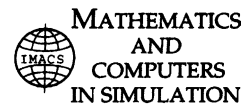




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# A hybrid chaotic genetic algorithm for short-term hydro system scheduling

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## Abstract

This paper proposes a novel hybrid chaotic genetic algorithm (HCGA) to solve the short-term generation scheduling of hydro system. The integration of chaotic sequence and genetic algorithm with a new self-adaptive error back-propagation mutation operator are developed, which can overcome premature and increase the convergence speed. Simulation results have demonstrated that the proposed approach is feasible and effective for the applications. © 2002 IMACS. Published by Elsevier Science B.V. All rights reserved.

**Keywords:** Chaos; Genetic algorithm; Short-term optimal dispatch; Hydroelectric system

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## 1. Introduction

From an economic point of view, an efficient hydropower scheduling plays an important role in the planning and operation of a power system. The purpose of short-term hydro system scheduling is to find the optimum hourly generation of hydro units by utilizing the limited water resource in a schedule horizon so as to optimize the total benefit of hydro generated energy, while satisfying various constraints. Cascaded hydroelectric plants are related to each other in both power and hydraulic aspects, so the short-term optimal dispatch of cascaded hydroelectric plants is a large-scale, dynamic with time delay and complicated constrained nonlinear optimization problem.

Various approaches have been proposed to solve the short-term optimal schedule problem of hydro generation system. The main methods include dynamic programming [1], maximum principle [2], network flow method and linear programming [3,4], nonlinear programming [5], functional analysis [6], mathematical decomposition and artificial neural networks [7,8]. These methods have one or another drawback such as dimensionality difficulty, large memory requirement or inability to handle a nonlinear cost function.

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### Nomenclature

$h_j^t$	net head of hydro plant $j$ at time $t$
$I_j^t$	natural inflow into reservoir $j$ at time $t$
$j$	plant index, $j = 1, 2, \dots, N$
$N$	number of hydro plants
$P_j^t$	power generation of hydro plant $j$ at time $t$
$P_{j \min}$	minimum power generation of hydro plant $j$
$P_{j \max}$	maximum power generation of hydro plant $j$
$Q_j^t$	water discharge of hydro plant $j$ at time $t$
$Q_{j \min}$	minimum water discharge of hydro plant $j$
$Q_{j \max}$	maximum water discharge of hydro plant $j$
$S_j^t$	water spillage of hydro plant $j$ at time $t$
$t$	time index, $t = 1, 2, \dots, 24$
$V_j^t$	water volume of reservoir $j$ at the end of hour $t$
$V_{j \min}$	minimum water volume of reservoir $j$
$V_{j \max}$	maximum water volume of reservoir $j$
$V_j^{\text{begin}}$	initial storage volume of reservoir $j$
$V_j^{\text{end}}$	final storage volume of reservoir $j$
$\tau_{j-1}$	water travel time from plant $j - 1$ to plant $j$

In recent years, a new optimization method called genetic algorithm (GA) has aroused intense interest, due to the flexibility, versatility and robustness in solving optimization problems, which above conventional optimization methods find difficult. It also has been given much attention to apply the short-term generation scheduling of hydro system [9]. However, there exist some flaws on GA. For example, it tends to converge prematurely and takes a large number of iterations to reach the global optimal solution, the optimization may get stuck at a local optimum. In order to overcome these flaws, it is necessary to adopt some improvements on GA to speed up the convergence and heighten the effectiveness of GA.

The chaotic optimization method based chaos theory has recently come into being as a new type of random search algorithm. Taking advantage of the universality, randomness and sensitivity dependence on initial conditions, it is more likely to acquire the global optimum solution. Although the chaotic optimization method reduces the search space by carrier wave and thus speeds up search, it makes no use of the experiential information previously acquired. As a result, search effect of chaotic optimization has its own limitation.

In order to overcome the shortcomings of both chaotic optimization method and GA, this paper presents a new optimization approach, i.e. hybrid chaotic genetic algorithm (HCGA) by integration of chaotic sequence and GA with a new self-adaptive error backpropagation mutation operator. Taking advantage of the sensitivity dependence on initial conditions makes it unlikely for the new population to trap simultaneously into the same local solution, while keep evolving without break. The optimum solution will be updated and premature convergence is restrained, thanks to the intervention of chaotic sequence and the introduction of new mutation operator. Finally, application of this algorithm to the short-term scheduling of cascaded hydroelectric system, simulation results have demonstrated the feasibility and effectiveness of the proposed approach for the practical applications.

