A Novel Meta-Heuristic Optimization Algorithm Inspired by the Spread of Viruses

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Abstract

According to the no-free-lunch theorem, there is no single meta-heuristic algorithm that can optimally solve all optimization problems. This motivates many researchers to continuously develop new optimization algorithms. In this paper, a novel nature-inspired meta-heuristic optimization algorithm called virus spread optimization (VSO) is proposed. VSO loosely mimics the spread of viruses among hosts, and can be effectively applied to solving many challenging and continuous optimization problems. In VSO, the ribonucleic acid (RNA) of the virus represents a solution to the problem at hand. Here, we devise a new representation scheme and viral operations that are radically different from all the previously proposed virus-based optimization algorithms. First, the viral RNA of each host in VSO denotes a potential solution for which different viral operations will help to diversify the searching strategies in order to largely enhance the solution quality. In addition, an imported infection mechanism, inheriting the searched optima from another colony, is introduced to possibly avoid the prematuration of any potential solution in solving complex problems. VSO has an excellent capability to conduct adaptive neighborhood searches around the discovered local and global optima for achieving better solutions. Furthermore, with a flexible infection mechanism, VSO is able to quickly escape from local optima so as to look for other globally (sub-)optimal solution(s). To clearly demonstrate both its effectiveness and efficiency, the newly proposed VSO is critically evaluated on a series of well-known benchmark functions. Moreover, VSO is validated on its applicability through two real-world examples including the financial portfolio optimization and optimization of hyper-parameters of support vector machines for classification problems. The experimental results show that VSO has attained superior performance in terms of solution fitness, convergence rate, scalability, reliability, and flexibility when compared to those results of the conventional as well as state-of-the-art meta-heuristic optimization algorithms.

Keywords: Virus Spread Optimization, Nature-Inspired Algorithms, Meta-Heuristic Optimization, Continuous Optimization

1. Introduction

Optimization techniques have been widely applied in many scientific and engineering applications. For instance, in the field of artificial intelligence, researchers often attempt to op-

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timize various machine learning models, e.g. tuning hyper-parameters of support vector machines (SVMs) [1] and optimizing deep neural network architecture [2, 3], to obtain a better performance. In the areas of industrial design and manufacturing, engineers always encounter numerous optimization problems for various products and scenarios, such as the optimization of aerodynamic shapes for aircraft, cars, bridges, etc. [4] and the optimization of supply chain management [5]. In finance, investors usually pursue an optimal portfolio aiming to maximize the return while minimizing the risk [6, 7]. There are many optimization problems in our daily lives like finding the shortest vehicle route to a destination [8], resource allocation to satisfy performance goals [9], and so on.

Since many real-world optimization problems are too complex to be solved with a good solution by conventional optimization approaches in a reasonable time, meta-heuristic optimization algorithms have recently captured much attention and achieved some success [10]. In the past decades, researchers have invented several nature-inspired meta-heuristic optimization algorithms to imitate some phenomena or behaviors of the nature. Such algorithms can be classified into five categories: evolution-based, swarm-intelligence-based, physics-based, chemistry-based and human-based algorithms. Evolutionary algorithms (EAs) are inspired by the biological evolutionary process. Genetic algorithm (GA) [11], evolution strategies (ES) [12] and differential evolution (DE) [13] can be regarded as representative algorithms in EAs. For the second category, swarm intelligence algorithms (SIs) imitate the intelligent behaviors of creatures in nature. Particle swarm optimization (PSO) is the most pioneering work of SIs [14]. Up to now, the research of SIs has been very active such that new algorithms are being proposed from time to time. Some well-known examples of SIs include: Ant colony optimization (ACO) [15], artificial bee colony (ABC) [16], social spider algorithm (SSA) [17], whale optimization algorithm (WOA) [18], grey wolf optimizer (GWO) [19], etc. For both physics-based and chemistry-based optimization algorithms, that are motivated by physical phenomena and chemical reactions, examples include simulated annealing (SA) [20], chemical reaction optimization (CRO) [21], nuclear reaction optimization (NRO) [22] and so on. Lastly, collective decision optimization algorithm (CDOA) [23] and queuing search algorithm (QSA) [24] are examples of the last category.

According to the no-free-lunch theorem (NFL), there is no single meta-heuristic algorithm that can optimally tackle all optimization problems [25]. Undoubtedly, this motivates researchers to continuously develop new algorithms for various applications. In particular, the proposed algorithm should be very competitive with the few existing successful optimization approaches such as PSO for solving the well-known benchmark functions as well as various real-world problems in terms of the solution quality, rate of convergence, scalability, reliability and flexibility, etc.

In this paper, we propose a novel, powerful and nature-inspired meta-heuristic algorithm namely the virus spread optimization (VSO) for tackling continuous optimization problems. VSO mimics the mighty spread of viruses among hosts. Here, we devise a new representation scheme and operations that are radically different from all the previously proposed virus-based optimization algorithms. First, the viral ribonucleic acid (RNA) of each host in VSO denotes a potential solution to the problem at hand for which different viral infection, mutation and recovery operations will help to diversify the searching strategies in order to largely enhance the solution quality. In addition, an imported infection mechanism, inheriting the searched optima from another colony, is introduced to possibly avoid the prematuration of any potential solution in solving complex problems. The VSO algorithm has an excellent capability to conduct adaptive neighborhood searches around the discovered local and global optima for achieving better solutions. Furthermore, with a flexible infection mechanism, VSO can quickly escape from local optima in order to look for other globally (sub-)optimal solution(s).

To evaluate the performance of the proposed optimization algorithm, experiments are conducted on a series of well-known benchmark functions including 16 classical examples listed in [26] [27] [28] and 30 problems specially designed by the IEEE CEC 2014 for competition [29]. In addition, VSO is applied to two real-world applications such as the financial portfolio optimization and optimization of hyper-parameters of SVMs for classification problems. To investigate the scalability, the algorithm was well-tested on the classical benchmark functions and portfolio optimization problems with different ranges of dimensions including: low (30 & 100 dimensions), medium (300 & 500 dimensions) and high (1,000 dimensions) for the benchmark functions, and different numbers as 30, 100 and 250 of stocks for portfolio optimization. A standardized running environment and settings are used for a fair comparison of the performance of the VSO algorithm with those of the conventional meta-heuristic algorithms including GA [11], DE [13], PSO [14], ABC [16], as well as state-of-the-art ones, i.e. SSA [17], WOA [18] and covariance matrix adaptation evolution strategy (CMA-ES) [30] with their outstanding performance reported in literature. The experimental results verify that VSO achieves impressive performances in terms of solution quality, convergence rate, scalability, reliability and flexibility when compared to those of the above conventional and state-of-the-art meta-heuristic algorithms.

To the best of our knowledge, two virus-based algorithms namely the virus colony search (VCS) [31] and virus optimization algorithm (VOA) [32] have been proposed to tackle various optimization problems. However, VSO is radically different from these two existing algorithms in their analogies, motivation, implementations and search behaviors. We will further reveal the details in Section 2.

In summary, the major contributions of this paper are as follows.

- A new meta-heuristic algorithm as a very competitive and potential approach is proposed to solve challenging and continuous optimization problems;
- The proposed optimization algorithm, combining the strengths of EAs and SIs, can achieve
 an excellent trade-off between exploitation and exploration by the unique design of the
 diversification of the search strategies. This makes the algorithm applicable to a wider
 range of problems in practice;
- The imported infection mechanism, as a novel search strategy cooperating with other metaheuristic algorithms, helps to significantly enhance the overall optimization algorithm for tackling more complex problems;
- The outstanding performance of the proposed algorithm is demonstrated not only on the solution quality but also the rate of convergence, scalability and reliability through performing a series of experiments on 46 well-recognized benchmark functions and two realworld optimization problems.

The rest of this paper is organized as follows. Section 2 describes the analogies, operations, implementations and work flow of the VSO algorithm in details. The experimental results and related discussion on the benchmark functions are presented in Section 3. The performances for two real-world applications including financial portfolio optimization and optimization of hyper-parameters of SVMs for classification problems are shown and discussed in Section 4 and Section 5 respectively. We conclude this work and shed lights on various potential future directions in Section 6.

2. The Virus Spread Optimization Algorithm

2.1. Analogies and Definitions

Considering the powerful spread of viruses with a great diversity of viral behaviors, VSO is proposed to simulate such process loosely. The analogies of VSO are listed in Table 1. The host and virus are essential components of the algorithm.

Table 1: Analogy of VSO

Terminology	Natural Meaning	Algorithmic Meaning
Viral Spread	To infect all hosts.	To search the solution space and find an optimal one.
Virus	A virus that contains an RNA which may mutate.	The RNA represents a solution to the problem.
Host	Organism (e.g. animals, humans) that is infected by the virus. The infected host may show symptoms of various degrees.	The symptom intensity generally represents the fitness of a feasible solution. The critical host denotes the best fitness.

In VSO, the population is composed of hosts. There are four types of hosts imitating the spread of viruses and the immunological differences in nature: *healthy, mild, severe* and *critical*. Each host including the healthy one carries a virus. In fact, many animals including humans may carry all kinds of non-infectious viruses in nature [33]. For instance, a healthy human may carry a few viruses like endogenous retrovirus (ERV) that are in fact beneficial to our immune system [34]. Besides, bats carry a lot of unknown viruses yet may not get sick from those viruses [35]. The main difference between *healthy* hosts and other hosts is that *healthy* hosts act as healthy carriers with non-infectious viruses while the infected (also called infectious) ones, i.e. *mild, severe, and critical* hosts, can infect the *healthy* hosts.

More importantly, there are different viral infection and mutation operations for each type of hosts in VSO to diversify the searching strategies so that the optimizing capability and flexibility can be largely enhanced. More definitions are provided as follows.

• Definition 1: A Viral RNA

Each host has a viral RNA that represents a possible solution as shown in (1).

$$X_i = [x_i^1, x_i^2, \dots, x_i^D]$$
 (1)

where X_i (vector) is the RNA of the virus denoting a possible solution to the problem at hand, i is the iteration number, and D is the dimensionality, i.e. the number of decision variables, of the problem.

• Definition 2: A Healthy Host

A *healthy* host is a host carrying a non-infectious virus whose RNA is generated randomly in every iteration. The host conducts a random search in the solution domain as listed in (2).

$$X_i = U(S) \tag{2}$$

where S is the whole search space while U is a random number generator function based on the uniform distribution of S.

• Definition 3: A Mild Host

A *mild* host is carrying an infectious virus. As shown in (3)-(4), the virus of this host can mutate with a mutation intensity *intensity*_i^M and also infect other *healthy* hosts with a rate R^M that is relatively low when compared to other infectious hosts.

$$intensity_i^M = \alpha * intensity_{i-1}^M + \gamma * rand(0,1) * (gbest_{i-1} - X_i)$$
(3)

$$X_{i+1} = X_i + intensity_i^M (4)$$

where $intensity_i^M$ (vector) is the mutation intensity of the mild host at the iteration $i, \alpha \in [0, 1]$ and $\gamma \in [1, 2]$ are the scaling factors, $gbest_{i-1}$ is the best solution obtained by the population at the iteration i-1, and rand(0, 1) is a random number between 0 and 1.

Definition 4: A Severe Host

As shown in (5)-(6), a *severe* host carries an infectious virus that can mutate with a mutation intensity *intensity*_i^S and also infect other *healthy* hosts with its own rate R^S . Overall speaking, its infectious ability is medium as compared to that of the *critical* host.

$$intensity_i^S = \delta_s * intensity_{i-1}^S$$
 (5)

$$X_{i+1} = X_i + Gaussian(0, intensity_i^S) * X_i$$
 (6)

where $intensity_i^S$ (scalar) is the mutation intensity of the *severe* host at the iteration i, $\delta_s \in (0, 1]$ is the decay rate, and $Gaussian(0, intensity_i^S)$ is the Gaussian function with the mean as 0 and the standard deviation as $intensity_i^S$.

• Definition 5: A Critical Host

In VSO, there is only one *critical* host which represents the currently most optimal solution obtained so far. As shown in (7), its viral mutation is paused yet with the highest infection rate R^C to carry its relatively good solution quality to other *healthy* hosts.

$$X_{i+1} = X_i \tag{7}$$

2.2. Operations

In VSO, the *initialization, selection, mutation, infection* and *recovery* are five essential operations while the *imported infection* serves as an additional operation to enhance the optimizing performance.

2.2.1. Initialization

At the starting point with the number of iterations as 0, the whole population is initialized as *healthy* hosts. The viral RNA of each host is randomly generated in the search space according to (8).

$$X_{i=0} = bound_l + rand(0, 1) * (bound_u - bound_l)$$
(8)

where i is the iteration number, $bound_l$ and $bound_u$ denote the lower and upper bounds of the corresponding domain of the variable being considered.

The mutation intensities *intensity*_i^S and *intensity*_i^M of the mild and *severe* hosts are initialized in (9) and (10) as below.

$$intensity_{i=0}^{M} = \frac{U(bound_l, bound_u)}{10}$$
(9)

$$intensity_{i=0}^{S} = \frac{1}{rand(0,1)}$$
(10)

Algorithm 1 shows the detailed initialization process.

Algorithm 1: Initialization

```
Input: Population size: N_{pop}
Searching bound: bound_l and bound_u
Random number generator based on the uniform distribution: U

Output: Newly created hosts: hosts

1 hosts \leftarrow 0;
2 i \leftarrow 0;
3 while (i < N_{pop}) do

4 | Initialize a new host h with a viral RNA according to (8);
5 | Initialize the mutation intensities of the host h according to (9) & (10);
6 h.type \leftarrow 'healthy';
7 | Insert h into hosts;
8 | i \leftarrow i + 1;
9 end
10 return hosts
```

2.2.2. Selection

In VSO, the host with the best solution will be selected as the *critical* host after calculating fitness for all hosts at each iteration. As presented in Algorithm 2, the host that has achieved the best solution up to current iteration will be designated as the *critical* host while the previous *critical* one will be downgraded to the *severe* host.

In nature, due to the complicated viral mutation, immune response and outside environment, some viruses infecting a *healthy* host may develop into deadly viruses shortly. Analogously, a *healthy* host conducting a random search will possibly become the *critical* one directly as well in VSO.

Algorithm 2: Selection

```
Input: Hosts: hosts

Current number of iterations: i

Output: critialHost

Get the host gBestHost with best solution from hosts;

gBestHost.type \leftarrow 'critical';

if (gBestHost \neq prev_gBestHost) then

prev_gBestHost.type \leftarrow 'severe';

gBest<sub>i</sub> \leftarrow gBestHost.virus.rna;

end

end
```

2.2.3. Mutation

The mutation behavior of the searching strategy is one of the key factors to the success of VSO. Depending on the type of hosts, the mutation operation will work according to (2)-(7). Algorithm 3 clearly shows the pseudo-code of the mutation operation. The viral RNAs of all hosts will be updated accordingly by the mutation operation at each iteration.

$2.2.4.\ In fection$

The main objective of the infection mechanism is to spread the viral information among all the hosts so as to empower the search effectiveness of the VSO algorithm. In the real world, the transmission route, such as direct contact, is necessary for the spread of many viral diseases [36]. We hereby design a three-step mechanism for the infection operation in VSO.

Algorithm 3: Mutation

```
Input: All hosts: hosts
   Output: All hosts with updated viral RNAs: hosts'
1 for each host in hosts do
        switch host.type do
2
             case healthy do
              Update host.virus.rna according to (2);
             end
                  Update host.virus.intensity<sup>M</sup> according to (3);
                  Update host.virus.rna according to (4);
             end
10
                  Update host.virus.intensity<sup>S</sup> according to (5);
11
12
                  Update host.virus.rna according to (6);
13
             end
14
             case critical do
15
                  Update host.virus.rna according to (7);
16
             end
17
        end
18 end
```

At first, every infectious host has one or more chances to contact *healthy* hosts at each iteration.

Secondly, we have to decide whether that contacted *healthy* host will be infected or not. Therefore, different infection rates are assigned to the hosts according to their types as shown in (11).

$$R_{infect} = [R^M, R^S, R^C] \tag{11}$$

where $0 < R^M \le R^S < R^C < 1$. They are the infection rates for *mild*, *severe*, and *critical* host, respectively. More specifically, the infection rate is the probability of an infectious host infecting a *healthy* host when they contact.

Lastly, in case of a *healthy* host infected by an infectious host successfully, it will become a *severe* or *mild* host at different probabilities. We hereby design a transformation matrix as illustrated in (12).

$$P_{trans} = \begin{bmatrix} P_{H->M}^{C} & P_{H->S}^{C} \\ P_{H->M}^{S} & P_{H->S}^{S} \\ P_{H->M}^{M} & P_{H->S}^{M} \end{bmatrix}$$
(12)

where P_{trans} is the matrix of transformation probabilities. For instance, $P_{H->M}^S$ is the conditional probability of a *healthy* host becoming the *mild* host given by being infected by a *severe* host. As there are only two events here that are mutually exclusive, i.e. becoming a mild or severe host, the summation of each row of probabilities is equal to 1.

At each iteration, the specific procedure of the infection is summarized as follows.

• As indicated in (11), the healthy host contacting with an infectious host will be infected

with probabilities R^C , R^S and R^M respectively as dependent on the type of the infectious host;

- During the infection, the *healthy* host may be infected as the severe or *mild* host according to the transformation probabilities as described in (12). Specially, the host infected by the *mild* host can become the *mild* host only so that the transformation probability $P_{H->M}^{M}$ is always 1 and $P_{H->S}^{M}$ is equal to 0;
- In addition, two solution sharing mechanisms may be performed during the infection process. When a *healthy* host (destination) infected by an infectious host (source) to become a *severe* host, the viral RNA of the source will be copied to the destination directly as shown in Figure 1. In addition, when a *healthy* host is infected by a *mild* host, each assigned value of the viral RNA of the destination will be randomly replaced by the source with a fixed probability of 0.5 as shown in Figure 2.

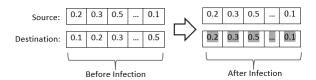


Figure 1: The host to be infected as the severe host

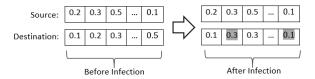


Figure 2: The host to be infected as the mild host

The implementation of the above viral infection is described in Algorithm 4. The infectious and *healthy* hosts are firstly sorted according to the ascending and descending order of their fitness values respectively. Since the VSO algorithm is designated for solving minimization problems, the smaller the fitness value, the better the solution. Thus, the infectious host with a better solution quality will be more likely selected to infect a *healthy* host. Conversely, a *healthy* host with a worse fitness value will be more likely to be infected. Moreover, an integer parameter *H* as mentioned above is used to limit the maximum number of *healthy* hosts to be contacted by each infectious host. This can help to avoid any premature convergence of the whole population to any local minimum.

2.2.5. Recovery

The recovery operation is another key mechanism of the VSO algorithm. Due to the powerful viral spread in the infection, all hosts may be infected very soon so that the searching capacity of the algorithm may still be quickly converged into a local minima even though with the aforementioned parameter *H* to restrain the maximum number of contacted hosts.

