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NOTE

EXTRACTION OF SCHOTTKY DIODE (AND *p–n* JUNCTION) PARAMETERS FROM *I–V* CHARACTERISTICS

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The analysis of I–V characteristics of Schottky barriers and p–n junctions is considered with a fitting method, using the MERLIN-multidimentional minimization system-program. The fitting method is applied to Schottky diodes and a comparison is made between this analysis and other known methods. The advantages of the MERLIN fitting method are: (a) it is able to calculate the diode parameters even when noise or random errors are existing during measurements and (b) the fitting method is accurate even when the ideality factor of a junction is not close to unity.

A Schottky diode I–V characteristic is usually described by the equation:

$$I_d = I_s[\exp(eV_d/nkT) - 1], \tag{1}$$

where I_s is the saturation current, n is the ideality factor, V_d is the applied voltage across the diode, T is the temperature, K is the Boltzmann's constant and e is the electronic charge. The saturation current I_s is given by the expression:

$$I_s = A^{**}ST^2 \exp(-e\phi_B/kT), \tag{2}$$

where A^{**} is the Richardson constant, S is the diode area and $e\phi_B$ is the barrier height of the diode. By taking into account the series resistance R_s of the diode, eqn (1) becomes:

$$I_d = I_s\{\exp[e(V - IR_s)/nkT] - 1\}. \tag{3}$$

The series resistance influences the current mostly at high voltages, while at low voltages the I–V characteristic is affected by the shunt resistance. A complete representation of a real I–V characteristic then is given by:

$$I = I_d + I_p = I_s\{\exp[e(V - IR_s)/nkT] - 1\} + G_p(V - IR_s), \tag{4}$$

where G_p is the parallel conductance and I_p the shunt current through the shunt resistance $R_p = 1/G_p$.

Norde[1] has proposed a method for evaluation of the series resistance R_s from forward I–V characteristics, considering an ideal diode with $n = 1$. His main idea was to determine the minimum of a function $F(V)$. The disadvantages of this method are: (a) it considers only ideal diodes ($n = 1$), (b) the minimum of the function $F(V)$ can not be determined accurately and (c) it is difficult to determine whether the minimum is caused by a series resistance only or by a non-ohmic back contact also.

Sato *et al.*[2] used a similar function $F(V)$ with Norde[1] taking into account also the ideality factor n in their method. The analysis requires measurements of the I–V characteristics at two different temperatures. Still, the problem of the exact determination of the minimum of the $F(V)$ function is existing and, on the other hand, the diode parameters n and R_s are not extracted from a single I–V measurement.

Cheung *et al.*[3] proposed another model and derived the diode parameters from plots, which are linear functions of the current density J. The plots are $dV/d(\ln J)$ vs J and $H(J)$ vs J, where $H(J)$ is a function that enables one to determine mainly the barrier height of a diode. According to the discussion of a previous work[4], this method is not so accurate. All the above mentioned models do not state at all about the influence of a possible parallel shunt resistance R_p on the I–V characteristics. This influence is mainly considerable for low applied forward biases.

Werner[4] proposed a method to evaluate the diode parameters by using eqn (4) and taking into account the possible existence of a parallel shunt resistance. In this method, the shunt conductance was evaluated from the reverse biased characteristics for large negative voltages. However, this evaluation can not give accurate values of G_p, since the real reverse I–V characteristics are not exactly linear. Werner uses three linear plots in his analysis: (A) G/I_d vs G, where $G = dI_d/dV$, (B) R_{rd} vs $1/I_d$, where R_{rd} is the differential resistance of the I–V curve and (C) I_d/G vs I_d. The final conclusion of the whole analysis is that case A is the most accurate method. Case C is actually the model of Cheung *et al.*[3], which was mentioned above. Besides the problem of evaluating the shunt conductance G_p, Werner in case A is disregarding some experimental points during the evaluation of the forward conductance G. The selection of experimental points for determining G, sometimes concludes to different results for slightly different sets of experimental points.

