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Short Communication

Accuracy of analytical expressions for solar cell fill factors

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Although the fill factor of a solar cell is a useful parameter in characterizing the cell performance, it cannot be expressed explicitly in terms of other cell parameters. The accuracy of previously published and new analytical approximations for this factor are compared and the most useful expressions are summarized, including the effects of temperature, ideality factor and series and shunt parasitic resistances.

The fill factor FF of a solar cell is defined as the ratio of the maximum power output of the cell to the product of its open-circuit voltage V_{oc} and its short-circuit current I_{sc}. Although this is a useful parameter both in solar cell design and in the interpretation of experimental measurements, it cannot be expressed explicitly in terms of other cell parameters. Accordingly, in several recent papers [1 - 3] approximate expressions for FF have been described in terms of these parameters for the case where a single-exponential model of the current–voltage curve of the cell is used. In most cases no mention has been made of the accuracy of these expressions. The purpose of the present paper is to compare the accuracy of these expressions as well as to describe new expressions which may be of use for specific accuracy requirements.

For cells where the shunt and series resistance are negligible the ideal FF depends only on the open-circuit voltage divided by nkT/q where kT/q is the thermal voltage and n is the cell ideality factor associated with the single-exponential model. If the symbol v_{oc} is used for this normalized voltage, which usually will lie in the range 10 - 50 V, a very simple empirical expression for this factor presented for the first time here is

$$\mathrm{FF} = \frac{v_{oc}}{v_{oc} + 4.7} \qquad (1)$$

This is accurate to better than 2% for $v_{oc} > 10$. It has its peak accuracy in the commonly used range $15 < v_{oc} < 25$ where its accuracy is better than 0.6%. A slightly more complex expression

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$$\mathrm{FF} = \frac{v_{oc} + 1}{1.015 v_{oc} + 5.7} \tag{2}$$

is accurate to better than three significant digits for $v_{oc} > 10$; this expression is also presented for the first time here.

Another range of expressions arises from the precise implicit relationship for FF:

$$\mathrm{FF} = \frac{v_m}{v_m + 1} \frac{v_{oc} - \ln(v_m + 1)}{v_{oc}\{1 - \exp(-v_{oc})\}} \tag{3}$$

where v_m is the normalized value of the voltage at the maximum power point of the cell. It is given implicitly by

$$v_m = v_{oc} - \ln(v_m + 1) \tag{4}$$

As a first approximation, v_{oc} could be used as an approximate value of v_m. If the term $\exp(-v_{oc})$, which is only important for $v_{oc} < 10$, is neglected, this gives [1]

$$\mathrm{FF} = \frac{v_{oc} - \ln(v_{oc} + a)}{v_{oc} + 1} \tag{5}$$

where $a = 1$. This is accurate to three significant digits for $v_{oc} > 15$. It has been pointed out [2] that setting $a = 0.72$ greatly increases the accuracy of this expression. In this case it is accurate to one digit in the fourth decimal place for $v_{oc} > 10$. This level of accuracy is more than adequate for most purposes. The inclusion of the $\exp(-v_{oc})$ term of eqn. (3) preserves an accuracy of better than one digit in the third decimal place for $v_{oc} > 3$.

A better approximation for v_m in eqn. (3) is $v_{oc} - \ln v_{oc}$, as has recently been suggested [3]. This gives very accurate results to better than five significant digits for $v_{oc} > 12$, regardless of whether the $\exp(-v_{oc})$ term of eqn. (3) is included or not.

For the ultimate in accuracy, an even more accurate approximation for v_m in eqn. (3) is $v_{oc} - \ln(v_{oc} + 1 - \ln v_{oc})$. This expression was obtained by inserting the approximation for v_m in the paragraph above into the right-hand side of eqn. (4). The insertion of this approximation into eqn. (3) gives an accuracy of better than five significant digits for $v_{oc} > 3$ and better than six significant digits for $v_{oc} > 6$.

