Measurement of Minority Carrier Lifetime and Surface Effects in Junction Devices*

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***Summary*—The characteristics of junction devices are influenced to a considerable degree by the lifetime of the minority carriers. Accordingly, methods for the measurement of this quantity are of considerable importance. Methods have been described for the measurement of the lifetime of minority carriers when these carriers are produced within the volume of a semiconductor. When the minority carriers are introduced near the surface of a semiconductor the resulting effective lifetime may be determined to a large extent by the nature of the surface. For most junction devices, it is the effective lifetime that is of primary importance.**

This paper describes a simple method for the measurement of effective lifetimes of injected minority carriers. The measurements may be applied to practical junction structures as, for example, an alloyed junction transistor. Measurements may be made on either completed or partially completed devices. The resulting data are potentially of value as quality controls during the fabrication of transistors and similar devices.

In many cases, the effective lifetime is a good indication of the surface conditions, and immediate evaluation of these conditions may be obtained at various stages of device processing. With selected geometries, the measurement method may be applied to determine absolute values of surface recombination velocities and should therefore be in studying surface conditions and treatments.

The measurement method is described in terms of junction devices using germanium as the semiconductor. However, the method is equally applicable to junction devices made with other semiconductor materials.

INTRODUCTION

An IMPORTANT material property which affects the performance of transistor devices is the lifetime of minority carriers in the semiconductor. This lifetime depends on the nature of the material and on the various treatments to which the material has been subjected. Electrically, it is a direct factor in many transistor parameters such as saturation current, current amplification factor, and others. It is, therefore, of considerable practical importance to be able to evaluate this factor directly on junction devices.

Earlier studies of minority carrier lifetimes have been mainly directed to evaluations as a property of the material[1] (volume lifetime) or as a property of a surface[2] (surface lifetime). As a result, the methods developed in these studies have not used the geometries of practical junction devices nor have they generally involved a *p-n* junction. This paper will describe a simple method[3] which is directly applicable to junction devices. Indeed' this method uses a *p-n* junction to inject minority carriers by means of a current pulse applied to the junction in the forward direction. The decay of the injected carriers is observed by open-circuiting the *p-n* junction and observing the junction voltage on an oscilloscope. A particular advantage for investigative work is that this measurement can be made on a single junction, thereby avoiding the more complex construction of a complete transistor. Furthermore, immediate evaluation can be made at various stages in the processing of junction units as a control in the fabrication of transistors or as a measurement in the study of process variations.

EXPERIMENTAL METHOD

The circuit of Fig. 1 shows an experimental arrangement for applying a constant current pulse in the forward direction through a *p-n* junction and, by means of a thermionic diode, open-circuiting the *p-n* junction at the termination of the current pulse. The open-circuited junction voltage is observed on an oscilloscope. Minority carriers are injected into the base region during the time the pulse is applied to the junction in the forward direction. Upon completion of the pulse, the thermionic diode effectively opens the circuit between the generator and the emitter. As a result, the junction voltage is a direct measure of what happens to the injected carriers. A typical open-circuited voltage wave form is also illustrated in Fig. 1 (voltage at *b-b'*).

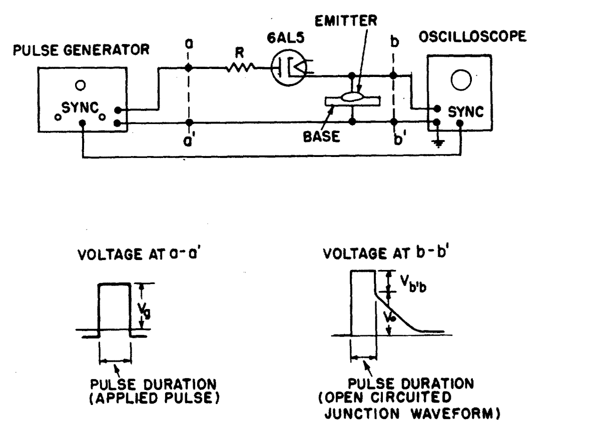

Fig. 1—Circuit illustration for applying constant current pulse to emitter-base junction and observing an open-circuited junction voltage upon termination of pulse.

It is observed that, after an initial drop due to an internal series resistance, the open-circuited junction voltage decays approximately linearly with time, and this linear decay is followed by an approximately exponen-

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1 L. B. Valdes, "Measurement of minority carrier lifetime in germanium," PROC. I.R.E., vol. 40, pp. 1420–1423; November, 1952.

