Large-signal and high-frequency analysis of nonuniformly doped or shaped pn-junction diodes

Anatoly A. Barybin[1] and Edval J. P. Santos[2,a)]

[1]*Electronics Department, Saint-Petersburg State Electrotechnical University, 197376, Saint-Petersburg, Russia.*
[2]*Laboratory for Devices and Nanostructures, Departamento de Eletrônica e Sistemas, Universidade Federal de Pernambuco, C.P. 7800, 50670-000, Recife PE, Brasil.*

An analytical theory of nonuniformly doped or shaped pn-junction diodes subjected to large signals at high frequencies is presented. The resulting expressions can be useful to evaluate the performance of semiconductor device modeling software. The transverse averaging technique is employed to reduce the three-dimensional charge carrier transport equations into the quasi-one-dimensional form, with all physical quantities averaged out over the longitudinally varying cross section. Although axial symmetry is assumed, this approach gives rise to useful analytic expressions for the static current–voltage characteristics, the diffusion conductance, and diffusion capacitance as a function of signal amplitude and cross-section nonuniformity. © *2011 American Institute of Physics*. [doi:10.1063/1.3592206]

I. INTRODUCTION

Depending on the application, different levels of device modeling are used to predict the behavior of electronic circuits. For circuits with many devices, compact models are required to reduce the simulation time. For circuit building blocks with a few devices, physical models may be used to get further insight. Physics-based modeling requires more computer resources, but it has the advantage of being valid at a wider operating range, and offers easier to interpret parameters.[1] When modeling a pn-junction, different operating conditions are possible, such as: steady-state small-signal, steady-state large-signal, DC, and transient.[1–4]

The *transverse averaging technique* (TAT) allows the general three-dimensional (3D) equations of semiconductor electronics to be converted into the so-called *quasi-one-dimensional* (quasi-1D) form. The quasi-1D equations involve all the physical scalar quantities (potential, charge density, etc.) and longitudinal components of the vector quantities (electric field, current density, etc.) in the form averaged over the longitudinally varying cross section $S(z)$ and dependent only on the longitudinal coordinate z.[5,6] The quasi-1D was first applied to derive analytical expressions for the depletion capacitance of the pn-junctions with nonuniform doping impurity profile and cross-sectional geometry peculiar to various real devices. It may also be useful to p-*i*-n diode modeling.[7] This work does not apply to avalanche type diodes, such as IMPATT, as no generation/ionization process is included in the theory at this point.[8]

Besides, the one-dimensional equations also include the contour integrals along interface lines which take into account the proper boundary conditions between different domains of the semiconductor structure. Such equations are completely equivalent to the initial three-dimensional equations and in this respect are accurate except that they deal with physical quantities averaged over the cross section of a nonuniform semiconductor structure.

This paper is devoted to the generalization of the large-signal and high-frequency theory of charge carrier transport developed previously for uniform PN-junctions,[5,6,9] to nonuniform structures. Application of the general TAT relations given in Ref. 5 to deriving the quasi-1D drift-diffusion equations for nonuniform junctions is performed in Sec. II. Spectral solution of the quasi-1D diffusion equations for nonuniform pn-junctions is set forth in Sec. III. Section IV deals with the derivation and analysis of the external circuit current, which serves as a basis for obtaining the static current–voltage characteristics (Sec. V) and the dynamic impedance (Sec. VI) of the pn-junction diodes with nonuniform cross section.

II. DERIVATION OF QUASI-ONE-DIMENSIONAL DRIFT-DIFFUSION EQUATIONS FOR NONUNIFORM pn-JUNCTIONS

The initial equations for derivation of the quasi-1D drift-diffusion equations of the nonuniform pn-junctions are the three-dimensional continuity equations and the current density expressions:[8]

• for holes

$$\frac{\partial p}{\partial t} + \frac{1}{q}\nabla \cdot \mathbf{j}_p = -\frac{p - p_n}{\tau_p},$$

$$\mathbf{j}_p = qp\mathbf{v}_p - q\nabla(D_p p), \quad \mathbf{v}_p = \mu_p(E)\mathbf{E};$$

(1)

• for electrons

$$\frac{\partial n}{\partial t} - \frac{1}{q}\nabla \cdot \mathbf{j}_n = -\frac{n - n_p}{\tau_n},$$

$$\mathbf{j}_n = qn\mathbf{v}_n + q\nabla(D_n n), \quad \mathbf{v}_n = \mu_n(E)\mathbf{E};$$

(2)

a)Author to whom correspondence should be addressed. Electronic mail: edval@ee.ufpe.br.

with the normal components of currents that obey the following surface-recombination boundary conditions:[8]

$$\mathbf{n} \cdot \mathbf{j}_p|_L = qR_{ps}, \quad R_{ps} = s_p(p_s - p_n), \quad (3)$$

and

$$\mathbf{n} \cdot \mathbf{j}_n|_L = qR_{ns}, \quad R_{ns} = s_n(n_s - n_p). \quad (4)$$

Here $p_n, n_p, p,$ and n are the equilibrium and nonequilibrium densities of minority carriers in the n and p regions, and $D_{p(n)}, \mu_{p(n)},$ and $\tau_{p(n)}$ are the diffusion constant, mobility, and lifetime for holes (electrons).

Integration of Eqs. (1) and (2) over the cross section area $S(z)$ by using the TAT relations[5] produces the linear integrals along the contour $L(z)$ bounding $S(z)$, see Fig. 1 (from Ref. 5). For holes, calculating the average and applying the 3D extension of Green's Theorem in Eq. (1), yields

$$\frac{\partial(\bar{p}S)}{\partial t} + \frac{1}{q}\frac{\partial(\bar{j}_{pz}S)}{\partial z} + \frac{1}{q}\oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{j}_p}{\cos\theta} dl = -\frac{\bar{p} - p_n}{\tau_p}S, \quad (5)$$

$$\frac{1}{q}\bar{j}_{pz}S = \bar{p}\bar{\mu}_p\bar{E}_zS - \frac{\partial(\bar{D}_p\bar{p}S)}{\partial z} - \oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{e}_z}{\cos\theta} D_p p \, dl. \quad (6)$$

Here p_s and n_s are the nonequilibrium minority-carrier volume densities taken at surface points of the structure, and $s_{p(n)}$ is the surface recombination velocity for holes (electrons).

