Analysis of surface acoustic wave propagation in a two-dimensional phononic crystal

Yong Li,[1,2] Zhilin Hou,[2,a)] Mourad Oudich,[3] and M. Badreddine Assouar[3,4]

[1]*Laboratory of Modern Acoustics, MOE, Institute of Acoustics, Department of Physics, Nanjing University, Nanjing 210093, China*

[2]*Department of Physics, South China University of Technology, Guangzhou 510640, China*

[3]*Institut Jean Lamour, UMR 7198, Nancy University-CNRS, BP 70239, Bd des Aiguillettes, 54506 Vandoeuvre-ls-Nancy Cedex, France*

[4]*International Joint Laboratory (UMI 2958), CNRS-GIT, Georgia Institute of Technology, Atlanta, Georgia 30332, USA*

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In this paper, we present a numerical technique to calculate the surface acoustic wave (SAW) in a two-dimensional phononic crystal (PC). By the technique, the SAW in the system, which is obtained by adding an additional composite surface layer on the *xy*-cut surface of a two-dimensional PC, is investigated. Result shows that the behavior of SAW in the studied system is mainly determined by the residual penetration depth of the SAW into the PC structure. Based on this understanding, we show that the SAW in the band gap of the PC can be controlled efficiently by changing the structure of the surface layer. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4740050]

I. INTRODUCTION

In the last decades, a great deal of works has been devoted to the study of phononic crystals (PCs), which are composed of two or three different acoustic materials arranged periodically.[1–7] An important feature of these structures is the possible existence of a range of frequency, called absolute band gap (ABG), in which elastic waves cannot propagate. Based on this property, numerous potential applications such as the noise shielding, filtering, focusing, or waveguiding are expected. Stimulated by the study of the bulk wave propagation in an infinite PC structure, the investigation of the Lamb wave in a thin PC plate[8–12] as well as the surface acoustic wave (SAW) in a half-infinite or thick enough periodic structures was also conducted.[13–26] In a thin PC plate, the confined elastic wave (i.e., Lamb wave) will be scattered not only by the embedded scatterers but also by the surfaces of the plate, resulting the isolated permitted modes. ABGs for Lamb modes were found in different structures,[11,12] based on which the waveguiding[9,11] and negative refraction[10] for Lamb wave were also suggested. As the plate goes thick enough, more and more permitted modes appear, Lamb modes will act more like bulk mode in the infinite PC. However, because of the existing of the free surfaces, another kind of confined modes, i.e., SAW, appears. The important feature of the SAW is that their energy is mainly confined on the surface, which can lead to many potential and appealing applications since SAW can be experimentally excited and detected. Both theoretical and experimental investigations showed that the dispersion curves of SAW can be strongly adjusted by a periodic arrangement on a surface of a solid. We notice that, in most of those works, only the systems with a two-dimensional PC truncated across the z axis (we define it as the structure with z-cut surface)[15–21] or a periodic layer deposits on a uniform substrate[13,14,22,23] were studied.

We know that the key feature of the PC structure is the existing of the Bragg scattering, by which the wave propagation can be strongly modulated. However, we find that this key feature, as we will show below, cannot be fully used by the SAWs in this kind of system. In fact, the problem in this kind of system is that, when the frequency is high enough so that the Bragg scattering happens, the periodic composite surface, on the one hand, plays a positive role on modifying strongly the propagating property of the SAW. On the other hand however, it will also scatter inevitably the SAW into the substrate, producing a so called Brekhovskikh attenuation of the SAW.[23] From the dispersion curve point of view, we know that SAW can only appears under the sound line (which is defined as the lowest bulk wave velocity permitted in the bulk elastic material) in a uniform material.[8,13,14] However, in a periodic structure, the dispersion curves will be folded at the boundary of the Brillouin zone because of the periodicity of the structure, so that a crossover between the bulk mode and the folded surface mode appears, producing their interaction. Such an interaction leads then to a "leaky" or "pseudosurface" SAW mode.[23]

Being aware of this mechanism, one can only modulate the SAW mode under the sound line (also called as sound cone in a two-dimensional structure) in the structure mentioned above, which limits the modification effects of the periodic surface structure. Besides the structures mentioned above, another SAW structure, which is obtained by truncating a two-dimensional PC across the *x* axis or *y* axis (we defined it as the structure with *xy*-cut surface, for which the active surface is parallel to the *z* axis), has also been studied in Ref. 22. Therein, a special kind of SAW, which exists in

a)Author to whom correspondence should be addressed. Electronic mail: phzlhou@scut.edu.cn.

the frequency region of the ABG, was reported. Unfortunately, the physics behind it was not fully discussed in that reference. We find out that the mechanism behind such kind of SAW can be fully understood if we are aware of the fact that the PC can behave like a special kind of substrate.

