Integrated Radiation Optimization: Inspired by the Gravitational Radiation in the Curvature of Space-time

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***Abstract*—A novel method for evolutionary optimization, called integrated radiation optimization (IRO), is proposed for solving nonlinear multidimensional optimization problems. Many modern optimization techniques explore the search space by sharing information they have found. In this study, the concept of gravitational radiation in Einstein's theory of general relativity is utilized as a fundamental theory for searching optimal solution in the search space. The idea of developing the algorithm and its detailed procedures are introduced. This work applied the proposed IRO to find the minimum value of a static polynomial function, and some applications that are known to be difficult. The preliminary experimental results show that the performance of the proposed IRO is promising, and IRO shows great performance in solving other NP-hard search and optimization problems.**

I. Introduction

IN RECENTLY DECADES, numerous optimization techniques were proposed by tens of thousands of scientists around the world. Some methods are extremely famous and have been widely applied to many aspects of applications, and most methods among them are based on heuristic optimization approach, such as simulated annealing (SA), genetic algorithm (GA), immune algorithm (IA), ant colony optimization (ACO), and particle swarm optimization (PSO). Heuristic based approaches usually organized multiple search agents into a swarm of particles spreading in the search space. Given a problem, each agent in the swarm has power to direct others' movement based on the quality of the represented solution that it has found. Therefore, the search agent that reaches the best objective value has the highest power in directing other agents' movement. When the global movements of the swarm are decreased to a certain threshold, the swarm is regarded as being convergent, and the optimal solution is obtained.

The history of the development of modern heuristic based optimization approaches can be traced back to 1980s, in which simulated annealing (SA) was proposed by Kirkpatrick *et al.* (1983) [1]. SA is a general purpose probabilistic based algorithm for the global optimization problems. It generalized Monte Carlo method and natural analogy with the statistical physics of random systems to simulate the cooling process of a material, such as crystals. Some other SA related methods have been developed to overcome local minimum problem, such as quantum annealing (QA). Unlike SA uses the concept of thermodynamics, QA uses quantum fluctuations to explore the search space. For more information about QA, please refer to Das and Chakrabarti (2005) [2]. Genetic algorithm (GA) and immune algorithm (IA) are optimization techniques that were proposed by Holland (1975) [3] and Mori *et al.* (1993) [4], respectively. These approaches were used to find the near optimal solution subjected to a given problem. They are also heuristic based optimization approaches inspired by genetics, molecular biology, and immunology. The data structure of GA and IA is more complex than other approaches, which involves more effort in maintaining the data structure of the solutions and the search space. Ant colony optimization (ACO) algorithm was proposed by Dorigo *et al.* (1996) [5]. ACO is a graphical model, and it was inspired by the behaviors of biological ants that how do they find the shortest path from one node to another. ACO can quickly converge to a near optimal solution, but an improper initialization to the algorithm would have a large chance to produce a local optimal solution instead. Particle swarm optimization (PSO) algorithm is one of the swarm intelligence based approach that has been proven useful in many research literatures of various fields. PSO was introduced by Kennedy and Eberhart (1995) [6]. There are several parameters of PSO have to be cleverly defined in *prior* to ensure both of good optimization result and the robustness of PSO. Hsiao *et al.* (2005) also proposed an optimal searching approach, called space gravitational optimization (SGO) algorithm [7]. SGO is a general purpose search technique for multidimensional and multiobjective optimization problems. The search agents of SGO are encouraged to aggressively search those undiscovered regions in the search space without quickly gathering in a certain region. SGO has been tested through several optimization problems and outputted promising results that overcomes the difficulties encountered in classical and modern optimization approaches, such as ACO and PSO. But what happen if we allow the search agents of SGO to communicate with each other for sharing information? This question is what we want to investigate in this study.

In this study, a novel optimization algorithm, called integrated radiation optimization (IRO) algorithm, is proposed to find the optimal solution to a nonlinear problem. In IRO, search space can be imagined as a hyperspace with a number

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of search agents distributing randomly in the inside of it. The solution that was found to have better objective value is assumed to have a supernova with imperfect symmetrical shape expanding at the place. For such a case, gravitational radiation would be given off from the massive supernova. If a search agent stands alone in the hyperspace, the direction that detects a large quantity of gravitational radiation can directly imply to have one or a group of massive stars locating at that direction. The search agent will move toward the location of the massive stars, and explore the undiscovered regions on the path to it. Finally, a global optimal solution can be produced. In order to test the validity and efficiency of the proposed algorithm, IRO has been applied to optimization problems that are known to be difficult. In this work, some preliminary experimental results are demonstrated for comparisons with other optimization approaches.

This paper is organized as follows: the concept of Einstein's general theory of relativity is briefly introduced in Section II. The detailed procedures of IRO and its searching strategy are proposed in Section III. The performance of IRO is provided in Section IV with comparison with other optimization approaches. Finally, a short discussion and preliminary conclusions are drawn in the last section.

