The diode quality factor of solar cells under illumination

P Mialhe†, J P Charles‡, A Khoury§ and G Bordure‡

† Université de Dakar, Ecole Normale Supérieure, B.P. 50 36, Dakar Fann, Sénégal
‡ Centre d'Electronique de Montpellier, Université des Sciences et Techniques du Languedoc, Place E. Bataillon, 34060 Montpellier, France
§ Lebanese University II, Mansourieh, B.P. 72, Lebanon

Abstract A review of the methods of determination of solar cell quality factor is discussed from a theoretical point of view; experimental results are compared. Temperature effects, light and bias dependencies of the parameters and the models are considered in order to specify the limits of application of each method.

1. Introduction

Recent developments in photovoltaic device technology have furthered the considerable current interest in the analysis and understanding of the electrical characteristics of these devices. A description of the current–voltage characteristics based upon the diode equation is given by

$$I = I_{ph} - (V + R_S I)/R_{SH} - I_S\{\exp[e(V + R_S I)/(AkT)] - 1\} \tag{1}$$

where I_{ph} is the light-generated current, I_S is the diode saturation current, R_S and R_{SH} are the series and shunt resistances and A is the diode quality factor. These parameters I_{ph}, I_S, R_S, R_{SH} and A are determined so that equation (1) gives a good description of the experimental characteristics.

The diode quality factor has been introduced for a p–n junction solar cell after consideration of the physical phenomena that occur in the diode. Several theories have been published which considered independently the mechanisms occurring in the device, to point out the particular effect. Taking into account generation/recombination in the space-charge layer, the analysis by Sah *et al* (1957) predicted $A \leqslant 2$. According to the diffusion theory by Shockley (1949) based on minority carrier diffusion, A should equal 1. Assuming that traps are situated in a localised region of the depletion layer, Faulkner and Buckingham (1968) proposed a theory which gives values of A between 1 and 2, that was confirmed by their experiments with p–n junctions; the model has also been discussed by Nussbaum (1973), who pointed out the effective change in the recombination current caused by the variation in potential across the junction, aiming to explain the diode quality factor values they had measured. Results published later by Ashburn *et al* (1975) show the increase of A from 1 to 2 as the density of recombination centres is increased. At high injection levels (I_{ph} is a few A cm^{-2}, that is with concentrators) assuming that

the recombination in the depletion layer is negligible, Hall (1952) found $A = 2$. At low injection level, values of A close to 2 are likely to be due to recombination. High values of A (>2) have been observed that could be due to various phenomena such as shunt resistance effects (Stirn 1972) or non-uniformities in the distribution of recombination centres (Shockley and Queisser 1961). It may also be possible to have A values smaller than unity (Hall 1981) in a junction under high-level injection conditions with a saturation current determined by Auger recombinations.

The introduction of the diode quality factor for a proper fit of Schottky barrier I–V data is also standard practice. Ponpon and Siffert (1976) pointed out that the open-circuit voltage can be increased either by increasing the diode quality factor or by using higher barriers at the metal–semiconductor contact; the diode quality factor is related to the thickness of the interfacial layer and depends on the interface states (Card and Rhoderick 1971, Olsen and Bohara 1975). A study by Sen and Tyagi (1984) explained the observed increase of diode quality factor values of polycrystalline MIS solar cells with decreasing grain size as due to interface charge on the I–S surface and to voltge drop at the grain boundaries in polycrystalline base semiconductors. Furthermore it has been shown (Green 1977, Mialhe *et al* 1983) that an increase of A leads to a degradation of the cell efficiency. It is doubtful whether complete independence of A and I_S can exist (Mialhe *et al* 1983, Wolf *et al* 1977) and in general high values of I_S are associated with conduction mechanisms exhibiting large values of A.

This literature review shows that the diode quality factor is related both to internal properties of the elements of solar cells and to the experimental conditions that define the operating point. Therefore it is of interest to consider its experimental determinations.

In this paper we discuss all the published works related to the experimental methods of A determination; experiments with commercial solar cells are described and enable one to determine the limitation of each approximate method.

