

# Precise experimental determination of electrical equivalent circuit parameters for ultrasonic piezoelectric ceramic transducers from their measured characteristics

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## ARTICLE INFO

### Keywords:

Ultrasonic transducer  
Piezoelectric ceramic  
Electrical equivalent circuit  
Impedance characteristic  
Resonant frequency

## ABSTRACT

The letter is devoted to a method for determining electric equivalent circuit parameters of piezoelectric ceramic transducers from their resonance characteristics. These parameters represent equivalents of mechanical damping, effective weight of the oscillating element, compliance of the mechanical element and static capacitance associated with the transducer material. This method is considered cost efficient, focuses on transducers commonly available on the market and is presented on a specific transducer – transmitter. The acquired parameters are important for performing subsequent simulations and defining their suitability for a particular application. However, these parameters are not included in the usual technical documentation of the commercially available piezoelectric ceramic transducers. The key part of this letter is an iterative process of finding the parameters. Before starting the iteration process, it is very important to set default values of these parameters. Then, a linearized system of equations, based on expressing a real part of the impedance, is used to determine these parameters. The advantage of this method is its simplicity to measure only the two above-mentioned characteristics, from which the required parameters can be then calculated. Thus, there is no need for expensive circuit analyzers, which are able to determine these parameters directly. Although the iterative process includes a large number of steps, and therefore can be relatively time consuming, it can be performed on a commercially available computer technology.

## 1. Introduction

The electronic component market offers piezoelectric ceramic transducers that can be used for various applications. A specific application use depends on their dimensions, material, resonance frequency and design in which the transducer will work. Ultrasonic piezoelectric ceramic transducers are most commonly used in sensor technology applications. An example of such transducers is shown in Fig. 1 [1,2].

This type of transducer can work as a transmitter, receiver or it can perform both functions. In some applications, it is represented by an electrical equivalent circuit. Its simplified schematic diagram is in Fig. 2. This diagram shows the most commonly used Butterworth-Van-Dyke unloaded circuit, which is also considered in this letter [3].

Values of the individual components in the diagram  $R_S$ ,  $L_S$ ,  $C_S$  and  $C_P$  have its meaning, which is described in detail in [4–8]. This simplified model consists of two branches, mechanical and electrical. The

mechanical part (mechanical oscillator) is represented by  $R_S$ ,  $L_S$  and  $C_S$  components.  $R_S$  represents mechanical damping,  $L_S$  represents an effective weight of the mechanical oscillator and  $C_S$  represents mechanical compliance. The electrical part in this circuit is the  $C_P$  component. The dielectric of this capacitor is the piezoelectric material of the transducer.

It is important to know the values of these parameters when developing applications for sensors. When using these parameters, it is then possible to create a model, analyze its behavior in the whole system, and determine the suitability of a particular transducer for a particular application. For commercially available transducers, their technical documentation does not include the above-mentioned parameters [9]. This letter focuses on determination of the electrical equivalent circuit parameters of the piezoelectric ceramic transducer from its measured characteristics.

Possible algorithm for determining and optimizing the above-

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<https://doi.org/10.1016/j.ultras.2020.106341>

Received 31 August 2020; Received in revised form 26 November 2020; Accepted 14 December 2020

Available online 7 January 2021

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Fig. 1. An example of piezoelectric ceramic transducers.

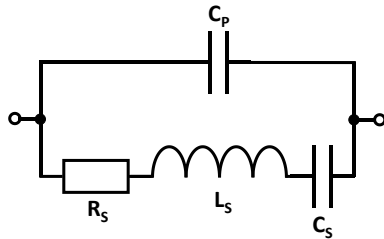


Fig. 2. Schematic diagram of an electrical equivalent circuit of the piezoelectric ceramic transducer.

mentioned individual parameters was published in [10]. When optimizing the individual parameters, there is a need to use a specialized tool in the MATLAB® development environment, called Optimization Toolbox. Before implementation of this tool, it is necessary to adjust the initial iteration conditions, it is not possible to use the calculated parameters directly from the relevant formulas. As some of the above-mentioned parameters are very low, there is a divergence in the calculation. The derived parameters are then meaningless.

The new algorithm presented in this letter offers a different approach for calculating these parameters. Then, these parameters can be used directly in the optimization algorithm. There is no need for a specialized tool for the optimization algorithm, thus, the presented algorithm is simple in terms of technical implementation. The presented algorithm also shows significantly better results in the parameters of the serial and parallel resonant frequencies of the derived model.