2 D. Navon, R. Bray, and H. Y. Fan, "Lifetime of injected carriers in germanium," PROC. I.R.E., vol. 40, pp. 1342–1347; November, 1952.

3 A related development has been described by B. R. Gossick, "Post-injection barrier electromotive force of *p-n* junction," *Phys. Rev.*, vol. 91, pp. 1012–1013; August 15, 1953.

tial decay. As will be shown below, this linear portion of the voltage variation lends itself very readily to computation of a minority carrier lifetime, which is here designated as an *effective lifetime* since it results from the combined effect of volume and surface lifetimes.

In Fig. 2 there is shown a flexible circuit for use in connection with a suitable pulse generator and an oscilloscope for observing either the reverse bias (to be described subsequently) or the open-circuited junction

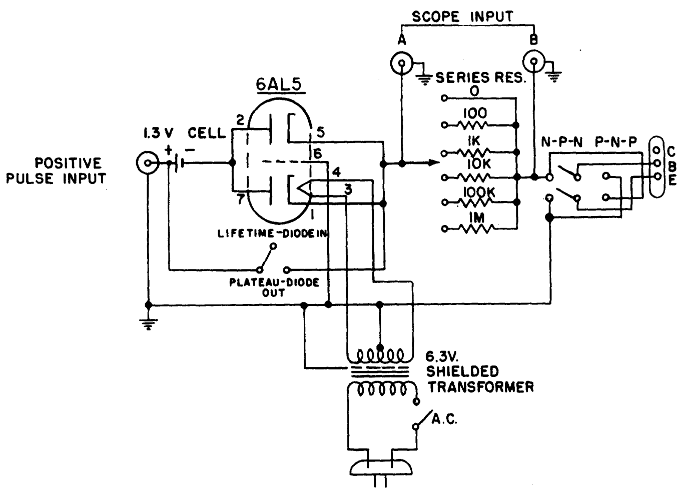

Fig. 2—Circuit used for the measurement of effective lifetime and related characteristics.

voltage. The pulse rise time and more important, the pulse decay time of the pulse generator, should be reasonably small—1/10 of the effective lifetimes to be measured should be adequate. Pulse length and repetition frequency usually used are 10 μsec and 3,000 p/sec, but the exact values employed are not important. The pulse amplitude and generator output impedance are also of no great importance. The oscilloscope response should be at least comparable to the pulse generator decay time mentioned above. It is important that the vertical amplification and horizontal trace speeds be calibrated. A differential oscilloscope connected as shown in Fig. 2 is a convenient means for measuring the pulse current flowing through the junction device. The 1.3-volt battery is inserted in series with the diode to eliminate a spurious voltage arising from the thermal velocities of the cathode-emitted electrons. A reversing switch is provided to accommodate both n-type and p-type devices with a single socket arrangement. A transparent alignment device with radial lines engraved thereon can be used for measuring effective lifetimes easily and quickly from oscilloscope displays of open-circuited junction voltages. When the vertical deflection sensitivity and horizontal sweep time are suitably adjusted, the effective lifetime is read directly by aligning one of the radial lines with the linear portion of the open-circuit junction-voltage waveform.

Theoretical Development

A theoretical interpretation of the observed junction voltage can be made on the basis of simple but approximate junction theory. A p-n junction in which the conductivity of the p-region is much greater than that of the n-region, as in an alloyed junction of indium on germanium, will be considered. In such a junction, the current flow across the transition region of the junction is predominantly a hole flow, and holes are injected into the n-type germanium. The results, however, will apply with equal validity to a junction in which the conductivity of the n-region is much greater than that of the p-region. In this case, electrons would be injected into the p-type region.

Let p_n be the hole density present in the n-region under themal equilibrium conditions, and Δp be the additional injected hole density in the n-region at the boundary of the junction transition region. The total hole density at the junction boundary will be

$$p = p_n + \Delta p. \tag{1}$$

From the theory of the p-n junction,[4] the hole density in the n-region at the junction boundary is given by

$$p = p_n e^{qV/kT}, \tag{2}$$

where V is the junction voltage. Combining (1) and (2) the solving for the voltage,

$$V = \frac{kT}{q} \ln\left(1 + \frac{\Delta p}{p_n}\right). \tag{3}$$

If the assumption is made that the excess carrier concentration, Δp, decays exponentially according to a single effective lifetime, τ_e, then

$$\Delta p = \Delta p_0 e^{-t/\tau_e}, \tag{4}$$

where Δp_0 is excess carrier concentration at the termination of the forward current pulse. Eq. (4) can be placed in (3). The constant $(1+\Delta p_0/p_n)$ can be readily evaluated in terms of the junction voltage, V_0, at $t=0$ (this is the junction voltage immediately before and immediately after the removal of the forward pulse—see Fig. 1), since

