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EFFECT OF PARTIAL ILLUMINATION ON THE OPEN-CIRCUIT VOLTAGE OF A SOLAR CELL

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Summary

A distributed parameter model has been proposed to account for the effect of partial illumination on the open-circuit voltage of a solar cell. The theory helps one to understand the recent experimental results of Cuevas and Romero.

1. Introduction

In a recent paper, Cuevas and Romero [1] gave experimental evidence and a possible theoretical explanation to show that the open-circuit voltage of a non-uniformly illuminated solar cell may be much less than that of a uniformly illuminated cell for the same value of short-circuit current. This problem has also been addressed by Dhariwal *et al.* [2] in an earlier paper. However, the results obtained from the theoretical modelling reported in these two papers do not seem to agree. We would contend that this is because the physical situations described in the two papers are quite different. The model of Dhariwal *et al.* [2] refers to a solar cell with a very fine grid structure, as is usually encountered in concentrator solar cells. The non-uniformities of illumination are assumed to be on a scale much larger than the grid spacing. In this situation one can assume the entire top surface to be equipotential. Details of the electrostatic potential variations in the regions between the grid lines are not important. A lumped parameter model can be used to analyse such a situation. The solar cell can be considered as two p–n junctions, one illuminated and the other dark, connected in parallel by external electrodes.

The situation considered by Cuevas and Romero [1] is altogether different. Figure 1 of the present paper is a reproduction of Fig. 2 of their paper which describes the configuration used in their study. Here the problem refers to non-uniform illumination within the grid spacing. The current flows within the emitter of the solar cell and the details of the electrostatic potential across this semiconducting layer now become important. The situation is of great practical interest as it provides details of potential

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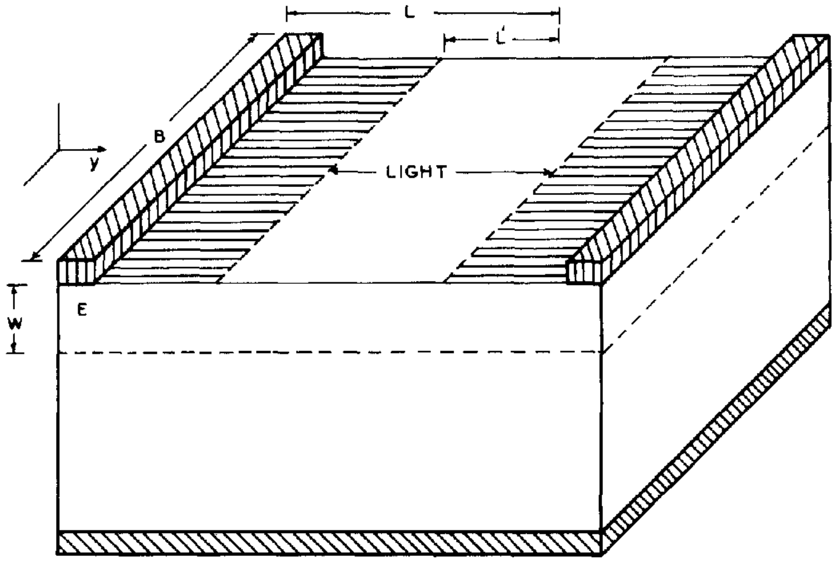

Fig. 1. The configuration under study.

variations due to the sheet resistance. We find that a distributed parameter model is more appropriate for such a configuration. A continuity equation for current in the sheet has been given by Luckovsky [3]; we make our calculations here using this equation. However, Luckovsky does not discuss the case of a solar cell of the configuration considered here. We find that the solution of Luckovsky's equation under suitable approximations and boundary conditions provides the correct answer to the problem. For completeness of the argument it will be worthwhile first to give a brief derivation of Luckovsky's equation, which will help us in understanding the basic mechanism underlying the flow of current.

2. Luckovsky's continuity equation

In Fig. 2 we consider a volume element of the sheet lying between the junction boundary and the grid structure of the solar cell. The length, width and breadth of this volume element are δy, W and B respectively. The density of current entering at the junction boundary is $\{J_{sc}(y) - J_d(y)\}$. Continuity of current requires that

$$WB\{J(y + \delta y) - J(y)\} = \delta y B\{J_{sc}(y) - J_d(y)\} \tag{1}$$

where we have set $J = 0$ for the top surface because current does not leave the solar cell in the open-circuit condition. The terms J_{sc}, J_d, etc. have their usual meanings. Equation (1) can be rewritten as

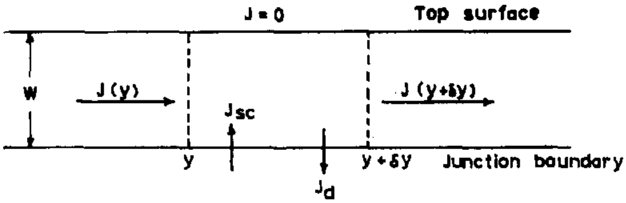

Fig. 2. Direction of various current components for setting up the current continuity equation in the sheet connecting the p–n junction with the metallic grid.

