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Hip-DE: Historical population based mutation strategy in differential evolution with parameter adaptive mechanism

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ABSTRACT

Differential Evolution (DE) was a powerful population-based evolutionary algorithm for global optimization, and it achieved great success in both evolutionary computation competitions and engineering applications. Despite the excellent performance of the state-of-the-art DE variants, there are still two main weaknesses existing within them: one is the weakness in a given mutation strategy and the other is the weakness in the corresponding parameter control (of the mutation strategy). By reviewing the existing mutation strategies in the recent state-of-the-art DE variants, it can be seen that all of them have insufficient use of the knowledge obtained during the evolution because the historical information of the population is not taken into consideration, which inevitably leads to a bad perception of the landscapes of the objectives. Moreover, the adaptations of the control parameters including F and CR in these state-of-the-art DE variants are interlaced with one another. A bad F and a good CR may produce a good trial vector candidate, then the bad F is of misuse in the parameter control and vice versa. In this paper, a novel DE variant, called Hip-DE, meaning the latest fashion of DE, with historical population based mutation strategy was proposed to tackle the above mentioned weaknesses. Moreover, novel parameter adaptive mechanisms for control parameters F and CR as well as a platform based step-decrease scheme of population size were proposed to enhance capacity of the mutation strategy. By incorporating these three advancements, the novel Hip-DE algorithm secured an overall better performance on the tested benchmarks in comparison with the recent proposed state-of-the-art DE variants.

1. Introduction

Differential Evolution (DE) was a population based stochastic optimization algorithm proposed by Storn and Price in 1995 [34]. Like other Evolutionary Algorithms (EAs) [31,23,40,24,22], DE algorithm employed nature-inspired operations such as mutation, crossover and selection in the simulation of the evolution process of organisms [19], and only the superior individuals between the parents and their offsprings were survived during the evolution [36], which mimicked the Darwin's Principle "survival of the fittest". Due to the simplicity and its excellent performance, DE algorithms have been widely used in fields such as science and engineering in the past decades [6,39].

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The original DE was derived from a so-called Genetic Annealing Algorithm (GAA) [34] which combined the Genetic Algorithm (GA) [14] and Simulated Annealing (SA) [17], therefore, the operations like mutation, crossover, and selection used in GA also existed in the DE algorithm, though the sequences of these operations were different [7]. Accordingly, there were three control parameters, namely the scale factor F, the crossover rate CR and the population size PS, restricting these operations [30]. The scale factor F determined the amplification ratio of the difference vectors: a small F meant that the algorithm preferred exploitation to exploration in each generation, and consequently it might converge into some local optima, while a large F meant a better exploitation capability and consequently it might be of slow speed in convergence [11,43,10]; The crossover rate CR reflects how much information in the mutant vector was retained in the trial vector [44], and a larger CR meant more parameters in the donor vector were retained in the trial vector while a smaller CR meant more parameters in the target vector were retained in the trial vector [28]; The population size PS represented the total number of individuals participating in each generation, and each individual was a potential candidate of the solutions in the optimization [2,33,42,47]. If the number of maximum function evaluation was determined, a larger PS in each generation meant a less number of generations while a smaller PS meant more iterations of the evolution [32]. Generally, the DE researchers should make a tradeoff between the population size PS and the number of generations when tackling optimization applications under the fixed cost or maximum number of function evaluation $nfe_{\max}$ criterion [8]. The list of notations used in this study is shown in Table 1.

After the control parameters F, CR and PS are determined, the donor (also the mutant) vector can be generated according to a certain mutation strategy, e.g. DE/rand/1/bin in the canonical DE algorithm [36]. As it is known to all that DE belongs to Evolutionary Computation (EC), which presents evolutionary methodologies and approaches to address and tackle complex optimization problems. As a general purpose optimization tool, it searches the solution space by individuals' evolution, to be more exactly, the search of the solution space is determined by mutation strategy which dominates the search direction and step size during each generation. However, the state-of-the-art DE variants still existed many weaknesses which have already mentioned in the abstract. Here in this paper we mainly focus on improving the overall performance of DE algorithm

Table 1
The list of notations.

Notations	Descriptions
F	Scale factor
CR	Crossover rate
PS	Population size
$PS_{\min}$	The minimum (also the terminal) population size
PS_{ini}	The initial population size
nfe	The current number of function evaluations
$nfe_{\max}$	The maximum number of function evaluations
μ_F	Location parameter of F in the corresponding distribution
μ_{CR}	Mean value of CR in the corresponding distribution
$C(\mu_F, 0.1)$	Cauchy distribution of scale factor F
$N(\mu_{CR}, 0.1)$	Normal distribution of crossover rate CR
r^{arc}	The ratio of the external archive to population
$\mathbf{P}$	The matrix of the population
$\mathbf{A}$	The set of inferior individuals in the external archive
$\mathbf{H}$	The set of historical individuals in the external archive
$X_{i,G}$	The i^{th} individual in the G^{th} generation
$x_{ij,G}$	The j^{th} parameter in $X_{i,G}$
$x_{\max_j}$	The upper bound of $x_{ij,G}$
$x_{\min_j}$	The lower bound of $x_{ij,G}$
$U_{i,G}$	The trial vector of $X_{i,G}$
$X_{r_1,G}$	A randomly selected individual from the population in the G^{th} generation
$X_{best,G}$	The best individual of the population in the G^{th} generation
$X^p_{best,G}$	A randomly selected individual of from the top $p\%$ of the population in the G^{th} generation
$\widetilde{X}_{r_2,G}$	A randomly selected individual from $\mathbf{P} \cup \mathbf{A}$
$\widehat{X}_{r_2,G}$	A randomly selected individual from $\mathbf{P} \cup \mathbf{H}$
$\mathbf{S}$	The set of successful individuals of the current generation
$\mathbf{S_F}$	The set of successful scale factors of the current generation
$\mathbf{S_{CR}}$	The set of successful crossover rates of the current generation
$mean_L(\mathbf{S_F})$	The Lehmer mean of the set $\mathbf{S_F}$
$mean_A(\mathbf{S_{CR}})$	The arithmetic mean of the set $\mathbf{S_{CR}}$
$mean_W L(\mathbf{S_F})$	The weighted Lehmer mean of the set $\mathbf{S_F}$
$mean_W L(\mathbf{S_{CR}})$	The weighted Lehmer mean of the set $\mathbf{S_{CR}}$
$\overline{T}_0$	The mean run time of basic arithmetic expressions in CEC2013 test suite
$\overline{T}_1$	The mean time consumption of 200000 function evaluations on f_{14} for 30D optimization
$\overline{T}_2$	The mean time of the overall cost of a certain algorithm optimizing f_{14}
$mean_W L(\mathbf{S_{CR}})$	The weighted Lehmer mean of the set $\mathbf{S_{CR}}$

from both of the two aspects including parameter control and mutation strategy. A novel historical population based mutation strategy was proposed to make a better perception of the landscape of the objectives by exploiting as much knowledge as possible from the evolution especially those reflecting the relationship between the current population and the historical population. Furthermore, some novel parameter adaptive mechanisms were also proposed in the paper in order to tackle the weaknesses existing in the famous DE variants like LSHADE and jSO. Both the novel mutation strategy and the adaptation schemes of control parameters can be considered as further extension of our early research [26,27,25]. Moreover, the novel Hip-DE algorithm can also be considered as further improvement of JADE, LSHADE, and jSO. The main contributions of the paper can be summarized as follows:

1. A novel mutation strategy with historical population was proposed to make a better balance between exploitation and exploration of DE by incorporating the knowledge reflecting the landscape of objective during the evolution.
2. A novel parameter adaptive mechanism for control parameters F and CR based on grouping strategy was proposed, and the successful parameter values were retained to further guide the generation of control parameters with a certain probability in the next evolution.
3. A platform based step-decrease scheme of population size was proposed, and this strategy allowed more individuals to explore the search space in the early stage of evolution, which could get a better perception of the whole landscape of the objective and consequently achieve an overall better performance.

The rest of the paper is organized as follows: Section 2 presents the literature review of DE algorithm, and in this part, the advantages and disadvantages of the famous DE variants are reviewed and discussed. Section 3 introduces the basic differential evolution algorithm. Section 4 provides detailed implementations of several well-known DE variants including JADE, LSHADE, jSO, PalmDE, LPalmDE and HARD-DE which performed very well on the commonly used single-objective benchmarks. In Section 5, the novel Hip-DE algorithm is depicted in detail from two aspects: the mutation strategy and the parameter control. Section 6 presents the experiment analysis of the novel Hip-DE algorithm under the CEC2013 test suite for real-parameter single objective optimization and finally, the conclusion is given in Section 7, which includes some limitations of our work and some future directions.

2. Related work

As it was known to all that the overall performance of the DE algorithm was closely related to the choice of proper trial vector generation strategy and the corresponding control parameters [3,46,15,45,35,41,13,12,5], therefore, many researchers proposed different trial vector generating strategies with different associated parameter control schemes [9,16,1] for different optimization applications. Generally, these algorithms can be divided into three categories: DE variants with single mutation strategy [3,46,15,45], DE variants with multi-mutation strategy [35,41,13], and DE variants with new frameworks/techinques that improved the performance of former proposed DE variants [12,5].

For the first category, Brest et al. [3] introduced a DE variant, called jDE, which had a remarkable new feature that the control parameters F and CR were adaptively changed during the evolution though it employed the same mutation strategy as the canonical DE algorithm. A novel idea was advanced in the adaptive parameter control scheme of jDE algorithm that better control parameters were often associated with better trial vectors, in turn, these better trial vectors were considered to be generated by better control parameters, and these better parameters should be retained in the propagation of control parameters in the next generation. This feedback mechanism was the corner stone of some later DE variants that won front ranks in optimization competitions, such as JADE [46], LSHADE [38], and jSO [4] etc. Besides these algorithms, Islam et al. [15], Yu et al. [45] and Meng et al. [29] also proposed new mutation strategies "DE/target-to-gr_best/1","DE/lbest/1", "DE/pbest/2" and "DE/target-to-pbest/1" with hierarchical archive respectively, and all of them obtained competitive performance, however, these mutation strategies were in a slow convergence speed in some benchmarks and the adaptation of population size could be further polished. For the second category, Qin et al. [35] proposed a SaDE algorithm with several mutation strategies recorded in a candidate pool. The key thought of SaDE was that these mutation strategies were dynamically selected according to the performance of each strategy in each generation. Wang et al. [41] proposed a CoDE algorithm with fixed mutation strategies and fixed control parameters. In each generation of the CoDE algorithm, each mutation strategy was randomly composed with a pair of control parameters. Gong et al. [13] proposed a two-component mechanism to dynamically select the mutation strategies (with archive and without archive) proposed in JADE, and obtained an improved performance. However, all these multi-mutation strategy based DE variants had a serious weakness that the improper choice of a mutation strategy might cause a disaster in the final performance. For the last category, Gong and Cai [12] proposed a ranking-based framework, and some of the vectors in the mutation strategy were proportionally selected from the ranked individuals of the population. Cai et al. [5] proposed a social learning based technique to enhance the former DE variants. The social network of individuals in this algorithm was built according the social influence of the individuals. However, these frameworks mainly focused on DE variants with fixed population size, and recent research in the literature revealed that DE variants with population size reduction scheme usually outperformed the DE variants with fixed population size. Nevertheless, it doesn't mean that any DE variants with fixed population size can be improved by incorporating a linear population size reduction scheme.

DE variant with single mutation strategy was actually the basement of the variants in the other two categories, and here in the paper we focused on the DE variants with single mutation strategy. As we know that the mutation strategy "DE/target-to-pbest/1/" proposed in JADE was proven to be an excellent mutation strategy, and this strategy implemented a great balance between exploration and exploration for a larger number of objectives by incorporating the elite-thought and the external archive storing inferior solutions. Both the elite-thought and the inferior solutions in the external archive can be considered as "knowledge" extracted from the evolution. Generally, the more knowledge was exploited during the evolution, the better overall optimization performance can be obtained by the algorithm. By reviewing the literature, it can be seen that all the recent proposed state-of-the-art DE variants have insufficient use of the knowledge obtained during the evolution because the historical information of the population is not taken into consideration, which inevitably leads to a bad perception of the landscapes of the objectives in different stages of the evolution. The mutation strategy in this paper presents an approach to tackling this weakness, furthermore, some novel parameter adaptive mechanisms are also proposed in the paper in order to tackle the weaknesses mentioned above in the former state-of-the-art DE variants.

3. The canonical differential evolution

DE is a famous and powerful branch of EAs aiming at tackling different kinds of optimization problems. Generally, the procedure of solving a minimum optimization problem by DE can be illustrated by the following steps: initialization and the circle of evolution including mutation, crossover and selection before termination.

3.1. Initialization

PS is used for denoting the size of the population of DE here, and G is used for denoting the generation number, then the whole population $\mathbf{P}$ can be defined as follows: $\mathbf{P} = \{X_{1,G}, X_{2,G}, \cdots, X_{ps,G}\}$, where $X_{i,G}$ denotes the i^{th} individual of the population $\mathbf{P}$ in the G^{th} generation, and $X_{i,G}$ is a D-dimensional vector with its j^{th} parameter denoted by $x_{ij,G}$, then $X_{i,G}$ can be presented like this: $X_{i,G} = (x_{i1,G}, x_{i2,G}, \cdots, x_{ij,G}, \cdots, x_{iD,G})$. Generally, $x_{ij,0}$ is initialized by uniform distribution according to Eq. (1):

$$x_{ij,0} = x_{\min_j} + rand(0,1) \cdot \left(x_{\max_j} - x_{\min_j}\right) \tag{1}$$

where $x_{\max_j}$ and $x_{\min_j}$ denotes the upper bound and the lower bound of the j^{th} parameter respectively.

3.2. Circle of evolution

The first operation in the circle of evolution is mutation which is used for generating mutant vectors, the mutation strategy in the canonical DE is "DE/rand/1" shown in Eq. (2):

$$V_{i,G} = X_{r_0,G} + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right) \tag{2}$$

where $X_{r_0,G}, X_{r_1,G}$ and $X_{r_2,G}$ are three randomly selected individuals from the population with the restriction: $r_0 \neq r_1 \neq r_2 \neq i$, and F is the scale factor. Besides the mutation strategy "DE/rand/1", there are still other well-known mutation strategies, such as:

- "DE/best/1": $V_{i,G} = X_{best,G} + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right)$
- "DE/rand/2": $V_{i,G} = X_{r_0,G} + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right) + F \cdot \left(X_{r_3,G} - X_{r_4,G}\right)$
- "DE/best/2": $V_{i,G} = X_{best,G} + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right) + F \cdot \left(X_{r_3,G} - X_{r_4,G}\right)$
- "DE/target-to-best/1": $V_{i,G} = X_{i,G} + F \cdot \left(X_{best,G} - X_{i,G}\right) + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right)$
- "DE/target-to-pbest/1": $V_{i,G} = X_{i,G} + F \cdot \left(X^{p}_{best,G} - X_{i,G}\right) + F \cdot \left(X_{r_1,G} - X_{r_2,G}\right)$

Generally, the mutation strategy "DE/target-to-pbest/1" with external archive secured better optimization performance in recent competitions, and the mutation strategy in this paper can also be considered as an extension of this strategy.

Crossover is the second operation in the circle of evolution, and there are mainly two crossover schemes in DE, one is exponential crossover, a combined 1-point crossover and 2-point crossover, and the other is binomial crossover. These two crossover schemes are actually to determine how many parameters in the trial vector are inherited from the mutant vector and the target vector. Furthermore, there are some other schemes, e.g. disable crossover, line recombination, and intermediate recombination, etc., used as alternatives of these two crossover schemes. What should be addressed more is that the state-of-the-art DE variants mainly employ the binomial crossover in their competitions.

The last operation of the procedure is selection, and this operation mimics the biological evolution "survival of the fittest", and the detailed implementation of selection operation in minimum optimization by DE is presented in Eq. (3):

$$X_{i,G+1} = \begin{cases} U_{i,G}, & \text{if } f(U_{i,G}) < f(X_{i,G}) \\ X_{i,G}, & \text{otherwise} \end{cases} \tag{3}$$

4. Several powerful DE variants

In this section, we mainly have a brief review of the JADE algorithm [46], the LSHADE algorithm [38], the jSO algorithm [4], the PalmDE algorithm [26], the LPalmDE algorithm and the HARD-DE algorithm [25] for real-parameter single objective optimization as our novel Hip-DE algorithm can be considered as further improvement of these algorithms.

4.1. The JADE algorithm

The JADE algorithm was proposed by Zhang and Sanderson in 2009, and a new mutation strategy "DE/target-to-*p*best/1/bin" was proposed in this algorithm. This mutation strategy had two modes in the original JADE algorithm: the mutation strategy with and without external archive. Because the mutation strategy with external archive achieved great success in the offspring state-of-the-art DE variants, we mainly focused on the mutation strategy with external in this part. There were mainly two novel ideas incorporated into this mutation strategy, one was the "elite thought" for the guidance of evolution, and the other was the "external archive" recording inferior solution aiming at increasing the diversity of the trial vectors. By incorporating these two novel ideas, JADE made a better balance between exploration and exploitation, and consequently obtained an overall better performance in the optimization competition. The detailed mutation strategy with external archive of JADE is given in the following Eq. (4):

$$V_{i,G} = X_{i,G} + F \cdot \left(X^{p}_{best,G} - X_{i,G}\right) + F \cdot \left(X_{r_1,G} - \widetilde{X}_{r_2,G}\right) \tag{4}$$

where $\widetilde{X}_{r_2,G}$ denotes a randomly selected individual from the union $\mathbf{P} \cup \mathbf{A}$ with index restriction ($r_1 \neq r_2 \neq i$), and $\mathbf{A}$ denotes the external archive recording the inferior individuals.

For the parameter control of the JADE algorithm, scale factor F of each individual obeyed Cauchy distribution, $F \sim C(\mu_F, 0.1)$, while crossover rate CR obeyed Guassian distribution, $CR \sim N(\mu_{CR}, 0.1)$. The recommended initial settings of μ_F and μ_{CR} were $\mu_F = \mu_{CR} = 0.5$, and they were dynamically changed during the evolution. JADE extended the key thought of jDE to be a new rule that better control parameters should be retained to guide the distribution of themselves. In the adaptation of μ_F and μ_{CR}, if the trial vector of an individual obtained a better fitness value, then the individual was labeled as "s" individual, otherwise, it was labeled as "f" individual. All the "s" individuals were recorded in the set **S**, meanwhile the corresponding F and CR values of the "s" individuals were recorded in sets $\boldsymbol{S_F}$ and $\boldsymbol{S_{CR}}$ respectively. The detailed adaptation schemes of μ_F and μ_{CR} are listed below in Eq. (5) and Eq. (6) respectively:

$$\begin{cases} mean_L(\boldsymbol{S_F}) = \dfrac{\sum_{F \in \boldsymbol{S_F}} F^2}{\sum_{F \in \boldsymbol{S_F}} F} \\ \mu_F = (1-c) \cdot \mu_F + c \cdot mean_L(\boldsymbol{S_F}) \end{cases} \tag{5}$$

$$\begin{cases} mean_A(\boldsymbol{S_{CR}}) = \dfrac{\sum_{CR \in \boldsymbol{S_{CR}}} CR}{|\boldsymbol{S_{CR}}|} \\ \mu_{CR} = (1-c) \cdot \mu_{CR} + c \cdot mean_A(\boldsymbol{S_{CR}}) \end{cases} \tag{6}$$

where $mean_L(\boldsymbol{S_F})$ denotes the Lehmer mean of $\boldsymbol{S_F}$, $mean_A(S_{CR})$ denotes the arithmetic mean of $\boldsymbol{S_{CR}}$, $|\boldsymbol{S_{CR}}|$ denotes the size of the set $\boldsymbol{S_{CR}}$ and c is a balance parameter with the default setting $c = 0.1$. Moreover, the population size PS is set constant value during the whole evolution and it satisfies:

$$PS = \begin{cases} 30, & \text{if } D \leqslant 10 \\ 100, & \text{if } D = 30 \\ 400. & \text{if } D = 100 \end{cases} \tag{7}$$

4.2. The LSHADE algorithm

The LSHADE algorithm was actually an improved algorithm of JADE though it was directly developed from SHADE [37], and an H-entry pool was employed in the parameter control, which enhanced the robustness of the adaptation of control parameters during the evolution. The control parameters F and CR in LSHADE obeyed the same distribution as JADE, and each individual employed a randomly chosen entry that recorded μ_F–μ_{CR} pairs from the pool in generating the corresponding F and CR. The key thought of JADE was also employed in LSHADE and these successful F and CR values were also employed in renewing their corresponding μ_F and μ_{CR}. The detailed adaptation schemes are presented in Eq. (8) and Eq. (9) respectively:

$$\begin{cases} \Delta f_j = f(X_{j,G}) - f(U_{j,G}) \\ w_s = \frac{\Delta f_j}{\sum_{s=1}^{|\boldsymbol{S_F}|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_F}) = \frac{\sum_{s=1}^{|\boldsymbol{S_F}|} w_s \cdot \boldsymbol{S}_{\boldsymbol{F}}^2(s)}{\sum_{s=1}^{|\boldsymbol{S_F}|} w_s \cdot \boldsymbol{S_F}(s)} \\ \mu_{F,h} = \begin{cases} mean_{WL}(\boldsymbol{S_F}), & if\ \boldsymbol{S_F} \neq \emptyset \\ \mu_{F,h}. & otherwise \end{cases} \end{cases} \tag{8}$$

$$\begin{cases} \Delta f_j = f(X_{j,G}) - f(U_{j,G}) \\ w_s = \frac{\Delta f_j}{\sum_{s=1}^{|\boldsymbol{S}|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_{CR}}) = \frac{\sum_{s=1}^{|\boldsymbol{S_{CR}}|} w_s \cdot \boldsymbol{S}_{\boldsymbol{CR}}^2(s)}{\sum_{s=1}^{|\boldsymbol{S_{CR}}|} w_s \cdot \boldsymbol{S_{CR}}(s)} \\ \mu_{CR,h} = \begin{cases} mean_{WL}(\boldsymbol{S_{CR}}),\ if\ \boldsymbol{S_{CR}} \neq \emptyset \\ \mu_{CR,h}.\ otherwise \end{cases} \end{cases} \tag{9}$$

where the same symbols have the same meanings as the ones in Eq. (5) and Eq. (6). Moreover, Δf_j denotes the fitness difference of the j^{th} individual in the population and it is also the s^{th} individual in the set $\mathbf{S}$, $mean_{WL}$ denotes the weighted Lehmer mean, $\mu_{F,h}$ and $\mu_{CR,h}$ denotes the μ_F–μ_{CR} pair recorded in the h^{th} entry, $h \in [1, H]$, and only one entry can be updated in each generation. By the way, readjustments are also necessary steps for both scale factor F and crossover rate CR before calculating the trial vectors, and the detailed readjustments are shown in Eq. (10) and Eq. (11) respectively:

$$F = \begin{cases} randc\left(\mu_{F,h}, 0.1\right), while\ F \leqslant 0 \\ 1.\ if\ F > 1 \end{cases} \tag{10}$$

$$CR = \begin{cases} 0,\ if\ \mu_{CR} = 0 \\ randn_i\left(\mu_{CR,h}, 0.1\right).\ otherwise \end{cases} \tag{11}$$

then CR should be truncated into $[0, 1]$ before calculating the trial vectors in the LSHADE algorithm.

There was also a sparking point that the LSHADE algorithm first proposed a novel linear population size reduction scheme which was proven to be an effective way to improve the overall performance of DE algorithm. The detailed scheme is presented in the following Eq. (12):

$$PS_{G+1} = round\left[\frac{PS_{\min} - PS_{ini}}{nfe_{\max}} \cdot nfe\right] + PS_{ini} \tag{12}$$

where PS_{ini} denotes the initial population size; $PS_{\min}$ denotes the minimum population size; $nfe_{\max}$ denotes the maximum number of function evaluations and nfe denotes the current number of function evaluations; $round[\cdot]$ means rounding to the nearest natural number. It is worth noting that the size of external archive $\mathbf{A}$ in LSHADE is dynamically changed according to $|\mathbf{A}| = r^{arc} \cdot PS$ due to the reduction scheme.

