received copies are identical. Otherwise, $Y_j=0$ and the signal is not accepted at node j. The signal is always accepted when it is stored for a unit time. The indicator function is then

$$\phi_{ij}(Z) = Y_j \sum_{\substack{i_1<i_2<\dots<i_k \\ i_l\in i^*,\, i_l\neq j}} \eta_{i_1j}(Z)\eta_{i_2j}(Z)\cdot\cdots\cdot\eta_{i_kj}(Z) + \underset{\text{if } j\in i^*}{\eta_{jj}(Z)}$$

where

$$P\{Y_j = 1\} = b_j \qquad P\{Y_j = 0\} = 1 - b_j.$$

If each incoming copy consists of m bits, and if ρ is the probability of an error in each bit of each copy, we have

$$b_j \geq ((1-\rho)^k + \rho^k)^m.$$

That is, the bit at each position is received either correctly or incorrectly for all m positions if exactly k copies are received. If more than k copies are received, the probability of matching is larger.

It is possible to represent coupling between nodes which governs reception. Consider the case in which a successful reception at node j is possible only if node j and neighboring nodes $l_1, l_2, \cdots, l_j$ are operational; we then have

$$\phi_{ij}(Z) = Y_j Y_{l_1} Y_{l_2}\cdot\cdots\cdot Y_{l_j} \sum_{k\in i^*} \eta_{kj}(Z)$$

where the arrival of one copy is sufficient for a successful reception.

For rules which are "one-edge," where loss may take place in the channels, the node variable may be used to approximate the queuing process which takes place before the signal is transmitted. Assume the signal may be transmitted only if it is at the head of the queue. At each interval its position in the queue is indicated by Y_j. If the signal is at the head of the queue and is next to be transmitted, $Y_j=1$; otherwise, $Y_j=0$ and the signal is stored for a unit of time. Thus, Y_j is used to generate a zero-memory waiting time. If the arrival of one copy is sufficient for a successful reception, the indicator function becomes

$$\phi_{ij}(Z) = \sum_{k\in i^*} Y_k \eta_{kj}(Z) + \underset{\text{if } j\in i^*}{(1 - Y_j)}$$

A signal having arrived at node j waits a time t_j, until it is transmitted. The distribution of t_j is

$$P\{t_j = k\} = b_j(1-b_j)^k \qquad k = 0, 1, 2, \cdots.$$

The value of b_j may be selected to approximate the queuing conditions at node j.

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Carrier Mobilities in Silicon Empirically Related to Doping and Field

***Abstract*—Equations are presented which fit the experimental dependence of carrier mobilities on doping density and field strength in silicon. The curve-fitting procedures are described.**

Introduction

Analytic expressions for mobility dependence on doping density and field strength have many applications in the design and analysis of semiconductor devices. The expressions given in the following were derived empirically from published experimental data[1]–[4] for use by one of the present authors in the simulation of whole devices.[6]

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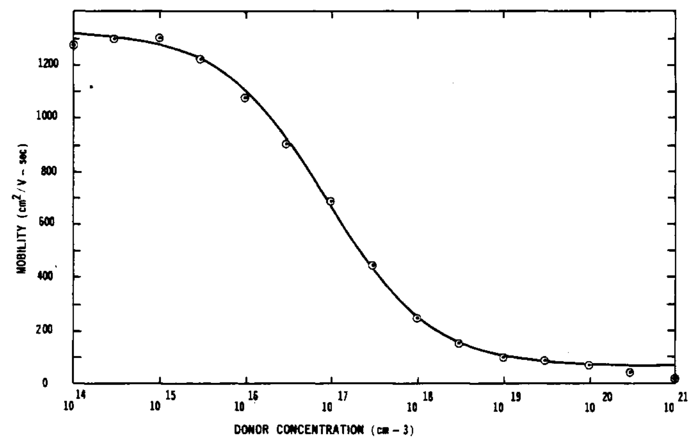

Fig. 1. Hole-mobility variation with doping.

