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Algorithm of Marriage in Honey Bees Optimization Based on the Wolf Pack Search

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Abstract

Marriage in Honey Bees Optimization (MBO) is swarm-intelligence methods. In this paper, the author proposed a new swarm-intelligence method, named as Wolf Pack Search (WPS), which is abstracted from the behavior feature of the wolf pack. Utilizing the WPS algorithm into the local search process of Marriage in Honey Bees Optimization algorithm, the paper gives a new algorithm, Wolf Pack Search-Marriage in Honey Bees Optimization (WPS-MBO), and proves its global convergence characteristic with the probability being 1 by using the Markov Chain theory. Some simulations are done based on some popular complex Evaluation Functions and Traveling Salesman Problem (TSP). By comparing WPS-MBO with original MBO and Genetic Algorithm, the results show that WPS-MBO has better convergence performance.

1. Introduction

Swarm intelligence is a new research area. It studies the behavior of social insects and uses their models to solve problems. Recently, Based on the marriage process of honey bees, the new technique of Marriage in Honey Bees Optimization (MBO) was proposed by Jason Teo and Hussein A. Abbass[1][2] and has been updated by Jason Teo, Hussein A. Abbass [3] and Omid Bozorg Haddad et al [4][5].

The objective of this paper is to propose a new swarm-intelligence method and utilize it to increase the performance of MBO.

The paper is organized as follows. As the basis of the study, Marriage in Honey Bees Optimization (MBO) algorithm and Markov Chain with some basic theorems and definitions are reviewed respectively in Section 2 and Section 4. In Section 4, a new algorithm related to the wolf pack is proposed. In Section 5 and Section 6, the proposed algorithm, that is Wolf Pack Search-Marriage in Honey Bees Optimization (WPS-MBO) algorithm, and its convergence analysis are presented. Finally, some simulations are done and conclusion is given.

2. Marriage in honey bees optimization

The behavior of honey-bees shows many features like cooperation and communication, so honey-bees have aroused great interests in modeling intelligent behavior these years.

Marriage in Honey Bees Optimization (MBO) is a kind of swarm-intelligence method. And such swarm-intelligence has some successful applications. Ant colony is an example and the search algorithm is inspired by its behavior. Mating behavior of honey-bees is also considered as a typical swarm-based optimization approach. The behavior of Honey-bees is related to the product of their genetic potentiality, ecological and physiological environments, the social conditions of the colony, and various prior and ongoing interactions among these three [1][2].

3. Wolf pack search

As a member of the flesh-eater and social animals in the nature, wolf gives people a uniformed impression. A pack of wolves can cooperate well with each other and it is always strong enough to attack the outside.

The arrest activity of a wolf group can be described as the following process. First, the wolves walk around. When the odor is left by the quarry, the wolves begin searching the smell in the direction of the thickest odor. Also the coat and egesta the quarry left along the road also help the wolves to track the quarry. The thicker the odor is becoming, the nearer the wolves from the quarry. The wolves will separate into several sub-groups. Each sub-group will approach the quarry in different routes, and towards the directions of the thickest smell. If there is an emergency, wolves will call together the other wolf group, and they will

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attack the quarry. Finally, one wolf will bite into the throat of the quarry till it falls down. And then the wolves will share in the food.

Based on the above description of a wolf pack, we abstract the process to a search method and name it as Wolf Pack Search (WPS). The frame of the Wolf Pack Search is shown below.

1) Initialize Step;
2) Initialize randomly a pack of wolves;
3) Compare and decide the best wolf $GBest$ and its fitness $GBFit$;
4) Circulate and update the formula below, until the states of all the wolves are the same or the iteration time reaches the limit;

$$wolf_{new}=wolf+step*(GBest-wolf)/|GBest-wolf| \quad (1)$$

5) If the fitness of $wolf_{new}$ is optimizer than $GBFit$, replace $GBest$ and $GBFit$ with $wolf_{new}$ and its fitness respectively.

The process of WPS is random and heuristic. And because only when the new solution is more optimal than the best solution so far, can it be saved and seen as the best one so far. So the solution sequence is monotone decreasing.

