# Determination of diode parameters of a silicon solar cell from variation of slopes of the *I–V* curve at open circuit and short circuit conditions with the intensity of illumination

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### **Abstract**

An analytical method of determination of all the four diode parameters of the single exponential model of a silicon solar cell, namely shunt resistance  $R_{\rm sh}$ , series resistance  $R_s$ , diode ideality factor n and reverse saturation current  $I_0$  from the variation of slopes of the I-V curve of the cell near short circuit and open circuit conditions with intensity of illumination in a small range of intensity, is presented for the first time. In a suitable range of intensity the variation of dI/dV at short circuit enables determination of  $R_{\rm sh}$ , whereas the variation of dI/dV at open circuit enables determination of  $R_s$ , n and  $I_0$ . The diode parameters of a silicon solar cell were determined with this method using I-V characteristics of the cell in 40-125 mW cm<sup>-2</sup> intensity range of a simulated AM1.5 solar radiation. Theoretical I-V curves generated using so determined values of the diode parameters matched well with the experimental I-V curves of the cell obtained under various intensities of illumination in the above range.

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## Introduction

The steady state *I–V* characteristics of a p–n junction silicon solar cell are often described based on a single exponential model [1–4]:

$$I = -I_{\rm ph} + I_0 \left( e^{\frac{q(V - IR_s)}{nkT}} - 1 \right) + \frac{V - IR_s}{R_{\rm sh}}.$$
 (1)

In equation (1)  $I_{\rm ph}$  is the light-generated current, q is the electron charge, k is the Boltzmann constant, T is the temperature,  $R_{\rm sh}$  is the shunt resistance,  $R_s$  is the series resistance, n is the diode ideality factor and  $I_0$  is the reverse saturation current of the cell.  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  are diode parameters of the cells.

The performance of a solar cell is monitored through four performance parameters, i.e. open circuit voltage ( $V_{\rm oc}$ ), short circuit current ( $I_{\rm sc}$ ), curve factor (CF) and efficiency

 $(\eta)$  of the cell but the diode parameters decide the values of the performance parameters at a given intensity  $P_{\rm in}$  and temperature. Despite the diode parameters remaining constant, the values of performance parameters change as the intensity changes. In general,  $I_{\rm sc}$  increases linearly, whereas  $V_{\rm oc}$  increases logarithmically with  $P_{\rm in}$  [5–7]. However, CF and  $\eta$  first increase with  $P_{\rm in}$  and, then, decrease after a certain value of  $P_{\rm in}$  [5–7].

Many analytical methods have been developed to determine one or more of the diode parameters [1, 8–13]. However, none of the above analytical methods is able to determine all the diode parameters of a cell. Therefore, one has to generally use three or four methods to determine all the diode parameters. But since each method has a different physical basis they are rarely compatible. Consequently the values of  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  obtained using them are not likely to be correct values of the diode parameters to describe the I-V characteristics of the cell accurately.

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Some numerical or curve fitting techniques have been applied to extract all diode parameters from a single I–V curve obtained under illumination conditions [14–18]. Each of these methods [15–18] uses a single I–V curve and requires special computational knowledge to determine the values of all the diode parameters of a cell. These methods have a severe drawback in that using two close I–V curves (such as those obtained at two slightly different intensities of illumination), they give quite different values of the diode parameters and one is not sure about the correct values [18].

In this paper a new analytical method of measurement of all four diode parameters  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  of a single exponential model of a solar cell is presented. It is based on variation of the slope  ${\rm d}I/{\rm d}V$  of the I-V curve at short circuit and open circuit conditions with the change of incident light intensity and enables determination of not only  $R_{\rm sh}$  and  $R_s$  but also n and  $I_0$  of the cell accurately.

### Theoretical details

Let us assume that the values of diode parameters  $R_{\rm sh}$ ,  $R_{\rm s}$ , n and  $I_0$  remain constant with I and V. Then differentiating equation (1), we get

$$\frac{\mathrm{d}I}{\mathrm{d}V} = \frac{\left[\frac{1}{R_{\rm sh}} + \frac{q}{nkT} \left\{ I_{\rm ph} + I_0 - I \left( 1 + \frac{R_s}{R_{\rm sh}} \right) - \frac{V}{R_{\rm sh}} \right\} \right]}{1 + R_s \left[ \frac{1}{R_{\rm sh}} + \frac{q}{nkT} \left\{ I_{\rm ph} + I_0 - I \left( 1 + \frac{R_s}{R_{\rm sh}} \right) - \frac{V}{R_{\rm sh}} \right\} \right]}.$$
(2)

For the short circuit condition (V = 0,  $I = I_{sc}$ ), equation (1) gives

$$I_{\rm ph} + I_0 - I_{\rm sc} \left( 1 + \frac{R_s}{R_{\rm sh}} \right) = I_0 e^{\frac{qI_{\rm sc}R_s}{nkT}}$$
 (3)

and for the open circuit condition ( $V = V_{oc}$ , I = 0) it gives

$$I_{\rm ph} + I_0 - \frac{V_{\rm oc}}{R_{\rm sh}} = I_0 e^{\frac{qV_{\rm oc}}{nkT}}.$$
 (4)

