A quantum-based sine cosine algorithm for solving general systems of nonlinear equations

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Abstract

In this paper, a quantum-based sine cosine algorithm, named as Q-SCA, is proposed for solving general systems of nonlinear equations. The Q-SCA hybridizes the sine cosine algorithm (SCA) with quantum local search (QLS) for enhancing the diversity of solutions and preventing local optima entrapment. The essence of the proposed Q-SCA is to speed up the optimum searching operation and to accelerate the convergence characteristic. The proposed Q-SCA works in twofold: firstly, an improved version of SCA based on tuning the search space around the destination solution dynamically, so that the search space is shrunken gradually as the optima are attained. In addition, a new mechanism to update the solutions is introduced using bidirectional equations. Secondly, QLS is incorporated to improve the quality of the obtained solutions by the SCA phase. By this methodology, the proposed Q-SCA can achieve high levels of exploration/exploitation and precise stable convergence to high-quality solutions. The performance of the proposed algorithm is assessed by adopting twelve systems of nonlinear equations and two electrical applications. Furthermore, the proposed Q-SCA algorithm is applied on expensive large-scale problems including CEC 2017 benchmark and realistic optimal power dispatch (OPD) to confirm its scalability. Experimental results affirm that the Q-SCA is performs steadily, and it has a promising overall performance among several compared algorithms.

Keywords Sine cosine algorithm · Quantum strategy · Systems of nonlinear equations · Power system applications

1 Introduction

The systems of nonlinear equations (SNLEs) are an important class of problems in the area of optimization, because many real-world applications arising in engineering, economics, chemistry, and physics can be modeled as systems of nonlinear equations (Hoffman 2001; Kelley 2003; Yuan and Lu 2008). Thus, finding the solutions to SNLEs has been a great concern in view point of applications. However, SNLEs have some difficulties with

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complex systems, such as difficulty in calculating the derivatives, sensitivity to the initial conditions and requirement of a large memory.

There are tremendous traditional techniques such as Newton method, Secant method and Muller's method (Goyel 2007) are not capable of efficiently handling SNLEs with a complex nature, because they are extremely sensitive to the choice of initial guess of the solution and may come into oscillatory conduct or may come into divergence sate, especially, when the good initial value about the candidate root hasn't picked. Furthermore, they are incapable specifically, for a large scale system, where they require a high memory. In addition, most of these techniques are derivative based and the condition of derivative may not be fulfilled for some functions.

To dispose of awkward aspects related to the traditional techniques, many approaches based on metaheuristic algorithms have been developed (Rizk 2018a, b, c, d; Rizk-Allah 2018; Rizk-Allah et al. 2019). Metaheuristic algorithms, such as genetic algorithm (GA) (Holland 1975), differential evolution (DE) (Das and Suganthan 2011), particle swarm optimization (PSO) (Kennedy and Eberhart 1995; Wang et al. 2014), ant colony optimization (ACO) (Dorigo et al. 1996; Rizk-Allah 2014), firefly algorithm (FA) (Yang 2010; Rizk-Allah 2016a), and fruit fly optimization algorithm (FOA) (Pan 2012; Rizk-Allah 2016b), have offered promising efficiency for solving complex optimization problems and among others (El-Sawy et al. 2013; El-Sawy et al. 2013; Rizk-Allah et al. 2013; Rizk 2017).

Recently, metaheuristic algorithms have attracted much attention to solve systems of nonlinear equations (SNLEs) (Wu et al. 2011; Jaberipour et al. 2011; Wu and Kang 2003; Dai et al. 2008; Ouyang et al. 2009; Luo et al. 2008; Mo et al. 2009; Wolpert and Macready 1997). Wu et al. (2011) hybridized social emotional optimization algorithm and metropolis principle while Jaberipour et al. (2011) introduced a novel approach based on PSO to deal with SNLEs. Wu and Kang (2003) presented a parallel elite-subspace evolutionary algorithm to solve SNLEs. Dai et al. (2008) proposed a combined algorithm of genetic algorithm and quasi-Newton method for solving SNLEs. Ouyang et al. (2009) hybridized PSO and Nelder–Mead simplex method to solve SNLEs, while, Luo et al. (2008) embedded the chaotic optimization to improve the performance of gradient method. Mo et al. (2009) proposed a conjugate direction particle swarm optimization with the aim of reducing high-dimensionality of the problem. However, no free lunch (NFL) theorem (Wolpert and Macready 1997) proves that no single optimization algorithm is the prominent for all problems, so it is necessary to introduce more efficient algorithms to get a higher-quality solution for the SNLEs.

Sine cosine algorithm (SCA) is one of the recently developed metaheuristic algorithms (Seyedali Mirjalili 2016). It was presented by Seyedali (Seyedali Mirjalili 2016) for solving optimization problems. SCA is a mathematical model technique that simulates the behaviors of sine and cosine functions to obtain the best solution (destination). In SCA, a set of solutions are created randomly, then they are updated based on sine and cosine functions by fluctuating these solutions whether outwards or towards the destination to create new solutions. However, as a recent optimization method, SCA contains some disadvantages that deteriorate its performance characteristic. The first one is that the guidance strategy through four some random parameters may impedes the diversity of solutions and leads to the stucking on local optima. The second is SCA does not have efficient strategy to improve the quality of solutions during each generation which may produce poor quality of solutions. Besides, to the best of our knowledge no endeavors have been reported in the literature to introduce SCA for solving system of nonlinear equations and its applications in power system.

It is worth mentioning here that the author has developed some works based on SCA (Rizk 2018b, d) recently. However, the methodology developed in this study is completely different in terms of the inspiration-based quantum search, and real-world application. For instance, the presented work in Ref. (Rizk 2018b) introduces the SCA-based multi-orthogonal search strategy using discretization mechanism to get the orthogonal arrays. The multi-arrays are performed and then invoked in sequential form to improve the position of best so far solution. This algorithm is applied on eighteen benchmark functions of unimodal and multimodal, and four constrained engineering design problems. In Ref. (Rizk 2018d), an improved variant of SCA was proposed based on two improvements. Firstly, an opposition strategy is introduced to enhance the diversity of solutions, while the second one develops the multiple-orthogonal arrays strategy which is invoked in a parallel form to improve the quality of the final solution. Also the proposed method is benchmarked on 10 unconstrained optimization problems, 4 constrained optimization and 6 engineering design applications. The proposed study introduces an improved variant of SCA based on quantum local search for the first time. Furthermore, from the application point of view, the presented method is applied to solve systems of nonlinear equations, CEC 2017 real problems as well as realistic optimal power dispatch (OPD) for the first time.

In this paper, a new hybrid algorithm called a quantum-based sine cosine algorithm (named as Q-SCA) to cope with SNLEs. The proposed algorithm hybridizes the features of the sine cosine algorithm with quantum computing. The main features of the proposed methodology consist in tuning the search space around the destination solution dynamically, so that the search space is shrunken gradually with increasing number of iterations. Further, quantum computing scheme is incorporated as a local search scheme, namely quantum local search (QLS), in order to further strengthen and improve the solutions obtained in the sine cosine algorithm stage. The inherent features of this hybridization can increase the diversification and intensification of the Q-SCA and then can prevent trapping into the local optimum. The proposed algorithm is tested on twelve SNLEs, two electrical applications, CEC 2017 benchmark and realistic OPD problem. Experimental results proved that Q-SCA can get the more powerful solutions than the comparative algorithms.

The main contributions of this approach are to:

(1) Introduce a novel quantum-based sine cosine algorithm (Q-SCA) for solving SNLEs. In Q-SCA, an updating strategy is designed and implemented randomly around the current solution and destination to improve the diversity of solutions efficiently.
(2) Integrate intelligently the merits of two phases namely; sine cosine algorithm (SCA) and quantum local search (QLS), then it can avoid the trapping into the local optima.
(3) Improve the exploration capabilities of the SCA phase to seek the overall search space while incorporate QLS phase as a counterpart in sequential form to enhance the exploitation tendency and refine the quality of solutions.
(4) The integration between SCA and QLS improve the quality of solutions and speed up the convergence to the global solution.
(5) The effectiveness of Q-SCA is validated and demonstrated through twelve case studies from literature and two real applications of power system to evaluate the bus variables.
(6) The scalability test of the proposed Q-SCA algorithm is confirmed by applying it on expensive large-scale problems including CEC 2017 benchmark and realistic optimal power dispatch (OPD) problem.

The novelty of this study resides primarily in implementing and using an improved variant of sine cosine algorithm (SCA), named quantum-based SCA (Q-SCA), through three amendments to overcome the shortages of conventional SCA variant: (1) Non-linear bridging mechanism is developed to perform comparatively better transition from the exploration phase to exploitation phase rather than the linear transition. (2) A modified searching equation is introduced to enrich the population diversity and prevent the skipping of promising regions. (3) A quantum local search (QLS) strategy is presented to enhance neighborhood-informed capacities and avoid falling in local optimum. (4) The scalability test is strictly investigated by applying the proposed method for solving large-scale practical optimization tasks such as CEC 2017 datasets and optimal power dispatch (OPD) optimization problem. Additionally, the proposed Q-SCA is implemented and employed to solve systems of nonlinear equations and two electrical applications for the first time to effectively envisage an accurate, reliable and efficient alternative while dealing with realistic systems of nonlinear equations.

The main frame of the paper is organized as follows. In Sect. 2, the preliminaries for the system of nonlinear equations and the basics of SCA are described. Section 3 presents the proposed Q-SCA in detail. Experimental results obtained for different systems are discussed in Sect. 4. Finally, Sect. 5 delivers the conclusion of this work and the future research.

2 Preliminaries

2.1 System formulation

The general form of the system of nonlinear equations (SNLEs) can be stated as follows:

$$\text{SNLEs}: \begin{cases} f_1(x_1, x_2, ..., x_i, ..., x_n) = 0 \\ f_2(x_1, x_2, ..., x_i, ..., x_n) = 0 \\ \quad\vdots \\ f_i(x_1, x_2, ..., x_i, ..., x_n) = 0 \\ \quad\vdots \\ f_n(x_1, x_2, ..., x_i, ..., x_n) = 0 \end{cases} \tag{1}$$

where n is the number of dimensions, x_i is the ith decision variable and $f_i(.)$ is the ith equation.

Definition 1 If $\forall i$, $i = \{1, 2, ..., n\}$, $f_i(.) = 0$, then the solution $(x_1^*, x_2^*, ..., x_i^*, ..., x_n^*)$ is called the optimal solution of the SNLEs.

In order to solve SNLEs, it is usually transformed into an equivalent single optimization problem as follows:

$$\min f(\mathbf{x}) = \sum_{i=1}^{n} f_i^2(\mathbf{x}) , \quad \mathbf{x} = (x_1, x_2, ..., x_i, ..., x_n) \tag{2}$$

where $f(\mathbf{x})$ is the objective function that will be minimized.

2.2 An overview of sine cosine algorithm (SCA)

SCA is a population-based optimization algorithm that is established based on the mathematical sine and cosine functions. Similar to other metaheuristic algorithms, SCA starts the search process by creating set of solutions randomly. Afterwards, these solutions are evaluated according to objective function, while the best one denoted by the destination point is stored and evolved during each iteration. Then the solutions are updated to create new ones according to sine and cosine functions (see Eq. (3)). Finally, the algorithm stops the optimization process when maximum number of iterations is satisfied.

$$x_{j,t+1} = \begin{cases} x_{j,t} + r_1 \times \sin(r_2) \times |r_3 P_{j,t} - x_{j,t}| \; ; \; r_4 < 0.5 \\ x_{j,t} + r_1 \times \cos(r_2) \times |r_3 P_{j,t} - x_{j,t}| \; ; \; r_4 \geq 0.5 \end{cases}$$
$$j = 1, 2, ..., n \tag{3}$$

where n is the number of dimensions, $x_{j,t}$ is the location of the current solution in jth dimension at tth iteration and the $P_{j,t}$(destination) is the location of the best solution so far at tth iteration. The parameters r_1, r_2, r_3, r_4 are random numbers where $|.|$ denotes the absolute value. The solution for each search agent is denoted by $\mathbf{x}_i = (x_{i,1}, x_{i,2}, ..., x_{i,n})$, $i = 1, 2, ..., PS$, PS is population size.

As shown in Eq. (3), the SCA contains four parameters. The first parameter r_1 is responsible for determining the candidate region. The second parameter, r_2, dictates how far the movement should be towards or outwards the destination. The third parameter, r_3, assigns random weights of destination, so it is responsible for emphasizing if $r_3 > 1$ or deemphasizing if $r_3 < 1$. Finally, the parameter r_4 is responsible for switching between the sine term and cosine term in Eq. (3). Further, the parameter r_1 is adjusted adaptively to balance between exploration/exploitation capabilities by using Eq. (4):

$$r_1 = a - \frac{a \times t}{T} \tag{4}$$

where a is a constant number, t is the current iteration, T is the maximum number of iterations. The flowchart of the original SCA is illustrated in Fig. 1.

2.3 Quantum computing

More recently, quantum computing (QC) has been flourished due to its superiority to classical computing on many specialized tasks (Wolpert and Macready 1997). It is inspired by principles and concepts of quantum theory like superposition of quantum states, entanglement and intervention (Jaeger 2006). QC is capable of executing huge sates of quantum simultaneously. In addition, plenty of researches have been progressed to integrate metaheuristic algorithms with QC (Xi et al. 2008; Zouache et al. 2016; Oguz Emrah Turgut 2014). Turgut et al. (Oguz Emrah Turgut 2014) presented a novel PSO based on chaotic quantum integration for solving SNLEs. Although this algorithm provides an effective algorithm than that developed in (Sun et al. 2005) through introducing chaotic maps, however this algorithm suffers from trapping into local optimum for some cases. Also, it takes long time for computation.
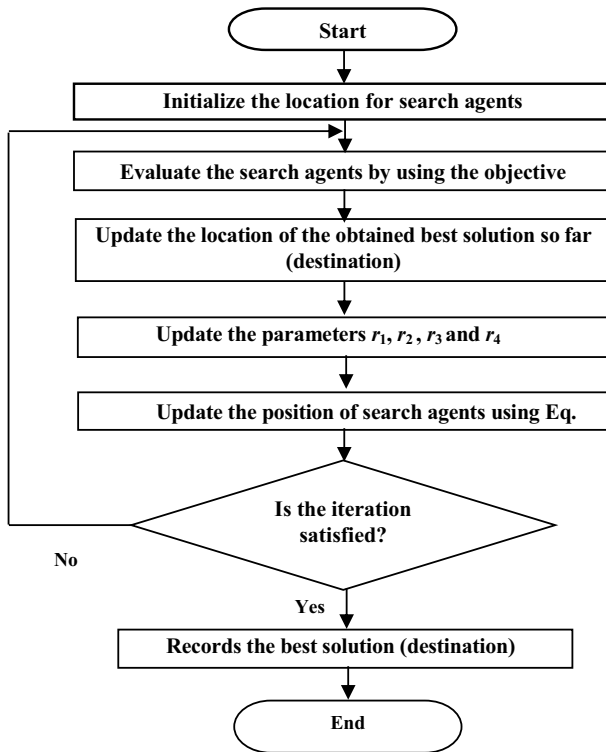
Fig. 1 The basic SCA algorithm

2.4 Motivation aspects

Although SCA provides an efficient accuracy while dealing with optimization tasks and it can exposes competitive results in comparisons with other well-known population-based optimization techniques, it does not has enough fitting and efficient quality for highly complex tasks of optimization and is still suffer from the dilemma of getting falling in local optima. To overcome these challenges and improve its performance characteristic regarding search capability, a newly integrated version based on SCA and quantum local search (QLS) is proposed to solve real applications of electrical power station. The proposed integrated version is detonated as Q-SCA. In the proposed Q-SCA, the updating mechanism of SCA is improved by searching around the current solution and destination in random manner with the aim to maintain the diversity. Furthermore, the QLS working in a sequential aspect as guidance phase to achieve the promising region and provide the solution quality. Accordingly, by the proposed methodology, it is intended to enhance the performance of convergence, balance the diversification and intensification searches, and improve the seeking process instead of running notable generations without any improvement. The effectiveness of Q-SCA has been demonstrated and investigated through comprehensive experiments of different case studies and two real applications of electrical power station. Experimental results

and comparisons affirm that the proposed Q-SCA is a promise searching technique for handling nonlinear system applications of optimization.

3 The proposed Q-SCA

Until now, there was no other improved variant for SCA in reputable literature. With standard scenario, SCA updates its agents towards the candidate solution based on Eq. (3). However, the SCA, which is an efficient algorithm, is still inclined to convergence in local optima on some cases. Hence, SCA's problems of immature convergence and obtaining incompetent results can still be experienced. In some cases, the standard SCA is not capable of performing a seamless transition from the exploration to exploitation phases through the use of Eq. (4). In this regard, the basic SCA needs improved operators to tackle problems with more exploration/exploitation capabilities. To relive the above mentioned concerns, quantum local search (QLS) is introduced as a local search stage. QLS can help SCA to search based on deeper exploration/exploitation patterns as an alternative. Using this concept, it can be ensured that SCA can handle global searching tasks more efficiently and not be locked in local optima. Hence, hybridization of SCA with quantum local search, namely quantum sine–cosine algorithm (Q-SCA), attains better searching quality. For this purpose, three main modifications are proposed: (1) a new equation for parameter r_1 is introduced to tune the search space dynamically, so that the search space is shrunken gradually as the optima are attained. (2) A new solution generating method is introduced based on exploiting the destination to enhance the quality of solution. (3) QLS is incorporated with the aim of improving the solution quality through generating new solutions around the best obtained solutions so far to achieve an effective exploration and exploitation. Further, QLS can avoid the running of the proposed algorithm without any improvement in the solutions. Based on the above phases, it can be concluded that the proposed Q-SCA is a robust approach and has powerful searching quality. We start the explanation of the Q-SCA as follows.

