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Orca predation algorithm: A novel bio-inspired algorithm for global optimization problems

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ABSTRACT

A novel bio-inspired algorithm called Orca Predation Algorithm (OPA) is proposed in this paper. OPA simulates the hunting behavior of orcas and abstracts it into several mathematical models: including driving, encircling and attacking of prey. The algorithm assigns different weights to the phases of prey driving and encircling through parameter adjustment to balance the exploitation and exploration stages of the algorithm. In the attacking phase, after considering the positions of several superior orcas and some randomly selected orcas, the optimal solution can be approached without losing the diversity of the particles. In order to estimate the performance of OPA, 67 unconstrained benchmark functions were first employed, and then the efficiency of the algorithm was further evaluated on five constrained engineering optimization problems. Besides, the computational complexity, parameter sensitivity and four qualitative metrics of OPA were analyzed to evaluate the applicability of the algorithm. The experimental results demonstrate that OPA can generate more promising results with superior performance relative to other test algorithms on different search landscapes.

1. Introduction

Optimization is the process of finding the optimal solution to specific problems (Jain, Singh, & Rani, 2019). There are various types of optimization problems in reality, such as discrete, continuous, static, dynamic, single-objective, and multi-objective problems (Dhiman & Kumar, 2018). These problems are usually non-convex, with nonlinear constraints, large solution space and high computational cost, and become more difficult to solve with increasing dimensionality (Dhiman & Kumar, 2019). Solution of these problems with traditional methods is time-consuming, and the obtained solutions are usually local optimal solutions, which can hardly meet the requirement of feasibility and accuracy for actual problems (Nematollahi, Rahiminejad, & Vahidi, 2017; Li, Wang & Alavi, 2020). Therefore, meta-heuristic algorithms have been proposed as an alternative and competitive optimization tool owing to their advantages of simplicity, easy operation and wide application range (Hammouri et al., 2020).

Since the meta-heuristic algorithm utilizes stochastic methods, it is independent of the reciprocal information or the mathematical characteristics of the objective landscape. That is, the algorithm is not affected by the nature of the problem itself (Heidari et al., 2019). The problem is treated as a black box to solve, without considering the nonlinear type of the constraint and search space of the problem (Mirjalili et al., 2017). Meta-heuristic algorithms have attracted increasing research attention due to these unique advantages. In recent years, numerous meta-heuristic algorithms have been proposed, which can be divided into four types, including evolutionary algorithms, physics-based algorithms, human-based algorithms and swarm intelligence algorithms (Chen et al., 2021; Dhiman & Kumar, 2018).

Evolutionary algorithms are inspired by the evolution of living things in nature, among which genetic algorithm (GA) (Holland, 1992) proposed by Holland is the most popular. Each chromosome in GA corresponds to a candidate solution, and the chromosome evolution process is executed by simulating the selection, crossover, mutation and other operation processes. In this way, the candidate solutions are continuously improved until the loop termination condition is reached.

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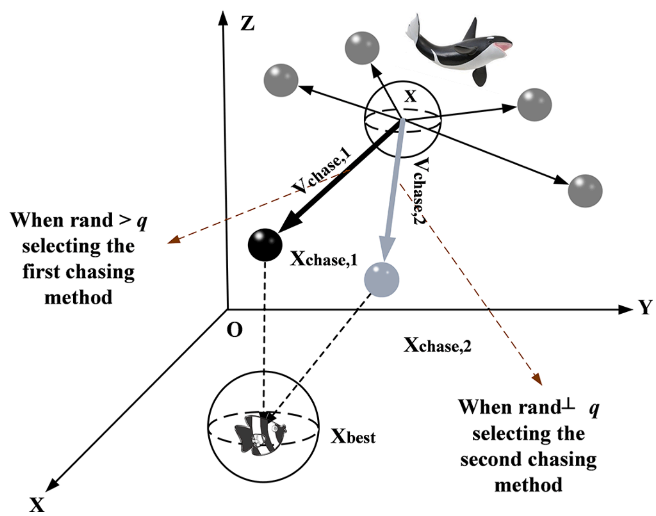

Fig. 1. The model of orca driving preys.

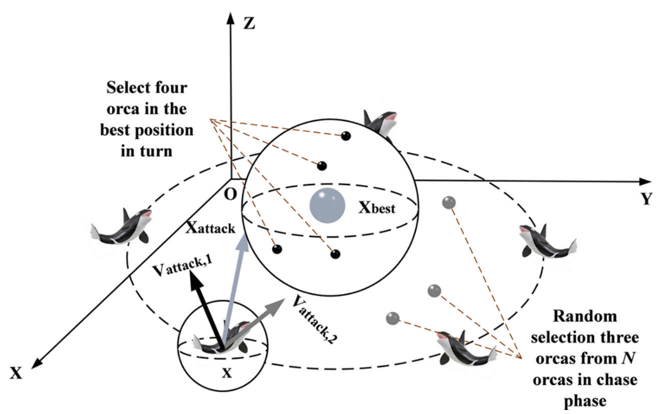

Fig. 2. The model of orca attacking preys.

Differential Evolution (DE) (Storn & Price, 1997) is another evolution algorithm developed by Storn and Price in 1997 on the basis of GA. This algorithm retains the evolution strategy of selection, crossover and mutation in GA; but different from GA, the mutation vector is generated by the parent differential vector in the DE algorithm, and crossover operation is performed on the parent individual to form a new individual.

Physics-based algorithms are a kind of algorithms inspired by the laws of physics. Gravity Search Algorithm (GSA) uses the law of gravitation between two objects to guide the motion of each particle to search for the optimal solution (Rashedi, Nezamabadi-pour, & Saryazdi, 2009). Simulated Annealing (SA) algorithm (Dowsland, 1993) is derived from the annealing process in thermodynamics. The temperature is gradually reduced through adjustment of the temperature parameters after the initial temperature is set, so that the system will eventually tend to a lower energy equilibrium state. Besides, Charged System Search (CSS) (Kaveh & Talatahari, 2010) and Galaxy-based Search Algorithm (GbSA)

Table 1
Parameter settings for algorithms.

Algorithms	Search Agents	Number of iterations	Parameters
ABC	40	1000,5000	Limit = 100
DE	40	1000,5000	Scaling Factor $F = 0.5$; Crossover Probability $CR = 0.8$ CR = 0.8;%交叉概率
SOA	40	1000,5000	Control Parameter $(A) = [0,2]$; $fc = 2$
GWO	40	1000,5000	Control Parameter $(\vec{a}) = [0, 2]$
WOA	40	1000,5000	a_1 in Location updating = [0, 2]; a_2 in Location updating = [−2, −1]; $b = 1$
SSA	40	1000,5000	$C_1 = [0, 1]$; $C_2 = [0, 1]$
HHO	40	1000,5000	The initial energy $E = [-1, 1]$; jump strength $(J) = [0, 2]$
PSO	40	1000,5000	Inertia Coefficient = 0.75; Cognitive and Social Coeff = 1.8, 2
MFO	40	1000,5000	Logarithmic Spiral = 0.75; Convergence Constant = [−1,−2]
GSA	40	1000,5000	Gravitational Constant = 100; Alpha Coefficient = 20
OPA	40	1000,5000	Probability of choice $(p1, p2) = [0,1]$; Probability of injury or death $(a) =$ 0.005 or 0.01

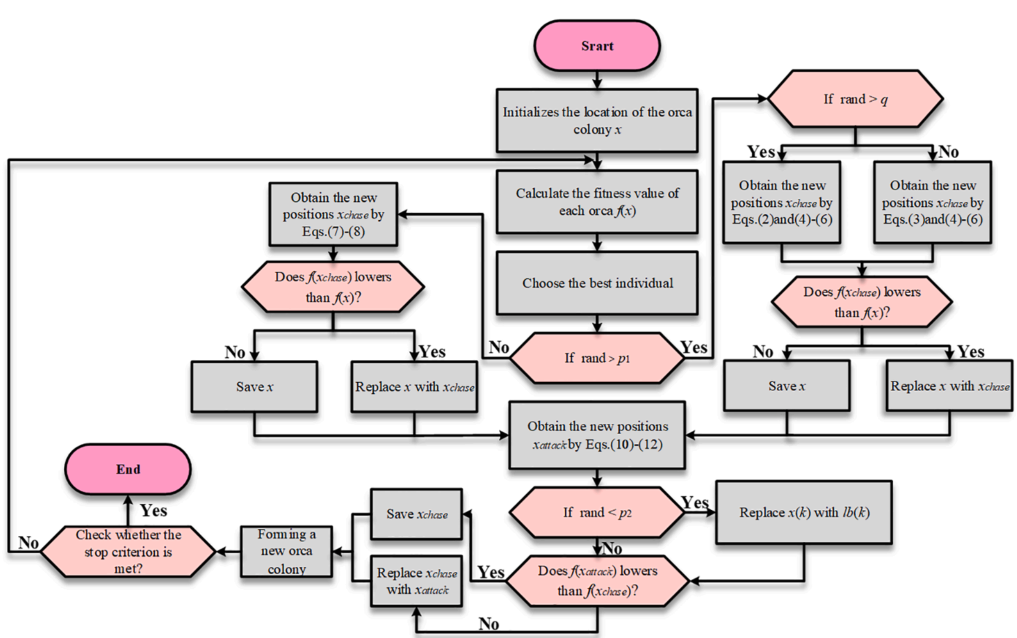

Fig. 3. Flow chart of the proposed algorithm.

Table 2
Experimental results of the applied algorithms on unimodal function with 30*D*.

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F1	Mean	**0.00E + 00**	7.73E-179	4.67E-08	3.72E-117	1.00E + 03	5.78E-27	1.29E-08	3.77E-169	5.81E-18	1.27E-11	3.54E-12
	Optimal	**0.00E + 00**	3.43E-213	6.35E-11	8.43E-123	1.59E-05	2.56E-30	6.42E-09	8.95E-185	1.86E-18	1.56E-12	1.37E-13
	Std	**0.00E + 00**	**0.00E + 00**	2.02E-07	1.54E-116	3.05E + 03	1.45E-26	3.96E-09	**0.00E + 00**	2.08E-18	1.27E-11	7.29E-12
	Rank	1	2	10	4	11	5	9	3	6	8	7
F2	Mean	**0.00E + 00**	1.25E-96	1.67E + 00	3.41E-67	4.17E + 01	6.30E-18	8.37E-01	2.49E-109	6.90E-09	6.79E-07	1.79E-07
	Optimal	**0.00E + 00**	6.26E-107	3.89E-06	3.46E-70	1.00E + 01	2.40E-19	2.77E-03	4.29E-119	3.26E-09	1.12E-07	4.00E-08
	Std	**0.00E + 00**	4.19E-96	3.79E + 00	4.08E-67	1.97E + 01	8.34E-18	1.02E + 00	1.22E-108	2.03E-09	3.14E-07	1.15E-07
	Rank	1	3	10	4	11	5	9	2	6	8	7
F3	Mean	**0.00E + 00**	3.59E-156	4.07E-02	8.73E-54	1.47E + 04	3.37E-14	2.86E + 02	5.54E + 00	4.45E-06	1.39E + 04	2.96E + 02
	Optimal	**0.00E + 00**	7.97E-188	4.55E-03	2.17E-62	5.59E + 02	1.17E-18	7.82E + 01	6.22E-05	1.78E-18	1.07E + 04	1.18E + 02
	Std	**0.00E + 00**	1.86E-155	4.29E-02	4.16E-53	1.11E + 04	9.61E-14	1.41E + 02	1.04E + 01	2.43E-05	1.63E + 03	1.49E + 02
	Rank	1	2	6	3	11	4	8	7	5	10	9
F4	Mean	**0.00E + 00**	2.41E-88	2.15E-02	2.37E-37	6.97E + 01	3.18E-08	8.65E + 00	4.25E-02	1.83E-09	6.43E + 00	1.23E + 01
	Optimal	**0.00E + 00**	8.75E-107	5.55E-03	9.29E-40	4.35E + 01	1.19E-10	2.70E + 00	3.96E-08	1.07E-09	5.01E-01	2.19E + 00
	Std	**0.00E + 00**	1.32E-87	1.50E-02	2.93E-37	8.29E + 00	8.42E-08	4.19E + 00	1.18E-01	4.28E-10	5.37E + 00	5.20E + 00
	Rank	1	2	6	3	11	5	9	7	4	8	10
F5	Mean	**7.16E-06**	3.88E-03	1.21E + 02	6.33E + 00	2.68E + 06	2.81E + 01	1.33E + 02	5.94E + 00	7.90E + 00	8.29E + 00	4.27E + 01
	Optimal	**3.18E-17**	1.39E-05	4.25E + 00	5.23E + 00	1.82E + 00	2.65E + 01	2.30E + 01	5.35E + 00	5.28E + 00	4.19E + 00	1.85E + 01
	Std	**2.24E-05**	5.31E-03	5.49E + 02	6.02E-01	1.46E + 07	5.38E-01	2.14E + 02	4.73E-01	1.30E + 01	2.51E + 00	3.09E + 01
	Rank	1	2	9	4	11	7	10	3	5	6	8
F6	Mean	**1.51E-30**	4.08E-05	3.17E-08	8.38E-03	2.01E + 03	3.20E + 00	1.21E-08	2.00E-05	5.52E-18	1.17E-11	1.17E-12
	Optimal	**0.00E + 00**	6.85E-08	7.84E-11	4.13E-07	1.34E-05	2.49E + 00	6.97E-09	4.23E-06	1.77E-18	5.77E-13	1.16E-13
	Std	**4.69E-30**	4.74E-05	6.93E-08	4.59E-02	4.87E + 03	3.49E-01	3.38E-09	1.31E-05	2.24E-18	1.03E-11	1.16E-12
	Rank	1	8	6	9	11	10	5	7	2	4	3
F7	Mean	1.36E-04	**9.98E-05**	8.67E-03	3.08E-04	4.53E + 00	8.02E-04	1.07E-01	8.25E-04	7.94E-03	9.47E-03	3.04E-01
	Optimal	1.67E-05	**3.72E-06**	2.43E-03	4.92E-05	6.57E-02	1.63E-04	4.73E-02	3.22E-05	1.89E-03	4.46E-04	1.17E-01
	Std	**9.78E-05**	1.04E-04	4.87E-03	2.48E-04	7.69E + 00	5.51E-04	4.02E-02	9.40E-04	4.39E-03	7.72E-03	1.03E-01
	Rank	2	1	7	3	11	4	9	5	6	8	10

(Shah-Hosseini, 2011) are also typical examples of physics-based algorithms.

Human-based algorithms simulate human behavior and can be divided into two types: one type is based on the working mode of human brain such as Neural Network Algorithm (NNA) (Sadollah, Sayyaadi, & Yadav, 2018), and the other type simulates the group social behavior of humans, such as Imperialist Competitive Algorithm (ICA) (Xing & Gao, 2014), and Teaching-Learning-Based Optimization (TLBO) (Rao, Savsani, & Vakharia, 2012). ICA divides countries into colonies and imperialist countries, in which the empires compete with each other for more colonies. A more powerful empire has a higher probability of occupying more colonies, and a weaker empire gradually loses its colonies. The algorithm ends when all colonies are occupied by an empire. TLBO simulates the learning and teaching model in the classroom. The learning level of students in the class is improved through teacher guidance (teacher phase) and interaction among students (learner phase).

The last class of meta-heuristic algorithms are swarm intelligence algorithms, which simulate the social behavior of biological groups. One of the most commonly used algorithms is Particle Swarm Optimization (PSO) based on the social behavior of birds (Kennedy & Eberhart, 1995). In PSO, each particle constantly updates its position by tracking the individual optimum and the global optimum until the loop termination condition is reached. Another famous swarm intelligence algorithm is Artificial Bee Colony (ABC), whose intuitive background is derived from the nectar-collecting behavior of bees. The algorithm divides the artificial bee colony into three categories, and the bees share and exchange colony information according to their respective division of labor, so as

Table 3
Experimental results of the applied algorithms on multimodal function with 30*D*.

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F8	Mean	**−1.26E + 04**	**−1.26E + 04**	−2.49E + 03	−2.67E + 03	−8.70E + 03	−5.49E + 03	−7.66E + 03	−3.70E + 03	−1.60E + 03	−1.22E + 04	−5.98E + 03
	Optimal	**−1.26E + 04**	**−1.26E + 04**	−3.10E + 03	−3.29E + 03	−1.06E + 04	−7.83E + 03	−9.25E + 03	−4.19E + 03	−2.15E + 03	−1.24E + 04	−8.06E + 03
	Std	3.43E-01	**1.93E-01**	2.96E + 02	3.28E + 02	1.01E + 03	7.63E + 02	7.37E + 02	5.90E + 02	2.70E + 02	1.30E + 02	7.04E + 02
	Rank	1.5	1.5	10	9	4	7	5	8	11	3	6
F9	Mean	**0.00E + 00**	**0.00E + 00**	7.11E + 00	**0.00E + 00**	1.57E + 02	9.53E-01	6.04E + 01	**0.00E + 00**	1.10E + 01	1.01E + 00	1.55E + 02
	Optimal	**0.00E + 00**	**0.00E + 00**	1.99E + 00	**0.00E + 00**	8.95E + 01	**0.00E + 00**	2.69E + 01	**0.00E + 00**	2.98E + 00	3.23E-02	1.07E + 02
	Std	**0.00E + 00**	**0.00E + 00**	5.58E + 00	**0.00E + 00**	3.85E + 01	4.83E + 00	1.66E + 01	**0.00E + 00**	2.70E + 00	5.27E-01	2.31E + 01
	Rank	2.5	2.5	7	2.5	11	5	9	2.5	8	6	10
F10	Mean	**8.88E-16**	**8.88E-16**	1.91E-04	5.27E-15	1.46E + 01	2.00E + 01	1.86E + 00	3.85E-15	3.20E-09	3.15E-05	4.47E-07
	Optimal	**8.88E-16**	**8.88E-16**	1.59E-05	4.44E-15	1.16E + 00	2.00E + 01	2.29E-05	**8.88E-16**	1.86E-09	6.18E-06	9.36E-08
	Std	**0.00E + 00**	**0.00E + 00**	1.73E-04	1.53E-15	7.05E + 00	1.29E-03	7.17E-01	2.30E-15	7.20E-10	1.66E-05	5.75E-07
	Rank	1.5	1.5	8	4	10	11	9	3	5	7	6
F11	Mean	**0.00E + 00**	**0.00E + 00**	9.41E-02	1.25E-02	1.81E + 01	5.95E-03	7.39E-03	2.32E-02	8.37E-02	5.41E-04	2.38E-03
	Optimal	**0.00E + 00**	**0.00E + 00**	3.69E-02	**0.00E + 00**	5.68E-05	**0.00E + 00**	4.55E-08	**0.00E + 00**	**0.00E + 00**	1.55E-07	3.26E-13
	Std	**0.00E + 00**	**0.00E + 00**	4.26E-02	1.41E-02	3.67E + 01	1.65E-02	1.02E-02	5.87E-02	1.05E-01	2.06E-03	4.64E-03
	Rank	1.5	1.5	10	7	11	5	6	8	9	3	4
F12	Mean	**1.69E-31**	1.91E-06	4.02E-09	3.90E-03	7.22E-01	3.28E-01	6.25E + 00	8.73E-04	1.33E-19	5.10E-12	5.24E-02
	Optimal	**1.57E-32**	6.32E-09	1.68E-12	1.59E-07	1.02E-04	1.50E-01	1.48E + 00	3.80E-06	5.23E-20	1.84E-14	5.98E-14
	Std	**4.62E-31**	4.96E-06	7.93E-09	7.94E-03	9.28E-01	1.39E-01	2.69E + 00	4.23E-03	3.96E-20	9.91E-12	1.36E-01
	Rank	1	5	4	7	10	9	11	6	2	3	8
F13	Mean	**4.53E-20**	3.31E-05	3.70E-08	2.96E-02	1.37E + 07	1.94E + 00	5.61E-01	9.08E-04	2.38E-04	4.26E-11	8.76E-02
	Optimal	**1.35E-32**	1.39E-07	8.37E-11	6.06E-07	1.26E-04	1.53E + 00	6.65E-10	3.38E-05	2.85E-19	3.24E-13	1.14E-13
	Std	**1.81E-19**	6.04E-05	8.11E-08	5.85E-02	7.49E + 07	2.28E-01	2.45E + 00	2.78E-03	1.30E-03	9.07E-11	4.19E-01
	Rank	1	4	3	7	11	10	9	6	5	2	8

to find the nectar source with the largest amount of nectar, namely the optimal solution to the problem (Karaboga & Basturk, 2007). Besides, cuckoo search, a novel population-based optimization technique proposed by Xin-She Yang and Suash Deb, is inspired by the parasitic nest behavior of cuckoos and the Levy flight behavior of some birds and fruit flies (Yang & Deb, 2009). Moreover, numerous other swarm intelligence algorithms with effective performance have emerged. The polar bear optimization algorithm proposes a mathematical model for the foraging and hunting of polar bears under harsh arctic conditions (Połap & Woźniak, 2017). The red fox optimization algorithm simulates the habits of red fox, including finding food, hunting and developing population while fleeing from hunters (Połap & Woźniak, 2021). Firefly algorithm (FA) simulates the luminous behavior of fireflies (Yang, 2010). Some other SI algorithms include the Whale Optimization Algorithm (Mirjalili & Lewis, 2016), Grey Wolf Optimizer (Mirjalili, Mirjalili, & Lewis, 2014), Cheetah Based Optimization Algorithm (Klein, Mariani, & dos Santos Coelho, 2018), Barnacles Mating Optimizer (Sulaiman, Mustaffa, Saari, & Daniyal, 2020), Cultural Coyote Optimization Algorithm (Pierezan, Maidl, Massashi Yamao, dos Santos Coelho, & Cocco Mariani, 2019), Tunicate Swarm Algorithm (Kaur, Awasthi, Sangal, & Dhiman, 2020), Chameleon Swarm Algorithm (Braik, 2021), Manta Rray Foraging Optimization (Zhao, Zhang, & Wang, 2020), Modified Crow Search Approach (Dos Santos Coelho, Richter, Mariani, & Askarzadeh, 2016), and Owl Optimization Algorithm (de Segundo, 2019).

