THEORETICAL TEMPERATURE DEPENDENCE OF SOLAR CELL PARAMETERS

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Summary

A simple formulation has been derived for the temperature dependence of cell parameters for any solar cell material. Detailed calculations have been performed for high-quality monocrystalline GaAs, Si and Ge cells. Preliminary experimental data for GaAs and Si cells are close to the calculated values. In general, the higher the energy gap of a material, the smaller is the temperature dependence of its solar cell parameters.

1. Introduction

Although solar cell parameters are generally measured at 20-30 °C, flat plate modules normally operate at 40-50 °C under terrestrial conditions, and even higher temperatures are used for some concentrator cell applications. Therefore, it is interesting to calculate the dependence of cell parameters on temperature. In this paper we have derived a simple formulation for obtaining the temperature dependence of open-circuit voltage V_{oc}, short-circuit current density J_{sc}, fill factor FF, and conversion efficiency η, for materials of different bandgap E_g. As an illustration, the cell parameters at five temperatures between 273 and 373 K have been calculated for high-quality monocrystalline GaAs, Si and Ge cells. In addition, the variation of $(1/V_{oc})(\partial V_{oc}/\partial T)$ with V_{oc} has been calculated for the three types of cell at 300 K.

2. General formulation

Assume the dark saturation current density, J_{00}, for a solar cell is given [1] by

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$$J_{00} = K'T^{3/n} \exp\left(\frac{-E_g}{mkT}\right) \tag{1}$$

and V_{oc} is given [1] by

$$V_{oc} = \frac{AkT}{q} \ln\left(\frac{XJ_{sc}}{J_{00}}\right) \tag{2}$$

where K' is an empirical parameter, T the absolute temperature, E_g the energy gap, k is Boltzmann's constant, A the diode factor, X the concentration ratio and m, n are empirical parameters depending on the quality of the cell material and junction. Differentiating eqns. (1) and (2) with respect to T gives

$$\frac{\partial J_{00}}{\partial T} = \frac{J_{00}}{T}\left[\frac{3}{n} - \frac{1}{mkT}\left(T\frac{\partial E_g}{\partial T} - E_g\right)\right] \tag{3}$$

$$\frac{\partial V_{oc}}{\partial T} = \frac{V_{oc}}{T} + \frac{AkT}{qJ_{sc}}\frac{\partial J_{sc}}{\partial T} - \frac{AkT}{qJ_{00}}\frac{\partial J_{00}}{\partial T} \tag{4}$$

By substituting eqns. (1) and (3) into eqn. (4) we obtain

$$\frac{\partial V_{oc}}{\partial T} = \frac{1}{T}\left[V_{oc} - \frac{AE_g}{mq} - \frac{3AkT}{nq}\right] + \frac{A}{mq}\frac{\partial E_g}{\partial T} + \frac{AkT}{qJ_{sc}}\frac{\partial J_{sc}}{\partial T} \tag{5}$$

This equation is a general expression and should be applicable to cells made from amorphous, polycrystalline or monocrystalline materials.

3. High-quality monocrystalline GaAs, Si and Ge cells

For high-quality monocrystalline cells, the parameters m, n and A are all close to unity, and eqn. (5) becomes

$$\frac{\partial V_{oc}}{\partial T} = \frac{1}{T}\left[V_{oc} - \frac{E_g}{q} - \frac{3kT}{q}\right] + \frac{1}{q}\frac{\partial E_g}{\partial T} + \frac{kT}{qJ_{sc}}\frac{\partial J_{sc}}{\partial T} \tag{6}$$

For GaAs, Si and Ge, the measured values of E_g are well represented by the equation [2]

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{(T+\beta)} \tag{7}$$

where $E_g(0) = E_g$ at $T = 0$ K, and α and β are empirical parameters. Substituting eqn. (7) into eqn. (6) gives

$$\frac{\partial V_{oc}}{\partial T} = \frac{1}{T}\left[V_{oc} - \frac{E_g}{q} - \frac{3kT}{q}\right] - \frac{\alpha T}{q}\frac{(T+2\beta)}{(T+\beta)^2} + \frac{kT}{qJ_{sc}}\frac{\partial J_{sc}}{\partial T} \tag{8}$$

Table 1 lists the values of $E_g(0)$, α, and β for GaAs, Si and Ge [2].

