

# Correlation spectrum analyzer for direct measurement of device current noise

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This article analyzes the realization and the performance of a correlation spectrum analyzer specifically conceived to directly measure the current noise produced by electronic devices with maximum sensitivity. The text describes in detail and gives the design rules of the instrument input amplifiers taking into consideration noise, dynamic range, stability, and bandwidth, together with the effects that a device under test (DUT) having complex impedance introduce. This article shows that the proposed scheme may allow current noise measurements with a sensitivity improved by few orders of magnitude with respect to a standard spectrum analyzer and to a correlation analyzer in voltage scheme whenever the DUT has an impedance larger than few 10 k $\Omega$ . Such a sensitivity makes the proposed instrument ideal for the characterization of advanced devices, such as ultrashort channel metal-oxide-semiconductor field effect transistors, mesoscopic junctions, or spin dependent electron transfer devices where it may be necessary to detect noise levels as low as fA/ $\sqrt{\text{Hz}}$ .

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## I. INTRODUCTION

In a previous article<sup>1</sup> we have discussed the striking advantages of using a correlation spectrum analyzer to measure very small noises generated by electronics devices with respect to using a standard analyzer. In that article two possible measuring schemes based on the correlation principle over two channels were introduced, one for voltage noise measurements and one for current noise measurements. As most of the primary physical noise sources in the electronic devices are in the form of current, the current scheme should be the one of choice: the direct sensing of the current scheme corresponds more closely to physical intuition, eases interpretation of the results and, by avoiding voltage conversion, often simplifies the measuring setup leading to an improvement in the overall performance.

In this work we take a very close look at this current noise measurement scheme, analyzing in detail all the aspects involved in the instrument construction, in its use and in the analysis of the results in order to reach the best sensitivity in all possible applications. The article is organized as follows: Section II recalls briefly the principles of correlation spectrum analysis of current signals in comparison to a standard commercial spectrum analyzer; Sec. III describes in detail the design of the front-end transimpedance amplifiers and gives practical rules on how to obtain maximum sensitivity; Sec. IV explains the effects on the instrumental sensitivity when measuring devices of complex impedance; Sec. V compares the performance of the correlation scheme with the one of a standard analyzer. After showing that nonuniformities between the two channels (Sec. VI) and quantization of the signal (Sec. VII) produce negligible effects, we summarize in Sec. VIII the advantages of the current scheme in

comparison with a voltage scheme, before drawing some conclusions.

## II. PRINCIPLE OF CORRELATION SPECTRUM ANALYSIS OF CURRENT SIGNALS

Figure 1 shows for comparison the basic schemes of a standard spectrum analyzer (a) and of a correlation spectrum analyzer—CSA—(b) both set for current noise measurements.

A commercially available spectrum analyzer, typically having voltage input, needs an external transimpedance amplifier to convert into voltage the current from the device under test (DUT). The analyzer is schematically summarized by its input voltage amplifier followed by a circuit that selects, with a given bandwidth (BW), the frequency at which the spectral density should be calculated, and a final stage that computes the mean squared value, proportional to the DUT's current noise power density for the selected frequency. The sensitivity (minimum detectable DUT signal) of this configuration depends directly on the added transimpedance amplifier: only DUT signals with spectral density greater than the noise of transimpedance amplifier can be measured.<sup>2</sup>

In the correlation spectrum analyzer [Fig. 1(b)] the DUT is connected between the virtual grounds of two transimpedance amplifiers. The current signal produced by the DUT is processed in phase by the two frequency selectors to give at the output of the multiplier a signal with a mean value proportional to the DUT's noise power density ( $S_{\text{DUT}}$ ) at the selected frequency. In first approximation, the noises produced by each transimpedance amplifier are uncorrelated with each other. They are multiplied with random phase and at the output of the multiplier they give a signal with zero mean value and standard deviation equal to the amplifiers noise.

