PHYSICAL REVIEW B **94**, 121201(R) (2016)

Magnetophonon oscillations caused by acoustic phonons in bulk conductors

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(Received 22 June 2016; revised manuscript received 15 August 2016; published 6 September 2016)

The interaction of electrons with acoustic phonons under a magnetic field leads to a remarkable kind of magnetophonon oscillation of transport coefficients, recently discovered in two-dimensional electron systems. The present study shows that similar oscillations exist in bulk conductors and provides a theory of this phenomenon for the case of spherical Fermi surfaces. The resonance peaks occur when the product of the Fermi surface diameter by the sound velocity is a multiple of the cyclotron frequency. Theoretical predictions may facilitate the experimental observation of the phenomenon.

DOI: 10.1103/PhysRevB.94.121201

The magnetophonon oscillations [1] (MPOs) of transport coefficients due to resonance scattering of electrons by optical phonons under Landau quantization had been predicted [2] and soon thereafter discovered in semiconductors [3] more than half a century ago. Since then, it was believed for a long time that only optical phonons [4] can be responsible for magnetophonon resonances. Acoustic phonon modes, with much smaller energies and a significant dispersion, were not considered in this connection. In the basic theoretical works on quantum magnetotransport [2,5–8], the interaction of electrons with acoustic phonons was treated either in the elastic approximation or under conditions when the characteristic phonon frequency defined as a product of electron wave number k by the sound velocity s was smaller than the cyclotron frequency ω_c.

It came as a surprise when MPOs with maxima under a condition of *acoustic magnetophonon resonance* $2k_F s = n\omega_c$ (k_F is the Fermi wave number and n is integer) were observed in the resistance [9] and in the thermopower [10] of high-mobility two-dimensional electrons. The unusual phenomenon of acoustic MPOs is a subject of intensive experimental and theoretical studies [9–23] as part of the currently developing physics of transport phenomena in high Landau levels [22]. Due to the smallness of acoustic phonon energies compared to the Fermi energy ε_F, acoustic magnetophonon resonances occur under conditions when many Landau levels are occupied. In two-dimensional systems based on semiconductor quantum wells, these conditions correspond to relatively weak magnetic fields, $B < 1$ T, so the observation of acoustic MPOs requires high-quality samples to minimize the broadening of the Landau levels. Acoustic MPOs in the resistance of two-dimensional electron systems are known as phonon-induced resistance oscillations (PIROs).

In this Rapid Communication, it is shown that the phenomenon of acoustic MPOs is not specific for two-dimensional electrons; it exists in bulk conductors as well. Consider first the origin of MPOs in two dimensions. The MPOs due to optical phonons naturally occur because the frequency of such phonons is a constant. For acoustic MPOs, in spite of the dispersion of the phonons, there exists a selected resonance frequency $2k_F s$. This selectivity is a consequence of the kinematics of electron scattering near the Fermi surface [circle in Fig. 1(b)]. As the electron velocity is much larger than s, the scattering is nearly elastic. The number of phonons satisfying the energy and momentum conservation laws with wave numbers in the interval dq is proportional to $|d\varphi/dq|_q = 1/\sqrt{(2k)^2 - q^2}$, where φ is the angle of phonon momentum. Thus, the density of phonons participating in the scattering (and, therefore, the scattering probability of electrons) has an inverse square root divergence at $q/2 = k \simeq k_F$, which means that the corresponding phonon frequency $sq = 2k_F s$ is resonant.

In three dimensions, the number of phonons satisfying the conservation laws is determined by the derivative of the solid angle of phonon momenta, $|d\Omega/dq|_q$. For a simple spherical Fermi surface this quantity is a constant, π/k_F. This property means that there is no selected phonon frequency in the absence of magnetic field and seemingly denies the existence of acoustic MPOs. However, a more careful consideration taking into account the presence of the magnetic field B (directed, by convention, along the z axis) leads to a different conclusion. Indeed, the electrons in the magnetic field form Landau subbands described by the discrete number n and including the states with continuous wave numbers along the z axis, k_z. The density of states in each subband diverges at $k_z = 0$. The transitions of electrons between highly populated states with small k_z and k'_z are assisted by phonons with small wave numbers $|q_z| = |k'_z - k_z| \ll k_F$. This means that the phonons contributing to resonance transitions between Landau subbands are effectively two dimensional, and the arguments above can be applied to them, suggesting that $2k_F s$ is again the resonance phonon frequency. Therefore, acoustic MPOs can exist in bulk conductors. The remaining part of this Rapid Communication provides a theory of this phenomenon by means of calculation and analysis of the transverse magnetotransport coefficients.

