Ultrasensitive low noise voltage amplifier for spectral analysis

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Recently we have proposed several voltage noise measurement methods that allow, at least in principle, the complete elimination of the noise introduced by the measurement amplifier. The most severe drawback of these methods is that they require a multistep measurement procedure. Since environmental conditions may change in the different measurement steps, the final result could be affected by these changes. This problem is solved by the one-step voltage noise measurement methodology based on a novel amplifier topology proposed in this paper. Circuit implementations for the amplifier building blocks based on operational amplifiers are critically discussed. The proposed approach is validated through measurements performed on a prototype circuit. © 2008 American Institute of Physics. [DOI: 10.1063/1.2967339]

INTRODUCTION

Low frequency noise measurements (LFNM) have been extensively used since the late 1960s as a tool for the characterization of the quality of electronic devices and for the study of the charge conduction mechanisms in solid state devices. $^{1-4}$ In particular, the measurement of the 1/f noise component associated with the drain and the gate currents in complementary metal oxide semiconductor devices allows the evaluation of the defect density at the semiconductordielectric interface and inside the dielectric layer. 5-15 Moreover, the measurement of the shot noise component associated with the gate current has been used for distinguishing between charge transport mechanisms due to pure tunneling, characterized by full shot noise, and charge transport mechanisms due to trap-assisted tunneling, characterized by suppressed shot noise. 16,17 Unfortunately, in spite of the wealth of information provided by the LFNM for the device characterization, their use is not so diffuse as they deserve to be. Especially at very low frequencies (f < 1 Hz), accurate noise measurements are hampered by the spurious effects which superimpose to the signal (the noise) to be detected. While external interferences could be, at least in principle, completely removed by proper electrical, mechanical, and thermal shielding, the intrinsic measurement instrumentation noise may become the limiting factor because it limits the minimum detectable signal level. At very low frequencies, thermal noise does not usually play an important role. It is the flicker noise, particularly the one introduced by the active devices used in electronic instrumentation, which plays the dominant role. The best strategy to tackle this problem consists of employing high-sensitivity noise measurement methods. Among these methods the most widely used is the crosscorrelation technique 18-21 in which the device under test (DUT) noise is amplified by two identical preamplifiers and it is estimated by taking the cross spectrum of the outputs of the two channels. The advantage of this method with respect to the use of conventional single-channel preamplifier is the elimination of the preamplifier noise sources which produce uncorrelated effects at the outputs of the two channels. The limit of this technique is, however, imposed by not eliminating the noise sources which produce correlated effects at the two outputs.

Recently, we proposed a class of measurement methods, which consist of a multistep measurement procedure, to overcome this problem for both voltage and current noise measurements. The principal limitation of these methods is the multistep measurement procedure because environmental conditions (e.g., temperature, mechanical vibrations) may change in the different measurement steps and the final result could be affected by these changes. The solution in the case of current noise measurements was proposed in Ref. 27. This solution employs a one-step measurement procedure based on a four channel measurement amplifier. The goal of this paper is to propose a solution in the case of voltage noise measurements.

The remainder of this work is organized as follows. Section II briefly describes the cross-correlation method for voltage noise measurements and its limitations. Section III describes how to measure the equivalent input current noise (EICN) of the measurement preamplifier, which is the residual and the limiting factor of the cross-correlation method. Section IV illustrates the proposed method, which combines the cross-correlation method (Sec. II) and the measure of the EICN (Sec. III). In Sec. V, electrical implementations for voltage and transimpedance amplifiers will be analyzed. Measurements on a prototype circuit are reported in Sec. VI. Finally, in Sec. VII, the main results are summarized.

