

A NEW EXPERIMENTAL DETERMINATION OF THE RELATIONSHIP BETWEEN THE HALL MOBILITY AND THE HOLE CONCENTRATION IN HEAVILY DOPED *p*-TYPE SILICON

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Abstract—The relationship between the Hall mobility and the hole concentration in *p*-type silicon were experimentally investigated. Boron and gallium were used as dopants; their doping was done by either the ion implantation or the diffusion technique. It was found that the relationships for boron-doped specimens and for gallium-doped specimens for the heavily doped range are considerably different from each other. Both of these relationships are also different from the so-called Irvin curve which is well known as a standard relationship between the mobility of carriers and the impurity concentration, but the relationship for boron doping is almost consistent with that of Thurber *et al.*, if the Hall mobility factor is reasonably taken into account.

1. INTRODUCTION

The carrier mobility of semiconductors is one of the most important parameters for designing semiconductor devices and integrated circuits. The characteristic curves of resistivity at 300 K vs impurity concentration, one for *p*-type and another for *n*-type silicon, have been valuable for designing silicon devices. These curves were obtained by Irvin[1] in 1962 by adjusting various experimental data and judged by himself to be the most reasonable compromise of the mass of published and unpublished data until 1961. He described in his paper that 75% of the data points did not scatter more than 10% of the determined values of the curves. The values of impurity concentration were determined by several methods such as the photometric carmine technique, neutron activated gamma-ray spectrometry and the Hall effect, on the assumption that the Hall factor *r* was equal to unity. The meaning of *r* is μ_H/μ , where μ_H is the Hall mobility and μ is the conductivity mobility.

An analytical expression for the carrier mobilities as a function of impurity concentration was given by Caughey and Thomas[2] in order to describe the Irvin curves. The expression is as follows:

$$\mu = \mu_{\min} + \frac{\mu_{\max} - \mu_{\min}}{1 + (N/N_{\text{ref}})^\alpha} \quad (1)$$

where *N* is the impurity concentration. The values of the parameters μ_{\min} , μ_{\max} , α and N_{ref} were determined so as to fit the Irvin mobility data.

The mobilities of holes and electrons depend on the carrier scattering by the impurity ions as well as by the lattice vibration. The carrier mobility of heavily doped ($N > 10^{17} \text{ cm}^{-3}$) silicon at room temperature depends mainly on the scattering by the impurity ions. For $N > 10^{19} \text{ cm}^{-3}$, the impurity levels spread onto the intrinsic band edge, and the carriers are conducted mainly in the hybrid band of impurity and intrinsic conduction. Since the carriers have a greater chance of passing through the vicinity of ionized impurities as the doping level becomes heavier, the scattering rate and the carrier mobility in the heavily doped semiconductor can be varied by using impurity species which have somewhat different scattering potentials from each other.

For *n*-type silicon, the Irvin curve was modified experimentally by Mousty *et al.*[3] and the parameters of the Caughey and Thomas expression were revised by Baccarani and Ostojja[4]. Their mobility values are larger than the values given by the Caughey and Thomas expression by about 10–35% for the heavier doped region ($> 10^{19} \text{ cm}^{-3}$) and by about 20% for the doping range between 10^{16} and 10^{17} cm^{-3} . However, the difference is small for the doping range of 10^{18} – 10^{19} cm^{-3} . For *p*-type silicon, the value of N_{ref} in eqn (1) which had been proposed by Caughey and Thomas was modified by Wagner[5]. He adopted a new value of N_{ref} in order that the total carriers of the boron implanted and furnace annealed surface layers could reasonably agree with the boron dose. The new value is three times larger than the original value by Caughey and Thomas.

A new relationship between the drift mobility and the hole concentration in the boron doped silicon was obtained by Thurber *et al.*[6]. They obtained the carrier concentration in the range of boron doping between 10^{14} and $3 \times 10^{17} \text{ cm}^{-3}$ by capacitance-

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voltage (C - V) measurements on n^+p junction diodes and in the range between 8×10^{18} and $1.5 \times 10^{20} \text{ cm}^{-3}$ by Hall effect measurements. The carrier concentrations of some samples were checked with the boron concentrations determined by the nuclear track technique (NTT)[7]. The drift mobility was calculated from the resistivity and the carrier concentration. The expression for the drift mobility of holes μ_p as a function of hole concentration was shown as follows:

$$\mu_p = A \exp(-p_c/p) + \frac{\mu_{\max}}{1 + (p/p_{\text{ref}})^\alpha}, \quad (2)$$

where p is the hole concentration, A , p_c , μ_{\max} , p_{ref} and α are constants. Their results were in agreement with the Wagner curve and considerably different from that of Irvin. However, their data points in the doping range between 3×10^{17} and $8 \times 10^{18} \text{ cm}^{-3}$ were not enough, being only two, and those were determined by the NTT, or by a different technique from those used in both the higher and lower doping ranges.

