

Electron and Hole Mobilities in Silicon as a Function of Concentration and Temperature

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Abstract—An analytical expression has been derived for the electron and hole mobility in silicon based on both experimental data and modified Brooks-Herring theory of mobility. The resulting expression allows one to obtain electron and hole mobility as a function of concentration up to $\sim 10^{20} \text{ cm}^{-3}$ in an extended and continuous temperature range (250–500 K) within ± 13 percent of the reported experimental values.

INTRODUCTION

THE ELECTRON and hole mobilities in silicon as a function of dopant concentration and temperature are important parameters for device design and analysis. These mobilities have been measured [1]–[8] at room temperature for concentrations up to $4 \times 10^{20} \text{ cm}^{-3}$. However, very little data are available [6], [8] at higher temperatures and for higher concentrations. Based on classical theory, Li and Thurber [9], [10] have calculated mobilities for concentrations up to 10^{19} cm^{-3} and temperatures up to 500 K and have found that the calculated values agree well with the experimental values reported in the literature. For concentrations higher than 10^{19} cm^{-3} the classical theory, however, fails to predict correct values—it gives values of mobility much lower than those measured experimentally. Various theories [8], [11]–[13] have been developed to explain the anomaly between the theoretical and experimental values, but they are too complicated to be used for device analysis and design work where an analytical expression is more appropriate.

In this paper, we present a single empirical relationship for mobility as a function of temperature and concentration which can be used for temperatures up to 500 K and concentration up to about 10^{20} cm^{-3} .

MOBILITY FORMULATION METHOD

The procedure adopted to arrive at the analytical expression follows. The values of mobility for concentration up to $5 \times 10^{18} \text{ cm}^{-3}$ were calculated by appropriately combining the lattice and ionized impurity scattering mobilities. The

effect of electron-electron scattering in the case of electron mobility and hole-hole scattering in the case of hole mobility was incorporated empirically for dopant densities greater than $2 \times 10^{16} \text{ cm}^{-3}$ following the approach of Li and Thurber [9], [10]. For higher concentrations, experimental values of mobility were taken from the literature. These mobility values, both calculated and experimental (henceforth called experimental for the sake of distinguishing them from those calculated from the empirical relation to be derived) were then fitted into the following expression for the mobility:

$$\mu = \mu_{\min} + \frac{\mu_0}{1 + (N/N_{\text{ref}})^\alpha} \quad (1)$$

which is similar to the one used by Caughey and Thomas [14]. Here μ_{\min} is the minimum mobility value expected, μ_0 is the difference between the maximum and minimum mobility expected, N_{ref} is a reference concentration, while α is an exponential factor that controls the slope around $N = N_{\text{ref}}$. All four parameters are taken to be constant at a particular temperature. To define these parameters at a fixed temperature we have used an optimization technique [15] which minimizes the error

$$\delta = \sqrt{\frac{1}{M} \sum_{j=1}^M (\mu_j - \mu'_j)^2 / \mu_j^2} \quad (2)$$

where μ_j represents the experimental mobility while μ'_j represents the mobility calculated by (1) pertaining to the j th concentration N . In this way, four parameters in (1) can be calculated at different temperatures. Thus knowing the temperature variation of these parameters we arrive at analytical expressions, for both electron and hole mobilities similar to that of (1).

RESULTS AND DISCUSSIONS

A. Electron Mobility

The experimental data of Lang [3] and Norton *et al.* [4] for lightly doped n-type silicon in the temperature range 200–500 K were used to calculate the lattice scattering mobility μ_L . It is found that their data can be fitted very well to the following expression:

$$\mu_L = 8.56 \times 10^8 T^{-2.33} \quad (3)$$

where T is the temperature in Kelvins.

The ionized impurity scattering mobility μ_I is calculated

Manuscript received August 16, 1981; revised September 28, 1981. This work was supported in part under a Research Grant from SERI, Golden, CO.

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using a modified Brooks-Herring formulation [9] which takes into account the anisotropic scattering effects and can be written as

$$\mu_I = \frac{7.3 \times 10^{17} T^{3/2}}{N_I G(b)} \quad (4)$$

where N_I is the number of ionized impurity atoms and $G(b)$ is a function given by

$$G(b) = \ln(b+1) - \frac{b}{b+1} \quad (5)$$

where

$$b = \frac{1.52 \times 10^{15} T^2}{n'} \quad (6)$$

and $n' = n[2 - (n/N)]$ which assumes the acceptor concentration to be zero, n being electron density per cubic centimeter.

