A simple and efficient solar cell parameter extraction method from a single current-voltage curve

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In this work, a simple and efficient method for the extraction of all the parameters of a solar cell from a single current-voltage (*I-V*) curve under the constant illumination level is proposed. With the help of the Lambert W function, the explicit analytic expression for I is obtained. By reducing the number of the parameters, the expression for I only depends on the ideality factor n, the series resistance R_s, and the shunt resistance R_{sh}. This analytic expression is directly used to fit the experimental data and extract the device parameters. This simple solar cell parameter extraction method can be directly applied for all kinds of solar cells whose *I-V* characteristics follow the single-diode model. The parameters of various solar devices including silicon solar cells, silicon solar modules, dye-sensitized solar cells, and organic solar cells with standalone, tandem, and multi-junction structures have been successfully extracted by using our proposed method. © *2011 American Institute of Physics*. [doi:10.1063/1.3632971]

I. INTRODUCTION

Solar cells are promising devices for clean electric power generation. Various intensive research efforts have been devoted to their continuous performance improvement. To estimate solar cell performance and thus further to simulate, design, fabricate, and quality control solar cells, an accurate knowledge of their parameters from experimental data is of vital importance. The electrical characteristic of a solar cell can be described by the equivalent circuit of the single-diode model, the two-diode model,[1] or the three-diode model.[2,3] Among these circuit models, the single-diode model has the simplest form as shown in Fig. 1. Although the single-diode mode is simple, it can well describe the characteristics of various solar cells, satisfy most of the applications, and thus becomes the most widely used circuit model.[4–21] In the single-diode model, the relation of the current I and the voltage V is given as

$$I = I_0\left(\exp\left(\frac{q(V - R_sI)}{nk_BT}\right) - 1\right) + \frac{V - R_sI}{R_{sh}} - I_{ph}, \qquad (1)$$

where I_0 is the saturation current, I_{ph} the photocurrent, R_s the series resistance, R_{sh} the shunt resistance, n the ideality factor, q the electron charge, k_B the Boltzmann constant, and T the temperature. The methods to determine the unknown parameters of I_0, I_{ph}, n, R, and R_{sh} have been the subject of many studies.[4–21] Some methods use the measurements of illuminated *I-V* characteristics at different illumination levels[4–6] and some utilize dark and illuminated measurements.[7–10] However, it should be noted that the device parameters are widely influenced by the different illumination levels.[11] Therefore, it is very important to estimate all the parameters from a single *I-V* curve measured under the condition of one constant illumination level. This is essentially important for the recently emerging organic solar cells, whose parameters have been shown to be greatly influenced by different illumination levels.[12]

Recently, Ishibashi *et al.* introduced one method to extract all the parameters of a solar cell under one constant illumination level.[13] However, their method needs to calculate the differential value dV/dI from the experimental data, which requires a very smooth *I-V* curve. Thus, the polynomial approximation or other method to smooth the experimental curve is inevitable. Furthermore, in their method, only a part of the experimental data can be used to extract the parameters because the differential dV/dI will have a very large error when I is close to the short circuit current (I_{SC}). Till now, the most used way to extract the solar cell parameters is still the curve fitting approach.[14–17] The least-squares method, which is the most commonly used method in the curve fitting, extracts the parameters by minimizing the squared error between the calculated target variable and the experimental data. However, in the conventional curve fitting methods, the expressions for the target variable are usually implicit functions and include the independent and dependent variables at the same time. For example, in Eq. (1), the expression for I includes both V and I. This implicit nature of the target variable expression increases the complexity and difficulty of the parameter extraction. With the help of the Lambert W function, the explicit analytic expressions for I or V can be obtained.[18–21] In one previous work from Jain *et al.*,[18] Lambert W function has been used to study the properties of solar cells. However, their study is validated only on simulated *I-V* characteristics instead of the parameter extraction from the experimental data. Another work from Ortiz-Conde *et al.*[21] proposed an efficient method to extract the solar cell parameters from the *I-V*

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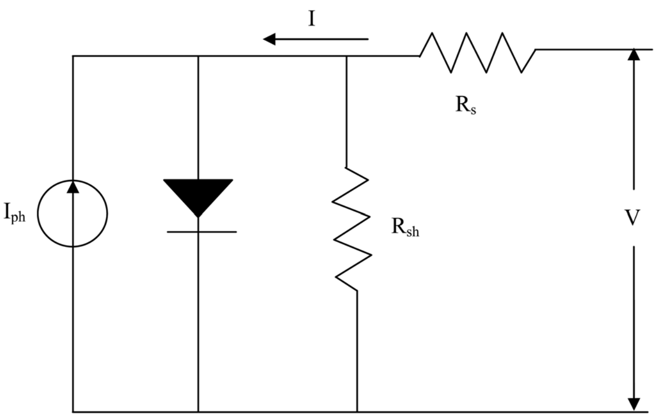


FIG. 1. Generic solar cell equivalent circuit (single-diode model).

characteristics based on the Lambert W function. Because the explicit analytic expressions directly derived from Eq. (1) still remain unsuitable for the purpose of extracting the model parameters, there they first calculated the Co-content (CC) function from the exact explicit analytical expressions and then extracted the device parameters by fitting.[21] However, the CC is still a function of I and V, and thus the fitting process is a bi-dimensional fitting progress.

