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A modified equilibrium optimizer using opposition-based learning and novel update rules

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ABSTRACT

Equilibrium Optimizer (EO) is a newly developed physics-based metaheuristic algorithm that is based on control volume mass balance models, and has shown competitive performance with other state-of-the-art algorithms. However, the original EO has the disadvantages of a low exploitation ability, ease of falling into local optima, and an immature balance between exploration and exploitation. To address these shortcomings, this paper proposes a modified EO (m-EO) using opposition-based learning (OBL) and novel update rules that incorporates four main modifications: the definition of the concentrations of some particles based on OBL, a new nonlinear time control strategy, novel population update rules and a chaos-based strategy. Based on these modifications, the optimization precision and convergence speed of the original EO are greatly improved. The validity of m-EO is tested on 35 classical benchmark functions, 25 of which have variants belonging to multiple difficulty categories (Dim = 30, 100, 300, 500 and 1000). In addition, m-EO is used to solve three real-world engineering design problems. The experimental results and two different statistical tests demonstrate that the proposed m-EO shows higher performance than original EO and other state-of-the-art algorithms.

1. Introduction

Optimization is applied in many fields worldwide; it is the process of using a search mechanism to find the best solution such that certain constraints are satisfied (Chen et al., 2017; Ning et al., 2018; Ochoa et al., 2020). As technology has developed, increasingly complex optimization problems have continually emerged. However, traditional mathematical optimization methods are too inefficient and inaccurate to meet the needs of many current problems (Hu et al., 2020; Sánchez et al., 2020; Vanneschi et al., 2017). Therefore, the use of metaheuristic algorithms to solve optimization problems has attracted the attention of a large number of researchers (Bernal et al., 2020; Ghahremani-Nahr et al., 2019; Li et al., 2018).

Metaheuristic algorithms find the best solution by simulating random phenomena in nature. Such algorithms can be categorized into evolutionary, swarm-based and physics-based algorithms (Lin & Middendorf, 2013). Evolutionary algorithms (EAs) simulate the laws of evolution in nature, and the genetic algorithm (GA) is one of the most popular examples of EAs. GA simulates the natural selection of evolution and the biological evolutionary process of genetic mechanisms (Wang et al., 2020). Similarly, differential evolution (DE) simulates crossover, mutation and other mechanisms in genetics and is an efficient global optimization algorithm (Zhao et al., 2019). Inspired by natural biogeography, a new EA for biogeography-based optimization (BBO) has been proposed (Ma et al., 2016). The learner performance-based behaviour (LPB) algorithm is inspired by the process of accepting high school graduates across university departments (Kaveh et al., 2020). The geometric probabilistic evolutionary algorithm (GPEA) is composed of a Bernoulli reflection search operator and a Cauchy distributed inversion search operator (Segovia-Domínguez et al., 2020).

Swarm-based algorithms imitate various physical or biological behaviours observed in nature, such as those of moths, bees, birds and

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wolves (Cui & Gao, 2012). Particle swarm optimization (PSO) is inspired by the foraging behaviour of birds (Kennedy & Eberhart, 1995), and ant colony optimization (ACO) simulates the behaviour of ants when finding food (Dorigo et al., 1996). The grey wolf optimizer (GWO) imitates the behaviour of grey wolves searching for prey (Mirjalili et al., 2014). Recently, several new swarm-based algorithms have been proposed. The salp swarm algorithm (SSA) is inspired by the navigation and foraging behaviour of salps (Mirjalili et al., 2017). The coyote optimization algorithm (COA) is designed to mimic the behaviour of Canis latrans (Pierezan & Dos Santos Coelho, 2018). The pathfinder algorithm (PFA) mimics the behaviour of fauna in finding the best source areas for food (Yapici & Cetinkaya, 2019). The sailfish optimizer (SFO) is inspired by the group-hunting behaviour of sailfish (Shadravan et al., 2019). The tunicate swarm algorithm (TSA) imitates the group behaviour of tunicates during navigation and foraging (Kaur et al., 2020). The slime mould algorithm (SMA) is inspired by the spreading and foraging behaviour of slime moulds (Li et al., 2020). The side-blotched lizard algorithm (SBLA) simulates the polymorphic population of the eponymous lizard (Maciel et al., 2020).

Physics-based algorithms emulate the physical laws of the universe. The most famous of these algorithms is simulated annealing (SA), the idea of which originated from the phenomenon of annealing in metallurgy. The gravitational search algorithm (GSA) is inspired by the law of gravity and mass interactions (Rashedi et al., 2009). The sine cosine algorithm (SCA) uses the fluctuations of the sine and cosine functions to find the optimal solution (Mirjalili, 2016). Additionally, a new algorithm based on physical behaviour, called henry gas solubility optimization (HGSO), mimics the behaviour described by Henry's law (Hashim et al., 2019). The queueing search (QS) has been proposed based on the physical queueing behaviour of humans (Zhang et al., 2018). Atomic search optimization (ASO) is inspired by atomic dynamics and simulates atomic interactions in nature (Zhao et al., 2019). The billiards-inspired optimization algorithm (BOA) imitates the game of billiards based on the laws of conservation of energy and momentum (Kaveh et al., 2020). The gradient-based optimizer (GBO) is inspired by the gradient-based Newton search method (Ahmadianfar et al., 2020). Although most of these algorithms have been widely applied to optimization problems, it is still very common for them to fall into local optima or suffer from slow convergence (Dokeroglu et al., 2019; Lopes Silva et al., 2018; Mahdavi et al., 2015).

Moreover, the no-free-lunch (NFL) theorem suggests that no algorithm can be valid for all optimization problems; consequently, there is always a need for further innovation in optimization algorithms. At the same time, this theorem has prompted many researchers to start improving existing algorithms. Frumen et al. proposed a new adaptive method based on dynamic parameters for PSO using interval type-2 fuzzy logic (Olivas et al., 2016). Castañeda was inspired by ancient metallurgical technology to propose orikaeshi tanren SA (OTSA) by adding two new operators (folding and reheating) to the original SA (Morales-Castañeda et al., 2019). Guo et al. proposed a hybrid SCA and improved the population quality by using optimal neighbourhood and quadratic interpolation strategies (Guo et al., 2020). Elaziz et al. proposed an improved opposition-based SCA (OBSCA) for global optimization, in which opposition-based learning (OBL) is used to select the better position between the original position and its opposite (Morales-Castañeda et al., 2019). Valdez et al. improved the harmony search algorithm by using the concept of fuzzy functions and dynamically adjusting the main parameters of the algorithm throughout the iterative process (Valdez et al., 2020). Hammouri et al. proposed a binary dragonfly algorithm using a new updating mechanism (Hammouri et al., 2020). In addition, Dhargupta et al. used OBL to improve the exploration capacity of GWO, thereby proposing selective opposition-based GWO (SOGWO) (Dhargupta et al., 2020).

Recently, a novel Equilibrium Optimizer (EO) has been proposed by Faramarzi, inspired by control volume mass balance models. Compared with GA, PSO, GWO, GSA, SSA and the covariance matrix adaptation evolution strategy (CMA-ES), EO has been proven to exhibit excellent performance (Faramarzi et al., 2020a). Nevertheless, the NFL theorem prompts us to improve this latest physics-based algorithm. Although EO is competitive with other popular algorithms, it still suffers from slow convergence, low accuracy and ease of falling into local optima in some cases. These drawbacks are mainly caused by the low quality of the randomly generated initial population, unbalanced exploitation and exploration capabilities in the search space, and the low possibility of large spatial jumps during the process of iteratively updating the population. Therefore, this paper proposes a modified EO (m-EO) using OBL and novel update rules, which offers four novel contributions. The first contribution is the introduction of OBL, which distinguishes this algorithm from others. OBL is used to solve for the opposite points of particles, except for the four particles with the highest concentrations. Second, a new balanced exploration and exploitation strategy is proposed to facilitate an effective transition from exploration to exploitation. Third, several new functions are proposed to redefine the update rules to avoid falling into local optima while accelerating convergence. Finally, chaotic mapping is introduced to prevent stagnation of the search process by finding other valuable search areas through mutation and the generation of new solutions.

To verify the performance of the proposed m-EO, 35 well-known benchmark functions with different numbers of dimensions (Dim = 30, 100, 300, 500, 1000) and 3 real-world engineering design problems are used. All optimization results are compared with those of 8 other state-of-the-art algorithms, and the results are evaluated by means of the Wilcoxon rank-sum test (García et al., 2010) and the Friedman test (Theodorsson-Norheim, 1987). The comprehensive results prove that the proposed m-EO is superior to its competitors. In summary, the main contributions of this paper are as follows. 1) OBL and chaotic mapping are introduced to improve the population quality and increase the diversity of the solutions. 2) A new nonlinear time control strategy and novel update rules are proposed to improve search efficiency and avoid premature convergence. 3) The effectiveness and efficiency of m-EO are tested on 35 benchmark functions of different difficulty levels and 3 real-world engineering design problems. 4) The superiority of the proposed m-EO is proven by two different statistical tests.

The remainder of this paper is structured as follows: Section 2 introduces the original EO, and in Section 3, the proposed m-EO is described in detail. The benchmark function tests and a discussion of the results are presented in Section 4. In Section 5, the proposed m-EO is applied to three real-world engineering problems. Finally, conclusions and future research directions are presented in Section 6.

2. Equilibrium Optimizer (EO)

The design of EO is based on the dynamic mass balance of a control volume, in which the particle concentration is described by the mass balance equation in physics (Faramarzi et al., 2020b). In EO, the particle concentration update rule is mainly described in terms of an equilibrium pool ($\overrightarrow{C}_{eq,pool}$), an exponential term ($F$) and a generation rate (G).The details of these three components of the algorithm are described below.

For population initialization in EO, random concentrations of particles are generated in the d-dimensional search space. The initial concentration of each particle is described as follows:

$$\overrightarrow{C}_i^{initial} = Lb + rand_i \otimes (Ub - Lb) \quad i = 1, 2, \ldots..n \tag{1}$$

where $\overrightarrow{C}_i^{initial}$ is the i_{th} particle concentration; Ub and Lb are the maximum and minimum boundaries, respectively, in the search space; n is the number of particles; and $rand_i$ is a random vector in the interval of $[0, 1]$. After population initialization, the particle concentrations are evaluated, and four particles with high fitness are selected in preparation for the formation of the equilibrium pool.

The equilibrium pool, which provides candidates for the search process, consists of the four particles with the highest concentrations and an average particle. The average particle is calculated as follows:

$$\overrightarrow{C}_{eq\_ave} = \frac{\overrightarrow{C}_{eq1} + \overrightarrow{C}_{eq2} + \overrightarrow{C}_{eq3} + \overrightarrow{C}_{eq4}}{4} \tag{2}$$

where $\overrightarrow{C}_{eq1} \sim \overrightarrow{C}_{eq4}$ are the four particles with the highest concentrations, and $\overrightarrow{C}_{eq\_ave}$ is the average particle. These five particle vectors construct the equilibrium pool, which is defined as follows:

$$\overrightarrow{C}_{eq,pool} = \left\{\overrightarrow{C}_{eq1},\ \overrightarrow{C}_{eq2},\ \overrightarrow{C}_{eq3},\ \overrightarrow{C}_{eq4},\ \overrightarrow{C}_{eq\_ave}\right\} \tag{3}$$

In the equilibrium pool ($\overrightarrow{C}_{eq,pool}$), the first four particles give EO a better exploration capability, while the last particle assists in exploitation. The particles in the equilibrium pool are updated in each iteration, so the candidates also change.

$$\overrightarrow{C}_{eq} = randi\left\{\overrightarrow{C}_{eq,pool}\right\} \tag{4}$$

where $\overrightarrow{C}_{eq}$ is randomly selected from among the five candidate vectors in the equilibrium pool with equal probability. This helps generate the optimal solution in each iteration.

The balance between exploration and exploitation determines the performance of the algorithm, and the exponential term F helps EO achieve a reasonable balance during the optimization process. F is calculated using the following equation:

$$\overrightarrow{F} = a_1 \mathrm{sign}(\overrightarrow{r} - 0.5) \otimes [e^{-\overrightarrow{\lambda} t} - 1] \tag{5}$$

where a_1 is a constant value, $\overrightarrow{r}$ and $\overrightarrow{\lambda}$ are random vectors in the interval of $[0,1]$, $sign(\cdot)$ represents the sign function, and the time t is the only nonlinear coefficient that changes with the number of iterations. It is calculated as follows:

$$t = \left(1 - \frac{Iter}{Max\_iter}\right)^{\left(a_2 \frac{Iter}{Max\_iter}\right)} \tag{6}$$

where a_2 is a constant value, *Iter* represents the current iteration, and *Max_iter* is the maximum number of iterations. It can be seen that t has a significant impact on the balance between exploration and exploitation. Experiments show that when $a_1 = 2$ and $a_2 = 1$, EO has the strongest optimization ability (Faramarzi et al., 2020b).

However, the most important component of EO is the generation rate (G), which affects the performance of the algorithm in obtaining the optimal solution. G represents the exploitation ability of EO throughout the whole iterative optimization process and is defined as follows:

$$\overrightarrow{G} = \overrightarrow{GCP} \otimes (\overrightarrow{C}_{eq} - \overrightarrow{\lambda}\overrightarrow{C}) \otimes \overrightarrow{F} \tag{7}$$

where $\overrightarrow{C}$ is the current particle concentration, and $\overrightarrow{GCP}$ is a control parameter defined as:

$$\overrightarrow{GCP} = \begin{cases} 0.5r_1 & r_2 \geqslant GP \\ 0 & r_2 < GP \end{cases} \tag{8}$$

where $GP = 0.5$ is a constant value, and r_1 and r_2 are random vectors in the interval of $[0,1]$. As seen above, the generation rate (G) controls the renewal mechanism for all particles.

In summary, the update rule of EO is defined as follows:

$$\overrightarrow{C} = \overrightarrow{C}_{eq} + \left(\overrightarrow{C} - \overrightarrow{C}_{eq}\right) \cdot \overrightarrow{F} + \frac{\overrightarrow{G}}{\overrightarrow{\lambda} V}(1 - \overrightarrow{F}) \tag{9}$$

where V is considered to be a unit volume, and the other variables are as shown above. The first term of the update rule is the equilibrium concentration that is randomly selected from the equilibrium pool, and the exponential term (F) is used to adjust the difference between it and the current concentration, which is helpful for improving the exploration ability of the algorithm. In the last term, small changes in particle concentration can help guide the local search process and improve the exploitation ability.

In addition, the particle's memory saving mechanism is used to compare the fitness of each particle with its previous fitness and save the higher concentration of the particle. The pseudocode of the original EO is provided in Algorithm 1.

Algorithm 1 The Equilibrium Optimizer (EO)

Initialization {
  initialize the particle population $\overrightarrow{C}_i^{initial} (i = 1, 2, \ldots, n)$
  define t, $a_1 = 2$, $a_2 = 1$ and $GP = 0.5$}
Main loop
  while ($Iter < Max\_iter$)
    calculate the fitness of each particle
    find $\overrightarrow{C}_{eq1}$, $\overrightarrow{C}_{eq2}$, $\overrightarrow{C}_{eq3}$ and $\overrightarrow{C}_{eq4}$
    calculate the average particle $\overrightarrow{C}_{eq\_ave}$ and construct the equilibrium pool $\overrightarrow{C}_{eq,pool}$
    perform memory saving and particle concentration updating ($Iter > 1$)
    update t
    for each particle agent
      update all positions according to Eq. (9)
    end for
      $Iter = Iter + 1$
    end while
  print($\overrightarrow{C}_{eq1}$)

3. Proposed m-EO

Compared with other metaheuristic algorithms, EO is very effective in solving optimization problems. However, in some cases, EO still becomes trapped in local optima or suffers from unbalanced development and low accuracy. In this section, several modifications of EO are proposed. These four modifications are explained in detail below.

