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CARRIER MOBILITIES IN SILICON SEMI-EMPIRICALLY RELATED TO TEMPERATURE, DOPING AND INJECTION LEVEL

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Abstract—From a review of different publications on the carrier mobilities in silicon, the authors propose an approximated calculation procedure which permits a quick and accurate evaluation of these mobilities over a large range of temperatures, doping concentrations and injection-levels.

The proposed relations are well adapted to semiconductor device simulation because they allow short computation times.

NOTATION

A, B	parameters of the Brooks–Herring formula
N	effective doping (cm^{-3})
T	absolute temperature (K)
n	electron concentration (cm^{-3})
p	hole concentration (cm^{-3})
μ	resultant mobility
μ_I	impurity mobility
μ_L	lattice mobility
μ_{ccs}	carrier–carrier scattering mobility at high injection level
μ_{Iccs}	mixed scattering mobility resulting from μ_I and μ_{ccs}
μ_n	electron mobility ($cm^2 . V^{-1} . s^{-1}$)
μ_p	hole mobility ($cm^2 . V^{-1} . s^{-1}$)

INTRODUCTION

The physical mechanisms which govern carrier mobilities in a monocrystalline semiconductor have been studied and understood for a long time. However if interest is focused on electron device models the various theoretical formulae proposed in the literature for the calculation of mobility are too complicated to use when a quick estimation of order of magnitude is required and they are too costly in computation time when used in device behaviour simulation.

Different attempts have been made at obtaining a simple calculation formula. That of Caughey and Thomas[1], particularly, had a lot of success due to its ease of use and its accuracy. This formula however was established for a temperature of 300 K and only accounts for the doping dependence of mobilities. More recently the temperature and injection level dependence have also been introduced[2, 3]. In the same spirit, but following different lines, this paper proposes, from a review of various theoretical and experimental studies which have appeared on the subject, a more accurate procedure which allows the simultaneous appreciation of the variation of the mobilities with doping, with temperature and with the injection level. This calculation procedure involves simple formulae only, which are adapted for manual calculation or computer use with a short time of execution, but permanently supported by physical considerations.

(1) *Review of the mechanisms which influence mobility*

Free carrier mobilities in semiconductors are determined by various kinds of scattering mechanisms to which the carriers are subjected under the effect of thermal agitation. Each scattering mechanism gives rise to a particular mobility component. Generally the following components can be distinguished:

(a) Lattice mobility μ_L which is connected to scattering due to acoustic phonons. This partial mobility decreases with increasing temperature. It can be written[4]:

$$\mu_L = \mu_{L_0}\left(\frac{T}{T_0}\right)^{-\alpha} \tag{1}$$

where μ_{L_0} and α depend on the nature of the carriers and the temperature range under consideration. This component is largely responsible for the behaviour of over-all mobility with temperature.

(b) Impurity mobility μ_I which is connected to the interactions between the carriers and the ionized impurities. This partial mobility increases as the temperature increases or the doping concentration decreases. The relationship which we use in the calculation of the μ_I component is that of Brooks and Herring[5]. Since, in most practical cases, it is impossible to distinguish the concentrations of both types of impurity, we assume that the concentration of ionized impurities is simply equal to the effective doping N:

$$\mu_I = \frac{AT^{3/2}}{N}\left[\ln\left(1+\frac{BT^2}{N}\right)-\frac{BT^2}{N+BT^2}\right]^{-1} \tag{2}$$

where A and B are parameters which depend on the nature of the carriers.

(c) Neutral impurity scattering mobility which will be ignored here since in general the concentration of neutral impurities cannot be known in a real device. It can however be noted that this mobility only becomes of any importance at very high concentrations of neutral impurities ($> 10^{18}\ cm^{-3}$).

(d) Carrier–carrier scattering mobility μ_{ccs} which

becomes very influential when carriers of both types are at high concentrations. This is the case in devices subjected to a high level of injection. The main reference remains that of Chapman and Cowling[6]. As pointed out by Fletcher[7], their results on the mutual diffusion of two groups of charged gaseous particles apply, following a classical approximation, to the quasi-neutral electron-hole plasma when the injection level is high. However, instead of Fletcher's adaptation of Chapman's result, we use here a slightly different expression proposed by Choo[8], whole validity range as a function of the injection level is somewhat larger:

$$\mu_{ccs} = \frac{2\times 10^{17} T^{3/2}}{\sqrt{(pn)}} [\ln\{1 + 8.28\times 10^{8} T^{2}(pn)^{-1/3}\}]^{-1} \quad (3)\dagger$$

p or n expressed in cm^{-3}.