Here, we propose a simple fitting method to estimate all the parameters of a solar cell just from a single *I-V* curve under one constant illumination level based on the Lambert W function. In this method, an accurate analytic expression for I only depends on three parameters of n, R_s, and R_{sh}, and is directly used to fit the experimental data. Because this analytic expression is only a function of V, the fitting is only one-dimensional and, thus, a much easier method. The proposed method has been used to analyze various solar devices, including Si solar cells, Si solar modules, standalone organic solar cells, tandem organic solar cells, multi-junction organic solar cells, and dye-sensitized solar cells (DSSCs).

II. THEORY

Eq. (1) gives the most popular description of the *I-V* characteristics of a solar cell under a given illumination level. It is an implicit equation and cannot be solved analytically. However, with the help of the Lambert W function, the solution can be given as

$$I = \frac{V}{R_s} - \frac{R_{sh}(R_sI_{ph} + R_sI_0 + V)}{R_s(R_{sh} + R_s)} + \frac{nk_BT}{qR_s} lambertw \times \left[\frac{qR_sI_0R_{sh}}{(R_s + R_{sh})nk_BT} \exp\left(\frac{R_{sh}q(R_sI_{ph} + R_sI_0 + V)}{nk_BT(R_s + R_{sh})}\right)\right], \quad (2)$$

where *lambertw* is the Lambert W function, which can be easily solved numerically. The reason we can derive Eq. (2) is that there is only one exponential term in Eq. (1). If there are two or more exponential terms in the equation, it will become very difficult to derive the expression for I based on the Lambert W function. This is another important reason why we choose the single-diode model in this work. Eq. (2) has an analytical form and it is very convenient to be used in computer programs to reproduce the *I-V* curves of a solar cell when knowing all the parameters. However, this expression is still unsuitable for the purpose of extracting the device parameters.[21] When it is used in the curve fitting to extract the device parameters, very large errors can be introduced. This is mainly caused by the very large value difference between I_0 and I_{ph}. Although both I_0 and I_{ph} are current and have the same unit, the difference between their values is usually larger than six orders. Thus, Eq. (2) must be well dealt before it can be used for the device parameter extraction.

At the short circuit condition ($I = -I_{SC}$ and $V = 0$) and the open circuit condition ($I = 0$ and $V = V_{OC}$, the open circuit voltage), from Eq. (1), it gives

$$-I_{SC} = I_0\left\{\exp\left(\frac{qR_sI_{SC}}{nk_BT}\right) - 1\right\} + \frac{R_sI_{SC}}{R_{sh}} - I_{ph}, \quad (3a)$$

$$0 = I_0\left\{\exp\left(\frac{qV_{OC}}{nk_BT}\right) - 1\right\} + \frac{V_{OC}}{R_{sh}} - I_{ph}. \quad (3b)$$

Combining Eqs. (3a) and (3b), it can be derived that

$$I_0 = \frac{\left(I_{SC} + \frac{R_sI_{SC} - V_{OC}}{R_{sh}}\right)\exp\left(\frac{-qV_{OC}}{nk_BT}\right)}{1 - \exp\left(\frac{q(R_sI_{sc} - V_{OC})}{nk_BT}\right)}, \quad (4a)$$

$$I_{ph} + I_0 = \frac{I_{SC} + \frac{R_sI_{SC} - V_{OC}}{R_{sh}}}{1 - \exp\left(\frac{q(R_sI_{sc} - V_{OC})}{nk_BT}\right)} + \frac{V_{OC}}{R_{sh}}. \quad (4b)$$

Substituting Eqs. (4a) and (4b) into Eq. (2), gives

$$I = \frac{V}{R_s} - \frac{R_{sh}\left(R_s\frac{I_{SC} + \frac{R_sI_{SC} - V_{OC}}{R_{sh}}}{1 - \exp\left(\frac{q(R_sI_{sc} - V_{OC})}{nk_BT}\right)} + \frac{R_sV_{OC}}{R_{sh}} + V\right)}{R_s(R_s + R_{sh})} + \frac{nk_BT}{qR_s} lambertw\left[\frac{qR_s}{nk_BT}\frac{\left(I_{SC} - \frac{V_{OC}}{R_s + R_{sh}}\right)\exp\left(\frac{-qV_{OC}}{nk_BT}\right)}{1 - \exp\left(\frac{q(R_sI_{sc} - V_{OC})}{nk_BT}\right)} \times \exp\left(\frac{R_{sh}q\left(R_s\frac{I_{SC} + \frac{R_sI_{SC} - V_{OC}}{R_{sh}}}{1 - \exp\left(\frac{q(R_sI_{sc} - V_{OC})}{nk_BT}\right)} + \frac{R_sV_{OC}}{R_{sh}} + V\right)}{nk_BT(R_s + R_{sh})}\right)\right]. \quad (5)$$

