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A New Metaheuristic Football Game Inspired Algorithm

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Abstract— **Metaheuristics are high level strategies for exploring the search space by using different methods to solve global optimization problems. In this paper, Football Game Algorithm has been proposed as a new metaheuristic algorithm based on the simulation of football players' behavior during a game for finding best positions to score a goal under supervision of the team coach. Simulation of humans' intelligences who are working together as a team to reach a specific goal instead of simulating the intelligence of various animal swarms in the nature is the most important distinction of the proposed algorithm to other existing algorithms that also introduces a new approach for making balance between diversification and intensification. Football Game Algorithm is a nature inspired, population base algorithm with ability in finding multiple global optimums. We have studied general football game tactics and idealized its characteristics to formulate Football Game Algorithm. We have then compared the proposed algorithm with other metaheuristics, including standard and modified particle swarm optimization and bat algorithm. The result of comparison studies show that the proposed Algorithm outperforms other algorithms and also has more robust performance. Finally, we have discussed and concluded by pointing out special attributes of the Football Game Algorithm.**

Keywords-optimization; metaheuristics; swarm intelligence; diversification; intensification

I. INTRODUCTION

Metaheuristics are high level strategies for exploring the search space by using different methods to solve global optimization problems [1]. The main purposes of development of modern metaheuristic algorithms are to solve problems faster, solve large complex problems, and obtain robust solutions. Obviously, the choice of the optimization algorithms largely depends on the type of the problem of interest and the expected quality of solution. For a specific category of problems, some algorithms may produce better results faster and more efficiently [2]. So the objective is to design better algorithms for most types of problems, not for all the problems [3].

Two major components of any metaheuristic algorithms are: intensification and diversification, or exploitation and exploration. Diversification means to generate diverse solutions so as to explore the search space on the global scale, while intensification means to focus on the search in a local region by exploiting the information that a current good solution is found in this region [3]. The balance between diversification and intensification is so important, on one side to quickly identify regions in the search space with high quality solutions and on the other side not to waste too much times in the regions of the search space witch are either already explored or do not provide high quality solutions [1]. The good combination of these two major components will usually ensure that the global optimality is achievable [3]. Different algorithms apply different combinations of these two major components due to their strategies that are dependent on the philosophy of the metaheuristics themselves.

The most successful and promising metaheuristic algorithms have been inspired from the natural systems. For example, Genetic algorithm (GA) and Deferential Evolution (DE) were inspired from biological systems and Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO) were developed based on the swarm behavior of animals. Other new nature inspired algorithms are also emerging recently, including Harmony Search (HS), Bee Algorithms (BA), Firefly algorithm (FA), Cuckoo Search (CS), and Bat-inspired Algorithm (BA) [4-12]. There are also other novel proposed algorithms in the literature which inspired from sport games like football [13-15]. Football Optimization Algorithm (FOA) is one of these newly proposed algorithms which despite its similarity to the Football Game Algorithm in terms of the titles of the algorithm and its parameters, comprises complete different approaches for exploration and exploitation strategies. In addition FOA has not discussed clearly and the algorithm formulations are vague without any pseudo code to make it possible to compare FOA functions against FGA, and also its comparison study for four simple benchmark problems has not shown any special superiority of FOA against other existing algorithms [13].

In this paper Football Game Algorithm (FGA) has been introduced as a new metaheuristic for solving continues global optimization problems. The algorithm imitates football players' behavior during a game for finding best positions to score a goal under supervision of the team coach. Since the movements of the players are totally purposeful and the presence of the team coach as a higher level supervisor helps better and more efficient team working, FGA performs better than other existing algorithms. FGA uses different strategies to make a

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balanced combination of diversification and intensification against other existing algorithms with considerable distinction. Simulation of humans' intelligences who are working together as a team to reach a specific goal instead of simulating the intelligence of various animal swarms in the nature is the most important distinction of the proposed algorithm to other existing algorithms that also introduces a new approach for making balance between diversification and intensification. We will first describe the most important football tactics in a real game and then Football Game Algorithm will be formulated through general movement and coaching parts. The performance of the algorithm will be studied and compared with other algorithms and finally we will discuss the special characteristics of FGA and research topics for further studies.

