Regular Paper

# Soccer league competition algorithm: A novel meta-heuristic algorithm for optimal design of water distribution networks

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## A B S T R A C T

Water distribution networks are one of the most important elements in the urban infrastructure system and require huge investment for construction. Optimal design of water systems is classified as a large combinatorial discrete non-linear optimization problem. The main concern associated with optimization of water distribution networks is related to the nonlinearity of discharge-head loss equation, availability of the discrete nature of pipe sizes, and constraints, such as conservation of mass and energy equations. This paper introduces an efficient technique, entitled Soccer League Competition (SLC) algorithm, which yields optimal solutions for design of water distribution networks. Fundamental theories of the method are inspired from soccer leagues and based on the competitions among teams and players. Like other meta-heuristic methods, the proposed technique starts with an initial population. Population individuals (players) are in two types: fixed players and substitutes that all together form some teams. The competition among teams to take the possession of the top ranked positions in the league table and the internal competitions between players in each team for personal improvements are used for simulation purpose and convergence of the population individuals to the global optimum. Results of applying the proposed algorithm in three benchmark pipe networks show that SLC converges to the global optimum more reliably and rapidly in comparison with other meta-heuristic methods.

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## 1. Introduction

The fundamental goal of water distribution network optimization is to minimize the costs while satisfying the performance and hydraulic constraints required by the design codes and specifications. This involves determining the commercial diameter for each pipe in the network while satisfying the minimum head pressure at each node. The optimal design cost is the minimum option out of numerous possibilities.

The nonlinear relationships between pipe discharges and head loss along with the presence of pipe diameters in design optimization makes this task highly challenging. Over the last two decades many evolutionary optimization techniques have been successfully applied to water network optimal design, such as genetic algorithms [1–4]; simulated annealing [5]; harmony search [6]; shuffled frog leaping algorithm [7]; ant colony optimization [8]; particle swarm optimization [9]; cross entropy [10]; scatter search [11]; differential evolution [12,13] and Self-Adaptive Differential

Evolution [14]. In an effort to achieve better optimal solutions and reduce the computational effort with high degree of reliability in optimizing complex water distribution networks, a new evolutionary algorithm entitled "Soccer League Competition (SLC) algorithm", is introduced in the present study. Fundamental ideas of the method are inspired from soccer leagues and based on the competitions among teams and players. Like other meta-heuristic methods, the proposed technique starts with an initial population. Population individuals called player are in two types: fixed players and substitutes that all together form some teams. The competition among teams to take the possession of the top ranked positions in the league table and the internal competitions between players in each team for personal improvements are used for simulation purpose and convergence of the population individuals to the global optimum.

In this work, we examine SLC for three benchmark Water Distribution Networks (WDNs) available in the literature, and finally compare it with other meta-heuristic algorithms documented in the literature.

The optimization problems addressed herein are solved through linking the SLC algorithm with the Global Gradient Algorithm (GGA) for minimizing the design cost of water distribution systems, with pipe diameters as discrete decision variables.

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**Nomenclature**

| | |
|---|---|
| $A_{nn}$ | coefficient matrix in GGA |
| $A_{pp}$ | diagonal matrix in GGA |
| $A_{np}$, $A_{pn}$, $A_{p0}$ | topological incidence sub-matrices in GGA |
| $C$ | objective function |
| $CM$ | Hazen–Williams coefficient |
| $CP$ | Penalty function |
| $c_i$ | cost of ith pipe |
| $\mathbf{d}_n$ | column vector of demands |
| $D_i$ | diameter of ith pipe |
| $\mathbf{D}$ | diagonal matrix |
| $\mathbf{D}_{nn}$ | diagonal matrix of derivatives of the pressure-demand with respect to $\mathbf{H}_n$ |
| $\mathbf{D}_{pp}$ | diagonal matrix of derivative of head losses with respect to $\mathbf{Q}_p$ |
| $\mathbf{F}_n$ | temporary matrix used in GGA |
| $FP$ | solution vector of fixed player |
| $G$ | average value of fixed player's solution vectors |
| $h_f$ | head-loss |
| $H_{min}$ | minimum required pressure head |
| $\mathbf{H}_0$ | column vector of known nodal heads |
| $\mathbf{H}_n$ | column vector of unknown nodal heads |
| $iter$ | counter for iterations |
| $K$ | number of candidate diameters |
| $L$ | lower triangular matrix |
| $L_i$ | length of ith pipe |
| $M$ | pre-conditioner of the iterative method |
| $n_l$ | number of loops |
| $n_n$ | number of nodes with unknown heads |
| $n_p$ | number of pipes |
| $n_0$ | number of nodes with known heads |
| $nT$ | number of teams |
| $nPlayer$ | number of players |
| $nFP$ | number of fixed players |
| $pp(i,j)$ | power of player $j$ in team $i$ |
| $PV$ | probability of victory |
| $q$ | external demand |
| $\mathbf{Q}_p$ | column vector of unknown pipe flow rates |
| $Q_{in}$ | flow into the node |
| $Q_{out}$ | flow out of the node |
| $RP$ | solution vector of random player |
| $S$ | solution vector of substitute |
| $SP$ | solution vector of star player |
| $SSP$ | solution vector of super star player |
| $TP(i)$ | power of team i |
| $\Delta H$ | difference between nodal heads |
| $\lambda$ | penalty multiplier |
| $\chi$, $\tau$ | random numbers with uniform distribution |

GGA is used for solving the mass and energy conservation equations [15]. The remainder of this paper is arranged as follows:

Section 2 briefly presents the characteristics of the discrete pipe network optimization problems. In Section 3, the basic concepts of SLC are defined. In Section 4, we compare SLC algorithm with other meta-heuristic algorithms in terms of number of function evaluations and number of success for finding global solution in large number of runs. Finally, the conclusions are given in Section 5.

## 2. Problem formulation

A Water Distribution Network (WDN) is a collection of many components such as pipes, reservoirs, pumps and valves which are combined together to provide water to consumers. The optimal design of such network can be defined as determining the best combination of component sizes and settings (e.g., pipe size diameters, pump characteristic curve, pump locations and maximum power, reservoir storage volumes, etc.) that gives the minimum cost for the given layout of network, such that hydraulic laws for conservation of mass and energy are maintained and constraints on quantities and pressures at the consumer nodes are fulfilled. In this paper, water distribution network design is formulated as a least-cost optimization problem with a selection of pipe diameters as the decision variables, while pipe layout and its connectivity, nodal demand, and minimum pressure requirements are imposed. The optimization problem can be stated mathematically as [4,7,16,17]:

$$Min\ C = \sum_{i=1}^{n_p} c_i(D_i) \times L_i \tag{1}$$

where $c_i(D_i) \times L_i$ is the cost of pipe $i$ with length $L_i$ and diameter $D_i$, and $n_p$ is the number of pipes in the network. This objective function (1) is minimized under the following constraints:

(i) Continuity equation constraint
   For each junction node, a continuity equation should be satisfied,

$$\sum Q_{in} - \sum Q_{out} = q_n, \quad \forall n \in n_n \tag{2}$$

where $Q_{in}$ and $Q_{out}$ are flow into and out of the node, respectively, and $q_n$ is the external demand (consumption) at the node n, and $n_n$ is the number of nodes.

