Online automatic tuning of a proportional integral derivative controller based on an iterative learning control approach

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Abstract: A new approach is proposed for closed-loop automatic tuning of a proportional integral derivative (PID) controller based on an iterative learning control (ILC) approach. The method does not require the control loop to be detached for tuning, but it requires the input of a periodic reference signal. Such a reference signal can be the natural reference signal of the control system when it is used to execute a repetitive sequence, or it can be an excitation signal purely for tuning the PID controller. A modified ILC scheme iteratively changes the control signal by adjusting the reference signal only. Once a satisfactory performance is achieved, the PID controller is then tuned by fitting the controller to yield close input and output characteristics of the ILC component. Simulation and experimental results are furnished to illustrate the effectiveness of the proposed tuning method.

1 Introduction

Proportional integral derivative (PID) controllers are now widely used in various industrial applications where the tracking and regulation of time-continuous variables is necessary. The strong affinity with industrial applications is largely because of its simplicity and the satisfactory level of control robustness that it offers. Apart from possible minor structural differences, the distinct factor governing how well the controller performs is the tuning method adopted. To date, many different approaches are available for tuning the PID controller [1–3]. In more recent times, automatic tuning methods have evolved [4–6], where the user of the industrial controller only needs to provide simple performance specifications, initiate the tuning process with a push of a button and the PID controller can be tuned satisfactorily. These tuning approaches can generally be classified under offline and online approaches. In the latter case, the controller is tuned while it is still performing the control function with no loss in production time. From economical, practical usage and application domain viewpoints, the closed-loop online approach is the more attractive approach.

To date, however, a specific PID tuning approach has been typically limited to certain classes of systems only. It also typically requires a linear model of the system, in an implicit or explicit form, based on which the controller is tuned. It is unrealistic to assume that the system of concern will fit the assumed model well, as all systems encountered in practice are nonlinear in nature. As a result, the final control performance can be rather limited and unacceptable when the user requirements become stringent. Under this situation, one response may be to develop a more complex version of the PID controller. An adaptive PID controller based on the model reference technique has been proposed in Gjamadam and Blankenship [7] and a direct adaptive PID control scheme has been proposed by Badreddine and Lin [8] for both offline and online tuning of PID parameters. A learning-enhanced nonlinear PID controller has been developed specifically for nonlinear systems by Tan *et al.* [9] and three optimal-tuning PID controller design schemes has been presented for industrial control systems by Liu and Daley [10], in which detailed calculations are introduced. Central to all these works is a model that becomes progressively more complicated and unwieldy in order to yield the incremental performance needed. Correspondingly, the entire control design procedure also becomes complicated.

This paper presents a new scheme to tune the PID control parameters based on an iterative learning control (ILC) approach. The basic idea was to use ILC to derive the ideal control signal for the system to track a periodic reference sequence. This reference sequence can be the natural reference signal for the control system when it is executing repetitive operations, for example, a servo-mechanical system executing repeated pick and place operations. It can also be a deliberate periodic sequence purely for tuning the PID controller, after which the natural reference sequence can be applied. This deliberate excitation signal can be derived, for example, by subjecting the closed-loop system to relay feedback and inducing a steady-state oscillation signal at an appropriate frequency for the closed system. In this paper, deliberate effort was put into ensuring that the tuning was done online, that is, the tuning procedure was carried out while closed-loop operation was in progress. To this end, the ILC deviates from the usual configuration [11], by iteratively changing the reference signal rather than the control signal. Most industrial control systems do not allow the control signals to be changed, although the reference signal can be subject to user specifications. Thus, the method represents an approach

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that could be more readily incorporated into existing closed-architecture systems.

Once the ILC yields a satisfactory overall control signal, as far as a selected function of the tracking error is concerned, the PID controller was ready to be tuned. A system identification approach was adopted where the PID parameters were adjusted such that the best-fit to the overall input and output signals of the ILC-augmented control system was obtained. For this purpose, it was possible to adopt a higher-order controller instead of the PID controller. The proposed method was a model-free approach as no model, implicit or explicit, was assumed, and at no time, was the controller detached from the system. In this paper, the proposed method was subjected to a simulation-based evaluation as well as real-time experimental tests on a piezoelectric linear motor.

2 Proposed approach

In this section, the proposed tuning of the PID controller using an ILC approach will be elaborated. The entire procedure was essentially carried out over two phases. In the first phase, a modified ILC procedure was carried out to yield the ideal input and output signals of the overall ILC-augmented control system. The second phase used these signals to identify the best-fitting PID parameters with a standard least-squares (LS) algorithm.

