

# A new Kho-Kho optimization Algorithm: An application to solve combined emission economic dispatch and combined heat and power economic dispatch problem

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## ABSTRACT

In this article, a new optimization technique known as Kho-Kho optimization (KKO) algorithm is presented. This proposed technique is a population based meta-heuristic method which is inspired from the strategies used by players in a well known tag-team game played in India, i.e. Kho-Kho. The performance and superiority of the proposed method with respect to other existing methods is evaluated using twenty nine benchmark functions and real-time optimization problems related to power system i.e. combined emission economic dispatch and combined heat and power economic dispatch problem.

## 1. Introduction

Optimization algorithms have wide applications in almost all the disciplines such as engineering, science, economics, etc. Researchers are always trying to develop new optimization algorithms so that these large and complex problems can be solved effectively. Numerical methods have difficulties in global optimality (Storn and Price, 1997). Nowadays, population-based heuristic and meta-heuristic algorithms have special attention. Many such algorithms are inspired by nature, animal behavior, etc.

The classical algorithms such as Quadratic Programming (QP) (Frank and Wolfe, 1956), Dynamic Programming (DP) (Bellman, 2013) and Lagrangian approach (Bertsekas, 1999) fail to search the global optima. These algorithms use to stuck in local optima. The drawbacks of the numerical methods are the main source of inspiration to develop some alternatives and many nature-inspired heuristic and meta-heuristic optimization algorithms. These are based on different natural processes and intelligence of animals in food searching, prey hunting, etc. A population-based lion optimization algorithm (LOA) is introduced in Yazdani and Jolai (2016), which is based on the lifestyle of lions and their co-operation characteristics. A meta-heuristic whale optimization algorithm (WOA) is presented in Mirjalili and Lewis (2016), which is inspired by the social behavior of humpback whales. In Zheng (2015), a water wave optimization (WWO) is introduced which is inspired by the shallow water wave theory. The herding behavior of elephants group is taken as an inspiration source to develop a new search method, i.e elephant herding optimization (EHO) in Deb et al. (2015). In Meng et al. (2014), the hierarchal order and behaviors of the chicken swarms are the source of developing new chicken

swarm optimization (CSO) algorithm. It is also evident that the natural processes can also be a good source for developing new optimization techniques. A new algorithm named as mine blast optimization (MBO) based on mine bomb explosion concept is developed by the researches in Sadollah et al. (2013). In Yang (2012), a flower pollination algorithm is introduced where the source of inspiration is the pollination process of the flowers.

In recent year, many optimization techniques have been developed and used to solve many optimization problems. Based on the source of inspiration, they can be classified into different groups. One of the most interesting and widely used optimization group is population-based optimization algorithms (Karaboga and Akay, 2009). This group can be further divided into two groups which are evolutionary algorithms (Eiben et al., 2003) and swarm intelligence (Eberhart et al., 2001). Evolutionary algorithm include techniques such as genetic algorithm (GA) (Whitley, 1994), genetic programming (GP) (Banzhaf et al., 1998), evolutionary programming (EP) (Yao et al., 1999), etc. In recent years, swarm intelligence have also influenced many researchers. According to Bonabeau et al. (1999), swarm intelligence algorithms can be defined as the designing of intelligent agent algorithm taking into the account of the collective behavior of insects or animals. Several techniques based on swarm intelligence have been developed which include particle swarm optimization (PSO) inspired by social behavior of bird flocking (Kennedy and Eberhart, 1995), artificial bee colony (ABC) inspired by the cooperative behavior of bees (Karaboga and Akay, 2009), krill herd (KH) inspired by the herding behavior of krill individuals (Gandomi and Alavi, 2012), social spider optimization (SSO) inspired by social behavior of spiders (Cuevas et al., 2013), Firefly

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(FF) method inspired by the mating behavior of firefly insects (Yang, 2010), Locust search (LS) inspired by the behavior of swarm of locusts (Cuevas et al., 2016), etc. Another interesting optimization group is music-based meta-heuristic optimization (Yang, 2009). This group includes technique such as harmony search (HS) which is inspired by the observation that aims to find the best music by a perfect combination of harmony (Yang, 2009), an improved harmony search (IHS) which is developed to improve the fine tuning characteristic of HS (Mahdavi et al., 2007), etc.

## 2. Novelty of the paper

In this paper, the authors have proposed a new population-based meta-heuristic optimization algorithm known as Kho-Kho optimization (KKO) algorithm. The proposed KKO algorithm is inspired by a well-known tag-team game played in India, i.e. Kho-Kho. The game is played between two teams, one is known as chasing team and other one is known as running team. During a match, a set of three players from the running team runs within the field and try to avoid being touched by the chasing team. These three players of the running team are termed as 'runners'. The players of the chasing team try to touch the runners within the permissible duration of time. For this, a unique strategy is used by the chasing team. Among the players of the chasing team, one player is selected as a chaser who tries to touch the runners. Remaining players of the team sit in rows with alternate players facing the opposite side. A chaser selects a new chaser from the sitting players so that the runner is close to him and it can be touched. Once the allotted time is over or all the players of the running team are touched, the second inning begins. In the second inning of the game, the role of chasing team and running team are interchanged. At the end of the second inning, who so ever score more points is declared as the winner. The proposed KKO algorithm mimics the unique strategy used by the chasing team to touch the runners (global best solution).

The performance of the proposed KKO algorithm is tested with twenty-nine benchmark functions. Next, the technique is used to solve large and highly non-linear problem related to power system i.e. a combined emission economic dispatch (CEED) and combined heat and power economic power dispatch (CHPED) problem.

## 3. Kho-Kho optimization (KKO) algorithm

### 3.1. Game information

Kho-Kho is a well-known tag-team game played in India. The game is played between the two teams, a chasing team and a running team. Both the team includes twelve players each. Among which only nine players from both the team play this game. The game includes two sections known as innings. During one inning, one team acts as a chasing team and other as a running team. During the second inning, the roles are interchanged i.e. running team chases and chasing team runs.

During an inning, a set of three players from the running team occupy random positions on the field. These players are termed as runners and their role is to avoid being touched by the opposition team. These players can move freely within the field in any direction. On the other hand, one player is selected as a chaser from the chasing team and rest players sit in rows on the field as shown in Fig. 1. This sitting position is such that two adjacent players of the chasing team face opposite to each other. This unique sitting position helps the chasing team in switching the side and direction of movement as the chaser cannot switch his direction and side. Chaser can select any sitting player as the new chaser if he finds that a sitting player faces opposite to the chaser at a more optimal position concerning the runner. If this happens, the chaser takes the position of the new chaser on the field and the new chaser can decide to switch direction based on the position of the runners. The new chaser has to move his facing direction. This

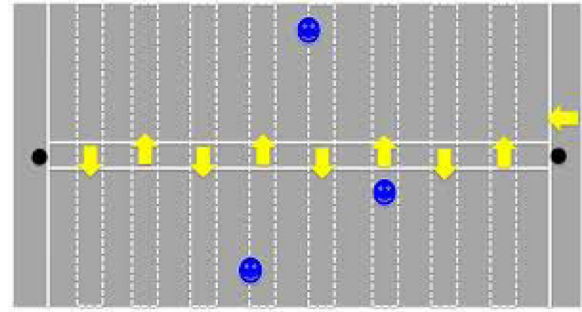


Fig. 1. Facing of chaser's.

selection of new chaser is important and helps in touching the runners. Once a chaser touches a runner, the runner is replaced by a new runner. Once all the runners are touched or the chasing team runs out of time, the inning is stopped. At the end of two-innings, the team with less taken to out all the players or less number of players out in a fixed time wins the game.

The proposed KKO mimics the strategies used by the chasing team to touch a runner (global optimal solution).

### 3.2. Explanation to chasing strategy

In this section, the strategy used by the chasers to catch the runner is explained. According to the game, the chasers used to settle down in a straight line of chasing blocks in the center of the court in which two neighboring chasers are facing towards the two opposite sidelines. The chasers sitting at an even-numbered sitting block face towards one side of the center line and the chasers sitting at an odd-numbered sitting block face towards the other side of the centerline. This facing property categorizes the chasers in two categories: even position chasers and odd position chasers. The chasers which are facing towards the runner are the possible candidate to be selected as a chaser. The centerline divides the playing court into two halves. Throughout the game, the runner keeps moving randomly in the court. The even position chasers manage to chase and touch the runners in one half of the court while on the other half is managed by the odd position chasers. The interesting rule of the game is that only one chaser can stand up and attempt to chase and catch only when it satisfies the following conditions: 1. The chaser must face towards the current position of the runner, 2. The chaser must be sitting closest to a runner position.

Once a chaser is selected, he tries to chase and touch the runner. As per rules, a runner can move on the other side to avoid touching. In the case of a chaser, this cannot happen. Therefore, the present chaser passes its activeness to a sitting player-facing towards the other side of the court. For the selection of new chaser, the criterion remains the same. Once a new chaser is selected, the old chaser replaces the sitting position of the new chaser. This process is continued till a chaser touch the runner. Therefore, the chasers keep on changing their sitting blocks.

### 3.3. Mathematical model of chasing strategy

The behavior of both runners and a chaser on the court and strategies followed by the chaser to chase and touch out a runner can be mathematically modeled in this section.

#### 3.3.1. Identification of runner position and type of the sitting players

To start the chase, the position of the runner should be known exactly. In the proposed algorithm, the fittest candidate serves as the runner position ( $\vec{X}_r(t)$ ). In the beginning, all the chasers occupy space in a line of sitting blocks in which two consecutive players of the chasing team face opposite to each other. This opposite facing of the chasers

generates two sets of the chasers. The classification of players in two sets is based upon a number at which the player is sitting in the line of chasing blocks. Therefore, the chasers are classified according to an even or an odd-numbered position. The set of even-numbered chasers is represented as:

$$C_j (j = 2, 4, 6, \dots, N) \subset C_i, (i = 1, 2, 3, \dots, N), \quad (1)$$

and the set of odd numbered chasers is represented as:

$$C_j (j = 1, 3, 5, \dots, N-1) \subset C_i, (i = 1, 2, 3, \dots, N), \quad (2)$$

### 3.3.2. Selection of a chaser

A chaser is one who stands up and chases a runner. To execute the chasing, a chaser must be selected among the players which are facing towards the runner's current position. In the proposed algorithm, a chaser ( $\bar{C}_b(t)$ ) is the fittest candidate which virtually represent the runner position ( $\bar{X}_r(t)$ ) on the court.

### 3.3.3. Chase and touch out (exploration and exploitation)

In the proposed algorithm, we have classified players in two sets which are even and odd-numbered position set. When a runner randomly moves in the court, both set of chasers reacts with different strategies. The selection of a chaser's strategy is totally dependent upon the position of a runner. If a runner is in front of any set, a player (chaser) out of the players of this set chases the runner. This process is named as chasing strategy. On the other hand, the players of the other set remain settled down. This strategy of settling down is named unmoved strategy. Both the strategies i.e. chasing and unmoved strategy happens simultaneously. Depending on the position of a runner, two possibilities can happen:

Possibility 1: The odd-numbered position chasers may follow the strategy of chasing and the even-numbered position chasers follow the strategy of settling down.

Possibility 2: The vice-versa of the possibility 1.

To select randomly anyone out of the two possibilities, an operator  $m$  is defined. Depending on the value of  $m$ , strategy for both the sets is decided. If  $m$  is less than or equal to 0.5, even-set follows the strategy of chasing and odd-set follows strategy of unmoved. If  $m$  is greater than, 0.5, odd-set follows the strategy of chasing and even-set follows strategy of unmoved.

The updating rule according to the strategy of chasing is mathematically represented as follows:

$$\bar{v}(t+1) = \bar{v}_0(t) + (\bar{C}_b(t) - \bar{C}_b(t)_{old}), \quad (3)$$

$$\bar{C}_f(t+1) = \bar{C}_b(t) - [g \cdot [e \cdot \bar{C}_b(t) - \bar{C}_f(t)]] + \bar{v}(t+1), \quad (4)$$

where  $\bar{C}_f(t+1)$  represents the updated position of the chaser which are facing towards runner,  $\bar{C}_f(t)$  is the position vector of the chasers facing towards the runner,  $\bar{C}_b(t)$  is the position vector for active chaser or best chaser which is also representing the runner's position,  $v(t)$  represents a feedback position of the runner from previous time instant,  $v(t+1)$  is the updated feedback position,  $t$  is the present instant of time,  $t+1$  is the next instant of time, scalar coefficient ( $g$ ) is a gap randomizing operator and the operator  $e \in [2, 0]$ , an endurance factor indicating the runner endurance level.

The updating rule according to the strategy of unmoved is mathematically represented as follows:

$$\bar{C}_o(t+1) = \bar{C}_o + g \cdot [e \cdot \bar{C}_b(t) - \bar{C}_o(t)], \quad (5)$$

where  $\bar{C}_o(t)$  are the position vectors for the chaser which are facing opposite to the runner.

The gap randomizing operator ( $g$ ) is dependent on the endurance factor ( $e$ ) of the chaser. This can be modeled as follows:

$$g = [(2 \cdot e \cdot \text{rnd}) - e], \quad (6)$$

### Kho-Kho Optimization (KKO) Algorithm

Generate initial chasers population  $X_i (i = 1, 2, \dots, N)$

Chasers settling in Kho-Kho formation

Odd position chasers,  $C_i (i = 1, 3, 5, \dots, N-1)$

Even position chasers,  $C_i (i = 2, 4, 6, \dots, N)$

Calculate the fitness of each chaser

Initialize runner position ( $\bar{X}_r(t)$ ), emdurance ( $e$ ) and gap randomizing factor ( $g$ )

Initialize the best chaser position ( $\bar{C}_b$ )

while  $t < \text{max iteration}$

for each solution

Update ( $\bar{X}_r(t)$ ), ( $e$ ), ( $g$ ) and ( $m$ )

if ( $m \leq 0.5$ )

Runner is in front of even position chasers

Runner is in opposite of odd position chasers

Update the **positions and speed** of the current **even position** chasers

Update the **positions Only** of the current **odd position** chasers

else

Runner is in front of odd position chasers

Runner is in opposite of even position chasers

Update the **positions and speed** of the current **odd position** chasers

Update the **positions only** of the current **even position** chasers

end if

end for

Check if any solution goes beyond the search space and reject it.

Calculate the fitness of the solution.

Update ( $\bar{C}_b$ ), if there is a more satisfactory solution

$t = t + 1$

end while

return  $\bar{X}_1$ .

Fig. 2. Algorithm of proposed KKO algorithm.

where 'rnd' is a random number within the range [0, 1]

$$e = [e_T - \delta_e \cdot e_T], \quad (7)$$

where the operator  $e_T$  is the total endurance of the runner, which a fresh runner possesses in the beginning of its run. The rate of declination in the endurance level of a runner  $\delta_e(t)$  is represented as:

$$\delta_e(t) = (t/t_R)^2. \quad (8)$$

where  $t$  represents the present instant of time and  $t_R$  is the run-time of the runner.

### 3.4. Endurance factor

The game of Kho-Kho is about running and chasing. The endurance factor is a very crucial physical factor which affects a runner performance. As the game progresses, there comes fatigue which leads to a declination in the endurance of a runner which affects its position on the court. Therefore, an endurance factor  $e \in [2, 0]$  is associated with the runner position. The endurance level is considered as high for  $e = 2$  and zero for zero levels. The endurance factor values with 1 to 2 represent moderate-high and moderate low for 0 to 1 value. As the runner starts running on the court, it is a natural phenomenon that the endurance level of the runner keeps on reducing at an increasing rate till he/she quits running and  $\delta(e)$  is the rate of declination in endurance. During any chase, there exists a certain separating distance (gap) between the runner and a chaser. The chase is successful only when this separating distance minimizes towards zero. This gap keeps on varying throughout the game and it depends upon a few physical factors such as fatigue, obstacles, speed variation, decreased endurance etc. Therefore, an operator named as gap randomizing factor ( $g$ ) is introduced and the value of  $g$  depends on the endurance ( $e$ ) of the runner. All these above mention operators are quite necessary to mimic the situation of the game and behavior of the players.

### 3.5. Discussion for global optimality

The optimization procedure of proposed KKO algorithm begins with the selection of a runner (best particle) and chasers (candidate particles). The basic idea of this algorithm is to chase the best particle and identify the global best solution as the termination criteria satisfy. Here, chasing the best particle implies that keeping a track of movement (variations) of the best particle and improving the other candidate particles according to it. In a good optimization algorithm, the variations in the parent particle (best particle) should reflect effectively into the other candidate particles. In the proposed KKO, there are two cases of the candidate particles improvement (learning) are present. In both the case, candidate particles are improving themselves according to the movement of the best particle (chaser) but in the first case where the runner is in the front half of the court, an additional velocity improvement is also there. The velocity improvement is the variations in the best particle position concerning the old position of the best particle itself, whereas the primary improvement is the variations in the best particle concerning the other candidate particles. Therefore, the knowledge of both variations assists the other candidate particle vary effectively to improve. Now, in an optimization algorithm, the variations or improvement should be random within a defined search space and should be diminishing in nature. The constant variations will never lead the optimization algorithm to the global solution. In the proposed KKO, the random nature of gap varying factor ( $g$ ) and the diminishing nature of endurance factor ( $e$ ) leads the algorithm to achieve the global best solution without getting stuck into the local optimal solutions. In this algorithm, more emphasis is on exploration and less number of iterations are available for the exploitation as compared to the exploration. If the gap varying operator ( $|g| < 1$ ), the search is converging towards the solution and search is diverging from the solution if ( $|g| \geq 1$ ). More emphasis is given to the condition ( $|g| \geq 1$ ) as it makes the algorithm capable of utilizing the search space very efficiently. One last important thing is the termination criterion. The termination criterion or condition is the stage which declares any optimal solution as a global optimal. As the proposed algorithm is based on the chasing and making the runner touch out. Therefore, the termination criterion for the proposed algorithm is the condition when an active chaser chases the runner until its endurance reduces to nil and the chaser makes the runner touch out. The zero endurance ( $e$ ) is the terminating criterion for the proposed KKO algorithm. This is the real game condition when the runner becomes exhausted due to continuous running and the runner has no endurance to run further and then the chaser makes him out.

