

# A new approach to the extraction of single exponential diode model parameters

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**Abstract:** A new method is presented for the extraction of the parameters of a single exponential diode model with series resistance from the measured forward  $I$ - $V$  characteristics. The extraction is performed using auxiliary functions based on the integration of the data which allow to isolate the effects of each of the model parameters. Measured and simulated data are used to verify the applicability of the proposed method.

**Keywords:** diode model, single-exponential, parasitic series resistance, function  $D$ , function  $G$ .

## 1. INTRODUCTION

Parameter extraction in diode and solar cells has been an active research topic for many years [1-15]. Some methods are based on optimization of the measured current-voltage characteristics [2,5] while other methods have proposed new functions [1,3,12,16-22], which eliminate the effects of one parameter. Some of these functions [3,12, 16-22] are based on integration and as a means to reduce the extraction uncertainties arising from the probable presence of noise in the measured data. On the other hand, graphical methods have been used for the evaluation of series resistance in solar cells [23,24] as well as for the calculation of majority carrier density of semiconductors with multiple donors and acceptors [25,26].

In this article, we present a new method to evaluate the parameters of an idealized single-exponential diode with a parasitic series resistance. This method is based on functions which isolate the effects of each parameters. Physical insight about the validity of the model is also obtained by using the proposed determination of the parameters.

## 2. INTEGRATION OF THE MEASURED FORWARD BIAS DATA

Consider a single-exponential diode with a parasitic series resistance  $R$  whose  $I$ - $V$  characteristics may be described by [1,12]:

$$I = I_0 \left[ \exp \left( \frac{V - RI}{nV_{th}} \right) - 1 \right] \quad (1)$$

where  $V$  is the terminal voltage,  $I_0$  is the reverse saturation current,  $n$  is the so-called diode quality factor, and  $v_{th} = k_B T/q$  is the thermal voltage. The current can be explicitly solved from implicit equation (1) making use of the Lambert  $W$  function [6,15], while the terminal voltage can be explicitly solved using elementary functions [2,5,12]:

$$V = RI + n v_{th} \ln \left( 1 + \frac{I}{I_0} \right) . \quad (2)$$

Following the pioneering idea of using numerical integration for parameter extraction [3], the drain current may be integrated by parts using the voltage expression in (2):

$$\int_0^V I dV = VI - \int_0^I V dI = \frac{R}{2} I^2 + n v_{th} I - I_0 V . \quad (3)$$

For values of forward current  $I \gg I_0$  the effect of  $I_0$  on the integral is negligible, so that the last term of the RHS of (3) may be neglected. Thus, the integral of the forward current turns out to be approximately described by a simple second order polynomial on  $I$  whose coefficients are directly and independently determined by two of the diode's parameters:  $R$  and  $n$  [17,18]:

$$\int_0^V I dV \approx \frac{R}{2} I^2 + n v_{th} I . \quad (4)$$

The two coefficients of this second order polynomial defined by the RHS of (4) may be adjusted by optimization to fit the numerical integral of the forward current data specified by the LHS of (4). Such optimization would directly extract the values of two of the diode's parameters:  $R$  and  $n$ .

On the other hand, it is also possible to remove the effect of the parasitic series resistance  $R$  using integration of the measured forward bias data. This may be easily done through the use of the "Integral Difference Function," which is defined as [19,20]:

$$D(V, I) \equiv \int_0^I V dI - \int_0^V I dV = IV - 2 \int_0^V I dV , \quad (5)$$

where  $D$  has units of "electric power." Applying function  $D$  to the case of a single-exponential diode model with series resistance and restricting the analysis to values of forward current  $I \gg I_0$ , substitution of (2) into (5) yields [19,20]:

$$D = (I + 2I_0) n v_{th} \ln(I/I_0 + 1) - 2n v_{th} I \approx I n v_{th} [\ln(I/I_0) - 2] , \quad (6)$$

which is an expression that contains two of the diode's parameters:  $I_0$  and  $n$ , but does not contain  $R$ . Dividing (6) by the current yields another auxiliary function, called  $G$ :

$$G \equiv D/I \equiv V - \frac{2}{I} \int_0^V I dV \approx n v_{th} [\ln(I/I_0) - 2] . \quad (7)$$

It is interesting to note that function  $G$  expressed by (7) is very similar to the expression for the voltage of an intrinsic diode ( $R=0$ ) which according to (2) is:  $n v_{th} \ln(I/I_0 + 1)$ .

