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Logarithmic derivative method and system for capacitance measurement

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A novel method based on logarithmic derivative is introduced to analyze multi-lifetime decay. As the discharge voltage signal of a *RC* circuit is a special kind of multi-lifetime exponential decay, the logarithmic derivative method can be used to measure single capacitance and multiple capacitances. With the logarithmic derivative method, a $\log(t)$ curve strongly peaked at precisely $\log(\tau)$ is obtained, where the lifetime τ equals to *RC*. In a measurement system, if the resistance *R* is known, then the capacitance under test can be calculated. A logarithmic derivative curve fitting method is also presented, which has better anti-noise capability than the method that simply finds the maximum data on the peak. The curve fitting method can also be used for multiple capacitors measurement. To measure small capacitances, a large enough time window of the measuring instrument is required. Based on a field programmable gate array and a high speed analog-to-digital converter, a measurement system is developed. This system can provide the 16-bit resolution with sampling rate up to 250 MHz, which has a large enough time window for measuring lifetime shorter than 10^{-8} s. To reduce the amount of data needed to be stored and the noise due to the derivative treatment of transient data, the interpolation and noise-filter algorithms are employed. Experiments indicate that the logarithmic derivative method and system are suitable for the measurement of capacitances discharge and other exponential decay processes. [http://dx.doi.org/10.1063/1.4928061]

I. INTRODUCTION

Capacitive sensors are widely used in the measurement of a variety of physical parameters, such as proximity, acceleration, and pressure.[1–4] The accurate capacitance measurement is the key of the application of these capacitive sensors. Several methods are used to measure capacitance, including charge and discharge methods,[5,6] switched-capacitor method,[7,8] four terminal-pair coaxial bridge,[9] capacitance-to-frequency conversion,[10] capacitance-to-phase angle conversion,[11,12] current compensation method,[13] and the method of direct interface to microcontroller without any analog circuit in signal path.[14]

In this article, a new method based on logarithmic derivative is introduced. The logarithmic derivative method was employed in the analysis of multiple non-exponential decay data.[15] A single-lifetime exponential decay is a special multiple non-exponential decay that just has one lifetime, so the logarithmic derivative method can be applied to a single-lifetime exponential decay. A field programmable gate array (FPGA) and high speed analog-to-digital converter (ADC) based measurement system are developed to record a first-order *RC* circuit discharge data, which are a kind of single-lifetime exponential decay data and to execute the logarithmic derivative data processing. In the rest of the paper, more details of the method will be discussed. The design of the measurement system and the application examples are also presented and discussed.

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II. THEORETICAL ANALYSIS

A. Logarithmic derivative method

Using the logarithmic derivative method, a decay data $N = N(t)$ can be calculated as

$$K(t) = -\frac{dN(t)}{d\log(t)}. \quad (1)$$

The obtained $\log(t)$ derivative curve $K(t)$ shows strongly multi-peaked structure characterizing the main features of the decay. $K(t)$ and $N(t)$ have the same unit. For any decays, the apparent lifetime distribution and the derivative decay quantities of individual lifetimes are shown visually with the $\log(t)$ curves.

B. Single-lifetime decay process

A single-lifetime decay can be written as

$$N(t) = N_0 + N_1 \cdot \exp\left(-\frac{t}{\tau}\right), \quad (2)$$

where N_0 is the decay bottom line, N_1 is the initial value of the decay, and τ is the lifetime.

Substituting Eq. (2) into Eq. (1) results in

$$\begin{aligned} K(t) &= -\frac{dN(t)}{d\log(t)} \\ &= -t \cdot \ln 10 \cdot \frac{dN(t)}{dt} \\ &= \left(\frac{t}{\tau}\right) \cdot \ln 10 \cdot N_1 \cdot \exp\left(-\frac{t}{\tau}\right). \end{aligned} \quad (3)$$

For the case $t = \tau$, the maximum value of $K(t)$ equal to $K(\tau) = \ln 10 \times N_1 \times e^{-1} = 0.847 \times N_1$. It is noticed that there