## 4. Validation with benchmark functions

### 4.1. Introduction

The performance of a new optimization algorithm can be judged from its exploitation, exploration, global convergence and local optima avoidance capabilities. Therefore, to validate these capabilities of the proposed KKO algorithm, twenty-nine benchmark functions (BF's) (García et al., 2009) are used. These functions comprise of three groups (i) uni-modal, (ii) multi-modal and (iii) composite function. Uni-modal functions include seven different functions ( $F_1$  to  $F_7$ ) which have unique global optimal points. Using these functions, the exploitation capabilities of the proposed KKO algorithms can be evaluated. Next group i.e. multi-modal group includes sixteen different function ( $F_8$  to  $F_{23}$ ). These functions have a single global optimal point along with multiple local minima. These functions are therefore used to evaluate the exploration capabilities of the proposed algorithm. At last, six composite functions ( $F_{24}$  to  $F_{29}$ ) are used to test the balance between exploration and exploitation. This also gives information about the local minima avoidance capability.

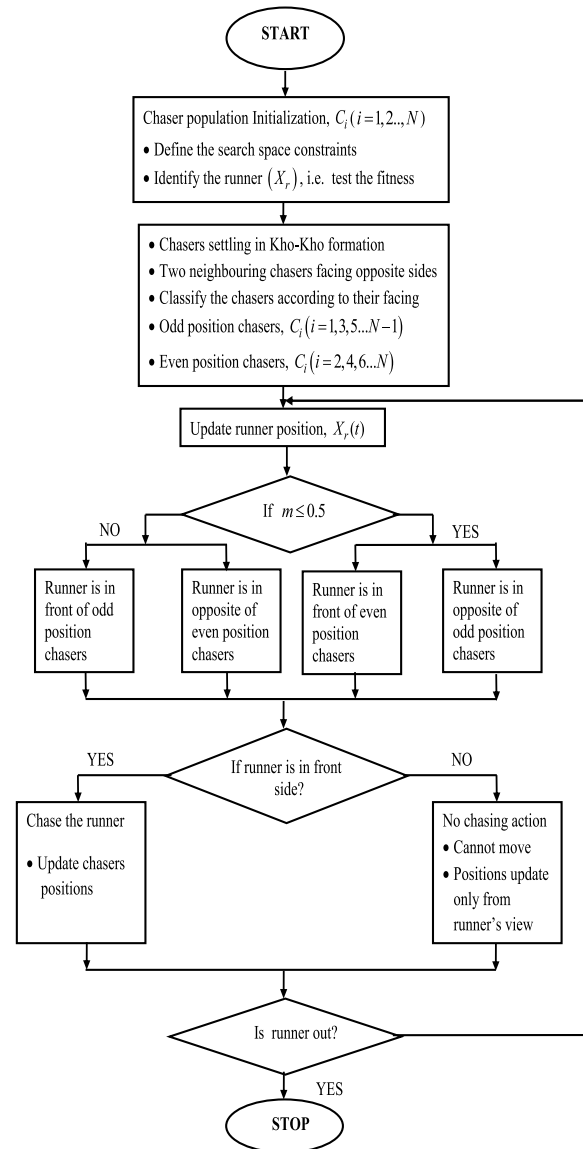
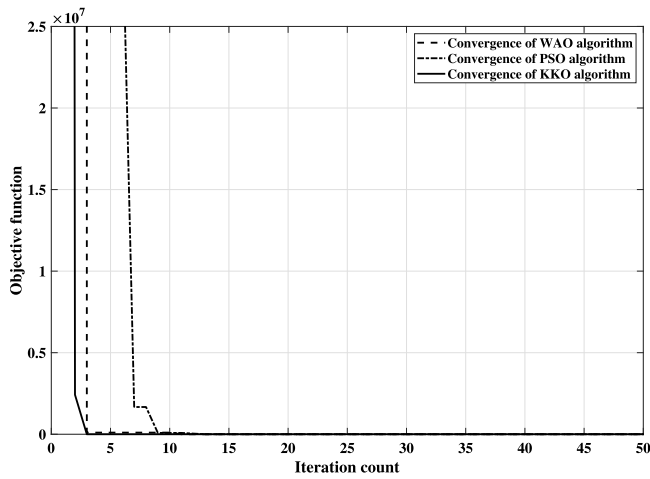
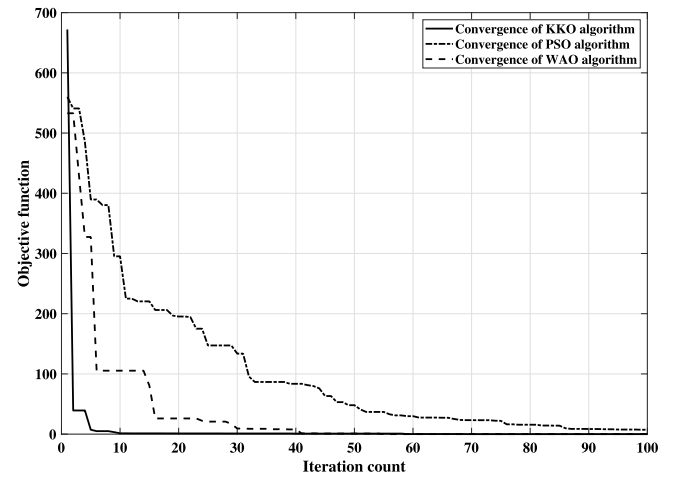


Fig. 3. Flowchart of proposed KKO algorithm.

### 4.2. Result and discussions

According to Zhang et al. (2019), optimization algorithms may not provide a globally optimal solution every time because of the initial value of random population (candidate solution). But, they can provide a sub-optimal solution close to an optimal solution in a short computational time. Therefore, each function is simulated for fifteen times. This helps in computation of important parameters such as minimum value (best result), maximum value (worst result), average value (AVG), standard deviation (STD) and  $p$ -value. Average value provides the average results for the fifteen simulations while standard deviation helps to analyze the deviation in the results. As the objective while solving these functions is to minimize the function, it is better to get smaller average values. Also, the deviation for every case should be as small as possible. All fifteen simulations are simulated with an iteration count of 500. Player population is considered to be 50. The result obtained for the three groups of benchmark functions is discussed as follows.

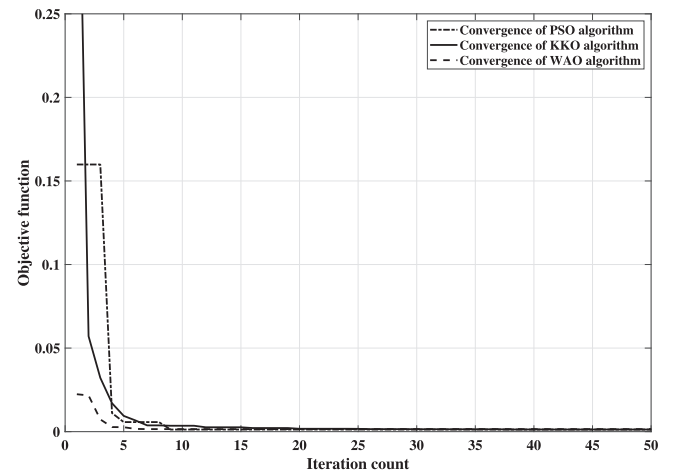
Fig. 4. Convergence profile of  $F_2$ .Fig. 5. Convergence profile of  $F_{11}$ .

#### 4.2.1. Test with uni-modal benchmark functions

The average and standard deviation obtained for the uni-modal functions is presented in Table 2 along with the existing results for DE, WAO, GSA, PSO, etc. It is observed that for this group of functions, KKO provides the best result for functions  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_7$ . The deviation in the result is also small as compared to other algorithms used for comparison for functions  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_7$ . This proves that the proposed KKO exhibits better exploitation capabilities.

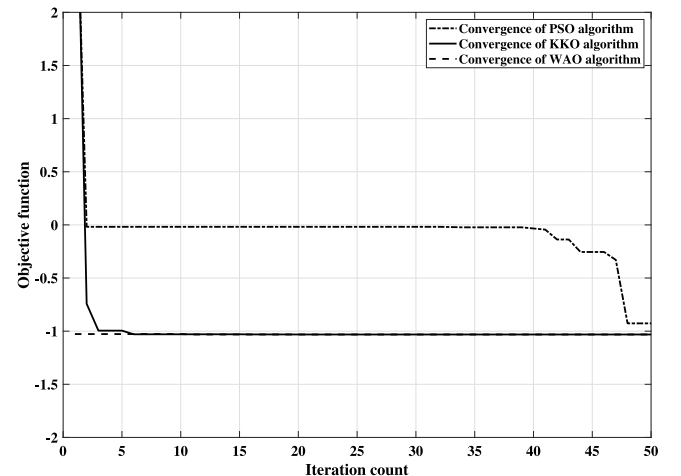
#### 4.2.2. Test with multi-modal benchmark functions

Table 2 also presents the average and standard deviation value of the sixteen multi-modal functions along with existing results for DE (Storn and Price, 1997), WAO (Mirjalili and Lewis, 2016), GSA (Rashedi et al., 2009), and PSO (Kennedy and Eberhart, 1995). It is observed that the proposed KKO provide the optimal results for functions  $F_9$ ,  $F_{11}$ , and  $F_{17}$ . KKO provides third optimal results for functions  $F_{10}$  and forth best for other functions. From this analysis, it can be inferred that the proposed algorithm exhibits good exploration capabilities. (See Table 1.)

Fig. 6. Convergence profile of  $F_{15}$ .

#### 4.2.3. Test with composite functions

Composite functions are used to test the balance between exploration and exploitation. The average and standard deviation obtained for the six composite functions is presented in Table 2. For comparison, existing results of DE (Storn and Price, 1997), WAO (Mirjalili and Lewis, 2016), GSA (Rashedi et al., 2009), and PSO (Kennedy and Eberhart, 1995) are also presented in Table 2. It is observed that for functions  $F_{24}$  to  $F_{29}$  the proposed KKO provides better result compared to existing techniques. The deviation for these functions is also less. These test dues prove that the proposed technique exhibits a good balance between exploration and exploitation.

Fig. 7. Convergence profile of  $F_{16}$ .

#### 4.2.4. Convergence analysis for KKO algorithm

To analyze the convergence behavior of the proposed KKO algorithm, convergence profile of the functions  $F_2$ ,  $F_{11}$ ,  $F_{15}$ ,  $F_{16}$  and  $F_{24}$  are examined. The convergence profile of WAO and PSO is used for the comparison. The convergence plot for these functions are presented in Figs. 4–8 respectively. These figures show that the proposed algorithm exhibits a faster and better convergence profile that the existing techniques WAO and PSO.

## 5. Asymptotic analysis of the proposed algorithm

In the asymptotic analysis, the run time performance of an optimization algorithm is computed in terms of input parameter size. The proposed KKO algorithm includes some input parameters which



**Table 1**

Statistical data for benchmark function from fifteen simulation.

| BF's     | Best result | Worst result | AVG         | STD       | p-value (WAO) | p-value (PSO) |
|----------|-------------|--------------|-------------|-----------|---------------|---------------|
| $F_1$    | 6.83E-55    | 1.33E-33     | 1.30E-35    | 3.74E-35  | 0.0271        | 0.6627        |
| $F_2$    | 6.12E-29    | 1.48E-22     | 3.60E-23    | 6.41E-23  | 0.0271        | 0.1858        |
| $F_3$    | 3.15E-72    | 1.90E-34     | 1.58E-35    | 5.48E-35  | 0.7618        | 0.1858        |
| $F_4$    | 4.81E-33    | 7.29E-18     | 6.05E-19    | 2.01E-18  | 0.3790        | 0.008         |
| $F_5$    | 28.8566     | 28.9732      | 28.90336    | 0.038939  | 0.0905        | 0.0024        |
| $F_6$    | 4.38972     | 7.1798       | 5.788447    | 0.911463  | 0.0014        | 4.9752E-11    |
| $F_7$    | 1.486E-06   | 0.00025862   | 8.77456E-05 | 8.29E-05  | 0.8766        | 1             |
| $F_8$    | -4176.42    | -2935.58     | -3758.63    | 408.5173  | 0.0064        | 0.0064        |
| $F_9$    | 0           | 0            | 0           | 0         | 1.1148E-14    | 1.6853E-14    |
| $F_{10}$ | 0           | 20.4577      | 11.24042    | 9.234612  | 0.0773        | 1             |
| $F_{11}$ | 0           | 0            | 0           | 0         | 0.4204        | 0.3790        |
| $F_{12}$ | -0.8471     | 0.160425     | -0.30359    | 0.360173  | 0.1023        | 0.4825        |
| $F_{13}$ | 2.461       | 4.22067      | 3.302702    | 0.729056  | 0.4290        | 8.1014E-10    |
| $F_{14}$ | 1.01987     | 1.99837      | 1.28144     | 0.478666  | 0.333         | 0.333         |
| $F_{15}$ | 0.001069    | 0.003337     | 0.002303    | 0.001076  | 0.8456        | 0.1463        |
| $F_{16}$ | 1.03136     | -0.99652     | -1.02271    | 0.01186   | 0.6667        | 0.6667        |
| $F_{17}$ | 0.39868     | 0.440983     | 0.412356    | 0.017307  | 0.4655        | 0.4655        |
| $F_{18}$ | 3.01404     | 3.63975      | 3.206278    | 0.227625  | 0.6667        | 0.6667        |
| $F_{19}$ | -3.7312     | -3.42052     | -3.59924    | 0.12549   | 0.7           | 1             |
| $F_{20}$ | -2.51678    | -1.57962     | -2.13964    | 0.31007   | 0.132         | 0.156         |
| $F_{21}$ | -4.35158    | -1.05939     | -2.90419    | 1.482463  | 0.0286        | 0.9408        |
| $F_{22}$ | -5.67592    | -2.90984     | -4.14019    | -1.093177 | 0.0286        | 0.8857        |
| $F_{23}$ | -4.55522    | -2.62289     | -3.97328    | 0.674575  | 0.0154        | 0.0286        |
| $F_{24}$ | 0.295574    | 41.4243      | 3.94335     | 8.39575   | 0.8494        | 0.9448        |
| $F_{25}$ | 0.096249    | 54.1581      | 4.4041      | 8.91397   | 0.6548        | 0.3484        |
| $F_{26}$ | 0.005098    | 124.0245     | 6.3271      | 23.8971   | 0.1456        | 0.7461        |
| $F_{27}$ | 0.82401     | 84.2746      | 8.76504     | 30.4813   | 0.4286        | 0.1876        |
| $F_{28}$ | 2.4954      | 73.2484      | 9.8846      | 36.2415   | 0.4481        | 0.8951        |
| $F_{29}$ | 1.21806     | 44.01587     | 4.13161     | 36.2415   | 0.8453        | 0.9842        |

**Table 2**

Comparative analysis of AVG and STD for benchmark function.

