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Paper:

Spiral Dynamics Inspired Optimization

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We recently proposed a new multipoint search method for 2-dimensional continuous optimization problems based on an analogy of spiral phenomena called 2-dimensional spiral optimization. Focused spiral phenomena, which appear frequently in nature, are approximated to logarithmic spirals. Two-dimensional spiral optimization used a feature of logarithmic spirals. In this paper, we propose *n*-dimensional spiral optimization by extending the 2-dimensional one. The *n*-dimensional spiral model is constructed based on rotation matrices defined in *n*-dimensional space. Simulation results for different benchmark problems show the effectiveness of our proposal compared to other metaheuristics.

Keywords: metaheuristics, spiral phenomena, multipoint search, global optimization, evolutionary computation

1. Introduction

Metaheuristics is a heuristic approximation framework for continuous or discontinuous global optimization problems. A large number of metaheuristics methods are constructed based on the analogy of natural phenomena, such as biological evolution (Genetic Algorithm: GA [1]), birds flocking or fish schooling (Particle Swarm Optimization: PSO [2, 3]) and the behavior of ants seeking a path (Ant Colony Optimization: ACO [4, 5]). These methods can find good approximated global solutions without demanding strict conditions such as differentiability or fatal evaluation times of objective functions, so they are recognized as practical and versatile global optimization methods.

We recently proposed a new multipoint metaheuristics search method for 2-dimensional continuous optimization problems based on the analogy of spiral phenomena in nature, called 2-dimensional spiral optimization [6]. Focused spiral phenomena are approximated to logarithmic spirals, such as shown in **Fig. 1**, which frequently appear in nature, such as whirling currents, low pressure fronts, nautilus shells and arms of spiral galaxies. A remarked point about logarithmic spirals is that their discrete processes generating spirals can realize effective behavior in metaheuristics. Two-dimensional spiral optimization uses

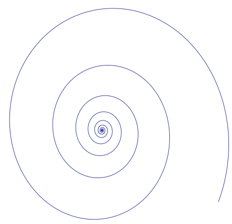


Fig. 1. Logarithmic spiral.

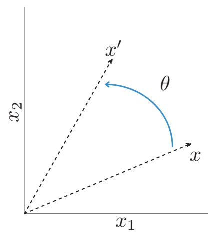


Fig. 2. Rotation on x_1-x_2 plane.

the feature of logarithmic spirals.

We propose *n*-dimensional spiral optimization using a design philosophy of 2-dimensional optimization and constructing an *n*-dimensional spiral model. This paper is organized as follows. Section 2 reviews 2-dimensional optimization where the representation of a 2-dimensional spiral model is redefined for constructing an *n*-dimensional one. Section 3 defines rotation in *n*-dimensional space and proposes *n* dimensional spiral optimization after constructing an *n* dimensional spiral model based on rotation. Section 4 verifies the effectiveness of our proposal through numerical experiments for various benchmark problems compared to two well-known methods PSO methods that are representative metaheuristics based on analogy. Section 5 presents the conclusions drawn from this paper.

2. Two-Dimensional Spiral Optimization

This section reviews 2-dimensional spiral optimization [6] for preparing an *n*-dimensional spiral optimization at the next section.

2.1. Two-Dimensional Spiral Model

Rotating point x in 2-dimensional orthogonal coordinates to the left around the origin by θ makes x' written as

$$x' = R^{(2)}_{1,2}(\theta)x$$

whose rotation matrix

$$R^{(2)}_{1,2}(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad \ldots\ldots\ldots (1)$$

The rotation image is shown in **Fig. 2**.