Although there are distinctive types of meta-heuristic algorithms, all these algorithms have two common stages: exploration and exploitation stage (Lin & Gen, 2008). The exploration stage ensures that the algorithm investigates different promising areas in a given feasible region, and determines the ability of the algorithm to avoid local optima. It will be easier for an algorithm with stronger exploration ability to avoid local optima. The exploitation stage ensures that the algorithm further searches for the global optimum in the promising area obtained at the exploration stage. A higher exploitation capability of the algorithm will bring about more accurate optimal solutions (Li, Chen, Wang, Heidari & Mirjalili, 2020). Therefore, it is of great significance to balance these two stages for the improvement of algorithm performance, which remains to be very challenging so far. In addition, based on the notion of ‘No Free

Table 4
Experimental results of the applied algorithms on fixed-dimension multimodal function.

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F14	Mean	**9.98E-01**	1.13E + 00	1.03E + 00	4.46E + 00	1.62E + 00	1.53E + 00	**9.98E-01**	1.46E + 00	3.48E + 00	**9.98E-01**	**9.98E-01**
	Optimal	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	1.00E + 00	**9.98E-01**	**9.98E-01**
	Std	**0.00E + 00**	3.44E-01	1.81E-01	4.27E + 00	1.31E + 00	8.92E-01	2.59E-16	8.54E-01	2.10E + 00	**0.00E + 00**	**0.00E + 00**
	Rank	2.5	6	5	11	9	8	2.5	7	10	2.5	2.5
F15	Mean	4.24E-04	3.67E-04	3.30E-03	6.35E-03	1.02E-03	1.20E-03	1.41E-03	5.75E-04	3.24E-03	5.65E-04	**3.46E-04**
	Optimal	**3.07E-04**	3.09E-04	3.08E-04	**3.07E-04**	5.63E-04	3.08E-04	3.37E-04	3.09E-04	1.17E-03	3.12E-04	**3.07E-04**
	Std	3.59E-04	1.70E-04	5.96E-03	9.33E-03	3.67E-04	1.73E-04	3.58E-03	2.72E-04	3.65E-03	1.76E-04	**8.90E-05**
	Rank	3	2	10	11	6	7	8	5	9	4	1
F16	Mean	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Optimal	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Std	6.78E-16	7.46E-11	6.52E-16	7.37E-09	6.78E-16	5.36E-07	6.91E-15	9.14E-11	**5.13E-16**	6.12E-16	6.78E-16
	Rank	6	6	6	6	6	6	6	6	6	6	6
F17	Mean	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Optimal	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Std	**0.00E + 00**	5.16E-07	**0.00E + 00**	5.97E-07	**0.00E + 00**	8.72E-05	3.31E-14	1.49E-06	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Rank	6	6	6	6	6	6	6	6	6	6	6
F18	Mean	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Optimal	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Std	1.16E-15	6.57E-09	**8.45E-16**	1.19E-05	1.71E-15	3.09E-05	7.43E-14	4.71E-05	2.33E-15	1.70E-15	1.60E-15
	Rank	6	6	6	6	6	6	6	6	6	6	6
F19	Mean	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**
	Optimal	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**
	Std	2.71E-15	2.95E-03	2.00E-03	2.39E-03	2.71E-15	2.70E-03	3.93E-14	3.38E-03	**2.29E-15**	2.55E-15	2.71E-15
	Rank	6	6	6	6	6	6	6	6	6	6	6
F20	Mean	−3.28E + 00	−3.11E + 00	−3.24E + 00	−3.26E + 00	−3.24E + 00	−2.93E + 00	−3.23E + 00	−3.28E + 00	**−3.32E + 00**	**−3.32E + 00**	−3.29E + 00
	Optimal	**−3.32E + 00**	−3.29E + 00	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**	−3.20E + 00	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**
	Std	5.83E-02	8.31E-02	1.22E-01	7.04E-02	5.98E-02	4.20E-01	4.91E-02	7.90E-02	1.39E-15	**1.36E-15**	5.11E-02
	Rank	4.5	10	7.5	6	7.5	11	9	4.5	1.5	1.5	3
F21	Mean	−9.90E + 00	−5.22E + 00	−6.56E + 00	**−1.02E + 01**	−5.81E + 00	−3.31E + 00	−8.22E + 00	−9.13E + 00	−6.83E + 00	**−1.02E + 01**	−9.40E + 00
	Optimal	**−1.02E + 01**	−9.99E + 00	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**
	Std	1.37E + 00	9.01E-01	3.33E + 00	2.35E-04	3.28E + 00	4.06E + 00	2.86E + 00	2.07E + 00	3.64E + 00	**5.44E-15**	2.00E + 00
	Rank	3	10	8	1.5	9	11	6	5	7	1.5	4
F22	Mean	−1.01E + 01	−5.26E + 00	−7.24E + 00	**−1.04E + 01**	−7.60E + 00	−5.48E + 00	−9.54E + 00	−8.05E + 00	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	Optimal	**−1.04E + 01**	−1.03E + 01	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	Std	1.39E + 00	9.48E-01	3.71E + 00	2.41E-04	3.55E + 00	4.78E + 00	2.28E + 00	2.95E + 00	**9.33E-16**	**9.33E-16**	1.40E-15
	Rank	5	11	9	2.5	8	10	6	7	2.5	2.5	2.5
F23	Mean	−9.61E + 00	−5.30E + 00	−7.45E + 00	**−1.05E + 01**	−6.24E + 00	−7.61E + 00	−9.11E + 00	−8.36E + 00	−9.91E + 00	**−1.05E + 01**	**−1.05E + 01**
	Optimal	**−1.05E + 01**	−1.02E + 01	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**
	Std	2.47E + 00	9.23E-01	3.45E + 00	2.08E-04	3.86E + 00	3.83E + 00	2.68E + 00	3.23E + 00	1.97E + 00	3.90E-06	**1.81E-15**
	Rank	5	11	9	2	10	8	6	7	4	2	2

Lunch’, no algorithm can always find the best solution in all fields (Wolpert & Macready, 1997). Therefore, different algorithms need to be proposed for specific problems. Based on the above two points, we propose a new meta-heuristic algorithm named as Orca Predator Algorithm (OPA), which simulates a variety of behaviors of orcas in their hunting process.

The main innovations of this study can be summarized as follows.

Table 5
Experimental results of the applied algorithms on CEC 2015 function with 30*D*.

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
CEC-15–1	Mean	**2.21E + 03**	7.81E + 06	5.44E + 05	2.63E + 07	9.51E + 07	4.32E + 07	1.79E + 06	5.66E + 07	1.12E + 06	3.23E + 06	3.52E + 05
	Optimal	**1.05E + 02**	2.93E + 06	4.01E + 04	2.90E + 06	8.79E + 06	8.19E + 06	5.63E + 05	1.37E + 07	5.15E + 05	1.31E + 06	9.01E + 04
	Std	**4.72E + 03**	3.91E + 06	1.87E + 06	1.43E + 07	8.24E + 07	2.57E + 07	8.71E + 05	2.90E + 07	5.70E + 05	1.09E + 06	2.13E + 05
	Rank	1	7	3	8	11	9	5	10	4	6	2
CEC-15–2	Mean	1.92E + 03	1.05E + 07	1.05E + 04	2.15E + 09	9.38E + 09	6.92E + 09	4.71E + 03	1.76E + 07	1.05E + 03	**9.72E + 02**	1.90E + 03
	Optimal	**2.01E + 02**	6.59E + 06	2.40E + 03	2.06E + 07	9.82E + 08	2.30E + 09	2.21E + 02	5.24E + 06	2.03E + 02	2.83E + 02	2.44E + 02
	Std	2.54E + 03	1.97E + 06	6.33E + 03	2.03E + 09	6.14E + 09	2.95E + 09	4.68E + 03	1.77E + 07	1.01E + 03	**6.70E + 02**	1.83E + 03
	Rank	4	7	6	9	11	10	5	8	2	1	3
CEC-15–3	Mean	**3.20E + 02**	**3.20E + 02**	3.21E + 02	3.21E + 02	**3.20E + 02**	3.21E + 02	**3.20E + 02**	**3.20E + 02**	**3.20E + 02**	**3.20E + 02**	3.21E + 02
	Optimal	**3.20E + 02**	**3.20E + 02**	**3.20E + 02**	3.21E + 02	**3.20E + 02**	3.21E + 02	**3.20E + 02**	**3.20E + 02**	**3.20E + 02**	**3.20E + 02**	3.21E + 02
	Std	3.67E-01	1.54E-01	1.90E-01	9.16E-02	1.80E-01	9.87E-02	1.14E-01	1.94E-01	**7.44E-05**	3.37E-02	6.72E-02
	Rank	4	4	9.5	9.5	4	9.5	4	4	4	4	9.5
CEC-15–4	Mean	**4.72E + 02**	6.26E + 02	5.91E + 02	5.04E + 02	6.07E + 02	5.77E + 02	5.42E + 02	6.95E + 02	6.14E + 02	4.77E + 02	5.73E + 02
	Optimal	**4.40E + 02**	5.68E + 02	5.35E + 02	4.52E + 02	5.06E + 02	5.30E + 02	4.60E + 02	6.04E + 02	5.58E + 02	4.57E + 02	5.48E + 02
	Std	2.42E + 01	3.39E + 01	2.76E + 01	2.55E + 01	5.76E + 01	2.68E + 01	4.71E + 01	6.68E + 01	2.98E + 01	**8.81E + 00**	1.06E + 01
	Rank	1	10	7	3	8	6	4	11	9	2	5
CEC-15–5	Mean	3.48E + 03	4.70E + 03	5.91E + 03	3.23E + 03	4.87E + 03	5.35E + 03	4.36E + 03	5.28E + 03	4.22E + 03	**2.48E + 03**	7.38E + 03
	Optimal	2.78E + 03	3.29E + 03	3.28E + 03	**1.95E + 03**	3.56E + 03	3.98E + 03	3.05E + 03	3.84E + 03	2.79E + 03	2.03E + 03	6.60E + 03
	Std	4.88E + 02	8.88E + 02	1.87E + 03	6.08E + 02	7.99E + 02	8.79E + 02	8.51E + 02	8.88E + 02	4.98E + 02	**2.07E + 02**	2.77E + 02
	Rank	3	6	10	2	7	9	5	8	4	1	11
CEC-15–6	Mean	**9.28E + 03**	7.89E + 05	1.92E + 04	1.44E + 06	2.23E + 06	1.34E + 06	1.25E + 05	2.03E + 06	2.02E + 05	8.82E + 05	1.67E + 04
	Optimal	2.21E + 03	9.00E + 04	**1.90E + 03**	4.43E + 04	3.79E + 04	1.61E + 05	3.91E + 03	4.26E + 05	6.88E + 04	1.48E + 05	3.52E + 03
	Std	**6.74E + 03**	4.57E + 05	2.95E + 04	1.62E + 06	3.49E + 06	1.12E + 06	8.89E + 04	1.46E + 06	1.05E + 05	4.69E + 05	1.13E + 04
	Rank	1	6	3	9	11	8	4	10	5	7	2
CEC-15–7	Mean	**7.07E + 02**	7.31E + 02	7.46E + 02	7.21E + 02	7.41E + 02	7.35E + 02	7.17E + 02	7.35E + 02	7.19E + 02	7.08E + 02	**7.07E + 02**
	Optimal	**7.04E + 02**	7.09E + 02	7.23E + 02	7.15E + 02	7.15E + 02	7.17E + 02	7.13E + 02	7.20E + 02	7.05E + 02	7.05E + 02	7.05E + 02
	Std	1.97E + 00	2.72E + 01	2.77E + 01	5.36E + 00	3.24E + 01	1.72E + 01	2.52E + 00	2.50E + 01	1.72E + 01	1.07E + 00	**8.59E-01**
	Rank	1.5	7	11	6	10	8.5	4	8.5	5	3	1.5
CEC-15–8	Mean	**1.85E + 03**	1.54E + 05	9.46E + 03	4.07E + 05	6.26E + 05	7.41E + 05	8.96E + 04	3.28E + 05	2.77E + 04	2.52E + 05	5.58E + 03
	Optimal	**1.12E + 03**	2.92E + 04	2.08E + 03	6.07E + 04	7.13E + 03	2.47E + 05	1.92E + 04	3.58E + 04	1.54E + 04	1.15E + 05	1.60E + 03
	Std	**4.05E + 02**	9.31E + 04	9.25E + 03	3.40E + 05	9.04E + 05	3.52E + 05	5.23E + 04	1.65E + 05	1.35E + 04	9.64E + 04	4.78E + 03
	Rank	1	6	3	9	10	11	5	8	4	7	2
CEC-15–9	Mean	**1.00E+03**	1.05E+03	1.10E+03	1.01E+03	1.05E+03	1.03E+03	1.02E+03	1.12E+03	1.03E+03	**1.00E+03**	**1.00E+03**
	Optimal	**1.00E+03**	**1.00E+03**	**1.00E+03**	**1.00E+03**	1.02E+03	1.01E+03	**1.00E+03**	1.01E+03	**1.00E+03**	**1.00E+03**	**1.00E+03**
	Std	3.36E-01	1.23E+02	1.67E+02	1.05E+01	1.47E+01	1.10E+01	5.90E+01	2.03E+02	8.44E+01	2.28E-01	**1.80E-01**
	Rank	2	8.5	10	4	8.5	6.5	5	11	6.5	2	2
CEC-15–10	Mean	**2.82E+03**	8.00E+05	1.65E+04	1.83E+06	4.48E+06	2.54E+06	1.72E+05	4.01E+06	5.60E+05	1.32E+06	5.79E+03
	Optimal	1.88E+03	1.82E+05	2.89E+03	1.10E+05	4.74E+04	1.06E+05	1.66E+04	2.48E+05	2.40E+05	5.83E+05	**1.53E+03**
	Std	**1.25E+03**	4.15E+05	1.54E+04	1.29E+06	8.38E+06	1.75E+06	1.27E+05	2.33E+06	2.09E+05	4.23E+05	3.95E+03
	Rank	1	6	3	8	11	9	4	10	5	7	2
CEC-15–11	Mean	1.93E+03	2.18E+03	2.42E+03	1.86E+03	2.17E+03	2.15E+03	1.96E+03	2.23E+03	**1.41E+03**	**1.41E+03**	1.50E+03
	Optimal	1.74E+03	1.42E+03	1.40E+03	1.43E+03	1.98E+03	1.55E+03	1.41E+03	1.42E+03	1.41E+03	**1.15E+03**	1.40E+03
	Std	8.56E+01	4.78E+02	7.39E+02	1.09E+02	1.13E+02	1.31E+02	2.13E+02	4.35E+02	**2.20E+00**	5.01E+01	3.54E+01
	Rank	5	9	11	4	8	7	6	10	1.5	1.5	3
CEC-15–12	Mean	**1.31E+03**	**1.31E+03**	**1.31E+03**	**1.31E+03**	1.34E+03	**1.31E+03**	**1.31E+03**	**1.31E+03**	**1.31E+03**	**1.31E+03**	**1.31E+03**
	Optimal	**1.30E+03**	1.31E+03	1.31E+03	1.31E+03	1.31E+03	1.31E+03	1.31E+03	1.31E+03	**1.30E+03**	1.31E+03	**1.30E+03**
	Std	6.79E-01	2.63E+00	9.16E+00	1.24E+00	1.35E+01	2.65E+00	1.71E+00	2.06E+00	1.13E+00	**4.43E-01**	5.26E-01
	Rank	5.5	5.5	5.5	5.5	11	5.5	5.5	5.5	5.5	5.5	5.5
CEC-15–13	Mean	**1.41E+03**	1.44E+03	1.83E+03	1.42E+03	1.43E+03	1.43E+03	1.42E+03	1.44E+03	3.06E+03	**1.41E+03**	1.42E+03
	Optimal	**1.39E+03**	1.44E+03	1.48E+03	1.40E+03	1.41E+03	1.42E+03	1.41E+03	1.43E+03	1.44E+03	1.40E+03	1.42E+03
	Std	6.77E+00	3.57E+00	3.96E+02	8.06E+00	6.45E+00	4.35E+00	6.94E+00	4.35E+00	1.97E+03	**2.45E+00**	2.49E+00
	Rank	1.5	8.5	10	4	6.5	6.5	4	8.5	11	1.5	4
CEC-15–14	Mean	3.51E+04	3.67E+04	3.65E+04	3.76E+04	3.89E+04	3.80E+04	3.57E+04	3.83E+04	**1.50E+03**	2.79E+04	3.35E+04
	Optimal	3.27E+04	3.28E+04	1.52E+03	3.43E+04	3.57E+04	3.48E+04	3.43E+04	3.46E+04	**1.50E+03**	3.17E+03	3.25E+04
	Std	1.56E+03	2.29E+03	7.30E+03	2.31E+03	1.61E+03	1.35E+03	9.61E+02	2.87E+03	**1.32E-07**	1.05E+04	9.34E+02
	Rank	4	7	6	8	11	9	5	10	1	2	3
CEC-15–15	Mean	**1.60E+03**	**1.60E+03**	1.61E+03	1.65E+03	2.69E+03	1.76E+03	**1.60E+03**	1.61E+03	**1.60E+03**	**1.60E+03**	**1.60E+03**
	Optimal	**1.60E+03**	**1.60E+03**	**1.60E+03**	1.62E+03	1.62E+03	1.63E+03	**1.60E+03**	1.61E+03	**1.60E+03**	**1.60E+03**	**1.60E+03**
	Std	4.50E-09	1.48E-01	1.01E+01	4.93E+01	2.33E+03	1.98E+02	3.37E-06	6.14E+00	2.69E-10	7.68E-13	**0.00E+00**
	Rank	3.5	3.5	7.5	9	11	10	3.5	7.5	3.5	3.5	3.5

Table 6
Experimental results of the applied algorithms on CEC 2017 function with 30*D*.