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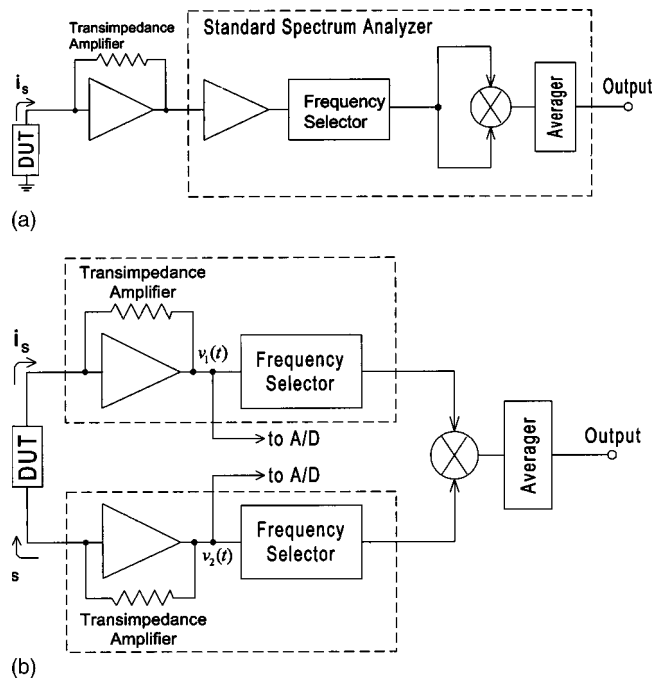


FIG. 1. Basic scheme (a) for current measurements using a traditional spectrum analyzer with voltage input, in which the current signal from the DUT is converted into voltage by an external transimpedance amplifier. Lower scheme (b) is the correlation spectrum analyzer. The signal from the DUT is fed to two distinct and independent input amplifiers operated in parallel.

The final averaging reduces these fluctuations to any low value by properly extending the measuring time  $T_m$  and allowing the desired DUT information to be evaluated with increasingly high precision. Assuming the uncorrelated amplifier noise ( $S_A$ ) to be much larger than the DUT noise ( $S_{DUT}$ ) to be measured, the ratio between correlated ( $S_C$ ) and uncorrelated ( $S_u$ ) signals at the output of the instrument, given by Ref. 1:

$$\frac{S_C}{S_u} = \sqrt{2 \cdot BW \cdot T_m} \cdot \frac{S_{DUT}}{S_A} \quad (1)$$

shows well how the sensitivity of the instrument can be improved at will by correspondingly extending the measuring time. The total measurement time, as the sum of the time  $T_m$  spent for each frequency point of the full spectrum, is longer for a larger amount of uncorrelated noises present on each channel and for smaller DUT noises to be detected. In addition to measuring time, the instrument sensitivity is affected by any other spurious correlated signal generated in the instrument which adds to the one produced by the DUT without being distinguishable. This crucial point will be discussed in detail in Sec. IV.

The actual instrument may be based on the digital processing of the signals available at the outputs of the transimpedance amplifiers [see Fig. 1(b)] to perform cross correlation analysis and averaging, leading to the same instrumental performance as the analog architecture.

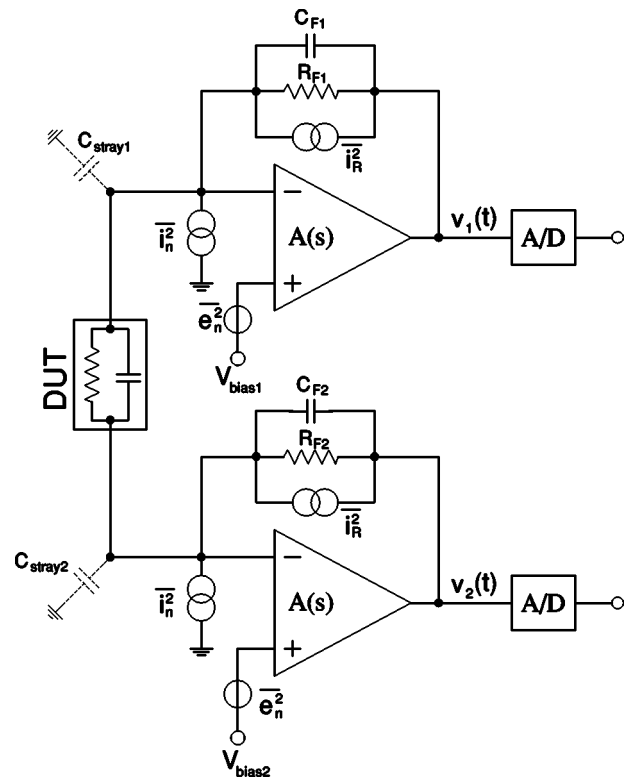


FIG. 2. Schematics of the instrument input stage. Noise sources and parasitic capacitances are also indicated.

### III. DESIGN RULES FOR THE FRONT-END TRANSIMPEDANCE AMPLIFIERS

Although not directly setting the sensitivity of the instrument as in a standard spectrum analyzer, the two transimpedance amplifiers of the CSA should be carefully designed to limit the measurement time, to minimize the correlated signal produced through the DUT, and to provide adequate instrument bandwidth. To analyze the role of each circuit component we will refer to the detailed schematic of the instrument input stage shown in Fig. 2.