Using this rotation matrix $R^{(2)}_{1,2}(\theta)$, we formulate the following discrete logarithmic spiral models which gen-

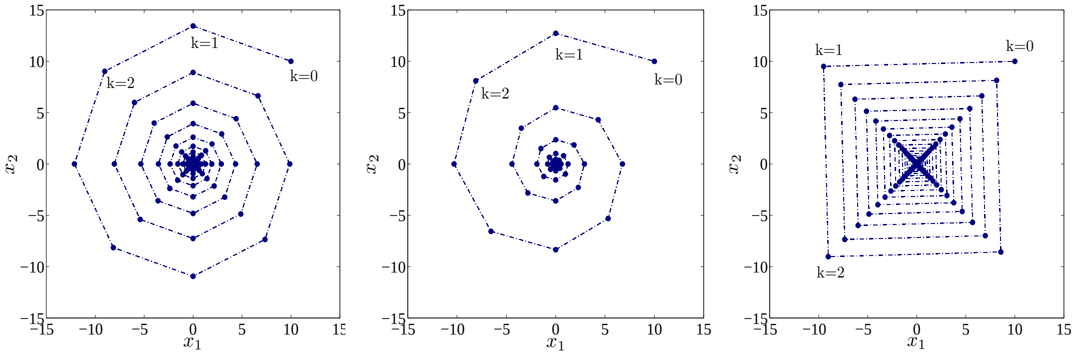


Fig. 3. Illustrations of Eq. (2).

Table 1. Parameters for illustrations.

Case (a)	$r = 0.95,\ \theta = \pi/4$
Case (b)	$r = 0.90,\ \theta = \pi/4$
Case (c)	$r = 0.95,\ \theta = \pi/2$

erates a point converging at the origin from arbitrary initial point x_0 on the x_1-x_2 plane while discretely drawing a logarithmic spiral.

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = rR^{(2)}_{1,2}(\theta) \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} := S_2(r,\theta)x(k),\ x(0) = x_0, \quad . \ . \ . \quad (2)$$

where $0 \leq \theta < 2\pi$ is a rotation angle around the origin at each k and $0 < r < 1$ is a convergence rate of distance between a point and the origin at each k. $S_2(r,\theta)$ is a stable matrix from the range of r and eigenvalues $\lambda_i \in \mathbb{C}$, $i = 1,2$ such that $|\lambda_i| = 1$ of rotation matrix $R^{(2)}_{1,2}(\theta)$.

Note that this spiral model has an expression different so that parameter roles are clearer than the model used in [6], but they are essentially the same and have the same behavior.

Equation (2) with parameters of **Table 1** is shown in **Fig. 3**. These show spiral model behavior and the roles of two parameters.

Spiral model Eq. (2) does not have enough flexibility for applications because it has the center only at the origin. We thus enhance the spiral model Eq. (2) to have the center at an arbitrary point $x^\star$ as follows:

$$x(k+1) = S_2(r,\theta)x(k) - (S_2(r,\theta) - I_2)x^\star, \quad . \ . \quad (3)$$

derived by translating the origin of Eq. (2) toward arbitrary point $x^\star$. The convergence of the trajectory at $x^\star$ is established because Eq. (3) is transformed into $e(k+1) = S_2(r,\theta)e(k)$ by using error variables $e(k) = x(k) - x^\star$.

2.2. Spiral Model Diversification and Intensification

We next show the possibility spiral model Eq. (3) has for being a good search model for optimization problems from the standpoint of a representative metaheuristics strategy.

In metaheuristics search strategies, a well-known effective strategy holds that search point dynamics should have diversification in the early phase and intensification in the late phase during a search [7]:

1. Diversification: Strategy for searching for better solutions by searching a wide region coarsely.
2. Intensification: Strategy for searching for better solutions by searching around a good solution intensively.

This strategy from diversification to intensification works especially well for practical problems that have a multipeak structure such that better solutions exist around good solutions. Namely, diversification in the early phase can find regions having a high possibility that better solutions exist, and intensification in the last stage can intensively search for much better solutions in the region found in the early stage.

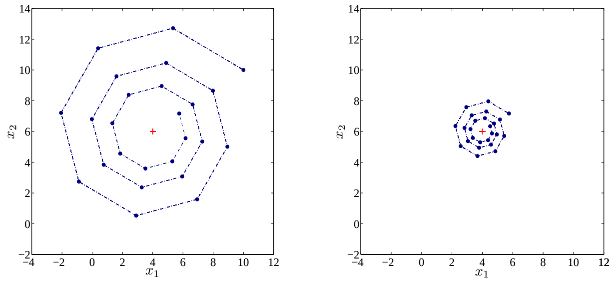
Diversification and intensification are opposite concepts and many in metaheuristics methods try to achieve this strategy by controlling or scheduling search situations to improve performance [7]. The spiral model Eq. (3) realizes this strategy naturally and individually by setting the arbitrary center as a good point because, with logarithmic spirals, by nature, the distance between a point and the center exponentially converges to zero churning rotation.

