

# MODELLING OF TANDEM CELL TEMPERATURE COEFFICIENTS

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## ABSTRACT

This paper discusses the temperature dependence of the basic solar-cell operating parameters for a GaInP/GaAs series-connected two-terminal tandem cell. The effects of series resistance and of different incident solar spectra are also discussed.

## INTRODUCTION

For both terrestrial and space applications, a detailed understanding of device temperature coefficients is vital if the results of simulator testing are to be used for quantitative prediction of performance in the field. This paper analyzes the temperature dependence of open-circuit voltage ( $V_{OC}$ ), short-circuit current ( $J_{SC}$ ), and fill factor ( $FF$ ) for a two-terminal series connected tandem cell, extending the work of Fan on single-junction cells [1]. For the purpose of quantitative discussion, the GaInP/GaAs system is considered for detailed examination. This particular system is of specific practical interest due to its demonstrated high efficiencies in one-sun, concentrator, and space applications [2-4].

## MODEL

The model used for the current-voltage ( $J$ - $V$ ) calculations has been described elsewhere [5]. For convenience, the essentials are reproduced very briefly here.

An idealized spectral response determined only by the cell absorption coefficients is assumed. All photons absorbed by the top cell, as determined by the top-cell thickness  $t$  and absorption coefficient  $\alpha(\lambda)$ , are assumed to contribute to the top-cell short-circuit current  $J_{SCt}$ :

$$J_{SCt} = \int_0^{\infty} e I_0(\lambda) (1 - \exp[-\alpha(\lambda)t]) d\lambda, \quad (1)$$

where  $I_0$  is the incident spectral intensity. The bottom cell is idealized to be infinitely thick, so that all photons transmitted through the top cell and above the band gap  $E_{gb}$  of the bottom cell are assumed to contribute to the bottom-cell short-circuit current  $J_{SCb}$ :

$$J_{SCb} = \int_0^{hc/E_{gb}} e I_0(\lambda) \exp[-\alpha(\lambda)t] d\lambda. \quad (2)$$

The absorption coefficient  $\alpha(\lambda)$  depends on the bandgap  $E_{gt}$  of the top cell; the form of Kurtz et al. [5] is used:

$$\alpha(\lambda) = 4.55 (E - E_{gt})^{1/2} + 2.05 (E - E_{gt} - 0.1)^{1/2}. \quad (3)$$

In the present work, the temperature dependence of the short-circuit currents is accounted for by assuming a linear temperature dependence of  $E_{gt}$  and  $E_{gb}$  in the above equations; any other temperature effects on the spectral response are neglected. For the GaInP top cell, we take  $dE_g/dT = -0.46$  meV/K, valid around 300 K [6]. For the GaAs bottom cell, we take  $dE_g/dT = -0.45$  meV/K, valid around 300 K [7]. The general conclusions drawn later do not depend on these precise numbers.

The voltage  $V_{tandem}$  for the tandem device is the sum of the top and bottom cell voltages:

$$V_{tandem} = V_t + V_b. \quad (4)$$

For the top and bottom cells, an ideal resistanceless solar cell has been assumed (except in the section discussing series resistance effects), with the top (bottom) cell voltage depending on the top-cell short-circuit current  $J_{SCt(b)}$  and dark current  $J_{0t(b)}$  by the relationship

$$V_{t(b)} = kT/e \ln [(J + J_{SCt(b)})/J_{0t(b)} + 1] \quad (5)$$

where we have set the junction ideality factor  $n=1$ .  $J_0$  depends strongly on the temperature  $T$  through its proportionality to the square of the intrinsic carrier concentration  $n_i$ . With the simplifying assumption of zero surface recombination velocities,  $J_0$  is given by

$$J_0 = e (L_e/\tau_e)(n_i^2/N_A) \tanh(x_p/L_e) + e (L_h/\tau_h)(n_i^2/N_A) \tanh(x_p/L_h), \quad \text{with} \quad (6)$$

