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A heuristic particle swarm optimization method for truss structures with discrete variables

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ABSTRACT

A heuristic particle swarm optimizer (HPSO) algorithm for truss structures with discrete variables is presented based on the standard particle swarm optimizer (PSO) and the harmony search (HS) scheme. The HPSO is tested on several truss structures with discrete variables and is compared with the PSO and the particle swarm optimizer with passive congregation (PSOPC), respectively. The results show that the HPSO is able to accelerate the convergence rate effectively and has the fastest convergence rate among these three algorithms. The research shows the proposed HPSO can be effectively used to solve optimization problems for steel structures with discrete variables.

1. Introduction

In the past 30 years, many algorithms have been developed to solve the structural engineering optimization problems. Most of these algorithms are based on the assumption that the design variables are continuously valued and the gradients of functions and the convexity of the design problem satisfied. However, in reality, the design variables of optimization problems such as the cross-section areas are discretely valued. They are often chosen from a list of discrete variables. Furthermore, the function of the problems is hard to express in an explicit form. Traditionally, the discrete optimization problems are solved by mathematical methods by employing round-off techniques based on the continuous solutions. However, the solutions obtained by this method may be infeasible or far from the optimum solutions [1].

Recently, some papers, published on the subject of the structural engineering optimization, are about the evolutionary algorithms such as the genetic algorithm (GA) [2], the particle swarm optimizer algorithm (PSO) [3,4] and other stochastic search techniques based on natural phenomena [5]. The PSO algorithm has fewer parameters and easier to implement, and it has shown a fast convergence rate than other evolutionary algorithms for some problems [6]. Most of the applications of the PSO algorithm to structural optimization problems are based on the assumption that the variables are continuous. Only in few papers PSO algorithm is used to solve the discrete structural optimization problems [7,8].

This paper presents a heuristic particle swarm optimizer (HPSO) algorithm, which is based on the standard particle swarm optimize (PSO) and the harmony search (HS) scheme, and is applied to the discrete valued structural optimization problems. The HPSO algorithm has all the advantages that belong to the PSO and the PSOPC algorithms. Furthermore, it has faster convergence rate than the PSO and the PSOPC algorithms, especially in the early iterations [9].

This paper introduces the formulation of the discrete valued optimization problems in Section 2. The PSO and the PSOPC algorithms for the discrete valued variables are presented in Sections 3 and 4, respectively. The HPSO algorithm for the discrete valued variables is introduced in Section 5, and it is tested on several examples in Section 6. Section 7 concludes this paper.

2. Mathematical model for discrete structural optimization problems

A structural optimization design problem with discrete variables can be formulated as a nonlinear programming problem. In the size optimization for a truss structure, the cross-section areas of the truss members are selected as the design variables. Each of the design variables is chosen from a list of discrete cross-sections based on production standard. The objective function is the structure weight. The design cross-sections must also satisfy some inequality constraints equations, which restrict the discrete variables. The optimization design problem for discrete variables can be expressed as follows:

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$$\begin{aligned} \text{min} \quad & f(x^1, x^2, \ldots, x^d), \quad d = 1, 2, \ldots, D \\ \text{subjected to}: \quad & g_q(x^1, x^2, \ldots, x^d) \leqslant 0, \quad d = 1, 2, \ldots, D, \\ & q = 1, 2, \ldots, M \\ & x^d \in S_d = \{X_1, X_2, \ldots, X_p\} \end{aligned}$$

where $f(x^1,x^2,\ldots,x^d)$ is the truss's weight function, which is a scalar function. And $x^1,x^2,\ldots,x^d$ represent a set of design variables. The design variable x^d belongs to a scalar S_d, which includes all permissive discrete variables $\{X_1,X_2,\ldots,X_p\}$. The inequality $g_q(x^1,x^2,\ldots,x^d) \leqslant 0$ represents the constraint functions. The letter D and M are the number of the design variables and inequality functions, respectively. The letter p is the number of available variables.

3. The particle swarm optimizer (PSO) algorithm for discrete variables

The PSO algorithm was first invented by Kennedy and Eberhart [6]. It is a population-based algorithm with fewer parameters to implement. The PSO algorithm is first applied to the optimization problems with continuous variables. Recently, it has been used to the optimization problems with discrete variables [7]. The optimization problem with discrete variables is a combination optimization problem which obtains its best solution from all possible variable combinations. The scalar S includes all permissive discrete variables arranged in ascending sequence. Each element of the scalar S is given a sequence number to represent the value of the discrete variable correspondingly. It can be expressed as follows:

$$S_d = \{X_1, X_2, \ldots, X_j, \ldots X_p\}, \quad 1 \leqslant j \leqslant p$$

A mapped function $h(j)$ is selected to index the sequence numbers of the elements in set S and represents the value X_j of the discrete variables correspondingly.

$$h(j) = X_j$$

Thus, the sequence numbers of the elements will substitute for the discrete values in the scalar S. This method is used to search the optimum solution, and makes the variables to be searched in a continuous space.

The PSO algorithm includes a number of particles, which are initialized randomly in the search space. The position of the ith particle in the space can be described by a vector x_i,

$$x_i = (x_i^1, x_i^2, \ldots, x_i^d, \ldots, x_i^D), \quad 1 \leqslant d \leqslant D, \quad i = 1, \ldots, n$$

where D is the dimension of the particle, and n is the sum of all particles. The scalar $x_i^d \in \{1, 2, \ldots, j, \ldots, p\}$ corresponds to the discrete variable set $\{X_1,X_2,\ldots,X_j,\ldots,X_p\}$ by the mapped function $h(j)$. Therefore, the particle flies through the continuous space, but only stays at the integer space. In other words, all the components of the vector x_i are integer numbers. The positions of the particles are updated based on each particle's personal best position as well as the best position found by the swarm at each iteration. The objective function is evaluated for each particle and the fitness value is used to determine which position in the search space is the best of the others. The swarm is updated by the following equations:

$$V_i^{(k+1)} = \omega V_i^{(k)} + c_1 r_1 (P_i^{(k)} - x_i^{(k)}) + c_2 r_2 (P_g^{(k)} - x_i^{(k)}) \tag{1}$$

$$x_i^{(k+1)} = INT(x_i^{(k)} + V_i^{(K+1)}) \tag{2}$$

$$1 \leqslant i \leqslant n$$

where $x_i^{(k)}$ and $V_i^{(k)}$ represent the current position and the velocity of each particle at the kth iteration, respectively, $P_i^{(k)}$ the best previous position of the ith particle (called *pbest*) and $P_g^{(k)}$ the best global position among all the particles in the swarm (called *gbest*), r_1 and r_2 are two uniform random sequences generated from $U(0,1)$, and ω the inertia weight used to discount the previous velocity of the particle persevered [10]. The object function and the constraint functions can be expressed by the scalar x_i^d as follows:

$$f(h(x_i^1), h(x_i^2), \ldots, h(x_i^d), \ldots, h(x_i^D))$$
$$g_q(h(x_i^1), h(x_i^2), \ldots, h(x_i^d), \ldots, h(x_i^D))$$