Note that for low injection levels ($pn \ll N^2$) μ_{ccs} as defined by eqn (3) takes extremely high values and therefore has no effect on the resulting mobility. However, at low injection levels, carrier–carrier scattering still exists but it acts as a perturbation of impurity and lattice scattering which are then the dominant mechanisms, and Chapman's analysis is no longer applicable. The influence of carrier–carrier scattering in the case of low injection levels has been discussed by various authors[9, 10] for like carrier collisions: the partial mobilities μ_L and μ_I are affected by a coefficient of between 1 and 0.88 for μ_L and between 1 and 0.632 for μ_I depending on the concentration of the majority carriers. The poor understanding of the exact variation law of these coefficients and the relative weights of the partial mobilities μ_L and μ_I, depending on whether the doping concentration is low or high, led us to leave μ_L uncorrected (coefficient 1 at low concentration) and to fully correct μ_I (coefficient 0.632 at high concentratons). μ_L does in fact play a preponderant role in the resulting mobility at low doping, whereas μ_I is preponderant at high doping.

In what follows, we shall assume that the same corrections hold for minority carriers submitted to non-like carrier scattering.

(2) *Mixed scattering mobility*

The manner in which the partial mobilities combine to give the resulting mobility depends on the kind of the energy dependence variation of the corresponding relaxation time. So, since the relaxation times associated with μ_I and μ_{ccs} have similar energy dependent expressions, these two partial mobilities can be combined by the following simple relationship:

$$\mu_{Iccs} = \left(\frac{1}{\mu_I} + \frac{1}{\mu_{ccs}}\right)^{-1}. \quad (4)$$

The resulting mobility μ_{Iccs} can be combined with the lattice scattering mobility using Debye nd Conwell's formula[9] which gives the global mobility μ of the carriers:

$$\mu = \mu_L f(X)$$
$$f(X) = 1 + X^2[\mathrm{Ci}(X)\cos(X) + \sin(X)\{\mathrm{Si}(X) - \pi/2\}]. \quad (5)$$
$$X = \sqrt{(6\mu_L/\mu_{Iccs})}$$

(3) *Simplified calculation procedure*

The calculation of μ using eqns (5) is not very easy due to the fact that rather complicated functions are involved such as $\mathrm{Ci}(X)$ (integral cosine) and $\mathrm{Si}(X)$ (integral sine). However the ranges of doping temperature and injection level usually met in the study of semiconductor devices allow the calculation of the function $f(X)$ to be restricted to a limited variation of the variable $X(0 \leq X \leq 9)$.

In this case, $f(X)$ can be obtained to within 2% using the following approximate formula:

$$f(X) = \frac{1.025}{1 + (X/1.68)^{1.43}} - 0.025. \quad (6)$$

Equation (6) is in fact very similar to the formula proposed by Caughey and Thomas[1] for calculation of the variation of carrier mobilities with doping:

$$\mu = \frac{\mu_{max} - \mu_{min}}{1 + (N/N_{ref})^{\alpha}} + \mu_{min}$$

where parameters μ_{max}, μ_{min}, μ_{ref} and α take on suitable values for each case. As a remark, it can be seen that eqns (5), by taking into account (6), practically lead to Caughey and Thomas' expression in the case of high doping and low injection level.

Thus we propose the calculation of carrier mobility in Silicon using the following series of simple formulae:

$$\mu_L = \mu_{L0}\left(\frac{T}{300}\right)^{-\alpha}$$
$$\mu_I = \frac{AT^{3/2}}{N}\left[\ln\left(1 + \frac{BT^2}{N}\right) - \frac{BT^2}{N + BT^2}\right]^{-1}$$
$$\mu_{ccs} = \frac{2\times 10^{7} T^{3/2}}{\sqrt{(pn)}} [\ln\{1 + 8.28\times 10^{8} T^{2}(pn)^{-1/3}\}]^{-1} \quad (7)$$
$$X = \sqrt{\left(\frac{6\mu_L(\mu_I + \mu_{ccs})}{\mu_I \mu_{ccs}}\right)}$$
$$\mu = \mu_L\left[\frac{1.025}{1 + (X/1.68)^{1.43}} - 0.025\right]$$