$$n_i^2 = 4M_c M_v (2\pi kT/h^2)^3 (m_e^* m_h^*)^{3/2} \exp(-E_g/kT), \quad (7)$$

and more weakly through temperature dependencies of materials parameters such as mobilities and minority-carrier lifetimes. Here  $L$  is the minority-carrier diffusion length

$$L_{e(h)} = (\tau_{e(h)} D_{e(h)})^{1/2}, \quad (8)$$

and  $D$  is the diffusion constant

$$D_{e(h)} = kT\mu_{e(h)}/e. \quad (9)$$

Any temperature dependence of mobilities  $\mu$ , minority-carrier lifetimes  $\tau$ , effective masses  $m^*$ , is ignored.

However, the temperature dependence of the bandgap  $E_g$  is taken into account as described above.

### $V_{OC}$ TEMPERATURE COEFFICIENT

Because the tandem voltage is the sum of the top- and bottom-cell voltages, the tandem  $V_{OC}$  temperature coefficient  $dV_{OC}/dT$  is simply the sum of the top- and bottom-cell temperature coefficients. Numerically calculating the GaInP top- and GaAs bottom-cell temperature coefficients  $dV_{OCt}/dT$  and  $dV_{OCb}/dT$  at 300 K, including the temperature dependence of minority-carrier diffusion constants and lifetimes as described above, gives  $dV_{OCt}/dT = -2.20$  mV/°C,  $dV_{OCb}/dT = -1.99$  mV/°C. The resulting tandem  $dV_{OC}/dT = -4.2$  mV/°C. Neither top- nor bottom-cell temperature coefficient varies significantly with temperature. A simple analytic expression for a single-junction (e.g., either a top or bottom subcell)  $dV_{OC}/dT$  may be obtained from  $V_{OC} \approx kT/e \ln[(J_{SC})/J_0]$  by neglecting the temperature dependence of  $J_0$  on materials parameters such as diffusion lengths and minority-carrier lifetimes, so that  $J_0(T) \approx \text{const} \times T^3 \exp(-E_g/kT)$ . Setting  $n=1$ , we get

$$\frac{dV_{OC}}{dT} \approx \frac{1}{T} \left[ V_{OC} - \frac{3kT}{e} + \frac{T dE_g}{e dT} - \frac{E_g}{e} \right] + \frac{kT}{e} \frac{1}{J_{SC}} \frac{dJ_{SC}}{dT} \quad (10)$$

which is Fan's eq. 6 [1]. This last term must be computed numerically; Fig. 1 shows that for GaInP/GaAs it is on the order of  $(10^{-3}kT/e)/K$ . This is roughly 0.02 mV/K at 300 K, which is negligible. It should be noted that the spectrum affects only this last term. With this last term neglected, Eq. (10) gives  $dV_{OCt}/dT = -2.25$  mV/°C,  $dV_{OCb}/dT = -2.02$  mV/°C, very close to the more precise calculation.

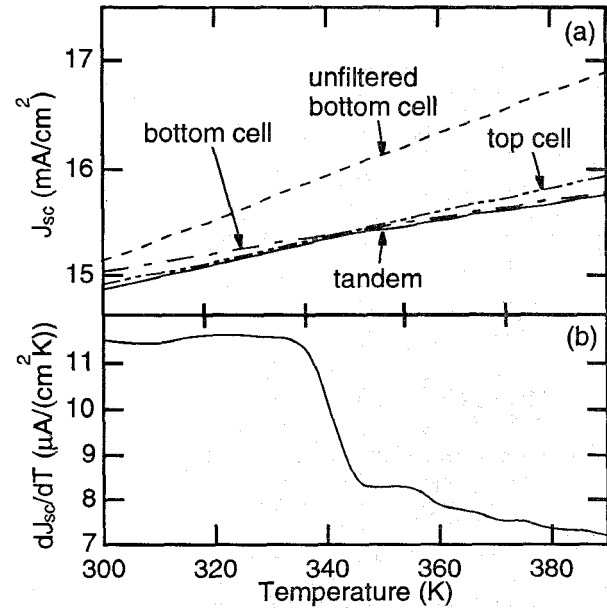
For real devices, the single-junction ideality factor  $n$  is generally somewhat greater than 1, resulting in an greater magnitude of  $dV_{OC}/dT$  compared to its magnitude for  $n=1$ .

