

Comparison of Mathematical Models of Photo-Voltaic (PV) Module and effect of various Parameters on its Performance

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Abstract- This paper presents a comparison of different electrical equivalent mathematical models for a PV cell/module/array based on Shockley diode equation. The presented different models include single diode model and double diode models. These models accept solar irradiance and temperature as input parameters and yield the I-V & P-V output characteristics. A control algorithm called MPPT (maximum power point tracking) algorithm is typically integrated with PV systems in order to extract maximum possible power from it. Thus, there is a need to select an efficient and simple PV model which can be used to realize efficient MPPT algorithm. In addition to this, other parameter should also be taken into account for finding the best model for use in the simulation. In this paper, the comparison of different models are performed on the basis of the MPP tracking and values of various parameters like peak current, peak voltage, open circuit voltage and short circuit current value achieved from the output I-V and P-V characteristics and their resemblance with manufacturer specified specifications. Using this analysis, the best (efficient as well as simple) model that can be used for simulation purpose is selected. Such a model is useful for professionals who require PV simulators for their design.

Additionally, the paper also illustrates the effect of various parameters like diode ideality factor, series resistance, input sun radiation and cell temperature on single diode model performance. Model evaluation is performed using a VIKRAM ELV-40 of 37W PV module. All mathematical models presented here are programmed using script file in the MATLAB environment.

Keywords- Solar cell model, PV module, photocurrent, single - double diode model, double diode model, ideality factor, series resistance.

I. INTRODUCTION

Dependence on fossil fuels is a major cause of concern for the energy community across the globe and has created a vast interest in solar energy as it is abundant, clean, distributed over the earth and offers zero fuel cost. With these benefits at hand, solar energy based power generation is still challenging due to low efficiency, intermittent power generation and high manufacturing cost of photovoltaic (PV) modules.

In the recent past, increasing industrial interest in solar energy has fuelled rich research activities in this domain in order to improve the efficiency of PV module. In order to make PV system efficient, a detailed modeling, simulation and analysis is required. Intermittency in power generation is a major problem when connected to electrical systems because of lack of control on produced power. These stated problems require selection of appropriate PV module and assessment of PV power potential [1-3]. In the literature, mathematical

modeling of PV system components was illustrated and simulated to better understand their performances [4].

The PV module modeling involves the estimation of its I-V and P-V characteristics curve. Among various mathematical model of PV module proposed in literature, simplest is the ideal single diode model. This model involve three parameters, namely, short circuit current, open circuit voltage, and diode ideality factor. However, without series resistance [9], it does not give accurate shape between open circuit voltage and maximum power point of P-V characteristic of a module. Further simplified single diode model which consider the effect of series resistance was proposed in [5], though this causes lack of accuracy when subjected to large temperature variations. The temperature sensitivity was improved by including additional shunt resistance in the simplified single diode model [5]. However, with the shunt resistance, the accuracy also deteriorates at low irradiance levels. In order to eliminate these drawbacks, two diode model was proposed in [6] which is more complex due to involvement of larger number of parameters. A three diode linear PV model was also proposed in literature, which account the behaviour of mismatched PV module, but it led to increased computational complexity due to presence of large number of model parameters. Recently, a simpler and more accurate new approach for PV modeling and simulation has been proposed in [7] and different PV models have also been compared in [13]. However, an integrated efficient and simple model of PV module for the purpose of simulation does not exist in literature. Simulation tools are required to develop PV systems for handling mathematical model simulations. The simulation enable development of new system like power converter and also helpful for realizing MPPT (maximum power point tracker) algorithm in an effective way by reducing cost and time.

This paper presents a comparison of different electrical equivalent mathematical models for a PV cell/module/array based on Shockley diode equation. The presented different models include single diode model and double diode model for PV cell, module & array, respectively. These models accept solar irradiance and temperature as input parameters and yield the I-V & P-V output characteristics. In this paper, the comparison of different models are performed on the basis of MPP tracking and values of the various parameters like peak current, peak voltage, open circuit voltage and short circuit current value achieved from output I-V and P-V characteristics and their resemblance with manufacturer specified specifications. Using this comparison analysis, the

best (efficient as well as simple) model that can be used for simulation purpose is selected. The selected model is useful for professionals who require simple and accurate PV simulators for their design. It is also useful in the analysis of MPPT technologies and in the design of Power converter system.

