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Technical Note

Exact analytical solutions of the forward non-ideal diode equation with series and shunt parasitic resistances

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Abstract

Exact closed form solutions based on the Lambert W-function are presented to express the forward current–voltage characteristics of non-ideal single-exponential diodes containing all possible combinations of series and shunt parasitic resistances. It is shown that these expressions could be useful for carrying out highly accurate computations at speeds almost as fast as those possible when using less precise approximate solutions based on common elementary functions. © 2000 Published by Elsevier Science Ltd. All rights reserved.

Keywords: Non-ideal diode; Parasitic resistance; Approximate diode model; Diode forward-current analytical solution; Lambert W-function

1. Introduction

Solving the equation for the current–voltage characteristics of non-ideal diodes with either (or both) series or shunt parasitic resistances has traditionally been approached through the use of iteration or analytic approximations [1–3] due to the absence of explicit solutions containing only common elementary functions. However, recently an exact analytical solution [4], based on the Lambert W-function, has been offered for the case of a non-ideal diode model comprised of a single exponential and a series parasitic resistance. The Lambert W-function, also labeled simply "W-function", originated from Lambert's work [5] on trinomial equations and was first discussed in 1779 by Euler [6]. This function, defined by the solutions of $W\exp(W) = x$, has not been frequently used in electronics problems but it appears in a variety of other fields of physics and mathematics. One recent such application is in the problem of photorefractive two-wave mixing [7]. Approximations have been proposed [8] to evaluate the Lambert W-function and efficient and accurate algorithms [9] are available to compute it.

In this note, we extend this type of approach to the more general case of diodes which require to be modeled by a single exponential expression with both series and parallel connected parasitic resistances. These solutions are exact and explicit and are easily differentiable. Therefore, they allow as well the explicit formulation and direct computation of the conductance at any point on the forward current–voltage characteristics. Consequently, they should also be useful for the extraction of the device model's parameters.

2. Exact explicit solutions

Consider a generalized situation where the diode contains a junction, modeled by a single exponential expression, with a parasitic series, R_S, and two parallel resistances: one connected across the junction, R_{P1}, to model possible shunt loss paths at the junction and another connected across the terminals, R_{P2}, to model shunt losses that might be present at the periphery of the

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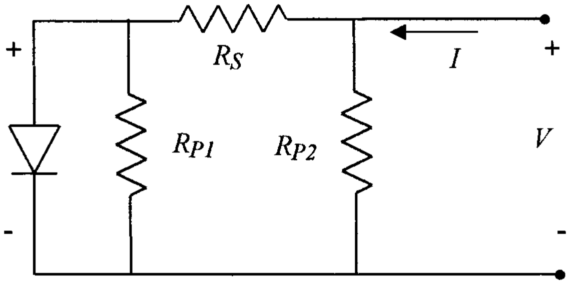

Fig. 1. Lumped circuit model of a single exponential junction, with parasitic series, R_S, and parasitic parallel resistances to represent shunt loss paths at the junction (R_{P1}) and shunt losses at the periphery (R_{P2}).

device, as shown in Fig. 1. The general describing equation for this circuit model is

$$I = I_0\left\{\exp\left[\frac{V\left(1+\frac{R_S}{R_{P2}}\right) - IR_S}{\eta V_T}\right] - 1\right\} + \frac{V - IR_S}{R_{P1}} + \frac{V}{R_{P2}} + \frac{VR_S}{R_{P1}R_{P2}}, \tag{1}$$

where I and V are the terminal current and voltage; I_0, the junction reverse current; η, the junction ideality factor, and V_T, the thermal voltage $= kT/q$. As is well known, this equation cannot be solved in general in terms of common elementary functions. However, explicit solutions for the current and the voltage can be found in terms of W-functions, as follows:

$$I = \frac{\eta V_T}{R_S} W\left\{\frac{I_0 R_S R_{P1}}{\eta V_T(R_{P1}+R_S)} \exp\left[\frac{R_{P1}(V + I_0 R_S)}{\eta V_T(R_{P1}+R_S)}\right]\right\} + \left(\frac{V - I_0 R_{P1}}{R_{P1}+R_S}\right) + \frac{V}{R_{P2}}, \tag{2}$$