(ii) Energy conservation constraint
   The total head loss or the accumulated energy loss around the closed loop (a closed loop is made by some pipes connecting together e.g., Fig. 2) should be equal to zero or the head loss along a loop between the two fixed head reservoirs (known heads) should be equal to the difference in water level of reservoirs

$$\sum_{i \in loop\ L} hf_i = \Delta H, \ \forall L \in n_l \tag{3}$$

where $hf_i$ is the head loss due to friction in the pipe i computed by the Hazen–Williams or Darcy–Weisbach formula; $n_l$=loop set; $\Delta H$=difference between nodal heads at both ends, and $\Delta H$=0, if the path is closed [12]. The Hazen–Williams formula which was used as the pressure head loss equation for pipe i of connecting nodes j and k is

$$hf_i = H_k - H_j = 10.667\ (L_i/(CM_i^{1.852} D_i^{4.871}))\ Q_i|Q_i|^{0.852} \tag{4}$$

where $CM_i$, $D_i$ and $L_i$ are pipe's Hazen–Williams coefficient (depending on the pipe material), diameter and length, respectively.

(iii) Minimum pressure constraint
   For each junction node in the network, the pressure head should be greater than the prescribed minimum pressure head.

$$H_k \geq H_k^{min}, \ \forall\ k \in n_n \tag{5}$$

where $H_k$ is the pressure head at node k, $n_n$ is the number of nodes, and $H_k^{min}$ is the minimum required pressure head.

(iv) Pipe size availability constraint
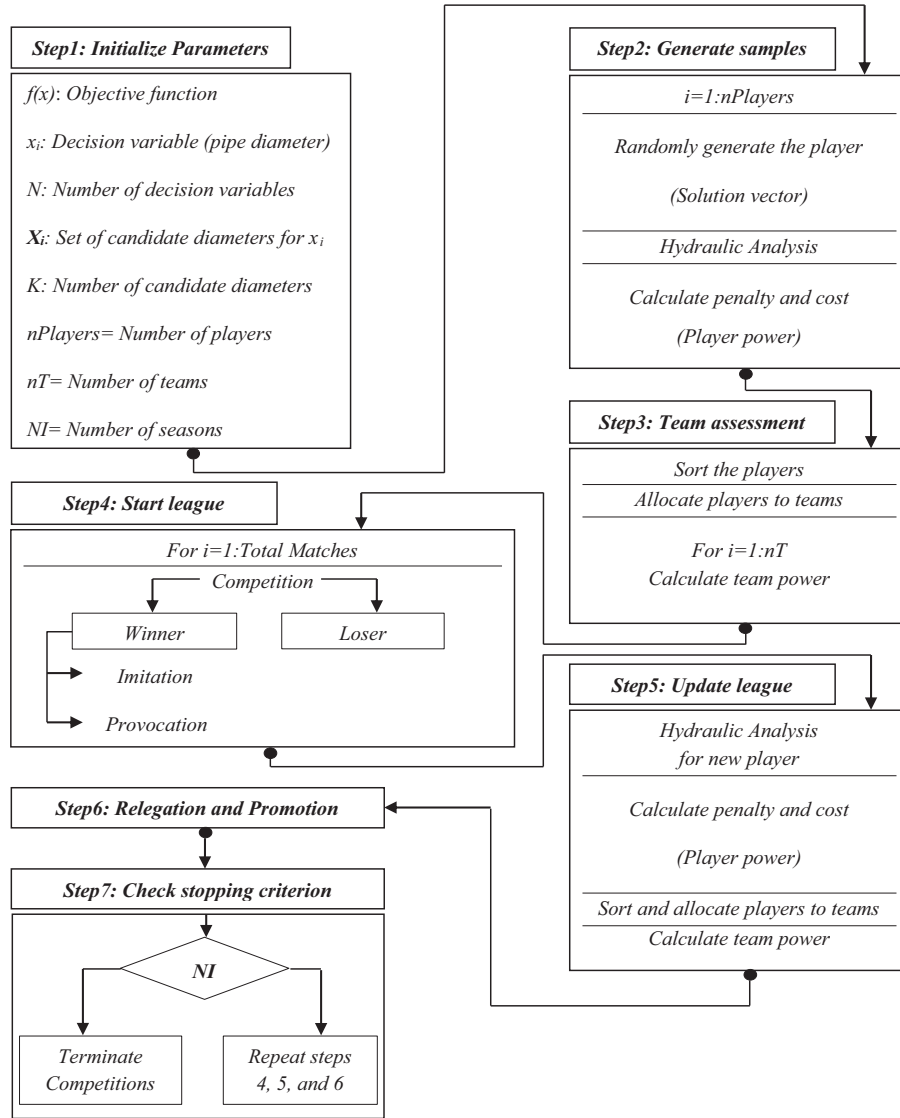   The diameter of the pipes should be available from a

**Fig. 1.** SLC procedure for water distribution network optimization.

commercial size set

$$D_i = \{D(1), D(2), ..., D(K)\}, \quad \forall i \in n_p \tag{6}$$

where K is the number of candidate diameters which can be fixed for all pipes, $D_i$ is the diameter size of ith pipe, and $n_p$ is the number of pipes.

In this study, a Global Gradient Algorithm (GGA) in MATLAB environment is applied for hydraulic modeling of network. GGA is described in the next section and satisfies the continuity and energy conservation constraints (i) and (ii). GGA calculates the pressure head $H_k$ (unknown heads) at each junction node and the flow rate $Q_i$ in each pipe. To satisfy minimum pressure constraint (iii), we apply a penalty function. We define the optimization process of WDNs in the following steps:

Step 1: because of nonlinear characteristics of hydraulic modeling, there is no direct dependency between size and discharge values of each pipe. Therefore, we must select diameters of all pipes and then perform a hydraulic analysis to satisfy continuity and energy conservation (i) and (ii). All nodal pressures will become known after analysis.

Step 2: we compare known nodal heads with minimum pressure head that is determined based on regulation of zone (minimum pressure head is known before the optimization process). If any nodal head was less than the minimum pressure, a penalty is added to objective function.

Step 3: a meta-heuristic algorithm selects different set of diameters for all pipes and does step 1 and step 2 to find the global minimum of function (1).

It is worth to note that decision variables in WDNs are discrete which increase complexity of optimization.

