Expert Systems With Applications 237 (2024) 121597

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Human memory optimization algorithm: A memory-inspired optimizer for global optimization problems

Donglin Zhu [a,1], Siwei Wang [a,2], Changjun Zhou [a,*,3], Shaoqiang Yan [b,4], Jiankai Xue [c,*,5]

[a] College of Mathematics and Computer Science, Zhejiang Normal University, Jinhua 321004, China
[b] Xi'an Research Institute of High Technology: PLA Rocket Force University of Engineering, Xi'an, Shaanxi 710025, China
[c] College of Information Sciences and Technology, Donghua University, Shanghai 201620, China

ARTICLE INFO

Keywords:
Heuristic algorithms
Human memory optimization algorithms
CEC 2013
Engineering optimization problems

ABSTRACT

With the progress of science and technology, optimization problems have become complex. Meta-heuristic algorithms have the advantages of high efficiency and strong global search ability in solving optimization problems, so more and more meta-heuristic algorithms are proposed and investigated deeply, and how to improve the universality of an algorithm is an important issue. In this paper, propose an optimization algorithm that simulates human memory behaviour, fully known as human memory optimization Algorithm, abbreviated as HMO, which simulates the way humans behave in production, stores human preferences for success and failure, simulates the way humans behave in their memory, gradually moving towards better directions and outcomes to find a reasonable optimal solution. The results were compared with other meta-heuristic algorithms in the CEC 2013 test set and showed that HMO has better optimization capabilities, and the feasibility of the algorithm was verified from convergence analysis and parametric analysis experiments. In three engineering optimization problems, HMO was able to find optimal solutions within a reasonable range of parameters, verifying the practicality of HMO.

1. Introduction

With the rapid development of technology at home and abroad, human life has become more colorful and therefore involves more optimization problems; how to use limited storage space and computational resources to solve these optimization problems more appropriately is the current focus of scientific research. In everyday life, optimization problems can be found everywhere. For example, how to find the shortest route in the path planning of drones while ensuring safety (Phung & Ha, 2021; Puente-Castro et al., 2022); how to plan rationally in production scheduling to improve efficiency and save human and material resources (Wei et al., 2021; Zhao et al., 2021); how to place nodes to maximize coverage in layout optimization (Yin, Deng & Zhang, 2022; Deepa & Venkataraman, 2021); and so on, such as transportation scheduling (Abosuliman & Almagrabi, 2021; Wu et al., 2021), and there are numerous optimization problems in areas such as image processing (Zhu et al., 2023; Liang, Qin & Zhou, 2022), neural networks (Chen et al., 2022; Zhang et al., 2022), etc. their time complexity grows exponentially with the size of the problem so that simple methods such as exhaustive search are no longer effective.

Over time, many scholars have been exploring suitable solutions to such problems. In the early days, they proposed some traditional optimization algorithms such as hill climbing, Newton's method, sequence comparison, gradient descent, etc. These algorithms require that the functions and parameters of the optimization problem must be specifically given. Although they are effective in solving convex optimization and simple linear or continuum problems, most real-world optimization problems are large, non-linear, and multi-modal in nature; traditional optimization methods have inadequate global search capabilities when solving such complex optimization problems, and suffer from

* Corresponding authors.
E-mail addresses: zhouchangjun@zjnu.edu.cn (C. Zhou), xuejiankai@mail.dhu.edu.cn (J. Xue).
[1] ORCID: 0000-0001-9868-8103.
[2] ORCID: 0009-0001-9661-6699.
[3] ORCID: 0000-0002-6787-5325.
[4] ORCID: 0000-0002-4076-4641.
[5] ORCID: 0009-0003-5468-2545.

https://doi.org/10.1016/j.eswa.2023.121597
Received 17 March 2023; Received in revised form 10 September 2023; Accepted 10 September 2023
Available online 16 September 2023

Fig. 1. Human memory.

shortcomings such as low solution accuracy and low optimization search efficiency (Wu et al., 2015).

Meta-heuristic algorithms originate from the study of the group behavior of natural animals such as ant colonies, bee colonies, and bird flocks (Seyyedabbasi & Kiani, 2023), and are characterized by collaborative cooperation, individual competition, self-organization, and self-adaptation. The characteristics of swarm intelligence have opened up new ways to solve optimization problems, and therefore many swarm intelligence optimization algorithms based on swarm intelligence theory have been born, and these algorithms have received a lot of attention and extensive research from researchers at home and abroad, and gradually become a common method for solving complex optimization problems. The advantages of swarm intelligence algorithms, which effectively integrate swarm intelligence and optimization theory, lie in the following aspects: 1) they are simple and easy to implement; 2) they have good stability and robustness; and 3) they are scalable. In addition, swarm intelligence algorithms do not require in-depth analysis of the solution problem, do not depend on the specific solution form, and have a better ability to find the best in solving the problem, while having strong parallelism and a certain degree of stability, the more classic are ant colony optimization (ACO) (Dorigo, Birattari & Stutzle, 2006), particle swarm optimization (PSO) (Kennedy & Eberhart, 1995), artificial bee colony algorithm (ABC) (Karaboga & Basturk, 2008) and so on. However, as the difficulty of solving complex optimization problems increases, these algorithms need to be improved in terms of their optimization ability.

In recent years, to better solve these complex optimization problems, scholars have proposed a series of optimization algorithms, such as the whale optimization algorithm (WOA) (Mirjalili & Lewis, 2016), the Grey Wolf Optimizer (Mirjalili, Mirjalili & Lewis, 2014), the sparrow search algorithm (SSA) (Xue & Shen, 2020), and the Harris hawks optimization (HHO) (Heidari et al., 2019), the Dung beetle optimizer (DBO) (Xue & Shen, 2023), the Manta ray foraging optimization (MRFO) (Zhao, Zhang & Wang, 2020), and others. All of these algorithms beat the classical ones in their respective original texts and are also evolving. Numerous studies have shown that all these algorithms have their shortcomings, for example, the whale optimization algorithm suffers from the problem of falling into local optimum and slow convergence speed, Sanjoy-Chakraborty et al. introduced Symbiotic Organisms Search (SOS) (Chakraborty et al., 2021) to enhance the ability of the whale algorithm to jump out of local optimum and improve the convergence speed; Chi Ma et al. introduced the idea of Aquila Optimizer (AO) to improve the GWO algorithm (Ma et al., 2022); the sparrow search algorithm has low convergence accuracy, and Jiankai Xue introduced neighborhood search and saltation learning to update the position of sparrow (Xue, Shen & Pan, 2023). Currently, more and more algorithms are being improved and proposed to deal with different complex problems. According to the theorem that there is no free lunch in the world (Wolpert & Macready, 1997), no single algorithm can show better optimization capability in any optimization problem, so proposing a universal algorithm with better applicability is the goal we are pursuing.

Taking the minimization problem as an example, this paper develops a heuristic algorithm inspired by human memory behavior. Human memory is behavior is multi-modal, and people are often impressed by failures as well as successes that guide future behavior, at the same time, humans will recall recent memories from time to time, but they will also forget recent memories. A specific schematic is shown in Fig. 1.The person in (a) searches everywhere for his mobile phone, but it turns out to be in his own hand, which is the process of temporary forgetting of human memory; the person in (b) has been bitten by a snake before, so it is very afraid of snakes, and that painful memory is very deep. Inspired by human memory behavior, this paper proposes a human memory optimization algorithm, referred to as HMO, which stores the memories of successes and failures generated by human behavior and recalls them through a human-specific mindset, thus continuously motivating people towards a better place. A comparison with various heuristic algorithms in CEC 2013 shows that HMO is more general and has better optimization capabilities; in addition, HMO has better optimization results in a variety of engineering cases, verifying the practicality and feasibility of HMO.

The paper is structured as follows: Section 2 focuses on the description and analysis of the HMO; Section 3 tests the HMO and other algorithms in the CEC 2013 test set; Section 4 tests each algorithm on three engineering optimization problems; and the final section provides an analytical discussion of the experiments in this paper and concludes with directions for future work.

2. Human memory optimization algorithm

In this section, a new swarm intelligence optimization method named the HMO algorithm is discussed, including the following two aspects: 1) Inspiration. 2) the mathematical model.

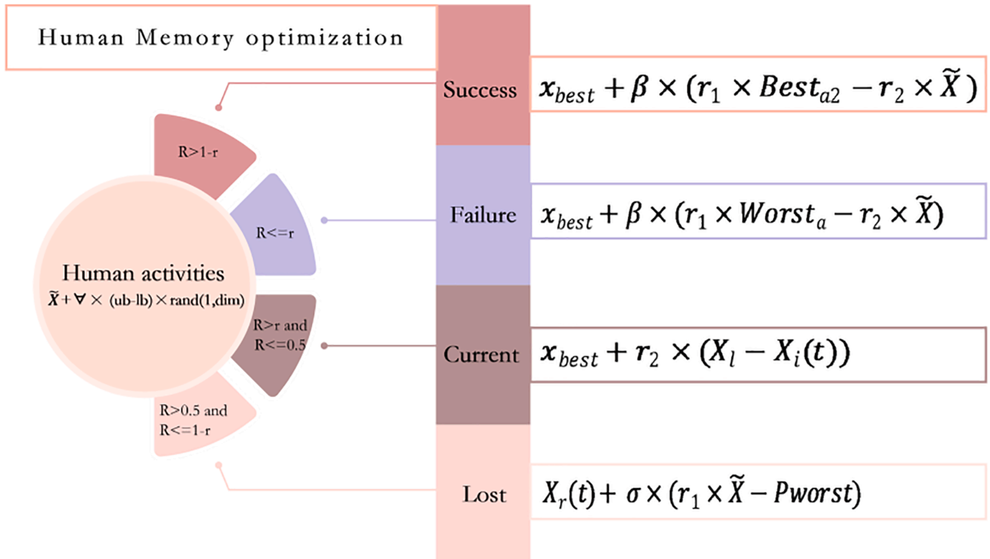

Fig. 2. Principle of the HMO.

2.1. Inspiration

As people go about their daily lives, constantly performing activities and producing records of events that form memories, it is also through the constant recollection of past events that we guide our present behavior. In most cases, mistakes made in the past we try to avoid making the same mistakes. On the other hand, what we have achieved in the past needs to be built upon. It has been shown that humans remember unpleasant as well as pleasant events (LaBar, 2007) and that such memories are retained in the brain for a long time, while in the recent past, we still miss them, and Kornell et al. also show that our memory is biased (Kornell & Bjork, 2009), especially in the recent ordinary time, which is more likely to be missed. life and behavior (Mahr & Csibra, 2020). It is because of these memories to influence that we are continually guided towards a better direction through the experiences generated by memory, working together to promote human development through alternating cycles of failure and success.

Based on the above ideas and inspiration,this paper propose a human memory optimization algorithm, which gradually approaches the optimal solution to the optimization problem by constantly updating new search direction through human behavioral activities and recall. The specific schematic diagram is shown in Fig. 2. The HMO model has the following main stages:

- Humans need to produce memories or events through productive activities.
- Failures and successes are especially remembered, and people will continue to recall these events. It is worth noting that failed events are those experiences that are painful and successful events are those that are joyful.
- Humans may be able to recall or forget current memories.
- The process of remembering is also a human activity that can also produce memories or events.
- The mood is different when recalling different events.

2.2. The proposed algorithm

2.2.1. Human activities

According to the above discussion, we know that people will create memories and store them all the time during production activities. In order to simulate this behavior of humans, the updating of the memory production rules of each person can be expressed as:

$$\begin{aligned} X_i(t+1) &= \widetilde{X} + \forall \times (ub - lb) \times rand(1, dim) \\ \forall &= k \times sin(1/k) \times b \\ \mathrm{b} &= 2 \times (1 - \mathrm{t}/\mathrm{T}) \end{aligned} \tag{1}$$

where $\widetilde{X}$ is the average position, T is the maximum number of iterations. t denotes the current iteration number, $\forall$ is composed of divergence functions that simulate the state of human behavior. k is a random constant number belonging to (0.1, 1.3), dim is the dimension of the optimization problem, lb and ub denote the lower and upper bounds of the entire problem space, respectively. The global exploration and exploitation capabilities of the algorithm are balanced by the $\widetilde{X}$ and $\forall$, allowing the algorithm to search flexibly in space.

In this paper, the process of human memory can be divided into successful memory and failure memory to simulate different storage activities in the real world (see Algorithm 1). Store the success and failure events generated each time. The matrix for memory storage is of finite capacity, set to $N*dim$, and when the newest memory is stored, it replaces the oldest memory one at a time if it exceeds the capacity, with N representing the number of populations. According to the nature of human memory, the storage of memories only stores better and worse events than the previous ones. It is worth noting that the global optimal and worst solutions are retained here, so there will be cases where the same solutions are stored.

Algorithm 1 Memory Storage Strategies

```
Input:
Pwrost:Worst position(memory) to be stored
Best:Best position(memory) to be stored
N:Number of populations
Output:
Mworst:Storage Matrix for Worst Memory
Mbest:Storage Matrix for Best Memory
Storage:
  W = 1;% W is the number of stored memories
  E = mod(W,N);
    If E==0
      Mworst (N,:) = Pwrost;
      Mbest (N,:) = Best;
    else
      Mworst(E,:) = Pwrost;
      Mbest(E,:) = Best;
    end
W = W + 1;
```

Table 1
Parameter settings.

Algorithms	parameters
DBO	$K = 0.1, b = 0.3, S = 0.5$
GWO	a decreases linearly from 2 to 0
HHO	–
PSO	$w_{\max} = 0.9, w_{\min} = 0.1, c_1 = 2, c_2 = 2$
SCA	r_1 decreases linearly from 2 to 0
SSA	$PD = 15, SD = 10$
WOA	a decreases linearly from 2 to 0
HMO	$r = 0.1$

2.2.2. Recollective behavior

As we all know, humans are constantly recalling successes and failures in the process of making memories. It is worth noting that this behavior plays an important role in human production activities. It's easy to remember the hard work that came before when achieved something, like an award speech. These tough times can be seen as failures. To mimic this behavior, we use the following update to represent the process of each person recalling a failed event:

$$\begin{aligned} X_i(t+1) &= x_{best} + \beta \times (r_1 \times Worst_a - r_2 \times \widetilde{X}) \\ \beta &= -1/\delta \times log(1-\phi) \end{aligned} \tag{2}$$

where δ is a constant value 4, ϕ is a random vector by size $1 \times \dim$ that follows normally distributed. x_{best} denotes historical optimal memory. a denotes the randomly selected a-th position inside the worst matrix. β denotes the state factor of emotion. r_1 and r_2 are both uniform random numbers of [0,1], and they represent is the proportion of extracted information. The storage scheme for failed memories is more slowly updated because the worst solution is locked in the early stages and it is more difficult to find the worst solution in the later stages. In this case, the position in Worst is more different from the current population position information and provides better global exploration capability.

Similarly, the representation of recalling a successful event is as follows:

$$X_i(t+1) = x_{best} + \beta \times (r_1 \times Best_{a2} - r_2 \times \widetilde{X}) \tag{3}$$

$Best_{a2}$ represents a randomly selected a_2-segment memory in the optimal memory matrix. In the recall of successful events, the difference between successes is rapidly becoming smaller, so the recall behaviour for successful events is a rapid shift from global exploration to local exploitation.

It should be mentioned that people often fail to remember when recalling the current event,which is referred to in this article as transient memory loss. In this case, lost memories can only be retrieved through impressive events. Therefore, we use the following update to simulate finding lost memories:

$$X_i(t+1) = X_r(t) + \sigma \times \left(r_1 \times \widetilde{X} - Pworst\right) \tag{4}$$

where σ represents a random number, represents the state factor at this time. *Pworst* denotes the most recent worst memory. $X_r(t)$ represents a randomly selected individual within the population. Note that the significance of this equation is the dissatisfaction with the current state of memory and the association of recent unlucky events, thus providing some guidance for future behavior. It can be seen from the Eq. (4) that the algorithm's optimization process does not take into account successful events, but is linked to the current population position, so the behaviour of memory loss is one that has some global exploration.

Finally, if the current memory is not lost, it can be represented by the following:

$$X_i(t+1) = x_{best} + r_2 \times (X_l - X_i(t)) \tag{5}$$

where X_l represents the current optimal solution, it represents the best solution in the current population. r_2 denotes the mood state at this time and is a uniform random number obeying [0,1]. The significance of this equation is to indicate that humans identify with the current memory state and thus develop it in accordance with the current behavior. From Eq. (5), it can be seen that the optimal positions are considered together with the locally optimal positions, and also they are fixed positions at the same number of iterations, thus enabling some local exploitation.

The process of recall is also a human activity, so the optimal and worst solutions it produces also need to be stored, as shown in Algorithm 1.

2.3. Algorithm flow

The HMO algorithm is divided into two main parts, one is the simulation of human activity behavior and the other is the simulation of memory behavior. Among them, memory behavior is divided into four parts. For simplicity, the selection of memory behavior will be performed by parameter r in this paper, where the selection probability of failure and success is the same; the probability of recalling the current event is also the same as the probability of the current event being lost. The selection probabilities of the other two approaches are also the same. The flow of the proposed HMO algorithm is depicted in Algorithm 2.

Algorithm 2 The framework of the HMO algorithm

```
Require: The maximum iterations T_max, the size of the population N.
Ensure: Optimal position Best and its fitness value Fbest.
Initialize the each individual i ← 1, 2, …, N and define relevant parameters
t = 1;
while (t=<Tmax) do
for i ← 1 to N do
Update the memory production's position by using (1);
end for
Obtain different memory storage values by Algorithm 1.
for j ← 1 to N do
  R = rand(1);;
    if R<=r then
    Update the process of each person recalling a failed event by (2);
    else if R > r and R <= 0.5 then
    Update the process of each person recalling recent events by (4);
    else if R > 0.5 and R<= 1-r then
    Update the process of finding lost memories by (5);
    else then
Update the process of each person recalling a successful event by (3);
end for
Obtain different memory storage values by Algorithm 1.
t = t + 1;
end while
Return Best and its Fbest
```

r is the value of the probability of recalling a failed event and is required to be set, where the probability of recalling a successful and failed event is the same, and the probability of recalling a current and memory loss is the same so that only one r is required to be able to adjust the choice of the four recall types.

2.4. Time complexity analysis

The HMO algorithm, like other meta-heuristic algorithms, has a time

Table 2
Table of the optimization results of each algorithm (D = 30).

