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Review and perspective on acoustic metamaterials: From fundamentals to applications

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ABSTRACT

In the past two decades, the research on acoustic metamaterials has flourished, which is also benefited from the development of additive manufacturing technology. The exotic physical phenomena and principles exhibited by acoustic metamaterials have attracted widespread attention from academia and engineering communities, which can be applied to noise reduction and acoustic nondestructive testing in industrial; invisible cloaking and camouflage in the military; medical ultrasound imaging in national health; acoustic stealth in defense security, detection in the ocean, communication, and other fields, i.e., acoustic metamaterials have important scientific research value and broad application prospects. This review summarizes the history and research status of acoustic metamaterials, focusing on the main research progress of metamaterials in nonlinear acoustic and acoustic coatings fields, including the research on acoustic coatings with cavities of our group. Finally, the future development direction of acoustic metamaterials is prospected, and the difficulties and challenges faced by the actual engineering of acoustic metamaterials are discussed, such as difficulties in mass production, hydrostatic pressure resistant property, omnidirectional wave control, high production costs, and so on.

I. INTRODUCTION

Acoustics, as an important branch of modern science, is widely used in engineering science, life science, earth science, and humanities. Among them, simple, efficient, and precise sound field regulation has always been a key requirement in the fields of underwater acoustics, ultrasonics, audio acoustics, and physical acoustics. Constrained by the inherent acoustic properties of natural conventional materials composed of atoms/molecules and chemical bonds, the control of sound waves using classical acoustic methods has encountered a bottleneck. The proposal of acoustic metamaterials brings the light to break through this bottleneck and provides a way to control and manipulate sound waves. The acoustic metamaterials rose to prominence in aerospace, national defense security, national health, medical ultrasound imaging, underwater countermeasures, ocean exploration, communication, and industrial nondestructive testing and becomes an emerging field at the frontier of modern acoustics.

Acoustic metamaterials, namely, periodic or random man-made structures composed of meta-atoms (subwavelength resonators—the size of which is bigger than the atomic scale but much smaller than the relevant radiated wavelength), whose concept is extended by artificial metamaterials. Artificial metamaterials with unique functions have been one of the research hotspots in academia and engineering communities in recent years and were awarded one of top ten breakthroughs in 2003.[1] In 1968, Russian physicist Veselago proposed the concept of metamaterials[2] for electromagnetic waves. Veselago believes that there may be metamaterials with simultaneously negative permittivity and magnetic permeability, accompanied by phenomena related to negative refractive index, and call them left-handed materials or double negative metamaterials.

Due to the lack of experimental verification, this work did not attract widespread attention in the scientific community at that time. Until three decades later, scientists experimentally verified the existence of double negative metamaterials,[3–5] and the research on metamaterials began to rise in various fields. In 1987, photonic crystals, artificial materials in the field of optics, were independently proposed by and Yablonovitch[6] and John[7] Photonic crystals are artificial microstructures that are periodically arranged by dielectric media with different refractive indices, and their periodicity is the same as the radiation wavelength. Due to the spatial periodicity of the permittivity, i.e., the periodic change in the spatial refractive index, the dispersion relationship of light waves will appear in a banded structure, and the frequency range where the wave cannot propagate is called photonic bandgap. Inspired by photonic crystals, phononic crystals were attracted the attention of scholars.[8–12] Based on a similar principle, phononic crystals are artificial materials made of periodic arrangement of solid scatterers embedded in a matrix, which could introduce the presence of band gaps for elastic waves due to Bragg scattering. Following phononic crystals, the concept of acoustic metamaterials was proposed by the group of Shen in 2000[13] by arranging inner local resonators in a cubic phononic crystal. Inner local resonators could introduce another bandgap at low frequencies, which is much lower than the bandgap caused by Bragg scattering. In the following two decades, acoustic metamaterials developed rapidly. Of course, this was also attributed to the advanced additive manufacturing technology,[14–17] such as 3D printing technology, which solves the problem of processing and preparing acoustic metamaterials in the laboratory, and the developed advanced measuring instruments and methods to meet the measurement requirements of acoustic metamaterials. Figure 1 presents a historical development of the artificial metamaterials. In this review, we browse the history and development status of acoustic metamaterials and detail the main research progress of nonlinear acoustic metamaterials and underwater acoustic metamaterials, including the research results of our group. Finally, the future development direction of acoustic metamaterials is discussed and prospected.