The mobility of holes in boron doped silicon were also obtained by Masetti *et al.*[8] from Hall effect measurement. However, the boron doping was restricted to the higher range of concentration.

In this paper, we show detailed relationships between the Hall mobility of holes and the concentration of holes for boron doped silicon and gallium doped silicon. The doping ranges are between 10^{17} and $3 \times 10^{20} \text{ cm}^{-3}$ for boron and 10^{17} and $2.3 \times 10^{19} \text{ cm}^{-3}$ for gallium. A revision of the Irvin curve and a comparison with the result of Thurber *et al.* are also described.

2. EXPERIMENTAL PROCEDURE

Samples were p -type silicon surface layers doped by diffusion technique or ion implantation with boron ions and by diffusion of gallium into n -type silicon crystals ($N_D < 10^{15}$). The boron diffusion was carried out using a BN wafer, or poly boron film (PBF) coated on a silicon wafer as the diffusion source, with a heat treatment at 950–1095°C for 10–40 min in dry nitrogen. The gallium diffusion was done in a closed evacuated quartz tube using pure gallium with a heat treatment of 900–1200°C for 1–36 h. For the ion implantation, the implant energy of boron was 35 KeV and the dose was $8 \times 10^{15} \text{ cm}^{-2}$. After the implantation, the wafers were annealed by a Q-switched argon laser pulse with energy of 2.0 J/cm² followed by a furnace annealing at 900°C for 0.5–90 h in order to eliminate residual defects and to make gentle the slope of impurity profiles.

The carrier mobility was determined by the following procedure. The sheet conductivity g_s and the Hall voltage V_H of the p -type surface layer were measured by the Van der Pauw method[9]; then the conducting surface layer was oxidized by anodic oxidation. The oxide layer so formed was removed by an HF solu-

tion and the remaining g_s and V_H were measured as a function of the removed depth z measured from the original surface. The removed thickness z was determined by the sum of the times of anodic oxidation under the regulated condition. This procedure was repeated until the n -type pristine silicon appeared. The total thickness of the removed p -type layer was measured with a multiple interference microscope and checked with the sum of the removed thicknesses determined as above. The Hall mobility $\mu_H(z)$ and the hole concentration $p(z)$ at the depth z were calculated by the following equations (Appendix):

$$\mu_H(z) = -\frac{1}{IB} \cdot \frac{d\{g_s(z)^2 \cdot V_H(z)\}}{dg_s(z)} \quad (3)$$

$$p(z) = \frac{1}{q\mu(z)} \cdot \frac{dg_s(z)}{dz}, \quad (4)$$

where $g_s(z)$ and $V_H(z)$ are the sheet conductivity and the Hall voltage when the surface of thickness z is removed, I is the electric current and B is the magnetic field applied perpendicularly to the surface. For the actual calculation of $p(z)$ by the eqn (4), $\mu(z)$ was substituted by $\mu_H(z)$.

3. RESULTS

The data points obtained for boron doped and gallium doped silicon are shown in Fig. 1 and show the relationship between the hole concentration p and the Hall mobility μ_H . The dashed lines are drawn to show the Irvin curves for n -type and p -type silicon as expressed by Caughey and Thomas[2] and the dot-dash line is drawn to show the relationship between the phosphorus concentration N_D and the electron mobility μ_n obtained by Mousty *et al.*[3] and analytically expressed by Baccarani and Ostojja[4]. There are large deviations of the present data from the Irvin curve for p -type silicon. Our μ_H values are larger than the Irvin curve in the region of the hole concentration p from 10^{17} to 10^{19} cm^{-3} and smaller in the region of the hole concentration larger than $1.2 \times 10^{19} \text{ cm}^{-3}$. The largest deviation appears around $1.5 \times 10^{18} \text{ cm}^{-3}$ where the present data are 1.5 times larger than Irvin's.

For the boron-doped silicon, the μ_H values obtained for the heavily doped region ($p > 10^{20} \text{ cm}^{-3}$) are nearly constant (38.0 cm²/V·s), and their values are 20% smaller than Irvin's. For gallium-doped silicon, the μ_H values decrease more rapidly as p increases beyond 10^{18} cm^{-3} compared to those in boron-doped silicon.

