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An efficient analytical approach for obtaining a five parameters model of photovoltaic modules using only reference data

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HIGHLIGHTS

- Availability of *I–V* curve is essential for reliable economic assessment of PV arrays.
- Determination of *I–V* curves generally depends on five parameters values.
- Generally, parameters evaluation requires at least five boundary conditions.
- Our model employs only reference data and pure analytical relationships.
- An effective and optimized resolution procedure to extract the parameters is proposed.

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ABSTRACT

Exploiting the equivalent one-diode circuit of a photovoltaic (PV) module, this paper proposes a novel and fully analytical model to predict the electrical performance upon solar irradiance intensity and PV module temperature. The model refers essentially to an equivalent circuit governed by five parameters and the extraction of them permits to describe the current–voltage curve of the PV panel and consequently permits to assess the energy output of PV modules. The proposed model extracts the five characteristic parameters using only exact analytical relationship and tabular data always available such as short-circuit current, open circuit voltage and the Maximum Power Point (MPP). The difference with other models consists in the complete absence of mathematical simplifications or other physical assumptions. All used equations were obtained with a transparent analytical procedure. A new resolution procedure for solving the equation that describes the equivalent one diode circuit system is also described. The procedure is based upon the Generalized Reduced Gradient (GRG) algorithm and transforms the extraction of the five parameters into a constrained non-linear optimization problem. The purely analytical model, the absence of data to be obtained from graphic methods or not always available in data-sheets, and a new optimized procedure to solve the system of equations lead to obtain values of the five parameters that perfectly fit the official tabular data. The suggested procedure of numerical solution of a local minimum problem allows converging towards the solution with the desired accuracy in a fast and effective way. Although in the scientific literature there are several models able to determine the value of these five parameters, these procedures are always affected by inevitable inaccuracies linked to various simplifications or due to the use of non-tabular data such as some graphic characteristics of the experimental *I–V* curve (moreover not always available). The model, as opposed to those already known in the literature, is exclusively based on analytical relationships and is free of any simplifications that may affect the reliability of the results. The proposed model allows a more accurate modeling of the PV modules based solely on reference data and the availability of decision support tools that may reliably predict the energy produced by a photovoltaic panel is essential in the design phase of the plant to avoid future problems related to incorrect sizing. Furthermore, reliable energy predictions lead to more correct economic analyses that can stimulate the diffusion of the PV technology.

1. Introduction

Growing interest in renewable energy resources has caused the photovoltaic (PV) power market to expand rapidly, especially in the area of distributed generation. For this reason, designers need a flexible and reliable tool to accurately predict the electrical power produced from PV arrays of various sizes [1,2]. The problem of energy production prediction of a PV cell system is particularly challenging due to the complicate dependence of the electrical current yield on weather related and environmental factors, such as temperature and the global irradiance.

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Nomenclature

AM	air mass	P_{max}	maximum electrical power (W)
G	solar irradiance (W/m^2)	R_L	electrical load (Ω)
G_{ref}	solar irradiance at STC (1000 W/m^2)	R_s	series resistance (Ω)
I	current generated by the panel (A)	R_{sh}	shunt resistance (Ω)
I_L	photocurrent (A)	T	temperature of the PV cell (°K)
$I_{L,ref}$	photocurrent at STC (A)	T_{ref}	temperature of the PV cell at STC (25 °C–298.15 °K)
I_{mpp}	current at the maximum power point (A)	V	voltage generated by the PV panel (V)
I_{SC}	short circuit current (A)	V_{mpp}	voltage at the maximum power point (V)
$I_{SC,ref}$	short circuit current at STC (A)	V_{OC}	open circuit voltage of the panel (V)
I_0	reverse saturation current (A)	α_G	ratio between the current irradiance and the irradiance at STC
K	thermal correction factor (Ω/°C)	μ_{Isc}	short circuit current thermal coefficient (A/°C)
n	ideality factor	μ_{Voc}	open circuit voltage thermal coefficient (V/°C)
P	electrical power (W)		

To design and assess the operation of a PV system, an accurate PV model should be with appropriate complexity that one can apply it to predict the reliable Current–Voltage (I–V) and Power–Voltage (P–V) output characteristics under real operating conditions [3]. Some detailed models such as the two-diode [4–7] and the three-diode [8] models can provide better accuracy especially for poly-Si cells by considering the effect of carrier recombination and the leakage current. Nevertheless, in order to design and assess *PV* devices, a single-diode model has been the first candidate with a suitable degree of complexity and a sufficient degree of precision [9–14]. This model is often called "five-parameters model" because of its five governing variables. Its equivalent circuit is composed of a photocurrent source I_L, a diode in parallel with a shunt resistance R_{sh}, and a series resistance R_s as shown in Fig. 1.

Numerous methods [15–23] have been developed to solve the five parameters model introducing several simplifications or approximations or using graphical imprecise data. For example in [22] the authors developed three numerical methods for solving the model proposing as simplification that the value of the photocurrent I_L is equal to the short circuit current I_{SC} and the series resistance is geometrically deduced from graphical analysis of the I–V curve. Even other works [2,23] propose the construction of the model for a PV panel using data-sheet parameters, but deducing the series resistance from a graphical analysis of the I–V curve.

Other authors in [24] developed a simplified explicit method by assuming that the photocurrent is equal to the short-circuit current and deriving value of the series resistance from thermal drift coefficient [1]. Zhou et al. in [25] introduced the concept of a Fill Factor (*FF*) to obtain the series resistance applying this correlation in case of maximum power-output. Rajapakse and Muthumuni in [26] developed a model based on the I–V relationship for the single diode within the popular electromagnetic transient simulation program PSCAD/EMTDC including a PV array model, a maximum power point tracking controller model, and a grid connected inverter. In [27] Campbell developed a circuit-based, piecewise linear PV device model, which is suitable for use with converters in transient and dynamic electronic simulation software. Other authors such as King [28,29] developed a model to reproduce the I–V curve using three important points: short-circuit, open-circuit, and MPP conditions on the curve, but requiring the empirical determination of the series resistance R_s and shunt resistance R_{sh}. Other authors completely neglected in [30] the value of shunt resistance. Furthermore, since the mathematical model of PV panels is always linked to a transcendental equation, certain studies preferred to apply numerical method to determine the I–V characteristics. In [31–34] the Newton–Raphson method is applied, but some simplifications are imposed; in [31] the value of the shunt resistance is correlated with the slope of the I–V curve; in [32] the value of I_{SC} is approximated. In [33,34] some exponential terms involved in the equation system solving procedure are neglected. These kinds of positions, although widely verified in the SRC, inevitably leads to an error of the evaluation of the produced current in conditions of low irradiance and small load, conditions that may occur in the very early morning or in the last period of the day when in presence of a Maximum Power Point Tracker device. Furthermore, this kind of simplifications, together with the fact that the nature of the employed equations is implicit and transcendental, leads to obtain values of the five parameters that, once replaced the main equation, identify an I–V curve that does not pass through I–V points used as inputs but only approaches these points.