After taking into account electron-electron scattering effects on μ_L and μ_I as is done by Li and Thurber [9], the two mobilities are combined according to the mixed scattering formula [16]

$$\mu = \mu_L \left[1 + x^2 \left\{ \text{Ci}(x) \cos x + \sin x \left(\text{Si}(x) - \frac{\pi}{2} \right) \right\} \right] \quad (7)$$

where $x^2 = 6\mu_L/\mu_I$, and $\text{Ci}(x)$, $\text{Si}(x)$ are cosine and sine integrals of x , respectively. Normally μ_L and μ_I are combined using Mathiessens' rule for the sake of simplicity, but this is known to give incorrect results [8], [16]. Thus electron mobility can be calculated at any temperature using (3)-(7), for concentration up to $5 \times 10^{18} \text{ cm}^{-3}$. For higher concentrations, experimental value of mobilities have been taken from the literature. However, at these concentrations (degenerate condition) the mobility depends upon the nature of the dopant [6], [8]. For phosphorus dopant, the mobility value is 10-15 percent higher than the corresponding value for arsenic dopant [6], [8]. In what follows, we have taken experimental values of the mobility for phosphorus dopant. Although there is a large spread (20-25 percent) in the mobility values reported in the literature, [5]-[8], [12], [17] there is consistency in the observation—the mobility becomes almost constant independent of concentration for concentration greater than about $2 \times 10^{19} \text{ cm}^{-3}$ and decreases slowly with increasing temperature. In our calculations, we have taken mobility value for concentrations greater than $2 \times 10^{18} \text{ cm}^{-3}$ from experimental data of Mousty *et al.* [5] and Finetti *et al.* [12] for room temperature and below as their data seem to be fairly accurate being within 2 percent of the experimental error. For temperatures above 300 K, the mobility is calculated from the resistivity data of Chapman *et al.* [6], for heavily doped phosphorus, and used in the present calculations, taking Mousty *et al.* [5] mobility value at room temperature as the base value. These mobility values for four values of temperature 200, 300, 400, and 500 K are shown as a continuous line in Fig. 1. Near an electron concentration of $5 \times 10^{18} \text{ cm}^{-3}$, the data are smoothed out to match between the theoretical and experimental values—the maximum difference observed was about 7 percent at $5 \times 10^{18} \text{ cm}^{-3}$. Each mobility curve

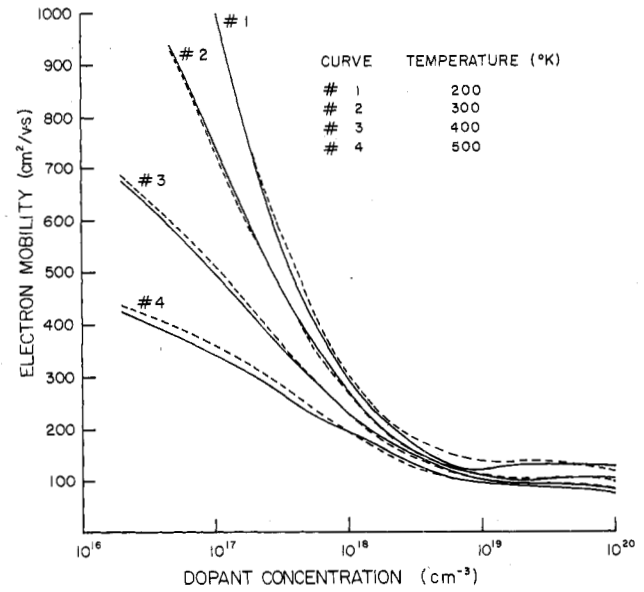


Fig. 1. The electron mobility in phosphorus-doped silicon as a function of concentration for four temperatures 200, 300, 400, and 500 K. The continuous lines are experimental values of the mobility (see text) while the dotted lines are the corresponding values calculated using (8).

