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"Big Crunch" Optimization Method

Moacir Kripka[1] and Rosana Maria Luvezute Kripka[2]

[1] Universidade de Passo Fundo, Programa de Pós-Graduação em Engenharia, 99001-970, Passo Fundo, RS, Brazil
mkripka@upf.br
[2] Universidade de Passo Fundo, Instituto de Ciências Exatas e Geociências, 99001-970, Passo Fundo, RS, Brazil
rkripka@upf.br

Abstract: In this article we present the Big Crunch Optimization Method (BCOM), a global optimization method developed by us upon the analogy of a cosmological theory known as Closed Universe. According to this, if the gravitational energy is greater than the energy generated by the Big Bang (kinetic energy) the expansion of the universe will stop, being followed by a contraction, which will reduce the universe to a single point with both infinite density and temperature (Big Crunch). Similarly, the optimization method starts from an initial population, which is being reduced due to the attraction of bodies (solutions) by other ones with greater mass (aptitude). The process is finished when just one body lies in the search space (which corresponds to the optimal solution). Therefore, the Big Crunch Optimization Method, among its particularities, works with a population of variable size, unlike usual procedures, which consider a single solution or a population with fixed size. In the following item of this paper the main theories related to the future of the universe are briefly described, making in the sequence the analogy between the Closed Universe Theory and our method of optimization. Aiming to illustrate the applicability of the proposed method to the structural optimization, the weight minimization of a plane truss with ten bars was carried, which consists of a classical example. This structure was analyzed considering the sections of elements as continuous variables. The obtained results are compared to the ones found in the technical literature.
Key words: Optimization, Heuristics, Big Crunch, Truss

1. Introduction

In many fields of knowledge, such as structural engineering, one of the reasons normally attributed to the little application of the optimization techniques to real problems consists of the complexity of the mathematic model generated, normally described by non-linear behavior functions and producing a non-convex space of solutions (several points of optimum), problems for which the resolution by traditional mathematical programming methods have proved to be little efficient. For the resolution of these kinds of problems the heuristic methods have played an important role, since they involve only values of functions in the process, regardless if there is unimodality or even continuity in their derivatives. Despite the great emphasis in the development of global optimization methods (a good reference for the study of these is found in *http://www.mat.univie.ac.at/~neum/glopt.html*), researchers are even far from the attainment of a method that can be applied with the same efficiency to any class of problems.

In this paper we present a heuristic method of optimization, developed in analogy with the Closed Universe Theory. According to this theory, in case the energy of bodies attraction overcomes the kinetic energy generated by the initial universe explosion (*Big Bang*) its expansion will stop, being followed by a contraction, or collapse, leading to an end very similar to the beginning, that is, until the universe comes to a single point with infinite temperature and density (*Big Crunch*). The method of optimization stems from an initial population, or universe, which is reduced as the bodies (or solutions) are attracted by other bodies with bigger matter (or bigger aptitude), either because of the matters or the relative distances. The process is concluded when a single body remains in the space, which will is supposed to constitute the optimal solution. The method presents as peculiar characteristic the fact that working with a population of variable size, unlike the most popular heuristics, which consider a single solution at a time as the Simulated Annealing [1] or a population of solutions as the Genetic Algorithms [2] or Ants Colony [3], among others.

In the following item of this paper the main theories related to the future of the universe are briefly described, making in the sequence the analogy between the Closed Universe Theory and our method of optimization. Aiming to illustrate the applicability of the proposed method to the structural optimization, the weight minimization of a plane truss with ten bars was carried, which consists of a classical example. This structure was analyzed considering the sections of elements as continuous variables. The obtained results are compared to the ones found in the technical literature.

2. Cosmological Theories

The finding that the galaxies move away from each other was mathematically formalized in 1929 by the American astronomer Edwin Hubble, known as the Hubble's Law. Up to those days, they believed that the Universe was something static in large scale, which is the same as saying that despite all the movements in its interior (from the moon spinning around the Earth to the galaxies rotation), Universe itself didn't alter. This model, of the static Universe, was privileged, in 1916, by Albert Einstein in his Theory of General Relativity. The equations found by Einstein did not confirm this idea, that's why he, arbitrarily, created the so-called Cosmologic Constant that, added to the final result of his equation, resulted in a universe without global movement. However, some years later, Einstein himself admitted it to have been the biggest scientific mistake of his life [4].