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
F1	Best	9.5895E-05	7.3659E + 03	3.0564E-04	2.6422E + 01	1.2865E-01	3.7358E + 00	2.2737E-13	5.1405E-05
	Worst	2.7102E-02	1.5827E + 04	3.7346E + 03	1.6458E + 02	7.5643E-01	8.7802E + 00	5.9804E-02	4.6362E-04
	Ave	6.7667E-03	1.0279E + 04	9.6663E + 02	7.4742E + 01	3.2184E-01	5.8748E + 00	3.2860E-03	1.8619E-04
	Std	7.8039E-03	1.9347E + 03	7.8764E + 02	3.4557E + 01	1.3221E-01	1.3936E + 00	1.1731E-02	8.2076E-05
	Rank	3	8	7	6	4	5	2	1
F2	Best	9.7817E + 05	6.9647E + 07	2.9029E + 06	2.3057E + 07	1.8033E + 07	5.7423E + 06	2.1336E + 06	1.7679E + 06
	Worst	5.5244E + 06	2.4672E + 08	4.2190E + 07	2.0596E + 08	6.3577E + 07	2.2801E + 07	8.1950E + 07	5.5244E + 06
	Ave	2.9528E + 06	1.4142E + 08	1.8782E + 07	6.8829E + 07	3.4400E + 07	1.0373E + 07	2.9871E + 07	3.3442E + 06
	Std	1.1471E + 06	4.6505E + 07	1.0854E + 07	3.4450E + 07	1.1293E + 07	3.6523E + 06	2.0994E + 07	1.0960E + 06
	Rank	1	8	4	7	6	3	5	2
F3	Best	4.9837E + 07	1.8977E + 10	1.6517E + 08	1.1642E + 10	2.3302E + 09	2.7066E + 08	5.0909E + 08	4.7959E + 07
	Worst	1.3701E + 09	7.1847E + 10	2.5892E + 10	1.9632E + 12	3.8501E + 10	5.1673E + 09	3.7105E + 10	5.7242E + 09
	Ave	5.2855E + 08	3.6554E + 10	5.8085E + 09	9.2234E + 10	1.1133E + 10	1.7702E + 09	1.3413E + 10	7.9463E + 08
	Std	4.3924E + 08	1.3367E + 10	5.0268E + 09	3.5357E + 11	7.9884E + 09	1.4375E + 09	1.0732E + 10	1.1008E + 09
	Rank	1	7	4	8	5	3	6	2
F4	Best	9.2329E + 01	2.1737E + 04	4.9400E + 02	5.7143E + 04	2.0094E + 04	2.7574E + 03	6.1253E + 03	9.2704E + 01
	Worst	4.7796E + 02	4.2743E + 04	4.4720E + 04	6.8499E + 04	1.0999E + 05	1.1650E + 04	5.0180E + 04	4.9400E + 02
	Ave	1.9902E + 02	3.3333E + 04	3.1646E + 04	6.3689E + 04	5.6291E + 04	6.7115E + 03	1.9224E + 04	2.5993E + 02
	Std	1.0377E + 02	5.5299E + 03	9.0963E + 03	3.3599E + 03	2.3772E + 04	1.7385E + 03	1.1600E + 04	1.1245E + 02
	Rank	1	6	5	8	7	3	4	2
F5	Best	1.7660E-02	1.2547E + 03	2.9552E-02	9.7823E + 01	3.8083E + 01	1.5654E + 00	5.6843E-13	2.0725E-02
	Worst	2.3508E-01	5.8803E + 03	3.3990E + 03	1.5974E + 03	1.2157E + 02	2.7457E + 00	3.7748E + 00	4.7068E-02
	Ave	5.1164E-02	2.1536E + 03	7.3734E + 02	2.3239E + 02	8.0432E + 01	2.2419E + 00	7.2607E-01	3.6832E-02
	Std	4.2328E-02	8.4421E + 02	6.0543E + 02	2.6181E + 02	2.1125E + 01	2.7316E-01	7.9420E-01	6.3383E-03
	Rank	2	8	7	6	5	4	3	1
F6	Best	1.0776E + 01	3.2066E + 02	5.0457E + 01	7.3147E + 01	2.9503E + 01	2.2634E + 00	1.6094E + 01	5.1978E + 00
	Worst	9.3816E + 01	8.2639E + 02	2.9944E + 02	2.4238E + 02	2.0361E + 02	1.4434E + 02	1.2061E + 02	1.0891E + 02
	Ave	5.4528E + 01	6.0079E + 02	1.4114E + 02	1.5667E + 02	1.1929E + 02	6.9183E + 01	6.2104E + 01	6.4354E + 01
	Std	2.8119E + 01	1.2591E + 02	5.7800E + 01	3.7486E + 01	4.4201E + 01	2.7922E + 01	2.8102E + 01	2.5053E + 01
	Rank	1	8	6	7	5	4	2	3
F7	Best	6.7633E + 01	1.1810E + 02	5.6670E + 01	1.6554E + 02	1.2185E + 02	9.8442E + 01	1.0433E + 02	2.5895E + 01
	Worst	2.3050E + 03	2.3633E + 02	3.6021E + 02	5.2575E + 03	3.1115E + 03	2.8557E + 04	3.7069E + 02	9.0972E + 01
	Ave	2.0332E + 02	1.6050E + 02	1.4997E + 02	5.1854E + 02	4.6530E + 02	1.7602E + 03	1.6573E + 02	5.4684E + 01
	Std	4.0013E + 02	2.6563E + 01	5.9827E + 01	9.2703E + 02	6.3684E + 02	5.3869E + 03	6.5428E + 01	1.7345E + 01
	Rank	5	3	2	7	6	8	4	1
F8	Best	2.0745E + 01	2.0797E + 01	2.0744E + 01	2.0866E + 01	2.0765E + 01	2.0777E + 01	2.0763E + 01	2.0737E + 01
	Worst	2.1028E + 01	2.1019E + 01	2.1043E + 01	2.1046E + 01	2.1005E + 01	2.0988E + 01	2.1013E + 01	2.1019E + 01
	Ave	2.0943E + 01	2.0941E + 01	2.0954E + 01	2.0955E + 01	2.0924E + 01	2.0896E + 01	2.0933E + 01	2.0941E + 01
	Std	6.9260E-02	5.7960E-02	6.5195E-02	4.3905E-02	6.2516E-02	5.9332E-02	5.7564E-02	6.4911E-02
	Rank	6	5	7	8	2	1	3	4
F9	Best	1.9276E + 01	3.5360E + 01	2.8016E + 01	3.0906E + 01	2.9158E + 01	3.1978E + 01	2.7395E + 01	1.4247E + 01
	Worst	3.7965E + 01	4.1230E + 01	4.1642E + 01	4.7895E + 01	4.0335E + 01	4.1727E + 01	3.9452E + 01	3.8139E + 01
	Ave	3.1369E + 01	3.9120E + 01	3.4721E + 01	4.0301E + 01	3.6449E + 01	3.5503E + 01	3.2968E + 01	2.0007E + 01
	Std	4.5713E + 00	1.5675E + 00	2.8624E + 00	3.6710E + 00	2.9163E + 00	2.7388E + 00	3.3485E + 00	4.2506E + 00
	Rank	2	7	4	8	6	5	3	1
F10	Best	5.2708E-01	9.6661E + 02	1.1649E + 00	7.9112E + 01	1.4568E + 01	3.2975E + 00	3.6146E-02	4.0746E-01
	Worst	1.9577E + 00	2.2726E + 03	7.1922E + 02	3.5824E + 02	1.2641E + 02	1.0558E + 01	3.6338E + 01	1.4929E + 00
	Ave	1.4093E + 00	1.6377E + 03	3.1869E + 02	1.6787E + 02	6.1110E + 01	6.0802E + 00	1.6412E + 01	1.0412E + 00
	Std	3.1246E-01	3.7893E + 02	1.7984E + 02	7.3964E + 01	2.9081E + 01	1.5005E + 00	1.1446E + 01	3.0289E-01
	Rank	2	8	7	6	5	3	4	1
F11	Best	2.4718E + 02	3.2164E + 02	2.4775E + 02	3.0369E + 02	2.2087E + 02	8.7766E + 01	6.9195E + 01	5.5180E + 01
	Worst	4.9151E + 02	4.1991E + 02	6.1487E + 02	5.6757E + 02	6.9322E + 02	2.3688E + 02	3.1175E + 02	4.4972E + 02
	Ave	3.2998E + 02	3.6735E + 02	4.2346E + 02	4.2125E + 02	4.6222E + 02	1.6628E + 02	1.6430E + 02	1.1178E + 02
	Std	6.6682E + 01	2.7620E + 01	9.0948E + 01	6.6853E + 01	1.0875E + 02	3.6881E + 01	7.0914E + 01	6.9319E + 01
	Rank	4	5	7	6	8	3	2	1
F12	Best	2.6699E + 02	3.0304E + 02	2.9453E + 02	3.9323E + 02	2.8539E + 02	2.6188E + 02	1.4905E + 02	3.9049E + 01
	Worst	7.5915E + 02	4.3494E + 02	7.5915E + 02	1.0157E + 03	7.6297E + 02	7.6847E + 02	4.9034E + 02	4.8454E + 02

(continued on next page)

Table 2 (*continued*)

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
	Ave	3.7873E + 02	3.7875E + 02	4.3814E + 02	6.8424E + 02	5.0014E + 02	5.7896E + 02	2.3843E + 02	1.2429E + 02
	Std	9.3648E + 01	3.3132E + 01	1.0244E + 02	1.7762E + 02	1.1378E + 02	1.0409E + 02	6.8255E + 01	8.7752E + 01
	Rank	3	4	5	8	6	7	2	1
F13	Best	2.6888E + 02	3.2807E + 02	2.5483E + 02	3.8277E + 02	3.0742E + 02	3.6018E + 02	2.1225E + 02	1.0139E + 02
	Worst	6.1231E + 02	4.4803E + 02	5.6670E + 02	1.0659E + 03	7.3384E + 02	7.8984E + 02	4.0253E + 02	4.1456E + 02
	Ave	4.4641E + 02	3.6834E + 02	4.0086E + 02	6.7630E + 02	4.8471E + 02	5.8731E + 02	2.7586E + 02	1.9591E + 02
	Std	8.1539E + 01	2.7138E + 01	7.6651E + 01	2.0089E + 02	9.5919E + 01	9.2877E + 01	4.4179E + 01	5.8430E + 01
	Rank	5	3	4	8	6	7	2	1
F14	Best	2.3356E + 03	6.5205E + 03	3.2853E + 03	4.3526E + 03	2.8581E + 03	1.4848E + 03	1.9256E + 03	1.2881E + 03
	Worst	5.5602E + 03	7.9047E + 03	5.5602E + 03	7.7490E + 03	7.3858E + 03	5.1779E + 03	5.0429E + 03	4.9129E + 03
	Ave	3.4542E + 03	7.1024E + 03	4.4291E + 03	5.6163E + 03	4.9943E + 03	2.7030E + 03	3.7102E + 03	3.1030E + 03
	Std	7.5401E + 02	3.2706E + 02	6.1867E + 02	9.1080E + 02	1.0667E + 03	7.8400E + 02	7.9386E + 02	8.2510E + 02
	Rank	3	8	5	7	6	1	4	2
F15	Best	3.1509E + 03	6.5247E + 03	3.0706E + 03	3.6427E + 03	3.9344E + 03	3.0142E + 03	3.3001E + 03	2.1748E + 03
	Worst	6.0820E + 03	7.9186E + 03	6.0820E + 03	8.0713E + 03	7.2056E + 03	6.2281E + 03	7.1838E + 03	7.1792E + 03
	Ave	4.4791E + 03	7.3819E + 03	4.7566E + 03	6.4043E + 03	5.7826E + 03	4.8150E + 03	5.1761E + 03	3.3729E + 03
	Std	7.3845E + 02	3.7109E + 02	6.7512E + 02	1.1077E + 03	8.1430E + 02	8.0178E + 02	9.4823E + 02	1.0090E + 03
	Rank	2	8	3	7	6	4	5	1
F16	Best	1.0095E + 02	1.0192E + 02	1.0077E + 02	1.0120E + 02	1.0073E + 02	1.0090E + 02	1.0080E + 02	1.0077E + 02
	Worst	1.0261E + 02	1.0298E + 02	1.0301E + 02	1.0451E + 02	1.0259E + 02	1.0289E + 02	1.0288E + 02	1.0218E + 02
	Ave	1.0165E + 02	1.0253E + 02	1.0242E + 02	1.0248E + 02	1.0183E + 02	1.0179E + 02	1.0218E + 02	1.0131E + 02
	Std	4.5873E-01	2.5516E-01	4.8989E-01	7.1426E-01	4.2653E-01	4.4360E-01	4.0651E-01	3.2825E-01
	Rank	2	8	6	7	4	3	5	1
F17	Best	2.1063E + 02	5.0615E + 02	3.8618E + 02	6.5697E + 02	4.2161E + 02	5.9303E + 02	1.7223E + 02	1.9211E + 02
	Worst	9.1475E + 02	6.4436E + 02	9.1475E + 02	1.0938E + 03	8.5427E + 02	9.4968E + 02	4.3620E + 02	4.4928E + 02
	Ave	4.4336E + 02	5.8382E + 02	6.8359E + 02	9.3344E + 02	6.6685E + 02	8.0627E + 02	2.8634E + 02	2.7683E + 02
	Std	2.0405E + 02	3.4719E + 01	1.5713E + 02	9.1068E + 01	1.1883E + 02	1.0977E + 02	7.1746E + 01	6.7189E + 01
	Rank	3	4	6	8	5	7	2	1
F18	Best	3.8666E + 02	5.1326E + 02	3.4389E + 02	7.9998E + 02	4.7144E + 02	6.4823E + 02	1.8903E + 02	2.1448E + 02
	Worst	7.4819E + 02	6.4666E + 02	8.5836E + 02	1.0863E + 03	9.0609E + 02	9.5315E + 02	5.1675E + 02	5.1596E + 02
	Ave	4.9177E + 02	5.7416E + 02	5.9237E + 02	9.6334E + 02	7.2287E + 02	8.2456E + 02	3.4950E + 02	3.5595E + 02
	Std	8.9828E + 01	3.6037E + 01	1.3776E + 02	8.6673E + 01	1.1008E + 02	8.7303E + 01	8.6196E + 01	4.9072E + 01
	Rank	3	4	5	8	6	7	1	2
F19	Best	1.0502E + 02	1.0663E + 03	1.0369E + 02	1.4752E + 02	1.2349E + 02	1.2488E + 02	1.0527E + 02	1.0959E + 02
	Worst	1.5487E + 02	8.9172E + 03	3.7204E + 03	6.1729E + 04	1.9780E + 02	1.6577E + 02	1.3355E + 02	1.6826E + 02
	Ave	1.1543E + 02	3.1090E + 03	4.3395E + 02	2.3253E + 03	1.5651E + 02	1.4432E + 02	1.1593E + 02	1.3803E + 02
	Std	1.3917E + 01	1.9631E + 03	8.5205E + 02	1.1222E + 04	1.9368E + 01	1.2337E + 01	8.2614E + 00	1.4715E + 01
	Rank	1	8	6	7	5	4	2	3
F20	Best	1.1402E + 02	1.1334E + 02	1.1239E + 02	1.1450E + 02	1.1402E + 02	1.1450E + 02	1.1151E + 02	1.0921E + 02
	Worst	1.1500E + 02	1.1451E + 02	1.1500E + 02	1.1500E + 02	1.1500E + 02	1.1500E + 02	1.1450E + 02	1.1500E + 02
	Ave	1.1466E + 02	1.1390E + 02	1.1440E + 02	1.1493E + 02	1.1477E + 02	1.1482E + 02	1.1311E + 02	1.1239E + 02
	Std	2.7250E-01	2.9429E-01	7.5200E-01	1.7032E-01	3.0133E-01	2.3936E-01	6.8475E-01	1.6296E + 00
	Rank	5	3	4	8	6	7	2	1
F21	Best	2.0139E + 02	1.4931E + 03	5.4354E + 02	5.5157E + 02	2.3341E + 02	2.5687E + 02	2.0000E + 02	4.0015E + 02
	Worst	5.4355E + 02	2.1825E + 03	1.7090E + 03	2.8243E + 03	5.4407E + 02	5.4453E + 02	5.4354E + 02	5.4354E + 02
	Ave	4.2392E + 02	1.9292E + 03	1.0304E + 03	7.4511E + 02	4.7667E + 02	4.6812E + 02	4.1497E + 02	4.7666E + 02
	Std	9.2227E + 01	1.8201E + 02	3.0762E + 02	4.4690E + 02	7.9734E + 01	7.6930E + 01	9.2495E + 01	7.2725E + 01
	Rank	2	8	7	6	5	3	1	4
F22	Best	3.1828E + 03	6.6167E + 03	3.0263E + 03	4.4979E + 03	3.9023E + 03	2.0697E + 03	2.7440E + 03	2.1353E + 03
	Worst	6.5066E + 03	8.3822E + 03	7.7285E + 03	8.8182E + 03	8.4802E + 03	5.8555E + 03	5.8418E + 03	5.5338E + 03
	Ave	4.8008E + 03	7.5814E + 03	6.0280E + 03	6.8747E + 03	6.2163E + 03	3.5515E + 03	4.4745E + 03	3.2697E + 03
	Std	9.5748E + 02	4.3140E + 02	1.0963E + 03	1.2069E + 03	1.1084E + 03	8.5577E + 02	8.2032E + 02	7.4635E + 02
	Rank	4	8	5	7	6	2	3	1
F23	Best	3.4673E + 03	6.5198E + 03	4.8026E + 03	5.1089E + 03	4.7628E + 03	3.7412E + 03	3.8439E + 03	1.8978E + 03
	Worst	6.5887E + 03	8.5522E + 03	7.3853E + 03	8.7862E + 03	8.0134E + 03	8.1771E + 03	8.1018E + 03	7.2015E + 03
	Ave	5.3234E + 03	7.8502E + 03	6.2107E + 03	7.1746E + 03	6.6062E + 03	6.3513E + 03	5.8432E + 03	4.0199E + 03
	Std	8.9427E + 02	4.3704E + 02	6.1512E + 02	9.8649E + 02	8.8058E + 02	1.0269E + 03	9.2228E + 02	1.3360E + 03
	Rank	2	8	4	7	6	5	3	1

(*continued on next page*)

Table 2 (*continued*)

