

Received April 17, 2021, accepted May 5, 2021, date of publication May 11, 2021, date of current version May 19, 2021.

Digital Object Identifier 10.1109/ACCESS.2021.3079161

Crystal Structure Algorithm (CryStAl): A Metaheuristic Optimization Method

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This work was supported in part by the University of Tabriz under Grant 1615, and in part by the University of Liverpool.

ABSTRACT Metaheuristics are computational procedures that intelligently lead the search process through the efficient exploration of the search space associated with an optimization problem. With the progressive outburst of problems with large data sets in various fields, there is an ongoing quest for enhancing existing metaheuristic algorithms as well as developing new ones with greater accuracy and efficiency. In general, a powerful and efficient metaheuristic algorithm is based on a rich inspiration source, implemented effectively through a precise mathematical model. Aiming to develop a highly efficient, nature-inspired optimization algorithm, here we propose a novel metaheuristic called Crystal Structure Algorithm (CryStAl). This method is chiefly inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points, which is a natural phenomenon that can be seen in the symmetric arrangement of constituents (i.e. atoms, molecules, or ions) in crystalline minerals such as quartz. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of this novel algorithm, 12 different classical and modern metaheuristic algorithms are selected from the literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly. The obtained results, further supported by a complete statistical analysis, demonstrated that the proposed algorithm is capable of providing very competitive results, outperforming the other metaheuristics in most cases.

INDEX TERMS Crystal Structure Algorithm (CryStAl), lattice, function, metaheuristic, optimization, statistical analysis.

I. INTRODUCTION

Many design problems in nature can be considered as optimization problems that demand appropriate optimization techniques and methods to be dealt with. Nowadays, design problems have become extremely complex for which classical optimization methods based on mathematical principles are incapable of providing satisfactory results in a reasonable period of time. Gradient-based methods, which utilize the gradient of the objective function for the configuration of the optimization problem, are a type of these mathematical methods. Over the past few decades, exploring the deficiencies of classical optimization methods and introducing new

The associate editor coordinating the review of this manuscript and approving it for publication was Emanuele Crisostomi^{ID}.

efficient optimization algorithms have been of great interest. Based on recent technological advances, there is a growing interest in introducing new optimization methods with enhanced efficiency, accuracy, and increased speed rate for tackling difficult optimization problems. Besides, some other concerns in dealing with some specific issues such as the local optima issues alongside the smoothness and convexity of the search spaces have been of great importance for a long period of time.

The presented concerns about the classical optimization algorithms have led optimization experts to a new methodology in solving different optimization problems called “Metaheuristic”. Glover [1] firstly proposed this term in 1986 which is comprised of the main word, i.e. heuristics, and a prefix, i.e. meta, which both have Greek origins. The

term “heuristic” comes from *heuriskein* which is an old Greek word meaning “to discover”, while “meta” means “beyond the normal or natural limits of something”. Metaheuristics are solution techniques that implement higher-level strategies into search processes in order to guide an optimization process to perform a powerful search into the search space with some special capabilities such as avoiding local optima.

As presented by Sørensen [2], the history of utilizing metaheuristics as the solution methods for dealing with real-world problems can generally be categorized into five distinct periods. In the first period which is named the “pre-theoretical” period (until 1940), there was not any formal presentation of heuristics and metaheuristics methods. Despite that, these methods had been used for solving some simple optimization problems in this period. In the second period which is from 1940 to 1980 and known as the “early” period, some studies were conducted on heuristics which was the first formal introduction and discussion in this field. In the third period which is called the “method-centric” period (1980 to 2000), multiple metaheuristics were proposed and developed for specific applications which extended the field of heuristics and metaheuristics. In the fourth period, which is from 2000 until now and known as the “framework-centric” period, the methodology of utilizing metaheuristics as frameworks alongside methods has been successfully presented with considerable growth of intuition in this field. In the fifth or last period which is named the “scientific” or “future” period, the design and introduction of new metaheuristics will turn into a matter of science rather than art. A summary of the abovementioned historical periods is presented in Table 1.

TABLE 1. Summary of the historical periods of metaheuristics evolution.

No.	Name	Duration	Details
1	Pre-theoretical	Until 1940	No formal presentation with limited applications.
2	Early	1940 to 1980	Heuristics were formally introduced and discussed.
3	Method-centric	1980 to 2000	Multiple metaheuristics were proposed and developed for specific applications.
4	Framework-centric	2000 to now	The methodologies of utilizing metaheuristics as frameworks alongside various methods have been successfully presented.
5	Scientific or future	Future	The design and introduction of new metaheuristics will turn into a matter of science rather than art.

Considering the development of various metaheuristic algorithms, four classifications can be made in terms of their inspiration. The first category is entitled “evolutionary algorithms” including the Memetic Algorithm (MA) [3], Genetic Algorithm (GA) [4], Differential Evolution (DE) [5], and the Evolution Strategies (ES) [6], which were developed based on the biological evolution and reproduction. The second category contains swarm intelligence-based algorithms that were formed based on the cooperative behavior of decentralized and self-organized natural or artificial systems. The Particle

Swarm Optimization (PSO) [7], Ant Colony Optimization (ACO) [8], Artificial Bee Colony (ABC) [9], Cat Swarm Optimization (CSA) [10], Firefly Algorithm (FA) [11], Krill Herd (KH) algorithm [12], and **Slap Swarm Algorithm** (SSA) [13] are some of the well-known methods in this category. The third category consists of algorithms motivated by physical laws. The Simulated Annealing (SA) [14], Magnetic Optimization Algorithm (MOA) [15], Gravitational Search Algorithm (GSA) [16], Charged System Search (CSS) algorithm [17], Ray Optimization Algorithm (ROA) [18], Colliding Bodies Optimization (CBO) [19], Multiverse Algorithm (MVO) [20], and the Sine Cosine Algorithm (SCA) [21] are some methods belonging to this category. Beyond these methods, some other metaheuristic algorithms were presented based on the lifestyle of humans and animals (the fourth category) such as the Harmony Search (HS) [22], Teaching–learning-based Optimization (TLBO) [23], **Creativity-Oriented Optimization Algorithm** (COOA) [24], **Human Behavior-Based Optimization** (HBBO) [25], and the Gaining Sharing Knowledge-based algorithm (GSK) [26]. In addition to these standard algorithms, some other challenges in developing, upgrading, or hybridizing standard algorithms have also been achieved [27]–[38]. A summary of these metaheuristic algorithms is presented in Table 2.

TABLE 2. Summary of the classification of the metaheuristic algorithms.

Classification	Algorithm	Year of Proposal
Evolution	Memetic Algorithm (MA) [3]	1989
	Genetic Algorithm (GA) [4]	1992
	Differential Evolution (DE) [5]	1997
	Evolution Strategies (ES) [6]	2002
Swarm intelligence	Particle Swarm Optimization (PSO) [7]	1995
	Ant Colony Optimization (ACO) [8]	1996
	Artificial Bee Colony (ABC) [9]	2006
	Cat swarm Optimization (CSA) [10]	2006
	Firefly Algorithm (FA) [11]	2010
	Krill Herd (KH) algorithm [12]	2012
	Slap Swarm Algorithm (SSA) [13]	2017
Physical laws	Simulated Annealing (SA) [14]	1983
	Magnetic Optimization Algorithm (MOA) [15]	2008
	Gravitational Search Algorithm (GSA) [16]	2009
	Charged System Search (CSS) algorithm [17]	2010
	Ray Optimization Algorithm (ROA) [18]	2012
	Colliding bodies Optimization (CBO) [19]	2014
	Multi-vers Algorithm (MVO) [20]	2016
	Sine Cosine Algorithm (SCA) [21]	2016
Lifestyle	Harmony Search (HS) [22]	2001
	Teaching–learning-based optimization (TLBO) [23]	2011
	Creativity-Oriented Optimization Algorithm (COOA) [24]	2015
	Human Behavior-Based Optimization (HBBO) [25]	2017
	Gaining Sharing Knowledge-based algorithm (GSK) [26]	2019

In this paper, a novel metaheuristic optimization method called Crystal Structure Algorithm (CryStAl) is proposed which is inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points. A total number of 239 mathematical functions which are categorized into four different groups are utilized to evaluate the overall performance of the proposed method. To validate the results of CryStAl, 12 different classical and modern metaheuristic algorithms are selected from the

literature. The minimum, mean, and standard deviation values alongside the number of function evaluations for CryStAl and the other metaheuristics for a specific tolerance are calculated and presented accordingly.

In general, the efficiency of novel metaheuristic algorithms in producing improved solutions to well-known optimization problems has been a significant research challenge for algorithm developers in recent decades. Considering the source of inspiration and the mathematical model as the two foundations of metaheuristic algorithms, this mission can generally be accomplished by utilizing solid mathematical models developed based on suitable inspirational concepts.

In this regard, this paper proposes CryStAl as a metaheuristic algorithm conceptualized based on the principles underlying the formation of crystal structures as a well-known physical paradigm in nature. This method is implemented using a fully-detailed mathematical model comprised of the details of crystalline configurations which have been established by crystallographers over the past few centuries. By developing a metaheuristic based on such a rich inspiration source followed by a precise mathematical model, we have shown that excellent results in dealing with different optimization problems can be achieved.

It should be also noted that the proposed approach, i.e. CryStAl, is a parameter-free metaheuristic algorithm in which there is no internal parameter to be determined throughout the optimization procedure. In other words, a notable feature of this algorithm is its parameter-free framework in which the exploitation and exploration phases of optimization are adjusted through the main loop of the algorithm. Besides, the position updating process of candidate solutions in this method is conducted in four separate phases in which the local and global searches of the entire search space are satisfied in a more precise way that results in excellent responses.

A summary of this paper is as follows. In section 2, the inspirational background of the proposed algorithm alongside the mathematical model of the new optimization algorithm is presented. In section 3, some mathematical functions with different characteristics are presented for further utilization in evaluating the proposed metaheuristic algorithm along with some other alternative approaches. In section 4, the selected alternative metaheuristic algorithms for comparative purposes are presented in detail. In section 5, the results of CryStAl alongside the other metaheuristics in dealing with mathematical test functions are presented. In section 6, a comprehensive statistical analysis is conducted to compare the results of the new algorithm with the other metaheuristic approaches. In section 7, the main findings of this paper including the conclusions alongside some suggestions for future challenges are presented accordingly.

II. CRYSTAL STRUCTURE ALGORITHM (CryStAl)

A. INSPIRATION

Solid minerals the constituent components (molecules, atoms, or ions) of which are regularly and repeatedly arranged

in three spatial directions or have a crystallographic order are called crystals. Crystalline solids are highly diverse and can have isotropic or anisotropic properties. The word crystal has Greek roots and means “frozen by cold”. They believed that if water was kept at very low temperatures for some time, it would become stable at high temperatures. “Crystal” is also an Arabic word derived from the Greek word “berlis” meaning emerald [39]. A representative example of a typical crystal is depicted in Fig. 1a.

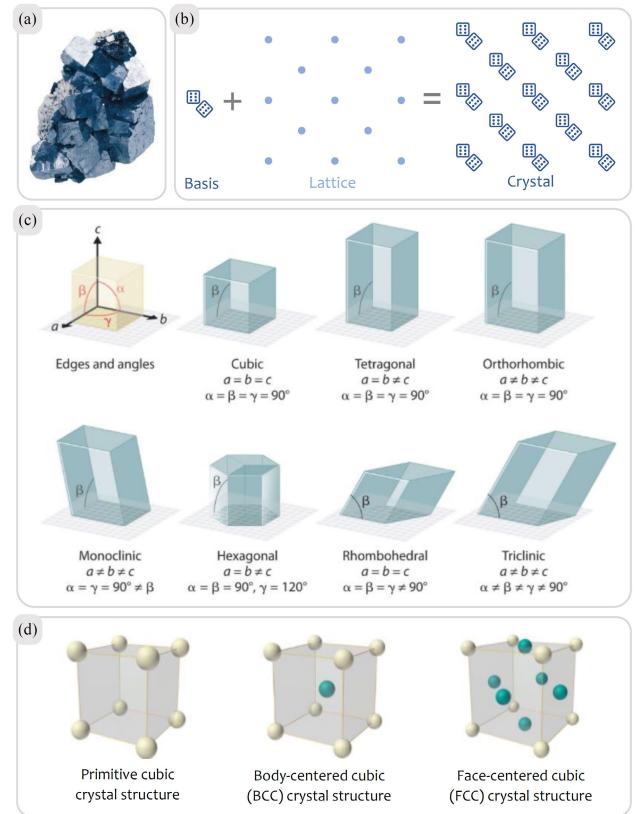


FIGURE 1. (a) An example of a natural crystal called Galena. (b) Definition of a crystal as a basis added to a lattice. (c) Various lattice configuration options. (d) Three common varieties of the cubic crystal system (Parts a, c, and d are adapted from [39]).

The earliest references to the regular arrangement of particles that make up crystals can be found in the works of Johannes Kepler in 1619 and Robert Hooke in 1665. Some time later in 1690, Christine Hogens studied the optical properties of calcite crystals and hypothesized that the crystals were made of very small particles with a definite shape. Since then, different physical and chemical formulations for crystals have been proposed and investigated experimentally [39]. Furthermore, crystals and their rich symmetries have inspired the conception and design of many man-made structures, mechanisms, and artworks [40]–[80].

The underlying component of a crystal is a “lattice” which represents a periodic array of points in predefined spaces, though it is not capable of defining the specific locations of atoms in the material. On the other hand, the location

of atoms in the structure of crystals is determined by the “basis” associated with each lattice point. Hence, crystals are determined by the combination of these two elements, i.e. the basis and the lattice, as illustrated in Fig. 1b.

Since the lattice determines only the overall shape of the crystal, different geometrical shapes can be composed considering the fact that infinite geometrical shapes are found in nature; however, here we consider some of the most well-known regular shapes, as represented in Fig. 1c.

For the basis, different configurations of atoms in the lattice can be considered in which the location of atoms can be in the corner points alongside other irregular patterns. In Fig. 1d, this aspect is represented in a simple cubic crystal system.

As a mathematical representation of these aspects should be defined for numerical investigations, the Bravais model [39] is considered in this paper for defining crystal configurations. In this model, a periodic crystal structure is defined by considering infinite lattice shape in which any lattice point is described by the location of their lattice point with a vector as follows:

$$\mathbf{r} = \sum n_i \mathbf{a}_i, \quad (1)$$

where n_i is an integer, \mathbf{a}_i is the shortest vector along the principal crystallographic directions, and i is the number of crystal corners.