$$V = -\frac{R_{P2}}{(R_{P2}+R_S)}\eta V_T W\left\{\frac{I_0 R_{P1}(R_{P2}+R_S)}{\eta V_T(R_{P1}+R_{P2}+R_S)} \times \exp\left[\frac{IR_{P1}R_{P2} + I_0 R_{P1}(R_{P2}+R_S)}{\eta V_T(R_{P1}+R_{P2}+R_S)}\right]\right\} + I\frac{R_{P2}}{(R_{P2}+R_S)}\left[R_S + \frac{R_{P1}R_{P2}}{(R_{P1}+R_{P2}+R_S)}\right] + I_0\frac{R_{P2}R_{P1}}{(R_{P1}+R_{P2}+R_S)}. \tag{3}$$

As expected from an explicit solution, the arguments of the W-functions in Eqs. (2) and (3) only contain the corresponding variable and the model's parameters. Let us now view some particular sub-cases of practical interest.

2.1. Only junction shunt loss and series resistance

When the significant shunt losses occur at the junction and a series parasitic resistance is also present, letting $R_{P2} \to \infty$, the describing equation reduces to

$$I = I_0\left[\exp\left(\frac{V - IR_S}{\eta V_T}\right) - 1\right] + \frac{V - IR_S}{R_{P1}}. \tag{4}$$

Explicit solutions for the current and the voltage in terms of W-functions are

$$I = \frac{\eta V_T}{R_S} W\left\{\frac{I_0 R_S R_{P1}}{\eta V_T(R_{P1}+R_S)} \exp\left[\frac{R_{P1}(V + I_0 R_S)}{\eta V_T(R_{P1}+R_S)}\right]\right\} + \left(\frac{V - I_0 R_{P1}}{R_{P1}+R_S}\right), \tag{5}$$

$$V = I(R_{P1}+R_S) + I_0 R_{P1} - \eta V_T W\left\{\frac{I_0 R_{P1}}{\eta V_T} \exp\left[\frac{(I + I_0)R_{P1}}{\eta V_T}\right]\right\}. \tag{6}$$

2.2. Only peripheral shunt loss and series resistance

When only shunt losses at the periphery are relevant, letting $R_{P1} \to \infty$, the describing equation becomes

$$I = I_0\left\{\exp\left[\frac{V\left(1+\frac{R_S}{R_{P2}}\right) - IR_S}{\eta V_T}\right] - 1\right\} + \frac{V}{R_{P2}}. \tag{7}$$

Again using the W-function, the explicit solutions for the current and the voltage are

$$I = \frac{\eta V_T}{R_S} W\left[\frac{I_0 R_S}{\eta V_T} \exp\left(\frac{V + I_0 R_S}{\eta V_T}\right)\right] - I_0 + \frac{V}{R_{P2}}, \tag{8}$$

$$V = IR_{P2} + I_0 R_{P2} - \frac{R_{P2}}{(R_{P2}+R_S)}\eta V_T W\left\{\frac{I_0(R_{P2}+R_S)}{\eta V_T} \times \exp\left[\frac{IR_{P2} + I_0(R_{P2}+R_S)}{\eta V_T}\right]\right\}. \tag{9}$$

2.3. Only series resistance, no shunt loss of any type

In the event that all shunt loss mechanisms could be neglected, letting $R_{P1} = R_{P2} \to \infty$ yields the case already studied by Banwell and Jayakumar [4],

$$I = I_0\left[\exp\left(\frac{V - IR_S}{\eta V_T}\right) - 1\right], \tag{10}$$

repeated here only for the sake of completeness, which can be expressed explicitly (Eq. (4) in Ref. [4]) as

$$I = \frac{\eta V_T}{R_S} W\left[\frac{I_0 R_S}{\eta V_T} \exp\left(\frac{(V + I_0 R_S)}{\eta V_T}\right)\right] - I_0. \tag{11}$$

Obviously the solution for the voltage can be easily obtained from Eq. (10) in terms of common elementary functions:

$$V = IR_S + \eta V_T \ln\left(\frac{I + I_0}{I_0}\right). \tag{12}$$

2.4. Only shunt loss, no series resistance

Conversely, if a series resistance is negligible, letting $R_S \to 0$ and denoting R_P the combination of any shunt losses present, yields directly the explicit solution for the current,

$$I = I_0\left\{\exp\left[\frac{V}{\eta V_T}\right] - 1\right\} + \frac{V}{R_P}. \tag{13}$$

