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Acoustic wave propagation in fluid metamaterial with solid inclusions

I.V. Lisenkov · R.S. Popov · S.A. Nikitov

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Abstract Acoustic wave propagation in a composite of water with embedded double-layered silicone resin/silver rods is considered. Approximate values of effective dynamical constitutive parameters are obtained. Frequency ranges of simultaneous negative constitutive parameters are found. Localized surface states on the interface between metamaterial and "normal" material are found. The Doppler effect in metamaterial is considered. The presence of anomalous modes is shown.

1 Introduction

Media with effective negative constitutive parameters have attracted attention for several decades due to unique properties of wave propagation in them. In pioneering works of Veselago [1] and Pendry [2] electromagnetic waves in media with simultaneous negative ϵ and μ were considered. Since that time, a lot of theoretical and experimental works have been done on this subject [3]. In fact, for acoustic waves metamaterials are possible too and for them both density and stiffness should be negative. Some proposals and experiments to achieve negative effective constitutive parameters for acoustic composites were made in the past, exploiting various techniques, such as embedding soft inclusions in fluids [4, 5], using Helmholtz resonators [6] and pipe-membrane structures [7].

I.V. Lisenkov (✉) · R.S. Popov · S.A. Nikitov
Kotel'nikov Institute for Radio-engineering and Electronics of RAS, Moscow, Russia
e-mail: lisenkov@cplire.ru

I.V. Lisenkov · S.A. Nikitov
Department for Physical and Quantum Electronics, Moscow Institute for Physics and Technology (State University), Moscow, Russia

The aim of this work is to show that it is possible to construct a metamaterial exhibiting negative constitutive parameters within ultrasonic range and demonstrate that acoustic waves can exhibit exotic behavior, namely presence of localized states at the boundary between metamaterial and "normal" material and reversed Doppler effect. In the mathematical model both longitudinal and shear modes are included in calculations. Also internal dissipation losses are taken in account.

2 Mathematical model

2.1 Effective constitutive parameters

In this work a composite consisting of fluid host with rods embedded in it is considered. Rods have a round cylindrical shape and consist of elastic materials. Acoustic waves propagate in host material (density ρ_0 and bulk modulus B_0) with wave vector perpendicular to the generatixes of the rods. A metamaterial should be quasi-isotropic, so the wavelength is much longer than the distance between the centers of cylinders (L) and their radii (R) [3]. All cylinders are considered identical and are placed at random at approximate equal distance between neighbors. The aim is to compute the dynamical effective constitutive parameters of a metamaterial on given frequency and dispersion of the propagating bulk acoustic wave.

To proceed the calculations a coherent potential approximation (CPA) [5, 8] is used. We used this method applied to fluid/solid composite taking into account longitudinal and shear polarizations in solid inclusions and acoustic damping in both inclusions and host.

Following the CPA procedure, one inclusion is surrounded by a cylinder of radius L (forming coated inclusion) and outside the cylinder some effective fluid is placed. The constitutive parameters of the effective fluid (ρ_e for effective density and B_e for effective bulk modulus) are unknown and should be estimated. Pressure ($\Psi(r,\phi,t)$) and radial component of velocity ($v_r(r,\phi,t)$) in host material and in effective material can be decomposed by cylindrical harmonics [9]:

$$\begin{aligned}\Psi &= e^{-i\omega t}\sum_{n=0}^{\infty} X_n^m J_n(k_m r)e^{in\phi} + Y_n^m H_n(k_m r)e^{in\phi}\\ v_r &= e^{-i\omega t}\frac{k_m}{\rho_m}\sum_{n=0}^{\infty} X_n^m J_n'(k_m r)e^{in\phi} + Y_n^m H_n'(kr)e^{in\phi},\end{aligned} \tag{1}$$

where $J_n()$ and $H_n()$ are Bessel and Hankel functions of the first kind, n-th order, X_n^m, Y_n^m are unknowns and m corresponds to e in effective media and for 0 in host material.