Here the average quantities $\bar{E}_z, \bar{j}_{pz},$ and $\bar{p}$ are introduced,[5] while the average mobility and diffusion constant for holes (and similar ones for electrons) are defined as

$$\bar{\mu}_p = \frac{\overline{pv_p(E)E_z/E}}{\bar{p}\bar{E}_z} \quad \text{and} \quad \bar{D}_p = \frac{\overline{D_p p}}{\bar{p}}.$$

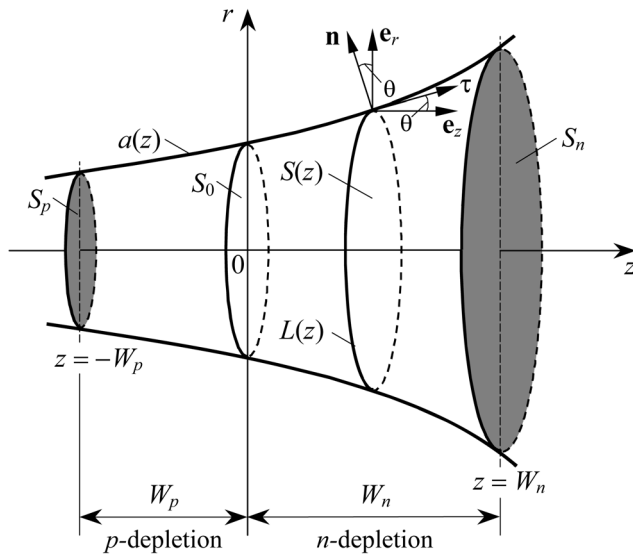
FIG. 1. Axially symmetric PN-junction; the depletion layer is situated between the cross sections $z = -W_p$ of area S_p and $z = W_n$ of area S_n. Reprinted with permission from A. A. Barybin and E. J. P. Santos, Semicond. Sci. Technol. **22**, 312 (2007).

For axially-symmetrical structures with $S(z) = \pi a^2(z)$, we have

$$\frac{\mathbf{n} \cdot \mathbf{e}_z}{\cos\theta} = -\tan\theta = -\frac{da(z)}{dz},$$

so that the contour integrals in Eqs. (5) and (6) with the boundary condition (3) assume the form

$$\frac{1}{q}\oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{j}_p}{\cos\theta} dl = 2s_p \frac{\sqrt{1 + (da/dz)^2}}{a(z)}(p_s - p_n)S, \quad (7)$$

$$\oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{e}_z}{\cos\theta} D_p p \, dl = -D_{ps}p_s \frac{dS}{dz}. \quad (8)$$

By substituting expressions (6)–(8) into Eq. (5) and introducing the effective lifetime $\bar{\tau}_p$ to allow for both the volume and surface recombination

$$\frac{1}{\bar{\tau}_p} = \frac{1}{\tau_p} + 2s_p \frac{\sqrt{1 + (da/dz)^2}}{a(z)}\frac{p_s - p_n}{\bar{p} - p_n},$$

we arrive at the quasi-1D drift-diffusion equation for holes injected into the n region:

$$\frac{\partial}{\partial t}(\bar{p}S) + \frac{\partial}{\partial z}(\bar{p}\bar{\mu}_p\bar{E}_zS) - \frac{\partial^2}{\partial z^2}(\bar{D}_p\bar{p}S) + \frac{\partial}{\partial z}\left(D_{ps}p_s\frac{dS}{dz}\right)$$
$$= -\frac{\bar{p} - p_n}{\bar{\tau}_p}S. \quad (9)$$

Similarly, from the initial Eqs. (2) and (4) we obtain the quasi-1D drift-diffusion equation for electrons injected into the p region:

$$\frac{\partial}{\partial t}(\bar{n}S) + \frac{\partial}{\partial z}(\bar{n}\bar{\mu}_n\bar{E}_zS) - \frac{\partial^2}{\partial z^2}(\bar{D}_n\bar{n}S) + \frac{\partial}{\partial z}\left(D_{ns}n_s\frac{dS}{dz}\right)$$
$$= -\frac{\bar{n} - n_p}{\bar{\tau}_n}S. \quad (10)$$

All the average quantities $\bar{\mu}_{p(n)}, \bar{D}_{p(n)},$ and $\bar{\tau}_{p(n)}$ will be considered as phenomenologically given parameters with dropping the bar sign over them for simplicity.

Quasi-one-dimensional Eqs. (9) and (10) are of the general form applicable for both the *pin*- and pn-diodes. For low level of injection in the pn-diodes we can assume $\bar{E}_z = 0$ for diode base so that Eqs. (9) and (10) take the simplified form:

$$\frac{\partial}{\partial t}(\bar{p}S) - \frac{\partial}{\partial z}\left(S\frac{\partial}{\partial z}(D_p\bar{p})\right) - \frac{\partial}{\partial z}\left[(D_p\bar{p} - D_{ps}p_s)\frac{dS}{dz}\right]$$
$$= -\frac{\bar{p} - p_n}{\tau_p}, \quad (11)$$

$$\frac{\partial}{\partial t}(\bar{n}S) - \frac{\partial}{\partial z}\left(S\frac{\partial}{\partial z}(D_n\bar{n})\right) - \frac{\partial}{\partial z}\left[(D_n\bar{n} - D_{ns}n_s)\frac{dS}{dz}\right]$$
$$= -\frac{\bar{n} - n_p}{\tau_n}. \quad (12)$$

Not having a specific knowledge of surface properties, we shall assume that $D_{ps}p_s = D_p\bar{p}$ and $D_{ns}n_s = D_n\bar{n}$. Then, from

equations (11) and (12), the *quasione-dimensional diffusion equations* follow for the excess concentrations of holes, $\Delta\bar{p}(z,t) = \bar{p}(z,t) - p_n$, and electrons, $\Delta\bar{n}(z,t) = \bar{n}(z,t) - n_p$, injected into the appropriate neutral parts of the pn-diode:[5]

$$\left(1 + \tau_p \frac{\partial}{\partial t}\right)\Delta\bar{p} - L_p^2 \frac{\partial^2 \Delta\bar{p}}{\partial z^2} - L_p^2 \frac{\partial \ln S}{\partial z}\frac{\partial \Delta\bar{p}}{\partial z} = 0, \quad (13)$$

$$\left(1 + \tau_n \frac{\partial}{\partial t}\right)\Delta\bar{n} - L_n^2 \frac{\partial^2 \Delta\bar{n}}{\partial z^2} - L_n^2 \frac{\partial \ln S}{\partial z}\frac{\partial \Delta\bar{n}}{\partial z} = 0, \quad (14)$$

where $L_{p(n)} = \sqrt{D_{p(n)}\tau_{p(n)}}$ is the diffusion length for holes (electrons). The additional last term involving $\partial \ln S / \partial z \equiv S'(z)/S(z)$ takes into account the cross-sectional nonuniformity.

III. SPECTRAL SOLUTION OF QUASI-ONE-DIMENSIONAL DIFFUSION EQUATIONS

In the general case, the voltage applied to the pn-junction consists of the dc bias voltage V_0 and the ac harmonic signal $V_\sim \cos \omega t$:

$$v(t) = V_0 + V_\sim \cos \omega t \equiv V_0 + [V_1(\omega)e^{i\omega t} + c.c.]. \quad (15)$$

Nonlinearity of electronic processes in the pn-junction produces the frequency harmonics $k\omega$ so that solutions of Eqs. (13) and (14) have the form of Fourier series:

$$\Delta\bar{p}(z,t) = \sum_{k=-\infty}^{\infty} \Delta\bar{p}_k(z)e^{ik\omega t}, \quad (16)$$

$$\Delta\bar{n}(z,t) = \sum_{k=-\infty}^{\infty} \Delta\bar{n}_k(z)e^{ik\omega t}. \quad (17)$$

Real values of $\Delta\bar{p}(z,t)$ and $\Delta\bar{n}(z,t)$ are provided with the following relations for the complex amplitudes: $\Delta\bar{p}_k = \Delta\bar{p}_{-k}^*$ and $\Delta\bar{n}_k = \Delta\bar{n}_{-k}^*$.