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
F24	Best	3.6815E + 02	4.0222E + 02	3.9379E + 02	3.9876E + 02	3.9714E + 02	3.7918E + 02	3.7712E + 02	3.4160E + 02
	Worst	4.3867E + 02	4.2435E + 02	4.4500E + 02	4.3853E + 02	4.3467E + 02	4.6248E + 02	4.1091E + 02	3.9415E + 02
	Ave	3.9838E + 02	4.1720E + 02	4.1525E + 02	4.2132E + 02	4.1233E + 02	4.2577E + 02	3.9497E + 02	3.5258E + 02
	Std	2.0078E + 01	5.1472E + 00	1.5931E + 01	1.0265E + 01	1.0104E + 01	1.6761E + 01	7.8008E + 00	1.1510E + 01
	Rank	3	6	5	7	4	8	2	1
F25	Best	3.9240E + 02	4.1370E + 02	4.1338E + 02	4.0891E + 02	4.0309E + 02	4.0815E + 02	3.8312E + 02	3.4977E + 02
	Worst	5.0534E + 02	4.3370E + 02	4.5756E + 02	4.5418E + 02	4.4550E + 02	4.6090E + 02	4.2937E + 02	4.1657E + 02
	Ave	4.3771E + 02	4.2631E + 02	4.3088E + 02	4.3167E + 02	4.2290E + 02	4.3547E + 02	4.0335E + 02	3.7387E + 02
	Std	2.8454E + 01	4.1501E + 00	1.0952E + 01	1.0363E + 01	1.1228E + 01	1.5239E + 01	1.0039E + 01	1.3000E + 01
	Rank	8	4	5	6	3	7	2	1
F26	Best	3.0006E + 02	3.0492E + 02	3.0007E + 02	3.0215E + 02	3.0058E + 02	3.0030E + 02	3.0014E + 02	3.0017E + 02
	Worst	5.0830E + 02	3.2215E + 02	5.0916E + 02	5.2086E + 02	5.1204E + 02	5.1199E + 02	4.9059E + 02	4.9094E + 02
	Ave	4.1233E + 02	3.1256E + 02	4.5355E + 02	4.7728E + 02	4.5710E + 02	4.5338E + 02	3.1332E + 02	4.0828E + 02
	Std	8.7756E + 01	3.6188E + 00	7.8397E + 01	7.0010E + 01	7.9680E + 01	8.6033E + 01	4.4854E + 01	6.6845E + 01
	Rank	4	1	6	8	7	5	2	3
F27	Best	1.0324E + 03	1.3611E + 03	1.1725E + 03	1.2733E + 03	1.2813E + 03	1.2297E + 03	1.0666E + 03	7.9999E + 02
	Worst	1.4627E + 03	1.5232E + 03	1.6214E + 03	1.7119E + 03	1.6214E + 03	1.6282E + 03	1.4580E + 03	1.2265E + 03
	Ave	1.2581E + 03	1.4624E + 03	1.3752E + 03	1.5122E + 03	1.4332E + 03	1.4434E + 03	1.2648E + 03	9.2500E + 02
	Std	1.1662E + 02	4.1961E + 01	1.0127E + 02	9.9286E + 01	7.3473E + 01	9.8443E + 01	1.0775E + 02	9.9195E + 01
	Rank	2	7	4	8	5	6	3	1
F28	Best	2.7325E + 03	2.4195E + 03	2.7175E + 03	2.2578E + 03	2.8121E + 03	4.0134E + 03	4.0000E + 02	6.4366E + 02
	Worst	5.3448E + 03	3.0211E + 03	5.3448E + 03	8.0465E + 03	5.4654E + 03	5.7993E + 03	1.6821E + 03	5.1936E + 03
	Ave	3.7228E + 03	2.7542E + 03	3.9927E + 03	4.7180E + 03	4.3077E + 03	4.8996E + 03	5.0656E + 02	1.3616E + 03
	Std	6.1337E + 02	1.4631E + 02	6.1128E + 02	1.4344E + 03	6.6779E + 02	4.3946E + 02	3.1251E + 02	7.6935E + 02
	Rank	4	3	5	7	6	8	1	2
Total rank		3.0000	6.0000	5.1786	7.1786	5.3929	4.7500	2.8571	1.6429

complexity that depends on the number of populations N, the maximum number of iterations T, and the variables of the problem D. HMO are divided into two main behavioral approaches, human activities, and recollective behavior. Their time complexity is therefore as follows:

$$\begin{aligned} O(HMO) &= O(T(O(Human\ activities) + O(Recollective\ behavior))) \\ &= O(T(ND + ND)) = O(TND) \end{aligned} \tag{6}$$

3. Performance tests

This subsection focuses on testing each algorithm in CEC 2013 (Liang et al., 2013), comparing the optimization results, and also analyzing the *r*.

3.1. Comparison with the basic algorithm

In order to verify the performance of HMO, this paper tested it on the CEC 2013 test function set and compared it with other heuristics, such as GWO, HHO, PSO, sine cosine algorithm (SCA) (Mirjalili, 2016), SSA, WOA DBO, and SSA are new algorithms with high recognition in recent years, and GWO, PSO, SCA and WOA are more classical algorithms, and the specific parameter settings are shown in Table 1. There are 28 functions in CEC 2013 and the theoretical optimum ranges from −1400 to 1400. In order to see the differences between the algorithms, each algorithm subtracts the theoretical optimum from the result of the final optimization search, so that the theoretical optimum becomes 0.

To ensure the fairness of the experiment, the computer configuration used for the simulation is AMD Ryzen 5 4600H with Radeon Graphics CPU@3.00 GHz, 16 GB (3200MHZ) RAM, 64-bit operating system, and Matlab2018a. The basic parameters are set as follows: population size $N = 50$, maximum number of evaluations are $MaxEval = dim \times 10^5$, number of independent runs are 30, and dimension (dim) 30. The best (Best), worst (Worst), average (Ave) and standard deviation (Std) were calculated, and finally each algorithm was ranked (Rank) using the average, where the standard deviation was considered if the averages were equal, and the results are shown in Table 2.

From Table 2, it can be seen that HMO ranks first in F1, F5, F7, F9, F10, F11, F12, F13, F15, F16, F17, F20, F22, F23, F24, F25, F27, a total of 16 functions, and ranks second in F2-F4, F14,F18,F28, and only F8 and F21 ranked close to the fourth among the 28 functions. Among the 28 functions, only F8 and F21 ranked fourth, while the rest of the functions ranked in the top three, showing a superior performance in finding the best. In the overall ranking of all functions, HMO is ranked first, with a ranking of 1.6429, followed by DBO and PSO. It can be seen that the performance of HMO in all kinds of complex optimization functions is more outstanding, and the algorithm has a strong comprehensive optimization-seeking ability and better generalization.

In order to verify the performance of the HMO algorithm in higher dimensions, this paper tested it again in a 50-dimensional function with the same parameters as above, and the results are shown in Table 3. From the 50-dimensional results, HMO still ranks first in most of the functions in higher dimensions, specifically in F1-F2, F4-F5, F7, F9-F14, F16-F17, F22-F25, F27-F28, despite the more complex computational dimensions. The overall ranking is still the first, with a specific value of 1.5, which again proves the superiority-seeking ability of HMO, while the other algorithms are less effective in comprehensive search, with the

Table 3
Table of the optimization results of each algorithm (D = 50).

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
F1	Best	8.0102E-01	2.1821E + 04	7.7735E + 02	3.7733E + 02	3.4513E + 00	1.9821E + 01	9.0949E-13	1.7071E-03
	Worst	5.2744E + 00	3.5948E + 04	1.1402E + 04	6.0047E + 04	2.3948E + 01	3.3586E + 01	1.0818E + 01	8.1600E-03
	Ave	2.3745E + 00	2.7997E + 04	3.6976E + 03	3.1785E + 03	9.5250E + 00	2.6155E + 01	1.4367E + 00	3.8281E-03
	Std	1.1621E + 00	3.5031E + 03	2.2239E + 03	1.0752E + 04	4.6513E + 00	3.6039E + 00	2.7808E + 00	1.3004E-03
	Rank	3	8	7	6	4	5	2	1
F2	Best	4.8144E + 06	2.2554E + 08	1.0860E + 07	9.9418E + 07	3.1295E + 07	1.4062E + 07	1.1576E + 07	2.9190E + 06
	Worst	1.1449E + 07	7.5507E + 08	1.2855E + 08	2.4839E + 08	9.4240E + 07	3.1214E + 07	1.9793E + 08	7.8838E + 06
	Ave	7.3996E + 06	4.4998E + 08	4.2822E + 07	1.4158E + 08	6.2374E + 07	2.0828E + 07	9.1886E + 07	5.3216E + 06
	Std	1.6296E + 06	1.2877E + 08	2.7227E + 07	3.3710E + 07	1.7296E + 07	4.1146E + 06	5.1804E + 07	1.3450E + 06
	Rank	2	8	4	7	5	3	6	1
F3	Best	1.2568E + 08	5.0143E + 10	7.5543E + 09	5.0055E + 10	1.6549E + 10	9.6663E + 08	1.7177E + 10	5.0296E + 08
	Worst	2.0201E + 09	1.5186E + 11	3.1408E + 10	1.1884E + 13	5.9836E + 10	9.0650E + 09	9.1775E + 10	1.4812E + 10
	Ave	6.6407E + 08	9.6301E + 10	1.7254E + 10	4.7346E + 11	2.9780E + 10	3.1064E + 09	5.2468E + 10	3.5758E + 09
	Std	3.9291E + 08	2.4327E + 10	6.4039E + 09	2.1556E + 12	9.6969E + 09	1.9472E + 09	1.8766E + 10	3.7244E + 09
	Rank	1	7	4	8	5	2	6	3
F4	Best	3.3976E + 02	5.1046E + 04	2.8867E + 04	8.7014E + 04	3.4630E + 04	6.4753E + 03	3.9046E + 04	4.0201E + 01
	Worst	1.1135E + 03	8.2367E + 04	6.0258E + 04	1.4201E + 05	6.5567E + 04	1.4079E + 04	7.8590E + 04	2.7835E + 02
	Ave	6.3712E + 02	6.2873E + 04	4.6777E + 04	1.1034E + 05	4.7805E + 04	9.7634E + 03	6.6142E + 04	1.2832E + 02
	Std	2.0284E + 02	7.1826E + 03	8.3893E + 03	1.2882E + 04	7.7225E + 03	1.8460E + 03	1.0510E + 04	4.8755E + 01
	Rank	2	6	4	8	5	3	7	1
F5	Best	7.3591E-01	2.2767E + 03	3.6822E + 02	3.4792E + 02	7.7178E + 01	5.8228E + 00	1.4274E + 00	9.3328E-02
	Worst	2.4101E + 00	6.4493E + 03	1.9490E + 03	1.0982E + 03	2.1310E + 02	8.7539E + 00	1.7303E + 01	4.6741E-01
	Ave	1.5595E + 00	3.2423E + 03	1.0055E + 03	6.5423E + 02	1.3700E + 02	7.1944E + 00	8.8528E + 00	2.5417E-01
	Std	3.9272E-01	7.9452E + 02	3.6483E + 02	1.6931E + 02	3.0185E + 01	8.4078E-01	4.1291E + 00	1.1075E-01
	Rank	2	8	7	6	5	3	4	1
F6	Best	3.7463E + 01	1.5250E + 03	1.6473E + 02	2.6882E + 02	4.7794E + 01	4.4821E + 01	4.4851E + 01	4.3467E + 01
	Worst	1.5796E + 02	2.4890E + 03	5.0992E + 02	1.0029E + 03	3.5574E + 02	1.6175E + 02	1.3286E + 03	1.5530E + 02
	Ave	7.2070E + 01	2.0169E + 03	2.9933E + 02	4.0259E + 02	1.6916E + 02	8.5630E + 01	1.4759E + 02	8.1751E + 01
	Std	3.1239E + 01	2.8902E + 02	8.2743E + 01	1.3805E + 02	6.8741E + 01	4.0772E + 01	2.3001E + 02	3.2629E + 01
	Rank	1	8	6	7	5	3	4	2
F7	Best	9.3157E + 01	1.3758E + 02	9.9684E + 01	1.4679E + 02	1.4619E + 02	1.4215E + 02	1.1060E + 02	4.8377E + 01
	Worst	1.7257E + 02	2.6445E + 02	1.5642E + 03	8.4844E + 03	2.0045E + 03	6.4304E + 02	2.5778E + 02	1.0648E + 02
	Ave	1.3582E + 02	1.9440E + 02	2.6019E + 02	7.3291E + 02	5.0363E + 02	2.8016E + 02	1.6205E + 02	6.7524E + 01
	Std	1.9290E + 01	2.8964E + 01	2.7760E + 02	1.5306E + 03	4.7679E + 02	1.0483E + 02	3.7977E + 01	1.3703E + 01
	Rank	2	4	5	8	7	6	3	1
F8	Best	2.1000E + 01	2.1070E + 01	2.1053E + 01	2.1055E + 01	2.1061E + 01	2.1078E + 01	2.1024E + 01	2.1034E + 01
	Worst	2.1191E + 01	2.1208E + 01	2.1223E + 01	2.1223E + 01	2.1200E + 01	2.1201E + 01	2.1197E + 01	2.1190E + 01
	Ave	2.1128E + 01	2.1150E + 01	2.1146E + 01	2.1160E + 01	2.1142E + 01	2.1140E + 01	2.1134E + 01	2.1137E + 01
	Std	4.3455E-02	3.1873E-02	4.1316E-02	3.8204E-02	3.6476E-02	3.2237E-02	4.4692E-02	4.0228E-02
	Rank	1	7	6	8	5	4	2	3
F9	Best	4.1430E + 01	7.0138E + 01	5.8001E + 01	6.4609E + 01	6.1262E + 01	5.7601E + 01	5.2823E + 01	3.0721E + 01
	Worst	6.5846E + 01	7.5913E + 01	7.3455E + 01	8.0840E + 01	7.6140E + 01	7.2232E + 01	7.1155E + 01	4.5730E + 01
	Ave	5.2155E + 01	7.3563E + 01	6.7241E + 01	7.5297E + 01	6.9573E + 01	6.7058E + 01	6.2369E + 01	3.9122E + 01
	Std	5.7569E + 00	1.5257E + 00	4.2278E + 00	3.9759E + 00	3.8952E + 00	3.7988E + 00	4.2157E + 00	4.0104E + 00
	Rank	2	7	5	8	6	4	3	1
F10	Best	3.7907E + 00	2.7366E + 03	3.3199E + 02	2.9491E + 02	9.2745E + 01	1.4084E + 01	5.3847E + 00	2.5854E + 00
	Worst	1.1261E + 01	5.6648E + 03	1.4870E + 03	4.7364E + 03	2.8363E + 02	3.8092E + 01	3.1075E + 03	8.4269E + 00
	Ave	6.3798E + 00	3.5556E + 03	7.6162E + 02	7.3765E + 02	1.8846E + 02	2.5752E + 01	2.5372E + 02	4.4513E + 00
	Std	1.6770E + 00	5.9036E + 02	2.9505E + 02	7.7081E + 02	5.2368E + 01	6.2004E + 00	6.3509E + 02	1.5260E + 00
	Rank	2	8	7	6	4	3	5	1
F11	Best	3.4249E + 02	5.9140E + 02	4.9269E + 02	5.2934E + 02	6.1356E + 02	3.3239E + 02	1.9093E + 02	1.6533E + 02
	Worst	6.7530E + 02	7.9687E + 02	9.0449E + 02	1.0719E + 03	9.5505E + 02	5.8626E + 02	5.8888E + 02	3.5838E + 02
	Ave	4.7284E + 02	6.9985E + 02	7.2967E + 02	8.1300E + 02	7.5736E + 02	4.4445E + 02	3.1043E + 02	2.3406E + 02
	Std	7.3380E + 01	4.7972E + 01	1.0325E + 02	1.3375E + 02	8.3445E + 01	5.5532E + 01	1.0815E + 02	3.9860E + 01

(continued on next page)

Table 3 (*continued*)

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
	Rank	4	5	6	8	7	3	2	1
F12	Best	4.7604E + 02	6.8631E + 02	5.9413E + 02	8.4347E + 02	7.0368E + 02	7.2645E + 02	3.0957E + 02	1.5937E + 02
	Worst	8.3233E + 02	8.5834E + 02	1.1086E + 03	1.3166E + 03	1.1576E + 03	1.1786E + 03	9.7076E + 02	5.2029E + 02
	Ave	6.2741E + 02	7.5209E + 02	8.6363E + 02	1.1895E + 03	9.6006E + 02	1.0082E + 03	5.6105E + 02	2.9544E + 02
	Std	8.9477E + 01	4.6444E + 01	1.4594E + 02	9.3562E + 01	9.9109E + 01	1.0109E + 02	1.5435E + 02	9.5851E + 01
	Rank	3	4	5	8	6	7	2	1
F13	Best	6.0834E + 02	6.1551E + 02	6.7394E + 02	8.6820E + 02	7.1744E + 02	8.0532E + 02	3.9514E + 02	2.7049E + 02
	Worst	9.2844E + 02	8.4271E + 02	1.1823E + 03	1.4169E + 03	1.2222E + 03	1.3077E + 03	7.4416E + 02	5.6731E + 02
	Ave	7.5529E + 02	7.4703E + 02	9.5167E + 02	1.2346E + 03	9.4216E + 02	1.0798E + 03	5.9129E + 02	3.8677E + 02
	Std	8.7332E + 01	5.0925E + 01	1.3254E + 02	1.6813E + 02	1.3689E + 02	1.2918E + 02	8.6239E + 01	7.2848E + 01
	Rank	4	3	6	8	5	7	2	1
F14	Best	4.2698E + 03	1.2739E + 04	5.0216E + 03	9.2229E + 03	5.8946E + 03	2.5155E + 03	4.0302E + 03	4.0241E + 03
	Worst	7.1061E + 03	1.4314E + 04	1.0006E + 04	1.3810E + 04	1.1836E + 04	9.4079E + 03	8.2654E + 03	1.1108E + 04
	Ave	5.8198E + 03	1.3515E + 04	7.2696E + 03	1.0700E + 04	9.3762E + 03	6.3205E + 03	7.0309E + 03	5.6563E + 03
	Std	7.2811E + 02	4.3787E + 02	1.0964E + 03	1.0511E + 03	1.5683E + 03	1.5158E + 03	9.0606E + 02	1.3321E + 03
	Rank	2	8	5	7	6	3	4	1
F15	Best	5.4710E + 03	1.3302E + 04	7.1997E + 03	9.9646E + 03	8.3120E + 03	8.2387E + 03	7.0186E + 03	5.4061E + 03
	Worst	1.0283E + 04	1.5046E + 04	1.1618E + 04	1.4853E + 04	1.4055E + 04	1.2221E + 04	1.4181E + 04	1.4978E + 04
	Ave	8.1941E + 03	1.4481E + 04	9.3128E + 03	1.2502E + 04	1.1077E + 04	1.0330E + 04	1.0541E + 04	9.0365E + 03
	Std	1.2223E + 03	4.5068E + 02	1.1890E + 03	1.3008E + 03	1.5109E + 03	9.1141E + 02	1.9817E + 03	3.5023E + 03
	Rank	1	8	3	7	6	4	5	2
F16	Best	1.0237E + 02	1.0270E + 02	1.0277E + 02	1.0160E + 02	1.0126E + 02	1.0129E + 02	1.0247E + 02	1.0109E + 02
	Worst	1.0389E + 02	1.0390E + 02	1.0413E + 02	1.0466E + 02	1.0357E + 02	1.0354E + 02	1.0375E + 02	1.0300E + 02
	Ave	1.0316E + 02	1.0349E + 02	1.0351E + 02	1.0335E + 02	1.0260E + 02	1.0249E + 02	1.0323E + 02	1.0187E + 02
	Std	3.9733E-01	3.0978E-01	3.2941E-01	6.8893E-01	5.8274E-01	5.5473E-01	3.5591E-01	4.8094E-01
	Rank	4	7	8	6	3	2	5	1
F17	Best	4.6763E + 02	9.2800E + 02	8.2434E + 02	1.2865E + 03	1.0194E + 03	1.1543E + 03	3.6855E + 02	3.2388E + 02
	Worst	8.4275E + 02	1.2221E + 03	1.3799E + 03	1.5861E + 03	1.4750E + 03	1.4117E + 03	7.5074E + 02	6.1636E + 02
	Ave	6.3365E + 02	1.0617E + 03	1.1879E + 03	1.4120E + 03	1.2397E + 03	1.2796E + 03	5.3029E + 02	4.4378E + 02
	Std	9.1396E + 01	8.4427E + 01	1.2689E + 02	8.2573E + 01	1.1327E + 02	6.8617E + 01	9.8636E + 01	7.6732E + 01
	Rank	3	4	5	8	6	7	2	1
F18	Best	6.1505E + 02	9.2314E + 02	7.5324E + 02	1.2549E + 03	9.8939E + 02	1.1322E + 03	3.7411E + 02	4.5938E + 02
	Worst	1.0766E + 03	1.2190E + 03	1.3436E + 03	1.5756E + 03	1.4380E + 03	1.4043E + 03	8.4237E + 02	7.2526E + 02
	Ave	8.5941E + 02	1.0615E + 03	1.1407E + 03	1.4926E + 03	1.2249E + 03	1.3018E + 03	6.2034E + 02	6.2473E + 02
	Std	1.2245E + 02	7.3562E + 01	1.5425E + 02	5.7148E + 01	1.1920E + 02	7.9769E + 01	1.2484E + 02	6.0462E + 01
	Rank	3	4	5	8	6	7	1	2
F19	Best	1.1672E + 02	1.4118E + 04	1.4794E + 02	5.5121E + 02	1.7741E + 02	1.5247E + 02	1.1295E + 02	1.4641E + 02
	Worst	1.3337E + 02	7.4004E + 04	6.3691E + 03	5.7295E + 03	3.1680E + 02	2.4327E + 02	1.6567E + 02	2.6996E + 02
	Ave	1.2564E + 02	2.8599E + 04	9.4881E + 02	2.1153E + 03	2.4277E + 02	1.8660E + 02	1.3501E + 02	1.8908E + 02
	Std	4.2644E + 00	1.5752E + 04	1.1548E + 03	1.2943E + 03	3.5799E + 01	2.2706E + 01	1.2888E + 01	2.8592E + 01
	Rank	1	8	6	7	5	3	2	4
F20	Best	1.2243E + 02	1.2310E + 02	1.2307E + 02	1.2450E + 02	1.2332E + 02	1.2412E + 02	1.2110E + 02	1.1836E + 02
	Worst	1.2454E + 02	1.2474E + 02	1.2451E + 02	1.2500E + 02	1.2500E + 02	1.2500E + 02	1.2450E + 02	1.2260E + 02
	Ave	1.2409E + 02	1.2404E + 02	1.2441E + 02	1.2459E + 02	1.2465E + 02	1.2451E + 02	1.2319E + 02	1.2111E + 02
	Std	5.3669E-01	4.1163E-01	2.9375E-01	1.8564E-01	3.4346E-01	1.1703E-01	9.6389E-01	9.2959E-01
	Rank	4	3	5	7	8	6	2	1
F21	Best	9.3662E + 02	3.5947E + 03	1.5012E + 03	1.4994E + 03	3.5210E + 02	9.3930E + 02	3.0543E + 02	9.3644E + 02
	Worst	1.2248E + 03	4.4704E + 03	3.5750E + 03	3.8919E + 03	1.2595E + 03	1.2375E + 03	1.2247E + 03	1.2224E + 03
	Ave	1.0517E + 03	3.9607E + 03	2.6422E + 03	2.8405E + 03	9.4525E + 02	1.1064E + 03	1.0154E + 03	1.0412E + 03
	Std	1.4304E + 02	1.5718E + 02	5.0462E + 02	5.1324E + 02	3.1432E + 02	1.4790E + 02	3.0935E + 02	1.4009E + 02
	Rank	4	8	6	7	1	5	2	3
F22	Best	5.8818E + 03	1.3187E + 04	8.3530E + 03	8.6988E + 03	8.4833E + 03	6.3339E + 03	5.9235E + 03	4.4461E + 03
	Worst	1.3014E + 04	1.5243E + 04	1.3067E + 04	1.5223E + 04	1.4972E + 04	1.1790E + 04	1.1022E + 04	8.9834E + 03
	Ave	9.4434E + 03	1.4310E + 04	1.0524E + 04	1.3078E + 04	1.2069E + 04	8.6784E + 03	8.4541E + 03	6.8833E + 03
	Std	1.7547E + 03	4.6426E + 02	1.3330E + 03	1.3665E + 03	1.5962E + 03	1.3008E + 03	1.3326E + 03	1.0203E + 03