$$V_0 = \frac{kT}{q} \ln\left(1 + \frac{\Delta p_0}{p_n}\right). \tag{5}$$

The open-circuited junction voltage as a function of time is then

$$V = \frac{kT}{q} \ln\left[1 + (e^{qV_0/kT} - 1)e^{-t/\tau_e}\right]. \tag{6}$$

For t/τ_e very small, and if, as usual, $V_0 \gg kT/q$ (6) may be simplified to

[4] William Shockley, "Electrons and Holes in Semiconductors," D. Van Nostrand Company, Inc., New York, p. 312; 1950.

$$V \cong V_0 - \frac{kT}{q} t/\tau_e. \tag{7}$$

The initial voltage variation is linear with time. The slope of the linear variation is a measure of the effective lifetime.

$$\tau_e = -\frac{\Delta t}{\frac{q}{kT}\Delta V} = -\frac{kT}{q} \times \frac{1}{\text{Slope of Linear Decay}} \cdot \tag{8}$$

The values of Δt and ΔV may be read directly with the use of a calibrated oscilloscope. Fig. 3 shows some typical voltage wave shapes and some typical calculations for effective lifetimes.

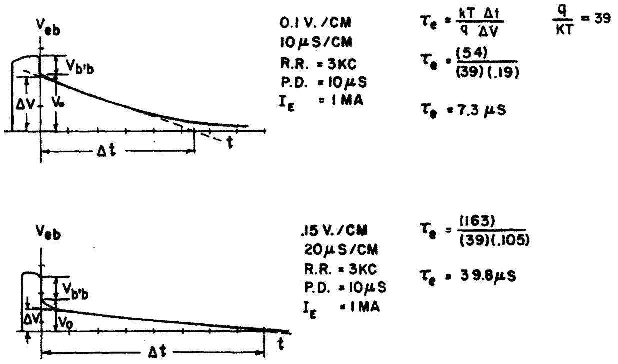

Fig. 3—Measurement of effective lifetime emitter-to-base open-circuited junction voltage.

The basic *p-n* junction theory applied above and based on (2) assumes that the injected minority carrier density is small compared with the majority carrier density. Accordingly, accurate measurements of τ_e should be made using small enough currents so that this assumption is valid. However, if the current is too small, a well-defined linear region is not obtained. For most of the devices that have been measured, a junction current of about 2 ma has been appropriate. When the junction current is increased so as to invalidate the assumption mentioned above, calculations similar to those above can be carried out, but the results are considerably more complex. The voltage decay for this case, as viewed on the oscilloscope, will exhibit a "hump" separating two regions of approximate linear decay. The latter decay corresponding to the region where the minority carriers are again small compared with the majority carriers can be used for measurement.

As is implied above, the preceding analysis does not possess a high degree of rigor. Only the life history of the holes has been considered, and the manner of their decay has been assumed without consideration of accompanying diffusion effects. In Appendix I this problem is examined in a more rigorous fashion. Both hole and electron carriers with independent lifetimes as well as diffusion effects are included. It is again assumed that the minority carrier density is small compared with the majority carrier density. A study of the solution indicates that as long as the portion of the junction current due to minority carriers (holes) is approximately equal to the total junction current (injection efficiency, γ, $=1$), then the resulting junction voltage decay will be that due to holes irrespective of the lifetime of the electrons. Further, it appears that the method of measuring lifetime discussed above should give results that are adequate for engineering purposes. As an additional check, the method of measuring lifetime discussed herein has been compared with another more involved method of measurement, and good agreement between the two methods of measurement has been obtained.[5]

Lifetime Measurements

Typical Measurements

Measurement of effective lifetimes for *p-n-p* junction transistors will give results generally ranging from 1 to 10 μsec.[6] Sample diodes made with materials having volume lifetimes of 1, 4, and 700 μsec gave effective lifetimes of 0.5, 3.4, and 39.8 μsec, respectively. Effective lifetimes as small as 0.01 μsec have been measured.

