Averaging the Unaverageable: Defining a Meaningful Local Series Resistance for Large-Area Silicon Solar Cells

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Abstract. From its definition based on lateral voltage drops, the local series resistance isn't an arithmetically averageable quantity because voltage isn't extensive; however, only such quantities can be averaged. Still, empirically it was found that averaging series images arithmetically provides rather reasonable results. Since the emitter of a large-area silicon solar cell is nearly an equipotential layer, one can linearize the description of the lateral voltage distribution (with respect to the forward-bias current and the emitter resistivity), from which we obtain an averageable local series resistance for the H-type grid geometry; its averageability stems from averaging the parallel paths of vertical current flow. By construction, the average obtained from this linear response theory is identical to the lumped value coming from the illumination intensity variation method, and it also shows the experimentally observed dependency on the forward-bias diode current.

INTRODUCTION

The most robust and widely accepted measurement technique to determine the lumped series resistance of any solar cell is the illumination intensity variation method (IIVM; or multi-light method [1]), originally introduced as a two-light-level method by Swanson in 1960 (cf. [2]). By its usage, the dependency of the series resistance on the solar cell's operating point can be obtained [1, 2]. For large-area silicon solar cells also spatially resolved measurements of the series resistance are relevant; from these, a lumped resistance value can be obtained by arithmetically averaging the resulting R_s image. However, using the arithmetic average is so far mostly done just in an ad-hoc manner; it was introduced this way in by Ramspeck *et al*. [3]. This is problematic because from its definition based on voltage differences, the local series resistance isn't an arithmetically averageable quantity, since voltage isn't an extensive property; yet only extensive quantities can be averaged. Instead, reciprocal resistances (*i.e*., conductances) of parallel paths can be averaged, giving the harmonic mean; this was consistently done by Trupke *et al*. [4].

Nevertheless, empirically it was found that taking the arithmetic average of local series resistances provides rather reasonable results [3, 5–10], even when the local series resistance was obtained by a method for which only taking the harmonic mean is justified. So, these experimental results give a clear hint that, despite the fundamental theoretical objection of not being an extensive property, local series resistances can still be arithmetically averaged – but why? In this contribution, we present a theory that can at least motivate this fact for a certain class of solar cells. The basics of this theory were originally presented in the context of analyzing CELLO photo current and photo voltage maps [11]; however, it has been found that they are of much wider applicability.

Here, we present a local series resistance concept that has the following properties: (a) It takes into account that, by nearly being an equipotential layer, the emitter of the solar cell leads to a strong lateral coupling; (b) it justifies the arithmetic averaging; (c) its arithmetic average is identical to the lumped value obtained (i) from the illumination intensity variation method and (ii) from the integrated Joule losses; (d) it includes the experimentally observed dependence of R_s on the cell's operating point, but in contrast to other works (cf. [1] and references given therein) by a sole dependency on the lumped forward-bias diode current, I_D; (e) it provides an analytical expression for the latter dependency; (f) it provides an analytical expression for the dependency of R_s on geometry and resistivity of the cell.

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THEORY: LOCAL SERIES RESISTANCE OF SILICON SOLAR CELLS

Here, we will derive our local series resistance concept from general physical principles by just using the fact that, since the emitter of "healthy" large-area silicon solar cells is of rather low resistivity, under standard operation conditions, lateral voltage differences are small for such solar cells. In the literature, such "healthy" large-area silicon solar cells are known as "properly designed" [12] or "economically reasonable" ones [13]. Although this seems to restrict our approach to such "healthy" cells, in the end it is completely left to the experiment to test "how valid" our approach is; only from the theoretical point of view our approach is of limited validity. So far, our measurements gave no hints as to contradict the application of our method also to multicrystalline silicon (mc-Si) cells.

Our starting point is a general description of the electrostatic potential and the currents in the emitter of a large-area silicon solar cell, combining Ohm's law for the lateral emitter current and the conservation law for the currents entering/leaving the emitter via the p–n junction. This is the standard approach found in the literature, both in 1D (cf., *e.g.*, [12, 14–22]) and in 2D (cf., *e.g.*, [23–25]). Note that in this description, due to the distributed nature of the emitter resistance there are only local voltages and the local sheet resistivity, but there is no local series resistance.

