Accurate expressions for solar cell fill factors including series and shunt resistances

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Together with open-circuit voltage and short-circuit current, fill factor is a key solar cell parameter. In their classic paper on limiting efficiency, Shockley and Queisser first investigated this factor's analytical properties showing, for ideal cells, it could be expressed implicitly in terms of the maximum power point voltage. Subsequently, fill factors usually have been calculated iteratively from such implicit expressions or from analytical approximations. In the absence of detrimental series and shunt resistances, analytical fill factor expressions have recently been published in terms of the Lambert W function available in most mathematical computing software. Using a recently identified perturbative relationship, exact expressions in terms of this function are derived in technically interesting cases when both series and shunt resistances are present but have limited impact, allowing a better understanding of their effect individually and in combination. Approximate expressions for arbitrary shunt and series resistances are then deduced, which are significantly more accurate than any previously published. A method based on the insights developed is also reported for deducing one-diode fits to experimental data. © *2016 AIP Publishing LLC*.
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As shown in Fig. 1(a), many solar cells from a range of technologies[1,2] are well described by the one-diode equation relating current, I, to voltage, V, the diode ideality factor n, the thermal voltage, kT/q (equal to 0.02569257 V at 25 °C), and parasitic series and shunt resistances, R_S and R_{SH}

$$I = I_L - I_{on}\left[e^{q(V+IR_S)/nkT} - 1\right] - \frac{V + IR_S}{R_{SH}}. \quad (1)$$

Here, I_L is the photogenerated current and I_{on} is the effective diode saturation current.

For some specific values of n, an equation of the above form can be derived from device physics. However, in general, this equation should be regarded as a parametric fit to experimental data, capable of reproducing it accurately in many cases (Fig. 1(a)). For example, the apparent shunt describing the slope of the organic cell in Fig. 1(a) near short-circuit and appearing in the tabulation of fitting parameters in Table I may not be due to an actual shunt but could be a distributed series resistance effect, whereby less current is collected from regions remote from the contact as the voltage at the contact increases.

Parameters also of interest are the open-circuit voltage, V_{OC}, and short-circuit content, I_{SC}, defining the extremities of the power generation curve, with cell power output at the maximum power point, P_{MP}, given by

$$P_{MP} = V_{OC} I_{SC} FF, \quad (2)$$

where FF is the fill factor of present interest.

In general, the expression for I_{on} in terms of more accessible parameters is quite cumbersome

$$I_{on} = I_{SC}(1 + R_S/R_{SH})e^{-qV_{oc}/nkT}\left[\frac{1 - (V_{OC}/I_{SC})/(R_S + R_{SH})}{1 - e^{-q(V_{OC} - I_{SC}R_S)/nkT}}\right], \quad (3)$$

where the term in square brackets approaches unity as device quality improves. Another important parameter in theoretical work is

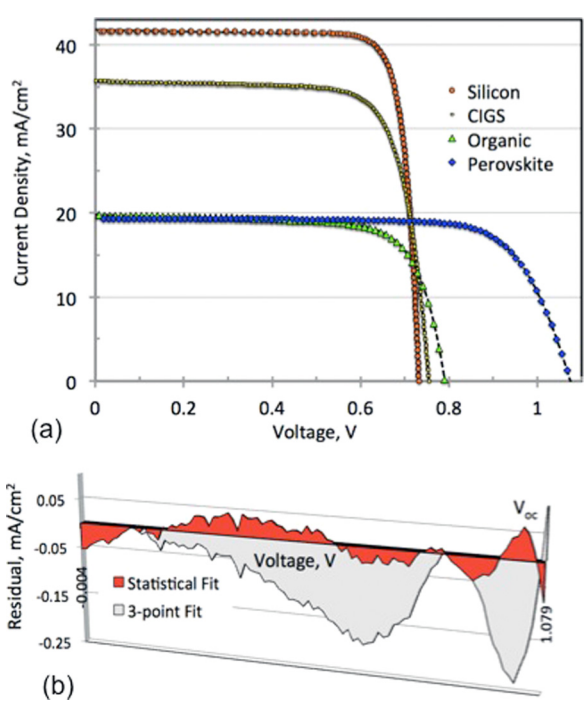

FIG. 1. (a) Experimental current–voltage curves of solar cells fabricated using a range of technologies described more fully elsewhere.[1,2] Also shown (dashed lines) is a one-diode fit to each characteristic, although mostly obscured by the experimental data points due to the quality of each fit. (b) Residuals (actual value minus fitted value) as a function of voltage for the least squares fit to the perovskite cell of (a). Also shown are the larger residuals from a simpler 3-point fit at V_{oc}, I_{sc}, and the maximum power point (CIGS: $CuIn_{1-x}Ga_xSe_2$).