Using equations (3) and (4) in equation (2) respectively and defining dI/dV at short circuit as  $m_{\rm sc}$  and at open circuit as  $m_{\rm oc}$  we obtain

$$m_{\rm sc} = \frac{\left[\frac{1}{R_{\rm sh}} + \frac{qI_0}{nkT} e^{\frac{qI_{\rm sc}R_{\rm s}}{nkT}}\right]}{\left[1 + R_{\rm s}\left\{\frac{1}{R_{\rm sc}} + \frac{qI_0}{nkT} e^{\frac{qI_{\rm sc}R_{\rm s}}{nkT}}\right\}\right]}$$
(5)

and

$$m_{\rm oc} = \frac{\left[\frac{1}{R_{\rm sh}} + \frac{qI_0}{nkT} e^{\frac{qV_{\rm oc}}{nkT}}\right]}{\left[1 + R_s \left\{\frac{1}{R_{\rm sh}} + \frac{qI_0}{nkT} e^{\frac{qV_{\rm oc}}{nkT}}\right\}\right]}.$$
 (6)

Equation (5) shows that  $m_{sc}$  is related to the diode parameters  $(R_{sh}, R_s, n \text{ and } I_0)$  and  $I_{sc}$  of the cell. Similarly equation (6) shows that  $m_{oc}$  is related to the above diode parameters and  $V_{oc}$  of the cell. Since both  $I_{sc}$  and  $V_{oc}$  depend on the intensity of illumination  $P_{in}$  we can expect both  $m_{sc}$  and  $m_{oc}$  to change with  $P_{in}$ , whether or not the diode parameters change with  $P_{in}$ . Thus, equations (5) and (6) can be used to determine the values of the above diode parameters of a cell by measuring the slopes  $m_{sc}$  and  $m_{oc}$  at different intensities in a small range of intensity, wherein the diode parameters remain constant.

In a solar cell, the excess carrier concentration (or in other words the injection level  $n_o$ ) increases linearly with  $P_{in}$ . Since

most cells are made using silicon of more than  $0.2 \Omega$  cm resistivity, the Auger recombination in them is negligible. In such cells the variations of  $\tau$  and  $D_n$  of minority carriers in p-base region of an n<sup>+</sup>-p-p<sup>+</sup> solar cell and hence of n and  $I_0$  with  $P_{\rm in}$  is expected to be negligible over one or two orders of magnitude of  $P_{\rm in}$  [19–24].

Because of the negligible contribution of the base region, the effect of  $P_{\rm in}$  on  $R_s$  of such cells is also insignificant. Therefore, it is fair to consider diode parameters as constants in a small  $P_{\rm in}$  range of less than one order of magnitude for cell operating under low-level conditions.

For some suitable  $P_{\rm in}$  range equations (5) and (6) may further be simplified if the following conditions are satisfied simultaneously:

$$\frac{qI_0}{nkT} e^{\frac{qI_{sc}R_s}{nkT}} \ll \frac{1}{R_{ch}} \tag{7}$$

and

$$\frac{qI_0}{nkT} e^{\frac{qV_{\text{oc}}}{nkT}} \gg \frac{1}{R_{\text{sh}}}.$$
 (8)

The left-hand side of inequality (7) stands for the diode conductance at short circuit condition, whereas, the left-hand side of inequality (8) stands for the diode conductance at open circuit condition.  $1/R_{\rm sh}$  represents the shunt conductance in both the inequalities. Thus, each of the above inequalities compares the diode conductance with the shunt conductance. In inequality (7) the diode conductance is much less than the shunt conductance, whereas in inequality (8) the diode conductance is much greater than the shunt conductance.

If we delineate the suitable  $P_{\rm in}$  range as  $P_{\rm in1} < P_{\rm in} < P_{\rm in2}$ , then, inequality (9) will define the higher intensity limit  $P_{\rm in2}$  and inequality (10) will define the lower intensity limit  $P_{\rm in1}$  of this suitable  $P_{\rm in}$  range. We will determine  $P_{\rm in1}$  and  $P_{\rm in2}$  values a little later.

Conditions (7) and (8) being valid equations (5) and (6) get simplified to give, respectively,

$$m_{sc}^{-1} = (R_{sh} + R_s) (9)$$

and

$$m_{\rm oc}^{-1} = \left[ R_s + \frac{nkT}{qI_0} \cdot e^{-\left(\frac{qV_{\rm oc}}{nkT}\right)} \right]. \tag{10}$$

For most cases of practical interests [6] one finds that

$$R_s/R_{\rm sh} \ll 1 \tag{11}$$

and

$$I_0 e^{\frac{qI_{\rm sc}R_s}{nkT}} \ll \left(I_{\rm sc} - \frac{V_{\rm oc}}{R_{\rm sh}}\right). \tag{12}$$

Using inequalities (11) and (12) equation (3) gives  $I_{sc} \approx I_{ph}$  and equations (3) and (4) yield

$$I_0 e^{\frac{qV_{\text{oc}}}{nkT}} = \left(I_{\text{sc}} - \frac{V_{\text{oc}}}{R_{\text{sh}}}\right). \tag{13}$$

Substituting equation (13) into equation (10) we get

$$m_{\rm oc}^{-1} = \left[ R_s + \frac{nkT}{q} \cdot \frac{1}{\left( I_{\rm sc} - \frac{V_{\rm oc}}{R_{\rm sh}} \right)} \right].$$
 (14)

Inequality (11) simplifies equation (9) to

$$R_{\rm sh} = m_{\rm sc}^{-1}$$
. (15)

Taking logarithm of both sides of equation (13) we also obtain the relation

$$V_{\rm oc} = \frac{nkT}{q} \cdot \ln\left(I_{\rm sc} - \frac{V_{\rm oc}}{R_{\rm sh}}\right) - \frac{nkT}{q} \cdot \ln(I_0). \tag{16}$$

Combination of equations (10) and (14)–(16) can be used to determine representative values of  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  of a cell from measurements of  $I_{\rm sc}$ ,  $V_{\rm oc}$ ,  $m_{\rm sc}$  and  $m_{\rm oc}$  at different intensities in a suitable  $P_{\rm in}$  range in which inequalities (7), (8), (11) and (12) are well satisfied.