$$W\frac{\mathrm{d}J}{\mathrm{d}y} = J_{sc}(y) - J_{d}(y) \tag{2}$$

We note that

$$J = -\frac{1}{\rho}\frac{\mathrm{d}V}{\mathrm{d}y} \tag{3}$$

where ρ is the resistivity of the sheet material. This gives

$$-\frac{W}{\rho}\frac{\mathrm{d}^2V}{\mathrm{d}y^2} = J_{sc}(y) - J_{d}(y) \tag{4}$$

This is the equation of continuity as given by Luckovsky [3].

3. Solution of the continuity equation

To solve eqn. (4) let us write J_d in terms of the forward voltage of the junction

$$J_d = J_{d0}\left\{\exp\left(\frac{qV}{kT}\right) - 1\right\} \tag{5}$$

and let us define an average potential V_0 and a variation δV such that

$$V = V_0 + \delta V \tag{6}$$

For $\delta V < kT/q$, we can write

$$J_d = \left\{J_{d0}\exp\left(\frac{qV_0}{kT}\right)\right\}\left(1 + \frac{\delta V}{kT/q}\right) - J_{d0} \tag{7}$$

V_0 can be defined as the open-circuit voltage of a uniformly illuminated solar cell, such that

$$2LJ_{d0}\left\{\exp\left(\frac{qV_0}{kT}\right) - 1\right\} = \int_{-L}^{L} J_{sc}(y)\,\mathrm{d}y \tag{8}$$

Next, let us consider the illumination condition as presented by Cuevas and Romero [1]

$$J_{sc}(y) \begin{cases} = J_{sc0} & 0 < |y| < L' \\ = 0 & L' < |y| < L \end{cases} \tag{9}$$

For this case, we obtain from eqn. (8)

$$J_{d0}\left\{\exp\left(\frac{qV_0}{kT}\right) - 1\right\} = fJ_{sc0} \tag{10}$$

where

$$f = \frac{L'}{L} \tag{11}$$

gives the fraction of the total surface area which has been illuminated by light. The fraction of the shaded area is given by

$$f_s = 1 - f \tag{12}$$

Using eqns. (7) and (10), eqn. (4) becomes

$$\frac{W}{\rho}\,\frac{d^2\delta V}{dy^2} - \frac{(fJ_{sc0} + J_{d0})}{(kT/q)}\,\delta V = -J_{sc}(y) + fJ_{sc0} \tag{13}$$

$J_{sc}(y)$ of eqn. (9) can be expressed as a Fourier series

$$J_{sc}(y) = J_{sc0}\left\{f + \sum_{m=1}^{\infty} \frac{2}{m\pi}\sin(m\pi f)\cos\left(m\pi\frac{y}{L}\right)\right\} \tag{14}$$

Thus we get from eqn. (13)

$$\frac{d^2\delta V}{dy^2} - \frac{\rho(fJ_{sc0} + J_{d0})}{W(kT/q)}\,\delta V = -\sum_{m=1}^{\infty} \frac{\rho 2J_{sc0}}{W}\,\frac{\sin m\pi f}{m\pi}\cos m\pi\frac{y}{L} \tag{15}$$

Since δV must follow the condition

$$J_y(\pm \mathrm{L}) = \frac{1}{\rho}\,\frac{d\delta V}{dy}\bigg|_{y=\pm L} = 0 \tag{16}$$

the solution of eqn. (15) will be of the form

$$\delta V = \sum_{m=1}^{\infty} \delta V_m \cos m\pi\frac{y}{L} \tag{17}$$

Substituting this in eqn. (15) we obtain

$$\sum_{m=1}^{\infty}\left\{\left(\frac{m\pi}{L}\right)^2+\frac{\rho(fJ_{sc0}+J_{d0})}{W(kT/q)}\right\}\delta V_m\cos m\pi\frac{y}{L}$$

$$=\sum_{m=1}^{\infty}\frac{2\rho J_{sc0}}{W}\frac{\sin m\pi f}{m\pi}\cos m\pi\frac{y}{L} \tag{18}$$

On comparing the various terms, we obtain

$$\delta V_m=\frac{(L\rho/WB)I_{sc}\{\sin(m\pi f)/m\pi f\}}{(m\pi)^2+\frac{1}{2}(L\rho/WB)\{q(I_{sc}+I_{d0})/kT\}} \tag{19}$$

where we have put

$$J_{sc0}f2LB=I_{sc} \tag{20a}$$

and

$$J_{d0}2LB=I_{d0} \tag{20b}$$

If we assume that the series resistance contribution of a solar cell comes mostly from the sheet resistance, we have [1]

$$R=\frac{1}{3}\frac{L\rho}{WB} \tag{21}$$

Equation (19) then gives

$$\delta V_m=\frac{3RI_{sc}\{\sin(m\pi f)/m\pi f\}}{(m\pi)^2+\{3(I_{sc}+I_{d0})R/2(kT/q)\}} \tag{22}$$

