$I - V$ characteristics of an open quantum dot with a Coulomb interaction: Extension of the Landauer formula with exact scattering eigenstates

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Through an extension of the Landauer formula, we study the electron transport in an open quantum dot system under a finite bias voltage. The system that we study is an interacting resonant-level model equipped with infinite two leads of a linearized dispersion relation. We explicitly construct many-electron scattering eigenstates whose incident states are free-fermionic. A remarkable feature of the scattering state is the appearance of many-body bound states after the scattering. Extending Landauer's idea, we calculate the average electric current flowing through the quantum dot. We use a renormalization-group technique to deal with the divergences that appear in the limit of the large bandwidth of the leads. As a result, the average electric current is brought to a universal form.

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I. INTRODUCTION

Mesoscopic and nanoscale systems have attracted considerable interest recently both from experimental and theoretical points of view. One of the reasons is quantum effects appearing in electron transport across the sample that is much smaller than the coherence length. In open quantum dot systems, the Coulomb interaction affects the electron transport. For example, the nonequilibrium Kondo effect is experimentally observed in semiconductor quantum dots[1–3] and molecular systems.[4–6] On the other hand, theoretical understanding of such electron transport through an interacting region is still in progress. One of the difficulties is how to realize the steady state in a given system out of equilibrium.

The Landauer formula is a well-known phenomenological approach to the transport properties of noninteracting electrons. It gives a valuable result describing experiments of mesoscopic systems if the electron density is low enough.[7–11] In the formula, it is important to introduce an open quantum system equipped with infinite two leads in place of a system consisting of a small sample sandwiched between two electron reservoirs. The formula tells us that the electrical conductance across the sample is proportional to the transmission probability of the scattering state in the open system. One of the advantages of the formula is that the problem of nonequilibrium systems such as the electron transport can be treated in the (time-independent) scattering theory of quantum mechanics.

One of the main purposes of the present article is to extend Landauer's idea to an interacting case. By extending Landauer's idea, we mean the following procedure:

(i) To construct a quantum-mechanical many-electron scattering eigenstate that has a free-fermionic plane wave as its incident state;

(ii) To calculate the quantum-mechanical expectation value of the current operator with respect to the many-electron scattering eigenstate;

(iii) To calculate the statistical-mechanical average of the current with respect to the different Fermi distributions in the electron reservoirs.

Meanwhile, the Keldysh formalism of nonequilibrium Green's functions is known as a standard approach to interacting cases.[12–14] The approach realizes the nonequilibrium steady state through the time evolution together with adiabatic switching-on of the perturbative term. The approach has succeeded in simple systems such as quantum point contacts and open quantum dots. For an open quantum dot described by the Anderson model, a formula for the electric current beyond the linear-response regime has been proposed.[15–18] Thereby it has been often said that the Landauer formula has been extended to the interacting case. In order to calculate the electric current in the interacting case practically, however, we need some perturbation technique, which is generally a hard task. Recently, numerical approaches based on the Keldysh formalism have been developed.[19]

The Bethe-ansatz technique[20–24] has been applied to the analysis of open systems out of equilibrium.[25–29] However, the application seems to have the following debatable points. In the study of electron transport in mesoscopic systems, we need to realize the nonequilibrium steady state between two electron reservoirs that are in the Fermi-degenerate states with given chemical potentials. The above mentioned Bethe-ansatz study of electron transport[25–29] constructed an equilibrium state of the electron reservoir by imposing the periodic boundary conditions to the Bethe-ansatz incident state, which is different from our extension of the Landauer formula. However, it is not easy to show that this Bethe-ansatz equilibrium state is the Fermi-degenerate state with the chemical potential since the

Bethe-ansatz incident state is characterized by quasimomenta, which are not real momenta. Actually, for the system with a linearized dispersion relation, it is impossible to relate the upper and lower limits of the quasimomentum to the chemical potential and the low energy cutoff at the same time only from the number of electrons. It is also reported that the Bethe-ansatz eigenfunctions adopted in Ref. 28 do not satisfy the matching conditions that are derived from the equation of motion and their result deviates from that of the Green's function approach.[30]

In the previous Letter,[31] we applied Landauer's idea to the calculation of the electric current in an open quantum dot system described by an interacting resonant level model (IRLM). We constructed exact many-electron scattering eigenstates of the open system and calculated the electric current flowing through the quantum dot. In a sense, our approach is a genuine extension of the Landauer formula. In order to prepare the Fermi-degenerate state with a given chemical potential, it was essential to construct many-electron scattering eigenstates whose incident states are free-electronic. It was remarkable that the set of wave numbers characterizing the incident plane wave is *not* conserved during the scattering; hence the scattering eigenstates is not in the form of the Bethe-ansatz state.[22,28,32] The plane wave is partially scattered to many-body bound states. By using the explicit form of the two- and three-electron scattering eigenstates, we calculated the contribution of the two-body bound states to the average electric current. As a result, we obtained the nonlinear current-voltage characteristics, which qualitatively agreed with that obtained by the other approaches[30,33–35] except for the Bethe-ansatz approach.

In the present article, we study the nonlinear current-voltage characteristics of the same open quantum dot system in more detail. In the previous Letter,[31] based on the results of two- and three-electron cases, we speculated the form of the quantum-mechanical expectation value of the electric current for general N-electron cases. In the present article, we explicitly construct the N-electron scattering eigenstates in the first order in the interaction parameter U and calculate the expectation value of the current. Then we find that the two- and three-electron scattering processes are essential for the calculation. The average electric current includes terms of linear and logarithmic divergences in the limit of the large bandwidth of the two leads. To deal with the divergences, we use a renormalization-group technique.[30,34] As a result, we obtain a universal form of the average electric current in the first order in U.

It would be valuable to show that our approach is equivalent to the current formula in the Green's function approach.[15–18] Indeed the former is based on quantum-mechanical scattering states satisfying appropriate boundary conditions, while the latter realizes the nonequilibrium steady state through the time evolution of the Keldysh formalism. We present the first analytic example that strongly suggests the equivalence.

The article consists of the following: in Sec. II, the two-lead IRLM is introduced. Through a transformation, the IRLM Hamiltonian is decomposed into even and odd parts. A general form of the N-electron state is given in the even-odd picture. The operator describing the electric current through the quantum dot is also introduced. To express the many-electron scattering eigenstates, we need the terms of the symmetric group, which is briefly explained in Appendix A. In Sec. III, the one- and two-electron scattering eigenstates are explicitly constructed by solving the Schrödinger equations under appropriate boundary conditions. The details of the construction of the two-electron scattering eigenstates are presented in Appendix B. The three-electron scattering eigenstates are also given in Appendix C. The N-electron scattering eigenstates are presented in the first order in the interaction parameter U. In Sec. IV, the two-electron scattering eigenstates are discussed in the framework of the Lippmann-Schwinger equation. The two-body bound states are interpreted in terms of two-electron propagators. In Sec. V, by using the many-electron scattering eigenstates, we calculate the quantum-mechanical expectation value of the electric current in the first order in the interaction parameter U. The detailed calculation is presented in Appendix D. Under the assumption that electrons are equilibrated in each reservoir before being emitted to the quantum dot, the average electric current for the system under a finite bias voltage is calculated. By using a renormalization-group technique, we arrive at a universal electric current.

II. OPEN QUANTUM DOT SYSTEM

A. Interacting resonant-level model

In order to study the quantum dot sandwiched between two electron reservoirs, we introduce an open quantum dot system described by the interacting resonant-level model (IRLM) with infinite two leads (see Fig. 1). The Hamiltonian is given by

$$H = \sum_{\ell=1,2} \left(\int_{-L/2}^{L/2} dx\, c_\ell^\dagger(x) \frac{1}{\mathrm{i}} \frac{d}{dx} c_\ell(x) + t_\ell c_\ell^\dagger(0) d + t_\ell^* d^\dagger c_\ell(0) \right) + \epsilon_d d^\dagger d + U \sum_{\ell=1,2} c_\ell^\dagger(0) c_\ell(0) d^\dagger d. \tag{1}$$

Here $c_\ell^\dagger(x)$ and $c_\ell(x)$ are the creation and annihilation operators of an electron on the lead ℓ, while $d^\dagger$ and d are those on the quantum dot. The parameter $t_\ell (\in \mathbb{C})$ is the transfer integral between the lead ℓ and the dot, $\epsilon_d (\in \mathbb{R})$ is the gate energy of the dot, and $U(>0)$ expresses the strength of the Coulomb repulsion around the quantum dot. The dispersion relation in each lead is linearized in the vicinity of the Fermi energy ϵ_F to be $\epsilon(k) = v_\mathrm{F}(k - k_\mathrm{F}) + \epsilon_\mathrm{F}$ under the assumption that $|t_\ell|$, $|\epsilon_d|$ and U are small compared with the Fermi energy ϵ_F.[20–22] For simplicity, we have set $v_\mathrm{F} = 1$, $k_\mathrm{F} = 0$, and $\epsilon_\mathrm{F} = 0$ in Eq. (1). We treat the system as an open system in the limit $L \to \infty$ to construct scattering eigenstates.

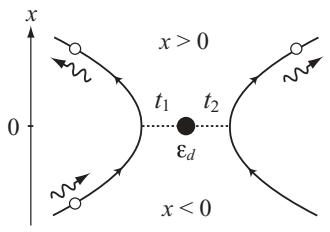

FIG. 1. The interacting resonant-level model.

To deal with the two-lead IRLM, it is convenient to decompose the Hamiltonian (1) into even and odd parts. After the transformation,

$$c_e(x) = \frac{1}{t}[t_1^* c_1(x) + t_2^* c_2(x)], \qquad c_o(x) = \frac{1}{t}[t_2 c_1(x) - t_1 c_2(x)], \tag{2}$$

with $t = \sqrt{|t_1|^2 + |t_2|^2}$, the Hamiltonian (1) is decomposed as

$$H = H_e + H_o, \tag{3a}$$

$$H_e = \int_{-L/2}^{L/2} dx\, c_e^\dagger(x) \frac{1}{\mathrm{i}} \frac{d}{dx} c_e(x) + t[c_e^\dagger(0) d + d^\dagger c_e(0)] + \epsilon_d d^\dagger d + U c_e^\dagger(0) c_e(0) d^\dagger d, \tag{3b}$$

$$H_o = \int_{-L/2}^{L/2} dx\, c_o^\dagger(x) \frac{1}{\mathrm{i}} \frac{d}{dx} c_o(x) + U c_o^\dagger(0) c_o(0) d^\dagger d. \tag{3c}$$

The even part H_e is essentially the Hamiltonian of the one-lead IRLM. Due to the commutation relations $[H, N_e + N_d] = [H, N_o] = 0$ for the number operators defined by

$$N_e = \int dx\, c_e^\dagger(x) c_e(x), \quad N_o = \int dx\, c_o^\dagger(x) c_o(x), \quad N_d = d^\dagger d, \tag{4}$$

the pair $(N_e + N_d, N_o)$ gives a good quantum number. Note that the odd part H_o is not decoupled from the even part H_e due to the term of the Coulomb interaction.

B. General form of scattering states

Let us consider the general form of N-electron states in the framework of the leads e and o introduced in Eqs. (3a), (3b), and (3c). Since the pair $(N_e + N_d, N_o)$ of the number operators gives a good quantum number, the N-electron state in the sector with $(N_e + N_d, N_o) = (\bar{n}, n) = (N - n, n)$ is expressed in the form,

$$|N,n\rangle = \left(\int dx dy\, g^{(n)}(x;y) |x_1, \ldots, x_{\bar{n}}; y_1, \ldots, y_n\rangle + \int dx dy\, e^{(n)}(x;y) |x_1, \ldots, x_{\bar{n}-1}, d; y_1, \ldots, y_n\rangle \right). \tag{5}$$

Here we have used the notation,

$$|x_1, \ldots, x_{\bar{n}}; y_1, \ldots, y_n\rangle = c_e^\dagger(x_1) \cdots c_e^\dagger(x_{\bar{n}}) c_o^\dagger(y_1) \cdots c_o^\dagger(y_n) |0\rangle,$$
$$|x_1, \ldots, x_{\bar{n}-1}, d; y_1, \ldots, y_n\rangle = c_e^\dagger(x_1) \cdots c_e^\dagger(x_{\bar{n}-1}) d^\dagger c_o^\dagger(y_1) \cdots c_o^\dagger(y_n) |0\rangle, \tag{6}$$

with the vacuum state $|0\rangle$ characterized by $c_1(x)|0\rangle = c_2(x)|0\rangle = d|0\rangle = 0$ and $\langle 0|0\rangle = 1$. The wave functions giving the N-electron state $|N,n\rangle$,

$$g^{(n)}(x;y) = g^{(n)}(x_1, \ldots, x_{\bar{n}}; y_1, \ldots, y_n), \qquad e^{(n)}(x;y) = e^{(n)}(x_1, \ldots, x_{\bar{n}-1}; y_1, \ldots, y_n), \tag{7}$$

are antisymmetric with respect to the variables $\{x_i\}$ and with respect to $\{y_i\}$. In terms of the wave functions, the square norm of the N-electron state $|N,n\rangle$ is calculated as

$$\langle N,n|N,n\rangle = \bar{n}!n! \int dx dy\, |g^{(n)}(x;y)|^2 + (\bar{n}-1)!n! \int dx dy\, |e^{(n)}(x;y)|^2. \tag{8}$$

The general N-electron state $|N\rangle$ is given by a linear combination of $|N;n\rangle$, $(0 \leqslant n \leqslant N)$ with coefficients $\{A^{(n)} \in \mathbb{C}\}$ as

$$|N\rangle = \sum_{n=0}^{N} A^{(n)} |N,n\rangle. \tag{9}$$

As we shall see below, the scattering eigenstate carrying electrons from one lead to the other lead is not in the form of the N-electron state $|N;n\rangle$ but in the form of the linear combination (9).