II. Einstein's General Theory of Relativity

General theory of relativity is a geometrical theory that was proposed by Albert Einstein [8-9] from 1915 to 1916. The main purpose of this famous theory in modern physics is to describe how do mass, energy and momentum distort the geometry of space-time. The curved geometry of space-time can be interpreted as the geometrical distribution of gravity. The foundation of Einstein's general theory of relativity, also called Einstein equivalence principle, states that the result of a local non-gravitational experiment in an inertial frame of reference is independent of its own velocity or location. The gravitational force observed locally by two reference frames, one is in a space free from gravitational fields, and another is with uniformly acceleration, are physically exactly equivalent. In other words, the effects of gravity are exactly equivalent to the effects of acceleration, and the theory also states that the acceleration of a freefalling laboratory cancels out the gravitational effect completely. So, in short, the trajectory of the freefalling laboratory in the space is directed by the gradient of gravitational field nearby it. An illustration of the geometry of curved space-time is shown in Fig. 1, and an example of the freefalling laboratory (as an asteroid in this case) experiment is depicted in Fig. 2 for ease of understanding the general idea of gravitational field. One of our previously study, SGO [7], was inspired by the concept of Einstein equivalence principle mentioned in above; and the performance of SGO is quite promising. Please note that the search agent (as an asteroid in this case) in SGO moves toward a better solution only based on the geometrical variance around it.

In this study, we try to extend SGO by adapting the concept

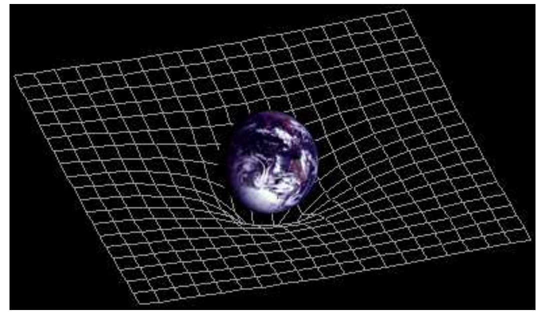

Fig. 1. Illustration of a curved space-time. The distortion at the center of the plot is created by the mass of the earth. Therefore, the geometrical gradient of the curved space-time represents the gravitational field formed by the earth. (The picture of the earth was captured from the website of National Space Science Data Center, NASA.)

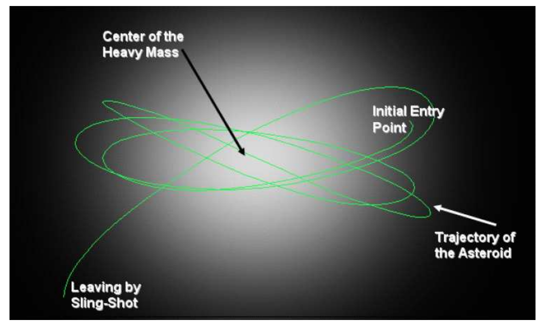

Fig. 2. Illustration of simulating the effect of gravitational field acts on directing the movement of an asteroid in a space with a massive star nearby. During the asteroid is shifting into the gravitation field, it cannot detect any acceleration acted on itself, but it can still observe accelerations of other reference frames.

of "*gravitational radiation*" to the development of a new optimization algorithm, so the search agents will be able to see more information in the entire search space rather than local geometrical variance around it. Therefore, it is essential to understand the theoretical concept of gravitational radiation. In modern physics, the gravitational wave propagates as a wave travelling outward from a moving object or system of objects. Gravitational radiation is an energized fluctuation in the curved space-time transported by the gravitational wave. The existence of gravitational radiation has been indirectly proven by Hulse and Taylor [10]. They detected pulsed radio emissions from a pulsar in a binary star system, and also discovered that the changing orbit of the binary star system is matched with the loss of energy due to give off gravitational radiation from the binary star system. They were also awarded by the 1993 Nobel Prize in Physics for this significant discovery.

Not all of massive stars in the universe can radiate gravitational wave. Gravitational radiation is emitted only by an accelerating object. A static object and an isolated non-spinning solid object moving at a constant velocity will not radiate. Nevertheless, gravitational radiation will be emitted in two general cases: two or more stars orbiting to each other, and an isolated supernova with non-axis-symmetrical expanding in the space. The strength of the gravitational radiation strongly depends on the mass of the star. A supernova is a stellar explosion that generates exceptional bright light, and then blasts inner stellar materials away due to re-invigoration of neutrino reactions surrounding the outer core of the collapsing star. The massive stellar materials that have blasted away are ejected into space during the supernova explosion. If the shape formation of the ejected material is not perfectly symmetric, the supernova will radiate gravitational radiation. Another case, the binary star system, or called pulsar (such as the Hulse-Taylor binary star system), usually gives off strong gravitational radiation. It is because stars in the system orbits around its companion star, and both of them are extremely massive. The combination of massive stars and small distance between them leads to significant intensive of gravitational radiation given off by the binary star system. The properties of gravitational radiation, such as amplitude, frequency, angular velocity and power, are not in the scope of this study. For detailed information about gravitational wave and gravitational radiation, please refer to [11-13]. The descriptions about how we utilize the properties of gravitational radiation in solving an optimization problem are given in Section III.

