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A new method to determine the optimum load of a real solar cell using the Lambert *W*-function

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ABSTRACT

An exact explicit solution based on the Lambert *W*-function is presented to express the optimum load of an illuminated solar cell containing a parasitic series resistance and a shunt resistance. The *W*-function expressions are derived using Matlab software. The method is validated by comparing the model-predicted results to the experimental data for three real solar cells. The impacts of the series resistance and shunt resistance on the optimum load are also studied and the results show a good consistency with the data reported in the literature.

1. Introduction

Maximum power can be extracted from a solar cell only when it is operating with an optimum load. In a solar cell operating under the normal conditions [1,2], even a small deviation of 1% from the optimum power transfer condition can cause a loss of output power by nearly 10%. The effect is even more pronounced at high intensities of illumination.

The classical maximum power theorem from the linear circuit theory states that the optimal load for a linear one-port has an impedance equal to the complex conjugate of the source impedance [3]. However, the current–voltage characteristic of a solar cell is nonlinear. The problem of determining the optimum load for a solar cell has been studied earlier [4–7]. For cases where the series resistance is zero and the shunt resistance is infinite, it has been shown [4,5] that the load impedance at maximum power transfer is given by

$$R_{\mathrm{Lmp}} = nV_{\mathrm{th}} e^{-V_{\mathrm{mp}}/nV_{\mathrm{th}}}/I_0 \tag{1}$$

where I_0 is the reverse saturation current; n is the ideality factor and $V_{\mathrm{th}} = kT/q$ is the thermal voltage.

Govil [6] used a "trial and error" method to obtain the maximum-power-transfer condition and has shown that for a solar cell with a series resistance R_s or a shunt resistance R_{sh}, respectively,

$$R_{\mathrm{Lmp}} = R_s + nV_{\mathrm{th}}/(I_{\mathrm{ph}} - I_{\mathrm{mp}}) \tag{2}$$

$$R_{\mathrm{Lmp}} = \frac{nV_{\mathrm{th}} + V_{\mathrm{mp}}}{I_{\mathrm{ph}} + (nV_{\mathrm{th}} - V_{\mathrm{mp}})/R_{\mathrm{sh}}} \tag{3}$$

where I_{ph} is the photo-generated current and I_{mp} is the current corresponding to the maximum-power-transfer condition.

Kothari et al. [7] have obtained the expression of optimum load which takes into account both R_s and R_{sh} using the Lagrange's method of undetermined multipliers:

$$R_{\mathrm{Lmp}} = R_s + \frac{R_{\mathrm{sh}}}{I_0 R_{\mathrm{sh}}/nV_{\mathrm{th}}\ \exp((V_{\mathrm{mp}} + I_{\mathrm{mp}}R_s)/nV_{\mathrm{th}}) + 1} \tag{4}$$

But this method needs the values of V_{mp} and I_{mp} in advance. If the values of V_{mp} and I_{mp} are obtained, it also means that the optimum load can be calculated directly using $R_{\mathrm{Lmp}} = V_{\mathrm{mp}}/I_{\mathrm{mp}}$.

To the best of our knowledge, no expression for the R_{Lmp} is available in literature which only contains the device model parameters n, V_{th}, I_0, R_s and R_{sh}, which is partly because the implicit transcendental equation of a solar cell I–V characteristic may not be solved explicitly in general for I or V using the common elementary functions. However, the exact explicit analytical solutions for I and V already exist [8] and these solutions have been used to study the organic solar cell [9], determine the diode ideality factor [10], evaluate the value of series resistance [11] and even calculate the solar array parameters [12]. These solutions make use of what is known as the Lambert *W*-function [13], a special function which is not

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expressible in terms of elementary analytical functions. In the present paper, using the Lambert W-function method, an expression for R_{Lmp} has been obtained, which takes into account both R_{s} and R_{sh} and contains only the device model parameters. The expression derived here is quite general, since it is based on the solution of the basic current–voltage characteristics of the cell, and should therefore hold for the various types of solar cells. We also report some comparison of model-predicted values to that of experimental results reported on measured R_{Lmp} for three solar cells.