Figures 4(a) and **(b)**, for example, show trajectories of the first 25 points and the last 25 points respectively when simulating Eq. (3) until 50 steps with the parameters in Case (a) in **Table 1**, $x_0 = (10,10)$ and $x^\star = (4,6)$. These figures show that the behavior of the spiral model Eq. (3) has diversification in the early phase and intensification at the center in the late phase.

We guess that this spiral model Eq. (3) can grow into a good search model that structurally suites a search by setting the arbitrary center as a good point.

2.3. Two-Dimensional Spiral Optimization Algorithm

Many metaheuristics methods, such as GA, PSO, and ACO, use multipoint search with interaction. Representa-



(a) First spiral (25 steps) (b) Last spiral (25 steps)

Fig. 4. Former and latter trajectories of Eq. (3) in 50 steps.

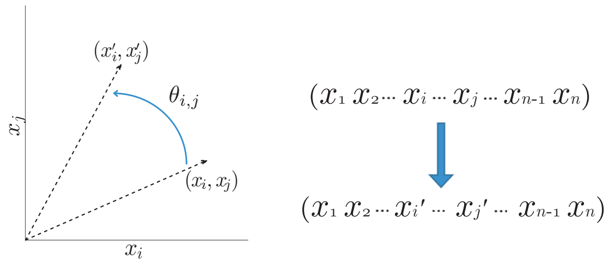


Fig. 5. Rotation on x_i-x_j plane in n-dimensional space.

tive interactions among search points are crossing in GA and using the global best solution in PSO. The search behavior generated by interactions contribute to achieving global and effective search. Extending the spiral model Eq. (3) to a multipoint search model with a proper interaction is thus a valid approach to constructing better metaheuristics.

The multipoint search model using Eq. (3) is formulated as follows:

$$x_i(k+1) = S_2(r,\theta)x_i(k) - (S_2(r,\theta) - I_2)x^\star, \quad . \tag{4}$$

$i = 1,2,\ldots,m$, with the common center $x^\star$ set as the best solution among all search points during a search. Namely, $x^\star$ becomes an interaction.

Using the multipoint and adding the interaction mean the following:

1. Interaction: Contribution to realizing intensification that should be done around a good solution.
2. Multipoint: Contribution to enhancing both intensification and diversification of the spiral model.

In a minimization problem, the algorithm based on the multipoint model Eq. (4) is shown below. Search points drawing spiral trajectories toward common center $x^\star$ that is the best solution, which can naturally realize the strategy from diversification to intensification, can be expected to search for a better solution.

The effectiveness of this method was confirmed by numerical experiments for 2-dimensional benchmark problems [6].

Algorithm of 2-Dimensional Spiral Optimization

Step 0: Preparation
Select the number of search points $m \geq 2$, parameters $0 \leq \theta < 2\pi$, $0 < r < 1$ of $S_2(r,\theta)$, and maximum iteration number $k_{\max}$. Set $k = 0$.

Step 1: Initialization
Set initial points $x_i(0) \in \mathbb{R}^2, i = 1,2,\ldots,m$ in the feasible region, and center $x^\star$ as $x^\star = x_{i_g}(0)$, $i_g = \arg\min_i f(x_i(0))$ $i = 1,2,\ldots,m$.

Step 2: Updating x_i
$x_i(k+1) = S_2(r,\theta)x_i(k) - (S_2(r,\theta) - I_2)x^\star$, $i = 1,2,\ldots,m$.

Step 3: Updating $x^\star$
$x^\star = x_{i_g}(k+1)$, $i_g = \arg\min_i f(x_i(k+1))$, $i = 1,2,\ldots,m$.

Step 4: Checking Termination Criterion
If $k = k_{\max}$ then terminate. Otherwise, set $k = k+1$, and return to Step 2.

3. *n*-Dimensional Spiral Optimization

Here we propose n-dimensional spiral optimization based on the philosophy that constructed 2-dimensional spiral optimization in Section 2.