### 2.1. Global gradient algorithm (GGA) for WDN modeling

Steady-state analysis of a hydraulic network including $n_p$ pipes with unknown flow rates, $n_n$ nodes with unknown heads (internal nodes) and $n_0$ nodes with known heads can be performed by solving the non-linear system based on energy and mass balance conservation equations as follows:

$$\begin{aligned} \mathbf{A}_{pp}\mathbf{Q}_p + \mathbf{A}_{pn}\,\mathbf{H}_n &= -\mathbf{A}_{p0}\mathbf{H}_0 \\ \mathbf{A}_{np}\mathbf{Q}_p - \mathbf{d}_n(\mathbf{H}) &= \mathbf{0}_n \end{aligned} \tag{7}$$
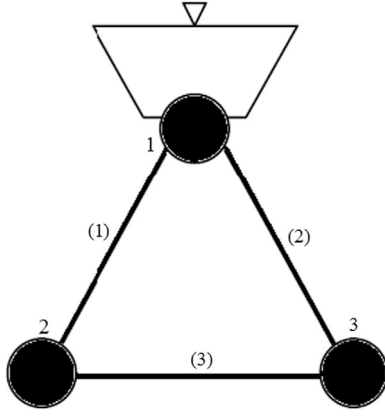
**Fig. 2.** Three pipes network.

where $\mathbf{Q}_p = [n_p, 1]$ column vector of unknown flow rates in pipes; $\mathbf{H}_n = [n_n, 1]$ column vector of unknown nodal heads; $\mathbf{H}_0 = [n_0, 1]$ column vector of known nodal heads; $\mathbf{d}_n = [n_n, 1]$ column vector of demands which may be a function of head status of the system (head/pressure driven analysis); $\mathbf{A}_{pn} = \mathbf{A}_{np}^{\mathrm{T}}$ and $\mathbf{A}_{p0} =$ topological incidence sub-matrices of size $[n_p, n_n]$ and $[n_p, n_0]$, respectively; $\mathbf{A}_{pp}\mathbf{Q}_p = [n_p, 1]$ column vector of pipe head losses caused by the pipe hydraulic resistance and any device (e.g., valves and pumps) installed on pipes. The solution of the nonlinear system in (7) leads to the hydraulic system state $(\mathbf{Q}_p; \mathbf{H}_n)$ after defining some boundary conditions, such as nodal demands; pump curves; hydraulic resistance in pipes; tank levels; status of control valves; etc. The steady-state solution of water system by GGA (7) can be obtained by iteratively solving the following [18–21]:

$$\mathbf{F}_n^{iter} = (\mathbf{A}_{np}\mathbf{Q}_p^{iter} - \mathbf{d}_n) - \mathbf{A}_{np}(\mathbf{D}_{pp}^{iter})^{-1}(\mathbf{A}_{pp}^{iter}\mathbf{Q}_p^{iter} + \mathbf{A}_{p0}\mathbf{H}_0)$$

$$\mathbf{H}_n^{iter+1} = [\mathbf{A}_{np}(\mathbf{D}_{pp}^{iter})^{-1}\mathbf{A}_{pn} + \mathbf{D}_{nn}^{iter}]^{-1}\mathbf{F}_n^{iter}$$

$$\mathbf{Q}_p^{iter+1} = \mathbf{Q}_p^{iter} - (\mathbf{D}_{pp}^{iter})^{-1}(\mathbf{A}_{pp}^{iter}\mathbf{Q}_p^{iter} + \mathbf{A}_{p0}\mathbf{H}_0 + \mathbf{A}_{pn}\mathbf{H}_n^{iter+1}) \quad (8)$$

where $iter =$ counter of iterations; $\mathbf{D}_{pp} =$ diagonal matrix with elements including derivatives of the head loss functions with reference to $\mathbf{Q}_p$ [21].

## 3. Soccer league competitions

Level one soccer league consists of teams (clubs) competing each other during a season. In this environment, some stronger teams aim to sit in the first positions of the league table while some weaker teams plan to survive in the level one league in order to prevent a crash out to the second level league. During the course of a season, teams play with their rivals, three points are given to the winners, while losers get no point. Teams are weekly ranked by total points and the club with the most point is crowned champion at the end of each season. The number of matches in each season depends on the team numbers [22]. For instance, in a league consisting of $nT$ teams the total number of matches is calculated as follows:

$$Total\ Match = (nT \times (nT-1))/2 \quad (9)$$

In this league, each team participates in $nT-1$ independent matches, and totally, $(nT \times (nT-1))/2$ competitions are being held during a season. There is always an intense competition between the teams at the bottom of the league table. As a rule, the two bottom table teams are crashed out to the second level soccer league (relegations spots) at the end of the season. In return, two first table teams of the second level league (promotions spots) are replaced with the relegated teams. Generally, promotions spots import new players to the league which may have potential of being a future star [22].

Each team consists of 11 fixed players (FP) and some substitutes (S). Team's power depends on the power of its players. Moreover, powerful teams have a higher chance of winning their matches. However, it is not possible to predict the exact winner of a specified match before the game ends. As well as the league competitions among teams, there is an internal competition in each team. Players compete with each other to attract the head coach's attention by improving their performance. This internal competition leads to a growth in the quality and power of a team.

In each team, there is a key player which is called Star Player (SP). SP has the best performance among other players in the team. Moreover, there is a unique player in each league which is called the Super Star Player (SSP). SSP is defined as the most powerful player in the league.

After every match, players in winner and loser teams adopt different strategies for improving their future performance. When a team wins a match, fixed players try to imitate the team's SP, and the SSP of the league (this strategy is simulated by imitation operator in this study). They aim to experience a promotion to the SP or, optimistically, occupy the place of SSP in the league. But, the main provocation of winner's substitutes is being a fixed player in the team. For this purpose, they try to have a performance approximately equal to the average level of fixed players in the team (this tendency is described by the provocation operator in this study). In other words, higher provocation for advancement gives them more chance of being a fixed player in the future.

Above mentioned strategies improve the overall performance of teams after each match. Therefore, team's powers progressively increase while all teams play much better at the end of the season. Obviously, players with a noticeable progression increase the winning chance of their team. As the first rank teams of each league have better financial affordance, they are able to recruit powerful players of other teams. This intensifies their power for future seasons. In the next section, the solving style of an optimization problem using the Soccer League Competition (SLC) algorithm is discussed.

### 3.1. Soccer league competition (SLC) algorithm

Competitions between teams in a soccer league for reaching success, and among players for being a SP or SSP can be simulated for solving optimization problems. Similar to a soccer league in which every player desires to be the best (SSP), in an optimization problem each solution vector seeks for the global optimum position. Therefore, each player in a league, Star Player (SP) in each team, and the Super Star Player (SSP) can be assumed as a solution vector, a local optimum, and the global optimum.

