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CHARACTERIZATION OF PV ARRAY OUTPUT USING A SMALL NUMBER OF MEASURED PARAMETERS

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Abstract—The purpose of the present study was to develop a sufficiently good fit for the measured I–V curve of a PV module and array using only three easily measurable parameters: —the open-circuit voltage (V_{oc}); —the short-circuit current (I_{sc}); —the maximum power (P_m). With an additional three parameters ($\partial V_{oc}/\partial T$; $\partial V_{oc}/\partial Q$; $\partial I_{sc}/\partial T$) it is possible to describe any I–V curve, taking into account cell temperature T and solar radiation Q. This method has been tested on various solar array panels as well as on a single 10 cm dia. solar cell. The difference between the real curve and the proposed fit was found to be less than 3 percent for a fixed temperature and radiation and about 6 percent for various combinations of temperature and radiation.

1. INTRODUCTION

A photovoltaic (PV) module is a rectangular matrix consisting of solar cells arranged in series and parallel connection. The output of the PV module is described by its current-voltage (I–V) curve, which varies as a function of temperature T and radiation intensity Q, such that $I = I(V, Q, T)$.

The manufacturer usually provides many I–V graphs corresponding to different radiation levels at standard temperature, and to different cell temperatures at standard radiation.

Since data in such form are not practical for calculations, several models of PV cells have been developed[1–5] with I–V curves that clearly approximate the measured I–V curve of the PV cell. However, these models involve a large number of parameters that are difficult to measure. Other models, such as that reported by Slonim[6], provide a linear fit which is simple and easily suited to calculations and corresponds sufficiently, except at the maximum power point.

Proposed herein is a phenomenological fit which also takes into account the data corresponding to the maximum power point. Using three basic parameters, the I–V curve at constant temperature and radiation is described, and, using three additional parameters, the influence of variable temperature and radiation intensity is determined. The main advantage of our fit is the very limited number of easily measurable parameters necessary to obtain sufficient accuracy. We believe that such an approach is probably more useful to the system designer than a basic theoretical approach.

2. DESCRIPTION OF THE I–V CURVE OF A SINGLE SOLAR CELL USING THREE PARAMETERS

The classical equation describing the I–V curve of a single solar cell is[7, 8]:

$$I = -I_0\left\{\exp\left[\frac{q}{AKT}(V + IR)\right] - 1\right\} + I_L \qquad (1)$$

where I is the current and V the voltage, I_0 the diode saturation current, I_L the light-generated current, R the series resistance, q the electronic charge, k the Boltzmann constant, T the temperature (°K) and A a dimensionless factor.

Equation (1) describes the I–V curve quite well, but the parameters cannot be measured in a simple manner. We therefore have developed a fit based on a smaller number of parameters which can be measured easily. These include (Fig. 1): —the open-circuit voltage (V_{oc}); —the short-circuit current (I_{sc}); —the maximum power (P_m).

Our model relies on eqn (1) but uses the following simplification:

$$\exp(q/AKT)V \gg 1; \quad I_L \simeq I_{sc}.$$

Substituting $\lambda = q/AKT$, the I–V curve can be expressed as:

$$I = I_{sc}\left[1 - \left(\frac{I_0}{I_{sc}}\right)\exp\lambda(V + IR)\right]. \qquad (2)$$

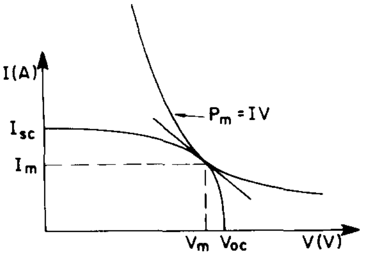

Fig. 1. I–V curve of a solar cell and hyperbola of a constant power (P_m) tangent to the curve.

