

A simple approach to determine the solar cell diode ideality factor under illumination

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Abstract

A simple approach, which can estimate the diode ideality factor of a high efficiency pn junction solar cell under illumination by using its current–voltage data, is explained. We have proposed that an analytical method based on Lambert W -function is sufficient for the extraction of the diode ideality factor of a solar cell modeled by double junction behavior with considerable compliance. Various illumination intensities are also considered in order to specify the reliable limit of the method. The dependence of the ideality factor and the reverse saturation current with light intensity has also been investigated in order to provide insight into the alteration of electrical conduction at junction interface at room temperature.

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1. Introduction

Research and development studies on polycrystalline heterojunction solar cells have still received a considerable attention because of the possibility of producing relatively high efficient photovoltaic devices more cheaply than ever before. Thus, an accurate calculation of solar cell parameters from experimental data is of vital importance for the optimized design of the high efficiency solar cell fabrication process. These parameters are principally the saturation current, the series and shunt parasitics, the ideality factor and the photocurrent. The ideality factor (n) is a real and an important parameter that can be used to decide whether the pn junction device behaves closely or apart from the ideal case. Thus, its dependence on both the temperature and illumination not only gives a good indication of the quality of the junction but also the electrical transport behavior. While the reverse saturation current density

(J_o) of the solar cell provides valuable information on the current transport mechanism across the pn junction interface, the ideality factor, depending onto the device operation conditions, i.e., temperature (T), voltage (V) and illumination, determines crucial information about the nature of the junction interface such that the presence of interface states and spatial inhomogeneity, generation–recombination, tunneling, etc. (Fahrenbruch and Bube, 1983).

The commonly used experimental methods to estimate the value of junction ideality factor is that n can be estimated directly from the slope of the straight-line regions of forward $\text{Log } J$ (Wolf and Rauschenbach, 1963) and $\text{Log}(J + J_{sc})$ vs. V plots (Quanxi and Enke, 1987), for dark and illuminated conditions, respectively where there are no series and shunt resistance effects. For these plots, J_o corresponds to the intercept on the current axis at zero bias voltage. Over the years several methods have also been suggested for extracting the value of ideality factor from both dark (Bayhan, 2009) and illuminated (Jain and Kapoor, 2005; Ortiz-Conde et al., 2006; Chegaar et al.,

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2004; Singh et al., 2009; Alfaramawi, 2010) current–voltage data with considerable series and shunt parasitics. An overview on the methods for the determination of the solar cell junction ideality factor has been written by Mialhe et al. (1986) and Bashahu and Nkundabakura (2007). The detailed examination of these methods reveals that the use of the exact explicit analytical solutions for current and voltage of the solar cell's JV characteristics, which are expressed in terms of Lambert W -function, is convenient and offers a simple and an efficient computational alternative for the purpose of extracting the value of the ideality factor.

Ordinarily, almost all of these published methods utilize the one – exponential diode model under single illumination intensity. However in this work, attempts have been made to estimate the values of n of a typical pn junction solar cell approximated in terms of a parallel connected double diode circuit model containing parasitic shunt and series resistance effects under different illumination intensities with an alternative method. Specifically, in the first part of this work, the standard experimental $J - V$ method was modified and applied as a measure of the validity of the proposed method. Then, the illumination intensity dependent values of n were estimated at room temperature by using a simple method based on the exact explicit analytic solution for the current–voltage relation expressed in terms of Lambert W -function, experimentally estimated parasitic series (R_s) and shunt resistances (R_{sh}), the diode saturation current density (J_o) and any value of current selected on the linearly varying $\text{Log}(J + J_{sc})$ vs. $V' (= V + J_{sc}R_s)$ plot. Another analytical method which also based on Lambert W -function was also considered to discuss the limitations of each approximate method under different illumination intensities. In the second part, the alteration trend of both extracted n and J_o values with illumination intensity were considered for the suggestion of the effect of illumination onto the room temperature dark electronic loss mechanism in the solar cell.