3.1 Phase 1: SCA

Step 1 Initialize the agents' locations

A population of agents are initialized randomly within the search space, where the algorithm assigns a random vector of n dimensional for the ith agent, $\mathbf{x}_i = (x_{i,1}, x_{i,2}, ..., x_{i,n})$,$i = 1, 2, ..., PS$.

Step 2 Evaluate the agents

Each agent is evaluated according to the quality of its location according to the desired objective function as shown in Eq. (2), where the best solution (destination) so far is recorded.

Step 3 Update the agent's locations

Agents create new locations inside the search space as follows: the parameters r_1, r_2, r_3, r_4 are updated firstly; then the new location of the candidate agent in *jth* dimension is created as follows:

$$x_{j,t+1} = \begin{cases} \begin{cases} x_{j,t} + r_1 \times \sin(r_2) \times |r_3 P_{j,t} - x_{j,t}| & r_4 < 0.5 \\ x_{j,t} + r_1 \times \cos(r_2) \times |r_3 P_{j,t} - x_{j,t}| & r_4 \geq 0.5 \end{cases} ; & r_5 > p_j \\ \begin{cases} P_{j,t} + r_1 \times \sin(r_2) \times |r_3 P_{j,t} - x_{j,t}| & r_4 < 0.5 \\ P_{j,t} + r_1 \times \cos(r_2) \times |r_3 P_{j,t} - x_{j,t}| & r_4 \geq 0.5 \end{cases} ; & r_5 \leq p_j \end{cases} \tag{5}$$

$$i = 1, ..., PS, \ j = 1, ..., n.$$

$$p_j = 0.4 - 0.4 \frac{t}{T}, \tag{6}$$

where p_i is a mutation probability for switching between the equations states according to the random parameter r_5.

As seen from the Eq. (5), a new generating mechanism is introduced by exploiting the destination in the updating mechanism. In Eq. (4), the new location is produced by picking the candidate region in linear manner, but this is unsatisfactory for some applications, where for large search space the algorithm needs extensive number of iterations to find the promising region. To overcome this drawback, the performance of the SCA is improved by tuning the parameter r_1 dynamically, so that the search space is shrunken gradually as the optima are approaching with increasing number of iterations as follows.

$$r_1 = \alpha . \left(a^{\max} - \frac{t \left(a^{\max} - a^{\min} \right)}{T} \right), \quad \alpha = 0.98^t \tag{7}$$

3.2 Phase 2: QLS

To enhance the searching quality and prevent trapping into local optimum, QLS is incorporated as a local search strategy. In the beginning, SCA phase operates to explore space and the obtained solutions from the SCA phase are taken as the starting point for the QLS phase. Afterwards the steps of the QLS are applied in order to reach high levels of exploration and exploitation and precise stable convergence to high-quality solutions. Thus, the hybrid mechanism improves the quality of solutions and saves the computational time for exploring a global optimal solution for the nonlinear system of equations. The steps of this phase can be outlined as follows:

Step 1 Receive the candidate solutions

This step receives a population of solutions, $\{\mathbf{x}_i\}_{i=1}^{PS}$, with its best location (destination), $\mathbf{x}^*$ is from the SCA phase.

Step 2 Determine the quantum search scope

In the quantum search scope, the state of a solution is depicted by wave function ψ, instead of random increment, where the random movement of the solution may yields a divergent behavior. The new location is determined by the probability density function, $|\psi|^2$. Let that at iteration t, the solution i moves in n-dimensional space with a δ potential well centered at P_{ij} on the jth dimension. Correspondingly, the wave function at iteration $t + 1$ is described as follows.

$$\psi(x_{ij}^{t+1}) = \frac{1}{\sqrt{L_{ij}^t}} \exp(-|x_{ij}^{t+1} - P_{ij}^t|/L_{ij}^t), \tag{8}$$

where L_{ij}^t is the quantum search scope (standard deviation) of the double exponential distribution which varies with iteration number t.

$$Q(x_{ij}^{t+1}) = |\psi(x_{ij}^{t+1})|^2 = \frac{1}{L_{ij}^t} \exp(-2|x_{ij}^{t+1} - P_{ij}^t|/L_{ij}^t), \tag{9}$$

Then the probability distribution function F can be written as follows:

$$F(x_{ij}^{t+1}) = 1 - \exp(-2|x_{ij}^{t+1} - P_{ij}^t|/L_{ij}^t), \tag{10}$$

According to Monte Carlo technique, we can determine the j dimension of the search agent $\mathbf{x}_i$ at iteration $t + 1$ as follows:

$$x_{ij}^{t+1} = P_{ij}^t + \frac{1}{2} L_{ij}^t \ln(1/u_{ij}^{t+1}), \tag{11}$$

where u_{ij}^{t+1} is a random number uniformly distributed over (0, 1) and L_{ij}^t is determined as

$$L_{ij}^t = 2\chi|C_j^t - x_{ij}^{t+1}|, \tag{12}$$

where C^t is known as the mean agents of the search agents defined as the mean of the positions of all search agents. That is

$$C^t = (C_1^t, C_2^t, ..., C_n^t) = \left(\frac{1}{PS} \sum_{i=1}^{PS} x_{i1}^t, \frac{1}{PS} \sum_{i=1}^{PS} x_{i2}^t, ..., \frac{1}{PS} \sum_{i=1}^{PS} x_{in}^t \right) \tag{13}$$

Step 3 Generate new solution: the position of the search agent is updatedaccording to the following equation

$$x_{ij}^{t+1} = P_{ij}^t \pm \beta|C_j^t - x_{ij}^{t+1}| \ln(1/u_{ij}^{t+1}), \tag{14}$$

$$\beta = \beta_{\max}.\exp\left(\log\left(\beta_{\min}/\beta_{\max}\right).(t/T)\right) \tag{15}$$

where P_{ij}^t is the local attractor solution that chosen randomly weather from the current population or the destination, β is the search radius in each iteration, $\beta_{\max}$ is the maximum radius, $\beta_{\min}$ is the minimum radius. Therefore we obtain a new population that is denoted by $\{\mathbf{x'}_i\}_{i=1}^{PS}$, and its best location (destination) is denoted by $\mathbf{x'}^*$.

Step 4 Update the destination

If the function $f(\mathbf{x'}^*) < f(\mathbf{x}^*)$, update the fitness value and best location (destination) as: $f(\mathbf{x}^*) = f(\mathbf{x'}^*), \mathbf{x}^* = \mathbf{x'}^*$.

Step 5 Stopping quantum search

If no improvement in the function $f(\mathbf{x}^*)$ for all t iterations, stop quantum search process and store $\mathbf{x}^*$ as the best solution. The flowchart of the proposed Q-SCA can be shown in Fig. 2.

4 Experiment and Results

In this section, twelve systems of nonlinear equations and two electrical applications have been utilized in order to demonstrate the efficiency and robustness of the proposed algorithm. The obtained results have been compared with the other known methods that use the same systems (Oguz Emrah Turgut 2014; Sun et al. 2005; Jäger and Ratz 1995; Sharma and Arora 2013; Grosan and Abraham 2008; Abdollahi et al. 2013, 2016; Oliveira and Petraglia 2013; Floudas et al. 1999; Wang et al. 2011). For these experiments, the algorithm is coded in MATLAB 7, running on a computer with an Intel Core I 5 (1.8 GHz) processor and 4 GB RAM memory. No commercial SCA-based tool was applied in this research. The statistical measures in terms of minimum value, mean deviation value, maximum value and standard deviation for each algorithm for each case are reported in (Oguz Emrah Turgut 2014). Referring to Derrac et al. (Garcia et al. 2009), statistical tests should be completed to judge the superiority of the evaluated algorithms. Statistical tests reflect each test's results and verify that the differences observed in results are statistically significant. To statistically assess the new Q-SCA compared to other algorithms, Wilcoxon rank-sum test at 5% significance degree is also executed here.

4.1 Parameters settings

To show the improvement of the proposed algorithm, the fair comparisons are conducting through unified the parameters regarding the compared algorithms. For all the studied cases, the parameters configurations for all implemented algorithms are set after perfuming a few trials and tabulated in Table 1. Also 30 consecutive algorithm runs is carried out for each case study to demonstrate the stability of the proposed algorithms regarding some of the statistical measures.

Case 1 The following system has been studied in (Oguz Emrah Turgut 2014; Sun et al. 2005) where the system equations are defined as follows

$$
\begin{aligned}
f_1(\mathbf{x}) &= 2x_1 + x_2 + x_3 + x_4 + x_5 = 0 \\
f_2(\mathbf{x}) &= x_1 + 2x_2 + x_3 + x_4 + x_5 = 0 \\
f_3(\mathbf{x}) &= x_1 + x_2 + 2x_3 + x_4 + x_5 = 0 \\
f_4(\mathbf{x}) &= x_1 + x_2 + x_3 + 2x_4 + x_5 = 0 \\
f_5(\mathbf{x}) &= x_1 x_2 x_3 x_4 x_5 - 1.0 = 0 \\
&-2 \le x_i \le 2, \; i = 1, 2, ..., 5.
\end{aligned}
\tag{16}
$$

This system contains five equations (f_1, f_2, f_3, f_4, f_5) that needed to be optimized or solved simultaneously, and five unknowns (x_1, x_2, x_3, x_4, x_5). In this context, the proposed optimization methodology aims to achieve the optimal solutions for these unknowns and then the values for the corresponding equations (f_1, f_2, f_3, f_4, f_5) are calculated, where the obtained results for these unknowns and values for the corresponding equations are recorded in
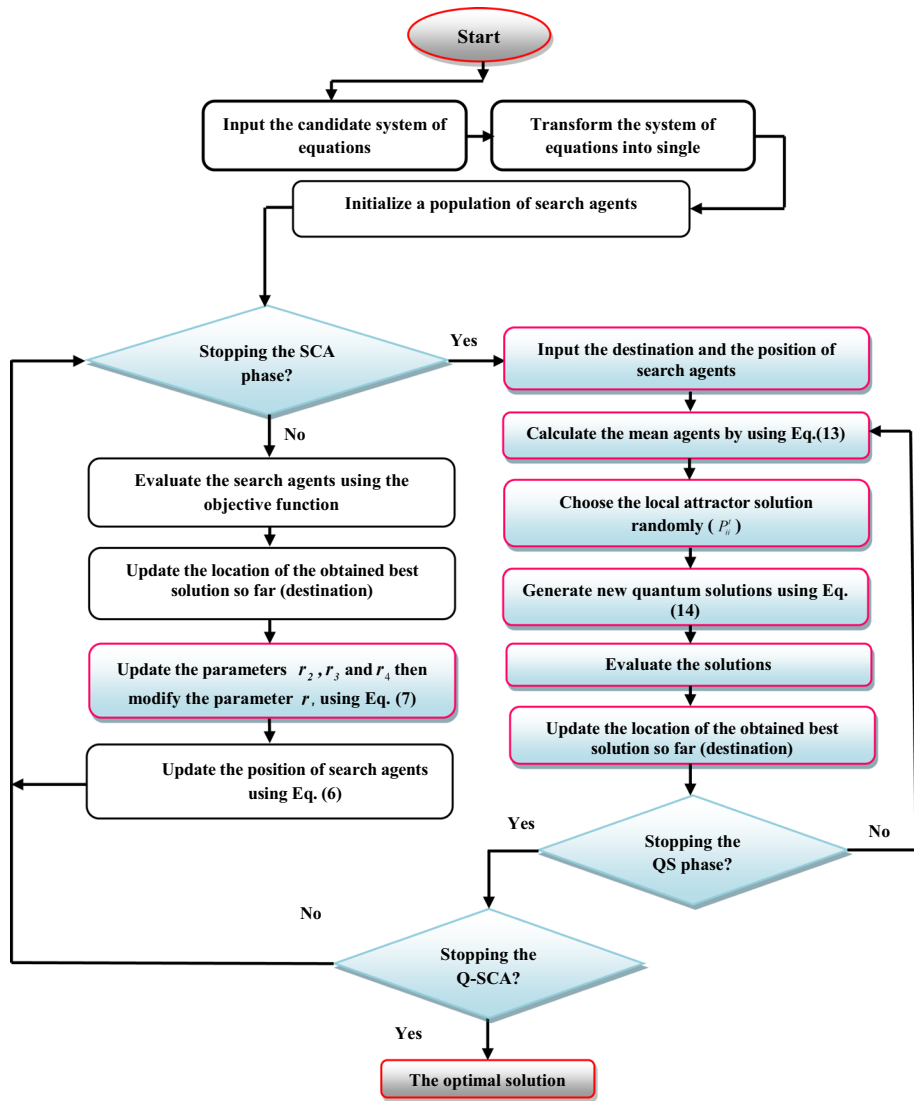
Fig. 2 Architecture of the proposed Q-SCA

Table 1 The parameters settings the implemented algorithms

Algorithm	Parameters
SCA	$PS = 30$; $T = 600$,$r_1 = 1 - 0$ (Linear decreasing); $r_2 \in [0, 2\pi]$, $r_3 \in [0, 2]$,$a = 1$
Q-SCA	$PS = 30$; $T = 600$,$r_1 = 1 - 0$ (Nonlinear decreasing); $r_2 \in [0, 2\pi]$, $r_3 \in [0, 2]$ $a^{\max} = 1$,$a^{\min} = 0.0005$

Table 2. Table 2 shows the optimum solutions obtained by the proposed Q-SCA and SCA without incorporating the quantum local search phase (i.e. SCA). The proposed Q-SCA is compared with other algorithms (Oguz Emrah Turgut 2014) such as chaotic quantum behaved particle swarm optimization algorithm (L-QPSO), quantum behaved particle swarm optimization (QPSO), gravitational search algorithm (GRAV), intelligent tuned harmony search (ITHS) algorithm and other literature studies. The proposed Q-SCA surpasses the other algorithms since it has better results than the other algorithms but is competitive with L-QPSO and Jäger and Ratz (1995).

Further, the convergence behavior for case 1 is shown in Fig. 3a.

Case 2 The system has been introduced in (Oguz Emrah Turgut 2014) and the system is stated as follows

$$
\begin{aligned}
f_1(\mathbf{x}) &= x_1 + \frac{x_2^4 x_4 x_6}{4} + 0.75 = 0 \\
f_2(\mathbf{x}) &= x_2 + 0.405 \exp(1 + x_1 x_2) - 1.405 = 0 \\
f_3(\mathbf{x}) &= x_3 - \frac{x_4 x_6}{2} + 1.5 = 0 \\
f_4(\mathbf{x}) &= x_4 - 0.605 \exp(1 - x_3^2) - 0.395 = 0 \\
f_5(\mathbf{x}) &= x_5 - \frac{x_2 x_6}{2} + 1.5 = 0 \\
f_6(\mathbf{x}) &= x_6 - x_1 x_5 = 0
\end{aligned}
\tag{17}
$$

Table 3 shows that the optimum solution obtained by the proposed Q-SCA, where the proposed Q-SCA outperforms other algorithms since it has better results than the other algorithms. The convergence behavior for case 2 is shown in Fig. 3b.

Case 3 The system has been developed in (Oguz Emrah Turgut 2014) and its formulation is stated as follows

$$
f_i(\mathbf{x}) = x_i - \cos\left(2x_i - \sum_{j=1}^{4} x_j\right) = 0,
\tag{18}
$$

$$
i = 1, 2, 3, 4.
$$

The simulation results for this case are shown in Table 4. Table 4 illustrates the results obtained for this case, where the simulations revealed that the proposed Q-SCA is competitive with some algorithms but it outperforms the GRAV. The convergence behavior for case 3 is depicted in Fig. 3c.

