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A global optimization algorithm inspired in the behavior of selfish herds

Fernando Fausto, Erik Cuevas*, Arturo Valdivia, Adrián González

Departamento de Electrónica, Universidad de Guadalajara, CUCEI, Av. Revolución 1500, Guadalajara, Jal, Mexico

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ABSTRACT

In this paper, a novel swarm optimization algorithm called the Selfish Herd Optimizer (SHO) is proposed for solving global optimization problems. SHO is based on the simulation of the widely observed selfish herd behavior manifested by individuals within a herd of animals subjected to some form of predation risk. In SHO, individuals emulate the predatory interactions between groups of prey and predators by two types of search agents: the members of a selfish herd (the prey) and a pack of hungry predators. Depending on their classification as either a prey or a predator, each individual is conducted by a set of unique evolutionary operators inspired by such prey-predator relationship. These unique traits allow SHO to improve the balance between exploration and exploitation without altering the population size. To illustrate the proficiency and robustness of the proposed method, it is compared to other well-known evolutionary optimization approaches such as Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Firefly Algorithm (FA), Differential Evolution (DE), Genetic Algorithms (GA), Crow Search Algorithm (CSA), Dragonfly Algorithm (DA), Moth-flame Optimization Algorithm (MOA) and Sine Cosine Algorithm (SCA). The comparison examines several standard benchmark functions, commonly considered within the literature of evolutionary algorithms. The experimental results show the remarkable performance of our proposed approach against those of the other compared methods, and as such SHO is proven to be an excellent alternative to solve global optimization problems.

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1. Introduction

The intelligent collective behavior of many species of animals and insects, have attracted the attention of researches for many years. Many animal species such as birds, ants, and fishes, which live in social animal groups such as flocks, colonies and schools respectively, exhibit particular aggregative conducts widely known as swarm behavior. Such collective phenomenon has been studied by entomologist in order to model the behavior of the many biological swarms. Computer science researchers have studied and adapted these models as frameworks for solving complex real-world problems, giving birth to a branch of artificial intelligence commonly addressed as swarm intelligence [REF]. As a result of this, many unique swarm optimization algorithms, which mimic the collective behavior of groups of animals or insects, have been developed to solve a wide variety of optimization problems. Some of these methods include well know state-of-the-art techniques such as Particle Swarm Optimization (PSO,) which emulates the social behavior of bird flocking and fish schooling (Kennedy and Eberhart, 1995), Artificial Bee Colony (ABC), which is based on the cooperative behavior of bee colonies (Karaboga and Basturk, 2008), Firefly Algorithm (FA) which mimics the mating behavior of firefly insects (Yang, 2010), and Cuckoo Search (CS), which draws inspiration from the cuckoo bird lifestyle (Rajabioun, 2011). Although most of these methods are widely used to solving complex optimization problems, they are known to suffer from some serious flaws, such as premature convergence and the difficulty to overcome local optima (Wang et al., 2011; Xiang and An, 2013), which prevent them from finding optimal solutions. The cause of such issues is usually related to the operators used to modify each individual's position. In the case of PSO, for example, the position of each search agent for the next iteration is updated yielding an attraction towards the best particle position seen so-far, while in the case of ABC, positions are updated with respect of some other randomly chosen individuals. As the algorithm evolves, those behaviors allow the entire population to, either rapidly concentrates around the current best particle or to diverge without control, which in return favors the premature convergence or a misbalance between exploration and exploitation respectively (Wang et al., 2013; Banharnsakun et al., 2011). In addition, most state of the art swarm algorithms only model

* Corresponding author.
E-mail address: erik.cuevas@cucei.udg.mx (E. Cuevas).

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individual entities that perform virtually the same behavior. Under such circumstances, the possibility of adding new and selective operators based on individual unique characteristics (such as task-responsibility, strength, size, sex, etc.) that could improve several important algorithm characteristics such as population diversity and searching capabilities.

While it is true that a wide range of organisms living in aggregations show distinctive cooperative behaviors, this is not true for every single animal species living in social units. In contrast to the popular hypothesis that social behavior is based on mutual benefits for the entire population, the widely accepted selfish herd theory proposed by William D. Hamilton in 1971 illustrates that actions among individuals within aggregations (referred as herds) exhibit an unusual degree of selfishness, particularly when members of such aggregations are endangered by the presence of predators (Hamilton, 1971). In fact, the selfish herd theory establishes that decisions made by any member of such herds do not only benefit the individual itself but also, in exchange, there are usually some negative repercussions for other members on said aggregation.

In this paper, a novel swarm optimization algorithm called Selfish Herd Optimizer (SHO) is proposed for solving optimization problems. Such algorithm is inspired in the behaviors described on Hamilton's selfish herd theory. The algorithm considers two different kinds of search agents: predators and prey. Each of these agents movements are conducted by a set of unique rules and operators based the observed natural behavior of individuals on a selfish herd while they are endangered by a pack of hungry predators. The rest of this paper is organized as follows: in Section 2, we address the selfish herd theory, as proposed by Hamilton (1971); in Section 3, we illustrate our proposed swarm optimization approach (SHO); in Section 4, we summarize all steps the proposed SHO algorithm; in Section 5, we open a discussion about SHO and its most distinctive traits in comparison to other similar methods; in Section 6, we illustrate our experimental setup and results; finally, in Section 7, conclusions are drawn.

2. The selfish herd theory

The selfish herd theory, as proposed by Hamilton (1971), is an antithesis to the common view of gregarious behavior as a way of seeking mutual benefits among members of a population or group of organism. In his paper, Hamilton proposed that gregarious behavior may be considered a form of cover-seeking, in which each individual attempts to reduce their chance of being caught by a predator. It is also stated that, during a predator attack, individuals within a population will attempt to reduce their predation risk by putting other conspecifics between themselves and the predator(s). The basic principle governing the selfish herd theory is that, in aggregations, predation risk increases among individuals in the periphery and decreases toward the center of such aggregation. It is also proposed that more dominant (stronger) animals within the population are easily able to obtain low-risk central positions among the aggregation, whereas subordinate (weaker) animals are usually forced into higher risk positions.

Hamilton illustrated his theory by modeling a circular lily pond in which a population of frogs (a group of prey) and a water snake (a predator) are sheltered. Upon appearance of the water snake, it is supposed that the frogs will scatter to the rim of the pond and that the water snake will most certainly attack the one nearest to it. In this model Hamilton suggests that the predation risk of each frog is related not only to how close they are from the attacking predator, but also with the relative position of all other frogs on the pond. Under these considerations, Hamilton proposes that each frog has a better chance of not being closest to, and thus, vulnerable to attack by the water snake, if other frogs are between them. As a result, modeled frogs attempt to reduce their predation risk by jumping to smaller gaps between other neighboring frogs in an attempt to use them as a "shield". Hamilton also went on to model two-dimensional predation by considering lions as examples. He proposed that movements which would lower an individual's domain of danger are largely based on the theory of marginal predation, which states that predators attack the closest prey (which are typically those at the periphery of an aggregation). From this, Hamilton suggested that, in the face of predation, there should be a strong movement of individuals toward the center of an aggregation. Research has also revealed that there exist several factors which may influence chosen movement rules, such as initial spatial position, population density, the predator's attack strategy, and vigilance. In particular, it has been observed that individuals holding initially central positions are more likely to be successful at remaining in the center of the aggregation, increasing their chances of surviving a predator attack (Morrell et al., 2010).

The selfish herd theory may also be applied to the situation of group escape, in which, the safest position, relative to predation risk, is not the central position but rather that in the front of the herd. In this sense, members at the back of the aggregation have the greatest domain of danger, and thus, suffer the highest predation risk. As the most likely targets for predation, these slower members must choose whether to stay with the herd, or to desert it, which may in turn entice the pursuit of the predator to such vulnerable individuals. This strategy, formally known as herd desertion, is mainly used by slower individuals among the aggregation in an attempt to escape from the sight of predators, although this in turn may signal their vulnerability and thus promote the predators to pursuit such individual (Eshel et al., 2011).

By considering this, it could be assumed that the escape route chosen by members in front of the herd may be greatly affected by the actions of the slowest members. For example, if the herd's leader chooses an escape route that promotes the dispersal of the slowest members of the group it may endanger itself due to the dissipations of its protective buffer. Also, it is known that the leader's chosen escape strategy is often affected by terrain particularities (Eshel et al., 2011).

Many examples of selfish herd behavior have been witnessed in nature. One of the most extensively studied examples is that of aggregations of fiddler crab, in which, dispersed groups are more likely to form an aggregate when subjected to a danger while at the same time individual members attempt to move toward the center of the forming group (Viscido and Wethey, 2002). Other selfish herd behavior examples include that of mammals living in open plains, such as wildebeest and zebras (which aggregations are likely associated with predation risk reduction), many species of fishes (such as minnows, which school to reduce their individual predation risk) (Orpwood et al., 2008), the Adelie Penguins (which frequently wait to jump into the water until they have formed an aggregate to form protective buffers against seal predation) (Alcock, 2001), and the Forest Tent Caterpillar (famous for foraging in groups as a strategy to reduce predation risk) (Mcclure and Despland, 2010).

3. The selfish herd optimizer algorithm

The selfish herd theory establishes that, in the face of predation, each individual within a herd of possible prey pursues to increases their chance of survival by aggregating with other conspecifics in ways which could potentially increase their chances of surviving a predator attack without regard of how such behavior affects other individuals' chances of survival (Hamilton, 1971).

In this paper, the selfish herd behavior particularities observed by several groups of organism has been used as guidelines for developing a new swarm optimization algorithm known as the Selfish

Herd Optimizer (SHO). The proposed method assumes that the entire search space is an open plain where groups of animals interact. This algorithm models two different types of search agents: a herd of prey living in aggregation (also referred as a selfish herd) and a pack of predators which hunts for the prey within said aggregation. Both kinds of search agents are individually conducted by a set of different evolutionary operators based on the unique behavioral aspects observed in such prey-predator relationship.

3.1. Initializing the population

Similar to other evolutionary algorithms, SHO is an iterative process which first step is to randomly initialize a population of animals (prey and predators). The algorithm starts by initializing a set $\mathbf{A}$ of N individual positions $\mathbf{a}_i$ ($\mathbf{A} = \{\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_N\}$), with N representing the total population size. Each of such positions is represented as an n-dimensional vector $\mathbf{a}_i = \left[a_{i,1}, a_{i,2}, \ldots, a_{i,n}\right]$, which further represents a possible solution for a given optimization problem. These positions are initialized by considering a random uniform distribution between a pair of pre-specified parameter bounds as given as follows:

$$\begin{aligned} a^0_{i,j} &= x^{\text{low}}_j + \text{rand}(0,1)\cdot(x^{\text{high}}_j - x^{\text{low}}_j) \\ i &= 1,2,\ldots,N; \quad j = 1,2,\ldots,n \end{aligned} \tag{1}$$

where x^{low}_j and x^{high}_j represent the decision space's lower and upper bounds, respectively. Furthermore, i and j denote the individual and parameter indexes corresponding to each animal, respectively. Also, rand $(0, 1)$ stand for randomly generated number, drawn from within the interval [0,1].

The selfish herd theory establishes a series of distinctive behaviors, resulting from the interaction between a group of prey and predators. With this in mind, SHO employs two different types of search agents: a group of prey (known as a herd while in an aggregation) and a group of predators (collectively known as a pack). With that being said, the entire population of animals $\mathbf{A}$ is divided in two sub-groups: a group $\mathbf{H} = \left\{\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_{N_h}\right\}$ formed by all individuals which belong to the herd of prey (with N_h denoting the number of individuals of $\mathbf{H}$) and a group $\mathbf{P} = \left\{\mathbf{p}_1, \mathbf{p}_2, \ldots, \mathbf{p}_{N_p}\right\}$ represented by the members of the pack of predators (with N_p denoting the number of individuals of $\mathbf{P}$), and such that $\mathbf{A} = \mathbf{H} \cup \mathbf{P}$ ($\mathbf{A} = \{\mathbf{a}_1 = \mathbf{h}_1, \mathbf{a}_2 = \mathbf{h}_2, \ldots, \mathbf{a}_{N_h} = \mathbf{h}_{N_h}, \mathbf{a}_{N_h+1} = \mathbf{p}_1, \mathbf{a}_{N_h+2} = \mathbf{p}_2, \ldots, \mathbf{a}_N = \mathbf{p}_{N_p}\}$). In nature, the number of animals within a herd of prey usually outnumbers those within most packs of predators. In SHO, the number of prey (herd's size) N_h is randomly selected within a range of between 70 and 90% of the total population N, while the remainder individuals are labeled as predators. As such, N_h and N_p are calculated by the following equations:

$$N_h = \text{floor}\,(N \cdot \text{rand}\,(0.7, 0.9)) \tag{2}$$

$$N_p = N - N_h \tag{3}$$

where rand $(0.7, 0.9)$ denotes a random number from within the interval [0.7, 0.9], while floor $(\cdot)$ maps a real number to an integer number. Furthermore, the number of predators (pack's size) N_p is simply computed as the complement between the total population N and the herd's size N_h.

3.2. Survival value assignation

In the biological metaphor of a common prey-predator interaction, each individual within a herd of prey or a pack of predator, depending on their survival capabilities, has a chance of surviving an attack or succeed on killing an animal, respectively. In the proposed approach, each animal $\mathbf{a}_i$ (irrespective of it being a prey or a predator) is assigned with a survival value $\text{SV}_{\mathbf{a}_i}$ which represents its survival aptitude (solution quality) relative to its current position within the solution space. In SHO, it is assumes that each animal's survival value $\text{SV}_{\mathbf{a}_i}$ is related to both the safest and the riskiest positions currently known by all members of the population, which in the context of a global optimization problem are represented by the current best and worst solutions found so far by the optimization process. As such, the survival value assigned to of each individual animal is calculated as follows:

$$\text{SV}_{\mathbf{a}_i} = \frac{f(\mathbf{a}_i) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}} \tag{4}$$

where $f(\mathbf{a}_i)$ denotes the fitness value corresponding to the evaluation of the objective function $f(\cdot)$ with regard to the individual's position $\mathbf{a}_i$, while f_{best} and f_{worst} stand for the best and worst fitness values found so far by the SHO algorithm's evolutionary process which, considering a maximization problem, are defined as follows:

$$f_{\text{best}} = \max_{j\in\{0,1,\ldots,k\}}\left(\left(\max_{i\in\{1,2,\ldots,N\}}(f(\mathbf{a}_i))\right)_j\right) \tag{5}$$

$$f_{\text{worst}} = \min_{j\in\{0,1,\ldots,k\}}\left(\left(\min_{i\in\{1,2,\ldots,N\}}(f(\mathbf{a}_i))\right)_j\right) \tag{6}$$

with k denoting the current iteration of SHO's evolutionary process.