Substitution of the required solutions (16) and (17) into Eqs. (13) and (14) with regard for the orthogonality of harmonics reduces to the following equations for the harmonic amplitudes:[5]

$$\frac{d^2 \Delta\bar{p}_k}{dz^2} + \frac{d \ln S}{dz}\frac{d\Delta\bar{p}_k}{dz} - \frac{\Delta\bar{p}_k}{L_{pk}^2} = 0, \quad (18)$$

$$\frac{d^2 \Delta\bar{n}_k}{dz^2} + \frac{d \ln S}{dz}\frac{d\Delta\bar{n}_k}{dz} - \frac{\Delta\bar{n}_k}{L_{nk}^2} = 0, \quad (19)$$

where

$$L_{pk} = \frac{L_p}{\sqrt{1 + ik\omega\tau_p}} \quad \text{and} \quad L_{nk} = \frac{L_n}{\sqrt{1 + ik\omega\tau_n}}.$$

As an analytical approximation to a mesalike structure, the special case of exponential change of the cross section $S(z) = S_0 \exp(2\alpha z)$ is considered and Eqs. (18) and (19) turn into the linear equations[5]

$$\frac{d^2 \Delta\bar{p}_k}{dz^2} + 2\alpha\frac{d\Delta\bar{p}_k}{dz} - \frac{\Delta\bar{p}_k}{L_{pk}^2} = 0, \quad (20)$$

$$\frac{d^2 \Delta\bar{n}_k}{dz^2} + 2\alpha\frac{d\Delta\bar{n}_k}{dz} - \frac{\Delta\bar{n}_k}{L_{nk}^2} = 0. \quad (21)$$

Representing the desired solution in the form of $e^{-\lambda z}$, we obtain the following characteristic equations for Eqs. (20) and (21):

$$\lambda^2 - 2\alpha\lambda - L_k^{-2} = 0, \quad \text{where} \quad L_k = \begin{cases} L_{pk} & \text{for holes,} \\ L_{nk} & \text{for electrons.} \end{cases} \quad (22)$$

The complex roots of Eq. (22) can be written as follows

$$\lambda_{1,2} = \begin{cases} \pm\Lambda_{pk}^\pm/L_p & \text{for the } n\text{-region} \quad (z \geq W_n), \\ \pm\Lambda_{nk}^\pm/L_n & \text{for the } p\text{-region} \quad (z \leq -W_p), \end{cases} \quad (23)$$

where we have introduced the following notations:

• for holes injected into the n region ($z \geq W_n$)

$$\Lambda_{pk}^\pm = (a_{pk}^\alpha A_p^\alpha \pm \alpha L_p) + ib_{pk}^\alpha A_p^\alpha, \quad A_p^\alpha = \sqrt{1 + (\alpha L_p)^2},$$

$$a_{pk}^\alpha = \frac{1}{\sqrt{2}}\sqrt{1 + \Theta_{pk}^\alpha}, \quad b_{pk}^\alpha = \frac{k\omega\tau_p^\alpha}{2a_{pk}^\alpha}, \quad (24)$$

$$\Theta_{pk}^\alpha = \sqrt{1 + (k\omega\tau_p^\alpha)^2}, \quad \tau_p^\alpha = \frac{\tau_p}{1 + (\alpha L_p)^2};$$

• for electrons injected into the p region ($z \leq -W_p$)

$$\Lambda_{nk}^\pm = (a_{nk}^\alpha A_n^\alpha \pm \alpha L_n) + ib_{nk}^\alpha A_n^\alpha, \quad A_n^\alpha = \sqrt{1 + (\alpha L_n)^2},$$

$$a_{nk}^\alpha = \frac{1}{\sqrt{2}}\sqrt{1 + \Theta_{nk}^\alpha}, \quad b_{nk}^\alpha = \frac{k\omega\tau_n^\alpha}{2a_{nk}^\alpha}, \quad (25)$$

$$\Theta_{nk}^\alpha = \sqrt{1 + (k\omega\tau_n^\alpha)^2}, \quad \tau_n^\alpha = \frac{\tau_n}{1 + (\alpha L_n)^2}.$$

The general solutions of Eqs. (20) and (21) have the following form:

• for holes injected into the n region ($z \geq W_n$)

$$\Delta\bar{p}_k(z) = C_{pk}^+ \exp\left(-\Lambda_{pk}^+ \frac{z - W_n}{L_p}\right) + C_{pk}^- \exp\left(\Lambda_{pk}^- \frac{z - W_n}{L_p}\right); \quad (26)$$

• for electrons injected into the p region ($z \leq -W_p$)

$$\Delta\bar{n}_k(z) = C_{nk}^+ \exp\left(-\Lambda_{nk}^+ \frac{z + W_p}{L_n}\right) + C_{nk}^- \exp\left(\Lambda_{nk}^- \frac{z + W_p}{L_n}\right). \quad (27)$$

In accordance with notations (24) and (25), always $\mathrm{Re}\Lambda_{pk}^\pm = a_{pk}^\alpha A_p^\alpha \pm \alpha L_p > 0$ and $\mathrm{Re}\Lambda_{nk}^\pm = a_{nk}^\alpha A_n^\alpha \pm \alpha L_n > 0$ so that for pn-diodes with thick bases we can assume $C_{pk}^- = 0$ and $C_{nk}^+ = 0$, which eliminates the necessity for boundary conditions on ohmic contacts. Taking into account formulas (26) and (27) with $C_{pk}^- = C_{nk}^+ = 0$, the general solutions (16) and (17) of Eqs. (13) and (14) are written in the form

$$\Delta\bar{p}(z,t) = \sum_{k=-\infty}^{\infty} C_{pk}^+ \exp\left(-\Lambda_{pk}^+ \frac{z - W_n}{L_p}\right)e^{ik\omega t}, \quad (28)$$

$$\Delta\bar{n}(z,t) = \sum_{k=-\infty}^{\infty} C_{nk}^{-} \exp\left(\Lambda_{nk}^{-} \frac{z+W_p}{L_n}\right) e^{ik\omega t}. \quad (29)$$

These expressions provide $\Delta\bar{p}(z,t) \to 0$ as $z \to \infty$ and $\Delta\bar{n}(z,t) \to 0$ as $z \to -\infty$ because of $\mathrm{Re}\Lambda_{pk}^{+} > 0$ and $\mathrm{Re}\Lambda_{nk}^{-} > 0$.

