A REVIEW OF CURVE FITTING ERROR CRITERIA FOR SOLAR CELL *I–V* CHARACTERISTICS

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Summary

Various methods for recovering solar cell lumped circuit model parameters from experimental characteristics are briefly reviewed. The advantages of extracting parameters from illuminated characteristics are highlighted. These include the availability of accurate analytical expressions developed recently. A commonly used method of parameter recovery by curve fitting minimises σ which is defined as the r.m.s. of the relative current errors between the experimental and theoretical characteristics. This method is demonstrated to be unreliable when used with characteristics which have been collected by linear analogue to digital systems, or which have certain data point distributions. A more reliable minimisation criterion ϵ is proposed. ϵ is based on the area difference between the experimental and theoretical characteristics. Computation experiments show that the use of ϵ results in much more accurate parameter recovery for both dark and illuminated characteristics, and that its accuracy is almost independent of data point distribution. ϵ also provides a good basis for comparing the quality of fit of theoretical models to experimental characteristics.

1. Introduction

The performance of solar cells can be conveniently and accurately simulated using lumped circuit models. The determination of model parameters can therefore be an important aspect of solar cell design and production, especially if the model's parameters can be closely associated with known physical processes. For example, measurement of the model parameters can be an important tool in the control of the solar cell manufacturing process, and may be a means of pinpointing causes of sudden deterioration in the performance of cells being produced. A detailed knowledge of the way in which parameter values vary with different working conditions (*e.g.* temperature and illumination) can also provide a powerful tool in the optimisation of solar cell performance.

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There have been two approaches in determining model parameters. One has been to extract model parameters from the dark current I–V characteristics. The advantage is that the dark characteristic can be measured more easily, without the problem of temperature variations which occur in cells under intense illumination. However the underlying assumption here is that the parameter values do not change with illumination level, and that solar cell behaviour can be predicted using model parameters determined from the dark characteristic. The alternative is to use the illuminated characteristic. This has an important advantage in that it allows the investigation of the variation of model parameters with illumination level. This advantage is balanced by the fact that the illuminated characteristic is more difficult to measure accurately than the dark characteristic. The open-circuit region in the illuminated characteristic, for example, has a combination of a steep slope and a small value of the current.

The algorithms for determining model parameters in solar cells can be grouped into two types: the first group makes use of selected parts of the characteristic while the second uses the whole characteristic. The first group of algorithms involves the solution of five equations derived from considering selected points of a characteristic, *e.g.* the open-circuit and short-circuit points, the maximum power points and the slopes at strategic portions of the characteristic [1, 2]. Although the exact solution of these equations requires iterative techniques, this method is still a much faster and simpler way of parameter determination in comparison with curve fitting. Moreover, it has been recently demonstrated that, by making judicious approximations, analytical expressions can be derived which give parameter values accurate to about 5% when compared with the exact solution [3, 4]. The disadvantage of this approach is that only selected points in the characteristic are used in determining parameter values.

Curve fitting methods have the advantage of taking all the experimental data into consideration, so giving the best overall fit. All these methods involve the minimisation of some error criterion in order to find the set of parameter values which, when inserted into the theoretical relationship, gives the best fit to the experimental characteristic. However, it has been pointed out that these methods should be used with circumspection [4, 5], and can sometimes give misleading results. There have been two suggestions regarding the error criterion which should be used in curve fitting. Otterbein *et al.* [6] suggested the use of the root of the sum of the square of current errors (*i.e.* the difference between the theoretical and experimental current values). Other workers, namely Wolf *et al.* [5] and Araujo *et al.* [7] have rightly pointed out that the use of this criterion essentially tests the fit over the high current regions only and neglects the quality of fit in the low current regimes, since the absolute value of the current errors would be larger in the high current regions. Some form of weighting is obviously necessary to give an even fit over the entire characteristic. Wolf then suggested the use of the r.m.s. value of the relative current errors as a more appropriate fitting criterion. This criterion can be expressed as

$$\sigma = \left\{ \frac{1}{N} \sum_{j=1}^{N} \left(\frac{(I_{\text{th}})_j - (I_{\text{exp}})_j}{(I_{\text{exp}})_j} \right)^2 \right\}^{1/2} \tag{1}$$

where N is the total number of data points in the characteristic, $(I_{\text{th}})_j$ is the theoretically generated current at voltage V_j and $(I_{\text{exp}})_j$ is the experimentally measured current at voltage V_j.