| BF's     | DE (Storn and Price, 1997) |                  | PSO (Kennedy and Eberhart, 1995) |          | GSA (Rashedi et al., 2009) |                 | WAO (Mirjalili and Lewis, 2016) |                 | KKO                |                 |
|----------|----------------------------|------------------|----------------------------------|----------|----------------------------|-----------------|---------------------------------|-----------------|--------------------|-----------------|
|          | AVG                        | STD              | AVG                              | AVG      | AVG                        | AVG             | AVG                             | STD             | AVG                | STD             |
| $F_1$    | 8.2E-14                    | 5.9E-14          | 0.000136                         | 0.000202 | 2.53E-16                   | 9.67E-17        | 1.41E-30                        | 4.91E-30        | <b>1.30E-35</b>    | <b>3.74E-35</b> |
| $F_2$    | 1.5E-09                    | 9.9E-10          | 0.042144                         | 0.045421 | 0.055655                   | 0.194074        | 1.06E-21                        | 2.39E-21        | <b>3.60E-23</b>    | <b>6.41E-23</b> |
| $F_3$    | 6.8E-11                    | 7.4E-11          | 70.12562                         | 22.11924 | 896.5347                   | 318.9559        | 5.39E-07                        | 2.93E-06        | <b>1.58E-35</b>    | <b>5.48E-35</b> |
| $F_4$    | <b>0</b>                   | <b>0</b>         | 1.086481                         | 0.317039 | 7.35487                    | 1.741452        | 0.072581                        | 0.39747         | 6.05E-19           | 2.01E-18        |
| $F_5$    | <b>0</b>                   | <b>0</b>         | 96.71832                         | 60.11559 | 67.54309                   | 62.22534        | 27.86558                        | 0.763626        | 28.90336           | 0.038939        |
| $F_6$    | <b>0</b>                   | <b>0</b>         | 0.000102                         | 8.28E-05 | 2.5E-16                    | 1.74E-16        | 3.116266                        | 0.532429        | 5.788447           | 0.911463        |
| $F_7$    | 0.00463                    | 0.0012           | 0.122854                         | 0.044957 | 0.089441                   | 0.04339         | 0.001425                        | 0.001149        | <b>8.77456E-05</b> | <b>8.29E-05</b> |
| $F_8$    | <b>-11080.1</b>            | 574.7            | -4841.29                         | 1152.814 | -2821.07                   | 493.0375        | -5080.76                        | 695.7968        | -3758.63           | 408.5173        |
| $F_9$    | 69.2                       | 38.8             | 46.70423                         | 11.62938 | 25.96841                   | 7.470068        | <b>0</b>                        | <b>0</b>        | <b>0</b>           | <b>0</b>        |
| $F_{10}$ | 9.7E-08                    | 4.2E-08          | 0.276015                         | 0.50901  | 0.062087                   | 0.23628         | 7.4043                          | 9.897572        | 11.24042           | 9.234612        |
| $F_{11}$ | <b>0</b>                   | <b>0</b>         | 0.009215                         | 0.007724 | 27.70154                   | 5.040343        | 0.000289                        | 0.001586        | <b>0</b>           | <b>0</b>        |
| $F_{12}$ | <b>7.9E-15</b>             | <b>8.E-15</b>    | 0.006917                         | 0.026301 | 1.799617                   | 0.95114         | 0.339676                        | 0.214864        | -0.30359           | 0.360173        |
| $F_{13}$ | <b>5.1E-14</b>             | <b>4.8E-14</b>   | 0.00675                          | 0.008907 | 8.899084                   | 7.126241        | 1.889015                        | 0.266088        | 3.302702           | 0.729056        |
| $F_{14}$ | <b>0.998004</b>            | <b>3.3E-16</b>   | 3.627168                         | 2.560828 | 5.859838                   | 3.831299        | 2.111973                        | 2.498594        | 1.28144            | 0.478666        |
| $F_{15}$ | <b>4.5E-14</b>             | <b>0.00033</b>   | 0.000577                         | 0.000222 | 0.003673                   | 0.001647        | 0.000572                        | 0.000324        | 0.002303           | 0.001076        |
| $F_{16}$ | -1.03163                   | 3.1E-13          | -1.03163                         | 6.25E-16 | <b>-1.03163</b>            | <b>4.88E-16</b> | -1.03163                        | 4.2E-07         | -1.02271           | 0.01186         |
| $F_{17}$ | 0.397887                   | 9.4E-09          | <b>0.397887</b>                  | <b>0</b> | <b>0.397887</b>            | <b>0</b>        | 0.397914                        | 2.7E-05         | 0.412356           | 0.017307        |
| $F_{18}$ | 3                          | 2E-15            | 3                                | 1.33E-15 | 3                          | 4.17E-15        | 3                               | 4.22E-15        | <b>3</b>           | <b>0</b>        |
| $F_{19}$ | NA                         | NA               | <b>-3.86278</b>                  | 2.58E-15 | -3.86278                   | <b>2.29E-15</b> | -3.85616                        | 0.002706        | -3.59924           | 0.12549         |
| $F_{20}$ | NA                         | NA               | <b>-3.26634</b>                  | 0.060516 | -3.31778                   | 0.023081        | -2.98105                        | 0.376653        | -2.13964           | 0.31007         |
| $F_{21}$ | <b>-10.1532</b>            | <b>0.0000025</b> | -6.8651                          | 3.019644 | -5.95512                   | 3.737079        | -7.04918                        | 3.629551        | -2.90419           | 1.482463        |
| $F_{22}$ | <b>-10.4029</b>            | <b>3.9E-07</b>   | -8.45653                         | 3.087094 | -9.68447                   | 2.014088        | -8.18178                        | 3.829202        | -4.14019           | -1.093177       |
| $F_{23}$ | -10.5364                   | 1.9E-07          | -9.95291                         | 1.782786 | <b>-10.5364</b>            | <b>2.6E-15</b>  | -9.34238                        | 2.414737        | -3.97328           | 0.674575        |
| $F_{24}$ | 6.75E-2                    | 1.11E-1          | 100                              | 81.65    | <b>6.63E-17</b>            | <b>2.78E-17</b> | 0.568846                        | 0.505946        | 3.94335            | 8.39575         |
| $F_{25}$ | 28.759                     | <b>8.6277</b>    | 155.91                           | 13.176   | 200.6202                   | 67.72087        | 75.30874                        | 43.07855        | <b>4.4041</b>      | 8.91397         |
| $F_{26}$ | 144.41                     | <b>19.401</b>    | 172.03                           | 32.769   | 180                        | 91.89366        | 55.65147                        | 21.87944        | <b>6.3271</b>      | 23.8971         |
| $F_{27}$ | 324.86                     | <b>14.784</b>    | 314.3                            | 20.066   | 170                        | 82.32726        | 53.83778                        | 21.621          | <b>8.76504</b>     | 30.4813         |
| $F_{28}$ | 10.789                     | <b>2.604</b>     | 83.45                            | 101.11   | 200                        | 47.14045        | 77.8064                         | 52.02346        | <b>9.8846</b>      | 36.2415         |
| $F_{29}$ | 490.94                     | 39.461           | 861.42                           | 125.81   | 142.0906                   | 88.87141        | 57.88445                        | <b>34.44601</b> | <b>4.13161</b>     | 36.2415         |

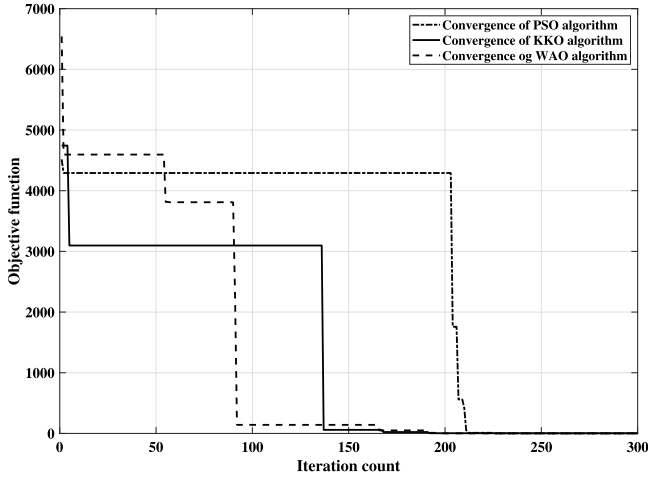
influences the running time performance. These input parameters in KKO algorithm includes number of iteration ( $T_{max}$ ), number of players ( $N_{max}$ ) and position of the players ( $P_{max}$ ). Depending on these input parameters, the operations to be executed by the proposed algorithm are presented as follows:

1.  $T_{max} + 1$
2.  $T_{max} \times N_{max} \times P_{max}$

3.  $T_{max} \times (N_{max} + 1)$
4.  $T_{max} \times (N_{max} + 1)$
5.  $T_{max} \times N_{max} \times P_{max}$

The total number of operations ( $O_{total}$ ) can be computed using the above operations as follows:

$$O_{total} = (T_{max} + 1) + (T_{max} \times N_{max} \times P_{max}) + (T_{max} \times (N_{max} + 1))$$

Fig. 8. Convergence profile of  $F_{24}$ .

**Table 3**  
Comparative time complexity analysis.

| Technique                        | Time complexity (under worst case) |
|----------------------------------|------------------------------------|
| PSO (Kennedy and Eberhart, 1995) | $O(n^4)$                           |
| MBCO (Das et al., 2018a)         | $O(n^4)$                           |
| CTO (Das et al., 2018b)          | $O(n^4)$                           |
| WAO (Mirjalili and Lewis, 2016)  | $O(n^3)$                           |
| KKO                              | $O(n^3)$                           |

$$+ (T_{max} \times (N_{max} + 1)) \\ + (T_{max} \times N_{max} \times P_{max}). \quad (9)$$

$$O_{total} = 2 \times (T_{max} \times N_{max} \times P_{max}) \\ + 2 \times (T_{max} \times (N_{max} + 1)) \\ + 3 \times (T_{max}) + 1. \quad (10)$$

Now, for the run time performance analysis, all the input parameters are considered to be equal for the worst case scenario. Hence, (10) can be written as a function of  $n$  as

$$f(n) = 2 \times n^3 + 2 \times n^2 + 3 \times n + 1 \quad (11)$$

Here, we assume that  $f(n)$  is big-O of  $n^3$ , i.e.  $g(n) = n^3$ . Hence,  $f(n) = O(g(n))$  if and only if, there exists two positive constants  $c$  and  $n_0$  such that

$$|f(n)| \leq c|g(n)| \quad (12)$$

for all  $n \geq n_0$  and  $f(n)$  is non-negative. Therefore, (12) can be written as

$$0 \leq f(n) \leq cg(n) \quad (13)$$

for all  $n \geq n_0$ .  $c$  can be computed by adding all coefficients of (14) as (8). Then, if  $n \geq n_0 = 1$ ,

$$2 \times n^3 + 2 \times n^2 + 3 \times n + 1 \leq 2 \times n^3 + 2 \times n^3 + 3 \times n^3 + n^3 \quad (14)$$

thus  $2 \times n^3 + 2 \times n^2 + 3 \times n + 1 = O(n^3)$ . Therefore,  $f(n) = O(g(n)) = O(n^3)$ . By this it can be concluded that as  $n$  is increased,  $f(n)$  will always be less than or equal to  $g(n)$ . For further understanding, Fig. 9 shows that as  $n$  is increased,  $f(n)$  is either equal or less than to  $g(n)$ . To further analyze the performance of KKO with respect to other existing technique on basis of time complexity analysis, Table 3 is presented. It is observed that the proposed KKO requires less number of operation and hence less time to compute the results.

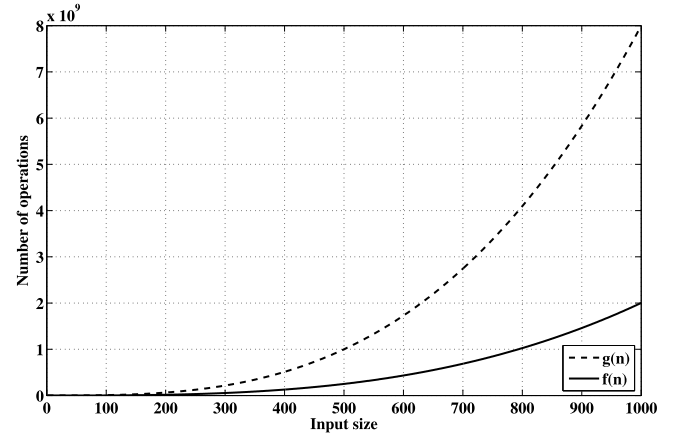


Fig. 9. Asymptotic behavior of KKO.

## 6. Convergence analysis of KKO

In this section, convergence of KKO is analyzed mathematically using differential equation. For this analysis some assumptions are taken into consideration. Let us consider that the problem space is one dimensional and the number player is one. Next, it is also assumed that endurance factor is 1. Now, the odd and even conditions are analyzed one by one.

Considering the odd condition along with the above assumptions, (5) can be simplified as:

$$C_o(t+1) = C_o(t) + g \cdot [C_b(t) - C_o(t)]. \quad (15)$$

$$\text{Let } y(t) = C_b(t) - C_o(t),$$

$$C_o(t+1) = C_o(t) + g \cdot y(t). \quad (16)$$

At  $(t+1)$  instant, (16) can be written as:

$$C_o(t+2) = C_o(t+1) + g \cdot y(t+1). \quad (17)$$

Considering (16) and the assumption  $y(t) = C_b(t) - C_o(t)$ ,  $y(t+1)$  can be written as:

$$y(t+1) = (1-g) \left( \frac{C_o(t+1) - C_o(t)}{g} \right). \quad (18)$$

Replacing  $y(t+1)$  from (18) to (17), (17) can be rewritten as:

$$C_o(t+2) + (2-g) \cdot C_o(t+1) + (g-1) \cdot C_o(t) = 0. \quad (19)$$

Making (19) to be continuous and a second order differential equation is derived as:

$$\frac{d^2 C_o}{dt^2} + \ln(e_1 \cdot e_2) \frac{dC_o}{dt} + \ln(e_1) \cdot \ln(e_2) C_o = 0. \quad (20)$$

where  $e_1$  and  $e_2$  are the roots of the quadratic equation (19) which can be further solved to  $e_1 = (g-2)/2 + \sqrt{(2-g)^2 + 4(1-g)}/2$  and  $e_2 = (g-2)/2 - \sqrt{(2-g)^2 + 4(1-g)}/2$ . Hence, the general solution of the differential equation (20) can be written as follows:

$$C_o(t) = K_1 \cdot e_1^t + K_2 \cdot e_2^t. \quad (21)$$

Further  $y(t)$  can be thus written as:

$$y(t) = \frac{K_1 \cdot e_1^t (e_1 - 1) + K_2 \cdot e_2^t (e_2 - 1)}{g}. \quad (22)$$

where  $K_1 = \frac{C_o(0)(e_2+1)+g \cdot y(0)}{e_1+e_2}$  and  $K_2 = \frac{C_o(0)(e_1-1)+g \cdot y(0)}{e_1+e_2}$

In a similar way, the even condition along with the assumptions can be solved and (4) can be simplified to a quadratic equation which is represented as follows:

$$C_f(t+2) + (2+g)C_f(t+1) - (1+g)C_f(t) - (2+g)v(t) = 0. \quad (23)$$

Here, for simplicity  $v(t)$  is assumed to be zero then (23) can be rewritten as:

$$C_f(t+2) + (2+g)C_f(t+1) - (1+g)C_f(t) = 0. \quad (24)$$

Making (24) to be continuous and a second order differential equation is derived as:

$$\frac{d^2 C_f}{dt^2} + \ln(p_1 \cdot p_2) \frac{dC_f}{dt} + \ln(p_1) \cdot \ln(p_2) C_f = 0. \quad (25)$$

where  $p_1$  and  $p_2$  are the roots of the quadratic equation (24) which can be further solved to  $p_1 = (g+2)/2 + \sqrt{(2+g)^2 + 4(1+g)}/2$  and  $p_2 = (g+2)/2 - \sqrt{(2+g)^2 + 4(1+g)}/2$ . Hence, the general solution of (20) can be written as follows:

$$C_f(t) = L_1 \cdot p_1^t + L_2 \cdot p_2^t. \quad (26)$$

Further  $y(t)$  can be thus written as:

$$y(t) = \frac{L_1 \cdot p_1^t (p_1 - 1) + L_2 \cdot p_2^t (p_2 - 1)}{g}. \quad (27)$$

where  $L_1 = \frac{-C_f(0)(e_2-1)+g \cdot y(0)}{e_1-e_2}$  and  $L_2 = \frac{C_f(0)(e_1-1)-g \cdot y(0)}{e_1-e_2}$

Now, from (21), (22), (26) and (27), if  $t$  tends to infinity, then  $C_o(t)$ ,  $C_f(t)$  and  $y(t)$  will tend to zero. This shows that KKO convergence to a global optima.

## 7. Advantage and disadvantage of proposed KKO

The uniqueness of the proposed KKO optimization technique with respect to other existing optimization scheme (Storn and Price, 1997; Mirjalili and Lewis, 2016; Rashedi et al., 2009; Kennedy and Eberhart, 1995; Gandomi and Alavi, 2012; Cuevas et al., 2013; Yang, 2010) is the use of two groups for the search of the global optimal solution. In KKO, the population of the players are divided into two groups. Players of each group update their position depending on the position of the runner (optimal solution) using different update rules. This increases the diversity of the search and improves the ability of finding the global optimal solution. This can also be observed from the study with the benchmark functions presented in Section 4. The proposed KKO technique provides the best average solution in twelve functions out of twenty-nine functions which surpasses the results of other obtained using other techniques (Storn and Price, 1997; Mirjalili and Lewis, 2016; Rashedi et al., 2009; Kennedy and Eberhart, 1995). This proves that the searching diversity of KKO is better in comparison to other techniques apart from good exploitation, exploitation, convergence capabilities. It is also observed that this approach also improves the convergence profile of the KKO techniques which can be concluded from the Figs. 4–8.

Further, it is also observed that the standard deviation for the twenty-nine benchmark functions obtained for KKO is not the best. In case of standard deviation, differential evolution provides a much stable result in comparison to KKO technique. This concludes that KKO has more deviation in the results. But, still the this is better compared to PSO, WAO etc.

## 8. KKO to solve combined emission economic dispatch (CEED) problem

### 8.1. Introduction

Increasing environmental problem due to the burning of fossil fuel in thermal power plants is a big concern for the power industries. Burning of fossil fuels in these industries emits harmful gases like carbon dioxide, sulfur dioxide, nitrogen dioxide, etc. These gases are not only harmful to the environment but also for the human being. To handle this problem power industries changed their objective from an economic load dispatch problem to a CEED. The objective while solving a CEED problem is to minimize the overall generation cost as well

as minimize the emission rate. Along with these objectives, different constraints should also be fulfilled.

Numerous methodologies have been developed to solve this highly non-linear and complex multi-objective problem more effectively. Some of the recent works in this domain are lightning flash algorithm (Kheshti et al., 2017), stochastic fractal search algorithm (Alomoush and Oweis, 2018), interior search algorithm (Karthik et al., 2019), parallel hurricane optimization algorithm (Rizk-Allah et al., 2018), multi-objective hybrid bat algorithm (Liang et al., 2018), enhanced moth-flame optimizer (Elsakaan et al., 2018), bio-inspired algorithms (Dey et al., 2019), chaotic improved harmony search algorithm (Rezaie et al., 2019), reinforcement learning based on non-dominated sorting genetic algorithm (Bora et al., 2019), Quantum-behaved bat algorithm (Mahdi et al., 2018), Double weighted particle swarm optimization (Kheshti et al., 2018), Floating search space (Amiri et al., 2018), Chaotic self-adaptive interior search algorithm (Rajagopalan et al., 2019), modified shuffle frog leaping algorithm (Elattar, 2019), improved TLBO (Joshi and Verma, 2019), new sine cosine algorithm (Gonidakis and Vlachos, 2019). Though many optimization algorithms are developed and used to solve the CEED problem. However, new techniques are always entertained to have more efficient and effective results.