Algorithm 4: Infection

```
Input: All hosts: hosts
            The maximum number of healthy hosts to be contacted by each infectious host: H
            Problem dimensionality: D
   Output: All hosts with possibly updated viral RNAs: hosts'
   Select infectiousHosts from hosts;
 2 Sort infectiousHosts by the ascending order of fitness values;
 3 for each infectious Host \in infectious Hosts do
         Select healthyHosts from hosts;
         if (|healthyHosts| \ge H) then
 5
              Sort healthyHosts by the descending order of fitness values;
 7
              contactedHosts \leftarrow \{ the first \ H \ hosts \} \subseteq healthyHosts;
              for each healthvHost \in contactedHosts do
 8
                   infected \leftarrow false;
                   T \leftarrow infectiousHost.type;
10
                   Get the infection rate R^T from (11);
11
                   if rand(0,1) \le R^T then
12
                       infected \leftarrow true;
13
                   end
14
15
                   if (infected) then
                         Get the transformation probabilities \langle P_{H->M}^T, P_{H->S}^T \rangle from (12);
16
                         if (0 \le rand(0, 1) \le P_{H->M}^T) then
17
                             to\_be\_infected\_type \leftarrow M;
18
19
                         else
                              // P_{H->M}^T + P_{H->S}^T = 1
20
                             to\_be\_infected\_type \leftarrow S;
21
22
                         end
23
                         switch to_be_infected_type do
                             case M do
24
25
                                   healthyHost.type \leftarrow 'mild';
                                   for idx = 0 to D - 1 do
26
                                        if rand(0, 1) \le 0.5 then
27
                                             healthyHost.virus.rna[idx] \leftarrow infectiousHost.virus.rna[idx];
                                        end
29
30
                                   end
31
                              end
                             case S do
32
                                   healthyHost.type \leftarrow 'severe';
33
34
                                   healthyHost.virus.rna \leftarrow infectiousHost.virus.rna;
35
                              end
36
                         end
37
                   end
38
              end
39
         end
40 end
```

Thus, in case all the hosts are infected, the recovery operation will be performed to carefully reset some of the infected hosts to continue with the exploration process. We have not adopted the simple random or scheduled restart approaches used by many algorithms such as [37, 38, 39]. Instead, an interesting mechanism to gradually downgrade the infected hosts is devised as inspired by the nature in which an infected host has to recover gradually. Likewise, each infected host will be downgraded to the less severe host type of the VSO framework. For example, a *severe* host will be recovered to the *mild* host while a *mild* host will become the *healthy* host. As

the searching restrictions will be relaxed for the "recovered" host types, the searching capacity of the algorithm will be enhanced gradually as well so as to explore the other parts of the search space.

Furthermore, a parameter *recPercent* called the recovery rate is used to specify the percentage of the infected hosts with the worst solution quality to be recovered. This can help to avoid losing all the search information accumulated so far during the search process. The detailed implementation is given in Algorithm 5.

Algorithm 5: Recovery

```
Input: Infectious hosts: infectiousHosts
           Population size: N_{pop}
           Recovery percentage: revPercent
   Output: Hosts recovered from in fectious Hosts
  Sort infectiousHosts by the descending order of fitness values;
2 if (|infectiousHosts| = N_{pop}) then
         revNum \leftarrow N_{pop} * revPercent;
3
4
         RH \leftarrow \{\text{the first } revNum \text{ hosts}\} \subseteq infectiousHosts;
         for each host h \in RH do
5
              Initialize h referring to Algorithm 1;
7
              switch h.type do
                   case severe do
8
                       h.type \leftarrow 'mild';
                   end
10
                   case mild do
11
                        h.type \leftarrow 'healthy';
12
13
                   end
14
              end
15
        end
16 end
```

2.2.6. Imported Infection

As inspired by the possible migration of hosts from one place to another that may increase the spread of a viral disease in the real world, the concept of "imported infection" is introduced as an additional operation of the VSO framework to enhance its search performance for solving complex optimization problems.

Accordingly, a new colony is developed through the DE algorithm to construct some potentially better solution to the whole population of VSO. However, this simple heuristic operation may break the searching patterns of the concerned VSO algorithm, thus possibly leading to a poorer performance. Therefore, an adaptive probability is predefined to export the DE colony to the whole population of VSO in a probabilistic manner as illustrated in Algorithm 6.

As an additional operation, the imported infection may help to improve the search performance in some complex cases yet it will also increase the overall computational complexity of the VSO algorithm. Hence, we may flexibly skip this additional operation in some cases. More importantly, this novel design provides a useful interface for researchers or users to integrate their own algorithms for some specific problems.

2.3. The Algorithmic Flow of VSO

Figure 3 manifests the algorithmic flow of VSO. Firstly, the concerned parameters of the algorithm and the features of the problem are provided as the input to start the execution of VSO.

Algorithm 6: Imported Infection

```
Input: Critial host: criticalHost

DE algorithm: DE with the populaztion size as N_{im} (refer to [40])

Infection probability: P_{im}

Current number of iterations: i

Total number of iterations: j

Output: A critical host with the updated viral RNA: criticalHost

1 \langle bestSolution, bestFitness \rangle \leftarrow DE;

2 if (rand(0, 1) \leq (P_{im} * i/j)) then

3 | if (bestFitness < criticalHost.fitness) then

4 | criticalHost.virus.rna \leftarrow bestSolution

5 | end

6 end
```

Then, the involved operations as described in Section 2.2 are performed successively.

2.4. A Detailed Analysis on the Search Behavior

Exploitation and exploration are the two cornerstones of search techniques in solving optimization algorithms. If the exploitation ability is too strong, the algorithm may easily fall into local optima. On the other hand, the algorithm may not be able to converge to any possible solution of a relatively high fitness value in case it solely relies on a very powerful exploration mechanism.

With the novel design of VSO as clearly explained in the previous subsections, it is obvious that VSO combines both advantages of SIs and EAs in order to achieve an excellent balance between exploitation and exploration. The search behavior of the VSO algorithm is summarized as follows.

2.4.1. Exploitation

- As the *critical* host representing the best solution obtained so far, it has the highest infection rate. Thus, it is more likely to infect *healthy* hosts to become the *severe* hosts in the next iteration. To perform such infection operation, the viral RNA of the *critical* host will be directly replicated to the newly infected host according to Section 2.2.4. This implies that an increasing number of *healthy* hosts may acquire the valuable search information of the currently best solution to become *severe* hosts. On the other hand, since the mutation intensity of the *severe* hosts will be decreased rapidly as described in Section 2.2.3, the *severe* hosts will conduct neighborhood searches around the locally optimal solution. Overall speaking, this will surely help to enhance the exploitation ability of VSO to improve its solution quality;
- Each time when a better solution is found, the previous *critical* host will be automatically downgraded to a *severe* host to continue its neighborhood search around the previous best solution for a certain duration as seen in Section 2.2.4 before any possible transformation to another host type. Meanwhile, the downgraded host is able to infect other healthy hosts to search this area together.

2.4.2. Exploration

• All *healthy* hosts of the VSO algorithm perform random exploration to try to find a better solution of the whole search space;

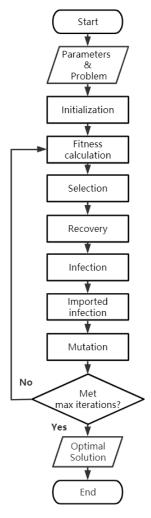


Figure 3: The Algorithmic Flow of VSO

- The main role of *mild* hosts is to improve the exploration capacity of the VSO algorithm. When a *healthy* host is infected to become a *mild* host, the viral RNA of the infectious one will not be replicated directly to the *healthy* host. Instead, a uni-directional infection mechanism as presented in Section 2.2.4 is performed, that is different from the two-sided crossover operation used in EAs. Moreover, a *mild* host can always mutate with a higher degree of freedom as guided by the computed intensity. This infection scheme empowers the VSO algorithm with an outstanding exploration ability;
- Due to the recovery mechanism, the infected hosts will be recovered and re-initialized from time to time. The recovery mechanism helps to escape from any local minimum for a better exploration;
- The imported infection mechanism hybridizes the whole population of the VSO algorithm

with another new colony using a totally different searching approach. This may possibly enlarge the search scope of the VSO algorithm for tackling more complex optimization problems.

As illustrated in Figure 4, the red cross denotes the globally optimal solution of the specific function while the only red dot represents the *critical* host as the best solution obtained so far. Clearly, this *critical* host infect several *severe* hosts as denoted by gray dots around the central circle to look better solutions whereas the *mild* hosts as represented by the orange dots will continue to search toward the red dot that is very likely to achieve a near optimal solution.

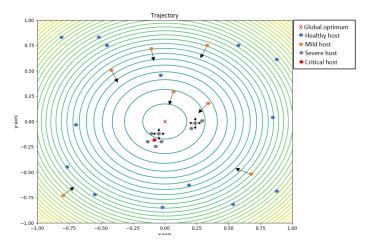


Figure 4: The Searching Pattern of VSO

2.4.3. Parameters Setting

Below is a few basic rules for the parameters setting of the VSO algorithm.

- The higher the value of R^M , $P^C_{H->M}$ or γ , the better the global search ability of the VSO algorithm, and vice versa;
- On the other hand, the larger the value of R^C , R^S , $P^C_{H->S}$, $P^S_{H->S}$, or α , or the smaller the value of δ_s , the better the local search capability of the VSO algorithm, and vice versa;
- R and revPercent are the conflicting parameters to balance the convergence of the algorithm. R should not be very large, and generally depends on the population size N_{pop} of the VSO algorithm. For instance, R can be set to 1 for a specific problem with the population size as 50 to be discussed in the subsequent section;
- A larger value of P_{im} may sometimes help to get some quick improvement in solving specific complex optimization problems. Yet for a relatively large value of P_{im} , it may also break the good searching patterns. From the empirical observations, $P_{im} \in (0, 0.5]$ is typically a good choice for most benchmark problem sets carefully examined in this work.

Because of the diverse searching strategies utilized in the VSO algorithm, the number of parameters is relatively larger than other popular meta-heurisite algorithms such as GA, PSO, etc.

Yet from the preliminary observations, the performance of the VSO algorithm is relatively robust when only a few of the aforementioned parameters are changed at the same time. Moreover, it is found that the VSO algorithm can flexibly tackle a variety of optimization problems using the same parameter settings without much tuning. As revealed in Section 3, the same parameter settings of the VSO algorithm are consistently used in all the following experiments.

3. Evaluations on Benchmark Functions

To validate both the efficiency and effectiveness of the proposed algorithm, the VSO algorithm is utilized to solve two benchmark function groups including the classical and IEEE CEC 2014 benchmark functions.

For the classical benchmark functions, a total of 16 well-known functions given in Table A.15 are used. These functions have been well-tested in all kinds of studies of meta-heuristic algorithms in previous research. Among the functions, F1 - F8 are uni-modal functions while F9 - F16 are multi-modal functions. Besides, all functions can be scalable from 2 to 1,000 dimensions so that the scalability of the concerned algorithms can be investigated. The motivation for testing these classical functions is outlined as follows:

- To quickly evaluate the searching capability of VSO when compared to those of other popular meta-heuristic algorithms, especially in terms of the solution quality;
- To evaluate the rate of convergence;
- To test the reliability of the algorithm;
- To investigate the scalability of the algorithm.

On the other hand, the IEEE CEC 2014 benchmark functions, as shown in Table B.16, are specially designed for evaluating the performance of meta-heuristic algorithms in the competition of single objective real-parameter numerical optimization problems. The functions (CEC1-CEC30) contain various novel characteristics such as shiftings and rotations, it is much more difficult to solve them than the classical set. Up to our understanding, no algorithm has solved all functions optimally. More details about these functions can be found in [29]. Despite the difficulty, we evaluate the effectiveness and robustness of the VSO algorithm on this set of challenging functions.

In the following experiments, all results are collected on the same computer with the Intel Core i9-7900X CPU running at $3.3 \sim 4.5$ GHz and 64 GB of RAM. All algorithms were implemented in *Python3*. Table 2 lists the parameter settings of each concerned algorithm according to the recommended values reported in the literature. Except for the population size, there are totally 11 unique parameters in VSO as listed in Table 2. In fact, for other parameters that are not listed, they can be derived according to the relationships mentioned in Section 2.2. For instance, since $P_{H->S}^C$ is set to 0.8, $P_{H->M}^C$ is 0.2. Furthermore, with the imported infection operation, the population size of the main process of the VSO algorithm is consistently set as 30 while that of the imported infection is 20. It is worth noting that the parameters of each algorithm remain unchanged in all experiments in order to evaluate the adaptability of the underlying algorithm with the same parameter settings on various problem sets for a fair comparison.

Table 2: Parameters Setting

Algorithm	Parameter	Value
ABC	Population size	50
	Elite bees num	16
	Onlooker bess num	4
	Patch size	5
	Patch factor	0.985
	Sites num	3
	Elite sites num	1
CMA-ES	Population size	4+3*log(dim)
	Initial mean	0
	σ	0.5
DE	Population size	50
	Crossover rate	0.3
	Differential weight,	0.5
GA	Population size	50
	Probability of mutation	0.001
	Selection tournsize	3
PSO	Population size	50
	Inertial weight	0.8
	Cognitive constant	0.5
	Social constant	0.5
SSA	Population size	50
	P_a	1
	P_c	0.7
	P_m	0.1
VSO	Population size	50
	R^{C}	0.8
	R^S	0.3
	R^{M}	0.3
	$P_{H \rightarrow S}^{C}$	0.8
	H->2	0.5
	$ \begin{array}{l} P_{H->S} \\ \delta_s \end{array} $	0.9
	α	0.1
	γ	2
	revPercent	0.8
	P_{im}	0.5
	H	1
WOA	Population size	50
1, 011	Initial a	2
	Probability of Spiral updating	0.5
	Constant of shape	1

3.1. Classical Benchmark Functions

The classical benchmark functions with 30 dimensions have been widely used for evaluating many meta-heurisitic algorithms like PSO, GA, etc., in many previous studies. In the following evaluation, each function is tested over 31 runs for each algorithm. The maximum number of iterations in each run is 10⁴. Table 3 shows the relevant results with the mean as the average value of the fitness values obtained over all runs. The standard deviation of the fitness values is calculated to examine the robustness of the algorithms. Furthermore, the best and worst results are carefully considered. To investigate the computational complexity, the average computational time in CPU seconds is recorded. Finally, two rankings in terms of the averaged fitness values and computational times are listed in order to make more precise and objective comparisons on

the different performance measures of the underlying algorithms.

- In respect of the uni-modal functions F1-F8, VSO consistently beats other algorithms in all the rankings. For multi-modal functions F9-F16, the VSO algorithm gets the first places for 6 functions as well. More importantly, VSO achieves the exact global optima for all the 12 functions, i.e. F1-F9, F11, F13, and F15. The standard deviations are 0 for all these cases, thus showing the excellent robustness of VSO;
- As for other algorithms, we can observe that the performance of CMA-ES and WOA are not bad for the uni-modal functions. Regarding multi-modal ones (F9-F16), it is clear that CMA-ES goes worse but WOA still works well;
- For both GA and ABC, their performances are not satisfied for multi-modal functions because they may not be good at solving these relatively high dimensional and complex problems;
- Regarding the DE and SSA algorithms, although they acquire very small errors in some functions, they cannot find the exact global optima;
- Due to the simple and efficient searching strategies, PSO is very fast. It ranks as the first place in computational time in 9 cases. Unfortunately, the performance of fitness is worst among all the algorithms.

Table 4 shows the summary of classical function evaluations where the average of the rankings in all functions for each algorithm is computed. VSO ranks as the first place with respect to the fitness values whereas it is ranked as the fourth place in terms of the computational time.

Table 3: Results of Classical Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
F1	Mean	3.18E+06	0.00E+00	3.33E-54	2.97E-11	4.40E+04	6.15E-40	0.00E+00	0.00E+00
	Std	3.08E+05	0.00E+00	2.46E-54	1.08E-11	2.57E+04	4.07E-40	0.00E+00	0.00E+00
	Best	2.23E+06	0.00E+00	3.29E-55	6.56E-12	1.18E+04	1.57E-40	0.00E+00	0.00E+00
	Worst	3.68E+06	0.00E+00	6.12E-54	5.69E-11	1.30E+05	2.19E-39	0.00E+00	0.00E+00
	Time(s)	11.31	222.32	72.33	44.29	4.98	5.67	9.49	31.53
	Fitness Rank	8	1	4	6	7	5	1	1
	Time Rank	4	8	7	6	1	2	3	5
F2	Mean	2.26E-02	0.00E+00	4.57E-53	5.15E-14	2.84E+01	9.53E-44	0.00E+00	0.00E+00
	Std	1.22E-01	0.00E+00	4.87E-53	1.26E-29	2.20E+01	7.65E-44	0.00E+00	0.00E+00
	Best	3.05E-131	0.00E+00	4.88E-55	5.15E-14	6.73E+00	7.26E-45	0.00E+00	0.00E+00
	Worst	6.79E-01	0.00E+00	1.22E-52	5.15E-14	7.42E+01	3.30E-43	0.00E+00	0.00E+00
	Time(s)	224.53	498.46	218.26	170.04	155.86	157.80	230.81	190.66
	Fitness Rank	7	1	4	6	8	5	1	1
	Time Rank	6	8	5	3	1	2	7	4
F3	Mean	1.15E-02	5.25E-284	1.97E-51	3.84E-09	1.38E+05	5.51E-41	0.00E+00	0.00E+00
	Std	6.17E-02	0.00E+00	9.36E-52	0.00E+00	1.49E+05	5.80E-41	0.00E+00	0.00E+00
	Best	7.31E-128	3.64E-289	9.97E-52	3.84E-09	8.93E+03	9.96E-42	0.00E+00	0.00E+00
	Worst	3.44E-01	2.60E-283	3.46E-51	3.84E-09	7.28E+05	2.17E-40	0.00E+00	0.00E+00
	Time(s)	108.83	413.26	167.20	78.65	55.84	102.46	152.77	121.10
	Fitness Rank	7	3	4	6	8	5	1	1
	Time Rank	4	8	7	2	1	3	6	5
F4	Mean	6.08E+01	0.00E+00	1.37E-54	2.30E-13	1.50E+01	1.09E-41	0.00E+00	5.32E-233
	Std	4.15E+00	0.00E+00	1.40E-54	7.78E-14	2.53E+00	7.21E-42	0.00E+00	0.00E+00
	Best	4.44E+01	0.00E+00	4.14E-55	1.17E-13	9.14E+00	2.72E-42	0.00E+00	1.77E-269