In addition to this, the paper also illustrates the effect of various parameters like diode ideality factor, series resistance, input sun radiation and cell temperature on single diode model performance. Model evaluation is performed using a VIKRAM ELV-40 of 37W PV module. All mathematical models presented here are programmed using script file in MATLAB environment. mathematical modelling of photovoltaic cell/module/array

A. Single Diode Model for PV Cell

A p-n junction fabricated in a thin wafer of semiconductor forms a solar/PV cell. These cells rely on photovoltaic effect in order to convert solar radiation into electricity. The photovoltaic phenomenon is due to the property of the semiconductor material, which enable absorption of specific type of photons from sunlight that have higher energy than the band-gap energy of the semiconductor. This leads to creation of some free electron-hole pairs in the cell that are directly proportional to input solar irradiance. The internal electric field of p-n junction separates these electron-hole pairs creating a photocurrent which consequently is also proportional to solar radiation. The output I-V and P-V characteristics of a PV cell are nonlinear and vary with the input solar radiation as well as the cell temperature [8].

The equivalent circuit representation for a PV cell is classified into three types of practical, simplified and ideal model as illustrated in Fig.1. The ideal model of PV cell includes a diode and current source connected in parallel. The amount of photo current generated by current source is proportional to input solar radiation. The diode determines the

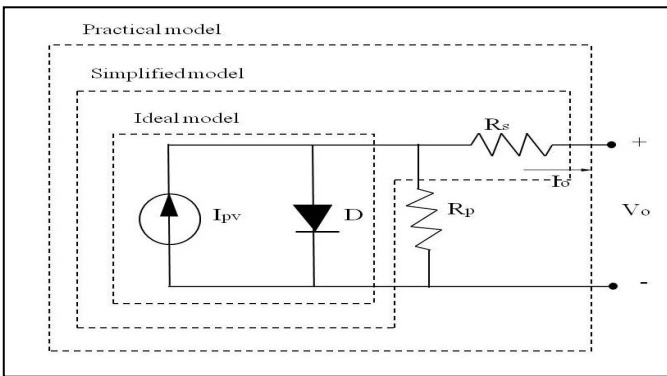


Figure 1 PV cell model using single diode equivalent circuit [8].

I-V characteristics of the cell. Performance of single diode model is improved by including temperature dependent diode reverse saturation current (\$I_r\$), temperature dependent photo current (\$I_{pv}\$), diode ideality factor (\$A\$), Series resistance (\$R_s\$) and Shunt resistance (\$R_p\$) [9]. The \$R_s\$ represent the internal losses due to current flow and gives accurate shape between the maximum power point and the open circuit voltage of P-V characteristics of a cell. The \$R_p\$ connected in parallel with the the diode, represents leakage current to the ground.

In an ideal case it is commonly assumed that \$R_s\$ and \$R_p\$ is equal to zero. Here, a simplified model is used which consider only \$R_s\$ and neglect \$R_p\$. Modeling of PV cell based on mathematical equation is used to find out the characteristic curves voltage versus current and power versus voltage. The mathematical equation for characterizing the I-V behavior for single diode model is expressed by following expression:

$$I_o = I_{pv} - I_r \left[\exp \left\{ \frac{q(V_o + I_o R_s)}{k T_{cell} A} \right\} - 1 \right] \quad (1)$$

Where \$I_{pv}\$ is the photons-generated current due to sunlight, \$I_r\$ is the diode reverse saturation current, \$q\$ is an electron charge (\$1.6 \times 10^{-19}\$), \$k\$ is the Boltzmann's constant (\$=1.38 \times 10^{-23}\$ J/K), \$V_o\$ is cell terminal voltage, \$I_o\$ is cell terminal current, \$T_{cell}\$ is working temperature of the cell in Kelvin, \$A\$ is the ideality factor, \$R_s\$ is the series resistance. The photo generated current mainly depends on solar radiation and working cell temperature, which is given by

$$I_{pv} = [I_{sc}(T_{ref}) + K_I(T_{cell} - T_{ref})]H \quad (2)$$

Where \$I_{sc}(T_{ref})\$ is short-circuit current of the cell at reference temperature (\$25^\circ\text{C}\$) and standard radiation \$1000 \text{ W/m}^2\$, \$K_I\$ is temperature coefficient of cell's short circuit current, \$T_{ref}\$ is a reference temperature of the cell, and \$H\$ is the solar insolation in \$\text{kW/m}^2\$. On the other hand, the diode reverse saturation current is a function of the cell temperature which is given as;

$$I_r = I_{rs} \left(\frac{T_{cell}}{T_{ref}} \right)^3 \exp \left[\frac{q E_G \left(\frac{1}{T_{ref}} - \frac{1}{T_{cell}} \right)}{k A} \right] \quad (3)$$

$$I_{rs} = \frac{I_{sc}(T_{ref})}{\left[\exp \left(\frac{q * V_{oc}}{k A T_{ref}} \right) - 1 \right]} \quad (4)$$

B. Single Diode Model for PV Module and Array

PV Cells in series-parallel configuration forms a module. Similarly two or more PV modules in series or parallel configuration form an array. Consider a PV module having (\$n_s\$) number of cells in series and (\$n_p\$) number of cells in parallel configuration. A single diode equivalent circuit model of PV module is illustrated in Fig.2.