The Lambert-W function is applied in [35] to approximate the value of the current. Another numerical method is applied in [36] to graphically fit the extracted parameters to the I–V curve with genetic algorithms optimization.

In this study, the authors propose a new five parameters model based only on exact analytical relationships and using only always available technical data that can be obtained from official datasheets or from authoritative database.

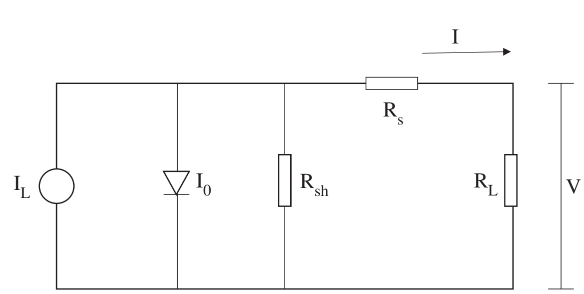

Fig. 1. Schema of one diode simplified equivalent circuit close on a resistive load RL.

1.1. The available reference data of PV modules

The necessary data to apply the model that we are proposing, can easily be found in freeware tools. In internet, it is possible to find a wide variety of databases that accurately collect data-sheets of several photovoltaic panels. The manufacturers of PV panels never provide the values of the five parameters, but sometimes a calculation procedure is available in these databases. Among the most authoritative free databases of PV panels, known for their completeness of data, there are the PHOTON database and the Solar Advisor Model (SAM) database.

The PHOTON free online database simply provides a very extensive collection of PV official data-sheets without supplying any tool to extract the five parameters. A simple and immediate web page provides the main data for a wide variety of PV cells and panels. Of course, the amount of technical data is different depending on the adopted manufactures policy.

SAM is a free tool distributed by the US Department of Energy (DOE) that provides three options for PV module performance models: the Sandia Performance Model proposed in [29], the five parameter model popularized by De Soto et al. [10] and a single-point efficiency model. In the SAM module page it is possible to choose among the following options:

- Sandia PV Array Performance Model with Module Database
- CEC Performance Model with Module Database
- CEC Performance Model with User Entered Specifications
- Simple Efficiency Module Model

The Sandia PV Array Performance Model with Module Database applied the procedure illustrated in [29].

The California Energy Commission (CEC) Performance Model with Module Database and with User Entered Specifications applies a model developed at the University of Wisconsin-Madison Solar Energy Laboratory and described in [10]. The CEC Performance Model with User Entered Specifications uses the same algorithms, but allows users to enter their own module specifications.

The Simple Efficiency Module Model is the least accurate of the previous models and is suggested when the other models required parameters are not available. On the other hand, the models adopted by SAM are characterized by some simplifications, as above described, that could make the results not completely reliable.

2. The PV solar cells and modules modeling

As previously introduced the electric behavior of a photovoltaic panel can be described by the electrical circuit of Fig. 1. Based on this circuit, the mathematical model of a photovoltaic cell can be defined in accordance with the following expression that permits to retrieve the *I–V* curve:

$$I = I_L - I_0\left(e^{\frac{V+I\cdot R_s}{nT}} - 1\right) - \frac{V + I \cdot R_s}{R_{sh}} \tag{1}$$

in which I_L is the photocurrent and it depends on the solar irradiance, I_0 is the diode reverse saturation current and is affected by the silicon temperature, n is the ideality factor and T is the cell temperature in Kelvin. In this case, to use the model of Fig. 1 it is necessary to determine the five unknown parameters (I_L, I_0, n, R_s and R_{sh}) that produce the best fit to a given photovoltaic cell operating under certain conditions.

As widely described in literature, when it is necessary forecasting the electrical behavior of a PV cell/module/array, the most difficult and important problem is to solve the equation system that represents the thermo-electrical behavior of a single PV panel. Once known the trend of current and voltage of the single module varying G and T, it is easy to determine the behavior of an entire photovoltaic plant/array, if it is known the series/parallel configuration of the single modules. To this aim, the core of calculation procedures used by widely disseminated software such as PVsyst or TRNSYS, employed for the design of complex PV systems, is based on numerical models applied to single panels. In PVsyst software, a simplified four parameters model proposed by Beckman et al. is used; in TRNSYS instead, the five parameters model proposed by De Soto et al. is employed.

The aim of this study is to develop a procedure to extract the five parameters of Eq. (1) without any simplification or assumptions, basing only on the common technical data (Fig. 2) at Standard Test Conditions (STC) when the cell temperature $T = 25\,°C$, the irradiance $G = 1000\ W/m^2$ and the air mass coefficient AM = 1.5 such as:

- Short-circuit current I_{SC}: the maximum value of the generated current of the PV module when the PV module terminal is shorted.
- Open-circuit voltage V_{OC}: the voltage across the PV module terminal when the leads are left open.
- MPP: the optimum operating point at $V = V_{mpp}$ and $I = I_{mpp}$; this is where the power generated by the module is the maximum: $P = P_{max}$.

To make the Eq. (1) best fit with the above introduced experimental technical data issued from manufactures and PV databases, the proper values of the five parameters must be calculated. For this purpose, it is necessary to set up five independent mathematical equations. The simultaneous resolution of these five equations permits to extract the five unknown parameters.

