SYSTEMS SCIENCE & CONTROL ENGINEERING: AN OPEN ACCESS JOURNAL
2020, VOL. 8, NO. 1, 22–34
https://doi.org/10.1080/21642583.2019.1708830

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A novel swarm intelligence optimization approach: sparrow search algorithm

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ABSTRACT
In this paper, a novel swarm optimization approach, namely sparrow search algorithm (SSA), is proposed inspired by the group wisdom, foraging and anti-predation behaviours of sparrows. Experiments on 19 benchmark functions are conducted to test the performance of the SSA and its performance is compared with other algorithms such as grey wolf optimizer (GWO), gravitational search algorithm (GSA), and particle swarm optimization (PSO). Simulation results show that the proposed SSA is superior over GWO, PSO and GSA in terms of accuracy, convergence speed, stability and robustness. Finally, the effectiveness of the proposed SSA is demonstrated in two practical engineering examples.

ARTICLE HISTORY
Received 2 November 2019
Accepted 20 December 2019

KEYWORDS
Swarm optimization; sparrow search algorithm; convergence speed; stability and robustness

1. Introduction

The optimization problems are common in engineering applications such as knapsack problems, data clustering, data classification, path planning, robot control, and so on. It is well known that the swarm intelligence (SI) optimization algorithms have been used as primary techniques to solve global optimization problems because of its simplicity, flexibility and high efficiency. It should be mentioned that, the SI optimization algorithms mainly introduce the randomness in the search process, which is different from deterministic approaches. Note that the deterministic algorithm is easy to get trapped in local optimal solutions in the complex situation. Therefore, it is of practical importance to employ the SI optimization algorithm so as to obtain an optimal solution to the global optimization problem.

In the past decades, the SI optimization algorithm has been developed rapidly and becomes a hotspot in many fields. So far, there have been many different types of optimization algorithms available in the existing literature. Among various optimization algorithms, the ant colony optimization (ACO) algorithm and particle swarm optimization (PSO) algorithm are representative that have received considerable attention. For example, the ACO algorithm has been proposed in Dorigo, Maniezzo, and Colorni (1996), which mimics the biological characteristics of ants in nature that mark the path by pheromone. As one of the well-known algorithms, the PSO algorithm has been proposed in Eberhart and Shi (2001) and Kennedy and Eberhart (1995), which mimics the cooperation and foraging behaviour of the bird flocks.

On the other hand, due to ACO and PSO were proven to be very competitive and have the strong global searching ability, the SI optimization algorithm has attracted increasing attention from scholars in this area. Many new algorithms started to be proposed, which imitates the social behaviour of organisms such as fishs, birds, or insects in nature. For example, the bat algorithm (BA) has been proposed in Yang and He (2013), which mimics the echolocation behaviour of bats. The grey wolf optimizer (GWO) algorithm (Mirjalili, Mirjalili, & Lewis, 2014) is another popular algorithm, mimicking the leadership and hunting behaviour of grey wolves. There are four types of grey wolves in the GWO algorithm: alpha, beta, delta, and omega wolves. The alpha wolves are responsible for making decisions to the pack, whereas beta and delta wolves should help the alpha wolves in decision making process. Furthermore, the rest of the recent SI optimization algorithms are: artificial bee colony (ABC) algorithm (Karaboga, 2005; Karaboga & Basturk, 2007), firefly algorithm (FA) (Yang, 2008, 2010a), cuckoo search (CS) algorithm (Yang & Deb, 2009), et al.

Except for the SI optimization algorithms, some algorithms are inspired by the concept of natural evolution or the physical rules. For example, the well-known genetic algorithm (GA) has been presented in Holland (1975, 1992), which is a powerful stochastic search

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algorithm based on the principles of natural selection and natural genetics. Generally, there are consists of three operators: selection, reproduction, and mutation, which makes GA an efficient global optimizer. Moreover, it should be pointed out that there are also algorithms usually proposed by simulating the physical rules such as gravitational search algorithm (GSA) (Rashedi, Nezamabadi-Pour, & Saryazdi, 2009), simulated annealing (SA) (Kirkpatrick, Gelatto, & Vecchi, 1983), etc.