## 2. Mathematical model for short-term cascaded hydro system scheduling

To formulate the problem and its solution mathematically, the following notation used in this paper is first introduced:

### 2.1. Objective function and constraints

Suppose the object studied is mainly power generation for cascaded hydroelectric systems. The short-term hydro scheduling problem can be stated as to find the water release from each reservoir and through each powerhouse over all the planning time intervals so as to maximize the total power generation while satisfying various constraints. Mathematically, the generation scheduling problem may be stated as follows:

$$\max f(V, Q) = \max \sum_{j=1}^N \sum_{t=1}^{24} P_j^t(Q_j^t, V_j^t, V_j^{t+1}) \quad (1)$$

subject to the following constraints.

#### 1. Hydro plant power generation limits

$$P_{j \min} \leq P_j^t \leq P_{j \max} \quad (2)$$

#### 2. Hydro plant discharge limits

$$Q_{j \min} \leq Q_j^t \leq Q_{j \max} \quad (3)$$

#### 3. Reservoir storage volumes limits

$$V_{j \min} \leq V_j^t \leq V_{j \max} \quad (4)$$

#### 4. Initial and terminal reservoir storage volumes

$$V_j^1 = V_j^{\text{begin}}, \quad V_j^{25} = V_j^{\text{end}} \quad (5)$$

#### 5. Water dynamic balance equation with travel time

$$V_j^t = V_j^{t-1} + I_j^t + Q_{j-1}^{t-\tau_{j-1}} + S_{j-1}^{t-\tau_{j-1}} - Q_j^t - S_j^t \quad (6)$$

### 2.2. Power generation characteristics

Power output of a hydro plant is a nonlinear function of effective reservoir head, turbine discharge and generating efficiency  $P_j^t = f(\eta_j^t, Q_j^t, V_j^t)$ . By approximating the plant generating characteristic at maximum head by a concave fourth degree polynomial function of turbine discharge and by adding a linear correction term for head losses, the following model is obtained:

$$P_j^t = a_{0j} + a_{1j} Q_j^t + a_{2j} (Q_j^t)^2 + a_{3j} (Q_j^t)^3 + a_{4j} (Q_j^t)^4 - \Delta H_j^t \quad (7)$$

Head losses  $\Delta H_j^t$  are modeled as a function with maximum head as reference

$$\Delta H_j^t = \beta_j \Delta h_j^t Q_j^t \quad (8)$$

In case power production depends on the upper reservoir level only, then

$$\Delta h_j^t = h_{\max j} - h_j^t \quad (9)$$

If power production depends on upper reservoir level as well as the level of the neighboring downstream reservoir,  $\Delta h_j^t$  is evaluated as

$$\Delta h_j^t = h_{\max j} - h_j^t + h_i^t - h_{\min i} \quad (10)$$

where  $a_{0j}, a_{1j}, a_{2j}, a_{3j}, a_{4j}, \beta_j$  are constants,  $h_{\max j}$  denote maximum water head at reservoir  $j$ ,  $h_{\min i}$  denote minimum water head at reservoir  $i$ , index  $i$  relates to the downstream reservoir.

Reservoir water heads are approximated as piecewise linear functions of the reservoir storage.

$$h_j^t = k_j^t + \alpha_j^t V_j^t = \min\{k_j^i + \alpha_j^i V_j^i\}, \quad i = 1, 2, \dots \quad (11)$$

### 3. Hybrid chaotic genetic algorithm (HCGA)

#### 3.1. Chaotic sequence

Chaos is apparently an irregular motion, seemingly unpredictable random behavior exhibited by a deterministic nonlinear system under deterministic conditions. Roughly speaking, a nonlinear system is said to be chaotic if it exhibits sensitive dependence on initial conditions and has an infinite number of different periodic responses. We must distinguish here between so-called random and chaotic motions. The former is reserved for problems in which we truly do not know the input forces or we only know some statistical measures of the parameters. Chaotic is reserved for those deterministic problems for which there are no random or unpredictable inputs or parameters. It may happen that small differences in the initial conditions produce very great ones in the final phenomena. A small error in the former will produce an enormous error in the latter.

