From junction to terminal: Extended reciprocity relations in solar cell operation

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The electronic and optoelectronic reciprocity relations for solar cells in their differential forms are extended to account for series-resistance effects. They are expressed in terms of the terminal current and voltage at any device-operating point. Three new reciprocity relations are derived for the *carrier collection efficiency density*, *current transport efficiency*, and the *current conversion efficiency*, whose definition and use are discussed. The potential usefulness of these relations in electroluminescence imaging, spectral response measurements, and solar cell modeling is outlined.

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I. INTRODUCTION

A *p*-*n* junction solar cell's operation can be summarized succinctly as a series of three processes: (1) the absorption of incident photons leading to the generation of charge carriers (electron-hole pairs), (2) the collection of minority carriers at the junction, and (3) the aggregation of junction currents into the terminal current. Each of these processes incurs a loss in that the output rate of the particles is lower than that at the input. In process (1), a portion of photons is lost as they escape out of the cell or are absorbed but do not create electron-hole pairs. In process (2), some photogenerated minority carriers recombine before they reach the junction. In process (3), the flow of current from the junction to the terminal encounters series resistance, which raises the junction voltage and, thus, the forward-bias current in a way that offsets the photogenerated current. While much work has been performed to describe and to formalize the theoretical basis of the first two processes,[1–6] there has been relatively scant attention given to the third. Part of the reason for this discrepancy is that the determination of the dark-to-photogenerated current ratio is a nonlinear and, thus, a more intractable mathematical problem. The systems of equations governing processes (1) and (2) are not only linear (or approximately so), but also lead to useful reciprocity relations. One of these is the electronic reciprocity of Donolato,[1] which spells out the relationship between the carrier collection efficiency f_c and the dark minority carrier distribution, and the other is the optoelectronic reciprocity proposed by Rau,[4] which ties together the external quantum efficiency (EQE) and the efficiency of the device to operate as a light-emitting diode. The aim of this paper is to bridge the gap in the mathematical description of the solar cell's operation by focusing on the differential changes and responses in the solar cell, for example, the small change in terminal current resulting from incremental changes in generation rates. As will be seen in the following sections, such a linearization in studying the rates of changes will lead to a reciprocity theorem related to the final process in the conversion of light into terminal current, the journey of the majority carriers from the junction to the terminal. The existing theories are also reformulated in relation to the terminal voltage and current, extending their usefulness to any device-operating point.

II. THE CURRENT TRANSPORT AND CURRENT CONVERSION EFFICIENCIES

It is easier to outline the concepts first within the simplified model of a solar cell as a network of diodes interconnected by resistors. Section IV will show that the properties of the defined terms have straightforward analogs within the construct of a three-dimensional solar cell with a more arbitrary geometry. Since the simplified network model is universally used in modeling and the interpretation of mapping data, for practical purposes, the reader may find it sufficient to read the results developed in this section only. Figure 1 shows the generic network model with the interconnecting resistors spanning the xy plane and diodes pointing in the z direction. The operating point of the solar cell can be characterized by the terminal voltage V_T and the light-induced current $I_L(x,y)$ at each node. We define the *current transport efficiency* f_T as

$$f_T(x,y) = \left.\frac{\partial I}{\partial I_L(x,y)}\right|_{\partial V_T=0}, \tag{1}$$

where I is the terminal current of the solar cell. In other words, $f_T(x,y)$ is the fraction of differential light-induced current generated at (x,y), which does not get lost in the form of increased forward-bias current and translates into current at the terminal. The following reciprocity theorem holds for f_T:

$$f_T(x,y) = \left.\frac{\partial V(x,y)}{\partial V_T}\right|_{\delta I_L=0}, \tag{2}$$

where $V(x,y)$ is the local voltage across the diode at node (x,y) in Fig. 1. Equation (2) states that the current transport efficiency $f_T(x,y)$ is equal to the change $V(x,y)$ normalized to the change in terminal voltage with I_L being held constant everywhere. It can be proven by citing the Lorentz reciprocity theorem in linear circuits[7] in the following way. Figure 2(a) shows a linearized representation of the network, in particular, drawing out the terminal and the node at position (x,y). The linearization consists of replacing the diode element at each node with a resistor R_p, whose value is equal to the derivative of the forward-bias current with respect to the diode voltage. In this way, the relationship between small changes in the network voltages and currents is accurate to first order. Figure 2(b) shows the same network but with the

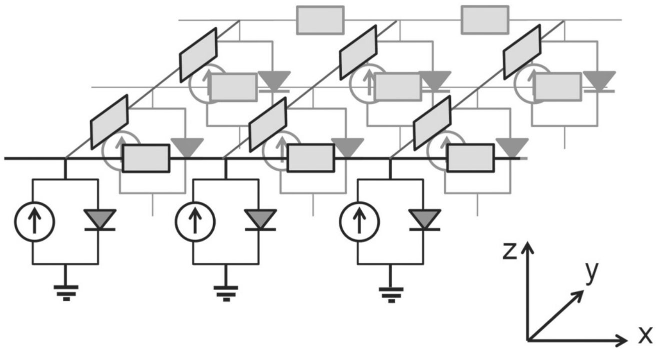

FIG. 1. Network model of the solar cell. Each node consists of the parallel combination of a current source and a diode, which is a generic element describing the local dark current-voltage characteristics. The nodes are connected via resistors (gray rectangles).