C. Current operator

We define the operator for the electric current through the quantum dot as

$$I = \alpha I_1 - (1-\alpha) I_2, \tag{10}$$

where I_ℓ is the current operator of the lead ℓ given by

$$I_\ell = \mathrm{i}[t_\ell c_\ell^\dagger(0) d - t_\ell^* d^\dagger c_\ell(0)], \tag{11}$$

and α is an arbitrary real parameter. We will calculate the quantum-mechanical expectation value $\langle\Psi|I|\Psi\rangle$ for a normalized scattering eigenstate $|\Psi\rangle$. The expectation value does not depend on the parameter α since the relation $\langle\Psi|I_1|\Psi\rangle = -\langle\Psi|I_2|\Psi\rangle$ holds for an arbitrary eigenstate $|\Psi\rangle$ due to the current conservation. Then, by choosing the parameter $\alpha = |t_2|^2/t^2$, the current operator I is simplified as

$$I = \frac{\mathrm{i}}{t}[t_1 t_2 c_o^\dagger(0) d - t_1^* t_2^* d^\dagger c_o(0)], \tag{12}$$

where the operators of the electrons on the lead e do not appear. For the general N-electron scattering state $|N\rangle$ in Eq. (9), the quantum-mechanical expectation value of the current is calculated as

$$\langle N|I|N\rangle = \frac{2}{t} \mathrm{Im}\left(t_1 t_2 \sum_{n=1}^{N} (-)^n \bar{n}!n! A^{(n)*} A^{(n-1)} \times \int dx dy\, g^{(n)*}(x;y)|_{y_n=0} e^{(n-1)}(x;y) \right), \tag{13}$$

which indicates that the expectation value is an overlap integral of the two states $|N,n-1\rangle$ and $|N,n\rangle$ included in the linear combination $|N\rangle$ in Eq. (9). Therefore, the expectation value can be nonzero only when both the coefficients $A^{(n-1)}$ and $A^{(n)}$ in the linear combination $|N\rangle$ are nonzero for some n. In what follows, we refer to the expectation value (13) for the N-electron scattering eigenstate as the N-electron current.

III. SCATTERING EIGENSTATES

A. One-electron scattering eigenstates

Let us first construct one-electron scattering eigenstates. Although the technique presented here is elementary, it will help us to understand the many-electron cases.

First, we consider one-electron eigenstates in the framework of the leads e and o. For $N = 1$, the eigenvalue problem

$H|1,n\rangle = k|1,n\rangle$, $(k \in \mathbb{R}, n = 0,1)$ is cast into a set of the following Schrödinger equations:

$$\left(\frac{1}{\mathrm{i}}\frac{d}{dx} - k\right) g^{(0)}(x) + te^{(0)}\delta(x) = 0, \tag{14a}$$

$$tg^{(0)}(0) + (\epsilon_d - k)e^{(0)} = 0, \tag{14b}$$

$$\left(\frac{1}{\mathrm{i}}\frac{d}{dx} - k\right) g^{(1)}(x) = 0. \tag{14c}$$

It is readily found that the eigenfunction $g^{(1)}(x)$ is just a plane wave with the wave number k. On the other hand, the eigenfunction $g^{(0)}(x)$ is a plane wave with the wave number k in the regions $x < 0$ and $0 < x$ but is discontinuous at $x = 0$. The jump of the eigenfunction $g^{(0)}(x)$ at $x = 0$ is determined by the matching condition,

$$g^{(0)}(0+) - g^{(0)}(0-) = -\mathrm{i}te^{(0)}, \tag{15}$$

which is derived by integrating the Schrödinger equation (14a) from $x = 0-$ to $x = 0+$. Since the value of the eigenfunction $g^{(0)}(x)$ at $x = 0$, which appears in Eq. (14b), is not determined by the Schrödinger equations, we assume

$$g^{(0)}(0) = \tfrac{1}{2}[g^{(0)}(0+) + g^{(0)}(0-)], \tag{16}$$

which should be consistent with physical intuition. By combining Eqs. (14b), (15), and (16), we have

$$g^{(0)}(0+) = \frac{k - \epsilon_d - \mathrm{i}t^2/2}{k - \epsilon_d + \mathrm{i}t^2/2} g^{(0)}(0-), \tag{17}$$

which gives the eigenfunction $e^{(0)}$ through Eqs. (14b) and (16) as

$$e^{(0)} = \frac{t}{k - \epsilon_d + \mathrm{i}t^2/2} g^{(0)}(0-). \tag{18}$$

Then the one-electron eigenfunctions are summarized as

$$\begin{aligned} g^{(0)}(x) = g_k(x) &:= \frac{1}{\sqrt{2\pi}} e^{\mathrm{i}kx}[\theta(-x) + e^{\mathrm{i}\delta_k}\theta(x)], \\ e^{(0)} = e_k &:= \frac{1}{\sqrt{2\pi}} \frac{t}{k - \epsilon_d + \mathrm{i}t^2/2}, \\ g^{(1)}(x) = h_k(x) &:= \frac{1}{\sqrt{2\pi}} e^{\mathrm{i}kx}, \end{aligned} \tag{19}$$

where $\theta(x)$ is the step function and $e^{\mathrm{i}\delta_k} = e_k/e_k^*$ is the scattering phase factor of the eigenfunction $g^{(0)}(x)$. The one-electron eigenstates $|1,n;k\rangle$, $(n = 0,1)$ with the energy eigenvalue $E = k$ are obtained by inserting the eigenfunctions (19) into the general form (5),

$$\begin{aligned} |1,0;k\rangle &= \left(\int dx\, g_k(x) c_e^\dagger(x) + e_k d^\dagger\right)|0\rangle, \\ |1,1;k\rangle &= \left(\int dx\, h_k(x) c_o^\dagger(x)\right)|0\rangle. \end{aligned} \tag{20}$$

The normalization constants of the eigenfunctions (19) are chosen so that the orthonormal relation,

$$\langle 1,n;k|1,m;l\rangle = \delta_{nm}\delta(k - l), \tag{21}$$

holds in the limit $L \to \infty$. Recall that the even part H_e of the Hamiltonian is equivalent to the IRLM with one lead. We remark that, only for the one-electron case, the eigenstate $|1,0;k\rangle$ has the same form as that constructed by the Bethe ansatz[22] although we never impose periodic boundary conditions.

Next, we construct one-electron scattering eigenstates whose incident states are a plane-wave state in the lead 1 or 2. We need to take a linear combination $|k\rangle = A|1,0;k\rangle + B|1,1;k\rangle$, which gives a general one-electron eigenstate with the energy eigenvalue $E = k$. The relations,

$$\begin{aligned} c_1^\dagger(x) &= \frac{1}{t}[t_1^* c_e^\dagger(x) + t_2 c_o^\dagger(x)], \\ c_2^\dagger(x) &= \frac{1}{t}[t_2^* c_e^\dagger(x) - t_1 c_o^\dagger(x)], \end{aligned} \tag{22}$$

tell us that, by choosing the linear combination as

$$\begin{aligned} |k\rangle_1 &= \frac{t_1^*}{t}|1,0;k\rangle + \frac{t_2}{t}|1,1;k\rangle, \\ |k\rangle_2 &= \frac{t_2^*}{t}|1,0;k\rangle - \frac{t_1}{t}|1,1;k\rangle, \end{aligned} \tag{23}$$

we obtain the scattering eigenstate $|k\rangle_\ell$, $(\ell = 1,2)$ whose incident plane wave is emitted only from the lead ℓ. For example, the scattering eigenstate $|k\rangle_1$, which is depicted in Fig. 1, is explicitly written as

$$|k\rangle_1 = \left(\int dx[g_k^{(+)}(x)c_1^\dagger(x) + g_k^{(-)}(x)c_2^\dagger(x)] + e_k d^\dagger\right)|0\rangle, \tag{24}$$

with the wave functions,

$$\begin{aligned} g_k^{(+)}(x) &= \frac{1}{\sqrt{2\pi}} e^{\mathrm{i}kx}\left(\theta(-x) + \frac{|t_1|^2 e^{\mathrm{i}\delta_k} + |t_2|^2}{t^2}\theta(x)\right), \\ g_k^{(-)}(x) &= \frac{1}{\sqrt{2\pi}} e^{\mathrm{i}kx}\frac{t_1^* t_2}{t^2}(e^{\mathrm{i}\delta_k} - 1)\theta(x). \end{aligned} \tag{25}$$

If we set $t_1 = 0$, the scattering state $|k\rangle_1$ is just a plane-wave state only on the lead 1. On the other hand, if we set $t_2 = 0$, the scattering state $|k\rangle_1$ is a plane-wave state on the lead 1 and is scattered at the quantum dot.

B. Two-electron eigenstates

We now consider the exact two-electron eigenstates, which are obtained by solving the following set of the Schrödinger equations:

$$\begin{aligned} &\left(\frac{1}{\mathrm{i}}(\partial_1 + \partial_2) - E\right) g^{(0)}(x_1,x_2) \\ &\quad - \frac{t}{2}[\delta(x_1)e^{(0)}(x_2) - e^{(0)}(x_1)\delta(x_2)] = 0, \\ &\left(\frac{1}{\mathrm{i}}\frac{d}{dx} + U\delta(x) + \epsilon_d - E\right) e^{(0)}(x) + 2tg^{(0)}(x,0) = 0, \\ &\left(\frac{1}{\mathrm{i}}(\partial_1 + \partial_2) - E\right) g^{(1)}(x_1;x_2) + t\delta(x_1)e^{(1)}(x_2) = 0, \\ &\left(\frac{1}{\mathrm{i}}\frac{d}{dx} + U\delta(x) + \epsilon_d - E\right) e^{(1)}(x) + tg^{(1)}(0;x) = 0, \\ &\left(\frac{1}{\mathrm{i}}(\partial_1 + \partial_2) - E\right) g^{(2)}(x_1,x_2) = 0, \end{aligned} \tag{26}$$

where $\partial_i = \partial/\partial x_i$, $(i = 1,2)$. Recall that the functions $g^{(0)}(x_1,x_2)$ and $g^{(2)}(x_1,x_2)$ are antisymmetric with respect to the exchange of the variables x_1 and x_2. The Schrödinger equations (26) imply that the eigenfunction $g^{(0)}(x_1,x_2)$ is discontinuous at $x_1 = 0$ and $x_2 = 0$, $g^{(1)}(x_1;x_2)$ at $x_1 = 0$, and $e^{(0)}(x)$ and $e^{(1)}(x)$ at $x = 0$. In a way similar to the one-electron case, we need to assume the values of the eigenfunctions $g^{(0)}(x_1,x_2)$, $g^{(1)}(x_1;x_2)$, $e^{(0)}(x)$, and $e^{(1)}(x)$ at the discontinuous points as

$$\begin{aligned} g^{(0)}(x,0) &= \tfrac{1}{2}[g^{(0)}(x,0+) + g^{(0)}(x,0-)], \\ e^{(0)}(0) &= \tfrac{1}{2}[e^{(0)}(0+) + e^{(0)}(0-)], \\ g^{(1)}(0;x) &= \tfrac{1}{2}[g^{(1)}(0+;x) + g^{(1)}(0-;x)], \\ e^{(1)}(0) &= \tfrac{1}{2}[e^{(1)}(0+) + e^{(1)}(0-)]. \end{aligned} \tag{27}$$

On the other hand, the function $g^{(2)}(x_1,x_2)$ should be a free-electronic eigenfunction. To establish an extension of the Landauer formula, we need the eigenfunction whose incident state is a free-electronic plane wave, that is,

$$\begin{aligned} g^{(n)}(x_1,x_2) &= \frac{1}{4\pi}\sum_P \mathrm{sgn}(P) e^{\mathrm{i}(k_{P_1}x_1 + k_{P_2}x_2)}, \quad (n = 0,2), \\ g^{(1)}(x_1;x_2) &= \frac{1}{2\pi} e^{\mathrm{i}(k_1x_1 + k_2x_2)}, \end{aligned} \tag{28}$$

in the region $x_1,x_2 < 0$. Here $P = (P_1,P_2)$ is a permutation of $(1,2)$ (see Appendix A). The eigenfunctions with the energy eigenvalue $E = k_1 + k_2$, $(k_1,k_2 \in \mathbb{R})$ are constructed as follows:

$$\begin{aligned} 2g^{(0)}_{k_1k_2}(x_1,x_2) &= \sum_Q \mathrm{sgn}(Q)\big[g_{k_1}(x_{Q_1})g_{k_2}(x_{Q_2}) \\ &\quad + ue^{\mathrm{i}Ex_{Q_2}} Z_{k_1k_2}(x_{Q_1Q_2})\theta(x_{Q_1})\big], \\ e^{(0)}_{k_1k_2}(x) &= \sum_P \mathrm{sgn}(P) g_{k_{P_1}}(x) e_{k_{P_2}} + \frac{u}{\mathrm{i}t} e^{\mathrm{i}Ex} Z_{k_1k_2}(-x), \\ g^{(1)}_{k_1k_2}(x_1;x_2) &= g_{k_1}(x_1)h_{k_2}(x_2) - ue^{\mathrm{i}Ex_2} X_{k_1}(x_{12})\theta(x_1), \\ e^{(1)}_{k_1k_2}(x) &= e_{k_1}h_{k_2}(x) + \frac{u}{\mathrm{i}t} X_{k_1}(-x)e^{\mathrm{i}Ex}, \\ 2g^{(2)}_{k_1k_2}(x_1,x_2) &= \sum_Q \mathrm{sgn}(Q) h_{k_1}(x_{Q_1})h_{k_2}(x_{Q_2}), \end{aligned} \tag{29}$$

where $P = (P_1,P_2)$ and $Q = (Q_1,Q_2)$ are permutations of $(1,2)$, $x_{ij} = x_i - x_j$, $u = 2U/(2 + \mathrm{i}U)$ and

$$\begin{aligned} Z_{kh}(x) &= -\frac{t}{\sqrt{2\pi}}(e_k - e_h)e^{\mathrm{i}(\epsilon_d - \mathrm{i}\frac{t^2}{2})x}\theta(-x), \\ X_k(x) &= \frac{t}{\sqrt{2\pi}} e_k e^{\mathrm{i}(\epsilon_d - \mathrm{i}\frac{t^2}{2})x}\theta(-x). \end{aligned} \tag{30}$$

The derivation is presented in Appendix B. The wave-number set $\{k_1,k_2\}$ in each of the eigenfunctions $g^{(0)}_{k_1k_2}(x_1,x_2)$ and $g^{(1)}_{k_1k_2}(x_1;x_2)$ is not conserved during the scattering; the plane wave with $\{k_1,k_2\}$ is partially scattered to that with $\{\epsilon_d - \mathrm{i}t^2/2, E - \epsilon_d + \mathrm{i}t^2/2\}$ in the region $x_1,x_2 > 0$. In this sense, they are not the Bethe eigenfunctions.[22,28,32]