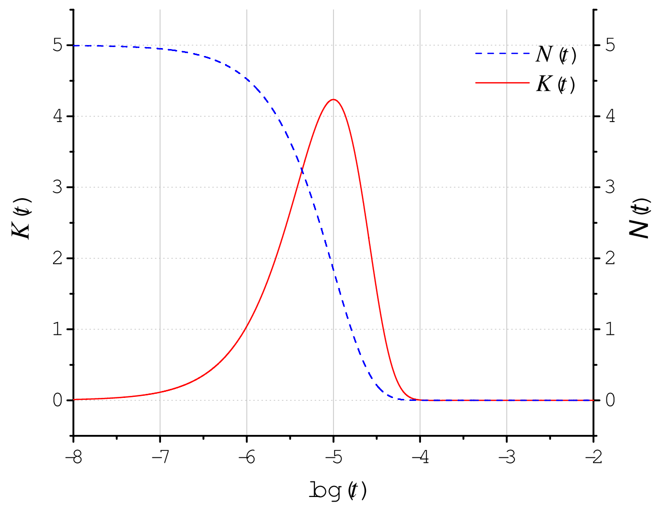

FIG. 1. A theoretical single-lifetime decay $N(t)-\log(t)$ curve (the dashed blue line) and its logarithmic derivative $K(t)-\log(t)$ curve (the solid red line), where $N_0=0$, $N_1=5$, and the lifetime $\tau=1.0\times10^{-5}$ s. As the specific application has not been assigned, the two vertical axes have no specific units. But $N(t)$ and $K(t)$ have the same unit.

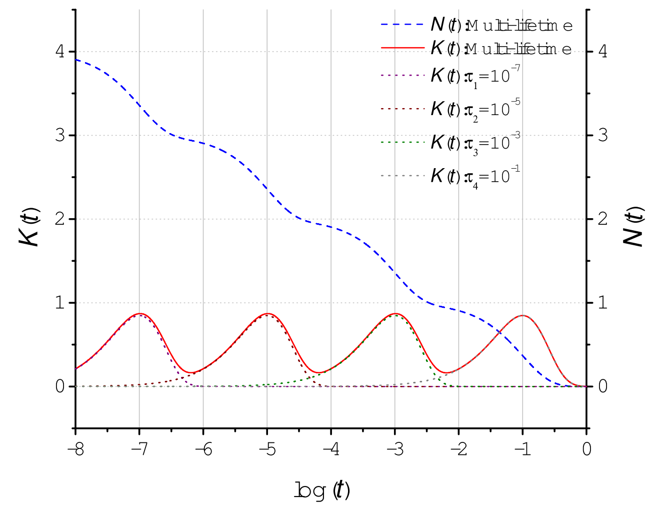

FIG. 2. A theoretical multi-lifetime decay $N(t)-\log(t)$ curve (the dashed blue line, $N(t)$:Multi-lifetime) and its logarithmic derivative $K(t)-\log(t)$ spectrum (the solid red line, $K(t)$:Multi-lifetime), where $N_0=0$, $N_1=N_2=N_3=N_4=1$, $\tau_1=1.0\times10^{-7}$ s, $\tau_2=1.0\times10^{-5}$ s, $\tau_3=1.0\times10^{-3}$ s, and $\tau_4=1.0\times10^{-1}$ s. The $K(t)-\log(t)$ spectrum shows strongly four peaked structures. There are four individual single-lifetimes $K(t)-\log(t)$ dotted curves (the purple dot $K(\mathrm{t}):\tau_1=1.0\times10^{-7}$ s line; the wine dot $K(t):\tau_2=1.0\times10^{-5}$ s line; the olive dot $K(\mathrm{t}):\tau_3=1.0\times10^{-3}$ s line; the gray dot $K(t):\tau_4=1.0\times10^{-1}$ s line).

is no N_0 item in Eq. (3); therefore, N_0 would not affect the results of $K(t)$.

As shown in Figure 1, a theoretical single-lifetime decay $N(t)$ and its logarithmic derivative $K(t)$ are plotted versus the logarithmic of time. The dashed blue $N(t)-\log(t)$ line decays from $N_1=5$ to $N_0=0$ with a lifetime of $\tau=1.0\times10^{-5}$ s. It shows that the solid red $K(t)-\log(t)$ curve is an asymmetrical single peak, the left slope of the peak is gentler than the right side, and the area beneath the peak curve is exactly N_1. It is obvious that the curve is strongly peaked at precisely $\log(\tau)$ with a peak value of

$$K(\tau)\,|_{\tau=1.0\times10^{-5}}=0.847\times N_1=4.235.$$

The full width at half maximum (FWHM) for the curve is 1.05. Therefore, theoretically, if the maximum value on a single-lifetime $K(t)-\log(t)$ curve is found, then the corresponding lifetime τ of the exponential decay would be obtained. The lifetime τ contains the characteristic of the decay.

