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Survey Paper

A review of opposition-based learning from 2005 to 2012

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ABSTRACT

Diverse forms of opposition are already existent virtually everywhere around us, and utilizing opposite numbers to accelerate an optimization method is a new idea. Since 2005, opposition-based learning is a fast growing research field in which a variety of new theoretical models and technical methods have been studied for dealing with complex and significant problems. As a result, an increasing number of works have thus proposed. This paper provides a survey on the state-of-the-art of research, reported in the specialized literature to date, related to this framework. This overview covers basic concepts, theoretical foundation, combinations with intelligent algorithms, and typical application fields. A number of challenges that can be undertaken to help move the field forward are discussed according to the current state of the opposition-based learning.

1. Introduction

This paper is inspired in part by the observation that opposites permeate everything around us, in some form or another. In the last 2500 years, its study has already attracted the attention of countless experts in the field. In a sense, the interplay between entities and opposite entities is fundamental for maintaining universal balance and harmony. Sometimes we unconsciously or consciously apply the opposition concept in our regular life. However, due to the lack of an accepted mathematical or computational model for opposition, until recently it has not been explicitly studied to any great length in fields outside of philosophy and logic (Tizhoosh and Ventresca, 2008).

Many machine intelligence algorithms inspired by different natural systems consider finding the solution of a given problem as function approximation. In many cases, the starting points are chosen randomly, such as weights of a neural network, initial population of soft computing algorithms, and action policy of reinforcement agents. If the starting point is close to the optimal solution, this results a faster convergence. On the other hand, if it is very far from the optimal solution, such as opposite location in worst case, the convergence will take much more time or even the solution can be intractable. Looking simultaneously for a better candidate solution in both current and opposite directions may help us to solve the present problem quickly and efficiently.

The basic concept of Opposition-Based Learning (OBL) was originally introduced by Tizhoosh (2005a). The main idea of this optimization is, for finding a better candidate solution, the simultaneous consideration of an estimate and its corresponding opposite estimate which is closer to the global optimum. In a very short period of time, it has been utilized in different soft computing areas. These efficient meta-heuristic methods mainly include Differential Evolution (DE), Particle Swarm Optimization (PSO), Reinforcement Learning (RL), Biogeography-Based Optimization (BBO), Artificial Neural Network (ANN), Harmony Search (HS), Ant Colony System (ACS) and Artificial Bee Colony (ABC). However, the comprehensive surveys published in the technical literature about opposition-based learning with other natural computation methods, especially in future trends and challenges, are relatively scarce (Al-Qunaieer et al., 2010a; Ergezer and Sikder, 2011; Imran et al., 2010). But they do not discuss further researches and challenges thoroughly, and several other approaches have arisen since the publication of those papers. The intention of the present work is to provide researchers an updated survey and the future research trends of theoretical and practical areas on OBL.

The review of the literature in this paper consists of 138 articles concerned with the theory and application of opposition-based learning. These papers are listed in the bibliography and are drawn from the period 2005–2012. The articles in this literature review have been from refereed journal articles and conferences proceedings from across a broad range of disciplines. Books (Rahnamayan, 2009; Tizhoosh and Ventresca, 2008) and dissertations (Malisia, 2007; Rahnamayan, 2007; Salama, 2007; Shokri, 2008) have generally not been included, although the tendency is to be inclusive when dealing with borderline cases. One of the major

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concerns here is that, these results and key contributions with rarely novel idea are usually the collection of previous results published in journal or conference. But it is undeniable that, as classics, two books authored by Rahnamayan and Tizhoosh have discussed the genuine beginning of thought, evolution and definition of the concept, variation of typical algorithms and their applications.

The remaining of this paper is organized as follows: some basic concepts and description of opposition-based learning are introduced briefly in Section 2. In Section 3, its mathematical analysis is provided which takes into consideration various performance metrics. Then, opposition-based computing with natural computation methods is summarized in Section 4, with special emphasis on the different ways of employing the opposition concept. In Section 5, a review of the applications of opposition-based learning in soft computing is conducted. Finally, the trends and challenges for further research are discussed in Section 6.

2. Basic concepts

2.1. The concept of opposition

The footprints of the opposition concept can be observed in many areas around us (Rahnamayan et al., 2012). This concept has sometimes been labeled with different names. Following are just some examples of opposition concept often mentioned in relevant research: opposite particles in physics, absolute or relative complement of an event in set theory, antithetic variables in simulation, antonyms in languages, opposite proverbs in culture, opposition parties in politics, subject and object in philosophy of science, theses and antitheses in dialectic and “Yin” and “Yang” in Chinese philosophy and Taoist religion.

Moreover, due to the omnipresence of opposition in the real world, regardless in what amount intensity and form we may encounter its diverse presence, the nature of entities and their opposite entities might be understood in different ways (Tizhoosh and Ventresca, 2008). A whole set of words are also invented to describe the diversity and complexity of oppositeness: antipodal, antithetical, contradictory, contrary, diametrical, polar, antipodean, adverse, disparate, negative, hostile, antagonistic, unalike, antipathetic, counter, converse, inverse, reverse, dissimilar, and divergent. All these words describe some notion of opposition and can be conveniently employed in different practical contexts to portray different relationships.

Therefore it seems that without using the opposition concept, the explanation of different entities in all cases will be very difficult, and maybe even impossible. When you are trying to explain an entity, a situation or an idea, it is sometimes easier to explain its opposite instead. In fact, opposition often manifests itself in a balance between completely different entities. For instance, the east, west, south and north cannot be defined alone, but only in terms of one another. The same is valid for many other objects, such as cold and hot, wet and dry. Imagination of the infinity is vague, but when we consider the limited, it then becomes more imaginable because its opposite is definable (Rahnamayan et al., 2012).

2.2. Opposition-based learning

In general, soft computing or, more generally, computational intelligence algorithms start from some initial solutions (initial population) and iteratively try to replace the current solutions by some better solutions toward some optimal solutions. In the absence of a priori information about the solution, starting with random guesses, generally with a uniform distribution on the entire range, is a common initialization. Many of the variables measured, such as computation time, memory usage and storage complexity, are related to the distance of these initial guesses from the optimal solution. If we simultaneously check a solution and its opposite solution, the closer (fitter) one (guess or opposite guess) can be chosen as an initial solution. In fact, according to probability theory, 50% of the time a guess is further from the solution than its opposite guess. Therefore, starting with the closer of the two guesses has the huge potential to accelerate convergence and improve the precision of the approximate methods. The same or similar approach can be applied not only to initial solutions, but also continuously to each solution in the current population as well (Rahnamayan et al., 2008b).

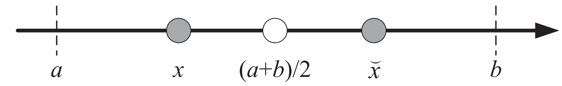

Fig. 1. Opposite point defined in domain $[a, b]$. x is a candidate solution and $\breve{x}$ is the opposite of x.