Kaminski et al [16] generalized this method in 1997 by allowing an arbitrary nonzero lower integration limit. In 2005 Tan et al [10] extended function  $G$  by including the presence of a parallel parasitic resistance.

### 3. EXTRACTION PROCEDURE

In the previous section we presented the expression for  $G$  that contains two of the diode's parameters,  $I_0$  and  $n$ , but does not contain  $R$ . Next, we will define a new function,

which we call  $\Delta G$ , as a means to also remove the effect of  $I_o$ , leaving only one parameter to be determined,  $n$ . The function is defined as the difference:

$$\Delta G(V, I) \equiv G(V, I) - G(V_R, I_R) \quad (8)$$

where  $(V_R, I_R)$  is a reference point of the I-V characteristic to be selected. Then, since parameters  $I_o$  and  $n$  of the model described by (1) are assumed to have constant values throughout the entire forward  $I$ - $V$  characteristics, and because (7) is a logarithmic function, substitution of (7) into (8) yields:

$$\Delta G(V, I) = n v_{th} \ln(I/I_R) \quad (9)$$

In order to use this  $\Delta G$  function to extract the diode's parameters, the procedure to follow is:

*First*: Function  $G$  is numerically calculated using the integration in (7).

*Second*, Function  $\Delta G$  is evaluated for some selected value of  $I_R$  using (8).

*Third*, Parameter  $n$  is obtained from (9) as:

$$n = \frac{\Delta G(V, I)}{v_{th} \ln(I/I_R)} \quad (10)$$

and plotted as a function of the forward current.

*Fourth*, After having found the value of  $n$ ,  $I_o$  is obtained using (7) as:

$$I_o = \frac{I}{\exp\left(\frac{G}{n v_{th}} + 2\right)} \quad (11)$$

and plotted as a function of the current.

*Fifth* and last, Parameter  $R$  is evaluated with (2) using the already extracted values of  $n$  and  $I_o$ :

$$R = \frac{V}{I} - \frac{n v_{th}}{I} \ln\left(\frac{I}{I_o} + 1\right) \approx \frac{V}{I} - \frac{n v_{th}}{I} \ln\left(\frac{I}{I_o}\right) \quad (12)$$

and plotted as a function of the current.

It is important to check that the resulting curves of  $n$ ,  $I_o$  and  $R$  as plotted versus  $I$  should approach constant values, indicating that the assumed model of a single-exponential diode with parasitic series resistance is an adequate description of the actual  $I$ - $V$  characteristics of the measured real device within the range of interest.

### 3.1. Verification of the procedure using simulations

The top part of figure 1 shows a simulated diode's  $I$ - $V$  characteristics using equation (1), with a 10 mV step size, and the parameters previously reported in [5,12]:  $n = 1.05$ , and  $I_o = 0.58$  nA and  $R = 33.4$   $\Omega$ . Function  $G$  is calculated using the numerical integration of the simulated data as described by (7). We note that  $G$  is linearly proportional to the logarithm of the current as soon as few points are calculated in the numerical integration, that is, as soon as the forward current  $I \gg I_o$ . Then, function  $\Delta G$  is numerically calculated using the definition in (8) and three selected values of  $I_R$ . The bottom part of figure 1 also presents functions  $\Delta G$ . We note that function  $\Delta G$  is function  $G$  shifted down by a constant value such that  $\Delta G = 0$  for  $I = I_R$ . Therefore, we recommend that the selected values of  $I_R$  be far from the two extreme of the data values.