The discharge voltage signal of a first-order *RC* circuit belongs to a kind of single-lifetime exponential decay and satisfies Eq. (2), where N_0 is the ultimate voltage of a discharge process, N_1 is the initial voltage, and $\tau=RC$. In the application of this method, a measurement system is designed to acquire a discharge voltage signal of first-order *RC* circuit, first. Then, using the logarithmic derivative method to obtain its $K(t)-\log(t)$ curve and find the position of the peak on the $\log(t)$ axis that equals to its lifetime τ. Finally, we substitute the obtained τ and the given precise resistance R into the equation $C=\tau/R$, then the capacitance value of the capacitor under test (CUT) can be calculated.

C. Multi-lifetime decay process

A multi-lifetime decay with the form of

$$N(t)=N_0+\sum_i N_i\cdot\exp\left(-\frac{t}{\tau_i}\right), \tag{4}$$

which is the superposition of individual exponential decays with single-lifetime τ_i. The $\log(t)$ derivative of Eq. (4) is equivalent to

$$K(t)=\sum_i\left(\frac{t}{\tau_i}\right)\cdot\ln 10\cdot N_i\cdot\exp\left(-\frac{t}{\tau_i}\right). \tag{5}$$

When Eq. (5) is plotted with respect to $\log(t)$, a logarithmic derivative spectrum can be obtained, which is the superposition of multiple single-lifetime $\log(t)$ derivative spectra.

For example, Figure 2 shows the $N(t)-\log(t)$ and $K(t)-\log(t)$ curves of a multi-lifetime decay. The decay is composed of four individual single lifetimes, and it can be written as

$$N(t)=\sum_{i=1}^{4}N_i\cdot\exp\left(-\frac{t}{\tau_i}\right), \tag{6}$$

where $N_1=N_2=N_3=N_4=1$, $\tau_1=1.0\times10^{-7}$ s, $\tau_2=1.0\times10^{-5}$ s, $\tau_3=1.0\times10^{-3}$ s, and $\tau_4=1.0\times10^{-1}$ s. It is obvious that there are four peaks on the red solid $K(t)-\log(t)$ multi-lifetime spectrum, and the four vertexes locate precisely at the four τ_i's in Eq. (6). There are four individual exponential decays with single-lifetime plotted as four dotted peaks, and their lifetimes are the same with the four τ_i's in Eq. (6). Superposing the four individual single-lifetime $K(t)-\log(t)$ curves, the superposition curve is identical with the red solid $K(t)-\log(t)$ multi-lifetime spectrum.

As shown in Figure 2, if the individual lifetimes are well separated, the $\log(t)$ derivative spectrum of multi-lifetime decay may show strongly multi-peaked structure. Each peak is associated with a decay channel. Therefore, by identifying the peaks on a $K(t)-\log(t)$ derivative spectrum of multi-lifetime decay, different lifetimes can be obtained at the same time. If a measurement system with well separated lifetimes

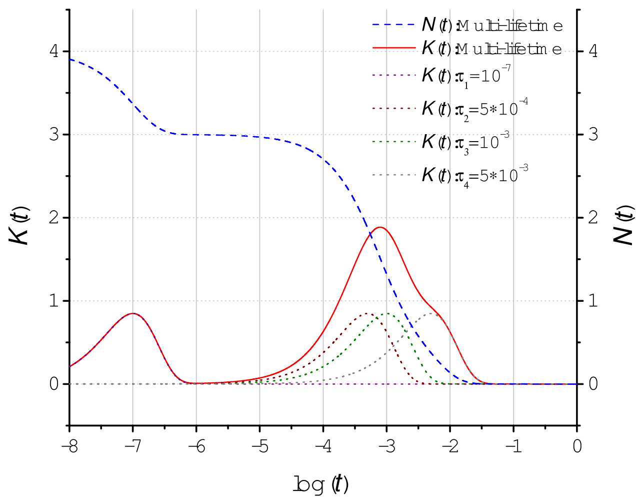

FIG. 3. Another example is the theoretical multi-lifetime decay $N(t)-\log(t)$ curve (the dashed blue line, $N(t)$:Multi-lifetime) and its logarithmic derivative $K(t)-\log(t)$ curve (the solid red line, $K(t)$:Multi-lifetime), where $N_0=0$, $N_1=N_2=N_3=N_4=1$, $\tau_1=1.0\times10^{-7}$ s, $\tau_2=5.0\times10^{-4}$ s, $\tau_3=1.0\times10^{-3}$ s, and $\tau_4=5.0\times10^{-3}$ s. There are four individual single-lifetimes $K(t)-\log(t)$ dotted curves (the purple dot $K(t):\tau_1=1.0\times10^{-7}$ s line; the wine dot $K(t):\tau_2=5.0\times10^{-4}$ s line; the olive dot $K(t):\tau_3=1.0\times10^{-3}$ s line; the gray dot $K(t):\tau_4=5.0\times10^{-3}$ s line).