In this section, the proposed KKO algorithm is tested with five test system of a CEED problem. Before moving towards the test systems, the mathematical model of the CEED problem is discussed in the next section.

### 8.2. Problem formulation

As stated earlier, the objective for a CEED problem is to optimize power generation of generating units such that the overall fuel cost and emission rates should be minimum. As there are two objective, CEED problem is considered as a multi-objective problem.

The overall fuel cost for the generating units is computed using the following expression:

$$F_c = \sum_{i=1}^m (a_i G_i^2 + b_i G_i + c_i) + |g_i \times \sin(h_i \times (G_i^{\min} - G_i))|, \quad (28)$$

where  $F_c$  is the total fuel cost;  $a_i$ ,  $b_i$ ,  $c_i$  are fuel cost coefficients of the  $i$ th generating unit;  $g_i$  and  $h_i$  are coefficients of the  $i$ th generator due to valve point effect;  $G_i$  is the real active power generated from the  $i$ th generating units and  $m$  is total number of generating units in the power system.

Similarly, for the evaluation of the emission rate for the  $m$  generating units, the mathematical equation used is given as.

$$F_E = \sum_{i=1}^m \alpha_i G_i^2 + \beta_i G_i + \gamma_i + \eta_i \times \exp(\delta_i \times G_i), \quad (29)$$

where  $E_C$  is the total emission,  $\alpha_i$ ,  $\beta_i$ ,  $\gamma_i$ ,  $\eta_i$ ,  $\delta_i$  are the emission coefficients of the  $i$ th generating units.

Hence, by combining the two objective functions, the objective for CEED problem is formulated as:

$$C = F_c + p * F_E, \quad (30)$$

where  $C$  is the objective function for a CEED problem to be minimized and  $p$  is referred as a penalty factor.

The penalty factor provides a ratio between the average fuel cost computed for maximum power capacity for each generating unit and average emission computed for maximum power capacity for the same. The computation of penalty factor  $p$  for a given load can be done as present in Abdelaziz et al. (2016a). It is used to convert a dual objective CEED problem to a single objective CEED problem.



### 8.3. Constraints

#### 8.3.1. Generation operating limits

According to this, all the generating units should operate within a prescribed limit of generation. Mathematically, it is given as:

$$G_i^{\min} \leq G_i \leq G_i^{\max}, i = 1, 2, 3, \dots, n, \quad (31)$$

where  $G_i^{\min}$  is the minimum power output of  $i$ th generating unit (in MW) and  $G_i^{\max}$  is the maximum output power of  $i$ th generating unit (in MW).

#### 8.3.2. Power balance criterion

During transmission and generation of power, losses are always associated. For this reason, it is important that the total power generation is generated in such a way that the following equation is fulfilled.

$$\sum_{i=1}^m G_i = D_p + L_p, \quad (32)$$

where  $D_p$  is the total power demand and  $L_p$  represents the power loss in transmission.

The total transmission loss is computed by:

$$L_p = \sum_{i=1}^m \sum_{j=1}^m G_i B_{ij} G_j, \quad (33)$$

where  $B_{ij}$  denotes the element in loss coefficient matrix.

### 8.4. Results and discussions

The proposed KKO algorithm is tested with five test systems. These test systems are used to evaluate the performance and effectiveness of proposed KKO to solve the CEED problem. The characteristics of these five systems are presented in Table 4. Each system is simulated for fifteen times with an iteration count of 500 and players population of 150. The statistical data obtained from the fifteen individual runs are presented in Table 5.

For test system V.i, the best result obtained is presented in Table 6. For a comparative analysis of the result, some existing results of well-known techniques like FA (Gherbi et al., 2016), BA (Gherbi et al., 2016), HYB (Gherbi et al., 2016), GA (Gherbi et al., 2016), PSO (Elattar, 2019), FPA (Abdelaziz et al., 2016b), and MSFLA (Elattar, 2019) are also presented in Table 6. It is observed that the proposed technique performs better than the existing techniques. Proposed KKO optimized the generator units to provide better fuel cost and emission rates with reduced losses. The convergence behavior of the objective function is presented in Fig. 10.

For test system V.ii, the optimal fuel cost and emission rate obtained using the proposed KKO algorithm is presented in Table 7. For comparative analysis of the result obtained, existing results of NSGA-II (Basu, 2011), PDE (Basu, 2011), SPEA-2 (Basu, 2011), GSA (Güvenç et al., 2012), ABC-PSO (Manteaw and Odero, 2012), EMOCA (Zhang et al., 2013), FPA (Abdelaziz et al., 2016a) and LFA (Kheshti et al., 2017) are used. These results are also presented in Table 7. On comparing the above mentioned techniques, it is observed that the proposed KKO provides the best emission rate whereas LFA provides the best fuel cost. The convergence behavior of the objective is presented in Fig. 11.

Test system V.iii is simulated under two simulation condition. In first condition, generators are optimized to provide best fuel cost only. Under this condition, the optimal power allocation that provides best fuel cost is presented in Table 8. For comparative analysis results of existing techniques like MHBA (Liang et al., 2018), FSBF (Liang et al., 2018), NSBF (Liang et al., 2018) are also presented in Table 8. It is observed that proposed KKO algorithm optimizes the generator units more effectively with reduced losses. For the second condition, generators are optimized to provide best emission rate. For this condition, the optimal power allocation that provides better emission is presented in Table 9. Results for MHBA (Liang et al., 2018), FSBF (Liang

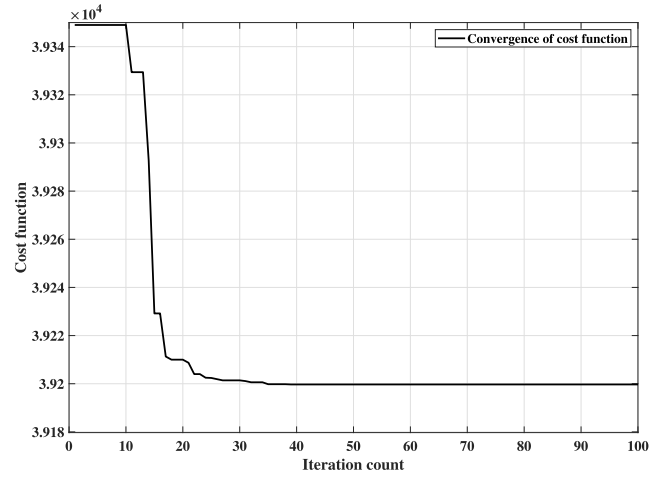


Fig. 10. Convergence profile for test system V.i.

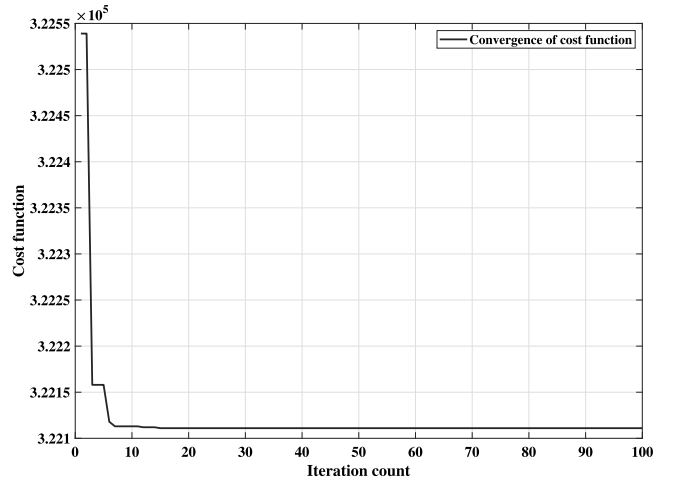


Fig. 11. Convergence profile for test system V.ii.

et al., 2018), NSBF (Liang et al., 2018) is also used in this case for comparative analysis. It is clear from the analysis that proposed KKO provides the best emission rate for this case. The convergence behavior of the objective function in case of best fuel and emission is presented in Figs. 12 and 13 respectively.

For test system V.iv, the results of existing technique like MODE (Basu, 2011), PDE (Basu, 2011), NSGA-II (Basu, 2011), SPEA-2 (Basu, 2011), GSA (Güvenç et al., 2012), MABC/D/C (Secui, 2015), MABC/D/Log (Secui, 2015), FPA (Abdelaziz et al., 2016b) and ISA (Karthik et al., 2019) along with the proposed KKO technique is presented in Table 10. It is observed that ISA provides the best result for this system whereas the proposed CSOA provides third best result for this case. The convergence of the objective function for this case is presented in Fig. 14.

For test system V.v, the optimal power allocation obtained for the fourteen generators is presented in Table 11. Techniques such as CSAISA (Rajagopalan et al., 2019), ISA (Rajagopalan et al., 2019), HSA (Jeddi and Vahidinasab, 2014), DE (Rajagopalan et al., 2019), PSO (Rajagopalan et al., 2019) and GA (Rajagopalan et al., 2019) are used for a comparative analysis. All the results are presented in Table 11. It is clear from the comparative analysis that the proposed KKO provides the best fuel cost and emission rate. Further, the convergence behavior of the objective function is presented in Fig. 15.

**Table 4**  
Characteristic of the ten test systems.

| Problem | System                            | No. of units | Power demand | Heat demand | VPE | Loss |
|---------|-----------------------------------|--------------|--------------|-------------|-----|------|
| CEED    | V.i (Gherbi et al., 2016)         | 3            | 500          | –           | ✓   | ✓    |
|         | V.ii (Abdelaziz et al., 2016b)    | 10           | 2000         | –           | ✓   | ✓    |
|         | V.iii (Liang et al., 2018)        | IEEE 30 Bus  | 2.834        | –           | ✓   | ✓    |
|         | V.iv (Kheshti et al., 2017)       | 40           | 10500        | –           | ✓   | –    |
|         | V.v (Rajagopalan et al., 2019)    | IEEE 118 Bus | 950          | –           | ✓   | ✓    |
| CHPED   | VI.i (Sun and Li, 2019)           | 4            | 200          | 115         | ✓   | –    |
|         | VI.ii (Haghras et al., 2016)      | 5            | 250          | 175         | ✓   | –    |
|         | VI.iii (Sun et al., 2017)         | 7            | 600          | 150         | ✓   | ✓    |
|         | VI.iv (Nazari-Heris et al., 2019) | 24           | 2350         | 1250        | ✓   | –    |
|         | VI.v (Basu, 2016)                 | 48           | 4700         | 2500        | ✓   | –    |

**Table 5**  
Statistical data for the ten test systems.

| System      | Method   | Best        | Worst       | AVG         | STD      |
|-------------|----------|-------------|-------------|-------------|----------|
| Case V.i    | KKO      | 39199.7     | 39200       | 39199.77    | 0.121106 |
|             | PSO      | 39210.20    | 39261       | 39229.02    | 23.29    |
|             | FPA      | 39210.15    | 39244.9     | 39216.49    | 15.48465 |
| Case V.ii   | KKO      | 311995      | 322464      | 322218.8    | 177.5696 |
|             | FPA      | 321927      | 322604      | 322201      | 356.4    |
|             | LFA      | 320914      | 322087      | 321278.66   | 701.15   |
| Case V.iii  | KKO      | 605.68      | 610.364     | 606.8468    | 2.106488 |
|             | MHBA     | 607.3897    | 607.5692    | 607.4492    | –        |
| Case V.iv   | KKO      | 10056       | 10185.2     | 10127.02    | 60.77781 |
|             | FPA      | 121074.5    | 121095.7    | 121196.3    | –        |
|             | ISA      | 120385.47   | 120403.21   | 120478.62   | –        |
| Case V.v    | KKO      | 251245      | 251345      | 251281.2    | 21.70714 |
|             | PSO      | 251268      | 251456      | 251298.1    | 35.6122  |
| Case VI.i   | KKO      | 9217.03     | 9527.75     | 9300.26     | 113.6207 |
|             | IGA-NCM  | 9257.075    | 9257.9014   | 9257.1553   | 0.1752   |
|             | TSCO     | 9257.07     | 9261.32     | 9258.09     | –        |
|             | CSA      | 9257.07     | 9327.72     | 9259.165    | 9.886    |
| Case VI.ii  | KKO      | 12113.4     | 12419.3     | 10798.44    | 942.7646 |
|             | IGA-NCM  | 12117.17    | 12117.1725  | 12117.1704  | 0.006    |
| Case VI.iii | KKO      | 10045.4     | 10063.5     | 10049.62    | 9.640211 |
|             | IGA-NCM  | 10107.9071  | 10108.6241  | 10108.04    | 0.1804   |
|             | EMA      | 10111.0732  | –           | 10111.0932  | –        |
|             | GWO      | 10111.24    | 10452.12    | 10194.91    | –        |
| Case VI.iv  | KKO      | 57735.6     | 60646.6     | 58491.46    | 1259.745 |
|             | IGA-NCM  | 57826.0802  | 57875.3621  | 57843.5046  | 10.4849  |
|             | GSO      | 58122.7     | –           | –           | –        |
|             | IGSO     | 58049.019   | 58219.141   | 58156.519   | –        |
|             | MPHS     | 57845.639   | –           | –           | –        |
| Case VI.V   | RCGA-IMM | 57927.69    | 58301.901   | 58066.635   | –        |
|             | KKO      | 115422      | 117172      | 116332      | 797.3646 |
|             | IGA-NCM  | 115685.1807 | 115828.4495 | 115767.6079 | 30.8627  |
|             | GSO      | 117098.4186 | 117109.97   | 117103.02   | –        |
|             | OGSO     | 116678.1987 | 116695.7408 | 116684.819  | –        |

**Table 6**  
Comparison analysis for test system V.i.

| Method                        | $G_1$    | $G_2$    | $G_3$    | $C$      | $F_C$   | $F_E$    | $L_P$   |
|-------------------------------|----------|----------|----------|----------|---------|----------|---------|
| FA (Gherbi et al., 2016)      | 128.8249 | 192.5856 | 190.2825 | 39209.93 | –       | 311.15   | 11.6936 |
| BA (Gherbi et al., 2016)      | 128.8280 | 192.5792 | 190.2858 | 39209.94 | –       | 311.15   | 11.6936 |
| HYB (Gherbi et al., 2016)     | 128.8343 | 192.5670 | 190.2918 | 39209.96 | –       | 311.15   | 11.6936 |
| GA (Gherbi et al., 2016)      | 128.997  | 192.683  | 190.11   | 39220    | –       | 311.27   | 11.6964 |
| PSO (Elattar, 2019)           | 128.984  | 192.645  | 190.063  | 39210.20 | –       | 311.150  | 11.6919 |
| FPA (Abdelaziz et al., 2016b) | 128.8074 | 192.5906 | 190.2958 | 39210.15 | –       | 311.155  | 11.6938 |
| MSFLA (Elattar, 2019)         | 128.338  | 191.964  | 191.389  | 39209.81 | –       | 311.1638 | 11.6927 |
| KKO                           | 129.011  | 192.303  | 190.274  | 39199.7  | 25490.5 | 311.013  | 11.6874 |

## 9. KKO to solve combined heat and power dispatch problem

### 9.1. Introduction

In past few decades, combined heat and power units have drawn attention of researchers. These units are quite popular in the field of energy saving and environmental protection. This is because the

efficiency of these units are high which can be up to 90% (Alipour et al., 2014; Haghras et al., 2016). They are capable enough to reduce the emission rate which can vary between 13%–18% (Vasebi et al., 2007). Unlike a economic load dispatch problem or a CEED problem the objective while solving a combined heat and power dispatch (CHPED) problem is same i.e. operate the units to fulfill the required demand

