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Modeling majority carrier mobility in compensated crystalline silicon for solar cells

Florian Schindler [a,*], Martin C. Schubert [a], Achim Kimmerle [a], Juliane Broisch [a], Stefan Rein [a], Wolfram Kwapil [b], Wilhelm Warta [a]

[a] Fraunhofer ISE Fraunhofer Institut für Solare Energiesysteme, Heidenhofstr. 2, 79110 Freiburg, Germany
[b] Freiburger Materialforschungszentrum, Albert-Ludwigs-Universität Freiburg, Stefan-Meier-Str. 21, 79104 Freiburg, Germany

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ABSTRACT

Carrier mobility in silicon plays a crucial role for photovoltaic applications. While the influence of doping on mobility in standard monocrystalline silicon is well understood, recent research has been focused on the effects of crystal defects in multicrystalline (mc) silicon and of the presence of both acceptors and donors in compensated silicon, both introducing additional scattering centers influencing carrier mobility. In this work measurements of the majority carrier mobility have been carried out in two blocks of compensated multicrystalline silicon. Confirming existing results we come to the conclusion that with increasing compensation level mobilities may be significantly lower than predicted by Klaassen's mobility model, which is basically suited for the description of mobilities in compensated silicon as it accounts for scattering on both acceptors and donors. However, as this model is based on mobility data from uncompensated silicon, a compensation-related reduction of screening is not taken into account sufficiently. To describe mobilities in compensated silicon, a modification of Klaassen's mobility model based on published mobility data in compensated silicon is suggested.

1. Introduction

As the minority carrier mobility is directly connected with the diffusion length it influences the cell efficiency. Moreover, for cell simulations and several lifetime measurement techniques, knowledge of exact majority and minority carrier mobility values is inevitable. Besides a variety of mobility models, e.g. [1], only the one of Klaassen [2] accounts for the presence of both acceptors and donors as scattering centers. As silicon obtained from low cost purification routes like upgraded metallurgical-grade silicon (umg-Si) typically features high amounts of both types of dopants (called compensated), Klaassen's model is currently used as the standard model for mobility predictions in silicon used for photovoltaic (PV) applications.

The focus of this work lies on the influence of compensation on the carrier mobility. Majority carrier mobilities measured in compensated multicrystalline (mc) silicon are found to be significantly lower than predicted by Klaassen's model. One reason could be the possible influences of crystal defects like grain boundaries or dislocations, as they are not taken into account in the model. However, in a recent publication it has been shown that such crystal defects as occurring in standard mc silicon appear to have only a minor impact on majority carrier mobilities which is insignificant for PV applications [3]. Therefore, lower mobilities can be attributed to compensation, which is also confirmed by the observations in several publications indicating that for an increasing compensation level Klaassen's model overestimates mobilities significantly [4–14]. To discuss the reasons for the deviation from Klaassen's mobility model, we compare our experimental results with published data, revealing an increasing deviation from Klaassen's model with increasing compensation level. It has been speculated if a new scattering mechanism occurring in compensated silicon might be the reason for lower mobilities [10–12]. However, all involved scattering partners are taken into account in Klaassen's model and there is no evidence for a further scattering mechanism so far. Thus, we favor a different approach to resolve the discrepancies between experimental results and Klaassen's model: As Klaassen's mobility model is based on the empirical approach of Caughey and Thomas [1], describing mobilities in uncompensated silicon, and their fit parameters are derived from mobility data from uncompensated silicon samples, respectively, the cause of the deficient prediction of mobilities in compensated

* Corresponding author. Tel.: +49 761 4588 5321; fax: +49 761 4588 9250.
E-mail addresses: florian.schindler@ise.fraunhofer.de (F. Schindler), martin.schubert@ise.fraunhofer.de (M.C. Schubert), achim.kimmerle@ise.fraunhofer.de (A. Kimmerle), juliane.broisch@ise.fraunhofer.de (J. Broisch), stefan.rein@ise.fraunhofer.de (S. Rein), wolfram.kwapil@fmf.uni-freiburg.de (W. Kwapil), wilhelm.warta@ise.fraunhofer.de (W. Warta).