Phonon-assisted resistivity. Assuming an interaction of electrons described by a parabolic energy spectrum $\varepsilon_{nk_z} = \hbar\omega_c(n + 1/2) + \hbar^2 k_z^2/2m$, with equilibrium phonons described by the spectrum $\omega_{\lambda\mathbf{q}}$ characterized by the mode index λ and wave vector $\mathbf{q} = (\mathbf{q}_\perp, q_z)$, one can write the conductivity as (see, e.g., Ref. [8])

$$\sigma_{xx} = \frac{e^2\ell^2 T}{\hbar^3} \int \frac{d\mathbf{q}}{(2\pi)^3} \sum_{nn'\lambda} q_\perp^2 \Phi_{nn'}(u) C_{\lambda\mathbf{q}} \omega_{\lambda\mathbf{q}}^{-2} \times F\left(\frac{\hbar\omega_{\lambda\mathbf{q}}}{2T}\right) \int d\varepsilon \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \left(f_{\varepsilon-\hbar\omega_{\lambda\mathbf{q}}} - f_\varepsilon\right) \times A_\varepsilon\left(\varepsilon_{nk_z}\right) A_{\varepsilon-\hbar\omega_{\lambda\mathbf{q}}}\left(\varepsilon_{n'k_z-q_z}\right), \tag{1}$$

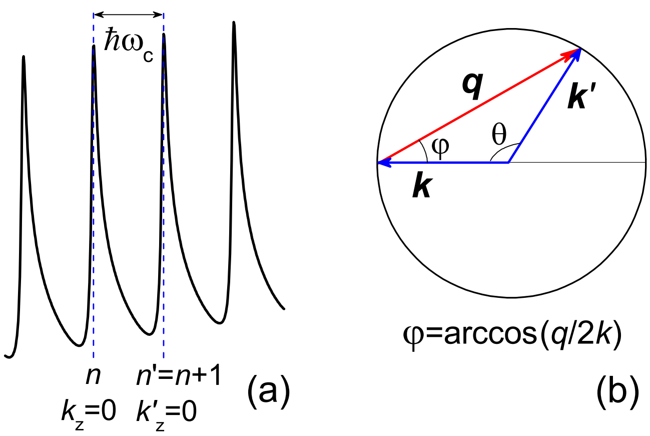

FIG. 1. (a) Density of states in a magnetic field. The transitions between peaks of Landau subbands involve phonons with small q_z only, so the resonance scattering is effectively two dimensional. (b) Kinematics of elastic scattering in two dimensions. The density of phonon states participating in the scattering is proportional to $|d\varphi/dq|$ and diverges at $q = 2k \simeq 2k_F$.

where the sums are taken over Landau level numbers n, n' running from 0 to infinity, $C_{\lambda \mathbf{q}}$ is the squared matrix element of the electron-phonon interaction, f_ε is the Fermi distribution function, $A_\varepsilon(\varepsilon_i) = (\hbar/2\pi\tau)/[(\varepsilon - \varepsilon_i)^2 + (\hbar/2\tau)^2]$ is the spectral function, τ is the quantum lifetime of the electron at the Fermi level, $\Phi_{nn'}(u) = (n!/n'!)u^{n'-n}e^{-u}[L_n^{n'-n}(u)]^2$, $L_n^m(u)$ are the Laguerre polynomials, $u = q_\perp^2 \ell^2/2$, ℓ is the magnetic length, and $F(x) = [x/\sinh(x)]^2$. In the undamped approximation, when $A_\varepsilon(\varepsilon_i)$ is reduced to the delta function $\delta(\varepsilon - \varepsilon_i)$, the integrals over ε and k_z can be taken analytically,

