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War Strategy Optimization Algorithm: A New Effective Metaheuristic Algorithm for Global Optimization

TUMMALA. S. L. V. AYYARAO[1], N. S. S. RAMAKRISHNA[1], RAJVIKRAM MADURAI ELAVARASAN[2], NISHANTH POLUMAHANTHI[3], M. RAMBABU[1], GAURAV SAINI[4], BASEEM KHAN[5], (Senior Member, IEEE), AND BILAL ALATAS[6]

[1]Department of Electrical and Electronics Engineering, GMR Institute of Technology, Rajam 532127, India
[2]Department of Electrical and Electronics Engineering, Thiagarajar College of Engineering, Madurai 625015, India
[3]Bharath Heavy Electricals Ltd., Hyderabad 530012, India
[4]School of Advanced Materials, Green Energy and Sensor Systems, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
[5]Department of Electrical and Computer Engineering, Hawassa University, Awassa 1530, Ethiopia
[6]Department of Software Engineering, Faculty of Engineering, Firat University, 23119 Elaziğ, Turkey

Corresponding author: Baseem Khan (baseem.khan04@gmail.com)

ABSTRACT This paper proposes a new metaheuristic optimization algorithm based on ancient war strategy. The proposed War Strategy Optimization (WSO) is based on the strategic movement of army troops during the war. War strategy is modeled as an optimization process wherein each soldier dynamically moves towards the optimum value. The proposed algorithm models two popular war strategies, attack and defense strategies. The positions of soldiers on the battlefield are updated in accordance with the strategy implemented. To improve the algorithm's convergence and robustness, a novel weight updating mechanism and a weak soldier's relocation strategy are introduced. The proposed war strategy algorithm achieves good balance of the exploration and exploitation stages. A detailed mathematical model of the algorithm is presented. The efficacy of the proposed algorithm is tested on 50 benchmark functions and four engineering problems. The performance of the algorithm is compared with ten popular metaheuristic algorithms. The experimental results for various optimization problems prove the superiority of the proposed algorithm.

INDEX TERMS Metaheuristic, optimization, war strategy, swarm optimization.

I. INTRODUCTION

The use of advanced technology in various fields of science is increasing the complexity of the problems to be solved. The shortcomings of traditional optimization techniques resulted in the emergence of the metaheuristic optimization algorithm for solving complex engineering problems. As a result, new optimization algorithms become a ray of hope.

Meta-heuristic methods: Meta-heuristics methods are considered to be global best optimization algorithms and possess several advantages, such as robustness, performance reliability, simplicity, ease of implementation, etc. Meta-heuristic algorithms have been classified into different literary categories, such as:

(a) Evolutionary-based algorithms: These algorithms are originated from the theory of evolution.

(b) Swarm-based algorithms: These algorithms emulate the social behavior and the collective decision-making of various social groups. In these algorithms, the explanation for reaching a given objective is usually based on bio-community intelligence and collective action.

(c) Physics-based algorithms: The physics-based algorithms have been influenced by the laws of natural physics

(d) Human behavior-based algorithms: Recently optimization algorithms inspired by human beings' social behavior have been proposed in the literature.

(e) Hybrid and advanced algorithms:

Hybrid algorithms combine features of two or more optimization algorithms to achieve better results.

Examples of various categories proposed in the literature are given in Table 1.

II. LITERATURE SURVEY

Every algorithm proposed in the literature has its own characteristics and uniqueness in order to achieve the desired

The associate editor coordinating the review of this manuscript and approving it for publication was Wei Liu.

TABLE 1. Classification of metaheuristic algorithms.

Category	Algorithm name	Reference
Evolutionary-based algorithms	Genetic Algorithm (GA)	[1]
	Differential Evolution	[2]
	Evolutionary Strategy (ES)	[3]
Swarm-based algorithms	Particle Swarm Optimization (PSO)	[4]
	Ant colony optimization (ACO)	[5]
	Bacterial Foraging Optimization	[6]
	Grey Wolf Optimizer (GWO)	[7]
	Whale Optimization Algorithm (WOA)	[8]
	Moth-flame optimization	[9]
	Salp Swarm Optimization (SSA)	[10]
	Grasshopper optimization algorithm	[11]
	Artificial bee colony algorithm	[12]
	Bat algorithm	[13]
	Firefly algorithm	[14]
	Cuckoo Search algorithm	[15]
	Spherical search algorithm	[16]
	Social Spider Optimization	[17]
	Marine Predators Algorithm	[18]
	Crow search algorithm	[19]
	Krill herd algorithm	[20]
	Chimp optimization algorithm	[21]
	Squirrel search algorithm	[22]
	Flower pollination algorithm	[23]
	Manta ray foraging optimization	[24]
	Sailfish Optimizer	[25]
	Emperor penguin optimizer	[26]
	Spotted hyena optimizer	[27]
	Coyote Optimization Algorithm	[28]
Human behavior-based algorithms	Group teaching optimization	[29]
	Imperialist competitive algorithm	[30]
	Teaching-Learning Based Optimization (TLBO)	[31]
	League champion algorithm	[32]
	Political optimizer	[33]
	Poor and rich optimization	[34]
	Hunger games search	[35]
Physics-based algorithms	Gravitational Search Algorithm (GSA)	[36]
	Simulated annealing	[37]
	Artificial Electric field optimization	[38]
	Sine Cosine Algorithm (SCA)	[39]
	Magnetic optimization algorithm	[40]
	Turbulent Flow of Water-based Optimization	[41]
	Henry gas solubility optimization	[42]
	Archimedes optimization algorithm	[43]
	Fireworks Algorithm	[44]
	Mine blast algorithm	[45]
Hybrid/advanced algorithms	Sine-cosine Harris hawks optimization	[46]
	Self-adaptive differential evolution	[47]
	Comprehensive learning particle swarm optimizer	[48]
	Locally Informed Particle Swarm optimizer	[49]
	Hybrid binary ant lion optimizer	[50]
	Sine–cosine and Spotted Hyena-based Chimp Optimization Algorithm	[51]
	Hybrid firefly algorithm with grouping attraction	[52]
	Improved Moth-Flame Optimization	[53]

objective. Features like adaptive mechanism [54], [55], Chaotics [56]–[58] learning mechanisms [59]–[62], novel mutation strategies [63], [64], fuzzy logic [65], [66], quantum computing [67] etc., are added to the basic algorithms to achieve better convergence and robustness. The original version of PSO has deficiencies when applied to complex functions, such as premature convergence, limiting to local optima, and slow convergence. Aside from slow convergence, GWO has low precision in the majority of problems. Algorithms such as PSO, Jaya, GWO updating mechanisms are based on a global best position, and thus these algorithms may converge to a local optima.

Some of the issues/challenges with the existing literature are as follows:

i. One of the major issues is that slower convergence and a high computational burden are required to achieve global optimum value.
ii. The majority algorithms lack good balance between exploration and exploitation capabilities [33].

iii. Some of algorithms prematurely converge to the local optimum and therefore are not suitable to real-world engineering problems.
iv. Another issue is the algorithm's vast number of algorithm-specific parameters and selecting appropriate values entails a significant computational burden.

As per No Free Lunch [NFL] [68], there is no single optimization algorithm which gives satisfactory results for all the optimization problems. Hence the research is still attractive in this domain which results into unearthing of new optimization algorithms by different authors' thought process. This paper proposes a new meta-heuristic optimization technique based on the war strategy. The concept is based on the dynamic movement of soldiers during wartimes. Each soldier continuously updates his position based on the positions of the King and the commander. This war strategy is modeled as an optimization process wherein each soldier dynamically moves towards the optimum value. Each soldier is assigned a rank and a weight. The soldier's weight is updated based on his or her success in improving the attacking force or fitness value. The proposed war strategy algorithm is first tested for the 50 benchmark test functions and four engineering problems. The results obtained are compared with popular metaheuristic algorithms.

The following are the contributions of the proposed algorithm:

i. This article proposes a new meta-heuristic algorithm named 'War Strategy Optimization' and in this algorithm, we have developed two war strategies, the first is concerned with attack strategy, while the second is concerned with a defense strategy.
ii. The proposed algorithm employs a unique (soldier) updating policy, in which the soldier's current position is determined by the war strategy
iii. A new policy for updating the weak soldiers (particles) has been incorporated.
iv. The developed algorithm includes an adaptive weight updating policy for each particle as well as a specific weight assignment for each particle.
v. The proposed algorithm's performance has been evaluated on 50 benchmark functions, and its results are compared with the popular meta-heuristic algorithms.
vi. The proposed algorithm is applied for the design of the engineering models.

The paper is organized is as follows: Section-2 gives an introduction to WSO. Section-3 details the mathematical model of the algorithm. Section-4 analyzes the performance of the algorithm on various benchmark functions and engineering problems. Finally, the concluding remarks with the future scope are given in the last section.

III. WAR STRATEGY OPTMIZATION

Ancient kingdoms maintained a military to fight themselves from attacks by other dynasties. The kingdom's army is comprised of various forces such as infantry, chariots, elephants, and so on. During the war, each kingdom devises a strategy known as ''Vyuha'' to attack the opposing army to win the battle and thus establish their supremacy. A Vyuha is a pattern or arrangement of various army troops used to conquer the opposing kingdom during a war [69]. To ensure that their army meets the intended targets and achieves the goal, the emperor and commanders of each unit will coordinate their forces in a specific pattern. The warfare strategy was formulated in light of the mission's objectives, threats, difficulties, and prospects. War strategy is a continuous dynamic process in which armed forces simply coordinate and fight the opposition. This strategy can adapt to changing conditions as the war progresses. The positions of the king and commander have a constant impact on the army soldier's position. The flags on top of the king's and army commander's chariots represent their location, which is observable to all soldiers. Soldiers on the team are trained to follow a strategy based on the sounds of a drum or another musical instrument. When one of the military commanders dies, the strategy changes, and every other commander must learn how to rebuild and continue the war strategy's establishment. The King's target is to conquer the opposing king/leader, whereas the army soldier's main objective is to attack the opposing team and progress in rank.

The various steps involved in the war strategy are as follows:

A. RANDOM ATTACK

In the battlefield, the army troops randomly distribute over the entire battle ground in a strategic manner and attack the opposite army. The strongest of the army personnel with highest attacking force is considered as the army chief or the Commander. The King is the leader of various such army chiefs.

B. ATTACK STRATEGY

The primary objective of this strategy is to attack the opposition. The King takes the lead and guides the army troops. Army troops identify the weak positions (promising search space) of the opponent and continue to attack. The King and the Commander travel in two different chariots strategically with flags at the top. The Soldiers dynamically change their position based on the positions of the king and the Commander. If a soldier is successful in improving his attacking force (fitness value), his rank will be improved. As the soldier advances, he will serve as a good example for the others. However, if the new position is not suitable to fight, the soldier moves back to his previous position. At the beginning of the war, army troops move in all directions and takes large steps to change their position.

C. SIGNALING BY DRUMS

The King changes the strategy dynamically based on the prevailing situation in the battle ground. Accordingly, a group of soldiers beat the drums with a rhythm. The soldiers will change their strategy and adjust their positions based on the rhythm of the drums.

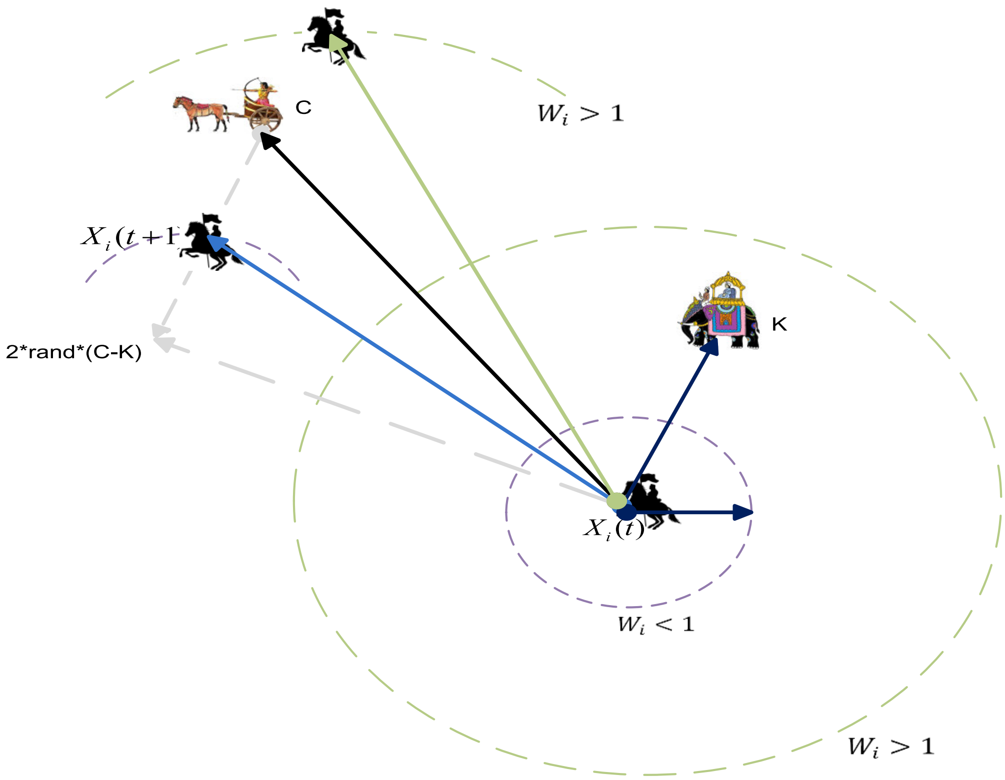

FIGURE 1. Attack strategy in WSO.

D. DEFENSE STRATEGY

The primary objective of this strategy is to protect the King without losing the battle. The commander or the Army chief takes the lead and forms like a chain and surround the King by using the army troops. Thus, every soldier changes the position based on the positions of the nearby soldier and the king position. Army troops try to explore a large area of war field (search space) during the war. To confuse the opposing army, the army dynamically changes its strategy from time to time.

E. REPLACEMENT/RELOCATON OF WEAK SOLDERS

During the battle, the soldier whose combat skills is lowest or an injured soldier can be treated as good as an enemy soldier. With his poor performance, the credibility of the Army altogether is at stake (algorithm efficiency). Few soldiers die during the war and this may impact the result of the war. Here there are two options available with the army. One is replacing the injured/weak soldiers with new soldiers. The second option is to relocate the weak soldier. Hence, he will be guided (mean position of all the soldiers) and insulated by all the other soldiers to protect him and thereby maintain the army morale and making the chances of winning in the war battle high.

F. TRAPS BY OPPOSITION

The opposing army employs a variety of strategies, depending on its capabilities, to force the former army to move in the wrong direction or to reach the wrong target (local optima).

IV. MATHEMATICAL MODEL OF THE WAR STRATEGY

At every iteration, all the soldiers have equal probability to become either King or Commander depending on their Combating Strength (Fitness Value). Both the King and the Commander act as Leaders in the War field. The movement of the King and the Commander in the War field will guide the rest of the soldiers. There is a possibility for either the King or the Commander to face stiff competition from the opponent's soldier (Local Optima) who has enough strength to trap the Leaders. To avoid this, soldiers in war will be guided not only by the King's or Commander's position, but also by their combined movement tactics.

A. ATTACK STRATEGY

We have modeled two war strategies. In the first case, every soldier updates his position based on the positions of the King and the Commander. This updating mechanism of the attack model is illustrated in Figure 1. The king assumes an advantageous position to launch a massive attack on the opposition. As a result, the soldier with the greatest attack force or fitness is regarded as the king. All soldiers will have the same rank and weight at the start of the war. If the soldier successfully executes the strategy, his rank rises. However, as the war progresses, the ranks and weights of all soldiers will be updated based on the strategy's success. As the war nears its conclusion, the position of the King, Army commander, and soldiers remain very close as they approach the target.

$$X_i(t+1) = X_i(t) + 2 \times \rho \times (C - K) + rand \times (W_i \times K - X_i(t)) \quad (1)$$

where, $X_i(t+1)$ is the new position, X_i is the previous C position, is the position of the commander, K is the position of the king, W_i is the weight.

The colored circles round the soldier in Figure 1 represents of locus points of $W_i \times k - X_i(t)$ based on the King position. If $W_i > 1$, then position of $W_i \times k - X_i(t)$ is beyond the king position and the hence the updated position of the soldier is beyond the commander position. If $W_i < 1$, then position of $W_i \times k - X_i(t)$ is in between the king position and the soldier current position. The updated position of the soldier is closer when compared to the previous case. If W_i tends to zero, then updated position of the soldier moves very close to the commander position which represents the final stage of the war.

B. RANK AND WEIGHT UPDATION

The position update of each search agent depends on the interaction of the position of the King, the Commander and the rank of each soldier. The rank of each soldier depends on his success history in the war field governed by equation (4) which will subsequently influence the weighing factor W_i. The rank of each soldier reflects how close the soldier (search agent) is to the target (fitness value). It can be noted that the weighing factors in other competitive algorithms like GWO, WOA, GSA, PSO will vary linearly whereas in the current proposed WSO algorithm, the weight (W_i) varies exponentially as a factor of α.

If the attack force (fitness) in the new position (F_n) is less than that of the previous position (F_p), the soldier takes the previous position.

$$X_i(t+1) = (X_i(t+1)) \times \left(F_n \geq F_p\right) + (X_i(t)) \times \left(F_n < F_p\right) \tag{2}$$

If the soldier updates the position successfully, the rank R_i of the soldier will be upgraded

$$R_i = (R_i + 1) \times \left(F_n \geq F_p\right) + (R_i) \times \left(F_n < F_p\right) \tag{3}$$

Based on the rank, the new weight is calculated as:

$$W_i = W_i \times \left(1 - \frac{R_i}{Max\_iter}\right)^{\alpha} \tag{4}$$

C. DEFENSE STRATEGY

The second strategy position update is based on the positions of King, the army head and a random soldier. Whereas the ranking and weight updating remains same.

$$X_i(t+1) = X_i(t) + 2 \times \rho \times (K - X_{rand}(t)) + rand \times W_i \times (c - X_i(t)) \tag{5}$$

This war strategy explores more search space when compared to the previous strategy as it involves the position of the random soldier. For large values of W_i, soldiers take large steps and update their position. For small values of, W_i soldiers take small steps while updating the position.