In this paper, we will investigate the SAW propagation in the two-dimensional PC substrate with a xy-cut surface, which is similar to the structure presented in Ref. 22, but with an additional periodic composite surface layer. We will first give an extensive understanding of the SAW in a system with uniform surface layer. Then, we will show that the SAW appearing in the ABG of the PC substrate can be efficiently controlled by replacing the uniform surface layer by a periodic composite one. The paper is organized as follows. In Sec. II, a new numerical method based on COMSOL Multiphysics software is presented. In Sec. III, the results and their discussions are given. A brief summery is finally given in Sec. IV.

II. NUMERICAL METHOD

As shows schematically in Fig. 1, to construct the structure, we use a two-dimensional PC, consisting of tungsten cylinder scatterers squarely embedded in silicon substrate, coated identically with an additional layer on each outer surface. The lattice constant and the radius of the scatterers in the PC structure are a and r_1, respectively. The additional surface layers can either be uniform or be periodic; the layer thickness and the radius of the scatterer in it (if any) are h and r_2, respectively. We implement the numerical calculation making use of the finite element method provided by COMSOL Multiphysics. In the flowing computations, the radius of scatterer r_1 in the PC substrate is fixed to $0.25a$ to obtain a wider ABG. The material density, the longitudinal and shear wave velocities of the materials are $\rho = 2420\,\mathrm{kg/m^3}$, $c_l = 8300\,\mathrm{m/s}$, $c_t = 5800\,\mathrm{m/s}$, respectively, for silicon; and $\rho = 19\,300\,\mathrm{kg/m^3}$, $c_l = 5100\,\mathrm{m/s}$, $c_t = 2800\,\mathrm{m/s}$, respectively, for tungsten.

Basically, to deal with SAW, we need to consider a semi-infinite system. However here, taking into account the fact that the 0th order Lamb waves (symmetric and anti-symmetric) can be degenerated into surface modes when the plate is thick enough,[19,27] we will consider a finite system with a supercell of $10a$ thickness in x direction instead. To ensure computing surface modes, we can check the behavior of the lowest symmetric and anti-symmetric Lamb waves: The surface mode will be their overlapping. To obtain the symmetric and anti-symmetric Lamb wave separately, we consider only one half of the supercell with suitable boundary condition: For the surface with additional layer, the free boundary condition is applied, but at the center line of the supercell, the symmetric and anti-symmetric boundary condition are used, respectively, in two different calculations.[7] This numerical technique can not only reduce the numerical calculation but also separate the symmetric and anti-symmetric modes. Bloch-Floquet periodic boundary condition is applied along y direction so that only one unit cell in y direction is needed to be considered. We can then obtain the dispersion relation of the system by changing the wave vector along the first Brillouin-Zone. One should note that the dispersion relation obtained by the technique given above is a mixed spectrum, in which both of the SAW and Lamb modes exist. To get the band structure of SAW, the limitation of the bulk propagating wave needs to be calculated. For the system with uniform substrate, this limitation is the so called sound cone: in which the bulk wave cannot exist so that the only modes left are the confined ones along the surface, i.e., the SAW. However, for our system with PC structure as substrate, besides the "sound cone," which is similar to the case for the system with uniform substrate, the frequency region in ABG is another bulk wave limitation, the modes within it should also be the SAW. The dispersion of the SAW given in the following is obtained by overlapping the mixed spectrum (including the SAW and Lamb modes) and the bulk wave band structure projected on the ΓX direction.

III. RESULTS AND DISCUSSIONS

To understand the mechanism behind SAW behavior in this kind of system, we first investigate the structure with a uniform surface layer. The band structures for $h = 0a$, $0.3a, 0.5a$, and $0.8a$ are plotted in Fig. 2, in which the open dots (red in color) represent the anti-symmetrical Lamb modes and the blue solid ones represent the symmetrical Lamb modes. We can see that they are completely overlapped when the normalized frequency verifies $\omega a/2\pi > 500$, which means that they are completely decoupled into surface modes.[27] The shaded region represents the projected bulk bands along the ΓX direction. The white regions represent the sound cone domain, and the full ABG of the PC (along the ΓMX direction), in which the bulk wave cannot exist.