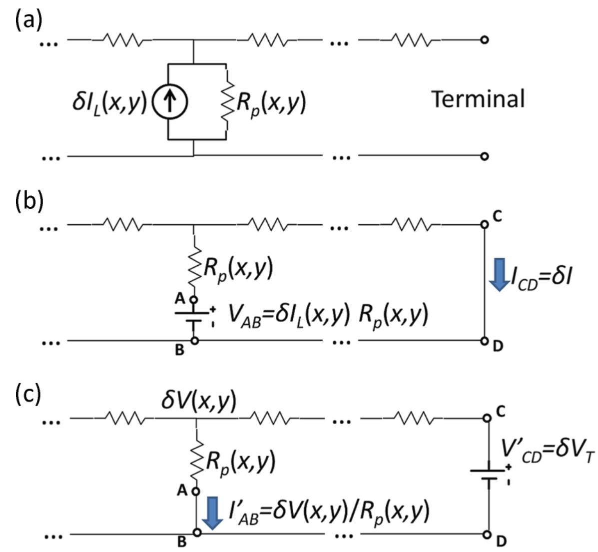

FIG. 2. (Color online) (a) A solar cell network in the linearized representation, valid for small changes and responses, showing the terminal and the diode at position (x,y). (b) The same network with the diode at (x,y) represented by its Thevenin equivalent. The change in terminal voltage δV_T is set to zero, which, in the linearized representation of the network, is indicated by a short circuit at terminal CD. (c) The same network now is excited by a change in terminal voltage δV_T. The change in the light-induced current $I_L(x,y)$ is set to zero, which, in the diagram, is indicated by a short circuit at branch AB.

node at (x,y) represented by its Thevenin equivalent where the parallel configuration of the current source $\delta I_L(x,y)$ and resistor $R_p(x,y)$ has been replaced by the serial connection of a voltage source $V_{AB} = \delta I_L(x,y)R_p(x,y)$ and the resistor $R_p(x,y)$. The change in terminal voltage δV_T is set to zero, which in the linearized representation, is indicated by a short-circuit condition. Therefore, the ratio of $\delta I_L(x,y)$ to the change in terminal current δI in this case would be equal to $f_T(x,y)$. Figure 2(c) shows a different situation where δV_T is nonzero as represented by placing a voltage source at the terminal, and $\delta I_L(x,y)$ is turned off, which in the Thevenin representation, amounts to shorting AB as shown in the figure. Lorentz reciprocity states that

$$V_{AB} I'_{AB} = V'_{CD} I_{CD}, \tag{3}$$

where V_{AB}, I'_{AB}, V'_{CD}, and I_{CD} are as labeled in Fig. 2. Simple substitution of $V_{AB} = \delta I_L(x,y)R_p(x,y)$, $I'_{AB} = \delta V(x,y)/R_p(x,y)$, $V'_{CD} = \delta V_T$, $I_{CD} = \delta I$, and rearrangement results in Eq. (2).

The reciprocity relation (2) closely resembles Donolato's analog, which governs the carrier collection efficiency,[1] which we write here in its differential form[8]

$$\hat{f}_c(x,y,z) = \frac{\partial u(x,y,z)}{\partial u_J(x,y)} = \frac{kT}{q} \exp\left(-\frac{qV(x,y)}{kT}\right) \frac{\partial u(x,y,z)}{\partial V(x,y)}. \tag{4}$$

As in Ref. 8, $\hat{f}_c$ is the differential carrier collection efficiency, defined as the ratio of increment in light-induced current density δj_L to the increment in generation rate δG at point (x,y,z) multiplied by the elementary charge q. The normalized excess carrier density u is defined as [9]

$$u(x,y,z) = \frac{p(x,y,z) - p_0(x,y,z)}{p_0(x,y,z)}, \tag{5}$$

where p is the minority carrier density and p_0 is its value at equilibrium. In Eq. (4), u_J is the value of u evaluated at the p-n junction, which, in the network model, varies with the local diode voltage as $u_J \propto \exp(qV/kT)$ for $qV \gg kT$. Equation (4) relates $\hat{f}_c$ to the rate of change in u with V at the operating point. Obviously, the product of $\hat{f}_c$ and f_T is the overall fraction of photogenerated electron-hole pairs, which is converted into charge flowing out of the terminal. We call this quantity the *current conversion efficiency* $f_I(x,y,z) = \hat{f}_c(x,y,z) f_T(x,y)$. Combining Eqs. (2) and (4) results in the following relationship:

$$f_I(x,y,z) = \frac{kT}{q} \exp\left(-\frac{qV(x,y)}{kT}\right) \frac{\partial u(x,y,z)}{\partial V_T}. \tag{6}$$

Equation (6) can be regarded as a sort of reciprocity relationship for f_I. In Sec. IV, we derive its analog for the general three-dimensional solar cell and examine the underlying assumptions involved in that case.

The external quantum efficiency EQE is traditionally defined as $\partial j_L(x,y)/q\,\partial\phi(x,y,\lambda)$ where $\phi(x,y,\lambda)$ is the photon flux of wavelength λ normal to the cell surface at (x,y). This definition is suitable in most measurements that set the solar cell in short-circuit condition where the terminal current I is very nearly a summation of all the light-induced currents $I_L(x,y)$. But if one were to study the device at some arbitrary operating point, for example, when the cell has a voltage bias, then one must extend the meaning of EQE in terms of the terminal current. We define the *terminal* EQE* as the ratio of the incremental gain in terminal current I to q times the rate of photons incident on the solar cell surface. Following Rau's analysis,[4]

$$\text{EQE}^*(x,y,\lambda) = \int T(x,y|\vec{r})\alpha(\vec{r},\lambda)\hat{f}(\vec{r})dr, \tag{7}$$

where α is the absorption coefficient. The integral in Eq. (7) is carried out along the optical path of a ray normally incident on the surface of the solar cell at (x,y), and $T(x,y|\vec{r})$ denotes the transmission probability for a photon in the ray to traverse from (x,y) to point $\vec{r}$ in the bulk. As in Ref. 4, the emitted photon flux density $\phi_{em}(x,y,\lambda)$ at the cell surface, in addition to the equilibrium emission in the normal direction, is expressed by performing a similar integration along the same optical path,

$$\phi_{em}(x,y,\lambda) = \int T(x,y|\vec{r})\alpha(\vec{r},\lambda)\phi_{bb}(\lambda)u(\vec{r})dr, \tag{8}$$

where $\phi_{bb}(\lambda)$ is Planck's black-body distribution. Differentiating Eq. (8) by V_T, substituting Eq. (6), and then comparing the result to Eq. (7) yields

$$\frac{\partial\phi_{em}(x,y,\lambda)}{\partial(qV_T/kT)} = \mathrm{EQE}^*(x,y,\lambda)\phi_{bb}(\lambda)\exp\left(\frac{qV(x,y)}{kT}\right). \tag{9}$$

