Pergamon

0038-1101(96)00179-4

Solid-State Electronics Vol. 39, No. 4, pp. 577–581, 1996

0038-1101/96 $15.00 + 0.00

DEPLETION LAYER RESISTANCE AND ITS EFFECT ON I–V CHARACTERISTICS OF FULLY- AND PARTIALLY-ILLUMINATED SILICON SOLAR CELLS

K. CHAKRABARTY and S. N. SINGH

Division of Materials, National Physical Laboratory, Dr K. S. Krishnan Road, New Delhi 100012, India

(Received 8 January 1995; in revised form 25 June 1995)

Abstract—The single exponential model is convenient to use, but in many cases it is not adequate to describe the I–V characteristics of a silicon solar cell and often a two exponential model has to be invoked. We have found that this is because, conventionally, the contribution of the depletion layer to the series resistance, which decreases with junction voltage, is ignored. If the contribution of the depletion region is taken into account, the single exponential model can also describe the solar cell characteristic adequately for a fully-illuminated solar cell. Still, it fails to describe the behavior of a partially-illuminated cell if the dark and the illuminated portions of the cell are assumed to be connected electrically through the metal contacts only. Our investigations reveal that the illuminated and the dark portions are electrically connected through the junction and, under steady state, the cell attains an equipotential surface along the junction. The theoretical curves obtained, using the single exponential model that considers the junction to have the same voltage everywhere and takes the contribution of the depletion region to the series resistance into account, show an excellent match with the experimental I–V characteristics of fully- or partially-illuminated solar cells.

1. INTRODUCTION

The steady state I–V characteristics of a uniformly illuminated p–n junction silicon solar cell are generally described by equation:

$$I = -I_L + I_0 \exp\left[\frac{q(V - IR_s)}{nkT} - 1\right] + \frac{V - IR_s}{R_{sh}}. \quad (1)$$

Equation (1) is commonly referred to as the single exponential model where all parameters except V and I are constants. In eqn (1), I_L is the light generated current, I_0 is the reverse saturation current, R_s is the series resistance, R_{sh} is the shunt resistance, n is the diode ideality factor and kT/q is the thermal voltage. For an n^+–p silicon solar cell, it is assumed that R_s consists of the resistance of the n^+ front region and the base region; the other contributions to R_s, if any, are often considered as negligible. Accordingly, R_s is expected to be constant for low level conditions[1–4]. However, in practice, R_s is generally found to decrease with the intensity of illumination and the output voltage of the cell even for the low level conditions[5]. Araujo *et al.*[5] developed an improved model which showed that the contribution of the gridded emitter to R_s is higher at low output voltages than at higher output voltages. However, this only provided a qualitative explanation of the decrease of R_s with junction voltage V_j. A model providing a quantitative explanation to the observed decrease of R_s with V_j is still not available in the literature.

It is a fact that in a solar cell the charge carriers have to cross the depletion layer, which is the highest resistivity region in the cell. Moreover, the resistivity of the depletion region is controlled by the voltage across it and decreases as the voltage increases. Therefore, if the depletion region were to make a significant contribution to R_s, this contribution should be expected to decrease with V_j. Surprisingly, however, the influence of the modulation of the depletion layer resistivity on the I–V characteristics of fully- or partially-illuminated silicon solar cells has not been investigated by earlier researchers[4,6–8]. In this paper, we consider for the first time the modulation of the depletion layer resistivity with the forward junction voltage V_j and derive an expression to determine the contribution of the depletion layer to R_s and its influence on the I–V characteristics of the fully- and partially-illuminated silicon solar cells. We also present our results of measurement of R_S at different bias voltages for a uniformly-illuminated silicon solar cell and the I–V characteristics of fully as well as partially-illuminated cells. We show that the experimental data fit into the theory excellently.

