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Effect of Coulomb interaction on current noise in open quantum dots

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We analyze the effect of Coulomb interaction on the noise of electric current through an open quantum dot. We demonstrate that the ensemble average value of the noise power acquires an interaction correction even for a dot coupled to the leads by reflectionless point contacts, when the ensemble average conductance is known to have no interaction corrections. To leading order, the correction to the noise originates from the formation of a nonequilibrium state of the Coulomb field describing the interaction between electrons. We find the dependence of the current noise power on the electron temperature, the applied voltage bias, and the strength of the Coulomb interaction.

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The Coulomb interaction in quantum dots is usually associated with the phenomenon of the Coulomb blockade,[1] which occurs in a quantum dot coupled to the leads through tunnel barriers. In the Coulomb blockade regime, electric current strongly depends on gate voltages due to the electrostatic energy cost to change the electron number in the dot. As contacts between the leads and the dot become more transparent, the Coulomb blockade becomes less pronounced. In the case of fully transmitting contacts (reflectionless contacts), the Coulomb interaction has no effect on the conductance of a quantum dot,[2–4] averaged over the unitary ensemble (i.e., in the presence of magnetic field), but it still manifests itself in the correlation functions of the conductance.[5]

Additional information about electron correlations can be gained by measuring other transport properties such as the shot noise.[6] Indeed, the study of the current noise in quantum dots is currently the subject of numerous experiments,[7–13] aimed at understanding the interplay between Fermi statistics and electron repulsion. This study may present opportunities to use this interplay in quantum information technologies. In most cases the experiments were performed in the regime of Coulomb blockade, when coupling between leads and the dot is weak. The question arises as to what happens to the power of the current noise as the contacts become transparent. In this paper, we investigate the effect of the Coulomb interaction on the noise power of electric current, averaged over a unitary ensemble, for quantum dots with reflectionless contacts. In particular, we are interested in understanding how relaxation in the dot affects the noise power.

A phenomenological description of the effects of interaction between electrons on the current noise through a quantum dot was developed within a model of "fictitious voltage probes."[14] The properties of such fictitious probes are assumed to represent the proper effect of interactions on electrons, and three main scenarios[6,15] were considered: (i) quasielastic scattering (phase relaxation), (ii) inelastic scattering (energy relaxation), and (iii) heating. Although this description is sufficient in many cases to characterize the interaction effects on current noise, it does not specify the relation between microscopic parameters of the system and the magnitude of the current noise. An alternative description of the interaction effects on current noise was developed in Refs. 16–18, based on the "effective action" analysis[19] of the cumulant generating function. The effect of inelastic scattering was considered, for quantum contacts, in Ref. 18; however, the configuration of the environment was taken to be the equilibrium state of the field, decoupled from the nonequilibrium electron system. Also, according to Ref. 17, the elastic interaction correction to the generating function vanishes for a quantum dot with reflectionless contacts for any state of the dot; the effect of the electron-electron interaction in the inelastic channel was not analyzed.

Here we show that, due to inelastic processes, in the stationary nonequilibrium state of the dot's collective excitations a finite correction to the average current noise arises, which survives even in the case of reflectionless channels, when the elastic contribution vanishes. For example, for a unitary ensemble of quantum dots coupled to the left (right) electron reservoirs by point contacts containing $N_l(N_r)$ reflectionless channels ($N_t=N_l+N_r\gg 1$), we obtain for the power of the shot noise at zero temperature $T=0$ and intermediate bias $1/\tau\ll|eV|\ll E_C N_t$ (here and below $\hbar=1$ and $k_B=1$)

$$S = 2e\mathcal{F}|I|\left(1+\frac{1-2\mathcal{F}}{N_t}(\ln|eV\tau|-1)\right), \tag{1}$$

where $I=\mathcal{G}V$ is the current in response to voltage V, $\mathcal{G}=(e^2/2\pi\hbar)N_lN_r/N_t$ and $\mathcal{F}=N_lN_r/N_t^2$ are the ensemble average conductance and Fano factor, respectively. The second term in Eq. (1) represents the $1/N_t$ correction due to the Coulomb interaction, $E_C=e^2/2C$ is the charging energy, $\tau=2\pi/(N_t\delta_1)$ is the electron dwell time in the dot, and δ_1 is the mean level spacing in the dot. As we employ the random matrix theory description of the dots' ensemble, we assume that the Thouless energy $E_T=1/\tau_f$, where τ_f is the electron flight time across the dot, is the largest energy scale, so that, for example, $E_T\gg 1/\tau$.

