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Lion Optimization Algorithm (LOA): A nature-inspired metaheuristic algorithm

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Abstract

During the past decade, solving complex optimization problems with metaheuristic algorithms has received considerable attention among practitioners and researchers. Hence, many metaheuristic algorithms have been developed over the last years. Many of these algorithms are inspired by various phenomena of nature. In this paper, a new population based algorithm, the Lion Optimization Algorithm (LOA), is introduced. Special lifestyle of lions and their cooperation characteristics has been the basic motivation for development of this optimization algorithm. Some benchmark problems are selected from the literature, and the solution of the proposed algorithm has been compared with those of some well-known and newest meta-heuristics for these problems. The obtained results confirm the high performance of the proposed algorithm in comparison to the other algorithms used in this paper.

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Keywords: Lion Optimization Algorithm (LOA); Global optimization; Metaheuristic

1. Introduction

Many engineering optimization problems are usually quite difficult to solve, and many applications have to deal with these complex problems. In these problems, search space grows exponentially with the problem size. Therefore, the traditional optimization methods do not provide a suitable solution for them. Hence, over the past few decades, many meta-heuristic algorithms have been designed to solve such problems. Researchers have shown good performance of meta-heuristic algorithms in a wide range of complex problems such as scheduling problems [1–6], data clustering [7,8], image and video processing [9–12], tuning of neural networks [13–15] and pattern recognition [16–18], etc.

For many years, human have utilized the guidance of nature in finding the most appropriate solution for problems. Hence, during the last decades, there has been a growing attempt in developing algorithms inspired by nature [19–21]. For example, Genetic algorithm was proposed by Holland [22], and

simulates Darwnian evolution concepts. Artificial Immune Systems [23], simulate biological immune systems for optimization. Ant Colony Optimization [24] was inspired by behavior of ants foraging for food. Particle Swarm Optimization [25] mimics the social behavior of a flock of migrating birds trying to reach an unknown destination. Marriage in Honey Bee Optimization Algorithm (MBO) was proposed by Abbass [26], and mimics processes of reproduction in the honey bee colony. Bacterial Foraging Algorithm [27] simulates search and optimal foraging of bacteria. The Shuffled Frog Leaping algorithm [28] was inspired by a frog population searching for food. The Cat Swarm algorithm [29] was developed based on the behavior of cats. Invasive weed optimization was proposed by Mehrabian and Lucas [30], and mimics the ecological behavior of colonizing weeds. Monkey Search [31] simulates a monkey in search for food resources. Water flow-like algorithm [32] was inspired by water flowing from higher to lower levels. Biogeography-based optimization algorithm was introduced by Simon [33], and inspired by biogeography which refers to the study of biological organisms in terms of geographical distribution (over time and space). The Fish

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School Search [34] was proposed based on the gregarious behavior of oceanic fish. Cuckoo Search [35] and Cuckoo optimization algorithm [36] are based on reproduction strategy of cuckoos. Bat-inspired Algorithm [37] was inspired by the echolocation behavior of bats. Firefly algorithm [38] simulates the social behavior of fireflies based on their flashing characteristics. Dolphin Partner Optimization [39] and Dolphin echolocation algorithm [40] were inspired by dolphins' behaviors. Flower pollination algorithm [41] mimics the pollination characteristics of flowering plants and the associated flower consistency of some pollinating insects. Krill herd [42] inspired by the herding behavior of krill individuals. Wolf search [43] and Grey Wolf Optimizer [44] are inspired by behaviors of wolves. Water cycle algorithm [45] was based on the observation of water cycle process and how rivers and streams flow to the sea in the real world. The Social spider optimization, inspired by the social behavior of a kind of spider, has been proposed recently [46]. Forest Optimization Algorithm [47] was inspired by few trees in the forests which can survive for many years, while other trees could live for a short time.

Aforementioned algorithms are widely applied by researchers in many different areas [48–51]. However, there is no particular algorithm to gain the most appropriate solution for all optimization problems. Some algorithms provide better solution for some particular problems compared with others. Therefore, pursuing for new optimization techniques is an open problem [52].

In this paper, an optimization algorithm based on lion's behavior and social organization, namely Lion Optimization Algorithm (LOA) is proposed. In the literature, Wang [53] and Rajakumar [54] proposed two algorithms inspired by few characters of lions. Rajakumar [54] described the main operator of Lion's Algorithm as "Mating that refers to deriving new solutions and Territorial Defense and Territorial Takeover intend to find and replace the worst solution by new the best solution". Like Lion's Algorithm, Lion pride optimizer [53] is based on fighting between individual and mating. But lions in addition of mating and fighting exhibit other behaviors such as special style of prey capturing, territorial marking, migration, difference between life style of nomad and resident lions. So, proposed algorithm is inspired by simulation of the solitary and cooperative behaviors of lions which are completely different from the previous algorithm.

After this introduction, the remainder of this paper is structured as follows: In Section 2 the proposed Lion Optimization Algorithm (LOA) is outlined, and its implementation steps are explained in details. Comparative study and experimental results are presented in Section 3 to verify the efficiency of the proposed algorithm. Finally, conclusions are presented in the last section.

2. Lion Optimization Algorithm (LOA)

In this section, the inspiration of the proposed meta-heuristic is first discussed. Then, Lion Optimization Algorithm (LOA) is presented.

2.1. Inspiration

Lions are the most socially inclined of all wild cat species which display high levels of cooperation and antagonism [55]. Lions are of particular interest because of their strong sexual dimorphism in both social behavior and appearance. The lion is a wild felid with two types of social organization: residents and nomads. Residents lives in groups, called pride [56]. A pride of lions typically includes about five females, their cubs of both sexes, and one or more than one adult males. Young males are excluded from their birth pride when they become sexually mature [56]. As mention before, the second organizational behavior is called nomads, who move about sporadically, either in pairs or singularly. Pairs are more seen among related males who have been excluded from their maternal pride. Notice that a lion may switch lifestyles; residents may become nomads and vice versa [56].

Unlike all other cats, Lions typically hunt together with other members of their pride. Several lionesses work together and encircle the prey from different points and catch the victim with a fast attack. Coordinated group hunting brings a greater probability of success in lion hunts. The male lions and some lionesses usually stay and rest while waiting for the hunter lionesses to return from the hunt [57]. Lions do mate at any time of the year, and the females are polyestrous (when females not rearing their cubs are receptive) [58]. A lioness may mate with multiple partners when she is in heat [59]. In nature, male and female lions mark their territory and elsewhere, which seems a good place with urine.

