

# Saturation Current as a Function of Sheet Resistance in Si

Luigi Abenante<sup>a)</sup> and Massimo Izzi

*ENEA, Italian National Agency for New Technologies, Energy and Sustainable Economic Development, ROMA (Italy)*

<sup>a)</sup>Corresponding author: luigi.abenante@enea.it

**Abstract.** We present the exact analytical solution to minority-carrier transport in the dark for non-uniformly doped Si regions in low-level injection, where bulk lifetime is inversely proportional to the square of doping density according to the Dziewior and Schmid model of Auger recombination and diffusivity is constant. The relevant expression for emitter saturation current density,  $J_{0em}$ , can be set as a function of sheet resistance,  $R_{SHEET}$ . Reported measured  $J_{0em}(R_{SHEET})$ -curves in  $n$ - and  $p$ -type metal-coated emitters are matched by the presented  $J_{0em}(R_{SHEET})$ -expression. To this aim, reported doping functions for band-gap narrowing are set as functions of  $R_{SHEET}$ . The application range of the presented solution is checked.

## INTRODUCTION

The defect-independent Auger recombination process at negligible Coulomb interaction among carriers sets the ultimate performance limits of Si devices. Such theoretical limits could serve as a reference to assess the performance of actual or theoretical devices. In the present work, an analytical approach to calculate exactly the saturation current,  $J_{0em}$ , in highly-doped Si emitters, where defect-dependent recombination and Coulomb interaction among carriers are negligible, is proposed. Such an approach is based on assuming a constant diffusivity in the case of pure Auger recombination.

The new exact  $J_{0em}$ -expression can be set as a function of sheet resistance,  $R_{SHEET}$ , which is a parameter measurable with non-destructive methods. We show, in addition, that the relevant effective intrinsic concentration can be set as a function of  $R_{SHEET}$  and emitter width,  $x_j$ , through a mean doping value,  $N_m$ , calculated over  $x_j$ .  $x_j$  is another parameter measurable with non-destructive methods.

It is maintained that  $R_{SHEET}$  is a poor parameter for the saturation current because it is easy to find two different doping profiles which have the same  $R_{SHEET}$ -value but different saturation current. The fact of including  $N_m$ , however, allows the exact  $J_{0em}(R_{SHEET})$ -function derived in this work to discriminate among emitter doping profiles of same  $R_{SHEET}$ . This is experimentally proven by comparisons between  $J_{0em}(R_{SHEET})$ -values calculated with the new expression and reported  $J_{0em}(R_{SHEET})$ -values measured in sets of metal-coated  $n$ - and  $p$ -type emitters, where "ambiguous" cases are present.

In the calculations of  $J_{0em}(R_{SHEET})$  with the new expression no parameter is adjusted besides the aforementioned constant diffusivity. To check the approach, the adjusted diffusivity values are successively used to calculate Auger diffusion-length curves to be compared to analogous curves calculated using reported doping functions for both Auger lifetime and diffusivity. Agreement is found for  $N_m > 5 \times 10^{17} \text{ cm}^{-3}$ , which is the doping range of validity for the approach presented in this work.

The new  $J_{0em}(R_{SHEET})$ -function only includes a single doping value, which is the aforementioned mean value  $N_m$  calculated over  $x_j$ . In all the modeled emitters, we found  $N_m > 10^{18} \text{ cm}^{-3}$ , which is inside the aforementioned doping range of validity for the modeling approach presented in this work. All the calculated  $J_{0em}(R_{SHEET})$ -values are therefore reliable. Since, in this work, more than 50 emitters with  $R_{SHEET}$  ranging from 11 to 290 ohm/square circa have been examined, it can be inferred that emitters with  $N_m < 5 \times 10^{17} \text{ cm}^{-3}$  are uncommon. The  $J_{0em}(R_{SHEET})$ -function

presented in this work can be then considered as general enough to be used for the simulation of maximum-quality emitters like the ones, which are considered in this work.

## DERIVATION

In a non-uniformly doped Si region under low-injection conditions, if diffusivity,  $D$ , is equal to a constant diffusivity,  $D_A$ , and lifetime,  $\tau_A$ , is modeled according to [1] as

$$\tau_A = 1/(C_{DS}N^2) \quad (1)$$

where  $N$  is the doping density and  $C_{DS}$  is the Dziewior and Schmid Auger coefficient suitable for that region [1], then, with  $y=N\Delta m$ , where  $\Delta m$  is the excess minority-carrier concentration, the diffusion equation, which describes minority-carrier transport, becomes

$$y'' - \frac{N'}{N} y' - \frac{C_{DS}N^2}{D_A} y = 0 \quad (2)$$

which is solved by

$$y = A \sinh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^x N da\right) + B \cosh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^x N da\right) \quad (3)$$

Under boundary conditions for emitter in the dark including band-gap narrowing,  $\Delta E_g$ , (3) gives the emitter saturation current density,  $J_{0em}$ , as