Table 3: Results of Classical Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	Worst	6.56E+01	0.00E+00	4.15E-54	4.81E-13	2.13E+01	2.81E-41	0.00E+00	1.60E-231
	Time(s)	11.67	229.63	71.92	43.76	5.06	8.95	9.68	30.90
	Fitness Rank	8	1		6	7	5	1	
	Time Rank	4	8	7	6	1	2	3	5
F5	Mean	3.95E-130	0.00E+00	1.04E-52	6.77E-13	1.27E+01	4.09E-43	0.00E+00	0.00E+00
	Std	1.28E-130	0.00E+00	1.21E-52	2.02E-28	6.41E+00	3.27E-43	0.00E+00	0.00E+00
	Best	1.76E-130	0.00E+00	1.22E-53	6.77E-13	5.29E+00	6.17E-44	0.00E+00	0.00E+00
	Worst	6.72E-130	0.00E+00	3.30E-52	6.77E-13	2.97E+01	1.09E-42	0.00E+00	0.00E+00
	Time(s)	90.23	328.04	128.58	97.28	61.20	38.05	101.42	82.87
	Fitness Rank	4	1	5	7	8	6	1	1
	Time Rank	4	8	7	5	2	1	6	3
F6	Mean	1.03E-24	1.04E-57	1.79E-15	8.88E-16	9.92E-08	2.15E-116	0.00E+00	0.00E+00
	Std	1.93E-24	1.16E-57	3.58E-15	1.97E-31	1.62E-07	1.43E-116	0.00E+00	0.00E+00
	Best	8.13E-28	1.40E-60	1.40E-34	8.88E-16	1.43E-09	5.82E-117	0.00E+00	0.00E+00
	Worst	9.12E-24	3.17E-57	8.95E-15	8.88E-16	7.92E-07	7.00E-116	0.00E+00	0.00E+00
	Time(s)	73.73	398.39	148.99	106.30	83.30	87.26	130.74	85.20
	Fitness Rank	5	4	7	6	8	3	1	1
	Time Rank	1	8	7	5	2	4	6	3
F7	Mean	1.95E+02	1.2e-322	3.33E-31	6.05E+00	1.31E+02	5.26E-03	0.00E+00	1.04E-20
	Std	3.99E+01	0.00E+00	3.36E-31	2.54E+00	6.93E+01	2.43E-03	0.00E+00	5.59E-20
	Best	1.20E+02	0.00E+00	6.92E-33	2.33E+00	2.71E+01	1.84E-03	0.00E+00	1.29E-60
	Worst	2.74E+02	5.93e-322	9.46E-31	1.22E+01	3.12E+02	1.35E-02	0.00E+00	3.12E-19
	Time(s)	60.39	368.70	137.53	63.75	40.19	69.38	115.98	53.24
	Fitness Rank	8	2	3	6	7	5	1	4
	Time Rank	3	8	7	4	1	5	6	2
F8	Mean	2.60E+04	3.14E-128	1.87E-34	1.83E+01	1.63E+04	6.07E+01	0.00E+00	4.14E-67
	Std	5.04E+03	5.31E-128	2.55E-34	8.94E+00	7.08E+03	1.65E+01	0.00E+00	2.23E-66
	Best	1.43E+04	6.57E-258	1.50E-35	3.12E+00	3.63E+03	2.65E+01	0.00E+00	1.32E-245
	Worst	3.54E+04	1.37E-127	6.91E-34	3.95E+01	3.12E+04	1.02E+02	0.00E+00	1.24E-65
	Time(s)	378.85	534.32	321.64	298.88	257.02	264.40	271.99	175.60
	Fitness Rank	8	2	4	5	7	6	1	
	Time Rank	7	8	6	5	2	3	4	1
F9	Mean	1.24E+01	4.36E+01	2.49E+01	3.80E+01	8.67E+01	2.25E-13	0.00E+00	0.00E+00
	Std	3.30E+00	9.55E+00	1.89E+00	9.42E+00	1.78E+01	7.95E-13	0.00E+00	0.00E+00
	Best	5.97E+00	3.18E+01	2.19E+01	1.79E+01	4.21E+01	0.00E+00	0.00E+00	0.00E+00
	Worst	1.89E+01	5.97E+01	2.69E+01	5.79E+01	1.30E+02	4.23E-12	0.00E+00	0.00E+00
	Time(s)	18.28	197.22	76.80	44.99	9.26	13.10	12.63	26.09
	Fitness Rank	4	7	5	6	8	3	1	1
	Time Rank	4	8	7	6	1	3	2	5
F10	Mean	1.72E+01	2.00E+01	8.97E-15	1.27E-07	9.47E+00	7.90E-15	4.44E-16	4.44E-16
	Std	8.01E-01	1.17E-02	2.84E-15	1.26E-08	1.23E+00	1.41E-15	0.00E+00	0.00E+00
	Best	1.48E+01	2.00E+01	7.55E-15	1.19E-07	7.36E+00	7.55E-15	4.44E-16	4.44E-16
	Worst	1.84E+01	2.00E+01	1.47E-14	1.71E-07	1.20E+01	1.47E-14	4.44E-16	4.44E-16
	Time(s)	21.05	112.15	83.13	47.54	15.59	18.52	23.63	40.88
	Fitness Rank	7	8	4	5	6	3	1	1
	Time Rank	3	8	7	6	1	2	4	5
F11	Mean	7.75E+00	1.97E-03	0.00E+00	6.93E-02	1.10E+00	0.00E+00	0.00E+00	0.00E+00
	Std	7.05E-01	3.94E-03	0.00E+00	1.37E-01	7.31E-02	0.00E+00	0.00E+00	0.00E+00
	Best	6.46E+00	0.00E+00	0.00E+00	5.77E-15	9.54E-01	0.00E+00	0.00E+00	0.00E+00
	Worst	9.02E+00	9.86E-03	0.00E+00	7.73E-01	1.33E+00	0.00E+00	0.00E+00	0.00E+00
	Time(s)	16.77	206.10	80.82	52.69	8.78	18.81	32.01	41.97
	Fitness Rank	8	5	1	6	7	1	1	1
	Time Rank	2	8	7	6	1	3	4	5
F12	Mean	-1.04E+03	-8.92E+02	-7.51E+02	-1.17E+03	-1.01E+03	-5.52E+02	-1.17E+03	-1.17E+03
	Std	2.75E+01	1.07E+02	0.00E+00	2.12E-13	3.76E+01	4.16E+01	6.48E+00	4.49E-05
	Best	-1.09E+03	-1.01E+03	-7.51E+02	-1.17E+03	-1.09E+03	-6.32E+02	-1.17E+03	-1.17E+03
	2000	1.071103	1.011103	7.511102	1.171103	1.071103	0.551102	1.171103	1.171103

Table 3: Results of Classical Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	Worst	-9.91E+02	-7.51E+02	-7.51E+02	-1.17E+03	-9.47E+02	-4.74E+02	-1.16E+03	-1.17E+03
	Time(s)	183.06	429.66	192.95	93.44	72.23	130.79	93.04	152.13
	Fitness Rank	4	6	7	1	5	8	3	2
	Time Rank	6	8	7	3	1	4	2	5
F13	Mean	0.00E+00	0.00E+00	1.92E-150	1.42E-13	4.05E-06	2.08E-106	0.00E+00	0.00E+00
	Std	0.00E+00	0.00E+00	3.77E-150	1.88E-13	4.15E-06	2.45E-106	0.00E+00	0.00E+00
	Best	0.00E+00	0.00E+00	7.65E-158	2.22E-22	1.05E-06	3.40E-108	0.00E+00	0.00E+00
	Worst	0.00E+00	0.00E+00	9.46E-150	4.71E-13	1.23E-05	5.73E-106	0.00E+00	0.00E+00
	Time(s)	16.77	98.05	47.45	39.50	4.21	13.32	22.69	33.79
	Fitness Rank	1	1	5	7	8	6	1	1
	Time Rank	3	8	7	6	1	2	4	5
F14	Mean	8.54E-12	1.71E-11	3.51E-12	7.38E-12	2.50E-11	5.17E-12	3.51E-12	3.51E-12
	Std	9.04E-13	1.33E-12	2.54E-27	8.82E-13	3.03E-12	3.99E-13	4.96E-26	2.69E-17
	Best	7.17E-12	1.46E-11	3.51E-12	5.70E-12	1.70E-11	4.34E-12	3.51E-12	3.51E-12
	Worst	1.12E-11	1.82E-11	3.51E-12	9.10E-12	3.00E-11	5.89E-12	3.51E-12	3.51E-12
	Time(s)	159.41	334.09	180.82	154.80	118.05	123.38	87.64	79.20
	Fitness Rank	6	7	1	5	8	4	2	3
	Time Rank	6	8	7	5	3	4	2	1
F15	Mean	2.24E-14	2.00E-12	3.60E-18	1.75E-06	7.26E+00	2.24E-03	0.00E+00	0.00E+00
	Std	7.69E-15	2.64E-13	2.56E-18	3.97E-07	4.70E+00	5.30E-04	0.00E+00	0.00E+00
	Best	9.66E-15	1.53E-12	6.39E-19	9.84E-07	1.08E+00	1.22E-03	0.00E+00	0.00E+00
	Worst	4.17E-14	2.25E-12	7.65E-18	2.51E-06	2.07E+01	3.48E-03	0.00E+00	0.00E+00
	Time(s)	123.42	362.59	144.51	112.47	82.66	77.79	121.00	112.19
	Fitness Rank	4	5	3	6	8	7	1	1
	Time Rank	6	8	7	4	2	1	5	3
F16	Mean	-2.77E+01	-1.86E+01	-2.40E+01	-2.62E+01	-1.73E+01	-4.33E+00	-2.88E+01	-1.25E+01
	Std	5.57E-01	1.60E+00	1.25E+00	6.03E-01	2.04E+00	6.37E-01	1.91E-01	1.86E+00
	Best	-2.86E+01	-2.03E+01	-2.64E+01	-2.73E+01	-2.05E+01	-6.17E+00	-2.91E+01	-1.57E+01
	Worst	-2.66E+01	-1.56E+01	-2.29E+01	-2.46E+01	-1.18E+01	-3.15E+00	-2.85E+01	-7.73E+00
	Time(s)	262.23	569.71	248.92	219.29	125.46	122.20	254.45	116.78
	Fitness Rank	2	5	4	3	6	8	1	7
	Time Rank	7	8	5	4	3	2	6	1

Table 4: Summary of Classical Benchmarking Function Evaluations

Avg Fitness	Avg Time	Overall Fitness	Overall Time
Rank	Rank	Rank	Rank
5.69	4.38	7	4
3.69	8.00	3	8
4.06	6.69	4	7
5.44	4.75	6	6
7.25	1.50	8	1
5.00	2.69	5	2
1.19	4.38	1	4
2.00	3.63	2	3
	Rank 5.69 3.69 4.06 5.44 7.25 5.00 1.19	Rank Rank 5.69 4.38 3.69 8.00 4.06 6.69 5.44 4.75 7.25 1.50 5.00 2.69 1.19 4.38	Rank Rank Rank 5.69 4.38 7 3.69 8.00 3 4.06 6.69 4 5.44 4.75 6 7.25 1.50 8 5.00 2.69 5 1.19 4.38 1

3.2. CEC benchmark Functions Test

Following the CEC 2014 recommendation [29], the dimension of problems is selected as 30 as well. Function *CEC*1-*CEC*3 are uni-modal functions with rotations. *CEC*4-*CEC*16 are simple multi-modal functions but with various shiftings and rotations. *CEC*17-*CEC*22 are hybrid

functions while *CEC23-CEC30* are the composition functions. As the global optimum of each function is different (from 100 to 3000), the fitness result is converted to the error as calculated in (13) to make the comparison more straightforward. In other words, when the result gets closer to 0, it implies that the best solution obtained by the algorithm is closer to the global optimum of the corresponding function.

$$fitness = f(x) - f(x^*)$$
(13)

where x is the best solution obtained by the algorithm while $f(x^*)$ is the real global optimum of the function.

Table 5 reports the results over 31 independent runs on each function for each algorithm. A few observations are specified as follows.

- For the uni-modal functions *CEC1-CEC3*, no algorithm is dominated. Because of the complicated rotation, the errors for *CEC1* are huge for all algorithms. Only GA and VSO perform relatively better. Both errors are on the same order of magnitude, i.e. 10⁶. In the case of *CEC2*, it is similar to *CEC1*, only SSA and VSO are on the smallest order (10²) of magnitude. Meanwhile, the best metric of all runs for VSO is 0, which demonstrates that only VSO has once achieved the exact global optima. Interestingly, WOA performs very well in the classical uni-modal functions but does not work in these complicated uni-modal cases. It is remarkable that only VSO is able to achieve the global optimum exactly (with fitness error 0) in *CEC3*;
- For multi-modal functions *CEC4-CEC*16, it is obvious that the VSO algorithm attains a better performance than those of other algorithms. In terms of the mean of fitness, VSO ranks the best for over half of the functions, including the *CEC*6, *CEC*9, *CEC*10, *CEC*13, *CEC*14, *CEC*15 and *CEC*16. SSA followed by VSO acquires the best performance of fitness in 4 functions;
- For 6 hybrid functions *CEC*17-*CEC*22, the VSO algorithm achieves the best performance in all functions except for *CEC*18 where it ranks the second place;
- For the composition functions of CEC23-CEC30, VSO outperforms all compared algorithms in the first 6 functions, i.e. CEC23-CEC28. It can also be observed that WOA obtains the best performance in CEC23 and CEC25. This verifies the outstanding optimizing capacity of VSO on such complex functions.

Table 6 indicates that VSO generally outperforms all other algorithms in terms of the fitness values. In addition, VSO with the imported infection operation powered by DE works well here yet the performance of the standalone DE is the worst of all.

Table 5: Results of CEC Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
CEC1	Mean	5.15E+08	5.83E+07	2.36E+09	2.07E+06	2.05E+08	6.49E+06	3.17E+06	3.67E+08
	Std	1.39E+08	2.90E+07	4.92E-07	1.35E+06	2.53E+08	1.68E+06	1.28E+06	1.85E+08
	Best	1.61E+08	9.84E+06	2.36E+09	3.56E+05	3.31E+07	2.68E+06	1.21E+06	1.11E+08
	Worst	7.82E+08	1.49E+08	2.36E+09	5.79E+06	1.40E+09	1.02E+07	6.67E+06	8.87E+08
	Time(s)	7.15	1568.44	66.78	47.22	3.69	5.90	10.40	14.70
	Fitness Rank	7	4	8	1	5	3	2	6
GE GA	Time Rank	3	8	7	6	1 225 10	2 200 02	4	5
CEC2	Mean	5.27E+10	1.12E+04	9.65E+10	9.99E+03	1.32E+10	3.28E+02	5.97E+02	2.64E+10
	Std	6.79E+09	8.97E+03	3.33E-05	1.16E+04	1.09E+10	4.64E+02	3.21E+03	6.55E+09
	Best Worst	3.85E+10	2.09E+02	9.65E+10	2.15E+01	5.28E+08 4.98E+10	1.29E-01	0.00E+00	1.48E+10
	Time(s)	6.43E+10 7.00	4.04E+04 182.04	9.65E+10 28.97	3.43E+04 24.01	4.96E+10 3.63	1.58E+03 5.80	1.79E+04 10.04	3.95E+10 32.17
	Fitness Rank	7.00	182.04	28.97	3	5.05	3.80		52.17
	Time Rank	3	8	6	5	1	2	4	7
CEC3	Mean	1.01E+05	7.61E+04	2.30E+07	1.76E+04	7.29E+04	2.48E+02	0.00E+00	7.97E+04
CLC3	Std	1.38E+04	8.26E+03	7.57E-09	1.40E+04	3.70E+04	3.32E+02	0.00E+00	6.30E+03
	Best	7.11E+04	5.75E+04	2.30E+07	1.80E+02	1.78E+04	9.16E+00	0.00E+00	6.25E+04
	Worst	1.38E+05	8.60E+04	2.30E+07	5.46E+04	2.07E+05	1.76E+03	0.00E+00	8.89E+04
	Time(s)	6.74	1605.94	65.76	46.67	3.41	5.57	9.89	31.85
	Fitness Rank	7	5	8	3	4	2	1	6
	Time Rank	3	8	7	6	1	2	4	5
CEC4	Mean	7.31E+03	7.83E+01	2.39E+04	7.34E+01	1.60E+03	2.56E+01	6.99E+01	3.34E+03
	Std	1.46E+03	3.41E+01	5.01E-12	4.53E+01	1.16E+03	2.12E+00	3.33E+01	1.25E+03
	Best	3.66E+03	8.65E+00	2.39E+04	5.98E-05	2.31E+02	1.74E+01	1.76E+00	1.50E+03
	Worst	1.01E+04	1.45E+02	2.39E+04	1.49E+02	5.80E+03	3.02E+01	1.37E+02	6.53E+03
	Time(s)	7.18	156.35	29.49	24.80	3.70	5.95	10.66	15.60
	Fitness Rank	7	4	8	3	5	1	2	6
	Time Rank	3	8	7	6	1	2	4	5
CEC5	Mean	2.00E+01	2.08E+01	2.09E+01	2.00E+01	2.01E+01	2.05E+01	2.00E+01	2.08E+01
	Std	4.87E-04	3.32E-01	4.60E-02	3.10E-05	1.53E-01	4.96E-02	8.10E-02	8.32E-02
	Best	2.00E+01	2.00E+01	2.08E+01	2.00E+01	2.00E+01	2.04E+01	2.00E+01	2.05E+01
	Worst	2.00E+01	2.10E+01	2.10E+01	2.00E+01	2.05E+01	2.06E+01	2.04E+01	2.09E+01
	Time(s)	14.33	375.54	29.43	47.13	7.16	6.11	21.05	32.31
	Fitness Rank	2	7	8	1		5	3	6
OF GC	Time Rank	3	8	5	7	2 000 01	1 405 01	4	6
CEC6	Mean	3.58E+01	2.84E+01	4.60E+01	2.03E+01	2.88E+01	1.40E+01	7.67E+00	3.91E+01
	Std	1.61E+00	1.05E+01	8.30E-02	3.18E+00	3.25E+00	1.42E+00	5.76E+00	3.21E+00
	Best	3.25E+01	1.51E+00	4.59E+01	1.40E+01	2.40E+01	1.14E+01	0.00E+00	2.91E+01
	Worst Time(s)	3.92E+01 94.91	4.42E+01 276.51	4.62E+01 63.42	2.59E+01 105.90	3.66E+01 38.86	1.66E+01 41.20	1.87E+01 59.62	4.51E+01 49.78
	Fitness Rank	94.91	4	8	103.90	58.80	41.20	39.02	49.78
	Time Rank	6	8	5	7	1	2	4	3
CEC7	Mean	4.92E+02	1.49E-02	9.89E+02	3.33E-02	1.83E+02	1.18E-05	1.39E-03	1.69E+02
CEC	Std	5.35E+01	2.09E-02	3.02E-13	2.82E-02	9.56E+01	1.14E-05	6.27E-03	5.68E+01
	Best	3.93E+01	1.64E-04	9.89E+02	1.93E-12	2.89E+01	3.80E-07	0.27E-03 0.00E+00	8.95E+01
	Worst	5.80E+02	8.44E-02	9.89E+02	1.01E-01	4.01E+02	5.21E-05	3.44E-02	2.75E+02
	Time(s)	7.64	64.66	30.02	24.93	3.96	6.28	11.20	15.44
	Fitness Rank	7.01	3	8	4	6	1	2	5
	Time Rank	3	8	7	6	1	2	4	5
CEC8	Mean	2.04E+02	1.67E+02	4.02E+02	3.38E+01	1.56E+02	1.03E-10	6.57E+00	2.47E+02
	Std	3.65E+01	2.82E+00	1.41E-13	9.00E+00	3.95E+01	7.87E-12	2.31E+00	2.75E+01
	Best	1.35E+02	1.62E+02	4.02E+02	1.99E+01	7.27E+01	8.95E-11	2.98E+00	2.03E+02
	Worst	2.64E+02	1.73E+02	4.02E+02	5.97E+01	2.68E+02	1.18E-10	1.29E+01	3.17E+02
	Time(s)	13.11	62.90	65.14	24.33	6.50	5.83	10.35	14.69
	Fitness Rank	6	5	8	3	4	1	2	7