It is important to observe that the effective lifetime of the units made from material having 700 μsec volume lifetime was measured as only 39.8 μsec. On the other hand, the effective lifetime measured on units which were made from low volume lifetime material was quite close to the volume lifetime. This seems reasonable assuming effective lifetime to be a measure of the combined effects of volume recombination and surface recombination. In accordance with calculations for simple geometries,[7] effective lifetime, τ_e, volume lifetime, τ_v, and surface lifetime, τ_s, are related as

$$\frac{1}{\tau_e} = \frac{1}{\tau_v} + \frac{1}{\tau_s}. \tag{9}$$

The surface lifetime, τ_s, will be dependent upon the geometry and upon the surface recombination velocity. For a fixed geometry, the effective lifetime together with the volume lifetime (measured by conventional methods on the bulk material) can be used for determining a surface lifetime which is directly related to the surface treatment. When the volume lifetime is much larger than the measured effective lifetime (as is usually the case in practical device geometries), effective lifetime is very nearly a measure of surface lifetime and can accordingly be used as an index of surface treatment.

[5] These measurements were carried out by Dr. A. R. Moore, RCA Laboratories and utilize the decay of photoconductivity following illumination with a pulsed light source. This technique has been described by D. T. Stevenson and R. J. Keyes, "Measurement of lifetimes and diffusion constants in germanium," *Phys. Rev.*, vol. 94, p. 1416; June 1, 1954.

[6] R. R. Law, C. W. Mueller, J. I. Pankove, and L. Armstrong, "A developmental germanium *p-n-p* junction transistor," Proc. I.R.E., vol. 40, pp. 1352–1357; November, 1952.

[7] W. Shockley, *op. cit.*, pp. 318–325.

TABLE I

CHANGE IN EFFECTIVE LIFETIME VALUES AS A RESULT OF ETCHING

Specimen	τ_v Volume Lifetime	τ_e After Fabrication and chemical etch $s=400$ cm/sec	τ_e Dipped in etch containing $Cu(NO_3)_2$ $s=7400$ cm/sec	τ_e Electrolytic etch 2 min 2 ma $s=250$ cm/sec	τ_e Electrolytic etch 5 min 3 ma $s=250$ cm/sec	τ_e Electrolytic etch 5 min 3 ma $s=250$ cm/sec
T-6	4 μsec	3.4 μsec	1.8 μsec			
T-61	700 μsec	37 μsec	4.1 μsec	40 μsec	58 μsec	58 μsec

Effects of Etching on Effective Lifetime

To observe the effect of surface treatment on effective lifetime, two germanium alloy junctions having 0.045-inch diameter emitter dots, base wafer-thickness of 0.005 inch, and volume lifetimes that were substantially different were first chemically etched and measured and then dipped in an etch containing copper nitrate for 15 seconds. This etch was chosen because of its ability to produce a high surface recombination velocity which has been reported to be approximately 7,400 cm/sec.[8] It was noticed upon removing the junction from the etch that copper was deposited on the dot and on the germanium surface. Following the etch treatment, the effective life was measured and indicated a substantial lower lifetime than before etching. The copper was next removed by an ammonia and hydrogen-peroxide solution, and the unit was washed in distilled water. The junction was then electrolytically etched in 1 per cent sodium hydroxide for two minutes at 2 ma current. Subsequent measurement of effective lifetime indicated a decided increase from its previous value. The effective lifetime was further increased by additional electrolytic etching; subsequent etching produced no further increase in τ_e. The measured data for the sequence of etching together with reported values of surface recombination velocities produced by these etching solutions on germanium are shown in Table I.[8] The data in Table I indicate that there is a close correlation between surface recombination velocity and effective lifetimes when the volume lifetime is large. With the aid of (9), $\tau_s=39.5$, 4.12, and 63.3 μsec are obtained for the chemical etch, containing copper nitrate, and electrolytic etches, respectively. If these surface lifetimes are proportionally related to surface recombination velocities as

$$\frac{1}{\tau_s} = Ks \tag{10}$$

values of the geometrical factor, K, can be computed as 63.2, 32.8, and 63.2 cm^{-1}. If the second value is discarded, a geometrical factor of 63.2 cm^{-1} is applicable for the units described above. Data similar to that shown in Table I can be used to obtain geometrical factors for different junction devices. After the geometrical factor of the unit has been determined, measurements of effective lifetime and volume lifetime can be used for determining the surface recombination velocity of completed or partially completed units. Often only a relative comparison of surface treatments is desired. In this case, the geometrical factor need not be determined. The surface lifetime serves as an index of comparison.

Absolute Determination of the Surface Recombination Velocity

It is sometimes necessary to make a direct determination of surface recombination velocity. Thus, the efficacy of the etching solution may be in question, or a new solution may need calibration.