Both Otterbein's and Wolf's criteria have the disadvantage that the error criterion is strongly dependent on the distribution of experimental data over the characteristic. The fit will therefore be weighted in favour of the parts of the characteristic which are well represented in the experimental data. It is suggested that part of the reason why curve fitting sometimes gives unreliable parameter values may be the use of σ as a minimising criterion. It will be shown in this paper that σ over-emphasises the errors in the low current part of the characteristic, and that the reliability of recovering model parameters by minimising σ is dependent on the distribution of experimental data along the characteristic. It will also be shown that the quantity ϵ as defined in the next section will be a better alternative minimisation criterion and that curve fitting carried out by minimising ϵ will be shown to be a more robust and reliable method of parameter recovery.

2. Proposed error area criterion

A curve fitting criterion based on the area difference between the experimental and theoretical characteristics (Fig. 1) is suggested as a more suitable minimisation criterion for recovering solar cell parameters. This quantity can be defined as

$$\bar{V}_{\text{area}} = \sum_{\substack{j=1 \\ j \neq m}}^{N-1} \left| \frac{(\Delta I_j + \Delta I_{j+1})\{(V_{\text{exp}})_{j+1} - (V_{\text{exp}})_j\}}{2} \right| + \left| \frac{\{(V_{\text{exp}})_{m+1} - (V_{\text{exp}})_m\}}{2} \; \frac{(\Delta I_m)^2 + (\Delta I_{m+1})^2}{|\Delta I_m| + |\Delta I_{m+1}|} \right| \tag{2}$$

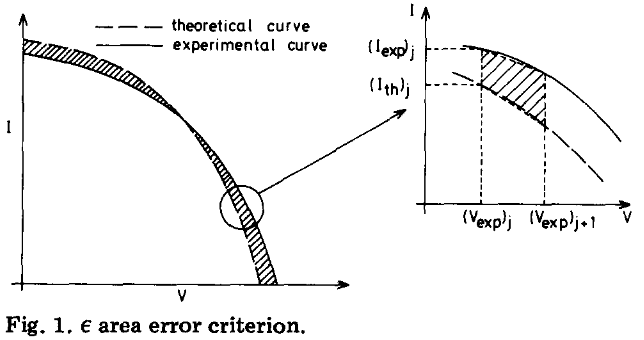

Fig. 1. ϵ area error criterion.

where $(V_{exp})_j$ is the experimental measured voltage at the jth point, $\Delta I_j = (I_{th})_j - (I_{exp})_{j+1}$, the current error ΔI changes sign between the mth and $(m + 1)$th point, and all other symbols are as defined in eqn. (1).

This criterion weights the current error in two ways. Firstly, it weights it at each point in the curve by the voltage interval between this point and the next point. In so doing, it weights the fit at each region of the curve inversely proportionally to the number of data points in that region. The parameters obtained by curve fitting will therefore be less dependent on the distribution of experimental points in the curve. Secondly, the shape of the I–V characteristic for solar cells is such that the use of this criterion will weight the current errors in the flat part of the curves more heavily. For example, if a dark experimental characteristic were to be sampled at constant voltage intervals so that the current error at each point got an equal weighting when applying the $\bar{V}_{area}$ criterion, the resulting data would contain many more points in the low current range than in the high current range. If the same procedure were to be carried out for an illuminated characteristic, there would be many points in the short-circuit region of the curve and relatively few points in the open-circuit part of the characteristic. In other words, this criterion puts more emphasis on the current errors in the flat part of the characteristic than on those in the steep part of the characteristic. This is convenient, as, all things being equal, the errors in the steep part of the characteristic are likely to be larger in absolute value, and are likely to dominate the total error unless a weighting factor is applied.

