J Glob Optim
DOI 10.1007/s10898-016-0444-2

A diverse human learning optimization algorithm

Ling Wang[1] · **Lu An**[1] · **Jiaxing Pi**[2] · **Minrui Fei**[1] ·
Panos M. Pardalos[2]

Received: 16 April 2015 / Accepted: 18 May 2016

Abstract Human Learning Optimization is a simple but efficient meta-heuristic algorithm in which three learning operators, i.e. the random learning operator, the individual learning operator, and the social learning operator, are developed to efficiently search the optimal solution by imitating the learning mechanisms of human beings. However, HLO assumes that all the individuals possess the same learning ability, which is not true in a real human population as the IQ scores of humans, one of the most important indices of the learning ability of humans, follow Gaussian distribution and increase with the development of society and technology. Inspired by this fact, this paper proposes a Diverse Human Learning Optimization algorithm (DHLO), into which the Gaussian distribution and dynamic adjusting strategy are introduced. By adopting a set of Gaussian distributed parameter values instead of a constant to diversify the learning abilities of DHLO, the robustness of the algorithm is strengthened. In addition, by cooperating with the dynamic updating operation, DHLO can adjust to better parameter values and consequently enhances the global search ability of the algorithm. Finally, DHLO is applied to tackle the CEC05 benchmark functions as well as knapsack problems, and its performance is compared with the standard HLO as well as the other eight meta-heuristics, i.e. the Binary Differential Evolution, Simplified Binary Artificial Fish Swarm Algorithm, Adaptive Binary Harmony Search, Binary Gravitational Search Algorithms, Binary Bat Algorithms, Binary Artificial Bee Colony, Bi-Velocity Discrete Particle Swarm Optimization, and Modified Binary Particle Swarm Optimization. The experimental results show that the presented DHLO outperforms the other algorithms in terms of search accuracy and scalability.

Keywords Human learning optimization · Gaussian distribution · Meta-heuristic · Global optimization · Computational experiments

✉ Ling Wang
wangling@shu.edu.cn

[1] Shanghai Key Laboratory of Power Station Automation Technology, School of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200072, China

[2] Center for Applied Optimization, Department of Industrial and Systems Engineering, University of Florida, Gainesville, FL 32611, USA

Published online: 24 May 2016

1 Introduction

Optimization problems widely exist in the real world, and therefore methods used to solve these problems have been being a hot topic. However, optimization problems are becoming more and more complicated with the development of science and technology, and traditional gradient-based methods are inefficient and inconvenient for such problems as they require substantial gradient information, depend on a well-define starting point, and need a large amount of enumeration memory. On the other hand, meta-heuristic algorithms, such as Genetic Algorithms (GAs) [1], Differential Evolution [2], Particle Swarm Optimization (PSO) [3], and Ant Colony Optimization (ACO) [4], have shown better results on various complex problems such as feature selection [5], the design of controllers [6], and the node placement of wireless sensor networks [7]. Encouraged by the achievements of meta-heuristics, more and more researchers devote themselves into the study of the design and application of meta-heuristics.

The well-known No Free Lunch theorem states that any two optimization algorithms are equivalent when their performance is averaged across all possible problems. It hints that some algorithm can be better than the others on a class of problems, which has been demonstrated by previous works. Thus developing new meta-heuristics for solving various problems more efficiently and effectively has drawn more and more attention, and because of the great success of GAs, PSO, and ACO, which are inspired by biological systems, exploring biologically inspired meta-heuristics has been one of hottest topics in evolutionary computation community. During the last decade, varieties of biosystem-based meta-heuristics, such as Artificial Fish Swarm Algorithms (AFSA) [8], Artificial Bee Colony Optimization (ABC) [9], Bat Algorithms (BA) [10], Hunting Search Algorithms [11], Harmony Search (HS) [12], Fruit Fly Optimization Algorithms (FOA) [13], Firefly Algorithms [14], Shuffled Frog-leaping Algorithms [15], and Cuckoo Search [16], have been developed and applied to different problems. As is known to all, human being is the smartest creature in the world because of the most powerful learning ability, and humans are able to tackle a large number of complicated problems that other living beings, such as birds and ants, cannot solve. Therefore, it is natural to presume that the meta-heuristic based on the learning mechanisms of human being may have advantages over other biological systems based algorithms on optimization problems in our daily life. Actually, many human learning activities are similar to the search process of meta-heuristics. For example, people repeatedly study and evaluate the performance of each practice to update their experience for guiding the following study to master a new skill better, which is analog to meta-heuristics iteratively yielding new candidate solutions and calculating the corresponding fitness values for adjusting their following search. Motived by this idea, Wang et al. [17] presented a new meta-heuristic algorithm called Human Learning Optimization (HLO) recently. However, HLO assumes that all the individuals have the same learning ability, which is not true. Herrnstein presented in his famous book “The bell curve” that Intelligence Quotient (IQ) scores followed Gaussian distribution [18], and the previous research results also showed that IQ test scores had significantly increased and would continue to rise with the development of society and technology [19,20]. Inspired by these facts, this paper proposes an improved HLO algorithm, called Diverse Human Learning Optimization (DHLO), in which the learning ability of individuals follows a Gaussian distribution and dynamically adjusts to improve the search ability of the algorithm.

The rest of the paper is organized as follows. Section 2 presents the concept, operators, and implementation of DHLO in details. Then the parameter study of DHLO is performed and discussed in Sect. 3. Section 4 verifies the performance of DHLO on benchmark functions as well as knapsack problems, and the results are compared with the standard HLO as well as the other eight meta-heuristic algorithms. Finally, conclusions are remarked in Sect. 5.

2 Diverse human learning optimization

Human learning process is extremely complicated of which the study is the part of neuropsychology, educational psychology, learning theory, and pedagogy. For the ease of implementation, DHLO, like HLO [17], uses three learning operators, i.e. the random learning operator, the individual learning operator, and the social learning operator, to update the population and search out the optimal solution, which emulates the behaviors of random learning, individual learning, and social learning in human learning activities. For example, when a person learns to play basketball, he or she may study new skills randomly because of lack of prior knowledge (random learning), learn from his or her former experience (individual learning), and find useful methods from his or her coach or related books (social learning).

DHLO adopts the binary-coding framework, that is, the individual of DHLO is represented as a binary string, in which each bit of solutions is analog to a basic element of the knowledge that humans need to learn. Assuming that there is no prior-knowledge of the problems at the beginning, an individual is initialized with “0” or “1” randomly as Eq. (1)

$$x_i = \left[x_{i1}\ x_{i2}\ \ldots\ x_{ij}\ \ldots\ x_{iN}\right],\ x_{ij} \in \{0, 1\}, 1 \le i \le M, 1 \le j \le N \tag{1}$$

where x_{ij} is the *jth* bit of the *ith* individual, and M and N denote the number of individuals in the population and the length of solutions, respectively.

2.1 Random learning operator

As Cziko [21] presented that human learning was the result of random variation and universal selection, randomness always exists in the process of human learning. At the beginning of learning, humans usually learn by their random acts since there is no prior knowledge of a new problem. With the proceeding of studying, people still perform random learning because of various factors such as forgetting, disturbance, and knowing partial knowledge about problems. Besides, human being keeps exploring new strategies to learn better in which random learning is unavoidable. DHLO performs the random learning operator to mimic these phenomena as Eq. (2),

$$x_{ij} = RE(0, 1) = \begin{cases} 0, & rand \le 0.5 \\ 1, & else \end{cases} \tag{2}$$

where *rand* is a stochastic number between 0 and 1.

2.2 Individual learning operator

Individual learning is the ability of humans to gain knowledge through the individual reflection on external stimuli [22]. People memorize the useful experience during their study and use it when they face the same or similar problems and therefore they can avoid mistakes and learn more efficiently. To simulate this learning behavior, each individual in DHLO stores its personal best solutions in the individual knowledge database (IKD) represented as Eq. (3)

$$IKD_i = \begin{bmatrix} ikd_{i1} \\ ikd_{i2} \\ \vdots \\ ikd_{ir} \\ \vdots \\ ikd_{iP} \end{bmatrix} = \begin{bmatrix} ik_{i11} & ik_{i12} & \cdots & ik_{i1j} & \cdots & ik_{i1N} \\ ik_{i21} & ik_{i22} & \cdots & ik_{i2j} & \cdots & ik_{i2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ ik_{ir1} & ik_{ir2} & \cdots & ik_{irj} & \cdots & ik_{irN} \\ \vdots & \vdots & & \vdots & & \vdots \\ ik_{iP1} & ik_{iP2} & \cdots & ik_{iPj} & \cdots & ik_{iPN} \end{bmatrix}, 1 \le r \le P \tag{3}$$

where IKD_i denotes the individual knowledge database of person i, ikd_{ir} stands for the *rth* best solution of person i, and P is the size of IKDs. When DHLO executes the individual learning operator, it chooses a random solution in the IKD and then copies the corresponding value as Eq. (4),

$$x_{ij} = ik_{irj} \tag{4}$$

where r is a random integer.

2.3 Social learning operator

However, when problems become extremely complicated, it would be impossible or very time-consuming for a single person to solve. In a social environment, humans directly or indirectly transfer their knowledge and therefore improve the efficiency and effectiveness of study by social learning [23]. The previous works demonstrate that population-based meta-heuristics have an advantage on complicated problems because of the sharing of knowledge among individuals. Therefore, social learning is simulated in DHLO to enhance the search ability of the algorithm and the best solutions found by all the individuals are archived in the social knowledge database (SKD) as Eq. (5) for sharing experience in the population,

$$SKD = \begin{bmatrix} skd_1 \\ skd_2 \\ \vdots \\ skd_s \\ \vdots \\ skd_Q \end{bmatrix} = \begin{bmatrix} sk_{11} & sk_{12} & \cdots & sk_{1j} & \cdots & sk_{1N} \\ sk_{21} & sk_{22} & \cdots & sk_{2j} & \cdots & sk_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ sk_{s1} & sk_{s2} & \cdots & sk_{sj} & \cdots & sk_{sN} \\ \vdots & \vdots & & \vdots & & \vdots \\ sk_{Q1} & sk_{Q2} & \cdots & sk_{Qj} & \cdots & sk_{QN} \end{bmatrix}, 1 \le s \le Q \tag{5}$$

where skd_s denotes the *sth* solution in the SKD and Q is size of the SKD. Based on the knowledge in the SKD, DHLO performs the social learning operator to generate a new solution as Eq. (6),

$$x_{ij} = sk_{sj} \tag{6}$$

where $s$is a random integer.

2.4 Gaussian-distribution and dynamic updating of the learning ability

DHLO, as well as HLO, generates new solutions by performing the random learning operator, the social learning operator, and the individual learning operator. In general, the implementation of these three learning operators can be formulated as Eq. (7),

$$x_{ij} = \begin{cases} RE(0,1), & 0 \le rand \le p_r \\ ik_{irj}, & pr < rand \le p_i \\ sk_{sj}, & else \end{cases} \tag{7}$$

where p_r and p_i are two control parameters used to determine the probabilities of running the operators. Specifically, p_r determines the probability of random learning while $(p_i - p_r)$ and $(1 - p_i)$ are the rates of individual learning and social learning, respectively. In the standard HLO these two parameters, i.e. p_r and p_i, are both set as constants and the recommended values are $5/M$ and $0.85+2/M$ where M is the length of solutions. Therefore, all the individuals of HLO have the same learning capabilities, which is not true in a real human population. For instance, the IQ scores of humans [24], as well as some other factors influencing human learning, follow Gaussian distribution, which results in different learning ability of people, and consequently the scores on an exam usually follow an approximately Gaussian distribution. In addition, Flynn points out that IQ test scores would rise. Inspired by these facts, the Gaussian-distributed learning ability and dynamic adjusting strategy are developed in the DHLO.

Taking a deep insight into the learning operators of HLO, it is obvious that the random learning operator performs a random search in which none of knowledge is taken into account. Considering that only two values, i.e. 0 and 1, exist in binary space, the function of the random learning operator is similar to the mutation operator of Genetic Algorithms. Thus it is sensible that the suggested value of p_r is very small since the contribution of the random learning operation is to keep the diversity of the population and perform a local search, otherwise the random search may impair the learning mechanisms of HLO and significantly spoils the performance of the algorithm. Compared with the random learning operator, the individual learning operator and the social learning operator are two main learning operators that update the population according to the individual experience and the knowledge of the population, respectively. Therefore, p_i plays a very important role since it directly determines the abilities of individual learning and social learning. For example, if $p_i = 1$, HLO would lose the ability of social learning and consequently the efficiency and effectiveness of the algorithm is ruined since the advantage from the knowledge sharing does not exist. On the other hand, if $p_i = p_r$, which means that individual learning is abandoned, HLO would be degraded to a local search around the global best solution. Unfortunately, the optimal p_i depends on problems and thus it is almost impossible to set the optimal value without prior knowledge. To tackle this problem, the Gaussian distribution and the dynamic updating of the parameter p_i are introduced in DHLO to tune p_i and improve the search ability.

First, when initializing the algorithm, each individual of DHLO is given a different personal p_i instead of the same one for all the individuals in HLO, which follows Gaussian distribution as Eq. (8),

$$p_i \sim N(\mu, \sigma^2) \tag{8}$$

where μ and σ are the mean and standard deviation, respectively. The advantages of using Gaussian distribution are: (1) a majority of values of p_i are yielded in the range determined by μ and σ, and therefore a fair performance of DHLO can be guaranteed; (2) compared with HLO using the only one value of p_i, the robustness of DHLO is enhanced by searching with various reasonable p_i values; (3) the difference of the performance of individuals will be shown due to using different p_i values, which can be used for dynamically updating p_i to improve the search ability further.