3.1. Opposition-based learning (OBL)

OBL is a new technique recently proposed by Tizhoosh in the field of computing (Tizhoosh, 2005). The core idea of OBL is to consider both estimates and counter-estimates to reach the optimal solution because any random solution and its opposite are better than two independent solutions (Rahnamayan et al., 2008). For any solution $\Gamma = \{\gamma_1, \gamma_2, \ldots, \gamma_d\}$ in a d-dimensional space, its opposite solution can be defined as $\overline{\Gamma} = \left\{\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_d\right\}$, which is calculated as follows:

$$\overline{\gamma}_j = Lb_j + Ub_j - \gamma_j \tag{10}$$

where Lb_j and Ub_j are the lower and upper bounds on the j_{th} solution for Γ. In recent years, a large number of researchers have used OBL to improve the initial population quality of metaheuristic algorithms (Abd Elaziz et al., 2017; Wang et al., 2007; Liang et al., 2019; Rahnamayan et al., 2012; Zhou et al., 2017), but in most cases, OBL has been used only to increase the diversity of the initial population without considering the selection of the individuals in the initial population.

In contrast to most algorithms, in this paper, we find the opposite positions corresponding to the concentrations of all particles except $\overrightarrow{C}_{eq1} \sim \overrightarrow{C}_{eq4}$ in each iteration. Since the four particles with the highest concentrations are selected to construct the equilibrium pool, OBL is used only to solve for the opposite concentrations of the other particles, thereby effectively improving the global search capabilities and convergence speed of the algorithm. The model for the opposite concentrations of the other particles is as follows:

$$\overrightarrow{OC}_{eq,i}^{others} = Lb + Ub - \overrightarrow{C}_{eq,i}^{others}(i = 5, 6, ..., n) \tag{11}$$

where $\overrightarrow{OC}_{eq,i}^{others}$ is the opposite concentration vector for the i_{th} particle (excluding $\overrightarrow{C}_{eq1} \sim \overrightarrow{C}_{eq4}$).

3.2. Nonlinear time parameter (t) strategy

As the discussion above shows, EO achieves a balance between exploration and exploitation by means of the exponential term F, allowing it to search for the most promising regions in the search space to reach the global optimum. According to Eq. (5) and Eq. (6), the time (t) continuously varies with the number of iterations and has an important influence on F. In addition, t is the only variable associated with the number of iterations. In this paper, t is redefined and expressed in a new form, as follows:

$$\theta = \frac{\pi}{2} \cdot \frac{Iter}{Max\_iter} \tag{12}$$

$$t = (t_{start} - t_{end}) \cdot \left[(1 - sin\theta) + \frac{cos\theta}{2}\right]^{\left(\frac{Iter}{Max\_iter}\right)} \tag{13}$$

where θ represents the amplitude of the change in time (t) with the number of iterations, t_{start} and t_{end} are the initial and final values of t, respectively. To verify the effectiveness of the parameters proposed in this paper, we compared them with the linear control parameter strategy proposed by Mirjalili (Mirjalili, 2016) and the nonlinear control parameter strategy used in the original EO, as shown in Fig. 1. The newly proposed nonlinear time control strategy places more emphasis on exploration, thereby enhancing the global search ability and reducing the risk of falling into local optima; however, as the number of iterations increases, it still facilitates quick convergence in the late phase of the algorithm.

3.3. Novel update rules

The update rule in EO rule is the most important part of the entire algorithm. As seen from Eq. (7)–(9), the updating of the particle concentrations is affected by three terms, but the vector $\overrightarrow{G}$ in the last term may become equal to zero as the vector $\overrightarrow{GCP}$ varies, which can easily cause EO to fall into local optima. The exploration and exploitation achieved during the optimization process of EO are also affected due to such sudden stagnation. Therefore, in this paper, we define the following new update rules for m-EO:

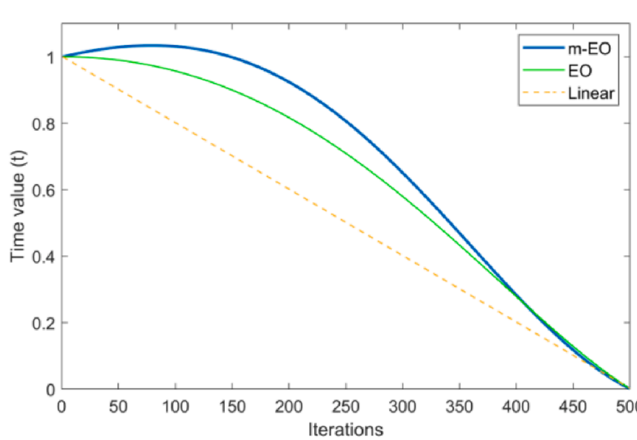

Fig. 1. Comparison of control parameters.

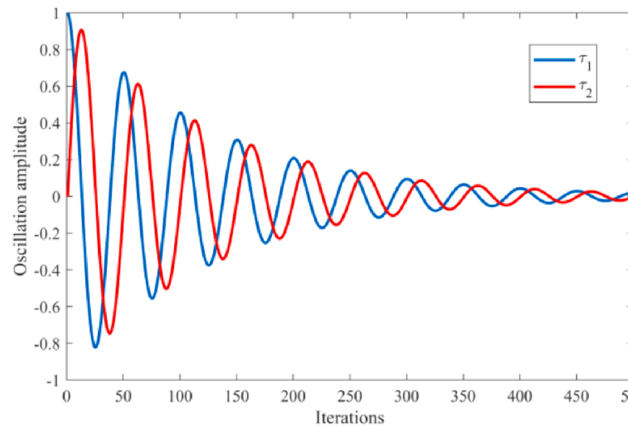

Fig. 2. Ranges of the oscillating parameters.

$$\overrightarrow{C} = \begin{cases} \overrightarrow{C}_{eq} + \left(\overrightarrow{C} - \overrightarrow{C}_{eq}\right) \cdot \overrightarrow{F} + \dfrac{\overrightarrow{G}}{\overrightarrow{\lambda} V}(1 - \overrightarrow{F}) & r_2 > GP \\ \left.\begin{array}{ll} \overrightarrow{\tau}_1 \cdot \overrightarrow{C}_{eq} + \left(\overrightarrow{C} - \overrightarrow{C}_{eq}\right) \cdot \dfrac{\overrightarrow{C}}{\left|\overrightarrow{C}_{best} + \overrightarrow{C}_{worst} - \overrightarrow{C}\right|} & r_\tau > 0.5 \\ \overrightarrow{\tau}_2 \cdot \overrightarrow{C}_{eq} + \left(\overrightarrow{C} - \overrightarrow{C}_{eq}\right) \cdot \dfrac{\overrightarrow{C}}{\left|\overrightarrow{C}_{best} + \overrightarrow{C}_{worst} - \overrightarrow{C}\right|} & r_\tau \leqslant 0.5 \end{array}\right\} & r_2 \leqslant GP \end{cases} \tag{14}$$

with:

$$\overrightarrow{\tau}_1 = cos\left[\frac{\pi}{100} \cdot (4 \cdot Iter)\right] \cdot e^{\left(\frac{\pi}{100} \cdot \frac{Iter}{4}\right)}$$
$$\overrightarrow{\tau}_2 = sin\left[\frac{\pi}{100} \cdot (4 \cdot Iter)\right] \cdot e^{\left(\frac{\pi}{100} \cdot \frac{Iter}{4}\right)} \tag{15}$$

where $\overrightarrow{\tau}_1$ and $\overrightarrow{\tau}_2$ are two oscillating vectors; $\overrightarrow{C}_{best}$ and $\overrightarrow{C}_{worst}$ represent the best and worst particle concentrations, respectively; and r_τ is a random number in $[0, 1]$. In the new update rules, the concentration of the current particle is changed by means of oscillating functions, as shown in Fig. 2. Moreover, the direct difference between the equilibrium and sample particles is weighted by a ratio in terms of three particle

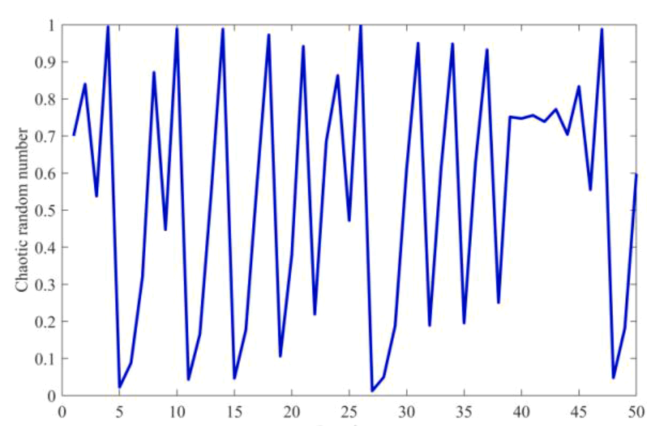

Fig. 3. Visualization of the logistic chaotic map.

Table 1
Descriptions of 35 benchmark functions.

Function	Dim	Range	f_{min}	Type
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30,100,300,500,1000	[−100,100]	0	Unimodal
$f_2 = \sum_{i=1}^{n} ix_i^2$	30,100,300,500,1000	[−10,10]	0	Unimodal
$f_3(x) = \sum_{i=1}^{n} ix_i^4$	30,100,300,500,1000	[−1.28,1.28]	0	Unimodal
$f_4(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1]$	30,100,300,500,1000	[−1.28,1.28]	0	Unimodal
$f_5(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30,100,300,500,1000	[−10,10]	0	Unimodal
$f_6(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j\right)^2$	30,100,300,500,1000	[−100,100]	0	Unimodal
$f_7(x) = max_i\{\lvert x_i \rvert, 1 \leqslant i \leqslant n\}$	30,100,300,500,1000	[−100,100]	0	Unimodal
$f_8(x) = x_1^2 + 10^4 \sum_{i=2}^{n} x_i^2$	30,100,300,500,1000	[−5,5]	0	Unimodal
$f_9(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	30,100,300,500,1000	[−30,30]	0	Unimodal
$f_{10}(x) = 10^4 \cdot x_1^2 + \sum_{i=2}^{n} x_i^2$	30,100,300,500,1000	[−5,5]	0	Unimodal
$f_{11} = (x_1 - 1)^2 + \sum_{i=2}^{n} i\left(2x_i^2 - x_{i-1}\right)^2$	30,100,300,500,1000	[−10,10]	0	Unimodal
$f_{12}(x) = \sum_{i=1}^{n} \lvert x_i \rvert^{i+1}$	30,100,300,500,1000	[−1,1]	0	Unimodal
$f_{13} = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{n} 0.5ix_i\right)^4$	30,100,300,500,1000	[−5,10]	0	Multimodal
$f_{14}(x) = \sum_{i=1}^{n} -x_i sin\left(\sqrt{\lvert x_i \rvert}\right)$	30,100,300,500,1000	[−500,500]	−418.9829 × n	Multimodal
$f_{15}(x) = \sum_{i=1}^{n} \left[x_i^2 - 10cos(2\pi x_i) + 10\right]$	30,100,300,500,1000	[−5.12,5.12]	0	Multimodal
$f_{16}(x) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)\right)$	30,100,300,500,1000	[−32,32]	0	Multimodal
$f_{17}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30,100,300,500,1000	[−600,600]	0	Multimodal
$f_{18}(x) = 0.1\left\{\begin{array}{c} sin^2(3\pi x_1) + \sum_{i=1}^{n} (x_i - 1)^2\left[1 + sin^2(3\pi x_i + 1)\right] \\ +(x_n - 1)^2\left[1 + sin^2(2\pi x_n)\right] \end{array}\right\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30,100,300,500,1000	[−50,50]	0	Multimodal
$f_{19}(x) = \sum_{i=1}^{n} \lvert x_i sin(x_i) + 0.1x_i \rvert$	30,100,300,500,1000	[−10,10]	0	Multimodal
$f_{20} = 0.5 + \frac{\left(sin\left(\sum_{i=1}^{n} x_i^2\right)\right)^2 - 0.5}{\left(1 + 0.001\left(\sum_{i=1}^{n} x_i^2\right)\right)^2}$	30,100,300,500,1000	[−100,100]	0	Multimodal
$f_{21}(x) = 0.1n - \left(0.1\sum_{i=1}^{n} cos(5\pi x_i) - \sum_{i=1}^{n} x_i^2\right)$	30,100,300,500,1000	[−1,1]	0	Multimodal
$f_{22}(x) = \sum_{i=1}^{n-1} \left(x_i^2 + 2x_{i+1}^2\right)^{0.25} \times \left[1 + sin\left(50\left(x_i^2 + x_{i+1}^2\right)^{0.1}\right)^2\right]$	30,100,300,500,1000	[−10,10]	0	Multimodal
$f_{23} = \sum_{i=1}^{n-1} \left[x_i^2 + 2x_{i+1}^2 - 0.3cos(3\pi x_i) - 0.4cos(4\pi x_{i+1}) + 0.7\right]$	30,100,300,500,1000	[−15,15]	0	Multimodal
$f_{24} = \sum_{i=2}^{n} \left(10^6\right)^{(i-1)/(n-1)} \cdot x_i^2$	30,100,300,500,1000	[−100,100]	0	Multimodal
$f_{25}(x) = 1 - cos\left(2\pi\sqrt{\left(\sum_{i=1}^{n} x_i^2\right)}\right) + 0.1\sqrt{\left(\sum_{i=1}^{n} x_i^2\right)}$	30,100,300,500,1000	[−100,100]	0	Multimodal
$f_{26}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6}\right)^{-1}$	2	[−65.536,65.536]	1	Multimodal
$f_{27}(x) = \sum_{i=1}^{11} \left[a_i - \frac{x_1\left(b_i^2 + b_i x_2\right)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	[−5,5]	0.0003	Multimodal
$f_{28}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[−5,5]	−1.0316	Multimodal
$f_{29}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)cosx_1 + 10$	2	[−5, 5]	0.398	Multimodal
$f_{30}(x) = \left[1 + (x_1 + x_2 + 1)^2\left(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2\right)\right] \times \left[30 + (2x_1 - 3x_2)^2 \times \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2\right)\right]$	2	[−2,2]	3	Multimodal
$f_{31}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	3	[0,1]	−3.86	Multimodal
$f_{32}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	6	[0,1]	−3.32	Multimodal
$f_{33}(x) = -\sum_{i=1}^{5} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,10]	−10.1532	Multimodal
$f_{34}(x) = -\sum_{i=1}^{7} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,10]	−10.4028	Multimodal
$f_{35}(x) = -\sum_{i=1}^{10} \left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0,10]	−10.5363	Multimodal

Table 2
Parameter settings for related algorithms.