# **Determination of suitable intensity range**

Let us denote  $V_{\text{oc}}$ ,  $I_{\text{sc}}$  values which correspond to low intensity limit  $P_{\text{in1}}$  as  $V_{\text{oc1}}$ ,  $I_{\text{sc1}}$  respectively and those which correspond to high intensity limit  $P_{\text{in2}}$  as  $V_{\text{oc2}}$ ,  $I_{\text{sc2}}$  respectively. Then using inequalities (7) and (8) we can write

$$\left[ \left( \frac{qI_0}{nkT} e^{\frac{qI_{sc2}R_s}{nkT}} \right) \middle/ \left( \frac{1}{R_{sh}} \right) \right] = \varepsilon_2$$
 (17)

and

$$\left[ \left( \frac{1}{R_{\rm sh}} \right) \middle/ \left( \frac{q I_0}{nkT} e^{\frac{q V_{\rm ocl}}{nkT}} \right) \right] = \varepsilon_1 \tag{18}$$

which imply that  $\varepsilon_1 \ll 1$  and  $\varepsilon_2 \ll 1$ .

In equation (18)  $\varepsilon_1$  represents the ratio of shunt conductance to the diode conductance at open circuit whereas  $\varepsilon_2$  represents the ratio of diode conductance at short circuit to the shunt conductance of the cell in equation (17). The value of  $\varepsilon_1$  indicates the fraction of the error that has been allowed in obtaining equation (9) from equation (5). Similarly, the value of  $\varepsilon_2$  indicates the fraction of the error that has been allowed in obtaining equation (10) from equation (6). Smaller the values of  $\varepsilon_1$  and  $\varepsilon_2$  better the validity of the method.

With assumptions (11) and (13), equation (3) yields  $I_{\rm ph} \approx I_{\rm sc}$  which indicates that the validity of the method is restricted to the range where  $I_{\rm sc}$  increases linearly with  $P_{\rm in}$ . The lower intensity limit  $P_{\rm in1}$  and the upper intensity limit  $P_{\rm in2}$  of suitable intensity range may also be defined in terms of  $I_{\rm sc1}$  and  $I_{\rm sc2}$  respectively.

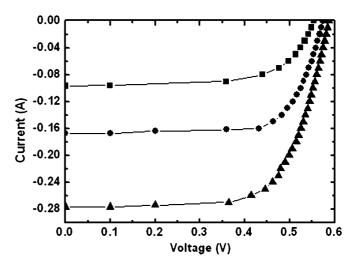
From equation (17) one can obtain  $I_{sc2}$  as

$$I_{\text{sc2}} = \frac{nkT}{qR_{\text{s}}} \cdot \ln \left[ \frac{1}{R_{\text{sh}}} \cdot \frac{nkT}{qI_0} \cdot \varepsilon_2 \right]. \tag{19}$$

Similarly combining equations (18) and (13) one can obtain  $I_{sc1}$  as

$$I_{\text{sc1}} = \frac{nkT}{qR_{\text{sh}}} \cdot \left[ \frac{1}{\varepsilon_1} + \ln \left( \frac{nkT}{qI_0\varepsilon_1R_{\text{sh}}} \right) \right]. \tag{20}$$

Thus, the suitable intensity range defined by  $P_{\rm in1} < P_{\rm in} < P_{\rm in2}$  may also be represented in terms of the valid  $I_{\rm sc}$  range as  $I_{\rm sc1} < I_{\rm sc} < I_{\rm sc2}$ .



**Figure 1.** Illuminated I-V characteristics of cell 1 at 25 °C under three different intensities of simulated AM1.5 solar radiations in the  $40-125 \text{ mW cm}^{-2}$  range.

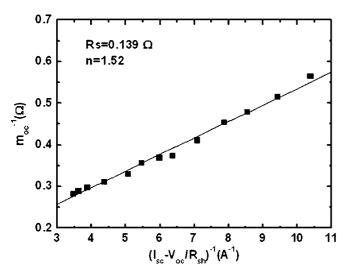
### **Experimental details**

The measurements for the present work were made on monocrystalline silicon (c-Si) solar cells of an  $\sim$ 8 cm<sup>2</sup> area which were fabricated using 300  $\mu$ m thick (100) oriented p-Cz silicon wafer of 1  $\Omega$  cm base resistivity and the p-n junction was made by P-diffusion using the POCl<sub>3</sub> liquid source. Front and back contacts were realized by screen printing Ag paste on front and Ag/Al paste on backsides of the cells. A single layer silicon nitride antireflection coating was given by a PECVD process using SiH<sub>4</sub>, NH<sub>3</sub> and N<sub>2</sub> gases. Illuminated I-V characteristics of the cells were measured at 25 °C under various intensities of illumination in 40–125 mW cm<sup>-2</sup> range of simulated AM 1.5 solar radiations. A Keithley 2420 system sourcemeter was used for measurement of I-V characteristics and the intensity was measured using a reference silicon solar cell obtained from PV Measurements, USA. In the following the results of measurements are reported for a silicon solar cell, cell 1 fabricated as described above.