Table 1
Number of publications on opposition-based learning in the period 2005–2012.

Year	2005	2006	2007	2008	2009	2010	2011	2012*	Total
Journal	–	1	1	4	3	6	12	19	46
Conference	2	4	12	9	22	20	16	7	92
Total	2	5	13	13	25	26	28	26	138

* papers published in the print edition in 2013 and downloaded before January 2013 are also included.

In order to explain easier opposition-based learning, we need to define clearly the concept of opposite numbers. An opposition-based number can be defined as follows. Fig. 1 illustrates $\breve{x}$ (Rahnamayan et al., 2008b).

Definition 1. Let $x \in [a, b]$ be a real number. The opposition number $\breve{x}$ is defined by

$$\breve{x} = a + b - x \tag{1}$$

Similarly, the opposite point in D-dimensional space can be defined as follows.

Definition 2. Let $P=(x_1, x_2,\ldots, x_D)$ be a point in D-dimensional space, where $x_1, x_2, \ldots, x_D \in R$ and $x_i \in [a_i, b_i]$, $\forall i \in \{1, 2, \ldots, D\}$. The opposite point $\breve{P}=(\breve{x}_1, \breve{x}_2, \ldots, \breve{x}_D)$ is completely defined by its coordinates

$$\breve{x} = a_i + b_i - x_i \tag{2}$$

Now, by employing the definition of opposite point, the opposition-based optimization can be defined as follows.

Definition 3. Let $P=(x_1, x_2,\ldots, x_D)$ be a point in D-dimensional space (i.e., candidate solution). Assume $f(\cdot)$ is a fitness function which is used to measure the candidate's fitness. According to the definition of the opposite point, $\breve{P}=(\breve{x}_1, \breve{x}_2,\ldots, \breve{x}_D)$ is the opposite of $P=(x_1, x_2,\ldots, x_D)$. Now if $f(\breve{P}) \geq f(P)$, i.e., $\breve{P}$ has a better fitness than P, then point P can be replaced with $\breve{P}$; otherwise, we continue with P. Hence, the point and its opposite point are evaluated simultaneously to continue with the fitter one.

The varieties of opposition-based learning include Quasi-Opposition-Based Learning (QOBL) (Rahnamayan et al., 2007c), Quasi-Reflection Opposition-Based Learning (QROBL) (Ergezer et al., 2009), Center-based Sampling (Rahnamayan and Wang, 2009), Generalized Opposition-Based Learning (GOBL) (Wang et al., 2009a) and Opposition-Based Learning using the Current Optimum (COOBL) (Xu et al., 2011a). For further details please read the references listed above.

2.3. Literature review and analysis

Table 1 demonstrates the published items about opposition-based learning in the period mentioned. It can be observed that high number of researchers have interest in this research area. Several factors influenced this increase from 2 articles in 2005 to 26 articles in 2012. Firstly, there is the transferal effect of advances in meta-heuristic methods that is subsequently extended into opposition-based learning models. Secondly, there exists a growing awareness during the decade of the existence and importance of opposition-based learning in various disciplines.

Another interesting observation arising from Table 1 is the number of articles published in refereed journals as opposed to conferences proceedings. At the beginning, the overwhelming majority of results are published in conferences before 2010. The unexpected thing is that great changes have taken place from 2011, and even the number of articles published in refereed journals is firstly more than in conferences in 2012. This fully indicates that, the novel idea and algorithm published in conferences for its unusual rapidity and its tradition of promote innovation, especially in computer science fields. On the other hand, this shows, from one aspect, that the models and technologies of opposition-based learning are approved as independent academic discipline by the profession. Until now, it has already developed and established the distinct theoretical basis, algorithm framework and application cases when compared with other intelligent algorithms, such as GA, PSO, DE and ANN.

The status in using six main opposition-based learning and its varieties are shown as Fig. 2. Obviously, the strategy of OBL is the most popular for the simplicity of algorithm design and presentation clarity. Another reason is that, it is proposed at the earliest in this field and then easy to be recognized by the following researchers. Another interesting thing is that the strategy of GOBL is widely used as a learning tool and appeared 13 times from 2009 to 2012. It mainly owes a great deal to the hardworking and tenacity of the establisher of GOBL, Dr. Wang when he studied as a Ph.D. candidate at Wuhan University, China. He acts as the first author in six and co-authors in two, while only five articles are the others' contributions.

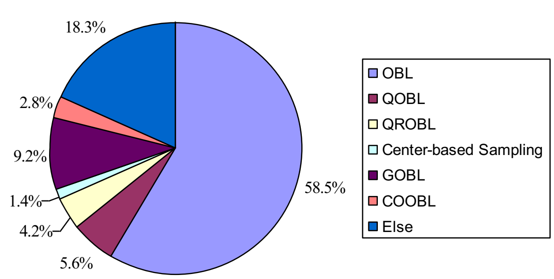

Fig. 2. Breakdown of articles by primary opposition-based learning model.

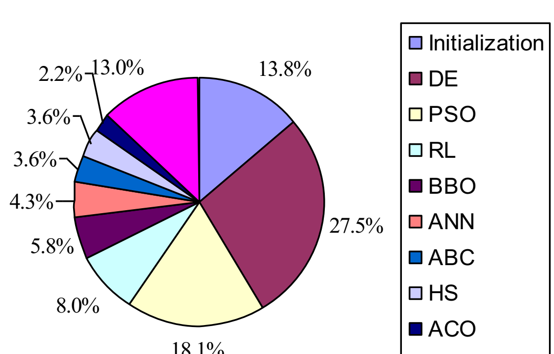

Fig. 3. Breakdown of articles by the primary meta-heuristic method.

Fig. 3 shows that 25% of the articles utilize differential evolution as the primary meta-heuristic, 18% particle swarm optimization, and 8% reinforcement learning. Simplicity and easy implementation are two main preferences of DE than other soft computing algorithms and a basic reason for widespread application of DE over several benchmark functions and real-world problems in recent years. The more important reason of its prevalence may be that, it is the meta-heuristic algorithm used in the pioneer and classic paper by Rahnamayan et al. (2006a, 2008b). In subsequent years, DE and its varieties are used as a basic meta-heuristic algorithm and then improved by others.

Besides 8 algorithms listed in Fig. 3, there are some other cases of the applications of this strategy to the soft computing algorithms. The meta-heuristic algorithms used in published literature so far include Genetic Algorithm (GA) (Lin and Wang, 2010; Tizhoosh, 2005a), Simulated Annealing (SA) (Ventresca and Tizhoosh, 2007b), Evolutionary Algorithm (EA) (Dong et al., 2009), Policy Gradient Reinforcement Learning (PGRL) (Kulk and Welsh, 2011), Estimation of Distribution Algorithm (EDA) (Gao et al., 2011b), Window Memoization (WM) (Khalvati et al., 2007), Bacterial Foraging Optimization Algorithm (BFOA) (Mai and Li, 2011), Gravitational Search Algorithm (GSA) (Shaw et al., 2012), Population-Based Incremental Learning (PBIL) (Ventresca and Tizhoosh, 2008a), Group Search Optimizer (GSO) (Wang et al., 2012a).