2. Theory

The current–voltage characteristics for a practical solar cell under illumination are given by

$$I = I_{\mathrm{ph}} - I_0\left(\exp\left(\frac{V+IR_{\mathrm{s}}}{nV_{\mathrm{th}}}\right) - 1\right) - \frac{V+IR_{\mathrm{s}}}{R_{\mathrm{sh}}} \tag{5}$$

Fig. 1 presents the equivalent circuit of such a model. Eq. (5) is transcendental in nature, hence it is not possible to solve it for V in terms of I and vice versa. However, the explicit solution for voltage can be expressed using the Lambert W-function:

$$\begin{aligned} V = {} & R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0-I) - IR_{\mathrm{s}} - nV_{\mathrm{th}}\,\mathrm{Lambert}\,W \\ & \times\left(I_0R_{\mathrm{sh}}\ \exp\left(\frac{R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0-I)}{nV_{\mathrm{th}}}\right)\Big/(nV_{\mathrm{th}})\right) \end{aligned} \tag{6}$$

The maximum-power-transfer condition could be written in the form

$$\frac{\mathrm{d}P}{\mathrm{d}I} = 0 \tag{7}$$

which is equivalent to the condition

$$\frac{\mathrm{d}V}{\mathrm{d}I}\Big|_{I=I_{\mathrm{mp}}} = -\frac{V}{I}\Big|_{I=I_{\mathrm{mp}}} \tag{8}$$

The explicit solution of the optimum load R_{Lmp} in terms of Lambert W-function can be derived by substituting Eq. (6) in Eq. (8):

$$R_{\mathrm{Lmp}} = R_{\mathrm{s}} + \frac{R_{\mathrm{sh}}}{1+\mathrm{Lambert}\,W(I_0R_{\mathrm{sh}}\exp(R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0-I_{\mathrm{mp}})/nV_{\mathrm{th}})/(nV_{\mathrm{th}}))} \tag{9}$$

(a) When $R_{\mathrm{s}}=0$ and $R_{\mathrm{sh}}=\infty$, i.e., for an ideal solar cell, Eq. (9) reduces to

$$R_{\mathrm{Lmp}} = \frac{nV_{\mathrm{th}}}{I_{\mathrm{ph}}+I_0-I_{\mathrm{mp}}} \tag{10}$$

Eq. (10) is equivalent to Eq. (1).

(b) When $R_{\mathrm{sh}}=\infty$ and R_{s} has a finite value, Eq. (9) reduces to

$$R_{\mathrm{Lmp}} = R_{\mathrm{s}} + \frac{nV_{\mathrm{th}}}{I_{\mathrm{ph}}+I_0-I_{\mathrm{mp}}} \tag{11}$$

If we accept the assumption $I_{\mathrm{ph}} \gg I_0$ [5], Eq. (11) is equivalent to Eq. (2).

(c) When $R_{\mathrm{s}}=0$ and R_{sh} has a finite value, Eq. (9) reduces to

$$R_{\mathrm{Lmp}} = \frac{nV_{\mathrm{th}}R_{\mathrm{sh}}}{nV_{\mathrm{th}}+R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0-I_{\mathrm{mp}})-V_{\mathrm{mp}}} \tag{12}$$

Substitution of the equation presented by Govil [6]

$$I_0\left(1+\frac{1}{nV_{\mathrm{th}}}\right)\exp\left(\frac{1}{nV_{\mathrm{th}}}\right) + \frac{2V_{\mathrm{mp}}}{R_{\mathrm{sh}}} = I_{\mathrm{ph}} \tag{13}$$

would result in Eq. (12) being equivalent to Eq. (3).

For calculating R_{Lmp} directly from the device model parameters, I_{mp} in Eq. (9) must be substituted by the equation which has the form of $I_{\mathrm{mp}} = f(R_{\mathrm{s}}, R_{\mathrm{sh}}, n, I_0, I_{\mathrm{ph}}, R_{\mathrm{Lmp}})$. Substituting the V in Eq. (5) by $V = I_{\mathrm{mp}}R_{\mathrm{Lmp}}$, the explicit solution for current I_{mp} can be expressed using the Lambert W-function as follows:

$$\begin{aligned} I_{\mathrm{mp}} = {} & \frac{R_{\mathrm{sh}}(I_0+I_{\mathrm{ph}})}{R_{\mathrm{Lmp}}+R_{\mathrm{s}}+R_{\mathrm{sh}}} - \frac{nV_{\mathrm{th}}}{R_{\mathrm{Lmp}}+R_{\mathrm{s}}}\mathrm{Lambert}\,W \\ & \times\left(\frac{(R_{\mathrm{Lmp}}+R_{\mathrm{s}})I_0R_{\mathrm{sh}}}{nV_{\mathrm{th}}(R_{\mathrm{sh}}+R_{\mathrm{Lmp}}+R_{\mathrm{s}})}\exp\left(\frac{R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0)(R_{\mathrm{Lmp}}+R_{\mathrm{s}})}{nV_{\mathrm{th}}(R_{\mathrm{sh}}+R_{\mathrm{Lmp}}+R_{\mathrm{s}})}\right)\right) \end{aligned} \tag{14}$$