For comparison with the calculated  $-4.2$  mV/°C value, a recent on-sun measurement [8] of  $dV_{OC}/dT$  for a GaInP/GaAs tandem under an approximately AM1.5 direct spectrum gave  $-3.9$  mV/°C, close to the calculated value, implying that  $n \approx 1$  for both subcells of this specific device.

### $J_{SC}$ TEMPERATURE COEFFICIENT

The dependence of the tandem  $J_{SC}$  temperature coefficient on the top and bottom cells and on temperature is more complex than the  $V_{OC}$  dependence. Figure 1 shows the top and bottom subcell currents  $J_{SCt}$  and  $J_{SCb}$  as a function of temperature for a tandem cell with a 1.15- $\mu\text{m}$ -thick top cell, calculated for the standard AM1.5 direct spectrum. (The effect of varying the spectrum will be discussed later.) For this spectrum and top-cell thickness,  $J_{SCt}$  is slightly less than  $J_{SCb}$  at 300 K. Because the top and bottom cells are in series, the tandem  $J_{SC}$  is limited to be the lesser of  $J_{SCt}$  and  $J_{SCb}$ . For the tandem illustrated in the figure, at 300 K the tandem is slightly top-cell current-limited. This is illustrated in the figure, which shows the tandem  $J_{SC}$  resulting from  $J_{SCt}$  and  $J_{SCb}$ .

Also shown is  $J_{SC}$  for a cell equivalent to a bottom cell but without a top cell filtering the incident light. The temperature variation  $dJ_{SCb}/dT$  of  $J_{SC}$  for the bottom cell as filtered by the top cell in the tandem structure is much smaller than for the unfiltered bottom cell. This difference is because the lowering of the tandem cell's bottom-cell

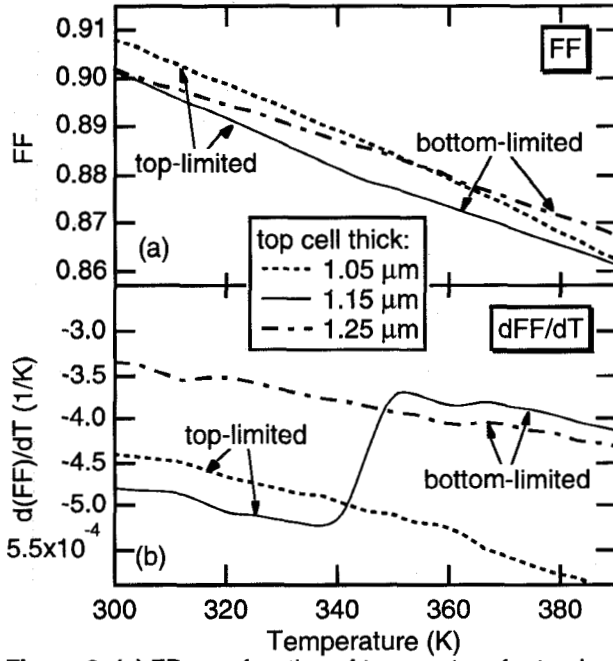


**Figure 1.** (a) Top and bottom subcell  $J_{SC}$ 's as a function of temperature for a tandem cell with a 1.15- $\mu\text{m}$ -thick top cell, calculated for the standard AM1.5 direct spectrum. Also shown is  $J_{SC}$  for an bottom cell unfiltered by a top cell. This unfiltered  $J_{SC}$  has been shifted down on the graph for easy comparison with the standard bottom-cell  $J_{SC}$ . (b) The resulting temperature coefficient  $dJ_{SC}/dT$  for the tandem cell.

