physica status solidi a pss

www.pss-a.com

What Do Apparent Series and Shunt Resistances in Solar Cell Estimated by *I*–*V* Slope Mean? Study with Exact Analytical Expressions

Kazuya Tada

The reciprocal slopes of a current–voltage curve at open-circuit- and short-circuit-conditions are frequently mentioned as apparent series (R_s) and shunt (R_p) resistances in emerging solar cell materials research. The relationship between such values and those in the one-diode equivalent-circuit model is studied by using exact analytical expressions of the current–voltage slope. It is found that the apparent R_s and R_p significantly deviate from true R_s and R_p.

1. Introduction

Solar cell is recognized as important clean energy source for the development of the future sustainable society. Since the printing technology is one of the cheapest technologies for production, huge efforts have been devoted to develop solar cells based on printable active materials such as organic and perovskite compounds, and nowadays the power conversion efficiency of them is steeply increasing.[1] Although the full description of solar cell characteristics requires the numerical solution of a set of differential equations,[2] such model is so complicated to be implemented into the electronic circuit design software. Suitable equivalent-circuit model of a solar cell is not only necessary for designing efficient electronic circuit powered by the solar cell, but also useful for understanding the loss mechanism taking place in the solar cell.[3–6]

The one-diode equivalent-circuit shown in **Figure 1** is the first-line model for solar cell research, and it is natural that series and shunt (parallel) resistances mentioned in a solar cell research article are expected to indicate R_s and R_p in the model, respectively, if not otherwise specified. Unlike the case of Si solar cell, it seems that the full physical descriptions of the circuit parameters in the model are not given at this stage when the model is adopted for the emerging solar cell research. Nevertheless, the model is capable of reproducing the current (I)–voltage (V) characteristic of the emerging solar cell. This fact clearly means that it is difficult to discriminate the material used in a solar cell by analyzing a single I–V curve of the solar cell. Such discrimination requires a systematic study including the collection of suitable set of data such as the dependence of estimated circuit parameters on experimental condition like temperature or illumination light intensity. For example, it has been reported that R_p in organic solar cell is inversely proportional to the illumination light intensity[4,5] while that in Si solar cell slightly depends on the light intensity,[7,8] reflecting the difference in the carrier mobilities in the active material. It is unquestionable that accurate estimation of circuit parameters is critically important for such purpose.

Since a single solar cell generally outputs relatively large current with low voltage, the evaluation of the series resistance is considered as especially important issue. While parameter extraction through the fitting of current–voltage curve with the model gives adequate value,[4–6,9] a variety of methods for the series resistance estimation without fitting had been proposed so far.[10] On the other hand, in the field of solar cell research using emerging materials such as organic and perovskite compounds, it is customary to mention the reciprocal slopes of a current–voltage curve at open-circuit- and short-circuit-conditions as apparent series and shunt resistances, respectively.[11–16] It is also noticed that similar method is occasionally mentioned in Si solar cell research.[17,18] Although it is obvious that the series and shunt resistances estimated by slope do not coincide with the relevant resistances in the equivalent-circuit model, the difference between them has been rarely discussed. This paper addresses this issue in terms of the exact analytical expressions for the slope of current–voltage curve of the equivalent-circuit model.

2. Theory

I–V characteristics of the one-diode model of solar cell shown in Figure 1 is described by the equation

$$I = I_s \cdot \left(\exp\left(\frac{V - I \cdot R_s}{n \cdot V_t} \right) - 1 \right) + \frac{V - I \cdot R_s}{R_p} - I_{ph} \quad (1)$$

where I_s and n denote the reverse saturation current and the ideality factor of the diode, respectively, and I_{ph} is the ideal

Dr. K. Tada
Department of Electrical Materials and Engineering
University of Hyogo
2167 Shosha, Himeji 671-2201, Japan
E-mail: tada@eng.u-hyogo.ac.jp

The ORCID identification number(s) for the author(s) of this article can be found under https://doi.org/10.1002/pssa.201800448.

DOI: 10.1002/pssa.201800448

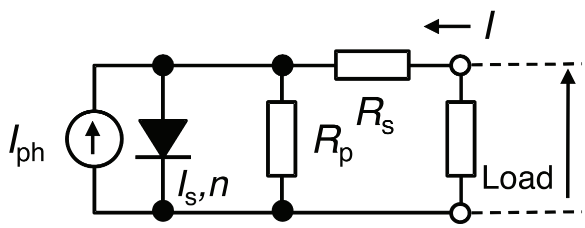

Figure 1. One-diode equivalent-circuit model of solar cell.

photocurrent. At room temperature, the thermal voltage V_t is $\approx$26 mV.