2. Experimental details

The small area (0.47 cm²) Al:ZnO/CdS/Cu(In,Ga)Se₂/Mo/Glass solar cell investigated here was grown in the Institut für Physikalische Electronic (IPE) at University of Stuttgart (Germany). The Cu(In,Ga)Se₂ thin layer of about 2 μm was deposited by co-evaporation of Cu, In, Ga and Se onto Mo-coated soda-lime glass substrates. CdS buffer layer of about 0.01 μm thick was deposited by chemical bath deposition and window ZnO layer (≈500 nm) was deposited by RF sputtering technique. The device structure was completed by the evaporation of Al metal grid onto ZnO layer. Details of the fabrication steps are described elsewhere (Stolt et al., 1995). The efficiency of the investigated device is about 13% and Ga content of the bulk of the absorber layer is $x = \text{Ga}/(\text{In} + \text{Ga}) = 34\%$ as determined by energy dispersive spectroscopy.

The dark and illuminated current–voltage ($I-V$) data of the solar cell were obtained at room temperature using an Oriel solar simulator (300 Watt Xenon lamp, model number 81160) at the University of Durham, UK. The illumination intensity was varied between 0.01 and 100 mW/cm² using neutral density filters.

3. Methods

The most widely used methods to estimate the diode ideality factor of a typical solar cell can generally classified into two depending upon whether they are graphical or they make use of a numerical procedure. This paper describes a useful modification for the direct graphical method and also proposes a simple analytical method for the extraction of double diode solar cell ideality factors from illuminated current–voltage characteristics. Another analytical method is also applied for further verification of the estimated n values and also to discuss the limits of these three different methods under different illumination intensities ranging from relatively low to high percentages.

All methods described here are based on an analytical description of a solar cell by the two diode model. The forward current density–voltage relation of a typical pn junction device which modeled by double exponential expression, with significant parasitic series and shunt resistances under a given uniform illumination intensity can be expressed by,

$$J = -J_{ph} + J_{d1} + J_{d2} + J_p$$

$$= -J_{ph} + J_{o1} \left[\exp \left(\frac{q(V - JR_s)}{n_1 k_B T} \right) - 1 \right] + J_{o2} \left[\exp \left(\frac{q(V - JR_s)}{n_2 k_B T} \right) - 1 \right] + \frac{V - JR_s}{R_{sh}}, \quad (1)$$

here J_{ph} , J_d , J_p and J_o being the photocurrent, the dark diode current, the total shunt current and the diode saturation current densities, respectively. Where the indices 1 and 2 are used for the first and the second junction interfaces, n is the diode ideality factor and k_B is the Boltzmann constant.

This equation may also be conveniently expressed as,

$$i = \sum_{j=1}^2 J_{oj} \left[\exp \left(\frac{q(V' - JR_s)}{n_j k_B T} \right) - 1 \right] + \frac{V' - JR_s}{R_{sh}}, \quad (2)$$

where $i = J + J_{ph}$ and $V' = V + J_{ph}R_s$.

3.1. The modified graphical method

The standard graphical method assumes that if R_s is small and R_{sh} is infinite then a plot of $\text{Log}(J + J_{sc})$ vs. V results in a straight line with slope $q/nk_B T$. However, it is worth pointing out that the device parameters of the solar cell usually depend on illumination and besides we may not simply ignore the relative parasitic resistance effects in most cases. Thus, at that point one may easily re-write Eq. (1) in a more convenient form as given in Eq. (2). Clearly, one

can safely suppose that the slope of the straight – line region of the typical $\log i$ vs. V' characteristic yields a more accurate approximation for the experimental estimation of the junction ideality factor under different illumination intensities. By using such a plot, the illumination dependent value of J_o for a given voltage region can also be estimated certainly from intercept of linear variation on the current axis at zero V' value.