Case 4 The Neurophysiology application (Oguz Emrah Turgut 2014) is utilized to test the efficiency of the proposed algorithm and system equations is described as follows

Table 2 The simulation results of the proposed Q-SCA and different approaches for case 1

	SCA	Q-SCA	LQPSO	QPSO
x_1	0.999996767571065	1.00000000000000000	0.91635458253338494	1.000144199216134
x_2	1.000000638890576	1.00000000000000000	0.91635458253338494	1.000120111355508
x_3	0.9999991349489244	1.00000000000000000	0.91635458253338494	1.000131866228283
x_4	0.9999997217143648	1.00000000000000000	0.91635458253338494	1.000129832805469
x_5	1.000016337257035	1.00000000000000000	1.41822708733075541	0.999369060482274
f_1	−9.2207736557270E−7	0.00000000000000000	0.00000000000000000	0.00003909089303800
f_2	2.949241472303E−6	0.00000000000000000	0.00000000000000000	0.000015001443173
f_3	−6.34015918699049E−6	0.00000000000000000	0.00000000000000000	0.000026576315949
f_4	−4.7250478285576E−7	0.00000000000000000	0.00000000000000000	0.000024722893135
f_5	2.31017406937539E−6	0.00000000000000000	0.00000000000000000	−0.00010533819589

	ITHS	GRAV	Jäger and Ratz (1995)		
x_1	0.9999999983928838	0.916566028481882	0.91635458253386	−0.5790430884942	1.00000000000
x_2	0.9999999984261559	0.916262407670433	0.916354582533851	−0.5790430884941	1.00000000000
x_3	0.9999999984646038	0.916981249858047	0.916354582533851	−0.5790430884941	1.00000000000
x_4	0.9999999984062309	0.916141646585271	0.916354582533851	−0.5790430884941	1.00000000000
x_5	1.00000000076182086	1.417478352612262	1.41822708733076	8.89521544247056	1.00000000000
f_1	−0.02990331537E−7	−0.00428631022E−3	0.33750779948E−13	−0.1403321903E−12	0.00000000000
f_2	−0.02657611024E−7	−0.30790712167E−3	0.23980817331E−13	−0.0399680288E−12	0.00000000000
f_3	−0.02273131903E−7	0.410935065933E−3	0.23980817331E−13	−0.0399680288E−12	0.00000000000
f_4	−0.02856860523E−7	−0.42866820683E−3	0.23980817331E−13	−0.0399680288E−12	0.00000000000
f_5	0.130808268661E−7	0.053287740231E−3	0.21982418875E−13	0.0617284001E−12	0.00000000000

(a) Convergence behavior case study 1

(b) Convergence behavior case study 2

(c) Convergence behavior case study 3

(d) Convergence behavior case study 4

(e) Convergence behavior case study 5

(f) Convergence behavior case study 6

(g) Convergence behavior case study 7

(h) Convergence behavior case study 8
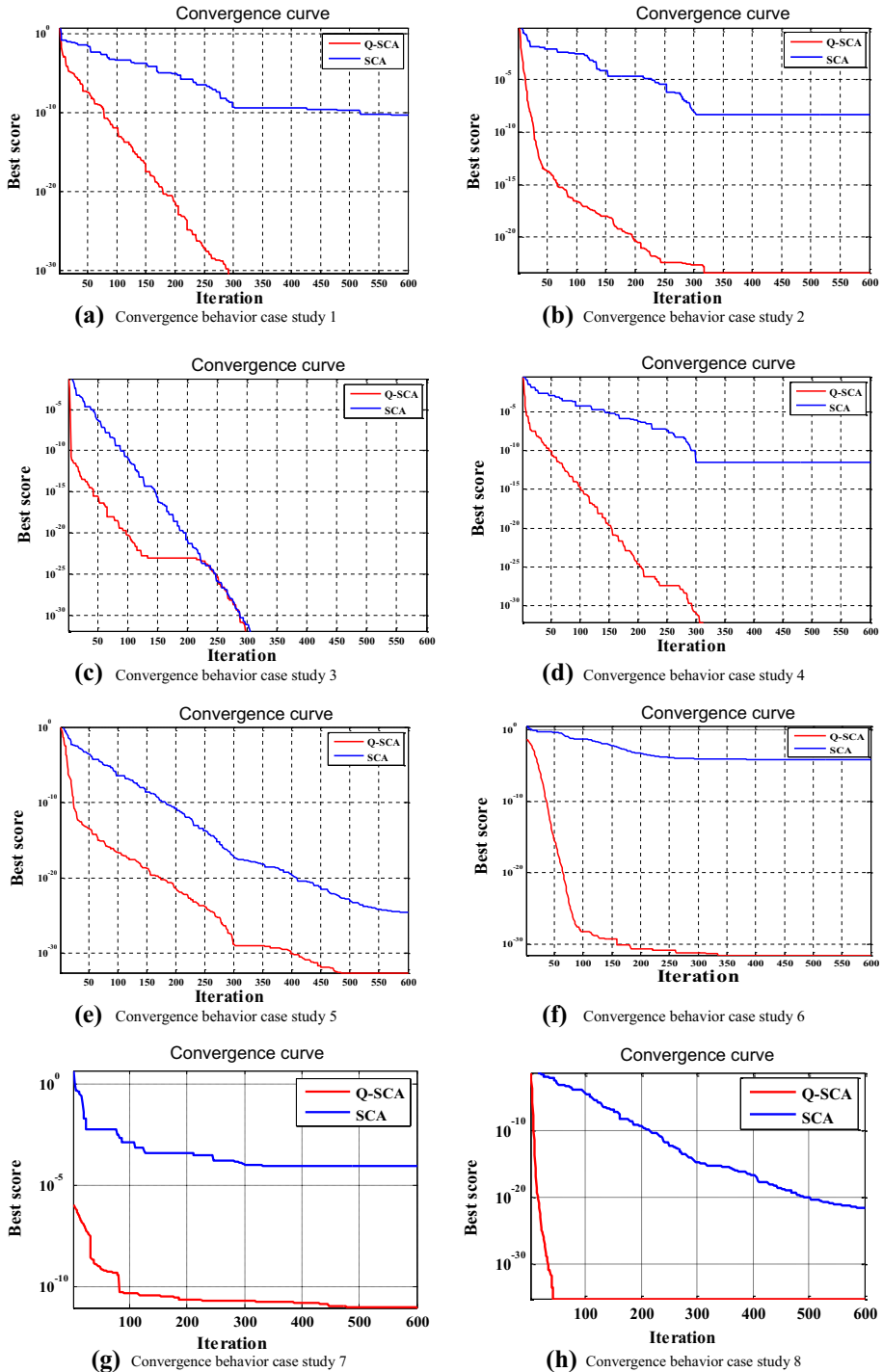
Fig. 3 The convergence behavior for all cases

(i) Convergence behavior case study 9

(j) Convergence behavior case study 10

(k) Convergence behavior case study 11

(l) Convergence behavior case study 12
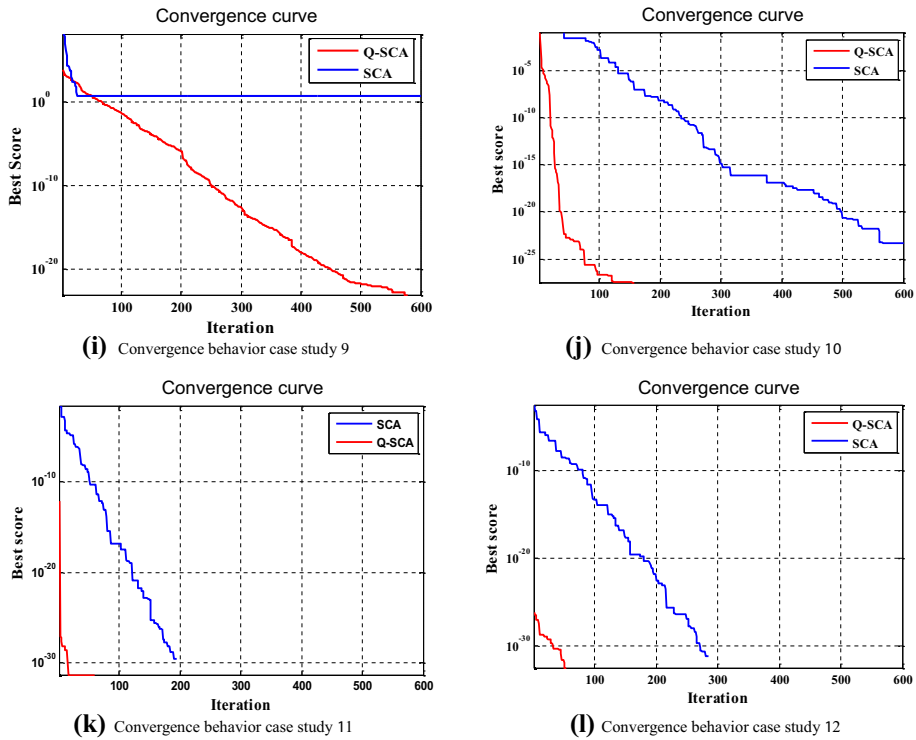
Fig. 3 (continued)

$$\begin{aligned}
f_1(\mathbf{x}) &= x_1^2 + x_3^2 = 1 \\
f_2(\mathbf{x}) &= x_2^2 + x_4^2 = 1 \\
f_3(\mathbf{x}) &= x_5 x_3^3 + x_6 x_4^3 = 0 \\
f_4(\mathbf{x}) &= x_5 x_1^3 + x_6 x_2^3 = 0 \\
f_5(\mathbf{x}) &= x_5 x_1 x_3^2 + x_6 x_4^2 x_2 = 0 \\
f_6(\mathbf{x}) &= x_5 x_1^2 x_3 + x_6 x_2^2 x_4 = 0 \\
&-10 \le x_i \le 10, \ i = 1, 2, ..., 6.
\end{aligned} \tag{19}$$

Table 5 shows that the optimum solution obtained by the proposed Q-SCA algorithm is competitive with LQPSO and QPSO algorithms but it outperforms SCA, ITHS, GRAV and Grosan et. al. (Grosan and Abraham 2008). Although the LQPSO and QPSO algorithms give the same results after 747 iterations, the proposed Q-SCA reaches the optimum solution after 308 iterations and then it outperforms all algorithms from the view point of convergence speed. The convergence behavior for case 4 is shown in Fig. 3d.

Case 5 This case describes the interval arithmetic problem that has been presented in (Oguz Emrah Turgut 2014). The nonlinear system is formulated as follows

$$f_1(\mathbf{x}) = x_1 - 0.25428722 - 0.18324757x_4x_3x_9 = 0$$
$$f_2(\mathbf{x}) = x_2 - 0.37842197 - 0.16275449x_1x_{10}x_6 = 0$$
$$f_3(\mathbf{x}) = x_3 - 0.27162577 - 0.16955071x_1x_2x_{10} = 0$$
$$f_4(\mathbf{x}) = x_4 - 0.19807914 - 0.15585316x_7x_1x_6 = 0$$
$$f_5(\mathbf{x}) = x_5 - 0.44166728 - 0.19950920x_7x_6x_3 = 0$$
$$f_6(\mathbf{x}) = x_6 - 0.14654113 - 0.18922793x_8x_5x_{10} = 0 \qquad (20)$$
$$f_7(\mathbf{x}) = x_7 - 0.42937161 - 0.21180476x_2x_5x_8 = 0$$
$$f_8(\mathbf{x}) = x_8 - 0.07056438 - 0.17081208x_1x_7x_6 = 0$$
$$f_9(\mathbf{x}) = x_9 - 0.34504906 - 0.19612740x_{10}x_6x_8 = 0$$
$$f_{10}(\mathbf{x}) = x_{10} - 0.42651102 - 0.21466544x_4x_8x_1 = 0$$

The simulation results are recorded in Table 6. From Table 6, the comparisons between the proposed Q-SCA with different algorithms (Oguz Emrah Turgut 2014; Grosan and Abraham 2008; Oliveira and Petraglia 2013) are conducted. Based on simulation results, it can conclude that the proposed Q-SCA algorithm is competitive with L-QPSO and QPSO and it outperforms the other algorithms.

Further Q-SCA has better convergence speed than other L-QPSO and QPSO since it reached the optimum solution after 474 iterations while the other converged after 487 iterations (Oguz Emrah Turgut 2014). The convergence behavior for case 5 is shown in Fig. 3e.

Case 6 This case presents the inverse position problem of the six-revolute joint application that taken from (Oguz Emrah Turgut 2014) and system equations are outlined as follows

$$
\begin{aligned}
f_{1i}(\mathbf{x}) &= x_i^2 + x_{i+1}^2 - 1 = 0 \\
f_{2i}(\mathbf{x}) &= a_{1i}x_1x_3 + a_{2i}x_1x_4 + a_{3i}x_2x_3 + a_{4i}x_2x_4 \\
&\quad + a_{5i}x_2x_7 + a_{6i}x_5x_8 + a_{7i}x_6x_7 + a_{8i}x_6x_8 \\
&\quad + a_{9i}x_1 + a_{10i}x_2 + a_{11i}x_3 + a_{12i}x_4 + a_{13i}x_5 \\
&\quad + a_{14i}x_6 + a_{15i}x_7 + a_{16i}x_8 + a_{17i} = 0
\end{aligned}
\qquad (21)
$$

The coefficients used in this system, a_{ij}, are presented in Table 7, where $1 \le i \le 17$, $1 \le j \le 4$.

Table 8 demonstrates the comparison between the proposed Q-SCA algorithm with different algorithms (Oguz Emrah Turgut 2014; Grosan and Abraham 2008) for case 6. The proposed Q-SCA outperforms the other algorithms in term of optimality. The convergence behavior for this case is portrayed in Fig. 3f.

Case 7 This case simulates the combustion problem that occurred at a temperature of 3000 °C and it has been studied in (Oguz Emrah Turgut 2014) and the general form of this system can be defined as follows

Table 3 The simulation results of the proposed Q-SCA and different approaches for case 2

	SCA	Q-SCA	LQPSO
x_1	−0.999941065644694	−0.999999999993204	−1.0000007185712
x_2	1.000029561511447	0.999999999995099	1.00000045091713
x_3	−1.000206522831925	−1.000000000009111	−0.9999992090087
x_4	0.999718579437908	0.999999999987575	1.00000957562795
x_5	−1.000021542701820	−1.000000000004006	−0.99999966616156
x_6	0.999939346668104	0.999999999997199	1.000000474541353
f_1	−1.18039326558739E−5	5.3856918924566E−13	9.037302239889E−8
f_2	4.14583935284263E−5	−1.6409096303960E−13	−2.27255014806E−8
f_3	−3.54944194784768E−5	−1.49791290482426E−12	7.493891107651E−8
f_4	−3.15537432804547E−5	−1.40099043477448E−12	4.63049154575E−10
f_5	−5.9958950928696E−6	−1.5498713423767E−13	−1.28890915052E−7
f_6	−2.32604088045152E−5	−1.121325254871E−14	8.980881838205E−8
	QPSO	ITHS	GRAV
x_1	−1.06006024401217	−1.00469360193614	−0.94769257629329
x_2	1.03789221092740	1.002699552010811	0.95991034374004
x_3	−0.96490791149742	−0.99763045117792	−1.03187245704251
x_4	1.04304617226693	1.002704645251826	0.96116900972767
x_5	−0.96783925322234	−0.99763694114471	−1.03086507223002
x_6	1.02588330512720	1.00262777388871	0.97633630789931
f_1	0.36034275521E−3	−0.63374448056E−3	0.00149483833089
f_2	−0.73227698937E−3	−0.28872794332E−3	−0.0018160678696
f_3	0.07026119979E−3	−0.30021434625E−3	−0.0010845581545
f_4	−0.14208871243E−3	−0.16590037166E−3	−0.00089244886256
f_5	−0.21739907838E−3	−0.30415100062E−3	0.00053726730918
f_6	−0.08460980828E−3	0.30832206541E−3	−0.00060686822131

Table 4 The simulation results of the proposed Q-SCA and different approaches for case 3

	SCA	Q-SCA	LQPSO	QPSO
x_1	0.514933264661129	0.514933264661129	0.5149332646611294	0.514933264661129
x_2	0.514933264661129	0.514933264661129	0.5149332646611294	0.514933264661129
x_3	0.514933264661129	0.514933264661129	0.5149332646611294	0.514933264661130
x_4	0.514933264661129	0.514933264661129	0.5149332646611294	0.514933264661129
f_1	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
f_2	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
f_3	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000
f_4	0.000000000000000	0.000000000000000	0.000000000000000	0.000000000000000

	ITHS	GRAV	Sharma and Arora (2013)
x_1	0.514933264661129	0.514931340034248	0.51493326466112941
x_2	0.514933264661129	0.514934600978757	0.51493326466112941
x_3	0.514933264661129	0.514923716523520	0.51493326466112941
x_4	0.514933264661129	0.514944432722957	0.51493326466112941
f_1	0.000000000000000	0.225940766129E−5	0.000000000000000
f_2	0.000000000000000	−0.00704139425E−5	0.000000000000000
f_3	0.000000000000000	0.770620340042E−5	0.000000000000000
f_4	0.000000000000000	−0.70946910128E−5	0.000000000000000

$$f_1(\mathbf{x}) = x_2 + 2x_6 + x_9 + 2x_{10} - 10^{-5} = 0$$
$$f_2(\mathbf{x}) = x_3 + x_8 - 3 \times 10^{-5} = 0$$
$$f_3(\mathbf{x}) = x_1 + x_3 + 2x_5 + 2x_8 + x_9 + x_{10} - 5 \times 10^{-5} = 0$$
$$f_4(\mathbf{x}) = x_4 + 2x_7 - 10^{-5} = 0$$
$$f_5(\mathbf{x}) = 0.5140437 \times 10^{-7}x_5 - x_1^2 = 0$$
$$f_6(\mathbf{x}) = 0.1006932 \times 10^{-6}x_6 - x_1^2 = 0$$
$$f_7(\mathbf{x}) = 0.7816278 \times 10^{-15}x_7 - x_4^2 = 0 \qquad (22)$$
$$f_8(\mathbf{x}) = 0.1496236 \times 10^{-6}x_8 - x_1x_3 = 0$$
$$f_9(\mathbf{x}) = 0.6194411 \times 10^{-7}x_9 - x_1x_2 = 0$$
$$f_{10}(\mathbf{x}) = 0.2089296 \times 10^{-14}x_{10} - x_1x_2^2 = 0$$

Table 9 demonstrates the comparison between the proposed Q-SCA with different algorithms (Oguz Emrah Turgut 2014; Grosan and Abraham 2008; Oliveira and Petraglia 2013) for case 7. The proposed Q-SCA outperforms the other algorithms in term of optimality. The convergence behavior for this case can be provided in Fig. 3g.