II. ACOUSTIC METAMATERIALS

In the past decades, acoustic metamaterials have extensively been exploited for precise programmable depth control and manipulation of waves due to their exotic properties that do not exist in nature. Currently, the effective parameters are still the main evaluation criteria for acoustic metamaterials. In the conventional elastic medium composed of atoms, molecules, and chemical bonds, the two key parameters governing sound waves propagating in them, namely, mass density and bulk modulus, are always positive. Due to the strong dispersion introduced by inner local resonance units in acoustic metamaterials, we can obtain exotic acoustic parameters that do not exist in natural materials. These effective acoustic parameters could be

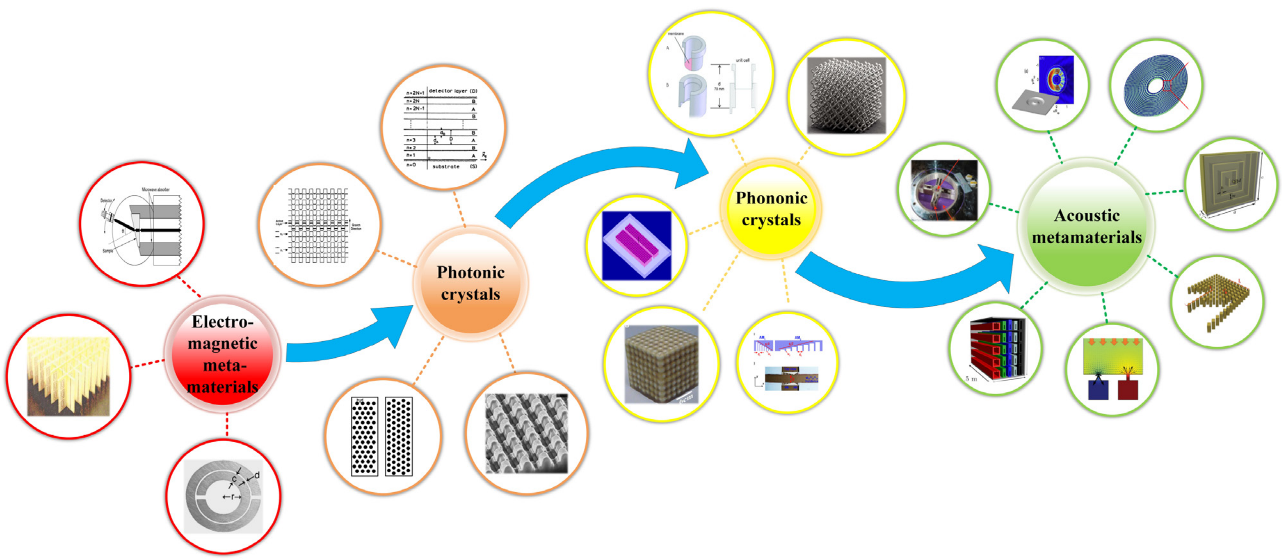

FIG. 1. Historical development of artificial metamaterials, including the emergence of electromagnetic metamaterials, photonic crystals, phononic crystals, and acoustic metamaterials. The images used from left to right: Reproduced with permission from Shelby *et al.*, Science **292**, 77–79 (2001).[3] Copyright 2001 AAAS Publishing. Reproduced with permission from Houck *et al.*, Phys. Rev. Lett. **90**, 137–401 (2003).[4] Copyright 2003 American Physical Society. Reproduced with permission from Phys. Rev. Lett. **84**, 4184–4187 (2003).[5] Copyright 2003 American Physical Society. Reproduced with permission from Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987).[6] Copyright 1987 American Physical Society. Reproduced with permission from Tamura *et al.*, Phys. Rev. B **38**, 1427 (1988).[8] Copyright 1988 American Physical Society. Reproduced with permission from Vasseur *et al.*, Phys. Rev. Lett. **86**, 3012 (2001).[12] Copyright 2001 American Physical Society. Reproduced with permission from Rill *et al.,* Nat. Mater. **7**, 543–546 (2008).[17] Copyright 2008 Springer Nature. Reproduced with permission from Mei *et al.*, Phys. Lett. A **373**, 2948–2952 (2009).[18] Copyright 2009 Elsevier. Reproduced with permission from Liu *et al.*, Science **289**, 1734 (2000).[13] Copyright 2000 AAAS Publishing. Reproduced with permission from Zhu *et al.*, Appl. Phys. Lett. **107**, 113501 (2015).[14] Copyright 2015 AIP Publishing. Reproduced with permission from Lee *et al.*, Phys. Lett. A **373**, 4464–4469 (2009).[19] Copyright 2009 Elsevier. Reproduced with permission from Hedayati *et al.*, Appl. Phys. Lett. **110**, 091905 (2017).[16] Copyright 2017 AIP Publishing. Reproduced with permission from Yang *et al.*, Phys. Rev. Lett. **101**, 204301 (2008).[20] Copyright 2014 Springer Nature. Reproduced with permission from Romero-García *et al.*, Sci. Rep. **6**, 19519 (2016).[21] Copyright 2016 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Herrero-Dura *et al.*, Appl. Sci. **10**, 1690 (2020).[22] Copyright 2020 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Li *et al.*, Phys. Rev. Lett. **106**, 084301 (2011).[23] Copyright 2011 American Physical Society. Reproduced with permission from Pelat *et al.*, J. Sound Vib. **476**, 115316 (2020);[24] Copyright 2020 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Zhang *et al.*, Phys. Rev. Lett. **106**, 024301 (2011).[25] Copyright 2016 AIP Publishing. Reproduced with permission from Li and Assouar, Appl. Phys. Lett. **108**, 063502 (2016).[26] Copyright 2016 AIP Publishing.