3.2. The proposed method

It is well known that the transcendental equation represented by Eq. (2) is implicit and may not be solved explicitly for current or voltage using ordinary elementary functions. However, nowadays exact, explicit and differentiable analytical solution based on Lambert W -function is frequently used. The Lambert W -function of a variable x , which can be indicated as Lambert $W(x)$, is a non linear function of x and it is the inverse function of $f(x) = xe^x$ (Lambert, 1758). Then the value of the Lambert W -function for an assigned value of the independent variable x can be obtained with the help of a simulation environment such as Matlab® and Mathematica®. This representation has been frequently used in a variety of fields of physics and mathematics. One of the recent such application is in the basic pn junction diode current equation which is transcendental in nature. Thus in a simple manner one can express the explicit solution of Eq. (2) for the current density i in terms of Lambert W -function as follows (Ortiz-Conde and Sánchez, 2005),

$$i = \sum_j n_j \frac{kT}{qR_s} \text{Lambert } W(x) + \frac{V' - J_{oj}R_{sh}}{R_s + R_{sh}} \quad (3)$$

$$x = \left[\frac{qR_sR_{sh}J_{oj}}{n_jkT(R_s + R_{sh})} \exp \left(\frac{qR_{sh}(V' + J_{oj}R_s)}{n_jkT(R_s + R_{sh})} \right) \right] \quad (4)$$

For a given diode (i.e., $j = 1, 2$) when the experimentally determined values of J_{oj} , R_s , R_{sh} , any value of $V'(i)$ on the linearly varying $\log i$ vs. V' plot and the value Lambert $W(x)$ (mathworld.wolfram.com) for a defined illumination intensity are inserted to Eq. (3), then an equation in the form of $i(n_j)$ can be determined. The value of $i(n_j)$ at selected $V'(i)$ thus corresponds to the expected value of the ideality factor for a given diode.

3.3. Jain and Kapoor's method

The illuminated current–voltage relation for a solar cell modeled by a single diode can be expressed in terms of Lambert W -function as follows (Jain and Kapoor, 2005)

$$i = -n \frac{kT}{qR_s} \text{Lambert } W \left[\frac{qR_sR_{sh}I_o}{nkT(R_s + R_{sh})} \exp \left(\frac{qR_{sh}(V_{oc} + I_oR_s + R_sI_{ph})}{nkT(R_s + R_{sh})} \right) \right] - \frac{V_{oc}}{R_s + R_{sh}} + \frac{R_{sh}(I_o + I_{ph})}{R_s + R_{sh}} \quad (5)$$

If the experimentally obtained parameter values of R_s , R_{sh} , I_o , open circuit (V_{oc}) and short circuit current (I_{sc}) are inserted into the Eq. (5) then one can obtain a collection of i vs. n data at a given temperature. The expected value of n corresponds to the value of n at which i is zero.

4. Results and discussion

In an attempt to conform the validity of the proposed method, a high efficiency n-ZnO/n-CdS/p-Cu(In,Ga)Se₂ solar cell modeled with double exponential dependence was chosen. As seen in Fig. 1, the room temperature dark forward current density–voltage characteristic of this solar cell contains two linearly varying voltage regions; $V < 0.40$ V and $V > 0.40$ V. As it is reported in the literature (Kasis and Saad, 2009), this behavior can be simply modeled as the sum of two non-ideal diode characteristics. The dark n and J_o values of the corresponding diodes were estimated experimentally from the slopes and the extrapolation to zero volt of the straight-line regions of the current–voltage characteristic where there were no series resistance and shunt conductance, see the inset of Fig. 1.

For an initial step, the illumination intensity dependent values of the ideality factor were estimated according to the direct graphical technique to use them for the validity of the proposed method. Fig. 2. demonstrates $\log i$ vs. V' plots at different illumination intensities varying from 0.01 to 100 mW/cm². As it is seen from this figure, there is a clear presence of two straight-line regions approximately lower ($0.4 \text{ V} < V' < 0.6 \text{ V}$) and higher than 0.6 V ($0.60 \text{ V} < V' < 0.8 \text{ V}$) indicating the presence of two independent routes for the current transport in the device still under different illumination intensities. Thus the equivalent circuit of the solar cell can be safely described by the two