As shown above, by using I_{SC} and V_{OC}, the number of unknown parameters is reduced from five (I_0, I_{ph}, n, R_s, and R_{sh}) to only three (n, R_s, and R_{sh}). This change makes Eq. (5) suitable to be used in the parameter extraction by the numerical fitting method. In the derivation of Eq. (5), no assumption or approximation is made. Therefore, Eq. (5) is the accurate expression for I. However, this expression seems a little complicated. It can be greatly simplified if we make the assumption

$$\Delta = \exp\left(\frac{q(R_s I_{SC} - V_{OC})}{nk_B T}\right) \ll 1. \quad (6)$$

Using this assumption, Eqs. (4a) and (4b) are reduced to

$$I_0 = \left(I_{SC} + \frac{R_s I_{SC} - V_{OC}}{R_{sh}}\right) \exp\left(\frac{-qV_{OC}}{nk_B T}\right), \quad (7a)$$

$$I_{ph} + I_0 = I_{SC} + \frac{R_s I_{SC}}{R_{sh}}. \quad (7b)$$

Thus, Eq. (5) is reduced to

$$I = \frac{nk_B T}{qR_s} lambertw\left[\frac{qR_s}{nk_B T}\left(I_{SC} - \frac{V_{OC}}{R_s + R_{sh}}\right) \exp\left(\frac{-qV_{OC}}{nk_B T}\right) \times \exp\frac{q}{nk_B T}\left(R_s I_{SC} + \frac{R_{sh} V}{R_{sh} + R_s}\right)\right] + \frac{V}{R_s} - I_{SC} - \frac{R_{sh} V}{R_s(R_{sh} + R_s)}. \quad (8)$$

Eq. (8) is much simpler than Eq. (5), but it requires that the assumption (6) is valid. Fortunately, this assumption is generally valid for various solar cells.[13]

Both Eqs. (5) and (8) are suitable to be used to extract the device parameters of n, R_s, and R_{sh} by employing the very mature least-squares method (for example, use the nonlinear least-squares function "*Lsqnonlin*" in Matlab, please refer to the supplemental materials[25]). Because there is only one independent variable V in the right side of Eqs. (5) and (8), the fitting process is only one dimensional and, thus, the whole parameter extracting process becomes simple. For example, in a Matlab environment, only a few lines of code are required (please refer to the supplemental materials).[25] After n, R_s, and R_{sh} are extracted, I_0 and I_{ph} can be calculated according to Eqs. (4) and (7).

The initial values for n, R_s, and R_{sh} are given as the following. From Eq. (1), dV/dI is expressed as

$$\frac{dV}{dI} = \frac{nk_B T/q}{I_{ph} + I_0 + I - (V - R_s I - nk_B T/q)/R_{sh}} + R_s. \quad (9)$$

Substituting (7b) into (9),

$$\frac{dV}{dI} = \frac{nk_B T/q}{I_{SC} + I - \frac{V}{R_{sh}} + \frac{nk_B T}{qR_{sh}} + \frac{(I + I_{SC})R_s}{R_{sh}}} + R_s. \quad (10)$$

At short circuit condition ($I = -I_{SC}$ and $V = 0$)

$$\left.\frac{dV}{dI}\right|_{I=I_{SC}, V=0} = R_{sh} + R_s \approx R_{sh}. \quad (11)$$

And around the open circuit condition ($I = 0$ and $V = V_{OC}$)

$$\frac{dV}{dI} = \frac{nk_B T/q}{I_{SC} + I - \frac{V}{R_{sh}} + \frac{nk_B T}{qR_{sh}} + \frac{(I + I_{SC})R_s}{R_{sh}}} + R_s \approx \frac{nk_B T/q}{I_{SC} + I - \frac{V}{R_{sh}}} + R_s. \quad (12)$$

The initial values of n and R_s are derived by the y-intercept and the slope of the plot of dV/dI as a function of $(I_{SC} + I - V/R_{sh})^{-1} k_B T/q$. In the derivation of Eqs. (11) and (12), the assumption that $R_{sh} \gg R_s$ has been used, which is generally a good approximation to set the initial values.

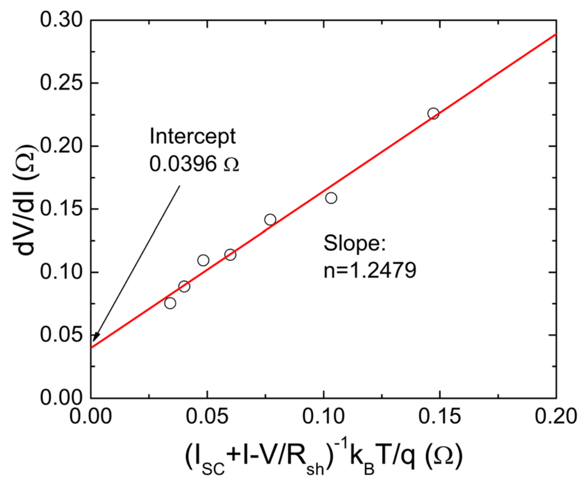
III. RESULTS AND DISCUSSION

From the introduction of the theory, it can be seen that our proposed method relies on I_{SC} and V_{OC} being known. Normally, the values of I_{SC} and V_{OC} can be obtained directly from the experimental data.[13] However, sometimes the *I-V* characteristics are only measured in the "active" quadrant of the solar cell without even measuring I_{SC} and V_{OC} directly or sometimes the density of the points are not sufficient to unambiguously determine both I_{SC} and V_{OC} although the *I-V* characteristics are measured in three quadrants. Under this condition, I_{SC} and V_{OC} can be obtained by the interpolation method or derived from the approximate *I-V* polynomial expression.[13] Thus, I_{SC} and V_{OC} can be determined accurately for most cases and then our method can be used widely.