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Since λ and R are unknown, two conditions are required to enable use of this fit: (a) If $I = 0$, then $V = V_{oc}$; (b) At the maximum power point the fit is tangent to the hyperbola $P_m = IV$. Condition (a) yields:

$$V_{oc} = V|_{I=0} = \frac{1}{\lambda}\ln\left(\frac{I_{sc}}{I_0}\right)$$

or:

$$\lambda = \frac{1}{V_{co}}\ln\left(\frac{I_{sc}}{I_0}\right). \quad (3)$$

It was found that a typical value for the ratio I_o/I_{sc} for a silicon cell at 25°C and 1 sun ranges from approx. 10^{-8}–10^{-10}. The quality of the fit is affected only slightly when this ratio varies within this range, due to compensation by the variations in the calculated values of R and λ (Fig. 2). Thus, in order to reduce the number of measurements, we may assume that $I_o/I_{sc} = 10^{-9}$. Substituting this value into eqns (2) and (3) then yields:

$$I = I_{sc}\left[1 - 10^{-9}\exp\frac{20.7}{V_{oc}}(V + IR)\right], \quad (4)$$

and eqn (4) yields:

$$V = V_{oc}\left[1 + \frac{1}{20.7}\ln\frac{I_{sc} - I}{I_{sc}}\right)\Bigg] - RI. \quad (5)$$

Condition (b) can be expressed as (Appendix A):

$$V|_{I=I_m} = \frac{P_m}{I_m} \quad (6)$$

$$\left.\frac{\partial V}{\partial I}\right|_{I=I_m} = \left.\frac{\partial}{\partial I}\left(\frac{P_m}{I}\right)\right|_{I=I_m} = \frac{P_m}{I_m^2}. \quad (7)$$

The current at the maximum power point I_m is unknown.

Substituting eqn (6) into eqn (5) we have:

$$\frac{P_m}{I_m} = V_{oc}\left[1 + \frac{1}{20.7}\ln\left(\frac{I_{sc} - I_m}{I_{sc}}\right)\right] - RI_m, \quad (8)$$

and differentiating eqn (5) according to eqn (7) yields:

$$\frac{P_m}{I_m^2} = \frac{V_{oc}}{20.7}\left(\frac{1}{I_{sc} - I_m}\right) + R. \quad (9)$$

Combining eqns (8) and (9) then yields:

$$I_m\left[1 + \frac{1}{20.7}\left(\frac{I_m}{I_{sc} - I_m} + \ln\frac{I_{sc} - I_m}{I_{sc}}\right)\right] - \frac{2P_m}{V_{oc}} = 0. \quad (10)$$

In order to determine the value of I_m, eqn (10) is solved numerically. Then R is calculated using eqn (9), and this value is substituted into eqn (4) to find the I–V curve of a single solar cell.

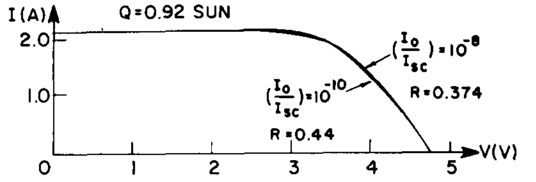

Fig. 2. I–V for Module TSG–P (AEG) when I_o/I_{sc} is taken as 10^{-8} and 10^{-10}. Note resulting different calculated values of R and λ.

3. I–V CURVE FOR SOLAR PANEL AND ARRAY

A solar panel (or module) is an environmentally protected matrix of solar cells arranged in series and parallel connection. If all the cells were identical, the I–V curve of the panel could be determined, in a uniform environment, by scaling the I–V curves of each individual cell (i.e. by multiplying the voltage by the number of rows and the current by the number of columns)[9, 10].

In practice, however, there is a distribution of the cellular parameters, such that synthesis of the I–V curves is difficult[11–13]. Our phenomenologic approach to characterization of a single cell can be used also to determine the I–V curve of the entire module. Using this approach, the solar module is considered as a two-terminal black box which has some measured I–V curve to which we try to find a fit. Because the I–V curves of the module and the single solar cell have a similar shape, the same fit is suitable for both. Thus, the same parameters should be measured, and no additional parameters are required (such as the number of cells in series and parallel connection).