Of course, there are already other local series resistance concepts in the literature: the independent diode model, serving as a basis in many different measurement schemes (references are given below), and the one derived from a perturbation-theoretical approach by Wong [26]. However, here we are not about to "re-invent the wheel:" In the independent diode model, the only physically sensible way to average the local resistances is to take their harmonic mean; within this model, there cannot be any dependency of the series resistance on the solar cell's operating point. In the approach of Wong there is a weighted arithmetic averaging rule. Yet his approach is just a unique R_s evaluation specification since it misses an explicit description of the actual voltage distribution on the solar cell; in his theory, the dependency of the series resistance on the solar cell's operating point is included but isn't given explicitly. So, there is still a need for a model that can explain in general why local series resistances can be averaged *arithmetically*, and that provides the *explicit* dependency of R_s on the operating point of the solar cell.

General Theory for the Local Emitter Voltage of a Large-Area Silicon Solar Cell

Derivation and Discussion of the Basic Equation

We consider the emitter of a large-area silicon solar cell as a 2D layer in the x–y plane with sheet resistivity ρ (measured in ohm/sqr). Lateral currents in the emitter are described by the sheet current density $\vec{j}$, current passing the p–n junction is given by the z component of the bulk current density, J_z, at $z = 0$. Let $U(x,y)$ be the electrostatic potential distribution in the emitter, measured relative to a back contact perfectly providing equipotential conditions. According to Ohm's law, all lateral current flow leads to lateral voltage differences according to $\rho\,\vec{j} = -\mathrm{grad}_2\,U$, with grad_2 being the two-dimensional gradient in the x–y plane. The current flow in the emitter sheet is altered by the junction currents (photo current and dark currents, *i.e.* diode forward current and shunt current), summed up as J_z. Current conservation is given by $\mathrm{div}_2\,\vec{j} = -J_z$, with div_2 being the two-dimensional divergence in the x–y plane. (Note that here, the sign for the junction current is chosen as to count the dark currents as positive.) Inserting Ohm's law in the current conservation law, one obtains $\mathrm{div}_2\,(\sigma\,\mathrm{grad}_2\,U) = J_z$ as the basic equation; here, $\sigma = 1/\rho$ is the sheet conductivity. If the latter is homogeneous, the basic equation simplifies to $\sigma\,\mathrm{div}_2\,(\mathrm{grad}_2\,U) = J_z$, or

$$\Delta_2\,U(x,y) = \rho\,J_z(x,y), \tag{1}$$

with Δ_2 being the two-dimensional Laplace operator in the x–y plane. (For this equation to hold for actual solar cells, Glatthaar *et al*. [25] have estimated that the lateral variation of the sheet resistivity has to be less than 10 Ω/mm while the junction current density has to remain below 35 mA/mm^2.)

It is worthwhile to discuss the basic equation (1) qualitatively in some depth, independent of any explicit dependency of the junction current density J_z on the local voltage. First of all, one notices that the local J_z determines the Laplacian of the local voltage, not the local voltage itself. This means that the local voltage is influenced by the *whole* current distribution all over the cell, because for solving this partial differential equation it has to be integrated over the whole cell area, thereby taking into account the boundary conditions not discussed so far (that determine whether the cell is *e.g.* at open circuit or short circuit condition, or at the maximum power point).

Having pointed that out, it also makes clear what "distributed series resistance" means in detail – namely, that the series resistance effects *"unfortunately … cannot be considered independently of the junction. Current flow in* *each [resistance] element … produces a bias which determines [the junction current] in the vicinity of the resistance element."* [15] Thus, it is rather astonishing that, in the standard equivalent circuit of a solar cell, the lumped series resistance and the ideal diode are separate elements, being completely independent of each other.

However, the standard equivalent circuit misses an important effect: the dependency of the lumped series resistance on the solar cell's operating point, becoming noticeable for larger forward voltages; it can be explained by the distributed nature of the series resistance [1]. This not only motivates to slightly modify the equivalent circuit (which, directly based on experimental findings, we already proposed in [27, 28]), it is also an effect that should be regarded by any local R_s concept; without it, it can only be of limited validity for large-area silicon solar cells.

