Frequency-dependence of the acoustic rectifying efficiency of an acoustic diode model

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For an acoustic diode (AD) model formed by coupling a superlattice with a nonlinear medium, the transmission of acoustic wave is investigated as a function of frequency in a numerical manner. The important phenomenon of rectifying effect of acoustic energy flux is identified in a series of frequency bands whose locations agree exactly with the analytical predictions. Within these frequency bands, the AD can exhibit extremely high rectifying efficiency and strong resistance to "reverse breakdown." These results are particularly of practical significance for the potential design and application of ADs. © *2010 American Institute of Physics*. [doi:10.1063/1.3447361]

As the device that has rectifying effect on current flux, electric diode had led to worldwide revolutions in various aspects. Many contributions have been devoted to the investigations on the significant rectifying phenomena of other forms of energy.[1–4] The "diode" effects have been previously identified by Li *et al.*[1,2] for thermal flux and later by Nesterenko *et al.*[3] for solitary waves. Recently Liang *et al.*[4] have presented the theoretical model of an "acoustic diode" (AD) to yield the rectifying effect on acoustic waves in one-dimensional (1D) systems. Their pioneering design promises potential applications in a variety of important situations where a special control of acoustic waves is required, e.g., acoustic one-way mirrors or the medical utility of focused ultrasound, etc. In the previous work of Liang the efficiency of the AD is only exemplified for a particular frequency. However, an important difference between an AD and its electric or thermal counterparts is that acoustic rectification could only be yielded within particular frequency ranges.[4] For the potential design and application of ADs, it should be crucial to learn how the effectiveness of the AD is affected by the change in the frequency of incident acoustic wave. Consequently, it is of academic as well as practical significance to further investigate the frequency-dependence of the rectifying efficiency (RE) for an AD model.

In this paper, we numerically study the RE as a function of frequency for a 1D AD model, which consist of a superlattice (SL) and a nonlinear medium, by using an extended transfer-matrix method that has been proved effective for calculating the nonlinear sound propagation in layered media.[4,5] The important rectifying phenomenon of acoustic energy flux has been identified within a series of frequency bands denoted as effective rectifying bands (ERBs). The locations of the ERBs agree exactly with the analytic results predicted by invoking the dispersive relationship of SL. As we manually adjust the structural parameters of the AD model within a wide range, the rectifying effect persists for acoustic waves whose frequencies fall in the ERBs. The parameter-sensitivity of the RE of AD is numerically investigated, and the results manifest that a substantially high rectifying ratio may be obtained within the ERBs. As compared with an electric diode, the AD exhibits extremely strong resistance to "reverse breakdown".

We consider the propagation of a longitudinal acoustic wave in an AD model immersed in a water tank large enough to neglect the wave reflection from its walls as shown in Fig. 1(a). The medium I and II are chosen as water and glass whose acoustical parameters are:[6] $\rho_{\mathrm{I}}=998\ \mathrm{kg/m^3}$, $c_{\mathrm{I}}=1483\ \mathrm{m/s}$ and $\rho_{\mathrm{II}}=2767\ \mathrm{kg/m^3}$, $c_{\mathrm{II}}=5784\ \mathrm{m/s}$, respectively. The acoustical parameters of medium III are identical with water except that medium III is assumed as a strong