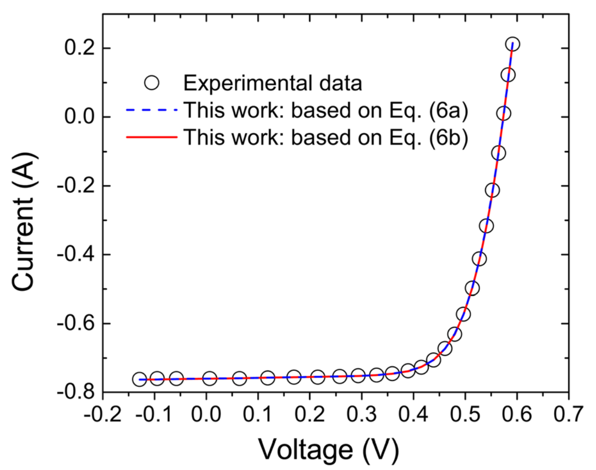
The single-diode model [Eq. (1)] can describe not only the *I-V* characteristics of standalone solar cells but also the electrical properties of tandem, multi-junction solar cells and modules where the cells are connected in series and/or parallel. Various types of solar cells have different properties. To test the validity of our proposed method, we will apply our method to extract the parameters from the experimental *I-V* curves of various solar devices in the following. These devices include a silicon solar cell, a silicon solar cell module, organic solar cells with different structures (standalone, tandem, and multi-junction organic solar cells), and a DSSC.

A. Application to a silicon solar cell

The proposed method is first applied to extract the parameters of a 57-mm-diameter commercial (R.T.C. France) silicon solar cell.[14] The value of dV/dI at short circuit condition gives the initial value of R_{sh} at about 36 Ω. Figure 2(a) shows the plot of dV/dI and $(I_{SC} + I - V/R_{sh})^{-1} k_B T/q$. The y-intercept and the slope give the initial values of R_s and n at about 0.0396 Ω and 1.25, respectively. Using these initial values, the parameters extracted by our proposed method are summarized in Table I. It is shown that the initial values are close to the extracted values and thus confirms the validity of the method to set the initial values. The corresponding *I-V* curves rebuilt with the extracted parameters are shown in Fig. 2(b). It is clearly shown that the rebuilt curves are in very good agreement with the experimental data (open circles). Dashed line represents the curve calculated using



(a)



(b)

FIG. 2. (Color online) (a) Plot of dV/dI as a function of $(I_{SC}+I-V/R_{sh})^{-1}k_BT/q$. The y-intercept and the slope of the plot of dV/dI give the initial values of n and R_s. (b) Experimental data of a silicon solar cell (R.T.C France) (open circles) and the I-V curves calculated using the value of the parameters derived by our method based on Eq. (5) (dashed line) and Eq. (8) (solid line).

the values of parameters extracted based on Eq. (5) where no assumption is needed. Solid line represents the curve calculated using the values of parameters extracted based on Eq. (8), which needs the assumption (6). The two lines are in excellent agreement and we cannot distinguish them from each other. This is because when the assumption (6) is satisfactory ($\Delta \ll 1$), Eq. (5) and Eq. (8) are well equivalent. Because the extracted parameters according to the two equations are almost the same, only one set of the extracted data is give in Table I. In the following, only the simplified Eq. (8) is used when $\Delta \ll 1$ because Eq. (8) is well equivalent to Eq. (5) under this condition.

As stated above, the initial values of n, R_s and R_{sh} in our proposed method are derived from Eqs. (10–12). They are only the approximate values and can never be known accurately. As is known, the results of many fitting algorithms greatly depend on the initial values. Here, to test the robustness of our proposed method to the initial values, the initial values of n, R_s, and R_{sh} are changed simultaneously from half to twice of the previous initial values used in Fig. 2 and then applied in our proposed method to extract the parameters. By using the same experimental data as in Fig. 2, the deviations of the extracted parameters for different initial values of n, R_s, and R_{sh} are obtained and shown in Fig. 3. It can be seen that the deviations are relatively small for a large change range of the initial values. This shows the robustness of our proposed method to the different initial values.

To further test the sensitivity of our method to the measurement error or noise, we add random noise with different relative intensity to the same experimental data as used in Fig. 2(b). By considering possible electronic noise or random errors during measurements, the current can be written as

$$I_{with\_noise} = I_{without\_noise}(1 + percent \times random). \qquad (13)$$

Thus, the data with noise are used in our method to extract device parameters. To extract the parameters, I_{SC} and V_{OC} are firstly obtained by the cubic spline interpolation method.

TABLE I. Extracted parameters by our method and the previous work of a silicon solar cell, a silicon solar module, and a DSSC.