Algorithm	Parameters
PSO	$c_1 = 2, c_2 = 2, w_{Max} = 0.9, w_{Min} = 0.2$
SCA	a = 2 (linearly decreased over iterations)
PFA	~
EO	$a1 = 2, a2 = 1, GP = 0.5, t = 1$(nonlinearly decreased over iterations)
HGSO	$\alpha = \beta = 1, M_1 = 0.1, M_2 = 0.2, l_1 = 5E-03, l_2 = 100, l_3 = 1E-02$
OBSCA	a = 2 (linearly decreased over iterations)
OTSA	$coolingRate = 0.2, maxFolds = 50$
SOGWO	a = 2 (linearly decreased over iterations)

Table 3
Comparison of results on benchmark functions with 30 dimensions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1	Mean	2.0286E-04	3.8052E-11	1.8422E + 01	5.4273E-41	1.6504E-190	8.5196E-11	1.6361E-10	1.2039E-27	**0.0000E + 00**
	Std	2.3355E-04	5.4702E-11	5.4636E + 01	1.0289E-40	0.0000E + 00	2.0804E-10	8.3002E-10	1.4388E-27	**0.0000E + 00**
F2	Mean	2.5926E-03	4.9597E-12	1.8645E + 00	3.0728E-42	2.7349E-184	1.1139E-11	7.0263E-14	1.0761E-28	**0.0000E + 00**
	Std	3.5651E-03	7.3558E-12	3.8688E + 00	3.8452E-42	0.0000E + 00	2.8149E-11	1.3672E-13	1.0373E-28	**0.0000E + 00**
F3	Mean	2.5652E-05	7.2010E-20	4.3913E-02	2.7322E-73	7.5616E-287	1.0225E-16	2.1048E-15	9.0148E-50	**0.0000E + 00**
	Std	4.0385E-05	1.6799E-19	1.4537E-01	7.5844E-73	0.0000E + 00	4.4281E-16	1.1520E-14	4.7608E-49	**0.0000E + 00**
F4	Mean	1.7371E-01	4.5306E-02	1.0485E-01	1.2906E-03	1.6022E-04	5.3889E-03	4.2497E-03	2.1130E-03	**1.5690E-04**
	Std	6.5108E-02	2.1571E-02	9.9095E-02	7.5563E-04	1.6778E-04	4.3718E-03	3.5880E-03	1.1643E-03	**1.2770E-04**
F5	Mean	3.9613E-02	7.0817E-08	2.7885E-02	5.9553E-24	3.9633E-94	2.7653E-07	1.3265E-12	1.1549E-16	**0.0000E + 00**
	Std	3.3550E-02	8.3328E-08	5.2299E-02	7.3780E-24	2.1704E-93	2.7824E-07	4.5835E-12	7.5755E-17	**0.0000E + 00**
F6	Mean	7.9639E + 01	2.7766E + 01	8.9287E + 03	1.7026E-08	1.1510E-155	6.3273E-11	2.9061E + 00	1.2503E-04	**0.0000E + 00**
	Std	3.4339E + 01	2.8134E + 01	5.0826E + 03	6.5601E-08	6.3041E-155	1.2124E-10	1.0428E + 01	2.6537E-04	**0.0000E + 00**
F7	Mean	1.0867E + 00	3.2608E + 00	4.1122E + 01	2.5633E-10	3.6877E-93	6.0117E-07	2.7850E-01	1.2303E-06	**0.0000E + 00**
	Std	2.1888E-01	1.1177E + 00	9.7663E + 00	4.1294E-10	1.9744E-92	4.9656E-07	4.5181E-01	1.2974E-06	**0.0000E + 00**
F8	Mean	1.9943E + 00	8.4486E-10	2.0870E + 02	2.0752E-39	9.9685E-179	6.3155E-12	1.1297E-11	4.9704E-26	**0.0000E + 00**
	Std	3.2849E + 00	1.5331E-09	3.1705E + 02	6.0627E-39	0.0000E + 00	1.0952E-11	3.2848E-11	1.1077E-25	**0.0000E + 00**
F9	Mean	8.3652E + 01	4.8131E + 01	1.0556E + 05	2.5375E + 01	2.8138E + 01	**7.4513E-12**	2.8631E + 01	2.7181E + 01	1.4337E-04
	Std	5.0930E + 01	4.0059E + 01	3.4268E + 05	1.6914E-01	4.6005E-01	**1.0986E-11**	2.5448E-01	6.1633E-01	1.6135E-04
F10	Mean	2.0042E-04	1.6211E-13	2.1132E-02	1.1259E-43	8.7673E-184	1.0515E-21	1.0694E-13	2.8709E-30	**0.0000E + 00**
	Std	2.0982E-04	1.8004E-13	4.0466E-02	1.5292E-43	0.0000E + 00	5.7593E-21	3.7620E-13	3.2778E-30	**0.0000E + 00**
F11	Mean	2.5195E + 00	9.3115E-01	1.0656E + 02	6.6667E-01	6.6667E-01	2.4946E-01	6.7025E-01	6.6668E-01	**2.4753E-01**
	Std	1.4934E + 00	8.7145E-01	2.1204E + 02	8.0650E-10	1.4294E-06	6.2398E-11	2.8983E-03	3.2151E-05	**6.7858E-03**
F12	Mean	1.5238E-04	7.7255E-35	2.9694E-04	2.1667E-131	8.5669E-278	6.9535E-26	3.4057E-24	8.4427E-95	**0.0000E + 00**
	Std	8.2700E-04	3.1866E-34	8.9195E-04	8.8172E-131	0.0000E + 00	3.7783E-25	1.5532E-23	4.5007E-94	**0.0000E + 00**
F13	Mean	8.0082E + 01	1.4764E + 01	3.7474E + 01	1.5272E-05	7.3833E-80	7.1574E-12	8.2399E + 01	1.2251E-07	**0.0000E + 00**
	Std	2.2839E + 01	9.9833E + 00	2.4658E + 01	3.6412E-05	4.0440E-79	2.0535E-11	4.1045E + 01	1.8118E-07	**0.0000E + 00**
F14	Mean	−4.9391E + 03	−8.5791E + 03	−3.7988E + 03	−8.9019E + 03	−6.4604E + 03	−1.0082E + 04	−3.8194E + 03	−6.0199E + 03	**−1.2569E + 04**
	Std	1.3979E + 03	5.5647E + 02	2.5732E + 02	4.7814E + 02	8.8703E + 03	1.6561E + 03	2.1205E + 02	1.3157E + 03	**2.1091E-01**
F15	Mean	5.9603E + 01	8.7861E + 01	3.2283E + 01	**0.0000E + 00**	**0.0000E + 00**	2.8535E-12	1.5939E-07	4.2560E + 00	**0.0000E + 00**
	Std	1.3003E + 01	1.9919E + 01	3.1021E + 01	**0.0000E + 00**	**0.0000E + 00**	4.4975E-12	5.6025E-07	5.6153E + 00	**0.0000E + 00**
F16	Mean	2.5678E-01	3.6626E + 00	1.2900E + 01	8.8226E-15	**8.8818E-16**	1.0274E-07	1.6402E + 00	1.0321E-13	**8.8818E-16**
	Std	5.3714E-01	5.6495E + 00	9.4019E + 00	2.2242E-15	**0.0000E + 00**	9.1965E-08	2.0687E + 00	1.4192E-14	**0.0000E + 00**
F17	Mean	6.0130E-03	1.3115E-02	1.0885E + 00	**0.0000E + 00**	**0.0000E + 00**	3.1353E-07	1.2274E-04	5.5644E-03	**0.0000E + 00**
	Std	7.4457E-03	1.2390E-02	1.1260E + 00	**0.0000E + 00**	**0.0000E + 00**	7.2712E-07	5.9859E-04	7.9129E-03	**0.0000E + 00**
F18	Mean	4.8438E-03	4.6288E-02	2.2582E + 05	1.2991E-02	2.7053E + 00	**3.9312E-16**	2.5454E + 00	7.4889E-01	2.3559E-06
	Std	8.9716E-03	1.2729E-01	6.7884E + 05	2.5922E-02	2.4018E-01	**1.3158E-15**	1.4673E-01	2.3567E-01	3.5363E-06
F19	Mean	2.9715E-02	1.7546E + 00	6.3515E-01	1.5510E-08	3.1504E-99	3.8583E-08	4.8976E-07	4.3364E-04	**0.0000E + 00**
	Std	3.8058E-02	8.2110E-01	9.8804E-01	8.4951E-08	1.7255E-98	4.0917E-08	1.5364E-06	6.3879E-04	**0.0000E + 00**
F20	Mean	2.2171E-02	4.7529E-02	2.4475E-01	3.1266E-03	1.2527E-03	3.3589E-06	3.6902E-01	3.7476E-03	**0.0000E + 00**
	Std	5.5406E-03	1.4591E-02	1.0647E-01	1.0993E-10	1.5605E-03	1.1979E-05	1.1064E-01	1.2595E-03	**0.0000E + 00**
F21	Mean	5.1664E-01	1.1829E + 00	2.7220E-03	**0.0000E + 00**	**0.0000E + 00**	5.6917E-14	5.4771E-16	**0.0000E + 00**	**0.0000E + 00**
	Std	2.4427E-01	3.6448E-01	4.5441E-03	**0.0000E + 00**	**0.0000E + 00**	4.7659E-14	2.9999E-15	**0.0000E + 00**	**0.0000E + 00**
F22	Mean	1.5839E + 01	3.4860E + 01	2.7539E + 00	2.8539E-11	1.2914E-48	4.4017E-03	3.2649E-06	5.0644E-07	**0.0000E + 00**
	Std	5.5871E + 00	8.5075E + 00	2.3568E + 00	1.6450E-11	5.0742E-48	2.3469E-03	5.9596E-06	3.5915E-07	**0.0000E + 00**
F23	Mean	2.8930E + 00	8.2722E + 00	2.3922E + 00	**0.0000E + 00**	**0.0000E + 00**	6.9264E-12	1.1807E-12	0.0000E + 00	**0.0000E + 00**
	Std	1.6555E + 00	2.4295E + 00	3.1241E + 00	**0.0000E + 00**	**0.0000E + 00**	1.0946E-11	3.6613E-12	0.0000E + 00	**0.0000E + 00**
F24	Mean	8.5264E + 00	8.4205E-08	2.8902E + 03	6.2344E-37	2.3113E-169	1.7570E-08	4.8192E-10	2.0419E-23	**0.0000E + 00**
	Std	1.2995E + 01	9.5750E-08	9.2525E + 03	2.5505E-36	0.0000E + 00	3.2225E-08	2.1811E-09	3.3268E-23	**0.0000E + 00**
F25	Mean	4.4322E-01	7.6987E-01	1.2064E + 00	9.9873E-02	9.6931E-02	1.6208E-07	1.3728E-01	1.9361E-01	**0.0000E + 00**
	Std	6.7883E-02	2.2614E-01	5.3129E-01	2.8695E-13	1.8329E-02	1.5240E-07	5.0149E-02	2.7380E-02	**0.0000E + 00**

Table 4
Comparison of results on benchmark functions with 100 dimensions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1	Mean	2.0978E + 01	1.1635E + 04	2.7320E + 00	3.7703E-29	2.1206E-170	4.1323E-11	2.5258E-02	3.2397E-12	**0.0000E + 00**
	Std	7.4099E + 00	7.8659E + 03	1.6391E + 00	4.9933E-29	0.0000E + 00	8.0671E-11	5.9119E-02	3.1222E-12	**0.0000E + 00**
F2	Mean	9.3758E + 02	4.0865E + 03	1.2119E + 00	1.4491E-29	2.4558E-180	4.2370E-12	2.2426E-02	4.6976E-13	**0.0000E + 00**
	Std	3.1363E + 02	2.3261E + 03	5.9957E-01	2.0200E-29	0.0000E + 00	7.5169E-12	4.9114E-02	3.2561E-13	**0.0000E + 00**
F3	Mean	1.5239E + 03	1.3272E + 02	2.0015E-04	1.0885E-50	**0.0000E + 00**	3.8351E-17	9.4993E-04	1.5296E-25	**0.0000E + 00**
	Std	2.0678E + 02	7.3030E + 01	2.3450E-04	2.8007E-50	**0.0000E + 00**	1.9755E-16	2.6982E-03	2.0154E-25	**0.0000E + 00**
F4	Mean	1.5163E + 03	1.3778E + 02	5.5031E-01	2.5360E-03	1.8121E-04	4.9235E-03	3.6168E-02	6.2651E-03	**1.6719E-04**
	Std	1.9149E + 02	5.6072E + 01	1.2843E-01	1.2941E-03	1.4762E-04	3.2921E-03	3.5304E-02	2.0296E-03	**1.3033E-04**
F5	Mean	3.9191E + 01	7.9336E + 00	3.0513E-01	2.1171E-17	1.7752E-85	5.2705E-07	7.3635E-06	4.6243E-08	**0.0000E + 00**
	Std	1.3075E + 01	6.0584E + 00	1.0243E-01	1.3134E-17	9.7229E-85	5.6925E-07	1.7285E-05	1.6914E-08	**0.0000E + 00**
F6	Mean	1.6380E + 04	2.3514E + 05	4.4357E + 04	9.9345E + 01	5.1531E-150	2.5660E-09	3.5013E + 04	1.0839E + 03	**0.0000E + 00**
	Std	3.8656E + 03	5.4524E + 04	1.2201E + 04	5.0552E + 02	2.8225E-149	3.5098E-09	2.1143E + 04	9.1858E + 02	**0.0000E + 00**
F7	Mean	1.2359E + 01	9.0429E + 01	4.3887E + 01	3.5032E-02	1.2297E-89	5.0040E-07	8.1765E + 01	1.0873E + 00	**0.0000E + 00**
	Std	1.9466E + 00	2.7344E + 00	1.8049E + 01	1.7666E-01	6.6784E-89	4.3751E-07	6.4488E + 00	8.4400E-01	**0.0000E + 00**
F8	Mean	1.6017E + 05	2.3879E + 05	6.6100E + 01	9.5700E-28	7.1869E-184	1.1923E-11	7.3405E-01	4.3461E-11	**0.0000E + 00**
	Std	4.6062E + 04	1.6626E + 05	2.9090E + 01	1.3851E-27	0.0000E + 00	2.3461E-11	1.4257E + 00	3.1842E-11	**0.0000E + 00**
F9	Mean	1.6177E + 04	1.2819E + 08	1.0142E + 03	9.6538E + 01	9.8686E + 01	**1.0985E-11**	2.7540E + 03	9.7687E + 01	2.8761E-04
	Std	6.2401E + 03	4.6457E + 07	3.1025E + 02	1.0357E + 00	2.3639E-01	**1.4243E-11**	5.0070E + 03	8.3130E-01	2.4627E-04
F10	Mean	2.1207E + 01	2.1446E + 01	1.1361E-02	2.8235E-31	4.9994E-177	5.7056E-21	1.0031E-04	4.6153E-15	**0.0000E + 00**
	Std	8.5492E + 00	1.4668E + 01	6.0378E-03	5.5882E-31	0.0000E + 00	2.6401E-20	3.6337E-04	3.0897E-15	**0.0000E + 00**
F11	Mean	1.9328E + 04	2.0785E + 06	5.3448E + 01	6.6667E-01	6.6667E-01	2.4995E-01	1.2082E + 02	6.8897E-01	**2.4863E-01**
	Std	6.5306E + 03	1.2022E + 06	1.8384E + 01	3.8498E-08	2.4973E-06	1.5186E-11	3.0940E + 02	8.4548E-02	**7.6455E-03**
F12	Mean	1.3403E + 00	2.0517E-01	3.5848E-18	9.0442E-129	3.6186E-287	1.1119E-26	2.4023E-04	4.1665E-46	**0.0000E + 00**
	Std	4.0875E-01	1.5447E-01	1.0822E-17	4.6662E-128	0.0000E + 00	6.0888E-26	3.7266E-04	2.2821E-45	**0.0000E + 00**
F13	Mean	4.1536E + 03	6.0215E + 02	1.8400E + 03	1.9530E + 02	1.4852E-21	2.2929E-10	9.5163E + 02	1.0966E + 02	**0.0000E + 00**
	Std	1.6278E + 03	1.5193E + 02	2.5726E + 02	1.1730E + 02	6.0225E-21	4.6427E-10	3.3635E + 02	4.9127E + 01	**0.0000E + 00**
F14	Mean	−1.0532E + 04	−6.7604E + 03	−2.3622E + 04	−2.5690E + 04	−4.6588E + 03	−3.5582E + 04	−7.0871E + 03	−1.6090E + 04	**−4.1898E + 04**
	Std	3.4695E + 03	5.0289E + 02	1.5333E + 03	1.5717E + 03	9.2547E + 02	6.0098E + 03	5.3787E + 02	2.9651E + 03	**6.6519E-01**
F15	Mean	5.9736E + 02	2.5420E + 02	4.8374E + 02	**0.0000E + 00**	**0.0000E + 00**	7.0107E-13	1.7696E-01	8.8015E + 00	**0.0000E + 00**
	Std	7.6429E + 01	1.1475E + 02	6.7931E + 01	**0.0000E + 00**	**0.0000E + 00**	2.0888E-12	6.5187E-01	9.1119E + 00	**0.0000E + 00**
F16	Mean	3.8121E + 00	1.9911E + 01	5.6915E + 00	3.4284E-14	1.0066E-15	1.6462E-07	1.1020E + 01	1.3244E-07	**8.8818E-16**
	Std	3.4966E-01	2.5686E + 00	5.8858E + 00	5.7210E-15	6.4863E-16	1.5135E-07	6.4251E + 00	5.5270E-08	**0.0000E + 00**
F17	Mean	3.9040E-01	1.0479E + 02	7.1187E-01	**0.0000E + 00**	**0.0000E + 00**	1.3350E-08	5.6459E-02	8.0690E-03	**0.0000E + 00**
	Std	8.6527E-02	8.1900E + 01	2.0366E-01	**0.0000E + 00**	**0.0000E + 00**	2.7218E-08	1.3105E-01	1.2655E-02	**0.0000E + 00**
F18	Mean	6.4619E + 01	5.7502E + 08	1.2858E + 02	5.8396E + 00	9.9461E + 00	**7.7289E-15**	2.8074E + 01	6.9189E + 00	3.6309E-06
	Std	1.8666E + 01	2.8110E + 08	2.1734E + 01	1.0362E + 00	4.1757E-02	**1.6615E-14**	1.4307E + 01	4.4227E-01	4.3391E-06
F19	Mean	2.0967E + 01	2.7391E + 01	4.1042E + 01	3.8967E-18	1.1694E-94	6.5451E-08	2.7177E-03	2.9241E-03	**0.0000E + 00**
	Std	4.0505E + 00	1.5150E + 01	5.0870E + 00	3.3860E-18	6.2960E-94	4.9528E-08	7.1162E-03	2.4272E-03	**0.0000E + 00**
F20	Mean	2.8255E-01	4.9929E-01	4.9911E-01	4.0559E-03	2.1946E-03	2.0698E-06	4.9959E-01	1.1013E-02	**0.0000E + 00**
	Std	2.8762E-02	9.2103E-04	3.8167E-04	1.4438E-03	1.4613E-03	1.0103E-05	3.1961E-04	2.0622E-03	**0.0000E + 00**
F21	Mean	2.9401E + 01	1.9128E + 00	5.1866E + 00	**0.0000E + 00**	**0.0000E + 00**	7.2712E-14	3.3206E-06	3.2863E-14	**0.0000E + 00**
	Std	5.3633E + 00	1.3093E + 00	1.0546E + 00	**0.0000E + 00**	**0.0000E + 00**	8.8678E-14	9.2528E-06	9.3735E-15	**0.0000E + 00**
F22	Mean	1.9687E + 02	4.1686E + 01	2.1485E + 02	4.5608E-08	2.3867E-42	1.9619E-02	7.1344E-03	2.5059E-01	**0.0000E + 00**
	Std	1.2995E + 01	2.5264E + 01	2.1727E + 01	1.7738E-08	1.3068E-41	1.0442E-02	9.5654E-03	8.7713E-02	**0.0000E + 00**
F23	Mean	1.2309E + 02	8.6364E + 02	3.9661E + 01	**0.0000E + 00**	**0.0000E + 00**	1.0767E-11	1.9866E-02	1.2997E-12	**0.0000E + 00**
	Std	1.8637E + 01	5.6524E + 02	7.0166E + 00	**0.0000E + 00**	**0.0000E + 00**	2.2437E-11	9.4581E-02	1.1779E-12	**0.0000E + 00**
F24	Mean	6.8778E + 05	1.6577E + 07	6.8987E + 03	2.1632E-25	2.4023E-162	7.3356E-08	6.0035E + 00	6.4660E-09	**0.0000E + 00**
	Std	3.4786E + 05	2.2122E + 07	3.5878E + 03	3.9494E-25	1.3150E-161	1.3645E-07	9.7240E + 00	5.7331E-09	**0.0000E + 00**
F25	Mean	1.8975E + 00	1.2028E + 01	4.8724E + 00	1.8654E-01	1.0021E-01	1.3210E-07	6.4229E-01	3.7321E-01	**0.0000E + 00**
	Std	1.0533E-01	3.4724E + 00	6.4907E-01	3.4575E-02	4.9744E-04	1.0887E-07	2.2470E-01	6.9149E-02	**0.0000E + 00**