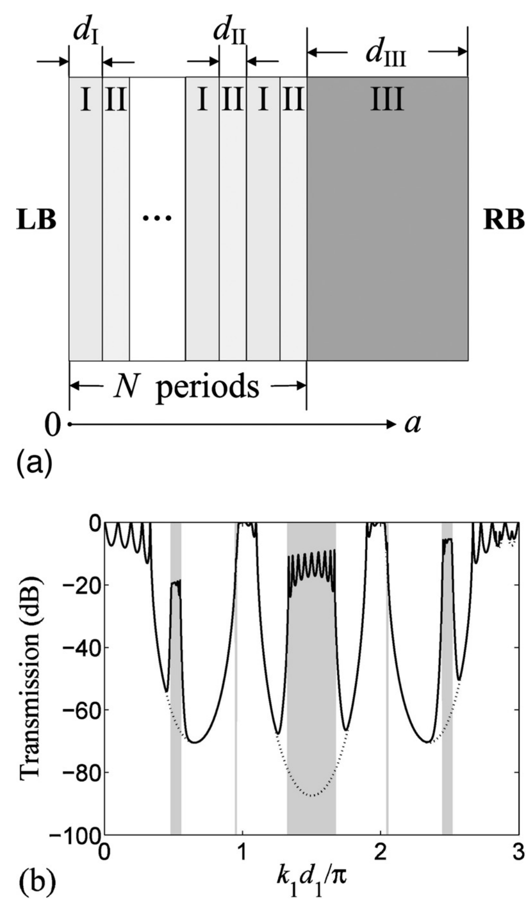

FIG. 1. (a) Schematic illustration of the AD model. (b) Transmissions of acoustic waves that incident from LB (dotted line) and RB (solid line).

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nonlinear medium described by employing the concept of equivalent nonlinearity parameter Γ widely used for evaluating the nonlinearity of inhomogeneous media.[7–10] In practice, the sample of medium III may be produced by utilizing a bubbly elastic soft medium that has particularly strong acoustic nonlinearity at low frequency region and almost the same acoustical parameters as water.[4,8,11] We use the definition of Γ derived by Ostrovsky[7] from the nonlinear Hooke's law to estimate the extraordinary "physical" nonlinearity of a bubbly soft medium caused by its inhomogeneous structure. For such a medium, the value of Γ may reach the magnitude of 10^3-10^5 while the nonlinear variation in sound velocity still remains small.[7,8] Therefore, we neglect the nonlinearity of media I and II. Unless otherwise stated, the system parameters are chosen as: $d_{\mathrm{II}}=1.3d_{\mathrm{I}}$, $d_{\mathrm{III}}=15d_{\mathrm{I}}$, $N=5$, and $\Gamma=10^4$. The amplitude of the pressure of incident wave is $p_0=0.04P_0$ with P_0 being the pressure of atmosphere. The abbreviations LB and RB refer to the leftmost and the rightmost boundaries of the system, respectively. In numerical simulations, we have neglected the effects of mode conversion and viscosity, the discontinuity due to the accumulation of nonlinear distortion, and the inherent nonlinearity of interfaces. In this model, the system symmetry is destroyed by introducing the nonlinear medium III, and the SL serves as an effective acoustic filter with different transmission properties for the fundamental wave (FW) and the second harmonic wave (SHW). As a result, this AD model is expected to pass some acoustic energy flux in only one direction but almost totally block the wave from the other direction. Such an AD effect could be achieved only for the acoustic wave whose frequency ω can satisfy a given rectifying condition that may be readily determined by invoking the dispersive relationship of band structure of the SL[12] as follows:

$$\begin{aligned}&\cos\chi_{\mathrm{I}}\cos\chi_{\mathrm{II}}-\cosh\eta\sin\chi_{\mathrm{I}}\sin\chi_{\mathrm{II}}>1,\\&\cos2\chi_{\mathrm{I}}\cos2\chi_{\mathrm{II}}-\cosh\eta\sin2\chi_{\mathrm{I}}\sin2\chi_{\mathrm{II}}<1,\end{aligned}\tag{1}$$

where $\eta=\ln(\rho_{\mathrm{I}}c_{\mathrm{I}}/\rho_{\mathrm{II}}c_{\mathrm{II}})$, $\chi_i=\omega d_i/c_i$ $(i=\mathrm{I},\mathrm{II})$.