4. Markov chain

A Markov chain is a sequence of random values whose probability at a time interval depends upon the value of the number at the previous time. The probabilities of a Markov chain are usually entered into a transition matrix indicating which state or symbol follows which other state or symbol.

Definition 1[6]: A square matrix is $A=\left[a_{ij}\right]_{n\times n}$.

$$\begin{array}{ll}
\text{(a) if } \forall i,j\in\{1,\dots n\}: a_{ij}>0, & \text{A is positive } (A>0);\\
\text{(b) if } \forall i,j\in\{1,\dots n\}: a_{ij}\geq 0, & \text{A is nonnegative } (A\geq 0);\\
\text{(c) if } A\geq 0 \text{ and } \exists m\in \mathrm{N}: A^m>0, & \text{A is primitive;}\\
\text{(d) if } A\geq 0 \text{ and } \forall i\in\{1,\dots n\}: \sum_{j=1}^{n} a_{ij}=1, & \text{A is stochastic.}
\end{array} \quad (2)$$

Definition 2[6]: If the state space S is finite ($|S|=n$), and the probability $p_{ij}(t)$ that transition from state $i\in S$ to state $j\in S$ at step t is independent from t . If $\exists i,j\in S,\exists u,v\in \mathrm{N}, p_{ij}(u)=p_{ij}(v)$, the Markov chain is said to be finite and homogeneous.

Theorem 1[6]: For a homogeneous finite Markov chain, with the transition matrix $P=(p_{ij})$, If $\exists m\in \mathrm{N}: P^m>0$, then this Markov chain is ergodic and with finite distribution. $\lim_{t\to\infty} p_{ij}(t)=\overline{p_j}, i,j\in S$ is the steady distribution of the homogeneous finite Markov Chain.

Theorem 2[6] (The basic limit theorem of Markov chain): If P is a primitive homogeneous Markov chain's transition matrix, then

(a) $\exists!\omega^T>0:\omega^T P=\omega^T$,

ω: a probability vector.

(b) $\forall\varphi_i\in S$: $\lim_{k\to\infty} g_i^T P^k=\omega^T$. (3)

(φ_i: initial state, g_i^T: probability vector of φ_i)

(c) $\overline{P}=\lim_{k\to\infty} P^k$, $\overline{P}: n\times n$, $\overline{P}$'s rows are same as ω^T .

Theorem 3[6]: Let P be a reducible stochastic matrix, where $C: m\times m$ is a primitive stochastic matrix and $R,T\neq 0$. Then

$$P^{\infty}=\lim_{k\to\infty}P^k=\lim_{k\to\infty}\begin{pmatrix} C^k & 0\\ \sum_{i=0}^{k-1}T^i RC^{k-i} & T^k\end{pmatrix}=\begin{pmatrix} C^{\infty} & 0\\ R^{\infty} & 0\end{pmatrix} \quad (4)$$

is a stable stochastic matrix.

5. Wolf pack-marriage in honey bees optimization (WPS-MBO)

5.1. Analysis of MBO local search ability

One of the most important advantages of MBO over Genetic Algorithm is MBO does a local search in each iteration. So MBO can avoid solely using crossover operator or mutation operator which is of worse local search ability.

But MBO algorithm chooses some simple and random local searching methods, such as random walk and random flip [1], which will reduce the probability of obtaining optimal solution. So such low efficiency of Worker in MBO badly influences the whole performance of MBO.

As talked in Section 3, Wolf Pack Search algorithm simulates the hunting process of a pack of wolves. And its solution list is monotone decreasing. Such character is just what the Worker need in MBO algorithm.

So we will apply the WPS algorithm as the local searching, and replace the Worker in MBO algorithm.

5.2. Improvement of MBO

Some studies related to MBO have been carried out in our research. One of them is to increase the convergence speed. Here we make some introduce about it, because the main work in this paper will based on such improved MBO algorithm.

In MBO algorithm, the probability of a drone makes with a queen is defined by the annealing function [1].

Not only the calculation of probability is complex, but also its calculation participants are complicated. So the whole process has a large computation burden.