Case 8 This system contains eight nonlinear equations as shown in (Oguz Emrah Turgut 2014) and it is defined as follows

Table 5 The simulation results of the proposed Q-SCA and different approaches for case 4

	SCA	Q-SCA	LQPSO	QPSO
x_1	−0.961293003159725	−0.627056578863954	0.446209184554328	−0.796616684320047
x_2	0.961292281140527	−0.627056578863954	−0.446209184554328	0.796616684320047
x_3	0.275526877701916	−0.778973713871934	0.894928691918726	−0.604484787453692
x_4	−0.275528088322011	−0.778973713871934	−0.894928691918726	0.604484787453692
x_5	0.321198489351863	−0.517463334921782	0.366779058332292	−0.343529649687506
x_6	0.321198718424798	0.517463334921782	0.366779058332292	−0.343529649687506
f_1	−7.0173999122680E−7	0.000000000000000	0.000000000000000	0.000000000000000
f_2	−1.42276526049478E−6	0.000000000000000	0.000000000000000	0.000000000000000
f_3	−9.335045290937E−8	0.000000000000000	0.000000000000000	0.000000000000000
f_4	−4.3942810534769E−7	0.000000000000000	0.000000000000000	0.000000000000000
f_5	2.0509470346000E−7	0.000000000000000	0.00000000000000	0.000000000000000
f_6	−2.9480439443785E−7	0.000000000000000	0.000000000000000	0.000000000000000
	ITHS	GRAV	Grosan and Abraham (2008)	Abdollahi et al. (2013)
x_1	0.757992217157792	0.835326847252122	0.045943625	−0.041096050919063
x_2	0.757995636725586	0.782860693935276	−0.1626952821	0.041096050919063
x_3	0.652290147139058	0.549753670392631	−0.9215324786	0.999155200456294
x_4	0.652305698905455	−0.622197020332733	0.9841530788	−0.999155200456294
x_5	0.026046699540825	−0.000000012589636	−0.6789794019	0.098733550533454
x_6	−0.026009939089497	0.000000014361731	−0.9070329917	0.098733550533454
f_1	0.3463732647923E−4	0.398503403642E−7	0.1489636110	0.0000000000000000
f_2	0.6011011956097E−4	−0.01780246392E−7	0.0049729625	0.0000000000000000
f_3	0.0968608629041E−4	−0.05551106317E−7	0.3332320690	0.0000000000000000
f_4	0.1585609322857E−4	−0.00447429924E−7	0.0038536711	0.0000000000000000
f_5	0.1141785526275E−4	0.011742031683E−7	0.1183698936	0.0000000000000000
f_6	0.1345652755927E−4	−0.10305918922E−7	0.0224932754	0.0000000000000000

$$f_1(\mathbf{x}) = 4.731 \times 10^{-3}x_1x_3 - 0.3578x_2x_3 - 0.1238x_1$$
$$+ x_7 - 1.637 \times 10^{-3}x_2 - 0.9338x_4 - 0.3571 = 0$$
$$f_2(\mathbf{x}) = 0.2238x_1x_3 + 0.7623x_2x_3 + 0.2638x_1$$
$$- x_7 - 0.007745x_2 - 0.6734x_4 - 0.6022 = 0$$
$$f_3(\mathbf{x}) = x_6x_8 + 0.3578x_1 + 4.731 \times 10^{-3}x_2 = 0$$
$$f_4(\mathbf{x}) = -0.7623x_1 + 0.2238x_2 + 0.3461 = 0 \qquad (23)$$
$$f_5(\mathbf{x}) = x_1^2 + x_2^2 - 1 = 0$$
$$f_6(\mathbf{x}) = x_3^2 + x_4^2 - 1 = 0$$
$$f_7(\mathbf{x}) = x_5^2 + x_6^2 - 1 = 0$$
$$f_8(\mathbf{x}) = x_7^2 + x_8^2 - 1 = 0$$

The comparisons between the proposed Q-SCA and the other algorithms are shown in Table 10, where the proposed Q-SCA is capable of attaining better solutions compared to the other algorithms. Further, the convergence curve for case 8 is portrayed in Fig. 3h.

Table 6 The simulation results of the proposed Q-SCA and different approaches for case 5

	SCA	Q-SCA	LQPSO	QPSO
x_1	0.2578333937002272	0.2578333937050357	0.2578333937050357	0.2578333937050357
x_2	0.3810971546602991	0.3810971546280674	0.3810971546280674	0.3810971546280674
x_3	0.2787450173464419	0.2787450173464036	0.2787450173464036	0.2787450173464036
x_4	0.2006689642217982	0.2006689642254358	0.2006689642254358	0.2006689642254358
x_5	0.4452514248830734	0.4452514248104163	0.4452514248104163	0.4452514248104163
x_6	0.1491839199969003	0.14918391996935457	0.14918391996935457	0.14918391996935457
x_7	0.4320096979737881	0.4320096989372027	0.4320096989372027	0.4320096989372027
x_8	0.0734027777767587	0.0734027777624865	0.0734027777624865	0.0734027777624865
x_9	0.3459668208675395	0.3459668268755431	0.3459668268755431	0.3459668268755431
x_{10}	0.4273262757593127	0.4273262759329052	0.4273262759329052	0.4273262759329052
f_1	−9.9549708754143E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_2	1.9351187392165E−13	0.000000000000000	0.000000000000000	0.000000000000000
f_3	−1.5988946278078E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_4	1.15115816184952E−13	0.000000000000000	0.000000000000000	0.000000000000000
f_5	3.7306529393488E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_6	2.2455128034782E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_7	5.9895664816789E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_8	−4.6666635889901E−13	0.000000000000000	0.000000000000000	0.000000000000000
f_9	−4.8662137687450E−14	0.000000000000000	0.000000000000000	0.000000000000000
f_{10}	−3.6721493901215E−14	0.000000000000000	0.000000000000000	0.000000000000000

	ITHS	GRAV	Grosan and Abraham (2008)	Oliveira and Petraglia (2013)
x_1	0.2546864410312621	0.2578399946926554	0.0464905115	0.2578333937005036
x_2	0.3785230047533339	0.3810792616668136	0.1013568357	0.3810971546028067
x_3	0.2765254683744490	0.2787378091727205	0.0840577820	0.2787450173464404
x_4	0.2018040332606634	0.2006767758291179	−0.1388460309	0.2006689642253436
x_5	0.443869219215441	0.4452515604096610	0.4943905739	0.4452514248410416

Table 6 (continued)

	ITHS	GRAV	Grosan and Abraham (2008)	Oliveira and Petraglia (2013)
x_6	0.147985685015705	0.1491855582343140	−0.0760685163	0.1491839199693546
x_7	0.432376554488803	0.432006811493179	0.2475819110	0.4320096989837203
x_8	0.0698716908186600	0.0734037127845558	−0.0170748156	0.0734027777624865
x_9	0.349297348759015	0.349650562291278	0.0003667535	0.3459668268755543
x_{10}	0.432318039408281	0.427333090362260	0.1481119311	0.4273262759932905
f_1	−0.003172703109397	0.0652503548955E−4	0.2077959240	3.295974604355E−17
f_2	−0.002550893777805	−0.1803340006456E−4	0.2769798846	−7.285838599102E−17
f_3	−0.002166744725415	−0.0716837992760E−4	0.1876863212	3.122502256758E−17
f_4	0.001185071568625	0.0773423072990E−4	0.3367887114	3.59551212651E−17
f_5	−0.001328103066942	0.0021227038120E−4	0.0530391321	−3.64291929955E−17
f_6	−0.001092587590357	0.0158576124970E−4	0.2223730535	2.949029909160E−17
f_7	0.000518467284375	−0.0279803050751E−4	0.1816084752	6.071532165918E−17
f_8	−0.003476285138268	0.0085020886210E−4	0.0874896386	−0.607153216591E−17
f_9	0.003371565377805	−0.0180713729210E−4	0.3447200366	1.301042606982E−17
f_{10}	0.005036117718070	0.0675152562420E−4	0.2784227489	−2.526191061891E−17

Table 7 Coefficients for case 6

a_{ij}	1	2	3	4
1	−0.24915068	0.125016350	−0.63555007	1.48947730
2	1.609135400	−0.686607360	−0.11571992	0.23062341
3	0.27942343	−0.119228120	−0.66640448	1.32810730
4	1.43480160	−0.719940470	0.11036211	−0.25864503
5	0.00000000	−0.432419270	0.29070203	1.16517200
6	0.40026384	0.000000000	1.2587767	−0.26908494
7	−0.80052768	0.000000000	−0.62938836	0.53816987
8	0.000000000	−0.864838550	0.58140406	0.58258598
9	0.074052388	−0.037157270	0.19594662	−0.20816985
10	−0.083050031	0.035436896	−1.2280342	2.68683200
11	−0.38615961	0.085383482	0.000000000	−0.69910317
12	−0.75526603	0.00000000	−0.079034221	0.35744413
13	0.50420168	−0.039251967	0.026387877	1.24991170
14	−1.0916287	0.00000000	−0.05713143	1.46773600
15	0.00000000	−0.432419270	−1.1628081	1.16517200
16	0.04920729	0.0000000000	1.2587767	1.07633970
17	0.04920729	0.013873010	2.162575	−0.69686809

Case 9 This case is devoted to find the optimal solution of a thin wall rectangle girder section (Oguz Emrah Turgut 2014), where the system is considered as follows

$$f_1(\mathbf{x}) = bh - (b - 2t)(h - 2t) = 165$$
$$f_2(\mathbf{x}) = \frac{bh^3}{12} - \frac{(b - 2t)(h - 2t)}{12} = 9369$$
$$f_3(\mathbf{x}) = \frac{2t(h - t)^2(b - t)^2}{h + b - 2t} = 6835$$

(24)

where *b* is the width of the section, *h* is the height of the section, and *t* is the section thickness. Table 11 shows the optimum solutions obtained by the proposed Q-SCA competitive with the LQPSO, Abdollahi et al. (2013), Mo et al. (2009) and Luo et al. (2008) algorithms but it outperforms the SCA, QPSO, ITHS, GRAV and Jaberipour et al. (2011). The convergence behavior for case 9 is shown in Fig. 3i.

Case 10 This case is taken from (Jaberipour et al. 2011; Abdollahi et al. 2013, 2016) to demonstrate the robustness of the proposed Q-SCA in handling the systems of nonlinear equations

Table 8 The simulation results of the proposed Q-SCA and different approaches for case study 6

	SCA	Q-SCA	LQPSO	QPSO
x_1	0.920601439413393	−0.799061438480658	0.590050131899	0.772231269942513
x_2	0.392377136803073	−0.601249380484688	0.807366609320	0.635092244414582
x_3	−0.917059006740100	−0.799061438480658	−0.59005013189	0.772345476746086
x_4	−0.405874687167377	0.601249380484688	0.807366609320	0.635464640410570
x_5	0.913151100791601	0.799061438480658	−0.59005013189	−0.771984058288123
x_6	0.071472430479573	−0.678638355137620	0.942931949016	0.501630602442857
x_7	0.409987235752483	0.943622119805113	0.266895066252	0.016544744588547
x_8	−0.925540199319822	−0.634899737330961	−1.22424777217	−1.199869634113995
f_1	0.001466827735788	0.000000000000	2.22044604E−16	−0.000316706807422
f_2	−0.005042960671084	2.22044604925031E−16	0.000000000000	−0.000140305634309
f_3	0.005731483526354	−2.22044604925031E−16	0.000000000000	0.000332844662273
f_4	−0.001420805439872	0.000000000000	0.000000000000	−0.000225304536866
f_5	0.000682398615040	6.938893903907E−18	−3.4000580E−16	0.000867394190697
f_6	−0.001651465365249	−7.4593109467003E−17	−6.0715321E−17	0.001365368860350
f_7	−0.000568527108298	0.000000000000	0.000000000000	0.000799271797368
f_8	0.001282415827330	0.000000000000	1.11022302E−16	0.000532099949980

	ITHS	GRAV	Grosan and Abraham (2008)	
x_1	−0.481187883256480	−0.418714637584636	−0.0625820337	−0.1564353525
x_2	0.855827688882007	0.908150863302484	0.7777446281	0.4507122320
x_3	0.489354396992578	−0.418496945673775	−0.0503725828	0.4622139796
x_4	0.856058114076982	0.908273850895166	0.3805368959	−0.8818348503
x_5	0.529021825853160	0.418286659161462	−0.5592587603	−0.6522824284
x_6	−0.413193459556378	−0.267011571005521	−0.6988338865	0.4082826235
x_7	−0.583759143369605	−0.589809505946614	0.3963927675	0.4718261386
x_8	−0.817013237700635	−0.998579648272644	0.0861763643	−0.5070478474
f_1	−0.036017187950030	0.0599382446380E−3	0.3911967824	0.7723864643
f_2	−0.028091241086912	−0.122315944674E−3	0.3925758963	0.5832167209
f_3	−0.027696779466991	0.1010817582113E−3	0.8526542737	0.0087255337
f_4	0.012699586906051	−0.074882547608E−3	0.5424213097	0.2031050697
f_5	−0.017478713588524	0.0652303303051E−3	0.7742116224	0.6056929403
f_6	−0.064389984564789	0.0321280002644E−3	0.1537105718	0.3663682493
f_7	−0.008180482707589	−0.001702726915E−3	0.9116019977	0.3532359802
f_8	−0.056480815532868	0.0226792051423E−3	0.1519175234	0.4646334692

Table 9 The simulation results of the proposed Q-SCA and different approaches for case 7

	SCA	Q-SCA	LQPSO	QPSO	ITHS
x_1	-0.0099887207520752498	0.0000002606188476	-5.9286450E-8	-4.88278463E-7	0.0072458787409306
x_2	-0.0314483969977582	-0.0010291322284942	-6.9427900E-5	6.47373030E-3	0.0101801769311163
x_3	0.0008157888277002	-0.049589305489299	-0.298022727	0.98868088610	-0.0029051737979654
x_4	0.0676494506616815	-0.00148099518353527	-8.8526040E-5	0.00688493552	-0.00232291581807
x_5	-0.912285232335751	-0.129302764364770	-0.4127268522	0.24930591960	-0.00257099288961
x_6	0.0315666358204635	-0.159125466988501	-0.0547120683	-0.00475443378	-0.00012851335424
x_7	-0.0331002760999990	0.00074549914848449	4.92534440E-5	-0.0034343749623	0.00107527109 7203
x_8	0.0074597207672120	0.049619305238496	0.2980527304	-0.98865126389	0.0028217171 7052936
x_9	3.671482250886807	0.0987218516 21468	0.9453385321	0.976976825720	0.0001706056 26642
x_{10}	-1.851864020353604	0.110284110644183	-0.417917503	-0.48696584421	-0.00493715824672
f_1	-0.0006070743388713	6.64789013728E-9	-3.718425434E-8	7.5793361E-12	2.09438979507E-4
f_2	0.0082454509044214	-2.5080258479E-10	3.403141215E-9	-3.77787894E-7	-1.13456743608E-4
f_3	0.0007444274920629	-8.5771902632E-10	-1.524300123E-8	3.476367522E-8	2.560031932362E-5
f_4	0.00143889841683 4	3.11337170457E-9	-1.915259325E-8	-5.69542591E-8	-1.82373623661E-4
f_5	-0.0000998218214 37719	-6.64679506342E-9	-2.121593269E-8	1.281644356E-8	5.250288608271E-5
f_6	-0.0019824775 52581	-2.13424937229329E-6	-1.514954299E-8	-8.38188468E-5	2.072720176124E-4
f_7	-0.004576448168757	-2.19334673362927E-6	-7.836859219E-9	-4.74023371E-5	5.395937897848E-6
f_8	0.0000081497974 42	2.034810829033E-8	2.196726011E-8	3.348260222E-7	2.105095828321E-5
f_9	-0.000314257157586	6.38344812371E-9	5.855288223E-8	6.367894304E-8	-7.37643136604E-5
f_{10}	0.0000099012232 06	-2.7579407E-13	-5.071595221E-16	2.04623346E-11	7.509383718631E-8

	GRAV	Oliveira et.al. (2013)	Grosan et. al. (2008)	
x_1	-0.00129788338220	6.05151659800073E-7	-0.0552429896	-0.338378558
x_2	0.00918114480811	-4.11586962641 0E-3	-0.0023377533	0.0185669333
x_3	0.351976543447875	-5.6525990199 17E-1	0.0455880930	0.0534924988
x_4	0.0134449786136748	5.84693914255 0E-3	-0.1287029472	0.0392783417
x_5	0.381060037378405	-1.30586435471E-1	0.0539771728	0.0183882247

Table 9 (continued)

	GRAV	Oliveira et al. (2013)	Grosan et. al. (2008)	
x_6	0.328340515466930	3.613537479299E−2	−0.0151036079	0.0005246892
x_7	−0.006722201015185	−2.918469098378E−3	0.1063159019	−0.1024269629
x_8	−0.351943591928842	5.652899283870E−1	0.0386267592	0.0500461848
x_9	−0.151595919400474	−5.400505135447E−1	−0.1144905135	−0.1013361102
x_{10}	−0.257144043111724	2.359528246328E−1	0.0872294353	0.0404252678
f_1	−3.18298965591E−5	1.5680469988E−8	0.0274133878	0.0008794626
f_2	2.951519453150E−6	2.639600000269E−8	0.0841848522	0.1035086837
f_3	1.215884436503E−4	1.292598473E−10	0.1482418893	0.0955626197
f_4	−4.234166956693E−7	9.457940001E−10	0.0839188567	0.2441423777
f_5	−1.664913122655E−6	−6.7130796532E−9	0.0030517851	0.0011449995
f_6	−1.685537378343E−4	−3.3877126976E−5	0.0000109317	0.0006894619
f_7	−1.808967471246E−4	−3.4186697336E−5	0.0165644486	0.0015427967
f_8	4.567718475998E−4	4.266486820E−7	0.0025184283	0.0018100789
f_9	1.190666480262E−5	3.096222308E−8	0.0001291515	0.0006282589
f_{10}	1.094030285101E−7	−1.025100078E−11	0.0000003019	0.0000116649

Table 10 The simulation results of the proposed Q-SCA and different approaches for case 8

	SCA	Q-SCA	LQPSO	QPSO	ITHS
x_1	0.6715542261827984	0.1644316665854327	0.1644316665854332	0.16443166585854	0.163641391023647
x_2	0.7409553788332084	−0.9863884768850967	−0.9863884768509	−0.98638847685	−0.98627002980711
x_3	0.9624929049443431	0.6830298948486491	0.9476237964080808	0.947623796408	0.95152343463466
x_4	−0.2713068519867200	−0.7303904179898823	−0.3193886981111	−0.31938869811	−0.30706727299400
x_5	0.9624352698422156	−0.9979018154051 38	−0.998427473970	−0.998427473937	−0.9968607179168
x_6	0.2715112366160490	0.0647453999922544	−0.0560587128612	−0.056058712861	0.03838334619587
x_7	0.4402172493887112	−0.5477895959039993	−0.2575850995046	−0.257585099504	−0.25765415743645
x_8	−0.8978912926073955	−0.8366616133709842	0.9662556165654937	0.9662556165493	−0.96464 64464758
f_1	2.338129689861E−13	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.01014171386634
f_2	−3.9660497108684E−12	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.01139457294335
f_3	−1.4267061490547 1E−11	2.60208521396521E−18	2.6020852139E−18	1.7347234759E−18	0.016858410227952
f_4	−8.8518081753364E−12	0.0000000000000000	0.00000000000000000	1.6653345369E−16	0.00062893495184
f_5	−4.7384318691000E−13	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.00049292344812
f_6	1.3880008253864E−12	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.00031284319707
f_7	−2.4588588880324E−12	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.00479542780918
f_8	−1.8673951274195E−12	0.00000000000000000000	0.00000000000000000	0.0000000000000000	−0.00306767514011