3.3. Structure of a selfish herd

In most selfish herds, a dominant member, known as the herd's leader, may be identified. Such individual distinguishes itself from the rest of the herd as the individual with the greatest survival aptitudes. During a predator attack, the herd's leader performs an important task in choosing the escape route or strategy to be employed by all other members of the herd, and as such, its leadership behavior heavily influences the movement patterns of the entire aggregation (Eshel et al., 2011). On the other hand, it is known that animals among an aggregation of prey will try to reduce their predation risk by putting other conspecifics between them and the attacking predators (Hamilton, 1971). Intuitively, since a given individual's chance of surviving a predator attack is related to its relative position among the herd aggregation, such selfish behavior is also necessarily related to the herdís current internal structure and movement patterns.

By considering this, SHO models several distinctive decision making behaviors by first dividing the herd's population with regard to three distinctive roles: 1. a herd leader tasked to guide the movement of the prey aggregation; 2. a group of herd followers which guide their moves by considering the positions and survival aptitudes of other herd members; and 3. a group of herd deserters which move independently of other prey individuals.

3.3.1. Leader of a selfish herd

The leader of a selfish herd is the strongest, wisest and most capable individual for survival among the members of the herd, and as such has an important role on guiding the movement of all of its conspecifics. The herd's leader is usually the individual whose position inside the herd aggregation promotes it to have the highest chances of surviving a predator attack (Eshel et al., 2011). Analogous to this, at each iteration k, the SHO algorithm designates a single individual $\mathbf{h}_i$ among the herd's population ($\mathbf{H}$) as the leader of the selfish herd. Such individual (designated as $\mathbf{h}_L$) is chosen by considering the current survival values possessed by each individual within the herd's aggregation as follows:

$$\mathbf{h}^k_L = \left(\mathbf{h}^k_i \in \mathbf{H}^k | \text{SV}_{\mathbf{h}^k_i} = \max_{j\in\{1,2,\ldots,N_h\}}\left(\text{SV}_{\mathbf{h}^k_j}\right)\right) \tag{7}$$

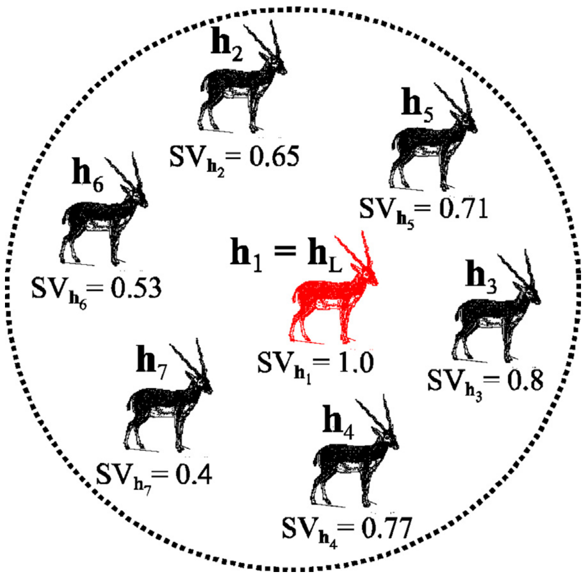


Fig. 1. The individual $\mathbf{h}_i$ holding the highest survival value from among all other members within the herd's population is designated as the herd's leader ($\mathbf{h}_L$).

In other words, the prey individual $\boldsymbol{h}_i^k$ (with $i \in [1, 2, \ldots, N_h]$) possessing highest survival value among all other members of $\mathbf{H}^k$ (with k denoting the current iteration) is assigned as the herd's leader (see Fig. 1).

3.3.2. Nearest best neighbors within a selfish herd

As previously stated, individuals within a selfish herd aims to increase their chances of surviving a predator attack by putting other members of the herd between them and the attacking predators (Hamilton, 1971). In this sense, individuals within a herd of prey move toward closer conspecifics among the aggregation, which they could potentially use to protect themselves from the attacking predators. Intuitively, in order to motivate such movement pattern, such closer individuals must first possess a relatively safer position respective to the predator's location. In this context, at each iteration k, SHO considers that the nearest best neighbor to any prey individual within the selfish herd is that which possess two important traits: 1. it is the nearest herd member to $\mathbf{h}_i$ (other than the herd's leader $\mathbf{h}_L$, as illustrated in Section 33.31); and 2. it has better survival aptitudes than said individual $\mathbf{h}_i$ (see Fig. 2). By considering this, the nearest best neighbor of $\mathbf{h}_i$ may be defined as follows:

$$\boldsymbol{h}_{c_i}^k = \left(\begin{array}{c} \mathrm{h}_j^k \in \boldsymbol{H}^k, \boldsymbol{h}_j^k \neq \left[\mathrm{h}_i^k, \boldsymbol{h}_L^k\right] | \mathrm{SV}_{\boldsymbol{h}_j^k} > \mathrm{SV}_{\boldsymbol{h}_i^k}, r_{i,j} \\ = \min\limits_{j \in \{1,2,\ldots,N_h\}} \left(\|\mathbf{h}_i^k - \mathbf{h}_j^k\| \right) \end{array} \right) \tag{8}$$

where $r_{i,j}$ denotes the Euclidean distance between the indexed herd members i and j ($\boldsymbol{h}_i^k$ and $\boldsymbol{h}_k^k$ respectively), and with k denoting the current iteration number.

3.3.3. Herd followers and herd deserters

Perhaps the most interesting behaviors observed in selfish herds is the decision taken by its members for either following the group's movement or to desert the aggregation and move independently of it (Eshel et al., 2011). The decision making criteria behind these behaviors is strongly related to the degree of safety experimented by each individual among the herd during a predators attack, which in turn is also related to each individual's relative position among the herd. By considering this, the SHO algorithm models a set of unique individual decision making operators which consider the individual survival capabilities of each member of the herd. Such

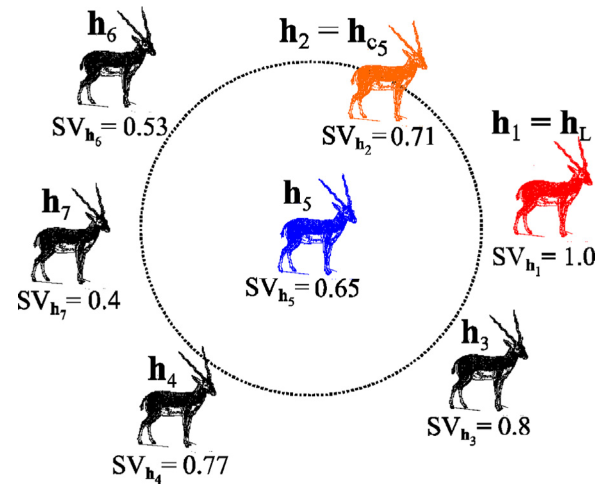


Fig. 2. The nearest best neighbor h_{c_i} to any given prey individual $\mathbf{h}_i$ (in this case illustrated as $\mathbf{h}_5$) is the herd member $\mathbf{h}_j$ (different to the herd's leader $\mathbf{h}_L$) which is both closer to $\mathbf{h}_i$ and that possess a higher survival value than $\mathbf{h}_i$ (represented as $\mathbf{h}_2$ in this case).

behaviors are modeled by further dividing the herd's population $\mathbf{H}$ in two subgroups: 1. a group of herd followers ($\mathbf{H}_F$), formed by those members who opt to follow the aggregation and 2. a group of herd deserters ($\mathbf{H}_D$), which comprises all prey individuals which decided to move independently of other members of the herd. In SHO, these two groups are defined for each iteration k as follows:

$$\boldsymbol{H}_F^k = \{\mathrm{h}_i^k \neq \boldsymbol{h}_L^k | \mathrm{SV}_{\boldsymbol{h}_i^k} \geq \mathrm{rand}(0, 1)\} \tag{9}$$

$$\boldsymbol{H}_D^k = \{\mathrm{h}_i^k \neq \boldsymbol{h}_L^k | \mathrm{SV}_{\boldsymbol{h}_i^k} < \mathrm{rand}(0, 1)\} \tag{10}$$

where $\mathbf{H}_F^k$ denotes the groups of herd followers whereas $\mathbf{H}_D^k$ stands for the group of herd deserters. Furthermore, rand (0, 1) denotes a random number from the interval [0,1].

In other words, each individual $\mathbf{h}_i$ on a selfish herd (other than the herdís leader $\mathbf{h}_L$ as illustrated in Section 3.3.1) is grouped in either $\mathbf{H}_F$ or $\mathbf{H}_D$ depending on its current survival value $\mathrm{SV}_{\mathbf{h}_i}$. In this sense, it is clear that for any given prey individual $\mathbf{h}_i \neq \mathbf{h}_L$, having a higher survival value $\mathrm{SV}_{\mathbf{h}_i}$ yields to higher chances of following the herd, whereas lower values increases the chance of deserting such aggregation (see Fig. 3).

Furthermore, prey individuals within $\mathbf{H}_F$ may be further divided into a set of dominant herd members ($\mathbf{H}_d$) and a set of subordinate herd members ($\mathbf{H}_s$) depending on their current survival capabilities. With this in mind, SHO divides the members of $\mathbf{H}_F$ as either dominant or subordinate members as follows:

$$\boldsymbol{H}_d^k = \{\mathrm{h}_i^k \in \boldsymbol{H}_F^k | \mathrm{SV}_{\boldsymbol{h}_i^k} \geq \mathrm{SV}_{\boldsymbol{H}_\mu^k}\} \tag{11}$$

$$\boldsymbol{H}_s^k = \{\mathrm{h}_i^k \in \boldsymbol{H}_F^k | \mathrm{SV}_{\boldsymbol{h}_i^k} < \mathrm{SV}_{\boldsymbol{H}_\mu^k}\} \tag{12}$$

where $\mathrm{SV}_{\boldsymbol{H}_\mu}$ represents the mean survival value of the herd's aggregation $\mathbf{H}$ as defined as follows:

$$\mathrm{SV}_{\boldsymbol{H}_\mu^k} = \frac{\sum_{i=1}^{N_h} \mathrm{SV}_{\boldsymbol{h}_i^k}}{N_h} \tag{13}$$

In other words, individuals $\mathbf{h}_i$ belonging to $\mathbf{H}_F$ are classified within $\mathbf{H}_d$ (dominant members) if their survival value $\mathrm{SV}_{\mathbf{h}_i}$ is equal to or greater than the mean survival value $\mathrm{SV}_{\mathbf{H}_\mu}$ of the entire herd aggregation, otherwise, they grouped within $\mathbf{H}_s$ (subordinated members). As stated in Eshel et al. (2011), in most cases, dominant members (individuals with higher survival aptitudes) are more frequently able to secure safer position within the herd aggregation

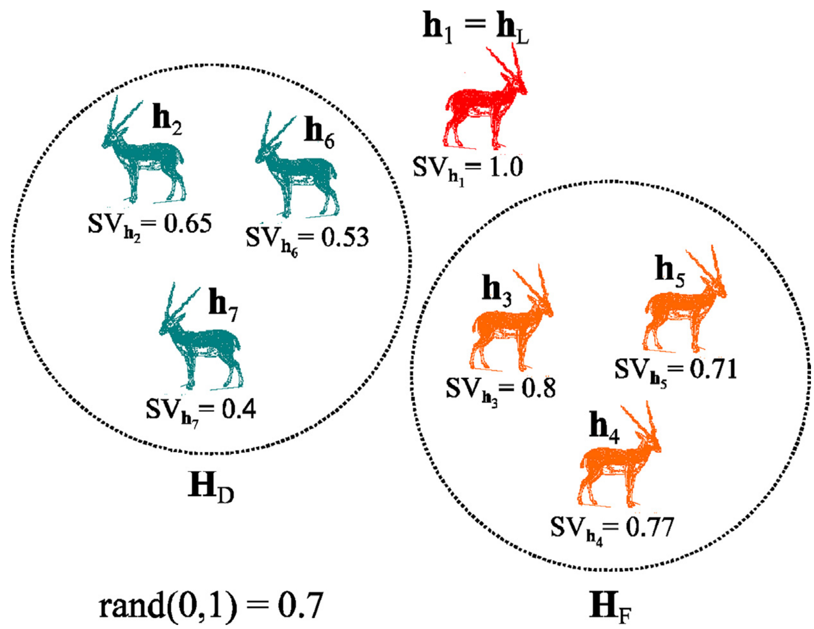


Fig. 3. Designation of herd followers ($\mathbf{H}_F$) and herd deserting members ($\mathbf{H}_D$) for a given value rand (0, 1).

while subordinate members (individuals with lower survival aptitudes) are usually forced to assume higher risk positions. This fact is specially considered in SHO to model several different movement rules, which depend on the survival value of each member currently following the herd aggregation.

3.3.4. Relative safer and riskier positions

In most cases, the predation risk of individuals within a selfish herd increases among individuals in the herd's periphery (where there are fewer animals to use as protection) and decreases toward the center of such aggregation. With this in mind, the SHO algorithm considers the existence of a relatively safer central position within the herd of prey. In this approach such location is given by the herd's population center of mass, as defined as follows:

$$\boldsymbol{h}_M^k = \frac{\sum_{i=1}^{N_h} \mathrm{SV}_{\boldsymbol{h}_i^k} \cdot \boldsymbol{h}_i^k}{\sum_{j=1}^{N_h} \mathrm{SV}_{\boldsymbol{h}_j^k}} \tag{14}$$

While the previous is useful to illustrate the location of a potentially safe location within a selfish herd, such assumption does not consider the presence of predators as the main source of danger to any individual within such aggregation. In a similar manner to Eq. (14), it is possible to define a position of relatively higher risk by considering both, the current locations and survival aptitudes of the attacking predators as follows:

$$\boldsymbol{p}_M^k = \frac{\sum_{i=1}^{N_p} \mathrm{SV}_{\boldsymbol{p}_i^k} \cdot \boldsymbol{p}_i^k}{\sum_{j=1}^{N_p} \mathrm{SV}_{\boldsymbol{p}_j^k}} \tag{15}$$

Furthermore, since both, $\mathbf{h}_M$ and $\mathbf{p}_M$ represent potential solutions within the solution space of a given optimization problem, a corresponding survival value $\mathrm{SV}_{\mathbf{h}_{cm}}$ and $\mathrm{SV}_{\mathbf{p}_{cm}}$ may be assigned to each of such positions by applying Eq. (4) (see Fig. 4).

3.4. Herd movement operators

In order to model the movement of each individual within a selfish herd, SHO considers two different sets of evolutionary operators: 1. a set of herd's leader movement operators, and 2. a set of herd's following and desertion movement operators. Such movement operators consider important characteristics shared by all members of a selfish herd, such as individual survival values and the distance to other members of the aggregation, in order to accurately model the movement patterns manifested by such prey aggregations.