Table 5: Results of CEC Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	Time Rank	4	7	8	6	2	1	3	5
CEC9	Mean	2.74E+02	1.86E+02	3.81E+02	1.18E+02	1.85E+02	5.60E+01	5.15E+01	3.00E+02
	Std	3.80E+01	3.57E+00	1.35E-01	3.13E+01	4.15E+01	6.30E+00	2.02E+01	2.81E+01
	Best	2.07E+02	1.84E+02	3.81E+02	7.16E+01	1.15E+02	4.27E+01	2.39E+01	2.48E+02
	Worst	3.56E+02	1.96E+02	3.81E+02	1.97E+02	2.53E+02	6.76E+01	1.08E+02	3.47E+02
	Time(s)	7.36	63.72	29.75	24.84	3.87	11.45	10.93	15.91
	Fitness Rank	6	5	8	3	4	2	1	7
	Time Rank	2	8	7	6	1	4	3	5
CEC10	Mean	3.99E+03	4.25E+03	8.05E+03	4.53E+02	4.17E+03	8.32E+01	7.70E+01	5.58E+03
	Std	5.10E+02	2.28E+02	1.15E-12	2.65E+02	8.11E+02	1.67E+01	7.60E+01	5.71E+02
	Best	2.92E+03	3.78E+03	8.05E+03	9.76E+00	2.20E+03	4.45E+01	2.51E+00	4.38E+03
	Worst	5.37E+03	4.74E+03	8.05E+03	1.03E+03	5.85E+03	1.14E+02	2.59E+02	6.80E+03
	Time(s)	7.25 4	64.04	65.94	24.16	3.72 5	5.98	10.36	32.68
	Fitness Rank Time Rank	3	6 7	8 8	3 5	1	2 2	1 4	7 6
CEC11	Mean	4.06E+03	4.33E+03	9.17E+03	3.08E+03	4.58E+03	2.45E+03	3.85E+03	6.40E+03
CECII	Std	4.74E+02	4.33E+03 2.03E+02	1.91E-12	6.08E+03	7.71E+02	2.43E+03 2.52E+02	7.35E+03	5.58E+02
	Best	2.67E+03	3.69E+03	9.17E+03	1.89E+03	2.97E+03	1.79E+03	1.86E+03	5.31E+03
	Worst	4.70E+03	4.62E+03	9.17E+03 9.17E+03	4.82E+03	5.95E+03	2.87E+03	5.09E+03	7.77E+03
	Time(s)	7.85	559.44	29.05	24.57	8.07	6.48	11.10	15.14
	Fitness Rank	4	5	8	2	6	1	3	7
	Time Rank	2	8	7	6	3	1	4	5
CEC12	Mean	1.39E-01	2.09E-01	4.02E-01	2.77E-01	8.11E-01	4.99E-01	7.57E-01	1.83E+00
02012	Std	4.70E-02	1.35E-01	2.10E-02	9.70E-02	2.57E-01	7.78E-02	2.61E-01	5.23E-01
	Best	7.40E-02	2.66E-02	3.35E-01	1.14E-01	3.77E-01	3.57E-01	3.15E-01	8.80E-01
	Worst	2.58E-01	6.48E-01	4.32E-01	5.37E-01	1.43E+00	6.91E-01	1.41E+00	2.70E+00
	Time(s)	20.72	681.08	38.33	33.46	13.30	15.46	47.79	51.98
	Fitness Rank	1	2	4	3	7	5	6	8
	Time Rank	3	8	5	4	1	2	6	7
CEC13	Mean	5.89E+00	4.73E-01	1.06E+01	5.17E-01	3.63E+00	2.82E-01	2.20E-01	4.36E+00
	Std	3.63E-01	4.53E-02	1.04E-13	1.16E-01	1.43E+00	3.08E-02	5.33E-02	7.00E-01
	Best	5.16E+00	3.19E-01	1.06E+01	2.65E-01	3.78E-01	2.29E-01	9.46E-02	2.92E+00
	Worst	6.49E+00	5.63E-01	1.06E+01	7.41E-01	7.30E+00	3.52E-01	3.29E-01	5.61E+00
	Time(s)	6.75	75.06	28.46	23.71	3.46	5.63	9.88	14.33
	Fitness Rank	7	3	8	4	5	2	1	6
	Time Rank	3	8	7	6	1	2	4	5
CEC14	Mean	1.61E+02	2.72E-01	3.76E+02	3.18E-01	5.76E+01	2.62E-01	2.40E-01	9.28E+01
	Std	2.99E+01	3.15E-02	1.34E-13	6.34E-02	3.15E+01	3.03E-02	2.89E-02	2.99E+01
	Best	7.08E+01	2.15E-01	3.76E+02	2.03E-01	1.23E+00	1.98E-01	1.44E-01	3.74E+01
	Worst	2.01E+02	3.32E-01 65.65	3.76E+02	4.46E-01	1.14E+02	3.09E-01	2.94E-01	1.45E+02
	Time(s) Fitness Rank	6.76 7	3	28.56	23.87	3.41	5.60	9.79 1	14.29
	Time Rank	3	8	8 7	4	5 1	2 2	4	6 5
CEC15	Mean	5.64E+05	0 1.89E+01	5.86E+05	2.96E+01	6.99E+04	7.31E+00	5.93E+00	1.16E+04
CECIS	Std	2.52E+05	1.05E+01	5.48E-01	1.13E+01	1.28E+05	7.70E-01	1.54E+00	6.90E+03
	Best	6.77E+04	1.03E+00 1.71E+01	5.86E+05	1.13E+01 1.27E+01	1.29E+03	5.35E+00	3.31E+00	2.58E+03
	Worst	1.12E+06	2.08E+01	5.86E+05	6.30E+01	6.04E+05	8.72E+00	9.90E+00	3.08E+04
	Time(s)	14.66	65.27	28.79	23.93	3.60	11.39	10.25	14.65
	Fitness Rank	7	3	8	23.93	5.00	2	10.23	5
	Time Rank	5	8	7	6	1	3	2	4
CEC16	Mean	1.27E+01	1.30E+01	1.38E+01	1.15E+01	1.23E+01	1.10E+01	1.09E+01	1.25E+01
CLC10	Std	2.52E-01	1.34E-01	1.47E-02	5.06E-01	4.56E-01	3.79E-01	6.59E-01	4.48E-01
	Best	1.19E+01	1.27E+01	1.37E+01	1.05E+01	1.15E+01	1.01E+01	8.79E+00	1.12E+01
	Worst	1.32E+01	1.32E+01	1.38E+01	1.25E+01	1.30E+01	1.15E+01	1.20E+01	1.32E+01
	WOISL								

Table 5: Results of CEC Benchmarking Functions

Function	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	Fitness Rank	6	7	8	3	4	2	1	5
	Time Rank	2	8	7	6	1	4	3	5
CEC17	Mean	9.39E+06	3.96E+06	8.83E+08	1.02E+06	3.75E+06	4.59E+05	1.85E+05	3.22E+07
	Std	6.20E+06	2.60E+06	2.16E-01	6.86E+05	3.17E+06	1.90E+05	1.07E+05	2.98E+07
	Best	4.28E+05	5.28E+05	8.83E+08	2.54E+05	3.43E+05	4.93E+04	5.94E+04	2.37E+06
	Worst	2.80E+07	1.08E+07	8.83E+08	2.75E+06	1.27E+07	8.58E+05	4.34E+05	1.07E+08
	Time(s)	7.98	1576.93	30.19	25.05	4.02	6.59	11.78	16.40
	Fitness Rank	6	5	8	3	4	2	1	7
	Time Rank	3	8	7	6	1	2	4	5
CEC18	Mean	2.28E+08	2.75E+07	1.47E+10	3.37E+03	2.20E+08	1.28E+02	3.07E+02	2.27E+08
	Std	2.49E+08	2.68E+07	2.23E-06	4.25E+03	3.42E+08	2.49E+01	2.45E+02	3.55E+08
	Best	4.95E+02	2.20E+06	1.47E+10	6.90E+01	6.36E+02	8.66E+01	8.54E+01	3.89E+04
	Worst	1.03E+09	1.29E+08	1.47E+10	2.10E+04	1.57E+09	1.97E+02	1.06E+03	1.29E+09
	Time(s)	7.71	1555.67	29.71	24.90	3.93	6.31	11.17	14.77
	Fitness Rank	7	4	8	3	5	1	2	6
	Time Rank	3	8	7	6	1	2	4	5
CEC19	Mean	1.93E+02	2.36E+01	8.10E+02	1.64E+01	1.32E+02	8.85E+00	5.66E+00	2.30E+02
	Std	4.91E+01	8.62E+00	7.92E-02	1.78E+01	7.27E+01	1.61E+00	1.00E+00	9.71E+01
	Best	1.14E+02	1.23E+01	8.10E+02	7.51E+00	1.88E+01	6.01E+00	3.83E+00	5.72E+01
	Worst	2.83E+02	5.78E+01	8.11E+02	7.02E+01	3.39E+02	1.18E+01	8.21E+00	4.61E+02
	Time(s)	17.64	1527.29	36.47	31.36	11.12	13.52	20.79	34.17
	Fitness Rank	6	4	8	3	5	2	1	7
	Time Rank	3	8	7	5	1	2	4	6
CEC20	Mean	3.69E+04	8.04E+04	2.50E+09	3.82E+04	3.43E+04	3.18E+03	1.62E+02	1.07E+05
02020	Std	1.30E+04	4.02E+04	5.18E-01	1.51E+04	3.78E+04	2.19E+03	8.41E+01	8.41E+04
	Best	1.10E+04	1.57E+04	2.50E+09	8.10E+03	3.49E+03	6.29E+02	4.08E+01	4.92E+03
	Worst	5.93E+04	2.05E+05	2.50E+09	6.54E+04	1.95E+05	8.82E+03	3.57E+02	3.80E+05
	Time(s)	7.36	1581.01	29.06	24.15	3.74	5.98	11.08	32.76
	Fitness Rank	4	6	8	5	3	2	1	7
	Time Rank	3	8	6	5	1	2	4	7
CEC21	Mean	1.94E+06	1.14E+06	2.26E+09	5.19E+05	1.61E+06	7.07E+04	4.41E+04	1.09E+07
CEC21	Std	1.38E+06	6.04E+05	7.34E-07	4.31E+05	3.91E+06	3.97E+04	3.22E+04	8.53E+06
	Best	1.42E+05	1.72E+05	2.26E+09	2.22E+04	1.68E+04	2.10E+04	6.85E+03	2.53E+05
	Worst	4.94E+06	2.76E+06	2.26E+09	1.59E+06	2.23E+07	2.19E+05	1.24E+05	3.27E+07
	Time(s)	7.30	1574.80	65.79	24.20	3.81	6.05	21.74	32.79
	Fitness Rank	6	4	8	3	5.61	2	1	7
	Time Rank	3	8	7	5	1	2	4	6
CEC22	Mean	8.47E+02	6.79E+02	4.71E+06	7.85E+02	8.53E+02	2.07E+02	1.86E+02	1.15E+03
CEC22	Std	2.15E+02	1.53E+02	3.30E-01	2.47E+02	2.42E+02	6.01E+01	8.46E+01	4.03E+02
	Best	2.13E+02 2.69E+02	3.59E+02	4.71E+06	1.90E+02	4.52E+02	8.41E+01	3.10E+01	5.50E+02
	Worst	1.20E+02	9.29E+02	4.71E+06 4.71E+06	1.90E+02 1.21E+03	1.33E+02	3.48E+02	3.10E+01 3.08E+02	2.54E+03
		9.00	1499.73	29.91	25.28			12.06	15.91
	Time(s)					4.87	7.10		
	Fitness Rank	5	3	8	4	6	2 2	1	7
CECO2	Time Rank	5.500.02	2.005.02	7	6 2.15E - 02	1 055.02		2.005.02	2.005.02
CEC23	Mean	5.50E+02	2.00E+02	2.00E+02	3.15E+02	4.05E+02	3.14E+02	2.00E+02	2.00E+02
	Std	5.93E+01	9.25E-14	0.00E+00	1.32E-03	5.83E+01	1.58E-05	0.00E+00	0.00E+00
	Best	4.14E+02	2.00E+02	2.00E+02	3.15E+02	3.40E+02	3.14E+02	2.00E+02	2.00E+02
	Worst	6.82E+02	2.00E+02	2.00E+02	3.15E+02	5.75E+02	3.14E+02	2.00E+02	2.00E+02
	Time(s)	13.54	73.17	34.07	29.25	8.20	10.74	17.22	19.78
	Fitness Rank	8	3	4	6	7	5	1	1
anc.	Time Rank	3	8	7	6	1	2	4	5
CEC24	Mean	3.75E+02	2.02E+02	2.00E+02	2.43E+02	3.05E+02	2.25E+02	2.00E+02	2.00E+02
	Std	2.03E+01	5.15E-01	2.77E-02	7.36E+00	2.73E+01	4.38E-01	0.00E+00	5.11E-03
	Best	3.35E+02	2.01E+02	2.00E+02	2.29E+02	2.55E+02	2.24E+02	2.00E+02	2.00E+02
	Worst	4.08E+02	2.03E+02	2.00E+02	2.59E+02	3.61E+02	2.26E+02	2.00E+02	2.00E+02

Table 5: Results of CEC Benchmarking Functions

CEC25	Time(s) Fitness Rank Time Rank Mean Std Best Worst Time(s) Fitness Rank Time Rank	15.21 8 4 2.38E+02 7.48E+00 2.21E+02 2.57E+02 12.24	70.55 4 8 2.00E+02 8.53E-14 2.00E+02 2.00E+02	32.01 3 7 2.00E+02 1.83E-13	27.17 6 6 2.14E+02	6.23 7 1	8.63 5 2	14.25 1	17.63 2
CEC25	Time Rank Mean Std Best Worst Time(s) Fitness Rank	4 2.38E+02 7.48E+00 2.21E+02 2.57E+02	8 2.00E+02 8.53E-14 2.00E+02	7 2.00E+02 1.83E-13	6 2.14E+02	1			
CEC25	Mean Std Best Worst Time(s) Fitness Rank	2.38E+02 7.48E+00 2.21E+02 2.57E+02	2.00E+02 8.53E-14 2.00E+02	2.00E+02 1.83E-13	2.14E+02		2	_	
CEC25	Std Best Worst Time(s) Fitness Rank	7.48E+00 2.21E+02 2.57E+02	8.53E-14 2.00E+02	1.83E-13			_	3	5
	Best Worst Time(s) Fitness Rank	2.21E+02 2.57E+02	2.00E+02			2.32E+02	2.02E+02	2.00E+02	2.00E+02
	Worst Time(s) Fitness Rank	2.57E+02		2.000	8.31E+00	1.46E+01	3.83E-01	0.00E+00	0.00E+00
	Time(s) Fitness Rank		2 00F±02	2.00E+02	2.04E+02	2.16E+02	2.01E+02	2.00E+02	2.00E+02
	Fitness Rank	12.24		2.00E+02	2.31E+02	2.73E+02	2.03E+02	2.00E+02	2.00E+02
			109.71	32.42	28.17	6.91	9.49	15.73	18.69
		8	3	4	6	7	5	1	1
CE CO		3	8	7	6	1 125 02	2	4	5
CEC26	Mean	1.06E+02	1.01E+02	2.00E+02	1.30E+02	1.13E+02	1.00E+02	1.00E+02	1.53E+02
	Std	6.01E-01	1.73E+00	8.30E-14	4.57E+01	5.15E+01	4.38E-02	7.81E-02	4.72E+01
	Best Worst	1.05E+02	1.00E+02 1.08E+02	2.00E+02 2.00E+02	1.00E+02 2.00E+02	1.01E+02 3.90E+02	1.00E+02	1.00E+02	1.04E+02 2.00E+02
	Time(s)	1.07E+02 74.22	473.23	75.57	120.39	50.07	1.01E+02 52.89	1.01E+02 76.10	62.31
	Fitness Rank	4.22	4/3.23	13.31	120.39	50.07	32.89	70.10	7
	Time Rank	4	8	5	7	1	2	6	3
CEC27	Mean	7.42E+02	2.00E+02	2.00E+02	9.10E+02	1.10E+03	4.17E+02	2.00E+02	3.82E+02
CLC21	Std	6.97E+01	6.29E-13	2.57E-13	6.43E+01	2.54E+02	4.90E+00	0.00E+00	4.36E+02
	Best	6.14E+02	2.00E+02	2.00E+02	8.04E+02	4.24E+02	4.08E+02	2.00E+02	2.00E+02
	Worst	9.07E+02	2.00E+02	2.00E+02	1.02E+03	1.37E+03	4.26E+02	2.00E+02	1.65E+03
	Time(s)	117.03	161.16	138.20	71.41	49.87	53.97	107.82	62.05
	Fitness Rank	6	3	2	7	8	5	1	4
	Time Rank	6	8	7	4	1	2	5	3
CEC28	Mean	3.78E+03	2.00E+02	2.00E+02	1.65E+03	2.11E+03	4.13E+02	2.00E+02	4.12E+02
	Std	3.94E+02	7.13E-13	1.71E-13	4.25E+02	5.66E+02	4.85E+00	0.00E+00	8.35E+02
	Best	3.03E+03	2.00E+02	2.00E+02	1.08E+03	1.24E+03	4.02E+02	2.00E+02	2.00E+02
	Worst	4.42E+03	2.00E+02	2.00E+02	2.48E+03	3.43E+03	4.24E+02	2.00E+02	4.38E+03
	Time(s)	20.21	83.19	38.12	33.32	12.56	15.21	23.38	24.35
	Fitness Rank	8	3	2	6	7	5	1	4
	Time Rank	3	8	7	6	1	2	4	5
CEC29	Mean	6.95E+07	2.00E+02	2.00E+02	2.81E+05	1.83E+07	2.09E+02	3.03E+02	2.88E+07
	Std	4.28E+07	8.53E-14	2.25E-13	1.51E+06	1.32E+07	9.25E-01	3.89E+02	3.53E+07
	Best	3.23E+03	2.00E+02	2.00E+02	7.99E+02	2.68E+04	2.07E+02	2.00E+02	2.00E+02
	Worst	1.33E+08	2.00E+02	2.00E+02	8.40E+06	6.13E+07	2.11E+02	1.88E+03	1.15E+08
	Time(s)	22.91	88.77	83.29	35.56	25.54	17.03	44.90	26.22
	Fitness Rank	8	1		5	6	3	4	7
GE GAO	Time Rank	2	8	7	5	3	1	6	4
CEC30	Mean	5.50E+05	2.00E+02	2.00E+02	3.30E+03	1.67E+05	3.33E+02	2.49E+02	7.25E+05
	Std Post	2.74E+05	3.91E-04	2.16E-04	9.40E+02	1.27E+05	5.25E+01	2.63E+02	6.08E+05
	Best Worst	1.35E+05 9.76E+05	2.00E+02 2.00E+02	2.00E+02 2.00E+02	1.66E+03 5.20E+03	1.05E+04 4.82E+05	2.66E+02 4.51E+02	2.00E+02 1.66E+03	2.00E+02 2.55E+06
	Time(s)	9.76E+03 12.98	73.72	2.00E+02 32.61	5.20E+03 27.96	4.82E+05 7.59	4.51E+02 10.15	1.00E+03 16.29	2.55E+06 18.98
	Fitness Rank	12.98	13.12	32.01		7.39 6	10.15	16.29	18.98
	Time Rank	3	8	7	6	1	2	4	5

Table 6: Summary of Evaluations for CEC Benchmarking Functions

Algorithm	Avg Fitness Rank	Avg Time Rank	Overall Fitness Rank	Overall Time Rank
ABC	6.07	3.23	7	3
CMA-ES	3.93	7.93	4	8
DE	6.60	6.73	8	7
GA	3.83	5.77	3	6
PSO	5.37	1.20	5	1
SSA	2.63	2.03	2	2
VSO	1.67	4.03	1	4
WOA	5.83	5.07	6	5

3.3. Convergence Test

In addition to the solution quality, we are also interested in the rate of convergence. Therefore, the convergence test on those classical benchmark functions is conducted.