bandgap with increasing temperature, which increases the amount of light the bottom cell absorbs, is partially compensated by the simultaneous lowering of the top-cell bandgap, which decreases the amount of light that is transmitted to the bottom cell. This effect has been observed experimentally for an AlGaAs-filtered GaAs cell [9]. The temperature dependence of the top cell, in contrast, is determined only by the temperature dependence of the top-cell band gap. As illustrated in Fig. 1, a tandem cell that is slightly top-cell current-limited at some starting temperature  $T_0$  will eventually become bottom-cell-limited at a crossover temperature  $T_x > T_0$ . As a result, as the temperature is raised through  $T_x$ , the tandem  $J_{SC}$  temperature coefficient  $dJ_{SC}/dT$  will abruptly change from  $dJ_{SC}/dT = dJ_{SCt}/dT$  to  $dJ_{SC}/dT = dJ_{SCb}/dT$ . This crossover occurs at  $T_x \approx 350$  K for the tandem cell illustrated in the figure.

### FF TEMPERATURE COEFFICIENT

The temperature coefficient  $dFF/dT$  of the tandem fill factor  $FF$  depends strongly on how close the top and bottom subcells are to current matching. Figure 2(a) shows  $FF$  calculated for GaInP/GaAs tandem cells with top-cell thicknesses of 1.05  $\mu\text{m}$ , 1.15  $\mu\text{m}$ , and 1.25  $\mu\text{m}$ . The tandem with the 1.05- $\mu\text{m}$  top cell is current-limited by the top cell in the temperature region shown in the figure, while the tandem with the 1.25- $\mu\text{m}$  top cell is current-limited by the bottom cell in this temperature region. However, as discussed in the preceding section, the tandem cell with the intermediate top-cell thickness of 1.15  $\mu\text{m}$  is top-cell-limited below the crossover temperature  $T_x \approx 350$  K, and becomes bottom-cell-limited above  $T_x$ .

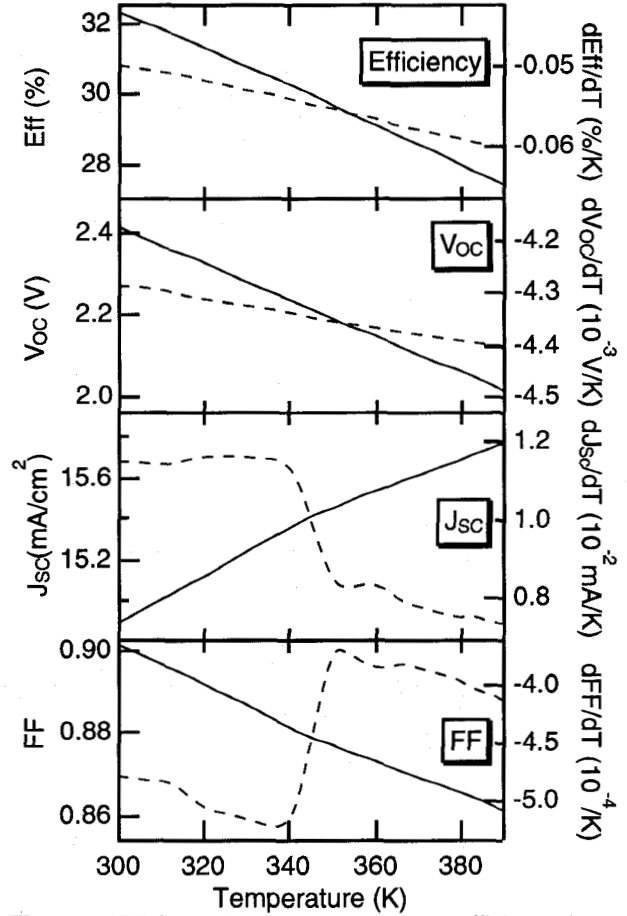


**Figure 2.** (a) FF as a function of temperature for tandem cells with top cells of various thicknesses, calculated for the standard AM1.5 direct spectrum. (b)  $d(FF)/dT$  for the FF's of panel (a).