Also deserves special mention is that 19 papers, 13.8% of total literatures, only employs opposition-based learning to generate initial population. From the point of experiments design and writing motivation, these papers can be divided into three classes. The first kind of literatures (Dong et al., 2012; Gao and Liu, 2011, 2012; Gao et al., 2012a, 2012b; Rahnamayan and Tizhoosh, 2008) compared the algorithm performance in two situations: opposition-based population initialization and random initialization. The second ones (Ali et al., 2009a, 2009b, 2012; Bao and Zeng, 2011; Gao et al., 2011a; Iqbal et al., 2010; Peng et al., 2008; Sharma and Pant, 2011a, 2011b), did not design the similar experiments and only compared the performance of the novel algorithm inserted all improvement measures. As a result, we cannot differentiate the contribution of opposition-based learning to algorithm performance.

The unique goal of the other four papers is to test the population initialization based on opposition-based learning. In Rahnamayan et al. (2007a), the opposition-based population initialization was firstly embedded in the classical DE to increase convergence speed. However, Wang et al. (2009c) proved that the generalized opposition-based population initialization outperforms traditional random initialization and opposition-based population initialization. Besides comparing the performance of Opposition-based Particle Swarm Optimization (O-PSO) with other variations of standard PSO, Jabeen et al. (2009) also tested the performance with 2-PSO, in which it selects best N particles from a random population of size $2N$ for optimization problem. In Gutiérrez et al. (2011), three different initialization strategies: the orthogonal array initialization, the chaotic technique initialization and the opposition-based initialization, had been considered and appropriately combined with binary and real PSO algorithm in order to solve the complex and high dimensional optimization problems. The authors proved that the most reliable initialization technique for two kinds of complex electromagnetic problems is the chaotic initialization.

3. Mathematical foundation of opposition-based learning

Up to now, one of the most widely used techniques, from the perspective of theory analysis, is the mathematical comparison between opposition-based sampling and random sampling. Ventresca and Tizhoosh (2007b) provided a theoretical foundation for opposition-based learning.

Theorem 1. *For any evaluation function f: $S \to R$ over solution space S and opposite evaluation function Φ: $S \to R$, $\Phi(s)$ yields at worst case an equal expected value to f(s).*

Please note that the traditional random sampling does not lead to much improvement since the $E[r_1]=E[r_2]$ for purely random solutions r_1, r_2. An important consequence of this theorem is that, it is reasonable to expect a more desirable outcome and lower variance using OBL. We can also conclude that, for a given optimization problem many possible opposite maps may exist and directly influence the opposite evaluation function. Furthermore, the best one, which considers properly properties such as symmetry or other a priori knowledge of the evaluation function, will provide the lowest expected evaluation.

One popular conclusion often mentioned and cited in this group is the Central Opposition Theorem, which is proved mathematically by Rahnamayan et al. (2008a) and then revised by Ventresca et al. (2010).

Theorem 2. *Assume that (a) $y=f(x)$, ($x \in [a, b]$) is an unknown function with at least one solution $x_s \in [a, b]$ for $f(x_s)=\alpha$; the solution can be anywhere in our search space (i.e. a black-box optimization problem), (b) x is the first uniform random guess and x_r is a second uniform random guess in [a, b]; candidate solutions should be uniform random numbers because all points have the same chance to be the solution, (c) opposite of $x \in [a, b]$ is defined as $\breve{x}=a+b-x$. Then, $Pr(|\breve{x}-x_s|<\min\{|x_r-x_s|, |x-x_s|\})>Pr(|x_r-x_s|<\min\{|x-x_s|, |\breve{x}-x_s|\})$. In other words, the probability that the opposite point is closer to the solution is higher than a second random guess, assuming the original guess is not closer (i.e. $|x-x_s|>\min(|\breve{x}-x_s|, |x_r-x_s|)$).*

When the first uniform random guess x and its opposite point $\breve{x}$ are given, the solution x_s and the second random guess x_r can form 16 different ways/combinations in the domain $[a, b]$. In order to establish an exhaustive proof, the authors calculated the probabilities of the first random guess x, the second random guess x_r, and the opposite point $\breve{x}$ being the closest to the solution x_s among $\{x, x_r, \breve{x}\}$, for each event. The theorem establishes that, in the case of an unknown function, considering the pair x and $\breve{x}$ has apparently a higher fitness probability than additional independent random points. The authors concluded that, in general, the opposition-based learning can be utilized to accelerate learning and optimization methods. And then, the mathematical proof is experimentally verified, demonstrating the feasibility and potential of such new learning approach.

However, the aforementioned proposed proof suffers from two shortcomings. First of all, only a one dimensional search space is considered and employed to establish that theorem. Secondly, it is not able to provide an instinctive explanation or additional comment for the observed results in that paper. In order to address these incurable issues, a much simpler mathematical proof is presented for high-dimensional function, and the philosophy behind the opposition concept is explained intuitively from the perspective of Euclidean distance to the optimal solution (Rahnamayan et al., 2012). Additionally the main theorem statement is confirmed by some computational experiments with varying problem dimensionality, solution location and ensemble size of the sample.

The other alternative method of evaluating the performance of opposition-based learning, is that the mathematical comparison among different strategies of opposition-based learning. For a black-box optimization problem in which the location of the ideal global solution is unknown in advance, Rahnamayan et al. (2007c) confirmed that the quasi-opposite point has a higher chance to be closer to the global solution and a better convergence rate than the opposite point. The proof is for a one-dimensional space mathematically, but experimental results demonstrate that the conclusion is the same for the higher dimensions.

Furthermore, Ergezer et al. (2009) proved how much quasi-opposition is better than opposition. Assuming that the domain of objective function is symmetric and the solution (including estimated solution, opposite solution, quasi-opposite solution, and quasi-reflected solution) is the uniform distribution, the authors calculated the expected probabilities of different types of opposite points for single dimensional space in six cases. As a result, the most cost effective opposition method with the highest probability of being closer to the solution is to create a quasi-reflected population and choose the fittest individuals among the original and reflected populations. In addition, the analysis and conclusion can easily be extended to higher dimensions through simulation experiments and data analysis.

Rahnamayan and Wang (2009) confirmed that the center point has the higher probability to be closer to an unknown solution than the opposite point for higher dimensions. Unfortunately this better property was only confirmed by computer simulation and then further study is needed by possible analytical and computational methods.