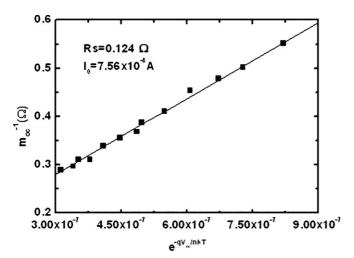
### **Result and discussion**

Three experimental I-V characteristics of cell 1 obtained at 25 °C under three different intensities of illumination are shown in figure 1. We note that the slope  $\mathrm{d}I/\mathrm{d}V$  of each curve near open circuit is distinctly different. A number of such I-V curves were obtained in 40–125 mW cm<sup>-2</sup> intensity range and were used to determine,  $m_{\rm sc}$ ,  $m_{\rm oc}$ ,  $V_{\rm oc}$  and  $I_{\rm sc}$  values for each curve which were in turn used to determine the diode parameters of the cell. The value of  $m_{\rm sc}$  was nearly invariant with intensity and thereby yielded a constant value of  $R_{\rm sh}$  according to equation (15). However, the values of  $R_{\rm s}$ , n and  $I_0$  could be determined from two different approaches A and B.

In approach A first the values of  $m_{\rm oc}^{-1}$  are plotted against  $(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  as shown in figure 2 and are fitted into a straight line represented by equation (14). The intercept of the straight line on the  $m_{\rm oc}^{-1}$  axis gave  $R_s$  and the slope of the line



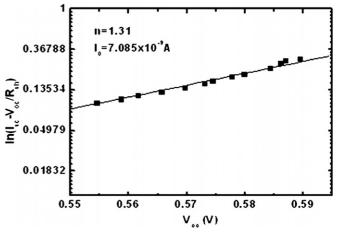
**Figure 2.** Plot of  $m_{\rm oc}^{-1}$  versus  $(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  for cell 1 at  $T=25\,^{\circ}{\rm C}$  and  $40 < P_{\rm in} < 125\,{\rm mW~cm^{-2}}$ . The solid line gives a straight line fit to the data. The intercept of the line on the  $m_{\rm oc}^{-1}$  axis gives  $R_s$  and the slope of the line from the  $(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  axis determines n.



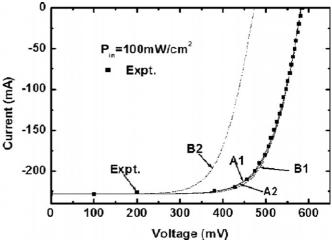
**Figure 3.** Plot of  $m_{\rm oc}^{-1}$  versus  ${\rm e}^{-qV_{\rm oc}/nkT}$  at  $T=25\,^{\circ}{\rm C}$  with n=1.52; n was obtained from figure 2. The solid line gives a straight line fit to the data. The slope of straight line with the  ${\rm e}^{-qV_{\rm oc}/nkT}$  axis determines  $I_0$  value.

from the  $(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  axis yielded the value of nkT/q. The value of n thus obtained was used in equation (10) and, then, the plot of the  $m_{\rm oc}^{-1}$  versus  ${\rm e}^{-qV_{\rm oc}/nkT}$  data and their subsequent fit into a straight line as shown in figure 3 yielded the value of  $I_0$  from its slope  $nkT/qI_0$  with the  ${\rm e}^{-qV_{\rm oc}/nkT}$  axis. The intercept of the  $m_{\rm oc}^{-1}$  versus  ${\rm e}^{-qV_{\rm oc}/nkT}$  line on the  $m_{\rm oc}^{-1}$  axis of figure 3 also gave a value of  $R_s$ . This approach determines values of all four diode parameters using  $m_{\rm sc}$  and  $m_{\rm oc}$  values.

In approach B we first determined  $R_{\rm sh}$  from  $m_{\rm sc}$  data using equation (15) as discussed earlier. Then, we determined  $I_0$  and n from a straight line obtained by plotting  $V_{\rm oc}$  against  $\ln(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})$  according to equation (16) as shown in figure 4. Subsequently, the value of n so obtained was used in equation (14) to determine  $R_s$  from the  $m_{\rm oc}^{-1}$  versus  ${\rm e}^{-q V_{\rm oc}/nkT}$  straight line graph similar to that of figure 3 (but not shown here) which also gave a value of  $I_0$  from the slope. Thus, in



**Figure 4.** Plot of the ln  $(I_{\rm sc}-V_{\rm oc}/R_{\rm sh})$  versus  $V_{\rm oc}$  axis for cell 1 at  $T=25~{\rm ^{\circ}C}$  and  $40 < P_{\rm in} < 125~{\rm mW~cm^{-2}}$ .  $I_0$  was determined from the intercept on the ordinate and n from the slope with the  $V_{\rm oc}$  axis.



**Figure 5.** Plot of experimental and theoretical I-V curves for cell 1 at  $T=25\,^{\circ}\text{C}$ . Theoretical curves  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  have been generated using the values of set  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  respectively given in table 2.

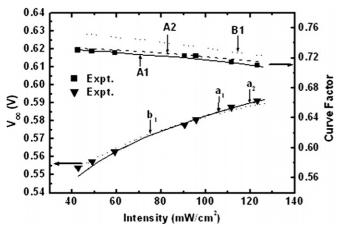
approach B, the  $m_{oc}$  data were not used for determination of n of the cell.

