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A dynamic metaheuristic optimization model inspired by biological nervous systems: Neural network algorithm

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ABSTRACT

In this research, a new metaheuristic optimization algorithm, inspired by biological nervous systems and artificial neural networks (ANNs) is proposed for solving complex optimization problems. The proposed method, named as neural network algorithm (NNA), is developed based on the unique structure of ANNs. The NNA benefits from complicated structure of the ANNs and its operators in order to generate new candidate solutions. In terms of convergence proof, the relationship between improvised exploitation and each parameter under asymmetric interval is derived and an iterative convergence of NNA is proved theoretically. In this paper, the NNA with its interconnected computing unit is examined for 21 well-known unconstrained benchmarks with dimensions 50–200 for evaluating its performance compared with the state-of-the-art algorithms and recent optimization methods. Besides, several constrained engineering design problems have been investigated to validate the efficiency of NNA for searching in feasible region in constrained optimization problems. Being an algorithm without any effort for fine tuning initial parameters and statistically superior can distinguish the NNA over other reported optimizers. It can be concluded that, the ANNs and its particular structure can be successfully utilized and modeled as metaheuristic optimization method for handling optimization problems.

1. Introduction

Among optimization approaches, metaheuristic optimization algorithms have shown their capabilities for finding near-optimal solutions to the numerical real-valued test problems. In contrast, analytical approaches may not detect the optimal solution within a reasonable computational time, especially when the global minimum is surrounded by many local minima.

Metaheuristic algorithms are usually inspired by observing phenomena and rules seen in nature such as the Genetic Algorithm (GA) [1], the Simulated Annealing (SA) [2], the Particle Swarm Optimization (PSO) [3], the Harmony Search (HS) [4], and so forth.

The GA is based on the genetic process of biological organisms [5]. Over many generations, natural populations evolve according to the principles of natural selections, i.e. survival of the fittest. In the GA, a potential solution to a problem is represented as a set of parameters. Each independent variable is represented by a gene. Combining the genes, a chromosome is produced which represents a solution (individual).

During the reproduction phase, individuals are selected from the population and recombined. Having selected two parents, their chromosomes are recombined, typically using a crossover mechanism. Also, in order to satisfy the population diversity, a mutation operator is applied to some individuals [1]. The GA has been utilized for solving various optimization problems in the literature and it is a well-known optimization method [6–9].

The origins of SA lay in the analogy of optimization and a physical annealing process [2]. Annealing refers to an analogy with thermodynamics, specifically with the way that metals cool and anneal. The SA is basically hill-climbing except instead of picking the best move, it picks a random move. If the selected move improves the solution, then it is always accepted. Otherwise, the algorithm makes the move anyway with some probability less than one. The probability decreases exponentially with the badness of the move, which is the amount of Δ by which the solution is worsened. A parameter T is also used to determine this probability. At higher values of T, uphill moves are more likely to occur. As T tends to zero, they become more and more unlikely. In a typical SA optimization, T starts with a high value and then, its value is gradually decreased according to an annealing schedule [2]. The SA is useful in finding global optima in the presence of large numbers of local

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optima and it is applied for solving a wide range of optimization problems [10–12].

The PSO is an evolutionary optimization method, developed by Kennedy and Eberhart [3], for solving global optimization problems. In the PSO, simple individuals, called particles, move in the search space of an optimization problem. The position of a particle represents a candidate solution to the optimization problem at hand. Each particle searches for better positions in the search space by changing its velocity according to rules (i.e., population cooperation and competition) originally inspired by behavioral models of bird flocking. Researchers found that the synchrony of animal's behavior was through maintaining optimal distances between individual members and their neighbors [13]. The PSO has proved its efficiency for handling real-life optimization problems and its variants have been developed considering design improvements for many engineering cases [14–16].

Geem et al. [4] developed the HS that reproduces the musical process of searching for a perfect state of harmony. The harmony in music is analogous to the optimum design, and the musicians' improvisation is analogous to local/global search schemes. The engineering applications of HS shows its popularity and excellent performance [17–20].

In spite of existing such metaheuristic methods, however, there are still gaps in developing efficient parameter free optimization methods having fast, matured convergence and capable of obtaining high solution quality. Being a user-parameter free optimization method is accounted as one of the most important aspects for every metaheuristic algorithm and it is a very significant area for future research, therefore, that deserves a lot more attention. In order to overcome those concerns which still exists in many optimization methods (especially when the number of design variable increases), this paper is thus motivated to focus on these aims.

This paper introduces a novel metaheuristic optimization algorithm for optimal problem solving. The proposed method is called neural network algorithm (NNA), which is inspired by the concepts of artificial neural networks (ANNs) and biological nervous systems. The NNA employs the structure and concept of ANNs to generate new candidate solutions and also other operators used in the ANNs. The proposed method is examined using 21 unconstrained benchmarks and several constrained engineering design problems, and its efficiency and superiority are highlighted against other reported optimizers along with statistical tests.

The remaining of this paper is organized as follows: In Section 2, brief introduction of ANNs and concepts of the proposed NNA are presented in detail. Validation and efficiency of the proposed method in finding optimal solutions are given in Section 3. In this section, 21 unconstrained benchmarks and several constrained engineering design problems have been examined and the obtained optimization results have been compared with the state-of-the-art and recent algorithms in the literature. Furthermore, sensitivity analysis over user parameters of used optimizers and computational complexity of the proposed optimizer have been given in this section. Finally, this paper ends with the conclusions section along with some future research lines. In Appendix A, the relationship between improvised exploitation and each parameter under asymmetric interval is proved mathematically.

2. Neural network algorithm

2.1. Basic idea

Artificial neural networks (ANNs) are computational models inspired by the structure and/or functional aspects of biological neural networks. The ANNs consist of dense interconnected computing units (i.e., artificial neurons) that are motivated by biological nervous systems [21]. As in nature, the connections among units largely determine the network function.

Depending on their connectivity pattern (architecture), the ANNs can be grouped into the following two categories: *a*) feed-forward neural networks: these are networks whose architecture has no loops. In general, the feed-forward networks are "static" because they produce only one set of output values rather than a sequence of values from a given input data set; *b*) recurrent networks: these are networks in which loops occur because of feedback connections, applying feedbacks means that a time parameter implicitly enters the model, and in this sense these neural networks are "dynamic" [22].

Recurrent networks have developed two kinds of feedback connections for neural networks: 1) local feedbacks: these are links that pass the output of a neuron to itself; 2) global feedbacks: these are links that pass the output of a neuron to other neurons in the same or lower layers in the multilayer network architecture [23,24]. Fig. 1 shows two typical architectures of ANNs for feed forward and recurrent neural networks. Other necessary information concerning ANNs are given at each related step of the proposed optimization method.

2.2. Proposed NNA

Similar to other metaheuristic optimization algorithms, the NNA begins with an initial population so called population of pattern solutions. As the ANNs mostly used for prediction purposes, it receives input data and target data and predicts the relationship among input and target data (i.e., mapping input date to target data). Usually, inputs of the ANNs are input values obtained by experiments, calculations, and so forth.

The ANNs, simply speaking, tries to map input data to the target data. Therefore, the ANNs tries to reduce the error (e.g., mean square error) among predicted solutions and target solutions using iteratively changing the values of weight functions (w_{ij}) (see Fig. 1b). However, in the optimization, the goal is to find the optimum solution, and a metaheuristic algorithm should search a feasible optimal solution using a defined strategy.

Therefore, inspired by the ANNs, in the NNA, the best obtained solution at each iteration (i.e., temporal optimal solution) is assumed as target data and the aim is to reduce the error among the target data and other predicated pattern solutions (i.e., moving other predicted pattern solutions towards the target solution). Based on the defined concept, the NNA is developed for minimization problems (i.e., minimizing the error among the target and pattern solutions). It is worth pointing out that this target solution has been updated at each iteration. In this paper, the structure of the ANNs, some parts of its mathematical formulations and concepts, all together are coming to develop a new optimization algorithm based on the neural network configuration. Detail descriptions and processes of the NNA are given in the following subsections.

2.2.1. Generating initial population

In order to solve an optimization problem, it may be necessary that the values of decision variables be represented as an array. Before explaining the NNA processes, the key terms used to describe this algorithm should be introduced. Each individual or agent, a set containing a value of each optimization variable, is called "pattern solution" (e.g., in the GA, this array is called "Chromosome"). In D dimensional optimization problem, a pattern solution is an array of $1 \times D$, representing input data in the NNA. This array is defined as given follows:

$$PatternSolution = [x_1, x_2, x_3, \ldots, x_D]. \tag{1}$$

Indeed, population of pattern solutions corresponds to input data in the ANNs. To start the optimization algorithm, a candidate

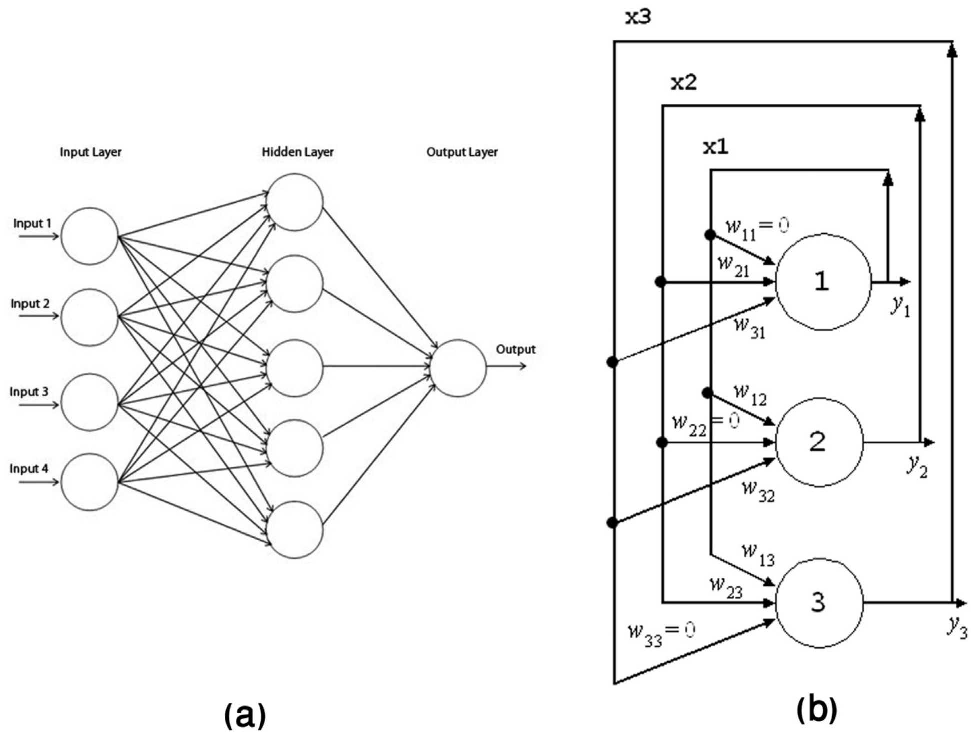

Fig. 1. Schematic view of an ANNs with: (a) feed forward neural networks, (b) recurrent neural networks.

of pattern solution matrix with size $N_{pop} \times D$ is generated. Hence, matrix X which is randomly generated between lower and upper bounds of a problem (i.e., lower and upper bounds are assumed to be defined by a decision maker), is given as follows (rows and column are the population size (N_{pop}) and dimension size (D), respectively):

$$Population \quad of \quad Pattern \quad Solutions = X$$
$$= \begin{bmatrix} x_1^1 \; x_2^1 \; x_3^1 & \cdots & x_D^1 \\ x_1^2 \; x_2^2 \; x_3^2 & \cdots & x_D^2 \\ \vdots \; \vdots \quad \vdots & \vdots & \vdots \\ x_1^{N_{pop}} \; x_2^{N_{pop}} \; x_3^{N_{pop}} & \cdots & x_D^{N_{pop}} \end{bmatrix}. \tag{2}$$

Each of the decision variable values ($x_1, x_2, \ldots, x_D$) can be represented as floating number (i.e., real values) or can be defined over a set of discrete variables. The cost (fitness for maximization problems) of a pattern solution is obtained by evaluating the cost function (fitness function) (C) at the corresponding pattern solution given as follows:

$$C_i = f(x_1^i, x_2^i, \cdots, x_D^i) \qquad i = 1, 2, 3, \ldots, N_{pop}. \tag{3}$$

where f is the objective function. In the entire paper, notations having a vector sign are corresponded as vector values (array), otherwise the rest of notations and parameters are considered scalar values. After calculating the cost function (fitness function) for all pattern solutions, then find the best pattern solution (in this paper, a candidate solution with the minimum objective function value) considered as the target solution.

The NNA resembles ANNs having N_{pop} input data having D dimension(s) and only one target data (response) (see Fig. 1a). After setting the target solution (X^{Target}) among the other pattern solutions, the target weight (W^{Target}), the weight corresponding to the target solution, has to be selected from the population of weight (weight matrix).

2.2.2. Weight matrix

In the ANNs, the artificial neurons or the processing units may have several input paths, corresponding to the dendrites. Using a simple summation, the unit combines the weighted values of these input paths. The result is an internal activity level for the unit [25].

The output path of a unit may be connected to the input path of other units through connection weights which correspond to the synaptic strength of the biological neural connections. Each connection has a corresponding weight (w) (see Fig. 1b), where the signals on the input lines to a unit are modified or weighted prior to being summed [26].

Initial weights in ANNs are random numbers and when the iteration number is increasing, they will be updated considering the calculated error of the network. Back to the NNA, initial weights are defined as given in the following equation:

$$W(t) = [W_1, W_2, \cdots, W_{N_{pop}}]$$
$$= \begin{bmatrix} w_1^1 & \cdots & w_1^i & \cdots & w_1^{N_{pop}} \\ w_2^1 & \cdots & w_2^i & \cdots & w_2^{N_{pop}} \\ \vdots & & \vdots & & \vdots \\ w_{N_{pop}}^1 & \cdots & w_{N_{pop}}^i & \cdots & w_{N_{pop}}^{N_{pop}} \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots & w_{i1} & \cdots & w_{N_{pop}1} \\ w_{12} & \cdots & w_{i2} & \cdots & w_{N_{pop}2} \\ \vdots & & \vdots & & \vdots \\ w_{1N_{pop}} & \cdots & w_{iN_{pop}} & \cdots & w_{N_{pop}N_{pop}} \end{bmatrix}, \tag{4}$$

where W is a square matrix ($N_{pop} \times N_{pop}$) which generates random numbers uniformly between zero to one during iterations, and t is an iteration index. The first subscript of weight relates to its pattern solution (e.g., $w_{2\times}$ relates to the second pattern solution) and the second subscript of weight is shared with the other pattern solutions (e.g., w_{23} is shared with the third pattern solution). Every pattern solution has its corresponding weight value which has been involved it for generating a new candidate solution.

However, there is a constraint for the weight values. The imposed constraint is the summation of weights for a pattern solu-

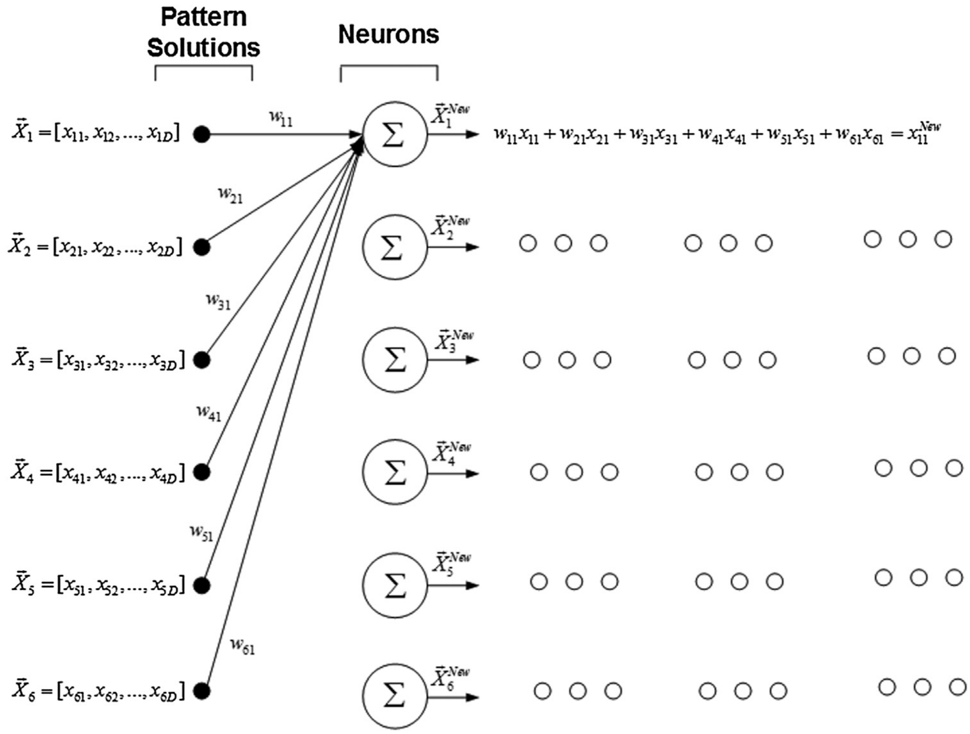

Fig. 2. Schematic view of generating new pattern solutions using Eqs. (7) and (8).

tion should not exceed one, mathematically, it can be defined as given follows:

$$\sum_{j=1}^{Npop} w_{ij}(t)=1, \quad i=1,2,3,\dots,N_{pop}\,. \tag{5}$$

$$w_{ij}\in U(0,1) \quad i,j=1,2,3,\dots,N_{pop} \tag{6}$$

Weight values are belonging to uniformly distributed random numbers between zero and one (Eq. (6)) where their summation for a pattern solution should not exceed one (Eq. (5)). Existence of such a constraint for weight values is due to control the bias of movement and generating new pattern solutions (new individuals). Without this constraint, the weight values tend to grow (i.e., values more than one) in a specific direction and therefore, the algorithm will be stuck in a local optimum point (For instance, act as pheromone parameter in the ant colony optimization (ACO) when a route has lots of pheromones for attracting other ants). Having this constraint gives the NNA's agents controlled movement with mild bias (varying from zero to one). After forming the weight matrix (W), new pattern solutions (X^{New}) are calculated using the following equation inspired by the weight summation technique used in the ANNs:

$$\vec{X}_j^{New}(t+1)=\sum_{i=1}^{Npop} w_{ij}(t)\times\vec{X}_i(t), \quad j=1,2,3,\dots,N_{pop}\,, \tag{7}$$

$$\vec{X}_i(t+1)=\vec{X}_i(t)+\vec{X}_i^{New}(t+1), \quad i=1,2,3,\dots,N_{pop}, \tag{8}$$

where t is an iteration index. Therefore, the new pattern solution has been updated for iteration $t+1$ using Eqs. (7) and (8). In the following, an example has been provided. For instance, if we have six pattern solutions (i.e., six neurons, population size of 6), updating the first new pattern solution can be calculated as given follows:

$$\vec{X}_1^{New}(t+1)=w_{11}\vec{X}_1(t)+w_{21}\vec{X}_2(t)+w_{31}\vec{X}_3(t)+w_{41}\vec{X}_4(t) +w_{51}\vec{X}_5(t)+w_{61}\vec{X}_6(t). \tag{9}$$

Also, for further clarification, Fig. 2 depicts how the NNA forms its new population of pattern solutions for D dimension(s).

After creating the new pattern solutions from the previous population of patterns, based on the best weight value so called "target weight", the weight matrix should be updated as well. The following equations suggest an updating equation for the weight matrix:

$$\vec{W}_i^{Updated}(t+1)=\vec{W}_i(t)+2\times rand\times(\vec{W}^{Target}(t) -\vec{W}_i(t)), \quad i=1,2,3,\dots,N_{pop}, \tag{10}$$

It is noted that weight matrix should always satisfy the constraints (5) and (6) during the optimization process

2.2.3. Bias operator

The bias current plays a vital role in the dynamics of the neural networks model. By virtue of its role, the bias current is always tied to a surrounding condition (e.g., noise), so as to make the output of each neuron respect the surrounding condition [27]. In the NNA, the bias operator modifies a certain percentage of the pattern solutions in the new population of pattern solutions ($X_i^{New}(t+1)$) and updated weight matrix ($\vec{W}_i^{updated}(t+1)$) (acting as a noise). In other words, the bias operator in the NNA is another way to explore the search space (exploration process) and it acts similar to the mutation operator in the GA.

In general, the bias operator prevents the algorithm from premature convergence (especially at early iterations) and modifies a number of individuals in the population. In fact, the bias operator acts as a noise to the new pattern solutions (Eq. (7)) and updated weight matrix (Eq. (10)). For this purpose, the Pseudo code given in Table 1 has been applied to new pattern solutions and updated weight matrix.

Looking at Table 1, LB and UB are lower and upper bounds of a problem, respectively. As can be seen in Table 1, β is a modification

Table 1
Suggested strategy for the bias operator applied to new input solutions and updated weight matrix.

```
For i = 1 to N_pop
  If rand ≤ β
    %% ------------- Bias for New Pattern Solution ---------------------------------------------
    N_b = Round (D×β)      % N_b: No. of biased variables in population of new pattern solution
    For j = 1: N_b
      X^Input (i, Integer rand [0, D]) = LB+(UB-LB) ×rand.
    End For
    %% ------------- Bias for Updated Weight Matrix --------------------------------------------
    N_wb = Round (N_pop×β)      % N_wb: No. of biased variables in updated weight matrix
    For j = 1: N_wb
      W^Updated (j, Integer rand [0, N_pop]) = U (0,1).
    End For
  End If
End For
```

factor, which determines the percentage of the pattern solutions that should be altered. The initial value of β is set to 1 (means 100 percentage chance to modify all individuals in population) and its value adaptively has been reduced at each iteration using any reduction formulation as suggested follows:

$$\beta(t+1) = \beta(t) \times 0.99 \qquad t = 1, 2, 3, \ldots, \mathrm{Max\_}Iteration \tag{11}$$

$$\beta(t+1) = 1 - \frac{t}{\max\_iteration} \qquad t = 1, 2, 3, \ldots, \mathrm{Max\_}Iteration \tag{12}$$

Both Eqs. (11) and (12), or any reduction equation can be used for this purpose. In this paper, Eq. (11) is used as reduction pattern for β. The bias operator adaptively is decreased to allow the algorithm searching for optimum solution near to the target solution and also avoid drastic changes in the pattern solutions at final iterations.

2.2.4. *Transfer function operator*

In the NNA, unlike ANNs, transfer function operator transfers the new pattern solutions in the population from their current positions in the search space to new positions in order to update and generate better quality solutions toward the target solution. The improvement of the solutions is made by moving the current new pattern solutions closer to the best solution (target solution). Therefore, the following equation is defined as a transfer function operator (TF) for the proposed method given as follows:

$$\vec{X}_i^*(t+1) = TF(\vec{X}_i(t+1)) = \vec{X}_i(t+1) + 2 \times rand \times (\vec{X}^{Target}(t) - \vec{X}_i(t+1)), \quad i = 1, 2, 3, \ldots, N_{pop}. \tag{13}$$

The constant value of two in Eq. (13) is logically chosen as for suggested in optimization updating equations (e.g., PSO and ICA). In order to transfer the updated pattern solution, value of two gives the chance of searching before and after target solution. For instance, if the value is chosen as one, the improved solution by transfer function operator moves toward the target solution from one side (between zero to one by *rand* operator), however it is not passing the target solution and exploring the other side of target solution. Therefore, the value of two is a constant value during optimization task.

Hence, using Eq. (13), the new pattern solution i^{th} ($\vec{X}_i(t+1)$) is transferred from its current position in the search space to its updated position ($\vec{X}_i^*(t+1)$). Table 2 describes the process and collaboration of both bias and TF operators in the NNA in detail.

As can be seen in Table 2, at early iterations, there exists more chances for the bias operator generating new pattern solutions (more opportunities for discovering unvisited pattern solutions) and also new weight values. However, when the iteration number is increasing, this chance decreases, and the TF operator plays more important roles in the NNA especially at final iterations.

2.2.5. *Steps of the NNA*

Taking the advantages of ANNs into account, the NNA is inspired by the structure and concept of ANNs. The whole procedures of the NNA are illustrated in Fig. 3 including all processes.

Looking at Fig. 3, "*W*", "Bias", and "TF" are the weight matrix, bias, and transfer function operators, respectively. Moreover, schematic view of the proposed NNA representing its functionality is shown in Fig. 4. By observing Fig. 4, the NNA has self-feedback (i.e., local feedback) in addition to having feedback to the other neurons (i.e., global feedback). By observing Fig. 4, local and global feedbacks are shown in dashed and solid lines, respectively. Furthermore, the weight matrix (see Eq. (10)) is modified at each iteration (i.e., the elements of the weight matrix change during the optimization process). In fact, the NNA is considered as a parallel sequential-batch learning method using global and local feedbacks.