TABLE 1

Energy gap parameters for GaAs, Si and Ge

	$E_g(0)$ (eV)	α (eV deg^{-1})	β (deg)
GaAs	1.519	5.405×10^{-4}	204
Si	1.170	4.730×10^{-4}	636
Ge	0.7437	4.774×10^{-4}	235

No analytical expression is available for $\partial J_{sc}/\partial T$. To evaluate this differential at a temperature T_0, J_{sc} values were calculated for T_0, $T_0 + 2$, and $T_0 - 2$, using the formulation presented elsewhere [3]. In calculating each J_{sc} value, the air mass one (AM 1) solar spectrum [4] was integrated to the value of E_g given by eqn. (7) for the appropriate temperature. The values of J_{sc} were then used to evaluate

$$\left(\frac{\partial J_{sc}}{\partial T}\right)_{T_0} = \frac{\left(\frac{\partial J_{sc}}{\partial T}\right)_{T_0+1} + \left(\frac{\partial J_{sc}}{\partial T}\right)_{T_0-1}}{2} \tag{9}$$

where

$$\left(\frac{\partial J_{sc}}{\partial T}\right)_{T_0+1} = \frac{(J_{sc})_{T_0+2} - (J_{sc})_{T_0}}{2} \tag{10}$$

and

$$\left(\frac{\partial J_{sc}}{\partial T}\right)_{T_0-1} = \frac{(J_{sc})_{T_0} - (J_{sc})_{T_0-2}}{2} \tag{11}$$

This procedure was used to obtain $(\partial J_{sc}/\partial T)_{T_0}$ for GaAs, Si and Ge cells at various values of T_0.

The value of K' appearing in eqn. (1) may be different for different materials. For our calculations, we have selected a single value, $K' = 0.02$. For AM 1, one-sun conditions at 300 K, this value of K' gives calculated V_{oc} values of 0.990 V for GaAs, 0.699 V for Si and 0.248 V for Ge. For GaAs, this value of V_{oc} is close to the best obtained for homojunction cells [5], although higher V_{oc} values have been obtained for heterostructure cells [6]. For Si, the calculated V_{oc} value is slightly higher than the values obtained for the best Si cells so far reported [7]. For Ge, the calculated V_{oc} value is significantly higher than the values so far obtained, which are in the range 0.15–0.20 V.*

*Range of values obtained at Lincoln Laboratory.

Values of $(\partial V_{oc}/\partial T)_{T_0}$ for GaAs, Si and Ge cells have been calculated from eqn. (8). In addition, the procedure used to calculate $(\partial J_{sc}/\partial T)_{T_0}$ was used to obtain $(\partial V_{oc}/\partial T)_{T_0}$ from values of V_{oc} calculated for different temperatures by using eqn. (2). The results obtained by the two methods were found to be the same, as expected. The procedure used to calculate $(\partial J_{sc}/\partial T)_{T_0}$ was also used to obtain values of $(\partial \mathrm{FF}/\partial T)_{T_0}$ from values of the fill factor FF calculated for different temperatures by using the equations [1]

$$\mathrm{FF} = \frac{V_m}{V_{oc}}\left[1 - \frac{\exp\left(\dfrac{qV_m}{kT}\right) - 1}{\exp\left(\dfrac{qV_{oc}}{kT}\right) - 1}\right] \tag{12}$$

where V_m is given by the relationship

$$\exp\left(\frac{qV_m}{kT}\right)\left(1 + \frac{qV_m}{kT}\right) = \frac{XJ_{sc}}{J_{00}} + 1 \tag{13}$$

Solar cell conversion efficiency η is given by

$$\eta = \frac{(V_{oc}) \times (J_{sc}) \times (\mathrm{FF})}{P_{in}} \tag{14}$$

where P_{in} is the insolation, which for AM 1 is 92.4 mW cm^{-2}. The temperature dependence of η is generally expressed as a relative variation

$$\left(\frac{1}{\eta}\right)\left(\frac{\partial \eta}{\partial T}\right) = \left(\frac{1}{V_{oc}}\right)\left(\frac{\partial V_{oc}}{\partial T}\right) + \left(\frac{1}{J_{sc}}\right)\left(\frac{\partial J_{sc}}{\partial T}\right) + \left(\frac{1}{\mathrm{FF}}\right)\left(\frac{\partial \mathrm{FF}}{\partial T}\right) \tag{15}$$

We have used the calculated values of V_{oc}, $(\partial V_{oc}/\partial T)$, J_{sc}, $(\partial J_{sc}/\partial T)$, FF and $(\partial \mathrm{FF}/\partial T)$ to evaluate the three relative quantities on the right side of eqn. (15) at $T = 273$, 300, 323, 353 and 373 K, and then to calculate η and $(1/\eta)(\partial \eta/\partial T)$. The results obtained for GaAs, Si and Ge cells are given in Tables 2, 3 and 4 respectively.

