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Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems

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ABSTRACT

A new efficient optimization method, called 'Teaching–Learning-Based Optimization (TLBO)', is proposed in this paper for the optimization of mechanical design problems. This method works on the effect of influence of a teacher on learners. Like other nature-inspired algorithms, TLBO is also a population-based method and uses a population of solutions to proceed to the global solution. The population is considered as a group of learners or a class of learners. The process of TLBO is divided into two parts: the first part consists of the 'Teacher Phase' and the second part consists of the 'Learner Phase'. 'Teacher Phase' means learning from the teacher and 'Learner Phase' means learning by the interaction between learners. The basic philosophy of the TLBO method is explained in detail. To check the effectiveness of the method it is tested on five different constrained benchmark test functions with different characteristics, four different benchmark mechanical design problems and six mechanical design optimization problems which have real world applications. The effectiveness of the TLBO method is compared with the other population-based optimization algorithms based on the best solution, average solution, convergence rate and computational effort. Results show that TLBO is more effective and efficient than the other optimization methods for the mechanical design optimization problems considered. This novel optimization method can be easily extended to other engineering design optimization problems.

1. Introduction

Engineering design can be characterized as a goal-oriented, constrained, decision making process to create products that satisfy well-defined human needs. Design optimization consists of certain goals (objective functions), a search space (feasible solutions) and a search process (optimization methods). The feasible solutions are the set of all designs characterized by all possible values of the design parameters (design variables). The optimization method searches for the optimal design from all available feasible designs.

Mechanical design includes an optimization process in which designers always consider certain objectives such as strength, deflection, weight, wear, corrosion, etc. depending on the requirements. However, design optimization for a complete mechanical assembly leads to a complicated objective function with a large number of design variables. So it is good practice to apply optimization techniques for individual components or intermediate assemblies rather than a complete assembly. For example, in an automobile power transmission system, the optimization of the gearbox is computationally and mathematically simpler than the optimization of the complete transmission system. Analytical or numerical methods for calculating the extremes of a function have long been applied to engineering computations. Although these methods perform well in many practical cases, they may fail in more complex design situations. In real design problems the number of design variables can be very large, and their influence on the objective function to be optimized can be very complicated, with a nonlinear character. The objective function may have many local optima, whereas the designer is interested in the global optimum. Such problems cannot be handled by classical methods (e.g. gradient methods) that only compute local optima. So there remains a need for efficient and effective optimization methods for mechanical design problems. Continuous research is being conducted in this field and nature-inspired heuristic optimization methods are proving to be better than deterministic methods and thus are widely used.

There are many nature-inspired optimization algorithms, such as the Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Ant Colony Optimization (ACO), Harmony Search (HS), the Grenade Explosion Method (GEM), etc., working on the principles of different natural phenomena. GA uses the theory of Darwin based on the survival of the fittest [1,2], PSO implements the foraging behavior of a bird for searching food [3,4], ABC uses the foraging behavior of a honey bee [5–7], ACO works on the behavior of an ant in searching for a destination from

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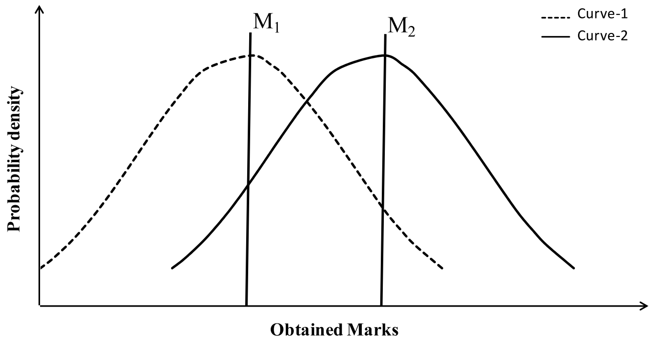

Fig. 1. Distribution of marks obtained by learners taught by two different teachers.

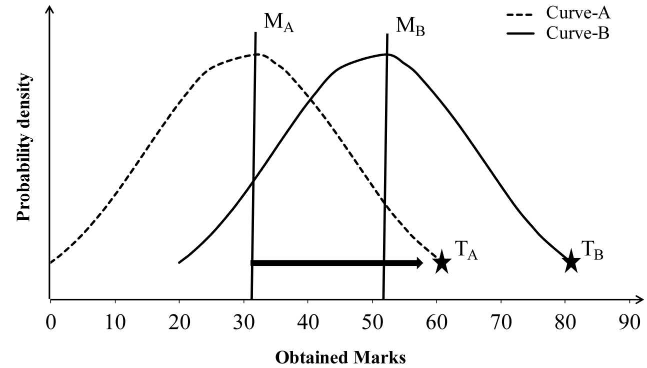

Fig. 2. Model for the distribution of marks obtained for a group of learners.

the source [8,9], HS works on the principle of music improvisation in music players [10] and GEM works on the principle of the explosion of a grenade [11]. These algorithms have been applied to many engineering optimization problems and proved effective in solving some specific kinds of problem.

The most commonly used evolutionary optimization technique is the genetic algorithm (GA). However, GA provides a near optimal solution for a complex problem having large number of variables and constraints. This is mainly due to the difficulty in determining the optimum controlling parameters such as population size, crossover rate and mutation rate. A change in the algorithm parameters changes the effectiveness of the algorithm. The same is the case with PSO, which uses inertia weight, social and cognitive parameters. Similarly, ABC requires optimum controlling parameters of number of bees (employed, scout, and onlookers), limit, etc. HS requires the harmony memory consideration rate, pitch adjusting rate, and the number of improvisations. Therefore, the efforts must be continued to develop a new optimization technique which is free from the algorithm parameters, i.e. no algorithm parameters are required for the working of the algorithm. This aspect is considered in the present work.

The main motivation to develop a nature-based algorithm is its capacity to solve different optimization problems effectively and efficiently. It is assumed that the behavior of nature is always optimum in its performance. In this paper a new optimization method, Teaching–Learning-Based Optimization (TLBO), is proposed to obtain global solutions for continuous non-linear functions with less computational effort and high consistency. The TLBO method is based on the effect of the influence of a teacher on the output of learners in a class. Here, output is considered in terms of results or grades. The teacher is generally considered as a highly learned person who shares his or her knowledge with the learners. The quality of a teacher affects the outcome of the learners. It is obvious that a good teacher trains learners such that they can have better results in terms of their marks or grades.

2. Teaching–learning-based optimization

Assume two different teachers, T_1 and T_2, teaching a subject with the same content to the same merit level learners in two different classes. Fig. 1 shows the distribution of marks obtained by the learners of two different classes evaluated by the teachers. Curves 1 and 2 represent the marks obtained by the learners taught by teacher T_1 and T_2 respectively. A normal distribution is assumed for the obtained marks, but in actual practice it can have skewness. The normal distribution is defined as

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{1}$$

where σ^2 is the variance, μ is the mean and x is any value for which the normal distribution function is required.

It is seen from Fig. 1 that curve-2 represents better results than curve-1 and so it can be said that teacher T_2 is better than teacher T_1 in terms of teaching. The main difference between both the results is their mean (M_2 for Curve-2 and M_1 for Curve-1), i.e. a good teacher produces a better mean for the results of the learners. Learners also learn from interaction between themselves, which also helps in their results.

Based on the above teaching process, a mathematical model is prepared and implemented for the optimization of a unconstrained non-linear continuous function, thereby developing a novel optimization technique called Teaching–Learning-Based Optimization (TLBO). Consider Fig. 2, which shows a model for the marks obtained for learners in a class with curve-A having mean M_A. The teacher is considered as the most knowledgeable person in the society, so the best learner is mimicked as a teacher, which is shown by T_A in Fig. 2. The teacher tries to disseminate knowledge among learners, which will in turn increase the knowledge level of the whole class and help learners to get good marks or grades. So a teacher increases the mean of the class according to his or her capability. In Fig. 2, teacher T_A will try to move mean M_A towards their own level according to his or her capability, thereby increasing the learners' level to a new mean M_B. Teacher T_A will put maximum effort into teaching his or her students, but students will gain knowledge according to the quality of teaching delivered by a teacher and the quality of students present in the class. The quality of the students is judged from the mean value of the population. Teacher T_A puts effort in so as to increase the quality of the students from M_A to M_B, at which stage the students require a new teacher, of superior quality than themselves, i.e. in this case the new teacher is T_B. Hence, there will be a new curve-B with new teacher T_B.

Like other nature-inspired algorithms, TLBO is also a population-based method that uses a population of solutions to proceed to the global solution. For TLBO, the population is considered as a group of learners or a class of learners. In optimization algorithms, the population consists of different design variables. In TLBO, different design variables will be analogous to different subjects offered to learners and the learners' result is analogous to the 'fitness', as in other population-based optimization techniques. The teacher is considered as the best solution obtained so far.

The process of TLBO is divided into two parts. The first part consists of the 'Teacher Phase' and the second part consists of the 'Learner Phase'. The 'Teacher Phase' means learning from the teacher and the 'Learner Phase' means learning through the interaction between learners (Fig. 3).

