Generalized Norde plot including determination of the ideality factor

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An extended Norde plot is described which makes it possible to determine the series resistance, barrier height, and ideality factor from one I-V measurement of a Schottky barrier diode. A theoretical derivation is performed followed with experimental data, which demonstrates the feasibility of the method.

The most common theory of a Schottky barrier diode is based on the thermionic emission model and according to this model the current-voltage relationship is given by

$$I = I_s[\exp(\beta V_d) - 1],$$

where $\beta = q/kT$ and V_d is the voltage across the junction and

$$I_S = AA^{**}T^2 \exp(-\beta\phi_B),$$

where A is the area of the diode, A^{**} the Richardsson constant, ϕ_B the barrier height, and T the absolute temperature. In the forward direction the barrier height can be determined by making a $\ln I$ vs V plot by extracting the saturation current from the extrapolated intercept with the zero voltage axis. However, this ideal behavior is seldom observed in experimental diodes and to get a better agreement between theory and experiment an ideality factor n has been introduced in the equation above. Another complication in the I-V characteristics is when the diode has a series resistance, associated with the bulk material in the semiconductor, which causes the voltage drop across the junction to be less than the applied voltage between the terminals of the diode. For the case with a diode with a high series resistance Norde[1] introduced a method which makes it possible to evaluate the barrier height and series resistance using a plot of the function,

$$F(V) = V/2 - 1/\beta[\ln(I/AA^{**}T^2)].$$

He showed that in the point where this function has a minimum the barrier height and series resistance can be determined. Several authors have in different ways introduced the ideality factor in this method.[2–4] In this communication a generalized Norde plot is proposed which makes it possible to determine the barrier height, series resistance, and the n-value from only one I-V measurement at a fixed temperature.

We assume that the forward I-V characteristic is described by

$$I = AA^{**}T^2 \exp(-\beta\phi_B)\exp[\beta(V - IR)/n], \tag{1}$$

where n is the ideality factor, V is the applied voltage across the terminals of the diode, and R is the series resistance. Defining a series of functions,

$$F(V,\gamma) = V/\gamma - 1/\beta[\ln(I/AA^{**}T^2)], \tag{2}$$

where γ is an arbitrary constant greater than n. Combining Eqs. (1) and (2) gives

$$F(V,\gamma) = (1/\gamma - 1/n)V + \phi_B + IR/n. \tag{3}$$

For an ideal diode the series resistance is zero and the function $F(V,\gamma)$ will be a straight line with the slope $(n-\gamma)/\gamma n$. If, on the other hand, there is only a resistance the function will be

$$F(V,\gamma) = V/\gamma - 1/\beta[\ln(V/RAA^{**}T^2)].$$

For large values of the voltage this function will approach a straight line with the slope $1/\gamma$. As long as γ is greater than n the function certainly will have a minimum. Differentiating Eq. (3) with respect to the voltage gives

$$\frac{dF}{dV} = \frac{1}{\gamma} - \frac{1}{n} + \frac{R}{n}\left(\frac{dI}{dV}\right), \tag{4}$$

and from Eq. (1) we get

$$\frac{dI}{dV} = \frac{\beta I}{n} - \beta RI\left(\frac{dI}{dV}\right)\Big/n.$$

Hence,

$$\frac{dI}{dV} = \frac{\beta I}{n}\Big/\left(1 + \frac{\beta RI}{n}\right). \tag{5}$$

