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# Current-voltage curve analysis for non-linear silicon solar cells and operation conditions beyond low injection

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## Abstract

This paper addresses the limitation of the double-diode model to low injection conditions. Assuming a constant carrier lifetime, we re-formulate the current of the first diode for general injection. The new parameter  $Y$  is introduced to quantify the share of the diode saturation current  $J_{01}$  that is due to lightly doped regions that go into high injection at higher voltages. The remaining  $J_{01}(1-Y)$  accounts for the highly doped regions that remain in low injection. When  $Y$  equals 0, the extended model is identical to the conventional model. A least square fit of the  $I_{sc}$ - $V_{oc}$  characteristics to the general-injection model makes the determination of  $J_{01}$  more reliable and indicates whether the recombination is dominated by the highly doped regions. We also propose to use a double-diode model with a variable  $J_{01}(V)$  to evaluate non-linear silicon solar cells. A  $J_{01}$  range between  $\sim 400$  mV and the maximum power point can be obtained from the bias-light dependent internal quantum efficiency (IQE). Based on the electrical and optical model that describes the IQE, the excess carrier concentration in the base when the solar cell is in short circuit is calculated for every bias light and used to attribute a voltage. The  $J_{01}(V)$  for larger voltages is extracted from the  $I_{sc}$ - $V_{oc}$  characteristics after an appropriate extrapolation of the parasitic currents through the second diode and the shunt.

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## 1. Introduction

The double-diode model is frequently used to simulate or evaluate the I-V curves of silicon solar cells. In several aspects, however, this model tends to be over-simplified such that care is required when using it to fit experimental data. Some of the model deficiencies are as follows [1]: (a) an inhomogeneous voltage over the solar cell is not well represented by the single constant series resistance; (b) shunts may behave more Schottky-like than ohmic; (c) the second diode current may show ideality factors of above  $n = 2$ ; and (d) the current of the first diode may show

ideality factors of greater than  $n = 1$ . In many cases, the curve-fitting fails to give a meaningful set of parameters, and a fill-factor analysis is then the better choice to evaluate the experimental curves [2].

Nevertheless, it is still worth analyzing the IV-characteristics to understand the composition of the current flow and check the consistency with the results of other measurement techniques. The motivation of this work is to get the recombination properties obtained from the internal quantum efficiency (IQE) measurement to agree with the I-V curve. A frequent disagreement is an open-circuit voltage that is clearly larger than expected from the IQE analysis. One possible reason for such differences is that the solar cell is not in low injection at  $V_{OC}$ . Another is that the solar cell is non-linear, i.e. the recombination parameters depend on the injection level. In this paper, we suggest a way in which such solar cells can be treated. Considering the  $I_{SC}$ - $V_{OC}$  curve eliminates the problem connected with the series resistance [3]. The focus is on the current of the first diode, which is usually considered ideal with  $n = 1$ .

### Nomenclature

$L_{eff}$	effective diffusion length
$n_i$	intrinsic carrier concentration
$n$	ideality factor
$N$	base doping concentration
$k$	Boltzmann constant
$T$	temperature
$q$	elementary charge
$J_{01}$	saturation current of the first diode
$Y$	share of $J_{01}$ attributed to lowly doped regions

## 2. Diode model for general injection

The transition between low and high injection is where the excess carrier concentration is equal to the base doping concentration  $N$ . Beyond low injection, the following assumptions underlying the derivation of the ideal diode equation are violated:

- The relation between excess minority carrier density and voltage becomes more complex.
- The minority carrier lifetime changes.
- The base contribution of the series resistance decreases because of the increased base conductance.
- An electric field between the excess electrons and holes prevents their spatial separation (Dember effect).