On the other hand, we have found that MBO with low speed need enough iteration times to approach optimization result. But several variables in MBO, such as energy, speed, can't make sure about this. As the process going, the mating probability becomes smaller, which neither help the calculation process put up, nor help converge globally. So based on the original MBO algorithm, we have done some improvement on the original MBO algorithm.

5.3. Wolf pack search-marriage in honey bees optimization algorithm

Here we further our research to improve the performance of MBO and propose an algorithm of Wolf Pack Search-Marriage in Honey Bees Optimization (WPS-MBO) by taking the WPS algorithm as the local search method.

The detail of WPS-MBO is shown below.

```
Define Q: the number of queens
         W: the number of workers
         D: the number of drone
         M: the sperm theca size
Initialize each worker with a unique heuristic
Initialize each queen's genotype at random
Apply WPS to improve the queen's genotype
While the stopping criteria is not satisfied (Cycle
Times bigger than Max Cycle Number or result is good
enough)
  for queen = 1 to Q
    for i=1 to M
      generate a drone randomly
      add its spermatozoa to the queen's sperm theca
      generate a brood by crossovering the queen's
genome with the selected sperm,
      mutate the generated brood's genotype
      use WPS to improve the drone's genotype
      if the new brood is better than the worst queen
        replace the least-fittest queen with the new brood
        refresh the queen list depending on fitness
      end if
    end for
  end for
end while
```

Figure 1. Wolf Pack Search- Marriage in Honey Bees Optimization algorithm (WPS-MBO).

In Figure 1, the algorithm is much easier than that in the MBO algorithm. And the number of the parameters is also less than the later. The whole process of WPS-MBO has fewer complex probability calculations which will help increase the computation speed.

In WPS-MBO, we define three operators: Crossover, Mutation and Heuristic. Crossover and Mutate are same as that in GA. But the Heuristic operator is a new one proposed in WPS-MBO.

1) Crossover: Crossover operator exchanges the pieces of genes between chromosomes.

2) Mutation: Mutation operation alters individual alleles at random locations of random chromosomes at a very probability.

3) Heuristic: Heuristic operator improves a set of broods. It help conduct local search on broods. For the good local convergence performance, we use local WPS as the heuristic operator.

6. Convergence analysis of WPS-MBO algorithm

In this section, we use Markov Chain to analysis the convergence of the Wolf Pack Search-Marriage in Honey Bees Optimization algorithm.

There are only three ways to change from one generation to another, is Crossover, Mutate and Heuristic. These operators depend only on the inputs and not restricted with time. Then we can get the following theorem.

Definition 3: The state space of WPS-MBO is

$$X = \{x = [t_1, t_2, ..., t_N] | t_i \in \{0,1\}, i = 1, ..., N\} \quad (5)$$

where $[t_1, t_2, ..., t_N]$ is the binary bit cluster listed in turn.

Define $f(x)$ as the fitness function based on X and y is the fitness. So the fitness aggregate Y is

$$Y = \{y | y = f(x), x \in X\} \quad (6)$$

Define

$$\forall x \in X, g = |Y| \quad (7)$$

we can get a ordered aggregate

$$\{y_1, y_2, \ldots, y_g\},\ y_1 > y_2 > \ldots > y_g \quad (8)$$

Crossover, Mutate and Heuristic operators lead to probable transition in the state space. And we use three transition matrix C, M and H to describe their infections respectively. Finally, we can get

$$Tr = C \cdot M \cdot H \quad (9)$$

where Tr is the transition matrix of the Markov chain of the WPS-MBO algorithm.

Theorem 4: WPS-MBO is a Markov Chain of finite and homogeneous.

Proof: The aggregate $\{x_1, x_2, \ldots, x_M\}$ is finite. So the Markov chain composed of $\{x_1, x_2, \ldots, x_M\}$ is finite. This finite space can also be said as a state space X.

With $\rho_i, \rho_j \in X$, the probability of transformation from the state ρ_i to the state ρ_j at step t only depends on ρ_i and is independent of time t. So the Markov chain of the WPS-MBO algorithm is homogeneous.

End.

Theorem 5: The transition matrixes of the crossover probability (C) and Heuristic probability (H) in the WPS-MBO algorithm are all stochastic.