TABLE 2

Calculated temperature dependence of GaAs solar cell parameters; one sun, AM 1

T (K)	E_g (eV)	V_{oc} (V)	$\frac{1}{V_{oc}}\frac{\partial V_{oc}}{\partial T}$ (% deg^{-1})	J_{sc} (mA cm^{-2})	$\frac{1}{J_{sc}}\frac{\partial J_{sc}}{\partial T}$ (% deg^{-1})	FF	$\frac{1}{\mathrm{FF}}\frac{\partial \mathrm{FF}}{\partial T}$ (% deg^{-1})	η (%)	$\frac{1}{\eta}\frac{\partial \eta}{\partial T}$ (% deg^{-1})
273	1.4345	1.047	−0.202	29.138	+0.033	0.894	−0.052	29.51	−0.221
300	1.4225	0.989	−0.217	29.399	+0.034	0.881	−0.057	27.73	−0.240
323	1.4120	0.940	−0.235	29.629	+0.037	0.869	−0.062	26.19	−0.269
353	1.3981	0.875	−0.255	29.932	+0.036	0.852	−0.069	24.14	−0.288
373	1.3887	0.830	−0.258	30.136	+0.032	0.840	−0.074	22.75	−0.307

TABLE 3

Calculated temperature dependence of Si solar cell parameters; one sun, AM 1

T (K)	E_g (eV)	V_{oc} (V)	$\frac{1}{V_{oc}}\frac{\partial V_{oc}}{\partial T}$ (% deg^{-1})	J_{sc} (mA cm^{-2})	$\frac{1}{J_{sc}}\frac{\partial J_{sc}}{\partial T}$ (% deg^{-1})	FF	$\frac{1}{FF}\frac{\partial FF}{\partial T}$ (% deg^{-1})	η (%)	$\frac{1}{\eta}\frac{\partial \eta}{\partial T}$ (% deg^{-1})
273	1.1312	0.750	−0.253	38.296	+0.0295	0.863	−0.074	26.83	−0.297
300	1.1245	0.699	−0.275	38.599	+0.0293	0.845	−0.083	24.67	−0.327
323	1.1185	0.654	−0.296	38.876	+0.0296	0.829	−0.092	22.80	−0.357
353	1.1104	0.595	−0.337	39.250	+0.0354	0.805	−0.107	20.33	−0.409
373	1.1048	0.555	−0.364	39.510	+0.0354	0.787	−0.119	18.67	−0.448

TABLE 4

Calculated temperature dependence of Ge solar cell parameters; one sun, AM 1

T (K)	E_g (eV)	V_{oc} (V)	$\frac{1}{V_{oc}}\frac{\partial V_{oc}}{\partial T}$ (% deg^{-1})	J_{sc} (mA cm^{-2})	$\frac{1}{J_{sc}}\frac{\partial J_{sc}}{\partial T}$ (% deg^{-1})	FF	$\frac{1}{FF}\frac{\partial FF}{\partial T}$ (% deg^{-1})	η (%)	$\frac{1}{\eta}\frac{\partial \eta}{\partial T}$ (% deg^{-1})
273	0.67366	0.302	−0.659	57.44	+0.0146	0.739	−0.240	13.88	−0.884
300	0.66339	0.248	−0.816	57.64	+0.0125	0.685	−0.329	10.60	−0.953
323	0.65444	0.201	−1.017	57.82	+0.0140	0.627	−0.451	7.89	−1.45
353	0.64253	0.139	−1.475	57.99	+0.0116	0.528	−0.710	4.62	−2.17
373	0.63446	0.099	−2.010	58.14	+0.0155	0.449	−0.916	2.79	−2.89

Comparing the results in Tables 2–4 shows that the higher E_g, the lower $(1/\eta)(\partial\eta/\partial T)$. The differences in the values of $(1/\eta)(\partial\eta/\partial T)$ for GaAs and Ge are quite large. In addition, the absolute magnitude of $(1/\eta)(\partial\eta/\partial T)$ is not independent of temperature, but increases with increasing temperature.

