# Spectrum analyzer with noise reduction by cross-correlation technique on two channels

M. Sampietro,[a] L. Fasoli, and G. Ferrari
*Dipartimento di Elettronica e Informazione, Politecnico di Milano, 20133 Milano, Italy*

(Received 10 September 1998; accepted for publication 26 January 1999)

A spectrum analyzer with a sensitivity better than few pV/$\sqrt{\text{Hz}}$ in voltage noise measurements and better than 1 fA/$\sqrt{\text{Hz}}$ in current noise measurements is presented. It has two distinct and independent input amplifiers in parallel, connected to the same device under test (DUT) and is based on the suppression of their uncorrelated noises. The instrument is modular with different front-end amplifiers conceived to optimize the measurement of low impedance or high impedance DUTs. The instrument can cover 8 decades of frequency span, from 10 mHz to 1 MHz. The improvement of sensitivity with respect to a traditional system and the simplicity in the connection and biasing of the DUT makes it perfectly suited to measure ultralow noise levels in semiconductor devices, like trapping noise, shot noise associated with tunneling in fractional quantum Hall systems, $1/f$ and channel noise in metal–oxide–semiconductor field effect transistors operated below threshold. © *1999 American Institute of Physics.* [S0034-6748(99)01805-5]

## I. INTRODUCTION

Ultralow current or voltage noise levels, as nowadays obtainable from semiconductor devices, are often well below the measuring possibility of conventional spectrum analyzers, whose minimum detectable signal (sensitivity) is limited by the noise level of their front-end amplifiers. The measurement of very low noise levels is therefore a challenging task that can be only partially overcome by ''interfacing'' the device under test (DUT) to the available instrument through a custom-designed extremely low-noise amplifier able to lower the intrinsic limits of the instrument.[1] To improve further the performance, an additional technique has been proposed to precisely evaluate and subtract the instrument noise, as reported in Ref. 2 where a power spectral density four times smaller than the power spectrum of the equivalent input generators of the amplifier has been measured. A different approach can be pursued by ''including'' the DUT in a custom-designed amplifier whose input active device is the DUT itself.[3] This solution allowed to measure junction field-effect-transistors (JFETs) series noise spectral densities as low as about 100 $pV/\sqrt{\text{Hz}}$ but it is obviously limited to the noise analysis of transistors operated in the active region.

When even lower noise levels are to be measured or when the DUT is a nonamplifying device [passive bipoles, $p$-$n$ junctions, metal–oxide–semiconductor field effect transistors (MOSFETs) operated in subthreshold regime, etc.], the only remaining possibility is the direct reduction of the instrument noise itself, as obtainable with a cross-correlation technique.[4] An instrument using this technique, which we call a correlation spectrum analyzer, is based on the processing of signals from two independent channels operated in parallel, and takes advantage of the uncorrelated properties of the noises of the two input stages. This concept has been proven to be effective in the experimental discovery of fractionally charged quasiparticles in quantum Hall systems[5,6] and is available in a few commercial digital spectrum analyzers[7] although limited to the measurement of voltage signals from a medium to low impedance source.

In this article we present a fully operational correlation spectrum analyzer that extends the capabilities of these latter systems by performing direct measurement of both voltage and current noise signals from devices of any impedance with great flexibility in the DUT biasing. After having introduced the working principle of this class of instruments, this article describes in detail the special features of the front-end setup for current and for voltage noise measurements and reports the instrument performance as a function of the time spent for the measurement and of the DUT impedance.

It is worth noting that lock-in noise reduction techniques cannot be used, as opposed to the correlation technique, to measure noise spectral densities from a generic DUT. The lock-in technique makes it possible to extract only deterministic signals from a noise background, whose frequency is known and is also available as an additional reference for demodulation purposes, but does not allow us to extract an unknown noise signal from a noise background to make a spectrum over any desired frequency range, as in our case.

## II. INSTRUMENT WORKING PRINCIPLE

Figure 1 shows a basic scheme of the correlation spectrum analyzer set for voltage measurements. The signal from the DUT is fed to two distinct and independent input amplifiers operated in parallel, followed by a frequency bandwidth selector circuit. A correlation stage multiplies the same frequency component of the two channels and a low pass filter averages the result.