II. Football Game

Football is the world's most popular ball game in numbers of participants and spectators. Simple in its principal rules and essential equipment, the sport can be played almost anywhere, from official football playing fields (pitches) to gymnasiums, streets, school playgrounds, parks, or beaches. Football's governing body, the Fédération Internationale de Football Association (FIFA), estimated that at the turn of the 21st century there were approximately 250 million football players and over 1.3 billion people "interested" in football; in 2010 a combined television audience of more than 26 billion watched football's premier tournament, the quadrennial month-long World Cup finals [16, 17].

The game of football is not a modern sport. It has been played in one form or another for at least 2,500 years. This timelessness and universality of football suggests that it may somehow be ingrained in what we take to be human nature. It embraces all those skills that were once essential for our survival as hunter -gatherers- a way of life that until only relatively recently was the norm for Homo sapiens and one which continues to shape much of our behavior today. Speed, agility and accuracy of aim, coupled with cooperation among men in groups, were the hallmarks of successful hunting tribes those 'teams' that would survive and prosper and pass on their genes to future generations. However talented an individual might have been, it was nigh impossible to hunt and kill prey alone, one needed to be in a team and devise, as a group, a strategic game plan in which individual talents could be harnessed and expressed and a goal achieved.

Football is a clear reminder of our origins of those attributes that contributed so much to our success as a species. That is one of the reasons why it plays such an important role in the lives of millions of people. When we watch a football match we do not simply observe a game, we witness a replay of our evolutionary heritage.

A. Football Tactics

There are various individual skills and team tactics needed to play effective football. Football is in theory a very simple game, as illustrated by Kevin Keegan's famous assertion that his tactics for winning a match were to "score more goals than the opposition". Every team uses various tactics and team formation against different opponent teams. Most of the time and in the first minutes of the game, a team tries to test and find the best ways to reach opposing team's goal. At the same time the coach of the team makes notes about the best positions and the ways to the goal. Sometimes this process takes a long time up to end of the first half of the game. According to the game situation, team willing, and those findings, the team coach guides the players to attack to the best position to increase the pressure on the opposing team and make better chances to score goals, Fig. 1.

Also he has some other choices like substitutions to change weaker or tired players of the team with fresh ones. The most tired players are generally substituted, but only if their substitutes are well trained to fill in the same role or if the formation is transformed at the same time to accommodate for the substitution. Coaches often refrain from substituting defensive players in order not to disrupt the defensive posture of the team. Instead, they often replace ineffective attackers or unimaginative midfielders in order to freshen up the attacking posture and increase their chances of scoring, Fig. 2.

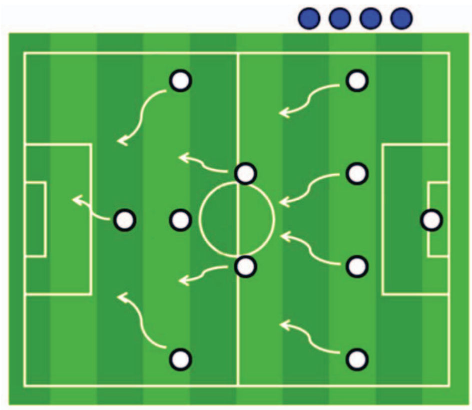

Figure 1. Attacking and increasing pressure on opposing team.

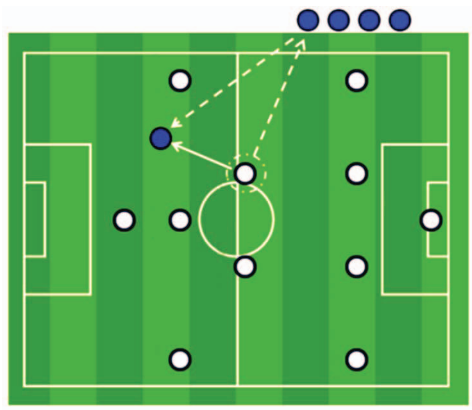

Figure 2. Substitutions to change weaker players with fresh ones in better positions.

For a team that is losing a game, a fresh striker can bring more benefit in circumventing an opposed defense line composed of relatively tired players. For a team that is winning a game, a fresh midfielder or a defender can bring more benefit in strengthening the defense against the opposition's attackers (who may be fresh substitutes themselves). In this situation, it is usually imaginative attacking flair players who are replaced by tough-tackling defensive midfielders or defenders.