2. Literature review

The determination of the diode quality factor of solar cells is achieved with methods that may be classified into two kinds depending upon whether they are graphical only, or whether they make use of a numerical procedure.

2.1. Determination through graphic techniques

As a first approximation the unity in equation (1) is small compared to the exponential term (that is true as soon as $V > 0.1$ V at 300 K for $A < 2$); then if R_S is small enough so that the term $R_S I$ can be ignored, and for the ideal case (R_{SH} infinite), equation (1) reduces to

$$I = I_{ph} - I_S \exp[eV/(AkT)] \tag{2}$$

A plot of $\ln(I_{ph} - I)$ versus V results in a straight line with a slope $e/(AkT)$, that gives a value of A; $\ln(I_S)$ is the corresponding intercept on the current axis. (This method is referred to in this paper as the direct measurement method, DMM).

Two particular cases of DMM have been discussed by Wolf and Rauschenbach (1963): (i) these authors considered the log–linear plot of the diode forward characteristic (under dark conditions) described by equation (2) with $I_{ph} = 0$ to determine A and I_S in the

same way; this has been used by several authors (Krammer and Ludington 1977, Agarwal *et al* 1981, Rohatgi *et al* 1980, Anderson *et al* 1977).

(ii) the measurements of the short-circuit current I_{SC} and of the open-circuit voltage V_{oc} may be performed for different light intensity settings; the plot of $\ln(I_{SC})$ versus V_{oc} yields A and I_S in the same way as DMM; (it is easy from equations (1) and (2) to find the relationship

$$I_{SC} = I_S \exp[eV_{oc}/(AkT)]$$

with the new approximation $\exp[R_S I_{SC}/(AkT)]$ close to unity); this has been used by several authors (Panayotatos and Card 1978, Tarr and Pulfrey 1979, Dhanasekaran and Gopalam 1981).

The determination of the diode quality factor from (i) or (ii) assumes that the values of the parameters are light independent. These experimental log–linear plots show two exponential parts: one for large bias and the other for small bias voltage. The transition region has been described (Wolf and Rauschenbach 1963) as the sum of two exponential terms. This leads to the double exponential solar cell equation:

$$I = I_{ph} - (V + R_S I)/R_{SH} - I_{S1}\{\exp[e(V + R_S I)/(kT)] - 1\}$$
$$- I_{S2}\{\exp[e(V + R_S I)/(AkT)] - 1\}. \tag{3}$$

This model has also been discussed by Wolf *et al* (1977), Neugroschel *et al* (1977), Shousha (1982) and Charles *et al* (1984).

An elegant graphic method has been developed by Warashina and Ushirokawa (1980): these authors showed from equation (1) that the curve $-dV/dI$ versus $(I_{SC} - I)^{-1}$ is a straight line with a slope AkT/e; (referred to as the graphic method slope GMS); it provides a direct determination of the quality factor from the I–V characteristic of the solar cell at one illumination level.

Mailhe and Charette (1983) derived an experimental method from measurements, for the same illumination level and for different values of a resistor R put in series with the cell; the plot of $\ln(I_{ph} - I)$, for a fixed value of the voltage V, against $(R_S + R)I$ is approximately linear with slope $e/(AkT)$; (referred to as the variable resistor series method VRSM).

2.2. Determination through the use of numerical analysis

Singal (1981) computed the changes in the maximum power point and in the curve factor due to the resistance (R_S) effect; their variation is tabulated for all V_{oc}/A values; the parameters R_S and A may then be simultaneously approximated with a dichotomic research; (referred to as the maximum power tabulation method MPTM); it makes use of the experimental determination of I_{ph} and of the maximum power point. Araujo and Sanchez (1982) introduced the two normalised parameters

$$v_{oc} = V_{oc} e/(AkT)$$

and

$$v_r = I_{ph} R_S e/(AkT)$$

then, from the single-exponential model equation they derived two equations depending on the maximum power point coordinates (I_m, V_m); the solution is the couple (v_{oc}, v_r),

from which R_S and A can be calculated; a graphic method is also proposed to obtain v_{oc} and v_r; (referred to as the normalised parameter coordinate method NPCM).