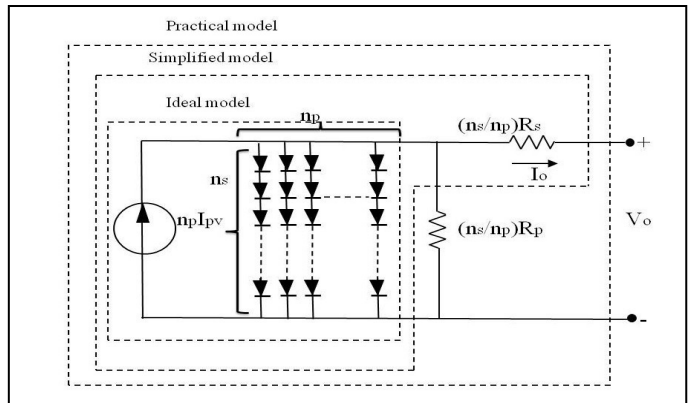


Figure 2 Model of a PV module using single diode equivalent circuit (\$n_p\$ cells in parallel, \$n_s\$ cells in series) [10].

The mathematical equation for characterizing the I-V behavior for simplified model is obtained and expressed as follows [10];

$$I_o = n_p I_{pv} - n_p I_r \left[\exp \left\{ \frac{q(n_p V_o + n_s I_o R_s)}{n_s n_p k T_{cell} A} \right\} - 1 \right] \quad (5)$$

The values of I_{pv} and I_r are given by Eqs. (2-3).

C. Double Diode Model for PV Cell [6][12]

The ideality factor is a function of voltage across the device. The diode ideality factor in case of a single diode model is considered to have a constant value. It is close to unity at higher voltage, when the recombination in the device is dominated by the surfaces and the bulk regions. It approaches the value of two at lower voltages, when the recombination in the junction or space charge region dominates other recombination. The junction recombination is modelled by adding a second diode in parallel with the first and setting the ideality factor typically to the value of two as shown in Fig.3.

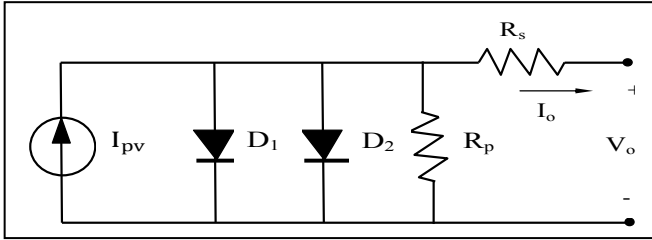


Figure 3 Double Diode Model of Polycrystalline PV Cell [8].

The mathematical equation for characterizing the I-V behavior for double diode model is expressed as follows:

$$I_o = I_{pv} - I_{r1} \left[\exp \left\{ \frac{q(V_o + I_o R_s)}{k T_{cell}} \right\} - 1 \right] - I_{r2} \left[\exp \left\{ \frac{q(V_o + I_o R_s)}{k T_{cell} A} \right\} \right] \quad (6)$$

where I_{r1} is the reverse saturation current of diode 1 due to recombination in the surfaces and bulk regions and I_{r2} is the reverse saturation current of diode 2 due to recombination in the junction. Here the value of R_p is assumed to zero for simplicity. Other parameters in Eq. (6) have already been explained in single diode model equations. Based on above equation the results and detailed analysis for PV system are presented in section III using Matlab function.

D. Double Diode Model for PV Module and Array

The electrical equivalent circuit for double diode model of a PV module and array can be drawn by replacing single diode equivalent circuit of a PV cell in Fig.2 with double diode equivalent model of cell. Similar to Eq. (5), the mathematical equation for characterizing the I-V behavior for double diode simplified model of a solar module and array is derived as shown below;

$$I_o = n_p I_{pv} - n_p I_{r1} \left[\exp \left\{ \frac{q(n_p V_o + n_s I_o R_s)}{n_s n_p k T_{cell} A} \right\} - 1 \right] - n_p I_{r2} \left[\exp \left\{ \frac{q(n_p V_o + n_s I_o R_s)}{n_s n_p k T_{cell} A} \right\} - 1 \right] \quad (7)$$

II. RESULT AND DISCUSSIONS

In order to compare PV models, & to see the effect of varying various parameters, a poly-crystalline VIKRAM ELV_40 solar photovoltaic module of 37W is considered. The manufacturer specifications of considered model are provided in Table I, is used for simulation study.