III. FOOTBALL GAME ALGORITHM

For translating a real football game to a simulated Football Game Algorithm we must idealize a sort of its characteristics. So we apply following idealized rules:

1. Pseudo time of the algorithm will be considered as the overall time of the match. The members of the population are considered as football players and search or design space is considered as the football pitch.

2. All the football players belongs to one team that is considered like a team that is losing the game or always wants to score more goals and they are attacking all long of the game.

3. It is considered that all the players of opposing team are positioned around the peaks of a minimization problem that makes these places unwilling for the attacking team (the coach and the players).

Initial population defines initial formation of the players in the pitch. After this initial condition, every player moves around his last position (random walk) biased toward the ball, as time goes by. Ball will be passed between the players and the players in the better position (having less fitness in a minimization problem) have more chance to receive the ball.

Corresponding fitness values to every players, shows the quality of their position. The team coach will memorize the best positions during the match and uses them to guide players and pushing them forward. Also he can use substitution option to change a defender in low quality position with a fresh striker in the best position to increase the chance of scoring.

These strategies will continue up to the end of the game or until the team scores a goal (this will be simulated by termination criteria).

A. General Movement of Players

Normally and without coaching effects on the players' movement, their movements composed of two terms. First term is a simple random walk, and second term is a movement toward the ball. Every player moves randomly from his last position to find a better position. Also he goes toward the player that has the ball to receive it and make a better chance for scoring a goal. So the new position of the player at time step t is given by:

$$\mathrm{X}_i^t = \mathrm{X}_i^{t-1} + \alpha_i \varepsilon + \beta\left(\mathrm{X}_{ball}^t - \mathrm{X}_i^{t-1}\right) \tag{1}$$

Where $\varepsilon \in [-1,1]$ and $\beta \in [0,1]$ are random numbers drawn from a uniform distribution and $\alpha > 0$ is the step size which should be related to the scales of the problem of interest. Furthermore, the random walk term can easily be extended to other distribution such as Levy fights. Also a further improvement on the convergence of the algorithm is to vary the randomization parameter α so that it decreases gradually as the optima are approaching. For example, we can use

$$\alpha_i = \alpha_0 \theta^t \tag{2}$$

Where $\theta \in (0,1]$ is the randomness reduction constant. α_0 is the initial randomization parameter which should be related to the scales of the problem of interest [3].

In (1), X_{ball}^t is the position of the player who has the ball at time step t. The ball will be passed randomly between the players and the players in the better positions have more chance to receive the ball.

B. Coaching

In addition to general movement of the players, the team coach uses his findings during the match to guide the players and put them in the better positions. The coach memory (CM) is actually the memory of the algorithm to save best positions and their corresponding fitness values. The coach memory size (CMS) will be chosen as a fraction of the population size.

As it was explained, there are two general strategies that can be applied by the team coach to increase the chance of scoring a goal (finding the global optima).

1) Strategy number 1: Attacking

In this strategy the coach pushes the back players like defenders and midfielders forward and toward the best positions. To increase the chance of success, the coach encourages players to go forward and increase the pressure on the opposing team.

Technically speaking, we consider an imaginary Hyper Sphere (HS) with Hyper Radius (HR) that the position of the best member of the population will be set as its center. As is shown in Fig. 3, a Hyper Radius Limitation Value (HRLV) will be defined that will be decreased gradually as the iterations proceed. On the other hand, every member of the population has a hyper distance (HD) from the best position. Hence, members with higher distance value in comparison with HRLV will be pushed toward the nearest best positions. According to the optimization terminology we can call this strategy *Hyper Radius Penalty* method.

So we have

$$HRLV^t = HRLV_{\min} + \left(HRLV^{t-1} - HRLV_{\min}\right)\cdot \gamma \tag{3}$$

Where $\gamma \in (0,1)$ is the reduction constant of HRLV .

2) Strategy number 2: Substitution

In this strategy the coach uses substitution option to change weaker players with better ones. Actually in this strategy weaker players that have high corresponding fitness values will be replaced with other players around the nearest best position according to the coach memory.