The above-mentioned electrical equivalent circuit of a piezoelectric ceramic transducer is simplified. In the complete electrical equivalent circuit of the provided transducer, there are other parallel branches to the mechanical part. These branches are serial resonant circuits representing element oscillations at higher frequencies, so-called harmonic frequencies. Piezoelectric ceramic transducers for sensor technology are often designed for their first harmonic frequency, so the model is simplified in this way [6]. It is also assumed that for future applications, the transducer would be only excited by signals with a frequency in close proximity of that frequency. Thus, the model will have a very narrow band.

## 2. Measured characteristics of the piezoelectric ceramic transducer

The MCUST16A40S12R0 transducer from MULTICOMP Company [9] was chosen as a sample for determining the required parameters. This transducer performs function of a transmitter. Significant characteristics of these circuits are resonance characteristics, which are possible to measure by an impedance analyzer. These characteristics, for the mentioned transducer, were measured by Analog Discovery 2 impedance analyzer from DIGILENT Company [11]. The measurement was performed in the laboratory at the temperature of 28 °C. The measured characteristics are shown in Fig. 3 and Fig. 4 (blue curve).

Technical documentation of the mentioned piezoelectric ceramic

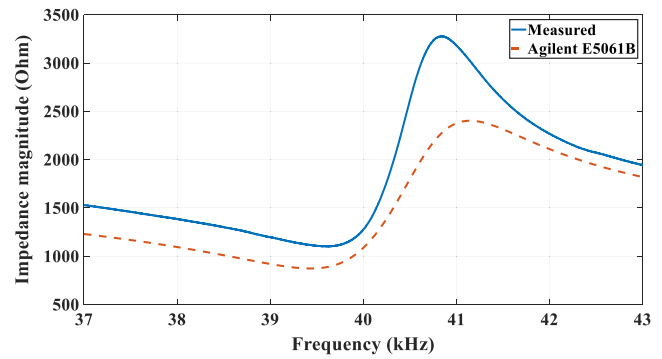


Fig. 3. Impedance magnitude resonance characteristic.

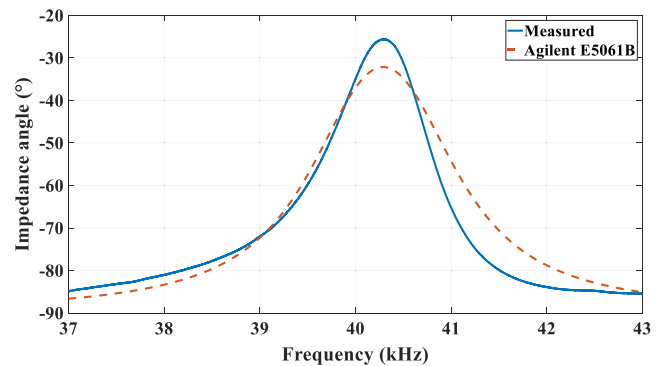


Fig. 4. Impedance angle resonance characteristic.

transducer states that it operates at the centre frequency of  $40 \pm 1$  kHz, so the characteristics were measured in the near range of 40 kHz. The other significant technical data are following: construction is an open structure, capacitance at 1 kHz is  $2.500 \text{ pF} \pm 25\%$ , directivity is  $50^\circ$ , operating and storage temperature range is from  $-35^\circ\text{C}$  to  $85^\circ\text{C}$ , detectable range is from 0.7 m to 18 m and housing material is aluminium. This transducer is in Fig. 1, on the right. The characteristics are distinguished by specific values or areas, which are described in detail in [12]. Significant points that can be used for further calculations are clear from the graph showing the magnitude resonance characteristic. The local minimum of this graph corresponds to the serial resonance. It is the resonance area of the mechanical oscillator - an oscillating element of a piezoelectric ceramic transducer. The local maximum of the graph corresponds to the parallel resonance. It is the area of resonance, where also a static capacitance has an impact. Shape of these graph curves and the position of the mentioned points, as well as the position of the maximum for impedance angle resonance characteristic, are influenced by some physical conditions. For example, temperature has a significant effect.