D. REPLACEMENT/RELOCATION OF WEAK SOLDIERS

For every iteration, identify the weak soldiers having worst fitness. We have tested multiple replacement approaches. One simplest approach is replacing the weak soldier with a random soldier as given in (6).

$$X_w(t+1) = Lb + rand \times (Ub - Lb) \tag{6}$$

The second approach is relocating the weak soldier closer to the median of entire army in a war field as given in (7). This approach improves the convergence behavior of the algorithm.

$$X_w(t+1) = -(1 - randn) \times (X_w(t) - median(X)) + K \tag{7}$$

E. SALIENT FEATURES OF THE PRPOSED ALGORTHM

i. The proposed algorithm achieves good balance between exploration and exploitation.
ii. Each solution (soldier) has a unique weight based on his rank.
iii. The weight of each soldier is updated if the soldier successfully improves his fitness in the updating step. Thus, the weight updating purely depends on the particle position relative to King's and commander position.
iv. The weights will vary nonlinearly. The weights vary in large values during the early iterations and vary in small values during the last iterations. This leads to faster convergence to the global optimum value.
v. The position updating process involves two stages. This improves the exploration capability to the global optimum solution.
vi. The proposed algorithm is simple and requires a less computational burden.

F. EXPLORATION AND EXPLOITATION

The exploration (for global optima) and exploitation (for convergence) are the two main criteria for any metaheuristic optimization algorithms [70]. A good trade-off between these two phenomena will make the algorithm more robust and efficient. Attack strategy represents the exploitation while defense strategy represents the exploration. The other major factors which influence the exploration and exploitation capability of the proposed algorithm are:

i. Firstly, the variable 'rand' which can take any value randomly between '0' to '1'. This 'rand' variable decides whether the soldier moves is to be exploration oriented or exploitation oriented.
ii. Secondly, the factor ρ_r helps the user in giving flexibility to choose a value depending on the objective function. From the experiments performed on different test functions, it is inferred that a low value of ρ_r in the range of (0-0.5) suits best for the unimodal functions and the values in the range of (0.5-1) suits best for multimodal functions.
iii. Thirdly, the movement of the search agent in the direction of X_{rand} makes the algorithm more explorative to

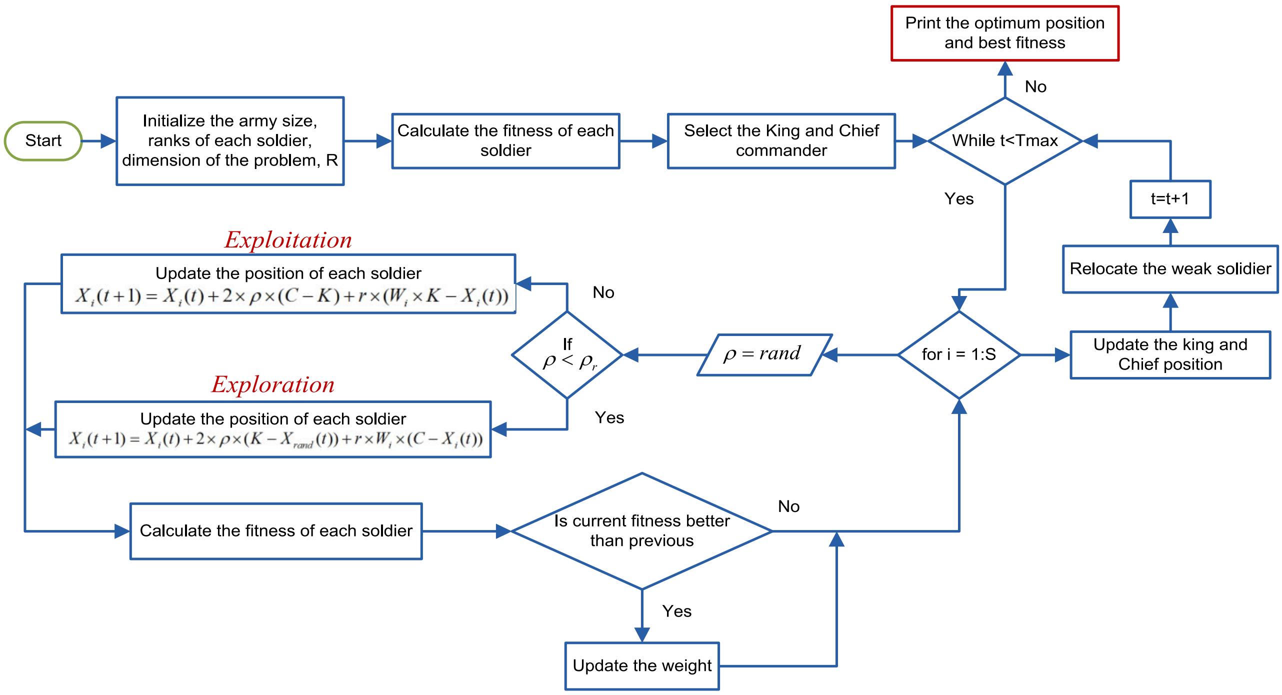

FIGURE 2. Flowchart of war strategy optimization algorithm.

search the prominent areas in the search space so as to settle at the global optima.

iv. Lastly, W_i factor influences the direction of the search agent towards the best possible location. W_i makes the search agents move globally and do exploration and as the search process advances and reaches final stage, it will make the search agents to be exploitative.

The flow chart for the proposed war strategy optimization algorithm is shown in Figure 2. The weights assigned to each soldier are adaptive and changes from iteration to iteration. The soldier with a large fitness level will have less weight and the soldier with less fitness will have a large weight. At the start of the war, every soldier takes large steps, and their weight varies in large steps. As the war nears its conclusion, the soldiers take small steps to reach the goal, and the weight varies in small steps. Because the strategy is chosen at random, the soldiers move in a random direction and do not precisely follow the king. This improves the algorithm's exploration capability. The target area is identified by army troops at the end of the war (prominent search space). Army troops surround the target as well as the King and Commander are very close to the target. Thus, from equations (1) and (5), the entire troop moves in small steps and converges to the target position. Thus, we can say that the algorithm possesses the exploitation feature also.

G. THE PSEUDO CODE OF WAR OPTIMIZATION ALGORITHM IS GIVEN AS FOLLOWS

See Algorithm 1.

V. RESULTS AND DISCUSSION

The robustness and convergence efficiency of the proposed war strategy optimization (WSO) algorithm was tested on 50 benchmark functions and four different engineering problems.

A. RESULTS ON BENCHMARK TEST FUNCTION

A comprehensive set of benchmark functions with a good combination of features such as Unimodal & multimodal, variable/fixed dimensions, separability, and continuity is used to assess the versatility of WSO. The proposed WSO algorithm is evaluated on the 50 benchmark test functions. Out of 50, first 25 functions are Unimodal function and remaining 25 are multimodal functions. The complete details of the functions are given in Table 12 and 13.

B. PARAMETER SELECTION

General setting for ρ_r is 0.5. However, for unimodal test functions like $\{F_1, F_2, F_3, F_4\}$ the parameter ρ_r is set to 0.1 and Multi-modal functions like $\{F_{26}, F_{28}, F_{29}, F_{30}\}$, ρ_r is set to be 0.95. The general setting of W_i is $2\times ones\ (1,\ S)$ and this can be adjusted on the need basis. The results are analyzed based on the features of exploitation, exploration, convergence, search history, trajectories etc,

C. COMPARISON WITH POPULAR METAHEURISTICS ALGORITHMS

To prove the efficacy and robustness of the proposed algorithm; it is compared with eleven state of the art and popular metaheuristic algorithms. The algorithms used for comparison are PSO [71], GA [1], DE [2], GWO [7], ALO [8], Chimp [21], MVO [72], JS [73], SSA [10], SDCS [74], DA [93]. The parameter settings for these eleven algorithms and maximum function evaluations are given in Table 2. For

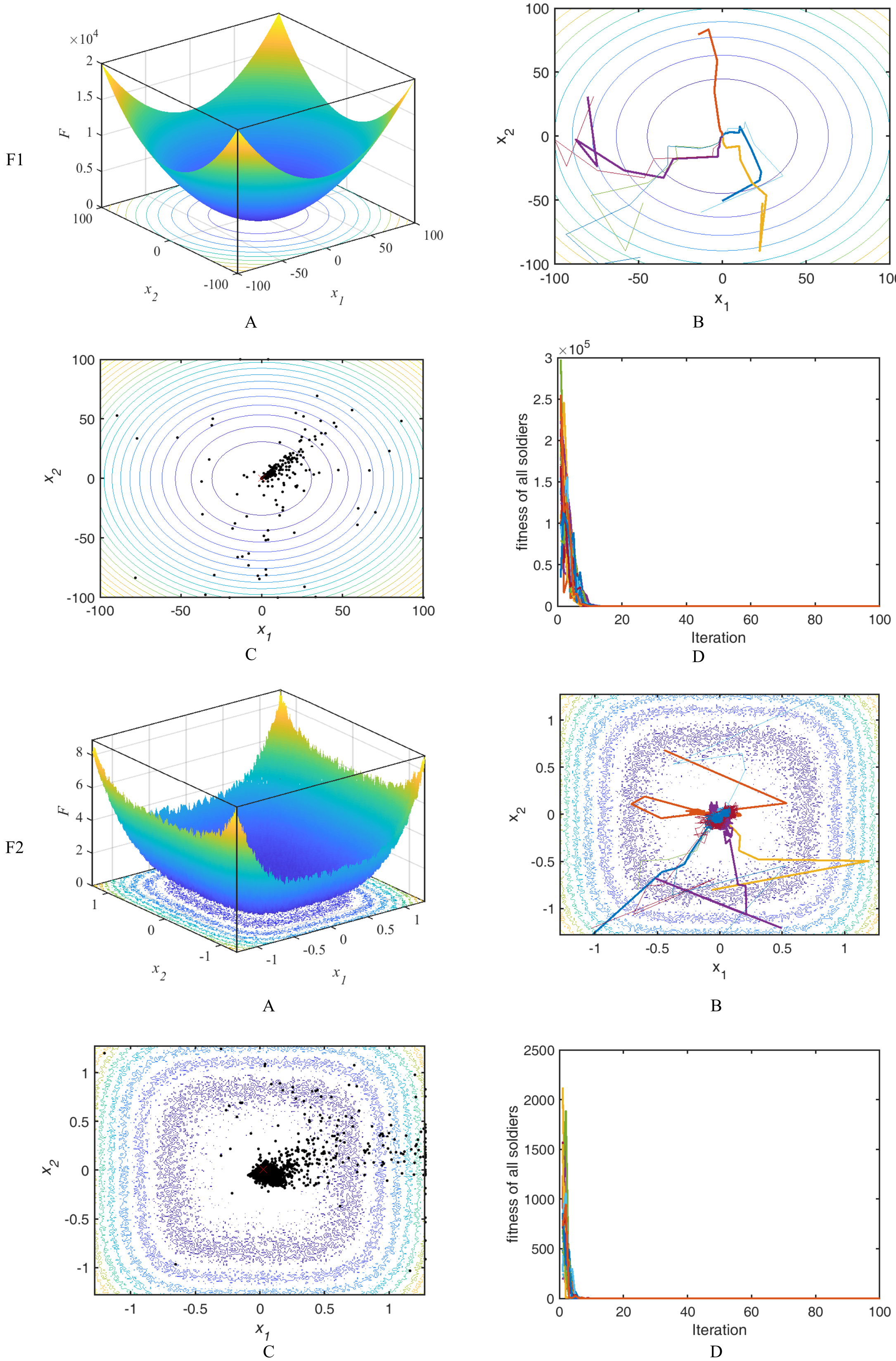

FIGURE 3. A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.

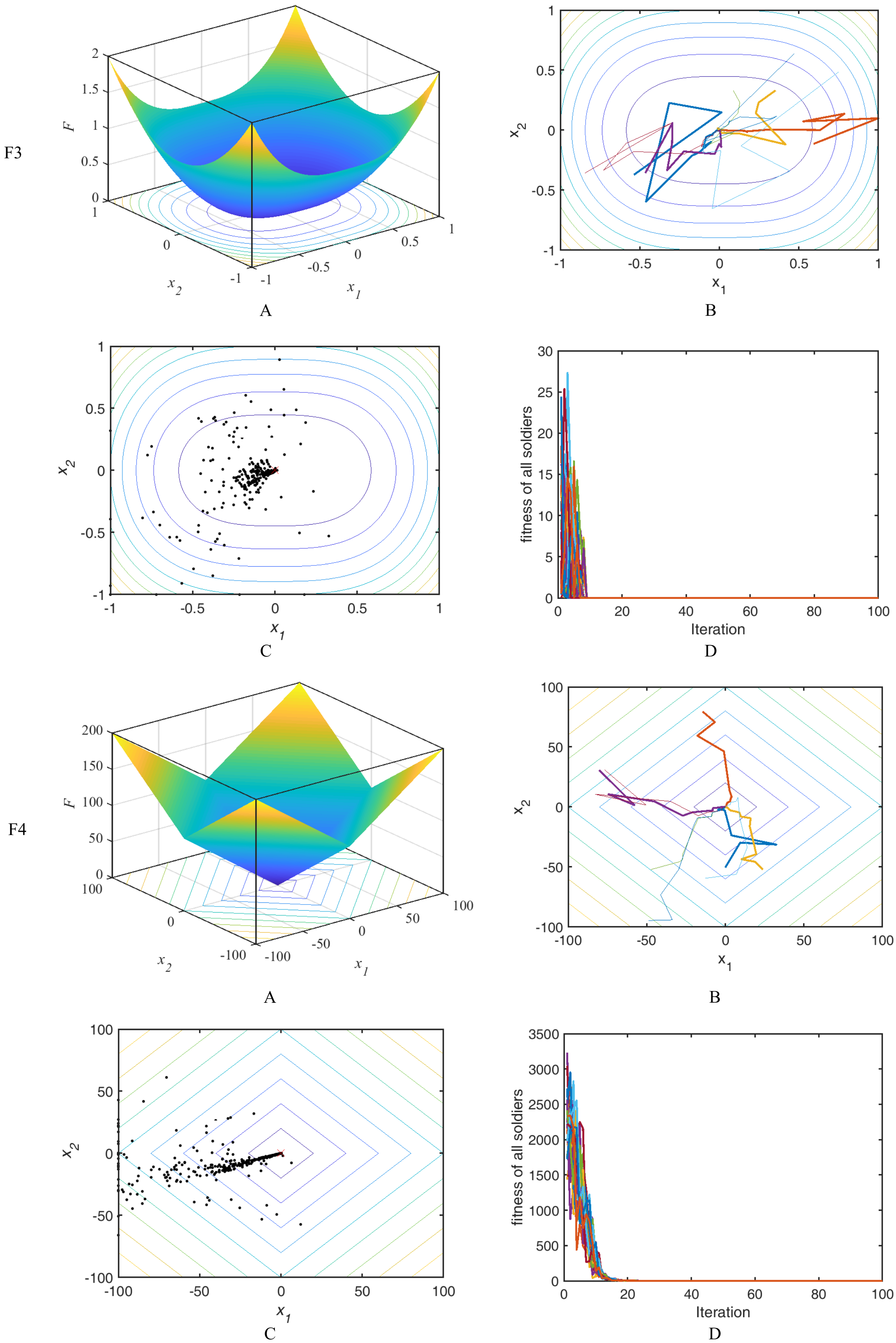

FIGURE 3. ***(Continued.)*** **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

