

# A new method of determination of series and shunt resistances of silicon solar cells

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## Abstract

A new method of measurement of series resistance  $R_s$  and shunt resistance  $R_{sh}$  of a silicon solar cell is presented. The method is based on the single exponential model and utilizes the steady state illuminated  $I$ – $V$  characteristics in third and fourth quadrants and the  $V_{oc}$ – $I_{sc}$  characteristics of the cell. It enables determination of values of  $R_{sh}$  and  $R_s$  with the intensity of illumination. For determination of  $R_s$  it does not require  $R_{sh}$  to be assumed infinite and realistic values of  $R_{sh}$  can be used. The method is very convenient to use and in the present study it has been applied to silicon solar cells having finite values of  $R_{sh}$ . We have found that  $R_{sh}$  is independent of intensity but the  $R_s$  decreases with both the intensity of illumination and the junction voltage.

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**Keywords:** Silicon solar cell; Series and shunt resistances;  $I$ – $V$  characteristics

## 1. Introduction

Series and shunt resistances in solar cells are parasitic parameters, which affect the illuminated  $I$ – $V$  characteristics and efficiency of an otherwise good cell. In an  $n^+$ – $p$  or  $n^+$ – $p$ – $p^+$  silicon solar cell the series resistance ( $R_s$ ) is mainly the sum of contact resistance on the front and back surfaces and the resistances of the bulk and the  $n^+$  diffused layer on the top. The shunt resistance ( $R_{sh}$ ) represents a parallel high-conductivity path across the  $p$ – $n$  junction or the cell edges and decreases the efficiency of the cells by increasing the leakage current that lowers the maximal output power ( $P_m$ ), the open-circuit voltage ( $V_{oc}$ ), and the curve factor (CF) [1,2]. In presence of series resistance, a low value of shunt resistance can affect the short-circuit current density as well.  $R_{sh}$  is crucial to  $PV$  performance, especially at reduced irradiance levels. When intensity level falls, as on cloudy days or when the sun is lower in the sky, low  $R_{sh}$  becomes an increasing concern [3]. Therefore,  $R_s$  and  $R_{sh}$  both need to be recognized and understood in order to analyze cell performance. Several methods are

available in the literature for the measurement of series [1,2,4,5] and shunt resistances [6,7] of a solar cell. Wolf and Rauschenbach [1] method gives the value of  $R_s$  using illuminated  $I$ – $V$  characteristics at two close intensities. In a method by Rohtagi et al. [4], the comparison of dark forward  $I$ – $V$  curve with the illuminated  $I$ – $V$  curve transferred from the fourth quadrant to the first quadrant gives the value of  $R_s$  that corresponds to open-circuit point. Agarwal et al. [5] used the nonlinearity in the short-circuit current at high intensity versus light intensity for determination of the  $R_s$  of the solar cell. This method [5] cannot be used at low intensities. Another limitation of this method [5] is that it presumes  $R_s$  to be independent of the intensity of illumination, which may not be valid. All these methods [1,2,4,5] are based on single exponential model of solar cell and assume that  $R_{sh}$  is infinite, an assumption that may not be valid for the cell having low  $R_{sh}$  values [8].

In this paper we present a new method for determination of  $R_{sh}$  and  $R_s$  of a solar cell using illuminated  $I$ – $V$  characteristics of the cell extending from fourth to third quadrant. It shows that  $R_{sh}$  is practically independent of the intensity of illumination. Unlike other methods it does not assume  $R_{sh}$  to be infinitely large and enables the

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determination of  $R_s$  as a function of junction voltage ( $V_j$ ) and the intensity using the practical values of  $R_{sh}$ .

## 2. Theoretical analysis

The illuminated  $I$ – $V$  characteristics of a p–n junction solar cell in fourth and third quadrants can most simply be described using single exponential model as

$$I = -I_{ph} + I_o(e^{q(V_j)/nKT} - 1) + V_j/R_{sh}, \quad (1a)$$

$$V_j = V - IR_s, \quad (1b)$$

where  $I_{ph}$  represents the photogenerated current,  $I_o$  is the reverse saturation current,  $V_j$  is the voltage developed or dropped across the junction,  $n$  is the ideality factor,  $k$  is the Boltzman constant,  $T$  is the temperature of the cell and  $V$  is the terminal voltage. In Eqs. (1a) and (1b) we note that  $I$  assumes a negative value in both the third and fourth quadrants. To derive a mathematical expression for  $R_s$  and  $R_{sh}$  it is convenient to consider all parameters including the variables  $I$ ,  $V$  and  $V_j$  as positive quantities. Then for the fourth quadrant we can write