$$\sigma_{xx} = \frac{e^2 T}{2\pi \hbar^4 \omega_c} \int \frac{d\mathbf{q}}{(2\pi)^3} \sum_{nn'\lambda} q_\perp^2 |q_z|^{-1} \Phi_{nn'}(u) C_{\lambda \mathbf{q}} \omega_{\lambda \mathbf{q}}^{-2} \times F\left(\frac{\hbar \omega_{\lambda \mathbf{q}}}{2T}\right) \left(f_{E_{nn'q_z} - \hbar\omega_{\lambda\mathbf{q}}/2} - f_{E_{nn'q_z} + \hbar\omega_{\lambda\mathbf{q}}/2}\right), \quad (2)$$

where

$$E_{nn'q_z} = \hbar\omega_c \kappa_{nn'} + \frac{\hbar^2 q_z^2}{8m} + \frac{m}{2q_z^2}\left[\omega_c(n - n') - \omega_{\lambda\mathbf{q}}\right]^2 \quad (3)$$

and $\kappa_{nn'} = (n + n' + 1)/2$.

Consider a degenerate electron gas, $T \ll \varepsilon_F$. Under an approximation of isotropic phonon modes, $\omega_{\lambda\mathbf{q}} = s_\lambda q$, and equipartition conditions, $\hbar\omega_{\lambda\mathbf{q}} \ll 2T$, an analytical evaluation of the conductivity is possible. For simplicity, consider an interaction of electrons with a single longitudinal acoustic mode ($\lambda = l$) via the deformation potential, $C_{\lambda\mathbf{q}} = \hbar\mathcal{D}^2 q/2\rho s_l$, where $\mathcal{D}$ is the deformation constant and ρ is the material density. Equation (2) then takes the form

$$\sigma_{xx} = \frac{e^2 \mathcal{D}^2 T}{(2\pi)^3 \hbar^2 \rho s_l^2 \omega_c} \int_0^\infty dq_\perp q_\perp^3 \sum_{nn'} \Phi_{nn'}(u) G_{nn'}(q_\perp),$$

$$G_{nn'}(q_\perp) = \int_0^\infty dq_z q_z^{-1} \left(-\frac{\partial f_\varepsilon}{\partial \varepsilon}\right)_{\varepsilon = E_{nn'q_z}}. \quad (4)$$

The nonelasticity of the scattering leading to MPO is described by the phonon frequency entering the last term of $E_{nn'q_z}$. A neglect of this frequency [7] leads to an elastic approximation when the behavior of σ_{xx} is the same as for any elastic scattering mechanism. On the other hand, retaining the phonon frequency in $E_{nn'q_z}$ imposes some constraints that facilitate further calculations. Specifically, since $\hbar\omega_c \ll \varepsilon_F$ and $\hbar\omega_{\lambda\mathbf{q}} \ll \varepsilon_F$, retaining the phonon frequency in Eq. (3) is justified only at $q_z \ll k_F$ and $|n - n'| \ll n + n'$. The first inequality allows us to replace $\omega_{\lambda\mathbf{q}}$ in Eq. (3) by $s_l q_\perp$, with the result

$$G_{nn'}(q_\perp) = \int d\varepsilon \left(-\frac{\partial f_\varepsilon}{\partial \varepsilon}\right) \frac{1}{\sqrt{(\varepsilon - \varepsilon_n^-)(\varepsilon - \varepsilon_{n'}^+)}}, \quad (5)$$

where $\varepsilon_n^\pm = \hbar\omega_c(n + 1/2) \pm \hbar s_l q_\perp/2$. The inequality $|n - n'| \ll n + n'$ leads to a convenient transformation of the integral over $q_\perp$ based on the observation that $\Phi_{nn'}(u) \simeq [2\pi\sqrt{\kappa_{nn'} u}\sqrt{1 - u/4\kappa_{nn'}}]^{-1}$ for $\kappa_{nn'} \gg 1$, and $\Phi_{nn'}(u)$ goes to zero at $u > 4\kappa_{nn'}$. Combining this property with the identity $\int du u \Phi_{nn'}(u) = 2\kappa_{nn'}$, one gets a relation

