

# Irradiance and temperature corrections of current-voltage curves—Quintessential nature and implications

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## ABSTRACT

Photovoltaic modules are rated at standard test conditions, STC, but in real life operation they are exposed to very different conditions. The power of a module and its device parameters are obtained from the current-voltage characteristic (I-V curve) of the module but the curve varies with irradiance and temperature conditions. The I-V curve is transformed from real operational conditions to STC and vice versa with translation equations. In this paper we investigate the fundamental nature of the translation equations and show that they introduce consistent bias in the power and device parameters, obtained from the translated I-V curves. We show that the nature of the translation equations forces the device parameters to transform from one to another set of irradiance and temperature conditions by algebraic transformations, which are derived in this paper. Validation of our findings is given for the series resistance. Our findings have significant implications for the assessment of module degradation.

## 1. Introduction

The power output of solar modules is strongly influenced by the amount of solar radiation incident on them and the temperature of the solar cells in them. The power and other characteristics of a module are specified at standard test conditions, STC, defined as: irradiance of  $1000 \text{ W m}^{-2}$ , solar cell temperature of  $25^\circ\text{C}$  and solar spectrum AM 1.5. The STC conditions are achievable in the laboratory but are almost never met in real life operation in the field. Predicting the performance of a photovoltaic (PV) system in field conditions involves transforming the current-voltage curve and the power of the solar modules from STC conditions to irradiance and temperature conditions encountered in the field. Module reliability and degradation assessments involve establishing changes in the STC power of a module after exposure to field conditions for some length of time. Uninstalling modules of a PV system for testing in the laboratory is frequently impracticable and therefore the current-voltage (I-V) characteristics and power of the modules are measured under field conditions and need to be transformed to STC conditions for comparison with the rated power.

The transformation of the I-V curve and power of a module from measured to a target set of irradiance and temperature conditions is achieved with algebraic equations, collectively called translation

equations. Many translation equations have been proposed (Sandstrom, 1967; Blaesser and Rossi, 1988; Blaesser, 1997; Anderson, 1996; Marion, 2002; Marion et al., 2004; Kratochvil et al., 2004; Tsuno et al., 2006; Tsuno and Hishikawa, 2012; Ding et al., 2014; Hishikawa et al., 2019) that, according to the authors of each, achieve high fidelity in reproduction of a module's I-V curve in the target irradiance and temperature conditions. Currently, three sets of translation equations of varying complexity and applicability are included in the international IEC 60891 (2009) standard for temperature and irradiance corrections to measured I-V characteristics. Some of the IEC 60891 translation procedures require several curve correction factors which are not provided by module manufacturers and present significant challenges for experimental determination.

The translation equations' applications in energy output modelling and in module reliability assessments raise the question of their performance. Duck et al. (2013) have used extensive outdoor data to compare the performance of the three IEC 60891 procedures over different ranges of irradiance and module temperature conditions. The authors obtained I-V curves at five irradiance and four temperature values, generating twenty reference I-V curves, each of which they translated to various outdoor conditions with the three IEC 60891 translation procedures. They calculated the maximum power from the

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translated curves and compared it with the maximum power of the corresponding outdoor measured I-V curves. The authors found that none of the translation procedures achieves highest accuracy, translating the reference curves from measured to target conditions, across all irradiance and temperature values and suggest that the selection of a translation procedure be based on the prevalent irradiance and temperature at the given location.

Pó et al. (2011) have assessed the sensitivity to variations in the inputs of two of the IEC 60891 translation procedures. They have divided the outdoor measured data into a set in which the short-circuit current is strictly linear (within 2%) with irradiance and another set in which such strict linearity condition is not met. The measured data set has been translated with the curve correction factors, required by the procedures, and without these curve correction factors. The results reveal systematic trends in the differences between measured and translated power in all cases regardless of whether curve correction factors are used or whether the linearity condition is met. The magnitude of the effect is smaller when the linearity condition is met and the curve correction factors are used. If the translation equations reproduced accurately an I-V curve in a given set of conditions, then the differences between quantities obtained from translated curves and from actually measured curves, should be random without any systematic trends. The results of Pó et al. (2011) seem to indicate the presence of a systematic bias in the power obtained from the translated I-V curves.

Hishikawa et al. (2016) have explored the transformation to target conditions of measured outdoor I-V curves by applying translation equations proposed by Tsuno et al. (2006). Their results also show systematic trend in the difference between the power obtained from the translated curves and the actually measured power. The trend appears to become more pronounced with increase in irradiance difference between STC and outdoor conditions.

Whitaker and Newmiller (1998) have assessed the translation equations suggested by Blaesser and Rossi (1988, 1997) and Anderson (1996) while developing energy rating procedures for modules. They have found that the module power can be accurately modelled in outdoor conditions only when the model coefficients are determined under the particular outdoor conditions. The translation equations they tested produced fairly good results whenever the conditions being translated from and to are not very different.

Another frequently used approach to obtaining I-V curves in particular irradiance and temperature conditions is based on iterative numerical methods. These use either the one- or two-diode model for the PV module, and applying Newton's method, Levenberg-Marquardt algorithm or others, iteratively adjust the device parameters to obtain the I-V curve in a particular set of conditions. The numerical methods output the maximum power, the short-circuit current, open-circuit voltage, series and shunt resistances and other parameters of the device in the process of iterative curve fitting. The numerical methods, perhaps due to their complexity, are perceived as more accurate in obtaining I-V curves at particular irradiance and temperature conditions than the algebraic translation equations but Hermann and Wiener (1996) have shown (in a round robin test) that neither the numerical nor the algebraic methods are superior in their accuracy of reproducing the measured outdoor curve or in reducing the measured curve to STC conditions.

It appears that the limitations of the algebraic and numerical methods for obtaining I-V curves in certain irradiance and temperature conditions originate in the nature of the I-V curves and the translation equations. In this paper we investigate the fundamental nature of the translation equations and its implications for the translation results. We demonstrate that even the device parameters can be transformed between an old and new set of irradiance and temperature conditions simply by algebraic transformations and derive the transformation laws. We justify why certain translation procedures may not produce viable results and why numerical methods for obtaining I-V curves, module power and device parameters may not offer advantages over the algebraic translation equations. Several of the most frequently used

translation equations are outlined in Section 2. The underlying mathematical nature of the translation equations, its implications for the transformations of the power and device parameters, and derivation of the parameter transformation laws are presented in Section 3. The agreement between our expectations for the behaviour of the device parameters, obtained from translated I-V curves, and their actual behaviour is demonstrated in Section 4. Our conclusions about the implications of the nature of the translation equations and the I-V curves for the assessment of module degradation or for module performance are presented in Section 5. Definitions of the mathematical concepts used in this paper and some of their important properties are given in Appendix A.