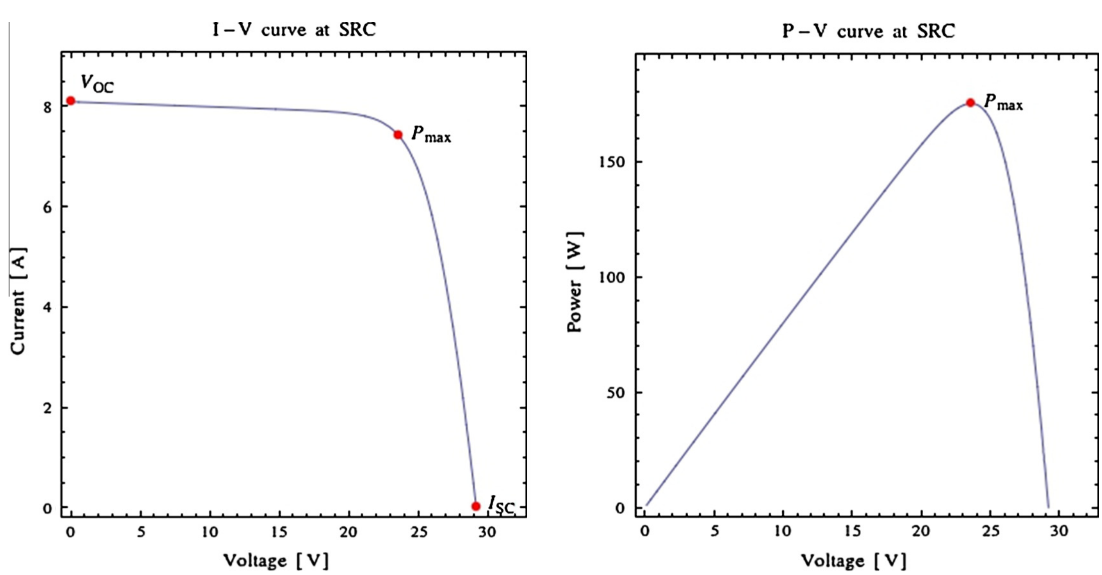

Fig. 2. A typical *I–V* curve and *P–V* curve of a PV panel.

The first equation can be derived from the values of I_{SC}. At the short circuit current point the voltage is null and substituting these values in the Eq. (1) it is possible to obtain:

$$f_1 : I_L + \left(1 - e^{\frac{I_{SC}R_s}{nT}}\right)I_0 - \frac{I_{SC}R_s}{R_{sh}} - I_{SC} = 0 \tag{2}$$

The second equation can be derived from the values of V_{OC}. At the open circuit voltage point, the current is null and substituting these values in the Eq. (1) it is possible to obtain:

$$f_2 : I_L + \left(1 - e^{\frac{V_{oc}}{nT}}\right)I_0 - \frac{V_{oc}}{R_{sh}} = 0 \tag{3}$$

The third equation can be derived from the knowledge of the MPP where $V = V_{mpp}$ and $I = I_{mpp}$; substituting these values in Eq. (1) it is possible to obtain:

$$f_3 : I_L + \left(1 - e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}}\right)I_0 - \frac{I_{mpp}R_s + V_{mpp}}{R_{sh}} - I_{mpp} = 0 \tag{4}$$

Stating that the power of the PV panel is $P = I \cdot V$ for each point of the I–V curve, the derivative of the power with respect to the voltage is:

$$\frac{dP}{dV} = \left(\frac{dI}{dV}\right)V + I \tag{5}$$

Of course, in the MPP $\frac{dP}{dV} = 0$ so that Eq. (5) becomes:

$$\left.\frac{dI}{dV}\right|_{P=P_{\max}} = -\frac{I}{V} \tag{6}$$

The Eq. (6) permits to state that in the MPP, where $V = V_{mpp}$ and $I = I_{mpp}$:

$$\left.\frac{dI}{dV}\right|_{P=P_{\max}} = -\frac{I_{mpp}}{V_{mpp}} \tag{7}$$

In other words, the slope of the tangent line to the I–V curve in the MPP is $-\frac{I_{mpp}}{V_{mpp}}$ as is possible to see in Fig. 3.

The derivative of Eq. (1) with respect to V is:

$$\frac{dI}{dV} = \frac{-nT - I_0R_{sh} \cdot e^{\frac{I \cdot R_s + V}{nT}}}{nT(R_s + R_{sh}) + I_0R_{sh} \cdot R_s \cdot e^{\frac{I \cdot R_s + V}{nT}}} \tag{8}$$

The fourth equation can be obtained substituting the Eqs. (7) into (8):

$$f_4 : \frac{-nT - I_0R_{sh} \cdot e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}}}{nT \cdot (R_s + R_{sh}) + I_0R_{sh} \cdot R_s \cdot e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}}} + \frac{I_{mpp}}{V_{mpp}} = 0 \tag{9}$$

Furthermore, knowing that the expression of the power is:

$$\begin{aligned} P &= V\left\{I_L - I_0\left(e^{\frac{V + I \cdot R_s}{nT}} - 1\right) - \frac{V + I \cdot R_s}{R_{sh}}\right\} \\ &= V \cdot I_L + V\left(1 - e^{\frac{P \cdot R_s + V^2}{V \cdot nT}}\right)I_0 - \frac{V^2 - P \cdot R_s}{R_{sh}} \end{aligned} \tag{10}$$

and that the derivative of the power with respect to the voltage is:

$$\frac{dP}{dV} = \frac{V \cdot nT(I_0R_{sh} + I_LR_{sh} - 2V) + I_0R_{sh} \cdot e^{\frac{P \cdot R_s + V^2}{V \cdot nT}}[P \cdot R_s - V(nT + V)]}{V[I_0R_sR_{sh} \cdot e^{\frac{PR_s + V^2}{V \cdot nT}} + nT(R_s + R_{sh})]} \tag{11}$$

It is possible to state that in the MPP (Fig. 4), where $V = V_{mpp}$ and $I = I_{mpp}$:

$$\left.\frac{dP}{dV}\right|_{P=P_{\max}} = 0 \tag{12}$$

The fifth equation can be obtained substituting the Eqs. (11) into (12):

$$f_5 : \frac{V_{mpp} \cdot nT(I_0R_{sh} + I_LR_{sh} - 2V_{mpp}) + I_0R_{sh} \cdot e^{\frac{I_{mpp}V_{mpp} \cdot R_s + V_{mpp}^2}{V_{mpp} \cdot nT}}[I_{mpp}V_{mpp} \cdot R_s - V_{mpp}(nT + V_{mpp})]}{V_{mpp}[I_0R_sR_{sh} \cdot e^{\frac{I_{mpp}V_{mpp}R_s + V_{mpp}^2}{V_{mpp} \cdot nT}} + nT(R_s + R_{sh})]} = 0 \tag{13}$$

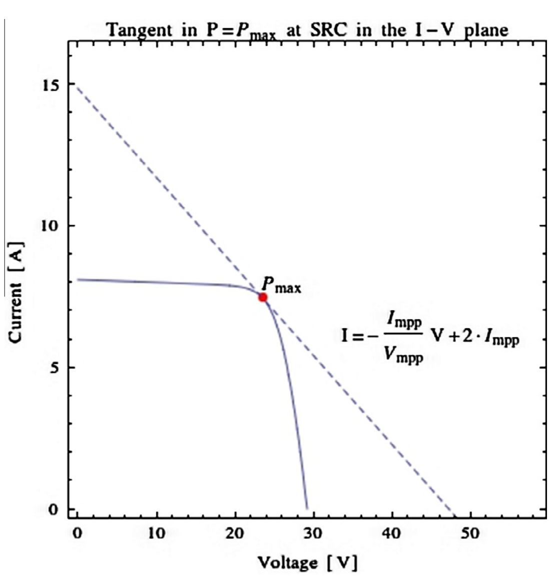

Fig. 3. Tangent line to the I–V curve in the MPP.