2.1. Teacher phase

As shown in Fig. 2, the mean of a class increases from M_A to M_B depending upon a good teacher. A good teacher is one who brings

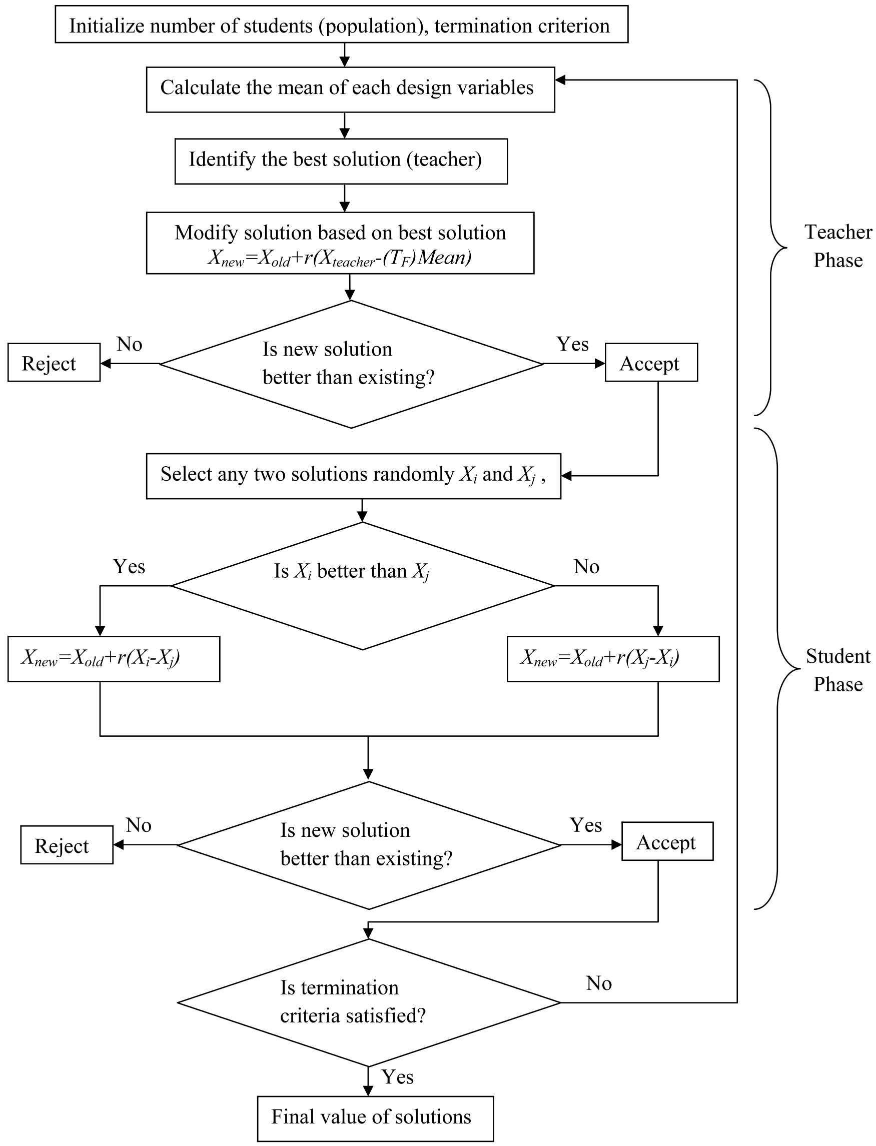

Fig. 3. Flow chart for Teaching–Learning-Based Optimization (TLBO).

his or her learners up to his or her level in terms of knowledge. But in practice this is not possible and a teacher can only move the mean of a class up to some extent depending on the capability of the class. This follows a random process depending on many factors.

Let M_i be the mean and T_i be the teacher at any iteration *i*. T_i will try to move mean M_i towards its own level, so now the new mean will be T_i designated as M_{new}. The solution is updated according to the difference between the existing and the new mean given by

$$\text{Difference\_Mean}_i = r_i\,(M_{\text{new}} - T_F M_i) \tag{2}$$

where T_F is a teaching factor that decides the value of mean to be changed, and r_i is a random number in the range [0, 1]. The value of T_F can be either 1 or 2, which is again a heuristic step and decided randomly with equal probability as $T_F = \text{round}[1 + \text{rand}(0, 1)\{2 - 1\}]$.

This difference modifies the existing solution according to the following expression

$$X_{\text{new},i} = X_{\text{old},i} + \text{Difference\_Mean}_i. \tag{3}$$

2.2. Learner phase

Learners increase their knowledge by two different means: one through input from the teacher and the other through interaction between themselves. A learner interacts randomly with other learners with the help of group discussions, presentations, formal communications, etc. A learner learns something new if the other learner has more knowledge than him or her. Learner modification is expressed as

For $i = 1 : P_n$
Randomly select two learners X_i and X_j, where $i \neq j$
If $f(X_i) < f(X_j)$
$X_{\text{new},i} = X_{\text{old},i} + r_i(X_i - X_j)$
Else
$X_{\text{new},i} = X_{\text{old},i} + r_i(X_j - X_i)$
End If
End For
Accept X_{new} if it gives a better function value.

3. Implementation of TLBO for optimization

The step-wise procedure for the implementation of TLBO is given in this section.

Step 1: Define the optimization problem and initialize the optimization parameters.

Initialize the population size (P_n), number of generations (G_n), number of design variables (D_n), and limits of design variables (U_L, L_L).

Define the optimization problem as: Minimize $f(X)$.

Subject to $X_i \in x_i = 1, 2, \ldots, D_n$

where $f(X)$ is the objective function, X is a vector for design variables such that $L_{L,i} \le x_{,i} \le U_{L,I}$.

Step 2: Initialize the population.

Generate a random population according to the population size and number of design variables. For TLBO, the population size indicates the number of learners and the design variables indicate the subjects (i.e. courses) offered. This population is expressed as

$$\text{population} = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,D} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,D} \\ \vdots & & \vdots & \vdots \\ x_{P_n,1} & x_{P_n,2} & \cdots & x_{P_n,D} \end{bmatrix}.$$

Step 3: Teacher phase.

Calculate the mean of the population column-wise, which will give the mean for the particular subject as

$$M_{,D} = [m_1, m_2, \ldots, m_D]. \tag{4}$$

The best solution will act as a teacher for that iteration

$$X_{\text{teacher}} = X_{f(X)=\min}. \tag{5}$$

The teacher will try to shift the mean from $M_{,D}$ towards $X_{,\text{teacher}}$, which will act as a new mean for the iteration. So,

$$M\_\text{new}_{,D} = X_{\text{teacher},D}. \tag{6}$$

The difference between two means is expressed as

$$\text{Difference}_{,D} = r\,(M\_\text{new}_{,D} - T_F M_{,D}). \tag{7}$$

The value of T_F is selected as 1 or 2. The obtained difference is added to the current solution to update its values using

$$X_{\text{new},D} = X_{\text{old},D} + \text{Difference}_{,D}. \tag{8}$$

Accept X_{new} if it gives better function value.

Step 4: Learner phase.

As explained above, learners increase their knowledge with the help of their mutual interaction. The mathematical expression is explained in Section 2.2.

Step 5: Termination criterion.

Stop if the maximum generation number is achieved; otherwise repeat from Step 3.

It is seen from the above steps that no provision is made to handle the constraints in the problem. Many types of constraint handling technique are available in the literature, such as incorporation of static penalties, dynamic penalties, adaptive penalties etc. Deb's heuristic constrained handling method [12] is used in the proposed TLBO method. This method uses a tournament selection operator in which two solutions are selected and compared with each other. The following three heuristic rules are implemented on them for the selection:

- If one solution is feasible and the other infeasible, then the feasible solution is preferred.
- If both the solutions are feasible, then the solution having the better objective function value is preferred.
- If both the solutions are infeasible, then the solution having the least constraint violation is preferred.

These rules are implemented at the end of Steps 2 and 3, i.e. at the end of the teacher phase and the learner phase. Instead of accepting solution X_{new}, if it gives better function value at the end of Steps 2 and 3, Deb's constraint handling rules [12] are used to select X_{new} based on the three heuristic rules.

4. Comparison of TLBO with other optimization techniques

Like GA, PSO, ABC, and HS, TLBO is also a population-based technique which implements a group of solutions to proceed to the optimum solution. Many optimization methods require algorithm parameters that affect the performance of the algorithm. GA requires the crossover probability, mutation rate, and selection method; PSO requires learning factors, the variation of weight, and the maximum value of velocity; ABC requires the limit value; and HS requires the harmony memory consideration rate, pitch adjusting rate, and number of improvisations. Unlike other optimization techniques TLBO does not require any algorithm parameters to be tuned, thus making the implementation of TLBO simpler. As in PSO, TLBO uses the best solution of the iteration to change the existing solution in the population, thereby increasing the convergence rate. TLBO does not divide the population like ABC. As in GA, which uses selection, crossover and mutation, and ABC, which uses employed, onlooker and scout bees, TLBO uses two different phases, the 'teacher phase' and the 'learner phase'. TLBO uses the mean value of the population to update the solution. TLBO implements greediness to accept a good solution, as in ABC.

5. Experimental studies

Different experiments have been conducted to check the effectiveness of TLBO against other optimization techniques. Different examples are investigated based on benchmark test functions, mechanical design benchmark functions and other mechanical design problems from the literature.

5.1. Constrained benchmark test functions

Five different constrained benchmark functions with different characteristics of objective functions and constraints (linear, non-linear, and quadratic) were experimented upon. These benchmark functions are given in Appendix A. The special features of these benchmark functions are discussed in detail in the following sub-sections.