The mobility values of boron-doped silicon in Fig. 1 have been obtained on many kinds of samples, i.e. the ion implanted and laser annealed samples having various thicknesses of conducting surface, those prepared by additive prolonged furnace annealing, and the samples prepared by boron diffusion with the use of several diffusion sources. The set of values of μ_H obtained from those various

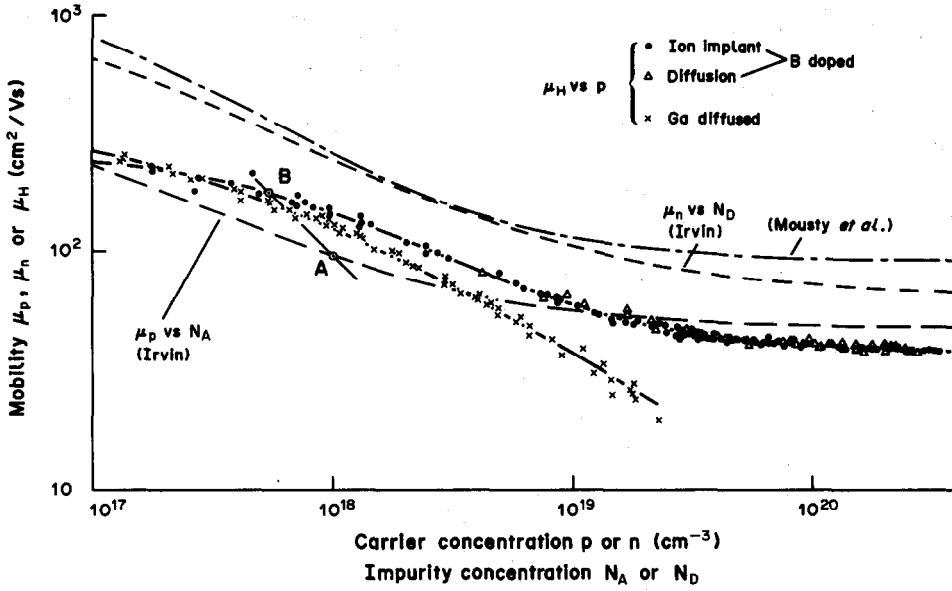


Fig. 1. Hall mobility μ_H vs hole concentration p , or conductivity mobility μ_p or μ_n vs impurity concentration N_A or N_D for silicon at room temperature. ●; Boron is doped by ion implantation. Δ; Boron is doped by diffusion. ×; Gallium is doped by diffusion. — The Caughey-Thomas fitting of Irvin's mobility. - - - The Baccarani-Ostojia fitting of the mobility of Mousty *et al.*[3].

specimens make a non-scattered single relationship as seen in the figure. Thus the μ_H values do not depend on the methods of sample preparation, but only depend on the carrier concentration. Therefore the relationship is reliable. The best fitting of the four parameters of eqn (1) to our experimental data are given in Table 1 by replacing N with the hole concentration p and μ with μ_H . The fitted curves are shown in Fig. 1 by the solid lines. The scatter of data is within $\pm 5\%$ of the experimental data for the boron doped silicon in the doping range of 1×10^{17} – $3 \times 10^{20} \text{ cm}^{-3}$ and within $\pm 10\%$ for gallium doped silicon in 1×10^{17} – $2.5 \times 10^{19} \text{ cm}^{-3}$.

In Fig. 2, the fitting curve of our mobility for the boron-doped silicon is shown by the solid line along with the curve obtained by Thurber *et al.*[6] (dot-dash line) and the Irvin curve (dashed line). The data points by Thurber *et al.*[6,7] are also plotted on the figure. Our μ_H values are 10–25% smaller than the values of the Thurber curve in the whole range of the hole concentration. It should be noticed that our curve is the Hall mobility vs the carrier concentration and the relation by Thurber *et al.* is the conductivity mobility vs the carrier concentration.

4. DISCUSSION

There are three problems to be examined about the causes of the discrepancy between the mobility values presently obtained and the Irvin's one. These are as follows.

- (1) How many acceptors are ionized?
- (2) Did the gradient of impurity concentration profile cause any error on the hole concentration measurement?
- (3) The measured values in the present experiment are the Hall mobilities and the Irvin's ones are a mixture of Hall and conductivity mobilities. How much difference is there between the Hall and conductivity mobilities?

(1) Our mobility values are plotted as a function of hole concentration but the Irvin curves are for impurity concentration. If the acceptors were not fully ionized, a difference between both plots should occur as Wolfstirn[10] pointed out. The hole concentration is given by:

$$p = N_v \exp\{-(F - E_v)/kT\}, \quad (5)$$

Table 1

	μ_{exp} [cm ² /V s]	μ_{fit} [cm ² /V s]	α	N_{ref} [cm ⁻³]
Present values				
(Boron doped)	330	37	0.823	5.0×10^{17}
(Gallium doped)	400	0	0.645	3.0×10^{17}
Caughey and Thomas[2]	495	47.7	0.76	6.3×10^{16}