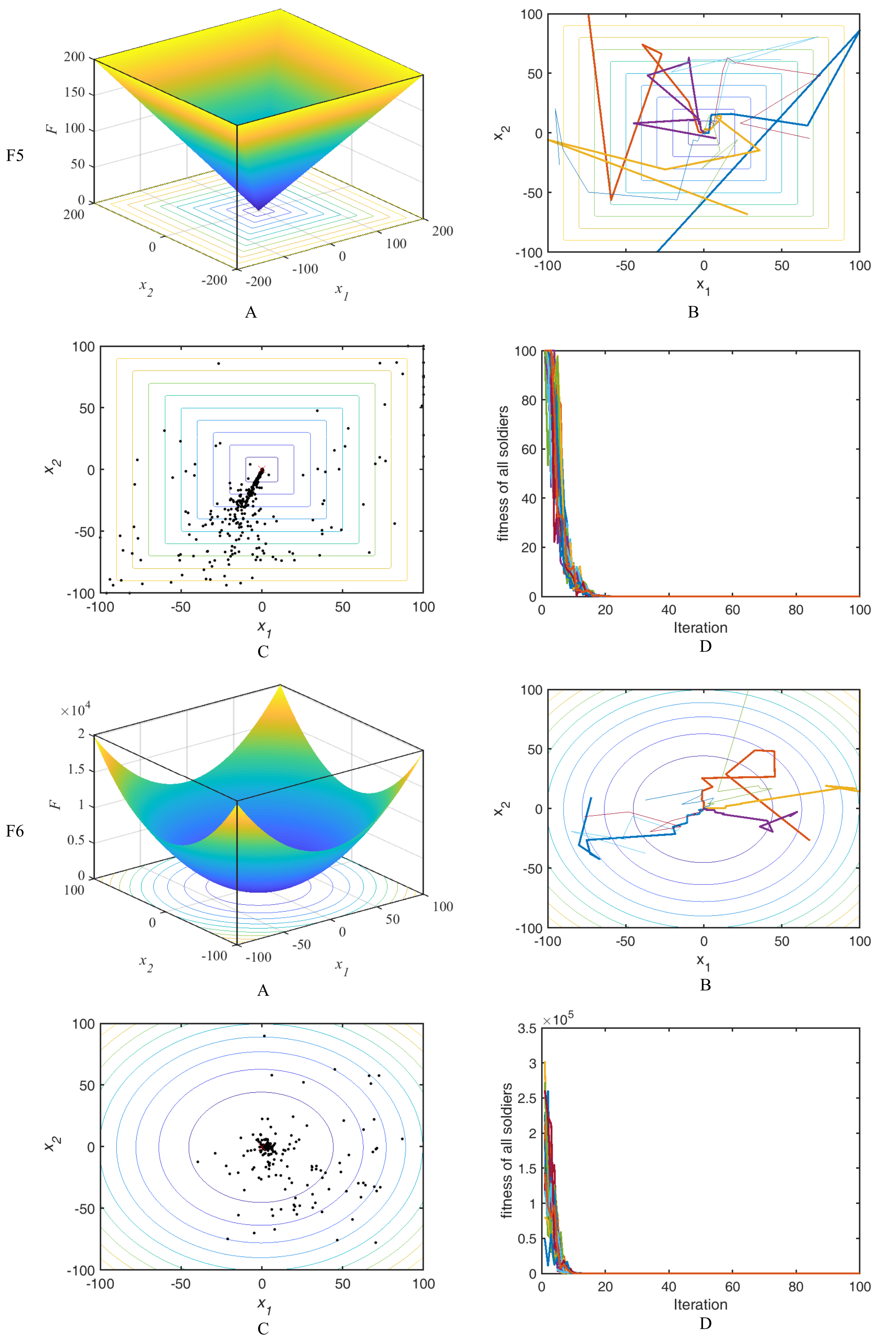

FIGURE 3. *(Continued.)* **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

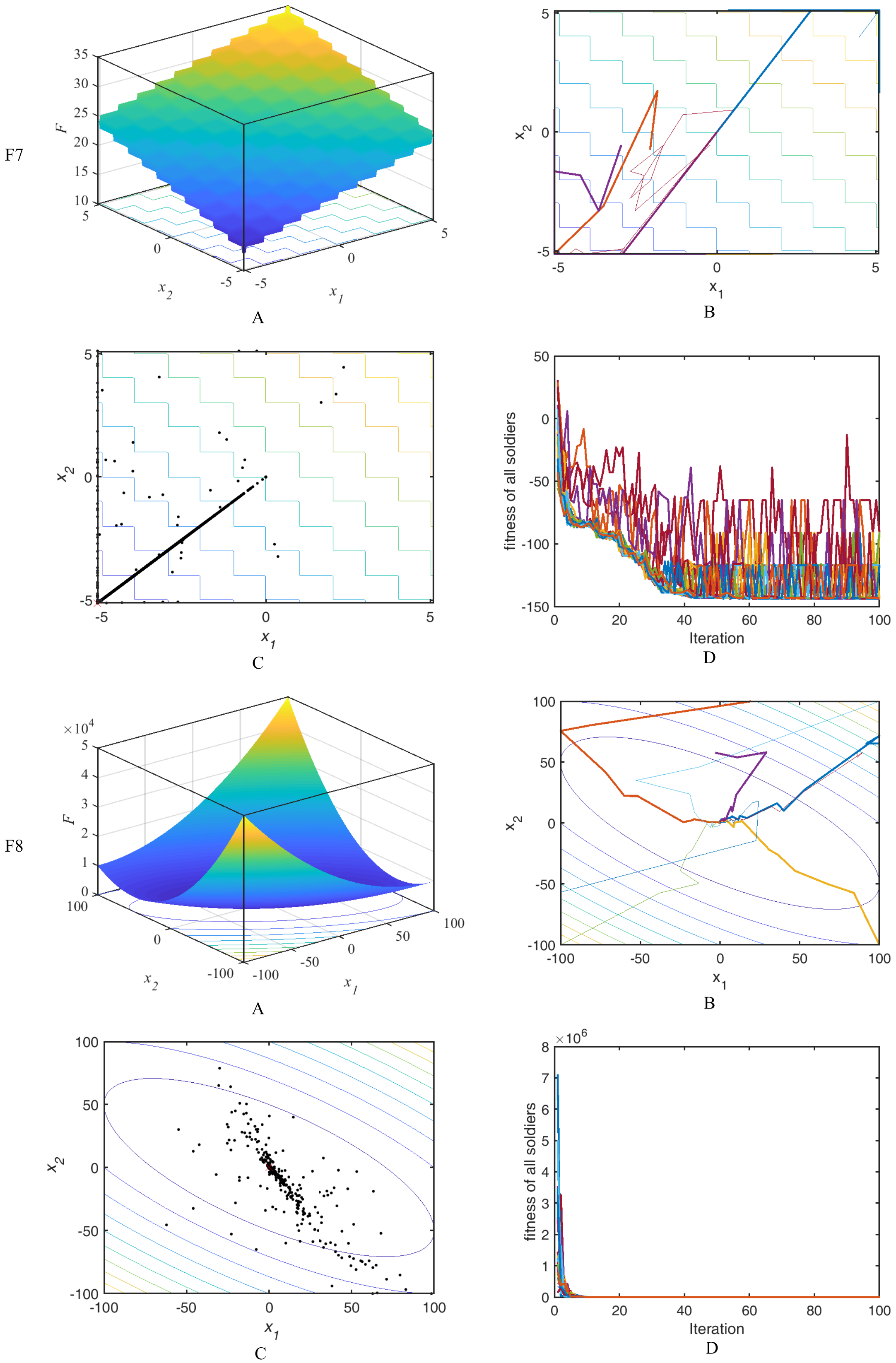

FIGURE 3. *(Continued.)* A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.

F27

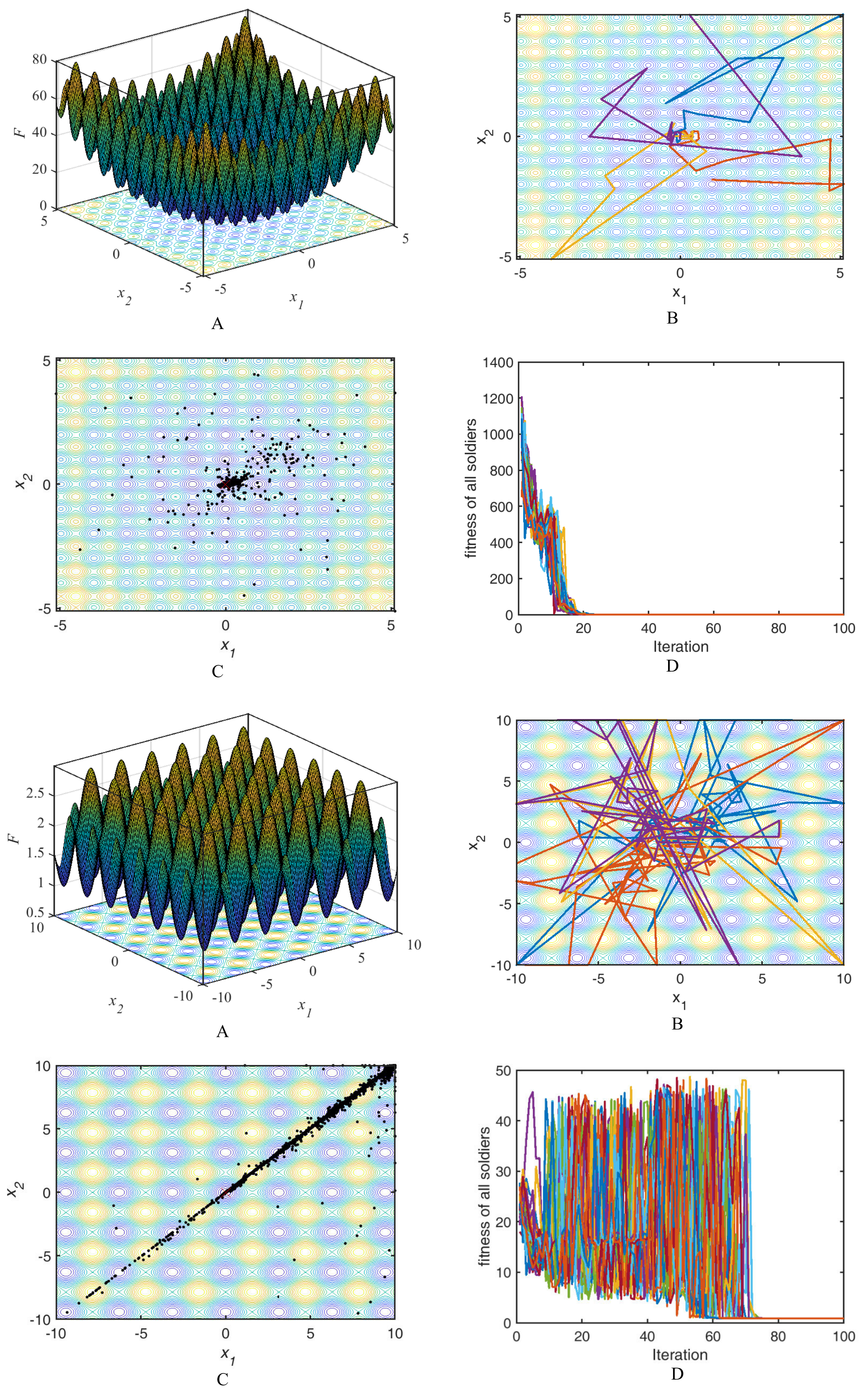

FIGURE 3. ***(Continued.)*** **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

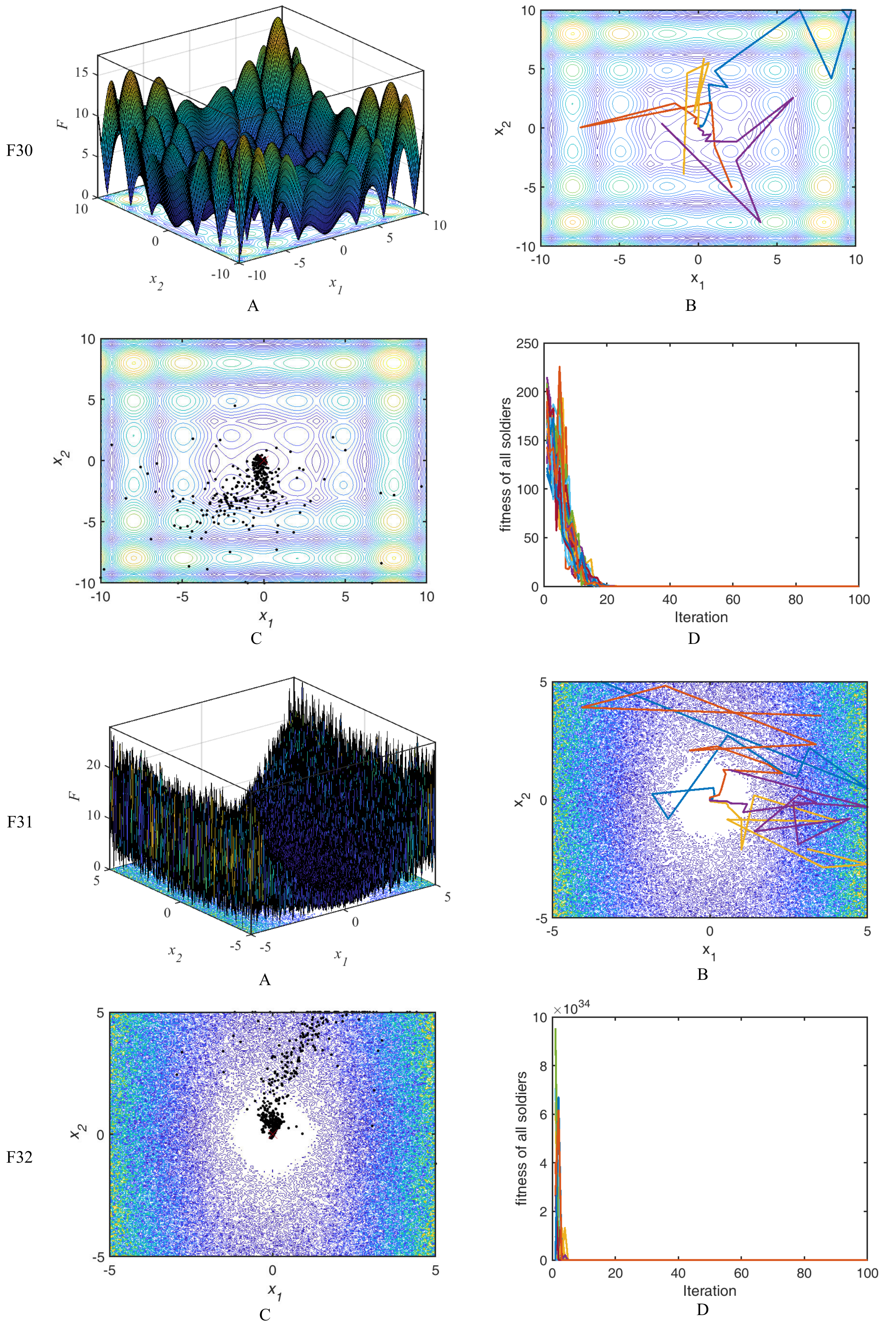

FIGURE 3. *(Continued.)* **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

F34

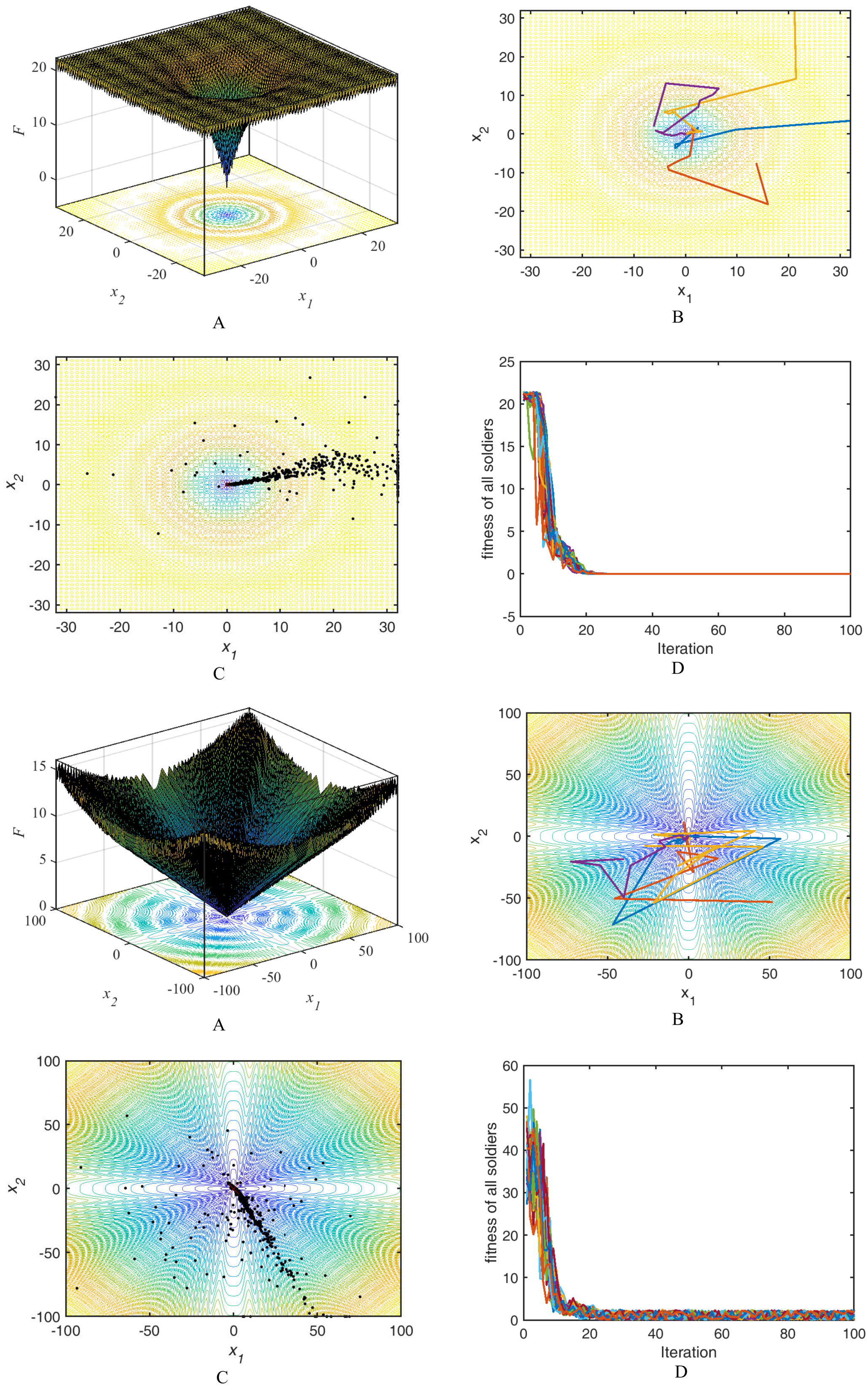

FIGURE 3. *(Continued.)* **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

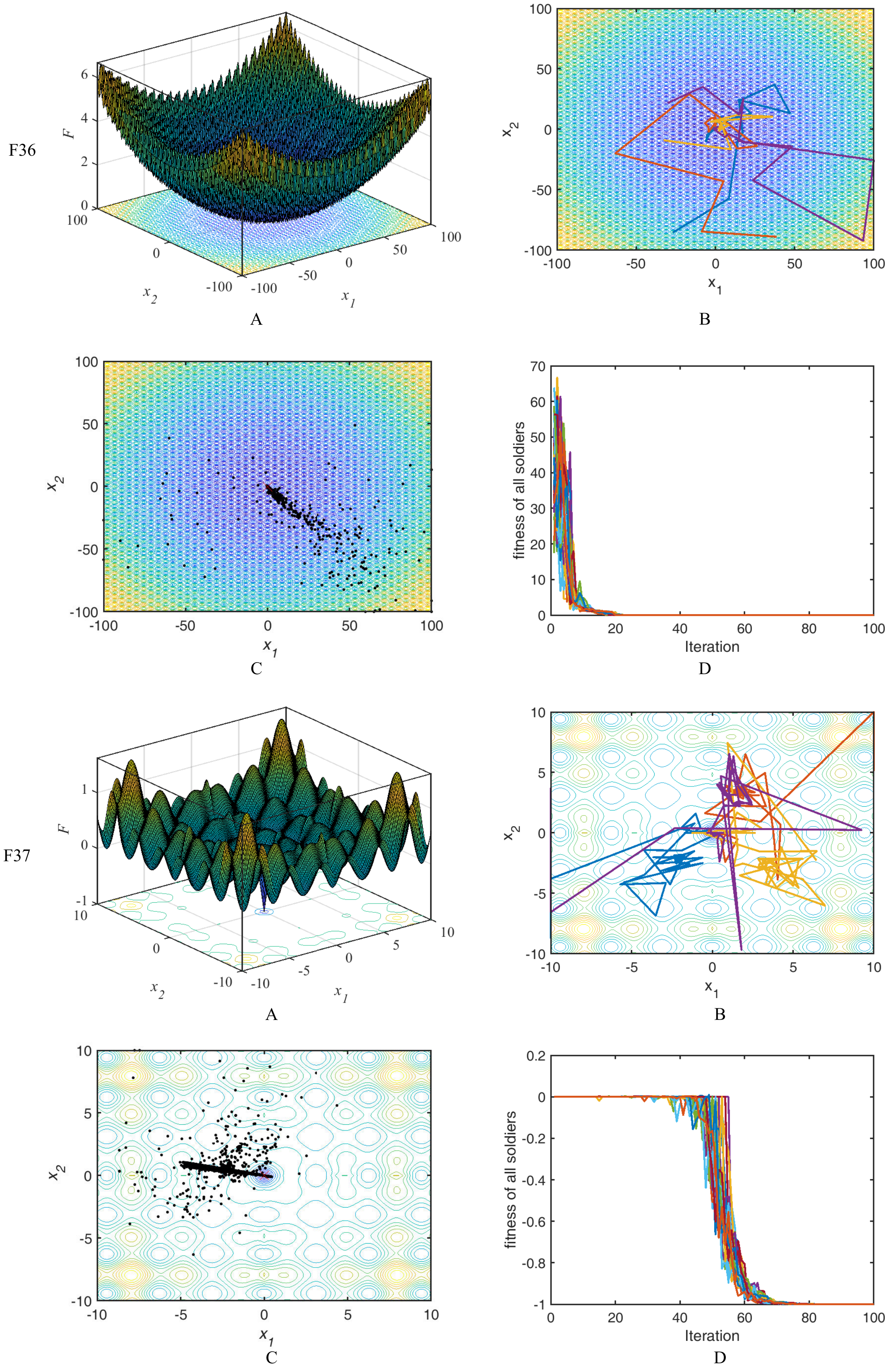

FIGURE 3. *(Continued.)* **A. Parameter space B. Trajectories for random soldiers C. Search history D. Fitness of all the soldiers for selected benchmark functions.**

Algorithm 1 War Strategy Optimization Algorithm

Initialize the soldier size, dimension of the war space (dimension of the problem), lower and upper bounds of the search space, Positions of the King (K), Army Commander(C), Attack forces of the King and the Army Commander, soldier size (S) are as follows:
(S) = 30; C = zeros (1, dim); K = zeros (1, dim); Max-iterations = 1000; $\rho_r = 0.5\rho_r = 0.5$
Initialize the parameters; R=zeros (1, soldier size); W = 2×ones (1, soldier size)
Randomly and uniformly distribute the soldiers in the war space (Random attack)
For 1: soldier_size
  Obtain the attack force for each soldier
End of for loop
Sort the fitness (attack force) of all soldiers
Select the soldier with best fitness as King and the second-best attack force as a commander
While t < Tmax (Max-iterations)
  For 1: soldier_size
    $\rho = rand$
    If $\rho < \rho_r$ *percentage signal given to follow the strategy*
      Update the position of each soldier using equation (5) (Exploration)
      Else update the position of each soldier using equation (1) (Exploitation)
    End of if condition
    Calculate the attack force for each soldier
    Sort the fitness of each soldier
    Update the position of every soldier based on the attack force of the current and previous positions using equation (2)
    Update the rank and weight of each soldier based on the success using equation (3)
  End of for loop
  Identify the weak soldier with worst fitness
  Relocate the weak soldier choosing a suitable relocation option
  Update the positions of the King and Commander
  $t = t + 1$
End of while loop
Display the attack force and position of the King

fair comparison the population size is set 30 and the number of iterations is set 1000 for all the algorithms. Median, mean and standard deviations for 10 independent runs are recorded in Table 3 and Table 4. The bold values in the tables represents the best optimum values obtained for a given benchmark function when compared to other algorithms. One can clearly visualize from the Table 3 that WSO dominates other metaheuristic algorithms in the list in the case of Unimodal functions. In the case of multimodal functions, the WSO algorithm performs similarly to the influential SDCS algorithm.