And last, but certainly not least, some researchers have discussed opposition-based learning in virtue of some particular algorithms. As known to all, estimating and determining a target's location within the potential target area is a crucial and difficult task for search and navigation problems. Oppositional Target domain Estimation (OTE) algorithm was presented as a new strategy to reduce the state of the environment to the smaller area including the target, and increase the efficiency and applicability of the search agent (Shokri et al., 2009). The main idea of employing OTE for state-space reduction for grid environment is, if taking opposite actions in opposite states leads to the same evaluative feedbacks, and if at least for one action we receive reward, then the target is located between those states and the rest of the search space can be disregarded. This idea is established in the form of the theorem, called the OTE Theorem, and proved completely. The experimental results show that, OTE algorithm can offer a considerable state-space reduction and then be performed quite efficiently. Overall, it is convinced both theoretically and experimentally that, OTE algorithm benefits from the concept of opposition.

Just like the evaluation function, the diversity of population is a crucial indicator to analyze population-based soft computing algorithms. It is typically measured by the user-defined distance between all-possible-pairs of samples in the population. As a population-based stochastic search and optimization process, PBIL is analyzed from the perspective of the diversity of population (Ventresca and Tizhoosh, 2008a). The authors described how the concept of opposition can be employed to improve algorithm performance and provided mathematical proofs to this effect.

Theorem 3. *Given a probability matrix $\mathbf{M}:=(m_i)_n$, a D-dimensional binary space B^D and sets R_1; R_2; $\breve{R}_1 \subset B^D$ with $|R_1|=|R_2|=|\breve{R}_1|=k$, if $G_1=R_1 \cup R_2$ and $G_2=R_1 \cup \breve{R}_1$, then the diversity of binary-valued opposite set $V(G_2) \geq V(G_1)$.*

Corollary 1. *Given a probability matrix $\mathbf{M}:=(m_i)_n$, a D-dimensional binary space B^D and sets R_1; R_2; $\breve{R}_1 \subset B^D$ (where R_1 and R_2 are i.i.d. samples based on* **M***), if the values of M converge (i.e. $m_i=0$ or 1), then for $k=|R_1|=|R_2|$ samples,*

$$\lim_{V(R_1 \to R_2) \to 0} V(R_1-\breve{R}_1)-V(R_1-R_2)=l\frac{k(k+1)}{2}$$

where the notation $V(R_1 \to R_2) \to 0$ represents the convergence of diversity of guesses based on $\mathbf{M}^t$ as $t=0, \ldots, \infty$ (i.e. as the values of **M** *converge).*

The theorem and corollary tell us that, the opposite guessing strategy can indeed increase diversity of population and show more advantages than traditional random sampling. Wang et al. (2010b) also experimentally showed the diversity changes during the sampling stages in ODE, which can offer some new evidence to support this conclusion.

Numerical condition is one of the most fundamental and important concepts, which affects the speed and accuracy, in the study of artificial neural networks because ill-conditioning is a common cause of slow and inaccurate results from backpropagation-like algorithms. Ventresca and Tizhoosh (2008b, 2009) proved that opposite transfer functions are symmetrical transformations in weight space which yield unique input–output mappings, and for a given problem, each of these neural networks will produce the minimum error with an equal probability. Experimental results show that an opposite network may actually yield a lower error than a trained network with a large probability, and also impact the accuracy and convergence rate of most numerical algorithms.

4. Opposition-based computing with natural computation methods

The idea of opposition-based learning is applicable to a wide range of computation methods. Although the proposed scheme is embedded in a classical DE, in practice it is general enough to be applied to all soft computing algorithms.

4.1. Differential evolution

As pioneers, Rahnamayan et al. (2006a) for the first time utilized opposite numbers to speed up the convergence rate of evolutionary algorithms. Their proposed Opposition-based DE (ODE) employs the OBL for population initialization and also for generation jumping. For population initialization, this is performed by initializing a random population P(n) and calculating the corresponding opposite population OP(n). Then, the fittest individuals are selected from union set of P(n) and OP(n). Unlike opposition-based initialization, generation jumping calculates the opposite population dynamically. Instead of using variables' predefined interval boundaries, generation jumping calculates the opposite of each variable based on minimum and maximum values of that variable in the current population, and then the fittest individuals are selected in the same manner. An extensive experimentation was performed in Rahnamayan et al. (2008b) using 58 benchmark functions in order to test the performance of ODE. Different sets of experiments are conducted independently to study the effect of dimensionality, opposite points, population size, mutation strategies and jumping rates on the ODE algorithm. This is beyond dispute that, as the pioneer algorithm, ODE is the most classical opposition-based algorithm with a powerful influence. What deserves special mention is that, in most cases, as you will see following others have utilized the same opposition-based initialization and generation jumping schemes to introduce new opposition-based methods.

Ahandani and Alavi-Rad (2012) proposed four modified versions of DE which utilize the OBL strategy in the Shuffled Differential Evolution (SDE), in which population is divided into several memeplexes and each memeplex is improved by the DE algorithm. The first modified version is called Shuffled Opposition-Based DE (SOBDE), which employs the opposition-based generation jumping after each iteration of the evolutionary process for each memeplex. The second modified version is called Shuffled Extended Opposition-Based DE (SEOBDE), which applies the opposition-based generation jumping as an extra stage to evolve the members of each memeplex. The third modified version, called Opposition-Based SDE (OBSDE), employs the opposition-based generation jumping after a complete iteration of SDE on the current population. The final modified version, called Opposition-Based Shuffled Extended Opposition-Based DE (OB-SEOBDE), applies the opposition-based generation jumping as an extra stage to improve the current offspring and also employs another opposition-based generation jumping stage after a complete iteration of algorithm on all members of the current population. Similarly, two population-based stochastic metaheuristic algorithms, FSDE (Free Search Differential Evolution) and CODEQ (Chaotic search, OBL, DE, Quantum mechanics) were proposed for global optimization over continuous spaces (Omran, 2010; Omran and Engelbrecht, 2009; Omran and Salman, 2009). The former is a hybrid of concepts and ideas learned in free search, DE and OBL, while the latter integrates concepts and tools from chaotic search, quantum mechanics, DE and OBL.

ODE introduces a new parameter, called jumping rate, to control the probability of generating opposite population, which is kept the same during the evolutionary process. In order to achieve higher convergence rates, a time varying policy to set this control parameter was introduced by Rahnamayan et al. (2007b). The authors introduced two types of time varying jumping rates, namely, linearly increasing and decreasing functions. The former has lower jumping rate during exploration and higher jumping rate during exploitation, and vice-versa for the latter. Experiments show that the linearly decreasing jumping rate achieves better performance than both constant and linearly increasing jumping rates. Put simply, this means that a higher jumping rate is more desirable during the exploration stage than during the exploitation stage.

In general, choosing suitable control parameter values is very much a problem-dependent task and requires previous experience of the user. Brest et al. (2006) presented a novel mechanism to adjust the control parameters, such as mutation constant and crossover rate, by a self-adaptive approach based on random procedures. Then, a new hybrid DE algorithm, called SAODE (Self-Adapting Opposition-based DE), was proposed by combining OBL and self-adaptive control parameters with classical DE (Miao et al., 2009). Moreover, GOjDE (DE with self-adapting control parameters and Generalized Opposition-based learning) proposed by Wang et al. (2013) employs the same self-adapting control parameters and GOBL strategy. It is worth noting that, this algorithm is run on multiprocessors of GPU (Graphics Processing Units) in a parallel manner, which is helpful to accelerate the evolution process and to reduce the computational time simultaneously. Furthermore, in Yüzgeç (2010), another control parameter jumping rate is also adjusted in the same way. However, unlike the previous adjustment scheme on mutation constant and crossover rate, the jumping rate is only adjusted once at each generation, but it is not adjusted for each individual in the generation during the evolutionary process.