Table 5
Comparison of results on benchmark functions with 300 dimensions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1	Mean	1.2723E + 03	9.5369E + 04	4.6363E + 03	3.1190E-24	2.8854E-161	2.3441E-13	1.3223E + 02	1.8226E-05	**0.0000E + 00**
	Std	1.2210E + 02	4.0227E + 04	8.9755E + 02	5.6620E-24	1.5805E-160	3.5840E-13	1.5542E + 02	6.3281E-06	**0.0000E + 00**
F2	Mean	1.1687E + 05	1.3106E + 05	5.9040E + 03	4.2510E-24	7.5423E-154	4.7802E-14	1.1619E + 02	1.5203E-05	**0.0000E + 00**
	Std	1.1122E + 04	5.1754E + 04	1.2165E + 03	6.3936E-24	4.1264E-153	8.1468E-14	2.4988E + 02	7.1499E-06	**0.0000E + 00**
F3	Mean	1.9458E + 04	4.4489E + 03	8.9093E + 00	6.2912E-41	0.0000E + 00	3.0997E-23	1.6868E + 01	1.2584E-13	**0.0000E + 00**
	Std	1.1409E + 03	1.0393E + 03	3.3120E + 00	2.1217E-40	0.0000E + 00	6.4425E-23	1.9499E + 01	1.1378E-13	**0.0000E + 00**
F4	Mean	1.8811E + 04	4.3194E + 03	1.2925E + 01	3.7470E-03	1.9963E-04	5.3867E-03	2.0321E + 01	2.4507E-02	**1.7295E-04**
	Std	1.3035E + 03	1.2856E + 03	3.1590E + 00	1.7306E-03	2.0288E-04	4.8432E-03	3.7162E + 01	7.0678E-03	**1.6836E-04**
F5	Mean	6.7436E + 02	5.4019E + 01	5.8381E + 01	1.0053E-14	1.2626E-80	1.2927E-07	2.3045E-03	5.8553E-04	**0.0000E + 00**
	Std	8.7146E + 01	3.1424E + 01	6.7916E + 00	5.7169E-15	6.9153E-80	1.4513E-07	3.0703E-03	1.4027E-04	**0.0000E + 00**
F6	Mean	1.9417E + 05	2.2487E + 06	6.9337E + 05	4.9262E + 03	8.2151E-148	9.3026E-10	4.2941E + 05	1.0755E + 05	**0.0000E + 00**
	Std	4.4085E + 04	4.4343E + 05	1.1702E + 05	7.8299E + 03	4.1864E-147	2.5795E-09	1.2591E + 05	3.7990E + 04	**0.0000E + 00**
F7	Mean	2.3715E + 01	9.8125E + 01	7.3263E + 01	6.0464E + 01	6.2461E-87	5.9674E-08	9.7836E + 01	4.6601E + 01	**0.0000E + 00**
	Std	1.7947E + 00	4.7884E-01	1.2842E + 01	2.2007E + 01	3.3807E-86	5.4650E-08	8.7642E-01	6.7620E + 00	**0.0000E + 00**
F8	Mean	6.0126E + 06	2.3561E + 06	1.1206E + 05	1.3371E-22	6.3827E-158	3.8648E-13	4.1805E + 03	3.2909E-04	**0.0000E + 00**
	Std	6.5704E + 05	7.9490E + 05	2.0538E + 04	1.9605E-22	3.2307E-157	9.0307E-13	6.5192E + 03	1.6785E-04	**0.0000E + 00**
F9	Mean	3.5758E + 06	9.2350E + 08	2.4697E + 06	2.9735E + 02	2.9874E + 02	**1.3917E-13**	3.8799E + 06	2.9774E + 02	2.9183E-03
	Std	7.0670E + 05	2.4533E + 08	9.3690E + 05	5.7984E-01	8.9523E-02	**2.1075E-13**	4.0275E + 06	4.2687E-01	4.4140E-03
F10	Mean	7.0756E + 02	1.5727E + 02	1.6338E + 01	1.9812E-26	8.4097E-179	2.1870E-130	1.8108E-01	2.8091E-08	**0.0000E + 00**
	Std	7.0010E + 01	7.9525E + 01	3.6299E + 00	2.8808E-26	0.0000E + 00	1.1979E-129	2.9341E-01	1.1600E-08	**0.0000E + 00**
F11	Mean	9.5922E + 06	6.5970E + 07	1.4377E + 05	6.6667E-01	6.6667E-01	**2.4999E-01**	3.7397E + 05	8.0222E-01	3.5360E-01
	Std	1.0072E + 06	1.8165E + 07	6.4188E + 04	4.2311E-07	2.7035E-06	**1.2715E-13**	6.3102E + 05	1.6452E-01	1.9293E-01
F12	Mean	2.0875E + 00	1.1216E + 00	3.7463E-11	1.5289E-86	2.7916E-284	5.5465E-103	9.6245E-02	4.9971E-08	**0.0000E + 00**
	Std	3.6433E-01	3.6286E-01	1.1955E-10	8.3740E-86	0.0000E + 00	3.0380E-102	9.7129E-02	1.9412E-07	**0.0000E + 00**
F13	Mean	3.8781E + 04	1.6875E + 04	8.1966E + 03	1.6926E + 03	5.1525E-03	1.0137E-10	6.6279E + 04	2.0633E + 03	**0.0000E + 00**
	Std	7.9504E + 03	5.8604E + 04	5.9316E + 02	4.9937E + 02	1.8226E-02	1.4709E-10	1.8181E + 05	3.4127E + 02	**0.0000E + 00**
F14	Mean	−2.0582E + 04	−1.1797E + 04	−6.0057E + 04	−5.5471E + 04	−7.3907E + 03	−1.0674E + 05	−1.2287E + 04	−3.8581E + 04	**−1.2569E + 05**
	Std	6.4072E + 03	6.3007E + 02	3.3160E + 03	3.5213E + 03	1.4917E + 03	1.8029E + 04	8.5340E + 02	6.4464E + 03	**1.2025E + 00**
F15	Mean	3.3218E + 03	7.6451E + 02	2.2160E + 03	1.5158E-14	**0.0000E + 00**	**0.0000E + 00**	1.3283E-01	4.2226E + 01	**0.0000E + 00**
	Std	2.6559E + 02	2.9487E + 02	1.7083E + 02	8.3025E-14	**0.0000E + 00**	**0.0000E + 00**	3.0203E-01	2.0573E + 01	**0.0000E + 00**
F16	Mean	8.7367E + 00	1.9577E + 01	1.3811E + 01	1.8587E-13	1.0066E-15	9.3312E-09	1.8135E + 01	2.3455E-04	**8.8818E-16**
	Std	2.7914E-01	3.1045E + 00	4.2422E + 00	7.3172E-14	6.4863E-16	7.0244E-09	4.8696E + 00	5.2189E-05	**0.0000E + 00**
F17	Mean	1.7436E + 01	7.4180E + 02	4.3169E + 01	3.7007E-18	**0.0000E + 00**	2.3067E-11	2.1349E + 00	6.6102E-03	**0.0000E + 00**
	Std	8.1440E + 00	3.3493E + 02	9.6642E + 00	2.0270E-17	**0.0000E + 00**	9.8411E-11	1.9321E + 00	2.0223E-02	**0.0000E + 00**
F18	Mean	1.8303E + 05	5.2086E + 09	1.2492E + 06	2.8797E + 01	2.9957E + 01	**5.8236E-14**	4.0084E + 07	2.7343E + 01	8.6395E-06
	Std	6.0283E + 04	1.1862E + 09	8.4311E + 05	3.8263E-01	3.2103E-02	**2.4949E-13**	6.0468E + 07	7.6545E-01	1.2144E-05
F19	Mean	2.5486E + 02	8.8070E + 01	2.5105E + 02	1.5754E-15	9.0644E-88	1.7311E-08	7.7868E-02	2.7005E-02	**0.0000E + 00**
	Std	1.7660E + 01	3.8439E + 01	1.8147E + 01	7.6644E-16	4.9380E-87	1.5230E-08	1.5491E-01	6.4301E-03	**0.0000E + 00**
F20	Mean	4.9895E-01	4.9998E-01	5.0000E-01	6.1190E-03	2.8399E-03	9.6686E-11	4.9998E-01	5.0198E-02	**0.0000E + 00**
	Std	3.1179E-04	1.1677E-05	1.4344E-06	1.2770E-03	9.6653E-04	1.4588E-10	1.5603E-05	8.5225E-03	**0.0000E + 00**
F21	Mean	1.1608E + 02	1.9599E + 01	2.1678E + 01	**0.0000E + 00**	**0.0000E + 00**	4.7204E-13	8.6459E-04	9.3777E-09	**0.0000E + 00**
	Std	3.8280E + 00	7.2427E + 00	1.8800E + 00	**0.0000E + 00**	**0.0000E + 00**	7.2584E-13	1.6322E-03	3.4402E-09	**0.0000E + 00**
F22	Mean	8.4469E + 02	1.8385E + 02	8.1554E + 02	1.2839E-06	7.3919E-43	1.1975E-02	1.0823E-01	6.5982E + 00	**0.0000E + 00**
	Std	2.9596E + 01	7.5477E + 01	3.8182E + 01	5.1441E-07	3.9903E-42	7.5657E-03	1.1506E-01	9.3654E-01	**0.0000E + 00**
F23	Mean	3.1207E + 03	6.3425E + 03	5.3677E + 02	**0.0000E + 00**	**0.0000E + 00**	3.5409E-14	2.0350E + 01	9.2410E-06	**0.0000E + 00**
	Std	2.5089E + 02	2.7738E + 03	8.3069E + 01	**0.0000E + 00**	**0.0000E + 00**	6.1602E-14	1.5385E + 01	4.0389E-06	**0.0000E + 00**
F24	Mean	4.4741E + 07	1.2185E + 09	1.6682E + 07	1.5551E-20	8.0685E-170	7.7648E-10	6.7068E + 04	2.9276E-02	**0.0000E + 00**
	Std	1.0470E + 07	8.3805E + 08	3.0670E + 06	1.7570E-20	0.0000E + 00	1.5324E-09	1.1426E + 05	1.0688E-02	**0.0000E + 00**
F25	Mean	8.6136E + 00	3.2940E + 01	2.3954E + 01	1.9987E-01	1.0040E-01	1.2965E-08	1.8728E + 00	7.5666E-01	**0.0000E + 00**
	Std	4.1074E-01	6.7044E + 00	1.4526E + 00	2.6261E-02	7.4349E-04	1.9622E-08	1.2090E + 00	1.0044E-01	**0.0000E + 00**