Then the dynamic updating of p_i is executed every DG generations where DG is a predefined constant. When performing dynamic updating, μ, i.e. the mean of the Gaussian distribution, is set as Eq. (9)

$$\mu = p_i^* \tag{9}$$

where p_i^* is the p_i value of the individual with the best fitness. The p_i value of each individual is adjusted as Eq. (10) if the global optima found by DHLO is updated in the latest DG generations,

$$p_{i,j} = p_{i,j} + rand \times (p_i^* - p_{i,j}) \tag{10}$$

where $p_{i,j}$ is the p_i value of the jth individual, and therefore the p_i of all individuals moves to a better value to improve the performance in the following search. Otherwise, all the values of p_i are re-initialized with σ and the updated μ.

2.5 Updating of the IKD and the SKD

After a new population is generated, the fitness of candidates is calculated according to the fitness function and used to update the IKDs and the SKD, which is analog to the process that humans evaluate their performance through practicing to refresh their knowledge for further studying. For the updating of the IKD, if the number of solutions in the current IKD is less than P, i.e. the pre-defined size of the IKD, the new candidate will be stored in the IKD no matter of its fitness. Otherwise the new candidate is reserved and used to replace the solution with the worst fitness in the IKD only when it has a better fitness. For the updating of the SKD, the same strategies as the updating of the IKD are applied. However, DHLO only permits to replace one solution in the SKD in each generation to keep the diversity and avoid the premature of the algorithm.

2.6 Implementation of DHLO

In summary, the procedure of DHLO can be concluded as follows:

Step 1: initialize the population randomly, yield the initial values of p_i for each individual following Gaussian distribution, and set the other parameters of DHLO such as p_r and the maximal generation;
Step 2: calculate the fitness of initial individuals and initialize the IKDs and SKD;
Step 3: yield new candidates by performing the three learning operators as Eq. (7);
Step 4: compute the fitness of all the new solutions;
Step 5: update the IKDs and SKD according to the updating rules;
Step 6: for every DG generations, set the mean μ of Gaussian distribution as Eq. (9), and then adjust the value of p_i of each individual as Eq. (10) if the global optima is updated; otherwise re-initialize the p_i of each individual with the updated μ;
Step 7: if the termination conditions are met, output the best solution; otherwise go to step 3.

3 Parameter analysis of DHLO

To apply the strategies of Gaussian distribution and dynamic updating efficiently, a parameter study on these two kinds of operation were carried out, and two functions, i.e. F2 and F9, chosen from the CEC05 benchmark functions [25] were adopted for testing. The characteristics of these two functions as well as the other 13 functions used as benchmarks for evaluating the DHLO in the next section are listed in Table 1.

Gaussian distribution includes two parameters, i.e. the mean μ and the standard deviation σ. In DHLO μ is dynamically adjusted by the algorithm, thus only the standard deviation σ need be manually set. As known to all, about 99.7 % of random numbers generated by Gaussian distribution are within three standard deviations of the mean, i.e. $[\mu - 3\sigma, \mu + 3\sigma]$. Therefore, 3σ was adopted as a variable for simplification in the parameter study. As for

Table 1 The CEC05 benchmark functions

	Name	Type	Dimension
F1	Shifted Sphere Function	Unimodal	2/30
F2	Shifted Schwefel's Problem 1.2	Unimodal	2/30
F3	Shifted Rotated High Conditioned Elliptic Function	Unimodal	2/30
F4	Shifted Schwefel's Problem 1.2 with Noise in Fitness	Unimodal	2/30
F5	Schwefel's Problem 2.6 with Global Optimum on Bounds	Unimodal	2/30
F6	Shifted Rosenbrock's Function	Multimodal	2/30
F7	Shifted Rotated Griewank's Function without Bounds	Multimodal	2/30
F8	Shifted Rotated Ackley's Function with Global Optimum on Bounds	Multimodal	2/30
F9	Shifted Rastrigin's Function	Multimodal	2/30
F10	Shifted Rotated Rastrigin's Function	Multimodal	2/30
F11	Shifted Rotated Weierstrass Function	Multimodal	2/30
F12	Schwefel's Problem 2.13	Multimodal	2/30
F13	Expanded Extended Griewank's plus Rosenbrock's Function	Multimodal	2/30
F14	Shifted Rotated Expanded Scaffer's F6	Multimodal	2/30
F15	Hybrid Composition Function	Hybrid	2/30

the dynamic updating strategy, the variable is DG. A set of 3σ and DG, i.e. $\{0.005, 0.01, 0.02, 0.05, 0.08, 0.1, 0.15\}$ and $\{100, 200, 500, 1000, 1500, 3000\}$, respectively, were used to solve the 2-dimensional and 30-dimeinsional F2 and F9. For the 2-dimensional functions, the population size was set to 50 and the maximal generation was 3000. For the 30-dimensional functions, the population size and the maximal generation number were increased to 100 and 6000, respectively. Each variable was encoded by 30 bits, and each function ran 50 times independently. The results, including the best fitness value (BFV), the mean fitness value (MFV), and the standard deviation (STD), are given in Tables 2 and 3. The best results of the algorithms are marked with bold-face in the corresponding tables.

Tables 2 and 3 show that the optimal 3σ and DG are dependent on problems and these two parameters also interact with each other. However, with a very big 3σ, for instance, a value bigger than 0.1, p_i will spread in a wide range and greatly deviate from the recommended value, which consequently spoils the exploration-exploitation trade-off of the algorithm. On the other hand, a very small 3σ is also improper since it reduces or even vanishes the advantage from Gaussian distribution. As for the DG, a large DG decreases the influence of dynamic updating since it reduces the chance of performing the operation, while a small DG can enhance the function of dynamic updating and improve the performance of the algorithm. For example, DHLO obtains better result on F2 with $3\sigma = 0.005$ and DG=100 than any other results yielded with $3\sigma = 0.005$ and DG > 100. However, setting a very small DG is risky as DHLO is very sensitive to 3σ and the algorithm is likely to be unstable in this situation. Due to the randomness of DHLO, the best solutions found during the search process might be far away from the real optimal solution, thus it is highly possible that the temporary best solutions might mislead the dynamic updating operation especially when a small DG is applied. Consequently, the performance of DHLO might become worse, which can be observed from the data of 30-dimensional F2.

Table 2 The results of parameter study on F2

3σ	DG	2-D			30-D		
		BFV	MFV	STD	BFV	MFV	STD
0.005	100	0.0000E+00	8.2991E−12	2.4389E−11	3.1227E+02	9.5391E+02	3.0919E+02
0.005	200	0.0000E+00	9.7771E−12	2.6459E−11	3.0651E+02	9.2786E+02	3.3428E+02
0.005	500	0.0000E+00	8.9244E−12	2.4564E−11	3.5799E+02	8.9014E+02	3.4509E+02
0.005	1000	0.0000E+00	9.9817E−12	2.6474E−11	1.5655E+02	9.2164E+02	3.8042E+02
0.005	1500	0.0000E+00	1.1767E−11	2.8117E−11	2.6805E+02	8.6314E+02	3.3858E+02
0.005	3000	0.0000E+00	5.7821E−08	4.0882E−07	2.9691E+02	8.5892E+02	3.6294E+02
0.01	100	0.0000E+00	4.7255E−12	2.5748E−11	3.7850E+02	9.2715E+02	3.3779E+02
0.01	200	0.0000E+00	4.6043E−12	1.6784E−11	4.4430E+02	9.3711E+02	3.7081E+02
0.01	**500**	**0.0000E+00**	**3.7187E−12**	**4.0877E−12**	3.9510E+02	9.1497E+02	3.5211E+02
0.01	1000	0.0000E+00	7.7307E−12	2.2537E−11	2.7718E+02	9.1341E+02	3.7205E+02
0.01	1500	0.0000E+00	3.7397E−10	2.5746E−09	2.4630E+02	9.2544E+02	3.7137E+02
0.01	3000	0.0000E+00	6.4233E−12	2.2093E−11	4.0178E+02	9.5819E+02	3.2382E+02
0.02	100	0.0000E+00	2.2283E−11	6.7594E−11	2.7716E+02	9.5646E+02	3.2335E+02
0.02	200	0.0000E+00	3.6954E−10	2.5752E−09	3.4924E+02	9.8575E+02	4.5684E+02
0.02	500	0.0000E+00	1.1653E−11	2.8188E−11	2.3698E+02	9.3180E+02	3.0237E+02

Table 2 continued

3σ	DG	2-D			30-D		
		BFV	MFV	STD	BFV	MFV	STD
0.02	**1000**	0.0000E+00	5.0022E−12	1.9343E−11	**2.0479E+02**	**8.5827E+02**	**3.3557E+02**
0.02	1500	0.0000E+00	1.6545E−09	9.3683E−09	2.8974E+02	9.8061E+02	3.4025E+02
0.02	3000	0.0000E+00	5.7827E−08	4.0882E−07	1.3463E+02	9.8287E+02	4.2442E+02
0.05	100	0.0000E+00	5.9104E−08	4.0874E−07	3.0758E+02	1.0351E+03	3.7101E+02
0.05	200	0.0000E+00	3.8995E−11	2.0276E−10	3.9330E+02	1.0197E+03	3.6646E+02
0.05	500	0.0000E+00	7.4607E−10	3.6024E−09	2.5536E+02	9.3131E+02	3.7072E+02
0.05	1000	0.0000E+00	3.6835E−11	2.0289E−10	2.8407E+02	9.1746E+02	3.5896E+02
0.05	1500	0.0000E+00	6.4801E−12	2.2114E−11	3.3599E+02	8.9742E+02	4.4076E+02
0.05	3000	0.0000E+00	6.3538E−12	1.5954E−11	2.6431E+02	8.6698E+02	3.4742E+02
0.1	100	0.0000E+00	1.7421E−07	6.9332E−07	4.0615E+02	1.2374E+03	5.3648E+02
0.1	200	0.0000E+00	5.8222E−08	4.0877E−07	4.3629E+02	1.2372E+03	4.6615E+02
0.1	500	0.0000E+00	1.7346E−07	6.9350E−07	5.4905E+02	1.0451E+03	5.2930E+02
0.1	1000	0.0000E+00	7.3328E−12	2.2320E−11	3.7816E+02	9.8642E+02	3.7427E+02
0.1	1500	0.0000E+00	8.1286E−12	2.4415E−11	2.9066E+02	1.0661E+03	3.7977E+02
0.1	3000	0.0000E+00	3.9523E−10	2.5794E−09	3.9709E+02	9.3255E+02	3.2765E+02
0.15	100	0.0000E+00	6.9575E−07	1.2462E−06	6.6679E+02	1.6108E+03	9.1353E+02
0.15	200	0.0000E+00	1.8281E−07	6.9304E−07	4.8301E+02	1.3355E+03	5.5684E+02
0.15	500	0.0000E+00	2.3384E−07	7.9155E−07	3.2138E+02	1.1938E+03	4.1749E+02
0.15	1000	0.0000E+00	2.0287E−10	1.1659E−09	5.4464E+02	1.1521E+03	4.2742E+02
0.15	1500	0.0000E+00	1.6577E−09	9.3674E−09	4.1114E+02	9.8562E+02	3.0481E+02
0.15	3000	0.0000E+00	3.9267E−10	2.5750E−09	2.4549E+02	1.1098E+03	4.4747E+02

Table 3 The results of parameter study on F9

3σ	DG	2-D			30-D		
		BFV	MFV	STD	BFV	MFV	STD
0.005	100	1.0232E−12	2.3012E−12	3.4452E−13	8.8007E−01	1.4364E+01	1.1425E+01
0.005	200	1.0232E−12	2.5580E−12	7.5728E−12	3.5472E+00	1.5983E+01	1.1951E+01
0.005	500	1.0232E−12	2.3874E−12	7.5225E−12	2.1242E+00	1.9027E+01	1.2266E+01
0.005	1000	1.0232E−12	5.7412E−12	1.4562E−11	1.4007E+00	1.6675E+01	1.2818E+01
0.005	1500	1.0232E−12	2.1600E−12	4.8979E−12	2.5328E+00	1.3609E+01	7.8969E+00
0.005	3000	1.0232E−12	3.4674E−12	1.0526E−11	2.4440E+00	1.5412E+01	1.0459E+01
0.01	100	1.0232E−12	2.5011E−12	7.5280E−12	2.0564E+00	1.8015E+01	1.5881E+01
0.01	200	1.0232E−12	2.3874E−12	7.5225E−12	2.2679E+00	1.4530E+01	9.5192E+00
0.01	500	1.0232E−12	3.4674E−12	1.0526E−11	1.9423E+00	1.6884E+01	1.2682E+01
0.01	**1000**	**1.0232E−12**	**1.3642E−12**	**3.4452E−13**	3.0669E+00	1.5269E+01	1.1245E+01
0.01	**1500**	**1.0232E−12**	**1.3642E−12**	**3.4452E−13**	1.5534E+00	1.5697E+01	1.0661E+01
0.01	3000	1.0232E−12	2.5580E−12	7.5516E−12	3.3921E+00	1.5839E+01	1.0829E+01
0.02	100	1.0232E−12	2.3874E−12	7.5225E−12	3.6315E+00	1.7372E+01	1.0589E+01
0.02	200	1.0232E−12	2.3874E−12	7.5225E−12	2.1257E+00	1.4499E+01	1.1026E+01
0.02	**500**	**1.0232E−12**	**1.3642E−12**	**3.4452E−13**	2.6935E+00	1.3798E+01	1.0846E+01
0.02	**1000**	**1.0232E−12**	**1.3642E−12**	**3.4452E−13**	2.8439E+00	1.3031E+01	9.2626E+00
0.02	1500	1.0232E−12	2.3874E−12	7.5225E−12	4.9393E+00	1.6337E+01	1.1059E+01
0.02	**3000**	**1.0232E−12**	**1.3642E−12**	**3.4452E−13**	3.4955E+00	1.5084E+01	1.2515E+01
0.05	100	1.0232E−12	1.1200E−08	7.9184E−08	2.3647E+00	1.5592E+01	1.7657E+01
0.05	200	1.0232E−12	3.4674E−12	1.0526E−11	2.2144E+00	1.9620E+01	1.6111E+01
0.05	500	1.0232E−12	2.3874E−12	7.5225E−12	3.0214E+00	1.6345E+01	1.2180E+01
0.05	1000	1.0232E−12	2.3874E−12	7.5225E−12	8.2458E−01	1.4104E+01	1.1054E+01