We consider a quantum dot, coupled by reflectionless point contacts to the electron reservoirs. Electron energy distribution is given by $\tilde{n}_l=n_F(\epsilon-eV)$ in the left reservoir and by $\tilde{n}_r=n_F(\epsilon)$ in the right reservoir, where $n_F(\epsilon)=1/[\exp(\epsilon/T)+1]$ is the Fermi distribution function. Taking into account the interaction between electrons in a quantum dot, we obtain[21] the following expression for the ensemble averaged noise power of the current through a quantum dot with reflectionless point contacts in terms of electron distribution functions in the leads $\tilde{n}_{l,r}(\epsilon)$ and in the dot $n(\epsilon)$:

$$S = 2\mathcal{G}\int d\epsilon\left(n(\epsilon)[1-n(\epsilon)] + \frac{N_r}{N_t}\tilde{n}_l(\epsilon)[1-\tilde{n}_l(\epsilon)] + \frac{N_l}{N_t}\tilde{n}_r(\epsilon)[1-\tilde{n}_r(\epsilon)]\right). \quad (2)$$

We note that Eq. (2) has the structure identical to that for the noise power in the absence of interaction. The interaction effects are completely hidden in the configuration of the distribution function $n(\epsilon)$ in the dot, and the further evaluation of the interaction corrections to the noise reduces to calculation of $n(\epsilon)$.

The distribution function $n(\epsilon)$ of electrons in the dot is found by solving a kinetic equation. The derivation of the kinetic equation for electrons in quantum dots can be found in Ref. 20. Here we just present the explicit form of the kinetic equation for the quantum dot with reflectionless contacts in a steady state:

$$\frac{n(\epsilon)}{\tau} = \frac{\delta_1}{2\pi}[N_l\tilde{n}_l(\epsilon) + N_r\tilde{n}_r(\epsilon)] + \hat{\mathrm{St}}_{(\mathrm{in})}\{n,\tilde{n}_n,N^\alpha\} + \hat{\mathrm{St}}_{(\mathrm{vb})}\{n,\tilde{n}_n\}. \quad (3)$$

The collision integral in the right hand side of Eq. (3) is written as the sum of three terms. The first term describes the relaxation of $n(\epsilon)$ due to the electron exchange between the leads and the dot. The other two terms of the collision integral, $\hat{\mathrm{St}}_{(\mathrm{in})}\{n,\tilde{n}_n,N^\alpha\}$ and $\hat{\mathrm{St}}_{(\mathrm{vb})}\{n,\tilde{n}_n\}$, represent the effect of electron-electron interaction in the Coulomb channel. We identify $\hat{\mathrm{St}}_{(\mathrm{in})}\{n,\tilde{n}_n,N^\alpha\}$ as the inelastic collision integral, corresponding to the processes in which an electron changes its energy due to emission or absorption of a bosonic collective excitation, while $\hat{\mathrm{St}}_{(\mathrm{vb})}\{n,\tilde{n}_n\}$ takes into account electron-electron scattering via the exchange of virtual bosons.

