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Simple Method for the Determination of Series Resistance and Maximum Power of Solar Cell

Masatoshi WARASHINA and Akio USHIROKAWA

Institute of Space and Aeronautical Science, University of Tokyo

A simple method is proposed to determine the series resistance R_s and the perfection factor n of solar cells from the measurement of the tangents to the current I—voltage V curve at constant light intensity. The values of R_s and n thus determined are then used to predict the maximum power P_m and the voltage V_m at which the power gets maximum. This new method is applied successfully to the conventional cell, BSF (Back Surface Field) cell and the textured BSF Si cell. A good agreement is obtained between the calculated- and the directly measured values of V_m and P_m.

§1. Introduction

It is well known that the series resistance R_s is a power loss factor that decreases the maximum power out-put P_m. Hence, R_s is an important key parameter to be fed back into the fabrication process of the solar cell. To determine R_s, the previous methods[1,2] required the exposing of the cell to different light intensities. With these methods the conventional N/P or P/N Si cell,[3] which consisted of the same contact grids with six fingers for 2×2 cm^2 cell area as our experiments, were found to have 0.20–0.25 ohm series resistance. However, it will be shown that such a value of R_s is too large to satisfy the measured I-V curve under illumination. The discrepancy is due to the uncertainties involved in determining R_s with different light intensities. The purpose of the present paper is to show a more distinct method in which the R_s and n can be determined under the condition of constant light intensity including a concentrated illumination. Moreover, the present analysis provides a simple formula for V_m and P_m[4,5] in terms of R_s and n which are distinctly determined from the experimental I-V curves. The new method are applied successfully to the prediction of V_m and P_m for the conventional cell, BSF (Back Surface Field) cell and the textured BSF Si cell.

§2. Theory

Under illumination, the relation between current I and voltage V is well known as

$$I=I_L-I_s[\exp\{q(V+R_sI)/(nkT)\}-1]-(V+R_sI)/R_{sh}, \quad (1)$$

where I_L is the light generated current, I_s is the diode saturation current, (kT/q) is the thermal voltage V_{th} and R_{sh} is shunt resistance. By neglecting R_{sh} and using the conditions of both $I=I_{sc}$ at $V=0$ and $I=0$ at $V=V_{oc}$, eq. (1) yields

$$I_s=I_{sc}[\{\exp(V_{oc}/V_{th})\}-\{\exp(R_sI_{sc}/V_{th})\}]^{-1}, \quad (2)$$

where I_{sc} is short circuit current and V_{oc} is open circuit voltage. In the case of $\exp(R_sI_{sc}/V_{th})\ll\exp(V_{oc}/V_{th})$, we obtain the following relations

$$I_s=I_{sc}\exp(-V_{oc}/V_{th}), \quad I_{sc}=I_L+I_s. \quad (3)$$

Substituting eq. (3) into eq. (1) and solving for V, we obtain the following relation

$$V=V_{oc}-R_sI+(nkT/q)\ln[(I_{sc}-I)/I_{sc}]. \quad (4)$$

Differentiating eq. (4) with respect to I yields the following equation

$$-\mathrm{d}V/\mathrm{d}I=R_s+(nkT/q)(I_{sc}-I)^{-1}. \quad (5)$$

If the value of $(-\mathrm{d}V/\mathrm{d}I)$ on the left hand is known at each current I, R_s and n can be determined by using the property of the straight line for $(I_{sc}-I)^{-1}$. Figure 1 shows how to determine the value of $(-\mathrm{d}V/\mathrm{d}I)$ by drawing tangents to the I-V curve at point "a". A simple relationship of V_m and P_m is derived from eq. (4). Equation (4) becomes

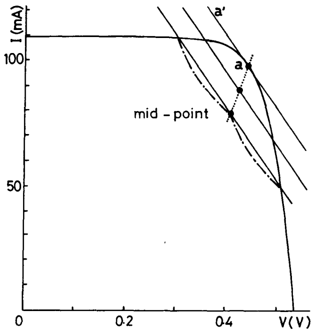

Fig. 1. How to draw the tangent to the I-V curve at point "a".

$$I=I_{sc}[1-\exp\{(V-V_{oc})/V_{th}\}]/[1+(R_sI_{sc}/V_{th})\exp\{(V-V_{oc})/V_{th}\}], \quad (6)$$

on the assumption of $R_sI<V_{th}$. The power out-put is defined as

$$P=IV=I_{sc}V[1-\exp\{(V-V_{oc})/V_{th}\}]/[1+(R_sI_{sc}/V_{th})\exp\{(V-V_{oc})/V_{th}\}]. \quad (7)$$