The signal to be measured from the DUT is therefore processed in phase by the two channels and multiplied frequency by frequency to give at the output of the multiplier a

[a]Electronic mail: sampietr@elet.polimi.it

 

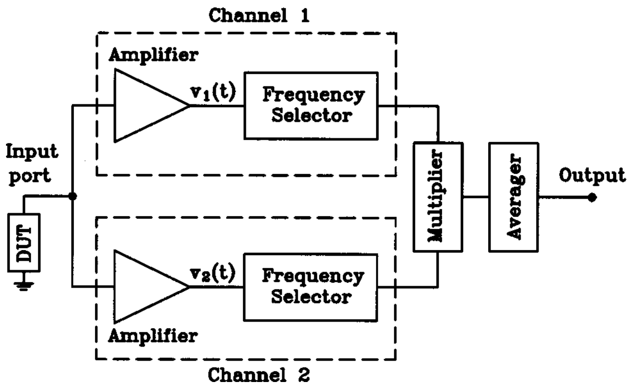


FIG. 1. Schematics of the building blocks of the correlation spectrum analyzer.

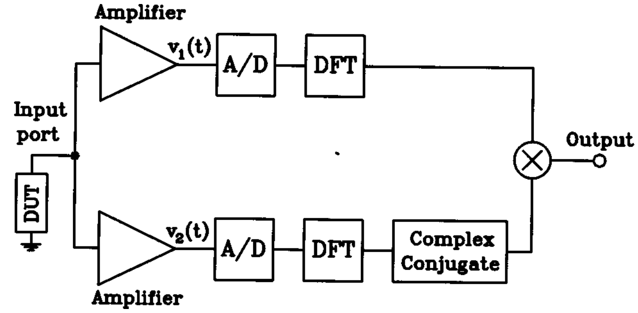


FIG. 2. Schematics of the building blocks of our correlation spectrum analyzer performing the suppression of the uncorrelated input noises by a digital processing of sampled data.

signal with mean value $\overline{S_{\mathrm{DUT}}}$ proportional to the DUT input signal (in our case the DUT noise power density) at the selected frequency. The frequency components of the noises of the two input amplifiers, instead, are uncorrelated to each other (out of phase) and, after having followed the same path as the DUT signal, give a signal at the output of the multiplier with zero mean value and standard deviation equal to the input amplifier noise power density at the selected frequency. The final averaging will reduce these fluctuations to any low value by properly extending the measuring time and allow us to evaluate $\overline{S_{\mathrm{DUT}}}$ (that is the desired DUT information) with increasingly high precision. Note that the noise power spectral density of the input amplifiers, which in a traditional single channel instrument is summed to the DUT signal power and therefore directly sets the minimum detectable DUT signal, in a two channel correlation instrument defines only the amplitude of the fluctuations around the DUT level.

The ideal instrument, performing an ideally long measurement, will measure the correlated signal and reject completely the uncorrelated noise introduced by the two amplifiers. The improvement in sensitivity is not infinite indeed but limited in a real instrument by the finite measuring time and by the residual correlation of the noises between the two channels, as will be discussed in detail in Secs. V and IV. The accuracy of the measurement, instead, is only limited by the precision of the calculation of the system gain and of its frequency response.

## III. DESCRIPTION OF THE DIGITAL CORRELATOR

The frequency selection in each channel and the following multiplication and averaging stage have been implemented in our instrument by a digital processing section whose two inputs contain the stream of digitized samples from the output $v_1(t)$ and $v_2(t)$ of the analog amplifiers (see Fig. 2). The Appendix shows that an estimate $\widetilde{S}_{\mathrm{DUT}}(f)$ of the frequency spectrum of the DUT signal can be obtained by multiplying the discrete Fourier transform (DFT) $V_1(f)$ of the output of one channel with the complex conjugate of the DFT $V_2^*(f)$ of the output of the other channel and by taking its real part:[8]

$$\widetilde{S}_{\mathrm{DUT}}(f)=\frac{1}{N}\cdot\mathcal{R}\{V_1(f)\cdot V_2^*(f)\},$$

where $N$ is the number of samples. The estimate $\widetilde{S}_{\mathrm{DUT}}(f)$ is improved increasing the total measurement time $T_{\mathrm{m}}$ by repeating M times the procedure with new streams of digitized data and by averaging them. The features of the measurement in terms of resolution bandwidth (RBW) and frequency span are set by the parameters of the digitalization. By recalling that a stream of N samples taken at the sampling frequency $f_s$ would give a DFT defined in N frequencies equally spaced by $\Delta f=f_s/N$, we chose the values of $f_s$ and of $N$ in order to set the desired frequency span from $f_{\mathrm{min}}=f_s/N$ to $f_{\mathrm{max}}=f_s/2$, therefore defining our resolution bandwidth to $\mathrm{RBW}=f_s/N$.

