Extraction of voltage-dependent series resistance from I-V characteristics of Schottky diodes

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A method for extracting the bias dependent behaviour of the series resistance of a Schottky barrier diode from experimental I-V data is presented. It was assumed that the behaviour of the Schottky barrier is well defined by thermionic emission theory. Relative merit of the method was determined by applying the method on some artificial sets of I-V data corresponding to known values of series resistances and comparing the results with existing methods.
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Behaviour of a Schottky diode is characterized by parameters such as the barrier height Φ_B, series resistance R_s, ideality factor n, and saturation current I_0. Having knowledge of these parameters is very important for analysis and design of electronic circuits containing Schottky diodes. It is generally accepted that all these parameters depend on applied bias potential.[1–3] However, in most of the studies, these parameters are assumed to be independent of the bias potential for the sake of simplicity.[4–11] These methods mostly utilize existing experimental current-voltage (I-V) measurements including standard approach using slope of the linear region of ln I vs. V curve,[4,5] methods based on the minima of Norde's function and similar functions[6–9] and methods of Lien *et al.* and Werner in which the conductance G for low amplitude signals is used.[8–11] In a few studies, authors discussed the methods for obtaining these parameters as a function of the bias voltage. Maeda *et al.*,[12] Ishida and Ikoma,[13] and Türüt *et al.*[14] obtained the effective barrier height and/or the ideality factor as a function of the bias voltage. Also Lyakas *et al.*[2] developed a slow and cumbersome method to determine the series resistance of a Schottky contact. Mikhelashvili and Eisenstein[3] proposed a method based on a power exponent and its change with bias potential under the assumption that the series resistance is a constant. However, they apply the method to obtain the series resistance as a function of bias voltage.

In this letter, we present a simple and robust method for determining the series resistance of a Schottky diode as a function of bias voltage from the measured I-V data. We assume that the current mechanism in Schottky diode is dominated by thermionic emission (TE) process under forward bias. Therefore, we employ TE theory to model I-V response of the diode. Also we assume that the logarithm of ideality factor (i.e., ln n) changes very slowly with the bias potential. Therefore, this method is not suitable for cases when there is strong voltage dependence in the ideality factor. However, such cases are seldom encountered and the present method can be used in most cases.

The I-V characteristic of a metal semiconductor junction under forward bias is given by

$$I = SA^*T^2\exp(-\frac{q\phi_B}{kT})\left\{\exp(\frac{qV}{kT}) - 1\right\}, \tag{1}$$

within TE theory,[4] where V, q, k, T, S, ϕ_B, and A^* are applied voltage, electron charge, Boltzmann constant, absolute temperature, contact area, effective barrier height, and Richardson constant, respectively. One may assume that the effective barrier height depends linearly on the bias voltage, i.e., $\phi_B = \phi_{B0} + \xi V$, where ϕ_{B0} is the zero bias barrier height and the coefficient ξ is positive, because ϕ_B always increases with forward bias[4] and the ideality factor n is defined by the relation $n^{-1} = 1 - \partial\phi_B/\partial V$, which is a semi-empirical parameter. When we consider the effect of series resistance, the net bias voltage on the metal semiconductor contact is reduced by IR_s. Then using the definition of ideality factor, the current through the contact can be written as

$$I = I_0\exp((V - IR_s)/n\beta)\{1 - \exp(-(V - IR_s)/\beta)\}, \tag{2}$$

where $\beta = kT/q$ and $I_0 = SA^*T^2\exp(-\phi_{B0}/\beta)$. For forward bias voltages larger than $V_0 = 3\beta$, one can ignore the second term in curly parentheses in Eq. (2) obtaining

$$I = I_0\exp(V - IR_s/n\beta). \tag{3}$$

In order to extract the physical parameters from I-V data, Mikhelashvili and Eisenstein[3] developed a method based on the analysis of two parameters defined by the relations

$$\alpha = d(\ln I)/d(\ln V), \quad \gamma = d(\ln\alpha)/d(\ln V), \tag{4}$$

where α is called as the power exponent. These parameters are very convenient to express and analyze experimental I-V curves. Therefore, we will use the same definitions.

Now we define argument of the exponential function in Eq. (3) as a variable $\Lambda(V)$, that is,

$$\Lambda(V) = V - IR_s/n\beta, \tag{5}$$

Using the definition of $\alpha(V)$ in Eq. (4) one may write

$$d\Lambda = d(V - IR_s/n\beta) = (\alpha(V)/V)dV. \tag{6}$$

Integrating this equation we obtain

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$$\Lambda(V) = \Lambda(V_0) + \int_{V_0}^{V} \frac{\alpha(V)}{V} dV = \Lambda_0 + \Lambda_1(V), \quad (7)$$

where Λ_0 is the value of $\Lambda(V)$ at V_0. This variable Λ proves to be a very convenient tool for the analysis of Schottky diodes. In order to obtain the series resistance R_s in terms of the parameters defined above, we take the derivative of Eq. (3) with respect to the bias voltage obtaining

$$\frac{\alpha}{V}(n\beta + IR) = 1 - I\frac{dR_s}{dV} - (V - IR_s)\frac{d(\ln n)}{dV}. \quad (8)$$