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silicon is not necessarily an additional scattering mechanism but simply an insufficient parameterization accounting for a compensation-dependent reduction of screening of charged impurities.

In this work we will outline this approach in detail and show that the introduction of a compensation-dependent term in the Caughey–Thomas mobility-expression is well suited to predict majority carrier mobilities in compensated silicon with Klaassen's mobility model satisfyingly. The parameters are obtained from a fit to mobility data in compensated silicon from this work and references [4,5,7–10].

2. Material and methods

For the evaluation of mobilities in compensated mc silicon we measured Hall mobilities μ_H, acceptor concentrations N_A, net doping concentrations p_0 and resistivities ρ (the latter two providing the conductivity hole mobility μ_C in p-type silicon) at different block positions of two multicrystalline upgraded metallurgical-grade (umg) silicon blocks. Both blocks were p-type, featuring an increasing compensation towards the block top without a type inversion to n-type. N_A was determined via iron-acceptor pairing [15], p_0 via the FTIR-FCA method [7] and ρ via 4-point-probe measurements. From ρ and p_0 the donor concentration is calculated as $N_D=N_A-p_0$ (assuming complete ionization). Hall factors were calculated from the measured Hall mobilities and the conductivity mobilities as $r_H=\mu_H/\mu_C$.

3. Theory

3.1. Empirical description of carrier mobility in silicon as a function of doping concentration

One of the first approaches for the description of carrier mobilities in silicon as a function of doping concentrations was published in 1967 by Caughey and Thomas [1]. As plots of measured mobilities versus the logarithm of the dopant density resemble a hyperbolic tangent, they fitted the data with the following expression:

$$\mu=\frac{\mu_{max}-\mu_{min}}{1+(N/N_{ref})^{\alpha}}+\mu_{min} \tag{1}$$

N denotes the doping concentration, and μ_{max}, μ_{min}, N_{ref}, and α are fit parameters, of which only μ_{max} can be interpreted as a physical parameter. For zero impurity concentration the mobility is limited by scattering on phonons only, thus μ_{max} describes the mobility limited by lattice scattering. This simple empirical mobility expression only holds for the presence of one type of dopant with concentration N.

3.2. A physics-based analytical model for carrier mobilities in silicon

In 1992, Klaassen published a physics-based analytical model [2] unifying the descriptions of majority and minority carrier mobility including screening of the impurities by charge carriers, electron-hole scattering, clustering of impurities and the temperature-dependence of carrier mobility [16]. As clustering only is important for doping concentrations higher than $10^{20}\,\mathrm{cm}^{-3}$, in the following we will discuss the basics of Klaassen's mobility model without clustering and temperature-dependence. Only the equations needed for the modifications suggested in this work are outlined, for a more detailed deduction see Ref. [2].

Klaassen's approach starts with the above mentioned mobility expression from Caughey and Thomas. In Eq. (1), lattice scattering and impurity scattering can be separated applying Matthiesen's rule

$$\frac{1}{\mu}=\frac{1}{\mu_L}+\frac{1}{\mu_I} \tag{2}$$

with $\mu_L=\mu_{max}$ denoting the mobility due to lattice scattering and μ_I denoting the mobility due to impurity scattering. Inserting Eq. (1) in (2) delivers for the impurity scattering

$$\mu_I=\frac{\mu_{max}^2}{\mu_{max}-\mu_{min}}\left(\frac{N_{ref}}{N}\right)^{\alpha}+\frac{\mu_{max}\mu_{min}}{\mu_{max}-\mu_{min}}=\mu_N\left(\frac{N_{ref}}{N}\right)^{\alpha}+\mu_c \tag{3}$$