A quality of fit parameter can be derived from the $\bar{V}_{area}$ by normalising it with respect to the total area. Thus

$$\epsilon = \frac{\bar{V}_{area} \times 100\%}{\text{Total area under experimental } I\text{–}V \text{ curve}} \tag{3}$$

ϵ makes possible a comparison of the quality of fit of theoretical models to different experimental characteristics. This is because ϵ is less dependent on the distribution of data points along the characteristic, and a valid comparison can be made between two different curves. For σ, however, a comparison is not always meaningful as this quantity is highly dependent on the distribution of data points.

3. Computational experiments

Some experiments were carried out to investigate the relative merits of the σ and ϵ criteria in recovering parameters. These were done by generating sets of dark and illuminated characteristics using known parameter values, and then attempting to recover these values by curve fitting, using first the σ and then the ϵ criterion.

Several different types of characteristics were generated to simulate the data that would be obtained by different data acquisition systems. The use of digital voltmeters (DVMs) gives high resolution but is much slower. This slower speed can result in temperature variations in the cell during data collection. The characteristic will therefore not be collected at a constant temperature. On the other hand, the use of analogue to digital (A/D) converters linked to a microcomputer gives high speed data collection (thus avoiding the heating problem) at the cost of lower resolution. Characteristics collected using DVMs can be assumed to have 4 - 5 digits resolution over the entire range of currents and voltages measured. Characteristics collected using fixed range A/D converters have a fixed absolute resolution limit. The first type of characteristic was simulated by truncating the computer-generated data to 5 significant digits, while to simulate the second type of characteristic, random noise related to the resolution limit was added to the computer-generated data. This fudging of the data was done according to the following formula which assumes the use of 12-bit converters

$$\begin{aligned} V1(N) &= V(N) + R(N) \times 0.6/(4096 \times 2) \\ I1(N) &= I(N) + R(N) \times I_{max}/(4096 \times 2) \end{aligned} \tag{4}$$

where $V(N)$ and $I(N)$ are the accurate computer-generated data, $R(N)$ are gaussian random numbers from a distribution having a mean of 0 and a variance of 1.0, I_{max} and 0.6 V are the current and voltage range of the A/D converters. For characteristics simulating A/D collected data, the data points were generated at intervals evenly distributed between the maximum and minimum resolvable current or voltage values.

The effect of a variation of the distribution of points in the characteristic was also investigated. Some curves were generated with evenly distributed data points, while some were generated at constant current intervals (weighting the points distribution heavily in favour of the steep part of the characteristic) and others were generated at constant voltage intervals (weighting the points distribution in favour of the flat part of the characteristic).

Three sets of characteristics were generated. Each set consisted of 6 curves, with each curve representing a permutation of the three types of point distributions and the two types of data resolution. The full details of the generated curves are given in Table 1. One set of dark and two sets of illuminated characteristics were generated. The two sets of illuminated curves represent one at near AM 1 light levels and the second at very low light levels. The model parameters chosen for curve generation are typical of the values encountered for 3 in silicon cells.

The curve fitting was carried out by computer using the ZMXWD routine from the IMSL Library [9]. This routine searches for the global minimum of the error criterion within a predetermined range of values. This is probably more time consuming than other recently suggested algorithms [7, 8], but the use of a sledge-hammer technique eliminates any doubts arising from possible deficiencies in less exhaustive techniques.

TABLE 1

Procedure for generating I–V characteristics in the computational experiments

A. Dark characteristics: curves DC1 - DC6

Model equation

$$I = I_{s1} \exp\frac{V - IR_s}{V_T} + I_{s2} \exp\frac{V - IR_s}{2V_T} + \frac{V - IR_s}{R_{sh}}$$

where $I_{s1} = 10^{-9}$ A, $I_{s2} = 10^{-5}$ A, $R_s = 20$ mΩ, $R_{sh} = 120$ Ω, $T = 50$ °C, $I_{max} = 1.0$ A

B. Illuminated characteristics I: curves IC1 - IC6

Model equation

$$I = I_{ph} - I_{s1} \exp\frac{V + IR_s}{V_T} - I_{s2} \exp\frac{V + IR_s}{2V_T} - \frac{V + IR_s}{R_{sh}}$$

where $I_{s1} = 10^{-9}$ A, $I_{s2} = 10^{-5}$ A, $R_s = 20$ mΩ, $R_{sh} = 120$ Ω, $I_{ph} = 1$ A, $T = 50$ °C, $I_{max} = 1.0$ A