The absolute calibration can be made by using a junction geometry amenable to analysis as carried out by Shockley.[9] Thus, for the geometry as shown in Fig. 4,

$$s = \frac{1}{\tau_s\left[\dfrac{1}{B} \quad \dfrac{1}{C}\right]}. \tag{11}$$

The dimensions of the sample are not critical. It has been convenient to use wafers whose dimensions are $2A=0.215$ inch, $2C=0.125$ inch and $2B=0.005$ inch.

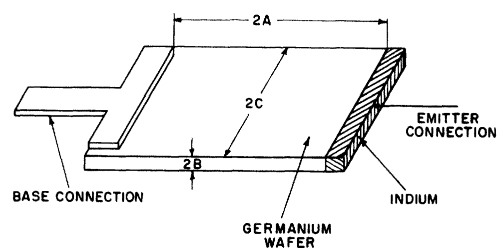

Fig. 4—Device for direct measurement of surface recombination velocity.

The $2A$ dimension should be chosen several times larger than the volume diffusion length. The $2B$ dimension should preferably be chosen so as to be the dominant term in (11). In this case then, (11) is valid as long as $s(2B/D)\leqq 1$ (D is the diffusion constant for the minority carriers under consideration).

As a typical example of the application of this method of direct determination, several specimens were made with germanium whose volume lifetime was $\tau_v=700$

[8] A. R. Moore and J. I. Pankove, "The effect of junction shape and surface recombination on transistor current gain," PROC. I.R.E., vol. 42, pp. 907–913; June, 1954.

[9] W. Shockley, "The theory of *p-n* junctions in semiconductors and *p-n* junction transistors," *Bell Sys. Tech. Jour.*, vol. 28, pp. 435–489; July, 1949. See also W. Shockley, *op. cit.*, pp. 318–325.

μsec. The effective lifetime for these specimens averaged 36.5 μsec. Accordingly, using (9), $\tau_s = 38.5$ μsec is computed. Finally, with the aid of (11), $s = 320$ cm/sec is computed. The surface treatment in question was an electrolytic etch so that this value of s is in good agreement with $s = 250$ cm/sec that has been previously used (see Table I).

Measurement of Base-Lead Resistance

In an alloyed junction device the base-lead resistance, $r_{bb'}$ is the majority carrier (ohmic) resistance of the semiconductor between the metallic contact to the semiconductor and the region near the actual *p-n* junction. It is an important factor in the performance of many junction devices.[10]

The method of measurement to be described below measures a diode base-lead resistance which is generally different from that of the corresponding device operating as a transistor. This difference is due to the dissimilar current distribution within the body of the semiconductor for diode and transistor operation.

The initial drop in the open-circuited junction voltage upon termination of the pulse can be used for the measurement of the resistance. The voltage, $V_{b'b}$, corresponding to the drop across $r_{bb'}$ is shown in Figs. 1 and 3. If the positive amplitude of the generator pulse, V_g, is measured, then $r_{bb'} = V_{b'b} R / V_g$, where R is the current limiting resistance in series with the pulse generator. This assumes that the voltage drop across the 6AL5 tube, the voltage across the junction, and the voltage of the series battery are negligible in comparison with the voltage drop across the current limiting resistor. If this assumption is not valid, the junction current just before the pulse is removed can be determined by measuring the appropriate voltage across R with the aid of the differential input to the oscilloscope (see circuit of Fig. 2).

Observations of Reverse Bias Waveform

During the course of this work experimental observations were made of the junction recovery voltage under conditions of applied reverse bias. In this case, a reverse bias is applied to the junction immediately after the termination of the forward pulse. The junction waveform under these conditions is observed on an oscilloscope. This type of switched junction operation has been investigated.[11–14] Since the interpretation of the observed waveform is somewhat more complex than that of the open-circuited case discussed above, this observation was not developed into a system for the determination of minority carrier lifetimes. However, qualitative observations made under these conditions may be quite valuable, and in some cases this method of operation is a more sensitive indication of whether or not minority carriers are being injected. A switch is included in the circuit of Fig. 2 to enable this observation to be made. This switch shorts out the 6AL5 diode and bias battery and is labeled "plateau-diode out." The reverse bias is supplied by a blocking condenser in the output of the generator. This condenser becomes charged during the forward pulse. After the forward pulse is terminated, the charged blocking condenser applies a reverse bias to the *p-n* junction.

Observations of the junction waveform under the conditions of a reverse bias following a forward pulse (see Fig. 5) show first an immediate drop in voltage after the termination of the pulse due to the base-lead resistance.