positive, negative, near-zero, or approaching-infinity. Inspired by the periodic mass–spring mechanical structure with effective mass density,[27–30] acoustic metamaterials with effective negative mass density could be achieved by a membrane-type system.[19,31,32] Acoustic metamaterials with effective negative bulk modulus are another corresponding branch.[31,33–38] Acoustic metamaterials with single effective negative mass density or bulk modulus can be used for the design of perfect sound absorbers,[39–45] since single effective negative parameter introduces pure imaginary wave vector and phase velocity, i.e., waves are evanescent. However, when mass density and bulk modulus are simultaneously negative, i.e., in double negative acoustic metamaterials, waves can propagate and the direction of group velocity and phase velocity is opposite. Many scholars have devoted to designing double negative acoustic metamaterials.[31,46–52] Here, we need to emphasize that the dissipative effect is a very important and non-negligible factor in the field of acoustic metamaterials, for example, Henríquez *et al.*[53] found that viscothermal losses could destroy the predicted behavior of double negative acoustic metamaterials designed by García-Salgado *et al.*,[54] and the dissipative effect could be used for the design of acoustic metamaterial absorbers.[21,55–59] In addition, near-zero refractive index acoustic metamaterials[60–65] with the characteristics of high transmission and high sound velocity are also an interesting research topic, which can be used for precise phase control and have potential applications in medical ultrasound and other fields.

Among all kinds of acoustic materials, the most demanded are sound-absorbing materials in vibration and noise reduction engineering. Unlike bulky traditional sound-absorbing materials, acoustic artificial sound-absorbing metamaterials have sub-wavelength dimensions, which can achieve perfect broadband sound absorption while reducing the volume. At present, sound-absorbing metamaterials can be mainly classified into membrane type,[20,42,66] Fabry–Pérot type,[26,67,68] Helmholtz type,[21,44,69] and ventilated type.[70–72] In addition to sound-absorbing metamaterials that can affect the battlefield adaptability of military equipment, acoustic cloaking technology is also an important technology valued by the military in the national defense engineering. In the field of acoustic metamaterials, coordinate transformation theory is currently the main method for realizing acoustic invisibility cloaking[73] and acoustic illusion cloaking.[74] In addition, the design of acoustic invisibility cloaking based on Parity-Time (PT) symmetry theory has emerged,[75] which consists of acoustic metasurfaces and loudspeaker arrays. As a special non-Hermitian system, PT symmetry theory can also be used to realize the asymmetric transmission of sound waves.[76–79] In addition, the study of breaking the reciprocity of acoustic systems also includes linear acoustic diodes[23,80] and nonlinear acoustic diodes[81–83] in classical Hermitian systems as well as acoustic analogues of topological insulators. Topological acoustic metamaterials could be used to break the time-reversal symmetry of acoustic systems.[84] Furthermore, there are types of topological acoustic metamaterials that do not break time-reversal symmetry.[85,86] Topological acoustics is also a rapidly developing field in the metamaterial community. For recent review papers on topological acoustics, the reader may refer to the works of Xue *et al.*,[87] Zangeneh-Nejad *et al.*,[88] and Ma *et al.*[89] In summary, due to the exotic properties and remarkable functionality of acoustic metamaterials over the past two decades, several applications, including perfect absorbers,[21,42] acoustic diodes,[23,81] acoustic topological systems[84,90,91] acoustic black holes[24,92–94] acoustic lenses for sub-diffraction imaging,[95] acoustic sound focusing based on gradient index lenses,[96–98] and acoustic cloaking,[25,99–103] and others, have been developed.

