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Chaos Game Optimization: a novel metaheuristic algorithm

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Abstract
In this paper, a novel metaheuristic algorithm called Chaos Game Optimization (CGO) is developed for solving optimization problems. The main concept of the CGO algorithm is based on some principles of chaos theory in which the configuration of fractals by chaos game concept and the fractals self-similarity issues are in perspective. A total number of 239 mathematical functions which are categorized into four different groups are collected to evaluate the overall performance of the presented novel algorithm. In order to evaluate the results of the CGO algorithm, three comparative analysis with different characteristics are conducted. In the first step, six different metaheuristic algorithms are selected from the literature while the minimum, mean and standard deviation values alongside the number of function evaluations for the CGO and these algorithms are calculated and compared. A complete statistical analysis is also conducted in order to provide a valid judgment about the performance of the CGO algorithm. In the second one, the results of the CGO algorithm are compared to some of the recently developed fractal- and chaos-based algorithms. Finally, the performance of the CGO algorithm is compared to some state-of-the-art algorithms in dealing with the state-of-the-art mathematical functions and one of the recent competitions on single objective real-parameter numerical optimization named “CEC 2017” is considered as numerical examples for this purpose. In addition, a computational cost analysis is also conducted for the presented algorithm. The obtained results proved that the CGO is superior compared to the other metaheuristics in most of the cases.

Keywords Metaheuristic · Statistical analysis · Chaos Game Optimization

1 Introduction

Most of the design problems in the nature can be considered as optimization problems which request some proper optimization techniques and algorithms to be dealt with. In these days, the design problems have become extremely complex in which the classical optimization algorithms based on the mathematical principles are incapable of providing some satisfactory results in a reasonable period of time. The gradient-based algorithms

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which utilize the gradient of the objective function for configuration of the optimization problem is one of these mathematical algorithms. Over the past few decades, dealing with the deficiencies of classical optimization algorithms and introducing new efficient optimization algorithms have been of great concern. Based on the recent advances of the technology, there is an increasing interest in introducing new optimization algorithms which prepare a high efficiency, great accuracy and increased speed rate in dealing with difficult optimization problems. Besides, some other concerns in dealing with other specific issues such as the local optima problems alongside the non-smoothness and non-convexity of the search spaces have been of great concerns in this regard.

These concerns about the optimization algorithms have led the researchers and experts to present new algorithms for solving different optimization problems called "Metaheuristic". Glover (1986) firstly proposed this term in 1986 which is comprised of one main word (Heuristics) and a suffix (Meta) which have origins from the Greek words. The term "heuristic" comes from the "heuriskein" which is an old Greek word with a meaning of discovering new rules (strategies) in dealing with different problems while the term "meta" denotes on some upper level methodologies in nature. The metaheuristics are some special kinds of solution techniques which use some higher-level strategies to perform a searching process considering some special capabilities (such as avoiding local optimum results) for finding appropriate solutions.

As presented by Sörensen et al. (2018), the history of utilizing metaheuristics as the solution algorithms for dealing with real world problems can be categorized into five distinct periods. In the first period which is named as the "pre-theoretical" period (until 1940), there was not any formal presentation of heuristics and metaheuristics algorithms. Despite that, these algorithms had been used for solving some simple optimization problems in this period. In the second period or "early" period, from 1940 to 1980, some studies had been conducted on heuristics which was the first formal introduction in this field. In the third period, so-called "method-centric" period (1980–2000), different metaheuristics had been proposed and developed for specific applications which extended the field of heuristics and metaheuristics. Started from 2000 as the "framework-centric" period, developing and applying metaheuristics as an important frameworks of many algorithms has been considered widely. In the fifth or last period, the "scientific" or "future" period, the design and introduction of different new metaheuristics will turn into a matter of science rather than art. A brief summary of these historical periods is presented in Table 1.

On the other hand, four classifications can be made in terms of these algorithms inspiration as "evolutionary algorithms", "swarm intelligence-based algorithms", "physical laws-based approaches" and "humans and animals life style-based techniques". A brief summary of these metaheuristic algorithms is presented in Table 2 while some other challenges in developing, upgrading or hybridizing these algorithms have also been achieved (Talatahari and Azizi 2020; Azizi et al. 2019a, b, 2020; Talatahari et al. 2019).

In this paper, a novel metaheuristic so-called Chaos Game Optimization (CGO) is proposed. The main concept of the CGO algorithm is based on some principles of chaos theory in which the configuration of fractals by chaos game methodology alongside the fractals self-similarity issues are in perspective. The fractals have been utilized as the main or sub concept of different metaheuristic algorithms such as Stochastic Fractal Search (SFS) algorithm proposed by Salimi (2015), Fractal-Based Algorithm (FBA) developed by Kaedi (2017), Fractal Decomposition-Based Algorithm (FDA) presented by Nakib et al. (2017), and Fractal Triangle Search (FTS) algorithm proposed by Rodrigues et al. (2018). In addition, some other improved versions of metaheuristic algorithms have been proposed in which the chaos theory is embedded in the general formulation of these algorithms with

Table 1 The brief summary of historical periods of metaheuristics evolution

No.	Name	Duration	Major concern
1	Pre-theoretical	Until 1940	No formal presentation with limited applications
2	Early	1940–1980	Heuristics was formally introduced and discussed
3	Method-centric	1980–2000	Multiple metaheuristics had been proposed and developed for specific applications
4	Framework-centric	2000 until now	The methodology of utilizing metaheuristics as the frameworks alongside the algorithms had been successfully presented
5	Scientific or future	Future	The design and introduction of different new metaheuristics will turn into a matter of science rather than art

Table 2 The brief summary of the metaheuristic algorithms

Classification	Metaheuristic algorithm	Main references
Evolution	Memetic algorithm (MA)	Moscato (1989)
	Genetic algorithm (GA)	Holland (1992)
	Genetic programming (GP)	Koza (1992)
	Differential evolution (DE)	Storn and Price (1997)
	Evolution strategies (ES)	Beyer and Schwefel (2002)
	Biogeography-based optimizer (BBO)	Simon (2008)
	Covariance matrix adaptation evolution strategy (CMA-ES)	Hansen et al. (2003)
Swarm intelligence	Particle swarm optimization (PSO)	Eberhart and Kennedy (1995)
	Ant colony optimization (ACO)	Dorigo et al. (1996)
	Artificial bee colony (ABC)	Basturk (2006)
	Cat swarm optimization (CSA)	Chu et al. (2006)
	Firefly algorithm (FA)	Yang (2010a)
	Krill Herd (KH) algorithm	Gandomi and Alavi (2012)
	Slap swarm algorithm (SSA)	Mirjalili et al. (2017)
	Yellow Saddle Goatfish Algorithm (YSGA)	(Zaldivar et al. 2018)

Table 2 (continued)

Classification	Metaheuristic algorithm	Main references
Physical laws	Simulated annealing (SA)	Kirkpatrick et al. (1983)
	HARMONY search (HS)	Geem et al. (2001)
	Big-Bang Big-Crunch (BBBC)	Erol and Eksin (2006)
	Small-world optimization algorithm (SWOA)	Du et al. (2006)
	Central force optimization (CFO)	Formato (2007)
	Magnetic optimization algorithm (MOA)	Tayarani-N and Akbarzadeh-T (2008)
	Gravitational search algorithm (GSA)	Rashedi et al. (2009)
	Charged system search (CSS)	Kaveh and Talatahari (2010)
	Galaxy-based search algorithm (GBSA)	Shah-Hosseini (2011)
	Artificial chemical reaction optimization algorithm (ACROA)	Alatas (2011)
	Curved space optimization (CSO)	Moghaddam et al. (2012)
	Ray optimization algorithm (ROA)	Kaveh and Khayatazad (2012)
	Black hole (BH) algorithm	Hatamlou (2013)
	Colliding bodies optimization (CBO)	Kaveh and Mahdavi (2014)
	Multi verse algorithm (MVO)	Mirjalili et al. (2016)

Table 2 (continued)

Classification	Metaheuristic algorithm	Main references
Life style	Bees algorithm (BA)	Pham et al. (2005)
	Imperialistic competitive algorithm (ICA)	Atashpaz-Gargari and Lucas (2007)
	Cuckoo search algorithm (CSA)	Yang and Deb (2009)
	Teaching–learning-based optimization (TLBO)	Rao et al. (2011)
	Grey wolf optimizer (GWO)	Mirjalili et al. (2014)
	Symbiotic organisms search (SOS)	Cheng and Prayogo (2014)
	Moth-flame optimization (MFO)	Mirjalili (2015)
	Sine cosine algorithm (SCA)	Mirjalili (2016)
	Whale optimization algorithm (WOA)	Mirjalili and Lewis (2016)

different purposes. Some of these approaches are Chaos-embedded Particle Swarm Optimization (C-PSO) (Alatas et al. 2009), Chaotic Krill Herd (C-KH) (Wang et al. 2014), Firefly Algorithm (FA) with chaos (Gandomi et al. 2013a), Chaotic Whale Optimization Algorithm (C-WOA) (Kaur and Arora 2018), Chaotic Harmony Search (C-HS) (Alatas 2010), Chaotic Grey Wolf Optimizer (C-GWO) (Yu et al. 2016), Chaotic Slap Swarm Algorithm (C-SSA) (Sayed et al. 2018), Imperialistic Competitive Algorithm (ICA) combined with chaos (Talatahari et al. 2012), Chaos-based Differential Evolution (C-DE) (Liang et al. 2006), Chaotic enhanced Colliding Bodies Optimization (C-CBO) (Kaveh et al. 2018), Chaotic Teaching–Learning-Based Optimization (C-TLBO) (He et al. 2016), Cuckoo Search Algorithm (CSA) with chaotic maps (Wang and Zhong 2015), Chaos-integrated Symbiotic Organisms Search (C-SOS) (Saha and Mukherjee 2018), Chaotic-based Big-Bang Big-Crunch (C-BB-BC) (Jordehi 2014), Chaos-enhanced Accelerated PSO (CA-PSO) (Gandomi et al. 2013b), Charged System Search (CSS) with chaos (Talatahari et al. 2011), and Chaotic Swarming of Particles (CSP) (Kaveh et al. 2014). Based on the provided literature review, it should be noted that the methodology of the CGO algorithm is completely different from the previous works. In the CGO algorithm, the chaos game theory is utilized as the main concept of the algorithm and the general formulation of the algorithm is based on the game theory.

In order to evaluate the performance of the CGO algorithm, a total number of 239 mathematical functions which are categorized into four different groups are collected. These mathematical functions have different characteristics and have been categorized based on the dimensions of the variables. The first group is consisting of 117 functions which have 2–10 dimensions, each of the second and third groups include 58 functions which have 50 and 100 dimensions, respectively and the fourth group is consisting of 6 composite and hybrid functions. In order demonstrate the capability of the novel CGO algorithm in dealing with different optimization problems, three comparative analysis with different characteristics are conducted. In the first approach, six different metaheuristic algorithms are selected from the literature while the minimum, mean and standard deviation values alongside the number of function evaluations for the CGO and these algorithms for a tolerance of 1×10^{-12} are calculated and compared. A complete statistical analysis is also conducted by utilization of the Kolmogorov–Smirnov (K–S) test, Mann–Whitney U (M–W) test, Wilcoxon signed-rank (W) test, Kruskal–Wallis (K–W) test and the post-hoc (P-H) analysis in order to provide a valid judgment on the performance of the novel CGO algorithm. As the second comparative step, the results of the CGO algorithm are compared to the algorithms which has been developed based on the fractals concept, containing three metaheuristics which use the chaos theory in their main framework and two other metaheuristics which have been recognized as the most competitive approaches in recent years. In the third step, the performance of the CGO algorithm is compared to some state-of-the-art algorithms in dealing with difficult mathematical functions of “CEC 2017” competition (Awad et al. 2016). In addition, a computational cost and complexity analysis is also conducted for the CGO algorithm by utilization of the “Big O notation”. A brief summary of this paper is as follows:

In Sect. 2, the inspiration of the proposed CGO algorithm alongside the mathematical model of the optimization algorithm based on the new algorithm are presented. In Sect. 3, some mathematical functions with different characteristics are presented for further utilization for testing the presented metaheuristic algorithm as well as some other approaches. In Sect. 4, a brief explanation of the selected metaheuristic algorithms are presented. The results are presented in Sect. 5 while a comprehensive statistical analysis is conducted in Sect. 6. Section 7 compares the results of the CGO algorithm with fractal- and chaos-based approaches. In Sect. 8, the results of the CGO algorithm for the CEC 2017 competition problems are presented. In Sect. 9, the results of the computational cost and complexity analysis for the CGO algorithm are presented and finally, the main findings of this paper including the conclusion alongside the suggestions for the future challenges are presented accordingly in Sect. 10.

2 Chaos Game Optimization (CGO)

In this section, the inspiration of the novel metaheuristic algorithm (CGO) and its mathematical model are presented.

2.1 Inspiration

Chaos theory is a branch of mathematics concentrating on the specific characteristics of dynamical systems which are extremely sensitive to initial conditions. Considering the randomness of these dynamical systems, chaos theory denotes on the existence of some primary patterns such as similar loops, repeated templates, fractals and multiple sub-systems in the behavior of these systems which represent them as self-similar and self-organized dynamical systems. The chaos theory demonstrates that some small changes in the initial conditions of a dynamical system will result in some extreme differences in the upcoming conditions of these systems due to the dependence of them to the initial conditions. Based on this theory, the present state of a system could determine the later state of this system while the approximate present state of a system does not approximately determine the later state of this system.

Most of the chaotic processes have graphical shapes of fractals. In mathematics, a fractal is a subset of Euclidian space in which a specific geometric pattern is repeated in multiple scales. Fractals have approximately similar shapes in different scales which represent them as self-similar systems. One of the famous fractals is the Mandelbrot set which represent an elaborate infinitive boundary in which multiple recursive details are progressively demonstrated in different scales. In Fig. 1, the self-similarity of Mandelbrot set is presented in different scales.

In mathematics, the chaos game is the methodology of creating fractals utilizing an initial polygon shape and a randomly selected initial point. The main purpose is to create a sequence of points in an iterative manner in order to achieve a sketch which has similar shape in different scales. In this regard, the vertices of a polygon which is considered as the main shape of the fractal should be firstly positioned properly. Then, an initial random point is selected as the starting point for creating fractal. Based on this initial point, the next point in the sequence is determined as a fraction of the distance between the initial point and one of the vertices of the polygon which is selected randomly in each iteration. By repeating this process continuously with consideration of the random initial point and the random vertex selection in each iteration, a fractal is created. By utilizing three vertices with the factor of 1/2, a Sierpinski triangle is created. As the number of initial vertices for the fractal is increased to N, a Sierpinski Simplex with N – 1 dimensions can be created.

As a simple example, the step by step creation of a Sierpinski triangle by the methodology of chaos game is presented. At first, three vertices are selected in order to create the main shape of the fractal which results in a triangle shape in this case. Each of the selected vertices is marked by one of the red, blue and green colors. A dice is taken

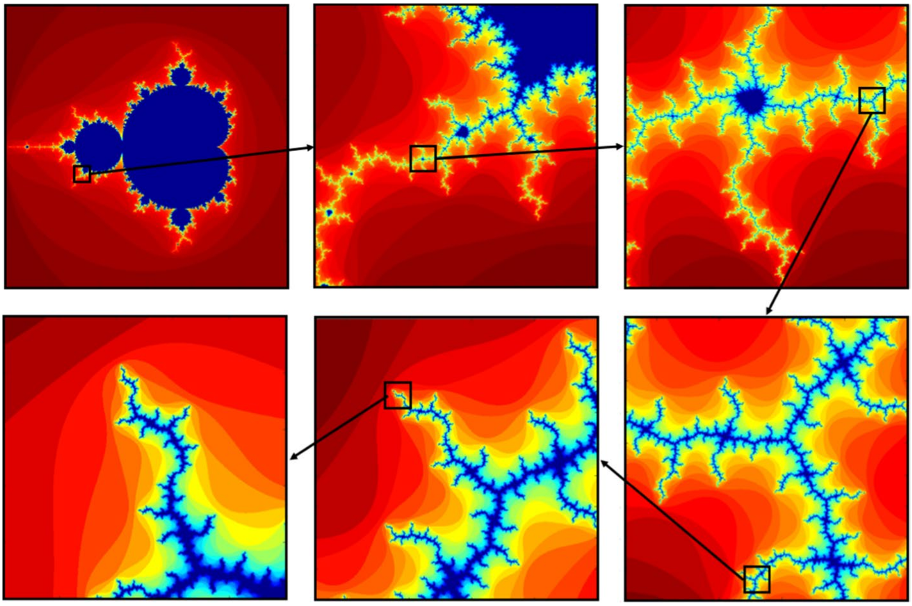

Fig. 1 Self-similarity of Mandelbrot set in different scales

which has two red faces, two blue faces and two green faces. An initial random point is selected as the starting point of the fractal which is considered as a seed in this example. As the dice is rolled, based on which color comes up, the seed in the initial point is moved toward the related vertex half the distance between the seed and the vertex. The new position of the seed is utilized in the next iteration as the starting point in which the dice is rolled again and the seed is moved to the intended vertex accordingly. By rolling the dice many times, the Sierpinski triangle is achieved as the final shape. The schematic view of the presented methodology is depicted in Fig. 2 while the final shape of the Sierpinski triangle and its self-similarity in different scales is presented in Fig. 3.

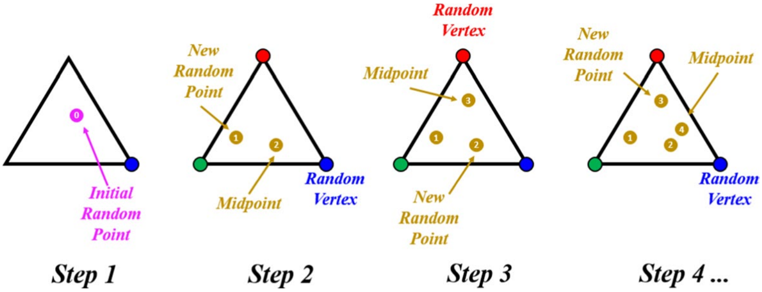

Fig. 2 The methodology of chaos game for creating Sierpinski triangle

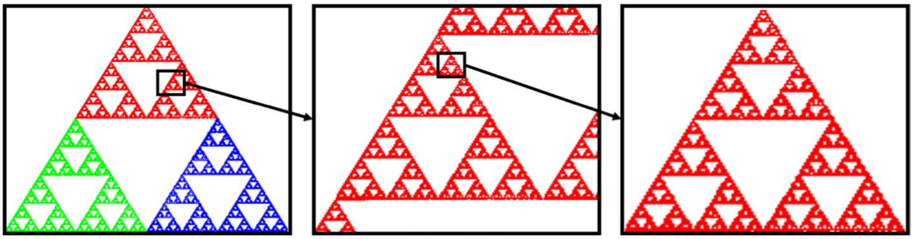

Fig. 3 The final shape and self-similarity of the Sierpinski triangle in different scales

2.2 Mathematical model

In this section, an optimization algorithm is proposed based on the presented principles of the chaos theory. The basic concepts of the fractals and chaos game are utilized in order to formulate a mathematical model for the CGO algorithm. Because of the fact that many of the natural evolution algorithms maintain a population of solutions which are evolved through random alterations and selection, the CGO algorithm considers a number of solution candidates (X) in this purpose which represent some eligible seeds inside a Sierpinski triangle. In this algorithm, each solution candidate (X_i) consists of some decision variables ($x_{i,j}$) which represent the position of these eligible seeds inside a Sierpinski triangle. The Sierpinski triangle is considered as the search space for solution candidates in the optimization algorithm. The mathematical presentation of these aspects is as follows:

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_i \\ \vdots \\ X_n \end{bmatrix} = \begin{bmatrix} x_1^1 & x_1^2 & \cdots & x_1^j & \cdots & x_1^d \\ x_2^1 & x_2^2 & \cdots & x_2^j & \cdots & x_2^d \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ x_i^1 & x_i^2 & \cdots & x_i^j & \cdots & x_i^d \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ x_n^1 & x_n^2 & \cdots & x_n^j & \cdots & x_n^d \end{bmatrix}, \quad \begin{cases} i = 1, 2, \ldots, n. \\ i = 1, 2, \ldots, d. \end{cases} \tag{1}$$

where n is the number of eligible seeds (solution candidates) inside the Sierpinski triangle (search space), and d is the dimension of these seeds.

The initial positions of these eligible seeds are determined randomly in the search space as follows:

$$x_i^j(0) = x_{i,min}^j + rand.\left(x_{i,max}^j - x_{i,min}^j\right), \quad \begin{cases} i = 1, 2, \ldots, n. \\ j = 1, 2, \ldots, d. \end{cases} \tag{2}$$

where $x_i^j(0)$ determines the initial position of the eligible seeds; $x_{i,min}^j$ and $x_{i,max}^j$ are the minimum and maximum allowable values for the *jth* decision variable of the *ith* solution candidate; *rand* is a random number in the interval of [0,1].

As previously described, the principles of chaos theory concern the existence of some primary patterns in the behavior of dynamical systems which represent them as the self-similar and self-organized systems. The created initial seeds (eligible seeds), represent the primary patterns of the dynamical systems based on the chaos theory. The eligibility of these seeds to be as the primary patterns (the self-similarity) can be modeled with solution candidates (X) for an optimization problem. The solution candidates with the highest and lowest eligibility levels are equivalent to the best and worst fitness values, respectively.

The main concept of this mathematical model is to create different eligible seeds inside the search space in order to complete the overall shape of a Sierpinski triangle. In this regard, the methodology of creating new seeds inside a Sierpinski triangle is utilized as well. For each of the eligible seeds in the search space (X_i), a temporary triangle is drawn with three seeds as follows:

- The position of the so far found Global Best (*GB*),
- The position of the Mean Group (MG_i),
- The position of the *ith* solution candidate (X_i) as the selected seed.

The *GB* refers to the so far found best solution candidate which have highest eligibility levels and the MG_i refers to the mean values of some randomly selected eligible seeds with equal probability of including the current considered initial eligible seed (X_i). The *GB* and MG_i alongside the selected eligible seed (X_i) are considered as three vertices of a Sierpinski triangle. As mentioned before, for each of the initial eligible seeds in the search space, a temporary triangle is created with the purpose of creating some new seeds inside the search space which could be considered as new eligible seeds for completing the Sierpinski triangle. The schematic view of the creating temporary triangles is depicted in Fig. 4a while the detailed schematic description of this aspect is presented in Fig. 4b.

The main purpose of creating temporary triangles is to create new eligible seeds in the search space as described in the following. To fulfill this aim, four approaches are developed. The *ith* temporary triangle (*i*th iteration) contains the *n* available eligible seeds obtained in the previous iteration and three vertices of a Sierpinski triangle [the *GB* (green seed), MG_i (red seed) and X_i (blue seed)]. In this temporary triangle, 3 seeds and a dice are utilized for creating new seeds regarding the chaos game methodology. The first seed is positioned in the X_i, the second one is located in the *GB* and the third one is placed in the MG_i. For the first seed, a dice with three green faces and three red faces is utilized. The dice is rolled and based on which color comes up (green or red), the seed in the X_i is moved toward the *GB* (green face) or the MG_i (red face). This aspect is modeled through some random integer generation function which creates only two integers as 0 and 1 for the possibility of selecting green or red faces. If the green face comes up, the seed positioned in the X_i is moving toward the *GB* but if the red face comes up, the seed positioned in the X_i is moving toward the MG_i. Regardless of the fact that each of the green or red faces have equal possibility of coming up in the game, the possibility of creating two similar random integers for both of the *GB* and the MG_i are also considered in which the seed in the X_i moves toward a point of the connected lines between the *GB* and the MG_i. Based on the fact that the movement of the seeds in the search space should be limited due to the chaos game methodology, some randomly generated factorials are utilized in this purpose in order to control this aspect. A schematic presentation of the described process for the first seed is presented in Fig. 5a while the mathematical presentation of this process is as follows:

$$Seed_i^1 = X_i + \alpha_i \times \left(\beta_i \times GB - \gamma_i \times MG_i\right), \quad i = 1, 2, \ldots, n. \tag{3}$$

where X_i is the *ith* solution candidate, *GB* is the so far found global best, and MG_i is the mean values of some selected eligible seeds. α_i is the randomly generated factorial for modelling the movement limitations of the seeds while each of the β_i and γ_i represent a random integer of 0 or 1 for modelling the possibility of rolling a dice.

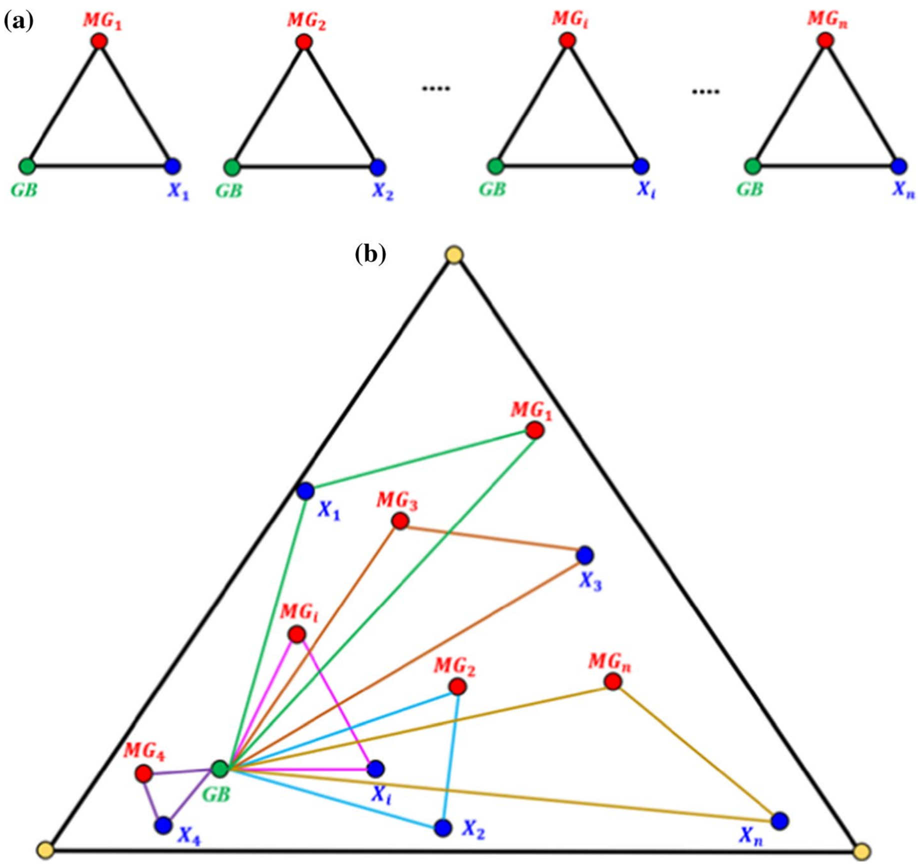

Fig. 4 **a** The schematic view of creating temporary triangles, **b** temporary triangles in the search space

For the second seed (GB), a dice with three blue and three red faces is utilized. The dice is rolled and based on which color comes up (blue or red), the seed in the GB is moved toward the X_i (blue face) or the MG_i (red face). This aspect is modeled as described for the first seed. If the blue face comes up, this seed is moving toward the X_i but if the red face comes up, the seed moves toward the MG_i. Similar to the moving process of the first seed, the second seed can move towards a point of the connected lines between the X_i and the MG_i and this movement is limited using some randomly generated factorials. A schematic presentation of the described process for the second seed is presented in Fig. 5b while the mathematical presentation of this process is as follows:

$$Seed_i^2 = GB + \alpha_i \times \left(\beta_i \times X_i - \gamma_i \times MG_i\right), \quad i = 1, 2, \ldots, n. \tag{4}$$

where α_i is the randomly generated factorial for modelling the movement limitations of the seeds while each of the β_i and γ_i represent a random integer of 0 or 1 for modelling the possibility of rolling a dice. The other parameters are as described for the first seed.

MG_i as the third seed, a dice has blue and green faces and it is rolled and based on which color comes up (blue or green), the seed moves toward the X_i (blue face) or the

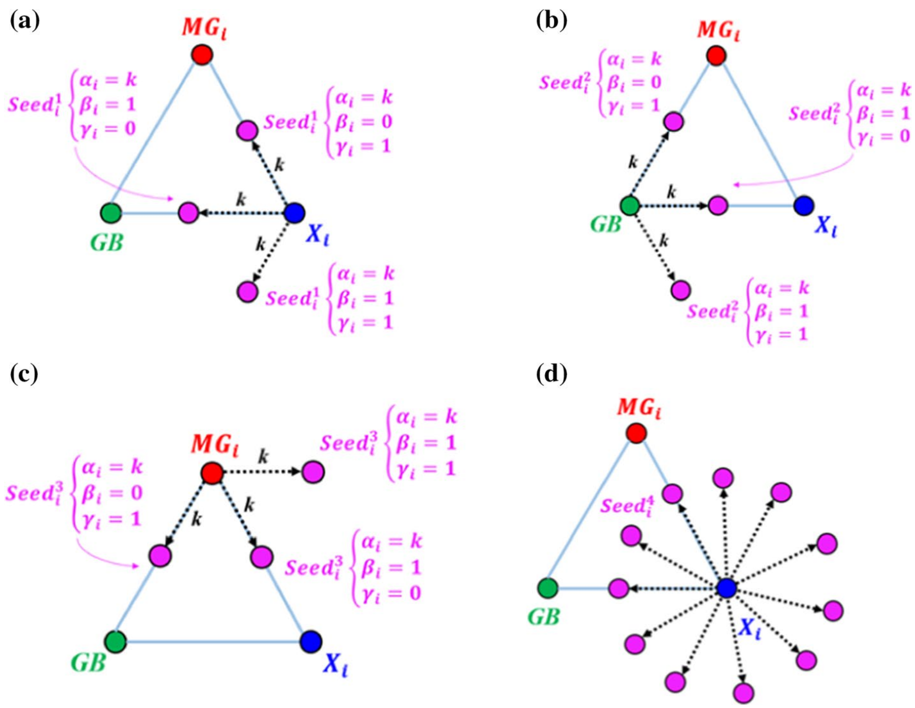

Fig. 5 The schematic view of position update for **a** first, **b** second, **c** third and **d** fourth seed in the search space

GB (green face). This aspect is modeled through some random integer generation function which creates only two integers as 0 and 1 for the possibility of selecting blue or green faces. It should be noted that the seed can move along the direction of the connected lines between the X_i and the *GB*, as well. Some random factorials are also utilized to fulfill this aim, as:

$$Seed_i^3 = MG_i + \alpha_i \times \left(\beta_i \times X_i - \gamma_i \times GB\right), \quad i = 1, 2, \ldots, n. \tag{5}$$

A schematic presentation of the described process for the third seed is presented in Fig. 5c.