Since the current values of design variables affect their next values (i.e., local and global feedbacks), the NNA is categorized as a dynamic optimization model. Hence, general behavior of the NNA can be described in the following equation:

$$\vec{X}_i(t+\Delta t) = f(\vec{X}_i(t), P(t)), \qquad i = 1, 2, 3, \ldots, N_{pop}, \tag{14}$$

Table 2
Combination of Bias and TF operators in the NNA.

```
For i = 1 to N_pop
  If rand ≤ β
  %% ---------------- Bias Operator ------------------------------------------
      Bias Operator (see subsection 2.2.3)
  Else (rand > β)
   %% ---------------- Transfer Function (TF) Operator --------------------
      Apply Eq. (13)
  End If
End For
```

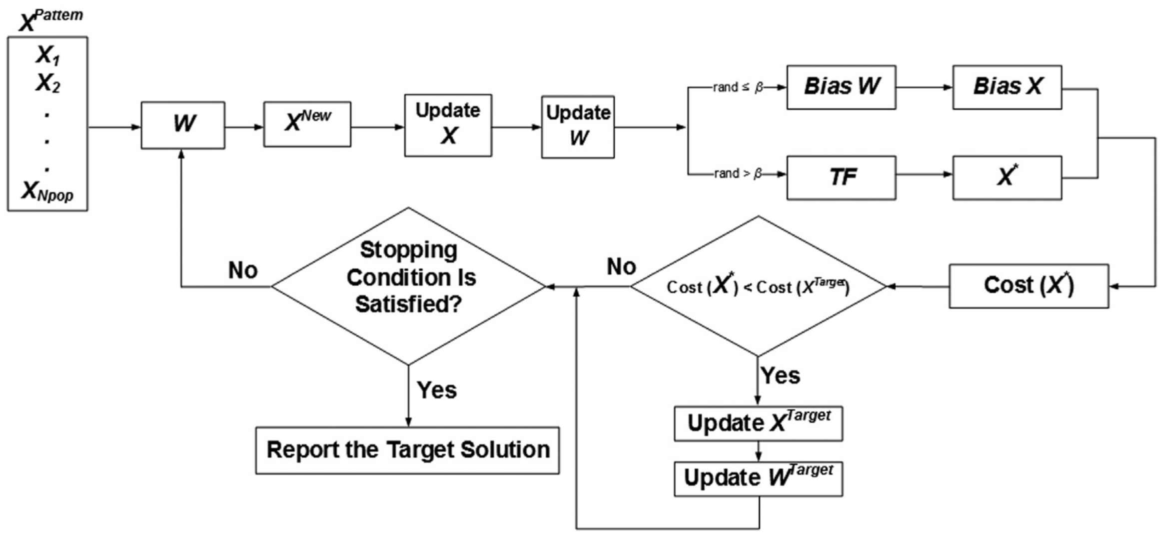

Fig. 3. Processes of the NNA.

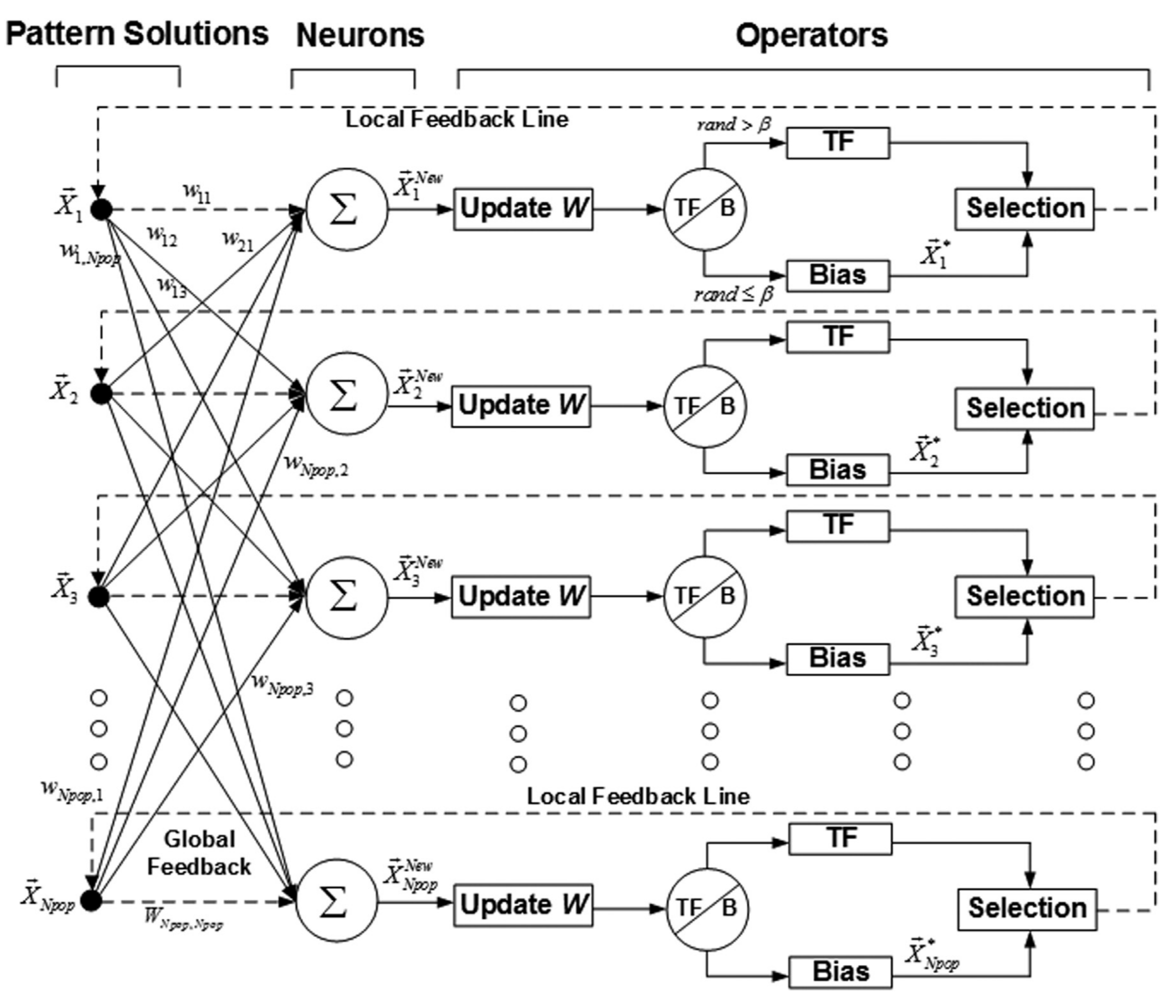

Fig. 4. Schematic view for the performance of the NNA.

where $X_i(t+\Delta t)$ and $X_i(t)$ are next and current locations of pattern solution i^{th}, respectively. $P(t)$ is population of pattern solutions with updated weights. Eq. (14) shows the general trend of NNA as a dynamic optimization model. It is worth pointing out that using the concepts of ANNs and its strategies used in the NNA, it is considered as associated memory based algorithm by using global and local feedbacks in its structure. Furthermore, the steps of the proposed method are summarized as follows:

Step 1: Choose the number of pattern solutions (i.e., population size) and maximum number of iterations (i.e., NFEs).

Step 2: Randomly generate an initial population of pattern solution between LB and UB.

Step 3: Calculate the cost of initial pattern solutions.

Step 4: Randomly generate the weight matrix (initialization phase) between zero and one considering the imposed constraint (see Eqs. (5) and (6)).

Step 5: Set target solution (X^{Target}) (the minimum value for minimization problems) and its corresponding target weight (W^{Target}).

Step 6: Generate new pattern solutions (X^{New}) and update the pattern solutions using Eqs. (7) and (8).

Step 7: Update the weight matrix (W) using Eq. (10) considering the applied constraints (see Eqs. (5) and (6)).

Step 8: Check the bias condition. If $rand \leq \beta$, performs the bias operator for both new pattern solutions and updated weight matrix (see Table 1).

Step 8: Otherwise ($rand > \beta$), apply the transfer function operator (TF) for updating new position of pattern solutions (X_i^*) using Eq. (13) (see Table 2).

Step 10: Calculate the objective function value for all updated pattern solutions.

Step 11: Update the target solution (i.e., temporal best solution) and its corresponding target weight.

Step 12: Update the value of β using any reduction formulation (e.g., Eq. (11))

Step 13: Check predefined stopping condition. If the stopping criterion is satisfied, the NNA stops. Otherwise, return to the Step 6.

Interestingly, the NNA can be easily applied for solving combinatorial optimization problem (i.e., discrete design variables). For this purpose, by choosing binary values of zero and one for the weight matrix (W), the NNA can be utilized to solve discrete optimization problems such as scheduling and assignment optimization problems.

Due to complex computation among neurons and input solutions in the NNA, improper selection of target solution and target weight (i.e., local optima) can be compensated with the complex performance of the neuron (inspired by the ANNs) during optimization process in the NNA (i.e., graceful degradation). In fact, this behavior underlies in the feature of ANNs model. This definition is extracted from the ANNs, and since the NNA is inspired by the ANNs structure and model, the same behavior can be expected. In fact, graceful degradation is the ability of a computer, electronic system or network to maintain limited functionality even when a large portion of it has been destroyed or rendered inoperative. In graceful degradation, the operating efficiency or speed declines gradually as an increasing number of components fail. Parallel structure and having local and global feedbacks through complex performance of the neurons give the NNA such a flexibility to ensure about the quality of the generated solutions even with the presence of low quality solutions.

Based on the ANNs terminology, the NNA is an adaptive unsupervised method for solving optimization problems. Unsupervised in NNA means there is no clue and information of global optimum and the solutions have been updated by learning from the environment.

The NNA is a single-layer perceptron optimization method having self-feedback. In the proposed method, there are no sensitive initial parameters (except the common user parameters: population size and maximum number of iteration or NFEs), therefore, a user does not require to fine tune or examine numerous set of initial parameters.

2.2.6. NNA vs. ANNs

The NNA is an optimization method for finding an optimum solution for a given problem. However, the ANNs is an information processing paradigm that is inspired by the biological nervous systems such as the brain. In fact, as an inspiring idea, the NNA has borrowed the concept and computational model of feed forward ANNs for optimization task. Furthermore, the following highlights are the differences between the NNA and ANNs in an itemized way:

- Feed forward ANNs is used for prediction, while the NNA (as a global optimizer) is utilized for finding optimum solution of an optimization problem.
- Unlike feed forward NNA, the NNA has local and global feedback.
- Transfer function used in the ANNs translates the input signals to output signals, while in the NNA, the transfer function operator is used for updating new positions of pattern solutions at each iteration.
- A bias in ANNs is an extra neuron added to each hidden layer that stores the value of one, unlike NNA which the bias operator is applied for having more chances for exploration phase in search space.
- Unlike ANNs, the NNA can be applied for optimal solving of constrained and multi-objective optimization problems.

3. Validation of the NNA

The proposed NNA was implemented and coded in MATLAB programming software. In order to have fair and significant comparison, the most applied optimizers from state-of-the-art approaches to modern and recent optimization methods in the literature have been taken into account. In this paper, thirteen optimization methods including the proposed method have been considered in the comparison pool.

3.1. Sensitivity analysis

In order to observe the effects of different values of common user parameters of the NNA (i.e., population size and maximum number of iteration), sensitivity analysis has been performed. Also, sensitivity analysis over user parameters of other reported optimizers has been given in this section. In this paper, the Taguchi method of design of experiment [28] is employed to evaluate the influence of selected values on the performance of the proposed optimizer.

The Taguchi method is a fractional factorial experiment introduced by Taguchi as an efficient alternative for full factorial experiments. The Taguchi method focuses on the level combinations of the control parameters to minimize the effects of the noise factors. In the Taguchi's parameter design phase, an experimental design is used to arrange the control and noise factors in the inner and outer orthogonal arrays, respectively. Afterward, the signal-to-noise (S/N) ratio is computed for each experimental combination. After the calculation of the S/N ratios, these ratios are analyzed to determine the optimal control factor for the level combination.

The Taguchi method categorizes an objective function into three groups: First, "smaller is better" for which the objective function is minimization. Second, "nominal is the best", for which the objective function has the modest variance around its target. Third, "bigger is better", where the objective function is maximization. Since, the objective functions used in this paper are minimization problems, the "smaller is better" option is appropriate, given as follows [28]:

$$S/N = -10\log_{10}\left(\frac{1}{n}\sum_{e=1}^{n} C_e^2\right), \tag{15}$$

where C_e is the objective function value of a given experiment e, and n is the number of times the experiment is performed. Minitab software has been utilized for this purpose. Two common user parameters of NNA that are required for calibrations are the population size (N_{pop}) and maximum number of iteration. Table 3 shows

Table 3
Considered parameters and their levels for fine tuning initial parameters of used optimizer.

Parameters	Factor Level		
NNA	1	2	3
N_{pop}	20	50	100
Max_Iteration	200	500	1000
GA	**1**	**2**	**3**
N_{pop}	20	50	100
P_c	0.7	0.8	0.9
P_m	0.05	0.1	0.3
Max_Iteration	200	500	1000
HS	**1**	**2**	**3**
HMS (N_{pop})	20	50	100
HMCR	0.8	0.85	0.95
PAR	0.1	0.3	0.5
Max_Iteration	200	500	1000
PSO	**1**	**2**	**3**
N_{pop}	20	50	100
C_1,C_2	1	1.5	2
w	0.7	0.8	0.9
Max_Iteration	200	500	1000
GSA	**1**	**2**	**3**
N_{pop}	20	50	100
G0	50	100	200
α	10	20	30
Max_Iteration	200	500	1000
SA	**1**	**2**	**3**
N_{pop}	20	50	100
Initial Temperature (T0)	50	100	200
Final Temperature (TF)	0.001	0.01	0.1
Max_Iteration	200	500	1000
TLBO	**1**	**2**	**3**
N_{pop}	20	50	100
Max_Iteration	200	500	1000
WCA	**1**	**2**	**3**
N_{pop}	20	50	100
N_{sr}	4	6	8
Max_Iteration	200	500	1000
DE	**1**	**2**	**3**
N_{pop}	20	50	100
Cross-over Rate	0.5	0.8	0.9
F	0.3	0.5	0.7
Max_Iteration	200	500	1000
ICA	**1**	**2**	**3**
N_{pop}	20	50	100
N_Empire	3	5	8
Revolution Rate	0.2	0.4	0.6
Max_Iteration	200	500	1000
CS	**1**	**2**	**3**
N_{pop}	20	50	100
Discovery Rate	0.1	0.25	0.5
Max_Iteration	200	500	1000

the algorithms' parameters, each at three levels with nine observations represented by L_9.

Benchmark *F1* (given in Section 3.2) has been examined using the given levels for all reported optimizers under 30 independent runs. Fig. 5 depicts the average *S*/*N* ratio plot for different parameter levels for all applied algorithms. By observing Fig. 5, the best parameter levels have the highest mean of *S*/*N* values. Therefore, based on the results seen from Fig. 5, Table 4 tabulates the optimal values of the initial parameters for all considered optimizers.

As it can be seen from Fig. 5 and Table 4, all optimizers obtained better solutions when the maximum number of iteration is large and population size is increasing. In fact, algorithms having more individuals (population size) and time (in terms of function evaluations) explore search space efficiently which causes finding better solutions. Talking about the NNA, the same trend can be seen for the population size and the maximum number of iteration.

Table 4
Optimal values of user parameters used in the reported optimizers.

Methods	Parameters	Optimal Values
GA	N_{pop}	100
	P_c	0.8
	P_m	0.3
HS	*HMS* (N_{pop})	50
	HMCR	0.95
	PAR	0.3
WCA	N_{pop}	100
	N_{sr}	8
DE	N_{pop}	100
	Cross-over Rate	0.8
	F	0.3
ICA	N_{pop}	100
	N_Empire	3
	Revolution Rate	0.6
GSA	N_{pop}	100
	G0	100
	α	10
PSO	N_{pop}	100
	C_1,C_2	2
	w	0.9
TLBO	N_{pop}	100
CS	N_{pop}	20
	Discovery Rate	0.25
SA	N_{pop}	100
	Initial Temperature (T0)	100
	Final Temperature (TF)	0.001
NNA	N_{pop}	100

3.2. NNA for solving unconstrained benchmark problems

In this research, 21 well-known benchmark minimization problems extracted from a special issue in the literature [29] have been investigated to demonstrate the efficiency and robustness of the proposed algorithm. The obtained solutions are compared with the results obtained from other optimization methods in terms of statistical results and statistical tests which are shown in tables and figures.

The dimensions of benchmark functions were 50–200. The optimal solution results, $f(x^*)$, were known for all benchmark functions. Properties of these functions are represented in Tables 5–7. Required explanations regarding hybrid composition functions (F_{12}-F_{21}) have been fully described in the literature [29]. It is worth pointing out that, in terms of complexity, functions F_{12} to F_{21} possess higher level of complication compared with their original shifted functions (F_1 to F_{11}).

Talking about maximum number of function evaluations (NFEs), considered as stopping condition in this paper, the predefined NFEs is 5000 multiples by dimension size (*D*) for each function [29]. The task of optimizing for each algorithm was executed in 30 independent runs.

Therefore, in addition to NNA, the applied algorithms, from the state-of-the-art algorithms to modern optimizers, include the GA [1], SA [2], PSO [3], HS [4], Imperialist Competitive Algorithm (ICA) [30], Gravitational Search Algorithm (GSA) [31], Water Cycle Algorithm (WCA) [32], Cuckoo Search [33], Teaching–Learning-Based Optimization (TLBO) [34], Differential Evolution [35], and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [36]. All the optimizers in this paper have been coded and implemented in MATLAB programming software. For having fair comparison, for all applied algorithm, population size was set to 50.

Regarding the initial parameters of reported optimizers, suggested user parameters given in Section 3.1 have been utilized for

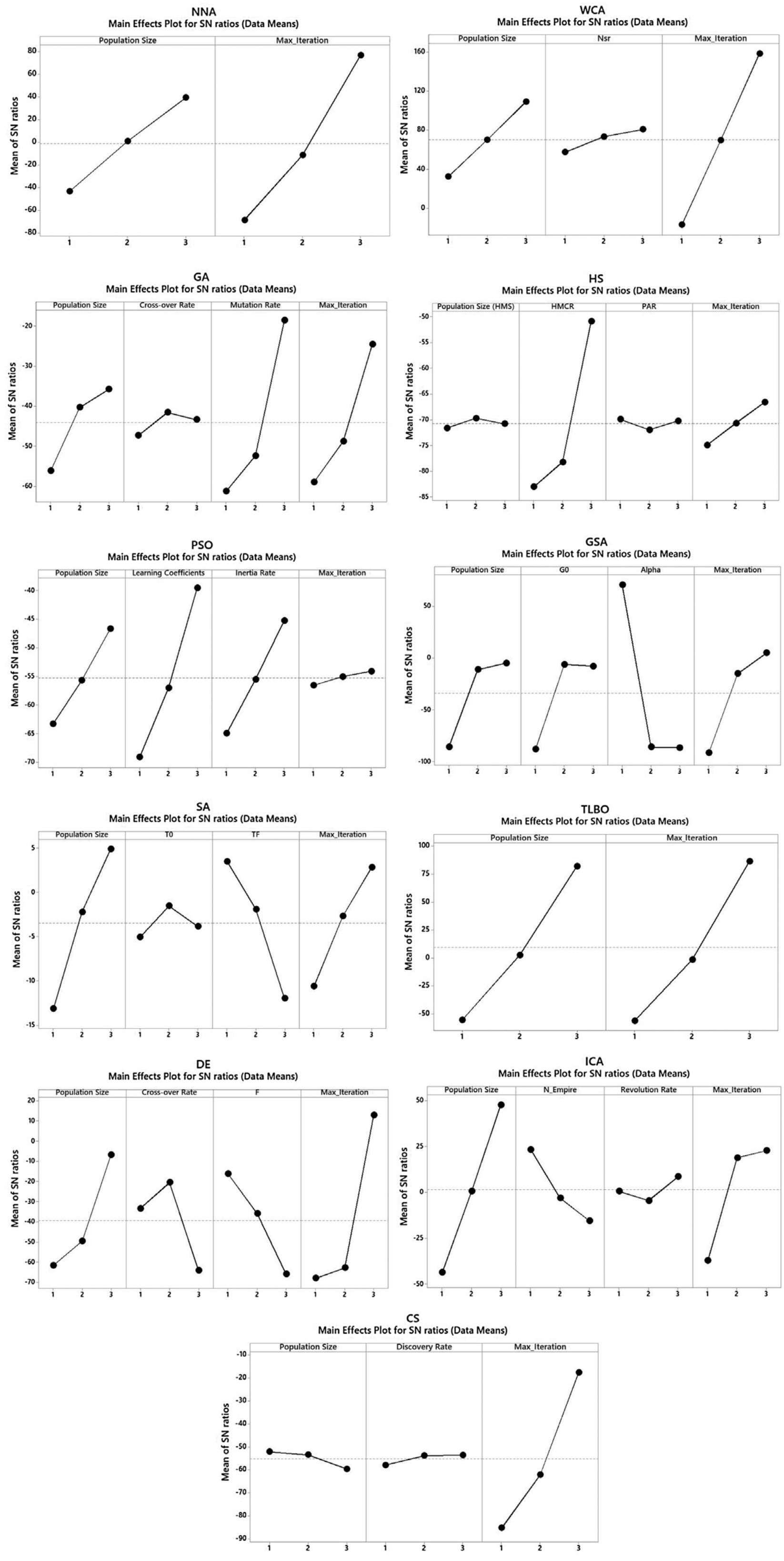

Fig. 5. Mean S/N ratio plot for each level of the factors for the considered optimizers (Horizontal axis stands for different levels for each parameter).

optimization task. To appropriately evaluate the performance of the proposed NNA compared with the other state-of-art algorithms, four quality indicators are utilized. The first one is the value-based method which is the solution quality in terms of four performance metrics including the best, average, median, worst, and standard deviation (SD).

The second metric is the rank-based method, which have been suggested by different authors in the literature. In this paper, the

Table 5
Benchmark functions F_1 to F_{11}.

Function	Name	Definition
F_1	Shifted Hyper Sphere	$\sum_{i=1}^{D} z_i^2 + f\_bias,\ \ z = x - o$ [a]
F_2	Shifted Schwefel 2.21	$\max_i \left\{ \lvert z_i \rvert, 1 \le i \le D \right\} + f\_bias,\ \ z = x - o$
F_3	Shifted Rosenbrock	$\sum_{i=1}^{D} \left(100(z_i^2 + z_{i+1})^2 + (z_i - 1)^2 \right) + f\_bias,\ \ z = x - o$
F_4	Shifted Rastrigin	$\sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10) + f\_bias,\ \ z = x - o$
F_5	Shifted Griewank	$\sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos\left(\frac{z_i}{\sqrt{i}}\right) + 1 + f\_bias,\ \ z = x - o$
F_6	Shifted Ackley	$-20\exp(-0.2\sqrt{\frac{\sum_{i=1}^{D} z_i^2}{D}}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i)) + 20 + e + f\_bias,\ \ z = x - o$
F_7	Schwefel 2.22	$\sum_{i=1}^{D} \lvert z_i \rvert + \prod_{i=1}^{D} \lvert z_i \rvert$
F_8	Schwefel 1.2	$\sum_{i=1}^{D} \left(\sum_{j=1}^{i} z_j \right)^2$
F_9	Extended f_{10}	$\left(\sum_{i=1}^{D-1} f_{10}(z_i, z_{i+1}) \right) + f_{10}(z_m, z_1),\ \ f_{10} = \left(x^2 + y^2\right)^{0.25} \left(\sin^2\left(50\left(x^2 + y^2\right)^{0.1}\right) + 1 \right)$
F_{10}	Bohachevsky	$\sum_{i=1}^{D-1} \left(z_i^2 + 2z_{i+1}^2 - 0.3\cos(3\pi z_i) - 0.4cos(4\pi z_{i+1}) + 0.7 \right)$
F_{11}	Schaffer	$\sum_{i=1}^{D-1} \left(z_i^2 + z_{i+1}^2\right)^{0.25} \left(\sin^2\left(50\left(z_i^2 + z_{i+1}^2\right)^{0.1}\right) + 1 \right)$

[a] Shifted point.

Table 6
Hybrid composition benchmark functions F_{12} to F_{21}.

Function	First Function		Second Function	Weight Factor
F_{12}	F_9	+	F_1	0.25
F_{13}	F_9	+	F_3	0.25
F_{14}	F_9	+	F_4	0.25
F_{15}	F_{10}	+	F_7	0.25
F_{16}	F_5	+	F_1	0.50
F_{17}	F_3	+	F_4	0.50
F_{18}	F_9	+	F_1	0.75
F_{19}	F_9	+	F_3	0.75
F_{20}	F_9	+	F_4	0.75
F_{21}	F_{10}	+	F_7	0.75

Table 7
Properties of F_1 to F_{21}. "U", "M", and "N/A" stand for unimodal and multimodal, respectively.

Function	Range	Optimum ($f(x^*)$)	U/M	Separable	Shifted	f_bias
F_1	$[-100,100]^D$	0	U	Yes	Yes	−450
F_2	$[-100,100]^D$	0	U	No	Yes	−450
F_3	$[-100,100]^D$	0	M	Yes	Yes	390
F_4	$[-5,5]^D$	0	M	Yes	Yes	−330
F_5	$[-600,600]^D$	0	M	No	Yes	−180
F_6	$[-32,32]^D$	0	M	Yes	Yes	−140
F_7	$[-10,10]^D$	0	U	Yes	No	–
F_8	$[-65.536,65.536]^D$	0	U	No	No	–
F_9	$[-100,100]^D$	0	U	No	No	–
F_{10}	$[-15,15]^D$	0	U	Yes	No	–
F_{11}	$[-100,100]^D$	0	U	Yes	No	–
F_{12}	$[-100,100]^D$	0	U	No	Yes	–
F_{13}	$[-100,100]^D$	0	M	No	Yes	–
F_{14}	$[-5,5]^D$	0	M	No	Yes	–
F_{15}	$[-10,10]^D$	0	U	Yes	No	–
F_{16}	$[-100,100]^D$	0	M	No	Yes	–
F_{17}	$[-10,10]^D$	0	M	Yes	Yes	–
F_{18}	$[-100,100]^D$	0	U	No	Yes	–
F_{19}	$[-100,100]^D$	0	M	No	Yes	–
F_{20}	$[-5,5]^D$	0	M	No	Yes	–
F_{21}	$[-10,10]^D$	0	U	Yes	No	–

Friedman test [37] which is a nonparametric statistical test is used to distinguish the differences among reported algorithms. Therefore, the average rankings of the algorithms according to the Friedman test are reported.