is constructed, it can be used to measure multiple capacitors discharge simultaneously.

Figure 3 shows another multi-lifetime decay example. The decay is composed of four individual single lifetimes that also follow Eq. (6), where $N_1 = N_2 = N_3 = N_4 = 1$, $\tau_1 = 1.0 \times 10^{-7}$ s, $\tau_2 = 5.0 \times 10^{-4}$ s, $\tau_3 = 1.0 \times 10^{-3}$ s, and $\tau_4 = 5.0 \times 10^{-3}$ s. Unlike Figure 2, there are no four distinct peaks on the $K(t) - \log(t)$ spectrum, as four individual lifetimes are not well separated. It represents the general case.

The $\log(t)$ derivative spectrum in Figure 2 is a special case. Generally, the individual lifetimes of multi-lifetime decay are not well separated, so it is impossible to obtain all individual lifetimes just by simply identifying the positions of peaks. However, if the number of individual lifetimes and the initial value N_i are known, using the superposition of multiple single-lifetime $K(t) - \log(t)$ curves could fit a multi-lifetime $\log(t)$ derivative spectrum, then obtain the lifetimes τ_i and equations like Eqs. (5) and (4). As shown in Figure 3, the four $K(t) - \log(t)$ dotted curves represent four single-lifetime decays, the superposition of the four curves is coincident with the solid multi-lifetime $K(t) - \log(t)$ spectrum.

For some multiple capacitors measurement cases, the values of capacitances are very close. By designing a specific measurement circuit with the same initial discharge voltage N_i and well separated resistances, the separated peaks on a multi-lifetime $\log(t)$ derivative spectrum can be identified. Then, the logarithmic derivative method can be employed to measure the multiple capacitors discharge simultaneously.

III. MEASUREMENT SYSTEM DESIGN

To measure small capacitances, a large enough time window of the measuring instrument is required. It is practical to make effort to measure the fast part of the transient instead of the slow part. The larger noise due to derivative treatment of transient data also needs to be reduced by increasing the resolution and the sampling rate of the transient data acquisition. This measurement system design is the integration of FPGA logic circuit design, Linux operating system, and application software development.

A. Hardware design

In this work, a ZedBoard, an AD9467 FMC card, and a first-order *RC* circuit compose the measurement system, as shown in Figure 4. The ZedBoard is a board based on the Xilinx ZynqTM-7000 All Programmable System on Chip (AP SoC).[16] Combining a dual Corex-A9 Processing System (PS) with 85 000 Series-7 Programmable Logic (PL) cells, the Zynq-7000 AP SoC is suitable for this application. This architecture enables implementation of ADC interface logic in the PL and customer software in the PS. The software includes some device drivers, a logarithmic derivative analysis application program, and a Linux operating system. The integration