$$I_f = I_{ph} - I_o(e^{q(V_j)/nKT} - 1) - V_j/R_{sh}, \quad (2a)$$

where

$$V_j = V_f + I_f R_s \quad (2b)$$

and for the third quadrant we can write

$$I_r = I_{ph} + I_o + (V_r - I_r R_s)/R_{sh}. \quad (3)$$

Here  $I$ ,  $V$  are represented by  $I_f$ ,  $V_f$  in fourth quadrant and  $I_r$ ,  $V_r$  in the third quadrant.

Eqs. (2a) and (2b) show that at short-circuit condition ( $V_f = 0$ ,  $I = I_{sc}$ ) the current  $I_{sc}$  is given by

$$I_{sc} = \frac{R_{sh}}{(R_s + R_{sh})} I_{ph}. \quad (4)$$

This shows that  $I_{sc} < I_{ph}$  when  $R_s$  is not zero.

Eq. (3) can be written as

$$I_r = \frac{R_{sh} I_{ph}}{(R_s + R_{sh})} + \frac{V_r}{(R_s + R_{sh})}, \quad (5)$$

where  $I_o \ll I_{ph}$ .

The slope of curve (5) from the  $V_r$  axis gives  $(R_s + R_{sh})^{-1}$ .

Let us take

$$(R_s + R_{sh}) = P. \quad (6)$$

It may be pointed out that the first term on the RHS of Eq. (5) represents the short-circuit current,  $I_{sc}$  of the cell. A combination of Eqs. (2)–(6) eliminates the theoretical parameter  $I_{ph}$  and yields a relation for  $R_s$  as

$$R_s = \frac{nkT}{qI_f} \ln \left\{ \frac{I_r P - (V_r + V_f + I_f P)}{I_o(P - R_s)} \right\} - \frac{V_f}{I_f}. \quad (7)$$

Eq. (7) can be used to determine  $R_s$  by iteration. However, if we consider  $R_s \ll P$ , which is generally valid for a practical solar cells, then we can obtain a simplified

relation for  $R_s$  as

$$R_s = \frac{nkT}{qI_f} \ln \left\{ \frac{I_r P - (V_r + V_f + I_f P)}{I_o P} \right\} - \frac{V_f}{I_f}. \quad (8)$$

For determining  $R_s$  from (7) or (8) the value of  $I_o$  and  $n$  have to be known besides  $P$ ,  $V_f$ ,  $V_r$ ,  $I_f$  and  $I_r$ . The values of  $I_o$  and  $n$  can be determined from  $V_{oc}$ – $I_{sc}$  characteristics of the cell where  $R_{sh}$  may be assumed equal to  $P$  since usually  $R_s \ll R_{sh}$ .

To correctly use Eq. (7) for calculation of  $R_s$  we first determine an approximate value of  $R_s$  using Eq. (8). Then, this value of  $R_s$  is used in Eq. (7) to determine correct value of  $R_s$  by iteration.  $I_f$  and  $I_r$  should be chosen such that

$$I_r P - (I_f P + V_r + V_f) > 0. \quad (9)$$

## 3. Experimental

Solar cells used in this study were based on  $n^+$ – $p$  structure and were fabricated from  $\langle 100 \rangle$  oriented,  $1 \Omega \text{cm}$  resistivity, p-type, Cz silicon wafers ( $10 \times 10 \text{cm}^2$  pseudo-square,  $350 \mu\text{m}$  thick) using  $\text{POCl}_3$  liquid diffusion source of phosphorous for creating an  $n^+$ – $p$  junction on the front side. The wafers were textured in a hot NaOH solution before junction fabrication. The contacts were made by screen printing of Ag paste on the front and Ag/Al paste on the back sides of the cells followed by sintering at  $\sim 700^\circ\text{C}$  in dry air. Subsequently, an antireflection coating of PECVD  $\text{Si}_3\text{N}_4$  was applied on the front side. First, the  $I$ – $V$  characteristics of the cells were measured under dark reverse bias conditions, after that the illuminated  $I$ – $V$  characteristics were measured under no bias and reverse bias conditions at different intensities of illumination. For all measurements reported in this work the illumination consisted of a simulated AM1.5 spectrum, only the intensity was varied when required. The values of  $I_o$  and  $n$  were determined using the  $V_{oc}$ – $I_{sc}$  [9] characteristics, which does not require  $R_s$  to be known. However, it requires the value of  $R_{sh}$  which in the first case can approximately be taken as equal to  $P$ . It will become clear in the next section that approximation  $R_{sh} = P$  does not introduce any significant errors because  $R_{sh}$  does not vary with the intensity of illumination and  $R_s$  is generally negligible in comparison with  $R_{sh}$ .