	GRAV	Floudas et al. (1999)	Abdollahi et al. (2013)	Wang et al. (2011)
x_1	0.671708013238769	0.1644	0.164431665854327	0.67154465
x_2	0.740773292936966	−0.9864	−0.9863884768850967	0.74097111
x_3	−0.253002500191632	−0.9471	0.718452601027603	0.95189459
x_4	−0.967496424017105	−0.3210	−0.695575919707312	−0.30643725
x_5	0.957898268905606	−0.9982	0.997964383970433	0.96381470
x_6	0.287126008079644	−0.0594	0.063773727557003	−0.26657405
x_7	−0.528417236794134	0.4110	−0.527809105283546	0.40463693
x_8	−0.848969584145896	0.9116	−0.849363025083964	0.91447470
f_1	−1.851983783159E−4	−8.8531E−5	2.7755575611562891E−016	−0.00000375

Table 10 (continued)

	GRAV	Floudas et al. (1999)	Abdollahi et al. (2013)	Wang et al. (2011)
f_2	−9.1791861881463E−5	3.5894E−5	− 1.110223024462515E−016	0.00001537
f_3	8.0477908869916E−5	6.6216E−6	1.734723475976807E−018	0.00000899
f_4	−1.5795553262093E−4	2.1560E−5	1.665334536937735E−016	0.00001084
f_5	−6.3273422149401E−5	1.2320E−5	0.000000000000000000000	0.00001039
f_6	5.9595891025122E−5	3.9410E−5	0.000000000000000000000	0.00000709
f_7	1.0438088108744442E−5	− 6.8400E−5	0.000000000000000000000	0.00000049
f_8	−2.5586905399671E−5	− 6.4440E−5	0.000000000000000000000	− 0.00000498

$$f_1(\mathbf{x}) = x_1^{x_2} + x_2^{x_1} - 5x_1x_2x_3 = 85$$
$$f_2(\mathbf{x}) = x_1^3 - x_2^{x_3} - x_3^{x_2} = 60 \tag{25}$$
$$f_3(\mathbf{x}) = x_1^{x_3} + x_3^{x_1} - x_2 = 2$$

The solution for this system was (4, 3, 1) as in (Jaberipour et al. 2011; Abdollahi et al. 2013, 2016). The proposed Q-SCA finds the global optimal solution for this case while the convergence behavior of the Q-SCA algorithm outperforms the other algorithms. Further, the proposed Q-SCA algorithm takes 120 iterations and 20 population size to reach the optimal solution whereas (Abdollahi et al. 2013, 2016) and (Jaberipour et al. 2011) had been reached to that answer with 200, 250, and 1000 iterations, respectively, where each of them has been used 250 agents for the population size. The results of the proposed algorithm are shown as in Table 12 and Fig. 3j.

Case 11 This case was given in (Jaberipour et al. 2011; Abdollahi et al. 2013, 2016)

$$f_1(\mathbf{x}) = x_1^3 - 3x_1x_2^2 - 1 = 0$$
$$f_2(\mathbf{x}) = 3x_1^2x_2 - x_2^3 + 1 = 0 \tag{26}$$

Table 13 demonstrates the optimum solution found by the proposed Q-SCA. From Table 13, we can see that the proposed Q-SCA outperforms the other algorithms in view of optimal value. In addition, the convergence behavior of case 11 is shown in Fig. 3k.

Case 12 This case was given in (Abdollahi et al. 2013, 2016; Floudas et al. 1999; Wang et al. 2011)

$$f_1(\mathbf{x}) = 0.5 \sin(x_1x_2) - 0.25x_2/\pi - 0.5x_1 = 0$$
$$f_2(\mathbf{x}) = (1 - 0.25/\pi)(\exp(2x_1) - e) + ex_2/\pi - 2ex_1 = 0 \tag{27}$$

The results for this case are shown in Table 14. The optimum solution obtained by the proposed Q-SCA is competitive with Abdollahi et al. (Abdollahi et al. 2013) and it outperforms the other algorithms in view of optimal value. Although Abdollahi et al. (Abdollahi et al. 2013) get the same optimal value but it has elapsed a large number of iteration (i.e. about 180), where the proposed Q-SCA has a less number of iterations (i.e., 63 iterations). In addition, the convergence behavior of case 12 is shown in Fig. 3l.

To evaluate the efficiency and feasibility of the proposed Q-SCA algorithm, the statistical measures for each case are reported in Table 15, such as best, mean, median and worst objective values, and standard deviations (SD) over 30 independent runs, where best result is given in bold font.

4.2 Wilcoxon signed-rank test

The Wilcoxon signed ranks test is a nonparametric procedure used in a hypothesis testing situation involving a design with two samples (Garcia et al. 2009). It is a pair-wise test that aims to detect significant differences between the behaviors of two methods. It is associated with *p*-value, where *p* is the probability of the null hypothesis being true. The result of the test is returned in $p < 0.05$ indicates a rejection of the null hypothesis, while $p > 0.05$ indicates a failure to reject the null hypothesis. The R^+ is the sum of positive ranks, while R^- is the sum of negative ranks. In Table 16, we present the results of the Wilcoxon signed-rank

Table 11 The simulation results of the proposed algorithm and different approaches for case 9

	b	h	t	f_1	f_2	f_3
Q-SCA	12.256519599348696	22.894938623626285	2.789817919538154	165	9369	6835
SCA	12.259200039815457	22.990287022276142	2.733791404575978	162.8350	9369.0100	6835.0038
LQPSO	12.25651961037963	22.894938922662796	2.7898177361173895	165	9369	6835
QPSO	12.25667460849311	22.903035265500499	2.7849857955969321	165.1859	9369.0052	6834.9887
ITHS	8.915790281648581	23.291452604339380	12.885353223417773	165.8741	9366.4924	6831.8042
GRAV	12.260243907887173	22.775634683577675	2.857554308400358	167.5713	9362.2016	6836.7276
Abdollahi et al. (2013)	8.943088778747601	23.271481879207862	12.912774291361677	165	9369	6835
Jaberipour et al. (2011)	43.155566052654329	10.128950202278199	12.944048457756352	709.2412	9369	6835
	−7.6029951984634	−24.541982377674	−11.576715672202	208.1851	9369	6835
Mo et al. (2009)	8.943089	23.271482	12.912774	165	9369	6835
Luo et al. (2008)	12.5655	22.8949	2.7898	166.7229	9369	6835
	−12.5655	−22.8949	−2.7898	166.7229	9369	6835
	8.943089	23.271482	12.912774	165	9369	6835
	−8.943089	−23.271482	−12.912774	165	9369	6835
	−2.3637	35.7564	3.0151	165	9369	6835
	2.3637	−35.7564	−3.0151	165	9369	6835

Table 12 The simulation results of the proposed algorithm for case study 10

Method	SCA	Q-SCA
x_1	3.999999999999975	4.000000000000000
x_2	3.000000000000050	3.000000000000000
x_3	1.000000000000083	1.000000000000000
f_1	8.8107299234252E−13	0.000000000000000
f_2	− 1.76214598468505E−12	0.000000000000000
f_3	7.1365136022905E−13	0.000000000000000

test for the different algorithms where the proposed Q-SCA is compared against LQPSO, QPSO, GRAV, ITHS, and SCA. We can conclude from Table 16 that the proposed Q-SCA is a significant algorithm and it is better than the other algorithms.

4.3 Power system applications

This section is devoted to validate the proposed algorithm from the view point of applications (Saadat 2004). Two electrical networks have been selected where one-line diagram for each network is shown in Fig. 4a shows the three bus system with generation at bus 1 (slack bus), where the voltage magnitude at bus 1 is adjusted to 1.05 per unit. The complete date of this system is marked in per unit on a 100-MVA base and the line charging susceptances are neglected as in Fig. 4a. On the other hand, Fig. 4b shows the one-line diagram for five bus system where complete system data is marked in per unit on a 100-MVA base as shown in Table 17.

The results for two cases are depicted in Tables 18 and 19 respectively, where the optimum solution for voltage at each node and the phase angle are reported. The obtained results demonstrate the effectiveness and robustness of the proposed algorithm in solving the power system applications. Further the convergence behavior is portrayed in Fig. 5.

From the reported results, we can see that the proposed Q-SCA is obviously superior to the reported algorithms. Thus we can conclude that the proposed algorithm is more environmentally friendly and cost-effective scheme in solving the nonlinear system equations from the view point of realization.

4.4 Convergence analysis

In order to analyze the convergence analysis of the proposed algorithm, statistical measures, and non parametric test namely, wilcoxon signed ranks (WSRs) test were developed. Based on the obtained simulation results as recorded in Table 15, we can see that the proposed Q-SCA has better searching quality, where the proposed Q-SCA outperforms other comparative algorisms for cases 2, 6, 7, and 9 and it is competitive with some algorithms for other cases. Moreover, the stability of the proposed Q-SCA is investigated as shown in Table 15, where the worst solutions found by the proposed Q-SCA is better than the best one found by the other algorithms. Further the non parametric WSRs is employed to offer the winner algorithm, where Table 16 showed that the proposed algorithm outperforms LQPSO, QPSO, GRAV, ITHS, and SCA algorithms in terms of the obtained p-value. Regarding the presented analyses, it can be concluded that the inherent characteristic of

Table 13 The simulation results of the proposed algorithm and different approaches for case study 11

Methods	x_1	x_2	f_1	f_2
Jaberipour et al. (2011)	−0.29051455550725	1.08421508149135	−6.77236045021345E−15	1.11022302462516E−15
Jaberipour et al. (2011)	−0.79370052525984100	−0.79370052525984100	8.88178419700125E−16	−8.88178419700125E−16
Abdollahi et al. (2013)	1.084215081491351	−0.290514555507251	0.000000000000000	1.88737914186277E−15
Abdollahi et al. (2013)	−0.79370052525984100	−0.79370052525984100	8.88178419700125E−16	−8.88178419700125E−16
Abdollahi et al. (2013)	−0.290514555507251	1.084215081491351	−1.88737914186277E−15	0.000000000000000
Mahdi et. al. (2016)	1.084215081563733	−0.2905145554550255	2.04554262417389E−9	2.99581126572690E−9
Mahdi et. al. (2016)	−0.793700525199129	−0.793700527168166	4.47548975657242E−9	2.96700342072853E−9
Mahdi et. al. (2016)	−0.290514556862374	1.084215081072923	3.64503893734991E−9	3.93069188309170E−9
Mahdi et. al. (2016)	1.0842150814913512220446049250313	−0.2905145555072514440892098500626	0.000000000000000	0.000000000000000
Mahdi et. al. (2016)	−0.79370052525984100	−0.79370052525984100	8.88178419700125E−16	−8.88178419700125E−6
Mahdi et. al. (2016)	−0.2905145555072514440892098500626	1.0842150814913512220446049250313	0.000000000000000	0.000000000000000
SCA	−0.290514555507251	1.084215081491351	0.000000000000000	0.000000000000000
Q-SCA	−0.290514555507251	1.084215081491351	0.000000000000000	0.000000000000000
Q-SCA	1.084215081491351	−0.290514555507251	0.000000000000000	0.000000000000000
Q-SCA	−0.79370052525984100	−0.79370052525984100	0.000000000000000	−4.44089209850063E−16

Table 14 The simulation results of the proposed algorithm and different approaches for case study 12

Methods	x_1	x_2	f_1	f_2
Abdollahi et al. (2013)	0.299448692495720	2.836927770471037	1.305289210051797e−012	2.284838984678572e−013
Abdollahi et al. (2013)	0.500000000000000	3.141592653589794	0	0
Floudas et. al. (1999)	0.29945	2.83693	NA	NA
Floudas et. al. (1999)	0.5	3.14159	NA	NA
Wang et. al. (2011)	0.50043285	3.14186317	−0.00023852	0.00014159
Mahdi et. al. (2016)	0.500000022686182	3.141593003459494	−1.4113503388202228e−007	2.046995830617959e−007
Mahdi et. al. (2016)	0.299448692490926	2.836927770458940	2.775557561562891e−017	0
Mahdi et. al. (2016)	0.500000000000000	3.141592653589794	0	0
SCA	0.500000000000000	3.141592653589794	0	0
Q-SCA	0.500000000000000	3.141592653589794	0	0

**Table 15** The simulation results of the proposed algorithm using the statistical measures

	Best	SD	Mean dev	Worst	Best	SD	Mean dev	Worst
	LQPSO				QPSO			
Case 1	0.00000000000	0.00000000000	0.00000000000	0.00000000000	1.19023842E−4	0.30615475322	0.19685961215	1.7729344742
Case 2	3.21075372E−8	0.07573068045	0.02546161341	0.75262288489	8.63494937E−4	0.22599042221	0.2284487031	1.3170466056
Case 3	0.00000000000	4.24594677E−33	1.4061158E−33	2.4651903E−32	0.00000000000	0.00760588253	0.00104357789	0.0736498679
Case 4	0.00000000000	2.9964896E−73	2.3914591E−74	3.7785053E−72	0.00000000000	0.00696221187	0.00276205661	0.0616852401
Case 5	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.14167213251	0.10273725469	0.5548308830
Case 6	6.47523361E−16	0.0592980616	0.02497783859	0.25783943858	0.00195450943	2.06200649958	1.46452419391	12.472963781
Case 7	6.20831492E−6	2.06229319E−4	6.44202906E−4	0.00113727036	9.62956081E−5	0.46025666089	0.31986584814	2.6750069648
Case 8	2.6020852E−18	3.1857439E−15	8.4696820E−16	1.9212860E−14	1.6654248E−16	0.17567273405	0.20739653915	0.7353425881
Case 9	1.6298215E−10	102.025240148	168.705483779	551.460632344	0.18607674831	176.052610218	228.515925917	767.75796004
	GRAV				ITHS			
Case 1	6.71035625E−4	0.00513514102	0.00621692452	0.01782211133	1.41577466E−8	0.016631394416	0.01236940795	0.1153152004
Case 2	0.00285697754	8.67200120E−4	0.00466824241	0.00626278271	8.88941958E−4	0.063627454049	0.08843564918	0.3014222498
Case 3	1.1483009E−10	7.16294837E−8	6.73570062E−8	3.07558863E−7	0.00000000000	0.002148256574	4.67800603E−4	0.0182132216
Case 4	4.15910112E−8	2.97523498E−4	2.55074433E−4	8.30651132E−4	7.39570981E−5	0.007209738538	0.00689635827	0.0541546139
Case 5	2.32151678E−5	9.27579035E−5	1.64723787E−4	3.81861144E−4	0.00864367972	0.012961479837	0.02848510526	0.0882327076
Case 6	2.00457355E−4	0.01458581301	0.01357994356	0.08536831725	0.10355348915	0.18684470848	0.39870851563	1.2277557987
Case 7	5.34589282E−4	0.00159989262	0.00264903045	0.00560918745	3.77206677E−4	0.023264433757	0.02010433014	0.1620341415
Case 8	2.87200515E−4	0.08897540132	0.13831108452	0.29177988077	0.02345292754	0.071914482939	0.15654934659	0.4159781524
Case 9	7.47089935933	17.8057247481	37.1556293303	130.470752028	4.15504414694	338.2946656533	312.651184712	5555.0798892

Table 15 (continued)

	SCA				Q-SCA			
Case 1	7.4368303E−6	4.19347826E−3	1.60827224E−3	1.70827953E−2	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 2	6.84881093E−5	0.11980219	7.18302878E−2	0.31506742	**2.12182949926E−12**	**1.8845978043E−13**	**2.2359388635E−12**	**2.4534659062E−12**
Case 3	0.00000000	0.10699189	4.98363814E−2	0.29267477	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 4	1.68744920E−6	4.95412781E−3	4.91325658E−3	2.30501674E−2	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 5	5.36993027E−13	1.54226154E−2	4.89904050E−3	7.13245623E−2	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 6	8.22267879E−3	0.15140164	0.15345153	0.71665997	**3.22831060112E−16**	**4.3810616777E−4**	**2.3972102938E−5**	**8.0066823814E−3**
Case 7	9.79613578E−3	1.45777691E−2	1.18312758E−2	7.59925301E−2	**3.06044832228E−6**	**2.3624151090E−6**	**3.7474558107E−6**	**1.30585466663E−5**
Case 8	1.75887758E−11	0.10203331	7.91390661E−2	0.43886205	2.60208523139E−18	1.65385776544E−17	5.12419466154E−18	1.11052791461E−16
Case 9	2.16499892	2.24298922E−15	2.16499892	2.16499892	0.00000000000	0.00000000000	0.00000000000	0.0
Case 10	1.27764134597E−3	4.58364669600008E−2	3.4324984020375E−2	1.72299626076878E−1	0.0	0.0	0.0	0.0
Case 11	1.31591268631539E−17	2.65619378978261E−15	1.1621070936037E−15	1.44112039845045E−14	1.02296660742877E−31	3.04040140553393E−32	4.93038065763132E−31	
Case 12	6.18210170945528E−14	2.45257069516594E−11	1.13697910715530E−11	9.96535298180761E−11	4.2787766233132E−31	1.67941091150567E−31	1.75105550543687E−30	

	SCA				Q-SCA			
Case 10	8.7008147181E−24	4.1140049356E−2	7.26404828231E−4	2.8320777481E−1	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 11	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000	0.00000000000
Case 12	3.97818378869E−34	2.5679065925E−34	7.70371977775E−34	7.70371977775E−34	0.00000000000	0.00000000000	0.00000000000	0.00000000000

Table 16 Wilcoxon test for comparison results in Table 15

Compared algorithms		Solution evaluations			
Algorithm 1	Algorithm 2	R^-	R^+	$p$value	Best algorithm
Q-SCA	LQPSO	6	0	0.108809	Q-SCA
Q-SCA	QPSO	15	0	0.043114	Q-SCA
Q-SCA	GRAV	45	0	0.007686	Q-SCA
Q-SCA	ITHS	36	0	0.011719	Q-SCA
Q-SCA	SCA	36	0	0.011719	Q-SCA