In our case we use analog to digital (A/D) converters with a variable sampling frequency (from $f_s=5$ Hz to $f_s=100$ MHz) and a buffer length $N=32k$ samples. This has allowed to reach values of $f_{\mathrm{min}}$ lower than 10 mHz and of $f_{\mathrm{max}}$ of about 10 MHz, limited by the bandwidth of our amplifiers. Because of the limited value of N, within the mentioned frequency span we are able to produce a direct spectral measurement covering 3 frequencies decades. A full spectrum on 8–9 decades can be obtained by simply placing the single results side by side.

## IV. INSTRUMENT FRONT ENDS

The characteristics of the preamplifiers forming the input stage of each channel are important to set the type of measurement (current noise spectra or voltage noise spectra) and to determine the ultimate performance of the instrument in term of sensitivity and covered bandwidth. In addition, the input electrical configuration allows the instrument to adapt to a wide variety of DUT bias schemes, thus covering all the requirements that can arise when testing the most advanced semiconductor devices. The following sections will describe in detail the test fixtures to perform current or voltage measurements.

### A. Current measurement front end

The configuration for current measurements is shown in Fig. 3. The DUT is connected between the inputs of two transimpedance amplifiers that convert the DUT current into a voltage output, $v_1(t)$ and $v_2(t)$. The amplifiers allow us to

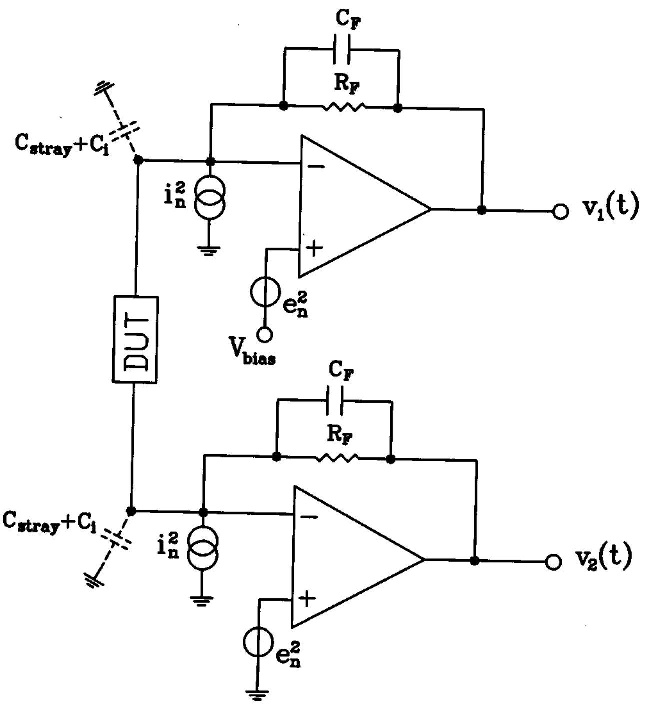


FIG. 3. Schematics of the active test fixture for current noise measurements.

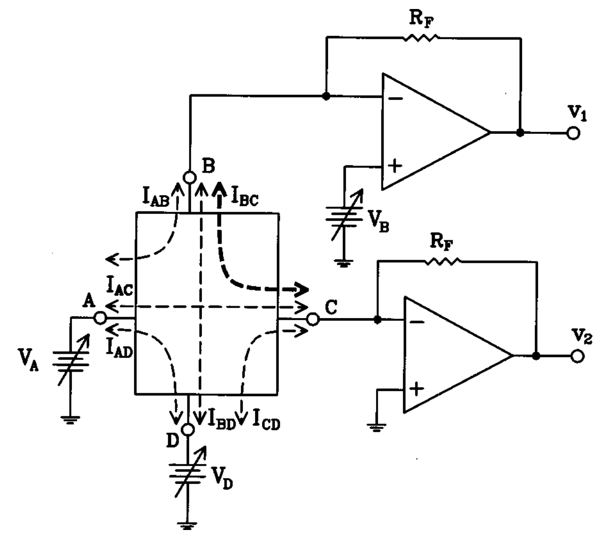


FIG. 4. Example of connection of a four-electrode DUT to perform the selective measurement of one current component excluding the others.

also define the bias voltage across the DUT to any desired dc value by setting $V_{\text{bias}}$. The noise characteristics of the amplifiers are summarized in their equivalent input noise generators, $i_n^2$ and $e_n^2$, and in their input capacitance $C_i$. $C_{\text{stray}}$ accounts for the stray capacitance of the text fixture to ground. In the case of our instrument $i_n^2 \cong (5\ \text{fA}/\sqrt{\text{Hz}})^2$, $e_n^2 = (3.3\ \text{nV}/\sqrt{\text{Hz}})^2$, $C_i = 5$ pF, and $C_{\text{stray}} = 25$ pF. $C_F$ is introduced to stabilize the amplifier and its value is chosen in order that $R_F C_F$ be about the same order of magnitude as $R_D(C_i + C_{\text{stray}})$, with $R_{\text{D}}$ the DUT resistance. The amplifier outputs are ac coupled (down to the mHz range) to the A/D converters (see Fig. 2).