Introduction of a Local Series Resistance

Although as such it is a passive component, the series resistance influence of the emitter depends on the pattern of the current flowing through it. Since the emitter sheet is closely linked to the p–n junction, this current flow pattern may change with the operating point and under varying illumination conditions, which means that "local series resistance of a large-area silicon solar cell" is a not-sufficiently-well-defined quantity right from the start. Thus, one should not expect to find a universally valid local series resistance for a large-area silicon solar cell.

Nevertheless, intuitively it is clear what a local series resistance should be good for: to serve as a means for describing the local voltage under various operating conditions, and to help clarify possible local-problems-related causes for a measured lumped value. Using the local series resistance, the local voltage is described via the difference to the one measured externally, *i.e.*, at the busbar. Therefore, for the calculation of the local series resistance, the voltage drop between the local position and the busbar is used – but then the question arises by which current this voltage has to be divided for obtaining a resistance; for this normalization, in the independent diode model the net local junction current density $J_z(x,y)$ is used, whereas in our approach the total external current I_{ext} is used. The consequences of these choices will be discussed below. (Wong's approach is an exception; see below.)

These are the only existing theoretical concepts to quantify the local series resistance. As mentioned below, there is a variety of different measurement methods related to the model of independent diodes, but they are all based on the same fundamental equations.

Linear Response Series Resistance Concept

Premise and Validity of the Linear Response Treatment for the H-Type Grid Geometry

The linear response model starts from the experimental observation that for commercial monocrystalline silicon (mono-Si) solar cells under standard operating conditions, lateral voltage variations are smaller than the thermal voltage $U_{\text{th}} = k_B T/e$ (cf., *e.g.*, luminescence measurements [3, 29, 30] or the Corescan method [31]); the applicability of our approach to multicrystalline (mc-Si) cells is discussed below. This allows to approximate the exponential function describing the diode forward current by just taking the constant and the linear term, and the p–n junction is considered to behave laterally homogeneously. Let $\bar{U}(x,y)$ be the voltage distribution under open-circuit condition, *i.e.* it fulfills Eq. (1) with the current distribution $\bar{J}_z(x,y) = \bar{J}_{\text{D}}(x,y) - \bar{J}_{\text{ph}}$; here, the photocurrent is assumed to be homogeneous, and without loss of generality, a one-diode model is used and shunts are neglected. Now, consider a deviation from open-circuit condition occurring due to an external current $\tilde{I}_{\text{ext}}$ causing a voltage variation; thus, the total voltage reads $U(x,y) = \bar{U}(x,y) + \tilde{U}(x,y)$. In linear order, this changes the diode current to

$$J_{\text{D}}(x,y) = J_0 \exp\left\{U(x,y)/U_{\text{th}}\right\} \cong \bar{J}_{\text{D}}(x,y)\left[1+\tilde{U}(x,y)/U_{\text{th}}\right] \approx \bar{J}_{\text{D}}(x,y) + K\tilde{U}(x,y)\,. \qquad (2)$$

In the last step, the slopes of all local current–voltage characteristics are replaced by the average one, $K = \langle \bar{J}_{\text{D}} \rangle / U_{\text{th}}$ (originally introduced in [11]), since under open-circuit condition, the current is rather homogeneous; with the total cell area A one further has $I_{\text{D}} = \langle \bar{J}_{\text{D}} \rangle A$ for the total diode current; thus, the conductivity of the p–n junction at the working point can be expressed by an effective diode resistance R_{D} as follows: $KA = I_{\text{D}} / U_{\text{th}} = 1/R_{\text{D}}$.

The total variational current on the r.h.s of Eq. (1) is given by $\tilde{J}_z(x,y) = K\tilde{U}(x,y) - \tilde{J}_{\text{ph}}$. By the substitution $U^*(x,y) = \tilde{U}(x,y) - \tilde{J}_{\text{ph}}/K$, the voltage offset due to the photo current variation is eliminated. (For later purposes, note that $\tilde{J}_{\mathrm{ph}} / K = \tilde{I}_{\mathrm{ph}} / (KA) = R_{\mathrm{D}} \tilde{I}_{\mathrm{ph}}$.) Using the linearized diode current, Eq. (2), in the basic differential equation for the total voltage, Eq. (1), the open-circuit terms cancel out, and one is left with a Helmholtz equation for U^*:

$$\Delta_2 U^*(x, y) = \rho K U^*(x, y) . \tag{3}$$

Although this equation is much simpler than Eq. (1), it still contains all physics relevant for the series resistance effects of "healthy" solar cells. Most importantly, it can be solved analytically for various solar cell geometries.