In this paper, a fitting method is used for evaluation of the Schottky diode parameters (n, R_s, G_p, I_s and $e\phi_B$) by using eqns (4) and (2). The method is based in fitting the experimental forward I–V characteristics using the MERLIN-multidimentional minimization system-program, version 2.1[5–8]. First, the fitting method is checked on a theoretical Schottky diode. An electronic noise of 1% is added by the computer to the values of the current. The Schottky diode parameters are extracted also with the methods used by Sato *et al.*[2], Cheung *et al.*[3] and Werner[4]. The influence of the noise to all methods is illustrated. Finally, all these methods are used to evaluate the diode parameters of PtSi/Si Schottky diodes and a comparison is made between them.

The characteristic parameters of the theoretical Schottky diode are given in Table 1. In the same Table, the extracted values of these parameters from the previous methods and from the fitting are also given. It is obvious that the extracted values of the diode parameters with the MERLIN minimization fitting method are almost identical with the theoretical ones. The only parameter that scatters is the shunt conductance G_p, a fact which is discussed below.

In order to check the accuracy of all methods, we added an electronic noise by the computer of 1% to the values of the current. In the analysis by Sato *et al.*[2], there is no influence of the noise on the $F(V)$ function, but the problem of determining the exact minimum of this function is still present [Fig. 1(a)]. Figure 1(b) shows the results according to the analysis by Cheung *et al.*[3]. The straight line

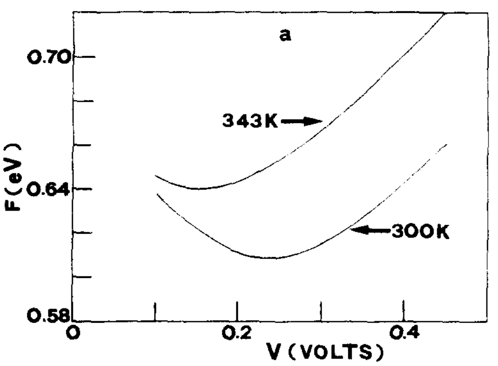
Table 1. Theoretical and extracted values of a diode with various methods

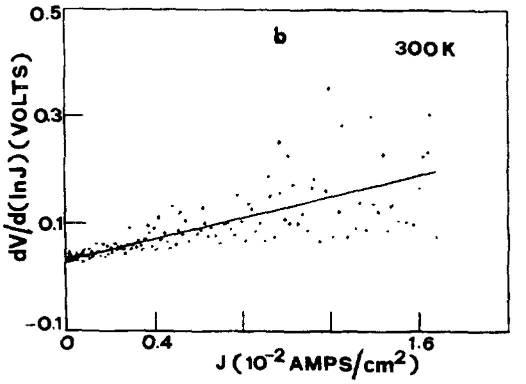
Diode parameters	Theoretical values	Sato *et al.* [2]	Cheung *et al.* [3]	Werner[4]	MERLIN
$S(\mathrm{cm}^2)$	3.14×10^{-2}				
$A^{**}(\mathrm{A/K \cdot cm^2})$	120				
$T_1(\mathrm{K})$	300				
$T_2(\mathrm{K})^a$		343			
n	1.200	1.171	1.197	1.195	1.200
$R_s(\Omega)$	10	10.1470	10.0210	10.0180	9.9999
$G_p(S)$	1×10^{-6}	Not considered	Not considered	Theoretical value was used	1.8×10^{-6}
$I_s(A)$	1.9756×10^{-6}	1.9757×10^{-6}	1.9000×10^{-6}	1.9000×10^{-6}	1.9756×10^{-6}
$e\phi_B(\mathrm{eV})$	0.670	0.674	0.670	0.671	0.670

$^a T_2$ is used only for the Sato *et al.* method[2].