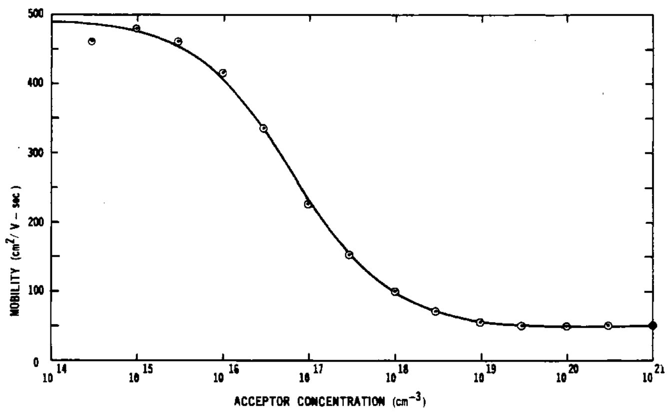

Fig. 2. Electron-mobility variation with doping.

TABLE I

	μ_{max} cm²/V·s	μ_{min} cm²/V·s	α	N_{ref} atoms/cm³
Holes	495	47.7	0.76	6.3×10^{16}
Electrons	1330	65	0.72	8.5×10^{16}

Doping Dependence of Mobility

Plots of measured mobility versus logarithm of doping density show a strong resemblance to the Fermi-Dirac function (or hyperbolic tangent) which can be expressed in the form

$$\mu = \frac{\mu_{\max} - \mu_{\min}}{1 + (N/N_{\text{ref}})^{\alpha}} + \mu_{\min} \tag{1}$$

where μ is mobility and N is doping density.

Figs. 1 and 2 show plots of electron and hole mobilities (circled dots) obtained from Irvin's[1] composite curves of silicon resistivity.[1] Equation (1) (solid line) gives an excellent fit to these data using the parameter values of Table I (discrepancies are less than 10 percent up to 10^{20} atoms per cubic centimeter). These parameter values were determined utilizing the following rearrangement of (1):

$$(N/N_{\text{ref}})^{\alpha} = (\mu_{\max} - \mu)/(\mu - \mu_{\min}). \tag{1a}$$

The values of $\mu_{\max}$ and $\mu_{\min}$ were derived by trial and error, being those giving the best fit of a straight line to a plot of $\log(\mu_{\max}-\mu)/(\mu-\mu_{\min})$ versus $\log N$, as indicated by (1a). The values of α and N_{ref} were then obtained from the slope and unity intercept of the straight line.

[1] Irvin[1] Fig. 1, p. 388, taking $\mu = 1/q\rho N$.

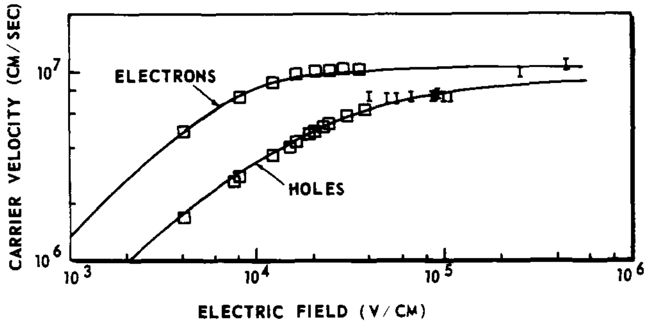

Fig. 3. Carrier-velocity variation with field strength.

TABLE II

	v_m cm/s	ε_c V/cm	β
Holes	9.5×10^6	1.95×10^4	1
Electrons	1.1×10^7	8×10^3	2

Field Dependence of Mobility

Several recent papers have reported improved measurement of carrier drift velocity dependence on field strength.[2]–[4] We have found that the following expression fits their results quite well:

$$v = v_m \frac{\varepsilon/\varepsilon_c}{[1 + (\varepsilon/\varepsilon_c)^\beta]^{1/\beta}}. \tag{2}$$