Table 3 continued

3σ	DG	2-D			30-D		
		BFV	MFV	STD	BFV	MFV	STD
0.05	**1500**	1.0232E−12	2.3874E−12	7.5225E−12	**1.4748E+00**	**1.1338E+01**	**7.8259E+00**
0.05	3000	1.0232E−12	2.3874E−12	7.5225E−12	2.7748E+00	1.3314E+01	7.9781E+00
0.1	100	1.0232E−12	1.1201E−08	7.9184E−08	2.1965E+00	2.1252E+01	1.6596E+01
0.1	200	1.0232E−12	1.1232E−08	7.9180E−08	3.0292E+00	1.8110E+01	1.1537E+01
0.1	500	1.0232E−12	1.1200E−08	7.9164E−08	3.1286E+00	1.6484E+01	1.1701E+01
0.1	1000	1.0232E−12	3.4674E−12	1.0526E−11	1.3348E+00	1.5700E+01	1.2115E+01
0.1	1500	1.0232E−12	3.6380E−12	1.0547E−11	1.9710E+00	1.5589E+01	1.1816E+01
0.1	3000	1.0232E−12	4.6043E−12	1.2756E−11	1.6165E+00	1.6570E+01	1.3708E+01
0.15	100	1.0232E−12	2.8784E−05	1.4202E−04	1.3495E+00	2.0629E+01	1.9112E+01
0.15	200	1.0232E−12	2.5988E−07	1.2996E−06	2.6312E+00	1.8964E+01	1.5822E+01
0.15	500	1.0232E−12	1.1202E−08	7.9184E−08	2.1207E+00	1.7127E+01	1.4905E+01
0.15	1000	1.0232E−12	1.5120E−11	9.0236E−11	2.2292E+00	1.6852E+01	1.3143E+01
0.15	1500	1.0232E−12	2.5011E−12	7.5280E−12	5.3991E−01	1.8998E+01	1.6433E+01
0.15	3000	1.0232E−12	2.3874E−12	7.5225E−12	3.2287E+00	1.5457E+01	9.0581E+00

In general, it is more reasonable to choose a moderate 3σ and a large DG so that the former can effectively improve the search ability and the latter can decrease the negative effect from the "wrong" best solutions. Based on the comprehensive analysis of the results in Tables 2 and 3, 0.02 and 1000 are chosen as the default values of 3σ and DG, respectively.

4 Experimental results and discussions

To evaluate the performance, DHLO, as well as the standard HLO [17] and the other eight binary-coded meta-heuristics, i.e. the Binary Differential Evolution algorithm (BDE) [26], the Simplified Binary Artificial Fish Swarm Algorithm (S_bAFSA) [27], the Adaptive Binary Harmony Search (ABHS) [28], the Binary Gravitational Search Algorithm (BGSA) [29], the Binary Bat Algorithm (BBA) [30], the Binary Artificial Bee Colony (BABC) [31], the Bi-Velocity Discrete Particle Swarm Optimization (BVDPSO) [32], and the Modified Binary Particle Swarm Optimization (MBPSO) [33], was applied to solve the 15 CEC05 benchmark functions listed in Table 1 and knapsack problems. For a fair comparison, the recommended parameter values of these algorithms were adopted, which are given in Table 4. As the CEC05 benchmarks and knapsack problems studied in this paper are the single-objective problems, the sizes of the IKDs and the SKD were both set to 1 as recommended in [17]. Besides, the IKDs of DHLO were re-initialized if the individual best solution was not updated in 100 successive generations to prevent the algorithm from being trapped in the local optima. The other parameters of DHLO, such as the population size and the maximal generation, were the same as those used in Sect. 3.

4.1 Benchmark functions

4.1.1 Low-dimensional functions

The numerical results and the Wilcoxon signed-rank test (W-test) results on the 2-dimensinal functions are given in Table 5, in which "1" denotes that DHLO significantly outperforms the compared algorithm at the 95 % confidence, "-1" represents that DHLO is significantly worse than the compared algorithm, and "0" indicates that the achieved results by DHLO and the compared algorithm are not statistically different. For clearly analyzing and comparing the performance, the rankings and the W-test results of all the algorithms are summarized in Tables 6 and 7, respectively.

Tables 6 and 7 show that DHLO has better performance on the low-dimensional functions. Specifically, DHLO achieves the best numerical results on all the functions. The performance ranking of all the algorithms sorted in the descending order is DHLO, HLO, BVDPSO, S_bAFSA, ABHS, BDE, BGSA, MBPSO, BBA, and BABC. The W-test results demonstrate that DHLO is significant better than HLO and the other eight algorithms on 12 and 14 out of 15 functions while it is inferior to them on none.

4.1.2 High-dimensional functions

The optimization results on the 30-dimensional functions are given in Table 8. Likewise, the rankings and the W-test results of all the algorithms are summarized in Tables 9 and 10 for clearly reviewing the performance of all the algorithms. The results of the high-dimensional functions also indicate that DHLO has an advantage over the other nine algorithms. Table 9 displays that DHLO obtains the optimal numerical result on 14 out of 15 functions and

Table 4 The recommended parameter values of all the algorithms

Algorithms	Parameters
DHLO	pr = 5/M, pi = 0.85, $3\sigma = 0.02$, DG = 1000
HLO	pr = 5/M, pi = 0.85 + 2/M
BDE	p = max(0.05, min(0.15, 10/n))
S_bAFSA	$\tau_1 = 0.1$, $\tau_2 = 0.9$, R = 100
ABHS	HMS = 30, NGC = 50, PAR = 0.2, C = 15
BGSA	$G_0 = 220$
BBA	a = 0.9, $\lambda = 0.9$, $-1 \leq \varepsilon \leq 1$, r = 0.5, A = 0.25, $F_{min} = 0$, $F_{max} = 2$
BABC	SN = 25, limit = 100
BVDPSO	$V_{max} = 1$, $V_{min} = 0$
MBPSO	$V_{max} = 4$, $c_1 = c_2 = 2$

is only inferior to S_bAFSA on F10. The performance of all the algorithms on the high-dimensional functions sorted in the descending order is DHLO, HLO, BDE, S_bAFSA, ABHS, BVDPSO, MBPSO, BBA, BGSA, and BABC. The W-test results in Table 10 indicate that DHLO significantly surpasses ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO on all the functions. Compared with HLO, BDE, and S_bAFSA, DHLO has significantly better results on 13, 12, and 13 out of 15 functions and yields statistically similar results on the other 2, 3, and 2 functions, respectively.

4.2 Knapsack problems

Previous work [34] show that the ranking of compared optimizers are sensitive to benchmark sets, and therefore the performance of DHLO is further evaluated on knapsack problems for a comprehensive comparison. Knapsack problems are combinatorial optimization problems and have been studied intensively in the last few decades as their simple structure, which, on the one hand, allows the exploitation of a number of combinatorial properties and, on the other hand, allows more complex optimization problems to be solved through a series of knapsack-type sub-problems [35]. Actually, many real application problems, such as cargo loading, cutting stock, project selection, and budget control, can be formulated as knapsack problems. In this work, DHLO and the other meta-heuristic algorithm are adopted to solve 0-1 knapsack problems (0-1 KP) and multidimensional knapsack problems (MKP).

4.3 0-1 knapsack problems

In a given set of N items, each of them has a weight w_j and a profit p_j. The 0-1 knapsack problem is to select a subset from the set of Nitems such that the overall profit is maximized without exceeding a preset weight capacity C, which can be mathematically formulated as Eq. (11)

$$Max\ f(\mathrm{x}) = \sum_{j=1}^{N} p_j x_j$$

$$s.t. \begin{cases} \sum_{j=1}^{N} w_j x_j \leq C \\ x_j = \{0, 1\}, \mathrm{j} = 1, 2, \ldots, \mathrm{N} \end{cases} \tag{11}$$

Table 5 The results of the 2-dimensional functions

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F1										
BFV	**0.00E+0**	0.00E+0	0.00E+0	0.00E+0	0.00E+0	6.74E−10	9.03E−10	4.96E−1	0.00E+0	0.00E+0
MFV	**1.59E−12**	1.44E−8	5.40E−1	2.02E−7	1.44E−7	1.27E−1	4.25E+0	1.15E+1	2.09E−7	1.17E+0
STD	**2.44E−12**	7.13E−8	3.68E+0	1.82E−7	1.97E−7	2.49E−1	9.70E+0	1.07E+1	4.68E−7	5.22E+0
W-test	/	1	1	1	1	1	1	1	1	1
F2										
BFV	**0.00E+0**	0.00E+0	8.07E−11	1.82E−12	2.89E−6	0.00E+0	4.46E−11	1.47E−1	1.71E−13	2.08E−9
MFV	**5.00E−12**	2.32E−6	8.96E−1	4.63E−4	5.65E−2	8.04E−1	1.18E+0	8.29E+0	4.72E−6	1.82E+0
STD	**1.93E−11**	9.22E−4	3.19E+0	1.28E−3	7.56E−2	2.68E+0	3.66E+0	1.64E+1	6.59E−6	4.32E+0
W-test	/	1	1	1	1	1	1	1	1	1
F3										
BFV	**1.21E−9**	1.21E−9	1.21E−9	1.21E−9	3.01E−4	1.21E−3	1.20E−1	4.00E+2	1.21E−09	1.78E−09
MFV	**3.02E−8**	1.13E+1	2.96E+4	3.57E+3	5.94E+2	4.83E+4	3.10E+4	4.50E+4	6.97E+01	3.13E+04
STD	**2.02E−7**	3.00E+1	3.74E+4	1.53E+4	7.95E+2	3.78E+04	3.78E+4	3.55E+4	2.89E+02	3.77E+04
W-test	/	1	1	1	1	1	1	1	1	1
F4										
BFV	**0.00E+0**	0.00E+0	0.00E+0	1.71E−13	1.00E−10	2.76E−7	2.29E−8	3.90E−1	1.82E−12	1.82E−12
MFV	**1.04E−11**	3.99E−4	1.18E+0	1.84E−4	4.87E−6	9.78E−1	4.24E−1	4.72E+0	3.10E−6	1.97E+0
STD	**2.66E−11**	2.33E−3	3.63E+0	7.76E−4	7.67E−6	2.84E+0	1.25E+0	4.26E+0	5.42E−6	4.06E+0
W-test	/	1	1	1	1	1	1	1	1	1
F5										
BFV	**0.00E+0**	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0
MFV	**0.00E+0**	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0
STD	**0.00E+0**	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0	0.00E+0
W-test	/	0	0	0	0	0	0	0	0	0

Table 5 continued

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F6										
BFV	**2.84E−13**	4.55E−9	5.64E−11	2.84E−13	7.97E−2	7.35E−03	0.00E+0	3.50E−1	2.39E−12	3.82E−04
MFV	**1.72E−4**	4.72E−3	3.56E+0	1.02E−1	1.41E+0	2.62E+1	1.50E+0	6.83E+1	1.29E−2	1.52E+1
STD	**4.59E−4**	2.07E−1	8.67E+0	2.85E−1	1.91E+0	3.80E+1	2.16E+0	9.26E+1	2.09E−2	2.77E+1
W-test	/	1	1	1	1	1	1	1	1	1
F7										
BFV	**1.56E−12**	1.56E−12	4.84E−8	5.47E−10	7.40E−3	8.53E−3	0.00E+0	9.32E−2	1.56E−12	2.49E−10
MFV	**2.66E−3**	3.53E−3	7.44E−2	1.26E−2	8.29E−2	9.36E−2	6.00E−2	6.65E−1	6.57E−3	6.73E−2
STD	**3.79E−3**	3.97E−3	8.17E−2	3.38E−2	8.00E−2	8.44E−2	9.00E−2	2.33E−1	7.00E−3	7.31E−2
W-test	/	1	1	1	1	1	1	1	1	1
F8										
BFV	**7.07E−6**	7.07E−6	7.07E−6	7.07E−6	7.07E−6	7.07E−6	7.07E−6	2.90E−1	7.07E−6	7.07E−6
MFV	**7.07E−6**	1.55E−3	2.62E+0	4.72E−3	7.67E−1	2.39E+0	6.21E+0	8.56E+0	5.66E−5	3.52E+0
STD	**5.74E−14**	6.13E−3	4.81E+0	2.34E−2	1.70E+0	4.13E+0	8.43E+0	6.23E+0	2.86E−4	6.22E+0
W-test	/	1	1	1	1	1	1	1	1	1
F9										
BFV	**1.02E−12**	1.02E−12	1.02E−12	1.02E−12	5.89E−10	8.30E−12	3.47E−5	8.02E−1	1.02E−12	7.11E−11
MFV	**1.36E−12**	7.19E−8	3.45E−4	3.16E−4	2.37E−7	9.33E−1	5.58E+0	5.88E+0	2.59E−4	1.32E+0
STD	**3.44E−12**	3.51E−7	3.61E−4	3.63E−4	3.10E−7	2.43E+0	1.35E+1	1.95E+0	3.47E−4	3.17E+0
W-test	/	1	1	1	1	1	1	1	1	1
F10										
BFV	**1.99E−12**	1.99E−12	1.99E−12	8.95E−10	1.46E−8	8.20E−5	1.32E−7	2.41E+0	5.16E−11	4.48E−9
MFV	**1.95E−9**	1.44E−4	6.84E−1	3.99E−1	6.39E−1	5.06E+0	1.37E+1	5.89E+0	1.20E−1	3.06E+0
STD	**1.30E−8**	3.08E−4	5.67E−1	5.16E−1	5.85E−1	6.06E+0	2.39E+1	2.76E+0	3.27E−1	3.57E+0
W-test	/	1	1	1	1	1	1	1	1	1