A test of validity of each approximate method may be the degree of accuracy which is achieved to describe the experimental characteristics with determined parameters. A practical method of analysis of the current–voltage characteristics of solar cells has been published by Charles *et al* (1981) (referred to as PMA); the macroscopic parameters R_S, R_{SH}, I_S, A and I_{ph} are computed, from experimental data for three parts of the characteristic, to give the best fit with experimental characteristics. The agreement between the calculated current and the experimentally observed current is within $\pm 0.5\%$ on average for the solar cells tested.

3. Results and discussion

The experiments have been performed on 2×2 cm^2 blue silicon solar cells. The cell under test was mounted on a copper block using water as the coolant. The temperature was measured with a digital thermometer and a small platinum resistance surface transducer. The illuminated I–V characteristics were measured under 2 Norma 100 W tungsten–iodine quartz bulbs and a 1 cm water filter; the light intensity of the source was monitored electronically with reference to the short-circuit current of a photovoltaic cell fixed near the bulbs.

Figure 1 illustrates how sensitive the diode quality factor is as far as the description of the characteristic is concerned. The full curve corresponds to the theoretical description, obtained with equation (1) using PMA, for measurements taken under 1 sun at $T = 307.1$ K; the values of the parameters are $I_{ph} = 0.1179$ A, $I_S = 0.25 \times 10^{-7}$ A,

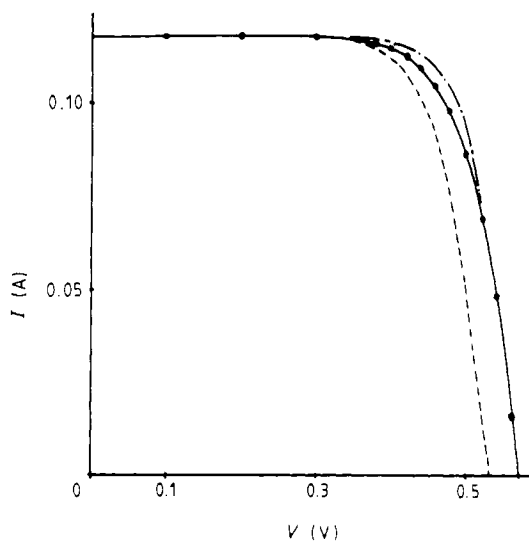**Figure 1.** The dependence of the characteristic upon the value of the diode quality factors. Circles: experimental points at $T = 307.1$ K under 1 sun. Full curve: theoretical curve with $R_S = 0.22\Omega$, $R_{SH} = 5827\Omega$, $I_{ph} = 0.1179$ A, $I_S = 0.25 \times 10^{-7}$ A and $A = 1.40$. Dashed curve: theoretical curve with $R_S = 0.22\Omega$, $R_{SH} = 5827\Omega$, $I_{ph} = 0.1179$ A, $I_S = 0.25 \times 10^{-7}$ A and $A = 1.30$. Chain curve: theoretical curve with $R_S = 0.22\Omega$, $R_{SH} = 5827\Omega$, $I_{ph} = 0.1179$ A, $I_S = 0.20 \times 10^{-8}$ A and $A = 1.20$.

$R_S = 0.22\ \Omega$, $R_{SH} = 5827\ \Omega$ and $A = 1.40$. Some experimental points are represented by open circles ($V_{oc} = 0.568$ V). There is no significant difference on the plot between the experimental and the calculated points. Taking $A = 1.30$ instead of $A = 1.40$ and the same values for the other parameters, the theoretical description represented by the broken curve is far from the experimental results. Taking $A = 1.20$ and the same values as earlier for R_S and R_{SH}, the theoretical characteristic passing through the ($I = I_{SC}$, $V = 0$) and ($I = 0$, $V = V_{oc}$) points is represented as a chain curve. It yields $I_S = 0.020 \times 10^{-8}$ A and shows a deviation from the experimental curve around its knee. These calculations take into account the intercorrelation of the cell parameters which can be represented from equation (1) when $I = 0$, $V = V_{oc}$ by:

$$V_{oc} = R_{SH}I_{ph} - R_{SH}I_S\{\exp[eV_{oc}/(AkT)] - 1\}. \tag{4}$$

The value of the quality factor obtained with PMA will be considered in the discussion as a reference (noted A_0) to allow for a comparison with all the methods.