The constants C_{pk}^{+} and C_{nk}^{-} appearing in Eqs. (28) and (29) can be found from the conventional injection boundary conditions:[8]

$$\Delta\bar{p}(W_n,t) = p_n f(t) \quad \text{for} \quad z = W_n, \quad (30)$$

$$\Delta\bar{n}(-W_p,t) = n_p f(t) \quad \text{for} \quad z = -W_p, \quad (31)$$

where for the applied voltage $v(t)$ of the form (15) we have introduced the function

$$f(t) = \exp\left(\frac{qv(t)}{\kappa T}\right) - 1. \quad (32)$$

Substitution of expressions (28) and (29) into the boundary conditions (30) and (31) yields

$$\sum_{k=-\infty}^{\infty} C_{pk}^{+} e^{ik\omega t} = p_n f(t),$$
$$\sum_{k=-\infty}^{\infty} C_{nk}^{-} e^{ik\omega t} = n_p f(t). \quad (33)$$

By using the orthogonality property of harmonics in expansions (33) it is easy to get the desired constants

$$C_{pk}^{+} = p_n F_k \quad \text{and} \quad C_{nk}^{-} = n_p F_k, \quad (34)$$

where F_k is the Fourier amplitude of the kth harmonic for the function $f(t)$ given by formula (32), that is

$$F_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-ik\omega t} d\omega t. \quad (35)$$

Substitution of the function (32) into formula (35) gives

$$F_0 = I_0(\beta V_\sim) \exp(\beta V_0) - 1 \quad \text{for} \quad k = 0, \quad (36)$$

$$F_k = F_{-k} = I_k(\beta V_\sim) \exp(\beta V_0) \quad \text{for} \quad k \neq 0, \quad (37)$$

where we have used the modified Bessel functions of the first kind of order k ($k = 0, \pm 1, \pm 2, \ldots$) having the following integral representation (see formula 8.431.5 in Ref. 10):

$$I_k(\beta V_\sim) = \frac{1}{\pi} \int_{0}^{\pi} e^{\beta V_\sim \cos \omega t} \cos k\omega t\, d\omega t. \quad (38)$$

This function depends on $\beta V_\sim$, where $V_\sim$ is an amplitude of the signal applied to the pn-junction and $\beta = q/\kappa T$.

Thus, with allowing for Eq. (34) the general solutions (28) and (29) of the diffusion equations (13) and (14) take the final form of spectral expansions:

$$\Delta\bar{p}(z,t) = p_n \sum_{k=-\infty}^{\infty} F_k \exp\left(-\Lambda_{pk}^{+} \frac{z-W_n}{L_p}\right) e^{ik\omega t}, \quad (39)$$

$$\Delta\bar{n}(z,t) = n_p \sum_{k=-\infty}^{\infty} F_k \exp\left(\Lambda_{nk}^{-} \frac{z+W_p}{L_n}\right) e^{ik\omega t}. \quad (40)$$

These expressions allow us to obtain the spectral composition of the current flowing through the external circuit connected to the pn-diode.

IV. EXTERNAL CIRCUIT CURRENT FOR SEMICONDUCTOR DIODE WITH NONUNIFORM CROSS SECTION

The initial equation to derive an expression for the diode current is the law of total current conservation:

$$\nabla \cdot \left(\mathbf{j}_p + \mathbf{j}_n + \epsilon \frac{\partial \mathbf{E}}{\partial t}\right) = 0 \quad (41)$$

following from Maxwell's equation $\nabla \times \mathbf{H} = \mathbf{j}_p + \mathbf{j}_n + \epsilon \partial \mathbf{E}/\partial t$.

The transverse averaging technique applied to Eq. (41) gives

$$\frac{\partial}{\partial z}\left[\left(\bar{j}_{pz} + \bar{j}_{nz} + \epsilon \frac{\partial \bar{E}_z}{\partial t}\right)S\right] + \oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{j}}{\cos\theta} dl + \frac{\partial}{\partial t} \oint_{L(z)} \frac{\mathbf{n} \cdot \epsilon\mathbf{E}}{\cos\theta} dl = 0. \quad (42)$$

The similar equation outside the semiconductor, where currents are absent and $\nabla \cdot (\epsilon^o \mathbf{E}^o) = 0$, has the following form:

$$\frac{\partial}{\partial z}\left(\epsilon^o \frac{\partial \bar{E}_z^o}{\partial t} S^o\right) - \frac{\partial}{\partial t} \oint_{L(z)} \frac{\mathbf{n} \cdot \epsilon^o \mathbf{E}^o}{\cos\theta} dl = 0, \quad (43)$$

where S^o is an effective localization area of the fringe outside field $\mathbf{E}^o$ such that usually $S^o \ll S$ and $\epsilon^o < \epsilon$. The addition of Eqs. (42) and (43) gives

$$\frac{\partial}{\partial z}\left[(\bar{j}_{pz} + \bar{j}_{nz})S + \frac{\partial}{\partial t}\left(\epsilon \bar{E}_z S + \epsilon^o \bar{E}_z^o S^o\right)\right] + \oint_{L(z)} \frac{\mathbf{n} \cdot \mathbf{j}}{\cos\theta} dl + \frac{\partial}{\partial t} \oint_{L(z)} \frac{\mathbf{n} \cdot (\epsilon\mathbf{E} - \epsilon^o\mathbf{E}^o)}{\cos\theta} dl = 0. \quad (44)$$

If a semiconductor surface contains traps with the charge density ρ_s, there are the following boundary conditions on the contour $L(z)$:[11]

$$\mathbf{n} \cdot \epsilon^o \mathbf{E}^o = \mathbf{n} \cdot \epsilon\mathbf{E} + \rho_s \quad \text{and} \quad \frac{\partial \rho_s}{\partial t} = \mathbf{n} \cdot \mathbf{j}. \quad (45)$$

In this case, the two contour integrals in Eq. (44) cancel each other and with allowing for inequality $|\epsilon^o \bar{E}_z^o S^o| \ll |\epsilon \bar{E}_z S|$, formula (44) takes the form

$$\frac{\partial}{\partial z}\left[\left(\bar{j}_{pz}(z,t)+\bar{j}_{nz}(z,t)+\epsilon\frac{\partial \bar{E}_z(z,t)}{\partial t}\right)S(z)\right]=0. \quad (46)$$

The quantity in brackets of Eq. (46), being independent of z, defines the *external circuit current* equal to

$$J(t)=q(\mu_p\bar{p}+\mu_n\bar{n})\bar{E}_zS-q\left(D_p\frac{\partial\bar{p}}{\partial z}-D_n\frac{\partial\bar{n}}{\partial z}\right)S+\epsilon\frac{\partial\bar{E}_z}{\partial t}S. \quad (47)$$

Here we have used expressions (6) and (8) for the average hole current $\bar{j}_{pz}S$ and similar expressions for the average electron current $\bar{j}_{nz}S$ (with dropping the bar sign over $\bar{\mu}_{p(n)}$ and $\bar{D}_{p(n)}$). All the terms on the right of Eq. (47) depend on both z and t but taken together at any cross section $S(z)$ they yield the external circuit current $J(t)$ as a function of only time.