Originally, the solution was given for circular symmetry [11]. This choice has the advantage that general series resistance properties can be derived analytically and be discussed in depth [32]. Here, we consider the practically relevant case of an H-type grid geometry; we will show that our approach leads to the standard text-book lumped R_s expressions, with just a slight modification added. Considering a simplified bus bar geometry (as for "smart wire" connections), Eq. (3) can be solved via a product approach, the general solutions then being hyperbolic functions:

$$U^*(x, y) = c \cosh(\alpha x) \cosh(\beta y) , \text{ with } \alpha^2 + \beta^2 = \rho K . \tag{4}$$

Taking the grid geometry, the boundary conditions, and Ohm's law into account, a transcendental equation for β can be derived; the details are given elsewhere [33, 34]. By definition, $KU^*(x, y) = K\tilde{U}(x, y) - \tilde{J}_{\mathrm{ph}} = \tilde{J}_{\mathrm{D}}(x, y) - \tilde{J}_{\mathrm{ph}}$, so the integral of KU^* over the whole solar cell area equals $\tilde{I}_{\mathrm{D}} - \tilde{I}_{\mathrm{ph}} = \tilde{I}_{\mathrm{ext}}$; from this, the normalization constant c can be determined as to provide current conservation. Expanding the hyperbolic cosine function in a power series and considering only the lowest non-trivial order terms, one obtains

$$U_1(x, y) = c_1 \left(1 + \frac{\alpha_1^2}{2} x^2 + \frac{\beta_1^2}{2} y^2 \right) , \text{ with } \alpha_1^2 + \beta_1^2 = \rho K . \tag{5}$$

Taking again the grid geometry, the boundary conditions, and Ohm's law into account, one further finds the relation $\alpha_1^2 = 2d\beta_1^2 r/\rho$, where $2d$ is the distance between neighboring grid fingers and r is the finger line resistivity. Taking into account that $r \approx 0.02\ \Omega$/mm and $\rho \approx 50\ \Omega_{\mathrm{sq}}$, as a very good approximation one finds $\alpha_1^2 \approx 2drK$ and $\beta_1^2 \approx \rho K$. As for the general solution above, the normalization constant c_1 follows from current conservation.

Before going into details, from Eq. (5) it becomes obvious already that (i) U_1 gives the voltage distribution in linear order in the sheet and in the finger resistivity (apart from additional terms due to c_1), and (ii) the relevant voltage distribution is parabolic in x and y directions, the equipotential lines being ellipses. Then, a histogram analysis (cf., *e.g.*, [13, 35]) results in a linear dependence of the voltage on the cell area, which can easily be checked experimentally (*e.g.*, by luminescence measurements): Such a linear dependence of voltage vs. area indicates that higher order terms (in the sheet and finger resistivity) are negligible.

This test can also be done for mc-Si cells; typically, even for such cells a linear dependence is found [35], indicating the applicability of this approach also for mc-Si cells. From the theoretical side this can be understood at least for such mc-Si cells where the (systematic) variations in the series resistance are not spatially correlated with the (more or less random) fluctuations in the material inhomogeneities. This comes from the fact that a "healthy" large-area silicon solar cell is intrinsically a 2D device, laterally strongly coupled, for which the variation of the voltage over the area depends only on the average of the local vertical currents; averaging over uncorrelated effects strongly reduces their influence [32, 36].