By differentiating eq. (7) with respect to V and setting the result to be equal to zero, a relation involving V_m is obtained as

$$(V_m-V_{oc})/V_{th}=-\ln[1+(R_sI_{sc}/V_{th})]-\ln[1+V_m/V_{th}], \quad (8)$$

on the assumption that the term of $\exp[2(V_m-V_{oc})/V_{th}]$ is negligible compared with the term of $\exp[(V_m-V_{oc})/V_{th}]$ and that

$$(R_sI_{sc}/V_{th})\exp[(V_m-V_{oc})/V_{th}]<1. \quad (9)$$

In order to solve eq. (8) for V_m, the V_m is defined to be

$$V_m=V_{oc}-x, \quad (10)$$

where x is assumed to be smaller than V_{oc}. Substituting eq. (10) into eq. (8) and then solving for x, we obtain the following equation

$$x=[\ln\{(1+R_sI_{sc}/V_{th})(1+V_{oc}/V_{th})\}]/[V_{th}^{-1}+(V_{oc}+V_{th})^{-1}], \quad (11)$$

on the approximation of $\ln(1+V_m/V_{th})=-x/(V_{th}+V_{oc})+\ln(1+V_{oc}/V_{th})$. Consequently, substituting eq. (11) into eq. (10) yields the relation

$$V_m=V_{oc}-[\ln\{(1+R_sI_{sc}/V_{th})(1+V_{oc}/V_{th})\}]/[V_{th}^{-1}+(V_{oc}+V_{th})^{-1}]. \quad (12)$$

Substituting V_m into eq. (7) yields the maximum power

$$P_m=\frac{I_{sc}V_m[1-\exp\{(V_m-V_{oc})/V_{th}\}]}{1+(R_sI_{sc}/V_{th})\exp\{(V_m-V_{oc})/V_{th}}. \quad (13)$$

In most cases, eqs. (12) and (13) are approximated as

$$V_m\cong V_{oc}-V_{th}\ln(1+V_{oc}/V_{th})-R_sI_{sc} \cong V_{oc}-V_{th}\ln(V_{oc}/V_{th}), \quad (14)$$

and

$$P_m\cong I_{sc}V_m[1-\exp\{(V_m-V_{oc})/V_{th}\}], \quad (15)$$

respectively, on the assumption of $R_sI_{sc}/V_{th}<1$ and $(V_{oc}+V_{th})^{-1}\ll V_{th}^{-1}$.

§3. Experiment

The samples (#1, #2 and #3) used in our experiment are $10\,\Omega\cdot$cm N/P Si cells with 300 μm thickness. The samples of #2 and #3 are a BSF and a textured BSF type, respectively. The number of the contact grid lines is six and the cell area is 2×2 cm^2. The I-V measurement at 28°C is carried out under the condition of 140 mW/cm^2 using Xe solar simulator as shown in Fig. 2. Figure 3 shows the straight lines between the tangent ($-\mathrm{d}V/\mathrm{d}I$) as ordinate and $(I_{sc}-I)^{-1}$ as abscissa with satisfying eq. (5). Consequently, R_s, n, V_m and P_m are obtained as shown in Table I. The results indicate that a good agreement is obtained between theory and experiment. However, if the R_s of the conventional cell is assumed to be 0.25 Ω, the I-V curve differs from the measured solid curve as shown in Fig. 2. As shown in Table I, the n value of 1.89 and 1.94 which are close to 2 is most likely due

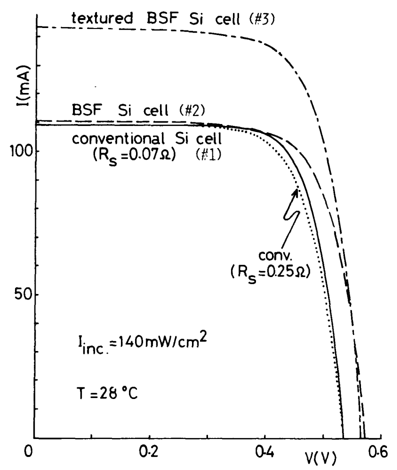

Fig. 2. Measured I-V curves.