With the research foundation of our team, here, we will discuss in detail two topics: "nonlinear acoustic metamaterials" and "underwater acoustic metamaterials."

A. Nonlinear acoustic metamaterials

Until now, acoustic metamaterials have mostly been designed to manipulate linear wave. Nevertheless, the presence of nonlinearity could be easily introduced at high acoustic levels[104] and this phenomenon appears generically in practice. After the concept of nonlinear metamaterials was proposed by Lapine *et al.* in the field of electromagnetism[105] in 2003, the study of nonlinear acoustic metamaterials has been receiving increasing attention in acoustician communities, for example, nonlinear acoustic lenses,[106] bifurcation-based acoustic switching and rectification,[107] nonreciprocal acoustic transmission,[81] the nonlinear self-demodulation effect,[108–110] waves coupling in nonlinear metamaterials,[111] discrete breathers,[112] the nonlinear dispersion relation,[113] and the formation of acoustic solitons of various types—pulse-like[36,114] and envelope ones,[36,115] and so on.

In the field of nonlinear acoustic metamaterial, dispersion, nonlinearity, and dissipation effect are three key factors. Combining analytical calculations, numerical simulations, and experimental results, Zhang *et al.* has systematically consider the interplay between all the above phenomena.[37,38,115–117] Dispersion effect introduced by the locally resonant building blocks and the periodicity of the acoustic metamaterials could tailor the source and the harmonics generated during the nonlinear wave propagation, for instance, we can observe acoustic diodes[110] or the beatings of generated harmonics due to mismatched phases.[116] A perfect balance between dispersion and nonlinearity could give rise to the emergence of solitons, namely, robust localized waves propagating undistorted. Depending on the properties of the acoustic metamaterials and especially on the dispersion relation, different types of solitons were supported in the system, for example, envelope (bright,[115] gap,[115] black,[38] and gray[38]) ones. The propagation direction of higher harmonics generated by the nonlinear effect can also be adjusted by the dispersion relation of acoustic metamaterials, for example, backward traveling generated harmonic could be achieved in effective double negative acoustic metamaterials. Inspired by the works of Shadrivov *et al.*[118] in the field of optics, the design of a nonlinear acoustic mirror that can completely convert the incident waves into reflected second harmonic waves could be a research topic in the future.

Although losses are non-negligible and naturally play a key role, only a few works have considered the dissipative effect in nonlinear acoustic metamaterials. Our group investigates the dissipation-induced dynamics of dark solitons in nonlinear acoustic metamaterials.[38] However, we only discussed weak dissipation associated with linear losses.[37,38,115] There are some other works[119–121] that consider the influence of losses when studying acoustic higher harmonic generations, but none of them highlight the effect of nonlinear losses. In acoustic structures, due to geometrical discontinuities, nonlinear losses may easily appear and should not be ignored.[122,123] Zhang *et al.* provided a quite accurate analytical model to capture the effect of such nonlinear losses[116] in both the scattering (reflection, transmission, and absorption) coefficients and the second-harmonic generation with its beating phenomenon. More specifically, we experimentally and analytically study nonlinear wave propagation in acoustic metamaterial made by an air-filled waveguide periodically side-loaded by holes with sharp edges. In this structure, linear losses are due to the radiation and viscothermal boundary layers. When the acoustic wave amplitude is sufficiently high, nonlinear losses appear due to jet and vortex formation at the location of geometrical discontinuities, i.e., side holes. Results of Zhang *et al.* clearly evidence that nonlinear losses play an important role in the proposed device, which could even transform the nature of this acoustic metamaterial from a reflective system to a sound absorbing one, suggesting the possibility to design a perfect nonlinear absorber (Fig. 2).