The second term of each of the first four eigenfunctions (29) is originated from the Coulomb interaction. The imaginary part of the wave numbers, $\mathrm{i}t^2/2$, in $Z_{kh}(x)$ and $X_k(x)$ indicates the appearance of *a two-body bound state* $e^{-\frac{t^2}{2}|x_{12}|}$, which exponentially decreases as the two electrons separate from each other. The Coulomb interaction is a necessary condition of the appearance of the two-body bound state and the strength of binding is determined by the transfer integrals t_1 and t_2 between each lead and the quantum dot. Similar two-body bound states have been found in an open quantum dot with the spin degrees of freedom described by the Anderson model.[36]

By inserting the eigenfunctions (29) into the general form (5), we obtain the two-electron eigenstates with the energy eigenvalue $E = k_1 + k_2$:

$$\begin{aligned} |2,0;k_1,k_2\rangle &= \int dx_1dx_2\, g^{(0)}_{k_1k_2}(x_1,x_2)|x_1,x_2;\rangle \\ &\quad + \int dx\, e^{(0)}_{k_1k_2}(x)|x,d;\rangle, \end{aligned} \tag{31a}$$

$$\begin{aligned} |2,1;k_1,k_2\rangle &= \int dx_1dx_2\, g^{(1)}_{k_1k_2}(x_1;x_2)|x_1;x_2\rangle \\ &\quad + \int dx\, e^{(1)}_{k_1k_2}(x)|d;x\rangle, \end{aligned} \tag{31b}$$

$$|2,2;k_1,k_2\rangle = \int dx_1dx_2\, g^{(2)}_{k_1k_2}(x_1,x_2)|;x_1,x_2\rangle. \tag{31c}$$

Recall that $|x_1,x_2;\rangle = c^\dagger_e(x_1)c^\dagger_e(x_2)|0\rangle$ and so on as is given in Eq. (6). We notice that, by exchanging k_1 and k_2 in the eigenstate $|2,1;k_1,k_2\rangle$ in Eq. (31b), we have another eigenstate $|2,1;k_2,k_1\rangle$ with the same energy $E = k_1 + k_2$. The four eigenstates satisfy the orthonormal relations,

$$\begin{aligned} &\langle 2,0;k_1,k_2|2,0;l_1,l_2\rangle = \langle 2,2;k_1,k_2|2,2;l_1,l_2\rangle \\ &\quad = \delta(k_1 - l_1)\delta(k_2 - l_2) - \delta(k_1 - l_2)\delta(k_2 - l_1), \\ &\langle 2,1;k_1,k_2|2,1;l_1,l_2\rangle = \delta(k_1 - l_1)\delta(k_2 - l_2), \\ &\langle 2,n;k_1,k_2|2,m;l_1,l_2\rangle = 0 \quad \text{for} \quad n \neq m, \end{aligned} \tag{32}$$

in the limit $L \to \infty$.

C. Two-electron scattering eigenstates

Now we construct the two-electron scattering eigenstates. In order to extend the Landauer formula to the interacting case, we need scattering eigenstates whose incident states are free-electronic plane-wave states in the framework of the original leads 1 and 2. To establish this, we take a linear combination of the two-electron eigenstates (31a), (31b), and (31c) with the same energy eigenvalue $E = k_1 + k_2$,

$$\begin{aligned} |k_1,k_2\rangle &= A|2,0;k_1,k_2\rangle + B_1|2,1;k_1,k_2\rangle \\ &\quad - B_2|2,1;k_2,k_1\rangle + C|2,2;k_1,k_2\rangle, \end{aligned} \tag{33}$$

and choose the coefficients A, B_1, B_2, and C as follows:

(i) $A = \dfrac{t_1^{*2}}{t^2}$, $B_1 = B_2 = \dfrac{t_1^*t_2}{t^2}$, $C = \dfrac{t_2^2}{t^2}$,

(ii) $A = \dfrac{t_1^*t_2^*}{t^2}$, $B_1 = -\dfrac{|t_1|^2}{t^2}$, $B_2 = \dfrac{|t_2|^2}{t^2}$, $C = -\dfrac{t_1t_2}{t^2}$,

(iii) $A = \dfrac{t_2^{*2}}{t^2}$, $B_1 = B_2 = -\dfrac{t_1t_2^*}{t^2}$, $C = \dfrac{t_1^2}{t^2}$.

The case (i) indeed gives the scattering state whose incident state is a two-electron plane wave on the lead 1. Similarly, the

case (iii) gives the scattering state whose incident state is a two-electron plane wave on the lead 2. The case (ii) corresponds to the scattering state whose incident state is a plane wave with the wave number k_1 on the lead 1 and a plane wave with k_2 on the lead 2. These coefficients are systematically extracted from the relations,

$$\begin{aligned}
c_1^\dagger(x_1)c_1^\dagger(x_2) &= \frac{t_1^{*2}}{t^2}c_e^\dagger(x_1)c_e^\dagger(x_2) + \frac{t_1^* t_2}{t^2}c_e^\dagger(x_1)c_o^\dagger(x_2) \\
&\quad - \frac{t_1^* t_2}{t^2}c_e^\dagger(x_2)c_o^\dagger(x_1) + \frac{t_2^2}{t^2}c_o^\dagger(x_1)c_o^\dagger(x_2), \\
c_1^\dagger(x_1)c_2^\dagger(x_2) &= \frac{t_1^* t_2^*}{t^2}c_e^\dagger(x_1)c_e^\dagger(x_2) - \frac{|t_1|^2}{t^2}c_e^\dagger(x_1)c_o^\dagger(x_2) \\
&\quad - \frac{|t_2|^2}{t^2}c_e^\dagger(x_2)c_o^\dagger(x_1) - \frac{t_1 t_2}{t^2}c_o^\dagger(x_1)c_o^\dagger(x_2), \qquad (34) \\
c_2^\dagger(x_1)c_2^\dagger(x_2) &= \frac{t_2^{*2}}{t^2}c_e^\dagger(x_1)c_e^\dagger(x_2) - \frac{t_1 t_2^*}{t^2}c_e^\dagger(x_1)c_o^\dagger(x_2) \\
&\quad + \frac{t_1 t_2^*}{t^2}c_e^\dagger(x_2)c_o^\dagger(x_1) + \frac{t_1^2}{t^2}c_o^\dagger(x_1)c_o^\dagger(x_2),
\end{aligned}$$

as is similar to the one-electron cases.

Let us present the explicit form of the two-electron scattering state (33) in the case (i). By introducing the eigenfunctions,

$$\begin{aligned}
2F^{(0)}(x_1,x_2) &= \langle 0|c_1(x_2)c_1(x_1)|k_1,k_2\rangle, \\
F^{(1)}(x_1;x_2) &= \langle 0|c_2(x_2)c_1(x_1)|k_1,k_2\rangle, \qquad (35) \\
2F^{(2)}(x_1,x_2) &= \langle 0|c_2(x_2)c_2(x_1)|k_1,k_2\rangle,
\end{aligned}$$

we have

$$\begin{aligned}
2F^{(0)}(x_1,x_2) &= \sum_Q \mathrm{sgn}(Q)\Big[g_{k_1}^{(+)}(x_{Q_1})g_{k_2}^{(+)}(x_{Q_2}) \\
&\quad + u\frac{|t_1|^2}{t^2}e^{\mathrm{i}Ex_{Q_2}}Z_{k_1k_2}(x_{Q_1Q_2})\theta(x_{Q_1})\Big], \\
F^{(1)}(x_1;x_2) &= \sum_P \mathrm{sgn}(P)g_{k_{P_1}}^{(+)}(x_1)g_{k_{P_2}}^{(-)}(x_2) \qquad (36) \\
&\quad - u\frac{t_1^* t_2}{t^2}e^{\mathrm{i}Ex_1}Z_{k_1k_2}(x_{21})\theta(x_2), \\
2F^{(2)}(x_1,x_2) &= \sum_Q \mathrm{sgn}(Q)g_{k_1}^{(-)}(x_{Q_1})g_{k_2}^{(-)}(x_{Q_2}),
\end{aligned}$$

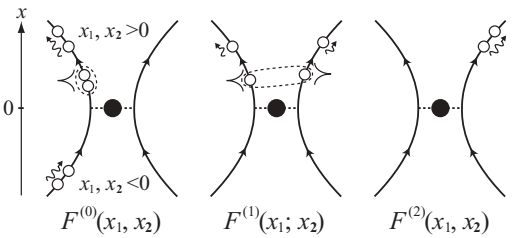

FIG. 2. A two-electron scattering eigenstate whose incident state is a two-electron plane-wave state in the left lead.

where $P=(P_1,P_2)$ and $Q=(Q_1,Q_2)$ are permutations of (1,2) and $g_k^{(\pm)}(x)$ are eigenfunctions of the one-electron scattering states introduced in Eq. (25). We find that $F^{(1)}(x_1;x_2)=F^{(2)}(x_1,x_2)=0$ for $x_1,x_2<0$. It is interesting that the two-body bound states appear only in the first two eigenfunctions $F^{(0)}(x_1,x_2)$ and $F^{(1)}(x_1;x_2)$ (see Fig. 2).

In what follows, we denote the scattering states in the three cases (i), (ii), and (iii) by $|k_1,k_2\rangle_{11}$, $|k_1,k_2\rangle_{12}$, and $|k_1,k_2\rangle_{22}$, respectively. In the limit $L\to\infty$, they satisfy the orthonormal relations,

$$\begin{aligned}
&{}_{\ell\ell}\langle k_1,k_2|l_1,l_2\rangle_{\ell\ell} \\
&\quad = \delta(k_1-l_1)\delta(k_2-l_2) - \delta(k_1-l_2)\delta(k_2-l_1), \qquad (37) \\
&{}_{12}\langle k_1,k_2|l_1,l_2\rangle_{12} = \delta(k_1-l_1)\delta(k_2-l_2),
\end{aligned}$$

which are readily derived from the relations (32).

D. *N*-electron scattering eigenstates

In principle, we can construct N-electron scattering eigenstates for arbitrary N in the same way as the two-electron cases. We present the three-electron eigenfunctions in Appendix C, in which "three-body bound states" appear.[31] Here we present N-electron scattering eigenstates in the first order in the interaction parameter U, which include only the two-body bound states. We remark that, in the case $N_o=0$, we have obtained the N-electron scattering eigenstates for all orders in U.

First, we present the N-electron eigenstates $|N,n;k\rangle$ whose incident state is a free-fermionic plane wave characterized by the wave numbers $k=(k_1,\ldots,k_{\bar n},k_{\bar n+1},\ldots,k_N)\in\mathbb{R}^N$ in the sector with $(N_e,N_o)=(\bar n,n)=(N-n,n)$. The Schrödinger equations in the sector are given as follows:

$$\begin{aligned}
&\left(\sum_{j=1}^{\bar n}\frac{1}{\mathrm{i}}\partial_{x_j}+\sum_{j=1}^{n}\frac{1}{\mathrm{i}}\partial_{y_j}-E\right)g^{(n)}(x;y)+\frac{t}{\bar n}\sum_{j=1}^{\bar n}(-)^{\bar n-j}\delta(x_j)e^{(n)}(\ldots,x_{j-1},x_{j+1},\ldots;y)=0, \quad (0\leqslant n\leqslant N-1), \\
&\left(\sum_{j=1}^{N}\frac{1}{\mathrm{i}}\partial_{y_j}-E\right)g^{(N)}(;y)=0, \qquad (38) \\
&\left[\sum_{j=1}^{\bar n-1}\left(\frac{1}{\mathrm{i}}\partial_{x_j}+U\delta(x_j)\right)+\sum_{j=1}^{n}\left(\frac{1}{\mathrm{i}}\partial_{y_j}+U\delta(y_j)\right)+\epsilon_d-E\right]e^{(n)}(x;y)+t\bar n g^{(n)}(x_1,\ldots,x_{\bar n-1},0;y)=0.
\end{aligned}$$

We denote the eigenfunctions with the energy eigenvalue $E=\sum_{i=1}^N k_i$ by $g_k^{(n)}(x;y)$ and $e_k^{(n)}(x;y)$. We expand them with respect to U as

$$\begin{aligned}
g_k^{(n)}(x;y) &= g_{k,0}^{(n)}(x;y)+Ug_{k,1}^{(n)}(x;y)+O(U^2), \\
e_k^{(n)}(x;y) &= e_{k,0}^{(n)}(x;y)+Ue_{k,1}^{(n)}(x;y)+O(U^2).
\end{aligned} \qquad (39)$$

