## Forward /-V plot for nonideal Schottky diodes with high series resistance

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In this work we present an extension of Norde's forward I-V plot. This modified method allows us to obtain reliable values for three different parameters  $(n, R, and I_s)$  in nonideal Schottky barrier diodes with high series resistance.

Traditionally, it has been possible to calculate the barrier height  $(\phi_B)$  of a nearly ideal  $(n \ge 1)$  Schottky diode through the determination of the saturation current  $(I_s)$  using the thermionic-emission or diffusion models. The ln I vs V plot for a diode without series resistance shows a linear region whose extrapolated intercept with the zero voltage axis gives  $I_s$  and its slope gives the ideality factor (n). I-Vcharacteristics for Schottky diodes with ideality factor greater than 1.1 cannot give us information about  $\phi_R$ . However, we have found very frequently in the literature that it is necessary to determine the parameters  $I_s$  and n in nonideal Schottky diodes.  $^{3,4}$  This is possible by approximating the I-Vcharacteristic of a nonideal Schottky diode as

$$I \simeq I_s [\exp(\beta V_D/n) - 1], \tag{1}$$

where

 $V_D$  = voltage across the diode,

$$\beta = q/KT$$
.

The diodes are usually made with highly resistive materials such as undoped hydrogenated amorphous silicon (a-Si:H). These diodes have a very high series resistance that cause the traditional method to fail. Norde<sup>5</sup> has developed a different kind of plot that allows the accurate determination of  $\phi_B$  or  $I_s$  and R for nearly ideal Schottky diodes, even with high series resistance. We will show below that it is possible to modify this plot in order to obtain the value of three parameters  $(n, R, \text{ and } I_s)$  for nonideal Schottky diodes with high series resistance.

The failure of the traditional method to determine  $I_{i}$ and n for highly resistive diodes can be easily understood if we consider three different regions in the ln I vs V characteristics. The first region can be defined as the region where these characteristics differ from linearity due to the nonexponential behavior of Schottky diodes for low voltages. This region, extends from the zero voltage axis to a voltage  $V_{MIN}$ , defined as the voltage where the relative error for nonlinear-

$$e_m = \frac{I_s \exp(\beta V_D/n) - I_s \left[ \exp(\beta V_D/n) - 1 \right]}{I_s \left[ \exp(\beta V_D/n) - 1 \right]}.$$

From this we can find

$$V_D = \frac{n}{\beta} \ln \left( \frac{e_{m+1}}{e_m} \right).$$

Making  $e_m = 0.01$  for  $V_D = V_{D\,\mathrm{MIN}}$ ; T = 300 K, and neglecting the voltage drop in the series resistance  $(V_R \triangleleft V_D)$  we obtain for  $V_{MIN}$ :

$$V_{\text{MIN}} \simeq V_{D,\text{MIN}} = 0.115 \, n. \tag{2}$$

Equation (2) shows that the upper limit of the first region has a strong dependence on the value of the ideality factor. This means, as shown below, that for high enough values of n the size of the second region (linear region) can be seriously affected. However, the most serious limitation to the extension of the linear region comes from the third region. This one is defined as the region where the series resistance causes the In I-V curve to differ from the linear one. The lower limit for this region is indicated by a voltage  $V_{\text{MAX}}$ , as the voltage at which the nonlinearity relative error reaches the value:

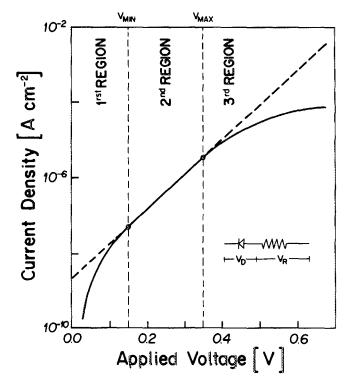


FIG. 1. Current density vs voltage characteristic for a nonideal Schottky barrier diode with series resistance (full line) and the linear extrapolation

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$$e_{M} = \frac{I_{s} \left[ \exp(\beta V/n) - 1 \right] - I_{s} \left\{ \exp[\beta (V-IR)/n] - 1 \right\}}{I_{s} \left\{ \exp[\beta (V-IR)/n] - 1 \right\}}.$$

From this we obtain

$$\exp(\beta IR/n) = \frac{e_M + 1}{1 + e_M \exp(-\beta V/n)}.$$
We can see, since we always take  $e_M < 1$ , that in Eq. (3)

 $1 > e_M \exp(-\beta V/n)$ 

for any positive value of V. Neglecting this term in Eq. (3) and solving for the current

$$I = \frac{n}{\beta R} \ln(e_M + 1) \tag{4}$$

but as the I-V relation is given by (1), we obtain from Eq. (4)

$$V_D \simeq \frac{n}{\beta} \ln \left[ 1 + \frac{n}{\beta RI_c} \ln(e_M + 1) \right].$$

Taking  $e_M = 0.01$  for  $V_D = V_{DMAX}$ ; T = 300 K, and neglecting  $V_R$  ( $V_R \lt V_D$  for  $e_M \lt 1$ ) we obtain for  $V_{MAX}$ 

$$V_{\text{MAX}} \simeq V_{D \text{MAX}} \simeq 0.025 \ n \ln \left( 1 + \frac{2.5 \times 10^{-4} n}{RI_s} \right).$$
 (5)

Equation (5) gives for the lower limits of the third region, an equivalent expression as that given by Eq. (2) for the upper limit of the first region.