## 4. Results and discussion

The present method was applied to determine  $R_s$  and  $R_{sh}$  values of a number of silicon solar cells fabricated as described in the preceding section. However, in this section we give detailed description of the measurements and results for only one cell, viz.  $S_1$ . Fig. 1 shows  $\ln(I_d)$  versus  $V_j$  curve of cell  $S_1$  obtained from the  $V_{oc}$ – $I_{sc}$  characteristics [9].  $I_d$  is the ideal diode current that was determined using the relation

$$I_d = \left( I_{sc} - \frac{V_{oc}}{R_{sh}} \right) = I_o e^{q(V_{oc})/nKT}, \quad (10)$$

where  $V_{oc} = V_j$ .

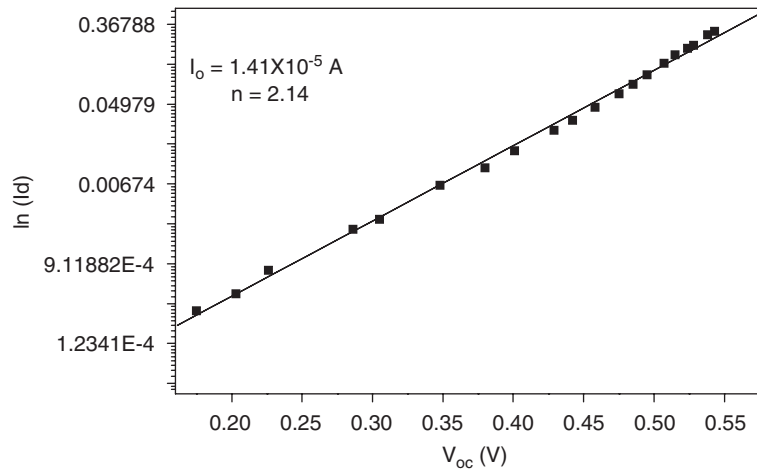


Fig. 1.  $\ln(I_d)$  versus  $V_j$  curve for cell  $S_1$  obtained from  $I_{sc}$ – $V_{oc}$  characteristics at room temperature ( $25^\circ\text{C}$ ).

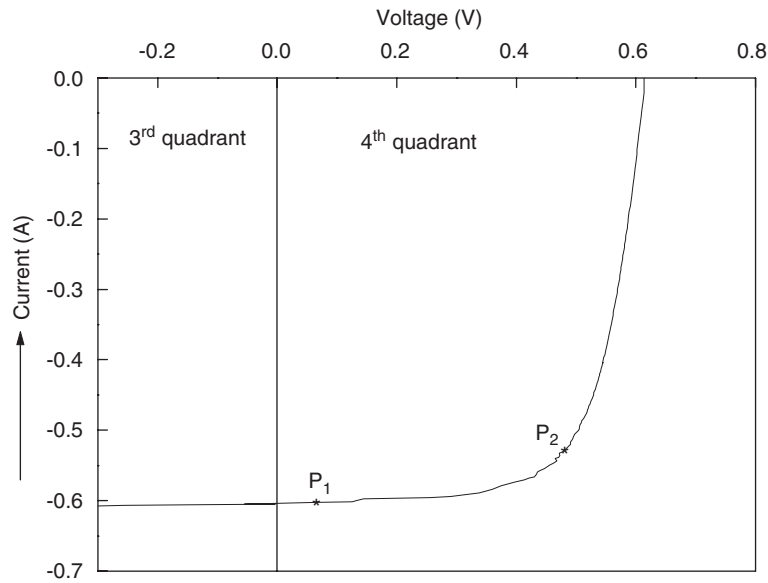


Fig. 2.  $I$ – $V$  curves without bias (fourth quadrant) and with reverse bias (third quadrant) at room temperature ( $25^\circ\text{C}$ ) under a simulated AM1.5 solar irradiance of  $100\text{ mW}/\text{cm}^2$  intensity.

