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TEMPERATURE, ELECTRIC FIELD, AND DOPING DEPENDENT MOBILITIES OF ELECTRONS AND HOLES IN SEMICONDUCTORS

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Abstract—A new formula for electron and hole mobilities in semiconductors is presented. Although empirical, it is accurate and widely applicable to a number of semiconductors, such as Si, Ge, GaAs, InP, etc. The formula is simple, and yet predicts temperature and field dependence of electron and hole mobilities very well. To our knowledge, the present model is more general than any other model (both empirical and theoretical) available in the literature. Because of very simplistic nature, it promises to be highly useful for analytically modeling the current–voltage characteristics of transistors.

1. INTRODUCTION

The electron and hole mobilities of semiconductors constitute important parameters of electronic devices. These parameters are essential tools for theoretical modeling of current–voltage characteristics both by analytical methods, and by two- and/or three-dimensional numerical simulations. Electron and hole mobilities vary with temperature and doping concentrations, and are functions of electric field. The field dependence arises when carriers, heated up by the intense electric field, are no longer in equilibrium with the crystal lattice. The heating occurs, for example, when short-channel MOS devices are subjected to large drain–source applied voltages.

Theoretically, electron and hole mobilities in semiconductors depend on the band structures of the semiconductors concerned, and on various scattering mechanisms, such as acoustic phonon, polar optic phonon, nonpolar optic phonon (holes only), piezoelectric, ionized impurity, carrier–carrier, and plasmon scattering mechanisms[1,2]. Very accurate calculations of semiconductor band structures, and of various scattering mechanisms is exceedingly difficult. Often, it requires knowledge of tensorial characteristics of carrier transport, and of the nonparabolicity of the band structure of the semiconductor concerned. Consideration of complex mathematical formulas based, for example, on Pauli exclusion principle and many-body methods of quantum mechanics becomes essential for the solution of problem. To circumvent this difficulty, various authors[5–7] resorted to one or other kind of approximation. Modeling of carrier mobilities following these approximations provides some understanding of the physics involved in the transport processes. These hardly lead, however, to simple and precise analytical formulas of mobilities involving electric field, temperature, and doping concentration.

The great difficulties associated with the calculations of carrier mobilities using first-principle formulas, and the discrepancies between the experiments and the theoretical results thus obtained, suggest that we look for an alternate approach, which even being empirical, is simple, precise, highly predictive, and useful for modeling the current–voltage characteristics of transistors. During the past years a number of such models have been tried and reported in the literature[2–13]. Among them, one due to Arora *et al.*[10] has appeared to be very successful. Although confined to silicon, it is applicable for a wide range of temperatures and doping concentrations. It does not, however, take the field dependence of mobilities into account. Another model, due to Hilsum[13], considers the doping dependence, but not the field and temperature dependence of mobilities. It is, however, useful for several semiconductors, such as silicon, germanium, gallium arsenide, indium phosphide, etc. The main problems with empirical formulas of Arora *et al.* and Hilsum are that they possess forms too complex to be used for analytical investigation of current–voltage characteristics of transistors.

The field dependence of electron and hole mobilities of silicon and germanium has been studied very widely by the Italian school at Modena[14,15]. All these studies indicate that carrier mobilities decrease in general with increase in electric field.

Our aim in this investigation is to present an empirical formula for electron and hole mobilities in semiconductors, which will promise wide applicability, and will involve field, temperature, and doping concentration.

2. FORMULATION OF THE MOBILITY

2.1. Low-field mobility

In a recent investigation Fischetti[2] calculated the effect of various scattering mechanisms on the electron and hole mobilities in silicon. This author noted that, among all the scattering mechanisms, the ionized impurity scattering gives rise to more than 90% of the mobilities. If the effect of electron–electron (e–e) scattering is taken into account, the two together leads even to better prediction. Considering that impurity atoms are fully ionized, and that ionized impurity scattering involves both isotropic and anisotropic scattering effects, the Brooks–Herring formula[16] for the ionized impurity scattering mobility μ_{IS} may be given by:

$$\mu_{IS} = \frac{\mu_{IS0}}{NG(\omega)} \tag{1}$$

where

$$\mu_{IS0} = \frac{2^{7/2}\epsilon_s^2(kT)^{3/2}}{\pi^{3/2}q^3m_e^{1/2}} \times 10^{-2}, \tag{2}$$

where ϵ_s is the dielectric constant of the semiconductor, k is the Boltzmann constant, T is the temperature in absolute scale, q is the electronic charge, m_e is the conductivity effective mass of electrons, and N is the density of ionized impurity atoms. If h is the Planck's constant, the parameter ω is given by:

$$\omega = \frac{\omega_0}{N} \tag{3a}$$

where

$$\omega_0 = \frac{24\pi m_e\epsilon_s(kT)^2}{q^2h^2} \times 10^{-6}. \tag{3b}$$

The function $G(\omega)$, corresponding to the variable ω, is:

$$G(\omega) = \ln(\omega + 1) - \omega/(\omega + 1). \tag{4}$$

Note that ω is an unitless parameter, and the unit of ω_0 is identical to that of N. As shown in Fig. 1, the function $G(\omega)$ may very well be approximated by:

$$(G(\omega))^{-1} = C_0 + \sum_{v=1}^{6} \frac{C_v}{[\omega]^{a_v}}, \tag{5}$$

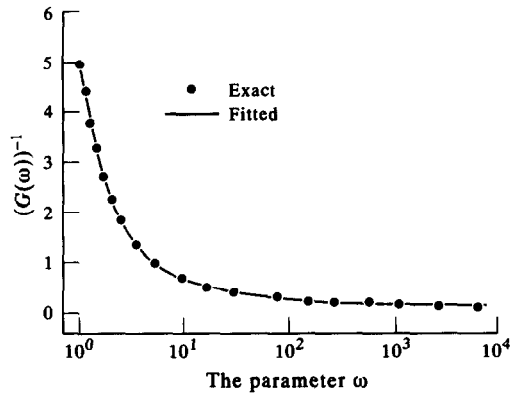


Fig. 1. Comparison of the exact and fitted curves for $G(x)$ as a function of x.

where C_v and a_v are various parameters listed in Table 1. Equations (1) and (5) yield:

$$\mu_{IS} = \frac{\mu_{IS0}}{\omega_0} \sum_{v=0}^{6} \frac{C_v}{[(N/\omega_0)]^{s_v}}, \tag{6}$$

where $s_0 = 1$, and $s_v = 1 - a_v$ ($v = 1$ to 6).

Making use of time-independent Hartree–Fock theory, Luong and Shaw[17] have analyzed the effect of e–e scattering on the ionized impurity mobility in the framework of single-particle-like approximation. It was noted that, when the effect of e–e scattering is taken into account, the effective mobility μ_{ISE} may be given by:

$$\mu_{ISE} = 0.632\,\mu_{IS}. \tag{7}$$

An inspection of terms in eqns (6) and (7) indicates that low-field electron and hole mobilities μ_{LF} in a semiconductor may be expressed as a summation series of N^{-1}. In order for this summation series to be widely applicable and highly predictive, the series must involve contributions from other scattering mechanisms as well. And, when the involvement is made, the numerical values of the coefficients of the series would be different from C_v and a_v ($v = 1$ to 6) listed in Table 1. The formula would also turn to be empirical having a form:

$$\mu_{LF} = \mu_{LF0}\left[D_0 + \sum_{v=1}^{6} \frac{D_v T_e^m}{(N/N_0)^{b_v}}\right], \tag{8}$$

where $N_0 = 10^{17}\,\text{cm}^{-3}$, D_v and b_v ($v = 1$ to 8) are the appropriate parameters listed in Table 1, μ_{LF0} is a normalization factor, having values different for different semiconductors (see Table 2), $T_e = (300/T)$,

Table 1. Various parameters used for the present modeling

v	C_v	a_v	D_v	b_v
0	81.499991×10^{-2}	0.00	70.273396	0.00
1	16.574423×10^{4}	0.76	-16.015068×10^{4}	0.55
2	65.861794×10^{4}	0.77	72.088255×10^{4}	0.60
3	-44.433304×10^{5}	0.78	-12.521139×10^{5}	0.65
4	77.307452×10^{5}	0.79	10.621290×10^{5}	0.70
5	-55.949152×10^{5}	0.80	-44.311247×10^{4}	0.75
6	14.831434×10^{5}	0.81	73.047665×10^{4}	0.80

Table 2. List of low-field normalization mobility values of various semiconductors

Semiconductor	Carrier type: Electron/hole	Majority/minority	μ_{LF0} (cm²/V · s)
Si	Electron	Majority/minority	1.4×10^3
Si	Hole	Majority/minority	5.4×10^2
Ge	Electron	Majority	4.2×10^3
InP	Electron	Majority	6.0×10^3
GaAs	Electron	Majority	9.0×10^3
GaAs	Electron	Minority	6.5×10^3
GaAs	Hole	Majority	5.0×10^2
GaAs	Hole	Minority	2.5×10^2

and $m = 1.7$. Note that μ_{LF0} of eqn (8) is equivalent to (μ_{IS0}/ω_0) of eqn (6). Both of them vary from one semiconductor to the other, with doping concentration N, and with temperature T.