Determination of the Lumped LR-R_s Expression According to the Illumination Intensity Variation Method

The IIVM requires to pick, on each of two current–voltage characteristics differing in the illumination intensity, points with the same total dark current I_{D}. Then, the lumped series resistance belonging to that dark current is given by the differences in external voltage and in external current between these two points, *i.e.*, $R_s = \Delta U_{\mathrm{ext}} / \Delta I_{\mathrm{ext}}$ (cf., *e.g.*, [1, 2, 35]). Since in general it holds that $\Delta I_{\mathrm{ext}} \equiv \tilde{I}_{\mathrm{ext}} = \tilde{I}_{\mathrm{D}} - \tilde{I}_{\mathrm{ph}}$, our linear response formalism is ideally suited for applying this IIVM expression of the lumped series resistance. To that end, one needs (i) to fulfill the condition of constant dark current, $\tilde{I}_{\mathrm{D}} = 0$, *i.e.*, to change both the illumination and the external current in a way that $\tilde{I}_{\mathrm{ext}} = -\tilde{I}_{\mathrm{ph}}$, and (ii) to determine the corresponding variation in external voltage, $\tilde{U}_{\mathrm{ext}} = \tilde{U}(b, d)$, located at that position due to the chosen grid geometry ($2b$ is the distance between neighboring busbars): From the definition of U^* we have that $\tilde{U}(b,d) = U^*(b,d) + \tilde{J}_{\mathrm{ph}}/K$; with the auxiliary function $R(x,y) := U^*(x,y)/\tilde{I}_{\mathrm{ext}}$, this is $\tilde{U}_{\mathrm{ext}} = R(b,d)\tilde{I}_{\mathrm{ext}} + R_{\mathrm{D}}\tilde{I}_{\mathrm{ph}}$. Inserting the result of (i), we obtain $\tilde{U}_{\mathrm{ext}} = R(b,d)\tilde{I}_{\mathrm{ext}} - R_{\mathrm{D}}\tilde{I}_{\mathrm{ext}}$, so that

$$R_{\mathrm{s}} = \tilde{U}_{\mathrm{ext}} / \tilde{I}_{\mathrm{ext}} = R(b,d) - R_{\mathrm{D}} . \tag{6}$$

Note that this resembles the standard equivalent circuit, where $R(b,d)$, giving the maximum of $R(x,y)$ for the chosen geometry, represents the total cell resistance, and R_{D} replaces the diode. (However, this expression is not equivalent to the standard equivalent circuit; see [32] for a detailed discussion.) Now, to use Eq. (6) in an actual measurement of the lumped series resistance, one needs to be able to obtain R_{D} without knowing R_{s}. To that end, we come back to the current conservation relation from above: Since the integral of KU^* over the whole solar cell area equals $\tilde{I}_{\mathrm{ext}}$, it follows that the integral of the auxiliary function R over the whole solar cell area equals $1/K$. Hence, the arithmetic average of R over the whole solar cell area (*i.e.*, the integral of R divided by the area A) equals R_{D}.

This is remarkable since it amounts to averaging a voltage-based quantity, which above was declared impossible as such; however, in this case it is not problematic since it is directly derived from current averaging (and thus is related to current conservation). It further enables us to arithmetically average even $\tilde{U}(x,y) = R(x,y)\tilde{I}_{\mathrm{ext}} + R_{\mathrm{D}}\tilde{I}_{\mathrm{ph}}$, with the result that $\langle\tilde{U}\rangle = R_{\mathrm{D}}(\tilde{I}_{\mathrm{ext}} + \tilde{I}_{\mathrm{ph}})$, and since $\langle\tilde{U}\rangle$, $\tilde{I}_{\mathrm{ext}}$, and $\tilde{I}_{\mathrm{ph}}$ can be measured without having to know R_{s}, R_{D} can be obtained as required.

Using the explicit expression for the full solution in linear order, Eq. (5) (cf. [33, 34]; see also [38]), we find explicitly the variation of the series resistance with the forward bias, as expressed by the effective diode resistance:

$$\frac{1}{R_{\mathrm{s}}} = \frac{1}{R_{\mathrm{s},\infty}} + \frac{1}{2R_{\mathrm{D}}} , \text{ with } R_{\mathrm{s},\infty} = \frac{1}{3}\left(\rho\frac{d}{b} + 2br\right). \tag{7}$$

Note that this series resistance only refers to the part of a single grid finger that is directly connected to the nearest bus bar, collecting current at both of its sides (thus the explicit factor 2 in the finger contribution to $R_{\mathrm{s},\infty}$), each up to a distance d into the emitter sheet, so the total cell area connected to this grid finger is $2bd$. To get the total series resistance of the solar cell, the relevant number of such basic symmetry elements has to be considered in parallel.