Each team consists of 11 fixed players (defined by principal solution vectors in the SLC algorithm) and some substitutes (described by reserved solution vectors in SLC algorithm). For each player, an objective function is calculated which stands for the power of its corresponding player. In a minimization problem, smaller values of objective functions (cost function) illustrate Power of Players (PP).

$$PP(i,j) = 1/C(i,j)\ i \in team,\ j \in player,\ C = objective\ function \quad (10)$$

The total power of a team is defined as the average power value of its fixed players. The following formula shows how a Team's Power (TP) is calculated.

$$TP(i) = (1/nFP)\sum_{j=1}^{nFP} PP(i,j) \quad (11)$$

$nFP$ is the total number of fixed players in the $i$th team. In each match, the team with more power has a higher chance of winning.

The probability of victory for each team in a match is given by

$$Pv(k) = TP\,(k)/(TP\,(i) + TP\,(k)) \tag{12}$$

$$Pv(i) = TP\,(i)/(TP\,(i) + TP\,(k)) \tag{13}$$

*Pv* stands for the probability of victory. It should be noted that the sum of $Pv(k)$ and $Pv(i)$ equals 1. After each match, the winner and the loser are noticed and some players (solution vectors), including fixed and substitute, experience changes. These changes, which are aimed to improve performance of both players and teams, are simulated with the following operators:

Imitation operator, provocation operator.

The steps in the procedure of SLC are shown in Fig. 1. They are as follows:

Step 1. Initialize problem and algorithm parameters.
Step 2. Samples generation.
Step 3. Teams assessment.
Step 4. League start.
Step 5. League update.
Step 6. Relegation and promotion.
Step 7. Check the stopping criterion.

### 3.1.1. Step 1. Initialize the problem and algorithm parameters

In step 1, the discrete optimization problem is specified as follows:

$$Min\ f(x)$$

$$Subject\ to\ x = \{x_1, x_2, ..., x_N\} \in X, \tag{14}$$

where $f(x)$ is an objective function; $x$ is a set of decision variable $x_i$; $N$ is the number of decision variables; $\boldsymbol{X}$ is the set of the possible range of values for each decision variable, that is, $\boldsymbol{X} = \{D(1), D(2), ..., D(K)\}$ for discrete decision variables $(D(1) < D(2) < ... < D(K))$; and $K$ is the number of possible values for the discrete decision variables (see Eq. (6)).

In pipe network design, the objective function is the pipe cost function; the pipe diameter is the decision variable; the number of decision variables $N$ is the number of pipes in the network ($n_p$); the set of decision variable values is the range of possible candidate diameters.

To clarify the operations of each step in SLC algorithm, we present an example of 3-pipe network in Fig. 2. Assume the set of decision variable values or the range of possible candidate diameters to be {100 mm, 200 mm, 300 mm, 400 mm} and the corresponding cost set is {$100, $200, $300, $400} [6]. In this network, the number of possible values for the decision variables $K$ equals 4. Also, the number of teams included in the league ($nT$), the number of fixed players ($nFP$), and the number of substitutes ($nS$) are assumed to be 2, 2 and 2, respectively.

### 3.1.2. Step 2. Samples generation

The total number of players in a league is calculated by the following formula:

$$nPlayers = nTx(nFP + nS) \tag{15}$$

In this step, randomly solution vectors are generated as many as the number of players in the league and each vector is devoted to a specified player. Hence the matrix TEAM which is generated

randomly is given as below

$$TEAM = \begin{bmatrix} FP_1 \\ FP_2 \\ . \\ FP_{nFP} \\ RP_1 \\ RP_2 \\ . \\ RP_{nS} \end{bmatrix} = \begin{bmatrix} xF_1^1 & xF_2^1 & . & . & . & . & xF_N^1 \\ xF_1^2 & xF_2^2 & . & . & . & . & xF_N^2 \\ . & & . & & & & . \\ xF_1^{nFP} & . & & . & & . & xF_N^{nFP} \\ xR_1^1 & xR_2^1 & . & . & . & . & xR_N^1 \\ xR_1^2 & xR_2^2 & . & . & . & . & xR_N^2 \\ . & & . & & & & . \\ xR_1^{nS} & . & . & . & . & . & xR_N^{nS} \end{bmatrix} \tag{16}$$

next, an objective function (design cost) including *a* penalty cost relating to each solution vector (player's power) is calculated.

Considering the three pipes network shown in Fig. 2, eight players are randomly generated as follows:

| | D(1) | D(2) | D(3) | Cost |
|---|---|---|---|---|
| Player 1 | 300 | 300 | 400 | 1000 |
| Player 2 | 100 | 200 | 100 | 400 |
| Player 3 | 200 | 200 | 300 | 700 |
| Player 4 | 300 | 400 | 100 | 800 |
| Player 5 | 400 | 100 | 100 | 600 |
| Player 6 | 400 | 200 | 400 | 1000 |
| Player 7 | 300 | 200 | 300 | 800 |
| Player 8 | 100 | 400 | 200 | 700 |

To satisfy minimum pressure constraints, the solutions (players) undergo a hydraulic analysis and a penalty cost is added to the total design cost when results violate the minimum requirements. The penalty function has the form

$$CP = \begin{cases} 0 & if\ H_k \geq H_{min}\ \ \forall k = 1, ..., nn \\ \lambda \times\ max\left|\left(\frac{H_k}{H_{min}} - 1\right)\right| & otherwise \end{cases} \tag{17}$$

where $CP$ is the penalty function, $H_k$ and $H_{min}$ are simulated pressure and minimum allowable pressure at node $k$, respectively, and $\lambda$ is the penalty multiplier.

As the examined 3-pipe network in Fig. 2 is not a real network and there is no available hydraulic analysis data, we do not calculate the penalty function at this stage for the matter of simplicity.

### 3.1.3. Step 3. Teams assessment

In this step, all players are arranged according to their calculated power and are devoted to teams. Each team's power is calculated by Eq. (11). For examined 3-pipe network in Fig. 2, this process is performed as follows:

| Team 1 | | | | | |
|---|---|---|---|---|---|
| | D(1) | D(2) | D(3) | Cost | Player power |
| Fixed players | 100 | 200 | 100 | 400 | 1/400=0.0025 |
| | 400 | 100 | 100 | 600 | 1/600=0.0017 |
| Substitutes | 300 | 400 | 100 | 800 | 1/800=0.00125 |
| | 300 | 200 | 300 | 800 | 1/800=0.00125 |
| | | Team power | | 0.0042 | =0.0025+0.0017 |
| | | Chance of victory | | 0.60* | |

*This value is calculated by formula 12 as follows:
0.6=0.0042/(0.0042+0.0028)

|  | Team 2 | | | | |
| --- | --- | --- | --- | --- | --- |
|  | $D(1)$ | $D(2)$ | $D(3)$ | Cost | Player power |
| Fixed players | 200 | 200 | 300 | 700 | $1/700 = 0.0014$ |
|  | 100 | 400 | 200 | 700 | $1/700 = 0.0014$ |
| Substitutes | 300 | 300 | 400 | 1000 | $1/1000 = 0.001$ |
|  | 400 | 200 | 400 | 1000 | $1/1000 = 0.001$ |
|  |  | Team power |  | 0.0028 | $= 0.0014 + 0.0014$ |
|  |  | Chance of victory |  | 0.40* | |

*This value is calculated by formula 13 as follows:
$0.4 = 0.0028/(0.0042 + 0.0028)$

A shown in above table, players are arranged based on their powers into fixed and substitute positions of the team. Team power equals the inverse value of fixed players cost summation.