Table 6
Comparison of results on benchmark functions with 500 dimensions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1	Mean	5.8611E + 03	1.8931E + 05	3.2792E + 04	1.0153E-22	3.3413E-169	2.3949E-13	9.7312E + 02	2.2315E-03	**0.0000E + 00**
	Std	4.7854E + 02	6.7653E + 04	5.5953E + 03	1.0485E-22	0.0000E + 00	4.5053E-13	2.3931E + 03	8.4677E-04	**0.0000E + 00**
F2	Mean	6.6101E + 05	4.7504E + 05	6.9530E + 04	3.6162E-22	3.1950E-149	9.7398E-14	1.0067E + 03	3.4222E-03	**0.0000E + 00**
	Std	4.2922E + 04	1.3622E + 05	1.3511E + 04	5.3755E-22	1.7500E-148	1.5877E-13	1.9804E + 03	1.3205E-03	**0.0000E + 00**
F3	Mean	5.7590E + 04	1.5254E + 04	2.1661E + 02	1.5785E-36	9.2219E-308	1.6559E-22	2.2111E + 02	6.3109E-10	**0.0000E + 00**
	Std	2.1288E + 03	3.2256E + 03	5.5726E + 01	8.4348E-36	0.0000E + 00	7.2590E-22	1.8755E + 02	4.3011E-10	**0.0000E + 00**
F4	Mean	5.8176E + 04	1.4617E + 04	1.8750E + 02	3.8182E-03	2.1205E-04	5.6039E-03	1.5620E + 02	4.6184E-02	**2.0090E-04**
	Std	1.9922E + 03	3.4608E + 03	6.4919E + 01	1.3010E-03	1.5089E-04	4.5658E-03	1.7470E + 02	1.2663E-02	**1.4593E-04**
F5	Mean	2.3187E + 46	9.3877E + 01	2.6825E + 02	7.0663E-14	8.6729E-92	1.7278E-07	1.5231E-01	1.1345E-02	**0.0000E + 00**
	Std	1.2696E + 47	5.6754E + 01	4.5539E + 01	3.7783E-14	4.7456E-91	1.2401E-07	7.4268E-01	2.3092E-03	**0.0000E + 00**
F6	Mean	5.6705E + 05	7.2657E + 06	2.1101E + 06	3.2735E + 04	2.4292E-130	1.8935E-09	1.5831E + 06	3.4094E + 05	**0.0000E + 00**
	Std	1.5021E + 05	1.3994E + 06	2.9236E + 05	5.4739E + 04	1.3304E-129	2.9757E-09	6.3082E + 05	7.2821E + 04	**0.0000E + 00**
F7	Mean	2.7823E + 01	9.8956E + 01	7.8412E + 01	7.1329E + 01	1.4393E-80	5.2468E-08	9.9109E + 01	6.5172E + 01	**0.0000E + 00**
	Std	1.4417E + 00	2.4591E-01	1.0084E + 01	1.8499E + 01	5.4776E-80	6.1589E-08	2.6717E-01	5.7290E + 00	**0.0000E + 00**
F8	Mean	1.7499E + 07	5.2511E + 06	8.1352E + 05	4.6120E-21	1.4474E-175	1.8459E-13	3.4227E + 04	3.5017E-02	**0.0000E + 00**
	Std	8.3477E + 05	2.0732E + 06	1.2550E + 05	7.6081E-21	0.0000E + 00	3.2960E-13	4.5827E + 04	1.2329E-02	**0.0000E + 00**
F9	Mean	3.0651E + 07	1.9686E + 09	2.8121E + 07	4.9759E + 02	4.9870E + 02	**1.9221E-13**	2.8300E + 07	4.9808E + 02	3.3648E-03
	Std	3.4255E + 06	4.1647E + 08	6.5275E + 06	3.1390E-01	1.4966E-01	**3.2485E-13**	2.9517E + 07	3.5973E-01	3.2368E-03
F10	Mean	1.9005E + 03	3.1632E + 02	1.0683E + 02	3.4530E-25	5.6235E-168	9.7088E-130	1.4460E + 00	4.5733E-06	**0.0000E + 00**
	Std	1.4575E + 02	1.5524E + 02	2.0137E + 01	4.6936E-25	0.0000E + 00	5.3177E-129	2.0313E + 00	1.5940E-06	**0.0000E + 00**
F11	Mean	7.3527E + 07	2.1110E + 08	3.1273E + 06	6.6667E-01	6.6667E-01	**2.5000E-01**	4.0364E + 06	9.3312E-01	5.0355E-01
	Std	5.7585E + 06	5.5499E + 07	1.2390E + 06	1.0176E-06	3.6882E-06	**8.7428E-14**	5.4726E + 06	1.6491E-01	2.0583E-01
F12	Mean	2.5122E + 00	1.9409E + 00	3.2437E-09	4.9902E-11	2.0828E-288	1.0951E-179	2.8943E-01	1.3389E-04	**0.0000E + 00**
	Std	5.3421E-01	4.6928E-01	1.3523E-08	2.7275E-10	0.0000E + 00	0.0000E + 00	2.0242E-01	5.3202E-04	**0.0000E + 00**
F13	Mean	9.4558E + 04	4.3414E + 06	1.4331E + 04	2.8342E + 03	4.1259E-01	8.9544E-10	1.9612E + 08	3.9091E + 03	**0.0000E + 00**
	Std	2.3512E + 04	1.1194E + 07	6.8733E + 02	6.1302E + 02	8.8321E-01	1.2292E-09	4.9188E + 08	4.5106E + 02	**0.0000E + 00**
F14	Mean	−2.0956E + 04	−1.5147E + 04	−8.4758E + 04	−7.6495E + 04	−1.0135E + 04	−1.8186E + 05	−1.6247E + 04	−5.3734E + 04	**−2.0949E + 05**
	Std	8.1974E + 03	1.0422E + 03	3.4312E + 03	5.3535E + 03	2.1251E + 03	3.0049E + 04	1.2218E + 03	1.3899E + 04	**2.7221E + 00**
F15	Mean	6.5002E + 03	1.2422E + 03	4.3885E + 03	1.2127E-13	**0.0000E + 00**	**0.0000E + 00**	5.3860E + 00	7.1376E + 01	**0.0000E + 00**
	Std	4.7197E + 02	4.6479E + 02	2.4932E + 02	3.1445E-13	**0.0000E + 00**	**0.0000E + 00**	1.4866E + 01	2.5814E + 01	**0.0000E + 00**
F16	Mean	1.1855E + 01	1.9144E + 01	1.8614E + 01	5.2207E-13	1.0066E-15	1.2008E-08	1.6024E + 01	2.0348E-03	**8.8818E-16**
	Std	3.2137E-01	3.7992E + 00	2.0526E + 00	2.5864E-13	6.4863E-16	1.0498E-08	7.6697E + 00	3.7117E-04	**0.0000E + 00**
F17	Mean	7.6630E + 01	1.7253E + 03	3.0619E + 02	9.9920E-17	**0.0000E + 00**	3.6386E-12	8.3978E + 00	2.6593E-02	**0.0000E + 00**
	Std	1.0751E + 01	9.4240E + 02	4.4242E + 01	3.3876E-17	**0.0000E + 00**	8.0416E-12	1.2045E + 01	5.1693E-02	**0.0000E + 00**
F18	Mean	4.4482E + 06	1.0136E + 10	4.2022E + 07	4.9088E + 01	4.9961E + 01	**5.0825E-11**	1.6062E + 08	5.1341E + 01	1.7525E-05
	Std	1.0534E + 06	1.8055E + 09	1.3285E + 07	2.8476E-01	3.0516E-02	**7.3200E-11**	1.8960E + 08	1.4696E + 00	2.8588E-05
F19	Mean	5.8269E + 02	1.5010E + 02	5.2570E + 02	1.1367E-14	1.5250E-92	2.0232E-08	1.2772E-01	7.3892E-02	**0.0000E + 00**
	Std	3.2264E + 01	4.9871E + 01	3.8159E + 01	6.0745E-15	5.8691E-92	2.2673E-08	1.3291E-01	1.8825E-02	**0.0000E + 00**
F20	Mean	4.9987E-01	5.0000E-01	5.0000E-01	7.3485E-03	3.1662E-03	2.5626E-10	4.9999E-01	1.5596E-01	**0.0000E + 00**
	Std	2.4527E-05	2.3419E-06	1.2824E-07	1.7088E-03	5.4002E-05	3.6164E-10	7.8284E-06	2.5583E-02	**0.0000E + 00**
F21	Mean	2.0161E + 02	3.8910E + 01	4.1768E + 01	**0.0000E + 00**	**0.0000E + 00**	6.4304E-13	7.1039E-03	1.2279E-06	**0.0000E + 00**
	Std	3.7337E + 00	1.1010E + 01	3.5274E + 00	**0.0000E + 00**	**0.0000E + 00**	1.9142E-12	1.1704E-02	3.2781E-07	**0.0000E + 00**
F22	Mean	1.5297E + 03	2.9940E + 02	1.4371E + 03	4.0799E-06	4.9207E-47	1.8090E-02	3.2862E-01	2.1994E + 01	**0.0000E + 00**
	Std	2.5472E + 01	1.1016E + 02	4.1708E + 01	1.0077E-06	2.4670E-46	1.1485E-02	3.6546E-01	2.5946E + 00	**0.0000E + 00**
F23	Mean	1.1574E + 04	1.2701E + 04	2.5465E + 03	**0.0000E + 00**	**0.0000E + 00**	7.3867E-14	7.5702E + 01	1.1249E-03	**0.0000E + 00**
	Std	5.8651E + 02	4.2546E + 03	4.2437E + 02	**0.0000E + 00**	**0.0000E + 00**	1.3281E-13	6.3167E + 01	3.5278E-04	**0.0000E + 00**
F24	Mean	1.7199E + 08	3.8192E + 09	1.6763E + 08	5.0697E-19	1.5098E-172	1.5445E-09	9.3473E + 05	3.8709E + 00	**0.0000E + 00**
	Std	2.8984E + 07	2.2046E + 09	3.2692E + 07	4.2305E-19	0.0000E + 00	2.1058E-09	1.1598E + 06	1.4212E + 00	**0.0000E + 00**
F25	Mean	1.6527E + 01	4.5760E + 01	4.2495E + 01	2.1987E-01	1.0008E-01	1.4767E-08	3.3051E + 00	1.1665E + 00	**0.0000E + 00**
	Std	5.7710E-01	1.0072E + 01	2.1067E + 00	4.0684E-02	2.4389E-04	9.2525E-09	1.4457E + 00	1.2685E-01	**0.0000E + 00**

Table 7
Comparison of results on benchmark functions with 1000 dimensions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1	Mean	4.1743E + 04	4.7395E + 05	2.3919E + 05	3.9680E-21	2.2572E-172	2.1809E-13	6.4184E + 03	3.5464E-01	**0.0000E + 00**
	Std	1.8642E + 03	1.2612E + 05	2.5716E + 04	2.5318E-21	0.0000E + 00	3.7679E-13	8.8627E + 03	1.0249E-01	**0.0000E + 00**
F2	Mean	4.8910E + 06	2.2900E + 06	9.9793E + 05	2.9234E-20	4.9349E-171	3.6860E-13	1.5994E + 04	1.0399E + 00	**0.0000E + 00**
	Std	2.2103E + 05	8.1391E + 05	1.3453E + 05	2.2614E-20	0.0000E + 00	1.0098E-12	1.8267E + 04	3.0132E-01	**0.0000E + 00**
F3	Mean	2.4507E + 05	6.6282E + 04	4.9813E + 03	9.7780E-34	1.45216483277000e-311	1.0524E-23	2.7172E + 03	4.7776E-06	**0.0000E + 00**
	Std	5.4651E + 03	1.5778E + 04	9.9815E + 02	2.8441E-33	0.0000E + 00	2.4287E-23	1.8852E + 03	3.4815E-06	**0.0000E + 00**
F4	Mean	2.4198E + 05	6.7423E + 04	5.0567E + 03	5.4355E-03	2.4359E-04	6.3949E-03	3.1508E + 03	1.4513E-01	**1.9449E-04**
	Std	5.4662E + 03	1.4623E + 04	1.2366E + 03	2.3660E-03	1.3679E-04	5.3977E-03	2.0069E + 03	2.9142E-02	**1.3456E-04**
F5	Mean	1.4112E + 03	NaN	NaN	4.0447E-13	NaN	4.2525E-07	NaN	6.7626E-01	**0.0000E + 00**
	Std	7.4167E + 01	NaN	NaN	1.8357E-13	NaN	3.9547E-07	NaN	4.3486E-01	**0.0000E + 00**
F6	Mean	2.3910E + 06	2.7647E + 07	8.1313E + 06	2.9974E + 05	7.7031E-142	1.5054E-08	7.4813E + 06	1.6039E + 06	**0.0000E + 00**
	Std	6.0468E + 05	6.4846E + 06	1.2135E + 06	3.3489E + 05	4.2191E-141	1.7230E-08	3.0311E + 06	3.1443E + 05	**0.0000E + 00**
F7	Mean	3.3347E + 01	9.9608E + 01	8.8140E + 01	8.0969E + 01	7.0418E-79	4.8761E-08	9.9579E + 01	7.8692E + 01	**0.0000E + 00**
	Std	1.7063E + 00	1.0479E-01	7.3862E + 00	1.2345E + 01	3.8561E-78	4.3532E-08	1.2220E-01	4.0937E + 00	**0.0000E + 00**
F8	Mean	5.2371E + 07	1.2684E + 07	5.9953E + 06	1.7334E-19	1.4620E-166	1.1870E-12	1.4649E + 05	6.6189E + 00	**0.0000E + 00**
	Std	2.0275E + 06	4.1667E + 06	5.7263E + 05	2.9533E-19	0.0000E + 00	1.5092E-12	1.5329E + 05	2.1980E + 00	**0.0000E + 00**
F9	Mean	2.8308E + 08	4.1193E + 09	4.1735E + 08	9.9752E + 02	9.9871E + 02	**3.9607E-13**	1.8877E + 08	1.0634E + 03	1.0747E-02
	Std	3.0920E + 07	1.0937E + 09	9.3964E + 07	1.1785E-01	1.2330E-01	**6.5742E-13**	1.5950E + 08	3.1831E + 01	1.2948E-02
F10	Mean	5.3606E + 03	9.5383E + 02	7.0280E + 02	1.6932E-23	1.2519E-175	1.1691E-134	1.8415E + 01	6.4043E-04	**0.0000E + 00**
	Std	2.2462E + 02	4.8876E + 02	1.0278E + 02	1.6875E-23	0.0000E + 00	6.4035E-134	3.8646E + 01	1.9081E-04	**0.0000E + 00**
F11	Mean	6.6384E + 08	1.0747E + 09	8.0582E + 07	7.0000E-01	6.6667E-01	**2.5000E-01**	3.6745E + 07	2.3954E + 01	5.5703E-01
	Std	4.3154E + 07	1.9494E + 08	1.6909E + 07	1.0171E-01	4.3220E-06	**6.1954E-13**	2.0968E + 07	1.0456E + 01	1.8560E-01
F12	Mean	3.0412E + 00	2.5943E + 00	1.3142E-06	3.8227E-01	1.1263E-283	2.4855E-148	1.8576E + 00	8.7754E-03	**0.0000E + 00**
	Std	6.3555E-01	6.4502E-01	6.5919E-06	7.4186E-01	0.0000E + 00	1.3614E-147	5.5846E-01	2.1536E-02	**0.0000E + 00**
F13	Mean	3.2953E + 05	1.3249E + 14	3.0107E + 04	5.8401E + 03	3.1058E + 00	2.0190E-08	1.2383E + 17	7.9289E + 03	**0.0000E + 00**
	Std	4.8894E + 04	4.6620E + 14	1.4360E + 03	1.6287E + 03	5.1314E + 00	4.7300E-08	3.9346E + 17	7.2878E + 02	**0.0000E + 00**
F14	Mean	−3.3405E + 04	−2.2117E + 04	−1.3019E + 05	−1.1290E + 05	−1.3926E + 04	−3.5976E + 05	−2.2376E + 04	−8.9535E + 04	**−4.1898E + 05**
	Std	1.2433E + 04	1.6361E + 03	6.5027E + 03	8.3233E + 03	2.6409E + 03	6.0232E + 04	1.6203E + 03	1.4042E + 04	**6.1795E + 00**
F15	Mean	1.4729E + 04	2.3990E + 03	1.0529E + 04	1.8190E-13	**0.0000E + 00**	**0.0000E + 00**	2.2239E + 01	1.9287E + 02	**0.0000E + 00**
	Std	5.1143E + 02	9.4408E + 02	5.6915E + 02	5.5503E-13	**0.0000E + 00**	**0.0000E + 00**	7.5669E + 01	4.0137E + 01	**0.0000E + 00**
F16	Mean	1.5903E + 01	1.9325E + 01	1.9985E + 01	2.0732E-12	1.0066E-15	7.7459E-09	1.9218E + 01	1.9230E-02	**8.8818E-16**
	Std	3.0640E-01	3.1468E + 00	7.2669E-01	9.9937E-13	6.4863E-16	7.4496E-09	5.0224E + 00	3.7135E-03	**0.0000E + 00**
F17	Mean	2.7475E + 02	4.1927E + 03	2.1451E + 03	1.5173E-16	**0.0000E + 00**	1.1839E-12	4.7519E + 01	1.5943E-01	**0.0000E + 00**
	Std	1.9531E + 01	1.5253E + 03	2.3921E + 02	5.4416E-17	**0.0000E + 00**	2.9765E-12	5.7893E + 01	1.2032E-01	**0.0000E + 00**
F18	Mean	8.3798E + 07	2.2135E + 10	1.1200E + 09	9.9379E + 01	9.9967E + 01	**6.3305E-11**	1.1428E + 09	1.2372E + 02	4.7248E-05
	Std	1.0979E + 07	3.9911E + 09	4.7772E + 08	1.9263E-01	9.4180E-03	**1.5582E-10**	7.5160E + 08	1.0916E + 01	7.2671E-05
F19	Mean	1.5088E + 03	2.4637E + 02	1.3507E + 03	1.1998E-13	1.5017E-85	4.6280E-08	4.7808E-01	8.6743E-01	**0.0000E + 00**
	Std	4.2202E + 01	8.4113E + 01	6.1356E + 01	6.9100E-14	8.2248E-85	3.8365E-08	7.6244E-01	1.5447E + 00	**0.0000E + 00**
F20	Mean	4.9998E-01	5.0000E-01	5.0000E-01	9.2872E-03	3.0509E-03	6.2910E-10	5.0000E-01	4.3565E-01	**0.0000E + 00**
	Std	3.3628E-06	5.2409E-07	4.8054E-08	1.9649E-03	5.7860E-04	1.3416E-09	7.2956E-06	2.5823E-02	**0.0000E + 00**
F21	Mean	4.1375E + 02	7.1902E + 01	1.1783E + 02	**0.0000E + 00**	**0.0000E + 00**	9.2560E-13	4.9278E-01	1.7338E-04	**0.0000E + 00**
	Std	4.4738E + 00	2.4870E + 01	7.3795E + 00	**0.0000E + 00**	**0.0000E + 00**	1.7963E-12	1.4195E + 00	4.0321E-05	**0.0000E + 00**
F22	Mean	3.2940E + 03	5.2034E + 02	3.1047E + 03	1.5613E-05	3.1808E-42	4.0960E-02	1.0077E + 00	9.3632E + 01	**0.0000E + 00**
	Std	6.0078E + 01	2.8791E + 02	6.6749E + 01	4.8012E-06	1.2830E-41	2.5311E-02	2.3121E + 00	5.2302E + 00	**0.0000E + 00**
F23	Mean	4.9916E + 04	3.3510E + 04	1.7090E + 04	**0.0000E + 00**	**0.0000E + 00**	1.0352E-13	5.0736E + 02	2.8367E + 00	**0.0000E + 00**
	Std	1.6828E + 03	1.2285E + 04	1.7850E + 03	**0.0000E + 00**	**0.0000E + 00**	2.6684E-13	4.9557E + 02	2.2953E + 00	**0.0000E + 00**
F24	Mean	8.7164E + 08	1.5296E + 10	2.0420E + 09	5.0813E-17	4.9734E-138	3.9333E-09	1.9761E + 07	7.0344E + 02	**0.0000E + 00**
	Std	9.5783E + 07	7.4917E + 09	3.3381E + 08	8.5270E-17	2.7241E-137	9.0179E-09	2.3469E + 07	1.5498E + 02	**0.0000E + 00**
F25	Mean	3.2168E + 01	6.8728E + 01	8.1252E + 01	2.3987E-01	1.0025E-01	1.9004E-08	7.1328E + 00	2.0499E + 00	**0.0000E + 00**
	Std	8.5028E-01	1.5259E + 01	3.0060E + 00	4.9827E-02	8.8981E-04	1.4519E-08	2.4647E + 00	1.7171E-01	**0.0000E + 00**