4. The PSOPC algorithm for discrete valued variables

It is known that the PSO may outperform other evolutionary algorithms in the early iterations, but its performance may not be competitive when the number of the iterations increases [11]. He and Wu have improved the particle swarm optimizer with passive congregation (PSOPC), which can improve the convergence rate and accuracy of the PSO efficiently [12]. The PSOPC algorithm is first used in optimization problems with continuous variables [13]. By modification it can be used to the optimization problems with discrete variables [8,14]. The formulations of the PSOPC algorithm for discrete variables can be expressed as follows:

$$\begin{aligned} V_i^{(k+1)} = {} & \omega V_i^{(k)} + c_1 r_1 (P_i^{(k)} - x_i^{(k)}) + c_2 r_2 (P_g^{(k)} - x_i^{(k)}) \\ & + c_3 r_3 (R_i^{(k)} - x_i^{(k)}) \end{aligned} \tag{3}$$

$$x_i^{(k+1)} = INT(x_i^{(k)} + V_i^{(k+1)}) \tag{4}$$

$$1 \leqslant i \leqslant n$$

where R_i is a particle selected randomly from the swarm, c_3 the passive congregation coefficient, and r_3 a uniform random sequence in the range $(0,1)$: $r_3 - U(0,1)$.

5. The heuristic particle swarm optimizer (HPSO) for discrete variables

The heuristic particle swarm optimizer (HPSO) algorithm, which is based on the PSOPC algorithm and the harmony search (HS) scheme, is introduced by Li [9] and is first used in continuous variable optimization problems. The HPSO algorithm presented by Li [9] makes it possible that the particle meets the demand of constraints' boundary or the variables' boundary for each fly. Similarly, The HPSO algorithm for the discrete valued variables can be expressed as follows:

$$\begin{aligned} V_i^{(k+1)} = {} & \omega V_i^{(k)} + c_1 r_1 (P_i^{(k)} - x_i^{(k)}) + c_2 r_2 (P_g^{(k)} - x_i^{(k)}) \\ & + c_3 r_3 (R_i^{(k)} - x_i^{(k)}) \end{aligned} \tag{5}$$

$$x_i^{(k+1)} = INT(x_i^{(k)} + V_i^{(k+1)}) \tag{6}$$

$$1 \leqslant i \leqslant n$$

where x_i is the vector of a particle's position, and x_i^d is one component of this vector. After the (k + 1)th iterations, if $x_i^d < x^d(LowerBound)$ or $x_i^d > x^d(UpperBound)$, the scalar x_i^d is regenerated by selecting the corresponding component of the vector from *pbest* swarm randomly, which can be described as follows:

$$x_i^d = (P_b)_t^d, \quad t = INT(rand(1, n))$$

where $(P_b)_t^d$ denotes the dth dimension scalar of *pbest* swarm of the tth particle, and t denotes a random integer number. The pseudocode for the HPSO algorithm is listed in Table 1.

6. Numerical examples

In this section, the HPSO algorithm is tested by five truss structures. The algorithm proposed is coded in FORTRAN language and executed on a Pentium 4, 2.93 GHz machine.

The PSO, the PSOPC and the HPSO algorithms for discrete variables are applied to all these examples and the results are compared in order to evaluate the performance of the HPSO

Table 1
The pseudo-code for HPSO

Set $k = 1$;
Randomly initialize positions and velocities of all particles;
FOR (each particle i in the initial population)
WHILE (the constraints are violated)
Randomly re-generate the current particle x_i
END WHILE
END FOR
WHILE (the termination conditions are not met)
FOR (each particle i in the swarm)
Generate the velocity and update the position of the current particle (vector) $x_i^{(k)}$
Check feasibility stage I: Check whether each component of the current vector violates its corresponding boundary or not (Check whether each cross-section is one of the given cross-sections). If it does, select the corresponding component of the vector from *pbest swarm* randomly.
Check feasibility stage II: Check whether the current particle violates the problem specified constraints or not (Check whether the response of the structure with the selected cross-sections under the external loads violate the design constrains). If it does, reset it to the previous position $x_i^{(k-1)}$
Calculate the fitness value $f(x_i^{(k)})$ of the current particle.
Update *pbest*: Compare the fitness value of *pbest* with $f(x_i^{(k)})$. If the $f(x_i^{(k)})$ is better than the fitness value of *pbest*, set *pbest* to the current position $x_i^{(k)}$.
Update *gbest*: Find the global best position in the swarm. If the $f(x_i^{(k)})$ is better than the fitness value of *gbest*, *gbest* is set to the position of the current particle $x_i^{(k)}$.
END FOR
Set $k = k+1$
END WHILE

algorithm for discrete variables. For all these algorithms, a population of 50 individuals are used, the inertia weight ω, which starts at 0.9 and ends at 0.4, decreases linearly, and the value of acceleration constants c_1 and c_2 are set to 0.5 [15]. The passive congregation coefficient c_3 is set to 0.6 for the PSOPC and the HPSO algorithms. All these truss structures have been analyzed by the finite element method (FEM). The maximum velocity is set as the difference between the upper and the lower bounds, which ensures that the particles are able to fly across the problem-specific constraints' region. Different iteration numbers are used for different optimization structures, with smaller iteration number for smaller variable number structures and larger one for large variable number structures.