Considering all these effects in the diode model would result in a numerical simulation. We are instead looking for a simple modification to the diode model that helps us to see whether an  $I_{SC}$ - $V_{OC}$  curve is still in low injection. The lifetime spectrum is therefore not treated as a high-injection effect at this point and assumed to be constant. We only take the dominant high-injection effect into account: the altered relationship between the excess minority carrier density ( $\Delta n$ ) and voltage ( $V$ ). It results from the mass action law  $p n = n_i^2 \exp(qV/kT)$  and can be written as follows:

$$\Delta n = \frac{N}{2} \left( \sqrt{1 + \frac{4n_i^2}{N^2} e^{qV/kT}} - 1 \right) \quad (1)$$

With these simplifications, the diode current  $J_1$  can be assumed to be simply proportional to the excess carrier concentration. The proportionality constant is identified from the low-injection case, for which the first diode current is that of the ideal diode:  $J_1 = J_{01}(\exp(qV/kT) - 1)$ . In this case Equation 1 simplifies to  $\Delta n = n_i^2 / N \exp(qV/kT)$ . Thus, we can re-formulate the first diode current for general injection as follows:

$$J_1 = J_0 \frac{N^2}{2n_i^2} \left( \sqrt{1 + \frac{4n_i^2}{N^2} e^{\frac{qV}{kT}}} - 1 \right) \quad (2)$$

Figure 1 shows a PC1D simulation [4] of the  $I_{SC}$ - $V_{OC}$  curve of a solar cell where all recombination happens in the lowly doped base with an injection-independent Shockley-Read-Hall lifetime. Under these conditions, Equation 2 correctly describes the transition from low to high injection, where the ideality factor approaches a value of  $n = 2$ .

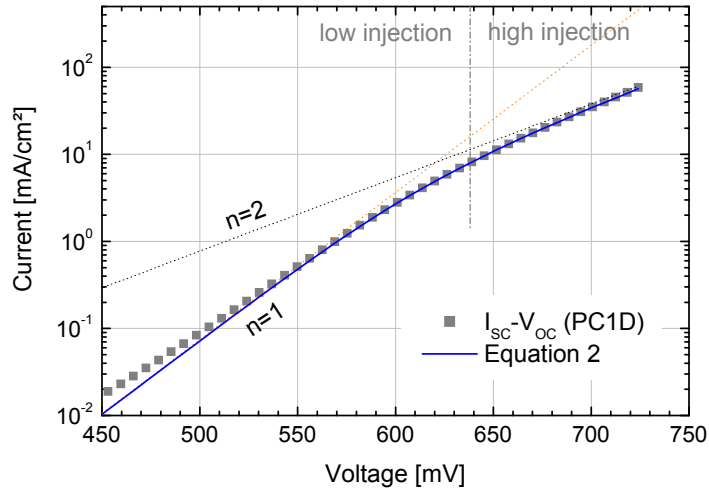


Fig. 1. PC1D simulation of a solar cell on 3  $\Omega$ cm n-type silicon with a constant carrier lifetime of 1000  $\mu$ s. Recombination in the emitter and at surfaces was set negligibly small. Equation 2 describes the transition from low to high injection well.

The emitter and back-surface field are highly doped regions that remain lowly injected while the base goes into high injection. We therefore separate  $J_{01}$  into two contributions: a fraction  $J_{01} Y$  accounts for the recombination in the lowly doped base volume and at surfaces with constant surface recombination velocity. The remaining fraction  $J_{01} (1-Y)$  stands for the recombination in highly doped regions. The first diode current in the modified double-diode model then takes the following form:

$$J_1 \cong J_{01} \left[ (1-Y) e^{\frac{qV}{kT}} + Y \frac{N^2}{2n_i^2} \left( \sqrt{1 + \frac{4n_i^2}{N^2} e^{\frac{qV}{kT}}} - 1 \right) - 1 \right] \quad (3)$$

To account for the injection dependence of the bulk lifetime and surface recombination velocity, the fixed value of  $J_{01}$  needs to be replaced with a voltage-dependent  $J_{01}(V)$  as described in Section 3.

### 2.1. Application of the extended model

Since we ignored the injection dependence of the bulk lifetime, the model needs to be applied with care. Of course, it is not to be used to obtain the base doping or the emitter saturation current from a least square fit to an I-V curve. The following are samples of meaningful applications or conclusions:

- Least square fits of  $I_{SC}$ - $V_{OC}$  curves of high-efficiency solar cells from lowly doped substrates usually need to be restricted to low voltages where there is still a significant fraction of the parasitic currents involved. When the extended model is implemented, the voltage range can extend to higher voltages, making the determination of the  $J_{01}$  more reliable.
- Toggling the value for  $Y$  between 0 and 1 visualizes at what voltage the solar cell departs from low injection.
- A fit result of  $Y = 0$  for a solar cell in high injection indicates that recombination in highly doped regions is dominant.
- The extended model can be used to calculate the open circuit voltage that corresponds to the  $J_{01}$  obtained from an IQE-analysis of a lowly doped solar cell.