Table 8
Comparison of results on fixed-dimensional benchmark functions.

F(x)		PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F26	Mean	3.0622E + 00	1.8602E + 00	1.7245E + 00	**9.9800E-01**	1.8226E + 00	**9.9800E-01**	2.4441E + 00	3.8085E + 00	**9.9800E-01**
	Std	3.0810E + 00	9.9785E-01	1.2153E + 00	**1.6493E-16**	7.2558E-01	1.5475E-13	2.9311E + 00	3.8029E + 00	7.3803E-10
F27	Mean	9.2902E-04	1.0528E-03	3.8495E-03	2.3552E-03	3.8130E-04	1.6744E-03	9.5035E-04	3.1212E-03	**3.0878E-04**
	Std	2.6213E-04	3.9748E-04	7.5216E-03	6.1076E-03	5.6531E-05	4.8044E-10	2.6586E-04	6.8836E-03	**1.5588E-06**
F28	Mean	**−1.0316E + 00**	**−1.0316E + 00**	**−1.0316E + 00**	**−1.0316E + 00**	−1.0315E + 00	8.2267E-10	**−1.0316E + 00**	**−1.0316E + 00**	**−1.0316E + 00**
	Std	**6.3877E-16**	5.4195E-05	4.4335E-10	**6.2532E-16**	1.4438E-04	2.8181E-09	3.8878E-06	1.7484E-08	7.1999E-08
F29	Mean	**3.9789E-01**	3.9899E-01	**3.9789E-01**	**3.9789E-01**	3.9978E-01	8.4467E-01	4.0000E-01	**3.9789E-01**	**3.9789E-01**
	Std	**0.0000E + 00**	1.3252E-03	1.4011E-08	**0.0000E + 00**	1.7033E-03	1.3165E-09	2.0165E-03	7.3934E-07	4.5724E-06
F30	Mean	**3.0000E + 00**	**3.0001E + 00**	**3.0000E + 00**	**3.0000E + 00**	3.0001E + 00	3.2685E + 01	**3.0000E + 00**	**3.0000E + 00**	**3.0000E + 00**
	Std	1.4331E-15	8.1948E-05	2.6058E-08	1.1662E-15	1.6841E-04	2.7464E-12	6.4001E-05	5.1603E-05	**4.5168E-16**
F31	Mean	**−3.8628E + 00**	−3.8544E + 00	**−3.8628E + 00**	−3.8623E + 00	−3.8554E + 00	−1.8997E + 00	−3.8574E + 00	−3.8613E + 00	**−3.8628E + 00**
	Std	**2.6823E-15**	2.6548E-03	7.9399E-08	1.9996E-03	3.9729E-03	8.8471E-11	5.0848E-03	2.8241E-03	7.9279E-06
F32	Mean	−3.2705E + 00	−2.8238E + 00	−3.2744E + 00	−3.2659E + 00	−3.0583E + 00	−1.1698E + 00	−3.1064E + 00	−3.2386E + 00	**−3.3218E + 00**
	Std	5.9923E-02	4.2055E-01	5.9242E-02	6.7088E-02	9.7720E-02	1.2574E-10	3.4000E-02	7.7548E-02	**1.1597E-04**
F33	Mean	−7.6364E + 00	−2.4486E + 00	−5.8009E + 00	−8.5475E + 00	−4.7074E + 00	−9.9848E + 00	−9.2301E + 00	−9.0637E + 00	**−1.0151E + 01**
	Std	3.0228E + 00	1.8163E + 00	3.2827E + 00	2.5310E + 00	1.8457E-01	9.2244E-01	1.8503E + 00	2.5221E + 00	**1.0675E-03**
F34	Mean	−9.2666E + 00	−3.9382E + 00	−6.1915E + 00	−9.8714E + 00	−4.7506E + 00	−1.0126E + 01	−1.0129E + 01	−1.0047E + 01	**−1.0401E + 01**
	Std	2.3455E + 00	1.4889E + 00	3.3581E + 00	1.6218E + 00	1.9408E-01	1.0693E + 00	2.1294E-01	1.3481E + 00	**1.3008E-03**
F35	Mean	−9.5966E + 00	−4.0108E + 00	−6.6127E + 00	−9.5447E + 00	−4.7315E + 00	−1.0000E + 01	−1.0281E + 01	−1.0534E + 01	**−1.0535E + 01**
	Std	2.1488E + 00	1.8283E + 00	3.9995E + 00	2.6082E + 00	1.9742E-01	1.6357E + 00	1.6977E-01	1.2215E-03	**9.4323E-04**

concentrations ($\vec{C}_{best}$, $\vec{C}_{worst}$ and $\vec{C}$), which is more conducive to the optimization process of the algorithm. In this way, the novel update rules effectively avoid falling into local optima and accelerate the speed of global convergence.

3.4. Chaos-based strategy

Chaos theory is the study of chaotic dynamical systems. Due to its sensitivity, ergodicity and randomness, chaos theory has been widely applied in the random search strategies of metaheuristic algorithms in recent years (Demir et al., 2020; Gupta et al., 2020b; Mirjalili & Gandomi, 2017; Xu et al., 2019). Although the nonlinear time parameter strategy and the oscillating functions improve the optimization ability of m-EO to allow the algorithm to find the most promising region in the search space, it still faces the risk of the stagnation of the iterative process in some cases. Therefore, a new phase is also added to m-EO to increase the particle concentrations after the update rules are applied. A logistic chaotic map (Z. Guo et al., 2006) is used to mutate the particles and retain particles with higher concentrations. The mathematical model of the proposed chaos-based strategy is as follows:

$$\varphi_{l+1} = \omega \cdot \varphi_l \cdot (1 - \varphi_l) \tag{16}$$

where ω is a control parameter equal to 4 (Mirjalili & Gandomi, 2017). The logistic chaotic distributions over 50 iterations are shown in Fig. 3.

With this chaos-based strategy, the particle concentration update model is as follows:

$$\vec{C}_{Chaos} = \varphi_l \cdot \left(\vec{C}_{best} - \vec{C}_{worst}\right) + \vec{C} \tag{17}$$

In summary, the pseudocode for m-EO with all of the improvement strategies proposed above is presented in Algorithm 2. It can be seen from Algorithm 2 that the introduced OBL approach improves the concentration diversity of the remaining particles in the initial phase, the proposed novel population update rules change the position of each particle during the intermediate phase of optimization, and the additional chaos-based strategy is used to generate spatial jumps in the late phase to prevent stagnation of the optimization process. In addition, the newly proposed nonlinear time control strategy can better balance exploration and exploitation throughout the whole iterative optimization process. These four modifications can effectively improve the optimization ability and efficiency of the original EO.

3.5. Computational complexity

The computational complexity of an algorithm is an important criterion for analysing its performance. In this paper, big-O notation is used to represent complexity (Fan et al., 2021; Gupta et al., 2020a). According to the above pseudocode, the complexities of the original EO and the proposed m-EO are calculated as follows:

3.5.1. The original EO

- The original EO initializes the concentration of each particle in $O(n \times d)$ time, where n represents the number of particles and d is the dimensionality of the problem.
- The fitness evaluation of each particle requires $O(n)$ time.
- The selection of the particles with the highest concentrations requires $O(n)$ time.
- The update of the particle concentrations in the original EO requires $O(n \times d)$ time.

In summary, the total computation time of the original EO is equivalent to $O(n \times d \times Max\_iter)$ for Max_iter iterations.

Table 9
Statistical results of the Wilcoxon rank-sum test.

m-EO VS.		F1-F25(Dim = 30)	F1-F25(Dim = 100)	F1-F25(Dim = 300)	F1-F25(Dim = 500)	F1-F25(Dim = 1000)	F26-F35(Fix Dim)
Wilcoxon's rank sum test (+/=/-)	PSO	25/0/0	25/0/0	25/0/0	25/0/0	25/0/0	8/0/2
	SCA	25/0/0	25/0/0	25/0/0	25/0/0	25/0/0	10/0/0
	PFA	25/0/0	25/0/0	25/0/0	25/0/0	25/0/0	5/0/5
	EO	21/4/0	21/4/0	21/2/2	22/2/1	21/2/2	9/0/1
	HGSO	18/5/2	18/5/2	18/5/2	17/4/4	17/4/4	10/0/0
	OTSA	25/0/0	24/0/1	23/1/1	23/1/1	22/2/1	10/0/0
	OBSCA	24/0/1	25/0/0	25/0/0	25/0/0	25/0/0	10/0/0
	SOGWO	23/2/0	25/0/0	25/0/0	25/0/0	25/0/0	8/0/2
	Sum	186/11/3	188/9/3	187/8/5	187/7/6	185/8/7	70/0/10

Table 10
Statistical results of the Friedman test.

			PSO	SCA	PFA	EO	HGSO	OTSA	OBSCA	SOGWO	m-EO
F1-F25	Dim = 30	Friedman value	8.0233	8.7767	6.6100	3.2533	2.2817	4.8767	5.6267	4.3500	**1.2017**
		Friedman rank	8	9	7	3	2	5	6	4	**1**
	Dim = 100	Friedman value	8.0133	8.5900	7.0000	3.3300	2.2000	3.8200	6.3633	4.4900	**1.1933**
		Friedman rank	8	9	7	3	2	4	6	5	**1**
	Dim = 300	Friedman value	7.7700	8.3667	7.1033	3.5500	2.2933	3.2633	6.5000	4.9467	**1.2067**
		Friedman rank	8	9	7	4	2	3	6	5	**1**
	Dim = 500	Friedman value	7.8100	8.2467	7.1533	3.5667	2.2817	3.2417	6.4700	5.0200	**1.2100**
		Friedman rank	8	9	7	4	2	3	6	5	**1**
	Dim = 1000	Friedman value	7.3833	8.2600	7.2433	3.4400	2.7150	3.1583	6.5967	4.9700	**1.2333**
		Friedman rank	8	9	7	4	2	3	6	5	**1**
F1-F35	All dimensions	Friedman value	7.8000	8.4480	7.0220	3.4280	2.3543	3.6720	6.3113	4.7553	**1.2090**
		Friedman rank	8	9	7	3	2	4	6	5	**1**

3.5.2. The proposed m-EO

- The initialization of the particle concentrations in m-EO requires $O(n \times d)$ time, where n represents the number of particles and d is the dimensionality of the problem.
- The fitness evaluation of each particle requires $O(n)$ time.
- The selection of the particles with the highest concentrations requires $O(n)$ time.
- The calculation of the particles with opposite concentrations requires $O(n_{op} \times d)$ time, where $n_{op} = (n-4)$.
- The update of the particle concentration in m-EO requires $O(n \times d)$ time.
- The chaos-based strategy requires $O(n)$ time.

In summary, the total computation time of m-EO is equivalent to $O(n \times d \times Max\_iter)$ for Max_iter iterations. Thus, in terms of time complexity, the original EO and the proposed m-EO are the same.

4. Numerical experiments and discussion

To evaluate the performance of the m-EO proposed in this paper, various experiments are reported in this section, including optimization for problems of 5 different dimensionalities (Dim = 30, 100, 300, 500, 1000). In addition, the experimental results of m-EO are compared with 8 state-of-the-art algorithms, as described in detail below.