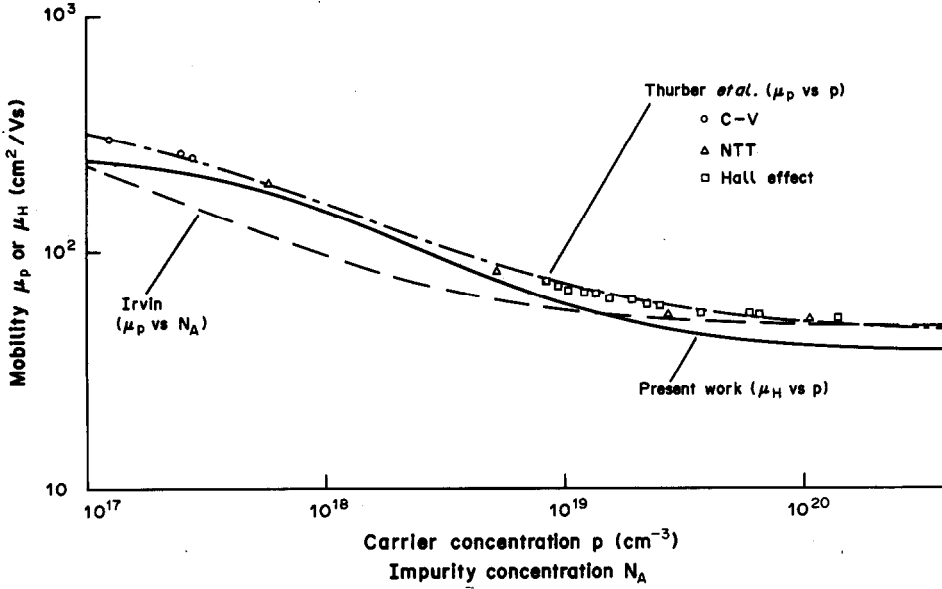


Fig. 2. Hall mobility μ_H and/or conductivity mobility μ_p vs hole concentration p and/or impurity (boron) concentration N_A for silicon at room temperature. The solid line is the result of the present work. The dashed line is the Caughey-Thomas fitting of Irvin's mobility. The dot-dash line is the conductivity mobility obtained by Thurber *et al.*[3] with the C-V measurements (○), the NTT (Δ) and the Hall effect measurements under the assumption of $r = 0.8$ (□).

and the ionized acceptor concentration is:

$$N_A^- = \frac{N_A}{1 + g \exp\{(E_A - F)/kT\}}, \quad (6)$$

where N_v is the effective density of states in the valence band, F the Fermi energy, E_v the energy at the top of the valence band, k the Boltzmann's constant, T the absolute temperature, N_A the total acceptor concentration, g the degeneracy factor and E_A the energy level of the acceptor. The degeneracy factor g is 4 for acceptor levels in silicon[11]. When we eliminate F from the above two equations, we have:

$$p = \frac{N_v}{2g} \exp\left(-\frac{\Delta E_A}{kT}\right) \left(\left\{ 1 + \frac{4gN_A}{N_v} \times \exp\left(\frac{\Delta E_A}{kT}\right) \right\}^{1/2} - 1 \right), \quad (7)$$

where $\Delta E_A = E_A - E_v$. For silicon, N_v is $1.15 \times 10^{19} \text{ cm}^{-3}$ and ΔE_A for boron impurity is 0.045 eV at 300 K. If $N_A = 1 \times 10^{18} \text{ cm}^{-3}$, eqn (7) gives the p value as $4.9 \times 10^{17} \text{ cm}^{-3}$, namely a half of the acceptors is not ionized. So the Irvin curve in Fig. 1 should be properly shifted to the left along the horizontal axis in order to replace the quantity of abscissa from the impurity concentration N_A to the hole concentration p for the comparison with the present data. However, the effective value of ΔE_A depends on N_A and becomes smaller with an increase of N_A , the actual ionization rate may be somewhat larger than the values calculated by eqn (7)[10]. In another experiment, we measured that the resistivity vs $1/T$ curve on

a boron doped silicon with a concentration of $1 \times 10^{19} \text{ cm}^{-3}$ did not show any gradient of a thermal excitation at liquid helium temperature; this fact means ΔE_A is nearly zero. Thus the effective Irvin curve for p as the abscissa has almost no need of shift from the original curve in Fig. 1, because the effective curve has the value of only 5% lower than the original one at $1 \times 10^{17} \text{ cm}^{-3}$ if ΔE_A is 0.045 eV.

(2) If the gradient of acceptor concentration among the Debye length L_D was fairly large, holes might diffuse to the lower concentration side and the hole profile might deviate from the acceptor profile. This is not the case for the following reason.

The Debye length decrease in inverse proportion to the square root of the carrier concentration and the value of the length is about 40 Å in silicon at the hole concentration $p = 10^{18} \text{ cm}^{-3}$, at which the differences of our mobility and Irvin's become largest. The total thicknesses of the conductive layers of our samples varied from 0.3 to 7 μm, and the thickness of the removed layers on each stripping procedures was controlled in the range from 160 Å to several 1000 Å so as to limit the ratio of hole concentrations of the both faces within about 2. As the thickness of any removed layer was much larger than L_D , the difference of the hole concentration among L_D was very small. Therefore, the obtained values of μ_H and p should substantially agree with the values for the uniformly doped samples. Furthermore, despite the fact that the measurements were made with successive sheet strippings of various thicknesses, the plots of μ_H vs p lie almost on a single curve as shown in Fig. 1.