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
CEC-17–1	Mean	**1.22E + 02**	2.59E + 07	2.95E + 07	2.92E + 09	8.35E + 09	1.03E + 10	8.06E + 03	7.82E + 08	8.13E + 08	2.53E + 03	4.87E + 02
	Optimal	**1.00E + 02**	1.41E + 07	3.96E + 04	4.14E + 08	3.01E + 04	3.72E + 09	1.11E + 02	3.08E + 08	3.30E + 03	7.16E + 02	**1.00E + 02**
	Std	**2.52E + 01**	5.42E + 06	1.61E + 08	2.67E + 09	4.21E + 09	4.09E + 09	8.63E + 03	3.92E + 08	6.09E + 08	1.23E + 03	1.61E + 03
	Rank	1	5	6	9	10	11	4	7	8	3	2
CEC-17–3	Mean	**3.00E + 02**	2.43E + 04	2.13E + 03	4.80E + 04	1.16E + 05	5.60E + 04	2.46E + 04	6.87E + 04	9.96E + 04	8.89E + 04	8.50E + 04
	Optimal	**3.00E + 02**	1.44E + 04	4.36E + 02	2.64E + 04	4.78E + 04	3.58E + 04	6.18E + 03	4.34E + 04	7.97E + 04	7.73E + 04	5.91E + 04
	Std	**2.33E-03**	5.48E + 03	1.33E + 03	1.24E + 04	4.68E + 04	1.27E + 04	1.02E + 04	1.25E + 04	9.30E + 03	6.44E + 03	1.15E + 04
	Rank	1	3	2	5	11	6	4	7	10	9	8
CEC-17–4	Mean	**4.29E + 02**	5.42E + 02	4.84E + 02	5.92E + 02	1.16E + 03	1.00E + 03	4.64E + 02	7.44E + 02	6.39E + 02	4.49E + 02	4.51E + 02
	Optimal	**4.06E + 02**	4.45E + 02	4.14E + 02	4.71E + 02	4.97E + 02	5.86E + 02	4.17E + 02	5.93E + 02	5.53E + 02	4.18E + 02	4.20E + 02
	Std	2.35E + 01	4.06E + 01	3.52E + 01	1.10E + 02	8.11E + 02	3.92E + 02	2.22E + 01	1.06E + 02	8.91E + 01	1.51E + 01	**9.23E + 00**
	Rank	1	6	5	7	11	10	4	9	8	2	3
CEC-17–5	Mean	**5.81E + 02**	7.47E + 02	6.90E + 02	6.07E + 02	6.97E + 02	6.99E + 02	6.51E + 02	8.03E + 02	7.29E + 02	6.09E + 02	7.00E + 02
	Optimal	**5.45E + 02**	6.41E + 02	6.26E + 02	5.67E + 02	6.06E + 02	6.35E + 02	5.76E + 02	6.66E + 02	6.71E + 02	5.76E + 02	6.60E + 02
	Std	2.16E + 01	3.44E + 01	2.93E + 01	3.56E + 01	4.89E + 01	3.23E + 01	4.25E + 01	6.89E + 01	2.82E + 01	1.62E + 01	**1.34E + 01**
	Rank	1	10	5	2	6	7	4	11	9	3	8
CEC-17–6	Mean	**6.00E + 02**	6.57E + 02	6.50E + 02	6.09E + 02	6.41E + 02	6.46E + 02	6.45E + 02	6.70E + 02	6.55E + 02	**6.00E + 02**	**6.00E + 02**
	Optimal	**6.00E + 02**	6.38E + 02	6.38E + 02	6.03E + 02	6.20E + 02	6.33E + 02	6.24E + 02	6.54E + 02	6.49E + 02	**6.00E + 02**	**6.00E + 02**
	Std	8.63E-03	5.72E + 00	8.66E + 00	4.06E + 00	1.05E + 01	6.06E + 00	1.21E + 01	1.01E + 01	3.63E + 00	1.17E-02	**8.10E-04**
	Rank	2	10	8	4	5	7	6	11	9	2	2
CEC-17–7	Mean	**7.98E + 02**	1.34E + 03	1.18E + 03	8.85E + 02	1.11E + 03	1.14E + 03	9.06E + 02	1.33E + 03	9.92E + 02	8.45E + 02	9.36E + 02
	Optimal	**7.71E + 02**	1.23E + 03	1.03E + 03	8.19E + 02	8.42E + 02	1.03E + 03	8.32E + 02	1.05E + 03	9.03E + 02	8.22E + 02	9.07E + 02
	Std	2.01E + 01	6.22E + 01	7.60E + 01	4.24E + 01	2.19E + 02	5.20E + 01	3.94E + 01	1.00E + 02	5.04E + 01	1.41E + 01	**1.08E + 01**
	Rank	1	11	9	3	7	8	4	10	6	2	5
CEC-17–8	Mean	**8.83E + 02**	1.01E + 03	9.71E + 02	8.98E + 02	9.93E + 02	9.89E + 02	9.50E + 02	1.08E + 03	9.92E + 02	9.17E + 02	1.00E + 03
	Optimal	**8.30E + 02**	9.60E + 02	9.00E + 02	8.55E + 02	9.40E + 02	9.34E + 02	8.87E + 02	9.69E + 02	9.60E + 02	8.75E + 02	9.77E + 02
	Std	2.38E + 01	2.50E + 01	4.01E + 01	3.67E + 01	3.78E + 01	2.86E + 01	3.26E + 01	6.04E + 01	1.90E + 01	1.43E + 01	**1.05E + 01**
	Rank	1	10	5	2	8	6	4	11	7	3	9
CEC-17–9	Mean	9.73E + 02	1.37E + 04	9.60E + 03	2.59E + 03	6.41E + 03	7.72E + 03	6.08E + 03	1.55E + 04	7.19E + 03	3.95E + 03	**9.00E + 02**
	Optimal	9.07E + 02	7.80E + 03	5.25E + 03	1.15E + 03	2.90E + 03	3.21E + 03	2.10E + 03	7.72E + 03	5.27E + 03	2.24E + 03	**9.00E + 02**
	Std	5.89E + 01	2.46E + 03	2.96E + 03	1.11E + 03	2.73E + 03	1.88E + 03	2.61E + 03	5.37E + 03	7.95E + 02	9.31E + 02	**8.49E-02**
	Rank	2	10	9	3	6	8	5	11	7	4	1
CEC-17–10	Mean	3.95E + 03	5.29E + 03	8.01E + 03	4.24E + 03	5.27E + 03	6.97E + 03	4.95E + 03	6.82E + 03	5.13E + 03	**3.58E + 03**	8.33E + 03
	Optimal	**2.84E + 03**	4.14E + 03	4.83E + 03	2.95E + 03	4.08E + 03	5.40E + 03	4.03E + 03	4.64E + 03	3.61E + 03	3.04E + 03	7.68E + 03
	Std	6.70E + 02	5.43E + 02	1.62E + 03	5.37E + 02	7.48E + 02	7.81E + 02	5.73E + 02	9.09E + 02	5.64E + 02	**2.57E + 02**	3.50E + 02
	Rank	2	7	10	3	6	9	4	8	5	1	11
CEC-17–11	Mean	1.23E + 03	1.45E + 03	1.32E + 03	2.44E + 03	6.71E + 03	3.11E + 03	1.45E + 03	2.16E + 03	5.59E + 03	6.97E + 03	**1.19E + 03**
	Optimal	**1.16E + 03**	1.24E + 03	1.20E + 03	1.53E + 03	1.24E + 03	1.59E + 03	1.25E + 03	1.41E + 03	3.05E + 03	3.35E + 03	1.17E + 03
	Std	5.04E + 01	1.22E + 02	6.89E + 01	7.62E + 02	7.50E + 03	1.27E + 03	9.92E + 01	3.42E + 02	1.60E + 03	2.11E + 03	**9.44E + 00**
	Rank	2	4.5	3	7	10	8	4.5	6	9	11	1
CEC-17–12	Mean	**4.59E+03**	3.34E+07	2.94E+06	1.75E+08	6.01E+08	8.76E+08	3.37E+07	2.07E+08	9.99E+06	2.81E+05	2.55E+04
	Optimal	**2.04E+03**	1.07E+07	2.19E+05	4.73E+06	7.07E+05	6.73E+07	2.12E+06	5.53E+07	6.91E+05	7.07E+04	8.31E+03
	Std	**2.39E+03**	2.02E+07	2.09E+06	2.32E+08	8.84E+08	9.29E+08	2.24E+07	1.41E+08	1.70E+07	1.37E+05	2.28E+04
	Rank	1	6	4	8	10	11	7	9	5	3	2
CEC-17–13	Mean	1.43E+03	2.36E+05	1.69E+04	1.74E+07	7.77E+04	1.66E+08	5.82E+04	6.73E+05	2.35E+04	2.40E+03	**1.41E+03**
	Optimal	**1.34E+03**	9.28E+04	6.21E+03	3.14E+04	1.87E+04	4.61E+05	1.02E+04	9.30E+04	1.39E+04	1.52E+03	1.37E+03
	Std	8.13E+01	1.08E+05	9.01E+03	5.40E+07	5.56E+04	2.19E+08	6.46E+04	7.99E+05	7.12E+03	8.94E+02	**1.52E+01**
	Rank	2	8	4	10	7	11	6	9	5	3	1
CEC-17–14	Mean	1.54E+03	7.84E+05	1.28E+04	4.57E+05	6.15E+05	8.19E+05	1.60E+05	2.52E+06	4.54E+05	8.25E+04	**1.49E+03**
	Optimal	**1.44E+03**	3.05E+04	1.75E+03	5.65E+04	1.75E+03	3.72E+04	8.81E+03	7.95E+04	5.59E+04	8.17E+03	1.48E+03
	Std	6.00E+01	8.30E+05	2.67E+04	1.34E+06	1.96E+06	1.84E+06	1.05E+05	2.62E+06	3.65E+05	8.66E+04	**7.20E+00**
	Rank	2	9	3	7	8	10	5	11	6	4	1
CEC-17–15	Mean	1.85E+03	9.98E+04	1.05E+04	5.89E+06	2.56E+07	1.75E+07	4.34E+04	1.33E+06	9.72E+03	3.58E+03	**1.59E+03**
	Optimal	1.57E+03	2.31E+04	2.60E+03	7.18E+03	4.73E+03	1.52E+05	8.46E+03	1.35E+05	4.64E+03	1.79E+03	**1.56E+03**

(*continued on next page*)

Table 6 (*continued*)

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
	Std	3.18E+02	5.79E+04	7.06E+03	1.65E+07	1.40E+08	2.07E+07	3.42E+04	2.36E+06	3.34E+03	9.86E+02	**1.30E+01**
	Rank	2	7	5	9	11	10	6	8	4	3	1
CEC-17–16	Mean	**2.37E+03**	3.50E+03	3.02E+03	2.57E+03	3.17E+03	3.05E+03	2.81E+03	4.02E+03	3.55E+03	2.39E+03	3.25E+03
	Optimal	**1.73E+03**	2.52E+03	2.36E+03	2.16E+03	2.07E+03	2.38E+03	2.20E+03	2.81E+03	2.83E+03	2.10E+03	2.65E+03
	Std	3.10E+02	5.29E+02	3.03E+02	3.61E+02	4.13E+02	3.55E+02	3.02E+02	5.26E+02	3.34E+02	**1.53E+02**	2.30E+02
	Rank	1	9	5	3	7	6	4	11	10	2	8
CEC-17–17	Mean	**1.84E+03**	2.65E+03	2.58E+03	2.03E+03	2.56E+03	2.31E+03	2.26E+03	2.65E+03	2.88E+03	1.98E+03	2.11E+03
	Optimal	**1.73E+03**	2.15E+03	1.91E+03	1.80E+03	2.07E+03	1.89E+03	1.91E+03	2.12E+03	2.36E+03	1.77E+03	1.79E+03
	Std	1.06E+02	3.32E+02	3.65E+02	1.96E+02	2.62E+02	1.91E+02	2.23E+02	2.92E+02	2.66E+02	**8.34E+01**	1.83E+02
	Rank	1	9.5	8	3	7	6	5	9.5	11	2	4
CEC-17–18	Mean	**1.16E+04**	2.27E+06	9.35E+04	1.78E+06	6.77E+06	1.85E+06	8.23E+05	7.81E+06	5.66E+05	4.84E+05	2.70E+04
	Optimal	**2.82E+03**	5.19E+04	4.35E+04	5.18E+04	1.15E+05	2.91E+05	7.03E+04	2.73E+05	8.25E+04	1.27E+05	5.90E+03
	Std	**8.18E+03**	3.20E+06	4.62E+04	4.53E+06	1.37E+07	1.79E+06	5.68E+05	7.88E+06	3.90E+05	1.91E+05	1.93E+04
	Rank	1	9	3	7	10	8	6	11	5	4	2
CEC-17–19	Mean	4.90E+03	6.13E+05	2.05E+04	6.17E+06	3.57E+06	1.82E+07	2.22E+06	1.15E+07	1.60E+05	5.40E+04	**1.95E+03**
	Optimal	**1.92E+03**	4.99E+04	2.21E+03	1.18E+04	2.12E+03	3.51E+05	2.01E+05	1.31E+05	1.67E+04	7.43E+03	1.93E+03
	Std	4.41E+03	3.55E+05	1.92E+04	2.22E+07	1.39E+07	2.47E+07	1.44E+06	7.98E+06	9.46E+04	3.01E+04	**6.94E+00**
	Rank	2	6	3	9	8	11	7	10	5	4	1
CEC-17–20	Mean	**2.26E+03**	2.83E+03	2.99E+03	2.43E+03	2.67E+03	2.62E+03	2.56E+03	2.82E+03	3.11E+03	2.36E+03	2.39E+03
	Optimal	**2.04E+03**	2.43E+03	2.50E+03	2.17E+03	2.24E+03	2.23E+03	2.32E+03	2.35E+03	2.72E+03	2.21E+03	2.06E+03
	Std	1.66E+02	2.16E+02	2.57E+02	1.69E+02	2.38E+02	2.01E+02	1.53E+02	2.23E+02	1.97E+02	**6.82E+01**	2.30E+02
	Rank	1	9	10	4	7	6	5	8	11	2	3
CEC-17–21	Mean	**2.35E+03**	2.56E+03	2.53E+03	2.41E+03	2.50E+03	2.49E+03	2.43E+03	2.60E+03	2.63E+03	**2.35E+03**	2.50E+03
	Optimal	2.32E+03	2.43E+03	2.47E+03	2.36E+03	2.43E+03	2.41E+03	2.35E+03	2.50E+03	2.56E+03	**2.23E+03**	2.47E+03
	Std	4.32E+01	4.36E+01	3.35E+01	4.25E+01	4.71E+01	3.28E+01	3.88E+01	5.40E+01	3.22E+01	7.90E+01	**1.08E+01**
	Rank	1.5	9	8	3	6.5	5	4	10	11	1.5	6.5

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
CEC-17–22	Mean	2.81E+03	7.07E+03	7.14E+03	4.45E+03	6.11E+03	8.28E+03	4.27E+03	7.14E+03	7.31E+03	2.35E+03	**2.30E+03**
	Optimal	**2.30E+03**	2.37E+03	2.31E+03	2.41E+03	2.68E+03	6.30E+03	**2.30E+03**	2.47E+03	6.19E+03	2.32E+03	**2.30E+03**
	Std	1.33E+03	1.22E+03	2.25E+03	1.49E+03	1.38E+03	8.54E+02	2.18E+03	2.02E+03	5.66E+02	2.44E+01	**3.66E-05**
	Rank	3	7	8.5	5	6	11	4	8.5	10	2	1
CEC-17–23	Mean	**2.71E+03**	3.21E+03	3.40E+03	2.76E+03	2.84E+03	2.86E+03	2.78E+03	3.07E+03	3.89E+03	2.74E+03	2.85E+03
	Optimal	**2.69E+03**	2.99E+03	3.09E+03	2.70E+03	2.78E+03	2.79E+03	2.72E+03	2.91E+03	3.38E+03	2.70E+03	2.83E+03
	Std	1.61E+01	1.13E+02	1.65E+02	4.54E+01	3.68E+01	3.32E+01	3.60E+01	1.06E+02	2.19E+02	3.21E+01	**9.54E+00**
	Rank	1	9	10	3	5	7	4	8	11	2	6
CEC-17–24	Mean	**2.87E+03**	3.43E+03	3.48E+03	2.94E+03	2.99E+03	2.99E+03	2.94E+03	3.24E+03	3.50E+03	**2.87E+03**	3.01E+03
	Optimal	2.84E+03	3.16E+03	3.28E+03	2.87E+03	2.93E+03	2.92E+03	2.87E+03	3.02E+03	3.35E+03	**2.62E+03**	2.99E+03
	Std	4.03E+01	1.66E+02	1.18E+02	6.54E+01	3.54E+01	3.29E+01	3.75E+01	1.08E+02	9.15E+01	1.87E+02	**9.31E+00**
	Rank	1.5	9	10	3.5	5.5	5.5	3.5	8	11	1.5	7
CEC-17–25	Mean	**2.89E+03**	2.94E+03	2.94E+03	2.99E+03	3.21E+03	3.25E+03	2.93E+03	3.08E+03	2.99E+03	**2.89E+03**	**2.89E+03**
	Optimal	**2.88E+03**	2.89E+03	2.90E+03	2.93E+03	2.90E+03	3.04E+03	2.89E+03	2.99E+03	2.96E+03	**2.88E+03**	2.89E+03
	Std	1.39E+01	1.80E+01	2.21E+01	5.15E+01	3.30E+02	1.63E+02	2.99E+01	4.31E+01	2.19E+01	2.82E+00	**1.97E-01**
	Rank	2	5.5	5.5	7.5	10	11	4	9	7.5	2	2
CEC-17–26	Mean	4.45E+03	7.38E+03	8.30E+03	4.93E+03	6.04E+03	5.77E+03	4.86E+03	8.35E+03	7.97E+03	**3.41E+03**	5.41E+03
	Optimal	4.04E+03	2.92E+03	3.76E+03	3.92E+03	4.95E+03	5.14E+03	**2.90E+03**	4.39E+03	6.17E+03	**2.90E+03**	4.97E+03
	Std	3.11E+02	1.58E+03	1.33E+03	5.10E+02	6.52E+02	3.81E+02	8.64E+02	1.17E+03	5.59E+02	5.44E+02	**1.40E+02**
	Rank	2	8	10	4	7	6	3	11	9	1	5
CEC-17–27	Mean	3.23E+03	3.44E+03	4.33E+03	3.26E+03	3.25E+03	3.29E+03	3.26E+03	3.41E+03	5.02E+03	**3.22E+03**	**3.22E+03**
	Optimal	3.21E+03	3.28E+03	3.95E+03	3.22E+03	3.20E+03	3.23E+03	3.22E+03	3.26E+03	4.19E+03	3.20E+03	**3.19E+03**
	Std	1.78E+01	1.62E+02	2.85E+02	2.39E+01	2.08E+01	4.75E+01	2.96E+01	9.15E+01	3.94E+02	**7.37E+00**	1.22E+01
	Rank	3	9	10	5.5	4	7	5.5	8	11	1.5	1.5

(*continued on next page*)

Table 6 (*continued*)

F		OPA	HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
CEC-17–28	Mean	**3.14E+03**	3.32E+03	3.27E+03	3.43E+03	3.98E+03	4.73E+03	3.27E+03	3.51E+03	3.58E+03	3.24E+03	3.21E+03
	Optimal	**3.10E+03**	3.26E+03	3.21E+03	3.28E+03	3.32E+03	3.43E+03	3.23E+03	3.38E+03	3.33E+03	3.23E+03	3.19E+03
	Std	5.21E+01	2.98E+01	2.35E+01	9.62E+01	7.31E+02	1.23E+03	2.85E+01	9.36E+01	2.15E+02	**1.29E+01**	1.61E+01
	Rank	1	6	4.5	7	10	11	4.5	8	9	3	2
CEC-17–29	Mean	**3.63E+03**	4.65E+03	4.74E+03	3.85E+03	4.13E+03	4.34E+03	4.29E+03	5.21E+03	5.23E+03	3.64E+03	4.04E+03
	Optimal	**3.31E+03**	4.05E+03	3.85E+03	3.56E+03	3.50E+03	3.93E+03	3.69E+03	4.37E+03	4.82E+03	3.49E+03	3.70E+03
	Std	2.24E+02	4.90E+02	3.66E+02	1.93E+02	3.11E+02	2.52E+02	3.10E+02	5.15E+02	2.62E+02	**8.20E+01**	1.63E+02
	Rank	1	8	9	3	5	7	6	10	11	2	4
CEC-17–30	Mean	**8.01E+03**	3.78E+06	1.94E+05	7.41E+06	7.24E+05	2.69E+07	6.09E+06	3.11E+07	3.07E+06	1.02E+05	2.40E+04
	Optimal	**5.31E+03**	1.06E+06	4.53E+04	1.15E+06	1.90E+04	1.11E+06	4.50E+05	1.35E+06	1.31E+06	4.40E+04	1.49E+04
	Std	**2.90E+03**	2.30E+06	1.46E+05	4.67E+06	1.18E+06	2.08E+07	3.64E+06	3.38E+07	1.73E+06	5.29E+04	6.52E+03
	Rank	1	7	4	9	5	10	8	11	6	3	2

- A new meta-heuristic algorithm OPA is proposed, which is inspired by the predatory behavior of orcas, and the models of driving, encircling, attacking and adjusting are established in the algorithm.
- Twenty-three well-known unconstrained benchmark test functions, modern CEC2015 and 2017 benchmark function as well as five constrained engineering design problems are employed to evaluate the performance of OPA.
- The statistical results of OPA and eight well-established algorithms are compared, and the parameter sensitivity and four qualitative metrics of the OPA algorithm are analyzed.

The rest of the paper is arranged as follows: Section 2 puts forward the OPA algorithm, describes the execution process in detail and analyzes its principle; Section 3 describes the experimental results and data; Section 4 presents the application of OPA to engineering problems; Section 5 is discussion of the results; and finally the conclusions and future directions are listed in Section 6.

2. Orca predation algorithm

2.1. Inspiration

Orca is a very intelligent carnivore in the dolphin family with a high degree of socialization. They live in closely-knit family groups comprising a few to more than 30 members. A typical orca group consists of 20% calves, 20% mature males and 60% females, and a larger group may have more adult males. Females stay within the group for life, but when the group becomes too large, the males will leave to find a new group. Individuals of a group swim within 100 m from each other, and share preys and rarely leave the group for more than a few hours. The vision of orcas is fairly good, but their vision is useless for preying or navigating in dark underwater environment. Like other toothed whales, orcas heavily depend on sonar to sense the aquatic environment. They can make three different types of sounds: clicks, whistles and pulsed calls, among which the main function of clicks are navigation, prey search, surrounding environment exploration and social interactions (Ford, 2018).

Similar to wolves, orcas have unique hunting methods in packs. When coming across with a school of fish, they tend to use their sonar to communicate with each other and plan their tactics instead of lunging at them and gulping quantities of fish. Members of the pack coordinate their moves, herding a mass of fish to the surface and encircle them into a manageable ball. Then, everyone takes turns to blow bubbles, flash their white bellies and whip their tails against the ball, stunning or killing the fish. Once the shoal is under control, the orca beats the edge of the shoal with its tail to get the food. The detailed information about the hunting behavior is available in the following link: https://www.nationalgeographic.com/magazine/article/orcas-feeding-cooperative-hunting -killer-whales.

Based on this process, we modeled the social aggregation, echolocation and hunting behavior of orcas and proposed the Orca Predation Algorithm (OPA).

2.2. Establishment of a colony of orcas

In OPA, we set a group of orcas with N members. The orca can swim in 1-D, 2-D, 3-D or extra-dimensional space. The corresponding mathematical model of the orca group is established as follows:

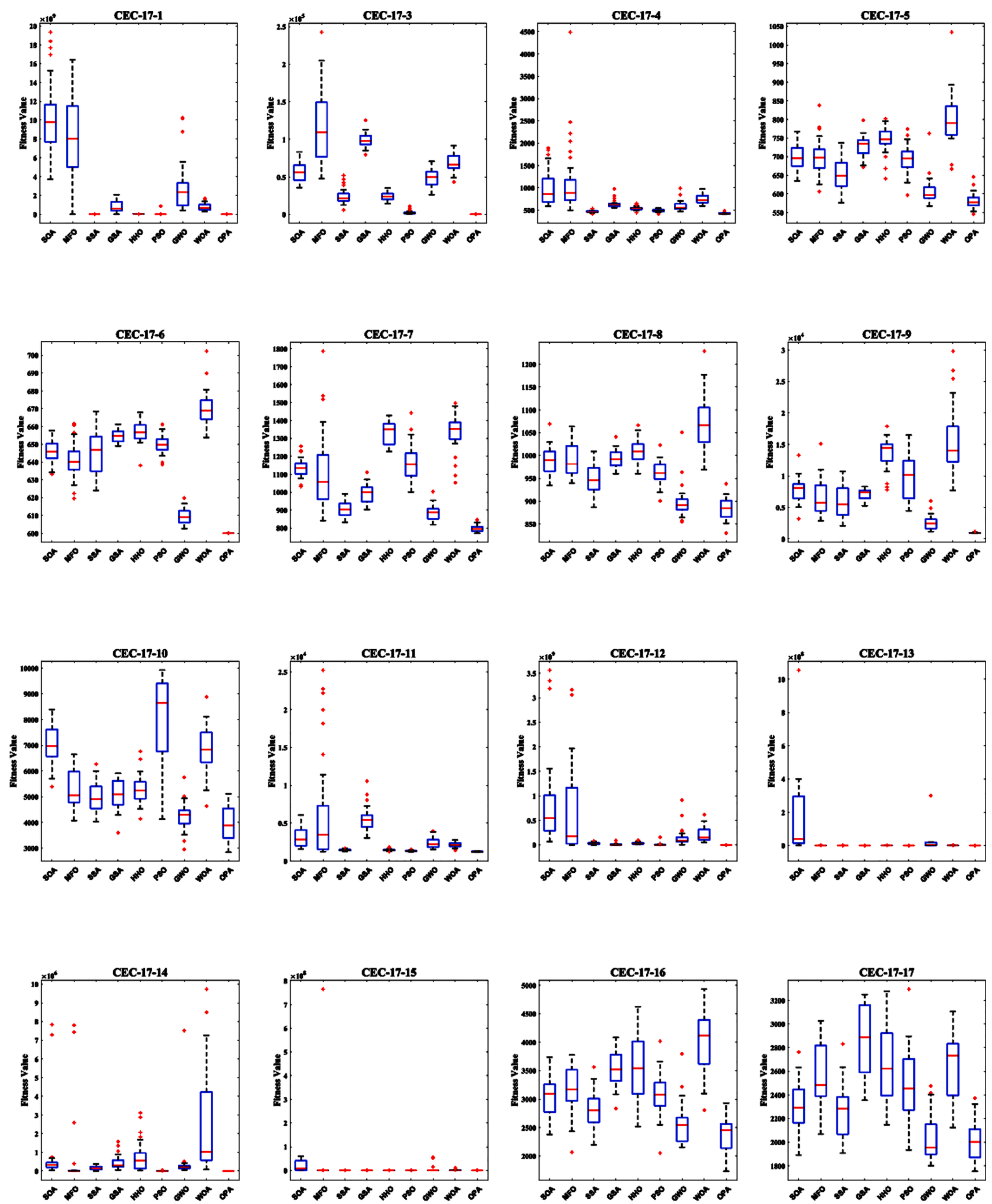

Fig. 4. Boxplot obtained on CEC 2017 benchmark test functions for different algorithms.