The general departing of galaxies agreed to the solutions that the Dutch astronomer William de Sitter (and after him, with more complete solutions, the Russian mathematician Alexander Friedman) had found for the equations of Einstein without the Cosmologic Constant, in 1917. However, the notion that the Universe should always be presented as it can be seen currently made appear a model known as the Stationary Universe, proposed by the astronomer Fred Hoyle, in 1948. This Universe admitted the departing of galaxies, adding to this proved fact the creation of new galaxies (and mass in general). Thus, despite the distances between the galaxies increased always, due to the departing, its aspect remained unchanged, this being the reason for its name. The Stationary Universe model was dismissed, for one of its bases, creation of matter from nothing, could never be explained.

In 1927, the Belgian astrophysicist Georges-Henri Lemaitre had already concluded that the expansion of the Universe meant that, in its beginning, this very Universe was much smaller. Thus, upon turning back the time, it would be possible to get to a time in which the size of the Universe would be so small that all the matter that constitutes it would suffer an incredible pressure (at a pretty high temperature). Lemaitre called this very small body of cosmic egg. Under these extreme conditions the Laws of Physics would be quite different, which would make possible the creation of mass from energy. In 1948, contemporarily to Hoyle's model, George Gamow, American physicist, suggested that this initial egg would have begun its expansion in a violent way, like in a burst. This cosmological theory, undoubtly the most famous, is known as the *Big Bang* (a name coined by Hoyle himself as a way to disbelieve in this idea). The credibility of this model was reinforced by something known as bottom radiation, detected for the first time in 1964 by American physicists Arno Penzias and Robert Wilson.

Whether the expansion of the Universe will take place forever or will stop some day, it depends on the quantity of matter it has. The energy discharged by the initial explosion, kinetic energy, is counterbalanced by the energy of bodies attraction, gravitational. If there is enough mass so that the latest is bigger than the first when a critic density is reached, the expansion will stop and the Universe will start to contract, leading to an end very similar to its beginning, named by the scientists as the *Big Crunch* (Great Implosion). This hypothesis is known as Closed Universe. But if the total matter of the Universe isn't sufficient to stop its expansion, it will take place forever. In this case, the Universe will walk slowly to an icy end, in which the galaxies will be endlessly distant from each other. This would be the Open Universe. These two theories are illustrated in Figure 1, in which *D (t)* represents the distance among the galaxies, which is the function of time *t*.

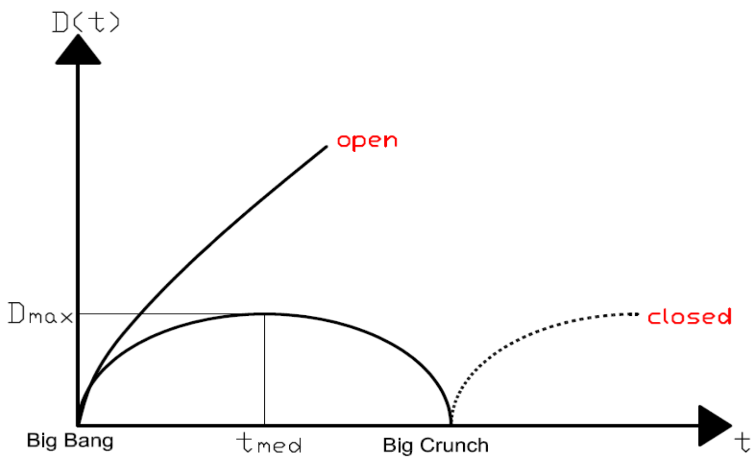

Figure 1. Open and Closed Universe Theories

To corroborate the model of the Closed Universe, the existence of a kind of matter that cannot be observed would be necessary, but that acts gravitationally. Other problems in Astronomy (for instance, the galaxy rotation) evoked the postulation of something similar, which was named of dark matter. The existence of such matter would confirm the Closed Universe, introducing in a subtle way a temporal symmetry (if there is a beginning, there must be an end). In short, the quantity of matter in the Universe with the resulting force of its gravity is the core factor in response to these questions. The gravity is the weakest forces in nature when examined in an atomic perspective, but in the astronomic scale it is the dominant force.