Figure 5 displays the convergence results based on the median fitness of all trials. The results are given as below.

- VSO generally converges faster than other algorithms and hence possesses superior convergence capability for such optimization problems;
- For the uni-modal functions, it seems that almost all algorithms can quickly converge. This is because most algorithms can achieve small errors as stated in Table 3. However, only VSO achieves the exact global optima for these functions which has been discussed in Section 3.1;
- With the more complicated multi-modal functions F9-F16, it is obvious that the VSO performs very well with respect to the rate of convergence. The convergence rates of some other algorithms decrease, such as GA, DE and SSA on F9; DE, ABC and SSA on F10; and CMA-ES and DE on F16. WOA has a fast rate of convergence as well.

3.4. Reliability test

Figure 6 plots a series of box plots through all runs for the classical benchmark functions for each algorithm. From the obtained results, the following observations can be drawn.

- For the uni-modal functions, the reliability of VSO is impressive over other algorithms. For example, the performances of both ABC and PSO are quite unstable;
- For the multi-modal functions, VSO can constantly generate stable results. The only exception is *F*12 where DE achieves the best reliability but it fails to acquire a good solution. The reliability of WOA is followed by VSO. But it becomes much worse in *F*16.

3.5. Scalability Test

In addition to the above low dimensional benchmark functions, a series of evaluations are performed on the medium and high dimensional classical benchmark problems, including 100, 300, 500 and 1000 dimensions to test the scalability of VSO. To make a thorough comparison, we also employ other algorithms in this evaluation. As aforementioned, the parameter settings of each algorithm are the same as the test on the 30-D problems.

From the results listed in Table 7 we have the following observations:

- VSO achieves the best performance in almost all functions with all dimensions except for *F*12 and *F*14 with 100 dimensions. In other words, the VSO algorithm ranks first on 59 out of the total 64 cases (≈ 92.19%). More importantly, VSO attains the exact globally optimal solutions for most of the cases. Take *F*4 with 500 dimensions and *F*8 with 1000 dimensions as examples. Only VSO obtains the globally optima in both cases. For the latter one, other algorithms except for WOA not only fail to get the exact globally optimal solution but also their values are very bad. Similar cases include *F*4 with 100 to 1000 dimensions, *F*7 with 100 to 1000 dimensions, *F*8 with 100 to 1000 dimensions, etc., in which VSO is the only algorithm with the fitness error as 0. The findings demonstrate the excellent scalability of our proposed algorithm;
- More specifically, VSO shows the advantage of the computational time for some 1000-*D* high dimensional problems. For instance, the ranking of VSO for the computational time goes up to the second or even the first place in *F*2, *F*3, *F*5, *F*6, *F*12, *F*16;
- As for other algorithms, the solution qualities drop down with increasing dimensions. For instance, for CMA-ES, the mean of fitness of 30-D F3 is 5.25E-284 as reported in Table 3. Nevertheless, the values are 1.74E + 01, 2.24E + 04, 1.14E + 05, and 6.38E + 05 for 100, 300, 500, and 1000-D problems, respectively. Likewise, the mean value of SSA for F9 with 30-D is 2.25E 13 while the values become 1.77E + 02, 1.42E + 03, 2.93E + 03 and 6.98E + 03 for 100, 300, 500, and 1000-D problems, accordingly;
- The only competitor is WOA probably due to its sophisticated design of the searching strategies as inspired by searching for prey and attacking the prey of the whales [18].

Table 8 summaries the results of the scalability test.

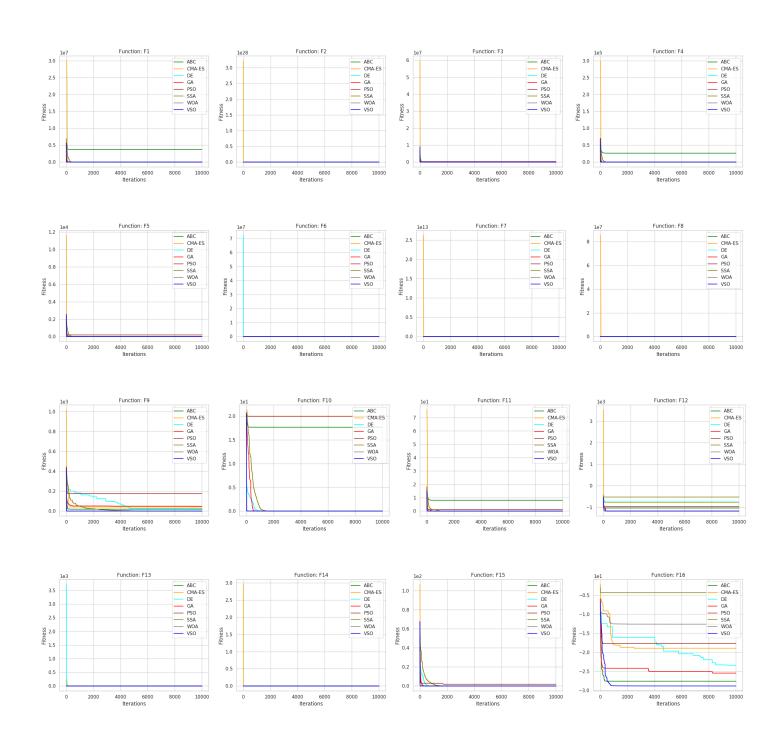


Figure 5: Convergence Test of Results of Classic Benchmarking Functions

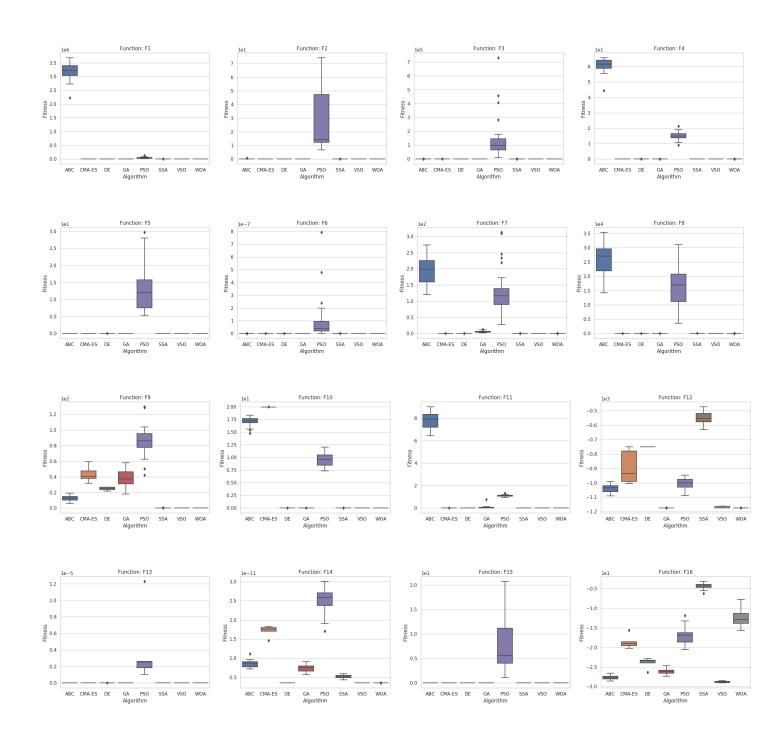


Figure 6: Box Plot of Results of Classic Benchmarking Functions

Table 7: Results of Scalability Test

Function	Dimension	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
F1	100	Mean	1.99E+07	2.17E-145	2.85E-05	7.78E+01	7.12E+05	8.46E-21	0.00E+00	0.00E+00
		Time(s)	12.00	1284.26	208.53	98.58	6.51	11.59	22.12	73.90
		Fitness Rank	8	3	5	6	7	4	1	1
		Time Rank	3	8	7	6	1	2	4	5
	300	Mean	7.61E+07	1.59E-61	5.41E+00	6.08E+05	3.09E+06	1.15E-05	0.00E+00	0.00E+00
		Time(s)	8.02	1767.10	678.84	240.96	10.88	13.55	35.24	138.95
		Fitness Rank	8	3	5	6	7	4	1	1
	500	Time Rank	1	8	7	6	2	3	4	2005
	500	Mean	1.36E+08	1.16E-36	5.54E+01	4.89E+06	5.06E+06	6.52E-01	0.00E+00	0.00E+00
		Time(s)	14.57	7001.47	1084.15	625.01	15.10	28.86	48.09	310.53
		Fitness Rank	8	3	5	6	7	4	1	1
	1000	Time Rank	1	8	7	6 2.14F - 07	2	3	0.005.00	0.005.00
	1000	Mean	2.90E+08	5.65E-16	4.54E+02	3.14E+07	1.04E+07	9.98E+03	0.00E+00	0.00E+00
		Time(s)	17.95	10230.70	1440.88	841.15	26.28	52.01	80.67	625.77
		Fitness Rank Time Rank	8 1	3 8	4 7	7 6	6 2	5 3	1 4	1
F2	100	Mean	1.14E+45	5.58E-148	2.03E-09	8.44E-04	8.66E+02	1.22E-24	0.00E+00	0.00E+00
ГΖ	100	Time(s)	752.81	2519.08	734.01	348.47	370.57	532.59	651.63	623.33
		Fitness Rank	8	3	754.01	6	7	4	1	023.33
		Time Rank	7	8	6	1	2	3	5	
	300	Mean	1.29E+130	2.26E-62	1.36E+01	6.32E+00	2.02E+04	9.86E-07	0.00E+00	0.00E+00
	500	Time(s)	1223.19	14659.60	2247.05	1936.23	1589.16	878.93	1670.29	1832.11
		Fitness Rank	8	3	6	5	7	4	1	1002.11
		Time Rank	2	8	7	6	3	1	4	
	500	Mean	1.05E+164	2.58E-35	1.44E+02	5.79E+01	8.83E+05	2.84E+16	0.00E+00	0.00E+00
		Time(s)	2010.94	13776.97	3501.35	1779.98	2669.66	2694.56	3575.00	3063.28
		Fitness Rank	8	3	5	4	6	7	1]
		Time Rank	2	8	6	1	3	4	7	4
	1000	Mean	5.76E+15	3.98E-10	1.59E+03	6.44E+02	2.39E+09	1.08E+18	0.00E+00	0.00E+00
		Time(s)	7514.75	16009.25	7239.32	3585.26	5302.06	5352.70	4131.05	5799.34
		Fitness Rank	7	3	5	4	6	8	1	1
		Time Rank	7	8	6	1	3	4	2	4
F3	100	Mean	4.36E+04	1.74E+01	7.32E-08	2.24E+00	2.21E+05	1.19E-21	0.00E+00	0.00E+00
		Time(s)	259.54	1844.19	535.13	310.23	322.51	328.64	495.33	377.94
		Fitness Rank	7	6	4	5	8	3	1	1
		Time Rank	1	8	7	2	3	4	6	5
	300	Mean	2.09E+07	2.34E+04	1.56E+03	2.47E+04	2.78E+07	1.13E-06	0.00E+00	0.00E+00
		Time(s)	780.02	6295.56	1640.89	1201.25	863.22	978.61	1296.99	1119.25
		Fitness Rank	7	5	4	6	8	3	1	1
	500	Time Rank	1	8	7	5	2	3	6	2 2 2 2 2
	500	Mean	8.52E+07	1.14E+05	3.84E+04	3.65E+05	6.18E+06	5.65E-02	0.00E+00	0.00E+00
		Time(s)	1823.73	6440.02	2697.62	2045.81	1616.01	1138.56	1339.03	1010.65
		Fitness Rank	8	5	4	6	7	3	1	1
	1000	Time Rank	5 2.21E+09	6 205 - 05	1.500.00	6	4	2 (9(E + 02	0.005.00	0.005.00
	1000	Mean	3.21E+08	6.38E+05	1.50E+06	6.30E+06	1.66E+07	6.86E+02	0.00E+00	0.00E+00
		Time(s) Fitness Rank	2966.12 8	26706.69 4	5408.11 5	4317.29 6	2119.22 7	3012.57	2305.81 1	3725.88
		Time Rank	3	8	3 7	6	1	4	2	
F4	100	Mean	8.71E+01	2.63E-148	9.22E-09	7.68E-01	2.54E+01	1.11E-22	0.00E+00	2.34E-234
17	100	Time(s)	10.82	1086.66	115.86	7.08E-01 77.10	6.51	8.06	12.09	69.00
		Fitness Rank	8	3	5	6	7	4	12.09	09.00
		Time Rank	3	8	7	6	1	2	4	4
	300	Mean	9.53E+01	2.29E-64	6.85E+00	6.28E+03	3.08E+01	1.16E-07	0.00E+00	8.95E-226
	550	Time(s)	13.83	2197.76	673.23	313.06	7.14	17.55	18.97	193.64
		Fitness Rank	7	3	5	8	6	17.55	10.77	1/5.0-

Table 7: Results of Scalability Test

unction	Dimension	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
		Time Rank	2	8	7	6	1	3	4	5
	500	Mean	9.72E+01	1.84E-39	5.52E+01	4.89E+04	3.15E+01	6.72E-03	0.00E+00	2.38E-214
		Time(s)	8.68	2924.74	1086.18	593.71	10.21	24.05	47.52	317.68
		Fitness Rank	7	3	6	8	5	4	1	2
		Time Rank	1	8	7	6	2	3	4	5
	1000	Mean	9.86E+01	7.61E-19	4.66E+02	3.15E+05	3.54E+01	9.90E+01	0.00E+00	1.16E-220
		Time(s)	10.37	6599.45	2189.51	784.09	23.12	51.56	79.31	320.46
		Fitness Rank	5	3	7	8	4	6	1	2
		Time Rank	1	8	7	6	2	3	4	5
F5	100	Mean	4.07E+02	3.03E-128	6.43E-08	8.69E-02	8.16E+02	1.38E-23	0.00E+00	0.00E+00
		Time(s)	165.08	2408.31	414.52	307.95	142.14	171.29	273.60	200.76
		Fitness Rank	7	3	5	6	8	4	1	j
		Time Rank	2	8	7	6	1	3	5	4
	300	Mean	1.35E+05	1.26E-20	3.80E+02	1.90E+03	1.17E+04	3.69E-08	0.00E+00	0.00E+00
		Time(s)	842.31	6270.31	1264.79	552.57	595.20	605.67	914.20	740.26
		Fitness Rank	8	3	5	6	7	4	1	1
		Time Rank	5	8	7	1	2	3	6	
	500	Mean	5.75E+05	8.65E-04	1.03E+04	2.58E+04	3.26E+04	3.52E-03	0.00E+00	0.00E+00
	200	Time(s)	1412.46	7125.18	2080.14	1530.79	927.43	1012.26	1235.48	1253.48
		Fitness Rank	8	3	5	6	727.43	4	1233.40	1255.40
		Time Rank	5	8	7	6	1	2	3	
	1000	Mean	3.14E+06	1.97E+01	1.82E+05	3.62E+05	1.38E+05	1.16E+02	0.00E+00	0.00E+0
	1000	Time(s)	1720.69	17176.86	4114.95	3028.92	1333.80	2025.61	1431.53	2474.72
		Fitness Rank	1720.09	3	4114.93	3028.92 7	1333.60	2023.01	1431.33	24/4./.
		Time Rank	3	8	7	6	1	4	2	
F6	100			o 2.67E-15		8.88E-16			0.00E+00	0.00E+00
го	100	Mean	7.07E-09		1.93E+09		2.95E-07	2.41E-106		
		Time(s)	390.29	7590.62	465.27	383.19	183.48	209.95	408.66	198.68
		Fitness Rank	6	5	8	4	7	3	1	1
	200	Time Rank	5	8 2.54E.00	7	4	1	3	6	0.000
	300	Mean	6.03E-03	3.54E-08	1.28E+59	8.88E-16	8.82E-06	6.75E-79	0.00E+00	0.00E+0
		Time(s)	1158.10	11463.29	1478.97	1136.27	816.13	830.36	1138.49	1025.1
		Fitness Rank	7	5	8	4	6	3	1	
		Time Rank	6	8	7	4	1	2	5	
	500	Mean	2.72E-02	3.97E-07	2.18E+137	8.88E-16	2.23E-08	1.01E-72	0.00E+00	0.00E+00
		Time(s)	1562.68	9426.44	2413.17	1739.14	1374.66	1370.46	982.74	1239.92
		Fitness Rank	7	6	8	4	5	3	1	1
		Time Rank	5	8	7	6	4	3	1	2
	1000	Mean	1.98E-01	3.00E+290	inf	8.88E-16	4.97E-08	6.01E-57	0.00E+00	0.00E+00
		Time(s)	2700.88	26362.53	4685.92	3835.83	2771.86	2807.65	2118.34	3427.74
		Fitness Rank	6	7	8	4	5	3	1	-
		Time Rank	2	8	7	6	3	4	1	:
F7	100	Mean	1.54E+03	6.73E-64	3.40E-01	6.64E+02	2.74E+03	8.36E+02	0.00E+00	6.07E+02
		Time(s)	319.38	2939.20	244.27	328.23	224.54	126.26	342.64	290.04
		Fitness Rank	7	2	3	5	8	6	1	4
		Time Rank	5	8	3	6	2	1	7	
	300	Mean	1.16E+16	3.47E+04	7.63E+13	4.19E+03	3.00E+04	7.25E+03	0.00E+00	4.36E+03
		Time(s)	939.04	4376.30	847.62	661.57	640.94	620.11	929.80	851.48
		Fitness Rank	8	6	7	2	5	4	1	
		Time Rank	7	8	4	3	2	1	6	
	500	Mean	2.23E+19	5.79E+04	1.62E+17	7.33E+03	8.29E+04	1.23E+04	0.00E+00	7.22E+0
	200	Time(s)	1552.68	5983.16	2156.84	1633.77	625.13	750.54	1344.41	1408.3
		Fitness Rank	1332.08	5985.10	7	3	6	750.54 4	1344.41	
		Time Rank	5	8	7	6	1	2	3	2
	1000		2.79E+22	1.03E+05	4.55E+20	1.51E+04	2.94E+05	3.14E+04	0.00E+00	1.56E+0
	1000	Mean	2.19E+22	1.03E±03	4.55E+20	1.51E±04	2.94E±03	J.14E±04	$0.00E \pm 00$	1.50E+04