On the other hand, we found that although each algorithm has its advantages, there are also shortcomings by a deeper investigation. For instance, the ACO algorithm has the disadvantage of slow search speed, and the PSO algorithm has the disadvantage of easy premature convergence. Therefore, it is very important to enhance the current optimization algorithm. According to the no-free-lunch (NFL) theorem (Wolpert & Macready, 1997), the expected performance of each algorithm is the same for solving all optimization problems. In other words, an optimization algorithm may perform well in a series of problems and show poor performance in a different series of problems. Obviously, we can solve the different problems by proposing new optimization algorithms. At the same time, the newly proposed optimization algorithm provides a new solution to solve a complex global optimization problem.

In response to the above discussions, in this paper, we aim to propose a novel swarm intelligence optimization technique which is called sparrow search algorithm (SSA). The main contributions of this paper are summarized as follows: (1) a new SI technique, i.e. the SSA is proposed inspired by the sparrow population's foraging and anti-predation behaviours; (2) by using the proposed SSA, both the exploration and the exploitation of the search space of the optimization are improved to some extent; and (3) the proposed SSA is successfully applied in two practical engineering problems. Finally, in order to test the effectiveness and performance of the proposed algorithm in this paper, some comparative experiments are carried out. The simulation results show that the proposed SSA is superior to other existing algorithms in terms of searching precision, convergence rate, stability and the avoidance of local optimal value.

The remainder of the paper is organized as follows. Section 2 introduces the SSA in detail. Section 3 is the verification and comparison of the SSA. Section 4 applies the SSA to the two practical engineering problems and further tests the performance of the algorithm. In Section 5, we draw the conclusion of this paper and discuss the next work.

2. Sparrow search algorithm (SSA)

In this section, we discuss the inspiration of the SSA. Then, the mathematical model and the SSA are described in details.

2.1. Biological characteristics

The sparrows are usually gregarious birds and have many species. They are distributed in most parts of the world and like to live in places where the human life. Moreover, they are omnivorous birds and mainly feed on seeds of grains or weeds. It is well known that the sparrows are common resident birds. In contrast with many other small birds, the sparrow is strongly intelligent and has a strong memory. Note that there are two different types of captive house sparrows, both the producer and the scrounger (Barnard & Sibly, 1981). The producers actively search for the food source, while the scroungers obtain food by the producers. Furthermore, the evidence shows that the birds usually use behavioural strategies flexibly, and switch between producing and scrounging (Barta, Liker, & Mónus, 2004; Coolen, Giraldeau, & Lavoie, 2001; Koops & Giraldeau, 1996; Liker & Barta, 2002). It can also be said that, in order to find their food, the sparrows usually use the strategy of both the producer and the scrounger (Barnard & Sibly, 1981; Johnson, Grant, & Giraldeau, 2001; Liker & Barta, 2002).

The studies have shown that the individuals monitor the behaviour of the others in the group. Meanwhile, the attackers in the bird flock, which want to increase their own predation rate, are used to compete food resources of the companions with high intakes (Bautista, Alonso, & Alonso, 1998; Lendvai, Barta, Liker, & Bokony, 2004). In addition, the energy reserves of the individuals may play an important role when the sparrow chooses different foraging strategies, and the sparrows with low energy reserves scrounge more (Lendvai et al., 2004). It is worth mentioning that the birds, which located on the periphery of the population, are more likely to be attacked by predators and constantly try to get a better position (Budgey, 1998; Pomeroy & Hepner, 1992). Note that the animals, which located on the centre, may move closer to their neighbours in order to minimize their domain of danger (Hamilton, 1971; Pulliam, 1973). We also know that all sparrows display the natural instinct of curiosity about everything, and at the same time they are always vigilant. For example, when a bird does detect a predator, one or more individuals give a chirp and the entire group flies away (Pulliam, 1973).

2.2. Mathematical model and algorithm

According to the previous description of the sparrows, we can establish the mathematical model to construct the sparrow search algorithm. For simplicity, we idealized the following behaviour of the sparrows and formulated corresponding rules.