D. PERFORMANCE ANALYSIS OF THE WSO ALGORTHM

The performance of the algorithm is analyzed by testing the algorithm under various conditions to project the salient features/uniqueness of the proposed WSO algorithm. As discussed in section 3.4, one of the main reasons for faster convergence is adaptive weight mechanism assigned to each soldier. To understand the importance of this weight assign mechanism, we have examined the algorithm with multiple operating cases.

i. A constant weight for all soldiers for all iterations
ii. A linearly varying weight which is same for all soldiers
iii. A nonlinear varying weight which varies from soldier to soldier based on the success in the position updating process

For case-1 for test function F26, the average function value for 30 runs is 1.38E-01. Whereas for case-2, the average function value is 4.57E-03. However, in the third case, it is 1.273E-05.

Another unique feature of the WSO algorithm is replacement/relocation of weak soldiers. To examine this feature, we have tested the algorithm for three cases.

i. Algorithm without replacement feature
ii. Algorithm with replacement feature given in (6)
iii. Algorithm with replacement feature given in (7)

The average function values with 20 runs on test function F26 for the three cases are 2.89E+02, 1.27E-05, 2.68E+02 respectively. The test results on test function F31 gives the function values for the above three cases as 4.89E-05, 3.18E-12, 7.77E-62 respectively. The test results on test function F1 gives the function values for the above three cases as

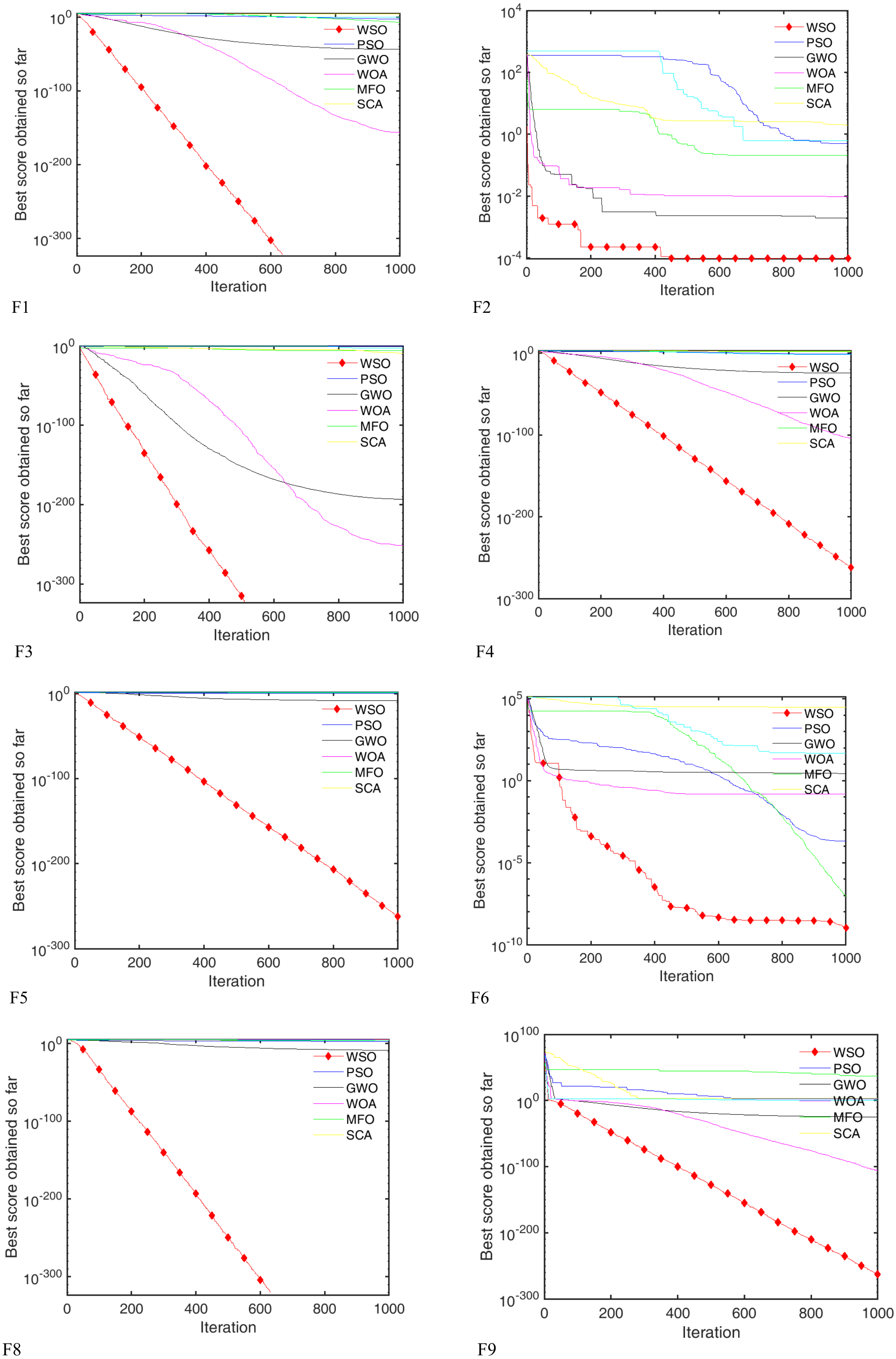

FIGURE 4. Convergence plots for various benchmark functions.

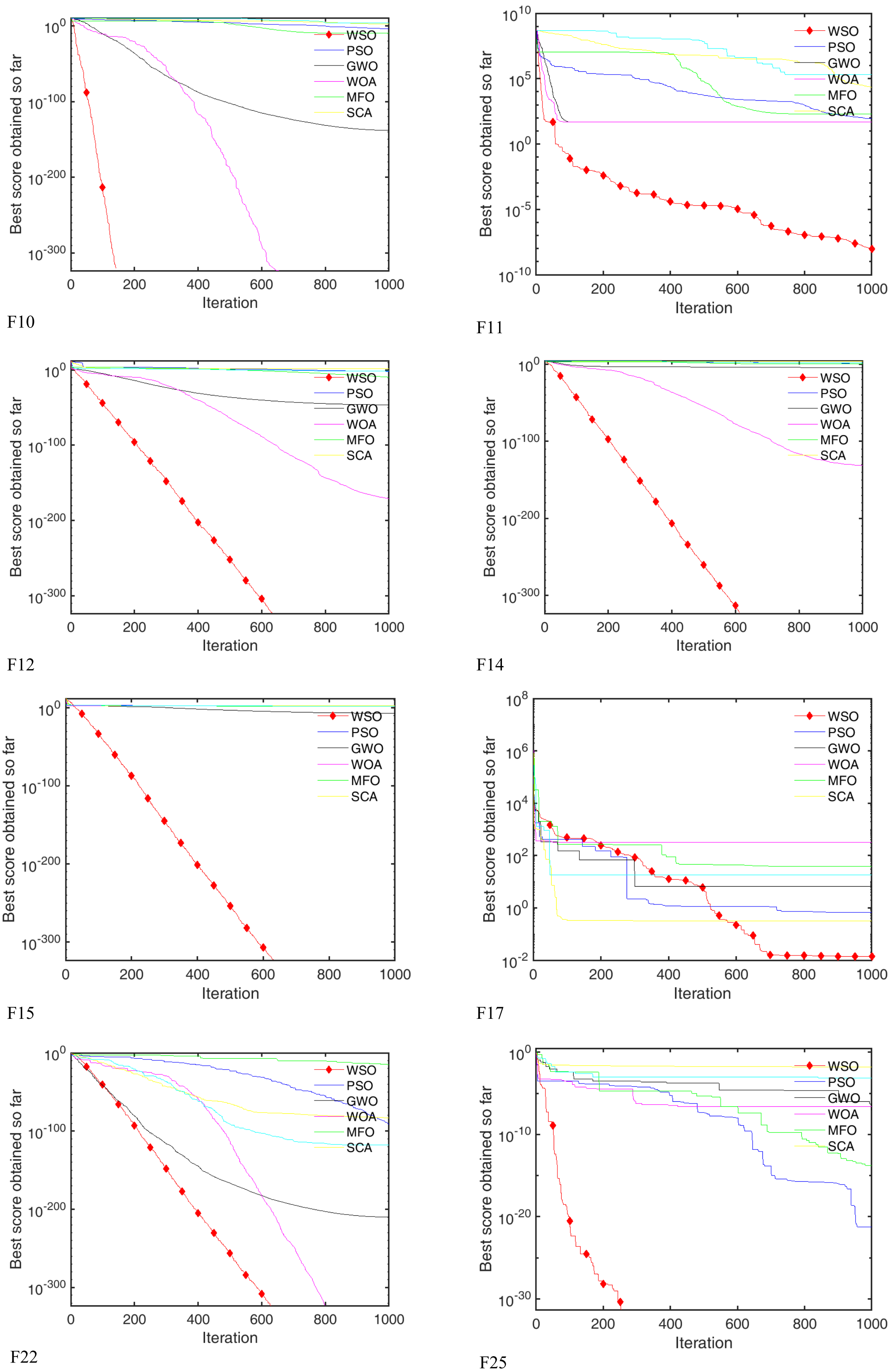

FIGURE 4. ***(Continued.)*** **Convergence plots for various benchmark functions.**

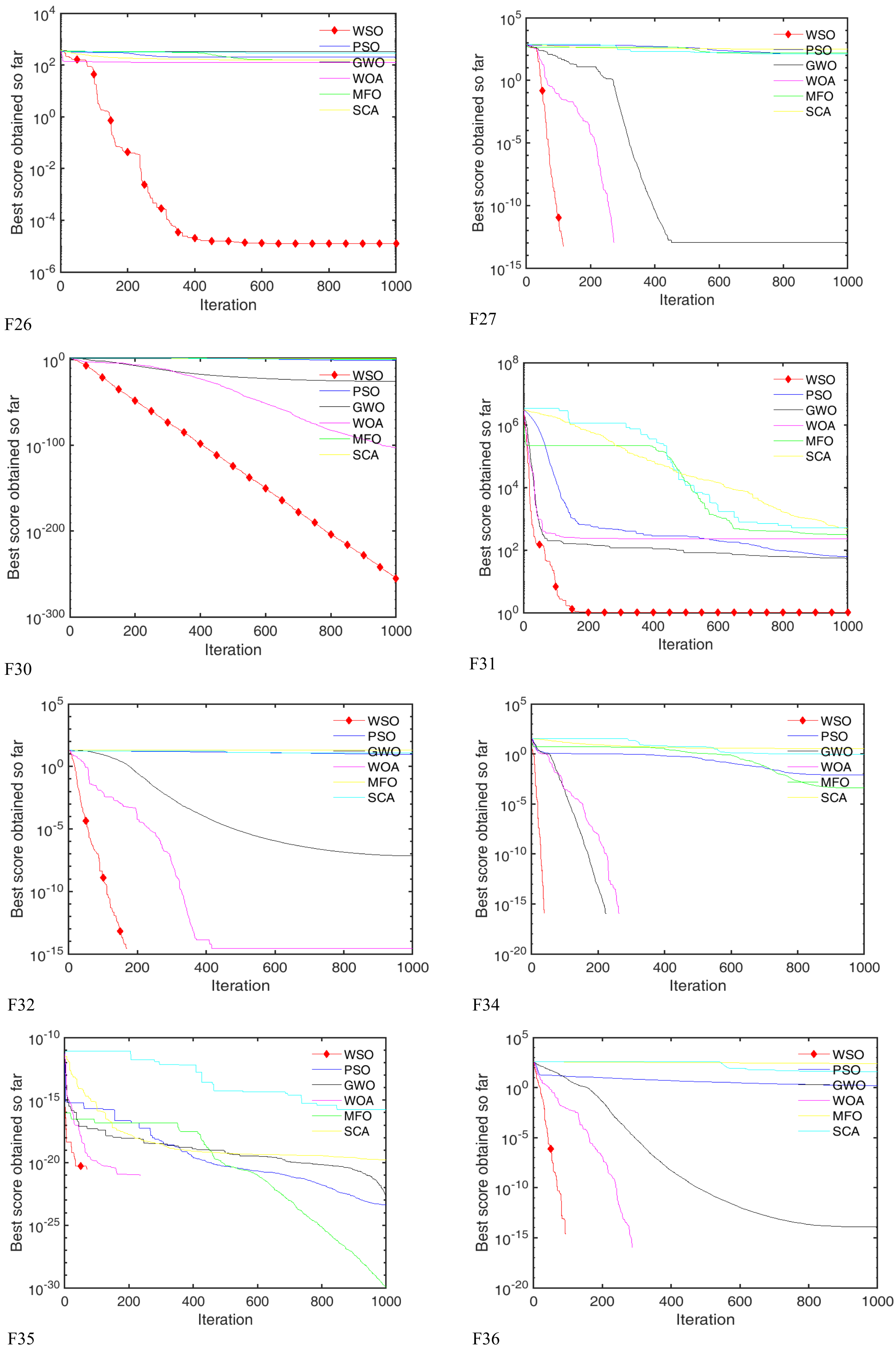

FIGURE 4. *(Continued.)* Convergence plots for various benchmark functions.

TABLE 2. Parameter settings of algorithms used for comparison with WSO.

Name of the algorithm	Parameters	Max-Fes
PSO	c_1=2, c_2=2 and ω=0.3	30000
GA	Mutation =0.1, crossover fraction =0.8	30000
GWO	A ∈ [2 0]	30000
WOA	a_1 ∈ [2 0], a_2 ∈ [-2 -1], b=1	30000
DA	C ∈ [0.1 0], ω=0.9	30000
MVO	WEP ∈ [1 0.2]	30000
Chimp	a ∈ [2 0]	30000
JS	A_r∈ [1 0]	30000
SSA	c ∈ [2 0]	30000
DE	scaling factor =0.5, crossover probability=0.5	30000
SDCS	J = 0.3, Se = 1	30000

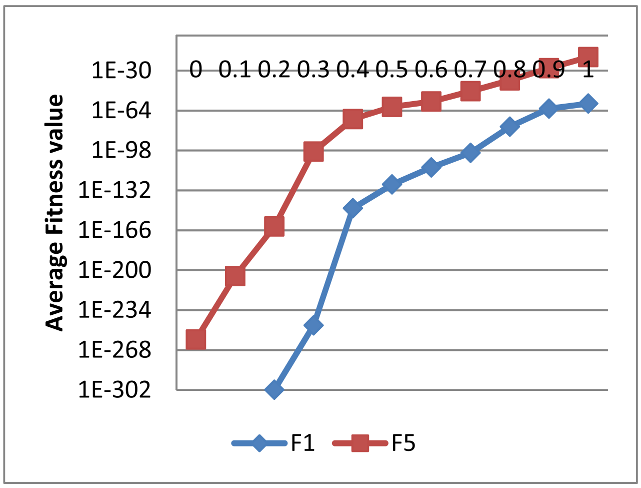

FIGURE 5. Average fitness values of two test functions F1 & F5 with variation in parameter ρ_r.

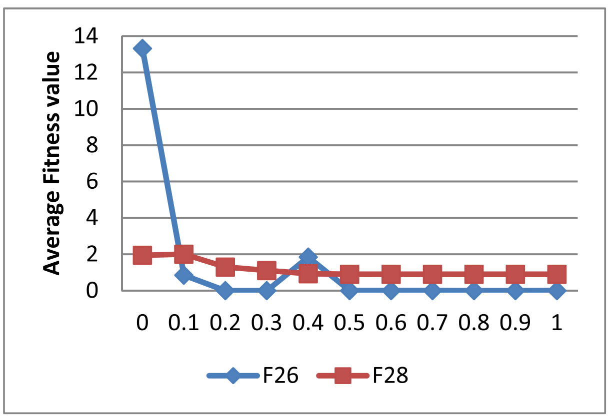

FIGURE 6. Average fitness values of two test functions F26 & F28 with variation in parameter ρ_r.

3.78E-123, 0.00E+00, 5.97E-255 respectively. This proves that replacement strategy is another unique feature of the algorithm. For most of the test function, replacement strategy given in (7) works superior. Research engineers who apply this algorithm for design of practical systems must wisely choose the relocation strategy.

E. SENSITIVITY OF CONTROL PARAMETERS

Selection of suitable algorithm specific parameters is important as it decides the overall performance of the algorithm. Now the performance of the WSO algorithm is analyzed with different parameter variations. For optimization functions like {F3, F8, F15}, the impact of the parameter variations is negligible. However, on certain functions the impact is high. The most dominant parameter which impacts the global optimum selection is ρ_r. The impact of variation of the parameter ρ_r is shown in Figure 5 and Figure 6. From this we can clearly understand for low values of ρ_r (attack strategy) represents the exploitation phase and higher values of ρ_r (defense strategy) represents the exploration phase. For robust performance of the algorithm requires a good balance between exploration and exploitation. The impact of other parameters on the performance of the algorithm is negligible.