4.3. The jSO algorithm

The jSO algorithm was actually a further development of the LSHADE algorithm, and an inertia weight was incorporated into the mutation strategy. Then the mutation strategy can be denoted by "DE/target-to-*p*best-*w*/1/bin" with the detailed equation shown as follows in Eq. (13):

$$V_{i,G} = X_{i,G} + iw \cdot F \cdot \left(X_{best,G}^p - X_{i,G}\right) + F \cdot \left(X_{r_1,G} - \widetilde{X}_{r_2,G}\right) \tag{13}$$

where the same symbol has the same meanings as Eq. (4), and the inertia weight iw follows:

$$iw = \begin{cases} 0.7,\ if\ nfe < 0.2 \cdot nfe_{\max} \\ 0.8,\ if\ nfe < 0.4 \cdot nfe_{\max} \\ 1.2.\ otherwise \end{cases} \tag{14}$$

Moreover, some improvements are also made when updating the control parameters in comparison with the LSHADE algorithm, and these modifications are presented in Eq. (15) and Eq. (16) respectively.

$$\begin{cases} \Delta f_j = f(X_{j,G}) - f(U_{j,G}) \\ w_s = \frac{\Delta f_j}{\sum_{s=1}^{|S|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_F}) = \frac{\sum_{s=1}^{|S_F|} w_s \cdot \boldsymbol{S_F^2}(s)}{\sum_{s=1}^{|S_F|} w_s \cdot \boldsymbol{S_F}(s)} \\ \mu_F = \begin{cases} (mean_{WL}(\boldsymbol{S_F}) + \mu_F)/2, \text{ if } \boldsymbol{S_F} \neq \emptyset \\ \mu_F. \text{ otherwise} \end{cases} \end{cases} \tag{15}$$

$$\begin{cases} \Delta f_j = f(X_{j,G}) - f(U_{j,G}) \\ w_s = \frac{\Delta f_j}{\sum_{s=1}^{|S|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_{CR}}) = \frac{\sum_{s=1}^{|S_{CR}|} w_s \cdot \boldsymbol{S_{CR}^2}(s)}{\sum_{s=1}^{|S_{CR}|} w_s \cdot \boldsymbol{S_{CR}}(s)} \\ \mu_{CR} = \begin{cases} (mean_{WL}(\boldsymbol{S_{CR}}) + \mu_{CR})/2, \text{ if } \boldsymbol{S_{CR}} \neq \emptyset \\ \mu_{CR}. \text{ otherwise} \end{cases} \end{cases} \tag{16}$$

where the same symbols have the same meanings as the LSHADE algorithm. The scale factor *F* and crossover rate *CR* are also generated according to Eq. (10) and Eq. (11) respectively. Furthermore, readjustments of control parameters *F* and *CR* are also incorporated into the jSO algorithm before calculating the trial vectors in order to better fit the CEC2017 benchmarks, and the detailed readjustments are given in Eq. (17) and Eq. (18) respectively:

$$F_{i,G} = \begin{cases} min(F_{i,G}, 0.7), \text{ if } nfe < 0.6 \cdot nfe_{\max} \\ 0.7. \text{ otherwise} \end{cases} \tag{17}$$

$$CR_{i,G} = \begin{cases} max(CR_{i,G}, 0.7), \text{ if } nfe < 0.25 \cdot nfe_{\max} \\ max(CR_{i,G}, 0.6), \text{ if } nfe < 0.5 \cdot nfe_{\max} \\ CR_{i,G}. \text{ otherwise} \end{cases} \tag{18}$$

For the population size *PS*, jSO algorithm employed the same reduction scheme as LSHADE which was ready presented in Eq. (12).

4.4. The PalmDE algorithm

Parameters with adaptive learning mechanism DE (PalmDE) was proposed in 2018 [26], and this algorithm aimed at tackling the misleading interaction weakness between control parameters *F* and *CR* in the above mentioned state-of-the-art DE variant. As we know that jDE, JADE, LSHADE and jSO all employed the same key thought that better control parameters produced better trial vectors and in turn, better trial vectors were produced by better control parameters, then these parameters should guide the adaptation of themselves during the evolution. However, these control parameters were not treated separately in these DE variants, and there were the cases that a better *F* and a worse *CR* (or a worse *F* and a better *CR*) together produced a better trial vector, then the worse *CR* (or *F*) was considered to be the right one which was consequently employed in the renewing process of itself. This was called misleading interaction in the PalmDE algorithm. In order to eliminate this weakness, a novel parameter with adaptive learning mechanism was proposed, and control parameters *F* and *CR* were renewed separately.

The whole population of individuals were divided into *K* groups by stochastic universal selection [27], and each group had a unique μ_F–CR pair, which meant that the individuals in the same group share the same μ_F and *CR* values. In the initialization stage, the location parameter of Cauchy distribution that the scale factor of each individual obeyed in each group was the same, $\mu_{F_1} = \mu_{F_2} = \ldots = \mu_{F_k} = \ldots = \mu_{F_K} = \mu_F$, and the crossover rate CR_k in each group obeyed Gaussian distribution, $CR_k \sim N(\mu_{CR}, 0.1)$. Another parameter $p(k)$ was introduced into the PalmDE algorithm in order to denote the probability that a certain individual was classified into the k^{th} group, and it was initialized according to $\{p(k)|p(1) = p(2) = \ldots = p(k) = \ldots = p(K) = 1/K, k \in \{1, 2, \ldots, K\}\}$.

After initialization, the location parameters μ_{F_k} in the k^{th} group and the crossover rate *CR* were renewed independently, that was also the reason why PalmDE tackled the mis-interaction weakness between control parameters *F* and *CR*. The detailed adaptation schemes of μ_F and μ_{CR} are given in Eq. (19) and Eq. (20) respectively:

$$\begin{cases} \Delta f_j = f(U_{j,G}) - f(X_{j,G}) \\ w_s = \frac{\Delta f_j}{\sum_{F \in \boldsymbol{S}_{\boldsymbol{F}_k}} \Delta f_j} \\ mean_{WL}(\boldsymbol{S}_{\boldsymbol{F}_k}) = \frac{\sum_{F \in \boldsymbol{S}_{\boldsymbol{F}_k}} w_s \cdot F^2}{\sum_{F \in \boldsymbol{S}_{\boldsymbol{F}_k}} w_s \cdot F} \\ \mu_{F_k} = \begin{cases} mean_{WL}(\boldsymbol{S}_{\boldsymbol{F}_k}), \ if\ \boldsymbol{S}_{\boldsymbol{F}_k} \neq \emptyset \\ \mu_{F_k}. \ otherwise \end{cases} \end{cases} \tag{19}$$

$$\begin{cases} ns = \sum_{k=1}^{K} ns_k, \\ r_k = \begin{cases} \frac{ns_k^2}{ns \cdot (ns_k + nf_k)}, \ if\ ns_k > 0 \\ \epsilon, \ otherwise \end{cases} \\ p(k) = \frac{r_k}{\sum_{k=1}^{K} (r_k)} \\ \mu_{CR} = \frac{\sum_{k=1}^{K} p(k) \cdot CR_k^2}{\sum_{k=1}^{K} p(k) \cdot CR_k} \end{cases} \tag{20}$$

where $\boldsymbol{S}_{\boldsymbol{F}_k}$ denotes the set of scale factors of the “s” individuals in the k^{th} group, ns_k denotes the number of “s” individuals in the k^{th} group, ns denotes the number of “s” individuals in the population, nf_k denotes the number of “f” individuals in the k^{th} group, ϵ is assigned a very small constant value, e.g. $\epsilon = 0.001$, which is used in avoiding possible null value of possibility. Then the scale factor F and the crossover rate in the k^{th} group can be generated according to Eq. (21) and Eq. (22) respectively:

$$F = randc\left(\mu_{F_k}, 0.2\right) \tag{21}$$

$$CR_k = randn(\mu_{CR}, 0.1) \tag{22}$$

For the mutation strategy, PalmDE employed a similar mutation strategy to the one in JADE, and the only difference between these two strategies was that a time-stamp based mechanism was employed in the strategy of PalmDE. The incorporation of this mechanism was to avoid too old inferior solutions to have been archive-residents for a long period of the evolution. When the threshold T_0 of the time stamp equaled to zero, the mutation strategy in PalmDE was degraded into the mutation strategy without external archive in JADE, and when the threshold was larger than $gen_{\max}/r_d$ ($gen_{\max}$ denoted the maximum number of generation, r_d denoted the decay rate in each generation), the mutation strategy in PalmDE was degraded into the mutation strategy with external archive in JADE. Therefore, it can be considered as a balance between mutation strategy with archive and without archive, and by setting a proper threshold of the time stamp, the PalmDE algorithm maintained a better diversity of trial vectors and consequently obtained an overall better performance.

By the way, the population size was kept fixed during the whole evolution in the PalmDE algorithm, and in the same paper, a further extension of the PalmDE algorithm, called LPalmDE algorithm was also presented by incorporating a population size reduction scheme as well as a slight change of the adaptation of scale factors.

4.5. The LPalmDE algorithm

The LPalmDE algorithm was an upgraded version of PalmDE, and a linear population size reduction as well as a slight change in the adaptation of scale factor was incorporated into LPalmDE. The population size reduction scheme in LPalmDE was the same as the one in LSHADE (see Eq. (12)), therefore, the number of individuals in the K groups would inevitably decrease to a very small number which was unable to update μ_{F_k} properly according to Eq. (19). That was the reason why the LPalmDE algorithm proposed a new adaptation scheme for μ_F by combining all the K groups into a union, and the details of the adaptation was shown in Eq. (23) and Eq. (24):

$$\begin{cases} w_s = \frac{\Delta f_j}{\sum_{s=1}^{|\boldsymbol{S}|} \Delta f_j} \\ \boldsymbol{S}_{\boldsymbol{F}} = \bigcup_{k=1}^{K} \boldsymbol{S}_{\boldsymbol{F}_k} \\ mean_{WL}(\boldsymbol{S}_{\boldsymbol{F}}) = \frac{\sum_{s=1}^{|\boldsymbol{S}_{\boldsymbol{F}}|} w_s \cdot \boldsymbol{S}_{\boldsymbol{F}}^2(s)}{\sum_{s=1}^{|\boldsymbol{S}_{\boldsymbol{F}}|} w_s \cdot \boldsymbol{S}_{\boldsymbol{F}}(s)} \end{cases} \tag{23}$$

$$\begin{cases} \mu_F = \begin{cases} mean_{WL}(\boldsymbol{S_F}), \ if \ \boldsymbol{S_F} \neq \emptyset \\ \mu_F. \ otherwise \end{cases} \\ \mu_{F_k} = \mu_F, \ k \in [1, K]. \end{cases} \tag{24}$$

where the same symbols have the same meanings as the ones in Eq. (15). Moreover, the adaptation of crossover rate μ_{CR} in LPalmDE keeps the same as PalmDE (see Eq. (20)).

4.6. HARD-DE

The HARD-DE algorithm was actually a further extension of both the LPalmDE algorithm and the LSHADE algorithm [25]. There were three enhancements involved into the HARD-DE algorithm, the first improvement was that a novel depth information based mutation strategy was proposed in it. Unlike the above mentioned mutation strategies, the relationship between individuals in the current generation and the individuals in the former generations was firstly incorporated into the mutation strategy because historical information of the population, from a certain perspective of view, could reflect the landscape of the objective, which might be useful in the generation of trial vectors. The detailed equation of the mutation strategy is presented in Eq. (25):

$$V_{i,G} = X_{i,G} + F \cdot \left(X^p_{best,G} - X_{i,G}\right) + F_1 \cdot \left(X_{r_1,G} - \widetilde{X}_{r_2,G}\right) + F_2 \cdot \left(X_{r_1,G} - \widehat{X}_{r_3,G}\right) \tag{25}$$

where the same symbols have the same meaning as Eq. (4) except for F_1, F_2 and $\widehat{X}_{r_3,G}$. The relationships between F_1, F_2 and F are given in Eq. (26):

$$\begin{cases} F_1 = 0.9 \cdot F \\ F_2 = 0.7 \cdot F \end{cases} \tag{26}$$

$\widehat{X}_{r_3,G}$ denotes a randomly selected individual from the union $\mathbf{P} \cup \mathbf{B}$ where $\mathbf{P}$ denotes the current population and $\mathbf{B}$ denotes the set recording the former populations of the evolution.

The second improvement was that a novel adaptation scheme for CR was proposed in the HARD-DE algorithm though the adaptation scheme for F was the same as LSHADE. In HARD-DE, the whole population was also classified into K groups by stochastic universal selection according to the initial selection probability of each group, $\{p(k)|p(1) = p(2) = \ldots = p(k) = \ldots = p(K) = \frac{1}{K}, k \in \{1, 2, \ldots, K\}\}$, which was the same as LPalmDE algorithm. The scale factor μ_F and the crossover rate μ_{CR} were initialized the same for the K groups, $\mu_{F_1} = \mu_{F_2} = \ldots = \mu_{F_k} = \ldots = \mu_{F_K} = \mu_F$ and $\mu_{CR_1} = \mu_{CR_2} = \ldots = \mu_{CR_k} = \ldots = \mu_{CR_K} = \mu_{CR}$. After initialization, the scale factor and the crossover rate of each individual in the k^{th} group were generated according to Eq. (27) and Eq. (28) respectively:

$$\begin{cases} F = randc\left(\mu_{F_k}, 0.1\right), \\ F = \begin{cases} randc\left(\mu_{F,h}, 0.1\right), while \ F \leqslant 0 \\ 1. \ if \ F > 1 \end{cases} \end{cases} \tag{27}$$

$$\begin{cases} CR = \begin{cases} 0, \ if \ \mu_{CR_k} = 0 \\ randn\left(\mu_{CR_k}, 0.1\right), \ otherwise \end{cases} \\ CR = \max(0, CR), \\ CR = \min(1, CR). \end{cases} \tag{28}$$

After the generation of trial vector and the consequent selection operation, the selection probability of the k^{th} group was updated at the end of each generation according to Eq. (29):

$$\begin{cases} ns = \sum_{k=1}^{K} ns_k, \\ r_k = \begin{cases} \frac{ns_k^2}{ns \cdot (ns_k + nf_k)}, \ if \ ns_k > 0, \\ \epsilon, \ otherwise. \end{cases} \\ p(k) = \frac{r_k}{\sum_{k=1}^{K}(r_k)}. \end{cases} \tag{29}$$

where the same symbols had the same meanings as the ones in Eq. (20). Moreover, the adaptation scheme of CR in each generation of the HARD-DE algorithm was presented in Eq. (30):

$$\begin{cases} w_s = \frac{\Delta f_j}{\sum_{s=1}^{|\mathbf{S}|} \Delta f_j} \\ \Delta f_j = f(X_{j,G}) - f(U_{j,G}) \\ mean_{WL}(\boldsymbol{S_{CR}}) = \frac{\sum_{s=1}^{|\boldsymbol{S_{CR}}|} w_s \cdot \boldsymbol{S}^2_{\boldsymbol{CR}}(s)}{\sum_{s=1}^{|\boldsymbol{S_{CR}}|} w_s \cdot \boldsymbol{S_{CR}}(s)} \\ idx \equiv \arg\min_{k \in \Omega} p(k) = \{k^* \in \Omega | p(k^*) \leqslant p(k), \forall k \in \Omega\}, \\ \qquad \Omega = \{1, 2, \ldots, K\} \\ \mu_{CR_{idx}} = \begin{cases} mean_{WL}(\boldsymbol{S_{CR}}), \ if\ \boldsymbol{S_{CR}} \neq \varnothing \& \max(\boldsymbol{S_{CR}}) > 0 \\ 0, \ if\ \boldsymbol{S_{CR}} \neq \varnothing \& \mu_{CR_{idx}} = 0 \\ \mu_{CR_{idx}}. \ otherwise \end{cases} \end{cases} \tag{30}$$

where *idx* denoted the index of the group with the minimum selection probability, and if there were more than one indices, *idx* was a randomly index selected from these indices.

The third improvement was that a novel parabolic population size reduction scheme was proposed in the HARD-DE algorithm. The detailed reduction scheme was presented in Eq. (31):

$$PS_{G+1} = round\left[\frac{PS_{\min} - PS_{ini}}{(nfe_{\max} - PS_{ini})^2} \cdot (nfe - PS_{ini})^2 + PS_{ini}\right] \tag{31}$$

where the same symbols had the same meanings as the ones in Eq. (12). Generally, a slow reduction of population was able to get a better perception of the landscape of the objective, and this was the advantage. Nevertheless, the slow reduction also led to a less number of generations because of the fixed maximum number of function evaluations, which weakened the exploitation capability of the algorithm. Therefore, a pivot based population size reduction was further proposed in the HARD-DE algorithm, and the details of this reduction scheme was presented in Eq. (32).

$$PS_{G+1} = \begin{cases} \lceil \frac{y - PS_{ini}}{x - PS_{ini}} \cdot (nfe - PS_{ini})^2 + PS_{ini} \rceil, \ if\ nfe \leqslant x \\ \lfloor \frac{y - PS_{ini}}{x - PS_{ini}} \cdot (nfe - nfe_{\max}) + PS_{ini} \rfloor. \ otherwise \end{cases} \tag{32}$$

where x and y were the coordinates of the pivot $p, p = (x, y)$. The default value of the pivot in HARD-DE was $p = (\frac{2}{3} \cdot nfe_{\max}, \frac{1}{3} \cdot PS_{ini})$. Actually, the pivot based reduction scheme made a balance between exploration and exploitation of the HARD-DE algorithm.

5. The novel Hip-DE algorithm

In this section, we present the novel historical population based DE in details, and the whole algorithm can be separated into two subsections including the novel mutation strategy and the parameter adaptive mechanism.

5.1. Historical population based mutation strategy

As we all know, mutation strategy is one of the most important parts in DE, because it determines the generation of trial vectors together with control parameters during the evolution. A good mutation strategy should be able to make full use of the knowledge obtained during the evolution and then get good perception of the landscape of objectives. Consequently, an overall better optimization performance can be secured by DE variant employing such a mutation strategy. The mutation strategy in the canonical DE algorithm, DE/rand/1/bin, only incorporated the relationship between individuals in the current generation, and there was no historical knowledge used in the generation of trial vectors during the evolution. The mutation strategy in JADE algorithm, DE/target-to-*p*best/1/bin, incorporated not only the relationship between individuals in the current generation but also the relationship between individuals in the current generation and the inferior individuals in the past, and a great success was achieved by this DE variant, nevertheless, some historical information was omitted in the mutation strategy. Here in the paper, a historical population based mutation strategy is advanced with the details shown in Eq. (33):

$$V_{i,G} = X_{i,G} + F \cdot \left(X^p_{best,G} - X_{i,G}\right) + F \cdot \left(X_{r_1,G} - \widehat{X}_{r_2,G}\right) \tag{33}$$

where the same symbols have the same meanings as the ones in Eq. (25), and $\widehat{X}_{r_2,G}$ denotes a randomly selected vector from the union $\mathbf{P} \cup \mathbf{H}$. $\mathbf{H}$ records historical populations in the past generations, and it is the same as archive $\mathbf{B}$ in HARD-DE with the size equalling to $r^{arc} \cdot PS$. Fig. 1 presents the relationships of the vectors in the mutation strategy, and we can see that there are three different relationships involved into the mutation strategy: the first is the relationship among common individuals in the current generation, the second is the relationship between elite individuals and target individuals in the current generation (both the first and the second relationships can be considered as relationships among individuals in the current generation), and the third is the relationship between individuals in the current generation and the individuals in the past. The reason why we incorporate the historical population is that, to some extent, the relationship between current population and the historical population can reflect the landscape of the objective and the knowledge extracted from the relationship is definitely beneficial to guide the evolution

5.2. The parameter adaptive mechanism

This part mainly presents the parameter adaptive mechanism in the novel Hip-DE algorithm, and this section involves three components: the first is about the adaptation of scale factor F, the second is the adaptation of crossover rate CR, and the last is the adaptation of population size PS. As is mentioned in the abstract, the adaptations of the control parameters including F and CR in the recently proposed state-of-the-art DE variants are interlaced with one another. There are the cases that a bad F and a good CR produce a good trial vector candidate, then the bad F is considered as the good value and improperly used in the parameter control and vice versa. These inevitably lead to bad performance of the DE variants, and the novel parameter adaptive mechanism is proposed on this observation.

In the initialization stage, the whole population is categorized into K groups by stochastic universal selection according to the initial selection probability of each group, $\{p(k)|p(1) = p(2) = \ldots = p(k) = \ldots = p(K) = 1/K, k \in \{1, 2, \ldots, K\}\}$, which is the same as LPalmDE and HARD-DE. The scale factors of all the individuals obey the same Cauchy Distribution, $F \sim C(\mu_F, 0.1)$, and the crossover rates of the individuals in the same group obey the same Gaussian distribution, $CR \sim N\left(\mu_{CR_k}, 0.1\right)$, where μ_{CR_k} denotes the corresponding μ_{CR} value in the k^{th} group. Furthermore, the mean values of the distributions of the crossover rates are initialized the same for the K groups, $\mu_{CR_1} = \mu_{CR_2} = \ldots = \mu_{CR_k} = \ldots = \mu_{CR_K} = \mu_{CR}$, and the standard deviations of crossover rates in all the K groups are kept fixed during the whole evolution. After initialization, the algorithm enters into the circle of evolution before reaching the maximum number of function evaluations. In each generation of the evolution, if an individual generates a better trial vector, then it is labeled as "s" individual; otherwise, it is labeled as "f" individual. Then μ_F can be updated according to Eq. (34):

$$\begin{cases} \boldsymbol{S} = \bigcup_{k=1}^{K} \boldsymbol{S_k} \\ w_s = \frac{\Delta f_j}{\sum_{s=1}^{|\boldsymbol{S}|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_F}) = \frac{\sum_{s=1}^{|\boldsymbol{S_F}|} w_s \cdot \boldsymbol{S_F^2}(s)}{\sum_{s=1}^{|\boldsymbol{S_F}|} w_s \cdot \boldsymbol{S_F}(s)} \\ \mu_F = \begin{cases} (1-c) \cdot \mu_F + c \cdot (mean_{WL}(\boldsymbol{S_F})), & if\ S \neq \varnothing \\ \mu_F. & otherwise \end{cases} \end{cases} \tag{34}$$

where the same symbols have the same meanings as the ones in Eq. (23), and c is a constant balance parameter. Consequently, the scale factor F of each individual, e.g. the scale factor of the i^{th} individual $F_{i,G+1}$, can be generated according to Eq. (35):

$$\begin{cases} F_{tmp} = randc(\mu_F, 0.1), \\ F_{tmp} = \begin{cases} randc(\mu_F, 0.1), while\ F \leqslant 0 \\ 1. if\ F > 1 \end{cases} \\ F_{i,G+1} = \begin{cases} F_{tmp}, & if\ rand_1 < \tau_1 \\ F_{i,G}, & otherwise \end{cases} \end{cases} \tag{35}$$

where $rand_1$ is a randomly generated number in the range $[0, 1]$, and τ_1 is a fixed probability with its default setting $\tau_1 = 0.9$. Further discussion of parameter τ_1 is presented in the experiment section.

In each generation of the evolution, the selection probability of each group is also dynamically changed according to Eq. (29), and only the μ_{CR} of the group with smallest selection probability need to be updated according to Eq. (36):

$$\begin{cases} w_s = \frac{\Delta f_j}{\sum_{s=1}^{|S|} \Delta f_j} \\ mean_{WL}(\boldsymbol{S_{CR}}) = \frac{\sum_{s=1}^{|\boldsymbol{S}|} w_s \cdot \boldsymbol{S_{CR}^2}(s)}{\sum_{s=1}^{|\boldsymbol{S}|} w_s \cdot \boldsymbol{S_{CR}}(s)} \\ \mu_{CR_{idx}} = \begin{cases} mean_{WL}(\boldsymbol{S_{CR}}), & if\ S \neq \varnothing \\ \mu_{CR_{idx}}, & otherwise \end{cases} \end{cases} \tag{36}$$

where idx has the same meaning as the one in Eq. (30). Then the crossover rate CR of the i^{th} individual in the population that is categorized into the k^{th} group can be generated according to Eq. (37) in each generation:

$$CR_{i,G+1} = \begin{cases} C\left(\mu_{CR_k}, 0.1\right), & if\ rand_2 < \tau_2 \& \mu_{Cr} \neq 0 \\ 0, & if\ rand_2 < \tau_2 \& \mu_{CR} = 0 \\ CR_{i,G}. & otherwise \end{cases} \tag{37}$$

where $rand_2$ is also a randomly generated number in $[0, 1]$, and τ_2 is also a fixed probability with its default setting $\tau_2 = 0.9$, which is further discussed in the experiment section.

A novel population size reduction scheme is also proposed in the Hip-DE algorithm, and in this new scheme, the population size is kept fixed value in some early generations of the evolution, then the population size is dynamically changed according to a step-decrease scheme with regard to number of function evaluation. The details of the population size during the whole evolution is given in Eq. (38):

$$PS = \begin{cases} PS_{ini}, \ if \ nfe \leqslant nfe_{st} \\ \lceil \frac{PS_{\min} - PS_{ini}}{nfe_{\max} - nfe_{st}} \cdot (nfe - nfe_{st}) + PS_{ini} \rceil, otherwise \end{cases} \tag{38}$$

where nfe_{st} denotes the number of function evaluations allowed in the early generations of the evolution that employs fixed population size, $nfe_{st} = N \cdot PS_{ini}$; other same symbols have the same meanings as the ones in Eq. (31). The pseudo code of the Hip-DE algorithm is given in Algorithm 1.