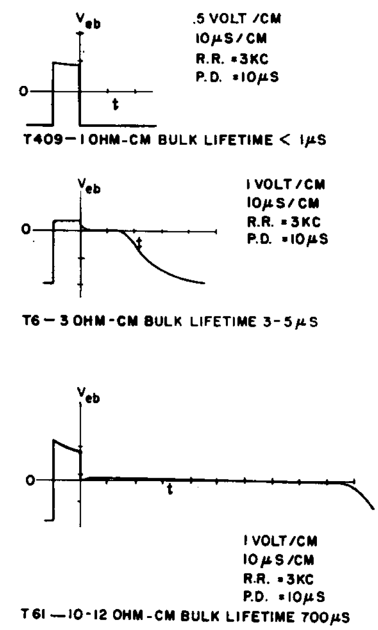

Fig. 5—Variation in plateau length of emitter-base voltage for different base wafer lifetimes.

This is similar to that discussed above in the case of the open-circuited junction voltage. This immediate drop in voltage is generally followed by an extended period of approximately zero voltage after which the reverse voltage across the junction gradually increases in magnitude as the injected carriers recombine and permit the junction to be biased in the reverse direction. The existence of the zero-voltage plateau indicates that minority carrier injection has taken place. These observations can be utilized in a qualitative manner to check for minority carrier injection and as a qualitative observation of the effective lifetime. These effects are illustrated by the experimental observations shown in Fig. 5. This figure shows the experimental waveforms under the reverse bias conditions observed on junction diodes made from

[10] L. J. Giacoletto, "Study of *p-n-p* alloy junction transistor from dc through medium frequencies," *RCA Rev.*, vol. 15, pp. 506–562; December, 1954.

[11] E. M. Pell, "Recombination rate in germanium by observation of pulsed reversed characteristics," *Phys. Rev.*, vol. 90, pp. 278–279; April 15, 1953.

[12] R. G. Shulman and M. E. McMahon, "Recovery currents in germanium *p-n* junction diodes," *Jour. Appl. Phys.*, vol. 24, pp. 1267–1272; October, 1953.

[13] R. H. Kingston, "Switching time in junction diodes and junction transistors," Proc. I.R.E., vol. 42, pp. 829–834; May, 1954.

[14] B. Lax and S. F. Neustadter, "Transient response of a *p-n* junction," *Jour. Appl. Phys.*, vol. 25, pp. 1148–1154; September, 1954.

germanium having different volume lifetimes. It is seen that, for the unit made from germanium having a volume lifetime of less than 1 microsecond, there is essentially no plateau region. An appreciable plateau region is observed in the second case for the unit having a volume lifetime between 3 and 5 microseconds. Finally, a rather extended plateau is observed in the third case for a unit made from material having a volume lifetime of 700 microseconds.

Appendix I: Open-Circuited Junction Voltage

This appendix contains the solution for the open-circuited junction voltage following operation in the forward direction when both holes and electrons with independent lifetimes are considered, and when the movement of these carriers is governed by the one-dimensional continuity equation. The material in this appendix is the work of Dr. D. O. North, RCA Laboratories, Princeton, New Jersey.

The p-n junction is operated in a forward direction until a steady-state condition is reached, and at time $t=0$, the forward bias is removed and the open-circuited junction voltage determined as a function of time. The solution for the open-circuited junction voltage when displacement currents are neglected and when the minority carrier density is small compared with the majority carrier density is

$$\frac{e^{\Lambda V(t)}-1}{e^{\Lambda V_0}-1} = \frac{J_p}{J_p-J_n}\left[1-\operatorname{erf}\sqrt{\frac{t}{\tau_p}}\right] - \frac{J_n}{J_p-J_n}\left[1-\operatorname{erf}\sqrt{\frac{t}{\tau_n}}\right] + \frac{\sqrt{J_nJ_p}}{J_p-J_n}\sqrt{A}e^{-Bt}\left[\operatorname{erf}\sqrt{\frac{J_n}{J_p}A\frac{t}{\tau_p}} - \operatorname{erf}\sqrt{\frac{J_p}{J_n}A\frac{t}{\tau_n}}\right], \tag{12}$$

where

$$A = \frac{J_pJ_n(\tau_p-\tau_n)}{J_p^2\tau_p - J_n^2\tau_n}, \tag{13}$$

$$B = \frac{J_p^2 - J_n^2}{J_p^2\tau_p - J_n^2\tau_n}, \tag{14}$$

and the various quantities have the following meaning:

$\Lambda = \frac{q}{kt}$ of suitable sign so that ΛV_0 is a positive quantity,

$V(t)$ = open-circuited junction voltage following $t=0$,

V_0 = forward junction voltage at $t=0$.