After substituting Eq. (14) into Eq. (9), the equation which can directly calculate the optimum load for a solar cell from the device model parameters is obtained:

$$R_{\mathrm{Lmp}} = R_{\mathrm{s}} + \cfrac{R_{\mathrm{sh}}}{1+\mathrm{Lambert}\,W\left(\cfrac{I_0R_{\mathrm{sh}}}{nV_{\mathrm{th}}}\exp\left(\cfrac{R_{\mathrm{sh}}}{nV_{\mathrm{th}}}\left(\begin{array}{c} I_{\mathrm{ph}}+I_0-\cfrac{R_{\mathrm{sh}}(I_0+I_{\mathrm{ph}})}{R_{\mathrm{Lmp}}+R_{\mathrm{s}}+R_{\mathrm{sh}}}+\cfrac{nV_{\mathrm{th}}}{R_{\mathrm{Lmp}}+R_{\mathrm{s}}} \\ \times\mathrm{LambertW}\left(\cfrac{(R_{\mathrm{Lmp}}+R_{\mathrm{s}})I_0R_{\mathrm{sh}}}{nV_{\mathrm{th}}(R_{\mathrm{sh}}+R_{\mathrm{Lmp}}+R_{\mathrm{s}})}\exp\left(\cfrac{R_{\mathrm{sh}}(I_{\mathrm{ph}}+I_0)(R_{\mathrm{Lmp}}+R_{\mathrm{s}})}{nV_{\mathrm{th}}(R_{\mathrm{sh}}+R_{\mathrm{Lmp}}+R_{\mathrm{s}})}\right)\right)\end{array}\right)\right)\right)} \tag{15}$$

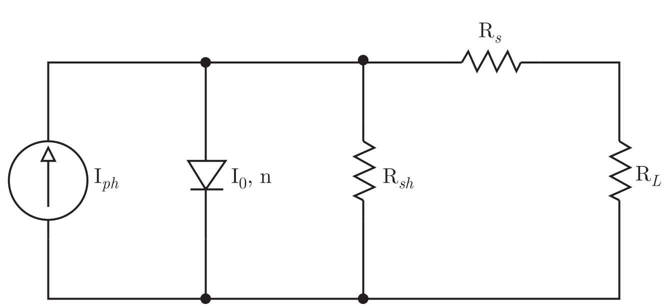

Fig. 1. Generic solar cell equivalent circuit including parasitic series resistance and shunt resistance.

3. Calculations

The Lambert W-function method is applied to the experimental solar cell I–V characteristics using the model with a single exponential and series resistance and junction shunt loss, and the results are summarized in Table 1. We evaluate R_{Lmp} for two solar cells (namely blue solar cell and grey solar cell[2]), using the data of Phang et al. [14] and Charles et al. [15] and compare it to the results predicted by models using the Lambert W-function method and the other methods presented in literature. All of the above calculations are performed in the Matlab environment using a 20-digit precision.

The calculated results are compared with the experimental data and the relative accuracies are also calculated.

Plastic solar cells have become an intensive field of research due to the possibilities of low material cost and easy solution processing

[2] A high-quality silicon solar cell, blue type, square, of area 4 cm^2, bought from SAT, 41rue Cantagrel, 75624 Paris Cedex 13, France; a low-quality silicon solar cell, grey, round, of area 25.8 cm^2, bought from Radio M.J., 19 rue Claude Benard, 75005 Paris, France.

[17–20]. The major limitation to the possible large-scale application of plastic solar cells has been the low efficiency compared to inorganic solar cells [20]. In general, plastic solar cells have large series resistance as well as too small shunt resistance, which tends to reduce the efficiency [16]. Plastic solar cells therefore represent an extreme case, where the effects of parasitic series resistance and shunt resistance are notably significant.