Algorithm 1. Pseudo code of the novel Hip-DE algorithm. **Input:** Bound constraints $\left[R^D_{min}, R^D_{max}\right]$, maximum number of function evaluations $nfe_{\max}$, objective $f(X)$;
Output: Global best value $f(X_{gbest})$, best individual X_{best}, number of function evaluations nfe;
  Initialize the population size PS, position of individuals
  $X = \{X_1, X_2, \ldots, X_{PS}\}, \mu_F = 0.6, \mu_{Cr} = 0.8, F_{best} = 0.5, CR_{best} = 0.9, K = 6, P(j) = \frac{1}{K}, PS = 15 \cdot D \sim K,$
  $\tau_F = \tau_{Cr} = 0.9, c = 0.1, p = 0.25 \sim 0.05, r^{rac} = 5, H = \varnothing, G = 1;$
  for $i = 1; i \leqslant PS; i++$ **do**
    $X_{i,G} = X_i$;
    Calculate the fitness value $f(X_{i,G})$
  end for
  $nfe = PS$;
  Find the global best $X_{gbest,G}$ and fitness value $f(X_{gbest,G})$;
  while $nfe < nfe_{\max}$ **do**
    Generate the top p superior individuals of the target individuals and historical population;
    Randomly divide the whole population into K groups, the same as HARD-DE algorithm;
    Generate F and CR of all individuals according to Eq. 35 and Eq. 37;
    for $i = 1; i \leqslant PS; i++$ **do**
      Generating donor vector $V_{i,G}$ according to Eq. 33;
      Generating trial vector $f(U_{i,G})$ by crossover operation;
      Calculate fitness value $f(U_{i,G})$;
    end for
    $nfe = nfe + PS$;
    for $i = 1; i \leqslant PS; i++$ **do**
      if $f(U_{i,G}) \leqslant f(X_{i,G})$; **then**
        $X_{i,G+1} = U_{i,G}$;
        $F_{best_i} = F_i$;
        $CR_{best_i} = CR_i$;
      else
        $X_{i,G+1} = X_{i,G}$;
      end if
    end for
    if $S_F \neq \varnothing$ **then**
      Update $P(\cdot)$ according to Eq. 20;
      Update μ_F and $\mu_{CR_{idx}}$ according to Eq. 34 and Eq. 36 respectively;
    end if
    Adjust archive **H**;
    Update X_{gbest} and $f(X_{gbest})$;
    Adjust population size PS according to Eq. 38;
    $G = G + 1$;
end while
return $f(X_{gbest}), X_{gbest}, nfe$;

---

6. Experiment analysis

In this section, we first present the recommended parameter settings of all these contrasted algorithms including JADE [46], LSHADE [38], jSO [4], LPalmDE [26], HARD-DE [25] and our novel Hip-DE algorithm. Then we present the optimization performance comparison of these DE variants under the commonly used test suite in the second part of this section. All the experiments are conducted on a PC with Inter(R) core(TM) i7-7700HQ CPU @ 2.80 GHz on Windows 10 Operating System, and all the algorithms are implemented in Matlab 2016b version. Fitness errors that are smaller than "eps" (eps = 2.2204E-016) are considered as zeros in our paper.

Third, the detailed parameter analysis of the novel Hip-DE algorithm is given in the last three subsequent parts of this section, and finally, the complexity analysis is presented in the last part.

6.1. Parameter settings of the contrasted algorithms

The recommended parameter settings of these contrasted state-of-the-art DE variants are summarized here in Table 2. For the JADE algorithm, the scale factor F obeyed Cauchy distribution in which the location parameter of the distribution was denoted by μ_F and scale parameter was denoted by σ_F. The location parameter μ_F was assigned the value $\mu_F = 0.5$ at the beginning of the evolution and it was dynamically changed during the evolution while the scale parameter σ_F was kept fixed, $\sigma_F = 0.1$, during the whole evolution. The crossover rate CR obeyed Gaussian distribution in which the mean of the distribution was denoted by μ_{CR} and the standard deviation was denoted by σ_{CR}. μ_{CR} was assigned the value $\mu_{CR} = 0.5$ at the beginning, and it was also dynamically changed during the evolution while σ_{CR} was set fixed value, $\sigma_{CR} = 0.1$. The proportion p defining the top superior individuals to the whole population, the balance parameter c in the update equation of μ_F and μ_{CR}, and the population size PS were all set constant values during the whole evolution, $p = 0.05, c = 0.1$, and $PS = 100$ in the JADE algorithm.

For the LSHADE algorithm, the scale factor F and crossover rate CR were initialized the same values and obeyed the same distributions as the ones in JADE, however, the detailed update schemes were different because a control parameter entry pool with H-entry was introduced into the LSHADE algorithm. In each entry, there was a control parameter μ_F–μ_{CR} pair recorded within it, and each individuals of the population in LSHADE was randomly assigned an entry of the H-entry pool. By employing the associated control parameters of the entry, the trial vector of the individual was consequently calculated, and then μ_F and μ_{CR} were able to be updated according to the predefined adaptation scheme. It was worth noting that only one entry was allowed to be updated in each generation in LSHADE. Furthermore, a linear population size reduction scheme was proposed in LSHADE, and the initial population size was set to $PS = 18 \cdot D, D$ denoted the dimension number, while the terminal population size was $PS_{\min} = 4$. The proportion p defining the top superior individuals, the size H of the entry pool, and the archive factor r^{arc} of the external archive were set different values, $p = 0.11, H = 6$, and $r^{arc} = 2.6$ in the LSHADE algorithm.

The parameter settings of the jSO algorithm were more or less the same as the ones in LSHADE because jSO was a further development of LSHADE. In the jSO algorithm, the distribution of μ_F and μ_{CR} were the same as the ones in LSHADE but with different initial values, $\mu_F = 0.3$ and $\mu_{Cr} = 0.8$. A linear reduction of the proportion p that defined top superior individuals

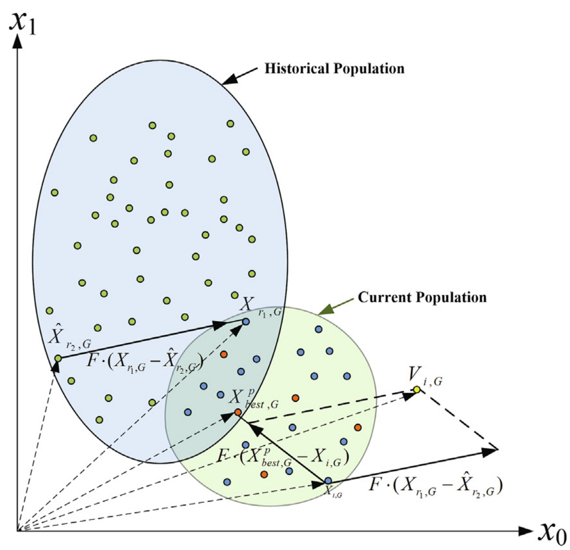


Fig. 1. Illustration of the relationships in the mutation strategy of the novel Hip-DE algorithm.

Table 2
The detailed initial parameter settings of the state-of-the-art DE variants.

DE Variants	Parameter initial settings
JADE	$\mu_F = \mu_{CR} = 0.5, F \sim C(\mu_F, 0.1), CR \sim N(\mu_{CR}, 0.1), p = 0.05, c = 0.1, PS = 100$
LSHADE	$\mu_F = \mu_{CR} = 0.5, F \sim C(\mu_F, 0.1), CR \sim N(\mu_{CR}, 0.1), p = 0.11, H = 6, PS = 18 \cdot D, PS_{\min} = 4, r^{arc} = 2.6$
jSO	$\mu_F = 0.3, \mu_{CR} = 0.8, F \sim C(\mu_F, 0.1), CR \sim N(\mu_{CR}, 0.1), p = 0.25 \sim 0.125, H = 5, ps = 25 \cdot ln(D) \cdot \sqrt{D} \sim 4$
LPalmDE	$\mu_{F_k} = \mu_{CR_k} = 0.5, F \sim C\left(\mu_{F_k}, 0.2\right), CR \sim N\left(\mu_{CR_k}, 0.1\right), p = 0.11, K = 6, PS = 23 \cdot D \sim K, r^{arc} = 1.6, T_0 = 70$
HARD-DE	$\mu_F = 0.3, \mu_{Cr} = 0.8, F \sim C(\mu_F, 0.1), CR \sim N(\mu_{CR}, 0.1), p = 0.11, K = 4, PS = 25 \cdot ln(D) \cdot \sqrt{D} \sim K, r^{arc} = 3$
our approach	$\mu_F = 0.6, \mu_{CR} = 0.8, F \sim C(\mu_F, 0.1), CR \sim N(\mu_{CR}, 0.1), p = 0.2 \sim 0.05, K = 6, PS = 15 \cdot D \sim K, r^{arc} = 5,$ $\tau_F = \tau_{Cr} = 0.9, c = 0.1, N = \lceil r^p \cdot nfe_{\max}/PS \rceil$

was also proposed in jSO. Moreover, the initial proportion p, the size of the entry pool H and the initial population size PS_{ini} were set different values, $p \in [0.25, 0.125], H = 5$, and $PS_{ini} = 25 \cdot lnD \cdot \sqrt{D}$.

In the LPalmDE algorithm, the distribution of F and CR were similar to LSHADE except that a larger scale parameter, $\sigma_F = 0.2$, of the Cauchy distribution was employed in the generation of scale factor F. It was different from LSHADE that all the individuals were categorizing into K groups by stochastic universal selection, and individuals in the same group has the same μ_F of the Cauchy distribution and shared the same CR value. CR values of all the K groups obey Gaussian distribution, $CR \sim N(\mu_{CR}, 0.1)$. Moreover, the proportion p, the number of groups K, the initial population size PS_{ini}, the factor of external archive r^{arc} and the threshold of time stamp T_0 were set fixed values, $p = 0.11, K = 6, PS_{ini} = 23 \cdot D, r^{arc} = 1.6$ and $T_0 = 70$.

The HARD-DE algorithm was actually a further improvement of the jSO algorithm and LPalmDE algorithm. In the HARD-DE algorithm, the distributions of F and CR were the same as the jSO algorithm, and the grouping strategy of individuals was the same as LPalmDE algorithm. Moreover, a hierarchical external archive was also employed in the mutation strategy, and there were two memories in the external archive: one was archive **A** recording the inferior solutions, and the other was archive **B** recording the historical solutions. The fixed parameter settings of p, K, and r^{arc} were $p = 0.11, K = 4$, and $r^{arc} = 3$ respectively.

6.2. Optimization performance comparison

Here we mainly focus on real-parameter single-objective optimization which can be formally represented as finding the optimal solution X^* from the following set:

$$\Omega^* \equiv \arg\min_{X \in \Omega} f(X) = \{X^* \in \Omega : f(X^*) \leqslant f(X), \forall X \in \Omega\} \tag{39}$$

where X is a D-dimensional vector and $\Omega \subseteq \mathbb{R}^D$ is the solution space. The experiment of optimization performance comparison was conducted on CEC 2013 test suite [18] for real-parameter single-objective optimization, and all the benchmarks in the test suite are considered as black-box objectives. The test suite contains 28 commonly used benchmarks which were categorized into three groups: the basic unimodal group f_1–f_5, the basic multi-modal group f_6–f_{20}, and the composition group f_{21}–f_{28}. In our experiment, the parameter settings of our novel Hip-DE algorithm were already listed in Table 2, and we conduct 51 runs on each benchmark with the fixed maximum number of function evaluations ($nfe_{\max} = 10000 \cdot D$) for all the tested algorithms. We summarize the comparison results on 10-D, 30-D, and 50-D optimization comparison in Table 3, Table 4 and Table 5 respectively. The mean ("Mean") and the standard deviation ("Std") of the 51-run fitness errors $\Delta f, \Delta f = f - f*$, are listed in these tables and the symbols "$>$", "$\approx$"and "$<$" in the parentheses behind the values "Mean/Std" denote " Better Performance", "Similar Performance" and "Worse Performance" under Wilcoxon signed rank test with the significance level $\alpha = 0.05$ in comparison with our novel Hip-DE algorithm.

For the 10-D optimization, we can see from Table 3 that the Hip-DE algorithm obtains better or similar performance on 24 benchmarks out of all the 28 benchmarks in comparison with JADE, obtains better or similar performance on 23 benchmarks out of all the 28 benchmarks in comparison with LSHADE, obtains better or similar performance on 21 benchmarks in comparison with jSO, obtains better or similar performance on 20 benchmarks in comparison with LPalmDE, and obtains better or similar performance on 21 benchmarks in comparison with HARD-DE on 10-D optimization. The novel Hip-DE algorithm outperforms JADE on 17 benchmarks out of the total 28 benchmarks, outperforms LSHADE on 15 benchmarks out of the total 28 benchmarks, and outperforms jSO, LPalmDE and HARD-DE on 13 benchmarks out of the total 28 benchmarks. The novel Hip-DE algorithm can also find the global optima on f_1, f_2, f_4, f_5 and f_{11}, which is the same as LSHADE, jSO and LPalmDE. Moreover, it obtains the best performance on $f_{12}, f_{13}, f_{17}, f_{18}, f_{20}, f_{26}$ and f_{27} among all these state-of-the-art DE variants except the benchmarks on which both the novel Hip-DE algorithm and other algorithm(s) can find the global optima.

For the 30-D optimization, we can see from Table 4 that the Hip-DE algorithm obtains better or similar performance on 24 benchmarks out of all the 28 benchmarks in comparison with JADE, obtains better or similar performance on 20 benchmarks out of all the 28 benchmarks in comparison with LSHADE, obtains better or similar performance on 23 benchmarks in comparison with jSO, obtains better or similar performance on 24 benchmarks in comparison with LPalmDE, and obtains better

Table 3
Algorithm comparisons between JADE, LSHADE, jSO, LPalmDE, HARD-DE and our novel Hip-DE algorithm on 10-D optimization

DE Variants: No.	JADE Mean/Std	LSHADE Mean/Std	jSO Mean/Std	LPalmDE Mean/Std	HARD-DE Mean/Std	Our approach Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_3	3.7651E+001/ 7.7285E+001(<)	1.9091E−001/ 9.7404E−001(<)	1.3992E−003/ 9.9919E−003(>)	4.1975E−003/ 1.6957E−002(<)	5.5967E−003/ 1.9375E−002(<)	2.7983E−003/ 1.3989E−002
f_4	2.2468E+002/ 1.2177E+003(<)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_5	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_6	4.4252E+000/ 4.9312E+000(<)	5.1948E+000/ 4.9465E+000(<)	1.3468E+000/ 3.4102E+000(>)	1.1544E+000/ 3.1929E+000(>)	0/0(>)	3.8480E+000/ 4.8384E+000
f_7	1.0759E−001/ 1.6301E−001(<)	1.4148E−005/ 2.3149E−005(>)	3.4153E−005/ 1.0479E−004(<)	2.0074E−005/ 3.7632E−005(>)	3.3804E−003/ 6.5293E−003(<)	2.4733E−005/ 7.2253E−005
f_8	2.0329E+001/ 6.4992E−002(<)	2.0230E+001/ 1.5088E−001(<)	2.0358E+001/ 8.3752E−002(<)	2.0079E+001/ 1.4596E−001(>)	2.0208E+001/ 1.2112E−001(<)	2.0140E+001/ 1.8624E−001
f_9	3.7562E+000/ 6.2099E−001(<)	2.3294E+000/ 1.6767E+000(<)	7.0117E−001/ 8.6243E−001(<)	4.0499E−001/ 6.3400E−001(>)	2.5237E+000/ 1.1282E+000(<)	5.6236E−001/ 7.1894E−001
f_{10}	1.8488E−002/ 8.3409E−003(<)	1.0334E−002/ 1.2181E−002(<)	1.7401E−003/ 3.8597E−003(<)	1.1630E−002/ 1.6007E−002(<)	5.4299E−006/ 3.8271E−005(>)	2.9004E−004/ 1.4499E−003
f_{11}	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	7.8020E−015/ 1.9755E−014(<)	0/0
f_{12}	4.2590E+000/ 1.3539E+000(<)	2.1393E+000/ 7.9528E−001(<)	2.3801E+000/ 8.2236E−001(<)	3.6872E+000/ 1.5579E+000(<)	2.1331E+000/ 8.6644E−001(<)	1.7406E+000/ 1.1592E+000
f_{13}	5.4677E+000/ 2.2796E+000(<)	2.0873E+000/ 1.1803E+000(<)	2.1201E+000/ 1.0770E+000(<)	3.1493E+000/ 1.9315E+000(<)	2.5298E+000/ 9.7643E−001(<)	1.4827E+000/ 7.1284E−001
f_{14}	1.2246E−002/ 2.7985E−002(>)	2.8166E−002/ 5.0451E−002(<)	3.6075E−002/ 4.3136E−002(<)	1.3327E+000/ 2.7736E+000(<)	6.5983E−013/ 4.0992E−013(>)	1.5920E−002/ 3.2657E−002
f_{15}	4.8334E+002/ 1.2449E+002(<)	3.0419E+002/ 1.1924E+002(>)	2.8348E+002/ 1.1125E+002(>)	4.7538E+002/ 1.7888E+002(<)	3.6552E+002/ 1.5347E+002(<)	3.1054E+002/ 1.4755E+002
f_{16}	1.1555E+000/ 2.1311E−001(≈)	2.9314E−001/ 1.6262E−001(<)	1.0942E+000/ 2.0347E−001(<)	1.1155E−001/ 1.1431E−001(>)	2.9639E−001/ 1.6617E−001(<)	1.1345E−001/ 1.0374E−001
f_{17}	1.0122E+001/ 7.9877E−015(≈)	1.0122E+001/ 7.9877E−015(≈)	1.0123E+001/ 8.5962E−004(≈)	1.0147E+001/ 9.3500E−002(<)	1.0122E+001/ 1.7940E−015(≈)	1.0122E+001/ 2.1895E−014
f_{18}	1.8796E+001/ 1.6743E+000(<)	1.3860E+001/ 1.2415E+000(<)	1.6434E+001/ 1.9716E+000(<)	1.5273E+001/ 2.6917E+000(<)	1.5253E+001/ 1.7140E+000(<)	1.3324E+001/ 1.9604E+000
f_{19}	3.3622E−001/ 4.2728E−002(<)	2.2556E−001/ 3.2446E−002(>)	2.7388E−001/ 4.9680E−002(>)	5.3609E−001/ 1.5695E−001(<)	2.4165E−001/ 5.5838E−002(>)	2.7771E−001/ 5.5077E−002
f_{20}	2.2740E+000/ 4.6883E−001(<)	1.9943E+000/ 3.8533E−001(<)	1.7248E+000/ 3.1470E−001(<)	1.7195E+000/ 4.0531E−001(<)	1.8424E+000/ 3.7433E−001(<)	1.5642E+000/ 4.0664E−001
f_{21}	4.0019E+002/0(≈)	4.0019E+002/0(≈)	3.9627E+002/ 2.8033E+001(>)	4.0019E+002/0(≈)	3.8842E+002/ 4.7573E+001(>)	4.0019E+002/0
f_{22}	3.2721E+000/ 4.2043E+000(>)	1.1607E+001/ 2.3918E+001(<)	6.5426E+000/ 4.8560E+000(<)	2.3400E+001/ 2.3195E+001(<)	3.4106E+000/ 3.8802E+000(>)	3.7749E+000/ 3.5976E+000
f_{23}	5.2172E+002/ 1.7317E+002(<)	2.8705E+002/ 1.5536E+002(<)	2.2153E+002/ 1.1252E+002(>)	4.1630E+002/ 2.0770E+002(<)	3.3960E+002/ 1.7191E+002(<)	2.3859E+002/ 1.5316E+002
f_{24}	1.9932E+002/ 9.7873E+000(>)	2.0074E+002/ 1.4231E+001(<)	1.9992E+002/ 1.1073E+001(>)	1.9812E+002/ 1.3405E+001(>)	2.0000E+002/ 9.4284E−004(≈)	2.0000E+002/0
f_{25}	2.0033E+002/ 5.1683E+000(<)	1.9963E+002/ 1.3963E+001(>)	2.0009E+002/ 6.3574E−001(<)	1.9813E+002/ 1.3359E+001(>)	1.9813E+002/ 1.3267E+001(>)	2.0000E+002/ 1.1170E−005
f_{26}	1.4147E+002/ 4.5620E+001(<)	1.6054E+002/ 4.7715E+001(<)	1.0258E+002/ 1.8040E+000(<)	1.1345E+002/ 2.8903E+001(<)	1.0278E+002/ 1.1268E+000(<)	1.0146E+002/ 9.8170E−001
f_{27}	3.0230E+002/ 1.5159E+001(<)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
f_{28}	2.9608E+002/ 2.8006E+001 (>)	2.9216E+002/ 3.9208E+001(>)	3.0000E+002/0(≈)	2.9608E+002/ 2.8006E+001(>)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l	4/7/17	5/8/15	7/8/13	8/7/13	7/8/13	–/–/–

or similar performance on 21 benchmarks in comparison with HARD-DE on 30-D optimization. The novel Hip-DE algorithm outperforms JADE on 22 benchmarks out of the total 28 benchmarks, outperforms LSHADE and jSO on 16 benchmarks out of the total 28 benchmarks, outperforms LPalmDE on 17 benchmarks out of the total 28 benchmarks, and outperforms HARD-DE on 14 benchmarks out of the total 28 benchmarks. The novel Hip-DE algorithm can also find the global optima on f_1 and f_{10} which is the same as jSO and HARD-DE algorithm. Moreover, it obtains the best performance on f_2,f_4,f_6,f_8,f_{11},f_{16},f_{17},f_{18},f_{20},f_{21},f_{24},f_{26} and f_{28} except the benchmarks on which both the novel Hip-DE algorithm and other algorithm(s) can find the global optima.