In order to implement the mutation phase in the position updates of the eligible seeds in the search space, another process is also utilized to generated the fourth seed. The position updates for this seed is conducted based on some random alterations in the randomly chosen decision variables. A schematic presentation of the described process for the fourth seed is presented in Fig. 5d while this aspect is mathematically modelled as follows:

$$Seed_i^4 = X_i\left(x_i^k = x_i^k + R\right), \quad k = [1, 2, \ldots, d]. \tag{6}$$

where k is a random integer in the interval of $[1, d]$ and R is a uniformly distributed random number in the interval of [0,1].

In order to control and adjust the exploration and exploitation rate of the CGO algorithm, four different formulations are presented for α_i which controls the movement limitations of the seeds:

$$\alpha_i = \begin{cases} Rand \\ 2 \times Rand \\ (\delta \times Rand) + 1 \\ (\varepsilon \times Rand) + (\sim \varepsilon) \end{cases} \tag{7}$$

where *Rand* is a uniformly distributed random number in the interval of [0,1], while δ and ε are random integers in the interval of [0,1].

Based on the self-similarity issues in the fractals, the eligibility of the available seeds alongside the new ones should be considered together in order to decide whether or not the new seeds should be included in the overall eligible seeds in the search space. The quality of new solution candidates is compared to the available ones and the better ones is kept and the seeds with worst fitness values is eliminated corresponding to the worst self-similarity levels. It should be noted that the substitution process in the mathematical approach is implemented in order to decrease the complexity of the mathematical model. In reality, all of the so far found eligible seeds in the search space is utilized in order to complete the overall shape of the Sierpinski triangle.

In order to deal with the solution variables (x_i^j) violating the boundary conditions of the variables, a mathematical flag is defined in which for the x_i^j outside the variables range, the flag orders a boundary change for the violating variables. The terminating criterion is considered based on the maximum number of iterations in which the optimization process is terminated after a fixed number of iterations. The step by step procedure of the CGO algorithm is as follows while the pseudo-code of the algorithm is presented in Fig. 6.

- *Step 1* The initial positions of solution candidates (X) or the initial eligible seeds in the search space are defined randomly.
- *Step 2* The fitness values of the initial solution candidates based on the self-similarity of the initial eligible seeds are calculated.
- *Step 3* The Global Best (GB) related to the seed with highest levels of eligibility is determined.
- *Step 4* For each eligible seed (X_i) in the search space, a Mean Group (MG_i) is determined.
- *Step 5* For each eligible seed (X_i) in the search space, a temporary triangle is determined with three vertices of X_i, GB, and MG_i.
- *Step 6* For each of the temporary triangles, α_i, β_i, and γ_i values are calculated.
- *Step 7* For each of the temporary triangles, four seeds are created based on the Eqs. 3–6.
- *Step 8* For the new seeds with ta variable outside the range, a boundary condition check is conducted.
- *Step 9* The fitness values of the new seeds based on the self-similarity issues are calculated.
- *Step 10* The available eligible seeds with worst fitness values corresponding to worst self-similarity levels are substituted by the new seeds.
- *Step 11* The terminating criterion is checked.