Each function is given by

$$\begin{aligned}
\bar{n}!n!g^{(n)}_{k,0}(x;y) &= \sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n})}_{k_P}(x_Q)\varphi^{(n)}_{k'_{P'}}(y_{Q'}),\\
\bar{n}!n!g^{(n)}_{k,1}(x;y) &= \frac{1}{2}\sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-2)}_{k_P}(x_Q)\varphi^{(n)}_{k'_{P'}}(y_{Q'})\hat{Z}_{k_{P_{\bar{n}-1}},k_{P_{\bar{n}}}}\left(x_{Q_{\bar{n}-1}},x_{Q_{\bar{n}}}\right)\\
&\quad - \sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-1)}_{k_P}(x_Q)\varphi^{(n-1)}_{k'_{P'}}(y_{Q'})\hat{X}_{k_{P_{\bar{n}}},k'_{P'_n}}\left(x_{Q_{\bar{n}}},y_{Q'_n}\right),
\end{aligned} \tag{40}$$

where $P,Q\in\mathfrak{S}_{\bar{n}}$ and $P',Q'\in\mathfrak{S}_n$ and

$$\begin{aligned}
(\bar{n}-1)!n!e^{(n)}_{k,0}(x;y) &= \sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-1)}_{k_P}(x_Q)\,e_{k_{P_{\bar{n}}}}\varphi^{(n)}_{k'_{P'}}(y_{Q'}),\\
(\bar{n}-1)!n!e^{(n)}_{k,1}(x;y) &= \frac{1}{2}\sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-3)}_{k_P}(x_Q)\,e_{k_{P_{\bar{n}-2}}}\varphi^{(n)}_{k'_{P'}}(y_{Q'})\hat{Z}_{k_{P_{\bar{n}-1}},k_{P_{\bar{n}}}}\left(x_{Q_{\bar{n}-2}},x_{Q_{\bar{n}-1}}\right)\\
&\quad + \frac{1}{2\mathrm{i}t}\sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-2)}_{k_P}(x_Q)\,\varphi^{(n)}_{k'_{P'}}(y_{Q'})\hat{Z}_{k_{P_{\bar{n}-1}},k_{P_{\bar{n}}}}\left(0,x_{Q_{\bar{n}-1}}\right)\\
&\quad + \sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-2)}_{k_P}(x_Q)\,\varphi^{(n-1)}_{k'_{P'}}(y_{Q'})e_{k_{P_{\bar{n}-1}}}\hat{X}_{k_{P_{\bar{n}}},k'_{P'_n}}\left(x_{Q_{\bar{n}-1}},y_{Q'_n}\right)\\
&\quad + \frac{1}{\mathrm{i}t}\sum_{P,P',Q,Q'} \mathrm{sgn}(PP'QQ')\psi^{(\bar{n}-1)}_{k_P}(x_Q)\,\varphi^{(n-1)}_{k'_{P'}}(y_{Q'})\hat{X}_{k_{P_{\bar{n}}},k'_{P'_n}}\left(0,y_{Q'_n}\right),
\end{aligned} \tag{41}$$

where $P\in\mathfrak{S}_{\bar{n}}$, $Q\in\mathfrak{S}_{\bar{n}-1}$, and $P',Q'\in\mathfrak{S}_n$. Here we have expressed the nth order symmetric group acting on $(1,2,\ldots,n)$ by $\mathfrak{S}_n$. (The symmetric group is explained in Appendix A.) We have also used

$$\begin{aligned}
\psi^{(n)}_{k_P}(x_Q) &= g_{k_{P_1}}\left(x_{Q_1}\right)\cdots g_{k_{P_n}}\left(x_{Q_n}\right)\theta\left(x_{Q_n},\ldots,x_{Q_1}\right),\\
\varphi^{(n)}_{k_P}(x_Q) &= h_{k_{P_1}}\left(x_{Q_1}\right)\cdots h_{k_{P_n}}\left(x_{Q_n}\right)\theta\left(x_{Q_n},\ldots,x_{Q_1}\right),\\
\hat{Z}_{k_1k_2}(x_1,x_2) &= e^{\mathrm{i}(k_1+k_2)x_2}Z_{k_1k_2}(x_{12})\theta(x_1),\\
\hat{X}_{k_1k_2}(x_1,x_2) &= e^{\mathrm{i}(k_1+k_2)x_2}X_{k_1}(x_{12})\theta(x_1),\\
\theta(x_n,\ldots,x_2,x_1) &= \theta(x_{n,n-1})\cdots\theta(x_{32})\theta(x_{21}),
\end{aligned}$$

with $(k'_1,\ldots,k'_n)=(k_{\bar{n}+1},\ldots,k_N)$. Note that the terms $\hat{Z}_{k_1k_2}(x_1,x_2)$ and $\hat{X}_{k_1k_2}(x_1,x_2)$ in the functions $g^{(n)}_{k,1}(x;y)$ and $e^{(n)}_{k,1}(x;y)$ give two-body bound states. We have proved this by inserting the eigenfunctions into the Schrödinger equations (38). These eigenfunctions in the first order in U correspond to the Born approximation for the Lippmann-Schwinger equation (45) discussed in the next section. For the N-electron eigenstates in the first order in U, we have confirmed the orthonormal relation,

$$\begin{aligned}
\langle N,n;k|N,m,l\rangle &= \delta_{nm}\sum_{\substack{P,R\\P',R'}}\mathrm{sgn}(PP'RR')\\
&\quad\times\prod_{i=1}^{\bar{n}}\delta\left(k_{P_i}-l_{R_i}\right)\prod_{j=1}^{n}\delta\left(k'_{P'_j}-l'_{R'_j}\right),
\end{aligned} \tag{42}$$

in the limit $L\to\infty$ within the first order in U.

Second, by taking a linear combination of the N-electron eigenstates $|N,n;k\rangle$ with the same energy $E=\sum_{i=1}^{N}k_i$, we construct N-electron scattering eigenstates in the framework of the lead 1 and 2,

$$|k\rangle = \sum_{n=0}^{N}\sum_{R}A^{(n)}_R|N,n;k_R\rangle, \tag{43}$$

where R is an element of the subset $\mathfrak{S}_N/(\mathfrak{S}_{\bar{n}}\times\mathfrak{S}_n)$. (The definition of $\mathfrak{S}_N/(\mathfrak{S}_{\bar{n}}\times\mathfrak{S}_n)$ is explained in Appendix A.) For the scattering state characterized by the N_1-electron plane wave with the wave numbers $k_1,\ldots,k_{N_1}$ emitted from the lead 1 and the N_2-electron plane wave with wave numbers $k_{N_1+1},\ldots,k_N$ emitted from the lead 2, we take the coefficients $\{A^{(n)}_R\}$ as

$$\begin{aligned}
A^{(n)}_R &= \left(\frac{t_1^*}{t}\right)^{\sharp\{R_i|R_i\leqslant N_1,\,i\leqslant\bar{n}\}}\left(\frac{t_2}{t}\right)^{\sharp\{R_i|R_i\leqslant N_1,\,\bar{n}<i\}}\\
&\quad\times\left(\frac{t_2^*}{t}\right)^{\sharp\{R_i|N_1<R_i,\,i\leqslant\bar{n}\}}\left(-\frac{t_1}{t}\right)^{\sharp\{R_i|N_1<R_i,\,\bar{n}<i\}},
\end{aligned} \tag{44}$$

where $\sharp S$ means the number of elements of the set S. The coefficients $\{A^{(n)}_P\}$ are determined by the following relation:

$$\begin{aligned}
&c_1^\dagger(x_1)\cdots c_1^\dagger(x_{N-s})c_2^\dagger(x_{N-s+1})\cdots c_2^\dagger(x_N)\\
&\quad=\sum_{n=0}^{N}\sum_{P\in\mathfrak{S}_N}\mathrm{sgn}(P)A^{(n)}_P c_e^\dagger\left(x_{P_1}\right)\cdots c_e^\dagger\left(x_{P_{\bar{n}}}\right)\\
&\qquad\times c_o^\dagger\left(x_{P_{\bar{n}+1}}\right)\cdots c_o^\dagger\left(x_{P_N}\right),
\end{aligned}$$

which is the same as the two-electron cases.

IV. LIPPMANN-SCHWINGER EQUATION

The Lippmann-Schwinger equation helps us to understand the two-body bound state that appears in the two-electron

scattering eigenstates. The equation also gives an alternative derivation of the two-electron scattering eigenstates.[36]

Let us review the Lippmann-Schwinger equation briefly. We decompose a Hamiltonian as $H = H_0 + H_1$. The parts H_0 and H_1 are referred to as nonperturbative and perturbative parts, respectively, although we will not develop the perturbation theory below. An N-electron eigenstate $|E\rangle$ of the total Hamiltonian H with an energy eigenvalue E is given by the solution of the (time-independent) Lippmann-Schwinger equation,

$$|E\rangle = |E\rangle_0 + \frac{1}{E - H_0 + \mathrm{i}0} H_1 |E\rangle. \tag{45}$$

Here, $|E\rangle_0$ is an eigenstate of the nonperturbative part H_0 with the same energy eigenvalue E as that of $|E\rangle$ and the resolvent operator $1/(E - H_0 + \mathrm{i}0)$ is defined in the N-electron sector. Equation (45) is equivalent to the Schrödinger equation, containing all orders of H_1.

We consider the even part (3b) of the IRLM Hamiltonian (1) for simplicity. We show that, when we take the decomposition,

$$\begin{aligned} H_0 &= \int dx\, c_e^\dagger(x) \frac{1}{\mathrm{i}} \frac{d}{dx} c_e(x) + t[c_e^\dagger(0) d + d^\dagger c_e(0)] + \epsilon_d d^\dagger d, \\ H_1 &= U c_e^\dagger(0) c_e(0) d^\dagger d, \end{aligned} \tag{46}$$

the two-electron eigenstate $|2,0;k_1,k_2\rangle$ that is given by Eq. (31a) with Eq. (29) is the solution $|E\rangle$ of the Lippmann-Schwinger equation (45) whose nonperturbative eigenstate $|E\rangle_0$ is a two-electron plane-wave state.

Generally, in interacting cases, it is hard to obtain the exact solution of the Lippmann-Schwinger equation for a given nonperturbative eigenstate. However, in the two-electron cases for the Hamiltonian (46), the equations (45) are easily solved in terms of the two-electron propagators.[37] In the two-electron sector, the interaction part H_1 is expressed as

$$H_1 = U |0,d;\rangle\langle 0,d;|. \tag{47}$$

Recall that $|x,d;\rangle = c_e^\dagger(x) d^\dagger |0\rangle$, which is defined in Eq. (6). Then the Lippmann-Schwinger equation is simplified as

$$|E\rangle = |E\rangle_0 + U \frac{1}{E - H_0 + \mathrm{i}0} |0,d;\rangle\langle 0,d;|E\rangle. \tag{48}$$

By applying $\langle x_1,x_2;| = \langle 0|c_e(x_2)c_e(x_1)$ and $\langle x,d;| = \langle 0|d c_e(x)$ from the left-hand side of Eq. (48), we have a set of equations,

$$\begin{aligned} 2g(x_1,x_2) &= 2g_0(x_1,x_2) + U G_0(x_1,x_2;0,d) e(0), \\ e(x) &= e_0(x) + U G_0(x,d;0,d) e(0). \end{aligned} \tag{49}$$

Here $G_0(x_1,x_2;x',d)$ and $G_0(x,d;x',d)$ are two-electron propagators (or retarded Green's functions) of the nonperturbative Hamiltonian H_0 defined by

$$\begin{aligned} G_0(x_1,x_2;x',d) &= \langle x_1,x_2;| \frac{1}{E - H_0 + \mathrm{i}0} |x',d;\rangle, \\ G_0(x,d;x',d) &= \langle x,d;| \frac{1}{E - H_0 + \mathrm{i}0} |x',d;\rangle. \end{aligned} \tag{50}$$

We have also denoted the nonperturbative eigenfunctions by $2g_0(x_1,x_2) = \langle x_1,x_2;|E\rangle_0$ and $e_0(x) = \langle x,d;|E\rangle_0$ and the eigenfunctions of the total Hamiltonian by $2g(x_1,x_2) = \langle x_1,x_2;|E\rangle$ and $e(x) = \langle x,d;|E\rangle$. The propagators appearing in Eq. (49) are explicitly given by

$$\begin{aligned} G_0(x_1,x_2;0,d) &= t\big[\theta(x_{21})\theta(x_1) e^{\mathrm{i}Ex_2 + \mathrm{i}(\epsilon_d - \mathrm{i}\frac{t^2}{2})x_{12}} \\ &\quad - \theta(x_{12})\theta(x_2) e^{\mathrm{i}Ex_1 + \mathrm{i}(\epsilon_d - \mathrm{i}\frac{t^2}{2})x_{21}}\big], \\ G_0(x,d;0,d) &= -\mathrm{i}\theta(x) e^{\mathrm{i}(E - \epsilon_d + \mathrm{i}\frac{t^2}{2})x}, \end{aligned} \tag{51}$$

through, for example, the Dyson equation. Notice here that the propagators have the term $e^{-\frac{t^2}{2}|x|}$. By setting $x = 0$ in the second equation in Eq. (49), we have

$$e(0) = \frac{e_0(0)}{1 - U G_0(0,d;0,d)}, \tag{52}$$

which leads to the solution of Eq. (49) as

$$\begin{aligned} 2g(x_1,x_2) &= 2g_0(x_1,x_2) + \frac{U e_0(0)}{1 - U G_0(0,d;0,d)} G_0(x_1,x_2;0,d), \\ e(x) &= e_0(x) + \frac{U e_0(0)}{1 - U G_0(0,d;0,d)} G_0(x,d;0,d), \end{aligned} \tag{53}$$

for given nonperturbative eigenfunctions $g_0(x_1,x_2)$ and $e_0(x)$.

We find that the forms of the solutions in Eq. (53) fit the first two eigenfunctions in Eq. (29). It is clear from $G_0(x_1,x_2;0,d) = 0$ for $x_1,x_2 < 0$ and $G_0(x,d;0,d) = 0$ for $x < 0$ that the nonperturbative eigenstate corresponds to the incident state that we have adopted in solving the Schrödinger equations (26). If we take the two-electron plane-wave state as the nonperturbative eigenstate $|E\rangle_0$, the solution (53) reproduces the two eigenfunctions $g^{(0)}_{k_1k_2}(x_1,x_2)$ and $e^{(0)}_{k_1k_2}(x)$ in Eq. (29). Thus the second terms of $g^{(0)}_{k_1k_2}(x_1,x_2)$ and $e^{(0)}_{k_1k_2}(x)$ are written in terms of the two-electron propagators. This implies that the decaying property of the two-body bound state is originated from that of the two-electron propagators. In a similar way, we can obtain the eigenstate $|2,1;k_1,k_2\rangle$ as the solution of the Lippmann-Schwinger equation.

It should be noted that it is possible to distinguish our scattering states from the Bethe eigenstates[28,32] in terms of nonperturbative eigenstates of the Lippmann-Schwinger equation. Indeed the Bethe eigenstates are the solutions of the Lippmann-Schwinger equation with a different nonperturbative eigenstate that depends on U even in the region $x_1,x_2 < 0$.

V. NONLINEAR CURRENT-VOLTAGE CHARACTERISTICS

A. N-electron current

We are now in a position to calculate the quantum-mechanical expectation value of the current operator I, Eq. (12), for the scattering states. We refer to the expectation value for the N-electron scattering eigenstate as the N-electron current.