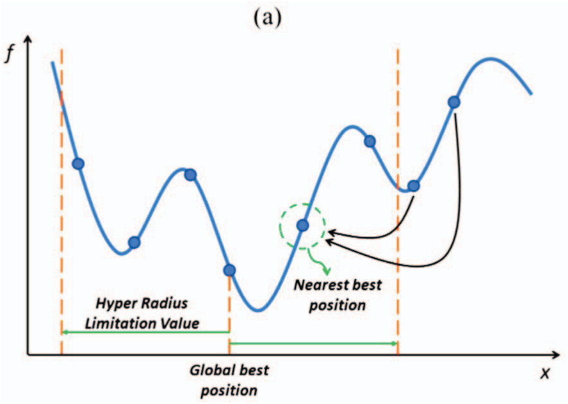

Figure 3. The schematic representation of Hyper Radius Penalty in the beginning, the of optimization process.

Technically speaking, we define a Fitness Limitation Value (FLV) that will be decreased accordingly as the iterations proceed. As can be seen in Fig. 4 every member of the population which has grater fitness value than FLV will be replaced with another one around the nearest best solution. Also we name this strategy Fitness Penalty method.

Now we have

$$FLV^{t} = FLV_{\min} + \left(FLV^{t-1} - FLV_{\min}\right)\cdot \lambda \tag{4}$$

Here λ has the same role as γ has for (3).

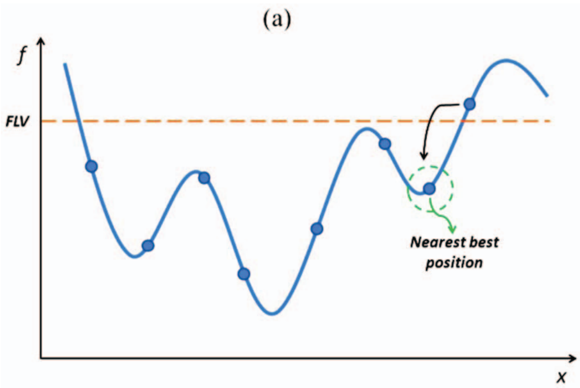

Figure 4. The schematic representation of Fitness Penalty in the beginning, the of optimization process.

Indeed coaching part is the local search part of the algorithm. After applying the strategies, the new position for players who are located beyond the limitation values will be achieved by using random walk from the nearest best solution to their old position:

$$X_{new} = X_{nearest-best} + \alpha_i \varepsilon \tag{4}$$

The basic steps of Football Game Algorithm can be summarized as the pseudo code shown in Fig. 5.

Football Game Algorithm

Define fitness function $f(X), X = (x_1, x_2, \ldots, x_{\dim})^T$

Initialize the team formation (players position) $X_i, (i=1,2,\ldots,N)$

Define algorithm parameters $\theta, \gamma, \lambda, CMS$

while (*t< max number of iterations*)

Compute fitness values

Rank the players and save best solutions in the CM

Identify the player who possess the ball X^{t}_{ball}

if ($f(x_i) > FLV^t$ OR $HD_i > HRLV^t$)

Generate a local solution around nearest best solution [equation (5)]

Update $\alpha_i, FLV, HRLV$ *[equations (2),(3),(4)]*

else

Generate new positions by general movement [equation (1)]

end

end while

Figure 5. The pseudo code of Football Game algorithm.

IV. VALIDATION AND COMPARISON

In order to verify the efficiency of the new algorithm, it was benchmarked using different benchmark functions for global optimization problems and the results have been compared against other metaheuristic algorithms, including standard and modified PSO and Bat Algorithm [6, 12, 18, 19]. PSO is one of the most popular metaheuristics that is accepted as a promising algorithm in global optimization. There is some strong evidence that PSO is better than traditional search algorithms and even better than genetic algorithms for many types of problems [3]. Also BA is one of the recent developments in the field of metaheuristic algorithms that is claimed to have better performance against other metaheuristics like PSO and GA [2, 12]. The benchmarks contain 12 unimodal and multimodal functions [18].

There are different ways to carry out the comparison of optimization algorithm performance. In this study we compare their ability and accuracy in finding global optima for a fixed number of function evaluations. For two dimensional problems we have considered 2,000 function evaluation with population size $n = 20$. Also we have used a fixed 10,000 function evaluation with population size $n = 50$ for multidimensional problems.