2.1 Phase 1: iterative refinement of control

Fig. 1 shows the system under PID feedback control (PID_1). The controller PID_1 is described by

$$u(t) = K_{\mathrm{p1}}e(t) + K_{\mathrm{i1}}\int_0^t e(t)\,\mathrm{d}t + K_{\mathrm{d1}}\frac{\mathrm{d}e(t)}{\mathrm{d}t} \qquad (1)$$

An ILC component was added to the basic control system to iteratively obtain enhanced control signals for the tracking of the periodic reference signal. Fig. 2 shows the configuration with the ILC augmentation. In Fig. 2, controller PID_1 was considered as the basic feedback controller; it was not expected that this controller would achieve perfect performance without tracking error. Instead of the usual approach of refining the control signal that may not be permitted in the typical closed-architecture control system, the ILC component modified the desired reference signal through successive trials to improve the tracking performance. The ILC component was used to modify the output of the controller PID_1 to enable the output x to approach the actual desired trajectory x_{d}, not x'_{d}. ILC has been proposed by Arimoto *et al.* [12] as a model-free approach to achieve a better system performance of repetitive systems over a finite time interval. The main idea associated with the use of the ILC was to enhance the system performance by using the information from the previous cycle in the next cycle over a period of time until the performance achieved was deemed satisfactory. In this paper, the P-type update law was adopted for ILC. Under the configuration shown in Fig. 2, during the ith iteration,

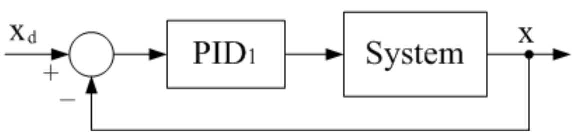

Fig. 1 *Basic PID feedback control system*

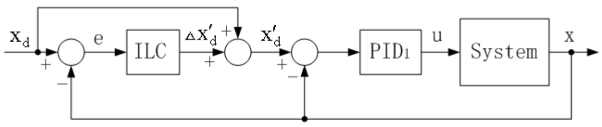

Fig. 2 *ILC block digram*

the modified desired trajectory $x'_{\mathrm{d},i}$ is given by

$$x'_{\mathrm{d},i}(t) = x_{\mathrm{d}}(t) + \Delta x'_{\mathrm{d},i}(t) \qquad (2)$$

where t is the discrete time index. The update law for the ILC is

$$\Delta x'_{\mathrm{d},i+1}(t) = \Delta x'_{\mathrm{d},i}(t) + \lambda e_i(t+1) \qquad (3)$$

where λ is the learning gain. For the P-type ILC, a sufficient condition for learning convergence can be found in Dou *et al.* [13], which also provides guidelines for the choice of λ. A more thorough analysis of convergence and stability for sampled-data ILC can be found in Huang *et al.* [14]. Under this ILC configuration, the tracking error and the output of the ILC during the previous cycle were used to update the output of the ILC during the present cycle. When the ILC convergence was obtained, the actual error $e_i = x_{\mathrm{d}} - x_i$ approaches zero and the ILC stops updating. $\Delta x'_{\mathrm{d},i+1}$ is maintained at the same value as $\Delta x'_{\mathrm{d},i}$.

Fig. 2 can be configured in the equivalent form as shown in Fig. 3*a*, where the ILC structure for enhancement of the reference signal can be viewed instead as a parallel learning controller to PID_1, comprising an ILC component and a PID_1 in series.

When a satisfactory level of control performance has been achieved, the ideal input e and output Δu for a cycle of the reference signal would have been available for the next phase.

2.2 Phase 2: identifying new PID parameters

In this phase, an equivalent PID control PID_2 was derived in place of the parallel ILC + PID_1 component, so that Fig. 3*b* would be as close to Fig. 3*a* as possible, as far as the response of the signal Δu to e is concerned.

The PID_2 controller can be expressed as

$$\Delta u(t) = K_{\mathrm{p2}}e(t) + K_{\mathrm{i2}}\int_0^t e(t)\,\mathrm{d}t + K_{\mathrm{d2}}\frac{\mathrm{d}e(t)}{\mathrm{d}t} \qquad (4)$$

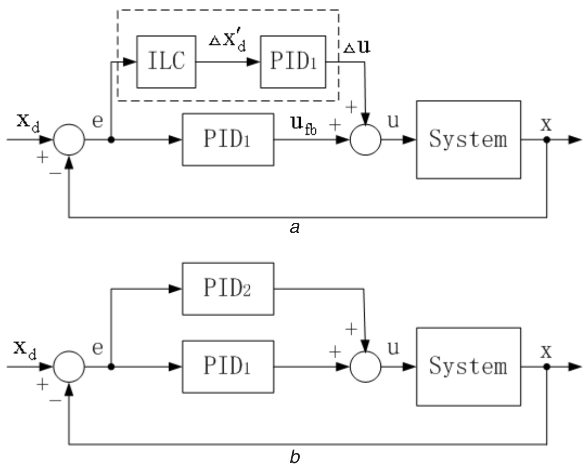

Fig. 3 *Derived equivalent PID_2 based PID controller*

a Equivalent representation of the ILC-augmented control system
b Approximately equivalent PID controller

The standard LS algorithm was used to obtain the parameters of PID_2. Equation (4) can be written in the linear regression form

$$\Delta u(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} \tag{5}$$

where $\boldsymbol{\theta} = [K_{p2}\ K_{i2}\ K_{d2}]^{\mathrm{T}}$ and $\boldsymbol{\varphi}^{\mathrm{T}}(t) = [e(t)\ \int_0^t e(t)\,\mathrm{d}t\ \mathrm{d}e(t)/\mathrm{d}t]$. In practical applications, the derivative signal is seldom obtained via direct measurement and measurement noise will be amplified if it is derived via direct differentiation. In this paper, the differential filter was used to derive the derivatives [15]. Fig. 4 shows the block diagram of the estimator with filters $H_f(p)$, where $p = \mathrm{d}/\mathrm{d}t$ represents the differential operator.