For the 50-D optimization, we can see from Table 5 that the Hip-DE algorithm obtains better or similar performance on 22 benchmarks out of all the 28 benchmarks in comparison with JADE, obtains better or similar performance on 16 benchmarks

Table 4
Algorithm comparisons between JADE, LSHADE, jSO, LPalmDE, HARD-DE and our novel Hip-DE algorithm on 30-D optimization

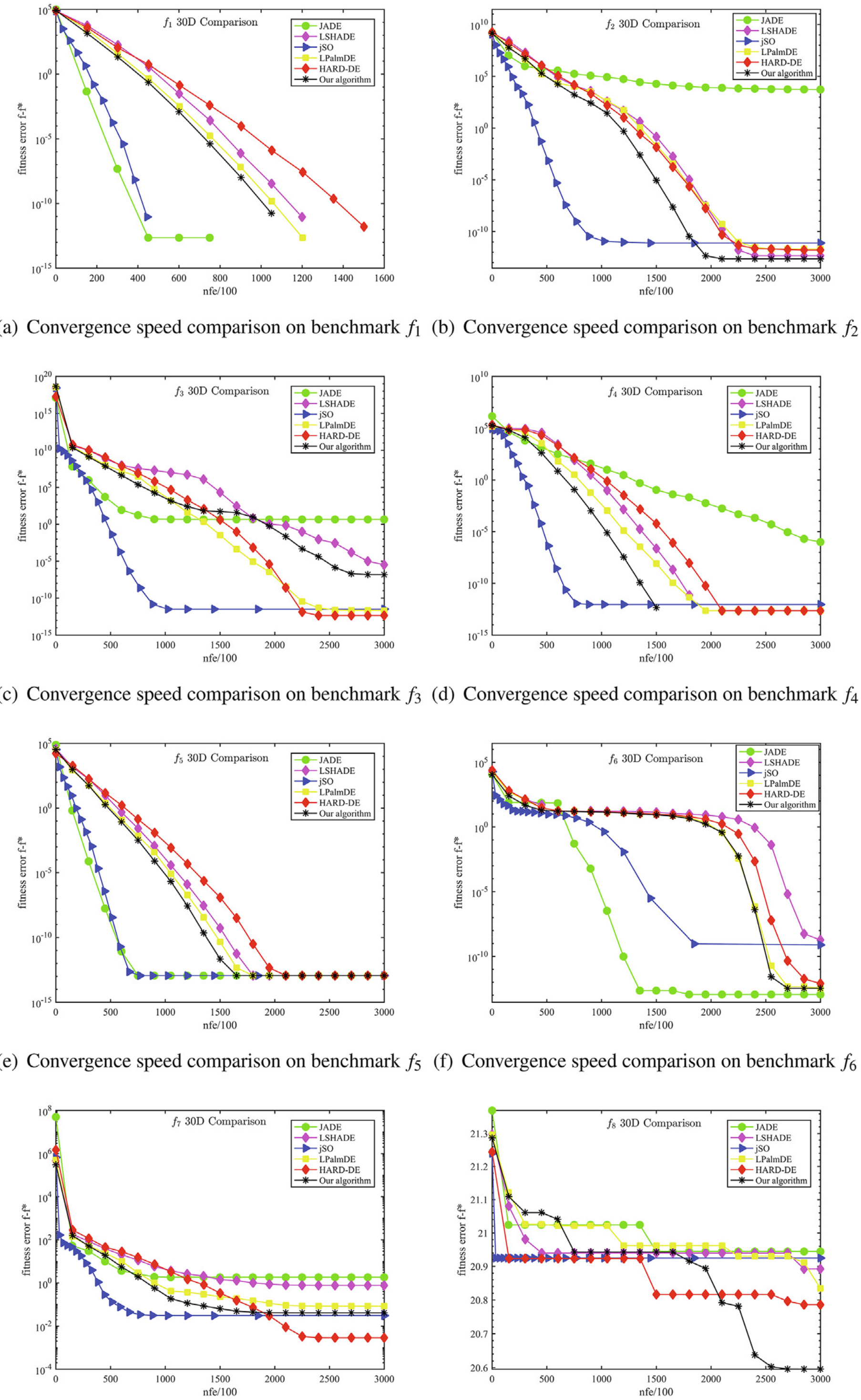
DE Variants: No.	JADE Mean/Std	LSHADE Mean/Std	jSO Mean/Std	LPalmDE Mean/Std	HARD-DE Mean/Std	Our approach Mean/Std
f_1	0/0 (≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	7.0462E+003/ 5.8745E+003(<)	2.8399E−012/ 1.2034E−011(<)	3.7115E−010/ 7.7174E−010(<)	3.5191E−011/ 1.8040E−009(<)	2.3362E−012/ 2.3555E−012(<)	4.5029E−013/ 4.9709E−013
f_3	3.1464E+005/ 1.096E5 + 006(<)	1.7272E−002/ 8.8023E−002(>)	2.3793E−009/ 1.5886E−008(>)	7.3958E−003/ 3.6215E−002(>)	2.2960E−012/ 3.8844E−012(>)	2.6383E−001/ 1.1828E+000
f_4	3.5640E+003/ 1.0965E+004(<)	1.6496E−013/ 1.0248E−013(<)	2.5457E−012/ 1.7191E−012(<)	3.2991E−013/ 2.1481E−013(<)	2.9425E−013/ 1.5283E−013(<)	4.0125E−014/ 8.7542E−014
f_5	1.1146E−013/ 1.5919E−014(>)	1.0923E−013/ 2.2287E−014(>)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0
f_6	2.0712E+000/ 7.1703E+000(<)	5.1779E−001/ 3.6978E+000(<)	1.0356E+000/ 5.1769E+000(<)	3.6853E−004/ 1.9134E−003(<)	1.1049E−009/ 4.2715E−009(<)	5.6478E−011/ 2.3019E−010
f_7	2.6358E+000/ 2.3447E+000(<)	6.7711E−001/ 3.8630E−001(<)	4.9068E−002/ 1.5007E−001(>)	1.0292E−001/ 1.2628E−001(>)	4.3935E−002/ 1.0781E−001(>)	1.3893E−001/ 1.8867E−001
f_8	2.0942E+001/ 5.3025E−002(<)	2.0890E+001/ 1.1297E−001(<)	2.0957E+001/ 4.8471E−002(<)	2.0717E+001/ 1.2151E−001(≈)	2.0833E+001/ 1.2659E−001(<)	2.0626E+001/ 1.9230E−001
f_9	2.6891E+001/ 1.8036E+000(<)	2.7053E+001/ 1.1305E+000(<)	2.3704E+001/ 2.6676E+000(≈)	1.5636E+001/ 3.7837E+000(>)	2.5028E+001/ 2.5150E+000(<)	2.3786E+001/ 3.4641E+000
f_{10}	5.2114E−002/ 2.4790E−002(<)	5.3153E−004/ 2.2212E−003(<)	0/0(≈)	9.6611E−004/ 3.1950E−003(<)	0/0(≈)	0/0
f_{11}	5.8169E+001/ 8.3865E+000(<)	7.8020E−014/ 3.4032E−014(<)	1.6496E−013/ 7.3021E−014(<)	3.8060E+000/ 2.2312E+000(<)	1.6161E−013/ 3.9989E−014(<)	2.1177E−014/ 2.7756E−014
f_{12}	2.4090E+001/ 4.8150E+000(<)	5.4969E+000/ 1.6086E+000(>)	9.0538E+000/ 2.4992E+000(<)	1.1296E+001/ 3.2566E+000(<)	1.1127E+001/ 1.6733E+000(<)	5.6900E+000/ 2.0596E+000
f_{13}	4.8920E+001/ 1.1661E+001(<)	5.3495E+000/ 2.7391E+000(>)	1.0715E+001/ 5.2616E+000(<)	2.2353E+001/ 1.0266E+001(<)	1.9049E+001/ 5.9231E+000(<)	6.5288E+000/ 3.3158E+000
f_{14}	2.7759E−002/ 2.3431E−002(>)	2.9395E−002/ 2.2487E−002(>)	8.4381E+000/ 4.4500E+000(<)	2.1041E+002/ 1.1899E+002(<)	8.5726E−003/ 1.3283E−002(>)	8.4502E−002/ 3.8706E−002
f_{15}	3.2918E+003/ 3.0778E+002(<)	2.6298E+003/ 2.8989E+002(>)	2.6816E+003/ 3.3684E+002(>)	3.1328E+003/ 4.4921E+002(<)	2.8906E+003/ 2.8315E+002(<)	2.7111E+003/ 3.9463E+002
f_{16}	1.9783E+000/ 6.2751E−001(<)	8.2157E−001/ 1.4440E−001(<)	2.3393E+000/ 3.0407E−001(<)	3.4416E−001/ 2.2456E−001(<)	7.3832E−001/ 4.4400E−001(<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0434E+001/ 8.0389E−015(≈)	3.0434E+001/ 1.3102E−006(≈)	3.0669E+001/ 1.0979E−001(<)	3.7692E+001/ 2.4068E+000(<)	3.0434E+001/ 4.4578E+000(≈)	3.0434E+001/ 9.4299E−007
f_{18}	7.5750E+001/ 7.6650E+000(<)	5.1783E+001/ 3.1302E+000(<)	5.6911E+001/ 6.2991E+000(<)	4.6128E+001/ 5.9005E+000(≈)	6.1596E+001/ 4.8775E+000(<)	4.6117E+001/ 6.7980E+000
f_{19}	1.4356E+000/ 1.2019E−001(<)	1.1878E+000/ 8.7252E−002(>)	1.2898E+000/ 1.0311E−001(>)	2.3760E+000/ 6.0265E−001(<)	1.1775E+000/ 9.3495E−002(>)	1.3100E+000/ 1.5268E−001
f_{20}	1.0161E+001/ 4.9428E−001(<)	1.1526E+001/ 2.1028E+000(<)	9.7472E+000/ 3.8117E−001(<)	9.2988E+000/ 5.5861E−001(<)	9.7054E+000/ 4.4241E−001(<)	8.9569E+000/ 4.7118E−001
f_{21}	3.1161E+002/ 7.0505E+001(<)	2.9608E+002/ 1.9604E+001(<)	3.0708E+002/ 5.8493E+001(<)	3.0281E+002/ 2.0100E+001(<)	2.9949E+002/ 5.3368E+001(<)	2.8627E+002/ 3.4754E+001
f_{22}	1.0324E+002/ 1.8223E+001(>)	1.0829E+002/ 2.5583E+000(<)	1.1976E+002/ 3.4337E+000(<)	2.1300E+002/ 6.6635E+001(<)	1.0601E+002/ 6.2829E−001(>)	1.0603E+002/ 6.5855E−001
f_{23}	3.4268E+003/ 3.8445E+002(<)	2.5185E+003/ 2.8743E+002(>)	2.3659E+003/ 3.3190E+002(>)	3.0503E+003/ 4.6513E+002(<)	2.9878E+003/ 2.9060E+002(<)	2.6137E+003/ 3.6526E+002
f_{24}	2.0857E+002/ 1.1226E+001(<)	2.0082E+002/ 1.7428E+000(<)	2.0003E+002/ 3.2709E−002(≈)	2.0003 + 002/ 3.3724E−002(≈)	2.0001E+002/ 7.2523E−003(≈)	2.0001E+002/ 1.3555E−002
f_{25}	2.8170E+002/ 6.3689E+000(<)	2.4080E+002/ 4.0815E+000(<)	2.3924E+002/ 6.8915E+000(<)	2.0806E+002/ 1.7642E+001(>)	2.0677E+002/ 1.5845E+001(>)	2.2050E+002/ 2.3163E+001
f_{26}	2.1581E+002/ 4.4406E+001(<)	2.0000E+002/0(≈)	2.0000E+002/0(≈)	2.0000E+002/0(≈)	2.0000E+002/0(≈)	2.0000E+002/ 1.8900E−013
f_{27}	6.6268E+002/ 2.4023E+002(<)	3.0201E+002/ 4.2899E+000(<)	3.0083E+002/ 9.8159E−001(<)	3.0180E+002/ 3.7048E+000(<)	3.0015E+002/ 1.4219E−001(>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/ 2.7047E−013(≈)	3.0000E+002/0
w/d/l:	3/2/22	8/4/16	5/7/16	4/7/17	7/7/14	–/–/–

out of all the 28 benchmarks in comparison with LSHADE, jSO and HARD-DE, and obtains better or similar performance on 21 benchmarks in comparison with LPalmDE on 50-D optimization. The novel Hip-DE algorithm outperforms JADE on 22 benchmarks out of the total 28 benchmarks, outperforms LSHADE, jSO and HARD-DE on 16 benchmarks out of the total 28 benchmarks, outperforms LPalmDE on 21 benchmarks out of the total 28 benchmarks. The novel Hip-DE algorithm can also find the global optima on f_1, and it can find tolerable optima near the global optima on f_4, f_5 and f_{11}. Moreover, it obtains the best performance on f_1, f_4, f_6, f_8, f_9, f_{16}, f_{20}, and f_{28}.

Table 5
Algorithm comparisons between JADE, LSHADE, jSO, LPalmDE, HARD-DE and our novel Hip-DE algorithm on 50-D optimization

DE Variants: No.	JADE Mean/Std	LSHADE Mean/Std	jSO Mean/Std	LPalmDE Mean/Std	HARD-DE Mean/Std	our approach Mean/Std
f_1	1.6942E−013/ 1.0008E−013(<)	2.2292E−014/ 4.8286E−014(<)	2.6750E−014/ 7.3986E−014(<)	6.2416E−014/ 1.0248E−013(<)	1.1146E−013/ 1.1480E−013(<)	0/0
f_2	2.7583E+004/ 1.5474E+004(<)	1.4036E+003/ 1.6680E+003(>)	2.2197E+002/ 1.6572E+002(>)	6.5623E+002/ 1.0106E+003(>)	1.9177E+002/ 6.0687E+002(>)	1.6761E+003/ 1.8242E+003
f_3	5.7894E+006/ 1.3374E+007(<)	6.6993E+004/ 1.7655E+005(<)	1.3171E+002/ 8.0003E+002(>)	7.6501E+004/ 4.4019E+005(<)	1.8254E+003/ 1.0164E+004(<)	9.9898E+002/ 4.0527E+003
f_4	6.2245E+003/ 1.7270E+004(<)	3.8029E−011/ 8.0764E−011(<)	4.7411E−009/ 7.7628E−009(<)	4.1006E−010/ 1.3387E−009(<)	1.3238E−009/ 3.1395E−009(<)	1.0615E−011/ 1.1188E−011
f_5	1.4267E−013/ 5.0038E−014(>)	1.8279E−013/ 5.6058E−014(>)	2.0954E−013/ 4.7545E−014(>)	1.7164E−013/ 6.1737E−014(>)	1.3821E−013/ 4.7224E−014(>)	2.2514E−013/ 4.8181E−014
f_6	4.3559E+001/ 7.9535E−001(<)	4.3447E+001/0(≈)	4.3447E+001/0(≈)	4.3447E+001/0(≈)	4.3447E+003/0(≈)	4.3447E+001/ 1.6078E−014
f_7	1.9311E+001/ 8.6851E+000(<)	1.6569E+000/ 1.2973E+000(<)	1.4960E−001/ 1.2100E−001(>)	2.1856E+000/ 1.5700E+000(<)	2.7876E−001/ 3.3636E−001(>)	5.2700E−001/ 4.4793E−001
f_8	2.1137E+001/ 3.8018E−002(<)	2.1091E+001/ 8.5035E−002(<)	2.1123E+001/ 4.4397E−002(<)	2.1026E+001/ 7.6054E−002(<)	2.1030E+001/ 1.1946E−001(<)	2.0880E+001/ 1.5854E−001
f_9	5.4444E+001/ 2.0762E+000(<)	5.3319E+001/ 1.9029E+000(<)	4.7942E+001/ 5.2101E+000(<)	4.3446E+001/ 4.8093E+000(<)	4.6825E+001/ 1.8323E+000(<)	3.6877E+001/ 1.1800E+001
f_{10}	3.6398E−002/ 3.0424E−002(<)	6.0887E−003/ 7.2157E−003(>)	4.5698E−014/ 2.2793E−014(>)	7.6337E−003/ 6.0158E−003(<)	4.4938E−003/ 5.8245E−003(>)	6.5219E−003/ 6.3611E−003
f_{11}	0/0(>)	8.3785E−011/ 9.8714E−011(<)	5.9902E−009/ 1.9493E−008(<)	2.5568E+001/ 7.2111E+000(<)	3.5221E−013/ 4.8260E−014(<)	2.1066E−013/ 8.3770E−014
f_{12}	5.6905E+001/ 8.7542E+000(<)	1.2194E+001/ 2.4613E+000(>)	1.5065E+001/ 3.4463E+000(<)	1.7738E+001/ 5.1633E+000(<)	2.5027E+001/ 3.4523E+000(<)	1.4929E+001/ 3.2994E+000
f_{13}	1.2639E+002/ 2.6752E+001(<)	1.6538E+001/ 6.5736E+000(>)	1.8860E+001/ 1.0259E+001(>)	4.2395E+001/ 2.0835E+001(<)	5.9910E+001/ 1.0352E+001(<)	3.1233E+001/ 9.9120E+000
f_{14}	4.4333E−002/ 2.4391E−002(>)	2.2459E−001/ 4.8029E−002(>)	5.9348E+001/ 1.4388E+001(<)	2.0103E+003/ 5.5732E+002(<)	4.0487E−002/ 1.8009E−002(>)	1.7268E+000/ 1.8325E+000
f_{15}	6.8822E+003/ 3.7540E+002(<)	6.2702E+003/ 3.6560E+002(<)	5.9500E+003/ 5.6668E+002(>)	6.8413E+003/ 8.9682E+002(<)	6.5896E+003/ 3.8730E+002(<)	6.1233E+003/ 6.6541E+002
f_{16}	2.2429E+000/ 7.3534E−001(<)	1.2660E+000/ 1.5780E−001(<)	3.1480E+000/ 3.3212E−001(<)	1.1265E+000/ 4.3821E−001(<)	1.1268E+000/ 5.1483E−001(<)	6.1917E−001/ 4.6484E−001
f_{17}	5.0786E+001/ 5.7856E−014(>)	5.0708E+001/ 2.0207E−003(>)	5.2434E+001/ 4.6550E−001(<)	1.0593E+002/ 1.2071E+001(<)	5.0786E+001/ 1.1487E−010(>)	5.0788E+001/ 3.4424E−003
f_{18}	1.4211E+002/ 9.3899E+000(<)	1.0623E+002/ 6.4447E+000(<)	1.1175E+002/ 8.8481E+000(<)	8.3629E+001/ 1.2214E+001(>)	1.1937E+002/ 9.2856E+000(<)	8.7549E+001/ 1.7289E+001
f_{19}	2.7351E+000/ 2.1437E−001(>)	2.5057E+000/ 1.7443E−001(>)	2.6998E+000/ 1.4630E−001(>)	5.0112E+000/ 8.4388E−001(<)	2.4163E+000/ 1.5435E−001(>)	2.8657E+000/ 4.0630E−001
f_{20}	1.9702E+001/ 5.9173E−001(<)	1.8545E+001/ 5.3900E−001(<)	1.9189E+001/ 4.2809E−001(<)	1.8735E+001/ 7.6499E−001(<)	1.8629E+001/ 3.2887E−001(<)	1.7619E+001/ 6.2504E−001
f_{21}	8.0741E+002/ 2.9372E+002(>)	8.5095E+002/ 4.2437E+002(<)	6.8389E+002/ 4.4920E+002(>)	7.4934E+002/ 4.4904E+002(>)	8.4663E+002/ 4.0890E+002(<)	8.2854E+002/ 4.1678E+002
f_{22}	1.5049E+001/ 1.4570E+001(<)	1.3566E+001/ 1.4444E+000(>)	5.9697E+001/ 1.2125E+001(<)	2.0521E+003/ 5.4303E+002(<)	1.1583E+001/ 6.0933E−001(>)	1.4504E+001/ 2.6816E+000
f_{23}	7.2688E+003/ 7.9041E+002(<)	5.3110E+003/ 4.4341E+002(>)	5.0592E+003/ 6.1531E+002(>)	5.9216E+003/ 9.4959E+002(>)	6.6357E+003/ 5.1804E+002(<)	6.1088E+003/ 6.6431E+002
f_{24}	2.5468E+002/ 2.6106E+001(<)	2.1139E+002/ 7.3310E+000(<)	2.0453E+002/ 1.0014E+001(<)	2.0916E+002/ 5.8199E+000(<)	2.0039E+002/ 7.6989E−001(>)	2.0121E+002/ 1.3316E+000
f_{25}	3.5901E+002/ 1.8605E+001(<)	2.7692E+002/ 5.5531E+000(>)	2.7417E+002/ 7.7462E+000(>)	2.7745E+002/ 5.9261E+000(>)	2.8358E+002/ 7.3162E+000(>)	2.8777E+002/ 6.7506E+000
f_{26}	3.3556E+002/ 1.0323E+002(<)	2.4876E+002/ 5.2279E+001(>)	2.3174E+002/ 4.7404E+001(>)	2.3260E+002/ 4.8714E+001(>)	2.3860E+002/ 5.0620E+001(>)	2.7945E+002/ 4.4587E+001
f_{27}	1.3112E+003/ 3.4144E+002(<)	4.5605E+002/ 9.9901E+001(<)	4.4705E+002/ 1.4816E+002(<)	3.7714E+002/ 4.3588E+001(<)	3.1654E+002/ 1.5274E+001(>)	3.1822E+002/ 1.4852E+001
f_{28}	4.5842E+002/ 4.1718E+002(<)	4.0000E+002/ 2.8433E−013(≈)	4.0000E+002/ 2.8705E−013(≈)	4.0000E+002/ 2.8705E−013(≈)	4.0000E+002/ 2.8159E−013(≈)	4.0000E+002/ 2.7035E−013
w/d/l:	6/0/22	12/2/14	12/2/14	7/2/19	12/2/14	–

The convergence comparisons of all these state-of-the-art DE variants under the 28 benchmarks are also given in this part by employing the convergence curve of the median value of the 51 runs on each benchmark. Here we only present the convergence curve on 30-D optimization because of page limitation and these comparisons on benchmarks f_1–f_{28} are summarized in four figures from Fig. 2–5. From these comparisons we can see that the novel Hip-DE algorithm outperforms the other state-of-the-art DE variants on benchmarks $f_2,f_4,f_8,f_{12},f_{13},f_{16},f_{18}$, and f_{20}, and it also shows competitiveness on



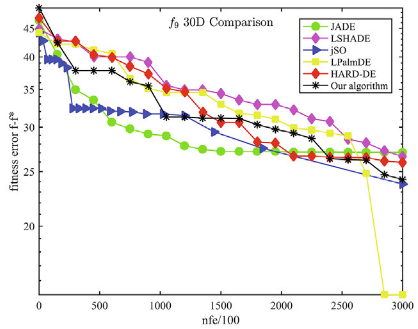
(a) Convergence speed comparison on benchmark f_1 (b) Convergence speed comparison on benchmark f_2

(c) Convergence speed comparison on benchmark f_3 (d) Convergence speed comparison on benchmark f_4

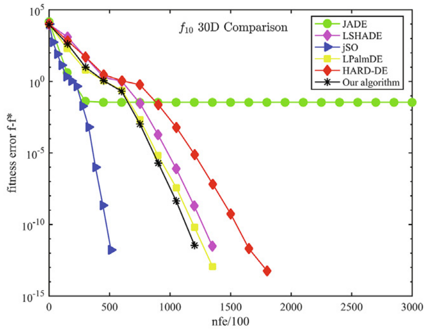
(e) Convergence speed comparison on benchmark f_5 (f) Convergence speed comparison on benchmark f_6

(g) Convergence speed comparison on benchmark f_7 (h) Convergence speed comparison on benchmark f_8

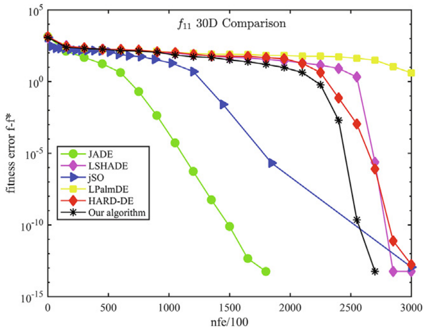
Fig. 2. Here presents the convergence speed comparison of 30-D optimization by employing the median value of the total 51 runs on each benchmark. Comparisons on f_1–f_8 are shown in this part.