THE DRAWBACK OF THE CROSS-CORRELATION METHOD

The cross-correlation method for noise measurements is based on the following idea: by amplifying the DUT signal by means of two independent amplifiers and by evaluating the cross correlation of their outputs, one can completely suppress the noise sources of the two measurement amplifiers which produce effects on the two outputs which are uncorrelated to one another. ^{18–21} It has been applied both for voltage and current noise measurements. A typical circuit

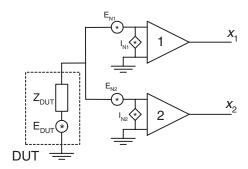


FIG. 1. The conventional cross-correlation method for the measurement of the DUT noise voltage. The DUT is connected to two voltage amplifiers and the DUT noise is estimated by taking the cross spectrum of the two outputs.

configuration, which allows the implementation of the crosscorrelation method for the voltage noise measurements, is reported in Fig. 1, where the DUT is connected to two voltage amplifiers. In the virtual short circuit approximation, it can be easily proven that the two input referred output voltages are given by

$$X_1 = E_{\text{DUT}} + E_{N1} + I_{N1}Z_{\text{DUT}} + I_{N2}Z_{\text{DUT}},$$

$$X_2 = E_{\text{DUT}} + E_{N2} + I_{N1} Z_{\text{DUT}} + I_{N2} Z_{\text{DUT}}, \tag{1}$$

where E_{DUT} represents the DUT voltage noise we want to measure, E_{N1} and E_{N2} are the equivalent input voltage noise (EIVN) of the two amplifiers, and I_{N1} and I_{N2} are their EICN. By assuming E_{Ni} uncorrelated with I_{Ni} the cross spectrum between the two outputs X_1 and X_2 is

$$S_{12} = E_{\text{DUT}}^2 + I_{N1}^2 |Z_{\text{DUT}}|^2 + I_{N2}^2 |Z_{\text{DUT}}|^2.$$
 (2)

Equation (2) suggests that through the cross-correlation method, it is possible to completely eliminate the effects of E_{N1} and E_{N2} of the measurement amplifiers, which give rise to output contributions which are uncorrelated to one another, but not the effects of I_{N1} and I_{N2} , whose effects are partially correlated. The inability of this method to eliminate the correlated noise contributions is its main drawback and sets the limit to the maximum sensitivity which can be obtained by applying the cross-correlation method. In the next section, we will propose a method for the measurement of the EICN, and in Sec. IV we will see how to combine the cross-correlation method and the measurement of the EICN to obtain the DUT voltage noise alone the as final result.

THE MEASUREMENT OF THE EICN

Several methods were employed in the past to measure the EICN of a general two port device. These methods are based on the knowledge of the DUT impedance $Z_{\rm DUT}$ and/or on the separate measure of the EIVN in an additional measurement step. In this section we will see a method for evaluating the EICN in a single step without the knowledge of $Z_{\rm DUT}$ and of the EIVN. For the analysis, Fig. 2 is referred. Amplifier 1 is a voltage amplifier with unity gain and we want to measure its EICN, which is I_{N1} . Amplifiers 3 and 4 are transimpedance amplifiers with Z_F transimpedance gain. The output of amplifier 3 is summed to its input by an addition block. Ideal models have been assumed for blocks in Fig. 2; therefore, voltage amplifier 1 and the addition node

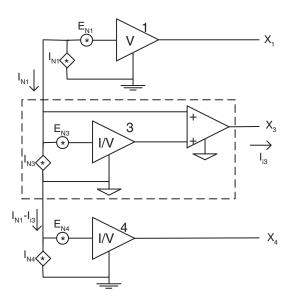


FIG. 2. Illustration of a method to measure the EICN of amplifier 1. Due to the separated ground of transimpedance amplifier 3, the current I_{N1} appears as input to transimpedance amplifiers 3 and 4, and it is estimated by taking the cross spectrum of their two outputs.

have infinite input impedance for each input, while the transimpedance amplifiers have zero input impedance. Unlike the addition node, each amplifier is reported with its noise model, that is, with its EICN and EIVN, which are assumed uncorrelated. Regarding the addition node, its EIVN can be included into the EIVN of the measurement voltage amplifier used to acquire X_3 , while its EICN can be included into I_{N3} .