Equation (4) can be expressed in a general form

$$\Delta u(t) = A(p)e(t) \tag{6}$$

where $A(p) = p + (1/p) + 1$. With the additional filters $H_f(p)$, (6) can be rewritten as

$$H_f(p)\Delta u(t) = H_f(p)A(p)e(t) \tag{7}$$

where the filter $H_f(p)$ is a stable transfer function.

Let

$$\begin{aligned} \Delta u_f(t) &= H_f(p)\Delta u(t) \\ e_f(t) &= H_f(p)e(t) \end{aligned} \tag{8}$$

Thus, (7) can be written as

$$\Delta u_f(t) = K_{p2}e_f(t) + K_{i2}\int_0^t e_f(t)\,\mathrm{d}t + K_{d2}\frac{\mathrm{d}e_f(t)}{\mathrm{d}t} \tag{9}$$

Hence, the parameter vector remains as $\boldsymbol{\theta} = [K_{p2}\ K_{i2}\ K_{d2}]^{\mathrm{T}}$. The regression vector becomes

$$\begin{aligned} \boldsymbol{\varphi}_f^{\mathrm{T}}(t) &= \left[e_f(t) \quad \int_0^t e_f(t)\,\mathrm{d}t \quad \frac{\mathrm{d}e_f(t)}{\mathrm{d}t} \right] \\ &= \left[H_f(p)e(t) \quad \frac{1}{p}H_f(p)e(t) \quad pH_f(p)e(t) \right] \end{aligned} \tag{10}$$

Define

$$\begin{aligned} \boldsymbol{U} &= [\Delta u_f(1) \quad \Delta u_f(2) \quad \cdots \quad \Delta u_f(N)]^{\mathrm{T}} \\ \boldsymbol{\Phi} &= \begin{bmatrix} \boldsymbol{\varphi}_f^{\mathrm{T}}(1) \\ \boldsymbol{\varphi}_f^{\mathrm{T}}(2) \\ \vdots \\ \boldsymbol{\varphi}_f^{\mathrm{T}}(N) \end{bmatrix} \end{aligned} \tag{11}$$

where N is the number of data used in the estimation. Thus, the LS estimates of the parameters can be determined efficiently as

$$\hat{\boldsymbol{\theta}} = (\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{U} \tag{12}$$

Once the best-fit PID_2 controller was identified, the final PID controller was the combination of PID_1 and PID_2 which can be written as

$$\begin{aligned} u(t) &= (K_{p1} + K_{p2})e(t) + (K_{i1} + K_{i2})\int_0^t e(t)\,\mathrm{d}t \\ &\quad + (K_{d1} + K_{d2})\frac{\mathrm{d}e(t)}{\mathrm{d}t} \\ &= K_p e(t) + K_i\int_0^t e(t)\,\mathrm{d}t + K_d\frac{\mathrm{d}e(t)}{\mathrm{d}t} \end{aligned} \tag{13}$$

where K_p, K_i and K_d are the three overall parameters of the final PID controller. In this way, the PID controller was tuned in the closed loop.

It should be noted that the approach is applicable to control systems where the natural reference signal may not be a repetitive signal. In these cases, a deliberate periodic sequence, at an appropriate frequency of the control system, can be injected purely for tuning the PID controller, and the aforementioned steps remain applicable. Thereafter, the deliberate signal can be replaced by the natural reference signal of the system.

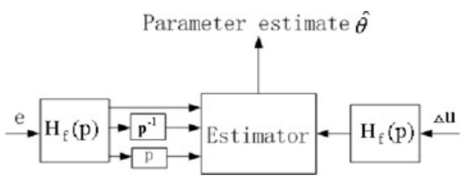

Fig. 4 *Block diagram of the estimator with filters, H_f*

3 Simulation results

In this section, a simulation study was conducted to verify the effectiveness of the proposed tuning method. The simulation example relates to the control of a permanent-magnet linear motor (PMLM) used for precise repetitive positioning applications.

The dynamics of a PMLM can be described by

$$u(t) = K_e\dot{x}(t) + Ri_q(t) + L\frac{\mathrm{d}i_q(t)}{\mathrm{d}t} \tag{14}$$

$$f(t) = K_f i_q(t) \tag{15}$$

$$f(t) = M\ddot{x}(t) + f_{\mathrm{friction}}(t) + f_{\mathrm{ripple}}(\dot{x}, x) + f_n(t) \tag{16}$$

where $u(t)$ and $i_q(t)$ are the time-varying motor terminal voltage and the equivalent armature current, respectively. $x(t)$ the motor position, $f(t)$ the developed force, and $f_{\mathrm{friction}}(t)$ and $f_{\mathrm{ripple}}(t)$ denote the frictional and ripple forces, respectively. The term $f_n(t)$ includes other residual uncertainties and disturbances in the system.