B. MATHEMATICAL MODEL

In this section, the mathematical model of CryStAl is presented in which the basic concepts of crystals are utilized with necessary modifications. In this model, each candidate solution of the optimization algorithm is considered as a single crystal in the space. For iterative purposes, a number of crystals are randomly determined for initialization.

$$\begin{aligned} Cr &= \begin{bmatrix} Cr_1 \\ Cr_2 \\ \vdots \\ Cr_i \\ \vdots \\ Cr_n \end{bmatrix} \\ &= \begin{bmatrix} x_1^1 & x_1^2 & \dots & x_1^j & \dots & x_1^d \\ x_2^1 & x_2^2 & \dots & x_2^j & \dots & x_2^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_i^1 & x_i^2 & \dots & x_i^j & \dots & x_i^d \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n^1 & x_n^2 & \dots & x_n^j & \dots & x_n^d \end{bmatrix}, \\ &\quad \begin{cases} i = 1, 2, \dots, n \\ j = 1, 2, \dots, d \end{cases} \end{aligned} \quad (2)$$

where n is the number of crystals (i.e., candidate solutions) and d is the dimension of the problem. The initial positions of these crystals are randomly determined in the search space

as follows:

$$x_i^j(0) = x_{i,\min}^j + \xi(x_{i,\max}^j - x_{i,\min}^j), \quad \begin{cases} i = 1, 2, \dots, n \\ j = 1, 2, \dots, d \end{cases} \quad (3)$$

where $x_i^j(0)$ determines the initial position of the crystals; $x_{i,\min}^j$ and $x_{i,\max}^j$ are the minimum and maximum allowable values, respectively, for the j^{th} decision variable of the i^{th} candidate solution; and ξ is a random number in the interval $[0,1]$.

Based on the concept of ‘basis’ in crystallography, all the crystals at the corners are considered as the *main crystals*, Cr_{main} , determined randomly by considering the initially-created crystals (candidate solutions). It should be noted that the random selection process for each step is determined by omitting the current Cr . The crystal with the *best* configuration is determined as Cr_b while the mean values of randomly-selected crystals are denoted by F_c .

To update the positions of the candidate solutions in the search space, basic lattice principles are considered in which four kinds of updating process are determined as follows:

(i) Simple cubicle:

$$Cr_{\text{new}} = Cr_{\text{old}} + rCr_{\text{main}}, \quad (4)$$

(ii) Cubicle with the best crystals:

$$Cr_{\text{new}} = Cr_{\text{old}} + r_1 Cr_{\text{main}} + r_2 Cr_b, \quad (5)$$

(iii) Cubicle with the mean crystals:

$$Cr_{\text{new}} = Cr_{\text{old}} + r_1 Cr_{\text{main}} + r_2 F_c, \quad (6)$$

(iv) Cubicle with the best and mean crystals:

$$Cr_{\text{new}} = Cr_{\text{old}} + r_1 Cr_{\text{main}} + r_2 Cr_b + r_3 F_c, \quad (7)$$

where, in the four equations above, Cr_{new} is the new position, Cr_{old} is the old position, and r , r_1 , r_2 and r_3 are random numbers.

It should be mentioned that exploration and exploitation, as two critical features of metaheuristics, have been considered in this algorithm through (4) to (7) in which local and global searches are conducted simultaneously. In order to deal with the solution variables x_i^j violating the boundary conditions of the variables, a mathematical flag is defined in which for the x_i^j outside the variables range, the flag orders a boundary change for the violating variables. The terminating criterion is considered based on the maximum number of iterations in which the optimization process is terminated after a fixed number of iterations. The pseudo-code of the algorithm is presented in Fig. 2.

III. MATHEMATICAL TEST FUNCTIONS

In this section, a number of mathematical functions are selected to be utilized as test functions for the performance evaluation of the proposed algorithm. A total number of 239 mathematical functions are tested which are categorized into four different groups based on their specific