B. Underwater acoustic metamaterials

Most of the work on acoustic metamaterials has focused on regulating the propagation of sound waves in air. There are relatively few theoretical and experimental studies on underwater acoustic metamaterials and their acoustic wave regulation. There are many objective factors restricting the development of underwater acoustic metamaterials, which are mainly reflected in the following aspects. The wavelength of the water sound wave is much longer, which is nearly five times that of the air sound wave of the same frequency.[124] Therefore, the underwater sound, especially the low-frequency underwater sound, is more difficult to control than the air sound of the same frequency. As to the Perspective of fluid structure coupling, on the one hand, the acoustic impedance of the water medium is much greater than that of the air medium, i.e., the structure can no longer be regarded as a rigid body and becomes an elastic body.[125] The complexity of designing materials is, therefore, increased. On the other hand, the fluid load effect of water on the metamaterials cannot be ignored due to the density of water is larger than that of the air.[126] Moreover, the current research on rubber-like medium with cavities takes little consideration of the influence of external hydrostatic pressure, that is, the structure does not deform under water, and the physical parameters of rubber material do not change. In fact, when the sound absorption structure is under high hydrostatic pressure,[127–129] the cavity will undoubtedly deformed, and the acoustic performance will be significantly affected, resulting in difficulties in underwater acoustic material research. From the experimental point of view, the underwater acoustic experiment is also much more complicated and expensive than the air acoustic experiment. Last but not the least, compared with the air, the application environment of underwater acoustic materials is more harsh. In addition to the acoustic properties, the underwater acoustic materials also need to have certain corrosion resistance, low temperature resistance, water tightness, and excellent mechanical properties.

FIG. 2. Nonlinear acoustic metamaterials. The images used from 0 o'clock in the clockwise direction: Reproduced with permission from Devaux *et al.*, Phys. Rev. Lett. **115**, 234301 (2015).[110] Copyright 2015 American Physical Society. Reproduced with permission from Khajehtourian and Hussein, AIP Adv **4**, 124308 (2014);[113] Copyright 2020 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Zhang *et al.*, Phys. Rev. E **96**, 022214 (2017).[115] Copyright 2014 American Physical Society. Reproduced with permission from Zhang *et al.*, Acta Acust. United Acust. **104**, 235–242 (2018);[37] Copyright 2018 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Zhang *et al.*, Appl. Sci. **8**(7), 1186 (2018);[38] Copyright 2018 Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Boechler *et al.*, Phys. Rev. Lett **104**, 244302 (2010).[112] Copyright 2010 American Physical Society. Reproduced with with permission from Zhang *et al.*, Appl. Phys. Lett. **118**, 104102 (2021),[116] Copyright 2021 AIP Publishing. Reproduced with permission from Appl. Phys. Lett. **104**, 014103 (2014).[106] Copyright 2014 AIP Publishing.

However, underwater acoustic metamaterials can provide important support for the development of marine equipment and have broad application and development prospects. Underwater acoustic metamaterials can help underwater vehicles, such as submarines, achieve acoustic cloaking[137,138] and improve acoustic detection performance.[139,140] The decoupling underwater acoustic metamaterials made of flexible viscoelastic material could directly be laid on the surface of the boat to isolate the transmission of the vibration of the underwater structure to the fluid medium,[136,141] thereby reducing the radiated noise level into the fluid and the probability of being discovered by enemy passive sonar, i.e., achieving acoustic cloaking. In order to take into account the requirements of sound absorption and sound insulation, scatterers, such as cavities, are often added to viscoelastic decoupling underwater acoustic metamaterial.[131,142] When the enemy uses active sonar for detection, absorbing underwater acoustic metamaterials can be laid to reduce the acoustic target strength and increase the concealment. At present, underwater sound-absorbing metamaterials can be mainly divided into local resonance type underwater sound-absorbing metamaterials[133,143–147] and non-resonant underwater sound-absorbing metamaterials (porous materials,[148–151] gradient index metamaterials,[130] and others[152,153]) The design of underwater acoustic metamaterials also contributes to the improvement of acoustic detection performance. Sound-transmitting underwater acoustic metamaterials are generally used for sound-transmitting windows of sonar domes.[139] Sound-reflecting underwater acoustic metamaterials are generally used in acoustic baffles of sonar array structures to improve the acoustic focusing/reflection capabilities,[154] thereby improving the sensitivity of sonar hydrophones. In addition to reflective underwater acoustic focusing metamaterials, there are diffractive[135,140,155–157] and refracting ones,[158–160] according to the different convergence principles of underwater acoustic focusing metamaterials for sound waves. Based on the regulation of refractive index, refractive underwater acoustic focusing metamaterials focus the incident acoustic waves to its rear focus, mainly including resonance type[160–162] and gradient index type[163–166] underwater acoustic focusing metamaterials (Fig. 3).

