Nonlocal effects in semiconductor nanostructure transport

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Abstract
A small quantum system, such as a quantum dot, can be considered to be mainly closed, in which case current flow follows by tunneling, or can be mainly open, in which case the conductance is several times $2e^2/h$, the quantum unit of conductance. Even in this case, however, there are trapped states within the quantum system which impact transport via phase space tunneling. When we have multiple systems, either an array of dots or of quantum point contacts, or of a combination of these, then there is the chance for a more nonlocal interaction to occur, and to affect the transport. Some of these cases are discussed in this paper.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Measurements of open quantum systems have been a main issue in quantum theory since its advent [1]. These are a basic ingredient of quantum information processing, which has become quite important in modern physics. In general, we distinguish between open systems and closed (or nearly closed) systems according to the size of the conductance relative to Landauer's unit of conductance $G_0 = 2e^2/h$ [2]. Generally, the behavior of an experimental system can fit nicely into one or the other of these two categories. But, in recent years, experimental structures have been created which include multiple quantum systems, in which complicated nonlocal behavior is exhibited in the conductance, and arising from interactions between these quantum systems. These can be arrays of quantum dots [3–5], a dot and a quantum point contact (QPC) [6–8], or multiple QPCs [9, 10]. By nonlocal here, we mean a quite generic description.

The most important meaning of nonlocal here refers to an action at a distance between these coupled quantum systems, and this may take the form, as we shall see, of tunneling or capacitance or perhaps the transport of quantum particles between the systems. But, we also refer to nonlocal as the situation when bipartite states evolve in the coupled system, where these states cannot be described within the tensor product of states of the individual systems. Hence, these latter nonlocal states are entangled states of the more complicated coupled systems, which may have usage in information processing, and this is the case in an array of quantum dots.

In this paper, we present a review of some of the more interesting forms in which this nonlocal interaction can appear. We will begin by discussing closed quantum dots, in which transport is described by single-electron tunneling into and out of the dot. By coupling these dots to a nearby QPC, an effective measure of the state filling within the closed dot can be obtained. We then turn to open quantum dot arrays, where new bipartite states are observed. We then return to QPCs and their coupling to other QPCs. Finally, we summarize with some unsolved questions which arise from these interactions.

2. Quantum dots

A quantum dot (QD) can arise from a great variety of structures [11]. These can be self-assembled dots, or defined through imposition of a self-consistent potential applied through confining gates, usually upon a surface. Here, we will focus only on the latter.

2.1. Single-electron closed dots

Originally, single-electron tunneling dots (SETs) were created through patterning metals, but then the use of surface gates began to appear, especially patterned into useful devices [12, 13]. It was found, however, that spectroscopy of the dot could be better done by creating a vertical dot [14].

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Here, a multi-layer heterojunction, such as a resonant tunneling diode, is grown first. Then, this structure is laterally patterned to form vertical pillars, with the small dot being the quantum well layer in the center of the resonant tunneling diode. For a sufficiently small dot area, current flows by single-electron tunneling into, and out of, the dot. More control of the tunneling process can be obtained by using a surround gate as a third terminal to modulate the dot population [15]. This approach has been used to fully deplete the quantum dot of all carriers with negative gate bias, and then to raise the bias so that individual electrons are allowed to charge the dot, providing a very sophisticated characterization of the real quantum dot energy levels [16]. Applying a magnetic field, the latter group was also able to study the splitting and the crossing of the energy levels as the field was varied, as well as studying the preferred spin occupation of the levels [17].

While gate control could be applied to surface dots with surface gates in order to do spectroscopy [18], assurance of being able to go to zero charge in the dot and subsequent effective spectral measurements awaited the coupling of the dot to a QPC [6, 7]. In this latter case, there was clear evidence of full depletion of the dot, so that the subsequent spectroscopy could be associated with energy levels within the dot. Here, the QPC uses a common defining gate with the dot, so that as the dot adds a charge, the self-consistent potential within the dot modulates the conductance of the QPC. It is this modulation that can then be used to monitor the addition or subtraction of charge within the dot. Thus, the interaction between the dot and the QPC is electrostatic in nature.

Nevertheless, evidence appeared in special double-gate Si MOS structures that the QD could be fully depleted and the four-state lowest (an additional factor of 2 for the two valleys) level could be observed [19]. Thus, the many-body effects not only split the two spins but also create a valley–valley interaction which is nonlocal in momentum space.