TABLE I. THE DIFFERENT ELECTRICAL PARAMETERS USED

Parameters	Specification
Maximum Power point (MPP) (P_{max})	37W
Voltage at peak power (V_{max})	17V
Current at peak power (I_{max})	2.25 A
Short circuit current (I_{sc})	2.4 A
Open circuit voltage (V_{oc})	21.8 V
Temperature coefficient of Short circuit current(K_I)	0.08 % / °C
Ideality Factor (A)	1.3

The Matlab programme calculate and analyze the output current and power of different PV module models using typical electrical parameters (I_{sc} , V_{oc} , K_I) of ELV_40 module. For calculating current I_o in Eq.(1), Newton Raphson method is used because of its simplicity.

A. I-V and P-V Curves for PV Module using Single Diode Model under Varying Solar Radiation

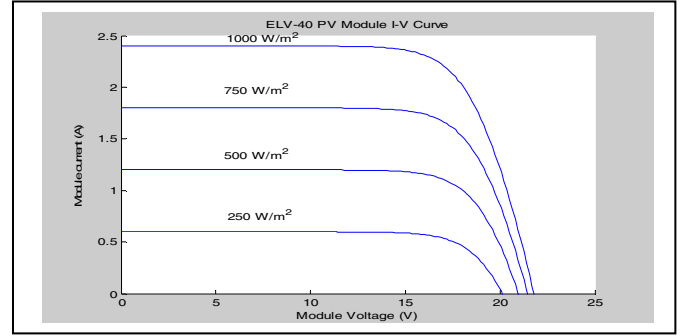


Figure 4 Matlab model I-V Curve for various irradiation level (ELV_40, Cell temp.=25°C)

Fig.4 and Fig.5 depict the typical I-V and P-V characteristics of a single diode ELV_40 PV module model under various irradiation levels. Fig.4 infers that with an increases of radiation, module short circuit current increases proportionally while there is an insignificant increment in the open circuit voltage. This leads to increase in maximum power as shown in Fig.5. It can also be justified mathematically after replacing the short circuit condition ($V_o=0$ and $I_o=I_{sc}$) and assuming the second term almost constant in Eq. (1), hence the equation is simplified as;

$$I_{sc} \alpha I_{pv} \quad (8)$$

Putting the open circuit condition ($V_o=V_{oc}$ and $I_o=0$) in Eq. (1) gives:

$$V_{oc} = \left(\frac{k T_{cell} A}{q} \right) * \ln \left[\frac{I_{pv}}{I_r} + 1 \right] \quad (9)$$

Hence, PV modules should be positioned perpendicular to the direction of sun in order to get maximum power from it.

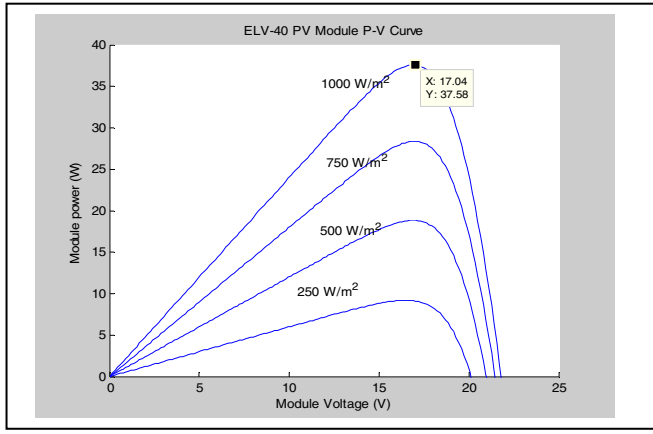


Figure 5 Matlab model P-V Curve for various irradiation level (ELV_40, Cell temp.=25°C)

B. I-V and P-V Curves for PV Module using Single Diode Model under Varying Temperature

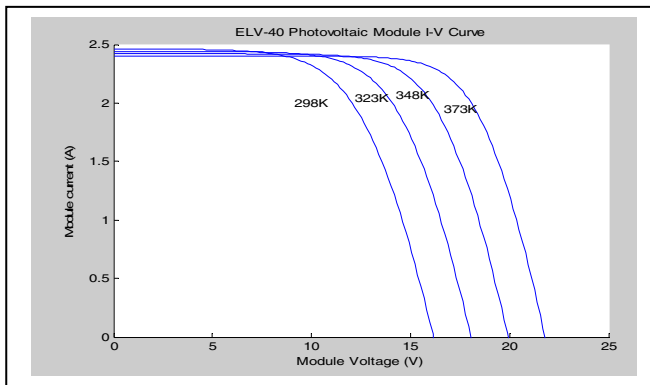


Figure 6 Matlab model I-V Curve for varying temperature (ELV_40, H=1kW/m²)