Generally speaking, the success of DE crucially depends on appropriately choosing trial vector generation strategies and their associated control parameters, significant efforts have been made to focus on these two issues because they obviously interact. Li (2012) proposed a new approach in attempt to enhance the ability of DE, in which different control parameter settings and mutation strategies are randomly combined to generate the trial vectors, and then opposition-based learning is introduced to select the better one as the offspring.

In the original ODE algorithm, the jumping rate value is a constant value, which is chosen based on the prior knowledge or observation of the user. For some benchmark functions, the opposition-based jumping is a wasted effort because there is extra computational cost in calculating and evaluating opposite individuals. Esmailzadeh and Rahnamayan (2011) also allowed the control parameter (jumping rate) to be a constant value for the entire search process just as in ODE. However, they intended to make generation jumping process of ODE protective, such that, when generation jumping is no longer helping the convergence, it is stopped for the rest of the search process to save the computation costs.

Recently, another strategy to apply the concept of opposition-based learning to DE is developed for handling global optimization tasks (Tang and Zhao, 2010; Xie and Yang, 2010). Opposition-based learning concept is applied on each individual instead of the whole population, which means that the opposite individual only competes with its parent individual with a probability, but not the individuals in the current and opposite population. If the opposite individual is better than the current individual in global optimization, then replace the current one with the opposite one; otherwise keep the current one unchangeable.

Rahnamayan et al. (2007c) enhanced the previous methods by replacing opposite points with quasi-opposite points. The authors proved that, for a black-box optimization problem a quasi-opposite point has a higher probability of being closer to the solution than an opposite point. The same procedure of population initialization and generation jumping that proposed in Rahnamayan et al. (2006a) is used, except that quasi-opposite points are used and a smaller jumping rate is employed. Experimental results clearly show that, QODE algorithm outperforms both ODE and classical DE in terms of number of function calls, success rate and success performance. Thus, both intuitively and statistically speaking, the quasi-opposite points are closer to the center-point compared to the opposite-points. By this way, QODE can be known as promising and reliable evidence to support center-based sampling theory presented by Rahnamayan and Wang (2009).

In Wang et al. (2009b), the random GOBL model was successfully applied to DE for population initialization and producing new candidates in the evolutionary generations. However, the embedded strategy of GODE is very different from ODE. In the ODE, the classical DE is performed for every generation, and the opposite candidates is produced and evaluated with a probability-based approach. But in the GODE, the GOBL model is performed, or the classical DE is performed. Furthermore, in Wang et al. (2010a), when the GOBL strategy is not executed, the chaotic operator, like a mutation operator, will be introduced to help the classical DE jump out from local optima and then find global optimum more efficiently. Recently, Iacca et al. (2011) adapted and tested the GOBL strategy to cDE frameworks using the same embedded strategy in Rahnamayan et al. (2006a). An important finding of this study is that, GOBL tends to enhance performance when handling non-separable functions while it tends to worsen performance in the case of separable functions.

When the midpoint of the range of function domain is not the global optimum, opposite points may keep away from the global optimum, which leads to decrease the contribution of opposite points and to the poor algorithm performance. A novel algorithm, COODE (Opposition-based Differential Evolution using the Current Optimum) was proposed for function optimization in Xu et al. (2011a, 2011b). In the COODE, the optimum in the current generation is dynamically served as the symmetry point between an estimate and its corresponding opposite estimate. The distance between opposite numbers and the global optimum is short enough to keep a high rate of opposition population usage during the evolution process, especially in the later stage. Later, the effects of function dimension and population size on the speedup of COODE were investigated completely (Xu et al., 2013).

4.2. Particle swarm optimization

The OPSO is introduced by utilizing OBL to enhance swarm initialization, generation jumping and also improving the swarm's best individual (Han and He, 2007). First, swarms are initialized with random positions and velocities. The opposite swarm is calculated by computing the opposite of position and velocity, and then the fittest of swarm and opposite swarm is selected as the next generation population. The same procedure is applied to current generations using jumping rate and dynamic constriction factor, which is used to enhance the convergence speed, in the calculation of opposite points.

An OBL scheme is applied to the PSO during the population initialization and improving current populations (Wang et al., 2007). In every generation, a dynamic Cauchy mutation is applied to the global best particle found by all particles so far, if newly created global best is better after application of mutation operator then global best is replaced. It helps to decrease the probability of being trapped in a local optimum. In addition, the effects of the probability of the opposition-based method used in OPSO are investigated carefully.

Wu et al. (2008) extended the previously proposed Comprehensive Learning Particle Swarm Optimization (CLPSO) to Opposition-based CLPSO (OCLPSO). Again opposition-based concept is used in population initialization and also exemplar selection. In opposition-based exemplar selection, two particles are firstly chosen from the population. The fitness of the selected particles and their opposites are compared, and then the best fitter particle among the four candidates, instead of two, is used as the exemplar to learn from that dimension.

An improved Opposition-based PSO (iOPSO) is presented and successfully applied to feed-forward neural network training (Rashid and Baig, 2010). The concept of opposition-based learning is utilized in population initialization, generation jumping as well as velocity calculation. Their three differences from the previous approaches are personal best fitness of particles instead of current fitness during the generation jumping, opposite velocity for opposite numbers selected during the generation jumping, and no local searching operator around the global best in the form of Cauchy mutation or difference offspring.

With the help of the basic concept of OBL, a new procedure for PSO, called opposition-based disturbance, was introduced and utilized to disturb particles position when the personal best position is updated (Chi and Cai, 2010). According to velocity update equation and experimental evidence, such operation will not only increase the population diversity, but also enhance the global search ability of PSO.

In all algorithms mentioned above, which used the OBL in the PSO algorithm, several difficult tuning parameters are added. Omran and Al-Sharhan (2008) used the OBL to improve the performance of original PSO without adding any extra parameter. Three variants of opposition-based PSO are investigated carefully, yet the ideas proposed in this paper can also be utilized with any PSO variant. The first variant (OPSO) uses opposition concepts just for population initialization as described in Rahnamayan et al. (2006a). The second variant (iOPSO) utilizes opposition for every iteration by replacing the particle with lowest strength of fitness by its opposite. The third variant (iPSO) is the same as iOPSO, but without opposition-based population initialization. Similarly, another interesting approach is OMPSO (PSO with Opposition Mutation) with the same tuning parameters as standard PSO (Chen and Li, 2008a, 2008b). The opposition-based mutation is adapted to accelerate the learning and searching process. In each iteration, some selected particles are suffered the mutation process, and then the best position of personal particle is replaced.