## 2. Translation equations

For most practical purposes the relationship between the current,  $I$ , and voltage,  $V$ , of a PV module, is given by the single-diode equation, Eq. (1),

$$I = I_{ph} - I_0 \left( e^{\frac{q(V+IR_s)}{nkT}} - 1 \right) - \frac{V + IR_s}{R_{sh}} \quad (1)$$

where  $I_{ph}$  is the light generated current,

$I_0$  is the dark saturation current,  
 $R_s$  is the series resistance,  
 $R_{sh}$  is the shunt resistance,  
 $n$  is the ideality factor,  
 $T$  is the absolute temperature and  
 $k$  is the Boltzmann constant.

The graphical representation of the current-voltage relationship, Eq. (1), is the current-voltage curve (I-V curve) of the module, shown in Fig. 1, and  $I_{ph}$ ,  $I_0$ ,  $R_s$ ,  $R_{sh}$  and  $n$  are the device parameters of the module. The parameters are not directly measurable but are extracted from the I-V curve. The maximum power of a module,  $P_{mpp}$ , at voltage  $V_{mpp}$  and current  $I_{mpp}$ , is also obtainable from the I-V curve. The values of  $R_s$ ,  $I_0$  and  $n$  are related to the curvature of the I-V curve between  $V_{mpp}$  and  $V_{oc}$ , while the values of  $R_{sh}$  and  $I_{ph}$  are related to the curvature between  $I_{sc}$  and  $I_{mpp}$ . The translation equations use the coordinates,  $(V, I)$ , of points on the I-V curve, measured at certain irradiance and temperature conditions, the values of the irradiance and the temperature, and the values

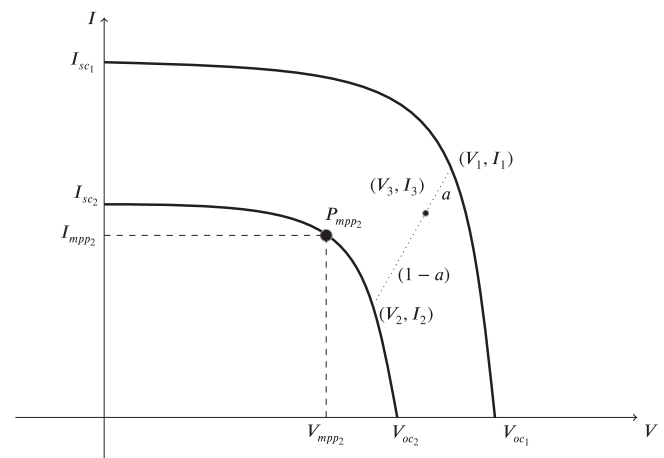


Fig. 1. The solid lines are the I-V curves in different sets of irradiance and temperature conditions,  $(G_1, T_1)$  and  $(G_2, T_2)$ . The dotted line segment between the two I-V curves connects points for which  $I_2 - I_1 = I_{sc2} - I_{sc1}$ , as required in the Correction procedure 3 of IEC 60891. The line segment is divided at a certain ratio by  $a$ , obtained from Eq. (15) or Eq. (16).

of current, voltage and power temperature coefficients to transform the available I-V curve to an I-V curve in a target irradiance and temperature conditions.

### 2.1. Anderson translation equations

The translation equations suggested by Anderson (1996) for transforming the current and the voltage, are reproduced here by Eqs. (2)–(5)

$$I_{sc2} = \frac{I_{sc1}}{[1 + \alpha(T_1 - T_2)] \left( \frac{G_1}{G_2} \right)} \quad (2)$$

$$V_{oc2} = \frac{V_{oc1}}{[1 + \beta(T_1 - T_2)] \left( 1 + \delta \ln \left( \frac{G_1}{G_2} \right) \right)} \quad (3)$$

$$V_2 = V_1 \frac{V_{oc2}}{V_{oc1}} \quad (4)$$

$$I_2 = I_1 \frac{I_{sc2}}{I_{sc1}} \quad (5)$$

where  $T$  denotes temperature,  $G$  denotes irradiance,  $\alpha$  is the relative short-circuit-current temperature coefficient in units of  $^{\circ}\text{C}^{-1}$ , given by Eq. (6)

$$\alpha = \frac{1}{I_{sc}} \left( \frac{\partial I}{\partial T} \right) \bigg|_{G, I_{sc}}, \quad (6)$$

$\beta$  is the relative open-circuit-voltage temperature coefficient in  $^{\circ}\text{C}^{-1}$ , given by Eq. (7)

$$\beta = \frac{1}{V_{oc}} \left( \frac{\partial V}{\partial T} \right) \bigg|_{G, V_{oc}}, \quad (7)$$

$\delta$  is a dimensionless irradiance correction factor which assumes different values for the different PV technologies, subscript 1 indicates quantities in the old set of conditions (usually measured) and subscript 2 indicates the corresponding quantities in the new (target) set of conditions. The temperature coefficients are usually provided in the data sheet of a module but if they are to be determined independently, Anderson (1996) has argued that using either  $I_{sc2}$  or  $I_{sc1}$  as normalising factor for  $\alpha$  and either  $V_{oc2}$  or  $V_{oc1}$  as normalising factor for  $\beta$  results in a negligibly small error in the temperature coefficients.

Anderson (1996) suggests two equations for the transformation of the maximum power, given by Eq. (8) and Eq. (9), which he considers equivalent.

$$P_{max2} = \frac{P_{max1} \left( \frac{G_2}{G_1} \right)}{(1 + \alpha(T_1 - T_2))(1 + \beta(T_1 - T_2)) \left( 1 + \delta \ln \left( \frac{G_1}{G_2} \right) \right)} \quad (8)$$

$$P_{max2} = \frac{P_{max1} \left( \frac{G_2}{G_1} \right)}{(1 + \gamma(T_1 - T_2)) \left( 1 + \delta \ln \left( \frac{G_1}{G_2} \right) \right)} \quad (9)$$

The coefficient  $\gamma$ , in units  $^{\circ}\text{C}^{-1}$ , is the relative temperature coefficient for the maximum-power-point and is given by Eq. 10

$$\gamma = \frac{1}{P_{max}} \left( \frac{\partial P}{\partial T} \right) \bigg|_{G, P_{max}}. \quad (10)$$

### 2.2. IEC 60891 translation equations

The international IEC 60891 (2009) standard provides three procedures for temperature and irradiance corrections of measured I-V

curves. Correction procedure 1 is based on work by Sandstrom (1967), Correction procedure 2 is based on the one-diode model and is empirically obtained while Correction procedure 3 is based on the works of Marion et al. (2004) and Tsuno et al. (2006). The first and second correction procedures in IEC 60891 require curve correction factors and parameters of the PV device, the determination of which can be achieved only in highly specialised laboratories. The main pre-requisite for the application of the IEC 60891 translation equations is for the devices' response to be linear with irradiance and temperature. The IEC 60891 translation procedures do not provide equations for translation of the maximum power point so that the maximum power on the translated curves must be obtained by other means. We replicate here only two of the three sets of translation equations in IEC 60891.