The chaotic sequence can usually be produced by the following well-known one-dimensional logistic map defined by

$$t_{k+1} = \lambda t_k (1 - t_k); \quad t_k \in (0, 1), \quad k = 0, 1, 2, \dots \quad (12)$$

where  $t_k$  is the value of the variable  $t$  at the  $k$ th iteration,  $t_k$  in the interval  $[0, 1]$ ,  $\lambda$  is a so-called bifurcation parameter of the system.

For certain values of the parameter  $\lambda$ , of which  $\lambda = 4$  is one, the above system exhibits chaotic behavior.

#### 3.2. Implementation steps of hybrid chaotic genetic algorithm

Consider the following optimization problem:

$$\min f(x_i); \quad x_i \in [a_i, b_i], \quad i = 1, 2, \dots, n \quad (13)$$

where  $x_i$  is an unknown decision variable,  $f(x_1, x_2, \dots, x_n)$  is the objective function,  $a_i$  and  $b_i$  are the bounds of  $x_i$ ,  $i$  is the variable numbers index,  $i = 1, 2, \dots, n$ .

### 3.2.1. Initialization

The  $m$  initial values  $y_{j,i}(0) \in [0, 1]$  are randomly generated in the range 0 and 1. And then use above logistic map (12) to produce  $l$  chaotic variables.

$$\{y_{j,i}(k)\}; \quad j = 1, 2, \dots, m, \quad i = 1, 2, \dots, n, \quad k = 1, 2, \dots, l$$

### 3.2.2. Generating family members $x_{j,i}(k)$ from the chaotic variables $y_{j,i}(k)$

$$x_{j,i}(k) = a_i + (b_i - a_i)y_{j,i}(k) \quad (14)$$

### 3.2.3. Floating-point number encoding

Since the short-term optimization dispatch of hydro system is a large-scale complex nonlinear constrained optimization problem, the floating point numbers encoding technique is appropriate to apply. In the floating-point numbers representation each chromosome vector is coded as a vector of floating point numbers of the same length as the solution vector. Each element is initially selected as to be within the desired domain. In addition, the floating-point numbers representation is capable of representing quite large domains (or cases of unknown domains). Also, it is easier to handle constraints.

Considering the features of short-term generation scheduling of cascaded hydroelectric system, encoded parameters adopt plants' water discharge  $Q_j^t (j = 1, 2, \dots, N; t = 1, 2, \dots, 24)$ . Each chromosome string contains  $N \times 24$  genes to represent the solution for the hourly water discharge schedule of the  $N$  plants in 24 h scheduling horizon.

### 3.2.4. Selection

According to the fitness value  $f(x_{j,i}(k))$ , an individual of the highest fitness value which had been seek out from each family has chance to become the parent individual.

### 3.2.5. Crossover

New parent individuals are produced by crossover operation. Uniform arithmetical crossover are usually used for the floating-point numbers encoding individuals, i.e. the offspring individuals are produced by the linear combination of the parent individuals. Suppose two parent individuals that have been selected from the  $t$ th generation population are  $s_v^t = \langle v_1, v_2, \dots, v_n \rangle$ ,  $s_w^t = \langle w_1, w_2, \dots, w_n \rangle$ , respectively, offspring individuals that are produced by parent individuals are

$$s_v^{t+1} = \beta s_v^t + (1 - \beta) s_w^t, \quad s_w^{t+1} = \beta s_w^t + (1 - \beta) s_v^t \quad (15)$$

where  $\beta$  is a constant between 0 and 1.