(*continued on next page*)

Table 3 (*continued*)

F	Index	PSO	SCA	GWO	SSA	WOA	HHO	DBO	HMO
	Rank	4	8	5	7	6	3	2	1
F23	Best	9.1125E + 03	1.4316E + 04	8.8634E + 03	1.1567E + 04	1.0063E + 04	1.1225E + 04	8.6726E + 03	6.1295E + 03
	Worst	1.3606E + 04	1.5807E + 04	1.4525E + 04	1.7279E + 04	1.5925E + 04	1.4846E + 04	1.5200E + 04	1.4754E + 04
	Ave	1.0925E + 04	1.5150E + 04	1.1834E + 04	1.4420E + 04	1.3205E + 04	1.2793E + 04	1.0793E + 04	8.1477E + 03
	Std	1.0962E + 03	4.0953E + 02	1.3341E + 03	1.4979E + 03	1.3340E + 03	1.0435E + 03	1.4156E + 03	1.6946E + 03
	Rank	3	8	4	7	6	5	2	1
F24	Best	4.2757E + 02	5.1331E + 02	4.8638E + 02	4.8836E + 02	4.7513E + 02	4.8136E + 02	4.4421E + 02	3.8447E + 02
	Worst	4.9939E + 02	5.3745E + 02	5.5151E + 02	5.3811E + 02	5.4823E + 02	5.6913E + 02	5.1373E + 02	4.2509E + 02
	Ave	4.6809E + 02	5.2456E + 02	5.1853E + 02	5.1083E + 02	5.0753E + 02	5.2714E + 02	4.7602E + 02	4.0449E + 02
	Std	1.8950E + 01	5.8386E + 00	1.7638E + 01	1.1545E + 01	1.5649E + 01	2.4270E + 01	1.6968E + 01	1.0255E + 01
	Rank	2	7	6	5	4	8	3	1
F25	Best	4.8036E + 02	5.3607E + 02	5.0861E + 02	5.0164E + 02	4.8598E + 02	5.1525E + 02	4.6808E + 02	4.1012E + 02
	Worst	7.0054E + 02	5.6057E + 02	5.7780E + 02	5.9191E + 02	5.6339E + 02	6.3628E + 02	5.1122E + 02	4.8075E + 02
	Ave	5.5509E + 02	5.4708E + 02	5.4819E + 02	5.5025E + 02	5.2218E + 02	5.5276E + 02	4.9486E + 02	4.4672E + 02
	Std	6.2330E + 01	6.1742E + 00	1.6947E + 01	1.8460E + 01	1.8141E + 01	3.1941E + 01	1.1308E + 01	1.6720E + 01
	Rank	8	4	5	6	3	7	2	1
F26	Best	5.2586E + 02	3.2266E + 02	3.0057E + 02	3.0877E + 02	3.0253E + 02	5.5790E + 02	3.0071E + 02	3.0148E + 02
	Worst	5.7364E + 02	6.0063E + 02	5.9098E + 02	6.1197E + 02	6.0042E + 02	6.1912E + 02	5.7255E + 02	5.2410E + 02
	Ave	5.4800E + 02	4.5981E + 02	5.5822E + 02	5.8386E + 02	5.5536E + 02	5.8117E + 02	4.3269E + 02	4.8018E + 02
	Std	1.1152E + 01	1.2552E + 02	7.0638E + 01	5.2757E + 01	8.5825E + 01	1.2641E + 01	1.3041E + 02	6.1003E + 01
	Rank	4	2	6	8	5	7	1	3
F27	Best	1.7630E + 03	2.2238E + 03	2.0571E + 03	2.1299E + 03	2.0701E + 03	2.1195E + 03	1.8497E + 03	1.2528E + 03
	Worst	2.3995E + 03	2.5144E + 03	2.6628E + 03	2.6792E + 03	2.5567E + 03	2.6753E + 03	2.2520E + 03	1.7388E + 03
	Ave	2.0858E + 03	2.4236E + 03	2.3073E + 03	2.4543E + 03	2.3135E + 03	2.4349E + 03	2.0455E + 03	1.4652E + 03
	Std	1.6281E + 02	5.5826E + 01	1.7099E + 02	1.3655E + 02	1.2604E + 02	1.4508E + 02	1.0461E + 02	1.3110E + 02
	Rank	3	6	4	8	5	7	2	1
F28	Best	5.0989E + 02	3.8214E + 03	3.6432E + 03	2.6281E + 03	4.5447E + 03	6.9630E + 03	5.0013E + 02	8.7778E + 02
	Worst	8.3681E + 03	5.9815E + 03	1.0403E + 04	1.1902E + 04	9.9683E + 03	1.0090E + 04	4.3897E + 03	4.1820E + 03
	Ave	6.1863E + 03	4.9243E + 03	7.5503E + 03	8.1261E + 03	7.4986E + 03	8.8023E + 03	2.4281E + 03	2.0483E + 03
	Std	1.3527E + 03	5.7600E + 02	1.4086E + 03	2.4515E + 03	1.4386E + 03	7.5172E + 02	1.7957E + 03	1.2091E + 03
	Rank	4	3	6	7	5	8	2	1
Total rank		2.8214	6.1071	5.3929	7.1786	5.1429	4.8214	3.0357	1.5000

DBO algorithm ranking second in 30 dimensions and third in 50 dimensions, which shows that DBO is somewhat affected at higher dimensions. In general, HMO can show strong advantages in both 30 and 50 dimensions, which further validates their competitiveness and value.

To further see how the HMO convergence speed varies, the average convergence plots for each algorithm in a 30-dimensional function are shown in Fig. 3. As can be seen from the convergence plots, HMO converges relatively quickly in the early stages, reaching the same level as the other compared algorithms after a very small number of iterations, and shows excellent local search ability in the middle and late stages, constantly jumping out of the local optimum and converging further towards the optimal value. As can be seen from Fig. 3, HMO converges to a better level than the other algorithms in most of the functions, that is, the search results show a strong ability of balanced exploration and development, especially in the later stages when it keeps getting rid of the attraction of the local optimum trap and converges to a better solution. Meanwhile, in order to further verify the effectiveness and generalizability of the HMO and visualize the optimization performance of the algorithms, Fig. 4 shows the comprehensive ranking radar plot of each algorithm in each function in 30 dimensions, with the smaller area around indicating a higher ranking.

As can be seen in Fig. 4, the HMO are coloured red, the HMO are tightly around the centre and are the closest, i.e. the most highly ranked, of most of the functions, and their enclosed area is much smaller than that of the other algorithms, demonstrating superior merit-seeking ability.

To test the difference and superiority of HMO to other algorithms, all algorithms were subjected to the Wilcoxon rank sum test against HMO based on the results of 30 runs, and the results are shown in Table 4. According to the Wilcoxon rank sum test rule, if the result is <0.05 then it can be concluded that there is a significant difference between the two compared algorithms, and to further highlight the HMO, the "+", "=", "−" indicate that HMO is better than, equal to, or inferior to a comparison algorithm respectively, and from the results, HMO is significantly different from the other algorithms, and at this point, combined with the rankings presented in the above table of arithmetic results, HMO is clearly superior to the other algorithms.

3.2. Comparison with variants of other algorithms

To further validate the effective competitiveness of HMO, HMO is compared with the variants of other algorithms proposed in recent

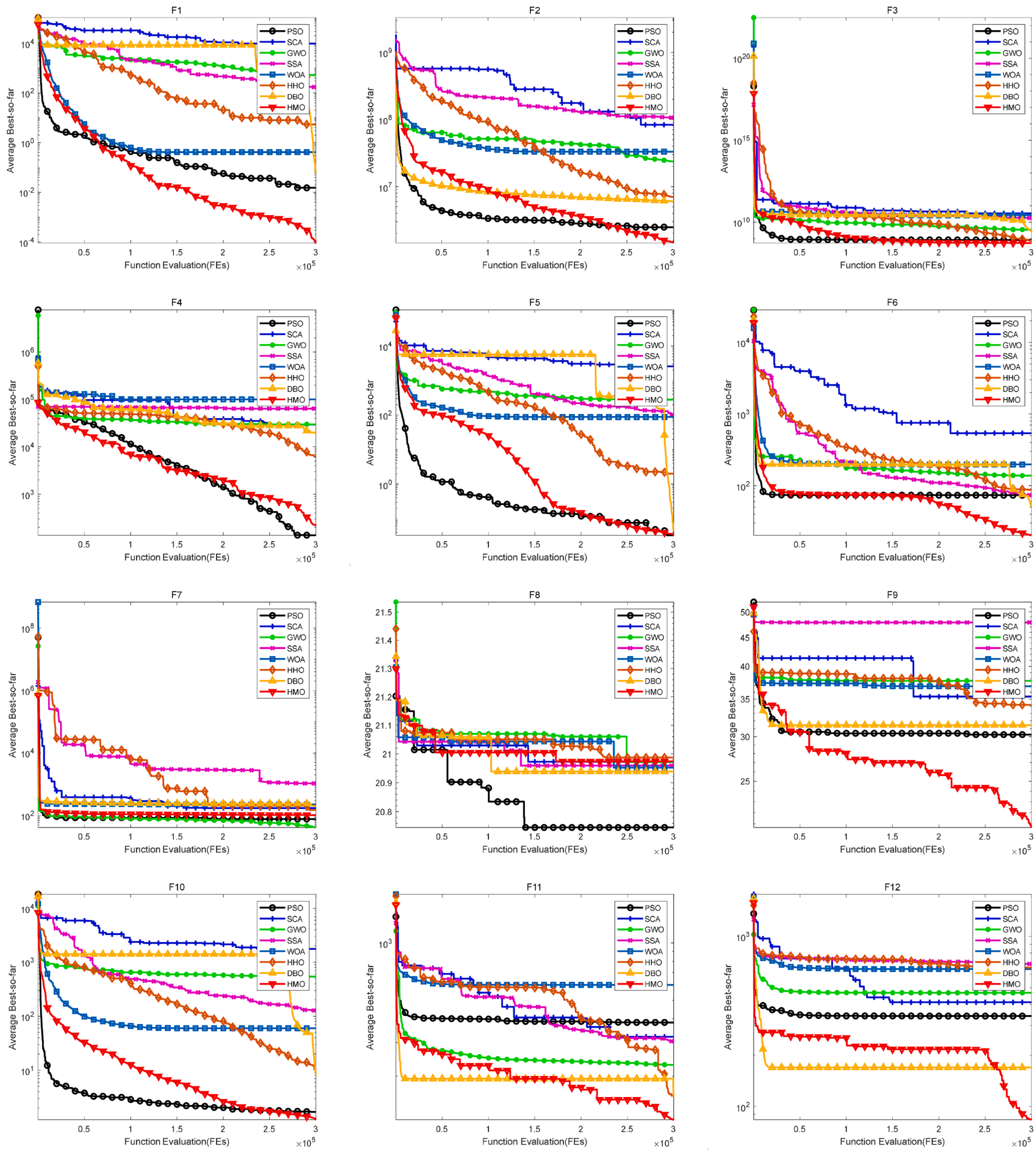

Fig. 3. Average convergence plot for each algorithm.

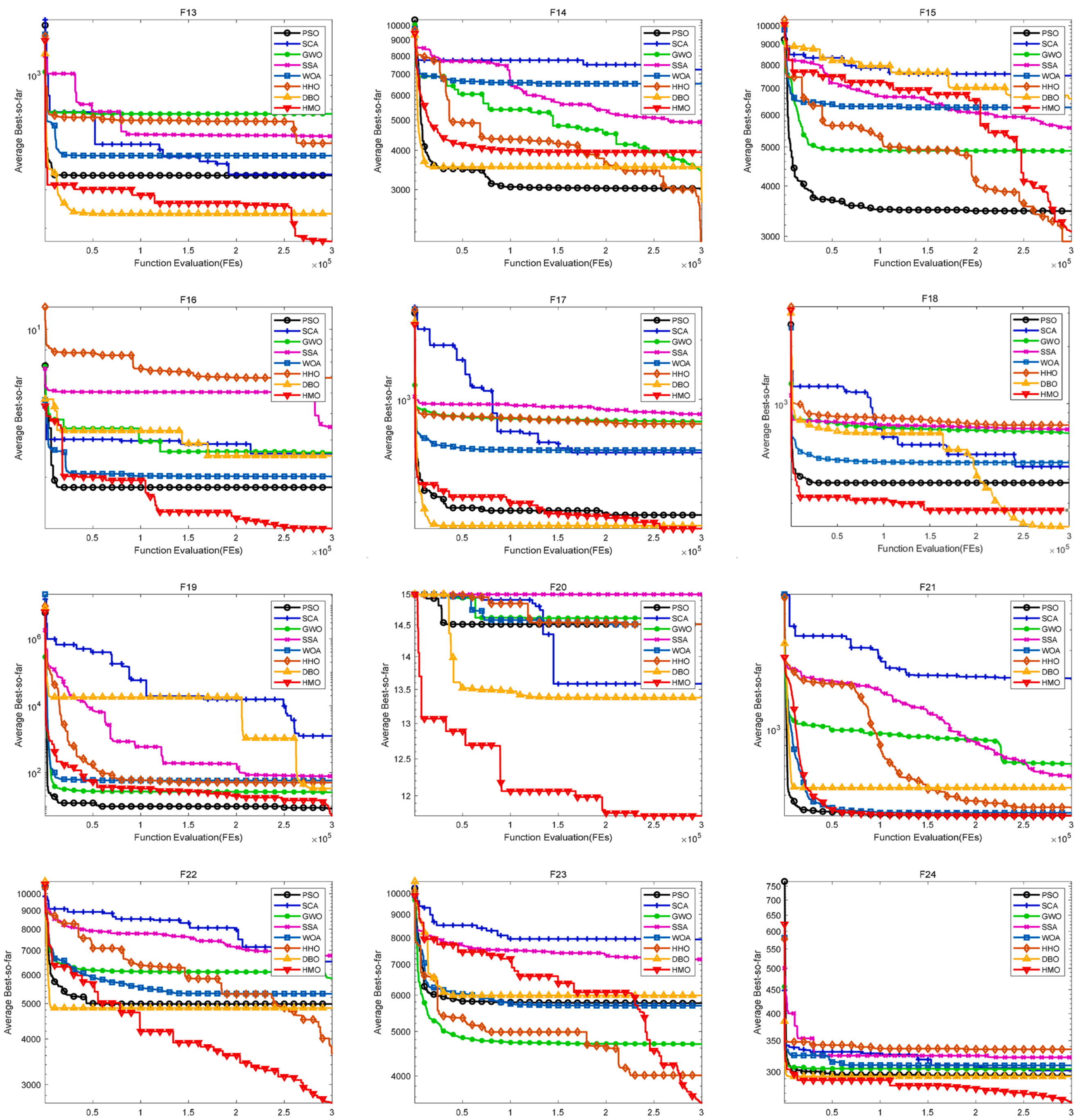

Fig. 3. (*continued*).