Proof: The square matrix C is $C = \left[c_{ij}\right]_{n\times n}$. Then

$$\forall i,j \in \{1,\ldots n\}: c_{ij} \geq 0 \text{ and } \forall i \in \{1,\ldots n\}: \sum_{j=1}^{n} c_{ij} = 1 \quad (10)$$

So C is stochastic.

The square matrix H is $H = [h_{ij}]_{n\times n}$. Then

$$\forall i,j \in \{1,...,n\}: h_{ij} \geq 0 \text{ and } \forall i \in \{1,...,n\}: \sum_{j=1}^{n} h_{ij} = 1 \quad (11)$$

So H is stochastic. End

Theorem 6: The transition matrix of the WPS-MBO with mutation probability (M) is stochastic and positive.

Proof: $M = [m_{ij}]_{n\times n}$ is a square matrix. Then

$$\forall i,j \in \{1,\ldots n\}: m_{ij} \geq 0 \text{ and } \forall i \in \{1,\ldots n\}: \sum_{j=1}^{n} m_{ij} = 1 \quad (12)$$

So M is stochastic.

And the mutation has an influence on every position of a state vector. We can easily know $\forall x_i, x_j \in X$. Each position of x_i can mutate to the value of x_j. So the probability of x_i mutate to x_j is positive.

So M is positive. End

Theorem 7: The Markov Chain of the WPS-MBO (Tr) is ergodic and with finite distribution. $\lim_{t\to\infty} tr_{ij}(t) = \overline{tr_j} > 0, i,j \in X$.

Proof: According to Theorem 5, Theorem 6 and (9), Tr is positive. And according to Theorem 1, this proposition is proved. End

Definition 4: The fitness of one generation is the largest one of the individuals in this generation.

$$f\left(\{x_1, x_2, \ldots, x_K\}\right) = \max_{i\in 1,2,\ldots\ldots K} \{f(x_i)\} \quad (13)$$

Define $X_i = \left\{x_1, x_2, \ldots, x_K \middle| f\left(\{x_1, x_2, \ldots, x_K\}\right) = y_i, x_1, x_2, \ldots, x_K \in X\right\}$, y_i are defined in (8), that is, the fitness of all the individuals in X_i is equal to y_i.

Definition 5: For an arbitrary initial generation $X(0)$, y_1 is of the largest fitness,

$$\lim_{t\to\infty} P_r(f(X(t)) = y_1) = 1 \quad (14)$$

Then the algorithm is global convergence.

Theorem 8: The WPS-MBO converges to the global optimum.

Proof: We can define

$$TX = \{X_i | i \in N\} \quad (15)$$

For Definition 4 and Theorem 4, TX is a Markov Chain. In the same time, we define

$$P(X_i) = P_r\{iX \in X_i\} \quad (16)$$

We can see that $P(X_i) > 0$ and $\sum_{1}^{n} P(X_i) = 1$

Define $P(X_i, X_j)$ is the probability state X_i go to X_j, we can get

$$P(X_i, X_j) = \sum_{ni=1}^{Ni} \sum_{nj=1}^{Nj} P(x_{ni}, x_{nj}), x_{ni} \in X_i, x_{nj} \in X_j \quad (17)$$

Because WPS-MBO saves the best individual at every generation, $P(X_i, X_j) = 0, i < j$.

And the transition matrix of TX's Markov Chain can be write as follows:

$$P = \begin{bmatrix} P(X_1, X_1) & \cdots & P(X_1, X_n) \\ \vdots & \cdots & \vdots \\ P(X_n, X_1) & \cdots & P(X_n, X_n) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ P(X_2, X_1) & P(X_2, X_2) & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ P(X_n, X_1) & \cdots & \cdots & P(X_n, X_n) \end{bmatrix} \quad (18)$$