4.1. Benchmark functions and parameter settings

To verify the performance and efficiency of m-EO, 35 well-known benchmark functions were collected from the original paper introducing EO (Faramarzi et al. 2020b) and several other references (Gupta et al., 2020b; Long et al., 2018; Mirjalili et al., 2014; Sun et al., 2018), including 12 unimodal functions, 13 multimodal functions and 10 fixed-dimensional multimodal functions. The unimodal functions F1-F12 each have only one global optimal value, so they can be used to study the local search ability of the algorithm, while the multimodal functions F13-F35, with more than two optimal values, can be used to evaluate its global exploration ability. In addition, to verify the accuracy and robustness of the m-EO algorithm proposed in this paper, functions F1-F25 were used for testing not only in their common 30-dimensional versions (Cai et al., 2019; Gupta & Deep, 2020; Mahdavi et al., 2015) but also in 100, 300, 500 and 1000 dimensions (Fan et al., 2020a, 2020b; Gupta & Deep, 2020; Hussain et al., 2019; Sun et al., 2019). Higher-dimensional test functions present a greater difficulty of optimization and can be used to better evaluate the performance of the proposed algorithm (Long et al., 2019). The descriptions of these test functions are shown in Table 1.

The optimization results of m-EO were compared with those of eight other state-of-the-art algorithms: PSO (Kennedy & Eberhart, 1995), SCA (Mirjalili, 2016), PFA (Yapici & Cetinkaya, 2019), EO (Faramarzi et al., 2020b), HGSO (Hashim et al., 2019), OTSA (Morales-Castañeda et al., 2019), OBSCA (Morales-Castañeda et al., 2019) and SOGWO (Dhargupta et al., 2020), which are described in Section 1. To ensure fair comparisons, all experiments were conducted in the same environment, and all parameters were selected in accordance with the suggestions given by the developers of the algorithms in the works cited above, as shown in Table 2. The population size and number of iterations were the same for all algorithms, equal to 30 and 500, respectively, and the results were recorded for 30 independent experiments. The mean value and the standard deviation (Std) were calculated to evaluate the performance of these algorithms, and the best results are shown in bold.

4.2. Comparison and analysis of optimization results

The optimization results for benchmark functions F1-F25 with dimensionalities of Dim = 30, 100, 300, 500 and 1000 are shown in Tables 3–7 respectively, and the results for the fixed-dimensional benchmark functions are shown in Table 8. It can be seen from these tables that m-EO shows superior performance on most test problems compared to the other algorithms.

Specifically, for the optimization of the unimodal functions F1-F12, compared with the other 8 algorithms, m-EO yields 11, 11, 10, 10 and

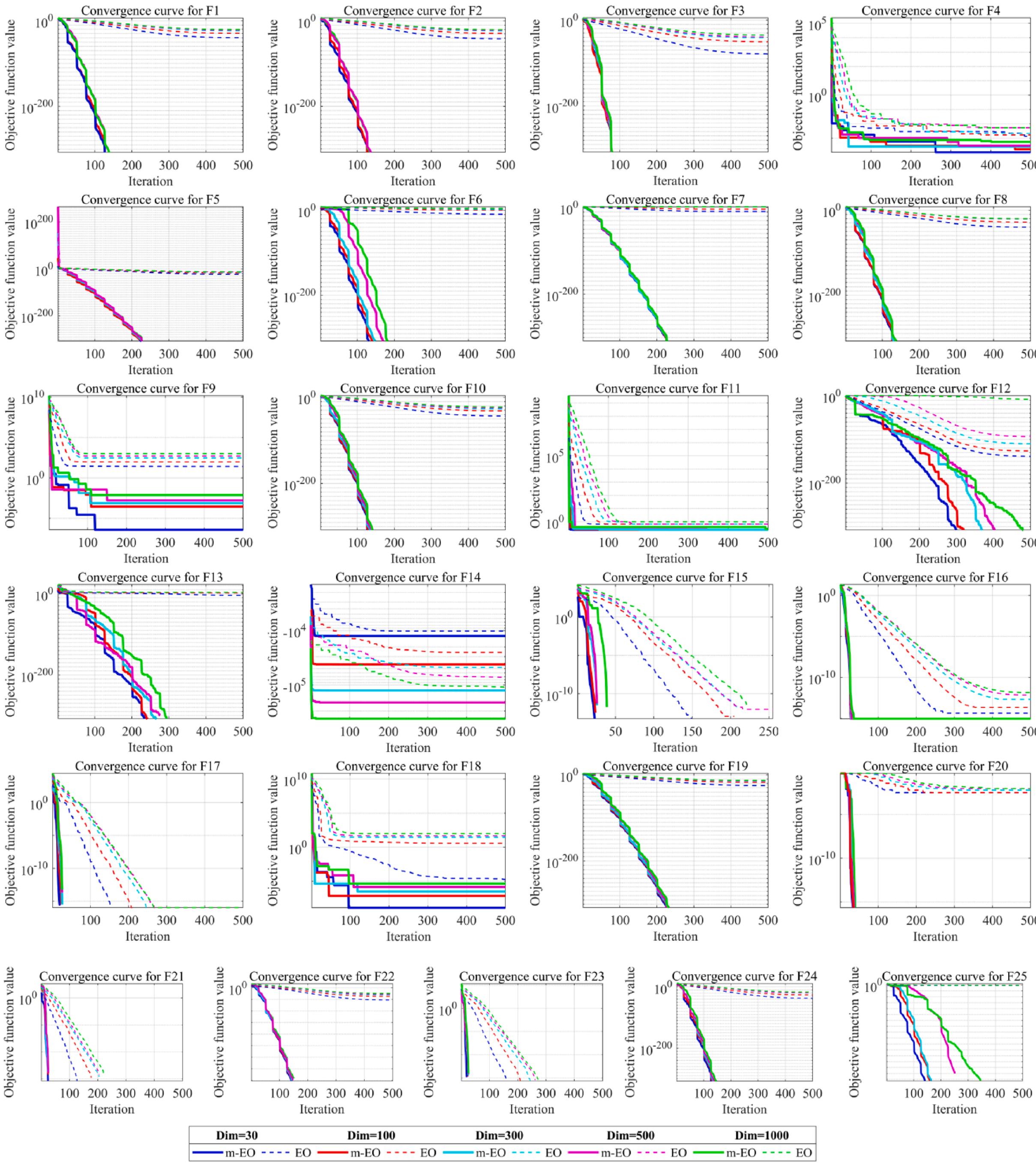

Fig. 4. Convergence curves of EO and m-EO on 25 test functions in 5 different dimensional cases.

10 optimal results in the 5 different dimensional cases. Moreover, regardless of the test dimensions, the proposed m-EO can achieve the theoretical optimal (0) for F1-F3, F5-F8, F10 and F12 within 500 iterations, proving that the algorithm proposed in this paper has a good exploitation ability. In addition, the standard deviation value is equal to 0 when solving these problems, which shows that the proposed m-EO has strong robustness. Although the proposed m-EO does not achieve the theoretically optimal solution for function F4, its optimal value is better than those of the other algorithms considered for comparison. When Dim = 30 and 100, m-EO can obtain the optimal solution for function F11; as the dimensionality increases, OTSA becomes superior to m-EO, but the optimization accuracy of the proposed m-EO is still higher than that of the original EO. Although m-EO also does not yield the optimal solution for function F9 in the five different dimensional cases, its optimal value is still better than that of the original EO. It is worth noting that when Dim = 1000, SCA, PFA, HGSO and OBSCA can no longer solve function F5; however, the proposed m-EO can still obtain stable theoretically optimal values.

For the multimodal functions F13-F25, in the five different dimensional cases, the proposed m-EO is superior to the other algorithms considered for comparison on all functions except F18. Moreover, the proposed algorithm obtains the theoretically optimal solution (0) for functions F13, F15, F17 and F19-F25. When Dim = 30, the proposed m-EO and the original EO yield the same optimal solution (0) for functions F15, F17, F21 and F23, but as the dimensionality increases, the original EO can no longer reach the optimal solutions, whereas the results of m-

Algorithm 2 The modified Equilibrium Optimizer (m-EO)

Initialization {
  initialize the particle population $\vec{C}_i^{initial}(i = 1, 2, ..., n)$
  define t, τ_1, τ_2, $a_1 = 2$, $a_2 = 1$ $GP = 0.5$ and $\omega = 4$}
Main loop
  while ($Iter < Max\_iter$)
    for $i = 1 : n$
      reinitialize the particle concentration beyond the search space
      calculate the fitness of the i_{th} particle
      if $fit(\vec{C}_i) < fit(\vec{C}_{eq1})$
        update $\vec{C}_{eq1}$ to $\vec{C}_i$ and $fit(\vec{C}_{eq1})$ to $fit(\vec{C}_i)$
      else if $fit(\vec{C}_i) > fit(\vec{C}_{eq1})$ && $fit(\vec{C}_i) < fit(\vec{C}_{eq2})$
        update $\vec{C}_{eq2}$ to $\vec{C}_i$ and $fit(\vec{C}_{eq2})$ to $fit(\vec{C}_i)$
      else if $fit(\vec{C}_i) > fit(\vec{C}_{eq1})$ && $fit(\vec{C}_i) > fit(\vec{C}_{eq2})$ && $fit(\vec{C}_i) < fit(\vec{C}_{eq3})$
        update $\vec{C}_{eq3}$ to $\vec{C}_i$ and $fit(\vec{C}_{eq3})$ to $fit(\vec{C}_i)$
      else if $fit(\vec{C}_i) > fit(\vec{C}_{eq1})$ && $fit(\vec{C}_i) > fit(\vec{C}_{eq2})$ && $fit(\vec{C}_i) > fit(\vec{C}_{eq3})$ && $fit(\vec{C}_i) < fit(\vec{C}_{eq4})$
        update $\vec{C}_{eq4}$ to $\vec{C}_i$ and $fit(\vec{C}_{eq4})$ to $fit(\vec{C}_i)$
      end if
    end for
    calculate the average particle $\vec{C}_{eq\_ave}$ via Eq. (2) and construct the equilibrium pool $\vec{C}_{eq.pool}$ via Eq. (3)
    perform memory saving and particle concentration updating ($Iter > 1$)
    calculate the opposite concentrations of all particles except $\vec{C}_{eq1} \sim \vec{C}_{eq4}$ via Eq. (11)
    merge the concentrations of all particles $\left[\vec{C}_{eq1}, \vec{C}_{eq2}, \vec{C}_{eq3}, \vec{C}_{eq4}, \vec{OC}_{eq}^{others}\right]$
    update t via Eq. (12) and Eq. (13) and update τ_1 and τ_2 via Eq. (15)
    for $i = 1 : n$
      pick a random candidate from the concentration pool via Eq. (4)
      calculate the values of vectors $\vec{F}$ and $\vec{G}$ via Eq. (5), Eq. (7) and Eq. (8)
      update the concentration of the particle via Eq. (14)
      obtain a new concentration $\vec{C}_{Chaos}$ by means of the chaos-based strategy via Eq. (16) and Eq. (17)
      if $fit(\vec{C}_{Chaos}) < fit(\vec{C}_i)$
        replace concentration $\vec{C}_i$ with $\vec{C}_{Chaos}$
      end if
    end for
    $Iter = Iter + 1$
  end while
print($\vec{C}_{eq1}$)

EO remain stable. Although OTSA yields the optimal solution for function F18, the proposed m-EO algorithm is still superior to EO in all five dimensional cases. These findings verify that the four modifications proposed in this paper can help m-EO effectively avoid becoming trapped in local optima and improve its exploration ability. From the above analysis, it can be seen that compared with the other state-of-the-art algorithms, m-EO is the best algorithm for solving large optimization problems.

Compared with the high-dimensional multimodal functions, the fixed-dimensional benchmark functions have fewer local optima. Since the optimal values for these test functions (F26-F35) are not equal to zero, these functions can be used to verify the ability of m-EO to maintain a proper balance between exploration and exploitation. The results obtained on the fixed-dimensional benchmark functions are presented in Table 8. As seen from the mean values in this table, m-EO can find the best solution for all functions, while PSO, SCA, PFA, EO, HGSO, OTSA, OBSCA, and SOGWO obtain the optimal solutions 4, 2, 4, 4, 0, 1, 2 and 3 times, respectively. By analysing the mean and standard deviation values, we can know that the 9 algorithms have obtained 2, 0, 0, 3, 0, 0, 0, 0 and 6 of the optimal values, and the proposed m-EO still performs well. In addition, for functions F27 and F30-F35, the m-EO algorithm proposed in this paper provides better solutions than the original EO. Moreover, it can be seen from F27, F30 and F32-F35 that unlike the other state-of-the-art algorithms, m-EO successfully finds the optimal solutions, thereby proving that the proposed m-EO can better balance exploitation and exploration.

Thus, it can be verified from Tables 3–8 that the modification strategies proposed in Section 4 all have an influence on improving the optimization mechanism in EO to obtain better solutions. The proposed m-EO algorithm has a strong exploitation ability and an efficient spatial exploration ability, allowing it to effectively solve optimization problems.

4.3. Statistical testing

To estimate the statistical significance of the differences between m-EO and the algorithms considered for comparison, the Wilcoxon rank-sum test (García et al., 2010) and the Friedman test (Theodorsson-Norheim, 1987) were used to test whether the performance differences are significant.

For the Wilcoxon rank-sum test, the significance level was set to 0.05, and the symbols “+/=/-” were adopted to indicate that m-EO performs better than, similar to or worse than the corresponding algorithm, respectively. Table 9 shows the statistical results in 6 different

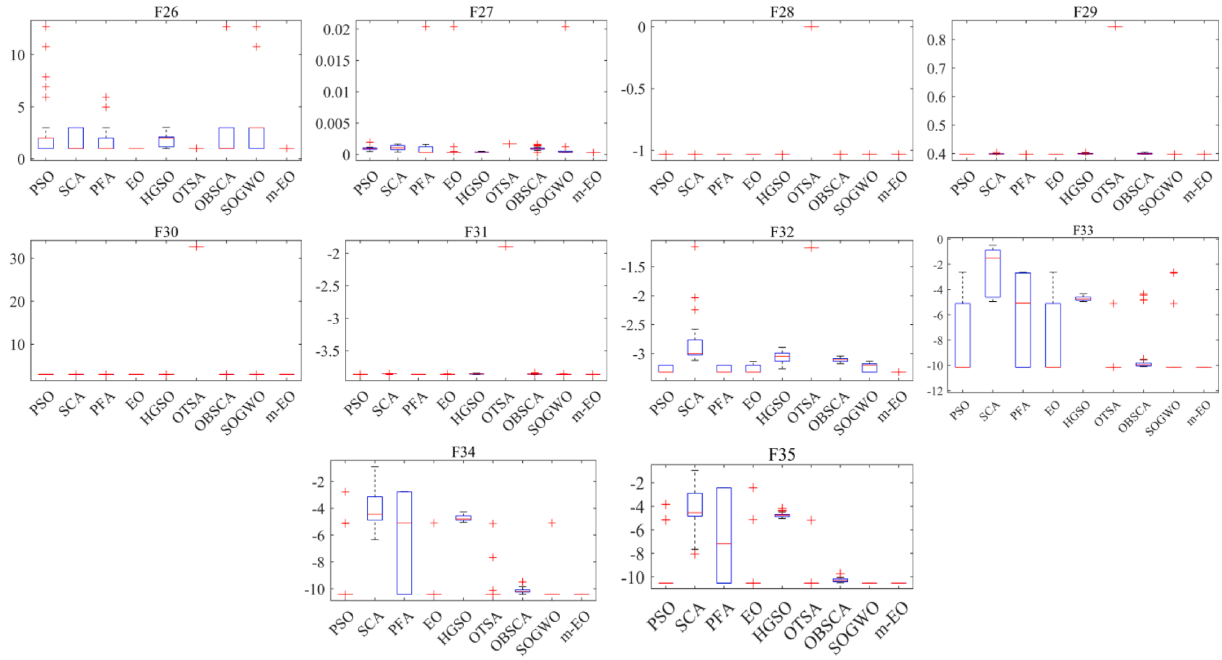

Fig. 5. Boxplot analysis for fixed-dimensional functions.

situations, in which the proposed m-EO algorithm obtains results of 186/11/3, 188/9/3, 187/8/5, 187/7/6, 185/8/7 and 70/0/10, thus proving that m-EO is superior to the other algorithms considered for comparison in most cases.