6.1. A 10-bar planar truss structure

A 10-bar truss structure, shown in Fig. 1 [14], has previously been analyzed by many researchers, such as Wu [2], Rajeev [16], and Ringertz [17]. The material density is 0.1 lb/in.^{3} and the modulus of elasticity is 10,000 ksi. The members are subjected to stress limitations of ±25 ksi. All nodes in both directions are subjected to displacement limitations of ±2.0 in. $P_1 = 10^5$ lbs, $P_2 = 0$. There are 10 design variables and two load cases in this example to be optimized. For case 1: the discrete variables are selected from the set $D = \{1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50\}$ (in.^{2}); for case 2: the discrete variables are selected from the set $D = \{0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 7.5, 8.0, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20.0, 20.5, 21.0, 21.5, 22.0, 22.5, 23.0, 23.5, 24.0, 24.5, 25.0, 25.5, 26.0, 26.5, 27.0, 27.5, 28.0, 28.5, 29.0, 29.5, 30.0, 30.5, 31.0, 31.5\}$ (in^2). A maximum iteration of 1000 steps is imposed.

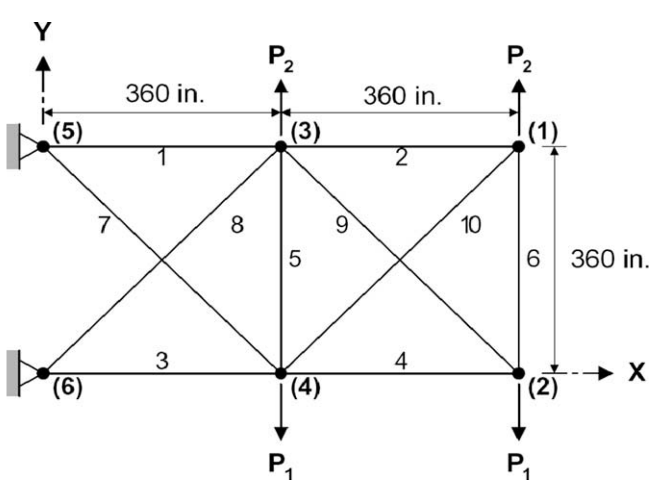

Fig. 1. A 10-bar planar truss structure.

Tables 2 and 3 give the comparison of optimal design results for the 10-bar planar truss structure under two load cases, respectively. Figs. 2 and 3 show the comparison of convergence rates for the 10-bar truss structure. From Tables 2 and 3, we find the re-

Table 2
Comparison of optimal designs for the 10-bar planar truss structure (case 1).

Variables (in.^{2})	Wu [2]	Rajeev [16]	PSO	PSOPC	HPSO
A_1	26.50	33.50	30.00	30.00	30.00
A_2	1.62	1.62	1.62	1.80	1.62
A_3	16.00	22.00	30.00	26.50	22.90
A_4	14.20	15.50	13.50	15.50	13.50
A_5	1.80	1.62	1.62	1.62	1.62
A_6	1.62	1.62	1.80	1.62	1.62
A_7	5.12	14.20	11.50	11.50	7.97
A_8	16.00	19.90	18.80	18.80	26.50
A_9	18.80	19.90	22.00	22.00	22.00
A_{10}	2.38	2.62	1.80	3.09	1.80
Weight (lb)	4376.20	5613.84	5581.76	5593.44	5531.98

Table 3
Comparison of optimal designs for the 10-bar planar truss structure (case 2).

Variables (in.^{2})	Wu [2]	Ringertz [17]	PSO	PSOPC	HPSO
A_1	30.50	30.50	24.50	25.50	31.50
A_2	0.50	0.10	0.10	0.10	0.10
A_3	16.50	23.00	22.50	23.50	24.50
A_4	15.00	15.50	15.50	18.50	15.50
A_5	0.10	0.10	0.10	0.10	0.10
A_6	0.10	0.50	1.50	0.50	0.50
A_7	0.50	7.50	8.50	7.50	7.50
A_8	18.00	21.0	21.50	21.50	20.50
A_9	19.50	21.5	27.50	23.50	20.50
A_{10}	0.50	0.10	0.10	0.10	0.10
Weight (lb)	4217.30	5059.9	5243.71	5133.16	5073.51

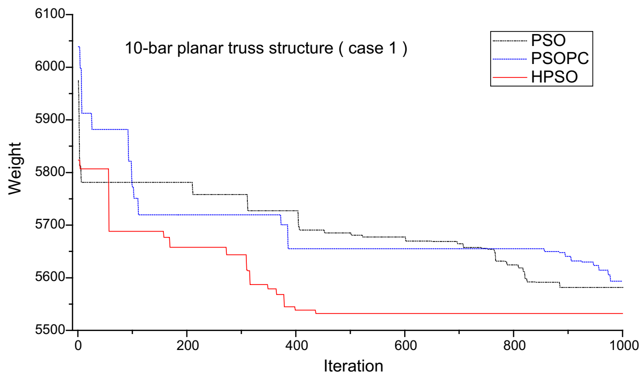

Fig. 2. Comparison of convergence rates for the 10-bar planar truss structure (Case 1).

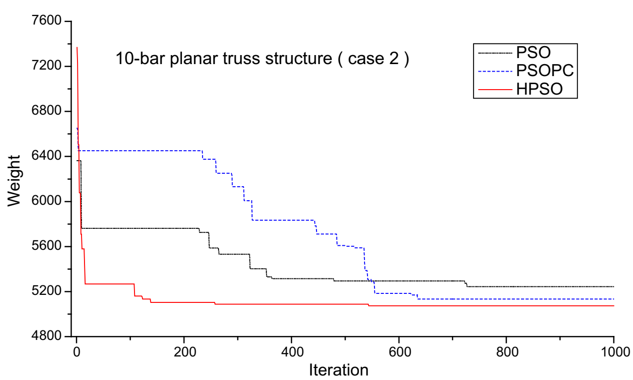

Fig. 3. Comparison of convergence rates for the 10-bar planar truss structure (Case 2).

sults obtained by the HPSO algorithm are larger than those of Wu [2]. However, it is found that Wu's results do not satisfy the constraints of this problem. It is believed that Wu's results need to be further valuated. For both cases of this structure, the PSO, PSOPC and HPSO algorithms have achieved the optimal solutions after 1000 iterations. But the latter is much closer to the best solution than the former in the early iterations.