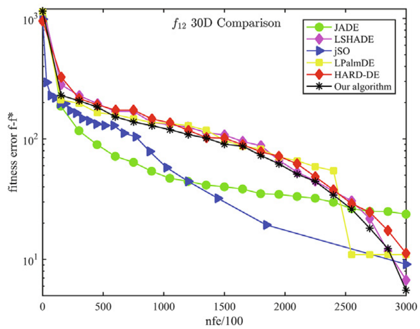
(a) Convergence comparison on benchmark f_9



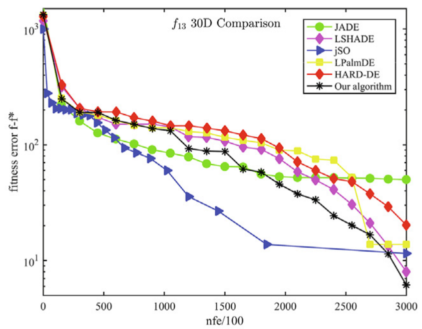
(b) Convergence comparison on benchmark f_{10}



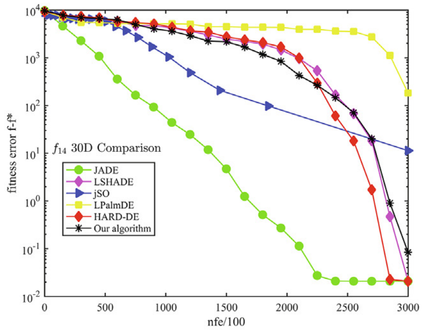
(c) Convergence comparison on benchmark f_{11}



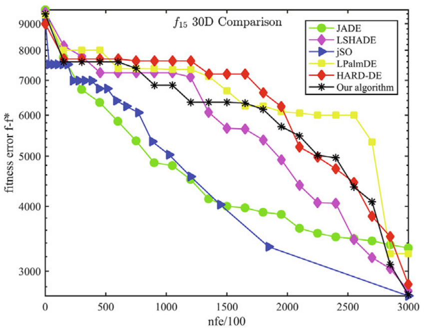
(d) Convergence comparison on benchmark f_{12}



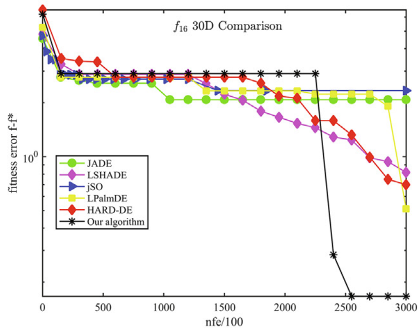
(e) Convergence comparison on benchmark f_{13}



(f) Convergence comparison on benchmark f_{14}

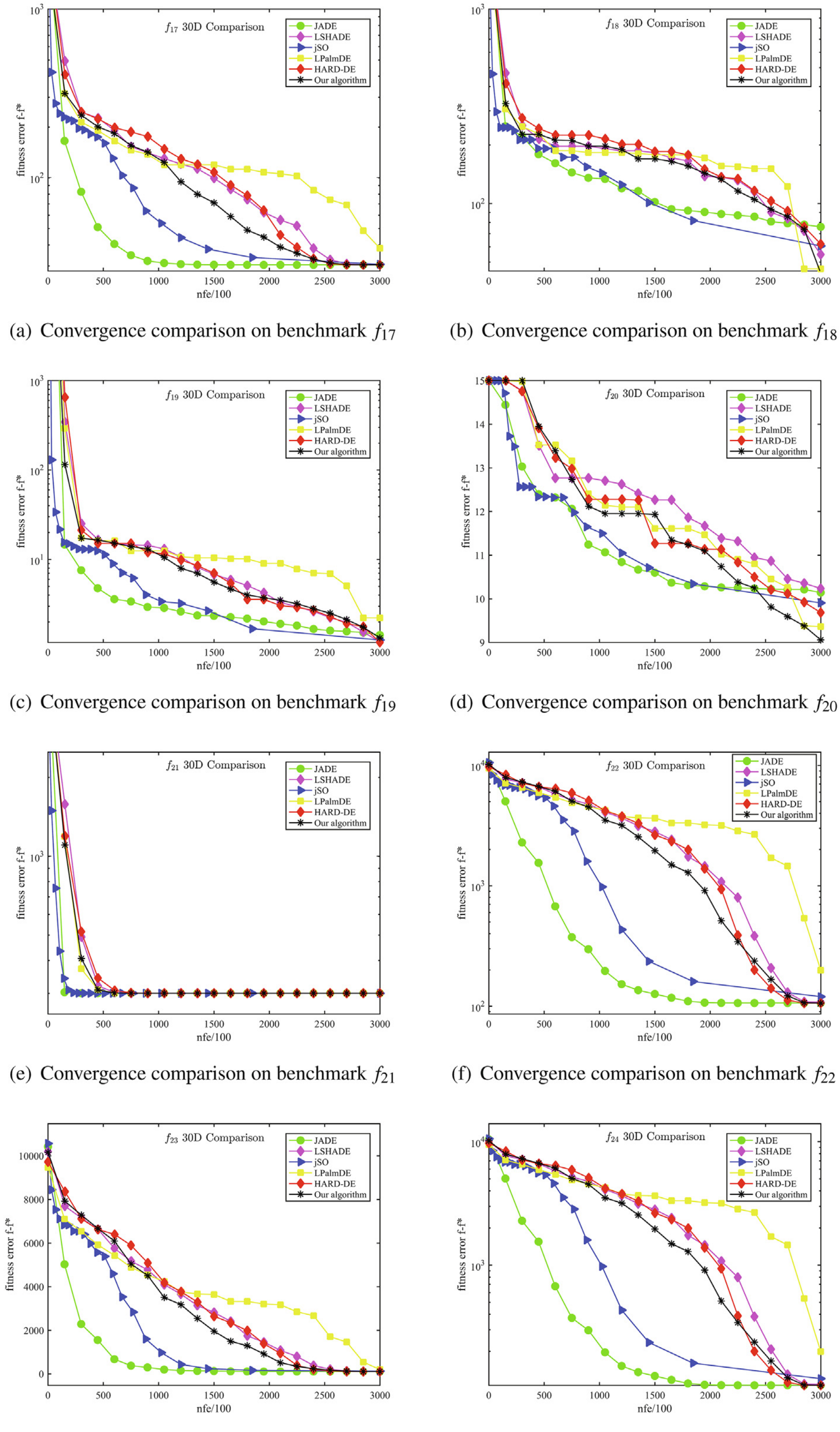


(g) Convergence comparison on benchmark f_{15}



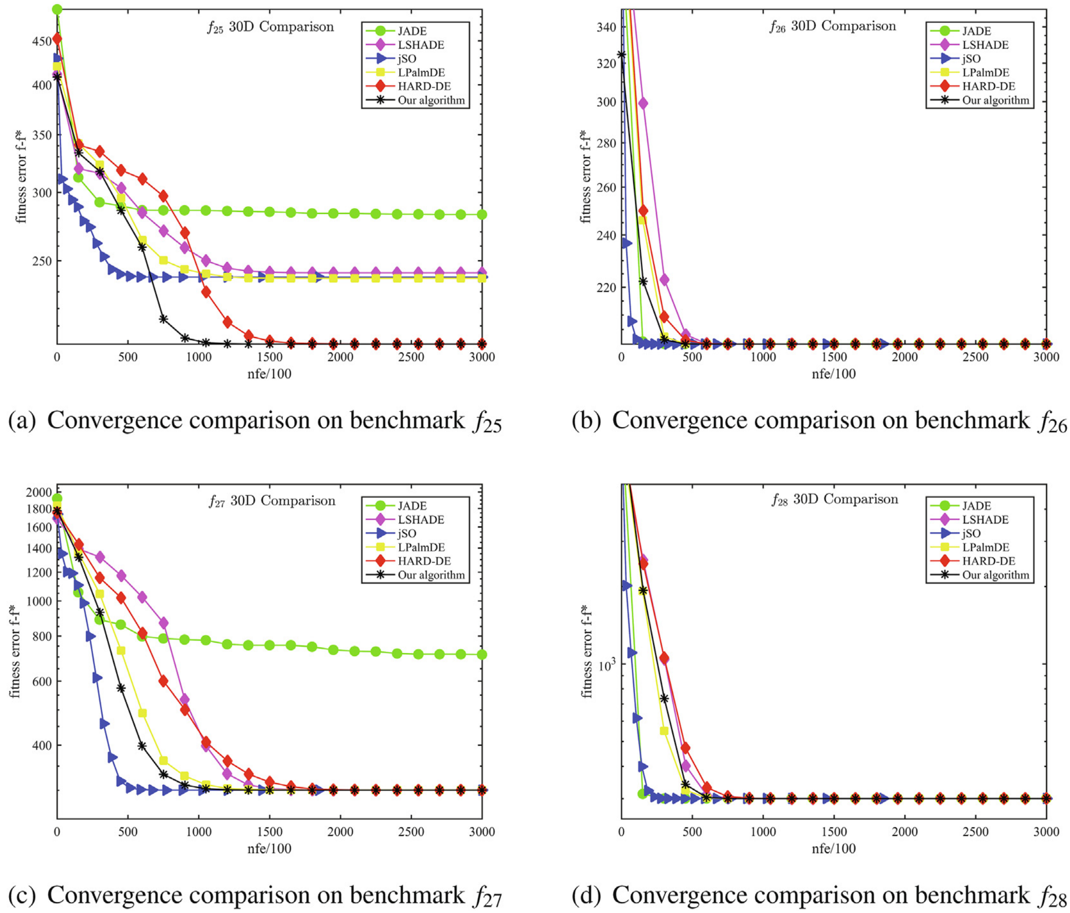
(h) Convergence comparison on benchmark f_{16}

Fig. 3. Continued from Fig. 2, comparisons on f_9–f_{16} are shown here as the second part.



(a) Convergence comparison on benchmark f_{17}
(b) Convergence comparison on benchmark f_{18}
(c) Convergence comparison on benchmark f_{19}
(d) Convergence comparison on benchmark f_{20}
(e) Convergence comparison on benchmark f_{21}
(f) Convergence comparison on benchmark f_{22}
(g) Convergence comparison on benchmark f_{23}
(h) Convergence comparison on benchmark f_{24}

Fig. 4. Continued from Fig. 3, comparisons on f_{17}–f_{24} are shown here as the third part.



(a) Convergence comparison on benchmark f_{25} (b) Convergence comparison on benchmark f_{26}

(c) Convergence comparison on benchmark f_{27} (d) Convergence comparison on benchmark f_{28}

Fig. 5. Continued from Fig. 4, comparisons on f_{25}–f_{28} are shown here as the last part.

benchmarks f_1,f_5,f_{10},f_{11},f_{17},f_{18},f_{21}, and f_{22}–f_{28} in comparison with other contrasted excellent algorithms from the convergence perspective of view.

6.3. The external archive in the Hip-DE algorithm

In the Hip-DE algorithm, an external archive $\boldsymbol{H}$ recording the historical populations of the evolution is employed in generating trial vectors, accordingly, there are mainly two concerns mentioned in this part:

1. Which is the better setting of the factor r^{arc} of the external archive?
2. Which is the better setting, recording the historical individuals or recording the inferior individuals in the external archive?

In order to tackle the first concern, we lists seven different cases that employ different factors of the external archive, $r^{arc} \in \{1, 2, \dots, 7\}$ into consideration. As is mentioned in the former part of the paper, the locations of the historical populations during the evolution, to some extent, can reflect the landscape of the objective, which determines that r^{arc} should not be a smaller value than $r^{arc} = 3$, in other words, at least three generations of the population are recorded in the external archive. Furthermore, if a large r^{arc} is employed, it is is inevitable that the convergence speed is very slow. Nevertheless, the state-of-the-art DE variants usually employed a relative small external archive size, therefore, we also take the smaller values like $r^{arc} = 1, r^{arc} = 2$ into consideration, and that's why we lists the 7 cases in our experiment. The experiment results obtained under these different r^{arc} settings are listed in Table 6, and we can see that the overall performance gradually becomes better from $r^{arc} = 1$ to $r^{arc} = 5$, then becomes worse from $r^{arc} = 5$ to $r^{arc} = 7$. In a word, the novel Hip-DE algorithm can obtain an overall better performance when employing the recommended setting, $r^{arc} = 5$, under the CEC2013 test suite on 30-D optimization.

Table 6
The factor r^{arc} of external archive comparisons on 30-D optimization under CEC2013 test suite.

	The novel Hip-DE algorithm						
r^{arc}: No.	$r^{arc}=1$ Mean/Std	$r^{arc}=2$ Mean/Std	$r^{arc}=3$ Mean/Std	$r^{arc}=4$ Mean/Std	$r^{arc}=6$ Mean/Std	$r^{arc}=7$ Mean/Std	$r^{arc}=5$ Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	4.5870E−008/ 2.4862E−007(<)	4.1997E−012/ 1.7084E−011(<)	8.5154E−013/ 1.7214E−012(<)	1.4712E−012/ 6.4443E−012(<)	4.3961E−012/ 2.8220E−011(<)	3.4639E−010/ 2.2448E−009(<)	4.5029E−013/ 4.9709E−013
f_3	3.0457E−002/ 1.4441E−001(>)	1.5617E−002/ 8.8565E−002(>)	8.7186E−002/5/ 9638E−001(>)	8.4889E−002/ 3.9385E−001(>)	4.6116E−001/ 2.4292E+000(<)	5.0276E+000/ 2.9605E+001(<)	2.6383E−001/ 1.1828E+000
f_4	7.2225E−013/ 4.1853E−014(<)	2.1400E−013/ 1.1515E−013(<)	9.3624E−014/ 1.1302E−013(<)	5.3500E−014/ 9.7408E−014(<)	3.1208E−014/ 7.9022E−014(>)	3.1208E−014/ 7.9022E−014(>)	4.0125E−014/ 8.7542E−014
f_5	1.2483E−013/ 3.4143E−014(<)	1.1815E−013/ 2.2287E−014(<)	1.1592E−013/ 1.5919E−014(<)	1.1369E−013/0 (≈)	1.1369E−013/0 (≈)	1.1369E−013/0 (≈)	1.1369E−013/0
f_6	1.2777E−009/ 6.1287E−009(<)	2.9871E−013/ 3.1173E−013(>)	2.8065E−012/ 1.5930E−011(>)	4.0794E−013/ 4.8189E−013(>)	1.9923E−009/ 1.2796E−008(<)	2.4216E−007/ 1.7226E−006(<)	5.6478E−011/ 2.3019E−010
f_7	1.8118E−001/ 2.3452E−001(<)	1.6087E−001/ 1.9573E−001(<)	1.4114E−001/ 1.9441E−001(<)	1.8529E−001/ 2.3076E−001(<)	1.0037E−001/ 1.0932E−001(>)	1.3422E−001/ 1.7046E−001(>)	1.3893E−001/ 1.8867E−001
f_8	2.0697E+001/ 2.1504E−001(<)	2.0654E+001/ 2.0363E−001(<)	2.0686E+001/ 1.7891E−001(<)	2.0632E+001/ 1.6802E−001(<)	2.0689E+001/ 1.6373E−001(<)	2.0655E+001/ 1.7805E−001(<)	2.0626E+001/ 1.9230E−001
f_9	2.4218E+001/ 3.0264E+000(<)	2.4676E+001/ 2.7719E+000(<)	2.3708E+001/ 3.0933E+000(>)	2.4831E+001/ 2.7045E+000(<)	2.3727E+001/ 3.7986E+000(>)	2.4016E+001/ 3.4848E+000(<)	2.3786E+001/ 3.4641E+000
f_{10}	8.7423E−003/ 8.8556E−003(<)	1.4987E−003/ 3.3108E−003(<)	5.8008E−004/ 2.0082E−003(<)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_{11}	3.4552E−014/ 2.8029E−014(<)	1.6719E−014/ 2.6158E−014(>)	1.7833E−014/ 2.6638E−014(>)	2.4521E−014/ 2.8433E−014(<)	2.7864E−014/ 2.8699E−014(<)	3.2323E−014/ 2.8433E−014(<)	2.1177E−014/ 2.7756E−014
f_{12}	8.5641E+000/ 2.1518E+000(<)	8.0186E+000/ 1.9149E+000(<)	7.3697E+000/ 1.9053E+000(<)	6.5928E+000/ 2.1338E+000(<)	5.9951E+000/ 1.7388E+000(<)	5.4588E+000/ 2.0372E+000(>)	5.6900E+000/ 2.0596E+000
f_{13}	1.2539E+001/ 5.3814E+000(<)	8.6961E+000/ 3.2271E+000(<)	7.5709E+000/ 3.0876E+000(<)	5.8934E+000/ 2.2773E+000(>)	5.9572E+000/ 2.6434E+000(>)	5.6954E+000/ 3.2276E+000(>)	6.5288E+000/ 3.3158E+000
f_{14}	9.5932E−002/ 3.1181E−002(<)	8.7359E−002/ 3.3832E−002(<)	8.8992E−002/ 3.3331E−002(<)	8.8276E−002/ 3.5719E−002(<)	9.2666E−002/ 3.9571E−002(<)	9.3891E−002/ 3.1539E−002(<)	8.4502E−002/ 3.8706E−002
f_{15}	2.8040E+003/ 3.0046E+002(<)	2.6807E+003/ 3.0960E+002(>)	2.7210E+003/ 3.2139E+002(<)	2.7673E+003/ 3.2468E+002(<)	2.7081E+003/ 3.1084E+002(>)	2.7432E+003/ 3.4428E+002(<)	2.7111E+003/ 3.9463E+002
f_{16}	4.7281E−001/ 3.2605E−001(<)	3.7471E−001/ 3.0658E−001(<)	3.1748E−001/ 2.7290E−001(<)	3.1771E−001/ 2.6178E−001(<)	3.5731E−001/ 2.5391E−001(<)	3.9620E−001/ 3.7258E−001(<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0434E+001/ 2.0225E−006(≈)	3.0434E+001/ 1.3202E−006(≈)	3.0434E+001/ 1.0674E−013(≈)	3.0434E+001/ 1.8285E−006(≈)	3.0434E+001/ 1.6003E−006(≈)	3.0434E+001/ 9.4299E−007(≈)	3.0434E+001/ 9.4299E−007
f_{18}	4.7335E+001/ 5.9477E+000(<)	4.6527E+001/ 6.5375E+000(<)	4.7717E+001/ 6.5008E+000(<)	4.5653E+001/ 6.6625E+000(>)	4.5114E+001/ 6.4138E+000(>)	4.6708E+001/ 7.5023E+000(<)	4.6117E+001/ 6.7980E+000
f_{19}	1.3612E+000/ 1.6421E−001(<)	1.3405E+000/ 1.6046E−001(<)	1.3074E+000/ 1.7428E−001(>)	1.3471E+000/ 1.8278E−001(<)	1.3170E+000/ 2.1115E−001(<)	1.3563E+000/ 1.9123E−001(<)	1.3100E+000/ 1.5268E−001
f_{20}	8.9335E+000/ 3.3858E−001(>)	9.0533E+000/ 4.1749E−001(<)	8.9987E+000/ 4.2445E−001(<)	8.9202E+000/ 4.3722E−001(>)	8.9293E+000/ 3.8058E−001(>)	8.9072E+000/ 3.8323E−001(>)	8.9569E+000/ 4.7118E−001
f_{21}	3.0452E+002/ 4.0192E+001(<)	3.0819E+002/ 5.0912E+001(<)	2.9412E+002/ 2.3764E+001(<)	3.0648E+002/ 3.7329E+001(<)	2.9583E+002/ 4.2502E+001(<)	2.9889E+002/ 2.8475E+001(<)	2.8627E+002/ 3.4754E+001
f_{22}	1.0628E+002/ 1.4120E+000(<)	1.0600E+002/ 4.3602E−001(>)	1.0603E+002/ 5.5704E−001(≈)	1.0598E+002/ 3.8965E−001(>)	1.0622E+002/ 9.2582E−001(<)	1.0614E+002/ 1.1348E+000(<)	1.0603E+002/ 6.5855E−001
f_{23}	2.5834E+003/ 4.3109E+002(>)	2.7075E+003/ 3.5210E+002(<)	2.6189E+003/ 3.2283E+002(<)	2.6018E+003/ 3.5953E+002(>)	2.6387E+003/ 3.3669E+002(<)	2.6947E+003/ 3.4300E+002(<)	2.6137E+003/ 3.6526E+002
f_{24}	2.0044E+002/ 4.4355E−001(<)	2.0014E+002/ 2.2402E−001(<)	2.0003E+002/ 3.8129E−002(<)	2.0001E+002/ 2.7148E−002(≈)	2.0001E+002/ 2.0999E−002(≈)	2.0000E+002/ 7.4886E−003(>)	2.0001E+002/ 1.3555E−002
f_{25}	2.0467E+002/ 1.4068E+001(>)	2.0003E+002/ 6.2873E−002(>)	2.0966E+002/ 1.8713E+001(>)	2.1957E+002/ 2.2961E+001(>)	2.2655E+002/ 2.1831E+001(<)	2.2774E+002/ 2.1006E+001(<)	2.2050E+002/ 2.3163E+001
f_{26}	2.0000E+002/ 1.8520E−013(≈)	2.0000E+002/ 1.5891E−013(≈)	2.0000E+002/ 1.7555E−009(≈)	2.0000E+002/ 1.9506E−013(≈)	2.0000E+002/ 1.3987E−013(≈)	2.0196E+002/ 1.4003E+001(<)	2.0000E+002/ 1.8900E−013
f_{27}	3.0845E+002/ 6.1051E+000(<)	3.0305E+002/ 3.4987E+000(<)	3.0180E+002/ 2.9894E+000(<)	3.0069E+002/ 1.1814E+000(<)	3.0014E+002/ 2.8917E−001(>)	3.0030E+002/ 7.3335E−001(>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0
w/d/l:	4/4/20	6/4/18	6/5/17	8/7/13	8/7/13	7/5/16	-/-/-

Table 7
The recommended settings of the factor of external archive in DE variants mentioned in this paper.

Algorithms:	JADE	LSHADE	jSO	LPalmDE	HARD-DE
r^{arc}	1	2.6	1	1.6	3

Table 8
Optimization results comparisons between external archive recording inferior individuals and external archive recording historical individuals on 30-D optimization under CEC2013 test suite.

The novel Hip-DE algorithm						
H	inferior individuals					historical individuals
r^{arc} No.	$r^{arc}=1$ Mean/Std	$r^{arc}=1.6$ Mean/Std	$r^{arc}=2.6$ Mean/Std	$r^{arc}=3$ Mean/Std	$r^{arc}=5$ Mean/Std	$r^{arc}=5$ Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	1.0588E−011/ 4.8647E−011(<)	3.9055E−012/ 1.8689E−011(<)	2.0954E−009/ 1.2731E−008(<)	4.5382E−006/ 3.2317E−005(<)	1.0149E−002/ 3.8139E−002(<)	4.5029E−013/ 4.9709E−013
f_3	2.1808E−001/ 9.3829E−001(>)	4.0052E−001/ 2.4403E+000(<)	1.2404E+001/ 5.5552E+001(<)	7.5622E−001/ 2.9642E+000(<)	8.0960E+000/ 2.2253E+001(<)	2.6383E−001/ 1.1828E+000
f_4	1.6050E−013/ 1.1409E−013(<)	6.6875E−014/ 1.0463E−013(<)	6.6875E−014/ 1.0463E−013(<)	3.1208E−014/ 7.9022E−014(>)	1.5158E−013/ 1.0825E−013(<)	4.0125E−014/ 8.7542E−014
f_5	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1815E−013/ 2.2287E−014(<)	1.1369E−013/0
f_6	1.9380E−011/ 1.3533E−010(>)	8.6440E−011/ 6.1441E−010(<)	9.8699E−009/ 6.5956E−008(<)	4.8531E−008/ 2.0095E−007(<)	3.2893E−002/ 7.7624E−002(<)	5.6478E−011/ 2.3019E−010
f_7	2.1506E−001/ 2.5475E−001(<)	1.9382E−001/ 2.1623E−001(<)	1.6996E−001/ 2.0915E−001(<)	1.5094E−001/ 1.8283E−001(<)	2.1064E−001/ 2.9524E−001(<)	1.3893E−001/ 1.8867E−001
f_8	2.0795E+001/ 1.2506E−001(<)	2.0811E+001/ 1.1580E−001(<)	2.0844E+001/ 1.0652E−001(<)	2.0829E+001/ 8.5500E−002(<)	2.0851E+001/ 9.7745E−002(<)	2.0626E+001/ 1.9230E−001
f_9	2.4796E+001/ 2.2844E+000(<)	2.4099E+001/ 2.7759E+000(<)	2.3437E+001/ 3.7374E+000(>)	2.4205E+001/ 2.2208E+000(<)	2.3474E+001/ 2.4485E+000(>)	2.3786E+001/ 3.4641E+000
f_{10}	4.6780E−003/ 5.5153E−003(<)	1.6916E−003/ 3.7699E−003(<)	2.9004E−004/ 1.4499E−003(<)	1.4502E−004/ 1.0357E−003(<)	0/0(≈)	0/0
f_{11}	2.1177E−014/ 2.7756E−014(≈)	2.5635E−014/ 2.8566E−014(<)	3.3437E−014/ 2.8254E−014(<)	3.7896E−014/ 2.7063E−014(<)	6.0187E−014/ 1.7655E−014(<)	2.1177E−014/ 2.7756E−014
f_{12}	7.3018E+000/ 2.0207E+000(<)	6.8990E+000/ 1.8474E+000(<)	6.3310E+000/ 2.1040E+000(<)	6.6321E+000/ 2.2721E+000(<)	8.5307E+000/ 2.2409E+000(<)	5.6900E+000/ 2.0596E+000
f_{13}	7.7599E+000/ 3.6889E+000(<)	7.0431E+000/ 3.5157E+000(<)	7.8188E+000/ 3.2115E+000(<)	6.6856E+000/ 3.3161E+000(<)	1.0446E+001/ 4.6763E+000(<)	6.5288E+000/ 3.3158E+000
f_{14}	8.6136E−002/ 2.6340E−002(<)	8.0828E−002/ 3.4247E−002(>)	7.3072E−002/ 2.9552E−002(>)	7.7878E−002/ 3.6612E−002(>)	8.9643E−002/ 3.5746E−002(<)	8.4502E−002/ 3.8706E−002
f_{15}	2.7397E+003/ 3.1965E+002(<)	2.6456E+003/ 3.3142E+002(>)	2.5808E+003/ 4.7109E+002(>)	2.5979E+003/ 3.4367E+002(>)	2.5720E+003/ 3.8237E+002(>)	2.7111E+003/ 3.9463E+002
f_{16}	7.8291E−001/ 3.2108E−001(<)	8.3545E−001/ 3.2784E−001(<)	9.2947E−001/ 3.3551E−001(<)	8.5176E−001/ 3.6219E−001(<)	8.8553E−001/ 3.6841E−001(<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0434E+001/ 9.0660E−014(≈)	3.0434E+001/ 1.3202E−006(≈)	3.0434E+001/ 1.6003E−006(≈)	3.0434E+001/ 9.4299E−007(≈)	3.0434E+001/ 1.3202E−006(≈)	3.0434E+001/ 9.4299E−007
f_{18}	4.8823E+001/ 6.8288E+000(<)	4.8019E+001/ 6.1008E+000(<)	4.7026E+001/ 5.1235E+000(<)	5.0455E+001/ 7.4756E+000(<)	5.1361E+001/ 7.2279E+000(<)	4.6117E+001/ 6.7980E+000
f_{19}	1.3013E+000/ 1.6356E−001(>)	1.3345E+000/ 1.4132E−001(<)	1.3231E+000/ 1.5013E−001(<)	1.3065E+000/ 1.4263E−001(>)	1.3643E+000/ 1.7613E−001(<)	1.3100E+000/ 1.5268E−001
f_{20}	8.9331E+000/ 4.4510E−001(>)	8.9937E+000/ 3.5377E−001(<)	9.1488E+000/ 4.6443E−001(<)	9.2082E+000/ 3.4206E−001(<)	9.5220E+000/ 4.4430E−001(<)	8.9569E+000/ 4.7118E−001
f_{21}	3.0085E+002/ 2.4726E+001(<)	3.0000E+002/0(<)	2.9608E+002/ 1.9604E+001(<)	2.9804E+002/ 1.4003E+001(<)	2.9693E+002/ 3.1662E+001(<)	2.8627E+002/ 3.4754E+001
f_{22}	1.0604E+002/ 5.0368E−001(<)	1.0604E+002/ 7.3641E−001(<)	1.0608E+002/ 5.4140E−001(<)	1.0601E+002/ 5.2521E−001(>)	1.0631E+002/ 1.3299E+000(<)	1.0603E+002/ 6.5855E−001
f_{23}	2.6195E+003/ 3.6073E+002(<)	2.4667E+003/ 3.9953E+002(>)	2.3826E+003/ 3.2352E+002(>)	2.4621E+003/ 3.3651E+002(>)	2.3308E+003/ 3.8801E+002(>)	2.6137E+003/ 3.6526E+002
f_{24}	2.0014E+002/ 1.9567E−001(<)	2.0004E+002/ 5.9372E−002(<)	2.0001E+002/ 1.2889E−002(≈)	2.0001E+002/ 1.9890E−002(≈)	2.0002E+002/ 1.3474E−001(<)	2.0001E+002/ 1.3555E−002
f_{25}	2.1206E+002/ 1.9942E+001(>)	2.2212E+002/ 2.2176E+001(<)	2.4292E+002/ 1.0358E+001(<)	2.4150E+002/9/ 8029E+000(<)	2.4766E+002/ 8.8831E+000(<)	2.2050E+002/ 2.3163E+001
f_{26}	2.0000E+002/ 1.9703E−013(≈)	2.0196E+002/ 1.4013E+001(<)	2.0000E+002/ 1.5141E−013(≈)	2.0000E+002/ 1.5141E−013(≈)	2.0000E+002/ 3.2640E−008(≈)	2.0000E+002/ 1.8900E−013
f_{27}	3.0362E+002/ 3.5025E+000(<)	3.0081E+002/ 1.1129E+000(<)	3.0057E+002/ 1.3482E+000(<)	3.0027E+002/ 1.1636E+000(>)	3.0016E+002/ 3.1061E−001(>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/ 6.4311E−014(≈)	3.0000E+002/0
w/d/l:	5/6/17	3/4/21	4/6/18	7/6/15	4/5/19	-/-/-