Table 5 continued

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F11										
BFV	**5.88E−4**	7.90E−3	6.38E−4	1.32E−1	1.39E−1	5.83E−1	5.21E−1	6.39E−1	1.32E−3	1.39E−1
MFV	**3.22E−1**	4.95E−1	4.83E−1	7.25E−1	4.92E−1	2.41E+0	2.93E+0	1.53E+0	5.47E−1	1.43E+0
STD	**3.05E−1**	3.34E−1	3.45E−1	3.67E−1	3.28E−1	9.64E−1	6.59E−1	4.74E−1	2.05E−1	8.36E−1
W-test	/	1	1	1	1	1	1	1	1	1
F12										
BFV	**1.42E−11**	1.42E−11	4.64E−11	4.64E−11	3.00E−10	2.38E−11	2.08E−3	2.79E+0	1.42E−11	5.80E−9
MFV	**8.58E−7**	8.67E−5	1.29E+1	2.18E−1	7.68E−1	7.43E−2	1.26E+3	2.26E+2	1.99E−1	2.84E+2
STD	**6.04E−6**	5.83E−5	8.73E+1	6.54E−1	1.18E+0	6.86E−2	3.24E+3	1.79E+2	4.54E−1	3.03E+2
W-test	/	0	1	1	1	1	1	1	1	1
F13										
BFV	**0.00E+0**	0.00E+0	0.00E+0	0.00E+0	0.00E+0	2.60E−4	0.00E+0	6.00E−3	0.00E+0	1.13E−7
MFV	**1.99E−4**	7.88E−4	1.41E−2	9.45E−3	1.03E−2	5.54E+3	5.00E−2	9.07E+1	5.57E−3	3.74E−2
STD	**1.40E−3**	2.52E−3	1.65E−2	1.00E−2	7.03E−3	1.31E+4	8.00E−2	6.44E+1	8.39E−3	4.04E−2
W-test	/	1	1	1	1	1	1	1	1	1
F14										
BFV	**0.00E+0**	9.09E−13	6.23E−6	9.63E−9	9.26E−3	1.94E−2	5.10E−5	2.00E−2	1.17E−9	5.62E−8
MFV	**7.81E−3**	1.10E−2	1.09E−1	1.36E−2	1.74E−2	5.95E−2	2.00E−2	1.90E−1	1.19E−2	6.62E−2
STD	**9.59E−3**	9.44E−3	1.41E−1	8.83E−3	4.55E−3	7.63E−2	2.00E−2	1.30E−1	9.39E−3	9.22E−2
W-test	/	0	1	1	1	1	1	1	1	1
F15										
BFV	**1.92E−11**	1.92E−11	1.92E−11	3.64E−9	1.92E−11	2.51E−6	3.08E−7	5.73E+1	1.92E−11	1.92E−11
MFV	**1.14E−10**	5.61E−2	8.36E+1	5.04E+1	4.79E+1	2.72E+2	2.82E+2	2.82E+2	1.17E−2	2.50E+2
STD	**5.16E−10**	2.65E−1	7.72E+1	5.93E+1	6.71E+1	1.82E+2	3.98E+2	1.35E+2	2.26E−2	1.91E+2
W-test	/	1	1	1	1	1	1	1	1	1

Table 6 The rankings of all the algorithms on the 2-dimensional functions

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F1	1	2	7	4	3	6	9	10	5	8
F2	1	2	7	4	5	6	8	10	3	9
F3	1	2	6	5	4	10	7	9	3	8
F4	1	5	8	4	3	7	6	10	2	9
F5	1	1	1	1	1	1	1	1	1	1
F6	1	2	7	4	5	9	6	10	3	8
F7	1	2	7	4	8	9	5	10	3	6
F8	1	3	7	4	5	6	9	10	2	8
F9	1	2	6	5	3	7	9	10	4	8
F10	1	2	6	4	5	8	10	9	3	7
F11	1	4	2	6	3	9	10	8	5	7
F12	1	2	7	5	6	3	10	8	4	9
F13	1	2	6	4	5	10	8	9	3	7
F14	1	2	9	4	5	7	6	10	3	8
F15	1	3	6	5	4	8	9	9	2	7
Average	**1.00**	2.40	6.13	4.20	4.33	7.07	7.53	8.87	3.07	7.33

Table 7 The summary of the W-test results between DHLO and the other meta-heuristics on the 2-dimensional functions

W-test	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1	12	14	14	14	14	14	14	14	14
0	3	1	1	1	1	1	1	1	1
−1	0	0	0	0	0	0	0	0	0

where the binary decision variable x_j takes values either 0 or 1 which represents the selection or rejection of the *j*th item. Without loss of generality, 0-1 KPs assume that all profits and weights are positive and all weights are smaller than C. As 0-1 KPs are constrained problems, infeasible solutions, of which the total weight exceeds the limit C, may be generated during the search process. Thus, the penalty function method as Eq. (12) is adopted to deal with infeasible solutions,

$$\begin{gathered} Max\ F(\mathrm{x}) = f(x) - \lambda \times \max(0, c) \\ c = \sum_{j=1}^{N} w_j x_j - C \end{gathered} \tag{12}$$

where the penalty coefficient λ is a big constant so that the fitness of infeasible solutions is inferior to that of feasible solutions, which can lead the algorithm to escape from the infeasible area and search in the feasible region.

A set of 0-1 KPs was generated according to [35,36] for the performance evaluation. The numbers of items were set to 50, 100, 250, 500, 800, 1000, 1200, 1500, 2000 and 2500, and three cases of each scale were yielded for achieving the comprehensive and exact

Table 8 The results of the 30-dimensional functions

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F1										
BFV	**1.11E+01**	3.51E+01	1.08E+01	5.38E+01	2.46E+02	3.21E+03	2.22E+03	5.33E+04	5.46E+01	9.35E+01
MFV	**1.13E+02**	2.05E+02	1.68E+02	2.12E+02	3.21E+02	5.76E+03	5.63E+03	6.44E+04	4.42E+02	5.04E+02
STD	**8.31E+01**	1.34E+02	1.18E+02	1.22E+02	7.93E+01	1.64E+03	2.39E+03	1.02E+04	2.83E+02	2.99E+02
W-test	/	1	0	1	1	1	1	1	1	1
F2										
BFV	**2.05E+02**	3.24E+02	5.01E+02	2.34E+03	2.32E+03	1.65E+04	2.72E+04	5.67E+04	2.40E+03	6.74E+03
MFV	**8.58E+02**	1.12E+03	1.62E+03	4.40E+03	3.42E+03	3.63E+04	3.49E+04	9.88E+04	7.83E+03	1.47E+04
STD	**3.36E+02**	4.61E+02	6.60E+02	1.27E+03	1.13E+03	1.12E+04	7.50E+03	1.76E+04	2.32E+03	5.37E+03
W-test	/	0	1	1	1	1	1	1	1	1
F3										
BFV	**4.83E+06**	8.34E+06	8.70E+06	1.10E+07	6.07E+07	4.27E+07	7.02E+07	7.34E+08	1.01E+07	1.94E+07
MFV	**1.50E+07**	2.12E+07	3.62E+07	2.62E+07	7.02E+07	1.77E+08	1.06E+08	9.64E+08	4.55E+07	5.72E+07
STD	**6.75E+06**	9.45E+06	1.48E+07	1.33E+07	7.12E+06	1.12E+08	4.38E+07	1.62E+08	2.08E+07	2.57E+07
W-test	/	1	1	1	1	1	1	1	1	1
F4										
BFV	**5.08E+02**	1.13E+03	7.85E+02	4.15E+03	3.89E+03	1.13E+04	2.12E+04	4.45E+04	6.03E+03	7.95E+03
MFV	**1.43E+03**	2.91E+03	2.36E+03	8.19E+03	6.84E+03	2.70E+04	5.36E+04	8.28E+04	1.19E+04	2.32E+04
STD	**5.44E+02**	1.25E+03	1.00E+03	3.18E+03	2.89E+03	9.28E+03	1.74E+04	2.25E+04	3.47E+03	8.33E+03
W-test	/	1	1	1	1	1	1	1	1	1
F5										
BFV	**1.66E+03**	1.67E+03	2.82E+03	3.28E+03	9.89E+03	5.67E+03	6.76E+03	2.69E+04	2.66E+03	3.15E+03
MFV	**4.37E+03**	4.39E+03	6.66E+03	4.76E+03	1.16E+04	1.05E+04	1.10E+04	3.34E+04	4.92E+03	6.75E+03
STD	**9.98E+02**	1.30E+03	1.97E+03	9.89E+02	1.60E+03	1.86E+03	3.19E+03	2.36E+03	1.13E+03	1.81E+03
W-test	/	0	1	0	1	1	1	1	0	1

Table 8 continued

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F6										
BFV	**1.73E+03**	4.05E+03	8.45E+04	1.16E+05	3.47E+06	1.03E+08	1.46E+08	1.45E+10	7.13E+05	6.81E+05
MFV	**7.84E+05**	2.89E+06	2.05E+06	3.20E+06	6.71E+06	9.57E+08	4.63E+08	3.58E+10	9.03E+06	1.34E+07
STD	**8.77E+05**	2.91E+06	2.01E+06	2.90E+06	2.99E+06	7.76E+08	3.08E+08	1.04E+10	6.60E+06	9.10E+06
W-test	/	1	0	1	1	1	1	1	1	1
F7										
BFV	**1.91E+00**	6.20E+00	7.17E+00	7.59E+00	1.28E+01	1.74E+02	2.20E+02	3.68E+03	4.70E+03	1.65E+01
MFV	**1.50E+01**	2.31E+01	2.28E+01	2.59E+01	2.48E+01	5.47E+02	5.21E+02	4.43E+03	4.70E+03	7.78E+01
STD	**9.72E+00**	1.23E+01	1.27E+01	1.58E+01	1.13E+01	2.07E+02	1.73E+02	5.17E+02	2.03E−01	5.06E+01
W-test	/	1	1	1	1	1	1	1	1	1
F8										
BFV	**2.06E+01**	2.07E+01	2.08E+01	2.08E+01	2.07E+01	2.09E+01	2.07E+01	2.09E+01	2.06E+01	2.06E+01
MFV	**2.08E+01**	2.08E+01	2.08E+01	2.09E+01	2.09E+01	2.11E+01	2.09E+01	2.11E+01	2.09E+01	2.09E+01
STD	7.68E−02	6.75E−02	9.64E−02	5.95E−02	1.32E−01	8.28E−02	6.49E−02	5.36E−02	6.98E−02	8.00E−02
W-test	/	1	1	1	1	1	1	1	1	1
F9										
BFV	**2.84E+00**	3.07E+00	4.01E+01	4.57E+00	6.86E+00	5.10E+02	2.94E+02	5.08E+02	7.97E+01	1.31E+02
MFV	**1.30E+01**	2.25E+01	5.50E+01	1.89E+01	1.43E+01	6.22E+02	3.48E+02	5.14E+02	1.63E+02	2.07E+02
STD	**6.34E+00**	1.85E+01	8.19E+00	7.79E+00	7.52E+00	5.23E+01	2.49E+01	8.61E+1	2.90E+01	3.61E+01
W-test	/	1	1	1	1	1	1	1	1	1
F10										
BFV	2.25E+02	2.54E+02	2.94E+02	**1.43E+02**	4.26E+02	9.37E+02	4.67E+02	9.70E+02	3.05E+02	3.26E+02
MFV	3.30E+02	3.51E+02	4.03E+02	**2.23E+02**	5.07E+02	1.23E+03	5.46E+02	9.77E+02	4.37E+02	4.81E+02
STD	5.82E+01	5.41E+01	6.48E+01	**4.52E+01**	6.85E+01	1.17E+02	6.20E+01	8.04E+0	6.14E+01	6.78E+01
W-test	/	1	1	0	1	1	1	1	1	1