Figure 2 displays experimental characteristics in a log–linear plot (curve A, same results as in figure 1; curve B, same device in the dark condition). Such a plot is not easy to use graphically: neglecting the high-voltage curvature (thermal effect) we use the DMM to take the slope at the point of inflexion around 500 mV (chain lines in figure 2) and obtain two extreme values: $A = 1.5$ and $A = 1.7$; note that the intercept of these straight lines on the current axis leads to a very imprecise determination of I_S; at low voltage the linear approximation yields $A = 3.8$. The use of such values of the diode quality factor to describe the experimental characteristic with equation (3) or with equation (1) yielded poor results; this is confirmed when comparing with $A_0 = 1.40$.
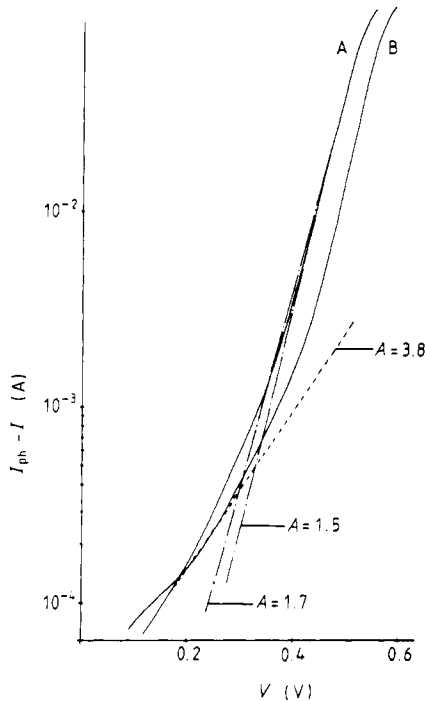

Figure 2. Determination of the diode quality factor with DMM; plot of $\ln|I_{ph} - I|$ versus V ($T = 307.1$ K), experimental results. Curve A: under illumination (1 sun); the dashed curve and the chain curve are linear approximations. Curve B: in the dark.
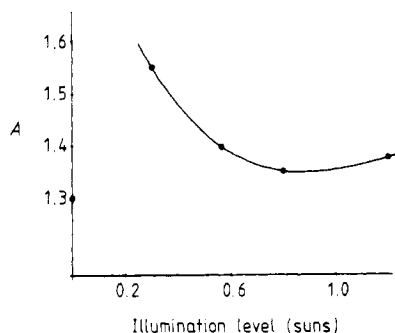
Figure 3. Determination of the diode quality factor from dark to light concentration ($T =$ 303 K) using PMA.
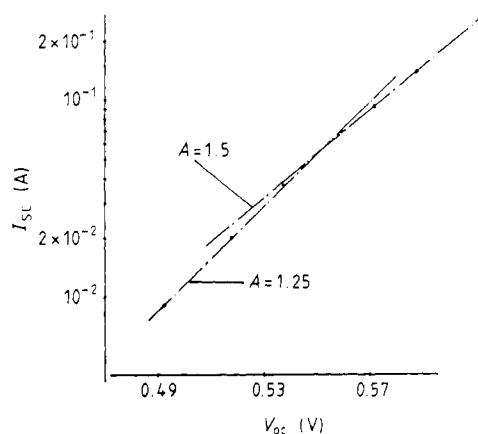

Figure 4. Linear approximation of the $\ln(I_{SC})$ versus V_{oc} plot; the points are experimental at $T =$ 303 K; measurements of the slopes yield the indicated A values.