TABLE I
BEST FITTING PARAMETERS OBTAINED FROM THE MOBILITY DATA
(SEE TEXT)

$T(^{\circ}\text{K})$	μ_{min} $\text{cm}^2/\text{V}\cdot\text{s}$	ν_0 $\text{cm}^2/\text{V}\cdot\text{s}$	$N_{\text{ref}} \times 10^{17}$ cm^{-3}	α
Electrons				
200	109.7	3103.6	0.503	0.947
300	88.3	1241.8	1.295	0.891
400	73.78	644.3	2.46	0.823
500	66.2	364.0	4.29	0.831
Holes				
300	54.3	406.9	2.35	0.88

is then fitted into (1) using the optimization technique as discussed earlier. The values of the four parameters thus obtained are shown in Table I. As can be seen from this table all four parameters vary with temperature. It is to be noted that the values of the four constants at 300 K are very close to those obtained by Baccarani *et al.* [7] based on measurements of electron mobility in phosphorus-doped silicon. Taking into account these temperature variations of the parameters, we arrive at the following empirical relation for the electron mobility as a function of temperature and concentration:

$$\mu_e = 88 T_n^{-0.57} + \frac{7.4 \times 10^8 T_n^{-2.33}}{1 + [N/(1.26 \times 10^{17} T_n^{2.4})] 0.88 T_n^{-0.146}} \quad (8)$$

where $T_n = T/300$. The values of electron mobility calculated from (8) are shown in Fig. 1 as dotted lines for the four tem-

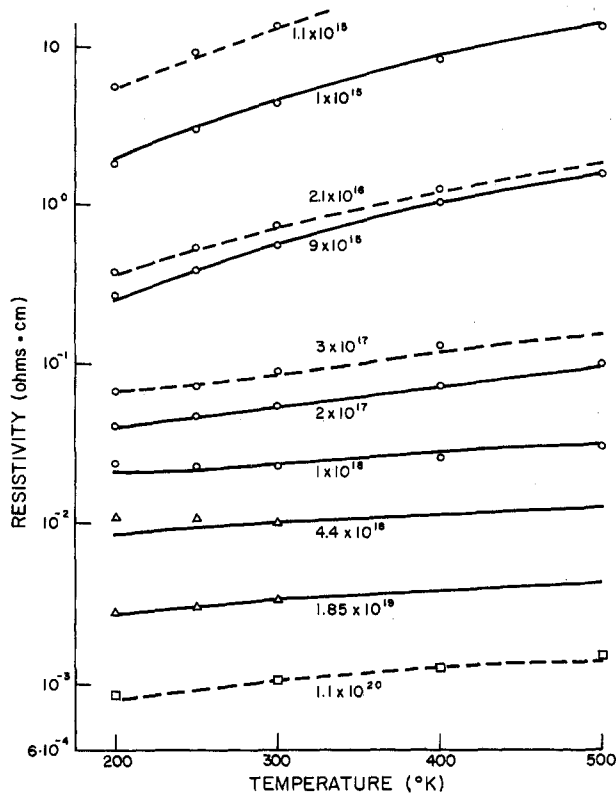


Fig. 2. Resistivity as a function of temperature for different phosphorus- (continuous line) and boron- (dotted line) doped silicon slices of different concentrations. The points are experimental data (\circ -Li and Thurber, Δ -Finetti *et al.*, and \square -Chapman *et al.*).

TABLE II
ROOT MEAN SQUARE DEVIATIONS AND MAXIMUM DEVIATIONS BETWEEN
THE EXPERIMENTAL MOBILITY AND THAT OBTAINED FROM (8)
(ELECTRON MOBILITY) OR (13) (HOLE MOBILITY)

T(°K)	Root mean square deviations (percent)	Maximum Deviation (percent)	Root mean square deviation (percent)	Maximum deviation (percent)
	Electrons		Holes	
250	7.03	12.17	7.2	12.9
300	4.06	8.62	4.43	8.9
400	4.67	9.6	5.4	13.2
500	6.13	10.2	-	-

peratures. As can be seen, the dashed curves are quite close to the continuous line. In fact, as reported in Table II, the root mean square deviation δ (2) is less than 7 percent while the maximum deviation is less than 12 percent in the temperature range 250–500 K. Although for the sake of clarity Fig. 1 is drawn for concentrations greater than 10^{16} cm^{-3} , (8) is valid for concentrations in the range 10^{13} – 10^{20} cm^{-3} .