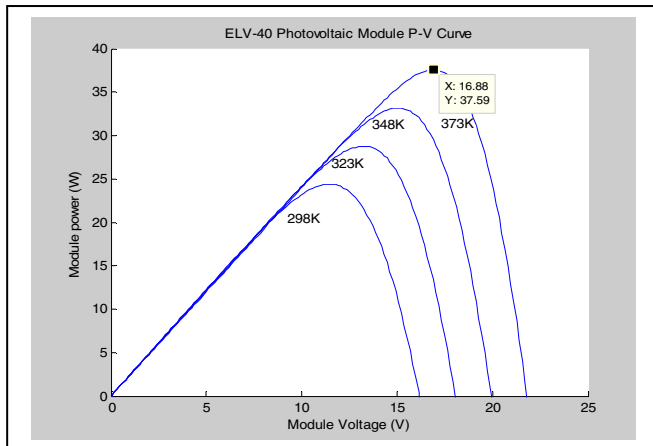


Figure 7 Matlab model P-V Curve for varying temperature (ELV_40, H=1kW/m²)

Figs. 6-7 show the variation of module current and power with module voltage for varying temperature. It is found that, V_{oc} decreases as temperature increases while there is an insignificant increment in module current.

C. I-V and P-V Curves for PV Arrays using Single Diode Model

Fig.8 represents I-V and P-V characteristics for a PV array (having two modules in parallel and one in series). The characteristics of the model are shown in Fig.8 and it is found

that I_{sc} get doubled (4.8 Amp.) and V_{oc} as 21.7V remains the same. This leads to double the value of maximum power.

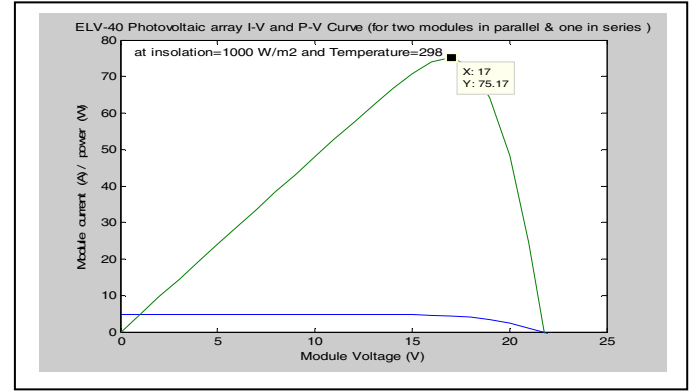


Figure 8 Matlab model I-V and P-V characteristics of array for ELV_40 model having two module in parallel and one in series at H=1kW/m² and Cell temperature=298 K

D. Effect of Ideality Factor on the I-V and P-V Curves of a PV Module using Single Diode Model

The diode ideality factor (A) is a fitting parameter and is used to match the practical behavior of diode with the theoretical ideal behavior. The value of ideality factor equals to unity indicate perfect matches to theory under STC (solar radiation=1000 W/m² and cell temperature=25°C). When recombination in space charge region dominates then the value of ideality factor increases to two. This value tends to decrease solar module output voltage and maximum power as shown in Fig.9 and Fig.10. The effect of changing ideality factor can be seen for a crystalline silicon solar cell in the I-V and P-V curves of Figs.9-10. Hence for getting more power from PV module, diode should have unity value of ideality factor.

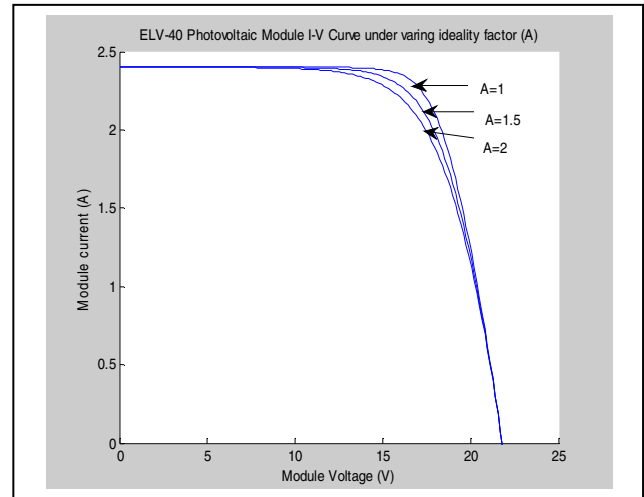


Figure 9 Matlab model I-V curves, for various diode Ideality factors (ELV_40, H=1kW/m², T=25°C and A=1, 1.5 and 2)

E. Effect of Series Resistance on the I-V and P-V Curves of a PV Module using Single Diode Model

The voltage drop between junction and terminal become high as the value of series resistance (R_s) increases. The effect of varying R_s on the I-V and P-V Curves of a PV module is shown in Figs.11-12. The results shows that there is a significant reduction in the terminal voltage V_o and a slight reduction in I_{sc} , the short-circuit current. The power loss in series resistance is given by $P_{loss} = V_{Rs} \cdot I_o = I_o^2 R_s$ and increase

proportionally with square of photo current. Therefore at higher solar illumination intensities series resistance losses are most important. Hence series resistance should have low value to get more power output from PV module.