(1) The producers typically have high levels of energy reserves and provide foraging areas or directions for all scroungers. It is responsible for identifying areas where rich food sources can be found. The level of energy reserves depends on the assessment of the fitness values of the individuals.
(2) Once the sparrow detects the predator, the individuals begin to chirp as alarming signals. When the alarm value is greater than the safety threshold, the producers need to lead all scroungers to the safe area.
(3) Each sparrow can become a producer as long as it searches for the better food sources, but the proportion of the producers and the scroungers is unchanged in the whole population.
(4) The sparrows with the higher energy would be acted as the producers. Several starving scroungers are more likely to fly to other places for food in order to gain more energy.
(5) The scroungers follow the producer who can provide the best food to search for food. In the meantime, some scroungers may constantly monitor the producers and compete for food in order to increasing their own predation rate.
(6) The sparrows at the edge of the group quickly move toward the safe area to get a better position when aware of danger, while the sparrows in the middle of the group randomly walk in order to be close to others.

In the simulation experiment, we need to use virtual sparrows to find food. The position of sparrows can be represented in the following matrix:

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & \cdots & x_{1,d} \\ x_{2,1} & x_{2,2} & \cdots & \cdots & x_{2,d} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{n,1} & x_{n,2} & \cdots & \cdots & x_{n,d} \end{bmatrix} \tag{1}$$

where n is the number of sparrows and d shows the dimension of the variables to be optimized. Then, the fitness value of all sparrows can be expressed by the following vector:

$$F_X = \begin{bmatrix} f([x_{1,1} & x_{1,2} & \cdots & \cdots & x_{1,d}]) \\ f([x_{2,1} & x_{2,2} & \cdots & \cdots & x_{2,d}]) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ f([x_{n,1} & x_{n,2} & \cdots & \cdots & x_{n,d}]) \end{bmatrix} \tag{2}$$

where n shows the number of sparrows, and the value of each row in F_X represents the fitness value of the individual. In the SSA, the producers with better fitness values have the priority to obtain food in the search process. In addition, because the producers are responsible for searching food and guiding the movement of the entire population. Therefore, the producers can search for food in a broad range of the places than that of the scroungers. According to rules (1) and (2), during each iteration, the location of the producer is updated as below:

$$X_{i,j}^{t+1} = \begin{cases} X_{i,j}^{t} \cdot \exp\left(\dfrac{-i}{\alpha \cdot \text{iter}_{\max}}\right) & \text{if } R_2 < ST \\ X_{i,j}^{t} + Q \cdot L & \text{if } R_2 \geq ST \end{cases} \tag{3}$$

where t indicates the current iteration, $j = 1, 2, \ldots, d$. $X_{i,j}^{t}$ represents the value of the jth dimension of the ith sparrow at iteration t. $\text{iter}_{\max}$ is a constant with the largest number of iterations. $\alpha \in (0, 1]$ is a random number. R_2 ($R_2 \in [0, 1]$) and ST ($ST \in [0.5, 1.0]$) represent the alarm value and the safety threshold respectively. Q is a random number which obeys normal distribution. L shows a matrix of $1 \times d$ for which each element inside is 1.

When $R_2 < ST$, which means that there are no predators around, the producer enters the wide search mode. If $R_2 \geq ST$, it means that some sparrows have discovered the predator, and all sparrows need quickly fly to other safe areas.

As for the scroungers, they need to enforce the rules (4) and (5). As mentioned above, some scroungers monitor the producers more frequently. Once they find that the producer has found good food, they immediately leave their current position to compete for food. If they win, they can get the food of the producer immediately, otherwise they continue to execute the rules (5). The position update formula for the scrounger is described as follows:

$$X_{i,j}^{t+1} = \begin{cases} Q \cdot \exp\left(\frac{X_{\text{worst}}^{t} - X_{i,j}^{t}}{i^2}\right) & \text{if } i > n/2 \\ X_P^{t+1} + |X_{i,j}^{t} - X_P^{t+1}| \cdot A^{+} \cdot L & \text{otherwise} \end{cases} \tag{4}$$

where X_P is the optimal position occupied by the producer. X_{worst} denotes the current global worst location. A represents a matrix of $1 \times d$ for which each element inside is randomly assigned 1 or -1, and $A^{+} = A^{T}(AA^{T})^{-1}$. When $i > n/2$, it suggests that the ith scrounger with the worse fitness value is most likely to be starving.