F. ANALYSIS ON EXPLORATION AND EXPLOITATION CAPABILITIES

Exploration refers to the search for the global optimum by exploring a large search space, whereas exploitation focuses on previously discovered local area possibilities for convergence into an optimal solution. A meta-heuristic algorithm will frequently start with more exploration and less exploitation. However, as the search progresses to the final moment, this feature reverses.

The exploitation capability of the WSO is evaluated with 25 unimodal test functions. For the first 16 functions i.e., F1-F16 outperform other algorithms for the variable dimension functions. Even for the fixed dimension problems,

TABLE 3. Comparison results for 25 Unimodal benchmark functions. The values in bold are the best optimum values.

BF	Stat	WSO	GWO	PSO	GA	WOA	DA	JS	SSA	Chimp	MVO	DE	SDCS
	Median	**0.00E+00**	4.44E-44	8.76E-04	2.09E+00	1.21E-156	7.47E+03	5.82E-58	1.58E-07	2.45E-09	4.20E+00	2.50E+02	4.06E-274
	Mean	**0.00E+00**	8.37E-44	1.04E-03	2.61E+00	1.43E-150	4.86E+03	5.8325e-59	9.76E-08	1.00E-09	2.65E+00	3.13E+01	3.76E-256
F1	STD	**0.00E+00**	1.20E-43	8.59E-04	1.83E+00	4.51E-150	1.54E+03	1.84E-58	2.80E-08	1.03E-09	7.54E-01	7.78E+01	**0.00E+00**
	Median	**3.88E-05**	1.08E-03	4.34E-01	5.80E+00	1.18E-03	4.05E+00	3.45E-04	3.71E-01	3.60E-03	8.76E-02	3.72E-01	7.00E-05
	Mean	**7.00E-05**	1.43E-03	4.70E-01	6.04E+00	2.44E-03	1.76E+00	1.65E-04	2.85E-01	1.60E-03	5.89E-02	2.82E-01	8.30E-05
F2	STD	**5.21E-05**	8.45E-04	1.55E-01	1.89E+00	2.83E-03	1.13E+00	9.55E-05	5.34E-02	1.41E-03	1.85E-02	7.84E-02	6.12E-05
	Median	**0.00E+00**	1.68E-192	7.22E-01	2.51E-01	2.28E-231	1.53E-04	1.74E-187	1.74E-06	2.96E-30	5.47E-07	1.01E-09	2.21E-306
	Mean	**0.00E+00**	1.93E-185	7.15E-01	6.58E+18	1.54E-223	3.74E-05	1.74E-188	5.44E-07	3.10E-31	2.57E-07	1.47E-10	3.74E-290
F3	STD	**0.00E+00**	0.00E+00	4.78E-01	2.08E+19	0.00E+00	4.56E-05	0.00E+00	4.51E-07	9.33E-31	1.86E-07	3.19E-10	**0.00E+00**
	Median	**2.53E-263**	3.35E-25	6.76E-02	1.35E+01	2.69E-105	5.69E+02	1.37E-25	5.30E+01	9.33E-06	2.18E+01	3.52E+01	1.80E-140
	Mean	**6.31E-259**	4.16E-25	1.47E-01	1.35E+01	2.40E-104	3.60E+02	1.55E-26	3.18E+01	2.04E-06	1.51E+01	6.10E+00	3.25E-136
F4	STD	**0.00E+00**	2.91E-25	1.42E-01	3.25E+00	4.15E-104	9.40E+01	4.27E-26	1.72E+01	2.81E-06	3.99E+00	1.04E+01	7.83E-136
	Median	**3.53E-262**	1.08E-09	2.08E+00	2.22E+00	4.13E+01	5.99E+01	3.99E-36	2.69E+01	4.13E+00	8.39E+00	3.35E+01	1.04E-121
	Mean	**9.27E-260**	1.66E-09	2.25E+00	2.17E+00	4.51E+01	3.57E+01	9.45E-37	1.90E+01	5.90E-01	6.10E+00	2.89E+01	1.59E-118
F5	STD	**0.00E+00**	2.00E-09	4.06E-01	4.09E-01	2.47E+01	1.11E+01	1.17E-36	4.44E+00	1.26E+00	1.96E+00	3.96E+00	5.02E-118
	Median	**8.206E-10**	2.63E+00	4.84E-04	8.25E+00	3.14E-01	5.32E+03	8.23E-07	1.28E-07	7.75E+00	3.58E+00	4.66E+00	**5.08E-22**
	Mean	**1.505E-08**	2.48E+00	1.21E-03	1.11E+01	3.75E-01	3.16E+03	1.06E-07	8.41E-08	7.25E+00	2.54E+00	1.78E+00	**8.77E-21**
F6	STD	**4.259E-08**	7.57E-01	1.29E-03	7.18E+00	1.95E-01	1.34E+03	2.54E-07	2.48E-08	6.55E-01	6.09E-01	1.47E+00	**1.68E-20**
	Median	**-2.75E+02**	-2.12E+02	-3.46E+04	-2.67E+02	**-2.75E+02**	-7.30E+01	-2.36E+02	-1.66E+02	-2.75E+02	-2.29E+02	-2.35E+02	**-2.75E+02**
	Mean	**-2.75E+02**	-2.12E+02	-3.45E+04	-2.59E+02	**-2.75E+02**	-1.17E+02	-2.55E+02	-2.12E+02	-2.75E+02	-2.40E+02	-2.40E+02	**-2.75E+02**
F7	STD	**0.00E+00**	1.21E+01	4.84E+02	3.10E+01	**0.00E+00**	3.16E+01	1.42E+01	2.01E+01	0.00E+00	7.23E+00	2.60E+00	**0.00E+00**
	Median	**0.00E+00**	8.55E-08	5.97E+02	3.14E+01	1.20E+05	7.72E+04	2.41E+02	8.30E+03	4.12E+02	2.02E+03	2.21E+03	9.69E-251
	Mean	**0.00E+00**	1.14E-06	6.19E+02	4.69E+01	1.20E+05	4.18E+04	4.84E+01	4.68E+03	1.15E+02	1.51E+03	1.22E+03	9.75E-228
F8	STD	**0.00E+00**	3.15E-06	1.29E+02	4.32E+01	3.22E+04	1.85E+04	9.29E+01	1.98E+03	1.24E+02	3.94E+02	5.40E+02	0.00E+00
	Median	**8.72E-261**	4.31E-25	1.68E+00	1.16E+01	1.60E-104	1.06E+03	6.56E-19	1.41E+32	1.67E-06	9.88E+50	1.37E+02	3.13E-138
	Mean	**6.47E-258**	5.67E-25	3.74E+00	1.20E+01	4.60E-103	4.76E+02	6.56E-20	1.41E+31	6.51E-07	9.88E+49	4.16E+01	4.32E-130
F9	STD	**0.00E+00**	4.03E-25	7.17E+00	3.20E+00	7.44E-103	2.73E+02	2.07E-19	4.45E+31	5.74E-07	3.12E+50	4.71E+01	1.35E-129
	Median	**0.00E+00**	2.09E-142	8.99E-03	6.66E-03	**0.00E+00**	1.26E+06	**0.00E+00**	2.27E-07	4.35E-22	4.27E-08	4.35E+02	**0.00E+00**
	Mean	**0.00E+00**	1.04E-136	2.72E-02	3.31E-02	**0.00E+00**	1.90E+05	**0.00E+00**	2.40E-08	5.00E-23	7.95E-09	9.00E+01	**0.00E+00**
F10	STD	**0.00E+00**	2.61E-136	5.35E-02	6.92E-02	**0.00E+00**	3.82E+05	**0.00E+00**	7.16E-08	1.37E-22	1.40E-08	1.46E+02	**0.00E+00**
	Median	1.95E-09	4.70E+01	1.30E+02	2.09E+02	4.77E+01	3.31E+06	3.62E-02	7.06E+02	4.89E+01	2.90E+03	1.88E+03	**6.61E-20**
	Mean	**1.15E-07**	4.68E+01	1.33E+02	2.22E+02	4.79E+01	1.15E+06	1.31E-02	1.65E+02	4.87E+01	8.55E+02	1.05E+03	9.60E+00
F11	STD	**2.61E-07**	6.02E-01	3.13E+01	9.51E+01	4.54E-01	9.54E+05	1.26E-02	1.94E+02	3.61E-01	9.72E+02	5.32E+02	2.02E+01
	Median	**0.00E+00**	9.33E-47	6.83E-03	5.62E+00	1.01E-158	6.32E+01	1.32E-29	5.95E-10	1.39E-12	9.35E-03	3.72E-03	7.04E-279
	Mean	**0.00E+00**	2.15E-46	8.55E-03	7.00E+00	4.74E-151	2.97E+01	1.32E-30	2.72E-10	2.42E-13	5.84E-03	1.47E-03	3.22E-266
F12	STD	**0.00E+00**	2.63E-46	7.33E-03	4.57E+00	1.49E-150	2.02E+01	4.18E-30	1.26E-10	4.25E-13	1.73E-03	1.29E-03	0.00E+00
	Median	**2.49E-01**	6.67E-01	7.44E+00	1.77E+02	6.67E-01	3.46E+04	2.42E-01	3.34E+01	1.00E+00	3.71E+01	4.84E+01	2.08E-01
	Mean	**2.49E-01**	6.67E-01	7.96E+00	1.81E+02	6.67E-01	1.27E+04	8.44E-02	8.05E+00	9.33E-01	1.46E+01	3.48E+01	2.51E-01
F13	STD	2.87E-04	**6.93E-07**	4.13E+00	8.86E+01	8.83E-05	8.45E+03	7.36E-02	9.68E+00	1.40E-01	1.16E+01	9.19E+00	1.47E-01
	Median	**0.00E+00**	3.06E-06	5.45E+00	2.70E+01	1.37E-27	4.70E+02	1.11E-12	9.45E+00	2.84E-04	9.59E+00	2.31E+01	8.14E-280
	Mean	**0.00E+00**	4.70E-06	2.03E+01	4.04E+01	7.21E-10	2.78E+02	1.14E-13	5.88E+00	4.60E-05	5.86E+00	3.58E+00	2.00E-267
F14	STD	**0.00E+00**	5.47E-06	2.76E+01	2.55E+01	2.28E-09	1.11E+02	3.51E-13	1.97E+00	9.19E-05	1.97E+00	6.92E+00	0.00E+00
	Median	**0.00E+00**	1.48E-08	2.94E+02	1.07E+03	8.73E+02	2.17E+03	2.00E+01	2.35E+02	9.01E+01	5.48E+00	2.81E+01	3.92E-188
	Mean	**0.00E+00**	1.02E-07	3.00E+02	8.81E+02	8.47E+02	1.36E+03	3.57E+00	1.48E+02	4.54E+01	3.34E+00	2.03E+01	1.79E-164
F15	STD	**0.00E+00**	1.99E-07	6.47E+01	5.65E+02	1.58E+02	4.52E+02	6.04E+00	4.57E+01	2.24E+01	1.11E+00	6.24E+00	0.00E+00
	Median	**0.00E+00**	2.01E-199	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.88E-120	1.56E-281	**0.00E+00**	9.83E-221	9.55E-160	**0.00E+00**
	Mean	**0.00E+00**	1.23E-179	**0.00E+00**	**0.00E+00**	-3.00E-01	**0.00E+00**	1.88E-121	1.56E-282	**0.00E+00**	1.01E-221	9.55E-161	**0.00E+00**
F16	STD	**0.00E+00**	0.00E+00	**0.00E+00**	**0.00E+00**	4.83E-01	**0.00E+00**	5.94E-121	0.00E+00	**0.00E+00**	0.00E+00	3.02E-160	**0.00E+00**
	Median	1.14E-01	1.51E+01	**1.02E-01**	3.18E+01	2.96E+02	1.68E+03	2.13E+00	3.66E+01	4.78E+03	2.09E+00	2.02E+01	1.32E+01
	Mean	4.39E-01	5.38E+01	**4.28E-01**	1.73E+02	5.55E+02	1.81E+02	6.50E-01	1.36E+01	2.54E+03	5.35E-01	8.53E+00	3.98E+01
F17	STD	9.23E-01	7.09E+01	**7.94E-01**	3.33E+02	6.65E+02	5.28E+02	6.54E-01	1.37E+01	2.16E+03	5.93E-01	5.47E+00	6.55E+01
	Median	**0.00E+00**	0.00E+00	1.56E-123	3.23E-11	6.13E-163	1.14E-06	8.61E-252	2.79E-15	2.27E-302	9.58E-08	5.25E-77	1.40E-292
	Mean	**0.00E+00**	0.00E+00	1.61E-117	8.01E-11	3.07E-149	2.54E-07	8.69E-253	1.16E-15	2.41E-303	3.52E-08	7.82E-78	1.69E-278
F18	STD	**0.00E+00**	0.00E+00	5.09E-117	1.24E-10	6.60E-149	4.33E-07	0.00E+00	9.53E-16	0.00E+00	3.07E-08	1.64E-77	0.00E+00
	Median	**0.00E+00**	2.05E-08	**0.00E+00**	3.89E-09	1.47E-11	3.69E-06	**0.00E+00**	1.03E-14	7.62E-01	7.62E-01	**0.00E+00**	**1.62E-13**
	Mean	**0.00E+00**	7.62E-02	**0.00E+00**	2.33E-01	7.87E-11	8.14E-07	**0.00E+00**	1.88E-15	7.62E+04	2.29E-01	**0.00E+00**	**2.12E-12**
F19	STD	**0.00E+00**	2.41E-01	**0.00E+00**	3.76E-01	2.02E-10	1.15E-06	**0.00E+00**	3.03E-15	2.41E-01	3.68E-01	**0.00E+00**	**4.05E-12**
	Median	**0.00E+00**	1.65E-07	**0.00E+00**	1.78E-10	1.73E-04	4.79E-05	**0.00E+00**	1.27E-13	3.78E-04	6.41E-07	**0.00E+00**	8.85E-13
	Mean	**0.00E+00**	1.85E-07	**0.00E+00**	4.23E-10	1.73E-04	5.90E-06	**0.00E+00**	3.71E-14	1.10E-04	2.01E-07	**0.00E+00**	1.29E-12
F20	STD	**0.00E+00**	1.68E-07	**0.00E+00**	5.66E-10	1.52E-04	1.49E-05	**0.00E+00**	4.05E-14	1.16E-04	2.00E-07	**0.00E+00**	1.39E-12
F21	Median	**1.38E-87**	**1.38E-87**	**1.38E-87**	1.23E-11	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**
	Mean	**1.38E-87**	**1.38E-87**	**1.38E-87**	4.29E-11	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**	**1.38E-87**
	STD	**2.35E-103**	**2.35E-103**	**2.35E-103**	7.30E-11	**2.35E-103**	**2.35E-103**	**2.35E-103**	**2.35E-103**	**2.35E-103**	**2.35E-103**	**2.35E-103**	**2.35E-103**
	Median	**0.00E+00**	3.36E-217	2.45E-90	3.13E-11	**0.00E+00**	3.06E-07	8.70E-172	5.45E-16	6.92E-131	6.42E-09	2.24E-66	2.46E-260
	Mean	**0.00E+00**	5.51E-205	4.11E-84	4.40E-11	**0.00E+00**	4.20E-08	1.18E-172	1.98E-16	6.96E-132	2.96E-09	5.33E-67	7.21E-243
F22	STD	**0.00E+00**	0.00E+00	1.30E-83	5.33E-11	**0.00E+00**	9.77E-08	0.00E+00	2.06E-16	2.19E-131	1.99E-09	8.13E-67	0.00E+00
	Median	**2.93E-01**	**2.93E-01**	**2.93E-01**	3.06E-01	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**
	Mean	**2.93E-01**	**2.93E-01**	**2.93E-01**	3.11E-01	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**	**2.93E-01**
F23	STD	**4.53E-17**	1.61E-07	6.92E-17	1.78E-02	7.11E-06	8.92E-08	5.55E-17	1.13E-14	7.19E-07	4.75E-07	2.00E-06	2.06E-06
	Median	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.99E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**
	Mean	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.92E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**	**1.91E+01**
F24	STD	**4.53E-17**	5.35E-06	2.05E-15	1.90E-09	1.42E-03	2.60E-01	1.67E-15	1.84E-10	4.69E-03	7.60E-04	4.74E-15	2.69E-11
	Median	**0.00E+00**	2.82E-07	4.50E-18	2.14E-03	4.10E-06	3.92E-03	3.66E-23	3.27E-15	3.57E-04	4.06E-08	**0.00E+00**	4.92E-21
	Mean	**0.00E+00**	3.53E-07	9.04E-17	1.90E-03	1.57E-05	7.91E-04	3.83E-24	5.17E-16	1.00E-04	1.54E-08	**0.00E+00**	3.23E-15
F25	STD	**0.00E+00**	3.66E-07	2.11E-16	1.14E-03	3.47E-05	1.65E-03	1.15E-23	9.75E-16	1.13E-04	1.60E-08	**0.00E+00**	1.02E-14

TABLE 4. Comparison results for the 25 Multimodal benchmark functions. The values in bold are the best optimum values.