For the Friedman test, the statistical results are shown in Table 10, and the proposed m-EO algorithm ranks first in all cases. In summary, based on the result of the Wilcoxon rank-sum test results and the Friedman test, m-EO performs significantly differently than the other state-of-the-art algorithms on most test problems and it is the best optimizer among them.

4.4. Convergence and stability analysis

To understand the optimization behaviour of m-EO, convergence and stability analyses are needed. Fig. 4 records the progress of m-EO and the original EO in solving the first 25 unimodal and multimodal functions. As seen from the figure, in all five different dimensional cases, the proposed algorithm converges faster than the original EO and provides the optimal solution. Thus, it is proven that the improvements proposed in this paper lead to a better balance between the exploration and exploitation capabilities of the original EO. Through these improvements, the proposed m-EO can achieve a higher search accuracy and faster convergence.

In addition, to verify the stability of the fixed-dimensional functions (F26-F35) as solved by m-EO and the other algorithms considered for comparison, a boxplot analysis was performed, as shown in Fig. 5, based on the results of 30 independent experiments. It can be seen that the proposed m-EO is very stable in most cases in terms of the median, minimum, maximum and other values. At the same time, it is worth noting that m-EO produces no outliers in the optimization process, thus showing its superiority.

In view of the above, the convergence and stability analyses prove that m-EO is superior to the other algorithms considered for comparison

Table 11
Comparison of the results for the three-bar truss design problem.

Algorithm	x_1	x_2	Optimum value
PSO	0.78780503	0.41071488	263.89640096
SCA	0.77246882	0.45680194	264.16737017
PFA	0.78830728	0.40930274	263.89588655
EO	0.78836232	0.40913375	263.89753955
HGSO	0.80440796	0.36627706	264.19246640
OTSA	0.78746687	0.41167658	263.89692377
OBSCA	0.79116815	0.40244864	264.02101092
SOGWO	0.78741859	0.41186362	263.90197031
m-EO	**0.78834565**	**0.40918256**	**263.89607783**

5. m-EO for engineering design problems

In the field of engineering design, mathematical models and meta-heuristic algorithms are widely used (H. Chen et al., 2020; Ewees & Elaziz, 2020; Gao et al., 2018). Therefore, in this section, m-EO is applied to three engineering problems: the design of a three-bar truss, the design of a tension/compression spring and the design of a multiple-disk clutch or brake.

5.1. Three-bar truss design (Ray & Saini, 2001)

This is a common optimization problem in civil engineering, in which the main goal is to minimize the weight of the bar. The volume of the truss is minimized by constraining the stress of each member. The mathematical description of the three-bar truss design problem is as

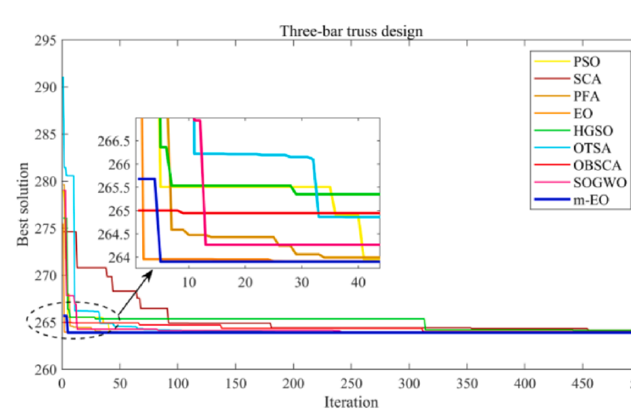

Fig. 6. Convergence curves for the three-bar truss design problem.

Table 12
Comparison of the results for the tension/compression spring design problem.

Algorithm	x_1	x_2	x_3	Optimum value
PSO	0.05159466	0.35445099	11.42311016	0.01266540
SCA	0.05220858	0.36925451	10.76001688	0.01284283
PFA	0.05181850	0.35971693	11.12085308	0.01267338
EO	0.05197805	0.36370991	10.89050915	0.01266675
HGSO	0.05104382	0.33480795	13.10937289	0.01318040
OTSA	0.05188686	0.36149369	11.01449983	0.01266610
OBSCA	0.05000000	0.31666775	14.19838338	0.01282376
SOGWO	0.05217225	0.36832956	10.64947180	0.01268201
m-EO	**0.05167583**	**0.35639954**	**11.30764601**	**0.01266524**

follows:

$$Minimize f(c) = \left(2\sqrt{2}x_1 + x_2\right) l$$

$$Subject\ to$$

$$g_1(\vec{x}) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leqslant 0$$

$$g_2(\vec{x}) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2} P - \sigma \leqslant 0$$

$$g_3(\vec{x}) = \frac{1}{x_1 + \sqrt{2}x_2} P - \sigma \leqslant 0$$

$$where,$$

$$0 \leqslant x_1 \leqslant 1$$

$$0 \leqslant x_2 \leqslant 1$$

$$l = 100cm, P = 2KN/cm^2, \sigma = 2KN/cm^2$$

Table 11 shows the minimum weights found by m-EO and the eight other algorithms when solving this problem and the optimal solutions in terms of the corresponding design variables. It can be seen from the table that the lowest cost for the three-bar truss design problem, $Minimize f(\vec{x}) = 263.89607783$, is obtained by the proposed m-EO at $\vec{x} = [0.78834565\ 0.40918256]$. In addition, Fig. 6 shows the convergence curves of the best solutions obtained by m-EO and the other algorithms considered for comparison. It is seen that m-EO converges quickly, thereby demonstrating that the improvements proposed in this paper effectively increase the diversity of the population and further balance the exploration and exploitation capabilities of EO.

5.2. Tension/Compression spring design (Belegundu & Arora, 1985)

This is a constrained minimization problem, the purpose of which is to optimize the weight of a tension or compression spring. This problem

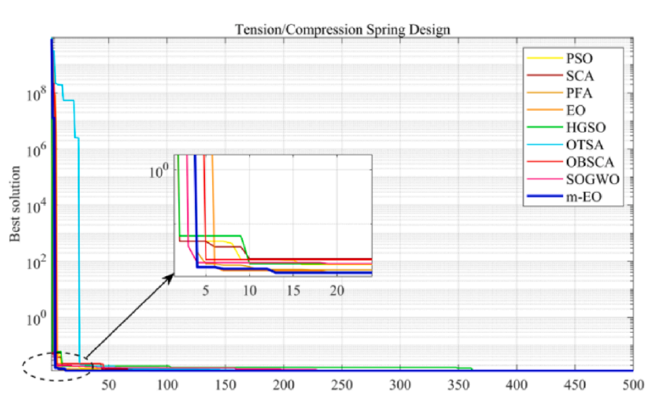

Fig. 7. Convergence curves for the tension/compression spring design problem.

has four constraints to that must met, and the objective function is calculated in terms of three design variables: the wire diameter (x_1), the mean coil diameter (x_2) and the number of active coils (x_3). The mathematical definition of this problem is as follows:

$$Minimze f(\vec{x}) = (x_3 + 2)x_2x_1^2$$

$$Subject\ to$$

$$g_1(\vec{x}) = 1 - \frac{x_2^3x_3}{71785x_1^4} \leqslant 0$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leqslant 0$$

$$g_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2x_3} \leqslant 0$$

$$g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \leqslant 0$$

$$where,$$

$$0.05 \leqslant x_1 \leqslant 2.00$$

$$0.25 \leqslant x_2 \leqslant 1.30$$

$$2.00 \leqslant x_3 \leqslant 15.0$$

The optimal solutions found by m-EO and the other 8 algorithms are shown in Table 12. The results show that m-EO is the best algorithm for solving the tension/compression spring design problem, and the optimal solution is $\vec{x} = [0.051675830.3563995411.30764601]$, with a corresponding optimal cost of $Minimize f(\vec{x}) = 0.01266524$. As shown in Fig. 7, m-EO can find the optimal solution fastest, proving that the proposed modifications can help the original EO avoid falling into the local optima and improve its convergence speed.

5.3. Multiple-disk clutch/brake design (Steven, 2002)

The purpose of this problem is to minimize the mass of a multiple-disk clutch or brake by determining the optimal values of five variables: the inner radius (x_1), the outer radius (x_2), the disk thickness (x_3), the force of the actuators (x_4), and the number of frictional surfaces (x_5). Mathematically, the multiple-disk clutch/brake design problem is expressed as follows:

$$Minimze f(\vec{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)\rho$$

$$Subject\ to$$

$$g_1(\vec{x}) = -p_{max} + p_{rz} \leqslant 0$$

$$g_2(\vec{x}) = p_{rz}V_{sr} - V_{sr,max}p_{max} \leqslant 0$$

$$g_3(\vec{x}) = \Delta R + x_1 - x_2 \leqslant 0$$

$$g_4(\vec{x}) = -L_{max} + (x_5 + 1)\ (x_3 + \delta) \leqslant 0 \quad g_5(\vec{x}) = sM_s - M_h \leqslant 0$$

$$g_6(\vec{x}) = T \geqslant 0$$

Table 13
Comparison of the results for the multiple-disk clutch/brake design problem.

Algorithm	x_1	x_2	x_3	x_4	x_5	Optimum value
PSO	**70.00000000**	**90.00000000**	**1.00000000**	**665.96414678**	**2.00000000**	**0.23524246**
SCA	69.99609399	90.00000000	1.00000000	12.51553241	2.00000000	0.23528266
PFA	70.00913959	90.01100461	1.00020480	402.37952849	2.00102273	0.23542243
EO	**70.00000000**	**90.00000000**	**1.00000000**	**1000.00000000**	**2.00000000**	**0.23524246**
HGSO	69.99991731	90.00000000	1.00000000	657.79153365	2.00000000	0.23524331
OTSA	70.00002732	90.00002804	1.00000000	548.54534562	2.00000000	0.23524255
OBSCA	69.99723154	90.00000000	1.00000000	1000.00000000	2.00000000	0.23527095
SOGWO	69.99983289	90.00000000	1.00000000	663.50521928	2.00000000	0.23524418
m-EO	**70.00000000**	**90.00000000**	**1.00000000**	**957.41470470**	**2.00000000**	**0.23524246**

$$g_7(\vec{x}) = -V_{sr,max} + V_{sr} \leqslant 0$$

$$g_7(\vec{x}) = T - T_{max} \leqslant 0$$

where,

$$60 \leqslant x_1 \leqslant 80$$

$$90 \leqslant x_2 \leqslant 110$$

$$1 \leqslant x_3 \leqslant 3$$

$$0 \leqslant x_4 \leqslant 1000$$

$$2 \leqslant x_5 \leqslant 9$$

$$M_h = \frac{2}{3}\mu x_4 x_5 \frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}\text{N.mm}$$

$$\omega = \frac{\pi n}{30}\text{rad/s}$$

$$A = \pi(x_2^2 - x_1^2)\text{mm}^2$$

$$p_{rz} = \frac{x_4}{A}\text{N/mm}^2$$

$$V_{sr} = \frac{\pi R_{sr} n}{30}\text{mm/s}$$

$$R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2 x_1^2}\text{mm}$$

$$T = \frac{I_z \omega}{M_h + M_f}$$

$$\Delta R = 20\text{mm}, L_{max} = 30\text{mm}, \mu = 0.6$$

$$V_{sr,max} = 10\text{m/s}, \delta = 0.5\text{mm}, s = 1.5$$

$$T_{max} = 15\text{s}, n = 250\text{rpm}, I_z = 55\text{Kg}\cdot\text{m}^2$$

$$M_s = 40\text{Nm}, M_f = 3\text{Nm}, \ \rho = 0.0000078, \ p_{max} = 1$$

Table 13 shows the optimal solutions obtained by the 9 algorithms when solving the multiple-disk clutch/brake design problem. As seen from this table, the minimum cost $Minimize f(\vec{x}) = 0.23524246$ is achieved by the m-EO algorithm proposed in this paper and is obtained at $\vec{x} = [70.00000000\ 90.00000000\ 1.00000000\ 957.41470470\ 2.00000000]$. Although the m-EO algorithm finds the same optimal value as PSO and EO, it can be seen from Fig. 8 that the proposed m-EO converges to the optimal value most quickly, thus showing the superior performance of m-EO in solving engineering design problems.

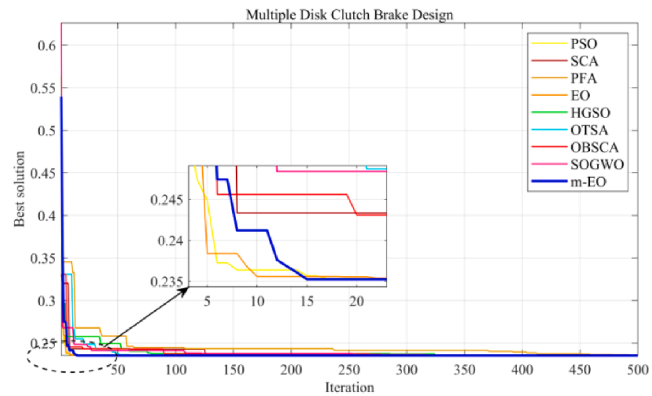

Fig. 8. Convergence curves for the multiple-disk clutch/brake design problem.

6. Conclusions and future directions

This paper presents four modifications to improve the performance of the original EO when solving global optimization problems. First, OBL is applied to obtain the opposite concentrations of all particles except $\vec{C}_{eq1} \sim \vec{C}_{eq4}$ to improve the population diversity. Second, a nonlinear time parameter strategy is formulated to balance exploration and exploitation. Third, novel update rules are proposed to avoid falling into local optima and accelerate convergence. Fourth, at the end of the optimization process, a chaotic map is used to further improve the performance of the original EO. To evaluate the effectiveness of the proposed m-EO, 35 benchmark test functions are used, including optimization in 5 different dimensional cases (Dim = 30, 100, 300, 500, 1000). The experimental results and statistical tests demonstrate that m-EO shows superior performance and is competitive with 8 state-of-the-art algorithms in terms of accuracy, convergence speed, and stability. Moreover, m-EO is applied to three engineering optimization problems, and the results and comparisons prove the effectiveness of the proposed algorithm in solving real-world problems.

In summary, the proposed m-EO offers stable search performance, a fast convergence speed and high convergence accuracy. Even in high dimensions, m-EO still has strong robustness. Therefore, in future research, we will attempt to apply this algorithm to high-dimensional complex single-objective engineering optimization problems. In addition, we will study a multi-objective optimization algorithm based on m-EO to solve multi-objective optimization problems such as workshop scheduling, feature selection and process parameter optimization. Furthermore, future research will study a hybrid binary version to enhance the search performance of the existing operators and further improve the performance of m-EO.

CRediT authorship contribution statement

Qingsong Fan: Conceptualization, Writing - original draft, Methodology, Formal analysis, Writing - review & editing, Software. **Haisong Huang:** Supervision, Conceptualization, Writing - review & editing. **Kai**

Yang: Software, Visualization. **Songsong Zhang:** Methodology. **Liguo Yao:** Data curation. **Qiaoqiao Xiong:** Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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