Table 8 continued

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F11										
BFV	**3.27E+01**	3.46E+01	3.20E+01	3.66E+01	4.01E+01	3.82E+01	3.66E+01	4.00E+01	3.48E+01	3.63E+01
MFV	**3.69E+01**	3.77E+01	3.75E+01	3.85E+01	4.16E+01	4.47E+01	4.37E+01	4.22E+01	3.77E+01	4.13E+01
STD	**1.84E+00**	1.33E+00	2.55E+00	1.13E+00	1.36E+00	2.26E+00	3.19E+00	1.28E+00	1.35E+00	1.99E+00
W-test	/	1	0	1	1	1	1	1	1	1
F12										
BFV	**3.45E+04**	6.66E+04	6.84E+04	7.21E+04	7.50E+04	1.22E+06	9.03E+05	1.40E+06	2.44E+05	4.07E+05
MFV	**1.18E+05**	1.42E+05	1.61E+05	1.38E+05	1.49E+05	1.97E+06	1.53E+06	2.53E+06	5.62E+05	9.32E+05
STD	**5.20E+04**	4.53E+04	4.85E+04	3.41E+04	5.95E+04	3.70E+05	3.21E+05	2.14E+05	1.59E+05	2.60E+05
W-test	/	1	1	1	1	1	1	1	1	1
F13										
BFV	**1.24E+00**	1.77E+00	1.45E+01	3.07E+00	2.20E+00	4.96E+06	3.97E+06	4.55E+02	6.18E+04	3.10E+05
MFV	**3.37E+00**	6.29E+00	2.01E+01	6.56E+00	5.47E+00	8.20E+06	5.93E+06	4.35E+02	7.15E+05	1.68E+06
STD	**1.22E+00**	3.56E+00	3.02E+00	3.49E+00	2.35E+00	1.63E+06	1.21E+06	4.02E+02	4.27E+05	8.41E+05
W-test	/	1	1	1	1	1	1	1	1	1
F14										
BFV	**1.16E+01**	1.13E+01	1.18E+01	1.31E+01	1.28E+01	1.29E+01	1.24E+01	1.31E+01	1.27E+01	1.20E+01
MFV	**1.25E+01**	1.26E+01	1.28E+01	1.34E+01	1.33E+01	1.35E+01	1.33E+01	1.38E+01	1.33E+01	1.30E+01
STD	**4.16E−01**	3.47E−01	4.20E−01	1.37E−01	3.54E−01	3.15E−01	3.29E−01	1.59E−01	2.17E−01	3.80E−01
W-test	/	1	1	1	1	1	1	1	1	1
F15										
BFV	**2.37E+02**	2.39E+02	3.77E+02	2.63E+02	3.04E+02	8.77E+02	8.73E+02	1.05E+03	4.49E+02	5.05E+02
MFV	**2.90E+02**	3.15E+02	4.17E+02	3.24E+02	3.21E+02	9.80E+02	9.63E+02	1.17E+03	6.07E+02	6.57E+02
STD	**3.02E+01**	4.17E+01	2.20E+01	3.09E+01	1.34E+01	4.25E+01	5.02E+01	4.59E+01	5.41E+01	6.49E+01
W-test	/	1	1	1	1	1	1	1	1	1

Table 9 The rankings of all the algorithms on the 30-dimensional functions

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
F1	1	3	2	4	5	9	8	10	6	7
F2	1	2	3	5	4	9	8	10	6	7
F3	1	2	4	3	7	9	8	10	5	6
F4	1	3	2	5	4	8	9	10	6	7
F5	1	2	5	3	9	7	8	10	4	6
F6	1	3	2	4	5	9	8	10	6	7
F7	1	3	2	5	4	8	7	9	10	6
F8	1	3	2	5	4	9	8	10	7	6
F9	1	4	5	3	2	10	8	9	6	7
F10	2	3	4	1	7	10	8	9	5	6
F11	1	3	2	5	7	10	9	8	3	6
F12	1	3	5	2	4	9	8	10	6	7
F13	1	3	5	4	2	10	9	6	7	8
F14	1	2	3	8	5	9	5	10	5	4
F15	1	2	5	4	3	9	8	10	6	7
Average	1.07	2.73	3.40	4.07	4.80	9.00	7.93	9.40	5.87	6.47

Table 10 The summary of the W-test results between DHLO and the other meta-heuristics on the 30-dimensional functions

W-test	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1	13	12	13	15	15	15	15	15	15
0	2	3	2	0	0	0	0	0	0
−1	0	0	0	0	0	0	0	0	0

results. The weight w_j and the profit p_j were produced randomly from 5 to 20 and from 50 to 100, respectively. The weight capability C was correspondingly set to 600, 1200, 3000, 6000, 10,000, 12,000, 15,000, 18,000, 25,000, and 30,000. For low-dimensional instances, in which the number of decision variables is less than 1000, the population size and maximum generation were set to 100 and 5000, respectively. For high-dimensional problems of which the items are no less than 1000, the population and maximum generation were set to 300 and 10,000, respectively. The experimental results are listed in Tables 11 and 12, and the summary results of the ranking and W-test are given in Tables 13 and 14.

Tables 11 and 12 show that DHLO searches out the best known results on all the 0-1 KPs while HLO, BDE, S_bAFSA, ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO find 19, 6, 6, 20, 5, 5, 3, 9 and 22 best known solutions out of 30 instances, respectively. Specifically, DHLO has the equal search ability as HLO and ABHS, on small-scale problems since all of them can find the best-known values on 50.1, 50.2, 50.3, 100.1, 100.2, 100.3 and 250.3 cases with 100 % success rate. However, DHLO displays an advantage over the other meta-heuristics as the dimension of problems increases. Table 12 illustrates that only DHLO can reach all the best fitness values when the item of 0-1 KPs is more than 1000.

Table 11 The results of low-dimensional 0-1 knapsack problems

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
50.1										
BFV	**3680**	**3680**	**3680**	3680	**3680**	3680	3680	3680	3680	3680
MFV	**3680**	**3680**	**3680**	3678.6	**3680**	3679.6	3679.4	3678.8	3679.2	3676.8
STD	**0.0**	**0.0**	**0.0**	2.3	**0.0**	1.5	1.6	2.1	0.4	6.1
W-test	/	0	0	0	0	0	0	0	0	0
50.2										
BFV	**3684**	**3684**	**3684**	**3684**	**3684**	**3684**	3684	3684	3684	3684
MFV	**3684**	**3684**	**3684**	**3684**	**3684**	**3684**	3681.2	3683.6	3683.2	3682.8
STD	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**	1.8	0.9	1.1	1.1
W-test	/	0	0	0	0	0	0	0	0	0
50.3										
BFV	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**
MFV	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**	**3732**
STD	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
W-test	**/**	0	0	0	0	0	0	0	0	0
100.1										
BFV	**7641**	**7641**	7641	7641	**7641**	7641	7641	5171	**7641**	**7641**
MFV	**7641.0**	**7641.0**	7516.8	7593.2	**7641.0**	7470.7	7534.0	4830.3	**7641.0**	**7641.0**
STD	**0.0**	**0.0**	54.7	35.8	**0.0**	98.4	84.4	132.2	**0.0**	**0.0**
W-test	/	0	1	1	0	1	1	1	0	0
100.2										
BFV	**7675**	**7675**	7675	7675	**7675**	7675	7675	5197	**7675**	**7675**
MFV	**7675.0**	**7675.0**	7569.2	7618.0	**7675.0**	7494.4	7508.3	4904.5	**7675.0**	**7675.0**
STD	**0.0**	**0.0**	49.7	36.7	**0.0**	79.8	109.9	156.2	**0.0**	**0.0**
W-test	/	0	1	1	0	1	1	1	0	0

Table 11 continued

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
100.3										
BFV	**7207**	**7207**	7207	7207	**7207**	7144	7152	5045	**7207**	**7207**
MFV	**7207.0**	**7207.0**	7113.3	7172.1	**7207.0**	7042.9	7028.3	4512.4	**7207.0**	**7207.0**
STD	**0.0**	**0.0**	40.3	26.7	**0.0**	69.4	86.9	173.0	**0.0**	**0.0**
W-test	/	0	1	1	0	1	1	1	0	0
250.1										
BFV	**18,761**	18,761	17,760	16,974	18,761	16,604	16,370	11,393	18,761	18,761
MFV	**18,759.9**	18,750.3	17,619.4	16,632.9	18,748.9	16,175.2	16,054.0	10,916.6	18,758.0	18,753.6
STD	**1.4**	17.5	109.1	145.8	15.4	254.2	218.7	251.1	2.2	12.8
W-test	/	1	1	1	0	1	1	1	1	1
250.2										
BFV	**18,374**	18,374	17,561	16,770	18,374	16,314	16,208	11,597	18,372	18,374
MFV	**18,363.4**	18,361.1	17,366.5	16,335.4	18,364.3	15,737.0	15,810.4	10,877.3	18,350.8	18,352.3
STD	**18.7**	17.2	101.3	167.5	19.0	260.1	242.3	303.5	22.1	25.1
W-test	/	0	1	1	0	1	1	1	1	0
250.3										
BFV	**18,526**	**18,526**	17,536	16,455	**18,526**	16,220	16,166	11,625	**18,526**	**18,526**
MFV	**18,526.0**	**18,526.0**	17,283.0	16,233.2	**18,526.0**	15,728.9	15,636.9	10,755.3	**18,526.0**	**18,526.0**
STD	**0.0**	**0.0**	152.3	114.8	**0.0**	262.1	276.8	299.9	**0.0**	**0.0**
W-test	/	0	1	1	0	1	1	1	0	0
500.1										
BFV	**36,828**	36,828	30,546	29,375	36,828	28,335	28,474	21,572	36,828	36,828
MFV	**36,827.6**	36,822.7	30,185.9	28,836.1	35,848.0	27,727.7	27,621.1	20,541.3	36,681.6	36,816.6
STD	**0.9**	5.9	154.7	212.3	148.5	404.1	433.7	395.8	127.7	11.1
W-test	/	1	1	1	1	1	1	1	1	1

Table 11 continued

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
500.2										
BFV	**37,078**	37,050	30,943	29,616	37,046	28,505	28,764	21,768	37,016	37,027
MFV	**37,041.3**	37,000.0	30,519.1	29,143.3	36,925.0	27,842.1	28,019.0	20,795.3	36,944.4	36,989.5
STD	**19.0**	32.0	204.1	263.5	23.9	389.4	392.0	432.8	74.2	36.5
W-test	/	1	1	1	1	1	1	1	1	1
500.3										
BFV	**36,728**	36,727	30,625	29,468	36,719	28,542	28,672	21,354	36,720	36,728
MFV	**36,713.4**	36,646.8	30,280.8	28,934.5	36,303.5	27,650.3	27,886.9	20,646.8	36,599.4	36,621.5
STD	**21.9**	58.2	199.3	302.1	85.4	367.7	455.8	402.4	56.9	55.3
W-test	/	1	1	1	1	1	1	1	1	1
800.1										
BFV	**60,095**	60,095	46,052	44,506	60,095	42,866	43,640	33,552	57,856	60,095
MFV	**60,095.0**	60,020.8	45,466.5	43,955.5	59,095.6	42,241.4	42,223.1	32,940.2	57,289.1	60,008.6
STD	**0.0**	64.7	333.4	330.8	112.5	329.7	657.9	369.8	269.0	88.2
W-test	/	1	1	1	1	1	1	1	1	1
800.2										
BFV	**59,954**	59,954	45,743	44,418	59,914	42,989	42,642	33,218	57,628	59,954
MFV	**59,954.0**	59,849.7	45,381.5	43,652.1	59,854.0	42,004.9	42,050.4	32,603.1	57,069.9	59,869.9
STD	**0.0**	87.4	281.9	388.1	75.5	561.2	475.2	309.8	226.3	63.7
W-test	/	1	1	1	1	1	1	1	1	1
800.3										
BFV	**59,813**	59,813	46,264	44,101	59,813	42,723	42,240	34,416	57,442	59,813
MFV	**59,813.0**	59,719.2	45,216.1	43,491.7	59,613.0	41,893.5	41,816.0	32,643.5	56,989.3	59,724.4
STD	**0.0**	87.4	417.7	354.4	153.5	530.3	398.1	595.6	269.5	66.8
W-test	/	1	1	1	1	1	1	1	1	1

Table 12 The results of high-dimensional 0-1 knapsack problems

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1000.1										
BFV	**74,322**	74,204	55,905	54,493	74,308	54,040	54,220	41,691	74,143	74,252
MFV	**74,276.9**	74,129.2	55,230.3	53,650.5	74,001.2	53,166.9	53,477.3	40,824.9	74,032.7	74,103.5
STD	**22.4**	54.0	317.0	337.9	104.4	518.6	497.0	360.0	78.5	88.9
W-test	/	1	1	1	1	1	1	1	1	1
1000.2										
BFV	**74,948**	74,924	56,294	54,180	74,947	54,246	54,243	42,647	74,783	74,889
MFV	**74,900.9**	74,828.3	55,475.2	53,869.1	73,905.0	53,380.3	53,389.3	41,105.7	74,557.4	74,822.4
STD	**15.5**	52.8	448.1	186.5	96.9	536.9	378.7	353.9	176.9	50.6
W-test	/	1	1	1	1	1	1	1	1	1
1000.3										
BFV	**73,867**	73,867	55,046	54,388	73,853	53,696	52,988	41,602	73,595	73,867
MFV	**73,867.0**	73,834.5	54,511.5	53,166.3	73,667.0	52,814.5	52,391.9	40,352.6	73,289.6	73,836.4
STD	**0.0**	28.4	306.0	377.7	102.5	570.0	443.3	420.3	207.3	30.6
W-test	/	1	1	1	1	1	1	1	1	1
1200.1										
BFV	**89,197**	89,197	64,723	63,367	89,197	63,717	63,616	50,113	87,695	89,197
MFV	**89,197.0**	89,193.8	63,433.8	62,542.0	89,079.0	62,294.0	62,201.6	48,487.1	86,972.4	89,194.5
STD	**0.0**	14.5	3729.6	422.4	96.6	773.7	593.3	683.6	373.0	11.4
W-test	/	0	1	1	1	1	1	1	1	0
1200.2										
BFV	**89,365**	89,365	64,847	63,316	89,346	63,668	63,445	49,517	87,408	**89,365**
MFV	**89,365.0**	89,361.8	64,037.2	62,600.0	89,325.0	62,199.6	62,166.9	48,664.0	86,925.9	**89,365.0**
STD	**0.0**	14.3	380.4	327.1	20.5	871.1	635.0	467.8	277.1	**0.0**
W-test	/	0	1	1	1	1	1	1	1	0