Evaluating both equations for typical values of R,  $I_s$ , and n, we found that it is common for nonideal diodes with high series resistance to obey the relation

$$V_{\text{MAX}} \leq V_{\text{MIN}}$$
.

It can be seen from Fig. 1, that the ln I vs V plot for diodes which satisfy this relation do not present the linear region. Norde's method overcomes this problem, by plotting a function F[V(I)] (where V and I are the voltage and current measured over the sample) which has a minimum point for a characteristic voltage  $V_0$  and a current  $I_0$ . From these characteristic values and the function I(V), it is possible to obtain analytical relations which determine the parameters. In order to deal with nonideal diodes, we have modified the function defined by Norde and take instead the form:

$$F(V) = V - V_A \ln I, \tag{6}$$

where  $V_A$  is defined as an arbitrary voltage independent of V

From Eq. (1), we can approximate the I(V) function as

$$I \simeq I_{*} \exp[\beta (V - IR)/n] \tag{7}$$

for  $V > V_{MIN}$ .

Replacing Eq. (7) in Eq. (6) gives

$$F(V) = V\left(1 - \frac{\beta V_A}{n}\right) - V_A \ln I_s + \frac{\beta V_A}{n} IR.$$

From this we can see that for low enough voltages the first term dominates  $(V_R \triangleleft V)$ , but as V increases, the last term becomes the most important one. Taking this into account, we find that F(V) will have a minimum point  $F(V_0,I_0)$  only when the first term is negative:

$$1 - \frac{\beta V_A}{n} < 0.$$

This is true only for

$$V_A > n/\beta. \tag{8}$$

If we make dF/dV = 0, we can find the current  $I_0$  at the minimum point of F(V):

$$I_0 = V_A / R - n / \beta R. \tag{9}$$

This show us that  $I_0$  depends linearly on  $V_A$ .

The linear behavior of the function  $I_0(V_A)$  is confined to a limited interval of values of  $V_A$ , for which the approximation of Eq. (7) is valid. That means

$$V_D(I_0) \geqslant V_{DMIN} \tag{10}$$

but from Eq. (7)

$$V_D(I_0) \simeq \frac{n}{\beta} \ln(I_0/I_s). \tag{11}$$

Replacing Eq. (9) in Eq. (11) and this in Eq. (10) we have

$$V_{DMIN} < \frac{n}{\beta} \ln \left( \frac{V_A - n/\beta}{RI_s} \right).$$
 (12)

Solving Eq. (12) for  $V_A$ :

$$V_A > RI_s \exp\left(\frac{\beta V_{DMIN}}{n}\right) + n/\beta$$
.

From Eq. (2) and choosing T = 300 K:

$$V_A \geqslant 99.5 I_s R + n/\beta. \tag{13}$$

Comparing this with Eq. (8), we can see that F(V) always has a minimum for values of  $V_A$  which satisfy Eq. (13).

A plot of  $I_0(V_A)$  for these values of  $V_A$ , will give us R from the slope of the straight line and n from the zero voltage axis intercept. With these parameters, it is easy to obtain  $I_s$ from Eq. (1). In order to get sufficient accuracy in the value of these parameters, a least square fit of several points in  $I(V_A)$  should be recommended. Experimental  $I_0(V_A)$  plots

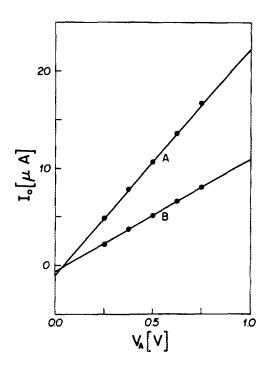


FIG. 2. Experimental plots of  $I_0(V_A)$  for two different Schottky diodes (P<sub>2</sub>Si-a-Si:H). (A) n = 2.33,  $R = 43 \text{ k}\Omega$ ,  $I_s = 1.9 \times 10^{-8} \text{ A}$ . (B) n = 1.7,  $R = 87 \text{ k}\Omega$ ,  $I_s = 5 \times 10^{-9} \text{ A}$ .

for different Schottky diodes are shown in Fig. 2.

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