As will be more clear in the following, EIVN sources of the voltage amplifiers of the acquisition system used to acquire X_1 , X_3 , and X_4 produce a null effect. Transimpedance amplifier 3 has a separate ground with respect to the rest of the circuit. I_{i3} is the EICN of the measurement amplifier used to acquire X_3 . Due to the separate ground, the current I_{N1} goes out from the dashed box, and it is amplified by the transimpedance amplifier 3 and by the transimpedance amplifier 4. The outputs X_1 , X_3 , and X_4 are

$$X_{1} = E_{N1} + E_{N3} + E_{N4},$$

$$X_{3} = I_{N1}Z_{F} + E_{N3} + E_{N4} + I_{N3}Z_{F},$$

$$X_{4} = I_{N1}Z_{F} + I_{N4}Z_{F} - I_{i3}Z_{F}.$$
(3)

Because I_{N1} is the only source that produces correlated effects at the outputs of amplifiers 3 and 4, we can evaluate I_{N1} by simply taking the cross spectrum between X_3 and X_4

$$S_{34} = I_{N1}^{2} |Z_F|^2. (4)$$

It is apparent from Eq. (4) that we do not need the knowledge of E_{N1} but only the knowledge of the transimpedance gain Z_F . Moreover it is evident that EIVN sources of the voltage amplifiers of the acquisition system used to acquire X_1 , X_3 , and X_4 produce a null effect because they are uncorrelated.

THE PROPOSED METHOD

In this section we propose a method for the measurement of the DUT noise in a single measurement step. Basically we

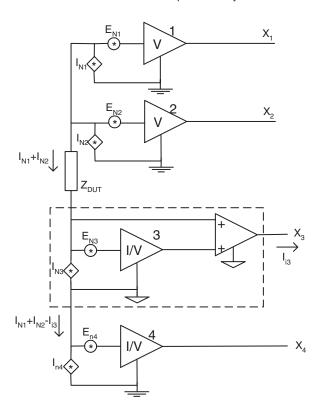


FIG. 3. The proposed method, which combines the cross correlation to cancel the EIVN (Fig. 1), and the method of Sec. III (Fig. 2) to measure the residual EICN. The DUT noise is estimated by measuring the cross spectra S_{12} , S_{13} , S_{14} , and S_{34} [Eq. (9)].

combine the cross-correlation process illustrated in Sec. II with the EICN measurement technique discussed in the previous section. For the analysis, Fig. 3 is referred. The circuit is similar to that of Fig. 2, but in this case we have also the DUT and another voltage amplifier (2) with a unity gain as amplifier 1. Amplifiers 1 and 2 play the same role that they have in Fig. 1. Since the limitation of the method of Fig. 1 is the EICN of the two amplifiers, in this case we measure the sum $I_{N1}+I_{N2}$ rather than I_{N1} alone. The concept is the one illustrated in the previous section: I_{N1} and I_{N2} are amplified by both amplifiers 3 and 4, and they should be evaluated by taking the cross spectrum between X_3 and X_4 . Indeed, in this case the presence of $Z_{\rm DUT}$ makes the analysis a bit more complicated because I_{N1} and I_{N2} produce a voltage drop over $Z_{\rm DUT}$. The outputs X_1 , X_2 , X_3 , and X_4 are

$$X_{2} = -I_{N1}Z_{DUT} - I_{N2}Z_{DUT} + E_{N1} + E_{N3} + E_{N4} + E_{DUT},$$

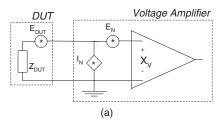
$$X_{1} = -I_{N1}Z_{DUT} - I_{N2}Z_{DUT} + E_{N2} + E_{N3} + E_{N4} + E_{DUT},$$

$$X_{3} = I_{N1}Z_{F} + I_{N2}Z_{F} + I_{N3}Z_{F} + E_{N3} + E_{N4},$$

$$X_{4} = I_{N1}Z_{F} + I_{N2}Z_{F} + I_{N4}Z_{F} - I_{i3}Z_{F},$$
(5)

and the cross spectra are given by

$$\begin{split} S_{12} &= I_{N1}^2 |Z_{\text{DUT}}|^2 + I_{N2}^2 |Z_{\text{DUT}}|^2 + E_{N3}^2 + E_{N4}^2 + E_{\text{DUT}}^2, \\ S_{13} &= -I_{N1}^2 Z_{\text{DUT}} Z_F^* - I_{N2}^2 Z_{\text{DUT}} Z_F^* + E_{N3}^2 + E_{N4}^2, \\ S_{14} &= -I_{N1}^2 Z_{\text{DUT}} Z_F^* - I_{N2}^2 Z_{\text{DUT}} Z_F^*, \end{split}$$