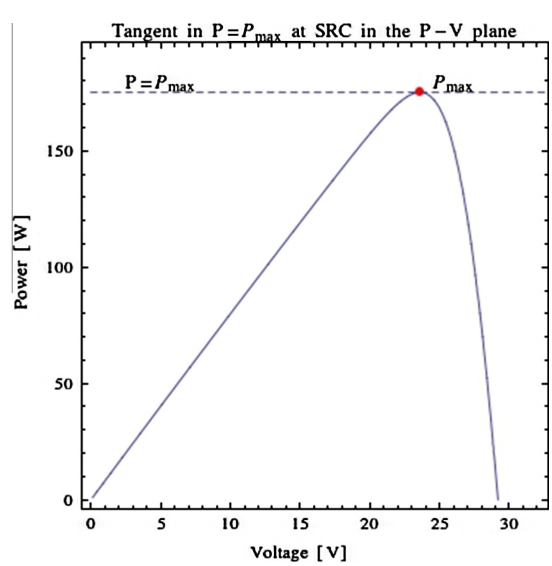

Fig. 4. Tangent line to the P–V curve in the MPP.

We have obtained five independent equations derived only from analytical relationships and from always available official data such as I_{SC}, V_{OC}, I_{mpp} and V_{mpp}. Unlike other models already available in the literature, no simplification was made. In addition, no equation that assumes knowledge of data implying the graphical evaluation of the I–V curve (for example the reciprocal of the slopes of the I–V curve in the short circuit and in the open circuit points, often called R_{sh0} and R_{s0}) or approximated relations derived from the thermal drift coefficients have been employed.

3. Evaluating the five parameters

The direct resolution of the system of equations

$$\begin{cases} f_1 = 0 \\ f_2 = 0 \\ f_3 = 0 \\ f_4 = 0 \\ f_5 = 0 \end{cases} \tag{14}$$

is not possible because of the implicit and transcendental nature of Eq. (1). The five equations above explained can be solved simultaneously to extract the five parameters using a root-finding algorithm such as the Bisection Method or the Newton's Method.

The Bisection Method is the simplest root-finding algorithm and works when f is a continuous function. It requires previous knowledge of two initial guesses, a and b, such that $f(a)$ and $f(b)$ have opposite signs. Although it is reliable, it converges slowly, gaining one bit of accuracy with each iteration. The Newton's Method assumes that the function f has a continuous derivative. The disadvantage of this method is that it may not converge if started too far away from a root. However, when it does converge, it is faster than the Bisection Method. These two methods give a general way to find numerical approximations to the system of Eq. (14).

To apply the above-mentioned root-finding algorithms, a more convenient form of the system of Eq. (14) is requested. This new form implies the direct expressions of the five unknowns starting from original equations.

The Eq. (2) can be used to obtain the photocurrent:

$$f_1 = 0 \Longleftrightarrow I_L = I_{SC} + \left(e^{\frac{I_{SC}R_s}{nT}} - 1\right)I_0 + \frac{I_{SC}R_s}{R_{sh}} \tag{15}$$

The Eq. (3) can be used to obtain the ideality factor:

$$f_2 = 0 \Longleftrightarrow nT = \frac{V_{oc}}{\ln\left(\frac{I_L}{I_0} - \frac{V_{oc}}{I_0 R_{sh}} + 1\right)} \tag{16}$$

The Eq. (4) can be used to obtain the diode reverse saturation current:

$$f_3 = 0 \Longleftrightarrow I_0 = \frac{R_{sh}(I_L - I_{mpp}) - I_{mpp}R_s - V_{mpp}}{\left(e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}} - 1\right)R_{sh}} \tag{17}$$

The Eq. (9) can be used to obtain the shunt resistance:

$$f_4 = 0 \Longleftrightarrow R_{sh} = \frac{V_{mpp}nT - I_{mpp}nT \cdot R_s}{I_{mpp}nT + e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}} I_0 R_s I_{mpp} - e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}} I_0 V_{mpp}} \tag{18}$$

Finally, the Eq. (13) can be used to obtain the series resistance:Eqs.

$$f_5 = 0 \Longleftrightarrow R_s = \frac{e^{-\frac{I_{mpp}R_s + V_{mpp}}{nT}}\left\{-I_L nT \cdot R_{sh} + 2nT \cdot V_{mpp} + I_0 R_{sh}\left[nT\left(e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}} - 1\right) + V_{mpp} e^{\frac{I_{mpp}R_s + V_{mpp}}{nT}}\right]\right\}}{I_{mpp} I_0 R_{sh}} \tag{19}$$

(15)–(19) can be used to apply a root-finding algorithm such as the Bisection Method or the Newton's Method. On the other hand, these methods are not enough robust: they are generally slow, not easy to code in software, may not converge over real solutions and their accuracy depends on how much closer are the roots with respect to initial guesses.

Another more efficient and faster procedure to solve simultaneously the five equations is to transform this problem as a constrained non-linear optimization problem. Given the equations:

$$F_1(I_L, I_0, R_s, R_{sh}, n) = 0, \ldots, f_5(I_L, I_0, R_s, R_{sh}, n) = 0$$

we can form the following objective function:

$$\Gamma(I_L, I_0, R_s, R_{sh}, n) = \sum_{i=1}^{5} f_i^2(I_L, I_0, R_s, R_{sh}, n) \tag{20}$$

Now it is simple to feed the Eq. (20) to a local minimum-finding algorithm, by starting at a random point and going through its neighbors. The initial guesses of I_L, I_0, R_s, R_{sh}, n may be chosen from a range of arbitrary values also relatively large and the procedure can be repeated until the residuals for the equations $f_1(I_L, I_0, R_s, R_{sh}, n) = 0, \ldots, f_5(I_L, I_0, R_s, R_{sh}, n) = 0$ are practically zero. An automatic procedure to facilitate the use of a non-linear optimization solver is nowadays available even in any spreadsheet software (also open source) and it does not require coding. These solvers generally use the Generalized Reduced Gradient (GRG) algorithm that is one of the most robust non-linear programming methods to solve optimization problem [37]. The adoption of this method allows to easily solve the system of Eq. (14) also using a common office application without writing a line of code.