The DUT current is read by both amplifiers and reaches the two outputs completely correlated. On the contrary, the current noise $i_n^2$ of each amplifier and the noise of the feedback resistor $R_{\text{F}}$ ($i_{R_F}^2 = 4kT/R_F$) are read only by the channel that generate them thanks to the very low input impedance of a transimpedance amplifier. These noises are uncorrelated over the two channels and can therefore be reduced by a properly long measurement. On the contrary, the voltage noise $e_n^2$ of each amplifier produces a current through the DUT, which is thus completely correlated over the two channels and therefore sets the lowest sensitivity limit of the instrument, as will be investigated in detail in Sec. VI

The choice of the value of the feedback resistor $R_F$ is a compromise between the following two competing needs: (1) high $R_F$ to maximize the amplification of the DUT signal and to minimize its own current noise: both these effects allow shorter measurements; (2) small $R_F$ to prevent the dc bias current in the DUT from saturating the output $v_1(t)$ and $v_2(t)$ of the amplifiers. A small $R_F$ also maximizes the bandwidth $f_{\text{max}} \cong 1/(2\pi R_F C_F)$ of the measurement. In practice, $R_F$ is chosen to satisfy the practical conditions defined in (2), that is dc bias current and bandwidth. This choice does not affect the sensitivity of the instrument, the only consequence being a variation of the measuring time necessary to reach the desired level of sensitivity. On the contrary, in a traditional one channel voltage spectrum analyzer that uses the same transimpedance amplifier in front of its input port to perform noise current measurements, the value of $R_F$ also directly sets the sensitivity of the instrument to the value $i_n^2 = 4kT/R_F$. This has strong consequences when high sensitivity measurements are performed because the necessary choice of a high value $R_{\text{F}}$ would drop the bandwidth and the capability of handling dc currents to very low values.

The current front end is well suited for direct current noise measurements on semiconductor devices. Figure 4 shows, as an example, the connection to the instrument input ports of a generic four-electrodes DUT, in which two electrodes [indicated with (B) and (C) in the figure] are directly biased by the instrument itself and the others can be biased by independent voltage sources. In addition to the ease and flexibility in the biasing of the device under test, Fig. 4 highlights a specific feature of the correlation spectrum analyzer, not available in a traditional instrument with only one channel: the possibility of extracting the current component ($I_{\text{BC}}$) that flows between the two terminals, (B) and (C), connected to the instrument irrespective of the presence in the same terminals of other current components ($I_{\text{AB}}$, $I_{\text{CD}}$, $I_{\text{AC}}$, $I_{\text{BD}}$) from the other terminals of the DUT. This peculiarity of the correlation spectrum analyzer has many practical applications in the characterization of semiconductor devices. For example it makes possible a selective and precise measurement of the current in the channel of a MOSFET when the current from the bulk is not negligible.

## B. Voltage measurement front end

The measurement of a voltage noise spectrum can be performed by the front-end scheme of Fig. 5. The signal from the DUT is read by two independent voltage amplifiers operated in parallel whose characteristics are summarized in their equivalent noise generators, $i_n^2$ and $e_n^2$, and in their

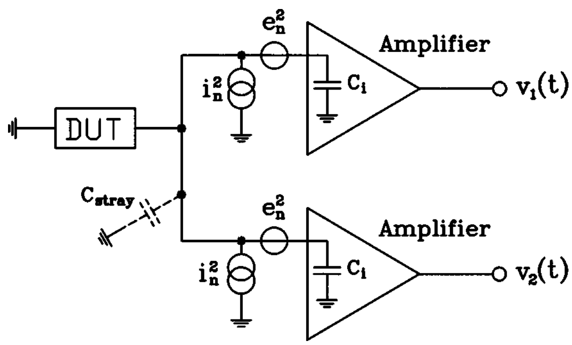


FIG. 5. Schematics of the active test fixture for voltage noise measurements.