Fig. 4 illustrates the stability of these three algorithms based on 100 independent calculation results. The standard difference for the PSO, PSOPC and HPSO is 664.07891, 12.84174 and 3.8402, respectively. It can be seen that the HPSO has the best stability.

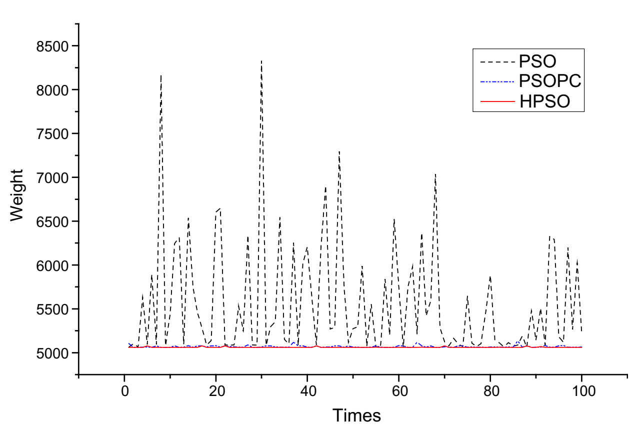

Fig. 4. Comparison of the stability of three algorithms.

6.2. A 15-bar planar truss structure

A 15-bar planar truss structure, shown in Fig. 5, has previously been analyzed by Zhang [18]. The material density is 7800 kg/m^3 and the modulus of elasticity is 200 GPa. The members are subjected to stress limitations of ±120 MPa. All nodes in both directions are subjected to displacement limitations of ±10 mm. There are 15 design variables in this example. The discrete variables are selected from the set $D = \{113.2, 143.2, 145.9, 174.9, 185.9, 235.9, 265.9, 297.1, 308.6, 334.3, 338.2, 497.8, 507.6, 736.7, 791.2, 1063.7\}$ (mm^2). Three load cases are considered: Case 1: $P_1 = 35$ kN, $P_2 = 35$ kN, $P_3 = 35$ kN; Case 2: $P_1 = 35$ kN, $P_2 = 0$ kN, $P_3 = 35$ kN; Case 3: $P_1 = 35$ kN, $P_2 = 35$ kN, $P_3 = 0$ kN. A maximum iteration step of 500 is imposed.

Table 4 and Fig. 6 give the comparison of optimal design results and convergence rates of 15-bar planar truss structure respec-

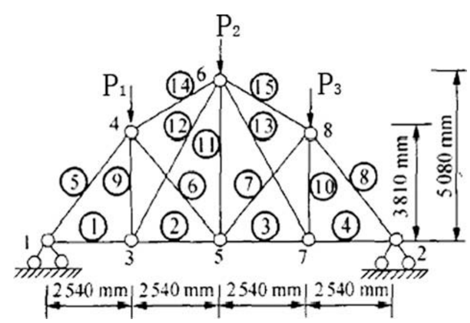

Fig. 5. A 15-bar planar truss structure.

Table 4
Comparison of optimal designs for the 15-bar planar truss structure.

Variables (mm^2)	Zhang [18]	PSO	PSOPC	HPSO
A_1	308.6	185.9	113.2	113.2
A_2	174.9	113.2	113.2	113.2
A_3	338.2	143.2	113.2	113.2
A_4	143.2	113.2	113.2	113.2
A_5	736.7	736.7	736.7	736.7
A_6	185.9	143.2	113.2	113.2
A_7	265.9	113.2	113.2	113.2
A_8	507.6	736.7	736.7	736.7
A_9	143.2	113.2	113.2	113.2
A_{10}	507.6	113.2	113.2	113.2
A_{11}	279.1	113.2	113.2	113.2
A_{12}	174.9	113.2	113.2	113.2
A_{13}	297.1	113.2	185.9	113.2
A_{14}	235.9	334.3	334.3	334.3
A_{15}	265.9	334.3	334.3	334.3
Weight (kg)	142.117	108.84	108.96	105.735

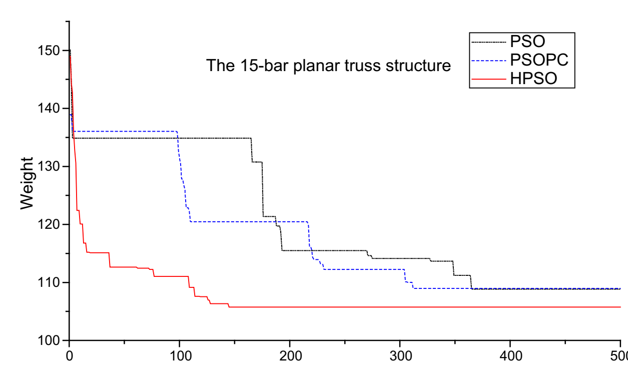

Fig. 6. Comparison of convergence rates for the 15-bar planar truss structure.

tively. It can be seen that, after 500 iterations, three algorithms have obtained good results, which are better than the Zhang's. The Fig. 6 shows that the HPSO algorithm has the fastest convergence rate, especially in the early iterations.

6.3. A 25-bar spatial truss structure

A 25-bar spatial truss structure, shown in Fig. 7, has been studied by Wu [2], Rajeev [16], Ringertz [17], and Lee [19]. The material density is 0.1 lb/in.^{3} and the modulus of elasticity is 10,000 ksi. The stress limitations of the members are ±40,000 psi. All nodes in three directions are subjected to displacement limitations of ±0.35 in. The structure includes 25 members, which are divided into eight groups, as follows: (1) A_1, (2) A_2–A_5, (3) A_6–A_9, (4) A_{10}–A_{11}, (5) A_{12}–A_{13}, (6) A_{14}–A_{17}, (7) A_{18}–A_{21} and (8) A_{22}–A_{25}. There are three optimization cases to be implemented. Case 1: The discrete variables are selected from the set D = {0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4} (in.^{2}). The loads are shown in Table 5; Case 2: The discrete variables are selected from the set *D* = {0.01, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6, 4.0, 4.4, 4.8, 5.2, 5.6, 6.0} (in.^{2}). The loads are shown in Table 6. Case 3: The discrete variables are selected from the American Institute of Steel Construction (AISC) Code, which is shown in Table 7. The loads are shown in Table 6. A maximum iteration of 500 is imposed for three cases.

