

How many photons does it take to form an image?

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Steven D. Johnson,  Paul-Antoine Moreau,  Thomas Gregory,  and Miles J. Padgett^{a)} 

AFFILIATIONS

SUPA, School of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United Kingdom

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^{a)}Author to whom correspondence should be addressed: miles.padgett@glasgow.ac.uk

ABSTRACT

If a picture tells a thousand words, then we might ask ourselves how many photons does it take to form a picture? In terms of the transmission of the picture information, then the multiple degrees of freedom (e.g., wavelength, polarization, and spatial mode) of the photon mean that high amounts of information can be encoded such that the many pixel values of an image can, in principle, be communicated by a single photon. However, the number of photons required to transmit the image information is not necessarily, at least technically, the same as the number of photons required to image an object. Therefore, another equally important question is how many photons does it take to measure an unknown image?

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The literature reports many different types of imaging systems based on low numbers of photons ranging from: the use of entangled photon sources to give sub-shot noise images¹ and sensing,^{2,3} imaging and manipulation of correlations and entanglement,^{4–12} single-photon 3D imaging,¹³ imaging with fewer than one detected photon per pixel,¹⁴ indirect 3D imaging based on first-photon arrival,¹⁵ and object tracking outwith the direct line of sight.¹⁶ However, in this short Perspective, we wish to discuss the more basic principle: given near perfect technology, how many detected photons are required to infer an image? Here, an image is defined as a spatially dependent intensity structure, the structure of which can be described in terms of an array of pixels.

There are some constrained circumstances where an entire image could be determined from a single photon. In recent years, the spatial transverse modes of both classical beams and single photons have been considered as a degree of freedom for space-division multiplexing within communication systems^{17,18} and as a high-dimensional Hilbert space for quantum information processing.¹⁹ Significant within these studies has been the orbital angular momentum of light typified by the Laguerre–Gaussian modes;²⁰ however, the fundamental opportunities and limitations apply to any orthogonal complete modal set.²¹ It has been shown that even at the level of single-photons, it is possible to separate these modes so that each single mode can be directed to a different element of a detector array.²² In principle, an N -element array, where each element is sensitive to a single photon, can distinguish one mode from all others with 100% efficiency. In essence, this is equivalent

to a spectrometer that can distinguish the wavelength of individual photons using a prism and a multi-element detector array. This perfect separation is possible because the complex amplitude distributions of the modes are orthogonal to each other.

Complex images cannot be reconstructed from an individual photon measurement, even if the object is illuminated with a coherent source; the phase of the backscattered light is usually ill controlled, and the image information is contained purely in the intensity information alone. Different real-world images spatially overlap and therefore are not orthogonal to each other, and the modal separation optics cannot therefore distinguish images from each other at the level of a single photon. There are potentially exceptions to this where two images may contain regions that are orthogonal to each other, such as a position wherein one image is bright and the other is dark, and vice versa.²⁴ If the measurement is restricted to these regions, then two images could be distinguished by a single photon and 2^N images can be readily distinguished from each other by N photons.

A distinction must be made at this point between the numbers of photons that need to be detected to compose or identify an image and the number of photons that needs to probe an object or a scene in order to acquire a full image. This distinction becomes particularly important when considering the quantum phenomenon. On one hand, by using quantum states of light, one can reduce both the number of photons required to probe a sample and the number of photons required to constitute its image compared to classical schemes for an equivalent image quality.

Such quantum schemes can be realized using either an entangled photon source^{1,25,26} or equivalently using a source with a well-defined number of photons (Fock states) to obtain sub-shot noise images.²⁷ But on the other hand, one can somehow decouple the probing of an object and the detection of the image by harnessing the wave-particle behavior of the light. An example of this is given by quantum imaging with undetected photons,^{28–30} where the photons that interact with the object need not be the same ones that are measured for an image to be obtained. A similar situation arises with interaction-free measurements,^{31,32} in which the effective number of photons necessary to probe and detect an image are decoupled.³³ Ultimately, it can be shown that through a modification of this experiment, a binary object can in theory be probed without any photons ever reaching it.³⁴