#### A. Noise and dynamic range considerations

Thanks to the virtual ground of each transimpedance amplifier, the current noise sources  $i_n^2$  of the operational amplifier and  $i_R^2$  of the feedback resistor remain confined to their own channel and contribute to the uncorrelated amplifier noise,  $S_A$ . The current noise of the feedback resistor  $R_F$  ( $i_R^2 = 4kT/R_F$ ) imposes a value of  $R_F$  as high as possible to correspondingly reduce the measuring time. When  $R_F$  is allowed to be very high (in our instrument about 2.5 G $\Omega$ ), the current noise  $i_n^2$  of the operational amplifier may become important. In practice the value of  $R_F$  is limited by the following two aspects. The first is that all the dc current produced by the DUT flows through  $R_F$  and one should prevent the amplifier from reaching saturation by limiting  $R_F$ . Second, as will be discussed in detail in the next paragraph, high values of  $R_F$  correspondingly reduce the bandwidth of the amplifier.

To conclude, we note that if  $R_F$  gets too small (about 1 k $\Omega$  in our instrument), the voltage noise of the following

stage may become the dominant contribution of uncorrelated noise.

### B. Stability and bandwidth considerations

The feedback network of the transimpedance amplifier also defines, in connection with the actual impedance of the DUT, the instrument bandwidth. Stability considerations are indeed the reasons for the introduction of the feedback capacitance  $C_F$  (see Fig. 2). If the feedback would be made only by  $R_F$  without  $C_F$ , the poles of the operational amplifier  $A(s)$  and the pole introduced by the DUT would possibly drive the circuit into instabilities. By adding  $C_F$ , we add a zero in the loop gain

$$G_{\text{loop}}(s) = A(s) \left[ \frac{R_D}{R_D + R_F} \right] \times \left[ \frac{1 + sR_FC_F}{1 + s(R_D \parallel R_F)(C_F + C_{\text{stray}} + C_D)} \right], \quad (2)$$

where  $R_D$  is the equivalent resistance of the DUT and  $R_D \parallel R_F$  indicates the parallel between the feedback resistor and the equivalent DUT resistance.

If the loop gain is sufficiently high, the bandwidth of the transimpedance amplifier is then practically given by  $f_{\text{BW}} \cong 1/2\pi R_FC_F$ .

The choice of  $R_F$  indeed defines the loop gain, the position of the pole, and the instrument bandwidth. In order to set the value of  $R_F$ , taking into account these three conditions, let us refer to Fig. 3 which shows a plot of  $G_{\text{loop}}(s)$  in the case of a single pole unconditionally stable amplifier  $A(s)$  having a gain-bandwidth product (GBWP).

If the pole of  $G_{\text{loop}}(s)$

$$f_p = \frac{1}{2\pi(R_D \parallel R_F)(C_F + C_{\text{stray}} + C_D)} \quad (3)$$

is at a higher frequency than  $f_{\text{BW}}$ , Fig. 3(a) guides us in the calculation of the maximum value of  $R_F$ :

$$R_{F_{\text{MAX}}} = \frac{R_D}{f_{\text{BW}} \cdot G_{\text{min}}} (\text{GBWP} - f_{\text{BW}} \cdot G_{\text{min}}) \quad (4)$$

that can be used providing that a minimum loop gain,  $G_{\text{min}}$ , is ensured over the full instrument bandwidth. If, instead, the pole  $f_p$  of  $G_{\text{loop}}(s)$  is at a lower frequency than  $f_{\text{BW}}$  [Fig. 3(b)], the constraint on  $R_F$  becomes more severe due to the higher slope of the Bode plot. In this case, the value of  $R_F$  is given by

$$\hat{R}_F = \frac{\text{GBWP} - f_{\text{BW}} \cdot G_{\text{min}}}{2\pi \cdot f_{\text{BW}}^2 (C_{\text{stray}} + C_D) G_{\text{min}}} < R_{F_{\text{MAX}}} \quad (5)$$

Stability is ensured providing  $G_{\text{min}} > 1$ .