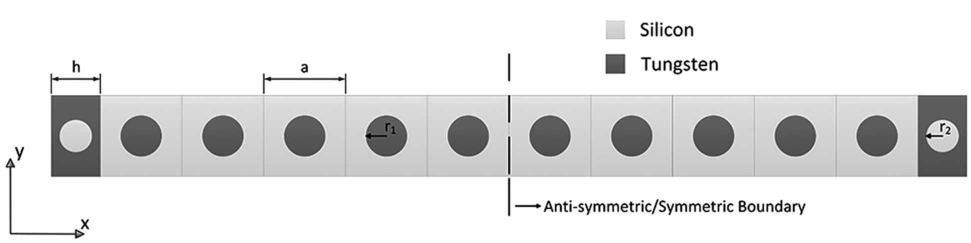

FIG. 1. Schematic view of the supercell of the studied structure with a surface layer deposited on a two-dimensional square lattice phononic crystal slab. The system is finite in x direction and periodic in y direction, a is the lattice constant of the phononic crystal. r_1 (r_2) is the radius of the scatterers in phononic crystal (surface layer). The thickness of the surface layer and the phononic crystal in x direction is h and $10a$, respectively. Because of the symmetry in x direction, a symmetric/anti-symmetric boundary condition can be used at the center of the supercell (labeled by dash line) so that only a half of the system is needed for the calculation. Symmetric (anti-symmetric) Lamb mode can be obtained when the symmetric (anti-symmetric) boundary condition is used.

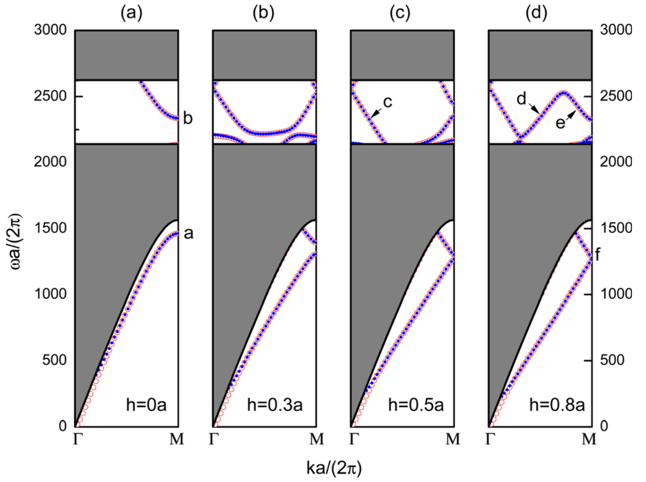

FIG. 2. SAW band structure of the system with different thickness of the uniform surface layer. The shaded area means the region where the bulk wave can exist. (a)-(d) are for the system with $h = 0a, 0.3a, 0.5a$, and $0.8a$, respectively. The empty (full) dot is the 0th anti-symmetric (symmetric) Lamb mode, which is obtained by using the anti-symmetric (symmetric) boundary condition on the middle plane of the supercell (shown in Fig. 1). The SAW mode is then the overlapping of the two Lamb modes. The wave distribution of the modes labeled by *a–f* is shown in Fig. 3.

In Fig. 2, we can see that some new modes appear both in the sound cone and in the ABG. To show that those new modes are indeed surface wave, the displacement field distributions, which are defined as $\sqrt{|u_x|^2 + |u_y|^2}$, where u_x and u_y are the x and y component of the displacement, respectively, of some modes marked in Fig. 2 are shown in Fig. 3. We can then clearly see that the field of these modes is mainly concentrated near the surface. As it can be deduced from Fig. 2, the height of the uniform layer h is an important parameter that can change drastically the band structure of SAW. We first discuss the modes in the sound cone. In this region, only one SAW mode can be found. Its dispersion curve is almost a straight line when $\omega a/2\pi < 1200$, which means that the PC structure can be regarded as an effective uniform medium. As the frequency increases, the Bragg scattering becomes more involved and, as a result, a SAW band gap (SBG) appears. Furthermore, the width of the SBG becomes narrower and narrower as h increases. Then, no SBG can be found when h increases to $0.8a$. This behavior can be understood from the fact that the SAW has a finite penetration depth into the substrate (usually taking the same order as the working wavelength) from the free surface. In the studied system, when the penetration is deep enough, the SAW can penetrate through the additional surface layer and reaches the PC substrate. The residual penetration depth, which is defined as the difference between the total penetration depth and the thickness of the uniform layer h, will then have a nonzero value, so the SAW can be scattered by the PC substrate. For long wavelength, although the residual depth will be large, the wave is not sensitive to the detail of the structure so that the scattering effect become small and the PC acts like a uniform medium. This means that, to control the propagation of such kind of SAW modes, we have to change the effective elastic parameters of the PC substrate (by changing the size of the scatterers in PC, for example). However, when the wavelength is of the order of the lattice constant of the PC, and the thickness of the surface layer is smaller than the penetration depth, the nonzero residual depth can drastically affect the SAW because of the strong Bragg scattering so that a SBG appears. For the modes with zero residual depth, which can be the case when the surface layer is thicker than the penetration depth, the SAW will be completely confined in the surface layer and the PC substrate will no longer affect the modes. To emphasize our understanding, we calculated the field distribution of the mode labeled by *a-f* in Fig. 2. The results are shown in Figs. 3(a)–3(f). We can then clearly see that for the structure with $h = 0$, the field of mode a (see Fig. 3(a)) penetrates into the PC substrate about two lattice parameter depth, leading to a large SBG. But for the system with $h = 0.8a$, the residual penetration depth of mode f (see Fig. 3(f)) is almost zero, which means that the SAW is mainly concentrated in the additional surface layer so that no SBG can be found.