Table 12 continued

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1200.3										
BFV	**89,654**	89,654	64,954	63,452	89,654	63,598	63,484	50,040	87,902	89,654
MFV	**89,653.6**	89,651.5	64,482.7	62,834.0	89,634.7	62,409.2	62,273.9	48,650.1	87,376.4	89,651.4
STD	**2.0**	11.2	306.5	298.4	31.5	490.3	628.1	774.8	345.1	11.6
W-test	/	0	1	1	1	1	1	1	1	0
1500.1										
BFV	**110,630**	110,630	78,128	76,408	110,630	75,965	75,884	60,625	105,900	110,630
MFV	**110,627.6**	110,608.4	77,016.5	75,453.1	110,590.3	74,756.8	74,778.0	59,688.1	104,769.5	110,599.9
STD	**9.1**	31.0	452.2	446.5	27.6	598.9	714.4	557.8	486.4	36.3
W-test	/	1	1	1	1	1	1	1	1	1
1500.2										
BFV	**112,172**	112,108	79,207	76,838	112,154	77,318	77,217	62,075	107,102	112,132
MFV	**112,136.0**	112,050.0	78,132.1	76,302.3	112,108.2	75,681.9	75,923.8	60,457.5	106,098.5	112,062.4
STD	**22.8**	44.9	524.4	364.1	23.1	716.7	671.7	678.5	521.0	50.0
W-test		1	1	1	1	1	1	1	1	1
1500.3										
BFV	**111,862**	111,862	78,664	76,587	111,862	76,750	76,494	61,490	106,722	111,862
MFV	**111,859.0**	111,838.0	77,987.7	76,066.2	111,826.8	75,579.9	75,447.9	60,380.6	106,010.4	111,846.6
STD	**13.4**	45.9	342.1	312.8	68.5	596.3	683.5	672.9	366.3	36.6
W-test	/	0	1	1	1	1	1	1	1	0
2000.1										
BFV	**148,823**	148,773	100,349	98,641	148,813	98,272	98,454	80,555	135,438	148,823
MFV	**148,817.0**	148,663.7	99,782.1	97,790.2	148,723.0	96,737.3	97,244.9	79,560.3	134,659.1	148,663.2
STD	**22.6**	111.3	379.3	394.2	85.6	913.1	555.4	681.8	409.3	133.2
W-test	/	1	1	1	1	1	1	1	1	1

Table 12 continued

	DHLO	HLO	BDE	S_BAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
2000.2										
BFV	**148,120**	148,063	99,919	98,309	148,120	98,590	97,984	79,898	135,168	148,067
MFV	**148,113.5**	147,932.9	99,175.7	97,529.6	147,520.6	96,963.0	96,365.0	78,991.6	134,358.5	147,906.6
STD	**23.0**	114.4	395.7	457.5	185.3	710.0	615.7	523.6	539.0	104.9
W-test	/	1	1	1	1	1	1	1	1	1
2000.3										
BFV	**148,118**	148,068	100,242	98,974	148,118	97,777	98,035	79,813	134,802	148,068
MFV	**148,109.5**	147,896.4	99,344.3	97,424.2	146,196.2	96,610.7	96,586.7	79,073.9	133,877.7	147,936.5
STD	**25.4**	123.8	510.5	507.8	143.7	691.2	759.8	559.3	434.7	94.2
W-test	/	1	1	1	1	1	1	1	1	1
2500.1										
BFV	**148,823**	148,773	100,349	98,641	148,813	98,272	98,454	80,555	135,438	148,823
MFV	**148,817.0**	148,663.7	99,782.1	97,790.2	148,723.0	96,737.3	97,244.9	79,560.3	134,659.1	148,663.2
STD	**22.6**	111.3	379.3	394.2	85.6	913.1	555.4	681.8	409.3	133.2
W-test	/	1	1	1	1	1	1	1	1	1
2500.2										
BFV	**148,120**	148,063	99,919	98,309	148,120	98,590	97,984	79,898	135,168	148,067
MFV	**148,113.5**	147,932.9	99,175.7	97,529.6	147,520.6	96,963.0	96,365.0	78,991.6	134,358.5	147,906.6
STD	**23.0**	114.4	395.7	457.5	185.3	710.0	615.7	523.6	539.0	104.9
W-test	/	1	1	1	1	1	1	1	1	1
2500.3										
BFV	**148,118**	148,068	100,242	98,974	148,118	97,777	98,035	79,813	134,802	148,068
MFV	**148,109.5**	147,896.4	99,344.3	97,424.2	146,196.2	96,610.7	96,586.7	79,073.9	133,877.7	147,936.5
STD	**25.4**	123.8	510.5	507.8	143.7	691.2	759.8	559.3	434.7	94.2
W-test	/	1	1	1	1	1	1	1	1	1

The ranking results in Table 13 show that the performance of all the 10 algorithms sorted in the descending order is DHLO, HLO, MBPSO, ABHS, BVDPSO, BDE, S_bAFSA, BGSA, BBA, and BABC, and the W-test results in Table 14 claim that DHLO is significant better than MBPSO, HLO, ABHS, BVDPSO, BDE, S_bAFSA, BGSA, BBA, and BABC on 18, 18, 21, 23, 27, 27, 27, 27, 27 out of 30 instances while it is worse than them on no one.

4.3.1 Multidimensional knapsack problems

The multidimensional knapsack problem (MKP) is a multi-constrained problem. The objective of MKPs is still to find out an optimal subset for the maximum total profit but with multiple constrains instead of only one constrain in the basic 0-1 knapsack problem, which can be formulated as Eq.(13):

$$max\ f(\mathrm{x}_1, \mathrm{x}_2, \ldots, \mathrm{x}_\mathrm{N}) = \sum_{j=1}^{\mathrm{N}} p_j x_j$$

$$s.t. \begin{cases} \sum\limits_{j=1}^{\mathrm{N}} r_{ij} x_j \le c_i, & i \in \{1, 2, \ldots, \mathrm{M}\} \\ x_j \in \{0, 1\}, & \mathrm{j} \in \{1, 2, \ldots, \mathrm{N}\} \end{cases} \tag{13}$$

where N is the number of items, M is the number of constrains, p_j is the profit of the *j*th item, c_i is the capacity of the *i*th knapsack, and r_{ij} is the weight of the *j*th item in the *i*th knapsack with capacity constrain c_i.

The MKP is well known to be much more difficult than the basic single-constrained 0-1 knapsack problem, thus various powerful local search or repair strategies have been to developed and introduced into meta-heuristics for fixing infeasible solutions and improving results. However, the real performance of meta-heuristics would be concealed with these additional heuristic operators, and therefore the penalty function strategy is still adopted in MKPs. Previous work [37] indicates that the penalty function method, called *pCOR,* has the best results on solving MKPs, and thus *pCOR* is adopted in this paper which can be described as Eqs. (14–15),.

$$pCOR(x) = \frac{p_{\max} + 1}{r_{\min}} \times \max\{CV(x, i)\} \tag{14}$$

$$CV(x, i) = \max\left(0, \sum r_{ij} x_j - c_j\right) \tag{15}$$

where *pCOR(x)* is the penalty coefficient used in the penalty function for infeasible solutions, p_{max} is the maximum profit coefficient, r_{min} is the minimum resource consumption, and CV(x,i) is the amount of constraint violation for constraint i.

For a comprehensive comparison, six problem sets from the OR-Library, i.e. Pet, Sento, HP, 5-100, 10-100, and gk, of which the number of item ranges from 6 to 2500, are adopted to test the performance of DHLO as well as the other meta-heuristics. For the problems in which the number of items is less than 1000, the population size and the maximum generation of all the algorithms are set to 100 and 5000. Otherwise, the cases are regarded as high-dimensional problems and the population size and the maximum generation of the meta-heuristics increase to 300 and 10,000. The numerical results are given in Tables 15, 16, 17, and the ranking and W-test results on all the instances are summarized in Tables 18 and 19, respectively.

The results in Tables 15, 16, 17 indicate that MKPs are much complicated than the basic 0-1 KPs, and therefore most algorithms with the penalty function method can only find the

Table 13 The rankings of all the algorithms on the 0-1 knapsack problems

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
50.1	1	1	1	9	1	5	6	8	7	10
50.2	1	1	1	1	1	1	10	7	8	9
50.3	1	1	1	1	1	1	1	1	1	1
100.1	1	1	8	6	1	9	7	10	1	1
100.2	1	1	7	6	1	9	8	10	1	1
100.3	1	1	7	6	1	8	9	10	1	1
250.1	1	4	6	7	5	8	9	10	2	3
250.2	2	3	6	7	1	9	8	10	5	4
250.3	1	1	6	7	1	8	9	10	1	1
500.1	1	2	6	7	5	8	9	10	4	3
500.2	1	2	6	7	5	9	8	10	4	3
500.3	1	2	6	7	5	9	8	10	4	3
800.1	1	2	6	7	4	8	9	10	5	3
800.2	1	4	6	7	3	9	8	10	5	2
800.3	1	3	6	7	4	8	9	10	5	2
1000.1	1	2	6	7	5	9	8	10	4	3
1000.2	1	2	6	7	5	9	8	10	4	3
1000.3	1	3	6	7	4	8	9	10	5	2
1200.1	1	3	6	7	4	8	9	10	5	2
1200.2	1	3	6	7	4	8	9	10	5	1
1200.3	1	2	6	7	4	8	9	10	5	3
1500.1	1	2	6	7	4	9	8	10	5	3
1500.2	1	4	6	7	2	9	8	10	5	3
1500.3	1	3	6	7	4	8	9	10	5	2
2000.1	1	3	6	7	2	9	8	10	5	4
2000.2	1	2	6	7	4	8	9	10	5	3
2000.3	1	3	6	7	4	8	9	10	5	2
2500.1	1	3	6	7	4	8	9	10	5	2
2500.2	1	2	6	8	3	7	9	10	5	4
2500.3	1	2	6	7	4	8	9	10	5	3
Average	1.03	2.27	5.63	6.60	3.20	7.77	8.27	9.53	4.23	2.90

Table 14 The summary of the W-test results between DHLO and the other meta-heuristics on 0-1 knapsack problems

W-test	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1	18	27	27	21	27	27	27	23	18
0	12	3	3	9	3	3	3	7	12
−1	0	0	0	0	0	0	0	0	0

Table 15 Results of the Pet and Sento problem sets

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
Pet1 (3800)	BFV	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**
	MBV	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**	**3800**
	STD	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**
	W-test	/	0	0	0	0	0	0	0	0	0
Pet2 (87,061)	BFV	**87,061**	87,061	87,061	**87,061**	87,061	87,061	87,061	**87,061**	87,061	87,061
	MBV	**87,061**	87,033	84,713	**87,061**	84,906	84,861	81,337	**87,061**	85,329	84,389
	STD	**0**	125	3165	**0**	2610	1860	4369	**0**	2197	1832
	W-test	/	0	1	0	1	1	1	0	1	1
Pet3 (4015)	BFV	**4015**	**4015**	**4015**	**4015**	**4015**	**4015**	**4015**	4015	**4015**	**4015**
	MBV	**4015**	**4015**	**4015**	**4015**	**4015**	**4015**	**4015**	3980	**4015**	**4015**
	STD	**0**	**0**	**0**	**0**	**0**	**0**	**0**	37	**0**	**0**
	W-test	/	0	0	0	0	0	0	1	0	0
Pet4 (6120)	BFV	**6120**	**6120**	**6120**	**6120**	6120	6120	**6120**	6070	**6120**	6120
	MBV	**6120**	**6120**	**6120**	**6120**	6106	6095	**6120**	5922	**6120**	6112
	STD	**0**	**0**	**0**	**0**	26	22	**0**	103	**0**	15
	W-test	/	0	0	0	1	1	0	1	0	1
Pet5 (12,400)	BFV	**12,400**	**12,400**	12,400	**12,400**	12,370	12,350	12,400	12,040	**12,400**	12,400
	MBV	**12,400**	**12,400**	12,399	**12,400**	12,246	12,224	12,398	11,764	**12,400**	12,381
	STD	**0**	**0**	3	**0**	73	77	4	185	**0**	28
	W-test	/	0	0	0	1	1	0	1	0	0
Pet6 (10,618)	BFV	**10,618**	10,618	10,618	10,618	10,586	10,523	10,588	10,344	10,618	10,618
	MBV	**10,605**	10,598	10,596	10,583	10,391	10,378	10,551	10,093	10,599	10,549
	STD	**15**	23	25	25	73	90	25	122	15	53
	W-test	/	0	0	1	1	1	1	1	0	1