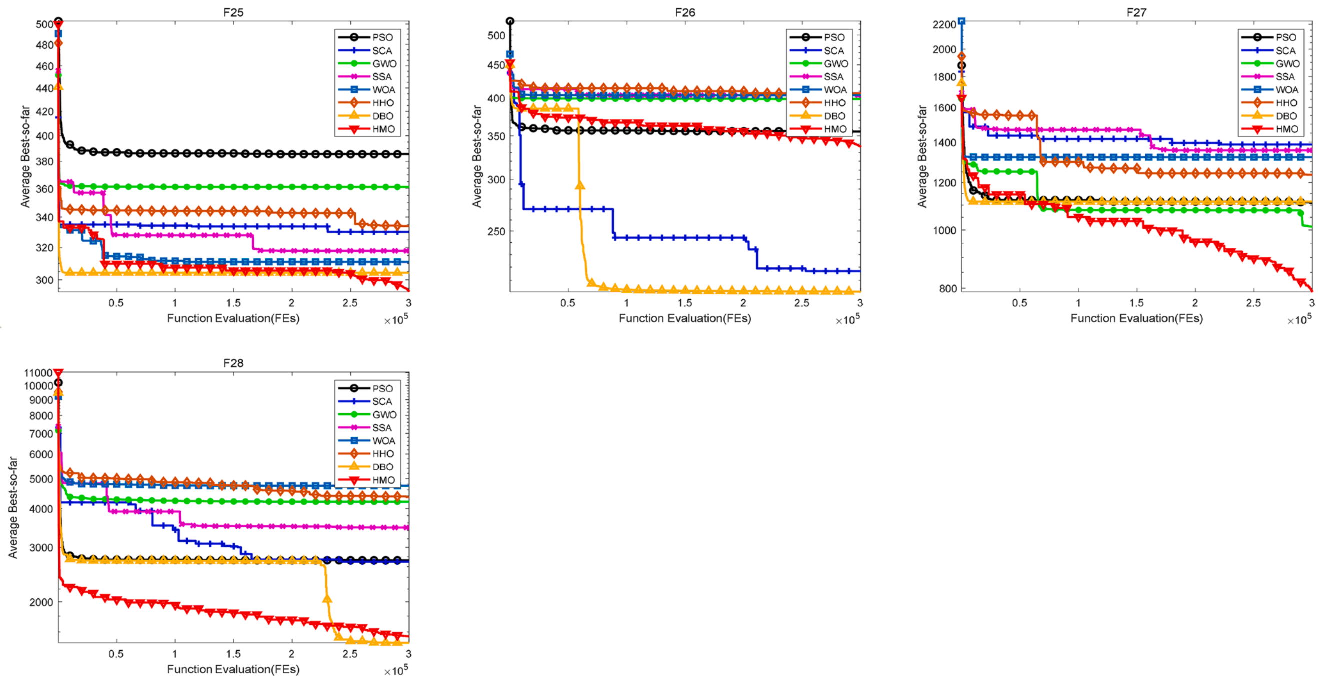

Fig. 3. (*continued*).

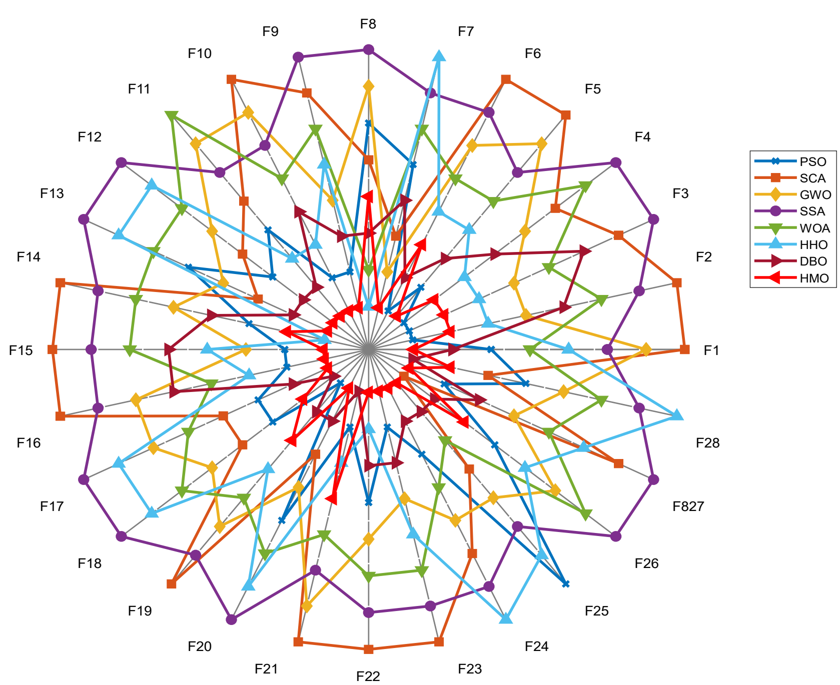

Fig. 4. Algorithmic ranking radar chart.

years, which include QMESSA (Wu et al., 2023), FACL (Peng et al., 2021), MPSO (Liu, Zhang & Tu, 2020; Meng et al., 2022), CSSA (Zhang & Ding, 2021), EAPSO (Zhang, 2023), VPPSO (Shami et al., 2023), each of which has been validated in the CEC test set and has a strong global optimization capability. The internal parameters of each algorithm are shown in Table 5. The optimization results of each algorithm in 30 and 50 dimensions are shown in Tables 6–7 above to measure the differences between the algorithms, and the average metrics of each algorithm are used to perform the Friedman test (Demšar, 2006) to get the overall ranking.

From Tables 5–6, it is intuitively clear that HMO has a certain optimization advantage in comparison with other variant algorithms of recent years, with an average ranking of 3.9286 in the 30-dimension and 3.9464 in the 50-dimension, and an overall ranking of fourth in both

Table 4
Table of test results for each algorithm.

F	PSO	SCA	GWO	SSA	WOA	HHO	DBO
F1	4.90E-06	3.02E-11	3.51E-11	3.02E-11	3.02E-11	3.02E-11	6.57E-05
F2	1.52E-01	3.02E-11	2.79E-09	3.02E-11	3.02E-11	3.02E-11	3.82E-10
F3	3.91E-01	3.02E-11	1.63E-08	3.02E-11	7.39E-11	2.68E-04	3.82E-10
F4	2.56E-02	3.02E-11	3.17E-11	3.02E-11	3.02E-11	3.02E-11	3.02E-11
F5	6.05E-01	3.02E-11	4.00E-10	3.02E-11	3.02E-11	3.02E-11	1.39E-06
F6	3.22E-01	3.02E-11	4.40E-10	1.61E-10	1.73E-07	5.79E-01	8.07E-01
F7	1.33E-10	3.02E-11	1.39E-10	3.02E-11	3.02E-11	3.02E-11	3.02E-11
F8	6.41E-01	9.00E-01	2.31E-01	5.89E-01	1.91E-01	1.68E-03	3.71E-01
F9	2.92E-09	6.70E-11	4.40E-10	6.70E-11	2.15E-10	3.16E-10	4.62E-10
F10	1.69E-05	3.02E-11	1.15E-10	3.02E-11	3.02E-11	3.02E-11	3.20E-09
F11	5.07E-10	5.57E-10	2.05E-10	2.37E-10	1.46E-10	2.78E-07	4.46E-04
F12	4.62E-10	5.57E-10	2.26E-10	3.69E-11	1.09E-10	4.08E-11	1.73E-07
F13	6.70E-11	4.62E-10	2.05E-10	3.69E-11	6.70E-11	3.34E-11	3.35E-08
F14	1.30E-01	3.02E-11	7.38E-08	1.96E-10	2.83E-08	3.15E-02	4.23E-03
F15	5.19E-07	6.70E-11	6.80E-08	6.72E-10	1.86E-09	1.36E-07	2.83E-08
F16	3.94E-03	4.08E-11	1.94E-09	4.18E-09	5.46E-06	2.77E-05	4.57E-09
F17	1.11E-04	3.02E-11	4.73E-11	3.02E-11	3.34E-11	3.02E-11	5.40E-01
F18	9.76E-10	3.34E-11	8.48E-10	3.02E-11	3.34E-11	3.02E-11	4.64E-01
F19	5.80E-07	3.02E-11	2.87E-01	8.99E-11	2.25E-04	7.73E-02	5.09E-08
F20	1.22E-05	7.28E-04	1.95E-05	5.50E-09	2.64E-07	2.05E-07	9.46E-03
F21	6.90E-01	3.02E-11	5.23E-11	3.02E-11	1.77E-03	4.84E-02	5.46E-06
F22	1.25E-07	3.02E-11	4.00E-10	4.98E-11	1.46E-10	1.96E-01	2.49E-06
F23	4.35E-05	5.49E-11	1.11E-07	1.41E-09	1.31E-08	1.87E-07	2.32E-06
F24	1.46E-10	3.02E-11	3.51E-11	3.02E-11	3.02E-11	3.34E-11	1.21E-10
F25	6.07E-11	3.34E-11	3.88E-11	3.69E-11	9.92E-11	4.50E-11	1.41E-09
F26	1.67E-01	1.95E-03	1.36E-04	9.06E-08	8.88E-06	9.79E-05	1.60E-03
F27	2.61E-10	3.02E-11	3.51E-11	3.02E-11	3.02E-11	3.02E-11	1.61E-10
F28	5.07E-10	5.57E-10	4.84E-10	2.15E-10	4.62E-10	2.87E-10	4.57E-09
+/=/-	8/0/20	1/0/27	2/0/26	1/0/27	1/0/27	3/0/25	4/0/24

Table 5
Internal parameter settings for each variant of the algorithm.

Algorithms	parameters
QMESSA	ST = 0.8, PD = 0.2, SD = 0.2,ED = 0.2
FACL	$\alpha = 0.5, \beta_{min} = 0.2, \beta = 1, \gamma = 1$
MPSO	$iw \in [0.4, 0.9], cw = 4 \bullet r \bullet (1 - r)$
CSSA	ST = 0.8, PD = 0.2, SD = 0.2
VPPSO	$\alpha = 0.3$, $N_1 = 15, N_2 = 15$

dimensions. In 30 dimensions, the best function of HMO's optimization ability is F4, F9, F11-13, F23, and the worse function is F16, F24-26. In 50 dimensions, the best function of optimization ability is F4, F7, F9, and the worse function is F16, F20, F24, F26. In the other functions, HMO is able to beat individual algorithms as well as lag behind other algorithms. MPSO and VPPSO have a better ability to find the optimal, HMO beats them relatively few times, while compared with FACL, CSSA, and EAPSO, HMO beats more than half of them, especially for the 30-dimensional EAPSO, HMO beats it 21 times, on the whole, HMO has a certain degree of competitiveness in the comparison of the 6 variants of algorithms, although it is behind the individual algorithms, but it is also stronger than some algorithms, which also verifies that HMO has some development and research value.

3.3. Parametric analysis

To further discuss the rationality of the parameter r setting, this subsection provides a parametric analysis of R. Since the probabilities of successful and failed memories are the same, as well as the behavior of recent memories, they sum to 1. Therefore, the values of r were taken as 0.1, 0.2, 0.3, and 0.4 for the experiments, respectively, and the other environmental and internal parameters were consistent as above, and the optimal values were bolded. The specific results are shown in Table 8.

It can be seen from Table 5 that HMO has the most optimal values for the optimization performance, especially in the functions F1, F4, F5, F7, F8, F9, F11-14, F17-18, F22-F23, F27 where the advantage is more significant. It can be seen that the HMO is better optimized in CEC 2013 when $r = 0.1$, and further validates the feasibility of the HMO. Some of the functions are also better when r is taken for other probabilities, for example, $r = 0.2$ gives better optimization results in F2 and F21, thus illustrating that r is a very important parameter and the value of r needs to be set reasonably according to different optimization problems.

4. Engineering optimization examples

This section focuses on the application of each algorithm to three engineering optimization problems to verify the practicality of the HMO. Assume that the number of parameters to be optimized is d and the maximum number of evaluations is d*10000.The parameters are consistent as described above.Each algorithm was run independently 10 times.The statistical indicators and optimal parameters for the results of the 10 runs are shown in each table. It is worth noting that the precision in this section is retained to six decimal places.

4.1. Three-bar truss design problem

The objective of this problem is to minimize the volume of a three-bar truss by adjusting the cross-sectional area (x_1, x_2) for each truss member subject to stress constraints. (Pathak & Srivastava, 2022). The table with optimization results is shown in Table 9 and the average convergence results for each algorithm are plotted in Fig. 5.

Table 9 shows the results of HMO with PSO, GWO, WOA, MRFO, HHO, SSA and DBO in solving the Three-bar truss design problem. As

Table 6
Optimization results of HMO and variant algorithms (D = 30).

F	Index	QMESSA	FACL	MPSO	CSSA	EAPSO	VPPSO	HMO
F1	Best	4.5475E-13	9.1542E-01	6.5938E-11	2.4826E-12	5.2907E + 02	2.1787E-08	5.1405E-05
	Worst	1.3642E-12	2.1738E + 00	4.5594E + 01	6.1385E-10	1.7778E + 03	5.8043E-08	4.6362E-04
	Mean	6.4423E-13	1.4131E + 00	1.6166E + 00	7.5607E-11	1.1667E + 03	3.6924E-08	1.8619E-04
	Std	2.4680E-13	2.9850E-01	8.3093E + 00	1.2900E-10	3.5625E + 02	7.5949E-09	8.2076E-05
	contest	1.9495E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F2	Best	7.5247E + 05	2.5512E + 06	6.8999E + 05	1.7974E + 06	4.0759E + 07	1.3068E + 06	1.7679E + 06
	Worst	3.7638E + 06	1.1260E + 07	3.7031E + 07	7.8640E + 06	1.3779E + 08	5.5341E + 06	5.5244E + 06
	Mean	2.0310E + 06	4.5447E + 06	9.5307E + 06	4.4585E + 06	7.7020E + 07	2.5854E + 06	3.3442E + 06
	Std	9.5069E + 05	1.7715E + 06	9.6291E + 06	1.3497E + 06	2.8022E + 07	9.8944E + 05	1.0960E + 06
	contest	1.1058E-04(-)	4.4272E-03(+)	1.6687E-01(=)	2.4994E-03(+)	3.0199E-11(+)	1.3271E-02(-)	
F3	Best	2.3975E + 07	5.0333E + 07	2.5725E + 07	4.5708E + 07	6.3105E + 09	2.9773E + 07	4.7959E + 07
	Worst	6.0281E + 09	7.9213E + 09	9.3847E + 09	1.7833E + 09	4.1823E + 10	2.0559E + 09	5.7242E + 09
	Mean	5.6394E + 08	2.3471E + 09	9.1914E + 08	5.9462E + 08	1.9664E + 10	4.9853E + 08	7.9463E + 08
	Std	1.0845E + 09	2.2723E + 09	1.6636E + 09	4.9021E + 08	8.2902E + 09	4.9357E + 08	1.1008E + 09
	contest	1.8575E-03(-)	8.5000E-02(=)	8.7710E-02(=)	2.8129E-02(-)	3.3384E-11(+)	2.8913E-03(-)	
F4	Best	6.4745E + 03	1.7552E + 04	2.2878E + 02	5.9025E + 03	8.8486E + 04	2.8305E + 03	9.2704E + 01
	Worst	1.9965E + 04	4.0771E + 04	2.0817E + 03	1.6669E + 04	1.8129E + 05	1.0234E + 04	4.9400E + 02
	Mean	1.3331E + 04	3.0349E + 04	7.1954E + 02	9.0880E + 03	1.2464E + 05	5.8521E + 03	2.5993E + 02
	Std	3.4534E + 03	5.3348E + 03	3.8826E + 02	2.4963E + 03	2.4391E + 04	1.8366E + 03	1.1245E + 02
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F5	Best	2.1600E-11	4.7157E-01	8.2838E-05	2.2668E-06	1.6975E + 02	2.7977E-03	2.0725E-02
	Worst	4.3078E-09	1.1436E + 00	4.5740E + 01	7.1820E-05	3.6558E + 02	3.9872E-03	4.7068E-02
	Mean	7.3869E-10	7.3946E-01	5.0418E + 00	1.4941E-05	2.3832E + 02	3.4571E-03	3.6832E-02
	Std	1.0291E-09	1.4975E-01	9.5054E + 00	1.4915E-05	4.8090E + 01	2.6887E-04	6.3383E-03
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F6	Best	6.6178E + 00	3.2662E + 00	1.6550E + 01	6.5461E + 00	1.1797E + 02	1.5762E + 01	5.1978E + 00
	Worst	8.6730E + 01	1.2512E + 02	1.2520E + 02	1.0426E + 02	2.8094E + 02	9.4421E + 01	1.0891E + 02
	Mean	3.3045E + 01	5.9170E + 01	4.9469E + 01	5.1118E + 01	1.8104E + 02	5.8640E + 01	6.4354E + 01
	Std	2.7354E + 01	2.8795E + 01	2.7786E + 01	2.6717E + 01	4.5540E + 01	2.5076E + 01	2.5053E + 01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	
F7	Best	8.5728E + 01	7.4555E + 01	3.3345E + 01	6.7498E + 01	8.0565E + 01	4.4460E + 01	2.5895E + 01
	Worst	6.0324E + 02	1.7948E + 02	9.9902E + 01	4.0103E + 02	2.7063E + 02	1.1473E + 02	9.0972E + 01
	Mean	1.7443E + 02	1.2389E + 02	5.3659E + 01	1.7851E + 02	1.4215E + 02	7.6330E + 01	5.4684E + 01
	Std	8.9589E + 01	2.1627E + 01	1.5283E + 01	6.7898E + 01	4.2364E + 01	1.9721E + 01	1.7345E + 01
	contest	4.1825E-09(+)	8.4848E-09(+)	8.4848E-09(+)	2.2273E-09(+)	6.5183E-09(+)	8.4848E-09(+)	
F8	Best	2.0892E + 01	2.0768E + 01	2.0858E + 01	2.0916E + 01	2.1063E + 01	2.0756E + 01	2.0737E + 01
	Worst	2.1045E + 01	2.0927E + 01	2.1023E + 01	2.1057E + 01	2.1229E + 01	2.1006E + 01	2.1019E + 01
	Mean	2.0971E + 01	2.0859E + 01	2.0954E + 01	2.0989E + 01	2.1159E + 01	2.0931E + 01	2.0941E + 01
	Std	4.2769E-02	4.3352E-02	3.5222E-02	3.6864E-02	4.5673E-02	6.3955E-02	6.4911E-02
	contest	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	
F9	Best	2.8964E + 01	2.8212E + 01	1.3062E + 01	2.6744E + 01	2.5486E + 01	1.5923E + 01	1.4247E + 01
	Worst	4.4126E + 01	3.7999E + 01	2.9513E + 01	4.0342E + 01	3.9343E + 01	2.7553E + 01	3.8139E + 01
	Mean	3.7664E + 01	3.3743E + 01	2.2527E + 01	3.4278E + 01	3.2419E + 01	2.2079E + 01	2.0007E + 01
	Std	3.2005E + 00	2.5837E + 00	4.2614E + 00	3.0715E + 00	3.3413E + 00	2.6861E + 00	4.2506E + 00
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F10	Best	6.8984E-02	1.3697E + 00	8.8676E-01	7.8043E-02	1.8426E + 02	1.9691E-02	4.0746E-01
	Worst	8.6552E-01	3.7407E + 00	3.3473E + 00	1.0626E + 00	5.4449E + 02	1.7011E-01	1.4929E + 00
	Mean	1.8935E-01	2.0961E + 00	1.6077E + 00	3.0476E-01	3.3896E + 02	6.3415E-02	1.0412E + 00
	Std	1.4207E-01	5.8669E-01	5.8163E-01	2.2179E-01	9.7279E + 01	3.8945E-02	3.0289E-01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F11	Best	3.8803E + 01	1.6214E + 02	1.7009E + 01	1.4128E + 02	2.2346E + 02	7.5552E + 01	5.5180E + 01
	Worst	1.0248E + 02	4.1682E + 02	6.1938E + 01	4.1091E + 02	2.9967E + 02	2.1789E + 02	4.4972E + 02
	Mean	6.4374E + 01	2.9809E + 02	3.4305E + 01	2.5053E + 02	2.5755E + 02	1.3136E + 02	1.1178E + 02
	Std	1.5669E + 01	6.7356E + 01	1.0535E + 01	6.9033E + 01	2.0850E + 01	3.8628E + 01	6.9319E + 01
	contest	2.9047E-01(=)	8.1014E-10(+)	4.1191E-01(=)	1.8500E-08(+)	4.5726E-09(+)	1.0576E-03(+)	