C. Illuminated characteristics II: curves IC7 - IC12

Model equation identical to that for B; parameter values are changed to

$I_{s1} = 10^{-9}$ A, $I_{s2} = 10^{-5}$ A, $R_s = 20$ mΩ, $R_{sh} = 20$ Ω, $I_{ph} = 0.1$ A, $T = 50$ °C, $I_{max} = 0.1$ A

Curve code	*Curve properties*
DC1, IC1, IC7	100 points evenly distributed over I–V curve, truncated to 5 significant figures
DC2, IC2, IC8	100 points generated at constant current intervals, truncated to 5 significant figures
DC3, IC3, IC9	100 points generated at constant voltage intervals, truncated to 5 significant figures
DC4, IC4, IC10	100 points evenly distributed over I–V curve, with random noise added
DC5, IC5, IC11	100 points generated at constant current intervals, with random noise added
DC6, IC6, IC12	100 points generated at constant voltage intervals, with random noise added

4. Results and discussion

4.1. Dark I–V characteristics

The results of the curve fitting for dark characteristics are presented in Table 2. The results show that the σ criterion recovered DC1 - DC3 without any problems. Difficulties arose with DC4 and DC6 and parameter recovery was impossible in the latter case. ϵ, on the other hand, was able to recover all the parameters for all six curves to at least two significant digit accuracy.

The causes of the failure with σ can be seen clearly in the results in Table 2. It can be noted that the value of the residue σ in DC1 - DC3 varied by over 50% even though these were identical curves which differed only in their points distribution. This confirms that σ is a strong function of points distribution. By comparison, it can be seen that the value of residue ϵ did not vary significantly over the three curves. Moreover, the σ criterion strongly

TABLE 2

Model parameters recovered from dark characteristics

Curve	*Criterion*	$I_{s1} \times 10^{-9}$ (A)	$I_{s2} \times 10^{-5}$ (A)	R_s (mΩ)	R_{sh} (Ω)	*Residue* (%)[a]
Parameter values used in curve generation						
		1.0	1.0	20	120	
Extracted parameter values						
DC1	σ	0.9999	1.000	19.99	120.0	0.5559×10^{-2}
	ϵ	0.9999	1.000	19.99	120.0	0.5328×10^{-2}
DC2	σ	0.9999	1.000	19.99	120.04	0.6779×10^{-2}
	ϵ	0.9999	1.000	19.99	120.04	0.5487×10^{-2}
DC3	σ	1.000	0.9999	20.0	120.0	0.4490×10^{-2}
	ϵ	1.000	0.9999	20.0	119.99	0.5564×10^{-2}
DC4	σ	0.9639	1.027	19.24	125.72	4.847
	ϵ	0.9909	1.009	19.88	122.47	0.2029
DC5	σ	1.002	0.9978	20.03	118.74	0.1875
	ϵ	1.003	0.9966	20.04	117.68	0.1705
DC6	σ[b]	0.9066	1.049	17.49	127.89	9.378
	ϵ	1.003	0.9982	20.11	119.93	0.1769

[a] Residue represents either $\sigma \times 100\%$ or ϵ as defined in eqn. (3).
[b] Values could not converge to 4 significant digits.

emphasises errors in the low current region, and thus when it encounters a curve with most of its points in the low current regions it will neglect the fit over the high current regions. This becomes a problem in the case of DC6 which has the majority of its data in the low current regions, and in which random noise was introduced to simulate quantisation errors in a 12-bit A/D converter. Since the quantisation errors are proportionately much more serious at low current values, and σ emphasises the fit in the low current regions, the less noisy data in the higher current regions will be neglected, thus causing the failure of the parameter recovery process. This problem does not arise in DC5 which has most of its points in the higher current regions. The practical implication of this result is that σ is not reliable as a minimisation criterion for fitting dark characteristics which have been collected using fixed range A/D converters at constant voltage intervals.