BF	Stat	WSO	GWO	PSO	GA	WOA	DA	JS	SSA	Chimp	MVO	DE	SDCS
F26	Median	**1.27E-05**	2.33E+02	2.09E+02	4.04E+02	1.19E+02	2.92E+02	1.61E+02	2.15E+02	3.46E+02	1.84E+02	7.60E+01	**1.27E-05**
	Mean	1.43E-05	2.30E+02	2.17E+02	4.04E+02	9.39E+01	2.74E+02	9.72E+01	1.84E+02	3.33E+02	1.63E+02	6.19E+01	**1.27E-05**
	STD	4.04E-06	1.66E+01	3.89E+01	2.06E+00	4.80E+01	1.68E+01	4.30E+01	2.50E+01	7.27E+00	1.29E+01	8.58E+00	**0.00E+00**
F27	Median	**0.00E+00**	0.00E+00	1.31E+02	6.42E+01	**0.00E+00**	3.90E+02	1.07E-02	1.46E+02	8.55E+00	3.28E+02	8.76E+01	**0.00E+00**
	Mean	**0.00E+00**	1.00E-01	1.29E+02	6.21E+01	**0.00E+00**	3.25E+02	3.28E-03	9.12E+01	3.34E+00	2.44E+02	5.18E+01	**0.00E+00**
	STD	**0.00E+00**	3.16E-01	1.61E+01	1.43E+01	**0.00E+00**	3.79E+01	3.93E-03	2.43E+01	2.59E+00	4.84E+01	1.69E+01	**0.00E+00**
F28	Median	**9.00E-01**	1.57E+00	1.00E+00	1.76E+00	**9.00E-01**	1.33E+01	1.38E+01	1.00E+00	1.36E+01	1.03E+00	2.56E+00	**9.00E-01**
	Mean	**9.00E-01**	2.45E+00	1.00E+00	1.80E+00	1.05E+00	1.06E+01	1.26E+01	1.00E+00	1.34E+01	1.02E+00	1.92E+00	**9.00E-01**
	STD	**1.17E-16**	2.71E+00	7.41E-04	3.60E-01	2.50E-01	2.03E+00	1.25E+00	4.68E-10	2.19E-01	3.51E-03	3.44E-01	**1.17E-16**
F29	Median	3.66E+03	9.78E+03	2.85E-02	6.36E+02	6.96E+03	2.61E+09	1.18E+03	**1.68E+02**	3.20E+04	6.05E+03	2.05E+07	4.75E+03
	Mean	3.83E+03	1.01E+04	5.77E-02	1.04E+03	6.54E+03	1.58E+09	4.00E+02	**6.70E+01**	2.70E+04	4.36E+03	2.50E+06	4.72E+03
	STD	6.02E+02	1.86E+03	5.93E-02	9.65E+02	1.35E+03	7.90E+08	4.04E+02	**4.26E+01**	2.50E+03	9.03E+02	6.38E+06	1.11E+03
F30	Median	**2.22E-261**	2.89E-21	3.72E-02	2.95E+00	1.99E-106	5.87E+01	9.57E-03	1.24E+01	1.79E-03	1.86E+01	9.26E-01	4.11E-140
	Mean	**2.30E-259**	4.26E-04	8.96E-02	2.91E+00	4.33E-104	3.69E+01	2.70E-03	7.58E+00	4.92E-04	1.34E+01	1.26E-01	2.57E-13
	STD	**0.00E+00**	6.71E-04	1.71E-01	1.62E+00	1.01E-103	1.71E+01	3.12E-03	2.89E+00	6.71E-04	3.48E+00	2.82E-01	6.19E-13
F31	Median	3.15E-70	1.52E-56	9.99E+02	2.82E+06	5.54E-14	3.02E+07	2.07E-14	1.31E+04	4.30E-07	5.02E+02	4.87E+01	**4.77E-136**
	Mean	7.77E-39	5.34E-36	1.63E+05	3.33E+13	3.93E-04	3.24E+06	2.19E-15	2.79E+03	4.31E-08	1.51E+02	1.82E+01	**1.14E-122**
	STD	2.46E-38	1.69E-35	4.13E+05	1.05E+14	1.02E-03	9.50E+06	6.51E-15	5.00E+03	1.36E-07	1.94E+02	1.66E+01	**3.60E-122**
F32	Median	**-8.88E-16**	3.11E-14	9.62E-01	2.71E+00	2.66E-15	1.27E+01	2.66E-15	4.40E+00	2.00E+01	3.47E+00	7.01E+00	**-8.88E-16**
	Mean	**-8.88E-16**	3.11E-14	8.70E-01	2.67E+00	1.95E-15	1.12E+01	1.24E-15	3.25E+00	2.00E+01	2.27E+00	4.81E+00	**-8.88E-16**
	STD	**0.00E+00**	4.10E-15	7.31E-01	4.99E-01	2.80E-15	1.18E+00	1.83E-15	6.09E-01	3.31E-04	6.57E-01	1.37E+00	**0.00E+00**
F33	Median	**1.00E+00**	6.56E+01	4.79E+01	3.50E+01	1.07E+02	1.30E+05	**1.00E+00**	5.07E+02	4.04E+02	6.80E+02	1.94E+03	**1.00E+00**
	Mean	**1.00E+00**	6.88E+01	5.02E+01	3.98E+01	1.25E+02	1.04E+05	**1.00E+00**	3.81E+02	1.99E+02	5.92E+02	5.64E+02	**1.00E+00**
	STD	4.90E-03	8.89E+00	1.59E+01	1.63E+01	3.41E+01	2.27E+04	1.83E-06	7.94E+01	1.06E+02	4.30E+01	5.04E+02	**0.00E+00**
F34	Median	4.09E-106	2.00E-01	6.50E-01	8.00E-01	9.99E-02	1.01E+01	9.99E-02	3.30E+00	2.00E-01	1.50E+00	4.60E+00	**8.08E-131**
	Mean	2.73E-42	1.90E-01	6.70E-01	8.10E-01	1.10E-01	8.79E+00	9.99E-02	2.68E+00	1.32E-01	1.25E+00	3.19E+00	**1.46E-126**
	STD	8.65E-42	3.16E-02	8.23E-02	1.60E-01	7.37E-02	1.27E+00	1.12E-09	3.85E-01	4.74E-02	1.72E-01	7.71E-01	**2.33E-126**
F35	Median	**-1.96E+03**	-1.39E+03	-1.79E+03	-1.30E+03	-1.96E+03	-1.46E+03	**-1.96E+03**	-1.60E+03	-7.32E+02	-1.59E+03	-1.73E+03	**-1.96E+03**
	Mean	**-1.96E+03**	-1.39E+03	-1.74E+03	-1.32E+03	-1.92E+03	-1.55E+03	**-1.96E+03**	-1.68E+03	-8.66E+02	-1.65E+03	-1.77E+03	**-1.96E+03**
	STD	5.02E-08	7.28E+01	5.28E+01	3.34E+01	7.15E+01	5.12E+01	6.16E-04	5.21E+01	7.09E+01	4.22E+01	3.08E+01	**2.40E-13**
F36	Median	**0.00E+00**	**0.00E+00**	3.72E-03	7.16E-02	0.00E+00	3.20E+00	**0.00E+00**	1.98E-02	1.30E-02	2.74E-01	1.86E-01	**0.00E+00**
	Mean	**0.00E+00**	**0.00E+00**	4.96E-03	8.17E-02	9.61E-03	2.37E+00	**0.00E+00**	5.21E-03	1.30E-03	2.08E-01	6.60E-02	**0.00E+00**
	STD	**0.00E+00**	**0.00E+00**	5.92E-03	3.89E-02	3.04E-02	5.04E-01	**0.00E+00**	6.68E-03	4.10E-03	3.81E-02	5.85E-02	**0.00E+00**
F37	Median	**-1.00E+00**	7.20E-24	8.83E-24	1.11E-20	2.40E-21	9.37E-20	4.05E-22	3.81E-30	1.20E-15	6.69E-23	1.41E-21	**-1.00E+00**
	Mean	**-1.00E+00**	8.60E-24	2.06E-23	1.72E-20	-2.00E-01	5.15E-20	1.15E-22	1.87E-30	4.92E-16	3.89E-23	5.38E-22	**-1.00E+00**
	STD	**0.00E+00**	5.22E-24	3.18E-23	1.67E-20	4.22E-01	2.85E-20	1.38E-22	9.40E-31	4.56E-16	1.27E-23	4.14E-22	**0.00E+00**
F38	Median	**1.21E-20**	1.38E-14	4.98E-20	4.93E-20	1.24E-20	3.00E-15	2.00E-20	8.90E-19	1.74E-19	6.99E-18	3.13E-18	**1.21E-20**
	Mean	**1.22E-20**	3.20E-11	4.88E-20	6.97E-20	1.38E-20	1.34E-15	1.34E-20	3.46E-19	1.71E-19	1.29E-18	1.03E-18	**1.22E-20**
	STD	2.52E-22	9.73E-11	7.69E-21	5.74E-20	4.35E-21	1.08E-15	2.55E-21	2.71E-19	1.64E-21	2.04E-18	9.14E-19	**1.57E-33**
F39	Median	9.05E-08	1.83E+00	1.10E-02	4.08E-01	6.85E-01	1.78E+06	3.16E-07	8.39E+01	4.89E+00	9.87E-01	4.15E+02	**5.30E-25**
	Mean	2.32E-07	1.86E+00	8.69E-03	4.17E-01	7.54E-01	6.77E+05	6.37E-08	6.57E+01	4.62E+00	4.89E-01	1.49E+02	**9.01E-24**
	STD	2.72E-07	3.93E-01	5.16E-03	1.79E-01	3.20E-01	8.57E+05	1.03E-07	1.62E+01	1.67E-01	2.77E-01	1.26E+02	**1.64E-23**
F40	Median	5.34E-12	8.16E-02	6.42E-06	3.64E-01	9.29E-03	9.76E+04	1.29E-08	2.00E+01	7.77E-01	6.18E+00	9.23E+02	**1.56E-27**
	Mean	8.81E-11	7.69E-02	1.24E-02	3.71E-01	1.04E-02	3.27E+04	2.03E-09	1.14E+01	6.07E-01	3.74E+00	1.25E+02	**6.83E-27**
	STD	2.28E-10	2.25E-02	2.62E-02	1.07E-01	7.55E-03	3.99E+04	3.88E-09	5.00E+00	1.24E-01	1.16E+00	2.91E+02	**1.10E-26**
F41	Median	**0.00E+00**	0.00E+00	1.95E-124	1.08E-09	6.75E-239	1.80E-06	2.10E-250	8.43E-14	9.28E-273	9.19E-07	2.51E-77	4.27E-298
	Mean	**0.00E+00**	0.00E+00	2.13E-123	1.90E+00	1.37E-219	4.52E-07	2.15E-251	1.53E-14	9.28E-274	3.49E-07	5.26E-78	3.81E-287
	STD	**0.00E+00**	0.00E+00	4.44E-123	4.00E+00	0.00E+00	7.79E-07	0.00E+00	2.56E-14	0.00E+00	2.85E-07	8.29E-78	0.00E+00
F42	Median	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**
	Mean	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**	**-1.96E+02**
	STD	**3.00E-14**	2.52E-08	**3.00E-14**	3.05E-09	5.82E-08	5.71E-04	**3.00E-14**	6.17E-13	2.07E-05	7.28E-06	**3.00E-14**	7.71E-11
F43	Median	**-2.02E+00**	**-2.02E+00**	-3.59E+03	-2.02E+00	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**
	Mean	**-2.02E+00**	**-2.02E+00**	-3.57E+03	-1.92E+00	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**	**-2.02E+00**
	STD	4.68E-16	8.91E-13	4.82E+01	3.09E-01	4.68E-16	**3.85E-16**	4.68E-16	4.68E-16	1.21E-10	3.72E-11	4.68E-16	2.56E-16
F44	Median	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-8.73E+01**	**-1.07E+02**	**-1.07E+02**
	Mean	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.03E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.07E+02**	**-1.01E+02**	**-1.07E+02**	**-1.07E+02**
	STD	**9.47E-15**	2.01E-05	1.06E-14	8.20E+00	4.13E-06	9.64E-04	1.06E-14	2.59E-13	8.00E-03	9.40E+00	1.89E-14	3.07E-06
F45	Median	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**
	Mean	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**
	STD	**0.00E+00**	4.86E-09	7.40E-17	5.29E-10	3.54E-11	2.71E-08	**0.00E+00**	1.23E-14	7.47E-06	1.07E-07	1.28E-16	5.10E+00
F46	Median	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.99E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Mean	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	STD	**0.00E+00**	1.02E-07	**0.00E+00**	3.05E-10	1.52E-06	4.78E-07	**0.00E+00**	2.92E-15	2.92E-04	2.45E-08	**0.00E+00**	**0.00E+00**
F47	Median	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+01**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**
	Mean	**3.00E+00**	**1.11E+01**	**3.00E+00**	**1.38E+01**	**3.00E+00**	**3.00E+00**	5.70E+00	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	5.70E+00
	STD	1.32E-15	2.56E+01	2.96E-16	1.39E+01	1.10E-05	1.11E-05	8.54E+00	7.67E-14	4.04E-05	1.20E-06	**4.44E-16**	8.54E+00
F48	Median	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.85E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**
	Mean	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.63E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**
	STD	**9.00E-16**	2.48E-03	9.36E-16	3.73E-01	1.59E-03	8.12E-05	9.36E-16	2.66E-14	2.41E-03	2.33E-07	9.36E-16	2.84E-09
F49	Median	**-3.32E+00**	-3.20E+00	-3.32E+00	-3.26E+00	**-3.32E+00**	-3.30E+00	**-3.32E+00**	-3.19E+00	-1.92E+00	-3.20E+00	-3.20E+00	**-3.32E+00**
	Mean	-3.27E+00	-3.22E+00	-3.27E+00	-3.26E+00	-3.28E+00	-3.32E+00	**-3.32E+00**	-3.25E+00	-2.64E+00	-3.24E+00	-3.30E+00	**-3.32E+00**
	STD	6.14E-02	7.99E-02	6.14E-02	6.27E-02	6.09E-02	8.42E-03	**3.63E-16**	6.35E-02	3.42E-01	5.76E-02	5.01E-02	2.30E-04
F50	Median	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**
	Mean	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**	**-2.06E+00**
	STD	4.68E-16	5.05E-09	4.68E-16	2.53E-11	9.60E-10	2.36E-10	4.68E-16	1.57E-15	2.72E-06	2.26E-08	**4.44E-16**	4.68E-16

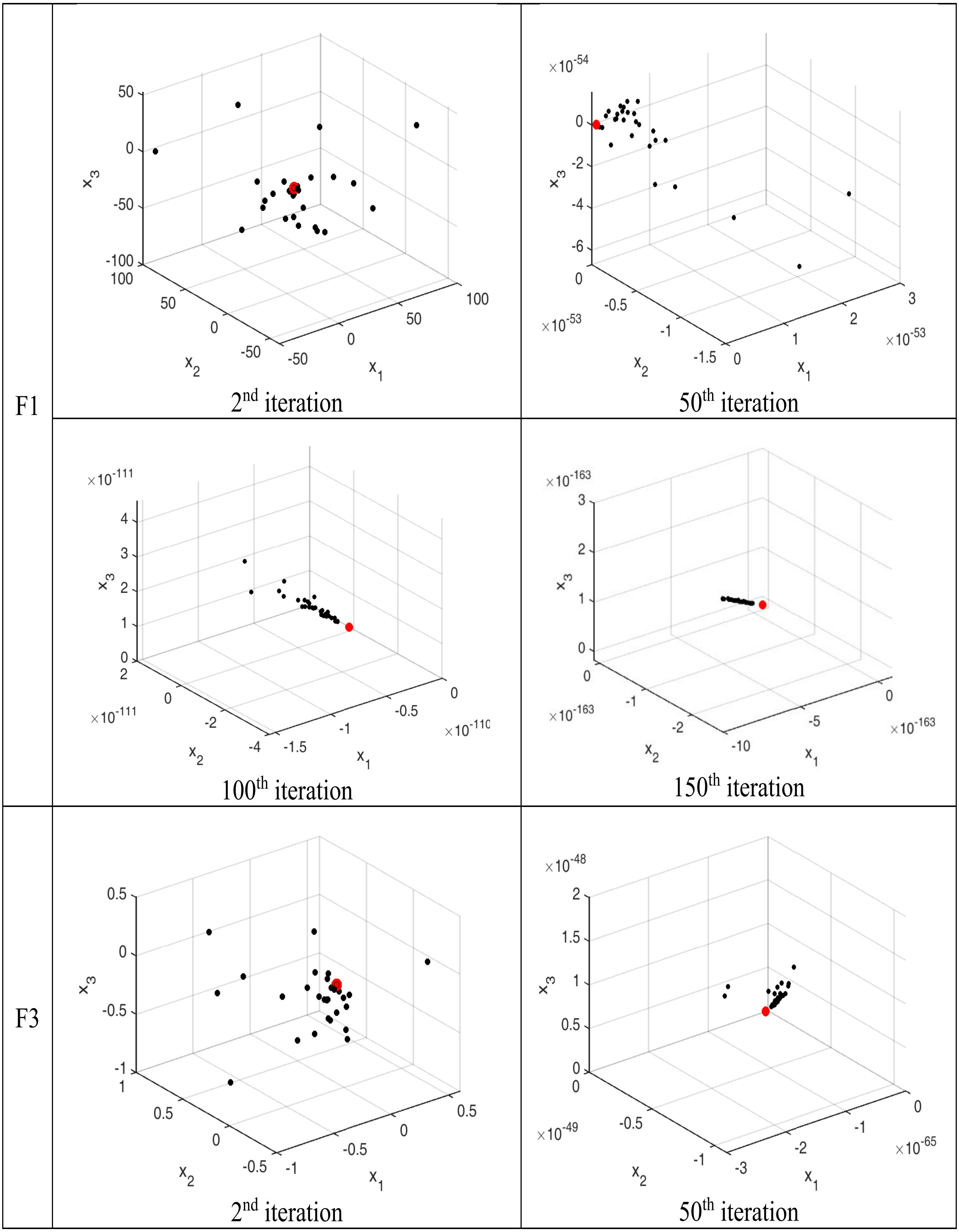

FIGURE 7. Search history to prove the convergence of the algorithm.

