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# Nonlinear Minimization Algorithm for Determining the Solar Cell Parameters with Microcomputers

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A nonlinear least-squares optimization algorithm based on the Newton model modified with Levenberg parameter, is described for the extraction of the five illuminated solar cell parameters from the experimental data. An initialization routine based on the reduced nonlinear least-squares technique in which only two parameters have to be initialized, is introduced to overcome the difficulty in initializing the cell parameters. The program when incorporated into a microcomputer-based data acquisition software, allows an *in-situ* theoretical modelling of solar cells in laboratories. Results obtained for a commercial solar cell and a module are given.

## INTRODUCTION

The simulation and design calculations of photovoltaic systems require an accurate knowledge of the parameters which describe the nonlinear electrical model of solar cells. These parameters could be determined from a few selected points on the solar cell $I$–$V$ curve, namely, the open-circuit voltage $V_{oc}$, the short circuit current $I_{sc}$, the current $I_m$ and voltage $V_m$ at the maximum power point MPP and the slopes at $I = 0$ and $V = 0$ [1–4]. Although it seems simpler, the uncertainties prevailing in measuring $V_{oc}$ and $I_{sc}$, in locating the MPP and in graphically determining the two slopes, impede an

accurate solution for the parameters. Besides, a fit thus constructed may not represent accurately the $I-V$ curve over its whole range.

The proper approach is then to apply the least-squares techniques [5–13] by taking into account the data over the entire range of the $I-V$ curve. As the solar cell model is nonlinear, a suitable nonlinear algorithm should be applied in order to minimize the sum of squared errors. While generalized nonlinear optimization routines are available for large computer program libraries, it is of great convenience for the photovoltaic research groups in laboratories to have a more simplified, well-adapted and a readily accessible version which could be incorporated into a microcomputer-based data acquisition software, enabling them an *in-situ* theoretical modelling solar cells in reasonable time.

In this respect, searching the minimum by means of direct and gradient methods could be tiresomely long especially when the strongly differing cell parameters in weight, enter the error function. The reduced nonlinear least-squares techniques [9–13], in which the linearly dependant parameters on the cell equations are directly obtained by linear regression, still involve direct search methods and/or parametric fixation and evaluation for the nonlinearly entering parameters. Gauss–Newton method has also been proposed [6, 8] for the determination of solar cell parameters. Among these methods, in view of its quadratic convergence and enhanced accuracy, Newton's method remains attractive with the number of variables being limited to five and their partial derivatives easily obtainable.

In this paper, we elaborate on an optimization algorithm, intended for microcomputers, which uses Newton's method to determine the five parameters of the single diode model of illuminated solar cells. The difficulty in initializing the parameters is overcome by an initialization routine based on the reduced nonlinear least-squares technique making use of Newton's method. The problem of undesired oscillations and an eventual overflow in the objective function, is eliminated by modifying the Newton model with Levenberg's parameter $\lambda$. The second-order Newton method is then used in the unidimentional minimization of the objective function in question with respect to $\lambda$. The algorithm allows to enlarge the region of convergence of the two parameters that have to be initialized and ensures convergence to the solution.

## LEAST-SQUARES ESTIMATION

The current-voltage relationship of the single diode lumped parameter equivalent electrical circuit of solar cells under illumination is given by:

$$I = I_{ph} - I_s[\exp(q(V + IR_s)/(NkT)) - 1] - G_{sh}(V + IR_s) \quad (1)$$

where $I_{ph}$ photocurrent, $I_s$ saturation current, $N$ diode quality factor, $R_s$ series resistance and $G_{sh}$ shunt conductance are the model parameters and $T$ is the absolute temperature. Although multiple diodes model would improve the fit [14], the single diode model considered here is rather simple, computationally efficient and sufficiently accurate for the photovoltaic simulation and design work. The single diode model can also be used to fit solar modules where the cells are connected in series and/or parallel [3], provided that the cell to cell variations are not that important. It should be noted, however, that the parameters eventually determined, in the case of solar modules, will lose somewhat their physical meanings.