When setting the instrument for a new measurement, one can first compute  $R_{F_{\text{MAX}}}$ , then check the position of  $f_p$  with respect to the desired bandwidth  $f_{\text{BW}}$  and, eventually, compute  $\hat{R}_F$  again on the basis of Eq. (5). At this point the only reasons to chose a smaller value for  $R_F$  are the dynamic problem imposed by the dc current from the DUT, as mentioned in the previous paragraph, and the practical consideration that  $C_F$  may not be made smaller than 0.5 pF.

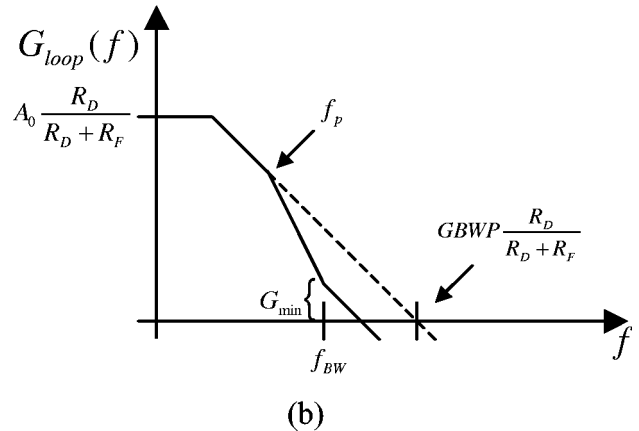
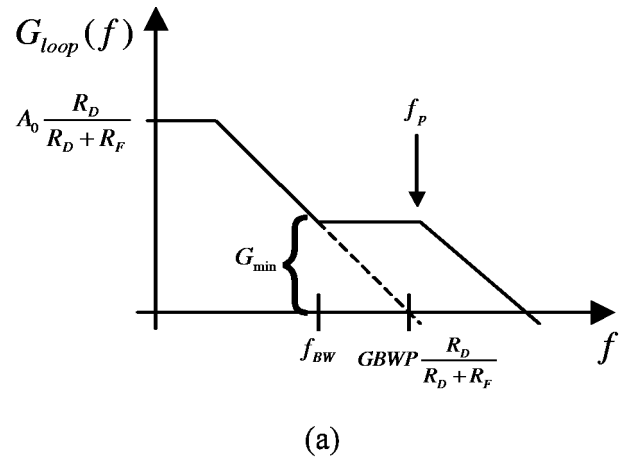


FIG. 3. Loop gain of the transimpedance amplifier for a single pole stable amplifier  $A(s)$  in the case  $f_{\text{BW}} < f_p$  (a) and  $f_{\text{BW}} > f_p$  (b).

### C. AC coupling of the DUT

The simplest way to set the feedback resistance independently of the DUT biasing current could be to ac couple the DUT to the transimpedance amplifiers, as shown in Fig. 4. This setup introduces new sources of noise from the voltage supply  $V_{cc}$  and the biasing resistors  $R_P$ , that only produce uncorrelated contributions on two channels in a similar way as  $R_F$ .

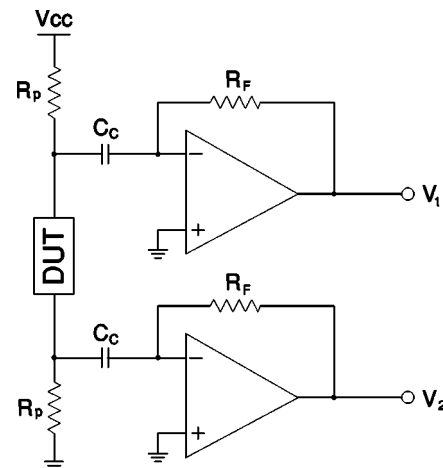


FIG. 4. Schematics of the ac coupling of the DUT to the correlation spectrum analyzer.

Although now  $R_F$  can be chosen at will, to benefit in terms of noise (and, consequently, in terms of measuring time) from this ac coupling, the resistance  $R_P$  should be larger than the value of  $R_F$  in the previous dc connection. This is possible by correspondingly increasing  $V_{CC}$  to many times the saturation voltage of the operational amplifier  $A(s)$ . For example, if this latter is 10 V and  $V_{CC}$  is set to 100 V the uncorrelated noise  $S_A$  is reduced by a factor of 5, and the measuring time [see Eq. (1)] improves by a factor of 25!