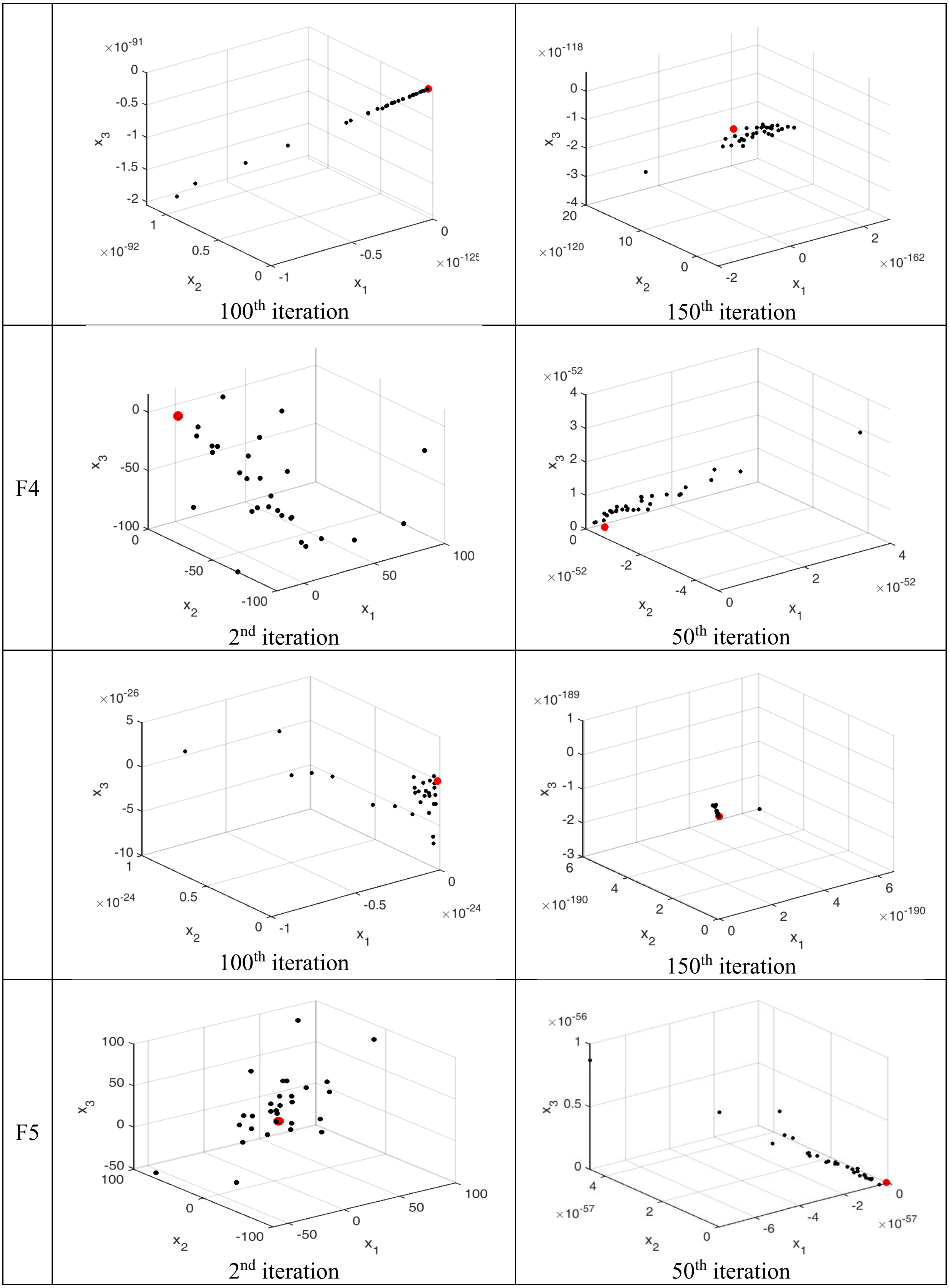

FIGURE 7. *(Continued.)* **Search history to prove the convergence of the algorithm.**

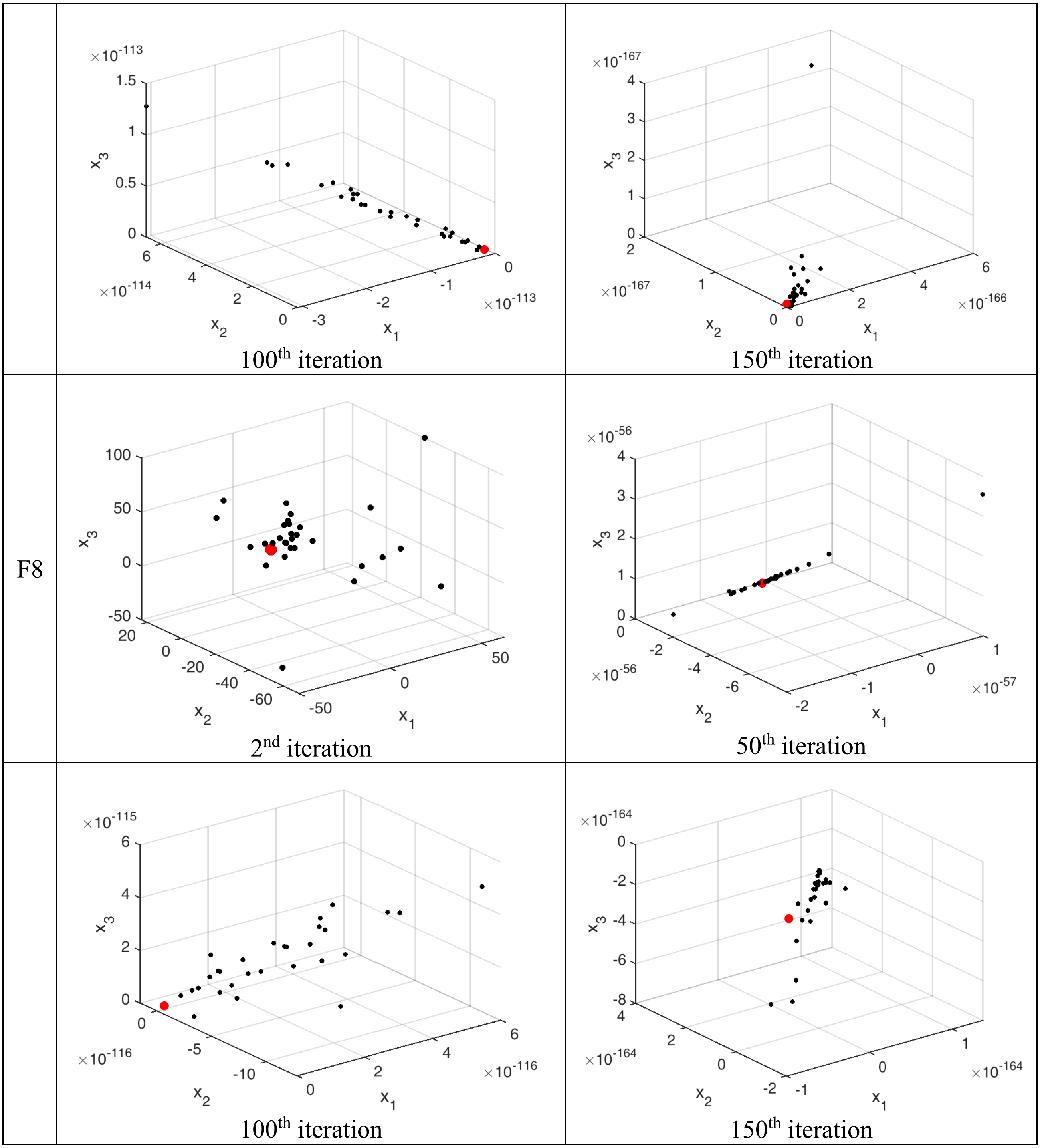

FIGURE 7. ***(Continued.)*** **Search history to prove the convergence of the algorithm.**

WSO has shown consistent performance. The objective function values for various functions with WSO are far better than the values obtained other algorithms. The comparison results with other algorithms for unimodal functions have shown the superior exploitation capability of the WSO algorithm.

The exploration capability of the WSO is evaluated with 25 multimodal functions and the performance of the WSO is compared with other algorithms. WSO has shown superior performance for variable dimension multimodal functions i.e., F26-F40 with first rank except for functions F29, F31, F39. For functions F31, F39 it stood with second rank. For fixed dimension multimodal functions (F41-F50) it has shown consistent performance. Thus, from the comparison results we can conclude that WSO possess good exploration capability. Random selection of war strategy, relocation of weak soldier, multiple position updation strategies are key

TABLE 5. Results of WSO for various dimensions.

BF	Stat	Dim = 50	Dim =500	Dim =1000
F1	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F2	Median	3.88E-05	8.529E-05	8.846E-05
	Mean	7.00E-05	9.890E-05	8.763E-05
	STD	5.21E-05	7.630E-05	4.628E-05
F3	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F4	Median	2.53E-263	3.202E-260	1.626E-260
	Mean	6.31E-259	1.693E-258	5.008E-255
	STD	0.00E+00	0.000E+00	0.000E+00
F5	Median	3.53E-262	1.349E-263	3.140E-262
	Mean	9.27E-260	2.059E-259	6.853E-258
	STD	0.00E+00	0.000E+00	0.000E+00
F6	Median	6.30E-07	2.846E-09	6.517E-07
	Mean	1.15E-06	1.309E-07	3.824E-05
	STD	1.31E-06	3.908E-07	8.756E-05
F7	Median	-2.75E+02	-2.975E+03	-5.975E+03
	Mean	-2.75E+02	-2.975E+03	-5.975E+03
	STD	0.00E+00	0.000E+00	0.000E+00
F8	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F9	Median	8.72E-261	2.577E-257	4.798E-253
	Mean	6.47E-258	4.079E-254	2.538E-244
	STD	0.00E+00	0.000E+00	0.000E+00
F10	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F11	Median	1.95E-09	1.761E-09	8.932E-09
	Mean	1.15E-07	1.825E-07	4.564E-08
	STD	2.61E-07	3.337E-07	7.122E-08
F12	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F13	Median	2.49E-01	6.250E-01	4.781E-01
	Mean	2.49E-01	6.250E-01	5.956E-01
	STD	2.87E-04	3.952E-01	3.744E-01
F14	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F15	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F26	Median	1.27E-05	1.277E-05	1.283E-05
	Mean	1.43E-05	9.170E-05	1.694E-05
	STD	4.04E-06	1.693E-04	9.211E-06
F27	Median	0.00E+00	0.000E+00	0.000E+00

TABLE 5. *(Continued.)* Results of WSO for various dimensions.

	STD	4.04E-06	1.693E-04	9.211E-06
F27	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F28	Median	9.00E-01	9.000E-01	9.000E-01
	Mean	9.00E-01	9.000E-01	9.000E-01
	STD	1.17E-16	1.170E-16	1.170E-16
F29	Median	8.78E+03	1.023E+07	8.245E+07
	Mean	8.71E+03	1.020E+07	8.242E+07
	STD	6.59E+02	7.975E+04	2.829E+05
F30	Median	2.22E-261	2.394E-262	1.206E-262
	Mean	2.30E-259	3.169E-259	1.911E-256
	STD	0.00E+00	0.000E+00	0.000E+00
F31	Median	3.15E-70	1.007E-37	2.233E-34
	Mean	7.77E-39	3.781E-26	6.371E-28
	STD	2.46E-38	1.196E-25	2.014E-27
F32	Median	-8.88E-16	-8.882E-16	-8.882E-16
	Mean	-8.88E-16	-8.882E-16	-8.882E-16
	STD	0.00E+00	0.000E+00	0.000E+00
F33	Median	1.00E+00	1.000E+00	1.005E+00
	Mean	1.00E+00	1.000E+00	1.004E+00
	STD	4.90E-03	5.797E-06	9.854E-04
F34	Median	4.09E-106	0.000E+00	0.000E+00
	Mean	2.73E-42	0.000E+00	0.000E+00
	STD	8.65E-42	0.000E+00	0.000E+00
F35	Median	-1.96E+03	-1.958E+04	-3.917E+04
	Mean	-1.96E+03	-1.958E+04	-3.917E+04
	STD	5.02E-04	1.001E-02	3.798E-05
F36	Median	0.00E+00	0.000E+00	0.000E+00
	Mean	0.00E+00	0.000E+00	0.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F37	Median	-1.00E+00	-1.000E+00	-1.000E+00
	Mean	-1.00E+00	-1.000E+00	-1.000E+00
	STD	0.00E+00	0.000E+00	0.000E+00
F38	Median	1.21E-20	4.466E-215	0.000E+00
	Mean	1.22E-20	6.319E-215	0.000E+00
	STD	2.52E-22	0.000E+00	0.000E+00
F39	Median	9.05E-08	2.598E-10	1.416E-09
	Mean	2.32E-07	6.497E-07	5.986E-09
	STD	2.72E-07	2.026E-06	1.014E-08
F40	Median	5.34E-12	1.63E-12	7.659E-13
	Mean	8.81E-11	9.87E-12	4.775E-12
	STD	2.28E-10	3.78E-12	7.113E-12

factors embedded in the algorithm for improving the exploration capability of the WSO algorithm. To illustrate the exploration capability of the proposed algorithm, search history is recorded for selected multimodal function as depicted in **Figure 3** C. The search history of various functions visualizes the exploration capability of WSO algorithm.

G. ANALYSIS CONVERGENCE BEHAVIOUR

Convergence is one of the prominent features in evaluating the performance of any population based meta-heuristic algorithm. The positions of the soldiers take large steps in the proposed WSO algorithm and this helps in improving the exploration of the large search space. As the iterations

TABLE 6. p-values of the Wilcoxon rank-sum test with 0.05 significance for WSO against other algorithms. The p-values are corrected according to Bonferroni-Holm.

Function type	WSO Vs PSO	WSO Vs GA	WSO Vs WOA	WSO Vs DA	WSO Vs JS	WSO Vs SSA	WSO Vs Chimp	WSO vs MVO	WSO Vs DE	WSO vs GWO
Unimodal	5.07E-10	3.73E-15	8.56E-07	8.80E-16	7.44E-05	1.52E-11	9.36E-10	8.11E-14	5.19E-11	9.56E-07
Corrected p value	2.54E-09	3.36E-14	2.57E-06	8.80E-15	7.44E-05	1.06E-10	3.74E-09	6.49E-13	3.11E-10	1.91E-06
Multimodal	1.17E-03	1.08E-05	1.82E-02	2.06E-06	4.16E-02	5.20E-05	1.67E-04	1.66E-05	2.87E-04	9.19E-03
Corrected p value	4.68E-03	9.72E-05	3.64E-02	2.06E-05	4.16E-02	3.64E-04	1.00E-03	1.33E-04	1.44E-03	2.76E-02

TABLE 7. Comparison results for the tension spring design problems.

Algorithm	Statistical Analysis				Designed parameters		
	Worst	Best	Mean	STD	x1	x2	x3
WSO	1.095788E-02	1.095788E-02	1.095788E-02	1.828559E-18	5.000000E-02	3.489146E-01	1.056224E+01
RO [75]	NA	1.267880E-02	NA	NA	5.137000E-02	3.490960E-01	1.176279E+01
CGWO [76]	1.217910E-02	1.195980E-02	1.217490E-02	1.039000E-05	5.279600E-02	8.043800E-01	2.000000E+00
NMDE [77]	NA	1.266523E-02	NA	NA	5.168929E-02	3.567231E-01	1.128865E+01
MoDE [78]	1.267400E-02	1.266500E-02	1.266500E-02	2.000000E-06	5.168800E-02	3.566920E-01	1.129048E+01
GEO [79]	NA	1.266580E-02	NA	NA	5.184990E-02	3.605987E-01	1.106507E+01
RFO [80]	NA	1.321000E-02	NA	NA	5.189000E-02	3.614200E-01	1.158436E+01
LAPO [81]	NA	1.265722E-02	NA	NA	5.190386E-02	3.618909E-01	1.128885E+01
NMPSO [82]	1.263300E-02	1.263020E-02	1.263140E-02	8.7375e007	NA	NA	NA
WSA [83]	1.267781E-02	1.266523E-02	1.267061E-02	4.06282E−06	5.168626E-02	3.566505E-01	1.129292E+01
PRO [34]	NA	1.266500E-02	NA	NA	5.181000E-02	3.598730E-01	1.106372E+00
TCSS [84]	1.283300E-02	1.264700E-02	1.275600E-02	2.300000E-05	5.224700E-02	3.707480E-01	1.049687E+01
SBO [85]	1.272300E-02	1.266530E-02	1.268734E-02	1.76705E−05	5.159767E-02	3.545231E-01	1.141880E+01
MSCS [86]	NA	1.266500E-02	NA	NA	5.169000E-02	3.567500E-01	1.128710E+01
MHS–PCL [87]	1.266520E-02	1.266520E-02	1.266520E-02	3.6008e−11	5.168910E-02	3.567207E-01	1.128879E+01
LSA [88]	NA	1.272045E-02	NA	NA	5.027598E-02	3.236795E-01	1.352541E+01
SA-MFO [89]	NA	1.267520E-02	NA	NA	5.000000E-02	3.174020E-01	1.403093E+01

TABLE 8. Comparison results for Welded beam problem.

Algorithm	Statistical Analysis				Designed parameters			
	Worst	Best	Mean	STD	x1	x2	x3	x4
WSO	1.724848E+00	1.724848E+00	1.724848E+00	1.812987E-16	2.057364E-01	3.470343E+00	9.036624E+00	2.057296E-01
GA([90])	2.590000E+00	2.430000E+00	2.503333E+00	8.082904E-02	2.489000E-01	6.173000E+00	8.178900E+00	2.533000E-01
WOA	NA	1.730499E+00	1.732000E+00	2.250000E-02	2.053950E-01	3.484292E+00	9.037425E+00	2.062750E-01
CGWO [76]	2.435700E+00	1.725450E+00	2.428900E+00	1.357800E+00	3.438910E-01	1.883570E+00	9.031330E+00	2.121210E-01
WWO [91]	NA	1.968420E+00	NA	NA	2.221400E-01	3.678120E+00	8.849650E+00	2.348900E-01
GWO	2.913600E+00	1.726240E+00	2.859400E+00	2.690800E+00	2.056760E-01	3.478377E+00	9.036810E+00	2.057780E-01
RO [75]	NA	1.735344E+00	NA	NA	2.036800E-01	3.528460E+00	9.004233E+00	2.072410E-01
NMDE [77]	NA	2.377135E+00	NA	NA	2.450054E-01	6.284511E+00	8.199110E+00	2.450054E-01
MoDE [78]	1.725000E+00	1.725000E+00	1.725000E+00	1.0E−15	2.057300E-01	3.470489E+00	9.036624E+00	2.057300E-01
GEO [79]	NA	1.865360E+00	NA	NA	2.443688E-01	3.063020E+00	8.291483E+00	2.443689E-01
NMPSO [82]	1.733339E+00	1.726373E+00	1.724717E+00	3.500000E-03	2.058300E-01	3.468338E+00	9.036624E+00	2.057300E-01
WSA [83]	3.823218E+00	1.724853E+00	2.125751E+00	6.52914E−01	2.057296E-01	3.470490E+00	9.036624E+00	2.057296E-01
TCSS [84]	1.735656E+00	1.724860E+00	1.730212E+00	2.000000E-06	2.057310E-01	3.470459E+00	9.036574E+00	2.057310E-01
RFO [80]	NA	1.866120E+00	NA	NA	2.184600E-01	3.510240E+00	8.872540E+00	2.249100E-01
SBO [85]	1.743486E+00	1.699215E+00	1.719856E+00	1.482600E-02	1.750550E-01	3.305377E+00	9.248898E+00	2.061370E-01
MHS–PCL [87]	1.724852E+00	1.724852E+00	1.724852E+00	8.11e−10	2.057300E-01	3.470489E+00	9.036624E+00	2.057300E-01
SA-MFO [89]	NA	1.724500E+00	NA	NA	2.057000E-01	3.475200E+00	9.036700E+00	2.057000E-01

increases the soldiers take small steps and move towards the global optimum point. This concept is visualized from the Figure 3 D where it shows the fitness of all soldiers vary in large values in the initial iterations and takes small values as the iterations increase. Figure 3 B depicts the trajectories of eight random soldiers. This figure clearly shows that search

TABLE 9. Comparison results for Speed reducer design.