Table 7
Results of Wilcoxon's rank sum test for unimodal functions, multimodal functions and fixed-dimension multimodal functions with 30*D*.

No		HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F1	p-value	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.20389E-12	1.21178E-12	1.21079E-12	1.21178E-12	1.21178E-12
	H	1	1	1	1	1	1	1	1	1	1
F2	p-value	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21079E-12	1.21079E-12
	H	1	1	1	1	1	1	1	1	1	1
F3	p-value	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.2098E-12	1.21178E-12	1.21178E-12
	H	1	1	1	1	1	1	1	1	1	1
F4	p-value	1.21178E-12	1.21178E-12	1.21079E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.20586E-12	1.21178E-12	1.21178E-12
	H	1	1	1	1	1	1	1	1	1	1
F5	p-value	6.36917E-11	3.01797E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F6	p-value	3.00287E-11	5.43298E-11	3.00475E-11	3.00475E-11	3.00475E-11	2.9991E-11	3.00287E-11	3.00475E-11	3.00287E-11	3.00475E-11
	H	1	1	1	1	1	1	1	1	1	1
F7	p-value	0.04132254	3.01986E-11	0.002891325	3.01986E-11	1.54652E-09	3.01986E-11	0.00022539	3.01986E-11	3.33839E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F8	p-value	0.001766564	3.01797E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F9	p-value	NaN	4.32458E-12	NaN	1.21178E-12	0.081522972	1.21079E-12	0.333710696	1.11648E-12	1.21178E-12	1.21178E-12
	H	0	1	0	1	0	1	0	1	1	1
F10	p-value	NaN	1.96873E-13	1.96873E-13	1.21178E-12	1.21178E-12	1.20487E-12	3.13589E-08	1.2098E-12	1.21079E-12	1.21079E-12
	H	0	1	1	1	1	1	1	1	1	1
F11	p-value	NaN	1.21178E-12	1.27173E-05	1.21178E-12	0.041926168	1.21178E-12	0.005584312	6.22584E-10	1.21178E-12	1.20684E-12
	H	0	1	1	1	1	1	1	1	1	1
F12	p-value	2.99534E-11	0.000326169	2.99534E-11	2.99534E-11	2.99534E-11	2.99534E-11	2.99346E-11	2.98971E-11	2.99346E-11	2.99534E-11
	H	1	1	1	1	1	1	1	1	1	1
F13	p-value	2.99158E-11	2.43706E-10	2.99346E-11	2.99346E-11	2.99346E-11	2.96541E-11	2.99346E-11	7.32072E-10	2.99346E-11	2.99346E-11
	H	1	1	1	1	1	1	1	1	1	1
F14	p-value	0.04177393	NaN	2.99555E-07	0.001358167	4.89087E-11	NaN	0.000646294	1.21178E-12	NaN	NaN
	H	1	0	1	1	1	0	1	1	0	0
F15	p-value	2.26588E-08	3.03615E-10	1.85227E-09	5.53906E-09	1.69558E-09	1.57907E-08	1.58225E-08	6.792E-12	2.47727E-08	0.004189833
	H	1	1	1	1	1	1	1	1	1	1
F16	p-value	NaN	NaN	1.87148E-10	NaN	1.21178E-12	NaN	NaN	NaN	NaN	NaN
	H	0	0	1	0	1	0	0	0	0	0
F17	p-value	1.93937E-09	NaN	1.21079E-12	NaN	1.21178E-12	NaN	4.78841E-08	NaN	NaN	NaN
	H	1	0	1	0	1	0	1	0	0	0
F18	p-value	0.020468529	0.160741998	1.2377E-09	0.160741998	1.2377E-09	0.160741998	1.10724E-08	0.160741998	0.160741998	0.160741998
	H	1	0	1	0	1	0	1	0	0	0
F19	p-value	1.21178E-12	0.081404216	1.21178E-12	NaN	1.21178E-12	NaN	1.21178E-12	NaN	NaN	NaN
	H	1	0	1	0	1	0	1	0	0	0
F20	p-value	1.16939E-10	0.237117087	1.80316E-05	0.014505344	1.24547E-11	4.95306E-07	3.75311E-05	0.000285265	0.000285265	0.267805564
	H	1	0	1	1	1	1	1	1	1	0
F21	p-value	4.56182E-11	4.25189E-06	4.56182E-11	1.5002E-06	5.82424E-12	0.004379462	4.10401E-11	0.000259286	0.333710696	0.184638602
	H	1	1	1	1	1	1	1	1	0	0
F22	p-value	4.56182E-11	0.027654087	4.56182E-11	0.000791908	1.01449E-11	0.169865764	4.10401E-11	0.333710696	0.333710696	0.333710696
	H	1	1	1	1	1	0	1	0	0	0
F23	p-value	2.71565E-09	0.981827293	4.2827E-06	0.000807139	9.6102E-08	0.589290609	4.59959E-07	0.452829547	0.078282105	0.021569281
	H	1	0	1	1	1	0	1	0	0	1

Table 8
Results of Wilcoxon's rank sum test for CEC 2015 with 30*D*.

No		HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F1	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F2	p-value	3.01986E-11	1.55808E-08	3.01986E-11	3.01986E-11	3.01986E-11	0.010314672	3.01986E-11	0.549326784	0.830255284	0.27718919
	H	1	1	1	1	1	1	1	0	0	0
F3	p-value	6.72195E-10	0.000110577	0.371077032	4.61591E-10	0.000200581	1.01733E-10	1.17374E-09	3.01986E-11	5.57265E-10	0.93519197
	H	1	1	0	1	1	1	1	1	1	0
F4	p-value	3.01797E-11	3.01797E-11	3.0797E-08	3.33631E-11	3.01797E-11	3.19514E-09	3.01797E-11	3.01797E-11	0.000117446	3.01797E-11
	H	1	1	1	1	1	1	1	1	1	1
F5	p-value	7.69496E-08	7.11859E-09	0.145319127	5.96731E-09	1.46431E-10	1.63506E-05	3.15889E-10	2.67842E-06	3.68973E-11	3.01986E-11
	H	1	1	0	1	1	1	1	1	1	1
F6	p-value	3.01986E-11	0.283778048	3.01986E-11	3.01986E-11	3.01986E-11	5.57265E-10	3.01986E-11	3.01986E-11	3.01986E-11	0.002265781
	H	1	0	1	1	1	1	1	1	1	1
F7	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	4.44405E-07	0.0351366	0.118817344
	H	1	1	1	1	1	1	1	1	1	0
F8	p-value	3.01986E-11	8.15274E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	0.000556111
	H	1	1	1	1	1	1	1	1	1	1
F9	p-value	3.69E-11	3.02E-11	4.50E-11	3.02E-11	3.02E-11	6.70E-11	3.02E-11	1.02E-05	3.02E-11	0.055545693
	H	1	1	1	1	1	1	1	1	1	0
F10	p-value	3.01986E-11	1.61323E-10	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	0.001370333
	H	1	1	1	1	1	1	1	1	1	1
F11	p-value	0.001952677	0.027086318	0.017649028	1.17374E-09	2.0338E-09	0.023243447	0.00039881	3.01986E-11	3.01986E-11	7.85657E-12
	H	1	1	1	1	1	1	1	1	1	1
F12	p-value	7.38908E-11	1.61323E-10	8.88288E-06	3.33839E-11	6.06576E-11	8.99341E-11	3.68973E-11	0.01563812	8.14648E-05	0.002499392
	H	1	1	1	1	1	1	1	1	1	1
F13	p-value	3.01986E-11	3.01986E-11	0.125970193	1.35943E-07	3.49711E-09	4.63897E-05	4.07716E-11	3.01986E-11	2.37682E-07	8.352E-08
	H	1	1	0	1	1	1	1	1	1	1
F14	p-value	0.007617064	0.003848068	7.22083E-06	1.17374E-09	5.0922E-08	0.032650939	1.60621E-06	1.21178E-12	3.33839E-11	0.000132495
	H	1	1	1	1	1	1	1	1	1	1
F15	p-value	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.21178E-12	1.1387E-12	1.21178E-12	NaN	NaN	NaN
	H	1	1	1	1	1	1	1	0	0	0

$$X = [x1 \quad , \quad x2 \quad , \cdots \quad xN] = \begin{bmatrix} x1,1 & x1,2 & \cdots & x1,D \\ x2,1 & x2,2 & \cdots & x2,D \\ \vdots & \vdots & & \vdots \\ xN,1 & xN,2 & \cdots & xN,D \end{bmatrix} \tag{1}$$

where X represents the orca population that corresponds to the set of all candidate solutions of the problem, x_N indicates the position of the N^{th} individual orca that corresponds to the N^{th} candidate solution of the problem, and $x_{N,D}$ stands for the position of D^{th} dimension of the N^{th} orca that corresponds to the value of the D^{th} dimension of the N^{th} candidate solution for the problem.

2.3. Chasing phase

When orcas encounter a school of fish, they do not simply swarm to hunt, but communicate and cooperate with each other through sonar. A group of orcas will disperse, driving the school of fish to the surface and then pushing them into a controlled enclosure. Based on this behavior, we abstract the chasing phase in the hunting process of orcas into two kinds of behaviors: driving of prey and encircling of prey. Here, parameter p_1 is introduced to adjust the probability of the orca to perform these two processes separately. p_1 is set as a constant between [0, 1], and a number between [0, 1] is randomly generated as well. When the value of the number is greater than p_1, the driving phase will be carried out; otherwise the encircling phase will be carried out.

2.3.1. Driving of prey

After spotting a school of fish, the orcas need to chase the shoal to the surface. If the school of orcas is small, the spatial dimension of swimming is low, or when the hunting environment is relatively simple, the orcas can quickly and accurately locate the position of the prey. On the contrary, if the school of orcas is large, the spatial dimension of swimming is higher or the hunting environment is relatively complicated, the orca swimming is easy to disperse and it will be difficult to accurately reach the desired position. At this time, in addition to enabling

Table 9
Results of Wilcoxon's rank sum test for CEC 2017 with 30*D*.

No		HHO	PSO	GWO	MFO	SOA	SSA	WOA	GSA	ABC	DE
F1	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	1.09367E-10	3.01986E-11	3.01986E-11	3.01986E-11	0.026077477
	H	1	1	1	1	1	1	1	1	1	1
F2	p-value	3.01986E-11	3.01986E-11	3.01986E-11	1.17374E-09	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F3	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.80526E-07	1.28704E-09	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	2.88633E-11
	H	1	1	1	1	1	1	1	1	1	1
F4	p-value	8.48477E-09	3.68973E-11	3.01986E-11	1.17374E-09	3.01986E-11	3.01986E-11	0.911708975	1.46431E-10	3.01986E-11	2.9991E-11
	H	1	1	1	1	1	1	0	1	1	1
F5	p-value	6.69552E-11	8.99341E-11	3.01986E-11	0.001679756	7.11859E-09	0.000189162	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F6	p-value	0.011710684	1.4918E-06	3.01986E-11	1.63506E-05	1.77691E-10	3.01986E-11	0.00946827	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F7	p-value	9.91863E-11	3.33839E-11	3.68973E-11	1.09367E-10	6.06576E-11	3.01986E-11	0.059427915	3.33839E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	0	1	1	1
F8	p-value	3.82016E-10	0.003501167	4.97517E-11	0.014412183	0.569220164	0.025101283	1.95678E-10	0.695215399	4.50432E-11	1.21178E-12
	H	1	1	1	1	0	1	1	0	1	1
F9	p-value	3.01986E-11	0.000952074	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	4.07716E-11	3.01986E-11	3.01986E-11	2.87158E-10
	H	1	1	1	1	1	1	1	1	1	1
F10	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.08105E-08	3.01986E-11	3.01986E-11	3.01986E-11	3.33839E-11	4.50432E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F11	p-value	3.01986E-11	3.01986E-11	3.01986E-11	1.72941E-07	5.18568E-07	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F12	p-value	3.01986E-11	3.01986E-11	3.82016E-10	3.01986E-11	0.11536236	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	2.51259E-11
	H	1	1	1	1	0	1	1	1	1	1
F13	p-value	5.53286E-08	3.01986E-11	3.82016E-10	1.42942E-08	4.99795E-09	3.01986E-11	0.010314672	2.60985E-10	3.01986E-11	2.33306E-11
	H	1	1	1	1	1	1	1	1	1	1
F14	p-value	3.01986E-11	3.01986E-11	1.77691E-10	5.57265E-10	3.49711E-09	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	2.67909E-11
	H	1	1	1	1	1	1	1	1	1	1
F15	p-value	1.52917E-05	1.95678E-10	4.97517E-11	2.87158E-10	1.20567E-10	3.68973E-11	0.773119942	3.52006E-07	3.01986E-11	1.9534E-10
	H	1	1	1	1	1	1	0	1	1	1
F16	p-value	0.001301665	1.25408E-07	3.01986E-11	2.67842E-06	8.15274E-11	8.15274E-11	0.000238848	0.773119942	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	0	1	1
F17	p-value	3.33839E-11	3.01986E-11	4.07716E-11	8.10136E-10	3.01986E-11	3.01986E-11	8.15274E-11	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F18	p-value	3.01986E-11	3.01986E-11	8.89099E-10	2.37147E-10	4.80107E-07	3.01986E-11	2.66947E-09	3.01986E-11	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F19	p-value	2.37682E-07	0.019883076	4.07716E-11	1.41098E-09	4.61591E-10	8.15274E-11	9.83289E-08	0.026077477	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F20	p-value	0.000178356	3.35195E-08	3.01986E-11	7.77255E-09	3.68973E-11	3.01986E-11	0.795845542	0.000158461	3.01986E-11	3.01986E-11
	H	1	1	1	1	1	1	0	1	1	1
F21	p-value	9.87003E-11	9.87003E-11	2.78047E-07	2.29603E-09	2.18216E-11	0.000452456	1.6145E-10	2.9627E-11	8.29264E-07	0.373481788
	H	1	1	1	1	1	1	1	1	1	0
F22	p-value	3.01986E-11	3.01986E-11	1.06657E-07	3.01986E-11	3.01986E-11	8.15274E-11	3.01986E-11	3.01986E-11	0.003338612	3.01986E-11
	H	1	1	1	1	1	1	1	1	1	1
F23	p-value	3.01986E-11	3.01986E-11	1.72903E-06	3.68973E-11	3.33839E-11	1.28704E-09	3.01986E-11	3.01986E-11	0.190730333	3.01986E-11
	H	1	1	1	1	1	1	1	1	0	1
F24	p-value	1.06657E-07	9.06321E-08	3.49711E-09	6.72195E-10	3.01986E-11	8.352E-08	3.01986E-11	8.10136E-10	1.06657E-07	1.06657E-07
	H	1	1	1	1	1	1	1	1	1	1
F25	p-value	2.57212E-07	7.04298E-07	4.08395E-05	4.97517E-11	4.97517E-11	3.15727E-05	1.32885E-10	3.01986E-11	2.60151E-08	6.69552E-11
	H	1	1	1	1	1	1	1	1	1	1
F26	p-value	3.68973E-11	3.01986E-11	5.97056E-05	0.003338612	3.35195E-08	9.79171E-05	4.50432E-11	3.01986E-11	0.000124771	0.012732115
	H	1	1	1	1	1	1	1	1	1	1
F27	p-value	3.0123E-11	9.89507E-11	3.0123E-11	3.0123E-11	3.0123E-11	4.96298E-11	3.0123E-11	3.0123E-11	2.14946E-10	9.52E-07
	H	1	1	1	1	1	1	1	1	1	1
F28	p-value	6.06576E-11	8.15274E-11	0.005828168	4.80107E-07	1.77691E-10	2.66947E-09	3.01986E-11	3.01986E-11	0.283778048	2.02829E-07
	H	1	1	1	1	1	1	1	1	0	1
F29	p-value	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	3.01986E-11	4.50432E-11
	H	1	1	1	1	1	1	1	1	1	1

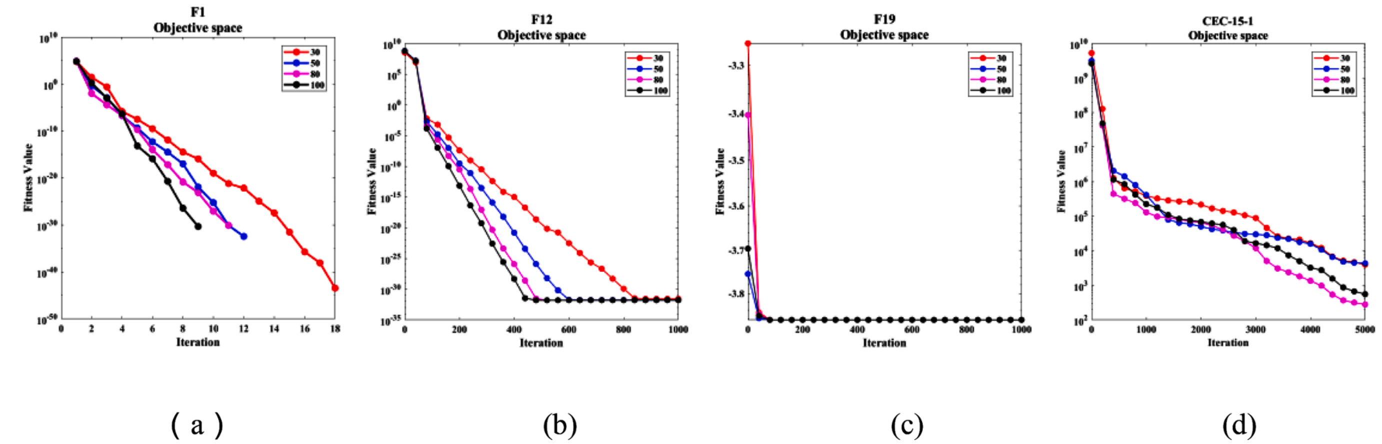

Fig. 5. Sensitivity analysis of OPA's control parameters: *N*.

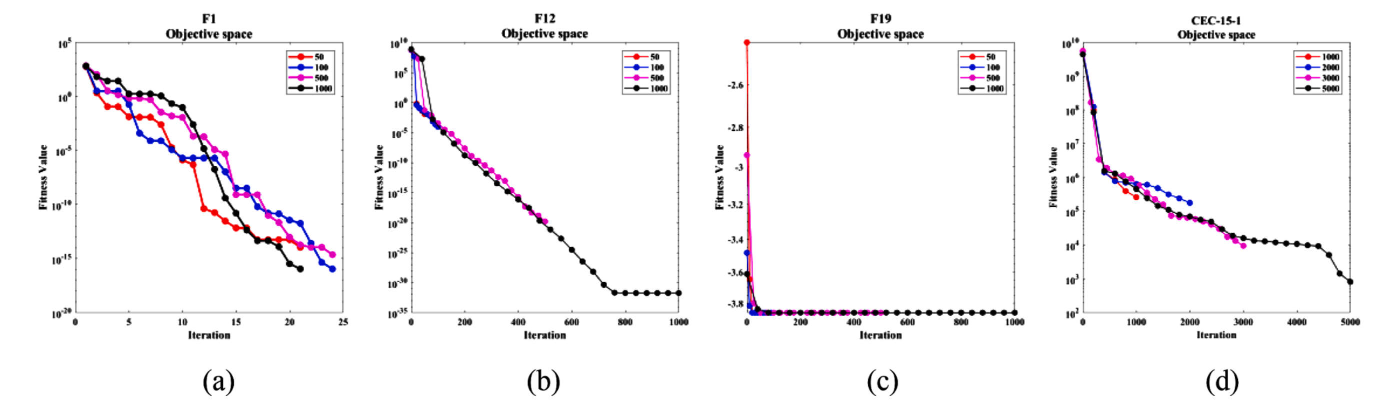

Fig. 6. Sensitivity analysis of OPA's control parameters: *Max_iter*.

individual orcas to get closer to the prey, it is also necessary to control the center position of the orca group to keep it close to the prey, so as to avoid the deviation of the orca group from the target. Based on the size of the orca population, we abstract two chasing methods. When the orca group is large (rand $> q$), the first method is taken; when the orca group is small (rand $\leq q$), the second method is taken. The process of prey driving of orcas is shown in Fig. 1.

