Schottky diode: Comments concerning the diode parameters determination from the forward *I-V* plot

J.-C. Manifacier, N. Brortryb, R. Ardebili, and J.-P. Charles
Centre d'Electronique de Montpellier, Université des Sciences et Techniques du Languedoc, 34060 Montpellier Cedex, France

(Received 7 December 1987; accepted for publication 26 April 1988)

Some methods, which have recently been proposed, allow the determination of the diode parameters even in the presence of a high series resistance. A similar but much simpler treatment is presented here. Using the *I-V* plot, a straightforward determination of the series resistance R and the ideality factor n is obtained. These methods should be used cautiously anyway as in certain circumstances they can lead to erroneous results.

I. INTRODUCTION

The voltage-current relationship of a Schottky diode is usually represented by the following equation:

$$I = I_s(\exp\{\beta[(V-RI)/n]\}-1), \qquad (1)$$

with

$$I_s = AA^*T^2\exp(-\beta\psi). \qquad (2)$$

R is the series resistance, $\beta = q/kT$, A the area of the diode, A^* the modified Richardson constant, n is the ideality factor, ψ the barrier height, and I_s is the saturation current.

To determine the four diode parameters ψ, n, A^*, and R the normal procedure is to use two graphical plots. The linear portion of the ln (I) vs (V) plot allows the determination of n, and from the departure of linearity at high current, the value of R can be determined. Extrapolation of the current at $V=0$ gives I_s. A plot at different temperatures of ln (I_s/T^2) vs $(1/T)$ gives then the parameters ψ and A^*.

In practice the linear portion of the $\ln(I)$ vs(V) plot is restricted to voltage values such that $\beta V/n \gg 1$ and current values such that $RI \ll V$. If the series resistance is large the linear portion of the plot can disappear and the above graphical method becomes useless. In that case, it is necessary to rely on more elaborate methods such as a numerical curve-fitting procedure. In the model corresponding to Eq. (1), the diode parameters are usually assumed to be constant. Complications may arise when these parameters are dependent on temperature or current.

Recently some methods were proposed[1-6] to circumvent the presence of a high series resistance, and to allow the determination of the diode parameters even when a straight portion of the $\ln(I)$ vs (V) was missing. In the original method, Norde[1] introduced an auxiliary function F:

$$F(V) = \frac{V}{2} - \frac{1}{\beta}\ln\left(\frac{I}{AA^*T^2}\right). \qquad (3)$$

He considered a simplified case where $n=1$ and A^* was known. The plot of this function shows a minimum and from the position of this minimum R and ψ can be determined. The original method was extended by other authors[3-6] to cases where the ideality factor was greater than one and unknown.

We present here an alternative and simpler approach, using a single *I-V* plot, to determine the values of n, R, I_s, and then ψ, if A^* is known. If A^* is not known then two *I-V* plots at two temperatures are necessary to determine both A^* and ψ. We will show that this method leads to a graphical procedure for the evaluation of n and R. Both these parameters can be temperature dependent, but must be current independent.

II. THE METHOD

We start with the simple function $F(I)$:

$$F(I) = V - R_0 I, \qquad (4)$$

where R_0 is an adjustable parameter. Looking at Fig. 1 it is clearly seen that this function will present a maximum for a certain value I_M of the diode current I as long as the R_0 value is chosen such as to have an intersection point with the diode *I-V* plot. This can be done by simple inspection. This intersection point A, see Fig. 1, is obtained for a R_0 value such that

$$R_{\mathrm{dif}(I=0)} > R_0 > R, \qquad (5)$$

where R is the diode series resistance, see Eq. (1), and $R_{\mathrm{dif}(I=0)}$ is the diode differential resistance in the limit of zero current:

$$R_{\mathrm{dif}(I=0)} = R + n/\beta I_s.$$

For voltage such that $\beta(V-RI)/n \geqslant 3$, we have

$$\ln(I/I_s) \simeq \beta(V-RI)/n, \qquad (6)$$

and

$$F(I) \simeq (n/\beta)\ln(I/I_s) + (R-R_0)I. \qquad (7)$$

The restriction for the analysis to voltage above the ohmic part of the *I-V* characteristic is not, in practice, a problem as at low voltage there are usually excess or "leakage" currents which are not part of the high-current model corresponding to Eq. (1). The maximum F value is obtained for

$$\frac{dF}{dI} = \frac{n}{\beta}\frac{1}{I} + (R-R_0) = 0,$$

leading to

$$I_M = (n/\beta)[1/(R_0-R)]. \qquad (8)$$

Using two different values for R_0 which satisfy Eq. (5), the values of n and R are easily obtained:

$$R = (R_{02}I_{M2} - R_{01}I_{M1})/(I_{M2}-I_{M1}),$$
$$n = \beta I_{M1}I_{M2}[(R_{01}-R_{02})/(I_{M2}-I_{M1})]. \qquad (9)$$