For the tandems that are current-mismatched (either top-cell-limited or bottom-cell-limited), FF depends smoothly on temperature. However, for a device which passes through current-matching at a crossover temperature  $T_x$  (350 K for the tandem with the 1.15- $\mu\text{m}$  top cell in Fig. 2).  $dFF/dT$  changes in the region of  $T_x$ . This is because, for top and bottom cells with given fill factors, the tandem FF will be determined by the current-limiting subcell. Therefore, as the temperature is raised through the current-matching temperature  $T_x$ , the device transitions from top-cell-limited to bottom-cell-limited, and  $dFF/dT$  changes accordingly. Therefore,  $dFF/dT$  becomes less negative as the temperature is raised through  $T_x$ . This behavior is illustrated in Fig. 2(b), which shows the derivatives  $dFF/dT$  of the FF's of Fig 2(a).

#### EFFICIENCY

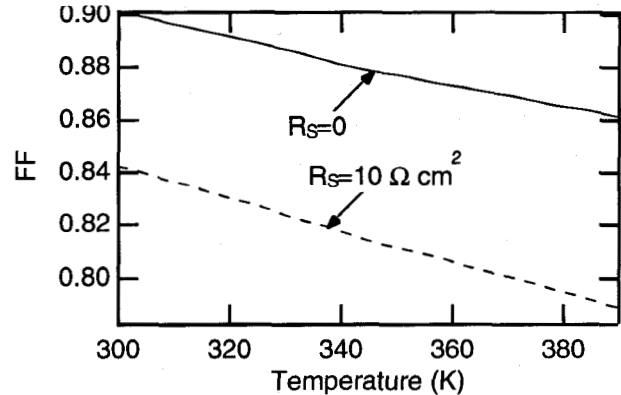
The efficiency is proportional to  $V_{OC}J_{SC}FF$ . Figure 3 shows FF,  $V_{OC}$ ,  $J_{SC}$ , and the resulting efficiency (solid lines, left axis) and their temperature derivatives (dashed lines, right axis) for a tandem with a 1.15- $\mu\text{m}$ -thick top cell. The increase in  $J_{SC}$  with increasing temperature is less than the decrease of  $V_{OC}FF$  with temperature, so the efficiency decreases with temperature. Both  $J_{SC}$  and FF show a small but perceptible change in their temperature dependencies as the temperature is raised through the point at which top and bottom cells are current-matched, as illustrated in Figs. 1 and 2 above. These changes are in opposite directions:  $dJ_{SC}/dT$  decreases, while  $dFF/dT$  increases, as the temperature is raised through the current-matching temperature. As a result the  $J_{SC}FF$  product, and hence the efficiency, do not show a significant change at the current-matching temperature.



**Figure 3.** FF,  $V_{OC}$ ,  $J_{SC}$ , and the resulting efficiency (solid lines, left axis) and their temperature derivatives (dashed lines, right axis) for a tandem with a 1.15- $\mu\text{m}$ -thick top cell, calculated for the standard AM1.5 direct spectrum.