This fact shows that the variety of the thickness and the concentration gradient of holes of each removed layers does not influence the relationship of μ_H and p obtained in the present experiment.

(3) The Hall coefficient is related to the carrier concentration n as $R_H = r/qn$ where q is the electronic charge. The Hall mobility factor r is the ratio of the Hall mobility to the conductivity mobility μ_H/μ and equal to $\langle \tau^2 \rangle / \langle \tau \rangle^2$ where τ is the relaxation time of the carrier. Generally, r varies between about 1 and 2 for a simple band and phonon dispersion according to the mechanism of the predominant scattering, and also with the degree of degeneracy of the relevant energy band. As it is well known, $r = 3\pi/8 = 1.18$ for phonon scattering and $r = 315\pi/512 = 1.93$ for ionized impurity scattering[12] in case of the non-degenerate single band.

The dashed line in Fig. 3 (curve 1) shows the relationship between the Hall mobility factor r and the electron concentration n of the phosphorus doped n -type silicon obtained by Mousty *et al.*[3], using their experiment on the conductivity mobility as well as the Hall mobility. The variation of r is seen in the doping range of $N_D < 10^{19} \text{ cm}^{-3}$ and the maximum value of r is 1.3 at $N_D = 10^{18} \text{ cm}^{-3}$, while $r = 1$ is expected for a simple degenerate semiconductor. For p -type silicon, the Hall mobility factor r may also vary with the hole concentration p . If $r > 1$, the values of hole concentration obtained by eqn (4) are apparent ones and must be smaller than the true values by the factor $1/r$ because μ of eqn (4) was substituted by μ_H of eqn (3) in the present analysis. We denote such an apparent concentration by p_H . Therefore, in order to put our data points in Fig. 1

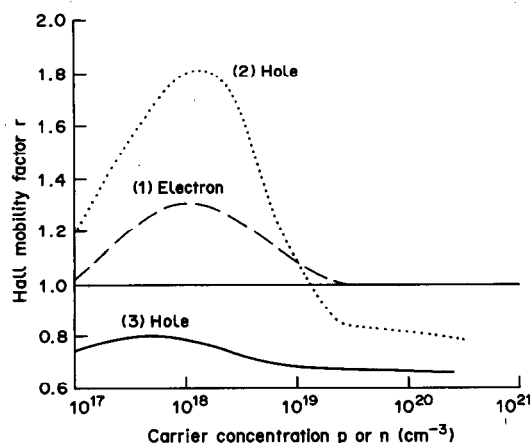


Fig. 3. Hall mobility factor r in silicon at room temperature vs carrier concentration. The curve (1) is the experimental result for electrons reported by Mousty *et al.*[3]. The curve (2) is estimated for holes under the assumption that the Irvin's mobilities are the real conductivity mobility and ours are the Hall mobility. The curve (3) is estimated in a similar way under the assumption that the mobility values obtained by Thurber *et al.*[6] are the real conductivity one and r equals not 0.8 but 0.7.

on the right positions corresponding to the true values, it was necessary to divide the ordinate values in the figure by r and to multiply the abscissa values by r .

In order to evaluate r as a function of p , our mobility curve for the born doped silicon was compared with another curve which represented the relationship between the conductivity mobility μ_p and the hole concentration p . The procedure of evaluation was as follows. At first, under the assumption that Irvin's relation of μ vs N_A (approximately p) was the correct one of μ_p vs p , the point A which corresponded to the pair of μ_p and p on the Irvin curve in Fig. 1, as an example, was connected to the point B which corresponded to the pair of μ_H and p_H on our curve by the straight line having the gradient of 45° from the ordinate as shown in the figure. The ratio of μ_H and μ_p (or p and p_H) gives the value of r , because the above line makes the corrections of multiplying $1/r$ to the ordinate and multiplying r to the abscissa in the logarithmic scales. The value of r is obtained as a function of p is shown in Fig. 3 by the dotted line (curve 2).