†If reference is made to Chapman's work, which remains a basis for the formulation of μ_{ccs}, it would seem logical to use the sum $\hat{p} + \hat{n}$ of excess carrier concentrations rather than a $2\sqrt{pn}$ term as done in equation (3). It should be noticed that this formula was established with a view to an essentially practical use, especially in programmes of numerical simulation of devices. In these programmes it is often the case (and it is so in those that we use) that the pn product is one of the pertinent and immediately accessible quantities, while the use of $\hat{p} + \hat{n}$ requires more time consuming calculations. At high injection levels, where carrier–carrier scattering is dominant $2\sqrt{pn} \simeq \hat{p} + \hat{n}$, and at low injection levels μ_{ccs} is in both cases so large that it does not influence the resulting mobility. At intermediate injection levels $2\sqrt{pn}$ is admitted by different from $\hat{p} + \hat{n}$ and difference can be noted, which remain however low enough as compared to experimental and theoretical uncertainties.

where μ_{L_0}, α, A an B are parameters which depend on the nature of the carriers.

(4) *Electron mobility*

Electron mobility μ_n has been widely studied and is the subject of numerous publications. The data concerning these carriers can, at present, be considered reliable.

We propose for the μ_{L_0} and α coefficients of formula (1) the following values for $T_0 = 300$ K:

$$\mu_{L_0} = 1430 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$
$$\alpha = 2.20.$$

Under these circumstances formula (1) gives values for mobility μ_L which agree with these given by Norton *et al.*[11] to within 3% over the temperature range 250–500 K. Over the range 200–600 K reasonable accuracy is still obtained ($\leq 10\%$).

For the formula which gives μ_I, the value of B used is that proposed by Li and Thurber[12]. A is from the same authors but corrected by the factor 0.632 proposed by Luong and Shaw[10] which accounts for the electron–electron scattering, whence:

$$A = 4.61 \times 10^{17} \text{ cm}^{-1} \cdot \text{V}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-3/2}$$
$$B = 1.52 \times 10^{15} \text{ cm}^{-3} \cdot \text{K}^{-2}.$$

The results for the overall mobility μ_n are plotted in Figs. 1–3 which show the influence of the main parameters: doping, temperature and injection level represented by the pn product.

As concerns the influence of doping temperature the results are in good agreement with those presented by Li and Thurber, the average deviation being of the order of 5%. Certain discrepancies appear at heavy doping ($N > 3 \times 10^{18}$ cm^{-3}), probably due to the fact that neutral impurities are ignored, and also at low temperatures, around 200 K, at which the values of μ_L are no longer obtained with high accuracy.

(5) *Hole mobility*

Since the theoretical analysis of hole mobility μ_p is greatly complicated by the particular structure of the valence band of silicon, the results concerning the

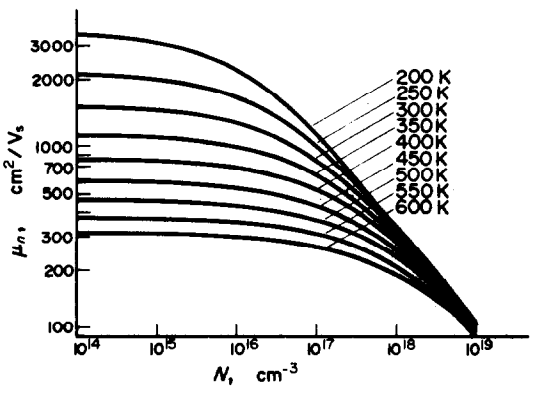

Fig. 1. Electron mobility vs doping at various temperatures, for a low injection level.

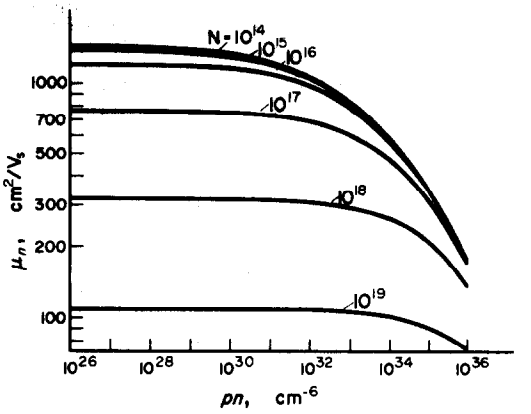

Fig. 2. Electron mobility vs injection level at various doping concentrations. $T = 300$ K.