(*continued on next page*)

Table 6 (*continued*)

F	Index	QMESSA	FACL	MPSO	CSSA	EAPSO	VPPSO	HMO
F12	Best	1.2238E + 02	1.4680E + 02	3.9809E + 01	2.3437E + 02	2.3220E + 02	7.3627E + 01	3.9049E + 01
	Worst	5.3825E + 02	4.3397E + 02	1.1267E + 02	9.0935E + 02	3.2340E + 02	2.4675E + 02	4.8454E + 02
	Mean	2.8645E + 02	3.0394E + 02	7.6206E + 01	4.1871E + 02	2.8156E + 02	1.3017E + 02	1.2429E + 02
	Std	9.0656E + 01	7.8055E + 01	1.8444E + 01	1.2538E + 02	2.1380E + 01	4.0928E + 01	8.7752E + 01
	contest	4.1997E-10(+)	1.9568E-10(+)	6.3088E-01(=)	3.0199E-11(+)	3.0199E-11(+)	7.2884E-03(+)	
F13	Best	2.3463E + 02	2.0420E + 02	7.6451E + 01	2.6254E + 02	2.3800E + 02	9.3423E + 01	1.0139E + 02
	Worst	5.8064E + 02	3.9604E + 02	2.1495E + 02	5.8726E + 02	3.0435E + 02	2.9415E + 02	4.1456E + 02
	Mean	3.8188E + 02	3.0332E + 02	1.4568E + 02	4.2470E + 02	2.7764E + 02	1.9197E + 02	1.9591E + 02
	Std	8.8203E + 01	4.7615E + 01	3.0516E + 01	8.1560E + 01	1.6594E + 01	4.7971E + 01	5.8430E + 01
	contest	9.0632E-08(+)	1.4932E-04(+)	8.5641E-04(=)	6.7220E-10(+)	8.6844E-03(+)	2.9047E-01(=)	
F14	Best	2.3305E + 02	3.1719E + 03	9.1429E + 02	1.8912E + 03	4.8856E + 03	1.9574E + 03	1.2881E + 03
	Worst	1.7667E + 03	5.5204E + 03	2.5917E + 03	3.7744E + 03	8.5643E + 03	4.7823E + 03	4.9129E + 03
	Mean	7.2377E + 02	4.4196E + 03	1.3969E + 03	2.8658E + 03	6.9941E + 03	3.0422E + 03	3.1030E + 03
	Std	3.4500E + 02	5.8412E + 02	4.2224E + 02	5.1938E + 02	7.6201E + 02	6.8274E + 02	8.2510E + 02
	contest	3.0199E-11(-)	2.8129E-02(+)	3.0199E-11(-)	1.4294E-08(-)	4.0772E-11(-)	2.3168E-06(-)	
F15	Best	3.4088E + 03	3.1527E + 03	2.4703E + 03	3.1416E + 03	6.2762E + 03	2.1155E + 03	2.1748E + 03
	Worst	5.5122E + 03	5.7040E + 03	5.4442E + 03	5.8622E + 03	8.8045E + 03	5.6984E + 03	7.1792E + 03
	Mean	4.4485E + 03	4.5951E + 03	3.6011E + 03	4.5574E + 03	7.9297E + 03	3.4757E + 03	3.3729E + 03
	Std	6.2735E + 02	6.0928E + 02	7.3754E + 02	6.5470E + 02	5.2191E + 02	7.0384E + 02	1.0090E + 03
	contest	9.4696E-01(=)	3.4783E-01(=)	2.9590E-05(-)	4.4642E-01(=)	3.0199E-11(+)	6.5261E-07(-)	
F16	Best	6.8698E-01	1.3918E + 00	8.4989E-01	4.6722E-01	2.8686E + 00	8.1858E-02	1.0077E + 02
	Worst	3.6244E + 00	2.4784E + 00	2.6688E + 00	2.3191E + 00	4.9557E + 00	1.1888E + 00	1.0218E + 02
	Mean	1.8726E + 00	2.0464E + 00	2.0458E + 00	1.2630E + 00	3.7767E + 00	3.8193E-01	1.0131E + 02
	Std	7.1008E-01	2.6451E-01	5.0549E-01	5.2387E-01	5.5237E-01	2.4623E-01	3.2825E-01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F17	Best	8.2640E + 01	2.4687E + 02	5.6777E + 01	2.1143E + 02	2.5323E + 02	6.9264E + 01	1.9211E + 02
	Worst	1.7414E + 02	4.6701E + 02	1.1772E + 02	7.2702E + 02	3.9752E + 02	1.6632E + 02	4.4928E + 02
	Mean	1.3324E + 02	3.5683E + 02	8.6382E + 01	4.0462E + 02	3.4266E + 02	1.2366E + 02	2.7683E + 02
	Std	2.7347E + 01	6.0605E + 01	1.7173E + 01	1.4002E + 02	3.4767E + 01	2.5023E + 01	6.7189E + 01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	6.0658E-11(+)	3.0199E-11(+)	3.0199E-11(-)	
F18	Best	2.2957E + 02	2.8801E + 02	6.5537E + 01	2.5603E + 02	2.8453E + 02	7.1462E + 01	2.1448E + 02
	Worst	6.5770E + 02	4.6323E + 02	1.2921E + 02	7.9541E + 02	3.9937E + 02	2.3084E + 02	5.1596E + 02
	Mean	4.2568E + 02	3.6456E + 02	9.6558E + 01	4.8044E + 02	3.3850E + 02	1.2590E + 02	3.5595E + 02
	Std	1.1896E + 02	4.8012E + 01	1.6905E + 01	1.4721E + 02	2.6676E + 01	3.1771E + 01	4.9072E + 01
	contest	3.6897E-11(+)	3.0199E-11(+)	3.0199E-11(-)	5.0723E-10(+)	3.0199E-11(+)	3.0199E-11(-)	
F19	Best	4.5256E + 00	2.5428E + 01	2.7088E + 00	7.7211E + 00	2.7994E + 01	3.1288E + 00	1.0959E + 02
	Worst	1.8115E + 01	2.0206E + 02	9.8266E + 00	2.6435E + 01	9.6439E + 01	1.4917E + 01	1.6826E + 02
	Mean	1.0095E + 01	1.0295E + 02	4.6657E + 00	1.4718E + 01	5.0753E + 01	7.6572E + 00	1.3803E + 02
	Std	2.7486E + 00	4.1443E + 01	1.4576E + 00	4.7506E + 00	1.8965E + 01	2.9331E + 00	1.4715E + 01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F20	Best	1.2870E + 01	1.2685E + 01	1.0350E + 01	1.3136E + 01	1.3495E + 01	1.0322E + 01	1.0921E + 02
	Worst	1.5000E + 01	1.5000E + 01	1.3854E + 01	1.5000E + 01	1.5000E + 01	1.3979E + 01	1.1500E + 02
	Mean	1.4227E + 01	1.4923E + 01	1.2614E + 01	1.4821E + 01	1.4924E + 01	1.1767E + 01	1.1239E + 02
	Std	5.0007E-01	4.2263E-01	7.0160E-01	4.2826E-01	3.0544E-01	7.8143E-01	1.6296E + 00
	contest	2.5190E-11(-)	1.3678E-12(-)	2.5206E-11(-)	5.2724E-12(-)	1.8917E-12(-)	2.5206E-11(-)	
F21	Best	1.0000E + 02	2.1932E + 02	2.3893E + 02	3.0000E + 02	6.1121E + 02	1.0000E + 02	4.0015E + 02
	Worst	4.4354E + 02	4.4372E + 02	1.8749E + 03	4.4354E + 02	1.0972E + 03	4.4354E + 02	5.4354E + 02
	Mean	3.5554E + 02	3.5219E + 02	1.0017E + 03	3.6220E + 02	8.7702E + 02	3.6323E + 02	4.7666E + 02
	Std	8.6171E + 01	6.3503E + 01	5.0314E + 02	7.2347E + 01	1.2903E + 02	9.6446E + 01	7.2725E + 01
	contest	2.5206E-11(-)	3.0199E-11(-)	2.8378E-01(+)	3.0199E-11(-)	9.0632E-08(+)	3.0199E-11(-)	
F22	Best	4.6273E + 02	4.1421E + 03	3.9580E + 02	1.8752E + 03	5.8339E + 03	2.0385E + 03	2.1353E + 03
	Worst	2.0340E + 03	7.6029E + 03	1.7643E + 03	6.1712E + 03	8.7953E + 03	5.2023E + 03	5.5338E + 03
	Mean	1.0520E + 03	6.0618E + 03	9.6303E + 02	3.5702E + 03	7.5179E + 03	3.4933E + 03	3.2697E + 03
	Std	3.8927E + 02	9.6756E + 02	3.9743E + 02	8.9913E + 02	6.7180E + 02	6.6138E + 02	7.4635E + 02
	contest	3.0199E-11(-)	1.0233E-01(=)	3.0199E-11(-)	1.0937E-10(+)	2.9590E-05(+)	4.0772E-11(+)	

(continued on next page)

Table 6 (*continued*)

F	Index	QMESSA	FACL	MPSO	CSSA	EAPSO	VPPSO	HMO
F23	Best	4.3443E + 03	4.4255E + 03	3.0507E + 03	3.5630E + 03	6.8764E + 03	1.9578E + 03	1.8978E + 03
	Worst	8.1119E + 03	7.6450E + 03	6.1628E + 03	8.4689E + 03	9.5055E + 03	5.1273E + 03	7.2015E + 03
	Mean	6.6113E + 03	6.2457E + 03	4.3626E + 03	5.7654E + 03	8.2714E + 03	4.0233E + 03	4.0199E + 03
	Std	1.0130E + 03	7.5851E + 02	7.7174E + 02	1.1275E + 03	6.4974E + 02	7.3730E + 02	1.3360E + 03
	contest	6.3088E-01(=)	7.6171E-03(+)	1.3289E-10(+)	2.3885E-04(+)	9.2603E-09(+)	3.0199E-11(+)	
F24	Best	2.8185E + 02	2.6650E + 02	2.3247E + 02	2.8282E + 02	2.6735E + 02	2.5053E + 02	3.4160E + 02
	Worst	4.2190E + 02	3.0864E + 02	2.8281E + 02	3.1963E + 02	3.0627E + 02	2.9575E + 02	3.9415E + 02
	Mean	3.0709E + 02	2.9088E + 02	2.5320E + 02	3.0439E + 02	2.9315E + 02	2.7396E + 02	3.5258E + 02
	Std	2.5893E + 01	1.2671E + 01	1.2956E + 01	9.4504E + 00	8.7352E + 00	1.1111E + 01	1.1510E + 01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F25	Best	3.0100E + 02	2.7766E + 02	2.3344E + 02	2.8723E + 02	2.9436E + 02	2.7642E + 02	3.4977E + 02
	Worst	3.5731E + 02	3.5138E + 02	3.1281E + 02	3.4651E + 02	3.2378E + 02	3.0387E + 02	4.1657E + 02
	Mean	3.2296E + 02	3.3296E + 02	2.7640E + 02	3.1359E + 02	3.0944E + 02	2.8910E + 02	3.7387E + 02
	Std	1.4123E + 01	1.5906E + 01	1.5393E + 01	1.5306E + 01	7.9396E + 00	7.8752E + 00	1.3000E + 01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F26	Best	2.0006E + 02	2.0010E + 02	2.0005E + 02	2.0011E + 02	2.0308E + 02	2.0003E + 02	3.0017E + 02
	Worst	4.0705E + 02	2.0030E + 02	3.5490E + 02	4.0665E + 02	3.9441E + 02	3.6631E + 02	4.9094E + 02
	Mean	3.2033E + 02	2.0018E + 02	2.5284E + 02	3.5411E + 02	3.5639E + 02	2.2739E + 02	4.0828E + 02
	Std	9.0262E + 01	4.9250E-02	7.0015E + 01	7.8590E + 01	6.1193E + 01	6.0966E + 01	6.6845E + 01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F27	Best	1.0814E + 03	8.5108E + 02	6.1380E + 02	1.0476E + 03	1.0304E + 03	7.7189E + 02	7.9999E + 02
	Worst	1.5291E + 03	1.2388E + 03	9.8724E + 02	1.4113E + 03	1.3278E + 03	1.1608E + 03	1.2265E + 03
	Mean	1.3227E + 03	1.0805E + 03	8.0388E + 02	1.2311E + 03	1.1750E + 03	9.4950E + 02	9.2500E + 02
	Std	8.9867E + 01	9.2439E + 01	9.5341E + 01	1.1781E + 02	7.9786E + 01	9.7829E + 01	9.9195E + 01
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F28	Best	3.0000E + 02	1.3784E + 02	1.0696E + 02	1.0000E + 02	1.2611E + 03	1.0000E + 02	6.4366E + 02
	Worst	4.0553E + 03	3.2512E + 03	2.8374E + 03	5.5021E + 03	2.2128E + 03	2.9058E + 03	5.1936E + 03
	Mean	1.7136E + 03	2.0998E + 03	3.7536E + 02	2.9304E + 03	1.5322E + 03	5.7024E + 02	1.3616E + 03
	Std	1.4417E + 03	6.5430E + 02	4.6708E + 02	1.5445E + 03	1.7828E + 02	6.4799E + 02	7.6935E + 02
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.8249E-09(+)	3.0199E-11(+)	3.0199E-11(-)	
+/=/−		9/3/16	18/3/7	10/5/13	15/1/12	21/0/7	6/1/21	
rank		3.7143	4.8929	2.6786	4.6429	5.7143	2.4286	3.9286

seen from the table, the optimal value $f(x) = 263.852346$ obtained from the HMO optimization, corresponding to the optimal solution $x = (0.788415, 0.408114)$, ranks first. It is worth noting that the mean value of HMO's solution is 263.852347 with a variance of 0. This proves that HMO has good stability in solving the Three-bar truss design problem compared to the other seven algorithms. The convergence curve in Fig. 5 also shows that HMO has a faster convergence rate compared to the other seven algorithms.

4.2. Compression Spring design

The objective of this problem is to minimize the weight of the tension/compression spring design problem. Three design variables are considered in this problem: (1) the wire diameter d (x_1); (2) the average coil diameter D (x_2); and (3) the number of active coils P (x_3) (Nadimi-Shahraki et al., 2020).The specific optimization results are shown in the Table 10 and a graph of the average convergence results for each algorithm shown in Fig. 6.

4.3. Welded beam design

The objective of this problem is to minimize the manufacturing cost of welded beams. This problem includes the following design variables: (1) the height of the reinforcement t (x_1); (2) the thickness of the weld h (x_2); (3) the thickness of the reinforcement b (x_3); and (4) the length of the reinforcement l (x_4) (Coello, 2000). A table with optimization results is shown in Table 11, and a graph of the average convergence results for each algorithm is shown in Fig. 7.

4.4. Comprehensive analysis

Among the three engineering optimization problems mentioned above, it can be seen that HMO optimizes better, while the optimized parameters are within a reasonable constraint. In particular, the optimal solutions found by the HMO for problems Three-bar truss design and Compression Spring Design have a high accuracy and are also very stable. In the average convergence plots of the three optimization

Table 7
Optimization results for HMO and variant algorithms (D = 50).