The general problem of fitting the $I$–$V$ curve to the collection data involves minimizing the objective function $S$ with respect to the set of parameters $\theta$:

$$S(\theta) = \sum_{i=1}^{m} [I_i - I(V_i, \theta)]^2 \quad (2)$$

where $\theta$ is the set of unknown parameters $\theta = (I_{ph}, I_s, N, R_s, G_{sh})$ and $(I_i, V_i)$ are respectively the measured current and voltage at the $i^{th}$ point among $m$ data points. The Eq. (1) is implicit in $I$ and one way of simplifying the computation of $I(V_i, \theta)$ is to substitute both $I_i$ and $V_i$ in the Eq. (1) [11].

$$I(V_i, \theta) = I_{ph} - I_s[\exp(q(V_i + I_iR_s)/(NkT)) - 1] - G_{sh}(V_i + I_iR_s) \quad (3)$$

The Eq. (3) is nonlinear in its parameters and hence the resulting set of normal equations $F(\theta)$, derived from multivariate calculus for a minimum to occur, will also be nonlinear and no exact solution can be generally found. Newton's method can be used to obtain an approximation to the exact solution for the nonlinear set of equations $F(\theta) = 0$. The Newton functional iteration procedure evolves from:

$$[\theta_k] = [\theta_{k-1}] - [J(\theta_{k-1})]^{-1}[F(\theta_{k-1})] \quad (4)$$

where $[J(\theta)]$ is the Jacobian matrix. Although Newton procedure converges rapidly, it has two major difficulties inherently present in converging to the solution:

i) a sufficiently accurate starting point for $\theta$ should be found.
ii) even when well-initialized, during the process an oscillatory behaviour could develop and may cause overflow in $S(\theta)$.

We describe in the following sections how these problems have been overcome with respect to the determination of illuminated solar cell parameters from the experimental data.



## INITIAL VALUES

Choosing reasonably good starting values for the five cell parameters which differ largely in magnitude between them, is rather a difficult task. The approximate solutions obtained from numerical and/or graphical methods [4, 5, 8] may not be close enough so as to be used as initial values for the Newton method. In fact, in many cases reasonably good initial approximations however, closer to the solution, particularly for the parameter $I_s$, lead to oscillations and eventually to overflow as a result of large Newton steps computed in the region near the optimum. A significant improvement in the convergence is obtained by the addition of Levenberg parameter $\lambda$ which will be discussed in the next section.

An alternate appealing idea is to reduce the number of parameters which should be initialized. An initialization subroutine based on the reduced nonlinear least-squares technique is introduced, in which only the two nonlinearly entering parameters $N$ and $R_s$ have to be initialized. With these values fixed, the linearly entering parameters $I_{ph}$, $I_o$ and $G_{sh}$ are estimated by linear regression [13]. Keeping these three parameters constant, $S$ is now minimized with respect to $N$ and $R_s$ by multivariate calculus and the resulting two nonlinear normal equations are subsequently solved using Newton's method, in which the iterations for $N$ and $R_s$ are continued till the relative accuracy for each of them becomes less than 0.5%. The steps are then repeated with newly determined values of $N$ and $R_s$, till the relative difference between two

consecutive values of $S$ computed soon after each linear regression, becomes smaller than 0.5%.

The initialization routine converges rapidly at the beginning and after some iterations $S$ diminishes with a smaller rate. This behaviour may be intuitively explained by the fact that the parameters have not been externally influenced as successively computed values are reinjected to obtain new values. In contrast, the reduction techniques in refs. [10–13] search the absolute minimum by varying $N$ and $R_s$ using direct search methods which could be expensive in time unless the number of nonlinearly entering parameters is reduced to one [11, 12]. The intention of the initialization routine here is not to attain the minimum, but to bring the five parameter values within the domaine of convergence, making use of the faster converging property of the routine at the beginning. It is found that this way of handling ensures a complete convergence when the parameters obtained are assumed as initial values to the Newton method modified with Levenberg's parameter.

Apart from eliminating the need to initialize all five parameters the routine enlarges substantially the region of convergence for $N$ and $R_s$ and thus reduces the number of initial trials that have to be made. However, sufficiently good initial approximations for $N$ and $R_s$ are determined from the high voltage region of the experimental data using the method given in [15].

## LEVENBERG PARAMETER

The problem of undesired oscillations and an eventual overflow results from a larger Newton step computed at some subsequent iteration even though it appears at the outset that the function decreases rapidly. An optimum Newton step is determined at each iteration using the Levenberg parameter $\lambda$, by which large Newton steps are limited and conversely short steps are increased. The new set of parameters $\theta$ is now obtained from:

$$[\theta_k] = [\theta_{k-1}] - \lambda[J(\theta_{k-1})]^{-1}[F(\theta_{k-1})] \tag{5}$$

In order to determine $\lambda$ from the unidimentional minimization of $S(\lambda)$ with $\theta$ fixed, that results from the substitution of Eq. (5) in Eq. (2), a line search method has been proposed in [8] where the