Since  $R_{sh}$  was not known its value, was approximately taken as equal to  $P$ . The slope of the  $\ln(I_d)$  versus  $V_j$  is equal to  $(q/nkT)$  which gives the value of  $n$ . The intercept of the curve on  $Y$ -axis gives the value of  $I_0$ . The values of  $I_0$  and  $n$  for cell  $S_1$  were found to be  $I_0 = 1.41 \times 10^{-5}\text{ A}$  and  $n = 2.14$ , respectively, when  $R_{sh} = P$  was assumed.

Illuminated  $I$ – $V$  curve for cell  $S_1$  in third and fourth quadrants under a simulated AM 1.5 solar radiation of  $100\text{ mW}/\text{cm}^2$  intensity is shown in Fig. 2. Series resistance  $R_s$  was calculated near the two points,  $P_1$  and  $P_2$ , on the illuminated  $I$ – $V$  characteristics of fourth quadrant.  $P_1$  is near the short-circuit point and thus corresponds to a low  $V_j$  value, whereas,  $P_2$  is near the maximum power point and therefore corresponds to a higher  $V_j$  value. Tables 1 and 2 provide values of  $R_s$  and  $R_{sh}$  for five different intensities of illumination and the various other parameters used for the determination of  $R_s$  and  $R_{sh}$ .

The variation of  $R_{sh}$  and  $(R_s + R_{sh})$  with intensity is shown in Fig. 3 and that of  $R_s$  in Fig. 4. These figures show that  $R_{sh}$  and  $(R_s + R_{sh})$  are almost invariant with intensity and the values of  $R_s$  are negligibly small in comparison with  $R_{sh}$ . This shows that for determining the values of  $I_0$  and  $n$  from the  $V_{oc}$ – $I_{sc}$  [9] characteristics of the cell the approximation that  $R_{sh}$  is nearly equal to  $P$  and does not depend on the intensity of illumination is fairly valid. However, Fig. 4 shows that the values of  $R_s$  corresponding to both near the short-circuit point  $P_1$  and near the maximum power point  $P_2$  decrease with intensity. Also the values of  $R_s$  are found to be smaller at  $P_2$  than at  $P_1$ . This is because at  $P_2$  the junction voltage was higher than at  $P_1$ . These observations are in agreement with those of Chakrabarty and Singh [10] who showed that  $R_s$  decreases with the junction voltage.

Table 1  
The values of various parameters used for determination of  $R_s$  and  $R_{sh}$  near the short circuit point  $P_1$  for five different intensities of illumination for cell  $S_1$

Intensity (mW/cm <sup>2</sup> )	$V_f$ (V)	$I_f$ (A)	$I_r$ (A)	$P$ (Ω)	$R_s$ (Ω)	$R_{sh}$ (Ω)
10	0.061	0.051	0.052	148.4	1.38	147.0
16	0.062	0.081	0.082	148.5	0.827	147.7
23	0.064	0.119	0.121	148.2	0.490	147.7
31	0.067	0.16	0.161	148.6	0.355	148.2
100	0.065	0.602	0.604	148.7	0.293	148.4

$I_o = 1.41 \times 10^{-5}$  A,  $n = 2.14$ ,  $V_r = 0.067$  V.

Table 2  
The values of various parameters used for determination of  $R_s$  and  $R_{sh}$  near the maximum power point  $P_2$  for five different intensities of illumination for cell  $S_1$

Intensity (mW/cm <sup>2</sup> )	$V_f$ (V)	$I_f$ (A)	$I_r$ (A)	$P$ (Ω)	$R_s$ (Ω)	$R_{sh}$ (Ω)
10	0.315	0.04	0.052	148.4	1.11	147.3
16	0.322	0.069	0.082	148.5	0.622	147.9
23	0.365	0.100	0.121	148.2	0.305	147.9
31	0.354	0.144	0.161	148.6	0.196	148.4
100	0.421	0.568	0.604	148.7	0.013	148.7

$I_o = 1.41 \times 10^{-5}$  A,  $n = 2.14$ ,  $V_r = 0.067$  V.

$P = R_s + R_{sh}$ .