In the simulation experiment, we assume that these sparrows, which are aware of the danger, account for 10% to 20% of the total population. The initial positions of these sparrows are randomly generated in the population. According to rules (6), the mathematical model can be expressed as follows:

$$X_{i,j}^{t+1} = \begin{cases} X_{\text{best}}^{t} + \beta \cdot |X_{i,j}^{t} - X_{\text{best}}^{t}| & \text{if } f_i > f_g \\ X_{i,j}^{t} + K \cdot \left(\dfrac{|X_{i,j}^{t} - X_{\text{worst}}^{t}|}{(f_i - f_w) + \varepsilon} \right) & \text{if } f_i = f_g \end{cases} \quad (5)$$

where X_{best} is the current global optimal location. β, as the step size control parameter, is a normal distribution of random numbers with a mean value of 0 and a variance of 1. $K \in [-1, 1]$ is a random number. Here f_i is the fitness value of the present sparrow. f_g and f_w are the current global best and worst fitness values, respectively. ε is the smallest constant so as to avoid zero-division-error.

For simplicity, when $f_i > f_g$ indicates that the sparrow is at the edge of the group. X_{best} represents the location of the centre of the population and is safe around it. $f_i = f_g$ shows that the sparrows, which are in the middle of the population, are aware of the danger and need to move closer to the others. K denotes the direction in which the sparrow moves and is also the step size control coefficient.

Based on the idealization and feasibility of the above model, the basic steps of the SSA can be summarized as the pseudo code shown in Algorithm 1.

Algorithm 1 The framework of the SSA.

Input:
G: the maximum iterations
PD: the number of producers
SD: the number of sparrows who perceive the danger
R_2: the alarm value
n: the number of sparrows
Initialize a population of n sparrows and define its relevant
parameters.
Output: X_{best}, f_g.

while $(t < G)$
 Rank the fitness values and find the current best individual and the current worst individual.
 $R_2 = rand(1)$
 for $i = 1 : PD$
  Using equation (3) update the sparrow's location;
 end for
 for $i = (PD + 1) : n$
  Using equation (4) update the sparrow's location;
 end for
 for $l = 1 : SD$
  Using equation (5) update the sparrow's location;
 end for
 Get the current new location;
 If the new location is better than before, update it;
 $t = t + 1$
end while
return X_{best}, f_g.

3. Validation and comparison

In this part, there are nineteen standard test functions verifying the feasibility and effectiveness of the proposed SSA, and the test results are compared with PSO, GSA and GWO. Integrated development environment is Matlab 2014a for all the experimentations. Operating System: Window7. The standard test functions (Fateen & Bonilla-Petriciolet, 2014; Jamil & Yang, 2013; Rashedi et al., 2009; Yang, 2010b) are unimodal test functions, multimodal test functions and fixed-dimension test functions corresponding to Tables 1–3, respectively. The size of each dimension is 30 ($DIM = 30$) for the functions of Tables 1 and 2, and the dimension of the fixed-dimension test functions can be seen in Table 3.

Table 1. Unimodal test functions ($Dim = 30$).

Function	Initial range	$F_{\min}$
$F_1(x) = \sum_{i=1}^{n} x_i^2$	[−100,100]	0
$F_2(x) = \sum_{i=1}^{n} \|x_i\| + \prod_{i=1}^{n} \|x_i\|$	[−10,10]	0
$F_3(x) = \sum_{i=1}^{n} (\sum_{j=1}^{i} x_j)^2$	[−100,100]	0
$F_4(x) = \max_i\{\|x_i\|, 1 \leq i \leq n\}$	[−100,100]	0
$F_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	[−30,30]	0
$F_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	[−100,100]	0
$F_7(x) = \sum_{i=1}^{n} i x_i^4 + \text{random}[0, 1)$	[−1.28,1.28]	0

Figures 1 and 2 show the trajectories of sparrows in the different test functions. We can clearly see that most sparrows aggregate towards the global optimum in Figure 1. Nevertheless, it should be pointed out that in the Damavandi function of Figure 2, although the most sparrows are clustered at the local minimums, some sparrows still are able to avoid local minimums to move towards the global best (2, 2).

In order to make the algorithm more convincing, in all cases, we run 30 times independent trials on each test function. The maximum number of the iterations is 1000 and the population size is set to 100 ($n = 100$) in each trial. The parameters of the GWO are arranged as follows: $\vec{a}$ value is linearly decreased from 2 to 0 and r_1, r_2 are random vectors in [0, 1]. The parameters of the PSO

Table 2. Multimodal test functions ($Dim = 30$).