The inelastic collision integral

$$\hat{\mathrm{St}}_{(\mathrm{in})}(\epsilon) = \frac{\delta_1^2}{4\pi^2}\sum_{n=l,r}N_n\int\frac{d\omega}{\omega} \times \mathrm{Re}\{(1+F^\rho)\mathcal{L}^\rho_\omega\tilde{\Upsilon}^\rho_n(\epsilon,\omega) - \mathcal{L}^g_\omega\tilde{\Upsilon}^g_n(\epsilon,\omega)\} \quad (4)$$

can be written as a difference between a charge boson part (ρ) and a ghost part (g). The details about the need and meaning of this separation can be found in Ref. 22. The propagators $\mathcal{L}^\alpha_\omega$ and coupling constants $F^{\rho,g}$ are given by

$$\mathcal{L}^\alpha_\omega = \frac{1}{-i\omega + (1+F^\alpha)/\tau}; \quad F^\rho = \frac{4E_c}{\delta_1}; \quad F^g = 0. \quad (5)$$

The functions $\tilde{\Upsilon}^\alpha_n(\epsilon,\omega)$ describe the differences between absorption and emission rates of bosonic excitations in lead $n=\{l,r\}$:

$$\tilde{\Upsilon}^\alpha_n(\epsilon,\omega) = (N^\alpha_\omega+1)\tilde{n}_n(\epsilon)[1-\tilde{n}_n(\epsilon-\omega)] - N^\alpha_\omega[1-\tilde{n}_n(\epsilon)]\tilde{n}_n(\epsilon-\omega). \quad (6)$$

The charge and ghost boson occupation numbers $N^{\rho,g}_\omega$ satisfy the steady-state kinetic equations[20]

$$0 = \frac{1+F^\alpha}{2\omega}\sum_{n=l,r}N_n\int d\epsilon[\tilde{\Upsilon}^\alpha_n(\epsilon,\omega) + \Upsilon^\alpha(\epsilon,\omega)] \quad (7)$$

with $\Upsilon^\alpha(\epsilon,\omega)$ obtained by replacing $\tilde{n}_n \to n$ in Eq. (6). These equations give $N^{\rho,g}_\omega = N_\omega$ as functionals of $n(\epsilon)$ and $\tilde{n}_n(\epsilon)$:

$$N_\omega = \frac{1}{2\omega}\int d\epsilon\sum_{n=l,r}\frac{N_n}{N_t} \times\{n(\epsilon)[1-n(\epsilon-\omega)] + \tilde{n}_n(\epsilon)[1-\tilde{n}_n(\epsilon-\omega)]\}. \quad (8)$$

Note that the identity of the charge and ghost boson occupation numbers also results in $\tilde{\Upsilon}^{\rho,g}_n(\epsilon,\omega) = \tilde{\Upsilon}_n(\epsilon,\omega)$ and $\Upsilon^{\rho,g}(\epsilon,\omega) = \Upsilon(\epsilon,\omega)$.

The second collision integral in Eq. (3) describes electrons interacting by exchanging virtual bosons:

$$\hat{\mathrm{St}}_{(\mathrm{vb})} = -\frac{\delta_1^2 N_t}{2\pi^2}\int\frac{d\omega}{\omega}\frac{\Upsilon(\epsilon,\omega)}{2+F^\rho}\mathrm{Re}[(1+F^\rho)\mathcal{L}^\rho_\omega - \mathcal{L}^g_\omega]. \quad (9)$$

We emphasize that this form of $\hat{\mathrm{St}}_{(\mathrm{vb})}$ given in terms of $\Upsilon(\epsilon,\omega)$, which contains the boson occupation number N_ω, is valid only in the steady state of the boson fields. In a general time-dependent state, $\hat{\mathrm{St}}_{(\mathrm{in})}$, Eq. (4), contains the boson occupation numbers $N^{\rho,g}_\omega$, which can be found as solutions to the corresponding nonstationary counterpart of Eq. (7), while $\hat{\mathrm{St}}_{(\mathrm{vb})}$ is still given in terms of the function $\Upsilon(\epsilon,\omega)$ in which N_ω is *defined* by Eq. (8) as a combination of electron distribution functions $n(\epsilon)$ and $\tilde{n}_{l,r}(\epsilon)$.