In order to tackle the second concern, experiment is designed to compared the performance between external archive recording historical populations and external archive recording inferior solutions. As we all know that the first external archive based DE variant was JADE proposed in 2009. Because of the great success obtained by the mutation strategy with

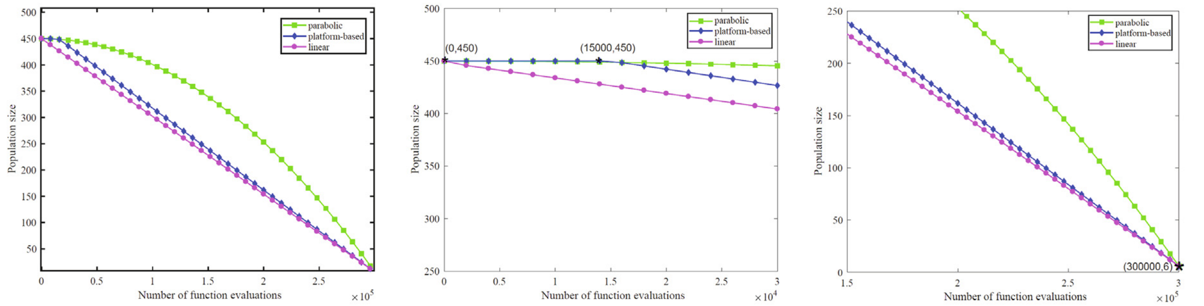
Table 9
Initial population size comparison of the Hip-DE algorithm on 30-D optimization under CEC2013 test suite.

ps_{ini} NO.	$11 \cdot D$ Mean/Std	$12 \cdot D$ Mean/Std	$13 \cdot D$ Mean/Std	$14 \cdot D$ Mean/Std	$16 \cdot D$ Mean/Std	$17 \cdot D$ Mean/Std	$15 \cdot D$ Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	1.6731E−004/7.7079E−004(<)	1.6289E−008/1.0321E−007(<)	1.5564E−011/4.6265E−011(<)	1.3053E−010/7.9563E−010(<)	4.2800E−013/5.6368E−013(>)	2.5412E−013/1.8554E−013(>)	4.5029E−013/4.9709E−013
f_3	7.9209E+000/3.9406E+001(<)	1.5159E+001/7.3713E+001(<)	1.5741E+000/6.3763E+000(<)	5.6488E−001/2.1857E+000(<)	9.5892E+000/6.8247E+001(<)	6.8412E+001/4.0155E+002(<)	2.6383E−001/1.1828E+000
f_4	1.8279E−013/1.0188E−013(<)	1.1592E−013/1.1480E−013(<)	9.8083E−014/1.1373E−013(<)	4.4583E−014/9.1172E−014(<)	3.5666E−014/8.3512E−014(>)	8.9166E−015/4.4574E−014(>)	4.0125E−014/8.7542E−014
f_5	1.1592E−013/1.5919E−014(<)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0
f_6	2.8756E−013/2.6257E−013(>)	7.8912E−013/2.2463E−012(>)	2.0107E−012/1.0133E−011(>)	7.2534E−011/5.1287E−010(<)	1.7684E−009/1.2207E−008(<)	2.6643E−010/1.0399E−009(<)	5.6478E−011/2.3019E−010
f_7	2.3797E−001/2.5922E−001(<)	1.7012E−001/2.4052E−001(<)	1.8004E−001/1.8797E−001(<)	1.5163E−001/1.7543E−001(<)	1.2599E−001/1.7015E−001(>)	9.4846E−002/1.1894E−001(>)	1.3893E−001/1.8867E−001
f_8	2.0550E+001/1.6544E−001(>)	2.0648E+001/2.0421E−001(<)	2.0674E+001/1.87033−001(<)	2.0720E+001/1.8255E−001(<)	2.0652E+001/1.9452E−001(<)	2.0704E+001/1.7033E−001(<)	2.0626E+001/1.9230E−001
f_9	2.3075E+001/4.3594E+000(>)	2.4076E+001/2.7088E+000(<)	2.4288E+001/3.7439E+000(<)	2.3471E+001/3.0776E+000(>)	2.3609E+001/3.6288E+000(>)	2.4111E+001/2.7351E+000(<)	2.3786E+001/3.4641E+000
f_{10}	3.8651E−004/1.9938E−003(<)	4.3506E−004/1.7576E−003(<)	0/0(≈)	1.4502E−004/1.0357E−003(<)	0/0(≈)	0/0(≈)	0/0
f_{11}	2.8020E−015/1.9755E−014(>)	6.6875E−015/1.8497E−014(>)	1.2260E−014/2.3612E−014(>)	2.0062E−014/2.7435E−014(>)	3.4552E−014/2.8029E−014(<)	3.6781E−014/2.7435E−014(<)	2.1177E−014/2.7756E−014
f_{12}	6.1048E+000/1.9639E+000(<)	6.3760E+000/1.9163E+000(<)	5.9771E+000/2.0119E+000(<)	6.4964E+000/1.8607E+000(<)	6.1117E+000/2.2211E+000(<)	5.8213E+000/2.1276E+000(<)	5.6900E+000/2.0596E+000
f_{13}	7.5578E+000/3.3874E+000(<)	6.8333E+000/3.3416E+000(<)	6.3750E+000/2.5386E+000(>)	6.4561E+000/2.6547E+000(>)	6.4222E+000/3.9549E+000(>)	5.9720E+000/2.9141E+000(>)	6.5288E+000/3.3158E+000
f_{14}	5.4293E−002/3.1181E−002(>)	7.1439E−002/3.3116E−002(>)	6.5724E−002/3.3408E−002(>)	7.5929E−002/2.9684E−002(>)	1.0695E−001/4.1434E−002(<)	1.1637E−001/3.7727E−002(<)	8.4502E−002/3.8706E−002
f_{15}	2.6566E+003/3.1195E+002(>)	2.6363E+003/3.5784E+002(>)	2.6256E+003/3.3882E+002(>)	2.6824E+003/4.0013E+002(>)	2.8014E+003/3.6256E+002(<)	2.7849E+003/4.1150E+002(<)	2.7111E+003/3.9463E+002
f_{16}	3.6059E−001/3.3907E−001(<)	2.8330E−001/2.4598E−001(<)	3.4240E−001/2.7482E−001(<)	2.7684E−001/2.2318E−001(>)	3.2601E−001/2.7327E−001(<)	4.1355E−001/3.9370E−001(<)	2.8301E−001/2.4921E−001
f_{17}	3.0434E+001/9.4299E−007(≈)	3.0434E+001/6.6738E−014(≈)	3.0434E+001/1.3202E−006(≈)	3.0434E+001/1.3202E−006(≈)	3.0434E+001/1.8285E−006(≈)	3.0434E+001/9.2167E−005(≈)	3.0434E+001/9.4299E−007
f_{18}	4.2220E+001/4.7943E+000(>)	4.3284E+001/5.7276E+000(>)	4.4751E+001/6.4152E+000(>)	4.6052E+001/6.7077E+000(>)	4.8341E+001/7.4680E+000(<)	5.0423E+001/7.3519E+000(<)	4.6117E+001/6.7980E+000
f_{19}	1.2382E+000/1.6265E−001(>)	1.2202E+000/1.5117E−001(>)	1.2411E+000/1.7678E−001(>)	1.2829E+000/1.7955E−001(>)	1.3692E+000/1.6992E−001(<)	1.3616E+000/1.5901E−001(<)	1.3100E+000/1.5268E−001
f_{20}	8.9587E+000/4.4021E−001(<)	8.9996E+000/3.9045E−001(<)	8.8979E+000/3.6058E−001(>)	8.8951E+000/3.8475E−001(>)	8.8547E+000/3.8399E−001(>)	9.0137E+000/3.8092E−001(<)	8.9569E+000/4.7118E−001
f_{21}	2.9412E+002/2.3764E+001(<)	2.9693E+002/3.1662E+001(<)	2.9693E+002/3.1662E+001(<)	2.9608E+002/1.9604E+001(<)	2.9608E+002/1.9604E+001(<)	2.9412E+002/2.3764E+001(<)	2.8627E+002/3.4754E+001
f_{22}	1.0574E+002/4.5586E−001(>)	1.0604E+002/1.1925E+000(<)	1.0596E+002/5.2432E−001(>)	1.0598E+002/6.5374E−001(>)	1.0625E+002/1.3930E+000(<)	1.0631E+002/1.3228E+000(<)	1.0603E+002/6.5855E−001
f_{23}	2.5234E+003/4.1285E+002(>)	2.5314E+003/3.1166E+002(>)	2.6460E+003/3.6696E+002(<)	2.6743E+003/3.5165E+002(<)	2.6652E+003/3.6903E+002(<)	2.5976E+003/3.3402E+002(>)	2.6137E+003/3.6526E+002
f_{24}	2.0005E+002/6.4769E−002(<)	2.0003E+002/5.0797E−002(<)	2.0003E+002/6.7212E−002(<)	2.0001E+002/2.0381E−002(≈)	2.0001E+002/2.2122E−002(≈)	2.0000E+002/4.2784E−003(>)	2.0001E+002/1.3555E−002
f_{25}	2.2652E+002/2.3646E+001(<)	2.1722E+002/2.1877E+001(>)	2,2250E+002/2.3472E+001(<)	2.2204E+002/2.2240E+001(<)	2.2402E+002/2.3265E+001(<)	2.2372E+002/2.3054E+001(<)	2.2050E+002/2.3163E+001
f_{26}	2.0196E+002/1.4003E+001(<)	2.0000E+002/1.7629E−013(≈)	2.0000E+002/1.6450E−013(≈)	2.0000E+002/1.4171E−013(≈)	2.0000E+002/1.3895E−013(≈)	2.0000E+002/1.3895E−013(≈)	2.0000E+002/1.8900E−013
f_{27}	3.0167E+002/2.2766E+000(<)	3.0124E+002/2.6402E+000(<)	3.0085E+002/1.9373E+000(<)	3.0068E+002/1.5881E+000(<)	3.0013E+002/2.6694E−001(>)	3.0022E+002/5.0552E−001(>)	3.0042E+002/8.8539E−001
f_{28}	3.0000E+002/0(≈)	3.0000E+002/0A(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l:	10/3/15	8/5/15	9/6/13	10/6/12	7/7/14	7/6/15	-/-/-

external archive recording inferior solutions in JADE, many following state-of-the-art DE variants also employed the same mutation strategy with external archive that recorded inferior solutions. Table 7 presents the recommended factors r^{arc} of these DE variants, and we can see that all these DE variants mainly employed a small r^{arc} setting. Therefore, all these archive factor settings in Table 7 and the recommended setting, $r^{arc} = 5$, of the external archive that records inferior solutions are also incorporated into our experiment for the second concern. The comparison results are listed in Table 8, and we can see that the external archive recording historical population of the evolution secures the best performance under the CEC2013 test suite on 30-D optimization in comparison with the external archive recording inferior solutions under recommended r^{arc} settings of several state-of-the-art DE variants. By the way, if the external archive in the novel Hip-DE algorithm records the inferior solutions, the mutation strategy in Hip-DE algorithm degrades to the mutation strategy of LSHADE shown in Eq. (4).

6.4. Population size in the Hip-DE algorithm

The initial population size and its associated reduction scheme also play very important role in the overall performance of a DE variant, because a large population size in each generation usually associates with a better understanding of the landscape of the objective, nevertheless, a large population size in each generation also means less number of generations when employing a fixed maximum number of function evaluation. The novel Hip-DE algorithm employs a similar population size reduction scheme in comparison with the other state-of-the-art DE variant, therefore, we just need to tune the initial pop-



(a) A general view of the three reduction scheme. (b) Zoom in to the beginning of the evolution. (c) Zoom in to the terminal of the evolution.

Fig. 6. Comparison of three population size reduction schemes.

Table 10
Experiment results obtained by the novel Hip-DE algorithm employing three different population size reduction schemes.

Scheme NO.	Linear Mean/Std	Parabolic Mean/Std	Platform-based Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0
f_2	7.4900E−013/1.3354E−012(<)	5.7958E−014/1.0008E−013(>)	4.5029E−013/4.9709E−013
f_3	1.3523E+000/4.6375E+000(<)	2.0776E−001/1.4652E+000(>)	2.6383E−001/1.1828E+000
f_4	5.3500E−014/9.7408E−014(<)	4.4583E−015/3.1839E−014(>)	4.0125E−014/8.7542E−014
f_5	1.1592E−013/1.5919E−014(<)	1.1369E−013/0(≈)	1.1369E−013/0
f_6	5.3141E−010/3.7623E−009(<)	1.2580E−006/8.9411E−006(<)	5.6478E−011/2.3019E−010
f_7	1.6984E−001/1.7067E−001(<)	7.9384E−002/1.1562E−001(>)	1.3893E−001/1.8867E−001
f_8	2.0661E+001/1.9116E−001(<)	2.0731E+001/1.6535E−001(<)	2.0626E+001/1.9230E−001
f_9	2.3407E+001/3.8388E+000(>)	2.4362E+001/2.9989E+000(<)	2.3786E+001/3.4641E+000
f_{10}	0/0(≈)	0/0(≈)	0/0
f_{11}	1.5604E−014/2.5620E−014(>)	6.9104E−014/2.3612E−014(<)	2.1177E−014/2.7756E−014
f_{12}	6.4360E+000/1.8285E+000(<)	7.6709E+000/2.5086E+000(<)	5.6900E+000/2.0596E+000
f_{13}	6.3469E+000/2.9458E+000(>)	7.8933E+000/3.7497E+000(<)	6.5288E+000/3.3158E+000
f_{14}	8.4093E−002/3.4073E−002(>)	3.6101E−001/1.6002E−001(<)	8.4502E−002/3.8706E−002
f_{15}	2.7117E+003/3.4739E+002(<)	3.1300E+003/4.0443E+002(<)	2.7111E+003/3.9463E+002
f_{16}	3.3379E−001/2.6299E−001(<)	4.1665E−001/3.8885E−001(<)	2.8301E−001/2.4921E−001
f_{17}	3.0434E+001/1.3202E−006(≈)	3.0434E+001/6.0815E−004(≈)	3.0434E+001/9.4299E−007
f_{18}	4.7300E+001/6.5535E+000(<)	6.0812E+001/1.0119E+001(<)	4.6117E+001/6.7980E+000
f_{19}	1.2695E+000/1.8459E−001(>)	1.5829E+000/2.2271E−001(<)	1.3100E+000/1.5268E−001
f_{20}	9.0326E+000/3.7259E−001(<)	9.1556E+000/4.2491E−001(<)	8.9569E+000/4.7118E−001
f_{21}	2.9301E+002/3.6909E+001(<)	2.9216E+002/2.7152E+001(<)	2.8627E+002/3.4754E+001
f_{22}	1.0602E+002/5.2063E−001(>)	1.0785E+002/1.9112E+000(<)	1.0603E+002/6.5855E−001
f_{23}	2.5496E+003/3.4890E+002(>)	2.9331E+003/4.2554E+002(<)	2.6137E+003/3.6526E+002
f_{24}	2.0001E+002/3.2559E−002(≈)	2.0000E+002/8.7686E−003(>)	2.0001E+002/1.3555E−002
f_{25}	2.2615E+002/2.2377E+001(<)	2.2014E+002/2.1758E+001(>)	2.2050E+002/2.3163E+001
f_{26}	2.0196E+002/1.4004E+001(<)	2.0000E+002/1.7845E−013(≈)	2.0000E+002/1.8900E−013
f_{27}	3.0024E+002/5.2993E−001(>)	3.0012E+002/4.6009E−001(>)	3.0042E+002/8.8539E−001
f_{28}	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l:	8/5/15	7/6/15	–/–/–

Table 11
Group number comparison in the novel Hip-DE algorithm.

K NO.	$K=1$ Mean/Std	$K=2$ Mean/Std	$K=3$ Mean/Std	$K=4$ Mean/Std	$K=5$ Mean/Std	$K=7$ Mean/Std	$K=8$ Mean/Std	$K=6$ Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	5.0825E−013/ 8.3677E−013 (<)	6.1525E−013/ 1.5200E−012 (<)	2.9425E−013/ 2.0486E−013 (>)	3.8787E−013/ 4.5085E−013 (>)	7.8020E−013/ 2.4679E−012 (<)	2.4788E−012/ 1.3831E−011 (<)	2.9732E−011/ 1.8834E−010 (<)	4.5029E−013/ 4.9709E−013
f_3	1.0919E−001/ 4.4401E−001 (>)	3.3491E+000/ 1.6176E+001 (<)	2.2026E−001/ 7.5069E−001 (>)	8.4558E−002/ 3.0849E−001 (>)	3.5281E+000/ 2.3924E+001 (<)	6.3435E+000/ 4.5184E+001 (<)	7.5710E−001/ 4.2375E+000 (<)	2.6383E−001/ 1.1828E+000
f_4	2.6750E−014/ 7.3986E−014 (>)	4.4583E−014/ 9.1172E−014 (<)	3.1208E−014/ 7.9022E−014 (>)	3.1208E−014/ 7.9022E−014 (>)	4.9041E−014/ 9.4449E−014 (<)	4.0125E−014/ 8.7542E−014 (≈)	5.3500E−014/ 9.7408E−014 (<)	4.0125E−014/ 8.7542E−014
f_5	1.1369E−013/ 0(≈)	1.1369E−013/ 0(≈)	1.1592E−013/ 1.5919E−014 (<)	1.1369E−013/ 0(≈)	1.1369E−013/ 0(≈)	1.1592E−013/ 1.5919E−014 (<)	1.1592E−013/ 1.5919E−014 (<)	1.1369E−013/ 0
f_6	1.3894E−011/ 5.8158E−011 (>)	3.8475E−012/ 1.8499E−011 (>)	1.4044E−012/ 7.4211E−012 (>)	2.8338E−010/ 1.9333E−009 (<)	2.8591E−010/ 1.6561E−009 (<)	2.7796E−008/ 1.9502E−007 (<)	1.0020E−011/ 6.4924E−011 (>)	5.6478E−011/ 2.3019E−010
f_7	1.5795E−001/ 1.9341E−001 (<)	1.8090E−001/ 2.0783E−001 (<)	1.2752E−001/ 1.7152E−001 (>)	1.3430E−001/ 1.6530E−001 (>)	1.3377E−001/ 1.9402E−001 (>)	1.3662E−001/ 1.6362E−001 (>)	1.2492E−001/ 1.7494E−001 (>)	1.3893E−001/ 1.8867E−001
f_8	2.0695E+001/ 1.9960E−001 (<)	2.0642E+001/ 1.9406E−001 (<)	2.0667E+001/ 1.8624E−001 (<)	2.0683E+001/ 1.8492E−001 (<)	2.0668E+001/ 1.8919E−001 (<)	2.0712E+001/ 1.8735E−001 (<)	2.0745E+001/ 1.5239E−001 (<)	2.0626E+001/ 1.9230E−001
f_9	2.1704E+001/ 4.0147E+000 (>)	2.3065E+001/ 3.3232E+000 (>)	2.4097E+001/ 2.6099E+000 (<)	2.4309E+001/ 2.3292E+000 (<)	2.3812E+001/ 3.2735E+000 (<)	2.2454E+001/ 5.0288E+000 (>)	2.2442E+001/ 4.7970E+000 (>)	2.3786E+001/ 3.4641E+000
f_{10}	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	1.4502E−004/ 1.0357E−003 (<)	0/0(≈)	0/0(≈)	0/0
f_{11}	5.7958E−014/ 7.9597E−015 (<)	4.4583E−014/ 2.6207E−014 (<)	2.8979E−014/ 2.8699E−014 (<)	2.4521E−014/ 2.8433E−014 (<)	2.4521E−014/ 2.8433E−014 (<)	2.6750E−014/ 2.8655E−014 (<)	3.1208E−014/ 2.8566E−014 (<)	2.1177E−014/ 2.7756E−014
f_{12}	7.8231E+000/ 2.3714E+000 (<)	7.9208E+000/ 2.3957E+000 (<)	6.9492E+000/ 2.1883E+000 (<)	6.3133E+000/ 1.7526E+000 (<)	6.5991E+000/ 1.6847E+000 (<)	6.5200E+000/ 2.0292E+000 (<)	5.9253E+000/ 1.6913E+000 (<)	5.6900E+000/ 2.0596E+000
f_{13}	8.4199E+000/ 3.9206E+000 (<)	7.5824E+000/ 3.5349E+000 (<)	6.8354E+000/ 3.0138E+000 (<)	6.7983E+000/ 3.9766E+000 (<)	6.0613E+000/ 2.5741E+000 (>)	5.7804E+000/ 2.7008E+000 (>)	6.1496E+000/ 2.4829E+000 (>)	6.5288E+000/ 3.3158E+000
f_{14}	2.8668E+000/ 4.5242E+000 (<)	8.6403E−001/ 2/2273E+000 (<)	1.6482E−001/ 4.1809E−001 (<)	8.0197E−002/ 3.8100E−002 (>)	9.4299E−002/ 3.2615E−002 (<)	8.9809E−002/ 3.3178E−002 (<)	1.0001E−001/ 3.7479E−002 (<)	8.4502E−002/ 3.8706E−002
f_{15}	2.7976E+003/ 3.7139E+002 (<)	2.6935E+003/ 3.4370E+002 (>)	2.6836E+003/ 3.7473E+002 (>)	2.6771E+003/ 3.7065E+002 (>)	2.6351E+003/ 4.0036E+002 (>)	2.8196E+003/ 4.4202E+002 (<)	2.9499E+003/ 4.3384E+002 (<)	2.7111E+003/ 3.9463E+002
f_{16}	2.1447E−001/ 1.1778E−001 (>)	2.4285E−001/ 2.6017E−001 (<)	2.6116E−001/ 2.1423E−001 (>)	2.3077E−001/ 1.7019E−001 (>)	2.9486E−001/ 2.4411E−001 (<)	3.7551E−001/ 3.3393E−001 (<)	3.7624E−001/ 2.1259E−001 (<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0448E+001/ 3.5222E−002 (<)	3.0434E+001/ 1/3202E−006 (≈)	3.0434E+001/ 1.3202E−006 (≈)	3.0434E+001/ 9.4299E−007 (≈)	3.0434E+001/ 9.4299E−007 (≈)	3.0434E+001/ 1.8285E−006 (≈)	3.0434E+001/ 1.6003E−006 (≈)	3.0434E+001/ 9.4299E−007
f_{18}	4.3188E+001/ 5.7817E+000 (>)	4.2458E+001/ 4.4826E+000 (>)	4.4000E+001/ 7.6389E+000 (>)	4.3834E+001/ 6.4714E+000 (>)	4.4544E+001/ 6.4353E+000 (>)	4.5810E+001/ 7.3321E+000 (>)	4.4777E+001/ 7.7776E+000 (>)	4.6117E+001/ 6.7980E+000
f_{19}	1.7088E+000/ 3.2223E−001 (<)	1.5350E+000/ 2.7307E−001 (<)	1.4110E+000/ 3.1367E−001 (<)	1.4664E+000/ 2.4176E−001 (<)	1.3358E+000/ 1.8314E−001 (<)	1.3098E+000/ 1.7137E−001 (>)	1.3504E+000/ 1.6046E−001 (<)	1.3100E+000/ 1.5268E−001
f_{20}	8.9832E+000/ 4.7054E−001 (<)	8.8823E+000/ 3.5605E−001 (>)	8.9067E+000/ 3.5433E−001 (>)	8.9810E+000/ 3.6113E−001 (<)	8.8717E+000/ 3.7898E−001 (>)	8.9554E+001/ 4.1745E−001 (>)	8.9427E+000/ 4.6919E−001 (>)	8.9569E+000/ 4.7118E−001
f_{21}	3.0000E+002/ 0(<)	2.9804E+002/ 1.4003E+001 (<)	2.9216E+002/ 2.7152E+001 (<)	3.0085E+002/ 2.4726E+001 (<)	2.9804E+002/ 1.4003E+001 (<)	2.9889E+002/ 2.8475E+001 (<)	2.9889E+002/ 2.8475E+001 (<)	2.8627E+002/ 3.4754E+001
f_{22}	1.1361E+002/ 5.1506E+000 (<)	1.0769E+002/ 5.3992E+000 (<)	1.0611E+002/ 7.6549E−001 (<)	1.0624E+002/ 1.3184E+000 (<)	1.0594E+002/ 5.4579E−001 (>)	1.0608E+002/ 8.5049E−001 (<)	1.0641E+002/ 1.4899E+000 (<)	1.0603E+002/ 6.5855E−001
f_{23}	2.7706E+003/ 4.3143E+002 (<)	2.7518E+003/ 4.0537E+002 (<)	2.6867E+003/ 3.5703E+002 (<)	2.4940E+003/ 2.9647E+002 (>)	2.6069E+003/ 3.7971E+002 (>)	2.6593E+003/ 4.1448E+002 (<)	2.7127E+003/ 4.1506E+002 (<)	2.6137E+003/ 3.6526E+002