For Theorem 3,

$$C = 1, T = \begin{bmatrix} P(X_2, X_2) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ P(X_n, X_2) & \cdots & P(X_n, X_n) \end{bmatrix}, R = \begin{bmatrix} P(X_2, X_1) \\ \vdots \\ P(X_n, X_1) \end{bmatrix} \quad (19)$$

$$P^\infty = \lim_{k\to\infty} P^k = \lim_{k\to\infty} \begin{pmatrix} C^k & 0 \\ \sum_{i=0}^{k-1} T^i R C^{k-i} & T^k \end{pmatrix} = \begin{pmatrix} C^\infty & 0 \\ R^\infty & 0 \end{pmatrix} \quad (20)$$

For Theorem 7 and Theorem 1, P^∞ is a stable random matrix, so $R^\infty = 1$.

$$R^\infty = \lim_{k\to\infty} R^k = \begin{bmatrix} \lim_{k\to\infty} \left(P(X_2, X_1)\right)^k \\ \vdots \\ \lim_{k\to\infty} \left(P(X_n, X_1)\right)^k \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad (21)$$

So every state in TX will go to X_1, if the iteration number is big enough, this proposition is proved. End

7. Simulation

To test the convergence performance of WPS-MBO, we choose original MBO algorithm and GA for comparison. We do the simulation on two parts, some popular complex Evaluation Functions and Traveling Salesman Problem (TSP).

7.1. Traveling salesman problem (TSP)

Given a number of nodes and their distances of each other, an optimal travel route is calculated so that starting from a node and visiting every other node only once with the total distance covered minimized. Here the TSP based on the data from TSPLIB is solved by WPS-MBO, MBO and GA respectively. The solution of the 16 nodes and the 48 nodes are shown in Figure 2 and Figure 3.

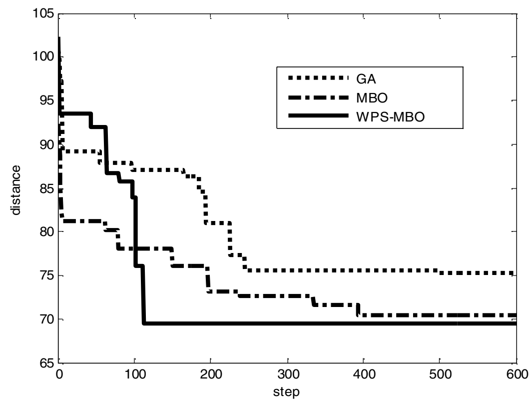

Figure 2. **TSP with 16 nodes solved by WPS-MBO and GA respectively.**

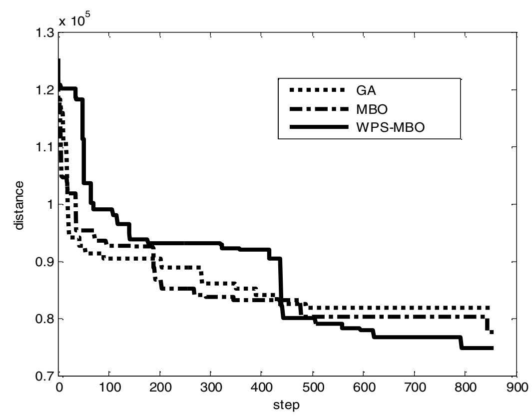

Figure 3. **TSP with 48 nodes solved by WPS-MBO and GA respectively.**

From Figure 2 and Figure 3, we can see that WPS-MBO can exceed the MBO and GA and approach a better value.

7.2. Evaluation functions

Four popular evaluation functions are chosen. The initial value is generated randomly. Simulation results are shown in Figure 4, Figure 5, Figure 6 and Figure 7.

1) Evaluation Function1: Generalized Rosenbrock's Problem

$$f_1(X)=\sum_{i=1}\left[100(x_{i+1}-x_i^{\,2})^2+(1-x_i)^2\right],\quad |x_i|\le 30 \tag{22}$$