Tables 8–10 show the comparison of optimal design results for the 25-bar spatial truss structure under three load cases. While Figs. 8–10 show comparison of convergence rates for the 25-bar spatial truss structure under three load cases. For all load cases of this structure, three algorithms can achieve the optimal solution after 500 iterations. But Figs. 8–10 show that the HPSO algorithm has the fastest convergence rate.

Fig. 11 is the stability curves based on 100 independent calculation results. The standard difference for the PSO, PSOPC and HPSO are 256.7491, 1.04208 and 0.02664, respectively. It is shown that the stability of the HPSO is the best among these three algorithms.

6.4. A 52-bar planar truss structure

A 52-bar planar truss structure, shown in Fig. 12, has been analyzed by Wu [2] and Lee [19]. The members of this structure are divided into 12 groups: (1) A_1–A_4, (2) A_5–A_6, (3) A_7–A_8, (4) A_9–A_{10}, (5) A_{11}–A_{14}, (6) A_{15}–A_{18}, and (7) A_{19}–A_{22}. The material density is 7860.0 kg/m^3 and the modulus of elasticity is 2.07×10^5 MPa. The members are subjected to stress limitations of ±180 MPa. Both of the loads, P_x = 100 kN, P_y = 200 kN are considered. The discrete variables are selected from Table 7. A maximum iteration step of 3000 is imposed.

Table 11 and Fig. 13 give the comparison of optimal design results and convergence rates of 52-bar planar truss structure, respectively. From Table 11 and Fig. 13, it can be observed that only the HPSO algorithm achieves the good optimal result. For the PSO and PSOPC algorithms, they do not get optimal results when the maximum number of iterations is reached.

6.5. A 72-bar spatial truss structure

A 72-bar spatial truss structure, shown in Fig. 14, has been studied by Wu [2] and Lee [19]. The material density is 0.1 lb/in.^{3} and

Table 5
The load case 1 for the 25-bar spatial truss structure.

	Load cases	Nodes	Loads		
			P_x (kips)	P_y (kips)	P_z (kips)
Case 1	1	1	1.0	−10.0	−10.0
		2	0.0	−10.0	−10.0
		3	0.5	0.0	0.0
		6	0.6	0.0	0.0

Table 6
The load case 2 and case 3 for the 25-bar spatial truss structure.

	Load cases	Nodes	Loads		
			P_x (kips)	P_y (kips)	P_z (kips)
Cases 2 and 3	2	1	0.0	20.0	−5.0
		2	0.0	−20.0	−5.0
	3	1	1.0	10.0	−5.0
		2	0.0	10.0	−5.0
		3	0.5	0.0	0.0
		6	0.5	0.0	0.0

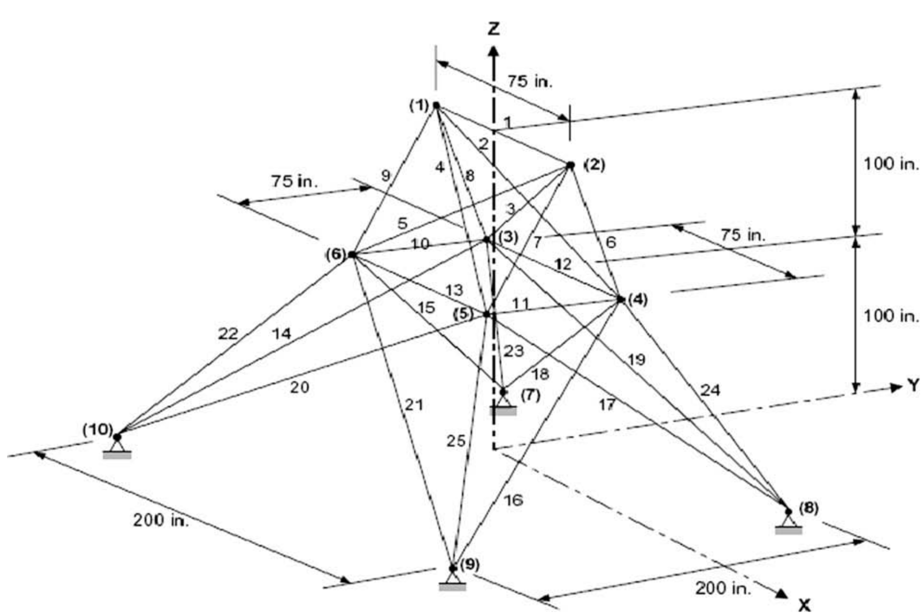

Fig. 7. A 25-bar spatial truss structure.

Table 7
The available cross-section areas of the ASIC code.

No.	in.^{2}	mm^2	No.	in.^{2}	mm^2
1	0.111	71.613	33	3.840	2477.414
2	0.141	90.968	34	3.870	2496.769
3	0.196	126.451	35	3.880	2503.221
4	0.250	161.290	36	4.180	2696.769
5	0.307	198.064	37	4.220	2722.575
6	0.391	252.258	38	4.490	2896.768
7	0.442	285.161	39	4.590	2961.284
8	0.563	363.225	40	4.800	3096.768
9	0.602	388.386	41	4.970	3206.445
10	0.766	494.193	42	5.120	3303.219
11	0.785	506.451	43	5.740	3703.218
12	0.994	641.289	44	7.220	4658.055
13	1.000	645.160	45	7.970	5141.925
14	1.228	792.256	46	8.530	5503.215
15	1.266	816.773	47	9.300	5999.988
16	1.457	939.998	48	10.850	6999.986
17	1.563	1008.385	49	11.500	7419.340
18	1.620	1045.159	50	13.500	8709.660
19	1.800	1161.288	51	13.900	8967.724
20	1.990	1283.868	52	14.200	9161.272
21	2.130	1374.191	53	15.500	9999.980
22	2.380	1535.481	54	16.000	10322.560
23	2.620	1690.319	55	16.900	10903.204
24	2.630	1696.771	56	18.800	12129.008
25	2.880	1858.061	57	19.900	12838.684
26	2.930	1890.319	58	22.000	14193.520
27	3.090	1993.544	59	22.900	14774.164
28	1.130	729.031	60	24.500	15806.420
29	3.380	2180.641	61	26.500	17096.740
30	3.470	2238.705	62	28.000	18064.480
31	3.550	2290.318	63	30.000	19354.800
32	3.630	2341.931	64	33.500	21612.860

Table 8
Comparison of optimal designs for the 25-bar spatial truss structure (case 1).