#### 2.2.1. Correction procedure 2 of IEC 60891

This procedure is recommended for irradiance corrections larger than 20%. The translation equations of Correction procedure 2 are given by Eqs. (11) and (12)

$$I_2 = I_1 (1 + \alpha_{rel}(T_2 - T_1)) \frac{G_2}{G_1} \quad (11)$$

$$V_2 = V_1 + V_{oc1} \left( \beta_{rel}(T_2 - T_1) + a \ln \frac{G_2}{G_1} \right) - R'_s(I_2 - I_1) - \kappa' I_2(T_2 - T_1) \quad (12)$$

where index 1 indicates old (measured) conditions and index 2 indicates new (target) set of conditions, so that:

$V_1, I_1$	are coordinates of points on the measured I-V curve;
$V_2, I_2$	are coordinates of the corresponding points on the translated I-V curve;
$G_1$	is the measured irradiance;
$G_2$	is the target irradiance for the translated I-V curve;
$T_1$	is the measured temperature of the device under test;
$T_2$	is the target temperatures of the device under test;
$V_{oc1}$	is the open circuit voltage at test conditions;
$\alpha_{rel}$ and $\beta_{rel}$	are relative short-circuit and relative open-circuit temperature coefficients, respectively,
	in units $^{\circ}\text{C}^{-1}$ which have been determined at irradiance of $1000 \text{ W m}^{-2}$ ;
$a$	is irradiance correction factor for open circuit voltage, dimensionless;
$R'_s$	is the series resistance of the device under test;
$\kappa'$	is temperature coefficient of the series resistance $R'_s$ .

The five correction parameters  $\alpha_{rel}$ ,  $\beta_{rel}$ ,  $a$ ,  $R'_s$  and  $\kappa'$  must be determined by I-V curve measurements at various irradiance and temperature conditions, as prescribed in IEC 60891 (2009).

#### 2.2.2. Correction procedure 3 of IEC 60891

This procedure does not require curve correction coefficients or fitting parameters. This correction procedure requires at least two measured I-V curves that span the target temperature and irradiance conditions. The translation equations are given by Eqs. (13) and (14)

$$V_3 = V_1 + a \cdot (V_2 - V_1) \quad (13)$$

$$I_3 = I_1 + a \cdot (I_2 - I_1) \quad (14)$$

where

$V_1, I_1$	are coordinates of points on measured curve at irradiance $G_1$ and temperature $T_1$ ;
$V_2, I_2$	are coordinates of points on another measured curve at irradiance $G_2$ and temperature $T_2$ ;
$V_3, I_3$	are coordinates of corresponding points on the translated curve at target irradiance $G_3$ and temperature $T_3$ ;
$a$	is the transformation coefficient that is related to irradiance and temperature through Eq. (15) or Eq. (16)

$$G_3 = G_1 + a \cdot (G_2 - G_1) \quad (15)$$

$$T_3 = T_1 + a \cdot (T_2 - T_1). \quad (16)$$

The target irradiance and temperature  $G_3$  and  $T_3$  are not independent—once the target irradiance is selected, the target temperature is fixed by the value of  $a$ , or if the target temperature is selected, then the target irradiance is fixed by the value of  $a$ . The points  $(V_1, I_1)$  on the first I-V curve and  $(V_2, I_2)$  on the second I-V curve must be chosen so that  $I_2 - I_1 = I_{sc2} - I_{sc1}$ , as shown in Fig. 1. The translation procedure is called interpolation (between the two measured curves) when  $0 < a < 1$  and extrapolation when  $a > 1$  or  $a < 0$ . This method requires measured data beyond  $V_{oc}$  for the translation of a complete I-V curve.

### 3. Nature of the translation equations

The nature of the translation equations can be understood on the basis of convex sets, affine transformations and convex/concave functions. Extensive treatment of these concepts can be found in the books by Giaquinta and Modica (2011), Boyd and Vandenberghe (2004) or Abstract Algebra textbooks. We have provided the most important concepts in Appendix A. In terms of these concepts, the region bounded by the current axis from  $I = 0$  to  $I = I_{sc}$ , the voltage axis from  $V = 0$  to  $V = V_{oc}$ , and the I-V curve is a convex set (Definition 1, Appendix A). The extreme points of this set are the points on the I-V curve and the origin. The I-V curve is the graph of a concave function. Linear transformations like stretching, shrinking, rotation together with translation constitute affine transformations (Appendix A.2). An affine transformation maps the convex set  $\{(V, I) \mid 0 \leq V \leq V_{oc}, 0 \leq I \leq I_{sc}, V \text{ and } I \in \mathbb{R}_{\geq 0}\}$  into a similarly convex set, the extreme points of the set into extreme points, and the I-V curve into a similarly concave curve. The equations Eqs. (4) and (5) can be written as a scaling transformation (Appendix A.2) of the current and voltage in the first set of conditions (index 1), given by equations Eq. (17)

$$\begin{aligned} V_2 &= \frac{V_1}{a} \\ I_2 &= \frac{I_1}{b} \end{aligned} \quad (17)$$

where the scaling factors  $a$  and  $b$  are given by equations Eq. (18)

$$\begin{aligned} a &= (1 + \beta(T_1 - T_2)) \left( 1 + \delta \ln \left( \frac{G_1}{G_2} \right) \right) \\ b &= (1 + \alpha(T_1 - T_2)) \frac{G_1}{G_2}. \end{aligned} \quad (18)$$

Now, considering that the relationship between the current and the voltage, Eq. (1), is valid for all sets of operating conditions, then the equation of the I-V curve in the first set of conditions will be given by Eq. (19)

$$I_1 = I_{ph1} - I_{01} \left( \exp \left( \frac{q(V_1 + I_1 R_{s1})}{n_1 k T} \right) - 1 \right) - \frac{V_1 + I_1 R_{s1}}{R_{sh1}}. \quad (19)$$

Scaling  $a$  units in the horizontal direction and  $b$  units in the vertical direction, transforms Eq. (19) into Eq. (20)

$$bI_2 = I_{ph1} - I_{01} \left( \exp \left( \frac{q(aV_2 + bI_2 R_{s1})}{n_1 k T} \right) - 1 \right) - \frac{aV_2 + bI_2 R_{s1}}{R_{sh1}}. \quad (20)$$

Now, taking  $a$  outside the brackets, dividing by  $b$  and rearranging further so that the equation looks like a typical I-V equation, we obtain Eq. (21)

$$\begin{aligned} bI_2 &= I_{ph1} - I_{01} \left( \exp \left( \frac{aq(V_2 + \frac{b}{a}I_2 R_{s1})}{n_1 k T} \right) - 1 \right) - \frac{a(V_2 + \frac{b}{a}I_2 R_{s1})}{R_{sh1}} \\ I_2 &= \frac{I_{ph1}}{b} - \frac{I_{01}}{b} \left( \exp \left( \frac{q(V_2 + \frac{b}{a}I_2 R_{s1})}{\frac{n_1}{a} k T} \right) - 1 \right) - \frac{(V_2 + \frac{b}{a}I_2 R_{s1})}{\frac{bR_{sh1}}{a}} \\ I_2 &= I_{ph2} - I_{02} \left( \exp \left( \frac{q(V_2 + I_2 R_{s2})}{n_2 k T} \right) - 1 \right) - \frac{(V_2 + I_2 R_{s2})}{R_{sh2}} \end{aligned} \quad (21)$$

where

$$\begin{aligned} I_{ph2} &= \frac{I_{ph1}}{b} \\ I_{02} &= \frac{I_{01}}{b} \\ n_2 &= \frac{n_1}{a} \\ R_{sh2} &= \frac{bR_{sh1}}{a} \\ R_{s2} &= \frac{b}{a}R_{s1} \end{aligned} \quad (22)$$

The translation of the I-V curve Eq. (19) into Eq. (21) is an affine transformation from which follow the algebraic transformations Eq. (22) for the device parameters.