We look for an iterative solution to the kinetic equation Eq. (3) in the form

$$n(\epsilon) = n_0(\epsilon) + \delta n(\epsilon); \quad n_0(\epsilon) = \frac{N_l\tilde{n}_l(\epsilon) + N_r\tilde{n}_r(\epsilon)}{N_t}, \quad (10)$$

where $\delta n(\epsilon)$ represents the lowest order correction to the distribution function due to the Coulomb interaction. Here we assume that the dominant scattering mechanism is the escape from the dot; therefore δn contains the additional small factor $1/N_t$. As a result, we can write the expression for the power of low-frequency current noise in the form

$$S = S_0 + \delta S; \quad S_0 = 4\mathcal{G}T + \mathcal{G}\mathcal{F}Y(eV,0);$$
$$\delta S = 2\mathcal{G}\int d\epsilon\,\delta n(\epsilon)[1-2n_0(\epsilon)]. \quad (11)$$

Here S_0 is the known result for the ensemble average value of current noise power in the absence of interaction. We introduced the function

$$Y(U,\omega) = \sum_\pm (U\pm\omega)\coth\frac{U\pm\omega}{2T} - 2\omega\coth\frac{\omega}{2T}, \quad (12)$$

which determines the power of the shot noise at frequency ω through a point contact.[23] For the low-frequency noise power S_0 only the limit $Y(U,\omega\to 0)$ is needed, but $Y(U,\omega)$ for arbitrary ω will also appear below.

To calculate the lowest order contribution to $\delta n(\epsilon)$ due to the Coulomb interaction, we replace $n(\epsilon)$ by $n_0(\epsilon)$ in Eqs. (8) and (9). As a result, we find the boson occupation numbers

$$N_\omega = N_\omega^P + \frac{\mathcal{F}Y(eV,\omega)}{4\omega}, \quad N_\omega^P = \frac{1}{e^{\omega/T}-1}. \tag{13}$$

Here, the first term is the equilibrium value of N_ω, represented by the Planck function N_ω^P, while the second term describes the nonequilibrium contribution due to finite bias V. The connection between $N_\omega - N_\omega^P$ and the power of shot noise at frequency ω is not occasional, as the noise measurement at frequency ω has the physical interpretation[24] of the number of electromagnetic quanta detected by the measurement circuit.

Substituting $n_0(\epsilon)$, Eq. (10), and N_ω, Eq. (13), into Eqs. (4) and (9), we find a correction to the electron distribution function $\delta n(\epsilon)$ as the first iteration solution to the kinetic equation, Eq. (3):

$$\begin{aligned}\delta n(\epsilon) = \mathcal{F}\int \frac{d\omega}{\omega}\frac{\delta_1}{2\pi}\mathrm{Re}[(1+F^\rho)\mathcal{L}_\omega^\rho - \mathcal{L}_\omega^g] \\ \times\Bigg\{\frac{F^\rho}{2+F^\rho}\frac{Y(eV,\omega)}{4\omega}[n_0(\epsilon)-n_0(\epsilon-\omega)] \\ -\frac{2}{2+F^\rho}[(N_\omega^P - N_{\omega-eV}^P)[\tilde{n}_l(\epsilon)-\tilde{n}_r(\epsilon-\omega)] \\ +(N_\omega^P - N_{\omega+eV}^P)[\tilde{n}_r(\epsilon)-\tilde{n}_l(\epsilon-\omega)]]\Bigg\}.\end{aligned} \tag{14}$$

Thanks to the identity

$$\frac{\delta_1}{2\pi}\mathrm{Re}[(1+F^\rho)\mathcal{L}_\omega^\rho - \mathcal{L}_\omega^g] = \frac{1}{N_t}\frac{\omega^2\tau^2F^\rho(2+F^\rho)}{(\omega^2\tau^2+1)[\omega^2\tau^2+(1+F^\rho)^2]}, \tag{15}$$

it becomes evident that the correction to the distribution function $\delta n(\epsilon)$ due to the electron-electron interaction has a small factor $1/N_t$. All other factors in the expression for $\delta n(\epsilon)$ are of the order of unity; in particular, the combinations of the divergent Plank's functions $N^P(\omega)$ and the Fermi functions $n_F(\epsilon)$ are such that the integrand for $\delta n(\epsilon)$ does not have divergences at either frequency $\omega=0, \pm eV$.