Table 15 continued

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
Pet7 (16,537)	BFV	**16,537**	16,537	16,537	16,524	16,203	16,270	16,397	15,842	16,537	16,517
	MBV	**16,516**	16,513	16,516	16,493	16,122	16,038	16,327	15,448	16,503	16,414
	STD	31	52	32	27	98	146	50	205	32	59
	W-test	/	1	0	1	1	1	1	1	0	1
Sento 1 (7772)	BFV	**7772**	7772	7772	7772	6582	6334	7238	5904	7772	7761
	MBV	**7759**	7741	7736	7746	4839	4944	6791	5129	7759	7677
	STD	**17**	43	30	39	1001	824	222	394	20	79
	W-test	/	1	0	0	1	1	1	1	0	1
Sento 2 (8722)	BFV	**8722**	8722	8722	8715	8423	8477	8513	7934	8722	8711
	MBV	**8712**	8708	8712	8703	8118	8156	8437	7141	8707	8668
	STD	**8**	29	8	11	295	233	52	302	12	34
	W-test	/	1	0	1	1	1	1	1	0	1
HP 1 (3418)	BFV	**3418**	3418	3418	3418	3418	3418	3418	3344	3418	3418
	MBV	**3416.6**	3412.6	3409.8	3409.8	3405.6	3361.8	3403.8	3326.2	3412.4	3387.6
	STD	**3.3**	7.4	7.5	7.5	27.7	43.6	10.6	17.8	7.7	37.0
	W-test	/	0	0	0	0	1	0	1	0	1
HP 2 (3186)	BFV	**3186**	3186	3186	**3186**	3186	3186	3168	3059	3186	3157
	MBV	**3186.0**	3180.4	3178.6	**3186.0**	3169.0	3155.0	3147.2	3005.8	3164.4	3124.4
	STD	**0.0**	13.1	10.1	**0.0**	31.3	30.8	29.5	32.0	29.6	35.6
	W-test	/	0	0	0	0	1	1	1	1	1

Table 16 Results of the 5.100 and 10.100 problem sets

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
5.100.00 (24,381)	BFV	**24,186**	24,073	24,095	24,132	21,112	20,828	21,451	20,474	24,073	23,623
	MBV	**24,016**	23,785	23,700	23,768	19,598	19,577	20,134	19,686	23,725	23,072
	STD	**119**	182	207	230	928	847	843	338	165	343
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.01 (24,274)	BFV	**24,130**	23,997	23,948	23,941	21,082	21,015	20,907	19,535	24,077	23,095
	MBV	**23,857**	23,625	23,536	23,457	19,450	19,397	19,745	19,288	23,604	22,645
	STD	**154**	187	164	257	940	629	592	195	253	302
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.02 (23,551)	BFV	**23,494**	23,292	23,187	23,344	19,498	19,279	20,293	19,570	23,322	22,811
	MBV	**23,258**	22,930	22,797	22,986	18,327	18,432	19,047	18,896	22,985	22,053
	STD	**126**	439	245	198	749	589	687	330	189	413
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.03 (23,534)	BFV	**23,393**	23,186	23,188	23,268	21,104	20,511	20,339	20,206	23,088	22,720
	MBV	**23,120**	22,914	22,871	22,891	19,592	19,332	19,597	19,549	22,818	22,105
	STD	**135**	401	208	211	823	775	380	297	159	418
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.04 (23,991)	BFV	**23,821**	23,522	23,634	23,627	20,029	21,062	20,745	20,063	23,779	23,375
	MBV	**23,618**	23,432	23,276	23,343	19,326	19,445	19,791	19,441	23,289	22,772
	STD	**127**	169	186	187	577	832	474	332	200	247
	W-test	/	1	1	1	1	1	1	1	1	1

Table 16 continued

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
5.100.05 (24,613)	BFV	**24,613**	24,204	24,263	24,240	21,253	21,048	21,278	20,405	24,431	23,507
	MBV	**24,213**	23,944	23,837	23,898	20,853	19,723	20,067	19,861	23,947	22,909
	STD	**131**	324	203	197	345	833	709	253	286	327
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.06 (25,591)	BFV	**25,388**	25,153	25,110	25,219	21,107	21,736	21,737	21,117	25,176	24,999
	MBV	**25,208**	24,864	24,715	24,853	20,723	20,474	20,755	20,492	24,883	24,012
	STD	**142**	410	268	194	435	697	636	307	152	427
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.07 (23,410)	BFV	**23,330**	22,975	22,920	23,053	20,065	19,613	20,079	19,617	23,044	22,550
	MBV	**23,005**	22,619	22,574	22,660	19,272	18,871	19,144	19,087	22,616	21,874
	STD	**213**	205	252	239	757	555	434	341	256	333
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.08 (24,216)	BFV	**24,100**	23,917	23,652	23,933	20,601	20,870	20,861	20,390	23,979	23,658
	MBV	**23,854**	23,645	23,433	23,575	19,707	19,314	19,719	19,543	23,527	22,753
	STD	**157**	262	191	243	771	776	413	377	273	396
	W-test	/	1	1	1	1	1	1	1	1	1
5.100.09 (24,411)	BFV	**24,246**	23,987	23,954	24,099	21,039	20,889	21,235	20,564	24,042	23,698
	MBV	**24,024**	23,745	23,679	23,732	19,805	19,681	19,885	19,894	23,712	23,042
	STD	**118**	122	212	235	768	792	435	273	172	376
	W-test	/	1	1	1	1	1	1	1	1	1

Table 16 continued

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
10.100.00 (23,064)	BFV	**22,959**	22,677	22,638	22,726	20,015	21,844	19,423	19,188	22,449	22,667
	MBV	**22,693**	22,344	22,302	22,360	18,679	21,144	18,404	18,573	22,158	21,702
	STD	**125**	234	197	250	758	546	446	280	179	351
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.01 (22,801)	BFV	**22,566**	22,457	22,276	22,547	19,865	20,288	18,834	18,834	22,457	22,054
	MBV	**22,244**	21,826	21,824	21,986	17,997	18,206	17,919	18,327	21,790	21,264
	STD	**154**	236	267	273	937	1018	385	273	402	391
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.02 (22,131)	BFV	**21,800**	21,766	21,784	21,753	18,569	18,905	19,437	18,460	21,671	21,296
	MBV	**21,525**	21,361	21,270	21,407	17,538	16,511	17,684	17,913	21,241	20,448
	STD	233	448	265	199	583	5755	553	307	247	477
	W-test	/	1	1	0	1	1	1	1	1	1
10.100.03 (22,772)	BFV	**22,463**	22,456	22,213	22,377	20,338	19,513	19,471	19,400	22,427	21,989
	MBV	**22,245**	21,949	21,869	21,985	18,518	18,293	18,815	18,939	21,895	21,236
	STD	**135**	378	263	256	874	1585	353	273	198	389
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.04 (22,751)	BFV	**22,361**	22,342	22,403	22,139	18,813	19,415	18,954	18,985	22,242	21,559
	MBV	**22,136**	21,803	21,775	21,793	18,066	17,993	18,097	18,366	21,808	20,981
	STD	**142**	259	271	380	443	778	540	309	288	340
	W-test	/	1	1	1	1	1	1	1	1	1

Table 16 continued

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
10.100.05 (22,777)	BFV	**22,461**	22,406	22,442	22,308	19,642	19843	19,175	19,542	22,129	22,039
	MBV	**22,190**	21,874	21,907	21,854	18,855	18,531	18,523	18,731	21,859	21,269
	STD	**113**	242	216	286	467	864	412	369	210	406
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.06 (21,875)	BFV	**21,731**	21,720	21,502	21,729	19,040	19,029	18,555	17,932	21,692	21,075
	MBV	**21,484**	21,047	21,039	21,062	17,126	17,005	17,257	17,306	21,110	20,094
	STD	**163**	204	229	406	675	958	576	328	283	404
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.07 (22,635)	BFV	**22,368**	22,316	22,035	22,352	19,160	18,720	18,872	19,081	22,111	21,609
	MBV	**22,061**	21,831	21,721	21,849	18,106	18,006	17,853	18,342	21,691	21,008
	STD	278	264	209	222	602	743	569	383	233	398
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.08 (22,511)	BFV	**22,337**	22,085	21,898	22,165	18,621	18,337	18,358	18,487	22,058	21,572
	MBV	**22,004**	21,465	21,589	21,765	17,465	17,310	17,580	17,708	21,703	20,943
	STD	205	190	185	229	609	870	478	402	143	367
	W-test	/	1	1	1	1	1	1	1	1	1
10.100.09 (22,702)	BFV	**22,391**	22,249	22,213	22,285	19,438	19,403	18,557	19,144	21,995	21,777
	MBV	**22,123**	21,857	21,806	21,814	18,095	18,085	17,996	18,375	21,713	20,933
	STD	**164**	271	228	280	338	671	466	294	186	378
	W-test	/	1	1	1	1	1	1	1	1	1

Table 17 Results of the gk problem set

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
gk 01 (3766)	BFV	**3746**	3739	3735	3746	3644	3628	3668	3610	3737	3713
	MBV	**3730**	3730	3718	3727	3618	3606	3647	3578	3717	3688
	STD	**8**	6	14	9	10	9	11	11	10	14
	W-test	/	1	1	0	1	1	1	1	1	1
gk 02 (3958)	BFV	**3942**	3934	3927	3933	3844	3837	3854	3798	3934	3894
	MBV	**3919**	3917	3906	3911	3806	3809	3835	3775	3909	3867
	STD	10	10	12	14	13	13	8	13	9	16
	W-test	/	0	0	0	1	1	1	1	0	1
gk 03 (5650)	BFV	**5609**	5607	5592	5580	5477	5468	5508	5427	5595	5573
	MBV	**5590**	5587	5574	5556	5441	5433	5477	5403	5566	5539
	STD	**10**	12	13	14	15	14	16	11	15	18
	W-test	/	0	1	1	1	1	1	1	1	1
gk 04 (5764)	BFV	**5725**	5709	5699	5694	5595	5594	5614	5574	5698	5671
	MBV	**5696**	5683	5684	5659	5570	5565	5594	5538	5666	5642
	STD	12	13	8	21	13	12	8	12	13	19
	W-test	/	1	1	1	1	1	1	1	1	1
gk 05 (7557)	BFV	**7489**	7480	7465	7445	7313	7313	7329	7262	7458	7436
	MBV	**7467**	7465	7440	7412	7277	7265	7308	7230	7429	7383
	STD	16	10	13	21	19	23	12	14	14	27
	W-test	/	0	1	1	1	1	1	1	1	1

Table 17 continued

Instances (best known)		DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
gk 06 (7672)	BFV	**7612**	7604	7586	7563	7454	7473	7485	7435	7564	7551
	MBV	**7587**	7574	7561	7523	7426	7420	7459	7393	7548	7521
	STD	15	16	17	19	14	18	16	18	13	17
	W-test	/	1	1	1	1	1	1	1	1	1
gk 07 (19,215)	BFV	**18,981**	18,951	18,792	18,811	18,571	18,552	18,582	18,440	18,815	18,795
	MBV	**18,921**	18,900	18,746	18,750	18,475	18,442	18,536	18,389	18,777	18,712
	STD	35	41	19	44	41	41	23	26	21	45
	W-test	/	1	1	1	1	1	1	1	1	1
gk 08 (18,801)	BFV	**18,579**	18,574	18,281	18,456	18,280	18,281	18,322	18,226	18,468	18,448
	MBV	18,529	18,534	18,197	18,360	18,210	18,214	18,272	18,168	18,417	18,396
	STD	**32**	23	48	31	34	35	19	20	26	34
	W-test	/	0	1	1	1	1	1	1	1	1
gk 09 (58,085)	BFV	**57,286**	57,264	56,562	56,713	56,164	56,112	56,142	55,967	56,767	56,649
	MBV	**57,163**	57,088	56,485	56,551	55,993	55,971	56,081	55,864	56,678	56,520
	STD	67	91	32	108	95	89	44	44	52	64
	W-test	/	1	1	1	1	1	1	1	1	1
gk 10 (57,292)	BFV	**56,567**	56,514	56,105	56,104	56,154	55,856	55,808	55,728	56,216	56,146
	MBV	**56,405**	56,404	56,062	56,001	55,981	55,687	55,758	55,622	56,120	56,078
	STD	66	51	23	66	101	84	21	44	49	42
	W-test	/	0	1	1	1	1	1	1	1	1
gk 11 (95,231)	BFV	**94,061**	93,937	93,597	93,494	93,299	93,326	93,246	93,110	93,674	93,674
	MBV	**93,941**	93,858	93,529	93,404	93,113	93,150	93,213	93,050	93,482	93,601
	STD	62	42	28	40	89	80	14	30	107	53
	W-test	/	1	1	1	1	1	1	1	1	1

Table 18 The rankings of all the algorithms on multidimensional knapsack problems

	DHLO	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
Pet1	1	1	1	1	1	1	1	1	1	1
Pet2	1	4	8	1	6	7	10	1	5	9
Pet3	1	1	1	1	1	1	1	10	1	1
Pet4	1	1	1	1	8	9	1	10	1	7
Pet5	1	1	5	1	8	9	6	10	1	7
Pet6	1	3	4	5	8	9	6	10	2	7
Pet7	1	3	1	5	8	9	7	10	4	6
Sento 1	1	4	5	3	10	9	7	8	1	6
Sento 2	1	3	1	5	9	8	7	10	4	6
HP 1	1	2	4	5	6	9	7	10	3	8
HP 2	1	3	4	1	5	7	8	10	6	9
5.100.00	1	2	5	3	9	10	7	8	4	6
5.100.01	1	2	4	5	8	9	7	10	3	6
5.100.02	1	4	5	2	10	9	7	8	3	6
5.100.03	1	2	4	3	8	10	7	9	5	6
5.100.04	1	2	5	3	10	8	7	9	4	6
5.100.05	1	3	5	4	7	10	8	9	2	6
5.100.06	1	3	5	4	8	10	7	9	2	6
5.100.07	1	3	5	2	7	10	8	9	4	6
5.100.08	1	2	5	3	8	10	7	9	4	6
5.100.09	1	2	5	3	9	10	8	7	4	6
10.100.00	1	3	4	2	8	7	10	9	5	6
10.100.01	1	3	4	2	9	8	10	7	5	6
10.100.02	1	3	4	2	9	10	8	7	5	6
10.100.03	1	3	5	2	9	10	8	7	4	6
10.100.04	1	3	5	4	9	10	8	7	2	6
10.100.05	1	3	2	5	7	9	10	8	4	6
10.100.06	1	4	5	3	9	10	8	7	2	6
10.100.07	1	3	4	2	8	9	10	7	5	6
10.100.08	1	5	4	2	9	10	8	7	3	6
10.100.09	1	2	4	3	8	9	10	7	5	6
gk 01	1	1	4	3	8	9	7	10	5	6
gk 02	1	2	5	3	9	8	7	10	4	6
gk 03	1	2	3	5	8	9	7	10	4	6
gk 04	1	3	2	5	8	9	7	10	4	6
gk 05	1	2	3	5	8	9	7	10	4	6
gk 06	1	2	3	5	8	9	7	10	4	6
gk 07	1	2	5	4	8	9	7	10	3	6
gk 08	2	1	9	5	8	7	6	10	3	4
gk 09	1	2	6	4	8	9	7	10	3	5
gk 10	1	2	5	6	7	9	8	10	3	4
gk 11	1	2	4	6	9	8	7	10	5	3
Average	1.02	2.48	4.12	3.31	7.81	8.60	7.17	8.57	3.48	5.83