2.2. *High-field mobility*

If E is the electric field, v_s is the saturation value of the carrier drift velocity v at high electric field, and μ_{HF} is the high-field mobility of electrons and holes, then $v \equiv v_T(E)$ may be given by[18]:

$$v_T(E) = \mu_{HF} E = \frac{\mu_{LF} E}{[1 + (\mu_{LF} E/v_S)^{\beta}]^{(1/\beta)}}, \qquad (9)$$

where β is an adjustable parameter dependent on temperature for both electrons and holes in silicon[19]. Some recent investigations[11,12] indicate that eqn (9) predicts high-field mobilities μ_{HF} of electrons and hole in Si and Ge indeed very well. This equation is not, however, a very good fit for the μ_{HF} vs E curves of compound semiconductors, such as GaAs and InP[20,21]. For these semiconductors, and possibly for others, the v vs E curves possess peaks with values higher than that at the saturation limit.

The v vs E curves for all semiconductors seem to possess the general form shown schematically in Fig. 2. For example, the curve H, which represents the common form of the variation of the drift velocity v of a semiconductor as a function of the electric field E, would assume the form of curve A if the said semiconductor involves very high negative differential mobility. It will assume the form of some other intermediate curve (e.g. curve B or C or D) if the negative differential mobility is lower. The equation representing the general form for v vs E curve would be:

$$v(E) = \mu_{HF} E = v_T(E) + \lambda_{PX} v_{PX} \qquad (10)$$

where

$$\lambda_{PX} = \{2e^{-\alpha E_N} - e^{-2\alpha E_N}\} \qquad (11a)$$

$$E_N = [E^{0.47} - E_0^{0.51}]^2 \qquad (11b)$$

$$v_{PX} = v_P - v_T(E_0), \qquad (11c)$$

where v_P is the peak value of the drift velocity v at $E = E_0$, and α is a parameter defining the curvature around the peak value of the curve. Note that at high electric field, λ_{PX} would be negligibly small, and the carrier drift velocity $v = (\mu_{HF} E)$ would reach the saturation value v_S. At $E = E_0$, λ_{PX} would tend to be unity, and hence v would be identical to v_P. At very low electric field, E_N would be approximately equal to E_0, and as a result, λ_{PX} would be marginally small. For Si and Ge, there is no peak in the v vs E curves at small electric field E. For these semiconductors, the peak of the velocity would occur at the saturation limit, and hence v_P would be identical to v_S at quite large values of the electric field E.

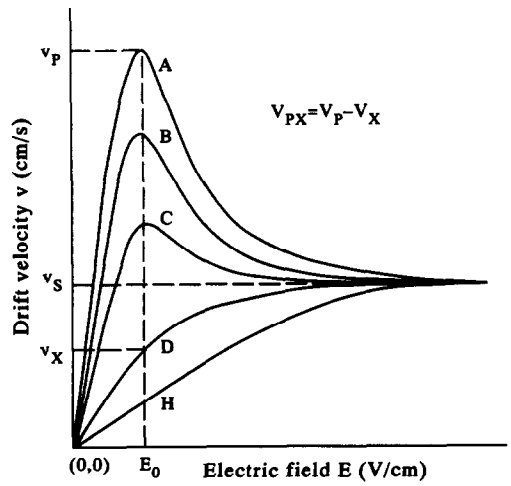
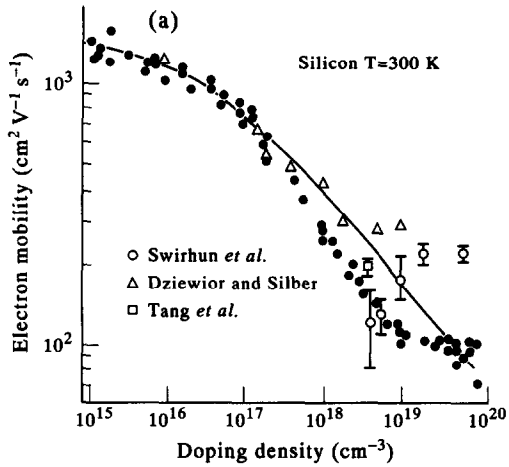


Fig. 2. Schematic representation of the general form of the μ_{HF} vs E curves. Curves A, B, C and D are modified forms for the curve H depending on the magnitude of correction for the negative differential mobility for the semiconductor. v_P is the peak values of v of these modified curves at $E = E_0$, and $v_X = v_T(E_0)$.