(a) Three bus system
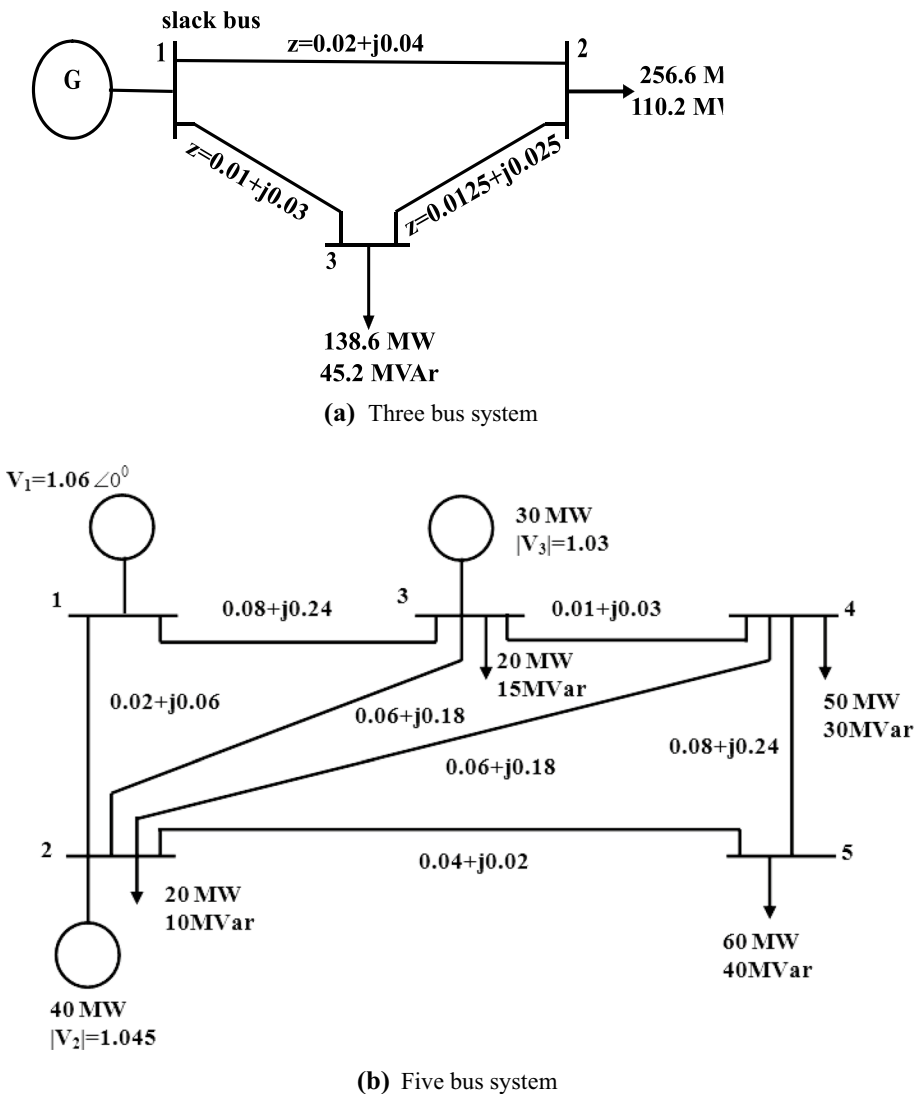
(b) Five bus system

Fig. 4 One-line diagram for the two systems. **a** three bus system **b** five bus system

Table 17 The complete data for the five bus system

Line (L_{ij})		Impedance		Admitance
1–2		0.02 + j0.06		j0.030
1–3		0.08 + j0.24		j0.025
2–3		0.06 + j0.18		j0.020
2–4		0.06 + j0.18		j0.020
2–5		0.04 + j0.12		j0.015
3–4		0.01 + j0.03		j0.010
4–5		0.08 + j0.24		j0.025
Node	Generator data	Load data	$_V(o)$	Node Type
1	0 + j0	0 + j0	1.06 + j0.0	Slack
2	0.4 + jQ	0.2 + j0.1	1.045 + j0	V. Controlled
3	0.3 + jQ	0.2 + j0.15	1.03 + ja	V. Controlled
4	0 + j0	0.5 + j0.3	1.0 + j0	Load
5	0 + j0	0.6 + j0.4	1.0 + j0	Load

Table 18 The results for the three bus system

	SCA	Q-SCA	Ref. (Saadat 2004)
$|V_1|$	1.05	1.05	1.05
$|V_2|$	0.981838079377364	0.981835016690686	0.98183
$|V_3|$	1.001251283783532	1.001249219725039	1.00125
δ_1	0.0000	0.0000	0.0000
δ_2	− 3.502382236093232	− 3.503531644784462	− 3.5035
δ_3	− 2.861380189597466	− 2.862405226111747	− 2.8624
f_1	− 5.05150474218397E−4	8.8817841970013E−16	
f_2	− 4.88402153445477E−4	4.4408920985006E−16	
f_3	1.78159569490965E−4	− 2.44249065417534E−15	
f_4	1.54114917028381E−4	2.66453525910038E−15	
f	5.492059049451215e−007	1.405158487424927E−29	

this improvement is contained in incorporating the quantum strategy as a local search strategy that accelerates the convergence behavior and avoids the systematic running of the algorithm without any improvements in the outcomes. It can be concluded that the proposed Q-SCA has a significant performance, and the immature convergence inaccuracies of SCA phase is mitigated efficiently.

In this subsection, a comparative study has been performed to evaluate the performance of the proposed Q-SCA algorithm regarding the integrated scheme, closeness to optimal solution and computational time. On one hand, pure techniques are still suffering from reaching an optimal solution in a suitable time. Also, the sinking into stagnation and premature convergence may be occurs in some of pure techniques. Consequently, the integrated sine cosine algorithm with quantum local search has twofold features; avoiding the premature convergence and improving the performance by using the properties of quantum behavior. On the

Table 19 The results for the five bus system

	SCA	Q-SCA
$|V_1|$	1.06	1.06
$|V_2|$	1.056	1.056
$|V_3|$	1.03	1.03
$|V_4|$	1.018435127204843	1.018630577870106
$|V_5|$	0.989538210278804	0.990098992150422
δ_1	0.0000000	0.0000000
δ_2	-1.944789603009603	-1.782148822376761
δ_3	-2.919882018480426	-2.663767586987853
δ_4	-3.511490409163109	-3.242881574384807
δ_5	-4.675411331717214	-4.404773706610992
f_1	0.011471071353534	-0.279221090693227
f_2	0.017876378728437	-0.374561492932912
f_3	0.018260614359347	-0.400235400377369
f_4	0.015992420160200	-0.316413562018170
f_5	-0.000366016023354	-0.026645352591004
f_6	-0.000323620656330	0.034972025275692
f	0.001040596631860	4.804996647868887E$-$027
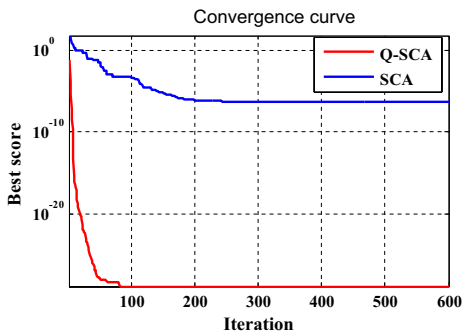
(a) The convergence history for 3-bus system
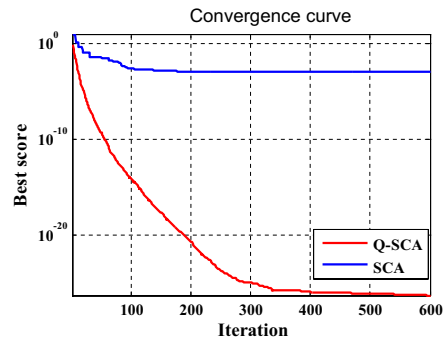
(b) The convergence history for 5-bus system

Fig. 5 The convergence behavior for two systems

other hand, the proposed Q-SCA algorithm is highly competitive when comparing it with the other methods in terms of calculating the statistical measures. So the use of the quantum behavior has a great potential for solving nonlinear systems and their applications.

4.5 Application of the Q-SCA in real real-world optimization problems

In this section, the proposed methodology is implemented for solving CEC 2017 benchmark problems and optimal power dispatch (OPD) problem as large-scale real-world optimization tasks.

4.5.1 Real-world problems from CEC 2017

For further investigating the performance of the proposed Q-SCA, it is applied to solve CEC2017 problems which are more competitive suits. The description of the CEC2017 problems is listed in Table 20. Table 20 exhibits the nature of each problem (Unimodal (U), Multimodal (M), Hybrid (H), and Composition (C) functions) with the global optimum (F^*) and the bounds for each dimension. As the CEC2017 problems involve highly complex composite and hybrid natures (Aydilek 2018), robust optimizers to achieve a suitable accuracy with fast rate are needed. In this context, the proposed Q-SCA algorithm is investigated on the CEC2017 suites with 30 dimensions (30D) and 100 dimensions (100D) to affirm its scalability and robustness. The obtained results by the Q-SCA and SCA are compared with other methods reported from the literature (Aydilek 2018) including PSO (Aydilek 2018), FA (Aydilek 2018), FFPSO (Aydilek 2018), HPSOFF (Aydilek 2018), HFPSO (Aydilek 2018), and HGSO (Hashim et al. 2019). Table 21 lists the statistical measures for the 30D and100D, respectively. Based on the recorded results, it can be seen that the Q-SCA is better than SCA for all problems. On the other hand, Tables 22 and 23 records the mean and standard deviation (Std. dev.) for the optimal fitness value for each problem achieved by the proposed Q-SCA and the other competitors for 30D and 100D respectively. As shown from Table 22, Q-SCA outperforms the other competitive methods for 30D except HGSO for $F_{CEC-8}, F_{CEC-11}, F_{CEC-21}, F_{CEC-24}, F_{CEC-25}, F_{CEC-27}$, and F_{CEC-29} problems, while for 100D, the proposed Q-SCA outperforms the other comparative methods. For clarity, the results of the best algorithms among all compared algorithms are marked in boldface. Moreover, the convergence behaviors for some selective problems are depicted in Fig. 6.

4.5.2 OPD problem-based economical operation

To further investigate the effectiveness of the Q-SCA, it is applied to solve the realistic optimal power dispatch (OPD) problem and comparisons with the recent methods from the literature.

The basis of formulation of the OPF can be described as follows:

$$\begin{aligned} Minimize &: \ F(\pmb{x}, \pmb{u}) \\ Subject\ to &: \ \begin{cases} G(\pmb{x}, \pmb{u}) \leq 0 \\ H(\pmb{x}, \pmb{u}) = 0 \end{cases} \end{aligned} \qquad (28)$$

where the function of $F(\pmb{x}, \pmb{u})$ denotes the objective function, $\pmb{x}$ indicates the vector of dependent variables, $\pmb{u}$ defines the vector of control variable, $G(\pmb{x}, \pmb{u})$ represents the inequality constraint, and $G(\pmb{x}, \pmb{u})$ defines the equality constraint.

In this work, the objective function is to minimize the total fuel cost (\$/h), F, that is expressed as follows:

Table 20 Characteristic of CEC 2017 benchmark problems

ID	Nature	Description	Optimum (F^*)	Search Range
F_{CEC-1}	U	Shifted and Rotated Bent Cigar Function	100	$[-100, 100]$
F_{CEC-2}	U	Shifted and Rotated Sum of Different Power Function	200	$[-100, 100]$
F_{CEC-3}	U	Shifted and Rotated Zakharov Function	300	$[-100, 100]$
F_{CEC-4}	M	Shifted and Rotated Rosenbrock's Function	400	$[-100, 100]$
F_{CEC-5}	M	Shifted and Rotated Rastrigin's Function	500	$[-100, 100]$
F_{CEC-6}	M	Shifted and Rotated Expanded Scaffer's F6 Function	600	$[-100, 100]$
F_{CEC-7}	M	Shifted and Rotated Lunacek Bi-Rastrigin Function	700	$[-100, 100]$
F_{CEC-8}	M	Shifted and Rotated Non-Continuous Rastrigin's Function	800	$[-100, 100]$
F_{CEC-9}	M	Shifted and Rotated Lévy Function	900	$[-100, 100]$
F_{CEC-10}	M	Shifted and Rotated Schwefel's Function	1000	$[-100, 100]$
F_{CEC-11}	H	Hybrid Function 1 (N=3)	1100	$[-100, 100]$
F_{CEC-12}	H	Hybrid Function 2 (N=3)	1200	$[-100, 100]$
F_{CEC-13}	H	Hybrid Function 3 (N=3)	1300	$[-100, 100]$
F_{CEC-14}	H	Hybrid Function 4 (N=4)	1400	$[-100, 100]$
F_{CEC-15}	H	Hybrid Function 5 (N=4)	1500	$[-100, 100]$
F_{CEC-16}	H	Hybrid Function 6 (N=4)	1600	$[-100, 100]$
F_{CEC-17}	H	Hybrid Function 6 (N=5)	1700	$[-100, 100]$
F_{CEC-18}	H	Hybrid Function 6 (N=5)	1800	$[-100, 100]$
F_{CEC-19}	H	Hybrid Function 6 (N=5)	1900	$[-100, 100]$
F_{CEC-20}	H	Hybrid Function 6 (N=6)	2000	$[-100, 100]$
F_{CEC-21}	C	Composition Function 1 (N=3)	2100	$[-100, 100]$
F_{CEC-22}	C	Composition Function 2 (N=3)	2200	$[-100, 100]$
F_{CEC-23}	C	Composition Function 3 (N=3)	2300	$[-100, 100]$
F_{CEC-24}	C	Composition Function 4 (N=4)	2400	$[-100, 100]$
F_{CEC-25}	C	Composition Function 5 (N=4)	2500	$[-100, 100]$
F_{CEC-26}	C	Composition Function 6 (N=5)	2600	$[-100, 100]$
F_{CEC-27}	C	Composition Function 7 (N=6)	2700	$[-100, 100]$
F_{CEC-28}	C	Composition Function 8 (N=6)	2800	$[-100, 100]$
F_{CEC-29}	C	Composition Function 9 (N=3)	2900	$[-100, 100]$
F_{CEC-30}	C	Composition Function 10 (N=3)	3000	$[-100, 100]$

$$F = \sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \tag{29}$$

where P_{G_i} is the ith bus generated real (active) power, a_i, b_i and c_i denote the coefficients of the fuel cost for the ith generator and NG denotes the number of generators.

The objective function can be optimized under some equality and inequality constraint equations. The equality constraints aim to give the power balanced of load flow that can be described as follows.

$$P_{Gi} - P_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \cos(\delta_{ij}) + B_{ij} \sin(\delta_{ij})] = 0 \, \forall i \in NB \tag{30}$$

Table 21 Statistical measures achieved by the proposed algorithms for CEC 2017 benchmark problems

ID	Method	30D statistical results				100D statistical results			
		Best	Mean	Worst	SD	Best	Mean	Worst	SD
F_{CEC-1}	SCA	9.2380E+09	1.4301E+10	1.9247E+10	3.7155E+09	1.6889E+11	1.8280E+11	1.9595E+11	7.5305E+09
	Q-SCA	1.0000E+02	**1.0000E+02**	1.0000E+02	1.4337E-08	1.0918E+02	**5.0620E+03**	1.3679E+04	5.1006E+03
F_{CEC-2}	SCA	2.9718E+30	7.6719E+34	5.2771E+35	1.7090E+35	1.5312E+140	5.7601E+154	7.1999E+155	1.2419E+355
	Q-SCA	2.0000E+02	**2.2167E+02**	2.6800E+02	3.1105E+01	1.0000E+30	**1.0000E+30**	1.0000E+30	2.8879E+14
F_{CEC-3}	SCA	4.8774E+04	5.6400E+04	6.9174E+04	6.4769E+03	2.6815E+05	3.4778E+05	4.2804E+05	4.8803E+04
	Q-SCA	3.0000E+02	**2.4412E+04**	1.0261E+05	4.4755E+04	1.3861E+04	**2.6757E+04**	3.7933E+04	7.6071E+03
F_{CEC-4}	SCA	1.4349E+03	2.1633E+03	2.6461E+03	4.1426E+02	2.4103E+04	3.1827E+04	4.1073E+04	4.2695E+03
	Q-SCA	4.0000E+02	**4.4543E+02**	4.6412E+02	2.7852E+01	5.4181E+02	**6.3089E+02**	7.0264E+02	3.4358E+01
F_{CEC-5}	SCA	7.7187E+02	8.1025E+02	8.7784E+02	3.5927E+01	1.8240E+03	1.8984E+03	1.9632E+03	3.7245E+01
	Q-SCA	5.1496E+02	**5.2090E+02**	5.2641E+02	4.1016E+00	6.3382E+02	**6.6992E+02**	7.0904E+02	1.8752E+01
F_{CEC-6}	SCA	6.4704E+02	6.5661E+02	6.6203E+02	5.2250E+00	6.8631E+02	6.9344E+02	7.0237E+02	4.4793E+00
	Q-SCA	6.0000E+02	**6.0005E+02**	6.0010E+02	2.9761E-02	6.0038E+02	**6.0300E+02**	6.0515E+02	1.3585E+00
F_{CEC-7}	SCA	1.1307E+03	1.1824E+03	1.2332E+03	3.9343E+01	3.3471E+03	3.5973E+03	3.7940E+03	1.2821E+02
	Q-SCA	7.4685E+02	**7.5457E+02**	7.6263E+02	5.6908E+00	1.0072E+03	**1.0740E+03**	1.1767E+03	4.4298E+01
F_{CEC-8}	SCA	1.0315E+03	1.0720E+03	1.0948E+03	2.1987E+01	2.1645E+03	2.2700E+03	2.3678E+03	6.2395E+01
	Q-SCA	8.1509E+02	**8.2194E+02**	8.2710E+02	4.1162E+00	9.2397E+02	**9.7249E+02**	1.0001E+03	1.9190E+01
F_{CEC-9}	SCA	5.6745E+03	6.7727E+03	8.3755E+03	9.7909E+02	6.1647E+04	7.2685E+04	8.3970E+04	6.8942E+03
	Q-SCA	9.0081E+02	**9.1462E+02**	9.4821E+02	1.5623E+01	1.2010E+03	**1.9624E+03**	2.8894E+03	4.4848E+02
F_{CEC-10}	SCA	8.0697E+03	8.3025E+03	8.6352E+03	2.1420E+02	3.0735E+04	3.1804E+04	3.2573E+04	5.5169E+02
	Q-SCA	2.6134E+03	**2.7727E+03**	2.9411E+03	1.3908E+02	1.3107E+04	**1.3996E+04**	1.4825E+04	4.5805E+02
F_{CEC-11}	SCA	2.4899E+03	2.6636E+03	2.9202E+03	1.6758E+02	7.8640E+04	9.6547E+04	1.1876E+05	1.1616E+04
	Q-SCA	1.1099E+03	**1.1434E+03**	1.1888E+03	3.2206E+01	1.9215E+03	**7.2769E+03**	5.4259E+04	1.5623E+04
F_{CEC-12}	SCA	1.2148E+09	1.5508E+09	2.1887E+09	3.2878E+08	4.7062E+10	6.3541E+10	8.7652E+10	9.8112E+09
	Q-SCA	2.0320E+03	**5.3407E+03**	1.9202E+04	5.0215E+03	1.4118E+05	**5.1230E+05**	8.6097E+05	2.0394E+05
F_{CEC-13}	SCA	2.9587E+08	6.2093E+08	9.1596E+08	1.9541E+08	7.9366E+09	1.1003E+10	1.3824E+10	1.6413E+09