Equation (9) is a minor modification to the optoelectronic reciprocity theorem relating the electroluminescence (EL) intensity of a solar cell to its quantum efficiency of Rau,[4] Kirchartz and Rau,[5] and Kirchartz *et al.*[6] As it is expressed in a differential form, it is rigorous, in general, for nonlinear solar cells with injection-level-dependent carrier lifetimes.

III. APPLICATIONS

Having defined f_T, f_I, and EQE* and having cast them in their respective reciprocity relationships (2), (6), and (9) in terms of the terminal voltage V_T, we are now poised to explore the usage of these terms and equations in characterization and modeling. Naturally, these applications deal with the solar cell operating at near the maximum power point at which there are significant dark currents and voltage variations throughout the device.

A. Adaptation of f_T in electroluminescence imaging

Series-resistance mapping of solar cells usually deals with the position-dependent series resistance $R_s(x,y)$, which has been defined variously.[10–15] In this paper, we define R_s as

$$R_s(x,y) = \left.\frac{\delta V_T - \delta V(x,y)}{\delta j(x,y)}\right|_{\delta j_L=0}, \tag{10}$$

where j is the current density at position (x,y) of the network, pointed in the opposite direction as j_L. From Eqs. (10) and (2), one sees that the current transport efficiency f_T and R_s are related through

$$\frac{1}{f_T(x,y)} = 1 + R_s(x,y)\frac{\partial j}{\partial V}(x,y). \tag{11}$$

We propose that f_T can be adapted to supplement R_s for two main reasons. First, f_T has a very intuitive meaning: It is the fraction of incremental light-induced current $\delta I_L(x,y)$ that is translated into terminal current. Second and more importantly, f_T can be easily obtained by comparing two EL images at slightly different bias voltages in a similar approach as that used by Hinken and co-workers.[12] In the luminescence imaging field, the EL intensity at (x,y) is often expressed as[10–12,14–16]

$$\phi_{em}(x,y) = C(x,y)\exp\left(\frac{qV(x,y)}{kT}\right), \tag{12}$$

where $C(x,y)$ is a position-dependent calibration constant. Rearranging Eq. (12) results in the following expression for $V(x,y)$:

$$V(x,y) = \frac{kT}{q}\ln\left(\frac{\phi_{em}(x,y)}{C(x,y)}\right). \tag{13}$$

Differentiating Eq. (13) with respective to V_T and then applying the reciprocity relation (2) results in

$$f_T = \frac{\partial\ \ln\ \phi_{em}(x,y)}{\partial(qV_T/kT)} - \frac{\partial\ \ln\ C(x,y)}{\partial(qV_T/kT)} \approx \frac{\partial\ \ln\ \phi_{em}(x,y)}{\partial(qV_T/kT)}, \tag{14}$$

where the approximation in Eq. (14) can be performed if $C(x,y)$ is not strongly dependent on the injection level. Equation (14) shows that f_T is approximately the rate of change in the logarithm of the EL intensity with respect to qV_T/kT, which is very accessible experimentally. It is similar to Eq. (7) of Ref. 12, except there, R_s is defined differently from this paper (as the authors pointed out, the said equation is also inexact, as it did not account for the dependence of the equivalent resistor values on the terminal voltage).

The ease of determination of f_T is in contrast with the inherent difficulty in extracting R_s for which the methods proposed often make assumptions to trade off speed for generality.[10–16] Moreover, R_s is often presented in the context that the diode network can be transformed into parallel nodes joining only at the terminal. This equivalent circuit representation can mislead one to assume that R_s is a linear element: R_s is, in fact, dependent on the operating point and the way that it is changed. For example, R_s, which is evaluated by changing V_T [as in Eq. (10)] and another, which is evaluated by changing the illumination intensity while holding V_T constant, give rise to different values. Working with f_T removes these confusions and concessions to use assumptions.

B. Voltage-biased spectral response measurement

Typically, the spectral response of nonlinear solar cells (whose effective lifetimes are injection-level dependent) is determined by differential EQE measurements at a certain light bias under short-circuit conditions. The resultant EQE is usually in good consistency with the solar cell's J_{sc}. There might be situations where one would wish to perform this measurement at a certain voltage bias instead. For example, certain types of solar cells, such as thin film *p-i-n* devices, have voltage-dependent spectral responses.[17] By measuring the rate of change in the terminal current with the probe-beam intensity, the EQE* can be deduced. Independent measurements of $\partial V(x,y)/\partial V_T$, either using the EL method described in Sec. III A or using physical probes, yields f_T. Then, the EQE can be obtained by using the approximate relation,

$$\mathrm{EQE} \approx \frac{\mathrm{EQE}^*}{f_T}. \tag{15}$$

Thus, the auxiliary determination of f_T in addition to the voltage-dependent spectral response measurement can account for the fraction of differential light-induced current, which is offset by increased forward-bias current.