Equations (11) and (12) of the second IEC 60891 procedure for transforming the I-V curve are mathematically equivalent to a translation in the horizontal direction (Eq. (23)) and a stretch/shrink in the vertical direction (Eq. (24)) (according to Appendix A.2):

$$V_2 = V_1 + c \quad (23)$$

$$I_2 = dI_1 \quad (24)$$

where  $c$  and  $d$  denote the following (Eq. (25))

$$\begin{aligned} c &= Voc_1 \left[ \beta_{rel}(T_2 - T_1) + a \ln \left( \frac{G_2}{G_1} \right) \right] - R'_s(I_2 - I_1) - \kappa' I_2(T_2 - T_1) \\ d &= (1 + \alpha_{rel}(T_2 - T_1)) \frac{G_2}{G_1}. \end{aligned} \quad (25)$$

Applying the transformations Eqs. (23) and (24) to the I-V equation Eq. (1), leads to Eq. (26)

$$\frac{I_2}{d} = I_{ph1} - I_{01} \left( \exp \left( \frac{q \left( (V_2 - c) + \frac{I_2}{d} R_{s1} \right)}{n k T} \right) - 1 \right) - \frac{(V_2 - c) + \frac{I_2}{d} R_{s1}}{R_{sh1}}. \quad (26)$$

Similarly to before, after successive rearrangements, Eq. (26) can be rewritten as a typical I-V equation in the second set of conditions, Eq. (27)

$$\begin{aligned} \frac{I_2}{d} &= I_{ph1} - I_{01} \left( \exp \left( \frac{q \left( V_2 + I_2 R_{s1} \left( \frac{1}{d} - \frac{c}{I_2 R_{s1}} \right) \right)}{n k T} \right) - 1 \right) - \frac{V_2 + I_2 R_{s1} \left( \frac{1}{d} - \frac{c}{I_2 R_{s1}} \right)}{R_{sh1}} \\ I_2 &= I_{ph2} - I_{02} \left( \exp \left( \frac{q(V_2 + I_2 R_{s2})}{n k T} \right) - 1 \right) - \frac{V_2 + I_2 R_{s2}}{R_{sh2}} \end{aligned} \quad (27)$$

where

$$\begin{aligned}
I_{ph2} &= I_{ph1} d \\
I_{02} &= I_{01} d \\
n_2 &= n_1 = n \\
R_{sh2} &= \frac{R_{sh1}}{d} \\
R_{s2} &= R_{s1} \left( \frac{1}{d} - \frac{c}{I_2 R_{s1}} \right) = \frac{R_{s1}}{d} \left( 1 - \frac{c}{I_1 R_{s1}} \right)
\end{aligned} \quad (28)$$

Equations Eq. (28) also constitute algebraic transformations of the device parameters from one set of conditions to another set of conditions. It is shown in Appendix A.2.1 that the translation equations of IEC 60891 Correction procedure 1 are also affine transformations of the I-V curve, resulting in algebraic transformations of the device parameters.

By considering the current and voltage as ordered pairs, equations Eqs. (13) and (14) of the third IEC 60891 procedure can be written as Eq. (29)

$$\begin{pmatrix} V_3 \\ I_3 \end{pmatrix} = (1-a) \begin{pmatrix} V_1 \\ I_1 \end{pmatrix} + a \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \quad (29)$$

For  $0 < a < 1$ , Eq. (29) represents the convex combination (Definition 2, Appendix A) of the points  $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$  and  $\begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$  on the first and second I-V curve, respectively. The transformation Eq. (29) is named interpolation (for  $0 < a < 1$ ) in the IEC 60891 (2009) documentation. The ratio at which the line segment, connecting the points  $\begin{pmatrix} V_1 \\ I_1 \end{pmatrix}$  and  $\begin{pmatrix} V_2 \\ I_2 \end{pmatrix}$ , is split is determined by the choice of target irradiance  $G_3$  from  $a = \frac{G_3 - G_1}{G_2 - G_1}$  or target temperature  $T_3$  from  $a = \frac{T_3 - T_1}{T_2 - T_1}$ . Since the non-negative weighted sum of concave functions  $I_i = g_i(V)$ ,  $i = 1, \dots, n$  is a concave function (Appendix A.2), it follows that the functional relationship between the current and voltage with index 3 is the same as the functional relationship between the current and voltage with index 1 and, similarly, between the current and voltage with index 2, i.e. the functional relationship between the points with index 3 is an I-V curve.

The convex combination of two I-V curves is the sum of Eq. (30a) and Eq. (30b)

$$\begin{aligned}
(1-a)I_1 &= (1-a)I_{ph1} - (1-a)I_{01} \left( \exp \left( \frac{(1-a)q(V_1 + I_1 R_{s1})}{n_1 k T} \right) - 1 \right) - (1-a) \frac{V_1 + I_1 R_{s1}}{R_{sh1}} \\
aI_2 &= aI_{ph2} - aI_{02} \left( \exp \left( \frac{aq(V_2 + I_2 R_{s2})}{n_2 k T} \right) - 1 \right) - a \frac{V_2 + I_2 R_{s2}}{R_{sh2}}
\end{aligned} \quad (30a)$$

$$\quad (30b)$$

which, after retention of only first order terms in the Taylor expansion of the exponents, results in Eq. (31)

$$I_3 = I_{ph3} - I_{03} \left( \exp \left( \frac{q(V_3 + I_3 R_{s3})}{n_3 k T} \right) - 1 \right) - \frac{V_3 + I_3 R_{s3}}{R_{sh3}} \quad (31)$$

with parameter transformations given by Eq. (32)

$$\begin{aligned}
I_{ph3} &= (1-a)I_{ph1} + aI_{ph2} \\
I_{03} &= (1-a)I_{01} + aI_{02} \\
n_3 &= (1-a)n_1 + an_2 \\
R_{sh3} &= R_{sh1} = R_{sh2} \\
R_{s3} &= R_{s1} = R_{s2}
\end{aligned} \quad (32)$$

The condition  $I_2 - I_1 = I_{sc2} - I_{sc1}$  ensures that only the end points of line segments (Fig. 1) that are split into the same ratio (Appendix A.1) by the factor  $a$ , are used in the I-V curve transformation. The convex combination of the points on the two I-V curves ensures that the resultant I-V curve is closer to the I-V curve whose conditions are closer to the target irradiance and temperature. The linearity of the device response with

irradiance and temperature is crucial for this method to work.