Kaucic (2013) presented an interesting multi-start PSO algorithm, which consists of three steps. In the initialization phase, the opposition-based learning is performed to improve the search efficiency. Then an adaptive velocity based on the differential operator is proposed to enhance some additional exploration capability of the algorithm. Finally, a restart technique based on two swarm diversity measures is acted in order to tackle the problem of premature convergence and stagnation in suboptimal regions. Meanwhile, the super-opposition paradigm is used to re-initialize particles in the swarm.

In Wang et al. (2009a), the GOBL model was firstly introduced and combined with PSO for optimization problem. Four typical GOBL models with different values of k are considered thoroughly (namely, GOBL-SS, GOBL-SI, OBL and GOBL-R). The comparison experimental results show that the GOBL-R model works better than the other three models in many benchmark function problems. Subsequently, Wang and Wu (2011) presented a method to accelerate the algorithm convergence rate by using Graphic Processing Unit in parallel.

Also, Wang et al. (2011b) presented an enhanced PSO algorithm, called GOPSO, which borrows the ideas of GOBL from Wang et al. (2009a) and Cauchy mutation from Wang et al. (2007). In addition, to solve large scale problems, the original GOPSO is modified by introducing a dynamic population mechanism, which can increase or decrease the number of particles in terms of the search status of the current population.

Two enhanced versions of opposition-based PSO, called EOPSO (Enhanced Opposition-based PSO) and QCLPSO (Quasi-oppositional Comprehensive Learning PSO) respectively, were proposed (Tang and Zhao, 2009; Zhang et al., 2009). Instead of calculating traditional opposition point, the both proposed algorithms calculate quasi-opposite particle, which is generated by uniform distribution from the interval between median and opposite position of the particle. When priori information is not available for any reason, the quasi-opposite particles may have more chances of being closer to global optimum than opposite particles.

In addition, the OBL strategy was applicable in some modified PSO algorithms (Munoz et al., 2011; Ning et al., 2009; Shahzad et al., 2009; Wang et al., 2011c; Yang and Xie, 2010; Yu et al., 2009). Generally speaking, prominent results have been achieved using the proposed modification to traditional PSO.

4.3. Reinforcement learning

An enhancement to reinforcement learning based on opposition based computation was proposed by Tizhoosh (2005a, 2005b). The simplest and most popular reinforcement algorithm, namely the Q-Learning (QL), is selected as the parent algorithm to demonstrate the opposition-based extension. The main idea of the algorithm is to consider actions and opposite actions and/or opposite states. This makes the traversal of the states of the environment shorter, which mean faster convergence time.

Consequently, Tizhoosh (2006) tested Opposition-based QL (OQL) by introducing three versions of that. The first one considers the opposite action and opposite reward for each taken action with reward. The second variant uses a second learning step, which is defined as a decreasing function of the number of episodes. The third algorithm considers the opposite actions only for a limited number of episodes. Algorithm 2 achieves the best results among the three proposed variants and is much better than original QL in term of convergence speed.

However, the above mentioned methods may still not be efficient enough for large problems with huge state-action spaces since it may be too expensive to find the global optimal solution with available computing resources. In order to prevent the "curse of dimensionality" problem, Wu et al. (2007) presented an active exploratory Q-Learning, which core principle is that the agent does not rush to the next state. Instead, the action that returns the greatest immediate reward is selected from a number of actions attempted at the current state firstly. And then, the state resulting from performing the action is considered as the next state. Moreover, four major active exploration algorithms are proposed for good actions: random search, opposition-based random search, cyclical parameter adjustment search, and opposition-based cyclical parameter adjustment search. Obviously, the previous methods proposed by Tizhoosh (2005a, 2005b, 2006) were integrated into random search.

Also in order to solve the large state space problem much efficiently, an Opposition-based $Q(\lambda)$ algorithm ($OQ(\lambda)$) is proposed by introducing the concept of opposition traces, which are eligibility traces for the actions and opposite actions (Shokri et al., 2006). The updating mechanism for opposite action is the same as mechanism used in (Tizhoosh, 2005a, 2005b), but the new algorithm punishes/rewards the eligible trace and opposite trace instead of the action and opposite action.

However, this approach cannot work with non-deterministic environments, since the next state of the opposite action cannot be detected or known by the agent. This issue was solved in Shokri et al. (2007) by introducing non-Markovian update of the opposition traces, in which a parameter $W \in [0, 1]$ is introduced. For some applications which have no clear definition of opposite action, the weight is set low at the initial stage and it is gradually increased as the agent explores the actions and the opposite actions. On the other hand, in problems which have clear definition of opposite action the weight is set to 1.

In order to find the tradeoff between exploration and exploitation in the processes of non-Markovian update of the opposite actions in a dynamic environment, Shokri et al. (2008) suggested an increasing function of the weight. Thus, the value of W is increasing as learning progresses and the number of steps increases. The rationale behind this is that, it has more positive effects on the Q-value updates for opposite actions in later stages. The simple case of the elevator control problem is selected for the experiments as a dynamic and non-deterministic environment.

Recently, Shokri (2011) carefully reviewed and extended some algorithms above mentioned. Finally, the author presented a number of challenges that can be undertaken to help move the field forward.

4.4. Biogeography-based optimization

Oppositional BBO (OBBO) was firstly proposed by Ergezer et al. (2009) by utilizing quasi-reflection opposition-based learning to accelerate the convergence speed. The opposite population is computed and evaluated during both population initialization and generation jumping operation as described in Rahnamayan et al. (2008b). The oppositional algorithm is further revised by the addition of dynamic domain scaling and weighted reflection.

In order to speed the optimization process, an Improved Opposition-Based Biogeography Optimization (IOBBO) method was proposed by Yang et al. (2011). The approach integrates three technologies: use of COOBL instead of QROBL, use of multiple-points to generate opposite points simultaneously, and divide the range of values into several areas to make initial population more evenly distributed.

Recently, Ergezer and Sikder (2011) presented the effects of four existing oppositional algorithms on continuous-domain optimization problems: OBL, QOBL, QROBL and Center-based Sampling. Furthermore, a new oppositional algorithm, fitness-ranking-based central opposition, is introduced for the first time, in which the reflection weight is utilized to produce the opposite point. Experiment results show that, the new algorithm has the highest success rate amongst all the oppositional algorithms.

4.5. Artificial neural network

As a preliminary attempt, two ways incorporating OBL strategies into ANN were firstly discussed by Tizhoosh (2005a): opposite weight, where a subset of weights are selected and set to their opposite, and opposite net, where all weights of the network are made opposite.