Another evaluation of r was done in a similar way by the use of μ_H (or p_H) of ours and μ_p (or p) of Thurber *et al.*[6]. After several trials, the curve 3 in Fig. 3 was obtained in the following way. Thurber *et al.*[6] obtained μ_p and p by using three measuring techniques. Here, we denote their μ_p and p by μ_T and p_T respectively for convenience. Their values of μ_T and p_T in the doping range higher than $8 \times 10^{18} \text{ cm}^{-3}$ were obtained by using the Hall measurements. They chose the numerical value of 0.8 for r in order to get μ_T and p_T from their values of μ_H and p_H as $\mu_T = \mu_H/r$ and $p_T = r \cdot p_H$. The value 0.8 they chose was after the theoretical work by Nakagawa and Zukotynski[13] who calculated the drift mobility and Hall mobility factor of holes in germanium and silicon taking into account the band structure and acoustic phonon, optical phonon and impurity scatterings; their calculation was done in the impurity range up to 10^{15} cm^{-3} , while the experimental data by Thurber *et al.*[6] extended over 10^{20} cm^{-3} . Therefore the choice of the r value had some ambiguity. If we adopted a numerical value r' different from 0.8, we should obtain $\mu' = (0.8/r') \mu_T$ and $p' = (r/0.8)p_T$ instead of μ_T and p_T respectively. By comparing these μ' and p' with our values of μ_H and $p_H (= 1/q\mu_H \cdot dg/dz)$, we determined the Hall mobility factor as shown by the curve 3 in Fig. 3 by adopting $r' = 0.7$. In this calculation, we did not use the values on the mobility curves which were formulated by Thurber *et al.* but used the values of their data points for μ_T and p_T , because there were a little difference between them. The Hall factor r in this curve is 0.68–0.66, i.e., nearly constant in the p -range larger than 10^{19} cm^{-3} . This r value is almost consistent with the above adopted r' value of 0.70. Another choice of r' such as 0.90 or 0.65 gave more limited p -range where r was constant. It is reasonable to get an approximately constant value

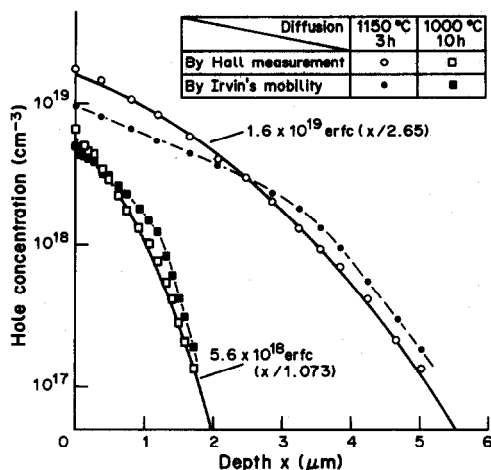


Fig. 4. Hole concentration profile in the gallium diffused silicon layer open circles and open squares are experimental results obtained by the Hall measurement. Closed circles and closed squares are obtained by using the Irvin's relationship[1] of the resistivity and the impurity concentration. Solid lines are the error function fitting on the former plots.

for r in the concentration range of $p > 10^{19} \text{ cm}^{-3}$, because the impurity scattering dominates in this range. The r' values of 0.80 as well as 0.70 gave almost the same consistency. The calculated r value by Nakagawa and Zukotynski[13] for the best set i.e. set II tends to decrease below 0.8 at high acceptor concentration. Thus we would prefer 0.7 as r' rather than the value 0.8 which was adopted by Thurber *et al.*[6], although the difference is not much. The curve 2 in Fig. 3 has a peak value of 1.8 around the p -value of 10^{18} cm^{-3} . This value is too high compared to the calculation by Nakagawa and Zukotynski, so that the Irvin curve is not quantitatively adequate.

The value of curve 1 in Fig. 3 takes the maximum value 1.3 at the n value of 10^{18} cm^{-3} . The value of the curve 3 is smaller than unity in the whole range of doping and also takes the maximum value 0.80 at the p value of $5 \times 10^{17} \text{ cm}^{-3}$. It is recognized that both curves of r , one for electrons and the other for holes, take the maximum values at the carrier concentration around 10^{18} cm^{-3} .

The mobility values of boron doped silicon and gallium doped one are nearly the same at the doping range lower than $1 \times 10^{18} \text{ cm}^{-3}$ showing the iso-electronic nature of both of dopants. In the heavier doped region, μ_H of gallium doped silicon deviates to smaller values than that of boron doped one as shown in Fig. 1. This discrepancy is attributed to the difference of potentials, masses and/or sizes of the impurity ions. There are possibly a few other possibilities[14]: (i) the effect of core potential[15], (ii) the effect of correlated impurity distribution at high concentration range where the distribution is affected, in somewhat different ways for different atomic weights, by the Coulomb repulsion between

ionized impurities during the solidifying or diffusing process[16] and (iii) the effect of strain field, if it may be small, caused by the impurity ion in different degrees by different ion sizes.

Finally, let us give another evidence showing the adequacy of our mobilities over Irvin's. Figure 4 shows the hole concentration profiles of the gallium diffused layer, in which the profiles indicated by open circles and open squares show the hole concentrations obtained by using our mobilities, and the other profiles indicated by closed circles and closed squares are obtained by using Irvin's. The former fits the complementary error function distribution shown by the solid lines but the latter does not. The reason is as follows. Gallium was diffused in the evacuated and sealed quartz tube at 1150°C , and the intrinsic carrier concentration of silicon n_i at 1150°C is $1.5 \times 10^{19} \text{ cm}^{-3}$. If the carrier concentration exceeded this value, the concentration of lattice vacancies might increase more than the thermal equilibrium value of the intrinsic silicon, and the diffusion of dopant atoms should be enhanced. For the measured hole concentration, this is not the case, and the diffusion is done with the constant diffusion coefficient under the constant surface concentration of the dopant. Theoretically this fact gives a complementary error function profile. As shown in Fig. 4, most of the data points are on the complementary error function curves. This fact implies that the Hall mobility factor r of holes for the gallium doped silicon is roughly constant and independent of the gallium concentration.