5.1.1. Benchmark function 1

This is a quadratic minimization problem with 13 design variables and 9 linear inequality constraints. The ratio of the feasible search space to the entire search space is approximately 0.0003% [13] and there are 6 active constraints at the optimum point. The optimum solution for this problem is at $x^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1)$ with objective function value $f(x^*) = -15$. This problem was solved by the different optimization methods: Multimembered Evolutionary Strategy (M-ES) [14], Particle Evolutionary Swarm Optimization (PESO) [15], Cultural Differential Evolution (CDE) [16], Co-evolutionary Differential Evolution (CoDE) [17] and Artificial Bee Colony (ABC) [18]. The results are shown in Table 1. The parameters for TLBO are set as: population size = 50, maximum number of generations = 500. It can be observed from Table 1 that TLBO finds the global optimum solution with better mean and worst solutions than PESO and CDE. Moreover TLBO requires approximately 89%, 92%, 75%, 90% and 89% fewer function evaluations than M-ES, PESO, CDE, CoDE and ABC respectively.

5.1.2. Benchmark function 2

This is a nonlinear maximization problem with 10 design variables and one nonlinear equality constraint. The ratio of the feasible search space to the entire search space is approximately 0.0000% [13] and there is 1 active constraint at the optimum point.

Table 1
Comparison of results obtained by different optimization methods for benchmark function 1.

Methods	Best	Mean	Worst	Function evaluations
M-ES	−15	−15	−15	240 000
PESO	−15	−14.710	−13	350 000
CDE	−15	−14.999996	−14.999993	100 100
CoDE	−15	−15	−15	248 000
ABC	−15	−15	−15	240 000
TLBO	−15	−15	−15	25 000

Table 2
Comparison of results obtained by different optimization methods for benchmark function 2.

Methods	Best	Mean	Worst	Function evaluations
M-ES	1	1	1	240 000
PESO	0.993930	0.764813	0.464	350 000
CDE	0.995413	0.788635	0.639920	100 100
ABC	1	1	1	240 000
TLBO	1	1	1	100 000

Table 3
Comparison of results obtained by different optimization methods for benchmark function 3.

Methods	Best	Mean	Worst	Function evaluations
M-ES	680.632	680.643	680.719	240 000
PESO	680.630	680.630	680.631	350 000
CDE	680.63006	680.63006	680.6301	100 100
CoDE	680.771	681.503	685.144	248 000
ABC	680.634	680.640	680.653	240 000
TLBO	680.630	680.633	680.638	100 000

The global maximum is at $x^* = (1/\sqrt{n}, 1/\sqrt{n}, 1/\sqrt{n}, \ldots)$ with objective function value $f(x^*) = 1$. The equality constraint is converted into inequality constraints as $|h| \leq \varepsilon$, where $\varepsilon = 0.001$. This problem was solved by the different optimization methods: Multimembered Evolutionary Strategy (M-ES) [14], Particle Evolutionary Swarm Optimization (PESO) [15], Cultural Differential Evolution (CDE) [16], and Artificial Bee Colony (ABC) [18]. The results are shown in Table 2. The parameters for TLBO are set as: population size = 50, maximum number of generations = 2000. It can be observed from Table 2 that TLBO finds the global optimum solution with better best, mean and worst solutions than PESO and CDE. Moreover, the results for TLBO are same as the results for M-ES and ABC, but TLBO requires approximately 58% fewer function evaluations than M-ES and ABC.

5.1.3. Benchmark function 3

This is a nonlinear minimization problem with 7 design variables and 4 nonlinear inequality constraints. The ratio of the feasible search space to the entire search space is approximately 0.5256% [13] and there are 2 active constraints at the optimum point. The optimum solution is at $x^* = (2.330499, 1.951372, -0.4775414, 4.365726, -0.6244870, 1.1038131, 1.594227)$ with objective function value $f(x^*) = 680.6300573$. This problem was solved by the different optimization methods: Multimembered Evolutionary Strategy (M-ES) [14], Particle Evolutionary Swarm Optimization (PESO) [15], Cultural Differential Evolution (CDE) [16], Co-evolutionary Differential Evolution (CoDE) [17] and Artificial Bee Colony (ABC) [18]. The results are shown in Table 3. The parameters for TLBO are set as: population size = 50, maximum number of generations = 2000. It can be observed from Table 3 that TLBO finds the global optimum solution with better best, mean and worst solutions than M-ES, CoDE and ABC. Moreover, the results for TLBO are the same as the results for PESO and CDE, but TLBO requires approximately 71%, fewer function evaluations than PESO and nearly same as CDE.

Table 4
Comparison of results obtained by different optimization methods for benchmark function 4.

Methods	Best	Mean	Worst	Function evaluations
M-ES	7051.903	7253.047	7638.366	240 000
PESO	7049.38	7205.5	7894.812	350 000
CDE	7049.2481	7049.2483	7049.2485	100 100
ABC	7053.904	7224.407	7604.132	240 000
TLBO	7049.2481	7083.6732	7224.4968	100 000

Table 5
Comparison of results obtained by different optimization methods for benchmark function 5.

Methods	Best	Mean	Worst	Function evaluations
M-ES	1	1	1	240 000
PESO	1	0.998875	0.994	350 000
CDE	1	1	1	100 100
CoDE	1	1	1	248 000
ABC	1	1	1	240 000
TLBO	1	1	1	50 000

5.1.4. Benchmark function 4

This is a linear minimization problem with 8 design variables and 3 nonlinear inequality and 3 linear inequality constraints. The ratio of feasible search space to entire search space is approximately 0.0005% [13] and there are 3 active constraints at the optimum point. The optimum solution is at $x^* = (579.3066, 1359.9709, 5109.9707, 182.0177, 295.601, 217.982, 286.165, 395.6012)$ with objective function value $f(x^*) = 7049.248021$. The equality constraint is converted into inequality constraints as $|h| \leq \varepsilon$ where $\varepsilon = 0.000001$. This problem was solved by the different optimization methods: Multimembered Evolutionary Strategy (M-ES) [14], Particle Evolutionary Swarm Optimization (PESO) [15], Cultural Differential Evolution (CDE) [16], and Artificial Bee Colony (ABC) [18]. The results are shown in Table 4. The parameters for TLBO are set as: population size = 50, maximum number of generations = 2000. It can be observed from Table 4 that TLBO finds the global optimum solution with better best, mean and worst solutions than M-ES, PESO and ABC. Moreover, the results from TLBO are the same as the results from CDE for the best solution, but CDE shows better results than TLBO for the mean and worst solutions.

5.1.5. Benchmark function 5

This is a quadratic maximization problem with 3 design variables and $9^3 = 729$ nonlinear inequality constraints. The ratio of feasible search space to entire search space is approximately 4.779% [13] and there are no active constraints at the optimum point. The optimum solution is at $x^* = (5, 5, 5)$ with objective function value $f(x^*) = 1$. This problem was solved by the different optimization methods: Multimembered Evolutionary Strategy (M-ES) [14], Particle Evolutionary Swarm Optimization (PESO) [15], Cultural Differential Evolution (CDE) [16], and Artificial Bee Colony (ABC) [18]. The results are shown in Table 5. The parameters for TLBO are set as: population size = 50, maximum number of generations = 1000. It can be observed from Table 5 that TLBO finds the global optimum solution with better mean and worst solutions than PESO. Moreover, the results from TLBO are the same as the results of M-ES, CDE, CoDE and ABC, but TLBO requires approximately 79%, 50%, 79%, and 79% fewer function evaluations than M-ES, CDE, CoDE and ABC respectively.

5.2. Constrained benchmark mechanical design problems

Four different constrained benchmark mechanical design problems with different characteristics of objective function and constraints (linear and nonlinear) were experimented upon. These

Table 6
Comparison of results obtained by different optimization methods for benchmark mechanical design problems 1–4.

Problem		$(\mu + \lambda)$-ES [19]	UPSO [20]	CPSO [21]	CoDE [17]	PSO-DE [13]	ABC [22]	TLBO
Welded Beam	Best	**1.724852**	1.92199	1.728	1.73346	**1.72485**	**1.724852**	**1.724852**
	Mean	1.777692	2.83721	1.74883	1.76815	**1.72485**	1.741913	1.72844676
	Evaluations	30 000	100 000	200 000	240 000	33 000	30 000	**10 000**
Pressure Vessel	Best	**6059.7016**	6544.27	6061.077	6059.734	6059.714	6059.714	6059.714335
	Mean	6379.938	9032.55	6147.1332	6085.23	**6059.714**	6245.308	**6059.71434**
	Evaluations	30 000	100 000	200 000	240 000	42 100	30 000	**10 000**
Tension Compression Spring	Best	0.012689	0.01312	0.012674	0.01267	**0.012665**	**0.012665**	**0.012665**
	Mean	0.013165	0.02294	0.01273	0.012703	**0.012665**	0.012709	**0.01266576**
	Evaluations	30 000	100 000	200 000	240 000	24 950	30 000	**10 000**
Gear train	Best	**2996.348**	NA	**NA**	NA	**2996.348**	2997.058	**2996.34817**
	Mean	**2996.348**	NA	NA	NA	**2996.348**	2997.058	**2996.34817**
	Evaluations	30 000	NA	NA	NA	54 350	30 000	**10 000**

The data in bold indicate the best solution.

Table 7
Variation of $f(a)$ with a.

a	≤ 1.4	1.5	1.6	1.7	1.8	1.9	2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	≥ 2.8
$f(a)$	1	0.85	0.77	0.71	0.66	0.63	0.6	0.58	0.56	0.55	0.53	0.52	0.51	0.51	0.5

benchmark design problems are given in Appendix B. Some of the problems have mixed discrete-continuous design variables. These problems are used by many researchers to test the performance of different algorithms. All these problems are discussed in detail in the following sub-sections.