3.1. Rotation in *n*-Dimensional Space

Rotation of a point in n-dimensional orthogonal coordinates can be represented by composing 2-dimensional rotations defined in n-dimensional space [8]. A 2-dimensional rotation in n-dimensional space means the operation of only two elements on the x_i-x_j plane of a point to the left around the origin by $\theta_{i,j}$ and let other elements except x_i, x_j invariable as shown in **Fig. 5**. Each 2-dimensional rotation matrix in n-dimensional space [8] is defined as follows.

$$R^{(n)}_{i,j}(\theta_{i,j}) :=$$

$$\begin{array}{c} \\ i \\ \\ j \\ \\ \end{array}
\begin{array}{c} \begin{array}{ccccccccc} & & & i & & & j & & \end{array} \\
\begin{bmatrix}
1 & & & & & & & & \\
& \ddots & & & & & & & \\
& & 1 & & & & & & \\
& & & \cos\theta_{i,j} & & & -\sin\theta_{i,j} & & \\
& & & & 1 & & & & \\
& & & & & \ddots & & & \\
& & & & & & 1 & & \\
& & & \sin\theta_{i,j} & & & \cos\theta_{i,j} & & \\
& & & & & & & 1 & \\
& & & & & & & & \ddots \\
& & & & & & & & & 1
\end{bmatrix} \end{array} \tag{5}$$

whose blank elements are 0. From this definition, there are $n(n-1)$ kinds of rotation matrices at most, that is, in case of all permutations ${}_nP_2$ on selecting 2 axes from n

axes. In case of all combinations ${}_nC_2$, there are $n(n-1)/2$ rotation matrices. Various composition rotation matrices can be made by multiplying these rotation matrices Eq. (5) with each other.

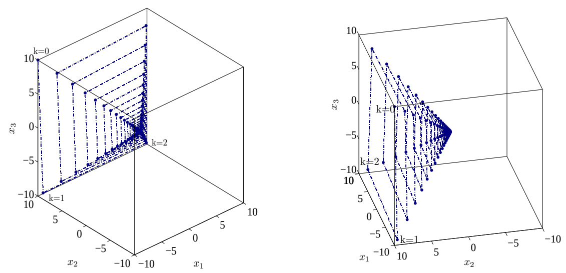
Example: For $n = 3$, there are 3 kinds of combination of 2 axes and rotation matrices are as follows:

$$R^{(3)}_{1,2}(\theta_{1,2}) = \begin{bmatrix} \cos\theta_{1,2} & -\sin\theta_{1,2} & 0 \\ \sin\theta_{1,2} & \cos\theta_{1,2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{(Rotation on } x_1\text{-}x_2 \text{ plane)}$$

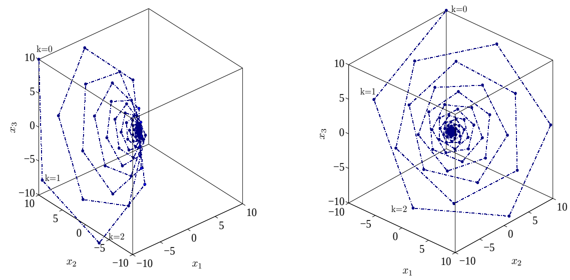
$$R^{(3)}_{1,3}(\theta_{1,3}) = \begin{bmatrix} \cos\theta_{1,3} & 0 & -\sin\theta_{1,3} \\ 0 & 1 & 0 \\ \sin\theta_{1,3} & 0 & \cos\theta_{1,3} \end{bmatrix} \text{(Rotation on } x_1\text{-}x_3 \text{ plane)}$$

$$R^{(3)}_{2,3}(\theta_{2,3}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{2,3} & -\sin\theta_{2,3} \\ 0 & \sin\theta_{2,3} & \cos\theta_{2,3} \end{bmatrix} \text{(Rotation on } x_2\text{-}x_3 \text{ plane)}$$

Given point $x \in \mathbb{R}^3$, for example, point $x' \in \mathbb{R}^3$ transferred by rotating x on the x_1-x_3 plane by $\theta_{1,3}$, then rotating it on the x_2-x_3 plane by $\theta_{2,3}$ is expressed as $x' = R^{(3)}_{2,3}(\theta_{2,3})R^{(3)}_{1,3}(\theta_{1,3})x$. The composition rotation matrix is therefore $R^{(3)}_{2,3}(\theta_{2,3})R^{(3)}_{1,3}(\theta_{1,3})$.