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procedure Chaos Game Optimization (CGO) Algorithm
        Create random values for initial positions (x_i^j) of eligible seeds (X_i)
        Evaluate fitness values for each eligible seed
        Find GB, So far found best eligible seed
        while (t < maximum number of iterations)
            for i=1: number of initial eligible seeds
            Find MG_i
            Create temporary triangles with X_i, GB, and MG_i
            Calculate the α_i, β_i, and γ_i values
            Create new seeds by Eqs. 3 to 6.
                if new seeds violate boundary conditions
                    Control the position constraints for new seeds and amend it
                end if
            Evaluate the fitness values for new seeds
                if new seeds have better fitness values than the worst initial eligible seeds
                    Substitute the worst initial eligible seeds by the new seeds
                end if
            Update GB if a better solution is found
            end for
            t=t+1
        end while
        return GB
end procedure
```

Fig. 6 The pseudo-code of the CGO algorithm

3 Mathematical test functions

In this section, almost all of well-known mathematical functions are considered in order to utilize as test functions for performance evaluation of the CGO algorithm. A total number of 239 mathematical functions categorized into four different groups based on their specific characteristics are utilized (Jamil and Yang 2013; Jamil et al. 2013; Momin and Yang 2013; Yang 2010a; Liang et al. 2005) in which different mathematical functions with different characteristics is reviewed and prepared for utilizing in validation of the present algorithm. Many available works in the optimization field utilize their algorithm for solving a high number of functions while, they select and report just the best one. In the other words, the worst results and problems were not reported and investigated at all and the advantage and disadvantages of algorithms remain unclear. Obviously, this is not a correct manner for evaluating an algorithm. We believe that for having a fair evaluation of the performance of an algorithm, different problems with various properties should be considered. As a result, we gathered and solved almost all of the classic and new mathematical optimization functions in this paper. Also, the bad results as well as the good ones are reported. Hopefully, this work opens new era for the future researches in presenting all results to discover all good or bad properties of algorithms.

In the first group, 117 mathematical functions are presented which have minimum and maximum dimensions of 2 and 10, respectively. Among these functions which are named as F_1 to F_{117}, the first 90 functions have 2 dimensions while the other 27 functions have dimensions of 3–10, accordingly. The second group of mathematical functions consist of

58 test functions in which the dimensions of these functions are variable due to the specific formulation and are called as N dimensional test functions. In this regard, the maximum dimensions of 50 is considered in dealing with these functions (fifty dimensional (50D) functions) as F_{118} to F_{175}. For the third group, the mathematical functions of the second group are considered with the maximum dimension of 100 (100D) as F_{175} to F_{233}. For the fourth group, three composite and three hybrid mathematical functions are considered which are named as F_{233} to F_{239}. The specific characteristics of the mentioned mathematical functions in these groups are all presented in Table 3. In this table, C, NC, D, ND, S, NS, Sc, NSc, U and M denote on Continuous, Non-Continuous, Differentiable, Non-Differentiable, Separable, Non-Separable, Scalable, Non-Scalable, Unimodal and Multi-modal respectively. In addition, R, D and Min represent the variables range, variables dimension and the global minimum of the functions.

4 Alternative metaheuristics for comparison

In order to evaluate the overall performance of the CGO algorithm, some different optimization algorithms ae utilized as alternative approaches which provide a valid comparative study. The utilized metaheuristics in this purpose are the FA, GWO, ICA, SOS, TLBO, and the WOA. Based on the fact that some of the selected optimization algorithms are recently proposed or developed for special purposes, the most recent and improved versions of these algorithms are used in this paper. It should be noted that the internal parameters of the optimization algorithms have the most vital role in the convergence performance of these algorithms. In this purpose, a parameter summary of the selected algorithms is presented in Table 4. The values of these parameters have been determined utilizing the reference-based parameter identification process in which the internal parameters of these algorithms are selected based on the previously published research papers. It should be noted that some of the utilized metaheuristics are parameter free such as the GWO, SOS, TLBO, and the WOA which are not mentioned in this table.

5 Numerical results

In this section, the obtained results of the optimization run for the CGO algorithm alongside the other metaheuristic approaches in dealing with the mathematical test functions are presented. The optimization problem in all of these metaheuristics are formulated in which the maximum population size is taken as 50 and the maximum number of Function Evaluation (FE) is selected as 150,000 for all of the metaheuristics. The maximum number of iterations in each algorithm is adjusted based on the selected maximum number of FE. Based on the fact that collecting quantitative results are of great importance in dealing with different optimization problems, each of the CGO and the other metaheuristic algorithms is used to solve the mathematical functions 100 times, separately and the mean and standard deviation (Std) of the final best solutions are reported. The tolerance of 1×10^{-12} is also determined as the terminating criterion in which the optimization run is stopped if so-far best result of the algorithm is in this tolerance of the Global Best of the problem. Then, the number of FEs is calculated based on the pre-selected tolerance. In this section, the minimum, mean and the standard deviation of the 100 optimization runs for the selected algorithms in dealing with different mathematical functions are presented. In Table 5, the

Table 3 Details of the 2D–100D mathematical functions

No.	Name	Type	R	D	Formulation	Min.
F_1	Ackley 2 Function	C, D, NS, Sc, M	[− 35, 35]	2	Jamil and Yang (2013)	− 200
F_2	Ackley 3 Function	C, D, NS, NSc, U	[− 32, 32]	2	Jamil and Yang (2013)	− 195.629
F_3	Adjiman Function	C, D, NS, NSc, M	[− 1, 2] and [− 1, 1]	2	Jamil and Yang (2013)	− 2.02181
F_4	Bartels Conn Function	C, ND, NS, NSc, M	[− 500, 500]	2	Jamil and Yang (2013)	1
F_5	Beale Function	C, D, NS, NSc, U	[− 4.5, 4.5]	2	Jamil and Yang (2013)	0
F_6	Becker–Lago function	S	[− 10, 10]	2	Jamil et al. (2013)	0
F_7	Biggs EXP2 Function	C, D, NS, NSc, M	[0, 20]	2	Jamil and Yang (2013)	0
F_8	Bird Function	C, D, NS, NSc, M	[− 2π, π]	2	Jamil and Yang (2013)	− 106.765
F_9	Bohachevsky 1 Function	C, D, S, NSc, M	[− 100, 100]	2	Jamil and Yang (2013)	0
F_{10}	Bohachevsky 2 Function	C, D, NS, NSc, M	[− 100, 100]	2	Jamil and Yang (2013)	0
F_{11}	Bohachevsky 3 Function	C, D, NS, NSc, M	[− 100, 100]	2	Jamil and Yang (2013)	0
F_{12}	Booth Function	C, D, NS, NSc, U	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{13}	Branin RCOS Function	C, D, NS, NSc M	[− 5, 10] and [0, 15]	2	Jamil and Yang (2013)	0.397887
F_{14}	Branin RCOS 2 Function	C, D, NS, NSc, M	[− 5, 15]	2	Jamil and Yang (2013)	5.559037
F_{15}	Brent Function	C, D, NS, NSc, U	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{16}	Bukin 4 Function	C, ND, S, NSc, M	[− 15, − 5] and [− 3, 3]	2	Jamil and Yang (2013)	0
F_{17}	Bukin 6 Function	C, ND, NS, NSc, M	[− 15, − 5] and [− 3, 3]	2	Jamil and Yang (2013)	0
F_{18}	Camel Function-Three Hump	C, D, NS, NSc, M	[− 5, 5]	2	Jamil and Yang (2013)	0
F_{19}	Camel Function-Six Hump	C, D, NS, NSc, M	[− 5, 5]	2	Jamil and Yang (2013)	− 1.0316
F_{20}	Carrom table function	NS	[− 10, 10]	2	Jamil et al. (2013)	− 24.1568
F_{21}	Chen Bird Function	C, D, NS, NSc, M	[− 500, 500]	2	Jamil and Yang (2013)	− 2000
F_{22}	Chen V Function	C, D, NS, NSc, M	[− 500, 500]	2	Jamil and Yang (2013)	− 2000
F_{23}	Chichinadze Function	C, D, S, NSc, M	[− 30, 30]	2	Jamil and Yang (2013)	− 42.9444
F_{24}	Cross-in-Tray Function	C, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 2.06261
F_{25}	Cube Function	C, D, NS, NSc, U	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{26}	Damavandi Function	C, D, NS, NSc, M	[0, 14]	2	Jamil and Yang (2013)	0
F_{27}	Deckkers–Aarts Function	C, D, NS, NSc, M	[− 20, 20]	2	Jamil and Yang (2013)	− 24,771.1

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{28}	Easom Function	C, D, S, NSc, M	[− 100, 100]	2	Jamil and Yang (2013)	− 1
F_{29}	El-Attar–Vidyasagar–Dutta Function	C, D, NS, NSc, M	[− 500, 500]	2	Jamil and Yang (2013)	1.7128
F_{30}	Egg Crate Function	C, D, NS, Sc, M	[− 5, 5]	2	Jamil and Yang (2013)	0
F_{31}	Exp 2 Function	S	[0, 20]	2	Jamil and Yang (2013)	0
F_{32}	Freudenstein Roth Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{33}	Giunta Function	C, D, S, Sc, M	[− 1, 1]	2	Jamil and Yang (2013)	0.060447
F_{34}	Goldstein Price Function	C, D, NS, NSc, M	[− 2, 2]	2	Jamil and Yang (2013)	3
F_{35}	Hansen Function	C, D, S, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 165.953
F_{36}	Himmelblau Function	C, D, NS, NSc, M	[− 5, 5]	2	Jamil and Yang (2013)	0
F_{37}	Hosaki Function	C, D, NS, NSc, M	[0, 5] and [0, 6]	2	Jamil and Yang (2013)	− 2.3458
F_{38}	Jennrich–Sampson Function	C, D, NS, NSc, M	[− 1, 1]	2	Jamil and Yang (2013)	124.3612
F_{39}	Keane Function	C, D, NS, NSc, M	[0, 10]	2	Jamil and Yang (2013)	− 0.67367
F_{40}	Leon Function	C, D, NS, NSc, U	[− 1.2, 1.2]	2	Jamil and Yang (2013)	0
F_{41}	Levy 3 Function	S	[− 10, 10]	2	Jamil et al. (2013)	− 176.542
F_{42}	Levy 5 Function	NS	[− 10, 10]	2	Jamil et al. (2013)	− 176.138
F_{43}	Matyas Function	C, D, NS, NSc, U	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{44}	McCormick Function	C, D, NS, NSc, M	[− 1.5, 4] and [− 3, 3]	2	Jamil and Yang (2013)	− 1.9133
F_{45}	Mexican hat Function	NS	[− 10, 10]	2	Jamil et al. (2013)	− 19.6683
F_{46}	Michaelewicz 2 Function	S	[0, π]	2	Jamil et al. (2013)	− 1.8013
F_{47}	Mishra 3 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 0.18465
F_{48}	Mishra 4 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 0.19941
F_{49}	Mishra 5 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 1.01983
F_{50}	Mishra 6 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 2.28395
F_{51}	Mishra 8 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	0
F_{52}	Mishra 10 Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{53}	Parsopoulos Function	C, D, S, Sc, M	[−5, 5]	2	Jamil and Yang (2013)	0
F_{54}	Pen Holder Function	C, D, NS, NSc, M	[−11, 11]	2	Jamil and Yang (2013)	−0.96354
F_{55}	Periodic Function	S	[−10, 10]	2	Jamil and Yang (2013)	0.9
F_{56}	Price 1 Function	C, ND, S, NSc, M	[−500, 500]	2	Jamil and Yang (2013)	0
F_{57}	Price 2 Function	C, D, NS, NSc, M	[−10, 10]	2	Jamil and Yang (2013)	0.9
F_{58}	Price 3 Function	C, D, NS, NSc, M	[−500, 500]	2	Jamil and Yang (2013)	0
F_{59}	Price 4 Function	C, D, NS, NSc, M	[−500, 500]	2	Jamil and Yang (2013)	0
F_{60}	Quadratic Function	C, D, NS, NSc	[−10, 10]	2	Jamil and Yang (2013)	−3873.72
F_{61}	Ripple 1 Function	NS	[0, 1]	2	Jamil and Yang (2013)	−2.2
F_{62}	Ripple 25 Function	NS	[0, 1]	2	Jamil and Yang (2013)	−2
F_{63}	Rosenbrock Modified Function	C, D, NS, NSc, M	[−2, 2]	2	Jamil and Yang (2013)	34.3712
F_{64}	Rotated Ellipse Function	C, D, NS, NSc, U	[−500, 500]	2	Jamil and Yang (2013)	0
F_{65}	Rotated Ellipse 2 Function	C, D, NS, NSc, U	[−500, 500]	2	Jamil and Yang (2013)	0
F_{66}	Rump Function	C, D, NS, NSc, U	[−500, 500]	2	Jamil and Yang (2013)	0
F_{67}	Scahffer 1 Function	C, D, NS, NSc, U	[−100, 100]	2	Jamil and Yang (2013)	0
F_{68}	Scahffer 2 Function	C, D, NS, NSc, U	[−100, 100]	2	Jamil and Yang (2013)	0
F_{69}	Scahffer 3 Function	C, D, NS, NSc, U	[−100, 100]	2	Jamil and Yang (2013)	0.001567
F_{70}	Scahffer 4 Function	C, D, NS, NSc, U	[−100, 100]	2	Jamil and Yang (2013)	0.292579
F_{71}	Schwefel 2.6 Function	C, D, NS, NSc, U	[−100, 100]	2	Jamil and Yang (2013)	0
F_{72}	Schwefel 2.36 Function	C, D, S, Sc, M	[0, 500]	2	Jamil and Yang (2013)	−3456
F_{73}	Table 1/Holder Table 1 Function	C, D, S, NSc, M	[−10, 10]	2	Jamil and Yang (2013)	−26.9203
F_{74}	Table 2/Holder Table 2 Function	C, D, S, NSc, M	[−10, 10]	2	Jamil and Yang (2013)	−19.2085
F_{75}	Table 3/Carrom Table Function	C, D, NS, NSc, M	[−10, 10]	2	Jamil and Yang (2013)	−24.1568
F_{76}	Testtube Holder Function	C, D, S, NSc, M	[−10, 10]	2	Jamil and Yang (2013)	−10.8723
F_{77}	Trecanni Function	C, D, S, NSc, U	[−5, 5]	2	Jamil and Yang (2013)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{78}	Trefethen Function	C, D, NS, NSc, M	[− 10, 10]	2	Jamil and Yang (2013)	− 3.30687
F_{79}	Tripod Function	C, D, NS, NSc, M	[− 100, 100]	2	Jamil and Yang (2013)	0
F_{80}	Ursem 1 Function	S	[− 2.5, 3] and [− 2, 2]	2	Jamil and Yang (2013)	− 4.81681
F_{81}	Ursem 3 Function	NS	[− 2, 2] and [− 1.5, 1.5]	2	Jamil and Yang (2013)	− 2.5
F_{82}	Ursem 4 Function	NS	[− 2, 2]	2	Jamil and Yang (2013)	− 1.5
F_{83}	Ursem Waves Function	NS	[− 0.9, 1.2] and [− 1.2, 1.2]	2	Jamil and Yang (2013)	− 7.307
F_{84}	Venter Sobiezcczanski-Sobieski Function	C, D, S, NSc	[− 50, 50]	2	Jamil and Yang (2013)	− 400
F_{85}	Wayburn Seader 1 Function	C, D, NS, Sc, U	[− 500, 500]	2	Jamil and Yang (2013)	0
F_{86}	Wayburn Seader 2 Function	C, D, NS, Sc, U	[− 500, 500]	2	Jamil and Yang (2013)	0
F_{87}	Wayburn Seader 3 Function	C, D, NS, Sc, U	[− 500, 500]	2	Jamil and Yang (2013)	21.35
F_{88}	Zettl Function	C, D, NS, NSc, U	[− 5, 10]	2	Jamil and Yang (2013)	− 0.00379
F_{89}	Zirilli or Aluffi-Pentini's Function	C, D, S, NSc, U	[− 10, 10]	2	Jamil and Yang (2013)	− 0.3523
F_{90}	Zirilli Function 2	C, D, S, S, M	[− 500, 500]	2	Jamil and Yang (2013)	0
F_{91}	Biggs EXP3 Function	C, D, NS, NSc, M	[0, 20]	3	Jamil and Yang (2013)	0
F_{92}	Gulf Research Problem	C, D, NS, NSc, M	[0.1, 100] and [0, 25.6] and [0, 6.5]	3	Jamil and Yang (2013)	0
F_{93}	Hartman 3 Function	C, D, NS, NSc, M	[0, 1]	3	Jamil and Yang (2013)	− 3.86278
F_{94}	Helical Valley	C, D, NS, Sc, M	[− 10, 10]	3	Jamil and Yang (2013)	0
F_{95}	Meyer–Roth Function	NS	[0, 1]	3	Jamil et al. (2013)	4.00E−05
F_{96}	Mishra 9 Function	C, D, NS, NSc, M	[− 10, 10]	3	Jamil and Yang (2013)	0
F_{97}	Wolfe Function	C, D, S, Sc, M	[0, 2]	3	Jamil and Yang (2013)	0
F_{98}	Biggs EXP4 Function	C, D, NS, NSc, M	[0, 20]	4	Jamil and Yang (2013)	0
F_{99}	Colville Function	C, D, NS, NSc, M	[− 10, 10]	4	Jamil and Yang (2013)	0
F_{100}	Corana Function	DC, ND, S, Sc, M	[− 500, 500]	4	Jamil and Yang (2013)	0
F_{101}	DeVilliers Glasser 1 Function	C, D, NS, NSc, M	[1, 100]	4	Jamil and Yang (2013)	0
F_{102}	Gear Function	NS	[12, 60]	4	Jamil et al. (2013)	2.70E−12

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{103}	Kowalik Function	NS	[−5, 5]	4	Jamil et al. (2013)	0.000308
F_{104}	Miele Cantrell Function	C, D, NS, NSc, M	[−1, 1]	4	Jamil and Yang (2013)	0
F_{105}	Shekel 5	C, D, NS, Sc, M	[0, 10]	4	Jamil and Yang (2013)	−10.1532
F_{106}	Shekel 7	C, D, NS, Sc, M	[0, 10]	4	Jamil and Yang (2013)	−10.4029
F_{107}	Shekel 10	C, D, NS, Sc, M	[0, 10]	4	Jamil and Yang (2013)	−10.5364
F_{108}	Biggs EXP5 Function	C, D, NS, NSc, M	[0, 20]	5	Jamil and Yang (2013)	0
F_{109}	DeVilliers Glasser 2 Function	C, D, NS, NSc, M	[1, 60]	5	Jamil and Yang (2013)	0
F_{110}	Dolan Function	C, D, NS, NSc, M	[−100, 100]	5	Jamil and Yang (2013)	−529.871
F_{111}	Langerman-5 Function	C, D, NS, Sc, M	[0, 10]	5	Jamil and Yang (2013)	−0.965
F_{112}	Biggs EXP6 Function	C, D, NS, NSc, M	[−20, 20]	6	Jamil and Yang (2013)	0
F_{113}	Hartman 6 Function	C, D, NS, NSc, M	[0, 1]	6	Jamil and Yang (2013)	−3.32236
F_{114}	Trid 6 Function	C, D, NS, NSc, M	[−36, 36]	6	Jamil and Yang (2013)	−50
F_{115}	Ann-XOR Function	NS	[−1, 1]	9	Jamil et al. (2013)	0.95979
F_{116}	Paviani Function	C, D, NS, Sc, M	[2.0001, 10]	10	Jamil and Yang (2013)	−45.778
F_{117}	Trid 10 Function	C, D, NS, NSc, M	[−100,100]	10	Jamil and Yang (2013)	−210
F_{118}	Ackley 1 Function	C, D, NS, Sc,M	[−35, 35]	50	Jamil and Yang (2013)	0
F_{119}	Alpine 1 Function	C, ND, S, NSc,U	[−10, 10]	50	Jamil and Yang (2013)	0
F_{120}	Brown Function	C, D, NS, Sc, U	[−1, 4]	50	Jamil and Yang (2013)	0
F_{121}	Chung Reynolds Function	C, D, PS, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{122}	Csendes Function	C, D, S, Sc, M	[−1, 1]	50	Jamil and Yang (2013)	0
F_{123}	Deb 1 Function	C, D, S, Sc, M	[−1, 1]	50	Jamil and Yang (2013)	−1
F_{124}	Deb 3 Function	C, D, S, Sc, M	[0, 1]	50	Jamil and Yang (2013)	−1
F_{125}	Dixon and Price Function	C, D, NS, Sc, U	[−10, 10]	50	Jamil and Yang (2013)	0
F_{126}	Extended Easom Function	C, D, S, NSc, M	[−2π, 2π]	50	Jamil and Yang (2013)	−1
F_{127}	Exponential Function	C, D, NS, Sc, M	[−1, 1]	50	Jamil and Yang (2013)	−1

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{128}	Griewank Function	C, D, NS, Sc, M	[−100,100]	50	Jamil and Yang (2013)	0
F_{129}	Holzman 2 Function	S	[−10, 10]	50	Jamil et al. (2013)	0
F_{130}	Hyper-ellipsoid Function	C, U	[−500, 500]	50	Jamil et al. (2013)	0
F_{131}	Inverted cosine wave Function	NS	[−10, 10]	50	Jamil et al. (2013)	−49
F_{132}	Levy 8 Function	NS	[−10, 10]	50	Jamil et al. (2013)	0
F_{133}	Mishra 1 Function	C, D, NS, Sc, M	[0, 1]	50	Jamil and Yang (2013)	2
F_{134}	Mishra 2 Function	C, D, NS, Sc, M	[0, 1]	50	Jamil and Yang (2013)	2
F_{135}	Mishra 7 Function	C, D, NS, NSc, M	[−10, 10]	50	Jamil and Yang (2013)	0
F_{136}	Mishra 11 Function	C, D, NS, NSc, M	[−10, 10]	50	Jamil and Yang (2013)	0
F_{137}	Pathological Function	C, D, NS, NSc, M	[−100, 100]	50	Jamil and Yang (2013)	0
F_{138}	Pint´er Function	C, D, NS, Sc, M	[−10, 10]	50	Jamil and Yang (2013)	0
F_{139}	Powell Singular Function	C, D, NS, Sc, U	[−4, 5]	50	Jamil and Yang (2013)	0
F_{140}	Powell Singular 2 Function	C, D, NS, Sc, U	[−4, 5]	50	Jamil and Yang (2013)	0
F_{141}	Powell Sum Function	C, D, S, Sc, U	[−1, 1]	50	Jamil and Yang (2013)	0
F_{142}	Rastrigin Function	C, D, S, M	[−5.12, 5.12]	50	Yang (2010b)	0
F_{143}	Qing Function	C, D, S, Sc, M	[−500, 500]	50	Jamil and Yang (2013)	0
F_{144}	Quintic Function	C, D, S, NSc, M	[−10, 10]	50	Jamil and Yang (2013)	0
F_{145}	Rosenbrock Function	C, D, NS, Sc, U	[−30, 30]	50	Jamil and Yang (2013)	0
F_{146}	Salomon Function	C, D, NS, Sc, M	[−100, 100]	50	Jamil and Yang (2013)	0
F_{147}	Schumer Steiglitz Function	C, D, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{148}	Schwefel Function	C, D, PS, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{149}	Schwefel 1.2 Function	C, D, NS, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{150}	Schwefel 2.4 Function	C, D, S, NSc, M	[0, 10]	50	Jamil and Yang (2013)	0
F_{151}	Schwefel 2.20 Function	C, ND, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{152}	Schwefel 2.21 Function	C, ND, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{153}	Schwefel 2.22 Function	C, D, NS, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{154}	Schwefel 2.23 Function	C, D, NS, Sc, U	[−10, 10]	50	Jamil and Yang (2013)	0
F_{155}	Schwefel 2.25 Function	C, D, S, NSc, M	[0, 10]	50	Jamil and Yang (2013)	0
F_{156}	Schwefel 2.26 Function	C, D, S, Sc, M	[−500, 500]	50	Jamil and Yang (2013)	−418.983
F_{157}	Sphere Function	C, D, S, Sc, M	[0, 10]	50	Jamil and Yang (2013)	0
F_{158}	Step Function	DC, ND, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{159}	Step 2 Function	DC, ND, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{160}	Step 3 Function	DC, ND, S, Sc, U	[−100, 100]	50	Jamil and Yang (2013)	0
F_{161}	Stepint Function	DC, ND, S, Sc, U	[−5.12, 5.12]	50	Jamil and Yang (2013)	−275
F_{162}	Streched V Sine Wave Function	C, D, NS, Sc, U	[−10, 10]	50	Jamil and Yang (2013)	0
F_{163}	Sum Squares Function	C, D, S, Sc, U	[−10, 10]	50	Jamil and Yang (2013)	0
F_{164}	Styblinski–Tang Function	C, D, NS, NSc, M	[−5, 5]	50	Jamil and Yang (2013)	−1958.3
F_{165}	Trid Function	C, D, NS, NSc, U	[−D^2, D^2]	50	Jamil and Yang (2013)	−22,050
F_{166}	Trigonometric 1 Function	C, D, NS, Sc, M	[0, π]	50	Jamil and Yang (2013)	0
F_{167}	Trigonometric 2 Function	C, D, NS, Sc, M	[−500, 500]	50	Jamil and Yang (2013)	1
F_{168}	W/Wavy Function	C, D, S, Sc, M	[−π, π]	50	Jamil and Yang (2013)	0
F_{169}	Xin-She Yang (Function 1)	DC, ND, NS, Sc, M	[−20, 20]	50	Momin and Yang (2013)	−1
F_{170}	Xin-She Yang (Function 2)	DC, ND, NS, Sc, M	[−10, 10]	50	Yang (2010b)	0
F_{171}	Xin-She Yang (Function 3)	DC, ND, NS, Sc, M	[−2π, 2π]	50	Yang (2010b)	0
F_{172}	Xin-She Yang (Function 4)	DC, ND, NS, Sc, M	[−5, 5]	50	Yang (2010b)	0
F_{173}	Xin-She Yang (Function 5)	DC, ND, NS, Sc, M	[−10, 10]	50	Yang (2010b)	−1
F_{174}	Xin-She Yang (Function 6)	DC, ND, NS, Sc, M	[−5, 5]	50	Yang (2010b)	0
F_{175}	Zakharov Function	C, D, NS, Sc, M	[−5, 10]	50	Jamil and Yang (2013)	0
F_{176}	Ackley 1 Function	C, D, NS, Sc,M	[−35, 35]	100	Jamil and Yang (2013)	0
F_{177}	Alpine 1 Function	C, ND, S, NSc,U	[−10, 10]	100	Jamil and Yang (2013)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{178}	Brown Function	C, D, NS, Sc, U	[− 1, 4]	100	Jamil and Yang (2013)	0
F_{179}	Chung Reynolds Function	C, D, PS, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{180}	Csendes Function	C, D, S, Sc, M	[− 1, 1]	100	Jamil and Yang (2013)	0
F_{181}	Deb 1 Function	C, D, S, Sc, M	[− 1, 1]	100	Jamil and Yang (2013)	− 1
F_{182}	Deb 3 Function	C, D, S, Sc, M	[0, 1]	100	Jamil and Yang (2013)	− 1
F_{183}	Dixon and Price Function	C, D, NS, Sc, U	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{184}	Extended Easom Function	C, D, S, NSc, M	[− 2π, 2π]	100	Jamil and Yang (2013)	− 1
F_{185}	Exponential Function	C, D, NS, Sc, M	[− 1, 1]	100	Jamil and Yang (2013)	− 1
F_{186}	Griewank Function	C, D, NS, Sc, M	[− 100,100]	100	Jamil and Yang (2013)	0
F_{187}	Holzman 2 Function	S	[− 10, 10]	100	Jamil et al. (2013)	0
F_{188}	Hyper-ellipsoid Function	C, U	[− 500, 500]	100	Jamil et al. (2013)	0
F_{189}	Inverted cosine wave Function	NS	[− 10, 10]	100	Jamil et al. (2013)	− 99
F_{190}	Levy 8 Function	NS	[− 10, 10]	100	Jamil et al. (2013)	0
F_{191}	Mishra 1 Function	C, D, NS, Sc, M	[0, 1]	100	Jamil and Yang (2013)	2
F_{192}	Mishra 2 Function	C, D, NS, Sc, M	[0, 1]	100	Jamil and Yang (2013)	2
F_{193}	Mishra 7 Function	C, D, NS, NSc, M	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{194}	Mishra 11 Function	C, D, NS, NSc, M	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{195}	Pathological Function	C, D, NS, NSc, M	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{196}	Pint´er Function	C, D, NS, Sc, M	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{197}	Powell Singular Function	C, D, NS, Sc, U	[− 4, 5]	100	Jamil and Yang (2013)	0
F_{198}	Powell Singular 2 Function	C, D, NS, Sc, U	[− 4, 5]	100	Jamil and Yang (2013)	0
F_{199}	Powell Sum Function	C, D, S, Sc, U	[− 1, 1]	100	Jamil and Yang (2013)	0
F_{200}	Rastrigin Function	C, D, S, M	[− 5.12, 5.12]	100	Yang (2010b)	0
F_{201}	Qing Function	C, D, S, Sc, M	[− 500, 500]	100	Jamil and Yang (2013)	0
F_{202}	Quintic Function	C, D, S, NSc, M	[− 10, 10]	100	Jamil and Yang (2013)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{203}	Rosenbrock Function	C, D, NS, Sc, U	[− 30, 30]	100	Jamil and Yang (2013)	0
F_{204}	Salomon Function	C, D, NS, Sc, M	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{205}	Schumer Steiglitz Function	C, D, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{206}	Schwefel Function	C, D, PS, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{207}	Schwefel 1.2 Function	C, D, NS, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{208}	Schwefel 2.4 Function	C, D, S, NSc, M	[0, 10]	100	Jamil and Yang (2013)	0
F_{209}	Schwefel 2.20 Function	C, ND, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{210}	Schwefel 2.21 Function	C, ND, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{211}	Schwefel 2.22 Function	C, D, NS, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{212}	Schwefel 2.23 Function	C, D, NS, Sc, U	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{213}	Schwefel 2.25 Function	C, D, S, NSc, M	[0, 10]	100	Jamil and Yang (2013)	0
F_{214}	Schwefel 2.26 Function	C, D, S, Sc, M	[− 500, 500]	100	Jamil and Yang (2013)	− 418.983
F_{215}	Sphere Function	C, D, S, Sc, M	[0, 10]	100	Jamil and Yang (2013)	0
F_{216}	Step Function	DC, ND, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{217}	Step 2 Function	DC, ND, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{218}	Step 3 Function	DC, ND, S, Sc, U	[− 100, 100]	100	Jamil and Yang (2013)	0
F_{219}	Stepint Function	DC, ND, S, Sc, U	[− 5.12, 5.12]	100	Jamil and Yang (2013)	− 575
F_{220}	Streched V Sine Wave Function	C, D, NS, Sc, U	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{221}	Sum Squares Function	C, D, S, Sc, U	[− 10, 10]	100	Jamil and Yang (2013)	0
F_{222}	Styblinski–Tang Function	C, D, NS, NSc, M	[− 5, 5]	100	Jamil and Yang (2013)	− 3916.6
F_{223}	Trid Function	C, D, NS, NSc, U	[− D^2, D^2]	100	Jamil and Yang (2013)	− 171,600
F_{224}	Trigonometric 1 Function	C, D, NS, Sc, M	[0, π]	100	Jamil and Yang (2013)	0
F_{225}	Trigonometric 2 Function	C, D, NS, Sc, M	[− 500, 500]	100	Jamil and Yang (2013)	1
F_{226}	W/Wavy Function	C, D, S, Sc, M	[− π, π]	100	Jamil and Yang (2013)	0
F_{227}	Xin-She Yang (Function 1)	DC, ND, NS, Sc, M	[− 20, 20]	100	Momin and Yang (2013)	− 1

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{228}	Xin-She Yang (Function 2)	DC, ND, NS, Sc, M	[− 10, 10]	100	Yang (2010b)	0
F_{229}	Xin-She Yang (Function 3)	DC, ND, NS, Sc, M	[− 2π, 2π]	100	Yang (2010b)	0
F_{230}	Xin-She Yang (Function 4)	DC, ND, NS, Sc, M	[− 5, 5]	100	Yang (2010b)	0
F_{231}	Xin-She Yang (Function 5)	DC, ND, NS, Sc, M	[− 10, 10]	100	Yang (2010b)	− 1
F_{232}	Xin-She Yang (Function 6)	DC, ND, NS, Sc, M	[− 5, 5]	100	Yang (2010b)	0
F_{233}	Zakharov Function	C, D, NS, Sc, M	[− 5, 10]	100	Jamil and Yang (2013)	0
F_{234}	Basic Functions: Sphere Function $f_1, f_2, f_3, \ldots, f_{10}$ = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots, 5/100]$	Composite	[− 5, 5]	10	(Liang et al. 2005)	0
F_{235}	Basic Functions: Griewank Function $f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/100, 5/100, 5/100, \ldots, 5/100]$	Composite	[− 5, 5]	10	(Liang et al. 2005)	0
F_{236}	Basic Functions: Griewank Function $f_1, f_2, f_3, \ldots, f_{10}$ = Griewank Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [1, 1, 1, \ldots, 1]$	Composite	[− 5, 5]	10	(Liang et al. 2005)	0
F_{237}	Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions f_1, f_2 = Ackley Function f_3, f_4 = Rastrigin Function f_5, f_6 = Weierstrass Function f_7, f_8 = Griewank Function f_9, f_{10} = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [5/32, 5.32, 1, 1, 5/0.5, 5/0.5, 5/100, 5/100, 5/100, 5/100]$	Hybrid	[− 5, 5]	10	(Liang et al. 2005)	0

Table 3 (continued)

No.	Name	Type	R	D	Formulation	Min.
F_{238}	Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions f_1, f_2 = Rastrigin Function f_3, f_4 = Weierstrass Function f_5, f_6 = Griewank Function f_7, f_8 = Ackley Function f_9, f_{10} = Sphere Function $[\sigma_1, \sigma_2, \sigma_3, \ldots, \sigma_{10}] = [1, 1, 1, \ldots, 1]$ $[\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_{10}] = [1/5, 1/5, 5/0.5, 5/0.5, 5/100, 5/100, 5/32, 5/32, 5/100, 5/100]$	Hybrid	[− 5, 5]	10	(Liang et al. 2005)	0
F_{239}	Basic Functions: Ackley, Rastrigin, Weierstrass, Griewank, and Sphere Functions f1, f2 = Rastrigin Function f3, f4 = Weierstrass Function f5, f6 = Griewank Function f7, f8 = Ackley Function f9, f10 = Sphere Function $[\sigma1, \sigma2, \sigma3, \ldots, \sigma10] = [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1]$ $[\lambda1, \lambda2, \lambda3, \ldots, \lambda10] = [0.1 \times 1/5, 0.2 \times 1/5, 0.3 \times 5/0.5, 0.4 \times 5/0.5, 0.5 \times 5/100, 0.6 \times 5/100, 0.7 \times 5/32, 0.8 \times 5/32, 0.9 \times 5/100, 1 \times 5/100]$	Hybrid	[− 5, 5]	10	(Liang et al. 2005)	0

minimum and mean values of different optimization runs for 2D mathematical functions obtained by the CGO algorithm and the other selected metaheuristics are presented. The standard deviation of the final best result as well as the mean values of the maximum number of function evaluations are also presented in Table 6. In Table 7, the best and mean results of 50D mathematical functions are presented while their related standard deviations as well as the mean values of the maximum number of function evaluations are presented in Table 8. Tables 9 and 10 present these results for the 100D mathematical functions. For the composite mathematical functions, the mean values and standard deviation of different optimization runs are presented in Table 11.

6 Statistical analysis

In this section, the maximum error values of the optimization convergence data have been calculated and utilized for statistical analysis in which the difference between the known Global Best of the functions and the final obtained optimal values are considered as the error values. The provided details of different statistical analysis suggested by García et al. (2009) are utilized for this purpose. To fulfill this aim, five statistical tests have been conducted in which the Kolmogorov–Smirnov (K–S) test is conducted for normality issues, the Mann–Whitney U (M–W) test is implemented for comparing the summation of the ranks of the different metaheuristics in a one by one comparing manner, the Wilcoxon signed-rank (W) test is conducted for comparing the mean ranks of different metaheuristics, the Kruskal–Wallis (K–W) test is conducted for comparing the overall rankings of the metaheuristics by considering the mean of the ranks of algorithms, and the Post-Hoc (P-H) analysis is conducted based on the results of the K–W tests for additional purposes.

6.1 Kolmogorov–Smirnov test

There are two kinds of statistical tests which are applicable to the obtained statistical data from multiple application as the parametric and non-parametric statistical tests. One of the most important criteria which demonstrates the possibility of utilizing each algorithm in a specific situation is the Kolmogorov–Smirnov (K–S) test. This test shows that the distribution of data is either normal or non-normal in which the distribution of each sample among the statistical data are considered and checked accordingly. If the K–S test is rejected, the data are normally distributed, and there is the possibility of using parametric statistical tests for the research. Conversely, if the K–S test is confirmed, the data do not have a normal distribution, so the nonparametric tests can be used.

The Asymptotic Significance (Asymp. Sig.) values of the K–S test which are also known as p values in dealing with the error values of the minimum, mean, standard deviation and the maximum function evaluations of the optimization runs for the 2D, 50D, and the 100D functions are presented in Table 12. This test is conducted as a two-sample test in which the distributions of the CGO results are compared to the results obtained by the other metaheuristics. It should be noted that if the p values are less than 0.05, the presented data are not distributed normally so the non-parametric statistical tests should be conducted for further investigations. The obtained results of the K–S test demonstrate that the p- values are less than 0.05 in most of the cases and the non-parametric statistical tests can be utilized for further considerations. In this table, the maximum difference between the statistical results of the CGO and the other metaheuristics are also presented in order to

Table 4 Parameter summary of the alternative metaheuristic algorithms

Metaheuristic	Parameter	Description	Value
FA	N_{pop}	Number of fireflies (swarm size)	50
	γ	Light absorption coefficient	1
	β	Attraction coefficient base value	2
	α	Mutation coefficient	0.2
	α_{damp}	Mutation coefficient damping ratio	0.98
	δ	Uniform mutation range	±0.05
ICA	N_{pop}	Population size	50
	N_{emp}	Number of empires/imperialists	10
	α	Selection pressure	1
	β	Assimilation coefficient	1.5
	p_r	Revolution probability	0.05
	μ	Revolution rate	0.1
	ζ	Colonies mean cost coefficient	0.2

have a fair judgment about the obtained results of the CGO algorithm. The maximum and minimum differences of the results of the CGO algorithm with the other algorithms are presented as bolted and italicized, respectively. The bolded values demonstrate the algorithms which has the maximum difference with the CGO algorithm while the italicized values show the algorithms which has the minimum difference with the CGO algorithm among the other metaheuristics.

6.2 Mann–Whitney U test

The Mann–Whitney U (M–W) test is a nonparametric test that allows two groups of data to be compared in which the null hypothesis denotes that it is equally likely that a randomly selected value from one sample will be less than or greater than a randomly selected value from a second sample. This test can be used to investigate whether two independent samples were selected from populations having the same distribution. This test provides the summation of the ranks for two sets of statistical data which is considered for comparing analysis. As a main criterion, if the summation of the ranks for one sample has lower values than the other one, the one with smaller sum of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the M–W test for different mathematical functions based on the obtained results of utilized algorithms have been presented in Table 13. In these tables, the upper and lower values are the summation of the ranks related to the alternative metaheuristics and the CGO algorithm, respectively. Based on the statistical results, the CGO for the summation of the ranks in most of the cases are lower than the related values for the rest metaheuristics (bolded values in the table). The related p values of this test for different mathematical functions are also presented in Table 14.

It should be noted that the results of the CGO algorithm (summation of the ranks) are superior to the results of other metaheuristics considering the minimum, mean, standard deviation and function evaluation values in dealing with 2D functions while for the function evaluation, the results of the ACO and ICA are better than the results of the CGO which demonstrates the deficiencies of the CGO in this issue. For the 50D and 100D