Figure 1(b) illustrates the transmissions as functions of frequencies of the acoustic waves incident from the two sides of the AD model. The transmissions are defined as $T=(E_1+E_2)/E_0|_{a=0}$ and $T=(E_1+E_2)/E_0|_{a=D}$ for the waves from RB and LB, respectively. Here E_1, E_2, and E_0 refer to the time-averaged acoustic energy densities of the FW, the SHW, and the incident wave, respectively, a refers to the Lagrangian coordinates, and $D=N(d_{\mathrm{I}}+d_{\mathrm{II}})+d_{\mathrm{III}}$ is the total thickness of system. Generally the time-averaged energy density of the nth-order term of acoustic waves is defined as $E_n=\rho_m[|u_n|^2+|p_n|^2/(\rho_m c_m)]/4$. Here p_n and u_n refer to the sound pressure and the particle velocity, the subscripts $n=1,2$, refer to FW and SHW, respectively, and the subscripts $m=\mathrm{I}$, II, and III refer to media I, II, and III, respectively. The gray regions denote the frequency ranges in which the rectifying condition given by Eq. (1) is satisfied. The acoustic wave that incidents from LB will hit the SL first. Since the acoustic parameters of media I and III are identical, the transmission property of the AD model is virtually identical with that of the SL. However, the acoustic wave from RB will be partially converted to SHW within medium III before it enters the SL. When its frequency satisfies Eq. (1), the generated SHW can pass through the SL almost totally but the original FW is blocked. Figure 1(b) shows that within the gray regions the acoustic transmissions from RB are extremely large as compared with that from LB. These regions are the ERBs where the system exhibits the significant rectifying effect on acoustic waves. The amplitudes of incident waves from RB have been cautiously controlled to guarantee the validity of perturbation technique; nevertheless an additional enhancement of transmission may be expected if we further increase their amplitude.[4]

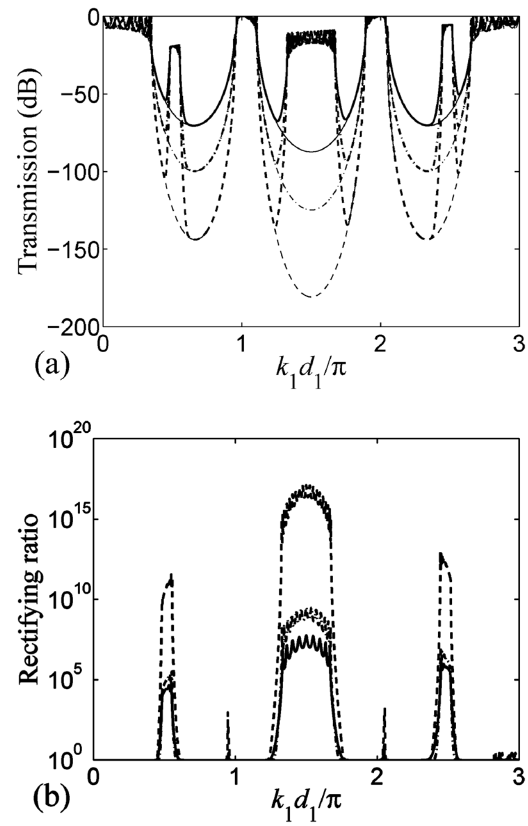

FIG. 2. (a) The comparison between the transmissions of waves that incident from LB (thin lines) and RB (thick lines) and (b) the rectifying ratios of the AD model for three particular cases: $N=5$ (solid line), 7 (dashed-dotted line) and 10 (dashed line).

The efficiency of the AD model is determined by the discrepancy between the acoustic transmissions along two opposite directions, which depends on the selection of system parameters. Figure 2(a) illustrates the comparison between the transmissions of waves from the two sides of system for three particular cases: $N=5$, 7, and 10. In Fig. 2(a) and the following Fig. 3, the thick and the thin lines refer to the transmissions of waves from RB and LB, respectively. The increase in N notably enhances the acoustic attenuations within the stop bands but hardly affects the transmissions within the pass bands. This results in a much higher RE of the AD. For a quantitative estimation of the RE, we introduce the parameter of rectifying ratio defined as $\sigma=T_+/T_-$ and plot the frequency-dependences of σ in Fig. 2(b). Here T_+ and T_- are the acoustic transmissions along the "positive" and the "negative" directions that are generally defined as the propagation directions of waves incident from the sides of medium III and SL, respectively. The values of σ can reach the magnitude of $10^{15}-10^{17}$ due to the extremely small values of T_-. It is generally unnecessary however, to obtain such an excessively high RE at the cost of greatly enlarging the size of an AD. It may be more interesting that a slight in-