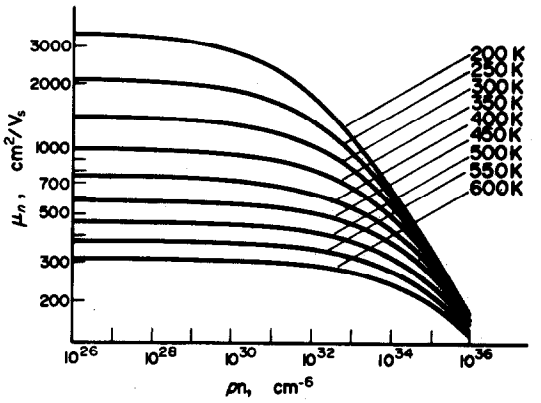

Fig. 3. Electron mobility vs injection level at various temperatures. $N = 10^{14}$ cm^{-3}.

various parameters of relationships (1) and (2) are, at present, questionable. Furthermore, the experimental data concern temperature ranges which often do not correspond to the field of temperatures usually fund in the use of semiconductor devices; at least the data published by various authors[13, 17], are not in very close agreement.

The insufficiency of the theoretical results and the dispersion of the experimental results restricted our choice of parameters: *a priori* we used the value of 2.2 for the α parameter of mobility $\mu_L(T)$ proposed by Canali *et al.*[14] and the value of B proposed by Elstner[4].

Then μ_{L_0} and A were adjusted for the best possible agreement with the results of Irwin at 300 K, which gave rise to the following parameters:

$$\mu_{L_0} = 495 \text{ cm}^2 \cdot \text{V}^{-1} \cdot \text{s}^{-1}$$
$$\alpha = 2.2$$
$$A = 1 \times 10^{17} \text{ cm}^{-1} \cdot \text{V}^{-1} \cdot \text{s}^{-1} \cdot \text{K}^{-3/2}$$
$$B = 6.25 \times 10^{14} \text{ cm}^{-3} \cdot \text{K}^{-2}.$$

The findings of the calculations of the resultant mobility μ_p are given in Figs. 4–6 which show the influence of the main parameters.

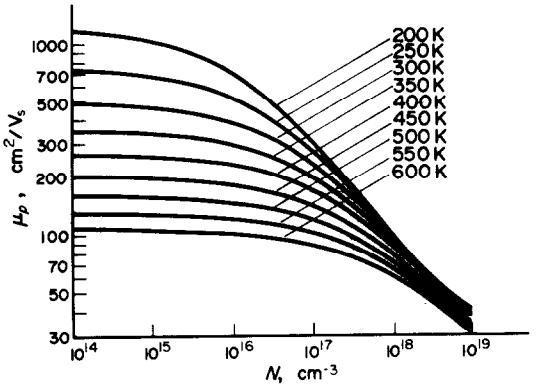

Fig. 4. Hole mobility vs doping at various temperatures, for a low injection level.

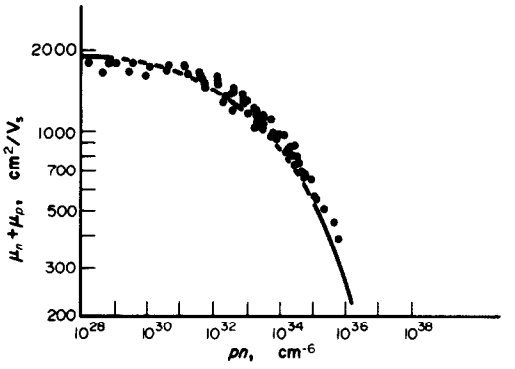

Fig. 8. ● Points measured by Krausse and Dannhauser. — Curve obtained through calculation using eqns (7).