Table 5 The minimum and mean values of the 2D mathematical functions for different metaheuristics

Fun. no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_1	−199.99	−199.923	−200	−200	−200	−200	−200	−200	−200	−200	−200	−200	−200	−200
F_2	−195.629	−195.627	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629
F_3	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181
F_4	1.5259	30.37011	1	1	1	1	1	1	1	1	1	1	1	1
F_5	7.68E−09	3.87E−07	9.71E−12	2.95E−09	0	0	0	0	0	0	0	2.86E−14	0	0
F_6	6.55E−08	5.35E−06	1.24E−10	2.60E−08	0	0	0	0	0	0	0	4.08E−10	0	0
F_7	1.05E−09	1.05E−06	1.31E−11	2.29E−09	0	0	0	0	0	0	0	9.01E−12	0	0
F_8	−106.765	−106.764	−106.765	−106.765	−106.765	−106.181	−106.765	−106.765	−106.765	−106.765	−106.765	−106.765	−106.765	−106.765
F_9	0.001451	0.416159	0	0	0	0	0	0	0	0	0	0	0	0
F_{10}	0.004088	0.285438	0	0	0	0	0	0	0	0	0	0	0	0
F_{11}	0.002944	0.224658	0	0	0	0	0	0	0	0	0	0	0	0
F_{12}	5.01E−09	1.29E−05	2.56E−11	7.56E−09	0	0	0	0	0	0	2.92E−09	8.51E−07	0	0
F_{13}	0.397887	0.397892	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887
F_{14}	5.558987	5.634131	5.559037	6.651694	5.559037	5.952089	5.559037	5.559037	5.559037	5.846894	5.559037	6.481648	5.559037	5.559037
F_{15}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{16}	2.27E−07	1.43E−05	1.07E−08	8.44E−07	0	0	0	0	0	0	2.00E−12	4.20E−07	0	0
F_{17}	0.035705	0.186095	0.04515	0.103461	0.000451	0.028893	0.012301	0.07921	0.004926	0.035614	0.00047	0.028643	0.000526	0.028888
F_{18}	5.14E−09	4.19E−07	0	0	0	0	0	0	0	0	0	0	0	0
F_{19}	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316
F_{20}	−24.1568	−24.1567	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568
F_{21}	−1000	−1000	−1999.8	−1027.89	−1000.03	−1000	−1000.57	−1000.01	−1000	−1000	−2000	−1999.96	−2000	−2000
F_{22}	−999.98	−91.357	−1999.77	−1017.98	−1000	−1000	−999.999	−951.279	−1000.01	−999.834	−1000	−1000	−2000	−1968.92
F_{23}	−42.9443	−42.9406	−42.9444	−42.9423	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444	−42.9056	−42.9444	−42.9444
F_{24}	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261
F_{25}	1.57E−07	4.73E−05	8.62E−11	2.11E−08	2.86E−10	0.00048	0	0	0	0	0	1.49E−08	0	0
F_{26}	0.001012	1.876688	4.20E−08	1.561132	2	2	0	0.004784	0	0.166709	0	7.64E−08	0	0.04

Table 5 (continued)

Fun. no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{27}	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1	−24,771.1
F_{28}	−0.9976	−0.74896	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
F_{29}	16.69464	9813.009	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128
F_{30}	1.34E−07	8.29E−06	0	0	0	0	0	0	0	0	0	0	0	0
F_{31}	2.36E−08	1.02E−06	1.59E−11	2.04E−09	0	0	0	0	0	0	0	1.79E−11	0	0
F_{32}	1.47E−06	0.000271	1.92E−08	9.461207	0	0	0	0	0	0	6.12E−11	5.65E−05	0	0
F_{33}	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447
F_{34}	3	3.000002	3	3	3	3	3	3	3	3	3	3	3	3
F_{35}	−165.953	−165.953	−165.953	−164.664	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953
F_{36}	3.61E−08	7.81E−06	1.50E−09	9.13E−07	0	0	0	0	0	0	0	4.13E−09	0	0
F_{37}	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458
F_{38}	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3688	124.3622	124.3622
F_{39}	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367
F_{40}	1.63E−10	9.51E−09	4.17E−10	1.56E−08	0	1.84E−06	0	0	0	0	0	7.34E−09	0	0
F_{41}	−176.542	−176.53	−176.542	−175.012	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542
F_{42}	−176.137	−176.132	−176.138	−172.956	−176.138	−174.867	−176.138	−176.138	−176.138	−175.821	−176.138	−176.138	−176.138	−176.138
F_{43}	8.29E−09	4.30E−07	0	0	0	0	0	0	0	0	0	0	0	0
F_{44}	−1.91322	−1.91322	−1.91322	−1.91322	−1.9133	−1.91323	−1.91322	−1.91322	−1.91322	−1.91322	−1.91322	−1.91322	−1.91322	−1.91322
F_{45}	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683	−19.6683
F_{46}	−1.8013	−1.8013	−1.8013	−1.79329	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.8013	−1.78527	−1.8013	−1.8013
F_{47}	−0.18425	−0.18119	−0.18434	−0.15555	−0.18465	−0.14896	−0.18465	−0.18459	−0.18465	−0.18463	−0.18465	−0.17718	−0.18465	−0.18465
F_{48}	−0.19912	−0.19628	−0.19911	−0.1778	−0.19941	−0.16462	−0.19941	−0.19937	−0.19941	−0.19938	−0.19941	−0.19151	−0.19941	−0.19941
F_{49}	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983	−1.01983
F_{50}	−2.28395	−2.28394	−2.28395	−2.27136	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395	−2.28395

Table 5 (continued)

Fun. no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{51}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{52}	0	8.71E−10	0	2.34E−11	0	0	0	3.20E−11	0	0	0	0	0	0
F_{53}	6.89E−09	3.77E−07	2.14E−12	1.41E−06	0	0	0	5.47E−11	0	0	0	2.72E−10	0	0
F_{54}	−0.96353	−0.96353	−0.96353	−0.96353	−0.96354	−0.96354	−0.96353	−0.96353	−0.96353	−0.96353	−0.96353	−0.96353	−0.96353	−0.96353
F_{55}	0.9	0.900005	0.9	0.9	0.9	0.918	0.9	0.9	0.9	0.9	0.9	0.913	0.9	0.9
F_{56}	0.006839	7.265038	1.91E−10	2.71E−08	0	0	0	0	0	0	0	1.38E−10	0	0
F_{57}	0.9	0.900005	0.9	0.9	0.9	0.918	0.9	0.9	0.9	0.9	0.9	0.913	0.9	0.9
F_{58}	0.65479	75,037.07	8.69E−11	0.002818	0	0	0	0	0	0.000167	0	0.001734	0	0.001928
F_{59}	0.000885	47,660.31	0	6.49E−12	0	2.20E−07	0	2.30E−12	0	1.09E−11	0	1.79E−07	0	0
F_{60}	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72	−3873.72
F_{61}	−2.2	−2.19999	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.2	−2.19912	−2.2	−2.2
F_{62}	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
F_{63}	34.3712	35.95635	34.3712	69.24454	34.3712	64.09282	34.3712	34.3712	34.3712	53.78931	34.3712	34.3712	34.3712	34.3712
F_{64}	5.306884	212.2401	0	0	0	0	0	0	0	0	0	0	0	0
F_{65}	0.080101	24.566	0	0	0	0	0	0	0	0	0	0	0	0
F_{66}	0.549318	81,320.38	0	7.21E−09	0	0	0	0	0	1.73E−12	0	1.59E−07	0	0
F_{67}	4.86E−11	6.23E−05	0	0	0	0	0	0	0	0	0	0	0	0
F_{68}	8.72E−09	0.000364	0	0	0	0	0	0	0	0	0	0	0	0
F_{69}	0.001584	0.002703	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567	0.001568	0.001567	0.001567
F_{70}	0.292492	0.293369	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579
F_{71}	0.004079	0.286967	3.14E−06	7.24E−05	0	2.81E−09	0	0	0	0	9.80E−05	0.01327	0	0
F_{72}	−3455.48	−703.783	−3456	−3456	−3456	−3456	−3456	−3456	−3456	−3283.2	−3456	−2315.52	−3456	−3456
F_{73}	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203	−26.9203
F_{74}	−19.2085	−19.2084	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085	−19.2085

Table 5 (continued)

Fun. no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{75}	−24.1568	−24.1567	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568
F_{76}	−10.8723	−10.8723	−10.8723	−10.8643	−10.8723	−10.8648	−10.8723	−10.8723	−10.8723	−10.8723	−10.8723	−10.867	−10.8723	−10.8723
F_{77}	1.80E−08	6.06E−07	0	1.65E−09	0	0	0	0	0	0	0	5.65E−09	0	0
F_{78}	−3.30679	−3.28198	−3.30687	−3.25464	−3.30687	−3.22729	−3.30687	−3.30687	−3.30687	−3.30684	−3.30687	−3.26623	−3.30687	−3.29403
F_{79}	0.02442	0.276014	1.54E−05	0.651549	0	0.06	0	0.017727	0	0	7.60E−06	0.441524	0	0.01
F_{80}	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681	−4.81681
F_{81}	−2.49999	−2.49991	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5	−2.5
F_{82}	−1.49999	−1.49994	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5	−1.5
F_{83}	−7.307	−7.307	−7.307	−7.30576	−7.307	−7.307	−7.307	−7.3062	−7.307	−7.30684	−7.307	−7.30431	−7.307	−7.30631
F_{84}	−400	−399.997	−400	−400	−400	−400	−400	−400	−400	−400	−400	−400	−400	−400
F_{85}	3.223916	3.5E+08	1.35E−09	4.41E−06	0	7.01E−12	0	0	0	0	1.07E−05	0.005194	0	0
F_{86}	18.48875	23,150.16	0	1.92E−08	0	1.68E−09	0	0	0	0	0	7.23E−06	0	0
F_{87}	21.63851	1416.965	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35	21.35
F_{88}	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379	−0.00379
F_{89}	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523
F_{90}	0.106125	20.70758	0	0	0	0	0	0	0	0	0	0	0	0
F_{91}	8.22E−07	1.91E−05	3.51E−10	1.13E−08	0	0	0	0	0	0	0	1.11E−10	0	0
F_{92}	2.10E−05	0.000346	1.29E−08	1.77E−06	1.38E−09	2.21E−05	0	0	0	0	6.66E−07	0.002308	0	0
F_{93}	−3.86278	−3.86278	−3.86278	−3.8623	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.86205	−3.86278	−3.86278
F_{94}	1.26E−07	7.91E−05	3.01E−11	5.71E−08	0	4.02E−06	0	1.33E−08	0	9.76E−14	0	8.40E−06	0	0
F_{95}	0.024536	0.024536	0.024536	0.024536	4.51E−05	0.000148	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536	0.024536
F_{96}	1.02E−11	3.44E−07	0	1.53E−06	0	0	0	6.84E−11	0	0	0	4.20E−08	0	0
F_{97}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{98}	8.45E−06	0.000171	3.62E−09	0.001005	1.36E−09	0.003382	0	0	0	0	1.23E−06	0.008834	0	0

Table 5 (continued)

Fun. no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{99}	0.000817	0.065175	1.25E−07	0.52241	2.98E−07	0.163367	0	0	0	0.03676	2.11E−05	5.092655	0	0
F_{100}	383,413.3	1,501,111	0	0	0	0	0	0	0	0	0	0	0	0
F_{101}	735.4406	3914.083	0.000786	15.93805	0	2.21E−12	0	0.023545	0	1.54E−07	3.880078	9790.16	0	7.22E−09
F_{102}	2.70E−12	2.24E−11	2.70E−12	5.14E−11	2.70E−12	1.94E−09	2.70E−12	1.19E−10	2.70E−12	4.18E−11	2.70E−12	5.02E−10	2.70E−12	1.01E−10
F_{103}	0.000346	0.000584	0.000308	0.004165	0.000308	0.000509	0.000308	0.000308	0.000308	0.000309	0.000309	0.000515	0.000308	0.000419
F_{104}	0	0	0	0	0	3.99E−14	0	0	0	0	0	0	0	0
F_{105}	−10.1524	−9.96915	−10.1532	−9.49407	−10.1532	−9.6743	−10.1532	−9.74536	−10.1532	−10.1532	−10.1532	−10.1532	−10.1532	−9.89921
F_{106}	−10.4028	−10.394	−10.4029	−10.2966	−10.4029	−9.61895	−10.4029	−10.2966	−10.4029	−10.2966	−10.4029	−10.3497	−10.4029	−10.1771
F_{107}	−10.5357	−10.5275	−10.5364	−10.3746	−10.5364	−9.46476	−10.5364	−10.4282	−10.5364	−10.417	−10.5364	−10.4282	−10.5364	−10.1995
F_{108}	3.29E−05	0.005536	3.22E−06	0.008677	1.63E−06	0.013395	1.98E−07	0.001867	2.79E−05	0.007951	2.32E−06	0.01277	0	0.004049
F_{109}	12.50226	199.6639	0.250057	1604.8	0	107.7299	2.52E−07	11.49758	4.28E−06	16.54835	40.63317	4196.345	0	1.122848
F_{110}	−529.485	−524.122	−529.87	−519.428	−529.871	−529.871	−529.871	−529.87	−529.871	−529.865	−529.871	−529.18	−529.871	−529.865
F_{111}	−0.965	−0.965	−0.965	−0.93382	−0.965	−0.79213	−0.965	−0.95994	−0.965	−0.95817	−0.965	−0.7702	−0.965	−0.95343
F_{112}	0.003661	0.008535	2.04E−05	0.004182	1.72E−05	0.005065	2.48E−06	0.001033	1.71E−05	0.002266	1.55E−05	0.028219	0	0.003141
F_{113}	−3.32236	−3.29177	−3.32236	−3.27028	−3.32236	−3.31759	−3.32236	−3.26991	−3.32236	−3.31822	−3.32236	−3.24167	−3.32236	−3.27706
F_{114}	−49.7715	−48.7025	−50	−50	−50	−50	−50	−50	−50	−50	−50	−50	−50	−50
F_{115}	0.95979	0.964521	0.95979	0.96812	0.95979	0.95979	0.95979	0.965342	0.95979	0.963269	0.95979	0.967555	0.95979	0.96519
F_{116}	−45.776	−45.7725	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778
F_{117}	−36.1947	468.218	−210	−187.832	−210	−210	−210	−210	−210	−210	−210	−210	−210	−210

Table 6 The standard deviation (Std. Dev.) and mean of function evaluations (Fun. Evl.) for 2D mathematical functions

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_1	0.043876	150,000	0	2126.5	0	7085.809	0	8112	0	6657	0	27191	0	4739.639
F_2	0.001822	150,000	4.57E−13	21,551	4.57E−13	1131.947	4.57E−13	3548	4.57E−13	2312	4.57E−13	1972.5	4.57E−13	2783.422
F_3	2.32E−13	150,000	8.32E−14	150,000	3.57E−15	119.0143	2.23E−15	150,000	2.23E−15	150,000	1.85E−15	150,000	2.23E−15	150,000
F_4	27.92913	150,000	0	1842	0	5521.321	0	6942	0	5513	0	24,476.5	0	4078.376
F_5	3.75E−07	150,000	3.08E−09	150,000	0	35,139.11	0	10,734	0	6353	2.04E−13	45,705.5	0	6183.991
F_6	6.04E−06	150,000	2.28E−08	150,000	0	2618.315	0	18,780	0	7550	1.36E−09	11,2853	0	29,731.28
F_7	1.25E−06	150,000	2.45E−09	150,000	0	2975.358	0	7086	0	3138	2.71E−11	108,212.5	0	3669.285
F_8	9.37E−05	150,000	1.95E−06	150,000	3.335263	150,000	5.58E−14	150,000	5.56E−14	150,000	9.29E−09	150,000	5.47E−14	150,000
F_9	0.213791	150,000	0	1265	0	3544.74	0	5004	0	4122	0	7994.5	0	2888.703
F_{10}	0.131701	150,000	0	1277	0	4785.739	0	5222	0	4610	0	9272.5	0	3258.189
F_{11}	0.147322	150,000	0	1448	0	58,536.7	0	6532	0	5749	0	27,831.5	0	3748.997
F_{12}	1.25E−05	150,000	8.25E−09	150,000	0	13,480.6	0	8832	0	4050	7.65E−07	150,000	0	4574.699
F_{13}	5.30E−06	150,000	6.65E−08	150,000	1.06E−15	150,000	1.06E−15	150,000	1.06E−15	150,000	5.43E−09	150,000	1.06E−15	150,000
F_{14}	0.298728	102,682.9	0.540671	142,156	0.578127	49,056.45	7.14E−15	7536	0.529347	43,274	0.571001	110,567.5	7.14E−15	7529.078
F_{15}	0	1256.098	0	101	0	164.6278	0	202	0	103	0	100	0	50.13369
F_{16}	9.02E−06	150,000	9.30E−07	150,000	0	4124.607	0	10,876	0	4569	5.78E−07	150,000	0	5486.631
F_{17}	0.088395	150,000	0.041041	150,000	0.016875	150,000	0.044072	150,000	0.015599	150,000	0.014523	150,000	0.017325	150,000
F_{18}	3.92E−07	150,000	0	987.5	0	3126.879	0	3784	0	3293	0	5832.5	0	2360.795
F_{19}	1.56E−15	64,646.34	1.56E−15	4581.5	1.56E−15	714.6103	1.56E−15	2186	1.56E−15	1551	1.56E−15	1232	1.56E−15	1775.234
F_{20}	0.000132	150,000	2.21E−06	149,997.5	3.57E−15	156.2391	3.57E−15	9958	3.57E−15	4093	1.46E−06	108,026.5	3.57E−15	9456.217
F_{21}	0	150,000	171.9491	150,000	0.003296	150,000	0.058308	150,000	3.08E−09	150,000	0.096132	150,000	9.14E−13	66,227.11
F_{22}	199.916	150,000	140.9966	150,000	5.87E−07	150,000	135.5187	150,000	1.162674	150,000	3.78E−06	150,000	171.2863	13,9636.4
F_{23}	0.003496	150,000	0.010369	150,000	5.01E−14	150,000	5.00E−14	150,000	5.00E−14	150,000	0.121991	150,000	5.00E−14	150,000
F_{24}	9.12E−07	150,000	2.14E−10	150,000	2.68E−15	150,000	2.68E−15	150,000	2.68E−15	150,000	1.32E−12	150,000	2.68E−15	150,000

Table 6 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{25}	4.00E−05	150,000	2.88E−08	150,000	0.001539	150,000	0	63,534	0	30,414	3.98E−08	146,135	0	24,286.76
F_{26}	0.466089	150,000	0.777145	150,000	0	150,000	0.036065	114,958	0.360281	148,065	2.27E−07	149,117	0.281411	46,829.88
F_{27}	0.022115	90,658.54	0	921	0	607.1304	0	1660	0	1147	0	813.5	0	1465.408
F_{28}	0.368956	150,000	1.47E−08	150,000	0	1440.231	0	11,130	0	7309	3.16E−10	139,885	0	6023.061
F_{29}	21,626.32	150,000	2.45E−15	140,972	2.45E−15	9795.351	2.45E−15	5646	2.45E−15	3431	2.45E−15	36,047.5	2.45E−15	3776.07
F_{30}	7.85E−06	150,000	0	1089.5	0	2543.341	0	4376	0	3910	0	4771	0	2684.158
F_{31}	1.07E−06	150,000	1.76E−09	150,000	0	2946.522	0	7018	0	3143	6.43E−11	103,976	0	3663.269
F_{32}	0.000288	150,000	19.63336	150,000	0	7443.901	0	9918	0	7170	0.000179	150,000	0	6806.15
F_{33}	4.60E−10	150,000	2.08E−10	150,000	8.91E−17	150,000	8.59E−17	150,000	1.04E−16	150,000	1.78E−11	150,000	1.04E−16	150,000
F_{34}	1.99E−06	150,000	5.63E−07	150,000	0	3774.904	0	9088	0	4282	3.24E−07	150,000	0	4804.813
F_{35}	0.000267	81,073.17	5.090788	31,677	5.71E−14	867.7036	5.71E−14	7710	5.71E−14	5641	5.71E−14	1813	5.71E−14	9650.234
F_{36}	8.13E−06	150,000	6.81E−06	150,000	0	3728.242	0	15,438	0	7885	1.26E−08	130,172.5	0	23,950.37
F_{37}	2.68E−15	11,975.61	2.68E−15	71,088	2.68E−15	566.2356	2.68E−15	1734	2.68E−15	1020	2.68E−15	1604.5	2.68E−15	916.4439
F_{38}	5.44E−06	150,000	9.60E−07	150,000	1.57E−13	150,000	1.45E−13	150,000	1.44E−13	150,000	0.00667	150,000	1.44E−13	150,000
F_{39}	2.92E−11	150,000	1.79E−09	150,000	1.12E−06	150,000	8.93E−16	150,000	8.93E−16	150,000	6.58E−08	150,000	8.93E−16	150,000
F_{40}	1.02E−08	150,000	1.73E−08	150,000	7.26E−06	14,6731	0	40,970	0	18,887	2.32E−08	143,110	0	15,120.82
F_{41}	0.010656	150,000	7.930173	150,000	2.38E−13	150,000	1.69E−09	150,000	2.32E−13	150,000	6.69E−07	150,000	2.45E−13	150,000
F_{42}	0.006086	150,000	9.591822	143,906	6.255525	13,378.36	3.43E−13	7854	3.161241	5832	3.43E−13	9985	3.43E−13	6647.727
F_{43}	4.18E−07	150,000	0	1131.5	0	19,759.52	0	4728	0	3856	0	2431.5	0	2896.725
F_{44}	6.47E−08	150,000	1.29E−09	150,000	1.52E−05	144,013.1	4.02E−15	150,000	4.02E−15	150,000	1.42E−11	150,000	4.02E−15	150,000
F_{45}	3.21E−14	2048.78	3.21E−14	279	3.21E−14	382.7333	3.21E−14	558	3.21E−14	491	3.21E−14	259.5	3.21E−14	297.7941
F_{46}	1.56E−15	35,865.85	0.08013	141,591	1.56E−15	1030.758	1.56E−15	3004	1.56E−15	1609	0.112747	8374	1.56E−15	1773.73
F_{47}	0.001612	150,000	0.052615	150,000	0.031644	13,4968.5	0.000103	98,226	9.95E−05	61,574	0.018672	144,838.5	5.58E−17	66,274.73

Table 6 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{48}	0.001653	150,000	0.04844	150,000	0.037396	125,801.8	6.17E−05	109,696	7.65E−05	85,096	0.020491	149,222.5	4.18E−16	55,840.91
F_{49}	3.35E−10	150,000	1.43E−11	150,000	8.93E−16	465.0472	1.79E−15	150,000	1.79E−15	150,000	1.79E−15	150,000	1.79E−15	150,000
F_{50}	1.30E−05	150,000	0.071941	150,000	2.72E−15	150,000	2.68E−15	150,000	2.68E−15	150,000	1.39E−08	150,000	2.68E−15	150,000
F_{51}	0	2317.073	0	336.5	0	211.2898	0	570	0	422	0	334	0	332.8877
F_{52}	1.91E−09	14,4646.3	2.13E−10	113,626.5	0	807.41	1.37E−10	118,782	0	39,046	0	21,375	0	16,217.75
F_{53}	4.42E−07	150,000	1.01E−05	150,000	0	2198.357	5.42E−10	58,166	0	12,603	8.70E−10	111,612	0	44,189.84
F_{54}	1.93E−07	150,000	1.36E−06	150,000	1.34E−15	110.6257	2.34E−15	150,000	2.34E−15	150,000	2.40E−10	150,000	2.34E−15	150,000
F_{55}	4.64E−06	150,000	7.81E−16	3254.5	0.038612	31,492.14	7.81E−16	11,290	7.81E−16	13,714	0.0338	27,126.5	7.81E−16	12,715.91
F_{56}	6.217954	150,000	2.90E−08	150,000	0	3646.452	0	18,196	0	8077	4.90E−10	108,033.5	0	19,505.51
F_{57}	4.64E−06	150,000	7.81E−16	3254.5	0.038612	31,492.14	7.81E−16	11,290	7.81E−16	13,714	0.0338	27,126.5	7.81E−16	12,715.91
F_{58}	12,9654.2	150,000	0.003617	150,000	0	46,593.32	0	24,288	0.001057	20,408	0.003289	106,303.5	0.003269	54,750
F_{59}	174,515.5	150,000	1.46E−11	109,673	1.99E−06	44,659.73	8.07E−12	89,620	3.08E−11	106,511	1.00E−06	11,5472.5	0	18,658.26
F_{60}	0.000557	150,000	1.09E−08	150,000	8.25E−12	150,000	8.23E−12	150,000	8.23E−12	150,000	4.12E−08	150,000	8.23E−12	150,000
F_{61}	8.34E−06	150,000	2.09E−06	150,000	4.02E−15	5360.888	4.02E−15	28,688	4.02E−15	12,290	0.001442	150,000	4.02E−15	20,657.59
F_{62}	2.05E−08	150,000	5.86E−09	150,000	0	3486.543	0	10,676	0	5929	3.10E−10	144,795.5	0	6994.652
F_{63}	7.804749	12,560.98	12.94271	132,403.5	17.24623	112,709.2	3.57E−14	4612	19.91024	75,139	3.57E−14	2030.5	3.57E−14	2977.941
F_{64}	227.1377	150,000	0	1381	0	9813.702	0	5232	0	4300	0	8152.5	0	3092.246
F_{65}	21.71972	150,000	0	1293.5	0	7901.608	0	4864	0	4033	0	6499.5	0	2925.301
F_{66}	262,168.6	150,000	1.53E−08	51,927	0	4075.323	0	18,858	1.55E−11	44,961	1.51E−06	135,264	0	20,969.42
F_{67}	0.000206	150,000	0	644	0	1879.063	0	3404	0	3240	0	1740.5	0	2014.372
F_{68}	0.000857	150,000	0	825	0	20,608.88	0	5136	0	4749	0	10,822	0	2930.816
F_{69}	0.001055	150,000	9.50E−09	150,000	2.60E−17	150,000	2.72E−17	150,000	1.81E−17	150,000	3.94E−06	150,000	1.60E−17	150,000
F_{70}	0.000993	144,707.3	0	788.5	0	3114.296	0	3188	0	3015	0	2643	0	1990.809

Table 6 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{71}	0.139554	150,000	4.38E−05	150,000	1.82E−08	91,328.56	0	23,456	0	9066	0.014428	150,000	0	10,989.81
F_{72}	1348.218	150,000	2.45E−06	150,000	0	69.73086	0	12,372	757.0123	13,903	1633.242	150,000	0	8890.207
F_{73}	7.37E−05	150,000	1.23E−06	150,000	3.57E−15	161.482	4.19E−14	150,000	4.12E−14	150,000	8.15E−07	150,000	4.12E−14	150,000
F_{74}	7.28E−05	148,073.2	0.000423	148,882.5	2.86E−14	116.9172	2.86E−14	8074	2.86E−14	3535	2.86E−14	5793	2.86E−14	9033.59
F_{75}	0.000132	150,000	2.21E−06	149,997.5	3.57E−15	156.2391	3.57E−15	9958	3.57E−15	4093	1.46E−06	108,026.5	3.57E−15	9456.217
F_{76}	2.62E−05	150,000	0.009727	78,742.5	0.009663	60,748.69	1.79E−15	15,966	1.79E−15	13,928	0.008838	58,813.5	1.79E−15	17,636.53
F_{77}	6.05E−07	150,000	4.94E−09	32,487.5	0	2340.965	0	4800	0	4735	2.95E−08	68,515	0	8035.929
F_{78}	0.027406	150,000	0.107615	150,000	0.11077	150,000	2.77E−15	150,000	0.000293	150,000	0.090871	150,000	0.03337	150,000
F_{79}	0.142678	150,000	0.716219	150,000	0.238683	13,738.55	0.125755	39,488	0	26,018	0.623702	150,000	0.1	35,728.28
F_{80}	3.45E−08	150,000	6.36E−09	150,000	1.16E−14	2376.617	1.16E−14	6598	1.16E−14	3036	6.92E−09	146,051.5	1.16E−14	3322.36
F_{81}	4.58E−05	150,000	0	1864	0	5074.1	0	7406	0	6390	0	21,929	0	4522.059
F_{82}	3.36E−05	150,000	0	1817.5	0	5299.546	0	7020	0	6020	0	22,152.5	0	4272.894
F_{83}	4.38E−08	150,000	0.001371	150,000	1.07E−14	1647.85	0.001209	99,026	0.000587	27,552	0.000386	148,999	0.001194	79,580.21
F_{84}	0.003509	150,000	0	1127	0	2618.315	0	4852	0	4210	0	5573	0	2996.491
F_{85}	1.96E+09	150,000	3.00E−05	150,000	6.60E−11	102,906	0	21,958	0	10,803	0.016538	150,000	0	21,629.18
F_{86}	40,729.86	150,000	9.46E−08	149,993.5	9.62E−09	138,874.5	0	19,144	0	9807	3.61E−05	141,033	0	13,265.37
F_{87}	2530.442	150,000	4.28E−14	410	4.28E−14	698.8815	4.28E−14	1130	4.28E−14	1009	4.28E−14	908	4.28E−14	637.7005
F_{88}	1.07E−06	144,878	6.97E−18	867.5	6.97E−18	1616.393	6.97E−18	2300	6.97E−18	1729	6.97E−18	2014	6.97E−18	1518.048
F_{89}	3.35E−16	48,792.68	3.35E−16	678	3.35E−16	603.9846	3.35E−16	1382	3.35E−16	1025	3.35E−16	814.5	3.35E−16	787.0989
F_{90}	19.68525	150,000	0	1278.5	0	3641.734	0	4696	0	3919	0	9159.5	0	2741.31
F_{91}	1.22E−05	150,000	8.48E−09	150,000	0	37,803.04	0	12,676	0	6131	2.53E−10	141,195.5	0	7370.655
F_{92}	0.000209	150,000	2.52E−06	150,000	5.17E−05	150,000	0	64,252	0	31,121	0.002065	150,000	0	22,263.87
F_{93}	1.07E−08	150,000	0.001838	150,000	5.79E−15	150,000	5.77E−15	150,000	5.75E−15	150,000	0.002047	150,000	5.74E−15	150,000

Table 6 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{94}	8.44E−05	150,000	6.57E−08	150,000	3.40E−05	77,858.44	1.06E−07	139,428	5.59E−13	60,606	4.50E−05	141,912	0	70,003.68
F_{95}	6.08E−15	150,000	1.04E−12	150,000	6.88E−05	150,000	1.05E−17	150,000	1.05E−17	150,000	6.31E−18	150,000	1.05E−17	150,000
F_{96}	6.29E−07	150,000	9.27E−06	148,532	0	1772.108	6.35E−10	73,988	0	24,941	4.15E−07	121,938.5	0	46,074.36
F_{97}	0	2829.268	0	155	0	370.6746	0	200	0	101	0	102	0	50.13369
F_{98}	9.59E−05	150,000	0.00707	150,000	0.009727	150,000	0	41,874	0	25,056	0.014423	150,000	0	26,218.92
F_{99}	0.037731	150,000	1.255673	150,000	0.514559	150,000	0	88,972	0.3676	53,513	9.346715	150,000	0	42,200.53
F_{100}	711,166.5	150,000	0	1795.5	0	7417.686	0	3546	0	3795	0	13,098.5	0	3409.592
F_{101}	811.3904	150,000	75.068	150,000	1.14E−11	105,211.3	0.186754	105,128	9.74E−07	77,353	11,958.47	150,000	4.05E−08	11,0008.4
F_{102}	8.81E−11	119,292.7	1.93E−10	103,094.5	3.33E−09	147,734	2.88E−10	109,172	1.70E−10	74,164	6.33E−10	129,992.5	2.47E−10	111,937
F_{103}	0.000122	150,000	0.007888	147,687	0.000234	149,870	1.63E−19	39,658	8.63E−06	53,334	0.000325	150,000	0.000302	41,811.5
F_{104}	0	7146.341	0	804.5	2.32E−13	85,236.28	0	1608	0	1042	0	2315.5	0	459.2246
F_{105}	1.02275	150,000	1.713648	150,000	1.547523	150,000	1.390021	150,000	0.000184	150,000	1.21E−05	150,000	1.112699	150,000
F_{106}	0.00491	150,000	0.74788	149,975.5	2.053979	23,168.47	0.747881	15,140	0.747881	12,130	0.531522	49,464	1.118363	14,738.8
F_{107}	0.004752	150,000	0.924479	150,000	2.412304	29,241.87	0.760923	14,186	0.843924	10,921	0.760921	86,482	1.345665	18,152.91
F_{108}	0.006858	150,000	0.006924	150,000	0.004784	150,000	0.004814	150,000	0.007312	150,000	0.009761	150,000	0.006554	13,2420.1
F_{109}	81.17334	150,000	2646.08	150,000	1077.299	17,732.09	18.46546	150,000	31.49175	150,000	3816.799	150,000	4.750317	148,610.8
F_{110}	2.980047	150,000	35.64045	150,000	1.37E−12	1114.121	0.011818	86,958	0.062868	82,913	1.2937	146,796.5	0.025498	79,443.35
F_{111}	7.06E−06	7158.537	0.076281	102,969.5	0.159289	116,325.8	0.015794	22,588	0.016525	48,922	0.182781	127,875.5	0.020829	61,147.56
F_{112}	0.001729	150,000	0.002264	150,000	0.007437	150,000	0.001708	150,000	0.002285	150,000	0.028604	150,000	0.00279	149,507.7
F_{113}	0.051999	109,146.3	0.067312	149,422.5	0.023476	9324.013	0.059466	72,696	0.021089	28,826	0.119349	97,058.5	0.058149	59,152.74
F_{114}	0.527168	150,000	3.74E−06	150,000	5.94E−13	114,349.7	0	17,468	0	10,897	1.47E−07	150,000	0	9504.846
F_{115}	0.002829	117,402.4	0.005267	138,605.5	8.93E−16	2817.546	0.003035	12,4816	0.00299	91,631	0.003681	145,747	0.003405	120,581.6
F_{116}	0.001423	150,000	6.43E−14	149,844.5	6.43E−14	3134.219	6.43E−14	7058	6.43E−14	5896	6.43E−14	36,549	6.43E−14	5325.702

Table 6 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{117}	176.0505	150,000	40.11871	150,000	5.49E−06	150,000	0	27,788	1.68E−12	96,074	0.000224	150,000	1.19E−12	32,827.04

Table 7 The minimum and mean values of the 50D mathematical functions for different metaheuristics

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{118}	20.381235	20.50565	0	0	5.04E−11	1.12E−08	0	0	0	0.426474	0	0	0	0
F_{119}	5.4233419	8.546599	0	1.15E−07	0	1.11E−07	0	0	0	0	0	0	0	0
F_{120}	0.0360813	0.050099	0	0	0	0	0	0	0	0	0	0	0	0
F_{121}	1.087E+09	1.72E+09	0	0	0	0	0	0	0	0	0	0	0	0
F_{122}	3.27E−12	1.11E−11	0	0	0	0	0	0	0	0	0	0	0	0
F_{123}	−0.993424	−0.990587	−0.999702	−0.910992	−1	−0.997473	−0.93148	−0.882864	−0.999889	−0.898661	−0.999859	−0.858739	−0.890149	−0.840595
F_{124}	−0.999628	−0.997335	−0.999725	−0.92839	−1	−1	−0.976282	−0.916823	−1	−0.996733	−0.999997	−0.943559	−0.901522	−0.858192
F_{125}	264.90199	490.3996	0.6666667	0.666667	0.6666667	3.949027	0.6666667	0.666667	0.6666667	0.666667	0.6666669	0.66667	0.6666667	0.666667
F_{126}	−0.208962	−0.01718	−1.03E−10	−1.03E−12	−1	−0.39	−1	−0.950159	−3.41E−23	−1.02E−24	−0.999641	−0.990955	−1	−0.970096
F_{127}	−0.999744	−0.999668	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
F_{128}	9.2437303	11.31371	0	0.000304	0	0.022023	0	0	0	0	0	0.000762	0	0
F_{129}	25.451383	82.16855	0	0	0	0	0	0	0	0	0	0	0	0
F_{130}	2.29E+18	8.73E+18	0	0	633,739.93	19,547,079	0	0	0	0	0	0	0	0
F_{131}	−13.54793	−10.03256	−49	−48.45589	−32.97777	−27.96647	−49	−45.24404	−35.63975	−29.11456	−49	−49	−49	−40.64257
F_{132}	1.836757	2.660342	1.452947	2.284753	0	0.000895	0.1790565	0.593572	0.6266978	1.265034	0.0006725	0.027819	0	0.323198
F_{133}	2.0193696	2.024891	2.0600683	1137.398	2	2	2	2	2	2	2	2	2	2
F_{134}	2.01904	2.02534	2.0788286	1412.075	2	2	2	2	2	2	2	2	2	2
F_{135}	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128	9.25E+128
F_{136}	2.69E−07	8.95E−07	0	1.01E−12	0	0	0	2.08E−05	0	9.92E−15	0	0	0	0
F_{137}	11.706461	13.12133	8.4603009	17.52603	10.694567	12.63216	6.4003225	8.64812	8.8077372	10.67097	0	0.00327	0	0.740721
F_{138}	4403.9404	5395.454	0	0	1422.6431	6768.277	0	0	0	0	0	0	0	0
F_{139}	4.2539444	9.935475	1.53E−11	3.36E−06	0.0237504	0.045578	0	0	0	5.70E−08	0	7.22E−09	0	0
F_{140}	19.103824	27.68775	0	0	1.54E−08	2.17E−05	0	0	0	0	0	0	0	0
F_{141}	1.39E−09	1.28E−08	0	0	0	0	0	0	0	0	0	0	0	0
F_{142}	112.68122	155.6537	0	0	54.723095	91.89453	0	0	0	13.6032	0	0	0	0.000235
F_{143}	2.672E+11	3.74E+11	3149.4241	7897.126	0	0	0	0	1.89E−12	6.58E−06	37.994795	412.4954	0	12.14012

Table 7 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{144}	116.11819	150.026	16.167052	37.32237	0	1.20E−11	3.28E−09	0.037474	1.82E−06	0.075992	2.5175229	12.92886	9.28E−08	0.169231
F_{145}	968,483.34	186,4485	44.89665	46.58408	2.2465077	139.5909	33.166918	36.48892	36.836742	40.74561	44.998504	45.61452	29.900335	36.01464
F_{146}	18.116479	21.19853	0.0998733	0.150418	0.8998733	1.268004	0.0998733	0.099873	0.0998733	0.148452	0	0.125882	0	0.055929
F_{147}	53,770,073	1.11E+08	0	0	0	0	0	0	0	0	0	0	0	0
F_{148}	1.29E+27	5.52E+27	0	0	0	0	0	0	0	0	0	0	0	0
F_{149}	5.35E−11	2.54E−05	2.01E−12	4.39E−08	0	0	9.64E−11	3.81E−06	0	1.65E−06	0	0	0	5.64E−11
F_{150}	1.7913223	2.475727	5.3483658	13.42373	1.92E−05	5.19E−05	5.46E−07	0.000108	0.0017685	0.022515	0.0045706	5.206693	9.35E−12	6.43E−10
F_{151}	979.14111	1096.032	0	0	0	1.55E−11	0	0	0	0	0	0	0	0
F_{152}	71.563828	83.56584	0	0	1.3707849	2.601344	0	0	0	0	6.81E−08	44.72021	0	0
F_{153}	1.58E+51	3.62E+56	0	0	2.80E−12	1.11E−10	0	0	0	0	0	0	0	0
F_{154}	0.3598278	7.798554	0	0	0	0	0	0	0	0	0	0	0	0
F_{155}	1.8337912	2.573681	6.8771565	13.47726	4.33E−05	0.00011	9.14E−07	0.00146	0.0034109	0.067705	0.0047254	4.97904	2.22E−11	2.13E−09
F_{156}	− 156.5671	− 133.7004	− 231.2476	− 181.6286	− 380.0135	− 309.7157	− 384.589	− 316.6897	− 304.6938	− 256.6237	− 418.9827	− 409.6193	− 289.1935	− 260.9953
F_{157}	0.0540401	0.078156	0	0	0	0	0	0	0	0	0	0	0	0
F_{158}	981	1076.81	0	0	0	0	0	0	0	0	0	0	0	0
F_{159}	30,456	41,443.82	0	0	0	0	0	0	0	0	0	0	0	0
F_{160}	31,922	41,086.85	0	0.01	0	0	0	0.07	0	0.22	0	0	0	0
F_{161}	− 275	− 273.9	− 254	− 231.15	− 275	− 275	− 275	− 272.73	− 275	− 272.2	− 275	− 275	− 275	− 275
F_{162}	20.076561	27.23315	0.0024305	0.074509	10.187035	13.05764	0.0005524	0.017441	0.0574294	0.218473	0	0.366565	0	1.2087
F_{163}	114.47094	153.5335	0	0	0	0	0	0	0	0	0	0	0	0
F_{164}	− 1867.043	− 1770.141	− 1678.529	− 1443.208	− 1774.531	− 1677.129	− 1958.3	− 1949.524	− 1816.941	− 1690.969	− 1958.3	− 1949.453	− 1958.306	− 1937.367
F_{165}	43,784,778	63,850,282	− 1046.745	− 345.1535	− 13,175.21	14,770.75	− 4475.024	− 3084.629	− 2853.152	− 2224.325	− 18,926.11	− 13,919.19	− 21,988.17	− 18,301.1
F_{166}	0.3701119	3.836854	0	0	2.2137688	66.35352	0	0	0	0	0	0	0	0
F_{167}	2,482,350.9	3,002,533	31.917057	53.58512	1	1.426401	13.722952	73.97508	19.386726	31.21802	5.4566182	76.41341	20.281137	37.93278

Table 7 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{168}	0.318961	0.39861	0	0	0.140724	0.20046	0	0	0.07501	0.177383	0	0	0	0.074077
F_{169}	0	0	0	6.76E−217	0	0	0	0	0	0	−1	−0.22	−1	−0.167991
F_{170}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{171}	0	8.89E−14	0	2.74E−13	0	0	0	0	0	0	0	0	0	0
F_{172}	0.0005211	2.869414	1.48E−12	7.32E−06	330,772.96	1.96E+15	0	4.78E−08	1.84E−11	4.07E−08	0	6.74E+17	0	3.34E−06
F_{173}	4.08E−21	5.62E−21	2.07E−25	3.71E−24	1.56E−44	1.06E−36	4.39E−46	4.04E−33	1.82E−33	2.18E−23	−1	−0.22	−1	−0.676587
F_{174}	9.05E−07	0.000714	7.00E−07	0.000629	1.01E−05	0.000664	3.02E−06	0.000571	1.09E−05	0.00064	1.24E−06	0.000752	3.94E−05	0.000802
F_{175}	43.057427	99.40362	0	0	1.046464	8.283508	0	0	0	0	588.59277	834.1466	0	0

Table 8 The standard deviation (Std. Dev.) and mean of function evaluations (Fun. Evl.) for 50D mathematical functions

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{118}	0.0502242	150,000	0	15,589	7.26E−08	150,000	0	21,110	1.2778323	33,729	0	35,359	0	44,081.45
F_{119}	1.5091166	150,000	1.15E−06	17,454.5	7.06E−07	149,988.5	0	20,338	0	15,893	0	33,364.5	0	33,463.66
F_{120}	0.0055062	150,000	0	8806.5	0	67,845.51	0	10,690	0	8339	0	21,161.5	0	15,810.78
F_{121}	303,655,817	150,000	0	6968.5	0	46,243.62	0	8408	0	6451	0	12,861	0	12,523.81
F_{122}	5.55E−12	150,000	0	4288.5	0	25,494.23	0	3746	0	2811	0	4093.5	0	5020.05
F_{123}	0.0011142	150,000	0.0695635	150,000	0.0165183	63,556.8	0.0162111	150,000	0.0979795	150,000	0.0721503	150,000	0.0188715	150,000
F_{124}	0.0057492	150,000	0.0660064	150,000	0	4419.783	0.0246438	150,000	0.0089068	149,697	0.0584204	150,000	0.0189725	150,000
F_{125}	102.69093	150,000	1.74E−07	150,000	2.7936929	150,000	4.63E−15	150,000	1.44E−12	150,000	5.42E−06	150,000	1.49E−15	150,000
F_{126}	0.0502911	150,000	1.03E−11	150,000	0.4902071	134,058.9	0.2183445	105,020	5.84E−24	150,000	0.0048422	150,000	0.1708999	107,827.1
F_{127}	3.13E−05	150,000	0	7914	0	55,353.2	0	9696	0	7436	0	18,365	0	14,496.24
F_{128}	0.9322215	150,000	0.0023868	12,752	0.0269142	124,642.1	0	11,976	0	9179	0.0037646	30,607	0	18,972.43
F_{129}	21.963345	150,000	0	7298.5	0	50,846.38	0	7350	0	5581	0	11,684	0	10,655.39
F_{130}	2.24E+18	150,000	0	15,071	25,555,416	150,000	0	21,866	0	16,776	0	35,850	0	33,477.44
F_{131}	1.2734135	150,000	2.981015	27,450	2.3406331	150,000	7.9192416	66,992	3.2433096	150,000	0	27,622	7.1608731	131,423.6
F_{132}	0.3589449	150,000	0.3017913	150,000	0.0089528	119,584.2	0.2093464	150,000	0.2665134	150,000	0.0600299	150,000	0.1598958	149,339.6
F_{133}	0.0022817	150,000	4475.1206	150,000	0	7383.083	0	44,948	0	57,472	0	197.5	0	142.8571
F_{134}	0.0022788	150,000	7945.5179	150,000	0	7401.957	0	46,684	0	58,078	0	195.5	0	142.8571
F_{135}	7.73E+113	150,000	7.73E+113	150,000	7.73E+113	150,000	7.73E+113	150,000	7.73E+113	150,000	1.21E+114	150,000	2.71E+114	150,000
F_{136}	2.37E−07	150,000	2.13E−12	149,743.5	0	24,247.47	0.000208	20,082	9.92E−14	7458	0	258.5	0	255.6391
F_{137}	0.4582672	150,000	2.4164374	150,000	0.8432095	150,000	1.0860518	150,000	0.8644548	150,000	0.0126245	130,393	1.3275984	82,187.97
F_{138}	411.36205	150,000	0	12,483	2289.4916	150,000	0	14,712	0	11,486	0	28,595	0	22,032.58
F_{139}	2.8237858	150,000	8.08E−06	150,000	0.0100069	150,000	0	14,230	1.56E−07	13,7572	5.84E−08	58,066.5	0	25,556.39
F_{140}	3.2700471	150,000	0	16,862	7.62E−06	150,000	0	12,512	0	9608	0	26,631.5	0	18,754.39
F_{141}	9.39E−09	150,000	0	2776.5	0	11,760.4	0	3576	0	2860	0	5493	0	6754.386
F_{142}	15.836323	150,000	0	13,689	15.18284	150,000	0	16,548	11.299551	107,789	0	27,595.5	0.0023517	36,982.46
F_{143}	3.781E+10	150,000	2285.1069	150,000	0	110,279.1	0	94,522	3.87E−05	150,000	333.85131	150,000	62.594012	147,562.7

Table 8 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{144}	13.014069	150,000	9.8463404	150,000	1.94E−11	149,840.1	0.1692307	150,000	0.4710474	150,000	7.4427843	150,000	1.495165	150,000
F_{145}	435,984.7	150,000	0.8583632	150,000	118.44994	150,000	1.4995873	150,000	1.0381569	150,000	0.2641631	150,000	1.459143	150,000
F_{146}	0.8220206	150,000	0.049999	150,000	0.1523753	150,000	4.54E−10	150,000	0.0498513	150,000	0.0645244	142,321.5	0.0498256	126,602.8
F_{147}	18,359,676	150,000	0	8175.5	0	60,235.93	0	8398	0	6375	0	16,516.5	0	12,340.85
F_{148}	2.72E+27	150,000	0	4598	0	25,769.49	0	5330	0	4131	0	4587	0	7656.642
F_{149}	7.88E−05	150,000	1.12E−07	150,000	0	1288.71	6.65E−06	150,000	3.34E−06	145,840	0	108,944.5	2.84E−10	86,010.03
F_{150}	0.266992	150,000	2.7424309	150,000	2.62E−05	150,000	0.0003368	150,000	0.0443137	150,000	8.6844085	150,000	1.48E−09	150,000
F_{151}	46.30883	150,000	0	15,086	8.29E−11	148,686.1	0	22,916	0	17,816	0	35,281.5	0	35,953.63
F_{152}	4.7471357	150,000	0	28,445	0.7344186	150,000	0	27,604	0	20,469	34.196865	150,000	0	42,665.41
F_{153}	9.27E+56	150,000	0	15,292	3.05E−10	150,000	0	23,818	0	18,281	0	35,218.5	0	39,768.17
F_{154}	5.4097314	150,000	0	5214	0	32,977.46	0	4086	0	3041	0	5108.5	0	5667.92
F_{155}	0.3082883	150,000	2.7609375	150,000	4.40E−05	150,000	0.0118753	150,000	0.2756654	150,000	8.3596718	150,000	3.68E−09	150,000
F_{156}	6.3690482	150,000	26.967055	150,000	24.835349	150,000	23.103259	150,000	20.841874	150,000	21.293816	150,000	11.119107	150,000
F_{157}	0.0109852	150,000	0	6018.5	0	24,807.41	0	930	0	3862	0	176	0	661.6541
F_{158}	39.589738	150,000	0	2802.5	0	21,020.97	0	3194	0	2509	0	4724	0	4503.759
F_{159}	3587.5477	150,000	0	3604	0	39,728.24	0	3690	0	2876	0	3721.5	0	5016.291
F_{160}	3547.0089	150,000	0.1	10,773	0	41,537.05	0.2564324	32,866	0.4623174	54,652	0	5950	0	26,840.85
F_{161}	0.745356	149,195.1	9.2467576	150,000	0	10,550.33	1.469247	141,308	3.931227	109,593	0	204.5	0	9127.82
F_{162}	2.7633682	150,000	0.4066735	150,000	1.1510244	150,000	0.0103176	150,000	0.6609536	150,000	1.0231986	91,658.5	1.6991519	116,977.4
F_{163}	15.234952	150,000	0	9869.5	0	80,473.09	0	12,472	0	9553	0	26,869	0	18,340.85