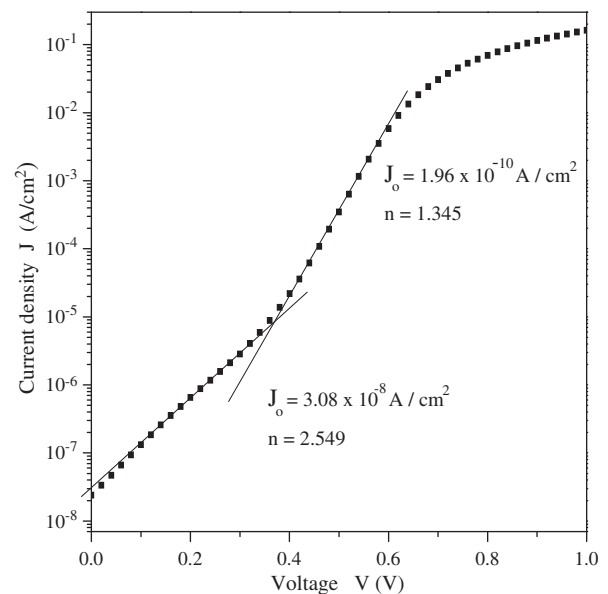


Fig. 1. The dark $J - V$ characteristics of a CdS/Cu(In,Ga)Se₂ solar cell at room temperature.

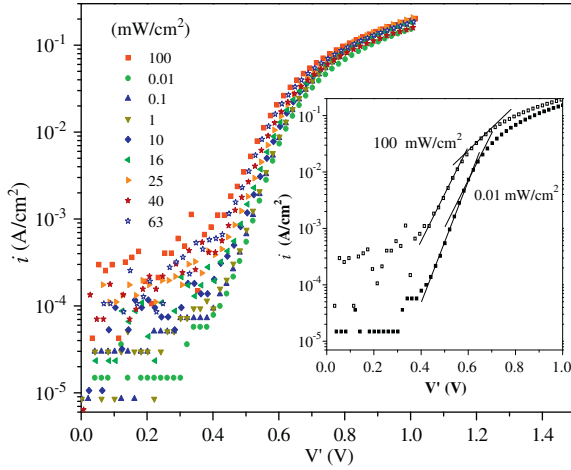


Fig. 2. i vs. V' characteristics under different illumination intensities.

diode model for all different illumination circumstances. At first, the value of n at a given illumination intensity was calculated by using the improved graphical technique as described in Section 3.1 for $V' > 0.6$ V and $V' < 0.6$ V and results were illustrated in Fig. 3.

Since the method that we have proposed here was based on Eq. (3), the illumination and voltage dependent device parameters namely area related parasitic series R_s and shunt resistances R_{sh} , reverse saturation current density J_o and the parameter $i(V')$ were determined by following the methods as described below;

The values of J_o for $V' > 0.6$ V and $V' < 0.6$ V were found from intercept of linear $\text{Log } i = (J + J_{sc})$ vs. $V' = (V + J_{ph}R_s)$ variations separately on the current axis at zero. The value of R_s was estimated initially from the fit of experimental illuminated current–voltage data to the theoretical formula given by Eq. (1). To get more accurate estimation to its value, the two diode method proposed by Chan and Phang (1987) was also used. We have found that both approaches determine the value of R_s almost constant at about $0.47 \Omega \text{ cm}^2$ independent on the illumination

intensity. The value of the other circuit parameter R_{sh} was estimated from the slope at short circuit point following the expression $dJ/dV|_{V=0} = I/R_{sh}$. The accuracy of this value was further checked by a simple fit of measured current–voltage data to Eq. (1). We have found that the value of R_{sh} is approximately equal to $4.7 \times 10^4 \Omega \text{ cm}^2$ and independent on the intensity of the illumination.

According to the method proposed in this work, the parameter $i(V')$ can be chose as any value on the linearly varying $\text{Log } i$ vs. V' characteristic for a given illumination intensity and the voltage region under consideration. In these calculations, we have assumed that the approximation $J_{ph} \cong |J_{sc}|$ introduces no significant errors in the subsequent calculations described here.