Alternatively, we can scrutinize the minimum number of photons that are needed to compose the detected image or signal needed for the image acquisition to be performed. In this low photon-number regime, the noise is Poissonian rather than Gaussian, and at these low photon numbers, the anticipated noise in the bright and dark areas of the image needs to be accounted for in any merit function optimization to avoid incorrect image reconstructions, such as overfitting to the bright areas of the image.^{35,36} As we will discuss later, the number of photons required to compose an image will depend on the complexity of the scene and consequently on the prior knowledge we have and can use about this scene. But in the most general case, we can assume that there is no strong knowledge about the scene other than as any realistic object it might exhibit some form of sparsity in certain domain. In this context, a reasonable starting assumption is that an image comprising of N independent pixels could be probed with N photons, the end result being a binary image with no grey-scale information. This rational also applies similarly to classical sources where the expectation value is set at one photon per pixel, which yields a signal-to-noise figure of order unity.³⁷ By making the assumptions of an intensity correlation between adjacent pixels, one can apply smoothing algorithms to improve the signal-to-noise ratio and the hence the number of grey-scale levels in the image, albeit at the expense of spatial resolution. It is interesting to note that although smoothing the image does allow resolution to be sacrificed in order to improve the signal-to-noise value of the resulting image, in general, the process results in

an overall loss of information. Figure 1 shows one example of taking a binary photon sparse image and applying image processing techniques²³ to obtain a grey-scale image, albeit at a reduced spatial resolution.

Rather than applying smoothing, a more sophisticated approach is to assume sparsity in the spatial frequency domain or any other appropriate basis, it is then possible to constrain the image solution to one that is both statistically compatible with the measured data and also favors the sparsity constraint.^{38–40} These concepts are central to the field of compressed sensing and depending upon the strength of the restriction applied to the likely image solutions, the number of photons per pixel can be reduced below one.⁴¹ The degree of compression or denoising that can be applied depends critically on the complexity of the scene and the strength of the image prior that can be applied. For example, if the image is known to comprise a single white disk on a black background, then the full image can be characterized by three variables: the x-y position and the radius of the disk. Such a prior if applied to the image reconstruction would allow a very high degree of compression of the measurements. Indeed, a similar prior could be applied to center of mass localization microscopy techniques such as PALM and STORM.⁴² More typically for generalized scenes, a convenient prior is to assume a sparsity on the spatial frequency domain. This sparsity assumption is the same image property that lies at the heart of JPEG image compression. In such situations, it is often the case that the number of measurements M that is required to estimate an N pixel can be reconstructed from K -sparse vectors which can be reconstructed with a high probability using just $M \geq O(K \log(N/K))$ random measurements.⁴³ Such systems might again be based on an N element detector array with efficient single-photon sensitivity, placed in the focal plane of an imaging lens. Such single-photon sensitive detector arrays are themselves not without challenges.

The detectors for single-photon imaging took the significant step in moving from single element detectors such as a photomultiplier tube or a single-photon avalanche photodiode (SPAD)⁴⁴ to SPAD array detectors that offer timing within the pixel to measure the arrival time of photons over the whole pixel array.⁴⁵ Alternatively, charge-coupled device (CCD) cameras have demonstrated single-photon

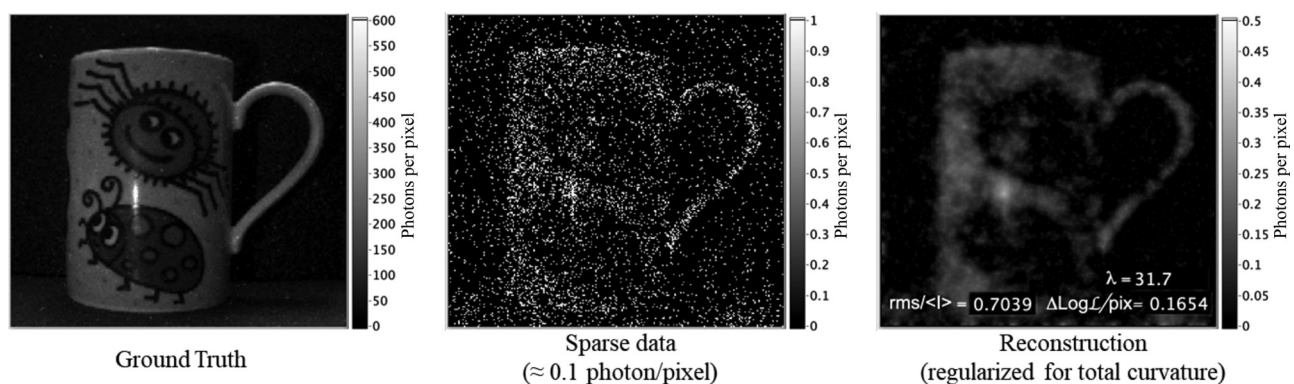


FIG. 1. A demonstration of reconstructing an image from sparse photon measurements. The ground truth is sampled in a simulated photon counting approach, producing a sparsely sampled image. The image is reconstructed based on a bootstrapping regularizer algorithm,²³ where the spatial resolution has been sacrificed for the benefit of image similarity.

sensitivity with the addition of electron multiplication registers to make EMCCDs,⁴⁶ the drawback being these devices suffer from read-out noise in the form of clock-induced charge, leading to a significant false positive detection rate. The competing technology of CMOS imaging sensors now has readout noise that is almost competitive with the current generation of EMCCDs, with sensors now showing significant improvement in room temperature detection of single photons.⁴⁷