The same observations may be made about using the DMM and the dark characteristics. The shift observed from $(I\text{--}V)$ measurements (figure 2) in the dark and under illumination (also observed by Panayotatos and Card (1980)) confirms that the parameters are light dependent. Using PMA we obtained results in figure 3 for the same device at $T =$ 303 K. The light dependence of the quality factor was observed by Panayotatos and Card (1978), Dhanasekarian and Gopalam (1982) and later by Charles *et al* (1984). Such results show that the DMM applied to (I_{SC}, V_{oc}) measurements under different illumination levels can only give a mean value; this is confirmed by experimental results in figure 4 which we obtained at 303 K for an illumination level varying from 1.2 suns to 0 sun; a comparison with figure 3 shows opposite results for the variation of A with the illumination level. The log–linear curves are close to straight lines when the short-circuit current is small i.e. $\exp[eR_SI_{SC}/(AkT)]$ is close to unity and when R_S values are small, i.e. the series resistance effect does not affect the shape of the characteristic: in these cases DMM yields the proper result since the graphic interpretation is straightforward. The DMM is based upon the assumption that the temperature change is negligible

during the measurements; the slope of the linear fit takes the form

$$e/(AkT) = \Delta \ln(I)/\Delta V. \tag{5}$$

Then knowing that the temperature coefficient of the current (Ak^{-1}) is small compared to the temperature coefficient of the voltage (Vk^{-1}) we obtain for the conditions of these experiments

$$A \sim A\Delta T/100. \tag{6}$$

When I_{ph} increases, a variation of temperature $\Delta T = 5$ K is commonly observed. This leads to an error $\Delta A = 0.07$ for $A = 1.50$. Consequently it appears necessary to control the temperature very carefully while taking measurements for a determiantion of A. This temperature effect has been discussed by Cape and Zehr (1980) and later, Sanchez and Araujo (1982) determined a thermal resistance from the $I_{SC} - V_{oc}$ measurements that helps one to obtain the real $I_{SC} - V_{oc}$ characteristics of a solar cell.

Using the GMS we plot $(-dV/dI)$ versus $(I_{SC} - I)^{-1}$ in figure 5 for the same experimental values as in figure 1 (1 sun, $T = 307.1$ K). The two linear approximations of the experimental curve yield $A = 1.79$ at low voltage and $A = 1.43$ at high voltage; this should be compared with $A_0 = 1.40$ (note that the graphic precision of the measurements is about 15% for low voltages and becomes 8% at high voltages). The good results at high voltages where I differs largely from I_{SC} confirm the good description of the characteristic by the model. For small bias $(I_{ph} - I)$ is very small and the characteristic corresponds to the exponential term only. The following calculation gives a better idea of the approximations involved in this method: consider equation (1) in the form

$$f(I, V) = 0. \tag{7}$$

It is easy to calculate

$$dV/dI = -(\partial f/\partial I)/(\partial f/\partial V). \tag{8}$$

Then with the assumption $1/R_{SH} \ll 1$ we find

$$-dV/dI = R_S + AkT/e[I_{ph} - I - (V + R_S I)/R_{SH}]^{-1}. \tag{9}$$

The GMS identifies I_{ph} and I_{SC}, and does not take into account the contribution $(V + R_S I)/R_{SH}$, which is a large approximation where $(I_{ph} - I)$ is close to zero.
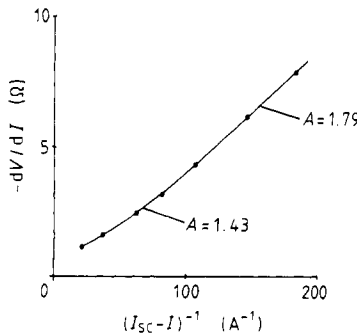

Figure 5. Plot of $(-dV/dI)$ versus $(I_{SC} - I)^{-1}$; circles represent experimental points; slope measurements yield the indicated values.
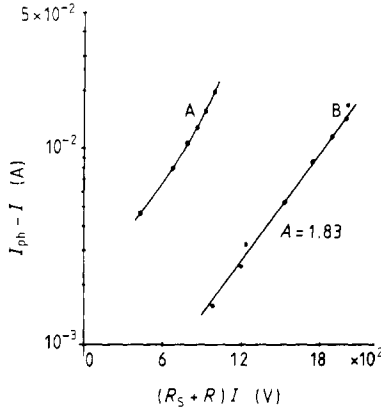
Figure 6. Plot of $\ln(I_{ph} - I)$ versus $(R_s + R)I$ for an arbitrary fixed voltage ($T = 305$ K, 1 sun illumination) and the computed parameters: Curve A, $R_s = 0.22\Omega$, $R_{SH} = 5230\Omega$, $I_s = 4 \times 10^{-8}$ A, $I_{ph} = 0.120$ A, $A_0 = 1.40$; Curve B, $R_s = 0.20\Omega$, $R_{SH} = 3230\Omega$, $I_s = 7 \times 10^{-6}$ A, $I_{ph} = 0.420$ A, $A_0 = 1.90$. The circles represent experimental points.