However, solving for the voltage involves a transcendental equation whose explicit solution may again be obtained in terms of W-functions:

$$V = (I + I_0)R_P - \eta V_T \mathrm{W}\left\{\frac{I_0 R_P}{\eta V_T}\exp\left[\frac{(I + I_0)R_P}{\eta V_T}\right]\right\}. \tag{14}$$

3. Discussion

Electronic circuit simulation requires the computation, usually a large number of times, of the current and voltage of real semiconductor junctions which exhibit non-ideal series and/or shunt parasitic resistance effects. In order to circumvent the fact that the describing equations (1) do not in general permit to develop exact explicit solutions in terms of common elementary functions, two alternatives have been used traditionally: iterative methods and approximate solutions. Although iteration can produce results as accurate as needed, it does so at the expense of speed. On the other hand, using approximate type solutions can considerably speed computation, which is desirable, but at the expense of loosing accuracy, which is not. The use of the Lambert W-function, as proposed by Banwell and Jayakumar [4], provides a very attractive alternative to answer this problem since efficient and accurate algorithms are available today [8] to compute this special function. Additionally, the use of exact explicit solutions of this type, which are easily differentiable, allows to express the conductance in an explicit form and to compute it directly at any point on the current–voltage characteristics.

However, in order to seriously consider the use of this type of solutions as substitutes for approximations or iteration, their advantage over the previous methods must be assessed at both accuracy and speed. To this end, we have made a brief comparison between the use of W-function type solutions versus iteration and approximate solution methods. The current corresponding

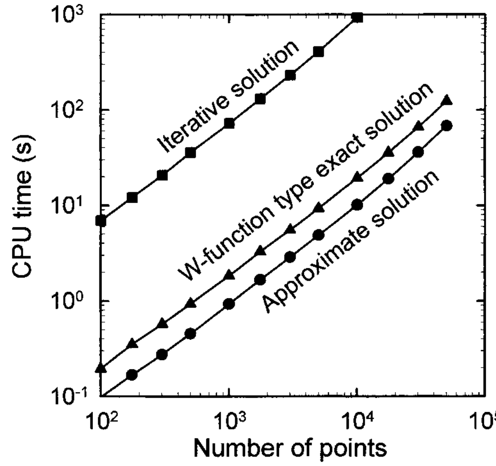

Fig. 2. CPU times required by each of the three types of solutions, as a function of the number of points calculated from 0 to 1 V.

to forward voltages up to 1 V, of a hypothetical junction with series resistance and only junction shunt loss, was calculated to 10 significant digits using a MAPLE V release 5.1 environment in a PC with a 450 MHz Pentium II and 256 MB of RAM. The parameters used for this test where $I_0 = 10^{-12}$A, $\eta = 1$, $R_S = 1$ KΩ, and $R_{P1} = 1$ MΩ. Fig. 2 shows the CPU times required by each of the three types of solution as a function of the number of points calculated from 0 to 1 V. Eq. 4 was used in the case of iterative solutions, whereas exact solutions were obtained from the explicit W-function expression of Eq. (5), and approximate solutions were calculated using the expression presented in Ref. [3]. The results confirm that computation with W-function type solutions is a significantly faster method than iteration, typically 40 times faster under the above-stated conditions. The results also make clear that the approximate solution is still the fastest alternative, about twice as fast as the W-function type solution in this case. However, approximate solutions available up to now present the serious disadvantage of producing appreciable error (about 3% maximum) [3]. Therefore, the usefulness of the W-function type solution rests upon its computational speed, since its results can be as accurate as those of direct iteration.

4. Conclusions

We have presented exact explicit analytical solutions for the forward current of non-ideal single-exponential diode models containing the possible combinations of series and shunt parasitic resistances. The solutions are based on the use of the Lambert W-function which allow highly accurate computations at speeds almost comparable to those of the less accurate approximate solutions. The brief test presented, although not exhaustive, indicates that W-function type of solutions are attractive alternatives to be considered for diode models that are to be used for circuit simulation purposes.

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