TABLE I. One-diode parameters for the solar cell curves giving the lowest residual sum of squares. Normalised to the cell I_{sc}, the square root of the mean value of the squared residual is less than 1%, even for the simple 3-point fit.

Cell	Measured efficiency (%)	I_L (mA/cm^2)	n	J_{0n} (fA/cm^2)	R_S (Ω cm^2)	R_{SH} (Ω cm^2)	NRMSD[a] (%)
Silicon	25.2	41.80	1.000	17.30	0.1630	929.5	0.28
CIGS	21.0	35.81	1.494	98.45	0.5953	582.3	0.29
Organic	11.5	19.78	1.901	1,740	0.3696	717.3	0.24
Perovskite	15.6	19.37	2.098	416.7	2.923	2676.5	0.15
Perovskite[b]	15.6	19.30[b]	2.473[b]	880.4[b]	2.078[b]	∞[b]	0.56[b]

[a]NRMSD: Normalised Root Mean Square Deviation.
[b]Parameters of 3-point fit.

$$(I_L + I_{on}) = I_{SC}\left(1 + R_S/R_{SH}\right) \times \left[\frac{\left(1 - e^{-q(V_{oc} - I_{sc}R_s)/nkT}(V_{OC}/I_{SC})/(R_S + R_{SH})\right)}{1 - e^{-q(V_{OC} - I_{SC}R_S)/nkT}}\right]. \tag{4}$$

In most practical cases, the term in square brackets approaches unity to a large number of significant digits. To simplify subsequent algebra while maintaining full rigour, it is given by the symbol f.

Other parameters of interest are the maximum power point voltage and current, V_{MP} and I_{MP}. Voltage and current factors, VF and IF, can be defined as

$$VF = V_{MP}/V_{OC}; \quad IF = I_{MP}/I_{SC}. \tag{5}$$

Eq. (1) is an implicit expression for I, since I appears on both sides of the equation. Following related work by Banwell and Jayakumar,[3] Jain and Kapoor[4] have shown that I can be expressed explicitly in terms of V (and conversely) using the Lambert W function $W(x)$ defined in an inverse sense as the function satisfying the equation[5]

$$W(x)\, e^{W(x)} = x. \tag{6}$$

An asymptotic approximation to $W(x)$ for $x \geq$ e, as in the cases of present interest, is[5,6]

$$W(x) = L_1 - L_2 + \sum_{\ell=0}^{\infty}\sum_{m=1}^{\infty} \frac{(-1)^{\ell}\begin{bmatrix} \ell + m \\ \ell + 1 \end{bmatrix}}{m!} L_1^{-\ell-m} L_2^m = L_1 - L_2 + \frac{L_2}{L_1} + \frac{L_2(-2 + L_2)}{2L_1^2} + \frac{L_2\left(6 - 9L_2 + 2L_2^2\right)}{6L_1^3} + \frac{L_2\left(-12 + 36L_2 - 22L_2^2 + 3L_2^3\right)}{12L_1^4} + \cdots, \tag{7}$$

where $L_1 = \ln x$ and $L_2 = \ln(\ln x)$ (the unusual term in square brackets is a non-negative Stirling number of the first kind[7]).