represents the theoretical curve. The noise has spread out the measurements so much that it is impossible to make any evaluation of the diode parameters. Figure 1(c) shows the results according to the analysis by Werner[4] and the straight line represents the theoretical curve. This method is also unable to compute the diode parameters in the case when some noise is existing in the measurements. With the MERLIN minimization method, we obtained the diode parameters: $n = 1.203$, $R_s = 9.982\,\Omega$, $I_s = 1.9973 \times 10^{-6}$ A, $e\phi_B = 0.67$ eV and $G_p = 0.5 \times 10^{-6}$ S. These values are in excellent agreement with their theoretical values, defined in Table 1. There is a small deviation between the determined value and the theoretical value of G_p only. This is probably due to the fact that the influence of G_p is only significant at very low voltages ($V < 3kT/e$) where eqn (4) is not valid. The above results indicate that the fitting method is able to make an estimation of G_p even when electrical noise or random errors are present during measurements.

The experimental I–V characteristics of two PtSi/Si diodes of different quality were analyzed with the various methods mentioned above. The selection of diodes of different quality was done in order to verify the accuracy of each method in cases that the ideality factor is close to unity or not. The extracted values of the diode parameters are given in Table 2. In Fig. 2(a), the closed circles are the experimental results for the diode 1 and the solid lines are the theoretical curves derived from eqn (4) with the parameters shown in Table 2, which were obtained with various methods. Figure 2(b) shows similar results for diode 2. The inserts of Fig. 2(a) and (b) show the experimental I–V characteristics. It must be emphasized that the fitting to the experimental I–V characteristics is made for $V > 3kT/e$, since for lower voltages the theoretical model presented by





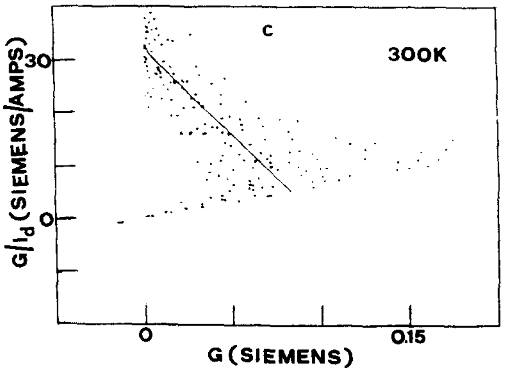


Fig. 1. Analysis of the theoretical diode, with various known methods, before and after introducing electronic noise of 1%. (a) With the model of Sato *et al.*[2]. No influence of the noise on the $F(V)$ function was found. (b) With the model of Cheung *et al.*[3]. (c) With the model of Werner[4]. In this case as G_p the theoretical value was used. In the last two cases it is impossible to calculate the diode parameters, when noise is present.

Table 2. Calculated values of two PtSi/Si diodes with various methods

Diode	Method	T(K)	n	$R_s(\Omega)$	I_s(A)	$e\phi_B$(eV)	$G_p(S)$
1	Cheung[3]	300	1.035	46.66–46.70	5.85×10^{-10}	0.843	Not considered
	Werner[4]	300	1.038–1.040	46.726	6.20×10^{-10}	0.841	1.05×10^{-8}
	MERLIN	300	1.051	43.202	7.43×10^{-10}	0.836	7.09×10^{-8}
2	Cheung[3]	300	1.042	36.71–36.95	2.46×10^{-10}	0.867	Not considered
	Werner[4]	300	1.050	36.738	2.36×10^{-10}	0.868	4.55×10^{-8}
	MERLIN	300	1.139	22.837	6.17×10^{-10}	0.841	2.35×10^{-7}