F_{164}	48.112958	150,000	82.121655	150,000	43.925051	150,000	10.575646	138,762	49.17714	150,000	33.090213	139,289	33.272057	143,714.3
F_{165}	6,532,202.5	150,000	184.38304	150,000	19,442.377	150,000	590.36613	150,000	211.58439	150,000	1312.2997	150,000	2598.8445	150,000
F_{166}	3.2023366	150,000	0	7714.5	45.844919	150,000	0	960	0	2754	0	179.5	0	730.5764
F_{167}	203,135.86	150,000	9.9327909	150,000	1.4897637	145,242.6	25.571569	150,000	5.122057	150,000	36.883061	150,000	20.422169	150,000

Table 8 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{168}	0.0313466	150,000	0	15,023	0.022912	150,000	0	26,304	0.0436488	150,000	0	22,232.5	0.0574611	138,759.4
F_{169}	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000	0.4163332	128,166	0.3735783	139,037.6
F_{170}	0	1219.512	0	50	0	52.42922	0	200	0	100	0	50	0	125.3133
F_{171}	3.68E−13	96,743.9	1.63E−12	56,106.5	0	1137.19	0	2580	0	191	0	103.5	0	125.3133
F_{172}	10.93318	150,000	2.23E−05	150,000	1.93E+16	150,000	2.27E−07	141,998	9.06E−08	150,000	4.64E+18	146,100	9.78E−06	147,280.7
F_{173}	6.27E−22	150,000	2.09E−23	150,000	7.38E−36	150,000	4.01E−32	150,000	1.77E−22	150,000	0.4163332	124,637	0.2711994	137,743.1
F_{174}	0.0006792	150,000	0.0005986	150,000	0.0006581	150,000	0.0005028	150,000	0.0006213	150,000	0.0008318	150,000	0.0008051	150,000
F_{175}	21.948499	150,000	0	28,495	5.1567128	150,000	0	99,998	0	116,969	145.36549	150,000	0	115,467.4

Table 9 The minimum and mean values of the 100D mathematical functions for different metaheuristics

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{176}	20.586604	20.718846	0	0	1.431615	12.31205	0	0	0	1.143617	0	0	0	0
F_{177}	50.405563	64.109332	0	4.56E−07	0.002751	0.028801	0	0	0	0	0	0	0	0
F_{178}	0.4772714	0.6804868	0	0	0	3.34E−13	0	0	0	0	0	0	0	0
F_{179}	1.78E+10	2.286E+10	0	0	0	0	0	0	0	0	0	0	0	0
F_{180}	6.10E−06	3.20E−05	0	0	0	0	0	0	0	0	0	0	0	0
F_{181}	−0.96588	−0.950316	−0.9197	−0.85527	−1	−0.99489	−0.80803	−0.76742	−0.97195	−0.80501	−0.99991	−0.8224	−0.76477	−0.72196
F_{182}	−0.997313	−0.990482	−0.98654	−0.90918	−1	−1	−0.99612	−0.83238	−0.99993	−0.97698	−1	−0.93357	−0.79327	−0.74487
F_{183}	29,900.156	55,885.031	0.666667	0.666667	2.831224	24.74766	0.666667	0.666667	0.666667	0.666667	0.666667	0.66667	0.666667	0.666667
F_{184}	0	0	0	0	0	0	0	0	0	0	−0.99151	−0.95805	0	0
F_{185}	−0.996165	−0.993911	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
F_{186}	34.352598	38.765115	0	0.000207	0	0.008733	0	0	0	0	0	0.000593	0	0
F_{187}	6058.917	13693.944	0	0	0	0	0	0	0	0	0	0	0	0
F_{188}	2.58E+33	1.03E+34	0	0	5.24E+18	1.76E+21	0	0	0	0	0	0	0	0
F_{189}	−15.43269	−10.68408	−99	−98.287	−41.5804	−35.5744	−99	−85.1954	−62.571	−51.7626	−99	−98.6672	−99	−81.9432
F_{190}	35.389796	57.928454	5.350569	6.133826	19.01878	77.51377	1.969623	3.258554	2.32782	3.71216	0.0184	0.10947	1.074365	2.193779
F_{191}	2.0982297	2.1162261	228438.4	2.05E+22	2	2	2	2	2	2	2	2	2	2
F_{192}	2.1052274	2.1175193	12975012	4.68E+20	2	2	2	2	2	2	2	2	2	2
F_{193}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{194}	1.11E−05	3.20E−05	0	0	0	0	0	9.64E−07	0	1.92E−08	0	0	0	0
F_{195}	27.907562	30.328962	26.20246	41.39556	29.62579	32.504	17.7514	22.89792	20.31474	24.0459	0	0.014734	0	1.042993
F_{196}	42,154.219	50,103.324	0	0	35,735.5	50,791.51	0	0	0	0	0	0	0	0
F_{197}	117.6082	206.06059	3.89E−10	2.68E−06	0.331522	0.665042	0	0	0	0	0	0	0	0
F_{198}	284.41418	471.64742	0	0	2.71E−06	0.000209	0	0	0	0	0	0	0	0
F_{199}	3.73E−08	1.50E−07	0	0	0	0	0	0	0	0	0	0	0	0
F_{200}	522.08913	610.80144	0	0	284.5577	462.9585	0	0	0	0.054343	0	0	0	0
F_{201}	6.567E+11	8.868E+11	80,312.3	121,658.9	1.87E−07	4.28E−06	0.000376	23.54445	2.142336	1056.635	3026.633	13,397.27	0.008689	4650.467

Table 9 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{202}	613.33009	1013.8289	124.1739	157.1621	4.97E−06	0.103038	0.616641	7.63817	0.486121	9.675635	13.14432	31.25526	0.190459	4.484061
F_{203}	36,057,720	61,493,505	95.06887	96.9346	124.0419	347.5509	84.29788	88.95276	91.02669	93.11738	95.09095	95.67857	82.73053	87.48213
F_{204}	35.728656	39.577622	0.099873	0.197873	2.299873	2.924874	0.099873	0.099873	0.199873	0.199873	0	0.131882	0	0.056928
F_{205}	538,327,823	698,836,295	0	0	0	0	0	0	0	0	0	0	0	0
F_{206}	5.64E+30	1.22E+31	0	0	0	0	0	0	0	0	0	0	0	0
F_{207}	2.90E−09	4.35E−05	4.23E−12	6.78E−08	0	0	8.54E−10	5.48E−06	9.78E−11	4.00E−06	0	1.60E−13	0	4.27E−10
F_{208}	15.437678	19.820979	34.57359	44.35066	0.062136	0.137295	0.238105	1.595233	1.226237	3.795858	0.282725	14.60447	0.007558	0.079592
F_{209}	2779.6739	2976.7993	0	0	8.69E−07	1.87E−05	0	0	0	0	0	0	0	0
F_{210}	92.82625	95.754961	0	0	15.44394	24.65842	0	0	0	0	0.004299	66.81846	0	0
F_{211}	5.34E+118	8.37E+126	0	0	0.000492	0.667995	0	0	0	0	0	0	0	0
F_{212}	113,932.46	777,593.76	0	0	0	0	0	0	0	0	0	0	0	0
F_{213}	16.003313	19.521313	33.08182	43.6676	0.085316	0.184241	0.248299	1.674584	1.074701	4.063953	0.736897	14.51532	0.006548	0.085683
F_{214}	−108.9406	−96.67194	−203.5	−166.817	−275.176	−247.671	−323.521	−243.441	−279.378	−228.29	−418.982	−407.921	−220.435	−191.575
F_{215}	0.5276357	0.6688825	0	0	0	0	0	0	0	0	0	0	0	0
F_{216}	2681	2936.95	0	0	0	0.01	0	0	0	0	0	0	0	0
F_{217}	12,9887	149,515.44	0	0	0	0.28	0	0	0	0	0	0.01	0	0
F_{218}	13,0660	148,673.85	0	0.1	0	0.26	0	2.19	0	2.98	0	0.03	0	2.08
F_{219}	−563	−557.62	−452	−394.95	−575	−575	−572	−561.03	−575	−561.19	−575	−575	−575	−575
F_{220}	70.470914	82.893324	0.005697	0.053734	24.53558	29.99663	2.32E−06	0.020105	0.033113	0.066364	0	0.00031	0	1.838743
F_{221}	2382.5386	3193.669	0	0	4.09E−11	9.99E−10	0	0	0	0	0	0	0	0
F_{222}	−3505.919	−3347.853	−2805.38	−2476.86	−3478.38	−3305.2	−3817.19	−3682.41	−3534.51	−3300.96	−3916.6	−3904.09	−3765.07	−3592.18
F_{223}	1.996E+09	2.39E+09	−654.126	−208.763	147,572.8	1,095,234	−2994.74	−2325.87	−1825.32	−1418.01	−48,365.5	−39,875.4	−23,507	−13,483.4
F_{224}	23.702927	103.74066	0	0	128.3375	615.7201	0	0	0	0	0	0	0	0
F_{225}	5,588,992.9	6,702,583.6	112.2197	162.2476	23.00566	67.61354	86.12778	260.447	52.15774	81.48437	66.26364	183.3127	55.26998	103.272

Table 9 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean	Minimum	Mean
F_{226}	0.4708199	0.541511	0	0	0.380988	0.435385	0	0	0.121418	0.258575	0	0	0	0.068812
F_{227}	0	0	0	0	0	0	0	0	0	0	−1	−0.01	−1	−0.11
F_{228}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{229}	0	0	0	0	0	0	0	0	0	0	0	0	0	0
F_{230}	58,691,612	6.901E+14	0	7.32E−06	3.87E+20	7.70E+38	0	1.34E−08	1.12E−12	1.69E−08	5.65E−11	1.78E+48	0	8.20E−07
F_{231}	9.53E−41	5.52E−40	1.14E−42	3.06E−41	1.14E−42	1.14E−42	1.14E−42	1.14E−42	1.14E−42	1.30E−42	−1	−0.25	−1	−0.54305
F_{232}	2.35E−05	0.0014437	2.65E−05	0.001191	2.81E−05	0.001734	1.51E−05	0.001508	8.77E−06	0.001628	2.30E−07	0.001156	3.41E−05	0.001812
F_{233}	603.08682	831.89671	0	0	86.79895	241.4952	2.21E−09	4.20E−06	7.47E−05	0.001364	1333.962	1691.183	0	6.01E−07

Table 10 The standard deviation (Std. Dev.) and mean of function evaluations (Fun. Evl.) for 100D mathematical functions

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{176}	0.0339569	150,000	0	22,349	7.557597	150,000	0	22,166	1.924839	54,872	0	35,901.5	0	46,119.05
F_{177}	5.932246	150,000	3.23E−06	23,989.5	0.048226	150,000	0	21,390	0	16,682	0	34,011.5	0	34,293.23
F_{178}	0.101094	150,000	0	13,474	8.85E−13	144,647.5	0	11,506	0	8932	0	22,504.5	0	17,195.49
F_{179}	2.033E+09	150,000	0	10,894.5	0	97,618.49	0	9124	0	6931	0	14,405.5	0	13,299.5
F_{180}	2.69E−05	150,000	0	7711.5	0	60,046.14	0	4102	0	3037	0	4022	0	5457.393
F_{181}	0.0068974	150,000	0.0269	150,000	0.034602	132,728.8	0.019791	150,000	0.110787	150,000	0.102061	150,000	0.017099	150,000
F_{182}	0.0070677	150,000	0.052672	150,000	0	2004.893	0.062164	150,000	0.023255	150,000	0.060665	150,000	0.019847	150,000
F_{183}	13,236.319	150,000	3.43E−08	150,000	9.926055	150,000	1.20E−11	150,000	4.66E−11	150,000	1.84E−06	150,000	1.88E−11	150,000
F_{184}	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000	0.025808	150,000	0	150,000
F_{185}	0.0011067	150,000	0	12,215.5	0	117,743.4	0	10,494	0	7960	0	19,782	0	15,392.23
F_{186}	1.6862662	150,000	0.001457	16,981.5	0.015275	149,552.3	0	12,616	0	9585	0.004218	28,442.5	0	19,347.12
F_{187}	3880.8462	150,000	0	11,963	0	109,146.6	0	8084	0	6023	0	13,310.5	0	11,476.19
F_{188}	2.80E+33	150,000	0	27,181	3.82E+21	150,000	0	33,416	0	25,514	0	41,346.5	0	51,765.66
F_{189}	1.50344	150,000	4.429801	31,074.5	2.892269	150,000	26.17784	65,630	6.373321	150,000	3.327879	29,384	12.20145	140,394.7
F_{190}	11.455529	150,000	0.41391	150,000	26.33093	150,000	0.507665	150,000	0.540065	150,000	0.149237	150,000	0.382965	150,000
F_{191}	0.0066603	150,000	2.04E+23	150,000	0	12,063.44	3.84E−13	93,804	5.20E−13	111,463	0	214.5	0	159.1479
F_{192}	0.0051334	150,000	2.69E+21	150,000	0	12,394.79	3.11E−13	93,178	4.49E−13	114,755	0	215.5	0	157.8947
F_{193}	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000
F_{194}	1.33E−05	150,000	0	120,699	0	46,941.98	9.64E−06	70,544	1.11E−07	89,143	0	259.5	0	328.3208
F_{195}	0.7153455	150,000	3.508229	150,000	1.281088	150,000	1.88645	150,000	1.210444	150,000	0.101184	121,770.5	2.163753	85,948.62
F_{196}	2908.1072	150,000	0	18,288	7284.559	150,000	0	15,906	0	12,312	0	29,865.5	0	24,180.45
F_{197}	43.234391	150,000	9.25E−06	150,000	0.170813	150,000	0	13,312	0	12,003	0	27,131.5	0	19,788.22
F_{198}	90.610204	150,000	0	19,732.5	0.00013	150,000	0	13,416	0	10,228	0	27,793	0	20,045.11
F_{199}	8.03E−08	150,000	0	6479	0	18,130.02	0	3740	0	2849	0	5733	0	7660.401
F_{200}	35.616087	150,000	0	18,339	55.72744	150,000	0	15,778	0.543433	15,536	0	28,237	0	26,860.9
F_{201}	6.969E+10	150,000	15,685.24	150,000	1.92E−05	150,000	139.3809	150,000	2162.643	150,000	5299.9	150,000	7502.695	150,000

Table 10 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{202}	298.45565	150,000	15.65079	150,000	1.002643	150,000	5.382007	150,000	5.728631	150,000	14.4403	150,000	4.294693	150,000
F_{203}	11,188,416	150,000	0.880912	150,000	217.5971	150,000	1.604963	150,000	1.153092	150,000	0.311334	150,000	1.593047	150,000
F_{204}	0.9873711	150,000	0.034757	150,000	0.245105	150,000	7.20E−10	150,000	1.26E−08	150,000	0.06174	142,418.5	0.049694	123,912.3
F_{205}	58,432,546	150,000	0	13,107	0	125,095.6	0	9098	0	6791	0	17,249	0	13,086.47
F_{206}	3.24E+30	150,000	0	7557	0	55,765.29	0	5884	0	4506	0	4754	0	8278.195
F_{207}	8.61E−05	150,000	1.45E−07	150,000	0	1400.384	9.53E−06	150,000	7.25E−06	150,000	8.19E−13	12,0767	1.26E−09	119,036.3
F_{208}	1.451369	150,000	3.758966	150,000	0.034201	150,000	1.248085	150,000	1.381343	150,000	16.07992	150,000	0.135719	150,000
F_{209}	68.734707	150,000	0	21,382.5	6.40E−05	15,0000	0	24,222	0	18,900	0	36,039.5	0	38,451.13
F_{210}	1.031102	150,000	0	53,689	4.078716	150,000	0	30,258	0	21,672	29.8041	150,000	0	44,120.3
F_{211}	2.42E+127	150,000	0	21,521.5	3.362426	150,000	0	24,768	0	19,165	0	35,990	0	41,540.1
F_{212}	497,836.36	150,000	0	9376.5	0	70,436.04	0	4576	0	3286	0	5619	0	5984.962
F_{213}	1.4601584	150,000	4.184353	150,000	0.041578	150,000	1.388724	150,000	1.840684	150,000	15.83864	150,000	0.14199	150,000
F_{214}	3.8393077	150,000	18.06028	150,000	14.00817	150,000	22.72475	150,000	24.66386	150,000	18.47507	150,000	9.652136	150,000
F_{215}	0.0580323	150,000	0	9243	0	56,510.31	0	1592	0	5561	0	191	0	1195.489
F_{216}	76.98529	150,000	0	4826	0.1	63,363.86	0	3516	0	2700	0	4115	0	4946.115
F_{217}	7204.3127	150,000	0	6355	0.514045	122,390.8	0	4042	0	3071	0.1	5855.5	0	5511.278
F_{218}	7049.1499	150,000	0.362372	32,044	0.504925	118,454.9	1.447359	14,3534	2.34835	14,6539	0.171447	11,636	1.685829	13,8130.3
F_{219}	1.9631967	150,000	15.8378	150,000	0	16,580.22	5.580368	150,000	11.00936	147,936	0	220	0	48,085.21
F_{220}	4.8369888	150,000	0.129323	150,000	2.192558	150,000	0.022185	150,000	0.028711	150,000	0.001771	49,846	2.526459	128,730.6
F_{221}	380.13745	150,000	0	151,45	1.83E−09	150,000	0	13,658	0	10,376	0	28,206	0	20,423.56
F_{222}	62.421035	150,000	128.5537	150,000	58.28286	150,000	49.86395	150,000	86.09474	150,000	53.78904	145,875	108.3805	150,000
F_{223}	157,878,662	150,000	101.7254	150,000	663,038.7	150,000	300.9213	150,000	132.6016	150,000	3293.196	150,000	3424.112	150,000
F_{224}	55.168355	150,000	0	11,434	325.9464	150,000	0	1654	0	5494	0	192.5	0	1246.867
F_{225}	337,357.68	150,000	15.48844	150,000	23.12218	150,000	50.6982	150,000	9.529613	150,000	54.11003	150,000	70.13358	150,000

Table 10 (continued)

Fun no.	Alternative metaheuristic algorithms													
	FA		GWO		ICA		SOS		TLBO		WOA		CGO	
	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.	Std. Dev.	Fun. Evl.
F_{226}	0.0242259	150,000	0	18,422	0.024887	150,000	0	22,506	0.073102	150,000	0	22,684.5	0.0538	136,917.3
F_{227}	0	150,000	0	150,000	0	150,000	0	150,000	0	150,000	0.1	148,963.5	0.314466	140,294.5
F_{228}	0	1219.512	0	50	0	150,000	0	200	0	100	0	50	0	125.3133
F_{229}	0	1219.512	0	99	0	106.9556	0	200	0	100	0	99	0	125.3133
F_{230}	4.86E+15	150,000	2.64E−05	144,137	4.63E+39	150,000	7.15E−08	144,764	3.82E−08	150,000	1.76E+49	150,000	1.57E−06	139,121.6
F_{231}	4.11E−40	150,000	7.09E−41	150,000	1.28E−48	150,000	2.23E−45	150,000	8.20E−43	150,000	0.435194	121,244.5	0.327413	133,388.5
F_{232}	0.0011401	150,000	0.001172	150,000	0.001664	150,000	0.001142	150,000	0.001349	150,000	0.001268	150,000	0.001756	150,000
F_{233}	74.331983	150,000	0	67,208	68.90903	150,000	9.54E−06	150,000	0.001661	150,000	177.0925	150,000	1.41E−06	136,701.8

Table 11 The mean and standard deviation (Std. Dev.) of the composite mathematical functions for different metaheuristics

Fun no.		ABC	DE (Mirjalili et al. 2014)	FA (Mirjalili 2015)	GA (Mirjalili 2015)	GWO (Mirjalili et al. 2014)	PSO (Mirjalili 2015)	WOA (Mirjalili and Lewis 2016)	CGO
F_{234}	Mean	8.813585	6.75E−02	175.9715	92.13909	43.83544	137.7789	0.568846	50.00025
	Std. Dev	16.62675	1.11E−01	86.928	27.90131	69.86146	116.3128	0.505946	64.35362
F_{235}	Mean	71.4911	28.759	353.6269	96.70927	91.80086	166.6643	75.30874	49.56733
	Std. Dev	16.32421	8.6277	103.423	9.703147	95.5518	164.3894	43.07855	67.33909
F_{236}	Mean	280.0985	144.41	308.0516	369.1036	61.43776	394.507	55.65147	149.7654
	Std. Dev	29.50073	19.401	37.435	42.84275	68.68816	121.949	21.87944	43.30166
F_{237}	Mean	347.8401	324.86	548.5276	450.829	123.1235	468.3534	53.83778	308.7539
	Std. Dev	11.24064	14.784	162.8993	31.54446	163.9937	67.31685	21.621	57.15233
F_{238}	Mean	41.11309	10.789	175.1975	95.92017	102.1429	256.5258	77.8064	31.58148
	Std. Dev	15.01647	2.604	83.15078	53.79146	81.25536	200.3816	52.02346	44.96027
F_{239}	Mean	548.5086	490.94	829.5929	523.7037	43.14261	790.1284	57.88445	489.6279
	Std. Dev	23.92378	39.461	157.2787	22.92001	84.48573	189.4915	37.4460	31.1774

functions, the results of the CGO algorithm are superior to the results of other metaheuristics considering the minimum, mean, standard deviation and function evaluation values except for the function evaluation of the 100D functions that the results of the WOA is better than the results of the CGO.

6.3 Wilcoxon singed-rank test

The Wilcoxon (W) singed-rank test is a statistical nonparametric test for examining the differences between different samples in a one by one manner. As a main criterion, if the mean ranks for one sample has lower values than the other one, the sample with smaller mean of ranks has better statistical results and the utilized metaheuristic is superior to the other one. The results of the W test for different mathematical functions based on the obtained results of the optimization runs have been presented in Table 15. In this table, the upper and lower values are the mean of the ranks related to the CGO algorithm and the other metaheuristics, respectively. Based on the statistical results, related vales for the mean of the ranks for the CGO are lower than those of the other metaheuristics in most of the cases. The related *p* values of this test for different mathematical functions are also presented in Table 16.

6.4 Kruskal–Wallis test

The Kruskal–Wallis (K–W) test is a non-parametric algorithm for testing whether or not different statistical samples are originated from the same distribution. It is used for comparing two or more independent samples of equal or different sample sizes. It extends the Mann–Whitney U test, which is used for comparing only two groups. A significant K–W test indicates that at least one sample stochastically dominates one other sample. This test provides the mean of the ranks for multiple sets of statistical data which are considered for comparing analysis. As a main criterion, if the mean of the ranks for one sample has lower values than the other ones, the one with smaller mean of ranks has better statistical results. The results of the K–W test for different mathematical functions are presented in Table 17. Based on the statistical results, the superiority of the CGO algorithms is proved. In this table, the bolded values are related to the better metaheuristic while the vales related to the CGO algorithm are all italicized. It should be noted that the CGO algorithm is the best in the ranking in dealing with 2D functions for the minimum, mean and standard deviation while it is second for the function evaluation. Considering 50D functions, the CGO algorithm is on the first place in the ranking considering the minimum, mean and function evaluation while for the standard deviation, the CGO is second best algorithm. For the 100D functions, the proposed CGO algorithm is on the first place in the ranking determining the minimum and mean while for the standard deviation and the function evaluation, the CGO is second best algorithm.

6.5 Post-hoc analysis

Post-hoc is a Latin phrase, meaning "after this" or "after the event". In a scientific study, Post-hoc (P-H) analysis consists of statistical analyses that were not specified before the data was seen. A P-H analysis involves looking at the data after a study has

Table 12 The K–S test results including difference between data (Diff.) and p values for different algorithms

Main algorithm	Function type	Data type	Alternative metaheuristic algorithms											
			FA		GWO		ICA		SOS		TLBO		WOA	
			p values	Diff.	p values	Diff.	p values	Diff.	p values	Diff.	p values	Diff.	p values	Diff.
CGO	2D	Min.	4.80E−22	**0.6410**	6.92E−07	0.3504	1.00E+0	*0.0427*	1.00E+0	0.0427	9.97E−01	0.0513	4.04E−02	0.1795
		Mean	8.43E−20	**0.6068**	3.30E−08	0.3846	4.43E−01	0.1111	9.97E−01	*0.0513*	7.68E−01	0.0855	6.92E−07	0.3504
		Std.	7.49E−29	**0.7350**	3.36E−15	0.5299	1.85E−02	0.1966	9.97E−01	*0.0513*	6.58E−01	0.0940	1.77E−12	0.4786
		Fun. Evl.	8.26E−24	**0.6667**	3.20E−11	0.4530	1.14E−01	0.1538	1.14E−01	0.1538	6.58E−01	*0.0940*	3.30E−08	0.3846
	50D	Min.	1.09E−15	0.7586	3.37E−02	0.2586	1.92E−02	0.2759	4.54E−01	0.1552	1.46E−01	0.2069	7.65E−01	*0.1207*
		Mean	1.54E−09	0.5862	4.54E−01	0.1552	3.24E−01	0.1724	9.99E−01	*0.0690*	7.65E−01	0.1207	9.77E−01	0.0862
		Std.	5.31E−08	**0.5345**	7.65E−01	0.1207	6.07E−01	0.1379	9.77E−01	0.0862	9.99E−01	*0.0690*	8.99E−01	0.1034
		Fun. Evl.	5.30E−15	**0.7414**	1.92E−02	0.2759	1.05E−02	0.2931	7.65E−01	0.1207	1.46E−01	0.2069	9.77E−01	*0.0862*
	100D	Min.	4.87E−13	0.6897	2.22E−01	0.1897	2.85E−03	0.3276	6.07E−01	0.1379	3.24E−01	0.1724	1.00E+0	*0.0517*
		Mean	5.20E−09	0.5690	7.65E−01	0.1207	3.37E−02	0.2586	9.99E−01	*0.0690*	8.99E−01	0.1034	9.77E−01	0.0862
		Std.	1.61E−07	**0.5172**	8.99E−01	0.1034	5.70E−02	0.2414	9.99E−01	*0.0690*	9.77E−01	0.0862	7.65E−01	0.1207
		Fun. Evl.	4.87E−13	**0.6897**	2.22E−01	0.1897	1.35E−04	0.3966	3.24E−01	0.1724	1.46E−01	0.2069	7.65E−01	*0.1207*

Table 13 The M–W test results (summation of the ranks) for 2D, 50D and 100D mathematical functions

Main algorithm	Function type	Data type	Alternative algorithms					
			FA	GWO	ICA	SOS	TLBO	WOA
CGO	2D	Min.	18,321.00	15,962.00	14,017.00	14,053.50	14,074.00	14,960.00
			9174.00	**11,533.00**	**13,478.00**	**13,441.50**	**13,421.00**	**12,535.00**
		Mean	18,147.00	16,284.00	14,607.00	13,966.50	14,307.50	16,027.50
			9348.00	**11,211.00**	**12,888.00**	**13,528.50**	**13,187.50**	**11,467.50**
		Std.	18,558.00	16,998.00	14,943.00	14,017.50	14,318.00	16,658.00
			8937.00	**10,497.00**	**12,552.00**	**13,477.50**	**13,177.00**	**10,837.00**
		Fun. Evl.	18,534.00	16,040.00	**13,391.00**	14,438.00	13,922.00	16,029.00
			8961.00	**11,455.00**	14,104.00	**13,057.00**	**13,573.00**	**11,466.00**
CGO	50D	Min.	4734.00	3840.00	3852.00	3593.50	3714.00	3540.50
			2052.00	**2946.00**	**2934.00**	**3192.50**	**3072.00**	**3245.50**
		Mean	4568.00	3655.00	3697.00	**3392.00**	3555.00	3459.00
			2218.00	**3131.00**	**3089.00**	3394.00	**3231.00**	**3327.00**
		Std.	4423.00	3563.00	3619.00	**3313.00**	3446.00	3464.00
			2363.00	**3223.00**	**3167.00**	3473.00	**3340.00**	**3322.00**
		Fun. Evl.	4657.00	3608.00	3940.00	3468.00	3547.00	3420.00
			2129.00	**3178.00**	**2846.00**	**3318.00**	**3239.00**	**3366.00**
CGO	100D	Min.	4708.00	3717.00	3963.50	3610.00	3678.00	3431.50
			2078.00	**3069.00**	**2822.50**	**3176.00**	**3108.00**	**3354.50**
		Mean	4547.50	3549.50	3845.50	3472.50	3554.50	3447.00
			2238.50	**3236.50**	**2940.50**	**3313.50**	**3231.50**	**3339.00**
		Std.	4404.00	3486.50	3804.50	**3393.00**	3482.50	3516.00
			2382.00	**3299.50**	**2981.50**	**3393.00**	**3303.50**	**3270.00**
		Fun. Evl.	4523.00	3577.00	4086.00	3509.00	3523.00	**3361.00**
			2263.00	**3209.00**	**2700.00**	**3277.00**	**3263.00**	3425.00

Table 14 The M–W test results (p values) for 2D, 50D and 100D mathematical functions

Main algorithm	Function type	Data type	Alternative Algorithms					
			FA	GWO	ICA	SOS	TLBO	WOA
CGO	2D	Min.	3.66E−21	5.72E−07	4.58E−01	3.99E−01	3.68E−01	2.72E−03
		Mean	2.70E−18	2.09E−07	5.90E−02	6.22E−01	2.14E−01	2.74E−06
		Std.	5.44E−22	1.24E−11	5.94E−03	5.05E−01	1.70E−01	9.46E−10
		Fun. Evl.	4.99E−23	5.93E−06	4.90E−01	1.81E−01	7.36E−01	8.62E−06
	50D	Min.	1.39E−14	3.38E−03	2.61E−03	1.56E−01	2.93E−02	2.86E−01
		Mean	5.34E−11	1.24E−01	7.62E−02	9.98E−01	3.39E−01	6.92E−01
		Std.	8.20E−09	3.15E−01	1.85E−01	6.27E−01	7.53E−01	6.70E−01
		Fun. Evl.	8.19E−15	2.27E−01	2.09E−03	6.78E−01	3.90E−01	8.83E−01
	100D	Min.	6.59E−14	3.51E−02	4.60E−04	1.52E−01	6.39E−02	7.90E−01
		Mean	1.04E−10	3.53E−01	9.26E−03	6.37E−01	3.42E−01	7.48E−01
		Std.	1.28E−08	5.73E−01	1.67E−02	1E+00	5.93E−01	4.60E−01
		Fun. Evl.	8.69E−13	2.99E−01	6.11E−05	5.15E−01	4.64E−01	8.60E−01

been concluded, and trying to find patterns that were not primary objectives of the study. In these kinds of analyses, a new analyze for new objectives, which were not planned before the experiment is planned. In this section, the P-H analysis is conducted in order to derive the overall rankings of the metaheuristic algorithms for all of the 2D, 50D and the 100D mathematical functions based on the achieved results of the K–W test. The overall rankings of the metaheuristics obtained by the P-H analysis are presented in Table 18. It should be noted that the CGO algorithms provides a success estimation of 100% in outranking the other metaheuristics which demonstrates the superiority of the proposed novel optimization algorithm. It should be noted that the CGO algorithm is the best one in the ranking in dealing with 2D, 50D and 100D functions for the minimum, mean, standard deviation and function evaluation.

Table 15 The W test results (mean of the ranks)

Main algorithm	Function type	Data type	Alternative algorithms					
			FA	GWO	ICA	SOS	TLBO	WOA
CGO	2D	Min.	4656	1953	104	45	55	565
			0	**0**	**86**	**0**	**0**	**30**
		Mean	5394	3347	1370	402	417	2724
			492	**139**	**341**	**264**	**286**	**357**
		Std.	5192	3219	1172	**327**	**366**	2703
			694	**267**	**368**	339	375	**457**
		Fun. Evl.	4631	4162	3028	3790	2431	4532
			25	**494**	**2123**	**866**	**2225**	**124**
CGO	50D	Min.	1510	353	393	118	238	91
			30	**25**	**103**	**53**	**62**	**45**
		Mean	1622	534	587	251	382	269
			31	**96**	**154**	**184**	**114**	**196**
		Std.	1404	381	453	**176**	**207**	278
			249	**249**	**288**	259	289	**187**
		Fun. Evl.	1081	548	1091	**414**	676	563
			0	**533**	**184**	667	**452**	**518**
CGO	100D	Min.	1462	281	503	184	237	91
			23	**19**	**127**	**69**	**63**	**80**
		Mean	1459	401	690	266	358	238
			26	**95**	**171**	**169**	**138**	**197**
		Std.	1315	305	559	**197**	242	**215**
			170	**191**	**302**	238	**254**	220
		Fun. Evl.	903	486	875	413	503	480
			0	**417**	**115**	**490**	**400**	**466**

Table 16 The W test results (p values)

Main algorithm	Function Type	Data type	Alternative algorithms					
			FA	GWO	ICA	SOS	TLBO	WOA
CGO	2D	Min.	1.78E−17	7.58E−12	7.17E−01	3.91E−03	1.95E−03	4.80E−06
		Mean	5.79E−14	3.28E−13	6.79E−05	2.78E−01	3.23E−01	3.75E−09
		Std.	5.44E−12	2.07E−11	7.57E−04	9.25E−01	9.48E−01	4.06E−08
		Fun. Evl.	3.90E−17	2.06E−11	1.25E−01	9.16E−08	7.07E−01	8.01E−16
	50D	Min.	5.64E−10	8.14E−05	4.49E−03	1.57E−01	1.19E−02	2.34E−01
		Mean	2.61E−10	3.34E−04	1.69E−03	4.69E−01	8.64E−03	4.53E−01
		Std.	4.47E−06	2.80E−01	2.32E−01	3.70E−01	4.22E−01	3.49E−01
		Fun. Evl.	3.52E−09	9.35E−01	1.20E−05	1.67E−01	2.36E−01	8.06E−01
	100D	Min.	5.83E−10	1.82E−04	2.07E−03	6.19E−02	1.29E−02	8.11E−01
		Mean	6.86E−10	2.72E−03	7.72E−04	2.94E−01	3.11E−02	6.58E−01
		Std.	8.25E−07	2.64E−01	9.59E−02	6.58E−01	9.06E−01	9.57E−01
		Fun. Evl.	1.65E−08	6.66E−01	9.22E−06	6.30E−01	5.20E−01	9.33E−01

7 Comparing to fractal- and chaos-based algorithms

In this section, the results of the CGO algorithm in dealing with the 2D, 50D and 100D mathematical functions are compared to Stochastic Fractal Search (SFS) developed based on the fractals concept (Salimi (2015)) as well as three other metaheuristics containing Chaos-embedded Particle Swarm Optimization (C-PSO) proposed by Alatas et al. (2009), Firefly Algorithm (FA) with chaos (C-FA) proposed by Gandomi et al. (2013a, b), and Imperialistic Competitive Algorithm (ICA) combined with chaos (C-ICA) proposed by Talatahari et al. (2012), in which the chaos theory is included in the general formulation of these algorithms. In addition, two other metaheuristics as Covariance Matrix Adaptation Evolution Strategy (CMA-ES) (Hansen et al. 2003) and Yellow Saddle Goatfish Algorithm (YSGA) (Zaldivar et al. 2018) are also utilized for comparative purposes. In Table 19, the minimum values of the selected mathematical functions obtained by the mentioned metaheuristics and the proposed CGO algorithm are presented. It is clear that the results of the CGO algorithm is better than the results of other metaheuristic.

8 Comparing to CEC 2017 competition results

In order to evaluate the overall performance of the proposed CGO algorithm with the state-of-the-art algorithms in dealing with the state-of-the-art mathematical test functions, one of the recent competitions on single objective real-parameter numerical

Table 17 The K–W test results (mean of the ranks)

Rankings	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of ranks	Algorithms	Mean of ranks	Algorithms	Mean of ranks	Algorithms	Mean of ranks
	2D							
1	***CGO***	***333.9103***	***CGO***	***317.2265***	***CGO***	***298.8248***	**ICA**	**317.0427**
2	ICA	349.9872	SOS	327.9829	SOS	311.7991	*CGO*	*325.6496*
3	SOS	353.2265	TLBO	349.7564	TLBO	331.6453	TLBO	336.9444
4	TLBO	354.8932	ICA	370.1068	ICA	369.359	SOS	359.2735
5	WOA	404.9487	WOA	447.0256	WOA	465.2479	WOA	447.2051
6	GWO	457.4274	GWO	465.1538	GWO	489.641	GWO	471.765
7	FA	615.6068	FA	592.7479	FA	603.4829	FA	612.1197
Chi-sq.	169.1482		133.7584		180.959		153.2064	
Prob > Chi-sq.	6.82E−34		2.08E−26		2.12E−36		1.62E−30	
	50D							
1	***CGO***	***153.2241***	***CGO***	***169.569***	**SOS**	**168.931**	***CGO***	***164.1552***
2	WOA	169.3103	SOS	169.6724	*CGO*	*178.1552*	WOA	171.2241
3	SOS	174.7759	WOA	179.7414	TLBO	183.8621	SOS	176.4052
4	TLBO	189.8879	TLBO	187.5086	WOA	188.0862	TLBO	185.6293
5	GWO	206.181	GWO	200.5086	GWO	197.6379	GWO	195.3966
6	ICA	208.75	ICA	205.5086	ICA	205.6034	ICA	224.1293
7	FA	322.3707	FA	311.9914	FA	302.2241	FA	307.5603
Chi-sq.	96.05921		67.92103		56.77908		69.04072	
Prob > Chi-sq.	1.66E−18		1.09E−12		2.03E−10		6.43E−13	
	100D							
1	***CGO***	***156.0862***	***CGO***	***168.0086***	**SOS**	**172.6034**	**WOA**	**165.0862**
2	WOA	159.6293	SOS	175.0345	*CGO*	*173.6983*	*CGO*	*165.2069*
3	SOS	178.6724	WOA	175.4569	TLBO	182.7845	SOS	181.431
4	TLBO	186.8793	TLBO	185.5259	GWO	184.5603	TLBO	183.8276

Table 17 (continued)

Rankings	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of ranks	Algorithms	Mean of ranks	Algorithms	Mean of ranks	Algorithms	Mean of ranks
5	GWO	196.5172	GWO	188.0259	WOA	189.2586	GWO	190.9828
6	ICA	223.5431	ICA	221.2241	ICA	222.9914	ICA	242.2241
7	FA	323.1724	FA	311.2241	FA	298.6034	FA	295.7414
Chi-sq.	97.43246		69.34553		56.74194		67.15812	
Prob > Chi-sq.	8.61E−19		5.57E−13		2.06E−10		1.56E−12	

Table 18 The P-H analysis results for all of the mathematical functions

Rankings	2D and 50D and 100D							
	Min.		Mean		Std.		Fun. Evl.	
	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks	Algorithms	Mean of Ranks
1	***CGO***	***643.2575***	***CGO***	***662.2232***	***CGO***	***661.4678***	***CGO***	***651.7082***
2	SOS	714.0472	SOS	682.2768	SOS	662.0429	TLBO	703.3369
3	WOA	727.9077	TLBO	732.5022	TLBO	708.2253	SOS	711.97
4	TLBO	740.5987	WOA	791.1094	ICA	805.0472	WOA	780.8348
5	ICA	795.618	ICA	805.2296	WOA	828.1803	ICA	783
6	GWO	851.1288	GWO	840.7554	GWO	857.3197	GWO	869.9871
7	FA	1239.442	FA	1197.903	FA	1189.717	FA	1211.163
Chi-sq.	309.102		223.3791		235.0263		240.0594	
Prob > Chi-sq.	9.16E−64		1.98E−45		6.47E−48		5.45E−49	

Table 19 Minimum values of the mathematical functions obtained by the fractal- and chaos-based algorithms

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_1	−200	−200	−200	−200	−200	−200	−200
F_2	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629	−195.629
F_3	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181	−2.02181
F_4	1	1	1	1	1	1	1
F_5	0	0	0	0	0	0	0
F_6	0	1.70E−08	0	0	0	0	0
F_7	0	2.21E−10	0	0	0	0	0
F_8	−106.765	−106.765	−106.765	−106.765	−106.765	−106.765	−106.765
F_9	0	0	0	9.83E−12	0	0	0
F_{10}	0	0	0	6.75E−11	0	0	0
F_{11}	0	0	0	2.09E−11	0	0	0
F_{12}	0	0	0	0	0	0	0
F_{13}	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887	0.397887
F_{14}	5.559037	6.810589	5.681669	6.810589	5.559037	5.559037	5.559037
F_{15}	0	0	0	0	0	0	0
F_{16}	0	0	0	4.63E−11	0	2500	0
F_{17}	1.32E−05	229.1788	0.000489	0.023954	0.000158	229.1788	0.000526
F_{18}	0	0	0	0	0	0	0
F_{19}	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316	−1.0316
F_{20}	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568	−24.1568
F_{21}	−2000	−1000	−1999.99	−1000	−1000	−1000	−2000
F_{22}	−2000	−999.999	−1999.97	−1000	−2000	−1993.22	−2000
F_{23}	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444	−42.9444
F_{24}	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261	−2.06261
F_{25}	0	5.00E−12	0	0	0	0	0
F_{26}	0	4.64E−06	2	2	2	0	0
F_{27}	−24771.1	−24771.1	−24771.1	−24771.1	−24771.1	−24771.1	−24771.1
F_{28}	−1	−1	−1	−1	−1	−1	−1
F_{29}	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128	1.7128
F_{30}	0	0	0	0	0	0	0
F_{31}	0	0	0	0	0	0	0
F_{32}	0	0	0	0.07796	0	0	0
F_{33}	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447	0.06447
F_{34}	3	3	3	3	3	3	3
F_{35}	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953	−165.953
F_{36}	0	2.38E−08	0	0	0	0	0
F_{37}	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458	−2.3458
F_{38}	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622	124.3622
F_{39}	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367	−0.67367
F_{40}	0	4.58E−08	0	0	0	0	0
F_{41}	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542	−176.542
F_{42}	−176.138	−176.138	−176.138	−176.138	−176.138	−176.138	−176.138
F_{43}	0	0	0	0	0	0	0
F_{44}	−1.91322	−1.91051	−1.91322	−1.91322	−1.9133	−1.91051	−1.91322

Table 19 (continued)

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_{45}	− 19.6683	− 19.6683	− 19.6683	− 19.6683	− 19.6683	− 19.6683	− 19.6683
F_{46}	− 1.8013	− 1.8013	− 1.8013	− 1.8013	− 1.8013	− 1.8013	− 1.8013
F_{47}	− 0.18465	− 0.18465	− 0.18465	− 0.18426	− 0.18465	− 0.18465	− 0.18465
F_{48}	− 0.19941	− 0.19923	− 0.19938	− 0.16649	− 0.19941	− 0.19941	− 0.19941
F_{49}	− 1.01983	− 1.01983	− 1.01983	− 1.01983	− 1.01983	− 1.01983	− 1.01983
F_{50}	− 2.28395	− 2.28395	− 2.28395	− 2.28395	− 2.28395	− 2.28395	− 2.28395
F_{51}	0	0	0	0	0	0	0
F_{52}	0	0	0	0	0	0	0
F_{53}	0	2.17E−09	0	0	0	0	0
F_{54}	− 0.96353	− 0.96353	− 0.96353	− 0.96353	− 0.96354	− 0.96353	− 0.96353
F_{55}	0.9	0.9	0.9	0.9	0.9	0.9	0.9
F_{56}	0	4.50E−10	0	1.70E−10	0	0	0
F_{57}	0.9	0.9	0.9	0.9	0.9	0.9	0.9
F_{58}	0	0	2.15E−12	1.48E−09	0	0	0
F_{59}	0	0	0	1.03E−12	0	0	0
F_{60}	− 3873.72	− 3873.72	− 3873.72	− 3873.72	− 3873.72	− 3873.72	− 3873.72
F_{61}	− 2.2	− 2.19915	− 2.2	− 2.2	− 2.2	− 2.2	− 2.2
F_{62}	− 2	− 2	− 2	− 2	− 2	− 2	− 2
F_{63}	34.3712	34.3712	34.3712	74	36.26407	34.3712	34.3712
F_{64}	0	0	1.82E−12	3.75E−10	0	0	0
F_{65}	0	0	0	8.97E−11	0	0	0
F_{66}	0	0	6.35E−10	1.20E−08	0	0	0
F_{67}	0	0	0	0	0	0	0
F_{68}	0	0	0	0	0	0	0
F_{69}	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567	0.001567
F_{70}	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579	0.292579
F_{71}	0	8.01E−05	3.09E−07	2.35E−06	0	2.00E−07	0
F_{72}	− 3456	− 3456	− 3456	− 3456	− 3456	− 3456	− 3456
F_{73}	− 26.9203	− 26.9203	− 26.9203	− 26.9203	− 26.9203	− 26.9203	− 26.9203
F_{74}	− 19.2085	− 19.2085	− 19.2085	− 19.2085	− 19.2085	− 19.2085	− 19.2085
F_{75}	− 24.1568	− 24.1568	− 24.1568	− 24.1568	− 24.1568	− 24.1568	− 24.1568
F_{76}	− 10.8723	− 10.8723	− 10.8723	− 10.8723	− 10.8723	− 10.8723	− 10.8723
F_{77}	0	0	0	0	0	0	0
F_{78}	− 3.30687	− 3.30687	− 3.30687	− 3.30687	− 3.30687	− 3.30687	− 3.30687
F_{79}	0	0	8.06E−08	2.000001	0	0	0
F_{80}	− 4.81681	− 4.81681	− 4.81681	− 4.81681	− 4.81681	− 4.81681	− 4.81681
F_{81}	− 2.5	− 2.5	− 2.5	− 2.5	− 2.5	− 2.5	− 2.5
F_{82}	− 1.5	− 1.5	− 1.5	− 1.5	− 1.5	− 1.5	− 1.5
F_{83}	− 7.307	− 7.30426	− 7.307	− 6.37186	− 7.307	− 7.30426	− 7.307
F_{84}	− 400	− 400	− 400	− 400	− 400	− 400	− 400
F_{85}	0	1.99E−08	3.17E−12	2.30E−10	0	0	0
F_{86}	0	2.23E−09	0	1.27E−11	0	0	0
F_{87}	21.35	21.35	21.35	21.35	21.35	21.35	21.35
F_{88}	− 0.00379	− 0.00379	− 0.00379	− 0.00379	− 0.00379	− 0.00379	− 0.00379

Table 19 (continued)

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_{89}	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523	−0.3523
F_{90}	0	0	0	3.66E−11	0	0	0
F_{91}	0	0	0	0	0	0	0
F_{92}	0	0	0	0	0	0	0
F_{93}	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278	−3.86278
F_{94}	0	6.75E−08	0	0	0	0	0
F_{95}	0.024536	0.024536	0.024536	0.024536	4.36E−05	0.024536	0.024536
F_{96}	0	1.70E−06	0	0	0	0	0
F_{97}	0	0	0	0	0	0	0
F_{98}	0	0	0	0	0	0	0
F_{99}	0	4.72E−08	5.17E−12	2.11E−11	0	1.24E−09	0
F_{100}	0	0	0	0	0	0	0
F_{101}	0	1659.363	43.36581	6739.838	0	0	0
F_{102}	2.70E−12	2.70E−12	2.70E−12	2.31E−11	2.70E−12	2.70E−12	2.70E−12
F_{103}	0.000308	0.000308	0.000308	0.000436	0.000308	0.000308	0.000308
F_{104}	0	0	0	0	0	0	0