A possible disadvantage of the ac coupling may be the bandwidth in the low frequency range, limited to the value of

$$f_{\min} = \frac{1}{2\pi \frac{C}{2} ((2R_P) \parallel R_D)}. \quad (6)$$

AC coupling of the DUT becomes unavoidable when the biasing voltage across the DUT should be higher than that which can be set directly by the input nodes of the transimpedance amplifier, as is the case, for example, in thick compound semiconductor devices that require 100 V bias.<sup>3</sup>

#### IV. DUT IMPEDANCE EFFECT ON THE MAXIMUM SENSITIVITY

The instrument sensitivity is limited by the spurious correlated signals present in the two channels. One source of spurious correlation could be voltage supply fluctuations. Fortunately, their effect can be easily eliminated by using independent batteries on the two channels.<sup>4</sup> Another source could be electromagnetic interferences. Also in this case, their effect can be very effectively reduced by properly shielding the full setup.<sup>5,6</sup>

The major limitation in the sensitivity appears to be the coupling of the two instrument channels directly through the DUT. In fact, the series noise of the two amplifiers (indicated with  $\overline{e_n^2}$  in Fig. 2) is applied across the DUT and produces a current flowing in phase on both channels. This correlated component of current adds to the DUT signal and would not be eliminated by the instrument.

To quantify this contribution, let us calculate the signal  $\hat{V}_1$  at the output of channel 1 due to the voltage series noise  $e_{n1}$  of channel 1:

$$\hat{V}_1(j\omega) = e_{n1} \left[ 1 + \frac{R_{F1}}{R_{D1}} + j\omega(C_D + C_{\text{stray}})R_{F1} \right], \quad (7)$$

where  $C_D$  is the DUT capacitance across the two terminals connected to the instrument inputs and  $R_{D1}$  is the DUT resistance as seen by channel 1.  $C_{\text{stray1}}$  summarizes all capacitances to ground at the input of channel 1, that is, strays of the amplifier, strays of the connection to the DUT, and capacitances internal to the DUT seen by channel 1 other than  $C_D$ .

The signal  $\hat{V}_2$  at the output of channel 2 due again to the same voltage noise,  $e_{n1}$ , of channel 1 is instead given by

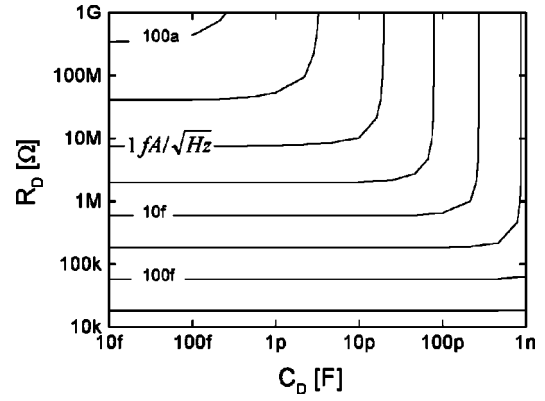


FIG. 5. Minimum measurable current signal as a function of the capacitance  $C_D$  and resistance  $R_D$  of the DUT. Curves refer to  $\overline{e_{n1}^2} = \overline{e_{n2}^2} = 4.5 \text{ nV}/\sqrt{\text{Hz}}$ ,  $C_{\text{stray}} = 20 \text{ pF}$ ,  $R_F = 10 \text{ M}\Omega$  and are calculated at 1 kHz.

$$\hat{V}_2(j\omega) = e_{n1} \left[ \frac{R_{F2}}{R_{D1}} + j\omega C_D R_{F2} \right]. \quad (8)$$

The components in phase between  $\hat{V}_1$  and  $\hat{V}_2$  give the correlated amount generated by  $e_{n1}$ :

$$\overline{V_{\text{corr1}}^2} = \overline{e_{n1}^2} \left[ \left( 1 + \frac{R_{F1}}{R_{D1}} \right) \frac{R_{F2}}{R_{D1}} + \omega^2 R_{F1} R_{F2} (C_D + C_{\text{stray}}) C_D \right] \quad (9)$$

while the quadrature components would be eliminated by the instrument.

The same calculations can be done for the voltage noise  $e_{n2}$  of channel 2 to obtain  $\overline{V_{\text{corr2}}^2}$ .