μ_N and μ_c are abbreviations for the terms depending on μ_{max} and μ_{min} and without physical meaning. As the mobility due to impurity scattering is proportional to the inverse product of the impurity concentration N and the collision cross section σ, the collision cross section can be expressed using (3) as

$$\sigma\propto(N\mu_I)^{-1}=(\mu_N(N_{ref})^{\alpha}(N)^{1-\alpha}+\mu_cN)^{-1} \tag{4}$$

The theoretical collision cross section, assuming a Coulomb potential for each charged scattering center, diverges for small scattering angles. This divergence was avoided by Conwell and Weisskopf by arbitrarily cutting off the Coulomb field at a radius equal to half the mean distance between the scattering centers [17]. In the approach of Brooks and Herring [18], an arbitrary cutting radius was avoided by introducing screening: At high carrier concentrations, free carriers tend to screen impurities which causes the scattering potential to fall off with distance more rapidly than a pure Coulomb potential would. But as the Brooks–Herring approach fails in the limit of zero screening carriers, here the Conwell–Weisskopf formula becomes the only viable replacement. In the statistical screening theory of Ridley [19,20], the approaches of Conwell–Weisskopf and Brooks–Herring are combined: At low concentrations the collision cross section becomes a function of the impurity concentration N and at high concentrations a function of the carrier concentration c. As in Eq. (4) the first term is predominant for low N and the second for high N, Klaassen replaces the impurity concentration N in the second term by the carrier concentration c. This results in the following expression for μ_I:

$$\mu_I=\mu_N\left(\frac{N_{ref}}{N}\right)^{\alpha}+\mu_c\left(\frac{c}{N}\right) \tag{5}$$

The limit of $c=N$ holds for uncompensated silicon, assuming complete ionization. Here, screening is most effective, and we reobtain from Eq. (5) the Caughey–Thomas expression (3). In compensated silicon, free carrier concentration decreases with a simultaneous increase of the ionized dopant concentration with increasing compensation level $C_I=(N_A+N_D)/(N_A-N_D)$, which means that screening decreases dramatically. The weakness of the Klaassen-model is in our view, that in Eq. (5) this is only accounted for in the second term, while the first term still stems from the Caughey–Thomas expression for uncompensated silicon. As the first term is predominant for impurity concentrations $N\le N_{ref}=2.23\times10^{17}\,\mathrm{cm}^{-3}$, the decreased screening effect is not taken into account adequately in Klaassen's model in this range of dopant concentrations. This is where our approach sets in to correctly describe mobilities in compensated silicon.

4. Results

4.1. Majority carrier mobility in compensated mc silicon

Measurements of doping concentrations in two compensated mc silicon blocks (*mc umg* 1 and *mc umg* 2) reveal a higher total dopant concentration N_A+N_D as well as a higher net doping concentration $p_0=N_A-N_D$ in block *mc umg* 2. Despite the

difference in doping concentrations, conductivity mobility (μ_C) and Hall mobility (μ_H) measurements show that surprisingly both blocks feature virtually identical mobilities. Thus, mobility values μ_C of the two blocks plotted against the acceptor concentration N_A are shifted only along the x-axis (Fig. 1). Searching a proper representation of the data, we note, that in contrast to doping concentrations the compensation level $C_l=(N_A+N_D)/p_0$ of both blocks is very similar. In a plot of mobilities against the compensation level the shift along the x-axis disappears (Fig. 2). Hence, the compensation level C_l appears to have a stronger influence on the mobility than predicted by Klaassen's model, which will be discussed in more detail in Sections 4.2 and 4.3. In this interpretation, the mobility-reducing influence of the larger amount of ionized scattering centers in block *mc umg* 2 appears to be compensated by a higher net doping concentration leading to a more effective screening of the ionized dopants and therefore to mobility values as high as in block *mc umg* 1.