The Lambert W-function, which is defined as the solution of the transcendental equation $W(x) \times \exp(W(x)) = x$, is useful to obtain explicit expressions of current–voltage characteristic of electronic circuits with diode.[19,20] As mentioned by Jain and Kapoor,[21] the current–voltage characteristic of the equivalent-circuit can be explicitly described as follows

$$V(I) = -n \cdot V_t \cdot W_0(P(I)) + I \cdot (R_s + R_p) + (I_s + I_{ph}) \cdot R_p \tag{2}$$

with

$$P(I) = \frac{I_s \cdot R_p}{n \cdot V_t} \cdot \exp\left(\frac{(I + I_s + I_{ph}) \cdot R_p}{n \cdot V_t}\right) \tag{3}$$

or

$$I(V) = \frac{n \cdot V_t}{R_s} \cdot W_0(Q(V)) + \frac{V - (I_s + I_{ph}) \cdot R_p}{R_s + R_p} \tag{4}$$

with

$$Q(V) = \frac{I_s \cdot R_s \cdot R_p}{n \cdot V_t \cdot (R_s + R_p)} \times \exp\left(\frac{(V + (I_s + I_{ph}) \cdot R_s) \cdot R_p}{n \cdot V_t \cdot (R_s + R_p)}\right) \tag{5}$$

In these expressions, $W_0(x)$ denotes the principal branch of $W(x)$ defined for $x \in [-\exp(-1), +\infty]$. Although $W_0(x)$ is not as common as the natural logarithmic function or the trigonometric functions, compact and efficient computational algorithms for $W_0(x)$ are available.[22,23]

From Equations (2) and (4), the exact analytical expressions for the reciprocal slope of the current–voltage characteristics (dV/dI) are obtained as follows

$$\frac{\partial V(I)}{\partial I} = R_s + R_p \cdot \left(\frac{1}{1 + W_0(P(I))}\right) \tag{6}$$

and

$$\left(\frac{\partial I(V)}{\partial V}\right)^{-1} = \frac{R_s + R_p}{1 + \frac{R_p}{R_s} \cdot \left(\frac{W_0(Q(V))}{1 + W_0(Q(V))}\right)} \tag{7}$$

Since $W_0(x)$ is a monotone increasing function with $W_0(0) = 0$, the reciprocal slope approaches R_s and $R_s + R_p$ when $I \to +\infty$ and $V \to -\infty$, respectively, as expected by the inspection of the circuit. However, the slopes at open-circuit- and short-circuit-conditions are not trivial.

Equation (6) is essentially the same as Equation (7) in ref. [18]. In principle, the reciprocal slope at open-circuit condition is calculated by substituting $I = 0$ in Equation (6). However, the expressions with $P(I)$ including Equation (6) do not seem to be very useful, because $P(I)$ easily exceeds the upper limit ($\approx 1.8 \times 10^{308}$ $\sim$exp(710)) of the double-precision binary floating-point format defined in IEEE754, which is also used in standard spreadsheet software, causing an overflow in computation. For example, If we assume $I_{ph} = 20$ mA, $R_p = 10\,k\Omega$, $I_s = 1$ nA, and $n = 1.5$, which correspond to a state-of-the-art organic solar cell with 1 cm^2 active area under AM 1.5G 100 mW cm^{-2} irradiation showing $\approx$10% of power conversion efficiency with reasonable shape of the current–voltage curve, the calculation of $P(0)$ fails because the argument in the exponential function in Equation (3) is $\approx$6410. The same reason limits the calculation of open-circuit voltage (V_{oc}) by substituting $I = 0$ in Equation (2).

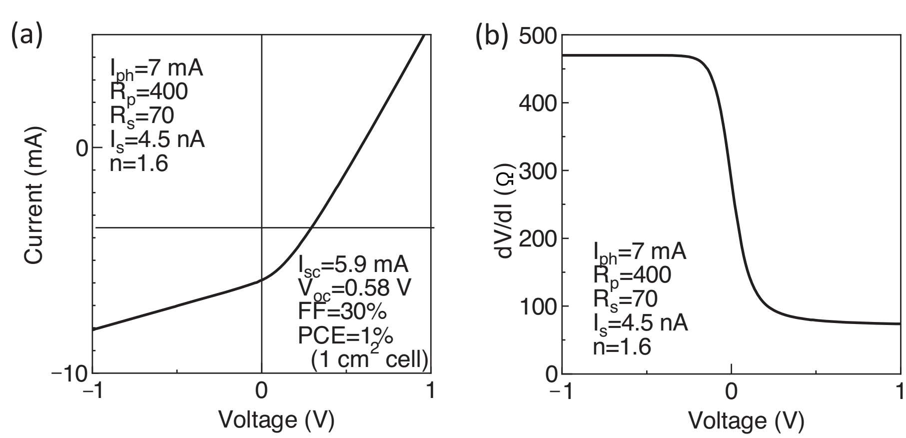

Figure 2. a) I–V and (b) dV/dI–V curves calculated for a hypothetical solar cell with 1 cm^2 active area with 1% of power conversion efficiency.