3. RESULTS AND DISCUSSIONS

In order to study the applicability of our theoretical model, eqn (8), we have carried out numerical calculations of the low-field mobility μ_{LF} of electrons and/or holes of Si, Ge, GaAs, and InP at 300 K. The results are depicted in Figs 3–6. As may be noted, all the parameters of eqn (6), except μ_{LF0}, are fixed parameters. The values of μ_{LF0} vary from semiconductor to semiconductor, from majority to minority electrons, and from majority to minority holes. These are presented in Table 2. For the sake of comparison, available experimental data for various semiconductors are also presented in Figs 3–6. For Si, experimental majority mobility from Hall measurements, and/or from empirical fitting (shown as small solid circles) are due to Arora *et al.*[10], Thurber *et al.*[22], and D'Avanzo *et al.*[23], and the experimental minority electron mobility are due to Swirhun *et al.*[24],



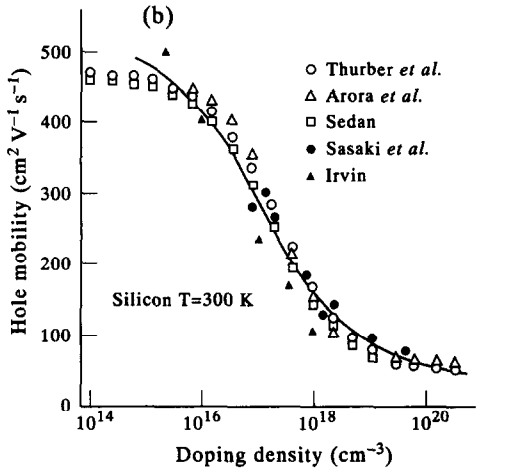


Fig. 3. (a) Comparison of the calculated present results (solid line) with the experimental results for low-field electron mobility μ_{LF} of silicon as a function of the doping concentration N. The vertical bars represent the uncertainty in the measured data, and the solid curve represents the present result. (b) Comparison of the calculated present results (solid curve) with the results from other theoretical calculations and experimental measurements for low-field hole mobility in silicon as function of doping concentrations.

Dziewior and Silber[25], and Tang *et al.*[26]. Unlike the data for the Hall mobility, those for the minority electron mobility cover only a narrow range of doping concentrations. This represents a limitation to our effort of providing with separate theoretical curves for the majority and minority electron mobilities respectively, in Si. As mentioned above, these curves would correspond to different values of the normalization mobility μ_{LF0}. We have used a compromised value of μ_{LF0} to obtain the solid-line curve presented in Fig. 3(a). This curve agrees well with the available experiments. Similar curves from Fig. 3(b) for majority hole mobility in Si, from Fig. 4 for majority electron mobility in Ge, and from Fig. 5 for majority electron mobility in InP agree also with the corresponding theoretical and experimental results. In the case of Si, the theoretical results are due to

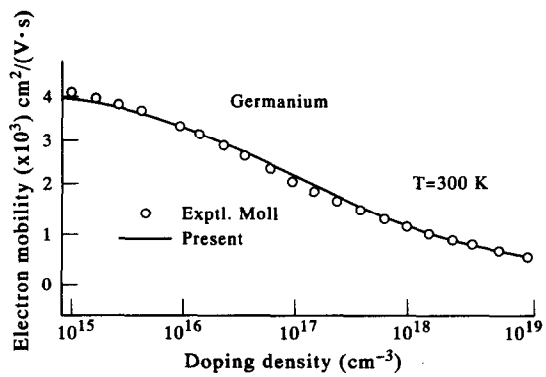


Fig. 4. Comparison of the calculated present results (the solid line) and the experimental results for electron mobility for germanium as function of the doping concentrations.

Klassen[8,9], Arora *et al.*[10], Thurber *et al.*[22], and D'Avanzo *et al.*[23], and the experimental results are due to Sasaki *et al.*[27], and Irvin[28]. In the case of Ge, the experimental data are from Moll[29], and in the case of InP these are from Madelung and Schulz[30] and from Rode[31]. In contrast to Si, Ge, and InP, sufficient experimental mobility data for both majority and minority electrons, and majority and minority holes are available for GaAs. These results prompted us to try to generate separate μ_{LF} vs N curves for these mobilities. The results are presented in Fig. 6(a)–(d). For the sake of comparison, experimental data of Meyer and Bartoli[32] (see Fig. 6 of this reference) for the majority electron mobility, experimental data of Ito and Ishibashi[33] (see Fig. 2 of this reference for details), Tiwari and Wright[34], Furuta and Tomizawa[35], and Lovejoy *et al.*[36] for the minority electron mobility, the experimental data of Wiley[37] (see Fig. 31 of this reference for details), Furuta and Tomizawa[35], and Klausmeier-Brown *et al.*[38] for the majority hole mobility, and the