(a) Trajectory of Eq. (8) with parameters Case (a) in **Table 1**



(b) Trajectory of Eq. (8) with parameters Case (c) in **Table 1**

Fig. 6. Illustrations of trajectories based on Eq. (8).

3.2. *n*-Dimensional Spiral Model

As stated, many rotation matrices can exist for Eq. (5) and realize various rotations by composing themselves in n-dimensional space. We propose the following general n-dimensional spiral model using composition rotation matrix $R^{(n)}$ which consists of rotation matrices Eq. (5) based on all combination of 2 axes.

$$x(k+1) = rR^{(n)}(\theta_{1,2}, \theta_{1,3}, \ldots, \theta_{n-1,n})x(k) \quad . \; . \quad (6)$$

where

$$\begin{aligned} &R^{(n)}(\theta_{1,2}, \theta_{1,3}, \ldots, \theta_{n,n-1}) := R^{(n)}_{n-1,n}(\theta_{n-1,n}) \\ &\times R^{(n)}_{n-2,n}(\theta_{n-2,n})R^{(n)}_{n-2,n-1}(\theta_{n-2,n-1}) \times \cdots \times R^{(n)}_{2,n}(\theta_{2,n}) \\ &\times \cdots \times R^{(n)}_{2,3}(\theta_{2,3})R^{(n)}_{1,n}(\theta_{1,n}) \times \cdots \times R^{(n)}_{1,3}(\theta_{1,3})R^{(n)}_{1,2}(\theta_{1,2}) \\ &= \prod_{i=1}^{n-1}\left(\prod_{j=1}^{i} R^{(n)}_{n-i,n+1-j}(\theta_{n-i,n+1-j})\right) \quad . \; . \; . \; . \; . \quad (7) \end{aligned}$$

whose $0 \le \theta_{i,j} < 2\pi$ are rotation angles for each plane around the origin at every k, and $0 < r < 1$ is the convergence rate of distance between a point and the origin at each k. $rR^{(n)}$ is a stable matrix from the range of r and rotation matrix $R^{(n)}$ eigenvalues $\lambda_i \in \mathbb{C}, i = 1, 2, \ldots, n$ that are $|\lambda_i| = 1$.

For this general n-dimensional spiral model, we can individually set $\theta_{i,j}$ on each plane , but this causes more difficulty to adjust $\theta_{i,j}$ when n is larger whereas search potential becomes higher.

We investigate the following simple n-dimensional spiral model, which has the same rotation angles $\theta = \theta_{i,j}$ on each plane in Eqs. (6) and (7):

$$x(k+1) = rR^{(n)}(\theta)x(k) := S_n(r,\theta)x(k). \quad . \; . \; . \quad (8)$$

This lets the distance between the point and the origin converge at zero and rotates the point around the origin by θ in the sense of Eq. (7).

Figure 6 shows Eq. (8) with parameters of Case (a) and Case (c) in **Table 1** in for a 3-dimension from two viewpoints. These figures show that the point generated by the spiral model Eq. (8) has diversification in the early phase and intensification around the center in the late phase as well as 2-dimensional case.

For applications, we extend the spiral model Eq. (8) to have the center at arbitrary point $x^\star$ as follows:

$$x(k+1) = S_n(r,\theta)x(k) - (S_n(r,\theta) - I_n)x^\star, \quad . \; . \quad (9)$$

which is obtained by moving the origin of Eq. (8) to arbitrary point $x^\star$. The convergence of the trajectory to $x^\star$ is proved from the fact that Eq. (9) can be described as $e(k+1) = S_n(r,\theta)e(k)$ by introducing error variables $e(k) = x(k) - x^\star$.

3.3. *n*-Dimensional Spiral Optimization Algorithm

Based on the same philosophy as 2-dimensional optimization, we propose the following multipoint search model using m of Eq. (9):

$$x_i(k+1) = S_n(r,\theta)x_i(k) - (S_n(r,\theta) - I_n)x^\star, \quad . \quad (10)$$

$i = 1, 2, \ldots, m$, with common center $x^\star$ set as the best solution among all search points during a search.