Variables (in.^{2})	Case 1					
	Wu [2]	Rajeev [16]	Lee [19]	PSO	PSOPC	HPSO
A_1	0.1	0.1	0.1	0.4	0.1	0.1
A_2–A_5	0.5	1.8	0.3	0.6	1.1	0.3
A_6–A_9	3.4	2.3	3.4	3.5	3.1	3.4
A_{10}–A_{11}	0.1	0.2	0.1	0.1	0.1	0.1
A_{12}–A_{13}	1.5	0.1	2.1	1.7	2.1	2.1
A_{14}–A_{17}	0.9	0.8	1.0	1.0	1.0	1.0
A_{18}–A_{21}	0.6	1.8	0.5	0.3	0.1	0.5
A_{22}–A_{25}	3.4	3.0	3.4	3.4	3.5	3.4
Weight (lb)	486.29	546.01	484.85	486.54	490.16	484.85

Table 9
Comparison of optimal designs for the 25-bar spatial truss structure (case 2).

Variables (in.^{2})	Case 2					
	Wu [2]	Ringertz [17]	Lee [19]	PSO	PSOPC	HPSO
A_1	0.4	0.01	0.01	0.01	0.01	0.01
A_2–A_5	2.0	1.6	2.0	2.0	2.0	2.0
A_6–A_9	3.6	3.6	3.6	3.6	3.6	3.6
A_{10}–A_{11}	0.01	0.01	0.01	0.01	0.01	0.01
A_{12}–A_{13}	0.01	0.01	0.01	0.4	0.01	0.01
A_{14}–A_{17}	0.8	0.8	0.8	0.8	0.8	0.8
A_{18}–A_{21}	2.0	2.0	1.6	1.6	1.6	1.6
A_{22}–A_{25}	2.4	2.4	2.4	2.4	2.4	2.4
Weight (lb)	563.52	568.69	560.59	566.44	560.59	560.59

the modulus of elasticity is 10,000 ksi. The members are subjected to stress limitations of ±25 ksi. The uppermost nodes are subjected to displacement limitations of ±0.25 in. both in x and y directions.

Table 10
Comparison of optimal designs for the 25-bar spatial truss structure (case 3).

Variables (in.^{2})	Case 3			
	Wu [2]	PSO	PSOPC	HPSO
A_1	0.307	1.0	0.111	0.111
A_2–A_5	1.990	2.62	1.563	2.130
A_6–A_9	3.130	2.62	3.380	2.880
A_{10}–A_{11}	0.111	0.25	0.111	0.111
A_{12}–A_{13}	0.141	0.307	0.111	0.111
A_{14}–A_{17}	0.766	0.602	0.766	0.766
A_{18}–A_{21}	1.620	1.457	1.990	1.620
A_{22}–A_{25}	2.620	2.880	2.380	2.620
Weight (lb)	556.43	567.49	556.90	551.14

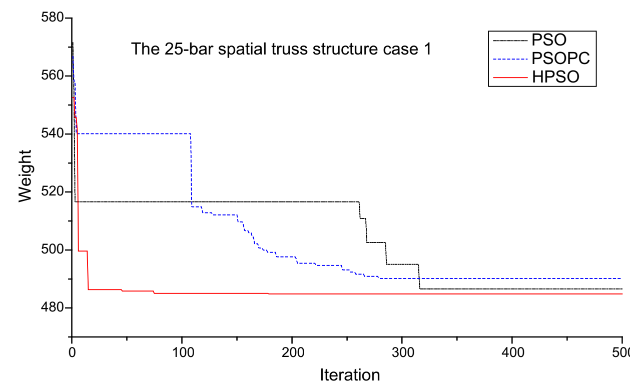

Fig. 8. Comparison of convergence rates for the 25-bar spatial truss structure (Case 1).

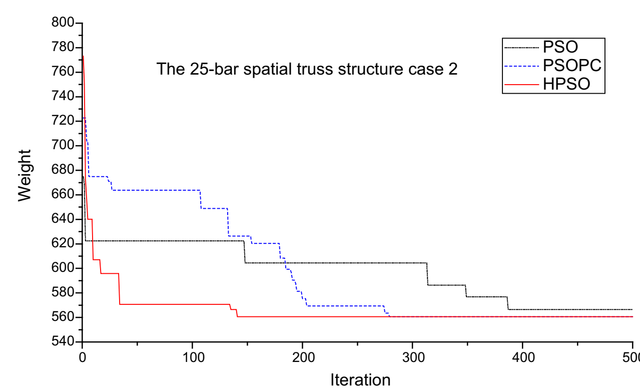

Fig. 9. Comparison of convergence rates for the 25-bar spatial truss structure (Case 2).

Two load cases are listed in Table 12. There are 72 members, which are divided into 16 groups, as follows: (1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30} (7) A_{31}–A_{34}, (8) A_{35}–A_{36}, (9) A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}–A_{58}, (14) A_{59}–A_{66} (15) A_{67}–A_{70}, (16) A_{71}–A_{72}. There are two optimization cases to be implemented. Case 1: The discrete variables are selected from the set $D = \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 2.9, 3.0, 3.1, 3.2\}$ (in.^{2}); Case 2: The discrete variables are selected from Table 7. A maximum iteration of 1000 is imposed.

Tables 13 and 14 are the comparison of optimal design results for the 72-bar spatial truss structure in two load cases. The Figs.

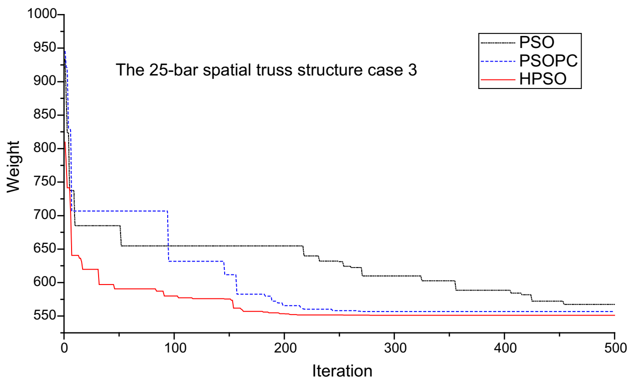

Fig. 10. Comparison of convergence rates for the 25-bar spatial truss structure (Case 3).