If the correction factor  $a$  in the third procedure of IEC 60891 (2009) is either  $a > 1$  or  $a < 0$ , then some of the coefficients in Eq. (29) will be negative making the combination not convex but an affine combination (Definition 3, Appendix A) of points on the two measured I-V curves. In this case, the concavity of the weighted sum is not preserved (Appendix A.2), i.e. the weighted sum of the functions, represented by the I-V curves with indices 1 and 2, need not be a function whose graph is an I-V curve. We expect that the extrapolation case in the third procedure of IEC 60891 (2009) will not produce satisfactory results. Tsuno et al. (2006) have reported unsatisfactory results with the extrapolation procedure.

Numerical methods that are employed to generate I-V curves at certain irradiance and temperature conditions and to extract parameters from the I-V curves, are perceived as more accurate alternatives to the algebraic translation equations, but they do not necessarily offer advantages. Since any convex/concave function on a convex set can be obtained through some affine transformation of another convex/concave function on the set, then any I-V curve is obtainable through an affine transformation of another I-V curve. Therefore, a numerically generated I-V curve can be seen as the affine transformation of another (albeit not a priori known) I-V curve. It is immaterial for the device parameters whether the I-V curve is obtained through numerical or algebraic methods and thus the parameters, obtained from an I-V curve generated with some numerical routines, are expected to exhibit the same tendencies as the parameters obtained from curves that are the result of algebraic translation equations.

#### 4. Implications of the translation equations for the device parameters

The validity of the theoretically obtained relations Eq. (22) was tested for the series resistance. A first order approximation to the series resistance, given by Eq. (22), can be obtained with the binomial expansion (Appendix A.3). The approximation is worked out in the Appendix from Eq. (A.11) to Eq. (A.14). The last factor in Eq. (A.14) can be reduced further, noting that for terrestrial non-concentrating applications it is smaller than unity by at least two orders of magnitude, i.e.

$\delta \ln \left( \frac{E_1}{1000} \right) \ll 1$ . Then Eq. (A.14) can be approximated by Eq. (33), which indicates that the series resistance in the new (target) conditions will vary linearly with the irradiance in the old (measured) conditions when the translation of the I-V curve is done with the equations by Anderson (1996),

$$R_{s2} \approx R_{s1} (1 + (\alpha - \beta)(T_1 - 25^\circ\text{C})) \frac{G_1}{1000} \quad (33)$$

The series resistance of a module is a parameter that is often investigated in respect of degradation assessment of the module (Carlsson and Brinkman, 2006). Even a modest increase in the series resistance can cause significant drop in the module's power. The series resistance of a module can be obtained from the I-V curve by various methods (Pysch et al., 2007; Wolf and Rauschenbach, 1963; Kennerud, 1969; Rajkanan and Shewchun, 1979; Aberle et al., 1993; Kunz and Wagner, 2004; Yordanov et al., 2010).

We performed outdoor I-V tracing measurements on a string of polycrystalline silicon modules, which is part of an operational PV system with location coordinates 22.5553° S, 17.0852° E. The measurements were performed within two hours around solar noon on a perfectly clear sky day with plane-of-array (POA) irradiance ranging from 826 W m<sup>-2</sup> to 1124 W m<sup>-2</sup>. The module temperature is the average of many measurements at various points on many modules of the string and ranges from 48.7°C to 69.6°C. The measured I-V curves were translated to STC conditions with Eqs. (2) to (5). The series resistance was extracted from the measured (this is  $R_{s1}$ ) and from the translated to STC conditions I-V curves (this is  $R_{s2}$ ) with



three different methods that we named curve fit, Wagner and Yordanov method. The curve fit method is a numerical method, based on Newton's algorithm, that applies iterative non-linear curve-fitting procedure to the one-diode model (routines for such are available from Schulte (2014)). The Wagner method, a revised version of which is published by Kunz and Wagner (2004), use one measured outdoors and one synthetic I-V curve to obtain the series resistance. The Yordanov method, proposed by Yordanov et al. (2010), is based on the observation that the semi-logarithmic plot of current versus voltage drop at the junction of a solar cell should yield a straight line; then the value of the series resistance is the one that produces best linearity. Detailed outlines of all three methods are given by Dobрева (2018). The series resistance obtained from the I-V curves was corrected for the resistance of the cabling. The values of the STC series resistance, obtained with the three methods, are not equal but they all show increasing with the measured POA irradiance trend, shown in Fig. 2. This is in good agreement with the expectations of the affine transformations approach, Eq. (33). The larger spread in the series resistance values obtained with the curve fit method is due to the nature of the non-linear numerical methods that can produce non-unique results (Appelbaum et al., 1993).

The series resistance, obtained with a particular method from the translated to STC conditions I-V curves, was regressed on the measured POA irradiance,  $R_{s,STC} = a + b \cdot POA$ , to test the validity of the linear trend. The regression for each case produced statistically significant slopes at the 5% significance level, indicating that the linear trend of the series resistance, obtained from translated I-V curves, with measured POA irradiance is not a chance occurrence. The regression coefficients and the  $p$ -values of the slopes are given in Table 1.

One of the most common use of the translation equations is for obtaining the maximum power in a set of conditions that is different from the field conditions in which measurements are done. Frequently, this means translating the field measured I-V curves to STC conditions and obtaining the maximum power from the translated curves. Most translation procedures do not provide expressions for translation of the power and the maximum power in the new set of conditions must be obtained from the translated I-V curves. The translation equations by Anderson (1996) offer two options for translation of the power, reproduced in this paper by Eq. (8) and Eq. (9). We used the two equations to translate the field measured power to STC power and assessed the deviations of the translated values from the expected values. We obtained the expected values using the STC maximum power from the data sheet of the modules and applied to it the degradation rate,  $r$ , suggested by the module manufacturer. The expected STC power after  $n$  years of degradation is equal to  $P_{STC,expect} = P_{STC}(1 - r)^n$ , where  $P_{STC}$  is the STC power from the data sheet. For the polycrystalline silicon modules in our measurements, the data sheet degradation rate is 0.7% p.a. The

**Table 1**

Regression results for the regression of STC series resistance on irradiance,  $R_{s,STC} = a + b \cdot POA$ . The column Method indicates the method used to extract the series resistance from STC translated I-V curves. The column  $p$ -value contains the  $p$ -values of the slopes of the regression lines. The regression lines are shown in Fig. 2.