$$\int_0^\infty du u \Phi_{nn'}(u) G_{nn'}(q_\perp) \simeq \frac{8\kappa_{nn'}}{\pi q_0^3} \int_0^{q_0} dq_\perp \frac{q_\perp^2}{\sqrt{1 - (q_\perp/q_0)^2}} G_{nn'}(q_\perp), \quad (6)$$

with $q_0 = 2\sqrt{2m\omega_c \kappa_{nn'}/\hbar}$, valid at arbitrary n and n'. By noticing that the factors $G_{nn'}(q_\perp)$ essentially depend on $q_\perp$ only when $n \simeq n' \simeq \varepsilon_F/\hbar\omega_c$, one may replace q_0 by $2k_F$. Introducing the scattering angle θ according to $q_\perp = 2k_F \sin(\theta/2)$, one finds

$$\frac{\sigma_{xx}}{\sigma_0} = \frac{3(\hbar\omega_c)^3}{8\varepsilon_F^2} \int_0^\pi \frac{d\theta}{\pi} (1 - \cos\theta) \sum_{nn'} \kappa_{nn'} G_{nn'}(q_\perp),$$

$$\sigma_0 = \frac{e^2 \mathcal{N} \nu_{\text{ph}}}{m\omega_c^2}, \quad \nu_{\text{ph}} = \frac{\sqrt{2} m^{3/2} \varepsilon_F^{1/2} \mathcal{D}^2 T}{\pi \hbar^4 \rho s_l^2}, \quad (7)$$

where σ_0 is the classical (Drude) conductivity presented through the electron density $\mathcal{N} = k_F^3/3\pi^2$ and phonon-assisted scattering rate ν_{ph}. The sums standing in Eq. (7) can be transformed to a double sum over cyclotron harmonics by using the Poisson rule of summation. One needs to retain only the terms which do not contain oscillating functions of ε and, therefore, survive the averaging over energy in Eq. (5). This approximation assumes $\hbar\omega_c \ll 2\pi^2 T$ and corresponds to a neglect of the Shubnikov–de Haas oscillations (SdHOs), which is justified in the MPO region since $\hbar s_l k_F \ll T$. Then

$$\frac{\sigma_{xx}}{\sigma_0} = 1 + \frac{3\hbar\omega}{8\varepsilon_F} \int_0^\pi \frac{d\theta}{\pi} (1 - \cos\theta) \sum_{j=1}^\infty j^{-1} \cos\left(\frac{2\pi j s_l q_\perp}{\omega_c}\right). \quad (8)$$

Calculation of the sum over j leads to the result

$$\frac{\sigma_{xx}}{\sigma_0} = 1 - \frac{3\hbar\omega_c}{16\varepsilon_F} \int_0^\pi \frac{d\theta}{\pi} (1 - \cos\theta) \times \ln\left\{2 - 2\cos\left[\epsilon \sin\frac{\theta}{2}\right]\right\}, \quad (9)$$

where $\epsilon = 4\pi k_F s_l/\omega_c$. Due to dispersion of the phonon spectrum, the acoustic MPOs do not have logarithmic divergences in the maxima, in contrast to the optical-phonon MPOs [2] in the undamped approximation.

The analytical method described above can be extended to include Landau level broadening. Starting from Eq. (1) and applying the approximations of isotropic single-mode phonons under equipartition conditions again leads to Eqs. (5) and (7) with a substitution

$$\frac{1}{\sqrt{(\varepsilon-\varepsilon_n^-)(\varepsilon-\varepsilon_{n'}^+)}} \to \left|\mathrm{Re}\frac{1}{\sqrt{\varepsilon-i\hbar/2\tau-\varepsilon_n^-}}\right|\left|\mathrm{Re}\frac{1}{\sqrt{\varepsilon-i\hbar/2\tau-\varepsilon_{n'}^+}}\right|.$$

As a result, the conductivity is given by Eq. (8) with an extra multiplier d^{2j} under the sum, and $d=\exp(-\pi/\omega_c\tau)$ is the Dingle factor. Instead of Eq. (9), one gets

$$\frac{\sigma_{xx}}{\sigma_0}=1-\frac{3\hbar\omega_c}{16\varepsilon_F}\int_0^\pi\frac{d\theta}{\pi}(1-\cos\theta) \times\ln\{1+d^4-2d^2\cos[\epsilon\sin(\theta/2)]\}. \quad (10)$$