### 3.1.4. Step 4. League start

In this step, competitions are started between all possible pairs of teams in the league, the winner and the loser of every match are determined, and the imitation and provocation operators act on fixed and substitute players of winner teams, respectively. In the next part, detailed description of operators is defined.

*3.1.4.1. Imitation operator.* Fixed players (FP) of the winner team, imitate both the Star Player (SP) in their own team and the Super Star Player (SSP) in the league to improve their future activities. Similarly, solution vectors relating to the fixed players in the winner team move toward the best solution of the own team and the best solution vector of the league. In the SLC algorithm, imitation is performed by the following formulas:

$$FP(i,j) = FP(i,j) + \tau_1(SSP - RP1(i)) \tag{18}$$

$$FP(i,j) = FP(i,j) + \tau_1(SP(i) - RP1(i)) \tag{19}$$

$$FP(i,j) = FP(i,j) + \tau_1(RP2(i) - RP1(i)) \tag{20}$$

where $\tau_1 \sim U(0.2, 0.8)$ is a random number with uniform distribution. $FP(i,j)$ stands for the $j$th fixed player of the $i$th team, $RP1(i)$ and $RP2(i)$ are arbitrary random players, and $SP(i)$ is the star player of the $i$th team.

First, solution vector of fixed players (FP) in the winner team experiences a move toward the resultant vector direction of SSP (Eq. (18)).

It is assumed that team 2 becomes the winner in 3-pipe network example shown in Fig. 2. Then, $SSP = \{100,200,100\}$; $SP = \{200,200,300\}$; $FP = \{100,400,200\}$; we assume $RP1 = \{200, 200,300\}$ $FPnew = \{100,400,200\} + \tau^*(\{100,200,100\} - \{200,200, 300\})$. We assume $\tau = 0.5$ then, $FPnew = \{100,400,200\} + 0.5^*$ $(\{100,200,100\} - \{200,200,300\})$.

$FPnew = \{100,400,200\} + \{-50,0,-100\} = \{50,400,100\}$. As 50 is not in our candidate diameters, we replace it with minimum size. Therefore, $FPnew = \{100,400,100\}$.

If the newly generated solution vector at this new position was better than the older solution vector, it is replaced with new solution vector (18). Otherwise, the solution vector experiences a move toward the resultant vector of SP (Eq. (19)). If this solution was better than the older one, it is replaced with new solution vector (19). Otherwise, the solution vector experiences a move toward the resultant vector of a random player of the team (Eq. (20)). If this solution was better than the older one, it is replaced with new solution vector (20). In the case that none of the

discussed movements gave a better solution vector, the player is kept in its position with no change.

*3.1.4.2. Provocation operator.* Substitutes of a winner team (S) have to prove a performance equal to the average performance level value of the fixed players in their team in order to be a fixed player. This process, which is performed by the provocation operator in SLC algorithm, is described by

$$S(i,j) = G(i) + \chi_1(G(i) - S(i,j)) \tag{21}$$

$$S(i,j) = G(i) + \chi_2(S(i,j) - G(i)) \tag{22}$$

where $\chi_1 \sim U(0.9,1)$, $\chi_2 \sim U(0.4,0.6)$ are random numbers with uniform distribution, and $G(i)$ is the average value of fixed player's solution vectors in the $i$th team. $S(i,j)$ is the $j$th substitute of the $i$th team.

Firstly, solution vector of the weakest substitute player in the winner team experiences a forward move toward the gravity center of fixed players (Eq. (21)). Assuming that team 1 becomes winner in 3-pipe network example shown in Fig. 2. Then, $G = \{(100+400)/2, (200+100)/2, (100+100)/2\} = \{250,150,100\}$. For Eq. (21) we assume $\chi_1 = 1$, therefore, $Snew = \{250,150,100\} + 1(\{250,150,100\} - \{300,200, 300\}) = \{250,150,100\} + \{-50,-50,-200\} = \{200,100,-100\}$.

We replace the candidate diameter with minimum size because –100 is not in the a logical value. Therefore, $Snew = \{200,100,100\}$.

If the newly generated solution vector relating to this new position was better than the last one, it is replaced with new solution vector (21). Otherwise, mentioned player will experience a backward movement toward the gravity center (Eq. (22)). If this movement gave a solution which was better than the weakest solution, it will be replaced with it 22. In the situation that none of the discussed movements gave a better solution vector for improving the weakest solution, a new vector is randomly generated and used instead of the old one. In an overall view, provocation operator acts on the weakest substitutes of winners. If advancement was evident in their performance, they are kept in the team. Otherwise, they are removed from the team while new random players (solution vectors) are entered for future games.

After each game, 2 discussed operators act on the players (solution vectors) and team's powers are updated according to the new solutions. Obviously, powerful teams are more likely to be successful in their future matches. This process continues to the end of the season and the Super Star Player (SSP) of the league yields the Global Optimum (best solution) for the optimization problem.

### 3.1.5. Step 5. League update

After each season, players are arranged taking into account their updated power. Before commencing a new league, top players are devoted to the best teams, medium players are allocated to the teams with an average performance, and the weakest players are transferred to the bottom teams in the league table.

### 3.1.6. Step 6. Relegation and promotion

In the real league the worst team (relegation spot) is removed from the league at the end of the season, but in SLC algorithm, to escape from local minimums and plateaus, worst teams are removed from the first level league, and in return, new teams (promotion spot) are imported to the league. In our algorithm, mentioned step is used when the best cost in each of three seasons are equal.
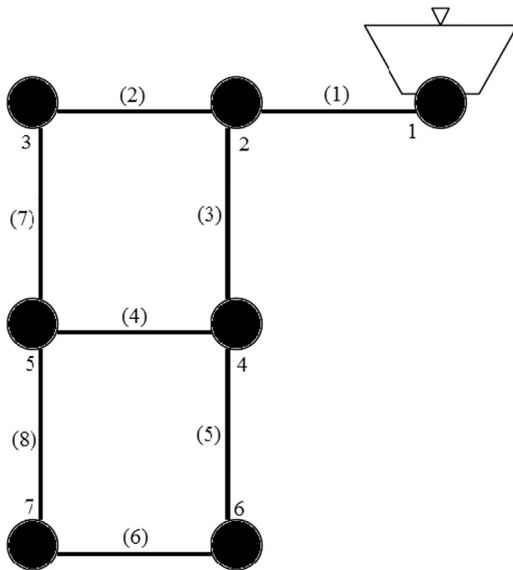
**Fig. 3.** Layout of two-loop network.