Table 11
Comparison of optimal designs for the 52-bar planar truss structure.

Variables (mm^2)	Wu [2]	Lee [19]	PSO	PSOPC	HPSO
A_1–A_4	4658.055	4658.055	4658.055	5999.988	4658.055
A_5–A_{10}	1161.288	1161.288	1374.190	1008.380	1161.288
A_{11}–A_{13}	645.160	506.451	1858.060	2696.770	363.225
A_{14}–A_{17}	3303.219	3303.219	3206.440	3206.440	3303.219
A_{18}–A_{23}	1045.159	940.000	1283.870	1161.290	940.000
A_{24}–A_{26}	494.193	494.193	252.260	729.030	494.193
A_{27}–A_{30}	2477.414	2290.318	3303.220	2238.710	2238.705
A_{31}–A_{36}	1045.159	1008.385	1045.160	1008.380	1008.385
A_{37}–A_{39}	285.161	2290.318	126.450	494.190	388.386
A_{40}–A_{43}	1696.771	1535.481	2341.93	1283.870	1283.868
A_{44}–A_{49}	1045.159	1045.159	1008.38	1161.290	1161.288
A_{50}–A_{52}	641.289	506.451	1045.16	494.190	792.256
Weight (kg)	1970.142	1906.76	2230.16	2146.63	1905.495

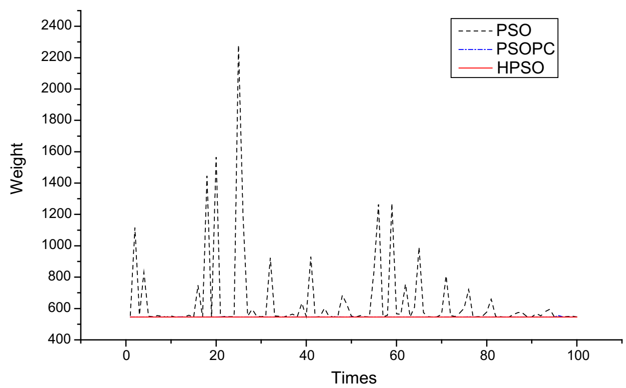

Fig. 11. Stability comparisons among three algorithms for the 25-bar spatial structure.

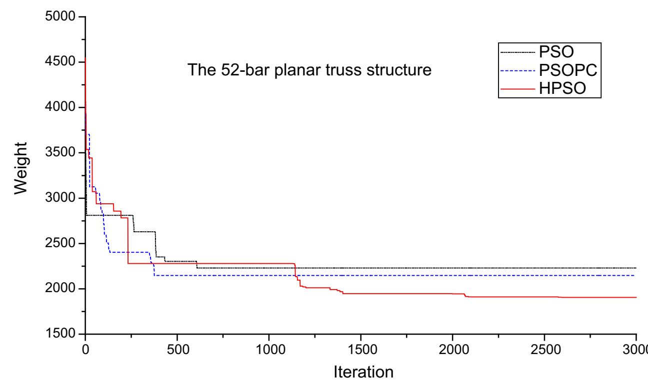

Fig. 13. Comparison of convergence rates for the 52-bar planar truss structure.

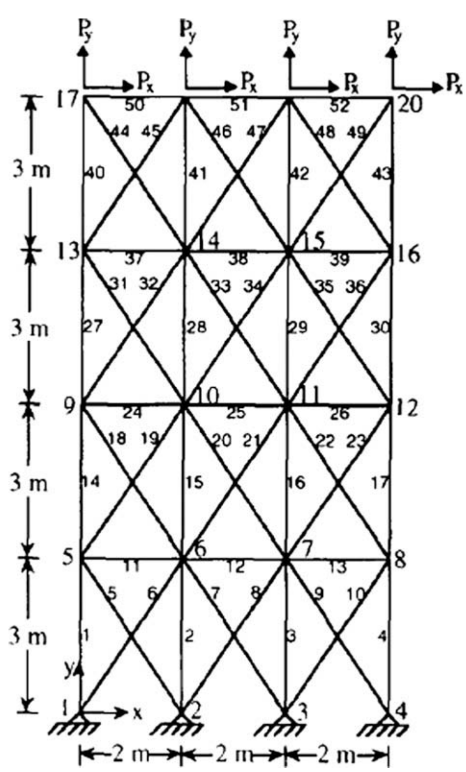

Fig. 12. A 52-bar planar truss structure.

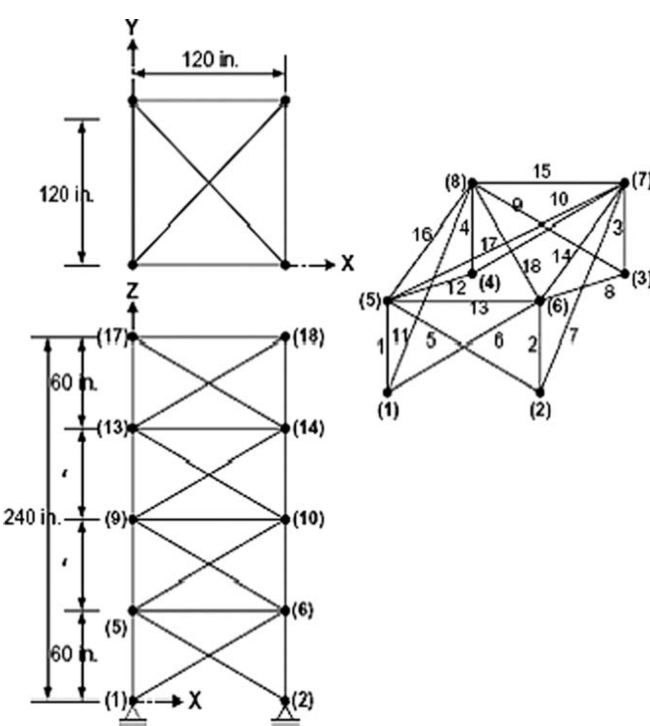

Fig. 14. The 72-bar spatial truss structure.

Table 12
The load cases for the 72-bar spatial truss structure.