Different shapes of these characteristics are in [13–16]. Based on these characteristics, the electrical equivalent circuit parameters  $R_s$ ,  $L_s$ ,  $C_s$ ,  $C_p$  were derived. There are other circuit analyzers that determine these parameters directly, however, after back-calculation, the characteristics did not match very well as is shown in Fig. 3 and Fig. 4 (red dashed line). Instrument E5061B [17] from Agilent Company was used for determination of these parameters. The instrument determined parameters are as follows:  $R_s = 1323 \Omega$ ,  $L_s = 0.1359 \text{ H}$ ,  $C_s = 1.1726 \cdot 10^{-10} \text{ F}$ ,  $C_p = 2.7005 \cdot 10^{-9} \text{ F}$ . In addition, these circuit analyzers are very expensive. This instrument was used only to demonstrate that even though there are instruments that determine these parameters directly, their accuracy is not sufficient as it is possible to see from their back-calculated characteristics. The relative errors for the significant points of their characteristics were chosen as a proof of the parameters determination accuracy, which are in Table 1. The basis for an analysis and

**Table 1**

Relative errors for significant points of the characteristics.

Case	$\delta_{ZS}$ (%)	$\delta_{fS}$ (%)	$\delta_{ZP}$ (%)	$\delta_{fP}$ (%)	$\delta_{\varphi m}$ (%)	$\delta_{f\varphi m}$ (%)
E5061B	20.73	0.48	26.70	0.73	25.39	0.00

results comparison were above mentioned resonance characteristics. These characteristics, represented by blue line in all graphs, were measured by the analog Discovery 2 impedance analyzer.

$\delta_{ZS}$  is the relative error of the impedance magnitude at the serial resonance frequency,  $\delta_{fS}$  is the relative error of the serial resonance frequency,  $\delta_{ZP}$  is the relative error of the impedance magnitude at the parallel resonance frequency,  $\delta_{fP}$  is the relative error of the parallel resonance frequency,  $\delta_{\varphi m}$  is the relative error of the maximum of the impedance angle,  $\delta_{f\varphi m}$  is the relative error of the frequency at the maximum of the impedance angle.

### 3. Algorithm for determination of the electrical equivalent circuit parameters

To derive the above-mentioned parameters, an equation for a real part of the impedance dependence on frequency  $R(\omega)$  [1] [12] was used.

$$R(\omega) = \frac{1}{\omega C_P} \frac{\omega \omega_S Q_m (\omega_P^2 - \omega_S^2)}{\omega^2 \omega_S^2 + Q_m^2 (\omega^2 - \omega_P^2)^2}, \quad (1)$$

where  $\omega$  is the angular frequency,  $C_P$  is the capacitance from the schematic diagram of the electrical equivalent circuit from Fig. 2,  $\omega_S$  is the serial angular resonance frequency corresponding to the minimum of the characteristic in Fig. 3,  $Q_m$  is the mechanical quality factor and  $\omega_P$  is the parallel angular resonance frequency corresponding to the maximum of the characteristic in Fig. 3. This equation was used as it enables, after some adjustment, to create a linearized equation system for further calculations in the easiest manner. The modified equation is then as follows:

$$\frac{1}{R(\omega)} = \frac{\omega^2 \omega_S C_P}{Q_m (\omega_P^2 - \omega_S^2)} + \frac{C_P Q_m (\omega^2 - \omega_P^2)^2}{\omega_S (\omega_P^2 - \omega_S^2)}, \quad (2)$$

The following substitution was used,  $C_P/Q_m = x$  and  $C_P Q_m = y$ , to linearize this equation. Then, the final linearized default system of equations is as follows:

$$\begin{aligned} \frac{1}{R(\omega(1))} &= \frac{(\omega(1))^2 \omega_S}{\omega_P^2 - \omega_S^2} x + \frac{((\omega(1))^2 - \omega_P^2)^2}{\omega_S (\omega_P^2 - \omega_S^2)} y, \\ \frac{1}{R(\omega(2))} &= \frac{(\omega(2))^2 \omega_S}{\omega_P^2 - \omega_S^2} x + \frac{((\omega(2))^2 - \omega_P^2)^2}{\omega_S (\omega_P^2 - \omega_S^2)} y, \\ &\vdots \\ \frac{1}{R(\omega(n))} &= \frac{(\omega(n))^2 \omega_S}{\omega_P^2 - \omega_S^2} x + \frac{((\omega(n))^2 - \omega_P^2)^2}{\omega_S (\omega_P^2 - \omega_S^2)} y. \end{aligned} \quad (3)$$

It is a system of  $n$  equations with two unknown, where  $x$  and  $y$  are the unknown in this case. Then,  $C_P$  and  $Q_m$  parameters were calculated from these values. Positive values of these parameters were calculated as negative values have no practical meaning. The number of equations in the system corresponded to the number of samples from the above characteristics, in the area of the serial resonance frequency. The area of bandwidth was selected for frequencies corresponding to the impedance magnitude of  $\sqrt{2Z_{min}}$  together with the proportional  $R(\omega)$ . Then, adjusted equation from [12] was used to calculate the capacitance  $C_S$ :

$$C_S = \frac{f_P^2 C_P - f_S^2 C_P}{f_S^2}, \quad (4)$$

where  $f_P$  is the parallel resonance frequency and  $f_S$  is the serial resonance

frequency.