The research on underwater acoustic metamaterials could date back to 1995. Milton and Cherkaev proposed a pentamode material,[167] a linear elastic 3D material for which five of the six possible modes of deformation cost no energy, with elastic properties similar to liquids. Subsequently, Norris proposed an acoustic cloaking theory based on pentamode materials.[168] Pentamode materials are considered to be one sort of underwater acoustic metamaterials with important potential applications in underwater acoustic environments.[164,165,169,170] In recent years, with the competition for sea control and ocean development between various countries, the research on various underwater acoustic metamaterials has also received much attention from researchers, for example, He *et al.* investigated topological phononic states for underwater sound using arrays of acoustic coupled ring resonators,[171] which could pave the way for designing topological acoustic transmission devices for sound guidance and switching; for focusing underwater sound, Calvo *et al.* created a Fresnel zone plate with thin acoustically opaque zones made of soft silicone rubber foam attached to a thin transparent rubber substrate.[140] Zhong *et al.*[134] proposes a type of composite underwater honeycomb-type acoustic metamaterial plate with the advantages of low-frequency broadband sound insulation and high hydrostatic pressure resistance. The proposed material is composed of a thin plate (rubber–steel–rubber) clamped between two layers of honeycomb plate, and the thickness of the whole structure is 20.25 mm. A structure for an underwater sound absorber with subwavelength thickness and a quasi-perfect absorption property at multiple frequency bands is reported which consists of a viscoelastic coating layer embedded with periodically distributed plate scatterers.[172] The embedded scatterers cannot only slow sound waves in the coating, leading to a down-shifted resonance frequency where the absorption is not only maximized but also introduce multiple local bending modes and local longitudinal modes in the coating. In order to save space in the absorbing material while absorbing sound waves, the researchers designed labyrinthines,[172,173] curls,[26,174] and spatial folding structures[175,176] to reduce thickness, making the physical size of the structure substantially shorter than the folded path. Yang *et al.*[67] conceived and implemented a sound absorption metamaterial, where a metamaterial unit is with 16 differently designed Fabry–Pérot channels. By using the synergistic effect provided by evanescent wave to interact with a small amount of highly dissipative media, broadband high sound absorption starting from 400 Hz can be achieved. Duan *et al.*[177] report a lightweight tunable acoustic metamaterial constructed by introducing a rubber coating and an embedded metallic neck into a metallic hexagonal honeycomb Helmholtz resonator. The material is with deep subwavelength thickness (e.g., $k = 300$) and strong load-bearing capability for underwater low-frequency and ultra-broadband acoustic perfect absorption. Recently, Shen *et al.* proposed impedance-matched and high-energy-density composite to achieve broadband underwater acoustic absorption with thin sample thickness.[132]

FIG. 3. Underwater acoustic metamaterials. The images used from 0 o'clock in the clockwise direction: Reproduced with permission from Wang *et al.*, J. Sound Vib. **479**, 115375 (2020).[128] Copyright 2020 Elsevier. Reproduced with permission from Naify *et al.*, Appl. Phys. Lett. **104**, 073505 (2014).[130] Copyright 2014 AIP Publishing. Reproduced with permission from Huang *et al.*, J. Sound Vib. **426**, 244–257 (2018).[131] Copyright 2018 Elsevier. Reproduced with permission from Qu *et al.*, Sci. Adv. **8**, 4206 (2022);[132] Copyright 2022Authors, licensed under a Creative Commons Attribution (CC BY) license. Reproduced with permission from Zhang *et al.*, J. Acoust. Soc. Am. **142**, 3722 (2017).[133] Copyright 2017 AIP Publishing. Reproduced with permission from Zhong *et al.*, Compos. Struct. **277**, 114603 (2021). (2021).[134] Copyright 2018 Elsevier. Reproduced with permission from Tarrazó-Serrano *et al.*, Appl. Acoust. **148**, 119–122 (2019).[135] Copyright 2019 Elsevier. Reproduced with permission from Huang *et al.*, Int. J. Mech. Sci. **175**, 105512 (2020).[136] Copyright 2020 Elsevier.