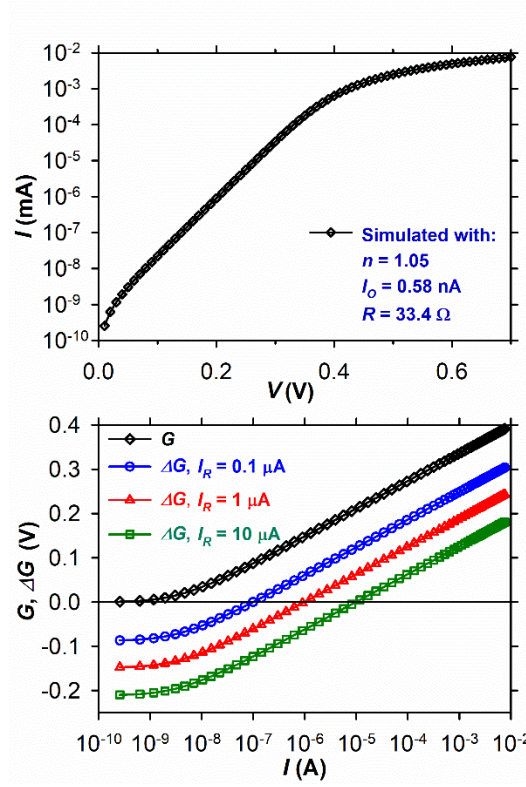


Fig. 1 Top: Simulated  $I$ - $V$  characteristics of a silicon diode. Bottom: Functions  $G$  and  $\Delta G$  as a function of the logarithm of the current calculated from the simulated  $I$ - $V$  characteristics. Notice that the curves become straight lines as soon as  $I \gg I_0$ .

The top part of Fig. 2 shows a plot of  $n$  as a function of the logarithm of the current, using (10), for the various selected values of  $I_R$ . We note that  $n$  tends to be a constant for large current which is almost independent of  $I_R$ . We think that the very weak dependence of  $n$  with respect to  $I_R$  is due to small errors of the numerical integration. We also observe in this figure peak values at  $I = I_R$  due to a mathematical artifact when the numerator and the denominator of (10) tends to zero at  $I = I_R$ . Therefore, these values of  $n$  at  $I = I_R$  must be ignored in the procedure.

The middle part of Fig. 2 shows a plot of  $I_0$  as a function of the logarithm of the current, using (11) and the estimated values of  $n$ , for the various selected values of  $I_R$ . We note that  $I_0$  tends to be a constant for large current which is almost independent of  $I_R$ .

The bottom part of Fig. 2 shows a plot of  $R$  as a function of the logarithm of the current, using (12) and the estimated values of  $n$  and  $I_0$ , for the various selected values of  $I_R$ . We note that  $R$  tends to be a constant for large current which is almost independent of  $I_R$ .

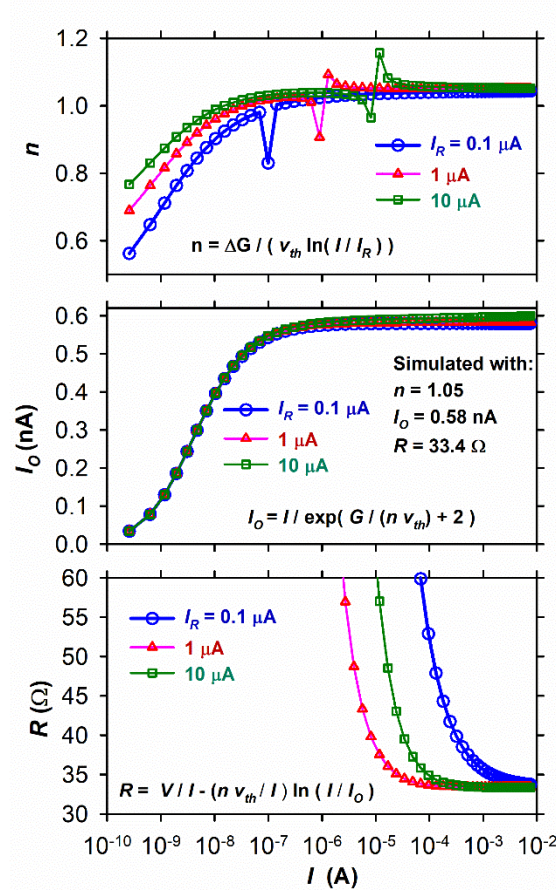


Fig. 2 Extracted parameters as a function of the logarithm of the current calculated from the previous simulated  $I$ - $V$  characteristics.