C. Solar cell sensitivity analysis

In Ref. 8, a perturbation theory was introduced to relate small changes in a solar cell's maximum power to small changes in device parameters, which affect generation and recombination rates. The theory relies on the deduction of "pseudosource terms" at the points of change and then the application of the reciprocity relation for $\hat{f}_c$, Eq. (4), to calculate the change in diode current resulting from these pseudosource terms. The perturbation theory can be made more rigorous in the presence of series-resistance effects by using the current conversion efficiency f_I and the reciprocity relation (6) so that the change in terminal current rather than diode current is calculated. This approach may not be very relevant within the network model because solving for f_I via Eq. (6) amounts to solving u over the entire spatial extent of the solar cell, which is prohibitively expensive computationally. However, if the subject of study was a small unit cell, then such a method might make sense. The extension of the perturbation theory to cells with significant series resistance will be discussed in detail in a future paper. In the next section, we re-derive the reciprocity relation for f_I in a general form that is applicable not only to the network model, but also to such a unit cell and to three-dimensional solar cells with arbitrary geometries.

IV. RECIPROCITY THEOREM FOR f_T IN THE GENERAL THREE-DIMENSIONAL SOLAR CELL

The network model divided up the solar cell into small diodes, interconnected at essentially the emitter layer only. The model does not consider the cross transport of carriers in the base of one diode to another. Likewise, the reciprocity relation (6), derived in the context of the network model, is not rigorous when such cross transport of carriers has been accounted for. Therefore, we must begin anew from the representation of the solar cell before reconsidering the properties of f_T in this broader case.

A. Division of the junction

Figure 3 illustrates a generic three-dimensional solar cell model. A key feature of the model is the p-n junction (dashed lines), which is treated as a continuum (or several, if there is more than one disconnected junction region) across which the voltage and current-density distributions are properly integrated into the systems of equations governing the device. Formally, we define the solar cell as consisting of an overall surface S and volume V. Internal junction regions are collectively treated as surfaces $\{S_{Jn}\} + \{S_{Jp}\}$, where $\{S_{Jn}\}$ are the n-side depletion edges and $\{S_{Jp}\}$ are the p-side depletion edges. Free surfaces are denoted by $\{S_{\text{free}}\}$, quasineutral regions by $\{V_{Q-N}\}$, and junction depletion regions by $\{V_{\text{dep}}\}$. Each free surface may have parts connected to a terminal, the voltage of which is set to V if the free surface is bounding a p-type quasineutral region, and zero if it is bounding an n-type region. The p-type terminals are denoted by $\{S_p\}$, and n-type terminals are denoted by $\{S_n\}$.

At the junction, we adopt the convention of defining each position by a point $\vec{r}_{Jn} \in \{S_{Jn}\}$ as shown in Fig. 4.

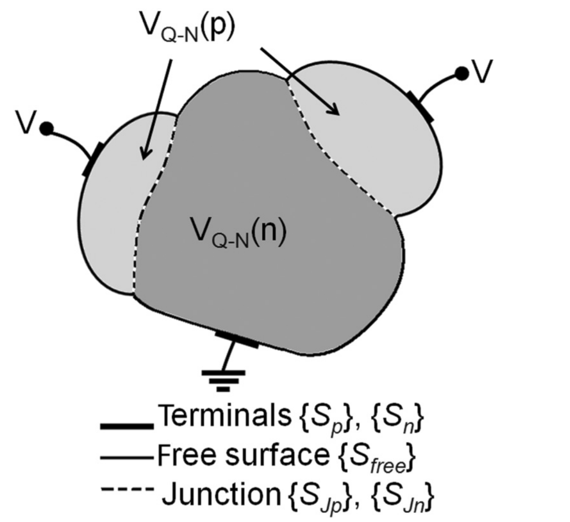

FIG. 3. The general three-dimensional solar cell representation.

Constructing the unit vector $\hat{n}_{n\to p}$ normal to the n-depletion region edge and pointing toward the p region, there is a corresponding point $\vec{r}_{Jp}(\vec{r}_{Jn}) \in \{S_{Jp}\}$, where $\vec{r}_{Jp} - \vec{r}_{Jn} = W_{\text{dep}}(\vec{r}_{Jn})\hat{n}_{n\to p}$ with W_{dep} being the depletion region width. The junction voltage and the current density normal to the junction surface are given by $V_J(\vec{r}_{Jn}) = V_J(\vec{r}_{Jp})$ and $j_J(\vec{r}_{Jn}) = j_J(\vec{r}_{Jp})$, respectively. As a convention, $j_J(\vec{r}_J)$ is positive when it is directed toward the p-type quasineutral region.

Now, consider a point $\vec{r}_{Jn_0} \in \{S_{Jn}\}$. A contiguous neighborhood P_n of $\vec{r}_{Jn_0}$ with a surface area $A(P_n)$ can be defined such that $\vec{r}_{Jn} \in P_n \in \{S_{Jn}\}$. Then, the collection of lines emanating from each point on P_n and normal to $\{S_{Jn}\}$ would trace out another neighborhood P_p of $\vec{r}_{Jp}(\vec{r}_{Jn_0})$ as shown in Fig. 4 with a surface area $A(P_p)$. If $W_{\text{dep}}(\vec{r}_{Jn})$ is very small compared to the radius of curvature of the depletion edge surface at $\vec{r}_{Jn}$, then approximately $A(P_n) = A(P_p)$. u is assumed to be constant along the line between $\vec{r}_{Jp}$ and $\vec{r}_{Jn}(\vec{r}_{Jp})$ on the junction and within the depletion region.