III. The Proposed Algorithm

A. General Description

In the last section, the Einstein's general theory of relativity, Einstein equivalence principle and gravitational radiations have been reviewed in general. The idea of utilizing these famous theories to develop a new optimization algorithm is explained in this section.

As mentioned in Section II, we know that the curvatures of space-time (also called gravity) are caused by massive stars in the space. According to Newton's law of universal gravitation, the gravity is proportional to the product of two masses, and inversely proportional to the square of the distance between the two masses, which can be formulated as follow:

$$F = G \cdot \frac{m_1 m_2}{r^2} \tag{1}$$

where F is the magnitude of the gravity, G is the gravitational constant (6.6742×10^{-11} m^{3}kg^{-1}s^{-2}), m_1 and m_2 are the first and second masses that attract each other, and r is the distance between them. Our previously proposed approach, SGO, basically treats the optimization problem as an asteroid travelling in space, and changes its movement direction simply according to the variation of gravity around the asteroid. However, according to Eq. (1), we can see that the strength of gravitational force F between two masses rapidly weakens when r significantly increases. Therefore, it might be difficult for the asteroid to find a global best solution if it travels through a local weak- or non-gravitational area in the space. An example of such event is depicted in Fig. 3.

In this study, a novel optimization algorithm called IRO is presented. In IRO, the role of search agents in search space is

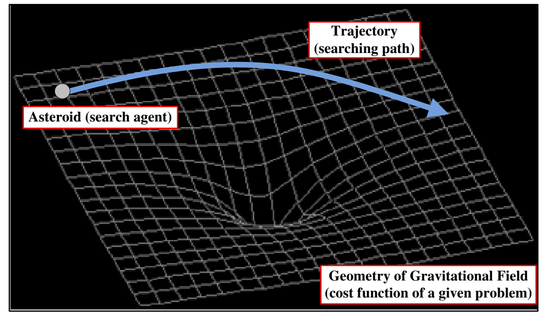

Fig. 3. Illustration of an asteroid misses the location of the local gravity source while traveling in space. This example shows that if we only consider the geometry variance of the cost function around the search agent to guide its movement, there will be a large possibility for the search agent that fails to discover actual optimal solutions in the search space.

modeled as massive binary star systems shifting moving in universe. The quality index of the solution found by the search agent is modeled as the mass of the binary star system. Therefore, the heavier the mass of the binary (star) system is, the better quality index of the solution corresponding to the search agent will be. An example is depicted in Fig. 4. The coordinates of two search agents are (34, 69) and (76, 23) locating in the search space, and their quality indices are 1 and 2, respectively. We can simply compute convolution of quality indices of solution in Fig. 4 and an inverse Gaussian distribution to simulate the curved space-time caused by the masses of binary star systems. The convolution result is depicted in Fig. 5. Gravitational radiations are emitted from the massive binary star systems along the curvature of time-space, as the red arrows in Fig. 5. The gravitational radiation is proportional to the mass of binary stars. Therefore, binary star system #1 in Fig. 5 will radiate more intensive gravitational radiation to the space. Therefore, the movement of a search agent in the search space can be directed simply by detecting the direction that receives the greatest quantity of gravitational radiations. Iteratively, search agents will converge and form one or more clusters in search space. The solution qualities of these search agents can be further accumulated to form a stronger source of gravitational radiation, as depicted in Fig. 6. This phenomenon is also agreed by the Einstein's general theory of relativity:

$$P_{GR} = \frac{dE}{dt} = -\frac{32}{\pi}\frac{G^4}{c^5}\frac{\left(m_1 m_2\right)^2\left(m_1 + m_2\right)}{R^5} \tag{2}$$

where P_{GR} is the power emitted by the two closing massive binary star systems, c is the speed of light, R is the distance between both of the binary star systems, m_1 and m_2 are the total masses of binary system #1 and #2, respectively. We can see that the P_{GR} is inverse proportional to R^5, which means that when R decreases, P_{GR} will have an exceptionally boosting rate.

Furthermore, we investigate the amount of power that is able to be detected by a static frame of reference with averaged distance R_t away from the source of P_{GR}. The power detected by the static frame of reference P_d can be calculated

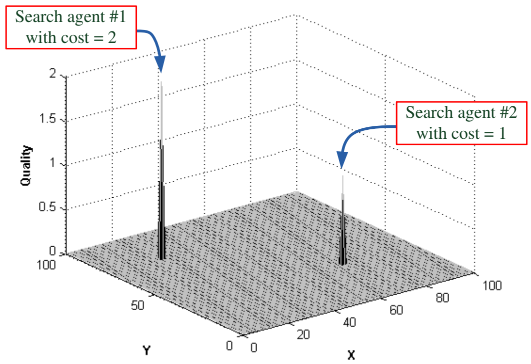

Fig. 4. Illustration of two search agents in search space, and the qualities of their solutions are evaluated by a presumed cost function. In this example, a higher quality represents a better solution.