Table 19 The summary of the W-test results between DHLO and the other meta-heuristics on multidimensional knapsack problems

W-test	HLO	BDE	S_bAFSA	ABHS	BGSA	BBA	BABC	BVDPSO	MBPSO
1	29	31	31	38	40	37	40	32	39
0	13	11	11	4	2	5	2	10	3
−1	0	0	0	0	0	0	0	0	0

best solutions of the first six instances of the simple problem set Pet, of which the number of items is no more than 39. As for the complicated problem sets like 5.100, 10.100, and gk, only DHLO successfully searches out the optimal solution on case 5.100.05. Specifically, DHLO, HLO, BDE, S_bAFSA, ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO find 12, 8, 11, 9, 6, 5, 4, 5, 10, and 6 best known solutions out of 42 instances, respectively, and DHLO achieves better fitness values on all the instances. Table 18 illustrates that the performance ranking of all the algorithms sorted in the descending order is DHLO, HLO, S_bAFSA, BVDPSO, BDE, MBPSO, BBA, ABHS, BABC, and BGSA, and the W-test results in Table 19 indicate that DHLO also has an advantage over the other algorithms on MKPs since it is superior to HLO, BDE, S_bAFSA, ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO on 29, 31, 31, 38, 40, 37, 40, 32, and 39 out of 42 instances, respectively.

In summary, based on the results of the benchmark functions and knapsack problems, it is fair to claim that the presented DHLO has better optimization performance in terms of search accuracy and scalability in comparison to HLO, BDE, S_bAFSA, ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO. In addition, the results on CEC05 benchmark functions and knapsack problems hints that the performance of algorithms is sensitive to problems. For example, BDE and S_bAFSA have better performance in high-dimensional numerical function problems than two PSO variants, i.e. MBPSO and BVDPSO, while these two binary PSO algorithms both surpass BDE and S_bAFSA on 0-1 KPs. As for MBPSO and BVDPSO, it can be found that MBPSO is superior to BVDPSO on 0-1 KPs while it is worse than BVDPSO on MKPs. PSO, DE, AFSA and the other algorithms are originally developed to tackle continuous or discrete problems, and therefore the operators of these algorithms need to be re-defined and modified for binary problems. However, these re-definitions or modifications are not always easy or natural, and varied strategies would change the search ability of algorithms and lead to different strengths and weakness, which causes the diverse performance of MBPSO and BVDPSO on 0-1 KPs and MKPs. Compared with the other meta-heuristics such as PSO, DE, and AFSA, HLO is an inborn binary-coding algorithm and the results of benchmark functions and knapsack problems show that HLO has more robust and steadier performance on binary problems. Therefore, it is reasonable that the presented DHLO gains an advantage over the other algorithms since it inherits excellent characteristics from HLO on binary problems and the developed dynamic adjusting strategy as well as the re-initialization of the IKDs can adaptively balance the exploitation and exploration ability and efficiently help the algorithm escape from the local optima.

5 Concluding remarks

Human Learning Optimization is a novel binary-coded meta-heuristic based on a simplified model of human learning. By mimicking random learning, individual learning, and social learning of human being, HLO develops three learning operators, i.e. the random learning operator, the individual learning operator, and the social learning operator, to search the optimal solution efficiently. However, all the individuals in the standard HLO share the same control parameters of learning operations, that is, all the individuals possess the same learning ability, which is not true in a real human population. Inspired by the fact that human IQ scores follow Gaussian distribution and increase with the development of technology, this paper presents an improved HLO algorithm, named Diverse Human Learning Optimization, in which the Gaussian distributed learning operator and dynamic adjusting strategy are introduced. Through yielding a set of control parameters of learning operators following Gaussian distribution, the robustness of the algorithm is strengthened. Besides, by cooperating with the dynamic updating operation, DHLO can adjust to the better parameter values and hence the global search ability of the algorithm is enhanced. The proposed DHLO is applied to solve CEC05 benchmark functions and knapsack problems to evaluate its performance against the standard HLO and the other eight meta-heuristics, i.e. BDE, S_bAFSA, ABHS, BGSA, BBA, BABC, BVDPSO, and MBPSO. The comparison results demonstrate that DHLO is superior to the other nine algorithms in terms of search accuracy and scalability.

As mentioned above, DHLO, as well as HLO, is based on a simplified human learning model while the real human learning is an extremely complicated process. During the last decades many achievements on cognitive science and learning theories have been reported. Therefore, one of our future work is to introduce these achievements on human learning into HLO to consummate the algorithm. Another important direction of the future work is to extend the applications of HLO for better understanding the characteristics of HLO as well as further improving its performance.

Acknowledgments This work is supported by National Natural Science Foundation of China (Grant No. 61304031, 61374044 & 61304143), Innovation Program of Shanghai Municipal Education Commission (14YZ007), Key Project of Science and Technology Commission of Shanghai Municipality under Grant No. 14JC1402200 and 14DZ1206302, Key Project of Shanghai Municipal Commission of Economy and Informatization (ZB-ZBYZ-02-14-0825), AirForce and DTRA grants (P. Pardalos, J. Pi), and a Paul and Heidi Brown Preeminent Professorship in Industrial and Systems Engineering, University of Florida.

References

1. Kim, H., Liou, M.S.: New fitness sharing approach for multi-objective genetic algorithms. J. Global Optim. **55**(3), 579–595 (2013)
2. Das, S., Suganthan, P.N.: Differential evolution: a survey of the state-of-the-art. IEEE Trans. Evol. Comput. **15**(1), 4–31 (2011)
3. Kaucic, M.: A multi-start opposition-based particle swarm optimization algorithm with adaptive velocity for bound constrained global optimization. J. Global Optim. **55**(1), 165–188 (2013)
4. Dorigo, M., Blum, C.: Ant colony optimization theory: a survey. Theor. Comput. Sci. **344**(2), 243–278 (2005)
5. Wang, L., Ni, H.Q., Yang, R.X., Pappu, V., Fenn, M.B., Pardalos, P.M.: Feature selection based on meta-heuristics for biomedicine. Optim. Methods Softw. **29**(4), 703–719 (2013)
6. Wang, L., Ni, H.Q., Zhou, W.F., Pardalos, P.M., Fang, J.T., Fei, M.R.: MBPOA-based LQR controller and its application to the double-parallel inverted pendulum system. Eng. Appl. Artif. Intell. **36**, 262–268 (2014)
7. Wang, L., Ye, W., Mao, Y.F., Georgiev, P.G., Wang, H.K., Fei, M.R.: The node placement of large-scale industrial wireless sensor networks based on binary differential evolution harmony search algorithm. Int. J. Innov. Comput. Inf. Control. **9**(3), 955–970 (2013)
8. Rocha, A.M.A., Costa, M.F.P., Fernandes, E.M.: A filter-based artificial fish swarm algorithm for constrained global optimization: theoretical and practical issues. J. Global Optim. **60**(2), 239–263 (2014)

9. Akay, B.: Synchronous and asynchronous Pareto-based multi-objective Artificial Bee Colony algorithms. J. Global Optim. **57**(2), 415–445 (2013)
10. Yang, X.S.: A New Metaheuristic Bat-Inspired Algorithm. Nature Inspired Cooperative Strategies for Optimization. Springer, Berlin (2010)
11. Oftadeh, R., Mahjoob, M.J., Shariatpanahi, M.: A novel meta-heuristic optimization algorithm inspired by group hunting of animals: hunting search. Comput. Math. Appl. **60**(7), 2087–2098 (2010)
12. Geem, Z.W., Kim, J.H., Loganathan, G.V.: A new heuristic optimization algorithm: harmony search. Simulation **76**(2), 60–68 (2001)
13. Pan, W.T.: A new fruit fly optimization algorithm: taking the financial distress model as an example. Knowl. Based Syst. **26**, 69–74 (2012)
14. Fister, I., Yang, X.S., Brest, J.: A comprehensive review of firefly algorithms. Swarm Evol. Comput. **13**, 34–46 (2013)
15. Elbeltagi, E., Hegazy, T., Grierson, D.: A modified shuffed frog-leaping optimization algorithm: applications to project management. Struct. Infrastruct. Eng. **3**(1), 53–60 (2007)
16. Yang, X.S.: Cuckoo Search and Firefly Algorithm. Springer, Berlin (2014)
17. Wang, L., Ni, H.Q., Yang, R.X., Fei, M.R., Ye, W.: A simple human learning optimization algorithm. Commun. Comput. Inf. Sci. **462**, 56–65 (2014)
18. Herrnstein, R.J., Murry, C.: The bell curve: intelligence and class structure in American life. The Free Press, New York (1994)
19. Flynn, J.R.: Are We Getting Smarter? Rising IQ in the Twenty-first Century. Cambridge University Press, New York (2012)
20. Flynn, J.R.: The mean IQ of Americans: massive gains 1932 to1978. Psychol. Bull. **95**(1), 29 (1984)
21. Cziko, G.: Without Miracles: Universal Selection Theory and the Second Darwinian Revolution. MIT Press, Cambridge (1997)
22. Forcheri, P., Molfino, M.T., Quarati, A.: ICT driven individual learning: new opportunities and perspectives. Educ. Technol. Soc. **3**, 51–61 (2000)
23. Andrews, K.M., Delahaye, B.L.: Influences on knowledge processes in organizational learning: the psychosocial filter. J. Manag. Stud. **37**(6), 797–810 (2002)
24. Zhong, J.G.: A review of studies on the individual difference of intelligence in the IQ-normal group. Psychol. Sci. **30**(2), 394–399 (2007). (in Chinese)
25. Suganthan, P.N., Hansen, N., Liang, J.J., Deb, K., Chen, Y., Auger, A., Tiwari, S.: Problem Definitions and Evaluation Criteria for the CEC 2005 Special Session on Real-Parameter Optimization, KanGAL Report 2005005 (2005)
26. Chen, Y., Xie, W., Zou, X.: A binary differential evolution algorithm learning from explored solutions. Neurocomputing **149**, 1038–1047 (2015)
27. Azad, M.A.K., Rocha, A.M.A.C., Fernandes, E.M.G.P.: A simplified binary artificial fish swarm algorithm for 0.1 quadratic knapsack problems. J. Comput. Appl. Math. **259**, 897–904 (2014)
28. Wang, L., Yang, R.X., Xu, Y., Niu, Q., Pardalos, P.M., Fei, M.R.: An improved adaptive binary Harmony Search algorithm. Inf. Sci. **232**, 58–87 (2013)
29. Yuan, X., Ji, B., Zhang, S., Tian, H., Hou, Y.: A new approach for unit commitment problem via binary gravitational search algorithm. Appl. Soft Comput. **22**, 249–260 (2014)
30. Mirjalili, S., Mirjalili, S.M., Yang, X.S.: Binary bat algorithm. Neural Comput. Appl. **25**(3–4), 1–19 (2014)
31. Chandrasekaran, K., Hemamalini, S., Simon, S.P., Padhy, N.P.: Thermal unit commitment using binary/real coded artificial bee colony algorithm. Electric Power Syst. Res. **84**, 109–119 (2012)
32. Shen, M., Zhan, Z.H., Chen, W.N., Gong, Y.J., Zhang, J., Li, Y.: Bi-velocity discrete particle swarm optimization and its application to multicast routing problem in communication networks. IEEE Trans. Ind. Electron. **61**(12), 7141–7151 (2014)
33. Bansal, J.C., Deep, K.: A modified binary particle swarm optimization for knapsack problems. Appl. Math. Comput. **218**(22), 11042–11060 (2012)
34. Liao, T., Molina, D., Stutzle, T.: Performance evaluation of automatically tuned continuous optimizers on different benchmark sets. Appl. Soft Comput. **27**, 490–503 (2015)
35. Zou, D., Gao, L., Li, S., Wu, J.: Solving 0–1 knapsack problem by a novel global harmony search algorithm. Appl. Soft Comput. **11**(2), 1556–1564 (2011)
36. Wang, L., Ni, H.Q., Yang, R.X., Pardalos, P.M., Du, X., Fei, M.R.: An adaptive simplified human learning optimization algorithm. Inf. Sci. **320**, 126–139 (2015)
37. Gottlieb, J.: On the feasibility problem of penalty-based evolutionary algorithms for knapsack problems. Lecture Notes in Computer Science, pp. 50–59 (2001)