Other numerical methods such as Newton–Raphson or Bisection Method could have some difficulties when applied to the problem of cell/module parameters extraction. Indeed, in this case, these methods must be applied consecutively to at least four equations. The initial conditions can lead the method to overshoot the root of each considered equation, then particular attention must be paid to the choice of initial values and to the progress of the calculation step. A large error in the initial estimate can contribute to non-convergence of the algorithm. These distinctive features require writing an "ad hoc" routine within a specialized software such as MATLAB or MATHEMATICA. The progress of the procedure

Table 1
Official data used to evaluate the five parameters of Kyocera KC175GHT-2 PV panel.

	V_{OC} (V)	I_{SC} (A)	V_{mpp} (V)	I_{mpp} (A)	$P_{max} = I_{mpp}V_{mpp}$ (W)
Temperature 25 °C Irradiance 1000 W/m²					
Kyocera issued values	29.2	8.09	23.60	7.42	175.112
Sanyo issued values	43.6	7.37	35.5	6.77	240.335

Table 2
Extracted values of the five parameters of Kyocera KC175GHT-2 and Sanyo HIT240HDE-4 modules.

	Kyocera	Sanyo
Temperature 25 °C		
Irradiance 1000 W/m²		
$\overline{I_0}$ (A)	$1.0660002452777384 \times 10^{-10}$	$8.258066972347851 \times 10^{-11}$
$\overline{I_L}$ (A)	8.117544842200639	7.392484839903704
$\overline{R_s}$ (Ω)	0.2836273332359883	0.4249742330120292
$\overline{R_{sh}}$ (Ω)	83.30217191557375	139.29652910089868
$\overline{nT}$ (V)	1.1674478842012481	1.7319149442241

must therefore be controlled and modified for each examined cell/module.

The advantage of the proposed GRG based procedure consists in using a single goal equation. Furthermore, the GRG is a resolving numerical algorithm reliable and widely available in open-source software. Moreover, in this case, a further advantage is that its application does not require any type of coding step and it can be implemented even in an Excel worksheet. Nevertheless, to achieve more precise results more sophisticated mathematical tools, capable of controlling the effective number of digits of precision in internal computation is suggested.

The above-described procedure is now applied to extract the five parameters values of two different modules: the Kyocera KC175GHT-2 and the Sanyo HIT240HDE-4. The official standard data of these panels can be retrieved from Photon database or SAM database or official data-sheet and they are reported in Table 1.

The GRG algorithm applied to these modules permits to obtain in few seconds the following results (see Table 2):

This is indeed a really good solution as the residuals are practically zero as showed in Table 3:

Finally, in Table 4 are reported the differences between calculated and issued data to test how the calculated *I–V* curve fits the issued data.

Results in Table 4 show a perfect fit with the official data.

Furthermore, we tested the proposed procedure on five anonymous PV panels whose experimental data were kindly provided by TÜV Rheinland PTL (Table 5), a world-renowned facility for PV technology testing located in Tempe, Arizona, USA. TÜV Rheinland PTL, is an ISO 17025 accredited facility by A2LA, the American Association for Laboratory Accreditation. They perform several tests of many critical standards including IEC 61215, IEC 61646, IEC 61730, IEC 62108, ANSI/UL 1703, PowerMark and California Energy Commission. In Table 6 are reported the extracted five parameters with the proposed procedure, of the respectively five PV panels described in Table 5.

Results in Tables 7 and 8 clearly show how the proposed model and the proposed resolution allow obtaining performances that no other previous five parameters models can achieve. The quality of the results and therefore the reliability of the model are due to the absence of any simplification and to the fully analytical development of mathematical equations derived from official and public data. Moreover, the suggested procedure of numerical solution of a local minimum problem allows converging towards the solution with the desired accuracy in a fast and effective way.

4. Temperature and irradiance impact

As a matter of fact it is necessary to make the model also able to represent the *I–V* characteristics in correspondence of irradiance and temperature values far from the STC. For this reason, the model must be sensitive to irradiance and temperature changes. This is an important problem already addressed in [14,38] by means of the following improved form of Eq. (1):

$$I(\alpha_G, T) = \alpha_G I_L(T) - I_0(\alpha_G, T)\left(e^{\frac{\alpha_G\left[V+K\cdot I(T-T_{ref})\right]+I\cdot R_s}{\alpha_G nT}} - 1\right) + \frac{\alpha_G\left[V + K\cdot I(T - T_{ref})\right] + I\cdot R_s}{\alpha_G R_{sh}} \quad (21)$$

in which the quantity $\alpha_G = \frac{G}{G_{ref}}$ denotes the ratio between the generic solar irradiance and the irradiance at STC (1000 W/m²) and K is a thermal correction factor similar to the curve correction factor described by the IEC 891.

The K factor can be calculated with the formal imposition that the *I–V* characteristic evaluated with Eq. (21) at G_{ref} and at the desired temperature T^* must contain the maximum power point issued by the manufacturer for the same T^*:

$$K = \frac{V_{mpp} - V^*_{mpp}}{I^*_{mpp}(T^* - T_{ref})} \quad (22)$$

The value of T^* to be used to determine K should be chosen by considering the maximum or the minimum expected working temperature of the PV module.

Table 3
Residuals obtained with the extracted values of the five parameters for Kyocera KC175GHT-2 and Sanyo HIT240HDE-4 modules.

Function	Residual (Kyocera)	Residual (Sanyo)
$\Gamma(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	$1.8371212627364313 \times 10^{-24}$	$2.18007529937378 \times 10^{-22}$
$f_1(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	$1.0935696792557792 \times 10^{-14}$	$1.6653345369377348 \times 10^{-16}$
$f_2(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	0 (less than 10^{-30})	0 (less than 10^{-30})
$f_3(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	0 (less than 10^{-30})	$-8.881784197001252 \times 10^{-16}$
$f_4(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	$5.551115123125783 \times 10^{-17}$	$-5.551115123125783 \times 10^{-17}$
$f_5(\overline{I_L}, \overline{I_0}, \overline{R_s}, \overline{R_{sh}}, \overline{n})$	$-3.637978807091713 \times 10^{-12}$	$-1.4551915228366852 \times 10^{-11}$

Table 4
Differences between issued and calculated data by using the extracted values of the five parameters for Kyocera KC175GHT-2 and Sanyo HIT240HDE-4 modules.

	V_{OC} (V)	I_{SC} (A)	V_{mpp} (V)	I_{mpp} (A)	$P_{max} = I_{mpp}V_{mpp}$ (W)
Temperature 25 °C					
Irradiance 1000 W/m²					
Issued values (Kyocera)	29.2	8.09	23.60	7.42	175.112
Calculated (Kyocera)	29.2	8.09	23.59999	7.42002	175.1123978
Difference (Kyocera)	0	0	0.00001	0.00002	0.0003978
Issued values (Sanyo)	43.6	7.37	35.5	6.77	240.335
Calculated (Sanyo)	43.6	7.37	35.4998	6.77004	240.335
Difference (Sanyo)	0	0	0.0002	0.00004	0

Table 5
TÜV Rheinland PTL experimental data used to evaluate the five parameters of five anonymous PV panels.