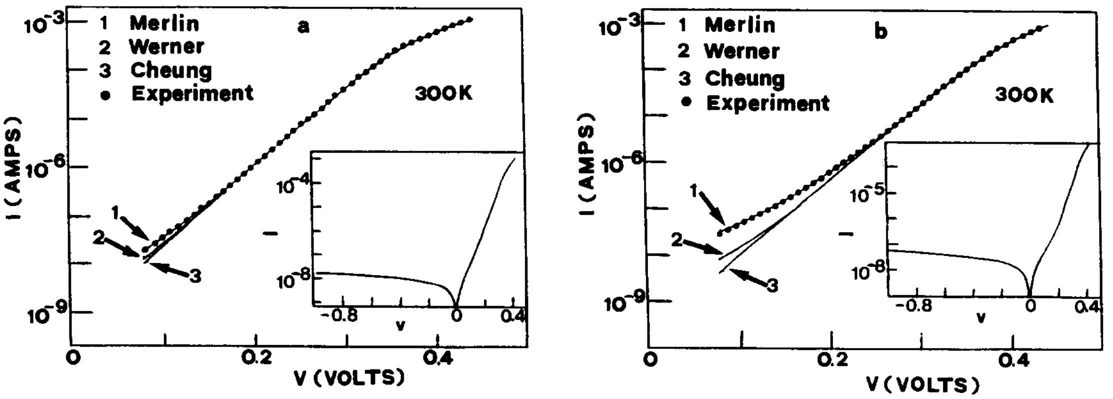


Fig. 2. Experimental data (●) of PtSi/Si diodes and theoretical I–V characteristics, derived with various methods. Only the MERLIN method shows a complete coincidence with the experimental curves for $V > 3kT/e$. The inserts are the experimental I–V characteristics. (a) For a high quality PtSi/Si diode. (b) For a lower quality PtSi/Si diode.

eqn (4) is not valid. From both figures, it is obvious that the methods of Werner [4] and Cheung *et al.* [3] are not so accurate at low voltages. This was expected for the method of Cheung *et al.*[3] because they do not consider the shunt conductance at all. The method proposed by Werner[4] also appears to have some inconsistency at low voltages. We believe, the main reason is that G_p is calculated for high reverse voltages assuming that the reverse I–V characteristic in this region is linear. However, from the inserts of Fig. 2(a) and (b) it is obvious that this consideration is not realistic. Figure 2(b) represents the I–V characteristic of diode 2, which is not so good as diode 1 [Fig. 2(a)]. By comparing the experimental with the theoretical I–V characteristics derived by the various methods, we observe that the methods of Werner[4] and Cheung *et al.*[3] are not reliable when the ideality factor n is not very close to unity. In a way, this is mentioned by Werner[4] for all the plots he considers, and case C of his analysis is actually the same with the model of Cheung *et al.*[3].

Since eqn (4) represents the I–V characteristics of p–n junctions, the above analysis can also be applied to p–n junctions. Of course, the barrier height in this case can not be calculated because eqn (2) is valid only for Schottky barriers (thermionic emission). All the other parameters (R_s, G_p, n and I_s) can be extracted very accurately.

In conclusion, the MERLIN minimization method is quite accurate even when electrical noise or random errors are existing in the I–V measurements. The fitting method can be used for any diode, even for diodes for which n is not close to unity. It can be used also for p–n junctions since eqn (4) is presenting both cases, the Schottky diodes and p–n junctions. The MERLIN minimization method is fast and does not require any kind of graphs for the analysis of the I–V characteristics, as other methods do.

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REFERENCES

1. H. Norde, *J. appl. Phys.* **50**, 5052 (1979).
2. K. Sato and Y. Yasumura, *J. appl. Phys.* **58**, 3655 (1985).
3. S. K. Cheung and N. W. Cheung, *Appl. Phys. Lett.* **49**, 85 (1986).
4. J. H. Werner, *Appl. Phys.* **A47**, 291 (1988).
5. G. A. Evangelakis, J. P. Rizos, I. E. Lagaris and I. N. Demetropoulos, *Comp. Phys. Commun.* **46**, 401 (1987).
6. C. S. Chassapis, D. G. Papageorgiou and I. E. Lagaris, *Comp. Phys. Commun.* **52**, 223 (1989).
7. D. G. Papageorgiou, C. S. Chassapis and I. E. Lagaris, *Comp. Phys. Commun.* **52**, 241 (1989).
8. D. G. Papageorgiou and I. E. Lagaris, *Comp. Phys. Commun.* **58**, 119 (1990).