2.2. Open quantum dots

We have demonstrated in the past few years that open quantum dots retain a set of stable resonant states, which are termed pointer states [20, 21], and which arise from the einselection process and connect to the classical trapped orbits within such a confining potential [22]. In this regard, the corresponding classical orbits are clearly associated with the wavefunction amplitude of the pointer states and live on a so-called Kolmogorov–Arnold–Moser (KAM) island in phase space [23, 24]. A new type of einselected state arises by coupling two or more quantum dots together, in the presence of a magnetic field, and to the environment. Here the coupling acts as a communication channel through which the states of the system may be perceived by the observer. We have defined such states as *bipartite-pointer states*, in that they cannot be presented by a linear combination of pointer states of the individual dots. By examining the classical electron dynamics in the array of open quantum dots, we find that the new states resemble trajectories connected to stable attractors, as opposed to KAM islands, which show that the einselection process takes place here as well.
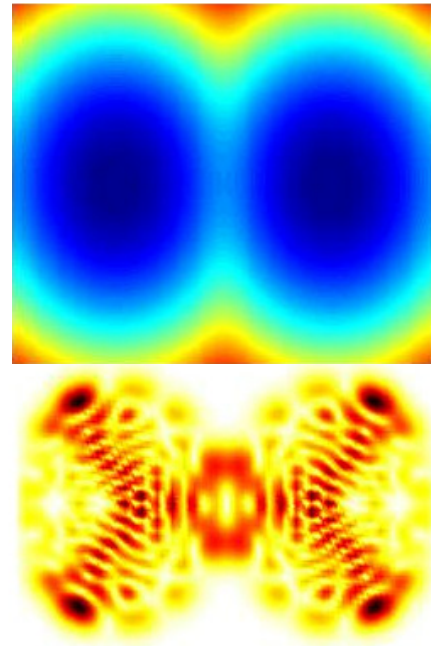

Figure 1. The confining potential for two quantum dots (top), and the bipartite-pointer state amplitude (bottom). Here, the Fermi energy is 8.63 meV and the magnetic field is 0.2 T. The total width of the image is 0.6 μm.

We have examined two dots in series as a representative example of an open dot array. In these calculations, the open system is fitted to what one would expect from a self-consistent calculation, which allows a controlled investigation of the coupling to the environment (reservoirs of electrons) and between the dots. In figure 1, we show the potential of the two dots and the wavefunction magnitude for such a bipartite confined state. The double parabolic confining potential has parameters that are defined in [24].

Hence, we have found the emergence of a new type of nonlocal multi-dot state due to a superselection process, and have defined this as a bipartite-pointer state. The results from the quantum-mechanical and classical calculations prove the stability and robustness of the state. Further, we find that the nonlocal bipartite-pointer state can *coexist* with single-dot pointer states. We point out that our work has been stimulated by the observation of corresponding resonances in experimentally studied quantum dot arrays [25]. While we have concentrated on arrays of electron billiards, we suggest that the bipartite-pointer states can be adapted to any multiply-coupled system.

3. Coupled quantum point contacts

Perhaps the earliest example of coupling of two quantum point contacts was the work of Eugster and del Alamo [9]. While the experiment was configured as two coupled (short) waveguides (see figure 2), it is the QPC behavior that is important. Tunneling between the two waveguides was certainly possible, but as one waveguide moved from one conductance plateau to the next, a peak in the tunneling current was observed. This
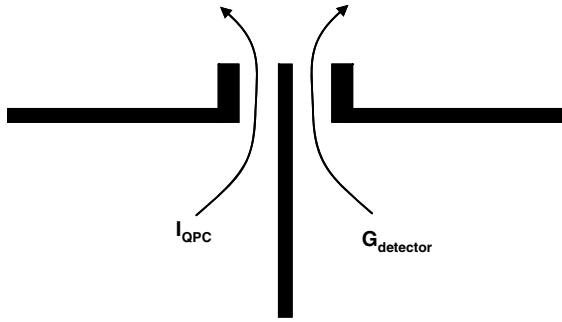
Figure 2. A pair of QPCs are coupled via the common wall.

Figure 3. Simulations taken from [26]. The QPC conductance is shown on the left, in units of $2e^2/h$, while the probe signal is the dotted line.

conductance peak is common to all of the nonlocal effects being discussed here. Generally, one thinks of the tunneling current being proportional to the density of states on each side of the barrier, but this peak in conductance is related to the change in the density of states as the additional conductance plateau arises. The original authors attribute the tunneling to the peaks in a one-dimensional density of states for the quantum waveguide itself, but such waveguides are seldom real one- dimensional structures, particularly as they are coupled to quasi-two-dimensional (Q2D) reservoirs. Rather, the transition from Q2D to QPC, and back, usually introduces some longitudinal resonances, and this has been shown to cause additional tunneling at the conductance step [26]. While these resonances may not show strongly in the conductance, the standing waves that are a result lead to the enhanced tunneling. In the simulation from [26], peaks can be seen in the QPC conductance as the plateaus are approached. These peaks arise from the resonances and lead to the signals in the probe conductance, as seen in figure 3.