The values in Tables 2–4 were calculated for high-quality cells. For lower quality cells, V_{oc} will generally be smaller and the temperature dependence of V_{oc} will also change. Figure 1 shows curves of $(1/V_{oc})(\partial V_{oc}/\partial T)$ *versus* V_{oc} calculated from eqn. (8) for GaAs, Si and Ge cells at 300 K. (These curves are approximate, since eqn. (8) assumes that A, n and m are all equal to one, a condition satisfied only for high-quality cells.) For each type of cell, the absolute magnitude of $(1/V_{oc})(\partial V_{oc}/\partial T)$ decreases as V_{oc} increases. For cells of comparable quality, the absolute magnitude of $(1/V_{oc})(\partial V_{oc}/\partial \mathrm{T})$ increases in the order GaAs, Si, Ge. However, the temperature dependence of V_{oc} can be lower for very good Si cells than for poor GaAs cells, and can be lower for very good Ge cells than for poor Si cells.

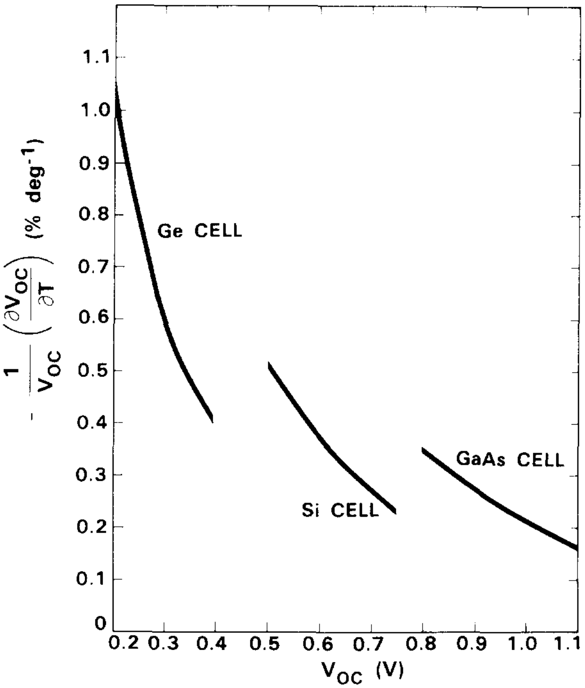

Fig. 1. Calculated curves of $(1/V_{oc})(\partial V_{oc}/\partial T)$ *vs.* V_{oc} for GaAs, Si and Ge cells at 300 K.

4. Experimental values

Experimental data on solar cell performance as a function of temperature are still quite limited, but there have been some measurements on GaAs and Si cells. For a GaAs cell with $V_{oc} = 0.96$ V at 300 K, we have measured $(1/V_{oc})(\partial V_{oc}/\partial T) = -0.25\%$ deg^{-1} at 300 K. This is close to the calculated value of -0.23% deg^{-1} shown in Fig. 1. In addition, a value of $(1/J_{sc})(\partial J_{sc}/\partial T) =$ 0.036 deg^{-1} has been measured at 300 K, which is close to the calculated value of 0.034% deg^{-1} (see Table 2).

For a Si cell with $V_{oc} = 0.67$ V at 300 K, $(1/V_{oc})(\partial V_{oc}/\partial T) = -0.30\%$ deg^{-1} and $(1/J_{sc})(\partial J_{sc}/\partial T) = 0.029\%$ deg^{-1} have been measured at 300 K.* The corresponding calculated values are -0.31% deg^{-1} (see Fig. 1) and $+0.029\%$ deg^{-1} (see Table 3).

5. Conclusions

A theoretical formulation is presented for the temperature dependence of solar cell parameters for materials with different energy gaps. Detailed

*Preliminary measurements by L. L. Kazmerski and associates at Solar Energy Research Institute, Golden, CO, on a Si solar cell similar to those reported in ref. 7.

calculations have been performed for monocrystalline GaAs, Si and Ge cells. Preliminary experimental values for GaAs and Si are close to the calculated values. For a given material, the higher the V_{oc} value, the smaller the absolute magnitude of $(1/V_{oc})(\partial V_{oc}/\partial T)$ and hence $(1/\eta)(\partial\eta/\partial T)$. If each material achieves its maximum V_{oc} values, then the higher energy gap material will always have $(1/V_{oc})(\partial V_{oc}/\partial T)$ and $(1/\eta)(\partial\eta/\partial T)$ values of smaller absolute magnitude.

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