To arrive at the final expression for the correction to the noise power, we substitute Eqs. (14) and (15) into Eq. (11) and obtain

$$\delta S = 2e\mathcal{F}|I|\frac{\chi(eV,T,F^\rho)}{N_t}; \tag{16a}$$

$$\chi = \int \frac{(F^\rho)^2K_1(eV,\omega) - 2F^\rho K_2(eV,\omega)}{(\omega^2\tau^2+1)[\omega^2\tau^2+(1+F^\rho)^2]}\frac{d\omega}{eV}, \tag{16b}$$

where

$$\frac{K_1(U,\omega)}{\tau^2} = Y(U,\omega)\left(\frac{Y(\omega,0)}{2} + \mathcal{F}(Y(\omega,U)-Y(\omega,0))\right),$$

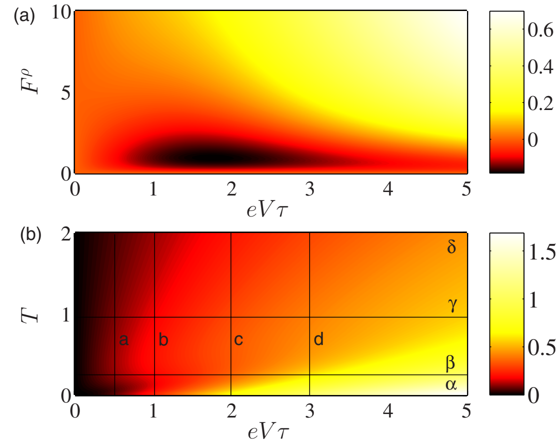

FIG. 1. (Color online). The correction to the noise power χ (a) as a function of bias and charging energy for $T=1/\tau$ and (b) as a function of bias and temperature for $F^\rho=100$.

$$\frac{K_2(U,\omega)}{\omega\tau^2} = \left[\sum_\pm N_{\omega\pm U}^P - 2N_\omega^P\right][Y(\omega,U)+Y(\omega,0)] + \left[\sum_\pm(\pm1)N_{\omega\pm U}^P\right]\left[\sum_\pm(\pm1)(\omega\pm U)\coth\frac{\omega\pm U}{2T}\right]. \tag{17}$$

Equations (16) and (17) are the main result of the paper. They are valid when random matrix theory is applicable and up to a large bias of order $V\sim N_t^2E_C/e$; above this bias, the assumption justifying the iterative solution in the form of Eq. (10) is not true anymore, although the kinetic equation approach is still viable.

The correction $\delta n(\epsilon)$ to the electron distribution function in the leads vanishes in equilibrium, when $V=0$. In this case the current noise contains only the Johnson-Nyquist component, which, in its turn, depends only on the linear conductivity of the system and for a dot with reflectionless point contacts has no interaction corrections.[2–4] At finite bias $V\neq 0$, the interaction correction to the noise appears. Below, we investigate the properties of δS for various relations between temperature T, voltage V, and interaction strength F^ρ.

We consider first the zero temperature limit $T=0$; in this case K_2 vanishes, and performing the frequency integration in Eq. (16) we find

$$\chi = \frac{F^\rho(1-2\mathcal{F})}{2+F^\rho}\left(\ln\sqrt{\frac{1+(\tau eV)^2}{1+(\tau_c eV)^2}} + \frac{\arctan eV\tau}{eV\tau} - \frac{\arctan eV\tau_c}{eV\tau_c}\right), \tag{18}$$

where $\tau_c=\tau/(1+F^\rho)$. For large $F^\rho \gg 1$ and intermediate values of bias V, $1/\tau \ll eV \ll E_CN_t$, Eq. (18) can be approximated by Eq. (1), while at larger bias, $eV\gg E_CN_t$, it has the asymptote $\chi=(1-2\mathcal{F})\ln(4E_C/\delta_1)$. Finally, at low bias, $eV\tau \ll 1$, we have