3.4.1. Selfish attraction and repulsion

When endangered by the presence of one of more predators, individuals within a selfish herd pursue to improve their chances of surviving a predator attack by moving in a way which could allow them to put other conspecifics between them and the attacking predators. This behavior could be effectively modeled as an attraction toward other individuals among the herd aggregation. Analogous to this, SHO assumes that each individual within the herd's population $\mathbf{H}$ (as illustrated in Section 3.1) is able to manifest a certain degree of attraction toward other members within such aggregation. This attraction depends on both, the relative distance toward a given individual and the current survival value possessed by said individual. By considering this, we may define an attraction factor experienced by a given herd member $\mathbf{h}_i$ toward any different member $\mathbf{h}_j$ as follows:

$$\psi_{\boldsymbol{h}_i,\boldsymbol{h}_j} = \mathrm{SV}_{\boldsymbol{h}_j} \cdot e^{-\|\mathbf{h}_i-\boldsymbol{h}_j\|^2} \tag{16}$$

Where $\mathrm{SV}_{\mathbf{h}_j}$ denotes for the survival value related to the herd member $\mathbf{h}_j$, whereas $\|\mathbf{h}_i - \boldsymbol{h}_j\|$ stands for the Euclidian distance between the prey individuals $\mathbf{h}_i$ and $\mathbf{h}_j$.

In this paper, the factor $\psi_{\boldsymbol{h}_i,\boldsymbol{h}_j}$ is known as selfish attraction. While it is possible to compute such value for virtually any pair of prey individuals, four specific relationships are of particular importance within the proposed approach:

1. The selfish attraction $\varphi_{\boldsymbol{h}_i,\boldsymbol{h}_L}$, which represents the attraction experimented by $\mathbf{h}_i$ toward the current herd's leader $\mathbf{h}_L$ (as illustrated in Section 3.1.1). This value represents the influence exerted by the herd's leader which is responsible of guiding the movements of all other members following the herd aggregation. Such attraction value is defined as follows:

$$\psi_{\boldsymbol{h}_i,\boldsymbol{h}_L} = \mathrm{SV}_{\boldsymbol{h}_L} \cdot e^{-\|\mathbf{h}_i-\boldsymbol{h}_L\|^2} \tag{17}$$

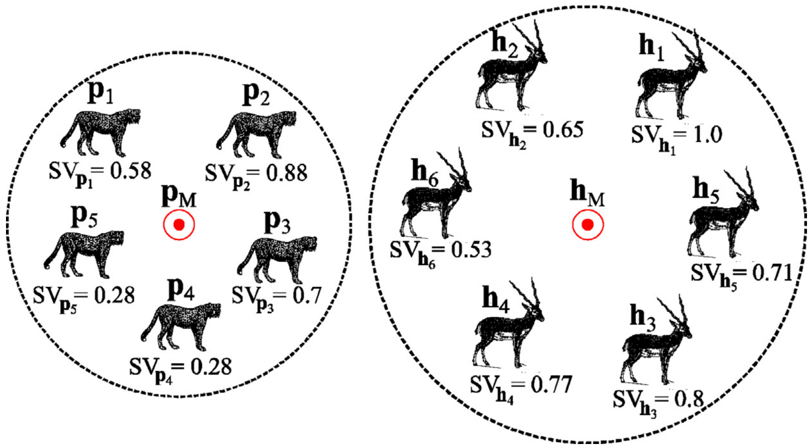


Fig. 4. Relatively safer and riskier positions represented by the herd's center of mass $\mathbf{h}_M$ and the predator's center of mass $\mathbf{p}_M$ respectively.

2. The selfish attraction $\varphi_{\boldsymbol{h}_i,\boldsymbol{h}_{c_i}}$ representing the attraction factor experimented by $\mathbf{h}_i$ toward its nearest best neighbor $\mathbf{h}_{c_i}$ (as illustrated in Section 3.1.2). Such attraction value is given as:

$$\psi_{\boldsymbol{h}_i,\boldsymbol{h}_{c_i}} = \mathrm{SV}_{\boldsymbol{h}_{c_i}} \cdot e^{-\|\mathbf{h}_i - \boldsymbol{h}_{c_i}\|^2} \quad (18)$$

3. The selfish attraction $\varphi_{\boldsymbol{h}_i,\boldsymbol{h}_{cm}}$ denoting the attraction experienced by $\mathbf{h}_i$ toward the herd's center of mass $\mathbf{h}_M$ (as illustrated in Section 3.1.4). Such attraction is expressed as:

$$\psi_{\boldsymbol{h}_i,\boldsymbol{h}_M} = \mathrm{SV}_{\boldsymbol{h}_M} \cdot e^{-\|\mathbf{h}_i - \boldsymbol{h}_M\|^2} \quad (19)$$

Furthermore, while the previous allows to represent the attraction experimented by $\mathbf{h}_i$ toward other herd members, it is also useful to define an attraction with respect to the safest position currently known by the whole aggregation as follows:

$$\psi_{\boldsymbol{h}_i,\boldsymbol{x}_{\text{best}}} = e^{-\|\mathbf{h}_i - \boldsymbol{x}_{\text{best}}\|^2} \quad (20)$$

where $\mathbf{x}_{\text{best}}$ stands for the best position found so far by during SHO's evolutionary process, which satisfies that:

$$f(\mathrm{x}_{\text{best}}) = f_{\text{best}} \quad (21)$$

with $f(\mathrm{x}_{\text{best}})$ denoting the fitness value corresponding to the evaluation of the objective function $f(\cdot)$ with regard to $\mathbf{x}_{\text{best}}$ and where f_{best} stands for the best fitness value found so far, as given by Eq. (5).

Furthermore, while Eq. (16) represents an attraction factor toward other individuals within the selfish herd, it is important to remember that predators represent the main source of danger, and as such the movement patterns of individuals within such aggregation are also subject of being influenced by their presence. By considering this, SHO also assumes that individuals within the selfish herd are also able to experience some degree of repulsion toward the pack of attacking predators. To model such behavior, a repulsion factor may be defined as follows:

$$\varphi_{\boldsymbol{h}_i,\boldsymbol{p}_M} = -\mathrm{SV}_{\boldsymbol{p}_M} \cdot e^{-\|\mathbf{h}_i - \boldsymbol{p}_M\|^2} \quad (22)$$

where $\mathrm{SV}_{\mathbf{p}_M}$ denotes the survival value related to the predators' center of mass $\mathbf{p}_M$ as given by Eq. (15).

In SHO, the value $\varphi_{\boldsymbol{h}_i,\boldsymbol{p}_M}$ is known as selfish repulsion. Moreover, while it is possible to define a repulsion value between any given pair of prey and predator individuals, SHO only considers the repulsion experimented toward the predators' mass $\mathbf{p}_M$ under the assumption that any prey individual will always try to get as far as possible from all attacking predators.

3.4.2. Herd's leader movement operators

Predation risk is the main motivation behind the individual decision-making behavior manifested by the members of a selfish herd. In this sense, while the leader of the herd is known to hold the safest position within the herd aggregation (and thus the highest chances of surviving a predator attack) this does not necessarily means that such individual is completely safe from the predators. With that being said, the leader of the selfish herd is able to manifest several different types of leadership behaviors depending on its current survival value. In SHO, the herd's leader position for the next iteration is updated as follows:

$$\boldsymbol{h}_L^{k+1} = \begin{cases} \mathrm{h}_L^k + \boldsymbol{c}^k & \text{if}\mathrm{SV}_{\boldsymbol{h}_L^k} = 1 \\ \boldsymbol{h}_L^k + \boldsymbol{s}^k & \text{if}\mathrm{SV}_{\boldsymbol{h}_L^k} < 1 \end{cases} \quad (23)$$

where k denoting the current iteration number.

The movement rule chosen by the herd's leader assuming that $\mathrm{SV}_{\mathbf{h}_L}^k = 1$ is called seemingly cooperative leadership. Such movement is performed under the assumption that the herd's leader $\mathbf{h}_L^k$ is located on either the current best location know so far by the selfish herd or an equally good position, thus granting it the highest possible survival value. In this sense, it can be shown from Eq. (4) that:

$$\text{iff}(\mathrm{h}_L^k) = f_{\text{best}} \rightarrow \mathrm{SV}_{\boldsymbol{h}_L}^k = 1 \quad (24)$$

It is important to recall that the movement of all other members within a selfish herd is strongly influenced by the decisions taken by the herd's leader. In this sense, a seemingly cooperative leadership aims to guide the movement of all other herd members in a way that could be potentially beneficial to the whole aggregation. In general, this is achieved by moving away of the pack of attacking predators (see Fig. 5). With that being said, the movement vector $\mathbf{c}^k$ could be expressed as:

$$\boldsymbol{c}^k = 2 \cdot \alpha \cdot \varphi_{\boldsymbol{h}_L,\boldsymbol{p}_M}^k \cdot (\mathrm{p}_M^k - \boldsymbol{h}_L^k) \quad (25)$$

where $\varphi_{\boldsymbol{h}_L,\boldsymbol{p}_M}^k$, as illustrated by Eq. (22), denotes the selfish repulsion experimented by the current herd's leader $\mathbf{h}_L^k$ toward the predators' center of mass $\mathbf{p}_M^k$ as given by Eq. (15), whereas α stand for random number within the interval [0,1].

On the other hand, if $\mathrm{SV}_{\mathbf{h}_L^k} < 1$, the herd's leader choses the movement rule known as openly selfish leadership. In the biological metaphor of the selfish herd behavior, each individual belonging to a selfish herd (including the herd's leader) will try at all cost to reduce their predation risk by moving to relatively safer positions. In an effort to improve its current survival value, the leader

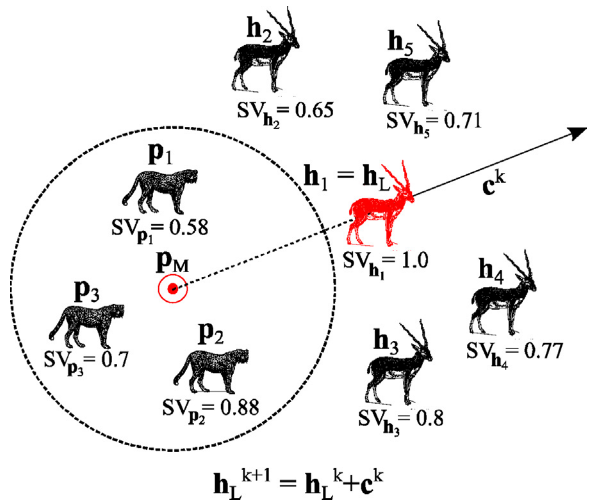


Fig. 5. Seemingly cooperative movement rule. Under such circumstance the herd's leader $\mathbf{h}_L$ (represented by $\mathbf{h}_1$) "altruistically" guides the herd aggregation away from the pack of attacking predators (represented by the predator's mass $\mathbf{p}_M$).

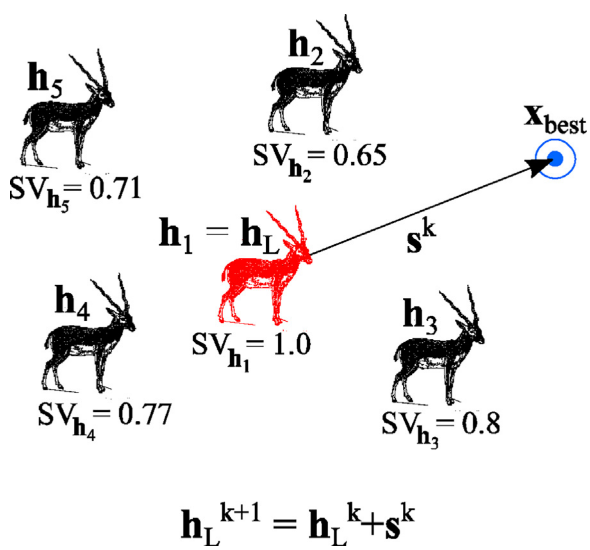


Fig. 6. Openly selfish movement rule. In such a case, the herd's leader $\mathbf{h}_L$ (shown as $\mathbf{h}_1$ in this case) aims to improve its own survival value by moving toward the currently known safest location (represented as $\mathbf{x}_{\text{best}}$).

of the herd will opt to move toward the safest position currently known by the herd's aggregation (see Fig. 6). By considering this, the movement vector $\mathbf{s}^k$ may be defined as:

$$\mathbf{s}^k = 2 \cdot \alpha \cdot \psi^k_{\boldsymbol{h}_L,\boldsymbol{x}_{\text{best}}} \cdot (\mathrm{x}^k_{\text{best}} - \boldsymbol{h}^k_L) \tag{26}$$

where $\psi^k_{\boldsymbol{h}_L,\boldsymbol{x}_{\text{best}}}$, as in Eq. (20), denotes the selfish attraction experienced by the herd's leader $\boldsymbol{h}^k_L$ toward the global best position $\boldsymbol{x}^k_{\text{best}}$ found so far during SHO's evolutionary process, whereas α stand for random number from within the interval [0,1].

From what was previously illustrated, the herd's leader selection criteria for a given movement rule may be summarized as follows: For any given iteration, if the leader of the herd $\mathbf{h}_L$ is the best individual among the members of the entire herd, the seemingly cooperative leadership movement rule is chosen; otherwise the openly selfish leadership movement rule is applied.

3.4.3. Herd's following and desertion movement operators

As illustrated in Section 3.3.3, depending on the current survival values of each individual within the selfish herd, SHO classifies the members of such aggregation in two distinctive groups: a group of herd followers ($\mathbf{H}_F$), and a group of herd deserters ($\mathbf{H}_D$). In SHO, the movement patterns manifested by the selfish herd are entirely dependent on the role assumed by each of its members. By considering this, at each iteration k, SHO computes the position update for each herd member as follows:

$$\boldsymbol{h}^{k+1}_i = \begin{cases} \mathrm{h}^k_i + \boldsymbol{f}^k_i & \text{if} \mathrm{h}^k_i \in \boldsymbol{H}^k_F \\ \boldsymbol{h}^k_i + \boldsymbol{d}^k_i & \text{if} \mathrm{h}^k_i \in \boldsymbol{H}^k_D \end{cases} \tag{27}$$

where $\mathbf{H}^k_F$ and $\mathbf{H}^k_D$ denote the sets of herd following and herd deserting members, as illustrated by Eqs. (9) and (10), respectively.