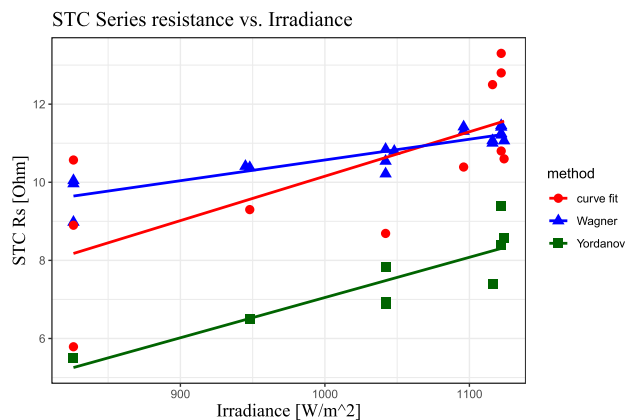
Method	Intercept, $a$ , in $\Omega$	Slope, $b$ , in $\Omega \text{ m}^2 \text{ W}^{-1}$	$p$ -value
Curve fit	−1.215	$1.137 \times 10^{-2}$	0.017
Wagner	5.262	$5.3097 \times 10^{-3}$	< 0.001
Yordanov	−3.255	$1.0307 \times 10^{-2}$	0.002

measurements were done four years after the modules were deployed in the field.

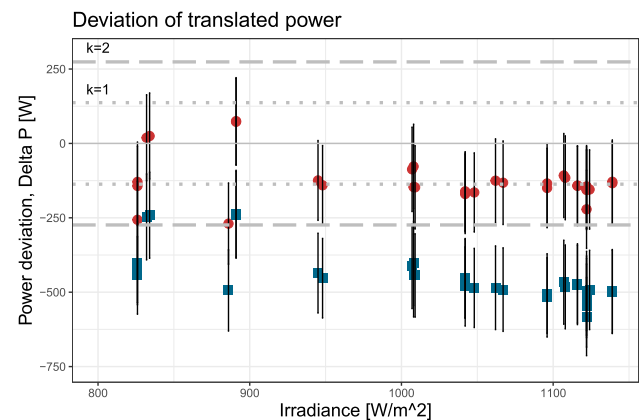
The deviations of the translated to STC conditions power from the expected STC power should spread randomly around zero within a range comparable with the translation uncertainty. The deviations should not exhibit any systematic trends. The uncertainty in the measured power is 4.9% and the uncertainty in the translated power, estimated by error propagation, is 5.5%. The deviations of the translated with Eq. (8) and Eq. (9) power from the expected are shown in Fig. 3. The expanded uncertainty coverage is indicated by the dotted lines (coverage factor  $k = 1$ ) and by the dashed lines (coverage factor  $k = 2$ ). The deviations in Fig. 3 show tendencies for being systematically negative and are not randomly spread around zero which is a clear indication of consistent bias in the translated values. The deviations are larger when the translations are done with Eq. (8) compared to when the translations are done with Eq. (9).

The deviations of the power when translations are done with Correction procedure 1 and Correction procedure 2 of IEC 60891 (2009) were investigated by Pó et al. (2011). Their results show that the deviations of the translated power from the carefully measured by them module power at STC, follow systematic trends with the measured irradiance, in agreement with our findings for the power deviations when translations are done with Anderson's equations.

Fig. 3 shows that non-negligible and systematic differences in the translated power might arise when the translations are done with different equations. One of the reasons for the different translated power values, depending on whether the translation is done with Eq. (8) or with Eq. (9), is that the two equations are not equivalent. Equations Eq. (8) and Eq. (9) will be equivalent if  $\gamma = (\alpha + \beta)$ . It can be shown that this is not the case, as follows:



**Fig. 2.** Variation of the series resistance with measured POA irradiance. Curve fit, Wagner and Yordanov indicate the series resistance extracted with the particular method from translated to STC conditions I-V curves. The regression lines are shown in the same colours as the method used for extraction of the series resistance.



**Fig. 3.** Deviations of the translated to STC conditions power from the expected power versus POA irradiance. The squares indicate translation with Eq. (8), while the circles indicate translation with Eq. (9). The dotted lines indicate uncertainty coverage factor  $k = 1$  and the dashed lines indicate uncertainty coverage factor  $k = 2$ .

$$\begin{aligned} \frac{1}{P_{max}} \left( \frac{\partial P}{\partial T} \right)_G \Big|_{P_{max}} &= \frac{1}{P_{max}} \left( \frac{\partial(IV)}{\partial T} \right)_G \Big|_{P_{max}} \\ &= \frac{1}{I_{max}} \left( \frac{\partial I}{\partial T} \right)_G \Big|_{P_{max}} + \frac{1}{V_{max}} \left( \frac{\partial V}{\partial T} \right)_G \Big|_{P_{max}} \end{aligned} \quad (34)$$

It is evident from Eqs. (6), (7) and (34) that  $\gamma \neq (\alpha + \beta)$ .

## 5. Conclusions

In this paper we have shown that the nature of the translation equations that transform I-V curves from one to another set of irradiance and temperature conditions is that of affine transformations of concave functions on convex sets. We demonstrated that as a result of this nature of the translation equations, the device parameters in the target irradiance and temperature conditions can be obtained by algebraic transformations of their values in the measured conditions and derived the transformation laws for them. This behaviour of the device parameters was tested for the series resistance and a good agreement between the properties expected by the derived in this paper transformation law for it and its actual behaviour was found. It is evident from our results and from studies, referenced in Section 1, that the values of the power and device parameters, obtained from translated I-V curves, will exhibit a bias that is due to the translation procedure that is applied and the initial conditions from which the translation is made. Changes in the power and device parameters cannot be reliably established from quantities obtained from translated I-V curves. Assessment of device degradation on the basis of translated I-V curves might lead to incorrect conclusions because even subtle variations in the shape of the I-V curve, arising from the translation, may lead to substantial change in the values of the parameters obtained from the translated curve. Such changes cannot necessarily be attributed to actual changes in the physical properties of the device.

We demonstrated that Correction procedure 3 of IEC 60891 represents either a convex combination (called interpolation in IEC 60891) or an affine combination (called extrapolation in IEC 60891) of points on two measured

I-V curves. We expect that the extrapolation procedure of Correction procedure 3 of IEC 60891 will not produce good results because affine combinations of concave functions (like the I-V curve) need not result in concave functions, i.e. need not result in an I-V curve.

Our findings on the fundamental nature of the translation equations and the I-V curves show that any I-V curve, even one that is obtained with numerical methods, can be represented as an affine transformation or as a convex combination of other I-V curves. Therefore, numerical iterative routines for generation of I-V curves at certain irradiance and temperature conditions and the device parameters, obtained from those curves, cannot be superior or more accurate than those obtained with algebraic translation equations.

Some of the commonly used translation equations like Anderson's or those of IEC 60891 present additional challenges either because they produce non-equivalent results, like Eq. (8) and Eq. (9), or because of the complexities in determination of the correction coefficients they require, like Correction procedure 1 and Correction procedure 2 of IEC 60891 (2009).