Measured conductivity mobilities were also compared to those predicted by the model of Klaassen. In Fig. 1 mobilities are plotted against the acceptor concentration, and different donor concentrations are taken into account in the Klaassen-model. For the sake of clarity, the Klaassen-model is shown only for the highest and lowest donor concentration occurring in the samples. All mobilities are lower than predicted by Klaassen, with a maximum deviation of 15%. Within one block, an increasing compensation level is correlated with an increasing deviation from Klaassen's model. Based on these measurements and mobility data published elsewhere [4,5,7–10], a modification of Klaassen's mobility model correctly accounting for screening in compensated silicon will be introduced in Section 4.3.

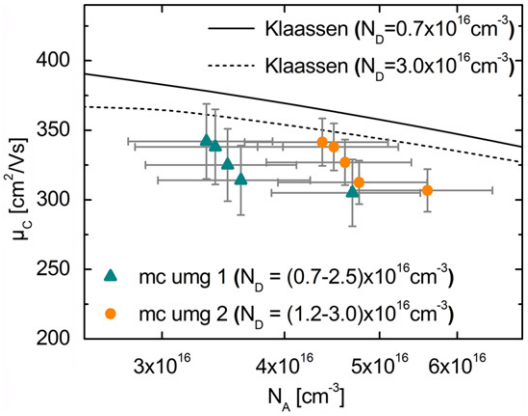

Fig. 1. Conductivity mobilities μ_C plotted against the acceptor concentration N_A for a comparison with the model of Klaassen. Measured mobilities are lower than predicted by the model.

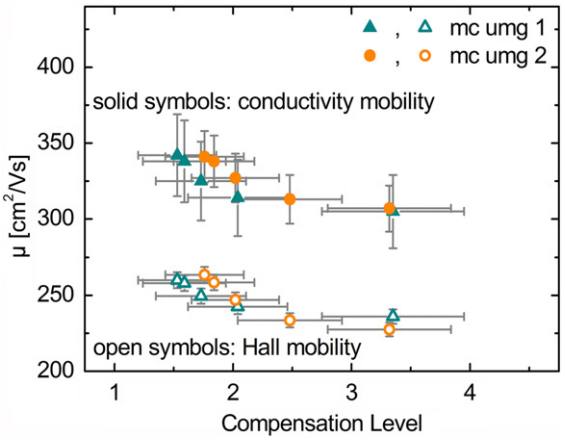

Fig. 2. Conductivity mobilities μ_C and Hall mobilities μ_H plotted against the compensation level C_l. This reveals that the compensation level influences the mobility significantly.

In a last step, Hall factors derived from the measured Hall mobility and conductivity mobility were compared with Lin's theoretical model [21] and with a model suggested in [3] (Fig. 3). While the majority of the Hall factors plotted versus the total dopant concentration is lower than predicted by Lin, good agreement is achieved with the model introduced in [3] based on measurements in uncompensated FZ silicon. Hall factors vary between 0.74 and 0.77. This confirms the general assumption that Hall factors are not affected by compensation [8,10].

4.2. Comparison of published majority carrier mobility data in compensated silicon with Klaassen's model

Several groups have recently been working on mobilities in compensated silicon [4–14]. The aim of this work is to review and compare published mobility data in compensated silicon and to draw conclusions for mobility modeling.

To understand published data, we have to explain in more detail the investigated materials of each research group, which are summarized in Table 1, and the different evaluation methods applied. For further details please see the corresponding references.

All investigated materials were p-type silicon, most of them doped with Boron as acceptors and Phosphorous as donors. Exceptions are the material investigated by Forster et al. [5], which was additionally co-doped with Gallium, and the material investigated by Veirman et al. [10], which was purely Boron doped. In the latter study, compensation was achieved by activating thermal donors. In this case, because it is supposed that one thermal donor (TD) provides two free electrons, the compensation level has been redefined as $C_l=([B]+2x[TD])/([B]-2x[TD])$. Also, because thermal donors are assumed to be double-ionized, their scattering power is enhanced by a factor of four compared to a single-charged donor impurity, which is why in the simulated mobility the donor concentration was set to $N_D=4x[TD]$ (for details see [10]).