Table 7: Results of Scalability Test

Function	Dimension	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
		Fitness Rank	8	5	7	2	6	4	1	3
		Time Rank	3	8	7	4	1	2	5	6
F8	100	Mean	3.41E+05	3.00E-66	3.19E-02	1.19E+04	2.28E+05	8.36E+04	0.00E+00	1.52E-25
		Time(s)	1930.00	5669.32	1553.35	875.01	1283.42	1367.57	1212.83	1374.31
		Fitness Rank	8	2	4	5	7	6	1	3
		Time Rank	7	8	6	1	3	4	2	5
	300	Mean	3.17E+06	1.67E+05	2.38E+05	3.98E+05	1.76E+06	1.22E+06	0.00E+00	2.80E-05
		Time(s)	11839.67	17670.98	8880.38	4825.99	4783.50	8349.41	7074.65	6134.58
		Fitness Rank	8	3	4	5	7	6	1	2
		Time Rank	7	8	6	2	1	5	4	3
	500	Mean	9.29E+06	1.75E+07	4.96E+06	1.67E+06	4.73E+06	3.90E+06	0.00E+00	4.32E-07
		Time(s)	18368.06	20186.46	15586.21	21407.55	21072.82	19527.32	17817.33	14203.77
		Fitness Rank	7	8	6	3	5	4	1	2
		Time Rank	4	6	2	8	7	5	3	1
	1000	Mean	3.26E+07	9.30E+07	1.29E+08	8.47E+06	1.77E+07	1.57E+07	0.00E+00	3.77E-12
	1000	Time(s)	76351.35	50589.75	66361.20	45854.39	51756.09	43272.60	56929.47	45467.26
		Fitness Rank	6	7	8	3	51730.05	43272.00	1	2
		Time Rank	8	4	7	3	5	1	6	2
FO	100									
F9	100	Mean	3.10E+02	2.64E+02	2.96E+02	1.64E+02	6.02E+02	1.77E+02	0.00E+00	0.00E+00
		Time(s)	20.61	999.59	123.13	119.20	11.77	17.38	28.81	31.02
		Fitness Rank	7	5	6	3	8	4	1	1
		Time Rank	3	8	7	6	1	2	4	5
	300	Mean	2.18E+03	1.14E+03	1.45E+03	7.53E+02	2.44E+03	1.42E+03	0.00E+00	0.00E+00
		Time(s)	27.90	3071.39	674.31	337.10	19.48	28.94	24.98	118.32
		Fitness Rank	7	4	6	3	8	5	1	1
		Time Rank	3	8	7	6	1	4	2	5
	500	Mean	5.12E+03	2.26E+03	1.48E+03	1.63E+03	4.42E+03	2.95E+03	0.00E+00	0.00E+00
		Time(s)	22.09	4024.56	1017.16	561.00	26.50	43.73	33.66	121.01
		Fitness Rank	8	5	3	4	7	6	1	1
		Time Rank	1	8	7	6	2	4	3	5
	1000	Mean	1.37E+04	5.43E+03	3.37E+03	5.91E+03	9.38E+03	6.98E+03	0.00E+00	0.00E+00
		Time(s)	51.59	11716.02	2154.50	727.77	47.42	56.84	57.03	218.97
		Fitness Rank	8	4	3	5	7	6	1	1
		Time Rank	2	8	7	6	1	3	4	5
F10	100	Mean	1.92E+01	2.00E+01	1.28E+00	1.43E-01	1.16E+01	2.87E-13	4.44E-16	4.44E-16
110	100	Time(s)	28.51	2127.15	228.14	92.33	13.32	21.11	37.82	83.52
		Fitness Rank	7	8	5	4	6	3	1	03.32
		Time Rank	3	8	7	6	1	2	4	5
	200									
	300	Mean	1.96E+01	2.00E+01	3.53E+00	6.66E+00	1.22E+01	1.72E-05	4.44E-16	4.44E-16
		Time(s)	35.79	1986.00	642.70	355.50	25.24	33.66	29.50	151.63
		Fitness Rank	7	8	4	5	6	3	1	1
		Time Rank	4	8	7	6	1	3	2	
	500	Mean	2.01E+01	2.03E+01	3.83E+00	1.10E+01	1.18E+01	1.15E+00	4.44E-16	4.44E-16
		Time(s)	27.35	6705.36	1111.68	592.61	33.01	46.70	37.97	328.41
		Fitness Rank	7	8	4	5	6	3	1	1
		Time Rank	1	8	7	6	2	4	3	5
	1000	Mean	2.07E+01	2.09E+01	4.33E+00	1.52E+01	1.19E+01	3.20E+00	4.44E-16	4.44E-16
		Time(s)	59.59	8052.04	2180.74	792.46	52.64	78.86	98.00	319.91
		Fitness Rank	7	8	4	6	5	3	1	1
		Time Rank	2	8	7	6	1	3	4	5
F11	100	Mean	5.06E+01	2.22E-17	8.21E-12	6.70E-02	2.78E+00	1.12E-13	0.00E+00	0.00E+00
•	~ ~	Time(s)	31.05	792.43	183.73	120.57	23.55	28.40	46.53	90.87
		Fitness Rank	8	3	5	6	7	4	1	1
		Time Rank	3	8	7	6	1	2	4	5
	200			o 4.66E-16		2.55E+00	8.73E+00			
	300	Mean	1.91E+02	4.00E-10	1.56E-02	2.33E+00	6./3E+00	2.47E-04	0.00E+00	0.00E+00

Table 7: Results of Scalability Test

Function	Dimension	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
		Time(s)	69.70	3351.32	709.85	366.91	47.71	56.63	72.30	230.70
		Fitness Rank	8	3	5	6	7	4	1	
		Time Rank	3	8	7	6	1	2	4	
	500	Mean	3.40E+02	1.38E-15	1.70E-01	1.32E+01	1.37E+01	7.68E-05	0.00E+00	0.00E+0
		Time(s)	60.36	4597.48	1051.91	638.26	57.24	59.64	136.39	372.1
		Fitness Rank	8	3	5	6	7	_ 4	1	
		Time Rank	3	8	7	6	1	2	4	
	1000	Mean	7.27E+02	1.45E-08	7.01E-01	7.93E+01	2.70E+01	1.69E-01	0.00E+00	0.00E+0
		Time(s)	160.82	7178.03	2283.78	1286.76	136.11	157.93	191.88	378.9
		Fitness Rank	8	3	5	7	6	4	1	
F10	100	Time Rank	3	8	7	6	1	2	4	2.025
F12	100	Mean	-3.32E+03	-2.82E+03	-2.50E+03	-3.92E+03	-2.70E+03	-1.55E+03	-3.81E+03	-3.92E+0
		Time(s)	591.35	5337.47	334.80	311.22	414.95	381.76	615.08	402.1
		Fitness Rank	4	5	7	2	6	8	3	
	200	Time Rank	6	8 125 . 02	2 7.51E : 02	1.165.04	5 075 - 02	3	7	
	300	Mean	-7.67E+03	-8.12E+03	-7.51E+03	-1.16E+04	-5.97E+03	-4.24E+03	-1.09E+04	-1.17E+0
		Time(s)	1759.03	7423.06	1903.20	1556.88	705.81	1274.47	872.73	1424.0
		Fitness Rank	5	4	6	2	7	8	3	
	500	Time Rank	1.025.04	1.255.04	7 1.25E : 04	5 1.97E - 04	0.105.02	3 (75E+02	1.775 - 04	1.00E+0
	500	Mean	-1.03E+04	-1.35E+04	-1.25E+04	-1.87E+04	-9.18E+03	-6.75E+03	-1.77E+04	-1.96E+0
		Time(s)	1680.95	10495.17	3082.19	1925.71	1386.75	2079.97	1447.51	1336.0
		Fitness Rank	6	4	5	2	7	8	3	
	1000	Time Rank	4 1.46E+04	8 2.74E+04	7 2.46E+04	5 2.24E+04	1.500.04	1.105.04	2.455.04	2.02E+0
	1000	Mean	-1.46E+04	-2.74E+04	-2.46E+04	-3.34E+04	-1.58E+04	-1.18E+04	-3.45E+04	-3.92E+0
		Time(s)	4708.50	26368.79	4191.13	5015.06	4143.72	4106.09	2885.12	2700.1
		Fitness Rank	7	4	5	3	6	8	2	
E12	100	Time Rank	6 2.87E 10	1 105 202	5 2.41E.06	7 1.55E 12	4 7 17E 02	3 5 OCE 45	0.005.00	0.005 . 0
F13	100	Mean	2.87E-10	1.18E-293	3.41E-06	1.55E-13	7.17E-02	5.96E-45	0.00E+00	0.00E+0
		Time(s)	12.29 6	1000.48	86.89	81.56	13.48	21.10	29.44	36.8
		Fitness Rank		3	7	5	8	4	1 4	
	200	Time Rank	1 245 : 00	8	1.025.00	6 2.24E.04	(105.00	1.550.07	· ·	0.00E+0
	300	Mean	1.24E+00	1.16E-123	1.03E+00	2.24E-04	6.19E+00	1.55E-07	0.00E+00	0.00E+0
		Time(s)	32.63 7	2376.43	313.42	184.45	23.50	26.51	55.79	100.7
		Fitness Rank		3	6	5	8	4	1	
	500	Time Rank	1.205.01	8 1 12F 76	7	6 4.55E.00	1.555.01	2.000.00	4	0.000
	500	Mean	1.28E+01	1.13E-76	1.02E+02	4.55E-02	1.55E+01	3.89E-06	0.00E+00	0.00E+0
		Time(s)	34.36	3571.98	421.67	358.88	26.47	45.91 4	58.30	192.5
		Fitness Rank	6	3	8	5	7		1	
	1000	Time Rank	2	8 C 42E 40	7	6 2.70E - 00	1 5.40E+01	3 2.41E.05	0.005.00	0.000
	1000	Mean Time(a)	9.09E+01	6.42E-40 7726.25	2.91E+03	2.79E+00	5.49E+01	3.41E-05	0.00E+00	0.00E+0 355.8
		Time(s) Fitness Rank	66.76		841.02 8	681.13	65.01	71.22	117.82 1	333.8
			7 2	3 8	8 7	5 6	6	4	4	
E14	100	Time Rank		8 2.51E-41			2.60E.24	5.36E-33	4.66E-42	4.66E-4
F14	100	Mean	1.24E-41		4.66E-42	1.10E-41	2.62E-24			
		Time(s)	507.83	2152.53	350.21	367.53	222.20 8	398.55 7	505.29	433.6
		Fitness Rank Time Rank	5 7	6 8	1 2	4 3	1	4	6	
	200				1.94E-128					1.94E-12
	300	Mean Time(s)	1.88E-114 1643.18	1.21E-127	1.94E-128 1014.32	2.09E-122 1481.20	5.16E-97	6.28E-72 1178.21	1.94E-128 1373.25	1.94E-12 1162.8
		Time(s) Fitness Rank		7054.96			657.57			1102.8
			6	4	2	5	7	8	1	
	500	Time Rank	7 5 20E 157	8 60E 200	4.60E.015	6 5 64E 102	2 27E 95	7.06E 104	5 4 47E 215	4.470.0
	500	Mean	5.30E-157	8.60E-209	4.62E-215	5.64E-192	2.37E-85	7.06E-104	4.47E-215	4.47E-21
		Time(s)	2741.59	11354.13	2997.58	2280.65	1935.71	1120.81	2323.48	2131.8
		Fitness Rank	6	4	3	5	8	7	1	
		Time Rank	6	8	7	4	2	1	5	

Table 7: Results of Scalability Test

Function	Dimension	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	1000	Mean	1.16E-213	8.41E-68	0.00E+00	0.00E+00	0.00E+00	6.75E-130	0.00E+00	0.00E+00
		Time(s)	4907.18	17610.50	5665.42	3848.17	3246.36	2213.49	4008.80	3944.58
		Fitness Rank	6	8	1	1	1	7	1	1
		Time Rank	6	8	7	3	2	1	5	4
F15	100	Mean	4.80E-01	7.73E-03	6.34E-03	3.65E-01	8.16E+01	2.13E-02	0.00E+00	3.23E-08
		Time(s)	208.79	1565.64	465.04	217.85	264.19	228.16	364.99	184.12
		Fitness Rank	7	4	3	6	8	5	1	2
		Time Rank	2	8	7	3	5	4	6	1
	300	Mean	2.47E+02	1.64E+01	2.04E+02	5.27E+01	3.88E+02	1.07E-02	0.00E+00	0.00E+00
		Time(s)	1107.84	5175.78	1394.85	1100.79	560.31	444.28	540.73	965.92
		Fitness Rank	7	4	6	5	8	3	1	1
		Time Rank	6	8	7	5	3	1	2	4
	500	Mean	7.17E+02	4.74E+01	4.40E+02	1.77E+02	3.38E+02	1.81E-02	0.00E+00	0.00E+00
		Time(s)	1838.47	6211.48	2327.62	1843.67	1295.99	1315.03	1743.18	1616.65
		Fitness Rank	8	4	7	5	6	3	1	1
		Time Rank	5	8	7	6	1	2	4	3
	1000	Mean	2.10E+03	1.41E+02	1.02E+03	7.08E+02	6.58E+02	1.10E+00	0.00E+00	0.00E+00
		Time(s)	3650.24	14997.81	4630.88	3696.85	1465.69	1437.00	1791.49	3235.66
		Fitness Rank	8	4	7	6	5	3	1	1
		Time Rank	5	8	7	6	2	1	3	4
F16	100	Mean	-8.31E+01	-5.34E+01	-4.02E+01	-8.57E+01	-4.57E+01	-8.00E+00	-9.06E+01	-2.82E+01
		Time(s)	855.81	5082.11	812.10	707.93	604.30	607.62	418.38	375.14
		Fitness Rank	3	4	6	2	5	8	1	7
		Time Rank	7	8	6	5	3	4	2	1
	300	Mean	-2.05E+02	-4.07E+01	-8.35E+01	-2.20E+02	-8.53E+01	-1.30E+01	-2.41E+02	-7.06E+01
		Time(s)	2541.31	14295.15	2454.61	1233.18	1800.00	1817.68	1233.30	1952.16
		Fitness Rank	3	7	5	2	4	8	1	
		Time Rank	7	8	6	1	3	4	2	5
	500	Mean	-2.92E+02	-5.12E+01	-1.28E+02	-2.82E+02	-1.29E+02	-1.77E+01	-3.64E+02	-1.11E+02
		Time(s)	4253.26	20391.82	3287.58	3453.88	2181.95	2966.42	3229.06	1809.69
		Fitness Rank	2	7	5	3	4	8	1	6
		Time Rank	7	8	5	6	2	3	4	1
	1000	Mean	-4.25E+02	-7.14E+01	-2.23E+02	-3.93E+02	-2.85E+02	-2.47E+01	-6.07E+02	-2.01E+02
		Time(s)	4910.91	16241.07	8069.27	5849.99	5975.85	5717.27	4097.36	5681.81
		Fitness Rank	2	7	5	3	4	8	1	6
		Time Rank	2	8	7	5	6	4	1	3

Table 8: Summary of Scalability Test

Algorithm	Avg Fitness Rank	Avg Time Rank	Overall Fitness Rank	Overall Time Rank
ABC	6.84	3.88	8	3
CMA-ES	4.42	7.91	3	8
DE	5.27	6.41	6	7
GA	4.72	4.92	4	6
PSO	6.36	2.06	7	1
SSA	4.81	2.88	5	2
VSO	1.13	3.89	1	4
WOA	1.69	4.06	2	5

3.6. Evaluation on VSO without Imported Infection

To investigate the performance of VSO without the imported infection operation, which is an additional function, another evaluation was conducted on the same set of classical and CEC benchmark functions. From Table 9 & 10, we can observe that:

Table 9: Results of Benchmarking Functions by VSO without Imported Infection

	Dimension	Mean	Std	Best	Worst	Time(s)
F1	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	10.15
F2	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	75.92
F3	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	52.46
F4	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	10.13
F5	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	35.65
F6	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	44.40
F7	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	40.06
F8	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	128.28
F9	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	11.86
F10	30	4.44E-16	0.00E+00	4.44E-16	4.44E-16	14.87
F11	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	14.94
F12	30	-1.00E+03	2.38E+01	-1.05E+03	-9.63E+02	63.79
F13	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	13.75
F14	30	3.51E-12	4.11E-26	3.51E-12	3.51E-12	58.31
F15	30	0.00E+00	0.00E+00	0.00E+00	0.00E+00	42.11
F16	30	-1.93E+01	2.73E+00	-2.34E+01	-1.48E+01	86.54
CEC1	30	1.92E+06	4.75E+05	1.13E+06	2.67E+06	11.39
CEC2	30	1.30E+04	1.42E+04	1.45E+03	3.45E+04	10.89
CEC3	30	5.25E-02	3.27E-02	9.03E-03	1.12E-01	10.93
CEC4	30	8.01E+01	3.71E+01	4.99E+00	1.42E+02	12.02
CEC5	30	2.00E+01	7.14E-02	2.00E+01	2.02E+01	11.74
CEC6	30	3.05E+01	4.08E+00	1.95E+01	3.45E+01	45.94
CEC7	30	1.52E-02	1.76E-02	1.02E-12	4.92E-02	11.85
CEC8	30	1.33E+02	2.09E+01	1.00E+02	1.65E+02	11.21
CEC9	30	1.85E+02	1.82E+01	1.64E+02	2.23E+02	11.77
CEC10	30	3.38E+03	7.10E+02	2.40E+03	4.44E+03	11.04
CEC11	30	4.41E+03	5.31E+02	3.85E+03	5.42E+03	11.50
CEC12	30	1.05E+00	3.46E-01	4.64E-01	1.67E+00	20.79
CEC13	30	5.18E-01	1.28E-01	2.80E-01	7.47E-01	10.51
CEC14	30	2.88E-01	4.45E-02	2.38E-01	3.84E-01	10.51
CEC15	30	3.22E+01	9.39E+00	1.70E+01	5.08E+01	11.10
CEC16	30	1.22E+01	4.07E-01	1.14E+01	1.29E+01	11.16
CEC17	30	9.86E+04	6.53E+04	1.58E+04	2.45E+05	12.53
CEC18	30	8.46E+03	1.06E+04	3.21E+02	2.62E+04	12.00
CEC19	30	1.68E+01	1.34E+00	1.37E+01	1.91E+01	18.34
CEC20	30	3.48E+02	7.91E+01	1.92E+02	4.86E+02	11.96
CEC21	30	4.98E+04	3.32E+04	7.41E+03	1.06E+05	11.40
CEC22	30	6.05E+02	1.20E+02	4.38E+02	7.86E+02	12.42
CEC23	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	16.59
CEC24	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	14.84
CEC25	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	15.40
CEC26	30	1.00E+02	1.64E-01	1.00E+02	1.01E+02	57.77
CEC27	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	58.32
CEC28	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	19.64
CEC29	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	22.62
CEC30	30	2.00E+02	0.00E+00	2.00E+02	2.00E+02	16.57

- Through a comparison of Table 3 and 9, the mean values of fitness by VSO without any imported infection operation are same as VSO with DE except for F12 and F16;
- As for the complicated CEC benchmark functions, the performances of two approaches are same for 5 cases, i.e. CEC23, CEC24, CEC25, CEC27, and CEC28. VSO without DE even achieves better in the cases of CEC1, CEC17, CEC29, and CEC30. For the remaining 21 functions, the VSO algorithm with the imported infection powered by DE is readily better;
- In terms of the computational time, the average time of running all 46 classical and CEC 30-D functions for VSO with and without DE are 50.84s and 26.69s, respectively. This means the introduction of such imported infection operation almost doubles the computational time;
- Taking 100-D classical benchmark functions as examples, the scalability of VSO without the imported infection mechanism is also impressive.