(*continued on next page*)

Table 11 (*continued*)

K NO.	$K=1$ Mean/Std	$K=2$ Mean/Std	$K=3$ Mean/Std	$K=4$ Mean/Std	$K=5$ Mean/Std	$K=7$ Mean/Std	$K=8$ Mean/Std	$K=6$ Mean/Std
f_{24}	2.0001E+002/ 4.0715E−002 (≈)	2.0001E+002/ 2.6242E−002 (≈)	2.0001E+002/ 2.0133E−002 (≈)	2.0001E+002/ 1.9443E−002 (≈)	2.0001E+002/ 2.3506E−002 (≈)	2.0001E+002/ 9.6064E−003 (≈)	2.0000E+002/ 7.1143E−003 (>)	2.0001E+002/ 1.3555E−002
f_{25}	2.2084E+002/ 2.2526E+001 (<)	2.2561E+002/ 2.2869E+001 (<)	2.1827E+002/ 2.1304E+001 (>)	2.1784E+002/ 2.1731E+001 (>)	2.1979E+002/ 2.2254E+001 (>)	2.2297E+002/ 2.2046E+001 (<)	2.2045E+002/ 2.2997E+001 (>)	2.2050E+002/ 2.3163E+001
f_{26}	2.0000E+002/ 1.4883E−013 (≈)	2.0000E+002/ 1.4262E−013 (≈)	2.0000E+002/ 1.4883E−013 (≈)	2.0000E+002/ 1.4708E−013 (≈)	2.0000E+002/ 1.4262E−013 (≈)	2.0196E+002/ 1.4005E+001 (<)	2.0000E+002/ 1.4171E−013 (≈)	2.0000E+002/ 1.8900E−013
f_{27}	3.0038E+002/ 7.1313E−001 (>)	3.0039E+002/ 7.1890E−001 (>)	3.0030E+002/ 5.5509E−001 (>)	3.0097E+002/ 2.6263E+000 (<)	3.0032E+002/ 7.9605E−001 (>)	3.0052E+002/ 9.7008E−001 (<)	3.0029E+002/ 6.1587E−001 (>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000E+002/ 0(≈)	3.0000E+002/ 0(≈)	3.0000E+002/ 0(≈)	3.0000E+002/ 0(≈)	3.0000E+002/ 0(≈)	3.0000E+000/ 0(≈)	3.0000E+002/ 0(≈)	3.0000E+002/ 0
w/d/l:	7/6/15	6/7/15	11/6/11	10/7/11	9/6/13	6/6/16	9/5/14	-/-/-

ulation size in a near range of the recommended settings of these DE variants. Moreover, our novel Hip-DE proposes a platform based linear reduction of population size which aims to balance the exploration (perception of the landscape of a benchmark in each generation) and exploitation (number of generations), therefore, a relative smaller initial population size is necessary in comparison with the initial values of other state-of-the-art DE variants. That's why we examine the seven different cases from $PS = 11 \cdot D$ to $PS = 17 \cdot D$ (with the interval equalling to D) in which $PS = 14 \cdot D$ and $PS = 15 \cdot D$ are relative smaller than $PS = 23 \cdot \ln D \cdot \sqrt{D}$ and $PS = 25 \cdot \ln D \cdot \sqrt{D}$, both of which are the recommended settings in LPlamDE and jSO. Table 9 presents the experiment results of the novel Hip-DE algorithm employing different initial population sizes, we can see that the recommended setting $PS = 15 \cdot D$ is a good choice for the novel Hip-DE algorithm.

In order to verify the adaptive scheme of population size in the novel Hip-DE algorithm, we also present the comparison among these schemes including the linear population size reduction scheme [38], the parabolic population size reduction scheme [25] and the novel platform based step-decrease scheme. All these different population size reduction schemes employed same initial population size and terminal population size, moreover, the maximum number of function evaluation is also the same, $nfe_{\max} = 10000 \cdot D$, in the comparison. Fig. 6 presents both a general view and a zoom in view of these three reduction schemes, and Table 10 presents the results obtained by the three reduction schemes. Again, we can see that the recommended setting, the platform based step-decrease scheme of population size, is a good choice for the novel Hip-DE algorithm.

As we all know that the terminal population size in the contrasted state-of-the-art DE variants with a population size reduction scheme all employed a very small value, e.g. $PS_{\min} = 4$ in LSHADE, jSO and HARD-DE, $PS_{\min} = 6$ in LPalmDE, and usually the optimization performances were not significantly different when employing such nearby settings of terminal population size. Nevertheless, the recommended setting in the novel Hip-DE of terminal population size is $PS_{\min} = K$, where K denotes the number of groups during the evolution. As the grouping strategy is very important in our Hip-DE algorithm, so is the parameter setting of K. In the Hip-DE, the recommended setting of K is $K = 6$, the same as the one in LPalmDE because Hip-DE can be considered as a further extension of LPlamDE. Besides this, we still conduct experiment to analyze the proper setting of K, and the results are summarized in Table 11.

From the table we can see that the recommended setting $K = 6$ is a good choice in the novel Hip-DE algorithm.

6.5. The ratio r^p of the platform-based evolution to the whole evolution

In the novel Hip-DE algorithm, the whole evolution is divided into two parts, the first part is platform-based evolution with fixed population size, and the second part is the population size reduction evolution during which the population size is adaptively changed according to the predefined step-decrease scheme. Therefore, a new parameter r^p defining the ratio of the first stage of the evolution to the whole evolution is introduced into the novel Hip-DE algorithm. Actually, when $r^p = 1$, the Hip-DE algorithm is degraded to an improved JADE algorithm employing fixed population during the whole evolution; when $r^p = 0$, the Hip-DE algorithm is degraded into an improved LSHADE with linear population size reduction during the whole evolution. Moreover, the LSHADE algorithm outperforms the JADE algorithm under the commonly used benchmarks, therefore, r^p should not be a large value in the range $[0, 1]$. We take $r^p = \frac{1}{40}, r^p = \frac{1}{30}, r^p = \frac{1}{20}, r^p = \frac{1}{10}, r^p = \frac{1}{5}, r^p = \frac{1}{2}$ into consideration, and the experiment results are summarized in Table 12.

We can see that when r^p equals to $\frac{1}{20}$, the Hip-DE algorithm can obtain an overall better performance. Then we can suppose that the number of generations N in the platform stage of the evolution is around $r^p \cdot nfe_{\max}/PS$ ($N \approx 33.3$). We also conduct a further experiment under different settings of N from $N = 31$ to $N = 36$ with the interval equalling to 1. The experiment results are summarized in Table 13, and we can see that $N = 34$ is a good choice.

Table 12
Comparison of the ratio of the platform stage to the whole evolution on 30-D optimization.

r^p NO.	$\frac{1}{40}$ Mean/Std	$\frac{1}{30}$ Mean/Std	$\frac{1}{10}$ Mean/Std	$\frac{1}{5}$ Mean/Std	$\frac{1}{2}$ Mean/Std	$\frac{1}{20}$ Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	7.2670E−013/ 1.6727E−012(<)	4.7704E−013/ 7.9841E−013(<)	2.9915E−011/ 2.1175E−010(<)	1.9617E−013/ 1.6409E−013(>)	7.5791E−014/ 1.0825E−013(>)	4.5029E−013/ 4.9709E−013
f_3	4.1548E−002/ 1.6682E−001(>)	1.1090E+001/ 6.8902E+001(<)	4.3320E+000/ 2.4577E+001(<)	1.7415E−001/ 8.0513E−001(>)	3.2002E−003/ 1.9755E−002(>)	2.6383E−001/ 1.1828E+000
f_4	3.1208E−014/ 7.9022E−014(>)	5.3500E−014/9/ 7408E−014(<)	2.2292E−014/ 6.8286E−014(>)	1.3375E−014/ 5.4032E−014(>)	0/0(>)	4.0125E−014/ 8.7542E−014
f_5	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1592E−013/ 1.5919E−014(<)	1.1592E−013/ 1.5919E−014(<)	1.1369E−013/0(≈)	1.1369E−013/0
f_6	6.7298E−012/ 2.4511E−011(>)	3.4374E−008/ 2.4547E−007(<)	4.4408E−008/ 3.1710E−007(<)	1.9298E−009/ 1.1040E−008(<)	4.6474E−006/ 2.5537E−005(<)	5.6478E−011/ 2.3019E−010
f_7	1.4850E−001/ 1.7898E−001(<)	1.1083E−001/ 1.5377E−001(>)	1.5210E−001/ 1.6638E−001(<)	7.6503E−002/ 1.0456E−001(<)	8.1976E−002/ 9.5232E−002(>)	1.3893E−001/ 1.8867E−001
f_8	2.0695E+001/ 1.7189E−001(<)	2.0673E+001/ 1.6465E−001(<)	2.0717E+001/ 1.8050E−001(<)	2.0723E+001/ 1.6976E−001(<)	2.0761E+001/ 1.4344E−001(<)	2.0626E+001/ 1.9230E−001
f_9	2.4278E+001/ 2.6695E+000(<)	2.4453E+001/ 2.7591E+000(<)	2.3475E+001/ 4.5981E+000(>)	2.4506E+001/ 2.6947E+000(<)	2.3950E+001/ 4.3282E+000(<)	2.3786E+001/ 3.4641E+000
f_{10}	1.4502E−004/ 1.0357E−003(<)	1.4502E−004/ 1.0357E−003(<)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_{11}	1.1848E−014/ 2.7063E−014(>)	2.5635E−014/ 2.8566E−014(<)	3.3437E−014/ 2.8254E−014(<)	5.1271E−014/ 1.7072E−014(<)	1.0254E−013/ 5.2122E−014(<)	2.1177E−014/ 2.7756E−014
f_{12}	6.2324E+000/ 2.0259E+000(<)	5.8470E+000/ 1.7391E+000(<)	6.4064E+000/ 1.9726E+000(<)	6.6511E+000/ 2.0345E+000(<)	7.7624E+000/ 2.3289E+000(<)	5.6900E+000/ 2.0596E+000
f_{13}	6.3506E+000/ 3.2369E+000(>)	6.9565E+000/ 3.2936E+000(<)	6.1152E+000/ 2.6641E+000(>)	5.7358E+000/ 3.2196E+000(>)	7.3361E+000/ 2.8590E+000(<)	6.5288E+000/ 3.3158E+000
f_{14}	8.0011E−002/ 3.2620E−002(>)	9.0217E−002/ 3.3224E−002(<)	1.0556E−001/ 3.7081E−002(<)	1.1544E−001/ 4.1800E−002(<)	6.1899E−001/ 5.5947E−001(<)	8.4502E−002/ 3.8706E−002
f_{15}	2.7290E+003/ 3.8862E+002(<)	2.6570E+003/ 3.7066E+002(>)	2.7713E+003/ 3.1899E+002(<)	2.7931E+003/ 3.8860E+002(<)	3.0867E+003/ 4.5276E+002(<)	2.7111E+003/ 3.9463E+002
f_{16}	3.2569E−001/ 2.3875E−001(<)	3.3635E−001/ 2.7359E−001(<)	3.4356E−001/ 2.6707E−001(<)	4.0211E−001/ 3.7649E−001(<)	4.6119E−001/ 3.5938E−001(<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0434E+001/ 1.3004E−013(≈)	3.0434E+001/ 1.6003E−006(≈)	3.0434E+001/ 1.8285E−006(≈)	3.0434E+001/ 1.5875E−004(≈)	3.0435E+001/ 2.6064E−003(<)	3.0434E+001/ 9.4299E−007
f_{18}	4.5050E+001/ 5.1579E+000(>)	4.4622E+001/ 7.5392E+000(>)	4.8887E+001/ 7.9904E+000(<)	5.1863E+001/ 1.0034E+001(<)	6.1935E+001/ 1.0778E+001(<)	4.6117E+001/ 6.7980E+000
f_{19}	1.3228E+000/ 1.6330E−001(<)	1.2606E+000/ 1.9835E−001(>)	1.3454E+000/ 1.9827E−001(<)	1.3737E+000/ 1.9995E−001(<)	1.6520E+000/ 2.4656E−001(<)	1.3100E+000/ 1.5268E−001
f_{20}	8.9490E+000/ 3.8635E−001(>)	9.0174E+000/ 4.0175E−001(<)	8.9378E+000/ 4.1264E−001(>)	8.9055E+000/ 3.8895E−001(>)	9.1581E+000/ 3.6532E−001(<)	8.9569E+000/ 4.7118E−001
f_{21}	2.9412E+002/ 2.3764E+001(<)	2.9301E+002/ 3.6909E+001(<)	2.9608E+002/ 1.9604E+001(<)	2.9693E+002/ 3.1662E+001(<)	3.0171E+002/ 3.4946E+001(<)	2.8627E+002/ 3.4754E+001
f_{22}	1.0615E+002/ 7.4123E−001(<)	1.0612E+002/ 1.1425E+000(<)	1.0643E+002/ 1.5062E+000(<)	1.0654E+002/ 1.5554E+000(<)	1.0987E+002/ 2.5814E+000(<)	1.0603E+002/ 6.5855E−001
f_{23}	2.6158E+003/ 3.6500E+002(<)	2.5697E+003/ 4.6593E+002(>)	2.6928E+003/ 3.4077E+002(<)	2.6500E+003/ 3.0635E+002(<)	3.1223E+003/ 4.4790E+002(<)	2.6137E+003/ 3.6526E+002
f_{24}	2.0001E+002/ 2.2824E−002(≈)	2.0001E+002/ 3.3472E−002(≈)	2.0001E+002/ 1.6074E−002(≈)	2.0000E+002/ 5.0560E−003(>)	2.0000E+002/ 3.1195E−003(>)	2.0001E+002/ 1.3555E−002
f_{25}	2.1749E+002/ 2.2279E+001(>)	2.1745E+002/ 2.2119E+001(>)	2.2515E+002/ 2.3311E+001(>)	2.2576E+002/ 2.1957E+001(<)	2.2336E+002/ 2.1588E+001(<)	2.2050E+002/ 2.3163E+001
f_{26}	2.0000E+002/ 1.4969E−013(≈)	2.0000E+002/ 1.4969E−013(≈)	2.OOOOE+002/ 1.4171E−013(≈)	2.0000E+002/ 9.4905E−006(≈)	2.0000E+002/ 1.8274E−013(≈)	2.0000E+002/ 1.8900E−013
f_{27}	3.0045E+002/ 9.8924E−001(<)	3.0081E+002/ 1.3911E+000(<)	3.0025E+002/ 6.4270E−001(>)	3.0049E+002/ 2.0325E+000(<)	3.0010E+002/ 4.3350E−001(>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000e + 002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l:	9/6/13	6/6/16	6/6/16	6/5/17	6/5/17	-/-/-

6.6. Analysis of the values of parameters τ, and c

In the adaptation schemes of control parameters μ_F and μ_{CR}, τ_1 and τ_2 proposed in jDE are also incorporated into the novel Hip-DE algorithm, and they are set the same value, $\tau_1 = \tau_2 = \tau$, which are the same as jDE. Instead of employing a random value in updating the control parameters F and CR in jDE, the novel Hip-DE algorithm employs Cauchy and Gaussian distribution in updating F and CR respectively. τ here denotes the probabilities when choosing the corresponding Cauchy/Gaussian distribution to update F and CR. Obviously, if τ equals to $0, F$ and CR are fixed constant during the whole evolution; if τ equals to 1, the adaptation schemes of control parameters degraded to the ones in HARD-DE. Due to the excellent perfor-

Table 13
Examine the number of generations in the platform stage of the evolution on 30-D optimization.

N NO.	$N = 31$ Mean/Std	$N = 32$ Mean/Std	$N = 33$ Mean/Std	$N = 35$ Mean/Std	$N = 36$ Mean/Std	$N = 34$
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	5.7066E−013/	3.3437E−013/	2.0062E−012/	3.2100E−013/	1.1280E−012/	4.5029E−013/
	1.3863E−012(<)	2.2897E−013(>)	1.2153E−011(<)	2.1415E−013(>)	4.6277E−012(<)	4.9709E−013
f_3	6.6159E−001/	1.9292E−002/	5.8456E−001/	1.5724E+000/	6.2332E+001/	2.6383E−001/
	2.8211E+000(<)	5.5982E−002(>)	2.0221E+000(<)	7.2228E+000(<)	4.4317E+002(<)	1.1828E+000
f_4	3.5666E−014/	4.9041E−014/	3.5666E−014/	3.5666E−014/	4.9041E−014/	4.0125E−014/
	8.3512E−014(>)	9.4449E−014(<)	8.3512E−014(>)	8.3512E−014(>)	9.4449E−014(<)	8.7542E−014
f_5	1.1815E−013/	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1592E−013/	1.1592E−013/	1.1369E−013/0
	2.2287E−014(<)			1.5919E−014(<)	1.5919E−014(<)	
f_6	7.1868E−012/	4.0854E−007/	3.4738E−010/	1.6112E−011/	4.1329E−010/	5.6478E−011/
	3.2950E−011(>)	2.9176E−006(<)	2.4436E−009(<)	9.8497E−011(>)	2.9217E−009(<)	2.3019E−010
f_7	1.4097E−001/	8.5204E−002/	1.3620E−001/	1.1231E−001/	1.5162E−001/	1.3893E−001/
	1.6286E−001(<)	1.4209E−001(>)	1.3940E−001(>)	1.4860E−001(>)	1.8452E−001(<)	1.8867E−001
f_8	2.0700E+001/	2.0674E+001/	2.0685E+001/	2.0662E+001/	2.0664E+001/	2.0626E+001/
	1.8710E−001(<)	1.6121E−001(<)	1.8384E−001(<)	1.9037E−001(<)	1.6446E−001(<)	1.9230E−001
f_9	2.4549E+001/	2.4574E+001/	2.3824E+001/	2.4651E+001/	2.4314E+001/	2.3786E+001/
	3.2031E+000(<)	3.8895E+000(<)	3.6054E+000(<)	2.8278E+000(<)	3.1214E+000(<)	3.4641E+000
f_{10}	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_{11}	2.1177E−014/	2.8979E−014/	2.2292E−014/	3.0094E−014/	2.6750E−014/	2.1177E−014/
	2.7756E−014(≈)	2.8699E−014(<)	2.8029E−014(<)	2.8655E−014(<)	2.8655E−014(<)	2.7756E−014
f_{12}	5.8270E+000/	5.9417E+000/	6.1065E+000/	6.5069E+000/	6.1898E+000/	5.6900E+000/
	1.7135E+000(>)	2.0281E+000(<)	2.1547E+000(<)	1.8977E+000(<)	1.9247E+000(<)	2.0596E+000
f_{13}	6.2983E+000/	6.0812E+000/	5.9974E+000/	6.6550E+000/	6.1881E+000/	6.5288E+000/
	3.0700E+000(>)	2.3792E+000(>)	2.3808E+000(>)	3.3429E+000(<)	3.4493E+000(>)	3.3158E+000
f_{14}	8.5318E−002/	8.5726E−002/	8.2869E−002/	8.7359E−002/	8.1242E−002/	8.4502E−002/
	3.7764E−002(<)	3.7854E−002(<)	3.3953E−002(>)	3.3575E−002(<)	3.4408E−002(>)	3.8706E−002
f_{15}	2.6824E+003/	2.8053E+003/	2.7019E+003/	2.6539E+003/	2.7291E+003/	2.7111E+003/
	3.5820E+002(>)	4.0927E+002(<)	3.7602E+002(>)	3.8469E+002(>)	3.4010E+002(<)	3.9463E+002
f_{16}	3.7690E−001/	2.8598E−001/	3.7738E−001/	3.3082E−001/	3.2632E−001/	2.8301E−001/
	3.0460E−001(<)	2.5028E−001(<)	3.2161E−001(<)	2.5084E−001(<)	2.6357E−001(<)	2.4921E−001
f_{17}	3.0434E+001/	3.0434E+001/	3.0434E+001/	3.0434E+001/	3.0434E+001/	3.0434E+001/
	7.3127E−014(≈)	6.4872E−005(≈)	1.3202E−006(≈)	1.3202E−006(≈)	1.3202E−006(≈)	9.4299E−007
f_{18}	4.6064E+001/	4.6929E+001/	4.6790E+001/	4.7305E+001/	4.5022E+001/	4.6117E+001/
	6.4892E+000(>)	7.3513E+000(<)	6.6126E+000(<)	7.2933E+000(<)	6.1837E+000(>)	6.7980E+000
f_{19}	1.2958E+000/	1.3272E+000/	1.3152E+000/	1.3597E+000/	1.3129E+000/	1.3100E+000/
	1.7053E−001(>)	1.6207E−001(<)	1.5821E−001(<)	2.0783E−001(<)	1.9021E−001(<)	1.5268E−001
f_{20}	8.9237E+000/	8.9216E+000/	8.9335E+000/	8.9798E+000/	8.9414E+000/	8.9569E+000/
	4.2328E−001(>)	4.3769E−001(>)	3.8198E−001(>)	4.3282E−001(<)	3.8892E−001(>)	4.7118E−001
f_{21}	2.9889E+002/	2.9608E+002/	2.9497E+002/	2.9412E+002/	2.9889E+002/	2.8627E+002/
	2.8475E+001(<)	1.9604E+001(<)	3.4443E+001(<)	2.3764E+001(<)	2.8475E+001(<)	3.4754E+001
f_{22}	1.0623E+002/	1.0593E+002/	1.0622E+002/	1.0618E+002/	1.0619E+002/	1.0603E+002/
	1.2344E+000(<)	5.1698E−001(>)	1.1250E+000(<)	1.1441E+000(<)	9.0841E−001(<)	6.5855E−001
f_{23}	2.5889E+003/	2.6110E+003/	2.6789E+003/	2.5738E+003/	2.6411E+003/	2.6137E+003/
	4.0638E+002(>)	4.1464E+002(>)	4.1430E+002(<)	3.4141E+002(>)	3.2605E+002(<)	3.6526E+002
f_{24}	2.0001E+002/	2.0001E+002/	2.0001E+002/	2.0001E+002/	2.0001E+002/	2.0001E+002/
	1.4911E−002(≈)	2.5794E−002(≈)	3.6932E−002(≈)	1.2077E−002(≈)	2.3634E−002(≈)	1.3555E−002
f_{25}	2.2892E+002/	2.2686E+002/	2.2347E+002/	2.1859E+002/	2.2252E+002/	2.2050E+002/
	2.1904E+001(<)	2.2174E+001(<)	2.3588E+001(<)	2.1756E+001(>)	2.2659E+001(<)	2.3163E+001
f_{26}	2.0000E+002/	2.0000E+002/	2.0000E+002/	2.0000E+002/	2.000E+002/	2.0000E+002/
	1.3987E−013(≈)	1.4080E−013(≈)	1.3987E−013(≈)	1.4796E−013(≈)	1.4080E−013(≈)	1.8900E−013
f_{27}	3.0054E+002/	3.0062E+002/	3.0080E+002/	3.0040E+002/	3.0047E+002/	3.0042E+002/
	1.3747E+000(<)	1.5327E+000(<)	1.9602E+000(<)	6.4476E−001(>)	8.3138E−001(<)	8.8539E−001
f_{28}	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l:	9/7/12	7/7/14	6/7/15	8/6/14	4/6/18	-/-/-

mance of HARD-DE algorithm, τ should be set a relative large value in the range $[0, 1]$. We also arrange 7 cases to test the assumption, $\tau = 0, 0.1, 0.3, 0.5, 0.7, 0.9, 1$, and from the summarized results in Table 14 we can see that $\tau = 0.9$ is a good choice.