Ventresca and Tizhoosh (2006) investigated yet another definition of "opposite" where they consider opposite transfer functions for a subset of neurons. In their approach, the opposite transfer function of $f(x)$ is defined as $\breve{f}(x)=f(-x)$, and then the opposite network is defined as a network which has similar weights as the original network and at least one neuron having the opposite transfer function. This is implemented by a probabilistic method whereby the probability is affected by whether the network error is improved or not. Subsequently, by theoretical proofs and simulation experiments, the authors investigated the potential impact of opposite transfer functions on numerical condition, which affects the speed and accuracy of neural network learning algorithms (Ventresca and Tizhoosh, 2008b). Two situations are carefully examined: network initialization and early stages of the learning process.

Ventresca and Tizhoosh (2009) utilized the opposite networks (via the same opposite transfer functions as before) to improve learning time and accuracy for large scale neural networks that have thousands of parameters. During the learning process, a set of neurons selected based on a probabilistic rule are assigned the opposite transfer function, and then the formed network represents the opposite network. In each iteration, both current and opposite networks are evaluated and the network that have best results is labeled as the current network. In addition, the impacts of layer size and the number of layers are examined on the performance of the proposed method.

4.6. Artificial bee colony

This concept of OBL was first incorporated into ABC algorithm through two steps: opposition-based population initialization and opposition-based generation jumping (El-Abd, 2011). Subsequently, the OABC algorithm was further extended to GOABC employed the concept of generalized opposition-based learning scheme (El-Abd, 2012).

An improved ABC algorithm, called Opposition-based ABC with dynamic Cauchy mutation (OCABC), was presented in Yang and Huang (2012). The OCABC embeds two strategies into the standard ABC algorithm. The first strategy, opposition-based learning, simultaneously considers the current candidate solutions and its opposite to generate high quality candidate solutions. And the later, dynamic Cauchy mutation, dynamically adjusts the mutation size during the search process to improve the global search ability.

Numerous empirical studies have shown that, for ABC algorithm, its performance is sensitive to its parameter "limit" controlling the behavior of scour. The primary purpose of scours is to avoid falling into local optimal. Therefore, to achieve the same goal, Bi and Wang (2011a, 2011b) presented a mutation strategy based on opposition-based learning instead of the traditional mutation, which uses the method of generating randomly a position. Besides, a new select scheme (sense-pheromone model) for onlookers is adopted to choose a food source.

4.7. Harmony search

Gao et al. (2010, 2012c) developed and explored a hybridization of the HS and OBL, called HS–OBL, in which the OBL is used to enhance the mutation operation of the HS. A very large number experiment results demonstrate its superiority over the regular HS.

The OBL approach is employed to generate initial Harmony memory (HM) in the new ODHS (HS algorithm based on Opposition and DE) algorithm to obtain fitter initial candidate solutions (Zhao, 2010). Furthermore, harmony search, which based on three rules: memory considering, pitch adjusting and "opposites" selection, and differential evolution, which also based on three rules: selection, mutation and crossover, are alternately used to update HM.

To deal with the major deficiencies in the original HS, a Dynamic Regional HS (DRHS) algorithm with opposition and local learning was proposed in Qin and Forbes (2011). DRHS initializes one half of the HM randomly within the solution space with another half obtained using OBL, which makes candidate solutions to better cover the entire solution space. Moreover, an opposition-based restarting is invoked to reawaken any convergent groups. For each group, two harmonies are simultaneously generated by the original HS operators and by applying opposition-based learning. Among these newly harmonies, the one with better quality is used to update the group memory.

4.8. Ant colony optimization

Malisia and Tizhoosh (2007) employed the OBL ideas to improve the quality of solutions and convergence rate of an ant colony system (ACS). The authors proposed five variants for employing opposition concept to extend the solution construction phase of ACS, namely, Synchronous Opposition, Free Opposition, Free Quasi-Opposition, Opposite Pheromone per Node (OPN), and Opposite Pheromone per Edge (OPE). Results on the application of these algorithms on TSP instances demonstrate that, only OPN method shows performance improvements that are statistically significant, and the other extensions show no clear improvement and even worse than the normal ACS. Subsequently, Banerjee and Tizhoosh (2010) initiated a model to apply OPN algorithm in ACO, and obtained the similar results.

Ma et al. (2010) employed the concept of OBL for pheromone updating of ACO to accelerate the evolutionary process. The opposite pheromone value proposed in this work is defined together by the initial pheromone, the current pheromone and the length of the path ant take across.

5. Applications

Any problem that involves the identification of optimal cost (minimum or maximum) is called optimization problem, which is an important field in all science and engineering fields. Because of its importance, many different algorithms, called optimization algorithms, have been proposed to solve optimization problems efficiently. Perhaps the most extensive application of the ideas of OBL (and the variant of OBL) is in the optimization problems, including large-scale optimization problem (Ahandani and Alavi-Rad, 2012; Gao et al., 2010, 2012c; Mai and Li, 2011; Rahnamayan and Wang, 2008a, 2008b; Wang and Wu, 2011; Wang et al., 2009b, 2011a, 2011b, 2013) (even 1000 dimensions), constrained optimization problem (Kaucic, 2013; Omran, 2010; Omran and Salman, 2009), optimization of noisy problem (Han and He, 2007; Rahnamayan et al., 2006b), multi-objective optimization problem (Dong and Wang, 2009; Dong et al., 2009; Leung et al., 2012).

Typically, only an approximate solution is possible due to the large search space size for large vertices (assuming a large number of edges as well). Some Traveling Salesman Problem (TSP) instances composed of between 51 and 198 cities are selected from the TSPLIB (http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/) to examine the ability of OACS (Malisia and Tizhoosh, 2007) and OPBIL (Ventresca and Tizhoosh, 2008a), respectively. Ergezer and Simon (2011) attempted to apply opposition-based learning into soft computing algorithm for solving discrete and combinatorial optimization problems. Two different methods of opposition are introduced to solve two different types of combinatorial optimization problems. The first technique is for open graph problems, where the final node in the graph does not have been connected to the first node, such as the graph-coloring problem. The latter technique is for closed walk problems, where the endpoints of a graph are linked, such as the well-known symmetric TSP. Wherein the main difficulty of these algorithm extensions is that how to define and evaluate the opposite numbers in discrete domain.

As we all know, population-based soft computing algorithms contain some parameters which are sensitive to specific problems and difficult to be predefined by users. The OBL strategy is successfully employed to assist with the solving of parameter control problem in DE (Zhang and Yuen, 2012) and MultiObjective Evolutionary Algorithms (MOEAs) (Leung et al., 2012). The parameters and their opposite parameters are used at the same time to create trial vectors. During the whole evolution process, fitness improvement at a generation serves as a performance filter to detect and locate proper parameters for a specific optimization problem. The proper parameters and their opposite parameters are both stored in pools, whereas the improper parameters and their opposite parameters are replaced by new randomly generated ones. In this way, the proposed approach can efficiently balance the exploration and exploitation abilities at the same time in one generation. The effects of population size and pool size are also investigated.