input capacitance $C_i$. $C_{\text{stray}}$ accounts for the stray capacitance of the test fixture. In the case of our instrument $i_n^2 \cong (5\ \text{fA}/\sqrt{\text{Hz}})^2$, $e_n^2 = (1.4\,\text{nV}/\sqrt{\text{Hz}})^2$, $C_i = 5$ pF, and $C_{\text{stray}} = 20$ pF. The voltage gain of the two channels is set to produce an output level, $v_1(t)$ and $v_2(t)$, within the dynamic range of the A/D converters. The value of the DUT resistance $R_D$ defines the maximum frequency that can be measured by the instrument to

$$f_{\max} = \frac{1}{2\pi R_D(C_{\text{stray}} + 2C_i)}.$$

If an infinite measuring time is used, the minimum DUT signal that can be measured by the instrument would be limited by the residual correlated components produced by the noises of each amplifier. These components, as will be investigated in detail in Sec. VI, are negligible and therefore the precision of the instrument in the voltage measurement is practically limited only by the time duration of the analysis that averages out the uncorrelated noise of the voltage noise sources of each preamplifier.

## V. INSTRUMENT PERFORMANCE

To quantify the improvement in sensitivity as a function of the time $T_m$ spent for the measurement, let us indicate with $s_n^2$ the power spectral density of the uncorrelated noises at the input of each channel (that is $s_n^2 = i_n^2$ in the current measuring front end of Fig. 3 or $s_n^2 = e_n^2$ in the voltage measuring front end of Fig. 5). As explained in Secs. II and III, the standard deviation $\sigma_{S_{\text{DUT}}}$ of the fluctuations around the DUT power density value at the output of the instrument is proportional to $s_n^2$. By averaging $M$ independent estimates $\tilde{S}_{\text{DUT}}(f)$ of the DUT power density, we reduce $\sigma_{S_{\text{DUT}}}$ by a factor $1/\sqrt{2M}$, giving

$$\sigma_{S_{\text{DUT}}} = s_n^2 \cdot \frac{1}{\sqrt{2M}}. \tag{1}$$

This is rigorously true when the input noise of each channel is higher than the noise to be measured from the DUT, which is exactly the situation in which the use of the correlation analyzer makes sense. By recalling that $T_m = M \cdot N/f_s = M/\text{RBW}$, Eq. (1) can be written as

$$\sigma_{S_{\text{DUT}}} = s_n^2 \cdot \frac{1}{\sqrt{2 \cdot \text{RBW} \cdot T_m}}, \tag{2}$$

where the dependence in the measurement time and in the resolution bandwidth are evidenced. The level of fluctuation given by Eq. (2) defines the minimum DUT signal that can be measured. Therefore, Eq. (2) gives the equivalent input noise of the instrument after a measuring time $T_m$. The sensitivities obtained with our instrument are summarized in Figs. 6(a) for voltage measurements and 6(b) for current measurements, where the square root of Eq. (2) is plotted as a function of the DUT resistance for different measuring times. The plots refers to a RBW=100 Hz and are calculated on the basis of the previously mentioned values of the circuit front-end components. Figure 6(a) shows that for low impedance noise sources (for which a voltage measurement is better suited) the exceptional noise level of tens of pV/$\sqrt{\text{Hz}}$ can be reached by properly extending the measurement time in the 1 day range. In the same time range a current noise measurement [see Fig. 6(b)] of a high impedance source would allow us to reach a noise level of few fA/$\sqrt{\text{Hz}}$. Figure 6(b) refers to a value of $R_F = 1$ MΩ. The performance of the instrument could be even better if a higher value of $R_F$ would be allowed by the bias current in the DUT. For ex-