In this work, some characters of lions are mathematically modeled in order to design an optimization algorithm. In the proposed algorithm, Lion Optimization Algorithm (LOA), an initial population is formed by a set of randomly generated solutions called Lions. Some of the lions in the initial population (%N) are selected as nomad lions and rest population (resident lions) is randomly partitioned into P subsets called prides. S percent of the pride's members are considered as female and rest are considered as male, while this rate(sex rate (%S)) in nomad lions is vice versa.

For each lion, the best obtained solution in passed iterations is called best visited position, and during the optimization process is updated progressively. In LOA, a pride territory is an area that consists of each member best visited position. In each pride, some females which are selected randomly go hunting. Hunters move towards the prey to encircle and catch it. The rest of the females move toward different positions of territory. Male lions in pride, roam in territory. Females in prides mate with one or some resident males. In each pride, young males are excluded from their maternal pride and become nomad when they reach maturity and, their power is less than resident males.

Also, a nomad lion (both male and female) moves randomly in the search space to find a better place (solution). If the strong nomad male invade the resident male, the resident male is driven out of the pride by the nomad lion. The nomad male becomes the resident lion. In the evolution, some resident females immigrate from one pride to another or switch their

lifestyles and become nomad and vice versa some nomad female lions join prides. Due to many factors such as lack of food and competition, weakest lion will die or be killed. Above process continues until the stopping condition is satisfied.

2.2. Proposed algorithm

2.2.1. Initialization

The LOA is a population-based meta-heuristic algorithm in which the first step is to randomly generate the population over the solution space. In this algorithm, every single solution is called "Lion". In a N_{var} dimensional optimization problem, a Lion is represented as follows:

$$Lion = [x_1, x_2, x_3, ..., x_{N_{var}}]$$
 (1)

Cost (fitness value) of each Lion is computed by evaluating the cost function, as:

fitness value of lion =
$$f(Lion) = f(x_1, x_2, x_3, ..., x_{N_{var}})$$
 (2)

In first step, N_{pop} solutions are generated randomly in search space. %N of generated solutions are randomly chosen as nomad lions. The rest of the population will be randomly divided into P prides. Every solution in this algorithm has a specific gender and remained constant during the optimization process. To emulating this fact, in each pride %S (%75-%90) of entire population formed in the last step are known as females and the rest as males. For nomad lions, this ratio is vice versa % (1-S). Over the searching process every lion marks its best visited position. According to these marked positions, every pride's territory is formed. So, for each pride, marked positions (best visited positions) by its members form that pride's territory.

2.2.2. Hunting

In each pride some female look for a prey in a group to provide food for their pride. These hunters have specific strategies to encircle the prey and catch it. In general, lions followed approximately the same patterns when hunting [60]. Stander [60] divided the lions into seven different stalking roles, shown in Fig. 1, grouping these roles into Left Wing, Centre and Right Wing positions. During hunting, each lioness corrects its position based on its own position and the positions of members of the group.

Due to this fact that during hunting some of these hunters encircle prey and attack from opposite position, we utilize

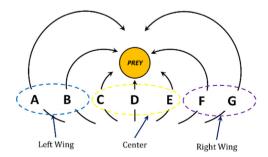


Fig. 1. A schematic of generalized lions hunting behavior [60].

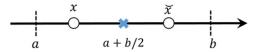


Fig. 2. Opposite point defined in domain [a, b]. x is a candidate solution and x is the opposite of x.

Opposition-Based Learning (OBL). The basic concept of Opposition-Based Learning (OBL) was proposed by Tizhoosh [61] and has been proven to be an effective method for solving optimization problems.

Definition. Let $X(x_1, x_2, x_3, ..., x_{N_{var}})$ be a point in N_{var} -dimensional space, where $x_1, x_2, x_3, ..., x_{N_{var}}$ are real numbers and $x_i \in [a_i, b_i]$, $i = 1, 2, 3, ..., x_{N_{var}}$. The opposite point of X is shown by $X(x_1, x_2, x_3, ..., x_{N_{var}})$ where $x_i = a_i + b_i - x_i$, $i = 1, 2, ..., N_{var}$. The principle of Opposition-Based Learning (OBL) is given in Fig. 2.

According to aforementioned facts, hunters are divided into three sub groups randomly. Group with highest cumulative members' finesses is considered as Center and the other two groups consider as two wings. A dummy prey (PREY) is considered in center of hunters ($PREY = \sum hunters(x_1, x_2, x_3, ..., x_{N_{var}})$ /number of hunters). During hunting, hunters are selected one after another randomly, and each selected hunter attack on dummy prey which this procedure will be defined later according to group that selected lion is belong to that. Throughout hunting, if a hunter improves its own finesses, PREY will escape from hunter and new position of PREY is obtained as follows:

$$PREY' = PREY + rand(0, 1) \times PI \times (PREY - Hunter)$$
 (3)

where PREY is current position of prey, Hunter is new position hunter who attack to prey and PI is the percentage of improvement in fitness of hunter (see Fig. 3).

The following formulas are proposed to mimic encircling prey by mentioned hunter groups. The new positions of hunters which are belong both left and right wing are generated as follows:

$$Hunter' = \begin{cases} rand((2 \times PREY - Hunter), PREY), & (2 \times PREY - Hunter) < PREY \\ rand(PREY, (2 \times PREY - Hunter)), & (2 \times PREY - Hunter) > PREY \end{cases}$$

$$\tag{4}$$

where *PREY* is current position of prey, Hunter is current position hunter and *Hunter'* is new position of hunter. Also, the new positions of Center hunters are generated as follows:

$$Hunter' = \begin{cases} rand(Hunter, PREY), & Hunter < PREY \\ rand(PREY, Hunter), & Hunter > PREY \end{cases}$$
 (5)

In above equations, rand (a, b) generates a random number between a and b, where a and b are upper and lower bounds, respectively. An example of encircling in LOA by Center lion and Wing lion is shown in Fig. 4. The proposed hunting mechanism has some advantages to achieve to the better

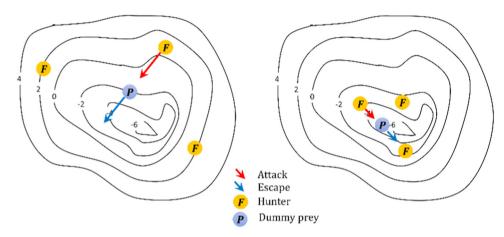


Fig. 3. An example attack and escape.

computed as:

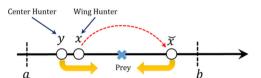


Fig. 4. An example of encircling in LOA.

solutions. Outstanding one is that this strategy provides a circle-shaped neighborhood around the prey, and let hunters to close to prey from different directions. Second, this strategy provides an opportunity for solutions to escape from local optima because some hunters use opposite position.