On the other hand, a more thorough investigation should be conducted in the future work on what specific condition(s) and how this additional operator can actually help to enhance the search performance of VSO in handling various complex real-world applications. Furthermore, other meta-heuristic algorithms can be studied as the algorithm in the imported infection operation.

Function	Dimension	Mean	Std	Best	Worst	Time(s)
F01	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	13.04
F02	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	240.49
F03	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	157.39
F04	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	12.95
F05	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	101.21
F06	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	129.01
F07	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	111.41
F08	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	634.53
F09	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	15.03
F10	100	4.44E-16	0.00E+00	4.44E-16	4.44E-16	18.14
F11	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	21.25
F12	100	-3.28E+03	4.98E+01	-3.39E+03	-3.22E+03	196.11
F13	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	20.28
F14	100	4.66E-42	1.41E-54	4.66E-42	4.66E-42	176.46
F15	100	0.00E+00	0.00E+00	0.00E+00	0.00E+00	119.50
F16	100	-4.71E+01	5.59E+00	-5.96E+01	-4.09E+01	269.93

Table 10: Results of Scalability Test by VSO without Imported Infection

4. Real-world Application I: Financial Portfolio Optimization

4.1. Problem Description

Portfolio optimization is one of the most important problems in finance. Investors usually want to maximize returns and minimize risks through allocating a fixed amount of capital into a collection of assets.

According to the mean-variance model, which is a well-known and widely-used portfolio optimization theory formulated by Markowitz [41], the variance is a risk measure. The optimization problem is presented in (14)-(15) as below.

$$\max E(R(x)) = \sum_{i=1}^{n} x_i u_i$$
 (14)

$$\min V(R(x)) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \sigma_{ij}$$
 (15)

subject to:
$$x \in X = \{x \in R \mid \sum_{i=1}^{n} x_i = 1, x_i \ge 0\}$$

where x_i is the proportion weight of the initial capital that will be allocated in the i^{th} asset, u_i is the return of the i^{th} asset, σ_{ij} stand for the covariance of returns of the i^{th} and j^{th} assets, E(R(x)) and V(R(x)) are the expected return and variance of the whole portfolio, respectively.

To optimize above two objectives simultaneously, we combine them into one single objective function as shown in (16).

$$\max SR = \frac{E(R(x)) - R_f}{V(R(x))} \tag{16}$$

where SR is called the sharpe ratio that represents the return and risk simultaneously of the portfolio, R_f is a risk-free rate.

Also, the sharpe ratio has been one of the most important measurement tools to evaluate the performance of investment portfolio in the real-world financial industry.

Since the VSO and other comparative algorithms are designated for solving minimization problems, the problem should be changed to the minimization problem as given in (17).

$$min\ fitness = \frac{1}{SR} \tag{17}$$

subject to:
$$SR = 10^{-10}$$
 if $SR \le 0$

As it is very possible that the return is zero or even negative in the financial market, a very small number 10^{-10} is assigned to the SR for this case.

In order to avoid handling the equality constraint, the solution can be converted to the unconstrained form as shown in 18.

$$x_i' = \frac{x_i}{\sum_{i=1}^n |x_i|} \tag{18}$$

4.2. Experimental Setting

Considering that the U.S. stock market is the biggest developed market and the Chinese stock market is the biggest emerging market all over the world, we select these two markets as our experimental targets. For the U.S. market, S & P500 represents 500 large companies listed on stock exchanges in the U.S. Likewise, CS I300 constituent stocks are the top 300 stocks traded on the Shanghai Stock Exchange and the Shenzhen Stock Exchange. As the lists of both S & P500 and CS I300 were adjusted from time to time, we selected the maximum number up to 250 stocks

in each group according to the order of their stock symbols to make a fair comparison. The full stocks list is illustrated in Table C.17.

Following the previous practice, the information of mean and covariance is acquired from the historical data. In the experiment, we calculated such values through the 5-year historical daily data of the candidate stocks, i.e. from 1 Jan 2015 to 31 Dec 2019 excluding non-transaction days. More specifically, the average daily return on the historical data is computed as the expected return of each stock.

We also tried different number of stocks, i.e. 10, 30, 100 and 250, to further investigate the scalability of the algorithm on this practical application. More specially, in addition to longing the stocks, Additionally, we studied a real-world scenario in which short-selling is allowed, i.e. x_i can be negative, which enlarges the searching space of the problem. The US 5-year treasury yield of 2.57% and China 3-year fixed deposit interest rate of 4.22% are performed as the risk-free rates for S & P500 and CS I300, accordingly.

We utilized the same parameters set for each algorithm as the benchmark functions test. But we set the maximum iteration as 10^3 due to a large amount of the data.

4.3. Results and Discussion

Different from the benchmark functions tests, the portfolio optimization is a maximization problem here. From Table 11-12, we can see that:

- For the group of *CS1*300, VSO achieves best in 6 cases. In particular, VSO gets the impressive sharpe ratio of 2.802 in the case of *CS1*300 Long/Short 250 stocks, which totally beat other algorithms (the second best 1.7184 is generated by ABC). In this group, the performance of SSA is also good. Regarding the case of Long 250 stocks, SSA obtains 1.8787 v.s. 1.6666 by VSO;
- For the group of *S&P*500, VSO performs best in 5 cases. Similarly, VSO acquires the result, i.e. 3.6203, that is much better than others for the case of Long/Short 250 stocks;
- Above phenomenon may imply that VSO is good at optimizing high-dimensional problems with large searching spaces;

Problem	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
CSI 300	Mean	1.224294	1.154355	1.224411	1.223784	1.181852	1.224364	1.224416	1.182489
Long 10 stocks	Std	0.000042	0.017363	0.000005	0.000555	0.021924	0.000017	0.000005	0.067856
-	Best	1.224359	1.170395	1.224418	1.224194	1.214714	1.224398	1.224423	1.224423
	Worst	1.224248	1.123408	1.224403	1.222685	1.149505	1.224348	1.224409	1.047452
	Time(s)	7.73	3.92	7.14	6.44	5.06	5.32	7.90	6.66
	Fitness Rank	4	8	2	5	7	3	1	6
	Time Rank	7	1	6	4	2	3	8	5
CSI 300	Mean	1.343552	1.294411	1.343901	1.339567	1.163706	1.344377	1.344898	1.199373
Long 30 stocks	Std	0.000223	0.012622	0.000253	0.005319	0.021636	0.000093	0.000070	0.104822
-	Best	1.343916	1.309900	1.344133	1.344161	1.192967	1.344537	1.344956	1.317878
	Worst	1.343266	1.280012	1.343485	1.329655	1.129238	1.344274	1.344801	1.039294
	Time(s)	7.76	75.79	10.74	8.55	5.05	5.45	8.02	7.83
	Fitness Rank	4	6	3	5	8	2	1	7
	Time Rank	3	8	7	6	1	2	5	4
CSI 300	Mean	1.628430	1.668617	1.418131	1.744238	1.142855	1.791720	1.777658	1.231332

Table 11: Results of Financial Portfolio Optimization

Table 11: Results of Financial Portfolio Optimization

Problem	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
Long 100 stocks	Std	0.013223	0.008872	0.023740	0.017892	0.018854	0.001025	0.011146	0.087356
	Best	1.650535	1.678654	1.447667	1.760895	1.168363	1.793024	1.790115	1.337276
	Worst	1.609472	1.654061	1.387057	1.710490	1.121737	1.790113	1.758177	1.110095
	Time(s)	247.62	558.56	406.05	338.75	189.69	213.90	221.46	310.89
	Fitness Rank	5	4	6	3	8	1	2	7
	Time Rank	4	8	7	6	1	2	3	5
CSI 300	Mean	1.270106	1.207605	1.075872	1.516896	0.924554	1.878767	1.666677	0.955822
Long 250 stocks	Std	0.023523	0.037647	0.014575	0.007608	0.011759	0.002942	0.018809	0.101130
	Best	1.296124	1.277362	1.092979	1.525166	0.940612	1.883268	1.696955	1.078950
	Worst	1.236049	1.166821	1.049480	1.505614	0.905254	1.875603	1.639783	0.850468
	Time(s)	283.03	15003.22	565.65	499.97	141.11	231.81	240.22	534.98
	Fitness Rank	4	5	6	3	8	1	2	7
	Time Rank	4	8	7	5	1	2	3	6
CSI 300	Mean	1.343369	1.337313	1.224411	1.342771	1.292502	1.344020	1.344057	1.301914
Long / Short 10 stocks	Std	0.000210	0.004408	0.000006	0.000824	0.017956	0.000048	0.000001	0.024955
	Best	1.343532	1.342048	1.224418	1.343786	1.315884	1.344052	1.344058	1.323401
	Worst	1.342999	1.329250	1.224400	1.341641	1.277836	1.343925	1.344056	1.258641
	Time(s)	4.27	5.12	7.14	6.45	5.04	5.42	7.92	6.62
	Fitness Rank	3	5	8	4	7	2	1	6
	Time Rank	1	3	7	5	2	4	8	6
CSI 300	Mean	1.717348	1.713387	1.343964	1.711504	1.445106	1.716730	1.717984	1.332379
Long / Short 30 stocks	Std	0.000273	0.001680	0.000138	0.003954	0.069596	0.000118	0.000104	0.035088
	Best	1.717570	1.714529	1.344153	1.714949	1.566683	1.716941	1.718096	1.396849
	Worst	1.716837	1.710087	1.343752	1.706446	1.368270	1.716580	1.717788	1.299481
	Time(s)	7.74	22.40	10.66	8.57	2.94	5.38	8.04	7.87
	Fitness Rank	2	4	7	5	6	3	1	8
	Time Rank	3	8	7	6	1	2	5	4
CSI 300	Mean	2.450529	2.553181	1.428805	2.541091	1.498058	2.527732	2.554013	1.331567
Long / Short 100 stocks	Std	0.010157	0.009911	0.006376	0.007563	0.143766	0.016540	0.013460	0.049987
	Best	2.466945	2.568735	1.437844	2.547923	1.713999	2.551748	2.564053	1.394569
	Worst	2.439633	2.538832	1.421629	2.527592	1.359364	2.504176	2.527447	1.285514
	Time(s)	220.08	661.09	530.60	350.70	160.47	160.98	278.92	336.33
	Fitness Rank	5	2	7	3	6	4	1	8
	Time Rank	3	8	7	6	1	2	4	5
CSI 300	Mean	1.718447	1.009201	1.075363	0.599596	0.520025	0.390616	2.802082	0.984467
Long & Short 250 stocks	Std	0.488650	1.218273	0.010703	1.142300	0.637036	0.781233	0.157465	0.124510
	Best	2.241016	2.582810	1.092765	2.882443	1.321030	1.953082	2.914946	1.191018
	Worst	0.838818	0.000000	1.061113	0.000000	0.000000	0.000000	2.495694	0.831104
	Time(s)	293.77	17083.33	760.85	500.59	192.89	231.38	288.08	550.75
	Fitness Rank	2	4	3	6	7	8	1	5
G.D. 500	Time Rank	4	8	7	5	1	2	3	6
S&P 500	Mean	1.328272	1.273155	1.328461	1.326898	1.306116	1.328419	1.328463	1.316488
Long 10 stocks	Std	0.000089	0.018868	0.000003	0.000963	0.019491	0.000014	0.000013	0.009451
	Best	1.328351	1.296125	1.328465	1.328222	1.327853	1.328437	1.328487	1.328310
	Worst	1.328109	1.239300	1.328457	1.325648	1.276763	1.328400	1.328448	1.302998
	Time(s)	7.68	3.64	7.15	6.42	4.83	5.39	4.39	6.84
	Fitness Rank	4	8	2	5	7	3	1	6
G 0 D 500	Time Rank	1 402210	1 447165	7	1 470420	1 200772	1 402060	1 404670	1 272401
S&P 500	Mean	1.483310	1.447165	1.483792	1.479429	1.298772	1.483968	1.484679	1.373401
Long 30 stocks	Std	0.000559	0.006446	0.000212	0.003340	0.051946	0.000099	0.000121	0.064840
	Best	1.484209	1.453680	1.484060	1.482190	1.360852	1.484078	1.484782	1.452699
	Worst	1.482517	1.435670	1.483520	1.472899	1.218479	1.483783	1.484444	1.261084
	Time(s)	7.77	80.05	10.72	8.46	4.93	5.64	7.98	7.74
	Fitness Rank	4	6	3	5	8	2	1	7
	Time Rank	4	8	7	6	1	2	5	3

Table 11: Results of Financial Portfolio Optimization

Problem	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
S&P 500	Mean	1.796196	1.834040	1.597832	1.904823	1.303445	1.945929	1.933929	1.478434
Long 100 stocks	Std	0.007650	0.009496	0.010696	0.015495	0.066804	0.001441	0.006118	0.079415
	Best	1.806243	1.841459	1.608517	1.927645	1.409888	1.947760	1.942193	1.629986
	Worst	1.783113	1.815415	1.580175	1.879508	1.212952	1.943413	1.923150	1.396049
	Time(s)	293.12	616.08	442.56	270.22	184.70	157.67	277.37	255.48
	Fitness Rank	5	4	6	3	8	1	2	7
	Time Rank	6	8	7	4	2	1	5	3
S&P 500	Mean	1.437987	1.375877	1.229892	1.678301	1.069901	2.023558	1.807794	1.444043
Long 250 stocks	Std	0.018157	0.029302	0.019513	0.014650	0.032413	0.006088	0.026674	0.232873
	Best	1.471751	1.409756	1.258616	1.701020	1.119266	2.030101	1.850081	1.849420
	Worst	1.419527	1.333188	1.198594	1.656095	1.018551	2.013455	1.766312	1.180427
	Time(s)	319.70	14666.58	588.99	478.51	153.36	228.73	263.82	531.59
	Fitness Rank	5	6	7	3	8	1	2	4
	Time Rank	4	8	7	5	1	2	3	6
S&P 500	Mean	1.484164	1.480492	1.328470	1.478947	1.416618	1.484277	1.484379	1.424352
Long / Short 10 stocks	Std	0.000052	0.002013	0.000017	0.007460	0.022674	0.000023	0.000012	0.033570
2,	Best	1.484250	1.482771	1.328504	1.483829	1.459236	1.484319	1.484392	1.472233
	Worst	1.484091	1.477682	1.328459	1.464090	1.390961	1.484256	1.484357	1.373205
	Time(s)	7.94	5.84	7.08	6.41	2.93	5.33	4.67	6.58
	Fitness Rank	3	4	8	5	7	2	1	6
	Time Rank	8	4	7	5	1	3	2	6
S&P 500	Mean	1.981001	1.980938	1.483602	1.974529	1.740378	1.981187	1.982164	1.581326
Long / Short 30 stocks	Std	0.000472	0.000462	0.000208	0.002906	0.038295	0.000151	0.000222	0.089208
8,	Best	1.981584	1.981570	1.483868	1.977541	1.816093	1.981377	1.982493	1.668017
	Worst	1.980143	1.980188	1.483235	1.969869	1.714468	1.980971	1.981798	1.418432
	Time(s)	4.24	21.08	10.58	8.91	4.97	5.37	7.96	4.40
	Fitness Rank	3	4	8	5	6	2	1	7
	Time Rank	1	8	7	6	3	4	5	2
S&P 500	Mean	2.866807	3.037614	1.581849	2.979065	1.995685	3.004576	3.004085	1.722598
Long / Short 100 stocks	Std	0.014769	0.004338	0.022500	0.011072	0.147366	0.004458	0.007368	0.110773
6,	Best	2.886660	3.042640	1.610707	2.990547	2.223260	3.010032	3.010575	1.905505
	Worst	2.848956	3.031767	1.556008	2.961458	1.761334	2.998492	2.990579	1.585622
	Time(s)	208.25	635.56	443.09	374.14	186.03	196.92	285.41	259.36
	Fitness Rank	5	1	8	4	6	2	3	7
	Time Rank	3	8	7	6	1	2	5	4
S&P 500	Mean	2.858795	3.292214	1.223152	3.545909	1.673160	3.253492	3.620369	1.331267
Long / Short 250 stocks	Std	0.028481	0.078095	0.004100	0.080018	0.120503	0.058260	0.015140	0.234846
01	Best	2.897627	3.388885	1.229140	3.634716	1.899798	3.344593	3.642228	1.769273
	Worst	2.811898	3.164605	1.216557	3.403792	1.558034	3.175441	3.596449	1.102636
	Time(s)	196.71	15869.66	1105.67	481.04	153.82	225.50	326.26	546.53
	Fitness Rank	5	3	8	2	6	4	1	7
	Time Rank	2	8	7	5	1	3	4	6

5. Real-world Application II: Optimization of Hyper-parameters of Support Vector Machines

5.1. Problem Description

SVMs are widely adopted machine learning algorithms particularly useful for some limited sample datasets within the framework of the statistical learning theory. According to the literature, SVMs have achieved impressive success in various applications, such as image classification [42], natural language processing [43], and financial prediction [44], etc.