The moving speed of the orca and the corresponding position after the moving can be depicted as follows:

$$v^t_{\text{chase},1,i} = a \times \left(d \times x^t_{\text{best}} - F \times \left(b \times M^t + c \times x^t_i\right)\right) \tag{2}$$

$$v^t_{\text{chase},2,i} = e \times x^t_{\text{best}} - x^t_i \tag{3}$$

$$M = \frac{\sum_{i=1}^{N} x^t_i}{N} \tag{4}$$

$$c = 1 - b \tag{5}$$

$$\begin{cases} x^t_{\text{chase},1,i} = x^t_i + v^t_{\text{chase},1,i} & \text{if} \quad \text{rand} > q \\ x^t_{\text{chase},2,i} = x^t_i + v^t_{\text{chase},2,i} & \text{if} \quad \text{rand} \leqslant q \end{cases} \tag{6}$$

where *t* represents the number of cycles, *vt* chase,1,*i* indicates the chasing speed of the i^{th} orca at time *t* after selecting the first chasing method, *vt* chase,2,*i* stands for the chasing speed of the i^{th} orca at time *t* after selecting the second chasing method, *M* represents the average position of the orca group, *xt* chase,1,*i* is the position of the i^{th} orca at time *t* after selecting the first chasing method, *xt* chase,2,*i* indicates the position of the i^{th} orca at time *t* after selecting the second chasing method, *a*, *b* and *d* are random numbers number between [0, 1] respectively, *e* is a random number between [0, 2], the value of *F* is equal to 2, and *q* is a number between [0, 1], which represents the probability of choosing a certain chasing method. After many experiments, OPA was found to have the best performance when $q = 0.9$.

2.3.2. Encircling of prey

After driving the shoal to the surface, orcas need to encircle the school of fish into a controlled ball. During the encircling, orcas communicate with each other through sonar, and determine their next moving position according to the position of the nearby orcas. Here, we assume that the orcas locate themselves based on the positions of three randomly selected orcas, and then the position after moving can be computed as follows:

$$x^t_{\text{chase},3,i,k} = x^t_{j1,k} + u \times \left(x^t_{j2,k} - x^t_{j3,k}\right) \tag{7}$$

$$u = 2 \times (\text{rand} - 1/2) \times \frac{Max\_iter - t}{Max\_iter} \tag{8}$$

where *Max_iter* represents the maximum number of iterations, j_1, j_2, j_3 indicate the three randomly selected orcas from *N* orcas, and $j_1 \neq j_2 \neq j_3$, *xt* chase,3,*i* stands for the position of the i^{th} orca after choosing the third chasing method at time *t*.

2.3.3. Adjustment of positions

Orcas can sense the position of the prey through sonar and adjust their own positions accordingly. If the orcas perceive the approaching of the fish in the chasing process, they will continue to chase along the updated position; otherwise, the orcas will stay at the original position. Their positions are adjusted using the following formula:

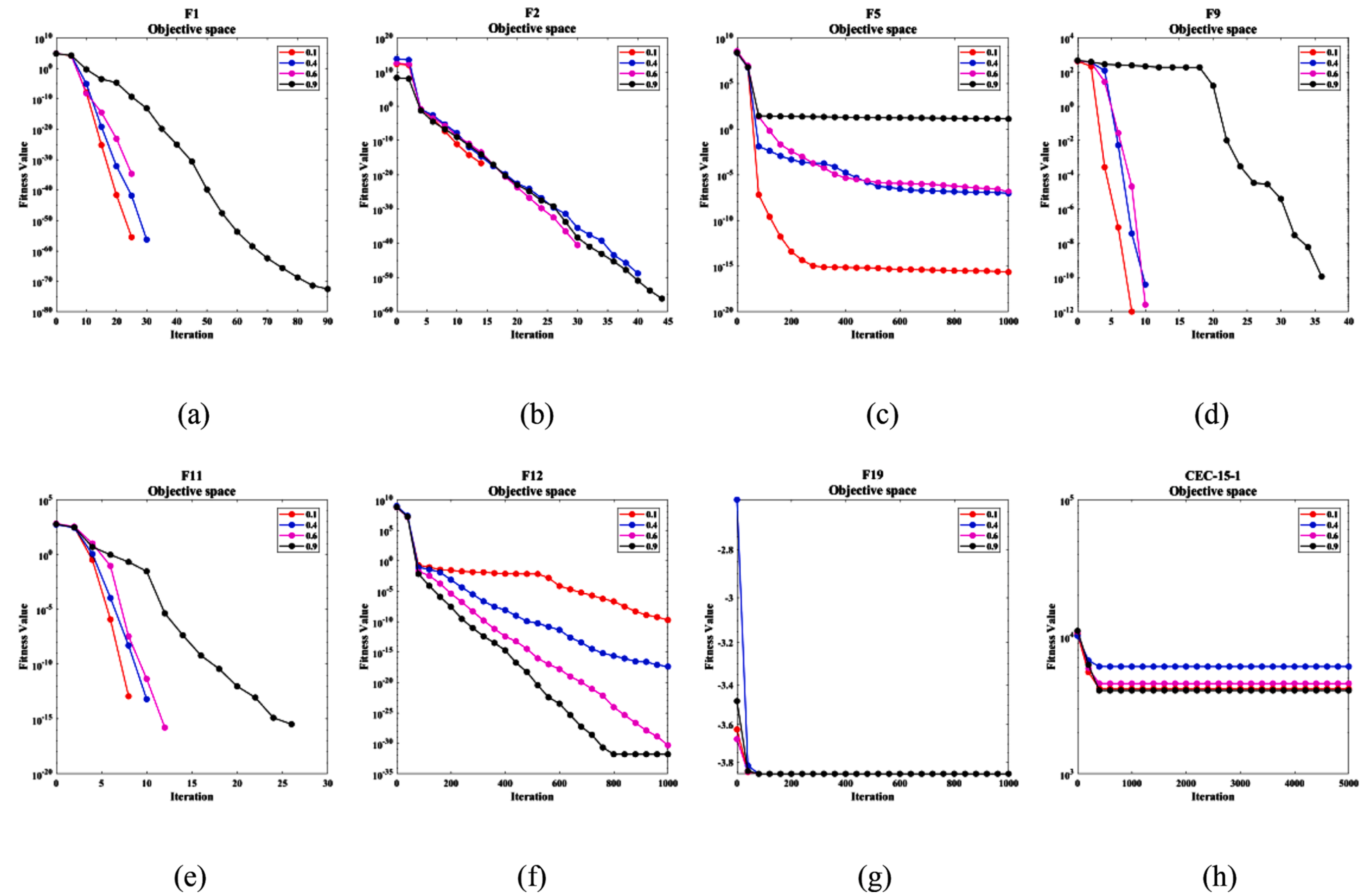

Fig. 7. Sensitivity analysis of OPA's control parameters: p_1.

$$\begin{cases} x^t_{\text{chase},i} = x^t_{\text{chase},i} & \text{if} \quad f\left(x^t_{\text{chase},i}\right) < f\left(x^t_i\right) \\ x^t_{\text{chase},t} = x^t_i & \text{if} \quad f\left(x^t_{\text{chase},i}\right) \geqslant f\left(x^t_i\right) \end{cases} \tag{9}$$

where *f*(*xt* chase,*i*) represents the fitness function value corresponding to *xt* chase,*i*, and *f*(*xt i*) indicates the fitness function value corresponding to *xt i*. For solving the minimum problem, the lower the fitness function value is, the better the corresponding position will be.

2.4. Attacking phase

2.4.1. Attacking preys

When orcas encircle their prey, they take turns to enter the enclosure to attack the prey, whipping their tails against the circle and eating the stunned fish, and then return to the original enclosure to replace another orca. We assume that there are four orcas which correspond to the four optimal positions of attacking in the circle (four orcas are chosen here because too few orcas will cause the moving of the particles in a single direction, while too many orcas will reduce the convergence speed of the algorithm). If other orcas want to enter the enclosure, they can move in the direction according to the position of the four orcas. If the orcas want to return to the enclosure to replace other orcas after feeding, the moving direction can be determined according to the position of randomly selected orcas nearby. The moving speed and position of the orca during the attacking process can be calculated according to the following formula and this process can be abstracted into the model as shown in Fig. 2.

$$v^t_{\text{attack},1,i} = \left(x^t_{\text{first}} + x^t_{\text{second}} + x^t_{\text{third}} + x^t_{\text{four}}\right)\Big/4 - x^t_{\text{chase},i} \tag{10}$$

$$v^t_{\text{attack},2,i} = \left(x^t_{\text{chase},j1} + x^t_{\text{chase},j2} + x^t_{\text{chase},j3}\right)\Big/3 - x^t_i \tag{11}$$

$$x^t_{\text{attack},i} = x^t_{\text{chase},i} + g1 \times v^t_{\text{attack},1,i} \quad + g2 \times v^t_{\text{attack},2,i} \tag{12}$$

where *vt* attack,1,*i* represents the speed vector of the i^{th} orca to hunt prey at time *t*, *vt* attack,2,*i* indicates the speed vector of the i^{th} orca to return to enclosure at time *t*, *xt* first, *xt* second, *xt* third, *xt* four represent the four orcas in the best position in turn, j_1,j_2,j_3 stands for the three randomly selected orcas from N orcas in the chasing phase and $j_1 \neq j_2 \neq j_3$, *xt* attack,*i* represents the position of the i^{th} orca at time *t* after the attacking phase, g_1 is a random number between [0, 2], and g_2 is a random number between [−2.5, 2.5].

2.4.2. Adjustment of positions

Similar to the chasing process, the orcas make use of sonar to locate the prey and adjust their positions. When the fish ball is under control, one orca will swim to the edge of the school of fish and beats the shoal with its tail to get food. The position of the orca is assigned with the minimum boundary value (*lb)* of the feasible range of the problem, which can be represented by the following pseudocode:

Position adjustment phase during an attack

```
If f (xt attack,i) < f (xt chase,i)
    xt + 1 i = xt attack,i
Else
    Q = rand;
    For k = 1: D
        if Q < p2
            x^{t+1}_{j,k} = lb(k)
        Else
```

(continued on next page)

(*continued*)

Position adjustment phase during an attack
9: $x_{j,k}^{t+1} = x_{\mathrm{chase},i,k}^{t}$
10: End
11: End
12: End

where p_2 is a constant between [0, 1], which selects different values according to specific problems, and u is a random number between [0, 1].

2.5. Steps and flowchart of the proposed OPA

In the following part, the steps for the implementation of OPA are presented, and Fig. 3 gives the flowchart of OPA.

Step 1: Initialization of the parameters and orca group. The parameters include number of population N, dimension of the problem D, maximum number of iterations Max_iter, selection probability p_1, p_2, lower bounds of design variable lb, upper bounds of design variable ub. The positions of orcas can be randomly defined based on lb and ub.

Step 2: Assessment of the orca group. The fitness value of each orca is calculated, and the best one is chosen as x_{best}.

Step 3: Position updating of the orca group at the chasing phase. At this stage, orcas choose to drive or encircle their prey to complete the process of chasing according to the selection factor p_1, during which they locate the prey and adjust their own positions through sonar. The orca updates its position according to Eqs. (2)–(9).

Step 4: Position updating of orca group at the attacking phase. Orcas attack the prey through Eqs (10)–(12), and update their positions through sonar, during which some orcas will beat the edge of the shoal and their positions are replaced by lower bounds lb.

Step 5: Construction of the new population. After completion of the attacking phase, the orcas will be reformed into a new group.

Step 6: Loop termination. It is determined whether the current iteration number reaches the maximum iteration number. If it does not reach the optimal output solution, the above process will be repeated from step 2.

2.6. Computation of complexity

For the assessment of the complexity, the newly proposed OPA is divided into four stages, and the complexity analysis and calculation process of each stage will be introduced in detail.

Phase 1: Initialization. Given that the group size is N and the orca is a vector of D dimension, and the initialization stage is to assign values to each dimension of each orca, the algorithm complexity can be calculated as $O(N \times D)$.

Phase 2: Assessment of the orca group. In the assessment of the orca group, the fitness value of each orca is calculated, and orcas and cycles are equal in number. Hence, the complexity of the algorithm is $O(N)$.

Phase 3: Chasing stage. This stage includes two sub-stages. In the

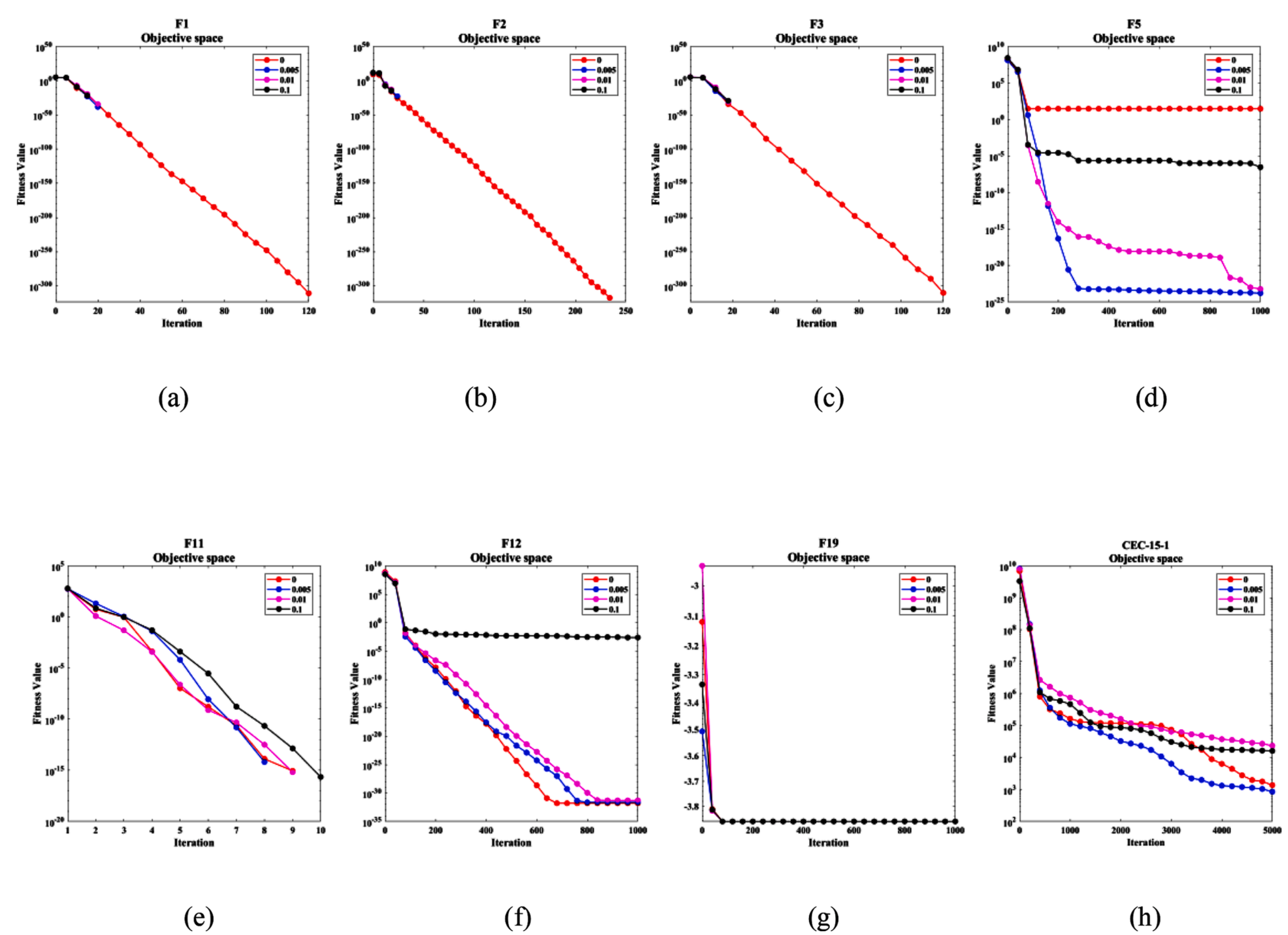

Fig. 8. Sensitivity analysis of OPA's control parameters: p_2.

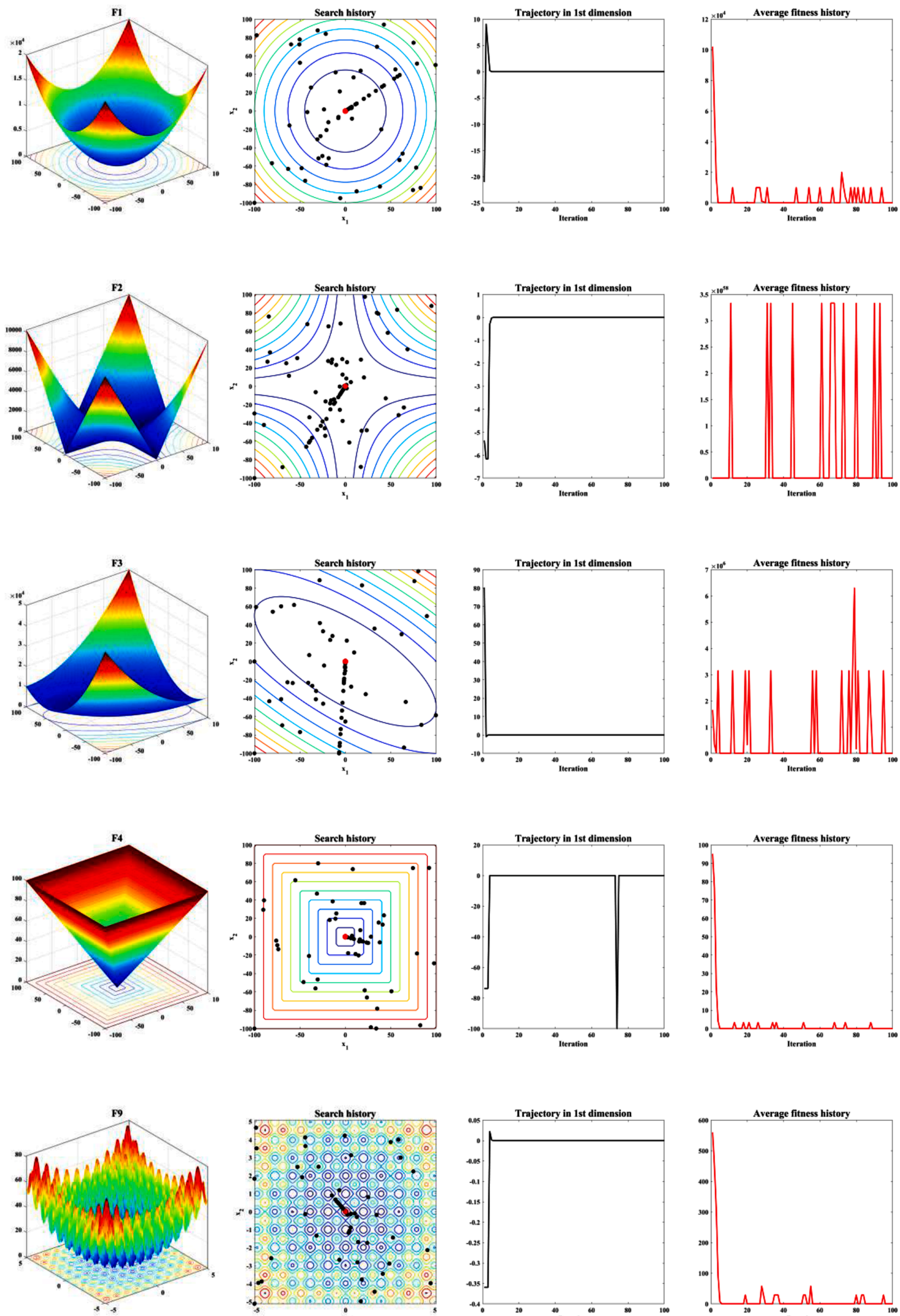

Fig. 9. Qualitative results on seven benchmark functions.

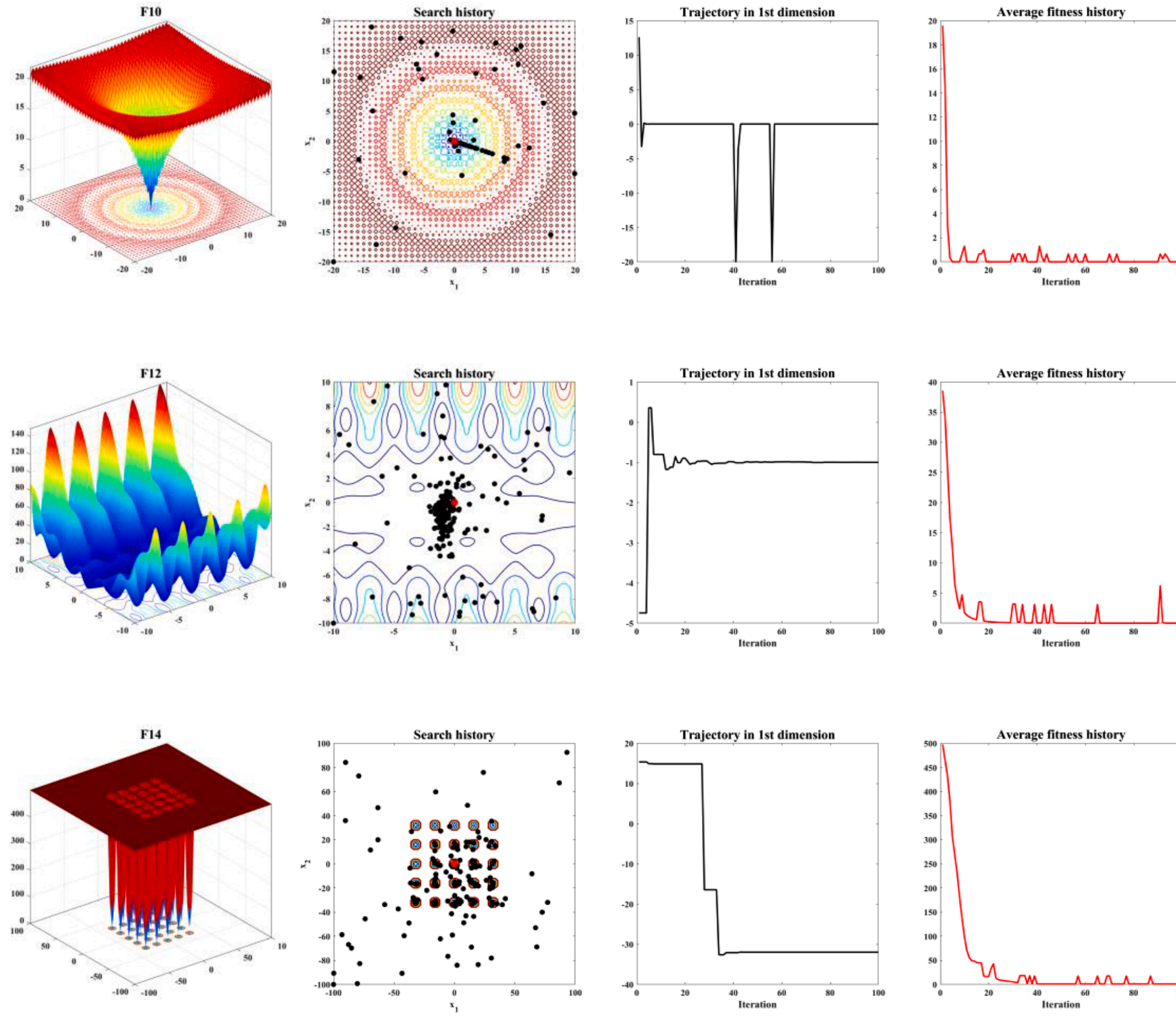

Fig. 9. (*continued*).

first sub-stage, there are two ways to update the positions of the orcas. If the orcas perform the position updating according to Eqs. (2)–(6), the updating is performed for each orca, and the complexity is $O(N)$; if the orcas adopt Eqs. (7)–(8) to update the position, the updating is performed for each dimension of each orca, and the complexity is $O(N \times D)$. In the second sub-stage, it is determined whether the orcas find better positions, which is only related to the number of orcas, and accordingly the complexity is $O(N)$.