	I_{SC} (A)	V_{OC} (V)	I_{mpp} (A)	V_{mpp} (V)	$P_{max} = I_{mpp}V_{mpp}$ (W)
Temperature 25 °C					
Irradiance 1000 W/m²					
Panel 1 (mono)	8.63	45.91	8.09	37.21	301.0289
Panel 2 (mono)	8.53	44.98	7.98	36.65	292.467
Panel 3 (poly)	8.69	44.67	8.05	34.70	279.335
Panel 4 (poly)	8.06	43.89	7.57	34.51	261.2407
Panel 5 (poly)	8.70	44.66	8.15	35.93	292.8295

Table 6
extracted five parameters of the five anonymous PV panels.

	$\overline{I_L}$ (A)	$\overline{I_0}$ (A)	$\overline{R_s}$ (Ω)	$\overline{R_{sh}}$ (Ω)	$\overline{nT}$ (V)
Temperature 25 °C					
Irradiance 1000 W/m²					
Panel 1 (mono)	8.64098	7.43943×10^{-11}	0.407507	320.269	1.80312
Panel 2 (mono)	8.57000	3.77791×10^{-18}	0.581457	123.988	1.06531
Panel 3 (poly)	8.76743	1.20593×10^{-19}	0.822487	92.306	0.977972
Panel 4 (poly)	8.07801	9.18612×10^{-13}	0.643256	287.849	1.47351
Panel 5 (poly)	8.73089	5.66977×10^{-15}	0.55971	157.626	1.27828

Table 7
Residual obtained with the extracted values of the five parameters of the five anonymous PV panels.

	$\overline{\Gamma}$	$\overline{f_1}$	$\overline{f_2}$	$\overline{f_3}$	$\overline{f_4}$	$\overline{f_5}$
Panel 1 (mono)	5.4609×10^{-25}	0	-4.21885×10^{-15}	-3.55271×10^{-15}	-8.32667×10^{-17}	0
Panel 2 (mono)	1.3333×10^{-24}	0	-1.17683×10^{-14}	-1.77635×10^{-15}	0	-1.45519×10^{-11}
Panel 3 (poly)	4.66674×10^{-24}	0	3.05311×10^{-14}	0	-2.77555×10^{-17}	-7.2759×10^{-12}
Panel 4 (poly)	6.33312×10^{-25}	0	1.38777×10^{-15}	8.88178×10^{-16}	-5.5511×10^{-17}	0
Panel 5 (poly)	1.46388×10^{-26}	0	-6.8833×10^{-15}	0	2.77555×10^{-17}	1.45519×10^{-11}

Table 8
Differences between issued and calculated data by using the extracted values of the five parameters for five unknown panels.

	I_{SC} (A)	V_{OC} (V)	I_{mpp} (A)	V_{mpp} (V)	$P_{max} = I_{mpp}V_{mpp}$ (W)
Temperature 25 °C					
Irradiance 1000 W/m²					
Panel 1 (mono)					
Issued values	8.63	45.91	8.09	37.21	301.0289
Calculated	8.63	45.91	8.09	37.21	301.029
Difference	0	0	0	0	0.0001
Panel 2 (mono)					
Issued values	8.53	44.98	7.98	36.65	292.467
Calculated	8.53	44.98	7.98	36.65	292.467
Difference	0	0	0	0	0
Panel 3 (poly)					
Issued values	8.69	44.67	8.05	34.70	279.335
Calculated	8.69	44.67	8.05	34.7	279.335
Difference	0	0	0	0	0
Panel 4 (poly)					
Issued values	8.06	43.89	7.57	34.51	261.2407
Calculated	8.06	43.89	7.57	34.51	261.241
Difference	0	0	0	0	0.0003
Panel 5 (poly)					
Issued values	8.70	44.66	8.15	35.93	292.8295
Calculated	8.7	44.66	8.14999	35.9301	292.83
Difference	0	0	0.00001	0.0001	0.0005

The photocurrent $I_L(T)$ can be evaluated with the following equation:

$$I_L(T) = I_{L,ref} + \mu_{I,sc}(T - T_{ref}) \tag{23}$$

where $I_{L,ref}$ is the value of the photocurrent at STC and $\mu_{I_{sc}}$ is the short circuit current temperature coefficient.

Table 9
Values of thermal drift coefficients of Kyocera KC175GHT-2 and Sanyo HIT240HDE-4 modules.

Coefficient	Kyocera	Sanyo
$\mu_{V_{oc}}$	$-0.1089\,[\frac{V}{°C}]$	$-0.109\,[\frac{V}{°C}]$
$\mu_{I_{sc}}$	$3.17937 \times 10^{-3}\,[\frac{A}{°C}]$	$2.21 \times 10^{-3}\,[\frac{A}{°C}]$
K	$-0.8753 \times 10^{-3}\,[\frac{V}{A \cdot °C}]$	$-1.12679 \times 10^{-3}\,[\frac{V}{A \cdot °C}]$

The current $I_o(\alpha_G, T)$ can be obtained by the following expression:

$$I_0(\alpha_G, T) = \alpha_G \left[\frac{I_L(T) - V_{oc}(\alpha_G, T)/R_{sh}}{e^{V_{oc}(\alpha_G, T)/nT} - 1} \right] \tag{24}$$

by interpolating, in a logarithmic mode, the value of $I_0(1, T)$ (for $G = 1000$ W/m^2) and $I_0(0.2, T)$ (for $G = 200$ W/m^2):

$$I_0(\alpha_G, T) = \exp\left[\left(\frac{\alpha_G - 0.2}{1 - 0.2} \right) \ln \frac{I_0(1, T)}{I_0(0.2, T)} + \ln I_0(0.2, T) \right] \tag{25}$$

The variation of the open circuit voltage V_{OC} and short circuit current I_{SC} with solar irradiance and temperature can be evaluated using the following expressions:

$$V_{OC}(\alpha_G, T) = V_{OC,ref}(\alpha_G) + \mu_{V_{OC}}(T - T_{ref}) \tag{26}$$

$$I_{SC} = I_{sc,ref} \cdot \alpha_G + \mu_{I_{SC}}(T_c - T_{ref}) \tag{27}$$

in which $V_{OC,ref}$ and $I_{SC,ref}$ are respectively the open circuit voltage and short circuit current at STC, $\mu_{V_{OC}}$ is the open circuit voltage temperature coefficients.