The experimental data recently published for a developmental plastic solar cell [11] are compared to the results predicted by the various models in Table 1. The calculated results again show that

Table 1
Device model parameters of solar cells

Parameters	Blue solar cell	Grey solar cell	Plastic solar cell
Experimental data of Charles et al.			
R_s	68.26 (mΩ)	77.69 (mΩ)	8.59 (Ω cm^2)
R_{sh}	1000.0 (Ω)	25.9 (Ω)	197.24 (Ω)
I_0	0.1036 (μA)	5.514 (μA)	13.6 (nA cm^{-2})
I_{ph}	0.1023 (A)	0.5610(A)	7.94(mA cm^{-2})
V_{th}	25.875 (mV)	26.479 (mV)	25.875 (mV)
n	1.5019	1.7168	2.31
V_{mp}	0.437 (V)	0.390 (V)	0.548 (V)
I_{mp}	0.0925 (A)	0.481 (A)	4.71 (mA cm^{-2})
R_{Lmp}	4.724 (Ω)	0.8108 (Ω)	116.35 (Ω cm^2)
Lambert *W*-function method			
R_{Lmp}	4.638 (Ω)	0.7985 (Ω)	116.18 (Ω cm^2)
Accuracy	1.821%	1.517%	0.146%
Loferski's method			
R_{Lmp}	4.9036 (Ω)	1.5498 (Ω)	458.37 (Ω cm^2)
Accuracy	3.801%	91.149%	293.958%
Govil's method			
Only R_s			
R_{Lmp}	4.0337 (Ω)	0.6459 (Ω)	27.10 (Ω cm^2)
Accuracy	14.612%	20.334%	76.712%
Only R_{sh}			
R_{Lmp}	4.6698 (Ω)	0.7951 (Ω)	111.22 (Ω cm^2)
Accuracy	1.147%	1.940%	4.411%
Kothari's method			
R_{Lmp}	4.2192 (Ω)	0.7414 (Ω)	115.39 (Ω cm^2)
Accuracy	10.686%	8.554%	0.821%

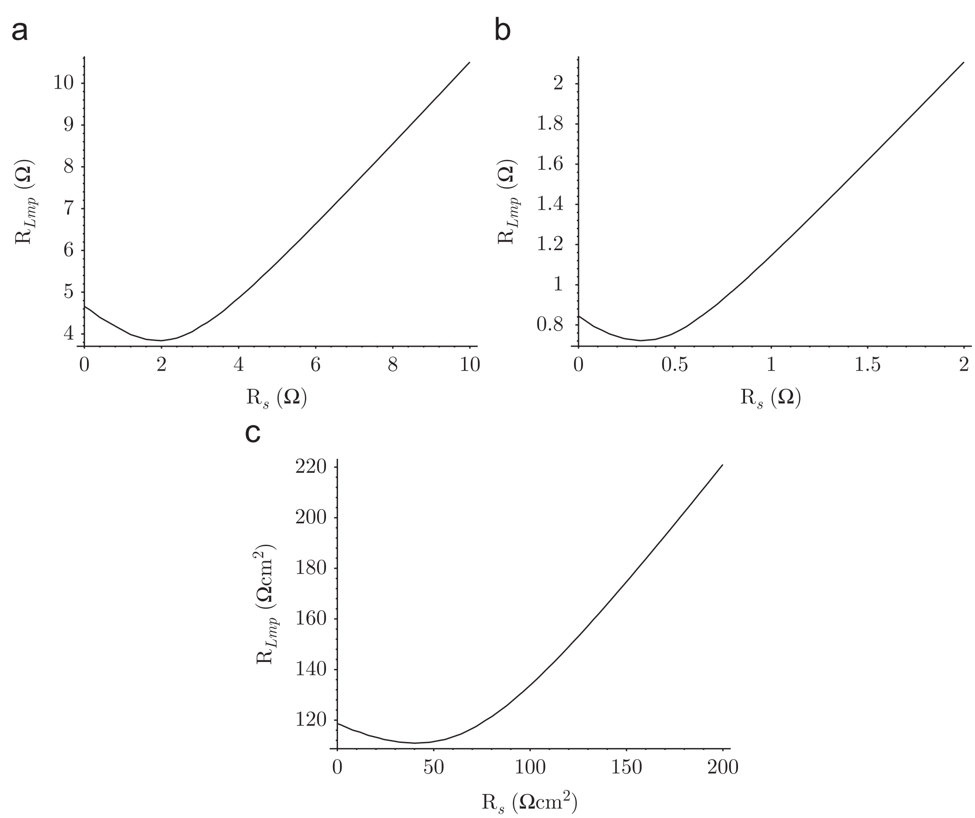

Fig. 2. Impact of the series resistance on the optimum load ((a) blue solar cell; (b) grey solar cell; (c) plastic solar cell).