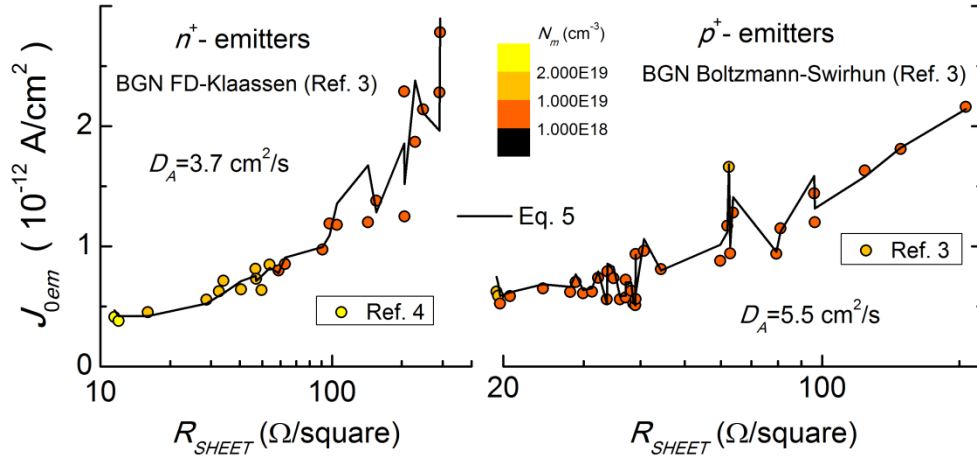
$$J_{0em} = \pm q D_A n_{ie}^2 \sqrt{\frac{C_{DS}}{D_A}} \times \left[ \sinh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^{x_j} N dx\right) + \frac{S}{D_A N_S} \cosh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^{x_j} N dx\right) \right] \times \left[ \cosh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^{x_j} N dx\right) + \frac{S}{D_A N_S} \sinh\left(\sqrt{\frac{C_{DS}}{D_A}} \int_0^{x_j} N dx\right) \right]^{-1} \quad (4)$$

where the sign depends on the doping type,  $n_{ie}$  is the effective intrinsic-carrier density,  $S$  is the surface recombination velocity,  $N_S$  is the surface doping,  $x_j$  is the junction depth, and  $q$  is the electron charge. At uniform doping,  $S=0$  cm/s, and  $\Delta E_g=0$  eV (4) reduces to the expression for  $J_0$  included in the expression for open-circuit voltage under low-injection conditions reported in [2]. Based on the definition of  $R_{SHEET}$ , (4) can be written as

$$J_{0em} = \pm q D_A n_{ie}^2 \sqrt{\frac{C_{DS}}{D_A}} \times \left[ \sinh\left(\frac{V_{th}}{q D_A R_{SHEET}} \sqrt{\frac{C_{DS}}{D_A}}\right) + \frac{S}{D_A N_S} \cosh\left(\frac{V_{th}}{q D_A R_{SHEET}} \sqrt{\frac{C_{DS}}{D_A}}\right) \right] \times \left[ \cosh\left(\frac{V_{th}}{q D_A R_{SHEET}} \sqrt{\frac{C_{DS}}{D_A}}\right) + \frac{S}{D_A N_S} \sinh\left(\frac{V_{th}}{q D_A R_{SHEET}} \sqrt{\frac{C_{DS}}{D_A}}\right) \right]^{-1} \quad (5)$$

$n_{ie}$  is such that  $n_{ie}^2 = n_i^2 \exp(\Delta E_g/kT)$ , where  $k$  is the Boltzmann constant and  $T$  is the absolute temperature.  $\Delta E_g$  can be set as a function of  $R_{SHEET}$ , if it is assigned the form [3]

$$\Delta E_g(N) = A \times \left[ \ln\left(\frac{N}{N_{ref}}\right) \right]^b \quad (6)$$



**FIGURE 1.** Comparison between  $J_{0em}(R_{SHEET})$ -values measured in [2,4] (symbols) and  $J_{0em}(R_{SHEET})$ -values calculated with (5) (lines) at the shown  $D_A$ -values and values for the remaining parameters taken from [3,4]. FD stands for Fermi-Dirac.

where the coefficients  $A$ ,  $b$ , and  $N_{ref}$  assume values depending on the considered statistics and carrier species [3], and  $N$  is assigned a mean emitter doping value,  $N_m$ , given by

$$N_m = \frac{V_{th}}{qx_j D_A R_{SHEET}} \quad (7)$$

so that (6) becomes

$$\Delta E_g(R_{SHEET}) = A \times \left[ \ln \left( \frac{N_m}{N_{ref}} \right) \right]^b \quad (8)$$

## APPLICATION

In [3,4], Yan and Cuevas report measured  $J_{0em}$ -data of metal-coated  $n$ - and  $p$ -type diffused emitters together with the relevant measured values of  $R_{SHEET}$ ,  $x_j$ , and  $N_S$ . In those works,  $S=3 \times 10^6$  cm/s and  $n_i=9.65 \times 10^9$  cm<sup>-3</sup> are assigned in all emitters [3,4].