3. The Big Crunch as an optimization method

Considering the Closed Universe Theory valid, two discrete phases must be taken into consideration. The first phase consists of the expansion of the Universe, following the *Big Bang*, until a critical density is reached in a way in which the gravitational energy equals the kinetic energy (situation illustrated in Figure 1 by the maximum distance *Dmax,* corresponding to a medium time *tmed*).The second phase consists of the contraction of the Universe through the mutual attraction among the bodies, culminating in the convergence to a single point, similar to the *Big Crunch*. Our analogy becomes evident when we employ the proper nomenclature: the universe (search space) is formed by galaxies, or heavenly bodies (solutions), interacting according to the matters (value of the objective function) and the relative distances. When one body enters (surroundings) another body of bigger matter orbit, it is attracted by it. In short, the proposed method can be described as it follows. Once the initial position of each body in the space is known (supposing the critical density was already reached, that is, the expansion phase concluded), it is believed that the attraction among the bodies will take place, according to the Universal Gravitation Law, derived by Newton, as:

$$Fg = \frac{GMm}{r^2} \tag{1}$$

being F_g the gravitational force between two bodies of masses M e m, and r the distance between these bodies. The letter G indicates a Constant of proportionality, without a meaning in our method, since we work only with the *relative* forces.
One body will, therefore, have its position in space influenced by the other bodies (and particularly by the nearest ones, in the case there aren't big differences between the matters). In the proposed method, the alteration in the position of the body with bigger matter is not granted resulting from the gravitational interaction, with the aim to preserve the best current solution. It is considered that the changing in the position of a body can also occur due to the translation and rotation movements, which would exist even in the hypothesis of a Stationary Universe. In the optimization method, these movements are represented by a random disturbance of the current solution, resulting in the exploration of the surroundings and aiming a local improvement. Thus, starting the analysis with the body of smaller matter, it may change its position either because of the translation and rotation, or also in the sense of the stronger gravitational force. If it enters another body with a bigger matter orbit, then it will be attracted by it, reducing the number of bodies of the universe (size of the population). Otherwise, it follows according to the increasing order of the matters, until the total contraction of the universe (that is, until the convergence to a single point). The steps to the application of the *Big Crunch* method to the maximization of the functions are next presented, pointing out in the sequence details regarding the implementation of the method.

3.1 Scheme of Method Application

Step 0: Parameters definition.

α = Fraction of the body distance to the centroid of the forces (step size).
θ = Maximum size of the perturbance for local improvement.
N_b = Number of bodies (population size).
N_{FE} max = Maximum number of function evaluations.
ε = Orbit radius of each body.

Step 1: Generate the Universe, composed by N_b solutions.

Step 2: Determine the mass of each body, listing them in an increasing order.

Step 3: For the body i (with i varying from 1 to $(N_b - 1)$), do :
Step 4: Calculate the gravitational force Fg held by each one of the Bodies upon the others.
Step 5: Calculate the centroid of Fg.
Step 6: Determine the new position of the Body i as being a fraction of distance (α) between its current position and the centroid of Fg and update the value of the Body i.

Step 7: For Body i (with i varying from 1 to $(N_b - 1)$) check if it entered into the orbit of the closest Body (ε smaller or equal to the distance between them). If so, do $N_b \rightarrow (N_b - 1)$

Step 8: Perform an exploratory movement (randomized) in each Body seeking a local improvement.

Step 9: Set the Bodies again in an increasing order.

Step 10: Increase $N_{FE} \rightarrow N_{FE} + 1$.
If $N_{FE} < N_{FE}$ max or $N_b > 1$ so return to step 3.
Otherwise, finish.

3.1.1 Universe generation (initial population)

Once defined the population size (or number of Bodies), the initial position of each body in space is generated randomly, being the inferior (lower) and superior (upper) limits known ($x^{\inf}$ and $x^{\sup}$, respectively) for each variable (*space* limits, or side constraints). Designating by x_{ij} the value of the *j-th* variable of the Body *i*, and *rand* a random number between zero and one:

$$x_{ij} = x^{\inf}{}_j + rand\left(x^{\sup}{}_j - x^{\inf}{}_j\right) \tag{2}$$

If we have a good start point (initial solution, or *Big Bang* point) this solution can be used (instead of the first $x^{\inf}{}_j$ in equation 2) to generate all the universe.

3.1.2 Mass calculation of each body

The mass corresponds to the aptitude of each body (or solution). Thus, it was chosen to consider the mass equal to the value of the objective function. Alternatively, it is possible to attribute a small initial mass to each body, aiming to make a more intense exploration of the search space.

In an analogous way, the bodies with smaller mass are potentially the most distant from the "center of the universe" or from the point to where the Big Crunch will converge (in other words, the optimum). This way, the bodies which violate any constraint of the problem may have their masses artificially reduced as a penalty.