Hence, the accuracy of analytical expressions for the ideal FF is related to computational complexity. Equation (5) with $a = 0.72$ appears to be the best compromise. Equations (1) and (2) are less accurate but can be evaluated with a four-function calculator. For the limited number of cases where an accuracy of better than four significant digits is required, the last approach gives the ultimate in accuracy.

Parasitic series resistance in the cell will reduce the FF below its ideal value. In this case [4], as well as depending on the normalized open-circuit voltage v_{oc}, the FFs depend on a normalized series resistance r_s. This normalized resistance is the actual cell series resistance R_s divided by the cell

TABLE 1

Summary of analytical expressions for solar cell fill factors

Parasitic resistance r_s	r_{sh}	Equation	Reference	Accuracy	Range
0	∞	$\mathrm{FF}_0 = \dfrac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1}$	[2]	One digit in the fourth decimal place	$v_{oc} > 10$
Finite	∞	$\mathrm{FF}_s = \mathrm{FF}_0(1 - 1.1r_s) + \dfrac{r_s^2}{5.4}$	This work	Four digits in the third decimal place	$v_{oc} > 10$ $r_s < 0.4$
0	Finite	$\mathrm{FF}_{sh} = \mathrm{FF}_0\left(1 - \dfrac{v_{oc} + 0.7}{v_{oc}}\dfrac{\mathrm{FF}_0}{r_{sh}}\right)$	[2]	One digit in the third decimal place	$v_{oc} > 10$ $r_{sh} > 2.5$
Finite	Finite	$\mathrm{FF} = \mathrm{FF}_s\left(1 - \dfrac{v_{oc} + 0.7}{v_{oc}}\dfrac{\mathrm{FF}_s}{r_{sh}}\right)$	[2], this work	Better than a few per cent	$v_{oc} > 10$ $r_s + \dfrac{1}{r_{sh}} < 0.4$

$$v_{oc} = \frac{V_{oc}}{nkT/q};\ r_s = \frac{R_s}{V_{oc}/I_{sc}};\ r_{sh} = \frac{R_{sh}}{V_{oc}/I_{sc}}.$$

characteristic resistance [4] R_{ch}, which is equal to V_{oc}/I_{sc}. A simple empirical expression for the effect of this resistance is [2]

$$\mathrm{FF}_s = \mathrm{FF}_0(1 - r_s) \tag{6}$$

where FF_0 is the ideal FF in the absence of series resistance. This expression is accurate to better than 2% for $v_{oc} > 10$ and $r_s < 0.4$. A slightly more complex expression described elsewhere [1] is less accurate over this range. A new expression, which is about three times more accurate than eqn. (6), is

$$\mathrm{FF}_s = \mathrm{FF}_0(1 - 1.1r_s) + \frac{r_s^2}{5.4} \tag{7}$$

This gives an error of up to four digits in the third significant place for $v_{oc} > 10$ and $r_s < 0.4$.

A far more complicated expression for this case has recently been described [3]. This gives a high accuracy over a small range of v_{oc} and r_s; however, the much simpler eqn. (7) is a better compromise over the range previously mentioned.

Parasitic shunt resistance also reduces the cell FF. For the case where series resistance is not important, it has been shown [2] that FF depends only on the normalized open-circuit voltage v_{oc} and the normalized shunt resistance r_{sh}. This normalized resistance is the actual resistance divided by the characteristic resistance (V_{oc}/I_{sc}). An empirical expression in this case accurate to about one digit in the third significant place for $v_{oc} > 10$ and $r_{sh} > 2.5$ is [2]

$$FF_{sh} = FF_0\left(1 - \frac{v_{oc} + 0.7}{v_{oc}}\,\frac{FF_0}{r_{sh}}\right) \tag{8}$$

For the case where both series and shunt resistances are important the preferred approach is to calculate FF_s using eqn. (7) and to substitute it for FF_0 wherever it appears in eqn. (8). This gives an accuracy of a few per cent in the worst case provided that $r_s + 1/r_{sh}$ is less than 0.4. The accuracy can be much better than this if one (or both) of r_s and $1/r_{sh}$ is small.

In conclusion, the most useful expressions for the cases described and their accuracy are summarized in Table 1.

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