Basically, there are two different methods for the determination of mobilities used in the reviewed literature. One is to measure directly the conductivity mobility $\mu_C=1/(q\rho p_0)$ via a measurement of the resistivity ρ and the net doping concentration p_0 (where q is the elementary charge). Alternatively, the Hall mobility μ_H can be measured. To deduce the conductivity

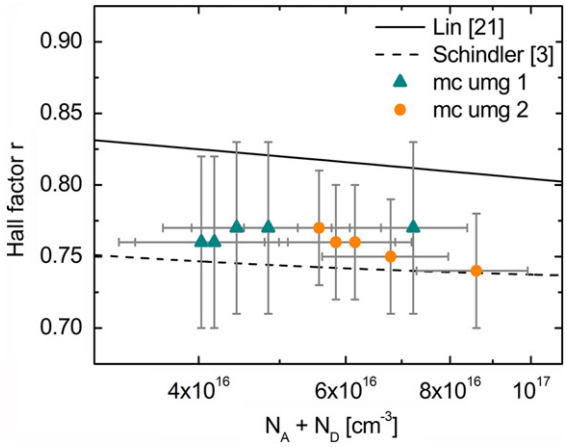

Fig. 3. Hall factors r_H plotted against the total dopant concentration N_A+N_D. For comparison, the theoretical model in uncompensated silicon proposed by Lin [21] is shown as well as the empirical model suggested in [3]. Good agreement is achieved with the latter one.

Table 1
Publications of mobilities in compensated silicon.

Publication	Material	Dopants	Measured Mobility	Deduced Mobility	Characteristics
Libal et al. [4]	Cz	B, P	μ_H	$\mu_C=\mu_H/0.65$	
Forster et al. [5]	Cz	B, Ga, P	μ_H	$\mu_C=\mu_H/0.7$	Ga co-doping
Geilker et al. [7]	Cz	B, P	μ_C		
Rougieux et al. [8]	Cz	B, P	μ_C, μ_H		
Schindler et al. [9]	mc	B, P	μ_C, μ_H		
Veirman et al. [10]	Cz	B	μ_H	$\mu_C=\mu_H/r_H$; $r_H=0.72-0.76$	Compensation by activation of thermal donors

mobility from the Hall mobility, the Hall scattering factor r_H has to be known, which links both mobilities via $\mu_C=\mu_H/r_H$. In order to uniformly represent mobilities, only conductivity mobilities are compared. In Refs. [7–9], the conductivity mobility was measured directly, Forster et al. [5] published conductivity mobilities deduced from Hall mobilities using a Hall factor of $r_H=0.7$, whereas Libal et al. [4] and Veirman et al. [10] published Hall mobilities. We recalculated the conductivity mobilities from Veirman's Hall mobilities using Hall factors of 0.76, 0.74 and 0.72, as indicated in the publication [10]. To achieve conductivity mobilities from the Hall mobilities published in reference [4], we used a Hall factor of 0.65. Here, some comments have to be made on this rather low Hall factor. Libal et al. propose a Hall factor of 0.74 in the same publication [4]. However, this does not agree with their published data. A comparison of the Hall mobility in the least compensated sample ($C_I=1.3$, virtually uncompensated, $\mu_H \approx 280\ \mathrm{cm^2\,V^{-1}\,s^{-1}}$) and the conductivity mobility predicted by Klaassen's model (which holds for uncompensated silicon) for the same sample ($\mu_C \approx 429\ \mathrm{cm^2\,V^{-1}\,s^{-1}}$, calculated from the dopant concentrations published in [4] for this sample), leads to a Hall factor of $r_H \approx 0.65$. As compensation does not seem to have a significant influence on the Hall factor (see for instance Section 4.1 in this work and references [8,10]), this Hall factor is used to deduce conductivity mobilities from the published Hall mobilities also for the compensated samples of Ref. [4]. Compared to Hall factors measured in this work and published in Refs. [3,10], a value of 0.65 seems to be rather low. However, it is comparable to the Hall factors measured in Ref. [8]. As Hall factors directly originate from measured Hall mobilities, a low Hall factor can be explained by an underestimated Hall mobility. As for example larger contacts on the Hall sample would lead to a lower measured Hall mobility, this could be the reason for such a low Hall factor.