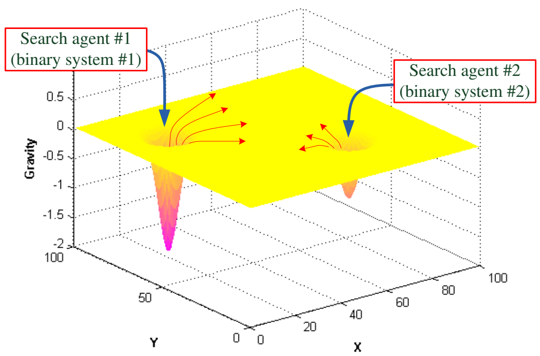

Fig. 5. The convolution result obtained by convoluting the qualities of solutions with an inverse Gaussian function. The caves in the figure represent curved space-time, and the red arrows represent gravitational radiation given off by the binary (star) system locating at the center of the caves.

by:

$$P_d = \frac{P_{GR}}{4\pi R_t^2} = \frac{dE}{dt} = -\frac{8}{\pi^2 R_t^2}\frac{G^4}{c^5}\frac{(m_1 m_2)^2 (m_1 + m_2)}{R^5} \quad (3)$$

We can see that the P_d is inverse proportional to the square of R_t.

Moreover, the search agent with good solution quality should have stronger resistance against directing by other search agents. Therefore, if the solution quality of a search agent is good enough, then its mass will be significantly heavier than others. Under such condition, the search agent with the best solution will be almost isolated from other search agents' influences. This phenomenon can be formulated into a basic equation in Newton' s laws of motion:

$$a_1 = \left(G \cdot \frac{m_1 m_2}{r^2}\right) \Big/ m_1 = G \cdot \frac{m_2}{r^2} \quad (4)$$

where a_1 is the acceleration rate of search agent #1. In an extreme case, if the quality index of search agent #1 is one million times better than that of search agent #2 (i.e., $m_1 = 1 \times 10^6\, m_2$), then the acceleration rate of search agent #1 will be one million times smaller than that acts on search agent #2 (i.e., $a_1 = 1 \times 10^{-6}\, a_2$).

In general, the movement of a search agent in IRO is directed by the accumulative gravitational radiation emitted from one or a set of search agents. Also, the search agents fight against directing by others according to the goodness of solutions represented by them. Detailed procedures of IRO are introduced in following subsections.

B. Initialization

Assume that there are m parameters to be optimized in a given problem. Reasonable ranges of m parameters have to be determined in advance, such as $[Min_m, Max_m]$. Since we have to convolute the search space with m-dimensional Gaussian distribution function, a section of memory space has to allocated, the required size of memory can be calculated by

$$Mem = size(SS) = \prod_{i=1}^{m} (res_m) \quad (5)$$

where Mem is the total amount of memory (in the unit of bytes for a double precision floating number) needed to be allocated for construction of the search space SS, and res_m is the number of grids in m-th dimension of the search space. A group of n search agents P_i, where $i = 1, 2, \ldots, n$, are randomly initialized in the m-dimensional hyperspace. Then, the cost of each search agent is calculated using the initial solution. Let the cost function (mass of the search agent) of a given problem to be F, and the position of search agent P_i in the search space can be denoted as $idx(P_i)$. Therefore, we treat the costs of all search agents as the spatial parameters and plant them into their corresponding position in search space as follow:

$$SS\left(idx(P_i)\right) = F(P_i) \quad (6)$$

The result of this operation shall be similar to the case with 2-dimensional search space that has been previously demonstrated in Fig. 4.

C. Preprocessing

In this step, the geometry of gravity field and the strength of gravitational radiation are estimated. Using the equations in astrophysics, we can estimate them in a very precisely way. However, the equations in astrophysics are high-order and very complex, it would increase the computation time when the number of heuristic iterations raises. In order to avoid those complex equations, Gaussian distribution function is utilized for fast approximation to the astrophysics equations.

An m-dimensional convolution of search space with Gaussian distribution function is computed to approximately estimate the geometry of gravity field and the strength of gravitational radiation. The Gaussian distribution function can be formulated as follow:

$$Gauss(x_1, x_2, \ldots, x_m) = A e^{-\left(\sum_{i=1}^{m}\left(\frac{x_i - x_0}{\sigma}\right)^2\right)} \quad (7)$$

where A is the amplitude, x_0 and σ are the center of the distribution and the spread of the blob in every dimension. An example of 2-dimensional Gaussian distribution function is