On the interface between the host fluid and the effective medium standard boundary conditions of velocity and pressure continuity should be satisfied. Due to cylindrical symmetry and isotropy of materials, we could satisfy boundary conditions for each of the cylindrical harmonics independently [9]:

$$\begin{aligned}&\begin{bmatrix} J_n(k_eL) & H_n(k_eL)\\ \frac{k_e}{\rho_e}J_n(k_eL) & \frac{k_e}{\rho_e}H_n(k_eL)\end{bmatrix}\cdot\begin{bmatrix}X_n^e\\ Y_n^e\end{bmatrix}\\ &\quad=\begin{bmatrix} J_n(k_0L) & H_n(k_0L)\\ \frac{k_0}{\rho_0}J_n(k_0L) & \frac{k_0}{\rho_0}H_n(k_0L)\end{bmatrix}\cdot\begin{bmatrix}X_n^0\\ Y_n^0\end{bmatrix},\end{aligned} \tag{2}$$

where k_e is the wave-number in effective medium, k_0 is the wave-number in host medium and $e^{in\phi}$ and $e^{-i\omega t}$ factors are dropped out.

The main CPA condition is an absence of scattering on the boundary, which means that there is a perfect matching between the coated inclusion and surrounding effective medium. Setting $X_n^e = 1$ and $Y_n^e = 0$ in (2) we obtain the CPA equation:

$$\begin{aligned}&-\frac{k_0}{B_0}J_n'(k_0L)J(k_eL) + \frac{k_e}{B_e}J_n(k_0L)J'(k_eL)\\ &\quad= \frac{Y_n}{X_n}\left[\frac{k_0}{B_0}H_n'(k_0L)J(k_eL) - \frac{k_e}{B_e}H_n(k_0L)J'(k_eL)\right]\end{aligned} \tag{3}$$

Exploiting the fact that metamaterial should be quasi-isotropic, in other words, the phase of the wave in host material should not change much on a distance between neighbor inclusions ($k_0L \to 0$ and $k_eL \to 0$). We use asymptotic formulas for the Bessel and Hankel functions for the near-zero argument limit. For small argument approximation only two, namely, the zeroth and first orders of Bessel and Hankel functions are significant. After some algebra we obtain approximate formulas for the effective constitutive parameters:

$$B_e = \frac{B_0}{1-S_0}, \qquad \rho_e = \rho_0\frac{1-S_1}{1+S_1} \tag{4}$$

where

$$S_n = \frac{D_n}{1+D_n}\frac{4}{\pi}\frac{1}{k_0^2L^2}\cdot i \tag{5}$$

and $D_n = Y_n/X_n$ is a scattering coefficient of the one embedded rod in an infinite fluid. In our case it is scattering of the acoustic wave in fluid on a round solid cylinder. This problem was widely investigated in past [10].

As is seen from formulas (4), dynamical effective constitutive parameters are frequency dependent and in the case of resonance inside inclusion their values can be changed dramatically and even become negative. Thus the speed of acoustic wave inside the inclusions should be much smaller than in a host fluid, to form a resonance frequency at which the wavelength in a host is much longer than the distance between inclusions.

Also we should notice that the bulk modulus depends on a monopolar scattering coefficient and effective density depends on the dipole coefficient. This is true, because the monopole mode is the uniform compression or expansion of the inclusion and dipole mode is the inclusion shift.

The refractive index of the medium is described by the following formula:

$$n(\omega) = \sqrt{1/B_e(\omega)}\cdot\sqrt{\rho_e(\omega)} \tag{6}$$

If the argument of the refractive index tends to zero the wave vector is pointing in the same direction as the Poynting vector. The argument's value close to $\pi/2$ means that the one of the constitutive parameters is negative and the other positive, thus the wave finally decays. The argument's value close to π means that wave vector and Poynting vector are directed oppositely and the wave have backward behavior.

2.2 Surface acoustic wave

Along the boundary between two ideal fluids any surface wave cannot propagate, because the boundary conditions cannot be satisfied. But if the one of materials has one negative constitutive parameter, boundary states can be excited [11]. This is a direct analogy to surface plasmon states in plasma with $\epsilon < 0$ [3].