The translation equations are essential in modelling the performance of solar modules and may be used to transform the I-V curve and power to conditions different from STC, but they inevitably lead to higher uncertainty in the predicted performance that needs to be taken into account.

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## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Appendix A

The concepts of convex sets, convex and affine combinations, convex and concave functions and affine transformations, used in this paper, can be found in the books by [Giaquinta and Modica \(2011\)](#) or [Boyd and Vandenberghe \(2004\)](#), or in books on Abstract Algebra.

### A.1. Convexity

**Definition 1.** A set  $S, S \subset \mathbb{R}^n$ , is *convex* if for any two elements  $x_1$  and  $x_2$  of  $S$  and for any real number  $\lambda \in \mathbb{R}$ , such that  $0 \leq \lambda \leq 1$ , the line segment  $\lambda x_1 + (1 - \lambda)x_2 \in S$

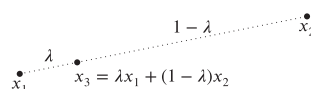
belongs to  $S$ . This means that a set is convex if the line segment, connecting any two points of the set, lies entirely in the set.

An *extreme point* of a convex set is a point of the set that cannot be between other points on the line segment connecting two points of the set. For example, a triangle with its interior is a convex set and the vertices are extreme points of the set.

**Definition 2.** The linear combination

$$\sum_{i=1}^k \lambda_i x_i$$

of points  $x_1, x_2, \dots, x_k \in \mathbb{R}^n$  with coefficients  $\lambda_1, \lambda_2, \dots, \lambda_k \geq 0$ , such that  $\sum_{i=1}^k \lambda_i = 1$ , is called a *convex combination* of  $x_1, x_2, \dots, x_k$ . The convex combination of two points is illustrated in [Fig. 4](#). A convex set contains all convex combinations of its points, i.e. it contains all line segments



**Fig. 4.** Convex combination of points  $x_1$  and  $x_2$ . The coefficient  $\lambda$  is  $0 \leq \lambda \leq 1$ . Point  $x_3$ , the result of the convex combination of  $x_1$  and  $x_2$ , is closer to the point with greater weight, i.e. the point with the larger coefficient in the convex combination expression).

connecting any two of its points.

**Definition 3.** The linear combination

$$\sum_{i=1}^k \lambda_i x_i$$

of points  $x_1, x_2, \dots, x_k \in \mathbb{R}^n$  with coefficients  $\lambda_1, \lambda_2, \dots, \lambda_k \in \mathbb{R}$ , such that  $\sum_{i=1}^k \lambda_i = 1$  is called an *affine combination* of  $x_1, x_2, \dots, x_k$ . The affine combination of points is the infinite line containing the points. An affine set is a set that contains all affine combinations of its points, i.e. it contains the infinite lines through any two points of the set.

**Definition 4.** A function  $f: S \rightarrow \mathbb{R}$  defined on a non-empty convex set  $S, S \in \mathbb{R}^n$ , is *convex* if for any  $x, y \in S$  and for every  $\lambda \in [0, 1]$

$$f(\lambda x + (1 - \lambda)y) \leq \lambda f(x) + (1 - \lambda)f(y)$$

This essentially means that the line segment between any two points on the graph of  $f$  is above the graph. Also, if  $f$  is convex and differentiable, then the tangent lines are below the graph of  $f$ . A simple example of a convex function is  $y = x^2$ . The opposite of a convex function is a *concave* function, i.e. if  $f$  is convex, then  $-f$  is concave. The line segment between any two points on the graph of a concave function is below the graph. For a differentiable concave function the tangent lines are above the graph of the function. A simple example of a concave function is  $y = -x^2$ .

## A.2. Affine transformations

If we consider the current and voltage as ordered pairs, then the transformation  $f: X \rightarrow Y$  from a set  $X$  of initial values at some initial irradiance and temperature conditions to values  $Y$  at some target conditions can be represented by the schematic in Fig. 5. The transformation  $f$ , which represents the translation equations, can be given by the affine transformation  $f(\vec{x}) = A\vec{y} + \vec{y}_0$ , where  $A$  is an invertible matrix. (The multivariate approach to the transformation  $f: \mathbb{R}^n \rightarrow \mathbb{R}^n$  can be explored in a future work.) Here, for simplicity, we adopt a single variable approach. An affine transformation is the combination of a linear transformation and a translation. Transformations like scaling (stretching or shrinking), or a rotation, or a reflection, or a shear are linear transformations. For a simple example of an affine transformation we can consider the transformation of a function of a single variable,  $f(x)$ , (where  $f: X \rightarrow Y$ ), given by

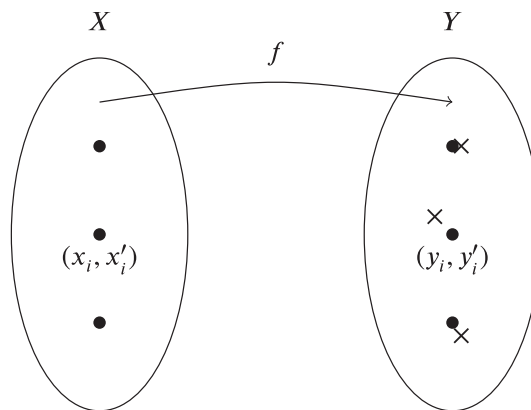
$$y = bf\left(\frac{(x+a)}{c}\right) + d \quad (\text{A.1})$$

where  $a, b, c, d$  are real numbers. The transformation, given by Eq. (A.1), amounts to: translation by  $a$  units in the horizontal direction, scaling by  $c$  units in the horizontal direction, scaling by  $b$  units in the vertical direction and translation by  $d$  units in the vertical direction.

Affine transformations preserve convexity, i.e. a convex set is mapped into a convex set, the extreme points of a convex set are mapped into extreme points of the image convex set. Affine transformations map a convex function into a convex function and a concave into a concave function if the scaling factor in the vertical direction is positive. If the scaling factor is negative, then a convex function is mapped into a concave function and vice versa.

The non-negative weighted sum, Eq. (A.2), of convex functions  $g_1, g_2, \dots, g_k$

$$\sum_{i=1}^k \omega_i g_i \quad (\text{A.2})$$



**Fig. 5.** Schematics of a transformation law  $f$ . The transformation law represents the translation equations. Set  $X$  indicates the values of current and voltage under initial irradiance and temperature conditions. Set  $Y$  indicates the current and voltage in the target irradiance and temperature conditions. The bullets in set  $Y$  indicate the results of the transformation  $f$  while the crosses indicate the true values of current and voltage in the target conditions.



where the weights are non-negative real numbers,  $\omega_1, \omega_2, \dots, \omega_k \geq 0$ , is also a convex function. The same applies to concave functions. The relationship between current and voltage of a solar cell is a concave function  $I = g(V)$  whose graph is the I-V curve. The weighted sum of several such functions (i.e. of several I-V curves) is a likewise concave function (i.e. I-V curve).