Therefore, hunting in each pride can be stated as Pseudo-code 1.

Divide hunters into three sub groups randomly
Generate a prey
For i=1:H (H is number of hunters)
Move ith hunter toward prey according to its relevant group
If new place of ith hunter is better than its last position
Prey escapes from hunter
End
End

2.2.3. Moving Toward Safe Place

As mentioned in last subsection, in each pride some females go hunting. Remained females go toward one of the areas of territory. Since territory of each prides consist of personal best so far positions of each member, and assists Lion Optimization Algorithm (LOA) to save the best solutions obtained so far over the course of iteration, it can be used as valuable and reliable information to improve solutions in LOA. Therefore, the new position for a female lion may be given as:

Female Lion' = Female Lion +
$$2D \times rand(0, 1)\{R1\}$$

+ $U(-1, 1) \times tan(\theta) \times D \times \{R2\}$
 $\{R1\}.\{R2\} = 0, ||\{R2\}|| = 1$ (6)

where *Female Lion* is current position of female lion, D shows the distance between the female lion's position and the selected point chosen by tournament selection among the pride's territory. $\{R1\}$ is a vector which its start point is the previous location of the female lion, and its direction is toward the selected position. $\{R2\}$ is perpendicular to $\{R1\}$. Now we

describe our tournament section strategy. First, we define the success of a lion if it improves his or her best position at last iteration of the LOA. In group P the success of lion i at iteration t is defined as:

$$S(i,t,P) = \begin{cases} 1 & Best_{i,P}^{t} < Best_{i,P}^{t-1} \\ 0 & Best_{i,P}^{t} = Best_{i,P}^{t-1} \end{cases}$$
 (7)

where $Best_{i,P}^t$ is the best position found by lion i until iteration t. A high number of successes indicate that the lions have converged to a point that is far from the optimum point. Similarly, a low number of success shows that the lions are swinging around the optimum solution without significant improvement. So this factor can be used as a useful elements for size of a tournament. Using the success values, $K_i(s)$ is

$$K_j(s) = \sum_{i=1}^n S(i, t, P) \quad j = 1, 2, ..., P$$
 (8)

where n is the number of lion in pride and $K_j(s)$ is the number of the lions in pride j which have had an improvement in their fitness in the last iteration. So tournament size in each pride is adaptive in every iteration. It means when success value decrease, tournament size is increased and it lead to increase diversity. Therefore, tournament size is calculated as follows:

$$T_j^{Size} = \max\left(2, ceil\left(\frac{K_j(s)}{2}\right)\right) \quad j = 1, 2, ..., P$$
 (9)

The Pseudo-code of this operator is as follow:

For i =1 to P(P is number of prides)
calculate tournament size for ith pride
For j =1 to R (R is number of remained female in ith pride)
Select a place among pride's territory by tournament selection
Move jth female toward selected place
End
End

An example of this strategy is shown in Fig. 5.

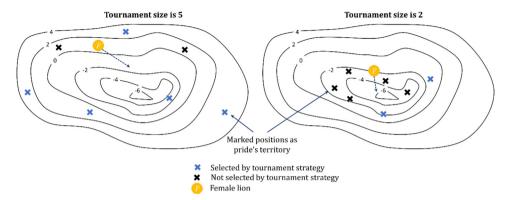


Fig. 5. Schematic view of different sizes of tournament.

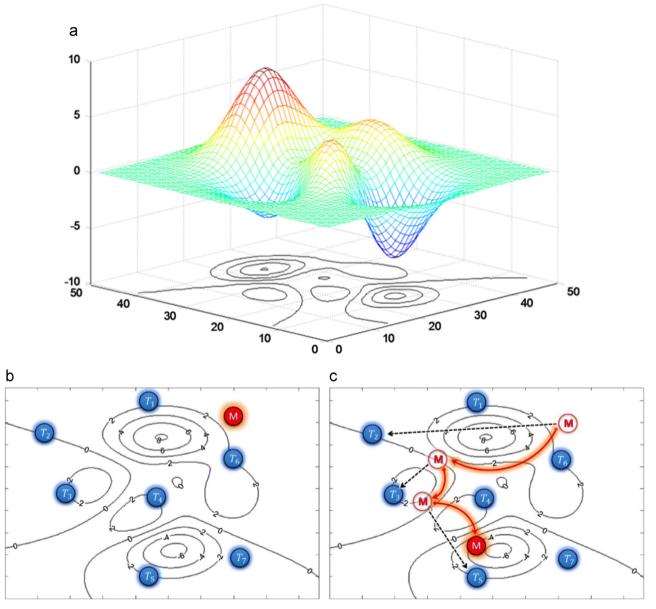


Fig. 6. Example of resident male roaming.

2.2.4. Roaming

Each male lion in a pride roams in that pride's territory due to some reasons. To emulate this behavior of resident males, % R of pride territory are selected randomly and are visited by that lion. Along roaming, if resident male visits a new position which is better than its current best position, update his best visited solution. This roaming is a strong local search and assists Lion Optimization Algorithm (LOA) to search around of a solution to improve it. This progress is shown in Fig. 6. As shown in Fig. 6, lion moves toward the selected area of territory by x units, wherein x is a random number with uniform distribution.

$$x \sim U(0, 2 \times d) \tag{10}$$

where d shows the distance between the male lion's position and the selected area of territory. The vector from the male lion's position to the selected area of territory shows the original direction of movement. To provide a chance for searching a wider area around current solution and adding intensification property to the method and to search for a wider area around current solution, the angle θ is added to this direction. It seems an angle which selected by uniform distribution among $-\pi/6$ (rad) and $\pi/6$ (rad) is adequate for this goal. In the Pseudo-code 3, the behavior of the male lion is shown:

For i = 1 to RM (RM is number of resident male)
Select %R of territory randomly to visit by ith male
For j = 1 to S (S is number of selected place in last step)
Go toward place jth
If new place of ith male better than its personal best visited position
Marking that place as territory (update best visited position)
End
End
select the best visited position by ith male as its current position
End

Also, a nomad lion (both male and female) moving randomly in search space as Pseudo-code 4.