The relative merits of σ and ϵ can also be seen in a plot of $I_{th} - I_{exp}$ *versus* voltage. The plots of DC1 - DC3 showed virtually no difference between those obtained by σ and those obtained by ϵ. However, the plots for DC4 and DC6 showed differences between the current errors obtained with σ and those obtained with ϵ. These plots are presented in Fig. 2, and show that ϵ gives a set of parameters which fit the experimental curve at least as well, if not better, than those given by σ. Apart from R_{sh} in curves DC4 - DC6, ϵ was able to recover all the parameters to better than 1% accuracy. R_{sh} poses a difficulty for both σ and ϵ. The reason is that this parameter is determined by a narrow part of the characteristic near the

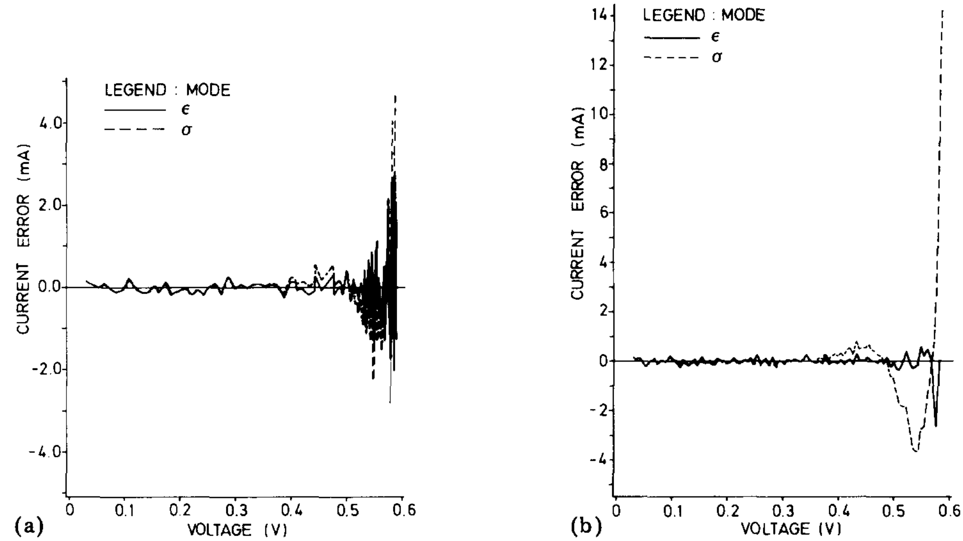

Fig. 2. Current errors ($I_{th} - I_{exp}$) *vs.* voltage for (a) dark characteristic DC4, (b) dark characteristic DC6.

$V = 0$ region, and this is the region which is most affected by the quantisation errors. However, even for R_{sh}, ϵ gives more accurate values than σ, and manages to recover R_{sh} to about 2% - 4% accuracy.

4.2. Illuminated I–V characteristics

The results of using σ and ϵ to fit illuminated characteristics are given in Table 3. The level of illumination is equivalent to a short-circuit current of 1 A, which for a 3 in cell is slightly lower than AM 1 light level. The results show that while ϵ recovered all the parameters effectively (the most difficult parameter R_{sh} could be recovered to less than 4% error), σ had difficulties with IC2, IC3, IC4, IC5 and IC6. In fact, σ could not accurately recover parameters from any of the curves which had quantisation noise introduced into them. The practical implication is that σ is an unreliable criterion to use for curve fitting experimental data which have been collected by 12-bit A/D systems.

The problem with σ here is similar to that for dark characteristics, namely, an over-emphasis on error currents in low current regions and a resulting strong dependence on data point distribution. For illuminated characteristics, this low current region is also an inherently more noisy region since it corresponds to the steep open-circuit region. Thus, σ tries to give a good fit over the $I = 0$ region at the expense of the fit over the $V = 0$ region. This can be seen from Fig. 3(e), (f) where the error current over the short-circuit region is substantial and can be described as a structural error rather than a noise error. (This is a common problem in curve fitting illuminated characteristics through minimising σ.) The strong dependence of