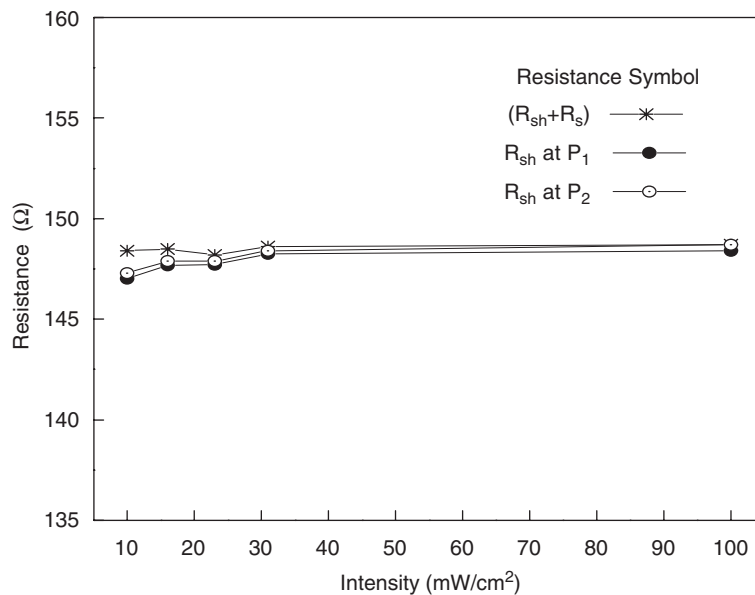


Fig. 3. Variation of  $(R_s + R_{sh})$  and  $R_{sh}$  at points  $P_1$  and  $P_2$  with intensity,  $P_1$  is near the short-circuit point and  $P_2$  is the near the maximum power point.

In our method we can determine  $R_s$  with  $V_j$  very easily using various  $I_f$ ,  $V_f$  values corresponding to different points between the short-circuit and open-circuit point on a single illuminated  $I$ - $V$  characteristics. To demonstrate the effect of  $V_j$  on  $R_s$  we have determined the values of  $R_s$  using Eq. (7) for the  $I$ - $V$  curve plotted for 100 mW/cm<sup>2</sup> intensity in Fig. 2. These values of  $R_s$  have been plotted against  $V_j$  in Fig. 5. The dependence of  $R_s$  on  $V_j$  shown in Fig. 5 is similar to that observed by Chakrabarty and Singh [10]. However, they had obtained the values of  $R_s$  at different  $V_j$  values by using the method of Ref. [4], which required

measurements of illuminated  $I$ - $V$  characteristics at various intensities, and then transferring each of these curves separately to the first quadrant to compare with the dark  $I$ - $V$  characteristics of the cell. This was more time consuming and also assumed  $R_{sh}$  to be infinitely large.

It may be pointed that it is reasonable to assume  $R_{sh} = P$  for determination of  $I_o$  and  $n$  values from the  $V_{oc}$ - $I_{sc}$  characteristics in the first case since  $R_{sh}$  is not known then. However, having determined  $I_o$  and  $n$  the nearly correct values of  $R_{sh}$  and  $R_s$  can be obtained using Eqs. (6) and (7).

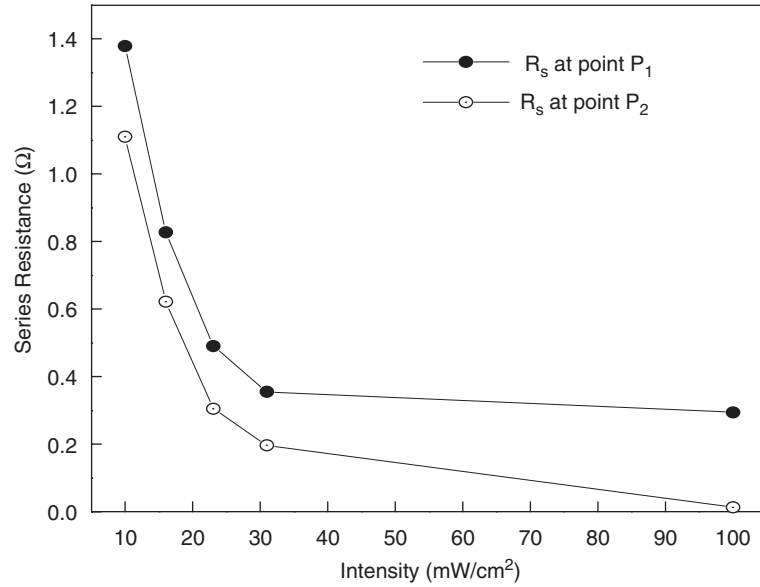


Fig. 4. Variation of series resistance ( $R_s$ ) at points  $P_1$  and  $P_2$  with intensity,  $P_1$  is near the short-circuit point and  $P_2$  is the near the maximum power point.