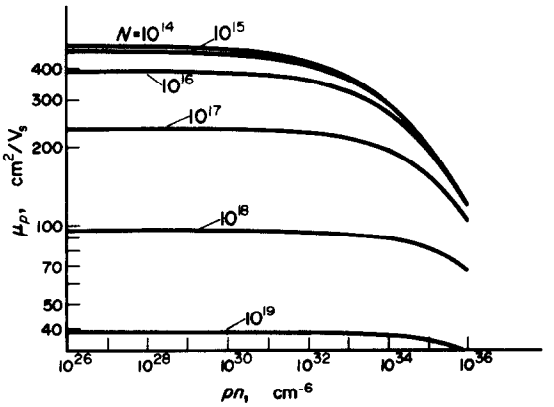

Fig. 5. Hole mobility vs injection level at various doping concentrations. $T = 300$ K.

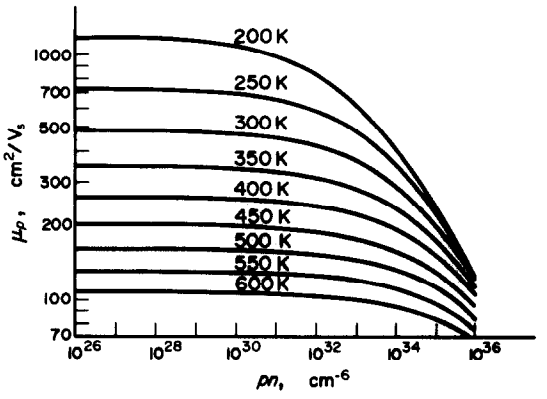

Fig. 6. Hole mobility vs injection level at various temperatures. $N = 10^{14}$ cm^{-3}.

As concerns the influence of doping at 300 K the results are in good agreement with those of Irwin, always within 5% except for doping at more than 2.10^{18} cm^{-3}. Due to lack of experimental results it is, at present, impossible to appreciate the accuracy of the calculation for the changes of the mobilities with temperature. It should be noted however that between 200 and 300 K our calculated values are compatible with those measured by Tsao and Sah[15] as shown in Table 1.

However Irwin's results do not quite agree with those given by Wagner[16] and more recently by Li[17] above all in the range of doping concentrations higher than 10^{16} cm^{-3}. This discrepancy, according to Li[17] results from the different nature of the doping material used by Irwin (Gallium doped silicon) and by Li (Boron doped silicon). However, our calculations can be brought to a good agreement with the results published by Li when the parameter A of formula (2) is taken equal to 2.8×10^{17} cm^{-1}. V^{-1}. s^{-1}. K$^{-3/2}$.

(6) *Influence of the carrier–carrier scattering at high injection level*

Few investigations have been carried out on the electron–hole scattering. Krausse[18] and Dannhauser[19] measured the variation of $\mu_n + \mu_p$ at 300 K as a function of the concentration of carriers injected into the weakly doped zone of a silicon P.I.N. diode. Once again the results of our calculations are in agreement with their experimental results as shown in Fig. 8.

CONCLUSION

The procedure (7) we propose for the calculation of electron and hole mobilities in silicon gives results which are in good agreement with the theoretical and experimental mobility values published to date. This procedure, which does not involve the use of complex functions is tractible over a large range of variation of the concerned parameters: doping, temperature and injection level. The proposed values of the parameters needed in the calculation could if necessary be refined if a more complete and reliable set of experimental results become available. Furthermore the procedure can be immediately applied, with suitable values of the parameters, to other indirect band-gap materials, in particular Germanium for which the mobility determining mechanisms are completely analogous to those of silicon.

The proposed procedure, which is here restricted to cases of weak electric field can be extended, with no

200 K				250 K			300 K		
N (cm^{-3})	μ_p ($cm^2v^{-1}s^{-1}$)			μ_p ($cm^2v^{-1}s^{-1}$)			μ_p ($cm^2v^{-1}s^{-1}$)		
	TSAO and SAH	our calculation	DEVIATION	TSAO and SAH	our calculation	DEVIATION	TSAO and SAH	our calculation	DEVIATION
9.67×10^{14}	1010	1033	2.3 %	600	674	12 %	445	466	4.7 %
5.74×10^{15}	820	792	3.4 %	510	568	11 %	367	415	13 %
1.08×10^{16}	730	680	6,8 %	460	511	11 %	344	385	5.2 %
2.93×10^{16}	540	502	7 %	390	408	4.6 %	311	326	4.8 %
4.85×10^{16}	470	409	13 %	360	348	3.3 %	285	288	1 %

major difficulty, to strong electric fields, following the same approach as Caughey and Thomas[1] or Canali *et al.*[14].

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