Note that in this context, the so-called quantum illumination protocols^{48,49} can be implemented to get rid of some of the camera single photon counting noise, together with potential external sources of noise that could pollute the image and prevent a low photon acquisition from being performed.^{8,9} We can use entangled photon pairs that exhibit spatial correlations to perform imaging in the presence of a spoofed scene that is illuminated by a thermal source. As may be seen in the schematic presented in Fig. 2, two spontaneous parametric downconversion beams are spatially separated and one of the beams acts as the reference beam and is detected on the camera, while the other beam probes the object. The spoof object is illuminated by a thermal light source and is detected on the same region of the camera as the object to be imaged. The classical image detected on the camera is the bird within its cage as the events registered by the camera may not be distinguished from their source. By performing a pixel-by-pixel AND-operation to the regions of the sensor that detect the reference beam and the probe beam for each of the acquired frames, the camera events resulting from the spatially correlated photon-pairs may be preferentially selected. By performing this AND-operation, the events that comprise the object probed by the quantum light (the bird) may be preferentially selected over those that originate from either sensor noise or the classically illuminated spoof object (the cage) and result in an image of a bird released from the cage in the quantum image, thereby illustrating the ability of the scheme to remove the noise present in an image.

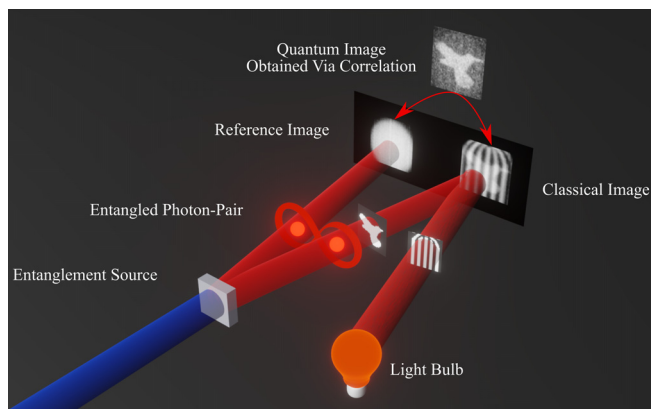


FIG. 2. Simplified experimental setup of the implementation of the quantum illumination protocol in the context of imaging.⁹ By exploiting the spatial correlations between the entangled photon-pairs, the image of the object illuminated by quantum light (the bird) may be recovered in the presence of both detector noise and a false image illuminated here by thermal light (the cage). The bird object is placed in the far-field of the crystal in which the probe beam and the reference beam are spatially separated and this plane is re-imaged on the camera sensor. The cage is imaged onto the region of the sensor on which the probe beam (which interacts with the bird object) is detected.

An alternative to using a detector array is to employ a single-pixel imaging system.⁵⁰ To reconstruct a photon-sparse image, a single-photon sensitive detector is used in combination with a way to recover spatial information by applying a series of binary (transmissive/opaque) intensity masks to the illumination. The system corresponds to that of a single-pixel camera⁴³ albeit one in which extreme temporal resolution reveals the time-of-flight between the source and object and hence enables depth imaging with a low number of photons.⁵¹ For a fully general image solution, one requires N orthogonal masks to fully measure an N -pixel image.

In the absence of Poissonian noise associated with finite photon number, the underlying strength of the signal associated with each mask is a measure of the overlap between the mask and the object. Assuming that the mask comprises N independent and randomly set pixels which overlap with the object $\approx 50\%$ of the time, then the average signal will be proportional to $N/2$ with a standard deviation of $\sqrt{N}/2$. It is this fluctuation in the signal, and not the average, which contains the image information. When measuring signal associated with each mask using only a finite number of photons, then it is reasonable to assume that the fluctuation in the mask signal must not be disguised by the shot noise related to the number of photons measured. This leads us to the conclusion that the number of photons P required per mask is of the order $P \approx N/2$. Hence, the number of photons required to fully measure a general image is $N^2/2$. This demonstrates that a single-pixel camera configuration is inherently less information efficient than an N -pixel focal-plane array detector, albeit without in general the timing and hence 3D capability of the single-pixel approach. Within the single-pixel approach, one can apply compressed sensing techniques to again take advantage of the sparsity of a typical in a user-chosen basis (e.g., sparsity in spatial frequencies) to either reduce the number of measurements required or improve the signal to noise of the resulting image. These techniques are typified by an optimization process based on the minimization of a cost function which combines the goodness of fit of the solution to data combined with a regularization function.