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procedure Crystal Structure Algorithm (CryStAl)
    Create random values for initial positions ( $x_i^j$ ) of initial
    crystals ( $Cr_i$ )
        Evaluate fitness values for each crystal
        while ( $t < \text{maximum number of iterations}$ )
            for  $i=1$ : number of initial crystals
                Create  $Cr_{main}$ 
                Create new crystals by Eq. 4
                Create  $Cr_b$ 
                Create new crystals by Eq. 5
                Create  $F_c$ 
                Create new crystals by Eq. 6
                Create new crystals by Eq. 7
                if new crystals violate boundary conditions
                    Control the position constraints for new crystals and
                    amend them
                end if
                Evaluate the fitness values for new crystals
                Update Global Best (GB) if a better solution is found
            end for
             $t = t + 1$ 
        end while
        Return GB
end procedure

```

FIGURE 2. The pseudo-code of the Crystal Structure Algorithm (CryStAl).

characteristics. These functions have been derived from various references [41]–[45] in which different mathematical functions with different characteristics had been reviewed and presented for utilization in the validation of novel metaheuristic algorithms.

In the first group, 117 mathematical functions are presented which have minimum and maximum dimensions of 2 and 10, respectively. Among these functions, which are named F_1 to F_{117} , the first 90 functions have 2 dimensions whereas the other 27 functions have dimensions of 3 to 10. In this paper, these functions are called the ‘two-dimensional (2D)’ test functions and are presented in Table 3. The second group of mathematical functions consists of 58 test functions in which the dimensions of functions are variable due to their specific formulations and are called the ‘ N -dimensional (ND)’ test functions. A maximum number of dimensions of 50 is considered in dealing with the functions of this group, called the 50-dimensional (50D) test functions, which are named F_{118} to F_{175} and presented in Table 4. For the third group, the mathematical functions of the second group are considered with the maximum dimension of 100 and are called the 100-dimensional (100D) test functions; these functions, named F_{175} to F_{233} , are presented in Table 5. For the fourth group, three composite and three hybrid mathematical functions are considered which are named F_{233} to F_{239} , presented in Table 6. In these tables, C, NC, D, ND, S, NS, Sc, NSc, U, and M denote Continuous, Non-Continuous, Differentiable, Non-Differentiable, Separable, Non-Separable, Scalable, Non-Scalable, Unimodal, and Multi-modal, respectively. Furthermore, R , D , and $Min.$ represent the variables

range, variables dimension, and the global minimum of the functions.

Based on the fact that a larger number of mathematical functions (239 functions) are considered in this paper, the 3D plots for some of these functions are presented in the following. The 3D plots for some of the 2D functions are shown in Fig. 3, while those of the 50D and 100D functions are depicted in Figs. 4 and 5, respectively. The complete mathematical formulations of these test functions are presented in Refs. [81]–[85].

IV. ALTERNATIVE METAHEURISTICS FOR COMPARISON

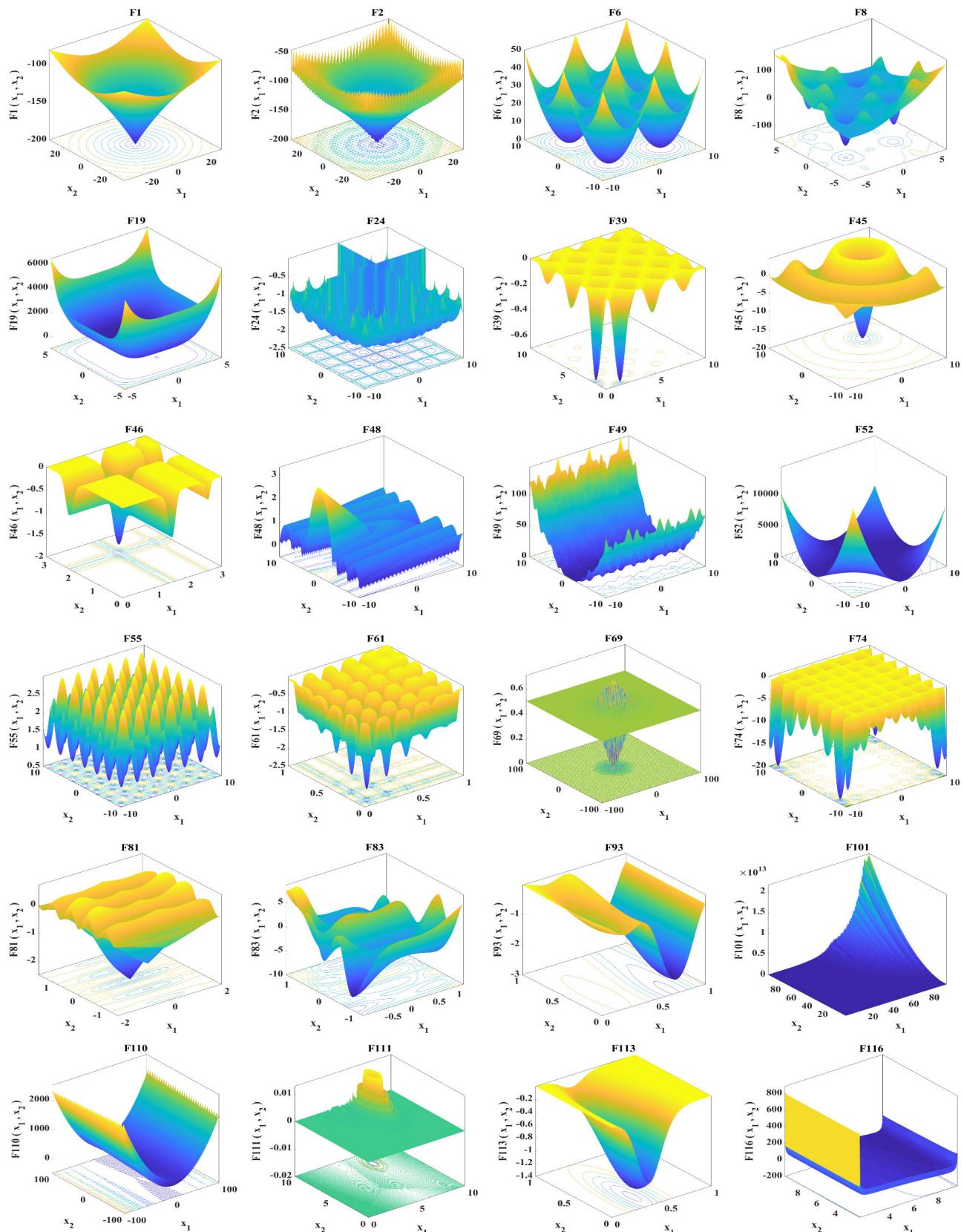
In order to evaluate the overall performance of the proposed algorithm, some different optimization algorithms are utilized as alternative approaches to provide a valid comparative study. The utilized metaheuristics for this purpose are the ABC, ACO, BA, FA, GA, HS, MFO, MVO, PSO, SA, SCA, and SSA. Based on the fact that some of the selected optimization algorithms are recently proposed or developed for special purposes, the most recent and improved versions of these algorithms are used in this paper. Knowing that the internal parameters of the optimization algorithms have the most vital role in their convergence performance, a parameter summary of the selected algorithms is presented in Table 7. The values of these parameters have been determined using the reference-based parameter identification process in which the internal parameters of these algorithms are selected based on relevant previously published research papers.

In many metaheuristic algorithms, some specific parameters are utilized for tuning the exploration and exploitation rates during the optimization process which are often problem-dependent parameters and so they should be tuned for each specific optimization problem. The mentioned parameters for the alternative algorithms in Table 7 were derived from the latest and most successful configurations of these algorithms available in the literature which resulted in acceptable optimum results in most of the previously considered optimization problems.

Knowing that such algorithms are potentially vulnerable to entrapment in local optima or even having convergence problems, we have proposed CryStAl as a simple algorithm without any internal or external parameters to be tuned. This characteristic can be considered as the major advantage of this algorithm over competing algorithms. In fact, as mentioned earlier in this section, CryStAl considers exploration and exploitation through (4) to (7) where local and global searches are performed simultaneously.

V. NUMERICAL RESULTS

In this section, the obtained results of the optimization run for CryStAl alongside the alternative metaheuristic approaches in dealing with the mathematical test functions are presented. The optimization problem is formulated with the maximum population size taken as 50 and the maximum number of Function Evaluations (FEs) selected to be 150000 for all of the metaheuristics. The maximum number of iterations in

**FIGURE 3.** The 3D plots of the 2D mathematical functions.

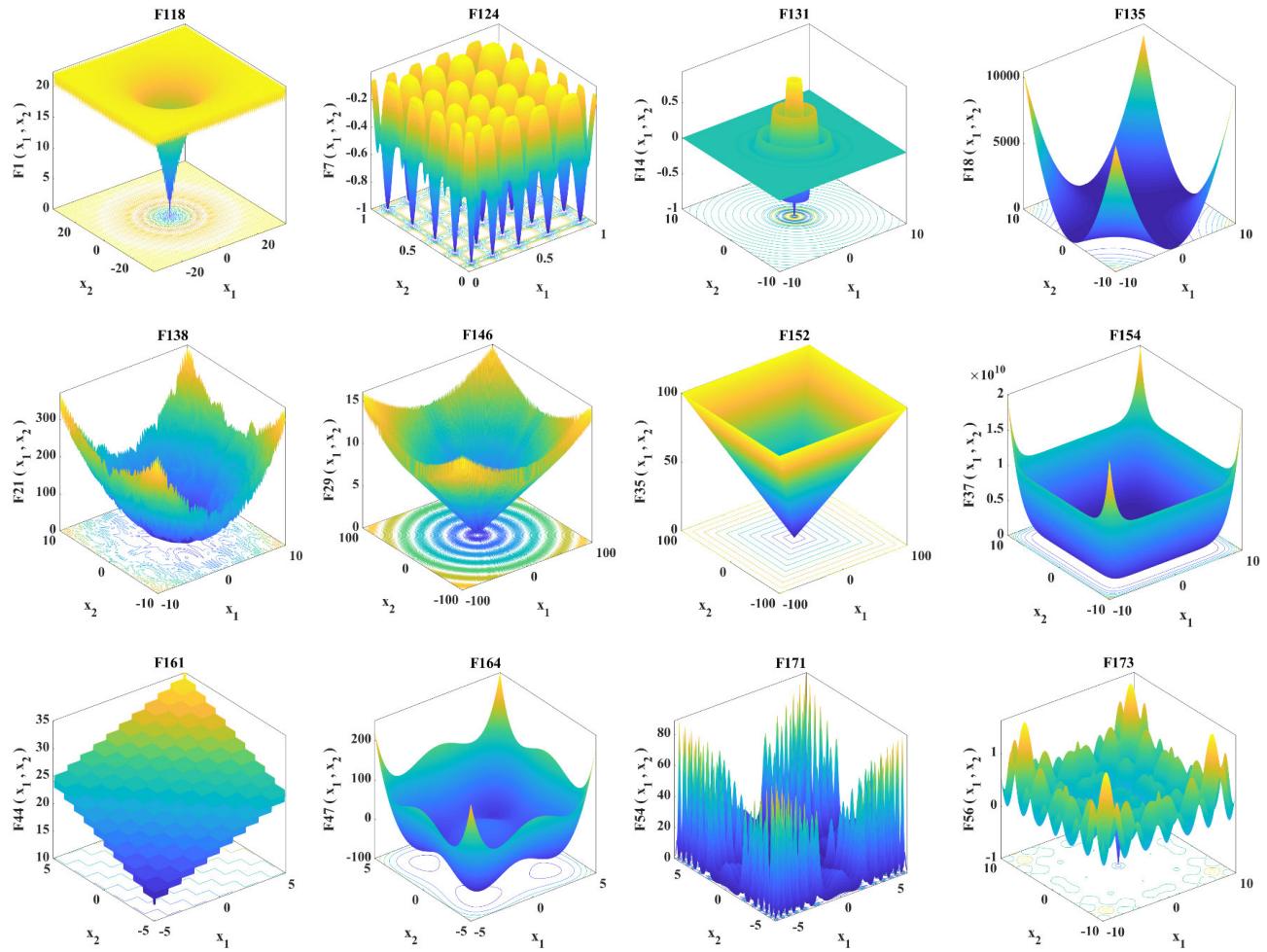


FIGURE 4. The 3D plots of the 50D mathematical functions.

each algorithm is adjusted based on the selected maximum number of FEs. As collecting quantitative results are of great importance in dealing with different optimization problems, CryStAl and the other algorithms are utilized 100 times with different initializations and the mean and standard deviation (std) of the best approximated solutions in the last iteration are reported. A tolerance of 1×10^{-12} is also considered for the convergence results of the algorithms in which the optimization runs are stopped at this tolerance of the Global Best (GB). It is assumed that the GB results are achieved by these optimization runs within this tolerance and the results of the GB are utilized instead of the final results of the optimization runs. The number of FEs are also calculated based on the selected tolerance. It should be noted that the above-mentioned is utilized as the stopping criterion in order to save time from a computational complexity perspective. In other words, if the algorithm reaches to this tolerance of the global best for the considered problem, the global best is reported as the final solution of the algorithm which requires less computational time. Therefore, the computational time for the considered 100 optimization runs will be reasonable.

Besides, the initial random state of each optimization run for each alternative algorithm has been selected equally in order to form a fair judgment about the performance of the proposed and alternative algorithms.

The detailed results of CryStAl and the other selected methods are presented in the Supplementary Materials which includes the convergence history of the proposed algorithm. It turned out that CryStAl can find the exact global results of 156 functions (65%); moreover, its result is very close to the global best result for 83 problems. Further investigations into the results of CryStAl compared to those of the other methods are performed in the next sections using some advanced statistical approaches. Moreover, the convergence curves of the proposed algorithm in dealing with some of the considered mathematical test functions are provided in the Supplementary Materials.

VI. STATISTICAL ANALYSIS

In this section, the maximum error values of the optimization convergence data have been calculated and utilized for statistical analysis. To this end, the difference between the Global

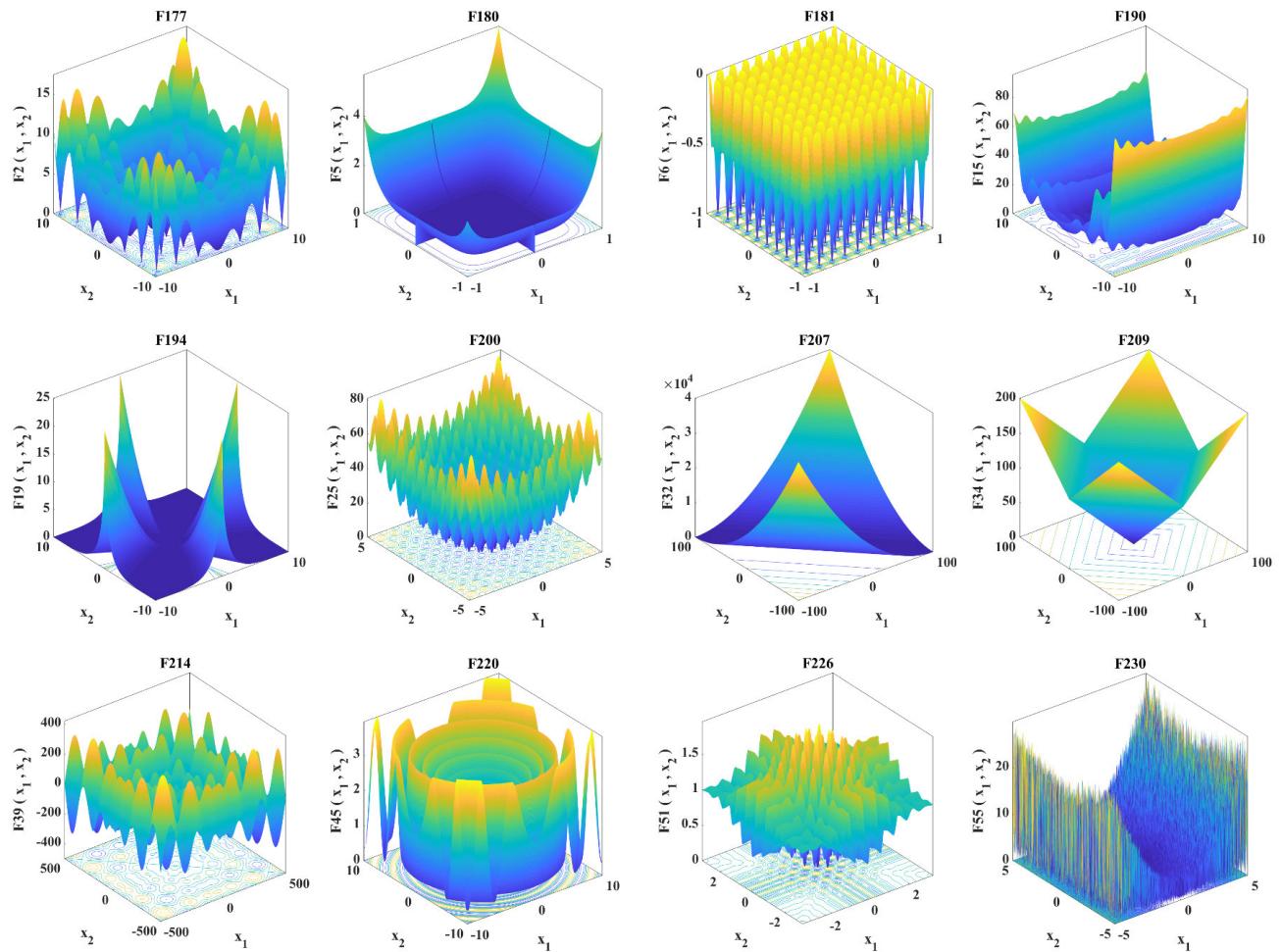


FIGURE 5. The 3D plots of the 100D mathematical functions.

Best (GB) of the functions and the obtained optimal values resulted from the optimization runs are considered as the error values. For statistical analysis purposes, four statistical tests have been conducted in which the Kolmogorov-Smirnov (K-S) test is utilized for normality issues, the Mann-Whitney U (M-W) test is implemented for comparing the summation of the ranks of different metaheuristics in a two-by-two comparing manner, the Kruskal-Wallis (K-W) test is conducted for comparing the overall rankings of the metaheuristics by considering the mean of their rankings, and the Post-Hoc (P-H) analysis is conducted based on the results of the K-W tests for further investigations.

A. KOLMOGOROV-SMIRNOV TEST

There are two kinds of statistical tests which are applicable to all of the obtained statistical data from multiple applications, known as the parametric and non-parametric statistical tests. One of the most important criteria which demonstrates the possibility of utilizing each method in a specific situation is the Kolmogorov-Smirnov test. This test shows that the distribution of data is either normal or non-normal in

which the distribution of each sample among the statistical data are considered and checked accordingly. If the K-S test is rejected, the data are normally distributed, and there is the possibility of using parametric statistical tests for the research. Conversely, if the K-S test is confirmed, the data do not have a normal distribution, so the nonparametric tests should be used in the study.

The results of the K-S test for the error values of the minimum, mean, standard deviation, and maximum function evaluations of the optimization runs for the 2D, 50D, and 100D functions are presented in Table 8. This test is conducted as a two-sample test in which the distributions of the CryStAl data are compared with the data obtained from other metaheuristics. It should be noted that if the Asymptotic Significance (Asymp. Sig.) value is less than 0.05, the presented data are not distributed normally, so the non-parametric statistical tests should be conducted for further investigations. The obtained results of the K-S test demonstrate that the Asymp. Sig. values in most of the investigated cases are less than 0.05, so the non-parametric statistical tests should be utilized for further considerations.

TABLE 3. Details of the 2D to 10D mathematical functions (First group).

No.	Name	Type	R	D	Min.	No.	Name	Type	R	D	Min.
F ₁	Ackley 2	C, D, NS, Sc, M	[-35, 35]	2	-200	F ₆₁	Ripple 1	NS	[0, 1]	2	-2.2
F ₂	Ackley 3	C, D, NS, NSc, U	[-32, 32]	2	-195.629	F ₆₂	Ripple 25	NS	[0, 1]	2	-2
F ₃	Adjiman	C, D, NS, NSc, M	[-1, 2] & [-1, 1]	2	-2.02181	F ₆₃	Rosenbrock Modified	C, D, NS, NSc, M	[-2, 2]	2	34.3712
F ₄	Bartels Conn	C, ND, NS, NSc, M	[-500, 500]	2	1	F ₆₄	Rotated Ellipse	C, D, NS, NSc, U	[-500, 500]	2	0
F ₅	Beale	C, D, NS, NSc, U	[-4.5, 4.5]	2	0	F ₆₅	Rotated Ellipse 2	C, D, NS, NSc, U	[-500, 500]	2	0
F ₆	Becker-Lago	S	[-10, 10]	2	0	F ₆₆	Rump	C, D, NS, NSc, U	[-500, 500]	2	0
F ₇	Biggs EXP2	C, D, NS, NSc, M	[0, 20]	2	0	F ₆₇	Scahffer 1	C, D, NS, NSc, U	[-100, 100]	2	0
F ₈	Bird	C, D, NS, NSc, M	[-2π, π]	2	-106.765	F ₆₈	Scahffer 2	C, D, NS, NSc, U	[-100, 100]	2	0
F ₉	Bohachevsky 1	C, D, S, NSc, M	[-100, 100]	2	0	F ₆₉	Scahffer 3	C, D, NS, NSc, U	[-100, 100]	2	0.001567
F ₁₀	Bohachevsky 2	C, D, S, NSc, M	[-100, 100]	2	0	F ₇₀	Scahffer 4	C, D, NS, NSc, U	[-100, 100]	2	0.292579