2. EXPERIMENTAL

The I–V characteristics of a few 100 mm diameter n^+–p solar cells fabricated using 1–2 Ωcm resistivity were measured under full- and partial-illumination conditions at a constant temperature of 25°C. The metal contact on the n^+ front surface of cell was in the form of a grid pattern which covered $\approx 10\%$ of the area of the cell. In the case of partial illumination, a certain percentage of the cell area was covered and the remaining portion was illuminated with radiation of a fixed intensity ($\approx 100\,\mathrm{mW\,cm^{-2}}$) under a tungsten halogen lamp. The value of R_{sh} was measured from dark reverse I–V characteristics, whereas I_0 and n were determined from V_{oc}–I_{sc} characteristics of the fully illuminated cell at 25°C. The value of R_s was measured from the comparison of the dark I–V curve with each of the illuminated I–V curves measured at 25°C under various intensities of illuminated and transferred subsequently from the 4th quadrant to the 1st quadrant [9]. The difference ΔV in the voltages for the dark and the transferred illuminated I–V curves corresponding to $I = I_{sc}$ gave R_s. This value of R_s corresponds to the $(V = V_{oc}, I = 0)$ point of the illuminated I–V curve of the cell. The intensity was low enough to ensure low level condition in the base region of the solar cell.

3. THEORETICAL

3.1. *Resistance of the depletion layer*

Consider an n^+–p silicon solar cell to have a step junction. The dependence of the depletion layer thickness d_s of the p–n junction, on the photovoltage V_j developed across it is given by eqn (2)[3]:

$$d_s = \left[\frac{2\epsilon\epsilon_0(N_A + N_D)(V_{bi} - V_j)}{qN_A N_D}\right]^{1/2}, \tag{2}$$

where V_{bi} is the built-in potential across the junction at equilibrium, ϵ_0 is the permitivity of free space, ϵ is the dielectric constant of silicon, $N_A(N_D)$ is the acceptor (donor) concentration on the $p(n)$ side. In the one-dimensional model the conductiviity $\bar{\sigma}$ of the depletion region can be determined using the expression:

$$\bar{\sigma} = \bar{\sigma}_p + \bar{\sigma}_n = \frac{\int_0^{d_s} (qn_d\mu_n + qp_d\mu_p)\,dx}{\int_0^{d_s} dx}, \tag{3}$$

where $\bar{\sigma}_n(\bar{\sigma}_P)$ is the average electron (hole) conductivity, $n_d(p_d)$ is the electron (hole) density and $\mu_n(\mu_p)$ is the electron (hole) mobility in the depletion layer. Both n_d and p_d are functions of x where x is the direction along the thickness of the cell, but μ_n and μ_p are assumed to be constant throughout the depletion layer, thickness. Relating n_d and p_d to the potential in the depletion layer, eqn (3) yields an expression for depletion layer resistivity $\bar{\rho} = 1/\bar{\sigma}$ as:

$$\bar{\rho} = \frac{(V_{bi} - V_j)\exp\left(\frac{-qV_j}{kT}\right)}{kT(\mu_n n_p + \mu_p p_n)\left[\exp\frac{q(V_{bi} - V_j)}{kT} - 1\right]}, \tag{4}$$

where $n_p(p_n)$ is the density of electrons (holes) in the $p(n^+)$ region at the edges of the depletion region. As will become clear a little later, the value of $\bar{\rho}$ is more realistically given by the equation:

$$\bar{\rho} = \frac{(V_{bi} - V_j)\exp\left(\frac{-qV_j}{n_2kT}\right)}{kT(\mu_n n_p + \mu_p p_n)\left[\exp\frac{q(V_{bi} - V_j)}{n_2kT} - 1\right]}. \tag{5}$$