As the electrical time constant is much smaller than the mechanical one, the delay due to the electrical response can be ignored. At this time, the following model can be obtained

$$\ddot{x}(t) = \frac{\begin{pmatrix} -(K_f K_e/R)\dot{x}(t) + (K_f/R)u(t) - f_{\mathrm{ripple}} \\ -f_{\mathrm{friction}} - f_n \end{pmatrix}}{M} \tag{17}$$

Let

$$a = \frac{K_f K_e}{RM} \tag{18}$$

$$b = \frac{K_f}{RM} \tag{19}$$

Thus, we have

$$\ddot{x}(t) = -a\dot{x}(t) + bu(t) - \frac{1}{M}(f_{\mathrm{ripple}} + f_{\mathrm{friction}} + f_n) \tag{20}$$

The force ripple can be represented as

$$\begin{aligned} f_{\mathrm{ripple}}(x) &= A_r\ \sin(\omega x + \varphi) \\ &= A_{r1}\sin(\omega x) + A_{r2}\ \cos(\omega x) \end{aligned} \tag{21}$$

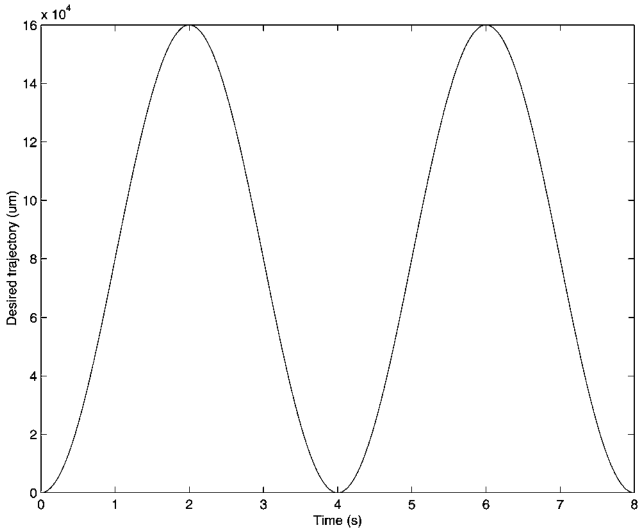

Fig. 5 *Desired trajectory*

and the friction model was considered as a combination of Coulomb and viscous friction. This friction model can be written as

$$f_{\text{friction}} = (f_c + f_v|\dot{x}|)\,\text{sgn}(\dot{x}) \tag{22}$$

where f_c is the minimum level of Coulomb friction and f_v the viscous friction parameter.

The following specific system model was used in the simulation

$$5.4\ddot{x} = -35.1\dot{x} + 8.1u - f_{\text{friction}} - f_{\text{ripple}} \tag{23}$$

The force ripple in (23) is described as a sinusoidal function with a period of 71.2 mm and an amplitude of 3 N, that is

$$f_{\text{ripple}} = 3\,\sin\left(\frac{2\pi x}{0.0712}\right) \tag{24}$$

According to the friction model in (22), the model parameters of the friction force used in the simulation study are given as

$$f_c = 3\,\text{N} \quad \text{and} \quad f_v = 10\,\text{N}$$

In the simulation study, a sinusoidal desired trajectory was chosen with a frequency of 0.25 Hz as shown in Fig. 5. The initial feedback controller PID_1 has parameters

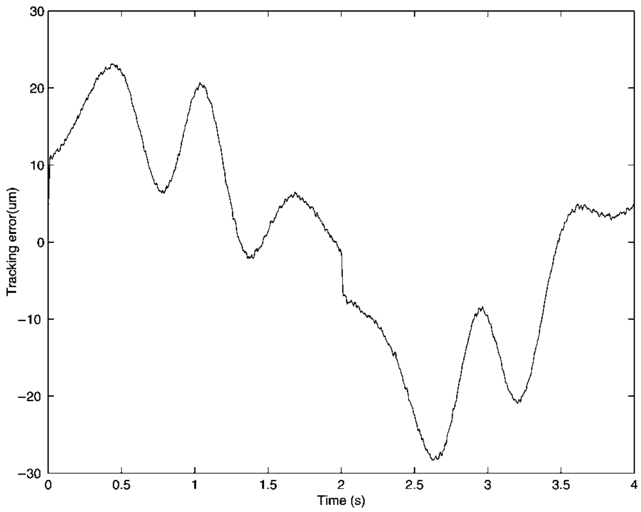

Fig. 6 *Tracking error with the feedback controller PID$_1$*

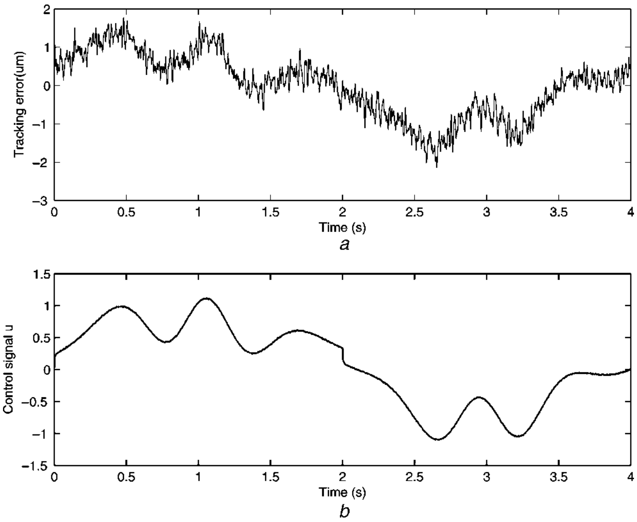

Fig. 7 *Tracking error during the 20th cycle*

a Tracking error (μm)
b Control signal Δu (V)

$K_{p1} = 0.045$, $K_{i1} = 0.03$ and $K_{d1} = 0.0002$. In order to test the robustness of the proposed tuning method under practical conditions, measurement noise was introduced to the system in the simulation study. Fig. 6 shows the tracking error incurred under the feedback controller PID_1.