F ₁₁	Bohachevsky 3	C, D, NS, NSc, M	[-100, 100]	2	0	F ₇₁	Schwefel 2.6	C, D, NS, NSc, U	[-100, 100]	2	0
F ₁₂	Booth	C, D, NS, NSc, U	[-10, 10]	2	0	F ₇₂	Schwefel 2.36	C, D, S, Sc, M	[0, 500]	2	-3456
F ₁₃	Branin RCOS	C, D, NS, NSc, M	[-5, 10] & [0, 15]	2	0.397887	F ₇₃	Table 1 / Holder Table 1	C, D, S, NSc, M	[-10, 10]	2	-26.9203
F ₁₄	Branin RCOS 2	C, D, NS, NSc, M	[-5, 15]	2	5.559037	F ₇₄	Table 2 / Holder Table 2	C, D, S, NSc, M	[-10, 10]	2	-19.2085
F ₁₅	Brent	C, D, NS, NSc, U	[-10, 10]	2	0	F ₇₅	Table 3 / Carrom Table	C, D, NS, NSc, M	[-10, 10]	2	-24.1568
F ₁₆	Bukin 4	C, ND, S, NSc, M	[-15, -5] & [-3, 3]	2	0	F ₇₆	Testtube Holder	C, D, S, NSc, M	[-10, 10]	2	-10.8723
F ₁₇	Bukin 6	C, ND, NS, NSc, M	[-15, -5] & [-3, 3]	2	0	F ₇₇	Trecanni	C, D, S, NSc, U	[-5, 5]	2	0
F ₁₈	Camel - 3 Hump	C, D, NS, NSc, M	[-5, 5]	2	0	F ₇₈	Trefethen	C, D, NS, NSc, M	[-10, 10]	2	-3.30687
F ₁₉	Camel - 6 Hump	C, D, NS, NSc, M	[-5, 5]	2	-1.0316	F ₇₉	Tripod	C, D, NS, NSc, M	[-100, 100]	2	0
F ₂₀	Carrom table	NS	[-10, 10]	2	-24.1568	F ₈₀	Ursem 1	S	[-2.5, 3] & [-2, 2]	2	-4.81681
F ₂₁	Chen Bird	C, D, NS, NSc, M	[-500, 500]	2	-2000	F ₈₁	Ursem 3	NS	[-2, 2] & [-1.5, 1.5]	2	-2.5
F ₂₂	Chen V	C, D, NS, NSc, M	[-500, 500]	2	-2000	F ₈₂	Ursem 4	NS	[-2, 2]	2	-1.5
F ₂₃	Chichinadze	C, D, S, NSc, M	[-30, 30]	2	-42.9444	F ₈₃	Ursem Waves	NS	[-0.9, 1.2] & [-1.2, 1.2]	2	-7.307
F ₂₄	Cross-in-Tray	C, NS, NSc, M	[-10, 10]	2	-2.06261	F ₈₄	Venter Sobiezczanski-	C, D, S, NSc	[-50, 50]	2	-400
F ₂₅	Cube	C, D, NS, NSc, U	[-10, 10]	2	0	F ₈₅	Wayburn Seader 1	C, D, NS, Sc, U	[-500, 500]	2	0
F ₂₆	Damavandi	C, D, NS, NSc, M	[0, 14]	2	0	F ₈₆	Wayburn Seader 2	C, D, NS, Sc, U	[-500, 500]	2	0
F ₂₇	Deckkers-Aarts	C, D, NS, NSc, M	[-20, 20]	2	-24771.1	F ₈₇	Wayburn Seader 3	C, D, NS, Sc, U	[-500, 500]	2	21.35
F ₂₈	Easom	C, D, S, NSc, M	[-100, 100]	2	-1	F ₈₈	Zettl	C, D, NS, NSc, U	[-5, 10]	2	-0.00379
F ₂₉	El-Attar-Vidyasagar-Dutta	C, D, NS, NSc, M	[-500, 500]	2	1.7128	F ₈₉	Zirilli or Aluffi-Pentini	C, D, S, NSc, U	[-10, 10]	2	-0.3523
F ₃₀	Egg Crate	C, D, NS, Sc, M	[-5, 5]	2	0	F ₉₀	Zirilli 2	C, D, S, M	[-500, 500]	2	0
F ₃₁	Exp 2	S	[0, 20]	2	0	F ₉₁	Biggs EXP3	C, D, NS, NSc, M	[0, 20]	3	0
F ₃₂	Freudenstein Roth	C, D, NS, NSc, M	[-10, 10]	2	0	F ₉₂	Gulf Research Problem	C, D, NS, NSc, M	[0.1, 100] & [0, 25.6]	3	0
F ₃₃	Giunta	C, D, S, Sc, M	[-1, 1]	2	0.060447	F ₉₃	Hartman 3	C, D, NS, NSc, M	[0, 1]	3	-3.86278
F ₃₄	Goldstein Price	C, D, NS, NSc, M	[-2, 2]	2	3	F ₉₄	Helical Valley	C, D, NS, Sc, M	[-10, 10]	3	0
F ₃₅	Hansen	C, D, S, NSc, M	[-10, 10]	2	-165.953	F ₉₅	Meyer-Roth	NS	[0, 1]	3	4.00E-05
F ₃₆	Himmelblau	C, D, S, NSc, M	[-5, 5]	2	0	F ₉₆	Mishra 9	C, D, NS, NSc, M	[-10, 10]	3	0
F ₃₇	Hosaki	C, D, NS, NSc, M	[0, 5] & [0, 6]	2	-2.3458	F ₉₇	Wolfe	C, D, S, Sc, M	[0, 2]	3	0
F ₃₈	Jennrich-Sampson	C, D, NS, NSc, M	[-1, 1]	2	124.3612	F ₉₈	Biggs EXP4	C, D, NS, NSc, M	[0, 20]	4	0
F ₃₉	Keane	C, D, NS, NSc, M	[0, 10]	2	-0.67367	F ₉₉	Colville	C, D, NS, NSc, M	[-10, 10]	4	0
F ₄₀	Leon	C, D, NS, NSc, U	[-1.2, 1.2]	2	0	F ₁₀₀	Corana	DC, ND, S, Sc, M	[-500, 500]	4	0
F ₄₁	Levy 3	S	[-10, 10]	2	-176.542	F ₁₀₁	DeVilliers Glasser 1	C, D, NS, NSc, M	[1, 100]	4	0
F ₄₂	Levy 5	NS	[-10, 10]	2	-176.138	F ₁₀₂	Gear	NS	[12, 60]	4	2.70E-12
F ₄₃	Matyas	C, D, NS, NSc, U	[-10, 10]	2	0	F ₁₀₃	Kowalik	NS	[-5, 5]	4	0.000308
F ₄₄	McCormick	C, D, NS, NSc, M	[-1.5, 4] & [-3, 3]	2	-1.9133	F ₁₀₄	Miele Cantrell	C, D, NS, NSc, M	[-1, 1]	4	0
F ₄₅	Mexican hat	NS	[-10, 10]	2	-19.6683	F ₁₀₅	Shekel 5	C, D, NS, Sc, M	[0, 10]	4	-10.1532
F ₄₆	Michalewicz 2	S	[0, π]	2	-1.8013	F ₁₀₆	Shekel 7	C, D, NS, Sc, M	[0, 10]	4	-10.4029
F ₄₇	Mishra 3	C, D, NS, NSc, M	[-10, 10]	2	-0.18465	F ₁₀₇	Shekel 10	C, D, NS, Sc, M	[0, 10]	4	-10.5364
F ₄₈	Mishra 4	C, D, NS, NSc, M	[-10, 10]	2	-0.19941	F ₁₀₈	Biggs EXP5	C, D, NS, NSc, M	[0, 20]	5	0
F ₄₉	Mishra 5	C, D, NS, NSc, M	[-10, 10]	2	-1.01983	F ₁₀₉	DeVilliers Glasser 2	C, D, NS, NSc, M	[1, 60]	5	0
F ₅₀	Mishra 6	C, D, NS, NSc, M	[-10, 10]	2	-2.28395	F ₁₁₀	Dolan	C, D, NS, NSc, M	[-100, 100]	5	-529.871
F ₅₁	Mishra 8	C, D, NS, NSc, M	[-10, 10]	2	0	F ₁₁₁	Langerman-5	C, D, NS, Sc, M	[0, 10]	5	-0.965
F ₅₂	Mishra 10	C, D, NS, NSc, M	[-10, 10]	2	0	F ₁₁₂	Biggs EXP6	C, D, NS, NSc, M	[-20, 20]	6	0
F ₅₃	Parsopoulos	C, D, S, Sc, M	[-5, 5]	2	0	F ₁₁₃	Hartman 6	C, D, NS, NSc, M	[0, 1]	6	-3.32236
F ₅₄	Pen Holder	C, D, NS, NSc, M	[-11, 11]	2	-0.96354	F ₁₁₄	Trid 6	C, D, NS, NSc, M	[36, 36]	6	-50
F ₅₅	Periodic	S	[-10, 10]	2	0.9	F ₁₁₅	Ann-XOR	NS	[-1, 1]	9	0.95979
F ₅₆	Price 1	C, ND, S, NSc, M	[-500, 500]	2	0	F ₁₁₆	Paviani	C, D, NS, Sc, M	[2,0001, 10]	10	-45.778
F ₅₇	Price 2	C, D, NS, NSc, M	[-10, 10]	2	0.9	F ₁₁₇	Trid 10	C, D, NS, NSc, M	[-100, 100]	10	-210
F ₅₈	Price 3	C, D, NS, NSc, M	[-500, 500]	2	0						
F ₅₉	Price 4	C, D, NS, NSc, M	[-500, 500]	2	0						
F ₆₀	Quadratic	C, D, NS, NSc	[-10, 10]	2	-3873.72						

In Table 9, the maximum difference between the statistical data of CryStAl and the other metaheuristics are also presented in order to have an initial judgment about the obtained results of the new algorithm. The maximum and minimum differences of CryStAl with the alternative algorithms are represented by bold font-weight and underlined font, respectively. The bolded values designate those algorithms which have the maximum difference with CryStAl among other metaheuristics, while the underlined values show the algorithms which have the minimum difference with CryStAl among other metaheuristics.

B. MANN-WHITNEY U TEST

The Mann-Whitney U (M-W) test is a non-parametric test that allows two groups of data to be compared in which the null hypothesis denotes that it is equally likely that a randomly-selected value from one sample will be less than or greater than a randomly-selected value from a second sample. This test can be used to investigate whether two independent samples were selected from populations having the same distribution. This test provides the summation of the ranks for two sets of statistical data considered for comparative analysis. As an essential criterion, if the summation of the

TABLE 4. Details of the 50D mathematical functions (Second group).

No.	Name	Type	R	D	Min.
F ₁₁₈	Ackley 1	C, D, NS, Sc, M	[-35, 35]	50	0
F ₁₁₉	Alpine 1	C, ND, S, NSc, U	[-10, 10]	50	0
F ₁₂₀	Brown	C, D, NS, Sc, U	[-1, 4]	50	0
F ₁₂₁	Chung Reynolds	C, D, PS, Sc, U	[-100, 100]	50	0
F ₁₂₂	Csendes	C, D, S, Sc, M	[-1, 1]	50	0
F ₁₂₃	Deb 1	C, D, S, Sc, M	[-1, 1]	50	-1
F ₁₂₄	Deb 3	C, D, S, Sc, M	[0, 1]	50	-1
F ₁₂₅	Dixon & Price	C, D, NS, Sc, U	[-10, 10]	50	0
F ₁₂₆	Extended Easom	C, D, S, NSc, M	[-2π, 2π]	50	-1
F ₁₂₇	Exponential	C, D, NS, Sc, M	[-1, 1]	50	-1
F ₁₂₈	Griewank	C, D, NS, Sc, M	[-100, 100]	50	0
F ₁₂₉	Holzman 2	S	[-10, 10]	50	0
F ₁₃₀	Hyper-ellipsoid	C, U	[-500, 500]	50	0
F ₁₃₁	Inverted cosine wave	NS	[-10, 10]	50	-49
F ₁₃₂	Levy 8	NS	[-10, 10]	50	0
F ₁₃₃	Mishra 1	C, D, NS, Sc, M	[0, 1]	50	2
F ₁₃₄	Mishra 2	C, D, NS, Sc, M	[0, 1]	50	2
F ₁₃₅	Mishra 7	C, D, NS, NSc, M	[-10, 10]	50	0
F ₁₃₆	Mishra 11	C, D, NS, NSc, M	[-10, 10]	50	0
F ₁₃₇	Pathological	C, D, NS, NSc, M	[-100, 100]	50	0
F ₁₃₈	Pint er	C, D, NS, Sc, M	[-10, 10]	50	0
F ₁₃₉	Powell Singular	C, D, NS, Sc, U	[-4, 5]	50	0
F ₁₄₀	Powell Singular 2	C, D, NS, Sc, U	[-4, 5]	50	0
F ₁₄₁	Powell Sum	C, D, S, Sc, U	[-1, 1]	50	0
F ₁₄₂	Rastrigin	C, D, S, M	[-5.12, 5.12]	50	0
F ₁₄₃	Qing	C, D, S, Sc, M	[-500, 500]	50	0
F ₁₄₄	Quintic	C, D, S, NSc, M	[-10, 10]	50	0
F ₁₄₅	Rosenbrock	C, D, NS, Sc, U	[-30, 30]	50	0
F ₁₄₆	Salomon	C, D, NS, Sc, M	[-100, 100]	50	0
F ₁₄₇	Schumer Steiglitz	C, D, S, Sc, U	[-100, 100]	50	0
F ₁₄₈	Schwefel	C, D, PS, Sc, U	[-100, 100]	50	0
F ₁₄₉	Schwefel 1.2	C, D, NS, Sc, U	[-100, 100]	50	0
F ₁₅₀	Schwefel 2.4	C, D, S, NSc, M	[0, 10]	50	0
F ₁₅₁	Schwefel 2.20	C, ND, S, Sc, U	[-100, 100]	50	0
F ₁₅₂	Schwefel 2.21	C, ND, S, Sc, U	[-100, 100]	50	0
F ₁₅₃	Schwefel 2.22	C, D, NS, Sc, U	[-100, 100]	50	0
F ₁₅₄	Schwefel 2.23	C, D, NS, Sc, U	[-10, 10]	50	0
F ₁₅₅	Schwefel 2.25	C, D, S, NSc, M	[0, 10]	50	0
F ₁₅₆	Schwefel 2.26	C, D, S, Sc, M	[-500, 500]	50	-418.98
F ₁₅₇	Sphere	C, D, S, Sc, M	[0, 10]	50	0
F ₁₅₈	Step	DC, ND, S, Sc, U	[-100, 100]	50	0
F ₁₅₉	Step 2	DC, ND, S, Sc, U	[-100, 100]	50	0
F ₁₆₀	Step 3	DC, ND, S, Sc, U	[-100, 100]	50	0
F ₁₆₁	Stepint	DC, ND, S, Sc, U	[-5.12, 5.12]	50	-275
F ₁₆₂	Stretched V Sine Wave	C, D, NS, Sc, U	[-10, 10]	50	0
F ₁₆₃	Sum Squares	C, D, S, Sc, U	[-10, 10]	50	0
F ₁₆₄	Styblinski-Tang	C, D, NS, NSc, M	[-5, 5]	50	-1958.3
F ₁₆₅	Trid	C, D, NS, NSc, U	[-D^2, D^2]	50	-22050
F ₁₆₆	Trigonometric 1	C, D, NS, Sc, M	[0, π]	50	0
F ₁₆₇	Trigonometric 2	C, D, NS, Sc, M	[-500, 500]	50	1
F ₁₆₈	W / Wavy	C, D, S, Sc, M	[-π, π]	50	0
F ₁₆₉	Xin-She Yang (1)	DC, ND, NS, Sc, M	[-20, 20]	50	-1
F ₁₇₀	Xin-She Yang (2)	DC, ND, NS, Sc, M	[-10, 10]	50	0
F ₁₇₁	Xin-She Yang (3)	DC, ND, NS, Sc, M	[-2π, 2π]	50	0
F ₁₇₂	Xin-She Yang (4)	DC, ND, NS, Sc, M	[-5, 5]	50	0
F ₁₇₃	Xin-She Yang (5)	DC, ND, NS, Sc, M	[-10, 10]	50	-1
F ₁₇₄	Xin-She Yang (6)	DC, ND, NS, Sc, M	[-5, 5]	50	0
F ₁₇₅	Zakharov	C, D, NS, Sc, M	[-5, 10]	50	0

ranks for one sample has lower values than the other one, the one with a smaller sum of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the M-W test for different mathematical functions based on the obtained results of the optimization runs are presented in Tables 10 to 12. In these tables, the upper and lower values are the summation of the ranks related to the alternative metaheuristics and CryStAl, respectively. Based on the statistical results, the related values of CryStAl for the summation of the ranks in most cases are lower than those of the other metaheuristics (bolded values in the table) which demonstrates the superiority of CryStAl to its competitors in dealing with optimization functions.

TABLE 5. Details of the 100D mathematical functions (Third group).

No.	Name	Type	R	D	Min.
F ₁₇₆	Ackley 1	C, D, NS, Sc, M	[-35, 35]	100	0
F ₁₇₇	Alpine 1	C, ND, S, NSc, U	[-10, 10]	100	0
F ₁₇₈	Brown	C, D, NS, Sc, U	[-1, 4]	100	0
F ₁₇₉	Chung Reynolds	C, D, PS, Sc, U	[-100, 100]	100	0
F ₁₈₀	Csendes	C, D, S, Sc, M	[-1, 1]	100	0
F ₁₈₁	Deb 1	C, D, S, Sc, M	[-1, 1]	100	-1
F ₁₈₂	Deb 3	C, D, S, Sc, M	[0, 1]	100	-1
F ₁₈₃	Dixon & Price	C, D, NS, Sc, U	[-10, 10]	100	0
F ₁₈₄	Extended Easom	C, D, S, NSc, M	[-2π, 2π]	100	-1
F ₁₈₅	Exponential	C, D, NS, Sc, M	[-1, 1]	100	-1
F ₁₈₆	Griewank	C, D, NS, Sc, M	[-100, 100]	100	0
F ₁₈₇	Holzman 2	S	[-10, 10]	100	0
F ₁₈₈	Hyper-ellipsoid	C, U	[-500, 500]	100	0
F ₁₈₉	Inverted cosine wave	NS	[-10, 10]	100	-99
F ₁₉₀	Levy 8	NS	[-10, 10]	100	0
F ₁₉₁	Mishra 1	C, D, NS, Sc, M	[0, 1]	100	2
F ₁₉₂	Mishra 2	C, D, NS, Sc, M	[0, 1]	100	2
F ₁₉₃	Mishra 7	C, D, NS, NSc, M	[-10, 10]	100	0
F ₁₉₄	Mishra 11	C, D, NS, NSc, M	[-10, 10]	100	0
F ₁₉₅	Pathological	C, D, NS, NSc, M	[-100, 100]	100	0
F ₁₉₆	Pint er	C, D, NS, Sc, M	[-10, 10]	100	0
F ₁₉₇	Powell Singular	C, D, NS, Sc, U	[-4, 5]	100	0
F ₁₉₈	Powell Singular 2	C, D, NS, Sc, U	[-4, 5]	100	0
F ₁₉₉	Powell Sum	C, D, S, Sc, U	[-1, 1]	100	0
F ₂₀₀	Rastrigin	C, D, S, M	[-5.12, 5.12]	100	0
F ₂₀₁	Qing	C, D, S, Sc, M	[-500, 500]	100	0
F ₂₀₂	Quintic	C, D, S, NSc, M	[-10, 10]	100	0
F ₂₀₃	Rosenbrock	C, D, NS, Sc, U	[-30, 30]	100	0
F ₂₀₄	Salomon	C, D, NS, Sc, M	[-100, 100]	100	0
F ₂₀₅	Schumer Steiglitz	C, D, S, Sc, U	[-100, 100]	100	0
F ₂₀₆	Schwefel	C, D, PS, Sc, U	[-100, 100]	100	0
F ₂₀₇	Schwefel 1.2	C, D, NS, Sc, U	[-100, 100]	100	0
F ₂₀₈	Schwefel 2.4	C, D, S, NSc, M	[0, 10]	100	0
F ₂₀₉	Schwefel 2.20	C, ND, S, Sc, U	[-100, 100]	100	0
F ₂₁₀	Schwefel 2.21	C, ND, S, Sc, U	[-100, 100]	100	0
F ₂₁₁	Schwefel 2.22	C, D, NS, Sc, U	[-100, 100]	100	0
F ₂₁₂	Schwefel 2.23	C, D, NS, Sc, U	[-10, 10]	100	0
F ₂₁₃	Schwefel 2.25	C, D, S, NSc, M	[0, 10]	100	0
F ₂₁₄	Schwefel 2.26	C, D, S, Sc, M	[-500, 500]	100	-418.98
F ₂₁₅	Sphere	C, D, S, Sc, M	[0, 10]	100	0
F ₂₁₆	Step	DC, ND, S, Sc, U	[-100, 100]	100	0
F ₂₁₇	Step 2	DC, ND, S, Sc, U	[-100, 100]	100	0
F ₂₁₈	Step 3	DC, ND, S, Sc, U	[-100, 100]	100	0
F ₂₁₉	Stepint	DC, ND, S, Sc, U	[-5.12, 5.12]	100	-575
F ₂₂₀	Stretched V Sine Wave	C, D, NS, Sc, U	[-10, 10]	100	0
F ₂₂₁	Sum Squares	C, D, S, Sc, U	[-10, 10]	100	0