As in the earlier parts of the paper, the solar cell is at some operating point determined by the terminal voltage V_T and a

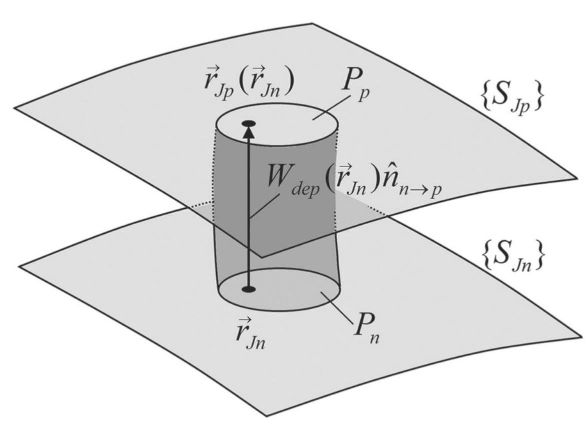

FIG. 4. The convention for junction position and depletion region. Each junction position is defined by a point $\vec{r}_{Jn}$ lying on the n-side depletion region edge. See text for details on the definition of the other terms.

distribution of carrier generation rate $G(\vec{r})$. We would like to deal with the differential system and study the relation between small changes in the cell's internal parameters, for example, δu. Let us consider for a moment the boundary conditions of δu on the depletion region edges to be $\delta u(\vec{r}_{Jn}) \equiv \delta u_J$ when $\vec{r}_{Jn} \in P$ and $\delta u(\vec{r}_{Jn}) = 0$ when $\vec{r}_{Jn} \notin P_n$ on $\{S_{Jn}\}$ and, similarly, $\delta u[\vec{r}_{Jp}(\vec{r}_{Jn})] = \delta u(\vec{r}_{Jn})$ on $\{S_{Jp}\}$. As far as the solution of δu is concerned, this setup is as though the junction region is confined to P_n and P_p, whereas, the regions $\{S_{Jn}\} - P_n$ and $\{S_{Jp}\} - P_p$ are free surfaces with infinite surface recombination velocity. In this case, we may define a *piecewise* differential carrier collection efficiency $\hat{f}_{c,P}(\vec{r})$, which denotes the ratio of the differential generation rate $\delta G(\vec{r})$ to the resultant sum of incremental electron flux on P_p and incremental hole flux on P_n. As the system of equations is linearized, Donolato's reciprocity theorem[1] states that

$$\hat{f}_{c,P}(\vec{r}) = \frac{\delta u_P(\vec{r})}{\delta u_J}, \tag{16}$$

with $\delta u_P(\vec{r})$ denoting the solution of u given the said boundary conditions at the junction and $\delta G(\vec{r}) = 0$ everywhere. Taking it further, we define the *differential carrier collection efficiency density* $\hat{f}_c'(\vec{r},\vec{r}_{Jn_0})$ as the limit of $\hat{f}_{c,P}(\vec{r})/A(P_n)$ when P_n becomes vanishingly small,

$$\hat{f}_c'(\vec{r},\vec{r}_{Jn_0}) \equiv \lim_{A(P_n)\to 0} \frac{\hat{f}_{c,P}(\vec{r})}{A(P_n)} = \lim_{A(P_n)\to 0} \frac{\delta u_P(\vec{r})/\delta u_J}{A(P_n)}. \tag{17}$$

We are now ready to express the differential junction current density $\delta j_J(\vec{r}_{Jn})$ in terms of the newly defined quantities by invoking the superposition principle,

$$\begin{aligned}\delta j_J(\vec{r}_{Jn}) = {} & q\int_V \delta G(\vec{r}) f_c'(\vec{r},\vec{r}_{Jn})d\vec{r} - \lim_{A(P_n)\to 0}[-qDp_0\nabla\delta u_P(\vec{r}_{Jn})\cdot\hat{n}_{n\to p} + qDn_0\nabla\delta u_P(\vec{r}_{Jp})\cdot\hat{n}_{n\to p}] \\ & - q\frac{\partial R_{\text{dep}}}{\partial u}(\vec{r}_{Jn})W_{\text{dep}}(\vec{r}_{Jn})\delta u(\vec{r}_{Jn}) = q\int_V \delta G(\vec{r}) f_c'(\vec{r},\vec{r}_{Jn})d\vec{r} \\ & - (-qDp_0\nabla f_c'(\vec{r},\vec{r}_{Jn})\cdot\hat{n}_{n\to p}|_{\vec{r}\to\vec{r}_{Jn}} + qDn_0\nabla f_c'(\vec{r},\vec{r}_{Jn})\cdot\hat{n}_{n\to p}|_{\vec{r}\to\vec{r}_P})\delta u(\vec{r}_{Jn}) \\ & - q\frac{\partial R_{\text{dep}}}{\partial u}(\vec{r}_{Jn})W_{\text{dep}}(\vec{r}_{Jn})\delta u(\vec{r}_{Jn})\end{aligned} \tag{18}$$

where D is the minority carrier diffusion coefficient. The first term is the differential light-induced current density, which we denote by $j_L(\vec{r}_J)$. The second term is the differential diffusion current density, and we denote the term in the brackets by $\partial j_D/\partial u_J(\vec{r}_J)$. The last term in Eq. (18) accounts for the recombination current in the depletion region with R_{dep} being the average recombination rate between the depletion edge points $\vec{r}_{Jn}$ and $\vec{r}_{Jp}(\vec{r}_{Jn})$. Thus far, we have derived the junction-position-specific analogs of the carrier collection efficiency, dark diffusion current, and solar cell diode equation.

B. Majority carrier current approximation

The network model makes an important implicit assumption that the current flowing in the network is entirely a majority carrier current, which is true for all practical purposes. The reason that this assumption is important is that voltage drops due to ohmic loss are actually proportionate to the majority carrier current density $\pm q\vec{\Gamma}_{\text{maj}}(\vec{r}_J)$ rather than the total current density $\vec{j}(\vec{r}_J)$, but the latter quantity is far easier to keep track of. In the general three-dimensional solar cell, we must state this approximation explicitly. For this purpose, we define the *altered Fermi level* $\bar{\varepsilon}_f$ in terms of the electron and hole Fermi levels ε_{fn} and ε_{fp} as

$$\bar{\varepsilon}_f = \begin{cases} \varepsilon_{fn}, & n \text{ regions}, \\ -\varepsilon_{fp}, & p \text{ regions}. \end{cases} \tag{19}$$