The spiral optimization algorithm based on Eq. (10) for minimization problem $\min_x f(x)$ is shown below. This algorithm can search for a better solution in n-dimensional space by using search points that draw spiral trajectories

Table 2. Benchmark problems.

Problem	Objective function $f(x)$	Search space	Optimal solution x^*	$f(x^*)$
Schwefel	$f(x)=\sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_j\right)^2$	$-5\le x_i\le 5$	$(0,0,\dots,0)$	0
2^nminima	$f(x)=\sum_{i=1}^{n}(x_i^4-16x_i^2+5x_i)$	$-5\le x_i\le 5$	$(-2.9,-2.9,\dots,-2.9)$	$\simeq -78n$
Rastrigin	$f(x)=\sum_{i=1}^{n}(x_i^2-10\cos 2\pi x_i+10)$	$-5\le x_i\le 5$	$(0,0,\dots,0)$	0
Griewank	$f(x)=\frac{1}{4000}\sum_{i=1}^{n}x_i^2+\prod_{i=1}^{n}\cos\frac{x_i}{\sqrt{i}}$	$-50\le x_i\le 50$	$(0,0,\dots,0)$	0

Table 3. Methods for simulations.

Method	Parameter settings
Spiral1	$r=0.95,\ \theta=\pi/4$
Spiral2	$r=0.95,\ \theta=\pi/2$
Spiral3	$r=0.99,\ \theta=\pi/4$
Spiral4	$r=0.99,\ \theta=\pi/2$
CM	$w=0.729,\ c_1=c_2=1.4955$
LDIWM	$w=0.9\rightarrow 0.4$ as $k=0\rightarrow k_{\max}$, $c_1=c_2=2.0$

toward common center $x^\star$ set as the best solution, which naturally realizes the strategy from diversification to intensification.

Algorithm of *n*-Dimensional Spiral Optimization

Step 0: Preparation
Select the number of search points $m\ge 2$, parameters $0\le\theta<2\pi$, $0<r<1$ of $S_n(r,\theta)$, and maximum iteration number $k_{\max}$. Set $k=0$.

Step 1: Initialization
Set initial points $x_i(0)\in\mathbb{R}^n, i=1,2,\dots,m$ in the feasible region at random and center $x^\star$ as $x^\star = x_{i_g}(0)$, $i_g=\arg\min_i f(x_i(0))$, $i=1,2,\dots,m$.

Step 2: Updating x_i
$x_i(k+1) = S_n(r,\theta)x_i(k) - (S_n(r,\theta)-I_n)x^\star$, $i = 1,2,\dots,m$.

Step 3: Updating $x^\star$
$x^\star = x_{i_g}(k+1)$, $i_g = \arg\min_i f(x_i(k+1))$, $i = 1,2,\dots,m$.

Step 4: Checking Termination Criterion
If $k=k_{\max}$ then terminate. Otherwise, set $k=k+1$, and return to Step 2.

4. Numerical Experiments

To verify the search performance and parameter properties, we conducted numerical experiments for various benchmark problems and compared them with PSO, which is a representative analogy-based metaheuristics for continuous optimization problems.

4.1. Conditions

Benchmark problems in **Table 2** are solved under dimensions $n=3,30$, and 100.

The four patterns of proposed methods, Spiral 1–Spiral 4 in **Table 3**, are executed. For comparison, we used two PSO methods – the constriction method (CM) [9, 10] and the linearly decreasing inertia weight method (LDIWM) [9] in **Table 3**.

Set number of search points $m=20$ and iteration numbers $k_{\max}=100$ and 1000.

The computational environment consists of Intel Corei7 3.3 GHz CPU, 4 GB memory, and Matlab software.