Nodes	Load case 1			Load case 2		
	P_x (kips)	P_y (kips)	P_z (kips)	P_x (kips)	P_y (kips)	P_z (kips)
17	5.0	5.0	−5.0	0.0	0.0	−5.0
18	0.0	0.0	0.0	0.0	0.0	−5.0
19	0.0	0.0	0.0	0.0	0.0	−5.0
20	0.0	0.0	0.0	0.0	0.0	−5.0

Table 13
Comparison of optimal designs for the 72-bar spatial truss structure (case 1).

Variables (in.^{2})	Wu [2]	Lee [19]	PSO	PSOPC	HPSO
A_1–A_4	1.5	1.9	2.6	3.0	2.1
A_5–A_{12}	0.7	0.5	1.5	1.4	0.6
A_{13}–A_{16}	0.1	0.1	0.3	0.2	0.1
A_{17}–A_{18}	0.1	0.1	0.1	0.1	0.1
A_{19}–A_{22}	1.3	1.4	2.1	2.7	1.4
A_{23}–A_{30}	0.5	0.6	1.5	1.9	0.5
A_{31}–A_{34}	0.2	0.1	0.6	0.7	0.1
A_{35}–A_{36}	0.1	0.1	0.3	0.8	0.1
A_{37}–A_{40}	0.5	0.6	2.2	1.4	0.5
A_{41}–A_{48}	0.5	0.5	1.9	1.2	0.5
A_{49}–A_{52}	0.1	0.1	0.2	0.8	0.1
A_{53}–A_{54}	0.2	0.1	0.9	0.1	0.1
A_{55}–A_{58}	0.2	0.2	0.4	0.4	0.2
A_{59}–A_{66}	0.5	0.5	1.9	1.9	0.5
A_{67}–A_{70}	0.5	0.4	0.7	0.9	0.3
A_{71}–A_{72}	0.7	0.6	1.6	1.3	0.7
Weight (lb)	400.66	387.94	1089.88	1069.79	388.94

Table 14
Comparison of optimal designs for the 72-bar spatial truss structure (case 2).

Variables (in^2)	Wu [2]	PSO	PSOPC	HPSO
A_1–A_4	0.196	7.22	4.49	4.97
A_5–A_{12}	0.602	1.80	1.457	1.228
A_{13}–A_{16}	0.307	1.13	0.111	0.111
A_{17}–A_{18}	0.766	0.196	0.111	0.111
A_{19}–A_{22}	0.391	3.09	2.620	2.88
A_{23}–A_{30}	0.391	0.785	1.130	1.457
A_{31}–A_{34}	0.141	0.563	0.196	0.141
A_{35}–A_{36}	0.111	0.785	0.111	0.111
A_{37}–A_{40}	1.800	3.09	1.266	1.563
A_{41}–A_{48}	0.602	1.228	1.457	1.228
A_{49}–A_{52}	0.141	0.111	0.111	0.111
A_{53}–A_{54}	0.307	0.563	0.111	0.196
A_{55}–A_{58}	1.563	1.990	0.442	0.391
A_{59}–A_{66}	0.766	1.620	1.457	1.457
A_{67}–A_{70}	0.141	1.563	1.228	0.766
A_{71}–A_{72}	0.111	1.266	1.457	1.563
Weight (lb)	427.203	1209.48	941.82	933.09

15 and 16 are comparison of convergence rates for the 72-bar spatial truss structure in two load cases. For both of the cases, it seems that Wu's results [2] achieve smaller weight. However, we discovered that both of these results do not satisfy the constraints, the results are unacceptable.

In case 1, the HPSO algorithm gets the optimal solution after 1000 iterations and shows a fast convergence rate, especially during the early iterations. For the PSO and PSOPC algorithms, they do not get optimal results when the maximum number of iterations is reached. In case 2, the HPSO algorithm gets best optimization result comparatively among three methods and shows a fast convergence rate.

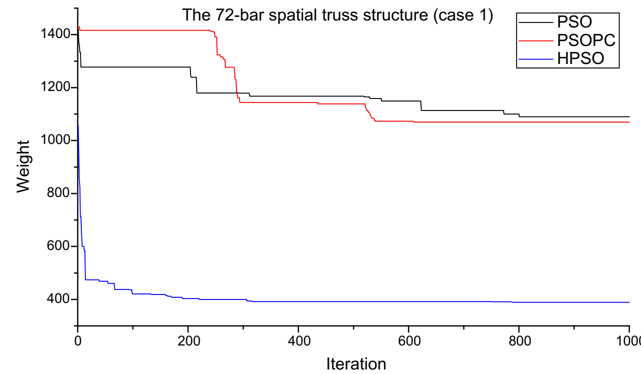

Fig. 15. Comparison of convergence rates for the 72-bar spatial truss structure (Case 1).

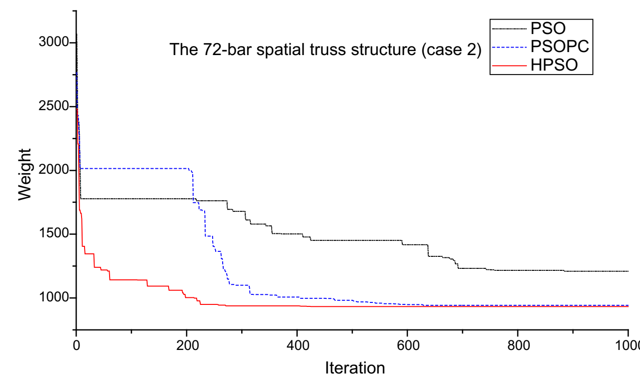

Fig. 16. Comparison of convergence rates for the 72-bar spatial truss structure (Case 2).

7. Conclusions

In this paper, a heuristic particle swarm optimizer (HPSO) handling discrete variables is presented. The HPSO algorithm for discrete variables has all the advantages that belong to the HPSO algorithms for continuous variables, and has faster convergence rate than the PSO and PSOPC for discrete variables. It is the most efficient one of these three algorithms.

The HPSO algorithm presented in this paper has been tested on five truss structure optimization problems. All the results show that the HPSO algorithm has better global/local search behaviour avoiding premature convergence while rapidly converging to the optimal solution.

Results from the tested cases using an HPSO illustrate the ability of the algorithm to find optimal results for discrete variables, which are better, or at the same level of the HPSO optimization methods for continuous variables.

Acknowledgements

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