Gauss–Newton model is applied. Instead, here the second order Newton method is used to solve the resulting single nonlinear normal equation $S'(\lambda) = 0$. The second order Newton method yields a quadratic convergence regardless of the multiplicity of root, which compensate amply the drawback of additional derivative calculations introduced, provided that $\lambda$ is initially well-guessed. To a certain extent, the difficulty in initializing $\lambda$ is solved by multiple trials beforehand. It has been observed that for the very first Newton step just after the parameter values have been transferred from the initialization routine, $\lambda$ is in the order of 0.10. For the subsequent steps $\lambda$ takes a value between 0.50 and 2.75, in general. With these notions on $\lambda$, for the first step $\lambda$ is initially assumed 0.10, for the second it is given 1.5 and for the subsequent steps $\lambda$ is initially approximated with the solution obtained from the precedent minimization of $S(\lambda)$.

The iterative process for $\lambda$ stops when the relative difference between successively computed objective function values becomes less than 0.1%. The test is made on the function value rather than on $\lambda$ itself since it is found that near optimum the reduction in the function value is small with respect to the change in $\lambda$. In this case, the program proceeds to the computation of the next Newton step. The number of iterations taken at each Newton step, in general, decreases from about 4 at the beginning to 1 in the end near the optimum.

The Newton steps are continued until the successively computed parameters are found to change by less than 0.5%. At this end, in order to test the quality of the fit to the experimental data, the standard deviation $\sigma$ is calculated.

$$\sigma = \left\{(1/m) \sum_{i=1}^{m} [I_{\text{cal},i}/I_i - 1]^2\right\}^{1/2} \tag{6}$$

where $I_{\text{cal},i}$ is determined for each $V_i$, by solving the implicit Eq. (3) with the optimized set of $\theta$. Also, the characteristic parameters of the solar cells $V_{oc}$, $I_{sc}$, $V_m$, $I_m$ and the fill factor $FF$ are deduced using Newton method.

## APPLICATION

As test examples, a 57 mm diameter commercial (R.T.C France) silicon solar cell and a solar module (Photowatt-PWP 201) in which

36 polycrystalline silicon cells are connected in series, are considered. Their experimental $I$–$V$ data is obtained under controlled conditions from an automated measuring system with a CBM 8096 microcomputer acting as the controller which once finishes the acquisition of data, starts executing the optimization program in order to determine the model parameters. The five cell and module optimal parameters and their characteristic points determined, are as follows:

| | | Cell (33 °C) | Module (45 °C) |
|---|---|---|---|
| $I_{ph}$ | $(A)$ | 0.7608 | 1.0318 |
| $I_s$ | $(\mu A)$ | 0.3223 | 3.2876 |
| $G_{sh}$ | $(\Omega^{-1})$ | 0.0186 | 0.00182 |
| $R_s$ | $(\Omega)$ | 0.0364 | 1.2057 |
| $N$ | | 1.4837 | 48.450 |
| $\sigma$ | (%) | 0.6251 | 0.7805 |
| $I_{sc}$ | $(A)$ | 0.7603 | 1.030 |
| $V_{oc}$ | $(V)$ | 0.5728 | 16.778 |
| $V_m$ | $(V)$ | 0.4507 | 12.649 |
| $I_m$ | $(A)$ | 0.6894 | 0.912 |
| $FF$ | | 0.7135 | 0.668 |

The experimental $(V_i, I_i)$, calculated $(I_{\text{cal},i})$ values and the deviations $D\{=I_{\text{cal},i}/I_i - 1\}$ for the two cases are given in Tables I and II and their corresponding plots are shown in Figures 1 and 2. As can be seen from the tables and optimal parameter values, the fits obtained for the two cases are good with $\sigma$ less than 1%. An improvement can be made in the fit by avoiding measurements at open circuit conditions where the current is not well-defined [5], and consequently leads to deviations of about 3% in the examples considered. It should be noted that in the case of modules although the theoretical model fits in with experimental data, the model parameters are not well-related to the physical phenomena which are intrinsically existing due to nonidentical cells which are connected in series and/or parallel to form modules.