The efficiency of every algorithm will be changed by tuning its parameters. Hence in order to do an accurate comparison the parameters of every algorithm have been set at the beginning and remain unchanged up to the end of the simulation. We have used Standard PSO with acceleration coefficients $\phi_1 = \phi_2 = 2.0$ but for Modified PSO (MPSO) the acceleration coefficients are set to $\phi_1 = \phi_2 = 1.5$ with inertia weight $\omega = 0.7$. We have also used the standard version of BA with loudness reduction constant $\alpha_b = 0.9$ and pulse rate expansion constant $\gamma_b = 0.9$ and the frequency is drawn uniformly from [0,2]. The parameters of the FGA are set as is shown in table 1.

Every algorithm was run 100 times for each problem. The best, worst, mean and standard deviation values for every problem are presented in table 2. as it can be seen, FGA outperforms other metaheuristics. Also it is seen that FGA is the most robust algorithm. The differences between worst and best, and standard deviation values in all cases illustrate how robust each algorithm is. We can see that MPSO in low dimension problems (up to 10) has comparable performance with FGA but even in this domain FGA shows more robustness. In the high dimensional cases FGA performs much better than other algorithm with a notable difference. As the number of problem dimension increases the performance of MPSO decreases dramatically and PSO obtains better results than MPSO. The unexpected results belong to BA. But same performances of BA have been reported in [20, 21]. Some of the studies indicate that it's advantageous to adopt "explore first, exploit later" approach to obtain necessary data about search space before exploitation is applied [20]. In BA we can see that this sequential approach is reverse and local search is more possible in the beginning of the simulation.

TABLE I. SET PARAMETERS OF FOOTBALL GAME ALGORITHM

Parameters	θ	γ	λ	CMS
Values	0.5	0.95	0.85	$n/2$

TABLE II. COMPARISON OF FGA WITH PSO, MPSO, AND BA

Function Dimension (Optimum)	**Statistical features**	**PSO**	**MPSO**	**BA**	**FGA**
Matyas 2D (0.0)	Best	6.2606853e-9	3.0799861e-15	6.2922518e-6	6.1373365e-11
	Worst	1.8489918e-5	4.7878640e-10	0.0091067	7.1298693e-8
	Mean	3.1685106e-6	**2.0766538e-11**	0.0010757	9.3071018e-9
	S.D.	3.3449076e-6	6.7308041e-11	0.0018361	1.0964718e-8
Beale 2D (0.0)	Best	2.2825886e-7	1.8163544e-13	1.0715264e-5	6.3557879e-9
	Worst	0.7621917	0.7620696	0.8106025	2.4992556e-5
	Mean	0.0381211	0.0076211	0.1021940	**4.7240806e-6**
	S.D.	0.1669302	0.0762069	0.2187558	5.2672754e-6
Camel 2D (0.0)	Best	7.7831913e-8	1.5198990e-15	1.8027111e-5	8.6960750e-9
	Worst	0.2986708	1.2561947e-10	0.0388887	3.6692275e-6
	Mean	0.0119554	**4.9620264e-12**	0.0051893	4.1697552e-7
	S.D.	0.0588170	1.8128849e-11	0.0074836	4.8149536e-7
Bohachevsky 1 2D (0.0)	Best	1.1801626e-4	2.5424107e-14	0.1359886	2.8120331e-7
	Worst	0.2476242	1.4224496e-7	27.2012711	4.8691945e-5
	Mean	0.0558603	**1.0037516e-8**	2.9959996	9.4753295e-6
	S.D.	0.0546137	1.9834850e-8	4.2018223	8.8291972e-6
Rastrigin 2D (-2.0)	Best	-1.9999994	-2	-1.9999692	-2
	Worst	-1.8786992	-1.7578013	-1.6782274	-1.8789006
	Mean	-1.9768461	**-1.9878898**	-1.9403828	-1.9854680
	S.D.	0.0477076	0.0403664	0.0595973	0.0395509
Becker and Lago 2D (0.0)	Best	8.6541668e-8	3.4751168e-16	1.1662412e-4	2.3762728e-16
	Worst	1.5999633e-4	1.0897582e-4	0.0452608	5.0490591e-8
	Mean	3.1554777e-5	1.0938218e-6	0.0063092	**6.7468555e-9**
	S.D.	3.3601780e-5	1.0897209e-5	0.0080865	9.5604706e-9
Ackley 1 5D (0.0)	Best	0.1040593	5.0789674e-9	5.3343711	3.6426862e-11
	Worst	1.0816385	2.3731040e-7	19.9817619	2.1384583e-10
	Mean	0.4797380	4.653993e-8	17.0178165	**1.0377800e-10**
	S.D.	0.1836619	3.609010e-8	2.2661967	3.7253305e-11
Rosenbrock 5D (0.0)	Best	0.6330208	0.0031365	4.8972034	0.0038337
	Worst	3.7493958e+2	1.0281065e+2	1.0143678e+5	4.4579706
	Mean	14.1239729	3.9780908	3.0990740e+3	**2.0916905**
	S.D.	44.1571198	11.6772594	1.2822490e+4	0.5568650
Schaffer 10D (0.0)	Best	1.8777251	0.4842178	2.1459835	0.3092560
	Worst	3.1200182	2.7227991	3.9962962	2.9531839
	Mean	2.5160015	1.9696978	3.2387578	**1.9525737**
	S.D.	0.2778752	0.3987023	0.3250216	0.4976083
Sphere 10D (0.0)	Best	0.0132862	5.1833637e-12	0.0503454	7.6856906e-12
	Worst	0.0462680	3.4875658e-8	1.0355286e+2	3.9825719e-11
	Mean	0.0268344	8.9755309e-10	27.4774265	**2.2599835e-11**
	S.D.	0.0071295	3.8033798e-9	29.2589455	6.9069694e-12
Griewank 30D (0.0)	Best	0.9381539	0.9946528	1.1610885	0.0037011
	Worst	1.0487634	7.6475492	11.0448122	0.6284755
	Mean	1.0090702	3.0219460	6.4779282	**0.1754222**
	S.D.	0.0198154	1.6503028	2.6020584	0.1607767
X.S. Yang 30D (0.0)	Best	0.9755871	31.1228924	1.6430391e+5	0.0026133
	Worst	1.9724320e+4	2.0394125e+9	4.292612e+12	0.1426933
	Mean	9.8521661e+2	23636424	1.580784e+11	**0.0395816**
	S.D.	2.8485032e+3	203782656	5.789848e+11	0.0241636