For the balance parameter c, it is used for shifting the μ_F from the current location to a new one, $mean_{WL}(\boldsymbol{S_F})$. The balance parameter c is recommended to be set $c = 0.1$ in JADE, and the DE variants mentioned in this paper can all be considered as further extension of JADE. Therefore, we also recommend to employ the same setting of c in the novel Hip-DE algorithm. Furthermore, we also arrange 7 cases of different settings of the balance parameter, $c = 0, 0.1, 0.3, 0.5, 0.7, 0.9, 1$, in this part, and summarize the optimization results in Table 15. We can see from the results that $c = 0.1$ is a good choice in our Hip-DE algorithm.

Table 14
Examine the probability τ on 30-D optimization under CEC2013 test suite.

τ No.	0 Mean/Std	0.1 Mean/Std	0.3 Mean/Std	0.5 Mean/Std	0.7 Mean/Std	1 Mean/Std	0.9 Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	2.0811E+003/2.4662E+003(<)	1.0675E+001/6.3500E+001(<)	4.1145E−004/2.0769E−004(<)	8.3950E−004/5.9901E−003(<)	5.6175E−012/3.7909E−011(<)	5.0031E−010/2.7182E−009(<)	4.5029E−013/4.9709E−013
f_3	4.9041E−014/9.4449E−014(>)	1.4275E+005/1.0169E+006(<)	2.4810E+002/1.5471E+003(<)	5.9125E+001/3.4143E+002(<)	1.3748E−001/7.7314E−001(>)	5.3273E+001/2.5950E+002(<)	2.6383E−001/1.1828E+000
f_4	4.5939E−002/4.7564E−002(<)	4.0125E−014/8.7542E−014(≈)	1.7833E−014/6.1737E−014(>)	2.7650E−014/7.3986E−014(>)	6.6875E−014/1.0463E−013(<)	2.6750E−014/7.3986E−014(>)	4.0125E−014/8.7542E−014
f_5	6.6875E−015/2.7016E−014(>)	1.5381E−013/5.4870E−014(<)	1.2706E−013/3.6993E−014(<)	1.2260E−013/3.0869E−014(<)	1.1369E−013/0(≈)	1.1369E−013/0(≈)	1.1369E−013/0
f_6	3.4421E−007/2.0458E−006(<)	8.3564E−002/1.8312E−001(<)	8.4850E−003/4.3401E−002(<)	1.0798E−004/7.5987E−004(<)	1.5537E−012/7.6617E−012(>)	8.8412E−007/4.1852E−006(<)	5.6478E−011/2.3019E−010
f_7	1.9053E−004/8.7993E−004(>)	1.4113E−001/3.6514E−001(<)	2.3085E−001/2.4304E−001(<)	2.5864E−001/2.6814E−001(<)	5.9214E−002/7.5396E−002(>)	1.7542E−001/1.9431E−001(<)	1.3893E−001/1.8867E−001
f_8	2.0951E+001/5.1288E−002(<)	2.0899E+001/8.4318E−002(<)	2.0801E+001/1.4026E−001(<)	2.0726E+001/1.2582E−001(<)	2.0656E+001/1.8219E−001(<)	2.0715E+001/1.5414E−001(<)	2.0626E+001/1.9230E−001
f_9	4.8063E+000/1.5594E+000(>)	9.7258E+000/3.6033E+000(>)	1.7365E+001/7.1422E+000(>)	2.4135E+001/4.6013E+000(<)	2.3122E+001/3.8176E+000(>)	2.5354E+001/2.9921E+000(<)	2.3786E+001/3.4641E+000
f_{10}	0/0(≈)	1.7882E−002/3.7884E−003(<)	1.3536E−003/2.9715E−003(<)	1.4502E−004/1.0357E−003(<)	0/0(≈)	4.8374E−004/1.9738E−003(<)	0/0
f_{11}	1.6189E+001/6.8321E+000(<)	3.1214E−001/6.1302E−001(<)	2.8979E−014/2.8699E−014(<)	1.1146E−014/2.2793E−014(>)	3.3437E−014/2.8254E−014(<)	1.7833E−014/2.6638E−014(>)	2.1177E−014/2.7756E−014
f_{12}	9.2621E+001/3.2545E+001(<)	6.8262E+000/2.2237E+000(<)	5.6717E+000/1.9138E+000(>)	5.6088E+000/1.4918E+000(>)	6.5751E+000/1.6899E+000(<)	5.6361E+000/1.5913E+000(>)	5.6900E+000/2.0596E+000
f_{13}	1.0447E+002/3.1199E+001(<)	5.9899E+000/3.6039E+000(>)	5.2965E+000/1.7536E+000(>)	5.6569E+000/2.5744E+000(>)	6.8667E+000/3.6898E+000(<)	5.5288E+000/2.8959E+000(>)	6.5288E+000/3.3158E+000
f_{14}	2.6476E+003/6.8504E+002(<)	2.1827E+001/1.0914E+002(<)	3.9597E−002/2.4370E−002(>)	4.1230E−002/2.5153E−002(>)	1.0736E−001/3.7791E−002(<)	5.8376E−002/3.2526E−002(>)	8.4502E−002/3.8706E−002
f_{15}	6.3646E+003/5.4538E+002(<)	2.8462E+003/3.5987E+002(<)	2.6718E+003/2.8720E+002(>)	2.7443E+003/2.6811E+002(<)	2.8958E+003/4.9986E+002(<)	2.6554E+003/2.9214E+002(>)	2.7111E+003/3.9463E+002
f_{16}	2.4425E+000/2.8004E−001(<)	1.4398E+000/5.9907E−001(<)	6.1892E−001/2.8139E−001(<)	3.9761E−001/2.2509E−001(<)	2.4532E−001/2.7830E−001(>)	3.9579E−001/2.9966E−001(<)	2.8301E−001/2.4921E−001
f_{17}	6.3129E+001/1.5933E+001(<)	3.0519E+001/6.0794E−001(<)	3.0434E+001/9.4299E−007(≈)	3.0434E+001/1.3202E−006(≈)	3.0434E+001/1.5875E−004(≈)	3.0434E+001/9.4299E−007(≈)	3.0434E+001/9.4299E−007
f_{18}	1.7301E+002/1.1744E+001(<)	4.8897E+001/5.1105E+000(<)	4.6733E+001/3.5821E+000(<)	4.6591E+001/3.8762E+000(<)	4.6847E+001/8.0821E+000(<)	4.7018E+001/6.1373E+000(<)	4.6117E+001/6.7980E+000
f_{19}	4.1322E+000/1.1775E+000(<)	2.1194E+000/4.7063E−001(<)	1.5056E+000/1.4236E−001(<)	1.3326E+000/1.5240E−001(<)	1.3507E+000/2.4518E−001(<)	1.2721E+000/2.0214E−001(>)	1.3100E+000/1.5268E−001
f_{20}	1.0744E+001/3.4332E−001(<)	8.9445E+000/3.6254E−001(>)	8.9328E+000/3.2480E−001(>)	8.9755E+000/4.0788E−001(<)	8.9540E+000/4.6928E−001(>)	8.9341E+000/3.7439E−001(>)	8.9569E+000/4.7118E−001
f_{21}	2.9216E+002/2.7152E+001(<)	3.0085E+002/2.4726E+001(<)	2.9804E+002/1.4003E+001(<)	2.9889E+002/2.8475E+001(<)	2.9216E+002/2.7152E+001(<)	2.9301E+002/3.6909E+001(<)	2.8627E+002/3.4754E+001
f_{22}	1.9534E+003/5.2678E+002(<)	1.1726E+002/7.3808E+001(<)	1.0578E+002/5.1174E−001(>)	1.0570E+002/5.9969E−001(>)	1.0610E+002/8.7966E−001(<)	1.0631E+002/1.9397E+000(<)	1.0603E+002/6.5855E−001
f_{23}	5.9350E+003/9.1485E+002(<)	2.6070E+003/4.0916E+002(>)	2.5133E+003/3.8862E+002(>)	2.6004E+003/3.1595E+002(>)	2.7307E+003/4.2900E+002(<)	2.5705E+003/4.4978E+002(>)	2.6137E+003/3.6526E+002
f_{24}	2.0000E+002/9.8002E−005(>)	2.0001E+002/1.2702E−001(≈)	2.0001E+002/1.2659E−001(≈)	2.0001E+002/2.1611E−001(≈)	2.0000E+002/9.2180E−003(>)	2.0003E+002/6.0721E−002(<)	2.0001E+002/1.3555E−002
f_{25}	2.3156E+002/1.6156E+001(<)	2.3509E+002/1.8115E+001(<)	2.2611E+002/2.2403E+001(<)	2.3285E+002/2.1778E+001(<)	2.1541E+002/2.1218E+001(>)	2.2594E+002/2.2214E+001(<)	2.2050E+002/2.3163E+001
f_{26}	2.0196E+002/1.4003E+001(<)	2.0000E+002/2.2534E−005(≈)	2.0196E+002/1.4007E+001(<)	2.0000E+002/1.5809E−013(≈)	2.0000E+002/1.3895E−013(≈)	2.0196E+002/1.4004E+001(<)	2.0000E+002/1.8900E−013
f_{27}	3.0000E+002/8.8805E−004(>)	3.0108E+002/1.3848E+000(<)	3.0363E+002/5.4322E+000(<)	3.0263E+002/4.0164E+000(<)	3.0030E+002/8.1699E−001(>)	3.0096E+002/1.7428E+000(<)	3.0042E+002/8.8539E−001
f_{28}	3.0000E+002/2.3561E−013(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0(≈)	3.0000E+002/0
w/d/l:	6/3/19	4/5/19	9/4/15	7/5/16	9/3/13	9/4/15	-/-/-

6.7. Complexity analysis

For the space complexity, the biggest memory cost is the maintaining of external archive that records the historical populations. The memory size is $r^{arc} \cdot PS$, where r^{arc} equals 5 and PS denotes the population size which obeys a combined reduction scheme (fixed value $15 \cdot D$ at the early stage and then step-decreased to be 6 around the termination of the evolution) in our Hip-DE algorithm. In other word, the memory cost is the biggest, $r^{arc} \cdot PS$, at the earlier stage of the evolution and then decreases gradually to 6 around the termination of the evolution. To summarize, the space complexity of the novel Hip-DE algorithm is $\mathcal{O}(PS)$ at the beginning and $\mathcal{O}(1)$ around the termination of the evolution.

Table 15
Examine the balance parameter c on 30-D optimization under CEC2013 test suite.

c No.	0 Mean/Std	0.3 Mean/Std	0.5 Mean/Std	0.7 Mean/Std	0.9 Mean/Std	1 Mean/Std	0.1 Mean/Std
f_1	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0(≈)	0/0
f_2	4.7258E−013/ 3.3982E−013(<)	3.5666E−013/ 3.3496E−013(>)	8.2033E−013/ 3.2406E−012(<)	3.9679E−013/ 3.1124E−013(>)	4.1908E−013/ 5.3105E−013(>)	4.3146E−013/ 5.1097E−013(>)	4.5029E−013/ 4.9709E−013
f_3	6.5091E−013/ 1.1167E−012(>)	4.3438E−001/ 2.9382E+000(<)	2.2841E+000/ 1.6028E+001(<)	3.2438E+000/ 2.3046E+001(<)	9.9278E−001/ 4.8581E+000(<)	1.4085E−001/ 7.7427E−001(>)	2.6383E−001/ 1.1828E+000
f_4	1.9171E−013/ 9.5091E−014(<)	3.1208E−014/ 7.9022E−014(>)	3.5666E−014/ 8.3512E−014(>)	2.6750E−014/ 7.3986E−014(>)	1.7833E−014/ 6.1737E−014(>)	4.4583E−014/ 9.1172E−014(<)	4.0125E−014/ 8.7542E−014
f_5	8.4708E−014/ 5.0038E−014(>)	1.1369E−013/0 (≈)	1.1369E−013/0 (≈)	1.1369E−013/0 (≈)	1.1592E−013/1/ 5919E−014(<)	1.1369E−013/0 (≈)	1.1369E−013/0
f_6	1.9171E−013/ 1.1921E−013(>)	1.3910E−012/ 3.6243E−012(>)	3.8103E−010/ 2.6876E−009(<)	1.6101E−011/ 7.6807E−011(>)	3.5733E−011/ 1.4603E−010(>)	1.1770E−012/ 3.2215E−012(>)	5.6478E−011/ 2.3019E−010
f_7	1.2233E−003/ 3.2603E−003(>)	2.1990E−001/ 2.5943E−001(<)	1.8184E−001/ 2.0951E−001(<)	1.9190E−001/ 2.8737E−001(<)	1.7319E−001/2/ 0366E−001(<)	1.8187E−001/ 2.2053E−001(<)	1.3893E−001/ 1.8867E−001
f_8	2.0837E+001/ 1.0833E−001(<)	2.0641E+001/ 1.9766E−001(<)	2.0633E+001/ 1.9727E−001(<)	2.0670E+001/ 1.9979E−001(<)	2.0696E+001/ 1.9106E−001(<)	2.0628E+001/ 1.9708E−001(<)	2.0626E+001/ 1.9230E−001
f_9	1.5987E+001/ 6.7853E+000(>)	2.4956E+001/ 3.2210E+000(<)	2.5113E+001/ 2.5639E+000(<)	2.4717E+001/ 3.4933E+000(<)	2.5747E+001/ 3.0262E+000(<)	2.4592E+001/ 2.7742E+000(<)	2.3786E+001/ 3.4641E+000
f_{10}	0/0(≈)	4.8332E−004/ 1.9731E−003(<)	2.9004E−004/ 1.4499E−003(<)	3.8685E−004/ 1.9339E−003(<)	3.3845E−004/ 1.7098E−003(<)	2.9004E−004/ 1.4499E−003(<)	0/0
f_{11}	6.9104E−014/ 2.6207E−014(<)	1.8948E−014/ 2.7063E−014(>)	2.2292E−014/ 2.8029E−014(<)	1.8948E−014/ 2.7063E−014(>)	1.5604E−014/ 2.5620E−014(>)	2.3406E−014/ 2.8254E−014(<)	2.1177E−014/ 2.7756E−014
f_{12}	1.4653E+001/ 5.5751E+000(<)	6.2888E+000/ 1.8215E+000(<)	6.4306E+000/ 1.9042E+000(<)	6.7510E+000/ 1.9571E+000(<)	7.1273E+000/ 2.3697E+000(<)	6.3610E+000/ 1.9374E+000(<)	5.6900E+000/ 2.0596E+000
f_{13}	3.1680E+001/ 1.8341E+001(<)	6.2019E+000/ 3.0770E+000(>)	6.5569E+000/ 2.6802E+000(<)	6.4351E+000/ 2.7023E+000(>)	7.0410E+000/ 3.8203E+000(<)	7.1831E+000/ 3.5089E+000(<)	6.5288E+000/ 3.3158E+000
f_{14}	2.1562E+000/ 3.8988E+000(<)	8.8992E−002/ 3.7723E−002(<)	9.3483E−002/ 3.7096E−002(<)	8.4502E−002/ 3.2895E−002(≈)	9.1003E−002/ 3.6050E−002(<)	9.1441E−002/ 3.6318E−002(<)	8.4502E−002/ 3.8706E−002
f_{15}	3.2114E+003/ 5.4141E+002(<)	2.7126E+003/ 3.9447E+002(<)	2.7326E+003/ 2.9809E+002(<)	2.7076E+003/ 3.8997E+002(>)	2.7144E+003/ 3.6176E+002(<)	2.7687E+003/ 3.7926E+002(<)	2.7111E+003/ 3.9463E+002
f_{16}	6.2089E−001/ 2.9516E−001(<)	3.1918E−001/ 2.3637E−001(<)	3.7840E−001/ 2.9803E−001(<)	3.6984E−001/ 2.1813E−001(<)	3.3004E−001/ 2.3111E−001(<)	3.8081E−001/ 2.6373E−001(<)	2.8301E−001/ 2.4921E−001
f_{17}	3.0440E+001/ 6.7948E−003(<)	3.0434E+001/ 1.3202E−006(≈)	3.0434E+001/ 9.4299E−007(≈)	3.0434E+001/ 1.6003E−006(≈)	3.0434E+001/ 1.3202E−006(≈)	3.0434E+001/ 1.6003E−006(≈)	3.0434E+001/ 9.4299E−007
f_{18}	5.3933E+001/ 8.7976E+000(<)	4.6156E+001/ 7.0281E+000(<)	4.4767E+001/ 6.3357E+000(>)	4.6303E+001/ 7.6934E+000(<)	4.4787E+001/ 6.1720E+000(>)	4.5286E+001/ 7.0314E+000(>)	4.6117E+001/ 6.7980E+000
f_{19}	1.3460E+000/ 2.5909E−001(<)	1.3661E+000/ 2.1370E−001(<)	1.3460E+000/ 1.3962E−001(<)	1.3600E+000/ 1.8747E−001(<)	1.3220E+000/ 1.8121E−001(<)	1.3820E+000/1/ 7548E−001(<)	1.3100E+000/ 1.5268E−001
f_{20}	9.6246E+000/ 5.5579E−001(<)	8.9883E+000/ 3.6864E−001(<)	8.8721E+000/ 4.0375E−001(>)	8.9991E+000/ 4.1967E−001(<)	9.0371E+000/ 4.1126E−001(<)	9.0849E+000/ 4.1880E−001(<)	8.9569E+000/ 4.7118E−001
f_{21}	3.0734E+002/ 4.4649E+001(<)	2.9608E+002/ 1.9604E+001(<)	2.9889E+002/ 2.8475E+001(<)	3.0000E+002/0 (<)	3.0367E+002/ 3.1788E+001(<)	2.9216E+002/ 2.7152E+001(<)	2.8627E+002/ 3.4754E+001
f_{22}	1.1023E+002/ 2.7198E+000(<)	1.0627E+002/ 1.5086E+000(<)	1.0595E+002/ 6.9048E−001(>)	1.0606E+002/ 6.9636E−001(<)	1.0633E+002/ 1.4026E+000(<)	1.0627E+002/ 1.2673E+000(<)	1.0603E+002/ 6.5855E−001
f_{23}	3.2343E+003/ 5.6978E+002(<)	2.6488E+003/ 3.7629E+002(<)	2.7016E+003/ 4.0906E+002(<)	2.6678E+003/ 4.0328E+002(<)	2.6533E+003/ 4.0461E+002(<)	2.6426E+003/ 4.0341E+002(<)	2.6137E+003/ 3.6526E+002
f_{24}	2.0000E+002/ 3.5014E−003(>)	2.0001E+002/ 1.7421E−002(≈)	2.0001E+002/ 1.9454E−002(≈)	2.0000E+002/ 1.1063E−002(>)	2.0001E+002/ 1.4219E−002(≈)	2.0001E+002/ 2.2182E−002(≈)	2.0001E+002/ 1.3555E−002
f_{25}	2.1810E+002/ 2.0365E+001(>)	2.2181E+002/ 2.2720E+001(<)	2.1927E+002/ 2.2576E+001(>)	2.1508E+002/ 2.0779E+001(>)	2.2094E+002/ 2.2658E+001(<)	2.1871E+002/ 2.2861E+001(>)	2.2050E+002/ 2.3163E+001
f_{26}	2.0000E+002/ 1.8485E−013(≈)	2.0000E+002/ 1.4171E−013(≈)	2.0000E+002/ 1.3987E−013(≈)	2.0000E+002/ 1.3895E−013(≈)	2.0000E+002/ 1.4080E−013(≈)	2.0000E+002/ 1.4352E−013(≈)	2.0000E+002/ 1.8900E−013
f_{27}	3.0005E+002/ 1.0983E−001(>)	3.0039E+002/ 9.3458E−001(>)	3.0041E+002/ 1.2366E+000(>)	3.0045E+002/ 8.7200E−001(<)	3.0047E+002/ 1.0856E+000(<)	3.0035E+002/ 9.2823E−001(>)	3.0042E+002/ 8.8539E−001
f_{28}	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0 (≈)	3.0000E+002/0
w/d/l:	8/4/16	6/6/16	6/6/16	8/6/14	5/5/18	6/6/16	-/-/-

For the time complexity, we follows the instruction of the Congress on Evolutionary Computation Competition test suite: three variables T_0, T_1 and T_2 are employed in the evaluation, where T_0 denotes the time consumption of basic arithmetic expressions in CEC2013 test suite, T_1 denotes the time consumption of 200000 function evaluations for 30D optimization on benchmark function f_{14} from CEC2013 test suit, and T_2 denotes the overall cost of a certain algorithm optimizing f_{14}. As many as 51 independent runs are conducted to get the average values of T_0, T_1 and T_2. Then, $\frac{\overline{T}_2-\overline{T}_1}{\overline{T}_0}$ is representing the complexity evaluation of each algorithm. The time complexity comparisons among these DE variants including JADE, LSHADE, jSO, LPalmDE, HARD-DE and Hip-DE are presented in Table 16. We can see that the proposed Hip-DE algorithm is more time

Table 16
Time complexity comparison of these algorithms on 30*D* optimization under benchmark f_{14} from CEC2013 test suite for real-parameter single-objective optimization.

Algorithms.	$\overline{T}_0$	$\overline{T}_1$	$\overline{T_2}$	$\frac{\overline{T}_2-\overline{T}_1}{\overline{T}_0}$
JADE			1.5362	7.95
LSHADE	0.0993	0.7470	1.6428	9.02
jSO			1.6385	8.98
LPalmDE			3.2698	25.40
HARD-DE			3.3235	25.95
Hip-DE			1.9941	12.56

consuming than JADE, LSHADE and jSO, however, it is necessary because of the big improvement of the overall optimization performance.

7. Conclusion

As it is known to all that DE has been a well-known and powerful stochastic optimization algorithm for many years since its inception in 1995, and the overall performance of it was mainly dependent on the generation of proper trial vectors. There were mainly two components affecting the generation of trial vectors, one was mutation strategy and the other was recombination scheme. Here in the paper, we advanced a novel Hip-DE algorithm which was actually a further development of several former state-of-the-art DE variants including JADE, LSHADE, jSO, LPalmDE, and HARD-DE.

In this novel Hip-DE algorithm, there were several highlights that contributed to the overall excellent performance. The first was that a novel historical population based mutation strategy was proposed in the novel Hip-DE algorithm. The reason why we incorporated the historical population into the mutation strategy was that we found the locations of the population in the evolution, to some extent, could reflect the landscape of the objective, then the knowledge extracted from the historical populations could definitely guide the following evolution. The second was that a grouping strategy was advanced in the adaptation of control parameters, which secured a better and robust change of control parameters. The third was that a novel platform-based step-decrease scheme was advanced in the reduction of population size, which made a great balance between the exploration and exploitation of our Hip-DE algorithm. All these three highlights contributed to the excellent performance of the novel Hip-DE algorithm.

As an evolutionary algorithm, the excellent performance of the novel Hip-DE algorithm is also dependent on the values of the parameters, which is also the main characteristics of algorithms in computational intelligence. Generally, a good framework is much more robust, to some extent, less sensitive to the initial values of the parameters, and the algorithms employing the recommended values of the parameters is much more likely to obtain better results due to the stochastic characteristics. Here we take the parameter, crossover rate *CR*, as an example. Empirically, the *CR* value in the framework employing binomial crossover is less sensitive than that employing exponential crossover, in other words, the proper *CR* values in exponential crossover are in a much narrower range, nevertheless, a DE variant employing exponential crossover can also outperform these former state-of-the-art DE variants (including LSHADE and jSO) employing binomial crossover, but the proper parameter values and the corresponding adaptation schemes of these control parameters are hard to be found. We found some useful adaptation schemes for control parameters in exponential framework, which will be discussed in a future work.

Moreover, by reviewing the recent proposed state-of-the-art DE variants especially those that can outperform the winner DE variants at CEC competitions including the novel Hip-DE algorithm in this paper, it can be seen that they all employed fitness-value based parameter adaptation schemes. Generally, the incorporation of fitness difference in the adaptation of control parameters, a DE variant can generate more accurate control parameters which consequently lead to better optimization performance. However, there are the optimization cases that the exact fitness values are unavailable. In these cases, all these fitness-difference based DE variants are useless. This issue will be addressed and tackled in our next paper in the near future, and some of these attempts can be found in [21,20].

CRediT authorship contribution statement

Zhenyu Meng: Conceptualization, Methodology, Supervision, Writing - review & editing. **Cheng Yang:** Software, Writing - original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, athttps://doi.org/10.1016/j.ins.2021.01.031.

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