Restricting our consideration to the pn-diodes with low injection, we can assume $\bar{E}_z=0$ in neutral parts of the p- and n-regions.[8] Then the external circuit current in Eq. (47) is determined only by the averaged diffusion currents taken at any cross section $S(z)$, for example, at $z=W_n$:

$$J(t)=-qD_p\frac{\partial\bar{p}(z,t)}{\partial z}\bigg|_{z=W_n}S(W_n)+qD_n\frac{\partial\bar{n}(z,t)}{\partial z}\bigg|_{z=W_n}S(W_n)$$
$$\equiv \bar{j}_{pz}(W_n,t)S(W_n)+\bar{j}_{nz}(W_n,t)S(W_n). \quad (48)$$

Neglecting recombination processes inside the pn-junction, which is true if $W=W_n+W_p\ll L_p$ and L_n,[8] we can write

$$\bar{j}_{nz}(W_n,t)S(W_n)=\bar{j}_{nz}(-W_p,t)S(-W_p). \quad (49)$$

Substitution of relation (49) into Eq. (48) gives the external circuit current (cf. Eq. (31) in Ref. 5)

$$J(t)=\bar{j}_{pz}(W_n,t)S(W_n)+\bar{j}_{nz}(-W_p,t)S(-W_p)$$
$$=-qD_p\frac{\partial\Delta\bar{p}(z,t)}{\partial z}\bigg|_{z=W_n}S(W_n)+qD_n\frac{\partial\Delta\bar{n}(z,t)}{\partial z}\bigg|_{z=-W_p}S(-W_p), \quad (50)$$

where $\Delta\bar{p}=\bar{p}-p_n$ and $\Delta\bar{n}=\bar{n}-n_p$ are the excess concentrations of injected carriers determined by formulas (39) and (40). Inserting these formulas into expression (50), we finally obtain the spectral representation for the external circuit current:

$$J(t)=J_{s,p}\sum_{k=-\infty}^{\infty}F_k\Lambda_{pk}^+e^{ik\omega t}+J_{s,n}\sum_{k=-\infty}^{\infty}F_k\Lambda_{nk}^-e^{ik\omega t}. \quad (51)$$

Here, the hole and electron contributions into the saturation current of a thick pn-junction are defined, as it is generally accepted,[8] in the form

$$J_{s,p}=\frac{qD_pp_n}{L_p}S_n \quad \text{and} \quad J_{s,n}=\frac{qD_nn_p}{L_n}S_p, \quad (52)$$

where

$$S_n\equiv S(W_n)=S_0\exp(2\alpha W_n),$$

$$S_p\equiv S(-W_p)=S_0\exp(-2\alpha W_p).$$

Expression (51) contains all the spectral components of the external circuit current including the dc current for $k=0$ and the ac current for $k=\pm1$ which are of most interest for our further consideration.

V. STATIC CURRENT–VOLTAGE CHARACTERISTIC OF PN-DIODE WITH NONUNIFORM CROSS SECTION

The term in series (51) numbered by $k=0$ corresponds to the dc current J_0, which by using Eq. (36) can be written in the form of the *static current–voltage characteristic*:

$$J_0(V_0,V_\sim)=J_s[I_0(\beta V_\sim)e^{\beta V_0}-1]. \quad (53)$$

Here we have introduced the saturation current for a nonuniform diode

$$J_s=J_{s,p}r_{p0}^\alpha+J_{s,n}r_{n0}^\alpha \quad (54)$$

and, in accordance with expressions (24) and (25) for $k=0$, have used the following notation:

$$r_{p0}^\alpha\equiv\Lambda_{p0}^+=A_p^\alpha+\alpha L_p=\sqrt{1+(\alpha L_p)^2}+\alpha L_p,$$
$$\quad (55)$$
$$r_{n0}^\alpha\equiv\Lambda_{n0}^-=A_n^\alpha-\alpha L_n=\sqrt{1+(\alpha L_n)^2}-\alpha L_n.$$

For the uniform p–n junction (with $\alpha=0$) we have $r_{p0}^\alpha=r_{n0}^\alpha=1$ and $S_n=S_p=S_0$ so that the static current–voltage characteristic retains the form of Eq. (53) with the usual saturation current

$$J_s=J_{s,p}^0+J_{s,n}^0\equiv\frac{qD_pp_n}{L_p}S_0+\frac{qD_nn_p}{L_n}S_0, \quad (56)$$

where, as before, the superscript 0 marks the cross-sectional uniformity (when $\alpha=0$ and $S_0=$ constant).

The modified Bessel function $I_0(\beta V_\sim)$ appearing in Eq. (53) is completely the same as that obtained in our previous paper[5] and it distinguishes our expression (53) from the similar formula given in the known literature on semiconductor electronics.[8] Coincidence between them occurs only for such small signals that $V_\sim\ll\kappa T/q$ and $I_0(\beta V_\sim)\simeq1$. The function $I_0(\beta V_\sim)\geq1$ reflects the effect of signal rectification, which provides the contribution into the dc current from a signal and results in upward shifts of curves $J_0(V_0)$ with increasing signal amplitude $V_\sim$.

VI. DYNAMIC IMPEDANCE OF PN-DIODE WITH NONUNIFORM CROSS SECTION

The first harmonic of the external current in the general expression (51) corresponds to terms numbered by $k=\pm1$ and equals

$$J_1(t)=J_1(\omega)e^{i\omega t}+c.c., \quad (57)$$

where $J_1(\omega) = F_1[J_{s,p}\Lambda_{p1}^+(\omega) + J_{s,n}\Lambda_{n1}^-(\omega)]$. The quantities Λ_{p1}^+, Λ_{n1}^-, and F_1 are respectively defined by formulas (24), (25), and (37) for $k=1$.

Expression (15) with $V_\sim = 2V_1(\omega)$ and Eq. (57) allow one to introduce the *dynamic admittance* of the pn-diode as a function of frequency:

$$Y(\omega) = \frac{J_1(\omega)}{V_1(\omega)} \equiv G_d(\omega) + i\omega C_d(\omega). \qquad (58)$$

The *dynamic (diffusion) conductance* G_d and the *dynamic (diffusion) capacitance* C_d are defined in a customary way.[8] After substituting Eq. (57) into Eq. (58) and some transformations with regard to Eq. (54), we obtain

$$G_d = gG_0\frac{J_{s,p}r_{p1}^\alpha + J_{s,n}r_{n1}^\alpha}{J_{s,p}r_{p0}^\alpha + J_{s,n}r_{n0}^\alpha} \equiv gG_0\left(r_{p1}^\alpha\frac{J_{s,p}}{J_s} + r_{n1}^\alpha\frac{J_{s,n}}{J_s}\right), \qquad (59)$$

$$C_d = \frac{gG_0}{\omega}\frac{J_{s,p}q_{p1}^\alpha + J_{s,n}q_{n1}^\alpha}{J_{s,p}r_{p0}^\alpha + J_{s,n}r_{n0}^\alpha} \equiv \frac{gG_0}{2}\left(q_{p1}^\alpha\frac{J_{s,p}\tau_p}{J_s} + q_{n1}^\alpha\frac{J_{s,n}\tau_n}{J_s}\right). \qquad (60)$$