For holes $\beta = 1$ is used (as did Trofimenkoff[7]), i.e.,

$$v = v_m \frac{(\varepsilon/\varepsilon_c)}{1 + (\varepsilon/\varepsilon_c)} \text{(holes)}. \tag{2a}$$

For electrons $\beta = 2$ gives a much better fit:[2]

$$v = v_m \frac{(\varepsilon/\varepsilon_c)}{\sqrt{1 + (\varepsilon/\varepsilon_c)^2}} \text{(electrons)}. \tag{2b}$$

Fig. 3 shows the agreement between (2a) and (2b) (solid lines) and the experimental hole and electron velocities plotted against field strength. The plot marks are the same as used in Gibbon's summary[5] of the references cited.[2]–[4] Note that the curves lie within the experimental accuracy (5 percent) for all but one of the plotted points.

Table II gives the parameter values used in Fig. 3. The values were obtained by placing templates for (2a) and (2b) over the experimental log-log plots.

The mobility field dependence can be obtained from (2). The cord mobility (ratio of velocity to field strength) is given by

$$\mu = \frac{\mu_0}{[1 + (\varepsilon/\varepsilon_c)^\beta]^{1/\beta}} \text{(chord)}. \tag{3}$$

The incremental mobility is the first derivative for (2) with respect to field:

$$\mu = \frac{\mu_0}{[1 + (\varepsilon/\varepsilon_c)^\beta]^{1+1/\beta}} \text{(incremental)} \tag{4}$$

where for both (3) and (4)

$$\mu_0 = v_m/\varepsilon_c.$$

Using Table II gives $\mu_0 = 487$ cm^2/V · s for holes and $\mu_0 = 1375$ cm^2/V · s for electrons, characteristic of higher resistivity material.

Conclusions

Analytic expressions for carrier mobility variation with doping and field have been presented for silicon, giving excellent fits to published experimental data. Separate relationships for doping dependence and field dependence were derived since little published information exists on the combined effects of doping and field.[3] However, a combined mobility relationship could take the form of (4), using (1) for μ_0. The parameter ε_c would then be a function of doping, i.e., $\varepsilon_c = v_m/\mu_0$, treating v_m as constant.

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[2] Nonintegral values of β could have been used, but the improvement in fit does not justify the increased complexity.

[3] The data used here to derive (1) and (2) were obtained with either low field or low doping density. Prior's combined data[8] have been largely superseded by the recent work. Seidel and Scharfetter[9] have studied hole velocity field dependence in *p*-silicon doped from 10^{14} to 10^{16} per cubic centimeter, obtaining results which fit (2) and Table II.

A Simple Method for Preparing "Sodium-Free" Thermally Grown Silicon Dioxide on Silicon

Abstract—**A simple, reproducible MOS process requiring few steps and no elaborate equipment was developed. A surface charge density of 1×10^{11}/cm^2 was obtained for 1000-Å thermal oxide on ⟨100⟩ 10-ohm · cm *p*-type silicon chips after mounting and bonding. Stability results after temperature-bias test (3 min at 300°C with a field of 10^6 V/cm) exhibited a flatband voltage shift of only −0.50 V.**

Introduction

Sodium ion migration in thermally grown silicon oxide is responsible for instability under temperature-bias stress and for poor device performance. The literature reports that "sodium-free" oxides can be prepared by minimizing the sodium concentration in the processing ambients, and that MOSFET devices fabricated from these "clean" oxides exhibit good device performance and excellent stability. However, processes for preparing "sodium-free" oxides are not common knowledge since the information on processing steps reported in the literature is vague and incomplete. This letter will report in detail each processing step and, when necessary, the reasons for using them. Conceivably, a semiconductor manufacturer can use some of these processing steps in his laboratory to obtain a "sodium-free" oxide without revamping or substantially altering his oxide process.

The approach taken to develop this process was to eliminate sodium from the processing ambients and to reduce surface states as much as possible.

Process for Preparing "Sodium-Free" Oxides

The silicon material, obtained from Texas Instruments Incorporated, consisted of a Czochralski-grown *p*-type boron-doped 10-ohm · cm silicon

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