For $(V_j/V_{bi}) \ll 1$, the contribution of depletion region to the series resistance R_{d_s} is determined using eqn (5) as:

$$R_{d_s} = \frac{\bar{\rho}d_s}{A} = C_1 - C_0V_j, \tag{6}$$

where

$$C_0 = \left[\frac{2\epsilon\epsilon_0(N_A + N_D)V_{bi}}{qN_A N_D}\right]^{1/2} \frac{\exp\left(-\frac{qV_{bi}}{n_2kT}\right)}{AkT(\mu_n n_p + \mu_p p_n)}, \tag{7a}$$

and

$$C_1 = C_0V_{bi}. \tag{7b}$$

Here, n_2 is a scaling factor whose value is expected to be greater than unity. Ignoring the contribution of the metal grids and the contact resistance, the total series resistance R_s of the cell is obtained as:

$$R_s = R_b + R_{d_s} + R_n, \tag{8}$$

where R_b and R_n are the contributions of the p-bulk region and the n^+ front region, respectively, to R_s. Further, $R_n = \rho_\square S^2/2AX_j$ and $R_b = \rho_b d_p/A$ where A is the area of the cell, d_p is the thickness of the quasi-neutral base region, $2S$ is the spacing between two successive grids of the front contact, $\rho_\square$ is the sheet resistance of the n^+ front region and ρ_b is the resistivity of the base region. Since R_b and R_n are independent of V_j, the dependence of R_s on V_j will be controlled by the dependence of R_{ds} on V_j. Therefore, R_s can be expected to vary linearly with V_j.

3.2. *Modelling of* I–V *characteristics of a partially illuminated solar cell*

The shadowed portion of a solar cell cannot generate photo current and hence acts as a drain of power produced in the illuminated region. We can consider R_s to be inversely proportional to the area of the cell[6]. For a partially shadowed cell, the individual series resistances of the two portions, i.e. R_{s1} of the illuminated portion and R_{s2} of the shadowed portion, will be inversely proportional to the fractional area (f) of the illuminated region and $(1-f)$ of the dark region. Sharma *et al.*[6] have assumed that there is also a surface resistance R_{sur} associated with the dark region which increases with the area of the dark region and is connected in series with R_{s2}. But even with this assumption they could not explain the I–V characteristics of the partially-illuminated cell adequately. The equivalent circuit of the Sharma

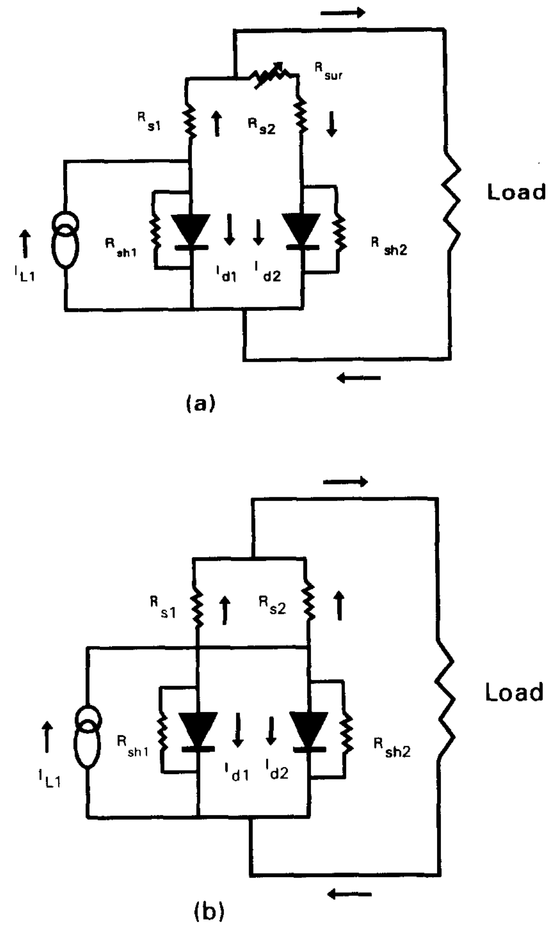

Fig. 1. (a) Equivalent circuit for Sharma model. (b) Equivalent circuit for equipotential model (this work).