The precision for VRSM is good if large variations of I_{SC} are obtained when the variable resistor increases from 0 to 5 R_S; this enables one to measure I at about $V = V_{oc}/3$, far from the knee of the characteristic; this is the case when $I_s \exp[5eR_sI_{SC}/(AkT)]$ is of the order of $I_{ph}/5$, that is for large values of I_s and I_{SC}. This is illustrated in figure 6 at 305 K for two cells under 1 sun conditions: with curve A no results can be determined for this cell whereas curve B gives the determination $A = 1.83$ compared to $A_0 = 1.90$; (the VRSM implies long experiments since it requires measurements for at least six different values of the variable resistor).

The starting point with the MPTM relies upon the use of two arbitrary A values (say $A = 1$, $A = 2$) to compute the first approximate R_S values; a linear interpolation of these R_S values as a function of A leads to the determination of an estimated A value; with each A value Singal (1981) showed how to calculate, along two routes, two R_S values R_1 and R_2: the correct A value is assumed to be obtained when the difference $|R_2 - R_1|$ is small. In practice it is commonly found that no estimated value of A can be obtained from the interpolation (this is the case for the characteristic in figure 1); in such cases the MPTM may be used with detailed calculations that consider each possible A value, one after another, as an estimated value. Figure 7 displays the
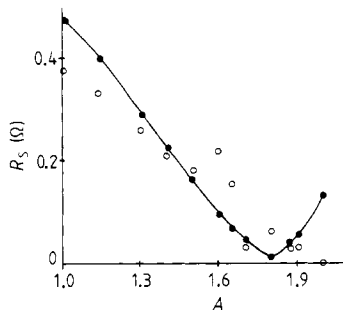

Figure 7. R_S values, R_1 (filled circles) and R_2 (open circles), computed for the corresponding A values: results from MPTM.

variations of R_1 and R_2 values computed for each A; according to these results the MPTM leads to three possible values: $A = 1.48$ with $R_S = 0.18\ \Omega$, $A = 1.69$ with $R_S = 0.05\ \Omega$ and $A = 1.87$ with $R_S = 0.04\ \Omega$, the last being the best with regard to the smaller $|R_2 - R_1|$ value. Such results come from the fact that MPTM considers A and R_S as equivalent parameters, but the influence of A in equation (1) is of first order and the influence of R_S is of second order! In the case of large R_S values only (when $R_S I_{SC}$ is comparable with V_m), some information may be obtained from MPTM.

With the experimental results of figure 1 and using NPCM we found the value 15.1 for the normalised parameter v_{oc}; this yields $A = 1.43$ (to be compared to $A_0 = 1.40$); NPCM gives good results when the maximum power intensity is very close to I_{ph} and (or) $(V_{oc} - (V_m + R_S I_m))$ is greater than AkT/e, which is the case of good cells with low R_S values.

4. Conclusion

We have discussed all the published techniques related to the determination of the diode quality factor of solar cells. The results show that only an exact description of the experimental curve enables one to state that the determined value of the quality factor is the proper value. The possibility of error when using dark currents as well as different illuminated conditions for a determination of the parameters is demonstrated. Further, since the physical meaning of A is related to conduction processes in the device, the proper determination of A should be simultaneous with the proper determination of the diode saturation current, that is consistent with the results of Neugroschel *et al* (1977).