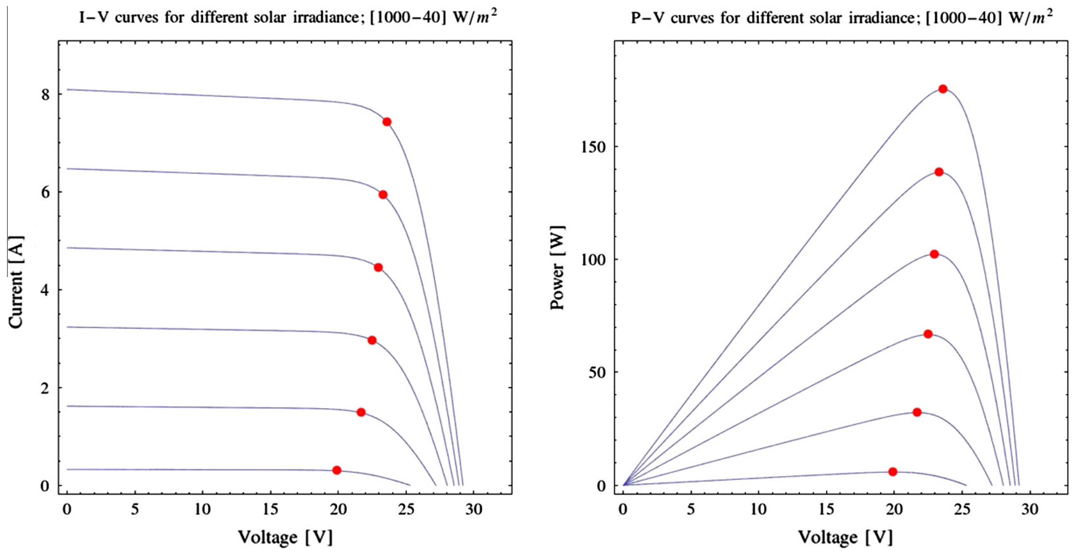

Fig. 5. Calculated *I–V* and *P–V* curves of Kyocera KC175GHT-2 at constant temperature (25 °C) varying solar irradiance.

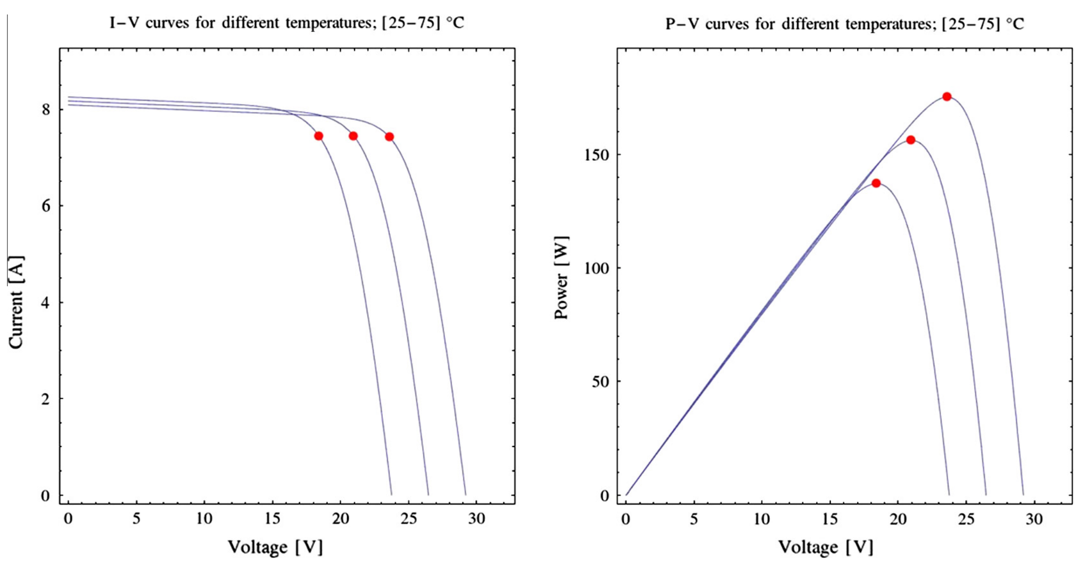

Fig. 6. Calculated *I–V* and *P–V* curves of Kyocera KC175GHT-2 at constant solar irradiance (1000 W/m^2) varying temperature.

Table 10
Calculated characteristic data values of Kyocera KC175GHT-2 at constant temperature (25 °C) varying solar irradiance.

	Solar irradiance (W/m^2)					
	1000	800	600	400	200	40
P_{mpp} (W)	175.112	138.418	102.236	66.7111	32.1507	5.8841
V_{mpp} (V)	23.6	23.3241	22.977	22.501	21.7091	19.9224
I_{mpp} (A)	7.41999	5.93455	4.4495	2.9648	1.48098	0.295351
V_{OC} (V)	29.2	28.9395	28.6036	28.1303	27.3211	25.4421
I_{SC} (A)	8.09	6.472	4.854	3.236	1.618	0.3236

Table 11
Calculated characteristic data values of Kyocera KC175GHT-2 at constant solar irradiance (1000 W/m^2) varying temperature.

	Cell temperature (°C)		
	25	50	75
P_{mpp} (W)	175.112	156.099	137.09
V_{mpp} (V)	23.6	20.975	18.4243
I_{mpp} (A)	7.41999	7.44215	7.44069
V_{OC} (V)	29.2	26.4771	23.7542
I_{SC} (A)	8.09	8.16948	8.24897

To better understand as the extraction of the five parameters can be used to retrieve the generic *I–V* curve at any condition of *G* and *T*, Eq. (21) is now applied to the two different silicon modules previously described in Section 3 (Kyocera KC175GHT-2 and Sanyo HIT240HDE-4). In Table 9 are indicated the thermal drift coefficients of the two modules.

In detail, once calculated the values of the five parameters, the proposed model permits to obtain the open circuit voltage, the maximum power output, the short circuit current as well as the whole representations of *I–V* and *P–V* curves for any different values of solar irradiance and operative temperature. For the Kyocera KC175GHT-2 module Figs. 5 and 6 show the *I–V* and *P–V* curves for different values of solar irradiance and operative temperature; Tables 10 and 11 collect the characteristic data values. Red circles identify the Maximum Power points.