In the elastic scattering approximation, Eq. (10) would give a monotonic dependence on B, $\sigma_{xx}/\sigma_0=1-(3\hbar\omega_c/8\varepsilon_F)\ln(1-d^2)$. For small Dingle factors, $d^2\ll 1$, Eq. (10) leads to

$$\frac{\sigma_{xx}}{\sigma_0}\simeq 1+\frac{3\hbar\omega_c}{4\varepsilon_F}d^2[J_0(\epsilon)-J_1(\epsilon)/\epsilon], \quad (11)$$

where $J_k(\epsilon)$ is the Bessel function. For small magnetic fields, when $\epsilon\gg 1$, the relative amplitude of the oscillations is of the order $(\hbar\omega_c/\varepsilon_F)d^2/\sqrt{\epsilon}$ and scales with B as $\omega_c^{3/2}\exp(-2\pi/\omega_c\tau)$. If the damping at $\omega_c\simeq 2k_F s_l$ is weak, the relative amplitude of the first magnetophonon peak is of the order $\hbar\omega_c/\varepsilon_F$. The inverse quantum lifetime τ^{-1} describing the damping is a sum of partial contributions from different scattering mechanisms.

Since the resistivity due to electron-phonon scattering is $\rho_{xx}=(m\omega_c/e^2\mathcal{N})^2\sigma_{xx}$, the quantity σ_{xx} calculated above determines the oscillating phonon-assisted part of the magnetoresistance. The magnetic field dependence of this contribution, $\rho_{xx}(B)$, normalized to its zero-field value $\rho_{xx}(0)$, is shown in Fig. 2. A good agreement between the results of Eqs. (4) and (9) confirms the reliability of the approximations used in the derivation of Eq. (9). Taking into account the other scattering mechanisms, one can describe the total resistivity ρ_{xx}^{tot} as a sum of different contributions including ρ_{xx}. At low T, the impurity-assisted contribution to ρ_{xx}^{tot} can be much larger than the phonon-assisted one. As a result, the relative amplitude of the magnetoresistance oscillations is strongly reduced because of large background resistance.

In the Bloch-Grüneisen temperature region, $T<\hbar s_l k_F$, the acoustic MPOs coexist with the SdHOs whose period is much smaller than the MPO period. The conductivity averaged over the SdHOs is given by the expression generalizing the result of Eq. (10),

$$\frac{\sigma_{xx}}{\sigma_0}\simeq\mathcal{R}_T-\frac{3\hbar\omega_c}{16\varepsilon_F}\int_0^\pi\frac{d\theta}{\pi}(1-\cos\theta)F\left(\frac{\hbar s_l k_F}{T}\sin\frac{\theta}{2}\right) \times\ln\{1+d^4-2d^2\cos[\epsilon\sin(\theta/2)]\}, \quad (12)$$

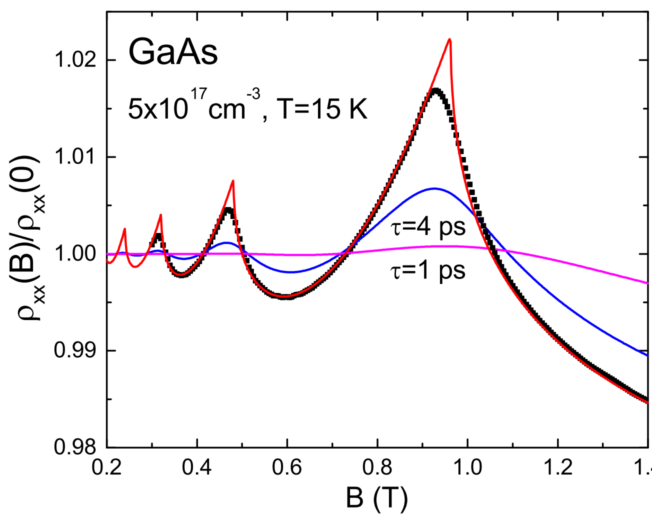

FIG. 2. Phonon-assisted resistivity of GaAs ($s_l=5.14$ km/s) with density $\mathcal{N}=5\times10^{17}$ cm^{-3} (Fermi energy $\varepsilon_F=34.3$ meV) at $T=15$ K. The unmarked plots correspond to the undamped approximation: the results of Eq. (9) (line) and Eq. (4) (points). Two plots marked by finite lifetimes $\tau=4$ ps and $\tau=1$ ps are the results of Eq. (10).