#### EFFECT OF SERIES RESISTANCE

In this section we consider the effect of a finite series resistance  $R_S$ , which in all the other sections is neglected. This effect is modelled by adding a  $JR_S$  term to the ideal  $V(J)$  relation:  $V(J) = V_{R=0}(J) + JR_S$ . This  $R_S$  has no effect on  $V_{OC}$ . Furthermore, for high-fill-factor devices such as the

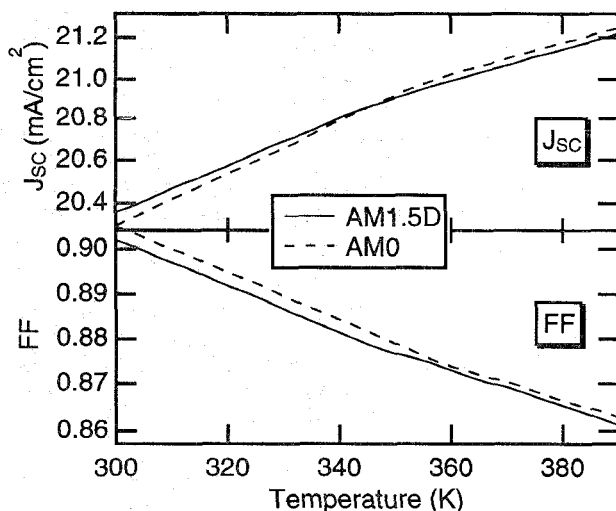


**Figure 4.** The effect of series resistance  $R_S$  on the temperature-dependent fill factor FF.

GaAs and GaInP cells, series resistance has a very small effect on  $J_{SC}$  as long as the series resistance does not dominate the J-V curve. Thus, the main effect of series resistance is on the fill factor FF. Figure 4 compares the temperature dependence of FF for  $R_S=0$  (as shown in Fig. 2) with FF calculated for  $R_S=10 \Omega \text{ cm}^2$ . The behavior of FF(T) is not qualitatively changed by the series resistance. The main difference is that the magnitude of the relative temperature coefficient  $(1/FF) d(FF)/dT$  is increased by the introduction of  $R_S$ .

### EFFECT OF SPECTRUM

As has been discussed above, the temperature dependence of  $J_{SC}$ ,  $J_{SC}(T)$  depends on the incident solar spectrum. This dependence in turn affects FF(T). Figure 5 shows  $J_{SC}(T)$  and FF(T) calculated for the AM1.5 direct spectrum compared with a calculation for the AM0 spectrum;  $V_{OC}(T)$  has been omitted because, as discussed above, its  $J_{SC}$  dependence and hence spectrum dependence is very weak. To make the spectrum comparison meaningful, the thicknesses of the top cells have been chosen so that in both cases, the devices are current-matched at approximately the same temperature of around 330 K. These thicknesses are 1.15  $\mu\text{m}$  for AM1.5 direct (as elsewhere in this paper) and 0.5  $\mu\text{m}$  for AM0. The figure shows that the qualitative behavior of the temperature coefficients is the same for the different solar spectra.



**Figure 5.**  $J_{SC}(T)$  and FF(T) calculated for the AM1.5 direct and AM0 spectra. Top cell thicknesses are 1.15  $\mu\text{m}$  for AM1.5 direct and 0.5  $\mu\text{m}$  for AM0. To facilitate comparison of the  $J_{SC}$  curves, the AM1.5-direct  $J_{SC}$  has been shifted upwards by 5.45  $\text{mA}/\text{cm}^2$  to overlay it with the AM0  $J_{SC}$ .

### CONCLUSIONS

The bottom-cell  $J_{SC}$  of the GaInP/GaAs series-connected tandem has a much smaller temperature variation than does an unfiltered GaAs cell. For this reason, the rate of increase of the full tandem  $J_{SC}$  with temperature becomes smaller as the temperature is raised through the crossover temperature at which the top and

bottom cells are current-matched. The transition from top-cell to bottom-cell-limited operation as the temperature is raised through the crossover temperature also results in a change in  $dFF/dT$  at the crossover temperature.  $V_{OC}$ , in contrast, remains essentially linear through the crossover temperature. The variations in  $dFF/dT$  and  $dJ_{SC}/dT$  at the crossover temperature cancel each other out when multiplied together, so that  $d(\text{Efficiency})/dT$  is a smooth, fairly linear function of T, even through the crossover temperature.

### ACKNOWLEDGMENTS

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