For a given illumination intensity, experimentally determined values of parasitic resistances and relevant J_o , Lambert $W(x)$ and V' values are inserted in Eq. (3) and then an equation in the form of $i(n)$ is obtained. For example, consider the percentage of the illumination is at about 100 mW/cm^2 , substituting $R_s = 0.47 \Omega \text{ cm}^2$, $R_{sh} = 4.7 \times 10^4 \Omega \text{ cm}^2$, $J_o (V' > 0.6 \text{ V}) = 8.253 \times 10^{-5} \text{ A/cm}^2$ and $J_o (V' < 0.6 \text{ V}) = 2.013 \times 10^{-7} \text{ A/cm}^2$, $i(0.6075 \text{ V})_{V' > 0.6} = 0.0249 \text{ A/cm}^2$ and $i(0.4767 \text{ V})_{V' < 0.6} = 0.002578 \text{ A/cm}^2$ in Eq. (3), one can obtain separate equations for $V' > 0.6$ V and $V' < 0.6$ V as;

$$i|_{V' > 0.6 \text{ V}} = nx0.055053 \text{ Lambert } W \left[\frac{1.49908 \times 10^{-3}}{n} \right] \times \exp \left(\frac{23.478026}{n} \right) - 6.96037 \times 10^{-5}$$

$$i|_{V' < 0.6 \text{ V}} = nx0.055053 \text{ Lambert } W \left[\frac{3.6564 \times 10^{-6}}{n} \right] \times \exp \left(\frac{18.4230}{n} \right) + 9.94115 \times 10^{-6} \quad (6)$$

Then, the values for the ideality factor at $i = 0.0249 \text{ A/cm}^2$ and $i = 0.002578 \text{ A/cm}^2$ can be found as 4.030 and 1.943 for $V' > 0.6$ V and $V' < 0.6$ V, respectively. The n values for different illumination intensities were estimated with the same procedure and found that these extracted values were in extremely well accordance with those calculated experimentally from the slopes of the straight-line regions of $\text{Log } i$ vs. V' plots. All estimates are plotted in Fig. 3.

In addition to the modified experimental method, another Lambert W -function (Jain and Kapoor, 2005) method was also used to compare the results. Although this method is originally proposed for a real solar cell approximated by a single diode under a single illumination level, here we have shown that this method is also available for extraction of n values if the useful parameters for the second junction are given. The additional device parameters used in these calculations are collected in Table 1 and the extracted n values for voltage regions $V' > 0.6$ V and $V' < 0.6$ V are plotted as a function of illumination intensity in Fig. 3.

All extracted n values estimated by using the methods explained here indicate a remarkable agreement even under

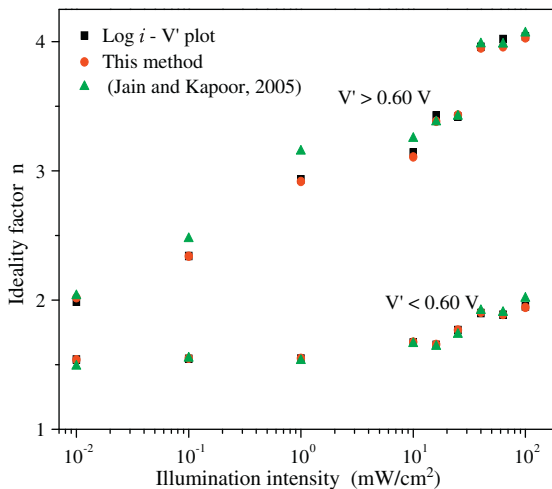


Fig. 3. Comparison of n values estimated by different techniques at various illumination intensities.

Table 1
The device parameters of the CIGS solar cell.

Illumination intensity (mW/cm ²)	V_{oc} (V)	J_{sc} (mA/cm ²)	FF	Eff. (%)	J_o (A/cm ²) Log i vs. V' plot	
					$V' > 0.6$ V	$V' < 0.6$ V
0.01	0.54	2.227	76.917	0.945	7.644×10^{-8}	1.81×10^{-9}
0.1	0.56	2.902	74.406	1.236	4.549×10^{-7}	2.444×10^{-9}
1.0	0.56	3.795	75.758	1.645	3.932×10^{-6}	2.723×10^{-9}
10	0.58	7.968	74.343	3.511	8.166×10^{-6}	1.096×10^{-8}
16	0.58	9.768	75.317	4.360	1.592×10^{-5}	1.139×10^{-8}
25	0.58	12.536	74.618	5.544	1.790×10^{-5}	3.027×10^{-8}
40	0.60	15.963	71.118	6.961	5.055×10^{-5}	8.894×10^{-8}
63	0.60	21.468	71.409	9.400	6.311×10^{-5}	1.089×10^{-7}
100	0.62	30.042	68.382	13.013	8.253×10^{-5}	2.013×10^{-7}