**Table 1**
Hydraulic data relevant to the two-loop network.

| Nodes | q (m³/h) | ELV (m) |
|---|---|---|
| 1 | −1120 | 210 |
| 2 | 100 | 150 |
| 3 | 100 | 160 |
| 4 | 120 | 155 |
| 5 | 270 | 150 |
| 6 | 330 | 165 |
| 7 | 200 | 160 |

*3.1.7. Step 7. Check the stopping criterion*

In this section, steps 3–5 are repeated until the termination criterion (nSeason) is satisfied.

## 4. Numerical examples

In this section, the performance of proposed algorithm is examined for three standard benchmark tests, the two-loop network, the Hanoi network, and the New York Tunnels network. All of computations were executed in MATLAB programming language environment with an Intel(R) Core(TM) 2Duo CPU P8700 @ 2.53 GHz and 4.00 GB RAM.

### 4.1. Example 1 (two-loop network)

The pipe network in this example, schematically shown in Fig. 3, is a hypothetical problem originally presented by Alperovits and Shamir [23]. The network has seven nodes and eight pipes with two loops, and is fed by gravity from a reservoir with a 210-m fixed head. The pipes are all 1000 m long with a Hazen–Williams coefficient of 130. The minimum pressure limitation is 30 m above ground level. There is 14 commercial pipe diameters to be selected. Although this two-loop network looks small, a complete enumeration comprises $14^8 = 1.48 \times 10^9$ different network designs, thus making this illustrative example difficult to solve [4,6,17]. Tables 1 and 2 provide the relevant hydraulic data and costs for each pipe size.

This problem was previously solved using different meta-heuristic optimization techniques, including genetic algorithms

**Table 2**
Pipe cost data for two-loop network.

| Diameter (in) | Diameter (mm) | Cost (dollar/meter) |
|---|---|---|
| 1 | 25.4 | 2 |
| 2 | 50.8 | 5 |
| 3 | 76.2 | 8 |
| 4 | 101.6 | 11 |
| 6 | 152.4 | 16 |
| 8 | 203.2 | 23 |
| 10 | 254 | 32 |
| 12 | 304.8 | 50 |
| 14 | 355.6 | 60 |
| 16 | 406.4 | 90 |
| 18 | 457.2 | 130 |
| 20 | 508 | 170 |
| 22 | 558.8 | 300 |
| 24 | 609.6 | 550 |

**Table 3**
Results obtained by different meta-heuristic algorithms for two-loop network.

| Algorithm | Average no. of evaluations | Cost ($) |
|---|---|---|
| Genetic algorithm (GA) [4] | 65,000 | 419,000 |
| Simulated annealing algorithm (SA) [5] | 25,000 | 419,000 |
| Shuffled leapfrog algorithm (SLA) [7] | 11,155 | 419,000 |
| Shuffled complex algorithm (SCA) [24] | 11,019 | 419,000 |
| Modified genetic algorithm [25] | 2440 | 419,000 |
| Particle swarm optimization (PSO) [17] | 5138 | 419,000 |
| Differential evolution (DE) [12] | 4750 | 419,000 |
| Harmony search (HS) [26] | 2891[a] | 419,000 |
| Scatter search (SS) [11] | 3215 | 419,000 |
| PSO+DE [17] | 3080 | 419,000 |
| Particle swarm harmony search (PSHS) [26] | 233[a] | 419,000 |
| Soccer league competition (SLC) | 2051 | 419,000 |

[a] Minimal number of evaluations.

(GA) [4], simulated annealing (SA) [5], shuffled frog leaping algorithm (SFLA) [7], harmony search (HS) [6], scatter search (SS) [11], differential evolution (DE) [12,13], particle swarm optimization (PSO) [17], PSO+DE [17], and particle swarm harmony search (PSHS) [26].

As a new method, The SLC algorithm is applied to solve this problem. SLC parameters for this problem are assumed to be: nT=8, nFp=3, and nS=3.

Table 3 gives the results of SLC and other meta-heuristic algorithms which were previously published for this example. The optimal solutions obtained using previous algorithms were exactly the same ($419,000). As shown in Table 3, the SLC algorithm is significantly more efficient than other techniques for finding the optimal solutions in terms of average number of function evaluations. As clearly shown, the average number of function evaluations required to find the global solution ($419,000) based on 50 different SLC algorithm runs was 2051, which is less than those required by other techniques except for PSHS algorithm. In SLC algorithm, minimum number of function evaluations equals 968.It is worth to mention that, in Table 3, SLC algorithm does not apply relegation and promotion and it can still find global optimum (the least cost of 419,000 units) with an average success probability of 40% (20 out of 50 different runs) while PSHS algorithm can reach global optimum with success probability of 11% (3 out of 27 different runs). Using SLC algorithm with relegation and promotion (the number of imported and removed teams in relegation and promotion step equals 6), the global optimum ($419,000) is found with an average success
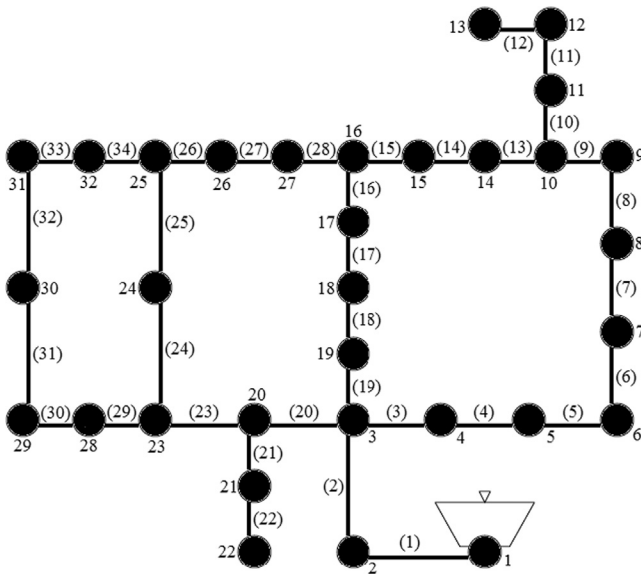
**Fig. 4.** Layout of Hanoi network.