et al.[6] model is shown in Fig. 1(a) and the relationship between current (I) and voltage (V) is given by eqn (9):

$$I = -I_L + I_0 f \exp\left[\frac{q(V + I_i R_{s1})}{nkT} - 1\right] + I_0(1-f)\exp\left[\frac{q(V - I_{d2}R_{s2})}{nkT} - 1\right] + \frac{1}{R_{sh}}[V - fR_{s1}I_i - (1-f) \times (R_{s2} + R_{sur})I_{d2}], \quad (9)$$

where $R_{s1} = R_s/f$ and $R_{s2} = R_s/(1-f) + R_{sur}$; I_i is the current passing through the illuminated region and I_d is that passing through the dark region. In eqn (9), I_L, I_i and I_{d2} stand for the absolute values. As is obvious from Fig. 1(a), the currents through R_{s1} in the illuminated region and R_{s2} in the dark region are in opposite directions. Therefore, under all conditions, the value of V_j for the illuminated portion of the cell should stay higher than that for the dark portion of the cell. However, in the actual case, as would be explained later, this should lead to a lateral flow of carriers from the illuminated region to the dark region and the cell would show a tendency of attaining an equipotential surface over the total area in due course of time. To take care of this flaw of the Sharma model[6], we have used a model which assumes, as is also the fact, that the dark and the illuminated portions of the cell area are connected in parallel through the junction. We also assume that, under steady-state conditions, they acquire the same voltage V_j under all stages of the operation of the cell. The equivalent circuit of such an equipotential model is shown in Fig. 1(b). In a partially illuminated cell, the shadowed portion has no contribution to I_L but makes full contribution to the diode current I_d. The I–V relationship for a fully or a partially illuminated solar cell is given by:

$$I = -I_L + I_d + \frac{V_j}{R_{sh}} = -I_L + I_0\left(\exp\frac{qV_j}{nkT} - 1\right) + \frac{V_j}{R_{sh}}, \quad (10)$$

where V_j is given by:

$$V_j = V - I\left(\frac{R_{s1}R_{s2}}{R_{s1} + R_{s2}}\right). \quad (11)$$

In eqn (11), suffixes 1 and 2 refer to the illuminated and dark portions of the solar cell, respectively. Further, $I_d = I_{d1} + I_{d2}$, $R_{s1} = R_s/f$ and $R_{s2} = R_s/(1-f)$, where f is the illuminated fraction of the cell area.

4. RESULTS AND DISCUSSION

The experimental values of the series resistance of a cell measured at different output voltages are plotted in Fig. 2. The theoretical values of R_s at different output voltages calculated using eqn (6) are also plotted in Fig. 2. For calculation of the theoretical values of R_s, the values of the parameters taken were $N_D = 2 \times 10^{19}\,cm^{-3}$, $N_A = 10^{16}\,cm^{-3}$, $V_{bi} = 0.886$ V, $\mu_n = 400\,cm^2\,V^{-1}\,s^{-1}$, $\mu_p = 200\,cm^2\,V^{-1}\,s^{-1}$, $\epsilon_0 = 8.85 \times 10^{-14}\,F\,cm^{-1}$, $d_p = 400\,\mu m$, $X_j = 0.5\,\mu m$, $2S = 0.25$ cm, $\rho_\square = 9.06\,\Omega/\square$ and $A = 78.6\,cm^2$. For this theoretical calculation the

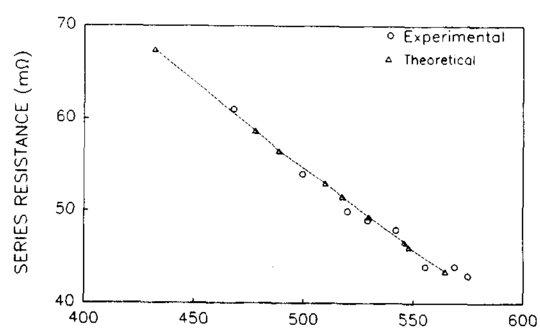

Fig. 2. Variation of experimental (○) and theoretical (△) series resistances of a silicon cell with the junction voltage of the cell.

value of n_2 is taken as 1.99. The above parameters were representative of the cells used for these investigations. It can be noted that R_S decreases linearly with V_j and the theoretical and the experimental curves are in excellent agreement.