It should be emphasized that in cases in which the shape of the modular I–V curve differs to a great extent from that of the curve of the single cell (as occurs in partially illuminated modules[15]), the fit developed for a single cell is unsuitable.

An assembly of solar panels or modules can be mechanically integrated, and combined with blocking diodes, shunt diodes and other components, to form a solar array. Also, in this case, it would be possible to find the characteristic I–V curve of the array by scaling the I–V curves of the single modules, and subtracting the decrease in voltage on the interconnections and blocking diodes; unforunately, the accuracy of this method is limited, because some mismatch must exist between the different modules. Using our approach, however, the I–V curve of the PV array is approximated according to the fit found for a single cell, so that the influence of the blocking diodes, interconnections and wiring resistance is included in the fit and affects the parameters (especially R). The shape of the resulting I–V curve of the array should, again, be similar to that of a single cell.

3.1 *Influence of variable temperature and illumination*

The I–V curves of both the solar cell and the PV array vary with solar radiation Q and temperature T. In nonconcentrated flat arrays, Q varies from 0 to 1 sun and T from 10 to 70°C. In order to use the fit described herein, the values of the open-circuit voltage V_{oc}, the short-circuit current I_{sc} and the maximum power P_m at any combination of Q and T (within the a range of their variations) are needed. Since direct measurement of these parameters as a function of Q and T requires many tests, another approach is offered.

Let us assume that the parameters at standard conditions ($Q = 1$ sun; $T = 25$°C) are known, and we want to find the parameters at other Q, T combinations. In this case, I_{sc}^{ST}, V_{oc}^{ST}, P_m^{ST} are defined as the short-circuit current, open-circuit voltage and maximum power at standard conditions. If $\Delta T = T - T^{ST}$ and $\Delta Q = Q - Q^{ST}$, then measuring Q at sun units yields:

$$\Delta Q = Q - 1.$$

Since I_{sc} is proportional to the light intensity[7], it is not a rapidly changing function of T[14] and can be approximated by:

$$I_{sc}(Q, T) \simeq I_{sc}^{ST} Q(1 + \alpha \Delta T). \tag{11}$$

A typical value for α is 0.0025 1/C^0.

V_{oc} is a logarithmic function of Q and decreases with T. Therefore, V_{oc} can usually be expressed by:

$$V_{oc}(Q, T) \simeq V_{oc}^{ST}(1 - \gamma \Delta T) \ln (l + \beta \Delta Q). \tag{12}$$

For a single silicon cell $\mathrm{d}V_0/\mathrm{d}T = 0.00288$ V/C^0[7] and a typical value of β is 0.5 1/sun. α, β, γ can be determined by taking several points for different values of Q and T.

Assuming that usually the shape of the I–V curve does not change, $P_m(Q, T)$ can be approximated by:

$$P_m(Q, T) \simeq P^{ST} \frac{I_{sc}(Q, T) V_{oc}(Q, T)}{I_{sc}^{ST} V_{oc}^{ST}}. \tag{13}$$

R is now calculated using eqns (11)–(13) and substituting the approximated parameters into eqns (9) and (10). (It is obvious that R and λ are functions of Q and T). By substituting the resulting values of R and λ into eqn (4), the characteristics of the I–V curve of the PV array as a function of Q and T are determined.

3.2 *Test and results*

In order to check the behavior of the fit when various values are taken for the ratio I_o/I_{sc}, the fit for AEG Module TSG–T was determined for $I_o/I_{sc} = 10^{-8}$ and $I_o/I_{sc} = 10^{-10}$. Results indicated that the influence of the variation of this ratio is very modest (Fig. 2).

To check the quality of the fit, the above result was compared to data sheets for a 10 cm diameter solar cell (Solar Power Corp., USA) and Panel 9200J (Solarex, USA), and to data measured at the Laboratory of Energy Conversion, Tel Aviv University for the following devices: Panel P2 (Solas, USA); Panel HE 60 (Solarex); Module TSG-P (AEG, Germany); String of four Panels HE 50 (Solarex); and Array of 10 Panels 785 (Solarex) and 5 panels (Solar Energy) with blocking diodes (Fig. 3). The array and string of panels had been mounted on the roof of the laboratory for about two years and were covered with dust. In all cases good matching was achieved between the measured curves and the fit (Figs. 4, 5).