First, we consider the two-electron cases. By using Eq. (13), we find that, for the two-electron eigenstate (33) with general coefficients A, B_1, B_2, and C, the two-electron current is

expressed by the overlap integral,

$$\langle k_1,k_2|I|k_1,k_2\rangle = -\frac{2}{t}\mathrm{Im}\left(t_1t_2\left\{\int dx\left(B_1^* g_{k_1k_2}^{(1)*}(x;0) - B_2^* g_{k_2k_1}^{(1)*}(x;0)\right)Ae_{k_1k_2}^{(0)}(x) - 2\int dx\, C^* g_{k_1k_2}^{(2)*}(x,0)\left[B_1 e_{k_1k_2}^{(1)}(x) - B_2 e_{k_2k_1}^{(1)}(x)\right]\right\}\right).$$

Inserting the eigenfunctions in Eq. (29), we have

$$\begin{aligned}\langle k_1,k_2|I|k_1,k_2\rangle = &-\frac{2}{\sqrt{2\pi}t}\mathrm{Im}\big(t_1t_2\{\big[\delta(0)-\delta(k_1-k_2)\big]\\ &\times\left[(B_2^*A+C^*B_1)e_{k_1}+(B_1^*A+C^*B_2)e_{k_2}\right]\\ &-\frac{u}{t^2}(k_1-k_2)\big[\big(B_2^*Ae_{k_2}^*+C^*B_1e_{k_2}\big)e_{k_1}^2\\ &-\big(B_1^*Ae_{k_1}^*+C^*B_2e_{k_1}\big)e_{k_2}^2\big]\}\big).\end{aligned} \tag{54}$$

Here the factor $\delta(0)$ has come from the integral $\int dx|e^{i0x}|^2/(2\pi)$. In what follows, we assume $k_1<k_2$, which gives $\delta(k_1-k_2)=0$. By specifying the coefficients A, B_1, B_2, and C as is described in Sec. III C, we obtain the two-electron current for the two-electron scattering states.

The results are summarized as follows: for the scattering state $|k_1,k_2\rangle_{\ell\ell'}$, $[(\ell,\ell')=(1,1),(1,2),(2,2)]$ whose incident state is a two-electron plane-wave state of an electron with the wave number k_1 in the lead ℓ and an electron with k_2 in the lead ℓ', the numerator of the two-electron current is

$$\langle k_1,k_2|I|k_1,k_2\rangle_{\ell\ell'} = \delta(0)[I_\ell(k_1)+I_{\ell'}(k_2)]+I'_{\ell\ell'}(k_1,k_2), \tag{55}$$

where

$$\begin{aligned}&I_1(k) = -I_2(k) = \lambda(k;t_1,t_2),\\ &I'_{11}(k_1,k_2) = \xi(k_1,k_2;t_1,t_2)+\xi(k_2,k_1;t_1,t_2),\\ &I'_{12}(k_1,k_2) = -\xi(k_1,k_2;t_1,t_2)+\xi(k_2,k_1;t_2,t_1),\\ &I'_{22}(k_1,k_2) = -\xi(k_1,k_2;t_2,t_1)-\xi(k_2,k_1;t_2,t_1),\end{aligned} \tag{56}$$

with

$$\begin{aligned}&\lambda(k;t_1,t_2) = -\frac{2|t_1t_2|^2}{\sqrt{2\pi}t^3}\mathrm{Im}(e_k),\\ &\xi(k,h;t_1,t_2) = \frac{|t_1t_2|^2}{\pi t^6}\mathrm{Im}\left[u\left(|t_1|^2\frac{e_k^*}{e_k}+|t_2|^2\right)(e_k-e_h)e_h\right].\end{aligned} \tag{57}$$

The factor $\delta(0)$ in Eq. (55) should be replaced with the system length L divided by 2π. Then the first term in Eq. (55) is of order L and is the two-electron current in the noninteracting cases, which is just the sum of the one-electron currents. The second term containing $I'_{\ell\ell'}(k_1,k_2)$ is considered as a contribution of the two-body bound state. Recall that the square norm of the two-electron scattering state is normalized on δ functions as $\langle k_1,k_2|k_1,k_2\rangle_{\ell\ell'}=\delta(0)^2=(L/(2\pi))^2$. Then the normalized two-electron current is given by

$$\frac{\langle k_1,k_2|I|k_1,k_2\rangle_{\ell\ell'}}{\langle k_1,k_2|k_1,k_2\rangle_{\ell\ell'}} = \frac{2\pi}{L}[I_\ell(k_1)+I_{\ell'}(k_2)]+\frac{4\pi^2}{L^2}I'_{\ell\ell'}(k_1,k_2). \tag{58}$$

Thus the effect of the interaction appears in the terms of order L^{-2} of the two-electron current and, in general, in the higher-order terms in L^{-1} of the N-electron current. In the Letter,[31] we speculated the form of the N-electron current and investigated the contribution of such higher-order terms in the limit $N\to\infty$.

Next, we turn to the N-electron current for general N. For simplicity, we consider the symmetric case $t_1=t_2=t/\sqrt{2}(\in\mathbb{R})$. We express an N-electron scattering eigenstate whose incident state consists of an N_1-electron plane wave with wave numbers $\{k_i\}$ on the lead 1 and an N_2-electron plane wave with $\{k'_i\}$ on the lead 2 by $|k\rangle=|k_1,k_2,\ldots,k_{N_1},k'_1,k'_2,\ldots,k'_{N_2}\rangle$. We assume $k_1<k_2<\cdots<k_{N_1}$ and $k'_1<k'_2<\cdots<k'_{N_2}$. The N-electron current is given as a function in the wave numbers $\{k_i\}$ and $\{k'_i\}$. From the result in the case $N=2$, we find that the overlap integral of the term e^{ik_1x} in the wave function $g_{k_1k_2}^{(1)}(x;0)$ and the term $e^{ik_1x}e_{k_2}$ in the wave function $e_{k_1k_2}^{(0)}(x)$ contributes a one-variable function in k_2 with the factor $2\pi/L$ to the two-electron current. On the other hand, the overlap integral of the term e^{ik_1x} in the wave function $g_{k_1k_2}^{(1)}(x;0)$ and the term $Z_{k_1k_2}(-x)e^{iEx}$ in the wave function $e_{k_1k_2}^{(0)}(x)$ contributes a two-variable function in k_1 and k_2 with the factor $4\pi^2/L^2$. Then we expect that the N-electron current is expanded as follows:

$$\begin{aligned}\frac{\langle k|I|k\rangle}{\langle k|k\rangle} = &\frac{2\pi}{L}\left(\sum_{i=1}^{N_1}I_1^{(1)}(k_i)+\sum_{i=1}^{N_2}I_2^{(1)}(k'_i)\right)\\ &+\frac{4\pi^2}{L^2}\left(\sum_{i\neq j}^{N_1}I_{11}^{(2)}(k_i,k_j)+\sum_{i=1}^{N_1}\sum_{j=1}^{N_2}I_{12}^{(2)}(k_i,k'_j)\right.\\ &\left.+\sum_{i\neq j}^{N_2}I_{22}^{(2)}(k'_i,k'_j)\right)+\frac{8\pi^3}{L^3}\left(\sum_{i\neq j\neq l(\neq i)}^{N_1}I_{111}^{(3)}(k_i,k_j,k_l)\right.\\ &+\sum_{i\neq j}^{N_1}\sum_{l=1}^{N_2}I_{112}^{(3)}(k_i,k_j,k'_l)+\sum_{i=1}^{N_1}\sum_{j\neq l}^{N_2}I_{122}^{(3)}(k_i,k'_j,k'_l)\\ &\left.+\sum_{i\neq j\neq l(\neq i)}^{N_2}I_{222}^{(3)}(k'_i,k'_j,k'_l)\right)+O(L^{-4}).\end{aligned} \tag{59}$$

The terms of order L^{-1} correspond to the electric current of free N electrons and the function $I_\ell^{(1)}(k)$ is determined by the one-electron scattering eigenstates. We have confirmed the form (59) in the first order in the interaction parameter U, which is presented in Appendix D. Each function in Eq. (59) is given by

$$\begin{aligned}&I_1^{(1)}(k) = -I_2^{(1)}(k) = \lambda(k),\\ &I_{11}^{(2)}(k,h) = -I_{22}^{(2)}(k,h) = \xi(k,h),\\ &I_{12}^{(2)}(k,h) = -\xi(k,h)+\xi(h,k),\end{aligned}$$

$$
\begin{aligned}
I^{(3)}_{111}(k,h,l) &= -I^{(3)}_{222}(k,h,l) = \eta(k,h,l) + \zeta(k,h,l),\\
I^{(3)}_{112}(k,h,l) &= -\eta(k,h,l) + \eta(h,l,k) + \eta(l,k,h)\\
&\quad -\zeta(k,h,l) - \zeta(h,l,k) - \zeta(l,k,h),\\
I^{(3)}_{122}(k,h,l) &= -\eta(k,h,l) - \eta(l,k,h) + \eta(h,l,k)\\
&\quad +\zeta(k,h,l) + \zeta(l,k,h) + \zeta(h,l,k),
\end{aligned}
\tag{60}
$$

with

$$
\begin{aligned}
\lambda(k) &= -\frac{t}{2\sqrt{2\pi}}\mathrm{Im}(e_k),\\
\xi(k,h) &= \frac{U}{8\pi}\mathrm{Im}\left[\left(\frac{e_k^*}{e_k}+1\right)(e_k-e_h)e_h\right] + O(U^2),\\
\eta(k,h,l) &= \frac{U}{8\pi}\mathrm{Re}\left[e_k - \left(\frac{e_k^*}{e_k}+1\right)e_h\right]e_h^* e_h \mathrm{Im}(e_l) + O(U^2),\\
\zeta(k,h,l) &= \frac{U}{8\pi}\mathrm{Re}(e_k)e_h^* e_h \mathrm{Im}(e_l) + O(U^2).
\end{aligned}
\tag{61}
$$

We remark that, in the calculation of the N-electron current in the first order in U, it is enough to take account of the terms of order lower than L^{-3}; the term in the first order in U does not appear in the terms of order higher than L^{-4}. We also find in the first order in U that the function $I^{(2)}_{\ell\ell'}(k,k')$ is the same as that appearing in the two-electron current in Eq. (55).

B. Average electric current

We have calculated the quantum-mechanical expectation value of the current. Extending Landauer's idea,[7–11] we now calculate the statistical-mechanical average of electric current for the system under a finite bias voltage. We call it the average electric current. Similarly to the previous subsection, we restrict our discussion to the symmetric case $t_1 = t_2 = t/\sqrt{2}$.

To employ the formula for the open quantum dot sandwiched between two electron reservoirs, we assume that

(i) each electron reservoir is always in the equilibrium state characterized by the zero-temperature Fermi distribution with a fixed chemical potential;

(ii) electrons are completely equilibrated in each electron reservoir before being emitted to the quantum dot.

The infinite leads of the IRLM play a role of electron reservoirs satisfying the assumption (ii) since the electrons scattered to the leads never come back to the quantum dot. Following the assumption (i), we assume that the wave numbers $\{k_i\}$ and $\{k'_i\}$ of the incident plane wave of the N-electron scattering state $|k\rangle$ follow the Fermi distribution functions $f_1(k) = \theta(\mu_1 - k)$ and $f_2(k') = \theta(\mu_2 - k')$ in the limit $N, L \to \infty$, respectively. We refer to the limit as the "reservoir limit." The bias voltage is given by the difference $V = \mu_1 - \mu_2$. In the reservoir limit of the N-electron current (59), the sum $\sum_{i=1}^{N_\ell}$ divided by $L/2\pi$ is replaced by the integral on k with the Fermi distribution function $f_\ell(k)$:

$$
\frac{2\pi}{L}\sum_{i=1}^{N_\ell} I(k_i) \to \int_{-\Lambda}^{\Lambda} dk\, f_\ell(k) I(k) = \int_{-\Lambda}^{\mu_\ell} dk\, I(k), \tag{62}
$$

where we have introduced the energy cutoff $\pm\Lambda$.

The resulting formula is regarded as an extension of the Landauer formula. In the original Landauer formula for noninteracting cases, the electrons are in a plane-wave state before and after they are scattered at the quantum dot. In the interacting case, as we have found in Secs. III B and III D, even when the incident wave is a plane wave, the reflected and transmitted waves of the scattering state $|k\rangle$ include the effect of the interaction in the form of many-body bound states. In the assumption (ii), we have assumed that the many-body bound states thus generated are absorbed into the electron reservoirs and equilibrated to a free-electronic state before emitted to the quantum dot again. We do not care about the mechanism of the equilibration in each electron reservoir in a way similar to the original Landauer formula.

The average electric current that is obtained in the reservoir limit of the N-electron current (59) is simplified as

$$
\begin{aligned}
\langle I\rangle &= \int_{\mu_2}^{\mu_1} dk\, \lambda(k) + \left(\int_{-\Lambda}^{\mu_1} dk + \int_{-\Lambda}^{\mu_2} dk\right)\int_{\mu_2}^{\mu_1} dh\, \xi(k,h)\\
&\quad + \left(\int_{-\Lambda}^{\mu_1} dk + \int_{-\Lambda}^{\mu_2} dk\right)\left(\int_{-\Lambda}^{\mu_1} dh + \int_{-\Lambda}^{\mu_2} dh\right)\\
&\quad \times \int_{\mu_2}^{\mu_1} dl\, \eta(k,h,l) + \int_{\mu_2}^{\mu_1} dk \int_{\mu_2}^{\mu_1} dh \int_{\mu_2}^{\mu_1} dl\, \zeta(k,h,l),
\end{aligned}
\tag{63}
$$

which we call an extension of the Landauer formula. Here the integrands $\lambda(k)$, $\xi(k,h)$, $\eta(k,h,l)$, and $\zeta(k,h,l)$ are given in Eq. (61). One finds that, at $U = 0$, the formula (63) is reduced to the original Landauer formula, which is just a single integral over the interval $[\mu_2, \mu_1]$. The integrals in the terms of order U can be explicitly done since each integrand $\xi(k,h)$, $\eta(k,h,l)$, and $\zeta(k,h,l)$ is written as a finite sum of the products of one-variable functions in k, h, or l. As a result, we obtain

$$
\begin{aligned}
\langle I\rangle &= J_0 + U\left(J_1^{(2)} + J_1^{(3)}\right) + O(U^2),\\
J_0 &= -\frac{\Gamma}{2\pi} j_-,\\
J_1^{(2)} &= \frac{\Gamma}{8\pi^2}\left[(j_- - j_{1,-})\log\left(\frac{\left(\epsilon_1^2+1\right)\left(\epsilon_2^2+1\right)}{\left(\epsilon_\Lambda^2+1\right)^2}\right)\right.\\
&\quad \left. - 2j_{2,-}(2\epsilon_\Lambda - \epsilon_1 - \epsilon_2 + j_+ - j_\Lambda)\right],\\
J_1^{(3)} &= \frac{\Gamma}{16\pi^3}\left[(j_{1,+} - j_{1,\Lambda})\log\left(\frac{\left(\epsilon_1^2+1\right)\left(\epsilon_2^2+1\right)}{\left(\epsilon_\Lambda^2+1\right)^2}\right)\right.\\
&\quad + 2(j_{2,+} - j_{2,\Lambda})(2\epsilon_\Lambda - \epsilon_1 - \epsilon_2 + j_+ - j_\Lambda)\\
&\quad \left. - \log\left(\frac{\epsilon_1^2+1}{\epsilon_2^2+1}\right) j_-\right] j_-,
\end{aligned}
\tag{64}
$$

where $\Gamma = t^2/2$ is the width of the energy level on the quantum dot in the noninteracting case and

$$
\begin{aligned}
&\epsilon_\ell = \frac{\epsilon_d - \mu_\ell}{\Gamma}, \quad (\ell = 1,2), \quad \epsilon_\Lambda = \frac{\epsilon_d + \Lambda}{\Gamma},\\
&j_\pm = \arctan(\epsilon_1) \pm \arctan(\epsilon_2), \quad j_\Lambda = 2\arctan(\epsilon_\Lambda),\\
&j_{s,\pm} = \frac{\epsilon_1^{2-s}}{\epsilon_1^2+1} \pm \frac{\epsilon_2^{2-s}}{\epsilon_2^2+1}, \quad j_{s,\Lambda} = \frac{2\epsilon_\Lambda^{2-s}}{\epsilon_\Lambda^2+1}, \quad (s = 1,2).
\end{aligned}
\tag{65}
$$

The current-voltage characteristics of the average electric current (64) is affected by the choice of the energy cutoff Λ. For example, if we take $\Lambda = V$, the current (in the first order

in U) increases with the increasing of U. We notice that the terms of order U include linear and logarithmic divergences in the limit $\Lambda \to \infty$. These divergences are taken care of in the next subsection.