Table 21 (continued)

ID	Method	30D statistical results				100D statistical results			
		Best	Worst	Mean	SD	Best	Worst	Mean	SD
F_{CEC-14}	Q-SCA	1.3280E+03	1.3978E+03	**1.3532E+03**	1.9940E+01	1.8865E+03	1.0581E+04	**5.0246E+03**	2.6688E+03
	SCA	9.0901E+04	4.7459E+05	2.0424E+05	1.1277E+05	1.3274E+07	7.5793E+07	3.7834E+07	1.8490E+07
F_{CEC-15}	Q-SCA	1.4221E+03	1.4449E+03	**1.4305E+03**	7.0897E+00	1.8739E+03	1.1892E+04	**3.9333E+03**	2.8825E+03
	SCA	4.2098E+06	4.5999E+07	2.3333E+07	1.2088E+07	2.2481E+09	6.2168E+09	3.5915E+09	1.1000E+09
F_{CEC-16}	Q-SCA	1.5086E+03	1.5907E+03	**1.5426E+03**	2.5953E+01	1.8640E+03	7.9142E+03	**3.1973E+03**	1.4826E+03
	SCA	3.2358E+03	4.0321E+03	3.7419E+03	2.3244E+02	1.2143E+04	1.4814E+04	1.3249E+04	6.5061E+02
F_{CEC-17}	Q-SCA	1.6232E+03	2.1989E+03	**1.9178E+03**	1.9802E+02	3.4914E+03	5.2569E+03	**4.5739E+03**	4.8915E+02
	SCA	2.3216E+03	3.0067E+03	2.5840E+03	2.2907E+02	9.9848E+03	1.8541E+04	1.2933E+04	2.1020E+03
F_{CEC-18}	Q-SCA	1.7312E+03	1.7654E+03	**1.7488E+03**	1.3386E+01	3.4563E+03	4.6499E+03	**4.0424E+03**	3.3743E+02
	SCA	1.2191E+06	1.2795E+07	6.5559E+06	3.7628E+06	3.6380E+07	9.1111E+07	6.0686E+07	1.8255E+07
F_{CEC-19}	Q-SCA	1.8268E+03	1.9728E+03	**1.8815E+03**	5.6046E+01	2.1571E+04	1.3046E+05	**5.1672E+04**	2.7505E+04
	SCA	9.4763E+06	5.0204E+07	3.3686E+07	1.1522E+07	2.1689E+09	4.6102E+09	2.8998E+09	6.1590E+08
F_{CEC-20}	Q-SCA	1.9082E+03	1.9619E+03	**1.9203E+03**	1.6841E+01	2.1627E+03	5.6568E+03	**3.1076E+03**	9.9462E+02
	SCA	2.4176E+03	2.9622E+03	2.6845E+03	1.6356E+02	6.7147E+03	8.0398E+03	7.4947E+03	3.7330E+02
F_{CEC-21}	Q-SCA	2.0330E+03	2.1856E+03	**2.0998E+03**	6.3689E+01	3.7533E+03	5.0352E+03	**4.5177E+03**	2.7525E+02
	SCA	2.5522E+03	2.6095E+03	2.5839E+03	2.2445E+01	3.7229E+03	4.0919E+03	3.9536E+03	8.0970E+01
F_{CEC-22}	Q-SCA	2.3091E+03	2.3218E+03	**2.3154E+03**	4.1403E+00	2.4563E+03	2.5236E+03	**2.4900E+03**	1.7606E+01
	SCA	8.6498E+03	1.0121E+04	9.7300E+03	4.2071E+02	3.2697E+04	3.4601E+04	3.3920E+04	4.8313E+02
F_{CEC-23}	Q-SCA	2.3000E+03	2.3000E+03	**2.3000E+03**	4.2874E-13	1.5428E+04	1.7404E+04	**1.6653E+04**	5.6876E+02
	SCA	2.9685E+03	3.0694E+03	3.0027E+03	3.0941E+01	4.7730E+03	5.1553E+03	4.9508E+03	1.0757E+02
F_{CEC-24}	Q-SCA	2.6474E+03	2.6751E+03	**2.6620E+03**	7.7862E+00	2.9464E+03	3.0098E+03	**2.9760E+03**	2.1779E+01
	SCA	3.1411E+03	3.2332E+03	3.1852E+03	2.7935E+01	6.0320E+03	6.8010E+03	6.4629E+03	1.9308E+02
F_{CEC-25}	Q-SCA	2.8256E+03	2.8413E+03	**2.8347E+03**	4.3033E+00	3.3875E+03	3.5192E+03	**3.4523E+03**	3.7318E+01
	SCA	3.2152E+03	3.5863E+03	3.3135E+03	1.1175E+02	1.4398E+04	2.1117E+04	1.6847E+04	1.6056E+03

Table 21 (continued)

ID	Method	30D statistical results				100D statistical results			
		Best	Mean	Worst	SD	Best	Mean	Worst	SD
	Q-SCA	2.8868E+03	**2.8869E+03**	2.8871E+03	1.0273E−01	3.1338E+03	**3.2650E+03**	3.3537E+03	6.4014E+01
F_{CEC-26}	SCA	6.6968E+03	7.1970E+03	7.4751E+03	2.6402E+02	3.2560E+04	3.5200E+04	3.8649E+04	1.7204E+03
	Q-SCA	3.5966E+03	**3.6963E+03**	3.7843E+03	5.7760E+01	6.9051E+03	**7.5998E+03**	8.1593E+03	3.3592E+02
F_{CEC-27}	SCA	3.3618E+03	3.4412E+03	3.5183E+03	4.9958E+01	6.7766E+03	7.3537E+03	8.0891E+03	3.6653E+02
	Q-SCA	3.1926E+03	**3.2025E+03**	3.2105E+03	6.3211E+00	3.3664E+03	**3.4458E+03**	3.5446E+03	4.2803E+01
F_{CEC-28}	SCA	3.6648E+03	4.0218E+03	4.2718E+03	1.9453E+02	1.7760E+04	2.0927E+04	2.3658E+04	1.7054E+03
	Q-SCA	3.1000E+03	**3.1361E+03**	3.2539E+03	5.9660E+01	3.3376E+03	**3.3864E+03**	3.4400E+03	3.0796E+01
F_{CEC-29}	SCA	4.3950E+03	4.9053E+03	5.2867E+03	2.5284E+02	1.5432E+04	1.7909E+04	2.1095E+04	1.5207E+03
	Q-SCA	3.3095E+03	**3.3592E+03**	3.3964E+03	2.6994E+01	4.6044E+03	**5.3800E+03**	6.1390E+03	4.2308E+02
F_{CEC-30}	SCA	4.5484E+03	4.9885E+03	5.3421E+03	2.8916E+02	4.7427E+09	7.2903E+09	1.1523E+10	1.4955E+09
	Q-SCA	3.3438E+03	**3.3793E+03**	3.4717E+03	3.4170E+01	5.9443E+03	**8.4790E+03**	1.8107E+04	3.8728E+03

Table 22 Comparison of Q-SCA against others in solving CEC 2017 benchmark problems under 30D

ID	Measures	PSO	FA	FFPSO	HPSOFF	HFPSO	HGSO	Q-SCA
F_{CEC-1}	Mean	5.5573E+13	3.3061E+14	1.1348E+15	6.0544E+13	1.4877E+13	5.39E+03	**1.0000E+02**
	SD	3.3893E+13	4.4031E+13	1.2767E+14	2.1716E+13	8.9773E+12	1.05E+03	1.4337E-08
F_{CEC-2}	Mean	3.9099E+36	1.8379E+43	1.0418E+61	3.1775E+38	4.2895E+34	NA	**2.2167E+02**
	SD	1.6788E+37	3.6984E+43	4.6498E+61	1.0461E+39	1.3632E+35	NA	3.1105E+01
F_{CEC-3}	Mean	1.3257E+09	2.1556E+09	6.7723E+13	1.3635E+09	1.3159E+09	5.64E+02	**2.4412E+04**
	SD	3.4533E+08	4.4961E+08	1.9830E+14	3.6125E+08	5.0828E+08	2.79E+02	4.4755E+04
F_{CEC-4}	Mean	9.0860E+06	6.8240E+07	4.4924E+08	1.2583E+07	6.8099E+06	4.64E+02	**4.4543E+02**
	SD	1.7332E+06	1.7720E+07	1.0769E+08	3.3792E+06	7.2977E+05	2.79E+00	2.7852E+01
F_{CEC-5}	Mean	8.0387E+06	8.7976E+06	1.1284E+07	8.1363E+06	7.4431E+06	6.01E+02	**5.2090E+02**
	SD	2.0464E+05	2.1895E+05	6.7520E+05	3.4695E+05	2.8310E+05	1.06E+01	4.1016E+00
F_{CEC-6}	Mean	6.5628E+06	6.7297E+06	7.2989E+06	6.7252E+06	6.5458E+06	6.04E+02	**6.0005E+02**
	SD	1.3442E+05	6.4935E+04	1.2755E+05	8.5569E+04	1.4862E+05	7.15E+00	2.9761E-02
F_{CEC-7}	Mean	1.1273E+07	1.7985E+07	2.9927E+07	1.1241E+07	1.0634E+07	8.20E+02	**7.5457E+02**
	SD	3.0590E+05	1.6329E+06	2.0330E+06	5.3149E+05	3.8217E+05	6.19E+01	5.6908E+00
F_{CEC-8}	Mean	1.0857E+07	1.1591E+07	1.3608E+07	1.0756E+07	1.0174E+07	**8.10E+02**	8.2194E+02
	SD	2.1442E+05	3.0228E+05	3.8288E+05	3.0109E+05	3.4938E+05	2.44E+01	4.1162E+00
F_{CEC-9}	Mean	6.6038E+07	1.3053E+08	3.0274E+08	9.0601E+07	9.0430E+07	1.64E+03	**9.1462E+02**
	SD	2.5645E+07	1.6887E+07	3.4821E+07	3.0384E+07	2.4238E+07	2.79E+02	1.5623E+01
F_{CEC-10}	Mean	9.2920E+07	9.4664E+07	1.0715E+08	8.9099E+07	7.4962E+07	5.64E+03	**2.7727E+03**
	SD	3.1888E+06	4.8533E+06	4.5678E+06	5.0987E+06	9.1080E+06	2.79E+02	1.3908E+02
F_{CEC-11}	Mean	3.6404E+07	1.1963E+08	3.4403E+08	4.5469E+07	2.2880E+07	**1.14E+03**	1.1434E+03
	SD	9.2311E+06	3.7161E+07	1.4283E+08	1.0526E+07	6.8090E+06	2.79E+01	3.2206E+01
F_{CEC-12}	Mean	6.4666E+12	3.0340E+13	2.7727E+14	3.3040E+12	5.9479E+11	4.88E+04	**5.3407E+03**
	SD	3.0524E+12	9.0341E+12	5.6036E+13	2.2407E+12	3.7560E+11	2.80E+04	5.0215E+03
F_{CEC-13}	Mean	1.9336E+12	1.1545E+13	2.7922E+14	2.7671E+11	5.0687E+10	6.52E+04	**1.3532E+03**
	SD	5.8827E+11	6.1763E+12	1.2029E+14	1.9963E+11	1.5852E+11	2.17E+03	1.9940E+01
F_{CEC-14}	Mean	1.1117E+10	1.8933E+10	5.6035E+11	1.1120E+10	6.6292E+09	1.51E+03	**1.4305E+03**

Table 22 (continued)

ID	Measures	PSO	FA	FFPSO	HPSOFF	HFPSO	HGSO	Q-SCA
	SD	7.8687E+09	1.8783E+10	3.5343E+11	1.1345E+10	9.6993E+09	2.52E+00	7.0897E+00
F_{CEC-15}	Mean	3.9118E+11	1.0725E+12	7.1411E+13	1.5676E+10	7.3512E+08	2.20E+03	**1.5426E+03**
	SD	1.8225E+11	6.1741E+11	3.1801E+13	1.1456E+10	2.3482E+08	4.27E+02	2.5953E+01
F_{CEC-16}	Mean	3.9514E+07	4.4164E+07	9.6352E+07	3.6383E+07	3.1951E+07	2.39E+03	**1.9178E+03**
	SD	4.2128E+06	2.9240E+06	2.6750E+07	4.0419E+06	4.2601E+06	3.35E+02	1.9802E+02
F_{CEC-17}	Mean	2.6556E+07	3.0044E+07	3.3347E+08	2.5254E+07	2.2984E+07	1.83E+03	**1.7488E+03**
	SD	2.3445E+06	1.8143E+06	4.9223E+08	2.5071E+06	2.1441E+06	1.21E+01	1.3386E+01
F_{CEC-18}	Mean	7.9043E+10	2.9281E+11	1.0554E+13	8.5589E+10	2.5984E+10	1.08E+04	**1.8815E+03**
	SD	6.2705E+10	1.9063E+11	7.7432E+12	9.7364E+10	2.1944E+10	5.98E+04	5.6046E+01
F_{CEC-19}	Mean	5.1700E+11	1.6613E+12	8.2628E+13	8.4341E+10	5.9487E+09	2.50E+03	**1.9203E+03**
	SD	2.9419E+11	6.2094E+11	3.4065E+13	9.5782E+10	5.7691E+09	2.89E+03	1.6841E+01
F_{CEC-20}	Mean	3.0177E+07	3.1126E+07	3.7975E+07	2.9083E+07	2.7685E+07	2.23E+03	**2.0998E+03**
	SD	2.1693E+06	1.3585E+06	2.1617E+06	2.5178E+06	1.8760E+06	1.19E+02	6.3689E+01
F_{CEC-21}	Mean	2.6009E+07	2.6405E+07	2.8880E+07	2.5982E+07	2.5087E+07	**2.21E+03**	2.3154E+03
	SD	2.2044E+05	2.6874E+05	6.5735E+05	5.7252E+05	2.9204E+05	1.86E+01	4.1403E+00
F_{CEC-22}	Mean	6.8425E+07	8.0003E+07	1.2117E+08	5.7433E+07	5.7966E+07	4.27E+03	**2.3000E+03**
	SD	3.8923E+07	1.8365E+07	3.2675E+06	3.1589E+07	3.2553E+07	8.29E+02	4.2874E-13
F_{CEC-23}	Mean	3.0819E+07	3.1542E+07	4.0749E+07	3.0969E+07	2.9597E+07	2.70E+03	**2.6620E+03**
	SD	1.0238E+06	4.9848E+05	2.8961E+06	9.0742E+05	7.4124E+05	6.78E+01	7.7862E+00
F_{CEC-24}	Mean	3.2139E+07	3.3056E+07	4.4529E+07	3.2719E+07	3.1742E+07	**2.62E+03**	2.8347E+03
	SD	7.7916E+05	5.1836E+05	2.5580E+06	9.8133E+05	1.0577E+06	8.27E+01	4.3033E+00
F_{CEC-25}	Mean	3.2145E+07	4.9480E+07	1.5958E+08	3.2320E+07	3.0623E+07	**2.81E+03**	2.8869E+03
	SD	6.7870E+05	4.0432E+06	3.1469E+07	1.4716E+06	7.0241E+05	3.32E+01	1.0273E-01
F_{CEC-26}	Mean	6.6461E+07	8.8041E+07	1.6163E+08	7.2386E+07	5.6590E+07	4.65E+03	**3.6963E+03**
	SD	1.1813E+07	6.0012E+06	1.5021E+07	1.6475E+07	1.5232E+07	1.02E+02	5.7760E+01

Table 22 (continued)

ID	Measures	PSO	FA	FFPSO	HPSOFF	HFPSO	HGSO	Q-SCA
F_{CEC-27}	Mean	3.3573E+07	3.6059E+07	5.8099E+07	3.4360E+07	3.3243E+07	**3.18E+03**	3.2025E+03
	SD	8.5636E+05	6.7467E+05	7.3079E+06	9.9726E+05	6.1360E+05	1.47E+02	6.3211E+00
F_{CEC-28}	Mean	3.6388E+07	5.6940E+07	1.3535E+08	3.7938E+07	3.3925E+07	3.41E+03	**3.1361E+03**
	SD	1.2870E+06	4.5852E+06	1.8714E+07	2.4230E+06	5.6285E+05	7.15E+01	5.9660E+01
F_{CEC-29}	Mean	4.9240E+07	5.7051E+07	9.3924E+08	4.8406E+07	4.4249E+07	**3.28E+03**	3.3592E+03
	SD	3.1984E+06	3.5852E+06	1.0965E+09	4.0875E+06	2.5350E+06	1.77E+02	2.6994E+01
F_{CEC-30}	Mean	4.8514E+11	1.7163E+12	4.1425E+13	2.5749E+11	4.6082E+10	9.06E+03	**3.3793E+03**
	SD	1.7476E+11	6.1288E+11	1.5608E+13	1.9168E+11	2.7679E+10	3.17E+03	3.4170E+01