For i = 1 to NN (NN is number of nomad lions)
Move ith nomad randomly
If new place of ith nomad is better than its personal best visited position
update ith nomad's best visited position
End
End

Nomad lions and their adaptive roaming assist proposed algorithm to search solution space randomly and avoid to trap in local optima. In the above procedure, new position of nomad lions is generated as follows:

$$\operatorname{Lion}_{ij}' = \begin{cases} \operatorname{Lion}_{ij} & \text{if } \operatorname{rand}_{j} > pr_{i} \\ \operatorname{RAND}_{j} & \text{otherwise} \end{cases}$$
 (11)

where $Lion_i$ is current position of ith nomad lion, j is dimension, rand_j is a uniform random number within [0, 1], RAND is random generated vector in search space, and pr_i is a probability that is calculated for each nomad lion independently as follows:

$$pr_i = 0.1 + \min\left(0.5, \frac{(\text{Nomad}_i - \text{Best}_{nomad})}{\text{Best}_{nomad}}\right) \ i = 1, 2, ...,$$
number of nomad lions (12)

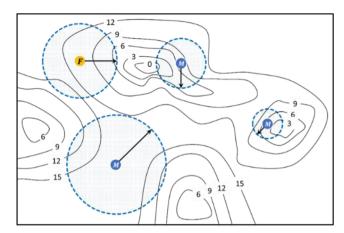


Fig. 7. The method of generating a new position for nomad lions according their fitness.

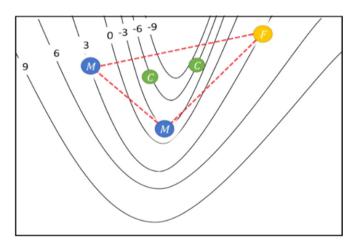


Fig. 8. Example of the mating operation: one female mate with two male.

where Nomad_i and Best_{nomad} are cost of current position of the *i*th lion in nomads and the best cost of the nomad lion, respectively. A clear visualization of how to generate new position of nomad lions is presented in Figs. 7 and 8. As shown in Fig. 7, this procedure provides an opportunity for the worst nomads to escape from unsuitable region with high probability.

2.2.5. *Mating*

Mating is an essential process that assures the lions' survival, as well as providing an opportunity for information exchange among members. In every pride, ${}^{0}\!\!/Ma$ of female lions mate with one or several resident males. These males are selected randomly from the same pride as the female to produce offspring. For nomad lions it's different in that a nomad female only mates with one of the males which are selected randomly. The mating operator is a linear combination of parents for producing two new offspring. According to the following equations, the new cubs are produced after selecting the female lion and male(s) for mating:

Offspring_j
$$1 = \beta \times \text{Female Lion}_j + \sum_{\substack{i=1 \ NR}} \frac{(1-\beta)}{\sum_{i=1}^{NR} S_i}$$

$$\times \text{MaleLion}_j^i \times S_i \tag{13}$$

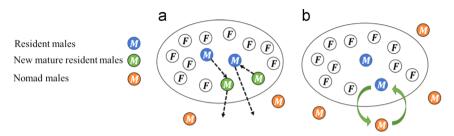


Fig. 9. (a) Defense against new mature resident males and (b) Defense against nomad males.

Offspring_j
$$2 = (1 - \beta) \times \text{Female Lion}_j + \sum_{i=1}^{\beta} \frac{\beta}{\sum_{i=1}^{NR} S_i}$$

$$\times \text{MaleLion}_i^i \times S_i \tag{14}$$

where j is dimension, S_i equals 1 if male i is selected for mating, otherwise it equals 0, NR is the number of resident males in a pride, β is a randomly generated number with a normal distribution with mean value 0.5 and standard deviation 0.1. One of two new offspring is randomly selected as male and the other one as female. A mutation is applied on each gene of one of the produced offspring with probability (%Mu). A random number replaces the value of gene. By Mating, LOA share information between genders while new cubs inherit character from both genders.

2.2.6. Defense

In a pride, male lions when mature, they become aggressive and fight other males in their pride. Beaten males abandon their pride and become a nomad. On the other hand, if a nomad male lion is strong enough to try to take over a pride by fighting its males, the beaten resident male lion is driven out of the pride and becoming a nomad. Defense operator in LOA divided into two main steps:

- I. Defense against new mature resident males.
- II. Defense against nomad males.

Therefore, defense against new mature resident males in each pride can be described as **Pseudo-code 5.**

Merge new mature males and old males

Sort all males according their fitness

Weakest males drive out of the pride and become nomad and remained males become resident males

The pseudo-code for defense against nomad males is given as below:

For i = 1 to number of nomad lions
BT [1, P] = Create binary template ([1×P]) and assign a randomly generated binary (0-1) to each cell (P is number of prides).
For j = 1 to P
If jth element of BT == 1
For z = 1 to NR (number of resident males in jth pride)
If ith nomad male is better than zth resident male in jth pride
zth resident male in jth pride drive out of the pride and become nomad and ith nomad male become resident
Go next i
End
End
End
End
End
End

Defense against new mature resident males and Defense against nomad males are depicted in Fig. 9. Defense operator assists Lion Optimization Algorithm (LOA) to retain powerful male lions as solutions that play an important role in LOA.

2.2.7. Migration

Inspired by the lion switch life and migratory behavior in the nature when one lion travels from one pride to another or switch its lifestyle and resident female become nomad and vice versa, it enhances the diversity of the target pride by its position in the previous pride. On the other hand, the lion's migration and switch lifestyle builds the bridge for information exchange.

In each pride, the maximum number of females is determined by S% of population of the pride. For migration operator, some female selected randomly and become nomads. Size of migrated female in each pride is equal to Surplus females in each pride plus %I of the maximum number of females in a pride. When selected females migrate from prides and become nomad, new nomad females and old nomad females are sorted according to their fitness. Then, the best females among them are selected randomly and distributed to prides to fill the empty place of migrated females. This procedure maintains the diversity of the whole population and share information among prides.

2.2.8. Lions' Population Equilibrium

Since there is always equilibrium in lions' population, at the end of each iteration, the number of live lions will be controlled. Respect to the maximum permitted number of each gender in nomads; nomad lions with the least fitness value will be removed. Fig. 10 depicts an example of migration operator and Lions' Population Equilibrium.

2.2.9. Convergence

For stopping condition, as commonly considered in optimization algorithms, the best result is calculated where the stopping condition may be assumed as the CPU time, maximum number of iterations, number of iterations without improvement etc.

The main steps of the LOA are summarized in the pseudo code shown in Fig. 11. To see how Lion Optimization Algorithm (LOA) is able to solve optimization problems, some points may be noted:

• In LOA, each solution has specific gender and each gender has its own strategy for searching. It assists Lion

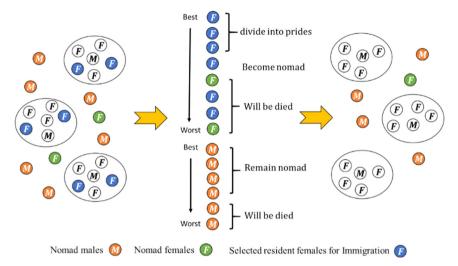


Fig. 10. Migration and Lions' Population Equilibrium.