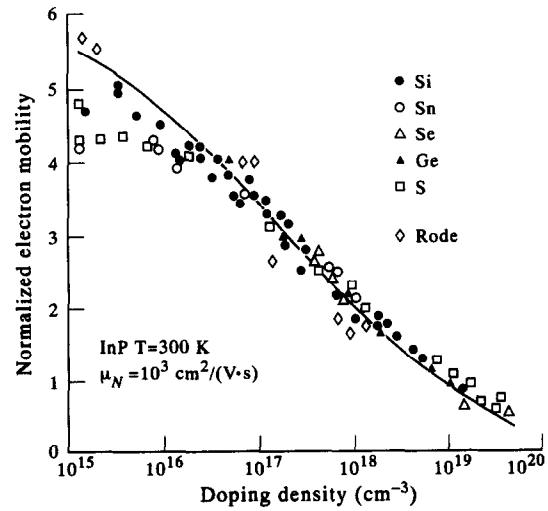


Fig. 5. Comparison of the calculated present results (solid line) and the experimental results for low-field electron mobility for InP as function of the doping concentrations.

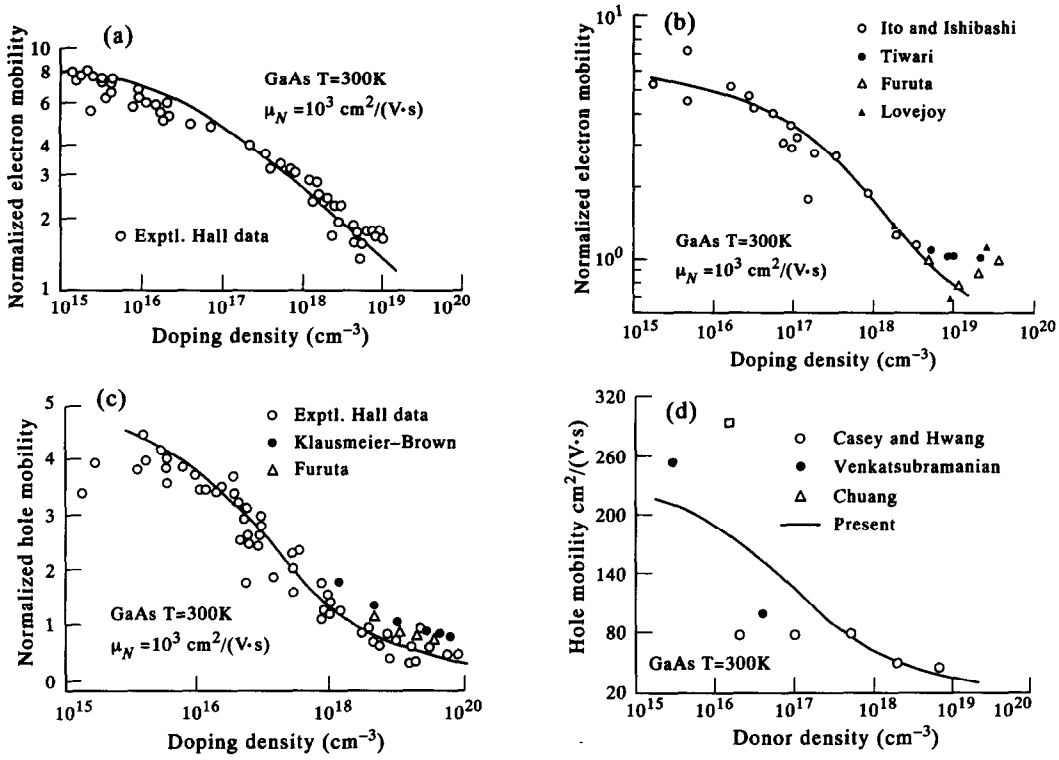


Fig. 6. (a) Comparison of the calculated present result (solid line) and the experimental results for low-field majority electron mobility of GaAs as function of the donor concentrations. The mobility normalization constant is denoted by μ_N. (b) Comparison of the calculated present results (solid line) and observed results for low-field minority electron mobility of GaAs as function of the acceptor concentrations. The mobility normalization constant is denoted by μ_N. (c) Comparison of the calculated present results (solid line) and observed results for low-field majority hole mobility of GaAs as function of the acceptor concentrations. The mobility normalization constant is denoted by μ_N. (d)The comparison of the calculated present results (solid line) and observed results for the low-field minority hole mobility in GaAs as function of the donor concentrations.

experimental data of Casey *et al.*[39], Hwang[40], Venkatsubramanian *et al.*[41], and Chuang *et al.*[42] for the minority electron mobility, are included in Fig. 6(a)–(d) respectively. As may be seen from these figures, our results for various GaAs mobilities are in good agreement with the corresponding experiments. A judicious choice of the values of μ_{LF0} for these mobilities may be credited for playing a significant role in the above-mentioned agreement.