The movement rule chosen by the herd member $\mathbf{h}^k_i$, assuming that it belongs to the group of herd following members ($\mathbf{h}^k_i \in \mathbf{H}^k_F$), is called herd following rule. This movement rule assumes that the herd member $\mathbf{h}^k_i$ opts to follow other members within the aggregation in an attempt to improve its own survival value, and as such, $\mathbf{f}^k_i$ denotes a movement vector computed with regard to the positions and survival aptitudes of other members within the herd. Furthermore, as previously stated, members within $\boldsymbol{H}^k_F$ may be further identified as either dominant members ($\mathbf{H}_d$) or subordinate members ($\mathbf{H}_s$), depending on how higher or lower their survival values are with respect to the mean survival value of the entire herd population. With this in mind, in SHO, the herd's following movement $\mathbf{f}^k_i$ performed by each herd member within $\mathbf{H}^k_F$ is calculated depending on whether it is a dominant or a subordinate member, as illustrated as follows:

$$\boldsymbol{f}^k_i = \begin{cases} 2 \cdot (\beta \cdot \psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_L} \cdot (\mathrm{h}^k_L - \boldsymbol{h}^k_i) + \gamma \cdot \psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_{c_i}} \cdot (\mathrm{h}^k_{c_i} - \boldsymbol{h}^k_i)) & \text{if} \mathrm{h}^k_i \in \boldsymbol{H}^k_d \\ 2 \cdot \delta \cdot \psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_M} \cdot (\mathrm{h}^k_M - \boldsymbol{h}^k_i) & \text{if} \mathrm{h}^k_i \in \boldsymbol{H}^k_s \end{cases} \tag{28}$$

where $\mathbf{H}^k_d$ and $\mathbf{H}^k_s$ denote the sets of dominant and subordinate herd members, as given by Eqs. (11) and (12), respectively, whereas the values β, γ, and δ each represent a random number drawn from the interval [0,1].

The movement rule performed by the herd's dominant members ($\mathbf{H}^k_d$) is known as the nearest neighbor movement rule, which represent the situation in which the herd member $\mathbf{h}^k_i$ considers the positions of both, its nearest best neighbor and the current herd's leader when deciding where to move (Hamilton, 1971). With that being said, $\mathbf{h}^k_L$ and $\mathbf{h}^k_{c_i}$ stands for the position of the current herd's leader and the nearest best neighbor to $\mathbf{h}^k_i$, respectively, both as defined by Eqs. (7) and (8). Furthermore, $\psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_L}$ and $\psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_{c_i}}$, as given by Eqs. (17) and (18), denote the selfish attractions experimented by the herd member $\mathbf{h}^k_i$ toward $\mathbf{h}^{it}_L$ and $\mathbf{h}^{it}_{C_i}$ respectively (see Fig. 7).

On the other hand, the movement rule chosen by the herd's subordinate members ($\mathbf{H}^k_s$) is known as crowded horizon movement. In this situation, each herd member considers both the location and survival values of all other members within the aggregation to guide its movement (Viscido et al., 2002). In SHO, such movement is performed by considering the herd's center of mass $\mathbf{h}^k_M$, as defined by Eq. (14), and its corresponding selfish attraction $\psi^k_{\boldsymbol{h}_i,\boldsymbol{h}_M}$, as given by according to Eq. (19) (see Fig. 8).

Finally, if the herd member $\mathbf{h}^k_i$ is instead identified as a herd deserting member ($\mathbf{h}^k_i \in \mathbf{H}^k_D$), said individual performs movement rule known in SHO as herd desertion. In this case, individuals grouped as herd deserting members are assumed to move independently to all other members of the herd (Eshel et al., 2011), and as such $\mathbf{d}^k_i$ denotes a movement performed without regard to any other individuals within the aggregation (see Fig. 9). By consider-

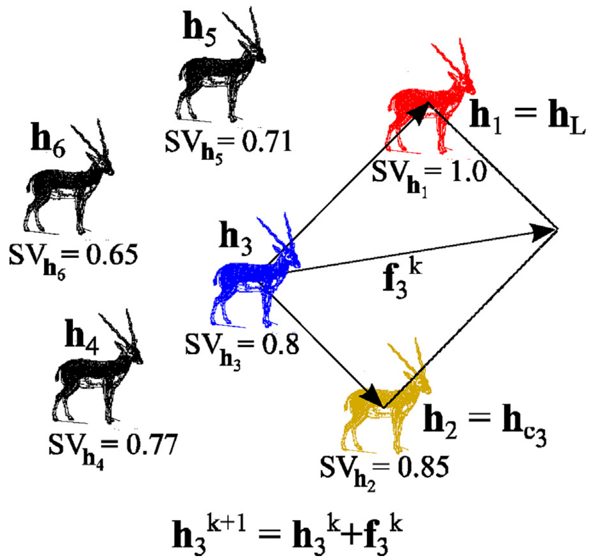


Fig. 7. Nearest neighbor movement rule for a dominant herd member (illustrated as $\mathbf{h}_3$). In this case, the dominant member considers the positions of both, its nearest best neighbor $\mathbf{h}_{c_3}$ (shown as $\mathbf{h}_2$) and the current herd's leader $\mathbf{h}_L$ (represented by $\mathbf{h}_1$).

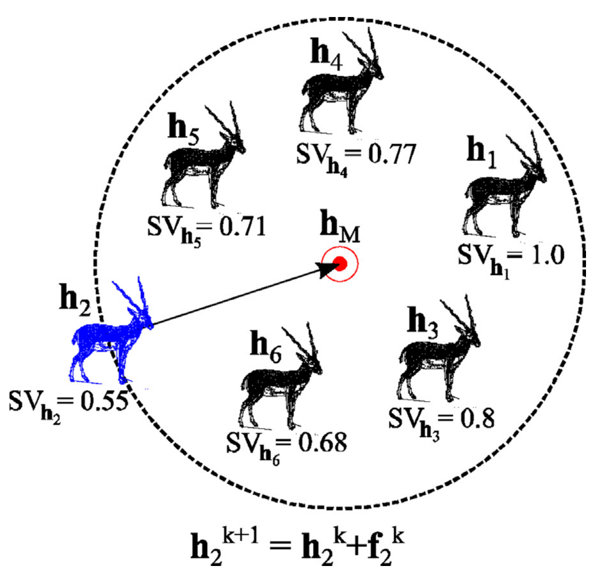


Fig. 8. Crowded horizon movement rule for a subordinated herd member (represented by $\mathbf{h}_2$). In this situation, the subordinated member moves toward the location of the herd's center of mass (shown as $\mathbf{h}_M$).

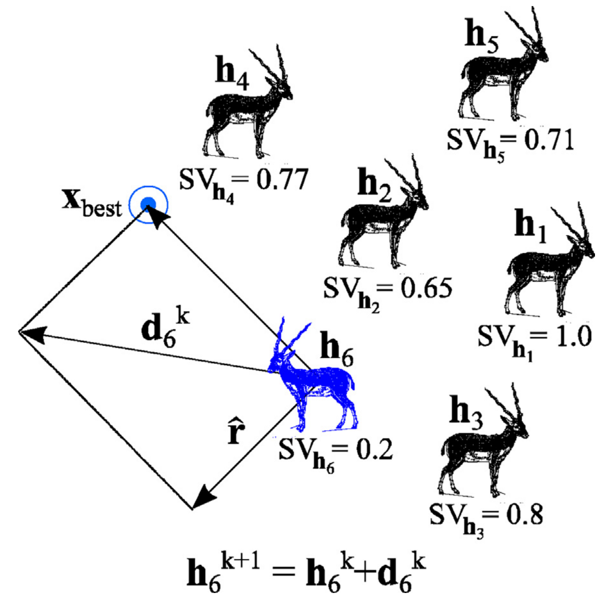


Fig. 9. Herd desertion movement rule. In such situation a given herd deserting member (shown as $\mathbf{h}_6$) move independently of all other members within the herd aggregation.

ing this, the herd desertion movement $\mathbf{d}_i^k$ may then be given as following:

$$\mathbf{d}_i^k = 2 \cdot (\beta \cdot \psi^k_{\boldsymbol{h}_i, \boldsymbol{x}_{\text{best}}} \cdot (\mathbf{x}^k_{\text{best}} - \boldsymbol{h}_i^k) + \gamma \cdot (1 - \text{SV}_{\boldsymbol{h}_i^k}) \cdot \hat{\mathbf{r}}) \tag{29}$$

where $\psi^k_{\boldsymbol{h}_i, \boldsymbol{x}_{\text{best}}}$, as in Eq. (20), denotes the selfish attraction experienced by the herd member $\mathbf{h}_i^k$ toward the current global best position ($\mathbf{x}^k_{\text{best}}$), whereas β and γ stand for random numbers drawn from the interval [0,1]. Furthermore $\hat{\mathbf{r}}$ denotes unit vector pointing to a random direction within the given n-dimensional solution space.

3.5. Predators movement operators

Herd members within an aggregation typically have lower predation risk in comparison to solitary individuals. This is because the effects of delusion and the confusion caused by the movement of many individuals influence the predator's decision to aim its attack toward a particular individual (Reluga and Viscido, 2005). However, these benefits are not homogenous among all members of a herd aggregation. As previously stated, the predation risk of any given member of a selfish herd is directly related to its current position within such aggregation (Hamilton, 1971). Intuitively, attacking predators take advantage of these apparent vulnerabilities when choosing a prey for pursuing. In addition, the relative position occupied by such predators with respect to the members of the herd is also an influential factor when deciding which herd member is going to be attacked.

By considering these facts, SHO models the movement of each individual within the pack of predators by considering both, the survival aptitudes of individuals within attacked herd and the distance which separate such individuals from the attacking predators.

3.5.1. Pursuit probabilities

In order to model the movement of individuals within the group of predators $\mathbf{P}$ (as defined in Section 3.1), it is first assumed that each member $\mathbf{h}_j$ within the herd $\mathbf{H}$ has a certain probability of being pursued by an attacking predator $\mathbf{p}_i$. In SHO, such pursuit probability is given as:

$$\mathcal{P}_{\boldsymbol{p}_i, \boldsymbol{h}_j} = \frac{\omega_{\boldsymbol{p}_i, \boldsymbol{h}_j}}{\sum_{m=1}^{N_h} \omega_{\boldsymbol{p}_i, \boldsymbol{h}_m}} \tag{30}$$

In this paper, the value $\omega_{\mathbf{p}_i, \mathbf{h}_j}$ is referred as the prey attractiveness between $\mathbf{p}_i$ and $\mathbf{h}_j$. Such value considers both, the survival aptitudes possessed $\mathbf{h}_j$ and the distance separating such individual from the attacking predator $\mathbf{p}_i$, as illustrated as follows:

$$\omega_{\mathbf{p}_i, \boldsymbol{h}_j} = (1 - \text{SV}_{\boldsymbol{h}_j}) \cdot e^{-\|\mathbf{p}_i - \boldsymbol{h}_j\|^2} \tag{31}$$

where $\text{SV}_{\boldsymbol{h}_j}$ denotes for the survival value related to $\mathbf{h}_j$ whereas $\|\mathbf{p}_i - \boldsymbol{h}_j\|$ stands for the Euclidian distance between $\mathbf{p}_i$ and $\mathbf{h}_j$. Intuitively, prey attractiveness $\omega_{\mathbf{p}_i, \mathbf{h}_j}$ yield to higher values toward members $\mathbf{h}_j$ possesing lower survival values $\text{SV}_{\boldsymbol{h}_j}$, whereas a higher survival value $\text{SV}_{\boldsymbol{h}_j}$ implies a lower attractiveness value. This is analogous to a predator's preference to attack apparently weaker prey over those who are seemingly stronger. Similarly, it can also be

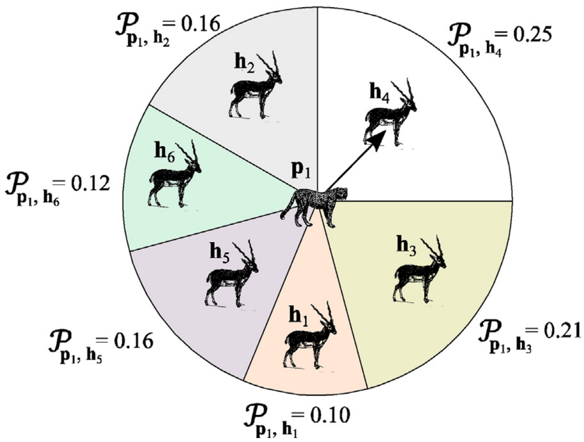


Fig. 10. By applying the roulette selection method with regard to the pursuit probabilities $\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j}$, related to $\mathbf{p}_i$ (illustrated as $\mathbf{p}_1$) and each individual $\mathbf{h}_j$ within the herd of prey (shown as $\mathbf{h}_1$–$\mathbf{h}_6$), a single member $\mathbf{h}_j$ is chosen to be pursued by $\mathbf{p}_i$.

observed that the value of $\omega_{\mathbf{p}_i,\mathbf{h}_j}$ increases as the distance $\|\mathrm{p}_i - \boldsymbol{h}_j\|$ between $\mathbf{p}_i$ and $\mathbf{h}_j$ decreases, while such value decreases as the distance gap between both individuals increases. Once again, this refers to a predator's preference to attack nearby preys rather than those occupying distant positions.

3.5.2. Predators position update

The pursuit probability $\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j}$ represents the probability for a given predator $\mathbf{p}_i$ to pursuit a certain herd member $\mathbf{h}_j$ given its perceived survival aptitudes and the distance separating them. With this in mind, at each iteration k, SHO models the movement of the each predator $\mathbf{p}_i$ within the pack of attacking predators $\mathbf{P}$ by considering the position of a particular herd member, as illustrated as follows:

$$\boldsymbol{p}_i^{k+1} = \mathrm{p}_i^k + 2 \cdot \rho \cdot (\mathrm{h}_r^k - \boldsymbol{p}_i^k) \tag{32}$$

where ρ denotes a random number between from the interval [0,1]. Furthermore, $\boldsymbol{h}_r^k = \boldsymbol{h}_j^k \in \boldsymbol{H}^k$ (with $r \in \{1, 2, \ldots, N_h\}$) denotes a herd member randomly chosen from among the members of the whole herd aggregation ($\mathbf{H}^k$) by applying the roulette selection method (Thomas, 1996) with regard to their individual pursuit probabilities $\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j}$ as given by Eq. (31) (see Fig. 10).

3.6. Predation phase

In nature, the biological interaction between groups of prey and predators, in which the former individuals are hunted by the latter, is known as predation. Such interactions often result in the death of prey and its eventual consumption by the predators. Typical hunting behavior suggest that there is a finite distance range over which a predator can launch a successful attack against a pursued prey. In the case of the selfish herd behavior described by Hamilton (1971), such finite range is described by the so called domain of danger which represents the area around a given prey in which, if a predator is present, there is a high change that such individual is attacked and possibly killed. Also, it is suggested that, if multiple domains of danger are invaded by attacking predator at the same time, there is usually a higher preference for killing the nearest prey; however other factors, such as the apparent survival aptitudes of each endangered prey, may also be an influential factor in the predator's final decision to attack.

In SHO, a computational procedure, called predation phase, is implemented to model such prey-predator interactions. In the predation phase, it is assumed that, after both the members of a selfish herd (the prey) and the pack of predators have performed a movement (according to the operators described in Sections 3.4 and 3.5), there is a chance for several herd members to be killed by the attacking predators, which further implies the exclusion of their respective solutions from within the given decision space.