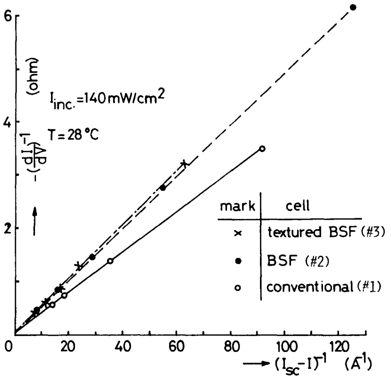

Fig. 3. Straight lines between $(-dV/dI)$ and $(I_{sc}-I)^{-1}$ determined from the measured I-V curve.

to space charge region recombination current.

Table I. Series resistance R_s, perfection factor n, maximum power P_m and voltage V_m at T=28 °C and $I_{inc.}$=140 (mW/cm²). The #3 is the textured BSF Si cell.

Type	R_s (ohm)	n	V_m (V)		P_m (mW)	
			cal.	exp.	cal.	exp.
Conv.	0.07	1.46	0.433	0.437	43.7	43.5
BSF	0.05	1.89	0.450	0.448	45.4	45.6
#3	0.06	1.94	0.449	0.448	57.1	57.2

The n value of 1.46 is attributed to the presence of both the recombination and the injection current.

As for the relative error of R_s and n, the following relations are obtained, respectively,

$$dn/n=[(I_{sc}-I_1)^{-1}+(I_2-I_1)^{-1}]\,dI_1+[(I_{sc}-I_2)^{-1}+(I_2-I_1)^{-1}]\,dI_2 \\ +(t_2-t_1)^{-1}(dt_2+dt_1), \quad (16)$$

$$dR_s/R_s=[\{t_1(I_{sc}-I_1)-t_2(I_{sc}-I_2)\}^{-1}t_1+(I_2-I_1)^{-1}]\,dI_1 \\ +[\{t_1(I_{sc}-I_1)-t_2(I_{sc}-I_2)\}^{-1}t_2+(I_2-I_1)^{-1}]\,dI_2 \\ +[t_1(I_{sc}-I_1)-t_2(I_{sc}-I_2)]^{-1}I_{sc}(dt_1+dt_2), \quad (17)$$

where I_1 and I_2 are the current at V_1 and V_2, t_1 and t_2 are the tangent $(-dV/dI)$ at I_1 and I_2. The errors of I_1, I_2, t_1 and t_2 are defined as dI_1, dI_2, dt_1 and dt_2, respectively. It is found that the relative errors of dR_s/R_s and dn/n increase as I_1 and I_2 approach to I_{sc} and as I_1 approaches to I_2, because the values of the denominator on the right hand of eqs. (16) and (17) become small.

Table II shows the temperature dependency of R_s, n, P_m and V_m of #2 cell. It is found that the series resistance decreases with increasing temperature. Table III illustrates the value of R_s, n, P_m and V_m of #2 cell as a function of light intensity. The values of R_s and n decrease as the light intensity decreases.

Table II. Temperature dependency of R_s, n, P_m and V_m of BSF Si cell at $I_{inc.}$=140 (mW/cm²).

T (K)	R_s (ohm)	n	V_m (V)		P_m (mW)	
			cal.	exp.	cal.	exp.
301	0.052	1.89	0.450	0.448	45.4	45.6
313	0.040	1.85	0.428	0.430	43.1	43.2
323	0.045	1.84	0.407	0.405	40.9	40.9
333	0.030	1.90	0.385	0.385	38.2	38.2
343	0.028	1.91	0.363	0.370	35.8	36.0

Table III. R_s, n, P_m and V_m of BSF Si cell at 28 °C as a function of light intensity (mW/cm²).

$I_{in.}$	R_s (ohm)	n	V_m (V)		P_m (mW)	
			cal.	exp.	cal.	exp.
140	0.052	1.89	0.450	0.448	45.4	45.6
120	0.052	1.80	0.449	0.450	39.2	39.3
100	0.030	1.79	0.447	0.450	32.4	32.4

§4. Conclusion

A simple method is proposed to determine the series resistance R_s and the perfection factor n, and to derive the maximum power P_m and the voltage V_m at which the power reaches P_m. The series resistance and the perfection factor are first determined by using the measured linear relationship between $(I_{sc}-I)^{-1}$ and $(-dV/dI)$ which is the tangent to the current I-voltage V curve at constant light intensity. The method then provides a simple formulae for V_m and P_m in terms of R_s and n. When this method is applied to the conventional cell, BSF (Back Surface Field) cell and the textured BSF Si cell it has permitted the unambiguous determination of R_s and n and also the successful prediction of P_m and V_m. It is further found that the series resistance of BSF cell decreases with increasing temperature and that the value of R_s and n decreases with decreasing the light intensity. The n-value of the BSF cell and the textured BSF cell is larger than that of the conventional cell.

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