5.2.1. Design of a pressure vessel

The objective is to minimize the total cost of a pressure vessel considering the cost of material, forming and welding. This problem has a nonlinear objective function with 3 linear and one nonlinear inequality constraints and two discrete and two continuous design variables.

5.2.2. Design of tension/compression spring

The objective is to minimize the weight of a tension/compression spring subjected to one linear and three nonlinear inequality constraints with three continuous design variables.

5.2.3. Design of welded beam

The objective is to design a welded beam for minimum cost. There are four continuous design variables with two linear and five nonlinear inequality constraints.

5.2.4. Design of gear train

The objective is to minimize the weight of a gear train with one discrete and six continuous design variables. There are 4 linear and 7 nonlinear inequality constraints. The peculiarity of this problem is that there are four active constraints at the best known feasible solution.

The aforementioned mechanical benchmark problems have been attempted by many researchers, but in this paper the effectiveness of the results of TLBO is compared with the results of research published since 2004. As PSO, DE, ES and ABC are well-known optimization algorithms, many researchers have tried to enhance the performance of the basic algorithms by modifications between the years 2005 and 2010. Efforts are ongoing to modify or hybridize these well-known algorithms to increase their effectiveness and efficiency. The aforementioned mechanical design problems were attempted by $(\mu+\lambda)$-Evolutionary Strategy (ES) [19], Unified Particle Swarm Optimization (UPSO) [20], Co-evolutionary Particle Swarm Optimization (CPSO) [21], Co-evolutionary Differential Evolution (CoDE) [17], Hybrid PSO-DE [13] and Artificial Bee Colony (ABC) [22].

All optimization algorithms require tuning of different algorithm parameters for their proper functioning. It should be noted that the selection of the algorithm parameters plays a very important role in the performance of an optimization algorithm. A small change in algorithm parameters may result in a large change in the performance of the algorithm. TLBO overcomes such difficulties as tuning and selecting the proper algorithm parameters. TLBO works such that it does not require any algorithm parameters. TLBO only requires the population size and maximum number of generations and these are set as 50 and 200 respectively.

TLBO is compared with the aforementioned methods for the best solution, mean solution and maximum number of function evaluations required to find the optimum solution. For TLBO, 100 independent runs are carried out to check the performance of the algorithm. The results for the comparison of all the methods are shown in Table 6. It is observed form the results that TLBO give the best solution for all the problems except for the pressure vessel problem, for which $(\mu + \lambda)$-ES has shown better performance. However, for the pressure vessel problem the average performance of TLBO is better than $(\mu + \lambda)$-ES. The average performance of TLBO and PSO-DE is same for all the problems, except for welded beam problem for which PSO-DE is better. TLBO requires 66%, 70% and 66% fewer function evaluations than $(\mu + \lambda)$-ES, PSO-DE and ABC respectively. So it can be concluded that TLBO require fewer function evaluations and that it also requires no algorithm parameters.

5.3. Constrained mechanical design problems from literature

Several further mechanical design problems were considered, including the Belleville spring, rolling bearing, thrust bearing, multiplate clutch disc brake, welded stiffened shells and robot gripper. All the considered problems have different natures of objective functions, constraints and design variables. The multiple disc clutch brake is a minimization problem with all discrete variables. The robot gripper problem has an objective function such that it has to select the minimum and maximum from the set of available values of objective function and it varies according to the harmonic function. The step-cone pulley is a minimization problem with three equality constraints and 8 inequality constraints. The hydrodynamic thrust bearing is a minimization problem with a logarithmically varying objective function and constraints. The rolling element bearing is a maximization problem with mixed discrete–continuous type design variables. The Belleville spring problem is a minimization problem in which one parameter existing in

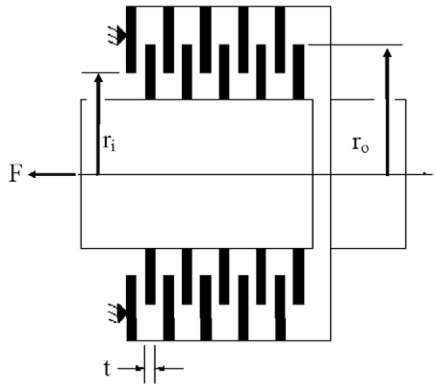

Fig. 4. Multiple disc clutch brake [23].

the constraints is to be selected according to the design variable ratios. All the problems are taken from the available literature, but the mathematical formulation is repeated to give completeness to this paper. The details are given in Appendix C.

5.3.1. Multiple disc clutch brake

This problem is taken from [23]. Fig. 4 shows a multiple disc clutch brake. The objective is to minimize the mass of the multiple disc clutch brake using five discrete variables: inner radius ($r_i = 60, 61, 62, \ldots, 80$), outer radius ($r_o = 90, 91, 92, \ldots, 110$), thickness of discs ($t = 1, 1.5, 2, 2.5, 3$), actuating force ($F = 600, 610, 620, \ldots, 1000$) and number of friction surfaces ($Z = 2, 3, 4, 5, 6, 7, 8, 9$).

5.3.2. Robot gripper

The objective is to minimize the difference between the maximum and minimum force applied by the gripper for the range of gripper end displacements. There are 7 continuous design variables (a, b, c, d, e, f, δ), as shown in Fig. 5. There are six different constraints associated with the robot gripper problem.

5.3.3. Step-cone pulley

The objective is to design a 4 step-cone pulley with minimum weight using 5 design variables, consisting of four design variables for the diameters of each step, with the fifth being the width of the pulley. Fig. 6 shows a step-cone pulley. It is assumed in this example that the widths of the cone pulley and belt are the same. There are 11 constraints, out of which 3 are equality constraints and the remainder are inequality constraints. The constraints are to assure the same belt length for all the steps, tension ratios, and power transmitted by the belt. The step pulley is designed to transmit at least 0.75 hp ($0.75 * 745.6998$ W), with an input speed of 350 rpm and output speeds of 750, 450, 250 and 150 rpm. The problem is taken from [25].

5.3.4. Hydrodynamic thrust bearing

The objective is to minimize the power loss. There are four design variables: bearing step radius (R), recess radius (R_o), oil viscosity (μ) and flow rate (Q). Fig. 7 shows a hydrodynamic thrust bearing. Seven different constraints are associated with the problem, based on load carrying capacity, inlet oil pressure, oil temperature rise, oil film thickness and physical constraints.

5.3.5. Rolling element bearing

The objective is to maximize the dynamic load carrying capacity of a rolling element bearing. A detailed discussion of the problem is given in [27]. The design variables are ball diameter (D_b), pitch diameter (D_m), inner and outer raceway curvature coefficients (f_i and f_o), and number of balls (Z), as shown in Fig. 8. Moreover, there are many parameters such as $K_{D\min}$, $K_{D\max}$, ε, e, and ζ that only appear in constraints and indirectly affect the internal geometry. Therefore, a total of 10 design variables is taken, out of which Z is the discrete design variable and the remainder are continuous design variables. Constraints are imposed based on kinematic and manufacturing considerations.

5.3.6. Belleville spring

The objective is to design a Belleville spring having minimum weight and satisfying a number of constraints. The problem has 4 design variables: external diameter of the spring (D_e), internal diameter of the spring (D_i), thickness of the spring (t), and the height (h) of the spring, as shown in Fig. 9. Of these design variables, t is a discrete variable and the remainder are continuous variables. The constraints are for compressive stress, deflection, height to deflection, height to maximum height, outer diameter, inner diameter, and slope.

5.3.7. Results of mechanical design problems from literature

All the problems are attempted in this work using TLBO and ABC with different population sizes and the maximum number of generations decided based on several trials. For ABC the modification rate is taken as 0.9 and the limit as '*population size* $*$ *number of designvariables* $*$ 5'. The results were obtained for 100 independent runs. Results are compared based on the best, mean and worst solutions in a predefined number of function evaluations, and the success rate of the algorithm. An algorithm is said to be successful if it finds the best optimum value. Moreover, comparison is done for the convergence rate of the algorithm. Here, graphs are obtained for the function value with number of generations. As TLBO and ABC are heuristic methods, function

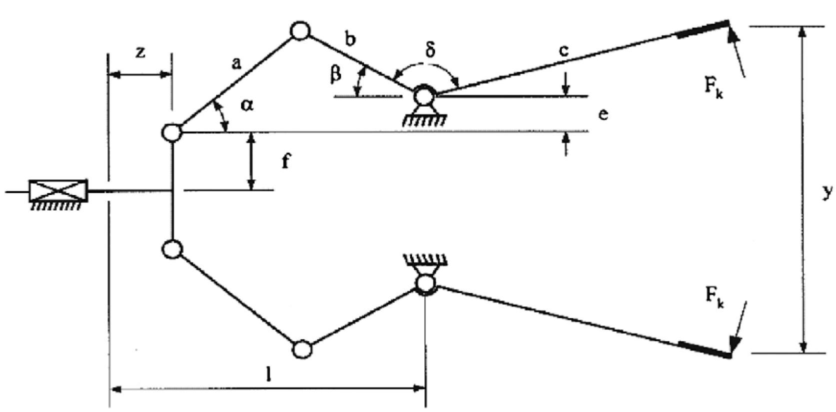

Fig. 5. Robot gripper [24].