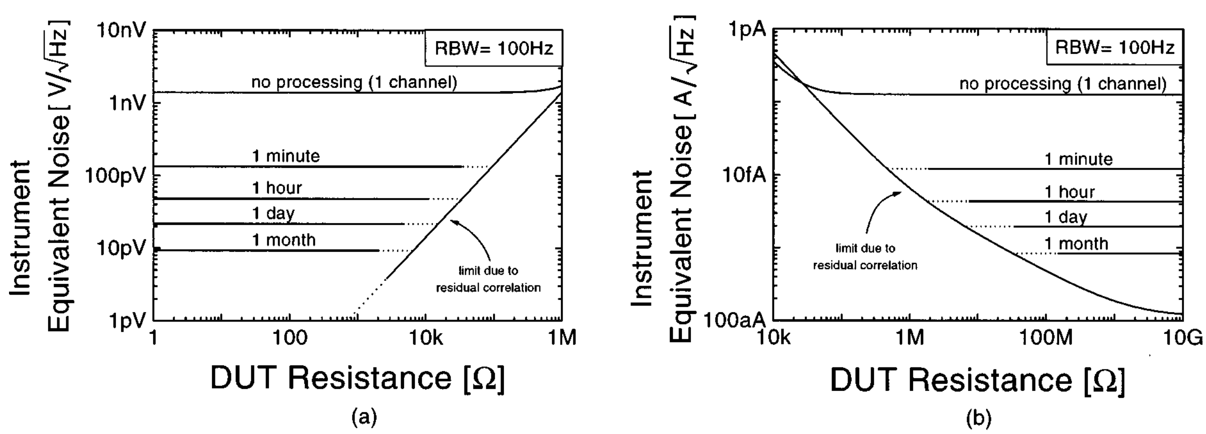


FIG. 6. Improvement of sensitivity of the instrument by increasing the duration of the measurement: (a) voltage measurements, (b) current measurements with $R_F = 1$ MΩ. The curve labeled ''no processing'' represents the limit of the instrument if only one channel would be used. The curves refer to the frequency of 10 kHz with a resolution bandwidth of 100 Hz. The figures also show the limit in sensitivity due to the residual correlation of the input amplifiers noises.

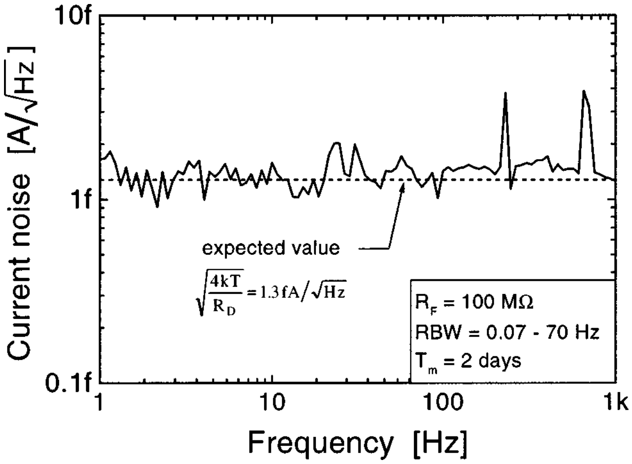


FIG. 7. Frequency spectrum of the current noise produced by a resistor of 10 GΩ. Peaks are probably due to an imperfect shielding from interferences that produce correlated signals.

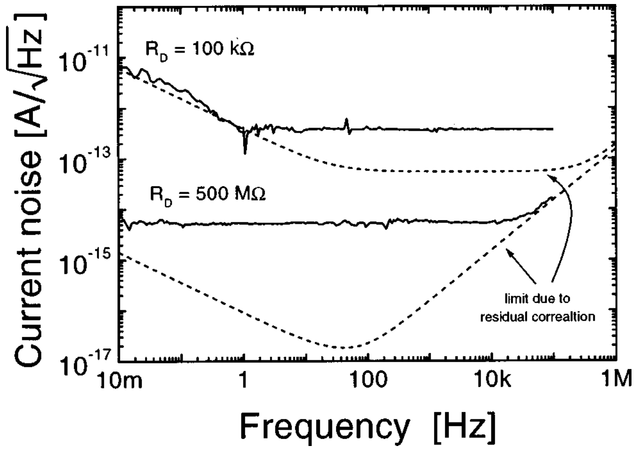


FIG. 9. Experimental frequency spectrum of the current noise from DUT resistances of 100 kΩ and 500 MΩ (continuous line) compared with the limits (dashed line) given by the instrument and set by residual correlated noise components.

ample, by using $R_F = 100$ MΩ, the curves would shift down by a factor of 10, therefore allowing us to easily measure sub-fA/$\sqrt{\mathrm{Hz}}$ DUT signals within a day.

The time needed to obtain a given sensitivity can be traded with the RBW as indicated by Eq. (2): a frequency resolution relaxed by a factor of 10 (that is RBW=1 kHz) would need ten times faster measurement for the same noise sensitivity. This, of course, implies that the low frequency section of a DUT spectrum would require a proportionally long measurement time.