3.6.1. Domain of danger

As previously illustrated, the domain of danger represents the area around a particular prey, in which, if a predator is present, there is a high chance for such prey to be attacked and killed. For the predation phase, SHO defines a finite domain of danger, represented as a circular area of finite radius around each prey as follows:

$$R = \frac{\sum_{j=1}^{n} |x_j^{\mathrm{low}} - x_j^{\mathrm{high}}|}{2 \cdot n} \tag{33}$$

where x_j^{low} and x_j^{high} stand for initial lower and upper bounds respectively, whereas n denotes the number of dimensions (for simplicity, SHO assumes that the radius of each domain of danger is the same for all prey).

3.6.2. Threatened prey

At the start of the predation phase, it is assumed that no prey has been hunted (killed) by the attacking predators. To represent this, SHO first initializes an empty set $\mathbf{K}$. That is:

$$\boldsymbol{K} = \{\emptyset\} \tag{34}$$

During the predation phase, such set $\mathbf{K}$ is used to group all herd members ($\mathbf{h}_j \in \mathbf{H}$) that are killed by the attacking predators ($\mathbf{p}_i \in \mathbf{P}$).

In SHO, a predator $\mathbf{p}_i$ is assumed to be able to hunt a certain herd member $\mathbf{h}_j$ if two specific conditions are met: 1. the member $\mathbf{h}_j$ has a lower survival value than $\mathbf{p}_i$, and 2. the distance between $\mathbf{p}_i$ and $\mathbf{h}_j$ is equal to or lower than the domain of danger radius R (as given by Eq. (33)), which implies that $\mathbf{p}_i$ has invaded $\mathbf{h}_j$'s domain of danger. Furthermore, it is also assumed that more than one herd member could be threatened by particular predator, assuming the previous two conditions have been met. With that being said, for each predator $\mathbf{p}_i$ we may represent a set of threatened prey as follows:

$$\boldsymbol{T}_{\mathbf{p}_i} = \{\mathrm{h}_j \in \boldsymbol{H} | \mathrm{SV}_{\mathbf{h}_j} < \mathrm{SV}_{\mathbf{p}_i}, \|\mathrm{p}_i - \boldsymbol{h}_j\| \le R, \boldsymbol{h}_j \notin \boldsymbol{K}\} \tag{35}$$

where $\mathrm{SV}_{\mathbf{p}_i}$ and $\mathrm{SV}_{\mathbf{p}_j}$ denote the survival values of $\mathbf{p}_i$ and $\mathbf{h}_j$, respectively, whereas $\|\mathrm{p}_i - \boldsymbol{h}_j\|$ denotes the Euclidian distance between the individuals $\mathbf{p}_i$ and $\mathbf{h}_j$. Furthermore, note from Eq. (36) that only members that are not currently grouped within $\mathbf{K}$ (which groups all herd members hunted during the predation phase) are able targeted by $\mathbf{p}_i$ as candidates to be hunted.

3.6.3. Probability of being hunted

Once a set threatened prey $\boldsymbol{T}_{\mathbf{p}_i}$ (as given by Eq. (35)) has been identified for a particular predator $\mathbf{p}_i$, one of such threatened individuals must be chosen, and then, killed. In SHO, such decision is taken based each threatened herd member probabilities of being hunted, as given as follows:

$$\mathcal{H}_{\mathbf{p}_i,\mathbf{h}_j} = \frac{\omega_{\mathbf{p}_i,\mathbf{h}_j}}{\sum_{(\mathrm{h}_m \in \boldsymbol{T}_{\mathbf{p}_i})} \omega_{\mathbf{p}_i,\mathbf{h}_m}}, \boldsymbol{h}_\mathrm{j} \in \boldsymbol{T}_{\mathbf{p}_i} \tag{36}$$

where $\omega_{\mathbf{p}_i,\mathbf{h}_j}$ denotes the prey attractiveness between $\mathbf{p}_i$ and $\mathbf{h}_j$, as given by Eq. (31).

Finally, by applying the roulette selection method (Thomas, 1996) with regard to the probabilities $\mathcal{H}_{\mathbf{p}_i,\mathbf{h}_j}$ of each threatened prey, one of such individuals is chosen, and then, considered to be killed by the attacking predator $\mathbf{p}_i$ (see Figs. 11 and 12). Furthermore, any member $\boldsymbol{h}_\mathrm{j} \in \boldsymbol{T}_{\mathbf{p}_i}$ selected by applying this method is grouped within the set of killed herd members $\mathbf{K}$. It should be noted that if $\boldsymbol{T}_{\mathbf{p}_i} = \{\emptyset\}$ for any given predator $\mathbf{p}_i$ (that is, there is no

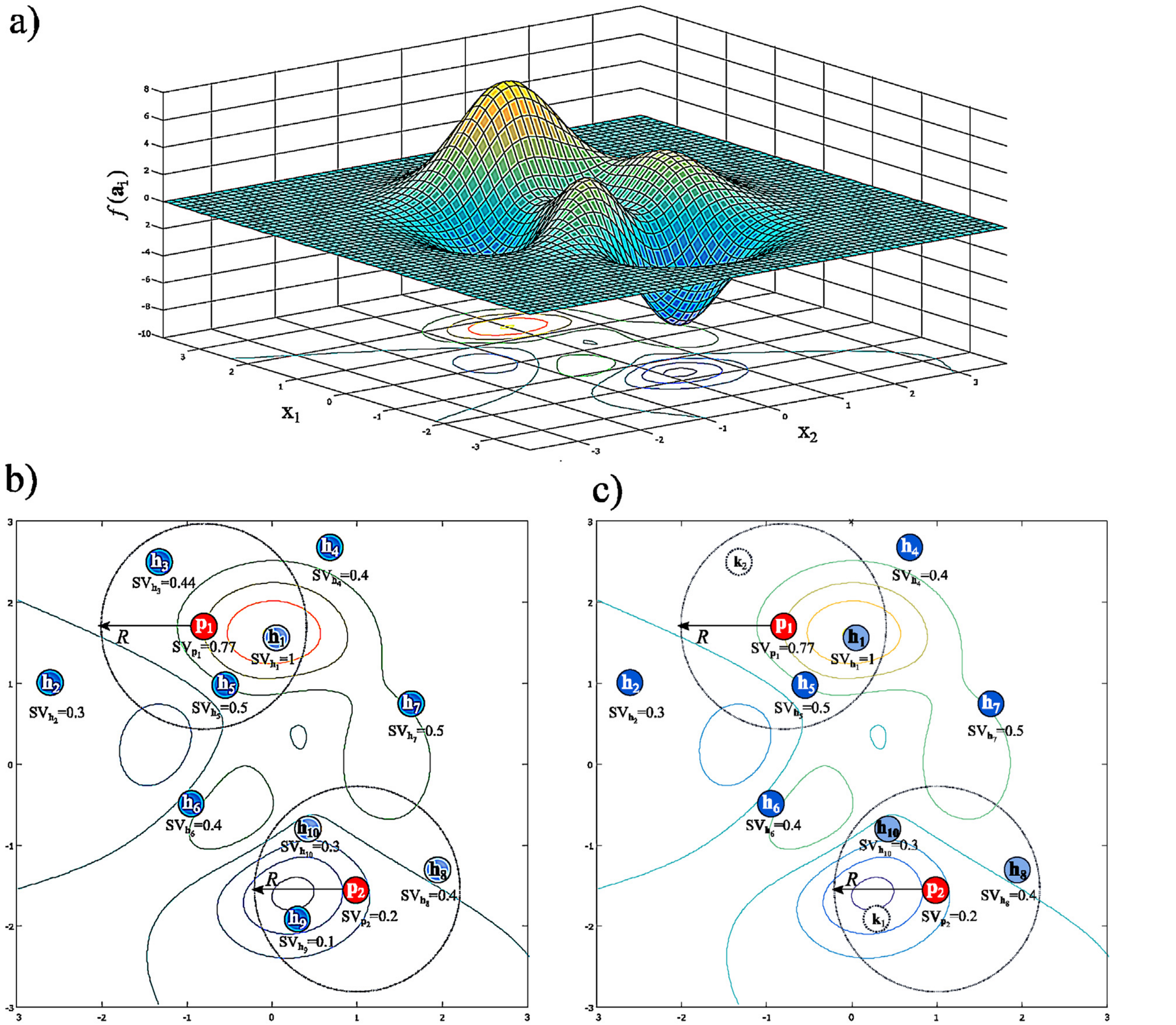


Fig. 11. Example of the predation phase procedure. **a)** Optimization problem, **b)** position's configuration for both prey (herd members) and predators, and **c)** outcome of the predation phase where the herd members $\mathbf{h}_3$ and $\mathbf{h}_9$ are assumed to be killed by the attacking predators $\mathbf{p}_1$ and $\mathbf{p}_2$, respectively.

prey that could be threatened by $\mathbf{p}_i$), then no prey is hunted by $\mathbf{p}_i$ and as such, no changes are made to $\mathbf{K}$ for that case. With the previous being said, at the end of the predation phase, the previously empty set $\mathbf{K}$ may group all herd members (if any) hunted by each of the attacking predators, such that:

$$\boldsymbol{K} = \{\mathbf{k}_i = (\mathbf{h}_j \in \boldsymbol{H})\}$$
$$\text{for} i = 1, \ldots, N_k, \quad j \in \{1, 2, \ldots, N_h\} \tag{37}$$

where N_k denote the total number of herd members $\mathbf{h}_j$ killed during the predation phase.

Furthermore, any solutions corresponding to a killed herd member $\mathbf{h}_j$ during the predation phase is further considered to be excluded from within the given decision space. However, as it will be illustrated in the next section, such excluded solutions will be replaced by some new solutions, generated by applying a special mating-like operator.

3.7. Restoration phase

In nature, the size of prey populations change dynamically through time as a result of predation, but in general, in well balanced biological systems, such change tends to be periodical. This means that even if a population of prey decreases as a result of the predation, the natural balance of the ecosystem will eventually allow the restoration of such population (Voltera, 1931).

Analogous to this phenomenon, SHO implements a computational procedure which allows the replacement of herd members killed during the predation phase (see Section 3.6). Such procedure, called restoration phase, employs a special mating-like operator to generate new solutions based on the survival aptitudes of the remaining herd members, and then such solutions are used to replace all solutions excluded as a result of such predation phase.

3.7.1. Mating probabilities

SHO's mating operation considers the positions and survival values of all herd members that weren't hunted during the predation

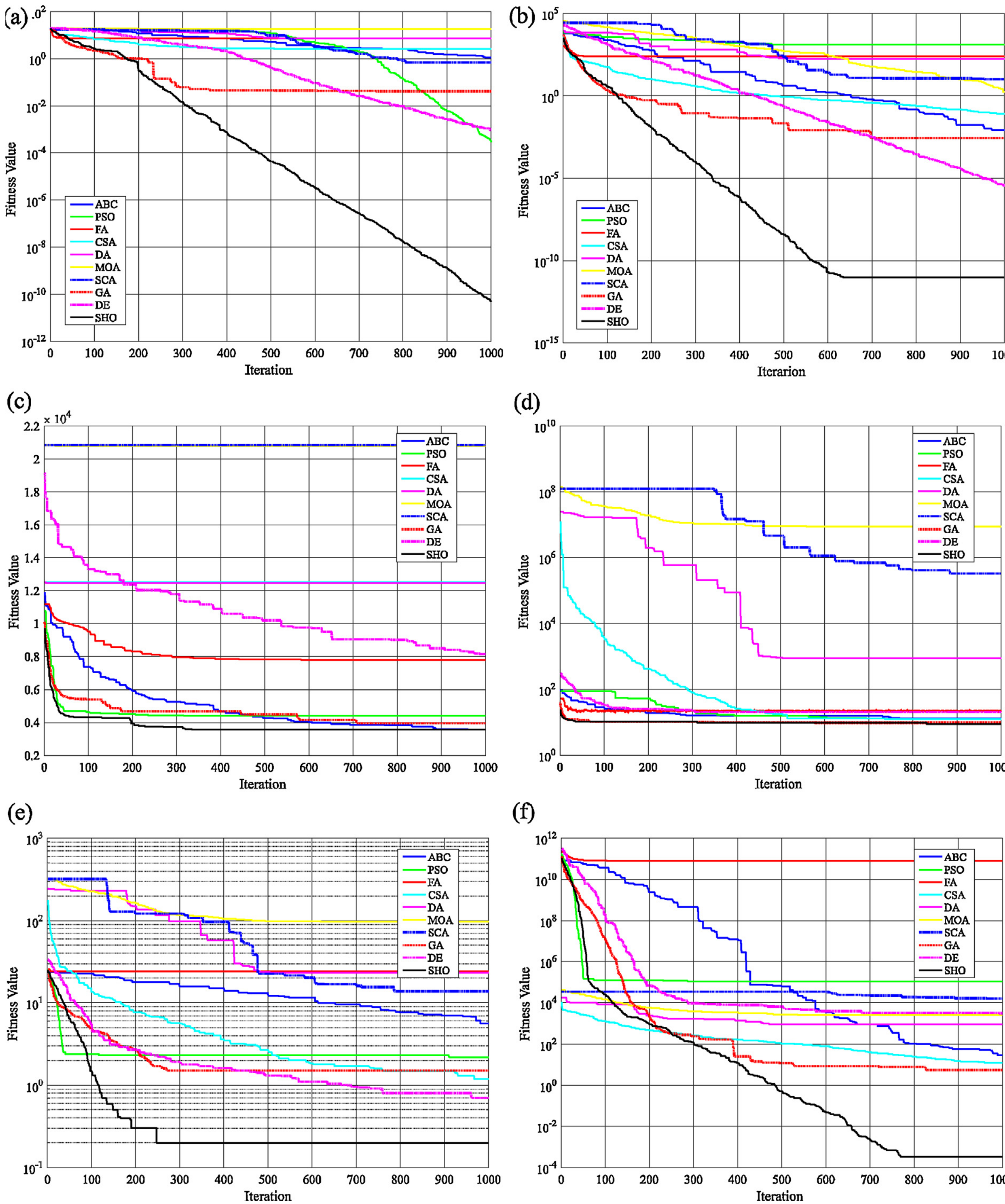


Fig. 12. Evolution curves for SHO, PSO, ABC, FA, DE, GA, CSA, DA, MOA and SCA considering as examples the functions (a) f_1, (b) f_3, (c) f_8, (d) f_{13}, (e) f_{14} and (f) f_{15} from the experimental set (see Appendix B).

phase. As such, we may first define a set of mating candidates as follows:

$$\boldsymbol{M} = \{\mathbf{h}_j \notin \boldsymbol{K}\} \tag{38}$$

where $\mathbf{K}$ denotes the set of herd members killed during the predation phase, as given by Eq. (37).