Experimental I–V curves of the solar cell in partially- and fully-illuminated conditions are depicted in Fig. 3. The bottom curve (E_1) is for the case when the total area of the cell was under illumination. The other three curves (E_2, E_3 and E_4) are for the cases when 67%, 55% and 25.2% of the cell area was under illumination, respectively. Theoretical I–V curves (S_1, S_2, S_3, and S_4) computed using the Sharma model [i.e. eqn (9)] assuming R_{sur} to be zero, and the theoretical I–V curves (K_1, K_2, K_3 and K_4) obtained with our model using eqn (8) in conjunction with eqn (10), are also plotted in this figure for comparison. For computation of the theoretical curves, the values of I_0, n, R_s and R_{sh} given in Table 1 were used; these values were determined experimentally. From Fig. 3 it is noted that for $R_{sur} = 0$ there is a substantially large mismatch between the theoretical (S_1, S_2, S_3 and S_4) and experimental curves for an operating voltage above 250 mV. The region of mismatch is more pronounced near the maximum power point, when the 100% of cell area is under illumination, but shifts towards the (V_{oc}, 0) point with the decrease in the percentage of the illuminated area of the cell. Even Sharma *et al.*[6] themselves had not obtained a good match between the experimental and the theoretical I–V characteristics of the cells. It may be noted that the theoretical curves (K_1, K_2, K_3 and K_4) also do not match well with the respective experimental curves. The region of mismatch remains confined by and large to the maximum power point and becomes less pronounced with the decrease in the percentage of the area of illumination. In this regard, it is important to point out that the theoretical curves of Fig. 3 were computed assuming R_s to be invariant

Table 1. The experimentally determined parameters of a typical silicon solar cell used in eqn (8) (area of the cell = 78.2 cm^2)

I_0 (10^{-6} A)	n	R_{sh} (Ω)	R_s (mΩ)
1.12	1.495	3.5	46.7

with V_j, although, as shown in Fig. 2, R_s decreases with V_j.

Figure 4 compares the experimental I–V curves (E_1, E_2, E_3 and E_4) along with the theoretical I–V curves (K_1, K_2, K_3 and K_4) based on our model and (S_1, S_2, S_3 and S_4) based on the Sharma model but assuming R_s to vary in every case with V_j according to eqn (8) in conjunction with the relation $R_{d_s} = \bar{p}d_s/A$, eqn (5) and $n_2 \approx 2$. It is noted that in Fig. 4 the theoretical I–V characteristics based on our model match well with the experimental curves for all cases of full- or partial-cell area illuminated, whereas those based on the Sharma model match only for the unshadowed case of partial illumination. This shows that the model presented in this work is adequate to describe the I–V characteristic of a solar cell whether any fraction of the cell is shadowed or not, provided that the influence of depletion layer on series reistance is duly taken into account.

The reason for failure of the Sharma model[6] to describe the I–V characteristic for the case of partial illumination can be attributed to its following drawback. According to the Sharma model[6], as pointed out earlier, the voltage across the junction has to be less in the dark portion than in the illuminated portion. This condition is not satisfied in a practical case because the junction of the illuminated and dark regions are connected directly with each other.