**Table 4**
Hydraulic data relevant to Hanoi network.

| Node | Demand (m³/h) | Node | Demand (m³/h) | Pipe | Length (m) | Pipe | Length (m) |
|------|------|------|------|------|------|------|------|
| 1 | − 19,940 | 17 | 865 | 1 | 100 | 18 | 800 |
| 2 | 890 | 18 | 1345 | 2 | 1350 | 19 | 400 |
| 3 | 850 | 19 | 60 | 3 | 900 | 20 | 2200 |
| 4 | 130 | 20 | 1275 | 4 | 1150 | 21 | 1500 |
| 5 | 725 | 21 | 930 | 5 | 1450 | 22 | 500 |
| 6 | 1005 | 22 | 485 | 6 | 450 | 23 | 2650 |
| 7 | 1350 | 23 | 1045 | 7 | 850 | 24 | 1230 |
| 8 | 550 | 24 | 820 | 8 | 850 | 25 | 1300 |
| 9 | 525 | 25 | 170 | 9 | 800 | 26 | 850 |
| 10 | 525 | 26 | 900 | 10 | 950 | 27 | 300 |
| 11 | 500 | 27 | 370 | 11 | 1200 | 28 | 750 |
| 12 | 560 | 28 | 290 | 12 | 3500 | 29 | 1500 |
| 13 | 940 | 29 | 360 | 13 | 800 | 30 | 2000 |
| 14 | 615 | 30 | 360 | 14 | 500 | 31 | 1600 |
| 15 | 280 | 31 | 105 | 15 | 550 | 32 | 150 |
| 16 | 310 | 32 | 805 | 16 | 2730 | 33 | 860 |
| | | | | 17 | 1750 | 34 | 950 |

probability of 100%. In this case, the average number of function evaluations equals 9540.

### 4.2. Example 2 (Hanoi network)

The Hanoi network, schematically shown in Fig. 4, first was presented by Fujiwara and Khang [27]. It consists of 32 nodes, 34 pipes, and 3 loops, and is fed by gravity from a reservoir with a 100-m (=328 ft) fixed head. The pipe lengths are shown in Table 4, and have a Hazen–Williams $C$ of 130. The minimum head limitation is 30 m (=98.4 ft) above ground level [6]. The solution space consists of $6^{34}=2.865 \times 10^{26}$ numbers of solutions, as there are 6 possible pipe diameters and 34 links in the system [12]. Table 5 lists the pipe cost per meter for the 6 available pipe diameters. SLC parameters for this problem are assumed to be: nT=8, nFp=8, nS=8, and the number of teams in relegation and promotion step equals 5.

Table 6 gives a result summary of the SLC algorithm and other techniques applied to the Hanoi network. Two different cases of SLC are used to optimize this network. As shown in Table 6, the second case of SLC algorithm found the global optimum with a

**Table 5**
Pipe cost data for Hanoi network.

| Diameter (mm) | Cost (dollar/meter) |
|------|------|
| 304.8 | 45.726 |
| 406.4 | 70.40 |
| 508.0 | 98.378 |
| 609.6 | 129.333 |
| 762.0 | 180.748 |
| 1016 | 278.28 |

success rate of 100% over the 50 different runs. Although, SLC algorithm does not apply relegation and promotion in the first case, it finds the least cost with an average success probability of 80%. In the second case, in which SLC algorithm uses relegation and promotion, global best solution is found in all 50 runs with an average number of 71,789 function evaluations. In this case, minimum number of function evaluation for finding the global optimum equals 16,909 which is less than minimum of those obtained by HS and PSHS algorithms (27,721 function evaluations for HS and 17,980 function evaluations for PSHS) [26].

### 4.3. Example 3 (New York City water supply tunnels network)

Schaake and Lai [25] first presented the New York City network, shown in Fig. 5. It consists of 20 nodes, 21 pipes and 1 loop, and is fed by gravity from a reservoir with a 300-ft fixed head. The objective of the problem is to add new pipes parallel to existing ones because the existing network cannot satisfy the pressure head requirements at certain nodes (nodes 16–20). The pipes lengths are shown in Table 7, and have a Hazen–Williams constant $C$ of 100 [6]. For each duplicate tunnel there are 16 allowable decisions including 15 available diameters and the 'do nothing'' option; therefore the search space of this optimization problem is $16^{21}=1.93 \times 10^{25}$ possible designs [6]. Fifteen commercial diameters are listed in Table 8. SLC parameters for this problem are assumed to be: nT=5, nFp=5, nS=5, and the number of teams in relegation and promotion step equals 4.

The results of the trial runs are presented in Table 9. In the first case, out of 20 trial runs, 16 trials gave a solution cost of $38.64 million with an average of 7821 function evaluations. In this case, SLC does not use relegation and promotion. In the second case, in which SLC applies relegation and promotion, best cost is found with an average success probability of 100%. Average number of function evaluations of HS and PSHS equal 5991 and 5923, respectively, that are less than this value for SLC algorithm, but there is no available data about success probability of these two algorithms.

As can be seen in Table 9, benefit of the suggested algorithm is on its high level of confidence in finding solutions. For instance, SLC reaches the global optimum in all executions of case 2 (with relegation and promotion).

## 5. Suggestions for improvement of SLC algorithm

SLC algorithm is multi-populous and applies different operators. To escape from a blind search, weights are devoted to teams and solution vectors. In this algorithm, first the winner team is found, then the best solution vectors (principal vectors) move toward the best points among populations. This process expedites the searching capability of the algorithm. To prevent the algorithm from being entrapped in local optimums, substitute solution vectors move towards other directions and change their position with the main solution vectors as soon as they reach a better

**Table 6**
Results obtained by different meta-heuristic algorithms for Hanoi network.

| Algorithm | Number of runs | Best cost ($) | Percent of trials with the best solution found (%) | Average cost ($) | Average no. of evaluations |
|---|---|---|---|---|---|
| MMAS [16] | 20 | 6,134,087 | 0 | 6.386 | 85,600 |
| PSO [17] | 2000 | 6,081,087 | 5 | 6.31 | NA |
| HD-DDS [28] | 50 | 6,081,087 | 8 | 6.252 | 100,000 |
| GENOME [29] | 10 | 6,081,087 | 10 | 6.248 | NA |
| GHEST [30] | 60 | 6,081,087 | 38 | 6.175 | 50,134 |
| SS [11] | 100 | 6,081,087 | 64 | NA | 43,149 |
| DE [12] | 300 | 6,081,087 | 80 | NA | 48,724 |
| SADE [14] | 50 | 6,081,087 | 84 | 6.09 | 60,532 |
| SLC[a] | 20 | 6,081,087 | 80 | 6.110 | 29,108 |
| SLC[b] | 50 | 6,081,087 | 100 | 6.081 | 71,789 |