**Table 7**  
Comparison analysis for test system V.ii.

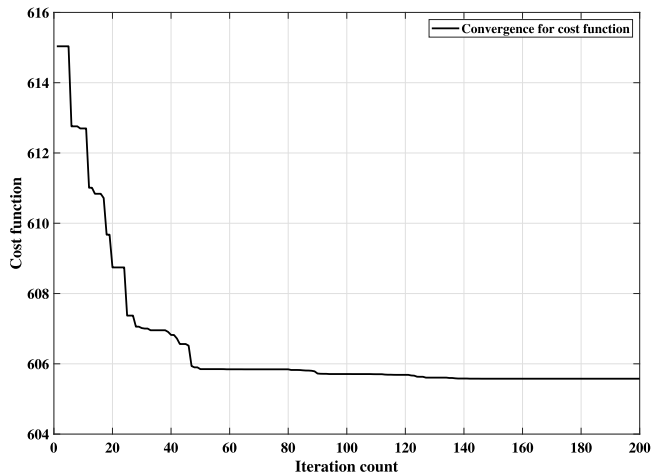
| Outputs  | NSGA-II (Basu, 2011) | PDE (Basu, 2011) | SPEA-2 (Basu, 2011) | GSA (Güvenç et al., 2012) | ABC-PSO (Manteau and Otero, 2012) | EMOCA (Zhang et al., 2013) | FPA (Abdelaziz et al., 2016a) | LFA (Kheshti et al., 2017) | KKO            |
|----------|----------------------|------------------|---------------------|---------------------------|-----------------------------------|----------------------------|-------------------------------|----------------------------|----------------|
| $G_1$    | 51.9515              | 54.9853          | 52.9761             | 54.9992                   | 55                                | 55                         | 53.188                        | 54.9920                    | 54.9923        |
| $G_2$    | 67.2584              | 79.3803          | 72.813              | 79.9586                   | 80                                | 80                         | 79.975                        | 78.7689                    | 78.8914        |
| $G_3$    | 73.6879              | 83.9842          | 78.1128             | 79.4341                   | 81.14                             | 83.5594                    | 78.105                        | 87.7168                    | 78.7946        |
| $G_4$    | 91.3554              | 86.5942          | 83.6088             | 85                        | 84.213                            | 84.6031                    | 97.119                        | 78.1055                    | 88.7479        |
| $G_5$    | 134.0522             | 144.4386         | 137.2432            | 142.1063                  | 138.3377                          | 146.5632                   | 152.74                        | 140.6272                   | 159.814        |
| $G_6$    | 174.9504             | 165.7756         | 172.9188            | 166.5670                  | 167.5086                          | 169.2481                   | 163.08                        | 157.0936                   | 160.555        |
| $G_7$    | 289.4350             | 283.2122         | 287.2023            | 292.8749                  | 296.8338                          | 300                        | 258.61                        | 299.9954                   | 262.174        |
| $G_8$    | 314.0556             | 312.7709         | 326.4023            | 313.2387                  | 311.5824                          | 317.3496                   | 302.22                        | 309.2219                   | 308.857        |
| $G_9$    | 455.6978             | 440.1135         | 448.8814            | 441.1775                  | 420.3363                          | 412.9183                   | 433.21                        | 439.32434                  | 430.307        |
| $G_{10}$ | 431.8054             | 432.6783         | 423.9025            | 428.6306                  | 449.1598                          | 434.3133                   | 466.07                        | 438.6947                   | 461.039        |
| $F_C$    | 1.13539              | 1.1351           | 1.1352              | 1.1349                    | 1.1342                            | 1.13445                    | 1.1337                        | <b>1.13246</b>             | 1.13481        |
| $F_E$    | 4130.2               | 4111.4           | 4109.1              | 4111.4                    | 4120.1                            | 4113.98                    | 3997.7                        | 4139.89                    | <b>3982.85</b> |
| $L_P$    | 84.25                | 83.9             | 84.1                | 83.9869                   | 84.1736                           | <b>83.56</b>               | 84.3                          | 84.37                      | 84.17          |

**Table 8**  
Comparison of  $C_{Fuel}$  for test system V.iii.

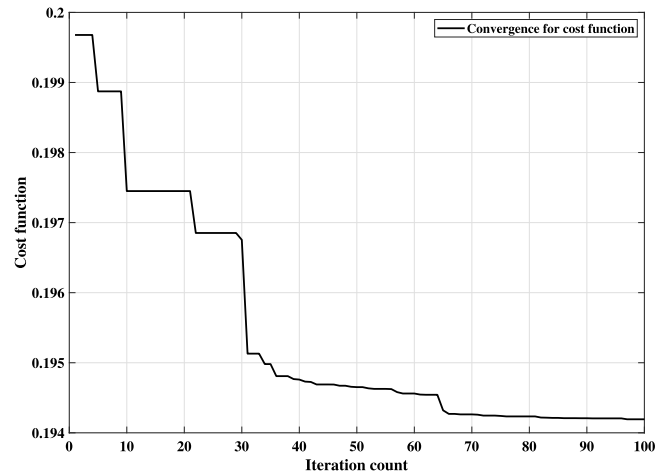
| Units | MHBA (Liang et al., 2018) | FSBF (Liang et al., 2018) | NSBF (Liang et al., 2018) | KKO            |
|-------|---------------------------|---------------------------|---------------------------|----------------|
| $G_1$ | 10.94                     | 19.43                     | 17.80                     | 0.129546       |
| $G_2$ | 29.85                     | 37.26                     | 33.66                     | 0.322445       |
| $G_3$ | 58.29                     | 68.57                     | 72.92                     | 0.541935       |
| $G_4$ | 99.48                     | 59.19                     | 59.08                     | 0.969029       |
| $G_5$ | 51.81                     | 60.85                     | 57.66                     | 0.529524       |
| $G_6$ | 36.20                     | 40.61                     | 44.74                     | 0.36548        |
| $F_C$ | 607.39                    | 619.3679                  | 619.6086                  | <b>605.68</b>  |
| $F_E$ | 0.2208                    | <b>0.2015</b>             | 0.2027                    | 0.217897       |
| $L_P$ | 3.204                     | 2.51                      | 2.46                      | <b>0.02396</b> |

**Table 9**  
Comparison of  $C_{Emission}$  for test system V.iii.

| Units | MHBA (Liang et al., 2018) | FSBF (Liang et al., 2018) | NSBF (Liang et al., 2018) | KKO             |
|-------|---------------------------|---------------------------|---------------------------|-----------------|
| $G_1$ | 40.94                     | 41.19                     | 40.47                     | 0.412812        |
| $G_2$ | 45.15                     | 46.62                     | 45.33                     | 0.461706        |
| $G_3$ | 53.30                     | 54.21                     | 54.39                     | 0.545594        |
| $G_4$ | 40.51                     | 38.48                     | 39.21                     | 0.386772        |
| $G_5$ | 54.25                     | 54.31                     | 54.54                     | 0.545754        |
| $G_6$ | 52.14                     | 51.60                     | 52.46                     | 0.516297        |
| $F_C$ | <b>643.3760</b>           | 645.6193                  | 644.4141                  | 646.457         |
| $F_E$ | 0.1942                    | 0.1942                    | 0.1942                    | <b>0.194185</b> |
| $L_P$ | <b>2.92</b>               | 3.01                      | 3                         | 0.03493         |



**Fig. 12.** Convergence profile of  $C_{Fuel}$  for test system V.iii.



**Fig. 13.** Convergence profile of  $C_{Emission}$  for test system V.iii.

and at the same time all constraints should be met. But the complexity for these system is high as more constraints are to be handled.

Considering the complexity and non-linearity of a CHPED problem, many techniques have been proposed to solve this problem more effectively and obtain more optimized results. Some of the techniques

include improved genetic algorithm with new constraints handling strategy (IGA-NCM) (Zou et al., 2019), group search optimization (GSO) (Davoodi et al., 2017), hybrid bat and artificial bee colony (Murgan et al., 2018), hybrid gravitational search algorithm and particle swarm optimization (Beigvand et al., 2017), integrated civilized swarm

**Table 10**

Comparison analysis for test system V.iv.

|          | NSGA-II (Basu, 2011) | SPEA-2 (Basu, 2011) | GSA (Güvenç et al., 2012) | MABC/D/C (Secui, 2015) | MABC/D/Log (Secui, 2015) | FPA (Abdelaziz et al., 2016b) | ISA (Karthik et al., 2019) | KKO     |
|----------|----------------------|---------------------|---------------------------|------------------------|--------------------------|-------------------------------|----------------------------|---------|
| $G_1$    | 113.8685             | 113.9694            | 113.9989                  | 110.7998               | 110.7998                 | 43.405                        | 43.567                     | 114     |
| $G_2$    | 113.6381             | 114                 | 113.9896                  | 110.7998               | 110.7998                 | 113.95                        | 113.56                     | 113.045 |
| $G_3$    | 120                  | 119.8719            | 119.9995                  | 97.3999                | 97.3999                  | 105.86                        | 105.76                     | 119.744 |
| $G_4$    | 180.7887             | 179.9284            | 179.7857                  | 174.5504               | 174.5486                 | 169.65                        | 169.43                     | 181.102 |
| $G_5$    | 97                   | 97                  | 97                        | 87.7999                | 97                       | 96.659                        | 96.62                      | 96.5081 |
| $G_6$    | 140                  | 139.2721            | 139.0128                  | 105.3999               | 105.3999                 | 139.02                        | 139.23                     | 139.796 |
| $G_7$    | 300                  | 300                 | 299.9885                  | 259.5996               | 259.5996                 | 273.28                        | 273.36                     | 299.686 |
| $G_8$    | 299.0084             | 298.2706            | 300                       | 284.5996               | 284.5996                 | 285.17                        | 285.15                     | 298.619 |
| $G_9$    | 288.8890             | 290.5228            | 296.2025                  | 284.5996               | 284.5996                 | 241.96                        | 241.54                     | 289.447 |
| $G_{10}$ | 131.6132             | 131.4832            | 130.3850                  | 130                    | 130                      | 131.26                        | 131.26                     | 131.386 |
| $G_{11}$ | 246.5128             | 244.6704            | 245.4775                  | 318.1921               | 318.2129                 | 312.13                        | 312.12                     | 247.114 |
| $G_{12}$ | 318.8748             | 317.2003            | 318.2101                  | 243.5996               | 243.5996                 | 362.58                        | 362.45                     | 318.381 |
| $G_{13}$ | 395.7224             | 394.7357            | 394.6257                  | 394.2793               | 394.2793                 | 346.24                        | 346.34                     | 395.689 |
| $G_{14}$ | 394.1369             | 394.6223            | 395.2016                  | 394.2793               | 394.2793                 | 306.06                        | 306.06                     | 393.82  |
| $G_{15}$ | 305.5781             | 304.7271            | 306.0014                  | 394.2793               | 394.2793                 | 358.78                        | 358.54                     | 305.891 |
| $G_{16}$ | 394.6968             | 394.7289            | 395.1005                  | 394.2793               | 394.2793                 | 260.68                        | 260.23                     | 394.283 |
| $G_{17}$ | 489.4234             | 487.9857            | 489.2569                  | 399.5195               | 399.5195                 | 415.19                        | 415.26                     | 489.706 |
| $G_{18}$ | 488.2701             | 488.5321            | 488.7598                  | 399.5195               | 399.5195                 | 423.94                        | 423.56                     | 487.897 |
| $G_{19}$ | 500.8                | 501.1683            | 499.2320                  | 506.1985               | 506.1716                 | 549.12                        | 549.03                     | 500.104 |
| $G_{20}$ | 455.2006             | 456.4324            | 455.2821                  | 506.1985               | 506.2206                 | 496.7                         | 496.74                     | 455.719 |
| $G_{21}$ | 434.6639             | 434.7887            | 433.4520                  | 514.1472               | 514.1105                 | 539.17                        | 538.76                     | 434.334 |
| $G_{22}$ | 434.15               | 434.3937            | 433.8125                  | 514.1455               | 514.1472                 | 546.46                        | 546.46                     | 434.86  |
| $G_{23}$ | 445.8385             | 445.0772            | 445.5136                  | 514.5237               | 514.5664                 | 540.06                        | 540.56                     | 446.6   |
| $G_{24}$ | 450.7509             | 451.8970            | 452.0547                  | 514.5386               | 514.4868                 | 514.5                         | 514.55                     | 451     |
| $G_{25}$ | 491.2745             | 492.3946            | 492.8864                  | 433.5196               | 433.5195                 | 453.46                        | 453.67                     | 491.259 |
| $G_{26}$ | 436.3418             | 436.9926            | 433.3695                  | 433.5195               | 433.5196                 | 517.31                        | 516.891                    | 435.771 |
| $G_{27}$ | 11.2457              | 10.7784             | 10.0026                   | 10                     | 10                       | 14.881                        | 14.345                     | 11.079  |
| $G_{28}$ | 10                   | 10.2955             | 10.0246                   | 10                     | 10                       | 18.79                         | 18.64                      | 10.3466 |
| $G_{29}$ | 12.0714              | 13.7018             | 10.0125                   | 10                     | 10                       | 26.611                        | 26.578                     | 12.2337 |
| $G_{30}$ | 97                   | 96.2431             | 96.9125                   | 97                     | 87.8042                  | 59.581                        | 59.565                     | 96.6001 |
| $G_{31}$ | 189.4826             | 190.0000            | 189.9689                  | 159.733                | 159.733                  | 183.48                        | 183.36                     | 189.436 |
| $G_{32}$ | 174.7971             | 174.2163            | 175                       | 159.733                | 159.7331                 | 183.39                        | 182.87                     | 175.188 |
| $G_{33}$ | 189.2845             | 190                 | 189.0181                  | 159.733                | 159.733                  | 189.02                        | 189.22                     | 189.992 |
| $G_{34}$ | 200                  | 200                 | 200                       | 200                    | 200                      | 198.73                        | 198.65                     | 199.679 |
| $G_{35}$ | 199.9138             | 200                 | 200                       | 200                    | 200                      | 198.77                        | 198.76                     | 199.89  |
| $G_{36}$ | 199.5066             | 200                 | 199.9978                  | 200                    | 200                      | 182.23                        | 182.45                     | 199.905 |
| $G_{37}$ | 108.3061             | 110                 | 109.9969                  | 89.1141                | 89.1141                  | 39.673                        | 39.635                     | 108.554 |
| $G_{38}$ | 110                  | 109.6912            | 109.0126                  | 89.1141                | 89.1141                  | 81.596                        | 81.525                     | 109.71  |
| $G_{39}$ | 109.7899             | 108.5560            | 109.4560                  | 89.1141                | 89.1141                  | 42.96                         | 42.91                      | 108.639 |
| $G_{40}$ | 421.5609             | 421.8521            | 421.9987                  | 506.1879               | 506.1951                 | 537.17                        | 537.15                     | 421.912 |
| $F_C$    | 1.2583               | 1.2581              | 1.2578                    | 1.24490903             | 1.24491161               | 1.23170                       | <b>1.23034</b>             | 1.25852 |
| $F_E$    | 2.1095               | 2.1110              | 2.1093                    | 2.56560267             | 2.56560267               | 2.0846                        | <b>2.0643</b>              | 2.10837 |

**Table 11**

Comparison analysis for test system V.v.

|          | CSAISA (Rajagopalan et al., 2019) | ISA (Rajagopalan et al., 2019) | HSA (Jeddi and Vahidinasab, 2014) | DE (Rajagopalan et al., 2019) | PSO (Rajagopalan et al., 2019) | GA (Rajagopalan et al., 2019) | KKO            |
|----------|-----------------------------------|--------------------------------|-----------------------------------|-------------------------------|--------------------------------|-------------------------------|----------------|
| $G_1$    | 102.6468                          | 100.3485                       | 100.3839                          | 100.5473                      | 100.7363                       | 100.8578                      | 65.43          |
| $G_2$    | 59.1816                           | 58.8270                        | 58.6583                           | 58.5372                       | 58.2314                        | 58.3547                       | 75.2154        |
| $G_3$    | 50.0599                           | 50.8309                        | 50.8302                           | 50.8474                       | 50.5242                        | 50.9943                       | 69.7721        |
| $G_4$    | 70.3498                           | 73.3932                        | 73.5292                           | 73.0932                       | 73.3238                        | 73.4352                       | 76.7522        |
| $G_5$    | 63.1042                           | 59.1153                        | 59.1846                           | 59.9323                       | 59.7272                        | 59.4636                       | 78.5975        |
| $G_6$    | 51.0080                           | 50.1468                        | 50.9231                           | 50.7397                       | 50.2726                        | 50.6254                       | 74.8284        |
| $G_7$    | 50.0000                           | 50.7470                        | 50.2832                           | 50.5360                       | 50.8362                        | 50.5363                       | 64.9154        |
| $G_8$    | 51.0166                           | 53.2494                        | 53.0220                           | 53.2324                       | 53.5242                        | 53.1321                       | 64.005         |
| $G_9$    | 83.9497                           | 85.1551                        | 85.8231                           | 85.4235                       | 85.4355                        | 85.3546                       | 73.8251        |
| $G_{10}$ | 98.1409                           | 92.1870                        | 92.6484                           | 92.5426                       | 92.0388                        | 92.7522                       | 66.2354        |
| $G_{11}$ | 58.6019                           | 61.3470                        | 61.9233                           | 61.5243                       | 61.6493                        | 61.4368                       | 68.8125        |
| $G_{12}$ | 119.1476                          | 121.8597                       | 121.4353                          | 121.6357                      | 121.7468                       | 121.2463                      | 65.6357        |
| $G_{13}$ | 50.0000                           | 50.0000                        | 50.4352                           | 50.5367                       | 50.6484                        | 50.7468                       | 64.8102        |
| $G_{14}$ | 50.0000                           | 50.0000                        | 50.5327                           | 50.6382                       | 50.8202                        | 50.4373                       | 50             |
| $L_P$    | 7.2070                            | <b>7.2069</b>                  | 7.3536                            | 7.6388                        | 7.8447                         | 7.4357                        | 8.854          |
| $F_C$    | 4352.39                           | 4353.57                        | 4366.27                           | 4387.44                       | 4427.26                        | 4453.85                       | <b>4304.62</b> |
| $F_E$    | 135.23                            | 136.46                         | 144.85                            | 153.42                        | 176.75                         | 184.38                        | <b>108.185</b> |

optimization with Powell pattern search method (Narang et al., 2017), cuckoo optimization algorithm (Mehdinejad et al., 2017), social cognitive optimization (SCO) (Sun and Li, 2019), SCO with tent map (Sun and Li, 2019), real coded genetic algorithm with improved Muhlenbein mutation (Haghray et al., 2016), effective cuckoo search algorithm (ECSA) (Nguyen et al., 2018), multi-player harmony search method (MPHS) (Nazari-Heris et al., 2019).

## 9.2. Problem formulation

For a CHPED problem, the objective is to optimize the heat and power units such that the operational cost is minimum. It is also necessary that the heat and power demands should satisfy. Taking these objectives into consideration, the objective function for a CHPED

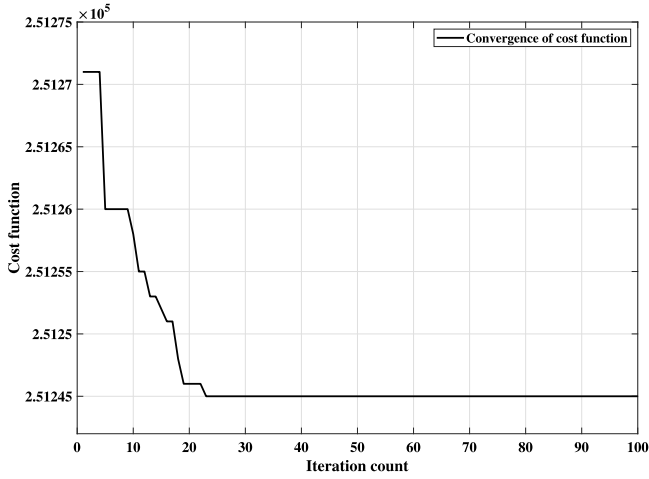


Fig. 14. Convergence profile for test system V.iv.