The third and fourth metrics are Kruskal-Wallis test [38] and Multiple Comparison Test [39]. Tables 8–13 tabulate the obtained optimization results using different optimizers for the benchmarks given in Tables 5 and 6 with dimensions of 50–200. Looking at Tables 8–13, performance of well-used optimizers such as GA, PSO, HS has not surpassed the results of recent optimizers for the most reported functions. Therefore, the competition is mostly among recent developed optimization methods.

Although, reporting the statistical results gives us a good sense of how an algorithm performs, the optimization results given in Tables 8–13 do not show the significance of an algorithm over the other. Therefore, in the following section, in addition to the optimizers given in Tables 8–13, four recent optimizers, i.e., Cuckoo Search (CS), Teaching-Learning-Based Optimization (TLBO), Differential

Table 8
Experimental optimization results for dimension 50 using reported optimizers for shifted function (F_1-F_{11}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 50	$\boldsymbol{F_1}$				$\boldsymbol{F_2}$				$\boldsymbol{F_3}$			
GA	1.71E-03	4.32E-03	9.43E-03	4.42E-03	7.84E-01	9.91E-01	1.30E+00	2.72E-01	5.17E+01	1.10E+02	3.75E+02	6.18E+01
PSO	1.00E+03	1.14E+03	1.29E+03	1.41E+02	2.19E+01	2.38E+01	2.52E+01	1.74E+00	5.97E+06	1.41E+07	2.47E+07	9.58E+06
SA	2.02E-01	2.45E-01	2.70E-01	3.76E-02	8.58E-01	1.01E+00	1.22E+00	1.91E-01	7.14E+01	9.20E+01	3.88E+02	2.40E+01
HS	2.49E-04	2.62E-04	2.72E-04	1.21E-05	1.73E+00	1.95E+00	2.19E+00	2.32E-01	1.17E+02	1.44E+02	3.82E+02	3.37E+01
ICA	3.13E-02	5.96E-02	1.14E-01	4.72E-02	4.64E+01	6.07E+01	8.00E+01	1.74E+01	4.95E+02	9.77E+02	1.60E+03	5.69E+02
GSA	5.68E-14	5.68E-14	5.68E-14	0.00E+00	3.43E-09	3.86E-09	4.55E-09	6.03E-10	4.07E+01	1.11E+01	4.14E+02	4.83E-01
WCA	1.71E-13	3.03E-13	5.12E-13	8.22E-14	1.24E+00	6.27E+00	2.44E+01	5.54E+00	4.36E+00	9.80E+01	4.22E+02	1.03E+02
NNA	4.95E-12	2.25E-10	1.39E-09	3.45E-10	4.52E-01	1.03E+00	1.65E+00	3.10E-01	5.39E-01	9.60E+01	3.35E+02	7.12E+01
D = 50	$\boldsymbol{F_4}$				$\boldsymbol{F_5}$				$\boldsymbol{F_6}$			
GA	1.46E-03	4.25E-03	7.88E-03	3.29E-03	1.76E-03	2.10E-03	2.75E-03	5.62E-04	7.92E-03	8.89E-03	1.06E-02	1.47E-03
PSO	9.39E+01	1.43E+02	1.81E+02	4.44E+01	1.09E+01	1.31E+01	1.52E+01	2.18E+00	1.11E+01	1.17E+01	1.28E+01	9.34E-01
SA	1.82E-01	2.01E-01	2.20E-01	1.88E-02	1.20E+00	1.21E+00	1.22E+00	1.01E-02	2.98E-01	3.41E-01	4.12E-01	6.22E-02
HS	3.88E-02	4.68E-02	5.20E-02	7.00E-03	9.87E-03	3.08E-02	5.29E-02	2.15E-02	8.27E-03	8.64E-03	9.16E-03	4.60E-04
ICA	2.50E+02	3.13E+02	3.92E+02	7.28E+01	4.47E-02	7.51E-02	1.26E-01	4.45E-02	1.82E+01	1.84E+01	1.86E+01	2.27E-01
GSA	2.29E+01	2.64E+01	2.98E+01	4.92E+00	3.69E-02	7.24E-02	1.08E-01	5.03E-02	3.02E-09	3.38E-09	3.73E-09	5.00E-10
WCA	1.01E+02	1.91E+02	3.64E+02	5.36E+01	2.56E-13	3.01E-02	5.24E-01	9.47E-02	9.05E-09	1.11E+01	1.88E+01	6.44E+00
NNA	1.99E+00	6.60E+00	1.19E+01	2.33E+00	6.76E-12	6.20E-02	1.61E-01	5.30E-02	1.21E-07	1.12E-06	5.27E-06	1.38E-06
D = 50	$\boldsymbol{F_7}$				$\boldsymbol{F_8}$				$\boldsymbol{F_9}$			
GA	1.19E-02	1.59E-02	2.19E-02	5.31E-03	2.15E-02	2.60E-02	3.05E-02	6.40E-03	9.25E+00	9.63E+00	3.01E+01	4.20E-01
PSO	2.58E+01	2.90E+01	3.32E+01	3.77E+00	1.20E+04	1.40E+04	1.60E+04	2.86E+03	2.48E+02	2.58E+02	2.70E+02	1.12E+01
SA	5.24E-01	5.43E-01	5.72E-01	2.59E-02	3.28E-01	3.69E-01	4.09E-01	5.72E-02	8.11E+00	8.52E+00	8.89E+00	3.89E-01
HS	5.50E-02	5.72E-02	5.98E-02	2.42E-03	4.47E-03	4.78E-03	5.09E-03	4.43E-04	2.77E+01	2.86E+01	2.92E+01	8.08E-01
ICA	8.07E+01	1.51E+02	1.87E+02	6.05E+01	1.55E+00	6.19E+03	1.24E+04	8.75E+03	3.70E+02	3.94E+02	4.20E+02	2.49E+01
GSA	3.15E-08	3.39E-08	3.62E-08	3.34E-09	2.42E-16	2.89E-16	3.36E-16	6.65E-17	4.67E+01	7.08E+01	9.48E+01	3.40E+01
WCA	1.39E-06	3.33E-01	1.00E+01	1.83E+00	3.58E-19	7.90E-17	1.01E-16	2.21E-17	2.08E+02	2.45E+02	3.29E+02	2.61E+01
NNA	1.78E-12	3.99E-11	1.44E-10	3.71E-11	6.80E-23	5.18E-20	3.08E-19	9.07E-20	1.07E-01	3.21E+00	1.10E+01	3.33E+00
D = 50	$\boldsymbol{F_{10}}$				$\boldsymbol{F_{11}}$							
GA	1.33E-03	2.75E-03	5.39E-03	2.29E-03	1.00E+01	1.06E+01	2.35E+01	7.86E-01				
PSO	1.22E+02	1.78E+02	2.33E+02	5.55E+01	2.16E+02	2.35E+02	2.65E+02	2.62E+01				
SA	3.41E-01	3.60E-01	3.73E-01	1.71E-02	7.63E+00	8.42E+00	9.21E+00	7.91E-01				
HS	9.80E-03	1.03E-02	1.12E-02	7.88E-04	2.41E+01	2.61E+01	2.90E+01	2.53E+00				
ICA	1.46E+01	1.53E+01	1.67E+01	1.18E+00	3.74E+02	3.91E+02	4.00E+02	1.42E+01				
GSA	0.00E+00	1.11E-16	6.27E-16	1.57E-16	5.90E+01	8.03E+01	1.02E+02	3.02E+01				
WCA	7.77E-16	5.64E-01	2.57E+00	7.29E-01	1.85E+02	2.49E+02	2.89E+02	2.17E+01				
NNA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.47E-02	5.80E+00	2.04E+01	5.54E+00				

Table 9
Experimental optimization results for dimension 50 using reported optimizers (F_{12}–F_{21}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 50	F_{12}				F_{13}				F_{14}			
GA	1.69E+00	2.44E+00	3.10E+00	7.14E-01	9.71E+01	1.35E+02	1.77E+02	3.99E+01	2.19E-01	5.64E-01	1.15E+00	5.08E-01
PSO	2.17E+02	2.55E+02	2.78E+02	3.29E+01	4.22E+05	5.16E+05	6.15E+05	9.65E+04	7.81E+01	8.03E+01	8.24E+01	2.14E+00
SA	2.05E+00	2.43E+00	2.68E+00	3.34E-01	3.57E+01	1.03E+02	1.61E+02	6.29E+01	7.99E-01	9.22E-01	1.01E+00	1.09E-01
HS	4.89E+00	6.22E+00	7.17E+00	1.19E+00	2.25E+02	6.59E+02	1.32E+03	5.86E+02	3.31E-01	1.25E+00	1.73E+00	8.00E-01
ICA	1.29E+02	1.34E+02	1.39E+02	5.22E+00	2.97E+02	2.55E+03	5.99E+03	3.03E+03	2.29E+02	2.47E+02	2.73E+02	2.29E+01
GSA	1.36E+01	2.08E+01	2.79E+01	1.01E+01	6.53E+01	6.80E+01	7.07E+01	3.84E+00	2.18E+01	2.28E+01	2.39E+01	1.54E+00
WCA	6.81E+01	1.03E+02	1.37E+02	1.66E+01	1.13E+02	2.27E+02	5.68E+02	1.38E+02	8.85E+01	1.44E+02	2.32E+02	3.67E+01
NNA	1.75E-01	1.64E+01	5.29E+01	1.23E+01	9.70E+00	1.46E+02	9.42E+02	2.49E+02	1.81E+00	6.83E+00	1.15E+01	2.57E+00
D = 50	F_{15}				F_{16}				F_{17}			
GA	8.08E-02	1.21E-01	1.45E-01	3.50E-02	5.15E-04	8.62E-04	1.16E-03	3.25E-04	1.94E-02	1.69E-01	4.57E-01	2.50E-01
PSO	5.95E+00	9.16E+00	1.24E+01	3.24E+00	7.73E+00	1.03E+01	1.29E+01	2.59E+00	5.13E+01	5.96E+01	6.43E+01	7.22E+00
SA	5.84E-01	6.01E-01	6.14E-01	1.52E-02	7.86E-02	1.17E-01	1.56E-01	3.88E-02	4.78E+00	1.32E+01	1.75E+01	7.33E+00
HS	2.24E-01	2.41E-01	2.73E-01	2.78E-02	1.08E-05	1.44E-05	2.03E-05	5.12E-06	2.02E+00	2.41E+00	3.02E+00	5.33E-01
ICA	1.86E+02	2.42E+02	2.95E+02	5.48E+01	4.81E-01	1.97E+00	3.50E+00	1.51E+00	2.99E+02	3.42E+02	3.77E+02	3.96E+01
GSA	3.44E-12	3.97E-12	4.51E-12	7.57E-13	5.18E-55	8.15E-51	1.63E-50	1.15E-50	8.95E+00	1.04E+01	1.19E+01	2.11E+00
WCA	1.32E-12	4.00E+00	6.00E+01	1.25E+01	0.00E+00	9.15E-01	1.93E+00	8.61E-01	1.69E+01	3.91E+01	1.33E+02	2.12E+01
NNA	1.48E-12	3.48E-11	2.75E-10	6.33E-11	4.56E-18	7.35E-11	2.01E-09	3.66E-10	4.63E-04	1.39E+00	9.60E+00	1.79E+00
D = 50	F_{18}				F_{19}				F_{20}			
GA	4.72E+00	4.97E+00	5.17E+00	2.29E-01	6.40E+00	9.49E+00	1.40E+01	4.02E+00	7.02E-01	1.77E+00	2.42E+00	9.33E-01
PSO	9.91E+01	1.12E+02	1.25E+02	1.31E+01	1.81E+02	1.88E+02	2.00E+02	1.05E+01	2.85E+01	3.03E+01	3.26E+01	2.10E+00
SA	5.91E+00	6.17E+00	6.54E+00	3.30E-01	6.37E+00	2.05E+01	2.81E+01	1.22E+01	1.88E+00	2.09E+00	2.39E+00	2.69E-01
HS	1.22E+01	1.28E+01	1.32E+01	5.65E-01	1.93E+01	3.55E+01	6.72E+01	2.74E+01	4.93E+00	7.11E+00	8.23E+00	1.89E+00
ICA	2.37E+02	2.45E+02	2.54E+02	8.54E+00	3.19E+02	3.33E+02	3.48E+02	1.46E+01	5.90E+01	6.45E+01	7.20E+01	6.74E+00
GSA	1.85E-07	1.85E-07	1.86E-07	9.15E-10	3.02E+01	3.43E+01	3.84E+01	5.83E+00	3.23E-05	4.83E-05	6.44E-05	2.27E-05
WCA	1.31E+02	2.14E+02	3.12E+02	3.90E+01	2.03E+02	3.08E+02	3.78E+02	3.89E+01	3.25E+01	4.68E+01	6.39E+01	8.29E+00
NNA	1.90E+00	1.14E+01	3.39E+01	7.23E+00	1.08E-01	4.21E+00	1.38E+01	3.78E+00	1.68E-03	2.62E+00	5.59E+00	1.53E+00
D = 50	F_{21}											
GA	1.11E-03	1.43E-03	1.67E-03	2.86E-04								
PSO	2.39E+01	2.73E+01	3.26E+01	4.63E+00								
SA	1.46E-01	1.80E-01	2.25E-01	4.03E-02								
HS	2.36E-03	3.43E-03	4.49E-03	1.07E-03								
ICA	1.15E+01	1.22E+01	1.31E+01	8.09E-01								
GSA	5.77E-36	8.95E-35	1.73E-34	1.18E-34								
WCA	4.32E-13	9.23E-01	4.61E+00	1.07E+00								
NNA	4.39E-72	8.85E-38	2.66E-36	4.85E-37								

Table 10
Experimental optimization results for dimension 100 using reported optimizers for shifted function (F_1-F_{11}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 100	$\boldsymbol{F_1}$				$\boldsymbol{F_2}$				$\boldsymbol{F_3}$			
GA	4.46E-03	5.93E-03	7.80E-03	1.71E-03	2.61E+00	3.38E+00	4.05E+00	7.24E-01	2.44E+02	2.49E+02	2.59E+02	8.57E+00
PSO	6.30E+03	8.13E+03	1.11E+04	2.63E+03	3.11E+01	3.43E+01	3.66E+01	2.86E+00	2.59E+08	3.81E+08	5.13E+08	1.27E+08
SA	4.37E-01	5.27E-01	6.47E-01	1.08E-01	2.29E+00	2.37E+00	2.45E+00	7.79E-02	8.65E+01	1.84E+02	2.41E+02	8.51E+01
HS	1.38E-03	1.44E-03	1.47E-03	5.51E-05	4.54E+00	4.78E+00	5.03E+00	2.47E-01	3.03E+02	4.21E+02	5.17E+02	1.09E+02
ICA	8.82E+01	9.76E+01	1.03E+02	8.14E+00	8.00E+01	8.00E+01	8.00E+01	0.00E+00	5.10E+05	8.51E+05	1.44E+06	5.14E+05
GSA	1.14E-13	1.33E-13	1.71E-13	3.28E-14	7.45E+00	7.96E+00	8.78E+00	7.15E-01	8.75E+01	8.75E+01	8.76E+01	7.89E-02
WCA	4.55E-13	6.75E-13	1.02E-12	1.25E-13	1.78E+01	2.94E+01	4.88E+01	6.57E+00	7.82E+01	3.33E+02	4.38E+03	7.72E+02
NNA	9.51E-11	1.40E-09	1.57E-08	2.87E-09	5.23E+00	7.42E+00	1.07E+01	1.40E+00	1.03E+02	2.29E+02	5.59E+02	1.08E+02
D = 100	$\boldsymbol{F_4}$				$\boldsymbol{F_5}$				$\boldsymbol{F_6}$			
GA	4.84E-03	5.80E-03	7.10E-03	1.17E-03	3.41E-03	7.72E-03	1.18E-02	4.21E-03	8.83E-03	9.35E-03	9.98E-03	5.85E-04
PSO	3.77E+02	4.31E+02	5.00E+02	6.27E+01	7.96E+01	8.94E+01	9.64E+01	8.79E+00	1.36E+01	1.39E+01	1.43E+01	3.81E-01
SA	4.88E-01	5.09E-01	5.29E-01	2.04E-02	1.41E+00	1.44E+00	1.47E+00	3.10E-02	6.17E-01	7.13E-01	8.35E-01	1.11E-01
HS	2.38E-01	2.50E-01	2.63E-01	1.25E-02	8.65E-02	9.92E-02	1.22E-01	2.01E-02	1.48E-02	1.51E-02	1.53E-02	2.80E-04
ICA	7.52E+02	7.70E+02	8.03E+02	2.82E+01	1.47E+00	1.68E+00	1.99E+00	2.72E-01	1.88E+01	1.92E+01	1.95E+01	4.12E-01
GSA	6.37E+01	7.01E+01	7.66E+01	9.15E+00	5.68E-14	5.73E-01	1.15E+00	8.10E-01	3.7E-09	4.02E-09	4.34E-09	4.55E-10
WCA	3.01E+02	4.41E+02	5.97E+02	6.53E+01	5.40E-13	7.97E-02	8.71E-01	2.04E-01	6.07E+00	1.74E+01	1.88E+01	2.25E+00
NNA	2.98E+01	4.58E+01	6.77E+01	8.90E+00	7.56E-11	6.02E-02	3.60E-01	8.34E-02	3.98E-07	3.92E-06	2.25E-05	4.86E-06
D = 100	$\boldsymbol{F_7}$				$\boldsymbol{F_8}$				$\boldsymbol{F_9}$			
GA	2.15E-02	2.26E-02	2.41E-02	1.33E-03	9.38E-02	1.44E-01	1.94E-01	7.10E-02	1.73E+01	2.11E+01	2.38E+01	3.38E+00
PSO	8.17E+01	9.95E+01	1.22E+02	2.09E+01	1.35E+05	1.71E+05	2.07E+05	5.15E+04	5.60E+02	6.03E+02	6.27E+02	3.74E+01
SA	9.40E-01	1.04E+00	1.20E+00	1.41E-01	9.12E-01	9.30E-01	9.49E-01	2.61E-02	1.69E+01	1.72E+01	1.77E+01	4.41E-01
HS	2.21E-01	2.30E-01	2.42E-01	1.07E-02	6.40E-02	6.60E-02	6.79E-02	2.69E-03	6.89E+01	7.02E+01	7.28E+01	2.22E+00
ICA	4.13E+02	4.23E+02	4.29E+02	8.79E+00	3.08E+04	5.01E+04	6.94E+04	2.73E+04	8.47E+02	8.79E+02	9.10E+02	3.15E+01
GSA	7.43E-08	7.81E-08	8.18E-08	5.32E-09	2.51E-15	2.64E-15	2.77E-15	1.84E-16	2.86E+02	3.18E+02	3.51E+02	4.58E+01
WCA	1.10E-09	6.67E-01	1.00E+01	2.54E+00	1.56E-18	7.33E-17	1.25E-16	2.28E-20	4.51E+02	5.04E+02	5.74E+02	3.49E+01
NNA	2.57E-11	2.65E-10	1.18E-09	2.65E-10	1.45E-20	4.52E-19	4.47E-18	9.27E-19	1.39E+01	4.23E+01	1.11E+02	2.10E+01
D = 100	$\boldsymbol{F_{10}}$				$\boldsymbol{F_{11}}$							
GA	6.03E-03	9.47E-03	1.22E-02	3.16E-03	1.99E+01	2.12E+01	2.22E+01	1.21E+00				
PSO	5.61E+02	6.82E+02	7.70E+02	1.09E+02	5.86E+02	6.06E+02	6.26E+02	2.01E+01				
SA	5.20E-01	5.75E-01	6.12E-01	4.89E-02	1.73E+01	1.85E+01	1.94E+01	1.05E+00				
HS	5.76E-02	5.93E-02	6.17E-02	2.14E-03	6.37E+01	6.78E+01	7.24E+01	4.38E+00				
ICA	7.27E+01	1.10E+02	1.62E+02	4.64E+01	8.30E+02	8.65E+02	8.93E+02	3.21E+01				
GSA	1.89E-15	2.44E-15	3.00E-15	7.85E-16	3.28E+02	3.39E+02	3.50E+02	1.55E+01				
WCA	4.05E-07	3.43E+00	9.45E+00	2.36E+00	4.46E+02	4.88E+02	5.46E+02	2.80E+01				
NNA	0.00E+00	2.59E-17	7.77E-16	1.42E-16	1.14E+01	4.82E+01	1.69E+02	3.72E+01				

Table 11
Experimental optimization results for dimension 100 using reported optimizers (F_{12}–F_{21}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 100	$\boldsymbol{F_{12}}$				$\boldsymbol{F_{13}}$				$\boldsymbol{F_{14}}$			
GA	3.28E+00	8.34E+00	1.67E+01	7.30E+00	1.65E+02	2.09E+02	2.81E+02	6.26E+01	4.64E-01	1.01E+00	1.65E+00	5.96E-01
PSO	9.49E+02	1.34E+03	1.57E+03	3.43E+02	3.40E+06	9.78E+06	1.63E+07	6.46E+06	2.31E+02	2.43E+02	2.65E+02	1.94E+01
SA	3.50E+00	4.25E+00	5.06E+00	7.79E-01	2.43E+02	2.63E+02	2.79E+02	1.83E+01	1.58E+00	1.76E+00	1.95E+00	1.87E-01
HS	1.19E+01	1.50E+01	1.67E+01	2.74E+00	1.27E+02	2.64E+02	3.66E+02	1.23E+02	3.19E+00	4.27E+00	4.91E+00	9.35E-01
ICA	3.04E+02	3.11E+02	3.24E+02	1.17E+01	6.96E+03	3.96E+04	7.79E+04	3.58E+04	5.23E+02	5.84E+02	6.47E+02	6.20E+01
GSA	1.18E+01	1.64E+01	2.11E+01	6.60E+00	1.70E+02	1.91E+02	2.12E+02	2.98E+01	5.50E+01	5.52E+01	5.53E+01	1.74E-01
WCA	1.69E+02	2.10E+02	2.39E+02	1.60E+01	2.69E+02	1.03E+03	1.33E+04	2.60E+03	2.76E+02	3.35E+02	4.42E+02	3.96E+01
NNA	5.60E+00	5.79E+01	1.01E+02	2.54E+01	8.44E+01	3.36E+02	1.14E+03	2.77E+02	2.36E+01	3.51E+01	4.76E+01	6.16E+00
D = 100	$\boldsymbol{F_{15}}$				$\boldsymbol{F_{16}}$				$\boldsymbol{F_{17}}$			
GA	2.09E-01	3.78E-01	5.68E-01	1.80E-01	1.17E-03	1.57E-03	2.16E-03	5.20E-04	3.07E-02	1.81E-01	4.00E-01	1.94E-01
PSO	1.09E+02	1.43E+02	2.10E+02	5.74E+01	8.03E+01	1.55E+02	2.81E+02	1.09E+02	1.88E+02	2.17E+02	2.42E+02	2.75E+01
SA	1.50E+00	1.80E+00	2.17E+00	3.41E-01	2.05E-01	2.81E-01	3.26E-01	6.64E-02	4.50E-01	7.69E+00	1.77E+01	8.93E+00
HS	3.64E+00	3.73E+00	3.85E+00	1.08E-01	1.50E-04	1.65E-04	1.91E-04	2.27E-05	1.84E-01	3.23E-01	4.18E-01	1.23E-01
ICA	5.29E+03	5.58E+03	5.82E+03	2.70E+02	3.27E-02	1.36E+00	3.96E+00	2.25E+00	7.72E+02	8.56E+02	1.02E+03	1.41E+02
GSA	8.97E-13	1.24E-12	1.58E-12	4.80E-13	2.61E-67	1.81E-66	3.35E-66	2.18E-66	2.49E+01	2.59E+01	2.69E+01	1.41E+00
WCA	2.56E-11	1.83E+01	3.30E+02	6.20E+01	3.16E-30	7.78E-01	4.34E+00	1.29E+00	9.45E+01	1.54E+02	2.86E+02	4.51E+01
NNA	1.62E-10	2.33E-09	1.83E-08	3.75E-09	7.51E-16	4.25E-12	3.74E-11	9.94E-12	1.12E-02	2.71E+00	2.12E+01	3.98E+00
D = 100	$\boldsymbol{F_{18}}$				$\boldsymbol{F_{19}}$				$\boldsymbol{F_{20}}$			
GA	1.41E+01	1.60E+01	1.84E+01	2.21E+00	1.35E+01	3.02E+01	5.74E+01	2.37E+01	1.59E+00	2.46E+00	3.84E+00	1.21E+00
PSO	2.39E+02	2.83E+02	3.09E+02	3.85E+01	4.14E+02	4.38E+02	4.74E+02	3.15E+01	6.30E+01	6.62E+01	6.79E+01	2.79E+00
SA	1.10E+01	1.13E+01	1.14E+01	2.72E-01	1.44E+01	2.43E+01	4.35E+01	1.67E+01	3.83E+00	4.21E+00	4.58E+00	3.71E-01
HS	3.49E+01	3.66E+01	3.91E+01	2.25E+00	4.47E+01	6.36E+01	8.45E+01	2.00E+01	1.56E+01	1.68E+01	1.85E+01	1.51E+00
ICA	6.09E+02	6.31E+02	6.47E+02	1.99E+01	6.85E+02	6.91E+02	6.99E+02	6.93E+00	1.34E+02	1.37E+02	1.40E+02	2.63E+00
GSA	2.06E+01	2.21E+01	2.36E+01	2.11E+00	3.07E+01	3.97E+01	4.87E+01	1.28E+01	3.04E-02	1.24E-01	2.18E-01	1.33E-01
WCA	3.52E+02	4.20E+02	5.25E+02	5.17E+01	5.47E+02	6.78E+02	7.99E+02	7.16E+01	6.95E+01	8.72E+01	1.31E+02	1.21E+01
NNA	3.09E+00	1.74E+01	4.87E+01	1.24E+01	6.23E+00	3.34E+01	9.63E+01	1.99E+01	1.58E+00	5.61E+00	1.23E+01	2.92E+00
D = 100	$\boldsymbol{F_{21}}$											
GA	1.76E-03	2.02E-03	2.21E-03	2.32E-04								
PSO	1.01E+02	1.21E+02	1.48E+02	2.42E+01								
SA	3.70E-01	4.08E-01	4.39E-01	3.53E-02								
HS	2.73E-02	2.76E-02	2.80E-02	3.98E-04								
ICA	3.49E+01	3.74E+01	3.97E+01	2.40E+00								
GSA	6.68E-37	5.18E-36	9.69E-36	6.38E-36								
WCA	7.14E-13	2.14E+00	1.45E+01	2.77E+00								
NNA	3.58E-85	1.13E-47	3.34E-46	6.09E-47								