The advantages to the definition of $\bar{\varepsilon}_f$ will be apparent in its use and in later discussions. The approximation to the current density becomes

$$\vec{j} = \begin{cases} -q\vec{\Gamma}_{\text{maj}} = \sigma\nabla\bar{\varepsilon}_f, & n \text{ regions}, \\ q\vec{\Gamma}_{\text{maj}} = -\sigma\nabla\bar{\varepsilon}_f, & p \text{ regions}, \end{cases} \tag{20}$$

where σ is the conductivity whether it be of a semiconductor, metal, or interface. Equation (20) is a poor approximation near the junctions but improves with distance from the depletion region edges and then becomes almost exact in the highly conductive layers of the device, which carries current laterally. The accuracy of relying on Eq. (20) for resistive loss calculations then depends on where most of the resistive losses occur. For example, Fig. 5(a) shows the majority carrier current densities, and Fig. 5(b) shows the total current densities in a bifacial solar cell. If the lateral distance that the current must flow to reach the metal contact is long, then most of the resistive voltage drops would occur along the heavily doped regions where the total and majority carrier current densities are almost equal. In this case, the penalty in using Eq. (20) is small. On the other hand, for problems in which resistive loss in the absorber region of the solar cell is significant, there will be an overestimation in the majority carrier current density and, thus, the resistive loss if Eq. (20) was used.

Equation (20) is advantageous in that it enables the decoupling of the equations governing majority and minority carriers. Under this approximation, the continuity equation for majority carriers in a quasineutral region reduces simply to the current continuity equation,

$$\nabla\cdot(\sigma\nabla\bar{\varepsilon}_f) = \pm\nabla\cdot\vec{j} = 0. \tag{21}$$

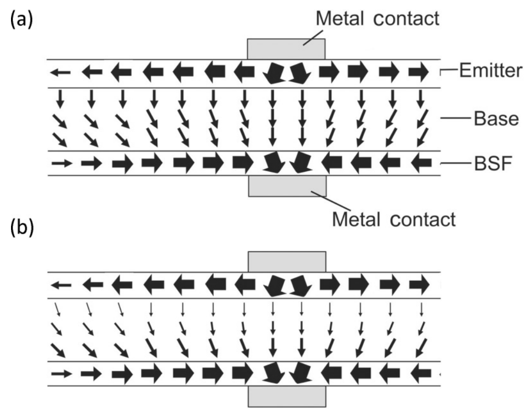

FIG. 5. (a) Total current densities and (b) majority carrier current densities in a bifacial solar cell, as represented by emitter, base, and back-surface-field (BSF) layers and metal contacts. The two types of current densities are dissimilar near the junction but become almost equal in the heavily doped regions.

At parts of the free surface not connected to the terminal, the boundary condition is

$$\sigma \nabla \bar{\varepsilon}_f \cdot \hat{n} = \pm \vec{j} \cdot \hat{n} = 0, \tag{22}$$

where $\hat{n}$ is a unit vector at the boundary point. Note that both Eqs. (21) and (22) are free of generation and recombination terms, which is a tremendous simplification, because if the recombination terms were present, then these equations would also depend on u. Meanwhile, at the depletion region edges, we may express the differential junction current density in terms of $\delta\bar{\varepsilon}_f$,

$$\sigma \nabla\, \delta\bar{\varepsilon}_f \cdot \hat{n} = \delta j_J(\vec{r}_{Jn}) = \delta j_L - \left(\frac{\partial j_D}{\partial u_J} + q\frac{\partial R_{\text{dep}}}{\partial u} W_{\text{dep}}\right) \times \exp\left(\frac{qV_J}{kT}\right) \frac{\delta\bar{\varepsilon}_f(\vec{r}_{Jn}) + \delta\bar{\varepsilon}_f(\vec{r}_{Jp})}{kT}, \tag{23}$$

where we have substituted the following expression for $\delta u(\vec{r}_{Jn})$:

$$\delta u(\vec{r}_{Jn}) = \delta\left[\exp\left(\frac{qV_J}{kT}\right)\right] = \frac{q\,\delta V_J}{kT}\exp\left(\frac{qV_J}{kT}\right) = \exp\left(\frac{qV_J}{kT}\right)\frac{\delta\bar{\varepsilon}_f(\vec{r}_{Jn}) + \delta\bar{\varepsilon}_f(\vec{r}_{Jp})}{kT}. \tag{24}$$

In Eq. (23), $\hat{n}$ is a unit vector at the boundary point, pointed *outward* from the quasineutral region of interest. Finally, at the terminals, the boundary condition for $\delta\bar{\varepsilon}_f$ is

$$\delta\bar{\varepsilon}_f = \begin{cases} q\ \delta V_T/2, & n\ \text{terminals}, \\ q\ \delta V_T/2, & p\ \text{terminals}. \end{cases} \tag{25}$$

Physically, the condition (25) can be achieved by setting the voltage at p-type terminals to be $\delta V_T/2$ and the voltage at n-type terminals to be $-\delta V_T/2$. The solar cell differential terminal current δI can be found by integrating the current density at either set of terminals $\{S_p\}$ or $\{S_n\}$, taking the outward flux as positive if integrating at $\{S_p\}$ and the inward flux as positive if integrating at $\{S_n\}$. Alternatively, δI can be expressed as

$$\delta I = \frac{1}{2}\left(\int_{\{Sp\}} \vec{j}\cdot\hat{n}\, d\vec{r} - \int_{\{Sn\}} \vec{j}\cdot\hat{n}\, d\vec{r}\right) = \frac{1}{2}\int_S -\sigma\nabla\delta\bar{\varepsilon}_f \cdot \hat{n}\, d\vec{r}. \tag{26}$$

Applying Stoke's theorem to obtain the integral form of Eq. (26), we can rewrite δI as

$$\delta I = -\frac{1}{2}\int_{\{V_{Q-N}\}} \nabla\cdot(\sigma\nabla\delta\bar{\varepsilon}_f)d\vec{r} + \frac{1}{2}\int_{\{S_{Jn}\}} \sigma\nabla\ \delta\bar{\varepsilon}_f\cdot\hat{n}\, d\vec{r} + \frac{1}{2}\int_{\{S_{Jp}\}} \sigma\nabla\ \delta\bar{\varepsilon}_f\cdot\hat{n}\, d\vec{r} = \int_{\{S_{Jn}\}} \delta j_J(\vec{r}_{Jn})d\vec{r}. \tag{27}$$

Equation (27) is intuitively clear: The solar cell current is the integral of the current densities over the junction surfaces.