The values of  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  determined by the two approaches for cell 1 are given in table 1. It shows that approach A gave two different values of  $R_s$ , whereas approach B gave two different values of  $I_0$ . From table 1 four sets of values of the four diode parameters could be formed. These sets of values of  $R_{\rm sh}$ ,  $R_s$ , n and  $I_0$  are listed in table 2.

Four theoretical I-V curves have been obtained using the four different sets of diode parameters of table 2 in equation (1) for  $I_{sc} = 228.6$  mA that corresponded to 100 mW cm<sup>-2</sup> intensity of the incident simulated AM1.5 solar radiation. These curves are plotted in figure 5 along with the experimental curve. Theoretical curves  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  correspond respectively to sets  $A_1$ ,  $A_2$ ,  $B_1$  and  $B_2$  of diode parameters listed in table 2. It can be noted that curves  $A_1$ ,  $A_2$  and  $B_1$  are quite close to the experimental curves, whereas curve  $B_2$  is far apart. This indicates that equations (10), (14)–(16) and the

Value Approach A Approach B Equation (14) Equation (10) Equation (16) Equation (10) Diode parameters Equation (15)  $(R_{\rm sh} = 998)$ (n = 1.52)Equation (15)  $(R_{\rm sh} = 998)$ (n = 1.31)998 998  $R_{\rm sh}\left(\Omega\right)$  $R_{\rm sh}\left(\Omega\right)$ 0.139 0.124 0.153 1.31 1.52  $7.56 \times 10^{-8}$  $1.90 \times 10^{-7}$  $7.09 \times 10^{-9}$  $I_0(A)$ 

**Table 1.** The values of diode parameters at T = 25 °C determined for cell 1 using approaches A & B.



**Figure 6.** Variation of experimental and theoretical  $V_{\rm oc}$  and CF at  $T=25\,^{\circ}{\rm C}$  for cell 1 with intensity in  $40 < P_{\rm in} < 125\,{\rm mW~cm^{-2}}$  range. Theoretical curves  $(a_1, A_1)$ ,  $(a_2, A_2)$  and  $(b_1, B_2)$  are based on sets  $A_1$ ,  $A_2$  and  $B_1$  of table 2.

assumptions made in obtaining them have been fairly valid in cases of the  $A_1$ ,  $A_2$  and  $B_1$  curves.

A closer look at the curves of figure 5 reveals that curve  $A_1$  matches best with the experimental curve. This is evident also from the comparison of  $V_{\rm oc}$  and CF values of the theoretical curves with the experimental curve of cell 1 at  $I_{\rm sc}=228.6$  mA and T=25 °C as given in table 3. From table 3 we observe that for curve  $B_2$  the value of CF is comparable with the experimental CF value. However, the theoretical  $V_{\rm oc}$  value for curve  $B_2$  is very small in comparison with the experimental  $V_{\rm oc}$  value. It shows that closeness of two I-V curves is revealed by  $V_{\rm oc}$  and CF values together. Matching only CF values without  $V_{\rm oc}$  may be misleading.

Similar observations were made by comparing theoretical and experimental I–V curves of cell 1 at other intensities. Figure 6 depicts variation of theoretical and experimental values of  $V_{\rm oc}$  and CF of cell 1 at T=25 °C with  $P_{\rm in}$  in the 40–125 mW cm<sup>-2</sup> range. There are three theoretical curves each for  $V_{\rm oc}$  and CF. The CF versus  $P_{\rm in}$  curves  $A_1$ ,  $A_2$  and  $B_1$  have been obtained using the diode parameters of sets  $A_1$ ,  $A_2$  and  $B_1$  respectively. Similarly, the  $V_{\rm oc}$  versus  $P_{\rm in}$  curves  $a_1$ ,  $a_2$  and  $b_1$  correspond to the diode parameters of set  $A_1$ ,  $A_2$  and  $B_1$  respectively. The  $V_{\rm oc}$  versus  $P_{\rm in}$  curves match reasonably well with the experimental values. Curves  $a_1$  and  $a_2$  are superposed. This is because for both these curves the values of the diode parameters  $R_{\rm sh}$ , n and  $I_0$  that together decide the value of  $V_{\rm oc}$  are same. Theoretical curve  $A_1$  matches closely with the

**Table 2.** Four sets of values of diode parameters of cell 1 at T = 25 °C formed using table 1.

Diode	Set						
parameters	$\overline{A_1}$	$A_2$	$\mathbf{B}_1$	$\mathbf{B}_2$			
$R_{sh}(\Omega)$	998	998	998	998			
$R_s(\Omega)$	0.139	0.124	0.153	0.153			
n	1.52	1.52	1.31	1.31			
$I_0 (A) \times 10^{-8}$	7.56	7.56	0.709	190			

experimental CF values. Curve  $A_2$  lies slightly above  $A_1$  indicating CF values that are not very significantly higher than the experimental values. On the other hand, curve  $B_1$  shows much higher CF values than the experimental ones.

From above it becomes clear that approach A which uses the slopes of I–V curves at short circuit and open circuit conditions along with the  $I_{\rm sc}$  values of the cell at different intensities provides most realistic values of diode parameters in the form of set  $A_1$  using combination of equation (15) for  $R_{\rm sh}$ , equation (14) for  $R_{\rm s}$  and n and equation (10) for  $I_0$ . Table 4 shows that the assumptions (i.e. inequalities (7), (8), (11) and (12)) have been fairly valid for cell 1 in the 40–125 mW cm<sup>-2</sup> intensity range.  $P_{\rm in}=40$  mW cm<sup>-2</sup> corresponded to  $\varepsilon_1=0.0027$ , whereas  $P_{\rm in}=125$  mW cm<sup>-2</sup> corresponded to  $\varepsilon_2=0.0053$ . Such small values of  $\varepsilon_1$  and  $\varepsilon_2$  in comparison with unity speak of excellent accuracy of the method in determining the diode parameters of cell 1.