The equation for δV becomes

$$\delta V(y)=\sum_{m=1}^{\infty}\frac{3RI_{sc}\{\sin(m\pi f)/m\pi f\}}{(m\pi)^2+\{3(I_{sc}+I_{d0})R/2(kT/q)\}}\cos m\pi\frac{y}{L} \tag{23}$$

The potential difference at the contact grids placed at $y=\pm L$ is given by

$$\delta V(L)=\sum_{m=1}^{\infty}\frac{(-1)^m 3RI_{sc}\{\sin(m\pi f)/m\pi f\}}{(m\pi)^2+\{3(I_{sc}+I_{d0})R/2(kT/q)\}} \tag{24}$$

Equation (24) gives an expression for the voltage reduction in a solar cell of which a fractional area f has been illuminated.

4. Results and discussion

Since we have assumed $\delta V < kT/q$ for the analytical solution to be possible, the same assumption gives us

$$\frac{3(I_{sc} + I_{d0})R}{2(kT/q)} \ll \pi^2$$

and we can write

$$\delta V(L) = \sum_{m=1}^{\infty} (-1)^m \frac{3RI_{sc}\{\sin(m\pi f)/m\pi f\}}{m^2\pi^2} \tag{25}$$

The above series can be summed to give

$$\delta V(L) = -0.25\, RI_{sc}(1 - f^2) \tag{26}$$

In Fig. 3 results obtained from eqn. (26) have been plotted against f for various values of RI_{sc}. We find that δV is a maximum for a very narrow line of illumination and becomes zero for uniform illumination.

It is important to note that in the above derivation we have made a linearization approximation in eqn. (7) which holds true for $\delta V < kT/q$. Such an approximation has been made in order to obtain an analytical expression for $\delta V(L)$. For higher voltages eqn. (4) becomes non-linear and

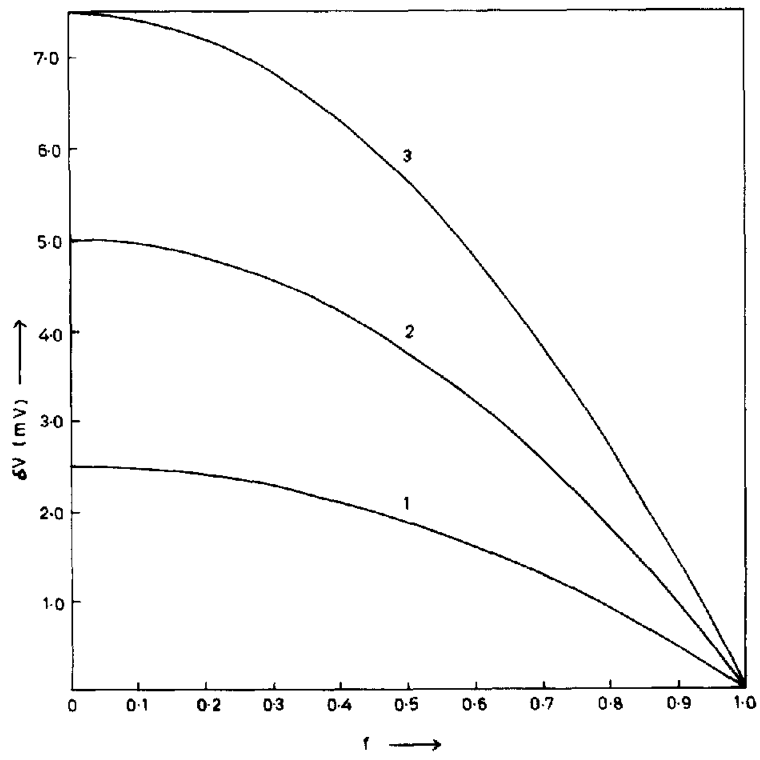

Fig. 3. Variation of the change in open-circuit voltage δV with the fraction f of the total surface area of the cell illuminated. Curves 1, 2 and 3 correspond to values of $I_{sc}R$ of 10, 20 and 30 mV respectively.

analytical solutions are difficult to obtain. Equation (26) is only valid for values of $\delta V(L)$ less than the thermal voltage (26 mV at room temperature).

Finally, we would like to point out that R itself is a voltage-dependent parameter [4] and due care must be taken in using the value of R in the above calculations. Referring to the values of R given in ref. 1, for the present analysis one should use a value of R corresponding to a uniformly illuminated solar cell. Also, the limitations of the one-dimensional analysis presented here only enable us to treat the open-circuit case. In due course we shall try to give a two-dimensional analysis which will reproduce the effect of non-uniform illumination over the entire I–V curve.

Acknowledgment

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