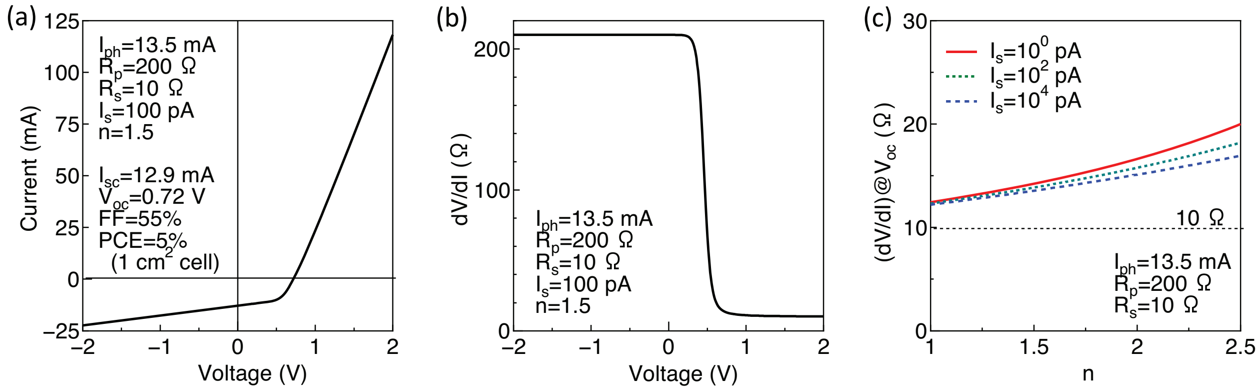

Figure 3. a) I–V and (b) dV/dI–V curves calculated for a hypothetical solar cell with 1 cm^2 active area with 5% of power conversion efficiency. c) Dependence of dV/dI at V_{oc} on n with various I_s.

On the other hand, the expressions using $Q(V)$ including Equation (7) have much less chance to cause an overflow than those using $P(I)$. To the best of the author's knowledge, Equation (7) has never been appeared in literature so far. Therefore, a practical procedure to calculate the reciprocal slope at open-circuit-condition is substituting $V = V_{oc}$ calculated by iteration such as Newton–Raphson method in Equation (7). The slope at short-circuit-condition is readily obtained by substituting $V = 0$ in Equation (7).

3. Results and Discussion

Although the parameters chosen for the following numerical examples target the organic solar cells, the results mentioned in the preceding section are inherent property of the one-diode equivalent circuit model of solar cell and is applicable to any type of solar cell as far as the I–V characteristic of the device is well fitted by the model.

Figure 2(a) and (b) show I–V and dV/dI–V curves calculated for a hypothetical solar cell with 1 cm^2 active area. The circuit parameters, which are listed in the figure, are chosen to reproduce an organic solar cell with ≈1% of power conversion efficiency under AM 1.5G 100 mW cm^{-2} irradiation, such as one reported in a preceding paper.[24] R_s and R_p in this example are 70 and 400 Ω, respectively. The reciprocal slope at short-circuit-condition or the apparent R_p is 285 Ω, which is significantly lower than true R_p, since dV/dI at short-circuit-condition is far from its upper and lower limits. The same is true for dV/dI at open-circuit-condition, resulting in the apparent R_s of 77 Ω, which is 10% higher than true R_s.

Figure 3(a) and (b) show I–V and dV/dI–V curves calculated for another hypothetical solar cell with ≈5% of power conversion efficiency under AM 1.5G 100 mW cm^{-2} irradiation, such as one reported in a preceding paper.[25] R_s and R_p in this example are 10 and 200 Ω, respectively. The reciprocal slope at short-circuit-condition is 210 Ω, which coincides with not R_p but $R_s + R_p$. In a solar cell with low fill-factor, the condition $R_s << R_p$ does not hold, resulting in a large discrepancy of the reciprocal slope at short-circuit-condition from R_p. On the other hand, the reciprocal slope at open-circuit-condition is 13.9 Ω, which is ≈40% higher than R_s. Although the reciprocal slope at high voltage such as 2 V is almost identical to R_s, the current at 2 V is ≈118 mA and it is a little bit questionable if the device survives such condition without major damage.