For the mating operation applied in SHO, each member $\mathbf{h}_j$ within the set of mating candidates $\mathbf{M}$ is considered to have a certain probability $\mathscr{M}_{\mathbf{h}_j}$ of being considered for the generation of new solution. Such probability depends on the survival aptitude of each mating candidate, as illustrated as follows:

$$\mathscr{M}_{\mathbf{h}_j} = \frac{\mathrm{SV}_{\boldsymbol{h}_j}}{\sum_{(\mathbf{h}_m \in \boldsymbol{M})} \mathrm{SV}_{\boldsymbol{h}_m}}, \ \boldsymbol{h}_j \in \boldsymbol{M} \tag{39}$$

From the previous, it is clear that mating candidates possessing higher survival values $\mathrm{SV}_{\boldsymbol{h}_j}$ will a have a higher chance to influence the generation of new solutions, whereas individuals with lower survival values are less likely to be considered for such a process.

3.7.2. Mating operation

In order to generate a new candidate solution, we first consider a set of n randomly chosen individuals $\{\mathbf{h}_{r_1}, \boldsymbol{h}_{r_2}, \ldots, \boldsymbol{h}_{r_n}\}$ (with $\boldsymbol{h}_{r_i} = \boldsymbol{h}_j \in \boldsymbol{M}$), selected by applying the roulette selection method (Thomas, 1996) with regard to the probabilities $\mathscr{M}_{\boldsymbol{h}_j}$ of each member within the set of mating candidates $\mathbf{M}$, as given by Eq. (39).

$$\boldsymbol{h}_{\text{new}} = \text{mix}([h_{r_1,1}, h_{r_2,2}, \ldots, h_{r_n,n}]) \tag{40}$$

where $h_{r_i,l}$ (with l = 1, 2, ..., n) correspond to the l-esim position element of the random candidate $\boldsymbol{h}_{r_i}$ and where n stands for the dimensionality of the solution space. Furthermore, the function mix $(\cdot)$ is applied to change the 'l' indexing of each element $h_{r_i,l}$, such that each of said elements is indexed on a different entry (dimension). An example of SHO's mating operation for generating a new candidate solution is illustrated in Table 1.

3.7.3. Replacement of excluded herd members

As previously stated, the mating operation in SHO is used to replace herd members (solutions) excluded as a result of the predation phase (as described in Section 3.6). With that being said, for each of member $\mathbf{h}_j \in \mathbf{K}$ (with $\mathbf{K}$ denoting the set of killed herd members, as given by Eq. (37)), we generate a new candidate solution $\mathbf{h}_{\text{new}}$ by applying the mating operation described by Eq. (40), and then such new solution is assigned to $\mathbf{h}_j$, such that:

$$\boldsymbol{h}_j = \boldsymbol{h}_{\text{new}}, \boldsymbol{h}_j \in \boldsymbol{K} \tag{41}$$

By applying this procedure, all previously excluded herd members $\mathbf{h}_j \in \mathbf{K}$ are replaced with new solutions, formed by considering the positions and survival aptitudes of the members which survived the predation phase.

4. Summary of the SHO algorithm

The Selfish Herd Optimizer (SHO) is an evolutionary algorithm developed to solve global optimization problems. The proposed method draws inspiration on the interesting selfish herd behavior observed in a wide variety of species subjected to a prey-predator relationship. Such method employs to kinds of search agents: a herd of prey and a pack predators. The movement of each of this agents within the solution space is performed by applying a set of unique evolutionary operators inspired in several movement rules observed in most selfish herds. Furthermore, the computational procedure known as predation phase emulates the situation in which prey belonging to a selfish herd are hunted down and killed by attacking predators. Such mechanism is employed to allow the exclusion of either bad or redundant solutions, found during SHO's the evolutionary process. Finally, the restoration phase allows the restitution of solutions previously excluded during the predation phase by generating a set of new candidate solution. Such procedure is reminiscent to the natural balance mechanism found in many biological systems and allows the diversification of the set of solutions found by SHO.

In general, the SHO algorithm's computational procedure may be summarized by the following steps:

Step 1: Initialize the animal population **A**.

Step 2: Split **A** in two groups: a group of prey **H** and a group of predators **P**.

Step 3: Calculate the survival values for each individual within **H** and **P**.

Step 4: Apply the herd movement operators to each individual of **H**.

Step 5: Apply the predators movement operators to each individual of **P**.

Step 6: Re-calculate the survival values for each individual within **H** and **P**.

Step 7: Perform predation phase.

Step 8: Perform restoration phase.

Step 9: If stop criterion is met, the process is finished; otherwise, return to **Step 4**.

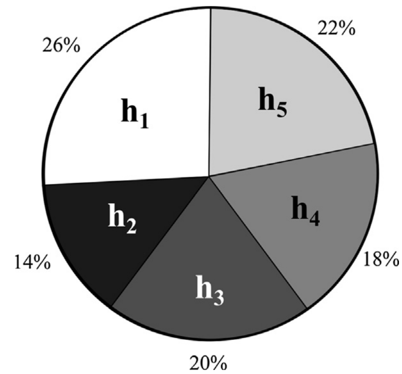
A more detailed explanation of the computational procedure employed by the proposed method may be found in Appendix A.

5. Discussion about the SHO algorithm

Within the framework of many Evolutionary Algorithms (EA), the dilemma concerning to the balance between exploration and exploitation of solutions has remained as an important topic for many years. In this sense, it is known that emphasizing to much on exploration increases the capacity of an EA to find new potential solutions; however, this usually yields to a degradation on the precision of such EA's evolutionary process. On the other hand, giving more importance to exploitation allows the refinement of currently existing solutions but, adversely, there is a tendency to drive the process toward local optima. With that being said, the ability of an EA to find a global optimum depends on its capacity to properly balance the exploration of the solutions and the exploitation of found-so-far elements. Furthermore, many EAs suffer from some common flaws, such as premature convergence and an inherent difficulty to overcome local optima (Wang et al., 2011; Xiang and An, 2013). These issues usually arise from the operators used to update the position of search agents. In the case of PSO, for example, search agents are usually attracted towards the position of the current best individual which inherently causes the entire population to concentrate around the best particle seen-so-far, and thus, favoring premature convergence (Wang et al., 2013). Also, many EAs, such as PSO, FA and CS, only model search agents with equal properties, and thus, restricting them to perform virtually the same behavior. Under these circumstances, these methods waste the possibility to add new and selective operators which could potentially improve some important traits, such as population diversity and search capabilities.

Different to other EA, the SHO algorithm models each individual of the entire population by first considering its role as either a prey belonging to a selfish herd or a hungry predator. Furthermore, individuals within such selfish herd manifest unique individual behaviors which depends not only on the aptitudes of better individuals, but also on their own aptitude with regard to a given optimization problem. This allows SHO to incorporate computational mechanisms that allow the avoidance of critical flaws commonly found in other EAs, such as premature convergence and inadequate balance between exploration and exploitation of solutions. From an optimization point of view, the use of selfish herd behaviors as a metaphor provide some interesting concepts to EAs: First of all, the division of the entire population into different search-agent categories and the implementation of selective and specialized operators allows SHO to improve the balance between exploration and exploitation without altering the total population size. Furthermore, the selfish herd behavior introduces an interesting computational scheme with three distinctive traits: 1. individuals are separately processed according to their classification as either prey or predators, with prey further manifesting a unique internal social structure; 2. although operators employed to modify the position of each individual differs depending of its type (as either a prey or a predator), they all use global information (such as the positions and aptitudes of other individuals); 3. the predation and restoration mechanisms allows the exclusion of potentially bad or redundant solutions, while at the same time enables the diversification of the whole set of solutions.

Table 1
Example of a mating operation used to generate a new herd member $\mathbf{h}_{new}$.

Mating Candidates ($h_j \in M$)	Positions $\left[h_{j,1}, h_{j,2}, h_{j,3}\right]$	$SV_{\mathbf{h}_j}$	$\mathcal{M}_{\mathbf{h}_j}$	$\mathbf{h}_{r_i}$	Roulette
$\mathbf{h}_1$	(0.3, 0.5, −0.8)	1	0.26	$\mathbf{h}_{r_1}$	
$\mathbf{h}_2$	(0.6, −0.4, 0.7)	0.55	0.14		
$\mathbf{h}_3$	(0.2, −0.5, 0.3)	0.76	0.20	$\mathbf{h}_{r_2}$	
$\mathbf{h}_4$	(0.1, 0.5, −0.7)	0.68	0.18		
$\mathbf{h}_5$	(−0.4, 0.3, 0.7)	0.83	0.22	$\mathbf{h}_{r_3}$	
Mixed Elements $[h_{5,3}, h_{1,1}, h_{3,1}]$		New position $\mathbf{h}_{new} = [0.7, 0.3, -0.5]$			

6. Experimental setup and results

We have applied SHO in the optimization of 15 benchmark functions, whose results have also been compared against those produced by Particle Swarm Optimization (PSO) (Kennedy and Eberhart, 1995), Artificial Bee Colony (ABC) (Karaboga and Basturk, 2008), Firefly Algorithm (FA) (Yang, 2010), Differential Evolution (DE) (Storn and Price, 1997), Genetic Algorithms (GA) (Mitchell, 1996), Crow Search Algorithm (CSA) (Askarzadeh, 2016), Dragonfly Algorithm (DA) (Mirjalili, 2016a), Moth-flame Optimization Algorithm (MOA) (Mirjalili, 2015), and Sine Cosine Algorithm (SCA) (Mirjalili, 2016b). For all comparisons, the population size is set to $N = 50$ individuals, while the maximum iteration number is set to $T = 1000$. Such stop criterion has been selected to keep consistency with other similar works currently reported on the literature (Ji et al., 2007; Rashedi et al., 2009). A detailed description of each implemented test functions is given in Appendix B.

The parameter setting for each of the compared methods is as follows:

1. PSO: The algorithm's learning factors are set to $c_1 = 2$ and $c_2 = 2$; also, the inertia weight w is set to decreases linearly from 0.9 to 0.2 as the process evolves.
2. ABC: The algorithm was implemented by setting the parameter limit = num Of Food Sources * dims, where num Of Food Sources = N (population size) and dims = n (dimensionality of the solution space).
3. FA: The parameters setup for the randomness factor and the light absorption coefficient are set to $\alpha = 0.2$ and $\gamma = 1.0$, respectively.
4. DE: The algorithm's differential weight is set to F = 1 while the crossover probability is set to CR = 0.2.
5. GA: The crossover and mutation probabilities are both set to $c_p = 0.8$ and $m_p = 0.2$ respectively.
6. CSA: The awareness probability and flight length are set to AP = 0.1 and fl = 2, respectively.
7. DA: The parameters are set as: w = 0.9 (inertia weight), s = 0.1 (separation weight), a = 0.1 (alignment weight), c = 0.7 (cohesion weight), f = 1.0 (food factor) and e = 1.0 (enemy factor).
8. MOA: The constant value used to model the moths' logarithmic spiral movement is set to b = 1.
9. SCA: The constant value used to generate the random value r_1 is set to a = 2.
10. SHO: The proposed method is tested by considering a herd population proportion randomly chosen from between a 70% and 90%, with the remaining individuals assigned as predators.

The previously illustrated sets of parameters were determined through exhaustive experimentation and as such represent the best possible configurations for each of the compared methods. The experimental setup aims to compare SHO's performance against those of PSO, ABC, FA, DE and GA. The reported results consider the following performance indexes: the Average Best-so-far (AB) solution, the Median Best-so-far (MB) and the Standard Deviation (SD) of the best-so-far solution. The averaged results corresponding to 30 individual runs are reported in Table 2, where the best outcome for each function is boldfaced. According to this table, for most cases, SHO's performance over the considered test functions is superior to those of the other compared methods. Such large difference in performance is intuitively related to a better trade-off between exploration and exploitation.

Also, the non-parametric statistical significance proof known as the Wilcoxon's rank sum test for independent samples (Wilcoxon, 2006; García et al., 2009) was conducted over the best fitness values found by each of the compared method on 30 independent test runs (30 samples per set). Table 3 reports the p-values produced by the Wilcoxon's test for the pair-wise comparison over two independent fitness samples (SHO vs. PSO, ABC, FA, GA, CSA, DA, MOA and SCA), by considering a 5% significance level. As a null hypothesis, it is assumed that there is a significant difference between mean values of two algorithms. On the other hand, the alternative hypothesis (rejection of the null hypothesis) considers that the difference between the mean values of both approaches is insignificant. As shown by all of the p-values reported in Table 2 there is enough evidence to reject the null hypothesis (this is that all values are less than 0.05, and as such satisfy the 5% significance level criteria). Such evidence indicates that the proposed method's results are statistically significant and that they had not occurred by coincidence (i.e. due to common noise contained in the process).

7. Conclusions

In this paper, a novel swarm optimization algorithm called Selfish Herd Optimizer (SHO) has been proposed to solve global optimization problems. SHO is based on the widely observed selfish herd behavior, manifested as a result of the predation risk inherently related to most prey-predator interactions. As such, the proposed method considers two types of search agents: the members of a selfish herd (prey) and the individuals which hunt for the individuals in such aggregation (predators). Depending on their designation (and its internal social structure in the case of the members of the herd), each individual moves around the solution space of a given optimization problem by considering a set of distinctive evolutionary operators, which mimic the different kind of behav-

Table 2
Minimization results for the benchmark functions in Table A1. Results were averaged from 30 individual runs, each by considering population size $N = 50$ and maximum number of iterations $T = 1000$.