F	Index	QMESSA	FACL	MPSO	CSSA	EAPSO	VPPSO	HMO
F1	Best	6.8212E-13	3.4298E + 00	7.1183E-07	5.5765E-06	9.0591E + 02	6.5731E-08	1.7071E-03
	Worst	2.0464E-12	3.0061E + 01	2.6821E + 03	3.3395E-04	2.6503E + 03	1.5129E-07	8.1600E-03
	Mean	1.1293E-12	6.2525E + 00	4.1900E + 02	1.3492E-04	1.7448E + 03	1.0199E-07	3.8281E-03
	Std	3.8919E-13	4.7117E + 00	8.7117E + 02	9.0100E-05	4.9406E + 02	1.9243E-08	1.3004E-03
	contest	2.4666E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F2	Best	1.6635E + 06	5.2400E + 06	5.6378E + 06	2.8077E + 06	7.1075E + 07	5.4653E + 05	2.9190E + 06
	Worst	4.4714E + 06	3.2279E + 07	8.1273E + 07	1.6339E + 07	1.8306E + 08	2.7527E + 06	7.8838E + 06
	Mean	3.1040E + 06	1.0104E + 07	3.7732E + 07	9.1885E + 06	1.2528E + 08	1.6656E + 06	5.3216E + 06
	Std	7.7394E + 05	5.1474E + 06	2.3822E + 07	3.1236E + 06	3.1866E + 07	6.0343E + 05	1.3450E + 06
	contest	2.8314E-08(-)	7.1186E-09(+)	1.0937E-10(+)	3.5201E-07(+)	3.0199E-11(+)	3.0199E-11(-)	
F3	Best	6.8500E + 07	1.1662E + 09	4.4452E + 08	1.7661E + 08	2.2258E + 10	1.5646E + 08	5.0296E + 08
	Worst	2.9112E + 09	1.7618E + 10	1.3891E + 10	3.8629E + 09	1.1361E + 11	2.5549E + 09	1.4812E + 10
	Mean	6.7386E + 08	6.1148E + 09	4.6205E + 09	1.2352E + 09	4.9612E + 10	1.0268E + 09	3.5758E + 09
	Std	5.5137E + 08	5.1306E + 09	3.0800E + 09	9.6635E + 08	1.6863E + 10	6.2616E + 08	3.7244E + 09
	contest	4.3106E-08(-)	7.2884E-03(+)	1.9883E-02(+)	4.9818E-04(-)	3.0199E-11(+)	4.9426E-05(-)	
F4	Best	1.1016E + 04	5.5931E + 04	2.5149E + 03	1.1921E + 04	9.1372E + 04	4.4507E + 03	4.0201E + 01
	Worst	3.2647E + 04	9.7704E + 04	9.9065E + 03	3.1934E + 04	2.8626E + 05	1.1697E + 04	2.7835E + 02
	Mean	2.1001E + 04	7.4991E + 04	5.9936E + 03	2.0089E + 04	1.6462E + 05	6.9337E + 03	1.2832E + 02
	Std	6.2119E + 03	1.1470E + 04	1.9979E + 03	4.9103E + 03	4.4480E + 04	1.5625E + 03	4.8755E + 01
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F5	Best	8.1745E-08	1.1865E + 00	8.7776E-04	6.1166E-03	1.8863E + 02	3.9978E-03	9.3328E-02
	Worst	1.6260E-06	4.1688E + 01	9.6153E + 01	4.1630E-02	4.2071E + 02	5.3457E-03	4.6741E-01
	Mean	4.6439E-07	3.2811E + 00	2.9878E + 01	1.6705E-02	3.0307E + 02	4.6400E-03	2.5417E-01
	Std	3.4927E-07	7.2689E + 00	2.9247E + 01	8.8374E-03	5.6486E + 01	3.4475E-04	1.1075E-01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F6	Best	2.0451E + 01	4.4556E + 01	4.6330E + 01	3.0855E + 01	1.2636E + 02	2.2526E + 01	4.3467E + 01
	Worst	1.5193E + 02	1.5264E + 02	1.9226E + 02	1.5447E + 02	3.4465E + 02	1.9948E + 02	1.5530E + 02
	Mean	7.2839E + 01	8.8593E + 01	9.2554E + 01	8.2838E + 01	2.1479E + 02	8.1479E + 01	8.1751E + 01
	Std	3.7538E + 01	2.4962E + 01	4.1408E + 01	3.5873E + 01	5.5068E + 01	3.3069E + 01	3.2629E + 01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	
F7	Best	1.0903E + 02	1.2271E + 02	4.9243E + 01	9.0234E + 01	1.3105E + 02	4.9800E + 01	4.8377E + 01
	Worst	4.8740E + 02	2.4302E + 02	9.9199E + 01	4.6763E + 02	2.2854E + 02	1.1990E + 02	1.0648E + 02
	Mean	1.8042E + 02	1.7053E + 02	7.3886E + 01	1.8925E + 02	1.7270E + 02	8.0257E + 01	6.7524E + 01
	Std	9.0533E + 01	2.9178E + 01	1.1140E + 01	6.7966E + 01	2.6853E + 01	1.5321E + 01	1.3703E + 01
	contest	5.5727E-10(+)	5.5727E-10(+)	5.5727E-10(+)	5.5727E-10(+)	5.5727E-10(+)	5.5727E-10(+)	
F8	Best	2.1088E + 01	2.0972E + 01	2.1026E + 01	2.1086E + 01	2.1162E + 01	2.1069E + 01	2.1034E + 01
	Worst	2.1211E + 01	2.1160E + 01	2.1205E + 01	2.1231E + 01	2.1352E + 01	2.1196E + 01	2.1190E + 01
	Mean	2.1150E + 01	2.1099E + 01	2.1136E + 01	2.1182E + 01	2.1278E + 01	2.1137E + 01	2.1137E + 01
	Std	2.9946E-02	3.9282E-02	4.4275E-02	3.4369E-02	4.6495E-02	3.1370E-02	4.0228E-02
	contest	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F9	Best	5.5819E + 01	5.6616E + 01	3.8259E + 01	4.8618E + 01	3.8807E + 01	3.1289E + 01	3.0721E + 01
	Worst	7.6904E + 01	7.3073E + 01	6.2240E + 01	7.2527E + 01	6.2247E + 01	6.0433E + 01	4.5730E + 01
	Mean	6.5759E + 01	6.6015E + 01	4.6944E + 01	6.5559E + 01	5.4376E + 01	4.5381E + 01	3.9122E + 01
	Std	5.9486E + 00	3.6656E + 00	5.2596E + 00	4.5769E + 00	5.0398E + 00	6.4536E + 00	4.0104E + 00
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F10	Best	6.8707E-02	4.0927E + 00	6.5672E + 00	1.8365E + 00	3.5888E + 02	1.4781E-02	2.5854E + 00
	Worst	1.0312E + 00	4.9774E + 01	1.1239E + 02	3.9527E + 00	1.2648E + 03	1.6267E-01	8.4269E + 00
	Mean	3.0681E-01	1.0815E + 01	2.9057E + 01	2.7834E + 00	6.9587E + 02	6.8003E-02	4.4513E + 00
	Std	2.4778E-01	1.0750E + 01	2.1749E + 01	5.6778E-01	1.8829E + 02	4.1277E-02	1.5260E + 00
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	
F11	Best	1.0845E + 02	4.1783E + 02	5.7882E + 01	3.0943E + 02	3.6575E + 02	1.9800E + 02	1.6533E + 02
	Worst	2.0894E + 02	9.2401E + 02	1.9442E + 02	7.5913E + 02	5.1911E + 02	5.9263E + 02	3.5838E + 02
	Mean	1.4669E + 02	6.7825E + 02	1.2083E + 02	5.5467E + 02	4.5134E + 02	2.8986E + 02	2.3406E + 02
	Std	2.8462E + 01	1.1481E + 02	3.5660E + 01	1.0580E + 02	4.0822E + 01	7.4598E + 01	3.9860E + 01
	contest	9.7555E-10(-)	5.4941E-11(+)	3.4742E-10(-)	4.5726E-09(+)	2.3168E-06(+)	7.4827E-02(=)	
F12	Best	3.2933E + 02	4.3395E + 02	1.0476E + 02	7.2432E + 02	4.0493E + 02	1.7312E + 02	1.5937E + 02
	Worst	7.5815E + 02	1.0277E + 03	2.7652E + 02	1.1113E + 03	5.7668E + 02	3.3032E + 02	5.2029E + 02
	Mean	5.2888E + 02	6.8920E + 02	1.8696E + 02	1.0324E + 03	5.0634E + 02	2.4835E + 02	2.9544E + 02
	Std	1.0748E + 02	1.1687E + 02	3.8860E + 01	7.4439E + 01	3.7040E + 01	3.9392E + 01	9.5851E + 01
	contest	3.1830E-01(=)	1.1738E-03(+)	3.0199E-11(-)	4.9752E-11(+)	2.8129E-02(+)	4.9752E-11(-)	
F13	Best	5.3893E + 02	5.5845E + 02	2.4554E + 02	7.0121E + 02	4.6268E + 02	3.2305E + 02	2.7049E + 02
	Worst	9.5396E + 02	9.0304E + 02	5.1574E + 02	1.3473E + 03	6.1369E + 02	5.5488E + 02	5.6731E + 02
	Mean	6.9984E + 02	7.3793E + 02	3.2584E + 02	9.9294E + 02	5.2611E + 02	4.1889E + 02	3.8677E + 02
	Std	1.1412E + 02	1.0083E + 02	6.4076E + 01	1.8771E + 02	4.1543E + 01	6.0562E + 01	7.2848E + 01
	contest	1.6238E-01(=)	7.1719E-01(=)	3.3384E-11(-)	1.8608E-06(+)	2.6695E-09(+)	5.4941E-11(+)	
F14	Best	3.6686E + 02	6.3360E + 03	1.7574E + 03	4.4141E + 03	9.0543E + 03	3.2141E + 03	4.0241E + 03
	Worst	2.5024E + 03	1.3858E + 04	5.6410E + 03	7.7065E + 03	1.3535E + 04	6.7560E + 03	1.1108E + 04
	Mean	1.3915E + 03	1.0989E + 04	3.6787E + 03	5.9756E + 03	1.1130E + 04	5.4796E + 03	5.6563E + 03
	Std	5.6483E + 02	2.8598E + 03	1.0762E + 03	7.8765E + 02	1.2065E + 03	9.2148E + 02	1.3321E + 03
	contest	3.0199E-11(-)	6.0459E-07(+)	6.6955E-11(-)	1.4298E-05(+)	6.6955E-11(+)	1.8731E-07(-)	
F15	Best	7.3772E + 03	7.8086E + 03	5.6428E + 03	6.4624E + 03	9.4870E + 03	4.5605E + 03	5.4061E + 03
	Worst	1.0555E + 04	1.4369E + 04	9.5291E + 03	9.4537E + 03	1.6103E + 04	8.8033E + 03	1.4978E + 04
	Mean	8.5882E + 03	1.2430E + 04	7.5706E + 03	8.1443E + 03	1.2998E + 04	6.7466E + 03	9.0365E + 03
	Std	7.7022E + 02	2.2368E + 03	8.9361E + 02	7.8904E + 02	2.1019E + 03	9.0744E + 02	3.5023E + 03

(*continued on next page*)

Table 7 (*continued*)

F	Index	QMESSA	FACL	MPSO	CSSA	EAPSO	VPPSO	HMO
	contest	1.7649E-02(-)	5.0912E-06(+)	4.8011E-07(-)	2.5306E-04(-)	3.8249E-09(+)	8.1014E-10(-)	
F16	Best	1.6055E + 00	2.4208E + 00	1.9504E + 00	7.7320E-01	2.9327E + 00	1.3889E-01	1.0109E + 02
	Worst	4.0992E + 00	3.3853E + 00	3.9866E + 00	3.1720E + 00	5.3088E + 00	8.1242E-01	1.0300E + 02
	Mean	2.7881E + 00	3.0303E + 00	3.0594E + 00	1.7922E + 00	4.3205E + 00	3.8626E-01	1.0187E + 02
	Std	6.8757E-01	2.6174E-01	5.3084E-01	6.0518E-01	6.2450E-01	1.7536E-01	4.8094E-01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F17	Best	1.9561E + 02	7.2637E + 02	1.3672E + 02	6.0791E + 02	5.4754E + 02	1.8979E + 02	3.2388E + 02
	Worst	4.0201E + 02	1.5312E + 03	2.5329E + 02	1.1451E + 03	6.6215E + 02	3.5701E + 02	6.1636E + 02
	Mean	2.7874E + 02	1.0942E + 03	1.8964E + 02	8.5845E + 02	6.0030E + 02	2.5872E + 02	4.4378E + 02
	Std	5.6553E + 01	1.9010E + 02	2.6828E + 01	1.5258E + 02	3.3980E + 01	4.2724E + 01	7.6732E + 01
	contest	3.0199E-11(-)	1.2541E-07(+)	3.0199E-11(-)	5.4941E-11(+)	3.0199E-11(+)	3.0199E-11(-)	
F18	Best	4.0334E + 02	8.3005E + 02	1.6578E + 02	6.5327E + 02	5.1295E + 02	1.8776E + 02	4.5938E + 02
	Worst	1.1559E + 03	1.5663E + 03	3.1698E + 02	1.2243E + 03	6.9741E + 02	3.2249E + 02	7.2526E + 02
	Mean	8.5400E + 02	1.1135E + 03	2.3073E + 02	1.0441E + 03	6.1143E + 02	2.6117E + 02	6.2473E + 02
	Std	1.8277E + 02	1.9601E + 02	4.0408E + 01	1.6097E + 02	3.9578E + 01	3.4879E + 01	6.0462E + 01
	contest	7.3891E-11(+)	8.3520E-08(+)	3.0199E-11(-)	1.4110E-09(+)	3.0199E-11(-)	3.0199E-11(-)	
F19	Best	1.4552E + 01	2.2391E + 02	6.3804E + 00	2.9494E + 01	4.7531E + 01	8.9631E + 00	1.4641E + 02
	Worst	7.2098E + 01	1.5951E + 03	1.9054E + 02	5.4498E + 01	3.4705E + 02	4.1205E + 01	2.6996E + 02
	Mean	2.8369E + 01	3.6168E + 02	2.0401E + 01	3.8124E + 01	1.2799E + 02	1.9643E + 01	1.8908E + 02
	Std	1.1324E + 01	2.4263E + 02	3.2663E + 01	6.9349E + 00	7.4938E + 01	6.8188E + 00	2.8592E + 01
	contest	3.0199E-11(-)	5.5727E-10(+)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F20	Best	2.3307E + 01	2.2213E + 01	2.1150E + 01	2.2087E + 01	2.3098E + 01	1.8569E + 01	1.1836E + 02
	Worst	2.5000E + 01	2.5000E + 01	2.4120E + 01	2.5000E + 01	2.5000E + 01	2.4017E + 01	1.2260E + 02
	Mean	2.4544E + 01	2.4711E + 01	2.2600E + 01	2.4417E + 01	2.4639E + 01	2.1843E + 01	1.2111E + 02
	Std	2.9026E-01	6.8519E-01	8.4470E-01	5.4320E-01	6.6430E-01	1.5351E + 00	9.2959E-01
	contest	3.0161E-11(-)	6.4789E-12(-)	3.0199E-11(-)	2.9822E-11(-)	9.4001E-12(-)	3.0199E-11(-)	
F21	Best	2.0000E + 02	2.3760E + 02	1.1998E + 03	8.3644E + 02	8.8057E + 02	2.0001E + 02	9.3644E + 02
	Worst	1.1224E + 03	1.1260E + 03	3.3119E + 03	1.1224E + 03	2.2815E + 03	1.1224E + 03	1.2224E + 03
	Mean	9.6765E + 02	7.2535E + 02	2.5288E + 03	9.8887E + 02	1.5055E + 03	1.0057E + 03	1.0412E + 03
	Std	2.0303E + 02	3.4271E + 02	5.4837E + 02	1.4502E + 02	3.7442E + 02	2.0158E + 02	1.4009E + 02
	contest	2.5416E-11(-)	3.0199E-11(-)	5.5329E-08(+)	3.0199E-11(-)	1.5798E-01(=)	3.0199E-11(-)	
F22	Best	1.0571E + 03	7.7077E + 03	1.0978E + 03	5.8001E + 03	9.9662E + 03	5.5393E + 03	4.4461E + 03
	Worst	3.3107E + 03	1.4732E + 04	6.1917E + 03	1.1507E + 04	1.5501E + 04	1.0111E + 04	8.9834E + 03
	Mean	2.1214E + 03	1.1341E + 04	4.0936E + 03	7.7845E + 03	1.2855E + 04	7.3003E + 03	6.8833E + 03
	Std	6.1949E + 02	2.3548E + 03	1.1423E + 03	1.3097E + 03	1.3000E + 03	1.2303E + 03	1.0203E + 03
	contest	3.0199E-11(-)	9.1171E-01(=)	3.0199E-11(-)	7.3803E-10(+)	5.6073E-05(+)	1.7769E-10(+)	
F23	Best	7.7923E + 03	8.1007E + 03	7.0032E + 03	8.4339E + 03	1.0661E + 04	4.7142E + 03	6.1295E + 03
	Worst	1.3819E + 04	1.5138E + 04	1.2405E + 04	1.3048E + 04	1.6439E + 04	1.2838E + 04	1.4754E + 04
	Mean	1.0900E + 04	1.2878E + 04	9.4667E + 03	1.0891E + 04	1.4675E + 04	7.6738E + 03	8.1477E + 03
	Std	1.2958E + 03	2.1808E + 03	1.4673E + 03	1.3710E + 03	1.4314E + 03	1.6366E + 03	1.6946E + 03
	contest	8.8829E-06(+)	1.8090E-01(=)	5.9673E-09(+)	1.4298E-05(+)	2.6784E-06(+)	1.6132E-10(-)	
F24	Best	3.6424E + 02	3.6746E + 02	2.7946E + 02	3.7595E + 02	3.2526E + 02	3.1489E + 02	3.8447E + 02
	Worst	4.2930E + 02	4.3854E + 02	3.3714E + 02	4.1751E + 02	3.9023E + 02	3.6949E + 02	4.2509E + 02
	Mean	3.9303E + 02	4.0433E + 02	3.1293E + 02	3.9200E + 02	3.5941E + 02	3.3708E + 02	4.0449E + 02
	Std	1.4980E + 01	1.8422E + 01	1.4439E + 01	1.0544E + 01	1.2400E + 01	1.4782E + 01	1.0255E + 01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F25	Best	3.8827E + 02	4.7193E + 02	3.3546E + 02	3.9197E + 02	3.7033E + 02	3.5461E + 02	4.1012E + 02
	Worst	4.6983E + 02	5.1450E + 02	3.9165E + 02	4.6400E + 02	4.2161E + 02	4.2204E + 02	4.8075E + 02
	Mean	4.3231E + 02	4.9132E + 02	3.6658E + 02	4.1778E + 02	3.9546E + 02	3.7865E + 02	4.4672E + 02
	Std	2.1130E + 01	1.0016E + 01	1.4925E + 01	1.9142E + 01	1.2936E + 01	1.6160E + 01	1.6720E + 01
	contest	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F26	Best	2.0036E + 02	2.0066E + 02	2.0081E + 02	2.0075E + 02	2.0832E + 02	2.0011E + 02	3.0148E + 02
	Worst	4.9748E + 02	2.7003E + 02	4.2666E + 02	4.9578E + 02	4.6266E + 02	4.5198E + 02	5.2410E + 02
	Mean	4.2480E + 02	2.1272E + 02	3.3566E + 02	4.6598E + 02	4.3251E + 02	3.9047E + 02	4.8018E + 02
	Std	1.0268E + 02	2.0383E + 01	9.4928E + 01	5.2130E + 01	4.3577E + 01	7.6752E + 01	6.1003E + 01
	contest	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	3.0199E-11(-)	
F27	Best	1.7856E + 03	1.9010E + 03	1.1803E + 03	1.7122E + 03	1.5616E + 03	1.2714E + 03	1.2528E + 03
	Worst	2.3418E + 03	2.3882E + 03	1.7554E + 03	2.3534E + 03	2.2111E + 03	2.0568E + 03	1.7388E + 03
	Mean	2.1386E + 03	2.1474E + 03	1.4420E + 03	2.1366E + 03	1.8153E + 03	1.6212E + 03	1.4652E + 03
	Std	1.3554E + 02	1.2607E + 02	1.2884E + 02	1.5445E + 02	1.3821E + 02	1.6741E + 02	1.3110E + 02
	contest	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(-)	3.0199E-11(+)	3.0199E-11(+)	3.0199E-11(+)	
F28	Best	4.0000E + 02	2.4093E + 03	4.4070E + 02	4.0006E + 02	7.5242E + 02	4.0000E + 02	8.7778E + 02
	Worst	5.7345E + 03	6.3719E + 03	4.5375E + 03	1.0060E + 04	4.4399E + 03	4.1912E + 03	4.1820E + 03
	Mean	1.7514E + 03	4.9717E + 03	1.6613E + 03	6.5058E + 03	1.7669E + 03	1.2163E + 03	2.0483E + 03
	Std	2.1126E + 03	1.1060E + 03	1.5691E + 03	2.4808E + 03	1.4481E + 03	1.5070E + 03	1.2091E + 03
	contest	4.0772E-11(-)	1.7769E-10(+)	3.0199E-11(-)	2.7726E-05(+)	3.0199E-11(-)	3.0199E-11(-)	
+/=/-		7/2/19	19/3/6	10/0/18	16/0/12	19/1/8	7/1/20	
rank		3.4286	5.3929	2.8929	4.6071	5.5357	2.1964	3.9464

Table 8
Optimization results for different parameters.