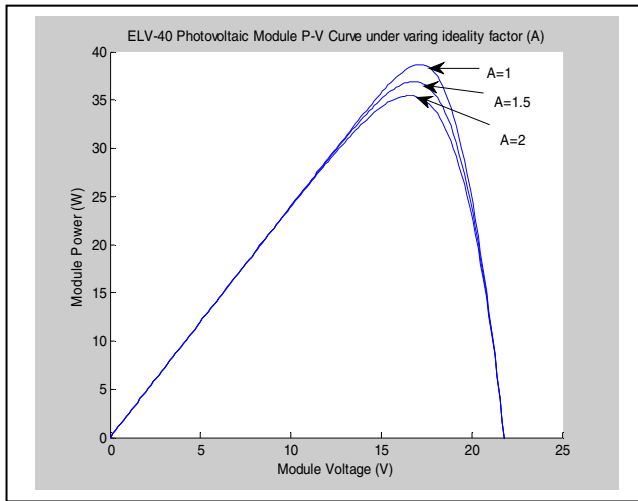


Figure 10 Matlab model P-V curves, for various diode Ideality factors (ELV_40, $H=1 \text{ kW/m}^2$, $T=25^\circ\text{C}$ and $A=1, 1.5$ and 2)

F. I-V and P-V Curves for PV Module using Double Diode Model

Figs. 13-14 show the results of I-V and P-V curve of double diode equivalent model for corresponding ideality factor for the first diode as 1.3 and for second diode as 2.

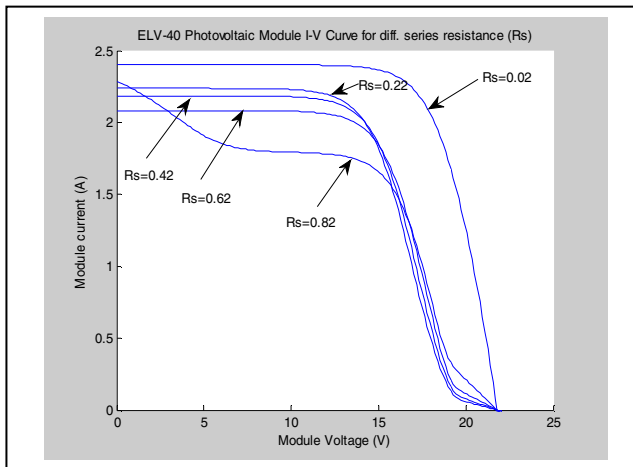


Figure 11 Matlab model I-V curves, for various series resistance (ELV_40, $H=1 \text{ kW/m}^2$, $T=25^\circ\text{C}$)

G. Comparison between Single and Double Diode Model for a PV Module using I-V and P-V Curves

Figs.15-16 show a comparison between single diode ($A=1.3$) and double diode equivalent model ($A_1=1.3$ and $A_2=2$). The corresponding maximum power, peak voltage, peak current, open circuit voltage and short circuit current values are given in Table II. The values of different parameters obtained from the Matlab function is listed in Table II for single diode model and double diode equivalent model of PV module and these values are compared with the parameter values specified by manufacturer as in Table I.

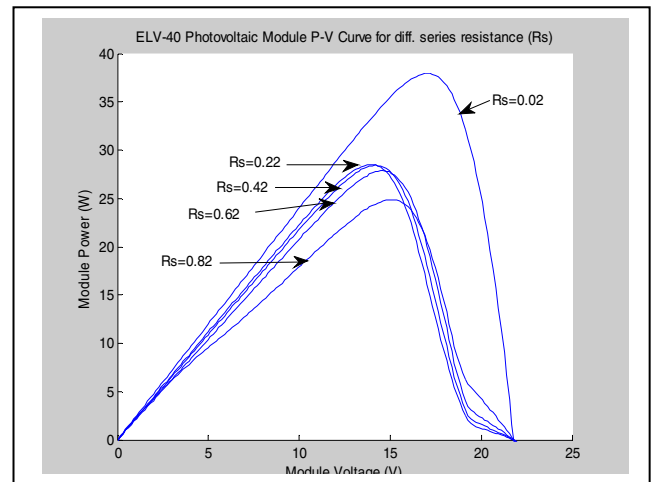


Figure 12 Matlab model P-V curves, for various series resistance (ELV_40, $H=1 \text{ kW/m}^2$, $T=25^\circ\text{C}$)