Let us consider a wave traveling on the interface between a half-space occupied by pure host fluid and a half-space occupied by a fluid with embedded rods in it. Matching boundary impedances of evanescent waves in both composite and pure fluid, one gets

$$\frac{\rho_0k_e^z - \rho_ek_0^z}{\rho_0k_e^z + \rho_ek_0^z} = 0, \tag{7}$$

where k_0^z and k_e^z are the projections of the wave vector perpendicular to the interface in a pure fluid and metamaterial, respectively. The dispersion equation for a surface wave has the following form:

$$k_s = \sqrt{\frac{(\rho_0\rho_e)(\rho_0/B_e - \rho_e/B_0)}{\rho_0^2 - \rho_e^2}}\omega. \tag{8}$$

We show below that the boundary states exist only if one of the constitutive parameters is negative.

3 Results

3.1 Bulk wave

In the model used for the calculations the rods are considered to be two-layered cylinders. Shell of the cylinder is soft silicone resin[1] ($\rho = 1.3 \times 10^3$ kg/m^3, $c_{11} = 5.2 \times 10^7$ N/m^2, $c_{44} = 3.25 \times 10^6$ N/m^2, Loss: 600 dB/m at 1 MHz), core of the cylinder is poly-crystal silver [12] ($\rho = 10.5 \times 10^3$ kg/m^3, $c_{11} = 1.397 \times 10^{11}$ N/m^2, $c_{44} = 2.7 \times 10^{10}$ N/m^2, Loss: 40 dB/m at 1 MHz). The outer radius of the rod is 3 mm, radius of the core is 0.5 mm, average distance between cylinders is taken 5 mm. Host material is water ($\rho = 10^3$ kg/m^3, $B = 2.25 \times 10^9$ N/m^2).

Figure 1 shows results of calculations for bulk wave in the composite. At low frequencies effective constitutive parameters tend to be close to "classic" values for composites [13]. But at frequencies close to resonances inside the inclusions the values of dynamical effective constitutive parameters are dramatically changed. In Fig. 1(a) two frequencies of monopolar resonances are seen, and close to the resonances there are two bands where the real part of compliance is negative (in this section we use compliance ($s = 1/B$) instead of stiffness to obtain symmetry between constitutive parameters in (6)).

In Fig. 1(b) frequency dependence of the dynamical effective density is shown. Due to the complex shear and longitudinal field structure inside inclusion, there are two families of resonances associated with shear waves and longitudinal waves (S and L in the figure, respectively). Thus the dynamical effective density has a number of frequency ranges where real part is negative. The amplitude of resonances associated with shear components becomes greater because of inertia of the core.

In Figs. 1(c) and 1(d) the dependence of the refractive index and its argument are plotted. At the low frequency less than all resonance frequencies of inclusions composite exhibits properties of "normal" medium (region 1). At frequency greater than the first monopolar resonance in the inclusions, the real part of $1/B$ becomes negative and the phase of the wave vector is shifted to $\pi/2$ (region 2). In this range the wave-number is close to pure imaginary, thus no oscillations can be excited and no wave is able to propagate. In such a way a band gap in the spectrum is formed. For a frequency greater than dipole resonance, the effective den-