Table 12: Summary of Evaluations for Financial Portfolio Optimization

Algorithm	Avg Fitness Rank	Avg Time Rank	Overall Fitness Rank	Overall Time Rank
ABC	3.94	4.06	3	3
CMA-ES	4.63	6.56	5	7
DE	5.75	6.94	6	8
GA	4.13	5.31	4	6
PSO	7.06	1.44	8	1
SSA	2.56	2.50	2	2
VSO	1.38	4.38	1	4
WOA	6.56	4.81	7	5

In practice, the performance of SVMs usually depends on its hyper-parameters. There are two major types of algorithms in SVMs: classification and regression. In this experiment, we apply SVMs to classify some real-world practical datasets.

The mathematical expressions of SVM is shown as in (19).

$$\max_{\alpha} \sum_{j} \alpha_{j} - \frac{1}{2} \sum_{j,k} \alpha_{j}, \alpha_{k} y_{j} y_{k} k(x_{j}, x_{k})$$

$$subject \ to: 0 \leq \alpha_{j} \leq C \ and \ \sum_{j} \alpha_{j} y_{j} = 0$$

$$(19)$$

where C is the tunable penalty factor and K is the kernal function. Due to the outstanding performance of RBF kernel function, it is used in this test as stated in (20).

$$k(x_j, x_k) = exp\left(-\frac{\|x_j - x_k\|^2}{2\sigma^2}\right)$$
 (20)

where σ is another tunable parameter. Using this kernel in the SVM classifier, we can get the decision function as shown in (21).

$$f(x) = \operatorname{sign}\left[\sum_{i} \alpha_{i} y_{i} exp\left(-\frac{\parallel x - x_{i} \parallel^{2}}{2\sigma^{2}}\right) + b\right]$$
 (21)

In this test, we have to optimize two hyper-parameters: the penalty factor C and σ for classification problems.

5.2. Experimental Setting

There are five datasets involved:

- Australian Credit Approval: A well-known dataset that concerns credit card applications approval in Australia [45];
- HCC Survival: HCC dataset was obtained at a University Hospital in Portugal and contais several demographic, risk factors, laboratory and overall survival features of 165 real patients diagnosed with HCC [46];
- Iris:This is perhaps the best known dataset to be found in machine learning. It is to classify type of iris plant [45];

- Somerville Happiness Survey: A dataset about life survey [47].
- Wine: This dataset is the results of a chemical analysis of wines grown in the same region in Italy but derived from three different cultivars [45];

All datasets as listed above are publicly available at [45]. As for the searching space, we set $C, \sigma \in [10^{-5}, 10^{5}]$. The accuracy of 10-fold cross validation is computed as the fitness in the evaluation. The maximum iteration is set as 500.

5.3. Results and Discussion

The detailed results are illustrated into Table 13 where the mean fitness represents the average classification accuracy of 30 runs. The findings are stated as follows.

- VSO outperforms over all other algorithms on 4 out of 5 datasets. For example, VSO achieves 83.83% of mean accuracy for the first dataset;
- The performance of CMA-ES is very bad in this test. On the other hand, ABC that performs unwell in previous benchmark functions tests becomes not bad here;
- Although VSO can get an enhancement of accuracy, it does not show a big advantage compared with other candidates in this low-dimensional problem optimization.

Table 13: Results of SVMs Optimization

Dataset	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
Australian Credit	Mean	0.8023	0.6058	0.7203	0.7119	0.7470	0.8075	0.8383	0.8203
Approval	Std	0.0159	0.0000	0.0000	0.0550	0.0628	0.0521	0.0020	0.0062
	Best	0.8203	0.6058	0.7203	0.8217	0.8246	0.8435	0.8406	0.8261
	Worst	0.7826	0.6058	0.7203	0.6812	0.6957	0.7043	0.8348	0.8101
	Time(s)	1136.98	82.23	815.05	789.18	814.23	798.28	989.86	823.65
	Fitness Rank	4	8	6	7	5	3	1	2
	Time Rank	8	1	5	2	4	3	7	6
HCC Survival	Mean	0.7196	0.4974	0.6489	0.6574	0.6985	0.7105	0.7317	0.7105
	Std	0.0126	0.0000	0.0000	0.0411	0.0482	0.0406	0.0241	0.0407
	Best	0.7316	0.4974	0.6489	0.7132	0.7438	0.7496	0.7438	0.7438
	Worst	0.6956	0.4974	0.6489	0.5875	0.6368	0.6610	0.6835	0.6603
	Time(s)	1271.23	114.26	952.46	864.63	872.66	923.54	1228.49	863.92
	Fitness Rank	2	8	7	6	5	4	1	3
	Time Rank	8	1	6	3	4	5	7	2
Iris	Mean	0.9720	0.4267	0.9733	0.9667	0.9740	0.9733	0.9773	0.9733
	Std	0.0027	0.0000	0.0000	0.0000	0.0055	0.0000	0.0068	0.0000
	Best	0.9733	0.4267	0.9733	0.9667	0.9800	0.9733	0.9867	0.9733
	Worst	0.9667	0.4267	0.9733	0.9667	0.9667	0.9733	0.9667	0.9733
	Time(s)	912.70	70.55	640.51	628.65	644.05	620.96	898.81	625.65
	Fitness Rank	6	8	3	7	2	3	1	3
	Time Rank	8	1	5	4	6	2	7	3
Somerville Happiness	Mean	0.5688	0.5114	0.5386	0.5114	0.5555	0.5386	0.5615	0.5386
Survey	Std	0.0222	0.0000	0.0000	0.0000	0.0208	0.0000	0.0126	0.0000
	Best	0.5871	0.5114	0.5386	0.5114	0.5810	0.5386	0.5810	0.5386
	Worst	0.5114	0.5114	0.5386	0.5114	0.5386	0.5386	0.5386	0.5386
	Time(s)	851.08	78.73	637.46	643.25	606.71	628.12	895.86	644.89
	Fitness Rank	1	7	4	7	3	4	2	4
	Time Rank	7	1	4	5	2	3	8	6
Wine	Mean	0.9618	0.7157	0.6513	0.9382	0.9557	0.9638	0.9644	0.9635
	Std	0.0055	0.0000	0.0000	0.0116	0.0075	0.0050	0.0024	0.0027

40

Table 13: Results of SVMs Optimization

Dataset	Metric	ABC	CMA-ES	DE	GA	PSO	SSA	VSO	WOA
	Best	0.9663	0.7157	0.6513	0.9497	0.9663	0.9663	0.9663	0.9663
	Worst	0.9497	0.7157	0.6513	0.9212	0.9497	0.9497	0.9608	0.9608
	Time(s)	925.41	108.37	654.10	632.19	649.95	637.86	911.85	626.50
	Fitness Rank	4	7	8	6	5	2	1	3
	Time Rank	8	1	6	3	5	4	7	2

Table 14: Summary of Evaluations for SVMs Optimization

Algorithm	Avg Fitness Rank	Avg Time Rank	Overall Fitness Rank	Overall Time Rank
ABC	3.4	7.8	4	8
CMA-ES	7.6	1.0	8	1
DE	5.6	5.2	6	6
GA	6.6	3.4	7	2
PSO	4.0	4.2	5	5
SSA	3.2	3.4	3	2
VSO	1.2	7.2	1	7
WOA	3.0	3.8	2	4

6. Conclusion

In summary, a novel and powerful meta-heuristic optimization algorithm called VSO is proposed for tackling challenging continuous optimization problems in many real-life applications. Inspired by the spread and behavior of viruses, the algorithm is carefully devised with different viral operations to diversify the searching strategies in order to highly improve its optimizing capacity.

In this paper, VSO is firstly evaluated on a total of 46 well-known benchmark functions covering many different types of optimization problems. The rate of convergence, scalability, and reliability of the algorithm are well-validated on all these benchmark functions. Moreover, VSO is used to solve two real-world applications including the financial portfolio optimization and optimization of hyper-parameters of SVMs for classification problems. All the obtained results are carefully compared and analyzed with those of classical algorithms such as GA, PSO, and DE as well as the state-of-the-art optimization approaches including CMA-ES, WOA, and SSA.

The results demonstrate the outstanding performance of our proposed algorithm in terms of solution fitness, convergence rate, scalability, reliability, and flexibility. Especially, VSO shows a unique potential for high-dimensional continuous optimization problems. Additionally, the algorithmic framework is much flexible to provide an interface to hybridization with other algorithms.

The drawbacks of VSO are summarized as follows. First, the number of algorithmic parameters is larger than those of the existing popular optimization approaches like GA. Second, the implementation is a bit complicated. Last but not least, the computational speed is not mostly in the dominant position due to its multiple searching strategies.

Concerning the future work, how to make the parameters of VSO to be self-adaptive is worth exploring. A more thorough investigation should be conducted on the imported infection operation. In addition, the applicability of VSO can be further investigated in various real-world applications. Lastly, VSO has a great potential to be extended for solving mixed continuous-discrete as well as multi-objective optimization problems.

Appendix A. Classical Benchmarking Functions

Table A.15: Classical Benchmarking Functions

Function	Name	Expression	Search Range	Global Optimum $f(x^*)$
F1	Sphere	$f(\mathbf{x}) = \sum_{i=1}^{D} x_i^2$	[-1000,1000]	0
F2	Brown	$f(\mathbf{x}) = \sum_{i=1}^{D-1} (x_i^2)^{(x_{i+1}^2 + 1)} + (x_{i+1}^2)^{(x_i^2 + 1)}$	[-1,4]	0
F3	Ellipsoid	$f(\mathbf{x}) = \sum_{i=1}^{D} \left[\left(1000 \frac{1}{D-1} x_i \right) \right]^2$	[-5.12,5.12]	0
F4	Schwefel 2.21	$f(\mathbf{x}) = \max_{i=1,\dots,D} x_i $	[-100,100]	0
F5	Weighted Sphere	$f(\mathbf{x}) = i \sum_{i=1}^{D} x_i^2$	[-5.12,5.12]	0
F6	Sum of Different Powers	$f(\mathbf{x}) = \sum_{i=1}^{D} x_i ^{i+1}$	[-1,1]	0
F7	Zakharov	$f(\mathbf{x}) = \sum_{i=1}^{D-1} x_i^2 + (\sum_{i=1}^{D} 0.5ix_i)^2 + (\sum_{i=1}^{D} 0.5ix_i)^4$	[-5,10]	0
F8	Schwefel 1.2	$f(\mathbf{x}) = \sum_{i=1}^{D} x_i^2 + (\sum_{i=1}^{D} 0.5ix_i)^2 + (\sum_{i=1}^{D} 0.5ix_i)^4$ $f(\mathbf{x}) = \sum_{i=1}^{D} \left(\sum_{j=1}^{i} x_j\right)^2$	[-100,100]	0
F9	Rastrigin	$f(\mathbf{x}) = 10d + \sum_{i=1}^{D} (x_i^2 - 10\cos(2\pi x_i))$	[-5.12,5.12]	0
F10	Ackley	$f(\mathbf{x}) = -20exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D}x_i^2})$	[-32,32]	0
		$-exp(\frac{1}{D}\sum_{i=1}^{D}cos(2x_i)) + 20 + exp(1)$		
F11	Griewank	$f(\mathbf{x}) = 1 + \sum_{i=1}^{D} \frac{x_i^2}{4000} - \prod_{i=1}^{D} cos(\frac{x_i}{\sqrt{t_i}})$	[-100,100]	0
F12	Styblinski-Tank	$f(\mathbf{x}) = \frac{1}{2} \sum_{i=1}^{D} (x_i^4 - 16x_i^2 + 5x_i)$	[-5,5]	-39.16599D
F13	Csendes	$f(\mathbf{x}) = \sum_{i=1}^{2} \sum_{i=1}^{D} x_i^{-1} (2 + \sin \frac{1}{x_i})$	[-1,1]	0
F14	Xin-She Yang N.2	$f(\mathbf{x}) = (\sum_{i=1}^{D} x_i) exp(-\sum_{i=1}^{D} sin(x_i^2))$	$[-2\pi, 2\pi]$	0
F15	Alpine N.1	$f(\mathbf{x}) = \sum_{i=1}^{L-1} i^{-1} x_i \sin(x_i) + 0.1x_i$	[-10,10]	0
F16	Michalewicz	$f(\mathbf{x}) = -\sum_{i=1}^{D} \sin(x_i) \sin^{20} \left(\frac{ix_i^2}{\pi} \right)$	$[0,\!\pi]$	-1.8013 (D=2)
	$D \in \{30, 100, 300, 500, 1000\}$	()		

Appendix B. CEC Benchmarking Functions

Table B.16: CEC Benchmark Functions

Function	Name	Dimension	Search Range	Global Optimum $f(x^*)$
CEC1	Rotated High Conditioned Elliptic Function	30	[-100,100]	100
CEC2	Rotated Bent Cigar Function	30	[-100,100]	200
CEC3	Rotated Discus Function	30	[-100,100]	300
CEC4	Shifted and Rotated Rosenbrocks Function	30	[-100,100]	400
CEC5	Shifted and Rotated Ackleys Function	30	[-100,100]	500
CEC6	Shifted and Rotated Weierstrass Function	30	[-100,100]	600
CEC7	Shifted and Rotated Griewanks Function	30	[-100,100]	700
CEC8	Shifted Rastrigins Function	30	[-100,100]	800
CEC9	Shifted and Rotated Rastrigins Function	30	[-100,100]	900
CEC10	Shifted Schwefels Function	30	[-100,100]	1000
CEC11	Shifted and Rotated Schwefels Function	30	[-100,100]	1100
CEC12	Shifted and Rotated Katsuura Function	30	[-100,100]	1200
CEC13	Shifted and Rotated HappyCat Function	30	[-100,100]	1300
CEC14	Shifted and Rotated HGBat Function	30	[-100,100]	1400
CEC15	Shifted and Rotated Expanded Griewanks plus Rosenbrocks Function	30	[-100,100]	1500
CEC16	Shifted and Rotated Expanded Scaffers F6 Function	30	[-100,100]	1600
CEC17	Hybrid Function 1	30	[-100,100]	1700
CEC18	Hybrid Function 2	30	[-100,100]	1800
CEC19	Hybrid Function 3	30	[-100,100]	1900
CEC20	Hybrid Function 4	30	[-100,100]	2000
CEC21	Hybrid Function 5	30	[-100,100]	2100
CEC22	Hybrid Function 6	30	[-100,100]	2200
CEC23	Composition Function 1	30	[-100,100]	2300
CEC24	Composition Function 2	30	[-100,100]	2400
CEC25	Composition Function 3	30	[-100,100]	2500
CEC26	Composition Function 4	30	[-100,100]	2600
CEC27	Composition Function 5	30	[-100,100]	2700
CEC28	Composition Function 6	30	[-100,100]	2800
CEC29	Composition Function 7	30	[-100,100]	2900
CEC30	Composition Function 8	30	[-100,100]	3000

Appendix C. Full Stocks List

Table C.17: Stocks List

				S&F	2500				
A	ALB	ATO	С	CMG	DAL	ED	FCX	GPN	HST
AAL	ALGN	ATVI	CAG	CMI	DD	EFX	FDX	GPS	HSY
AAP	ALK	AVB	CAH	CMS	DE	EIX	FE	GRMN	HUM
AAPL	ALL	AVGO	CAT	CNC	DFS	EL	FFIV	GS	IBM
ABBV	ALLE	AVY	CB	CNP	DG	EMN	FIS	GWW	ICE
ABC	ALXN	AWK	CBOE	COF	DGX	EMR	FISV	HAL	IDX
ABMD	AMAT	AXP	CBRE	COG	DHI	EOG	FITB	HAS	IEX
ABT	AMCR	AZO	CCI	COO	DHR	EQIX	FLIR	HBAN	IFF
ACN	AMD	BA	CCL	COP	DIS	EQR	FLS	HBI	ILM
ADBE	AME	BAC	CDNS	COST	DISCA	ES	FLT	HCA	INC
ADI	AMGN	BAX	CDW	COTY	DISCK	ESS	FMC	HD	INFO
ADM	AMP	BBY	CE	CPB	DISH	ETFC	FRC	HES	INTO
ADP	AMT	BDX	CERN	CPRI	DLR	ETN	FRT	HFC	INTU
ADS	AMZN	BEN	CF	CPRT	DLTR	ETR	FTI	HIG	IP
ADSK	ANET	BIIB	CFG	CRM	DOV	EVRG	FTNT	HII	IPG
AEE	ANSS	BK	CHD	CSCO	DRE	EW	GD	HLT	IPGF
AEP	ANTM	BKNG	CHRW	CSX	DRI	EXC	GE	HOG	IQV
AES	AON	BKR	CHTR	CTAS	DTE	EXPD	GILD	HOLX	IR
AFL	AOS	BLK	CI	CTL	DUK	EXPE	GIS	HON	IRM
AGN	APA	BLL	CINF	CTSH	DVA	EXR	GL	HP	ISRC
AIG	APD	BMY	CL	CTXS	DVN	F	GLW	HPE	IT
AIV	APH	BR	CLX	CVS	DXC	FANG	GM	HPQ	ITW
AIZ	APTV	BSX	CMA	CVX	EA	FAST	GOOG	HRB	IVZ
AJG	ARE	BWA	CMCSA	CXO	EBAY	FB	GOOGL	HRL	J
AKAM	ARNC	BXP	CME	D	ECL	FBHS	GPC	HSIC	JBH'
				CSI					
000001	000651	000983	002252	300144	600066	600252	600627	600900	6013
000001 000002	000651 000671	002007	002352 002385	300144	600068	600352 600362	600637	600958	6013
							600660		
800000	000686	002008	002415	300251	600085	600369	600663	600959	6013
000009	000709	002024	002424	300315	600089	600372	600674	600999	6015
000060	000718	002027	002456	600000	600100	600373	600685	601006	6016
000063	000725	002049	002465	600008	600104	600376	600688	601009	6016
000069	000728	002065	002466	600009	600109	600383	600690	601018	6016
000100	000738	002074	002470	600010	600111	600406	600703	601021	6016
000156	000750	002081	002475	600015	600115	600415	600704	601088	6016
000157	000768	002131	002500	600016	600118	600436	600705	601099	6016
000166	000776	002142	002508	600018	600150	600446	600718	601111	6016
000333	000783	002146	002555	600019	600170	600482	600737	601117	6016
000338	000792	002152	002594	600021	600177	600489	600739	601118	6016
000402	000793	002153	002673	600023	600188	600498	600741	601166	6016
000413	000826	002174	002714	600028	600196	600518	600795	601169	6017
000423	000839	002183	002736	600029	600208	600519	600804	601186	6017
000425	000858	002195	300017	600030	600221	600522	600816	601198	6017
000538	000876	002202	300024	600031	600233	600535	600820	601211	6018
000555	000895	002230	300027	600036	600256	600547	600827	601216	6018
000559	000917	002236	300033	600037	600271	600549	600837	601225	6018
000568	000938	002241	300059	600038	600276	600570	600871	601288	6018
000000	000959	002292	300070	600048	600297	600583	600886	601318	6018
	000707				600000	600505	(00007	(01220	(010
000623	000961	002299	300072	600050	600309	600585	600887	601328	6018
000623		002299 002304	300072 300124	600050 600060	600309	600585	600887	601328	6018

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