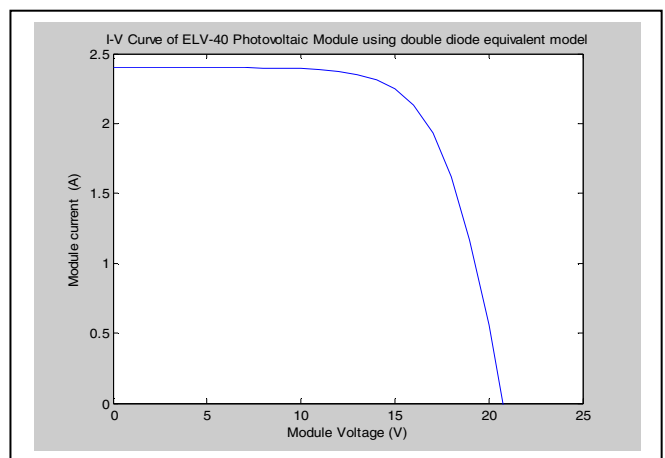


Figure 13 Matlab Model I-V curve, for double diode equivalent model (ideality factor for diode 1 is 1.3 and for diode 2 is 2).

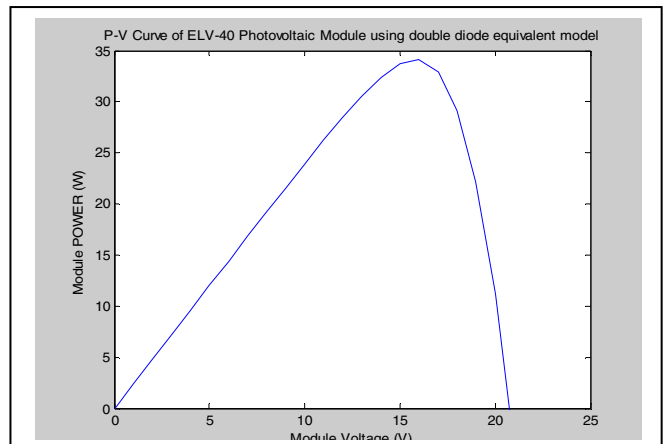


Figure 14 Matlab Model P-V curve, for double diode equivalent model (ideality factor for diode 1 is 1.3 and for diode 2 is 2).

For the single diode model with series resistance R_s , MPP is near to the value of manufacturer specified value, while in two diode model MPP deviates greatly from it. In this way single diode model is used to realize efficient MPPT algorithm and is simple in design. However, single diode model exhibits serious deficiencies when subjected to temperature variations.

TABLE II. THE DIFFERENT PARAMETERS VALUES FOR DIFFERENT MODELS

Parameters	Single diode Matlab model(A=1.3)	double diode Matlab model (A1=1.3,A2=2)	ELV_40 model (Manufacturer specification)
Maximum Power point (MPP)	37.58W	34.11W	37W
Voltage at peak power (VMPP)	17V	16V	17V
Current at peak power (IMPP)	2.21Amp	2.17Amp.	2.25 A
Open circuit voltage (V_{oc})	21.7 V	20.7V	21.8 V
Short circuit current (I_{sc})	2.4 Amp	2.4 Amp	2.4 Amp

The single diode model is significantly improved by using both the series and shunt resistance & is almost approximate to the actual PV module. This approach demands significant computing effort and its accuracy deteriorates at low irradiance. These drawbacks are totally eliminated in double diode model but it is more complex due to involvement of more number of parameters.

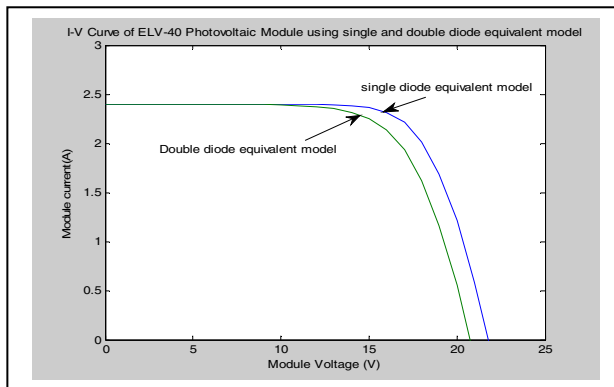


Figure 15 Matlab Model I-V curve, for single diode ($A=1.3$) and double equivalent model ($A_1=1.3$ and $A_2=2$).