To directly compare these spurious components with the DUT signal to be measured, we can refer  $\overline{V_{\text{corr1}}^2}$  and  $\overline{V_{\text{corr2}}^2}$  to the input as a current through the DUT

$$\begin{aligned} \overline{i_c^2} = \overline{e_{n1}^2} & \left[ \left( \frac{1}{R_{F1}} + \frac{1}{R_{D1}} \right) \frac{1}{R_{D1}} + \omega^2 (C_D + C_{\text{stray1}}) C_D \right] \\ & + \overline{e_{n2}^2} \left[ \left( \frac{1}{R_{F2}} + \frac{1}{R_{D2}} \right) \frac{1}{R_{D2}} + \omega^2 (C_D + C_{\text{stray2}}) C_D \right]. \end{aligned} \quad (10)$$

The value of  $\overline{i_c^2}$  gives the minimum DUT signal that can be measured and is valid for a generic passive multipole DUT. In the case of an active DUT, the calculation of  $\hat{V}_2$  takes into account the internal amplification factor. In the most common case of a DUT dipole, where  $\overline{e_{n1}^2} = \overline{e_{n2}^2} = \overline{e_n^2}$ ,  $R_{D1} = R_{D2} = R_D$ ,  $R_{F1} = R_{F2} = R_F$  and  $C_{\text{stray1}} = C_{\text{stray2}} = C_{\text{stray}}$  the equation reduces to the simpler expression

$$\overline{i_c^2} = 2 \cdot \overline{e_n^2} \left[ \left( \frac{1}{R_F} + \frac{1}{R_D} \right) \frac{1}{R_D} + \omega^2 (C_D + C_{\text{stray}}) C_D \right]. \quad (11)$$

Figure 5 shows the sensitivity limit of our instrument as a function of the DUT impedance. The figure indicates that a sensitivity around  $1 \text{ fA}/\sqrt{\text{Hz}}$  can be reached in practical situations. Note that at higher frequencies the reactance  $1/\omega C$  of the DUT is dominating over  $R_D$  and, therefore, the sensitivity is independent of  $R_D$ . By reducing the voltage



noise  $\overline{e_n^2}$  of the two transimpedance amplifiers and by minimizing the strays of the connection of the DUT, these limits can be further improved.

## V. COMPARISON WITH STANDARD SPECTRUM ANALYZER

The design considerations made so far on the transimpedance amplifiers allow us to state the differences in performance between a standard spectrum analyzer and the correlation spectrum analyzer (CSA) clearly. These differences are summarized in the following for the sake of clarity:

- *Sensitivity:*

In a standard instrument, the DUT signal should be larger than the instrumental noise; in CSA, DUT signal can be an order of a smaller magnitude than the instrumental noise and will be recovered at the expense of measuring time.

- *Dynamics:*

In a standard instrument the enhancement of sensitivity may be obtained by increasing the value of the feedback resistance  $R_F$ , at the expense of a correspondingly lower bias current in the DUT; in CSA,  $R_F$  is chosen to allocate for dc current without affecting sensitivity, at the expense of measuring time.

- *Bandwidth:*

In a standard instrument the enhancement of sensitivity (larger  $R_F$ ) correspondingly reduces the bandwidth of the measurement; in CSA the bandwidth is set independently from sensitivity at the expense of measuring time.

- *Stray capacitances:*

In a standard instrument, the signal generated by the input voltage noise through the DUT and the stray capacitances can be calculated to be

$$\overline{i_c^2} = \overline{e_n^2} \left[ \left( \frac{1}{R_F} + \frac{1}{R_D} \right)^2 + \omega^2 (C_D + C_{\text{stray}})^2 \right]. \quad (12)$$

In the CSA, the input voltage noise generates a correlated signal given by Eq. (11). Comparison of the two shows that the advantage of the CSA becomes more and more significant as  $C_D$  is smaller than the stray capacitances of the connections from the DUT to the instrument.

## VI. NONUNIFORMITIES BETWEEN THE TWO CHANNELS

The instrument is relatively insensitive to nonuniformity in gain and phase between the two channels. A phase difference  $\Delta\varphi$  between the two channels results in a product between correlated signals reduced by a factor  $\cos(\Delta\varphi)$ . The same phase difference does not affect the uncorrelated noise components. The effect of a phase difference is, therefore, to reduce the useful output signal (gain) without modifying the superimposed uncorrelated noise, with the consequence that the same sensitivity can be obtained by a further extension of the measuring time. Concerning different gains in the two

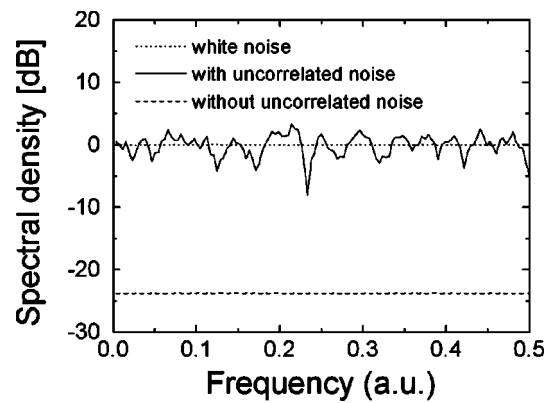


FIG. 6. Simulations of the quantization effects on white noise signal. The dotted line is the real value of spectral density; the continuous line is the spectrum reconstructed with a CSA when the uncorrelated noise is summed up, whereas the dashed line refers to the case without uncorrelated noise.

channels, they affect correlated and uncorrelated signals in the same way, and therefore do not play any role.