TABLE 3

Model parameters recovered from illuminated characteristics

Curve	*Criterion*	I_{ph} (A)	$I_{s1} \times 10^{-9}$ (A)	$I_{s2} \times 10^{-5}$ (A)	R_s (mΩ)	R_{sh} (Ω)	*Residue* (%)
Parameter values used in curve generation							
		1.000	1.0	1.0	20.00	120.00	
Extracted parameter values							
IC1	σ	0.9999	0.9993	1.002	19.98	121.88	0.1504×10^{-3}
	ϵ	0.9999	1.000	0.9997	20.00	120.02	0.6239×10^{-3}
IC2	σ	1.000	1.005	0.9832	20.10	101.35	0.6775×10^{-3}
	ϵ	1.000	0.9999	1.000	20.00	119.99	0.4951×10^{-3}
IC3	σ	1.000	1.000	0.9987	20.02	119.44	0.1461×10^{-4}
	ϵ	1.000	1.000	0.9993	20.01	119.84	0.5303×10^{-3}
IC4	σ	0.9977	0.9805	1.0583	19.00	150.44	0.7222×10^{-2}
	ϵ	1.000	1.002	0.9993	19.92	118.63	0.2075×10^{-1}
IC5	σ	0.9995	0.8860	1.325	17.10	249.55	0.1590×10^{-1}
	ϵ	1.000	0.9956	1.016	19.77	124.55	0.1338×10^{-1}
IC6	σ	1.000	0.9488	1.1562	18.62	182.95	0.1571×10^{-2}
	ϵ	0.9999	1.000	1.004	19.87	121.93	0.1673×10^{-2}

σ on the distribution of points can be deduced in the residue σ value for IC1 - IC3 (see Table 3) where σ varies over an order of magnitude in curves which are identical except for the distribution of points. (This dependence on data distribution seems more serious with illuminated data than with dark data where the residue σ for DC1 - DC3 (see Table 2) did not vary over as wide a range.)

The current error plots for ϵ show that the fit of the theoretical to the experimental characteristics was uniformly good, and current deviations were due more to noise rather than to any large structural errors. This demonstrated the ability of ϵ to give a good even fit over the entire characteristic.

It has also been suggested that the residue value of σ can be used to gauge whether the theoretical model fits the experimental data [5]. However, the strong dependence of σ on the distribution of data points makes this criterion an unreliable measure of the quality of fit. ϵ, because it is less dependent on data distribution, does offer some gauge of the quality of fit of a model to the experimental data and allows for a true comparison of fit between different experimental curves. A second advantage is that a criterion of the quality of the solution (*i.e.* the lowest possible value of ϵ) can be easily estimated for a given data collection system. For example, a 12-bit A/D converter with a full scale current of 1 A will have a resolution limit of 244 μA. Thus, assuming an average error in the current of half that value over the entire range, and assuming that the cell bias is set by an accurate voltage source, an estimate of the minimum value of ϵ for an illuminated curve will be given by

$$\frac{122\ \mu\text{A} \times V_{oc}}{\text{Total area under } I\text{–}V \text{ curve}} \tag{5}$$

A value of residue ϵ of the same order of magnitude as that given by the above expression would indicate a very good fit of the theoretical model to experimental data.

4.3. *Characteristics of cells under low illumination*

A third set of characteristics was generated to investigate the limits of parameter recovery at low illumination. The photocurrent was set at 100 mA