different illumination intensities, see Fig. 3. The modified graphical technique which includes the effects of parasitic series and shunt resistances seems to be quite simple and straight forward to get reliable n values and J_o values. However, it is obvious that the Lambert W -function method proposed here offer an attractive and reliable alternative way with relatively perfect accuracy for the estimation of diode ideality factors of a solar cell described by the double diode circuit model under different illumination intensities ranging from 0.01 to 100 mW/cm². The main advantage of this model proposed here is that the exact value for the diode ideality factor can be easily obtained from the $J - V$ data for a wide range of illumination intensities. Also, the diode quality factor of a pn junction solar cell modeled by single or double diode behavior can also be displayed clearly. Although, the method proposed by Jain and Kapoor (2005) originally proposed for single diode solar cells, here we have shown that this method can also be successfully used for the solar cells modeled with double diode structure.

Note however, for Jain and Kapoor's method, the strong precession of the device parameters i.e., R_s , R_{sh} , I_o , V_{oc} and I_{sc} are obviously important to get more accurate n values. It can be emphasized that the method proposed in this study is not based alone on the device parameters but also the illuminated JV characteristics. Therefore, it might be supposed to be the reason of advanced fit of all illumination dependent n values to those estimated by the improved graphical method.

5. Analysis of illuminated dependent n

In the second part of this work, the impact of illumination on the current transport properties of the solar cell was investigated through with the light intensity dependence of the ideality factor and the reverse saturation current. Fig. 4 clearly illustrates that for about $V' > 0.6$ V, the value of n increases from about 2 to 4 as the illumination intensity increases. In the literature, the analysis on CIGS (Cu(In,Ga)Se₂) based solar cells often reveals that when n exceeds 2 the tunneling enhanced interface recombination mechanism becomes the dominating route for the current transport (Nadenau et al., 2000). It can be also seen from the same plot and the tables that not only the value of n but

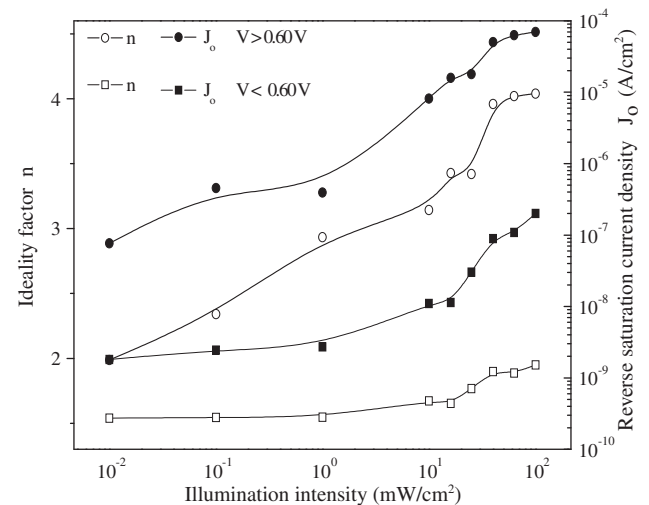


Fig. 4. Dependence of n and J_o on the illumination intensity.

also J_o values increase as the illumination intensity increases which probably indicates that the illumination activates the interface recombination of photo-generated carriers at junction interface (Saad and Kasis, 2003). As was reported by Zabierowski and Igalson (2000) and Nadenau et al. (2000), illumination of a typical CIGS based solar cell results in a metastable increase of the emission rate of the trapping states at junction interface and thus probably causes the shift of the Fermi level. One should expect also that this redistribution in the charge density influence the band bending in the absorber Cu(In,Ga)Se₂ layer at about hetero-interface (Zabierowski and Igalson, 2000). This probable re-arrangement and subsequent bending of the energy bands under illumination might possibly give rise to an apparent increase in the magnitude of the interface recombination current through the junction (Agostinelli et al., 2003). This also explains why the open circuit voltage, the short circuit current and the fill factor values of the solar cell decreases as the illumination intensity increases from 0.01 to 100 mW/cm², as it is seen in Table 1. In addition, when illumination intensity increases beyond 40 mW/cm², a relatively slight variation in the values of n and J_o is observed in this relatively high voltage region. Although illumination of the cell increases the density of charged defect states leading to a higher recombina-

tion current, the observed slight variation may possibly indicate that the interface recombination is limited by the excess free holes as proposed by Saad and Kasis (2003).