### 3. Voltage-dependent saturation current for non-linear solar cells

As long as low-injection is maintained, the diode saturation current  $J_{01}(V)$  can be found from the  $I_{SC}$ - $V_{OC}$  curve after subtracting the parasitic currents of the second diode ( $J_2$ ) and the shunt.

$$J_{01}(V) = \frac{J(V) - J_{shunt}(V) - J_2(V)}{e^{qV/kT}} \quad (4)$$

However, the parasitic currents are not easy to determine. A pronounced injection-dependence of  $J_{01}$  may look like a second diode [5]. Furthermore, the second diode frequently gives good fits only if the ideality factor is allowed to take values of  $n = 4$  rather than the expected  $n = 2$ . This makes the extrapolation of the corresponding current to higher voltages uncertain. Therefore, we prefer to determine the low-voltage range of  $J_{01}(V)$  from the IQE [6].

#### 3.1. $J_{01}$ determined from IQE measurements

The analysis of the IQE spectrum allows us to determine an effective diffusion length ( $L_{eff}$ ) in the base from which the base contribution of the diode saturation current can be calculated if the base doping concentration ( $N$ ) is known [7]. The emitter saturation current  $J_{0E}$  also needs to be known or be negligible. Under favourable conditions  $J_{0E}$  can be estimated by analysing the short-wavelength range of the IQE [8]. To obtain  $J_{01}$  at different injection conditions, QE measurements at several bias light levels are necessary, in principle. But it is sufficient and much faster to record the bias light dependence at a single suitable wavelength only and to measure the QE-spectrum at a single suitable bias intensity around 0.3 suns.

The quantum efficiency (QE) spectra are usually measured in short circuit with the injection level established by white bias light. The measurement of the QE under forward voltage is also possible and seems to be an obvious choice if the voltage-dependent  $J_{01}$  is wanted. However, experience has shown that the signal-to-noise ratio is better if the same injection level is established with the bias light at  $V = 0$  rather than under a forward voltage bias. Another difficulty arises when approaching  $V = V_{OC}$  where the QE signal from the monochromatic light is observed to decrease drastically.

#### 3.2. Conversion of bias light intensity to voltage

If the QE and reflectance spectra are fitted with an appropriate electrical and optical model [6], the necessary information to calculate the excess carrier concentration profile  $\Delta n(z)$  in the base of the solar cell in a short circuit becomes available. We use the average  $\Delta n$  to calculate the voltage using Equation 1.

It is important to remember that it is not the bias light intensity that is relevant for the optical generation: the QE measurement is a small-signal technique, i.e. the intensity of the modulated monochromatic test light is much smaller than the bias light. The measured QE therefore is a differential and not an absolute quantity [9]. This distinction is important for non-linear solar cells whose QE spectra depend on the bias light intensity. To obtain the absolute QE for one-sun illumination, a bias intensity of typically 0.3 suns is necessary [9]. This means that a differential QE spectrum measured at 1 sun bias light effectively corresponds to the injection conditions under more

than three suns. This intensity in short circuit conditions corresponds to a voltage close to the maximum power point under 1 sun illumination.

### 3.3. Combined evaluation of IQE and $I_{SC}$ - $V_{OC}$ curve

To provide good conditions for the extraction of  $J_{01}(V)$  from the  $I_{SC}$ - $V_{OC}$  curve, we propose the following procedure:

- 1) Subtract the first diode current obtained from the IQE analysis from the  $I_{SC}$ - $V_{OC}$  curve.
- 2) Fit the low-voltage range of the remaining parasitic current with a shunt resistance and a diode with variable  $n$ .
- 3) Subtract the 'fitted' parasitic current from the  $I_{SC}$ - $V_{OC}$  curve.
- 4) Calculate  $J_{01}(V)$  from the remaining first diode current using Equation 3.