Here we have introduced the new quantities:

$$r_{p1}^\alpha = a_{p1}^\alpha A_p^\alpha + \alpha L_p, \quad r_{n1}^\alpha = a_{n1}^\alpha A_n^\alpha - \alpha L_n, \qquad (61)$$

$$q_{p1}^\alpha = \frac{1}{a_{p1}^\alpha A_p^\alpha}, \quad q_{n1}^\alpha = \frac{1}{a_{p1}^\alpha A_p^\alpha}, \qquad (62)$$

and, in accordance with Eqs. (24) and (25) for $k=1$, used the following notation:

$$a_{p1}^\alpha = \frac{1}{\sqrt{2}}\sqrt{1+\Theta_{p1}^\alpha}, \quad a_{n1}^\alpha = \frac{1}{\sqrt{2}}\sqrt{1+\Theta_{n1}^\alpha},$$
$$\qquad (63)$$
$$\Theta_{p1}^\alpha = \sqrt{1+(\omega\tau_p^\alpha)^2}, \quad \Theta_{n1}^\alpha = \sqrt{1+(\omega\tau_n^\alpha)^2}.$$

Formulas (59) and (60) contain the differential conductance G_0 of the static current–voltage characteristic $J_0(V_0, V_\sim)$ defined as

$$G_0 = \frac{\partial J_0(V_0, V_\sim)}{\partial V_0}. \qquad (64)$$

For the static characteristic of the form given by Eq. (53), the differential conductance in Eq. (64) depends on both the bias voltage V_0 and the signal amplitude $V_\sim$:

$$G_0(V_0, V_\sim) = \frac{J_0 + J_s}{\kappa T/q} \equiv \frac{J_s\exp(\beta V_0)}{\kappa T/q}I_0(\beta V_\sim). \qquad (65)$$

Formulas (59) and (60) involve a quantity

$$g(V_\sim) = \frac{g_1(V_\sim)}{I_0(\beta V_\sim)} \leq 1 \quad \text{with} \quad g_1(V_\sim) = \frac{I_1(\beta V_\sim)}{\beta V_\sim/2} \qquad (66)$$

composed of the modified Bessel functions I_0 and I_1 and introduced before in Ref 5. The quantity depends on the sig-

nal amplitude $V_\sim$, so that $g(V_\sim) \simeq 1$ for small signals when $V_\sim \ll \kappa T/q$ and $g(V_\sim) \to 0$ as a function $2/(\beta V_\sim)$ when $V_\sim \to \infty$.

From Eqs. (65) and (66) it follows that

$$gG_0 = g_1\frac{J_s\exp(\beta V_0)}{\kappa T/q} \equiv \frac{I_1(\beta V_\sim)}{\beta V_\sim/2}\frac{J_s\exp(\beta V_0)}{\kappa T/q}. \qquad (67)$$

Substituting Eq. (67) into Eqs. (59) and (60), they assume the following form:

$$G_d(\omega) = g_1 G_{d0}\left(r_{p1}^\alpha(\omega)\frac{J_{s,p}}{J_s} + r_{n1}^\alpha(\omega)\frac{J_{s,n}}{J_s}\right), \qquad (68)$$

$$C_d(\omega) = g_1 C_{d0}\left(q_{p1}^\alpha(\omega)\frac{Q_p}{Q} + q_{n1}^\alpha(\omega)\frac{Q_n}{Q}\right), \qquad (69)$$

where, following Ref. 5, we have introduced the charges $Q_p \equiv J_{s,p}\tau_p = qL_p p_n S_n$, $Q_n \equiv J_{s,n}\tau_n = qL_n n_p S_p$, and similar to Eq. (54)

$$Q = Q_p r_{p0}^\alpha + Q_n r_{n0}^\alpha. \qquad (70)$$

The quantities G_{d0} and C_{d0} appearing in Eqs. (68) and (69) are defined, by analogy with those in Ref. 5, as

$$G_{d0}(V_0) = \frac{J_s\exp(\beta V_0)}{\kappa T/q} = \frac{J_{s,p}r_{p0}^\alpha + J_{s,n}r_{n0}^\alpha}{\kappa T/q}e^{\beta V_0}$$
$$\equiv \frac{q}{\kappa T}\left(\frac{qp_nD_p}{L_p}S_n r_{p0}^\alpha + \frac{qn_pD_n}{L_n}S_p r_{n0}^\alpha\right)e^{qV_0/\kappa T}, \qquad (71)$$

$$C_{d0}(V_0) = \frac{Q\exp(\beta V_0)}{2\kappa T/q} = \frac{Q_p r_{p0}^\alpha + Q_n r_{n0}^\alpha}{2\kappa T/q}e^{\beta V_0}$$
$$\equiv \frac{q}{\kappa T}\left(\frac{qp_nL_p}{2}S_n r_{p0}^\alpha + \frac{qn_pL_n}{2}S_p r_{n0}^\alpha\right)e^{qV_0/\kappa T}. \qquad (72)$$

An essential simplification of Eqs. (68)–(69) and (71)–(72) occurs in the case of the *one-sided* p^+n junction with highly doped emitter when $p_n \gg n_p$, $W_p \ll W_n$, $J_{s,n} \ll J_{s,p}$, $Q_n \ll Q_p$, so that Eqs. (54) and (70) yield $J_s \simeq J_{s,p}r_{p0}^\alpha$ and $Q \simeq Q_p r_{p0}^\alpha$. Then, the quantities (71) and (72) can be written in the simplified form:

$$G_{d0}(V_0) \simeq G_{d0}^0(V_0)r_{p0}^\alpha e^{2\alpha W_n}, \qquad (73)$$

$$C_{d0}(V_0) \simeq C_{d0}^0(V_0)r_{p0}^\alpha e^{2\alpha W_n}, \qquad (74)$$

where the newly introduced quantities (marked with superscript 0)

$$G_{d0}^0(V_0) = \frac{J_{s,p}^0\exp(\beta V_0)}{\kappa T/q} \equiv \frac{qS_0}{\kappa T}\frac{qp_nD_p}{L_p}e^{qV_0/\kappa T}, \qquad (75)$$

$$C_{d0}^0(V_0) = \frac{Q_p^0\exp(\beta V_0)}{2\kappa T/q} \equiv \frac{qS_0}{\kappa T}\frac{qp_nL_p}{2}e^{qV_0/\kappa T}, \qquad (76)$$

correspond, as before, to the cross-sectionally uniform structures (with $\alpha = 0$ and $S_0 = $ constant).