Table 12
Experimental optimization results for dimension 200 using reported optimizers for shifted function (F_1-F_{11}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 200	$\boldsymbol{F_1}$				$\boldsymbol{F_2}$				$\boldsymbol{F_3}$			
GA	1.24E-02	1.28E-02	1.33E-02	4.35E-04	1.33E+01	1.38E+01	1.41E+01	4.45E-01	2.86E+02	4.12E+02	5.12E+02	1.15E+02
PSO	3.74E+04	4.19E+04	4.83E+04	5.68E+03	3.53E+01	3.66E+01	3.91E+01	2.14E+00	2.87E+09	3.05E+09	3.37E+09	2.79E+08
SA	1.08E+00	1.18E+00	1.31E+00	1.18E-01	4.94E+00	5.43E+00	5.78E+00	4.37E-01	2.40E+02	2.67E+02	3.13E+02	4.00E+01
HS	6.04E+01	6.81E+01	7.41E+01	6.99E+00	1.47E+01	1.51E+01	1.59E+01	6.63E-01	3.28E+04	4.01E+04	4.80E+04	7.60E+03
ICA	1.27E+04	1.58E+04	2.02E+04	3.93E+03	9.43E+01	9.76E+01	1.02E+02	3.91E+00	8.38E+08	8.94E+08	9.26E+08	4.88E+07
GSA	3.41E-13	3.6E-13	3.98E-13	3.28E-14	1.60E+01	1.81E+01	1.93E+01	1.83E+00	1.82E+02	2.05E+02	2.28E+02	3.24E+01
WCA	1.19E-12	1.39E-12	1.88E-12	1.99E-13	4.77E+01	5.44E+01	6.28E+01	4.87E+00	2.63E+02	4.08E+02	5.71E+02	9.89E+01
NNA	6.04E-10	4.82E-07	7.03E-06	1.81E-06	1.56E+01	2.40E+01	2.83E+01	3.33E+00	3.28E+02	4.79E+02	9.35E+02	1.76E+02
D = 200	$\boldsymbol{F_4}$				$\boldsymbol{F_5}$				$\boldsymbol{F_6}$			
GA	7.17E-03	9.54E-03	1.18E-02	2.30E-03	4.81E-03	9.85E-03	1.58E-02	5.57E-03	1.04E-02	1.05E-02	1.06E-02	7.08E-05
PSO	1.31E+03	1.34E+03	1.35E+03	2.34E+01	3.80E+02	3.99E+02	4.26E+02	2.41E+01	1.51E+01	1.54E+01	1.58E+01	3.20E-01
SA	1.05E+00	1.08E+00	1.09E+00	2.10E-02	1.92E+00	1.93E+00	1.95E+00	1.50E-02	3.41E+00	3.44E+00	3.51E+00	5.44E-02
HS	8.70E+00	1.04E+01	1.18E+01	1.59E+00	1.59E+00	1.67E+00	1.81E+00	1.17E-01	1.05E+00	1.18E+00	1.43E+00	2.21E-01
ICA	1.85E+03	1.92E+03	2.01E+03	8.28E+01	1.08E+02	2.25E+02	3.21E+02	1.08E+02	1.97E+01	1.98E+01	1.99E+01	5.39E-02
GSA	1.48E+02	1.65E+02	1.82E+02	2.39E+01	9.11E-01	2.18E+00	3.46E+00	1.80E+00	5.10E-09	5.23E-09	5.35E-09	1.82E-10
WCA	7.37E+02	8.94E+02	1.03E+03	1.05E+02	1.53E-12	5.17E-03	1.48E-02	6.72E-03	1.76E+01	1.82E+01	1.87E+01	3.60E-01
NNA	1.93E+02	2.37E+02	2.95E+02	2.91E+01	3.95E-10	2.13E-02	1.32E-01	3.24E-02	2.72E-06	1.81E-04	1.56E-03	4.27E-04
D = 200	$\boldsymbol{F_7}$				$\boldsymbol{F_8}$				$\boldsymbol{F_9}$			
GA	5.60E-02	6.25E-02	6.62E-02	5.65E-03	4.04E-01	4.81E-01	5.57E-01	1.08E-01	4.14E+01	4.38E+01	4.70E+01	2.84E+00
PSO	2.60E+02	2.77E+02	2.88E+02	1.47E+01	1.66E+06	1.78E+06	1.91E + 06	1.72E+05	1.33E+03	1.34E+03	1.34E+03	7.66E+00
SA	2.18E+00	2.24E+00	2.29E+00	5.24E-02	2.46E+00	2.65E+00	2.84E+00	2.63E-01	3.49E+01	3.60E+01	3.76E+01	1.45E+00
HS	1.18E+00	1.30E+00	1.46E+00	1.44E-01	2.36E+03	2.51E+03	2.66E+03	2.09E+02	2.36E+02	2.42E+02	2.51E+02	8.04E+00
ICA	8.71E+02	9.10E+02	9.36E+02	3.40E+01	1.05E+06	1.42E+06	1.78E+06	5.18E+05	1.78E+03	1.84E+03	1.92E+03	7.35E+01
GSA	2.23E-07	2.46E-07	2.69E-07	3.26E-08	1.37E-14	1.45E-14	1.52E-14	1.05E-15	7.69E+02	7.84E+02	7.98E+02	2.02E+01
WCA	2.45E-09	2.12E+00	1.00E+01	4.17E+00	7.95E-18	2.54E-17	2.42E-16	7.61E-18	8.69E+02	1.02E+03	1.29E+03	1.42E+02
NNA	4.48E-10	2.04E-09	8.07E-09	1.98E-09	6.98E-19	9.9E-18	3.98E-17	1.02E-17	1.21E+02	2.09E+02	3.13E+02	5.56E+01
D = 200	$\boldsymbol{F_{10}}$				$\boldsymbol{F_{11}}$							
GA	1.51E-02	1.61E-02	1.79E-02	1.57E-03	4.20E+01	4.85E+01	5.51E+01	6.57E+00				
PSO	2.92E+03	3.30E+03	3.95E+03	5.61E+02	1.27E+03	1.30E+03	1.34E+03	3.47E+01				
SA	1.09E+00	1.19E+00	1.27E+00	9.25E-02	3.39E+01	3.54E+01	3.76E+01	1.97E+00				
HS	1.05E+01	1.07E+01	1.10E+01	2.67E-01	2.25E+02	2.34E+02	2.43E+02	9.14E+00				
ICA	9.95E+02	1.23E+03	1.43E+03	2.19E+02	1.86E+03	1.89E+03	1.93E+03	3.66E+01				
GSA	1.05E+00	2.10E+00	3.15E+00	1.48E+00	7.76E+02	7.93E+02	8.11E+02	2.50E+01				
WCA	8.83E+00	1.87E+01	3.66E+01	7.87E+00	9.15E+02	9.92E+02	1.14E+03	8.12E+01				
NNA	0.00E+00	4.44E-17	4.44E-16	1.17E-16	1.41E+02	2.11E+02	2.65E+02	3.75E+01				

Table 13
Experimental optimization results for dimension 200 using reported optimizers (F_{12}–F_{21}).

Methods	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD	Best Error	Average Error	Worst Error	Error SD
D = 200	$\boldsymbol{F_{12}}$				$\boldsymbol{F_{13}}$				$\boldsymbol{F_{14}}$			
GA	7.87E+00	8.12E+00	8.57E+00	3.96E-01	2.83E+02	4.18E+02	5.18E+02	1.22E+02	1.35E+00	1.51E+00	1.80E+00	2.53E-01
PSO	4.61E+03	6.06E+03	8.36E+03	2.02E+03	4.52E+07	5.46E+07	6.50E+07	9.96E+06	5.91E+02	6.55E+02	7.23E+02	6.58E+01
SA	8.89E+00	9.08E+00	9.25E+00	1.86E-01	2.51E+02	3.25E+02	3.67E+02	6.68E+01	3.41E+00	3.56E+00	3.78E+00	1.91E-01
HS	8.14E+01	8.62E+01	9.34E+01	6.34E+00	1.26E+03	1.52E+03	2.00E+03	4.14E+02	1.94E+01	2.36E+01	2.71E+01	3.89E+00
ICA	1.98E+03	2.45E+03	3.01E+03	5.22E+02	3.51E+07	4.40E+07	5.10E+07	8.16E+06	1.40E+03	1.44E+03	1.50E+03	5.08E+01
GSA	5.15E+01	6.34E+01	7.54E+01	1.69E+01	3.00E+02	3.34E+02	3.83E+02	4.74E+01	1.18E+02	1.20E+02	1.22E+02	2.90E+00
WCA	3.32E+02	4.07E+02	4.47E+02	3.36E+01	6.16E+02	7.23E+02	8.47E+02	7.14E+01	6.21E+02	7.19E+02	7.99E+02	5.65E+01
NNA	2.13E+02	2.72E+02	3.14E+02	3.29E+01	5.87E+02	8.26E+02	1.52E+03	2.53E+02	1.20E+02	1.60E+02	1.93E+02	2.10E+01
D = 200	$\boldsymbol{F_{15}}$				$\boldsymbol{F_{16}}$				$\boldsymbol{F_{17}}$			
GA	1.61E+00	2.04E+00	2.35E+00	3.82E-01	4.78E-03	5.34E-03	6.31E-03	8.44E-04	2.23E-01	3.61E-01	4.31E-01	1.20E-01
PSO	7.23E+02	8.49E+02	1.03E+03	1.58E+02	5.74E+02	1.12E+03	1.60E+03	5.16E+02	4.47E+02	4.84E+02	5.19E+02	3.59E+01
SA	5.05E+00	5.44E+00	5.96E+00	4.68E-01	5.33E-01	6.07E-01	6.95E-01	8.20E-02	5.14E+00	1.36E+01	1.79E+01	7.36E+00
HS	2.48E+01	2.67E+01	2.81E+01	1.74E+00	1.09E-03	1.23E-03	1.41E-03	1.65E-04	2.13E-01	1.68E+00	3.52E+00	1.68E+00
ICA	1.15E+08	1.39E+17	4.16E+17	2.40E+17	5.10E+01	9.48E+01	1.79E+02	7.33E+01	2.34E+03	2.47E+03	2.68E+03	1.87E+02
GSA	2.45E+00	2.89E+00	3.34E+00	6.33E-01	3.10E-110	2.6E-82	5.21E-82	3.68E-82	5.67E+01	5.87E+01	6.07E+01	2.81E+00
WCA	3.47E-08	3.20E+01	3.10E+02	9.77E+01	1.34E-27	1.43E+00	1.79E+00	7.55E-01	4.30E+02	6.14E+02	1.11E+03	2.26E+02
NNA	1.45E-08	1.40E-07	3.68E-07	1.29E-07	4.02E-14	3.43E-10	3.38E-09	8.77E-10	4.68E+00	1.02E+01	2.69E+01	5.54E+00
D = 200	$\boldsymbol{F_{18}}$				$\boldsymbol{F_{19}}$				$\boldsymbol{F_{20}}$			
GA	2.08E+01	2.32E+01	2.51E+01	2.20E+00	2.21E+01	2.70E+01	3.14E+01	4.68E+00	5.61E+00	6.63E+00	8.01E+00	1.24E+00
PSO	5.79E+02	5.98E+02	6.26E+02	2.50E+01	7.84E+02	8.65E+02	9.22E+02	7.21E+01	1.30E+02	1.36E+02	1.44E+02	7.33E+00
SA	2.54E+01	2.63E+01	2.70E+01	8.28E-01	2.97E+01	3.76E+01	4.91E+01	1.02E+01	7.79E+00	8.14E+00	8.44E+00	3.31E-01
HS	8.05E+01	9.30E+01	1.04E+02	1.17E+01	1.14E+02	1.47E+02	2.11E+02	5.50E+01	4.18E+01	4.56E+01	4.79E+01	3.35E+00
ICA	1.32E+03	1.38E+03	1.45E+03	6.25E+01	1.49E+03	1.52E+03	1.58E+03	4.91E+01	2.94E+02	2.95E+02	2.96E+02	8.23E-01
GSA	1.24E+02	1.30E+02	1.37E+02	9.07E+00	1.27E+02	1.39E+02	1.52E+02	1.77E+01	2.57E+00	3.48E+00	4.40E+00	1.30E+00
WCA	7.73E+02	8.42E+02	1.03E+03	7.85E+01	1.06E+03	1.31E+03	1.46E+03	1.23E+02	1.60E+02	1.75E+02	2.17E+02	1.74E+01
NNA	2.79E+01	4.60E+01	9.48E+01	1.60E+01	1.27E+02	2.29E+02	3.20E+02	5.76E+01	3.41E+00	2.30E+01	5.12E+01	1.24E+01
D = 200	$\boldsymbol{F_{21}}$											
GA	2.27E-03	3.63E-03	4.48E-03	1.19E-03								
PSO	2.79E+02	3.12E+02	3.54E+02	3.81E+01								
SA	7.40E-01	7.99E-01	8.70E-01	6.54E-02								
HS	1.41E-01	1.48E-01	1.51E-01	5.30E-03								
ICA	1.80E+02	1.94E+02	2.07E+02	1.36E+01								
GSA	1.05E+00	2.62E+00	4.20E+00	2.23E+00								
WCA	7.06E-08	9.45E+00	1.94E+01	5.77E+00								
NNA	3.04E-86	1.06E-23	1.59E-22	4.12E-23								

Table 14
Statistical test and optimization results obtained by reported optimizers ($D = 50$).

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test (Ranking)
1	**NNA**	4.95E-12	2.25E-10	7.48E-11	1.39E-09	3.45E-10	6.80
	RS	7.07E+04	9.02E+04	9.10E+04	1.01E+05	6.45E+03	13
	TLBO	6.25E-13	1.16E-09	3.33E-12	2.09E-08	4.17E-09	6.20
	ICA	9.16E-03	3.44E+01	4.17E-02	9.50E+02	1.73E+02	10.19
	CS	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.51
	GSA	5.68E-14	5.68E-14	5.68E-14	5.68E-14	0.00E+00	3.46
	WCA	1.71E-13	3.03E-13	2.84E-13	5.12E-13	8.22E-14	5
	HS	1.42E-04	2.36E-04	2.42E-04	2.72E-04	2.61E-05	8
	PSO	7.70E+00	2.36E+01	2.10E+01	5.51E+01	1.15E+01	11.93
	GA	4.56E-04	2.71E-03	2.31E-03	1.09E-02	2.03E-03	9.00
	SA	2.07E-01	3.21E-01	3.17E-01	4.57E-01	7.18E-02	10.86
	DE	0.00E+00	6.06E-14	5.68E-14	1.14E-13	2.08E-14	3.50
	CMA-ES	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.51
2	**NNA**	4.52E-01	1.03E+00	9.83E-01	1.65E+00	3.10E-01	2.57
	RS	7.91E+01	8.53E+01	8.54E+01	9.16E+01	2.81E+00	12.97
	TLBO	2.84E+01	4.18E+01	4.31E+01	5.11E+01	4.98E+00	10.37
	ICA	3.53E+01	5.54E+01	5.07E+01	9.52E+01	1.27E+01	11.57
	CS	2.52E+01	4.47E+01	4.53E+01	6.44E+01	9.21E+00	10.67
	GSA	2.70E-01	9.84E+00	1.00E+01	2.18E+01	5.14E+00	6.63
	WCA	1.24E+00	6.27E+00	4.15E+00	2.44E+01	5.54E+00	5.97
	HS	1.49E+00	2.21E+00	2.17E+00	2.98E+00	3.63E-01	5.10
	PSO	1.32E+01	2.62E+01	2.79E+01	3.84E+01	6.79E+00	8.40
	GA	8.50E-01	1.38E+00	1.27E+00	3.14E+00	4.51E-01	3.17
	SA	9.70E-01	1.45E+00	1.46E+00	1.84E+00	1.95E-01	3.63
	DE	1.30E+01	2.91E+01	2.84E+01	5.68E+01	1.01E+01	8.97
	CMA-ES	6.92E-10	1.2E-09	1.21E-09	1.79E-09	3.35E-10	1
3	**NNA**	5.39E-01	9.60E+01	8.26E+01	3.35E+02	7.12E+01	4.40
	RS	1.76E+10	3.72E+10	3.78E+10	4.78E+10	6.67E+09	13
	TLBO	8.63E+00	4.54E+02	9.50E+01	1.04E+04	1.88E+03	5.36
	ICA	2.89E+02	6.68E+04	6.19E+02	1.29E+06	2.55E+05	9.73
	CS	6.67E+02	7.87E+02	7.23E+02	8.89E+02	2.11E+02	12
	GSA	4.08E+01	1.02E+02	4.13E+01	1.01E+03	1.88E+02	3.83
	WCA	4.36E+00	9.80E+01	8.14E+01	4.22E+02	1.03E+02	4.03
	HS	2.96E+00	5.94E+02	1.53E+02	7.69E+03	1.58E+03	6.80
	PSO	3.31E+03	2.86E+04	1.88E+04	1.26E+05	3.07E+04	1.08
	GA	2.54E+01	3.83E+02	1.47E+02	5.41E+03	1.00E+03	6.46
	SA	9.51E+01	9.17E+02	1.69E+02	1.18E+04	2.57E+03	7.13
	DE	3.85E+01	5.48E+01	4.11E+01	1.12E+02	2.43E+01	3.06
	CMA-ES	2.04E+01	1.51E+03	2.17E+01	2.18E+04	4.51E+03	4.36
4	**NNA**	1.99E+00	6.60E+00	5.97E+00	1.19E+01	2.33E+00	2.63
	RS	5.87E+02	6.44E+02	6.48E+02	6.87E+02	2.56E+01	13
	TLBO	7.56E+01	1.08E+02	9.90E+01	1.50E+02	2.21E+01	7.30
	ICA	2.06E+02	3.49E+02	3.57E+02	4.32E+02	5.42E+01	11.8
	CS	1.51E+01	3.07E+01	2.97E+01	6.47E+01	1.05E+01	5.26
	GSA	2.39E+01	4.42E+01	4.48E+01	6.77E+01	1.07E+01	6
	WCA	1.01E+02	1.91E+02	1.90E+02	3.64E+02	5.36E+01	8.96
	HS	3.09E-02	4.36E-02	4.33E-02	5.44E-02	5.01E-03	1
	PSO	1.14E+02	1.57E+02	1.52E+02	2.04E+02	2.42E+01	8.40
	GA	1.99E+00	9.89E+00	9.46E+00	1.99E+01	4.64E+00	3.30
	SA	3.21E+00	8.25E+00	7.23E+00	1.62E+01	3.86E+00	3.16
	DE	1.62E+02	2.47E+02	2.58E+02	3.13E+02	4.07E+01	9.96
	CMA-ES	6.96E+00	2.67E+02	3.06E+02	3.31E+02	1.03E+02	10.20
5	**NNA**	6.76E-12	6.20E-02	5.26E-02	1.61E-01	5.30E-02	6.96
	RS	6.96E+02	8.36E+02	8.45E+02	1.00E+03	6.18E+01	13
	TLBO	9.09E-13	1.51E-01	7.36E-02	1.07E+00	2.15E-01	7.76
	ICA	1.82E-02	5.34E-01	6.58E-02	8.41E+00	1.71E+00	8.13
	CS	0.00E+00	5.49E-03	0.00E+00	4.16E-02	1.06E-02	2.80
	GSA	3.10E+00	2.63E+01	2.03E+01	6.72E+01	1.74E+01	12
	WCA	2.56E-13	3.01E-02	8.63E-03	5.24E-01	9.47E-02	5.13
	HS	7.24E-06	1.11E-02	8.64E-03	5.91E-02	1.49E-02	5.16
	PSO	1.08E+00	1.22E+00	1.17E+00	1.64E+00	1.22E-01	10.33
	GA	2.50E-04	7.68E-03	5.31E-03	2.12E-02	5.80E-03	5.06
	SA	1.14E+00	1.21E+00	1.20E+00	1.28E+00	3.23E-02	10.53
	DE	0.00E+00	2.47E-04	2.84E-14	7.40E-03	1.35E-03	2.73
	CMA-ES	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.36
6	**NNA**	1.21E-07	1.12E-06	4.97E-07	5.27E-06	1.38E-06	4.03
	RS	2.01E+01	2.04E+01	2.04E+01	2.05E+01	1.04E-01	12.97
	TLBO	6.60E+00	9.82E+00	1.01E+01	1.25E+01	1.42E+00	9.87
	ICA	4.09E+00	1.79E+01	1.89E+01	2.07E+01	3.72E+00	11.83
	CS	1.16E+00	1.92E+00	1.84E+00	3.29E+00	4.99E-01	8.13
	GSA	2.37E-09	3.18E-09	3.13E-09	3.91E-09	3.78E-10	3
	WCA	9.05E-09	1.11E+01	1.21E+01	1.88E+01	6.44E+00	9.73

Table 14 (*Continued*)