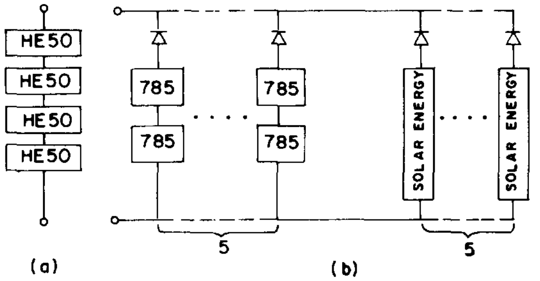

Fig. 3. (a) String composed of four panels HE 50 (Solarex). (b) Array composed of ten panels 785 (Solarex) and five panels (Solar Energy).

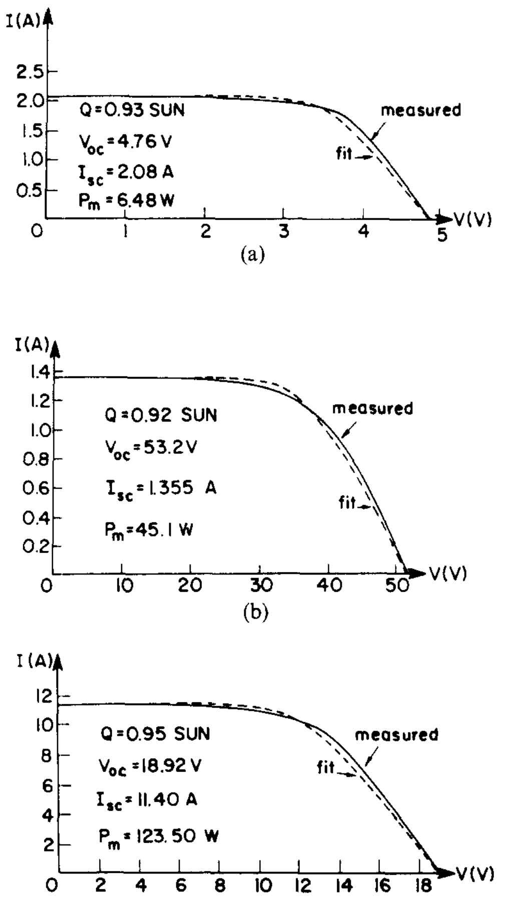

Fig. 4. Measured I–V curves compared to the fit: (a) For Module TSG–P (AEG); (b) For string shown in Fig. 3(a); (c) For array shown in Fig. 3(b).