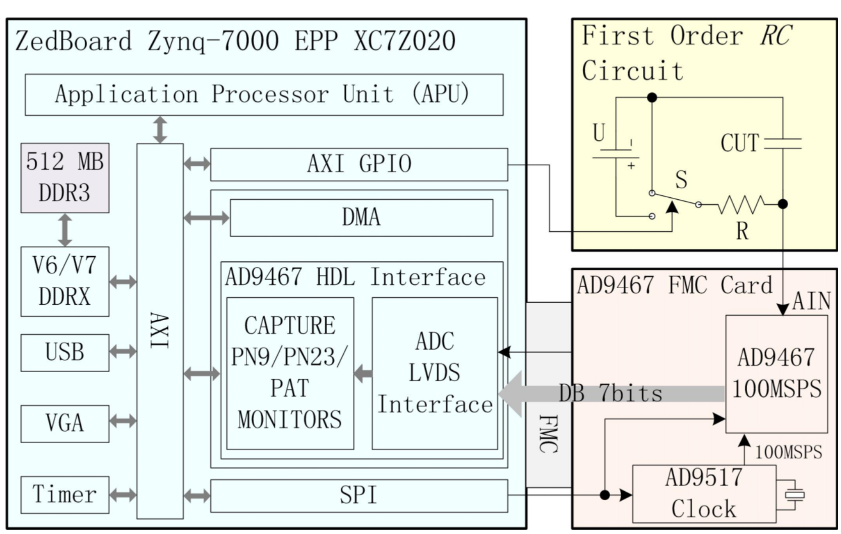

FIG. 4. Architecture of the measurement system. The USB, VGA, Timer, SPI, and V6/V7 DDRX are hard IP cores inside the PS. The analog switch S is connected with an AXI GPIO port in the PL of Zynq-7000, and then, the application software can control the states of the first-order *RC* circuit. Programing the AD9517 clock generator via a SPI Q6 interface. The AD9467 operates at a 100 MSPS conversion rate.

of the PS and PL simplifies the system design and improves data acquisition and processing speed.

The first-order *RC* circuit is composed of a CUT, a given precise resistor *R*, an analog switch S, and a battery U. As a kind of low-noise DC power, a battery is suitable as the capacitor charge power. At the discharge state, the total resistance of the *RC* circuit is equal to the sum of *R* and the ON-State Resistance (R_{ON}) of the analog switch S. So, the R_{ON} is one of the most critical parameters and should be as small as possible. The ADG839 is a device containing a single-pole, double-throw (SPDT) switch with small R_{ON} of less than 0.6 Ω. As the resistance *R* is much higher than the $R_{ON,}$ the effect of R_{ON} can be neglected. By controlling the states of switch S, the Zynq-7000 SoC can charge and discharge the *RC* circuit.

The AD9467 FMC card is a high-speed digitizer, which uses a FMC interface to stack on the Zedboard. The AD9467 is a 16-bit ADC with a conversion rate of up to 250 MSPS (sampling per second). The sampling rate can be adjusted by programming the onboard AD9517-4 clock chip through a serial peripheral interface (SPI) in the PL of Zynq-7000.

The ADC HDL interface intellectual property (IP) core in the PL consists of three functional modules, a low-voltage differential signaling (LVDS) interface, a PN9/PN23/PAT monitor, and a direct memory access (DMA) interface. The LVDS interface captures and buffers data from the AD9467. The DMA interface then transfers the data to the external DDR-DRAM (double data rate synchronous dynamic random access memory).[17] The capture is initiated by the application program.

As described in Figure 5, a summing circuit conform to Eq. (4) can be used to measure multiple capacitors. Note that for the voltage of battery U, it should be sure that the sum of all discharge signals has no overflow. Using this circuit to replace the first-order *RC* circuit in Figure 4, the measurement system can measure three capacitors simultaneously.

B. Software design and data processing

The application software contains three components: the AD9467 driver, the AD9517 driver, and the data processing program. The two drivers are from analog devices.[17] The AD9517 driver is used for programing the AD9517 clock output frequency and providing a differential clock signal to the AD9467 as sampling clock inputs. In this application, the AD9467's operation modes are configured through the AD9467 driver.

The data processing file is developed to interpolate data on the log(t) axis, filter noise and operate the logarithmic derivative algorithm. The sampling rate of AD9467 is set at 100 MHz, and it will output a 16-bit data per 10 ns. As shown in Figure 6, at a 100 MSPS conversion rate, the $N(t)$ data number on the left of the horizontal logarithmic axis is less and sparser than the right side. If we want to measure capacitances as small as possible, a large enough time window of the measuring instrument is required. It means that along with the increase of sampling time, a larger amount of data will be produced and most of the data will be concentrated in the rightmost intervals. Note that the increase of sampling rate will produce tremendous amount of data, which will consume huge storage and calculation resources. This requires high performance microprocessor and large-capacity high-speed memory storage on the measurement system.

On the other hand, for the logarithmic derivative algorithm, it does not need so much data for the rightmost intervals, but hopes to get more data on the leftmost intervals. However, for ordinary sampling process, this is contradictory. Therefore, an interpolation and rejection algorithm is employed to make all the intervals have the same data number, and all the intervals on the log(t) axis between adjacent data are the same. In this application, using an interpolation algorithm, the $N(t)$ data number in each interval is set to 1000, and then the log(t) axis interval between adjacent data is $\log(\Delta t) = 0.001$. To reduce the noise due to the derivative treatment, an adjacent-averaging method is used to filter noise on the $N(t)$ data. Then, we use Eq. (1) to calculate and obtain a $K(t) - \log(t)$ curve.