Lion Optimization Algorithm pseudo code

- 1. Generate random sample of Lions $N_{pop}(N_{pop}$ is number of initial population).
- 2. Initiate prides and nomad lions
 - i. Randomly select %N (Percent of lions that are nomad) of initial population as nomad lion. Partition remained lions into P (P is number of prides) prides randomly, and formed each pride's territory.
 - ii. In each pride %S (Sex rate) of entire population are known as females and the rest as males. This rate in nomad lions is inversed.
- 3. For each pride do
 - i. Some randomly selected female lion go hunting.
 - ii. Each of remained female lion in pride go toward one of the best selected position from territory.
- iii. In pride, for each resident male; %R (Roaming percent) of territory randomly are selected and checked. %Ma (Mating probability) of females in pride mate with one or several resident male. → New cubs become mature.
- iv. Weakest male drive out from pride and become nomad.
- 4. For Nomad do
 - i. Nomad lion (both male and female) moving randomly in search space.
 Ma (Mating probability) of nomad Female mate with one of the best nomad male. → New cubs become mature.
 - ii. Prides randomly attacked by nomad male.
- $5. \ For each pride do\\$
 - i. Some female with I rate ((Immigrate rate)) immigrate from pride and become nomad.
- 6. Do
 - i. First, based on their fitness value each gender of the nomad lions are sorted. After that, the best females among them are selected and distributed to prides filling empty places of migrated females.
 - ii. With respect to the maximum permitted number of each gender, nomad lions with the least fitness value will be removed.

If termination criterion is not satisfied, then go to step 3

Fig. 11. Pseudocode for Lion Optimization Algorithm.

Optimization Algorithm (LOA) to look for optimal point by different strategies.

- The general aim in using several prides is that each pride focuses on a specific region and balance between exploration and exploitation. Its character of LOA increases
- capability of it to fit for the optimization on multi-modal problems.
- Nomad lions and their adaptive roaming assist Lion Optimization Algorithm (LOA) to search solution space randomly and escape from local optima.

- Personal best so far positions of lions can provide valuable and reliable information found so far by the population. The proposed pride's territory assists Lion Optimization Algorithm (LOA) to save the best solutions obtained so far over the course of iteration.
- In LOA, by mating, lions share information between genders while new cubs inherit character from both genders.
- Resident males roaming assist Lion Optimization Algorithm (LOA) to exploit information from their respective pride's neighbors that hold valuable knowledge. This procedure can be considered as a strong local search.
- The proposed encircling mechanism during hunting has two advantages; first of all, provides a circle-shaped neighborhood around the solutions and let opportunity for hunter to close to prey from different directions and second provides an opportunity for solutions to escape from local optima.

3. Experimental results

To evaluate the performance of Lion Optimization Algorithm, a comprehensive set of 30 benchmark functions of the CEC 2014 competition on Single Objective Real-Parameter Numerical Optimization are selected from [62]. These functions are given in Table 1. Details definitions of the functions can be found in [62]. The proposed LOA algorithm

is compared with six recent popular metaheuristic methods (Invasive weed optimization (IWO) [30] algorithm, biogeography-based optimization (BBO) [33] algorithm, gravitational search algorithm (GSA) [63], hunting search (HuS) algorithm [64], bat algorithm (BA) [37], Water wave optimization (WWO) [65] algorithm). The results are given in Section 3.1. The recommended parameter settings of six first algorithms are as [65].

In all cases, population size is set to 50. The dimension is 30 (n=30), and a maximum number of function evaluation set as stopping condition; function evaluations are 150,000 for all functions. Parameters of the proposed algorithm are tuned by response surface methodology (RSM) [66] are described in Table 2.

The results, over 60 runs, are reported in Table 3 for functions. Following performance indexes are reported in

Table 2 Parameters values of compared algorithms.

Parameter	Value	Parameter	Value	
Number of pride	4	Sex rate	0.8	
Percent of nomad lions	0.2	Mating probability	0.3	
Roaming percent	0.2	Immigrate rate	0.4	
Mutate probability	0.2	-		

Table 1 Test functions used in the experimental study.

Type	ID	Function	f*
Unimodal	f1	Rotated high conditioned elliptic function	100
	f2	Rotated bent cigar function	200
	f3	Rotated discus function	300
Multimodal	f4	Shifted and rotated Rosenbrock function	400
	f5	Shifted and rotated Ackley's function	500
	f6	Shifted and rotated Weierstrass function	600
	f7	Shifted and rotated Griewank's function	700
	f8	Shifted Rastrigin function	800
	f9	Shifted and rotated Rastrigin's function	900
	f10	Shifted Schwefel function	1000
	f11	Shifted and rotated Schwefel's function	1100
	f12	Shifted and rotated Katsuura function	1200
	f13	Shifted and rotated HappyCat function	1300
	f14	Shifted and rotated HGBat function	1400
	f15	Shifted and rotated Expanded Griewank's plus Rosenbrock's function	1500
	f16	Shifted and rotated Expanded Scaffer's F6 function	1600
Hybrid	f17	Hybrid function1 (f 9, f 8, f 1)	1700
	f18	Hybrid function2 (f 2, f 12, f 8)	1800
	f19	Hybrid function3 (f 7, f 6, f 4, f 14)	1900
	f20	Hybrid function4 (f 12, f 3, f 13, f 8)	2000
	f21	Hybrid function5 (f 14, f 12, f 4, f 9, f 1)	2100
	f22	Hybrid function6 (f 10, f 11, f 13, f 9, f 5)	2200
Composition	f23	Composition function1 (f 4, f 1, f 2, f 3, f 1)	2300
	f24	Composition function2 (f 10, f 9, f 14)	2400
	f25	Composition function3 (f 11, f 9, f 1)	2500
	f26	Composition function4 (f 11, f 13, f 1, f 6, f 7)	2600
	f27	Composition function5 (f 14, f 9, f 11, f 6, f 1)	2700
	f28	Composition function6 (f 15, f 13, f 11, f 16, f 1)	2800
	f29	Composition function7 (f 17, f 18, f 19)	2900
	f30	Composition function8 (f 20, f 21, f 22)	3000

Table 3 Comparative results on unimodal benchmark functions.