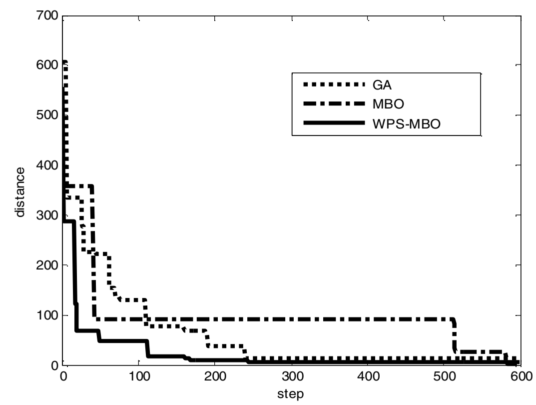

Figure 4. **Evaluation Function 1.**

2) Evaluation Function2: Schwefel's Problem 2

$$f_2(X)=\sum_{i=1}|x_i|+\prod_{i=1}|x_i|,\quad |x_i|\le 10 \tag{23}$$

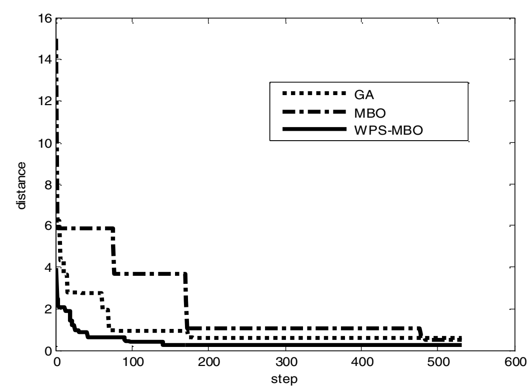

Figure 5. **Evaluation Function 2.**

3) Evaluation Function3: Schwefel's Problem 3

$$f_3(X)=\sum_{i=1}\left(\sum_{j=1}^{i}x_j\right)^2,\quad |x_i|\le 100 \tag{24}$$

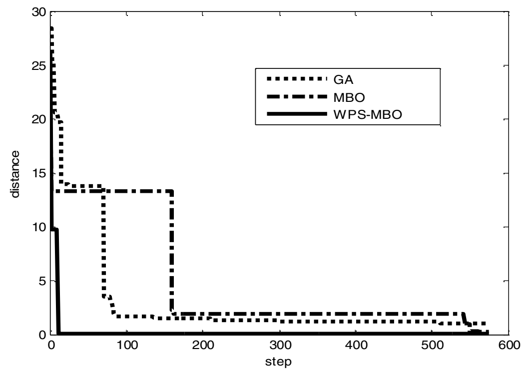

Figure 6. Evaluation Function 3.

4) Evaluation Function4: Generalized Rastrigin's Function

$$f_4(X)=\sum_i\left[x_i^2-10\cos\left(2\pi x_i\right)+10\right],\quad |x_i|\le 5.12 \tag{25}$$

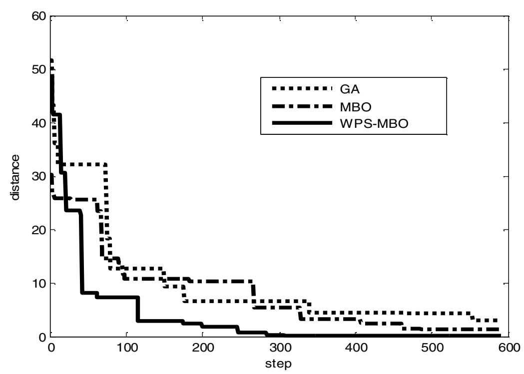

Figure 7. Evaluation Function 4.

The above results show that:

WPS-MBO is convergent and keeps good performance for all these test functions, though these test function are more complex than the normal ones and may have many local optimization points. WPS-MBO performs better than GA and MBO. WPS-MBO converges more quickly, especially at initial part. Particularly, even if the initial condition is worse, WPS-MBO can show finer result. MBO seems not perform steadily. Sometime, it keeps stay at a value for some time and then drop down. Sometime, it doesn't show better than GA.

8. Conclusions

Convergence performance is very important for optimization methods. This paper proposed a new swarm-intelligent method-Wolf Pack Search (WPS) and used it to improve the local search ability of Marriage in Honey Bees Optimization. By simulating on TSP and several evaluation functions, the proposed algorithm of WPS-MBO has shown better convergence ability than the original MBO and Genetic Algorithm. And its global convergence is preserved based on Markov Chain theory.

The algorithm still deserves research. And the research about WPS-MBO will be deeply studied and improved in the future.

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