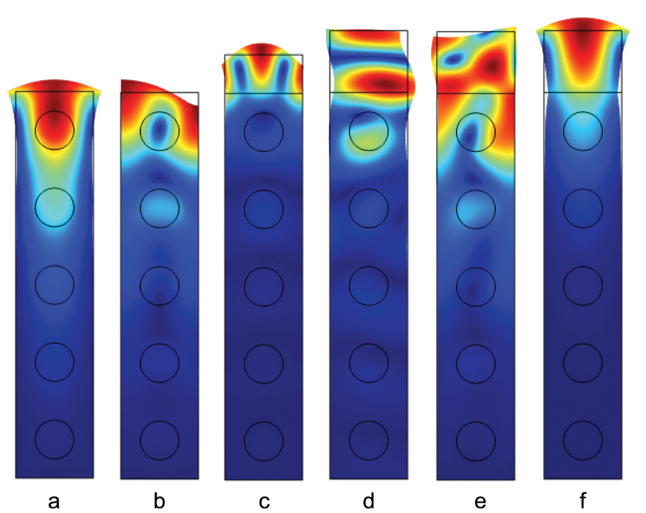

FIG. 3. Field distribution (calculated by $\sqrt{|u_x|^2 + |u_y|^2}$, where u_x and u_y are the x- and y-component of the displacement, respectively) of the modes labeled in Fig. 2. It can be found in (a) that the penetration depth of mode a (shown in Fig. 2(a)) is about $2a$, but it can be seen from (f) that the wave field of mode f (shown in Fig. 2(d)) is confined almost completely in the uniform layer.

For the SAW modes in the ABG, because the wave has a shorter wavelength in this region than the one under the sound cone, we can expect a shallower residual penetration depth in PC substrate, which means that the modifying ability of the PC substrate on the SAW in ABG is weaker than the one below the sound cone.

To modify more efficiently the SAW in ABG, the system with a composite surface layer is considered. In the following calculations, we fix the thickness of the surface layer as $h = 0.6a$. The band structures for $r_2 = 0.05a, 0.1a, 0.15a$, and $0.2a$ are calculated. The results are shown in Fig. 4. As it can be seen that, when r_2 is small ($r_2 = 0.05a$ and $0.1a$, for example), there is no SBG either below the sound cone or in the ABG. But as r_2 increases to $0.15a$, two SBGs are obtained. A narrow one appears below the sound cone, and a wide one appears in the ABG area. In addition, those two SBGs can be further enlarged as r_2 increases to $0.2a$,

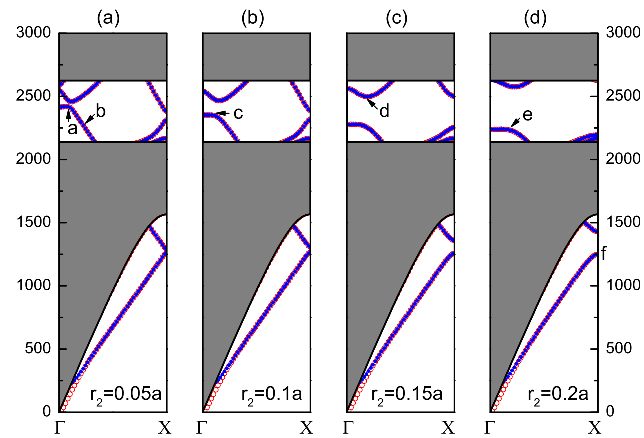

FIG. 4. SAW band structure of the system with periodic composite surface layer. The thickness of the surface layer is $h = 0.6a$, the radius of the scatterer in the surface layer is (a) $r_2 = 0.05a$, (b) $r_2 = 0.1a$, (c) $r_2 = 0.15a$, (d) $r_2 = 0.2a$. It can be seen that the SAW is strongly scattered by the scatterers in the surface layer so that full band gaps of SAW can be found in (c) and (d).