Table 8
Comparison of results for mechanical design problems obtained using the TLBO and ABC algorithms for examples 1–6.

	TLBO				ABC			
	Best	Mean	Worst	SR	Best	Mean	Worst	SR
Multiple disc clutch brake	0.313657	0.3271662	0.392071	0.67	0.313657	0.324751	0.352864	0.54
Robot gripper	4.247644	4.93770095	8.141973	0.56	4.247644	5.086611	6.784631	0.07
Step-cone pulley	16.63451	24.0113577	74.022951	0.34	16.634655	36.0995	145.4705	0.06
Hydrostatic thrust bearing	1625.443	1797.70798	2096.80127	0.19	1625.44276	1861.554	2144.836	0.05
Rolling element bearing	81 859.74	81 438.987	80 807.8551	0.66	81 859.7416	81 496	78 897.81	0.69
Belleville spring	1.979675	1.97968745	1.979757	0.45	1.979675	1.995475	2.104297	0.07

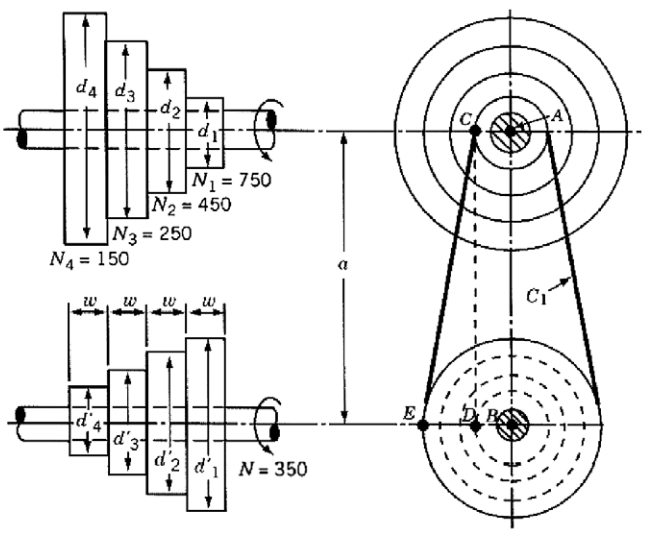

Fig. 6. Step-cone pulley [25].

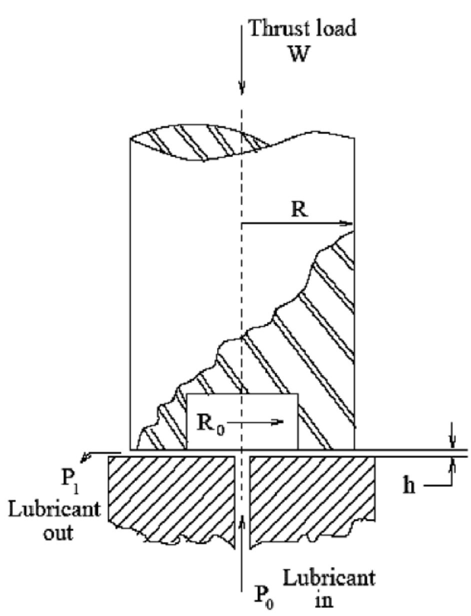

Fig. 7. Hydrodynamic thrust bearing [26].

values to be plotted in the convergence graphs are obtained by averaging the function values for each generation over 20 independent runs.

The problem of the multiple clutch brake was also attempted by Deb and Srinivasan [28] using NSGA-II. The value of minimum mass reported by them is 0.4704 kg, with $r_i = 70$ mm, $r_o = 90$ mm, $t = 1.5$ mm, $F = 1000$ N and $Z = 3$. The population size and number of generations are kept as 20 and 30 for both TLBO and ABC. The best value of the mass of the multiple disc clutch brake reported

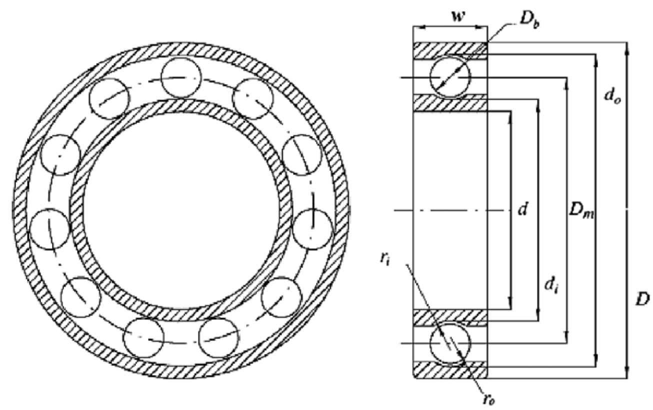

Fig. 8. Rolling element bearing [27].

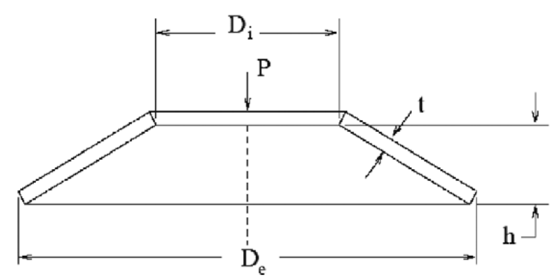

Fig. 9. Belleville spring [26].

by using TLBO is 0.313 kg, which is better than the previously published results, with $r_i = 70$ mm, $r_o = 90$ mm, $t = 1$ mm, $F = 810$ N and $Z = 3$. It is observed from Table 8 that for the multiple disc clutch brake both ABC and TLBO give better results than those available in the literature. The mean and worst solutions obtained by both the algorithms are almost same with ABC slightly better. It is also observed that the success rate of TLBO is 24.1% higher than that of ABC, which means that TLBO finds the optimum solution on more occasions than ABC. From Fig. 10 it can be seen that the convergence rate of TLBO is faster than ABC in earlier generations, but with an increase in the number of generations the convergence of both algorithms becomes nearly the same.

The robot gripper problem was attempted by Osyczka et al. [24] using GA with a population size of 400 and the number of generations as 400, i.e. requiring 160 000 function evaluations. The value of the objective function was 5.02 N with $a = 150, b = 131.1, c = 196.5, e = 12.94, f = 133.80, l = 175$ and $\delta = 2.60$. Now the same problem is attempted using TLBO and ABC with a population size of 50 and maximum number of generations as 500, requiring 25 000 function evaluations. The best obtained value using TLBO is 4.2476 N with $a = 150, b = 150, c = 200, e = 0, f = 150, l = 100$ and $\delta = 2.339539$. It is observed from Table 8 that the mean and worst values obtained using TLBO are better than ABC. Also the success rate of TLBO is approximately 700% higher than ABC. From Fig. 11 it can be seen that the convergence

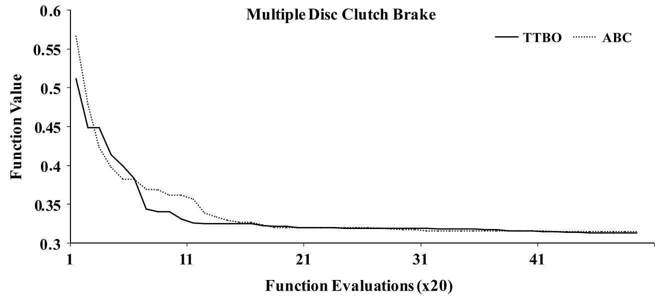

Fig. 10. Convergence plots for the multiple disc clutch brake using the TLBO and ABC algorithms (function value = weight of multiple disc clutch brake (kg)).

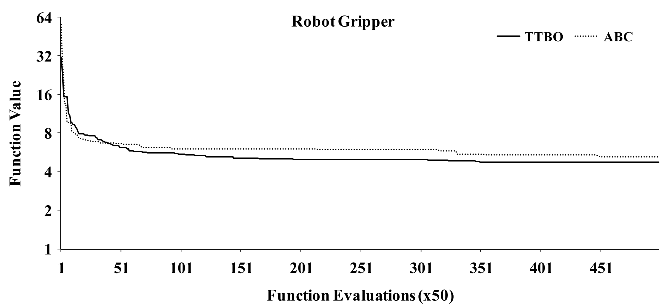

Fig. 11. Convergence plots for the robot gripper using the TLBO and ABC algorithms (function value = gripping force (N)).

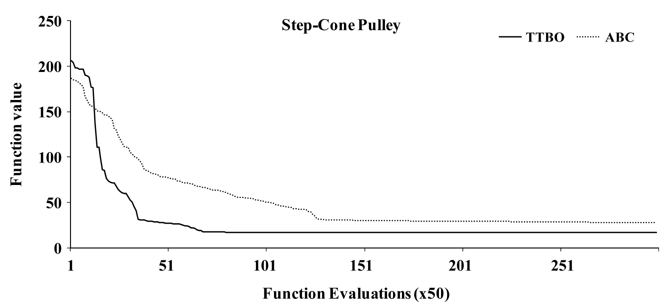

Fig. 12. Convergence plots for the step-cone pulley using the TLBO and ABC algorithms (function value = weight of step-cone pulley (kg)).

of TLBO and ABC is almost the same in the initial generations, but as the number of generations increase the convergence of TLBO is better than ABC.