F_{105}	−10.1532	−5.10071	−10.1532	−10.1532	−5.0552	−10.1532	−10.1532
F_{106}	−10.4029	−10.4027	−10.4029	−10.4029	−5.08767	−10.4029	−10.4029
F_{107}	−10.5364	−5.12848	−10.5364	−10.5364	−5.12848	−10.5364	−10.5364
F_{108}	0	0.000119	0.000323	0.015173	0	3.70E−07	0
F_{109}	0.333839	193.818	1048.496	3404.998	0	115.2022	0
F_{110}	−529.871	−529.834	−529.241	−517.305	−529.871	−529.871	−529.871
F_{111}	−0.965	−0.93981	−0.965	−0.965	−0.908	−0.965	−0.965
F_{112}	2.71E−08	4.01E−05	2.52E−05	0.005186	0	5.79E−06	0
F_{113}	−3.32236	−3.32236	−3.32236	−3.32236	−3.32236	−3.32236	−3.32236
F_{114}	−50	−50	−50	−50	−50	−50	−50
F_{115}	0.95979	0.965652	0.95979	0.973866	0.95979	0.95979	0.95979
F_{116}	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778	−45.778
F_{117}	−210	−210	−210	−210	−210	−210	−210
F_{118}	0.000461	19.98176	6.31E−05	9.35E−05	1.11E−08	0	0
F_{119}	0.159407	0	0.002818	0.019217	5.86E−08	0	0
F_{120}	3.12E−08	0	1.34E−10	2.90E−10	0	0	0
F_{121}	9.19E−12	0	0	0	0	0	0
F_{122}	0	0	0	5.66E−11	0	0	0
F_{123}	−0.98368	−0.94192	−1	−0.96155	−0.60948	−0.96507	−0.890149
F_{124}	−0.99456	−0.94984	−1	−0.98	−1	−0.98302	−0.901522
F_{125}	0.666774	0.666667	0.666977	0.886176	0.666667	0.666667	0.6666667
F_{126}	−1	−0.99991	−1	0	0	−0.99997	−1
F_{127}	−1	−1	−1	−1	−1	−1	−1
F_{128}	1.24E−07	0	6.05E−08	1.91E−07	0	0	0
F_{129}	3.83E−11	0	0	0	0	0	0
F_{130}	7721.368	0	8.34E+14	6.36E+15	1.37E+11	0	0
F_{131}	−33.7053	−21.3991	−23.976	−16.5792	−7.74406	−49	−49
F_{132}	5.95E−07	0.093212	1.05E−09	1.64E−05	0	0.000795	0

Table 19 (continued)

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_{133}	2.308767	2	2.139347	3.418983	2	2	2
F_{134}	2.35998	2	2.057896	2.950725	2	2	2
F_{135}	9.25E+128	9.25E+128	9.25E+128	9.25E+128	1.29E+118	9.25E+128	9.25E+128
F_{136}	3.64E−07	0	0	0	1.68E−11	0	0
F_{137}	14.49504	4.837566	17.39229	20.40403	21.45182	4.827224	0
F_{138}	1.165205	0	226.2443	782.9157	1.03E−11	0	0
F_{139}	0.013356	0	0.375701	1.039234	0.012943	0	0
F_{140}	5.16E−06	0	0.000639	0.179056	1.00E−09	0	0
F_{141}	0	0	1.62E−09	5.28E−08	3.71E−12	0	0
F_{142}	19.52312	0	28.85381	39.79834	2.984878	0	0
F_{143}	117.2885	0.55414	0.012039	0.041672	2.80E−10	0.513762	0
F_{144}	9.34938	5.185671	1.208168	10.68099	2.31E−06	3.749134	9.28E−08
F_{145}	27.40189	45.42774	45.14793	47.70977	32.41107	44.80675	29.900335
F_{146}	0.699873	0.099873	0.299873	0.499873	0.199873	0.099873	0
F_{147}	3.04E−08	0	0	0	0	0	0
F_{148}	0	0	0	0	0	0	0
F_{149}	0	0	0	0	4.15E−10	0	0
F_{150}	0.01691	7.17E−06	4.30E−09	0.000503	5.97E−10	3.86E−06	9.35E−12
F_{151}	0.003798	0	0.012102	0.137038	6.48E−07	0	0
F_{152}	2.096802	4.814532	0.021372	0.063408	9.87E−06	0.946046	0
F_{153}	0.113246	0	0.027789	0.169955	3.04E−06	0	0
F_{154}	0	0	0	0	0	0	0
F_{155}	0.037215	6.24E−05	2.90E−08	7.35E−05	1.65E−09	2.43E−06	2.22E−11
F_{156}	−402.159	−208.463	−280.802	−236.186	−418.983	−259.116	−289.1935
F_{157}	0.023207	0	1.10E−10	3.86E−10	0	0	0
F_{158}	0	0	0	0	0	0	0
F_{159}	0	0	0	0	0	0	0
F_{160}	0	0	0	0	0	0	0
F_{161}	−261	−275	−267	−262	−275	−275	−275
F_{162}	9.971167	18.32459	2.230903	4.057409	0.241673	3.468084	0
F_{163}	7.38E−07	0	0.000288	0.137184	0	0	0
F_{164}	−1958.3	−1707.2	−1831.08	−1760.39	−1958.3	−1944.1	−1958.306
F_{165}	−11856.9	−15337.3	−19040.4	−3161.65	−22050	−18027.5	−21988.17
F_{166}	8.62313	0	23.64303	59.97822	5.19E−06	0	0
F_{167}	22.30567	52.7867	11.2818	17.71482	1	29.55507	20.281137
F_{168}	0.058698	0.330326	0.192104	0.335612	0.725947	0	0
F_{169}	0	0	0	3.96E−302	0	0	−1
F_{170}	0	0	0	0	0	0	0
F_{171}	0	0	0	0	0	0	0
F_{172}	9.00E−07	0	7.55E−10	0.036634	9.90E−10	0	0
F_{173}	3.93E−22	7.11E−24	1.23E−30	1.99E−30	3.02E−36	6.70E−24	−1
F_{174}	0.544635	0.000578	9.31E−05	0.766646	1.96E−05	6.49E−06	3.94E−05
F_{175}	1.3852	4.000674	10.33676	56.3906	0	0.179843	0
F_{176}	0.08615	0	0.004096	0.007012	8.53E−05	0	0

Table 19 (continued)

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_{177}	4.379913	0	0.032822	0.200821	0.000429	0	0
F_{178}	0.007307	0	7.17E−05	0.001279	1.01E−09	0	0
F_{179}	0.015678	0	1.61E−08	1.59E−07	0	0	0
F_{180}	1.32E−09	0	1.66E−07	4.50E−07	0	0	0
F_{181}	−0.93484	−0.85014	−0.95442	−0.90918	−0.5065	−0.85639	−0.76477
F_{182}	−0.95923	−0.86829	−0.94893	−0.89262	−1	−0.89451	−0.79327
F_{183}	9.880318	0.666667	1.260284	1.575497	0.666671	0.666667	0.666667
F_{184}	0	−0.9457	0	0	0	−0.99262	0
F_{185}	−0.99999	−1	−0.99998	−0.99954	−1	−1	−1
F_{186}	0.001949	0	0.000317	0.000443	3.12E−07	0	0
F_{187}	0.004133	0	7.83E−06	4.71E−05	0	0	0
F_{188}	1.58E+19	0	2.22E+30	2.03E+31	9.08E+29	0	0
F_{189}	−42.7897	−35.5648	−40.0709	−33.6686	−8.85399	−99	−99
F_{190}	0.030605	109.573	0.000123	0.000182	6.66E−09	0.020334	1.074365
F_{191}	217.2778	2	14.49823	113.1912	2	2	2
F_{192}	183.2298	2	19.39233	245.8696	2	2	2
F_{193}	0	0	0	0	0	0	0
F_{194}	2.31E−05	0	0	0	1.71E−07	0	0
F_{195}	36.58503	9.919996	37.57346	42.38333	45.47596	9.840136	0
F_{196}	2307.331	0	2050.333	8585.416	35.22131	0	0
F_{197}	0.697279	0	2.473221	7.759918	0.017962	0	0
F_{198}	0.073885	0	0.0317	0.48086	6.61E−07	0	0
F_{199}	0	0	2.93E−09	1.92E−08	9.25E−11	0	0
F_{200}	133.2975	0	80.68107	183.111	19.0865	0	0
F_{201}	26335	735.0315	3.479846	24.13042	0.337952	10.89645	0.008689
F_{202}	58.43503	18.73577	15.72721	23.92143	0.028356	13.92349	0.190459
F_{203}	295.4997	95.00197	95.75061	95.75061	92.2158	94.88555	82.73053
F_{204}	2.599883	0.099873	0.599873	0.799873	0.299877	0.099873	0
F_{205}	2.193827	0	1.81E−08	2.09E−07	0	0	0
F_{206}	3.85E−06	0	0	0	0	0	0
F_{207}	0	7.76E−11	0	0	1.08E−09	0	0
F_{208}	17.92924	0.001787	0.027415	0.17144	2.21E−07	0.000883	0.007558
F_{209}	1.09529	0	0.558277	1.374292	0.007242	0	0
F_{210}	11.9985	48.88828	0.147203	0.246815	0.02252	37.61164	0
F_{211}	46.12763	0	0.825046	81.46816	0.031502	0	0
F_{212}	0.000817	0	0	0	0	0	0
F_{213}	23.95181	0.001401	0.03632	0.281402	2.35E−07	0.001056	0.006548
F_{214}	−329.162	−182.698	−246.843	−225.326	−418.983	−217.456	−220.435
F_{215}	5.84239	0	5.08E−05	0.000551	1.11E−09	0	0
F_{216}	0	0	0	0	0	0	0
F_{217}	0	0	0	0	0	0	0
F_{218}	0	0	0	0	0	0	0
F_{219}	−495	−575	−538	−527	−575	−575	−575
F_{220}	27.35396	13.20605	9.544515	17.33387	4.108229	0	0

Table 19 (continued)

No.	SFS	C-PSO	C-FA	C-ICA	CMA-ES	YSGA	CGO
F_{221}	0.042785	0	0.515799	1.148726	2.08E−06	0	0
F_{222}	− 3877.11	− 3378.76	− 3619.74	− 3393.55	− 3874.21	− 3548.52	− 3765.07
F_{223}	404,319.5	− 33,857.6	− 33,908.6	129,466.8	− 162,687	− 57,736.9	− 23,507
F_{224}	554.9544	0	479.2063	625.8056	4.847024	0	0
F_{225}	283.0328	141.2464	81.48305	143.8316	1.318782	104.7021	55.26998
F_{226}	0.229959	0	0.179456	0.268773	0.834146	0	0
F_{227}	0	0	0	0	0	0	− 1
F_{228}	0	0	0	0	0	0	0
F_{229}	0	0	0	0	0	0	0
F_{230}	0.000511	0	2.47E−05	0.061411	5.52E−08	0	0
F_{231}	1.39E−42	1.15E−42	1.14E−42	1.14E−42	3.68E−48	8.23E−44	− 1
F_{232}	0.241987	0.000867	0.013485	0.189038	2.39E−07	1.66E−05	3.41E−05
F_{233}	46.22346	466.6365	240.8251	316.8896	1663.062	225.3411	0

optimization named "CEC 2017" is considered in this section. In this regard, a list of 30 mathematical functions are considered and presented in Table 20 while the mathematical details of these functions have been presented by the CEC 2017 competition committee (Awad et al. 2016). These mathematical functions are consisting of 10 shifted and rotated functions, 10 hybrid functions and 10 composite functions. These test functions are considered in four dimensions of 10, 30, 50 and 100 and the optimization process is conducted for four different scenarios.

The statistical results of the CGO algorithm in dealing with these test functions with 10, 30, 50 and 100 dimensions are presented in Tables 21, 22, 23 and 24, respectively; the results of three other successful algorithms (Kumar et al. 2017; Awad et al. 2017; Sallam et al. 2017) are also presented. It should be noted that the error values are only considered in this competition and the statistical results are based on the best error values of 51 independent runs. The results show that the proposed CGO algorithm is capable of providing very acceptable results in dealing with these test functions with different dimensions.

In order to evaluate the computational time and complexity of the proposed CGO algorithm in dealing with the provided mathematical test functions by the CEC 2017 competition, the complexity assessment is also conducted for the CGO method based on the provided complexity scenario by Awad et al. (2016). In this scenario, four different computational times (T_0, T_1, T_2 and $\hat{T}_2$) are considered based on the four specific mathematical procedures. The T_0 refers to the running time of the mathematical procedure presented in Fig. 7.

The T_1 refers to the computational time for evaluation of the G_{18} test function considering 200,000 function evaluations for each of the 10, 30, 50 and 100 dimensions. The T_2 refers to the computational time of the considered metaheuristic algorithm (CGO in this paper) for evaluation of the G_{18} test function considering 200,000

Table 20 Summary of the CEC 2017 Test Functions (Awad et al. 2016)

Function type	Fun. no.	Function detail	Fun. Min.
Unimodal Functions	G_1	Shifted and Rotated Bent Cigar Function	100
	G_2	Shifted and Rotated Sum of Different Power Function	200
	G_3	Shifted and Rotated Zakharov Function	300
Simple Multimodal Functions	G_4	Shifted and Rotated Rosenbrock's Function	400
	G_5	Shifted and Rotated Rastrigin's Function	500
	G_6	Shifted and Rotated Expanded Scaffer's F6 Function	600
	G_7	Shifted and Rotated Lunacek Bi_Rastrigin Function	700
	G_8	Shifted and Rotated Non-Continuous Rastrigin's Function	800
	G_9	Shifted and Rotated Levy Function	900
	G_{10}	Shifted and Rotated Schwefel's Function	1000
Hybrid Functions	G_{11}	Hybrid Function 1 (N = 3)	1100
	G_{12}	Hybrid Function 2 (N = 3)	1200
	G_{13}	Hybrid Function 3 (N = 3)	1300
	G_{14}	Hybrid Function 4 (N = 4)	1400
	G_{15}	Hybrid Function 5 (N = 4)	1500
	G_{16}	Hybrid Function 6 (N = 4)	1600
	G_{17}	Hybrid Function 6 (N = 5)	1700
	G_{18}	Hybrid Function 6 (N = 5)	1800
	G_{19}	Hybrid Function 6 (N = 5)	1900
	G_{20}	Hybrid Function 6 (N = 6)	2000
Composition Functions	G_{21}	Composition Function 1 (N = 3)	2100
	G_{22}	Composition Function 2 (N = 3)	2200
	G_{23}	Composition Function 3 (N = 4)	2300
	G_{24}	Composition Function 4 (N = 4)	2400
	G_{25}	Composition Function 5 (N = 5)	2500
	G_{26}	Composition Function 6 (N = 5)	2600
	G_{27}	Composition Function 7 (N = 6)	2700
	G_{28}	Composition Function 8 (N = 6)	2800
	G_{29}	Composition Function 9 (N = 3)	2900
	G_{30}	Composition Function 10 (N = 3)	3000
Search range: $[-100,100]^D$			

function evaluations for each of the 10, 30, 50 and 100 dimensions. The $\hat{T}_2$ refers to the mean values of five different assessment of T_2. In Table 25, the comparative complexity results of the CGO method with the three other metaheuristics which have participated in the CEC 2017 competition are presented. The computational times are calculated and presented in seconds. It should be noted that the proposed CGO method is capable of providing very competitive results comparing to other metaheuristics.

Table 21 Comparative results of the CEC 2017 test functions with 10 dimensions

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_5	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.99E+00	1.99E+00	1.69E+00	7.53E−01
G_6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_7	1.04E+01	1.10E+01	1.05E+01	1.06E+01	1.75E−01	1.06E+01	1.29E+01	1.20E+01	1.20E+01	4.80E−01
G_8	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.70E−03	2.99E+00	1.99E+00	1.80E+00	7.71E−01
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_{10}	1.25E−01	2.17E+02	1.36E+01	3.72E+01	5.39E+01	3.71E−01	1.55E+02	1.51E+01	4.30E+01	5.57E+01
G_{11}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_{12}	0.00E+00	2.37E+02	1.18E+02	9.02E+01	7.44E+01	2.08E−01	2.38E+02	1.19E+02	1.01E+02	7.30E+01
G_{13}	0.00E+00	7.95E+00	3.13E−02	2.17E+00	2.53E+00	0.00E+00	8.32E+00	4.84E+00	3.66E+00	2.66E+00
G_{14}	0.00E+00	9.95E−01	0.00E+00	6.05E−02	2.36E−01	0.00E+00	9.95E−01	0.00E+00	7.80E−02	2.70E−01
G_{15}	3.81E−05	5.00E−01	3.07E−02	1.09E−01	1.74E−01	7.03E−06	5.00E−01	4.91E−01	3.24E−01	2.16E−01
G_{16}	2.62E−02	9.35E−01	4.43E−01	4.17E−01	1.98E−01	3.92E−03	1.08E+00	4.98E−01	5.37E−01	2.93E−01
G_{17}	1.00E−02	1.01E+00	4.94E−02	1.47E−01	2.03E−01	2.66E−03	2.63E+00	3.23E−01	3.07E−01	3.81E−01
G_{18}	3.92E−04	2.00E+01	4.09E−01	7.00E−01	2.77E+00	2.22E−04	2.05E+01	4.62E−01	3.86E+00	7.63E+00
G_{19}	0.00E+00	1.22E−01	1.79E−02	1.50E−02	1.88E−02	0.00E+00	1.50E+00	1.97E−02	4.47E−02	2.09E−01
G_{20}	0.00E+00	3.12E−01	0.00E+00	1.47E−01	1.57E−01	0.00E+00	6.24E−01	3.12E−01	2.57E−01	2.31E−01
G_{21}	1.00E+02	2.02E+02	1.00E+02	1.14E+02	3.52E+01	1.00E+02	2.04E+02	1.00E+02	1.46E+02	5.17E+01
G_{22}	2.17E+01	1.00E+02	1.00E+02	9.85E+01	1.10E+01	1.00E+02	1.00E+02	1.00E+02	1.00E+02	6.80E−02
G_{23}	3.00E+02	3.03E+02	3.00E+02	3.00E+02	7.07E−01	3.00E+02	3.05E+02	3.03E+02	3.02E+02	1.64E+00
G_{24}	1.00E+02	3.30E+02	1.00E+02	1.66E+02	9.97E+01	1.00E+02	3.32E+02	3.30E+02	3.16E+02	5.45E+01

Table 21 (continued)

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{25}	3.98E+02	4.43E+02	3.98E+02	4.12E+02	2.12E+01	3.98E+02	4.43E+02	4.43E+02	4.26E+02	2.24E+01
G_{26}	2.00E+02	3.00E+02	3.00E+02	2.65E+02	4.74E+01	3.00E+02	3.00E+02	3.00E+02	3.00E+02	0.00E+00
G_{27}	3.90E+02	3.95E+02	3.90E+02	3.92E+02	2.40E+00	3.84E+02	3.95E+02	3.89E+02	3.89E+02	1.96E+00
G_{28}	0.00E+00	5.84E+02	3.00E+02	3.07E+02	7.18E+01	3.00E+02	6.11E+02	3.00E+02	3.85E+02	1.19E+02
G_{29}	2.27E+02	2.45E+02	2.30E+02	2.31E+02	3.77E+00	2.26E+02	2.33E+02	2.28E+02	2.28E+02	1.72E+00
G_{30}	3.95E+02	4.43E+02	3.95E+02	4.07E+02	1.78E+01	3.39E+02	4.65E+05	4.07E+02	1.76E+04	8.61E+04
Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.37E−05	1.62E+01	3.29E−01	1.94E+00	3.52E+00
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+00	0.00E+00	7.84E−02	2.72E−01
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.26E−01	1.12E−05	5.46E−03	3.17E−02
G_5	0.00E+00	2.99E+00	9.95E−01	1.11E+00	7.35E−01	2.12E+00	2.09E+01	8.73E+00	8.85E+00	4.36E+00
G_6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.55E−05	1.17E−01	4.12E−03	1.13E−02	1.94E−02
G_7	1.04E+01	1.31E+01	1.15E+01	1.15E+01	6.71E−01	7.74E+00	3.50E+01	1.87E+01	2.00E+01	4.87E+00
G_8	0.00E+00	2.99E+00	9.95E−01	1.11E+00	9.68E−01	1.99E+00	1.99E+01	8.95E+00	8.48E+00	4.10E+00
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.73E+00	8.95E−02	5.47E−01	8.89E−01
G_{10}	2.50E−01	1.42E+02	6.83E+00	1.79E+01	3.64E+01	1.52E+01	9.70E+02	3.04E+02	3.47E+02	2.21E+02
G_{11}	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.85E−07	1.27E+01	2.99E+00	3.95E+00	2.93E+00
G_{12}	0.00E+00	2.64E+02	1.19E+02	1.02E+02	5.96E+01	7.28E+01	8.81E+03	6.24E+02	1.44E+03	1.82E+03

Table 21 (continued)

Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{13}	6.82E−05	9.63E+00	5.05E+00	4.19E+00	2.66E+00	3.28E+00	4.20E+01	1.17E+01	1.17E+01	6.17E+00
G_{14}	0.00E+00	1.03E+00	0.00E+00	8.80E−02	2.74E−01	2.59E−07	7.46E+00	2.15E+00	2.92E+00	1.95E+00
G_{15}	2.00E−05	5.00E−01	2.13E−02	6.71E−02	1.19E−01	6.66E−01	7.09E+00	2.51E+00	2.80E+00	1.56E+00
G_{16}	2.11E−02	8.80E−01	2.16E−01	2.53E−01	2.01E−01	4.56E−01	2.87E+02	3.81E+01	6.47E+01	7.57E+01
G_{17}	0.00E+00	3.76E−01	1.97E−02	5.64E−02	1.13E−01	5.68E−01	6.76E+01	1.78E+01	1.61E+01	1.50E+01
G_{18}	1.11E−04	2.00E+01	9.00E−02	9.69E−01	3.89E+00	2.18E+00	2.41E+01	8.25E+00	1.03E+01	6.10E+00
G_{19}	0.00E+00	1.94E−02	0.00E+00	3.80E−03	7.49E−03	1.08E−01	2.87E+00	1.18E+00	1.13E+00	7.91E−01
G_{20}	0.00E+00	6.24E−01	0.00E+00	6.73E−02	1.57E−01	0.00E+00	1.50E+01	9.95E−01	3.49E+00	5.03E+00
G_{21}	1.00E+02	2.03E+02	1.00E+02	1.04E+02	1.99E+01	1.00E+02	2.21E+02	1.00E+02	1.09E+02	3.01E+01
G_{22}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	6.88E−02	2.23E+01	1.10E+02	1.01E+02	1.00E+02	1.12E+01
G_{23}	1.00E+02	3.08E+02	3.03E+02	2.98E+02	2.84E+01	3.00E+02	3.44E+02	3.13E+02	3.14E+02	7.33E+00
G_{24}	1.00E+02	2.01E+02	1.00E+02	1.04E+02	1.97E+01	2.93E+01	3.56E+02	3.40E+02	3.07E+02	8.81E+01
G_{25}	3.98E+02	4.43E+02	3.98E+02	4.14E+02	2.19E+01	3.98E+02	4.50E+02	4.45E+02	4.34E+02	2.10E+01
G_{26}	2.00E+02	3.00E+02	3.00E+02	2.94E+02	2.38E+01	2.00E+02	4.46E+02	3.00E+02	3.25E+02	6.04E+01
G_{27}	3.89E+02	3.90E+02	3.90E+02	3.90E+02	1.22E−01	3.89E+02	4.73E+02	3.93E+02	3.96E+02	1.48E+01
G_{28}	3.00E+02	6.47E+02	3.00E+02	3.37E+02	1.02E+02	3.00E+02	6.12E+02	4.16E+02	4.61E+02	1.42E+02
G_{29}	2.30E+02	2.48E+02	2.34E+02	2.36E+02	4.19E+00	2.35E+02	3.46E+02	2.54E+02	2.63E+02	2.46E+01
G_{30}	3.95E+02	1.25E+06	3.95E+02	5.69E+04	2.34E+05	4.22E+02	8.18E+05	8.00E+02	1.91E+05	3.44E+05

EBO with CMAR: Effective Butterfly Optimizer with Covariance Matrix Adapted Retreat

LSHADE-cnEpSin: Ensemble Sinusoidal Differential Covariance Matrix Adaptation with Euclidean Neighborhood

MM_OED: Multi-Method Based Orthogonal Experimental Design

Table 22 Comparative results of the CEC 2017 test functions with 30 dimensions

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	6.41E+01	5.86E+01	5.65E+01	1.11E+01	3.48E+01	5.01E+01	4.25E+01	4.23E+01	3.07E+00
G_5	0.00E+00	7.96E+00	2.98E+00	2.78E+00	1.74E+00	5.55E+00	1.74E+01	1.25E+01	1.23E+01	2.34E+00
G_6	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_7	3.17E+01	3.60E+01	3.34E+01	3.35E+01	8.37E−01	3.89E+01	4.81E+01	4.33E+01	4.33E+01	2.17E+00
G_8	0.00E+00	5.97E+00	1.99E+00	2.02E+00	1.32E+00	5.44E+00	1.73E+01	1.34E+01	1.29E+01	2.86E+00
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_{10}	9.84E+02	1.94E+03	1.38E+03	1.41E+03	2.15E+02	8.44E+02	1.72E+03	1.42E+03	1.39E+03	2.10E+02
G_{11}	0.00E+00	6.40E+01	1.99E+00	4.49E+00	8.77E+00	1.72E+00	5.90E+01	3.96E+00	1.35E+01	1.94E+01
G_{12}	1.50E+01	1.10E+03	4.35E+02	4.63E+02	2.63E+02	1.32E+02	9.35E+02	3.44E+02	3.72E+02	2.01E+02
G_{13}	1.38E+00	2.66E+01	1.66E+01	1.49E+01	6.25E+00	2.03E+00	7.96E+01	1.71E+01	1.73E+01	1.02E+01
G_{14}	2.52E+00	2.80E+01	2.21E+01	2.19E+01	3.84E+00	7.61E+00	2.40E+01	2.20E+01	2.16E+01	2.26E+00
G_{15}	2.79E−01	1.02E+01	3.39E+00	3.69E+00	2.15E+00	5.75E−01	9.82E+00	2.86E+00	3.24E+00	1.98E+00
G_{16}	3.25E+00	2.57E+02	2.11E+01	4.26E+01	5.69E+01	3.98E+00	1.46E+02	1.45E+01	2.29E+01	3.07E+01
G_{17}	9.95E+00	4.32E+01	3.10E+01	2.98E+01	7.50E+00	1.58E+01	3.95E+01	2.88E+01	2.86E+01	5.56E+00
G_{18}	2.04E+01	2.55E+01	2.20E+01	2.21E+01	1.09E+00	2.02E+01	2.35E+01	2.13E+01	2.11E+01	7.52E−01
G_{19}	3.95E+00	1.33E+01	8.14E+00	8.04E+00	2.28E+00	2.96E+00	1.19E+01	5.84E+00	5.83E+00	1.92E+00
G_{20}	2.34E+01	6.56E+01	3.49E+01	3.57E+01	7.50E+00	1.31E+01	5.34E+01	3.04E+01	3.03E+01	7.35E+00
G_{21}	1.00E+02	2.07E+02	2.03E+02	1.99E+02	2.02E+01	2.07E+02	2.16E+02	2.12E+02	2.12E+02	2.56E+00
G_{22}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	0.00E+00	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.00E−13
G_{23}	3.45E+02	3.60E+02	3.51E+02	3.51E+02	3.51E+00	3.45E+02	3.65E+02	3.56E+02	3.56E+02	3.73E+00
G_{24}	1.00E+02	4.28E+02	4.24E+02	4.18E+02	4.55E+01	4.18E+02	4.34E+02	4.29E+02	4.28E+02	2.95E+00

Table 22 (continued)

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{25}	3.83E+02	3.87E+02	3.87E+02	3.87E+02	7.56E−01	3.87E+02	3.87E+02	3.87E+02	3.87E+02	8.90E−03
G_{26}	2.00E+02	9.28E+02	3.00E+02	5.37E+02	3.06E+02	8.46E+02	1.03E+03	9.55E+02	9.49E+02	4.60E+01
G_{27}	4.93E+02	5.11E+02	5.03E+02	5.02E+02	4.03E+00	4.88E+02	5.16E+02	5.05E+02	5.04E+02	6.70E+00
G_{28}	3.00E+02	4.14E+02	3.00E+02	3.08E+02	2.88E+01	3.00E+02	4.14E+02	3.00E+02	3.15E+02	3.86E+01
G_{29}	4.07E+02	4.60E+02	4.34E+02	4.33E+02	1.13E+01	4.18E+02	4.53E+02	4.35E+02	4.35E+02	7.36E+00
G_{30}	1.94E+03	2.10E+03	1.97E+03	1.99E+03	4.21E+01	1.94E+03	2.13E+03	1.97E+03	1.98E+03	4.17E+01
Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.73E−05	3.82E+03	7.04E+02	3.86E+02	8.71E+02
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	2.71E+04	8.26E+12	5.46E+11	6.31E+08	1.54E+12
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.41E+01	2.76E+02	8.63E+01	5.70E+01	7.15E+01
G_4	0.00E+00	6.41E+01	0.00E+00	1.17E+01	2.37E+01	1.41E−02	8.82E+01	5.91E+01	6.66E+01	2.65E+01
G_5	0.00E+00	2.49E+01	3.98E+00	4.23E+00	3.37E+00	2.69E+01	1.94E+02	5.99E+01	5.07E+01	3.23E+01
G_6	0.00E+00	1.37E−07	0.00E+00	2.68E−09	1.92E−08	3.46E−02	1.87E+00	5.53E−01	4.58E−01	4.07E−01
G_7	3.23E+01	4.14E+01	3.41E+01	3.44E+01	1.66E+00	7.14E+01	2.38E+02	1.38E+02	1.28E+02	5.02E+01
G_8	0.00E+00	9.95E+00	3.98E+00	4.57E+00	2.36E+00	2.98E+01	1.77E+02	6.21E+01	5.13E+01	3.59E+01
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	6.03E+00	2.78E+02	8.24E+01	5.31E+01	6.71E+01
G_{10}	5.54E+02	3.76E+03	1.82E+03	2.05E+03	6.48E+02	3.65E+03	7.18E+03	6.05E+03	6.33E+03	8.57E+02
G_{11}	1.99E+00	6.69E+01	6.97E+00	1.23E+01	1.59E+01	2.86E+01	1.99E+02	8.05E+01	7.46E+01	3.43E+01
G_{12}	3.74E+02	1.80E+03	1.03E+03	1.05E+03	3.66E+02	4.30E+03	7.16E+04	2.43E+04	2.04E+04	1.60E+04

Table 22 (continued)

Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{13}	2.99E+00	3.03E+01	1.85E+01	1.80E+01	6.32E+00	1.76E+02	6.05E+04	1.64E+04	4.17E+03	2.01E+04
G_{14}	3.98E+00	3.00E+01	2.40E+01	2.31E+01	4.86E+00	5.92E+01	1.18E+02	8.56E+01	8.37E+01	1.23E+01
G_{15}	1.49E+00	1.11E+01	4.47E+00	5.42E+00	2.71E+00	5.65E+01	5.31E+03	7.80E+02	2.07E+02	1.30E+03
G_{16}	2.78E+00	7.98E+02	2.67E+01	1.34E+02	1.71E+02	3.75E+01	1.31E+03	5.99E+02	5.79E+02	3.24E+02
G_{17}	2.15E+01	8.05E+01	4.57E+01	4.67E+01	1.14E+01	4.38E+01	4.32E+02	2.15E+02	2.06E+02	1.04E+02
G_{18}	1.42E+01	2.95E+01	2.31E+01	2.33E+01	2.28E+00	6.40E+02	1.88E+04	2.54E+03	1.85E+03	2.78E+03
G_{19}	4.78E+00	1.01E+01	7.23E+00	7.15E+00	1.30E+00	3.46E+01	4.56E+03	5.58E+02	6.71E+01	1.12E+03
G_{20}	1.69E+01	1.92E+02	3.97E+01	4.55E+01	2.99E+01	4.02E+01	4.10E+02	2.17E+02	1.92E+02	9.79E+01
G_{21}	1.00E+02	2.10E+02	1.00E+02	1.31E+02	4.87E+01	2.28E+02	3.31E+02	2.54E+02	2.48E+02	2.43E+01
G_{22}	1.00E+02	1.00E+02	1.00E+02	1.00E+02	1.40E−13	1.00E+02	1.05E+02	1.01E+02	1.00E+02	1.19E+00
G_{23}	3.47E+02	3.77E+02	3.56E+02	3.57E+02	6.14E+00	3.80E+02	5.27E+02	4.12E+02	4.07E+02	2.17E+01
G_{24}	2.00E+02	4.35E+02	4.30E+02	3.94E+02	8.45E+01	4.50E+02	5.90E+02	5.02E+02	4.94E+02	3.54E+01
G_{25}	3.87E+02	3.87E+02	3.87E+02	3.87E+02	2.84E−02	3.83E+02	4.25E+02	3.88E+02	3.88E+02	5.57E+00
G_{26}	3.00E+02	1.18E+03	1.00E+03	9.43E+02	2.23E+02	3.00E+02	2.32E+03	1.78E+03	1.77E+03	3.19E+02
G_{27}	4.92E+02	5.19E+02	5.08E+02	5.08E+02	5.04E+00	5.02E+02	5.80E+02	5.30E+02	5.26E+02	1.73E+01
G_{28}	3.00E+02	4.52E+02	3.00E+02	3.29E+02	5.00E+01	3.00E+02	4.83E+02	3.58E+02	3.17E+02	6.32E+01
G_{29}	4.14E+02	4.80E+02	4.40E+02	4.39E+02	1.50E+01	3.85E+02	1.22E+03	7.17E+02	6.99E+02	1.83E+02
G_{30}	1.94E+03	2.21E+03	1.97E+03	2.00E+03	6.62E+01	2.30E+03	1.27E+04	5.07E+03	4.65E+03	2.35E+03

Table 23 Comparative results of the CEC 2017 test functions with 50 dimensions

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	1.00E+01	1.00E+00	1.57E+00	1.93E+00
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	1.42E+02	2.85E+01	4.29E+01	3.32E+01	7.86E+00	1.40E+02	2.61E+01	5.14E+01	4.43E+01
G_5	2.98E+00	1.45E+01	6.96E+00	7.58E+00	2.42E+00	1.33E+01	3.56E+01	2.64E+01	2.52E+01	6.44E+00
G_6	0.00E+00	4.31E−07	4.79E−08	8.54E−08	1.14E−07	4.79E−08	6.15E−06	5.87E−07	9.16E−07	1.07E−06
G_7	5.53E+01	6.25E+01	5.79E+01	5.79E+01	1.53E+00	6.69E+01	8.80E+01	7.48E+01	7.66E+01	6.06E+00
G_8	3.98E+00	1.39E+01	7.96E+00	7.91E+00	2.47E+00	1.28E+01	3.61E+01	2.89E+01	2.63E+01	6.59E+00
G_9	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_{10}	2.14E+03	3.94E+03	3.16E+03	3.11E+03	4.01E+02	2.25E+03	3.75E+03	3.25E+03	3.20E+03	3.40E+02
G_{11}	1.93E+01	3.31E+01	2.62E+01	2.64E+01	3.36E+00	1.61E+01	2.56E+01	2.18E+01	2.14E+01	2.09E+00
G_{12}	1.23E+03	6.90E+03	1.78E+03	1.94E+03	8.34E+02	6.43E+02	2.26E+03	1.43E+03	1.48E+03	3.65E+02
G_{13}	7.96E+00	1.22E+02	4.49E+01	4.14E+01	2.48E+01	9.27E+00	1.28E+02	5.11E+01	6.94E+01	3.45E+01
G_{14}	2.50E+01	4.09E+01	3.09E+01	3.12E+01	3.52E+00	2.28E+01	3.57E+01	2.63E+01	2.65E+01	2.49E+00
G_{15}	9.59E+00	4.07E+01	2.93E+01	2.94E+01	5.20E+00	2.01E+01	3.94E+01	2.46E+01	2.56E+01	4.06E+00
G_{16}	1.25E+02	5.91E+02	3.44E+02	3.46E+02	1.46E+02	1.38E+02	4.81E+02	2.74E+02	2.75E+02	9.97E+01
G_{17}	6.28E+01	4.65E+02	2.61E+02	2.75E+02	8.63E+01	7.12E+01	3.85E+02	2.34E+02	2.07E+02	7.31E+01
G_{18}	2.25E+01	4.94E+01	3.05E+01	3.20E+01	5.99E+00	2.05E+01	3.16E+01	2.42E+01	2.43E+01	2.12E+00
G_{19}	1.71E+01	3.11E+01	2.44E+01	2.45E+01	3.94E+00	1.24E+01	2.30E+01	1.73E+01	1.74E+01	2.47E+00
G_{20}	5.76E+01	3.50E+02	1.21E+02	1.47E+02	7.44E+01	7.24E+01	2.88E+02	1.09E+02	1.14E+02	3.55E+01
G_{21}	2.02E+02	2.22E+02	2.10E+02	2.11E+02	4.06E+00	2.17E+02	2.41E+02	2.25E+02	2.27E+02	7.06E+00
G_{22}	1.00E+02	3.97E+03	1.00E+02	3.65E+02	9.24E+02	1.00E+02	3.95E+03	1.25E+02	1.59E+03	1.67E+03
G_{23}	4.15E+02	4.64E+02	4.34E+02	4.34E+02	8.16E+00	4.28E+02	4.56E+02	4.40E+02	4.39E+02	6.90E+00
G_{24}	4.99E+02	5.16E+02	5.06E+02	5.06E+02	3.85E+00	5.04E+02	5.23E+02	5.12E+02	5.13E+02	5.59E+00

Table 23 (continued)

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{25}	4.80E+02	5.63E+02	4.80E+02	4.89E+02	2.47E+01	4.80E+02	4.88E+02	4.80E+02	4.80E+02	1.08E+00
G_{26}	3.00E+02	1.31E+03	9.06E+02	7.06E+02	4.06E+02	9.79E+02	1.39E+03	1.24E+03	1.20E+03	1.19E+02
G_{27}	5.03E+02	5.40E+02	5.23E+02	5.22E+02	7.75E+00	5.08E+02	5.50E+02	5.25E+02	5.25E+02	9.21E+00
G_{28}	4.59E+02	5.08E+02	4.59E+02	4.67E+02	1.79E+01	4.53E+02	5.07E+02	4.56E+02	4.59E+02	1.19E+01
G_{29}	3.09E+02	4.06E+02	3.45E+02	3.47E+02	1.97E+01	3.33E+02	3.80E+02	3.53E+02	3.53E+02	9.78E+00
G_{30}	5.79E+05	6.89E+05	6.14E+05	6.18E+05	3.62E+04	5.78E+05	8.74E+05	6.46E+05	6.58E+05	7.24E+04
Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.14E−02	2.64E+04	3.73E+03	1.69E+03	4.87E+03
G_2	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	5.05E+17	3.58E+29	8.04E+27	2.77E+23	5.02E+28
G_3	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.20E+03	1.41E+04	7.24E+03	6.79E+03	2.48E+03
G_4	0.00E+00	1.42E+02	3.99E+00	3.75E+01	4.99E+01	9.77E+00	2.04E+02	9.11E+01	9.68E+01	5.02E+01
G_5	3.98E+00	2.39E+01	1.09E+01	1.12E+01	3.70E+00	7.76E+01	3.09E+02	1.42E+02	1.37E+02	3.96E+01
G_6	0.00E+00	4.85E−07	4.79E−08	7.44E−08	1.06E−07	1.39E+00	1.07E+01	3.23E+00	2.58E+00	1.85E+00
G_7	5.60E+01	6.41E+01	5.82E+01	5.89E+01	1.93E+00	1.90E+02	5.17E+02	2.91E+02	2.80E+02	7.43E+01
G_8	3.98E+00	3.28E+01	9.95E+00	1.03E+01	4.26E+00	8.46E+01	2.36E+02	1.40E+02	1.33E+02	3.15E+01
G_9	0.00E+00	8.95E−02	0.00E+00	1.76E−03	1.25E−02	4.10E+02	6.12E+03	2.38E+03	2.37E+03	1.21E+03
G_{10}	1.63E+03	6.74E+03	3.67E+03	3.82E+03	1.12E+03	9.52E+03	1.38E+04	1.20E+04	1.20E+04	9.12E+02
G_{11}	2.32E+01	5.90E+01	4.11E+01	4.04E+01	8.71E+00	6.49E+01	2.50E+02	1.60E+02	1.60E+02	4.28E+01
G_{12}	1.09E+03	3.31E+03	2.09E+03	2.14E+03	5.05E+02	3.34E+04	7.38E+05	2.90E+05	2.39E+05	1.89E+05

Table 23 (continued)

Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{13}	8.96E+00	1.27E+02	4.43E+01	4.64E+01	2.55E+01	1.19E+02	3.74E+04	6.28E+03	2.87E+03	8.44E+03
G_{14}	2.70E+01	4.59E+01	3.79E+01	3.71E+01	5.07E+00	1.63E+02	3.61E+02	2.51E+02	2.49E+02	4.72E+01
G_{15}	2.43E+01	6.10E+01	4.00E+01	4.09E+01	9.16E+00	1.19E+02	3.15E+04	7.91E+03	7.92E+03	6.56E+03
G_{16}	1.31E+02	1.59E+03	6.07E+02	6.77E+02	3.65E+02	2.63E+02	2.57E+03	1.38E+03	1.35E+03	4.43E+02
G_{17}	1.81E+02	1.13E+03	4.64E+02	4.81E+02	2.08E+02	5.00E+02	1.82E+03	9.70E+02	9.39E+02	2.69E+02
G_{18}	2.20E+01	7.12E+01	3.83E+01	3.86E+01	9.75E+00	1.35E+03	5.69E+04	1.58E+04	1.12E+04	1.36E+04
G_{19}	2.01E+01	8.15E+01	3.64E+01	4.12E+01	1.61E+01	1.15E+02	3.63E+04	1.46E+04	1.47E+04	1.01E+04
G_{20}	3.70E+01	9.86E+02	2.73E+02	3.02E+02	2.30E+02	9.36E+01	1.46E+03	7.04E+02	6.84E+02	3.35E+02
G_{21}	2.03E+02	2.24E+02	2.12E+02	2.12E+02	4.18E+00	2.64E+02	4.82E+02	3.29E+02	3.18E+02	3.93E+01
G_{22}	1.00E+02	5.52E+03	1.00E+02	6.80E+02	1.40E+03	1.00E+02	1.43E+04	1.01E+04	1.25E+04	5.07E+03
G_{23}	4.32E+02	4.68E+02	4.45E+02	4.45E+02	8.27E+00	5.17E+02	7.29E+02	5.95E+02	5.87E+02	4.27E+01
G_{24}	5.06E+02	5.27E+02	5.16E+02	5.17E+02	4.09E+00	5.93E+02	8.18E+02	7.02E+02	6.91E+02	6.36E+01
G_{25}	4.78E+02	4.92E+02	4.80E+02	4.82E+02	4.10E+00	4.61E+02	5.92E+02	5.33E+02	5.53E+02	3.95E+01
G_{26}	1.08E+03	1.42E+03	1.23E+03	1.24E+03	7.24E+01	3.00E+02	6.66E+03	3.02E+03	2.92E+03	1.05E+03
G_{27}	5.17E+02	6.40E+02	5.32E+02	5.40E+02	2.23E+01	5.15E+02	1.09E+03	7.80E+02	7.66E+02	1.56E+02
G_{28}	4.59E+02	5.08E+02	4.59E+02	4.82E+02	2.46E+01	4.59E+02	5.88E+02	4.97E+02	4.99E+02	2.66E+01
G_{29}	2.98E+02	4.25E+02	3.60E+02	3.62E+02	2.58E+01	6.38E+02	2.45E+03	1.08E+03	9.93E+02	3.43E+02
G_{30}	5.79E+05	8.41E+05	6.49E+05	6.64E+05	7.80E+04	6.25E+05	1.47E+06	8.55E+05	8.16E+05	1.85E+05

Table 24 Comparative results of the CEC 2017 test functions with 100 dimensions

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	5.15E−08	0.00E+00	1.33E−09	7.52E−09	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_2	0.00E+00	3.78E+11	95166	1.27E+10	5.99E+10	4.00E+00	4.40E+12	6.70E+03	9.57E+10	6.18E+11
G_3	7.61E−08	5.20E−06	1.86E−07	2.99E−07	7.04E−07	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_4	0.00E+00	2.18E+02	1.97E+02	1.93E+02	3.09E+01	1.80E+02	2.07E+02	1.93E+02	1.98E+02	8.30E+00
G_5	1.79E+01	4.28E+01	2.79E+01	2.87E+01	5.28E+00	4.46E+01	1.14E+02	5.51E+01	5.59E+01	9.91E+00
G_6	5.13E−06	3.41E−05	1.47E−05	1.63E−05	7.13E−06	2.31E−05	1.31E−04	5.55E−05	6.02E−05	2.18E−05
G_7	1.16E+02	1.38E+02	1.21E+02	1.22E+02	4.47E+00	1.42E+02	1.93E+02	1.62E+02	1.62E+02	7.91E+00
G_8	1.79E+01	4.68E+01	2.79E+01	2.97E+01	7.48E+00	4.21E+01	6.75E+01	5.39E+01	5.35E+01	5.39E+00
G_9	0.00E+00	8.95E−02	0.00E+00	1.76E−03	1.25E−02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
G_{10}	6.77E+03	1.34E+04	9.50E+03	9.91E+03	1.91E+03	8.50E+03	1.13E+04	1.04E+04	1.03E+04	5.21E+02
G_{11}	3.38E+01	1.42E+02	6.15E+01	6.56E+01	2.00E+01	2.17E+01	1.15E+02	3.41E+01	4.92E+01	3.02E+01
G_{12}	2.45E+03	6.06E+03	4.13E+03	4.19E+03	7.89E+02	2.93E+03	6.00E+03	4.63E+03	4.62E+03	6.48E+02
G_{13}	1.19E+02	5.50E+02	2.14E+02	2.45E+02	8.84E+01	5.13E+01	2.23E+02	1.28E+02	1.25E+02	3.65E+01
G_{14}	8.17E+01	2.06E+02	1.36E+02	1.38E+02	2.96E+01	3.71E+01	6.91E+01	4.80E+01	4.97E+01	8.17E+00
G_{15}	8.60E+01	2.70E+02	1.63E+02	1.65E+02	3.87E+01	4.62E+01	1.54E+02	8.32E+01	8.99E+01	2.83E+01
G_{16}	7.86E+02	2.53E+03	1.36E+03	1.41E+03	3.76E+02	7.12E+02	1.73E+03	1.23E+03	1.22E+03	2.36E+02
G_{17}	5.35E+02	1.69E+03	1.18E+03	1.21E+03	2.57E+02	5.82E+02	1.22E+03	9.44E+02	9.32E+02	1.74E+02
G_{18}	1.08E+02	4.05E+02	2.31E+02	2.37E+02	5.94E+01	4.46E+01	1.18E+02	7.63E+01	7.79E+01	1.99E+01
G_{19}	8.16E+01	1.72E+02	1.12E+02	1.15E+02	1.88E+01	4.24E+01	6.98E+01	5.59E+01	5.55E+01	6.05E+00
G_{20}	7.80E+02	2.09E+03	1.35E+03	1.36E+03	3.09E+02	4.75E+02	1.45E+03	1.12E+03	1.08E+03	2.16E+02
G_{21}	2.44E+02	3.15E+02	2.58E+02	2.60E+02	1.06E+01	2.61E+02	2.92E+02	2.77E+02	2.77E+02	6.94E+00
G_{22}	1.00E+02	1.44E+04	1.02E+04	1.02E+04	2.70E+03	8.72E+03	1.13E+04	1.05E+04	1.04E+04	5.30E+02
G_{23}	5.50E+02	6.23E+02	5.76E+02	5.77E+02	1.31E+01	5.76E+02	6.16E+02	5.98E+02	5.98E+02	7.69E+00
G_{24}	8.99E+02	9.74E+02	9.16E+02	9.19E+02	1.32E+01	8.99E+02	9.64E+02	9.14E+02	9.17E+02	1.34E+01

Table 24 (continued)

Fun.	Metaheuristics									
	EBO with CMAR (Kumar et al. 2017)					LSHADE-cnEpSin (Awad et al. 2017)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{25}	5.77E+02	7.74E+02	7.05E+02	7.16E+02	3.71E+01	5.76E+02	7.73E+02	6.97E+02	6.84E+02	4.34E+01
G_{26}	3.00E+02	3.53E+03	3.19E+03	2.77E+03	1.08E+03	2.92E+03	3.41E+03	3.11E+03	3.11E+03	1.22E+02
G_{27}	5.48E+02	6.19E+02	5.86E+02	5.88E+02	1.53E+01	5.59E+02	6.13E+02	5.87E+02	5.89E+02	1.31E+01
G_{28}	3.00E+02	5.77E+02	5.19E+02	5.10E+02	6.01E+01	4.77E+02	5.71E+02	5.16E+02	5.15E+02	2.20E+01
G_{29}	8.61E+02	1.84E+03	1.28E+03	1.28E+03	2.42E+02	8.99E+02	1.47E+03	1.11E+03	1.12E+03	1.49E+02
G_{30}	2.18E+03	2.79E+03	2.36E+03	2.40E+03	1.51E+02	2.08E+03	2.82E+03	2.35E+03	2.36E+03	1.44E+02
Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_1	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	3.20E+08	3.31E+09	1.16E+09	9.40E+08	6.68E+08
G_2	1.00E+00	8.57E+06	8.87E+04	5.21E+05	1.40E+06	3.41E+86	1.98E+102	6.39E+100	3.70E+94	2.96E+101
G_3	2.45E−06	2.24E+00	1.93E−02	1.46E−01	3.52E−01	1.19E+05	1.99E+05	1.57E+05	1.56E+05	1.79E+04
G_4	9.70E+01	2.50E+02	1.08E+02	1.18E+02	2.78E+01	5.00E+02	9.22E+02	6.81E+02	6.73E+02	1.00E+02
G_5	1.89E+01	5.17E+01	2.99E+01	3.01E+01	5.68E+00	4.57E+02	9.93E+02	6.25E+02	6.07E+02	1.11E+02
G_6	8.79E−07	1.67E−02	3.80E−04	1.48E−03	3.00E−03	2.70E+01	5.40E+01	3.95E+01	3.89E+01	5.34E+00
G_7	1.17E+02	1.31E+02	1.24E+02	1.24E+02	3.37E+00	1.09E+03	1.72E+03	1.37E+03	1.35E+03	1.45E+02
G_8	2.09E+01	4.38E+01	2.79E+01	2.91E+01	4.90E+00	4.34E+02	1.03E+03	6.16E+02	5.76E+02	1.25E+02
G_9	0.00E+00	3.36E+00	6.33E−01	9.58E−01	8.54E−01	2.20E+04	4.84E+04	2.92E+04	2.86E+04	7.32E+03
G_{10}	3.61E+03	1.12E+04	6.64E+03	6.80E+03	1.61E+03	2.46E+04	3.01E+04	2.82E+04	2.82E+04	1.24E+03
G_{11}	2.38E+02	5.94E+02	3.44E+02	3.52E+02	7.26E+01	4.43E+03	1.46E+04	7.05E+03	6.76E+03	1.80E+03
G_{12}	2.80E+03	5.81E+03	4.11E+03	4.07E+03	6.60E+02	5.12E+07	3.85E+08	1.69E+08	1.63E+08	5.91E+07

Table 24 (continued)

Fun.	Metaheuristics									
	MM_OED (Sallam et al. 2017)					CGO (present study)				
	Best	Worst	Median	Mean	Std	Best	Worst	Median	Mean	Std
G_{13}	2.00E+02	6.25E+02	3.15E+02	3.45E+02	9.41E+01	7.90E+03	5.11E+04	2.12E+04	1.93E+04	9.06E+03
G_{14}	1.82E+02	4.01E+02	2.64E+02	2.65E+02	4.11E+01	1.74E+05	1.51E+06	5.56E+05	5.25E+05	2.50E+05
G_{15}	1.41E+02	3.72E+02	2.72E+02	2.78E+02	5.62E+01	1.02E+03	1.65E+04	3.81E+03	2.33E+03	3.40E+03
G_{16}	3.17E+02	2.77E+03	9.90E+02	1.05E+03	4.61E+02	2.64E+03	7.48E+03	4.53E+03	4.40E+03	1.11E+03
G_{17}	2.64E+02	2.26E+03	1.05E+03	1.11E+03	4.81E+02	1.99E+03	4.45E+03	3.17E+03	3.29E+03	6.00E+02
G_{18}	1.82E+02	4.33E+02	2.80E+02	2.84E+02	5.79E+01	3.50E+05	1.77E+06	9.12E+05	8.39E+05	3.64E+05
G_{19}	1.18E+02	2.75E+02	2.07E+02	2.03E+02	3.69E+01	5.35E+02	1.31E+04	3.27E+03	1.95E+03	2.79E+03
G_{20}	3.13E+02	2.40E+03	8.96E+02	1.04E+03	5.34E+02	1.77E+03	4.45E+03	3.10E+03	3.11E+03	5.87E+02
G_{21}	2.41E+02	2.71E+02	2.58E+02	2.58E+02	6.53E+00	7.12E+02	1.32E+03	9.11E+02	8.74E+02	1.45E+02
G_{22}	3.93E+03	9.93E+03	5.84E+03	5.98E+03	1.31E+03	2.51E+04	3.09E+04	2.91E+04	2.93E+04	1.15E+03
G_{23}	5.65E+02	6.26E+02	5.83E+02	5.84E+02	1.18E+01	8.98E+02	1.34E+03	1.11E+03	1.12E+03	1.02E+02
G_{24}	8.90E+02	9.40E+02	9.17E+02	9.17E+02	1.17E+01	1.46E+03	2.07E+03	1.78E+03	1.74E+03	1.52E+02
G_{25}	6.15E+02	7.82E+02	7.01E+02	7.07E+02	4.01E+01	9.96E+02	1.50E+03	1.28E+03	1.30E+03	1.12E+02
G_{26}	3.10E+03	3.53E+03	3.27E+03	3.27E+03	1.12E+02	8.30E+03	1.97E+04	1.19E+04	1.12E+04	2.53E+03
G_{27}	5.00E+02	5.00E+02	5.00E+02	5.00E+02	1.38E−04	7.60E+02	1.27E+03	1.00E+03	9.84E+02	1.13E+02
G_{28}	4.88E+02	5.00E+02	5.00E+02	4.98E+02	4.76E+00	8.46E+02	1.51E+03	1.05E+03	1.04E+03	1.03E+02
G_{29}	1.01E+03	2.36E+03	1.45E+03	1.54E+03	3.53E+02	3.10E+03	5.71E+03	4.22E+03	4.32E+03	5.96E+02
G_{30}	2.24E+03	3.01E+03	2.41E+03	2.45E+03	1.77E+02	2.05E+05	2.05E+06	6.46E+05	6.08E+05	3.24E+05

Fig. 7 Mathematical process for T_0 assessment