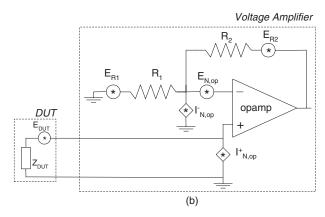


FIG. 4. Voltage amplifier with its noise model (a), electrical implementation with op-amp (b).

$$S_{34} = I_{N1}^{2} |Z_F|^2 + I_{N2}^{2} |Z_F|^2.$$
(6)

In this case, in addition to the contributions from I_{N1} and I_{N2} , we have also E_{N3} and E_{N4} in the cross spectrum S_{12} . From Eq. (6) we can simply evaluate all of these contributions by

$$\frac{|S_{14}|^2}{S_{34}} = I_{N1}^2 |Z_{\text{DUT}}|^2 + I_{N2}^2 |Z_{\text{DUT}}|^2 \tag{7}$$

and

$$S_{13} - S_{14} = E_{N3}^{2} + E_{N4}^{2}. (8)$$

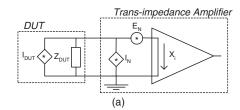
From Eqs. (6)–(8) we obtain the DUT noise alone

$$E_{\text{DUT}}^{2} = S_{12} - \frac{|S_{14}|^{2}}{S_{24}} - (S_{13} - S_{14}). \tag{9}$$

The effectiveness of this method is that the DUT noise is estimated in a single measurement step and that it is not necessary the knowledge of the noise coming from the active components or the knowledge of the DUT impedance. The ultimate limit is due to the assumption of uncorrelation between the EIVN and EICN of each amplifier.

CIRCUIT IMPLEMENTATION BASED ON OPERATIONAL AMPLIFIERS

In this section we propose and analyze circuit implementations for the building blocks of Fig. 3. It is very important, as discussed in the previous section, that EIVN and EICN of each block are uncorrelated. Let us start by implementing the voltage amplifiers 1 and 2 of Fig. 3. A general voltage amplifier model (a) and an operational-amplifier (op-amp) based implementation with all noise sources (b) are shown in Fig. 4. Notice that in Fig. 4(b), $E_{N,\text{op}}$ is the EIVN of the op-amp alone, and $I_{N+,\text{op}}$, $I_{N-,\text{op}}$ are the op-amp EICN associated with



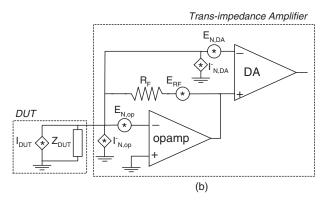


FIG. 5. Transimpedance amplifier noise model (a), electrical implementation with op-amp (b).

the not-inverting and inverting inputs, respectively. With reference to the general noise model [Fig. 4(a)], the input referred voltage noise X_V is

$$X_V = E_{\text{DUT}} + E_N + I_N Z_{\text{DUT}} \tag{10}$$

with reference to the electrical implementation [Fig. 4(b)], the input referred voltage noise is

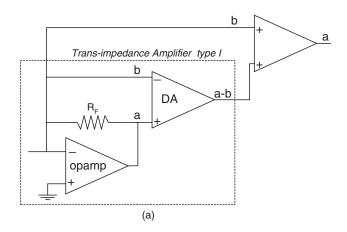
$$X'_{V} = E_{\text{DUT}} + E_{N,\text{op}} + I^{+}_{N,\text{op}} + I^{-}_{N,\text{OP}}(R_{1} \parallel R_{2}) + E_{R2} \frac{(R_{1} \parallel R_{2})}{R_{2}} + E_{R1} \frac{(R_{1} \parallel R_{2})}{R_{1}}.$$
(11)