TABLE I
Comparison of experimentally obtained values of load current and values computed from the model of the cell

| $V_i(V)$ | $I_i(A)$ | $I_{cal,i}(A)$ | $D(\%)$ |
|---|---|---|---|
| −0.2057 | 0.7640 | 0.7641 | −0.013 |
| −0.1291 | 0.7620 | 0.7627 | −0.091 |
| −0.0588 | 0.7605 | 0.7614 | −0.118 |
| 0.0057 | 0.7605 | 0.7602 | 0.039 |
| 0.0646 | 0.7600 | 0.7591 | 0.118 |
| 0.1185 | 0.7590 | 0.7580 | 0.131 |
| 0.1678 | 0.7570 | 0.7571 | −0.013 |
| 0.2132 | 0.7570 | 0.7561 | 0.119 |
| 0.2545 | 0.7555 | 0.7551 | 0.053 |
| 0.2924 | 0.7540 | 0.7537 | 0.040 |
| 0.3269 | 0.7505 | 0.7514 | −0.120 |
| 0.3585 | 0.7465 | 0.7473 | −0.107 |
| 0.3873 | 0.7385 | 0.7401 | −0.217 |
| 0.4137 | 0.7280 | 0.7274 | 0.082 |
| 0.4373 | 0.7065 | 0.7070 | −0.071 |
| 0.4590 | 0.6755 | 0.6754 | 0.015 |
| 0.4784 | 0.6320 | 0.6308 | 0.190 |
| 0.4960 | 0.5730 | 0.5722 | 0.140 |
| 0.5119 | 0.4990 | 0.4994 | −0.080 |
| 0.5265 | 0.4130 | 0.4134 | −0.097 |
| 0.5398 | 0.3165 | 0.3172 | −0.221 |
| 0.5521 | 0.2120 | 0.2125 | −0.236 |
| 0.5633 | 0.1035 | 0.1026 | 0.870 |
| 0.5736 | −0.0100 | −0.0097 | 3.000 |
| 0.5833 | −0.1230 | −0.1241 | −0.894 |
| 0.5900 | −0.2100 | −0.2093 | 0.334 |

## CONCLUSION

When the software is tested in CBM 8096 for number of samples, both solar cells and modules, it finds the absolute minimum with the desired accuracy, which leaves no doubt about the convergence. The time taken is in general 10 to 20 minutes in Basic language which is reduced to a few when $N$ and $R_s$ are well-initialized and it can be further reduced if the actual program is translated in Fortran or in any other compiled languages. For some cases of modules, the time taken to attain the minimum is longer which arises from the difficulty in finding a good initial approximation for $N$ and $R_s$. One exception is with the data taken for a solar panel in which one cell is

TABLE II
Comparison of experimentally obtained values of load current and values computed from the model of the module (Values corrected to the 4th significant decimal place)

| $V_i(V)$ | $I_i(A)$ | $I_{cal,i}(A)$ | $D$ (%) |
|---|---|---|---|
| −1.9426 | 1.0345 | 1.0326 | 0.184 |
| 0.1248 | 1.0315 | 1.0302 | 0.126 |
| 1.8093 | 1.0300 | 1.0282 | 0.175 |
| 3.3511 | 1.0260 | 1.0263 | −0.029 |
| 4.7622 | 1.0220 | 1.0244 | −0.235 |
| 6.0538 | 1.0180 | 1.0224 | −0.432 |
| 7.2364 | 1.0155 | 1.0199 | −0.433 |
| 8.3189 | 1.0140 | 1.0162 | −0.217 |
| 9.3097 | 1.0100 | 1.0102 | −0.020 |
| 10.2163 | 1.0035 | 1.0004 | 0.309 |
| 11.0449 | 0.9880 | 0.9844 | 0.364 |
| 11.8018 | 0.9630 | 0.9595 | 0.363 |
| 12.4929 | 0.9255 | 0.9230 | 0.270 |
| 13.1231 | 0.8725 | 0.8726 | −0.012 |
| 13.6983 | 0.8075 | 0.8074 | 0.012 |
| 14.2221 | 0.7265 | 0.7281 | −0.220 |
| 14.6995 | 0.6345 | 0.6366 | −0.331 |
| 15.1346 | 0.5345 | 0.5358 | −0.243 |
| 15.5311 | 0.4275 | 0.4289 | −0.327 |
| 15.8929 | 0.3185 | 0.3187 | −0.063 |
| 16.2229 | 0.2085 | 0.2078 | 0.336 |
| 16.5241 | 0.1010 | 0.0983 | 2.673 |
| 16.7987 | −0.0080 | −0.0082 | −2.500 |
| 17.0499 | −0.1110 | −0.1110 | 0.000 |
| 17.2793 | −0.2090 | −0.2091 | −0.048 |
| 17.4885 | −0.3030 | −0.3020 | 0.330 |

partially shadowed. Here, the execution ends up in overflow pointing out that the single diode model no longer corresponds to such cases where cell to cell variations are very important.