With diode parameter values  $R_{\rm sh}=998~\Omega$ ,  $R_s=0.139~\Omega$ , n=1.52 and  $I_0=7.56\times 10^{-8}$  A corresponding to curve  $A_1$ , the lower and upper intensity limits were calculated for  $\varepsilon_1=0.01$  and  $\varepsilon_2=0.01$  using equations (20) and (19) respectively.  $\varepsilon_1=0.01$  gave  $I_{\rm sc1}=4.37$  mA that corresponded to  $P_{\rm in1}=1.91$  mW cm<sup>-2</sup>. Likewise  $\varepsilon_2=0.01$  gave  $I_{\rm sc2}=467$  mA that corresponded to  $P_{\rm in2}=204$  mW cm<sup>-2</sup>. Thus, allowing  $\varepsilon_1=\varepsilon_2=0.01$  the valid intensity range for application of the present method is 1.91-204 mW cm<sup>-2</sup>. This shows that the  $P_{\rm in}$  range 40-125 mW cm<sup>-2</sup> used for measurement in this work has been well within the valid  $P_{\rm in}$  range. We shall henceforth refer approach A to determine the values of diode parameters as in set  $A_1$  of table 2 as our method.

## Comparison with other methods

Agarwal *et al* method [12] was not applicable for measurement of  $R_s$  for cell 1 in 40–125 mW cm<sup>-2</sup> range as this method [12] required  $I_{sc}$  to be significantly lower than  $I_{ph}$  which was not

**Table 3.** Comparison of values of  $V_{oc}$  and CF for experimental and theoretical I-V curves of cell 1 at 25 °C obtained under a simulated AM1.5 solar radiation of 100 mW cm<sup>-2</sup> intensity, using sets of diode parameters given in table 2.

$P_{\rm in}$	P <sub>in</sub> Value for curves					
$(mW cm^{-2})$	Parameter	Experimental	$A_1$	$A_2$	$\mathbf{B}_1$	$B_2$
100	V <sub>oc</sub> (mV) CF	584.4 0.718		583.1 0.721	584 0.731	471.5 0.691

**Table 4.** The validity of assumptions for cell 1 at T = 25 °C and  $40 < P_{\rm in} < 125$  mW cm<sup>-2</sup> intensity range.

	$R_S \ll$	$R_{ m sh}$	$\frac{qI_0}{nkT} e^{\frac{qI_{\rm Sc}R_s}{nkT}} \ll \frac{1}{R_{\rm sh}}$		$\frac{qI_0}{nkT} e^{\frac{qV_{\text{oc}}}{nkT}}$	$\Rightarrow \frac{1}{R_{\rm sh}}$	$I_0 e^{\frac{qI_{\rm Sc}R_{\rm S}}{nkT}} \ll \left(I_{\rm sc} - \right)$	$-\frac{V_{\rm oc}}{R_{\rm sh}}$
Curve	$R_s$ $(\Omega)$	$R_{ m sh}$ $(\Omega)$	$\frac{\frac{qI_0}{nkT}}{nkT}e^{\frac{qI_{SC}R_S}{nkT}}$ $(\Omega^{-1})$	$\frac{\frac{1}{R_{sh}}}{(\Omega^{-1})}$	$\frac{qI_0}{nkT} e^{\frac{qV_{\text{OC}}}{nkT}}$ $(\Omega^{-1})$	$\frac{\frac{1}{R_{sh}}}{(\Omega^{-1})}$	$I_0 e^{qI_{sc}R_s}_{nkT}$ (A)	$ \frac{\left(I_{\rm sc} - \frac{V_{\rm oc}}{R_{\rm sh}}\right)}{(A)} $
$A_1$ $A_2$ $B_1$	0.139 0.124 0.153	998 998 998	$\begin{array}{c} (2.71\times10^{-6}5.32\times10^{-6}) \\ (2.61\times10^{-6}4.77\times10^{-6}) \\ (3.24\times10^{-7}7.67\times10^{-7}) \end{array}$	0.001 0.001 0.001	(2.56–6.21) (2.56–6.21) (2.67–7.47)	0.001 0.001 0.001	$(1.06 \times 10^{-7} - 2.08 \times 10^{-7})$ $(1.03 \times 10^{-7} - 1.87 \times 10^{-7})$ $(1.1 \times 10^{-8} - 2.6 \times 10^{-8})$	(0.097–0.288) (0.097–0.288) (0.097–0.288)

**Table 5.** Measured diode parameters of cell 1 using other methods and sets of four diode parameters formed with their different combinations.