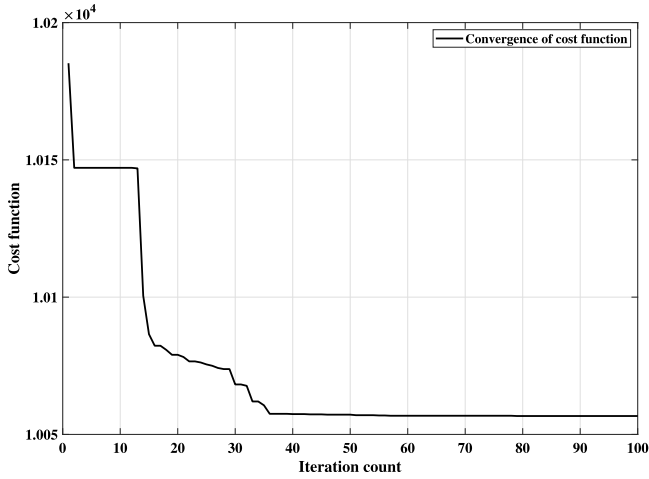


Fig. 15. Convergence profile for test system V.v.

problem is formulated as follows:

$$C = F_P + F_{(P,H)} + F_H \quad (34)$$

where  $F_P$  is the production cost for conventional thermal units;  $F_{(P,H)}$  is the production cost for co-generation units and  $F_H$  is the production cost for heat units. Production cost for these three generating units are given as follows:

The production cost for the conventional thermal units are computed as:

$$F_P = \sum_{i=1}^n (a_i G_i^2 + b_i G_i + c_i + |g_i \times \sin(h_i \times (G_i^{\min} - G_i))|); \quad (35)$$

where  $G_i$  is the active power generated by  $i$ th generating unit;  $a_i$ ,  $b_i$  and  $c_i$  are the fuel coefficients for  $i$ th generating unit;  $g_i$  and  $h_i$  are coefficients of the  $i$ th generator due to valve point effect;  $G_i^{\min}$  is the minimum generation capacity of  $i$ th generating unit.

The production cost for the co-generation units are calculated as follows.

$$F_{(P,H)} = \sum_{i=1}^m (a_i G_i^2 + b_i G_i + c_i + d_i H_i^2 + e_i H_i + f_i H_i P_i); \quad (36)$$

where  $P_i$  and  $H_i$  are the power and heat generated by  $i$ th co-generating unit respectively;  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$ ,  $e_i$  and  $f_i$  are the cost parameters of  $i$ th co-generating unit.

The production cost for heat thermal units are computed as follows.

$$F_H = \sum_{i=1}^o (a_i H_i^2 + b_i H_i + c_i); \quad (37)$$

where  $H_i$  is heat generated by  $i$ th heat unit;  $a_i$ ,  $b_i$ ,  $c_i$ ,  $d_i$ ,  $e_i$  and  $f_i$  are the cost parameters of  $i$ th heat unit.

### 9.3. Constraints

#### 9.3.1. Capacity limits

It is important that all the conventional thermal units, co-generating units and heat units operate in a prescribed generating limits.

For  $i$ th generating unit in a conventional thermal unit, this limits is given as follows:

$$P_i^{\min} \leq P_i \leq P_i^{\max}; \quad (38)$$

where  $P_i^{\min}$  is the minimum generating capacity and  $P_i^{\max}$  is the maximum generating capacity of  $i$ th generating unit.

Similarly, for a  $i$ th heat unit, the prescribed generating limits is given as follows:

$$H_i^{\min} \leq H_i \leq H_i^{\max}; \quad (39)$$

where  $H_i^{\min}$  is the minimum generating capacity and  $H_i^{\max}$  is the maximum generating capacity of  $i$ th heat unit.

In case of a  $i$ th co-generating unit, prescribed limits is given as

$$P_i^{\min}(H_i) \leq P_i \leq P_i^{\max}(H_i); \quad (40)$$

$$H_i^{\min}(P_i) \leq H_i \leq H_i^{\max}(P_i); \quad (41)$$

where  $P_i^{\min}(H_i)$  and  $P_i^{\max}(H_i)$  are the functions of generated heat  $H_i$  and represent the maximum and minimum power limits of  $i$ th co-generating unit;  $H_i^{\min}(P_i)$  and  $H_i^{\max}(P_i)$  are the functions of generated heat  $P_i$  and represent the maximum and minimum power limits of  $i$ th co-generating unit.

#### 9.3.2. Heat balance

It is important that the total heat produced by the heat and co-generating unit is equal to the heat demand. Mathematically, this is represented as follow:

$$\sum_{i=1}^n H_i + \sum_{i=1}^n H_i = H_D. \quad (42)$$

#### 9.3.3. Power balance

Similar to the heat balance, it is important that the total power generated by the thermal unit and co-generating unit is equal to the total power demand. This is represented as follows:

$$\sum_{i=1}^n P_i + \sum_{i=1}^n P_i = P_D \quad (43)$$

### 9.4. Results and discussions

To test the effectiveness of KKO algorithm to solve CHPED problem, five test systems are considered. The description of these five test systems are presented in Table 4. All the five systems are simulated for fifteen times with an iteration count and player population of 500 and 200 respectively. The statistical analysis of these simulation is presented in Table 5.

The power and heat scheduling for the test system VI.i is presented in Table 12. A comparative result analysis with existing techniques like RCGA-IMM (Haghray et al., 2016), EMA (Ghorbani, 2016), GWO (Jayakumar et al., 2016), MCSA (Nguyen et al., 2016), CSA (Nguyen et al., 2016), CSO (Meng et al., 2015), SCO (Sun and Li, 2019) and TSCO (Sun and Li, 2019) is presented in Table 12. The analysis shows that the proposed KKO technique schedules the units



**Table 12**

Comparison analysis for test system VI.i.

| Method                          | $G_1$  | $G_2$   | $G_3$   | $H_2$   | $H_3$   | $H_4$ | $C$      |
|---------------------------------|--------|---------|---------|---------|---------|-------|----------|
| RCGA-IMM (Haghray et al., 2016) | 0      | 160     | 40      | 40      | 75      | 0     | 9257.075 |
| EMA (Ghorbani, 2016)            | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| GWO (Jayakumar et al., 2016)    | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| MCSA (Nguyen et al., 2016)      | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| CSA (Nguyen et al., 2016)       | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| CSO (Meng et al., 2015)         | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| SCO (Sun and Li, 2019)          | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| TSCO (Sun and Li, 2019)         | 0      | 160     | 40      | 40      | 75      | 0     | 9257.07  |
| IGA-NCM (Zou et al., 2019)      | 0      | 160     | 40      | 40      | 75      | 0     | 9257.075 |
| KKO                             | 0.0282 | 155.015 | 44.9568 | 18.1301 | 96.8699 | 0     | 9217.03  |

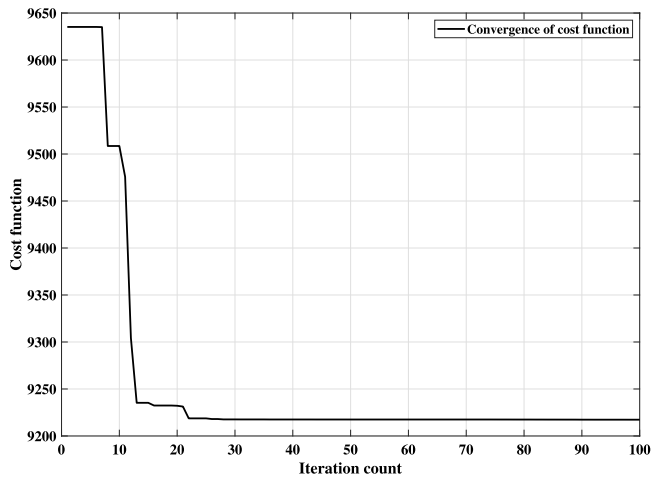


Fig. 16. Convergence profile for test system VI.i.

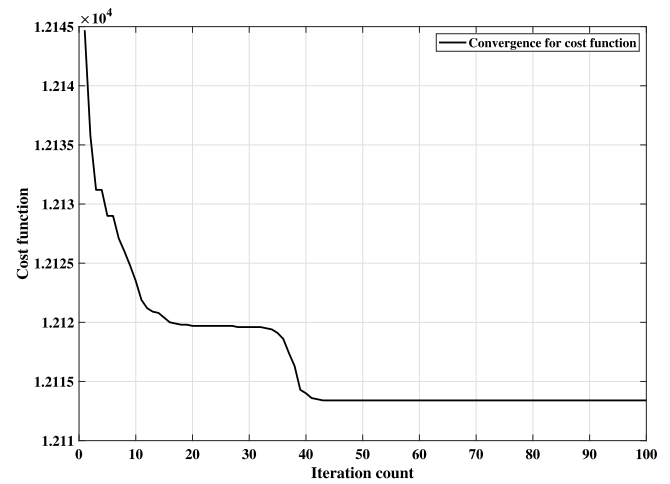


Fig. 17. Convergence profile for test system VI.ii.

more effectively and thus provides the best results a compared to existing techniques. The convergence behavior of the objective function is presented in Fig. 16.

For test system VI.ii, the result obtained using KKO is compared with existing results of RCGA-IMM (Haghray et al., 2016), MGSO (Davoodi et al., 2017), GSO (Davoodi et al., 2017), COA (Mehdinejad et al., 2017), BD (Abdolmohammadi and Kazemi, 2013), TVAC-PSO (Mohammadi-Ivatloo et al., 2013), GSA (Beigvand et al., 2016), PFCOA (Mellal and Williams, 2015), IWO (Jayabarathi et al., 2014) and IGA-NCM (Zou et al., 2019). All the results along with power and heat scheduling are presented in Table 13. It is observed that the KKO provides the best fuel cost. The convergence profile of the objective function is presented in Fig. 17.

Table 14 presents the best result obtained for KKO and some other existing techniques like RCGA (Haghray et al., 2016), AIS (Jordehi, 2015), ECSA (Nguyen et al., 2018), EMA (Ghorbani, 2016), TLBO (Roy et al., 2014), CSO (Meng et al., 2015), SCO (Sun and Li, 2019), TSCO (Sun and Li, 2019) for test system VI.iii. It is observed that the proposed KKO provides a fuel cost of 10045.4 which is better as compared to other techniques. This proves that KKO can also effectively solve this test system. The convergence profile for this system is presented in Fig. 18.

Best results of the proposed KKO for test system VI.iv is presented in Table 15. This table also includes the result for techniques like CPSO (Mohammadi-Ivatloo et al., 2013), TVAC-PSO (Mohammadi-Ivatloo et al., 2013), GSO (Haghray et al., 2014), IGSO (Haghray et al., 2014), OTLBO (Roy et al., 2014), GWO (Jayakumar et al., 2016), RCGA-IMM (Haghray et al., 2016) and MPHS (Nazari-Heris et al., 2019) which are used for comparison. It is observed that the proposed algorithm optimizes the generating units more effectively as the fuel cost obtained using the proposed KKO is less as compared to other techniques. The convergence behavior of the objective function for this test system is presented in Fig. 19.

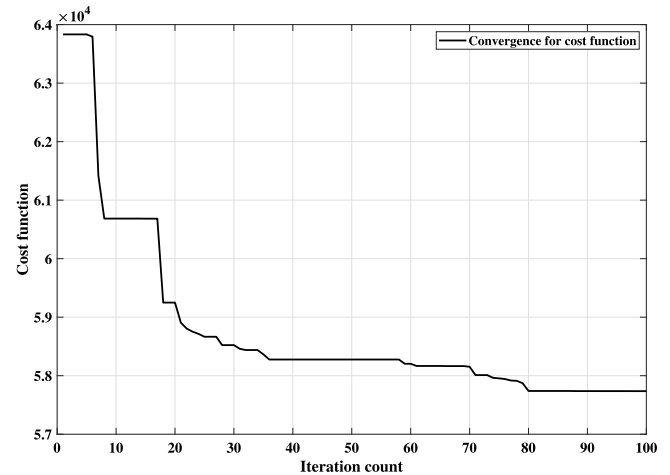


Fig. 18. Convergence profile for test system VI.iii.

For the last test system VI.v considered to test effectiveness of the proposed KKO algorithm, the optimal result obtained are presented in Table 16. The optimal results obtained using existing techniques like OGSO (Basu, 2015a), MPSO (Basu, 2015b), GSO (Basu, 2016) and IGA-NCM (Zou et al., 2019) are also presented in this table. From the comparison, it is clear that the proposed technique provides the best result as the fuel cost obtained using the KKO algorithm is minimum. The convergence behavior of the objective function is presented in Fig. 20.

**Table 13**

Comparison analysis for test system VI.ii.

| Method                                    | $G_1$    | $G_2$   | $G_3$   | $G_4$   | $H_2$   | $H_3$   | $H_4$   | $H_5$   | $C$          |
|---|----------|---------|---------|---------|---------|---------|---------|---------|--------------|
| RCGA-IMM (Haghras et al., 2016)           | 135.0000 | 40.0000 | 10.0000 | 65.0000 | 75.0000 | 40.0000 | 14.0595 | 45.9405 | 12104.868    |
| MGSO (Davoodi et al., 2017)               | 135      | 39.9999 | 9.9996  | 64.9003 | 75.0008 | 40.0002 | 14.4379 | 45.5609 | 12113.6041   |
| GSO (Davoodi et al., 2017)                | 134.9953 | 40.2832 | 10.0962 | 64.6251 | 71.7131 | 39.8592 | 6.1571  | 57.2704 | 12128.7805   |
| COA (Mehdinejad et al., 2017)             | 135      | 40      | 10      | 64991   | 75      | 40      | 14.4001 | 45.6    | 12116.6      |
| BD (Abdolmohammadi and Kazemi, 2013)      | 135      | 40      | 10      | 65      | 75      | 40      | 14.3226 | 45.6774 | 12116.601231 |
| TVAC-PSO (Mohammadi-Ivatloo et al., 2013) | 135      | 40.0118 | 10.0391 | 64.9491 | 74.8263 | 39.8443 | 16.1867 | 44.1428 | 12117.3895   |
| GSA (Beigvand et al., 2016)               | 135      | 39.9998 | 10      | 64.9807 | 74.9844 | 40      | 17.8939 | 42.1095 | 12117.37     |
| PFCOA (Mellal and Williams, 2015)         | 134.96   | 40      | 10      | 65      | 75      | 40      | 14.13   | 45.85   | 12115.91     |
| IWO (Jayabarathi et al., 2014)            | 134.59   | 40      | 10.94   | 64.47   | 75      | 38.98   | 8.81    | 52.21   | 12134.33     |
| IGA-NCM (Zou et al., 2019)                | 135      | 40      | 10      | 65      | 75      | 40      | 14.0595 | 45.9405 | 12117.1701   |
| KKO                                       | 135      | 40.0087 | 10      | 64.9913 | 78.0441 | 50.9347 | 2.92276 | 43.0985 | 12113.4      |

**Table 14**

Comparison analysis for test system VI.iii.

| Method | RCGA (Haghras et al., 2016) | AIS (Jordehi, 2015) | ECOA (Nguyen et al., 2018) | EMA (Ghorbani, 2016) | TLBO (Roy et al., 2014) | CSO (Meng et al., 2015) | SCO (Sun and Li, 2019) | TSCO (Sun and Li, 2019) | GWO (Zou et al., 2019) | IGA-NCM (Zou et al., 2019) | KKO     |
|--------|-----------------------------|---------------------|----------------------------|----------------------|-------------------------|-------------------------|------------------------|-------------------------|------------------------|----------------------------|---------|
| $G_1$  | 74.6834                     | 50.3610             | 53.7610                    | 52.6847              | 45.266                  | 45.4909                 | 58.7268                | 45.5231                 | 52.8074                | 45.155                     | 45.4842 |
| $G_2$  | 97.9578                     | 95.5552             | 98.5039                    | 98.5398              | 98.5479                 | 98.5398                 | 98.5398                | 98.5385                 | 98.5398                | 98.5398                    | 99.384  |
| $G_3$  | 167.2308                    | 110.7515            | 112.5996                   | 112.6734             | 112.6786                | 112.6734                | 112.6735               | 112.5798                | 112.6735               | 112.6735                   | 113.646 |
| $G_4$  | 124.9079                    | 208.7688            | 209.7993                   | 208.8158             | 209.8284                | 209.8158                | 209.8158               | 209.8159                | 209.8158               | 209.8158                   | 209.029 |
| $G_5$  | 98.8008                     | 98.8                | 93.0872                    | 93.8341              | 94.4121                 | 94.1838                 | 81                     | 94.2261                 | 93.8115                | 94.5549                    | 92.4743 |
| $G_6$  | 44.0001                     | 44                  | 40.2022                    | 40                   | 40.0062                 | 40                      | 40                     | 40                      | 40                     | 40                         | 40.7171 |
| $H_5$  | 58.0965                     | 19.4242             | 33.6571                    | 29.242               | 25.8365                 | 27.1786                 | 92.0061                | 28.6947                 | 29.3704                | 29.2388                    | 0       |
| $H_6$  | 32.4116                     | 77.0777             | 72.6890                    | 75                   | 74.9970                 | 75                      | 45.0590                | 74.9981                 | 75                     | 75                         | 97.1926 |
| $H_7$  | 59.4919                     | 53.4981             | 43.6539                    | 45.7579              | 49.1666                 | 47.8214                 | 12.9349                | 46.3072                 | 29.3704                | 45.7612                    | 52.8074 |
| $C$    | 10667                       | 10355               | 10121.9466                 | 10111.0732           | 10094.8384              | 10094.1267              | 10226.8556             | 10094.2351              | 10111.24               | 10107.9071                 | 10045.4 |
| $L_P$  | –                           | –                   | –                          | –                    | –                       | –                       | –                      | –                       | 0.76499                | –                          | 0.7346  |