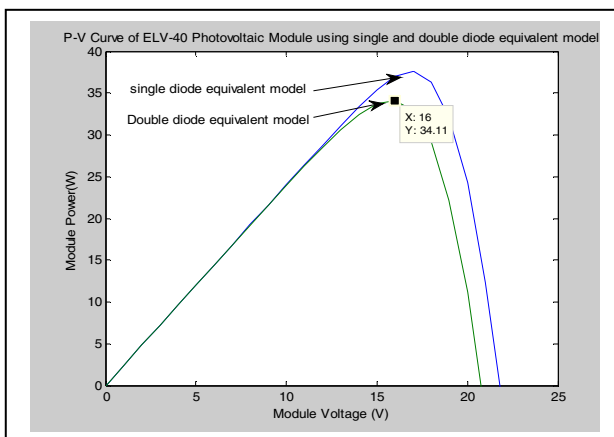


Figure 16 Matlab Model P-V curve, for single diode ($A=1.3$) and double equivalent model ($A_1=1.3$ and $A_2=2$).

III. CONCLUSIONS

For PV system development the need of simulation tools is essential, which handle mathematical simulations. Simulation is helpful for developing new system like power converter and

also helpful for realizing MPPT (maximum power point tracker) algorithm in an effective way by reducing cost and time.

This paper presents a comparison of different electrical equivalent mathematical models for a PV cell/module/array the presented different models include single diode model and double diode model. The comparison of different models are performed on the basis of the MPP tracking and value of various parameters like peak current, peak voltage, open circuit voltage and short circuit current value achieved from output I-V and P-V characteristics and their resemblance with manufacturer specified specifications. Using the comparison analysis, we conclude that the single diode model with minimum value of series resistance and with unity value of diode ideality factor is the best (efficient as well as simple) model that gives improved results as compare to the double diode model. As a result, maximum power point (MPP) can be tracked in an effective way using single diode model. The single diode model also take less simulation time due to the simplified simulation structure and is easy to implement. Thus, the presented single diode PV model can be used for modeling in the field of PV conversion system and MPPT technologies.

REFERENCES

- [1] J.P. Lee, B.D. Min, T.J. Kim, D.W. Yoo and J.Y. Yoo, "Design and Control of Novel Topology for Photo-Voltaic DC/DC Converter with High Efficiency under Wide Load Ranges", Journal of Power Electronics, vol.9, no.2, pp.300-307, Mar. 2009.
- [2] H.Altas, A.M. Sharaf, "A photovoltaic array simulation model for MATLAB-Simulink GUI Environment", Proc. of International Conference on Clean Electric Power, ICCEP'07, May 21-23, 2007.
- [3] M.A.G. Brito, L.G. Jnior, L.P. Sampaio, C.A. Canesin, "Evaluation of MPPT Techniques for Photovoltaic Applications", In Proc. of International Symposium on Industrial Electronics – ISIE, 2011.
- [4] W. Xiao, M.G.J. Lind, W.G. Dunford and A. Capel, "Real-time identification of optimal operating points in photovoltaic power systems", IEEE Trans. Ind. Electron., vol.53, no.4, pp. 1017-1026.
- [5] A.Gow and C. D. Manning, "Development of a photovoltaic array model for use in power electronics simulation studies", IEEE Proc. Elect. Power Appl., vol. 146, no. 2, pp. 193-200, 1999.
- [6] Zainal Salam, Kashif Ishaque and Hamed Taheri, "An Improved Two Diode PV Model for Photovoltaic (PV) System," IEEE, pp.1-5, 2010.
- [7] W.Xiao, H.H. Zeineldin, "A Simple Approach to Modeling and Simulation of Photovoltaic Modules", IEEE Tranction on Sustainable Energy, Vol.3, Iss.1, pp.185-186, 2012.
- [8] Huan-Liang Tsai, Ci-Siang Tu and Yi-Jie Su, "Development of Generalized Photovoltaic Model Using Matlab/Simulink," Proceedings of the World Congress on Engineering and Computer Science 2008, pp.1-6, October 2008.
- [9] G. Walker, "Evaluating MPPT converter topologies using a MATLAB PV model," Journal of Electrical & ElectronicsEngineering, Australia, IEAust, vol.21, No. 1, 2001, pp.49-56.
- [10] Chih-Hao Chang, Jia-Jun Zhu and Huan-Liang Tsai, "Model-Based Performance Diagnosis for PV Systems," SICE Annual Conference 2010, pp. 2139-2145, August 2010
- [11] Po-Wen Hsiao, Chih-Hao Chang and Huan-Liang Tsai, "Accuracy Improvement of Practical PV Model," SICE conference 2010, pp.2725-2730, 2010.
- [12] S.Chowdhury, S.P.Chowdhury, G.A.Taylor and Y.H.Song, "Mathematical Modelling and Performance Evaluation of a Stand-Alone Polycrystalline PV Plant with MPPT Facility," IEEE, pp.1-7, 2008.
- [13] Satarupa Bal, Anup Anurag and B. Chitti Babu, "Comparative Analysis of Mathematical Modeling of Photo-Voltaic (PV) Array," ISESEE conference 2011, pp.114 – 119, june 2011.