F ₂₂₂	Styblinski-Tang	C, D, NS, NSc, M	[-5, 5]	100	-3916.6
F ₂₂₃	Trid	C, D, NS, NSc, U	[-D^2, D^2]	100	-171600
F ₂₂₄	Trigonometric 1	C, D, NS, Sc, M	[0, π]	100	0
F ₂₂₅	Trigonometric 2	C, D, NS, Sc, M	[-500, 500]	100	1
F ₂₂₆	W / Wavy	C, D, S, Sc, M	[-π, π]	100	0
F ₂₂₇	Xin-She Yang (1)	DC, ND, NS, Sc, M	[-20, 20]	100	-1
F ₂₂₈	Xin-She Yang (2)	DC, ND, NS, Sc, M	[-10, 10]	100	0
F ₂₂₉	Xin-She Yang (3)	DC, ND, NS, Sc, M	[-2π, 2π]	100	0
F ₂₃₀	Xin-She Yang (4)	DC, ND, NS, Sc, M	[-5, 5]	100	0
F ₂₃₁	Xin-She Yang (5)	DC, ND, NS, Sc, M	[-10, 10]	100	-1
F ₂₃₂	Xin-She Yang (6)	DC, ND, NS, Sc, M	[-5, 5]	100	0
F ₂₃₃	Zakharov	C, D, NS, Sc, M	[-5, 10]	100	0

C. KRUSKAL-WALLIS TEST

The Kruskal-Wallis (K-W) test is a non-parametric method for testing whether or not different statistical samples are originated from the same distribution. It is used for comparing two or more independent samples of equal or different sample sizes. It extends the Mann-Whitney U test, which is used for comparing only two groups. A significant K-W test indicates that at least one sample stochastically dominates another sample. This test provides the mean of the ranks for multiple sets of statistical data which are considered for comparative analysis. As an important criterion, if the mean of the ranks for one sample has lower values than the other ones, the one

TABLE 6. Details of the composite and hybrid mathematical functions (Fourth group).

No.	Descriptions	R	D	Min.
<i>Basic Functions: Sphere Function</i>				
F ₂₃₄	$f_1, f_2, f_3, \dots, f_{10} = \text{Sphere Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [1, 1, 1, \dots, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [5/100, 5/100, 5/100, \dots, 5/100]$	[−5, 5]	10	0
<i>Basic Functions: Griewank Function</i>				
F ₂₃₅	$f_1, f_2, f_3, \dots, f_{10} = \text{Griewank Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [1, 1, 1, \dots, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [5/100, 5/100, 5/100, \dots, 5/100]$	[−5, 5]	10	0
<i>Basic Functions: Griewank Function</i>				
F ₂₃₆	$f_1, f_2, f_3, \dots, f_{10} = \text{Griewank Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [1, 1, 1, \dots, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [1, 1, 1, \dots, 1]$	[−5, 5]	10	0
<i>Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions</i>				
F ₂₃₇	$f_1, f_2 = \text{Ackley Function}$ $f_3, f_4 = \text{Rastrigin Function}$ $f_5, f_6 = \text{Weierstrass Function}$ $f_7, f_8 = \text{Griewank Function}$ $f_9, f_{10} = \text{Sphere Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [1, 1, 1, \dots, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [5/32, 5.32, 1, 1, 5/0.5, 5/0.5, 5/100, 5/100, 5/100, 5/100]$	[−5, 5]	10	0
<i>Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions</i>				
F ₂₃₈	$f_1, f_2 = \text{Rastrigin Function}$ $f_3, f_4 = \text{Weierstrass Function}$ $f_5, f_6 = \text{Griewank Function}$ $f_7, f_8 = \text{Ackley Function}$ $f_9, f_{10} = \text{Sphere Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [1, 1, 1, \dots, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [1/5, 1/5, 5/0.5, 5/0.5, 5/100, 5/32, 5/32, 5/100, 5/100]$	[−5, 5]	10	0
<i>Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions</i>				
F ₂₃₉	$f_1, f_2 = \text{Rastrigin Function}$ $f_3, f_4 = \text{Weierstrass Function}$ $f_5, f_6 = \text{Griewank Function}$ $f_7, f_8 = \text{Ackley Function}$ $f_9, f_{10} = \text{Sphere Function}$ [$\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_{10}] = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]$ [$\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_{10}] = [0.1 \times 1/5, 0.2 \times 1/5, 0.3 \times 5/0.5, 0.4 \times 5/0.5, 0.5 \times 5/100, 0.6 \times 5/100, 0.7 \times 5/32, 0.8 \times 5/32, 0.9 \times 5/100, 1 \times 5/100]$	[−5, 5]	10	0

with a smaller mean of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the K-W test for different studied functions based on the obtained results of the optimization runs have been presented in Tables 13 to 15. Based on the results, the CryStAl related values for the mean of the ranks in most of the cases are lower than the related values for the other metaheuristics which represents the superiority of CryStAl. In these tables, the bolded values are related to the metaheuristic which is superior to the other ones while the values related to CryStAl are all underlined.

D. POST-HOC ANALYSIS

Post-hoc is a Latin phrase, meaning “after this” or “after the event”. In a scientific study, a Post-Hoc (P-H) analysis

TABLE 7. Parameter summary of the alternative metaheuristic algorithms.

Metaheuristic	Parameter	Description	Value
ABC	N_{pop}	Colony Size	50
	N_o	Number of Onlooker Bees	50
	L	Abandonment Limit Parameter	60
	a	Acceleration Coefficient Upper Bound	1
ACO	N_{pop}	Archive Size	50
	N_s	Sample Size	50
	q	Intensification Factor	0.5
	ζ	Deviation-Distance Ratio	1
BA	N_{pop}	Number of Scout Bees	50
	N_{ss}	Number of Selected Sites	25
	N_{se}	Number of Selected Elite Sites	10
	N_{rs}	Number of Recruited Bees for Selected Sites	25
FA	N_{re}	Number of Recruited Bees for Elite Sites	50
	r	Neighborhood Radius	0.1
	r_{damp}	Neighborhood Radius Damp Rate	0.95
	N_{pop}	Number of Fireflies (Swarm Size)	50
GA	γ	Light Absorption Coefficient	1
	β	Attraction Coefficient Base Value	2
	α	Mutation Coefficient	0.2
	α_{damp}	Mutation Coefficient Damping Ratio	0.98
HS	δ	Uniform Mutation Range	± 0.05
	p_c	Crossover Percentage	0.8
	p_m	Mutation Percentage	0.3
	μ	Mutation Rate	0.02
PSO	β	Roulette wheel selection pressure	1
	HMS	Harmony Memory Size	50
	N_{new}	Number of New Harmonies	20
	$HMCR$	Harmony Memory Consideration Rate	0.9
SA	PAR	Pitch Adjustment Rate	0.1
	FW	Fret Width (Bandwidth)	± 0.02
	FW_{damp}	Fret Width Damp Ratio	0.995
	N_{pop}	Swarm Size	50
PSO	w	Inertia Weight	1
	w_d	Inertia Weight Damping Ratio	0.99
	c_1	Personal Learning Coefficient	2
	c_2	Global Learning Coefficient	2
SA	N_{pop}	Population Size	50
	M_{subit}	Maximum Number of Sub-iterations	15
	T_0	Initial Temperature	0.025
	α	Temperature Reduction Rate	0.99
PSO	N_m	Number of Neighbors per Individual	5
	μ	Mutation Rate	0.5
	σ	Mutation Range (Standard Deviation)	0.1

consists of statistical analyses that were not specified before the data was seen. A P-H analysis involves looking at the data after a study has been concluded, and trying to find patterns that were not the primary objectives of the study.

In this section, the P-H analysis is conducted in order to derive the overall rankings of the metaheuristic algorithms for all of the 2D, 50D, and 100D functions based on the achieved results of the K-W test. The overall rankings of the metaheuristics obtained by the P-H analysis are presented in Table 16. It should be noted that CryStAl provides a success estimation of 100 percent in outranking the other metaheuristics, which demonstrates the superiority of this proposed novel optimization algorithm.

TABLE 8. The K-S test results (Asymp. Sig.) for different algorithms.

Main Algorithm	Function Type	Data Type	Alternative Metaheuristic Algorithms											
			ABC	ACO	BA	FA	GA	HS	MFO	MVO	PSO	SA	SCA	SSA
CryStAl	2D	Min.	6.92E-07	8.66E-01	8.09E-11	1.79E-21	2.07E-01	3.51E-01	9.97E-01	2.35E-13	1E+00	3.61E-16	4.92E-10	2.72E-01
		Mean	1.30E-04	8.66E-01	1.11E-15	1.15E-16	2.91E-14	5.04E-03	8.19E-02	4.73E-12	4.04E-02	8.35E-14	1.56E-07	7.92E-03
		Std.	2.01E-10	2.07E-01	6.59E-21	5.04E-25	4.80E-22	6.95E-04	1.14E-01	8.43E-20	4.04E-02	1.26E-22	8.35E-14	2.09E-05
	50D	Fun. Evl.	6.47E-09	1.14E-01	9.99E-15	4.80E-22	4.73E-12	8.35E-14	2.07E-01	1.79E-21	2.76E-02	1.00E-18	3.20E-11	1.79E-21
		Min.	2.48E-14	1.09E-15	4.17E-17	3.23E-11	1.40E-03	1.12E-13	6.67E-04	4.69E-07	1.05E-02	2.17E-16	1.40E-03	3.06E-04
		Mean	9.37E-06	4.69E-07	1.69E-08	6.67E-04	6.07E-01	3.06E-04	5.76E-05	3.24E-01	7.65E-01	9.37E-06	2.22E-01	3.24E-01
	100D	Std.	3.06E-04	3.06E-04	1.32E-06	1.46E-01	9.99E-01	9.30E-02	6.67E-04	7.65E-01	7.65E-01	1.05E-02	4.54E-01	6.07E-01
		Fun. Evl.	1.40E-03	6.67E-04	1.35E-04	1.35E-04	2.22E-01	1.35E-04	8.99E-01	6.67E-04	7.65E-01	5.76E-05	1.46E-01	3.37E-02
		Min.	2.04E-12	4.87E-13	2.48E-14	1.21E-10	5.76E-05	8.27E-12	5.31E-08	1.61E-07	1.40E-03	1.12E-13	4.69E-07	3.06E-04
		Mean	3.58E-06	3.58E-06	1.61E-07	6.67E-04	6.07E-01	1.35E-04	2.37E-05	2.22E-01	3.24E-01	5.76E-05	5.58E-03	8.99E-01
		Std.	3.06E-04	3.06E-04	5.76E-05	3.37E-02	9.99E-01	5.58E-03	1.35E-04	8.99E-01	6.07E-01	2.85E-03	1.05E-02	6.07E-01
		Fun. Evl.	1.92E-02	1.05E-02	6.67E-04	6.67E-04	5.70E-02	6.67E-04	9.30E-02	6.67E-04	3.24E-01	6.67E-04	5.58E-03	5.70E-02

TABLE 9. The K-S test results (the overall difference between data) for different algorithms.

Main Algorithm	Function Type	Data Type	Alternative Metaheuristic Algorithms											
			ABC	ACO	BA	FA	GA	HS	MFO	MVO	PSO	SA	SCA	SSA
CryStAl	2D	Min.	0.3504	0.0769	0.4444	0.6325	0.1368	0.1197	0.0513	0.4957	0.0427	0.5470	0.4274	0.1282
		Mean	0.2821	0.0769	0.5385	0.5556	0.5128	0.2222	0.1624	0.4701	0.1795	0.5043	0.3675	0.2137
		Std.	0.4359	0.1368	0.6239	0.6838	0.6410	0.2564	0.1538	0.6068	0.1795	0.6496	0.5043	0.3077
	50D	Fun. Evl.	0.4017	0.1538	0.5214	0.6410	0.4701	0.5043	0.1368	0.6325	0.1880	0.5897	0.4530	0.6325
		Min.	0.7241	0.7586	0.7931	0.6379	0.3448	0.7069	0.3621	0.5000	0.2931	0.7759	0.3448	0.3793
		Mean	0.4483	0.5000	0.5517	0.3621	0.1379	0.3793	0.4138	0.1724	0.1207	0.4483	0.1897	0.1724
	100D	Std.	0.3793	0.3793	0.4828	0.2069	0.0690	0.2241	0.3621	0.1207	0.1207	0.2931	0.1552	0.1379
		Fun. Evl.	0.3448	0.3621	0.3966	0.3966	0.1897	0.3966	0.1034	0.3621	0.1207	0.4138	0.2069	0.2586
		Min.	0.6724	0.6897	0.7241	0.6207	0.4138	0.6552	0.5345	0.5172	0.3448	0.7069	0.5000	0.3793
		Mean	0.4655	0.4655	0.5172	0.3621	0.1379	0.3966	0.4310	0.1897	0.1724	0.4138	0.3103	0.1034
		Std.	0.3793	0.3793	0.4138	0.2586	0.0690	0.3103	0.3966	0.1034	0.1379	0.3276	0.2931	0.1379
		Fun. Evl.	0.2759	0.2931	0.3621	0.3621	0.2414	0.3621	0.2241	0.3621	0.1724	0.3621	0.3103	0.2414

TABLE 10. The M-W test results (summation of the ranks) for 2D mathematical functions.

Main Algorithm	Function Type	Data Type	Alternative Metaheuristic Algorithms											
			ABC	ACO	BA	FA	GA	HS	MFO	MVO	PSO	SA	SCA	SSA
CryStAl	2D	Min.	16014.00	14047.50	16845.50	18294.50	14779.00	14616.50	14146.50	16896.50	14064.00	17406.50	16912.50	14595.00
		11481.00	13447.50	10649.50	9200.50	12716.00	12878.50	13348.50	10598.50	13431.00	10088.50	10582.50	12900.00	
		Mean	15181.00	14235.50	17494.00	17703.00	17137.00	15344.00	14945.50	16680.00	14752.00	16954.00	16323.00	14893.00
	50D	12314.00	13259.50	10001.00	9792.00	10358.00	12151.00	12549.50	10815.00	12743.00	10541.00	11172.00	12602.00	
		Std.	16016.50	14528.00	17779.00	18090.50	17774.00	15501.00	14778.50	17393.00	14751.00	17583.50	16729.00	15552.50
		Fun. Evl.	15657.00	13306.00	17180.00	18550.00	16372.00	16576.00	14609.00	18323.00	14689.00	18335.00	17293.00	17633.00
		11838.00	14189.00	10315.00	8945.00	11123.00	10919.00	12886.00	9172.00	12806.00	9160.00	10202.00	9862.00	

TABLE 11. The M-W test results (summation of the ranks) for 50D mathematical functions.

Main Algorithm	Function Type	Data Type	Alternative Metaheuristic Algorithms											
			ABC	ACO	BA	FA	GA	HS	MFO	MVO	PSO	SA	SCA	SSA
CryStAl	50D	Min.	4570.00	4632.00	4753.00	4563.00	3995.00	4610.00	3765.00	4308.00	3625.00	4702.00	3870.00	4003.00
		2216.00	2154.00	2033.00	2223.00	2791.00	2176.00	3021.00	2478.00	3161.00	2084.00	2916.00	2783.00	
		Mean	4210.00	4292.50	4468.50	4089.50	3545.00	4060.00	4080.50	3681.00	3390.50	4268.00	3710.00	3539.00
	100D	2576.00	2493.50	2317.50	2696.50	3241.00	2726.00	2705.50	3105.00	3395.50	2518.00	3076.00	3247.00	
		Std.	3943.00	4002.00	4209.00	3795.00	3384.50	3760.50	3953.00	3530.50	3289.00	3938.00	3562.50	3387.50
		Fun. Evl.	3937.00	3968.00	4069.00	4063.00	3697.00	4048.00	3536.00	3983.00	3355.00	4079.00	3748.00	3816.00
		2849.00	2818.00	2717.00	2723.00	3089.00	2738.00	3250.00	2803.00	3431.00	2707.00	3038.00	2970.00	

VII. CEC 2017 COMPETITION RESULTS