Phase 4: Attacking stage. Similar to phase 3, the calculation of the complexity of this stage consists of two sub-stages. In the first sub-stage, position updating is performed for each individual, and the complexity is $O(N)$. In the second sub-stage, it is determined whether the orcas find better positions. Since the number of updated orcas is uncertain, the complexity of the algorithm cannot be determined too. The number of orcas to be updated is set as N', and correspondingly the complexity of the algorithm is $O(N' \times D)$. Because N' is smaller than N, the complexity of the algorithm is lower than $O(N \times D)$.

Overall, OPA shows a low computation complexity and thus a high computation speed.

2.7. Conceptual comparative analysis of OPA with other algorithms

Since most bio-inspired algorithms simulate the two distinct characteristics of adaptability and choice of the fittest, they tend to have similar appearances but with different internal search mechanism (Jain et al., 2019). This paper selects some algorithms to compare their search mechanism with that of the proposed OPA algorithm.

Since whale is one of the most intelligent species in the ocean, its lifestyle and hunting mechanism have attracted great research attention, and some bionic algorithms of whale have been proposed, one of which is the whale optimization algorithm (WOA) proposed by Mirjalili and Lewis. Besides, the grey wolf optimizer (GWO) is also a bionic predator algorithm that has received wide attention. Therefore, this paper chooses these two algorithms to compare with the proposed OPA.

The WOA simulates the bubble net search strategy of humpback whales, and uses the following formula to execute the bubble net attack and surround the prey.

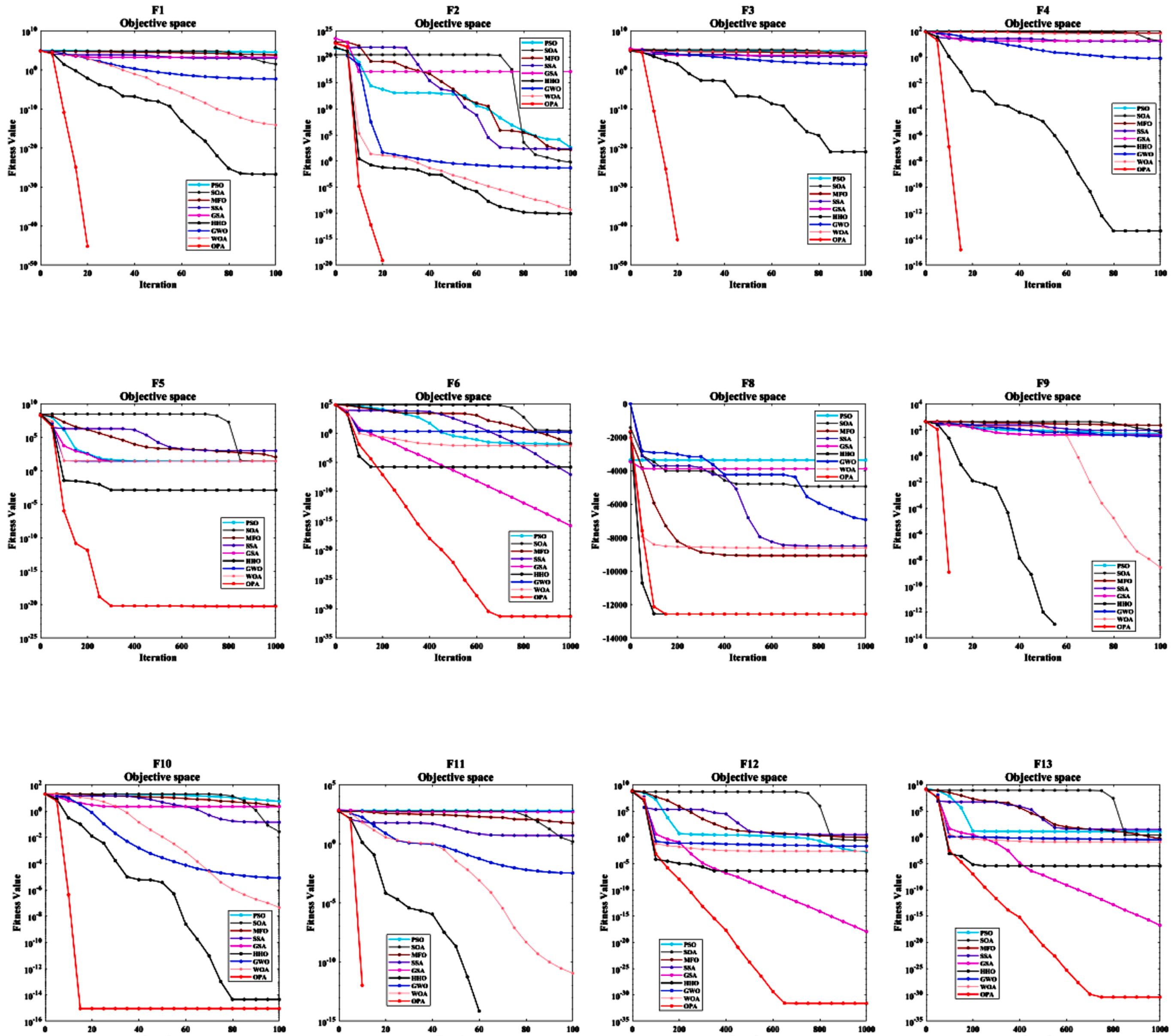

Fig. 10. Convergence curves of OPA and other compared algorithms.

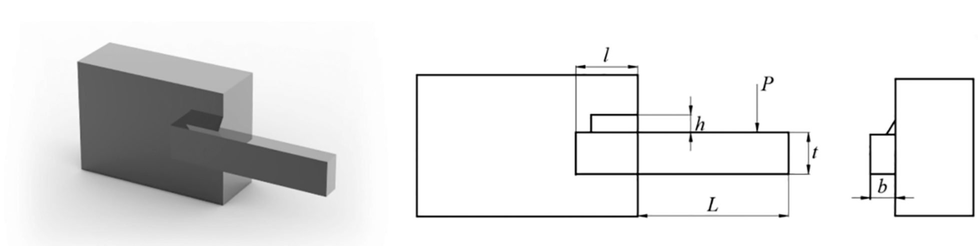

Fig. 11. Structure of welded beam design problem.

$$\vec{X}(t+1) = \begin{cases} \overrightarrow{X^*}(t) - \vec{A}\cdot\vec{D} & if \quad p < 0.5 \\ \overrightarrow{D'}\cdot e^{bl}\cdot cos(2\pi l) + \overrightarrow{X^*}(t) & if \quad p \geqslant 0.5 \end{cases} \tag{13}$$

Table 10
Statistical results of several algorithms for the welded beam design problem.

Algorithm	Optimal	Worst	Mean	Std	NFEs
SBM (Akhtar, Tai, & Ray, 2002)	2.4426	2.6315	2.5215	NA	19,259
BFOA (Montes, 2008)	2.3868	NA	2.404	0.016	48,000
SCA (Ray, 2003)	2.3854	6.3996	3.2551	0.9590	33,095
EA (Zhang, Liang, Huang, Wu, & Yang, 2009)	2.3816	2.38297	NA	0.00034	28,897
T-Cell (Aragón, Esquivel, & Coello, 2010)	2.3811	2.7104	2.4398	0.09314	320,000
FSA (Hedar & Fukushima, 2006)	2.3811	2.4889	2.4041	NA	56,243
IPSO (He, Prempain, & Wu, 2004)	23,810	NA	2.3819	0.00523	30,000
HSA-GA (Hwang & He, 2006)	2.2500	2.28	2.26	0.0078	26,466
CDE (Huang, Wang, & He, 2007)	1.7335	1.824105	1.768158	0.022194	240,000
CPSO (He & Wang, 2007)	1.7280	1.782143	1.748831	0.012926	200,000
OPA	**1.72485239014171**	**1.725013628697139**	**1.72486194246997**	**2.947062525011412e-05**	**10,800**

$$\overrightarrow{X}(t+1) \quad = \quad \overrightarrow{X_{rand}} - \overrightarrow{A}\cdot\overrightarrow{D} \tag{14}$$

In the exploitation phase (Eq. (13)) of WOA, the position of the whale is updated according to X^*. In OPA, through the parameter q, Eq. (2) or Eq. (3) is selected with a certain probability for a location update. In Eq. (2), in addition to the position of the optimal solution x_{best}, an intermediate position M is also introduced, which ensures that the population of orcas converges to the optimal solution as a whole, rather than the movement of single individuals. In Eq. (3), in order to increase the convergence speed, only the influence of x_{best} is considered.

In the exploration phase of WOA (Eq. (14)), the location is updated based on a randomly selected whale X_{rand}, while OPA randomly selects three orcas for cross reorganization.

WOA is based on the value of A in choosing to perform exploration or exploitation, and the assignment of A is random. OPA is based on the value of p_1, which is controlled by the user. Thus, different proportions can be assigned to the exploration and exploitation stages for different problems.

The GWO simulates the gray wolf leadership level, which is composed of alpha, beta, or omega and subordinate. The preying of wolves can be described by the following formula.

$$\overrightarrow{D}_{X=}\left|\overrightarrow{C}_1\cdot\overrightarrow{X}_X - \overrightarrow{X}\right|, \overrightarrow{D}_\beta = \left|\overrightarrow{C}_2\cdot\overrightarrow{X}_\beta - \overrightarrow{X}\right|, \overrightarrow{D}_\delta = \left|\overrightarrow{C_3}\cdot\overrightarrow{X}_\delta - \overrightarrow{X}\right| \tag{15}$$

$$\overrightarrow{X}_1 = \overrightarrow{X}_X - \overrightarrow{A}_1\left(\overrightarrow{D}_x\right), \overrightarrow{X}_2 = \overrightarrow{X}_\beta - \overrightarrow{A}_2\left(\overrightarrow{D}_\beta\right), \overrightarrow{X}_3 = \overrightarrow{X}_\delta - \overrightarrow{A}_3\left(\overrightarrow{D}_\delta\right) \tag{16}$$

$$\overrightarrow{X}(t+1) = \frac{\overrightarrow{X}_1 + \overrightarrow{X}_2 + \overrightarrow{X}_3}{3} \tag{17}$$

The moving direction of the wolf in GWO is affected by the cumulative effects of alpha, beta, and delta in Eq. (17). Unlike GWO, which has only one search mechanism, OPA has more abundant search mechanisms. In addition to the optimal position, the position of random particles is also introduced, and some positions are re-distributed with certain probabilities.

When forming a circling of the school of fish, an orca will swim to the edge and pat the school of fish with its tail to obtain food. Inspired by this, OPA has a new position adjustment mechanism, which ensures that the particles will always converge from the edge of the feasible region to the inside, which is not available in GWO.

3. Experimental results

In this section, we used 67 benchmark functions to evaluate the performance of the newly proposed OPA, and compared its performance with that of other ten algorithms. The detailed experimental process is described as follows.

3.1. Benchmark test functions and algorithms for comparison

We selected 67 benchmark functions to evaluate the performance of OPA, which could be divided into five categories: Unimodal, Multimodal, Fixed-dimension Multimodal (Chen, Zhang, Luo, Xu, & Zhang, 2020), CEC 2015 (Chen, Liu, Zhang, Liang, Suganthan, & Qu, 2015) and CEC 2017 functions (Wu, Mallipeddi, & Suganthan, 2016), which have their own unique characteristics and are used to evaluate different aspects of the algorithm performance. Unimodal function only has one global optimal solution, and thus is appropriate for evaluating the exploitation ability of the algorithm. Multimodal and Fixed-dimension Multimodal functions have multiple local optimal solutions. The dimension of Fixed-dimension Multimodal function is fixed and that of Multimodal function can be given arbitrarily. Both functions are used to test the exploration capability of the algorithm, and the algorithm with strong exploration capability can avoid local optimal solutions so as to find the global optimal solution (Faramarzi, Heidarinejad, Stephens, & Mirjalili, 2020). CEC 2015 and CEC 2017 include unimodal functions, simple multimodal functions, hybrid functions and composition functions, which combine multiple classic functions together through expansion, rotation, displacement and other ways for a comprehensive assessment of the exploration and exploitation capability of algorithms. For the test functions with uncertain dimension, we carried out experiments on 30 dimension (30*D*).

Besides, ten algorithms, including WOA (Mirjalili & Lewis, 2016), GWO (Mirjalili et al., 2014), SOA (Dhiman & Kumar, 2019), SSA (Mirjalili et al., 2017), HHO (Heidari et al., 2019), PSO (Kennedy & Eberhart, 1995), MFO (Mirjalili, 2015) and GSA (Rashedi et al., 2009), ABC (Karaboga & Basturk, 2007) and DE (Storn & Price, 1997), were selected to be compared with the proposed OPA, which are either well-established classic algorithms or new meta-heuristic algorithms with superior performance proposed in recent years. These algorithms are used in a variety of fields, with different focus on exploration and exploitation capability. Therefore, selection of these algorithms for comparison can more comprehensively and objectively evaluate the performance of OPA. The parameters of the algorithms are shown in Table 1. In particular, for Unimodal, Multimodal, Fixed-dimension Multimodal and CEC 2017, the maximum number of iterations taken in the paper is 1000. For CEC 2015, experiments showed that some algorithms cannot converge after 1000 iterations, and thus the maximum number of iterations was set as 5000.

3.2. Numerical analysis on benchmark function

3.2.1. Evaluation indictors

The optimal value, average value, standard deviation and rank are selected to analyze the experimental results. For the problem of solving the minimum value, a smaller optimal solution indicates higher quality of the results obtained by the algorithm. Similarly, a lower average value represents stronger comprehensive optimization ability of the algorithm, and a smaller standard deviation indicates a higher stability of the algorithm. The algorithm with ideal values of all the three indicators is naturally the best (Zhang & Jin, 2020). However, it is difficult to achieve this standard due to different emphasis on the performance of the

Table 11
Optimal results of several algorithms for the welded beam design problem.

Design variables	SBM	BFOA	SCA	EA	T-Cell	FSA	IPSO	HSA-GA	CDE	CPSO	OPA
$x_1(h)$	0.2407	0.2057	0.2444	0.2443	0.2444	0.2444	0.2444	0.2231	0.2031	0.2024	0.2057
$x_2(l)$	6.4851	3.4711	6.238	6.2201	6.1286	6.1258	6.2175	1.5815	3.543	3.5442	3.470489
$x_3(t)$	8.2399	9.0367	8.2886	8.294	8.2915	8.2939	8.2915	9.0335	9.04821	9.03513	9.03662
$x_4(b)$	0.2497	0.2057	0.2446	0.2444	0.2444	0.2444	0.2444	0.2444	0.2062	0.2057	0.2057
g_1	−7.19E + 03	−7.71E + 02	−7.05E + 03	−7.03E + 03	−6.94E + 03	−6.94E + 03	−7.03E + 03	1.05E + 04	−8.54E + 02	−8.01E + 02	−7.71E + 02
g_2	−2.72E + 02	3.82E + 00	−7.57E + 00	−2.21E + 01	−4.02E + 00	−2.14E + 01	−4.02E + 00	−4.73E + 03	−1.45E + 02	1.42E + 01	−1.32E-03
g_3	−9.00E-03	0.00E + 00	−2.00E-04	−1.00E-04	0.00E + 00	0.00E + 00	0.00E + 00	−2.13E-02	−3.10E-03	−3.30E-03	−2.27E-08
g_4	−2.97E + 00	−3.43E + 00	−3.02E + 00	−3.02E + 00	−3.03E + 00	−3.03E + 00	−3.02E + 00	−3.34E + 00	−3.42E + 00	−3.43E + 00	−3.43E + 00
g_5	−1.16E-01	−8.07E-02	−1.19E-01	−1.19E-01	−1.19E-01	−1.19E-01	−1.19E-01	−9.81E-02	−7.81E-02	−7.74E-02	−8.07E-02
g_6	−2.26E-01	−2.28E-01	−2.26E-01	−2.26E-01	−2.26E-01	−2.26E-01	−2.26E-01	−2.32E-01	−2.28E-01	−2.28E-01	−2.28E-01
g_7	−4.08E + 03	2.56E + 00	−3.51E + 03	−3.49E + 03	−3.49E + 03	−3.49E + 03	−3.49E + 03	−4.06E + 03	−4.63E + 01	3.24E + 00	−2.81E-05
$f(X)$	2.4426	2.3868	2.3854	2.3816	2.3811	2.3811	2.381	2.25	1.7335	1.728	**1.7249**

algorithm itself. In algorithm comparison, it is necessary to determine the measurement index according to the specific problems to for comparing the performance. For those problems that require high precision, the performance of the algorithms can be compared according to the optimal solution; if only rough results are needed, the performance can be compared according to the average value; if stable output results are required, more emphasis should be laid on the comparison of standard deviation.

To more intuitively present the quality of the solution, the tied rank (TR) method is applied. The method ranks the algorithm with the best average value as 1, that with the second best average value as 2, and that with the M best average value as M. The algorithms of the same level share the average rank (Zhang, Jin, & Chen, 2020). The four indicators in this experiment were obtained after the algorithm was tested on each function for 30 times.

3.2.2. Experimental results on unimodal function

The unimodal function has only one global optimum and F1-F7 unimodal functions were selected to evaluate the exploitation ability of OPA. Table 2 presents the statistical results solved by each algorithm when the dimension is 30 and the optimal results are marked in bold. For the results in Table 2, as a whole, OPA is superior to all other algorithms in all test functions except for F7. To be specific, OPA can obtain the global optimal solution for F1, F2, F3 and F4, while other algorithms for comparison are stuck at some local optimal solutions, and the standard deviation is 0, indicating that for the above four functions OPA has a 100% probability of finding the global optimal solution. Only for F7, OPA algorithm is slightly inferior to HHO algorithm, but the difference is insignificant. In conclusion, OPA has stronger local exploitation ability relative to other algorithms, which contributes to its obvious advantages in solving unimodal function problems.

3.2.3. Experimental results on multimodal function

The multimodal function has multiple local optimal solutions. Six multimodal functions (F8-F13) were selected to investigate the exploration ability of the algorithms. The statistical results presented in Table 3 demonstrate that OPA can obtain the optimal results in both the average value and the optimal value for all six functions. From the stability analysis, OPA has the lowest standard deviation on F9-F13 compared with other algorithms, confirming that it has the highest stability. The stability of OPA on F8 is only slightly poorer than that of HHO. In summary, when solving multimodal function problems, OPA can stably and accurately avoid local optima, with higher global exploration ability and competitiveness relative to other algorithms.

3.2.4. Experimental results on fixed-dimension multimodal functions

Fixed-dimension multimodal functions are multimodal functions with determined dimensions and can also be used to investigate the global exploration capability of the algorithm. The test results are depicted in Table 4. Generally, the differences in test results are not significant among these algorithms. The average value found by OPA is the first rank on five functions (F14, F16-F19). Superior performance could be found for the optimal value, on which OPA outperforms all other algorithms on all ten functions, indicating that OPA is an effective optimizer for fixed-dimension multimodal functions.

3.2.5. Experimental results on CEC 2015 functions

CEC 2015 includes shifted and rotated unimodal function, multimodal function, hybrid function and composition function, which are all defined in [−100, 100]. Table 5 shows the test results in 30 dimensions. As can be seen from Table 5, OPA excels all other algorithms on all functions except for F2, F5, F11 and F14.