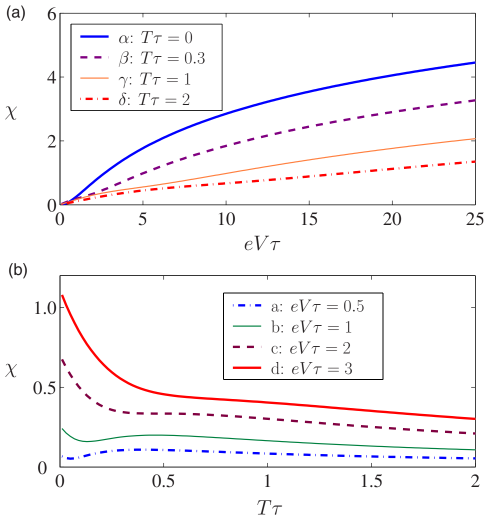

FIG. 2. (Color online). The correction to the noise power χ for $F^\rho=100$ (a) as a function of bias for different temperatures and (b) as a function of temperature for different biases. The labels refer to the cuts along the corresponding lines in Fig. 1(b).

$$\chi = \frac{1}{6}(1-2\mathcal{F})\left(\frac{F^\rho}{1+F^\rho}\right)^2 (eV\tau)^2, \tag{19}$$

whose quadratic dependence on the voltage can be explained by a standard phase-space argument. As the bias grows, this argument breaks down since the energy window to explore ($\sim eV$) becomes too big compared to the energy correlation in electron spectrum ($\sim 1/\tau$).

At finite temperature, the interaction correction to the noise power acquires a temperature dependence. Particularly, in the regime of validity of Eq. (1), when $1 \ll eV\tau \ll F^\rho$, the lowest order correction to the noise can be estimated as

$$\frac{\chi(T)-\chi(T=0)}{\chi(T=0)} = -\frac{\pi T\tau}{\ln|eV\tau|} \tag{20}$$

at small temperature $T \ll 1/\tau$.

The dependence of $\chi(eV,T,F^\rho)$ on the charging energy $E_C = F^\rho\delta_1/4$ and bias V is shown in Fig. 1(a) for finite $T = 1/\tau$. At small $F^\rho \lesssim 1$ and small bias $eV\tau \lesssim 1$ the correction to the noise can be negative. The negative values of χ at $F^\rho \lesssim 1$ originate from the K_2 term in Eq. (16b). However, typically $F^\rho \gg 1$, and the contribution of the K_1 term dominates; the interaction correction to the ensemble average noise power is then positive for any bias V, as illustrated in Fig. 2(a), where dependence of χ on the applied bias V is shown for $F^\rho = 100$.

We present in Fig. 1(b) the contour plot of $\chi(eV,T,F^\rho)$ as a function of bias V and temperature T at large $F^\rho = 100$. Although, in general, temperature suppresses the interaction correction to the noise power, this suppression is nonmonotonic. This nonmonotonic behavior is further illustrated in Fig. 2(b), where temperature dependence of χ at fixed V and F^ρ is shown. The slope of the initial temperature suppression can be calculated, at intermediate bias, using Eq. (20).

Finally we note that in the limit of strong Coulomb interaction, $F^\rho \gg 1$, the main contribution to the power of current noise comes from term K_1 in Eq. (16b). This term originates from the correction to the boson occupation number N_ω, which is given by the second term in Eq. (13) containing $Y(eV,\omega)$. This nonequilibrium correction is due to the coupling between the bosons and the electrons out of equilibrium. If the boson occupation number N_ω is not evaluated from the kinetic equations, but chosen to describe the equilibrium state at the temperature T of the electrons in the leads, the leading order contribution to the noise power vanishes.

In summary, we have considered the noise power of quantum dots with reflectionless contacts, averaged over the unitary ensemble. For such dots, the Coulomb interaction does not affect the average conductance, but only its mesoscopic fluctuations. Here we have shown that the average noise acquires an interaction correction in the nonequilibrium state. This correction is present at zero temperature and is suppressed as temperature increases; interestingly, the suppression can be a nonmonotonic function of temperature.

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