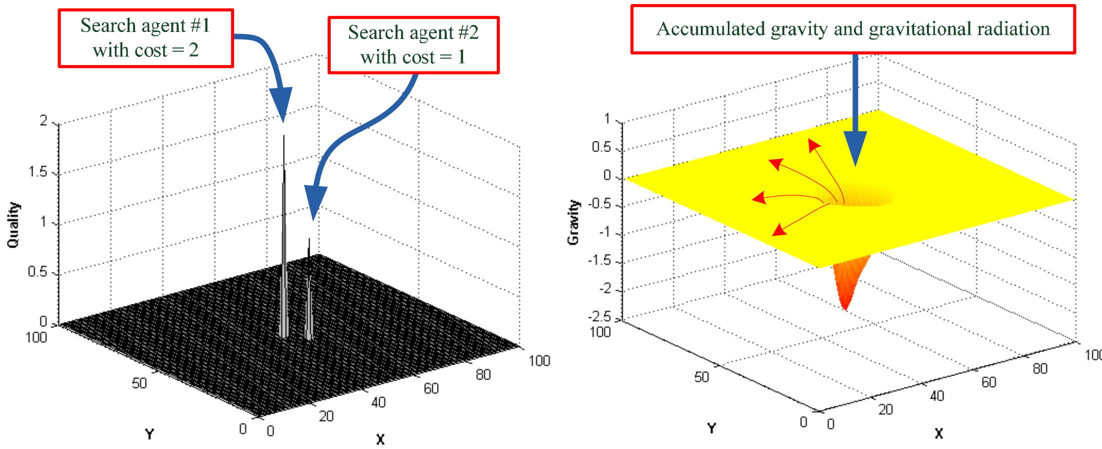

Fig. 6. An example of the accumulative effect of gravity and gravitational radiation occurs when two search agents are closing to each other, where (a) shows the qualities of their solutions and (b) shows that the gravitational radiations given off by those search agents are accumulated together to form a stronger gravitational radiation.

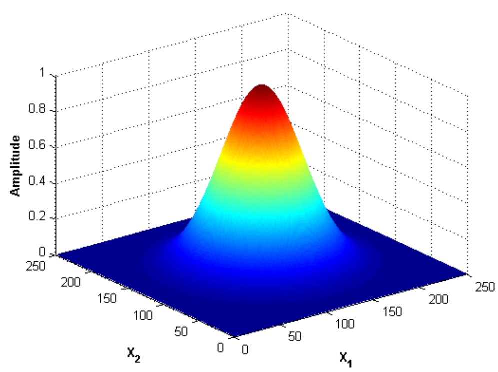

Fig. 7. Illustration of 2-dimensional Gaussian curve.

generated using $A = 1$, $x_0 = 125$, $\sigma = 50$, and depicted in Fig. 7. The power of gravitational radiation in the search space SS can be obtained by computing the following convolution:

$$GR = SS * Gauss$$
$$= -\sum_{k_1=-\infty}^{\infty} \cdots \sum_{k_m=-\infty}^{\infty} Gauss\left(x_1 - k_1, \ldots, x_m - k_m\right) SS\left(k_1, \ldots, k_m\right) \tag{8}$$

where GR is the approximated power of gravitational radiation, and the size of GR should be the same as SS. Therefore, the extended margin around GR caused by the computation of the m-dimensional convolution should be pruned.

D. Ranking and Movement

The percentiles of each search agent P_i in $SS(idx(P_i))$ and $GR(idx(P_i))$ are calculated, so

$$_{SS}Pct_i = percentile\left(SS\left(idx\left(P_i\right)\right)\right), \tag{9}$$

and

$$_{GR}Pct_i = percentile\left(GR\left(idx\left(P_i\right)\right)\right). \tag{10}$$

where $_{SS}Pct_i$ and $_{GR}Pct_i$ are the percentiles of P_i in $SS(idx(P_i))$ and $GR(idx(P_i))$, respectively. The cumulative distribution function (cdf) used in calculating $_{SS}Pct_i$ and $_{GR}Pct_i$ is Empirical (Kaplan-Meier) cdf. The Empirical cdf produces an estimate of nonparametric density and tries to adapt itself to the real data distribution. Please note that the direction of accumulation in computing the Empirical cdf depends on what type of the problem given. There is nothing different from normal computation method in calculating cdf if the given problem is to find out the maximum cost in the search space. If the objective of the optimization is to find the minimum cost, the cdf is changed to that of subtracting the original cdf from 1. The cdfs used in different purpose (finding maximum or minimum costs) are demonstrated in Fig. 8.