In order to evaluate the overall performance of the proposed algorithm, CryStAl, it is necessary to consider state-of-the-art mathematical test functions alongside

state-of-the-art algorithms. To this end, a recent competition on single-objective real-parameter numerical optimization named “CEC 2017” [86] is considered in this section. In this regard, a list of 30 mathematical functions are studied and

TABLE 12. The M-W test results (summation of the ranks) for 100D mathematical functions.

Main Algorithm	Function Type	Data Type	Alternative Metaheuristic Algorithms											
			ABC	ACO	BA	FA	GA	HS	MFO	MVO	PSO	SA	SCA	SSA
CryStAl	100D	Min.	4516.00	4555.00	4712.00	4571.00	4113.00	4610.00	4251.00	4381.00	3943.00	4650.00	4303.00	4047.00
			2270.00	2231.00	2074.00	2215.00	2673.00	2176.00	2535.00	2405.00	2843.00	2136.00	2483.00	2739.00
		Mean	4163.00	4193.00	4360.00	4069.00	3517.00	4145.00	4171.00	3688.00	3587.00	4208.00	3987.00	3469.00
			2623.00	2593.00	2426.00	2717.00	3269.00	2641.00	2615.00	3098.00	3199.00	2578.00	2799.00	3317.00
		Std.	3889.00	3918.00	4071.00	3756.00	3347.00	3865.00	4042.00	3485.00	3461.00	3916.00	3824.00	3325.00
		Fun. Evl.	3825.50	3858.00	3995.00	3995.00	3766.50	3989.00	3739.00	3989.00	3655.50	3997.00	3888.50	3775.50
			2960.50	2928.00	2791.00	2791.00	3019.50	2797.00	3047.00	2797.00	3130.50	2789.00	2897.50	3010.50

TABLE 13. The K-W test results (mean of the ranks) for 2D mathematical functions.

Ranking	2D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks						
1	CryStAl	558.8803	CryStAl	533.0342	CryStAl	491.7137	CryStAl	474.2222
2	<i>PSO</i>	595.5940	<i>ACO</i>	592.4744	<i>ACO</i>	580.1197	<i>ACO</i>	484.2607
3	<i>ACO</i>	601.7350	<i>SSA</i>	631.2607	<i>SSA</i>	622.9487	<i>PSO</i>	545.1838
4	<i>MFO</i>	608.3205	<i>ABC</i>	649.4615	<i>MFO</i>	629.0043	<i>MFO</i>	555.2906
5	<i>SSA</i>	648.7650	<i>PSO</i>	668.5684	<i>PSO</i>	630.4573	<i>HS</i>	698.1624
6	<i>HS</i>	659.7521	<i>MFO</i>	687.2735	<i>ABC</i>	685.3034	<i>G4</i>	700.3162
7	<i>GA</i>	682.7180	<i>HS</i>	727.0556	<i>HS</i>	701.8889	<i>ABC</i>	768.0342
8	<i>ABC</i>	793.7863	<i>SCA</i>	829.2863	<i>SCA</i>	843.2094	<i>SSA</i>	817.4274
9	<i>MVO</i>	875.8120	<i>MVO</i>	830.9573	<i>MVO</i>	860.1026	<i>SCA</i>	884.8205
10	<i>BA</i>	905.4060	<i>SA</i>	870.3333	<i>SA</i>	899.4744	<i>BA</i>	903.0769
11	<i>SCA</i>	927.8162	<i>GA</i>	897.7094	<i>GA</i>	931.0171	<i>MVO</i>	1000.7521
12	<i>SA</i>	950.4103	<i>FA</i>	983.9444	<i>FA</i>	992.2222	<i>SA</i>	1004.3590
13	<i>FA</i>	1084.0043	<i>BA</i>	991.6410	<i>BA</i>	1025.5385	<i>FA</i>	1057.0940
<i>Chi-sq.</i>	253.8161		168.4093		225.9612		332.7130	
<i>Prob>Chi-sq.</i>	2.1883E-47		1.0096E-29		1.3755E-41		6.1870E-64	

TABLE 14. The K-W test results (mean of the ranks) for 50D mathematical functions.

Ranking	50D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks						
1	CryStAl	193.3621	CryStAl	263.3879	PSO	290.0776	PSO	280.4397
2	<i>PSO</i>	244.8793	<i>PSO</i>	269.4224	<i>CryStAl</i>	307.8707	<i>CryStAl</i>	281.2414
3	<i>MFO</i>	297.6034	<i>GA</i>	296.2586	<i>GA</i>	308.6638	<i>MFO</i>	313.6552
4	<i>SCA</i>	299.5000	<i>SSA</i>	298.9310	<i>SSA</i>	312.3190	<i>GA</i>	351.0603
5	<i>GA</i>	300.1638	<i>MVO</i>	321.7931	<i>MVO</i>	337.1121	<i>SCA</i>	359.1983
6	<i>SSA</i>	309.4655	<i>SCA</i>	332.7845	<i>SCA</i>	348.0776	<i>SSA</i>	377.9138
7	<i>MVO</i>	348.3276	<i>HS</i>	389.2241	<i>HS</i>	380.3190	<i>ABC</i>	404.4828
8	<i>HS</i>	429.8621	<i>FA</i>	413.8190	<i>FA</i>	396.2500	<i>MVO</i>	406.4828
9	<i>FA</i>	449.7500	<i>MFO</i>	423.7672	<i>SA</i>	420.7500	<i>ACO</i>	411.5000
10	<i>SA</i>	489.8362	<i>SA</i>	450.6207	<i>MFO</i>	433.0259	<i>HS</i>	427.7328
11	<i>ABC</i>	490.7586	<i>ABC</i>	456.6638	<i>ABC</i>	434.9483	<i>FA</i>	429.3707
12	<i>ACO</i>	508.1121	<i>ACO</i>	473.9224	<i>ACO</i>	447.7500	<i>BA</i>	429.9569
13	<i>BA</i>	545.8793	<i>BA</i>	516.9052	<i>BA</i>	490.3362	<i>SA</i>	434.4655
<i>Chi-sq.</i>	189.8568		105.8629		61.5589		99.1510	
<i>Prob>Chi-sq.</i>	4.0193E-34		3.9241E-17		1.1716E-08		8.1644E-16	

presented in Table 17; the mathematical details of these functions have been presented by the CEC 2017 competition committee [86].

The statistical results of the CryStAl algorithm in dealing with these test functions (CEC 2017) with 10 dimensions are presented in the Supplementary Materials where the results of three other successful algorithms are also presented. It should be noted that the error values, rather than the global best values, of each run are considered in this competition and the statistical results are based on the best error values of 51 independent runs. The results show that the proposed CryStAl algorithm is capable of providing

eminently acceptable results in dealing with these test functions of different dimensions.

VIII. COMPUTATIONAL COST AND COMPLEXITY ANALYSIS

In this section, the computational cost and complexity of the proposed CryStAl method are examined and analyzed where three different approaches are considered to acquire a better understanding of these properties. In the first approach, the computational cost procedure of the CEC 2017 benchmark suite is determined while the results of three other state-of-the-art algorithms are also considered to form a fair

TABLE 15. The K-W test results (mean of the ranks) for 100D mathematical functions.

Ranking	100D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks						
1	<i>CryStAl</i>	171.7069	<i>CryStAl</i>	259.5517	<i>SSA</i>	298.1638	<i>CryStAl</i>	278.2328
2	<i>PSO</i>	274.6810	<i>SSA</i>	283.4741	<i>GA</i>	299.2241	<i>PSO</i>	340.7586
3	<i>SSA</i>	291.4397	<i>GA</i>	285.7414	<i>CryStAl</i>	305.3793	<i>MFO</i>	358.7500
4	<i>GA</i>	307.5172	<i>PSO</i>	304.1983	<i>PSO</i>	323.3362	<i>GA</i>	364.9741
5	<i>MVO</i>	342.7672	<i>MVO</i>	314.6724	<i>MVO</i>	327.7759	<i>SSA</i>	366.7328
6	<i>SCA</i>	349.0948	<i>SCA</i>	389.4483	<i>FA</i>	387.4310	<i>ABC</i>	375.9569
7	<i>MFO</i>	350.9914	<i>FA</i>	405.6379	<i>SCA</i>	400.7069	<i>ACO</i>	382.9741
8	<i>FA</i>	438.8103	<i>HS</i>	417.7241	<i>HS</i>	403.2586	<i>SCA</i>	389.6810
9	<i>HS</i>	451.3190	<i>MFO</i>	427.6121	<i>SA</i>	413.5862	<i>HS</i>	409.3276
10	<i>SA</i>	466.2414	<i>SA</i>	435.2241	<i>ABC</i>	420.7759	<i>MVO</i>	409.4569
11	<i>ABC</i>	472.2845	<i>ABC</i>	445.9655	<i>ACO</i>	426.6810	<i>BA</i>	410.1379
12	<i>ACO</i>	479.4483	<i>ACO</i>	452.9741	<i>MFO</i>	441.4052	<i>FA</i>	410.1897
13	<i>BA</i>	511.1983	<i>BA</i>	485.2759	<i>BA</i>	459.7759	<i>SA</i>	410.3276
<i>Chi-sq.</i>	149.9211		86.7540		50.0012		65.2723	
<i>Prob>Chi-sq.</i>	5.8829E-26		2.0918E-13		1.3965E-06		2.4278E-09	

TABLE 16. The P-H analysis results for all of the mathematical functions.

Ranking	2D & 50D & 100D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	<i>CryStAl</i>	1011.2554	<i>CryStAl</i>	1167.9185	<i>CryStAl</i>	1213.4700	<i>CryStAl</i>	1077.0343
2	<i>PSO</i>	1181.8820	<i>SSA</i>	1262.1524	<i>SSA</i>	1271.1373	<i>PSO</i>	1198.1524
3	<i>MFO</i>	1301.1180	<i>PSO</i>	1296.0794	<i>PSO</i>	1304.7704	<i>MFO</i>	1253.2403
4	<i>SSA</i>	1322.7361	<i>GA</i>	1463.3519	<i>ACO</i>	1467.9335	<i>ACO</i>	1306.1953
5	<i>GA</i>	1358.6159	<i>MVO</i>	1492.2876	<i>ABC</i>	1507.1803	<i>GA</i>	1403.3948
6	<i>SCA</i>	1552.6202	<i>ACO</i>	1520.1352	<i>GA</i>	1510.8391	<i>SSA</i>	1506.7189
7	<i>HS</i>	1575.1717	<i>ABC</i>	1523.9678	<i>HS</i>	1516.0665	<i>HS</i>	1507.9700
8	<i>ACO</i>	1577.6159	<i>SCA</i>	1542.6180	<i>MFO</i>	1517.6009	<i>ABC</i>	1555.5579
9	<i>MVO</i>	1591.6459	<i>MFO</i>	1543.5536	<i>MVO</i>	1526.1524	<i>SCA</i>	1632.6674
10	<i>ABC</i>	1679.0107	<i>HS</i>	1553.2382	<i>SCA</i>	1566.9571	<i>BA</i>	1739.8498
11	<i>SA</i>	1818.0408	<i>SA</i>	1697.1931	<i>SA</i>	1679.9893	<i>MVO</i>	1792.3734
12	<i>BA</i>	1847.7639	<i>FA</i>	1759.9850	<i>FA</i>	1738.1974	<i>SA</i>	1834.5622
13	<i>FA</i>	1877.5236	<i>BA</i>	1872.5193	<i>BA</i>	1874.7060	<i>FA</i>	1887.2833
<i>Chi-sq.</i>	270.5922		141.3699		125.5517		334.3474	
<i>Prob>Chi-sq.</i>	6.8410E-51		3.1677E-24		4.8084E-21		2.7999E-64	

judgment. In the CEC 2017 computational scenario, four different computational times, namely T_0 , T_1 , T_2 and \bar{T}_2 , are considered based on four specific mathematical procedures. T_0 refers to the running time of a predefined mathematical procedure [46], T_1 denotes the computational time for evaluation of the G_{18} test function considering 200000 function evaluations, T_2 represents the computational time of the considered metaheuristic algorithm (CryStAl in this paper) for evaluation of the G_{18} test function considering 200000 function evaluations, and \bar{T}_2 refers to the mean values of five different assessments of T_2 . The results of this scenario for the proposed and alternative algorithms are presented in Table 18 which demonstrates the capability of the proposed CryStAl algorithm in producing competitive results.

In computer science, “Big O notation” is a mathematical notation that determines the required running time and memory space of an algorithm by considering its growth rate in dealing with different inputs. In the following, the computational cost of the proposed CryStAl method is presented using this notation which is the second approach for testing the complexity of the proposed algorithm. For CryStAl, the random selection process in the initialization phase of the algorithm has a computational complexity of $O(NP \times D)$ where NP is the initial population size and D is the dimension of

the problem. The computational complexity of the objective function evaluation in the initialization phase of the algorithm is calculated as $O(NP) \times O(F(x))$ where $F(x)$ demonstrates the objective function value. After the initialization phase, the main loop of the algorithm is started based on the previously determined maximum number of iterations (MaxIter). By the consideration of the worst-case scenario, each line has a computational complexity of MaxIter in the main loop of the algorithm. In this loop, four new position vectors are created for each of the current vectors so the position updating process of the problem will have a computational complexity of $O(\text{MaxIter} \times NP \times D \times 4)$. In addition, the objective function evaluation in the main loop has a computational complexity of $O(\text{MaxIter} \times NP \times 4) \times O(F(x))$.

In general, the overall capacity of a metaheuristic algorithm depends on the balance between exploration and exploitation while the convergence speed is also an important factor in its evaluation. In order to demonstrate these properties for the proposed CryStAl algorithm, as the third complexity approach, the diversity graphs of CryStAl are plotted for functions F_1 , F_{61} , and F_{83} in the Supplementary Materials. As can be seen from these results, the population in the optimization process by CryStAl tends to localize the search for achieving better results.

TABLE 17. Summary of the CEC 2017 test functions [46].

Function type	Func. No.	Function details	Func. Min.
Unimodal functions	G_1	Shifted and Rotated Bent Cigar Function	100
	G_2	Shifted and Rotated Sum of Different Power Function	200
	G_3	Shifted and Rotated Zakharov Function	300
Simple multimodal functions	G_4	Shifted and Rotated Rosenbrock's Function	400
	G_5	Shifted and Rotated Rastrigin's Function	500
	G_6	Shifted and Rotated Expanded Schaffer's F6 Function	600
	G_7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
	G_8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	G_9	Shifted and Rotated Levy Function	900
	G_{10}	Shifted and Rotated Schwefel's Function	1000
	G_{11}	Hybrid Function 1 ($N = 3$)	1100
	G_{12}	Hybrid Function 2 ($N = 3$)	1200
	G_{13}	Hybrid Function 3 ($N = 3$)	1300
Hybrid functions	G_{14}	Hybrid Function 4 ($N = 4$)	1400
	G_{15}	Hybrid Function 5 ($N = 4$)	1500
	G_{16}	Hybrid Function 6 ($N = 4$)	1600
	G_{17}	Hybrid Function 6 ($N = 5$)	1700
	G_{18}	Hybrid Function 6 ($N = 5$)	1800
	G_{19}	Hybrid Function 6 ($N = 5$)	1900
	G_{20}	Hybrid Function 6 ($N = 6$)	2000
	G_{21}	Composition Function 1 ($N = 3$)	2100