For a comparison of the published mobility data with Klaassen's mobility model, a well suited and clear representation of the data is a plot of the relative deviation of measured mobilities from the mobility predicted by Klaassen $(\mu_{exp}-\mu_{Klaassen})/\mu_{Klaassen}$ against the compensation level (see Fig. 4).

Assuming a relative error of 10% on the mobility, for low compensation levels of up to 2–3 Klaassen's model appears to predict mobilities quite well. But it can be seen already that mobilities are systematically lower than predicted. With increasing compensation level, the deviation also increases, leading to a dramatic overestimation of mobilities for high compensation levels. For example, at a compensation level of about 30, the mobility is up to 60% lower than predicted by Klaassen. The reason for this overestimation and a modification in the model accounting for compensation will be discussed in the next section.

4.3. Modeling mobility in compensated silicon

Our perception of the reasons for the overestimation of mobilities in compensated silicon by Klaassen's mobility model has already been sketched in Section 3.2. An increasing

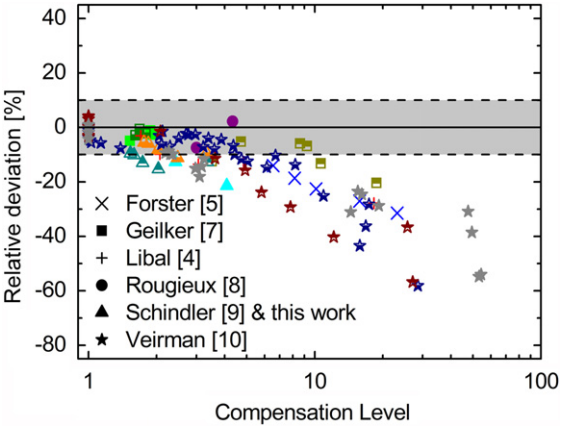

Fig. 4. Relative deviation of measured majority hole mobility in *p*-type compensated silicon from Klaassen's mobility model versus the compensation level. Measured mobilities are lower than modeled mobilities, with an increasing relative deviation with increasing compensation level. Solid and semi-open symbols of the same shape denote different samples or blocks investigated by the same author.

compensation level means an increase of ionized scattering centers, while the free carrier concentration decreases simultaneously leading to a much less effective screening of charged impurities. Although Klaassen's model is meant to include a screening-correction, this only holds for high impurity concentrations. For PV silicon as investigated here, total base-dopant concentrations are in the range of $1\times10^{15}\ \mathrm{cm^{-3}}$–$5\times10^{17}\ \mathrm{cm^{-3}}$. In this range of dopant concentrations, the first term in Eq. (5) is predominant. This term stems from the empirical Caughey–Thomas expression for mobility, which is derived from and consequently correctly describes the mobility for the case where the impurity concentration equals the free carrier concentration, i.e. the case of strongly screened impurities. With increasing compensation level the amount of screening carriers decreases, leading to a higher collision cross section of the charged impurities (or, in other words, a higher effective impurity concentration) and therefore to a lower mobility. This is not taken into account in Klaassen's model.