		IWO	BBO	GSA	HuS	BA	WWO	LOA
f1	Maximum	2.77E+06	8.09E+07	5.31E+07	1.26E+07	5.51E+08	1.17E+06	3.90E+05
	Minimum	3.44E + 05	5.75E + 06	4.56E + 06	1.61E + 06	1.18E + 08	1.44E + 05	2.43E + 04
	Median	1.42E + 06	2.14E + 07	8.37E + 06	5.10E + 06	3.10E + 08	6.26E + 05	1.45E + 05
	std	5.72E + 05	1.67E + 07	1.32E + 07	2.62E + 06	1.05E + 08	2.45E + 05	1.32E + 05
f2	Maximum	4.06E + 04	8.04E + 06	1.61E + 04	2.41E + 04	6.35E + 09	1.48E + 03	1.85E + 03
	Minimum	6.09E + 03	1.15E + 06	3.47E + 03	3.09E + 02	1.13E + 09	2.00E + 02	2.00E + 02
	Median	1.52E + 04	3.95E + 06	8.38E + 03	9.09E + 03	2.49E + 09	2.68E + 02	6.83E + 02
	std	8.67E + 03	1.55E + 06	2.90E + 03	6.01E + 03	7.55E + 08	2.02E + 02	4.96E + 02
f3	Maximum	1.50E + 04	5.07E + 04	7.58E + 04	3.36E + 03	1.11E+05	1.32E + 03	1.30E + 03
	Minimum	3.50E + 03	5.92E + 02	2.04E + 04	3.00E + 02	3.44E + 04	3.15E + 02	3.00E + 02
	Median	7.29E + 03	7.65E + 03	4.51E + 04	3.02E + 02	7.19E + 04	4.87E + 02	5.29E + 02
	std	2.69E + 03	1.28E + 04	1.04E + 04	5.41E + 02	1.75E + 04	1.85E + 02	3.20E + 02

Table 4 Comparative results on multimodal benchmark functions.

		IWO	ВВО	GSA	HuS	BA	wwo	LOA
f4	Maximum	5.45E+02	6.54E+02	8.49E+02	5.64E+02	1.26E+04	5.42E+02	5.45E+02
	Minimum	4.02E + 02	4.23E + 02	5.73E + 02	4.04E + 02	2.01E + 03	4.00E + 02	4.00E + 02
	Median	5.11E + 02	5.42E + 02	6.82E + 02	5.03E + 02	3.05E + 03	4.02E + 02	4.26E + 02
	std	2.88E + 01	3.84E + 01	5.15E + 01	3.66E + 01	1.97E + 03	3.64E + 01	4.81E + 01
f5	Maximum	5.20E + 02	5.20E + 02	5.20E + 02	5.21E + 02	5.21E + 02	5.20E + 02	5.10E + 02
	Minimum	5.20E + 02	5.20E + 02	5.20E + 02	5.21E + 02	5.21E + 02	5.20E + 02	5.00E + 02
	Median	5.20E + 02	5.20E + 02	5.20E + 02	5.21E + 02	5.21E + 02	5.20E + 02	5.03E + 02
	std	3.77E - 03	4.22E - 02	6.47E - 04	7.83E - 02	4.81E - 02	6.98E - 04	3.73E + 00
f6	Maximum	6.05E + 02	6.18E + 02	6.24E + 02	6.29E + 02	6.39E + 02	6.13E + 02	6.05E + 02
	Minimum	6.00E + 02	6.08E + 02	6.17E + 02	6.19E + 02	6.32E + 02	6.01E + 02	6.00E + 02
	Median	6.02E + 02	6.14E + 02	6.20E + 02	6.23E + 02	6.37E + 02	6.06E + 02	6.01E + 02
	std	1.12E + 00	2.35E + 00	1.83E + 00	2.18E + 00	1.56E + 00	2.62E + 00	2.17E + 00
f7	Maximum	7.00E + 02	7.01E + 02	7.00E + 02	7.00E + 02	9.63E + 02	7.00E + 02	7.00E + 02
	Minimum	7.00E + 02	7.01E + 02	7.00E + 02	7.00E + 02	8.19E + 02	7.00E + 02	7.00E + 02
	Median	7.00E + 02	7.01E + 02	7.00E + 02	7.00E + 02	9.12E + 02	7.00E + 02	7.00E + 02
	std	1.21E - 02	2.64E - 02	9.55E - 04	5.56E - 02	3.23E - 01	6.26E - 03	8.55E - 04
f8	Maximum	8.75E + 02	9.39E + 02	8.01E + 02	9.75E + 02	1.12E + 03	8.15E + 02	8.11E + 02
	Minimum	8.27E + 02	8.39E + 02	8.00E + 02	9.10E + 02	9.76E + 02	8.00E + 02	8.00E + 02
	Median	8.43E + 02	8.79E + 02	8.00E + 02	9.40E + 02	1.07E + 03	8.00E + 02	8.02E + 02
	std	1.01E + 01	2.07E + 01	2.06E - 01	1.27E + 01	2.56E + 01	2.34E + 00	3.81E + 00
f9	Maximum	9.78E + 02	9.84E + 02	1.10E + 03	1.09E + 03	1.34E + 03	9.84E + 02	9.10E + 02
	Minimum	9.30E + 02	9.35E + 02	1.02E + 03	9.59E + 02	1.15E + 03	9.35E + 02	9.00E + 02
	Median	9.46E + 02	9.49E + 02	1.06E + 03	1.01E + 03	1.25E + 03	9.61E + 02	9.03E + 02
	std	1.14E + 01	1.14E + 01	1.74E + 01	2.60E + 01	4.41E + 01	1.11E + 01	3.78E + 00
f10	Maximum	3.57E + 03	1.00E + 03	5.25E + 03	3.21E + 03	7.45E + 03	2.71E + 03	1.00E + 03
	Minimum	1.59E + 03	1.00E + 03	3.45E + 03	1.36E + 03	5.26E + 03	1.02E + 03	1.00E + 03
	Median	2.58E + 03	1.00E + 03	4.37E + 03	2.17E + 03	6.47E + 03	1.49E + 03	1.00E + 03
	std	3.80E + 02	6.80E - 01	3.61E + 02	4.33E + 02	5.19E + 02	3.62E + 02	9.00E - 02
f11	Maximum	3.80E + 03	4.51E + 03	6.35E + 03	4.23E + 03	8.75E + 03	3.89E + 03	1.12E + 03
	Minimum	1.48E + 03	2.12E + 03	3.70E + 03	2.20E + 03	7.20E + 03	2.49E + 03	1.10E + 03
	Median	2.92E + 03	3.32E + 03	4.99E + 03	3.24E + 03	8.24E + 03	3.38E + 03	1.11E + 03
	std	4.48E + 02	5.12E + 02	5.67E + 02	4.66E + 02	3.62E + 02	2.89E + 02	3.84E + 00
f12	Maximum	1.20E + 03						
	Minimum	1.20E + 03						
	Median	1.20E + 03	1.20E+03					
	std	1.48E - 02	5.62E - 02	1.00E - 03	7.77E - 02	3.34E - 01	5.61E - 02	2.30E - 02
f13	Maximum	1.30E + 03						
	Minimum	1.30E + 03						
	Median	1.30E + 03						
	std	6.50E - 02	1.06E - 01	6.65E - 02	6.50E - 02	5.48E-01	6.41E - 02	0.00E + 00
f14	Maximum	1.40E + 03	1.40E + 03	1.40E + 03	1.40E + 03	1.50E + 03	1.40E + 03	1.40E + 03
	Minimum	1.40E + 03	1.40E + 03	1.40E + 03	1.40E + 03	1.44E + 03	1.40E + 03	1.40E + 03
	Median	1.40E + 03	1.40E + 03	1.40E + 03	1.40E + 03	1.47E + 03	1.40E + 03	1.40E + 03