Let us compare the result with those obtained by the Green's function approaches in the first order in U.[30,34] In Ref. 30, the IRLM with two leads of the same linearized dispersion relation is studied, where both linear and logarithmic divergences appear in the average electric current. In the paper, a universal electric current is obtained, which we will discuss in the next subsection. On the other hand, in Ref. 34, the IRLM with two leads of a nonlinear dispersion relation is studied in the Keldysh formalism, where only the logarithmic divergence appears in the average electric current. We find that, at $\epsilon_d = 0$, the analytic expression of the average electric current that is presented in Ref. 34 agrees with our average current $\langle I\rangle' = J_0 + U J_1^{(2)}$ that is obtained from the two-electron current.

C. Universal electric current

We now bring the average electric current to a universal form. In the calculation of the average electric current, we have introduced the energy cutoff $-\Lambda$ to avoid the divergences in the limit $\Lambda \to \infty$. However, the electric current should not depend on an *ad hoc* choice of the energy cutoff. We find that, through a renormalization-group technique, the average electric current $\langle I\rangle' = J_0 + U J_1^{(2)}$ obtained by the two-electron current is brought to a universal form which does not depend on the energy cutoff. We speculate that the contribution $J_1^{(3)}$ of the three-electron current can be neglected in the universal regime in the first order in U.

We first show that the linear and logarithmic divergences are absorbed into a redefinition of the parameters Γ and ϵ_d through a renormalization-group technique.[30,34,38] We introduce the parameter D through the relation $D^2 = (\epsilon_d + \Lambda)^2 + \Gamma^2$ and consider the case $\Lambda \gg \Gamma, |\epsilon_d|$. In the case, the average electric current obtained from the two-electron current reads

$$\langle I\rangle' = -\frac{\Gamma}{2\pi} j_- + \frac{U\Gamma}{8\pi^2}\left[(j_- - j_{1,-})\log\left(\frac{(\epsilon_1^2+1)(\epsilon_2^2+1)}{(D/\Gamma)^4}\right) - 2j_{2,-}\left(2\frac{D}{\Gamma} - \epsilon_1 - \epsilon_2 + j_+ - \pi\right)\right]. \tag{66}$$

This satisfies the Callan-Symanzik equation, which is one of the renormalization-group equations,[30,34,38]

$$\left(D\frac{\partial}{\partial D} + \beta_\Gamma \frac{\partial}{\partial \Gamma} + \beta_{\epsilon_d}\frac{\partial}{\partial \epsilon_d}\right)\langle I\rangle' = 0, \tag{67}$$

in the first order in U. Here the beta functions β_Γ and β_{ϵ_d} are given by

$$\beta_\Gamma = -\frac{U}{\pi}\Gamma + O(U^2), \quad \beta_{\epsilon_d} = -\frac{U}{\pi}D + O(U^2). \tag{68}$$

The general solution of the Callan-Symanzik equation (67) is given in terms of a two-variable function J as

$$\langle I\rangle' = J\left(\Gamma D^{\frac{U}{\pi}}, \epsilon_d + \frac{U}{\pi}D\right). \tag{69}$$

Following the standard theory of the renormalization group, we consider D, Γ, and ϵ_d as "bare" parameters. By introducing a "physical" cutoff μ and changing the parameters Γ and ϵ_d as functions in μ as

$$\Gamma(\mu) = T_{\rm K}^{1+\frac{U}{\pi}}\mu^{-\frac{U}{\pi}}, \quad \epsilon_d(\mu) = E_d - \frac{U}{\pi}\mu, \tag{70}$$

the average electric current $\langle I\rangle'$ depends only on $T_{\rm K}$ and E_d, that is, $\langle I\rangle' = J(T_{\rm K}^{1+\frac{U}{\pi}}, E_d)$. Here $T_{\rm K}$ and E_d are constants that are determined by the bare parameters as $\Gamma = T_{\rm K}^{1+\frac{U}{\pi}} D^{-\frac{U}{\pi}}$ and $\epsilon_d = E_d - \frac{U}{\pi}D$. Thus both the linear and logarithmic divergences are absorbed into the redefinition of the parameters. We remark that, in Refs. 30 and 34, the Callan-Symanzik equation that has only the term of β_Γ is introduced in order to remove the logarithmic divergence.

Next, we investigate the current-voltage characteristics of the universal electric current specifically in the case $\mu_1 = -\mu_2 = V/2$ where V is the bias voltage. We confirm in the first order in U that, through the replacement of the parameters,

$$D \to \mu, \quad \Gamma \to \Gamma(\mu), \quad \epsilon_d \to \epsilon_d(\mu), \tag{71}$$

the average electric current $\langle I\rangle'$ takes the following form:

$$\langle I\rangle' = T_{\rm K}\tilde{I}\left(\frac{V}{T_{\rm K}}, \frac{E_d}{T_{\rm K}}\right). \tag{72}$$

We call this the universal electric current. In the case $E_d = 0$, the universal electric current is explicitly given as follows:

$$\tilde{I} = \frac{1}{\pi}\arctan\left(\frac{\tilde{V}}{2}\right) - \frac{U}{2\pi^2}\left[\arctan\left(\frac{\tilde{V}}{2}\right) - \frac{\frac{\tilde{V}}{2}}{\frac{\tilde{V}^2}{4}+1}\right]\log\left(\frac{\tilde{V}^2}{4}+1\right), \tag{73}$$

where $\tilde{I} = \langle I\rangle'/T_{\rm K}$ and $\tilde{V} = V/T_{\rm K}$ are scaling variables. In Fig. 3, we plot the current-voltage characteristics of the universal electric current (73) with respect to the scaling variables $\tilde{I}$ and $\tilde{V}$. The electric current decreases as the strength of the interaction is increased and negative differential conductance appears for large U. These are consistent with the results obtained by other approaches.[30,35]

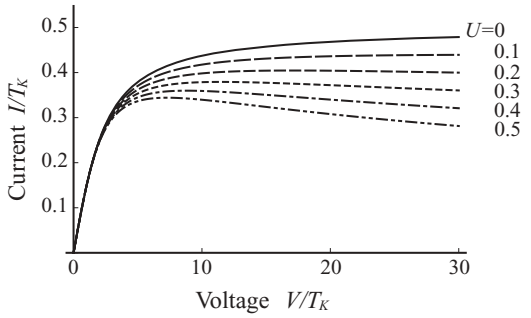

FIG. 3. $I-V$ characteristics for $E_d = 0$ and $U = 0$–0.5.

VI. SUMMARY AND CONCLUDING REMARKS

We have studied the nonlinear current-voltage characteristics in an open quantum dot system under a finite bias voltage. We have calculated the average electric current through an extension of the Landauer formula to the interacting case. In

the calculation, the exact solution of many-electron scattering eigenstate plays an essential role. The Coulomb interaction affects the average electric current through the many-body bound states appearing in the scattering eigenstates. In the interacting case, we have observed linear and logarithmic divergences in the average electric current in the limit of the large bandwidth. By dealing with the divergences with a renormalization-group technique, we have obtained the universal electric current.

The result in the first order in the interaction parameter U suggests that the equivalence between the extended use of the Landauer formula and the Keldysh formalism of the Green's function approach. In our approach with exact many-electron scattering eigenstates, the average electric current can be calculated in higher orders in U in principle. We believe that it enables us to study the electron transport in the strong coupling regime of the Coulomb interaction.

Our approach should be applicable to the case of finite temperatures by replacing the zero-temperature Fermi distribution functions with the finite-temperature one in Eq. (62). It is expected that the universal electric current at temperature T depends on the ratio $T/T_{\rm K}$ in addition to $V/T_{\rm K}$ and $E_d/T_{\rm K}$.[38] It is also interesting to investigate the IRLM system with asymmetric transfer integrals $t_1 \neq t_2$. In the case, the contribution of the higher-order terms in the interaction parameter U should play a significant role.

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APPENDIX A: SYMMETRIC GROUP

In the present Appendix, we briefly review the symmetric group. Let us denote the Nth order symmetric group acting on the set $\{1,2,\ldots,N\}$ by $\mathfrak{S}_N$. The element P of $\mathfrak{S}_N$ is called a permutation and is expressed as $P=(P_1,P_2,\ldots,P_N)$ when P maps i to P_i, $(1 \leqslant i \leqslant N)$. The transposition (i,j) is a special permutation that interchanges i and j. The symmetric group $\mathfrak{S}_N$ is generated by the adjacent transpositions $\{(i,i+1),1 \leqslant i \leqslant N-1\}$. Indeed, a permutation P is decomposed into a product of the transpositions as $P=(i_1,j_1)(i_2,j_2)\cdots(i_r,j_r)$. Although the decomposition is not unique, the parity of the number r of transpositions in all possible decompositions is unique. The sign function on $\mathfrak{S}_N$ is defined by $\mathrm{sgn}(P)=1$ if the number of transpositions in the decomposition of P is even and $\mathrm{sgn}(P)=-1$ otherwise.

We denote a subgroup of $\mathfrak{S}_N$ fixing the part $\{N-n+1,N-n+2,\ldots,N\}$ by $\mathfrak{S}_{N-n}$ since it is isomorphic to the $(N-n)$th order symmetric group. A subgroup of $\mathfrak{S}_N$ fixing the part $\{1,2,\ldots,N-n\}$ is also denoted by $\mathfrak{S}_n$. We introduce $\mathfrak{S}_N/(\mathfrak{S}_{N-n}\times\mathfrak{S}_n)$ as the set of elements $R\in\mathfrak{S}_N$ satisfying $R_1<R_2<\cdots<R_{N-n}$ and $R_{N-n+1}<R_{N-n+2}<\cdots<R_N$. Then any element P of $\mathfrak{S}_N$ is uniquely decomposed into the product $P=RP'P''$ with $R\in\mathfrak{S}_N/(\mathfrak{S}_{N-n}\times\mathfrak{S}_n)$, $P'\in\mathfrak{S}_{N-n}$, and $P''\in\mathfrak{S}_n$. Let us consider the case $N=3$ and $n=1$ as an example. Among the six elements in $\mathfrak{S}_3$, (1,2,3), (1,3,2), and (2,3,1) are elements of $\mathfrak{S}_3/\mathfrak{S}_2$, because they satisfy $1<2$, $1<3$, and $2<3$, respectively. Other elements in $\mathfrak{S}_3$ are decomposed as $(2,1,3)=(1,2,3)(2,1,3)$, $(3,1,2)=(1,3,2)(2,1,3)$, and $(3,2,1)=(2,3,1)(2,1,3)$.

APPENDIX B: CONSTRUCTION OF TWO-ELECTRON EIGENSTATES

We here show that the functions (29) solve the Schrödinger equations (26). A similar derivation is presented in Ref. 36 in the case of the Anderson model (see also Ref. 39). For simplicity, we consider the case $n=0$ only and omit the suffix (0) in the functions $g^{(0)}(x_1,x_2)$ and $e^{(0)}(x)$.

First, we connect the eigenfunction $g(x_1,x_2)$ in the region $x_1<x_2<0$ to the region $x_1<0<x_2$. The matching conditions for the eigenfunctions at the discontinuous points are

$$g(x,0+)-g(x,0-)+\mathrm{i}\frac{t}{2}e(x)=0, \tag{B1a}$$

$$e(0+)-e(0-)=-\mathrm{i}Ue(0). \tag{B1b}$$

By using the matching conditions with the assumptions (27), the second equation in Eq. (26) is expressed for $x\neq 0$ in the two forms:

$$\frac{d}{dx}e(x)=\mathrm{i}\left(E-\epsilon_d+\mathrm{i}\frac{t^2}{2}\right)e(x)-2\mathrm{i}tg(x,0-) \tag{B2a}$$

$$=\mathrm{i}\left(E-\epsilon_d+\mathrm{i}\frac{t^2}{2}\right)e(x)+2\mathrm{i}tg(0-,x). \tag{B2b}$$

For $x<0$, Eq. (B2a) is integrated as

$$e(x)=Ce^{\mathrm{i}(E-\epsilon_d+\mathrm{i}\frac{t^2}{2})x}-2\mathrm{i}t\int_0^\infty dz\, e^{\mathrm{i}(E-\epsilon_d+\mathrm{i}\frac{t^2}{2})z}g(x-z,0-), \tag{B3}$$

where C is an integration constant. We choose $C=0$ to avoid the divergence of $e(x)$ in the limit $x\to-\infty$. Since the function $g(x,0-)$ for $x<0$ is assumed in Eq. (28), we obtain

$$e(x)=\frac{1}{\sqrt{2\pi}}\sum_P \mathrm{sgn}(P)e^{\mathrm{i}k_{P_1}x}e_{k_{P_2}}. \tag{B4}$$

Then the matching condition (B1a) gives

$$g(x,0+)=\frac{1}{4\pi}\sum_P \mathrm{sgn}(P)e^{\mathrm{i}(k_{P_1}x+\delta_{k_{P_2}})}. \tag{B5}$$

Recall that $e^{\mathrm{i}\delta_k}=e_k/e_k^*=1-\mathrm{i}t\sqrt{2\pi}e_k$. Since the eigenfunction $g(x_1,x_2)$ in the region $x_1<0<x_2$ is at most a linear combination of plane waves $\{e^{\mathrm{i}(q_1x_1+q_2x_2)}\}$ whose wave numbers satisfy $q_1+q_2=E$, we have

$$g(x_1,x_2)=\frac{1}{4\pi}\sum_P \mathrm{sgn}(P)e^{\mathrm{i}(k_{P_1}x_1+k_{P_2}x_2+\delta_{k_{P_2}})}. \tag{B6}$$

Thus the eigenfunction $g(x_1,x_2)$ in the region $x_1,x_2<0$ has been connected to the region $x_1<0<x_2$.