[a] Without relegation and promotion.
[b] With relegation and promotion.
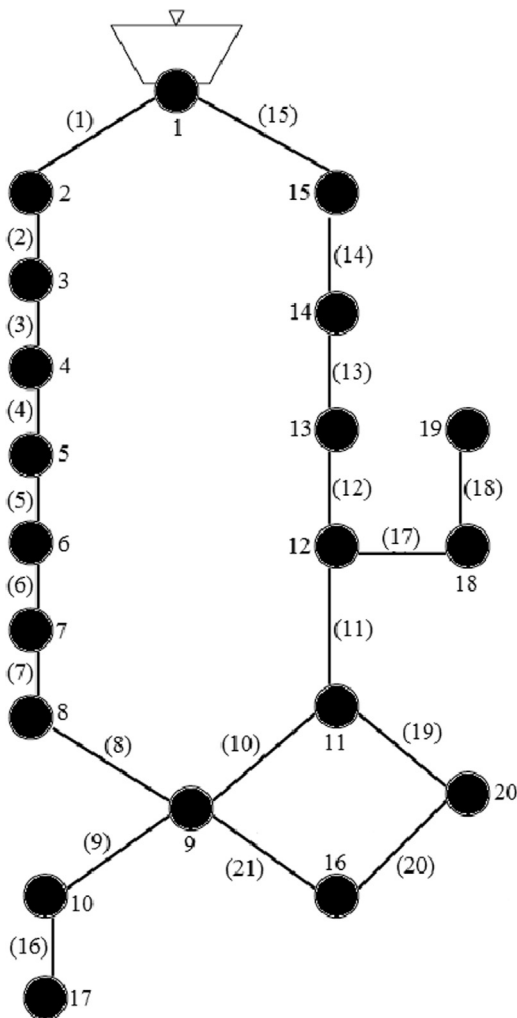


**Fig. 5.** Layout of New York Tunnels network.

**Table 7**
Hydraulic data relevant to New York Tunnels network.

| Node | Demand | Minimum head (ft) | Pipe | Length (ft) | Existing diameter (ft) |
|---|---|---|---|---|---|
| 1 | −22018 | 300 | 1 | 11,600 | 180 |
| 2 | 92.4 | 255 | 2 | 19,800 | 180 |
| 3 | 92.4 | 255 | 3 | 7300 | 180 |
| 4 | 88.2 | 255 | 4 | 8300 | 180 |
| 5 | 88.2 | 255 | 5 | 8600 | 180 |
| 6 | 88.2 | 255 | 6 | 19,100 | 180 |
| 7 | 88.2 | 255 | 7 | 9600 | 132 |
| 8 | 88.2 | 255 | 8 | 12,500 | 132 |
| 9 | 170 | 255 | 9 | 9600 | 180 |
| 10 | 1 | 255 | 10 | 11,200 | 204 |
| 11 | 170 | 255 | 11 | 14,500 | 204 |
| 12 | 117.1 | 255 | 12 | 12,200 | 204 |
| 13 | 117.1 | 255 | 13 | 24,100 | 204 |
| 14 | 92.4 | 255 | 14 | 21,100 | 204 |
| 15 | 92.4 | 255 | 15 | 15,500 | 204 |
| 16 | 170 | 260 | 16 | 26,400 | 72 |
| 17 | 57.5 | 272.8 | 17 | 31,200 | 72 |
| 18 | 117.1 | 255 | 18 | 24,000 | 60 |
| 19 | 117.1 | 255 | 19 | 14,400 | 60 |
| 20 | 170 | 255 | 20 | 38,400 | 60 |
|  |  |  | 21 | 26,400 | 72 |

**Table 8**
Pipe cost data for New York Tunnels network.

| Diameter (in) | Cost (dollar/ft) |
|---|---|
| 0 | 0 |
| 36 | 93.5 |
| 48 | 134 |
| 60 | 176 |
| 72 | 221 |
| 84 | 267 |
| 96 | 316 |
| 108 | 365 |
| 120 | 417 |
| 132 | 469 |
| 144 | 522 |
| 156 | 577 |
| 168 | 632 |
| 180 | 689 |
| 192 | 746 |
| 204 | 804 |

finding (better that the best solution vector). This process precisely searches neighbors of every single vector.

SLC algorithm performs optimized design of water distribution networks with a high level of confidence. But, to improve its performance and decrease the number of function evaluation times, it is possible to define operators for the fixed and reserved players of the loser teams. For example, the PSPH method, which is used for optimization of water distribution networks, has reliable operators which can be used for reserved players of the loser teams in SLC algorithm. Application of PSHS

operators helps to restore stagnated teams (loser teams) and solutions. Therefore, improved and arranged solution vectors increase the probability of success and gives the chance to all solution vectors to use a specific operator. In overall view, combination of different operators expedites the procedure,

**Table 9**
Results obtained by different meta-heuristic algorithms for New York Tunnels network.

| Algorithm | Number of runs | Best cost ($) | Percent of trials with the best solution found (%) | Average cost ($) | Average no. of evaluations |
|---|---|---|---|---|---|
| MMAS [16] | 20 | 38.64 | 60 | 38.84 | 30700 |
| PSO [17] | 2000 | 38.64 | 30 | NA | NA |
| HD-DDS [28] | 50 | 38.64 | 86 | NA | 47000 |
| GHEST [30] | 60 | 38.64 | 92 | NA | 11464 |
| SS [11] | 100 | 38.64 | 65 | NA | 57583 |
| DE [12] | 50 | 38.64 | NA | NA | 5494 |
| SADE [14] | 50 | 38.64 | 92 | NA | 6598 |
| SLC[a] | 20 | 38.64 | 80 | 38.81 | 7821 |
| SLC[b] | 50 | 38.64 | 100 | 38.64 | 15764 |

[a] Without relegation and promotion.
[b] With relegation and promotion.

increases the accuracy, precisely searches the neighbors, and raises the level of confidence.

Results of performing sensitivity analysis on nT, nFP and nS parameters showed that

1. Minimum of nFP and nS should be equal or greater than three.
2. By increasing nT, our algorithm will be more reliable, however, number of function evaluations will be increased too.
3. In WDN optimization, we suggest number of fixed players (nFP) and number of substitutes (nS) to be about one fourth of number of pipes.

## 6. Conclusions

This paper presented a new optimization technique called Soccer League Competition (SLC) algorithm. The basic concepts and ideas of the method are inspired from soccer leagues and based on competitions among teams and players. Each solution vector is called a player, and the best solution is called the Super Star Player (SSP). All players are devoted to some teams. Each team has a star, some fixed players, and some substitutes. The competition among teams to take the possession of the top ranked positions in the league table, and the competitions between players in each team for personal improvements results in the convergence of players to the global optimum. During the optimization process, some teams are replaced with new ones if there were no improvement in their best cost during three seasons. This replacement procedure is done to escape the algorithm from local minimums and plateaus. The overall optimization results indicate that SLC has the capability to optimize various pipe networks with lower computational efforts (measured as the number of function evaluations). As a result, SLC is a suitable alternative optimizer challenging other meta-heuristic methods especially in terms of computational efficiency and reliability in optimization of water distribution networks.

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