3.1.3 Calculation of gravitational force

Starting the process by the body with less mass, each body has its movement, or search direction, determinated because of its interaction with the other bodies, due to the attractive they hold upon each other. The gravitational force of interaction F_g (or force of attraction) is determined from equation 1, with *G* considered equal the unit. Thus, the gravitational force held by body *i* is calculated as:

$$F_{g_i} = \sum_{\substack{k=1 \\ k \neq i}}^{Nb} \left(\frac{M_i M_k}{D_{ik}^2} \right) = M_i \sum_{\substack{k=1 \\ k \neq i}}^{Nb} \left(\frac{M_k}{D_{ik}^2} \right) \tag{3}$$

being D_{ik} the distance between bodies *i* and *k* and where M_i and M_k represent the mass (aptitude) of solutions *i* and *k*, respectively.

The new position of body *i* ($x_{ij}{}^{new}$) will be determined from the calculation of the centroid (c_j), for each variable *j*, being dislocated a fraction of distance between the previous position and the centroid (α):

$$c_j = \left[\frac{\sum_{i=1}^{Nb} \left(x_{ij} F_{g_i} \right)}{\sum_{i=1}^{Nb} F_{g_i}} \right] \tag{4}$$

$$x_{ij}{}^{new} = x_{ij} + \alpha \left(c_j - x_{ij} \right) \tag{5}$$

The body will be dislocated to the new position even though this removal results in a reduction in the function value. Thus, only the body with more matter will have its position kept in order to preserve the best current solution.

3.1.4 Reduction in the number of Bodies (checking of the minimum orbit)

In 1850, Roche showed it was possible to calculate the minimum distance of a planet that a satellite can orbit stably. In an analogous way, designating by ε the orbit radius of each body, it is verified if the distance between the body being analyzed and the closest body to it (and with more mass) is smaller than the radius ε . In case it is, the body being analyzed will be attracted by it, with the resulting reduction in the number of bodies of the universe. Since the mass, in our case, represents the value of the objection function, it was chosen not to incorporate the mass of the eliminated body to the body that attracted it.
As it has already been pointed out, a gradual reduction in the population size consists of a differential of the proposed method, since we neither work with a population with fixed size nor with a single solution at a time. In proportion to the reduction in the population size it is verified the reduction of computational effort, while it is no longer operated with similar solutions, assuring, therefore, the option for the best one. The parameter ε , therefore, is directly connected to the convergence speed of the method. If the value attributed to ε is very high, we take the risk of premature convergence in a single body. Inferior values to the acceptable tolerance for the differences in the values of *x* (bodies positions in space), however, make the method unnecessarily slow.

3.1.5 Seek for local improvement

In order to make an attempt of improvement in the function value in each remaining point, a search in the neighborhood of each body is carried, starting from the definition of a maximum size of the step (θ). This way, after a little disturbance towards *j,* we have the new position of Body *i* determined by:

$$x_{ij}{}^{new} = x_{ij} + \theta \left(2rand - 1 \right) \tag{6}$$

Opposite the movement related to the attraction of bodies, the new position $x_{ij}{}^{new}$ is taken only when it results in an improvement in the current value of the function.

3.1.6 Stop criterion

Due to the method peculiarity, regarding the gradual reduction in the population size, a stop criterion, also particular, can be employed. Thus the optimization process may be finished when a single body remains in space (*Big Crunch*). It is believed that, for an initial population (universe) sufficiently big, the position of this last body is coincident with the global optimum (the method, due to its recent character, still lacks a convergence proof).

Other criteria of usual stop may also be employed, complementarily or in substitution to the aforementioned. In a complementary way, a maximum number of calculations N_{FE} max of the function value was defined.

4. Experimental Results

In this section we present the application of the Big Crunch optimization method to the optimization of a classical 10 bar truss, taken from Rajeev and Krishnamoorthy [5], here considering the cross sections of the members as continuous variables. The constraints imposed to the problem are the allowable stresses and the displacements on nodes. The obtained results are compared with those found on literature.

The BCOM was applied to minimize the weight W of a plane truss, described as follows:

$$f(x) = W = \sum_{i=1}^{10} \rho A_i L_i \tag{7}$$

being A_i and L_i , respectively, the area of cross section and the length of the i-th element. Additionally, the problem is subject to the following constraints, expressed in the normalized form as follows:

$$\frac{\sigma_i}{\sigma_a} - 1 \leq 0 \tag{8}$$

and $$\frac{u_i}{u_a} - 1 \leq 0 \tag{9}$$

being: σ_i the axial stress in element i, σ_a the admissible stress, u_i the maximum displacement of the i-th node (vertical or horizontal) and u_a the admissible displacement.

The consideration of constraints in computational implementation was done by penalizing the objective function:

$$F(x) = f(x)+P(x) \tag{10}$$

In the above expression, *P(X)* is the penalty function for the constraints not met for the current solution. According to this, even if the problem leaves from non-feasible solutions, little violations to the constraints are acceptable, by artificially increasing the mass of the correspondent body.