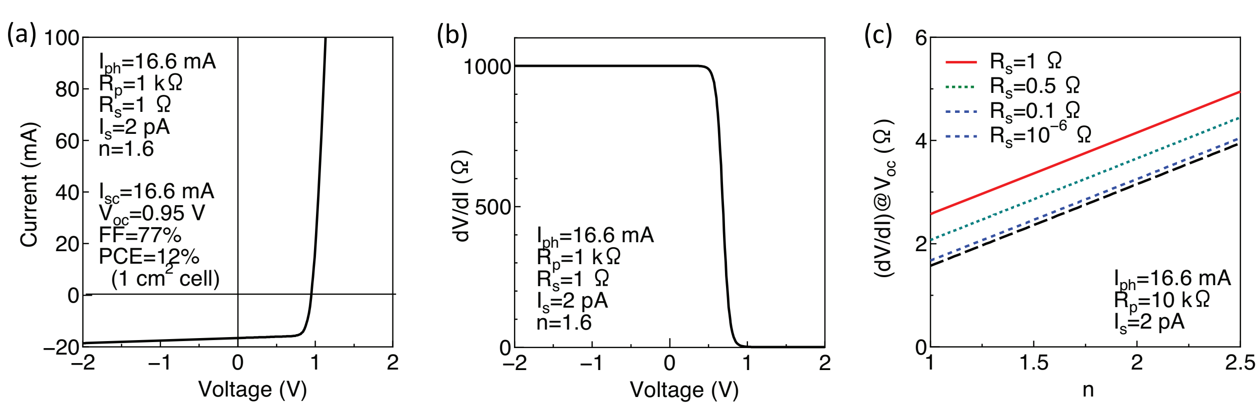

Figure 4. a) I–V and (b) dV/dI–V curves calculated for a hypothetical solar cell with 1 cm^2 active area with 12% of power conversion efficiency. c) Dependence of dV/dI at V_{oc} on n with various R_s.

Figure 3(c) shows the dependence of the reciprocal slope at open-circuit-condition on n. It is clearly observed that the reciprocal slope at open-circuit-condition strongly depends on n and its deviation from R_s decreases with decreasing n, while I_s has relatively minor effect. However, the deviation does not become negligible even for $n = 1$. The change of R_s in organic solar cells due to some treatment such as changing donor/acceptor ratio in an active layer is frequently attributed to the change of the carrier mobility in the active layer and/or the modulation of carrier injection property at interface. However, the present result suggests that the change in n, which is expected to represent the recombination mechanism in a solar cell, can also cause the change in the reciprocal slope at open-circuit-condition without changing true R_s.

Generally, the deviation of the reciprocal slope at open-circuit-condition from R_s becomes larger for a solar cell with higher power conversion efficiency with small R_s. **Figure 4**(a) and (b) show I–V and dV/dI–V curves calculated for another hypothetical solar cell, whose circuit parameters are chosen to reproduce a state-of-the-art organic photocell with 12% of power conversion efficiency.[26] dV/dI at open-circuit-condition is found to be $\approx 3\,\Omega$, which is three times of true $R_s = 1\,\Omega$. In this particular case, the reciprocal slope at open-circuit-condition becomes insensitive to R_s if $R_s \leq 0.1\,\Omega$ as shown in Figure 4(c), suggesting that it is governed by the characteristics of diode in the equivalent-circuit in considerable degree. This effect becomes more obvious in the case of solar cell with higher power conversion efficiency such as state-of-the-art perovskite solar cell or commercial Si solar cell in which true R_s is lower.

4. Conclusion

In this study, the relationship between the series and shunt resistances in solar cell estimated by current–voltage slope at open-circuit- and short-circuit-conditions, which are frequently used as apparent values in emerging solar cell materials research, and those in the one-diode equivalent-circuit model has been studied by using exact analytical expressions of the current–voltage slope. In order to avoid the problem in computation of known expression, the new expression is employed.

It is found that the apparent series/shunt resistances in a solar cell with poor performance significantly deviate from the true series/shunt resistances. In the case of solar cells with moderate power conversion efficiency, the reciprocal slope at short-circuit-condition is close not to R_p but to $R_s + R_p$. The difference should not be neglected for solar cell with low fill-factor, in which $R_s << R_p$ does not hold. On the other hand, the reciprocal slope at open-circuit-condition significantly deviates from R_s especially in the case of high performance solar cell. The results suggest that it is not suitable to use them as an apparent value of R_p and R_s.