However, it is possible to adopt a machine learning approach where the measurement masks that extract the maximum image information from the smallest number of measurements can be derived via a deep learning approach.⁵² The approach taken was to image with a low number of photons by using a deep learned set of patterns to sample the scene along with a reconstruction algorithm based on the trained neural network. These mask designs are derived from an extensive image library comprising of many thousands of typical images. Compared to a cost-function based regularization, the machine learning approach transfers the computation load from a post measurement optimization which has to be performed after every image acquisition to a pre-measurement learning based on prior information which, although extremely computationally intensive, only has to be performed once.⁵³ Ultimately, both approaches rely upon the image solution being sparse in one basis or other, but whereas the traditional denoising and compressed sensing typically require an explicit user-choice of a prior; the machine learning effectively establishes its own prior based upon a defined library. Following this machine learning approach, we were able to dramatically reduce the number of masks required to obtain a depth image, even though the masks themselves had been derived from a 2D image library. An example of the improvements in acquisition is shown in Fig. 3. This machine learning

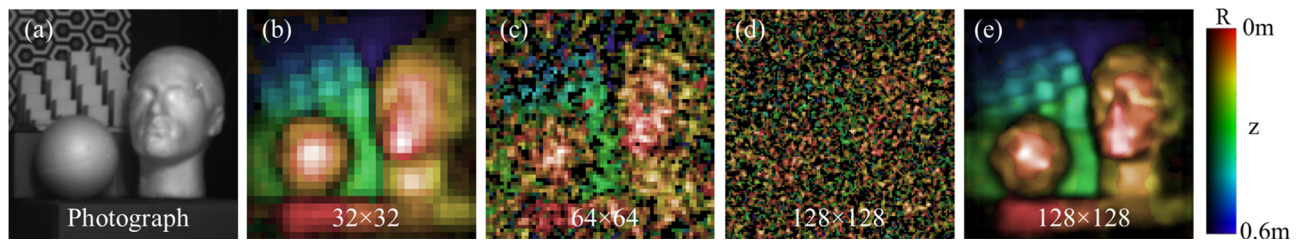


FIG. 3. A comparison of conventional single-pixel LIDAR imaging with a deep learning approach. (a) A photograph of the scene. The data captured via Hadamard patterns of resolution (b) 32×32 , (c) 64×64 , and (d) 128×128 , this is shown in comparison to (e) a deep learned pattern set and reconstruction for a 128×128 image. All measurements were measured over a total interval of 2 s. With a greater number of patterns, there is less time needed to acquire the signal for each pattern, and this decreases the signal to noise of the image. As deep learning requires fewer patterns, it can reduce this noise at higher resolutions. Reproduced with permission from Radwell *et al.*, "Deep learning optimized single-pixel LiDAR," *Appl. Phys. Lett.* **115**, 231101 (2019). Copyright 2019 AIP Publishing.

approach has also been applied to imaging with low numbers of photons⁵⁴ and phase detection.⁵⁵

Having considered how it is possible to reduce the number of measurements to be fewer than the number of image pixels, a second unexplored avenue is to consider whether the number of photons needed per measurement can itself be reduced. Within this avenue, one option might be to take advantage of the temporal bandwidth of the single-pixel detector and apply homodyne or heterodyne detection strategies where coherent illumination can be combined with a much more powerful local oscillator to make the measurement of a small signal possible.⁵⁶ However, although this might overcome the noise floor of a detector, it might not overcome the shot noise limit of intensity measurement itself. To overcome the shot noise, one needs to use an intensity-squeezed light source,^{57,58} however, this potential advantage is only easily accessible in a low loss modality due to the low repetition rate of the squeezed light source; the sensing is performed in a transmission rather than a backscatter configuration. These considerations are important for applications such as LIDAR where efficient detection demands that we fully utilize the minimal information available from the sparse number of photons received by the detector.^{59,60}

Historically, the question "how many photons does it take to form an image?" would have been answered by considering the noise floor of the detector. However, various types of detectors ranging from scientific CMOS cameras, gated intensified cameras, and SPAD arrays are all capable of detection at the single photon level, and the question as to the number of photons required is now a theoretical challenge. While complex spatial modes can be distinguished from each other by a single photon, most real images cannot. For intensity images, it seems that one detected photon per image pixel is a realistic guide, but this may be reduced by making further assumptions on the sparsity of an image in a chosen basis, such as spatial frequency. In this last respect, the advent of machine learning, knowledge-based reconstruction, and similar techniques alleviates the need for a user to explicitly define the sparse basis, but rather the prior is determined from a library of previously recorded images of a similar type. This machine learnt prior can then potentially be designed into the optimum measurement strategy. It seems likely therefore that future imaging systems will combine state-of-the-art single photon detectors with knowledge-based processing both in the design of the system itself and in the processing of the collected data to yield images or decisions based on these data on the basis of extremely low numbers of photons, potentially well below one photon per image pixel.

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DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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