Information about the value of  $Y$ , if relevant for step 4, can be obtained from the IQE analysis. Figure 2 shows an example of such a non-linearity analysis on an industrial multicrystalline solar cell with a pronounced non-linearity. Using this joint evaluation of  $I_{SC}$ - $V_{OC}$  and IQE, one can check whether a presumable second diode current is not actually caused by an injection-dependent recombination in the base.

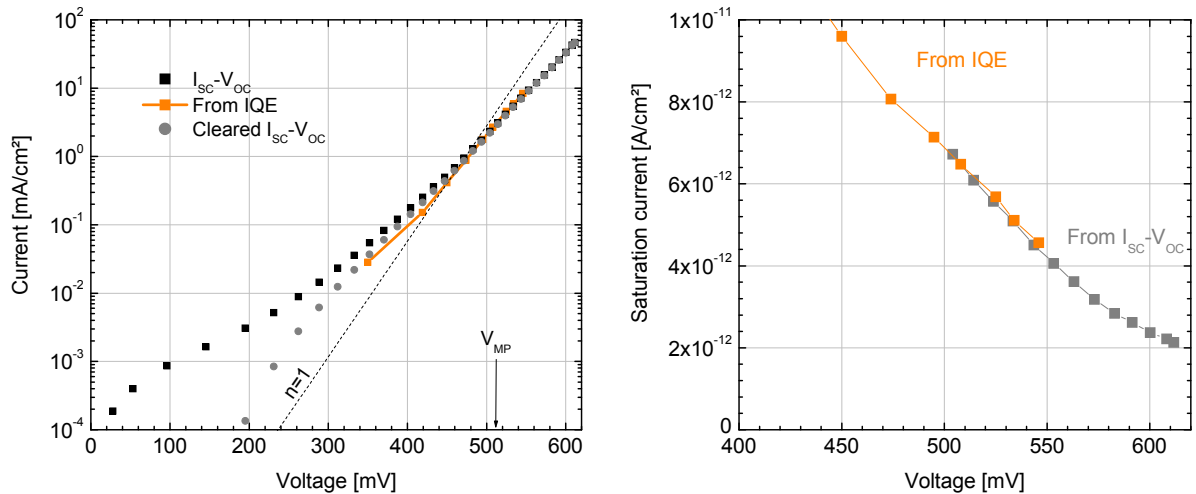


Fig. 2. Non-linearity analysis of an industrial multicrystalline silicon solar cell: Left:  $I_{SC}$ - $V_{OC}$  curve (black squares) with an ideality factor around 1.2 at higher voltages. The current resulting from the IQE analysis (orange squares) shows the same ideality factor and explains the experimental IV behaviour. The gray circles show the  $I_{SC}$ - $V_{OC}$  curve after subtracting the parasitic current. Right: The voltage-dependent diode saturation current is merged from the  $J_{01}(V)$  from the IQE analysis for lower voltages and from the  $I_{SC}$ - $V_{OC}$  characteristics in the upper voltage range.

There are several preconditions and limitations for the proposed method: 1) Some information about the emitter saturation current must be available, which is difficult to obtain on a finished solar cell. 2) It must be avoided to measure the IQE over an inhomogeneous area since the model underlying the evaluation then becomes invalid. On the other hand, for local IQE measurements it is necessary to make sure that the measured location is representative for the entire cell. 3) Generally, the determination of  $J_{01}$  from the IQE becomes less accurate if the  $L_{eff}$  is many times the cell thickness. The carrier collection is then almost complete, and it is actually the non-collection of carriers that provides the information about  $L_{eff}$  and thus  $J_{01}$ .

#### 4. Summary

We have modified the voltage dependence of the first diode current in the double-diode model to be able to examine  $I_{SC}$ - $V_{OC}$  curves beyond low injection. The diode saturation current  $J_{01}$  is split up into two parts: the fraction  $Y$  that accounts for the lowly doped base, and the fraction  $(1-Y)$  attributed to the highly doped layers, which stay in low-injection at high voltages. The known doping concentration and  $Y$  are the only extra parameters, making it easy to replace the familiar low-injection model since they are equivalent if  $Y = 0$ .

For the analysis of non-linear solar cells, we suggested seeking the voltage dependence of  $J_{01}$ . A method to obtain the  $J_{01}(V)$  spectrum is proposed that combines the information gained from the  $I_{SC}$ - $V_{OC}$  characteristics with that from the internal quantum efficiency.

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