### 3.2.6. Mutation

Mutation is an effective operator to increase and retain the population diversity, and is meanwhile an efficient method to escape the local optimum solution and to overcome the premature convergence. A large scale of mutation is good for acquiring the optimum solution in extensive search, but the search is rough and the solution precision is poor. On the other hand, if the precision is satisfactory, the solution will be got stuck at a local optimum or take too long time to converge. In view of overcoming these flaws, this paper adopts the self-adaptive error backpropagation mutation operator. The mutation rules

are shown as follows [10]:

$$\begin{aligned}
 x_j^i(k+1) &= x_j^i(k) + \eta \Delta x_j^i(k) + \alpha s x_j^i(k) \\
 \Delta x_j^i(k) &= [x_j^{\text{best}}(k) - x_j^i(k)] |N(0, 1)| \\
 s x_j^i(k+1) &= \eta \text{acc}^i(k) \Delta x_j^i(k) + \alpha s x_j^i(k) \\
 \text{acc}^i(k) &= \begin{cases} 1 & \text{if the current update has improved cost} \\ 0 & \text{otherwise} \end{cases}
 \end{aligned} \tag{16}$$

where  $\eta$  is the learning rate,  $\alpha$  the momentum rate,  $N(0, 1)$  denotes the normal distribution,  $|\cdot|$  denotes an absolute value,  $\Delta x_{j,i}(k)$  is the amount of change in an individual,  $s x_j^i(k)$  is the evolution tendency of previous evolution,  $x_j^i(k)$  is the  $j$ th variable of an  $i$ th individual at the  $k$ th generation.

### 3.2.7. Fitness function and constraint handling

GA adopts fitness value to direct the search. It means, for an individual with larger fitness value, it will appear in the next generation with larger possibility. In HCGA the value of  $Q_j^t$  satisfies its definition domain, so the water discharge constraint function is automatically satisfied. Other constraints may be handled through penalty function. Then the constrained optimization problem may be transformed to unconstrained optimization. After some treatment for constraints, the following objective functions can be obtained:

$$\min F = -f + \sum_{i=1}^3 \sigma_i \phi_i \tag{17}$$

$$\phi_1 = \sum_{j=1}^N |V_j^{25} - V_j^1| \tag{18}$$

$$\begin{aligned}
 \phi_2 &= \sum_{j=1}^N \sum_{t=1}^{24} |V_j^t - V_j^{\text{lim}}| \\
 V_j^{\text{lim}} &= \begin{cases} V_{j \max} & \text{if } V_j^t > V_{j \max} \\ V_{j \min} & \text{if } V_j^t < V_{j \min} \\ V_j^t & \text{otherwise} \end{cases}
 \end{aligned} \tag{19}$$

$$\begin{aligned}
 \phi_3 &= \sum_{j=1}^N \sum_{t=1}^{24} |P_j^t - P_j^{\text{lim}}| \\
 P_j^{\text{lim}} &= \begin{cases} P_{j \max} & \text{if } P_j^t > P_{j \max} \\ P_{j \min} & \text{if } P_j^t < P_{j \min} \\ P_j^t & \text{otherwise} \end{cases}
 \end{aligned} \tag{20}$$

where  $\sigma_i$  and  $\phi_i$  are the  $i$ th penalty coefficient and penalty functions, respectively.

The choice of the penalty term can be significant, for, if the penalty term is too harsh, infeasible strings that carry useful information for the GA, but lie outside the feasible region will largely be ignored and their information lost while if the penalty term is not strong enough, the GA may search only among infeasible strings, and miss out on the feasible solutions. Therefore, this paper chooses penalty factor  $\sigma_i = 1/T$ ,  $T = \alpha T$ , where  $\alpha$  is a constant between zero and one.  $\sigma_i$  chosen this way absorbs the idea of simulated annealing and forces  $T$  to decrease gradually, that is,  $\sigma_i$  increases gradually which thus ensures the satisfaction of constraints with the process of the evolution.

Aiming at the above objective functions, the fitness function FIT may be stated as follows:

$$\text{FIT} = F_{\max} - F + K(F_{\max} - F_{\min}) \quad (21)$$

where  $F_{\max}$ ,  $F_{\min}$  are, respectively, the maximum and the minimum of the objective functions for present populations;  $K$  the control parameter and its value choice ranges from 0.01 to 0.1.

#### 4. Numerical experiments

To investigate the effectiveness of the algorithms described above, in this section we present two examples to illustrate the application of the algorithms.

##### 4.1. Example 1

This example investigates the convergence speed and solution accuracy of the proposed approach on the following well-known benchmark test function.