The adjusted equation from [12] was again used to calculate the inductance  $L_S$ :

$$L_S = \frac{1}{4\pi^2 f_S^2 C_S}. \quad (5)$$

For calculation of the resistance  $R_S$ , another modified equation from [12] was applied:

$$R_S = \frac{2\pi f_S L_S}{Q_m}. \quad (6)$$

MATLAB® development environment was used to solve the linearized equation system and then also for further calculations. The resultant values of the electrical equivalent circuit parameters of the piezoelectric ceramic transducer – transmitter were as follows:  $R_S = 908 \Omega$ ,  $L_S = 0.0915 \text{ H}$ ,  $C_S = 1.7634 \cdot 10^{-10} \text{ F}$  and  $C_P = 2.8048 \cdot 10^{-9} \text{ F}$ . The red dashed curve in Fig. 5 and Fig. 6, depicts characteristics, which were calculated from acquired above-mentioned electrical parameters.

Original measured characteristics were plotted in blue color into the graph for comparison with the calculated parameters (red dashed line). The graph shows that the characteristics do not match, however, their shape is very similar. As a proof of accuracy of the individual parameter calculations for the electrical equivalent circuit, the relative errors of the characteristics significant points were evaluated again and they are in Table 2.

This fact was used for further calculations. These calculated parameters were set as the initial condition for the iterative algorithm to further determine the optimal electrical equivalent circuit parameters of the piezoelectric ceramic transducer – transmitter.

The initial and final iteration values and iteration steps were set for individual parameters  $R_S$ ,  $L_S$ ,  $C_S$  and  $C_P$ . At each step of the iterative algorithm, the array of impedance magnitude (based on the schematic diagram in Fig. 2) was calculated for the whole measured range of frequencies. The following formula was used to calculate the impedance magnitude:

$$Z = \text{abs} \left( \frac{1}{\frac{1}{R_S + jX_{LS} - jX_{CS}} + \frac{1}{-jX_{CP}}} \right), \quad (7)$$

where  $Z$  is the impedance magnitude,  $X_{LS}$  is the reactance corresponding to the inductance  $L_S$ ,  $X_{CS}$  is the reactance corresponding to the capacitance  $C_S$ ,  $X_{CP}$  is the reactance corresponding to the capacitance  $C_P$  and  $j$  is an imaginary unit.

Furthermore, an array of the impedance magnitude deviations from the measured values was calculated. Deviation of the impedance magnitude from the measured value was calculated by formula:

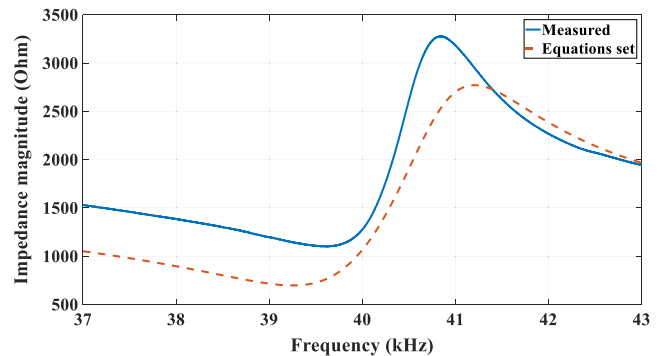
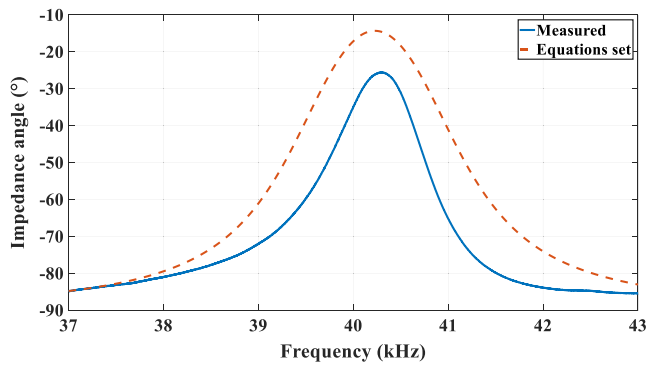


Fig. 5. Impedance magnitude resonance characteristic after calculation from the linearized equations system (red dashed line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 6.** Impedance angle resonance characteristic after the calculation from the linearized equations system (red dashed line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Table 2**

Relative errors for significant points of the characteristics.