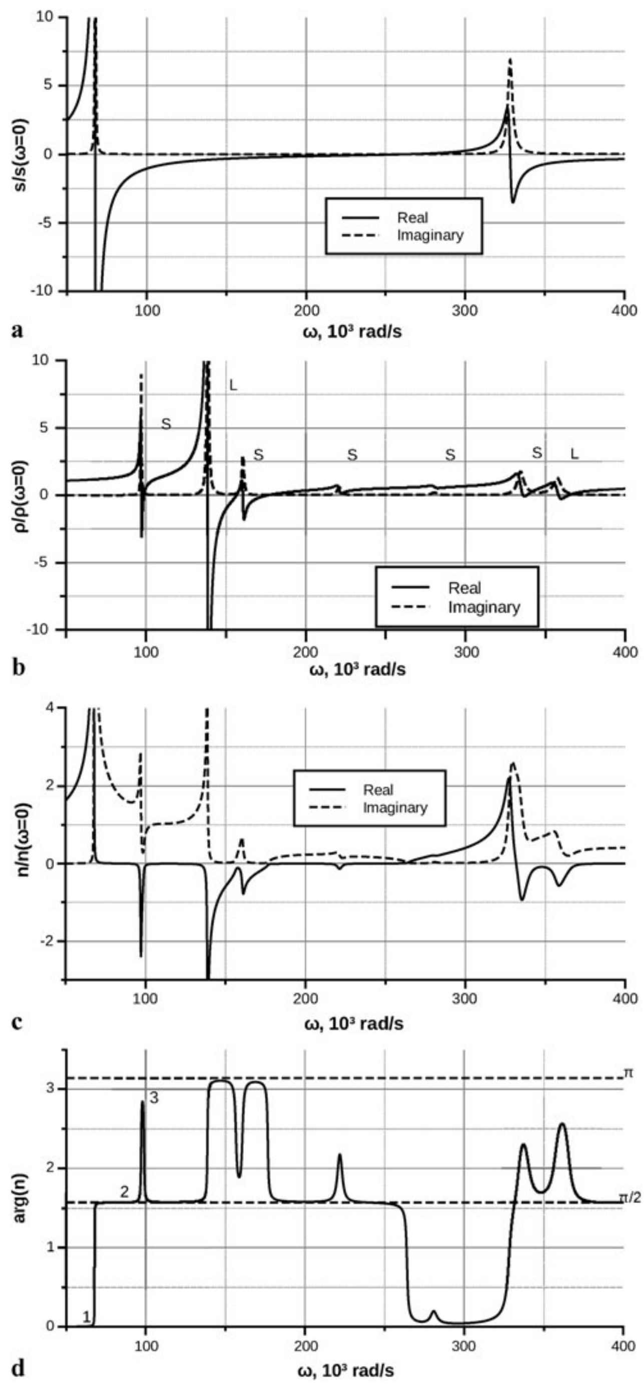

Fig. 1 Frequency dependence of effective compliance (**a**), density (**b**), refractive index (**c**) and the argument of the refractive index (**d**) for composite, normalized on zero-frequency limit

[1]There are various types of silicone resins, therefore our parameters are approximate [4].

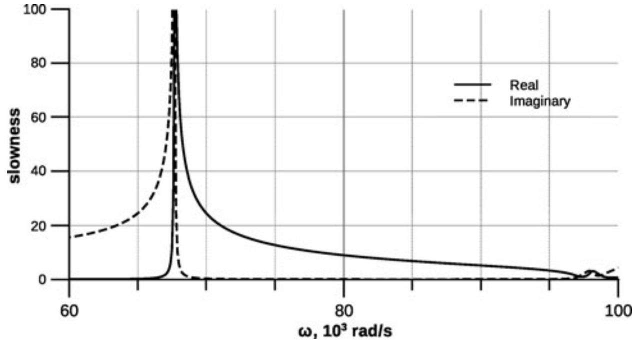

Fig. 2 Slowness (k_s/k_0) of surface acoustic wave at the interface between metamaterial and "normal" material

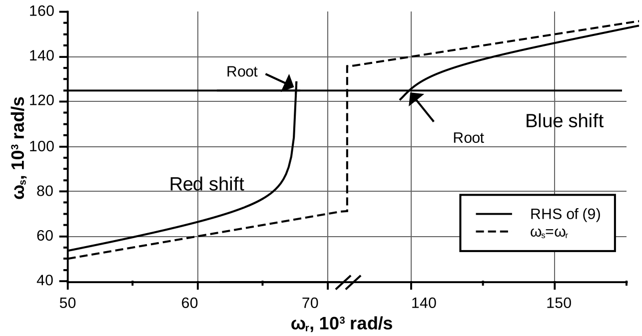

Fig. 3 Graphical solution for (9) in metamaterial

sity becomes negative and phase is shifted again for another $\pi/2$ and the wave-number becomes close to real negative. Thus phase velocity of the wave points backward and opposite to the Poynting vector. Also, the dependence of refractive index shows that in the frequency band where constitutive parameters have simultaneous significant negative parts, the imaginary part of the refractive index does not dominate and the wave is still able to propagate despite losses in inclusions.