The Rosenbrock function is given in (22),

$$\min f = 100 \times (x_1 - x_2)^2 + (1 - x_1)^2; \quad -2.048 \leq x_i \leq 2.048, \quad i = 1, 2 \quad (22)$$

The global optimum of the Rosenbrock function resides inside a long, narrow, and parabolic-shaped flat valley, which is difficult to follow.

Fig. 1 shows the evolutionary process of the proposed method.

From Fig. 1, the proposed method converged to the global optimum faster than the classical GA when applied to the above Rosenbrock function. While GA found a solution with an objective function value near  $10^{-5}$  after 1000 generations, the proposed algorithm converged to a solution around  $10^{-10}$

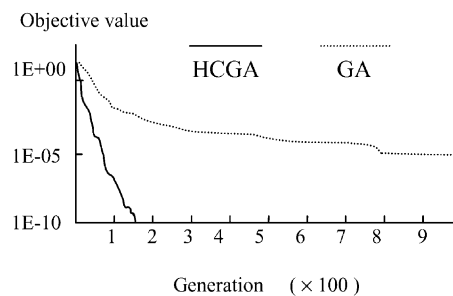


Fig. 1. Evolutionary process.

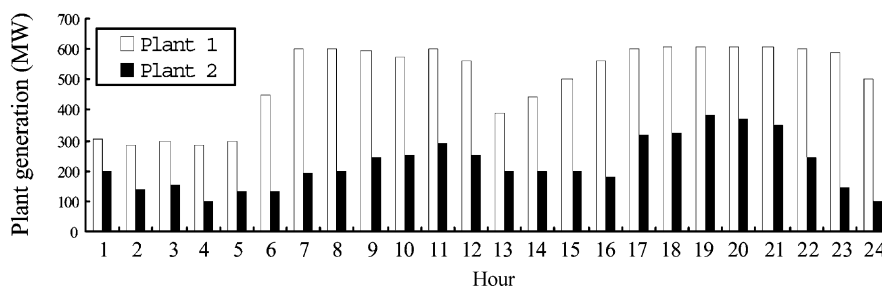


Fig. 2. Hourly hydro plant power generations.

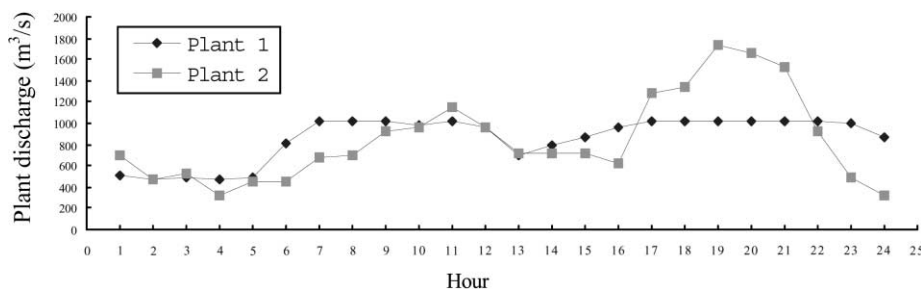


Fig. 3. Hourly hydro plant discharge trajectories.

within 200 generations. It is because that in HCGA exists chaotic sequence and a new self-adaptive error back-propagation mutation operator to direct search in the evolutionary process, the convergence speed is very faster than classical GA.

#### 4.2. Example 2

In this example, a hydro system with two reservoirs is used to demonstrate the feasibility and effectiveness of the proposed approach. The upstream reservoir is long-term adjustment while the downstream reservoir is daily adjustment reservoirs. The scheduling period is 24 h, with one hour time intervals. Each experiment was run 20 times, starting with a different random initial population, the best result is selected as the final result. The hourly hydro plant power generation and discharge trajectories are given in Figs. 2 and 3, respectively.

### 5. Conclusion

This paper presents a new HCGA. HCGA is thus formed by introducing the chaotic sequence and presenting a new self-adaptive error back-propagation mutation operator in the evolutionary process of GA. The algorithm not only retains the generality of GA, but also improves convergence speed and solution accuracy. The proposed method is applied to the short-term optimal dispatch of cascaded hydro



system. Simulation results show that the proposed approach is feasible and effective for the large-scale constrained nonlinear optimization problem.

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