The displacement of a search agent is governed by the gravitational radiation emitted from other search agents. The displacement can be calculated by following equation:

$$Disp\left(P_i\right) = \sum_{\forall j, j \neq i}^{m} \frac{_{SS}Pct_j}{\left\|\boldsymbol{u}_{ij}\right\|^2} \frac{_{GR}Pct_j}{\left\|\boldsymbol{u}_{ij}\right\|^2} \left(\boldsymbol{rnd}_j \cdot \vec{\boldsymbol{u}}_{ij}\right) \tag{11}$$

where $Disp(P_i)$ is the displacement of i-th search agent in an iteration, $\vec{u}_{ij}$ is the unit vector of the geometric vector $\boldsymbol{u}_{ij} = \boldsymbol{P}_j - \boldsymbol{P}_i$, and $\boldsymbol{rnd}_j$ is a $1 \times j$ random vector with each component generally a uniform random number between 0 and 1. The term $_{GR}Pct_j/\|\boldsymbol{u}_{ij}\|^2$ simulates the Eq. (3) and Eq. (4) that represents the strength of the gravitational radiation emitted by j-th search agent that arrives the position of i-th search agent. Another term $_{SS}Pct_j/\|\boldsymbol{u}_{ij}\|^2$ represents the acceleration rate of i-th search agent that is caused by the mass of j-th search agent. The random vector $\boldsymbol{rnd}_j$ allows IRO to have stochastic exploration capability, and to avoid IRO getting stuck at local maxima. Each search agent updates its position P_i by the equation shown below:

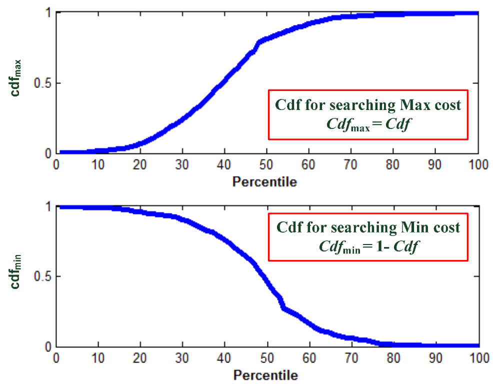

Fig. 8. Illustration of the cdfs used in different purpose, (a) is the cdf for finding maximum cost of a given problem and (b) is for finding minimum cost.

$$P_i = P_i + Disp\left(P_i\right) \tag{12}$$

Once the positions of all search agents are updated, an iteration of optimization is then completed. The search space *SS* will be reinitialized and the process will loop back to Eq. (4) till the maximum number of iteration is achieved.

IV. Experimental Results

A. 2-D Fixed Static Polynomials Function

In this subsection, IRO is applied to a 2-dimensional polynomial function to find the position where the global minimum value is located. The optimization problem is given as below:

$$\min \ f\left(x_1, x_2\right) = \sum_{i=1}^{2}\left(x_i^4 - 16x_i^2 + 0.5x_i\right) \tag{13}$$

$$\text{subject to } -50 \le x_1, x_2 \le 50$$

The reason of choosing this problem to test the performance of IRO is that it is a very typical problem that has been widely utilized to test many other optimization approaches. In this case, the objective space is highly nonlinear due to higher order terms in Eq. (13). The illustration of the polynomial function $f(x, x_2)$ is shown in Fig. 9. Some critical parameters of IRO used in the simulation are list as below:

<u>The primary properties (critical parameters) of IRO:</u>
Number of parameters to be optimized $m = 2$;
Number of search agents $n = 20$;
Reasonable range of solution space $[Min_m, Max_m]$ = [−50,50]
Resolution of search space in each dimension res_m = 1000
Number of maximum iteration = 100;

<u>The parameters of Gaussian distribution function *Gauss*:</u>
Size of the *Gaus* = 50 × 50;
Amplitude $A = 1$;
Center of the distribution $x_0 = 25$;
Spread of the blob $\sigma = 10$;

The geometrical distributions of the power of gravitational

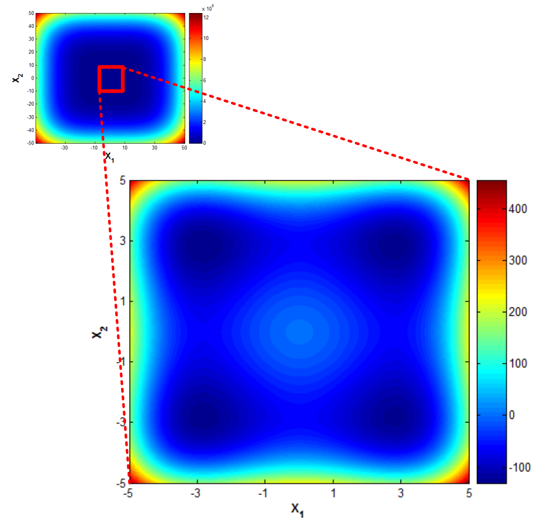

Fig. 9. Illustration of the 2-D polynomial function given in Eq. (13), and its zoomed in region that contains four minimum values. Please note that the grid values in x_1 and x_2 axis represent the value of x_1 and x_2.

radiation *GR* in IRO during the simulation are depicted in Fig. 10. In these graphs, dark blue regions indicate that intensive gravitational radiations are emitted from these regions. Finally, after 100 times of generations, almost all search agents are converged to the region with the global minimum value. The success rate (the error of IRO's output to actual minimum value of Eq. (13)) of each generation of evolution is illustrated in Fig. 11, which shows that IRO can find the desired solution in a very efficiency way. Nevertheless, the computation time used to complete the 100 epochs is around 1 minute.