A calibration of the instrument would compensate for the gain and phase variations in frequency and would ensure maximum accuracy over the full instrument bandwidth. Note that a delay  $\tau$  of one channel with respect to the other smaller than the sampling period (which is largely the case in this application), is equivalent to a phase shift  $\Delta\varphi = \omega\tau$  and, therefore, produces the same effects as above.

## VII. AUTOMATIC SUPPRESSION OF THE QUANTIZATION NOISE

The peculiarity of the CSA to measure DUT signals much smaller than the instrument noise imply that the dynamic range of the analog/digital (A/D) converters should be set in accordance to the instrument noise and that the DUT signal may, in some cases, be comparable to the least significant bit (LSB). This extreme situation does not introduce quantization errors nor prevent the measurement from being performed correctly. This is because the small DUT signal is summed equally (in the same direction) to both the large uncorrelated instrumental noise of the two channels. The large instrumental noise shifts the small DUT signal randomly across almost the full dynamic range of the A/D converter. The correlation procedure performed by the instrument sorts out the small DUT signal common to both channels. In addition, thanks to the random sweep of the analog-to-digital converter dynamics, the differential nonlinearities of the converter are averaged down by the number of quantization levels covered by the instrumental noise.<sup>7</sup>

Figure 6 shows the results of a simulation that verifies this point. It is the case of a small white noise whose rms value is equal to LSB/4 superimposed to an uncorrelated noise of 5LSB.

The results of the simulation of the reconstruction of the white noise (continuous line) sit well on the expected value of spectral density (dotted line), even though the noise was well below the quantization level. The dashed line shows the white noise spectral density as reconstructed by the simulation in the absence of uncorrelated noise. The value is lower

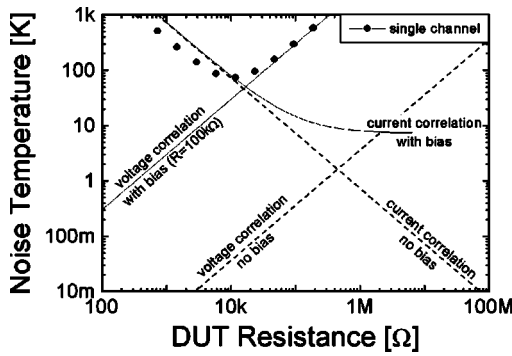


FIG. 7. Noise temperature of the correlation spectrum analyzer for voltage and for current measurement. The dashed lines indicate the sensitivity without bias applied, whereas the continuous lines report the case with bias. The dots indicate the sensitivity of a traditional single channel spectrum analyzer.

because most of the samples do not overcome the first quantization threshold, thus giving zero power.

### VIII. ADVANTAGES OF CURRENT NOISE MEASUREMENT WITH RESPECT TO VOLTAGE NOISE MEASUREMENT

A direct measurement of the current noise of an electronic device, in addition to the practical advantages in terms of the simplicity of the connection and biasing directly through the instrument, may have a breakthrough effect on the sensitivity if compared with a voltage sensitive scheme.

To compare the sensitivity of the voltage and the current scheme, we can refer to Fig. 7 where the noise temperature of the instruments is reported as a function of the DUT resistance. The dashed lines indicate the sensitivity limit in the measurement of devices where no bias is applied. The current correlation scheme should be chosen whenever the DUT resistance is larger than about 300 kΩ.

The range of applicability of the current scheme extends further to lower DUT resistances when a bias is applied to the DUT. The continuous curves in Fig. 7 report the best performance of the instrument in the case of a biasing network of 100 kΩ resistance, showing that the current scheme is already advantageous at 10 kΩ DUT resistance.