For performance comparison, the root-mean-square (RMS) error e_{RMS} and the maximum absolute error e_{MAX} was used as indicators to evaluate the tracking performance over the full cycle. As shown in Fig. 6, the initial PID controller (PID_1) alone yields $e_{\text{RMS}} = 12.9$ μm and $e_{\text{MAX}} = 28.3$ μm.

Next, the ILC scheme, as discussed in Section 2, was applied to the system to further reduce the tracking error, and in the process, yielded the input and output signals necessary for the tuning of the new PID controller. The learning gain was chosen as $\lambda = 0.14$. The result is shown in Fig. 7 after 20 iterations. The tracking performance is enhanced significantly with $e_{\text{RMS}} = 0.82$ μm and $e_{\text{MAX}} = 2.1$ μm. Fig. 8 shows the convergence performance of the ILC scheme over 20 cycles.

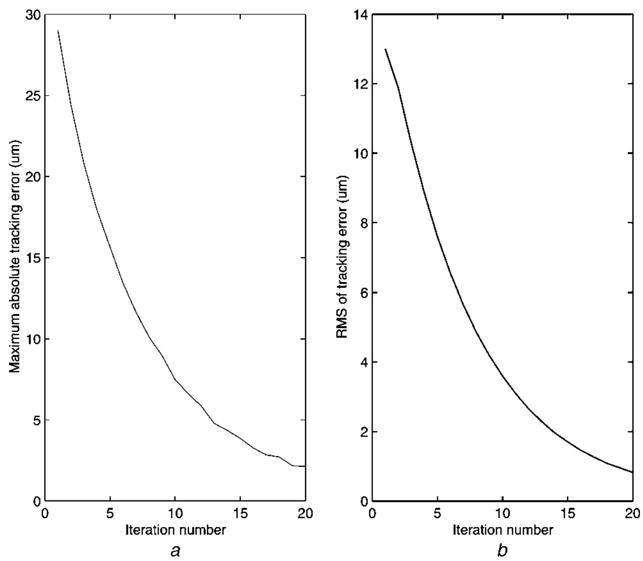

Fig. 8 *Iterative convergence performance*

a Maximum tracking error
b RMS tracking error

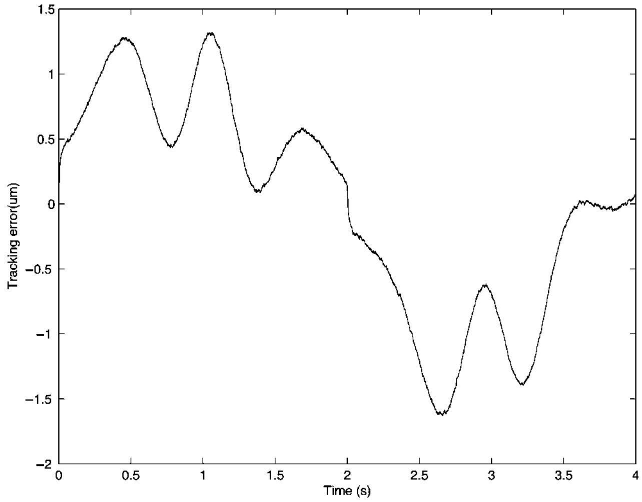

Fig. 9 *Tracking error with the tuned PID controller*

At this stage, we were ready to compute the parameters of PID_2. The tracking error $e(t)$ and the control signal $\Delta u(t)$, during the 20th cycle, were used as the input and the output signals to determine the parameters. These signals were first filtered using the low-pass filter $H_f(s)$. The filter $H_f(s)$ is designed as

$$H_f(s) = \frac{1600}{s^2 + 80s + 1600} \tag{25}$$

The LS algorithm was used to determine the estimates of the PID_2 parameters. The best-fitting parameters were calculated to be

$$K_{p2} = 0.7436, \quad K_{i2} = 0.2612 \quad \text{and} \quad K_{d2} = 0.0051 \tag{26}$$

Thus, the final PID controller was tuned with the parameters: $K_p = 0.7886$, $K_i = 0.2912$ and $K_d = 0.0053$. Fig. 9 shows the tracking performance when the tuned PID controller was applied to the system without any ILC component. A good performance with $e_{RMS} = 0.81$ μm and $e_{MAX} = 1.63$ μm was achieved. The maximum tracking error was reduced significantly, compared with the ILC scheme shown in Fig. 7. This was possible as the PID controller could suppress the noise in the system effectively, whereas the ILC was well known to be sensitive to noise and sharp changes in reference commands.