Table 4 (continued)

		IWO	BBO	GSA	HuS	BA	WWO	LOA
	std	1.19E-01	1.99E-01	4.23E-02	4.74E-02	1.39E-01	4.41E-02	0.00E+00
f15	Maximum	1.51E + 03	1.53E + 03	1.51E + 03	1.52E + 03	5.92E + 05	1.50E + 03	1.51E + 03
	Minimum	1.50E + 03	1.51E + 03	1.50E + 03	1.51E + 03	1.59E + 04	1.50E + 03	1.50E + 03
	Median	1.50E + 03	1.51E + 03	1.50E + 03	1.52E + 03	1.55E + 05	1.50E + 03	1.50E + 03
	std	8.48E - 01	4.30E + 00	7.30E - 01	3.27E + 00	1.40E + 05	7.75E - 01	3.51E+00
f16	Maximum	1.61E + 03						
	Minimum	1.61E + 03	1.61E+03	1.61E+03	1.61E+03	1.61E + 03	1.61E + 03	1.60E + 03
	Median	1.61E + 03	1.61E + 03	1.61E+03	1.61E + 03	1.61E + 03	1.61E + 03	1.60E + 03
	std	6.14E - 01	5.92E - 01	3.43E - 01	7.25E - 01	1.90E - 01	4.67E - 01	1.79E + 00

Table 5
Comparative results on hybrid benchmark functions.

		IWO	BBO	GSA	HuS	BA	WWO	LOA
f17	Maximum	3.50E+05	2.31E+07	1.14E+06	1.10E+06	9.90E+06	6.16E+04	1.80E+03
	Minimum	5.37E + 03	1.26E + 06	1.85E + 05	1.43E + 04	1.45E + 06	6.71E + 03	1.70E + 03
	Median	6.75E + 04	3.13E + 06	5.63E + 05	1.51E + 05	4.24E + 06	2.61E + 04	1.73E + 03
	std	6.85E + 04	4.19E + 06	2.20E + 05	1.61E + 05	1.79E + 06	1.24E + 04	3.10E + 01
f18	Maximum	1.80E + 04	1.03E + 05	4.20E + 03	1.09E + 04	3.64E + 08	2.73E + 03	1.85E + 03
	Minimum	2.26E + 03	6.74E + 03	2.02E + 03	2.02E + 03	1.33E + 07	1.85E + 03	1.80E + 03
	Median	4.35E + 03	2.28E + 04	2.13E + 03	2.73E + 03	8.54E + 07	2.01E + 03	1.82E + 03
	std	3.69E + 03	1.97E + 04	3.78E + 02	2.25E + 03	1.00E + 08	1.25E + 02	1.63E + 01
f19	Maximum	1.91E + 03	1.98E + 03	2.00E + 03	2.04E + 03	2.06E + 06	1.91E + 03	1.92E + 03
	Minimum	1.90E + 03	1.91E + 03	1.91E + 03	1.91E + 03	1.95E + 03	1.90E + 03	1.90E + 03
	Median	1.91E + 03	1.91E + 03	2.00E + 03	1.92E + 03	2.01E + 03	1.91E + 03	1.90E + 03
	std	1.65E + 00	2.77E + 01	3.43E + 01	3.31E + 01	2.03E + 01	1.38E + 00	7.12E + 00
f20	Maximum	5.34E + 03	8.62E + 04	6.82E + 04	6.03E + 04	4.44E + 04	1.58E + 04	2.00E + 03
	Minimum	2.30E + 03	8.64E + 03	2.32E + 03	2.22E + 04	5.40E + 03	2.14E + 03	2.00E + 03
	Median	2.74E + 03	2.72E + 04	1.77E + 04	3.68E + 04	1.63E + 04	4.25E + 03	2.00E + 03
	std	7.00E + 02	1.76E + 04	1.39E + 04	8.49E + 03	1.03E + 04	3.18E + 03	4.62E - 01
f21	Maximum	9.03E + 04	1.67E + 06	3.09E + 05	1.66E + 05	3.34E + 06	1.76E + 05	2.11E + 03
	Minimum	6.74E + 03	6.70E + 04	5.87E + 04	1.07E + 04	1.43E + 05	3.70E + 03	2.10E + 03
	Median	3.35E + 04	4.22E + 05	1.71E + 05	4.70E + 04	9.17E + 05	2.92E + 04	2.10E + 03
	std	2.30E + 04	3.35E + 05	6.53E + 04	4.24E + 04	7.51E + 05	3.50E + 04	2.06E + 00
f22	Maximum	2.52E + 03	3.28E + 03	3.63E + 03	3.67E + 03	3.56E + 03	2.85E + 03	2.21E + 03
	Minimum	2.23E + 03	2.25E + 03	2.63E + 03	2.37E + 03	2.72E + 03	2.22E + 03	2.21E + 03
	Median	2.36E + 03	2.71E + 03	3.15E + 03	3.08E + 03	3.14E + 03	2.48E + 03	2.21E + 03
	std	7.34E + 01	2.34E + 02	2.50E + 02	2.67E + 02	2.05E + 02	1.43E + 02	4.86E - 01

these tables: the best performance over 60 runs, "maximum" and "minimum" respectively denote the maximum and minimum fitness values of the algorithm, "Median" denotes the median of the result fitness values, "std" denotes standard deviation.

According to Table 3, LOA provides much better results than all algorithms on f1 in all criteria. LOA obtains the best minimum value on f2, and obtains the second best value for the other criteria in this function. Also, LOA provides the best maximum and minimum values on f3. To sum up, LOA is capable of handling these types of problems very effectively.