### 3.2. Verification of the procedure using measurements

The top part of figure 3 presents measurements of a silicon diode from Motorola [5,12] using a 10 mV increment. The bottom part of this figure presents the corresponding functions  $G$  and  $\Delta G$ . We note that the plot of  $G$  versus logarithm of  $I$  is not a straight line for either very small or very large current, which means that the assumed single-exponential diode model is not adequate at these extreme bias conditions. Therefore, we recommend that  $I_R$  be selected only where the model is valid; i.e., where function  $G$  is linearly proportional to the logarithm of the current.

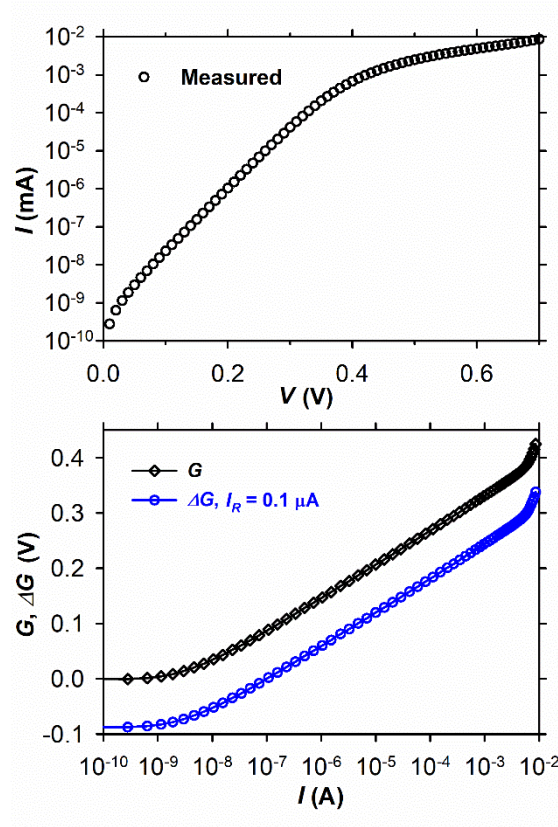


Fig. 3 Top: Measured  $I$ - $V$  characteristics of a silicon diode. Bottom: Functions  $G$  and  $\Delta G$  as a function of the logarithm of the current calculated from the previous measured  $I$ - $V$  characteristics.

The top part of Fig. 4 shows a plot of  $n$  as a function of the logarithm of the current, using (10), for  $I_R = 0.1 \mu\text{A}$ . We note that  $n$  tends to be a constant for currents up to approximately 5 mA and for larger currents  $n$  increases. In this plot, we find  $n=1.03$  for  $I=2.49$  mA (which correspond to  $V=0.5$  V) and  $n=1.04$  for  $I=4.89$  mA (which correspond to  $V=0.6$  V). The increase of  $n$  for currents approaching 10 mA implies that the model is starting to be invalid probably because of high injection.

The middle part of Fig. 4 shows a plot of  $I_O$  as a function of the logarithm of the current, using (11) for the two previous estimated values of  $n$  (1.03 and 1.04). From this plot, we estimate that  $I_O$  is 0.54 nA or 0.58 nA.

The bottom part of Fig. 4 shows a plot of  $R$  as a function of the logarithm of the current, using (12) for the estimated values of  $n$  and  $I_O$ . From this plot, we estimate that  $R$  is 34  $\Omega$  or 37  $\Omega$ . We note that the extraction of  $R$  for low current is meaningless because the effect of  $R$  is only noticeable at high current.