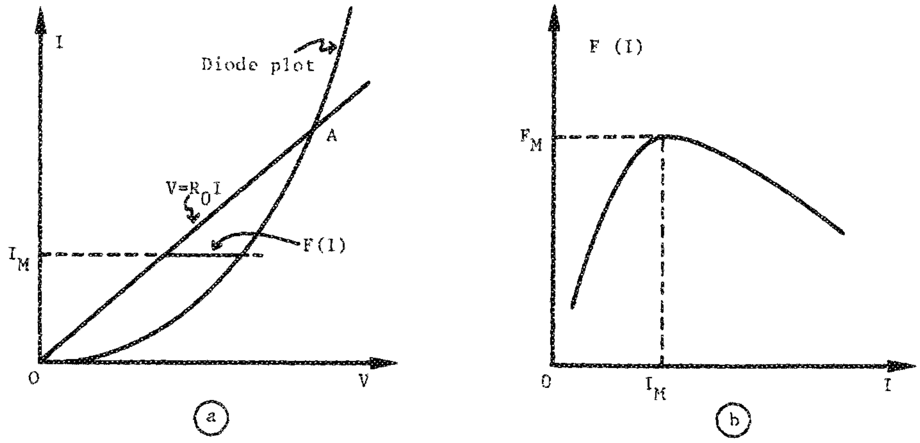

FIG. 1. Illustration of the method. (a) Determination of $F(I)$ from the intersect of the I-V characteristic by the line $V = R_0 I$. (b) Plot of the $F(I)$ function.

Once the n and R values are determined, the barrier height ψ is easily computed if A^* is known. Equation (7) gives

$$F_M = \frac{n}{\beta}\ln\left(\frac{I_M}{AA^*T^2}\right) + n\psi - \frac{n}{\beta}.$$

Hence,

$$\psi = \frac{F_M}{n} + \frac{1}{\beta}\left[1 - \ln\left(\frac{I_M}{AA^*T^2}\right)\right]. \qquad (10)$$

It is to be noted that the plot of the $F(I)$ function does not involve any logarithm calculation and, in that sense, is simpler than the Norde method.[1]

From Eq. (4), the maximum for F corresponds to

$$\left(\frac{dV}{dI}\right)_{I_M} = R_0. \qquad (11)$$

This remark leads with the use of Eqs. (9) to a very simple graphical determination of n and R, without recourse to a plot of the function $F(I)$. Looking at Fig. 2, for two points M_1 and M_2 of the I-V characteristic, one determines the slopes. This in turn gives R_{01} and R_{02} and then R and n through Eqs. (9). This graphical method involves uncertainties both in the measurements of I_M and R_0 contrary to the method based on the determination of the maximum of $F(I)$ for which R_{01} and R_{02} have known values. One can note too that the value I_M of the current which corresponds to the extremum of the function F can be obtained more elegantly by the use of the Lagrangian multipliers method.[7] F is the function to be maximized and the constraint corresponds to the I-V relation: see Eq. (6).

III. DISCUSSION

When, for a given temperature, the parameters n, R, and I_s are independent of the current, any of the above methods can be used to determine the diode parameters. These methods lead to false results anyway if any one of these parameters are a function of the current crossing the diode. For example, R is frequently a decreasing function of the current. This is due in particular to minority-carrier injection in the bulk of the semiconductor[8,9] (conductivity modulation). To give an example, for a single-carrier space-charge current, the bulk I-V characteristic has a V^2 dependence,[10,11] $I \sim V^2$ leading to $R \sim I^{-1/2}$. In that case a plot of F as a function of I or V will still show a maximum, but this maximum will lead to irrelevant results for n and R; in such a case only a curve-fitting method, once the proper current dependence is known for R, allows the determination of the diode parameters.

The simple graphical method [see Fig. 2 and Eqs. (9)]

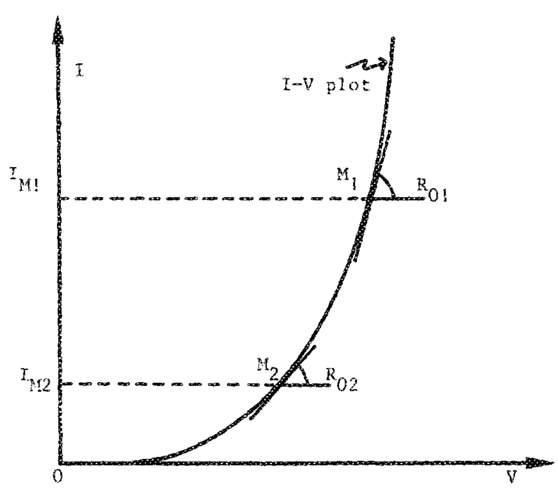

FIG. 2. Graphical determination of the ideality factor n and the series resistance R.

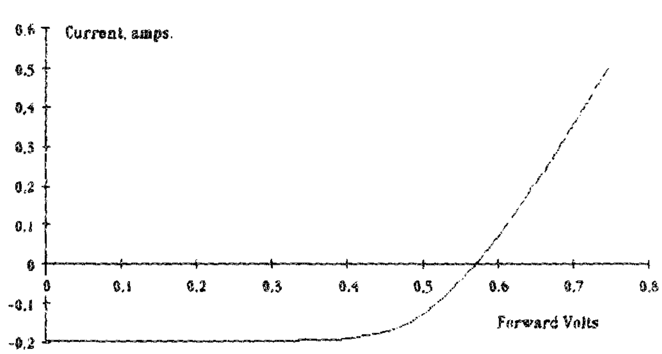

FIG. 3. Experimental $I(V)$ characteristic of an ITO-silicon cell under AM0 illumination at 303.9 K. ($I_{SC} = 0.197$ A).