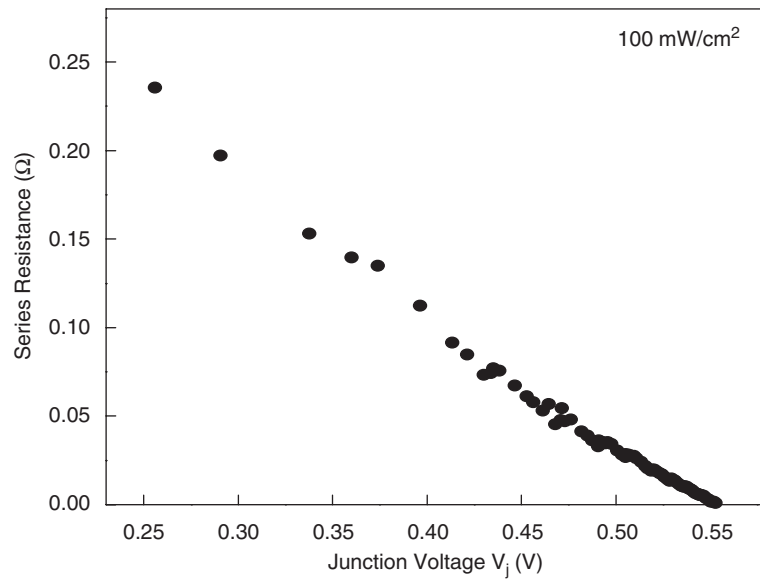


Fig. 5. Variation of  $R_s$  with the junction voltage determined from  $I$ - $V$  characteristics of cell  $S_1$  at room temperature ( $25^\circ\text{C}$ ) under a simulated AM 1.5 solar irradiance of  $100\text{ mW/cm}^2$  intensity.

Table 3

The values of  $I_o$ ,  $n$  and  $R_s$  determined with  $R_{sh} = P$  approximation (when  $R_{sh}$  was unknown) and with known value of  $R_{sh}$

Value of $R_{sh}$ used in Eq. (10)	$I_o$ (A)	$n$	$R_s$ (Ω) at $P_1$	$R_s$ (Ω) at $P_2$
$R_{sh} = P$	$1.41 \times 10^{-5}$	2.14	0.293	0.013
$R_{sh} = P - R_s$	$1.46 \times 10^{-5}$	2.17	0.295	0.019

The values of  $R_s$  correspond to  $100\text{ mW/cm}^2$  intensity of illumination.

For determination of  $R_{sh}$  from Eq. (6) for use in Eq. (10) the value of  $R_s$  corresponding to point  $P_1$  should be used. This value of  $R_{sh}$  will yield more realistic values of  $I_o$ ,  $n$  and  $R_s$  (at  $P_1$  and  $P_2$  points) than the earlier values. Table 3 compares the values of  $I_o$ ,  $n$  and  $R_s$  determined for cell  $S_1$

with  $R_{sh} = P$  approximation when  $R_{sh}$  was not known with those determined after  $R_{sh}$  became known as described above. The difference in the values of  $I_o$  and  $n$  is less than 3.5% in the two cases and their effect on the value of  $R_s$  at  $P_1$  is negligible and at  $P_2$  is small.

## 5. Conclusion

A theoretical expression given in Eq. (7) has been derived to determine series and shunt resistance values of a silicon solar cell using the illuminated  $I$ – $V$  characteristics of the cell in third and fourth quadrants for a given intensity. The method requires the values of the reverse saturation current  $I_0$  and the diode ideality factor  $n$ , which can be easily obtained using the  $V_{oc}$ – $I_{sc}$  characteristics of the cell. The method shows that  $R_{sh}$  of a silicon solar cell is nearly independent of intensity of illumination but the series resistance decreases with both the intensity of illumination and the junction voltage. The method is very convenient to use and enables determination of the series resistance with the intensity of illumination and the junction voltage using the measured finite value of the shunt resistance of the cell.

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