	Method			Sets of diode parameters				
Diode parameters	[1]	[9]	[10]	[13]	C [1, 9]	D [9, 10]	E [9, 13]	F [9, 10, 13]
$R_{\rm sh}\left(\Omega\right)$	- 0.265	998	12.91 0.354	- 0.130	998 0.265	12.91 0.354	998 0.130	12.91 0.130
$R_s(\Omega)$	- -	1.31	- -	1.405	1.31	1.31	1.405	1.405
$I_0(A) \times 10^{-8}$	_	0.709	_	_	0.709	0.709	0.709	0.709

the case. Similarly, the values of  $R_s$  could not be determined using Priyanka et al's method [9] since we have restricted the I-V measurement on cell 1 to the 4th quadrant and have not measured the I–V characteristics in the 3rd quadrant. However, we have applied the methods of [1, 9, 10, 13] to the I-V data of cell 1 in 40-125 mW cm<sup>-2</sup> intensity range to determine the diode parameters. These include the Araujo and Sanchez method [1] for  $R_s$ , Cueto method [13] for  $R_s$  and n, El-Adawi and Al-Nuaim method [10] for  $R_{sh}$  and  $R_s$  and Priyanka et al's method [9] for  $R_{sh}$ ,  $I_o$  and n. Following the authors [1, 10] we have also assumed  $R_{\rm sh} = \infty$  and n = 1 while applying the Araujo and Sanchez method [1] and  $R_{\rm sh} = \infty$ while applying the Cueto method [13]. On the other hand, we have assigned the values to n and  $I_0$  as 1.52 and 7.56  $\times$  10<sup>-8</sup> A, respectively (from table 2, set A of our method) when applying the El-Adawi and Al-Nuaim method [10]. The values of diode parameters determined by these methods [1, 9, 10, 13] have been listed in table 5.

A comparison of tables 5 and 2 (set A1) shows that the values of diode parameters obtained by the above methods [1, 9, 10, 13] are significantly different than the values ( $R_{\rm sh}$  = 998  $\Omega$ ,  $R_s$  = 0.139  $\Omega$ , n = 1.52 and  $I_0$  = 7.56 × 10<sup>-8</sup> A) obtained by our method. The only exception is the value of  $R_{\rm sh}$  determined by Priyanka *et al*'s method [9] which is same as obtained by our method. This is because the method of Priyanka *et al* [9] and our method are equivalent and use the same expression (as given by equation (15)) for determination

of  $R_{\rm sh}$  from the slope of illuminated I-V curve at the origin. On the other hand, the expressions used for determination of  $R_s$ ,  $I_o$  and n in Priyanka *et al*'s method [9] and all the diode parameters ( $R_{\rm sh}$ ,  $R_s$ ,  $I_o$  and n) in the other methods [1, 10, 13] are quite different than those used in our method.

The values of  $R_{\rm sh}$ ,  $R_{\rm s}$ ,  $I_o$  and n obtained by the above different methods and listed in table 5 were grouped to form four different sets of diode parameters. These sets (C, D, E, F) were then used in equation (1) to compute  $V_{\rm oc}$  and CF values corresponding to the  $I_{\rm sc}=228.6$  mA values of an experimental  $I\!-\!V$  curve of cell 1. The theoretical  $V_{\rm oc}$  and CF values obtained using sets C, D, E and F are listed in table 6 along with the experimental values and the values obtained with our method (approach A, set  $A_1$ ). It can be noted that all the sets C, D, E, F give values of  $V_{\rm oc}$  and CF which are too deviated from the experimental values, whereas our method gives the values that match excellently with the experimental  $V_{\rm oc}$  and CF values.

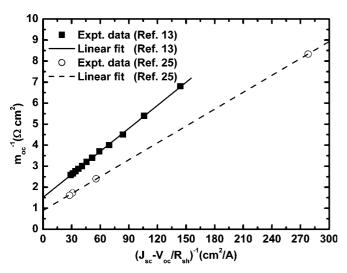
Thus, clearly our method (approach A, set  $A_1$ ) is more accurate in comparison with the other analytical methods [1, 9, 10, 13] and is superior to them because it determines all the four diode parameters independently and without depending on any other method.

### **Application of the method to other cells**

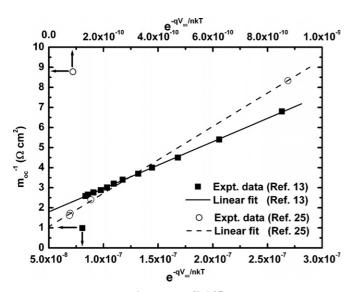
We have applied our method to determine the values of diode parameters of solar cells of a PV module 2577 of

**Table 6.** Comparison of values  $V_{oc}$  and CF for experimental curve and the theoretical I-V curves of cell 1 at 25 °C obtained using sets of diode parameters given in table 5 under a simulated AM1.5 solar radiation of 100 mW cm<sup>-2</sup> intensity.

Performance	Values for sets					
parameters	Experimental	A1	С	D	Е	F
V <sub>oc</sub> (mV) CF	584.4 0.718	583.2 0.717	581.9 0.696	575.3 0.572	623.8 0.745	616.1 0.620

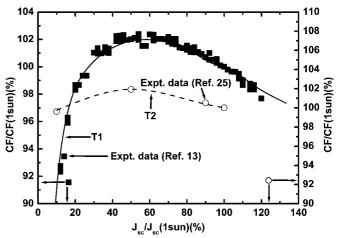


**Figure 7.** Plots of the  $m_{\rm oc}^{-1}$  versus  $(J_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  curves using data of [13] ( $\blacksquare$ ) and [25] ( $\circ$ ). The solid line gives a straight line fit to the data of 13 and the dashed line gives the straight line fit to the data of [25].



**Figure 8.** Plots of the  $m_{\text{oc}}^{-1}$  versus  $e^{-qV_{\text{oc}}/nkT}$  curves using the data of [13] ( $\blacksquare$ ) and [25] ( $\circ$ ). The solid line gives a straight line fit to the data of [13] and the dashed line gives the straight line fit to the data of [25].