	Silicon solar cells (33° C)					Silicon solar module (45° C)				DSSC (20° C)**		
Parameters	Previous work[a]	Previous work[b]	Previous work[c]	This work	This work (with noise)*	Previous work[a]	Previous work[b]	Previous work[c]	This work	Previous work[d]	Previous work[c]	This work
I_{sc} (A)	0.7608	—	0.76	0.7604	0.7602	1.030	—	1.0	1.0309	0.00206	0.0021	0.002057
V_{oc} (V)	0.5728	—	0.57	0.5737	0.5736	16.778	—	17	16.782	0.704	0.70	0.7065
I_{ph} (A)	0.7608	0.7609	0.77	0.7611	0.7618	1.0318	1.0359	1.0	1.0332	—	0.0021	0.002085
I_0 (μA)	0.3223	0.4039	0.20	0.2422	0.1195	3.2876	6.77	2.3	1.597	0.035	0.023	0.015143
N	1.484	1.504	1.4	1.4561	1.392	48.45	51.32	47	45.862	2.5	2.5	2.3865
R_s (Ω)	0.0364	0.0364	1.037	0.0373	0.0384	1.2057	1.146	1.3	1.313	43.8	42	44.7
R_{sh} (kΩ)	0.0538	0.0495	0.032	0.042	0.0205	0.549	0.2	0.83	0.6023	3.736	3.2	3.285
Δ	8.9×10^{-7}	—	5.6×10^{-7}	6.8×10^{-7}	3.6×10^{-7}	8.3×10^{-6}	—	6.3×10^{-6}	9.4×10^{-7}	6.0×10^{-5}	4.9×10^{-5}	$1.3e\times10^{-5}$

*Random noise with 5% relative intensity.

**No information on the temperature T in the reference.

[a]See Ref. 14.

[b]See Ref. 15.

[c]See Ref. 13.

[d]See Ref. 17.

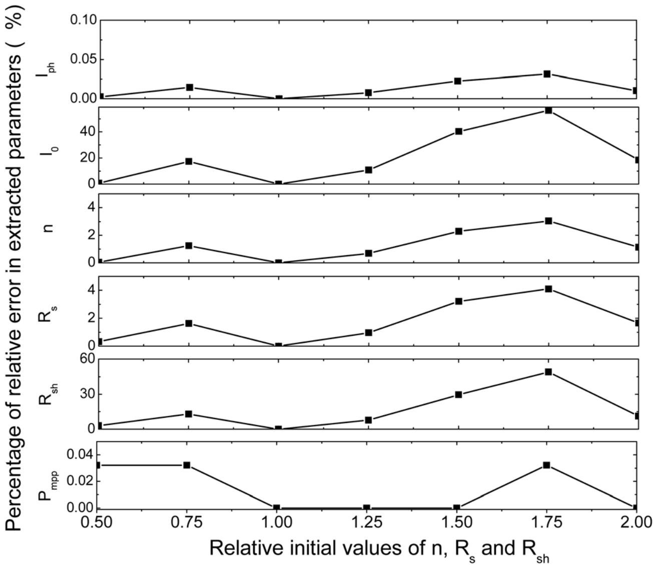


FIG. 3. Relative errors of the extracted parameters when the initial values of n, R_s, and R_{sh} are changed simultaneously from half to twice of the original values used in Fig. 2. P_{mpp} is the extracted maximum power.

By using this method, it can be seen that the obtained I_{SC} and V_{OC} are very accurate even when the relative intensity of a typical random noise is 5% as shown in Table I. For various levels of the typical random noise, the percentage of relative error in extracted parameters is shown in Fig. 4. It can be seen that the errors of the parameters extracted from the *I-V* data with different relative noise intensity turn out to be insignificant. When the noise relative intensity is 5%, the errors for n and R_s are less than 4% as shown in Fig. 4 and Table I, which are much smaller than the results from Ortiz-Conde *et al.*[21] This shows the stability of our proposed method. Especially, it is well known that R_s greatly affects the device performance and thus its determination becomes the target of many works.[4–10] The very small relative error of the extracted R_s under different noise intensity here shows the advantage of our method.

Compared with I_{ph}, n, and R_s, the errors in I_0 and R_s are relatively large as shown in Figs. 3 and 4. This is because the parameters are extracted by minimizing the squared error between the calculated I and the experimental I in our proposed method. As shown in Eq. (1), although I strongly depends on I_0, I_{ph}, n, R_s and R_{sh}, there are still some differences for the sensitivity of I on the five parameters. By comparing the five parameters, I is more sensitive to I_{ph}, n, R_s than I_0, R_{sh}. Thus, the determined error in I_0 and R_{sh} is relative large. Fortunately, this does not affect the validity of our proposed method. This can be seen from the error of the extracted maximum power (P_{mpp}). As shown in Figs. 3 and 4, the deviation of P_{mpp} is negligible. Because the most important parameter from the application point of view is the power derived from the solar cell, the very small deviation of P_{mpp} shows the validity and robustness of our proposed method.

In the above, we have discussed the sensitivity and robustness of the proposed method. And in the following, we will not discuss that again and just apply it to various devices.