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test (Ranking)
	HS	7.35E-03	8.55E-03	8.68E-03	9.63E-03	4.81E-04	5.27
	PSO	7.04E+00	1.20E+01	1.21E+01	1.77E+01	3.50E+00	10.13
	GA	7.14E-03	1.07E-02	1.04E-02	1.64E-02	2.08E-03	5.93
	SA	3.14E-01	4.26E-01	4.23E-01	5.99E-01	5.58E-02	7.10
	DE	5.68E-14	9.76E-14	5.68E-14	4.26E-13	8.06E-14	2
	CMA-ES	2.84E-14	2.84E-14	2.84E-14	2.84E-14	0.00E+00	1
7	**NNA**	1.78E-12	3.99E-11	3.01E-11	1.44E-10	3.71E-11	5.83
	RS	1.06E+08	1.10E+12	1.65E+11	1.03E+13	2.26E+12	13
	TLBO	5.42E-214	8.46E-213	6.04E-213	3.12E-212	0.00E+00	1
	ICA	2.42E+01	2.15E+07	1.56E+02	6.45E+08	1.18E+08	12
	CS	3.68E-26	1.79E-03	1.92E-22	5.38E-02	9.83E-03	2.57
	GSA	2.55E-08	3.36E-08	3.40E-08	4.98E-08	5.02E-09	6.90
	WCA	1.39E-14	3.33E-01	7.00E-13	1.00E+01	1.83E+00	4.73
	HS	5.29E-02	5.99E-02	6.07E-02	6.46E-02	3.18E-03	8.97
	PSO	3.07E-01	2.15E+00	1.82E+00	4.98E+00	1.21E+00	10.93
	GA	9.72E-03	1.67E-02	1.64E-02	3.41E-02	4.90E-03	7.93
	SA	4.48E-01	5.37E-01	5.39E-01	6.47E-01	5.45E-02	10
	DE	1.03E-22	5.10E-12	7.93E-19	1.32E-10	2.42E-11	3.07
	CMA-ES	1.6E-13	4.28E-13	3.74E-13	1.14E-12	2.15E-13	4.07
8	**NNA**	6.80E-23	5.18E-20	1.34E-20	3.08E-19	9.07E-20	5.93
	RS	6.72E+05	7.56E+05	7.54E+05	8.24E+05	3.71E+04	13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	ICA	2.35E-01	2.01E+00	1.22E+00	1.54E+01	2.85E+00	10.90
	CS	1.31E-44	1.50E-42	4.57E-43	1.17E-41	2.62E-42	2
	GSA	1.12E-16	3.58E-16	3.29E-16	6.91E-16	1.21E-16	7
	WCA	3.58E-29	7.90E-23	3.07E-24	1.01E-21	2.21E-22	4.20
	HS	4.04E-03	4.99E-03	4.91E-03	6.15E-03	5.40E-04	8.03
	PSO	6.36E+01	2.94E+02	2.53E+02	7.49E+02	1.72E+02	12
	GA	3.48E-03	3.73E-02	3.30E-02	8.51E-02	1.93E-02	8.97
	SA	2.54E-01	3.88E-01	3.89E-01	6.16E-01	8.83E-02	10.10
	DE	3.64E-36	2.63E-20	2.26E-29	7.67E-19	1.40E-19	3.40
	CMA-ES	6.54E-25	6.4E-24	5.57E-24	1.88E-23	4.41E-24	4.47
9	**NNA**	1.07E-01	3.21E+00	2.23E+00	1.10E+01	3.33E+00	4.77
	RS	4.52E+02	4.83E+02	4.85E+02	4.97E+02	1.04E+01	13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	ICA	3.30E+02	3.76E+02	3.78E+02	4.27E+02	2.31E+01	12
	CS	1.01E+02	1.72E+02	1.78E+02	2.17E+02	2.87E+01	9.53
	GSA	2.41E-03	9.06E-01	4.92E-02	8.99E+00	2.59E+00	3.60
	WCA	2.08E+02	2.45E+02	2.40E+02	3.29E+02	2.61E+01	11
	HS	2.09E+01	2.75E+01	2.78E+01	3.57E+01	3.16E+00	8
	PSO	1.40E+02	1.68E+02	1.70E+02	2.02E+02	1.57E+01	9.47
	GA	7.15E+00	1.08E+01	1.12E+01	1.44E+01	1.92E+00	6.67
	SA	8.09E+00	8.76E+00	8.61E+00	1.00E+01	5.06E-01	6.10
	DE	1.79E-03	6.19E-01	3.61E-01	3.71E+00	8.05E-01	3.87
	CMA-ES	1.28E-04	5.82E-04	2.16E-04	1.11E-02	1.99E-03	2
10	**NNA**	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.42
	RS	4.19E+03	5.18E+03	5.26E+03	5.71E+03	3.48E+02	13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.42
	ICA	1.10E+01	1.64E+01	1.57E+01	2.21E+01	2.99E+00	10.90
	CS	5.25E+00	1.07E+01	9.92E+00	1.76E+01	3.11E+00	10.10
	GSA	0.00E+00	7.00E-02	1.67E-16	1.05E+00	2.66E-01	4.55
	WCA	7.77E-16	5.64E-01	2.19E-01	2.57E+00	7.29E-01	7.32
	HS	6.28E-03	1.04E-02	1.05E-02	1.24E-02	1.12E-03	7.17
	PSO	2.92E+01	4.48E+01	4.52E+01	7.05E+01	1.05E+01	12
	GA	1.07E-03	3.36E-03	2.84E-03	7.96E-03	1.73E-03	6.17
	SA	1.77E-01	2.73E-01	2.65E-01	3.92E-01	5.92E-02	8.20
	DE	0.00E+00	1.87E-01	0.00E+00	1.05E+00	3.72E-01	4.35
	CMA-ES	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.42
11	**NNA**	6.47E-02	5.80E+00	4.41E+00	2.04E+01	5.54E+00	5.20
	RS	4.47E+02	4.69E+02	4.70E+02	4.89E+02	9.11E+00	13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	ICA	3.31E+02	3.71E+02	3.68E+02	4.23E+02	2.38E+01	12
	CS	1.12E+02	1.72E+02	1.73E+02	2.15E+02	2.70E+01	9.57
	GSA	1.25E-02	1.43E+00	6.11E-02	1.45E+01	3.64E+00	3.53
	WCA	1.85E+02	2.49E+02	2.45E+02	2.89E+02	2.17E+01	10.97
	HS	2.17E+01	2.66E+01	2.67E+01	3.36E+01	2.82E+00	8
	PSO	1.38E+02	1.64E+02	1.65E+02	1.88E+02	1.16E+01	9.47
	GA	7.19E+00	1.07E+01	1.04E+01	1.69E+01	2.38E+00	6.60
	SA	6.92E+00	8.45E+00	8.28E+00	1.05E+01	8.45E-01	5.87
	DE	1.06E-05	4.29E-01	3.18E-01	1.78E+00	4.21E-01	3.77
	CMA-ES	1.41E-04	1.86E-03	2.41E-04	3.23E-02	6.14E-03	2.03

Table 14 (*Continued*)

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test (Ranking)
12	**NNA**	1.75E-01	1.64E+01	1.59E+01	5.29E+01	1.23E+01	3.93
	RS	4.36E+04	5.70E+04	5.79E+04	6.91E+04	5.11E+03	13
	TLBO	3.41E+01	6.91E+01	6.80E+01	1.09E+02	1.64E+01	8.13
	ICA	1.01E+02	1.40E+02	1.37E+02	2.21E+02	2.25E+01	11.60
	CS	6.34E-02	1.90E+01	1.16E+01	5.80E+01	1.58E+01	3.97
	GSA	1.17E+01	3.50E+01	3.49E+01	6.16E+01	1.33E+01	6.30
	WCA	6.81E+01	1.03E+02	1.02E+02	1.37E+02	1.66E+01	9.70
	HS	7.59E+00	1.69E+01	1.40E+01	3.58E+01	7.81E+00	4.33
	PSO	8.20E+01	1.05E+02	1.02E+02	1.34E+02	1.31E+01	9.77
	GA	1.78E+00	1.45E+01	1.47E+01	3.21E+01	7.23E+00	3.70
	SA	2.48E+00	1.28E+01	1.25E+01	2.97E+01	6.51E+00	3.37
	DE	1.84E-02	9.78E+00	7.91E+00	2.63E+01	6.64E+00	2.43
	CMA-ES	6.31E+01	1.23E+02	1.28E+02	1.54E+02	2.17E+01	10.77
13	**NNA**	9.70E+00	1.46E+02	4.04E+01	9.42E+02	2.49E+02	3.07
	RS	1.13E+10	1.68E+10	1.71E+10	2.34E+10	2.60E+09	13
	TLBO	9.35E+01	2.15E+02	1.52E+02	1.11E+03	1.85E+02	6.17
	ICA	3.18E+02	1.41E+05	5.86E+02	1.82E+06	4.39E+05	9.50
	CS	2.60E+02	9.34E+09	1.00E+10	1.00E+10	2.50E+09	11.83
	GSA	6.09E+01	2.46E+02	7.60E+01	2.91E+03	5.47E+02	4.10
	WCA	1.13E+02	2.27E+02	1.56E+02	5.68E+02	1.38E+02	6.47
	HS	9.55E+00	1.01E+03	2.33E+02	7.94E+03	1.97E+03	6.63
	PSO	1.06E+03	8.35E+03	6.07E+03	3.15E+04	7.05E+03	10.53
	GA	3.35E+01	1.26E+02	1.23E+02	2.38E+02	5.92E+01	4.63
	SA	5.38E+01	1.62E+03	1.64E+02	1.84E+04	4.64E+03	5.90
	DE	1.40E+01	3.43E+01	2.28E+01	1.11E+02	2.70E+01	1.60
	CMA-ES	1.44E+02	1.39E+03	1.85E+02	1.60E+04	3.70E+03	7.57
14	**NNA**	1.81E+00	6.83E+00	7.26E+00	1.15E+01	2.57E+00	2.37
	RS	4.12E+02	4.60E+02	4.63E+02	5.02E+02	2.26E+01	13
	TLBO	4.81E+01	8.58E+01	8.62E+01	1.27E+02	2.00E+01	7.30
	ICA	1.46E+02	2.44E+02	2.51E+02	3.48E+02	4.61E+01	11.57
	CS	1.52E+01	2.79E+01	2.86E+01	4.64E+01	6.11E+00	5.20
	GSA	2.38E+01	3.55E+01	3.61E+01	4.97E+01	5.71E+00	5.90
	WCA	8.85E+01	1.44E+02	1.47E+02	2.32E+02	3.67E+01	8.93
	HS	3.76E-01	4.00E+00	3.78E+00	8.60E+00	1.96E+00	1.40
	PSO	6.20E+01	1.23E+02	1.23E+02	1.94E+02	2.72E+01	8.40
	GA	2.53E+00	1.15E+01	1.13E+01	2.22E+01	5.53E+00	3.33
	SA	3.20E+00	9.12E+00	8.02E+00	2.08E+01	4.28E+00	2.97
	DE	1.12E+02	1.79E+02	1.76E+02	2.36E+02	3.19E+01	10
	CMA-ES	7.27E+00	2.06E+02	2.21E+02	2.41E+02	5.54E+01	10.63
15	**NNA**	1.48E-12	3.48E-11	1.00E-11	2.75E-10	6.33E-11	4.87
	RS	1.40E+03	4.48E+04	4.22E+04	1.25E+05	3.11E+04	13
	TLBO	3.85E+01	3.49E+02	1.35E+02	2.67E+03	5.68E+02	1
	ICA	3.85E+01	3.49E+02	1.35E+02	2.67E+03	5.68E+02	12
	CS	2.25E-37	5.20E-36	2.56E-36	3.58E-35	6.89E-36	2
	GSA	1.97E-12	1.94E-11	8.55E-12	1.67E-10	3.21E-11	4.70
	WCA	1.32E-12	4.00E+00	3.99E-11	6.00E+01	1.25E+01	6.07
	HS	1.57E-01	2.42E-01	2.31E-01	4.03E-01	6.48E-02	8.77
	PSO	2.35E+00	7.61E+00	5.73E+00	3.18E+01	6.39E+00	10.90
	GA	4.12E-02	1.02E-01	9.51E-02	2.11E-01	3.80E-02	7.27
	SA	5.71E-01	6.96E-01	6.89E-01	9.09E-01	8.09E-02	9.87
	DE	6.83E-31	8.97E-30	2.02E-30	1.56E-28	2.81E-29	3.00
	CMA-ES	6.99E-02	1.23E-01	1.26E-01	2.15E-01	3.52E-02	7.57
16	**NNA**	4.56E-18	7.35E-11	1.05E-14	2.01E-09	3.66E-10	4.53
	RS	1.75E+04	2.49E+04	2.49E+04	2.87E+04	2.56E+03	13
	TLBO	1.26E-29	3.85E-01	2.41E-01	1.93E+00	5.71E-01	6.50
	ICA	1.43E-08	1.10E+00	4.81E-01	3.50E+00	1.04E+00	9.47
	CS	0.00E+00	1.12E-01	0.00E+00	1.93E+00	3.73E-01	2.82
	GSA	0.00E+00	3.21E-02	0.00E+00	4.81E-01	1.22E-01	2.47
	WCA	0.00E+00	9.15E-01	4.81E-01	1.93E+00	8.61E-01	7.73
	HS	7.40E-06	1.34E-05	1.25E-05	2.50E-05	4.16E-06	5.77
	PSO	4.45E-01	1.70E+00	1.65E+00	3.45E+00	8.11E-01	10.90
	GA	6.34E-05	8.22E-04	5.74E-04	3.46E-03	8.10E-04	6.77
	SA	7.43E-02	1.50E-01	1.46E-01	2.45E-01	4.34E-02	8
	DE	0.00E+00	3.21E-02	0.00E+00	4.81E-01	1.22E-01	2.62
	CMA-ES	2.89E-02	1.91E+00	1.96E+00	8.74E+00	2.16E+00	10.43
17	**NNA**	4.63E-04	1.39E+00	1.07E+00	9.60E+00	1.79E+00	2.50
	RS	5.05E+02	5.89E+02	5.93E+02	6.96E+02	4.87E+01	13
	TLBO	4.58E+01	9.46E+01	1.01E+02	1.45E+02	2.82E+01	8.93
	ICA	1.17E+02	2.32E+02	2.33E+02	3.37E+02	5.05E+01	11.67
	CS	9.95E-01	3.91E+00	2.89E+00	1.29E+01	3.08E+00	3.73
	GSA	6.96E+00	1.88E+01	1.59E+01	6.27E+01	1.12E+01	5.83

Table 14 (*Continued*)

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test (Ranking)
	WCA	1.69E+01	3.91E+01	3.58E+01	1.33E+02	2.12E+01	7.07
	HS	3.34E-03	7.11E-01	2.55E-01	4.18E+00	1.05E+00	1.80
	PSO	1.23E+02	2.12E+02	2.08E+02	3.60E+02	6.29E+01	11.23
	GA	5.10E-04	2.83E+00	1.26E+00	1.30E+01	3.72E+00	2.67
	SA	2.91E-01	8.53E+00	5.82E+00	2.65E+01	6.77E+00	4.57
	DE	3.57E+01	7.15E+01	7.04E+01	1.18E+02	2.29E+01	8.17
	CMA-ES	9.85E+01	1.20E+02	1.22E+02	1.38E+02	8.94E+00	9.83
18	**NNA**	1.90E+00	1.14E+01	1.07E+01	3.39E+01	7.23E+00	2.73
	RS	3.26E+02	3.51E+02	3.52E+02	3.73E+02	9.60E+00	13
	TLBO	1.28E+00	1.62E+01	1.62E+01	3.26E+01	8.29E+00	4.40
	ICA	1.87E+02	2.61E+02	2.68E+02	3.48E+02	2.85E+01	11.87
	CS	1.81E+01	5.58E+01	5.65E+01	9.87E+01	2.31E+01	8.57
	GSA	4.05E-01	1.72E+01	1.30E+01	7.79E+01	1.61E+01	3.87
	WCA	1.31E+02	2.14E+02	2.07E+02	3.12E+02	3.90E+01	11.13
	HS	1.53E+01	2.46E+01	2.18E+01	5.91E+01	8.48E+00	6.23
	PSO	8.90E+01	1.30E+02	1.32E+02	1.71E+02	1.77E+01	10
	GA	4.82E+00	1.61E+01	1.36E+01	3.48E+01	7.57E+00	4.27
	SA	7.36E+00	1.69E+01	1.53E+01	3.10E+01	6.60E+00	4.33
	DE	2.39E-01	1.08E+01	1.02E+01	2.15E+01	6.22E+00	2.90
	CMA-ES	2.27E+01	3.54E+01	3.45E+01	5.06E+01	7.25E+00	7.70
19	**NNA**	1.08E-01	4.21E+00	3.21E+00	1.38E+01	3.78E+00	2.83
	RS	3.81E+02	4.08E+02	4.11E+02	4.24E+02	1.18E+01	11.80
	TLBO	1.66E-08	1.58E+01	1.92E+00	1.69E+02	3.61E+01	3.07
	ICA	2.78E+02	3.59E+02	3.54E+02	4.72E+02	5.35E+01	11
	CS	1.16E+02	1.75E+02	1.76E+02	2.47E+02	2.93E+01	8.13
	GSA	3.77E-01	6.95E+01	7.52E+01	1.58E+02	3.59E+01	6.20
	WCA	2.03E+02	3.08E+02	3.09E+02	3.78E+02	3.89E+01	10.23
	HS	1.80E+01	4.71E+01	4.02E+01	1.18E+02	2.80E+01	5.63
	PSO	1.63E+02	2.06E+02	2.05E+02	2.50E+02	2.06E+01	8.83
	GA	3.69E+00	1.31E+01	8.48E+00	7.91E+01	1.44E+01	3.83
	SA	5.44E+00	4.93E+01	3.30E+01	1.37E+02	4.32E+01	5.37
	DE	1.85E-07	1.62E-02	1.07E-02	6.28E-02	2.02E-02	1.10
	CMA-ES	4.45E+02	5.18E+02	5.22E+02	5.85E+02	3.26E+01	12.97
20	**NNA**	1.68E-03	2.62E+00	2.86E+00	5.59E+00	1.53E+00	3.03
	RS	7.53E+01	8.00E+01	8.04E+01	8.41E+01	2.22E+00	13
	TLBO	0.00E+00	2.82E+00	2.65E+00	6.80E+00	1.72E+00	3.43
	ICA	5.13E+01	6.14E+01	6.15E+01	6.93E+01	4.98E+00	11.97
	CS	8.33E+00	1.57E+01	1.47E+01	2.62E+01	4.94E+00	8.83
	GSA	2.59E-02	3.20E+00	2.71E+00	1.02E+01	2.31E+00	3.40
	WCA	3.25E+01	4.68E+01	4.63E+01	6.39E+01	8.29E+00	10.93
	HS	3.85E+00	9.98E+00	9.61E+00	1.47E+01	2.67E+00	7.77
	PSO	2.55E+01	3.21E+01	3.19E+01	4.00E+01	3.52E+00	10.10
	GA	9.35E-01	4.52E+00	3.88E+00	1.06E+01	2.31E+00	4.87
	SA	2.03E+00	4.26E+00	4.12E+00	7.85E+00	1.56E+00	4.70
	DE	6.28E-12	2.48E+00	2.07E+00	5.83E+00	1.74E+00	2.83
	CMA-ES	3.25E+00	6.13E+00	5.92E+00	8.99E+00	1.61E+00	6.13
21	**NNA**	4.39E-72	8.85E-38	4.49E-49	2.66E-36	4.85E-37	3
	RS	1.17E+03	1.43E+03	1.45E+03	1.62E+03	1.12E+02	13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	ICA	4.67E+00	8.83E+00	8.40E+00	1.47E+01	2.50E+00	10.83
	CS	4.13E-01	5.17E+00	5.25E+00	1.05E+01	2.49E+00	10.12
	GSA	9.93E-38	8.77E-33	1.60E-34	1.28E-31	2.75E-32	4
	WCA	4.32E-13	9.23E-01	7.60E-01	4.61E+00	1.07E+00	7.85
	HS	2.82E-03	3.77E-03	3.75E-03	4.43E-03	3.92E-04	6.20
	PSO	1.50E+01	2.10E+01	2.04E+01	3.41E+01	4.27E+00	12
	GA	1.51E-04	1.70E-03	1.24E-03	5.61E-03	1.56E-03	5.47
	SA	1.18E-01	1.85E-01	1.82E-01	2.61E-01	3.25E-02	8.30
	DE	3.13E-149	2.09E-103	1.11E-125	4.20E-102	8.43E-103	2
	CMA-ES	3.43E-03	1.09E-02	9.74E-03	2.50E-02	5.50E-03	7.23

Evolution (DE) [35], and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) [36], are added to the comparison pool.

3.3. NNA comparing with other optimizers

Recently, an optimizer has been proposed in the literature with no initial parameters [34]. The idea of having a free-parameter optimizer is a challenging and interesting field in the optimization community, however, this important matter is hard to attain. Having user parameters gives an algorithm the maximum flexibly to adjust itself to a given problem having different level of complexity. However, working on user parameter free algorithms is highly appreciated in case of having design improvements.

Comparing the NNA with two different optimization method is focused on two aspects of NNA. Firstly, The NNA does not require to fine tune user parameters, therefore, a recently developed optimizer with no user parameters named as Teaching-Learning-Based optimization algorithm (TLBO) [34] is compared with the NNA. Regarding the second aspect of NNA, the exploration phase in NNA is based on random search between LB and UB. It is a simple explo-

Table 15
Sum of average ranking using Friedman test for applied optimization methods.

Methods	Total Average Ranking by Friedman Test (Rank)
NNA	**84.41 (1)**
RS	271.74 (13)
TLBO	103.21 (3)
ICA	232.52 (12)
CS	139.35 (8)
GSA	107.27 (4)
WCA	163.17 (10)
HS	126.03 (7)
PSO	206.81 (11)
GA	116.06 (5)
SA	140.05 (9)
DE	85.33 (2)
CMA-ES	125.25 (6)

ration approach which one can use in any optimizer, therefore, based on exploration phases of NNA, a simple random search (RS) is also compared with the NNA. One may claim that the NNA is based on random search or random walk, this simple comparison shows the huge gap between the results obtained by RS and NNA. Hence, we can conclude that this superiority (solution improvements) is rely on the unique structure (inspired by ANNs) and strategy of NNA for solving optimization problems. More comparisons with other optimizers can be included, however, the similarity of having no initial parameter and random search exploration were very close to TLBO and RS methods.

The NNA is designed to have no parameters with the aid of its unique structure. The TLBO algorithm is also an optimizer with the same characteristic in this matter. Random search method is a simple searching strategy looking for candidate solution between lower and upper bounds. Table 14 tabulates the obtained optimization results using the NNA, TLBO, RS, and the other modern optimizers for all considered benchmarks.

The Friedman test with the confidence level of 0.05 has been used for evaluating the significance level of differences among the reported methods. The Friedman test [37] is a nonparametric statistical test used to detect differences among algorithms. The Friedman test first ranks the j^{th} of k algorithms on the i^{th} of N datasets, and then calculates the average rank according to the F-distribution (i.e., distribution value) throughout all the datasets, and calculates the Friedman statistics.

By observing Table 14, the DE and TLBO have tight competition with the optimization results of NNA. For some cases, NNA has surpassed the DE and TLBO and vice versa. Last columns of Table 14 belongs to Friedman test with the confidence level of 0.05 (α). Table 15 shows the summation of all average ranking obtained from Friedman test for each algorithm (the lower the rank the better the performance).

As can be seen from Table 15, the NNA has been placed at first rank and the DE and TLBO have been located at the second and third ranks, respectively. The RS, as expected, is the worst algorithm for solving the reported benchmarks and could not even find acceptable results for all cases (see Table 14). Looking at huge gaps in the obtained optimization results by the NNA and RS, we can conclude that the NNA is not based on the simple random search even in its exploration phase.

With the aim of obtaining rigorous and fair conclusion, two other statistical tests have been carried out in this research including the Kruskal-Wallis H test [38] and Multiple Comparison Test [39]. These tests have been conducted for proving if there are significant differences in the results obtained by all methods and each method compared with the another.

The Kruskal-Wallis H test (sometimes also called the öne-way ANOVA on ranks̈) is a rank-based nonparametric test that can be used to determine if there are statistically significant differences between two or more groups of an independent variable on a continuous or ordinal dependent variable. Also, in statistics, the multiple comparison test occurs when one considers a set of statistical inferences simultaneously or infers a subset of parameters selected based on the observed values. Table 16 show the multiple comparison test for comparing two by two methods for reported optimization methods for the results obtained in Table 14.

The second column of Table 16 represent the methods which are compared. The third and fifth columns show lower and upper limits (LL and UL) for 95% confidence intervals for the true mean difference. The fourth column show the difference between the estimated method means. The sixth column (i.e., last column) contains the p-value for a hypothesis test that the corresponding mean difference is equal to zero obtained by Kruskal-Wallis H test with $\alpha = 0.05$. Figs. 6–9 depict the boxplot and multiple comparison plot among the considered optimizers for all considered benchmarks. In Figs. 6–9, vertical axis in Kruskal-Wallis H test is the value of objective function obtained by the studied optimizers

Looking at Figs. 6–9, the left side of the aforementioned figures are boxplots showing a standardized way of displaying the distribution of data based on the five number summary: the minimum value, first quartile, the median value, third quartile, and the maximum value. The right side of Figs. 6–9 demonstrates the multiple comparison test among reported optimizers.

From Figs. 6–9, right side, the red color lines are the methods which significantly differ with the proposed NNA (The blue lines belong to the NNA). The gray color lines are the method which are not significantly different with the proposed NNA. The obtained p-values in last column of Table 16 prove this comparison. For instance, looking at *F17* in Fig. 9, we can see the NNA statistically has outperformed six optimizers (i.e., TLBO, WCA, ICA, PSO, DE, and CMA-ES), while the optimization results obtained by the CS, GSA, HS, GA, and SA are the same with no significance difference. Lines right side of the NNA (i.e., blue lines) show the methods which are not significantly better than the NNA, while lines left side of the NNA show the methods which are statistically better than the NNA for a specific benchmark.

Furthermore, Table 17 tabulates the obtained optimization results along with their statistical tests and ranking for dimension 200 ($\alpha = 0.05$) for 1E + 06 NFEs. Due to the limitation of execution time, only the TLBO and GSA have been selected in the comparison pool.

From Table 17, the NNA obtained the lower total average ranking which shows its overall efficiency compared with the considered optimizers. For the most cases, the NNA has been placed in the first or second ranks. Now, it is interesting to see the performance of the NNA over few iterations. Therefore, Figs. 10 and 11 display the total cost reduction history of reported optimizers over only 200 iterations between selected optimizer given in Table 17. It is worth mentioning that all reported optimizers start with a random initial population between the lower and upper bounds.

By observing Figs. 10 and 11, the NNA has the slowest convergence rate as iteration is increasing. However, interestingly, around final iterations, the NNA catch the other optimization methods. For instance, looking *F1*, *F3*, *F13*, and *F16* (see Figs. 10 and 11), it can be seen, the NNA starts with high value of objective functions at early iterations and although it proceeds with the slow pace, however, with the acceptable reduction in cost values, it reaches around the cost function values attained by other optimizers, and even with more number of iterations, this reduction continues (see Table 17).

Note that starting with larger objective function values means better potential for searching wider region in the search space. In other words, it shows the NNA ability for exploration phase searching farther regions and leads to escape from local optima. The proof is the obtained results reported in Table 17. To name a few cases, looking at *F10*, *F12*, and *F14* shown in Figs. 10 and 11, although, after

Table 16
Statistical test using multiple comparison test for considered optimizers (Continued).