Table 23 Comparison of Q-SCA against others in solving CEC 2017 benchmark problems under 100D

| ID	Measures	PSO	FA	MPSO	NFA	SFLA	HFPSO	Q-SCA
F_{CEC-1}	Mean	6.23E+14	4.37E+15	5.62E+14	1.81E+15	4.57E+15	4.45E+14	**5.0620E+03**
F_{CEC-2}	Mean	1.37E+145	3.45E+184	3.77E+143	9.94E+166	2.02E+182	1.05E+137	**1.0000E+30**
F_{CEC-3}	Mean	5.38E+09	7.98E+09	5.57E+09	5.65E+09	8.34E+09	5.02E+09	**2.6757E+04**
F_{CEC-4}	Mean	8.62E+07	1.50E+09	7.3788E+3	3.69E+08	1.60E+09	4.90E+07	**6.3089E+02**
F_{CEC-5}	Mean	1.84E+07	2.62E+07	1.8696E+3	2.01E+07	2.68E+07	1.66E+07	**6.6992E+02**
F_{CEC-6}	Mean	6.89E+06	7.39E+06	6.89E+06	7.03E+06	7.40E+06	6.83E+06	**6.0300E+02**
F_{CEC-7}	Mean	2.97E+07	1.01E+08	3.05E+07	4.04E+07	1.03E+08	2.66E+07	**1.0740E+03**
F_{CEC-8}	Mean	2.21E+07	3.00E+07	2.18E+07	2.44E+07	3.11E+07	2.02E+07	**9.7249E+02**
F_{CEC-9}	Mean	7.07E+08	15,882E+5	7.37E+08	7.63E+08	1.62E+09	7.02E+08	**1.9624E+03**
F_{CEC-10}	Mean	3.29E+08	33,460	3.25E+08	2.91E+08	3.35E+08	2.76E+08	**1.3996E+04**
F_{CEC-11}	Mean	1.58E+09	3.53E+09	1.51E+09	2.14E+09	3.50E+09	1.34E+09	**7.2769E+03**
F_{CEC-12}	Mean	1.55E+14	1.86E+15	1.30E+14	6.99E+14	2.27E+15	5.38E+13	**5.1230E+05**
F_{CEC-13}	Mean	2.31E+13	4.28E+14	1.51E+13	8.31E+13	4.98E+14	2.25E+12	**5.0246E+03**
F_{CEC-14}	Mean	2.29E+11	1.82E+12	2.09E+11	2.03E+11	1.94E+12	8.74E+10	**3.9333E+03**
F_{CEC-15}	Mean	6.30E+12	1.74E+14	4.99E+12	1.78E+13	2.36E+14	9.46E+10	**3.1973E+03**
F_{CEC-16}	Mean	1.19E+08	2.11E+08	1.16E+08	1.46E+08	2.47E+08	8.03E+07	**4.5739E+03**
F_{CEC-17}	Mean	8.91E+07	1.83E+10	9.30E+07	4.09E+08	4.66E+10	6.23E+07	**4.0424E+03**
F_{CEC-18}	Mean	3.49E+11	3.30E+12	3.11E+11	3.00E+11	3.58E+12	1.13E+11	**5.1672E+04**
F_{CEC-19}	Mean	7.43E+12	1.83E+14	5.01E+12	1.38E+13	2.35E+14	5.79E+11	**3.1076E+03**
F_{CEC-20}	Mean	7.86E+07	8.41E+07	7.85E+07	6.71E+07	8.43E+07	6.42E+07	**4.5177E+03**
F_{CEC-21}	Mean	3.85E+07	4.70E+07	3.87E+07	4.34E+07	4.89E+07	3.60E+07	**2.4900E+03**
F_{CEC-22}	Mean	3.52E+08	3.63E+08	3.53E+08	3.17E+08	3.65E+08	3.11E+08	**1.6653E+04**
F_{CEC-23}	Mean	4.79E+07	6.17E+07	4.70E+07	5.46E+07	6.89E+07	4.62E+07	**2.9760E+03**
F_{CEC-24}	Mean	5.77E+07	9.73E+07	5.22E+07	7.47E+07	1.12E+08	5.40E+07	**3.4523E+03**
F_{CEC-25}	Mean	9.18E+07	7.49E+08	8.57E+07	1.81E+08	8.01E+08	6.36E+07	**3.2650E+03**
F_{CEC-26}	Mean	2.36E+08	5.77E+08	2.30E+08	4.04E+08	6.57E+08	2.37E+08	**7.5998E+03**
F_{CEC-27}	Mean	4.32E+07	1.10E+08	4.15E+07	6.37E+07	1.24E+08	4.15E+07	**3.4458E+03**
F_{CEC-28}	Mean	1.14E+08	5.03E+08	1.09E+08	2.33E+08	5.25E+08	8.79E+07	**3.3864E+03**
F_{CEC-29}	Mean	1.30E+08	5.57E+09	1.39E+08	2.37E+08	8.39E+09	1.10E+08	**5.3800E+03**
F_{CEC-30}	Mean	1.86E+13	2.93E+14	1.68E+13	5.60E+13	4.02E+14	3.07E+12	**8.4790E+03**

$$Q_{Gi} - Q_{Di} - V_i \sum_{j=1}^{NB} V_j [G_{ij} \sin(\delta_{ij}) - B_{ij} \cos(\delta_{ij})] = 0 \, \forall i \in NB \tag{31}$$

where $\delta_{ij} = (\delta_i - \delta_j)$ denotes the difference of voltage angles between the buses i and j, respectively, V_i denotes the voltage magnitude at bus i, and NB defines the number of buses. Here, P_D, Q_D denote the active and reactive load demands; G_{ij}, B_{ij} are the transfer conductance and the susceptance among the buses i and j, respectively.

On the other hand, the inequality constraints on the present equipment of the power system as well as the restrictions invoked on lines and load buses to acquire system stability and security can be formulated as follows. Generator constraints:

$$P_{G_i}^{\min} \leq P_{G_i} \leq P_{G_i}^{\max}, \, i = 1, 2, ..., NG \tag{32}$$

$$V_{G_i}^{\min} \leq V_{G_i} \leq V_{G_i}^{\max}, \, i = 1, 2, ..., NG \tag{33}$$
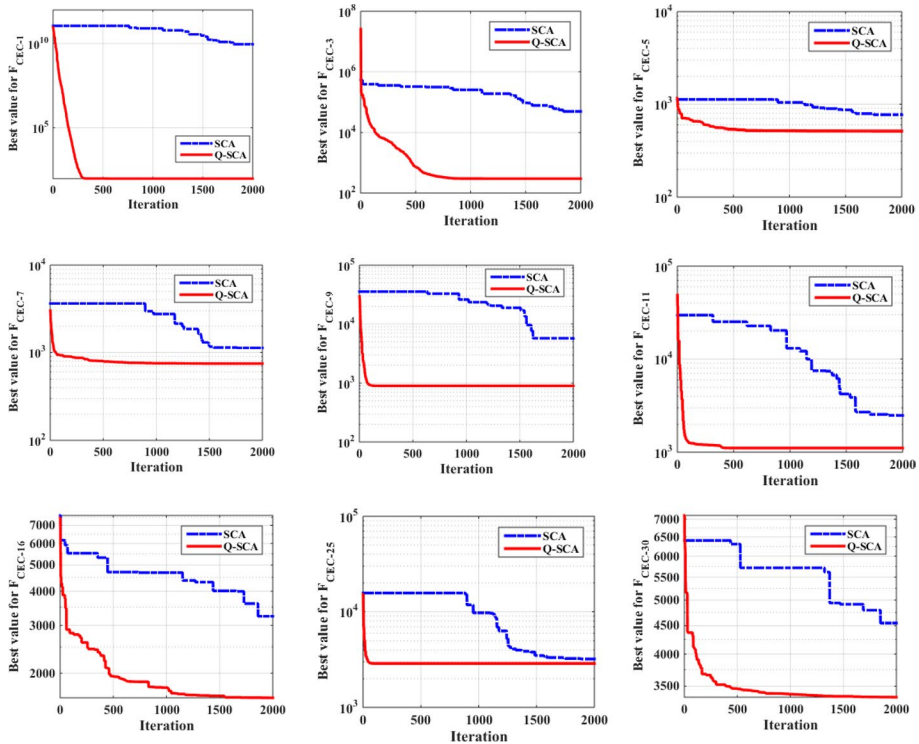
Fig. 6 Convergence curves of the proposed algorithms for 30D

$$Q_{G_i}^{\min} \leq Q_{G_i} \leq Q_{G_i}^{\max}, \ i = 1, 2, ..., NG \tag{34}$$

Shunt compensator constraints:

$$Q_{C_k}^{\min} \leq O_{C_k} \leq Q_{C_k}^{\max}, \ k = 1, 2, ..., NC \tag{35}$$

Transformer constraints:

$$T_j^{\min} \leq T_j \leq T_j^{\max}, \ j = 1, 2, ..., NT \tag{36}$$

Security constraints:

$$V_{L_h}^{\min} \leq V_{L_h} \leq V_{L_h}^{\max}, \ h = 1, 2, ..., NL \tag{37}$$

$$S_{l_p} \leq S_{l_p}^{\max}, \ p = 1, 2, ..., NTL \tag{38}$$

where *NT* is the number of transformers, *NC* is the number of shunt compensators, *NL* defines the number of load buses and *NTL* is the number of transmission lines.

The proposed SCA and Q-SCA have been implemented on the IEEE-30 bus system. Figure 7 exhibits the one-line diagram for the IEEE 30-bus test network that has following features (Bouchekara 2014; Biswas et al. 2018). The system involves 6 generator units located at buses 1, 2, 5, 8, 11, and 13 of the network, and 41 transmission lines. Also,

four transformers are located between the transmission lines 6–9, 6–10, 4–12, and 27–28, in voltage limits of (0.9–1.1). Reactive compensation sources in *MVAR* are placed at the 10, 12, 15, 17, 20, 21, 23, 24, and 29 load buses that are ranged from 0 to 5. Moreover, the voltages of *PV* buses are limited from 0.95 to 1.1 (*p.u.*). Operating ranges for the load buses are 0.95–1.05 (*p.u.*). The other characteristics of this system including the generator cost coefficients, bus data, and line data are detailed in (Bouchekara 2014).

The proposed Q-SCA and traditional SCA are applied to assess the total fuel cost to conduct the economic operation. The obtained results are recorded in Table 24. Based on these results, it can be observed that the achieved the control variables by Q-SCA can produce the lowest fuel cost than the SCA as well as the other methods. The convergence curve of optimizing the fuel cost is depicted in Fig. 8. Furthermore, the robustness and superiority of the proposed Q-SCA is further proved by the comparisons with other competing methods taken from the literature including ECHT-DE (Daryani et al. 2016), SP-DE (Daryani et al. 2016), SF-DE (Daryani et al. 2016), AGSO (Mohamed et al. 2017), MSA (Warid 2020), TLBO (Kumar and Premalatha 2015), ARCBBO (Pulluri et al. 2018), and SKH [58]. In this context, the results for these methods are provided in Table 25. Based on reported results, it is realized that the Q-SCA outperforms the other state-of-the-art methods as it can provide the lowest value for the fuel cost.

Thus from comprehensive experiments and comparisons on various optimization tasks including general systems of nonlinear equations, CEC 2017 benchmark suits, and realistic OPD problem, it can be concluded that the Q-SCA is promising approach and more fruitful than other existing metaheuristic methods.
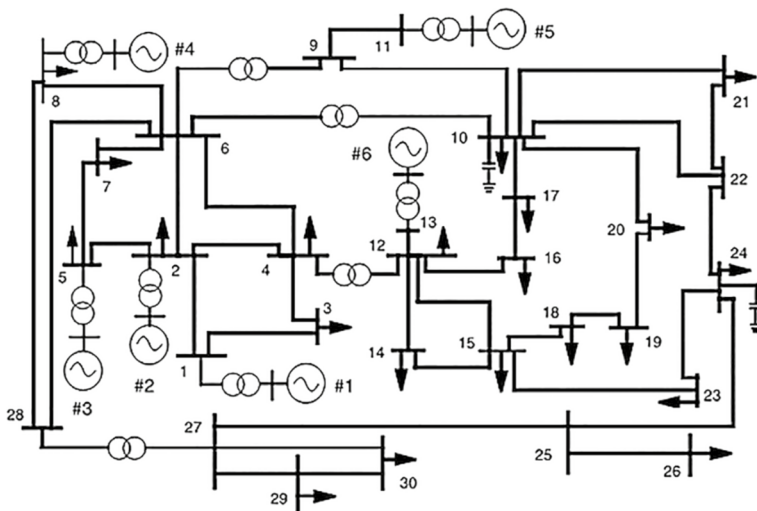
.

Fig. 7 IEEE 30-bus system

Table 24 Results of control variables obtained by SCA and Q-SCA for IEEE-30 bus

Variables	Boundaries		SCA	Q-SCA	Variables	Boundaries		SCA	Q-SCA
$P_{g1}(MW)$	50	200	178.7865	177.1535	$Q_{C12}(MVAr)$	0	5	3.8365	1.4620
$P_{g2}(MW)$	20	80	49.5346	48.7186	$Q_{C15}(MVAr)$	0	5	4.9395	4.0673
$P_{g5}(MW)$	15	50	18.6721	21.3834	$Q_{C17}(MVAr)$	0	5	4.5193	4.9965
$P_{g8}(MW)$	10	35	21.5638	21.2275	$Q_{C20}(MVAr)$	0	5	4.6674	4.1520
$P_{g11}(MW)$	10	30	12.8498	11.9185	$Q_{C21}(MVAr)$	0	5	4.9904	4.9793
$P_{g13}(MW)$	12	40	12.0182	12.0021	$Q_{C23}(MVAr)$	0	5	4.9912	2.8590
$V_1(P.U.)$	0.95	1.1	1.0409	1.0833	$Q_{C24}(MVAr)$	0	5	0.0227	4.9838
$V_2(P.U.)$	0.95	1.1	1.0152	1.0640	$Q_{C29}(MVAr)$	0	5	3.4244	2.3205
$V_5(P.U.)$	0.95	1.1	0.9742	1.0328	$T_{11}(P.U.)$	0.9	1.1	1.0064	1.0521
$V_8(P.U.)$	0.95	1.1	0.9940	1.0376	$T_{12}(P.U.)$	0.9	1.1	0.9166	0.9191
$V_{11}(P.U.)$	0.95	1.1	1.0996	1.0883	$T_{15}(P.U.)$	0.9	1.1	0.9588	0.9664
$V_{13}(P.U.)$	0.95	1.1	1.0521	1.0448	$T_{36}(P.U.)$	0.9	1.1	0.9394	0.9726
$Q_{C10}(MVAr)$	0	5	4.0474	0.5406	Fuel cost (\$/h)			803.8317	800.41229

Fig. 8 Convergence curve for fuel cost by the proposed algorithms
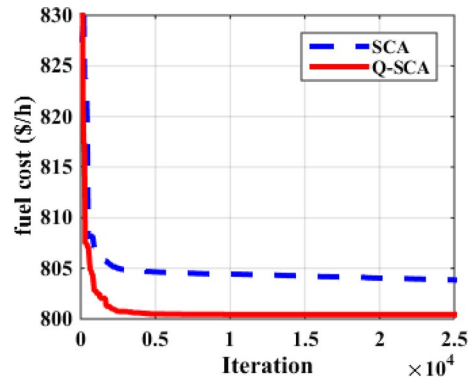
Table 25 The optimum fuel cost obtained by the proposed Q-SCA and other methods for IEEE 30-bus system

Algorithm	Fuel cost	Algorithm	Fuel cost
MSA	800.5099	ARCBBO	800.5159
ECHT-DE	800.4148	SKH	800.5141
SF-DE	800.4131	TLBO	800.4604
SP-DE	800.4293	SCA	803.8317
AGSO	801.75	Q-SCA	800.41229

5 Conclusions

In this paper, to overcome the unique SCA's problem of being converged to local optima and improving its exploration also exploitation tendencies, it is modified to redistribute search agents according to QLS concept. The experimental results show that the hybridization of SCA with QLS accelerates the convergence performance behavior and enhance the

quality of the solutions. The Q-SCA and SCA, L-QPSO, QPSO, GRAV, ITHS algorithm and other literature studies are compared based on the quality of the solutions in resolving 12 nonlinear systems of equations as test cases, two electrical applications, CEC 2017 benchmark and realistic OPD problem. The results verify that Q-SCA has better searching quality.

Careful evidences will discover the following benefits of the proposed Q-SCA.

(1) It can efficiently enrich the exploratory capabilities of SCA phase by introducing new updating rule based of the destination solution.
(2) The modification on the the parameter r_1 can assist SCA phase to be more exploitive in last iterations.
(3) QLS phase can assist the algorithm for regaining a correct balance amongst exploration and exploitation tendencies.
(4) It surpasses the other algorithms in terms of optimality.
(5) It can find the global optimal solution for the benchmark systems.
(6) It can deal with expensive large-scale tasks such as CEC 2017 datasets.
(7) It can improve the convergence and boost the performance through saving the computational time.
(8) The obtained results confirm that Q-SCA is capable to efficaciously escape from local optima in search space throughout optimization.

The future work will be concentrated on four trends: (i) developing new algorithms for these tasks; (ii) solving several complex systems in engineering and science; (iii) solving interval nonlinear system of equations; (iv) solving rough interval nonlinear system of equations. To end with, we hope that this work will motivate other researchers who are working on new metaheuristic algorithms and optimization of electrical power stations.

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