B. Application to a silicon solar module

The parameters of a silicon solar module (Photowatt-PWP 201)[14] in which 36 polycrystalline silicon cells are connected in series are also investigated by our proposed method and the results are shown in Table I. For a solar module, the ideality factor is very large as shown in Table I, which is because of the tunneling junctions which connect the subcells in the module. For these very large n values, the one diode model becomes only a mathematical model instead of a physical model. However, it still can well describe the module *I-V* characteristics. Figure 5 shows the experimental data of the solar module (open circles), the calculated curves using the extracted parameters in this work (solid line) and the rebuilt curves in the previous reports (dashed line and

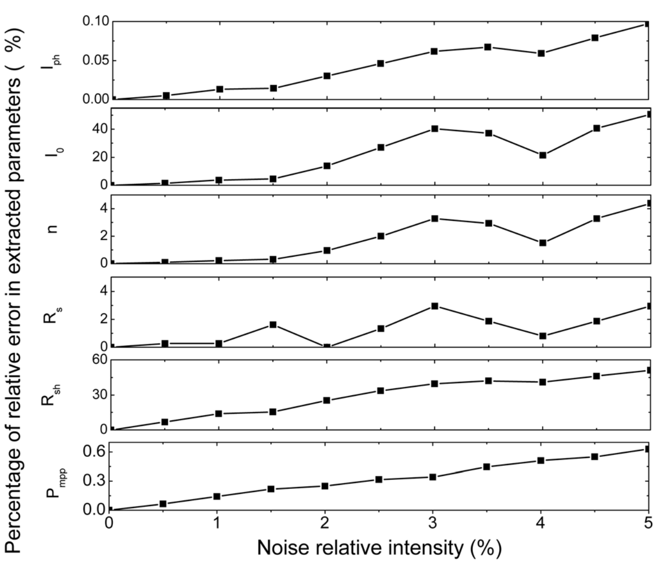


FIG. 4. Relative errors of the extracted parameters resulting as a function of the random noise with different relative intensity added to the experimental data.

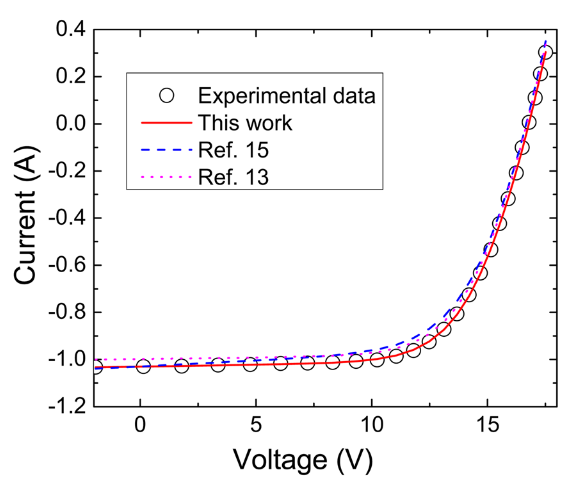


FIG. 5. (Color online) Experimental data of a silicon solar cell (open circles), the *I-V* curves calculated using the value of the parameters derived by our method (solid line) and the rebuilt *I-V* curves of the previous studies (Ref. 15) (dashed line) and (Ref. 13) (dotted line).

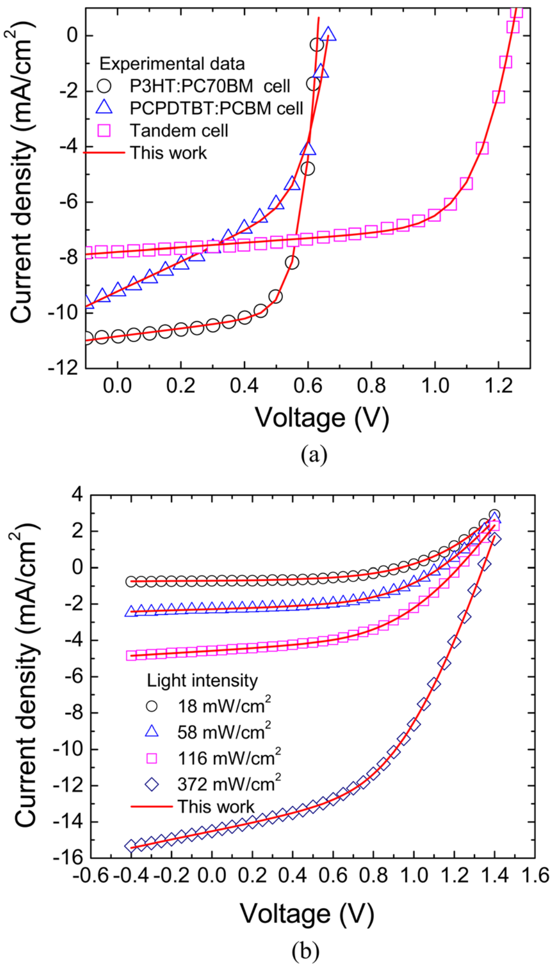


FIG. 6. (Color online) (a) Experimental data of a P3HT:PC$_{70}$BM polymer cell (open circles), a PCPDTBT:PCBM polymer cell (open triangles) and their corresponding tandem cell (open squares), and the *I*-*V* curves calculated using the value of the parameters derived by our method (solid line). (b) Experimental data of a CuPc/PTCBI multi-junction small molecule cell under the different illumination levels of 18 (open circles), 58 (open triangles), 116 (open squares), and 372 (open diamonds) mW/cm^2, and the *I*-*V* curves calculated using the value of the parameters derived by our method (solid line).