The geometry of the truss here employed is illustrated in Figure 2. To facilitate the comparisons with the results given by literature, original units were maintained. Thus, the employed data are: longitudinal elasticity modulus E = 10^4 ksi (6.89 x 10^4 MPa), specific weight of material ρ = 0.10 lb/in^3 (2,770 kg/m^3) and vertical loads of 100 kips (445.374 kN) applied from top to bottom on nodes 2 and 4. Admissible stresses are limited to $\pm$ 25 ksi (175.25 MPa), and displacements to 2 in (50.8 mm).

The areas of cross sections, considered as the design variables, may vary initially in a continuous way, within 0.10 and 33.5 in^2.

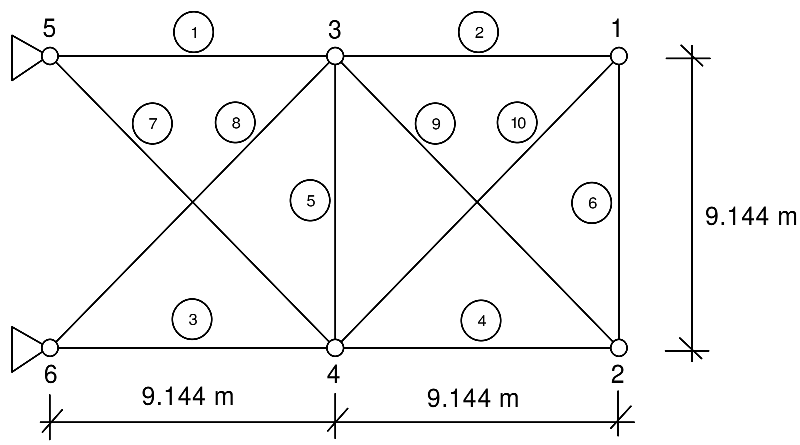

Figure 2. Plane truss with 10 elements.

The analysis of the structure was performed 100 times, varying the main parameters of the method. The lightest weight, with the correspondent transverse cross sections, is listed in Table 1, and was obtained with the following values: α = 0.1, θ = 1.0, N_b = 20 and ε = 0.001. Also in Table 1, this result (BCOM) is compared to the ones given by several other methods [6], listed below the same table.

Table 1. Plane truss with 10 elements (comparison among BCOM and diverse methods)

	W (lb)	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
1	5082.76	30.00	0.10	22.40	16.19	0.10	0.57	7.74	22.15	20.80	0.10
2	5563.32	25.28	1.90	24.87	15.83	0.10	1.75	16.76	19.73	20.98	2.51
3	5471.48	25.77	0.10	25.11	19.39	0.10	0.10	15.36	20.32	20.74	1.14
4	5932.21	30.69	2.37	31.62	11.66	0.10	3.71	21.71	20.90	13.97	3.26
5	5062.34	30.20	0.10	23.39	15.35	0.10	0.53	7.44	20.95	21.68	0.10

1-Genetic Algorithms (GA)
2-Feasible directions (CONMIN)
3-Feasible directions (OPTDYN)
4-External penalty (SUMT)
5-Big Crunch (BCOM, present work)

According to this table, a significant reduction in the final weight of the structure, in relation to other heuristic method (genetic algorithms) may be observed. Moreover, the results obtained were the best among those found in literature.

5. Conclusions and final considerations

In this paper, it was shown the Big Crunch optimization method (BCOM), a heuristic method developed in analogy to the Closed Universe theory. This method may constitute a good alternative for problems involving functions of difficult mathematical treatment, especially non-continuous functions or those with several local optimums. The method has shown to be competitive for several functions analyzed, and the application to the minimum weight of trusses led to even better results than the ones found in the literature.

The method shows as a peculiar characteristic, the fact of working with a population of variable size, on the contrary of other known heuristic methods, which consider a single solution (as Simulated Annealing) or a population of solutions (e.g. Genetic Algorithms or Ant Colony). Due to this, the convergence of the universe of solutions for a single point may also constitute an innovative stop criterion. It is believed that, for an initial population (universe) sufficiently big, the position of this last body is coincident with the global optimum. The method, due to its recent character, still needs a convergence proof.

In general, BCOM is a method of easy computational implementation that involves few control parameters, when compared to other popular heuristics. The adaptation of the method to the treatment of discrete variables may be done with a few modifications on the computational code.

A version of the developed software, in which diverse functions were implemented, among them, the ones presented here, is available from the authors.

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