V. Discussion and Conclusion

In this paper, Football Game Algorithm proposed and formulated for solving continues constrained optimization problems. Football Game Algorithm is a nature inspired, population base algorithm and can be also classified in a memory usage algorithm. FGA has been validated using several benchmark mathematical problems and compared against other metaheuristics like PSO and BA. Through this comparison study, we can see that the FGA is very promising algorithm and has the most robust performance in all cases.

Implementation of FGA is very simple and straightforward and user can simply understand the effect of every parameter on the process and adjust them easily to get his or her efficiency of interest.

Since exploitation component of FGA is supported by tactical strategies of the algorithm, it is possible to consider general movement of the players without ball biased term. Therefore the general movement is reduced to a simple multiple Markov Chain. This form of the algorithm has also been studied and the results showed the same performance to the main version.

Besides, the size of coach memory allows the players move toward more potential solutions against just one best solution that is formulated in PSO and BA in the way of the final convergence. This feature of the algorithm helps to sweep the search space better and more efficient than other metaheuristics. By choosing a higher fraction of population size for the coach memory size the ability of the algorithm for finding the global optimum increases but the accuracy decreases and vice versa. Also we can control the rate of convergence by adjusting the reduction constants of limitation values γ , λ .

On the other hand, Fitness and Hyper Radius Penalties are the special characteristics of Football Game algorithm that make it completely different from other metaheuristic algorithms. The coach uses his memory and these options to guide players toward the final convergence. It means that exploitation is coach's duty in the algorithm while the players just have to explore the search space with decreasing step size for random walk and around the best positions which is determined by the coach. In the FGA a team of players under supervision of a coach try to score a goal (to find the global optimum) through a team work.

The Considerable results of FGA make it an interesting subject for more study. More validation studies will be needed to discover how capable the algorithm is to deal with the optimization problems. Various comparison studies with other metaheuristics using much tough test functions and complicated constraint real world problems will illustrate the power and weakness of the algorithm. Also another important research topic can be sensitivity study of parameters θ, γ, λ and CMS and their effects on keeping an efficient balance between exploration and exploitation phases in the optimization process of the algorithm. Furthermore, extending other version of FGA for discrete or multi-objective optimization problems are other topics for further research.

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