Given an equation of the form

$$\ln(A + Bx) + Cx = D, \tag{8}$$

the Lambert W function gives the following solution:

$$x = -\frac{A}{B} + \frac{1}{C} W\left(\frac{C}{B} e^{\frac{AC}{B} + D}\right). \tag{9}$$

A limiting case of particular interest is when parasitic resistances are negligible ($R_S = 1/R_{SH} = 0$). In this case, analytical expressions for the above factors have been previously reported.[8] If voltages are normalized to (nkT/q) and normalized voltages indicated by lower-case symbols, such that $v_{oc} = V_{oc}/(nkT/q)$, the normalized maximum power point voltage v_{mp} is given implicitly by[9]

$$v_{mp} = v_{oc} - \ln(v_{mp} + 1). \tag{10}$$

This is in the standard form of Eq. (8). Defining $z_0 = exp(v_{oc} + 1)$, the solution for v_{mp} and hence VF becomes, in the absence of parasitic resistances (subscript 0 designates this case)

$$VF_0 = [W(z_0) - 1]/v_{oc0}. \tag{11}$$

Similarly,

$$IF_0 = \frac{v_{mp} f_0}{(v_{mp} + 1)} = \frac{[W(z_0) - 1] f_0}{W(z_0)}, \tag{12}$$

where $f_0 = (1 - e^{-v_{oc0}})^{-1}$ from Eq. (4) and equals unity to better than 4 decimal places for $v_{oc} > 10$, as is usually the case.

Finally, FF is given by

$$FF_0 = \frac{v_{mp}^2 f_0}{v_{oc0}(v_{mp} + 1)} = \frac{[W(z_0) - 1]^2 f_0}{v_{oc0} W(z_0)}, \tag{13}$$

where the exact expression in terms of the Lambert W-function is more compact than that given elsewhere.[8] The implicit solution for FF_0 in terms of v_{mp} was first derived by Shockley and Queisser[7] who also noted $(v_{mp} + 1)/f_0 = (v_{mp} + 1 - e^{-v_{mp}})$, allowing both FF_0 and v_{oc} (from Eq. (10)) to be calculated as a function of v_{mp}. In this way, they showed that FF_0 varied from 0.25 to 1.0 as v_{oc} varied from zero to infinity.[9]

The impact of series resistance can be calculated exactly in the limit of small R_S (with $1/R_{SH}$ again equal to zero) using the general relationship recently identified for small perturbations about the maximum power point of any solar cell[10]

$$\delta P/P = \delta V(I_{MP})/V_{MP} = \delta I(V_{MP})/I_{MP}. \tag{14}$$

Again, it is convenient to work in terms of normalised parameters. A suitable normalising resistance is the cell "characteristic resistance"[11–13] defined as $R_{CH} = V_{OC}/I_{SC}$, giving normalised series resistance $r_S = R_S/R_{CH}$ and

$$\delta V(I_{MP})/V_{MP} = I_{MP}R_S/V_{MP} = \frac{IF_0}{VF_0}r_s = \frac{v_{oc0}r_s f_0}{W(z_0)}. \quad (15)$$

Noting that R_S also has a small impact on I_{sc}, the final explicit fill factor expression in the presence of series resistance becomes

$$FF_s = FF_0\left[1 - \frac{v_{oc0}f_0}{W(z_0)}r_s\right]\frac{f_s}{f_0}, \quad (16)$$

where

$$f_s = [1 - e^{-v_{oc}(1-r_s)}]^{-1}, \quad (17)$$

f_s equals unity to better than 4 decimal places for $v_{oc}(1-r_s) > 10$.

Similarly, a finite shunt resistance R_{SH} in the absence of series resistance gives

$$\delta I(V_{MP})/I_{MP} = (V_{MP}/I_{MP})/R_{SH} = \frac{VF_0}{IF_0}\frac{1}{r_{sh}} = \frac{W(z_0)}{v_{oc0}r_{sh}f_0}. \quad (18)$$

Noting that R_{SH} has a small impact on V_{OC}, the explicit expression for the fill factor in the presence of shunt resistance becomes

$$FF_{sh} = FF_0\left[1 - \frac{W(z_0)}{f_0\,v_{oc0}}\frac{1}{r_{sh}}\right]\Big/\left(1 - \frac{1}{v_{oc0}r_{sh}}\right). \quad (19)$$