As Klaassen's model is currently used as the standard mobility model for device simulation and excellently agrees with experimental mobility data in uncompensated silicon, we basically follow Klaassen's approach for the description of mobilities, but with a modified empirical "Caughey–Thomas mobility expression" as a starting point. We modify the mobility expression of Eq. (1) by inserting a compensation-dependent term:

$$\mu=\frac{\mu_{max}-\mu_{min}}{1+(N/N_{ref})^{\alpha}+((C_I-1)/C_{I,ref})^{\beta}}+\mu_{min} \tag{6}$$

For uncompensated silicon, the third term in the denominator vanishes as the compensation level is equal to one and we reobtain the familiar mobility expression from Caughey–Thomas from Eq. (1). With increasing compensation level, this term accounts for a decreased screening leading to lower mobilities.

Using this empirical mobility expression as starting point for Klaassen's model, instead of Eq. (5) we obtain the following expression for mobility due to impurity scattering:

$$\mu_I = \mu_N \left(\left(\frac{N}{N_{ref}} \right)^{\alpha} + \left(\frac{C_I - 1}{C_{I,ref}} \right)^{\beta} \right)^{-1} + \mu_c \left(\frac{c}{N} \right) \quad (7)$$

The further calculations remain the same as in reference [2]. With this modified approach, we obtain two fit parameters in Klaassen's mobility model for the description of reduced screening, β and $C_{I,ref}$. These parameters are determined by fitting the model to the majority carrier mobility data in compensated silicon as summarized in Fig. 4. Minimizing χ^2 delivers

$$C_{I,ref} = 25 \pm 4$$
$$\beta = 0.81 \pm 0.04 \quad (8)$$

The relative deviation of measured mobilities from the mobility predicted by this "modified Klaassen-model" $(\mu_{exp} - \mu_{model})/\mu_{model}$ against the compensation level is shown in Fig. 5.

As can be seen in the graph, with the parameters (8) the systematic overestimation of mobilities in compensated silicon by the model description disappears as well as the trend of an increasing deviation with increasing compensation level. The majority of the data scatters in a band of $\pm 10\%$ around the modeled mobility, which is also estimated to be the uncertainty of the measured mobilities. Assuming this relative error of 10% for each data point, with the fit parameters (8) we obtain a reduced χ^2 of 0.89 ± 0.15 proving the goodness of the fit. For comparison, without the two introduced parameters $C_{I,ref}$ and β, Klaassen's model leads to $\chi^2_{red} \approx 10$. A few outliers at high compensation levels stem from samples with thermally activated donors, where the uncertainty is presumably much higher, as the values are based on the assumption that the thermal donors are doubly ionized. This affects the calculation of the donor concentration and thereby the compensation level. Furthermore, the modeled mobility for the samples from reference [10] depends on the assumption that the scattering power of the thermal donors is enhanced by a factor of four compared to a single-charged donor impurity. These additional assumptions clearly add to the uncertainty of the data from samples with thermally activated donors.

To summarize, by introducing a compensation-dependent correction term in the empirical mobility expression by Caughey–Thomas, we are able to predict majority carrier mobilities with Klaassen's model in good agreement with published data in compensated p-type silicon. It should be mentioned that predictions of mobilities in uncompensated silicon remain unaffected, as the correction term disappears for $C_I = 1$. This means that this correction can simply be implemented in standard programs without changing predictions for uncompensated silicon but with significant improvement for the prediction of majority carrier mobilities in compensated silicon.