If we assume that the logarithm of the ideality factor n is a smooth function of the bias voltage V, then we can ignore the third term on the right hand side so that Eq. (8) becomes

$$\frac{\alpha}{V}(n\beta + IR_s) = 1 - I\frac{dR_s}{dV}. \quad (9)$$

We note here that the conventional linear approximation also implies that $dn/dV = 0$. Equation (9) is an ordinary differential equation for the series resistance with voltage dependent factors and its solution can be obtained by using the experimental I-V data. In order to obtain R_s from this equation, we rewrite n in terms of Λ using Eq. (5), i.e., $n = (V - IR_s)/\beta\Lambda$. Substituting this n in Eq. (9) results in a first order ordinary differential equation for R_s. Solving this differential equation for R_s and after some algebraic manipulation we obtain

$$R_s(V) = \frac{\Lambda}{I}\int_{V_0}^{V} \frac{\Lambda - \alpha}{\Lambda^2} dV + R_0 \frac{\Lambda}{\Lambda_0} e^{-(\Lambda - \Lambda_0)}, \quad (10)$$

where R_0 is the series resistance when the bias voltage is equal to V_0. This equation describes the behaviour of R_s depending on the bias voltage. Here, parameter Λ_0 is a constant of integration, and effectively, it corresponds to a renormalization of the saturation current. The series resistance should be independent of this renormalization, and therefore we take $dR_s/d\Lambda_0 = 0$. Using this equation, one can obtain an expression for the second term on the right hand side of Eq. (10). Substituting this expression back into Eq. (10) we obtain

$$R_s(V) = \frac{\Lambda^2}{(\Lambda - \Lambda_0)I}\int_{V_0}^{V} \frac{(\Lambda - \Lambda_0)(\Lambda - \alpha) + \alpha\Lambda_0}{\Lambda^3} dV. \quad (11)$$

Now Eq. (11) contains only one free parameter, namely Λ_0, to be obtained from experimental data. Assuming that both the ideality factor n and series resistance R_s are constants, Mikhelashvili and Eisenstein obtained the following expression for the series resistance[3]

$$R_s = V(1 - \gamma)/I\alpha^2. \quad (12)$$

On the other hand, if we assume that both n and R_s are constants, then we can drop both the second and third terms from the right hand side of Eq. (8) obtaining $n\beta + IR_S = V/\alpha$. Now using the definition of $\Lambda(V)$, we obtain $V - IR_S = n\beta\Lambda$. Then the series resistance is

$$R_s = V(\Lambda - \alpha)/\alpha I(\Lambda - 1). \quad (13)$$

Because both the Eqs. (12) and (13) are obtained under the assumption that both n and R_s are constants, then these two equations should be equivalent. Therefore, equating the expressions for series resistances in Eqs. (12) and (13) we obtain

$$\Lambda_0 = (\alpha_0^2 + \gamma_0 - 1)/(\alpha_0 + \gamma_0 - 1), \quad (14)$$

where α_0 and γ_0 are the values of $\alpha(V)$ and $\gamma(V)$ at V_0, respectively.

Now, Eq. (11) along with Eqs. (4) and (14) completely defines an expression for determining the bias dependent behaviour of the series resistance from experimental I-V data.

This approach also provides a very useful method for determination of the saturation current I_0. Taking logarithm of both sides of Eq. (3) gives $\ln I = \ln I_0 + \Lambda$. It is clear that one can easily obtain the saturation current I_0 from a plot of $\ln I$ vs. Λ. When the saturation current is known, the zero bias barrier height can easily be found as $\phi_{B0} = \beta \ln(SA^*T^2/I_0)$.

In order to verify the proposed method and evaluate its relative merit, we have generated two sets of artificial I-V data for some known functions of series resistances R_s and effective barrier heights $\phi_B(V)$ displaying physically plausible bias dependent behaviours. However, chosen series resistance behaviours contain some unusual seldom observed maxima so that one can see whether the presented method can follow such structured behaviour or not. Using the known values of $R_s(V)$ and $\phi_B(V)$, we have calculated the I-V characteristics numerically from Eq. (2). We have used some reasonable constant values for S, A^*, T, and ϕ_{B0} parameters appearing in TE theory. However, these parameters appear only in the saturation current I_0 and their specific values are irrelevant for determination of the series resistance.