Reliable experimental data for temperature dependence of electron and hole mobilities are available only for a few pure semiconductors[11,12,19]. Arora *et al.*[10] extracted them for doped Si from various scattered measurements performed under varying experimental conditions. The variation of the present results for silicon at $T = 200$, 300, 400 and 500 K with doping concentrations are shown in Fig. 7. Experimental data obtained by the Italian school at Modena[8,10,11,18] for $N = 10^{14}$ cm^{-3}, and those extracted by Arora *et al.*[10] for higher doping concentrations are also presented in this figure. An inspection of these results indicates that the agreements between the two is quite reasonable.

Equation (10) is used to study the effect of high electric field on the electron and hole mobilities of various semiconductors. Experimental parameters, such as v_S, v_P, and E_0, and the fitting parameters, such as α and β used for these calculations are listed in Table 3. Calculated results were found to be in good agreement with available experiments. For the sake of brevity, these results only for GaAs are shown in Fig. 8. From this figure it may be noted that calculated results compare well with experiments[20,43–45]. It was observed that, in the case of GaAs, with hump in the v vs E curve, the end results are quite insensitive to the choice of β. These results in particular for Ge are, however, quite sensitive to the choice of β.

4. CONCLUSIONS

We have proposed a simple analytical model for the electron and hole mobilities in semiconductors. The model appears to predict both low- and high-field mobilities very well. Interestingly, only one value of m, viz., $m = 1.7$, suffices to reproduce experimental mobility data for all semiconductors with remarkable accuracy. D_v and b_v ($v = 1, 6$) are also found to be general enough for yielding the doping dependence of

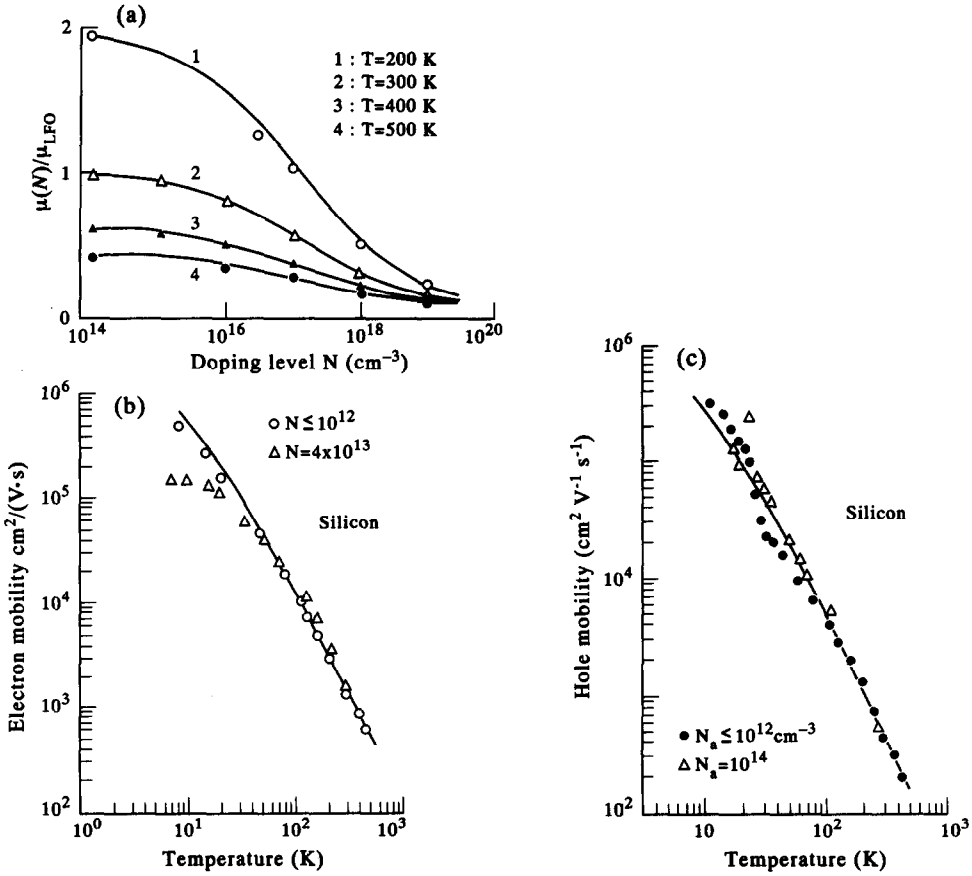


Fig. 7. (a) The variation of the calculated present results (solid line) and observed results for the low-field electron mobility in silicon at four different temperatures as function of temperature. The small solid circles, small solid triangles, small open triangles, and the small open circles correspond to the experimental results for $T = 500$, 400, 300 and 200 K, respectively. (b) The variation of the calculated present results (solid line) for low-field ($E = 1$ V/cm) electron mobility in pure silicon ($N = 5 \times 10^{13}$ cm^{-3}) as function of temperature. The small open circles and the small open triangles represent the observed results. (c) The variation of the calculated present results (solid line) and observed results for low-field ($E = 1$ V/cm) hole mobility in pure silicon ($N = 4 \times 10^{13}$ cm^{-3}) as function of temperature. The small solid circles and the small open triangles represent the observed results.