Substitution of Eqs. (73) and (74) into expressions (68) and (69) with $J_{s,n} \simeq 0$ and $Q_n \simeq 0$ converts them into the form appropriate to the one-sided junction:

$$\frac{G_d(\omega, V_\sim)}{G_{d0}^0(V_0)} \simeq F_G^\alpha(\omega) \frac{I_1(\beta V_\sim)}{\beta V_\sim/2}, \qquad (77)$$

$$\frac{C_d(\omega, V_\sim)}{C_{d0}^0(V_0)} \simeq F_C^\alpha(\omega) \frac{I_1(\beta V_\sim)}{\beta V_\sim/2}. \qquad (78)$$

Here we have defined the frequency-dependent factors:

$$F_G^\alpha(\omega) = r_{p1}^\alpha(\omega) e^{2\alpha W_n} \equiv [a_{p1}^\alpha(\omega) A_p^\alpha + \alpha L_p] e^{2\alpha W_n}, \qquad (79)$$

$$F_C^\alpha(\omega) = q_{p1}^\alpha(\omega) e^{2\alpha W_n} \equiv \frac{1}{a_{p1}^\alpha(\omega) A_p^\alpha} e^{2\alpha W_n}, \qquad (80)$$

where $A_p^\alpha = \sqrt{1 + (\alpha L_p)^2}$ and

$$a_{p1}^\alpha(\omega) A_p^\alpha = \frac{1}{\sqrt{2}} \sqrt{1 + (\alpha L_p)^2 + \sqrt{[1 + (\alpha L_p)^2]^2 + (\omega \tau_p)^2}}.$$

For the uniform pn-junction (with $\alpha = 0$) from Eqs. (79) and (80) it follows that

$$F_G^\alpha(\omega) \xrightarrow{\alpha \to 0} \frac{\sqrt{1 + \sqrt{1 + (\omega \tau_p)^2}}}{\sqrt{2}}, \qquad (81)$$

$$F_C^\alpha(\omega) \xrightarrow{\alpha \to 0} \frac{\sqrt{2}}{\sqrt{1 + \sqrt{1 + (\omega \tau_p)^2}}}, \qquad (82)$$

so that expressions (77) and (78) take a form identical to formulas (48) and (49) obtained in Ref. 5.

The dependence on the signal amplitude $V_\sim$ for the diffusion conductance $G_d(V_\sim)$ and capacitance $C_d(V_\sim)$ appearing in Eqs. (77) and (78) (as well as in the general formulas (68) and (69)) is expressed by the function $g_1(V_\sim) = I_1(\beta V_\sim)/(\beta V_\sim/2)$.

The frequency dependence of the diffusion conductance $G_d(\omega)$ and capacitance $C_d(\omega)$ is produced by the functions $r_{p1}^\alpha(\omega)$ and $q_{p1}^\alpha(\omega)$. These functions are defined by Eqs. (61)–(63) to appear in the factors $F_G^\alpha(\omega)$ and $F_C^\alpha(\omega)$ given by Eqs. (79) and (80). The curves $r_{p1}^\alpha(\omega)$ and $q_{p1}^\alpha(\omega)$ are plotted in Fig. 2 for different values of the nonuniformity parameter $\alpha L_p = 0, \pm 0.5, \pm 1, \pm 2, \pm 3$.

Two curves 1 corresponding to the uniform junction ($\alpha L_p = 0$) are fully the same as those shown in Fig. 23 of Chapter 2 in a book by Sze.[8] The cross-sectional nonuniformity ($\alpha L_p \neq 0$) changes the curves $r_{p1}^\alpha(\omega)$ and $q_{p1}^\alpha(\omega)$ in different ways. The function $q_{p1}^\alpha(\omega) < 1$ always, and it decreases with growing $|\alpha L_p|$ regardless of the sign of αL_p, as shown by the solid curves $2(2a), 3(3a), 4(4a)$, and $5(5a)$ in Fig. 2. By contrast, the function $r_{p1}^\alpha(\omega)$ increases for $\alpha > 0$ (solid curves $2, 3, 4, 5$) and decreases for $\alpha < 0$ (dashed curves $2a, 3a, 4a, 5a$) with growing $|\alpha L_p|$, as compared to curve 1 (for $\alpha = 0$).

At low frequencies such that $\omega \tau_p \ll 1$ and $a_{p1}^\alpha(\omega) \simeq a_{p1}^\alpha(0) = 1$, the functions $r_{p1}^\alpha(\omega)$ and $q_{p1}^\alpha(\omega)$ take constant values

$$r_{p1}^\alpha(\omega) \simeq r_{p1}^\alpha(0) = A_p^\alpha + \alpha L_p \quad \text{and} \quad q_{p1}^\alpha(\omega) \simeq q_{p1}^\alpha(0) = \frac{1}{A_p^\alpha}.$$

Then, the factors (79) and (80) become frequency independent and equal to

$$F_G^\alpha(\omega) \simeq F_G^\alpha(0) = r_{p1}^\alpha(0) e^{2\alpha W_n} \equiv (A_p^\alpha + \alpha L_p) e^{2\alpha L_p (W_n/L_p)}, \qquad (83)$$

$$F_C^\alpha(\omega) \simeq F_C^\alpha(0) = q_{p1}^\alpha(0) e^{2\alpha W_n} \equiv \frac{1}{A_p^\alpha} e^{2\alpha L_p (W_n/L_p)}. \qquad (84)$$

The dependence of the low-frequency factors (83) and (84) on the nonuniformity parameter αL_p is shown in Fig. 3 for three ratios $W_n/L_p = 0, 0.05, 0.1$. Such small values of W_n/L_p are chosen to ensure the condition $W_n \ll L_p$ for neglecting recombination processes in the depletion layer of width W_n.[8] The character of the cross-sectional nonuniformity (for $\alpha < 0$ with $S_p > S_n$ and for $\alpha > 0$ with $S_p < S_n$) exerts different influence on $F_G^\alpha(0)$ (or the conductance $G_d(0, V_\sim) = g_1(V_\sim) F_G^\alpha(0) G_{d0}^0$) and on $F_C^\alpha(0)$ (or the capacitance $C_d(0, V_\sim) = g_1(V_\sim) F_C^\alpha(0) C_{d0}^0$).

The dashed curves in Fig. 3 demonstrate that the low-frequency capacitance $C_d(0, V_\sim)$ for nonuniform structures (with $\alpha \neq 0$) is always less than C_{d0}^0 for uniform ones (with $\alpha = 0$) for both signs $\alpha > 0$ and $\alpha < 0$. But the low-frequency conductance $G_d(0, V_\sim)$ depicted by solid curves increases for $\alpha > 0$ (when $S_n > S_p$) and decreases for $\alpha < 0$ (when $S_n < S_p$).

In conclusion, it is pertinent to note that the initial differential equations (18) and (19) have been solved by using the exponential approximation $S(z) = S_0 \exp(2\alpha z)$ for the cross-sectional z-dependence. In this case, the sought
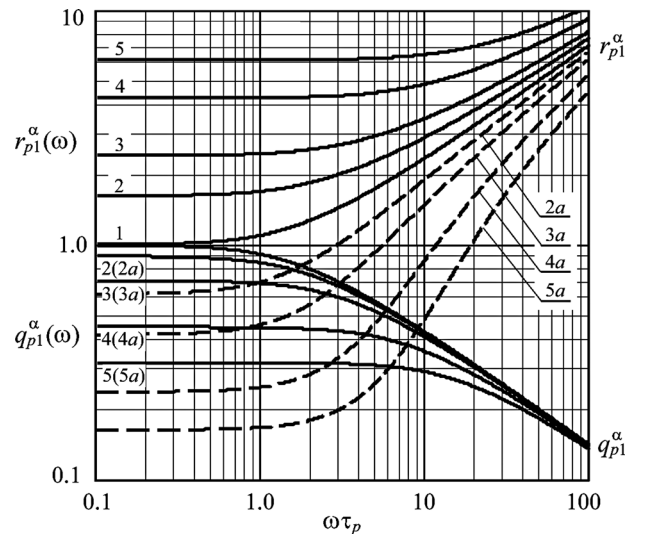