Function	Comparing	LL (95%)	Group Means	UL (95%)	p-value ($\alpha = 0.05$)
1	**NNA vs. TLBO**	−1.5304e+03	−9.3549e-10	1.5304e+03	1
	NNA vs. RS	−9.1758e+04	−9.0228e+04	−8.8697e+04	2.84e-07
	NNA vs. ICA	−1.5648e+03	−3.4409e+01	1.4960e+03	1
	NNA vs. CS	−1.5304e+03	2.2468e-10	1.5304e+03	1
	NNA vs. GSA	−1.5304e+03	2.2463e-10	1.5304e+03	1
	NNA vs. WCA	−1.5304e+03	2.2438e-10	1.5304e+03	1
	NNA vs. HS	−1.5304e+03	−2.3587e-04	1.5304e+03	1
	NNA vs. PSO	−1.5540e+03	−2.3575e+01	1.5068e+03	1
	NNA vs. GA	−1.5304e+03	−2.7148e-03	1.5304e+03	1
	NNA vs. SA	−1.5307e+03	−3.2074e-01	1.5301e+03	1
	NNA vs. DE	−1.5304e+03	2.2462e-10	1.5304e+03	1
	NNA vs. CMA-ES	−1.5304e+03	2.2468e-10	1.5304e+03	1
2	**NNA vs. TLBO**	−4.5988e+01	−4.0766e+01	−3.5545e+01	2.84e-07
	NNA vs. RS	−8.9498e+01	−8.4276e+01	−7.9055e+01	2.84e-07
	NNA vs. ICA	−5.9577e+01	−5.4356e+01	−4.9134e+01	2.84e-07
	NNA vs. CS	−4.8874e+01	−4.3653e+01	−3.8432e+01	2.84e-07
	NNA vs. GSA	−1.4031e+01	−8.8092e+00	−3.5878e+00	2.00e-06
	NNA vs. WCA	−1.0459e+01	−5.2377e+00	−1.6191e-02	4.84e-02
	NNA vs. HS	−6.3981e+00	−1.1766e+00	4.0448e+00	9.99e-01
	NNA vs. PSO	−3.0427e+01	−2.5206e+01	−1.9984e+01	2.84e-07
	NNA vs. GA	−5.5694e+00	−3.4797e-01	4.8735e+00	1
	NNA vs. SA	−5.6457e+00	−4.2423e-01	4.7972e+00	1
	NNA vs. DE	−3.3341e+01	−2.8119e+01	−2.2898e+01	2.84e-07
	NNA vs. CMA-ES	−4.1931e+00	1.0284e+00	6.2499e+00	9.99e-01
3	**NNA vs. TLBO**	−1.5820e+09	−3.5838e+02	1.5820e+09	1
	NNA vs. RS	−3.8760e+10	−3.7178e+10	−3.5597e+10	2.84e-07
	NNA vs. ICA	−1.5821e+09	−6.6734e+04	1.5819e+09	1
	NNA vs. CS	−1.1582e+10	−1.0000e+10	−8.4180e+09	2.84e-07
	NNA vs. GSA	−1.5820e+09	−5.9479e+00	1.5820e+09	1
	NNA vs. WCA	−1.5820e+09	−1.9716e+00	1.5820e+09	1
	NNA vs. HS	−1.5820e+09	−4.9764e+02	1.5820e+09	1
	NNA vs. PSO	−1.5820e+09	−2.8502e+04	1.5820e+09	1
	NNA vs. GA	−1.5820e+09	−2.8681e+02	1.5820e+09	1
	NNA vs. SA	−1.5820e+09	−8.2124e+02	1.5820e+09	1
	NNA vs. DE	−1.5820e+09	4.1224e+01	1.5820e+09	1
	NNA vs. CMA-ES	−1.5820e+09	−1.4167e+03	1.5820e+09	1
4	**NNA vs. TLBO**	−1.3502e+02	−1.0138e+02	−6.7749e+01	2.84e-07
	NNA vs. RS	−6.7095e+02	−6.3732e+02	−6.0368e+02	2.84e-07
	NNA vs. ICA	−3.7557e+02	−3.4193e+02	−3.0830e+02	2.84e-07
	NNA vs. CS	−5.7738e+01	−2.4104e+01	9.5306e+00	4.62e-01
	NNA vs. GSA	−7.1208e+01	−3.7574e+01	−3.9390e+00	1.34e-02
	NNA vs. WCA	−2.1790e+02	−1.8426e+02	−1.5063e+02	2.84e-07
	NNA vs. HS	−2.7076e+01	6.5590e+00	4.0194e+01	9.99e-01
	NNA vs. PSO	−1.8450e+02	−1.5086e+02	−1.1723e+02	2.84e-07
	NNA vs. GA	−3.6918e+01	−3.2836e+00	3.0351e+01	1
	NNA vs. SA	−3.5277e+01	−1.6425e+00	3.1992e+01	1
	NNA vs. DE	−2.7360e+02	−2.3997e+02	−2.0633e+02	2.84e-07
	NNA vs. CMA-ES	−2.9429e+02	−2.6066e+02	−2.2702e+02	2.84e-07
5	**NNA vs. TLBO**	−1.5325e+01	−8.9279e-02	1.5147e+01	1
	NNA vs. RS	−8.5093e+02	−8.3569e+02	−8.2046e+02	2.84e-07
	NNA vs. ICA	−1.5708e+01	−4.7225e-01	1.4764e+01	1
	NNA vs. CS	−1.5179e+01	5.6542e-02	1.5293e+01	1
	NNA vs. GSA	−4.1461e+01	−2.6225e+01	−1.0989e+01	1.17e-06
	NNA vs. WCA	−1.5204e+01	3.1940e-02	1.5268e+01	1
	NNA vs. HS	−1.5185e+01	5.0946e-02	1.5287e+01	1
	NNA vs. PSO	−1.6390e+01	−1.1537e+00	1.4082e+01	1
	NNA vs. GA	−1.5182e+01	5.4354e-02	1.5290e+01	1
	NNA vs. SA	−1.6384e+01	−1.1477e+00	1.4088e+01	1
	NNA vs. DE	−1.5174e+01	6.1786e-02	1.5298e+01	1
	NNA vs. CMA-ES	−1.5174e+01	6.2032e-02	1.5298e+01	1
6	**NNA vs. TLBO**	−1.1802e+01	−9.8179e+00	−7.8341e+00	2.84e-07
	NNA vs. RS	−2.2340e+01	−2.0356e+01	−1.8372e+01	2.84e-07
	NNA vs. ICA	−1.9888e+01	−1.7904e+01	−1.5920e+01	2.84e-07
	NNA vs. CS	−3.9009e+00	−1.9172e+00	6.6582e-02	7.00e-02
	NNA vs. GSA	−1.9837e+00	1.1150e-06	1.9837e+00	1
	NNA vs. WCA	−1.3086e+01	−1.1102e+01	−9.1182e+00	2.84e-07
	NNA vs. HS	−1.9923e+00	−8.5488e-03	1.9752e+00	1
	NNA vs. PSO	−1.4005e+01	−1.2022e+01	−1.0038e+01	2.84e-07
	NNA vs. GA	−1.9945e+00	−1.0723e-02	1.9730e+00	1
	NNA vs. SA	−2.4097e+00	−4.2592e-01	1.5578e+00	9.99e-01
	NNA vs. DE	−1.9837e+00	1.1182e-06	1.9837e+00	1
	NNA vs. CMA-ES	−1.9837e+00	1.1182e-06	1.9837e+00	1

Table 16 (*Continued*)

Function	Comparing	LL (95%)	Group Means	UL (95%)	p-value ($\alpha = 0.05$)
7	**NNA vs. TLBO**	−5.3682e+11	3.9874e-11	5.3682e+11	1
	NNA vs. RS	−1.6378e+12	−1.1010e+12	−5.6417e+11	2.85e-07
	NNA vs. ICA	−5.3684e+11	−2.1503e+07	5.3679e+11	1
	NNA vs. CS	−5.3682e+11	−1.7940e-03	5.3682e+11	1
	NNA vs. GSA	−5.3682e+11	−3.3564e-08	5.3682e+11	1
	NNA vs. WCA	−5.3682e+11	−3.3333e-01	5.3682e+11	1
	NNA vs. HS	−5.3682e+11	−5.9855e-02	5.3682e+11	1
	NNA vs. PSO	−5.3682e+11	−2.1495e+00	5.3682e+11	1
	NNA vs. GA	−5.3682e+11	−1.6662e-02	5.3682e+11	1
	NNA vs. SA	−5.3682e+11	−5.3683e-01	5.3682e+11	1
	NNA vs. DE	−5.3682e+11	3.4775e-11	5.3682e+11	1
	NNA vs. CMA-ES	−5.3682e+11	3.9446e-11	5.3682e+11	1
8	**NNA vs. RS**	−7.6495e+05	−7.5615e+05	−7.4736e+05	2.84e-07
	NNA vs. TLBO	−4.1939e+01	5.1853e-20	4.1939e+01	1
	NNA vs. ICA	−4.3951e+01	−2.0121e+00	3.9927e+01	1
	NNA vs. CS	−4.1939e+01	5.1853e-20	4.1939e+01	1
	NNA vs. GSA	−4.1939e+01	−3.5835e-16	4.1939e+01	1
	NNA vs. WCA	−4.1939e+01	5.1774e-20	4.1939e+01	1
	NNA vs. HS	−4.1944e+01	−4.9857e-03	4.1934e+01	1
	NNA vs. PSO	−3.3598e+02	−2.9405e+02	−2.5211e+02	2.24e-07
	NNA vs. GA	−4.1976e+01	−3.7315e-02	4.1902e+01	1
	NNA vs. SA	−4.2327e+01	−3.8808e-01	4.1551e+01	1
	NNA vs. DE	−4.1939e+01	2.5548e-20	4.1939e+01	1
	NNA vs. CMA-ES	−4.1939e+01	5.1847e-20	4.1939e+01	1
9	**NNA vs. RS**	−4.9176e+02	−4.8007e+02	−4.6838e+02	2.84e-07
	NNA vs. TLBO	−8.5149e+00	3.2132e+00	1.4941e+01	9.99e-01
	NNA vs. ICA	−3.8495e+02	−3.7322e+02	−3.6149e+02	2.24e-07
	NNA vs. CS	−1.8063e+02	−1.6890e+02	−1.5717e+02	2.24e-07
	NNA vs. GSA	−9.4205e+00	2.3076e+00	1.4036e+01	9.99e-01
	NNA vs. WCA	−2.5339e+02	−2.4166e+02	−2.2993e+02	2.24e-07
	NNA vs. HS	−3.6037e+01	−2.4309e+01	−1.2581e+01	2.25e-07
	NNA vs. PSO	−1.7700e+02	−1.6527e+02	−1.5354e+02	2.24e-07
	NNA vs. GA	−1.9360e+01	−7.6315e+00	4.0965e+00	6.03e-01
	NNA vs. SA	−1.7278e+01	−5.5503e+00	6.1778e+00	9.27e-01
	NNA vs. DE	−9.1336e+00	2.5945e+00	1.4323e+01	9.99e-01
	NNA vs. CMA-ES	−8.5155e+00	3.2126e+00	1.4941e+01	9.99e-01
10	**NNA vs. RS**	−5.2633e+03	−5.1806e+03	−5.0980e+03	2.84e-07
	NNA vs. TLBO	−2.7640e+00	0.00e+00	2.7640e+00	1
	NNA vs. ICA	−1.9133e+01	−1.6369e+01	−1.3605e+01	2.24e-07
	NNA vs. CS	−1.3424e+01	−1.0660e+01	−7.8962e+00	2.24e-07
	NNA vs. GSA	−2.8340e+00	−6.9986e-02	2.6941e+00	1
	NNA vs. WCA	−3.3281e+00	−5.6409e-01	2.1999e+00	9.99e-01
	NNA vs. HS	−2.7744e+00	−1.0391e-02	2.7536e+00	1
	NNA vs. PSO	−4.7516e+01	−4.4752e+01	−4.1988e+01	2.24e-07
	NNA vs. GA	−2.7674e+00	−3.3641e-03	2.7607e+00	1
	NNA vs. SA	−3.0375e+00	−2.7345e-01	2.4906e+00	1
	NNA vs. DE	−2.9510e+00	−1.8696e-01	2.5771e+00	1
	NNA vs. CMA-ES	−2.7640e+00	0.00e+00	2.7640e+00	1
11	**NNA vs. RS**	−4.7420e+02	−4.6347e+02	−4.5275e+02	2.84e-07
	NNA vs. TLBO	−4.9789e+00	5.8041e+00	1.6587e+01	8.40e-01
	NNA vs. ICA	−3.7615e+02	−3.6537e+02	−3.5459e+02	2.24e-07
	NNA vs. CS	−1.7742e+02	−1.6664e+02	−1.5586e+02	2.24e-07
	NNA vs. GSA	−6.4050e+00	4.3780e+00	1.5161e+01	9.75e-01
	NNA vs. WCA	−2.5376e+02	−2.4297e+02	−2.3219e+02	2.24e-07
	NNA vs. HS	−3.1613e+01	−2.0829e+01	−1.0046e+01	2.40e-07
	NNA vs. PSO	−1.6890e+02	−1.5811e+02	−1.4733e+02	2.24e-07
	NNA vs. GA	−1.5707e+01	−4.9235e+00	5.8595e+00	9.43e-01
	NNA vs. SA	−1.3430e+01	−2.6465e+00	8.1365e+00	9.99e-01
	NNA vs. DE	−5.4084e+00	5.3746e+00	1.6158e+01	8.98e-01
	NNA vs. CMA-ES	−4.9808e+00	5.8022e+00	1.6585e+01	8.40e-01
12	**NNA vs. RS**	−5.8150e+04	−5.6937e+04	−5.5725e+04	2.84e-07
	NNA vs. TLBO	−6.4901e+01	−5.2786e+01	−4.0670e+01	2.24e-07
	NNA vs. ICA	−1.3538e+02	−1.2326e+02	−1.1115e+02	2.24e-07
	NNA vs. CS	−1.4788e+01	−2.6729e+00	9.4424e+00	9.99e-01
	NNA vs. GSA	−3.0788e+01	−1.8672e+01	−6.5570e+00	3.05e-05
	NNA vs. WCA	−9.8942e+01	−8.6827e+01	−7.4711e+01	2.24e-07
	NNA vs. HS	−1.2633e+01	−5.1805e-01	1.1597e+01	1
	NNA vs. PSO	−1.0055e+02	−8.8435e+01	−7.6320e+01	2.24e-07
	NNA vs. GA	−1.0301e+01	1.8138e+00	1.3929e+01	1
	NNA vs. SA	−8.5844e+00	3.5309e+00	1.5646e+01	9.98e-01
	NNA vs. DE	−5.5294e+00	6.5859e+00	1.8701e+01	8.31e-01
	NNA vs. CMA-ES	−1.1906e+02	−1.0694e+02	−9.4829e+01	2.24e-07

Table 16 (*Continued*)

Function	Comparing	LL (95%)	Group Means	UL (95%)	p-value ($\alpha = 0.05$)
13	**NNA vs. RS**	−1.7679e+10	−1.6823e+10	−1.5966e+10	2.84e-07
	NNA vs. TLBO	−6.1005e+08	−6.1005e+08	6.1005e+08	1
	NNA vs. ICA	−6.1020e+08	−6.1020e+08	6.0991e+08	1
	NNA vs. CS	−9.9520e+09	−9.9520e+09	−8.7319e+09	2.24e-07
	NNA vs. GSA	−6.1005e+08	−6.1005e+08	6.1005e+08	1
	NNA vs. WCA	−6.1005e+08	−6.1005e+08	6.1005e+08	1
	NNA vs. HS	−6.1006e+08	−6.1006e+08	6.1005e+08	1
	NNA vs. PSO	−6.1006e+08	−6.1006e+08	6.1005e+08	1
	NNA vs. GA	−6.1005e+08	−6.1005e+08	6.1005e+08	1
	NNA vs. SA	−6.1006e+08	−6.1006e+08	6.1005e+08	1
	NNA vs. DE	−6.1005e+08	−6.1005e+08	6.1005e+08	1
	NNA vs. CMA-ES	−6.1006e+08	−6.1006e+08	6.1005e+08	1
14	**NNA vs. RS**	−4.7643e+02	−4.5351e+02	−4.3059e+02	2.84e-07
	NNA vs. TLBO	−1.0180e+02	−7.8923e+01	−5.6042e+01	2.24e-07
	NNA vs. ICA	−2.6038e+02	−2.3750e+02	−2.1462e+02	2.24e-07
	NNA vs. CS	−4.3946e+01	−2.1066e+01	1.8147e+00	1.05e-01
	NNA vs. GSA	−5.1555e+01	−2.8674e+01	−5.7941e+00	2.47e-03
	NNA vs. WCA	−1.6035e+02	−1.3747e+02	−1.1459e+02	2.24e-07
	NNA vs. HS	−2.0048e+01	2.8322e+00	2.5713e+01	1
	NNA vs. PSO	−1.3861e+02	−1.1573e+02	−9.2849e+01	2.24e-07
	NNA vs. GA	−2.7506e+01	−4.6252e+00	1.8255e+01	9.99e-01
	NNA vs. SA	−2.5166e+01	−2.2861e+00	2.0594e+01	1
	NNA vs. DE	−1.9519e+02	−1.7231e+02	−1.4943e+02	2.24e-07
	NNA vs. CMA-ES	−2.2179e+02	−1.9891e+02	−1.7603e+02	2.24e-07
15	**NNA vs. RS**	−5.2159e+04	−4.4783e+04	−3.7407e+04	2.84e-07
	NNA vs. TLBO	−1.3840e+02	3.4801e-11	1.3840e+02	1
	NNA vs. ICA	−4.8759e+02	−3.4919e+02	−2.1079e+02	2.24e-07
	NNA vs. CS	−1.3840e+02	3.4801e-11	1.3840e+02	1
	NNA vs. GSA	−1.3840e+02	1.5435e-11	1.3840e+02	1
	NNA vs. WCA	−1.4240e+02	−4.0000e+00	1.3440e+02	1
	NNA vs. HS	−1.3864e+02	−2.4237e-01	1.3816e+02	1
	NNA vs. PSO	−1.4601e+02	−7.6056e+00	1.3079e+02	1
	NNA vs. GA	−1.3850e+02	−1.0236e-01	1.3830e+02	1
	NNA vs. SA	−1.3910e+02	−6.9551e-01	1.3771e+02	1
	NNA vs. DE	−1.3840e+02	3.4801e-11	1.3840e+02	1
	NNA vs. CMA-ES	−1.3852e+02	−1.2311e-01	1.3828e+02	1
16	**NNA vs. RS**	−2.5479e+04	−2.4870e+04	−2.4262e+04	2.84e-07
	NNA vs. TLBO	−1.0591e+00	−3.8511e-01	2.8885e-01	7.79e-01
	NNA vs. ICA	−1.7741e+00	−1.1001e+00	−4.2619e-01	6.38e-06
	NNA vs. CS	−7.8628e-01	−1.1232e-01	5.6164e-01	9.99e-01
	NNA vs. GSA	−7.0606e-01	−3.2095e-02	6.4187e-01	1
	NNA vs. WCA	−1.5885e+00	−9.1459e-01	−2.4062e-01	5.68e-04
	NNA vs. HS	−6.7397e-01	−1.3415e-05	6.7395e-01	1
	NNA vs. PSO	−2.3788e+00	−1.7048e+00	−1.0308e+00	2.24e-07
	NNA vs. GA	−6.7478e-01	−8.2190e-04	6.7314e-01	1
	NNA vs. SA	−8.2435e-01	−1.5039e-01	5.2357e-01	9.99e-01
	NNA vs. DE	−7.0606e-01	−3.2095e-02	6.4187e-01	1
	NNA vs. CMA-ES	−2.5863e+00	−1.9123e+00	−1.2383e+00	2.24e-07
17	**NNA vs. RS**	−6.1238e+02	−5.8760e+02	−5.6281e+02	2.84e-07
	NNA vs. TLBO	−1.1569e+02	−9.3175e+01	−7.0656e+01	2.24e-07
	NNA vs. ICA	−2.5318e+02	−2.3066e+02	−2.0814e+02	2.24e-07
	NNA vs. CS	−2.5044e+01	−2.5250e+00	1.9994e+01	1
	NNA vs. GSA	−3.9903e+01	−1.7384e+01	5.1344e+00	3.25e-01
	NNA vs. WCA	−6.0182e+01	−3.7663e+01	−1.5145e+01	3.18e-06
	NNA vs. HS	−2.1843e+01	6.7580e-01	2.3195e+01	1
	NNA vs. PSO	−2.3317e+02	−2.1065e+02	−1.8813e+02	2.24e-07
	NNA vs. GA	−2.3961e+01	−1.4426e+00	2.1076e+01	1
	NNA vs. SA	−2.9663e+01	−7.1444e+00	1.5374e+01	9.96e-01
	NNA vs. DE	−9.2664e+01	−7.0145e+01	−4.7626e+01	2.24e-07
	NNA vs. CMA-ES	−1.4153e+02	−1.1901e+02	−9.6494e+01	2.24e-07
18	**NNA vs. RS**	−3.5426e+02	−3.3942e+02	−3.2457e+02	2.84e-07
	NNA vs. TLBO	−1.9874e+01	−4.8127e+00	1.0249e+01	9.96e-01
	NNA vs. ICA	−2.6433e+02	−2.4927e+02	−2.3421e+02	2.24e-07
	NNA vs. CS	−5.9528e+01	−4.4466e+01	−2.9405e+01	2.24e-07
	NNA vs. GSA	−2.0876e+01	−5.8146e+00	9.2470e+00	9.83e-01
	NNA vs. WCA	−2.1763e+02	−2.0257e+02	−1.8751e+02	2.24e-07
	NNA vs. HS	−2.8288e+01	−1.3226e+01	1.8353e+00	1.51e-01
	NNA vs. PSO	−1.3393e+02	−1.1887e+02	−1.0380e+02	2.24e-07
	NNA vs. GA	−1.9780e+01	−4.7186e+00	1.0343e+01	9.97e-01
	NNA vs. SA	−2.0567e+01	−5.5053e+00	9.5562e+00	9.89e-01
	NNA vs. DE	−1.4451e+01	6.1102e-01	1.5673e+01	1
	NNA vs. CMA-ES	−3.9128e+01	−2.4067e+01	−9.0053e+00	1.16e-05

Table 16 (*Continued*)

Function	Comparing	LL (95%)	Group Means	UL (95%)	p-value (α = 0.05)
19	**NNA vs. RS**	−4.3049e+02	−4.0415e+02	−3.7781e+02	2.84e-07
	NNA vs. TLBO	−3.8497e+01	−1.1606e+01	1.5285e+01	9.61e-01
	NNA vs. ICA	−3.8215e+02	−3.5526e+02	−3.2837e+02	2.24e-07
	NNA vs. CS	−1.9767e+02	−1.7078e+02	−1.4389e+02	2.24e-07
	NNA vs. GSA	−9.2211e+01	−6.5320e+01	−3.8429e+01	2.24e-07
	NNA vs. WCA	−3.3106e+02	−3.0417e+02	−2.7728e+02	2.24e-07
	NNA vs. HS	−6.9754e+01	−4.2863e+01	−1.5973e+01	1.24e-05
	NNA vs. PSO	−2.2835e+02	−2.0146e+02	−1.7457e+02	2.24e-07
	NNA vs. GA	−3.5814e+01	−8.9235e+00	1.7967e+01	9.95e-01
	NNA vs. SA	−7.1955e+01	−4.5064e+01	−1.8173e+01	3.01e-06
	NNA vs. DE	−2.2693e+01	4.1981e+00	3.1089e+01	1
	NNA vs. CMA-ES	−5.4063e+02	−5.1374e+02	−4.8685e+02	2.24e-07
20	**NNA vs. RS**	−8.0428e+01	−7.7368e+01	−7.4308e+01	2.84e-07
	NNA vs. TLBO	−3.2941e+00	−1.9853e-01	2.8970e+00	1
	NNA vs. ICA	−6.1910e+01	−5.8815e+01	−5.5719e+01	2.24e-07
	NNA vs. CS	−1.6149e+01	−1.3053e+01	−9.9578e+00	2.24e-07
	NNA vs. GSA	−3.6799e+00	−5.8434e-01	2.5112e+00	9.99e-01
	NNA vs. WCA	−4.7301e+01	−4.4206e+01	−4.1110e+01	2.24e-07
	NNA vs. HS	−1.0453e+01	−7.3577e+00	−4.2622e+00	2.24e-07
	NNA vs. PSO	−3.2574e+01	−2.9478e+01	−2.6383e+01	2.24e-07
	NNA vs. GA	−4.9925e+00	−1.8970e+00	1.1986e+00	6.91e-01
	NNA vs. SA	−4.7391e+00	−1.6436e+00	1.4520e+00	8.52e-01
	NNA vs. DE	−2.9531e+00	1.4244e-01	3.2380e+00	1
	NNA vs. CMA-ES	−6.6035e+00	−3.5079e+00	−4.1239e-01	1.14e-02
21	**NNA vs. RS**	−1.4598e+03	−1.4333e+03	−1.4068e+03	2.84e-07
	NNA vs. TLBO	−1.3751e+00	8.8501e-38	1.3751e+00	1
	NNA vs. ICA	−1.0209e+01	−8.8336e+00	−7.4585e+00	2.24e-07
	NNA vs. CS	−6.5445e+00	−5.1694e+00	−3.7943e+00	2.24e-07
	NNA vs. GSA	−1.3751e+00	−8.7725e-33	1.3751e+00	1
	NNA vs. WCA	−2.2983e+00	−9.2328e-01	4.5180e-01	5.53e-01
	NNA vs. HS	−1.3788e+00	−3.7665e-03	1.3713e+00	1
	NNA vs. PSO	−2.2369e+01	−2.0994e+01	−1.9619e+01	2.24e-07
	NNA vs. GA	−1.3768e+00	−1.7022e-03	1.3734e+00	1
	NNA vs. SA	−1.5597e+00	−1.8468e-01	1.1904e+00	1
	NNA vs. DE	−1.3751e+00	8.8501e-38	1.3751e+00	1
	NNA vs. CMA-ES	−1.3860e+00	−1.0903e-02	1.3642e+00	1

Table 17
Statistical test and optimization results obtained by recent optimizers for dimension 200.