As a clarifying example, let us suppose that we want to measure the drain current noise produced by a metal-oxide-semiconductor field effect transistor (MOSFET) biased with  $I_D = 100 \mu\text{A}$ . In the case of the current scheme [see Fig. 8(a)], the device can be simply inserted between the two virtual grounds of the instrument that set the drain to source voltage directly. To fully exploit the instrument sensitivity,  $R_F$  should be chosen as high as possible, about 100 kΩ in this example, to profit from the 10 V dynamic range of the amplifier. The equivalent instrument noise, given by Eq. (11), equaled  $4kT_N/R_D$  and neglecting DUT capacitance, gives the noise temperature as

$$T_N = \frac{2 \cdot e^2}{4k} \left( \frac{1}{R_D} + \frac{1}{R_F} \right), \quad (13)$$

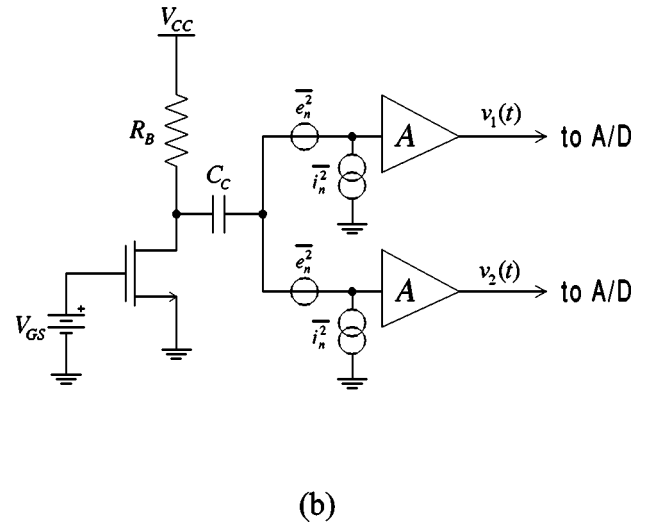
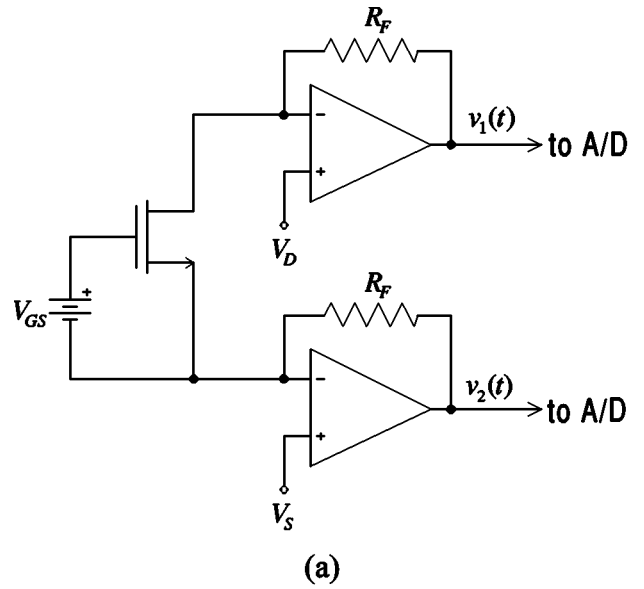


FIG. 8. Drain current noise measurement of a MOSFET with a current scheme (a) and a voltage scheme (b).

which is exactly the continuous line of Fig. 7 labeled “current correlation with bias.” In the limit of  $R_F \rightarrow \infty$ , one can obtain the corresponding dashed curve of Fig. 7.

If using a voltage correlation scheme [see Fig. 8(b)], about the same resistance of 100 kΩ is needed as a biasing element toward the power supply. The current noise  $\overline{i_{R_B}^2}$  introduced by  $R_B$  summed to the instrumental current noise  $\overline{i_n^2}$  produces a voltage signal in  $B$  that is correlated on the two channels. The instrument correlated output is therefore

$$v_c^2 = (r_0 \parallel R_B)^2 \cdot \left( \overline{i_n^2} + \frac{4kT}{R_B} \right) \cdot A^2. \quad (14)$$

By referring this signal to the input and equating to  $4kT_N/R_D$ ,

$$\frac{v_c^2}{A^2(R \parallel R_D)^2} = \frac{4kT_N}{R_D} \quad (15)$$

we obtain the noise temperature as

$$T_N = T \frac{R_D}{R} + \frac{\overline{i_n^2} R_D}{4k}. \quad (16)$$

This is exactly the straight continuous line of Fig. 7 labeled “voltage correlation with bias.” At the limit of no bias constraints ( $R_B \rightarrow \infty$ ) one obtains the corresponding dashed curve.

With commercial single-channel spectrum analyzers, the performance of the measurement would be the one indicated by the dots.

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