Here, we want to highlight underwater acoustic coatings with cavities, which could be used for designing submarine anechoic tiles. Until now, the acoustic performance of acoustic structures with different cavity shapes, such as spheroid,[178] cuboid,[179] horn,[180] cylinder,[181] and disk[182] for underwater acoustic coating applications, have investigated. In order to obtain the sound absorption mechanism of acoustic structure with cavities, many scholars have established a variety of effective acoustic models. Analytical and numerical models are developed by Sharma *et al.*[183] to study sound transmission through a periodically voided soft elastic medium submerged in water. A single layer of voids as well as multiple layers of voids along the direction of propagation of sound are considered. Attenuation of sound pressure is attributed to the monopole type resonance of the voids. Strong coupling of resonance of voids is shown to result in broadband attenuation of sound. Subsequently, Sharma *et al.*[184] investigated the acoustic performance of a model comprising a single layer of periodic voids in a soft elastic medium attached to a steel backing. A high absorption peak occurs at 200 Hz. Peaks of high sound absorption were attributed to Fabry–Pérot resonance created due to interference between scattered waves from the voids and reflected waves from the steel backing. A layer of periodic steel cylinder placed in front of the periodic cylinder cavity was found to exhibit very high sound absorption in a broad frequency range, not achieved using the corresponding designs comprising only voided or hard inclusions.[147]

Based on Sharma *et al.*'s inspiring research on cylindrical cavity type acoustic coating, our group has presented an acoustic model comprising periodic cylindrical cavities embedded in a soft elastic medium with gradient changes in radii and distances.[185] The main advantage of this model is that high sound absorption across a broadband frequency range can be realized. The basic mechanism for sound absorption is based on Fabry–Pérot resonance. Consider that the thickness of the acoustic model increases too much when the cylindrical cavity is stacked, we proposed an acoustic model with gradient changes of cavity numbers.[186] By means of higher loss factor of sound velocity and more content of elastic medium, this model significantly improves the sound absorption performance compared with the model with gradient radius. To further decrease the total thickness of the acoustic model, an acoustic model composed of cylindrical cavities distributed in the diagonal direction of a soft elastic medium is developed. The distances between cavities are designed as negative gradient so as to obtain a broadband absorption bandwidth. The finding confirm that the sound absorption bandwidth can be broadened effectively by arranging cavities in this particular way with the total thickness significantly decreased, which greatly reduces the sound range in the direction of the incident wave. In view of the problem that the cavity shape used in acoustic coatings is usually relatively simple, resulting in a poor sound absorption performance, the design of the composite shape cavity brings the dawn of enhancing the acoustic absorption ability.[187] Huang *et al.*[131] present an underwater metamaterial with a kind of composite shape cavity, including conical and cylindrical cavities, and the results demonstrate that the decoupling performance of acoustic coating can be significantly improved. To enhance the sound absorption property, our group has proposed a soft elastic medium embedded with cylindrical cavities with multi-coated rubbers in the direction perpendicular to the normal incident wave is proposed. The main advantage of the proposed model is that high sound absorption performance can be obtained in a low-frequency range. In addition, the absorption peak value can be maintained or enhanced while increasing the cavity radius to lower the frequency range. The sound absorption behavior of the proposed model is investigated using the finite element method. The absorption mechanism is studied based on Mie scattering, and the results show that the high sound absorption is attributed to the presence of solid-solid interfaces. Power dissipation density plots are presented to provide further insights into the absorption mechanism.

At present, underwater acoustic metamaterials are developing in the direction of low frequency, broadband, omnidirectional, large plane scale, subwavelength thickness, light weight, high pressure resistance, multi-functional composite type, and environmental adaptability.

