of this fact, the ideal load for a solar converter would be one that required a constant voltage; e.g., a storage battery. This unit also has a shunt resistance of 1000 ohms

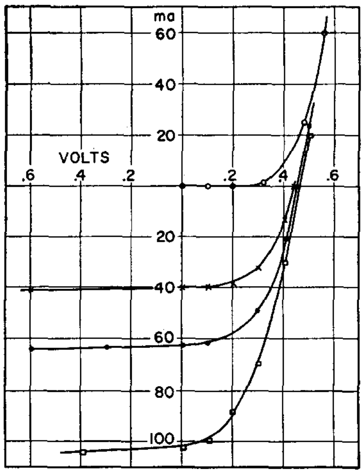

FIG. 10. Family of experimental $I-V$ characteristics of a solar energy converter for various light intensities.

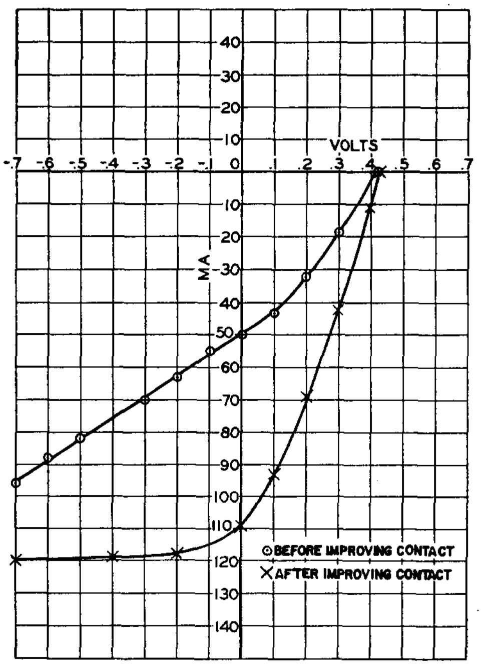

FIG. 9. Experimental $I-V$ characteristics of a converter before and after improving contact.

The $I-V$ characteristic for a series connection of 10 solar converters in bright sunlight is given in Fig. 11. The series resistance of this collection is 18.3 ohms or an average of less than two ohms per cell. The maximum power is obtained at a voltage equal to $\frac{2}{3}$ the open-circuit voltage and is equal to 0.2 watt. Since each unit has an area of 6.8–7.0 cm^2, the device is only about 3 percent efficient in the conversion of sunlight to electrical power. Individual units have been made with 5 percent to 6 percent conversion efficiencies.[3]

Figure 12 gives an experimental plot of the open-circuit voltage as a function of temperature. The straight line through the data has a slope of -0.00288 volt per degree centigrade in agreement with the value given by Eq. (13). The disagreement of the low-temperature point is probably due to the fact that the sample was cooled with the aid of dry ice and the temperature read by thermocouple system was not the effective average temperature of the device.

CONCLUSIONS

The experimental results given in the last section are in agreement with the design theory for boron diffused silicon solar converters. This fact leads us to believe that one should be able to produce units for conversion of incident solar radiant energy into electrical energy with efficiencies as high as 10 percent with modifications of present techniques. Cells have been made that have conversion efficiencies up to 6 percent.

Theoretical predictions and experimental confirmations indicate that the most important factor in the design of a solar energy converter is the series resistance

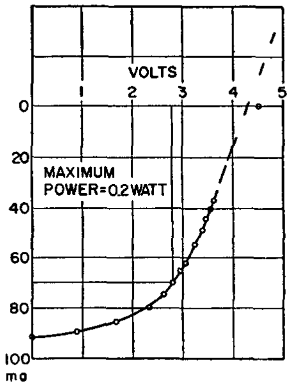

FIG. 11. $I-V$ characteristic of a series connection of ten solar energy converters in bright sunlight.

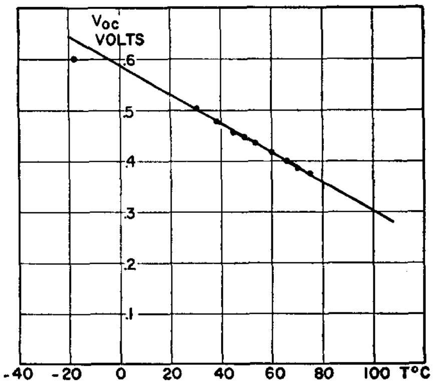

FIG. 12. Open-circuit voltage as a function of temperature.

of the device. Experimentally, units have been made with the series resistance being less than two ohms. Design theory indicates that units should be able to be made having a series resistance of less than one ohm. An experimental attainment of this value would improve the output of these units by 30 to 40 percent or to about 7 or 8 percent efficient.

Another method for possibly increasing the output of these devices appreciably would be the use of nonreflective coatings either of the $\frac{1}{4}\lambda$ type or by the use of various materials having intermediate indices of refraction between silicon ($n=4$) and air ($n=1$). With this thought in mind a unit was cemented to soft glass ($n=1.4$) with polystyrene cement ($n\cong 2$). The unit did not show any improvement. There are two possible explanations for this behavior. First, there is always a thin layer of quartz ($n=2$) on the silicon which makes the improvement by the addition of other films of negligible importance. The second possibility is that any improvement in the reflection loss is balanced by an absorbtion loss in the glass and cement. Probably the experimental facts are the result of a combination of these explanations.

Other possible means for increasing the output of these solar energy converters is by the use of reflectors (mirrors) or concentrators (lenses) whereby the intensity of radiation is increased. However these methods would be practical only in the case of small power supplies.

The author wishes to acknowledge several stimulating discussions with J. L. Moll, G. L. Pearson, D. M. Chapin, and C. S. Fuller. Much of the fabrication of the devices was done by T. J. Vasko.

JOURNAL OF APPLIED PHYSICS VOLUME 26, NUMBER 5 MAY, 1955

Effect of Air Damping on Transverse Vibrations of Stretched Filaments*

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(Received September 20, 1954)

The effect of air damping on the resonant frequencies of forced vibration of stretched filaments is studied by observing the shift in frequency when the surrounding medium is changed from air at atmospheric pressure to hydrogen at the same pressure, or to air at reduced pressures. For fibers of linear density less than about 10 micrograms per centimeter, the shift between atmospheric air and vacuum exceeds one cycle per second, becoming greater as the fibers become finer. The frequency shifts observed experimentally are compared with those predicted by certain formulas of Stokes. It is found that the theory works well enough to allow the effect of air damping to be taken into account in measurements of linear density by means of the vibroscope. However, the observed shift seems to be about one or two cycles per second higher than that calculated in the case of small shifts, and about 20 or 30 percent higher in the case of large shifts. A rigorous evaluation of the applicability of Stokes' theory would require further investigation.

INTRODUCTION

In the vibroscopic method of determining the mean cross-sectional area of filaments,[1-5] the elementary formulas of Mersenne and Galileo must be modified to take into account stiffness and nonuniformity. Studies of these effects are available.[6,7] With very fine filaments another effect, that of damping by the surrounding medium, becomes important. To study this effect experimentally, filaments of various linear densities were vibrated in air and in hydrogen at atmospheric pressure, and in air at reduced pressure, the shift in resonant frequency upon change of medium being observed. To get help from theory, the analysis of Stokes[8] for a related case has been applied.

* A portion of a thesis submitted by the first-named author to the School of Graduate Studies of Michigan State College in partial fulfillment of the requirements for the M.S. degree.

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