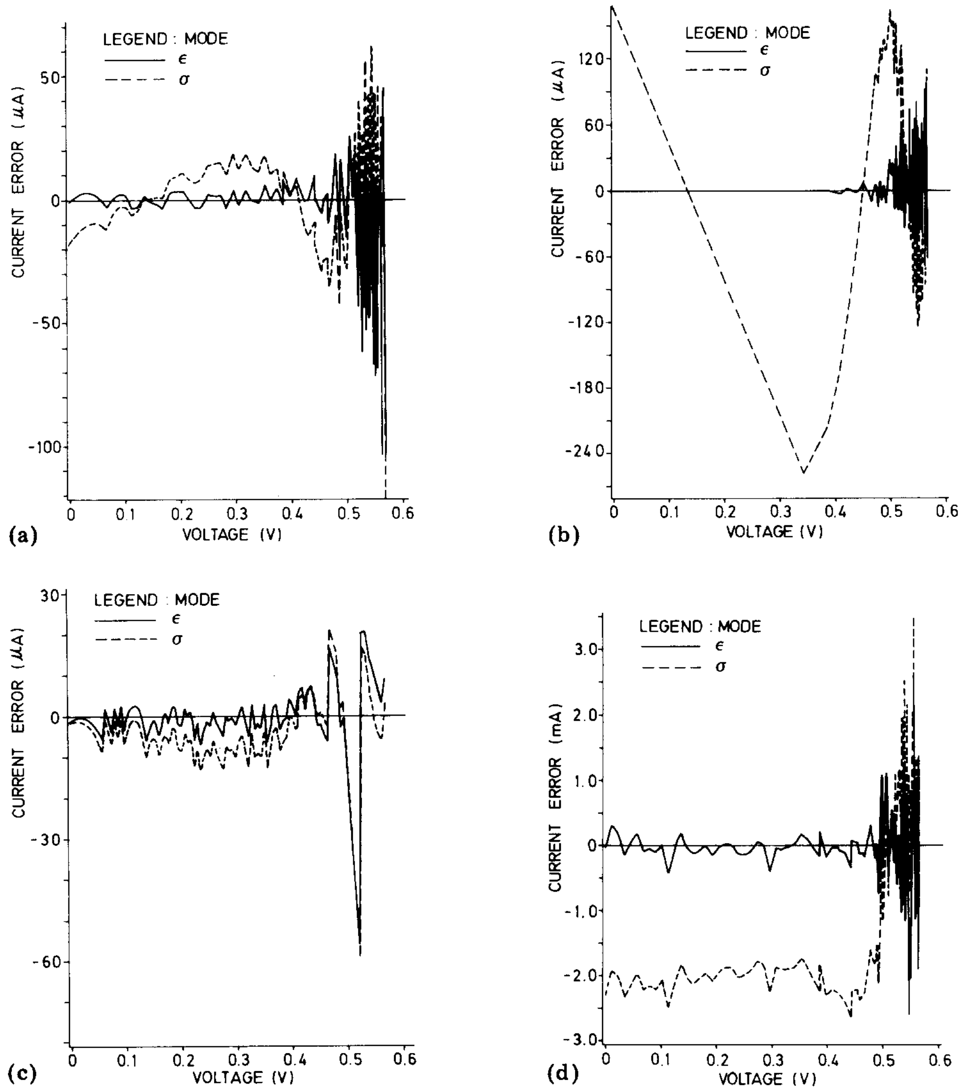

Fig. 3. (*continued*).