3.2.6. Experimental results on CEC 2017 functions

To further evaluate the exploitation and exploration capability of OPA, a more challenging test was implemented based on CEC2017,

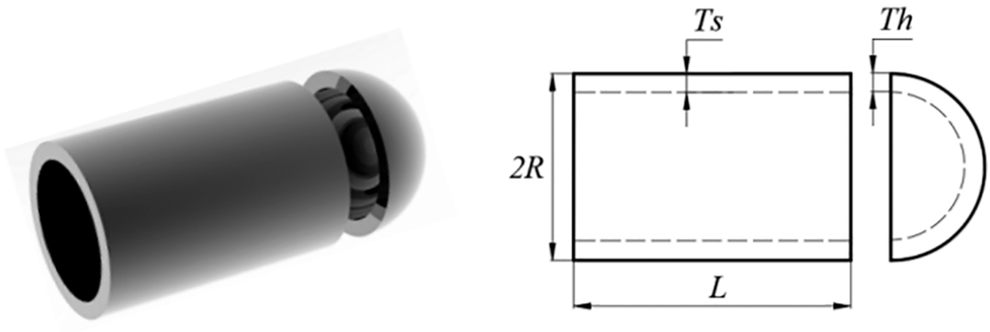

Fig. 12. Structure of pressure vessel design problem.

Table 12
Statistical results of several algorithms for the pressure vessel design problem.

Algorithm	Optimal	Worst	Mean	Std	NFEs
BFOA (E. Montes, 2008)	6060.46	NA	6074.625	156	48,000
HGA(2) (Bernardino et al., 2008)	6832.584	8012.612	7187.314	276	80,000
HGA(1) (Bernardino et al., 2008)	6065.821	8248.003	6632.376	515	80,000
CDE (Huang et al., 2007)	6059.734	6371.0455	6085.2303	43.013	240,000
CPSO (He & Wang, 2007)	6061.0777	6363.804	6147.1332	86.4545	200,000
HAIS-GA (Coello & Cortés, 2004)	6061.1229	7368.0602	6743.0848	457.99	150,000
DTS-GA (Coello & Montes, 2001)	6059.9463	6469.322	6177.2532	130.92	80,000
T-Cell (Aragón et al., 2010)	6390.554	7694.066	6737.065	357	80,000
ES (Mezura-Montes & Coello, 2008)	6059.746	7332.87	6850.00	426	25,000
OPA(Mixed variable)	**6059.714335**	**6059.714335**	**6059.714335**	**7.857E-08**	**9000**
OPA(Continuous)	**5885.33277**	**5885.33277**	**5885.33277**	**2.52E-12**	**9000**

where F2 was excluded due to the unstable behavior. Table 6 shows the test results of all algorithms in 30 dimensions. OPA achieves the first rank on most functions, and outperforms all other algorithms in optimal value on 24 functions except for F9, F15, F21, F26 and F27. In order to better understand the distribution of solutions, the data obtained from 30 runs were subjected to an ANOVA test, and the boxplots of OPA and other algorithms are given in Fig. 4. The results demonstrate that OPA has the lowest median value as marked by red and a narrower quartile interval, indicating that the distribution of OPA solutions is more concentrated than that of other algorithms. Overall, OPA is a robust and powerful optimizer that can well balance the exploitation and exploration stage.

3.3. Statistical analysis on benchmark functions

For a more rigorous comparison, Wilcoxon's rank-sum test was adopted with a significance level of 0.05 to evaluate the statistical performance of the algorithms. p and h are two important parameters in the test. When $p > 0.05$ or $h = 0$, there is no significant difference among the compared algorithms; and when $p < 0.05$ or $h = 1$, there is a significant difference. It is important to note that $p =$ NAN means that the two compared algorithms obtain the same results, while do not have the same performance.

The test results are depicted in Table 7–9. For unimodal functions, OPA is significantly different from all other algorithms. For multimodal functions, OPA finds the global optima on F9 and F11, and achieves the same performance with HHO on F10. In addition, OPA has significant differences from other algorithms. For most fixed-dimension multimodal functions, except for the functions with the same performance, there are also significant differences between OPA and the compared algorithms. For CEC 2015 and CEC 2017, similar conclusions can be drawn. In conclusion, for most functions, OPA can produce statistically better results than all the compared algorithms.

3.4. Sensitivity analysis of parameters in OPA

The parameters in OPA include the number of search agents (N), maximum number of iterations (Max_iter), and parameters p_1 and p_2. In order to analyze the effect of these parameters on the performance of OPA, different values for the parameters were selected to perform test on different types of functions.

(1) Number of search agents (N). In this experiment, N was set to 30, 50, 80 and 100, respectively, and then applied to the selected test functions (F1, F12, F19, CEC-15–1). The results in Fig. 5 reveal that the convergence speed of the algorithm is generally elevated with the increase of N such as F1, F12, CEC-15–1 (Fig. 5 (a), (b), (d)), but there are also some exceptions. For example, for F19 ((Fig. 5 (c)), the change in N has little effect on the convergence speed of OPA, and for CEC-15–1, when $N = 100$, the convergence speed is lower than that when $N = 80$.

(2) The maximum number of iterations (Max_iters). The setting of the Max_iters is determined by the complexity of the function. For complex functions, the algorithm is difficult to converge to the optimal solution with a small number of iterations, whereas the number of iterations can be smaller for simpler functions. In this experiment, Max_iter was set to 50, 100, 500 and 1000 for F1, F12 and F19 test functions, while 1000, 2000, 3000 and 5000 for CEC-15–1, respectively. The results are shown in Fig. 6. For F1 and F19 (Fig. 6 (a) and (c)), OPA can find the optimal solution with all the four selected Max_iters, indicating that the setting of Max_iters has little effect on the performance of OPA. For F12 and CEC-15–1 (Fig. 6(b) and (d)), when the number of iterations is small, the optimal solution cannot be obtained by the algorithm, and the algorithm converges to the optimal solution with increasing number of iterations.

(3) Value of p_1. The value of p_1 determines the difference in the emphasis of the algorithm on exploration and exploitation stages. A higher p_1 value indicates that the algorithm assigns more weight to the exploration stage, while a lower p_1 value represents

Table 13
Optimal results of several algorithms for the pressure vessel design problem.

Design variables	BFOA	HGA(2)	HGA(1)	CDE	CPSO	DTS-GA	T-Cell	ES	G-QPSO (Coelho, 2010)	OPA
$x_1(Ts)$	1.125	0.8125	0.8125	0.8125	0.8125	0.8125	0.8125	0.8125	0.8125	0.8125
$x_2(Th)$	0.5625	0.4375	0.4375	0.4375	0.4375	0.4375	0.4375	0.4375	0.4375	0.4375
$x_3(R)$	58.1267	42.098429	42.0492	42.091266	42.096394	42.097398	42.098087	42.098411	42.0984	42.0984456
$x_4(L)$	44.5941	190.787695	177.2522	176.7465	176.683231	176.654047	176.640518	176.63769	176.6372	176.6365958
g_1	−3.15E-03	−3.20E-07	−9.50E-04	−1.39E-04	−3.96E-05	−2.02E-05	−6.92E-06	−6.68E-07	−8.7999E-7	−4.65E-13
g_2	−7.97E-03	−3.59E-02	−3.64E-02	−3.59E-02	−3.59E-02	−3.59E-02	−3.59E-02	−3.59E-02	−3.5881E-2	−3.59E-02
g_3	3.14E + 00	−7.88E + 04	−2.45E + 01	−1.16E + 02	−1.18E + 02	−2.49E + 01	2.90E + 00	−3.71E + 00	−0.2179	−2.38E-07
g_4	−1.55E + 02	−9.21E + 00	−2.27E + 01	−2.33E + 01	−2.33E + 01	−2.33E + 01	−2.34E + 01	−2.34E + 01	−63.3628	−2.34E + 01
$f(X)$	6060.46	6832.583	6065.821	6059.734	6061.0777	6059.9463	6390.554	6059.746	6059.7208	**6059.714335**

that the algorithm assigns more weight to the exploitation stage. The value of p_1 was set as 0.1, 0.4, 0.6 and 0.9 respectively to analyze the effect of changes in the proportion of exploration and exploitation stage on the performance of OPA. Fig. 7 (a) - (e) show that when $p_1 = 0.9$, the convergence speed of OPA is significantly slower than that when $p_1 = 0.1$. This is because when $p_1 = 0.9$, the exploration stage takes a large proportion in the algorithm, leading to an increase in the diversity of the particle movement direction; while when $p_1 = 0.1$, the exploitation stage takes a significant proportion in the algorithm, so that the particles can converge quickly to the direction of the optimal solution. Therefore, for the unimodal function with only one global optimum, the value of p_1 should be low for a higher convergence speed of OPA. Fig. 7 (f) reveals that a decrease in the value of p_1 will result in worse optimal solutions obtained by OPA, which may be attributed to the property of multimodal functions. When the exploration ability of the algorithm is weak, the algorithm tends to mature prematurely. Therefore, for multimodal functions, the value of p_1 should be appropriately increased to enhance the global exploration capability of the algorithm.

(4) Value of p_2. The value of p_2 represents the probability of a particle to perform position assignment during the attacking phase in the algorithm. The value of p_2 was set as 0, 0.005, 0.01 and 0.1 and applied to the selected test functions. The experimental results in Fig. 8 show that for the unimodal function (Fig. 8(a)-(d)), the presence of p_2 has a significant impact on the convergence speed. When p_2 takes a value other than 0, the convergence speed is significantly enhanced, while for the multimodal function (Fig. 8 (e)-(f)), when the value of p_2 is too high, the performance of the algorithm will be affected and the convergence speed will also decrease. Taking all factors into consideration, a better result can be obtained when $p_2 = 0.005$.

3.5. Qualitative results and analysis of OPA

Four qualitative metrics were used to illustrate the exploitation and exploration of OPA and the results are demonstrated in Fig. 9 and Fig. 10. These well-known metrics include search history, trajectory, average fitness and convergence curves. This experiment reduced *N* and *Max_iter* to 30 and 100 respectively in order to for a clearer convergence process of the reaction particles.

(1) Search history. The search history images in Fig. 9 include all the historical positions of the 30 particles over the course of 100 iterations. Analysis of these images can help to further understand how OPA searches for the optimal solution. It can be seen that the particles in F1-F3, F9 and F10 have obvious linear motion trajectories, while the motion in F12 and F14 is relatively messy. This phenomenon shows that the particles have two motion modes, one is linear approximation, which can converge to the optimal solution faster, especially for unimodal functions; and the other is multi-directional movement, which can expand the algorithm search interval. For multimodal functions, it helps to prevent the algorithm from falling into local optima. These two search methods are interlinked so that OPA can better balance the exploration and exploitation stages.

(2) Trajectory. Trajectory represents the position of the first dimension of the first particle in each iteration. As shown in the third column of Fig. 9, the particle fluctuates during the initial iteration and then tends to be stable. However, in the process of stabilization, there may be a large mutation, possibly because the particle will have a new distribution of positions under a certain probability.

(3) Average fitness. Average fitness represents the variation of the average value of all particles in the solution process. As shown in

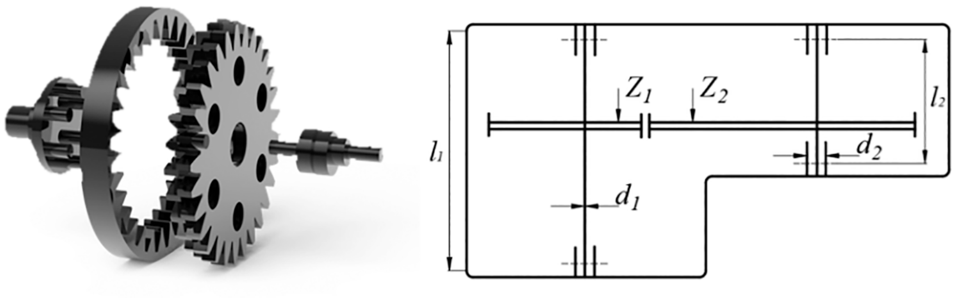

Fig. 13. Structure of speed reducer design problem.

Table 14
Statistical results of several algorithms for the speed reducer design problem.

Algorithm	Optimal	Worst	Mean	Std	NFEs
SC (Ray, 2003)	2994.744241	3009.964736	3001.758264	40	54,456
PSO-DE(Liu, Cai, & Wang, 2010)	2996.348167	2996.348204	2996.348174	0.0000064	54,350
DELC (Wang & Li, 2010)	2994.471066	2994.471066	2994.471066	1.9E-12	30,000
DEDS (Zhang, Luo, & Wang, 2008)	2994.471066	2994.471066	2994.471066	3.6E-12	30,000
HEAA (Wang, Cai, Zhou, & Fan, 2009)	2994.499107	2994.752311	2994.613368	0.07	40,000
FFA (Baykasoğlu & Ozsoydan, 2015)	2996.37	2996.669	2996.51	NA	50,000
MBA (Sadollah et al., 2018)	2997.4824	2999.65	2996.769	NA	6300
CSA (Gandomi, Yang, Alavi, & Talatahari, 2013)	3000.98	3009	3007.1997	NA	**5000**
PVS (Savsani and Savsani, 2016)	2996.348165	2996.348165	2996.348165	NA	54,350
WCA (Eskandar, Sadollah, Bahreininejad, & Hamdi, 2012)	2994.47166	2994.505578	2994.474392	0.0074	15,150
QS (Zhang, Xiao, Gao, & Pan, 2018)	2996.348165	2996.348165	2996.348165	NA	25,000
NNA (Sadollah et al., 2018)	2997.510246	3026.91991	3010.583596	6.6	10,500
TLBO (Rao et al., 2012)	2994.471074	2994.47179	2994.471141	0.00012	10,500
TLNNA (Zhang et al., 2020)	2994.471066	2994.474519	2994.471175	0.00054	10,500
OPA	**2994.47107**	**2994.471081**	**2994.471067**	**3.30E-06**	9000

Table 15
Optimal results of several algorithms for the speed reducer design problem.

Design variables	HHO	WOA	DE	SHO	GWO	PSO	MVO	SCA	GSA	OPA
$x_1(bs)$	3.5	3.500006335	3.506587775	3.50159	3.50669	3.500019	3.508502	3.508755	3.6	3.5
$x_2(ms)$	0.7	0.7	0.700165682	0.7	0.7	0.7	0.7	0.7	0.7	0.7
$x_3(ps)$	17	17	17.00880213	17	17	17	17	17	17	17
$x_4(l_{1s})$	7.3	7.3023721	7.483385723	7.3	7.380933	8.3	7.392843	7.8	7.8	7.3
$x_5(l_{2s})$	7.716066857	7.770700912	8.020559874	3.35127	3.357847	3.352412	3.358073	3.46102	3.369658	7.715319922
$x_6(d_{1s})$	3.352126011	3.351564426	3.359865704	3.35127	3.357847	3.352412	3.358073	3.46102	3.369658	3.350214666
$x_7(d_{2s})$	5.286654717	5.286673242	5.2936743	5.28874	5.286768	5.286715	5.286777	5.289213	5.289224	5.286654465
g_1	−7.39E-02	−7.39E-02	−7.66E-02	−7.43E-02	−7.57E-02	−7.39E-02	−7.62E-02	−7.62E-02	−9.96E-02	−7.39E-02
g_2	−1.98E-01	−1.98E-01	−2.01E-01	−1.98E-01	−2.00E-01	−1.98E-01	−2.00E-01	−2.00E-01	−2.20E-01	−1.98E-01
g_3	−5.00E-01	−4.99E-01	−4.67E-01	−5.00E-01	−4.87E-01	−2.66E-01	−4.85E-01	−4.64E-01	−4.03E-01	−4.99E-01
g_4	−9.05E-01	−9.03E-01	−8.94E-01	−9.92E-01	−9.92E-01	−9.92E-01	−9.92E-01	−9.91E-01	−9.92E-01	−9.05E-01
g_5	1.92E-10	−1.22E-10	−3.91E-03	−1.78E-03	−6.64E-04	−6.34E-04	−6.69E-04	−2.04E-03	−2.05E-03	−1.36E-10
g_6	−7.03E-01	−7.03E-01	−7.02E-01	−7.03E-01	−7.03E-01	−7.03E-01	−7.03E-01	−7.03E-01	−7.03E-01	−7.02E-01
g_7	0.00E + 00	−1.81E-06	−1.64E-03	−4.54E-04	−1.91E-03	−5.43E-06	−2.42E-03	−2.50E-03	−2.78E-02	−6.21E-11
g_8	−5.83E-01	−5.83E-01	−5.83E-01	−5.83E-01	−5.83E-01	−5.83E-01	−5.82E-01	−5.82E-01	−5.71E-01	−5.83E-01
g_9	−5.09E-02	−5.14E-02	−7.26E-02	−5.11E-02	−6.02E-02	−1.65E-01	−6.16E-02	−9.08E-02	−1.08E-01	−5.13E-02
g_{10}	−9.68E-05	−7.12E-03	−3.71E-02	1.30E + 00	1.30E + 00	1.30E + 00	1.30E + 00	1.23E + 00	1.29E + 00	−1.34E-09
g_{11}	−1.71E-03	−1.20E-03	−8.29E-03	−9.44E-04	−6.67E-03	−1.83E-04	−6.85E-03	−9.22E-02	−1.64E-02	−1.83E-10
$f(X)$	2994.974984	2996.066192	3014.6077	2998.5507	3001.288	3005.763	3002.928	3030.563	3051.12	**2994.47106676876**

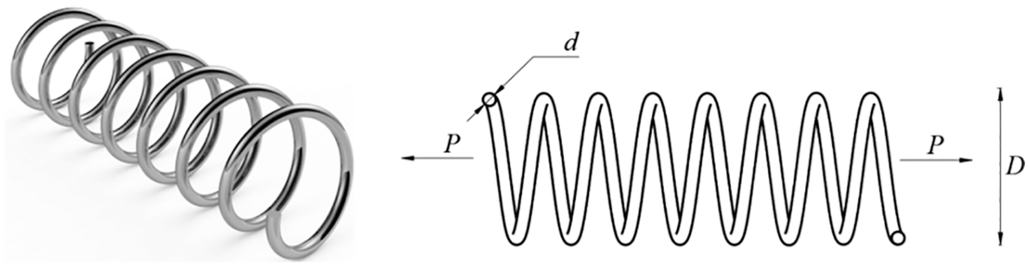

Fig. 14. Structure of tension/comparison spring design problem.

Table 16
Statistical results of several algorithms for the tension/comparison spring design problem.

Algorithm	Optimal	Worst	Mean	Std	Eval, No
SI (Ray & Saini, 2001)	0.013060	0.018992	0.015526	NA	20,000
GA(1) (Coello Coello, 2000)	0.012704	0.012822	0.012769	3.93E-05	N.A
CA (Coello Coello and Becerra, 2004)	0.012721	0.0151156	0.013568	8.40E-04	50,000
GA(2) (Coello and Montes, 2001)	0.012681	0.012973	0.012742	9.50E-05	80,000
CPSO (He & Wang, 2007)	0.012674	0.012924	0.012730	5.19E-05	200,000
BFOA (Montes and Ocana, 2008)	0.012671	NA	0.012759	1.36E-04	48,000
CDE (He & Wang, 2007)	0.012670	0.012790	0.012703	2.07E-05	240,000
SCA (Ray, 2003)	0.012669	0.016717	0.012922	5.92E-04	25,167
HGA (Bernardino, Barbosa, & Lemonge, 2007)	0.012668	0.016155	0.013481	NA	36,000
OPA	0.0126652577922432	0.0126852365100633	0.0126687340612818	4.73842931876093E-06	**18,000**

Table 17
Optimal results of several algorithms for tension/comparison spring design problem.

Design variables	SI	GA(1)	CA	GA(2)	CPSO	BFOA	CDE	SCA	HGA	OPA
$x_1(d)$	0.050417	0.05148	0.05	0.051989	0.051728	0.051825	0.051609	0.05216	0.051302	0.0517229
$x_2(D)$	0.321532	0.351661	0.317395	0.363965	0.367644	0.359935	0.354714	0.368159	0.347475	0.357533
$x_3(P)$	13.97991	11.632201	14.03179	10.890522	11.244543	11.107103	11.4108321	10.648442	11.852177	11.241324
g_1	−1.93E-03	−3.34E-03	9.28E-07	−1.26E-03	−8.71E-02	−1.89E-04	−3.87E-05	−1.91E-05	4.77E-06	−1.79E-07
g_2	−1.29E-02	−1.10E-04	−7.54E-05	−2.54E-05	2.25E-02	−1.39E-04	−1.83E-04	1.24E-05	−5.22E-06	−8.88E-08
g_3	−3.90E + 00	−4.03E + 00	−3.97E + 00	−4.06E + 00	−3.78E + 00	−4.06E + 00	−4.05E + 00	−4.08E + 00	−4.04E + 00	−4.06E + 00
g_4	−7.52E-01	−7.31E-01	−7.55E-01	−7.23E-01	−7.20E-01	−7.25E-01	−7.29E-01	−7.20E-01	−7.34E-01	−7.27E-01
$f(X)$	0.01306	0.012704	0.012721	0.012681	0.012674	0.012671	0.01267	0.012669	0.012668	**0.012665**

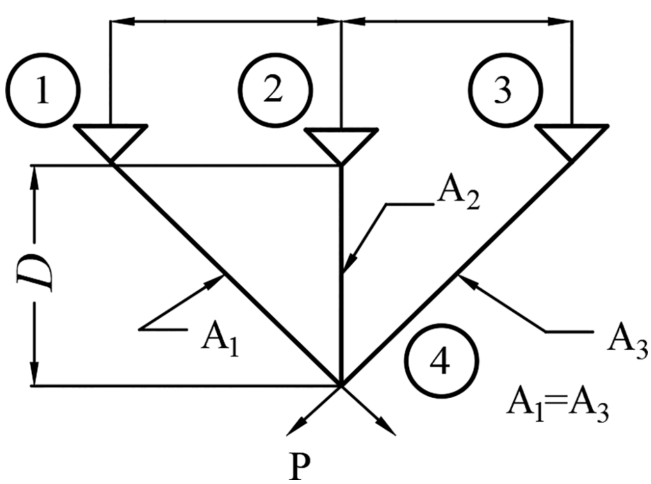

Fig. 15. Structure of three-bar truss design problem.

the fourth column of Fig. 9, the algorithm can quickly converge to the optimal solution, but random mutations occur in the process of convergence, because the new position distribution of particles during the attacking phase has a great impact on the average value of the overall fitness function.