Second, we connect the eigenfunction $g(x_1,x_2)$ in the region $x_1<0<x_2$ to the region $0<x_1<x_2$. For $0<x$, Eq. (B2b) is integrated as

$$e(x)=C'e^{i(E-\epsilon_d+i\frac{t^2}{2})x}+2it\int_0^x dz\,e^{i(E-\epsilon_d+i\frac{t^2}{2})z}g(0-,x-z). \tag{B7}$$

We keep the term with the integration constant C' since it is not divergent even in the limit $x\to\infty$. In fact, the term gives the two-body bound state that appears in the second term of the first two eigenfunctions in Eq. (29). By inserting the eigenfunction (B6), we obtain

$$e(x)=C''e^{i(E-\epsilon_d+i\frac{t^2}{2})x}-\frac{1}{\sqrt{2\pi}}\sum_P \mathrm{sgn}(P)e^{i(k_{P_2}x+\delta_{k_{P_2}})}e_{k_{P_1}}, \tag{B8}$$

where $C''=C'+(e_{k_1}-e_{k_2})/\sqrt{2\pi}$. Then the matching condition (B1a) gives

$$g(0+,x)=\frac{it}{2}C''e^{i(E-\epsilon_d+i\frac{t^2}{2})x}+\frac{1}{4\pi}\sum_P \mathrm{sgn}(P)e^{i(k_{P_2}x+\delta_{k_{P_1}}+\delta_{k_{P_2}})}, \tag{B9}$$

which leads to

$$g(x_1,x_2)=\frac{it}{2}C''e^{i[(\epsilon_d-i\frac{t^2}{2})x_1+(E-\epsilon_d+i\frac{t^2}{2})x_2]}+\frac{1}{4\pi}\sum_P \mathrm{sgn}(P)e^{i(k_{P_1}x_1+k_{P_2}x_2+\delta_{k_{P_1}}+\delta_{k_{P_2}})}. \tag{B10}$$

Thus we obtain the eigenfunction $g(x_1,x_2)$ in the region $0<x_1<x_2$.

Finally, we determine the constant C''. We employ the matching condition,

$$\left(1-\frac{iU}{2}\right)e(0-)=\left(1+\frac{iU}{2}\right)e(0+), \tag{B11}$$

which is given by combining the condition (B1b) with the assumption (27). Then we have

$$C''=\frac{1}{it}\frac{2U}{2+iU}(k_1-k_2)e_{k_1}e_{k_2}. \tag{B12}$$

In a similar way, the eigenfunctions $g^{(1)}(x_1;x_2)$ and $e^{(1)}(x)$ in Eq. (29) are constructed.

APPENDIX C: THREE-ELECTRON SCATTERING STATES

We here present the eigenfunctions of the three-electron eigenstates.[31] The construction is similar to the two-electron cases in Appendix B. For $(N_e+N_d,N_o)=(3,0)$, the eigenfunctions are

$$\begin{aligned}
3!g^{(0)}(x_1,x_2,x_3)&=\sum_P \mathrm{sgn}(P)g_{k_{P_1}}(x_1)g_{k_{P_2}}(x_2)g_{k_{P_3}}(x_3)+\frac{u}{2}\sum_{P,Q}\mathrm{sgn}(PQ)g_{k_{P_1}}(x_{Q_1})Z_{k_{P_2}k_{P_3}}(x_{Q_2Q_3})\,e^{i(k_{P_2}+k_{P_3})x_{Q_3}}\theta(x_{Q_2})\\
&\quad-\frac{u^2}{2i}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_1}}(x_{Q_2})Z_{k_{P_2}k_{P_3}}(x_{Q_1Q_3})\,e^{i(k_{P_2}+k_{P_3})x_{Q_3}}\theta(x_{Q_3Q_2})\theta(x_{Q_2Q_1})\theta(x_{Q_1}),\\
2!e^{(0)}(x_1,x_2)&=\sum_P \mathrm{sgn}(P)g_{k_{P_1}}(x_1)g_{k_{P_2}}(x_2)e_{k_{P_3}}+\frac{u}{2it}\sum_{P,R}\mathrm{sgn}(PR)g_{k_{P_1}}(x_{R_1})Z_{k_{P_2}k_{P_3}}(-x_{R_2})\,e^{i(k_{P_2}+k_{P_3})x_{R_2}}\\
&\quad+\frac{u}{2}\sum_{P,R}\mathrm{sgn}(PR)e_{k_{P_1}}Z_{k_{P_2}k_{P_3}}(x_{R_1R_2})\,e^{i(k_{P_2}+k_{P_3})x_{R_2}}\theta(x_{R_1})\\
&\quad-\frac{u^2}{2t}\sum_{P,R}\mathrm{sgn}(PR)h_{k_{P_1}}(x_{R_1})Z_{k_{P_2}k_{P_3}}(-x_{R_2})\,e^{i(k_{P_2}+k_{P_3})x_{R_2}}\theta(x_{R_2R_1})\theta(x_{R_1}),
\end{aligned} \tag{C1}$$

where $P=(P_1,P_2,P_3)$ and $Q=(Q_1,Q_2,Q_3)$ are permutations of (1,2,3) and $R=(R_1,R_2)$ is that of (1,2). Recall that $u=2U/(2+iU)$. The second terms of the eigenfunction $g^{(0)}(x_1,x_2,x_3)$, which are of second order in u, yield two-body bound states that are similar to those in the two-electron eigenfunctions. The third terms of the eigenfunction $g^{(0)}(x_1,x_2,x_3)$, on the other hand, are considered to be three-body bound states since the plain wave of $h_{k_{P_1}}(x_{Q_2})$ is always trapped between the two electrons forming a two-body bound state $Z_{k_{P_2}k_{P_3}}(x_{Q_1Q_3})$.

For $(N_e+N_d,N_o)=(2,1)$, the eigenfunctions are

$$\begin{aligned}
2!g^{(1)}(x_1,x_2;x_3)&=\sum_P \mathrm{sgn}(P)g_{k_{P_1}}(x_1)g_{k_{P_2}}(x_2)h_{k_3}(x_3)-u\sum_{P,Q}\mathrm{sgn}(PQ)g_{k_{P_1}}(x_{Q_1})X_{k_{P_2}}(x_{Q_23})\,e^{i(k_{P_2}+k_3)x_3}\theta(x_{Q_2})\\
&\quad+\frac{u}{2}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_3}(x_3)Z_{k_{P_1}k_{P_2}}(x_{Q_1Q_2})\,e^{i(k_{P_1}+k_{P_2})x_{Q_2}}\theta(x_{Q_1})
\end{aligned}$$

$$
\begin{aligned}
&+\frac{u^2}{\mathrm{i}}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_1}}(x_{Q_2})X_{k_{P_2}}(x_{Q_13})\,e^{\mathrm{i}(k_{P_2}+k_3)x_3}\theta(x_{3Q_2})\theta(x_{Q_2Q_1})\theta(x_{Q_1})\\
&+\frac{u^2}{2\mathrm{i}}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_3}(x_3)Z_{k_{P_1}k_{P_2}}(x_{Q_1Q_2})\,e^{\mathrm{i}(k_{P_1}+k_{P_2})x_{Q_2}}\theta(x_{Q_23})\theta(x_{3Q_1})\theta(x_{Q_1}),\\
e^{(1)}(x_1;x_3)&=\sum_P\mathrm{sgn}(P)g_{k_{P_1}}(x_1)e_{k_{P_2}}h_{k_3}(x_3)+\frac{u}{\mathrm{i}t}\sum_P\mathrm{sgn}(P)g_{k_{P_1}}(x_1)X_{k_{P_2}}(-x_3)\,e^{\mathrm{i}(k_{P_2}+k_3)x_3}\\
&+u\sum_P\mathrm{sgn}(P)e_{k_{P_1}}X_{k_{P_2}}(x_{13})\,e^{\mathrm{i}(k_{P_2}+k_3)x_3}\theta(x_1)+\frac{u}{\mathrm{i}t}h_{k_3}(x_3)Z_{k_1k_2}(-x_1)\,e^{\mathrm{i}(k_1+k_2)x_1}\\
&-\frac{u^2}{t}\sum_P\mathrm{sgn}(P)h_{k_{P_1}}(x_1)X_{k_{P_2}}(-x_3)\,e^{\mathrm{i}(k_{P_2}+k_3)x_3}\theta(x_{31})\theta(x_1)-\frac{u^2}{t}h_{k_3}(x_3)Z_{k_1k_2}(-x_1)\,e^{\mathrm{i}(k_1+k_2)x_1}\theta(x_{13})\theta(x_3),
\end{aligned}
\tag{C2}
$$

where $P=(P_1,P_2)$ and $Q=(Q_1,Q_2)$ are permutations of (1,2). For $(N_e+N_d,N_o)=(1,2)$, the eigenfunctions are

$$
\begin{aligned}
2!g^{(2)}(x_1;x_2,x_3)&=\sum_P\mathrm{sgn}(P)g_{k_1}(x_1)h_{k_{P_2}}(x_2)h_{k_{P_3}}(x_3)-u\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_3}}(x_{Q_3})X_{k_1}(x_{1Q_2})\,e^{\mathrm{i}(k_1+k_{P_2})x_{Q_2}}\theta(x_1)\\
&+\frac{u^2}{\mathrm{i}}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_3}}(x_{Q_2})X_{k_1}(x_{1Q_3})\,e^{\mathrm{i}(k_1+k_{P_2})x_{Q_3}}\theta(x_{Q_3Q_2})\theta(x_{Q_21})\theta(x_1),\\
2!e^{(2)}(x_2,x_3)&=\sum_P\mathrm{sgn}(P)e_{k_1}h_{k_{P_2}}(x_2)h_{k_{P_3}}(x_3)-\frac{u}{\mathrm{i}t}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_3}}(x_{Q_2})X_{k_1}(-x_{Q_3})\,e^{\mathrm{i}(k_1+k_{P_2})x_{Q_3}}\\
&+\frac{u^2}{t}\sum_{P,Q}\mathrm{sgn}(PQ)h_{k_{P_3}}(x_{Q_2})X_{k_1}(-x_{Q_3})\,e^{\mathrm{i}(k_1+k_{P_2})x_{Q_3}}\theta(x_{Q_3Q_2})\theta(x_{Q_2}),
\end{aligned}
\tag{C3}
$$

where $P=(P_2,P_3)$ and $Q=(Q_2,Q_3)$ are permutations of (2,3). The terms of second order in u in the eigenfunctions $g^{(1)}(x_1,x_2;x_3)$ and $g^{(2)}(x_1;x_2,x_3)$ are considered to be three-body bound states.

For $(N_e+N_d,N_o)=(0,3)$, the eigenfunction is free-fermionic as

$$
3!g^{(3)}(x_1,x_2,x_3)=\sum_P\mathrm{sgn}(P)h_{k_{P_1}}(x_1)h_{k_{P_2}}(x_2)h_{k_{P_3}}(x_3),
\tag{C4}
$$

where $P=(P_1,P_2,P_3)$ is a permutation of (1,2,3).

APPENDIX D: N-ELECTRON CURRENT IN THE FIRST ORDER IN U

In the present appendix, we calculate the quantum-mechanical expectation value of the current operator (10), which we call the N-electron current, in the first order in the interaction parameter U. For simplicity, we consider the case $t_1=t_2=t/\sqrt{2}$. Since the N-electron scattering eigenstate $|k\rangle$ is expressed by the linear combination (43), the corresponding eigenfunctions in the sector $(N_e+N_d,N_o)=(\bar{n},n)=(N-n,n)$ are given by

$$
\begin{aligned}
G_k^{(n)}(x;y)&=\sum_R A_R^{(n)}g_{k_R}^{(n)}(x;y),\\
E_k^{(n)}(x;y)&=\sum_R A_R^{(n)}e_{k_R}^{(n)}(x;y),
\end{aligned}
\tag{D1}
$$

where $R\in\mathfrak{S}_N/(\mathfrak{S}_{\bar{n}}\times\mathfrak{S}_n)$ (see Appendix A). The coefficient $A_R^{(n)}$ given in Eq. (44) reduces to

$$
A_R^{(n)}=(-)^{\sharp\{R_i|N_1<R_i,\bar{n}<i\}}2^{-\frac{N}{2}},
\tag{D2}
$$

in the case $t_1=t_2=t/\sqrt{2}(\in\mathbb{R})$. Then Eq. (D1) gives the N-electron scattering eigenstate $|k\rangle$ whose incident state consists of an N_1-electron plane wave in the lead 1 and an N_2-electron plane wave in the lead 2. In terms of these eigenfunctions, the N-electron current is given by

$$
\begin{aligned}
\langle k|I|k\rangle&=t\,\mathrm{Im}\Bigg(\sum_{n=1}^{N}(-)^n\bar{n}!n!\\
&\times\int dxdy\,G_k^{(n)*}(x;y)|_{y_n=0}E_k^{(n-1)}(x;y)\Bigg).
\end{aligned}
\tag{D3}
$$

We expand it in the first order in U by using the expansions,

$$
\begin{aligned}
G_k^{(n)}(x;y)&=G_{k,0}^{(n)}(x;y)+UG_{k,1}^{(n)}(x;y)+O(U^2),\\
E_k^{(n)}(x;y)&=E_{k,0}^{(n)}(x;y)+UE_{k,1}^{(n)}(x;y)+O(U^2).
\end{aligned}
\tag{D4}
$$