		SHO	PSO	ABC	FA	DE	GA	CSA	DF	MOA	SCA
$f_1(\boldsymbol{x})$	AB	3.30×10^{-13}	1.40×10^{-04}	2.90×10^{-01}	5.30×10^{00}	5.66×10^{-06}	1.16×10^{00}	3.25×10^{00}	6.36×10^{00}	1.88×10^{01}	1.12×10^{01}
	MB	7.50×10^{-10}	1.70×10^{01}	1.20×10^{00}	6.80×10^{00}	5.41×10^{-06}	1.34×10^{00}	3.35×10^{00}	5.67×10^{00}	1.91×10^{01}	1.84×10^{01}
	SD	2.80×10^{-09}	6.80×10^{00}	3.20×10^{-01}	7.10×10^{-01}	1.62×10^{-06}	8.01×10^{-01}	6.31×10^{-01}	1.83×10^{00}	1.20×10^{00}	9.61×10^{00}
$f_2(\boldsymbol{x})$	AB	1.00×10^{-13}	7.50×10^{-11}	1.70×10^{-05}	1.70×10^{-01}	1.27×10^{-13}	6.12×10^{-04}	4.07×10^{-05}	9.96×10^{-01}	1.48×10^{01}	3.81×10^{-01}
	MB	2.50×10^{-13}	1.70×10^{00}	1.40×10^{-04}	5.00×10^{-01}	$\mathbf{1.06\times10^{-13}}$	2.71×10^{-04}	3.53×10^{-05}	8.96×10^{-01}	3.90×10^{-01}	6.42×10^{-02}
	SD	1.40×10^{-14}	6.70×10^{00}	7.30×10^{-05}	2.70×10^{-01}	5.70×10^{-14}	8.89×10^{-04}	2.37×10^{-05}	7.58×10^{-01}	1.77×10^{01}	9.01×10^{-01}
$f_3(\boldsymbol{x})$	AB	7.20×10^{-12}	3.30×10^{-08}	2.20×10^{-03}	4.40×10^{00}	7.50×10^{-12}	7.30×10^{-02}	3.15×10^{-01}	6.12×10^{01}	2.37×10^{03}	1.74×10^{01}
	MB	1.90×10^{-12}	6.10×10^{02}	9.00×10^{-03}	1.90×10^{01}	6.58×10^{-12}	2.18×10^{-02}	2.64×10^{-01}	3.79×10^{01}	2.01×10^{03}	3.98×10^{00}
	SD	8.60×10^{-12}	5.00×10^{02}	6.10×10^{-03}	8.50×10^{00}	$\mathbf{3.36\times10^{-12}}$	1.21×10^{-01}	2.27×10^{-01}	5.46×10^{01}	2.25×10^{03}	3.63×10^{01}
$f_4(\boldsymbol{x})$	AB	1.20×10^{-03}	1.70×10^{02}	5.60×10^{-01}	3.00×10^{00}	8.05×10^{00}	7.30×10^{-02}	6.25×10^{-03}	1.02×10^{00}	2.13×10^{01}	3.56×10^{-01}
	MB	2.90×10^{-03}	1.70×10^{03}	3.40×10^{00}	1.50×10^{01}	7.19×10^{00}	2.18×10^{-02}	6.20×10^{-03}	7.22×10^{-01}	1.72×10^{01}	8.76×10^{-02}
	SD	1.10×10^{-03}	1.80×10^{03}	1.90×10^{00}	7.30×10^{00}	4.83×10^{00}	1.21×10^{-01}	3.01×10^{-03}	1.05×10^{00}	1.90×10^{01}	4.96×10^{-01}
$f_5(\boldsymbol{x})$	AB	1.10×10^{-13}	9.40×10^{-07}	3.60×10^{-04}	7.40×10^{-01}	1.91×10^{-12}	4.19×10^{-01}	3.36×10^{-01}	9.42×10^{-01}	2.25×10^{01}	4.60×10^{00}
	MB	3.40×10^{-13}	5.60×10^{00}	1.00×10^{-03}	1.70×10^{00}	1.76×10^{-12}	4.54×10^{-01}	3.20×10^{-01}	8.46×10^{-01}	2.08×10^{01}	4.45×10^{00}
	SD	2.90×10^{-13}	6.20×10^{00}	5.90×10^{-04}	6.40×10^{-01}	9.60×10^{-13}	4.24×10^{-01}	1.53×10^{-01}	4.67×10^{-01}	7.49×10^{00}	6.81×10^{-01}
$f_6(\boldsymbol{x})$	AB	1.80×10^{00}	7.30×10^{01}	2.20×10^{01}	5.70×10^{02}	2.99×10^{02}	7.05×10^{01}	3.10×10^{01}	1.08×10^{03}	1.60×10^{05}	8.69×10^{03}
	MB	4.30×10^{01}	2.00×10^{05}	6.80×10^{01}	2.40×10^{03}	2.65×10^{02}	7.18×10^{01}	2.89×10^{01}	1.04×10^{03}	8.24×10^{02}	5.12×10^{03}
	SD	3.60×10^{01}	1.00×10^{05}	$\mathbf{3.10\times10^{01}}$	1.60×10^{03}	1.19×10^{02}	4.46×10^{01}	1.17×10^{01}	9.04×10^{02}	3.57×10^{05}	1.34×10^{04}
$f_7(\boldsymbol{x})$	AB	1.60×10^{-09}	1.00×10^{04}	2.30×10^{00}	9.70×10^{04}	9.52×10^{-09}	2.37×10^{00}	2.14×10^{02}	2.25×10^{03}	8.85×10^{04}	3.95×10^{01}
	MB	7.40×10^{-09}	1.50×10^{06}	1.40×10^{01}	2.60×10^{05}	8.93×10^{-09}	9.74×10^{-01}	2.07×10^{02}	2.09×10^{03}	8.61×10^{04}	1.17×10^{01}
	SD	4.90×10^{-09}	1.20×10^{06}	8.60×10^{00}	9.80×10^{04}	5.30×10^{-09}	3.97×10^{00}	9.20×10^{01}	1.47×10^{03}	5.03×10^{04}	6.11×10^{01}
$f_8(\boldsymbol{x})$	AB	3.80×10^{-04}	2.50×10^{03}	6.60×10^{02}	9.20×10^{03}	3.80×10^{02}	1.20×10^{04}	1.25×10^{04}	1.25×10^{04}	2.08×10^{04}	2.09×10^{04}
	MB	6.60×10^{02}	4.80×10^{03}	1.10×10^{03}	1.00×10^{04}	$\mathbf{9.31\times10^{00}}$	1.20×10^{04}	1.25×10^{04}	1.25×10^{04}	2.08×10^{04}	2.09×10^{04}
	SD	4.40×10^{02}	8.10×10^{02}	2.30×10^{02}	3.40×10^{02}	6.52×10^{02}	2.90×10^{01}	2.56×10^{00}	$\mathbf{0.00\times10^{00}}$	3.70×10^{-12}	4.13×10^{00}
$f_9(\boldsymbol{x})$	AB	-4.50×10^{03}	1.10×10^{05}	5.30×10^{03}	4.30×10^{04}	4.65×10^{05}	-1.53×10^{03}	-1.39×10^{03}	5.23×10^{02}	5.08×10^{03}	4.60×10^{02}
	MB	-2.60×10^{03}	9.00×10^{05}	1.60×10^{04}	7.80×10^{04}	4.60×10^{05}	-1.53×10^{03}	-1.34×10^{03}	1.69×10^{02}	2.82×10^{03}	2.44×10^{02}
	SD	1.10×10^{03}	5.10×10^{05}	6.40×10^{03}	2.80×10^{04}	2.13×10^{05}	2.21×10^{01}	3.89×10^{02}	1.26×10^{03}	8.32×10^{03}	6.19×10^{02}
$f_{10}(\boldsymbol{x})$	AB	6.70×10^{-01}	6.70×10^{-01}	2.60×10^{00}	1.00×10^{01}	6.73×10^{00}	4.58×10^{00}	7.30×10^{01}	2.49×10^{06}	1.67×10^{09}	3.70×10^{07}
	MB	6.80×10^{-01}	1.40×10^{04}	6.80×10^{00}	4.00×10^{01}	6.67×10^{00}	4.47×10^{00}	3.03×10^{01}	4.72×10^{05}	7.92×10^{08}	1.97×10^{07}
	SD	6.50×10^{-02}	3.30×10^{04}	2.20×10^{00}	4.20×10^{01}	1.64×10^{-01}	2.83×10^{00}	9.83×10^{01}	5.42×10^{06}	2.74×10^{09}	4.84×10^{07}
$f_{11}(\boldsymbol{x})$	AB	2.40×10^{-10}	9.00×10^{-07}	1.30×10^{-01}	3.00×10^{03}	2.94×10^{-10}	4.50×10^{-02}	7.36×10^{02}	2.02×10^{05}	8.02×10^{06}	3.67×10^{04}
	MB	1.00×10^{-10}	3.70×10^{04}	3.10×10^{-01}	6.70×10^{03}	2.70×10^{-10}	1.94×10^{-02}	4.74×10^{02}	1.62×10^{05}	7.25×10^{06}	5.52×10^{03}
	SD	6.90×10^{-11}	3.10×10^{04}	1.80×10^{-01}	2.10×10^{03}	1.30×10^{-10}	7.57×10^{-02}	6.33×10^{02}	1.91×10^{05}	5.09×10^{06}	6.45×10^{04}
$f_{12}(\boldsymbol{x})$	AB	1.20×10^{01}	1.50×10^{02}	2.30×10^{02}	7.30×10^{01}	1.63×10^{03}	2.80×10^{02}	3.89×10^{00}	1.33×10^{03}	1.31×10^{04}	2.56×10^{03}
	MB	2.10×10^{01}	4.80×10^{02}	2.90×10^{02}	4.10×10^{03}	1.63×10^{03}	3.17×10^{02}	3.49×10^{00}	9.72×10^{02}	1.25×10^{04}	2.43×10^{03}
	SD	5.70×10^{00}	1.60×10^{02}	2.90×10^{01}	2.20×10^{04}	2.75×10^{02}	1.63×10^{02}	1.97×10^{00}	1.19×10^{03}	3.35×10^{03}	7.76×10^{02}
$f_{13}(x)$	AB	8.30×10^{00}	9.90×10^{00}	1.10×10^{01}	1.40×10^{01}	1.05×10^{02}	1.58×10^{01}	1.16×10^{01}	3.30×10^{03}	4.84×10^{06}	1.38×10^{05}
	MB	9.30×10^{00}	1.40×10^{01}	1.20×10^{01}	2.00×10^{01}	1.05×10^{02}	1.54×10^{01}	1.14×10^{01}	2.06×10^{03}	2.43×10^{06}	2.81×10^{04}
	SD	5.40×10^{-01}	4.50×10^{00}	6.20×10^{-01}	3.50×10^{00}	3.61×10^{00}	2.18×10^{00}	8.52×10^{-01}	4.75×10^{03}	6.11×10^{06}	2.05×10^{05}
$f_{14}(\boldsymbol{x})$	AB	1.00×10^{-01}	5.00×10^{-01}	3.10×10^{00}	2.30×10^{00}	4.11×10^{00}	5.67×10^{-01}	1.66×10^{00}	1.86×10^{01}	7.08×10^{01}	8.20×10^{00}
	MB	1.20×10^{-01}	1.70×10^{00}	4.00×10^{00}	3.60×10^{00}	4.01×10^{00}	6.00×10^{-01}	1.65×10^{00}	1.90×10^{01}	8.83×10^{01}	6.76×10^{00}
	SD	4.10×10^{-02}	2.90×10^{00}	4.80×10^{-01}	6.10×10^{-01}	3.01×10^{-01}	8.02×10^{-02}	3.19×10^{-01}	8.04×10^{00}	4.02×10^{01}	6.48×10^{00}
$f_{15}(\boldsymbol{x})$	AB	2.40×10^{-04}	7.60×10^{-04}	1.87×10^{01}	7.60×10^{06}	8.97×10^{02}	9.20×10^{-02}	1.61×10^{01}	4.22×10^{02}	1.08×10^{03}	1.92×10^{04}
	MB	6.10×10^{-04}	1.40×10^{-01}	5.94×10^{01}	6.20×10^{07}	9.03×10^{02}	3.89×10^{-02}	1.14×10^{01}	3.76×10^{02}	2.18×10^{01}	1.97×10^{04}
	SD	5.60×10^{-04}	3.00×10^{-01}	2.08×10^{01}	3.60×10^{07}	2.81×10^{02}	1.78×10^{-01}	1.28×10^{01}	2.76×10^{02}	1.68×10^{03}	1.69×10^{03}

Table 3
Wilcoxon's test comparison for SHO vs. PSO, ABC, FA, GA, CSA, DA, MOA and SCA. The table shows the resulting p-values for each pair-wise comparison.

Function	SHO vs PSO	SHO vs ABC	SHO vs FA	SHO vs DE	SHO vs GA	SHO vs CSA	SHO vs DA	SHO vs MOA	SHO vs SCA
$f_1(\boldsymbol{x})$	5.22×10^{-12}	3.01×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_2(\boldsymbol{x})$	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	5.86×10^{-06}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_3(\boldsymbol{x})$	2.89×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	6.52×10^{-09}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_4(\boldsymbol{x})$	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	5.86×10^{-06}	3.02×10^{-11}	3.02×10^{-11}	3.82×10^{-09}
$f_5(\boldsymbol{x})$	3.00×10^{-11}	3.01×10^{-11}	3.01×10^{-11}	1.09×10^{-10}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_6(\boldsymbol{x})$	6.68×10^{-11}	4.28×10^{-02}	3.02×10^{-11}	3.02×10^{-11}	2.32×10^{-02}	8.42×10^{-01}	6.72×10^{-10}	3.02×10^{-11}	7.38×10^{-11}
$f_7(\boldsymbol{x})$	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	4.08×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_8(\boldsymbol{x})$	2.96×10^{-11}	7.71×10^{-06}	2.99×10^{-11}	2.50×10^{-03}	3.01×10^{-11}	3.01×10^{-11}	1.21×10^{-12}	1.21×10^{-12}	3.01×10^{-11}
$f_9(\boldsymbol{x})$	3.01×10^{-11}	3.01×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	1.29×10^{-06}	4.61×10^{-10}	2.00×10^{-05}	4.20×10^{-10}
$f_{10}(\boldsymbol{x})$	1.01×10^{-09}	4.11×10^{-12}	4.11×10^{-12}	4.11×10^{-12}	4.17×10^{-08}	4.11×10^{-12}	4.58×10^{-12}	4.11×10^{-12}	4.11×10^{-12}
$f_{11}(\boldsymbol{x})$	2.97×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	1.29×10^{-09}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_{12}(\boldsymbol{x})$	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	1.11×10^{-06}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_{13}(x)$	9.92×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}
$f_{14}(\boldsymbol{x})$	2.70×10^{-11}	2.71×10^{-11}	2.71×10^{-11}	2.71×10^{-11}	2.67×10^{-11}	2.71×10^{-11}	5.07×10^{-10}	2.71×10^{-11}	2.71×10^{-11}
$f_{15}(\boldsymbol{x})$	8.15×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	4.08×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}	3.02×10^{-11}

iors manifested in said predatory interactions. In contrast to most existing swarm optimization algorithms, the SHO allows not only to emulate such interesting selfish behaviors, but also to incorporate computational mechanisms devised to avoid critical flaws commonly found on other similar methods, such as premature convergence and the inappropriate balance between the exploration and exploitation of solutions. The performance of SHO has been tested by considering a set of 15 benchmark optimization functions. Furthermore, the performance of the proposed approach was also compared against popular methods, such as Particle Swarm Optimization (PSO) Artificial Bee Colony (ABC, Firefly Algorithm (FA), Differential Evolution (DE), Genetic Algorithms (GA), Crow Search Algorithm (CSA), Dragonfly Algorithm (DA), Moth-flame Optimization Algorithm (MOA) and Sine Cosine Algorithm (SCA). The experimental results show SHO's a high performance in terms of solution quality. Such remarkable performance is associated with two different traits: 1. the use of operators which allow a better distribution within the search space, 2. the division of the population into different individual types which enable the proposed method to adopt different exploration and exploitation rates during the evolutionary process, and 3. The integration of unique predation based operators which allows the diversification of solutions.