For $V < 0.6$ V; as it is seen in Fig. 4, both n and J_o values are found to be almost invariant under low illumination intensities (<1 mW/cm²), however as the illumination intensity is increased further, relatively slightly increasing values are noted for n and J_o . Although with these parameters alone it is quite difficult to identify the dominating current transport mechanism certainly, the lowest saturation current J_o values and the range of the n values ($1.55 < n < 1.95$) could possibly indicate that the contribution of recombination in the space charge region dominates in this bias region. Nevertheless, since both n and J_o increases with increasing illumination intensity beyond 1 mW/cm², we can propose that although its contribution is illumination still activates the interface recombination of photo-generated carriers.

The model presented by Agostinelli et al. (2003) seems to be consistent with our results given above. According to this method that both light dependent recombination currents and the relative magnitudes of the light modulated majority carrier barriers ϕ_{CdS} (the distance between the electron quasi Fermi level and the CdS conduction band minimum) and $\phi_{absorber}$ (the distance between the hole quasi Fermi level and the absorber valence band minimum) have an influence on the light dependent current transport mechanisms in chalcogenide solar cells. They have proposed that interface recombination current dominates for the relatively high voltages, since for this case $\phi_{CdS} > \phi_{absorber}$. However for relatively low applied voltages, since $\phi_{absorber} > \phi_{CdS}$, the contribution of interface recombination to the $J - V$ characteristics have been reduced.

In order to demonstrate the effect of illumination on the current transport mechanism of the solar cell under consideration, the dark room temperature $J - V$ characteristics are also analysed. The values of the ideality factor n are estimated by using the ordinary graphical method and found as about 1.345 and 2.549 for $V > 0.4$ V and $V < 0.4$ V, respectively. For a usual CIGS based solar cell, such typical values for n could indicate that interface recombination has a pronounced contribution to the total current transport particularly for $V < 0.4$ V. Above than 0.4 V, recombination in the space charge layer could play a major role (Nadenau et al., 2000). These routes for the recombination are almost opposite of those proposed for the illuminated conditions. However, the presence of this discrepancy must be mainly attributed to a significant increase in loss current resulting from increase in the magnitude of interface recombination under illumination (Saad and Kasis, 2003; Nadenau et al., 2000).

6. Conclusion

A simple approach based on Lambert W -function is shown to be applicable for the determination of diode ideality factor values of a pn junction solar cell approxi-

mated by double junction behavior. Different illumination intensities in the range 0.01–100 mW/cm² are considered to specify the limit of the proposed method. The results indicate that the method proposed here enables excellent accuracy for the estimation of n in a wide range of illumination intensities for the typical double diode device considered. It can also be clearly seen that this method is very simple to use and gives highly consistent results with those obtained by modified graphical method. Additionally, we also propose that the plot of $\text{Log}(J + J_{sc})$ vs. $V' = (V + J_{ph}R_s)$ results a more accurate approximation for the graphical estimation of the junction ideality factor under different illumination intensities. Another simple method which also depends on the Lambert W -function has also been used for comparison. We have shown that this method can also be successfully used for the double diode solar cells under different illumination intensities.

In the second part of the study, the light intensity dependence of the ideality factor and the reverse saturation current was investigated in order to provide insight into the alteration of electrical conduction in the solar cell. The strong dependence of both n and J_o to the illumination intensity indicates that illumination activates the interfacial states. The contribution of interface recombination to the total current is found to be reduced as the illumination intensity decreased. The comparison between the illumination intensity dependent behavior of both ideality factor and the reverse saturation current in low and high voltage regions revealed that the re-arrangement and subsequent bending of the energy bands under illumination modifies the contribution of interface recombination to the total current transport across the solar cell.

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