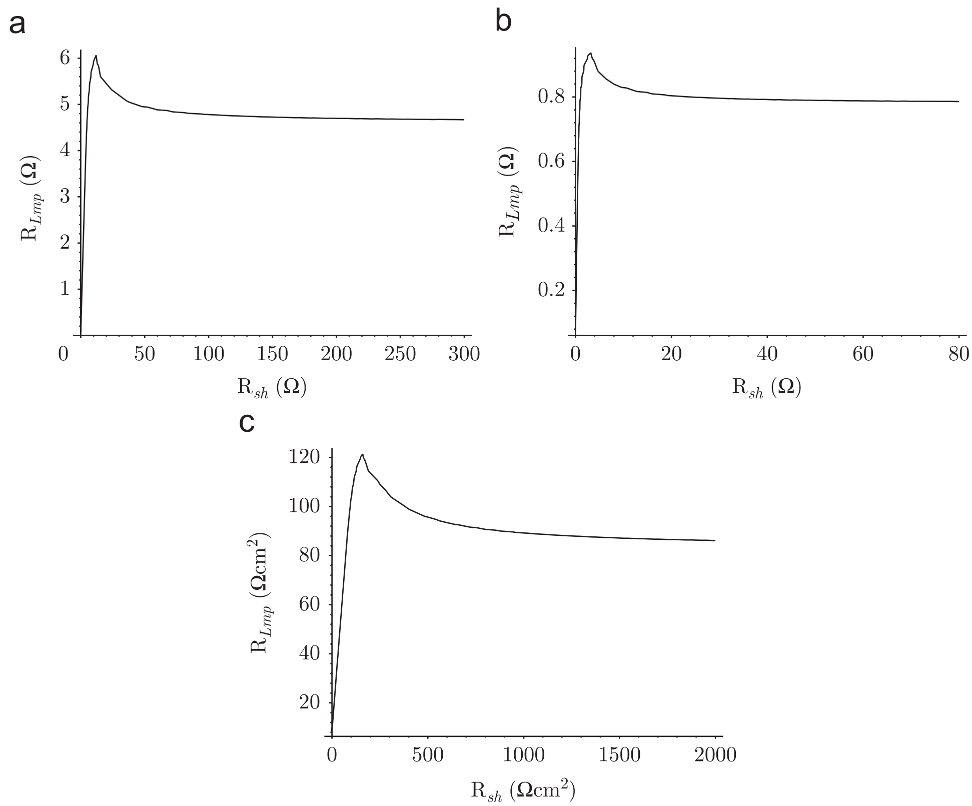

Fig. 3. Impact of the shunt resistance on the optimum load ((a) blue solar cell; (b) grey solar cell; (c) plastic solar cell).

the Lambert *W*-function method has the best accuracy for the three kinds of solar cell.

The errors of calculated optimum load from the experimental *I–V* data turn out to be insignificant. However, it is worth analyzing the dependence of the optimum load on the series resistance and shunt resistance of the solar cell. The methods presented in literature need the values of I_{mp} and V_{mp} to evaluate the value of R_{Lmp}. So it is impossible to use them to study the relation among the optimum load, series resistance and shunt resistance. However, it is very convenient to use the Lambert *W*-function method to perform this task.

Fig. 2 shows the impact of the series resistance on R_{Lmp}. From the plot it is clear that the results are consistent with those predicted by Kothari et al. [7], who concluded that the higher the series resistance, the greater its impact on the optimum load.

Fig. 3 shows the impact of the shunt resistance on R_{Lmp}. This is also consistent with the Kothari et al. [7] statement that the smaller the shunt resistance, the greater its impact on the optimum load.

4. Conclusion

A simple and accurate (without approximations) method using the Lambert *W*-function has been described, which gives the expression of the optimum load of the solar cell. Based on this expression, the optimum load could be calculated directly from the device model parameters. The impacts of the series resistance and shunt resistance on the optimum load are also studied and the results are found to be consistent with what has been previously reported in the literature.

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