C. Reciprocity relations

We define the current transport efficiency $f_T(\vec{r}_{Jn})$ as the ratio of majority carriers reaching the terminals to those emanating from the junction points $\vec{r}_{Jn} \in \{S_{Jn}\}$ and $\vec{r}_{Jp}(\vec{r}_{Jn}) \in \{S_{Jp}\}$. The reciprocity theorem for f_T can be derived by following exactly the same line of argument as that which governs the carrier collection efficiency f_c.[2] Here, we state it without proof. Suppose $\delta\bar{\varepsilon}_{fD}$ is the solution to the altered Fermi level with terminal boundary condition (25) and $\delta j_L = 0$ everywhere at the junction. In the general case, the reciprocity relation can be applied to the current equation (27) by replacing δj_J with $\delta j_L \delta\bar{\varepsilon}_{fD}/(qV_T/2)$ everywhere in the integrals,

$$\begin{aligned}\delta I &= \int_{\{S_{Jn}\}} \delta j_J(\vec{r}_{Jn})d\vec{r}_{Jn} \\ &= \frac{1}{2}\left(\int_{\{S_{Jp}\}} \delta j_J d\vec{r}_{Jn} + \int_{\{S_{Jn}\}} \delta j_J d\vec{r}_{Jn}\right) \\ &= \frac{1}{2}\left(\int_{\{S_{Jp}\}} \delta j_L \frac{\delta\bar{\varepsilon}_{fD}(\vec{r}_J)}{q(\delta V_T/2)} d\vec{r}_{Jn} + \int_{\{S_{Jn}\}} \delta j_L \frac{\delta\bar{\varepsilon}_{fD}(\vec{r}_J)}{q(\delta V_T/2)} d\vec{r}_{Jn}\right) \\ &= \frac{1}{2}\left(\int_{\{S_{Jn}\}} \delta j_L \frac{\delta\bar{\varepsilon}_{fD}(\vec{r}_{Jp}) + \delta\bar{\varepsilon}_{fD}(\vec{r}_{Jn})}{q\,(\delta V_T/2)} d\vec{r}_{Jn}\right) \\ &= \int_{\{S_{Jn}\}} \delta j_L \frac{\delta V_{JD}}{\delta V_T} d\vec{r}_{Jn},\end{aligned} \tag{28}$$

where δV_{JD} is the change in junction voltage corresponding to the solution of $\delta\bar{\varepsilon}_{fD}$. From Eq. (28), it is apparent that the carrier transport efficiency $f_T(\vec{r}_{Jn})$ is the coefficient to δj_L in the last integral,

$$f_T(\vec{r}_{Jn}) = \frac{\delta V_{JD}(\vec{r}_{Jn})}{\delta V_T}. \tag{29}$$

Equation (29) is identical to the reciprocity theorem (2) for f_T in the network model. Expanding the source term δj_L in Eq. (28) by substituting in the first integral term in Eq. (18), one obtains

$$\begin{aligned}\delta I &= \int_{\{S_J\}} \delta j_L f_T(\vec{r}_{Jn}) d\vec{r}_{Jn} \\ &= q \int_{\{S_J\}} f_T(\vec{r}_{Jn}) \int_V \delta G(\vec{r}) \hat{f}'_c(\vec{r},\vec{r}_{Jn}) d\vec{r}\, d\vec{r}_{Jn} \\ &= q \int_V \delta G(\vec{r}) \left(\int_{\{S_J\}} f_T(\vec{r}_{Jn}) \hat{f}'_c(\vec{r},\vec{r}_{Jn})\, d\vec{r}_{Jn} \right) d\vec{r}. \end{aligned} \tag{30}$$

Whereupon, it is apparent that the current conversion efficiency $f_I(\vec{r})$ is the coefficient to δG in the bracketed term of Eq. (30),

$$f_I(\vec{r}) = \int_{\{S_J\}} f(\vec{r}_{Jn}) \hat{f}'_c(\vec{r},\vec{r}_{Jn}) d\vec{r}_{Jn}. \tag{31}$$

Thus, we have derived $f_I(\vec{r})$ based on $f_T(\vec{r}_{Jn})$ and $\hat{f}'_c(\vec{r},\vec{r}_{Jn})$. We can rewrite Eq. (31) by substituting the expression for $f_T(\vec{r}_{Jn})$ by its reciprocity relation (29),

$$\begin{aligned} f_I(\vec{r}) &= \frac{1}{q\ \delta V_T/kT} \int_{\{S_{Jn}\}} \frac{q\ \delta V_{JD}(\vec{r}_{Jn})}{kT} \hat{f}'_c(\vec{r},\vec{r}_{Jn})\, d\vec{r}_{Jn} \\ &= \frac{\delta \bar{u}_D(\vec{r})}{q\ \delta V_T/kT}. \end{aligned} \tag{32}$$

Here, we have denoted the integral by a new variable $\delta\bar{u}_D$ that would cast Eq. (32) into a form which resembles a reciprocity relation for f_I. From Eq. (17) and the superposition principle, $\delta\bar{u}_D$ can be interpreted as the solution of δu given $\delta G = 0$ and the boundary condition,

$$\delta\bar{u}_D(\vec{r}_{Jn}) = \frac{q\ \delta V_{JD}(\vec{r}_{Jn})}{kT} = \frac{\delta\bar{\varepsilon}_{fD}(\vec{r}_{Jn}) + \delta\bar{\varepsilon}_{fD}(\vec{r}p)}{kT}. \tag{33}$$