For a first-order *RC* discharge process, which is a single-lifetime decay, it only has a peak on the $K(t) - \log(t)$ curve.

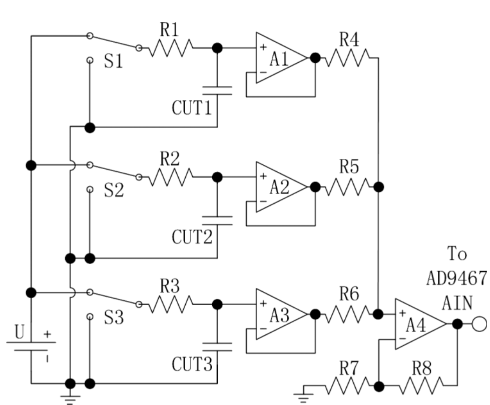

FIG. 5. A multiple first-order *RC* summing circuit example. Operational amplifiers A1, A2, and A3 compose three voltage followers. Amplifier A4 and R4-R8 sum the outputs of three first-order *RC* circuits via voltage followers, and their output is tied to the analog input of AD9467. As the sampling rate of AD9467 is set at 100 MSPS, the amplifier bandwidth of A1-A4 should be wider than 200 MHz at least.

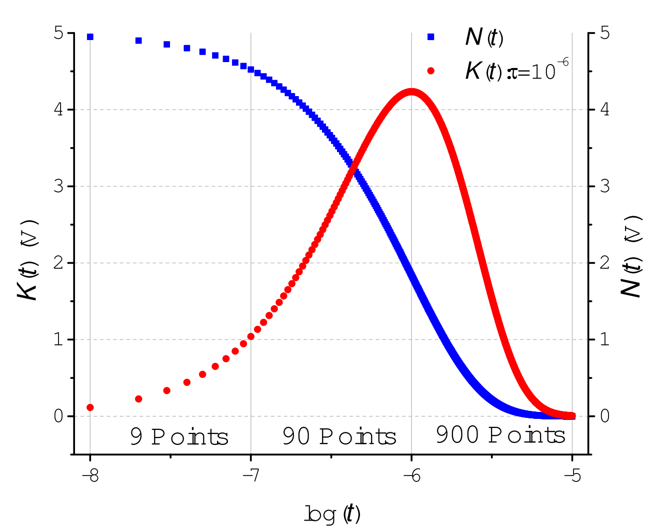

FIG. 6. A theoretical single-lifetime decay $N(t) - \log(t)$ sampling data (the rectangle blue dot) at 100 MSPS and its logarithmic derivative $K(t) - \log(t)$ data (the circular red dot), where $N_0 = 0$ V, $N_1 = 5$ V, and the lifetime $\tau = 1.0 \times 10^{-6}$ s. There are 9 points in the interval [−8, −7), 90 points in the interval [-7, -6), and 900 points in the interval [−6, −5).

The position value of the peak on the log(t) axis is equal to the lifetime τ. The capacitance of the CUT can be calculated with the equation $C = \tau/R$, as the values of τ and R are known. For a discharge process of multiple first-order RC summing circuit, a multi-lifetime decay, the curve is composed of multiple single-lifetime peaks. By finding the value of each lifetime τ_i, the corresponding capacitances would be calculated.

IV. EXPERIMENTAL RESULTS

Using the measurement system described above, the logarithmic derivative method is applied in the measurement of single CUT or multiple CUT's. A NI PXI-4072 6 1/2 digit FlexDMM and *LCR* meter is utilized to measure the given resisters and CUT's, and they are used as reference values to be compared with the values calculated by the logarithmic derivative method. As a *LCR* meter, the FlexDMM delivers 0.25% basic accuracy for capacitance. In order to achieve higher accuracy and stability, metal film resistors are chosen in the experiments.

A. Single capacitor measurement

Single capacitor measurement is conducted using a first-order RC circuit as shown in Figure 4. The test results are shown in Figure 7. With the simple method of finding the position of the maximum data on the peak, the lifetime $\tau = 10^{-3.960} = 1.096 \times 10^{-4}$ s, so, $C_{\mathrm{CUT1}} = \tau/R_r = 10.964$ nF. The inset in Figure 7 shows an enlargement of the $K(t)$ - log(t) peak, and it can be seen that there is some noise on the peak. Obviously, the peak value and its position are sensitive to noise. Therefore, the consistency of the measurement by finding the position of the maximum data on the peak is not very well.