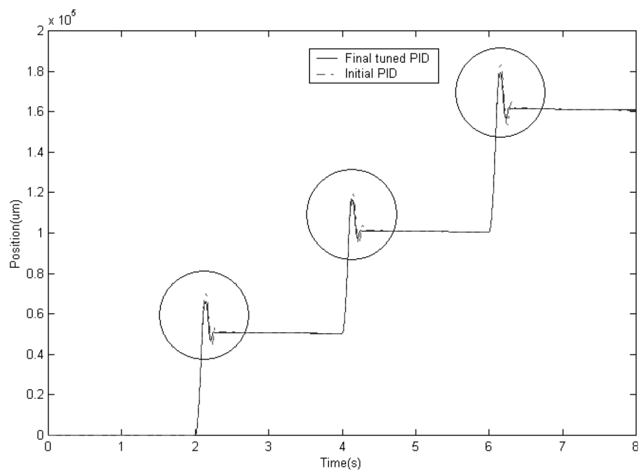

Fig. 10 *Comparison of performances for step changes in set point*

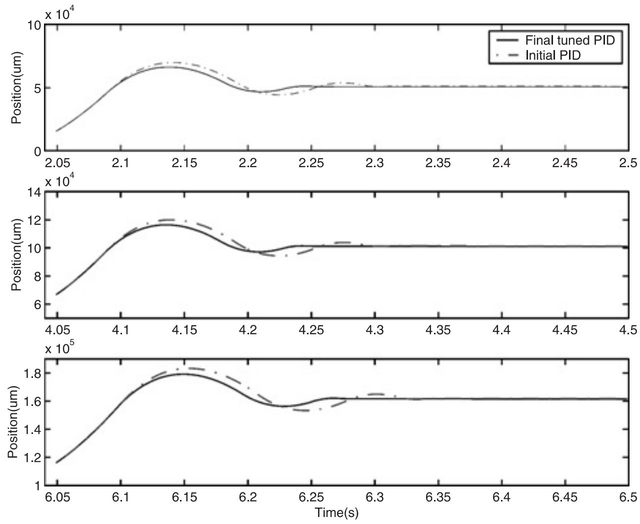

Fig. 11 *Magnified parts*

Next, a non-repetitive reference signal was used to simulate the case where the natural reference signal was a non-periodic one, such as step-types of reference signal for non-repetitive point-to-point positioning. Fig. 10 shows the performance comparison for the set point following. The circled parts in the figure are enlarged as shown in Fig. 11. From the figures, it can be observed that good performance could be achieved with the proposed PID tuning method, achieving a shorter rise time and settling time. Note that the vertical scale is in terms of 10^4 μm, so the improvement was significant.

4 Experimental study

In order to demonstrate the effectiveness of the proposed PID tuning method, experiments were conducted on a single axis stage manufactured by Steinmeyer. A SP-8 piezoelectric motor was used to drive the stage. Table 1 shows the specifications of the stage and the motor. The

Table 1: Specifications of piezoelectric linear motor

Travel, mm	Velocity (max), mm/s	Resolution, μm	Output force (max), N
200	250	0.1	40