3.2 Surface acoustic wave

Figure 2 shows slowness (k_s/k_0) for surface wave to bulk wave in water. As was predicted, the wave-number does not have a real part at low frequencies. Since the parameters of the metamaterial are positive, the wave cannot propagate along the interface. But at frequency greater than resonance (compare to Fig. 1(a)) one of the material parameters becomes negative and the wave-number is real and surface states appear. Surface wave is much slower than the bulk wave, thus the surface states are bounded to the interface.

3.3 Complex Doppler effect

It was shown that in electromagnetic media with simultaneous negative ϵ and μ, the Doppler effect should be reversed [1]. Similar effect can exist for acoustic metamaterials. Let us consider a source that travels inside a composite with velocity V. It transmits a signal with an angular frequency ω_s. We can assume that the source is moving in one line with the receiver. To find the frequency for the receiver (ω) we should solve the following equation:

$$\omega_s = \omega - V \times k(\omega) \tag{9}$$

For a dispersive medium like a metamaterial, the solution for (9) is not trivial [14].

A graphical solution for (9) is plotted in Fig. 3. We assume that damping is small and the wave-number is real. Angular frequency of source is taken as 1.22×10^5 rad/s and speed is 15 m/s. The source moves away from the receiver. For the pure host material the result is predictable, the frequency for the receiver is down-shifted. In contrast, for a composite, (9) has two roots and the two modes with down-shifted and up-shifted frequencies are present. Generation of two modes is possible, because modes have different phase velocities and one of the waves is forward and another is backward [14].

4 Conclusions

In this paper effective dynamic density and stiffness of composite consisting of water with embedded solid two-layered rods in terms of coherent potential approximation are calculated. It is shown that there are some frequency ranges in which dynamic constitutive parameters are simultaneously negative and the propagating wave becomes backward.

Dispersion of the surface acoustic wave at the interface between metamaterial and "normal" material is calculated. It is shown that there are frequency ranges in which the surface states are bounded to the interface.

The Doppler effect in the composite is considered. It is shown that in metamaterial the Doppler shift is reversed and several modes could be excited.

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References

1. V.G. Veselago, Phys. Usp. **10**, 509 (1968)
2. J.B. Pendry, D.R. Smith, Phys. Today **57**, 37 (2004)
3. C. Caloz, T. Itoh, *Electromagnetic Metamaterials: Transmission Line Theory and Microwave Applications* (Wiley-IEEE Press, Hoboken/New Jersey, 2005)
4. Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, P. Sheng, Science **289**, 1734–1736 (2000)

5. J. Li, C.T. Chan, Phys. Rev. E **70**, 055602 (2004)
6. N. Fang, D. Xi, J. Xu, M. Ambati, W. Srituravanich, C. Sun, X. Zhang, Nature Materials **5** (2006)
7. S.H. Lee, C.M. Park, Y.M. Seo, Z.G. Wang, C.K. Kim, Phys. Rev. Lett. **104**, 054301 (2010)
8. P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena*, 2nd edn. (Springer, Berlin, 2006)
9. D.T. Blackstock, *Fundamentals of Physical Acoustics* (Wiley, New York, 2000)
10. J.J. Faran Jr., J. Acoust. Soc. Am. **23**, 405 (1951)
11. M. Ambati, N. Fang, C. Sun, X. Zhang, Phys. Rev. B **75**, 195447 (2007)
12. B.A. Auld, *Acoustic Fields and Waves in Solids*, vol. 1 (Wiley, New York, 1973)
13. J.G. Berryman, J. Acoust. Soc. Am. **68**, 1809 (1980)
14. H. Berger, Am. J. Phys. **44**, 851 (1976)