The ρ-related part of $R_{\mathrm{s},\infty}$ is formally identical to the one obtained in the standard 1D approach (cf., *e.g.*, [17, 23, 38]). The contribution of the grid finger resistance is just added, as in a series connection. This is due to the simple current flow pattern holding in this order of the resistivities: Since the voltage is linear in ρ and r, from Ohm's law it follows that the currents are in zeroth (!) order. This means that the currents flow as if there were no series resistance; this implies that series resistance just leads to a voltage drop but not to a loss in current.

Local Series Resistance in the LR-R_s Concept

In analogy to Eq. (6), introducing a local series resistance by setting $R_{\mathrm{s}}(x,y) = R_{\max} - R(x,y)$ means that taking the average of this local R_{s} immediately results in the lumped R_{s} of the IIVM, Eq. (6) [hence also showing the variation with the forward bias, Eq. (7)]:

$$\langle R_{\mathrm{s}}(x,y)\rangle = R_{\max} - \langle R(x,y)\rangle = R_{\max} - R_{\mathrm{D}} = R_{\mathrm{s}} . \tag{8}$$

Note that here, this average can be taken since it is derived from the integral of KU^* over the whole solar cell area equaling $\tilde{I}_{\mathrm{ext}}$, *i.e.* it follows directly from the principle of current conservation (see above).

Altogether, in the LR-R_{s} concept, the local series resistance looks like this: $R_{\mathrm{s}}(x,y) = [U_1(b,d) - U_1(x,y)] / \tilde{I}_{\mathrm{ext}}$ (cf. [28]), *i.e.* the voltage drop is divided by the *total* external current. Note that this is a direct consequence of the R_{s} definition according to the IIVM. So, all local series resistance measurement methods can easily be switched from the model of independent diodes to the LR-R_{s} concept, just the local currents aren't involved but only their sum.

Since we have derived this result directly from the sole premise that the lateral voltage variation of a commercial solar cell is small, this makes plausible why it was empirically found reasonable to take the arithmetic average of local-voltage-based series resistance measurements (regardless of their underlying theory).

Discussion of Other Local Series Resistance Concepts

Independent Diode Model (aka Terminal Connected Diodes or Multi-Diode Model)

This model consists of a large number of “local equivalent circuits” being switched in parallel. It was first used by Mijnarends *et al*. [39] and Rau *et al*. [40] for the description of larger areas (classes of crystallites) of inhomogeneous solar cells, but only after its pixel-by-pixel application by Trupke *et al*. [4], this model has been very frequently applied in various spatially resolved measurement methods, even in some modified versions; cf., *e.g*., the detailed review by Bothe and Hinken [9]. Here, the method by Kampwerth *et al*. [31] will be discussed separately since it represents an important special case.

For various reasons, however, this model is unsuited for large-area silicon solar cells: Mainly, it disregards the fundamental property of the emitter of a silicon solar cell which, since it is nearly an equipotential layer, leads to a strong lateral coupling by lateral balancing currents. However, in this model the local voltage is interpreted as resulting only from the local net junction current (which is the meaning of the above-mentioned choice of the normalization denominator). This model can only hold when lateral currents play a negligible role, which obviously is not the case for a large-area silicon solar cell. Furthermore, a dependency of the series resistance on the operating point is excluded in this model since it contains standard ohmic resistors, each having a constant value.

As an aside, we point out that, to derive the model of independent diodes for the starting situation of Eq. (1), the following two conditions have to be considered: (i) The bulk diffusion length has to be rather small (according to [40, 41], models with independent diodes connected in parallel are valid only on lateral length scales larger than several bulk diffusion lengths); (ii) the electrical screening in the laterally conductive layer has to be rather strong (according to [42, 43], the screening length may range from millimeters to meters, and it depends strongly on the working point). All this underlines how far off this model is from the physics of large-area silicon solar cells.