especially, the SAW in the ABG is more sensitive to the size of the scatterers in the surface layer. This means that the periodic composite layer is an efficient way to control the propagating property for such kind of SAW. Based on the explanation for the system with uniform surface layer presented above, the phenomenon appearing here can be easily understood. As it has been already mentioned, for the system with $h = 0.6a$ and $r_2 = 0$, the SAW in the ABG is concentrated almost completely in the surface layer so that the structure of the PC substrate cannot affect its propagating properties. However, for the system with composite surface layer, the embedded scatterers will scatter strongly the SAW so that new SBGs appear. Note that because of this scattering effect, the acoustic field that cannot reach the PC substrate in the case of the uniform layer can then penetrate into the substrate again. So the SBGs creation is due to a combination of both the composite layer and the PC substrate scattering effects. To show further the understanding mentioned above, the acoustic field distributions of some modes, which are

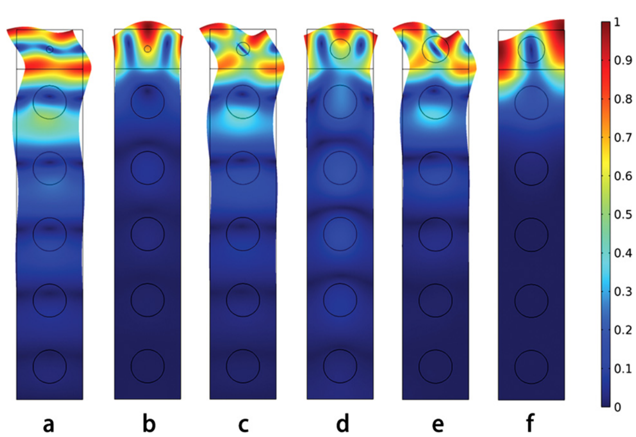

FIG. 5. Field distribution (calculated by $\sqrt{|u_x|^2 + |u_y|^2}$, where u_x and u_y are the x- and y-component of the displacement, respectively) of the modes labeled in Fig. 4. It can be found that the wave field of modes a, c, d, e, and f has been "pushed" into the PC substrate by the strong scattering effect. However, the wave field of mode b is still confined into the surface layer.

labeled as *a-f* in Fig. 4, are shown in Fig. 5. We can then see clearly that the wave field of the band-edge modes, i.e., modes a, c, d, e, and f, near where the strong scattering effect happens, has been "pushed" into the PC substrate, resulting on a nonzero residual penetration depth. But the wave field of mode b, which is away from the band edge, is still confined into the surface layer.

The above discussion can be expressed by a more simple way. The SAW in the surface layer is strongly scattered by the embedded scatterers because of the Bragg scattering. If the substrate is uniform, the scattered waves can be converted into bulk waves, leading to Brekhovskikh attenuation. However, when we use a PC structure as the substrate, the scattered waves will be reflected back into the surface layer so that the SAW is efficiently modified. The mechanism also shows that to use the Bragg scattering to modify the SAW, the lattice constant of the scatterers in the surface layer should be similar to the one in the PC substrate so that Bragg scattering can happen both in the surface layer and in the substrate at the same time.

IV. CONCLUSION

In summary, we have given a numerical technique to deal with the SAW in the PC structure using a semi-infinite substrate. The main idea of the method is to separately calculate the symmetric and anti-symmetric Lamb waves in a thick plate so that the SAW can be picked out by checking the overlapping of the 0th order of Lamb modes. By this technique, the SAW in the system with a uniform/composite surface layer deposited on PC structure is investigated. Results show that the SAW can exist both below the sound cone and in the bulk ABG. To explain the obtained results, a discussion based on the residual penetration depth is presented. Strong Bragg scattering occurs (and as a result, the SBGs appear) only when two conditions are satisfied in the same time, one of which is the residual penetration depth is nonzero and the other one is the working wavelength, is comparable to the lattice constant. By this mean, we show that the SAW in ABG can be efficiently controlled by changing the structure of the periodic surface layer.

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