5. CONCLUSIONS

(1) The relationship between the Hall mobility and the concentration of holes in the range of concentration between 10^{17} and $3 \times 10^{20} \text{ cm}^{-3}$ in p -type silicon was experimentally investigated. Boron and gallium was used as dopants and the doping was done by either the ion implantation or the diffusion technique into low carrier density n -type silicon crystals. The sheet conductivity and the Hall coefficient of the p -type conducting surface layer were measured by Van der Pauw method and the conducting depth was thinned successively by the anodic oxidation of surface and melting-off. Thus the Hall mobility of various depth was deduced as a function of the hole concentration.

The previously used Irvin curve[1], which shows the relationships between the mobility of electrons or holes and impurity concentration for both of n -type and p -type silicon, was revised by Mousty *et al.*[3] for n -type silicon. For p -type silicon the Irvin curve was also revised by Thurber *et al.*[6]. However, most of their data were restricted to the lower and higher doping ranges and the measuring technique in each range was different; they were obtained by using a C - V method in the lower range and by using Hall measurements in the higher range. Our Hall mea-

surements offered new data for the middle doping range between 3×10^{17} and $8 \times 10^{18} \text{ cm}^{-3}$, in which range they had only two data points, obtained by using NTT. In the higher doping range, we got about the same values as they did.

(2) The Hall mobility factor r was examined for boron doped silicon and it was suggested that the variation of r was small and showed a maximum value of 0.8 at a hole concentration of $5 \times 10^{17} \text{ cm}^{-3}$. The r value was about 0.7 for the higher concentration range of the dopant, above $5 \times 10^{18} \text{ cm}^{-3}$.

(3) If the mobility values of the Irvin's one for boron doped silicon are corrected by $r = 0.7$, these values are about 10% lower than the present result in the concentration range higher than 10^{20} cm^{-3} and 60% lower around the range of 10^{18} cm^{-3} . Therefore the Irvin curve is not adequate except the highest concentration range.

(4) Gallium doped silicon showed nearly the same Hall mobility values as that of boron doped one at the lower concentration range than 10^{18} cm^{-3} but deviated to lower values at higher concentrations, e.g. amounting to 63% at the concentration of $1 \times 10^{19} \text{ cm}^{-3}$.

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APPENDIX

An analytical method to determine the profiles of carrier concentration and mobility with the successive sheet stripping and the Hall measurement

A semiconductor plate of impurity doped by an ion implantation or a surface diffusion method has a non-uniform distribution of impurity concentration and mobility in the direction of the depth from the surface. The following describes an analytical method of determining the profiles of carrier concentration and mobility by the measurement of the sheet conductivity and the Hall voltage.

Figure 5(a) shows the configuration of the sample where an electric current I flows parallel to the surface (x -direction) and a magnetic field B is applied perpendicular to the surface (z -direction). As shown in Fig. 5(b), the sample plate is thought to be divided into thin layers; the thickness of each layer Δt_i is so thin that each layer has uniform conductivity along z -direction. Under the magnetic field B the current ΔI_i flowing through the i th layer gives rise to the Hall voltage V_{Hi} parallel to the y axis. The current ΔI_i and the Hall voltage V_{Hi} in each layer are given as follows:

$$\Delta I_i = \frac{\sigma_i}{g_s} I \cdot \Delta t_i \quad (\text{A1})$$

$$V_{Hi} = \frac{r}{q n_i} \cdot \frac{\Delta I_i}{\Delta t_i} \cdot B, \quad (\text{A2})$$