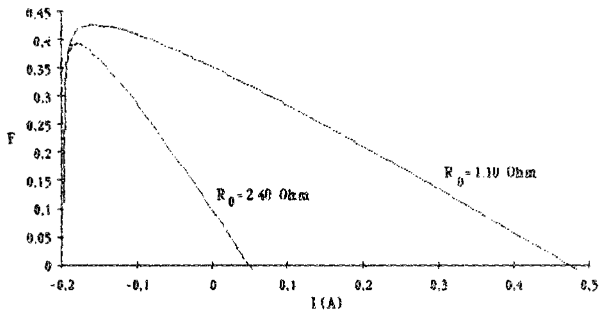

FIG. 4. $F(I)$ curves calculated for $R_{01} = 1.10\ \Omega$ ($I_{M1} = 0.045$ A) and $R_{02} = 2.40\ \Omega$ ($I_{M2} = 0.017$ A).

helps to check for the consistency of the simple Schottky diode model. By choosing several points on the I-V plot, one should obtain identical values for R up to the precision of the graphical measurements.

The preceding method was applied to Schottky contacts on germanium and to ITO-silicon solar cells from our laboratory. A typical $I(V)$ characteristic from an illuminated (AM0) cell is shown in Fig. 3. A set of $F(I)$ curves is shown in Fig. 4. The associated results are given in Table I, from Eq. (8) and from Eq. (11). This later determination has proved

TABLE I. Results from $I(V)$ measurements on an ITO-silicon solar cell, at 303.9 K, under AM0 illumination.

(1) Using Eqs. (8)–(9)
$R_{01} = 1.10\ \Omega$
$I_{M1} = 0.045$ A
$R_{02} = 2.40\ \Omega$
$I_{M2} = 0.017$ A
$R = 0.31\ \Omega$
$n = 1.36$
(2) Using Eqs. (9)–(11)
$I_{M1} = 0.671$ A
$R_{01} = 0.313\ \Omega$
$I_{M2} = 0.448$ A
$R_{02} = 0.339\ \Omega$
$R = 0.26\ \Omega$
$n = 1.35$

TABLE II. Comparison of the results of the graphical method according to the graphical choice of R_0 or I_M with numerical analysis. $I_M = I$(under illumination) + 0.197.

	R (Ω)	n
(1) Using Eqs. (8)–(9) $1.1 < R_0 < 1.9$ $0.017 < I_M < 0.045$	0.3 ± 0.3	1.4 ± 0.4
(2) Using Eqs. (9)–(11) $0 < I(\text{illum}) < 0.47$ $0.20 < I_M < 0.67$ $0.31 < R_0 < 0.44$	0.25 ± 0.05	1.4 ± 0.1
(3) Numerical analysis	0.26 ± 0.02	1.35 ± 0.05

to be more precise for it can be associated to operation in the high-current model condition of Eq. (1). The values obtained for the parameters n and R agree well with those obtained through a more elaborate curve-fitting technique.[12] The results are compared in Table II.

When A^* is not known, it is still possible to determine both ψ and A^*. It is then necessary to work at two different temperatures in order to separate the temperature dependence of $(\beta\psi)$ and (A^*T^2) in I_s; see Eq. (2). Expressions of ψ and A^* as a function of F_{M1}, F_{M2}, I_{M1}, and I_{M2} can be easily obtained.

[1] H. Norde, J. Appl. Phys. **50**, 5052 (1979).
[2] L. Stolt, K. Bohlin, P. A. Tove, and H. Norde, Solid-State Electron. **26**, 295 (1983).
[3] R. M. Cibils and R. H. Buitrago, J. Appl. Phys. **58**, 1075 (1985).
[4] K. Sato and Y. Yasumura, J. Appl. Phys. **58**, 3655 (1985).
[5] K. E. Bohlin, J. Appl. Phys. **60**, 1223 (1986).
[6] S. K. Cheung and N. W. Cheung, Appl. Phys. Lett. **49**, 85 (1986).
[7] G. Arfken, *Mathematical Methods for Physicists* (Academic, New York, 1985), p. 945.
[8] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (Wiley-Interscience, New York, 1981).
[9] H. K. Henisch, *Semiconductor Contacts* (Clarendon, Oxford, 1984).
[10] M. A. Lampert and P. Mark, *Current Injection in Solids* (Academic, New York, 1970).
[11] M. A. Lampert and R. B. Schilling, R. Baron, and J. W. Mayer, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic, New York, 1970), Vol. 6, Chaps. 1 and 4.
[12] J.-P. Charles, I. Mekkaoui-Alaoui, G. Bordure, and P. Mialhe, Solid-State Electron. **28**, 807 (1985).