In practice, if an n^+–p cell is partially illuminated, then the electrons (holes) generated by the incident light will cross the junction in the illuminated area from the $p(n^+)$ side to the $n^+(p)$ side, where they become the majority carriers and try to neutralize the built-in potential across the junction and develop a

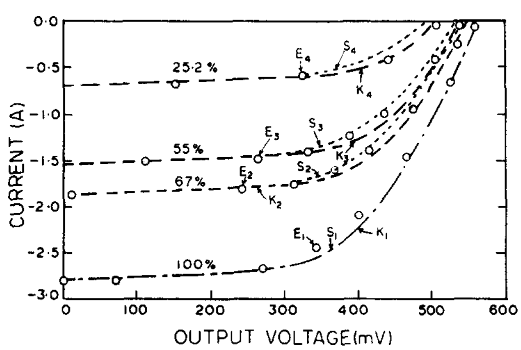

Fig. 3. Experimental and theoretical I–V characteristics of silicon solar cells with $R_{ds} = 0$. Three sets of I–V curves are drawn for 4 different conditions of illumination. Suffixes 1, 2, 3 and 4 indicate 100%, 67%, 55% and 25.2% of illumination, respectively. E_1, E_2, E_3 and E_4 (○) are the experimental curves. The theoretical curves S_1, S_2, S_3 and S_4 (····) are based on the Sharma model (with $R_{sur} = 0$) whereas curves K_1, K_2, K_3 and K_4 (–––) are based on this work.

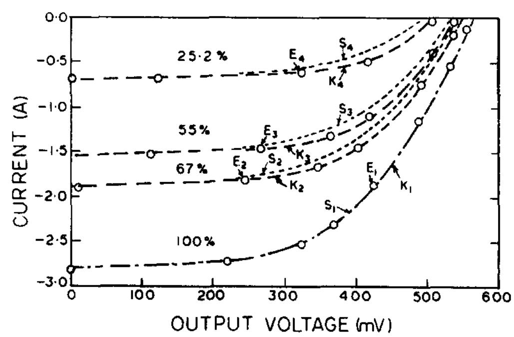

Fig. 4. Experimental and theoretical I–V characteristics of silicon solar cells with $R_{ds} > 0$. Three sets of I–V curves are drawn for 4 different conditions of illumination. Suffixes 1, 2, 3 and 4 indicate 100%, 67%, 55% and 25.2% of illumination respectively. E_1, E_2, E_3 and E_4 (○) are the experimental curves. The theoretical curves S_1, S_2, S_3 and S_4 (····) are based on the Sharma model (with $R_{sur} = 0$) whereas curves K_1, K_2, K_3 and K_4 (–––) are based on this work. For all these theoretical curves R_{ds} was computed using eqn (8).

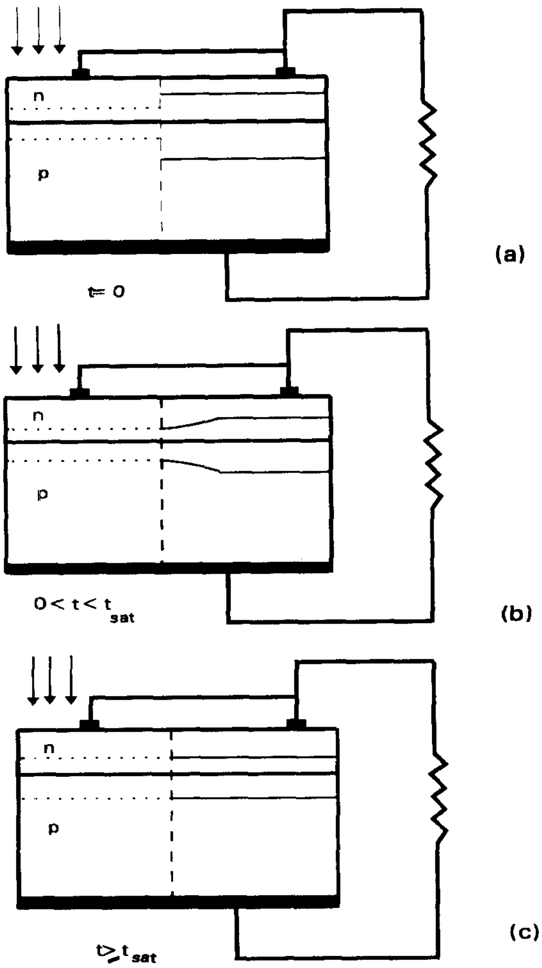

Fig. 5. Shrinkage of the depletion layer thickness of the cell in the illuminated and dark regions: (a) at $t = 0$, (b) for $0 < t < t_{sat}$ and (c) for $t \geqslant t_{sat}$.