Cueto [13] and a silicon solar cell of Mette *et al* [25]. The measurements of  $(m_{oc})^{-1}$ ,  $J_{sc}$ ,  $V_{oc}$  and CF by Cueto [13] on silicon PV modules and J-V characteristics by Mette *et al* [25] on silicon solar cells were done in 10–120 mW cm<sup>-2</sup> intensity range. Here, we determine the slope of J-V curves at open



**Figure 9.** Comparison of theoretical and experimental variations of normalized CF of a PV module 2577 of [13] and that of a silicon solar cell of [25] with their respective normalized  $J_{\rm sc}$  values. The values of CF and  $J_{\rm sc}$  have been normalized with respect to their values at 1sun. The 1 sun value of  $J_{\rm sc}$  was 30 mA cm<sup>-2</sup> for an average cell of PV module 2577 of [13] and 36.3 mA cm<sup>-2</sup> for cell of [25]. Symbol ( $\blacksquare$ ) denotes the experimental data of [13] and symbol ( $\circ$ ) denotes the experimental data of [25]. Theoretical curves T1 (——) and T2 (- – ) have been generated by applying the present method to the data of [13] and [25] respectively.

circuit and short circuit conditions where J is current density (A cm<sup>-2</sup>) Consequently, we obtain  $R_{\rm sh}$ ,  $R_{\rm s}$  in the units of  $\Omega$  cm<sup>2</sup> and reverse saturation current density  $J_0$  (in the units of A cm<sup>-2</sup>). The values of these diode parameters are listed in table 7. Following the observation of Cueto [13] the values of  $R_{\rm sh}$  were taken to be 1000  $\Omega$  cm<sup>2</sup> for the cells of PV module 2577, whereas for cells of [25]  $R_{\rm sh}$ was determined from the slope of J-V curve at short circuit conditions. Then values of  $R_s$  and n were determined from the  $m_{\rm oc}^{-1}$  versus  $(J_{\rm sc}-V_{\rm oc}/R_{\rm sh})^{-1}$  curves as plotted in figure 7 and the values of  $J_o$  were determined from the slopes of the  $m_{\rm oc}^{-1}$  versus  ${\rm e}^{-qV_{\rm oc}/nkT}$  curves shown in figure 8. Thus, determined values of the diode parameters  $(R_{sh}, R_s, n, J_o)$  were used to generate J-V curves theoretically and thereby calculate the values of CF in the two cases. The theoretical and experimental values of CF values have been plotted in figure 9 against the  $J_{\rm sc}$  values. Both the CF and  $J_{\rm sc}$  values have been normalized with respect to their 1 sun values. The 1 sun values of  $J_{\rm sc}$  were 30 mA cm<sup>-2</sup> for an average cell of PV module 2577 of [13] and 36.3 mA cm<sup>-2</sup> for cell of [25]. Theoretical curve T1 has been computed using the diode parameters determined by the present method using the data of an average cell of PV module 2577 of [13]. It can be noted that curve T1 matched very well the experimental CF data of PV module 2577 of [13] over the

**Table 7.** The values of diode parameters determined by our method using a silicon PV module no 2577 of [13] and a silicon solar cell of [25].

Silicon solar cell	$R_{\rm sh}$ $(\Omega~{ m cm}^2)$	$R_s$ ( $\Omega$ cm <sup>2</sup> )	n	$J_0  ({ m A \ cm^{-2}})$
[13]	1000	1.514	1.415	$2.441 \times 10^{-9}  3.215 \times 10^{-12}$
[25]	3300	0.894	1.043	

entire  $J_{\rm sc}$  range of the study. Similarly, the theoretical curve T2 matched well with the experimental CF data of [25] over the entire range of  $J_{\rm sc}$ . Thus, it is established that our method (approach A, set  $A_1$ ) of measurement of diode parameters is applicable equally well to the isolated cells and the cells of a PV module.

### Conclusion

We have evolved an analytical method of determining the representative values of all the diode parameters of a silicon solar cell using the values of slopes  $m_{\rm sc}$  and  $m_{\rm oc}$  of the I-Vcurves of cell at different intensities of the incident radiation in suitable intensity range. A combination of equations (15), (14) and (10) can be applied with high accuracy to determine values of  $R_{\rm sh}$ ,  $R_{\rm s}$ , n and  $I_0$  of a silicon solar cell analytically (with approach A, set  $A_1$ ) in the  $P_{in}$  intensity range wherein  $I_{\rm sc}$  increases linearly with  $P_{\rm in}$  and both  $\varepsilon_1$  and  $\varepsilon_2$  have values much smaller than unity. Smaller the values of  $\varepsilon_1$  and  $\varepsilon_2$  better the accuracy of application of the method. For  $\varepsilon_1 = \varepsilon_2 = 0.01$ the valid  $P_{\rm in}$  range for cell 1 consisted of 1.91–204 mW cm<sup>-2</sup>. Thus, the  $P_{\rm in}$  range 40–125 mW cm<sup>-2</sup> used for measurement of diode parameters of cell 1 in this work has been well within the valid  $P_{\rm in}$  range. The theoretical I-V curves and CF values obtained using diode parameters determined with this method matched excellently with the experimental I-V curves and CF values of the cells obtained at different intensities in the 40–125 mW cm<sup>-2</sup> range.

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