The step-cone pulley problem is taken from [25]. This is an interesting problem as there are 3 equality constraints along with inequality constraints, thus imposing complexity on the problem. This problem is attempted using TLBO and ABC with the population size of 50 and the number of generations as 300. The best obtained value of objective function using TLBO is 16.63451 kg with d_1 = 40 mm, d_2 = 54.7643 mm, d_3 = 73.01318 mm, d_4 = 88.42842 mm and w = 85.98624 mm. It is observed from Table 8 that TLBO gives better results than ABC for the best, mean and worst solutions. Moreover, TLBO has an approximately 470% better success rate than ABC. From Fig. 12 it is clear that the convergence rate of TLBO is considerably better than ABC for the step-cone pulley design problem.

The problem of the hydrostatic bearing was attempted by He et al. [29] using improved PSO, by Coello [26] using new constraint handling techniques, and by Deb and Goyal [30] using GeneAS. The best reported results are by He et al. [29] with the function value of 1632.2149 and R_o = 5.56868685, R_i = 5.389175395, μ = 5.40213310 and Q = 2.30154678 using

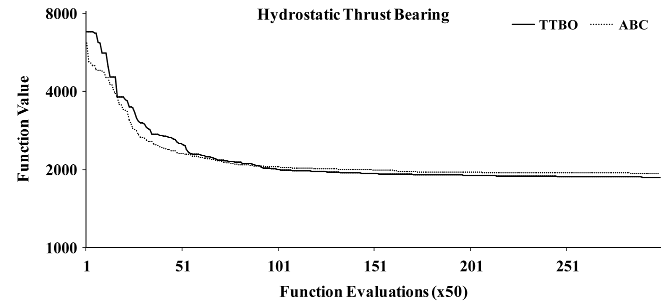

Fig. 13. Convergence plots for the hydrostatic thrust bearing using the TLBO and ABC algorithms (function value = power loss (ft lb/s)).

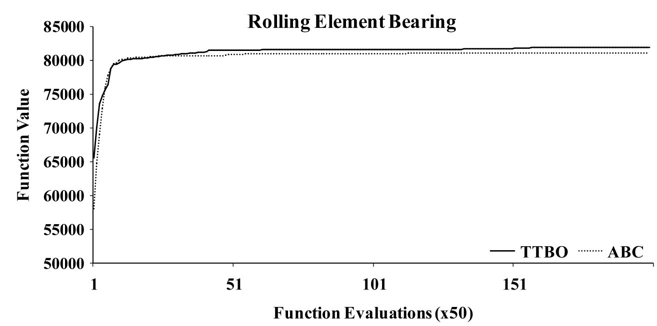

Fig. 14. Convergence plots for the rolling element bearing using the TLBO and ABC algorithms (function value = dynamic load carrying capacity (N)).

90 000 function evaluations. This problem is attempted using TLBO and ABC with the population size of 50 and the number of generations as 500, requiring 25 000 function evaluations. The best reported values are: function value = 1625.442764, R_o = 5.9557805026154158, R_i = 5.3890130519416788, μ = 0.0000053586972670629985, Q = 2.2696559728097379. This is a very interesting optimization problem because, out of 7 constraints, 6 are active constraints considering an accuracy of 3 decimal places, and all the design variables are highly sensitive. The accuracy of R_o, R_i, μ and Q is required up to 9, 9, 15 and 10 decimal places respectively. Considering R_o up to 8 decimal places violates constraints 2 and 7, R_i up to 8 decimal places violates constraint 1, μ up to 14 decimal places violates constraints 2, 3 and 7, and Q up to 9 decimal places violates constraint 1. So this example can be considered as a very good mechanical benchmark problem. As seen from Table 8, TLBO produces better results than ABC for the mean and worst solutions. Moreover, although the success rate of TLBO is only 0.19, it is still 280% better than ABC. Convergence rate of TLBO and ABC from Fig. 13 is nearly same but ability to find mean best solution is better for TLBO than ABC.

The problem of the rolling element bearing was presented by Gupta et al. [27]. The best reported function value is 81 843.3 with the design variables x = (125.7171, 21.423, 11, 0.515, 0.515, 0.4159, 0.651, 0.300043, 0.0223, 0.751). The number of function evaluations used by Gupta et al. [27] was 225 000. Here the problem is attempted using TLBO and ABC with a population size of 50 and the number of generations as 200. It is observed from Table 8 that the performance of TLBO and ABC is nearly same, with just a 4.5% worse success rate for TLBO than for ABC. From Fig. 14 it is observed that the convergence rate of ABC and TLBO is nearly same with a slightly higher mean searching capability for TLBO.

The problem for the Belleville spring was attempted by Coello [26] using a new constraint handling technique and Deb and Goyal [30] using GeneAS. The best function value reported by Coello [26] was 2.121964, with design variables x = (0.208, 0.2, 8751, 11.067). Here the same problem is attempted using

Table 9
Values of objective functions, design variables and constraints for mechanical design problems 1–6.

	Multiple disc clutch brake	Robot gripper	Step-cone pulley	Hydrostatic thrust bearing	Rolling element bearing	Belleville spring
x_1	70	150	40	5.95578050261541	21.42559	0.204143
x_2	90	150	54.7643	5.38901305194167	125.7191	0.2
x_3	1	200	73.01318	0.0000053586972670629	11	10.03047
x_4	810	0	88.42842	2.26965597280973	0.515	12.01
x_5	3	150	85.98624	–	0.515	–
x_6	–	100	–	–	0.424266	–
x_7	–	2.339539113	–	–	0.633948	–
x_8	–	–	–	–	0.3	–
x_9	–	–	–	–	0.068858	–
x_{10}	–	–	–	–	0.799498	–
$f(x)$	0.313656611	4.247643634	16.63451	1625.4427649821	81 859.74	1.979675
g_1	0	28.09283911	5.14E−09	0.0001374735	0	1.77E−06
g_2	24	21.90716089	1E−09	0.0000010103	13.15257	7.46E−08
g_3	0.91942781	33.64959994	1E−10	0.0000000210	1.5252	5.8E−11
g_4	9830.371094	16.35040006	0.986864	0.0003243625	0.719056	1.595857
g_5	7894.69659	79 999.998	0.99736	0.5667674507	16.49544	2.35E−09
g_6	0.702013203	9.8E−11	1.010154	0.0009963614	0	1.979527
g_7	37 706.25	0.00001	1.020592	0.0000090762	0	0.198966
g_8	14.2979868	–	698.5773	–	2.559363	–
g_9	–	–	475.8272	–	0	–
g_{10}	–	–	209.0369	–	0	–
g_{11}	–	–	1.05E−06	–	–	–

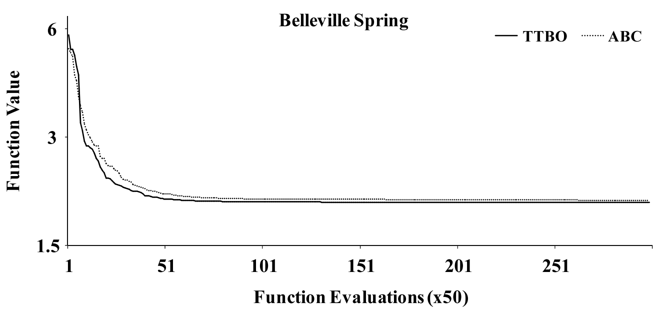

Fig. 15. Convergence plots for the Belleville spring using the TLBO and ABC algorithms (function value = weight of Belleville spring (lb)).

TLBO and ABC with a population size of 50 and the number of generations as 300. It is observed from Table 8 that TLBO outperforms ABC for the mean and the worst solutions. Also, TLBO has an 540% higher success rate than ABC for the Belleville spring problem. As shown in Fig. 15, the convergence rate of ABC and TLBO is nearly same with slight dominance of TLBO over ABC. Results showing the function value, design variables and constraint values are shown in Table 9. It is to be noted that the values reported in Table 9 are the best values obtained so far.

6. Conclusions

A novel optimization method, TLBO, is presented based on the philosophy of the teaching–learning process and its performance is checked by experimenting with different benchmark problems with different characteristics. The effectiveness of TLBO is also checked for different performance criteria, such as success rate, mean solution, average number of function evaluations required, convergence rate, etc. The results show the better performance of TLBO over other nature-inspired optimization methods for the constrained benchmark functions and mechanical design problems considered. Also, TLBO shows a better performance with less computational effort for large scale problems, i.e. problems of a high dimensionality. This novel method can be used for the optimization of engineering design applications.