Artificial neural networks are currently being used in a variety of applications with great success. One of the most important issues in the ANN applications is the architecture design, which can be formulated as an optimization problem, where each solution represents architecture. Large scale neural networks have many hundreds or thousands of adjustable parameters, which include the centers vectors, the widths of the basis functions and the parameters forms, and as a result tend to have very long training times for traditional learning algorithms. Hence the opposition-based learning is combined with some soft computing algorithms, including DE (Dhahri and Alimi, 2010a; Subudhi and Jena, 2011), PSO (Dhahri and Alimi, 2010b; Rashid and Baig, 2010; Yaghini et al., 2011, 2013) and BBO (Ovreiu and Simon, 2010), and then applied to artificial neural network training.

The data intensive nature of image processing, combined with the performance requirements of real time application, makes it both crucial and challenging to optimize the performance of digital image processing algorithms. Several enhanced image processing algorithms are proposed by utilizing the concept of opposition-based learning, and then applied in some challenging tasks in image processing and computer vision field. Among the challenging tasks, image segmentation (Sahba et al., 2007; Tizhoosh, 2009), morphological edge detector (Khalvati et al., 2007), image thresholding (Al-Qunaieer et al., 2010b; Rahnamayan and Tizhoosh, 2008; Tizhoosh and Sahba, 2009), classification (Ovreiu and Simon, 2010; Saki et al., 2010; Ventresca and Tizhoosh, 2006) and traffic congestion identification (Yang et al., 2012) are so far some good examples which have succeeded comparatively.

Economic Load Dispatch (ELD) problem, one of the major problems in power systems operation and planning, seeks the best generation schedule for the generating plants to supply the required demand plus transmission losses at minimum production cost. It is a large-scale, multimodal, non-differentiable and highly non-linear problem. In order to achieve better results, the opposition-based learning and its variation were adopted to BBO (Bhattacharya and Chattopadhyay, 2010b) and DE (Reghunathan and Baby, 2012), respectively. Similar to ELD, the OBBO algorithm had also been successfully applied to solve EELD (Bhattacharya and Chattopadhyay, 2010a) and Optimal Power Flow (OPF) (Roy and Mandal, 2012) problems while satisfying a set of non-linear equality and inequality constraints. When the environmental concerns that arise from the emissions produced by fossil-fueled electric power plants are combined with the ELD then the problem becomes CEED problem. The concept of OBL had also been employed to accelerate the HS algorithm (Chatterjee et al., 2012) and gravitational search algorithm (Shaw et al., 2012). The proposed algorithms are successfully applied for the solution of different CEED problems of power systems. Balamurugan and Subramanian (2009) presented an opposition-based self-adaptive DE for Emission-Constrained Dynamic Economic Dispatch (ECDED) problem. A multi-objective function is formulated by assigning the relative weight to each of the objective and then optimized by opposition-based self-adaptive DE. The convergence rate is improved by employing an OBL scheme and a self-adaptive procedure for control parameter settings.

The concept of OBL is, moreover, applied in other significant engineering fields. We briefly list some examples below, with absolutely no claim to exhaustiveness: industrial engineering (Chen and Li, 2008a; Gao et al., 2012c; Ma et al., 2009, 2010; Mahootchi et al., 2007; Omran, 2010; Omran and Salman, 2009; Shokri, 2011; Shokri et al., 2008; Subudhi and Jena, 2011; Yüzgeç, 2010), electronic and communications engineering (Gao et al., 2011b; Gutiérrez et al., 2011; Munoz et al., 2011; Samanta and Chandra, 2012; Ventresca and Tizhoosh, 2007a), aerospace engineering (Wang et al., 2012b), robotics (Kulk and Welsh, 2011; Shokri et al., 2009; Wu et al., 2007), life sciences (Koohi-Moghadam and Rahmani, 2012), economy and finance (Banerjee and Tizhoosh, 2010; Wu et al., 2009), linguistics (Ventresca and Tizhoosh, 2007a) and even athletic game (Boskovic et al., 2010).

6. Further work and conclusions

Compared with other well-known intelligent computing methods, research on the model and algorithm of opposition-based learning is just at the stage of discipline creation and preliminary exploration, and many challenges are yet to be discovered and overcome in theoretical models and engineering applications. According to the current situation, we propose some opportunities and challenges on the model and algorithm of OBL as follows.

Firstly, the concept of opposition-based learning is not very diverse and liberal in form, which is probably one of its weakest points. It is a well-known fact that, adequate understanding and accurate positioning the connotation and denotation of research object is chief and primary work and is also the first step to scientific research. According to literature research, the opposition-based learning is customary limited in the framework of algorithm design and improvement. Most of researchers emphasize solving problems using the correct tools while they ignore how to extract the properties and improve the tools. Hence, before researching the model of OBL and its applications, the connotation and denotation of opposition-based learning should be discussed deeply and comprehensively at the higher level. In philosophy, especially in natural dialectics, several ideas and conclusions about contradiction have prevalent meaning and much help to this project. For example, the struggle and identity of contradiction, contradictions exist everywhere, the internal contradiction is the fundamental cause of the development of a thing. These conclusions demonstrate fully the basic idea and principle of opposition-based learning, and also greatly enrich and promote its concept in general.

Secondly, mechanism and function of opposition-based learning is not clear so far, that is to say, know the hows but not the whys. Even though the opposition-based learning is a generally applicable method, the efficiency depend greatly on the matching degree between the definition of OBL and the solution space of the problem, and also the rationality of the combination of opposition-based learning and the meta-heuristic algorithm. Therefore it is useful to conduct new models and extent new applications, only when the solution landscape well corresponds to the definition of opposition and the optimization algorithm is suitable designed.

Thirdly, one must maintain the balance between theoretical analysis and practical application. At present, most researchers lay particular stress on practical application, and tend to ignore further study on the theoretical analysis of the model and algorithm of opposition-based learning. The better performance of the extended algorithms is, in most cases, shown by simulation results, not by mathematical analysis with some developed mathematical concepts and tools. The advanced research of the theoretical mode and functioning mechanisms of opposition-based learning is infrequent, and the distinct and integrated theoretical framework is not been conducted so far, and even the theoretical analysis, such as convergence, stability and diversity, of new algorithm is very limited. For this reason, the essential and fundamental development of opposition-based learning is hard to obtain.

Finally, developing new engineering applications for OBL is another valuable and interesting task. At present, most applications focus on traditional optimization fields, such as large-scale optimization problem, constrained optimization problem, optimization of noisy problem, multi-objective optimization problem. In the meantime, opposition-based soft computing algorithm seldom has been applied to discrete optimization problems such as the traveling salesman problem and knapsack problem. Up to now, opposition-based learning has not gained international recognition, and the reason may be just lack of inspiring results in fundamental, subversive and pioneer fields. What is more to the point, nobody considers carefully and deeply why not break through in such fields, and even summarizes the basic features of the model and algorithm of opposition-based learning.

In a word, based on the contradictory phenomena existing in nature and human society, the primary research on the model and its applications of opposition-based learning is to construct the uniform model of OBL, to establish the distinct and complete theoretical framework, to extend some original and competitive future directions, and then to solve some complex real-world problems in the near future.

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