#### A.2.1. Correction procedure 1 of IEC 60891 as affine transformation

Correction procedure 1 of IEC 60891 is given by Eqs. (A.3) and (A.4)

$$I_2 = I_1 + I_{sc1} \left( \frac{G_2}{G_1} - 1 \right) + \alpha(T_2 - T_1) \quad (\text{A.3})$$

$$V_2 = V_1 - R_s(I_2 - I_1) - \kappa I_2(T_2 - T_1) + \beta(T_2 - T_1) \quad (\text{A.4})$$

where the quantities with index 1 are the measured values, the quantities with index 2 are the target values,  $\alpha$  is the temperature coefficient of the short-circuit current in units  $\text{A } ^\circ\text{C}^{-1}$ ,  $\beta$  is the temperature coefficient of the open-circuit voltage in units  $\text{V } ^\circ\text{C}^{-1}$ ,  $\kappa$  is curve correction factor in units  $\Omega ^\circ\text{C}^{-1}$  and  $R_s$  is the series resistance of the device. Eqs. (A.3) and (A.4) can be written as translation transformations of the current and voltage, given by Eqs. (A.5)

$$\begin{aligned} I_2 &= I_1 + a \\ V_2 &= V_1 + b \end{aligned} \quad (\text{A.5})$$

where the translations  $a$  and  $b$  are presented by Eq. (A.6)

$$\begin{aligned} a &= I_{sc1} \left( \frac{G_2}{G_1} - 1 \right) + \alpha(T_2 - T_1) \\ b &= -R_s(I_2 - I_1) - \kappa I_2(T_2 - T_1) + \beta(T_2 - T_1). \end{aligned} \quad (\text{A.6})$$

Applying the translations Eq. (A.5) to the I-V curve equation, using similar approach to that in Section 3, results in Eq. (A.7)

$$\begin{aligned} I_2 - a &= I_{ph1} - I_{01} \left( \exp \left( \frac{q(V_2 - b) + (I_2 - a)R_{s1}}{n_1 kT} \right) - 1 \right) - \frac{(V_2 - b) + (I_2 - a)R_{s1}}{R_{sh1}} \\ I_2 &= I_{ph1} + a + \frac{aR_{s1} + b}{R_{sh1}} - I_{01} \left( \exp \left( \frac{q(V_2 + I_2 R_{s1})}{n_1 kT} \right) \exp \left( - \frac{q(aR_{s1} + b)}{n_1 kT} \right) - 1 \right) - \frac{V_2 + I_2 R_{s1}}{R_{sh1}} \end{aligned} \quad (\text{A.7})$$

The unity term within the brackets in the single diode equation can be neglected, and the single diode equation can be well approximated by Eq. A.8

$$I = I_{ph} - I_0 \exp \left( \frac{q(V + IR_s)}{nkT} \right) - \frac{V + IR_s}{R_{sh}}. \quad (\text{A.8})$$

Therefore, Eq. (A.7) can be written as a typical I-V equation for the quantities in the target conditions (index 2) as Eq. (A.9)

$$I_2 = I_{ph2} - I_{02} \exp \frac{q(V_2 + I_2 R_{s2})}{n_2 kT} - \frac{V_2 + I_2 R_{s2}}{R_{sh2}} \quad (\text{A.9})$$

where the parameters are given by Eq. (A.10)

$$\begin{aligned} I_{ph2} &= I_{ph1} + a + \frac{aR_{s1} + b}{R_{sh1}} \\ I_{02} &= I_{01} \exp \left( - \frac{q(aR_{s1} + b)}{n_1 kT} \right) \\ n_2 &= n_1 \\ R_{s2} &= R_{s1} \\ R_{sh2} &= R_{sh1}. \end{aligned} \quad (\text{A.10})$$

It has been shown that Correction procedure 1 of IEC 60891 is an affine transformation of the measured I-V curve and the device parameters can be obtained with algebraic transformations, Eq. (A.10).

#### A.3. Binomial series

A first order approximation to the series resistance in the new (STC in our case) conditions, given by the last equation in the set Eq. (22), can be obtained after rewriting it as Eq. (A.11) and using binomial expansion

$$R_{s2} = R_{s1} (1 + \alpha(T_1 - T_2)) \frac{G_1}{G_2} (1 + \beta(T_1 - T_2))^{-1} \left( 1 + \delta \ln \left( \frac{G_1}{G_2} \right) \right)^{-1}. \quad (\text{A.11})$$

Using the binomial series expansion Eq. (A.12), the two last factors in Eq. (A.11) can be approximated by Eq. (A.13a) and Eq. (A.13b)

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots, \quad \text{valid for } |x| < 1, \quad (\text{A.12})$$

$$(1 + \beta(T_1 - T_2))^{-1} = (1 - \beta(T_1 - T_2) + (\beta(T_1 - T_2))^2 + \dots) \approx (1 - \beta(T_1 - T_2)) \quad (\text{A.13a})$$

$$\text{valid for } |\beta(T_1 - T_2)| < 1,$$

$$\left(1 + \delta \ln\left(\frac{G_1}{G_2}\right)\right)^{-1} = \left(1 - \delta \ln\left(\frac{G_1}{G_2}\right) + \left(\delta \ln\left(\frac{G_1}{G_2}\right)\right)^2 + \dots\right) \approx \left(1 - \delta \ln\left(\frac{G_1}{G_2}\right)\right) \quad (\text{A.13b})$$

$$\text{valid for } \left|\delta \ln\left(\frac{G_1}{G_2}\right)\right| < 1$$

Translation to STC means  $T_2 = 25^\circ\text{C}$  and  $G_2 = 1000 \text{ W m}^{-2}$ . For the polycrystalline modules in our case  $\alpha = 0.00081^\circ\text{C}^{-1}$ ,  $\beta = -0.0037^\circ\text{C}^{-1}$  and  $\delta = 0.11$  (the values of  $\alpha$  and  $\beta$  are obtained from the data sheet while the value of  $\delta$  is as reported by Anderson (1996)), so that the convergence conditions on the binomial expansions are met for  $-245^\circ\text{C} < T_1 < 295^\circ\text{C}$  and  $G_1 < 8.8 \times 10^6 \text{ W m}^{-2}$ . All terrestrial applications of polycrystalline silicon modules are for temperatures and irradiances within these ranges.

After substituting the binomial expansions Eq. (A.13a) and Eq. (A.13b) in Eq. (A.11) and simplifying, the series resistance in the new (STC) conditions, to a first order approximation, is given by Eq. (A.14)

$$R_{s2} \approx R_{s1} (1 + (\alpha - \beta)(T_1 - 25)) \frac{G_1}{1000} \left(1 - \delta \ln\left(\frac{G_1}{1000}\right)\right). \quad (\text{A.14})$$

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