Appendix A. Constrained benchmark functions

A.1. Benchmark function 1

$$\mathrm{Min}\, f(x) = 5\sum_{i=1}^{4} x_i - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i$$

S.T. $g_1(x) = 2x_1 + 2x_2 + x_{10} + x_{11} - 10 \le 0,$
$g_2(x) = 2x_1 + 2x_3 + x_{10} + x_{12} - 10 \le 0,$
$g_3(x) = 2x_2 + 2x_3 + x_{11} + x_{12} - 10 \le 0,$
$g_4(x) = -8x_1 + x_{10} \le 0, \qquad g_5(x) = -8x_2 + x_{11} \le 0,$
$g_6(x) = -8x_3 + x_{12} \le 0,$
$g_7(x) = -2x_4 - x_5 + x_{10} \le 0, \qquad g_8(x) = -2x_6 - x_7 + x_{11} \le 0,$
$g_9(x) = -2x_8 - x_9 + x_{12} \le 0,$
$0 \le x_i \le 1, \quad i = 1, 2, 3, \ldots, 9,$
$0 \le x_i \le 100, \quad i = 10, 11, 12, \qquad 0 \le x_i \le 1, \quad i = 13.$

A.2. Benchmark function 2

$$\mathrm{Max}\, f(x) = (\sqrt{n})^n \prod_{i=1}^{n} x_i$$

$$\text{S.T. } h(x) = \sum_{i=1}^{4} x_i^2 - 1 = 0$$

where
$n = 10$ and $0 \le x_i \le 10 \quad (i = 1, 2, \ldots, n).$

A.3. Benchmark function 3

$$\begin{aligned}\mathrm{Min}\, f(x) &= (x_1 - 10)^2 + 5(x_2 - 12)^2 + x_3^4 + 3(x_4 - 11)^2 \\ &\quad + 10x_5^6 + 7x_6^2 + x_7^4 - 4x_6x_7 - 10x_6 - 8x_7\end{aligned}$$

S.T. $g_1(x) = -127 + 2x_1^2 + 3x_2^4 + x_3 + 4x_4^2 + 5x_5 \le 0,$
$g_2(x) = -282 + 7x_1 + 3x_2 + 10x_3^2 + x_4 - x_5 \le 0,$
$g_3(x) = -196 + 23x_1^{+}x_2^2 + 6x_6^2 - 8x_7 \le 0,$
$g_4(x) = 4x_1^2 + x_2^2 - 3x_1x_2 + 2x_3^2 + 5x_6 - 11x_7 \le 0,$

where
$-10 \le x_i \le 10 \quad (i = 1, 2, \ldots, 7)$.

A.4. Benchmark function 4

$\text{Min} f(x) = x_1 + x_2 + x_3$
S.T. $g_1(x) = -1 + 0.0025(x_4 + x_6) \le 0$,
$g_2(x) = -1 + 0.0025(x_5 + x_7 - x_4) \le 0$,
$g_3(x) = -1 + 0.01(x_8 - x_5) \le 0$,
$g_4(x) = -x_1 x_6 + 833.33252 x_4 + 100 x_1 - 83\,333.333 \le 0$,
$g_5(x) = -x_2 x_7 + 1250 x_5 + x_2 x_4 - 1250 x_4 \le 0$,
$g_6(x) = -x_3 x_8 + 1250\,000 + x_3 x_5 - 2500 x_5 \le 0$,
where
$-100 \le x_1 \le 10\,000, \qquad -1000 \le x_i \le 10\,000 \quad (i = 2, 3)$,
$-100 \le x_i \le 10\,000 \quad (i = 4, 5, \ldots, 8)$.

A.5. Benchmark function 5

$$\text{Max} f(x) = \frac{100 - (x_1 - 5)^2 - (x_2 - 5)^2 - (x_3 - 5)^2}{100}$$
S.T. $g(x) = (x_1 - p)^2 + (x_2 - q)^2 + (x_3 - r)^2 \le 0$
where
$0 \le x_i \le 10 \quad (i = 1, 2, 3)$
$p, q, r = 1, 2, 3, \ldots, 9$.

Appendix B. Constrained benchmark mechanical design problems

B.1. Design of pressure vessel

Minimize:
$f(x) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3$
Subject to:
$g_1(x) = -x_1 + 0.0193 x_3 \le 0$,
$g_2(x) = -x_2 + 0.00954 x_3 \le 0$,
$$g_3(x) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296\,000 \le 0,$$
$g_4(x) = x_4 - 240 \le 0$,
where
$0 \le x_1 \le 99, \qquad 0 \le x_2 \le 99$,
$10 \le x_3 \le 200, \qquad 10 \le x_4 \le 200$.

B.2. Design of tension/compression spring

Minimize:
$f(x) = (N + 2) D d^2$.
Subject to;
$$g_1(x) = 1 - \frac{D^3 N}{71785 d^4} \le 0,$$
$$g_2(x) = \frac{4D^2 - dD}{12566(Dd^3 - d^4)} + \frac{1}{5108 d^2} - 1 \le 0,$$
$$g_3(x) = 1 - \frac{140.45 d}{D^2 N} \le 0,$$
$$g_4(x) = \frac{D + d}{1.5} - 1 \le 0,$$
where
$0.05 \le x_1 \le 2, \qquad 0.25 \le x_2 \le 1.3, \qquad 2 \le x_3 \le 15$.

B.3. Design of welded beam design

Minimize:
$f(x) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2)$.
Subject to:
$g_1(x) = \tau(x) - \tau_{\max} \le 0$,
$g_2(x) = \sigma(x) - \sigma_{\max} \le 0, \qquad g_3(x) = x_1 - x_4 \le 0$,
$g_4(x) = 0.10471 x_1^2 + 0.04811 x_3 x_4 (14.0 + x_2) - 5.0 \le 0$,
$g_5(x) = 0.125 - x_1 \le 0$,
$g_6(x) = \delta(x) - \delta_{\max} \le 0, \qquad g_7(x) = P - P_c(x) \le 0$,
where
$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2},$$
$$\tau' = \frac{P}{\sqrt{2} x_1 x_2}, \qquad \tau'' = \frac{MR}{J}, \qquad M = P\left(L + \frac{x_2}{2}\right),$$
$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},$$
$$J = 2\left[\sqrt{2} x_1 x_2 \left\{\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right\}\right], \qquad \sigma(x) = \frac{6PL}{x_4 x_3^2},$$
$$\delta(x) = \frac{4PL^3}{E x_3^3 x_4},$$
$$P_c(x) = \frac{4.013 E \sqrt{\frac{x_3^2 x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right),$$
$P = 6000$ lb, $\qquad L = 14$ in., $\qquad E = 30e6$ psi,
$G = 12e6$ psi, $\qquad \tau_{\max} = 13\,600$ psi, $\qquad \sigma_{\max} = 30\,000$ psi,
$\delta_{\max} = 0.25$ in.
$0.1 \le x_1 \le 2.0, \qquad 0.1 \le x_2 \le 10.0$,
$0.1 \le x_3 \le 10.0, \qquad 0.1 \le x_4 \le 2.0$.

B.4. Design of gear train

Minimize:
$$f(x) = 0.7854 x_1 x_2^2 (3.3333 x_3^2 + 14.9334 x_3 - 43.0934) - 1.508 x_1 \times (x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x_4 x_6^2 + x_5 x_7^2)$$
Subject to:
$$g_1(x) = \frac{27}{x_1 x_2^2 x_3} - 1 \le 0, \qquad g_2(x) = \frac{397.5}{x_1 x_2^2 x_3^2} - 1 \le 0,$$
$$g_3(x) = \frac{1.93 x_4^3}{x_2 x_3 x_6^4} - 1 \le 0, \qquad g_4(x) = \frac{1.93 x_5^3}{x_2 x_3 x_7^4} - 1 \le 0,$$
$$g_5(x) = \frac{\sqrt{\left(\frac{745 x_4}{x_2 x_3}\right)^2 + 16.9e6}}{110 x_6^3} - 1 \le 0,$$
$$g_6(x) = \frac{\sqrt{\left(\frac{745 x_5}{x_2 x_3}\right)^2 + 157.5e6}}{85 x_7^3} - 1 \le 0,$$
$$g_7(x) = \frac{x_2 x_3}{40} - 1 \le 0, \qquad g_8(x) = \frac{5 x_2}{x_1} - 1 \le 0,$$
$$g_9(x) = \frac{x_1}{12 x_2} - 1 \le 0,$$

$$g_{10}(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0,$$
$$g_{11}(x) = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0,$$

where

$2.6 \le x_1 \le 3.6, \quad 0.7 \le x_2 \le 0.8, \quad 17 \le x_3 \le 28,$
$7.3 \le x_4 \le 8.3, \quad 7.8 \le x_5 \le 8.3,$
$2.9 \le x_6 \le 3.9, \quad 5.0 \le x_7 \le 5.5.$

Appendix C. Constrained mechanical design problems

C.1. Multiple disc clutch brake design

$$\text{Min } f(x) = \pi(r_0^2 - r_i^2)t(Z+1)\rho$$

Subject to:

$g_1(x) = r_o - r_i - \Delta r \ge 0,$
$g_2(x) = l_{\max} - (Z+1)(t+\delta) \ge 0, \quad g_3(x) = p_{\max} - p_{rz} \ge 0,$
$g_4(x) = p_{\max} v_{sr\max} - p_{rz} v_{sr} \ge 0,$
$g_5(x) = v_{sr\max} - v_{sr} \ge 0, \quad g_6(x) = T_{\max} - T \ge 0,$
$g_7(x) = M_h - sM_s \ge 0, \quad g_8(x) = T \ge 0,$

where

$$M_h = \frac{2}{3}\mu F Z \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}, \qquad p_{rz} = \frac{F}{\pi(r_o^2 - r_i^2)},$$
$$v_{sr} = \frac{2\pi n(r_o^3 - r_i^3)}{90(r_o^2 - r_i^2)}, \qquad T = \frac{I_z \pi n}{30(M_h + M_f)}.$$

$\Delta r = 20$ mm, $t_{\max} = 3$ mm, $t_{\min} = 1.5$ mm, $l_{\max} = 30$ mm, $Z_{\max} = 10$, $v_{sr\max} = 10$ m/s, $\mu = 0.5$, $s = 1.5$, $M_s = 40$ N m, $M_f = 3$ N m, $n = 250$ rpm, $p_{\max} = 1$ MPa, $I_z = 55$ kg mm^2, $T_{\max} = 15$ s, $F_{\max} = 1000$ N, $r_{i\min} = 55$ mm, $r_{o\max} = 110$ mm.