	G_{22}	Composition Function 2 ($N = 3$)	2200
	G_{23}	Composition Function 3 ($N = 4$)	2300
Composition functions	G_{24}	Composition Function 4 ($N = 4$)	2400
	G_{25}	Composition Function 5 ($N = 5$)	2500
	G_{26}	Composition Function 6 ($N = 5$)	2600
	G_{27}	Composition Function 7 ($N = 6$)	2700
	G_{28}	Composition Function 8 ($N = 6$)	2800
	G_{29}	Composition Function 9 ($N = 3$)	2900
	G_{30}	Composition Function 10 ($N = 3$)	3000

Search range: $[-100,100]^D$

IX. REAL-WORLD OPTIMIZATION PROBLEMS

In this section, the applicability of the proposed algorithm, CryStAl, is investigated by considering some real-world optimization problems which can be a great challenge for the proposed method. In this regard, we have considered six difficult power electronics problems on synchronous optimal pulse-width modulation (SOPWM) which is used to regulate medium-voltage (MV) drives. This approach provides a significant decrease of switching frequency without raising the distortion, which leads to the reduction of switching losses that enhances the performance of the inverter. Generally, switching angles are calculated by reducing the distortion of current. In this study, this problem is considered as a constrained optimization problem which is benchmarked by CEC 2020 [90] regarding real-world constrained optimization. In this paper, six configurations of this problem are determined and solved by the proposed CryStAl with a simple penalty approach for constrained handling purposes. A brief explanation of these problems is presented in Table 19 while the comparative results are provided in the Supplementary Materials. The findings of this study demonstrated that the

TABLE 18. Computational complexity results of CryStAl compared to other approaches.

Metaheuristics	Properties	Results (sec)
EBO with CMAR [47]	T_0	0.0413
	T_1	0.8218
	\hat{T}_2	7.5794
LSHADE-cnEpSin [48]	$(\hat{T}_2 - T_1) / T_0$	163.6223
	T_0	0.1093
	T_1	0.8391
MM-OED [49]	\hat{T}_2	2.1835
	$(\hat{T}_2 - T_1) / T_0$	12.30009
	T_0	2.157784
<i>CryStAl</i> (the present study)	T_1	0.146416
	\hat{T}_2	6.704923
	$(\hat{T}_2 - T_1) / T_0$	3.039417
<i>CryStAl</i> (the present study)	T_0	0.027387
	T_1	0.144345
	\hat{T}_2	5.378017
<i>CryStAl</i> (the present study)	$(\hat{T}_2 - T_1) / T_0$	191.10059

TABLE 19. Description of the investigated real-world design problems.

No. (CEC No.)	Name	D	g	h
M_1 (RC 45)	SOPWM for 3-level Inverters	25	24	1
M_2 (RC 46)	SOPWM for 5-level Inverters	25	24	1
M_3 (RC 47)	SOPWM for 7-level Inverters	25	24	1
M_4 (RC 48)	SOPWM for 9-level Inverters	30	29	1
M_5 (RC 49)	SOPWM for 11-level Inverters	30	29	1
M_6 (RC 50)	SOPWM for 13-level Inverters	30	29	1

proposed method is capable of producing eminently acceptable and even better results in dealing with these challenging problems.

Based on the presented results in this and previous sections, it can be concluded that the proposed algorithm produces excellent results in most of the considered cases. One of the key aspects of this study is the conducted statistical analysis to evaluate the capability of this algorithm in dealing with an extensive set of test problems. The employed benchmark test problems of CEC and the competitive results of CryStAl in dealing with these problems demonstrate that this algorithm can be considered as a successful metaheuristic approach.

X. CONCLUSION

This paper proposed a novel metaheuristic method called Crystal Structure Algorithm (CryStAl), inspired by the underlying principles of the formation of crystal structures from the addition of the basis to the lattice points. Four groups of mathematical test functions were selected in order to efficiently evaluate the performance of CryStAl with a total

number of 12 different metaheuristic algorithms. A complete statistical analysis was conducted to provide a valid judgment about the performance of this method. The most important findings of this paper are as follows:

- (i) CryStAl is superior to the other metaheuristics in converging to the global bests of the mathematical functions based on the selected tolerance.
- (ii) The results of the K-S test demonstrated that the maximum difference between CryStAl and the other metaheuristics is about FA and BA in most of the cases.
- (iii) The results of the M-W test showed that the summation of the ranks for CryStAl in most of the cases is lower than those of the other metaheuristics.
- (iv) The results of the K-W test manifested that CryStAl is 100% successful in outranking the other metaheuristics for the 2D functions in all of the cases such as the minimum, mean, and standard deviation values alongside the number of function evaluations.
- (v) The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values of the 50D test functions while the PSO outranks CryStAl in the standard deviation and function evaluation.
- (vi) The results of the K-W test showed that CryStAl has the first rank in the minimum and mean values alongside the number of function evaluations of the 100D test functions while the SSA and GA outrank CryStAl in the standard deviation values.
- (vii) The overall comparison of CryStAl and the alternative metaheuristics considering all of the 2D, 50D, and 100D test functions demonstrated that CryStAl is 100 percent successful in outranking the other metaheuristics in all of the cases.

As future challenges, different applications of CryStAl can be explored and its capabilities in dealing with difficult test problems can be examined. Besides, new configurations of this algorithm can be considered as other researchers may have different viewpoints on the presented methodology.

CODE AVAILABILITY

The MATLAB implementation of CryStAl is accessible at: <https://www.mathworks.com/matlabcentral/fileexchange/91850-crystal-structure-algorithm-crystal>

REFERENCES

- [1] F. Glover, “Future paths for integer programming and links to artificial intelligence,” *Comput. Oper. Res.*, vol. 13, no. 5, pp. 533–549, Jan. 1986.
- [2] K. Sørensen, M. Sevaux, and F. Glover, “A history of metaheuristics,” in *Handbook of Heuristics*. Cham, Switzerland: Springer, 2018. [Online]. Available: https://link.springer.com/content/pdf/10.1007%2F978-3-319-07124-4_4.pdf
- [3] P. Moscato, “On evolution, search, optimization, genetic algorithms and martial arts: Towards memetic algorithms,” *Caltech Concurrent Comput. Program*, vol. 826, p. 1989, Sep. 1989.
- [4] J. H. Holland, *Adaptation in Natural and Artificial Systems: An Introductory Analysis With Applications to Biology, Control, and Artificial Intelligence*. Cambridge, MA, USA: MIT Press, 1992.
- [5] R. Storn and K. Price, “Differential evolution—a simple and efficient heuristic for global optimization over continuous spaces,” *J. Global Optim.*, vol. 11, no. 4, pp. 341–359, 1997.
- [6] H. G. Beyer and H. P. Schwefel, “Evolution strategies—A comprehensive introduction,” *Natural Comput.*, vol. 1, no. 1, pp. 3–52, 2002.
- [7] R. Eberhart and J. Kennedy, “A new optimizer using particle swarm theory,” in *Proc. 6th Int. Symp. Micro Mach. Human Sci. (MHS)*, 1995, pp. 39–43.
- [8] M. Dorigo, V. Maniezzo, and A. Colomi, “Ant system: Optimization by a colony of cooperating agents,” *IEEE Trans. Syst., Man, Cybern., B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.
- [9] B. Basturk, “An artificial bee colony (ABC) algorithm for numeric function optimization,” in *Proc. IEEE Swarm Intell. Symp.*, Indianapolis, IN, USA, May 2006, pp. 181–184.
- [10] S. C. Chu, P. W. Tsai, and J. S. Pan, “Cat swarm optimization,” in *Proc. Pacific Rim Int. Conf. on Artif. Intell.* Berlin, Germany: Springer, Aug. 2006, pp. 854–858.
- [11] X. S. Yang, *Nature-Inspired Metaheuristic Algorithms*. Frome, U.K.: Luniver Press, 2010.
- [12] A. H. Gandomi and A. H. Alavi, “Kral herd: A new bio-inspired optimization algorithm,” *Commun. Nonlinear Sci. Numer. Simul.*, vol. 17, no. 12, pp. 4831–4845, Dec. 2012.
- [13] S. Mirjalili, A. H. Gandomi, S. Z. Mirjalili, S. Saremi, H. Faris, and S. M. Mirjalili, “Salp swarm algorithm: A bio-inspired optimizer for engineering design problems,” *Adv. Eng. Softw.*, vol. 114, pp. 163–191, Dec. 2017.
- [14] S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, “Optimization by simulated annealing,” *Science*, vol. 220, no. 4598, pp. 671–680, 1983.
- [15] M. H. N. Tayarani and M. R. T. Akbarzadeh, “Magnetic optimization algorithms new synthesis,” in *Proc. IEEE Congr. Evol. Comput., IEEE World Congr. Comput. Intell.*, Jun. 2008, pp. 2659–2664.
- [16] E. Rashedi, H. Nezamabadi-Pour, and S. Saryazdi, “GSA: A gravitational search algorithm,” *Inf. Sci.*, vol. 179, no. 13, pp. 2232–2248, Jun. 2009.
- [17] A. Kaveh and S. Talatahari, “A novel heuristic optimization method: Charged system search,” *Acta Mechanica*, vol. 213, nos. 3–4, pp. 267–289, Sep. 2010.
- [18] A. Kaveh and M. Khayatazad, “A new meta-heuristic method: Ray optimization,” *Comput. Struct.*, vols. 112–113, pp. 283–294, Dec. 2012.
- [19] A. Kaveh and V. R. Mahdavi, “Colliding bodies optimization: A novel meta-heuristic method,” *Comput. Struct.*, vol. 139, pp. 18–27, Jul. 2014.
- [20] S. Mirjalili, S. M. Mirjalili, and A. Hatamlou, “Multi-verse optimizer: A nature-inspired algorithm for global optimization,” *Neural Comput. Appl.*, vol. 27, no. 2, pp. 495–513, Feb. 2016.
- [21] S. Mirjalili, “SCA: A sine cosine algorithm for solving optimization problems,” *Knowl.-Based Syst.*, vol. 96, pp. 120–133, Mar. 2016.
- [22] Z. W. Geem, J. H. Kim, and G. V. Loganathan, “A new heuristic optimization algorithm: Harmony search,” *Simulation*, vol. 76, no. 2, pp. 60–68, Feb. 2001.
- [23] R. V. Rao, V. J. Savsani, and D. P. Vakharia, “Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems,” *Comput.-Aided Des.*, vol. 43, no. 3, pp. 303–315, Mar. 2011.
- [24] X. Feng, R. Zou, and H. Yu, “A novel optimization algorithm inspired by the creative thinking process,” *Soft Comput.*, vol. 19, no. 10, pp. 2955–2972, Oct. 2015.
- [25] S.-A. Ahmadi, “Human behavior-based optimization: A novel metaheuristic approach to solve complex optimization problems,” *Neural Comput. Appl.*, vol. 28, no. S1, pp. 233–244, Dec. 2017.
- [26] A. W. Mohamed, A. A. Hadi, and A. K. Mohamed, “Gaining-sharing knowledge based algorithm for solving optimization problems: A novel nature-inspired algorithm,” *Int. J. Mach. Learn. Cybern.*, vol. 18, pp. 1–29, Dec. 2019.
- [27] S. Talatahari and M. Azizi, “Chaos game optimization: A novel metaheuristic algorithm,” *Artif. Intell. Rev.*, vol. 22, pp. 1–88, Jun. 2020, doi: [10.1007/s10462-020-09867-w](https://doi.org/10.1007/s10462-020-09867-w).
- [28] S. Talatahari and M. Azizi, “Optimization of constrained mathematical and engineering design problems using chaos game optimization,” *Comput. Ind. Eng.*, vol. 145, Jul. 2020, Art. no. 106560, doi: [10.1016/j.cie.2020.106560](https://doi.org/10.1016/j.cie.2020.106560).
- [29] S. Talatahari and M. Azizi, “Optimal design of real-size building structures using quantum-behaved developed swarm optimizer,” *Struct. Des. Tall Special Buildings*, vol. 29, no. 11, p. e1747, Aug. 2020, doi: [10.1002/tal.1747](https://doi.org/10.1002/tal.1747).
- [30] S. Talatahari, P. Motamed, A. B. Farahmand, and M. Azizi, “Tribe-charged system search for parameter configuration of nonlinear systems with large search domains,” *Eng. Optim.*, vol. 53, no. 1, pp. 18–31, 2021, doi: [10.1080/0305215X.2019.1696786](https://doi.org/10.1080/0305215X.2019.1696786).

- [31] M. Azizi, R. G. Ejlali, S. A. M. Ghasemi, and S. Talatahari, "Upgraded whale optimization algorithm for fuzzy logic based vibration control of nonlinear steel structure," *Eng. Struct.*, vol. 192, pp. 53–70, Aug. 2019, doi: [10.1016/j.engstruct.2019.05.007](https://doi.org/10.1016/j.engstruct.2019.05.007).
- [32] M. Azizi, S. A. M. Ghasemi, R. G. Ejlali, and S. Talatahari, "Optimum design of fuzzy controller using hybrid ant lion optimizer and Jaya algorithm," *Artif. Intell. Rev.*, vol. 53, no. 3, pp. 1553–1584, Mar. 2020, doi: [10.1007/s10462-019-09713-8](https://doi.org/10.1007/s10462-019-09713-8).