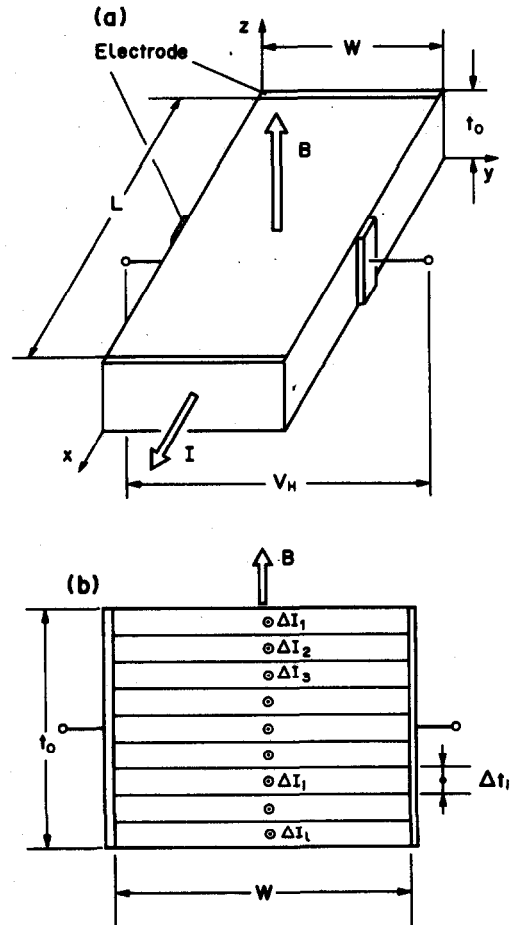


Fig. 5. Configuration of a sample for measuring the Hall voltage. (a): Geometrical arrangement of the electrodes and the magnetic field. (b): Sectional view of the conducting thin layer divided into thin sheets.

where σ_i is the conductivity of the i th layer, g , the overall sheet conductivity of the plate, r the Hall factor μ_H/μ and n_i the carrier concentration in the i th layer. σ_i is given by

$$\sigma_i = q \mu_i n_i \quad (\text{A3})$$

where μ_i is the carrier mobility in the i th layer. As μ_i and n_i are functions of z , g , is given by:

$$g = q \int_0^{t_0} \mu(z) n(z) dz \cong q \sum_{i=1}^1 \mu_i n_i \Delta t_i, \quad (\text{A4})$$

where t_0 is the total thickness of the conductive plate.

Using the eqn (A1)–(A3):

$$V_{Hi} = \frac{r I B}{g_i} \mu_i, \quad (\text{A5})$$

is obtained. If μ_i depends on the layer number, V_{Hi} also takes a different value from a layer to layer. Every layer is connected to the neighboring ones throughout the sample width, but no inter-layer current flows along z -direction except at the both ends, because the Hall voltage V_H and the Hall field dV_H/dy are common for all layers. This situation can be simulated by an equivalent circuit as shown in Fig. 6.

Then the measured Hall voltage V_H can be calculated by the Millman's circuitry theorem[17] in the following way,

$$V_H = \frac{\sum G_i V_{Hi}}{\sum G_i}, \quad (\text{A6})$$

where G_i is the sheet conductance of the i th layer along the y -direction and given by:

$$G_i = (qL/W) \mu_i n_i \Delta t_i, \quad (\text{A7})$$

where L is the length of the plate along the x -direction. Using the eqns (A5) and (A7), the following relation holds:

$$\sum_{i=1}^1 G_i V_{Hi} = \frac{q L I B}{g_i W} \sum_{i=1}^1 (r \mu_i^2 n_i \Delta t_i). \quad (\text{A8})$$

As the limit of

$$\Delta t_i \rightarrow 0, \quad \sum_{i=0}^1 (r \mu_i^2 n_i \Delta t_i)$$

is replaced by:

$$\int_0^{t_0} r \mu(z)^2 n(z) dz, \quad \sum_{i=1}^1 G_i$$

is replaced by $(L/W)g$, and the Hall voltage is expressed as:

$$V_H = \frac{q}{g_i^2} I B \int_0^{t_0} r \mu(z)^2 n(z) dz. \quad (\text{A9})$$

By the differentiation of eqns (A4) and (A9) with z , the depth z dependent carrier concentration $n(z)$ and the mobility $\mu(z)$ are obtained in the following expressions.

$$n(z) = \frac{1}{q \mu(z)} \frac{d g_s(z)}{d z}, \quad (\text{A10})$$

$$\begin{aligned} \mu_H(z) &= r \mu(z) \\ &= \frac{1}{I B} \frac{d \{g_s^2 \cdot V_H(z)\}}{d g_s(z)}, \end{aligned} \quad (\text{A11})$$

$g_s(z)$ and $V_H(z)$ are the overall sheet conductivity and Hall voltage respectively for the plate of thickness z , i.e. for the plate of which outer surface of the thickness $t_0 - z$ is removed, thus the profiles of the carrier concentration and the mobility are obtained by this successive sheet removing method with the use of eqns (A10) and (A11) by assuming $r = 1$.

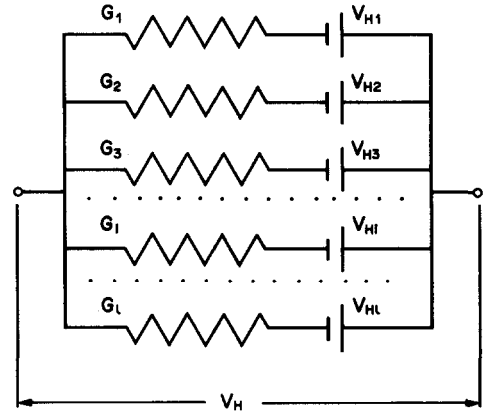


Fig. 6. Equivalent electric circuit of the conducting layer for calculating the effective Hall voltage V_H . G_i is the conductance of the i th layer along the y -direction and V_{Hi} is the Hall voltage induced in the layer.