Since $IF_0 > VF_0$, a given value of r_s will have a stronger effect than the same value of $1/r_{sh}$. If both series and shunt resistances are present, a two-step deviation from the ideal curve can be imagined, with the final result depending on the order the deviation is implemented with FF given by

$$FF = FF_0\frac{f_s}{f_0}\left\{1 - \frac{IF_0}{VF_0}r_s - \frac{VF_0}{IF_0}\frac{1}{r_{sh}} + \frac{r_s}{r_{sh}} \times \left[2 - Min.\left(\frac{IF_0}{VF_0}r_s\,,\ \frac{VF_0}{IF_0}\frac{1}{r_{sh}}\right)\right]\right\}\Big/\left(1 - \frac{1}{v_{oc0}r_{sh}}\right). \quad (20)$$

The above expressions are exact in the limit of small R_S and large R_{SH} but lose accuracy as parasitic resistance losses increase. Approximately 4 digit accuracy is maintained in technologically interesting cases, where losses are <5% for normalised $\nu_{oc} > 10$. Since the above approach is based on calculating power output for points on the actual I-V curve, the above expressions can never overestimate FF.

For larger losses, alternative approaches are required. A situation of practical interest is where R_S effects are large, while R_{SH} effects are negligible such as for cells under concentrated sunlight. Green[11,12] showed that FF in this case could be determined iteratively as

$$FF_s = IF[1 - IFr_s + \ln(1 - IF/f_s)/v_{oc})]|_{\max}, \quad (21)$$

where the maximum is determined for $0 < IF < 1$. The broad maximum reduces the accuracy required in determining the IF value maximizing Eq. (21) by over an order of magnitude compared to the accuracy required in FF.

In terms of normalised currents $i = I/I_{SC}$, the relation for the maximum power point parameters becomes

$$v_{mp} + i_{mp}v_{oc}r_s = v_{oc} - \ln(v_{mp} - i_{mp}v_{oc}r_s + 1). \quad (22)$$

This has the solution for ν_{mp} in terms of i_{mp} as

$$v_{mp} - i_{mp}v_{oc}r_s = W(z_s) - 1 \quad \text{where } z_s = e^{1+v_{oc}(1-2i_{mp}r_s)}. \quad (23)$$

Note that this relationship remains implicit. Substituting Eq. (22) into Eq. (1) gives the following relationship for i_{mp} and hence IF_s:

$$IF_s = i_{mp} = \frac{(v_{mp} - i_{mp}v_{oc}r_s)f_s}{(v_{mp} - i_{mp}v_{oc}r_s + 1)} = \frac{[W(z_s) - 1]f_s}{W(z_s)} = \left[1 - \frac{1}{W(z_s)}\right]f_s. \quad (24)$$

This retains the same general form as when $r_s = 0$, although the argument of the Lambert W function has changed. The final form of Eq. (24) also involves only i_{mp} as an unknown, allowing this parameter to be found iteratively, if desired.

Since the first term in the square brackets of Eq. (24) is much larger than the second, this reduces the accuracy required in determining $W(z_s)$ by about an order of magnitude compared to that required for IF_s and by about 2 orders of magnitude to that required for FF. Since i_{mp} is close to unity, a reasonable approximation for z_s is z'_s given by

$$z'_s = \exp[1 + v_{oc}\{1 - 2f_s\,[1 - 1/W\,(e^{1+v_{oc}(1-1.8r_s)})]\}]. \quad (25)$$

This equation uses $i_m = 0.9$ as a general purpose estimate, a better choice than unity since more accurate than for all but the very best cells, where corrections required are small. Using z'_s in Eq. (24) allows a reasonable estimate of IF_s and an even better estimate of FF_s.

This approach gives an accuracy to about 4 decimal places for $\nu_{oc} > 10$ even when fill factor loss due to R_S is as much as 40%, far more accurate than earlier approximations.[13–15] For example, for the most severe case tabulated elsewhere[14] ($\nu_{oc} = 15$, $r_s = 0.2$, $\nu_{oc}r_s = 3$), the above approach gives a value of FF of 0.600759, exact to 6 decimal places, compared to the value of 0.6006 calculated by the most accurate method previously assessed.[15] If higher accuracy is required, the value of i'_{mp} deduced using z'_s can be used to form the next approximation z''_s for z_s, with such iteration continued until the calculated value of FF_s is stable to the required number of digits.