Acknowledgments

This work was partially supported by the Iwatani Naoji Foundation's Research Grant and JSPS KAKENHI Grant Number JP18K04241.

Conflict of Interest

The author declares no conflict of interest.

Keywords

equivalent circuit model, exact analytical expressions, series resistance, shunt resistance, solar cell

Received: June 14, 2018
Revised: July 17, 2018
Published online:

[1] M. A. Green, Y. Hishikawa, E. D. Dunlop, D. H. Levi, J. Hohl-Ebinger, A. W. Y. Ho-Baillie, *Prog. Photovolt.* **2018**, *26*, 3.
[2] F. A. Lindholm, J. G. Fossum, E. L. Burgess, *IEEE Trans. Electron Dev.* **1979**, *26*, 165.
[3] B. Qi, J. Wang, *Phys. Chem. Chem. Phys.* **2013**, *15*, 8972.
[4] K. Tada, *Phys. Status Solidi A* **2017**, *214*, 1700018.
[5] K. Tada, *Org. Electron.* **2016**, *30*, 289.
[6] K. Tada, *Phys. Status Solidi A* **2015**, *212*, 1731.
[7] N. H. Reich, W. G. J. H. M. van Sark, E. A. Alsema, R. W. Lof, R. E. I. Schropp, W. C. Sinke, W. C. Turkenburg, *Sol. Ener. Mater. Sol. Cells* **2009**, *93*, 1471.
[8] F. Khan, S. N. Singh, M. Husain, *Sol. Energy Mater. Sol. Cells* **2010**, *94*, 1473.
[9] F. A. Shirland, *Adv. Energy Conv.* **1966**, *6*, 201.
[10] D. Pysch, A. Mette, S. W. Glunz, *Sol. Energy Mater. Sol. Cells* **2007**, *91*, 1698.
[11] T. Aernouts, W. Geens, J. Poortmans, P. Heremans, S. Borghs, R. Mertens, *Thin Solid Films* **2002**, *403–404*, 297.
[12] N. Moritake, Y. Fukui, M. Oonuki, K. Tanaka, H. Uchiki, *Phys. Status Solidi C* **2009**, *6*, 1233.
[13] W.-C. Su, C.-C. Lee, S.-W. Liu, C.-F. Lin, C.-C. Chou, B.-Y. Huang, C.-W. Cheng, *Jpn. J. Appl. Phys.* **2014**, *53*, 03CE02.
[14] W.-Y. Tan, R. Wang, M. Li, G. Liu, P. Chen, X.-C. Li, S.-M. Lu, H. L. Zhu, Q.-M. Peng, X.-H. Zhu, W. Chen, W. C. H. Choy, F. Li, J. Peng, Y. Cao, *Adv. Funct. Mater.* **2014**, *24*, 6540.
[15] R. Geethu, C. Sudha Kartha, K. P. Vijayakumar, *Sol. Energy* **2015**, *120*, 65.
[16] B. Xia, Z. Wu, H. Dong, J. Xi, W. Wu, T. Lei, K. Xi, F. Yuan, B. Jiao, L. Xiao, Q. Gong, X. Hou, *J. Mater. Chem. A* **2016**, *4*, 6295.
[17] K. Bouzidi, M. Chegaar, A. Bouhemadou, *Sol. Energy Mater. Sol. Cells* **2007**, *91*, 1647.
[18] T. J. Peshek, J. S. Fada, Y. Hu, Y. Xu, M. A. Elsaeiti, E. Schnabel, M. Kohl, R. H. French, *J. Vac. Sci. Technol. B* **2016**, *34*, 050801.
[19] T. C. Banwell, A. Jayakumar, *Electron. Lett.* **2000**, *36*, 291.
[20] A. Ortiz-Conde, F. J. Garcia Sanchez, J. Muci, *Solid-State Electron.* **2000**, *44*, 1861.
[21] A. Jain, A. Kapoor, *Sol. Energy Mater. Sol. Cells* **2004**, *81*, 269.
[22] R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, D. E. Knuth, *Adv. Comp. Math.* **1996**, *5*, 329.
[23] F. N. Fritsch, R. E. Shafer, W. P. Crowley, *Commun. ACM* **1973**, *16*, 123.
[24] K. Tada, *Sol. Energy Mater. Sol. Cells* **2012**, *100*, 246.
[25] K. Tada, *Sol. Energy Mater. Sol. Cells* **2015**, *143*, 52.
[26] S. Li, L. Ye, W. Zhao, X. Liu, J. Zhu, H. Ade, J. Hou, *Adv. Mater.* **2017**, *29*, 1704051.