Case	$\delta_{ZS}$ (%)	$\delta_{fS}$ (%)	$\delta_{ZP}$ (%)	$\delta_{fP}$ (%)	$\delta_{\varphi_m}$ (%)	$\delta_{f_{\varphi m}}$ (%)
Equations	36.64	0.94	15.44	0.86	44.14	0.16

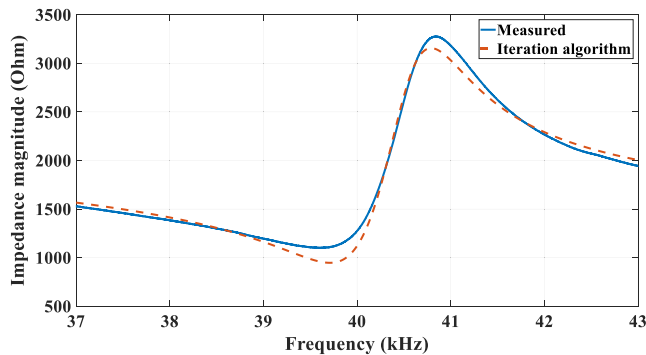
$$\sigma = \text{abs}(Z_i - Z), \quad (8)$$

where  $\sigma$  is the deviation of the impedance magnitude from the measured value,  $Z_i$  is the impedance magnitude for a particular iterative step and  $Z$  is the measured impedance magnitude.

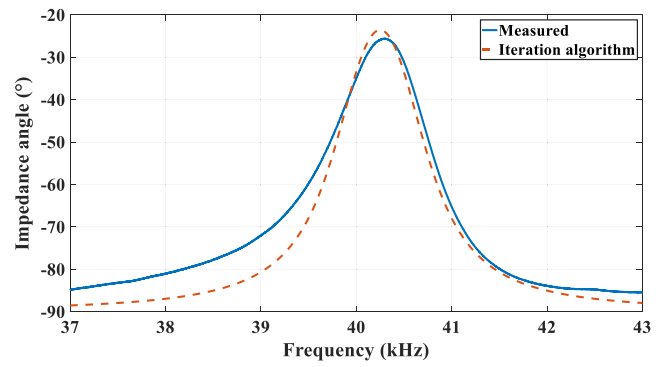
As a criterion for the best match of the results obtained by the iterative algorithm and the measured characteristics, a deviation sum minimum of the impedance magnitude from the measured value  $\min(\sum(\sigma))$  was chosen. This minimum was assigned to the parameters of the corresponding iteration step. The obtained parameters from the iterative algorithm are:

$R_S = 1330 \, \Omega$ ,  $L_S = 0.2300 \, \text{H}$ ,  $C_S = 6.9000 \cdot 10^{-11} \, \text{F}$  and  $C_P = 2.2700 \cdot 10^{-9} \, \text{F}$ . The parameters are again depicted in Fig. 7 and Fig. 8 for comparison, using red dashed line.

The above depicted characteristics show that the iterative algorithm has greatly improved the determination of the electrical equivalent circuit parameters of the piezoelectric ceramic transducer – transmitter. The characteristic courses calculated from the determined parameters by the iterative algorithm largely coincide with the original measured characteristics. As a proof of accuracy of the individual parameter calculations for the electrical equivalent circuit, the relative errors of the characteristics significant points were evaluated again and they are in Table 3.



**Fig. 7.** Impedance magnitude resonance characteristic after the iterative algorithm (red dashed line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 8.** Impedance angle resonance characteristic after the iterative algorithm (red dashed line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

**Table 3**

Relative errors for significant points of the characteristics.

Case	$\delta_{ZS}$ (%)	$\delta_{fS}$ (%)	$\delta_{ZP}$ (%)	$\delta_{fP}$ (%)	$\delta_{\varphi_m}$ (%)	$\delta_{f_{\varphi m}}$ (%)
Iteration	13.91	0.21	3.69	0.15	7.81	0.10

**Table 4**

Significant points of characteristics for the piezoelectric ceramic transducer – transmitter.