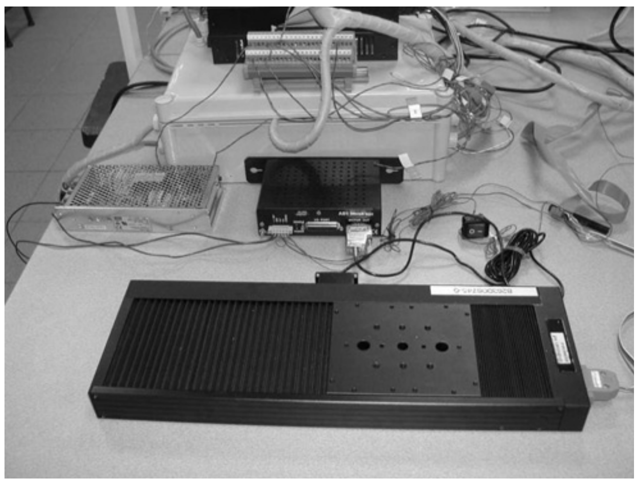

Fig. 12 *Setup of the linear piezoelectric motor*

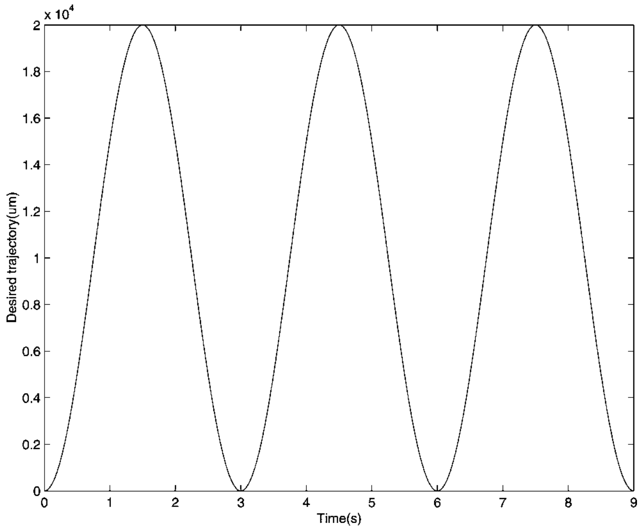

Fig. 13 *Desired trajectory used in the experimental study*

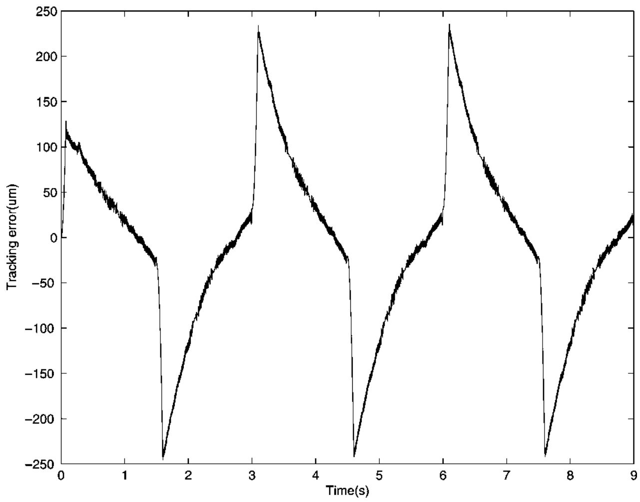

Fig. 14 *Tracking error with the initial feedback controller PID$_1$*

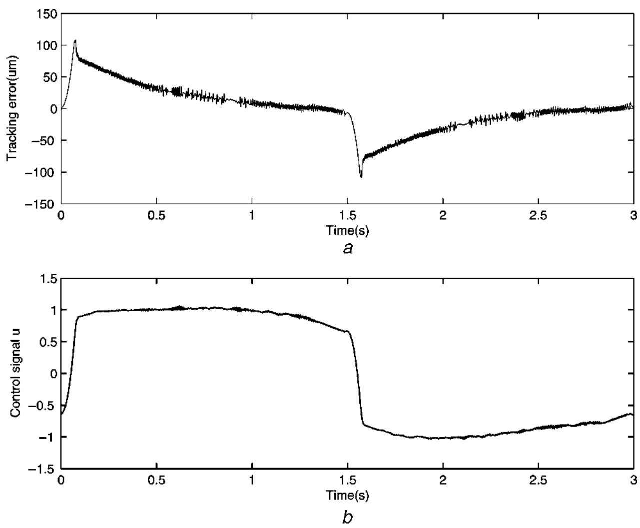

Fig. 15 *Tracking error during the 30th cycle*

a Tracking error (μm)
b Control signal Δu (V)

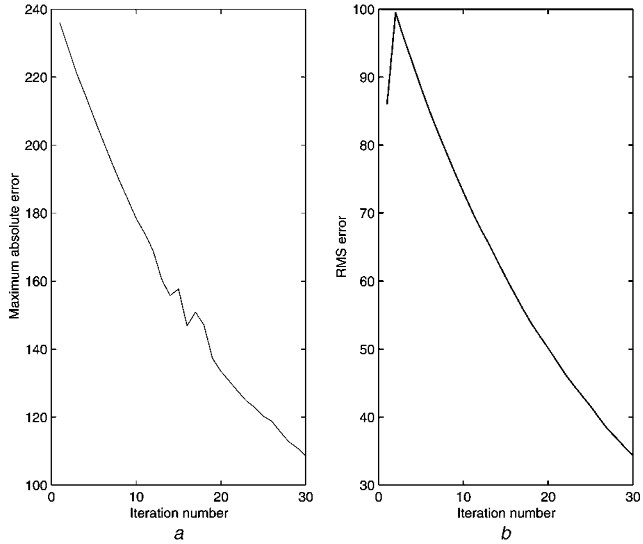

Fig. 16 *Iterative convergence performance*

a Maximum tracking error
b RMS tracking error

experimental studies were conducted on a dSPACE DS1102 control board. MATLAB and SIMULINK were the control platforms used for the experiment. The control scheme was implemented in the form of a C-coded S-function. A sampling frequency of 1 kHz was configured. Fig. 12 shows the experimental test platform.

A sinusoidal reference signal was used in the experimental study with a period of 3 s, shown in Fig. 13. The parameters of the initial feedback controller PID$_1$ were set as: $K_{p1} = 0.009$, $K_{i1} = 0.021$ and $K_{d1} = 0.000001$. Fig. 14 shows the tracking error incurred under the initial feedback controller PID$_1$. The above experimental result yielded $e_{RMS} = 102.99$ μm and $e_{MAX} = 245.71$ μm.