The arithmetic average, nowadays typically used for this model (following [3]), is incompatible with the basic physics behind this model. In the relevant literature, however, this problem is ignored; instead, hand-waving arguments are given why the model of independent diodes, together with arithmetic averaging, seems to work up to the maximum power point [9]. From our linear-response analysis, however, it becomes clear that this is due solely to the solar cell itself, functioning so good that the usage of the wrong model passes unnoticed because, as discussed above, local inhomogeneities are effectively averaged over.

Kampwerth’s Method

Starting from the independent diode model, Kampwerth *et al*. [31] use it in such a way that the most serious conceptional error of this model cancels: The measurement conditions are chosen in such a way that the local junction current is eliminated from the equations, with the lucky outcome that the resulting expression for the local series resistance corresponds to a local version of the IIVM. Therefore, this method allows to detect the dark-current dependency of the series resistance, and although it wasn’t found on a silicon solar cell [9], just recently such a dependency of R_s on the operating point was reported for a CdTe thin-film cell, with the expected behavior that the series resistance decreases for increasing forward bias [44].

However, this method doesn’t use luminescence images taken for a fixed working point but employs a variation of the cell voltage during the measurement of a single image, so it does not refer to a “natural” operating condition of a large-area solar cell. Moreover, due to this voltage variation it is also a significantly less robust method for the investigation of solar cells with locally severe series resistance problems than to keep the total dark current constant.

Since also Kampwerth’s local series resistances follow from the definition of the independent diode model, they must be averaged harmonically, not arithmetically. Yet as already discussed above, Bothe and Hinken [9] found also for Kampwerth’s method that arithmetic averaging is not that bad at all – for which the same plausibility argument applies as above. (Note, however, that this doesn’t mean that the independent diode model were acceptable.)

Wong’s Approach

Since the perturbation-theoretical R_s concept of Wong [26] is not based on local voltages, it constitutes a fundamental exception. His local R_s is formally based on the IIVM and on the use of elementary solar cell properties, described by certain response functions; it fulfills a well-defined, physically sensible weighted arithmetic averaging rule. This approach has the advantage that it also works, without any restrictions, for inhomogeneous solar cells.

However, the actual experimental accessibility is limited to taking *"the ratio of two luminescence images at the same illumination level and two slightly different forward biases"* [26], which means that (i) the result is prone to noise problems and that (ii) the measurement contradicts the IIVM (since it keeps I_D constant but not I_{ph}). This further means that (iii) the result might be distorted by the actual $R_s(I_D)$ dependency.

Moreover, in his work it remains completely unclear how to use the local series resistances for calculating a local voltage, because nothing is said in [26] about which current has to be used for that. Also, the dependency of the series resistance on the solar cell's operating point isn't given explicitly: The (remarkably simple) expression for the local series resistance consists of a ratio with both numerator and denominator becoming larger for increasing forward voltage (the denominator exactly equaling the present K), so that the overall trend remains unclear. Unfortunately, in the literature this method has not been referenced for any R_s measurement; instead, it was used to study solar cells on the basis of the underlying elementary response functions.

SUMMARY

On a large-area silicon solar cell, series resistance averaging is always due to combined effect of all current paths through the solar cell. In general, this gives a weighted arithmetic average, the local currents being the weights.

In the regime of small lateral voltage differences, being *linear* in the sheet resistivity, the whole solar cell area is homogeneously involved in the current distribution. For the lumped R_s this results in the standard expression resulting from the parabolic voltage distribution, and for the local R_s this gives the arithmetic averaging rule, Eq. (8).

Note that both of these results stem from the fact that the lateral current flow is identical to the one in the *zero* resistance case, because due to Ohm's law, the current is always one order lower in the resistivity than the voltage.

For increasing diode conductance, to cross the p–n junction is an alternative path for the lateral emitter currents, so effectively they travel shorter distances in the emitter. Hence, the resistance contribution of the emitter becomes lower and the lumped R_s value decreases. This effect can simply be described by the additional term $1 / (2R_D)$ for the inverse of the distributed series resistance, Eq. (7).

All of this is not contained in the model of independent diodes since to neglect the lateral current flow, bringing about these effects, is constitutive for this model. Luckily, it needs only a small step to get a physically sensible description of the local series resistance: Instead of dividing the local voltage drop by the local junction current (as in the model of independent diodes), one just has to use the total external current; this is a direct consequence of the series resistance definition according to the illumination intensity variation method.

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