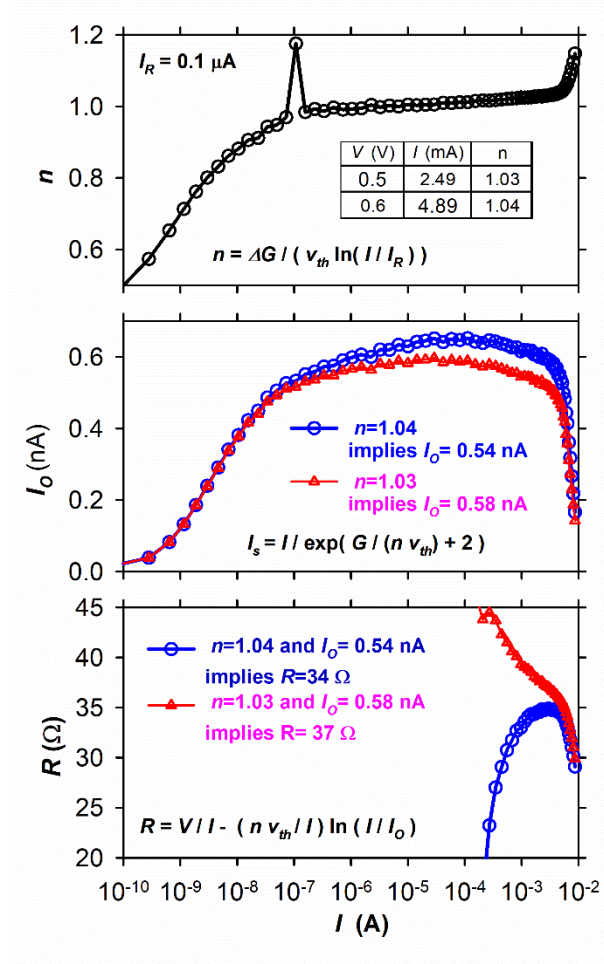


Fig. 4 Extracted parameters as a function of the logarithm of the current calculated from the previous measured  $I$ - $V$  characteristics.

Figure 5 presents the previous measurements of a silicon diode and simulations using the parameters extracted by lateral optimization [2,5,12] and by the present method. We note that the extracted values with the present method are very close to those previously obtained by lateral optimization [5,12].

Figure 6 presents the lateral errors of the previous simulations. We note that the three different simulations yields to comparable errors.



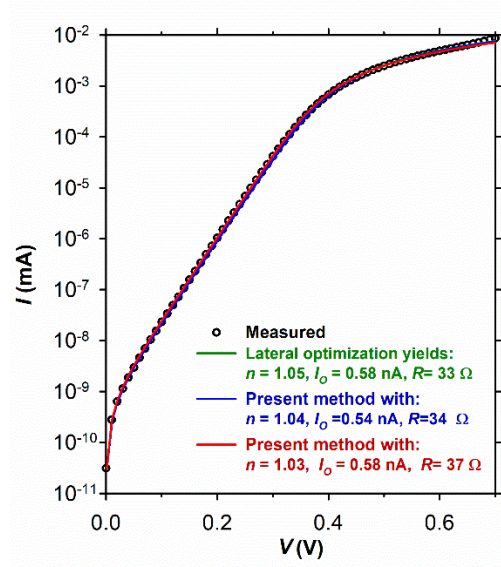


Fig. 5 Measured  $I$ - $V$  characteristics of a silicon diode and its simulation using the parameter values extracted by lateral optimization and by the present method. We note that the extracted values with the present method are very close to those previously obtained by lateral optimization [5,12].

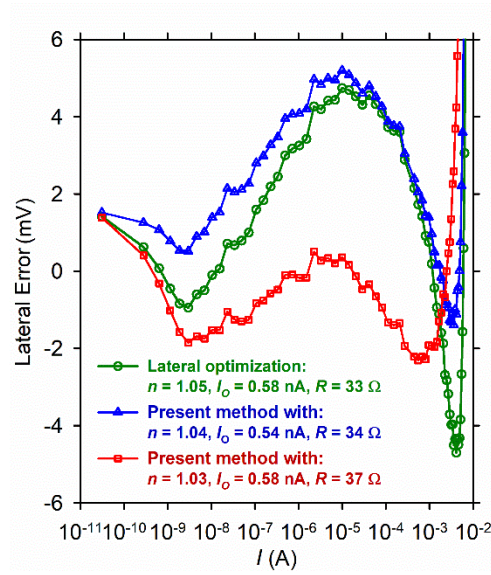


Fig. 6 Corresponding lateral error of the simulations presented in the previous figure. The three different simulations yields to comparable errors.



#### 4. CONCLUSIONS

A novel simple procedure has been proposed to extract the model parameter values of a single-exponential diode with series resistance. The procedure is based on using functions that isolate the effects of each of the parameters. The validity of the proposed procedure was confirmed by parameter extraction from experimental and simulated  $I$ - $V$  characteristics of diodes. Analogous procedures could be attempted to extract the parameters of other semiconductor devices.

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