Combining Eqs. (4) and (5) results in

$$\frac{dF}{dV} = \frac{(n - \gamma + \beta RI)}{[\gamma(n + \beta RI)]}$$

at the minimum point $dF/dV = 0$, hence,

$$I_0 = (\gamma - n)/(\beta R). \tag{6}$$

Using Eqs. (3) and (6) gives

$$F(V_0,\gamma) = (1/\gamma - 1/n)V_0 + \phi_B + (\gamma - n)/(\beta n),$$

where I_0 and V_0 are the corresponding values at the minimum point. As a conclusion we get

$$\phi_B = F(V_0,\gamma) + (1/n - 1/\gamma)V_0 - (\gamma - n)/(\beta n),$$

$$R = (\gamma - n)/(\beta I_0).$$

To solve this system of equations it is necessary to use at least two different γ values. Using two different γ gives a set of equations,

$$\phi_B = F(V_{01},\gamma_1) + (1/n - 1/\gamma_1)V_{01} - (\gamma_1 - n)/(n\beta), \tag{7}$$

$$\phi_B = F(V_{02},\gamma_2) + (1/n - 1/\gamma_2)V_{02} - (\gamma_2 - n)/(n\beta), \tag{8}$$

$$R = (\gamma_1 - n)/(\beta I_{01}), \tag{9}$$

$$R = (\gamma_2 - n)/(\beta I_{02}). \tag{10}$$

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TABLE I. Experimental determined values for a Ni–Si Schottky barrier diode.

γ_1	2	2.1	2.2	2.3	2.4	2.5
γ_2	2.1	2.2	2.3	2.4	2.5	2.6
ϕ_B (eV)	0.604	0.604	0.600	0.600	0.595	0.593
R (Ohm)	3.2	3.1	3.1	3.1	3.1	3.0
n	1.20	1.22	1.23	1.23	1.23	1.23

From Eqs. (7) and (8) we can determine the ideality factor n:

$$n = (V_{01} - V_{02} + \gamma_2/\beta - \gamma_1/\beta)/[F(V_{02},\gamma_2) - F(V_{01},\gamma_1) - V_{02}/\gamma_2 + V_{01}/\gamma_1], \quad (11)$$

or from Eqs. (9) and (10),

$$n = (\gamma_1 I_{02} - \gamma_2 I_{01})/(I_{02} - I_{01}). \quad (12)$$

The barrier height and series resistance can then be extracted from Eqs. (7)–(10). If the diode is described by a thermionic emission model the desired values can be evaluated over a large range in the I-V characteristic and hence there are no restrictions concerning the value of γ as long as it is greater than n. However, when a diode deviates from the ideal case, but still, in some regions, can be described by the theory used here, the method can be beneficial in that it makes it possible to monitor the changes in the parameters as functions of the applied voltage. Deviations from the simple theory are not unusual and may be due to different explanations like interfacial layers,[5] minority-carrier injection,[6] tunneling through the barrier, etc.

Experimental diodes were fabricated to test the method and for this purpose nickel was used as Schottky metal. As substrate 30-μm epitaxial n-type silicon was used with a doping concentration of 5×10^{14} cm^{-3}. The area of the diodes was 0.00785 cm^{-2}. The temperature during the measurements was 21.4 °C. In Table I the result is shown. The result shows that the diode fabricated deviates from the ideal behavior ($n = 1.2$) and hence the advantage of this method compared to the original one is clearly demonstrated.

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A fracture mechanics model of fragmentation

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A model of the fragmentation process is developed, based on the theory of linear elastic fracture mechanics, which predicts the average fragment size as a function of strain rate and material properties. This approach permits a unification of previous results, yielding Griffith's solution in the low-strain-rate limit and Grady's solution at high strain rates.

Grady[1,2] derived a simple model of dynamic fragmentation by balancing the available kinetic energy against the energy associated with the new surface created in the process. This model neglected the effect of stored elastic (strain) energy, an omission that was later accounted for by Glenn and Chudnovsky.[3] The revised model predicted that the strain energy should dominate for brittle materials, with low fracture toughness and high fracture-initiation stress. Evaluation with limited experimental data on brittle steels appeared to indicate that the strain energy is unimportant even when the kinetic energy density is small compared with the (potential) strain energy density. This result is puzzling in light of the well-known Rayleigh-wave limit on crack(s) propagation speed,[4] i.e., the time for body wave propagation (and consequent strain energy release) should normally be considerably less than the time for crack traversal of the specimen. Moreover, for quasistatic loading, it is clear that fragmentation is controlled entirely by the balance between

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