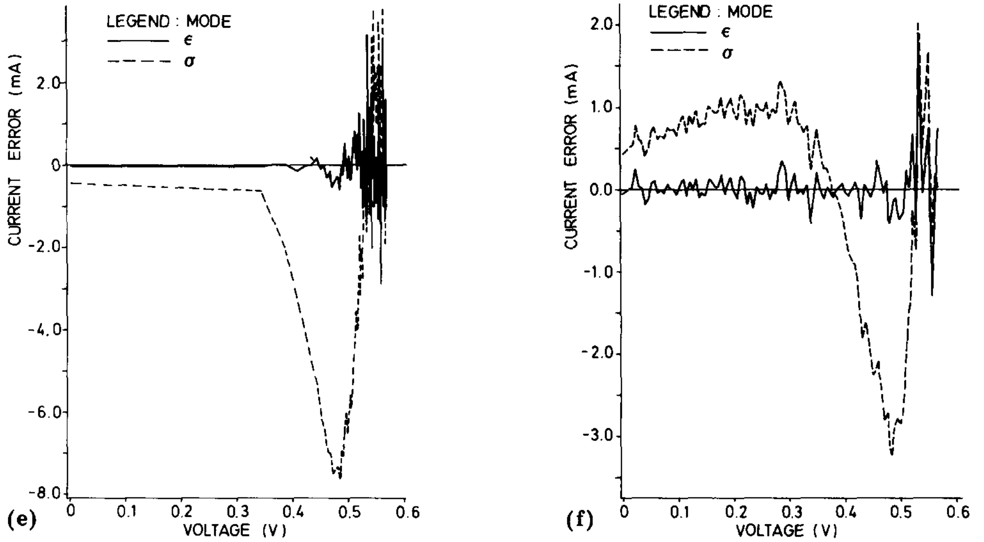

Fig. 3. Current errors ($I_{th} - I_{exp}$) *vs.* voltage for (a) illuminated characteristic IC1, (b) illuminated characteristic IC2, (c) illuminated characteristic IC3, (d) illuminated characteristic IC4, (e) illuminated characteristic IC5, (f) illuminated characteristic IC6.

and the model parameters used were similar to those in the first two sets of data except that R_{sh} was lowered to 20 Ω. The results of using σ and ϵ to recover parameters from these curves are presented in Table 4. The results show that for the first three curves IC7 - IC9 which simulated data collec-

TABLE 4

Model parameters recovered from low illumination characteristics

Curve	*Criterion*	I_{ph} (mA)	$I_{s1} \times 10^{-9}$ (A)	$I_{s2} \times 10^{-5}$ (A)	R_s (mΩ)	R_{sh} (Ω)	*Residue* (%)
Parameter values used in curve generation							
		100.0	1.0	1.0	20.00	20.00	
Extracted parameter values							
IC7	σ	99.99	0.9984	1.000	19.67	20.00	0.102×10^{-3}
	ϵ	99.99	0.9993	1.004	19.81	20.00	0.666×10^{-3}
IC8	σ	99.98	0.9914	1.005	18.52	20.02	0.313×10^{-3}
	ϵ	99.99	0.9999	1.000	19.91	19.99	0.561×10^{-3}
IC9	σ	99.99	0.9978	1.001	19.59	20.00	0.1835×10^{-4}
	ϵ	99.99	0.9988	1.000	19.77	20.00	0.684×10^{-3}
IC10	σ	100.1	1.116	0.9373	40.72	19.76	0.257×10^{-2}
	ϵ	100.0	1.021	0.9887	24.66	19.98	0.189×10^{-1}
IC11	σ	100.5	1.257	0.8564	58.94	19.13	0.901×10^{-2}
	ϵ	99.96	0.9687	1.018	12.18	20.04	0.196×10^{-1}
IC12	σ	100.13	1.097	0.9470	40.58	19.86	0.115×10^{-2}
	ϵ	99.98	0.9878	1.005	17.33	20.01	0.185×10^{-1}

tion by DVMs, parameter recovery was more accurate by minimising ϵ and less accurate by minimising σ. However, the curves which simulated data collected by A/D converters (IC10 to IC12) gave difficulties to both ϵ and σ. In fact, accurate determination of R_s was virtually impossible even though the residue ϵ value was very low and similar to the values obtained in IC4 - IC6. The cause of this difficulty is probably a combination of three factors: (a) the small effect R_s has on the characteristic at low illuminations, (b) the dominating effect of the small R_{sh} on the characteristic and (c) the quantisation noise of the 12-bit A/D system. (The accurate results in the unfudged curves DC1 - DC3 should be noted.) The result of these factors is that R_s cannot be recovered accurately while R_{sh} is recovered with little difficulty. This result demonstrates that for low illumination curves a good fit does not necessarily mean accurate parameter recovery.

5. Conclusions

The commonly used σ minimisation criterion has been shown to be unsuitable for recovering parameters from solar cell I–V characteristics which have been collected by fixed range A/D converter systems. This parameter has been shown to be particularly unreliable in the extraction of parameters from illuminated characteristics. A better alternative minimisation criterion ϵ is proposed which is able to recover model parameters from both dark and illuminated characteristics with different data point distributions. ϵ is also proposed as a more reliable measure of the quality of fit of a theoretical model to experimental data. It has also been demonstrated that parameter recovery at low illuminations can be inaccurate, even when good fits are obtained. This difficulty is more critical for the recovery of R_s, and is due to the inability of the data collection system to measure the small contribution R_s makes to the characteristic at low illuminations.

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