A theoretical logarithmic derivative curve F1 (the red solid line, $\tau = 1.096 \times 10^{-4}$ s) can be used to fit the experiment $K(t) - \log(t)$ curve, as shown in Figure 7. The T1 (the cyan dashed line) is the result of the $K(t) - \log(t)$ curve minus the F1 fitting line, and the residual sum of squares (RSS) between the F1 and the $K(t) - \log(t)$ lines is 18.69. However, it can be found that there are other fitting curves with different lifetimes

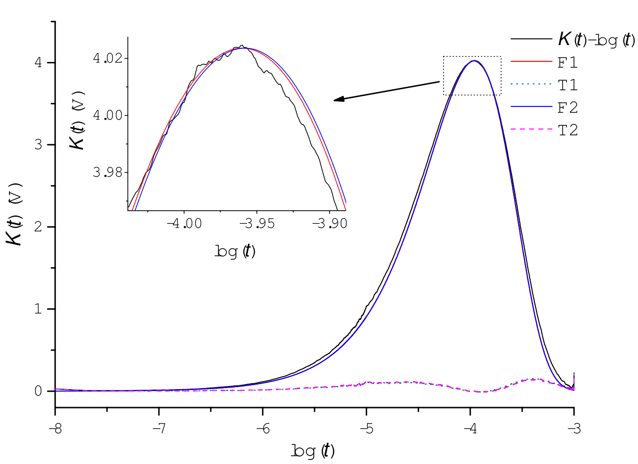

FIG. 7. The $K(t) - \log(t)$ curve (the black solid line) is a single CUT measurement data. Measured by a NI *LCR* meter, the reference values of $R_r = 10.0003$ kΩ and $C_r = 11.203$ nF. On the $K(t) - \log(t)$ curve, the peak position is $\tau = 10^{-3.960} = 1.096 \times 10^{-4}$ s.

whose RSS are smaller than F1. The F2 (the blue solid line, $\tau = 1.101 \times 10^{-4}$ s) curve has a minimum RSS of 18.59. The T2 (the magenta dashed line) is the subtraction of the F2 from the $K(t) - \log(t)$ curve, so it is the best fitting curve. Using lifetime $\tau = 1.101 \times 10^{-4}$ s, the calculated $C_{\mathrm{CUT2}} = 11.010$ nF, which is closer to C_r than C_{CUT1}. Its relative error is 1.72%. Compared with the method that simply finds the position of the maximum data on the peak, the logarithmic derivative curve fitting method has better anti-noise capability, which is more suitable for single capacitor measurement.

B. Multiple capacitors measurement

Using the multiple first-order RC summing circuit in Figure 5, an example of measuring three CUT's was done. In Figure 8, the black solid $K(t) - \log(t)$ curve shows the three capacitors measurement data. The SF curve (the magenta solid line) is the sum of three single-lifetime theoretical lines, and it fits the $K(t) - \log(t)$ curve. The dark yellow line T is the subtraction of the SF curve from the $K(t) - \log(t)$ curve, which can shows the fitting result directly.

By using the reference values of $R1_r$, $R2_r$, and $R3_r$ and the lifetimes of three single-lifetime fitting curves F1, F2, and F3, the calculated capacitance values of CUT1, CUT2, and CUT3 are 10.297 nF, 106.05 nF, and 606.25 nF, respectively. Their relative errors are 0.52%, 3.2%, and 4.5%, respectively. The example shows that the superposition of multiple single-lifetime $K(t) - \log(t)$ curves could fit a multi-lifetime $K(t) - \log(t)$ curve. For the logarithmic derivative curve fitting method, the same discharge initial voltage N_i could be obtained at first, as the same charge voltage source is employed in Figure 5. Then, by choosing different single lifetimes to construct a superposition $K(t) - \log(t)$ curve, when the minimum RSS data are obtained, all the lifetimes could be obtained.