It is quite obvious that the single exponential model, where the diode quality factor is a constant, cannot describe the characteristic in a wide voltage range. This consideration yields a bias dependent diode quality factor, the mathematical definition of which would take the form (Ashburn *et al* 1975):

$$A = (e/kT)[\mathrm{d}V/\mathrm{d}\,(\ln(I_{ph} - I))] \tag{9}$$

and would lead to a new approach to the electrical modelling of solar cells; it does not seem that any physical interpretation can be anticipated from such considerations.

References

Agarwal S K, Muralidharam R, Agarwala A, Tewary V K and Jain J C 1981 *J. Phys. D: Appl. Phys.* **14** 1643–6
Anderson W A, Davis J R and Delahoy A E 1977 *IEEE Trans. Electron Dev.*, **ED-23** 453–5
Araujo G L and Sanchez E 1982 *Solar Cells* **5** 377–86
Ashburn A, Morgan D V and Howes M J 1975 *Solid State Electron.* **18** 569–77
Cape J A and Zehr S W 1980 *Proc. 14th IEEE Plotovoltaic Specialists Conf.* pp 449–52
Card H C and Rhoderick E H 1971 *J. Phys. D: Appl. Phys.* **4** 1589–601
Charles J P, Abdelkrim M, Muoy Y H and Mialhe P 1981 *Solar Cells* **4** 169–78
Charles J P, Mekkaoui-Alaoui I, Bordure G and Mialhe P 1984 *Rev. Phys. Appl.* **19** 851–7
Dhanasekarian P C and Gopalam B S V 1981 *Solid State Electron.* **24** 1077–80
—— 1982 *Solid State Electron.* **25** 719–22
Faulkner E A and Buckingham M J 1968 *Electron. Lett.* **4** 359–60
Green M A 1977 *Solid State Electron.* **20** 265–6

Hall R N 1952 *Proc. IRE* **40** 1512–21
—— 1981 *Solid State Electron.* **24** 595–616
Kammer D W and Ludington M A 1977 *Am. J. Phys.* **45** 602–5
Mialhe P, Charette J and Charles J P 1983 *Proc. Int. AMSE Conf. Modelling and Simulation (Nice)* pp 97–103
Mialhe P and Charette J 1983 *Am. J. Phys.* **51** 68–70
Neugroschel A, Lindholm F A and Sah C T 1977 *IEEE Trans. Electron. Dev.* **ED–24** 662–71
Nussbaum A 1973 *Phys. Status Solidi* a **19** 441–50
Olsen L C and Bohara R C 1975 *Proc. 11th IEEE Photovoltaic Specialists Conf.* pp 381–90
Panayotatos P and Card H C 1978 *Proc. 13th IEEE Photovoltaic Specialists Conf.* pp 634–8
—— 1980 *Solid State Electron.* **23** 41–7
Ponpon J P and Siffert P 1976 *J. Appl. Phys.* **47** 3248–51
Rohatgi A, Davis J R, Hopkins R H, Rai-Choudhury P, McMullin P G and McCormick J R 1980 *Solid State Electron.* **23** 415–22
Sah C T, Noyce R N and Shockley W 1957 *Proc. IRE* **45** 1228–43
Sanchez E and Araujo G L 1982 *Solid State Electron.* **25** 817–9
Sen K and Tyagi B P 1984 *J. Appl. Phys.* **56** 1240–1
Shockley W 1949 *Bell Syst. Tech. J.* **28** 435–89
Shockley W and Queisser H J 1961 *J. Appl. Phys.* **32** 510–9
Shousha A H M 1982 *Solar Cells* **5** 67–73
Singal C M 1981 *Solar Cells* **3** 163–77
Stirn R J 1972 *Proc. 9th IEEE Photovoltaic Specialists Conf.* pp. 72–82
Tarr N G and Pulfrey D L 1979 *Proc. 2nd Eur. Communities Conf. on Photovoltaic Solar Energy, Berlin* pp 58–64
Warashina M and Ushirokawa A 1980 *Japan. J. Appl. Phys.* **19** 179–82
Wolf M and Rauschenbach H 1963 *Adv. Energy Convers.* **3** 455–79
Wolf M, Noel G T and Stirn R J 1977 *IEEE Trans. Electron Dev.* **ED-24** 419–28