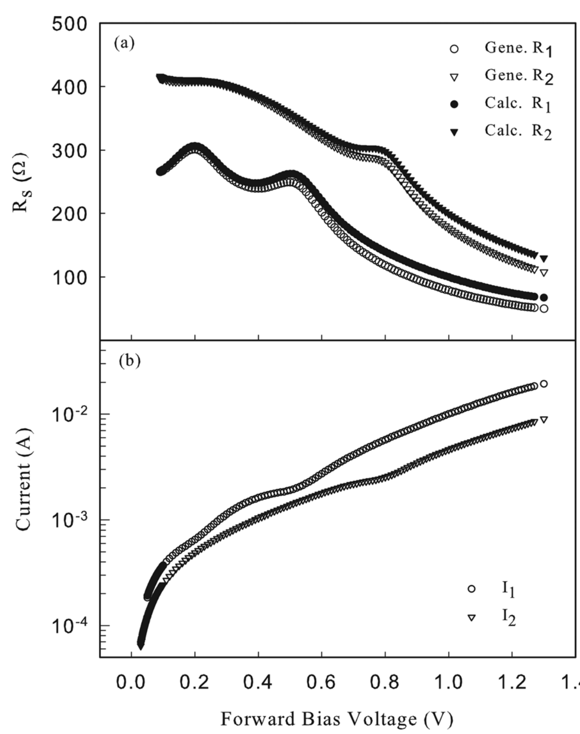

FIG. 1. (a) Series resistance values used in generation of artificial I-V data (empty symbols) and the resistance values extracted back from the I-V data (solid symbols) and (b) I-V characteristics generated from Eq. (2).

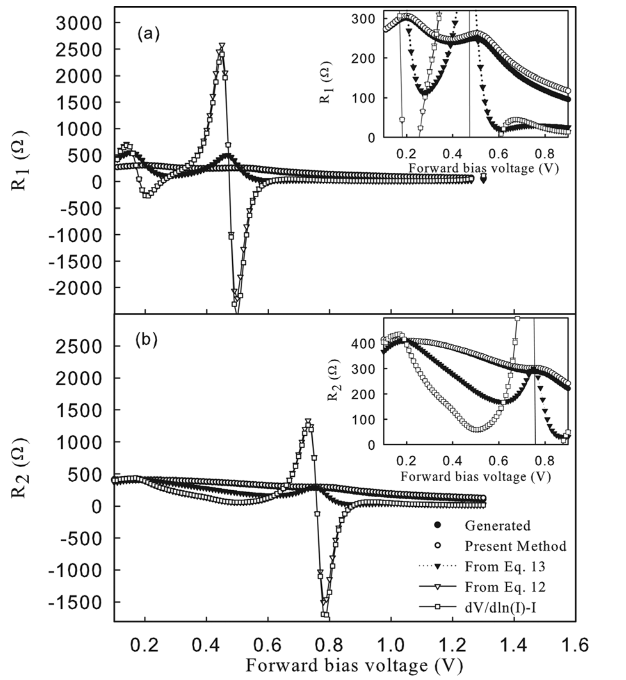

FIG. 2. Comparative plot of series resistance values extracted from I-V data by different methods. Panels (a) and (b) correspond to two different data sets.

Artificially generated series resistance $R_s(V)$ values are shown in Fig. 1(a) with open symbols. Corresponding artificially generated I-V characteristics for these two data sets are also shown in Fig. 1(b). The series resistance values calculated by Eq. (11) are also presented in Fig. 1(a) with solid symbols. For both data sets, Eq. (11) reproduces the voltage dependent behaviour of $R_s(V)$ over the whole range of bias voltages. However, we observe some small deviations as the bias voltage increases. This small difference is due to our assumption that the derivative of n with respect to bias voltage is zero.

There is no method reported in the literature to determine the bias dependent behaviour of the series resistance, R_s of a Schottky diode. However, in some studies, methods based on the assumption of R_s having a constant value were locally used to obtain a value for series resistance at each bias voltage.[3] For example, Eq. (12) can be used to obtain values of series resistance at different bias voltages. In order to evaluate the relative merit of the present method, we compare our results in Fig. 2 with the results obtained from local application of previous methods for both sets of artificial data. When locally applied, the standard method is equivalent to the method of Mikhelashvili and Eisenstein. In Fig. 2, solid circles indicate the original resistance values used to generate the artificial I-V data, empty circles show the resistance values calculated by Eq. (11), and empty squares are obtained from local application of standard $dV/d\ln I$ vs. I method. Solid triangles show the R_s values obtained from Eq. (13).

The results obtained from Eq. (12) and from local application $dV/d\ln I$ vs. I plots overlap with each other, the small differences observed between these results are due to the numerical errors. These resistance values tend to diverge in the regions where series resistance shows some structure. This behaviour might be considered to be normal because these approaches use second derivatives of the experimental data. Use of derivatives amplifies the experimental errors; therefore, these methods may produce even larger oscillations when applied to actual experimental data. It can be seen from Fig. 2 that in the regions where series resistance has a structure, Eq. (12) may also result in negative R_s values which is physically unacceptable. The results obtained from Eq. (13) seem to be more reasonable but they are still very far from actual resistance values and there are still some strong oscillations. In order to show the behaviour of actual resistance values, we have expanded the corresponding regions in the insets.

Also the present method provides a very efficient and robust approach to determine saturation current I_0 and thus zero-bias barrier height ϕ_{B0}. Existing methods to obtain the saturation current rely on an extrapolation of the linear region of $\ln I - V$ plot and the choice of this region may depend on the user. However, there is no need to choose a region when a plot of $\ln I$ vs. Λ is used.

Some of the previously used methods employ first[10] and sometimes second[3] derivatives of the experimental data. We note that differentiation process amplifies the experimental errors, whereas in the present method experimental errors are smoothed out because we use integration of the experimental data.

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