Appendix A. SHO algorithm computational procedure

Step 1: Considering N as the total number of n-dimensional animals, initialize the animal population $\mathbf{A}$ (section 3.1).

1.1. Randomly initialize each animal individual's position

for $(i = 1{:}N)$
    for $(j = 1{:}n)$
        $a_{i,j}^{0} = x_j^{\text{low}} + \text{rand}(0,1) \cdot (x_j^{\text{high}} - x_j^{\text{low}})$
    end for
end for

Step 2: Divide $\mathbf{A}$ in two groups: a group of herd members $\mathbf{H}$ and a group of predators $\mathbf{P}$ (section 3.1).

2.1. Define the number of herd members (N_h) and predators (N_p) within **A.**

$N_h = \text{floor}(N \cdot \text{rand}(0.7, 0.9))$
$N_p = N - N_h$

2.2. Assign N_h individuals from **A** as herd members.

for $(i = 1{:}N_h)$
    $\mathbf{h}_i = \mathbf{a}_i$
end for

2.3. Assign N_p individuals from **A** as predators.

for $(j = 1{:}N_p)$
    $\mathbf{p}_j = \mathbf{a}_{N_h+j}$
end for

Step 3: Calculate the survival values of each member within $\mathbf{H}$ and $\mathbf{P}$ (section 3.2).

3.1. Calculate survival values for member in **H.**

for $(i = 1{:}N_h)$
    $\text{SV}_{\mathbf{h}_i} = \frac{f(\mathbf{h}_i) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}}$
end for

3.2. Calculate survival values for each member in **P.**

for $(j = 1{:}N_p)$
    $\text{SV}_{\mathbf{p}_j} = \frac{f(\mathbf{p}_j) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}}$
end for

Step 4: Move all members in $\mathbf{H}$ by applying the herd movement operators (section 3.4)

4.1. Selfish herd movement operators.

for $(i = 1{:}N_h)$
    if $\text{SV}_{\mathbf{h}_i^k} = \max_{j \in \{1,2,\ldots,N_h\}} (\text{SV}_{\mathbf{h}_j})$
    4.1.1. $\mathbf{h}_i^k$ is the leader of the herd ($\mathbf{h}_i^k = \mathbf{h}_L^k$). Apply herd's leader movement operators.
        if $(\text{SV}_{\mathbf{h}_L^k} = 1)$
        4.1.1(A). Apply seemingly cooperative leadership movement operators.
            $\mathbf{c}^k = 2 \cdot \alpha \cdot \varphi_{\mathbf{h}_L,\mathbf{p}_M}^k \cdot (\mathbf{p}_M^k - \mathbf{h}_M^k)$
            $\mathbf{h}_L^{k+1} = \mathbf{h}_L^k + \mathbf{c}^k$
        else
        4.1.1(B). Apply openly selfish leadership movement operators.
            $\mathbf{s}^k = 2 \cdot \alpha \cdot \psi_{\mathbf{h}_L,\mathbf{x}_{\text{best}}}^k \cdot (\mathbf{x}_{\text{best}}^k - \mathbf{h}_L^k)$
            $\mathbf{h}_L^{k+1} = \mathbf{h}_L^k + \mathbf{s}^k$
        end if
    else
    4.1.2. Apply herd's following and desertion movement operators.
        if $\text{SV}_{\mathbf{h}_i^k} \geq \text{rand}(0,1)$
        4.1.2(A). $\mathbf{h}_i^k$ is a herd following member ($\mathbf{h}_i^k \in \mathbf{H}_F^k$). Apply herd following movement operators.
            if $\text{SV}_{\mathbf{h}_i^k} \geq \text{SV}_{\mathbf{h}_\mu^k}$
                $\mathbf{f}_i^k = 2 \cdot \left(\beta \cdot \psi_{\mathbf{h}_i,\mathbf{h}_L}^k \cdot (\mathbf{h}_L^k - \mathbf{h}_i^k) + \gamma \cdot \psi_{\mathbf{h}_i,\mathbf{h}_{c_i}}^k \cdot (\mathbf{h}_{c_i}^k - \mathbf{h}_i^k)\right)$
            else
                $\mathbf{f}_i^k = 2 \cdot \delta \cdot \psi_{\mathbf{h}_i,\mathbf{h}_M}^k \cdot (\mathbf{h}_M^k - \mathbf{h}_i^k)$
            end if
            $\mathbf{h}_i^{k+1} = \mathbf{h}_i^k + \mathbf{f}_i^k$
        else
        4.1.2(B). $\mathbf{h}_i^k$ is a herd deserting member ($\mathbf{h}_i^t \in \mathbf{H}_D^k$). Apply herd desertion movement operators.
            $\mathbf{d}_i^k = 2 \cdot \left(\beta \cdot \psi_{\mathbf{h}_i,\mathbf{x}_{\text{best}}}^k \cdot (\mathbf{x}_{\text{best}}^k - \mathbf{h}_i^k) + \gamma \cdot (1 - SV_{\mathbf{h}_i^k}) \cdot \hat{\mathbf{r}}\right)$
            $\mathbf{h}_i^{k+1} = \mathbf{h}_i^k + \mathbf{d}_i^k$
        end if

Step 5: Move all members in **P** by applying the predator movement operators (section 3.5)

5.1. Predators movement operators.

for $(i = 1 : N_p)$

5.1.1. For each member $\mathbf{p}_i$, compute the pursuit probabilities $\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j}$

for $(j = 1 : N_h)$

$$\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j} = \frac{\omega_{\mathbf{p}_i,\mathbf{h}_j}}{\sum_{m=1}^{N_h} \omega_{\mathbf{p}_i,\mathbf{h}_m}}$$

end for

5.1.2. Choose a random member $\mathbf{h}_r^k$ from **H** by applying the roulette selection method according to their pursuit probabilities $\mathcal{P}_{\mathbf{p}_i,\mathbf{h}_j}$ and update the position of $\mathbf{p}_i$.

$$\mathbf{p}_i^{k+1} = \mathbf{p}_i^k + 2 \cdot \rho \cdot (\mathbf{h}_r^k - \mathbf{p}_i^k)$$

end for

Step 6: Re-calculate the survival values of each member within **H** and **P**.

6.1. Re-calculate survival values for each member in **H**.

for $(i = 1 : N_h)$

$$\mathrm{SV}_{\mathbf{h}_i^{k+1}} = \frac{f(\mathbf{h}_i^{k+1}) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}}$$

end for

6.2. Re-calculate survival values for each member in **P**.

for $(i = 1 : N_p)$

$$\mathrm{SV}_{\mathbf{p}_i^{k+1}} = \frac{f(\mathbf{p}_i^{k+1}) - f_{\text{best}}}{f_{\text{best}} - f_{\text{worst}}}$$

end for

Step 7: Perform predation phase (section 3.7)

7.1. Calculate the domain of danger's radius R.

$$R = \frac{\sum_{j=1}^{n} \left| x_j^{\text{low}} - x_j^{\text{high}} \right|}{2 \cdot n}$$

7.2. Initialize the set of killed prey individuals **K** as an empty set.

$\mathbf{K} = \{\emptyset\}$

7.3. For each predator $\mathbf{p}_i \in \mathbf{P}$ choose a valid prey individual $\mathbf{h}_j \in \mathbf{H}$ to be hunted.

for $(i = 1 : N_p)$

7.3.1. Define a set of targeted prey $\mathbf{T}_{\mathbf{p}_i}$ for $\mathbf{p}_i$.

$$\mathbf{T}_{\mathbf{p}_i} = \left\{ \mathbf{h}_j \in \mathbf{H} \middle| \mathrm{SV}_{\mathbf{h}_j} < \mathrm{SV}_{\mathbf{p}_i},\ \left\| \mathbf{p}_i - \mathbf{h}_j \right\| \le R,\ \mathbf{h}_j \notin \mathbf{K} \right\}$$

end for

if $\mathbf{T}_{\mathbf{p}_i}$ is not empty $(\mathbf{T}_{\mathbf{p}_i} \neq \{\emptyset\})$

7.3.2. Compute probabilities of being hunted for each member in $\mathbf{T}_{\mathbf{p}_i}$

for $(j = 1 : |\mathbf{T}_{\mathbf{p}_i}|)$

$$\mathcal{H}_{\mathbf{p}_i,\mathbf{h}_j} = \frac{\omega_{\mathbf{p}_i,\mathbf{h}_j}}{\sum_{(\mathbf{h}_m \in \mathbf{T}_{\mathbf{p}_i})} \omega_{\mathbf{p}_i,\mathbf{h}_m}}, \quad \mathbf{h}_j \in \mathbf{T}_{\mathbf{p}_i}$$

end for

7.3.3. Choose a random member $\mathbf{h}_j \in \mathbf{T}_{\mathbf{p}_i}$ by applying the roulette selection method with regard to their respective probabilities $\mathcal{H}_{\mathbf{p}_i,\mathbf{h}_j}$ and group it within the set **K**.

$\mathbf{K} = \{\mathbf{K}, \mathbf{h}_j\}$

end if

Step 8: Perform restoration phase (section 3.8)

8.1. Define a set of mating candidates **M**.

$\mathbf{M} = \{\mathbf{h}_j \notin \mathbf{K}\}$

8.2. For each member within **M**, calculate its corresponding mating probability $\mathcal{M}_{\mathbf{h}_j}$.

for each $\mathbf{h}_j \in \mathbf{M}$

$$\mathcal{M}_{\mathbf{h}_j} = \frac{\mathrm{SV}_{\mathbf{h}_j}}{\sum_{(\mathbf{h}_m \in \mathbf{M})} \mathrm{SV}_{\mathbf{h}_m}}, \quad \mathbf{h}_j \in \mathbf{M}$$

end for

8.3. Replace each $\mathbf{h}_j \in \mathbf{K}$ with a new solution

for each $\mathbf{h}_j \in \mathbf{k}$

8.3.1. Generate a new solution $\mathbf{h}_{\text{new}}$ by applying SHO's mating operation.

$\mathbf{h}_{\text{new}} = \mathrm{mix}([h_{r_1,1}, h_{r_2,2}, \ldots, h_{r_n,n}])$

8.3.2. Assign $\mathbf{h}_{\text{new}}$ to $\mathbf{h}_j$.

$\mathbf{h}_j = \mathbf{h}_{\text{new}}$

end for

Step 9: If stop criterion is met, the process is finished; otherwise, go back to Step 4.

Table A1
Test functions used for our experiments. In the table, S indicates the subset of $\mathbb{R}^n$ which comprises the function's search space and n indicates the function's dimension. Also, the value $f_i(\mathbf{x}^*)$ indicates the optimum value of each function, while $\mathbf{x}^*$ indicates the optimum position.

Name	Function	S	n	Minimum
Ackley	$f_1(\mathbf{x}) = -20 \cdot \exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n} \cos 2\pi x_i\right) + 20 + \exp$	$[-32.8, 32.8]^n$	30	$f_1(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Sphere	$f_2(\mathbf{x}) = \sum_{i=1}^{n} x_i^2$	$[-100, 100]^n$	30	$f_2(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Sum of Squares	$f_3(\mathbf{x}) = \sum_{i=1}^{n} i x_i^2$	$[-10, 10]^n$	30	$f_3(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Powell	$f_4(\mathbf{x}) = \sum_{i=1}^{n/4}\left[(x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i} - x_{4i})^4\right]$	$[-4, 5]^n$	30	$f_4(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots 0)$
Levy	$f_5(\mathbf{x}) = \cos^2(\pi w_1) + \sum_{i=1}^{n-1}(w_i - 1)^2\,[1 + 10\sin \pi w_i + 1] + (w_n - 1)^2\left[1 + \sin^2 2\pi w_n\right]$ $w_i = 1 + \left(\frac{x_i + 1}{4}\right)$	$[-10, 10]^n$	30	$f_5(\mathbf{x}^*) = 0; \mathbf{x}^* = (1, \ldots, 1)$
Rosenbrock	$f_6(\mathbf{x}) = \sum_{i=1}^{n-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	$[-5, 10]^n$	30	$f_6(\mathbf{x}^*) = 0; \mathbf{x}^* = (1, \ldots, 1)$
Schwefel 2	$f_7(\mathbf{x}) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i} x_i\right)^2$	$[-100, 100]^n$	30	$f_7(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Schwefel 26	$f_8(\mathbf{x}) = 418.9829n - \sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$	$[-500, 500]^n$	30	$f_8(\mathbf{x}^*) = 0; \mathbf{x}^* = (420.968, \ldots, 420.968)$
Trid	$f_9(\mathbf{x}) = \sum_{i=1}^{n}(x_i - 1)^2 - \sum_{i=2}^{n} x_i(x_{i-1})$	$\left[-n^2, n^2\right]^n$	30	$f_9(\mathbf{x}^*) = -4500$
Dixon & Price	$f_{10}(\mathbf{x}) = (x_i - 1)^2 + \sum_{i=1}^{n} i\left(2x_i^2 - x_{i-1}\right)^2$	$[-10, 10]^n$	30	$f_{10}(x_i) = 0 x_i = 2^{-\frac{2^i - 2}{2^i}}$ for $i = 1, \ldots, n$
Rotated Hyper-Ellipsoid	$f_{11}(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{i} x_j^2$	$[-65.5, 65.5]^n$	30	$f_{11}(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Zakharov	$f_{12}(\mathbf{x}) = \sum_{i=1}^{n} x_i^2 + \left(\sum_{i=1}^{n} 0.5 i x_i\right)^2 + \left(\sum_{n=1}^{n} 0.5 i x_i\right)^4$	$[-5, 10]^n$	30	$f_{12}(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Quartic	$f_{13}(\mathbf{x}) = \sum_{i=1}^{n}\left[(i x_i)^4 + rand\right]$	$[-1.28, 1.28]^n$	30	$f_{13}(\mathbf{x}^*) = 0; \mathbf{x}^* = (0.5, \ldots, 0.5)$
Salomon	$f_{14}(\mathbf{x}) = -\cos\left(2\pi\sqrt{\sum_{i=1}^{n} x_i^2}\right) + 0.1\sqrt{\sum_{i=1}^{n} x_i^2 + 1}$	$[-100, 100]^n$	30	$f_{14}(\mathbf{x}^*) = 0; \mathbf{x}^* = (0, \ldots, 0)$
Qing	$f_{15}(\mathbf{x}) = \sum_{i=1}^{n}\left(x_i^2 - i\right)^2$	$[-500, 500]^n$	30	$f_{15}(\mathbf{x}^*) = 0; \mathbf{x}^* = \left(\pm\sqrt{1}, \pm\sqrt{2}, \ldots, \pm\sqrt{n}\right)$

Appendix B. List of benchmark functions

A comprehensive set of 15 functions, which were collected from Storn and Price (1997), Yang and Press (2008), Karaboga and Akay (2009) were used to test the performance of the proposed Selfish Herd Optimizer (SHO) approach. Such test functions are illustrated in Table A1.

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