In the coupling of the QD to the QPC, discussed above, modulation of the QPC conductance is seen as the dot is charged by single-electron tunneling [10]. In this case, it is felt that the variation in the self-consistent potential of the QD, as the charge is added/removed, affects the potential of the QPC itself, thus causing the conductance modulation. This is, in effect, a capacitive coupling between the QD and the QPC, which was shown to be extremely useful in counting the number of electrons entering, or leaving, the QD, as discussed above. Techniques such as these are discussed in the paper by Hohls *et al* (elsewhere in this issue), in their discussion of measuring noise in quantum dots.

Lüscher *et al* [10] studied a pair of QPCs in which the common gate structure was made sufficiently wide (80 nm) that tunneling was not evident in the characteristics. One QPC was used as a detector, sensitive to charge rearrangements in the other QPC, but no striking features were observed. However, in plotting the derivative $D = \mathrm{d}V_{\mathrm{eff}}/\mathrm{d}V_{\mathrm{QPC}}$, suppression of this quantity was observed at the transitions between plateaus of the QPC conductance. These dips in D are attributed to the presence of interactions and charge rearrangements in the QPC as each new subband begins to conduct.

More recently, Bird *et al* have studied a pair of coupled QPCs. At first, the two QPCs were separated with a barrier which contained a QD [8]. Then, they used a split barrier
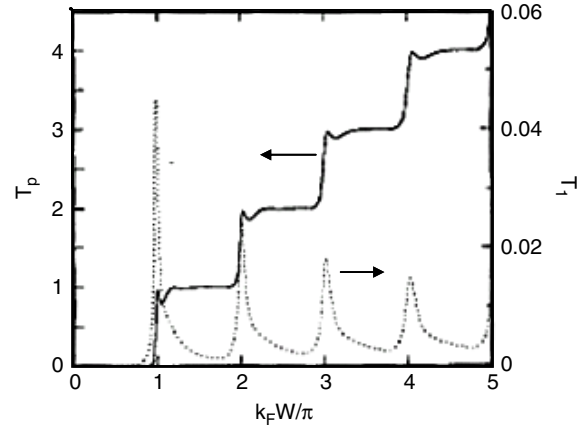
in which a gap was opened between the two QPCs [27]. Finally, each piece of the split barrier was replaced by a pair of nanowire gates to preclude any interactions, particularly electrostatic ones [28]. This latter arrangement gives 4 QPCs, any one of which can be used as the conducting QPC and any other one as the detector. (One form would be that of figure 2, with the central gate replaced by a parallel pair of gates.) In these measurements, they find a single peak in the detector which occurs *below* the first plateau onset. Presumably, the lack of other peaks arises from the separation which is too far for tunneling and specifically designed to avoid electrostatic interactions. But, the presence of the peak means that there are strong interactions between the two QPCs used in the measurement. As the results do not depend upon which QPC is used to study conductance and which is used as the detector, it can be assumed that structure and/or random defects do not play a role in the results. They interpret these results as the interaction between a plane wave in the detector and a particle trapped in a bound state within the swept QPC [29]. Such a bound state has been postulated by many authors in connection with the 0.7 plateau. Bird *et al* used a magnetic field to shift the peaks, and the results were consistent with a single electron occupying the bound state.