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test
1	**GSA**	3.41E-13	5.00E-13	4.97E-13	6.65E-13	9.81E-14	1.06
	TLBO	7.00E-01	4.20E+02	4.09E+01	5.40E+03	1.02E+03	3
	NNA	6.04E-10	4.82E-07	4.9E-09	7.03E-06	1.81E-06	1.93
2	**GSA**	4.16E+01	6.27E+01	6.12E+01	8.21E+01	1.38E+01	2.20
	TLBO	5.99E+01	6.88E+01	6.91E+01	7.71E+01	3.92E+00	2.80
	NNA	1.56E+01	2.40E+01	2.41E+01	2.83E+01	3.33E+00	1
3	**GSA**	3.38E+02	4.91E+02	4.83E+02	6.73E+02	8.83E+01	1.6
	TLBO	1.71E+04	6.35E+06	3.27E+05	7.95E+07	1.77E+07	3
	NNA	3.28E+02	4.79E+02	4.32E+02	9.35E+02	1.76E+02	1.40
4	**GSA**	2.78E+02	4.21E+02	4.26E+02	5.43E+02	7.83E+01	2
	TLBO	4.82E+02	6.19E+02	6.20E+02	8.14E+02	7.56E+01	2.93
	NNA	1.93E+02	2.37E+02	2.39E+02	2.95E+02	2.91E+01	1.06
5	**GSA**	7.32E+01	1.08E+02	1.09E+02	1.41E+02	2.22E+01	3
	TLBO	1.74E-01	3.89E+00	1.69E+00	2.98E+01	6.58E+00	2
	NNA	3.95E-10	2.13E-02	1.48E-02	1.32E-01	3.24E-02	1
6	**GSA**	1.63E+00	2.59E+00	2.64E+00	3.25E+00	4.21E-01	2
	TLBO	1.59E+01	1.66E+01	1.66E+01	1.71E+01	3.30E-01	3
	NNA	2.72E-06	1.81E-04	1.52E-05	1.56E-03	4.27E-04	1
7	**GSA**	2.02E-07	2.83E-07	2.89E-07	3.81E-07	5.43E-08	3
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	4.48E-10	2.04E-09	1.69E-09	8.07E-09	1.98E-09	2
8	**GSA**	1.17E-14	1.77E-14	1.79E-14	2.3E-14	3.45E-15	3
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	6.98E-19	9.9E-18	5.9E-18	3.98E-17	1.02E-17	2

Table 17 (*Continued*)

Function	Methods	Best	Average	Median	Worst	SD	Average Rankings Friedman Test
9	**GSA**	1.05E+02	1.56E+02	1.58E+02	2.06E+02	3.40E+01	2.13
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	1.21E+02	2.09E+02	1.89E+02	3.13E+02	5.56E+01	2.86
10	**GSA**	2.90E+00	3.20E+00	3.14E+00	4.14E+00	6.33E-01	3
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.40
	NNA	0.00E+00	4.44E-17	0.00E+00	4.44E-16	1.17E-16	1.60
11	**GSA**	1.31E+02	1.99E+02	2.04E+02	2.60E+02	4.30E+01	2.53
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	1.41E+02	2.11E+02	2.25E+02	2.65E+02	3.75E+01	2.46
12	**GSA**	1.79E+02	2.57E+02	2.48E+02	3.49E+02	4.82E+01	1.52
	TLBO	2.63E+02	4.49E+02	3.19E+02	2.26E+03	3.90E+02	2.60
	NNA	2.13E+02	2.72E+02	2.77E+02	3.14E+02	3.29E+01	1.86
13	**GSA**	3.26E+02	4.90E+02	5.10E+02	6.46E+02	9.12E+01	1
	TLBO	1.21E+03	4.99E+04	7.22E+03	5.13E+05	1.19E+05	2.93
	NNA	5.87E+02	8.26E+02	7.52E+02	1.52E+03	2.53E+02	2.06
14	**GSA**	1.75E+02	2.77E+02	2.81E+02	3.46E+02	4.84E+01	2
	TLBO	4.27E+02	4.95E+02	4.95E+02	5.64E+02	3.75E+01	3
	NNA	1.20E+02	1.60E+02	1.53E+02	1.93E+02	2.10E+01	1
15	**GSA**	1.40E-02	2.17E-02	2.24E-02	2.73E-02	4.08E-03	3
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	1.45E-08	1.40E-07	8.92E-08	3.68E-07	1.29E-07	2
16	**GSA**	1.01E-26	1.51E-26	1.55E-26	2.01E-26	3.37E-27	1
	TLBO	1.42E-20	1.19E-01	4.59E-15	1.79E+00	4.54E-01	2.20
	NNA	4.02E-14	3.43E-10	1.88E-12	3.38E-09	8.77E-10	2.80
17	**GSA**	8.95E+01	1.28E+02	1.30E+02	1.71E+02	2.51E+01	2
	TLBO	6.04E+02	8.83E+02	8.92E+02	1.28E+03	1.65E+02	3
	NNA	4.68E+00	1.02E+01	9.95E+00	2.69E+01	5.54E+00	1
18	**GSA**	8.91E+01	1.30E+02	1.35E+02	1.77E+02	3.11E+01	3
	TLBO	1.14E-03	2.64E+01	1.58E+01	7.95E+01	2.25E+01	1.20
	NNA	2.79E+01	4.60E+01	4.18E+01	9.48E+01	1.60E+01	1.80
19	**GSA**	1.09E+02	1.54E+02	1.47E+02	2.13E+02	3.15E+01	2.20
	TLBO	4.76E-07	3.35E+01	1.15E+00	4.34E+02	1.09E+02	1
	NNA	1.27E+02	2.29E+02	2.39E+02	3.20E+02	5.76E+01	2.80
20	**GSA**	4.18E+00	6.54E+00	6.91E+00	8.37E+00	1.26E+00	1.86
	TLBO	0.00E+00	3.76E+00	2.62E+00	1.73E+01	3.81E+00	1.20
	NNA	3.41E+00	2.30E+01	2.34E+01	5.12E+01	1.24E+01	2.93
21	**GSA**	1.05E+00	1.49E+00	1.46E+00	2.09E+00	2.92E-01	3
	TLBO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1
	NNA	3.04E-86	1.06E-23	2.07E-60	1.59E-22	4.12E-23	2
Total Average Ranking by Friedman Test	**GSA**	46.10			**Total Ranking**	(3)	
	TLBO	41.26				(2)	
	NNA	**38.56**				**(1)**	

200 iterations (i.e., 10,000 NFEs), the obtained values by the TLBO and GSA are better than the NNA, however, if iteration is increasing, the NNA can obtain better and statistically significance optimization results (see Table 17). Also, the steepest slope can be seen in the convergence of NNA in all cases.

3.4. Computational complexity

Furthermore, NNA complexity on both 10 and 30 dimensions has been evaluated following the guidelines provided in CEC'15 [40]. The value for T_0 has been calculated using the test program provided in the guidelines. The calculated computing time for the test program is $T_0 = 0.274$ s. Next, the average complete computing time, T_1, for all the benchmark functions is calculated. Finally, the algorithm complexity (T_1/T_0) for both dimensions has been measured and the NNA complexity has been tabulated in Table 18.

By observing Table 18, value of T_B/T_A equal to one shows the zero complexity from dimension 10 to 30 for the reported benchmarks. Values greater than one represent the complexity of computational time using the NNA. F_{12} to F_{21} categorized as hybrid functions (see Table 6) have shown higher T_B/T_A values due to their complication, especially for F_{19}. Relative computationally expensive problems are highlighted in bold in Table 18. For the rest of functions, the average T_B/T_A is almost equal to 3 which means the intricacy for dimension 10 to 30 is increased for almost 3 times.

3.5. NNA for solving real world constrained engineering design problems

Regarding the constrained benchmark problems, pressure vessel design, welded beam design, speed reducer design problem, three-bar truss design problem, and gear train design problem extensively used in the literature, have been investigated in this paper. Those constrained problems are recognized as well-known engineering benchmarks in the literature [41,42]. Mathematical formulation and required descriptions for all reported constrained engineering benchmark problems can be found in detail in the literature [41]. Using the feasible approach (i.e., direct approach) [43], the NNA has been equipped for solving constrained optimization problem.

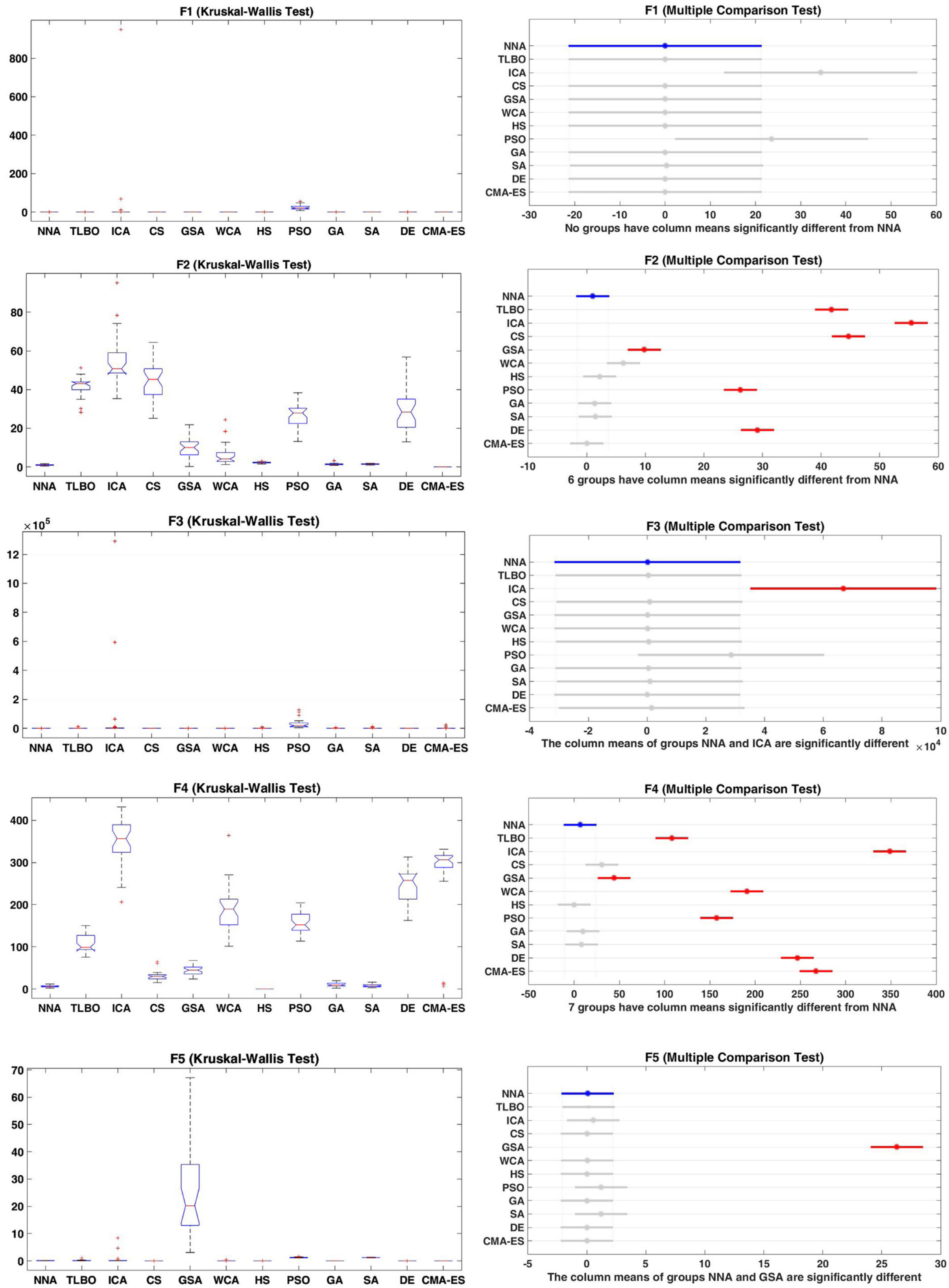

Fig. 6. Box-plot of objective function using the reported optimizers.

For the fixed predefined NFEs (i.e., 50,000), the aforementioned optimizers used in the previous sections have been considered and implemented for optimal solving of real world constrained engineering design problems. Statistical optimization results along with statistical tests (i.e., Friedman with $\alpha = 0.05$) obtained by the proposed optimizer and the other optimizers are tabulated in Table 19 for reported constrained engineering design problems. Table 20 shows overall ranking of reported

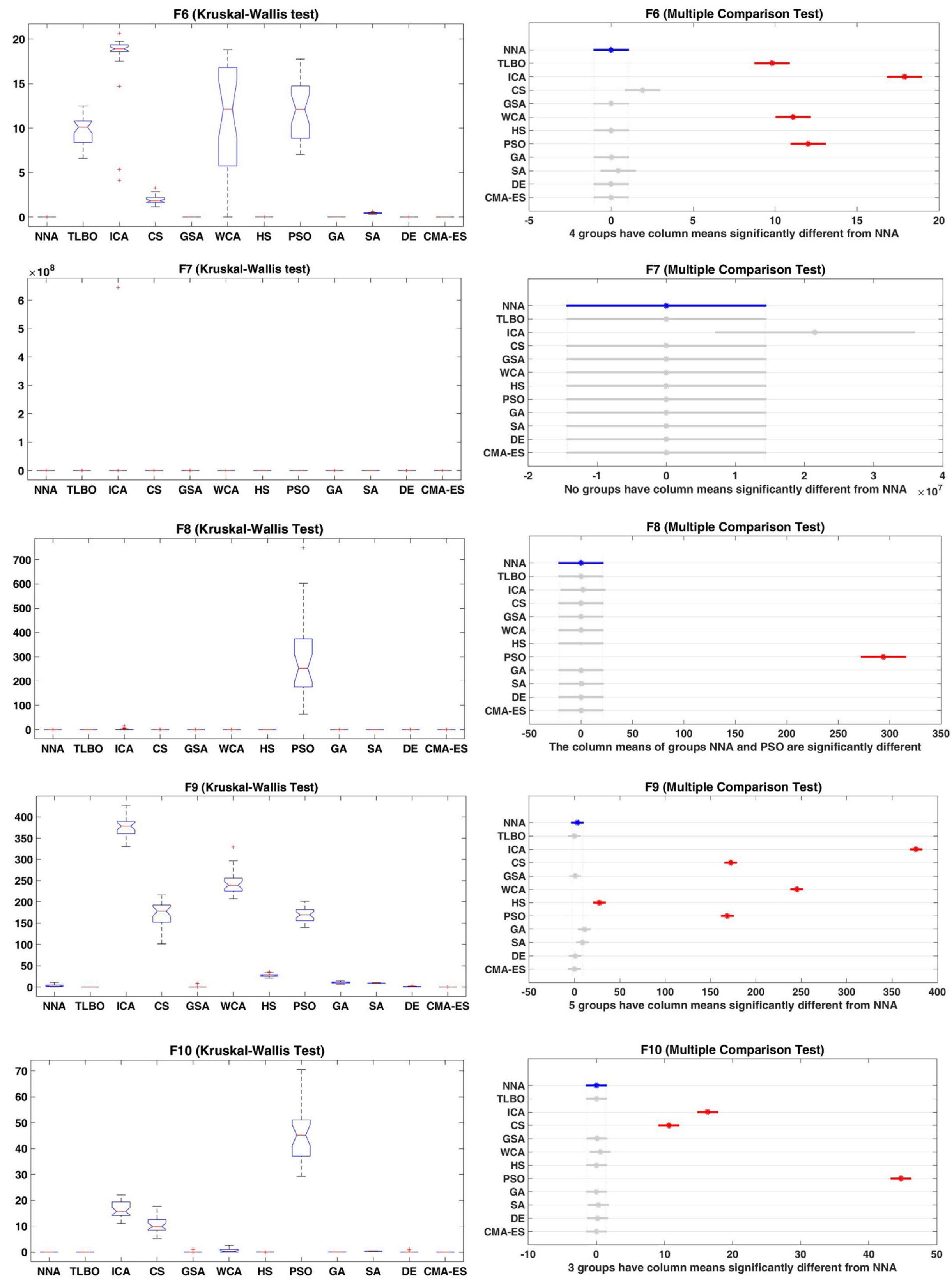

Fig. 7. Box-plot of objective function using the reported optimizers.

optimization methods for solving constrained engineering problems.

Looking at Tables 19 and 20, under pre-defined NFEs, in terms of statistical results and tests, the NNA has obtained lower average ranking using the Friedman test. As the performance of NNA has been shown in Sections 3.2 and 3.3, here the efficiency of NNA for handling constrained optimization problems has been tested and validated over the other optimizers. It shows that the proposed NNA

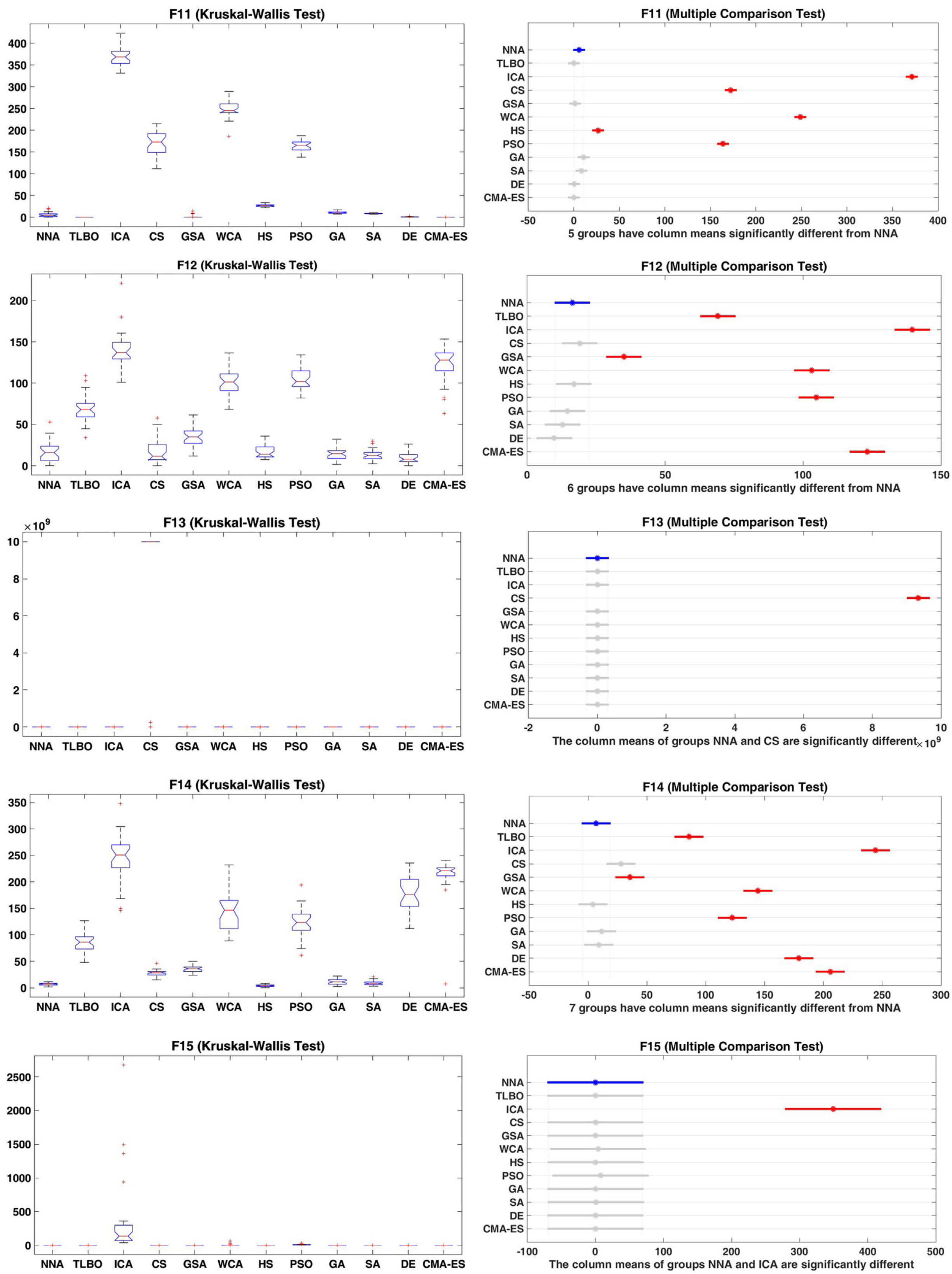

Fig. 8. Box-plot of objective function using the reported optimizers.

can be successfully applied for unconstrained and constrained optimization problems having many local optima. The NNA as for other optimizers can be used for solving optimization problems, where a problem is highly nonlinear and non-convex. Having mature convergence, being an algorithm without any effort for fine tuning initial parameters, and capability of finding quality solutions make the NNA attractive when one encounters NP-hard optimization problem.

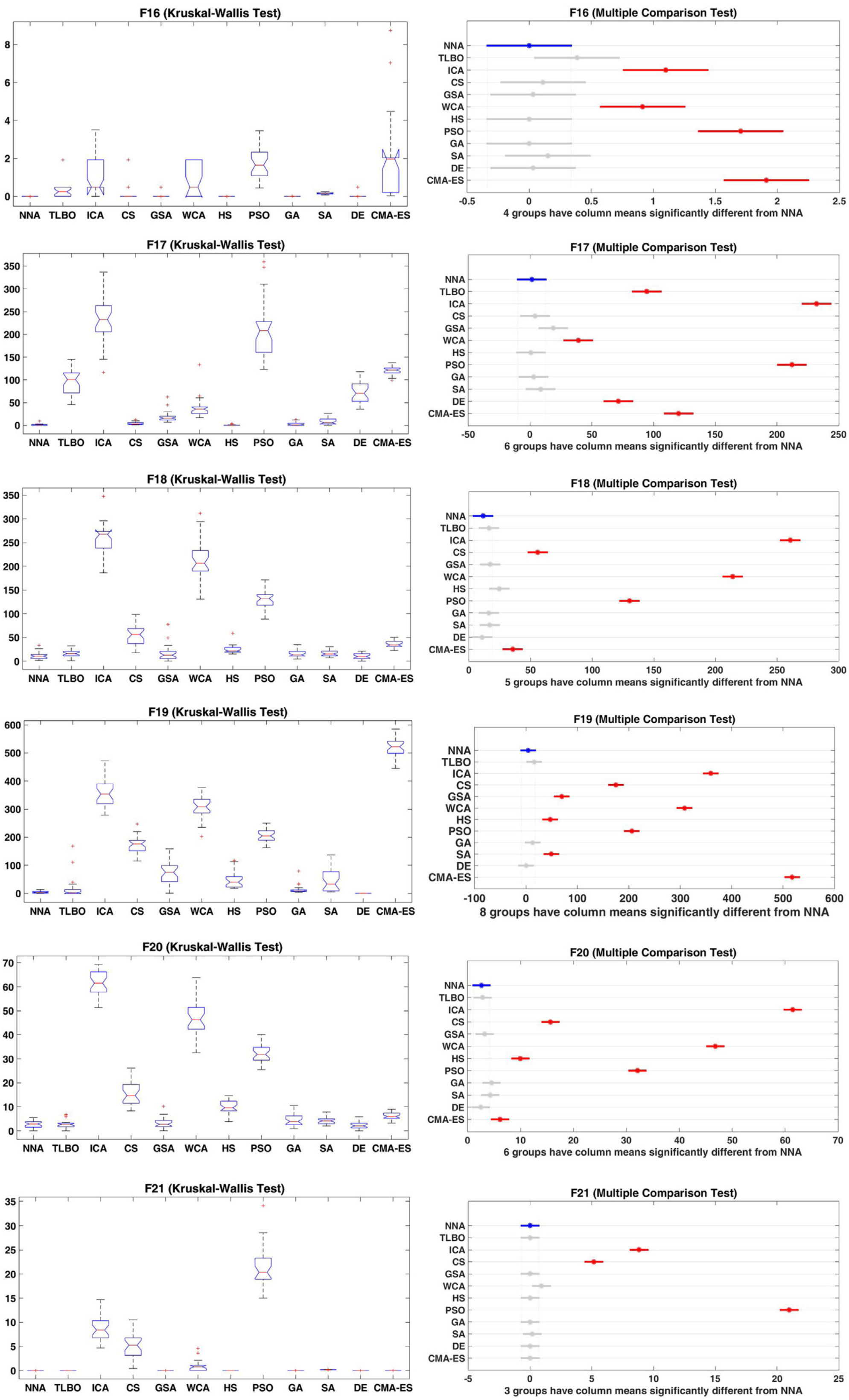

Fig. 9. Box-plot of objective function using the reported optimizers.