As an example of the capabilities of the instrument in measuring extremely low noise levels, we present the results of two different experiments. Figure 7 shows the frequency spectrum of the current noise produced by a resistor of $R_D = 10$ GΩ. Figure 7 proves that a 2 day experiment is long enough to measure with good precision the expected theoretical value of 1.3 fA/$\sqrt{\mathrm{Hz}}$. To optimize the measurement the data have been processed in order to produce a resolution bandwidth increasing proportionally with the frequency from a value of RBW=0.07 Hz at a frequency of 1 Hz until a value of RBW=70 Hz at the final frequency of 1 kHz. As a second example, we present in Fig. 8 the measurement of the voltage noise spectral density produced by resistors of different values. The upper square-shaped points are the values measured when only one channel is operated, that is when the instrument behaves like a traditional spectrum analyzer. In this case the sensitivity saturates to the limit given by the noise of the input stage, equivalent to about 1.4 nV/$\sqrt{\mathrm{Hz}}$ corresponding to the noise of an input resistor of about 100 Ω. The lower diamond-shaped points correspond to the measurement performed with both channels active. The decrease in the DUT noise, obtained by decreasing the DUT resistance, is correctly tracked by the instrument at least down to the value of 70 pV/$\sqrt{\mathrm{Hz}}$. Values of resistors lower than 0.25 Ω were not tested because of the stray resistances of the mounting.

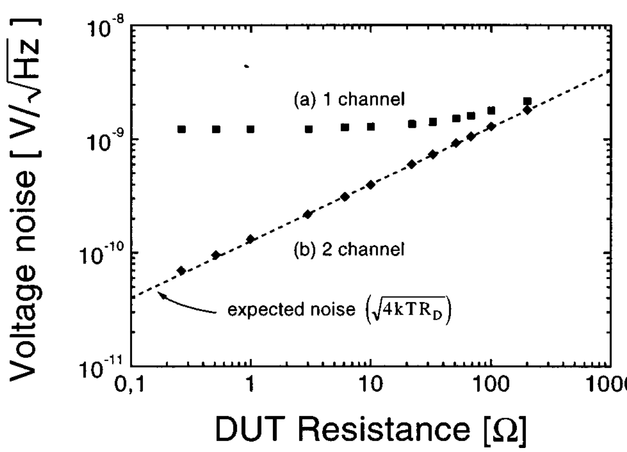


FIG. 8. Measurement of the noise spectral density of DUT resistors performed: (a) by using only one channel and (b) by using both channels and exploiting the peculiarity of the correlation technique. The dashed line indicates the theoretical noise values ($4\,kTR_D$) expected from the DUT resistors.

## VI. LIMITS DUE TO RESIDUAL CORRELATIONS BETWEEN THE TWO CHANNELS

As already mentioned, the ultimate performance of the instrument in term of sensitivity is set by those sources of noise in the input preamplifiers that produce a signal exactly in parallel to the one produced directly by the DUT. This correlated component is read by the two channels of the instrument the same way as the DUT component and can therefore not be removed.

For what concerns current noise measurements with the setup of Fig. 3, the correlated component is produced by the noise voltage sources $e_n^2$ and sets the minimum DUT signal that can be measured by the instrument as:

$$i_{\mathrm{corr}}^2 = 2e_n^2\left[\frac{1}{R_D}\left(\frac{1}{R_F}+\frac{1}{R_D}\right)+\omega^2 C_D(C_D+C_i+C_{\mathrm{stray}})\right], \tag{3}$$

where $R_D$ and $C_D$ are the equivalent resistance and capacitance of the DUT. The limits predicted by Eq. (3) in the case of our instrument and with a $C_D = 0.5$ pF are shown in Fig. 9 as a function of the frequency for two values of impedance $R_D$ of the DUT. Note that at low frequencies the $1/f$ noise

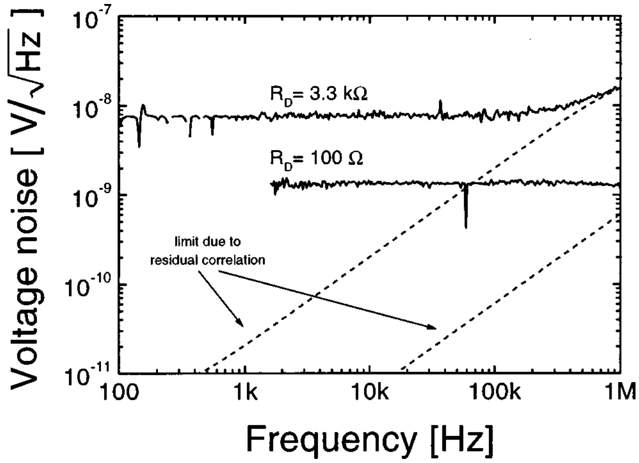


FIG. 10. Frequency spectrum of the voltage noise from DUT resistances of 3.3 kΩ and 100 Ω (continuous line) compared with the limits (dashed line) given by the instrument and set by residual correlated noise components.

component of $e_n^2$ is the limiting factor. At high frequencies, the second term in Eq. (3) increases and becomes the limiting factor when the impedance of the DUT is particularly large. The experimentally measured spectra of resistors (continuous lines) confirm the predictions. For the 100 kΩ resistor the frequency span where the noise measurement is correct is limited by $1/f$ correlated component of $e_n^2$, whereas for the 500 MΩ resistor it is limited by the mentioned effect of the input capacitances. Figure 9 shows that noise levels well below fA can be easily detected.