If finite but large R_{SH} is also included, the perturbative approach previously described can be used to estimate the impact, taking into account the changes in both V_{OC} and I_{SC} in this case. For more extreme values, Jain and Kapoor[4] have shown how the Lambert W function allows FF to be determined with a single level of iteration by solving the equation that, in present terminology, becomes

$$FF = IF[(f - IF)(r_s + r_{sh}) - W(z_{ff})/v_{oc}]|_{\max}, \quad (26)$$

where

$$z_{ff} = f_s v_{oc}(r_s + r_{sh} - 1)\exp[v_{oc}\,(f(r_s + r_{sh}) - IF\,r_{sh} - 1)]. \tag{27}$$

In this case, the condition satisfied by the maximum power point parameters is

$$v_{mp} + i_{mp}\,v_{oc}\,r_s = v_{oc} - \ln(v_{mp} - i_{mp}\,v_{oc}\,r_s + 1) + \ln\left[\frac{f(r_s + r_{sh}) - 2\,v_{mp}/v_{oc}}{f(r_s + r_{sh}) - 1}\right], \tag{28}$$

where f is close to unity, defined by Eq. (4).

Eq. (28) has a formal solution

$$v_{mp} - i_{mp}\,v_{oc}\,r_s = W(z) - 1 \quad \text{where} \quad z = \left[\frac{f(r_s + r_{sh}) - 2\,v_{mp}/v_{oc}}{f(r_s + r_{sh}) - 1}\right] e^{1 + v_{oc}\,(1 - 2 i_{mp}\,r_s)}. \tag{29}$$

In this more general case

$$IF = \left(1 + \frac{r_s}{r_{sh}}\right)\left[f - \frac{W(z) - 2}{v_{oc}(r_s + r_{sh})}\right] \Big/ \left(\frac{W(z)}{W(z) - 1} + \frac{2r_s}{r_{sh}}\right). \tag{30}$$

Again, IF and even more so FF are not strongly dependent on the accuracy of $W(z)$ estimates. The additional problem in the present case is that both i_{mp} and v_{mp} are needed to evaluate z. However, a good approximation to $W(z)$ can be found by expanding the final logarithmic term in Eq. (30) to first order in $(1/r_{sh})$. This approximate equation becomes exact as $1/r_{sh} \to 0$ and has the solution

$$v_{mp} - i_{mp}\,v_{oc}\,r_s = \frac{W(z')}{1 + 2/\left[(r_s + r_{sh})v_{oc}f\right]} - 1, \tag{31}$$

where

$$z' = \left[1 + \frac{2}{(r_s + r_{sh})v_{oc}f}\right]\exp\left[1 + \frac{2}{(r_s + r_{sh})v_{oc}f} + v_{oc}(1 - 2 i_{mp} r_s)\left(1 + \frac{1}{(r_s + r_{sh})v_{oc}f}\right)\right]. \tag{32}$$

z' involves only i_{mp} where a general purpose estimate of 0.9 again is a good choice, allowing $W(z'_s)$ to be estimated and hence $W(z) \approx W(z')/\{1 + 2/[(r_s + r_{sh})f]\}$ and IF from Eq. (30). Inserting into Eq. (26) then allows FF calculation to a high degree of accuracy. For a challenging example[12] discussed in the past literature ($v_{oc} = 17.3$, $r_s = 0.2$, $r_{sh} = 5$), this approach gives FF of 0.556460 compared to the actual value of 0.556465, far more accurate than the previously suggested approaches. As r_{sh} increases, the algorithm becomes identical to that used for the infinite r_{sh} case.

Again, since based on calculating the power output of a point on the actual I-V curves as close as possible to the maximum power point, the above approaches always underestimate the fill factor. If the Lambert W function is not available as a pre-programmed function, the final expansion of Eq. (7) allows fill factor calculation to an accuracy better than 1 part per 10^6 for $v_{oc} > 10$. The advantage of the present expressions is that they avoid the iteration required to solve Eq. (26) as well as giving more insight into underlying relationships.