The reported simulation and optimization results should not be taken to mean that the NNA is "better" than other population-based and evolutionary optimization algorithms. Such a general statement would be an oversimplification, especially in view of the no free lunch theorem [44]. However, the results presented in this paper show that the NNA provides superior performance than the utilized algorithms we tested for the particular unconstrained benchmarks and constrained engineering design problems. The

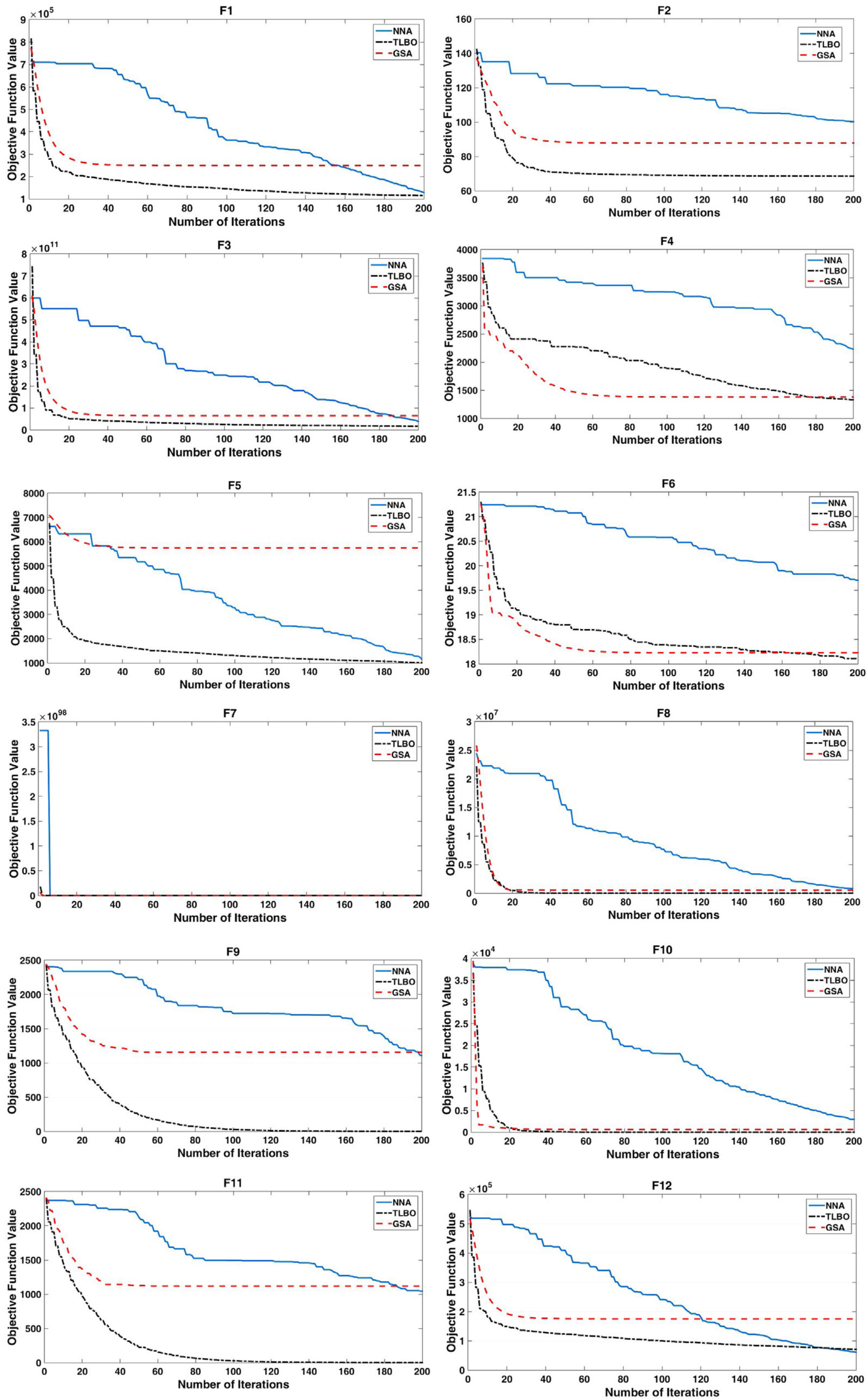

Fig. 10. Convergence history for considered functions with dimension 200.

proposed NNA in this paper is a simple search model inspired by ANNs. The purpose was to introduce the unique structure of ANNs as a new model for developing a metaheuristic optimization method and further developments in this area can be investigated. Furthermore, randomizing some of the control parameters [45] may have positive effects on the performance of the NNA.

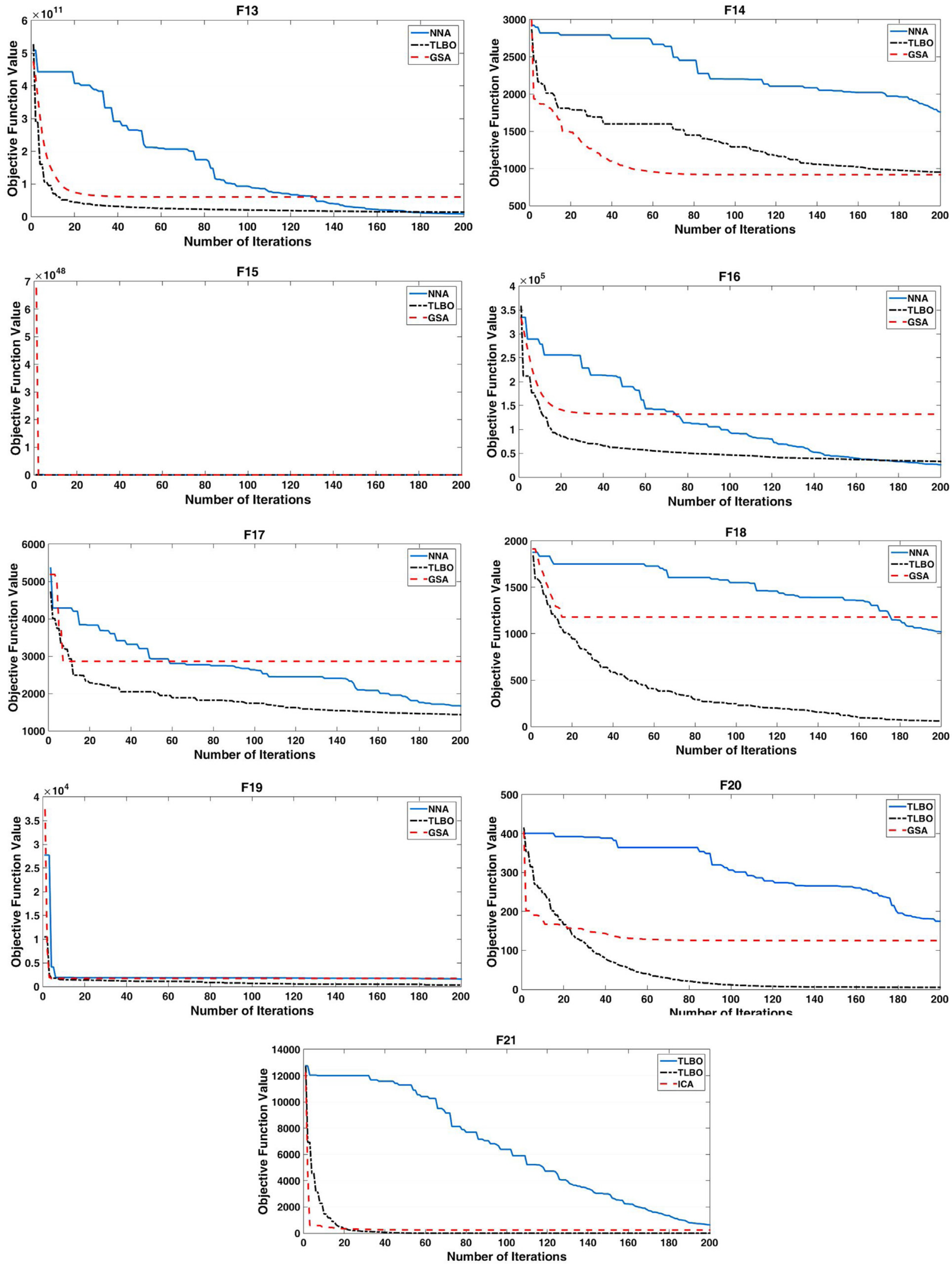

Fig. 11. Convergence history for considered functions with dimension 200.

4. Conclusions and future works

This paper presented a dynamic optimization model inspired by structure and concept of artificial neural networks (ANNs) and biological nervous systems, so called neural network algorithm (NNA). The NNA benefits from unique structure of ANNs along with search operators for solving complex optimization problems. The difficulty of tuning the initial parameters which is the most common

Table 18
Computational complexity measured in the NNA.

Function	$D=10$		$D=30$		T_B/T_A
	T_1	T_A=T_1/T_0	T_1	T_B=T_1/T_0	
F_1	5.707	20.775	16.649	60.607	2.917
F_2	5.804	21.128	16.503	60.076	2.843
F_3	6.080	22.133	16.618	60.495	2.733
F_4	5.744	20.910	17.215	62.668	2.997
F_5	6.181	22.500	18.470	67.236	2.988
F_6	5.546	20.189	17.669	64.321	3.185
F_7	5.061	18.423	15.895	57.863	3.140
F_8	6.478	23.582	21.683	78.933	3.347
F_9	5.611	20.425	22.298	81.172	**3.974**
F_{10}	5.498	20.014	17.369	63.228	3.159
F_{11}	5.394	19.635	22.391	81.510	**4.151**
F_{12}	7.447	27.109	39.200	142.701	**5.263**
F_{13}	5.666	20.626	22.358	81.390	**3.945**
F_{14}	7.821	28.471	41.768	152.049	**5.340**
F_{15}	8.004	29.137	38.654	140.713	**4.829**
F_{16}	7.185	26.155	27.326	99.475	**3.803**
F_{17}	6.366	23.174	21.320	77.611	3.349
F_{18}	9.340	34.000	51.172	186.283	**5.478**
F_{19}	8.652	31.496	51.719	188.274	**5.977**
F_{20}	9.309	33.887	53.224	193.753	**5.717**
F_{21}	8.294	30.192	43.839	159.588	**5.285**

Table 19
Comparison of statistical optimization results given by different methods for reported constrained benchmarks.

Methods	Worst	Average	Best	SD	Friedman Test (Ranking)
Pressure vessel design problem					
PSO	6946.41	6147.03	5906.30	2.08E+00	5.90 (6)
GA	7187.02	6405.38	6001.26	2.88E+00	7.16 (8)
SA	5885.44	5885.37	5885.33	2.67E+00	3.10 (3)
HS	7317.85	6794.55	6333.48	3.06E+00	8.30 (9)
DE	5885.32	5885.32	5885.32	9.25E-13	1.03 (1)
GSA	170036	902451	120366	5.39E+04	10 (10)
ICA	6596.42	5998.37	5885.33	1.60E+01	4.63 (4)
WCA	7319.00	6302.32	5885.33	5.36E+02	5.50 (5)
TLBO	5888.93	5885.53	5885.33	7.01E-01	2.53 (2)
NNA	7310.65	6501.62	5885.33	5.18E+02	6.83 (7)
Welded beam design problem					
PSO	1.72916	1.72524	1.72485	8.55E-04	3.80 (3)
GA	3.80378	2.86337	1.98404	4.65E-01	9.26 (7)
SA	1.76635	1.74870	1.73353	7.97E-03	6.56 (5)
HS	3.94678	2.81964	2.02476	4.80E-01	9.26 (7)
DE	1.72490	1.72490	1.72490	1.12E-15	3.80 (3)
GSA	2.81882	2.37691	2.05103	1.96E-01	8.46 (6)
ICA	1.91091	1.76397	1.72606	4.80E-02	6.36 (4)
WCA	1.75249	1.72596	1.72485	5.02E-03	3.80 (3)
TLBO	1.72485	1.72485	1.72485	1.12E-15	1.36 (1)
NNA	1.72613	1.72495	1.72485	2.71E-04	2.30 (2)
Speed reducer design problem					
PSO	3004.18	2994.47	2994.47	2.87E+00	6.10 (6)
GA	3853.02	3353.02	3019.19	3.12E+00	9.33 (9)
SA	2994.56	2994.50	2994.47	2.03E-02	6.83 (8)
HS	2994.50	2994.48	2994.47	7.64E-03	5.43 (5)
DE	2994.50	2994.50	2994.50	1.85E-12	6.76 (7)
GSA	4365.52	3617.63	3114.08	3.52E+02	9.66 (10)
ICA	2994.47	2994.47	2994.47	9.13E-11	2.51 (3)
WCA	3003.80	2996.33	2994.47	3.79E+00	4.83 (4)
TLBO	2994.47	2994.47	2994.47	1.85E-12	2.41 (2)
NNA	2994.47	2994.47	2994.47	1.48E-05	1.10 (1)
Three-bar truss design problem					
PSO	263.89	263.89	263.89	0.00E+00	2 (2)
GA	267.88	264.14	263.89	7.27E-01	8.36 (8)
SA	263.91	263.90	263.89	4.52E-03	6.96 (7)
HS	265.04	264.06	263.89	2.60E-01	8.50 (9)
DE	263.89	263.89	263.89	0.00E+00	6.45 (6)
GSA	264.09	263.96	263.89	5.53E-02	8.81 (10)
ICA	263.93	263.90	263.89	8.01E-03	5.90 (5)
WCA	263.89	263.89	263.89	9.18E-05	3.63 (4)
TLBO	263.89	263.89	263.89	1.01E-05	3.36 (3)
NNA	263.77	263.76	263.76	3.33E-03	1 (1)

Table 19 (*Continued*)

Methods	Worst	Average	Best	SD	Friedman Test (Ranking)
Gear train design problem					
PSO	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.83 (1)
GA	8.15E-14	2.75E-15	0.00E+00	1.48E-14	4.66 (2)
SA	2.97E-06	2.15E-07	5.92E-15	5.56E-07	9.96 (7)
HS	5.76E-17	6.26E-18	3.17E-21	1.16E-17	7.90 (5)
DE	5.63E-14	2.98E-15	0.00E+00	1.17E-14	6.46 (4)
GSA	2.27E-27	1.67E-28	1.73E-31	4.34E-28	5.83 (3)
ICA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.83 (1)
WCA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.83 (1)
TLBO	2.91E-12	2.24E-13	1.46E-21	5.78E-13	8.83 (6)
NNA	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.83 (1)

Table 20
Sum of average ranking using Friedman test for applied optimization methods.

Methods	Total Average Ranking by Friedman Test (Rank)
PSO	20.63 (4)
GA	38.77 (8)
SA	33.41 (7)
HS	39.39 (9)
DE	24.50 (6)
GSA	42.76 (10)
ICA	22.23 (5)
WCA	20.59 (3)
TLBO	18.49 (2)
NNA	**14.06 (1)**

and important part in almost all metaheuristic algorithms has been eliminated in the proposed model. The optimization results show that the NNA is able to find the global minimum of multimodal functions with the minimum possibility of getting trapped in local minima.

Sensitivity analysis over common user parameters of NNA which are population size and maximum number of iteration has been performed. Twenty-one unconstrained benchmarks have been considered and twelve well-known and recent optimizers have been compared for solving unconstrained and constrained benchmarks along with statistical results and tests. Computational optimization results obtained from several optimization problems clearly illustrate the attractiveness and competitiveness features of the proposed method for handling unconstrained and constrained real life problems with many design variables compared with recent and well-used optimizers.

As future research, there are many works to do. To name a few, for updating the weight matrix, in this paper, the same concept as for transfer function operator was used. However, based on the concept of learning and updating weights in ANNs, other learning approaches existed in ANNs such as Hebbian learning, competitive learning, reinforcement learning, and gradient descent learning can be considered for deriving different versions of NNA. In this paper, a simple form of NNA with simple weight updating approach have been considered. Also, the NNA can be mapped into a multi-objective optimizer handling many objectives using existing methods in the literature such as non-dominated sorting approach. Hybridizing of NNA with other population-based methods and utilizing its unique structure (inspired by the ANNs) in generating new solutions may be resulted developing powerful hybrid optimizers for tackling large-scale optimization problems. Diverse optimization problems emerging in transportation, scheduling, energy saving, sizing optimization, and so forth can be accounted as further studies.

Conflict of interest

The authors declare that they have no conflict of interest.

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Appendix A.

A Convergence analysis of NNA

In this section, a mathematical analysis of the underlying search mechanism of NNA is discussed. It is expected that such an analysis yields some important guidelines on the performance and validity of NNA. Convergence analysis shows the stability of the algorithm over the large number of iterations. The presented analysis is done for the case when $N\rightarrow\infty$, the evaluation of exact number of N steps and the quality of the solution is experimental aspect and it will vary form problem to problem. The stability model is developed in terms of linear equations, the functions such as *rand* are assumed to be uniform random variable and with theoretical calculation, their expected value is used for executing the proof. As proposed algorithm is stochastic in nature, therefore it is one of the mathematical way to deal with the stochastic terms. In order to make sure and guarantee that a pattern solution in the NNA finally converges to the target solution, the following theorem is stated and proved.

Theorem. A typical pattern solution of the NNA population converges to a stable point if the spectral radius of weight matrix is less than 1.

Proof. Let $X_i(t)$ be the any member of the population at a iteration t, where $i = 1, 2, 3, \ldots N_{pop}$. From Eq. (7), new pattern solution $X_i^{New}(t)$ can be written as given follows:

$$X_i\,(t+1) = \sum_{i=1}^{N_{pop}} \left(W_{ij}\,(t) \times X_i\,(t)\right) . \tag{A.1}$$

In general, all new pattern solutions on corresponding previous pattern solutions can be expressed as the following system of equations:

$$\begin{aligned}
X_1^{New}(t+1) &= w_{11}X_1(t) + w_{21}X_2(t) + \ldots + w_{i1}X_i(t) + \ldots + w_{N_{pop}1}X_{N_{pop}}(t)\\
X_2^{New}(t+1) &= w_{12}X_1(t) + w_{22}X_2(t) + \ldots + w_{i2}X_i(t) + \ldots + w_{N_{pop}2}X_{N_{pop}}(t)\\
&\vdots\\
X_{N_{pop}}^{New}(t+1) &= w_{1N_{pop}}X_1(t) + w_{2N_{pop}}X_2(t) + \ldots + w_{iN_{pop}}X_i(t) + \ldots + w_{N_{pop}N_{pop}}X_{N_{pop}}(t)
\end{aligned} \tag{A.2}$$

or in the same set of equation in matrix form can be expressed as given in the following formula:

$$\begin{bmatrix} X_1^{New}(t+1) \\ X_2^{New}(t+1) \\ \vdots \\ X_{N_{pop}}^{New}(t+1) \end{bmatrix} = \begin{bmatrix} w_{11} & w_{21} & \cdots & w_{N_{pop}1} \\ w_{12} & w_{22} & & w_{N_{pop}2} \\ \vdots & & \ddots & \vdots \\ w_{1N_{pop}} & w_{2N_{pop}} & \cdots & w_{N_{pop}N_{pop}} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_{N_{pop}}(t) \end{bmatrix} . \tag{A.3}$$

Let *rand* = r and after applying transfer function as described in Eq. (13), the set of Eq. (A.3) will becomes as:

$$\begin{aligned}
X_1(t+1) &= w_{11}\left(X_1(t) + 2\times r\left(X^{Target}(t) - X_1\right)\right) + w_{21}X_2(t) + \ldots + w_{N_{pop}1}X_{N_{pop}}(t)\\
X_2(t+1) &= w_{12}X_1(t) + w_{22}\left(X_2(t) + 2\times r\left(X^{Target}(t) - X_2\right)\right) + \ldots + w_{N_{pop}2}X_{N_{pop}}(t)\\
&\vdots\\
X_{N_{pop}}(t+1) &= w_{1N_{pop}}X_1(t) + \ldots + w_{N_{pop}N_{pop}}(X_{N_{pop}}(t) + 2\times r\left(X^{Target}(t) - X_{N_{pop}}(t)\right)
\end{aligned} \tag{A.4}$$

In order to solve the iterative system of equations shown in Eq. (A.4), the following system of equation can be established as given follows:

$$\begin{aligned}
X_1(t)(1 - w_{11}(1+2\times rand)) &= 2\times r\times\ w_{11}X^{Target} + w_{21}X_2(t) + \ldots + w_{N_{pop}1}X_{N_{pop}}(t)\\
X_2(t)(1 - w_{22}(1+2\times rand)) &= 2\times rand\, w_{22}X^{Target} + w_{12}X_2(t) + \ldots + w_{N_{pop}2}X_{N_{pop}}(t)\\
&\vdots\\
X_{N_{pop}}(t)\left(1 - w_{N_{pop}N_{pop}}(1+2\times rand)\right) &= 2\times r\, w_{N_{pop}N_{pop}}X^{Target} + w_{1N_{pop}}X_2(t) + \ldots + w_{N_{pop-1}N_{pop}}X_{N_{pop}-1}(t)
\end{aligned} \tag{A.5}$$

Eq. (A.5) is a system of linear equations which can be represented in matrix form for better understanding as shown in Eq. (A.6):

$$\begin{bmatrix} 1 - w_{11}(1+2r) & -w_{21} & -w_{31} & \cdots & -w_{N_{pop}1} \\ 1 - w_{22}(1+2r) & -w_{12} & -w_{32} & \cdots & -w_{N_{pop}2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 - w_{N_{pop}N_{pop}}(1+2r) & -w_{1N_{pop}} & -w_{2N_{pop}} & \cdots & w_{N_{pop-1}N_{pop}} \end{bmatrix} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_{N_{pop}}(t) \end{bmatrix} = 2\times rX^{Target} \begin{bmatrix} w_{11} \\ w_{22} \\ \vdots \\ w_{N_{pop}N_{pop}} \end{bmatrix} \tag{A.6}$$

The system of equations expressed in Eq. (A.6) is a stochastic in nature. The way the individual weights are defined (see Eqs. (5), (6) and (10)) can be assumed as a uniform random variable defined on zero to one, however, the components of weight matrix have to follow one more condition which is restated in Eq. (A.7):

$$\sum_{j=1}^{N_{pop}} w_{ij} = 1 \qquad j = 1, 2, 3\ldots, N_{pop}\,. \tag{A.7}$$

Without loss of generality, let us assume that $w_{11},\ w_{22}, \ldots w_{N_{pop}N_{pop}}$ are continuous uniform random variables defined in the range of zero and one, therefore the expected value of these variables can be evaluated as $E\left[w_{ii}\right] = \int_1^0 \frac{x}{1-0}dx = \frac{1}{2}$, similarly $E\left[r\right] = \int_1^0 \frac{x}{1-0}dx = \frac{1}{2}$, after substituting these values in Eq. (A.6), the system will change into the following system$AX = b$, where

$$A = \begin{bmatrix} 1 - w_{11}\left(1+2r\right) & -w_{21} & -w_{31} & \ldots & -w_{N_{pop}1} \\ 1 - w_{22}\left(1+2r\right) & -w_{12} & -w_{32} & \ldots & -w_{N_{pop}2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 - w_{N_{pop}N_{pop}}\left(1+2r\right) & -w_{1N_{pop}} & -w_{2N_{pop}} & \ldots & w_{N_{pop-1}N_{pop}} \end{bmatrix}, X = \begin{bmatrix} X_1\left(t\right) \\ X_2\left(t\right) \\ \vdots \\ X_{N_{pop}}\left(t\right) \end{bmatrix}, b = 2 \times rX^{Target} \begin{bmatrix} w_{11} \\ w_{22} \\ \vdots \\ w_{N_{pop},N_{pop}} \end{bmatrix} \tag{A.8}$$

Since the coefficient matrix A is stochastic as the values in the matrix are dynamically changing at each iteration, therefore to solve this system of equation (Eq. (A.6)), the Neumann Expansion method [46] is applied. Hence, the matrix A is treated as the sum of deterministic and stochastic matrices, symbolically it may be represented as $A = A_0 + \nabla A$, where A_0 is a deterministic matrix and ∇A is stochastic matrix, which is defined as given follows:

$$\nabla A = \alpha_1 A_1 + \alpha_2 A_2 + \ldots + \alpha_{N_{pop}} A_{N_{pop}}, \tag{A.9}$$

where,

$$A_0 = \begin{bmatrix} 1 - w_{11} & -w_{21} & -w_{31} & \ldots & -w_{N_{pop}1} \\ 1 - w_{22} & -w_{12} & -w_{32} & \ldots & -w_{N_{pop}2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 - w_{N_{pop},N_{pop}} & -w_{1N_{pop}} & -w_{2N_{pop}} & \ldots & w_{N_{pop-1},N_{pop}} \end{bmatrix}, \nabla A = \begin{bmatrix} -w_{11}(2r) & 0 & 0 & \ldots & 0 \\ -w_{22}(2r) & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -w_{N_{pop},N_{pop}}(2r) & 0 & 0 & \ldots & 0 \end{bmatrix} \tag{A.10}$$

Therefore, the solution of $(A_0 + \nabla A)X = b$ will be [46]:

$$X = \left(I + A_0^{-1}\nabla A\right)^{-1} A_0^{-1} b. \tag{A.11}$$

The term $\left(I + A_0^{-1}\nabla A\right)^{-1}$ can be expressed as Neumann series expansion, which gives $X = \left(I - B + B^2 - B^3 + \ldots\right) A_0^{-1} b$, where $B = A_0^{-1}\nabla A$. Therefore the random solution of the $AX = b$ can now be expressed in terms of the following series, $X = X_0 - BX_0 + B^2 x_0 - B^3 X_0 + \ldots$, where $X_0 = A_0^{-1} b$. The aforementioned solution series will be convergent if the spectral radius of the matrix B is less than 1. Therefore, the solution of $AX = b$, which are actually the pattern solution of the populations will also converge to a stable point.

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