```
Procedure T0 Assessment
        x=0.55;
        for i=1:1000000
            x=x+x;
            x=x/2;
            x=x×x;
            x=sqrt(x);
            x=log(x);
            x=exp(x);
            x=x/(x+2);
        end
end procedure
```

9 Computational cost and complexity analysis

In order to evaluate the overall performance of a new optimization algorithm from different points of view, the computational cost and complexity analysis can be conducted. In computer sciences, the "Big O notation" is a mathematical notation which represents the required running time and memory space of an algorithm by considering the growth rate in dealing with different inputs.

For the presented CGO algorithm, the random selection process in the initialization phase of the algorithm have computational complexity of O(NP*D) in which the NP is the initial population size related to the number of initial eligible seeds and D is the dimension of the problem. The computational complexity of the objective function evaluation in the initialization phase of the algorithm is calculated as O(NP)*O(F(x)) in which the F(x) demonstrates the objective function value. After the initialization phase, the main loop of the algorithm is started based on the previously determined maximum number of iterations (MaxIter). By consideration of a worst case scenario, each line has a computational complexity of MaxIter in the main loop of the algorithm. In this loop, four new seeds are created for each of the eligible seeds so the position updating process in this matter has computational complexity of O(MaxIter*NP*D*4). In addition, the objective function evaluation in the main loop has computational complexity of O(MaxIter*NP*4)*O(F(x)).

It should be noted that the CGO algorithm is a parameter-free metaheuristic approach in which there is no any internal parameter to be defined throughout the optimization process. In other words, one of the most remarkable aspects of this algorithm is its parameter-free framework in which the exploration and exploitation phases of the algorithm are adjusted through the main loop of the algorithm. In addition, the position updating process in this algorithm is conducted in four separate phases in which the global and local searching of the search space is satisfied in a more sensitive way which results in excellent responses.

Table 25 Computational complexity of the CGO method alongside other metaheuristics

Dim.	Metaheuristics															
	EBO with CMAR (Kumar et al. 2017)				LSHADE-cnEpSin (Awad et al. 2017)				MM_OED (Sallam et al. 2017)				CGO (present study)			
	T0	T1	$\hat{T}_2$	$\frac{\hat{T}_2 - T1}{T0}$	T0	T1	$\hat{T}_2$	$\frac{\hat{T}_2 - T1}{T0}$	T0	T1	$\hat{T}_2$	$\frac{\hat{T}_2 - T1}{T0}$	T0	T1	$\hat{T}_2$	$\frac{\hat{T}_2 - T1}{T0}$
10	0.0413	0.8218	7.5794	163.6223	0.1093	0.8391	2.1835	12.30009	2.157784	0.146416	6.704923	3.039417	0.022579	0.13228	3.168108	134.4536
30		1.1507	6.591	131.7264		1.0570	3.6724	23.92864		0.592848	20.84485	9.385555		0.539586	4.438477	172.6778
50		1.8792	8.7886	167.2978		1.4338	3.7066	20.79414		1.606688	38.51665	17.10549		1.251754	5.628126	193.8249
100		5.6887	18.4969	310.1259		3.0237	7.7564	43.30009		5.776893	72.62159	15.80554		5.310247	10.65569	236.7441

10 Conclusion

This paper develops Chaos Game Optimization algorithm based on some principles of chaos theory. The mathematical model of this algorithm is formulated based on chaos game methodology by considering the eligible configurations of fractals alongside the fractals self-similarity issues. Four groups of mathematical test functions have been selected in order to evaluate the performance of the new algorithm with a total number of six different metaheuristic algorithms. A complete statistical analysis is conducted in order to provide a valid judgment about the performance of the new algorithm. Some of the important results of this paper is as follows:

- The CGO algorithm is superior to other metaheuristics in converging to the global bests of the mathematical functions based on the selected tolerance.
- The results of the K–S test demonstrated that the maximum difference between the CGO algorithm and the other metaheuristics are about FA in most of the cases.
- The results of the M–W test showed that the summation of the ranks for the CGO algorithm in most of the cases are lower than the other metaheuristics.
- The results of the W test showed that the mean of the ranks for the CGO algorithm in most of the cases are lower than the other metaheuristics.
- The results of the K–W test proved that the CGO algorithm is successful in outranking the other metaheuristics for 2D functions in all of the cases such as the minimum, mean and standard deviation values except for the number of function evaluations.
- The results of the K–W test showed that the CGO algorithm has the first ranking in the minimum, mean and function evaluation values of the 50D test functions while the SOS outranks the CGO in the standard deviation values. For the 100D functions, SOS and WOA outranks the CGO in standard deviation and function evaluation respectively.
- The overall comparing of the CGO and the other metaheuristics considering all of the 2D, 50D and the 100D test functions demonstrate that the CGO the best algorithm in all of the cases.
- The comparative results of the CGO algorithm in dealing with the test functions provided by the CEC 2017 competition demonstrates that CGO is capable of providing very acceptable results in dealing with these test functions with different dimensions comparing to the other equipped algorithms.

For the future challenges, the different applications of the presented algorithm can be considered based on the fact that the capability of this algorithm should be checked in dealing with difficult other problems. Besides, the new configurations of this algorithm can be considered based on the fact that the researches may have different opinions about the presented methodology of the CGO.

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Compliance with ethical standards

Conflict of interest Authors have received research grant from the University of Tabriz (Grant Number 1105).

Appendix

Matlab code for the CGO algorithm.