For what concerns voltage noise measurements with the setup of Fig. 5, the correlated noise can be summarized by the equivalent voltage noise source $v_{\mathrm{corr}}^2$ whose value

$$v_{\mathrm{corr}}^2 = 2 \cdot i_n^2 R_D^2 + 2 \cdot e_n^2 \omega^2 R_D^2 (C_D + C_{\mathrm{stray}} + C_{\mathrm{in}}) C_{\mathrm{in}} \tag{4}$$

can be directly compared to the noise $v_{\mathrm{DUT}}^2$ produced by the DUT. The first term in the sum reduces to a negligible value thanks to the very small value of $i_n^2$ of amplifiers with a JFET input transistor. The second term, which increases with frequency, becomes significant only when the DUT resistance is high. Figure 10 shows the behavior of our instrument in which the limit set by Eq. (4) is visible below 1 MHz only when $R_D$ is higher than about 100 Ω.

Concerning the frequency bandwidth covered by the instrument, an improved version of the digital board (with higher sampling frequency of the A/D converters and longer buffers) could extend the maximum frequency to about 100 MHz. Spectra in the GHz region can also be done with the cross-correlation technique but the digital section should be substituted with a superheterodyne analog section performing the same cross-correlation analysis, as sketched in Fig. 1.

## ACKNOWLEDGMENTS

The authors wish to thank E. Gatti for having encouraged this work and for his continuous support, and G. Accomando and F. Ciani for the help given during the measurements.

## APPENDIX

Let us call $v_1(t) = s(t) + w_1(t)$ and $v_2(t) = s(t) + w_2(t)$ the samples at the output of the A/D converter, in which *s(t)* is the correlated component (mainly from the DUT) and $w_1(t)$ and $w_2(t)$ the uncorrelated components of each channel having zero-mean value. We are looking for the power spectral density $S_s(f)$ of *s(t)*, which is by definition the Fourier transform of the autocorrelation function ($r_s$) of the signal itself:[8]

$$S_s(f) = \mathfrak{F}\{r_s(\tau)\} = \sum_{\tau=-\infty}^{+\infty} r_s(\tau) \cdot e^{-j2\pi f \tau}.$$

It is easy to see that $r_s(\tau)$ is equal to the cross correlation of $v_1(t)$ and $v_2(t)$:

$$r_s(\tau) = r_{v_1 v_2}(\tau) = E[v_2(t) \cdot v_1(t+\tau)]$$

thanks to the uncorrelation among $w_1(t)$, $w_2(t)$, and $s(t)$. In the case of a stream of $N$ samples, an estimate of the cross correlation is

$$\tilde{r}_{v_1 v_2}(\tau) = \frac{1}{N} \cdot \sum_{t=0}^{N-1} v_2(t) \cdot v_1(t+\tau).$$

The corresponding estimate of $S_S(f)$ is therefore

$$\begin{aligned} \tilde{S}_s(f) &= \mathfrak{F}\{\tilde{r}_{v_1 v_2}(\tau)\} \\ &= \sum_{\tau=-(N-1)}^{N-1} \tilde{r}_{v_1 v_2}(\tau) \cdot e^{-j2\pi f \tau} \\ &= \frac{1}{N} \cdot \sum_{\tau=-(N-1)}^{N-1} \sum_{t=0}^{N-1} v_2(t) \cdot v_1(t+\tau) \cdot e^{-j2\pi f \tau} \\ &= \frac{1}{N} \cdot \sum_{t=0}^{N-1} v_2(t) \cdot V_1(f) \cdot e^{j2\pi f t} = \frac{1}{N} \cdot V_1(f) \cdot V_2^*(f), \end{aligned}$$

whose real part is indicated with $\tilde{S}_{\mathrm{DUT}}(f)$ in the text. Note that, by taking only the real part of $\tilde{S}_S(f)$, we discard half of the uncorrelated noise power and therefore improve the standard deviation $\sigma_{S_{\mathrm{DUT}}}$ of the fluctuations around the DUT power density value at the output of the instrument by a factor of $\sqrt{2}$, as reported in Eq. (1).

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