As an example, it is shown how they can be used to derive one diode parameters in the common case, such as for commercial or reasonable performance laboratory devices, where shunt effects are negligible under rated conditions, and similarly, the f_s parameter is unity. Normally, parameters V_{oc}, I_{sc}, V_{mp}, and I_{mp} are known, either from measurements or data sheets, allowing IF_s and VF_s to be calculated.

From Eq. (24), knowing IF_s, $W(z_s)$ is calculated as $1/(1 - IF_s)$ and then z_s from Eq. (6), giving one equation for v_{oc} and r_s from Eq. (23)

$$v_{oc}(1 - 2IF_s r_s) = \ln(z_s) - 1. \tag{33}$$

A second relationship follows from the first part of Eq. (24)

$$v_{oc}(VF_s - IF_s\,r_s) = IF_s/(1 - IF_s). \tag{34}$$

This gives

$$r_s = \frac{IF_s - (1 - IF_s)\left(\ln(z_s) - 1\right)VF_s}{IF_s\left[1 + IF_s - (1 - IF_s)\ln(z_s)\right]}, \tag{35}$$

with v_{oc} given by either Eqs. (34) or (35), allowing its value to be found. I_{on} and I_L are then given by Eqs. (3) and (4) with the bracketed terms unity.

Applying this procedure to the perovskite cell of Fig. 1(a) with $V_{oc} = 1.074$ V, $I_{sc} = 19.30$ mA/cm^2, $V_{mp} = 0.8687$ V, and $I_{mp} = 17.93$ mA/cm^2 gives $IF = 0.9290$, $VF = 0.8088$, and $FF = 0.7514$. Following the above procedure gives $W(z_s) = 14.085$, $z_s = 18\,431\,698$, $\ln(z_s) = 16.730$, $r_s = 0.03734$, $R_S = 2.078\,\Omega\,\text{cm}^2$, $v_{oc} = 16.903$, $\frac{nkT}{q} = 63.54$ mV, $n = 2.473$, $I_{on} = 0.8804$ pA/cm^2, and $I_L = 19.30$ mA/cm^2.

This procedure involves deducing one-diode parameters by fits at V_{oc}, I_{sc} and the maximum power point. As shown in Fig. 1(b), fitting errors are zero at these points but, although elsewhere tolerable since the fitted curve is visually indistinguishable from the experimental curve, are larger than the least squares error fit (Fig. 1(b)), largely due to the neglect of shunt resistance. However, the fit is more accurate near the maximum power point reproducing, as it must, the experimental fill factor (Table II), determined iteratively from Eq. (26). The least squares fit has more balanced residuals and

TABLE II. Comparison of fill-factor (FF) calculations for the perovskite solar cell. Values deduced from two different one-diode fits using different approaches are compared with the experimental value.

Source	Fill factor
Experimental	0.7514
Exact (S-fit[a])	0.7490
Approx. 1 [S-fit; Eq. (20)]	0.7471
Approx. 2 [S-fit; Eq. (31)]	0.7490
Exact (3-pt fit)	0.7514
Approx. 1 [3-pt fit, Eq. (16)]	0.7512
Approx. 2 [3-pt fit, Eq. (24)]	0.7514

[a]S-fit: Statistical one-diode fit.

also gives a reasonably accurate value of the fill factor (Table II). The simple expressions of Eqs. (16) and (20) give accurate estimates, even though this is the most challenging case for them in terms of accuracy for the four devices shown. The more complex non-iterative approach gives results indistinguishable from exact solutions.

Hence, the recommended procedure for determining accurate solar cell fill factors for one-diode models is to iteratively solve Eq. (26) (or the simpler Eq. (21) if shunt resistance is not an issue). For practical cells with relatively small resistive losses (less than 5% in total), the explicit Eq. (20) can be used with confidence (or the simpler Eq. (16) if shunt resistance is negligible). For more general cases where an accurate explicit solution is required, finding W(z) from W(z′) where z′ is given by Eq. (22) and inserting into an Eq. (30), then Eq. (26) gives accurate results (or the simpler combination of Eq. (25) with Eqs. (24) and (21) when shunt resistance is negligible).

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