To obtain it, we need to calculate the following three overlap integrals of the eigenfunctions (40) and (41):

$$
\begin{aligned}
(G0E0)&=\int dxdy\,G_{k,0}^{(n)*}(x;y)|_{y_n=0}E_{k,0}^{(n-1)}(x;y),\\
(G1E0)&=\int dxdy\,G_{k,1}^{(n)*}(x;y)|_{y_n=0}E_{k,0}^{(n-1)}(x;y),\\
(G0E1)&=\int dxdy\,G_{k,0}^{(n)*}(x;y)|_{y_n=0}E_{k,1}^{(n-1)}(x;y).
\end{aligned}
\tag{D5}
$$

Here the integral $(G0E0)$ corresponds to the N-electron current in the noninteracting case, whereas the integrals $(G1E0)$ and $(G0E1)$ are the contribution of the terms of order U. The overlap integrals are explicitly calculated as follows:

$$\bar{n}!n!\bar{n}!(n-1)!(G0E0) = (-)^{n-1}\frac{1}{\sqrt{2\pi}}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}}\delta\big(k_{P_i}-k_{R_i}\big)\prod_{j=2}^{n}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)e_{k_{R_{\bar{n}+1}}}, \tag{D6}$$

$$\begin{aligned}\bar{n}!n!\bar{n}!(n-1)!(G1E0) &= (-)^{n}\frac{\bar{n}(\bar{n}-1)}{4\pi\sqrt{2\pi}\,\mathrm{i}}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}-2}\delta\big(k_{P_i}-k_{R_i}\big)\prod_{j=2}^{n}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\\ &\times\frac{e_{k_{R_{\bar{n}-1}}}e_{k_{R_{\bar{n}}}}}{e^*_{k_{R_{\bar{n}-1}}}e^*_{k_{R_{\bar{n}}}}}\big(e^*_{k_{P_{\bar{n}-1}}}-e^*_{k_{P_{\bar{n}}}}\big)e^*_{k_{P_{\bar{n}-1}}+k_{P_{\bar{n}}}-k_{R_{\bar{n}}}}e_{k_{R_{\bar{n}+1}}}\Delta\big(-k_{P_{\bar{n}-1}}-k_{P_{\bar{n}}}+k_{R_{\bar{n}-1}}+k_{R_{\bar{n}}}\big)\\ &+(-)^{n}\frac{\bar{n}(n-1)}{2\pi\sqrt{2\pi}\,\mathrm{i}}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}-1}\delta\big(k_{P_i}-k_{R_i}\big)\prod_{j=2}^{n-1}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\\ &\times\frac{e_{k_{R_{\bar{n}}}}}{e^*_{k_{R_{\bar{n}}}}}e^*_{k_{P_{\bar{n}}}+k_{P_N}-k_{R_N}}e_{k_{R_{\bar{n}+1}}}\Delta\big(-k_{P_{\bar{n}}}-k_{P_N}+k_{R_{\bar{n}}}+k_{R_N}\big),\end{aligned} \tag{D7}$$

$$\begin{aligned}\bar{n}!n!\bar{n}!(n-1)!(G0E1) &= (-)^{n}\frac{\mathrm{i}\bar{n}(\bar{n}-1)}{4\pi\sqrt{2\pi}}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}-2}\delta\big(k_{P_i}-k_{R_i}\big)\prod_{j=2}^{n}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\\ &\times\frac{e^*_{k_{P_{\bar{n}-1}}}e^*_{k_{P_{\bar{n}}}}}{e_{k_{P_{\bar{n}-1}}}e_{k_{P_{\bar{n}}}}}\Big(e_{k_{R_{\bar{n}-1}}}-e_{k_{R_{\bar{n}}}}\Big)e_{k_{R_{\bar{n}-1}}+k_{R_{\bar{n}}}-k_{P_{\bar{n}}}}e_{k_{R_{\bar{n}+1}}}\Delta\big(-k_{P_{\bar{n}-1}}-k_{P_{\bar{n}}}+k_{R_{\bar{n}-1}}+k_{R_{\bar{n}}}\big)\\ &+(-)^{n}\frac{\mathrm{i}\bar{n}(n-1)}{2\pi\sqrt{2\pi}}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}-1}\delta\big(k_{P_i}-k_{R_i}\big)\prod_{j=2}^{n-1}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\\ &\times\frac{e^*_{k_{P_{\bar{n}}}}}{e_{k_{P_{\bar{n}}}}}e_{k_{R_{\bar{n}}}}e_{k_{R_{\bar{n}}}+k_{R_N}-k_{P_N}}e_{k_{R_{\bar{n}+1}}}\Delta\big(-k_{P_{\bar{n}}}-k_{P_N}+k_{R_{\bar{n}}}+k_{R_N}\big)\\ &+(-)^{n}\frac{\bar{n}}{4\pi t}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\prod_{i=1}^{\bar{n}-1}\big[\delta\big(k_{P_i}-k_{R_i}\big)-e^*_{k_{P_i}}e_{k_{R_i}}\big]\prod_{j=2}^{n}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\\ &\times\frac{e^*_{k_{P_{\bar{n}}}}}{e_{k_{P_{\bar{n}}}}}\Big(e_{k_{R_{\bar{n}}}}-e_{k_{R_{\bar{n}+1}}}\Big)e_{k_{R_{\bar{n}}}+k_{R_{\bar{n}+1}}-k_{P_{\bar{n}}}}-(-)^{n}\frac{n-1}{2\pi t}\sum_{P,R}\mathrm{sgn}(PR)A_P^{(n)*}A_R^{(n-1)}\\ &\times\prod_{i=1}^{\bar{n}}\big[\delta\big(k_{P_i}-k_{R_i}\big)-e^*_{k_{P_i}}e_{k_{R_i}}\big]\prod_{j=2}^{n-1}\delta\big(k_{P_{\bar{n}+j}}-k_{R_{\bar{n}+j}}\big)\,e_{k_{R_{\bar{n}+1}}}e_{k_{R_{\bar{n}+1}}+k_{R_N}-k_{P_N}},\end{aligned} \tag{D8}$$

where $P, R \in \mathfrak{S}_N$. Here we have used

$$\int_{x_2>x_1>0}dx_1dx_2\,e^{-\mathrm{i}(k_1+k_2)x_2}Z^*_{k_1k_2}(x_1-x_2)g_{k'_1}(x_1)g_{k'_2}(x_2) = \frac{\mathrm{i}}{2\pi}\frac{e_{k'_1}e_{k'_2}}{e^*_{k'_1}e^*_{k'_2}}\big(e^*_{k_1}-e^*_{k_2}\big)e^*_{k_1+k_2-k'_2}\Delta(-k_1-k_2+k'_1+k'_2),$$

$$\int_{y>x>0}dxdy\,e^{-\mathrm{i}(k_1+k_2)y}X^*_{k_2}(x-y)g_{k'_1}(x)h_{k'_2}(y) = -\frac{\mathrm{i}}{2\pi}\frac{e_{k'_1}}{e^*_{k'_1}}e^*_{k_1}e^*_{k_1+k_2-k'_2}\Delta(-k_1-k_2+k'_1+k'_2),$$

which are given by the formula,

$$\int_0^\infty dx\,e^{\mathrm{i}kx} = \pi\delta(k)+\mathrm{i}\mathcal{P}\frac{1}{k} =: \Delta(k), \tag{D9}$$

where $\mathcal{P}$ stands for Cauchy's principal value. We note that the apparent singularity in the terms of Cauchy's principal value is actually found to be regular.

Let us assume $k_1 < \cdots < k_{\bar{n}}$ and $k_{\bar{n}+1} < \cdots < k_N$. Since only the terms with $P = R$ contribute to the sum in Eq. (D6) then, we have

$$\begin{aligned}&\bar{n}!n!\bar{n}!(n-1)!(G0E0)\\ &\quad= (-)^{n-1}\delta(0)^{N-1}\sum_{P\in\mathfrak{S}_N}A_P^{(n)*}A_P^{(n-1)}\frac{1}{\sqrt{2\pi}}e_{k_{P_{\bar{n}+1}}}.\end{aligned} \tag{D10}$$

In a similar way, by combining Eqs. (D7) and (D8), we obtain

$$\begin{aligned}\bar{n}!n!\bar{n}!(n-1)!((G1E0)+(G0E1)) &= (-)^n\frac{\bar{n}}{2\pi t}\delta(0)^{N-2}\sum_{P\in\mathfrak{S}_N}A_P^{(n)*}A_P^{(n-1)}\frac{e^*_{k_{P_{\bar{n}}}}}{e_{k_{P_{\bar{n}}}}}\left(e_{k_{P_{\bar{n}}}}-e_{k_{P_{\bar{n}+1}}}\right)e_{k_{P_{\bar{n}+1}}}-(-)^n\frac{n-1}{2\pi t}\delta(0)^{N-2}\\ &\times\sum_{P\in\mathfrak{S}_N}A_P^{(n)*}\left(A_P^{(n-1)}e_{k_{P_{\bar{n}+1}}}-A^{(n-1)}_{P(\bar{n}+1,N)}e_{k_{P_N}}\right)e_{k_{P_{\bar{n}+1}}}+(-)^n\frac{\bar{n}(\bar{n}-1)}{2\pi t}\delta(0)^{N-3}\sum_{P\in\mathfrak{S}_N}A_P^{(n)*}\\ &\times A_P^{(n-1)}\left(\frac{e^*_{k_{P_{\bar{n}-1}}}}{e_{k_{P_{\bar{n}-1}}}}\left(e_{k_{P_{\bar{n}-1}}}-e_{k_{P_{\bar{n}}}}\right)+\frac{e_{k_{P_{\bar{n}-1}}}}{e^*_{k_{P_{\bar{n}-1}}}}\left(e^*_{k_{P_{\bar{n}-1}}}-e^*_{k_{P_{\bar{n}}}}\right)\right)e^*_{k_{P_{\bar{n}}}}e_{k_{P_{\bar{n}}}}e_{k_{P_{\bar{n}+1}}}-(-)^n\frac{\bar{n}(n-1)}{2\pi t}\\ &\times\delta(0)^{N-3}\sum_{P\in\mathfrak{S}_N}A_P^{(n)*}\left[A_P^{(n-1)}\left(e_{k_{P_{\bar{n}}}}+e^*_{k_{P_{\bar{n}}}}\right)e^*_{k_{P_{\bar{n}}}}e_{k_{P_{\bar{n}}}}-A^{(n-1)}_{P(\bar{n},N)}\left(e_{k_{P_{\bar{n}}}}+e^*_{k_{P_N}}\right)e^*_{k_{P_{\bar{n}}}}e_{k_{P_N}}\right]e_{k_{P_{\bar{n}+1}}}.\end{aligned}\tag{D11}$$

In the previous equation, the factors $\delta(0)$ should be replaced with the system length L divided by 2π. We remark that the terms of order higher than $\delta(0)^{N-4}$ do not appear in Eq. (D11). Indeed they cancel out each other because of the symmetry of the sums on $R\in\mathfrak{S}_N$. We find that each term in Eq. (D11) depends only on three different wave numbers. Thus the two- and three-electron scattering process is sufficient for calculating N-electron current in the first order in U.

We now specify an N-electron scattering eigenstate by taking the coefficients $\{A_P^{(n)}\}$ given by Eq. (D2). We find the relations,

$$2^N A_P^{(n)*}A_P^{(n-1)}=\kappa_{P_{m+1}},$$

$$2^N A_P^{(n)*}A^{(n-1)}_{P(m+1,N)}=\kappa_{P_N},\tag{D12}$$

$$2^N A_P^{(n)*}A^{(n-1)}_{P(m,N)}=\kappa_{P_m}\kappa_{P_{m+1}}\kappa_{P_N},$$

where $\kappa_i=1$ if $N_1\geqslant i$ and $\kappa_i=-1$ if $N_1<i$. Then, from Eq. (D10), we have

$$\begin{aligned}&2^N\bar{n}!n!\bar{n}!(n-1)!(G0E0)\\ &\quad=(-)^{n-1}(N-1)!\delta(0)^{N-1}\sum_{i=1}^N\kappa_i\frac{1}{\sqrt{2\pi}}e_{k_i}.\end{aligned}\tag{D13}$$

It is clear that this reproduces the original Landauer formula for the noninteracting case. In a similar way, by applying Eq. (D12) to Eq. (D11), we have

$$\begin{aligned}&2^N\bar{n}!n!\bar{n}!(n-1)![(G1E0)+(G0E1)]\\ &=(-)^n\frac{1}{2\pi t}\bar{n}(N-2)!\delta(0)^{N-2}\sum_{i\neq j}^N\kappa_j\frac{e^*_{k_i}}{e_{k_i}}\left(e_{k_i}-e_{k_j}\right)e_{k_j}-(-)^n\frac{1}{2\pi t}(n-1)(N-2)!\delta(0)^{N-2}\sum_{i\neq j}^N\kappa_j\left(e_{k_j}-e_{k_i}\right)e_{k_j}\\ &+(-)^n\frac{1}{2\pi t}\bar{n}(\bar{n}-1)(N-3)!\delta(0)^{N-3}\sum_{i\neq j\neq l(\neq i)}^N\kappa_l\left[\frac{e^*_{k_i}}{e_{k_i}}\left(e_{k_i}-e_{k_j}\right)+\frac{e_{k_i}}{e^*_{k_i}}\left(e^*_{k_i}-e^*_{k_j}\right)\right]e^*_{k_j}e_{k_j}e_{k_l}\\ &-(-)^n\frac{1}{2\pi t}\bar{n}(n-1)(N-3)!\delta(0)^{N-3}\sum_{i\neq j\neq l(\neq i)}^N\kappa_l\left[\left(e_{k_j}+e^*_{k_j}\right)-\kappa_i\kappa_j\left(e_{k_i}+e^*_{k_i}\right)\right]e^*_{k_j}e_{k_j}e_{k_l}.\end{aligned}\tag{D14}$$

Finally, by taking the sum on n such that the form (D3) is given, we obtain

$$\langle k|I|k\rangle=\delta(0)^{N-1}\sum_{i=1}^N\kappa_i\lambda(k_i)+\delta(0)^{N-2}\sum_{i\neq j}^N\kappa_j\xi(k_i,k_j)+\delta(0)^{N-3}\sum_{i\neq j\neq l(\neq i)}^N\kappa_l[\eta(k_i,k_j,k_l)+\kappa_i\kappa_j\zeta(k_i,k_j,k_l)]+O(U^2),$$

where $\lambda(k)$, $\xi(k,h)$, $\eta(k,h,l)$, and $\zeta(k,h,l)$ are defined in Eq. (61). This gives the N-electron current in the from (59).

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