To further check the validity of (8), the resistivity of n-type samples were calculated from the expression

$$\rho = \frac{1}{qN\mu_e} \quad (9)$$

In (9) μ_e is substituted from (8). The resistivity thus computed as a function of temperature is shown in Fig. 2 as a

continuous line for different concentrations. The points are experimental values of resistivity taken from the literature [5], [6], [9], [12]. It can be seen that the theoretically calculated resistivities are within 8 percent of those measured experimentally in the temperature range 200–500 K except for the concentration range 4 – $8 \times 10^{18} \text{ cm}^{-3}$ where the deviation becomes 26 percent at 200 K. The reason for this high deviation in this dopant range is because the experimentally measured resistivity changes its behavior [5], [6], [9], [12], [17] from decreasing resistivity, with decrease of temperature, to increasing resistivity through a constant value. This abnormal behavior of resistivity at lower temperatures is not taken into account fully in the analytical expression (8) for mobility which is used to calculate resistivity from (9). This also explains the reason for a larger error in the equation at lower temperatures.

B. Hole Mobility

The data given by Li [10] for the hole lattice scattering mobility in the temperature range 150–400 K can be fitted into the following expression:

$$\mu_L = 1.58 \times 10^8 T^{-2.23} \quad (10)$$

The hole ionized scattering mobility using the modified Brooks-Herring formulation can be written as

$$\mu_I = \frac{5.6 \times 10^{17} T^{3/2}}{N_I G(b)} \quad (11)$$

where $G(b)$ is given by (5)

$$b = \frac{2.5 \times 10^{15} T^2}{p'} \quad (12)$$

and

$$p' = p \left(2 - \frac{p}{N} \right)$$

p is the hole density (cm^{-3}) and the donor density is assumed to be zero.

Again taking into account hole-hole scattering effects on μ_L and μ_I as done by Li [10] and combining the two mobilities according to (7), hole mobilities are obtained for concentrations up to $2 \times 10^{18} \text{ cm}^{-3}$. For higher concentrations, experimental values of hole mobility have been taken. The hole mobilities thus obtained at room temperature are fitted into (1), again resulting in four parameters whose values are given in Table I. These parameters for hole mobility could not be calculated at higher temperature because of the lack of hole mobility data available at high temperatures and concentrations. However, since the three parameters μ_{\min} , N_{ref} , and α of (1) are mainly governed by μ_I and since factors affecting the temperature dependence of μ_I are the same for electrons and holes (see (4) and (11)), these temperature variations have been taken here to be the same for electrons and holes. Thus we arrive at the following expressions for the hole mobility as a function of temperature and concentration:

$$\mu_h = 54.3 T_n^{-0.57} + \frac{1.36 \times 10^8 T^{-2.23}}{1 + [N/(2.35 \times 10^{17} T_n^{2.4})] 0.88 T_n^{-0.146}} \quad (13)$$

The maximum deviation in the mobility calculated from (13) and those calculated from (7) and (10)–(12) for concentrations less than $2 \times 10^{18} \text{ cm}^{-3}$ are given in Table II. As in the case of n-type samples, (13) was used to calculate the resistivity of p-type boron-doped samples and compared with experimental value of resistivities reported in the literature [6], [10]. The dashed line in Fig. 2 are the resistivity calculated using (9) where μ_e is replaced by μ_h of (13) while the points are the corresponding experimental points. As can be seen, theoretically calculated resistivities are within 10 percent of these measured experimentally except in the resistivity range $5\text{--}9 \times 10^{-18} \text{ cm}^{-3}$ where resistivity shows an abnormal behavior as in the case of n-type samples. This further proves the point that the temperature variation of the parameters μ_{\min} , N_{ref} , and α are essentially the same for both electrons and holes. Thus it can be seen that (13) represents hole mobility variations with temperature and concentration within an error of less than 14 percent in the temperature range 200–400 K. This seems reasonable taking into consideration the spread in the mobility values reported in the literature.

CONCLUSIONS

In conclusion, an analytical expression has been given for electron and hole mobility in silicon as a function of concentration and temperature. The resulting expression allows one to obtain electron and hole mobility as a function of concentration in an extended and continuous temperature range within 13 percent of reported experimental values.

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