A bound state, as discussed above, can occur within a QPC that is biased below the first conductance plateau. The source of this bound state is still controversial, but appears in our own calculations in which the linear spin density approximation is used [30]. In these studies, the QPC is spin selective, passing first a single spin state. In figure 4, we plot the spin-dependent self-consistent potential of a single QPC. The transmitting spin state sees the dotted potential, while the non-transmitting state sees the solid potential. It is clear that there is a bound state in the potential of the non-transmitting spin state, as can also be seen in figure 5. Here, the saddle potential also shows a double barrier for the non-transmitted state, with the bound state located in the center of the QPC. The difference here, from that of the previous paragraph, is that this bound state is normally empty prior to the onset of transmission for this upper spin level. On the other hand, the transmitting spin state
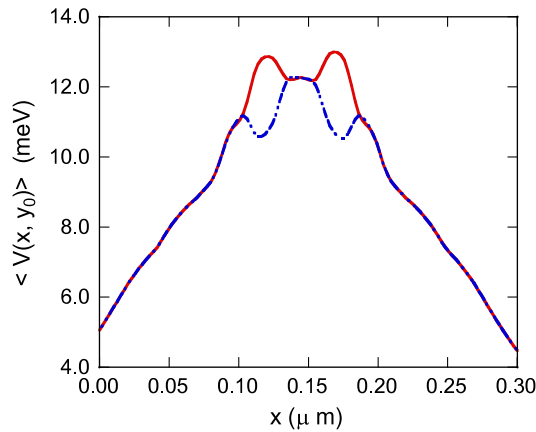
Figure 4. The self-consistent potentials seen along the central axis of a QPC. The transmitting spin state sees the dashed curve, while the reflected spin state sees the solid curve.
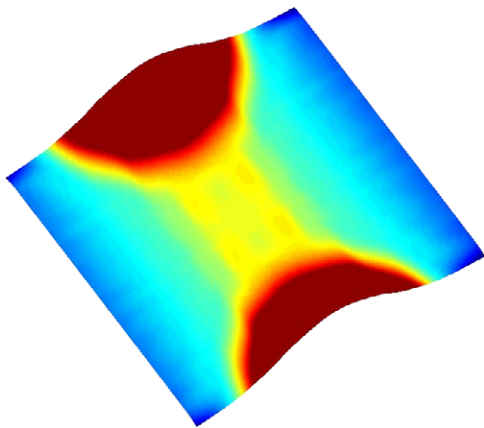

Figure 6. SGM scan of region within the saddle potential of a QPC. The ring structure, plus central peak are thought to be indicative of resonant tunneling through a local bound state [31]. Here, the device was biased well below the first plateau.

Figure 5. Local QPC potential, plus the spin-sensitive barriers for the non-transmitted spin level. The potential results from simulations detailed in [30].

shows a triple barrier potential, as can be seen in figure 4 above. In this case, the quasi-bound states are obviously occupied, as this state is transmitting, but some localization of wavefunction certainly occurs. Some confirmation of such a bound state is found in scanning gate microscopy studies of QPCs [31]. In such studies, a biased AFM tip is scanned over the QPC region and the change in conductance plotted versus tip position. In this way, variations can be correlated with density, which is sensitive to local potentials, such as those on the biased tip. Such a plot is shown in figure 6. A ring structure, plus a bright central peak, both of which are thought to be indicative of resonant tunneling through an (possibly) empty bound state localized within the QPC. However, this should not be viewed as precluding the filled state discussed here, and in the last paragraph, as the structure could arise from co-tunneling involving an electron in the bound state.

Another nonlocal interaction between QPCs has been reported by Khrapai *et al* [32], and discussed elsewhere in this issue. In this case, the barrier between the QPCs was firmly grounded and sufficiently thick to preclude tunneling. Then, the signal QPC was biased out of equilibrium, and a
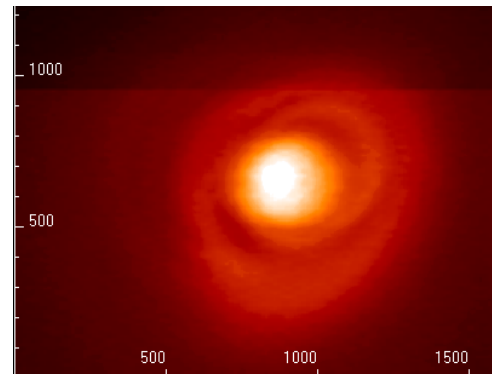
peak in the detector QPC occurred during the transition to the first plateau. In this case, the authors suggest that the non-equilibrium transport is accompanied by the emission of phonons that can be transported to the detector QPC. It is the detection of these phonons, which excite electrons, that gives rise to the detector response. Maximum sensitivity of the detector QPC seems to occur when it is biased at pinchoff.

4. Conclusions

Capacitive-coupled electrostatics, tunneling, interaction effects between bound and free carriers, emission of phonons by non-equilibrium carriers—the manner in which nonlocal effects can occur between two QPCs, or between a QD and a QPC, seems to encompass as many processes as can be conceived. In fact, it is quite apparent that these structures are a rich and fertile test bed in which to study a range of physical interactions. From this extensive set of interactions, considerable physics has been uncovered. Yet, it appears that more can still be discovered, and there remains a number of questions that must yet be addressed. Indeed, the range of explanations just for the so-called 0.7 plateau in the QPC itself has led to a recent special issue of this journal [33].

Here, we have reviewed a number of these observations, and compared them to results arising from our own calculations and measurements. While these results agree with most of the observations, it is still too early to know if this is the true situation, particularly with respect to e.g. the presence of bound states within the saddle potential of a QPC.

Acknowledgments

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