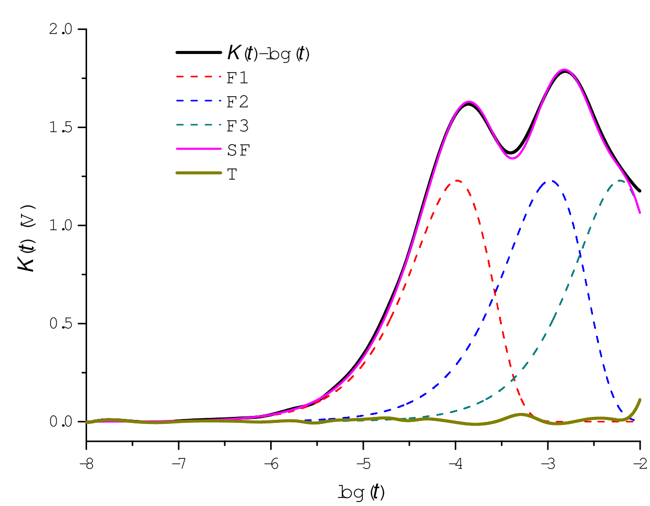

FIG. 8. A three CUT's $K(t) - \log(t)$ curve is obtained. As shown in Figure 5, the reference superposition values of the resistors and capacitors measured by a NI *LCR* meter are $R1_r = 10.003$ kΩ, $C_{\mathrm{CUT1r}} = 10.351$ nF, $R2_r = 10.005$ kΩ, $C_{\mathrm{CUT2r}} = 109.57$ nF, $R3_r = 9.9958$ kΩ, and $C_{\mathrm{CUT3r}} = 634.90$ nF. By choosing $N_1 = N_2 = N_3 = 1.45$ V, $\tau_1 = 1.038 \times 10^{-4}$ s, $\tau_2 = 1.061 \times 10^{-3}$ s, and $\tau_3 = 6.06 \times 10^{-3}$ s, the dashed lines of F1, F2, and F3 are three single-lifetime theoretical log(t) fitting curves. Summing the three fitting curves, the SF line is obtained, which is the best fitting curve with a minimum RSS of 1.20.

C. Measurement uncertainty analysis

There are several factors to be considered in the calculation of the uncertainty budget of the logarithmic derivative method, including the first-order *RC* summing circuit, the signal integration of the measurement system, the interpolation and rejection algorithm, and the filter algorithm. The first-order *RC* circuit suffers from the effect of stray capacitances, particularly in the measurement of small capacitances. Therefore, a drawback of current *RC* circuit is that the stray capacitances resistance ability is not well.

As show in Figure 7, the FWHM of $K(t) - \log(t)$ curve is slightly wider than the FWHM's of two fitting lines. The reason is the time accuracy error of the first sampling data. Due to the inherent limitation of the measurement system, compared to the real discharge start-time, the sampling time of first data may lag or advance. In the former situation, the $K(t) - \log(t)$ curve FWHM widens, as shown in Figure 7. On the contrary, it narrows. It increases the relative error of a $K(t) - \log(t)$ curve from a fitting one.

V. CONCLUSION

In this paper, a logarithmic derivative method is proposed to analyze multi-lifetime decays. As the discharge of first order *RC* circuit is a special kind of multi-lifetime decay, the logarithmic derivative method can be used to measure single capacitance and multiple capacitances. Based on a FPGA system on chip (SoC) and a high speed ADC, a measurement system is developed. This system can provide 16-bit data with sampling rate up to 250 MHz, which provides a large enough time-window for measuring lifetime shorter than 10^{-8} s. To reduce the amount of data needed to be stored and to improve the data processing speed, the measurement system automatically rejects parts of data with the increase of sampling time. The data processing program uses interpolation algorithm to make all the intervals have the same data number, and all the adjacent data on the horizontal $\log(t)$ axis are with the same interval. The lifetime τ could be found at the position of the maximum data on a single-lifetime $K(t) - \log(t)$ peak. The logarithmic derivative curve fitting method has a better anti-noise capability than the method by simply finding the maximum data on the peak. Furthermore, the logarithmic derivative curve fitting method can also be employed for the multiple capacitors measurement, so it is more suitable for the capacitance measurement. As the relative error of the measurement is not small enough yet, the future work will focus on determining the time of valid first acquisition data, reducing measurement system noise and optimizing data processing and curve fitting method. In order to measure small capacitance, the *RC* discharge circuit also needs to be refined to resist interference by stray capacitances.

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[17]See http://wiki.analog.com/resources/fpga/xilinx/fmc/ad9467 for Analog Devices Wiki, AD9467 Native FMC Card/Xilinx Reference Design.