Note that Eq. (33) is different from the boundary condition (24). Therefore, $\delta\bar{u}_D$ has no physical bearing with the real distribution of δu when the solar cell experiences a change δV_T in terminal voltage. There is nothing preventing one, however, from utilizing $\delta\bar{u}_D$ as a mathematical tool to deduce f_I. We have, thus, deduced a procedure whereby f_I can be found by solving a set of linear differential equations governing $\{\delta\bar{u}_D, \delta\bar{\varepsilon}_{fD}\}$. Other than the ancillary boundary condition (33), the volumetric differential equation and free surface boundary equation for $\delta\bar{u}_D$ remain identical to those that apply to δu with $\delta G = 0$. To recapitulate, the reciprocity theorem for f_I and the full system of linear equations for $\{\delta\bar{u}_D, \delta\bar{\varepsilon}_{fD}\}$ are recited below:

(1) Reciprocity relation,

$$f_I(\vec{r}) = \frac{\delta\bar{u}_D(\vec{r})}{q\ \delta V_T/kT} \qquad \text{[Eq.(32)]}.$$

(2) Continuity equations in the quasineutral regions $\{V_{Q-N}\}$,

$$\nabla\cdot(Dp_0\nabla\delta\bar{u}_D) + p_0\delta\bar{u}_D\hat{\tau}^{-1} - \int_{V_{Q-N}} \kappa_{eq}(\vec{r},\vec{r}')\delta\bar{u}_D(\vec{r}')d\vec{r}' = 0 \qquad \text{[Eq.(14) of Ref.8]},$$

$$\nabla\cdot(\sigma\nabla\delta\bar{\varepsilon}_{fD}) = 0 \qquad \text{[Eq.(21)]}.$$

(3) Boundary conditions at the free surfaces $\{S_{\text{free}}\}$,

$$Dp_0\nabla\ \delta\bar{u}_D\cdot\hat{n} - p_0\delta\bar{u}_D\hat{S} = 0 \qquad \text{[Eq.(15) of Ref.1]},$$

$$\sigma\nabla\ \delta\bar{\varepsilon}_{fD}\cdot\hat{n} = 0 \qquad \text{[Eq.(18)]}.$$

(4) Boundary conditions at the junction $\{S_J\}$

$$\begin{aligned}\sigma\nabla\ \delta\bar{\varepsilon}_f\cdot\hat{n} &= \delta j_J(\vec{r}_{Jn}) \\ &= -\left(\frac{\partial j_D}{\partial u_J} + q\frac{\partial R_{\text{dep}}}{\partial u}W_{\text{dep}}\right)\exp\left(\frac{qV_J}{kT}\right)\frac{\delta\bar{\varepsilon}_f(\vec{r}_{Jn}) + \delta\bar{\varepsilon}_f(\vec{r}_{Jp})}{kT} \qquad \text{[Eq.(31)]},\end{aligned}$$

with

$$\frac{\partial j_D}{\partial u_J}(\vec{r}_{Jn}) = -qDp_0\frac{\nabla\ \delta\bar{u}_D(\vec{r}_{Jn})}{\delta\bar{u}_D(\vec{r}_{Jn})}\cdot\hat{n}_{n\to p}|_{\vec{r}\to\vec{r}_{Jn}} + qDn_0\frac{\nabla\ \delta\bar{u}_D(\vec{r})}{\delta\bar{u}_D(\vec{r}_{Jp})}\cdot\hat{n}_{n\to p}|_{\vec{r}\to\vec{r}_{Jn}} \qquad \text{[Eq.(18)]},$$

$$\delta\bar{u}_{DJ}(\vec{r}_J) = \frac{\delta\bar{\varepsilon}_{fD}(\vec{r}_{Jn}) + \delta\bar{\varepsilon}_{fD}(\vec{r}_{Jp})}{kT} \qquad \text{[Eq.(31)]}.$$

(5) Boundary condition for $\delta\bar{\varepsilon}_f$ at the terminals,

$$\delta\bar{\varepsilon}_{fD} = q\ \delta V/2 \qquad \text{[Eq.(25)]}.$$

Equation (32) is strictly not a physically tractable reciprocity relation because $\delta\bar{u}_D$ is a mathematical construct and not a physical quantity. However, within the network model, one can show that $\delta\bar{u}_D = \delta u\ \exp[-qV(x,y)/kT]$, and Eq. (32) reduces to Eq. (6), which has a stronger physical grounding. Moreover, as mentioned in Sec. III C, the mathematical procedure to find f_I by solving one set of linear differential equations is a useful computational tool. The sensitivity of a solar cell's maximum power to any set of changes in device parameters can be computed very quickly once f_I has been solved.

V. CONCLUSIONS

Dealing with small changes and responses in a solar cell leads to the linearization of the entire system of equations from ones which describe the absorption of photons to ones which describe the flow of majority carrier current to the terminal. The reciprocity theorem for the carrier transport efficiency f_T completes the overall description of the solar cell's ability to convert photons into the terminal current and its ability to operate in reverse as a light-emitting diode, accumulating in a slightly modified version of Rau's optoelectronic reciprocity relation. While a reciprocity theorem for the carrier conversion efficiency f_I exists, in its general form, it is, however, not in terms of a physically tractable quantity. Nevertheless, it is a useful mathematical theorem related to the perturbation theory for solar cell efficiency. In experimental fields, we hope that f_T, f_I, and EQE* will be adapted and will be used in series-resistance mapping and measurements of differential responses at large voltage biases for their desirable properties in accounting for current loss due to increases in forward-bias current. In particular, the ease at which f_T can be determined by comparing two EL images at incrementally different voltage biases and its physically intuitive meaning should merit consideration of its usage in EL imaging.

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