**Table 15**

Comparison analysis for test system VI.iv.

| Output   | CPSO<br>(Mohammadi-<br>Ivatloo et al.,<br>2013) | TVAC-PSO<br>(Mohammadi-<br>Ivatloo et al.,<br>2013) | GSO (Hagh<br>et al., 2014) | IGSO (Hagh<br>et al., 2014) | OTLBO (Roy<br>et al., 2014) | GWO (Jayaku-<br>mar et al.,<br>2016) | RCGA-<br>IMM (Haghras<br>et al., 2016) | MPHS (Nazari-<br>Heris et al.,<br>2019) | IGA-<br>NCM (Zou<br>et al., 2019) | KKO        |           |            |         |
|----------|---|---|----------------------------|-----------------------------|-----------------------------|--------------------------------------|--|---|-----------------------------------|------------|-----------|------------|---------|
| $G_1$    | 680   | 538.5587  | 627.7455                   | 628.152                     | 538.5656                    | 538.8440                             | 448.8000                               | 628.3185                                | 628.3188                          | 575.764    |           |            |         |
| $G_2$    | 0   | 224.4608  | 76.2285                    | 299.4778                    | 299.2123                    | 299.3423                             | 299.9568                               | 299.1993                                | 299.1982                          | 356.429    |           |            |         |
| $G_3$    | 0   | 224.4608  | 299.5794                   | 154.5535                    | 299.1220                    | 299.3423                             | 299.2108                               | 299.1993                                | 299.1665                          | 49.4861    |           |            |         |
| $G_4$    | 180   | 109.8666  | 159.4386                   | 60.846                      | 109.9920                    | 109.9653                             | 109.8694                               | 109.8665                                | 109.8673                          | 69.8744    |           |            |         |
| $G_5$    | 180   | 109.8666  | 61.2378                    | 103.8538                    | 109.9545                    | 109.9653                             | 109.8679                               | 60                                      | 109.8662                          | 100.285    |           |            |         |
| $G_6$    | 180   | 109.8666  | 60                         | 110.0552                    | 110.4042                    | 109.9653                             | 159.7353                               | 109.8665                                | 60                                | 115.712    |           |            |         |
| $G_7$    | 180   | 109.8666  | 157.1503                   | 159.0773                    | 109.8045                    | 109.9653                             | 109.8684                               | 109.8665                                | 109.8608                          | 140.679    |           |            |         |
| $G_8$    | 180   | 109.8666  | 107.2654                   | 109.8258                    | 109.6862                    | 109.9653                             | 60.6545                                | 60                                      | 109.8237                          | 128.701    |           |            |         |
| $G_9$    | 180   | 109.8666  | 110.1816                   | 159.992                     | 109.8992                    | 109.9653                             | 159.7354                               | 159.7331                                | 109.8523                          | 124.517    |           |            |         |
| $G_{10}$ | 50.5304   | 77.5210   | 113.9894                   | 41.103                      | 77.3992                     | 77.6223                              | 75.8146                                | 40                                      | 40.0001                           | 59.0988    |           |            |         |
| $G_{11}$ | 50.5304   | 77.5210   | 79.7755                    | 77.7055                     | 77.8364                     | 77.6223                              | 40.1672                                | 40                                      | 77.0316                           | 82.4809    |           |            |         |
| $G_{12}$ | 55  | 120   | 91.1668                    | 94.9768                     | 55.2225                     | 55.0000                              | 92.6079                                | 55                                      | 55.0098                           | 82.6289    |           |            |         |
| $G_{13}$ | 55  | 120   | 115.6511                   | 55.7143                     | 55.0861                     | 55.0000                              | 92.4056                                | 91.9038                                 | 55                                | 94.9095    |           |            |         |
| $G_{14}$ | 117.4854  | 88.3514   | 84.3133                    | 83.9536                     | 81.7524                     | 83.4650                              | 83.0376                                | 81.0006                                 | 81.0035                           | 83.2419    |           |            |         |
| $G_{15}$ | 45.9281   | 40.5611   | 40                         | 40                          | 41.7615                     | 40.0000                              | 40.0071                                | 40.0079                                 | 40.0003                           | 41.4101    |           |            |         |
| $G_{16}$ | 117.4854  | 88.3514   | 81.1796                    | 85.7133                     | 82.2730                     | 82.7732                              | 81.4577                                | 81.0002                                 | 81.0003                           | 99.468     |           |            |         |
| $G_{17}$ | 45.9281   | 40.5611   | 40                         | 40                          | 40.5599                     | 40.0000                              | 41.6937                                | 40.0059                                 | 40.0001                           | 66.5692    |           |            |         |
| $G_{18}$ | 10.0013   | 10.0245   | 10                         | 10                          | 10.0002                     | 10.0000                              | 10.0042                                | 10.0317                                 | 10.0002                           | 10.3179    |           |            |         |
| $G_{19}$ | 42.1109   | 40.4288   | 35.0970                    | 35                          | 31.4679                     | 31.4568                              | 35.1058                                | 35                                      | 35.0003                           | 68.4258    |           |            |         |
| $H_{14}$ | 125.2754  | 108.9256  | 106.6588                   | 106.4569                    | 105.2219                    | 106.0991                             | 105.9431                               | 104.8003                                | 104.8013                          | 171.612    |           |            |         |
| $H_{15}$ | 80.1175   | 75.4844   | 74.9980                    | 74.998                      | 76.5205                     | 75.0000                              | 75.0059                                | 75.0068                                 | 75.0001                           | 114.465    |           |            |         |
| $H_{16}$ | 125.2754  | 108.9256  | 104.9002                   | 107.4073                    | 105.5142                    | 105.7890                             | 105.0550                               | 104.8                                   | 104.7995                          | 87.5713    |           |            |         |
| $H_{17}$ | 80.1174   | 75.484  | 74.9980                    | 74.998                      | 75.4833                     | 75.0000                              | 76.4619                                | 75.0051                                 | 74.9988                           | 131.198    |           |            |         |
| $H_{18}$ | 40.0005   | 40.0104   | 40                         | 40                          | 39.9999                     | 40.0000                              | 40.0007                                | 40.0136                                 | 39.9993                           | 36.5691    |           |            |         |
| $H_{19}$ | 23.2322   | 22.4676   | 19.7385                    | 20                          | 18.3944                     | 18.3782                              | 20.0477                                | 20                                      | 20.0001                           | 42.8038    |           |            |         |
| $H_{20}$ | 415.9815  | 458.7020  | 469.3368                   | 466.2575                    | 468.9043                    | 469.7337                             | 467.4871                               | 470.3751                                | 470.4096                          | 312.577    |           |            |         |
| $H_{21}$ | 60  | 60  | 60                         | 60                          | 59.9994                     | 60.0000                              | 59.9999                                | 60                                      | 60                                | 60         |           |            |         |
| $H_{22}$ | 60  | 60  | 60                         | 60                          | 59.9999                     | 60.0000                              | 59.9997                                | 60                                      | 60                                | 57.2295    |           |            |         |
| $H_{23}$ | 120   | 120   | 119.6511                   | 120                         | 119.9854                    | 120.0000                             | 119.9991                               | 119.9994                                | 120                               | 115.974    |           |            |         |
| $H_{24}$ | 120   | 120   | 119.7176                   | 119.8823                    | 119.9768                    | 120.0000                             | 119.9998                               | 119.9995                                | 119.9913                          | 119.999    |           |            |         |
| $C$      | 59736.26  | 35  | 58122.7460                 | 58225.74                    | 50                          | 58049.01                             | 97                                     | 57856.2676                              | 57846.84                          | 57927.6919 | 57845.639 | 57826.0902 | 57735.9 |

## 10. Conclusion

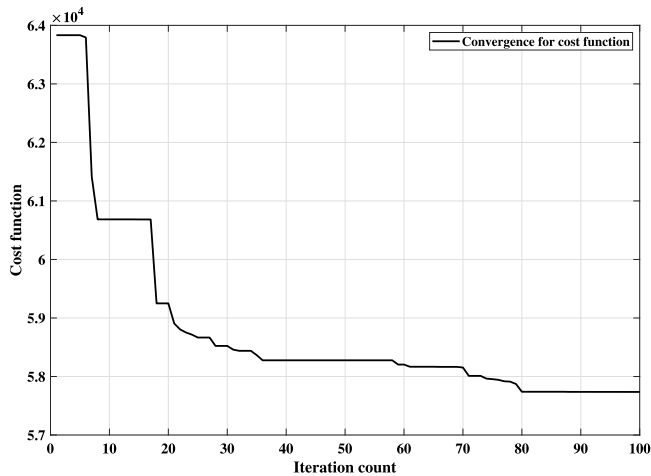
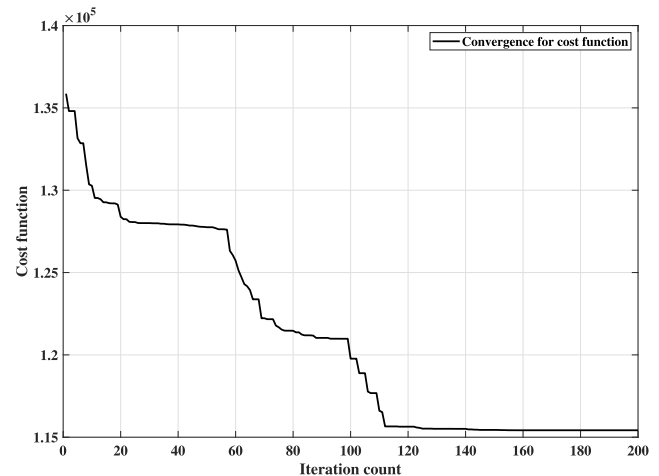
In this article, the authors have proposed a new population-based meta-heuristic optimization algorithm, i.e. Kho-Kho optimization algorithm. The proposed technique is inspired by a well-known game

played in India known as Kho-Kho. This technique mimics the strategies and behavior of players of a chasing team while they chase and touch the runner of a opposition team. The proposed algorithm is tested with twenty-nine benchmark functions, five test system for combined

**Table 16**

Comparison analysis for test system VI.v.

| Units    | OGSO (Basu, 2015a) | MPSO (Basu, 2015b) | GSO (Basu, 2016) | IGANCM (Zou et al., 2019) | KKO     | Units    | OGSO (Basu, 2015a) | MPSO (Basu, 2015b) | GSO (Basu, 2016) | IGANCM (Zou et al., 2019) | KKO     |
|----------|--------------------|--------------------|------------------|---------------------------|---------|----------|--------------------|--------------------|------------------|---------------------------|---------|
| $P_1$    | 628.291            | 179.7773           | 179.8745         | 538.5723                  | 371.36  | $P_{31}$ | 10.0213            | 10                 | 10.1191          | 10.15                     | 10.7027 |
| $P_2$    | 224.8131           | 299.9621           | 360              | 298.7715                  | 191.207 | $P_{32}$ | 45.0197            | 35.7983            | 35.1879          | 35.0002                   | 35.1253 |
| $P_3$    | 359.9981           | 290.3748           | 150.7185         | 299.4666                  | 256.636 | $P_{33}$ | 81.033             | 82.5995            | 96.8952          | 81.054                    | 81.8093 |
| $P_4$    | 160.311            | 179.5655           | 60               | 109.8936                  | 144.536 | $P_{34}$ | 47.3353            | 44.2751            | 44.8817          | 40.0192                   | 40.6326 |
| $P_5$    | 159.8826           | 160.0408           | 60.0648          | 159.7952                  | 164.287 | $P_{35}$ | 87.6544            | 91.0334            | 86.3425          | 81.0563                   | 81.1798 |
| $P_6$    | 160.6078           | 119.7704           | 159.8784         | 110.2276                  | 169.152 | $P_{36}$ | 56.2196            | 45.0766            | 44.867           | 40.3107                   | 40.6314 |
| $P_7$    | 159.98             | 162.9561           | 160.3713         | 159.7388                  | 114.192 | $P_{37}$ | 10.265             | 10.0002            | 10.0624          | 10.0033                   | 10      |
| $P_8$    | 160.6525           | 159.7361           | 177.5771         | 109.8456                  | 105.963 | $P_{38}$ | 35.5826            | 35.1411            | 35.1607          | 35.0039                   | 35.276  |
| $P_9$    | 110.0147           | 119.8672           | 120.0893         | 110.0311                  | 153.815 | $H_{27}$ | 115.5514           | 105.6169           | 112.5046         | 104.8189                  | 28.923  |
| $P_{10}$ | 40.0104            | 114.8494           | 115.373          | 40.2712                   | 103.196 | $H_{28}$ | 83.6316            | 78.8068            | 78.4728          | 75.0113                   | 58.9431 |
| $P_{11}$ | 40.0044            | 114.9528           | 114.9535         | 40.1973                   | 76.0931 | $H_{29}$ | 105.9949           | 113.8706           | 112.6499         | 105.013                   | 143.437 |
| $P_{12}$ | 119.9535           | 90.8763            | 94.5954          | 55.0122                   | 67.734  | $H_{30}$ | 79.3856            | 79.0298            | 79.0427          | 75.386                    | 129.366 |
| $P_{13}$ | 55.1371            | 92.7706            | 55.188           | 55.0817                   | 69.0932 | $H_{31}$ | 40.0003            | 40.0004            | 40.02            | 40.0493                   | 43.135  |
| $P_{14}$ | 0.0464             | 179.556            | 628.9382         | 538.5544                  | 329.117 | $H_{32}$ | 24.5077            | 20.3628            | 20.0605          | 19.9957                   | 34.3535 |
| $P_{15}$ | 151.8432           | 226.5089           | 360              | 299.234                   | 296.511 | $H_{33}$ | 104.7314           | 105.6984           | 113.5695         | 104.6892                  | 137.125 |
| $P_{16}$ | 151.3518           | 299.2174           | 299.4804         | 299.485                   | 299.199 | $H_{34}$ | 81.3557            | 8.7186             | 79.1803          | 75.0147                   | 135.491 |
| $P_{17}$ | 160.1601           | 159.9777           | 122.0289         | 159.8926                  | 137.854 | $H_{35}$ | 108.4749           | 110.4315           | 107.6768         | 104.8247                  | 149.79  |
| $P_{18}$ | 159.763            | 109.8667           | 110.0491         | 109.9319                  | 122.598 | $H_{36}$ | 89.033             | 79.4109            | 79.2272          | 75.2599                   | 97.1279 |
| $P_{19}$ | 159.9246           | 159.7483           | 60               | 110.0782                  | 137.979 | $H_{37}$ | 40.0628            | 40.0004            | 40.0098          | 39.9925                   | 15.8136 |
| $P_{20}$ | 160.3244           | 170.6698           | 87.0872          | 109.7738                  | 178.525 | $H_{38}$ | 20.2503            | 20.0641            | 19.9447          | 19.9972                   | 41.3296 |
| $P_{21}$ | 159.7561           | 160.6908           | 159.9218         | 109.7706                  | 81.5462 | $H_{39}$ | 448.2892           | 522.1121           | 435.2939         | 469.5234                  | 399.09  |
| $P_{22}$ | 159.8332           | 109.9663           | 60.0079          | 109.7815                  | 151.841 | $H_{40}$ | 60                 | 60                 | 59.9635          | 60                        | 59.8181 |
| $P_{23}$ | 114.8032           | 114.837            | 120              | 40.0047                   | 53.8394 | $H_{41}$ | 60                 | 59.9999            | 59.993           | 60                        | 58.9775 |
| $P_{24}$ | 115.8473           | 116.0113           | 40               | 40                        | 101.547 | $H_{42}$ | 119.9937           | 120                | 119.995          | 119.8133                  | 119.88  |
| $P_{25}$ | 55.0071            | 92.493             | 108.1572         | 56.0734                   | 74.8033 | $H_{43}$ | 119.9991           | 119.9996           | 119.8985         | 119.8159                  | 119.29  |
| $P_{26}$ | 119.967            | 92.4025            | 93.5594          | 55                        | 102.842 | $H_{44}$ | 438.8062           | 385.8784           | 462.5644         | 471.182                   | 370.001 |
| $P_{27}$ | 100.2424           | 82.4557            | 94.7653          | 81.043                    | 124.523 | $H_{45}$ | 59.998             | 59.9997            | 59.9983          | 59.9974                   | 59.1116 |
| $P_{28}$ | 50.009             | 44.3773            | 44.0478          | 40.0145                   | 47.5859 | $H_{46}$ | 60                 | 60                 | 59.9987          | 59.9938                   | 59.8754 |
| $P_{29}$ | 83.2757            | 97.1621            | 95.0884          | 81.4082                   | 96.5791 | $H_{47}$ | 119.9749           | 119.9995           | 120              | 119.7899                  | 119.886 |
| $P_{30}$ | 45.0583            | 44.6356            | 44.6682          | 40.4519                   | 40.4875 | $H_{48}$ | 119.9591           | 119.9996           | 119.9359         | 119.8319                  | 119.236 |
| —        | —                  | —                  | —                | —                         | —       | C        | 116678.1987        | 116918.9761        | 117098.4186      | 115685.1807               | 115422  |

**Fig. 19.** Convergence profile for test system VI.v.**Fig. 20.** Convergence profile for test system VI.v.

emission economic dispatch problem and five test systems for combined heat and power economic dispatch problem. The result obtained concludes that

- KKO exhibits good exploration capabilities.
- KKO exhibits good exploitation capabilities.
- KKO exhibits good balance between exploration and exploitation capabilities.
- KKO has a better convergence rate compared to some other existing well-known optimization techniques.
- KKO can effectively solve real-time optimization technique such as CEED and CHPED problem.