We have calculated  $J_{0em}(R_{SHEET})$  with (5) in the  $p^+$  and  $n^+$  Yan and Cuevas emitters at same assignments to  $S$ ,  $n_i$ ,  $R_{SHEET}$ ,  $x_j$ , and  $N_S$  as used in [3] and [4], respectively, and the  $D_A$ -values reported in Fig. 1. In this figure, calculated  $J_{0em}(R_{SHEET})$ -values are compared to measured  $J_{0em}(R_{SHEET})$ -values and the Yan and Cuevas  $\Delta E_g$ -functions [3] are indicated, from which the values for  $A$ ,  $b$ , and  $N_{ref}$  to be assigned in (8) are taken.

## CHECK AND CONCLUSION

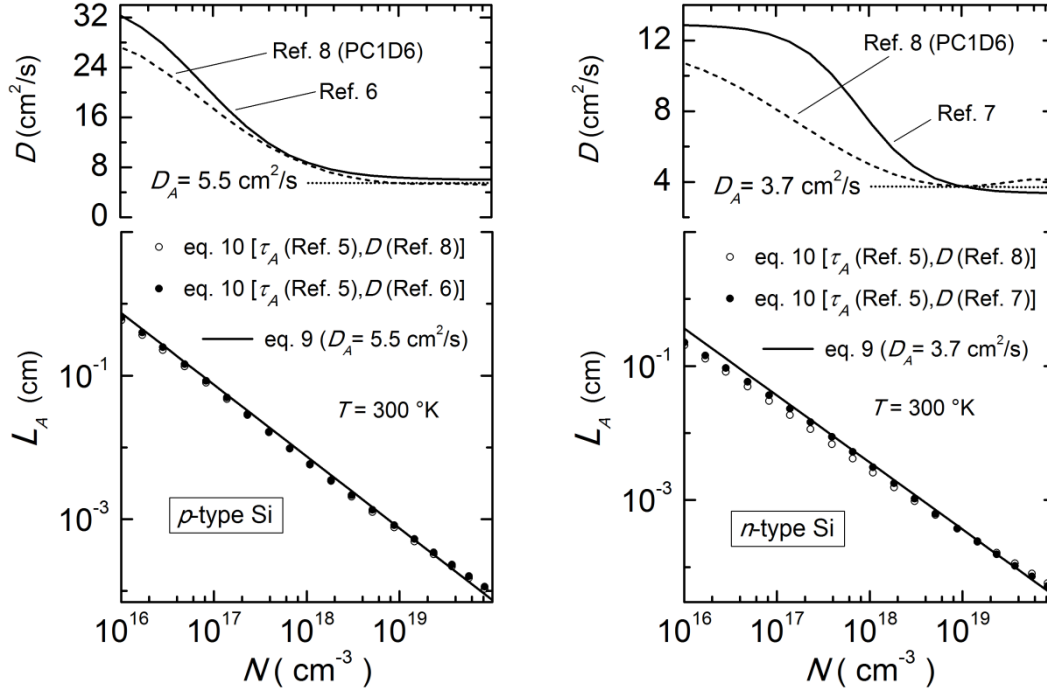
The previous derivation is based on the assumption that  $D=D_A$ , when Auger lifetime,  $\tau_A$ , is assigned (1). This assumption is only valid at  $N \geq 5 \times 10^{18}$  cm<sup>-3</sup> [1].

Assigning  $D=D_A$  and (1) implies an Auger diffusion length,  $L_A$ , given by

$$L_A = \frac{1}{N} \sqrt{\frac{D_A}{C_{DS}}} \quad (9)$$

As it is well known, however,  $L_A$  can also be calculated as

$$L_A = \sqrt{D\tau_A} \quad (10)$$



**FIGURE 2.** Comparisons between  $L_A$ -values calculated in  $p$ - and  $n$ -type Si with (9) at the shown  $D_A$ -values and with (10) at reported  $N$ -functions for  $\tau_A$  and  $D$ . The Klaassen [8] curves (dashed line) are obtained with the simulation program PC1D6 [9].

by using reported functions for  $\tau_A(N)$  and  $D(N)$ .

In Fig. 2,  $L_A$ -curves (lines) calculated with (9) are compared to  $L_A$ -values (circles) calculated with (10). In the same figure, the  $D_A$ -values and  $D(N)$ -functions used in the calculations are reported. Since (3) also solves

$$y'' + \frac{L_A'}{L_A} y' - \frac{1}{L_A^2} y = 0 \quad (11)$$

with  $L_A$  being given by (9) and, as can be seen in Fig. 2, (9) and (10) are practically equivalent for  $N > 5 \times 10^{17} \text{ cm}^{-3}$  in  $n$ -type Si and for  $N > 10^{17} \text{ cm}^{-3}$  in  $p$ -type Si, we can conclude that (3) and, hence, (4) and (5) are still applicable at  $N < 5 \times 10^{18} \text{ cm}^{-3}$ .

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