III. PERSPECTIVE

In conclusion, acoustic metamaterials has made great progress on fundamental research. The current research is developing toward the direction of industrialization of thinning, broadband, and high performance. While the theories behind acoustic metamaterials have been well-established for over 20 years (or even 30+ years including elastic-wave phononic crystals), there are still no commercial products available. The main reason is the limitation of the manufacturing process. The current processing capability of large-scale metamaterials is insufficient, especially the lack of research on large-scale sample fabrication and testing of underwater acoustic metamaterials. Underwater sound absorption tests are mostly conducted with small samples in tubes/pools which is full of water,[132] and their sound absorption performance is difficult to reflect the sound absorption capacity in real environments. There are a few reports of experimental studies on large-scale samples in lakes/seas. Common manufacturing methods for various acoustic metamaterials, include fused deposition modeling,[14] stereo lithography apparatus,[15] selected laser sintering/melting,[16] direct laser writing,[17] and polymer injection technology.[188] In order to accelerate the development of various acoustic metamaterials toward application, it is necessary to use interdisciplinary integration and combine manufacturing technology with electromagnetic, acoustics, physics, materials, and other disciplines to achieve large-scale manufacturing of acoustic metamaterials. In addition to the development of additive manufacturing technology, acoustic bio-metamaterials may facilitate mass production of acoustic metamaterials. For example, Ref. 189 shows the potential of cereal straws as low-cost acoustic bio-metamaterials. Bio-inspired acoustic metamaterial designs can reduce design costs, simplify the fabrication process, and improve the overall performance of metamaterials. Designing bio-inspired acoustic metamaterials can pave the way for commercialization and practical applications, showing large perspectives for future development.

In order to solve the practical application requirements, designing acoustic metamaterials that could integrate acoustic/mechanical/electromagnetic functions is also an important direction. For example, in the field of underwater acoustic coatings engineering, it is necessary to design acoustic metamaterials that can achieve decoupling, sound absorption, and sound focusing without increasing the thickness and weight of the material and without sacrificing the structural strength of the material. At present, research on sound absorption and acoustic focus are mainly concentrated in the range of middle and high frequency or even ultrasonic frequency, and the effect of low frequency or low frequency broadband is not ideal. Meanwhile, there are a few studies on low frequency, especially low frequency broadband acoustic focusing and sound absorption. The problem of low frequency and broadband acoustic wave regulation under 1 kHz needs to be solved urgently. It is still a difficult problem to design underwater acoustic metamaterials that can meet the requirements of low frequency, small size, light weight, and high hydrostatic pressure resistant at the same time. Up to now, the research about acoustic absorption and focusing has mainly focused on sound waves with normal incidence. Additionally, the incident acoustic waves can exist in different directions in practice. Thus, it is necessary to develop acoustic metamaterials with omnidirectional acoustic wave control capability. Futhermore, nonlinear tunable active acoustic metamaterials opens up possibilities for acoustic engineering and design,[190–192] providing valuable means for actively manipulating wave propagation, which can be used to create adaptive structures that can respond to changes in the acoustic environment. However, there are still challenges that need to be addressed, such as efficiency, the development of efficient control strategies, and the optimization of the material's structure.

Machine learning[193–197] and topology optimization[198–200] can be used to optimize the design of all kinds of acoustic metamaterials, improve the sound field control capability of artificial structures, and optimize the overall acoustic performance. At present, the structural design of acoustic metamaterials still mainly relies on experience to change various parameters of the structure and perform a large number of repeated iterations, which is inefficient. Using the neural network training method to optimize the design process of acoustic metamaterials and reverse design can greatly reduce the design cost. Finally, most of the current research on acoustic metamaterials investigate their acoustic and vibration characteristics in isolation and rarely consider complex boundary, load conditions, and constraints, such as size, quality, and environment, in practical engineering applications. Therefore, future research should be further promoted toward practical engineering applications.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Jiangyi Zhang: Conceptualization (equal); Investigation (equal); Resources (equal); Writing – original draft (equal). **Bo Hu:** Funding acquisition (equal); Supervision (equal); Writing – review & editing (equal). **Shibo Wang:** Investigation (equal); Resources (equal); Writing – original draft (equal); Writing – review & editing (equal).

DATA AVAILABILITY

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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