- [33] M. Azizi, S. A. M. Ghasemi, R. G. Ejlali, and S. Talatahari, "Optimal tuning of fuzzy parameters for structural motion control using multiverse optimizer," *Struct. Des. Tall Special Buildings*, vol. 28, no. 13, p. e1652, Sep. 2019, doi: [10.1002/tal.1652](https://doi.org/10.1002/tal.1652).
- [34] S. Talatahari and M. Azizi, "Optimum design of building structures using tribe-interior search algorithm," *Structures*, vol. 28, pp. 1616–1633, Dec. 2020, doi: [10.1016/j.struc.2020.09.075](https://doi.org/10.1016/j.struc.2020.09.075).
- [35] M. Azizi, S. A. M. Ghasemi, R. G. Ejlali, and S. Talatahari, "Optimization of fuzzy controller for nonlinear buildings with improved charged system search," *Struct. Eng. Mech.*, vol. 76, no. 6, pp. 781–797, Dec. 2020, doi: [10.12989/sem.2020.76.6.781](https://doi.org/10.12989/sem.2020.76.6.781).
- [36] S. Talatahari and M. Azizi, "Tribe-charged system search for global optimization," *Appl. Math. Model.*, vol. 93, pp. 115–133, May 2021, doi: [10.1016/j.apm.2020.12.007](https://doi.org/10.1016/j.apm.2020.12.007).
- [37] S. Talatahari and M. Azizi, "Optimal parameter identification of fuzzy controllers in nonlinear buildings based on seismic hazard analysis using tribe-charged system search," in *Advances in Structural Engineering-Optimization. Studies in Systems, Decision and Control*, vol. 326, S. M. Nigdeli, G. Bekdik, A. E. Kayabekir M. Yucel, Eds. Cham, Switzerland: Springer, 2020, doi: [10.1007/978-3-030-61848-3_4](https://doi.org/10.1007/978-3-030-61848-3_4).
- [38] M. Azizi, "Atomic orbital search: A novel Metaheuristic algorithm," *Appl. Math. Model.*, vol. 93, pp. 657–683, May 2021, doi: [10.1016/j.apm.2020.12.021](https://doi.org/10.1016/j.apm.2020.12.021).
- [39] B. A. Averill, P. Eldredge. (Jun. 23, 2019). *Chemistry: Principles, Patterns, and Applications*. Apr. 6, 2021. [Online]. Available: <https://chem.libretexts.org/go/page/3999>
- [40] P. Sareh, "The least symmetric crystallographic derivative of the developable double corrugation surface: Computational design using underlying conic and cubic curves," *Mater. Des.*, vol. 183, Dec. 2019, Art. no. 108128.
- [41] A. Zingoni, "Group-theoretic insights on the vibration of symmetric structures in engineering," *Phil. Trans. Roy. Soc. A, Math., Phys. Eng. Sci.*, vol. 372, no. 2008, Feb. 2014, Art. no. 20120037.
- [42] A. Zingoni, "Symmetry recognition in group-theoretic computational schemes for complex structural systems," *Comput. Struct.*, vols. 94–95, pp. 34–44, Mar. 2012.
- [43] A. Zingoni, "Use of symmetry groups for generation of complex space grids and group-theoretic vibration analysis of triple-layer grids," *Eng. Struct.*, vol. 223, Nov. 2020, Art. no. 111177.
- [44] M. Hammermesh and C. Flammer, "Group theory and its application to physical problems," *Phys. Today*, vol. 16, no. 2, pp. 62–64, Feb. 1963.
- [45] P. Sareh and S. D. Guest, "A framework for the symmetric generalisation of the Miura-ori," *Int. J. Space Struct.*, vol. 30, no. 2, pp. 141–152, Jun. 2015.
- [46] A. Zingoni, "Group-theoretic vibration analysis of double-layer cable nets of D4h symmetry," *Int. J. Solids Struct.*, vols. 176–177, pp. 68–85, Nov. 2019.
- [47] H. Necefoglu, "Crystallographic patterns in nature and turkish art," *Crystal Eng.*, vol. 6, no. 4, pp. 153–166, Dec. 2003.
- [48] Y. Chen, Q. Sun, and J. Feng, "Group-theoretical form-finding of cable-strut structures based on irreducible representations for rigid-body translations," *Int. J. Mech. Sci.*, vol. 144, pp. 205–215, Aug. 2018.
- [49] T. J. Healey, "A group-theoretic approach to computational bifurcation problems with symmetry," *Comput. Methods Appl. Mech. Eng.*, vol. 67, no. 3, pp. 257–295, Apr. 1988.
- [50] Y. Chen and J. Feng, "Generalized eigenvalue analysis of symmetric prestressed structures using group theory," *J. Comput. Civil Eng.*, vol. 26, no. 4, pp. 488–497, Jul. 2012.
- [51] P. Sareh and S. D. Guest, "Design of isomorphic symmetric descendants of the Miura-ori," *Smart Mater. Struct.*, vol. 24, no. 8, Aug. 2015, Art. no. 085001.
- [52] M. Frame, D. K. Washburn, and D. W. Crowe, "Symmetries of culture: Theory and practice of plane pattern analysis," *Afr. Arts*, vol. 22, no. 4, p. 82, Aug. 1989.
- [53] P. G. Glockner, "Symmetry in structural mechanics," *J. Struct. Division*, vol. 99, no. 1, pp. 71–89, Jan. 1973.
- [54] C. A. Zapatero, *Crystals and Life: A Personal Journey*. International University Line, San Diego, CA, USA, 2002.
- [55] Y. Chen, J. Yan, J. Feng, and P. Sareh, "A hybrid symmetry-PSO approach to finding the self-equilibrium configurations of prestressable pin-jointed assemblies," *Acta Mechanica*, vol. 7, pp. 1–7, Jan. 2020.
- [56] V. Gorshkov, V. Tereshchuk, and P. Sareh, "Heterogeneous and homogeneous nucleation in the synthesis of quasi-one-dimensional periodic core-shell nanostructures," *Cryst. Growth Des.*, vol. 21, no. 3, pp. 1604–1616, Feb. 2021.
- [57] V. N. Gorshkov, V. V. Tereshchuk, and P. Sareh, "Restructuring and breakup of nanowires with the diamond cubic crystal structure into nanoparticles," *Mater. Today Commun.*, vol. 22, Mar. 2020, Art. no. 100727.
- [58] Y. Chen, J. Yan, P. Sareh, and J. Feng, "Nodal flexibility and kinematic indeterminacy analyses of symmetric tensegrity structures using orbits of nodes," *Int. J. Mech. Sci.*, vol. 155, pp. 41–49, May 2019.
- [59] P. Sareh and S. D. Guest, "Designing symmetric derivatives of the Miura-ori," in *Advances in Architectural Geometry 2014*. Cham, Switzerland: Springer, 2015, pp. 233–241.
- [60] Y. Chen, L. Fan, Y. Bai, J. Feng, and P. Sareh, "Assigning mountain-valley fold lines of flat-foldable origami patterns based on graph theory and mixed-integer linear programming," *Comput. Struct.*, vol. 239, Oct. 2020, Art. no. 106328.
- [61] Y. Chen, P. Sareh, and J. Feng, "Effective insights into the geometric stability of symmetric skeletal structures under symmetric variations," *Int. J. Solids Struct.*, vols. 69–70, pp. 277–290, Sep. 2015.
- [62] M. L. A. N. De Las Peñas, M. L. Loyola, A. M. Basilio, and E. B. Santoso, "Symmetry groups of single-wall nanotubes," *Acta Crystallographica A, Found. Adv.*, vol. 70, no. 1, pp. 12–23, Jan. 2014.
- [63] M. L. Loyola, M. L. A. N. De Las Peñas, G. M. Estrada, and E. B. Santoso, "Symmetry groups associated with tilings on a flat torus," *Acta Crystallographica A, Found. Adv.*, vol. 71, no. 1, pp. 99–110, Jan. 2015.
- [64] M. L. A. N. De Las Peñas, A. Garciano, D. M. Verzosa, and E. Taganap, "Crystallographic patterns in Philippine indigenous textiles," *J. Appl. Crystallogr.*, vol. 51, no. 2, pp. 456–469, Apr. 2018.
- [65] P. Sareh, P. Chermprayong, M. Emmanuel, H. Nadeem, and M. Kovac, "Rotorigami: A rotary origami protective system for robotic rotorcraft," *Sci. Robot.*, vol. 3, no. 22, Sep. 2018, Art. no. eaah5228.
- [66] A. Zingoni, M. N. Pavlovic, and G. M. Zlokovic, "A symmetry-adapted flexibility approach for multi-storey space frames. Part 1: General outline and symmetry-adapted redundants," *Struct. Eng. Rev.*, vol. 2, no. 7, pp. 107–119, 1995.
- [67] A. Zingoni, M. N. Pavlovic, and G. M. Zlokovic, "A symmetry-adapted flexibility approach for multi-storey space frames. Part 2: Symmetry-adapted loads," *Struct. Eng. Rev.*, vol. 2, no. 77, pp. 121–130, 1995.
- [68] P. Sareh and Y. Chen, "Intrinsic non-flat-foldability of two-tile DDC surfaces composed of glide-reflected irregular quadrilaterals," *Int. J. Mech. Sci.*, vol. 185, Nov. 2020, Art. no. 105881.
- [69] A. Zingoni, "On the best choice of symmetry group for group-theoretic computational schemes in solid and structural mechanics," *Comput. Struct.*, vol. 223, Oct. 2019, Art. no. 106101.
- [70] B. Bai and L. Li, "Group-theoretic approach to enhancing the Fourier modal method for crossed gratings with square symmetry," *J. Opt. Soc. Amer. A, Opt. Image Sci.*, vol. 23, no. 3, p. 572, Mar. 2006.
- [71] V. Gorshkov, P. Sareh, N. Navadeh, V. Tereshchuk, and A. S. Fallah, "Multi-resonator metamaterials as multi-band metastructures," *Mater. Des.*, vol. 202, Apr. 2021, Art. no. 109522.
- [72] Y. Chen, P. Sareh, J. Yan, A. S. Fallah, and J. Feng, "An integrated geometric-graph-theoretic approach to representing origami structures and their corresponding truss frameworks," *J. Mech. Des.*, vol. 141, no. 9, pp. 1–22, Sep. 2019.
- [73] I. Hargittai and G. Lengyel, "The seventeen two-dimensional space-group symmetries in Hungarian needlework," *J. Chem. Educ.*, vol. 62, no. 1, p. 35, Jan. 1985.
- [74] M. Duda, A. Rafalska-Łasocha, and W. Łasocha, "Plane and frieze symmetry group determination for educational purposes," *J. Chem. Educ.*, vol. 97, no. 8, pp. 2169–2174, Aug. 2020.
- [75] K. S. Mamedov, "Crystallographic patterns," *Comput. Math. with Appl.*, vol. 12, nos. 3–4, pp. 511–529, May 1986.
- [76] A. Thalal, "Symmetry: Through the eyes of old masters," *Crystallogr. Rev.*, vol. 23, no. 3, pp. 229–231, Jul. 2017.
- [77] B. Grünbaum, Z. Grünbaum, and G. C. Shephard, "Symmetry in Moorish and other ornaments," in *Symmetry*. New York, NY, USA: Pergamon, Jan. 1986, pp. 641–653.
- [78] B. L. Bodner, "The planar crystallographic groups represented at the Alhambra," in *Proc. Bridges*, 2013, pp. 225–232.

- [79] M. F. Blanco and A. L. Harris, "Symmetry groups in the Alhambra," in *Visual Mathematics*, vol. 13. Belgrade, Serbia: Mathematical Institute SASA, 2011.
- [80] K. A. Kantardjieff, A. R. Kaysser-Pyzalla, and P. Spadon, "Crystallography education and training for the 21st century," *J. Appl. Crystallogr.*, vol. 43, no. 5, pp. 1137–1138, Oct. 2010, doi: 10.1107/S0021889810034357.
- [81] M. Jamil and X.-S. Yang, "A literature survey of benchmark functions for global optimization problems," Aug. 2013, arXiv:1308.4008. [Online]. Available: <http://arxiv.org/abs/1308.4008>
- [82] M. Jamil, X. S. Yang, and H. J. Zepernick, "Test functions for global optimization: A comprehensive survey," In *Swarm intelligence and Bio-inspired Computation*. Amsterdam, The Netherlands: Elsevier, Jan. 2013, pp. 193–222.
- [83] J. Momin and X. S. Yang, "A literature survey of benchmark functions for global optimization problems," *J. Math. Model. Numer. Optim.*, vol. 4, no. 2, pp. 150–194, 2013.
- [84] X.-S. Yang, "Test problems in optimization," Aug. 2010, arXiv:1008.0549. [Online]. Available: <http://arxiv.org/abs/1008.0549>
- [85] J. J. Liang, P. N. Suganthan, and K. Deb, "Novel composition test functions for numerical global optimization," in *Proc. IEEE Swarm Intell. Symp. (SIS)*, Jun. 2005, pp. 68–75.
- [86] N. H. Awad, M. Z. Ali, J. J. Liang, B. Y. Qu, and P. N. Suganthan, "Problem definitions and evaluation criteria for the CEC 2017 special session and competition on single objective bound constrained real-parameter numerical optimization," Nanyang Technol. Univ., Singapore, Tech. Rep. 201611, 2016.
- [87] A. Kumar, R. K. Misra, and D. Singh, "Improving the local search capability of effective butterfly optimizer using covariance matrix adapted retreat phase," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2017, pp. 1835–1842.
- [88] N. H. Awad, M. Z. Ali, and P. N. Suganthan, "Ensemble sinusoidal differential covariance matrix adaptation with Euclidean neighborhood for solving CEC2017 benchmark problems," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2017, pp. 372–379.
- [89] K. M. Sallam, S. M. Elsayed, R. A. Sarker, and D. L. Essam, "Multi-method based orthogonal experimental design algorithm for solving CEC2017 competition problems," in *Proc. IEEE Congr. Evol. Comput. (CEC)*, Jun. 2017, pp. 1350–1357.
- [90] A. Kumar, G. Wu, M. Z. Ali, R. Mallipeddi, P. N. Suganthan, and S. Das, "A test-suite of non-convex constrained optimization problems from the real-world and some baseline results," *Swarm Evol. Comput.*, vol. 56, Aug. 2020, Art. no. 100693.



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