Further, the proposed KKO technique can also be used to solve other large and complex optimization problems related to other fields

of engineering, mathematics, etc. In future, hybrid techniques using proposed KKO technique and other optimization techniques will be developed and tested to further improve the stability and convergence speed of the proposed the KKO technique. This could help to solve more complex problems with less computational time.

#### CRediT authorship contribution statement

**Abhishek Srivastava:** Conceptualization, Methodology, Writing - original draft . **Dushmanta Kumar Das:** Conceptualization, Methodology, Supervision, Writing - review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References

- Abdelaziz, A., Ali, E., Elazim, S.A., 2016a. Combined economic and emission dispatch solution using flower pollination algorithm. *Int. J. Electr. Power Energy Syst.* 80, 264–274.
- Abdelaziz, A., Ali, E., Elazim, S.A., 2016b. Implementation of flower pollination algorithm for solving economic load dispatch and combined economic emission dispatch problems in power systems. *Energy* 101, 506–518.
- Abdolmohammadi, H.R., Kazemi, A., 2013. A benders decomposition approach for a combined heat and power economic dispatch. *Energy Convers. Manage.* 71, 21–31.
- Alipour, M., Mohammadi-Ivatloo, B., Zare, K., 2014. Stochastic risk-constrained short-term scheduling of industrial cogeneration systems in the presence of demand response programs. *Appl. Energy* 136, 393–404.
- Alomoush, M.I., Oweis, Z.B., 2018. Environmental-economic dispatch using stochastic fractal search algorithm. *Int. Trans. Electr. Energy Syst.* 28 (5), e2530.
- Amiri, M., Khanmohammadi, S., Badamchizadeh, M., 2018. Floating search space: A new idea for efficient solving the economic and emission dispatch problem. *Energy* 158, 564–579.
- Banzhaf, W., Nordin, P., Keller, R.E., Francone, F.D., 1998. *Genetic Programming*. Springer.
- Basu, M., 2011. Economic environmental dispatch using multi-objective differential evolution. *Appl. Soft Comput.* 11 (2), 2845–2853.
- Basu, M., 2015a. Combined heat and power economic dispatch using opposition-based group search optimization. *Int. J. Electr. Power Energy Syst.* 73, 819–829.
- Basu, M., 2015b. Modified particle swarm optimization for non-smooth non-convex combined heat and power economic dispatch. *Electric Power Compon. Syst.* 43 (19), 2146–2155.
- Basu, M., 2016. Group search optimization for combined heat and power economic dispatch. *Int. J. Electr. Power Energy Syst.* 78, 138–147.
- Beigvand, S.D., Abdi, H., La Scala, M., 2016. Combined heat and power economic dispatch problem using gravitational search algorithm. *Electr. Power Syst. Res.* 133, 160–172.
- Beigvand, S.D., Abdi, H., La Scala, M., 2017. Hybrid gravitational search algorithm-particle swarm optimization with time varying acceleration coefficients for large scale CHPED problem. *Energy* 126, 841–853.
- Bellman, R., 2013. *Dynamic Programming*. Courier Corporation.
- Bertsekas, D.P., 1999. *Nonlinear Programming*. Athena scientific Belmont.
- Bonabeau, E., Dorigo, M., Marco, D.d.R.D.F., Theraulaz, G., Theraulaz, G., et al., 1999. *Swarm Intelligence: From Natural to Artificial Systems*, no. 1. Oxford university press.
- Bora, T.C., Mariani, V.C., dos Santos Coelho, L., 2019. Multi-objective optimization of the environmental-economic dispatch with reinforcement learning based on non-dominated sorting genetic algorithm. *Appl. Therm. Eng.* 146, 688–700.
- Cuevas, E., Cienfuegos, M., Zaldívar, D., Pérez-Cisneros, M., 2013. A swarm optimization algorithm inspired in the behavior of the social-spider. *Expert Syst. Appl.* 40 (16), 6374–6384.
- Cuevas, E., Cortés, M.A.D., Navarro, D.A.O., 2016. Optimization based on the behavior of locust swarms. In: *Advances of Evolutionary Computation: Methods and Operators*. Springer, pp. 101–120.
- Das, P., Das, D.K., Dey, S., 2018a. A modified bee colony optimization (MBCO) and its hybridization with k-means for an application to data clustering. *Appl. Soft Comput.* 70, 590–603.
- Das, P., Das, D.K., Dey, S., 2018b. A new class toppler optimization algorithm with an application to data clustering. *IEEE Trans. Emerg. Top. Comput.*
- Davoodi, E., Zare, K., Babaei, E., 2017. A GSO-based algorithm for combined heat and power dispatch problem with modified scrounger and ranger operators. *Appl. Therm. Eng.* 120, 36–48.
- Deb, S., Fong, S., Tian, Z., 2015. Elephant search algorithm for optimization problems. In: *Digital Information Management. ICDIM, 2015 Tenth International Conference on*. IEEE, pp. 249–255.
- Dey, B., Roy, S.K., Bhattacharyya, B., 2019. Solving multi-objective economic emission dispatch of a renewable integrated microgrid using latest bio-inspired algorithms. *Eng. Sci. Technol.* 22 (1), 55–66.
- Eberhart, R.C., Shi, Y., Kennedy, J., 2001. *Swarm Intelligence* (Morgan Kaufmann Series in Evolutionary Computation). Morgan Kaufmann Publishers.
- Eiben, A.E., Smith, J.E., et al., 2003. *Introduction to Evolutionary Computing*, Vol. 53. Springer.
- Elattar, E.E., 2019. Environmental economic dispatch with heat optimization in the presence of renewable energy based on modified shuffle frog leaping algorithm. *Energy* 171, 256–269.
- Elsakaan, A.A., El-Shehry, R.A., Kaddah, S.S., Elsaid, M.I., 2018. An enhanced moth-flame optimizer for solving non-smooth economic dispatch problems with emissions. *Energy* 157, 1063–1078.
- Frank, M., Wolfe, P., 1956. An algorithm for quadratic programming. *Nav. Res. Logist.* 3 (1–2), 95–110.
- Gandomi, A.H., Alavi, A.H., 2012. Krill herd: a new bio-inspired optimization algorithm. *Commun. Nonlinear Sci. Numer. Simul.* 17 (12), 4831–4845.
- García, S., Molina, D., Lozano, M., Herrera, F., 2009. A study on the use of non-parametric tests for analyzing the evolutionary algorithms behaviour: a case study on the CEC 2005 special session on real parameter optimization. *J. Heuristics* 15 (6), 617.
- Gherbi, Y.A., Bouzeboudja, H., Gherbi, F.Z., 2016. The combined economic environmental dispatch using new hybrid metaheuristic. *Energy* 115, 468–477.
- Ghorbani, N., 2016. Combined heat and power economic dispatch using exchange market algorithm. *Int. J. Electr. Power Energy Syst.* 82, 58–66.
- Gonidakis, D., Vlachos, A., 2019. A new sine cosine algorithm for economic and emission dispatch problems with price penalty factors. *J. Inf. Optim. Sci.* 40 (3), 679–697.
- Güvenç, U., Sönmez, Y., Duman, S., Yörükeren, N., 2012. Combined economic and emission dispatch solution using gravitational search algorithm. *Sci. Iran.* 19 (6), 1754–1762.
- Hagh, M.T., Teimourzadeh, S., Alipour, M., Aliasghary, P., 2014. Improved group search optimization method for solving CHPED in large scale power systems. *Energy Convers. Manage.* 80, 446–456.
- Haghighi, A., Nazari-Heris, M., Mohammadi-Ivatloo, B., 2016. Solving combined heat and power economic dispatch problem using real coded genetic algorithm with improved Mühlensbein mutation. *Appl. Therm. Eng.* 99, 465–475.
- Jayabarathi, T., Yazdani, A., Ramesh, V., Raghunathan, T., 2014. Combined heat and power economic dispatch problem using the invasive weed optimization algorithm. *Front. Energy* 8 (1), 25–30.
- Jayakumar, N., Subramanian, S., Ganesan, S., Elanchezian, E., 2016. Grey wolf optimization for combined heat and power dispatch with cogeneration systems. *Int. J. Electr. Power Energy Syst.* 74, 252–264.
- Jeddi, B., Vahidinasab, V., 2014. A modified harmony search method for environmental/economic load dispatch of real-world power systems. *Energy Convers. Manage.* 78, 661–675.
- Jordehi, A., 2015. A chaotic artificial immune system optimisation algorithm for solving global continuous optimisation problems. *Neural Comput. Appl.* 26 (4), 827–833.
- Joshi, P.M., Verma, H., 2019. An improved TLBO based economic dispatch of power generation through distributed energy resources considering environmental constraints. *Sustain. Energy Grids Netw.* 18, 100207.
- Karaboga, D., Akay, B., 2009. A comparative study of artificial bee colony algorithm. *Appl. Math. Comput.* 214 (1), 108–132.
- Karthik, N., Parvathy, A.K., Arul, R., 2019. Multi-objective economic emission dispatch using interior search algorithm. *Int. Trans. Electr. Energy Syst.* 29 (1), e2683.
- Kennedy, J., Eberhart, R., 1995. Particle swarm optimization (PSO). In: *Proc. International Conference on Neural Networks*, Perth, Australia. IEEE, pp. 1942–1948.
- Kheshti, M., Ding, L., Ma, S., Zhao, B., 2018. Double weighted particle swarm optimization to non-convex wind penetrated emission/economic dispatch and multiple fuel option systems. *Renew. Energy* 125, 1021–1037.
- Kheshti, M., Kang, X., Li, J., Regulski, P., Terzija, V., 2017. Lightning flash algorithm for solving non-convex combined emission economic dispatch with generator constraints. *IET Gener. Transm. Distrib.* 12 (1), 104–116.
- Liang, H., Liu, Y., Li, F., Shen, Y., 2018. A multiobjective hybrid bat algorithm for combined economic/emission dispatch. *Int. J. Electr. Power Energy Syst.* 101, 103–115.
- Mahdavi, M., Fesanghary, M., Damangir, E., 2007. An improved harmony search algorithm for solving optimization problems. *Appl. Math. Comput.* 188 (2), 1567–1579.
- Mahdi, F.P., Vasant, P., Abdullah-Al-Wadud, M., Kallimani, V., Watada, J., 2018. Quantum-behaved bat algorithm for many-objective combined economic emission dispatch problem using cubic criterion function. *Neural Comput. Appl.* 1–13.
- Manteaw, E.D., Otero, N.A., 2012. Combined economic and emission dispatch solution using ABC-PSO hybrid algorithm with valve point loading effect. *Int. J. Sci. Res. Publ.* 2 (12), 1–9.
- Mehdinejad, M., Mohammadi-Ivatloo, B., Dadashzadeh-Bonab, R., 2017. Energy production cost minimization in a combined heat and power generation systems using cuckoo optimization algorithm. *Energy Effic.* 10 (1), 81–96.
- Mellal, M.A., Williams, E.J., 2015. Cuckoo optimization algorithm with penalty function for combined heat and power economic dispatch problem. *Energy* 93, 1711–1718.
- Meng, X., Liu, Y., Gao, X., Zhang, H., 2014. A new bio-inspired algorithm: chicken swarm optimization. In: *International Conference in Swarm Intelligence*. Springer, pp. 86–94.
- Meng, A., Mei, P., Yin, H., Peng, X., Guo, Z., 2015. Crisscross optimization algorithm for solving combined heat and power economic dispatch problem. *Energy Convers. Manage.* 105, 1303–1317.
- Mirjalili, S., Lewis, A., 2016. The whale optimization algorithm. *Adv. Eng. Softw.* 95, 51–67.
- Mohammadi-Ivatloo, B., Moradi-Dalvand, M., Rabiee, A., 2013. Combined heat and power economic dispatch problem solution using particle swarm optimization with time varying acceleration coefficients. *Electr. Power Syst. Res.* 95, 9–18.

- Murugan, R., Mohan, M., Rajan, C.C.A., Sundari, P.D., Arunachalam, S., 2018. Hybridizing bat algorithm with artificial bee colony for combined heat and power economic dispatch. *Appl. Soft Comput.* 72, 189–217.
- Narang, N., Sharma, E., Dhillon, J., 2017. Combined heat and power economic dispatch using integrated civilized swarm optimization and powell pattern search method. *Appl. Soft Comput.* 52, 190–202.
- Nazari-Heris, M., Mohammadi-Ivatloo, B., Asadi, S., Geem, Z.W., 2019. Large-scale combined heat and power economic dispatch using a novel multi-player harmony search method. *Appl. Therm. Eng.* 154, 493–504.
- Nguyen, T.T., Nguyen, T.T., Vo, D.N., 2018. An effective cuckoo search algorithm for large-scale combined heat and power economic dispatch problem. *Neural Comput. Appl.* 30 (11), 3545–3564.
- Nguyen, T.T., Vo, D.N., Dinh, B.H., 2016. Cuckoo search algorithm for combined heat and power economic dispatch. *Int. J. Electr. Power Energy Syst.* 81, 204–214.
- Rajagopalan, A., Kasinathan, P., Nagarajan, K., Ramachandramurthy, V.K., Sengoden, V., Alavandar, S., 2019. Chaotic self-adaptive interior search algorithm to solve combined economic emission dispatch problems with security constraints. *Int. Trans. Electr. Energy Syst.* e12026.
- Rashedi, E., Nezamabadi-Pour, H., Saryazdi, S., 2009. GSA: a gravitational search algorithm. *Inf. Sci.* 179 (13), 2232–2248.
- Rezaie, H., Kazemi-Rahbar, M., Vahidi, B., Rastegar, H., 2019. Solution of combined economic and emission dispatch problem using a novel chaotic improved harmony search algorithm. *J. Comput. Design Eng.* 6 (3), 447–467.
- Rizk-Allah, R.M., El-Sehiemy, R.A., Wang, G.-G., 2018. A novel parallel hurricane optimization algorithm for secure emission/economic load dispatch solution. *Appl. Soft Comput.* 63, 206–222.
- Roy, P.K., Paul, C., Sultana, S., 2014. Oppositional teaching learning based optimization approach for combined heat and power dispatch. *Int. J. Electr. Power Energy Syst.* 57, 392–403.
- Sadollah, A., Bahreininejad, A., Eskandar, H., Hamdi, M., 2013. Mine blast algorithm: A new population based algorithm for solving constrained engineering optimization problems. *Appl. Soft Comput.* 13 (5), 2592–2612.
- Secui, D.C., 2015. A new modified artificial bee colony algorithm for the economic dispatch problem. *Energy Convers. Manage.* 89, 43–62.
- Storn, R., Price, K., 1997. Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces. *J. Global Optim.* 11 (4), 341–359.
- Sun, Y., Dong, W., Chen, Y., 2017. An improved routing algorithm based on ant colony optimization in wireless sensor networks. *IEEE Commun. Lett.*
- Sun, J., Li, Y., 2019. Social cognitive optimization with tent map for combined heat and power economic dispatch. *Int. Trans. Electr. Energy Syst.* 29 (1), e2660.
- Vasebi, A., Fesanghary, M., Bathaee, S., 2007. Combined heat and power economic dispatch by harmony search algorithm. *Int. J. Electr. Power Energy Syst.* 29 (10), 713–719.
- Whitley, D., 1994. A genetic algorithm tutorial. *Stat. Comput.* 4 (2), 65–85.
- Yang, X.-S., 2009. Harmony search as a metaheuristic algorithm. In: *Music-Inspired Harmony Search Algorithm*. Springer, pp. 1–14.
- Yang, X.-S., 2010. *Engineering Optimization: An Introduction with Metaheuristic Applications*. John Wiley & Sons.
- Yang, X.-S., 2012. Flower pollination algorithm for global optimization. In: *UCNC*. Springer, pp. 240–249.
- Yao, X., Liu, Y., Lin, G., 1999. Evolutionary programming made faster. *IEEE Trans. Evol. Comput.* 3 (2), 82–102.
- Yazdani, M., Jolai, F., 2016. Lion optimization algorithm (LOA): a nature-inspired metaheuristic algorithm. *J. Comput. Design Eng.* 3 (1), 24–36.
- Zhang, R., Zhou, J., Mo, L., Ouyang, S., Liao, X., 2013. Economic environmental dispatch using an enhanced multi-objective cultural algorithm. *Electr. Power Syst. Res.* 99, 18–29.
- Zhang, Q., Zou, D., Duan, N., Shen, X., 2019. An adaptive differential evolutionary algorithm incorporating multiple mutation strategies for the economic load dispatch problem. *Appl. Soft Comput.* 78, 641–669.
- Zheng, Y.-J., 2015. Water wave optimization: a new nature-inspired metaheuristic. *Comput. Oper. Res.* 55, 1–11.
- Zou, D., Li, S., Kong, X., Ouyang, H., Li, Z., 2019. Solving the combined heat and power economic dispatch problems by an improved genetic algorithm and a new constraint handling strategy. *Appl. Energy* 237, 646–670.