dotted line). One previous study[15] gave a smaller value of R_{sh} and a larger value of n (Table I). This makes that there is an obvious deviation between the calculated curve (dashed line in Fig. 5) and the experimental data around the maximum output point. This may be induced by their employed differential calculation which is sensitive to the measurement error. Another recent study[13] gave a low value of I_{ph} and thus a low value of I_{SC} (Table I and dotted line in Fig. 5). This may be because they only used a small part of the experimental data far from the short circuit point, and thus some information from the experimental data is lost. Compared to these previous studies, the curve obtained from our method has a better agreement with the experiment data as shown in Fig. 5, and thus our method shows the obvious advantage.

C. Application to organic solar cells with different structures

Organic solar cells generally have 2–4 orders of magnitude larger series resistance values and relatively smaller shunt resistance values than classical silicon solar cell.[22] Some conventional methods that are widely used in mature silicon solar cells, such as the assumption that R_{sh} is neglected, cannot be used in organic solar cells because they may lead to very large errors. However, our proposed method here can be directly used in organic solar cells.

An example is shown in Fig. 6(a). It is shown that the calculated *I*-*V* curves from the parameters (Table II) obtained by our method are in excellent agreement with the experimental data[13] of not only the standalone devices based on the different composites of poly[2,6-(4,4-*bis*-(2-ethylhexyl)-4 H-cyclopenta[2,1-b;3,4-b′]dithiophene)-alt-4,7-(2,1, 3-benzothiadiazole)][6]:-phenyl-C61 butyric acid methyl ester (PCPDTBT:PCBM) and poly(3-hexylthiophene)[6]:-phenyl-C71 butyric acid methyl ester (P3HT:PC$_{70}$BM), but also the organic solar cell with the tandem structure. Figure 6(b) shows another example. The investigated device is an organic solar cell with a multi-junction structure based on the

TABLE II. Extracted parameters by our method and the previous work of two standalone polymer solar cells, a tandem polymer cell, and a multi-junction small molecule cell (27° C was used in the parameter extraction).

	Standalone and tandem polymer cells						Multi-junction small molecule cell under different illumination				
	P3HT cell		PCPDTBT cell		Tandem cell		18 mW/cm^2	58 mW/cm^2	116 mW/cm^2		372 mW/cm^2
Parameters	Previous work[a]	This work	Previous work[a]	This work	Previous work[a]	This work	This work	This work	Previous work[b]	This work	This work
I_{sc} (mA/cm^2)	10.8	10.8	9.2	9.22	7.8	7.792	0.7091	2.2755	4.6	4.5728	17.509
V_{oc} (V)	0.63	0.63	0.66	0.66	1.24	1.24	0.9355	1.1294	1.2	1.2217	1.3431
I_{ph} (mA)	—	10.861	—	9.49	—	7.8273	0.7116	2.3162	4.7	4.7219	15.326
I_0 (mA)	—	3.48×10^{-6}	—	3.81×10^{-7}	—	8.84×10^{-7}	9.0×10^{-3}	1.7×10^{-3}	9.2×10^{-4}	1.04×10^{-3}	$4.04\text{e} \times 10^{-4}$
n	—	1.65	—	1.746	—	3.0243	8.611	6.3756	5.8	5.8847	5.162
R_s (Ωcm^2)	—	1.2	—	5.4	—	5.6	35.4	50.9	48	46.9	23.1
R_{sh} (kΩcm^2)	—	0.6833	—	0.1797	—	1.2357	17.196	2.9868	1.4	1.4024	0.4119
Δ	—	6.9×10^{-7}	—	1.8×10^{-6}	—	3.1×10^{-7}	1.8×10^{-2}	2.4×10^{-3}	1.1×10^{-3}	1.5×10^{-3}	1×10^{-3}

[a]See Ref. 23.
[b]See Ref. 13.

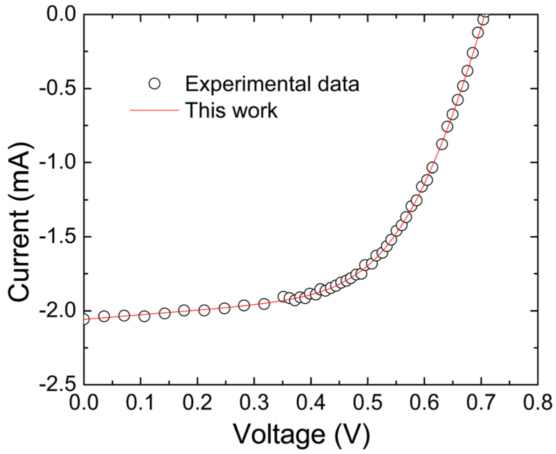


FIG. 7. (Color online) Experimental data of a DSSC (C4) (open circles) and the *I-V* curve calculated using the value of the parameters derived by our method (solid line).

materials of copper phthalocyanine/3,4,9,10-perylenetetracarboxylic bisbenzimid-azole (CuPc/PTCBI) and its *I-V* characteristics were measured under different illumination levels.[24] There is a very good agreement between the calculated curves and the experimental data [Fig. 6(b)]. As shown in Table II, the extracted parameters change with the different light intensity. This means that the conventional "different illumination level methods"[4–7] can cause a large error because these methods assume the device parameters are constant under different illumination levels.