Algorithm	Statistical Analysis				Designed parameters						
	Worst	Best	Mean	STD	x1	x2	x3	x4	x5	x6	x7
WSO	2.994470E+03	2.994470E+00	2.994470E+03	2.625485E-13	3.499998E+00	7.000000E-01	1.700000E+01	7.300000E+00	7.715318E+00	3.350215E+00	5.286653E+00
MoDE [78]	2.996390E+03	2.996357E+03	2.996367E+03	8.200000E-03	3.500010E+00	7.000000E-01	1.700000E+01	7.300156E+00	7.800027E+00	3.350221E+00	5.286685E+00
WSA [83]	2.996348E+03	2.996348E+03	2.996348E+03	9.10931E−06	3.500000E+00	0.000000E+00	1.700000E+01	7.300000E+00	7.800000E+00	3.350215E+00	5.286683E+00
TCSS [84]	2.999655E+03	2.996348E+03	2.997032E+03	4.723500E+00	3.500000E+00	7.000000E-01	1.700000E+01	7.300000E+00	7.800000E+00	3.350214E+00	5.286683E+00
MSCS[86]	NA	2.993750E+03	NA	NA	3.500000E+00	7.000000E+00	1.700000E+00	7.300000E+00	7.800000E+00	3.343364E+00	5.285351E+00
MHS–PCL[87]	2.994471E+03	2.994471E+03	2.994471E+03	7.142949e−06	3.500000E+00	7.000000E+00	1.700000E+00	7.300000E+00	7.715320E+00	3.350215E+00	5.286655E+00

TABLE 10. Comparison for Pressure vessel design.

Algorithm	Statistical Analysis				Designed parameters			
	Worst	Best	Mean	STD	x1	x2	x3	x4
WSO	5.885246E+03	5.885246E+03	5.885246E+03	4.287399E-13	7.781505E-01	3.846347E-01	4.031962E+01	2.000000E+02
CGWO [76]	6.188110E+03	5.034180E+03	5.783582E+03	2.545050E+02	1.187150E+00	6.000000E-01	6.970750E+01	7.798440E+00
NMDE [77]	NA	7.198005E+03	NA	NA	1.125000E+00	6.250000E-01	5.829016E+01	4.369266E+01
MoDE [78]	6.059702E+03	6.059702E+03	6.059702E+03	1.000000E-12	8.125000E-01	4.375000E-01	4.209845E+01	1.766360E+02
RFO [80]	NA	6.113320E+03	NA	NA	8.142500E-01	4.452100E-01	4.220231E+01	1.766215E+02
NMPSO [82]	5.960056E+03	5.930314E+03	5.946790E+03	9.161400E+00	8.036000E-01	3.972000E-01	4.163920E+01	1.824120E+02
SBO [85]	6.384858E+03	5.885333E+03	6.156403E+03	7.496350E+01	7.781686E-01	3.846490E-01	4.031962E+01	2.000000E+02
LAPO [81]	NA	5.916194E+03	NA	NA	7.862837E-01	3.916067E-01	4.075874E+01	1.942965E+02
TCSS [84]	6.100352E+03	6.059441E+03	6.062327E+03	1.036500E+01	8.125000E-01	4.375000E-01	4.210069E+01	1.766088E+02
WSA [83]	5.984757E+03	5.929622E+03	5.958410E+03	1.507840E+01	7.865429E-01	3.934884E-01	4.075268E+01	1.947806E+02
AAA [92]	7.197731E+03	7.197729E+03	7.197729E+03	4.153E−04	1.139055E+00	6.250000E-01	5.829016E+01	4.369266E+01
PRO [34]	NA	6.050713E+03	NA	NA	7.445000E-01	4.424000E-01	3.848998E+01	2.000000E+02
MSCS [86]	NA	6.059714E+03	NA	NA	8.125000E-01	4.375000E-01	4.209845E+01	1.766366E+02
MHS–PCL [87]	6.059714E+03	6.059714E+03	6.059714E+03	1.28120e−05	8.125000E-01	4.375000E-01	4.209845E+01	1.766366E+02
LSA [88]	NA	6.059946E+03	NA	NA	8.125000E-01	4.375000E-01	4.209740E+01	1.766541E+02
SA-MFO [89]	NA	6.059089E+03	NA	NA	8.125000E-01	4.375000E-01	4.210350E+01	1.675623E+02

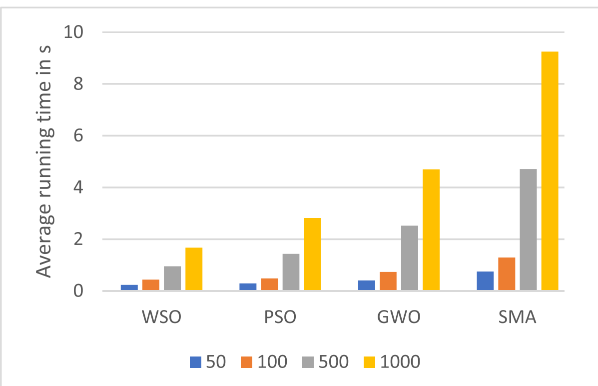

FIGURE 8. Average running time for various algorithms for different dimensions.

agents (soldiers) takes large steps initially and takes small steps as they converges to optimum position. The search history at 2nd iteration, 50th iteration, 100th iteration and 150th iteration for selected functions are illustrated in Figure. From these figures, we can understand that as iterations increases the search space scale down to the global optimum point. Convergence curves shown in Figure 4 for selected functions prove that WSO converges faster as compared to other algorithms. Nonlinear weight function is the main reason for

TABLE 11. Statistical results of Three-bar truss design.

Algorithm	Best	Designed parameters	
		x1	x2
WSO	263.89584	0.78868	0.40822
GEO [79]	263.91540	0.79369	0.39426
RFO [80]	268.51195	0.75356	0.55373
PRO [34]	263.89584	0.78865	0.40833
SBO [85]	263.89584	0.78868	0.40823

faster convergence of WSO algorithm. Thus we can conclude that the proposed WSO possess good convergence behavior.

H. SCALABLITY ANALYSIS

The efficacy of the proposed WSO is evaluated by testing the algorithm on variable dimension unimodal and multimodal functions with dimensions 50, 500 and 1000. The average, mean and standard deviation values are recorded in Table 5. After analyzing the results, it has been observed that the efficiency of the algorithm is same irrespective of the dimension of the problem and thus We can conclude that increasing the dimension has little impact on the algorithm's performance.

TABLE 12. Details of Unimodal benchmark functions.

Function	Function Definitions	F_{min}	Dim	Range
F1-Sphere	F1= $\sum_{i=1}^{n} {x_i}^2$	0	50	[-100, 100]
F2-Quartic Noise	F2= $\sum_{i=1}^{n} i{x_i}^4$+random[0,1]	0	50	[-1.28, 1.28]
F3-Powell Sum	F3= $\sum_{i=1}^{D} \|x_i\|^{i+1}$	0	50	[-1, 1]
F4-Schwefel's 2.20	F4= $\sum_{i=1}^{n} \|x_i\|$	0	50	[-100,100]
F5-Schwefel's 2.21	F5= max$_i$($\|x_i\|, 1 \le i \le n$)	0	50	[-100,100]
F6-Step	F6= $\sum_{i=1}^{n} ([x_i + 0.5])^2$	0	50	[-100,100]
F7-Stepint	F7= $25 + \sum_{i=1}^{n} ([x_i])$	25-6n	50	[-5.12,5.12]
F8-Schwefel's 1.20	F8=$\sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$	0	50	[-100, 100]
F9-Schwefel's 2.22	F9=$\sum_{i=1}^{n} \|x_i\|$+$\prod_{i=1}^{n} \|x_i\|$	0	50	[-100, 100]
F10-Schwefel's 2.23	F10=$\sum_{i=1}^{n} {x_i}^{10}$	0	50	[-10, 10]
F11-Rosenbrock	F11=$\sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	0	50	[-30, 30]
F12-Brown	F12=$\sum_{i=1}^{n-1} ({x_i}^2)^{(x_{i+1}^2+1)}$+ $(x_{i+1}^2)^{x_i+1}$	0	50	[-1, 4]
F13-Dixon and Price	F13=$(x_1 - 1)^2 + \sum_{i=2}^{D} i(2x_i^2 - x_{i-1})^2$	0	50	[-10, 10]
F14-Powell Singular	F14=$\sum_{i=1}^{D/4} (x_{4i-3} + 10x_{4i-2})^2$+$5(x_{4i-1} - x_{4i})^2$+$(x_{4i-2} - x_{4i-1})^4$+$10(x_{4i-3} - x_{4i})^4$	0	50	[-4, 5]
F15-Zakharov	F15=$\sum_{i=1}^{n} (x_i)^2 +$ $\left(\sum_{i=1}^{n} 0.5ix_i\right)^2$+$\left(\sum_{i=1}^{n} 0.5ix_i\right)^4$	0	50	[-5, 10]
F16-Xin-She Yang	F16=$exp\left(-\sum_{i=1}^{n} \left(\frac{x_i}{\beta}\right)^{2m}\right)$-2 exp$(-\sum_{i=1}^{n} (x_1)^2) \prod_{i=1}^{n} cos^2(x_i)$	-1	50	[-20, 20]
F17-Perm 0,D,Beta	F17=$\sum_{i=1}^{d} \left[\sum_{j=1}^{d} (j + \beta) \left(x_j^i - \frac{1}{j^i}\right)\right]^2$	0	5	[-d$_i$, d$_i$]
F18-Three-Hump Camel	F18=$2x_1^2 - 1.05x_1^4$+$\frac{x_1^6}{6} + x_1x_2 + x_2^2$	0	2	[-5, 5]
F19-Beale	F19=$(1.5 - x_1 + x_1x_2)^2$+$(2.25 - x_1 + x_1x_2^2)^2$+$(2.625 - x_1 + x_1x_2^3)^2$	0	2	[-4.5, 4.5]
F20-Booth	F20=$(x_1 + 2x_2 - 7)^2$+$(2x_1 + x_2 - 5)^2$	0	2	[-10, 10]
F21-Brent	F21=$(x_1 + 10)^2 + (x_2 + 10)^2$+$e^{-x_1^2-x_2^2}$	0	2	[-10, 10]
F22-Matyas	F22=$0.26(x_1^2 + x_2^2)$-$0.48x_1x_2$	0	2	[-10, 10]
F23-Schaffer N. 4	F23=0.5+$\frac{cos^2\left(sin(\|x^2-y^2\|)\right)-0.5}{(1+0.001(x^2+y^2))^2}$	0.29257	2	[-100, 100]
F24-Wayburn Seader 3	F24=$2\frac{x_1^3}{3}$- $8x_1^2$+ $33x_1$- x_1x_2+ $5 + [(x_1 - 4)^2 + (x_2 - 5)^2 - 4]^2$	21.35	2	[-500, 500]
F25-Leon	F25=$100(x_2 - x_1^2)^2$+$(1 - x_1)^2$	0	2	[-1.2, 1.2]

I. STATISTICAL ANALYSIS and COMPARISON

The performance of the WSO algorithm is compared with other algorithms using Wilcoxon rank sum test. The comparison results for Unimodal and multimodal functions are recorded in Table 6. The p-values are corrected to avoid type I errors following Bonferroni-Holm procedure [94], [95]. From the table it is evident that the p-values are less than 0.05. This clearly shows that WSO algorithm outperforms other algorithms.

J. TIME COMPLEXITY ANALYSIS

The proposed WSO algorithm is simple and takes lesser time for computing the optimized solution. The average run time is calculated for one benchmark function for different dimensions and the same is compared with different optimization algorithms. The average running time for different dimensions is illustrated in Figure 8. WSO runs faster to the compared algorithms. The run time increases with increase in dimension of the search space. The time complexity of the proposed WSO algorithm is calculated with big-O notation. The computational complexity of initialization is O(N×D), function evaluation is O(N) and position update is O((N+1)×D) and thus the overall complexity is O((N+1)×D×Max-iter).

K. ANALYSIS OF WSO FOR ENGINEERING DESIGN PROBLEMS

Many engineers nowadays use meta-heuristic algorithms to achieve optimum values for problem engineering

TABLE 13. Details of Multimodal benchmark functions.

Function	Type	F_{min}	Dim	Range
F26- Schwefel's 2.26	$F26 = 418.9829n - \sum_{i=1}^{n} -x_i sin\sqrt{\lvert x_i\rvert}$	0	50	[-500, 500]
F27-Rastrigin	$F27 = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	0	50	[-5.12, 5.12]
F28-Periodic	$F28 = 1 + \sum_{i=1}^{n} sin^2(x_i) - 0.1e^{(\sum_{i=1}^{n} x_i^2)}$	0.9	50	[-10,10]
F29-Qing	$F29 = \sum_{i=1}^{n} (x^2 - i)^2$	0	50	[-500,500]
F30-Alpine N.1	$F30 = \sum_{i=1}^{n} \lvert x_i sin(x_i) + 0.1x_i\rvert$	0	50	[-10,10]
F31-Xin She Yang	$F31 = \sum_{i=1}^{n} \epsilon_i \lvert x_i\rvert^i$	0	50	[-5,5]
F32-Ackley	$F32 = -20e^{\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right)} - e^{\left(\frac{1}{n\sum_{i=1}^{n}\cos(2\pi x_i)}\right)} + 20 + e$	0	50	[-32,32]
F33-Trignometric 2	$F33 = \sum_{i=1}^{n} 8sin^2(7\{x_i - 0.9\}^2) + 6sin^2(14\{x_i - 0.9\}^2) + (x_i - 0.9)^2$	1	50	[-500, 500]
F34-Salomon	$F34 = 1 - cos\left(2\pi\sqrt{\sum_{i=1}^{n} x_i^2}\right) + 0.1\sqrt{\sum_{i=1}^{n} x_i^2}$	0	50	[-100, 100]
F35-Styblinski-Tang	$F35 = \frac{1}{2}\sum_{i=1}^{n} (x_i^4 - 16x_i^2 + 5x_i)$	-39.16599 x n	50	[-5, 5]
F36-Griewank	$F36 = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	0	50	[-100, 100]
F37-Xin She YangN.4	$F37 = \left(\sum_{i=1}^{n} sin^2(x_i) - e^{\sum_{i=1}^{n} x_i^2}\right) e^{\sum_{i=1}^{n} sin^2(\sqrt{x_i})}$	-1	50	[-10, 10]
F38-Xin She Yang N.2	$F38 = \left(\sum_{i=1}^{n} \lvert x_i\rvert\right) e^{-\sum_{i=1}^{n} x_i^2}$	0	50	[-2π, 2π]
F39-Gen.Penalized	$F39 = 0.1\left[sin^2(3\pi x_i) + \sum_{i=1}^{n} (x_i - 1)^2\{1 + sin^2(3\pi x_i + 1)\} + (x_n - 1)^2\{1 + sin^2(2\pi x_n)\}\right] + \sum_{i=1}^{n} u(x_i, 5,100,4)$	0	50	[-50, 50]
F40-Penalized	$F40 = \pi/n\left[10sin(\pi x_i) + \sum_{i=1}^{n} (x_i - 1)^2\{1 + 10sin^2(\pi x_{i+1})\} + (x_n - 1)^2\right] + \sum_{i=1}^{n} u(x_i, 5,100,4)$	0	50	[-50, 50]
F41-Egg Crate	$F41 = x^2 + y^2 + 25(sin^2(x) + sin^2(y))$	0	2	[-5, 5]
F42-Ackley N.3	$F42 = -200e^{-0.2\sqrt{x^2+y^2}} + 5e^{\cos(3x)+\sin(3y)}$	-195.629	2	[-32,32]
F43-Adjiman	$F43 = \cos(x)\sin(y) - \frac{x}{y^2 + 1}$	-2.02181	2	[-1, 2]

TABLE 13. ***(Continued.)*** **Details of Multimodal benchmark functions.**

F44-Bird	$F44=\sin(x)\,e^{(1-\cos(y))^2}+\cos(y)\,e^{(1-\cos(x))^2}+(x-y)^2$	-106.7645	2	[-2π, 2π]
F45-Camel Six Hump	$F45=4x_1^2-2.1x_1^4+\frac{1}{3}x_1^6+x_1x_2-4x_2^2+4x_2^4$	-1.0316	2	[-5,5]
F46-Branin RCOS	$F46=\left(x_2-\frac{5.1}{4\pi^2}x_1^2-\frac{5}{\pi}x_1-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos(x_1)+10$	0.3978873	2	[-5, 10]
F47-Hartman 3	$F47=-\sum_{i=1}^{4}c_i e^{\left(\sum_{j=1}^{3}\alpha_{ij}\{x_j-p_{ij}\}^2\right)}$	-3.862782	3	[0, 1]
F48-Hartman 6	$F48=\sum_{i=1}^{4}c_i e^{\left(\sum_{j=1}^{6}\alpha_{ij}\{x_j-p_{ij}\}^2\right)}$	-3.32237	6	[0,1]
F49-Cross-in-tray	$F49=-0.0001\left(\left\|\sin(x)\sin(y)\exp\left(100-\frac{\sqrt{x^2+y^2}}{\pi}\right)\right\|+0.1\right)^{0.1}$	-2.062	2	[-10, 10]
F50-Bartels Conn	$F50=\|x^2+y^2+xy\|+\|\sin(x)\|+\|\cos(y)\|$	1	2	[-500 500]

designs/plans. In this section, we applied the WSO algorithm to four classic engineering design problems and compared its performance to that of other popular metaheuristic algorithms. For dealing with the constraints, we used a simple death penalty function-based approach.

i. Tension Spring Design Problem
The primary objective here is optimal spring design with three design variables and four constraints. The performance of WSO is compared with other popular metaheuristics algorithms in terms of worst, best, mean and standard deviation and the statistical results are presented in Table 7. From the results, we can understand that the proposed WSO algorithm outperform other algorithms,

ii. Welded Beam design Problem
In welded beam design problem, the key objective is to minimize the manufacturing cost of the welded beam. The simulation results for the welded design problem are shown in Table 8. The comparison results show that WSO algorithm rank first when compared to other algorithms.

iii. Speed Reducer Design Problem
Weight minimization is the objective of this problem. The comparison results with other metaheuristic algorithms are presented in Table 9. The comparison results show the efficacy of the proposed method when compared to other algorithms.

iv. Pressure vessel Design
The objective for this engineering problem is to minimize the cost in the design of a pressure vessel. The statistical results shown in Table show the superiority of the algorithm.

v. Three-bar truss design
The objective of this problem is to design a three-bar truss with minimum weight. The design includes a selection of two optimal parameters and the results are shown in Table 11.

VI. CONCLUSION

A new stochastic optimization algorithm 'War Strategy Optimizatio' inspired by the ancient war strategies has been proposed in this paper. In this algorithm, two war strategies have been developed to update the current position of the soldier. An adaptive weight mechanism has been introduced in the algorithm which varies from one solution (soldier) to another solution and is updated based on the rank achieved by the soldier during the updation stage. The proposed algorithm was tested with 50 benchmark test functions and has shown a significant performance when compared with the popular meta-heuristic algorithms in the literature. WSO algorithm achieves good tradeoff between exploration and exploitation stages. The proposed war strategy optimization algorithm can be developed with a multi-objective feature in future studies and can be applied for multi-objective functions.

APPENDIX

See Tables 12 and 13.

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