4.2. Results

Table 4 shows the results of numerical experiments after 100 runs. One run finish within a few seconds even for $n=100$ and $k_{\max}=1000$. Asterisks ($*$) in the table mark the best values obtained for each case. **Table 4** shows that:

1. Search performance: Our proposal is confirmed to have better performance than PSO because almost all patterns of our proposal are superior to PSO in each case.
2. Parameter θ: $\pi/2$ is confirmed to be superior to $\pi/4$ because the best methods for $k_{\max}=100$ and 1000 are Spiral 2 and Spiral 4.
3. Parameter r: The effectiveness of r depends on $k_{\max}$ because Spiral 1 and Spiral 2 are better when $k_{\max} = 100$ and Spiral 3 and Spiral 4 are better when $k_{\max} = 1000$. We guess that r should be selected as nearer to 1 as $k_{\max}$ is set larger to improve performance.

Table 4. Results of numerical experiments ($m = 20$).

			Maximum iteration number $k_{\max} = 100$						Maximum iteration number $k_{\max} = 1000$					
Problem	Dim.		CM	LDIWM	Spiral1	Spiral2	Spiral3	Spiral4	CM	LDIWM	Spiral1	Spiral2	Spiral3	Spiral4
Schwefel	3	Mean	0.12	0.12	0.0*	0.0*	0.2	0.3	0.02	0.01	0.0*	0.0*	0.0*	0.0*
		Best	0.0*	0.0*	0.0*	0.0*	0.01	0.02	0.0*	0.0*	0.0*	0.0*	0.0*	0.0*
		Worst	1.05	0.71	0.03	0.0*	0.66	2.0	0.3	0.06	0.03	0.0*	0.0*	0.0*
		Std. dev.	0.17	0.14	0.01	0.0*	0.13	0.2	0.04	0.01	0.0*	0.0*	0.0*	0.0*
	30	Mean	156	215	99	20*	226	154	131	150	95	17	13	3*
		Best	84	112	25	4.1*	107	67	69	81	22	2.9	0.8	1*
		Worst	257	455	265	91*	447	487	248	350	259	85	44	17*
		Std. dev.	40	69	51	16*	70	64	37	48	51	15	8	2*
	100	Mean	1697	2531	1293	255*	2915	1553	1338	1774	1235	225	731	73*
		Best	886	1123	618	88*	1267	875	682	974	571	75	291	38*
		Worst	3640	4486	2777	964*	7104	3203	2491	3966	2698	894	1821	243*
		Std. dev.	499	779	384	147*	972	456	335	516	376	137	297	35*
2^nminima	3	Mean	-226	-229*	-229*	-221	-224	-211	-232	-234*	-229	-221	-234*	-222
		Best	-235*	-235*	-235*	-235*	-235*	-234	-235*	-235*	-235*	-235*	-235*	-235*
		Worst	-182	-203*	-174	-178	-198	-168	-203	-206	-174	-178	-207*	-178
		Std. dev.	13	8	14.3	15.3	7*	16	7	4*	14	15	5	15
	30	Mean	-1285	-1088	-1798	-1846*	-1076	-1246	-1372	-1286	-1815	-1868	-1995*	-1989
		Best	-1594	-1345	-2007	-2038*	-1375	-1360	-1608	-1539	-2024	-2057	-2194	-2208*
		Worst	-1118	-793	-1563	-1649*	-682	-1080	-1152	-1002	-1568	-1671	-1802*	-1728
		Std. dev.	90	119	96	81	150	57*	90.5	-99	94	79	76	73*
	100	Mean	-2456	-2022	-4737*	-4724	-2001	-3816	-2779	-2282	-4864	-4775	-6317*	-5192
		Best	-3119	-3024	-5417*	-5074	-2703	-4141	-3633	-3232	-5553	-5108	-6744*	-5546
		Worst	-1452	-1181	-4055	-4336*	-1076	-3483	-1728	-1625	-4167	-4367	-5942*	-4805
		Std. dev.	338	344	267	159*	311	109	343	346	266	160	186	154*
Rastrigin	3	Mean	3.85	4.0	1.87	1.46*	3.86	5.7	2.1	2.0	1.8	1.4	0.7*	1.4
		Best	0.153	0.2	0.01	0.0*	0.23	1.6	0.02	0.0*	0.0*	0.0*	0.0*	0.0*
		Worst	11.7	11.2	7.96	5.98*	8.57	11	6.2	5.2	8.0	6.0	3.0*	6.0
		Std. dev.	2.1	2.4	1.39	1.15	0.23*	2.2	1.4	1.2	1.4	1.1	0.7*	1.1
	30	Mean	330	386	230	98*	380	272	308	327	209	71	149	55*
		Best	263	287	152	49*	304	238	239	252	133	27	69	23*
		Worst	393	460	391	189*	456	317	355	389	369	161	290	124*
		Std. dev.	25	33	40	26	35	15*	21*	25	39	26	39	21*
	100	Mean	1437	1588	1174	628*	1622	1032	1391	1558	1049	550	777	445*
		Best	1334	1434	954	516*	1503	950	1236	1384	810	432	596	345*
		Worst	1549	1736	1313	729*	1745	1093	1490	1691	1202	659	1000	595*
		Std. dev.	53	57	70	48	50	27*	47	65	72	48*	79	52
Griewank	3	Mean	0.14	0.16	0.06*	0.06*	0.12	0.13	0.08	0.07	0.1	0.1	0.03*	0.03*
		Best	0.02	0.01*	0.01*	0.04	0.03	0.04	0.0*	0.0	0.01	0.0*	0.0*	0.0*
		Worst	0.36	0.35	0.2	0.17*	0.23	0.3	0.3	0.2	0.2	0.2	0.1*	0.2
		Std. dev.	0.08	0.07	0.04*	0.04*	0.04*	0.1	0.05	0.04	0.04	0.04	0.02*	0.03
	30	Mean	3.4	4.4	1.6	1.0*	3.3	2.0	3.1	3.5	1.5	1.0	0.8	0.2*
		Best	2.5	2.7	1.1	0.7*	2.2	1.8	2.2	2.5	1.1	0.3	0.1	0.01*
		Worst	4.2	5.7	3.2	1.1*	4.7	2.1	4.0	4.3	3.2	1.1	1.5	0.9*
		Std. dev.	0.4	0.5	0.4	0.1*	0.5	0.07	0.3	0.4	0.4	0.2	0.4	0.2*
	100	Mean	13	17	8.6	1.8*	18	4.5	13	17	8.3	1.7	2.5	1.6*
		Best	11	15	4.6	1.5*	15	4.1	11	15	4.3	1.4*	1.3	1.4*
		Worst	15	20	12	2.0*	20	4.9	15	19	11	1.9*	4.9	1.9*
		Std. dev.	1.0	1.0	1.4	0.1*	1.1	0.1*	1	1	1.4	0.1*	1.0	0.1*