Function	Initial range	F_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i \sin(\sqrt{\lvert x_i \rvert})$	[−500,500]	−418.9829 n
$F_9(x) = \sum_{i=1}^{n} [x_i^2 - 10\cos(2\pi x_i) + 10]$	[−5.12,5.12]	0
$F_{10}(x) = -20\exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	[−32,32]	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[−600,600]	0
$F_{12}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\}$	[−50, 50]	0
$+ \sum_{i=1}^{n} u(x_i, 10, 100, 4)$		
$y_i = 1 + \frac{x_i + 1}{4}$		
$u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{cases}$		

Table 3. Fixed-dimension test functions.

Function	Dim	Initial range	F_{min}
$F_{13}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	[−5,5]	−1.0316
$F_{14}(x) = \left[1 - \left\lvert \frac{\sin[\pi(x_1 - 2)]\sin[\pi(x_2 - 2)]}{\pi^2(x_1 - 2)(x_2 - 2)} \right\rvert^5\right][2 + (x_1 - 7)^2 + 2(x_2 - 7)^2]$	2	[0,14]	0
$F_{15}(x) = -(\lvert e^{\lvert 100 - \sqrt{x_1^2 + x_2^2}/\pi \rvert}\sin(x_1)\sin(x_2)\rvert + 1)^{-0.1}$	2	[−10,10]	−1.0
$F_{16}(x) = [e^{-\sum_{i=1}^{n}(x_i/\beta)^{2m}} - 2e^{-\sum_{i=1}^{n}x_i^2}]\prod_{i=1}^{n}\cos^2(x_i), \beta = 15, m = 5$	2	[−20,20]	−1.0
$F_{17}(x) = \sum_{i=1}^{11}\left(a_i - \frac{x_1(b_i^2 + b_ix_2)}{b_i^2 + b_ix_3 + x_4}\right)^2$	4	[−5,5]	0.000307
$F_{18}(x) = -\sum_{i=1}^{4} c_i\exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	3	[0,1]	−3.86
$F_{19}(x) = -\sum_{i=1}^{4} c_i\exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	6	[0,1]	−3.32

are $c1 = c2 = 1.49445$, $w = 0.729$. The parameters of the GSA are $G_0 = 100$, $\alpha = 20$. The parameters of the SSA are set as follows: the number of the producers and SD accounts for 20% and 10%, respectively, and $ST = 0.8$. Finally, we get the best value, the mean value, and the standard deviation (Std) of the objective function values. With the same standard test function, the average value represents the convergence accuracy of the algorithm, and the standard deviation presents the stability of the algorithm. The solutions of the four different algorithms are shown in Table 4.

3.1. Unimodal test functions

On the unimodal test functions, it mainly reflects the good convergence property and exploitation capability of the algorithm. Generally speaking, these test functions are to make the algorithm concentrate to exploit during the optimization process to find the global optimum.

3.1.1. Analysis of the convergence accuracy

As shown in Table 4, the SSA obtains the optimal value for solving F_1–F_4. Although the SSA does not get the optimal value when solving F_5, it is also significantly better than GWO, PSO, and GSA. On the F_6 test function, the convergence accuracy of the PSO is higher than GWO, GSA and SSA. For the F_7 test function, the SSA is slightly better than the other three algorithms from the average values obtained.

3.1.2. Analysis of the stability

From the Std in Table 4, we can get the standard deviation of the SSA is zero on the F_1–F_4. This shows that our algorithm can yield more stable results than those of PSO,