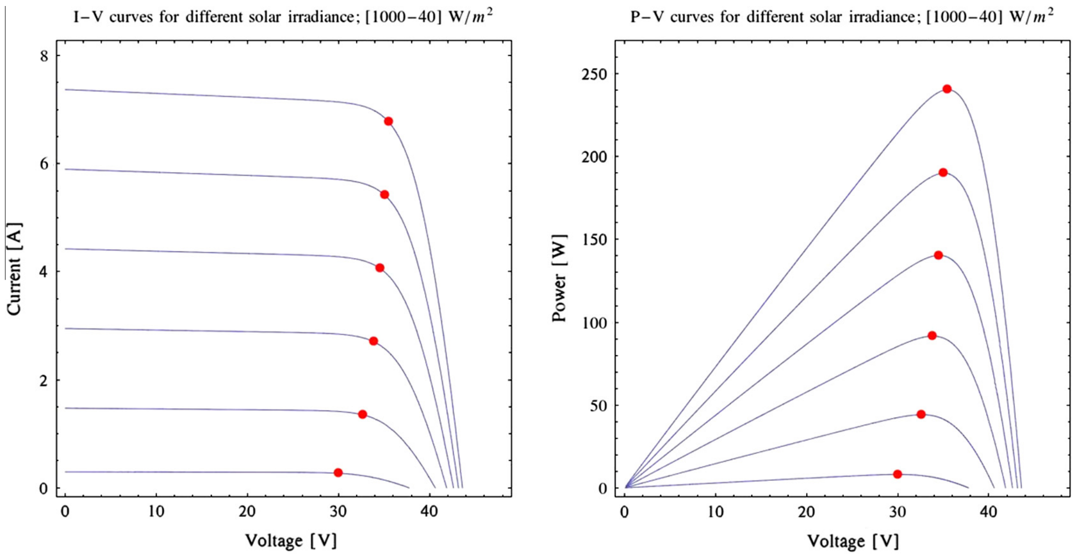

Fig. 7. Calculated *I–V* and *P–V* curves of Sanyo HIT240HDE-4 at constant temperature (25 °C) varying solar irradiance.

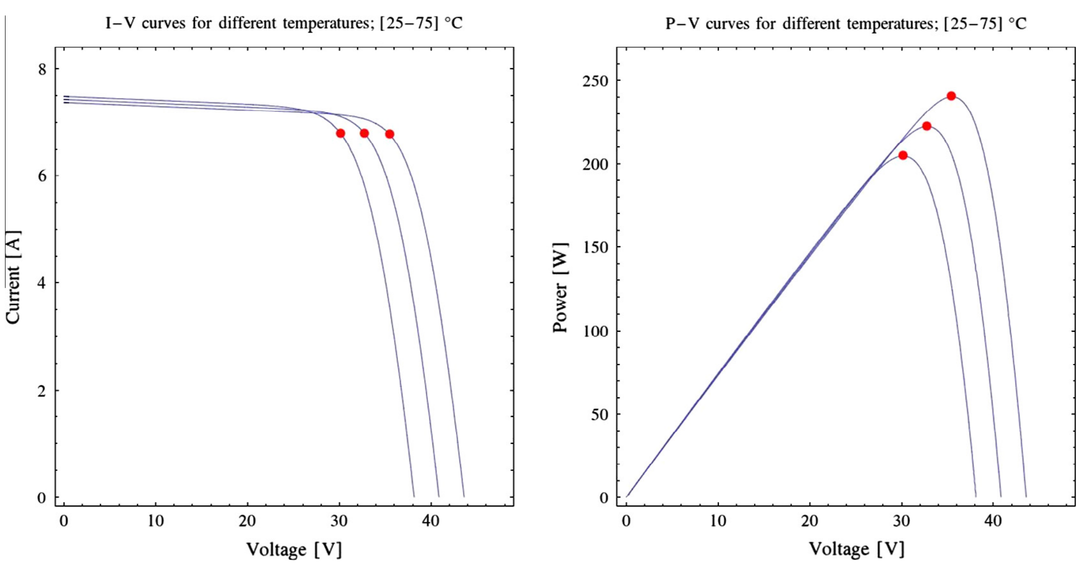

Fig. 8. Calculated *I–V* and *P–V* curves Sanyo HIT240HDE-4 at constant solar irradiance (1000 W/m^2) varying temperature.

Table 12
Calculated characteristic data values of Sanyo HIT240HDE-4 at constant temperature (25 °C) varying solar irradiance.

	Solar irradiance (W/m^2)					
	1000	800	600	400	200	40
P_{mpp} (W)	240.335	189.975	140.323	91.5759	44.1493	8.08932
V_{mpp} (V)	35.4998	35.0848	34.5641	33.8511	32.6691	30.0095
I_{mpp} (A)	6.77004	5.41475	4.0598	2.70526	1.35141	0.269559
V_{OC} (V)	43.6	43.2135	42.7153	42.0131	40.8126	38.0252
I_{SC} (A)	7.37	5.896	4.422	2.948	1.474	0.2948

Table 13
Calculated characteristic data values of Sanyo HIT240HDE-4 at constant solar irradiance (1000 W/m^2) varying temperature.

	Cell temperature (°C)		
	25	50	75
P_{mpp} (W)	240.335	222.492	204.759
V_{mpp} (V)	35.4998	32.7991	30.1697
I_{mpp} (A)	6.77004	6.78349	6.78691
V_{OC} (V)	43.6	40.875	38.15
I_{SC} (A)	7.37	7.42525	7.4805

Similarly, for the Sanyo HIT240HDE-4 module Figs. 7 and 8 show the *I–V* and *P–V* curves for different values of solar irradiance and operative temperature; Tables 12 and 13 collect the characteristic data values.

5. Conclusions

In this paper, the authors have presented a new five parameters model for descripting the *I–V* curve of a photovoltaic panel. The proposed model, as opposed to those already known in the literature, is exclusively based on analytical relationships and it is free of any simplifications that may affect the reliability of the results. The model uses only technical data that are always available in the official data-sheets or in the most popular and authoritative PV devices database. Concerning the numerical solution of the system of transcendental equations, two possible solutions are indicated: the resolution with root finding algorithms and a more advanced based on optimization algorithms. To facilitate the application of the root-finding method, a more useful form of the equations to be directly used is issued. As regards the transformation into an optimization problem, the expression of the objective function is provided. The model was finally tested on two modules whose data were taken from official data-sheets. Furthermore, the procedure was applied to five anonymous PV panels with electrical data kindly provided by TÜV Rheinland PTL.

The results clearly show that the absence of any simplifications or inaccurate data coming from graphical or empirical analyses together with a new more efficient approach to solve the equation system allows obtaining *I–V* and *P–V* curves that always perfectly fit the technical data for any value of cell temperature and solar irradiance.

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Appendix A. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.apenergy.2013.06.046.

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