The numerical procedure described herein for the extraction of the illuminated solar cell parameters, can be easily adapted into any microcomputer-based data acquisition software. The procedure can be also extended to two diodes model of solar cells. It thus allows real time characterization of solar cells and modules in laboratories or at the outlet of a solar cell or module production line, soon after the microcomputer accomplishes taking data.

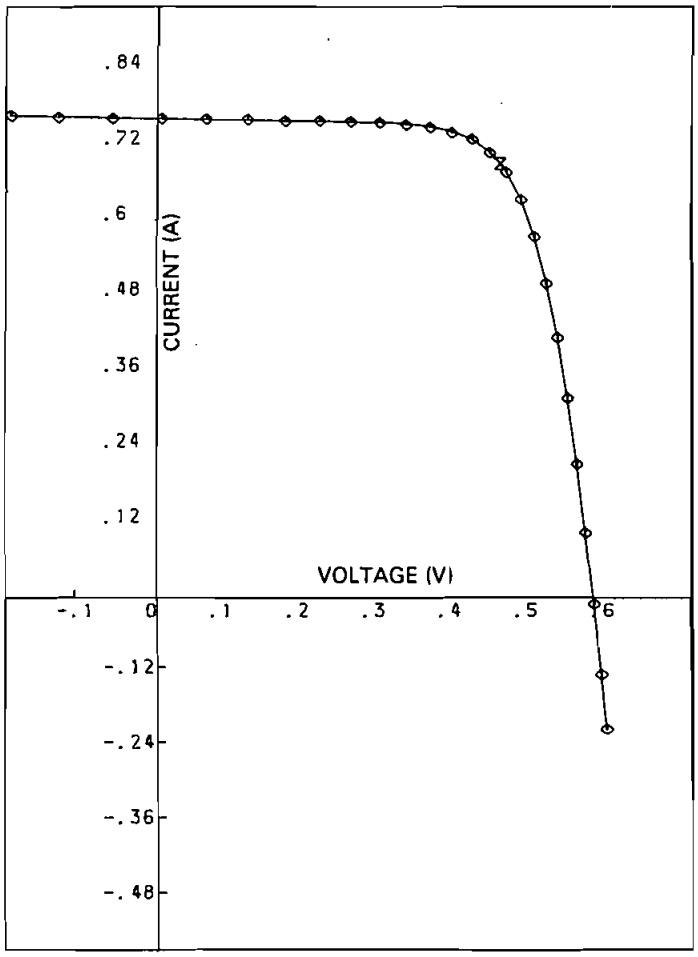


FIGURE 1 Experimental (◇◇◇) data and the fitted curve (——) for the commercial (R.T.C. France) solar cell. (X) denotes the maximum power point MPP.

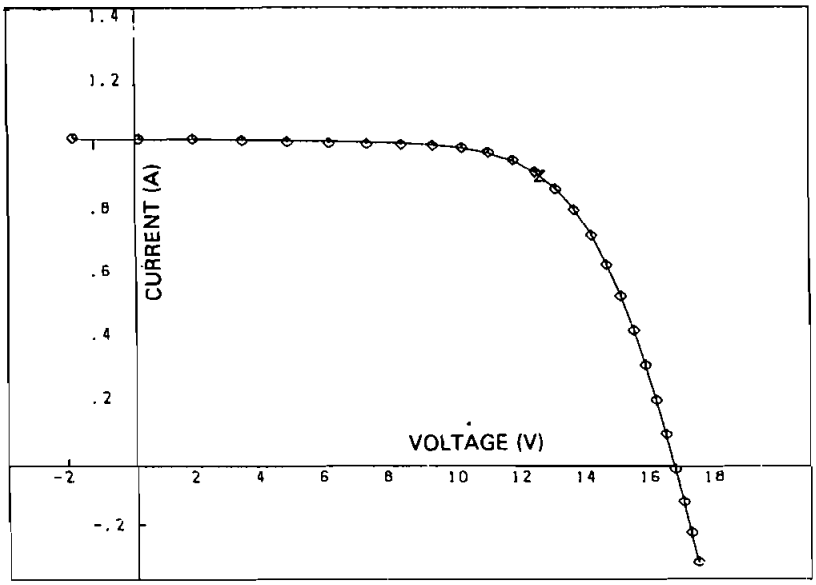


FIGURE 2 Experimental (◇◇◇) data and the fitted curve (——) for the commercial (Photowatt PWP 201) solar module. (X) denotes the maximum power point MPP.

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