The above results have shown that our proposed method cannot only be used in organic solar cells with different composites, but also can be used in organic solar cells with different structures. The good agreement between the calculation results and the experimental data indicates that our method is very suitable for organic solar cells.

D. Application to a DSSC

We again applied our method to a DSSC. The experimental data of a DSSC called C4 (Ref. 17) are shown in Fig. 7 (open circles). In the previous study,[17] their method requires that the parameter of n is fixed to some constant value and then the other parameters are extracted. But the method that requires fixing an unknown parameter can never yield accurate values of the parameters. Another method in a recent study[13] can extract all the parameters, but it requires that the experimental *I-V* curve must be smooth enough, and thus these data have to be approximated by a ninth-degree polynomial expression in their study.[13] Compared to these previous studies, our method can be directly used to extract the parameters from the raw data. As shown in Fig. 7, the calculated curve using the parameters (Table I) extracted by our method is in very good agreement with the experimental data. This shows the advantages of our method.

IV. CONCLUSION

In this work, we have proposed a simple method to extract all the parameters of a solar cell just from a single *I-V* curve under one constant illumination level. With the help of the Lambert W function, the explicit analytic expression for I can be obtained. By reducing the number of parameters, the expression for I only depends on three parameters of n, R_s and R_{sh}. This analytic expression for I is directly used in the numerical method to fit the experimental data and then determine the values of the parameters. It has been shown that our method can be easily used to analyze various solar devices, including silicon solar cells, silicon solar modules, dye-sensitized solar cells, and organic solar cells with stand-alone, tandem, and multi-junction structures.

[1] G. L. Araujo, E. Sanchez, and M. Marti, Sol. Cells **5**, 199 (1982).
[2] K. Nishioka, N. Sakitani, Y. Uraoka, and T. Fuyuki, Sol. Energy Mater. Sol.Cells **91**, 1222 (2007).
[3] B. Mazhari, Sol. Energy Mater. Sol.Cells **90**, 1021 (2006).
[4] M. Wolf and H. Rauschenbach, Adv. Energy Convers. **3**, 455 (1963).
[5] R. J. Handy, Solid-State Electron. **10**, 765 (1967).
[6] D. Pysch, A. Mette, and S. W. Glunz, Sol. Energy Mater. Sol.Cells **91**, 1698 (2007).
[7] R. Hussein, D. Borchert, G. Grabosch, and W. R. Fahrner, Sol. Energy Mater. Sol.Cells **69**, 123 (2001).
[8] E. Radziemska, Energy Convers. Manage. **46**, 1485 (2005).
[9] K. Rajkanan and J. Shewchun, Solid-State Electron. **22**, 193 (1979).
[10] J. D. Servaites, S. Yeganeh, T. J. Marks, and M. A. Ratner, Adv. Funct. Mater. **20**, 97 (2010).
[11] P. Mialhe, A. Khoury, and J. P. Charles, Phys. Status Solidi A **83**, 403 (1984).
[12] P. Schilinsky, C. Waldauf, J. Hauch, and C. J. Brabec, J. Appl. Phys. **95**, 2816 (2004).
[13] K. Ishibashi, Y. Kimura, and M. Niwano, J. Appl. Phys. **103**, 094507 (2008).
[14] T. Easwarakhanthan, J. Bottin, I. Bouhouch, and C. Boutrit, Int. J. Sol. Energy **4**, 1 (1986).
[15] M. Chegaar, Z. Ouennoughi, and A. Hoffmann, Solid-State Electron. **45**, 293 (2001).
[16] A. Kaminski, J. J. Marchand, and A. Laugier, Solid-State Electron. **43**, 741 (1999).
[17] M. Murayama and T. Mori, Jpn. J. Appl. Phys., Part 1 **45**, 542 (2006).
[18] A. Jain and A. Kapoor, Sol. Energy Mater. Sol. Cells **86**, 197 (2005).
[19] J. Ding and R. Radhakrishnan, Sol. Energy Mater. Sol. Cells **92**, 1566 (2008).
[20] A. Ortiz-Conde and F. J. Garcıa Sanchez, Solid-State Electron. **49**, 465 (2005).
[21] A. Ortiz-Conde, F. J. Garcıa Sanchez, and J. Muci, Sol. Energy Mater. Sol. Cells. **90**, 352 (2006).
[22] M. Chegaar, G. Azzoouzi, and P. Mialhe, Solid-State Electron. **50**, 1234 (2006).
[23] J. Y. Kim, K. Lee, N. E. Coates, D. Moses, T. Nguyen, M. Dante, and A. Heeger, Science **317**, 222 (2007).
[24] A. Yakimov and S. R. Forrest, Appl. Phys. Lett. **80**, 1667 (2002).
[25] See supplementary material at http://dx.doi.org/10.1063/1.3632971 for very simple rout to use our proposed method in matlab.