τ_n, τ_p = electron and hole lifetimes in p-type and n-type semiconductors, respectively.

$J_n = n_p\sqrt{\frac{D_p}{\tau_n}}$ = thermally generated electron current density in p-type semiconductor.

$J_p = p_n\sqrt{\frac{D^n}{\tau_p}}$ = thermally generated hole current density in n-type semiconductor.

n_p, p_n = electron and hole density present in p-type and n-type semiconductors respectively under equilibrium condition.

D_n, D_p = electron and hole diffusion constant in p and n semiconductors respectively.

$$\operatorname{erf} y = \frac{2}{\sqrt{\pi}}\int_0^y e^{-x^2}dx.$$

The solution given above is applicable to the general case where the n-type and p-type semiconductors have arbitrary characteristics. Certain special cases can now be considered.

1. If neither $\tau_n \to 0$ or $\tau_p \to 0$ and n_p and p_n remain finite, then respectively $J_n \to \infty$ or $J_p \to \infty$ and $V(t)=0$. This is the case when the minority carrier lifetime in either semiconductor approaches zero.

2. If $J_n \to 0$ by $n_p = 0$, then

$$\frac{e^{\Lambda V(t)}-1}{e^{\Lambda V_0}-1} = 1-\operatorname{erf}\sqrt{\frac{t}{\tau_p}}\cdot$$

Since $n_pp_p = n_i^2 =$ a constant, $n_p \to 0$ is the same as $p_p \to \infty$. This is the case of the conductivity of the p-type semiconductor being infinitely large. In this event the minority carrier lifetime, τ_n, can be arbitrarily small provided only that $J_n \to 0$. The same limit solution is obtained if $J_n \to 0$ by $\tau_n \to \infty$. Due to the symmetry of (12), the solution for $J_p \to 0$ is obtained by interchanging τ_n for τ_p.

3. If $\tau_p = \tau_n = \tau$, then

$$\frac{e^{\Lambda V(t)}-1}{e^{\Lambda V_0}-1} = 1-\operatorname{erf}\sqrt{\frac{t}{\tau}},$$

irrespective of the values of J_n and J_p.

4. If $J_n = J_p$, then

$$\frac{e^{\Lambda V(t)}-1}{e^{\Lambda V_0}-1} = 1 - \frac{1}{2}\left[\operatorname{erf}\sqrt{\frac{t}{\tau_p}} + \operatorname{erf}\sqrt{\frac{t}{\tau_n}}\right] - \frac{1}{2}\left[\operatorname{erf}\sqrt{\frac{t}{\tau_p}} - \operatorname{erf}\sqrt{\frac{t}{\tau_n}}\right] \cdot\left[\frac{\tau_p+\tau_n}{\tau_p-\tau_n} + \frac{4t}{\tau_p-\tau_n}\right] + \frac{2}{\sqrt{\pi}(\tau_p-\tau_n)}\left[\tau_n\sqrt{\frac{t}{\tau_n}}e^{-t/\tau_n} - \tau_p\sqrt{\frac{t}{\tau_p}}e^{-t/\tau_p}\right]$$

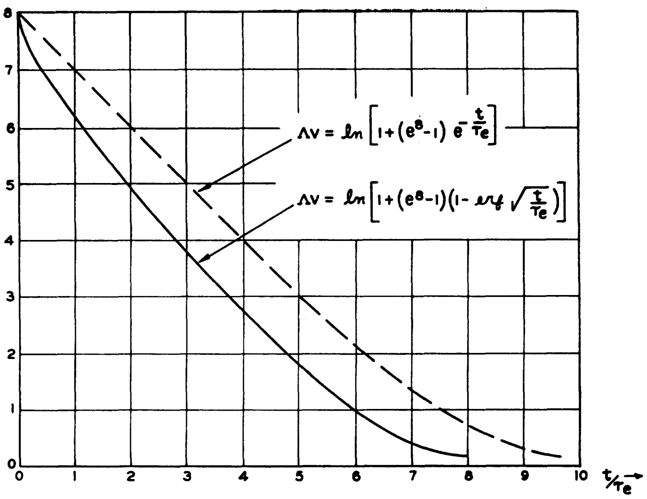

Fig. 6—Comparison of voltage decay for exponential and error function time dependency.