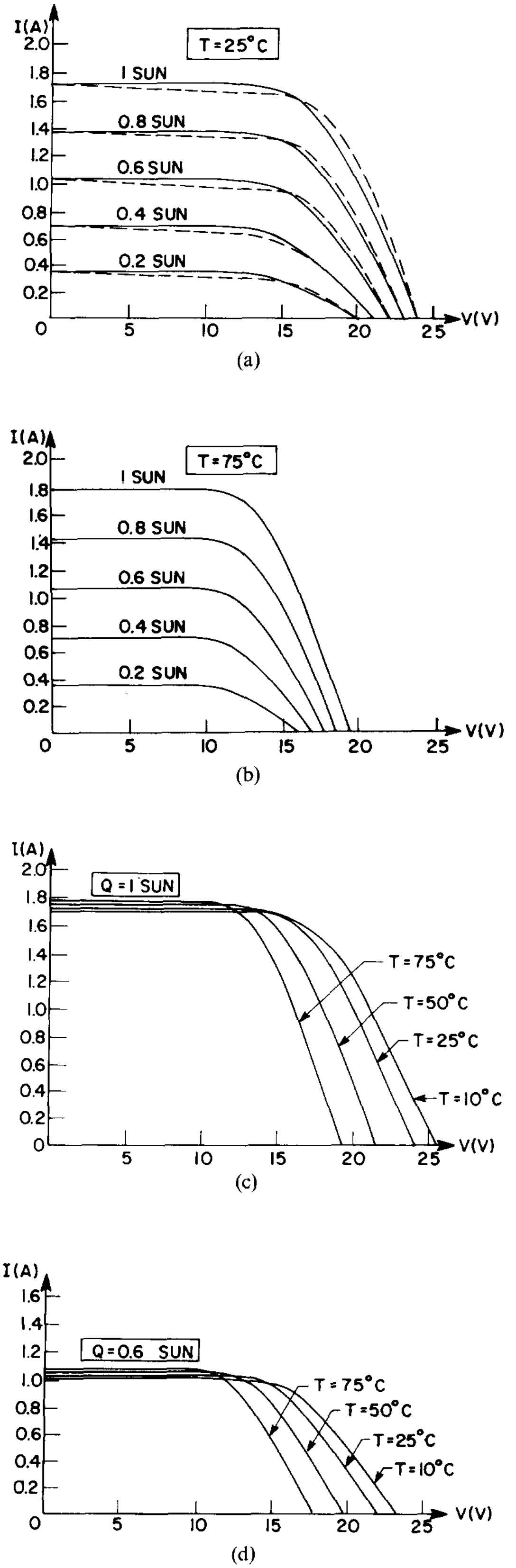

Fig. 5. I–V curves of Panel 9200J (Solarex) at various combinations of solar radiation Q and temperature T. (Information taken from manufactuer's data sheets.) (a) I–V curves at 25°C with varying Q. solid lines = fit; dashed lines = data sheets; (b) Fit at 75°C; (c) Fit at 1 sun with varying T; (d) Fit at 0.6 sun with varying T.

4. SUMMARY

A new method for fitting measured I–V curves of solar modules and arrays is described herein, based on only three classic parameters of the array. By determining the influence of an additional three parameters, I_{sc}, V_{oc} and P_m, good approximation can be obtained for varying temperature and solar radiation. Our model requires fewer measurements than models previously developed to describe the I–V curves of cells[1–5] and arrays, and the influence of the blocking diodes and wiring and interconnecting resistance of the array is included in the fit without any additional measurements.

When the model was tested for a single solar cell and for various types of panels, strings and arrays, the difference between the approximated and real curves was less than 3 per cent, and in some cases about 1 per cent, over the entire range, When T and Q were varied, the accuracy decreased at times to approx. 6 per cent. The same degrees of accuracy were achieved by Goldstein and Case[5] in a previous report.

Therefore, the parameters proposed for this model allow an exhaustive description of a given solar panel (or array) and a possible check of its I–V curve. It should be noted that the match between the fitted and measured curves is improved when the latter is flattened within the range of the short-circuit current and the "knee", and most newly developed devices fulfill this criterion. In addition, the I–V curve of a string of panels or an array is usually flatter than that of a single cell because the current through the cells connected in series is limited by the cell with the lowest short-circuit current, providing a narrower range.

We conclude that the accuracy of our fit is sufficient for most practical cases.

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APPENDIX A

$$P(I) = V(I) \cdot I.$$

If at $I_m I - P(I)$ is at its maximum value, then:

$$\left.\frac{\partial P}{\partial I}\right|_{I=I_m} = 0,$$

$$\left(\left.\frac{\partial V}{\partial I}\right|_{I=I_m}\right) \cdot I_m + V(I_m) = 0.$$

On the other hand,

$$P(I_m) = V(I_m) I_m = P_m,$$

so:

$$V(I_m) = \frac{P_m}{I_m},$$

$$\left.\frac{\partial V}{\partial I}\right|_{I=I_m} = \frac{V(I)}{I_m^2} = \frac{\partial}{\partial I}\left(\frac{P}{I}\right)\Big|_{I=I_m}.$$

Hence, the I–V curve is tangent to the hyperbola $V = P_m/I$.