```
% Chaos Game Optimization (CGO) source codes version 1.0
% Programmers and Authors: Siamak Talatahari & Mahdi Azizi
% E-mails: siamak.talat@gmail.com & mehdi.azizi875@gmail.com
clc; clear all;

%% Get Required Problem Information
ObjFuncName = @(x) Sphere(x); % @Cost Function
Var_Number = 10; % Number of variables;
LB = -10 *ones(1,Var_Number);  % Lower bound of variable;
UB = 10 *ones(1,Var_Number);  % Upper bound of variable;

%% Get Required Algorithm Parameters
MaxIter = 100; % Maximum number of generations;
Seed_Number = 25; % Maximum number of initial eligible points;

%% Outputs:
% BestSeed (Best solution);
% BestFitness (final Best fitness)
% Conv_History (Convergence History Curve)

%% Initialization
for i=1:Seed_Number
    % Initializing the Position of initial eligible points
    Seed(i,:)=unifrnd(LB,UB);
    % Initializing the fitness of initial eligible points
    Fun_eval(i,:)=feval(ObjFuncName,Seed(i,:));
end

%% Search Process of the CGO
for Iter=1:MaxIter
    for i=1:Seed_Number
        % Update the best Seed
        [~,idbest]=min(Fun_eval);
        BestSeed=Seed(idbest,:);
        %% Generate New Solutions
        % Random Numbers
        I=randi([1,2],1,12); % For Beta and Gamma
        Ir=randi([0,1],1,5); % For Alpha
        % Random Groups
        RandGroupNumber=randperm(Seed_Number,1);
        RandGroup=randperm(Seed_Number,RandGroupNumber);
        % Mean of Random Group
        MeanGroup=mean(Seed(RandGroup,:)).*(length(RandGroup)~=1)...
            +Seed(RandGroup(1,1),:)*(length(RandGroup)==1);
        % New Seeds
        Alpha(1,:)=rand(1,Var_Number);
        Alpha(2,:)= 2*rand(1,Var_Number)-1;
        Alpha(3,:)= (Ir(1)*rand(1,Var_Number)+1);
        Alpha(4,:)= (Ir(2)*rand(1,Var_Number)+(~Ir(2)));
        ii=randi([1,4],1,3);
        SelectedAlpha= Alpha(ii,:);
        NewSeed(1,:)=Seed(i,:)+SelectedAlpha(1,:).*(I(1)*BestSeed-
I(2)*MeanGroup);
        NewSeed(2,:)=BestSeed+SelectedAlpha(2,:).*(I(3)*MeanGroup-
I(4)*Seed(i,:));
```

```
        NewSeed(3,:)=MeanGroup+SelectedAlpha(3,:).*(I(5)*BestSeed-
I(6)*Seed(i,:));
        NewSeed(4,:)=unifrnd(LB,UB);
        for j=1:4
            % Checking/Updating the boundary limits for Seeds
            NewSeed(j,:)=bound(NewSeed(j,:),UB,LB);
            % Evaluating New Solutions
            Fun_evalNew(j,:)=feval(ObjFuncName, NewSeed(j,:));
        end
        Seed=[Seed; NewSeed];
        Fun_eval=[Fun_eval; Fun_evalNew];
    end

    % Update the best Seed
    [Fun_eval, SortOrder]=sort(Fun_eval);
    Seed=Seed(SortOrder,:);
    [BestFitness,idbest]=min(Fun_eval);
    BestSeed=Seed(idbest,:);
    Seed=Seed(1:Seed_Number,:);
    Fun_eval=Fun_eval(1:Seed_Number,:);

    % Store Best Cost Ever Found
    Conv_History(Iter)=BestFitness;

    % Show Iteration Information
    disp(['Iteration ' num2str(Iter) ': Best Cost = '
num2str(Conv_History(Iter))]);
end

%% Boundary Handling
function x=bound(x,UB,LB)
x(x>UB)=UB(x>UB); x(x<LB)=LB(x<LB);
end

%% Objective Function
function z=Sphere(x)
    z=sum(x.^2);
end
```

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