Case	$Z_S$ ( $\Omega$ )	$f_S$ (Hz)	$Z_P$ ( $\Omega$ )	$f_P$ (Hz)	$\varphi_m$ (°)	$f_{\varphi m}$ (Hz)
Measured	1100	39,624	3277	40,850	−25.6	40,292
E5061B	872	39,434	2402	41,147	−32.1	40,290
Equations	697	39,252	2771	41,203	−14.3	40,227
Iteration	947	39,709	3156	40,790	−23.6	40,250

**Table 5**

Relative errors for significant points of the characteristics.

Case	$\delta_{ZS}$ (%)	$\delta_{fS}$ (%)	$\delta_{ZP}$ (%)	$\delta_{fP}$ (%)	$\delta_{\varphi_m}$ (%)	$\delta_{f_{\varphi m}}$ (%)
E5061B	20.73	0.48	26.70	0.73	25.39	0.00
Equations	36.64	0.94	15.44	0.86	44.14	0.16
Iteration	13.91	0.21	3.69	0.15	7.81	0.10

Thus, the resonance frequencies and their corresponding impedance magnitudes, and impedance angle maximum and its corresponding frequency, were selected as significant points of the characteristics. For comparison, an example of these measured, computed and optimized points of the characteristics is shown in Table 4.

$Z_S$  is the impedance magnitude at the serial resonance frequency,  $f_S$  is serial resonance frequency,  $Z_P$  is the impedance magnitude at the parallel resonance frequency,  $f_P$  is parallel resonance frequency,  $\varphi_m$  is the maximum of impedance angle and  $f_{\varphi m}$  is the frequency at maximum of the impedance angle.

The result accuracy of the required parameters, using the instrument Agilent E5061B, the linearized equation set and the iterative algorithm, show relative errors of these significant points. For clear comparison and evaluation of the accuracy for individual measurements, calculations and optimizations, Table 5 was created showing the relative errors for significant points of the characteristics.

#### 4. Conclusion

This letter presented a method for determining electrical equivalent circuit parameters for a piezoelectric ceramic transducer. Derivation of these parameters was shown on a specific transducer–transmitter, which

is commonly available on the electronic devices market. The accuracy of these derived parameters was also analyzed and presented. The aim was to bring the derived characteristics as close as possible to the measured characteristics using back-calculation from the derived parameters. This method can be also used for other piezoelectric ceramic transducers that are commercially available on the market, performing the function of transmitter, receiver or both. The available technical documentation does not include the required parameters, however, these parameters are very important in the simulation processes to determine the suitability of their implementation in specific applications, or to determine their limitations.

The presented method also allows determining parameters of the transducers in more cost effective manner than circuit analyzers, using only resonance characteristics. There is no need for analyzers that determine these parameters directly. It was found that even expensive circuit analyzers cannot determine the parameters as well as the above-mentioned characteristics. The described method of calculating the electrical equivalent circuit parameters of a piezoelectric ceramic transducer for the initial iteration conditions and the presented optimization process show a different approach to the approach presented so far in the above-mentioned publications. From the calculated parameters, it was possible to choose the initial iteration conditions without significant modifications. No specialized tool was needed for the optimization process of determining the parameters. It was possible to come up with basic programming structures, so that the used algorithm was very simple and usable in any development environment. This simple algorithm also showed significantly better results than when applying the method from the mentioned publications. Only the relative errors of the serial and parallel resonance frequencies could be compared. Using the method from the mentioned publications, the relative error of the serial resonance frequency was 1.72% and the relative error of the parallel resonance frequency was 1.55%. Using the method presented in this letter, with the measured data, the relative error of the serial resonance frequency was 0.21% and the relative error of the parallel resonance frequency was 0.15%.

The computing processes in each step of the algorithm are not demanding, however, large number of iterative steps was executed. In this particular case presented in this letter, there were 145 955 706 steps. Performing such a number of computational steps is relatively time consuming, however, it is convenient as most commercially available computing technologies are suitable.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgments

The work presented in this letter was supported by the Czech

Republic Ministry of Defence – University of Defence development program “Research of Sensor and Control Systems to Achieve Battlefield Information Superiority”, and by the Czech Republic Ministry of Education, Youth and Sports – University of Defence student research program “Implementation of Modern Technologies in Avionic Systems”.

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