Next, the ILC scheme was applied to the system to reduce the tracking error further and achieve the input and output signals for the tuning of the new PID controller. The learning gain was chosen as $\lambda = 0.04$. Under the ILC scheme, the result is shown in Fig. 15 after 30 iterations. From the figure, it can be observed that the tracking performance was clearly enhanced with the learning scheme, compared with the initial feedback controller alone. In the 30th cycle, the performance indices of $e_{RMS} = 34.34$ μm and $e_{MAX} = 107.72$ μm were achieved. The tracking

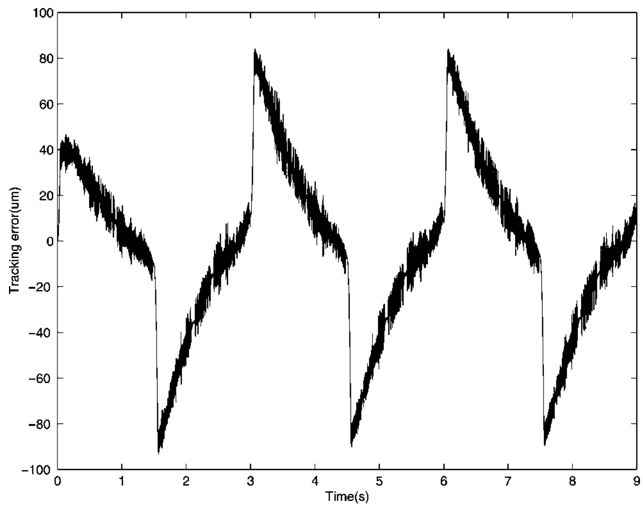

Fig. 17 *Tracking error with the tuned PID controller*

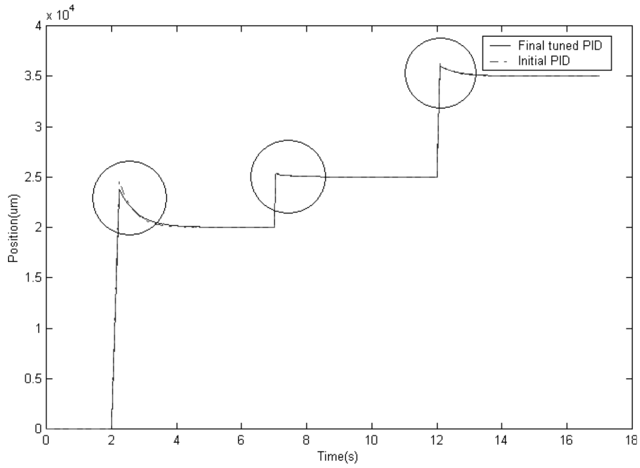

Fig. 18 *Performance comparison for step changes in set point*

convergence is shown in Fig. 16. Note that the blip in Fig. 16*b* was because of the initialisation transience associated with the ILC law.

With the information obtained from the 30th cycle, the parameters of PID_2 could be estimated. In the experimental study, the low-pass filter was still designed as in (26). The best-fitting parameters were determined as

$$K_{p2} = 0.0186, \quad K_{i2} = 0.0282 \quad \text{and} \quad K_{d2} = 0.0000105 \tag{27}$$

The final PID controller was thus obtained with $K_p = 0.0276$, $K_i = 0.0492$ and $K_d = 0.0000115$. Then, the final PID controller was applied to the system without the ILC component. The tracking performance is shown in Fig. 17 with $e_{RMS} = 39.3\ \mu m$ and $e_{MAX} = 93.4\ \mu m$. Improved tracking performance was obtained with the tuned PID controller. Similar to the phenomenon observed in the simulation study, the maximum tracking error was reduced significantly.

In the experimental study, tracking results for non-repetitive set point following were also observed. Fig. 18

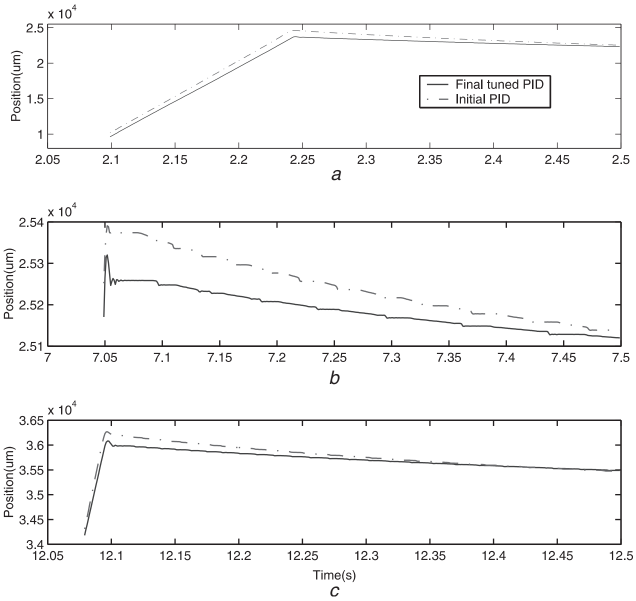

Fig. 19 *Magnified parts*

shows the performance for the set point following. In order to present the performance clearly, the circled parts in the figure are magnified and shown in Fig. 19. Improved performance was observed when compared with the initial PID controller. Note again that the vertical scale is in terms of 10^4 μm.

5 Conclusion

In this paper, a new approach for closed-loop automatic tuning of PID controller based on an ILC approach was proposed and developed. The method does not require the control loop to be detached for tuning purposes, but it requires the input of a periodic reference signal. Such a reference signal can be the natural reference signal of the control system when it is used to execute repetitive operational sequence, or it can be an excitation signal purely for tuning the PID controller. A modified ILC scheme iteratively changes the control signal by adjusting the reference signal only. Once a satisfactory performance was achieved, the PID controller was tuned by fitting the controller to yield close input and output characteristics of the ILC component via a standard LS algorithm. Simulation and experimental results verified the effectiveness of the proposed tuning method positively.

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