Table 5. Comparison between the proposed method and PSO.

	PSO	Spiral
Parameters	c_1, c_2, w	r, θ
Dynamic States	$v, w \in \mathbb{R}^n$	$x \in \mathbb{R}^n$
Randomness	$\circ$	$-$

5. Conclusions

In this paper, we have proposed spiral optimization for n-dimensional continuous optimization problems using a design philosophy of 2-dimensional optimization we proposed recently and constructing an n-dimensional spiral model.

We have confirmed our proposal's effectiveness and properties for some benchmark problems compared to two kinds of PSO which is representative metaheuristics based on analogy.

Our proposal has novelty about not only the analogy but also the structure that has no randomness and fewer design parameters and dynamic states against PSO as shown in **Table 5**.

The following tasks are to be considered:

1. Setting of parameter θ: We must investigate performance when using other angles beside $\pi/2, \pi/4$. We can set θ variously for each rotation matrix $R_{i,j}^{(n)}$ or search points although we set them uniformly in this paper. We would investigate an effective setting for such angles because search models with many parameters generally have higher search potential.

2. Setting of parameter r: From considering numerical results, we must investigate setting r corresponding to $k_{\max}$. From the search model structure, because we can adjust r each search point as $r_i,\ i = 1, 2, \ldots, m$, we must also investigate adjusting r_i.

3. Setting of center $x^\star$: In this paper, we used the best value of all as the common center, which gives interaction. It is important to study the setting of $x^\star$ because interaction generally affects search performance dramatically in multipoint search.

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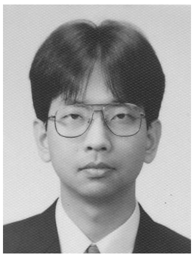

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