C.2. Robot gripper

$$\text{Minimize} \quad f(x) = \max_z F_k(x,z) - \min_z F_k(x,z)$$

Subject to:

$g_1(x) = Y_{\min} - y(x, Z_{\max}) \ge 0,$
$g_2(x) = y(x, Z_{\max}) \ge 0, \quad g_3(x) = y(x,0) - Y_{\max} \ge 0,$
$g_4(x) = Y_G - y(x,0) \ge 0, \quad g_5(x) = (a+b)^2 - l^2 - e^2 \ge 0,$
$g_6(x) = (l - Z_{\max})^2 + (a-e)^2 - b^2 \ge 0,$
$g_7(x) = l - Z_{\max} \ge 0,$

where

$$g = \sqrt{(l-z)^2 + e^2}, \qquad \alpha = \arccos\left(\frac{a^2 + g^2 - b^2}{2ag}\right) + \phi,$$
$$\beta = \arccos\left(\frac{b^2 + g^2 - a^2}{2bg}\right) - \phi,$$
$$\phi = \arctan\left(\frac{e}{l-z}\right) + \phi, \qquad F_k = \left(\frac{Pb\sin(\alpha+\beta)}{2c\cos(\alpha)}\right),$$
$$y(x,z) = 2(e + f + c\sin(\beta+\delta)).$$

$Y_{\min} = 50, \quad Y_{\max} = 100,$
$Y_G = 150, \quad Z_{\max} = 100, \quad P = 100,$
$10 \le a, b, f \le 150, \quad 100 \le c \le 200,$
$0 \le e \le 50, \quad 100 \le l \le 300, \quad 1 \le \delta \le 3.14.$

C.3. Step-cone pulley

$$\text{Minimize} \quad f(x) = \rho w\left[d_1^2\left\{1 + \left(\frac{N_1}{N}\right)^2\right\} + d_2^2\left\{1 + \left(\frac{N_2}{N}\right)^2\right\} + d_3^2\left\{1 + \left(\frac{N_3}{N}\right)^2\right\} + d_4^2\left\{1 + \left(\frac{N_4}{N}\right)^2\right\}\right]$$

Subject to:

$h_1(x) = C_1 - C_2 = 0, \quad h_2(x) = C_1 - C_3 = 0,$
$h_3(x) = C_1 - C_4 = 0,$
$g_{1,2,3,4}(x) = R_i \ge 2, \quad g_{5,6,7,8}(x) = P_i \ge (0.75 * 745.6998),$

where

C_i indicates the length of the belt to obtain speed N_i and is given by

$$C_i = \frac{\pi d_i}{2}\left(1 + \frac{N_i}{N}\right) + \frac{\left(\frac{N_i}{N} - 1\right)^2}{4a} + 2a \quad i = (1,2,3,4).$$

R_i is the tension ratio and is given by

$$R_i = \exp\left[\mu\left\{\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right\}\right\}\right] \quad i = (1,2,3,4).$$

P_i is the power transmitted at each step

$$P_i = stw\left[1 - \exp\left[-\mu\left\{\pi - 2\sin^{-1}\left\{\left(\frac{N_i}{N} - 1\right)\frac{d_i}{2a}\right\}\right\}\right]\right]\frac{\pi d_i N_i}{60}$$
$i = (1,2,3,4).$

$\rho = 7200$ kg/m^3, $a = 3$ m, $\mu = 0.35$, $s = 1.75$ MPa, $t = 8$ mm.

C.4. Hydrodynamic thrust bearing design

$$\text{Minimize:} \quad f(x) = \frac{QP_o}{0.7} + E_f$$

Subject to:

$g_1(x) = W - W_s \ge 0, \quad g_2(x) = P_{\max} - P_o \ge 0,$
$g_3(x) = \Delta T_{\max} - \Delta T \ge 0, \quad g_4(x) = h - h_{\min} \ge 0,$
$$g_5(x) = R - R_o \ge 0, \quad g_6(x) = 0.001 - \frac{\gamma}{gP_o}\left(\frac{Q}{2\pi Rh}\right) \ge 0,$$
$$g_7(x) = 5000 - \frac{W}{\pi(R^2 - R_o^2)} \ge 0,$$

where

$$W = \frac{\pi P_o}{2}\frac{R^2 - R_o^2}{\ln\frac{R}{R_o}}, \qquad P_o = \frac{6\mu Q}{\pi h^3}\ln\frac{R}{R_o},$$
$$E_f = 9336Q\gamma C\Delta T, \qquad \Delta T = 2(10^P - 560)$$
$$P = \frac{\log_{10}\log_{10}(8.122e6\mu + 0.8) - C_1}{n},$$
$$h = \left(\frac{2\pi N}{60}\right)^2 \frac{2\pi\mu}{E_f}\left(\frac{R^4}{4} - \frac{R_o^4}{4}\right)$$

where,

$\gamma = 0.0307, \quad C = 0.5, \quad n = -3.55, \quad C_1 = 10.04,$
$W_s = 101\,000, \quad P_{\max} = 1000, \quad \Delta T_{\max} = 50,$
$h_{\min} = 0.001, \quad g = 386.4, \quad N = 750.$
$1 \le R, R_o, Q \le 16, \quad 1e-6 \le \mu \le 16e-6.$

C.5. Rolling element bearing

Maximize $C_d = f_c Z^{2/3} D_b^{1.8}$ if $D_b \le 25.4$ mm
$C_d = 3.647 f_c Z^{2/3} D_b^{1.4}$ if $D_b > 25.4$ mm

Subject to:

$$g_1(X) = \frac{\phi_o}{2\sin^{-1}(D_b/D_m)} - Z + 1 \ge 0,$$

$$g_2(X) = 2D_b - K_{D\min}(D-d) \ge 0,$$

$$g_3(X) = K_{D\max}(D-d) - 2D_b \ge 0,$$

$$g_4(X) = \zeta B_w - D_b \le 0, \qquad g_5(X) = D_m - 0.5(D+d) \ge 0,$$

$$g_6(X) = (0.5+e)(D+d) - D_m \ge 0,$$

$$g_7(X) = 0.5(D - D_m - D_b) - \varepsilon D_b \ge 0,$$

$$g_8(X) = f_i \ge 0.515, \qquad g_9(X) = f_o \ge 0.515,$$

where

$$f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_o-1)}{f_o(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3},$$

$$\gamma = \frac{D_b\cos\alpha}{D_m}, \qquad f_i = \frac{r_i}{D_b}, \qquad f_o = \frac{r_o}{D_b},$$

$$\phi_o = 2\pi - 2 \times \cos^{-1}\frac{\left[\{(D-d)/2 - 3(T/4)\}^2 + \{D/2 - (T/4) - D_b\}^2 - \{d/2 + (T/4)\}^2\right]}{2\{(D-d)/2 - 3(T/4)\}\{D/2 - (T/4) - D_b\}}, \quad (20)$$

$$T = D - d - 2D_b,$$

$$D = 160, \qquad d = 90, \qquad B_w = 30.$$

$0.5(D+d) \le D_m \le 0.6(D+d)$, $0.15(D-d) \le D_b \le 0.45(D-d)$, $4 \le Z \le 50$, $0.515 \le f_i \le 0.6$, $0.515 \le f_o \le 0.6$, $0.4 \le K_{D\min} \le 0.5$, $0.6 \le K_{D\max} \le 0.7$, $0.3 \le \varepsilon \le 0.4$, $0.02 \le e \le 0.1$, $0.6 \le \zeta \le 0.85$.

C.6. Belleville spring

Minimize:

$$f(x) = 0.07075\pi(D_e^2 - D_i^2)t$$

Subject to:

$$g_1(x) = S - \frac{4E\delta_{\max}}{(1-\mu^2)\alpha D_e^2}\left[\beta\left(h - \frac{\delta_{\max}}{2}\right) + \gamma t\right] \ge 0,$$

$$g_2(x) = \left(\frac{4E\delta}{(1-\mu^2)\alpha D_e^2}\left[\left(h - \frac{\delta}{2}\right)(h-\delta)t + t^3\right]\right)_{\delta=\delta_{\max}} - P_{\max} \ge 0,$$

$$g_3(x) = \delta_l - \delta_{\max} \ge 0, \qquad g_4(x) = H - h - t \ge 0,$$

$$g_5(x) = D_{\max} - D_e \ge 0, \qquad g_6(x) = D_e - D_i \ge 0,$$

$$g_7(x) = 0.3 - \frac{h}{D_e - D_i} \ge 0,$$

where

$$\alpha = \frac{6}{\pi\ln K}\left(\frac{K-1}{K}\right)^2,$$

$$\beta = \frac{6}{\pi\ln K}\left(\frac{K-1}{\ln K} - 1\right), \qquad \gamma = \frac{6}{\pi\ln K}\left(\frac{K-1}{2}\right),$$

$$P_{\max} = 5400\text{ lb}, \qquad \delta_{\max} = 0.2\text{ in.}, \qquad S = 200\text{ kPsi},$$

$$E = 30e6\text{ psi}, \qquad \mu = 0.3, \qquad H = 2\text{ in.}, \qquad D_{\max} = 12.01\text{ in.},$$

$$K = \frac{D_e}{D_i}, \qquad \delta_l = f(a)a, \qquad a = h/t.$$

Values of $f(a)$ vary as shown in Table 7.

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