By comparing Eqs. (10) and (11) we obtain

$$\begin{cases} E_{N} = E_{N,\text{op}} + I_{N,\text{op}}^{-}(R_{1} \parallel R_{2}) + E_{R2} \frac{(R_{1} \parallel R_{2})}{R_{2}} + E_{R1} \frac{(R_{1} \parallel R_{2})}{R_{1}}, \\ I_{N} = I_{N,\text{op}}^{+}. \end{cases}$$
(12)

Equation (12) suggests that, if we start from the assumption that $E_{N,\text{op}}$, $I_{N+,\text{op}}$, and $I_{N-,\text{op}}$ are uncorrelated, then E_N and I_N are also uncorrelated. Therefore the electrical implementation of Fig. 4(b) is suitable to be used as a voltage amplifier in Fig. 3.

Now we are interested in the circuit implementation for the transimpedance amplifiers in Fig. 3. In Fig. 5(a) it is shown the general noise model for a transimpedance amplifier, and in Fig. 5(b) an op-amp based circuit implementation.²⁹ The output is taken across the feedback resistor R_F by a differential amplifier which has a unity gain. $I_{N-,DA}$ and $E_{N,DA}$ are the input referred noise sources of the differential amplifier. With reference to the general noise model [Fig. 5(a)] the input referred current noise X_I is



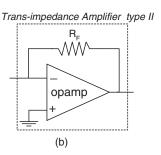


FIG. 6. Electrical implementation of the amplifier into the dashed box in Fig. 3(a), equivalent electrical implementation (b).

$$X_1 = I_{\text{DUT}} + \frac{E_N}{Z_{\text{DUT}}} + I_N \tag{13}$$

with reference to the electrical implementation [Fig. 5(b)], the input referred current noise $X'_{\rm I}$ is

$$X_{i}' = I_{\rm DUT} + \frac{E_{N,\rm op}}{Z_{\rm DUT}} + I_{N,\rm op}' + I_{N,\rm DA}' + \frac{E_{N,\rm DA}}{R_F} + \frac{E_{RF}}{R_F}. \tag{14}$$

By comparing Eqs. (13) and (14) we obtain

$$\begin{cases} E_N = E_{N,\text{op}} \\ I_N = I_{N,\text{op}} + I_{N,\text{DA}} + \frac{E_{N,\text{DA}}}{R_E} + \frac{E_{RF}}{R_E}. \end{cases}$$
(15)

Also in this case, under the assumption that $E_{N,\text{op}}$, $I_{N+,\text{op}}$, and $I_{N-,\text{op}}$ are uncorrelated, E_N and I_N are also uncorrelated. Therefore the electrical implementation of Fig. 5(b), which will be referred in the following as transimpedance amplifier type I, is suitable to be used as a transimpedance amplifier in Fig. 3.

In Fig. 6(a) an electrical implementation for the block included in the dashed box of Fig. 3, based on the previous discussed op-amp based transimpedance amplifier, is shown. It is evident that the differential amplifier and the addition node eliminate each other, and that the output a of the entire block is the same as the output of the op-amp. Therefore, the final electrical implementation for the content of the dashed box in Fig. 3 is shown in Fig. 6(b).

It consists of a simple op-amp based transimpedance amplifier, which will be referred in the following as transimpedance amplifier type II. Note that the type I amplifier is a type II amplifier with the output taken across the feedback resis-

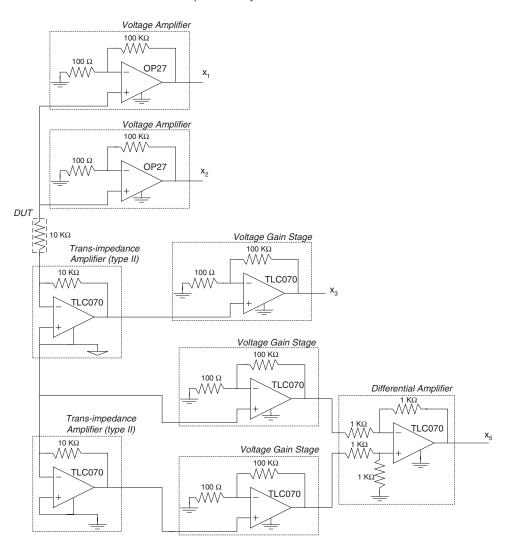


FIG. 7. Electrical implementation with op-amps of the circuit of Fig 3.