photovoltage V_j locally in the illuminated area of the cell. This will give rise to a lateral field at the interface of the illuminated and the dark regions which will traverse subsequently deeper into the dark region following the movement of the carriers farther into the dark region with time, as depicted in Fig. 5. This will continue until the steady state is established after a saturation time t (t_{sat}). At this stage, the entire junction area including the dark region would become an equipotential, as shown in Fig. 5(c), if the leakage current (given by V_j/R_{sh}) is not localized. Therefore, in an otherwise homogeneous singel crystal silicon solar cell, the zero excess concentration gradient can be achieved even if the cell is partially illuminated. In the absence of the dark region, the Sharma model reduces to the equipotential model of this work and that is why the theoretical I–V curve (S_1) obtained with the Sharma model also matches well with the experimental curves, as shown in Fig. 4.

5. CONCLUSION

We have found that, though the thickness of the depletion region is very small, it has an appreciable contribution to the series resistance because of its very large resistivity. The average resistivity of the depletion region can be computed using eqn (5), where n_2 is a scaling factor. Introduction of the scaling factor n_2 in eqn (5) helps to make a realistic evaluation of the resistivity of the depletion region. It is not equivalent to the n_2 factor of a two exponential model.

The resistivity of the depletion region, and hence the value of R_s, decreases linearly with increase in voltage across the junction and thereby also across the terminals of the cell. For low level conditions the decrease of R_s with V_j is linear for $V_j \ll V_{bi}$. Our study also shows that even for the case of partial illumination the voltage across the entire junction of a cell acquires a uniform value. This is because the creation of a favorable electric field at the interface of the illuminated and dark regions which causes photogenerated carriers to flow to the dark region. It penetrates subsequently into the dark region and leaves behind an equipotential surface at the junction. Under the assumption that the junction has the same voltage, the single exponential model, in conjunction with the contribution of the depletion layer to series resistance, describes the I–V characteristics of a partially illuminated silicon solar cell for low level conditions adequately.

Acknowledgements—The authors are thankful to Professor E. S. Raja Gopal, Director National Physical Laboratory, New Delhi for his permission to publish this paper and to the Ministry of Non-conventional Energy Sources for financial assistance to carry out these investigations.

REFERENCES

1. A. L. Fahrenbruch and R. H. Bube, *Fundamentals of Solar Cells*. Academic Press, New York (1983).
2. M. A. Green, *Solar Cells, Operating Principles, Technology and System Applications*. University of New South Wales, Kensington (1986).
3. S. M. Sze, *Physics of Semiconductor Devices*. John Wiley and Sons, New York (1981).
4. S. R. Dhariwal, R. K. Mathur and R. Gadre, *J. Physics D* **14**, 1325 (1981).
5. G. L. Araujo, A. Cuevas and J. M. Ruiz, *IEEE Trans. Electron Devices* **ED-33**, 391 (1986).
6. A. K. Sharma. R. Gopal, R. Dwivedi and S. K. Srivastav, *Solid-St. Electron.* **33**, 309 (1990).
7. R. W. Sandarson, D. T. O'Donnell and C. E. Backus, *Proc. 14th IEEE PVSC*, 431 (1980).
8. C. M. Garner and R. D. Nasby, *Proc. 14th IEEE PVSC*, 437 (1980).
9. A. Rohatgi, J. R. Davis, R. H. Hopkins, P. Rai-Choudhury, P. G. McMullin and J. R. McCormick, *Solid-St. Electron.* **23**, 415 (1980).