where $\mathcal{R}_T=4\int_0^1 dt t^3 F(\hbar s_l k_F t/T)$ is the factor describing a decrease of the background conductivity at $T<\hbar s_l k_F$. Since the backscattering of electrons is exponentially suppressed in the Bloch-Grüneisen region, the second term in Eq. (12) decreases with lowering T much faster than $\mathcal{R}_T$, so the acoustic MPOs are suppressed.

Phonon-drag thermopower. The drag of electrons by phonons appears because the phonon distribution, in the presence of temperature gradient ∇T, becomes asymmetric in the phonon momentum. The antisymmetric part of the phonon distribution function, determined from a kinetic equation for phonons in the relaxation time approximation, is given by

$$\delta N_{\lambda\mathbf{q}}=\frac{\partial N_{\omega_{\lambda\mathbf{q}}}}{\partial\omega_{\lambda\mathbf{q}}}\frac{\omega_{\lambda\mathbf{q}}}{T}\tau_\lambda\mathbf{u}_{\lambda\mathbf{q}}\cdot\nabla T, \quad (13)$$

where N_ω is the Planck distribution function, $\mathbf{u}$ is the group velocity of phonons, and τ_λ is the phonon lifetime. If ∇T is perpendicular to $\mathbf{B}$, the Seebeck component of phonon-drag thermopower is [24,25]

$$\alpha_{xx}=-\frac{1}{|e|\hbar\ell^2\mathcal{N}}\int\frac{d\mathbf{q}}{(2\pi)^3}\sum_{nn'\lambda}\tau_\lambda(q_\perp/q)^2\Phi_{nn'}(u)C_{\lambda\mathbf{q}} \times F\left(\frac{\hbar\omega_{\lambda\mathbf{q}}}{2T}\right)\int d\varepsilon\int_{-\infty}^{\infty}\frac{dk_z}{2\pi}(f_{\varepsilon-\hbar\omega_{\lambda\mathbf{q}}}-f_\varepsilon) \times A_\varepsilon(\varepsilon_{nk_z})A_{\varepsilon-\hbar\omega_{\lambda\mathbf{q}}}(\varepsilon_{n'k_z-q_z}). \quad (14)$$

In the isotropic single-mode approximation used above, $\alpha_{xx}=-(|e|s_l^2\tau_l\mathcal{N}/T)\rho_{xx}$, so the thermopower behaves in the same way as the resistivity.

Measurements of thermopower, as compared to resistance measurements, are more favorable for the detection of acoustic MPOs since the phonon-drag contribution to thermopower is typically not small compared to the other (diffusive) contribution, and no considerable enhancement of the nonoscillating background is expected.

Since only the transitions with small q_z contribute to acoustic MPOs, the oscillations of the longitudinal transport coefficients σ_{zz} and α_{zz} are expected to be small compared to the oscillations of the transverse transport coefficients considered above.

In summary, it is demonstrated that MPOs in bulk conductors can be caused by a resonance interaction of electrons with acoustic phonons. Experimental verification of the acoustic MPOs is expected to be difficult, mostly because of the exponential suppression of these oscillations due to Landau level broadening determined by the finite quantum lifetime of the electrons. At low temperatures, the lifetime is limited by electron-impurity scattering, while with increasing T, the inelastic mechanisms, electron-electron and electron-phonon scattering, become important. Semiconductor materials can be made pure enough—for example, studies of ultrapure GaAs crystals report [26] electron lifetimes of 25 ps for temperatures 10–20 K. However, in doped semiconductor crystals with an electron density of the order 10^{17}–$10^{18}\mathrm{cm}^{-3}$, the lifetime is expected to be much smaller, because increasing electron density is accompanied with an increasing number of scatterers. To make acoustic MPOs detectable, it is desirable to use materials with a higher Fermi energy without heavy doping, so the MPO region shifts to a higher magnetic field, and to keep the temperature high enough to enable phonon-assisted backscattering of the electrons. For example, if $\varepsilon_F/T \simeq \hbar/\tau m s^2$, which is consistent with the conditions typical for conductors, $\varepsilon_F \gg T > \hbar/\tau \gg m s^2$, no strong suppression of the oscillations is expected, because the Dingle exponent for the main MPO peak is of the order $-\sqrt{\hbar/\tau T}$. If electron-electron scattering dominates, $\hbar/\tau \sim T^2/\varepsilon_F$ and $\sqrt{\hbar/\tau T} \sim \sqrt{T/\varepsilon_F} \ll 1$, ensuring weak damping. The conditions discussed above can be realized in semimetals, such as bismuth crystals [27].