tor. Different from the case of the type I amplifier, for the type II amplifier, E_N and I_N are correlated and therefore it cannot be used to implement transimpedance amplifiers in Fig. 3.²⁹

EXPERIMENTAL VALIDATION

Noise measurements have been performed for ensuring the correctness and the validity of the proposed methodology. To this purpose we used the circuit whose electrical schematic is shown in Fig. 7. Amplifier stages have been implemented with op-amps as discussed in the previous section. To enhance the signal to noise ratio at the input of the acquisition system, the outputs X_1 , X_2 , X_3 , and X_4 have been amplified. For this reason, each voltage amplifier has a gain of 1001, and voltage gain stages with the same gain have been added at the output of the transimpedance amplifiers (type II). It can be easily verified that our circuit analysis remains valid also in the case of voltage amplifiers with gain different than 1. The differential amplifier has unity gain, while the gain of each transimpedance amplifier (type II) is 10 k Ω . Voltage amplifiers use the op-amp OP27, which is a bipolar input op-amp with I_N =0.6 pA/ $\sqrt{\text{Hz}}$ (100 Hz), E_N =3 nV/ $\sqrt{\text{Hz}}$ (100 Hz), while the other amplifiers use the TLC070 which is a metal oxide semiconductor field effect transistor (MOSFET) input op-amp with $I_N=1$ fA/ $\sqrt{\text{Hz}}$ (100 Hz), E_N =7nV/ $\sqrt{\text{Hz}}$ (100 Hz). The DUT is a 10 k Ω resistor. In Fig. 8 the experimental result is shown. The extracted DUT noise is flat, as expected from the thermal noise of a resistor, and the estimated voltage noise power in the bandwidth 10 Hz-1 kHz is $1.658 \times 10^{-16} \text{ V}^2/\text{Hz}$ with an error that is lower than 1%.

CONCLUSIONS

In this work, we have presented an original ultrasensitive four channel amplifier for the voltage noise measurement, which allows, at least in principle, the complete elimination

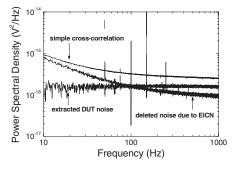


FIG. 8. Measurement result for a 10 k Ω resistor DUT. The figure shows the result of the simple cross correlation (Fig. 1), the deleted noise due to the EICN (Sec. III), and the extracted DUT noise [Eq. (7)].

of the noise introduced by the measurement amplifier itself without requiring either the knowledge of the noise coming from the active components or the knowledge of the DUT impedance. The main advantage with respect to previous high-sensitivity techniques is that the elimination of the measurement amplifier noise is obtained in a single measurement step. The proposed approach combines the conventional cross-correlation technique for the elimination of the contribution of the EIVN of the amplifier, with the measurement of the residual EICN after the cross-correlation process. The measurement of the EICN is performed by taking the cross correlation of the outputs of two transimpedance amplifiers which have the unknown EICN as input. We have presented operational-amplifier based circuit implementations for the voltage amplifiers and for the transimpedance amplifiers, which constitute the entire ultrasensitive amplifier. The method based on the proposed amplifier topology has been validated through voltage noise measurements performed with a prototype circuit. Obviously in the case of MOSFET input measurement amplifiers, the proposed solution has no practical advantages respect to the solution reported in Ref. 21. The aim of the proposed paper is not the measurement of a very low input noise but we propose a circuital configuration to enhance the signal to noise ratio of bipolar input measurement amplifiers for voltage noise measurements. The ultimate limit of the presented measurement technique is imposed by the assumed uncorrelation between the EIVN and the EICN of the amplifier stages.

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