On the second multimodal group of 13 functions, due to a large number of local optima, finding good solutions and scape from local optima is very hard. But, according to Table 4, LOA exhibits significant performance, and provides much better results than all algorithms on these functions.

On the third hybrid group of six functions, the variables are randomly divided into some subcomponents and then different basic functions are used for different subcomponents, which causes significant performance reduction of algorithms. As it can be seen in Table 5, the overall performance of LOA is significantly different from other algorithms on these types of functions, and almost on all functions its results are very much better than the other algorithms.

On the fourth composition group of eight functions, LOA ranks first on most of the functions (see Table 6). However, it must be mentioned that performance of LOA is a bit weak in f24, f28, f29. In summary, the overall performance of LOA is the best among the other five comparative algorithms on the benchmark suite, including unimodal, multimodal, hybrid, and composition functions. On several test functions among these 30 functions, the performance of LOA is not very satisfactory.

4. Conclusion

Over past decades, various metaheuristic optimization algorithms have been developed. Many of these algorithms are inspired by natural phenomena. In this study, a new

Table 6
Comparative results on composition benchmark functions.

		IWO	BBO	GSA	HuS	BA	WWO	LOA
f23	Maximum	2.62E+03	2.62E+03	2.65E+03	2.62E+03	2.88E+03	2.62E+03	2.74E+03
	Minimum	2.62E + 03	2.62E + 03	2.50E + 03	2.62E + 03	2.51E + 03	2.62E + 03	2.47E + 03
	Median	2.62E + 03	2.62E + 03	2.56E + 03	2.62E + 03	2.51E + 03	2.62E + 03	2.55E + 03
	std	7.95E + 02	1.32E + 00	6.45E + 01	8.45E + 01	1.28E + 02	1.45E + 01	8.93E + 01
f24	Maximum	2.63E + 03	2.65E + 03	2.60E + 03	2.71E + 03	2.60E + 03	2.63E + 03	2.67E + 03
	Minimum	2.60E + 03	2.63E + 03	2.60E + 03	2.63E + 03	2.60E + 03	2.62E + 03	2.60E + 03
	Median	2.62E + 03	2.63E + 03	2.60E + 03	2.66E + 03	2.60E + 03	2.63E + 03	2.62E + 03
	std	1.08E + 01	5.97E + 00	1.71E - 02	1.25E + 01	1.20E + 00	6.89E + 00	2.33E + 01
f25	Maximum	2.71E + 03	2.72E + 03	2.71E + 03	2.75E + 03	2.76E + 03	2.72E + 03	2.71E + 03
	Minimum	2.70E + 03	2.71E + 03	2.70E + 03	2.71E + 03	2.70E + 03	2.70E + 03	2.52E + 03
	Median	2.70E + 03	2.71E + 03	2.70E + 03	2.72E + 03	2.70E + 03	2.71E + 03	2.56E + 03
	std	8.08E - 01	3.01E + 00	1.32E + 00	6.27E + 00	1.50E + 01	2.00E + 00	6.93E + 01
f26	Maximum	2.70E + 03	2.80E + 03	2.80E + 03	2.80E + 03	2.70E + 03	2.70E + 03	2.61E + 03
	Minimum	2.70E + 03	2.70E + 03	2.80E + 03	2.70E + 03	2.70E + 03	2.70E + 03	2.60E + 03
	Median	2.70E + 03	2.70E + 03	2.80E + 03	2.80E + 03	2.70E + 03	2.70E + 03	2.61E + 03
	std	5.43E + 02	2.20E + 01	5.43E + 03	3.53E + 01	5.37E + 01	6.50E + 02	3.06E + 00
f27	Maximum	3.10E + 03	3.51E + 03	4.43E + 03	6.47E + 03	3.53E + 03	3.50E + 03	2.72E + 03
	Minimum	3.01E + 03	3.24E + 03	3.10E + 03	3.57E + 03	3.21E + 03	3.10E + 03	2.70E + 03
	Median	3.10E + 03	3.40E + 03	3.82E + 03	4.84E + 03	3.31E + 03	3.10E + 03	2.71E + 03
	std	3.38E + 01	6.35E + 01	3.51E + 02	6.83E + 02	6.46E + 01	5.90E + 01	5.79E + 00
f28	Maximum	3.85E + 03	4.27E + 03	6.92E + 03	6.65E + 03	6.10E + 03	5.39E + 03	7.09E + 03
	Minimum	3.56E + 03	3.61E + 03	3.76E + 03	4.70E + 03	3.01E + 03	3.10E + 03	3.15E + 03
	Median	3.69E + 03	3.79E + 03	5.43E + 03	5.36E + 03	4.52E + 03	3.78E + 03	4.25E + 03
	std	4.12E + 01	9.33E + 01	7.15E + 02	4.61E + 02	5.93E + 02	3.61E + 02	1.27E + 03
f29	Maximum	2.79E + 04	8.64E + 06	2.93E + 06	4.11E + 07	1.36E + 07	5.06E + 03	6.50E + 04
	Minimum	5.37E + 03	4.26E + 03	3.10E + 03	4.81E + 03	6.16E + 05	3.56E + 03	3.31E + 03
	Median	1.58E + 04	5.26E + 03	3.10E + 03	1.54E + 04	4.21E + 06	4.02E + 03	1.80E + 04
	std	5.14E + 03	1.11E+06	3.78E + 05	7.70E + 06	2.83E + 06	3.60E + 02	2.14E + 04
f30	Maximum	1.69E + 04	3.75E + 04	1.14E + 05	3.74E + 04	5.08E + 05	7.66E + 03	3.18E + 03
	Minimum	6.05E + 03	7.78E + 03	1.22E + 04	8.27E + 03	6.26E + 04	4.25E + 03	3.02E + 03
	Median	8.85E + 03	1.56E + 04	1.46E + 04	1.51E + 04	1.77E + 05	5.63E + 03	3.06E + 03
	std	2.08E + 03	6.08E + 03	1.84E + 04	6.58E + 03	9.11E + 04	7.38E + 02	5.31E+01

optimization algorithm that is called Lion Optimization Algorithm (LOA), is introduced. LOA is constructed based on simulation of the solitary and cooperative behaviors of lions such as prey capturing, mating, territorial marking, defense and the other behaviors. In order to evaluate performance of the introduced algorithm, we have tested it on a set of various standard benchmark functions. The results obtained by LOA in most cases provide superior results in fast convergence and global optima achievement, and in all cases are comparable with other metaheuristics.

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