(4) Convergence curves. The variation trend of the convergence curve in Fig. 10 shows that for the unimodal function F1-F4, the convergence speed of OPA at the beginning is significantly higher than that of other algorithms, and global optimal solutions can be achieved after about 20 iterations. The convergence speed of HHO is relatively fast at the beginning, but decreases significantly in the later stage, and there is a tendency of falling into local optima, and the optimal solution cannot be found smoothly. The convergence speed of other algorithms is too slow, and the fitness function value shows no significant change in 100 iterations. For all F5, F6, F7, F10, F12 and F13, OPA can quickly converge to satisfactory results, but other algorithms show different degrees of premature phenomenon, or due to their slow convergence speed, they cannot converge to the optimal solution within the specified number of iterations. In conclusion, OPA not only has significant competitiveness in terms of optimization accuracy, but also has a substantially higher convergence speed.

4. Engineering optimization test problems

In this section, the proposed OPA is applied to five well-known engineering design problems, and some other algorithms are selected for comparison with OPA to further validate its performance. In the process of solving these problems, the static penalty is used as the constraint processing strategy. If the constraint condition is violated, a sufficiently large penalty term is used in the fitness function to punish the violation

Table 18
Statistical results of several algorithms for the three-bar truss design problem.

Algorithm	Optimal	Worst	Mean	Std	NFEs
DEDS (Zhang, 2008)	**263.895843**	263.895849	**263.895843**	0.00000097	15,000
HEAA	**263.895843**	263.896099	263.895865	0.000049	15,000
PSO-DE (Liu et al., 2010)	**263.895843**	**263.895843**	**263.895843**	4.5E-10	17,600
BAT (Yang and Gandomi, 2012)	263.896248	263.9024677	263.90614	0.00353	15,000
CS (Yang & Deb, 2009)	263.97156	NA	264.0669	0.00009	15,000
BSA (Meng, Gao, Lu, Liu, & Zhang, 2016)	**263.895843**	263.895845	**263.895843**	0.000000264	13,720
CSA	**263.895843**	**263.895843**	**263.895843**	**1.01E-10**	25,000
OPA	**263.8958433775103**	263.895843971370	**263.895843465050**	1.19961148327635E-07	**13,500**

of constraints. For all the selected problems, the population size is set as 30 for OPA; the maximum number of iterations is selected according to the specific problem; and OPA run 30 times for each problem.

4.1. Welded beam design problem

The design problem is a common benchmark case targeted at minimizing the manufacturing expense of the welded beam. The structure of the problem is presented in Fig. 11, which illustrates the design of the welded beam and its corresponding parameters. The constraints of the design problem are as follows: bending stress (θ) in the beam; end deflection (δ) of the beam; shear stress (τ); and buckling load (P_c) on the bar. Besides, there are four variables including thickness of weld $h(x_1)$, length of the clamped bar $l(x_2)$, height of the bar $t(x_3)$, and thickness of the bar $b(x_4)$. The formulas are listed below:

Objective:

$$f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \tag{18}$$

Subject to:

$g_1(x) = \tau(x) - \tau_{max} \leqslant 0,\ g_2(x) = \sigma(x) - \sigma_{max} \leqslant 0,\ g_3(x) = x_1 - x_4 \leqslant 0$
$g_5(x) = 0.125 - x_1 \leqslant 0,\ g_6(x) = \delta(x) - \delta_{max} \leqslant 0,\ g_7(x) = P - P_c(x) \leqslant 0$

Variable ranges:

$$0.1 \leqslant x_1 \leqslant 2, 0.1 \leqslant x_2 \leqslant 10, \quad 0.1 \leqslant x_3 \leqslant 10, 0.1 \leqslant x_4 \leqslant 2$$

$$\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x2}{2R} + (\tau'')^2},\ \tau' = \frac{P}{\sqrt{2}x1x2},\ \tau'' = \frac{MR}{J}$$

$$M = P\left(L + \frac{x2}{2}\right),\ R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x1+x3}{2}\right)^2},\ J = 2\left\{\sqrt{2}x1x2\left[\frac{x_2^2}{4} + \left(\frac{x1+x3}{2}\right)^2\right]\right\}$$

$$\tau\text{max} = 13600\ psi, \quad \sigma\text{max} = 30000 psi$$

OPA provides the optimal fitness value of $f(x) = 1.72485239014171$ at $x^* = (0.205729616357273, 3.47048930461339, 9.03662412539957, 0.205729639034324)$, and the mean time taken by the algorithm is 0.4476 s. OPA achieves the optimal results in terms of the optimal solution, average value, worst solution and standard deviation as shown in Table 10–11, indicating that it outperforms other algorithms.

4.2. Pressure vessel design problem

This problem is another well-known benchmark case in which the objective is to minimize the fabrication cost. The schematic view for this design problem is shown in Fig. 12. Four design variables are considered, including thickness of the shell $Ts(x_1)$, thickness of the head $Th(x_2)$, inner radius $R(x_3)$, and length of the cylindrical section of the vessel L (x_4), and both thickness variables (T_s, T_h) must be integer multiple values of 0.0625 in. (Brajević & Ignjatović, 2019). The true globally minimal solution of the problem is $f$min = 6059.714335048436 at x*= (0.8125, 0.4375, 42.0984455958549, 176.6365958424394) (Yang, Huyck, Karamanoglu, & Khan, 2013). The mathematical formulation is given as follows:

Objective:

$$f(x) = 0.6224x1x3x4 + 1.7781x2x_3^2 + 3.1661x4x_1^2 + 19.84x3x_1^2 \tag{19}$$

Subject to:

$$g1(x) = -x1 + 0.0193x3 \leqslant 0 g2(x) = -x2 + 0.00954x3 \leqslant 0$$

$$g3(x) = -\pi x4x_3^2 - \frac{4}{3}\pi x_3^3 + 1296000 \leqslant 0 g4(x) = x_4 - 200 \leqslant 0$$

Variable ranges:

$$0 \leqslant x1 \leqslant 100,\ 0 \leqslant x2 \leqslant 100,\ 10 \leqslant x3 \leqslant 200,\ 10 \leqslant x4 \leqslant 200$$

Table 12–13 present the statistical results. Since this situation is also treated as a continuous problem by some researchers, the results of OPA are presented in both mixed variable and continuous form, but only the mixed variable is selected in the comparison algorithm.

OPA provides the optimal fitness value of $f(x)$ = 6059.714335052372 at x^*= (0.8125, 0.4375, 42.0984455958308, 176.636595842781) with a mean time of 1.4451 s, which is a very competitive result relative to other algorithms.

4.3. Speed reducer design problem

To minimize the weight of speed reducer, several constraints are considered in the problem, including surface stress, bending stress of the gear teeth, stress in the shafts, and transverse deflections of the shafts (Fig. 13). Seven variables are involved in this problem, including face width $b(x_1)$, module of teeth $m(x_2)$, number of teeth in the pinion $p(x_3)$, length of the first shaft between bearings $l_1(x_4)$, length of the second shaft between bearings $l_2(x_5)$, diameter of first shaft $d_1(x_6)$, and diameter of second shaft $d_2(x_7)$. The formulas of objective and eleven constraints are listed as follows:

Objective:

$$f(x) = 0.7854x_1x_2^2(3.3333x_3^2 + 14.9334x_3 - 43.0934) - 1.508x_1(x_6^2 + x_7^2) + 7.4777(x_6^3 + x_7^3) + 0.7854(x4x_6^2 + x5x_7^2) \tag{20}$$

Subject to:

$$g1(x) = \frac{27}{x1x_2^2x_3} - 1 \leqslant 0,$$

$$g3(x) = \frac{1.93x_4^3}{x2x_6^4x3} - 1 \leqslant 0,\ g4(x) = \frac{1.93x_5^3}{x2x_7^4x3} - 1 \leqslant 0$$

$$g5(x) = \frac{\left(\left(\frac{745x5}{x2x3}\right)^2 + 157.5\times10^6\right)^{1/2}}{85x_7^3} - 1 \leqslant 0$$

$$g6(x) = \frac{x2x3}{40} - 1 \leqslant 0,\ g7(x) = \frac{5x2}{x1} - 1 \leqslant 0,\ g_8(x) = \frac{x_1}{12x_2} - 1 \leqslant 0$$

$$g9(x) = \frac{1.5x_6 + 1.9}{x_4} - 1 \leqslant 0,\ g10(x) = \frac{1.1x7 + 1.9}{x5} - 1 \leqslant 0$$

$$g11(x) = \frac{\left(\left(\frac{745x4}{x2x3}\right)^2 + 16.9\times10^6\right)^{1/2}}{110x_6^3} - 1 \leqslant 0$$

Variable range:

Table 19
Optimal results of several algorithms for the three-bar truss design problem.

Design variables	HHO	DEDS	MVO	GOA	MFO	PSO-DE	SSA	MBA	CS	OPA
x_1	0.788662816	0.78867513	0.78860276	0.788897556	0.788244771	0.7886751	0.788665414	0.788565	0.79967	0.7886756
x_2	0.40828313	0.40824828	0.40845307	0.40761957	0.409466906	0.4082482	0.408275784	0.4085597	0.40902	0.408247
g_1	−1.31E-10	1.78E-08	−2.63E-08	−1.44E-08	−3.09E-10	1.43E-07	8.73E-10	1.42E-07	−2.39E-02	−6.67E-12
g_2	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.46E + 00	−1.48E + 00	−1.46E + 00
g_3	−5.36E-01	−5.36E-01	−5.36E-01	−5.35E-01	−5.37E-01	−5.36E-01	−5.36E-01	−5.36E-01	−5.49E-01	−5.36E-01
f(X)	263.8958434	263.8958434	263.8958499	263.8958815	263.8959797	263.8958433	263.8958434	263.8958522	263.9716	**263.8958434**

$2.6\leqslant x1\leqslant 3.6,\ 0.7\leqslant x2\leqslant 0.8,\ 17\leqslant x3\leqslant 28,\ 7.3\leqslant x4\leqslant 8.3,\ 7.3\leqslant x5\leqslant 8.3,\ 2.9\leqslant x6\leqslant 3.9,\ 5.0\leqslant x7\leqslant 5.5$

OPA provides the optimal fitness value of $f(x) = 2994.47106676876$ at $x^* = (3.50000000021722, 0.700000000000000, 17.0000000000816, 7.30000000895849, 7.71531992206150, 3.35021466631793, 5.28665446522391)$. The mean time is 0.5853 s. Table 14–15 show the statistical results. It can be seen that OPA outperforms other algorithms in lower values for mean, optimal, worst solution and standard deviation.

4.4. Tension/compression spring design problem

The tension/compression spring problem is another challenging engineering test problem (see in Fig. 14) with the main objective to minimize the tension/compression spring weight. There are three optimization constraints: shear stress; minimum deflection and surge frequency, and three design variables including wire diameter $d(x_1)$, mean coil diameter $D(x_2)$, and number of active coils $P(x_3)$. The formulas are listed below:

Objective:

$$f(x) = (x_3+2)x_2x_1^2 \tag{21}$$

Subject to:

$$g_1(x) = 1-\frac{x_2^3x_3}{71785x_1^4}\leqslant 0 g_2(x) = \frac{4x_2^2-x_1x_2}{12566(x_2x_1^3-x_1^4)}+\frac{1}{5108x_1^2}-1\leqslant 0$$

$$g_3(x) = 1-\frac{140.45x_1}{x_2^2x_3}\leqslant 0 g_4(x) = \frac{x_1+x_2}{1.5}-1\leqslant 0$$

Variable range:

$0.05\leqslant x1\leqslant 2.0,\ 0.25\leqslant x2\leqslant 1.3,\ 2.0\leqslant x3\leqslant 15.0$

OPA provides the optimal fitness value of $f(x) = 0.0126652577922432$ at $x^* = (0.0517229340052407, 0.357533140786723, 11.2413243865358)$, and the mean time taken by the algorithm is 0.3876 s. Table 16–17 present the experimental results of the selected algorithms, which show that OPA still produces the best results in all four evaluation indicators, demonstrating its higher competitiveness over other algorithms.

4.5. Three-bar truss design problem

A schematic diagram of the three-bar truss problem is shown in Fig. 15, which aims to minimize the volume while satisfying the stress constraints. The cross-section areas of member 1 and member 2 are used as the two design variables (x_1, x_2) for this problem (Brajevic & Tuba, 2013). The formulas of the problem are given below:

Objective:

$$f(x) = \left(2\sqrt{2}x_1+x_2\right)\times 1 \tag{22}$$

subject to:

$$g_1(x) == \frac{\sqrt{2}x_1+x_2}{\sqrt{2}x_1^2+2x_1x_2}p-\sigma\leqslant 0 g_2(x) = \frac{x_2}{\sqrt{2}x_1^2+2x_1x_2}p-\sigma\leqslant 0,$$

$$g_3(x) == \frac{1}{\sqrt{2}x_2+x_1}p-\sigma\leqslant 0$$

variable range:

$0\leqslant x_1, x_2\ \leqslant 1,$

where:

$1 = 100cm, \quad p = 2KN/cm^2, \quad \sigma = 2KN/cm^2$

OPA provides the optimal fitness value of $f(x)$ = 263.8958433775103 at x^*= (0.788675603652359, 0.408246963779195) with a mean time of 0.3084 s. Based on the results in Table 18–19, OPA significantly outperforms other compared algorithms, further demonstrating its capability of dealing with a constrained optimization problem.

5. Results and discussion

This paper selects a total of 67 functions, including unimodal functions, multimodal functions, fixed-dimensional multimodal functions, CEC 15 and CEC 17, and five engineering optimization problems for testing, and compares OPA with HHO, PSO, GWO, MFO, SOA, SSA, WOA, GSA, ABC and DE. Numerical analysis, statistical analysis, sensitivity analysis and qualitative analysis were performed on the test results. The experimental conclusions are summarized as follows.

In the numerical analysis, the performances of the 11 algorithms on the 67 functions are listed in Table 2–6, including the average value, optimal value, standard deviation and rank. Compared with other algorithms, OPA can attain the best results for most functions. Multiple global optimal solutions can be found on unimodal functions and precocity can be successfully avoided for multimodal functions, which supports the superior exploratory and exploitative strengths of OPA.

In the statistical analysis, the Wilcoxon's rank-sum test is used to further compare the algorithm performance in pairs. Table 7–9 show that OPA and other comparison algorithms have significant differences in most functions.

In the sensitivity analysis, the main parameters of the algorithm, including search agents (N), maximum number of iterations (Max_iter) and parameters p_1 and p_2, are selected for discussion. The experimental results are shown in Figs. 5–8. The size of search agents (N) and the maximum number of iterations (Max_iter) are usually determined according to the complexity of the problem. The more complex the problem is, the larger value will be selected, and vice versa. The size of p_1 will affect the weight of the exploration and exploitation stage. The higher the p_1 value is, the stronger the exploration capability will be, and a lower p_1 value indicates a stronger exploitation capability. p_2 has certain influence on the convergence speed. The experimental results show that the best result can be obtained when p_2 is set to 0.005.

In the qualitative results and analysis section, the qualitative metrics of search history, trajectory, average fitness and convergence curves are selected to illustrate the exploitation and exploration of OPA, and the results are demonstrated in Fig. 9 and Fig. 10. In particular, the iteration curve graph shows that OPA has obvious advantages in convergence speed and optimization accuracy.

In the section of engineering optimization problems, the results for five well-known constrained engineering optimization problems in Table 10–19 reveal that OPA is one of the top optimizers compared with several other selected methods. The results further prove that OPA has an effective mechanism to solve realistic constrained optimization problems.

The excellent performance of OPA algorithm in solving a given optimization problem can be ascribed to its three unique search mechanisms. The first mechanism is controlled by the parameter p_1. By adjusting the parameter p_1, the weight of the driving and encircling phases can be changed, thereby adjusting the exploration and exploitation capabilities of the algorithm. The second mechanism is about the predation phase. In this phase, the location is updated according to four better orcas and three randomly selected orcas. The number of orcas is selected based on the consideration that a too small number of choices will lead to the loss of the diversity of particles, while too many choices will reduce the convergence speed of the algorithm. The third mechanism is position adjustment. In order to help the orca to jump out of the local optima more effectively, the particles are redistributed with a certain probability through the parameter p_2. In addition, after multiple iterations, a large number of particles will gather near the optimal solution and only a few particles will be at the edge of the feasible region. Therefore, the lowest boundary of the feasible region is taken as the new position of the particles, so as to ensure that the particles always converge from the edge of the feasible region to the interior.

The above search mechanisms ensure that the algorithm can balance the exploration and exploitation stages, and thereby can effectively solve a variety of optimization problems with a strong potential to be applied in more fields.

6. Conclusion and future directions

Inspired by the hunting process of orcas, a new metaheuristic algorithm named as OPA is proposed in this paper. This algorithm consists of the chasing phase and attacking phase. The chasing phase includes two parts: driving of prey and encircling of prey. By changing the proportion of these two parts, the algorithm can vary its emphasis on the exploration and exploitation stage for different problems. In the attacking phase, the optimal solution is approached after considering the position of several superior orcas and some randomly selected ones. At total of 67 unconstrained benchmark problems and five engineering optimization problems were used to evaluate the performance of OPA. Comparison and analysis of the statistical results reveal that OPA has higher competitiveness over other algorithms in terms of better and more stable solutions.

In future research, OPA has some areas for further improvement and expansion. First of all, since OPA is sensitive to parameters, it can be developed to automatically adjust its parameters according to different nature of various problems. Secondly, different versions of OPA, such as binary and multi-target, may be developed. Thirdly, OPA can be employed to solve real-life engineering problems and extended to issues such as feature selection and combinatorial optimization.

CRediT authorship contribution statement

Yuxin Jiang: Methodology, Software, Writing – original draft. **Qing Wu:** Conceptualization, Supervision, Writing – review & editing. **Shenke Zhu:** Software, Software. **Luke Zhang:** Visualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Table A1
Unimodal benchmark functions.

Name	Function	D	Range	$f_{\min}$
Sphere	$F1(x) = \sum_{i=1}^{n} x_i^2$	30	[−100,100]	0
Schwefel 2.22	$F2(x) = \sum_{i=1}^{n} \lvert xi\rvert + \prod_{i=1}^{n} \lvert xi\rvert$	30	[−10,10]	0
Schwefel 1.2	$F3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} xj\right)^2$	30	[−100,100]	0
Schwefel 2.21	$F4(x) = max\{xi\rvert,\ 1\leqslant i\leqslant n\backslash\}$	30	[−100,100]	0
Rosenbrock	$F5(x) = \sum_{i=1}^{n-1} [100(xi+1-x_i^2)^2 + (xi-1)^2]$	30	[−30,30]	0
Step	$F_6(x) = \sum_{i=1}^{n} ([x_i+0.5])^2$	30	[−100,100]	0
Quartic	$F_7(x) = \sum_{i=1}^{n} ix_i^4 + random[0,1]$	30	[−1.28,1.28]	0

Table A2
Multimodal benchmark functions.

Name	Function	D	Range	$f_{\min}$
Generalized Schwefel 2.26	$F8(x) = \sum_{i=1}^{n} -x_i sin(\sqrt{\lvert x_i\rvert})$	30	[−100,100]	−418.9829 × 5
Generalized Rastrigin	$F9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10cos(2\pi x_i) + 10\right]$	30	[−5.12,5.12]	0
Ackley	$F10(x) = -20exp\left(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{30} x_i^2}\right) - exp\left(\frac{1}{30}\sum_{i=1}^{30} cos(2\pi x_i)\right) +20+e$	30	[−32,32]	0
Generalized Griewank	$F11(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	30	[−600,600]	0
Generalized penalized function No.01	$F12(x) = \frac{\pi}{n}\times\{10sin^2(\pi y_1) + \sum_{i=1}^{n-1}(y_i-1)^2[1+10sin^2(\pi y_{i+1})] +(y_n-1)^2\backslash\} + \sum_{i=1}^{n} u(x_i,a,k,m)$ $y_i = 1+\frac{1}{4}(x_i+1),$ $u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m & if\ \ x_i > a \\ 0 & if\ \ -a\leqslant x_i\leqslant a \\ k(-x_i-a)^m & if\ \ x_i < -a \end{cases}$	30	[−50,50]	0
Generalized penalized function No.02	$F13(x) = 0.1\times\{sin^2(3\pi x_1 + \sum_{i=1}^{n-1}(x_i-1)^2[1+sin^2(3\pi x_{i+1})] +(x_n-1)^2[1+sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i,a,k,m)$ $u(x_i,a,k,m) = \begin{cases} k(x_i-a)^m & if\ \ x_i > a \\ 0 & if\ \ -a\leqslant x_i\leqslant a \\ k(-x_i-a)^m & if\ \ x_i < -a \end{cases}$	30	[−50,50]	0

Table A3
Fixed-dimension multimodal benchmark functions.

Name	Function	D	Range	$f_{\min}$
Shekel's foxholes	$F14(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j+\sum_{i=1}^{2}(x_i-aij)^6}\right)^{-1}$	2	[−65.536,65.536]	0
Kowalik	$F15(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_i\left(b_i^2+b_i x_2\right)}{b_i^2+b_i x_3+x_4}\right]^2$	4	[−5,5]	0.0003075
Six-Hump Camel-Back	$F16(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	[−5,5]	−1.0316285
Branin	$F17(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1-\frac{1}{8\pi}\right)cosx_1 + 10$	2	[−5,5]	0.398
Goldstein-Price	$F18(x) = \left[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)\right] \times\left[30+(2z_1-3z_2)^2\times(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)\right]$	2	[−2,2]	3
Hartman's family	$F19(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j-p_{ij}\right)^2\right)$	3	[0,1]	−3.86
	$F20(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j-p_{ij}\right)^2\right)$	6	[0,1]	−3.86
Shekel's foxholes	$F21(x) = -\sum_{i=1}^{5}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.1532
	$F22(x) = -\sum_{i=1}^{7}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.4028
	$F23(x) = -\sum_{i=1}^{10}\left[(X-a_i)(X-a_i)^T+c_i\right]^{-1}$	4	[0,10]	−10.536

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