B. Optimization of Designing of PID Controller

The proposed IRO has been implemented to optimize the designing of PID controllers under various given plants. This problem is widely known to be difficult since the system involves with: gains of proportional, integral and derivative terms; and multiple objectives, e.g. maximum overshoot and rise time. A lot of previously proposed literatures tried to investigate the optimal approaches to determine these gains in designing of PID controllers, because the transfer function of a PID controller is highly nonlinear.

The transfer function of a PID controller is defined as:

$$G_C\left(s\right) = K_p + \frac{K_i}{s} + K_d \cdot s \tag{14}$$

where K_p, K_i and K_d are the proportional, integral and derivative gain, respectively. The gains K_p, K_i and K_d of the PID controller $G_C(t)$ are generated by IRO algorithm for a given plant $G_p(t)$. The output U(t) of the PID controller is:

$$U\left(t\right) = K_p e\left(t\right) + K_i \int_0^t e\left(\tau\right) d\tau + K_d \dot{e}\left(t\right) \tag{15}$$

where $e(t)$ is the error between the reference input $r(t)$ and the output $y(t)$ at time t. The parameters of the PID controller K_p, K_i and K_d can be manipulated to produce various response

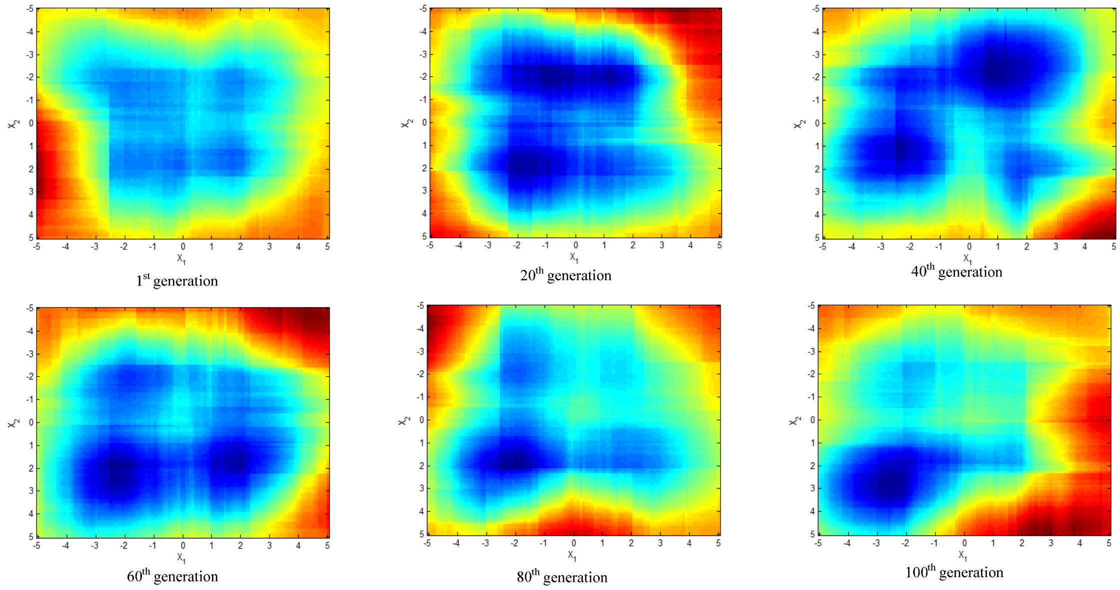

Fig. 10. Illustration of the geometrical distributions of the power of gravitational radiation *GR* in IRO during the simulation. The dark blue regions indicate that intensive gravitational radiations are emitted from these regions. Therefore, these regions are inferred as potential positions that the optimal solution would locate. Please note that the range of these graphs is the same as the zoomed in view plotted in Fig. 9. Please note that there are four optimal solutions existed in this objective function, and the qualities of them are equal to each other. The proposed IRO can always converge to one of the optimal solutions. In this figure, we demonstrate the case that IRO converges to the optimal solution located at the bottom-left region of the objective space.

curves from a given plant. For a given plant, the problem of designing a PID controller is to adjust the parameters K_p, K_i and K_d for getting a desired performance of the considered system. There are four performance indexes of the transient response are utilized to characterize the performance of the PID control system: maximum overshoot f_{mo}, rise time f_{rt}, settling time f_{st}, integral absolute control error f_{iae}. For more descriptions about these performance indexes, please refer to [14]. Therefore, the task is to find the parameters K_p, K_i and K_d that minimize the performance indexes on a given plant. Generally, design a PID controller is a multi-objective task that tries to obtain an overall best performance using an accumulated objective function, which is:

$$\min \ f = f_{mo} + f_{rt} + f_{st} + f_{iae} \tag{16}$$

All solutions that minimize this multi-objective function would be called as an optimal design of a PID controller. However, using weighted sum of all performance indexes is also a feasible approach when one or more indexes are considered to be relative important among all indexes. In order to generalize this study, the un-weighted Eq. (16) is utilized to be the multi-objective function of IRO. All of the simulations are conducted with MATLAB/Simulink on a Pentium M Core 2 Duo 1.83GHz computer with 2GB RAM. The key parameters of IRO used in the simulation are list as below:

<u>The primary parameters of IRO:</u>
Number of parameters to be optimized $m = 3$;
Number of search agents $n = 20$;
Reasonable range of solution space $[Min_m, Max_m] = [0, 5]$
Resolution of search space in each dimension $res_m = 100$
Number of maximum iteration = 100;

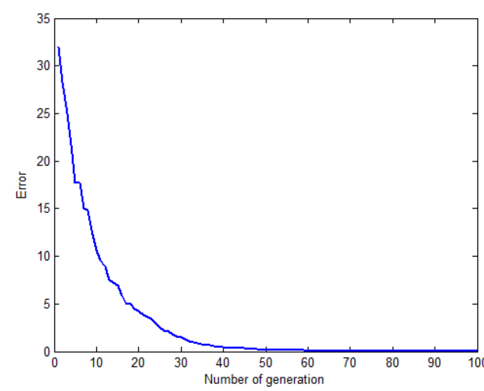

Fig. 11. Illustration of the success rate yielded by IRO.

<u>The parameters of Gaussian distribution function *Gauss*:</u>
Size of the *Gaus* = 25 × 25;
Amplitude $A = 1$;
Center of the distribution $x_0 = 12.5$;
Spread of the blob $\sigma = 7.5$;

We utilized three plants listed below to test the performance of IRO:

$$G_1(s) = \frac{e^{-0.5s}}{(s+1)^2} \tag{17}$$

$$G_2(s) = \frac{4.228}{(s+0.5)(s^2+1.64s+8.456)} \tag{18}$$

$$G_3(s) = \frac{27}{(s+1)(s+3)^3} \tag{19}$$

The results obtained by using Zirgler-Nichols [15], Kitamori [16], Fuzzy [17] and ACO [18] methods along with the result yielded by the proposed IRO are organized into Table I.

For $G_1(s)$, we can see that the rising time and settling time of the step response obtained by IRO are significantly better than other methods. The overall performance index f of IRO also outperforms the result produced by other methods.

TABLE I
Summary of simulation results with respect to the three cases shown in Eqs. (17)-(19).

Plants		Zeigler-Nichols	Kitamori	Fuzzy	ACO	IRO
$G_1(s)$	K_p	2.808	2.212		2.4911	2.8341
	K_i	1.712	1.085		0.8158	0.9231
	K_d	1.151	1.148		1.3540	1.4234
	f_{mo}	43%	15%	6%	4%	4%
	f_{rt}	0.75	1.02	3.09	1.01	0.91
	f_{st}	3.85	3.64		1.92	1.87
	f_{iae}	1.83	1.55	1.18	1.38	1.35
	f	6.86	6.36	4.33*	4.35	4.17
$G_2(s)$	K_p	2.190			4.5721	1.5012
	K_i	2.126			1.2351	0.8612
	K_d	0.565			2.2814	0.0203
	f_{mo}	16%		6%	6%	5%
	f_{rt}	0.73		5.01	0.80	1.11
	f_{st}	5.37			4.28	3.42
	f_{iae}	0.96		1.01	1.06	1.35
	f	7.22		6.08*	6.20	5.93
$G_3(s)$	K_p	3.072	2.236		3.1619	3.2591
	K_i	2.272	1.429		1.1032	0.9823
	K_d	1.038	0.976		1.6639	1.8109
	f_{mo}	33%	11%	2%	4%	5%
	f_{rt}	0.67	0.83	2.63	0.90	0.98
	f_{st}	3.72	2.30		1.52	1.53
	f_{iae}	1.12	0.88	0.81	1.09	1.12
	f	5.84	4.12	3.46*	3.55	3.68

*please note that f in Fuzzy approach excludes the term f_{st} in summation due to lack of information.

V. CONCLUSION

In this study, a novel optimization algorithm, called integral radiation optimization (IRO) algorithm, has been proposed. We utilized the concepts of the Einstein's general theory of relativity to design IRO. In general, IRO models the optimization problem into a simplified astrophysics model that simulates heavy masses in space searching for other massive objects by detecting the gravitational radiations emitted from them. By introducing random vectors into IRO, it allows IRO to have stochastic exploration capability. So IRO has the hill-climbing characteristic to avoid IRO from getting stuck at local optimum.

The proposed IRO has been applied to two kinds of typical problems to evaluate the performance of IRO. Besides the polynomials function case, IRO has been applied to optimize the design of PID controller, which is known as a tough nonlinear optimization problem. The results show that IRO significantly outperforms other methods. Therefore, we think that IRO is useful to solve difficult optimization problem with minimum computational costs.

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