mobility for various semiconductors. In this respect, Si is perhaps the only minor exception. The problem with this semiconductor seems to arise primarily from the non-availability of experimental data for minority electron mobilities over a range of 10^{15} to 10^{20} cm^{-3}. Once the problem is resolved, the present model would very likely be far more applicable to separately predict the doping dependence of majority and minority electrons in Si.

If the mobility at $E = E_0$ be denoted by μ_P, then μ_{HF} from eqn (10) may be expressed as:

$$\mu_{HF} = \frac{\mu_{LF}}{\{1 + (\mu_{LF} E/v_S)^\beta\}^{(1/\beta)}} + \frac{\lambda_{PX} E_0}{E}\left[\mu_P - \frac{\mu_{LF}}{\{1 + (\mu_{LF} E_0/v_S)^\beta\}^{(1/\beta)}}\right]. \quad (12)$$

Table 3. Experimental and fitting parameters used for the calculation of high-field electron and hole velocities in semiconductors at 300 K

		Experimental parameters			Fitting parameters	
Semicond.	Mobility type	E_0 (V/cm)	v_S ($\times 10^7$) (cm/s)	v_P ($\times 10^7$) (cm/s)	α (cm/V)	β
Si	Electron	6.5×10^4	1.00	1.00	0.6×10^{-4}	1.11
Si	Hole	3.0×10^5	1.00	1.00	0.2×10^{-4}	1.00
Ge	Electron	4.0×10^3	0.60	0.60	1.0×10^{-3}	2.50
Ge	Hole	2.0×10^4	0.56	0.56	1.0×10^{-3}	2.50
GaAs	Electron	3.8×10^3	0.70	2.02	1.0×10^{-3}	1.00
InP	Electron	9.0×10^3	0.50	2.00	0.5×10^{-3}	1.50

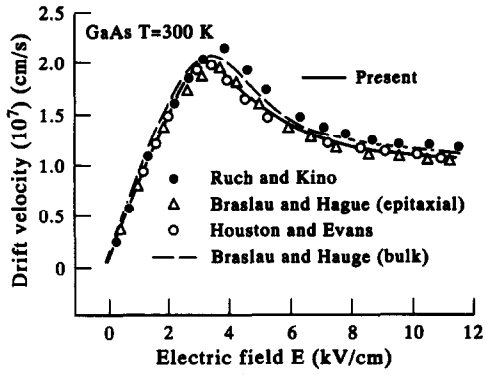


Fig. 8. Comparison of the calculated present results (solid line) and observed results for the high-field electron velocity in GaAs at $T = 300$ K.

Note that the first term of eqn (12) is the one due to Caughey and Thomas, and that the second term is a perturbation term, which leads to correction for negative differential mobility appearing in compound semiconductors, such as GaAs and InP.