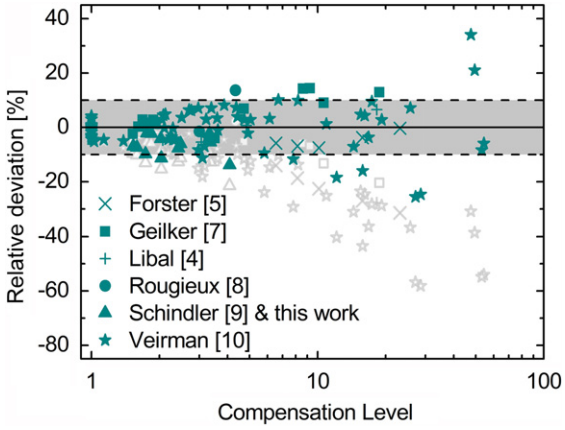

Fig. 5. Relative deviation of measured majority hole mobility in p-type compensated silicon from our "modified Klaassen-model" versus the compensation level (dark cyan solid symbols). For comparison, the deviation from the uncorrected Klaassen-model is also plotted (gray open symbols). After the correction, the systematic overestimation disappears. The data scatters in a band of $\pm 10\%$ around the modeled mobility except a few outliers for the samples with thermally activated donors. This is a typical uncertainty range of measured mobilities. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5. Discussion

The drop in mobility for increasing compensation has been discussed in several publications, see for instance Refs. [4,5,9,10,12,13]. In Ref. [12], a mobility correction term $\mu_{cor} \propto C_I^{-3/4}$ attributed to a specific compensation effect which is not taken into account in the existing mobility models is suggested, leading to the following mobility expression:

$$\frac{1}{\mu} = \frac{1}{\mu_{Klaassen}} + \frac{1}{\mu_{cor}} \quad (9)$$

Indeed, agreement with the published mobility data in compensated silicon and Eq. (9) is similar to the agreement achieved with the model presented in this work. However, adding a correction term according to Matthiesen's rule can only be justified with an additional scattering channel in compensated silicon (for example space-charge region scattering as suggested in [10]). As such a scattering mechanism has not been proven to occur in compensated silicon so far, we argue that the deviation from Klaassen's model simply arises from the fact that Klaassen's approach is based on an empirical mobility expression which had been established for uncompensated silicon and thus does not take into account a decreased screening of ionized scattering centers in compensated silicon sufficiently.

The approach presented here is based on mobility data of majority carriers in p-type compensated silicon. The parameters (8) have been derived for these conditions only. For the description of minority electron mobilities in compensated p-type silicon as well as majority and minority carrier mobilities in n-type silicon, the same approach is likely to apply, but this assumption still has to be validated. Further investigations in these directions are currently carried out.

6. Conclusions

In this work, we performed measurements of majority carrier conductivity mobilities as well as Hall mobilities in two compensated multicrystalline silicon blocks doped with different amounts of Boron and Phosphorous. Experimental results lead to the following conclusions:

- Mobilities in compensated silicon are lower than predicted by Klaassen's mobility model.
- A lower free carrier concentration in the block with the lower total dopant concentration leads to a less effective screening of ionized scattering centers and thus to mobilities as low as in the second block with both a higher concentration of ionized dopants and a higher concentration of free carriers.
- The compensation level C_I appears to have a stronger influence on the mobility than predicted by Klaassen's model.
- Measured Hall factors plotted against the total dopant concentration are comparable to Hall factors in uncompensated silicon with the same dopant concentration, confirming the

general assumption that Hall factors are not affected by compensation [8,10].

In a second part, majority carrier mobilities in compensated *p*-type silicon published by several research groups [4,5,7–10] are compared with our data and with Klaassen's mobility model. Plotting the relative deviation against the compensation level reveals a systematic overestimation of mobilities in compensated silicon by Klaassen's model, which increases with increasing compensation level.

Finally, we presented a modified mobility model based on the approach of Klaassen for the description of mobilities in compensated silicon. We argue that Klaassen's model does not account for decreased screening with increasing compensation level in the range of dopant concentrations typically occurring in compensated PV silicon, as the model is based on an empirical mobility expression for uncompensated silicon. In our approach, we introduce an additional compensation-dependent term into the mobility expression of Caughey–Thomas. Based on this modified expression, Klaassen's model predicts majority carrier mobilities in compensated *p*-type silicon in good agreement with experimental data. Predictions of mobilities in uncompensated silicon remain unaffected as the compensation-dependent term disappears for $C_l=1$. Further investigations have to validate this approach for minority carrier mobilities as well as mobilities in n-type compensated silicon.

Acknowledgments

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