Case 2 is the solution applicable to the operation considered herein. This solution differs from the solution given in (6) which states that

$$\frac{e^{\Delta V(t)} - 1}{e^{\Delta V_0} - 1} = e^{-t/\tau_p}.$$

When the injected minority carrier density is small, the voltage decay does have a form similar to that given by the error function solution as shown in Fig. 6 for an arbitrary case of $\Delta V_0 = 8$. For a somewhat larger minority carrier injection level, the voltage decay has more nearly the form of that given by the exponential solution also shown in Fig. 6 for comparison. At still larger minority carrier injection the voltage decay exhibits a "hump" as described in the text.

Correspondence

Understanding the Gyrator*

The gyrator, postulated by Tellegen[1] as a new nonreciprocal network element, is attracting the attention of network theorists nowadays. Shekel[2] has shown that a four-pole network with nonreciprocal admittance matrix $\|Y_{ij}\|$ can be separated into the parallel combination of a reciprocal network and a gyrator (Fig. 1) of gyrating admittance $\gamma = (Y_{12} - Y_{21})/2$, i.e.

$$\|Y_{ij}\| = \begin{Vmatrix} Y_{11} & Y_{12} - \gamma \\ Y_{21} + \gamma & Y_{22} \end{Vmatrix} + \begin{Vmatrix} 0 & \gamma \\ -\gamma & 0 \end{Vmatrix}.$$

Carlin[3] has found the necessary and sufficient conditions for the synthesis of nonreciprocal networks by means of reciprocal networks and real gyrators.

The gyrator's physical significance can be seen from the equivalent circuit of Fig. 2 with admittance matrix $\|Y_{ij}\|$; for an arbitrary value of the admittance Y_2 there follows:

$$Y_1 = Y_{11} - Y_2, \qquad Y_3 = Y_{22} - Y_2$$
$$I' = (Y_{12} + Y_2)V_2, \quad I'' = (Y_{21} + Y_2)V_1.$$

* Received by the IRE, January 6, 1955.

[1] B. D. H. Tellegen, "The Gyrator, a New Electric Network Element," Philips Res. Rep. 3, pp. 81–101; 1948.

[2] J. Shekel, "The gyrator as a 3-terminal element," Proc. I.R.E., vol. 42, pp. 1014–1016; August, 1953.

[3] H. J. Carlin, "Theory and Application of Gyrator Networks," Polytechnic Inst. of Bklyn., Res. Rep. 289; March, 1954.

Considering separately the system of two current generators it is seen that its total input power is Re $(Y_{12} + Y_2 + Y_{21}^* + Y_2^*) V_1^* V_2$.

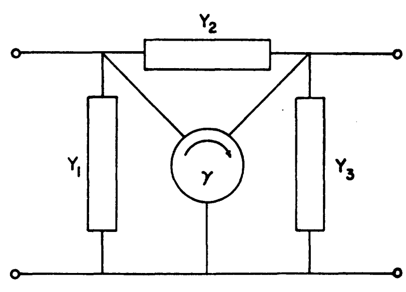

Fig. 1—Separation of a gyrator from a nonreciprocal network.

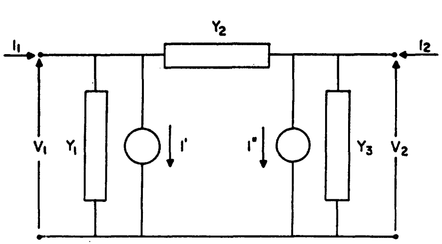

Fig. 2—π-equivalent circuit of a nonreciprocal network.

If in particular $Y_2 = -(Y_{12} + Y_{21})/2$, this power is zero and the system (I', I'') reduces to the gyrator.

Similarly, starting from the network's impedance matrix $\|Z_{ij}\|$ and assuming an equivalent circuit of the type of Fig. 3, there follows for an arbitrary value of the impedance Z_2

$$Z_1 = Z_{11} - Z_2 \qquad Z = Z_{22} - Z_2$$
$$V' = (Z_{12} - Z_2)I_2 \qquad V'' = (Z_{21} - Z_2)I_1.$$

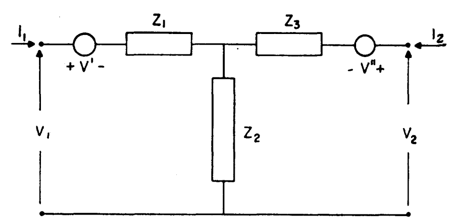

Fig. 3—T-equivalent circuit of a nonreciprocal network.

In particular, if $Z_2 = (Z_{12} + Z_{21})/2$, the system of two voltage generators reduces to a gyrator of gyrating impedance $\zeta = (Z_{21} - Z_{12})/2$.

These considerations suggest methods of simple realization of gyrators by means of current or voltage generators.

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