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REFERENCES

1. See, for example, D. Chattopadhyay and H. J. Queisser, *Rev. mod. Phys.* **53**, 745–768 (1981).
2. M. V. Fischetti, *Phys. Rev. B* **44**, 5527–5534 (1991).
3. S. Noor Mohammad, *Solid-St. Electron.* **35**, 1391–1396 (1992).
4. N. Shigyo, H. Tanimoto, M. Norishima and S. Yasuda, *Solid-St. Electron.* **33**, 727–731 (1990).
5. L. C. Linares and S. S. Li, *J. Electrochem. Soc.* **128**, 601–608 (1981).
6. S. S. Li and W. R. Thurber, *Solid-St. Electron.* **20**, 609–616 (1977).
7. S. S. Li, *Solid-St. Electron.* **21**, 1109–1117 (1978).
8. D. B. M. Klassen, *Solid-St. Electron.* **35**, 953–959 (1992).
9. D. B. M. Klassen, *Solid-St. Electron.* **35**, 961–967 (1992).
10. N. D. Arora, J. R. Hauser and D. J. Roulston, *IEEE Trans. Electron Devices* **ED-29**, 292–295 (1982).
11. M. A. Omar and L. Reggiani, *Solid-St. Electron.* **30**, 1351–1354 (1987).
12. M. A. Omar and L. Reggiani, *Solid-St. Electron.* **30**, 693–697 (1987).
13. C. Hilsum, *Electron. Lett.* **10**, 259–260 (1974).
14. C. Canali, C. Jacoboni, F. Nava, G. Ottaviani and A. Alberigi-Quaranta, *Phys. Rev. B* **12**, 2265–2284 (1975).
15. L. Reggiani (Editor), *Topics in Applied Physics: Hot-Electron Transport in Semiconductors*, Vol. 58. Springer, Berlin (1985).
16. H. Brooks, in *Advances in Electronics and Electron Physics* (Edited by L. Morton), Vol. 7, pp. 85–182. Academic Press, New York (1955).
17. M. Luong and A. W. Shaw, *Phys. Rev. B* **4**, 2436–2441 (1971).
18. D. M. Caughey and R. E. Thomas, *Proc. IEEE* **55**, 2192–2193 (1967).
19. C. Canali, G. Majni, R. Minder and G. Ottaviani, *IEEE Trans. Electron Devices* **22**, 1045 (1975).
20. J. Pozela and A. Reklaitis, *Solid-St. Electron.* **23**, 927–933 (1980).
21. G. H. Glover, *Appl. Phys. Lett.* **20**, 224–226 (1972).
22. W. R. Thurber, R. L. Mattis, Y. M. Liu and J. J. Filliben, *J. Electrochem. Soc.* **127**, 1807–1812 (1980).
23. D. C. D'Avanzo, M. Vanzi and R. W. Dutton, One-dimensional semiconductor analysis (SEDAN), Technical Report No. G-201-5, Stanford Electronics Laboratories, Oct. (1979).
24. S. E. Swirhun, D. E. Kane and R. M. Swanson, *IEDM Tech. Dig.*, pp. 298–301 (1988).
25. J. Dziewoir and D. Silber, *Appl. Phys. Lett.* **35**, 170–172 (1979).
26. D. D. Tang, F. F. Fang, M. Scheuermann, T. C. Chen and G. Sai-Halasz, *IEDM Tech. Dig.*, pp. 20–23 (1986).
27. Y. Sasaki, K. Itoh, E. Inoue, S. Kishi and T. Mitsuishi, *Solid-St. Electron.* **31**, 5–12 (1988).
28. J. C. Irvin, *Bell Syst. Tech. J.* **41**, 387–406 (1962).
29. J. L. Moll, *Physics of Semiconductors*, pp. 192–193. McGraw-Hill, New York (1964).
30. O. Madelung and M. Schulz (Editors), *Landolt–Bornstein Numerical Data and Functional Relationships in Science and Technology*, Vol. 22. Springer, Berlin (1988).
31. D. L. Rode, *Phys. Rev. B* **3**, 3287–3299 (1971).
32. J. R. Meyer and F. J. Bartoli, *Phys. Rev. B* **36**, 5989–6000 (1987).
33. H. Ito and T. Ishibashi, *J. appl. Phys.* **65**, 5197–5199 (1989).
34. S. Tiwari and S. L. Wright, *Appl. Phys. Lett.* **56**, 563–565 (1990).
35. T. Furuta and M. Tomizawa, *Appl. Phys. Lett.* **56**, 824–826 (1990).
36. M. L. Lovejoy, B. M. Keyes, M. E. Klausmeier-Brown, M. R. Melloch, R. K. Ahrenkiel and M. S. Lundstrom, in *Extended Abstracts for the 22nd Int. Conf. of Solid State Devices and Materials*, Sendai, Japan, pp. 613–616 (1990).
37. J. D. Wiley, in *Semiconductors and Semimetals* (Edited by Williardson and Beer), Vol. 10, p. 91. Academic Press, New York (1974).
38. M. E. Klausmeier-Brown, M. R. Melloch and M. S. Lundstrom, *Appl. Phys. Lett.* **56**, 160–162 (1990).
39. H. C. Casey Jr, B. I. Miller and E. Pinkas, *J. appl. Phys.* **44**, 1281–1287 (1973).
40. C. J. Hwang, *Phys. Rev. B* **6**, 1355–1359 (1972).
41. R. Venkatsubramanian, S. Bothra, S. K. Ghandhi and J. M. Borrego, in *12th IEEE Photovoltaic Specialists Conf.*, p. 689 (1988).
42. H. L. Chuang, M. E. Klausmeieir-Brown, M. R. Melloch and M. S. Lundstrom, *J. appl. Phys.* **66**, 273–277 (1989).
43. J. G. Ruch and G. S. Kino, *Phys. Rev.* **174**, 921–931 (1969).
44. P. A. Houston and A. G. R. Evans, *Solid-St. Electron.* **20**, 197–204 (1977).
45. N. Braslau and P. S. Hauge, *IEEE Trans. Electron Devices* **ED-17**, 616–622 (1970).