[1] Yu. A. Firsov, V. L. Gurevich, R. V. Parfeniev, and I. M. Tsidil'kovskii, in *Landau Level Spectroscopy*, edited by G. Landwehr and E. I. Rashba (North-Holland/Elsevier, Amsterdam, 1991), Chap. 20.
[2] V. L. Gurevich and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. **40**, 198 (1961) [Sov. Phys. JETP **13**, 137 (1961)].
[3] S. M. Puri and T. H. Geballe, Bull. Am. Phys. Soc. **8**, 309 (1963); Yu. A. Firsov, V. L. Gurevich, R. V. Parfeniev, and S. S. Shalyt, Phys. Rev. Lett. **12**, 660 (1964).
[4] In the crystals where electrons occupy several valleys, other high-energy phonon modes with wave vectors comparable to the size of the Brillouin zone can also contribute to magnetophonon resonances [1]. The physics of this phenomenon is not basically different from that of single-valley optical-phonon MPOs.
[5] P. N. Argyres, J. Phys. Chem. Solids **4**, 19 (1958).
[6] E. N. Adams and T. D. Holstein, J. Phys. Chem. Solids **10**, 254 (1959).
[7] A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskii, Zh. Eksp. Teor. Fiz. **48**, 204 (1965) [Sov. Phys. JETP **21**, 136 (1965)].
[8] J. R. Barker, J. Phys. C **5**, 1657 (1972).
[9] M. A. Zudov, I. V. Ponomarev, A. L. Efros, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **86**, 3614 (2001).
[10] J. Zhang, S. K. Lyo, R. R. Du, J. A. Simmons, and J. L. Reno, Phys. Rev. Lett. **92**, 156802 (2004).
[11] A. A. Bykov, A. K. Kalagin, and A. K. Bakarov, JETP Lett. **81**, 523 (2005).
[12] W. Zhang, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **100**, 036805 (2008).
[13] X. L. Lei, Phys. Rev. B **77**, 205309 (2008).
[14] A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **102**, 086808 (2009).
[15] A. A. Bykov and A. V. Goran, JETP Lett. **90**, 578 (2009).
[16] O. E. Raichev, Phys. Rev. B **80**, 075318 (2009).
[17] A. A. Bykov, A. V. Goran, and S. A. Vitkalov, Phys. Rev. B **81**, 155322 (2010).
[18] O. E. Raichev, Phys. Rev. B **81**, 165319 (2010).
[19] O. E. Raichev, Phys. Rev. B **81**, 195301 (2010).
[20] I. A. Dmitriev, R. Gellmann, and M. G. Vavilov, Phys. Rev. B **82**, 201311(R) (2010).
[21] A. T. Hatke, M. A. Zudov, L. N. Pfeiffer, and K. W. West, Phys. Rev. B **84**, 121301 (2011).
[22] I. A. Dmitriev, A. D. Mirlin, D. G. Polyakov, and M. A. Zudov, Rev. Mod. Phys. **84**, 1709 (2012).
[23] O. E. Raichev, Phys. Rev. B **91**, 235307 (2015).
[24] S. M. Puri, Phys. Rev. **139**, A995 (1965).
[25] S. K. Lyo, Phys. Rev. B **40**, 6458 (1989).
[26] M. Kozhevnikov, B. M. Ashkinadze, E. Cohen, and A. Ron, Phys. Rev. B **52**, 17165 (1995).
[27] S. G. Bompadre, C. Biagini, D. Maslov, and A. F. Hebard, Phys. Rev. B **64**, 073103 (2001).