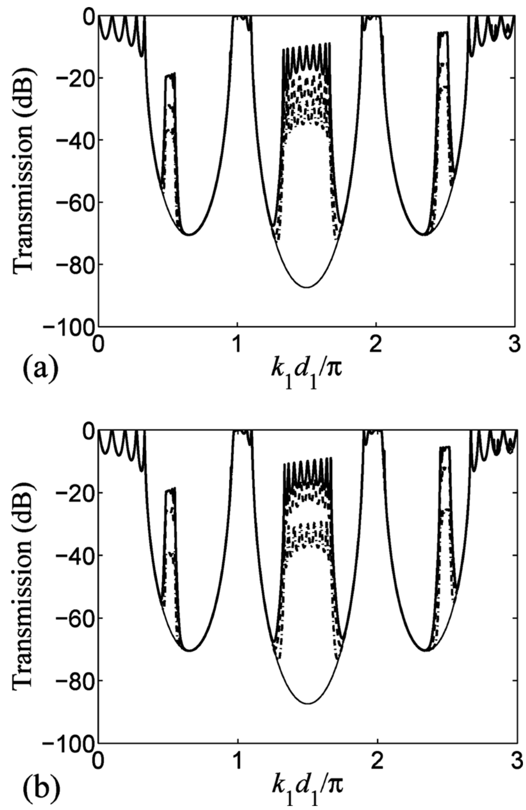

FIG. 3. The comparison between the acoustic transmissions from LB (thin lines) and RB (thick lines) for three particular values of (a) d_{III}: $2d_{\mathrm{I}}$ (dashed-dotted line), $5d_{\mathrm{I}}$ (dashed line), and $15d_{\mathrm{I}}$ (solid line) and (b) Γ: 10^3 (dashed-dotted line), 5×10^3 (dashed line), and 10^4 (solid line).

crease in period number can remarkably enhance the RE, and an AD containing as few as five periods yields a sufficiently high RE ($\sigma\sim10^7$), as compared with its electric or thermal counterparts. This should be of practical significance for the potential application of ADs.

Note that the RE may also be enhanced by generating more SHW within the ERBs. It is therefore important to study the frequency-dependence of the RE as we manually adjust the parameters of Γ and d_{III} which affect the amount of generated SHW. For a clear manifestation, we shall study both T_+ and T_- rather than merely the ratio of them. Figure 3 displays the frequency-dependences of acoustic transmissions for three particular values of (a) d_{III} and (b) Γ. It is shown that the thin lines overlap with each other. The change in the nonlinearity effect of medium III only affects the acoustic energy converted from FW to SHW (i.e., E_2/E_1), rather than the total transmitted energy (i.e., E_1+E_2). Therefore, T_- are independent of the nonlinearity property of medium III. For acoustic wave propagating along the positive direction, on the other hand, the energy converted to SHW increases as we increase Γ or d_{III}. Within the ERBs, therefore, the values of T_+ increase and an enhancement of RE can be expected. In Fig. 3 we choose $N=5$ for better contrasts between the curves denoting T_+ shown in the figures.

The one-way feature of an AD promises significant applications to a variety of practical situations, such as the important medical applications of ultrasound. In the medical ultrasonic imaging and therapy, as a simple but specific example, the ultrasound waves from sources can travel easily in both directions along a given path. Electric diodes may protect delicate devices from sudden reversals, and lasers may also be easily protected from their own reflections by utilizing polarizations. But there is no analogous method to protect ultrasound sources from the disturbance of backtracking acoustic waves without polarization. It might be possible now to build such an acoustic device practically significant for the ultrasonic medical applications. By studying the frequency-dependence of the RE of an AD, we can easily adjust the ERBs to cover the working frequency band of an ultrasound source. Moreover, such an acoustic device is expected to be very efficient because of the extremely small values of T_- shown in Figs. 2 and 3. As compared with an electric diode, an AD has an extremely strong resistance to reverse breakdown that may occur as the amplitude of acoustic waves propagating along negative direction is particularly large. This is particularly meaningful for the applications of high intensity focused ultrasound.

In summary, we have numerically investigated the frequency-dependence of RE of an AD model on acoustic energy flux. Acoustic rectification has been identified within a series of ERBs whose locations agree exactly with analytic predictions. The RE of AD is studied as a function of frequency for a wide range of system parameters. The numerical results indicate that the rectifying effect persists within the ERBs, and a substantially high rectifying ratio may be obtained by choosing the system parameters properly.

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