FIG. 2. Frequency dependencies of the quantities $r_{p1}^\alpha(\omega)$ and $q_{p1}^\alpha(\omega)$ for different values of the nonuniformity parameter $\alpha L_p = 0$ (solid curves 1), 0.5 (solid curves 2), 1 (solid curves 3), 2 (solid curves 4), 3 (solid curves 5); -0.5 (dashed curve 2a), -1 (dashed curve 3a), -2 (dashed curve 4a), -3 (dashed curve 5a).
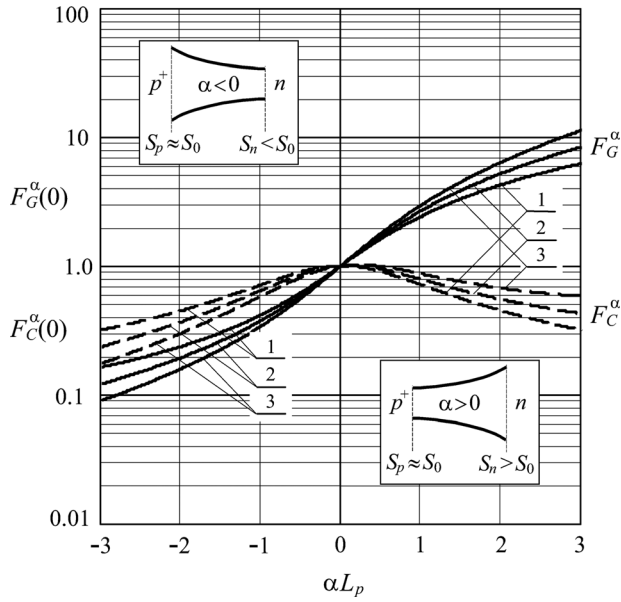
FIG. 3. Low-frequency values of the factors $F_G^{\alpha}(0)$ (solid curves) and $F_C^{\alpha}(0)$ (dashed curves) vs the nonuniformity parameter αL_p for three values of the ratio $W_n/L_p = 0$ (curves 1), 0.05 (curves 2), 0.1 (curves 3). The left and right insets qualitatively show the longitudinal geometry of a nonuniform structure with $\alpha < 0$ and $\alpha > 0$.

eigenfunctions are of exponential form $\exp(\pm\Lambda_{pk}^{\pm}z/L_p)$ and $\exp(\pm\Lambda_{nk}^{\pm}z/L_n)$ to yield the general solutions (26) and (27). If the power approximation $S(z) = S_0(\alpha z)^{2m}$ is more suitable, Eqs. (18) and (19) can be reduced to the following form (with $u(z) = \Delta\bar{p}_k(z)$ or $\Delta\bar{n}_k(z)$):

$$\frac{d^2u(z)}{dz^2} - \frac{2\nu}{z}\frac{du(z)}{dz} - c^2u(z) = 0,$$

whose solution is (see formula 8.494.9 in Ref. 10)

$$u(z) = z^{\nu+1/2}Z_{\nu+1/2}(icz), \tag{85}$$

where $Z_{\nu+1/2}$ is the Bessel function of the first or second kind. For equations (18) and (19) we have $\nu = -m$ and $c^2 = L_k^{-2}$, where L_k is defined by Eq. (22). Therefore, the sought solutions (26) and (27) include, instead of the exponential functions, the new functions given by Eq. (85), dependent on the complex argument (iz/L_k).

VII. CONCLUSION

This paper has demonstrated how to derive the explicit analytic form for the current–voltage characteristics of pn-junctions with nonuniformity in the cross section and doping impurity distribution by applying the transverse averaging technique (TAT). Application of the TAT to the three-dimensional transport equations of semiconductor electronics has converted them into the quasi-1D diffusion Eqs. (13) and (14) to analyze the minority-carrier transport processes in nonuniform pn-junctions.

Application of the spectral approach to the quasi-1D diffusion transport equations for the nonuniform pn-junctions under the action of arbitrary signal amplitude $V_\sim$ has given rise to changes in both the static current–voltage characteristic $J_0(V_0)$ and the dynamic characteristics – the diffusion conductance $G_d(\omega)$ and capacitance $C_d(\omega)$, as compared with the conventional theory of uniform junctions.[8] These changes are caused by both factors—the signal amplitude $V_\sim$ and the nonuniformity of $S(z)$.

The large-signal effects on the static and dynamic characteristics have proved to be completely identical to those obtained theoretically and corroborated experimentally for the uniform pn-junctions in our previous papers.[5,6] As a next step, these results should be compared to simulations.

The influence of the cross-sectional nonuniformity on the static current–voltage characteristic in Eq. (53) is exhibited in terms of the saturation current in Eq. (54). The similar influence on the diffusion conductance $G_d(\omega)$ and capacitance $C_d(\omega)$ is demonstrated by the novel formulas (68) and (69) and (77) and (78). The numerical calculations have been made for the exponential approximation $S(z) = S_0\exp(2\alpha z)$ of the cross-sectional z-dependence.

Until now, large-signal and high-frequency were treated separately to the best of our knowledge. This study may also have an impact on the understanding of distortion in high-frequency circuits. Further applications, limitations of the model, and carrier storage effects are the focus of further work.

[1] B. Schmithüsen, A. Schenk, I. Ruiz, and W. Fichtner, "Simulation of physical semiconductor devices under large and small signal conditions," (invited paper), Asia Pacific Microwave Conference, New Delhi (2004).

[2] A. T. Yang, Y. Liu, and J. T. Yao, IEEE Trans. Comput.-Aided Des. **13**, 231 (1994).

[3] R. B. Darling, IEEE Trans. Electron. Dev. **42**, 969 (1995).

[4] D. E. Root, M. Pirola, S. Fan, W. J. Anklam, and A. Cognata, IEEE Trans. Microwave Theory Tech. **41**, 2211 (1993).

[5] A. A. Barybin and E. J. P. Santos, Semicond. Sci. Technol. **22**, 312 (2007).

[6] E. J. P. Santos and A. A. Barybin, "Large-signal dynamic admittance of $p-n$-junctions," Proc. of the XVII Int. Symp. on Microelectronics Technology and Devices, Electrochem. Soc. Proc. **PV2002-8**, 237 (2002); E. J. P. Santos and A. A. Barybin, e-print arXiv:cond-mat/0204620.

[7] E. Gatard, R. Sommet, P. Bouysse, R. Quéré, M. Stanislawiak, and J.-M. Bureau, "High Power S Band Limiter Simulation with a Physics-Based Accurate PIN Diode Model," Proceedings of the 2nd European Microwave Integrated Circuits Conference, Munich, 8 to 10 October 2007.

[8] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (Wiley, New York, 1981).

[9] A. A. Barybin and E. J. P. Santos, Semicond. Sci. Technol. **22**, 1225 (2007).

[10] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products* (Academic Press, New York, 1980).

[11] A. A. Barybin, *Waves in Thin-Film Semiconductor Structures with Hot Electrons* (Nauka, Moscow, 1986) (in Russian).