F	Index	$r = 0.1$	$r = 0.2$	$r = 0.3$	$r = 0.4$
F1	Best	**5.1405E-05**	6.0386E-04	8.0600E-04	4.0858E-04
	Worst	**4.6362E-04**	2.7444E-03	4.9087E-03	3.3336E-03
	Ave	**1.8619E-04**	1.5682E-03	1.7330E-03	1.4622E-03
	Std	**8.2076E-05**	5.6817E-04	8.2708E-04	5.8919E-04
F2	Best	1.7679E + 06	**5.6456E + 05**	1.0800E + 06	7.8017E + 05
	Worst	5.5244E + 06	**4.5500E + 06**	4.7457E + 06	5.0594E + 06
	Ave	3.3442E + 06	**2.1082E + 06**	2.4230E + 06	2.4713E + 06
	Std	1.0960E + 06	9.8489E + 05	**8.2614E + 05**	1.0447E + 06
F3	Best	4.7959E + 07	**3.8904E + 07**	4.6303E + 07	5.5231E + 07
	Worst	5.7242E + 09	5.1254E + 09	3.4649E + 09	**3.1952E + 09**
	Ave	**7.9463E + 08**	1.3205E + 09	7.9499E + 08	9.7937E + 08
	Std	1.1008E + 09	1.2542E + 09	**7.6043E + 08**	9.0779E + 08
F4	Best	**9.2704E + 01**	1.6799E + 02	1.5987E + 02	1.1426E + 02
	Worst	**4.9400E + 02**	7.5593E + 02	1.2141E + 03	6.9094E + 02
	Ave	**2.5993E + 02**	3.6020E + 02	3.7547E + 02	3.3607E + 02
	Std	**1.1245E + 02**	1.4371E + 02	2.0668E + 02	1.5575E + 02
F5	Best	**2.0725E-02**	2.9741E-02	2.7399E-02	3.0745E-02
	Worst	**4.7068E-02**	6.2636E-02	6.2403E-02	6.6725E-02
	Ave	**3.6832E-02**	4.2086E-02	4.3925E-02	4.7617E-02
	Std	**6.3383E-03**	7.8156E-03	8.1707E-03	8.8451E-03
F6	Best	**5.1978E + 00**	6.7435E + 00	6.8504E + 00	1.7114E + 01
	Worst	1.0891E + 02	1.0407E + 02	1.3670E + 02	**8.6619E + 01**
	Ave	6.4354E + 01	6.0531E + 01	6.9356E + 01	**5.9352E + 01**
	Std	**2.5053E + 01**	2.8720E + 01	**2.5053E + 01**	2.3897E + 01
F7	Best	**2.5895E + 01**	8.6085E + 01	5.3085E + 01	8.2011E + 01
	Worst	**9.0972E + 01**	4.2269E + 04	2.1205E + 02	9.6721E + 02
	Ave	**5.4684E + 01**	1.6171E + 03	1.3896E + 02	2.1397E + 02
	Std	**1.7345E + 01**	7.6806E + 03	3.8329E + 01	1.8170E + 02
F8	Best	**2.0737E + 01**	2.0802E + 01	2.0835E + 01	2.0784E + 01
	Worst	**2.1019E + 01**	2.1029E + 01	2.1023E + 01	2.1033E + 01
	Ave	**2.0941E + 01**	2.0944E + 01	**2.0941E + 01**	2.0951E + 01
	Std	6.4911E-02	**5.1880E-02**	5.3758E-02	6.0663E-02
F9	Best	**1.4247E + 01**	2.8953E + 01	2.8659E + 01	2.7705E + 01
	Worst	**3.8139E + 01**	4.1575E + 01	4.2148E + 01	4.1062E + 01
	Ave	**2.0007E + 01**	3.4903E + 01	3.5385E + 01	3.4996E + 01
	Std	4.2506E + 00	3.1910E + 00	**3.1335E + 00**	3.1449E + 00
F10	Best	4.0746E-01	4.9603E-01	**2.9529E-01**	4.7929E-01
	Worst	1.4929E + 00	**1.3601E + 00**	1.5435E + 00	1.5112E + 00
	Ave	**1.0412E + 00**	1.0633E + 00	1.0556E + 00	1.0933E + 00
	Std	3.0289E-01	2.7088E-01	2.7628E-01	**2.4096E-01**
F11	Best	**5.5180E + 01**	2.9055E + 02	3.0349E + 02	2.0501E + 02
	Worst	**4.4972E + 02**	6.4474E + 02	6.8762E + 02	6.9149E + 02
	Ave	**1.1178E + 02**	4.6503E + 02	4.7896E + 02	4.7918E + 02
	Std	**6.9319E + 01**	9.3224E + 01	1.1811E + 02	1.0844E + 02
F12	Best	**3.9049E + 01**	2.8459E + 02	2.5276E + 02	1.8905E + 02
	Worst	**4.8454E + 02**	6.5867E + 02	8.0492E + 02	6.6066E + 02
	Ave	**1.2429E + 02**	4.7090E + 02	4.6562E + 02	4.3934E + 02
	Std	**8.7752E + 01**	9.5727E + 01	1.3221E + 02	1.0759E + 02
F13	Best	**1.0139E + 02**	2.3939E + 02	2.9502E + 02	3.1849E + 02
	Worst	**4.1456E + 02**	7.2160E + 02	5.9011E + 02	6.5204E + 02
	Ave	**1.9591E + 02**	4.7891E + 02	4.3807E + 02	4.5976E + 02
	Std	**5.8430E + 01**	1.0815E + 02	8.2095E + 01	9.4512E + 01
F14	Best	**1.2881E + 03**	3.0296E + 03	2.6282E + 03	2.2439E + 03
	Worst	**4.9129E + 03**	5.6090E + 03	5.6202E + 03	5.0215E + 03
	Ave	**3.1030E + 03**	4.3944E + 03	4.1382E + 03	4.1569E + 03
	Std	8.2510E + 02	**6.3086E + 02**	6.9603E + 02	6.3976E + 02
F15	Best	**2.1748E + 03**	2.7880E + 03	3.0276E + 03	3.2198E + 03
	Worst	7.1792E + 03	**5.6672E + 03**	6.6378E + 03	6.1267E + 03
	Ave	**3.3729E + 03**	4.4129E + 03	4.5360E + 03	4.3967E + 03
	Std	1.0090E + 03	7.4544E + 02	8.9990E + 02	**7.3218E + 02**
F16	Best	1.0077E + 02	**4.6396E-01**	7.3661E-01	7.0203E-01
	Worst	1.0218E + 02	2.3667E + 00	2.4835E + 00	**2.2797E + 00**
	Ave	1.0131E + 02	1.3850E + 00	1.4279E + 00	**1.3770E + 00**
	Std	**3.2825E-01**	4.9187E-01	4.4844E-01	4.2968E-01
F17	Best	**1.9211E + 02**	3.5001E + 02	4.1411E + 02	3.2372E + 02
	Worst	**4.4928E + 02**	8.6411E + 02	8.8606E + 02	8.0232E + 02
	Ave	**2.7683E + 02**	6.2831E + 02	6.5589E + 02	6.0297E + 02
	Std	**6.7189E + 01**	1.2687E + 02	1.3951E + 02	1.2538E + 02
F18	Best	2.1448E + 02	4.6872E + 02	2.8778E + 02	**2.1223E + 02**
	Worst	**5.1596E + 02**	8.2927E + 02	7.9047E + 02	8.3848E + 02
	Ave	**3.5595E + 02**	6.1740E + 02	5.4696E + 02	5.6834E + 02
	Std	**4.9072E + 01**	1.1145E + 02	1.1273E + 02	1.6474E + 02
F19	Best	1.0959E + 02	1.9594E + 01	**1.2743E + 01**	1.3033E + 01
	Worst	1.6826E + 02	**5.4925E + 01**	6.6187E + 01	5.7420E + 01

(*continued on next page*)

Table 8 (*continued*)

F	Index	$r = 0.1$	$r = 0.2$	$r = 0.3$	$r = 0.4$
	Ave	1.3803E + 02	3.5619E + 01	3.6282E + 01	**3.3464E + 01**
	Std	1.4715E + 01	**9.5482E + 00**	1.3493E + 01	1.1095E + 01
F20	Best	1.0921E + 02	1.2530E + 01	1.2727E + 01	**1.2363E + 01**
	Worst	1.1500E + 02	**1.5000E + 01**	**1.5000E + 01**	**1.5000E + 01**
	Ave	1.1239E + 02	**1.4535E + 01**	1.4592E + 01	1.4586E + 01
	Std	1.6296E + 00	5.9554E-01	**4.7212E-01**	5.4564E-01
F21	Best	4.0015E + 02	**3.0034E + 02**	3.0049E + 02	3.0045E + 02
	Worst	5.4354E + 02	**4.4354E + 02**	**4.4354E + 02**	**4.4354E + 02**
	Ave	4.7666E + 02	**3.6250E + 02**	3.6257E + 02	3.7684E + 02
	Std	7.2725E + 01	7.2081E + 01	**7.2020E + 01**	7.2528E + 01
F22	Best	**2.1353E + 03**	4.4952E + 03	3.9054E + 03	3.4223E + 03
	Worst	**5.5338E + 03**	8.2624E + 03	8.3591E + 03	8.3220E + 03
	Ave	**3.2697E + 03**	6.4974E + 03	6.2203E + 03	5.8332E + 03
	Std	**7.4635E + 02**	8.6634E + 02	9.8884E + 02	9.9193E + 02
F23	Best	**1.8978E + 03**	3.7015E + 03	3.8728E + 03	3.8236E + 03
	Worst	**7.2015E + 03**	7.9139E + 03	7.8715E + 03	8.5076E + 03
	Ave	**4.0199E + 03**	6.1976E + 03	6.3838E + 03	6.2687E + 03
	Std	1.3360E + 03	9.8905E + 02	**9.3602E + 02**	9.3771E + 02
F24	Best	3.4160E + 02	2.8939E + 02	2.8447E + 02	**2.8105E + 02**
	Worst	3.9415E + 02	**3.3277E + 02**	3.3996E + 02	3.4113E + 02
	Ave	3.5258E + 02	**3.0761E + 02**	3.1464E + 02	3.1277E + 02
	Std	**1.1510E + 01**	1.1866E + 01	1.4065E + 01	1.2205E + 01
F25	Best	3.4977E + 02	**2.9137E + 02**	3.0560E + 02	2.9784E + 02
	Worst	4.1657E + 02	**3.5176E + 02**	3.5336E + 02	3.6753E + 02
	Ave	3.7387E + 02	3.2666E + 02	3.3216E + 02	**3.3116E + 02**
	Std	1.3000E + 01	1.4539E + 01	**1.2939E + 01**	1.5275E + 01
F26	Best	3.0017E + 02	2.0004E + 02	**2.0003E + 02**	2.0004E + 02
	Worst	4.9094E + 02	4.1600E + 02	4.1185E + 02	**4.0721E + 02**
	Ave	4.0828E + 02	**3.4179E + 02**	3.6077E + 02	3.6029E + 02
	Std	**6.6845E + 01**	8.7213E + 01	7.3698E + 01	7.3381E + 01
F27	Best	**7.9999E + 02**	1.1436E + 03	1.0638E + 03	9.3860E + 02
	Worst	**1.2265E + 03**	1.5049E + 03	1.4592E + 03	1.4937E + 03
	Ave	**9.2500E + 02**	1.2882E + 03	1.2598E + 03	1.2814E + 03
	Std	9.9195E + 01	**9.1058E + 01**	1.0441E + 02	1.1305E + 02
F28	Best	**6.4366E + 02**	3.1483E + 03	3.4447E + 03	2.9054E + 03
	Worst	**5.1936E + 03**	5.8723E + 03	5.6247E + 03	5.8031E + 03
	Ave	**1.3616E + 03**	4.2935E + 03	4.1992E + 03	4.1262E + 03
	Std	7.6935E + 02	7.3674E + 02	**4.6140E + 02**	6.4794E + 02

Table 9
Three-bar truss design problem.

Algorithms	$f(x)$			x_1	x_2
	best	mean	std		
PSO	263.852350	263.852353	0.000002	0.788383	0.408196
GWO	263.853364	263.857184	0.004332	0.788888	0.406903
WOA	263.891120	264.487914	0.583780	0.795770	0.387699
MRFO	263.852347	263.852357	0.000010	0.788393	0.408176
HHO	263.867168	264.246701	0.396642	0.783957	0.420855
SSA	263.852352	263.853025	0.001256	0.788329	0.408356
DBO	263.852347	263.852383	0.000058	0.788433143	0.408064
HMO	263.852346	263.852347	0.000000	0.788415	0.408114

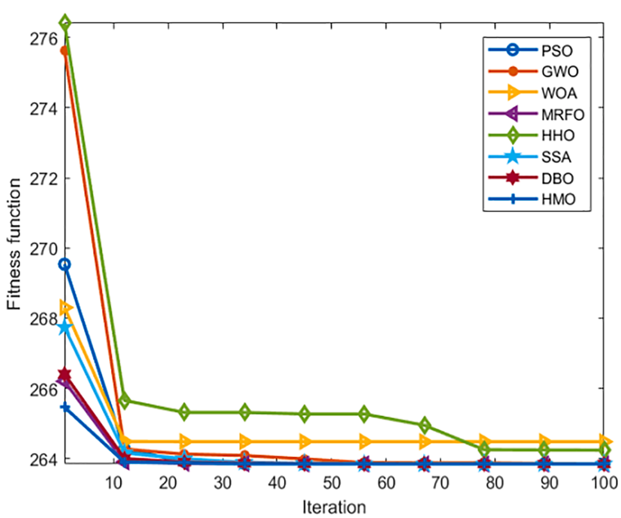

Fig. 5. Convergence effect of the three-bar truss design.

problems, the convergence of HMO has the advantage of being faster than the individual algorithms, further verifying the feasibility and practicality of the HMO.

5. Conclusion

To improve the generality of the optimization problems, this paper proposes a human memory optimization algorithm inspired by human memory patterns, which gradually searches for optimal solutions through human behavior and recall patterns. The HMO was verified to have better optimization capability than other algorithms in the CEC 2013 test set while demonstrating a high degree of superiority. In three engineering optimization problems, HMO was able to find good optimal solutions within a reasonable range of parameters, giving it an advantage over other algorithms. Although it can show better global optimization capabilities, the table still shows that the optimization metrics of HMO on individual functions are not yet satisfactory and there is room for improvement, HMO only beats some of the algorithms, and there is

Table 10
Optimization results for Compression Spring Design.

Algorithms	$f(x)$ best	mean	std	x_1	x_2	x_3
PSO	0.025130	0.025130	0.000000	0.064997	0.394951	9.712374
GWO	0.025130	0.025132	0.000002	0.064998	0.395026	9.698302
WOA	0.025148	0.026338	0.001058	0.065915	0.415109	8.850154
MRFO	0.025130	0.025130	0.000000	0.064981	0.394626	9.723339
HHO	0.025135	0.026415	0.001720	0.065419	0.404382	9.287695
SSA	0.025130	0.025177	0.000095	0.065006	0.395294	9.693592
DBO	0.025131	0.027757	0.003590	0.064792	0.390474	9.925062
HMO	0.025130	0.025130	0.000000	0.064982	0.394652	9.724669

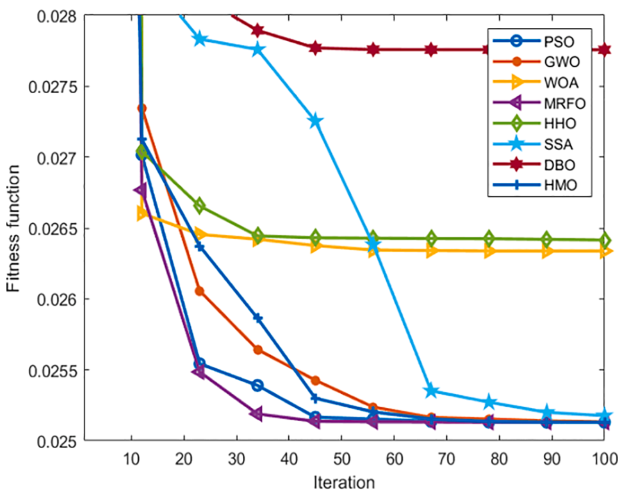

Fig. 6. Convergence effect of Compression Spring Design.

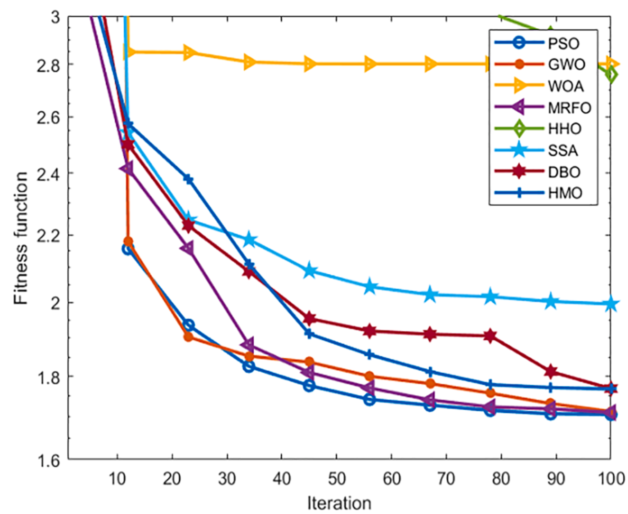

Fig. 7. Convergence effect of Welded Beam Design.

still a gap with individual variants of algorithms from recent years. On the other hand, the accuracy of HMO in solving engineering optimization problems is poor and converges more slowly than some algorithms.

In the recall probability *r*, depending on the different optimization problems it needs to be adjusted, as it will affect the convergence speed of the algorithm's search for an optimum. The algorithm searches in a way that relates only to particular (best, worst) individuals and communicates less with the interior of the population, so the ability of algorithm to search locally is somewhat flawed. In addition, in a general continuity optimization problem, the update of failure events is difficult and therefore the same information can easily appear in storage. In our future work, we will consider the following aspects:

- the settings of r can be changed adaptively, while the probabilities of the four recall types can be reprogrammed so that they can be adapted to different problems.
- Changes can be made to the storage, such as storing locally optimal information and locally worst information, so that the difference between the information in the storage matrix is a little more obvious.
- it can be used as a framework to be integrated into other meta-heuristic algorithms, so that the advantages can be complementary to each other and the performance of the algorithm can be improved.
- further mining complex practical problems to improve the application value of HMO, such as path planning problems, image processing, parameter identification and other problems.

Table 11
Welded Beam Design.

Algorithms	$f(x)$ best	mean	std	x_1	x_2	x_3	x_4
PSO	1.695896	1.705871	0.011498	0.205643	3.255246	9.034856	0.205835
GWO	1.701952	1.713001	0.011657	0.203617	3.311091	9.042565	0.205856
WOA	2.267928	2.802708	0.538191	0.309455	2.257199	7.994048	0.324537
MRFO	1.696197	1.710607	0.018472	0.205625	3.253793	9.042017	0.205742
HHO	2.093387	2.759564	0.688321	0.161215	5.735116	8.270343	0.245624
SSA	1.767023	1.994817	0.315341	0.225558	3.037960	8.630512	0.225642
DBO	1.699679	1.769363	0.146010	0.201230	3.335190	9.036630	0.205730
HMO	1.695641	1.768446	0.110076	0.205698	3.252767	9.040182	0.205714

CRediT authorship contribution statement

Donglin Zhu: Conceptualization, Methodology, Software, Data curation, Writing – original draft. **Siwei Wang:** Visualization, Investigation. **Changjun Zhou:** Conceptualization, Supervision, Funding acquisition. **Shaoqiang Yan:** Software, Methodology. **Jiankai Xue:** Software, Methodology, Writing – original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under Grant numbers 62272418 and 62002046, Basic Public Welfare Research Program of Zhejiang Province (No. LGG18E050011).

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