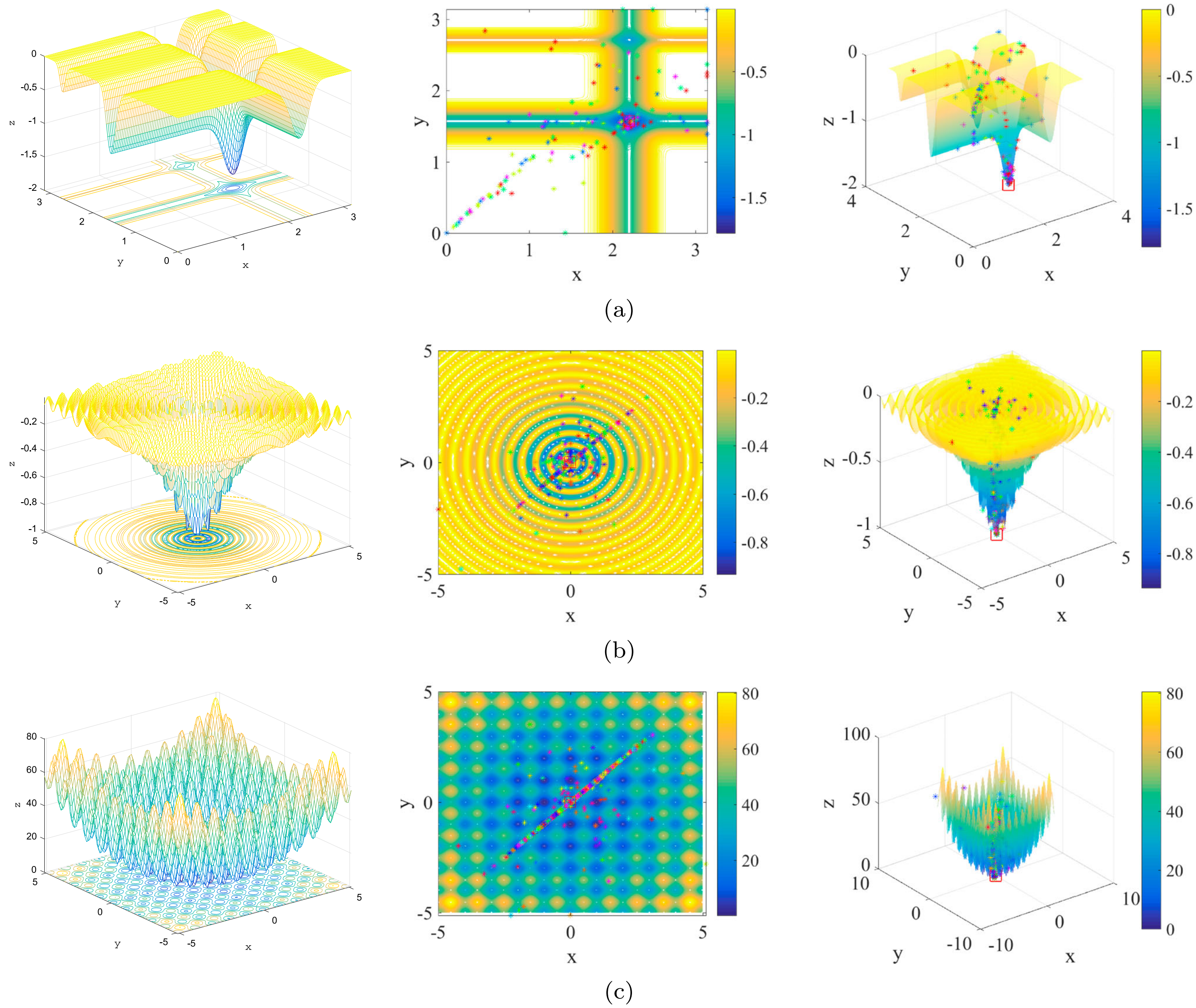

Figure 1. The paths of the SSA on the 2-D version of the test functions: (a) Michalewicz function, (b) Drop-wave function, (c) Rastrigin function.

GWO and GSA. On the F_5 test function, the SSA is the most stable, and PSO is the most unstable. When dealing with the F_6 test function, the stability of the SSA is worse than that of GSA and PSO, but it is much better than GWO. On the F_7 test function, the SSA is similar in the performance stability to the other three algorithms. In summary, the simulation experimental results show the satisfactory performance of the SSA in comparison with the other algorithms that have been reported.

3.1.3. *Analysis of the convergence speed*

In order to compare the convergence speed of the four algorithms intuitively, the fitness curves of the unimodal test functions are shown in Figure 3. It can be concluded that the SSA exhibits an absolute advantage on the F_1–F_4 test functions, which is obviously better than GWO, GSA and PSO. The SSA gets a good fitness value at the beginning and converges to a better value after about 400 iterations for the F_5 test function. In addition, It can be seen from the convergence curves of the F_6 and F_7 test functions that the proposed, SSA, not only enhances the convergence rate but also has strong competitiveness compared with other algorithms.

From the above, we draw a conclusion that the proposed SSA can quickly find the feasible solution and has the best performance in terms of efficiency and convergence in dealing with unimodal test functions.

3.2. *Multimodal test functions*

On the multimodal test functions, each of the functions is characterized by multiple local optimal solutions, which makes the algorithm easy to fall into local optimal points.

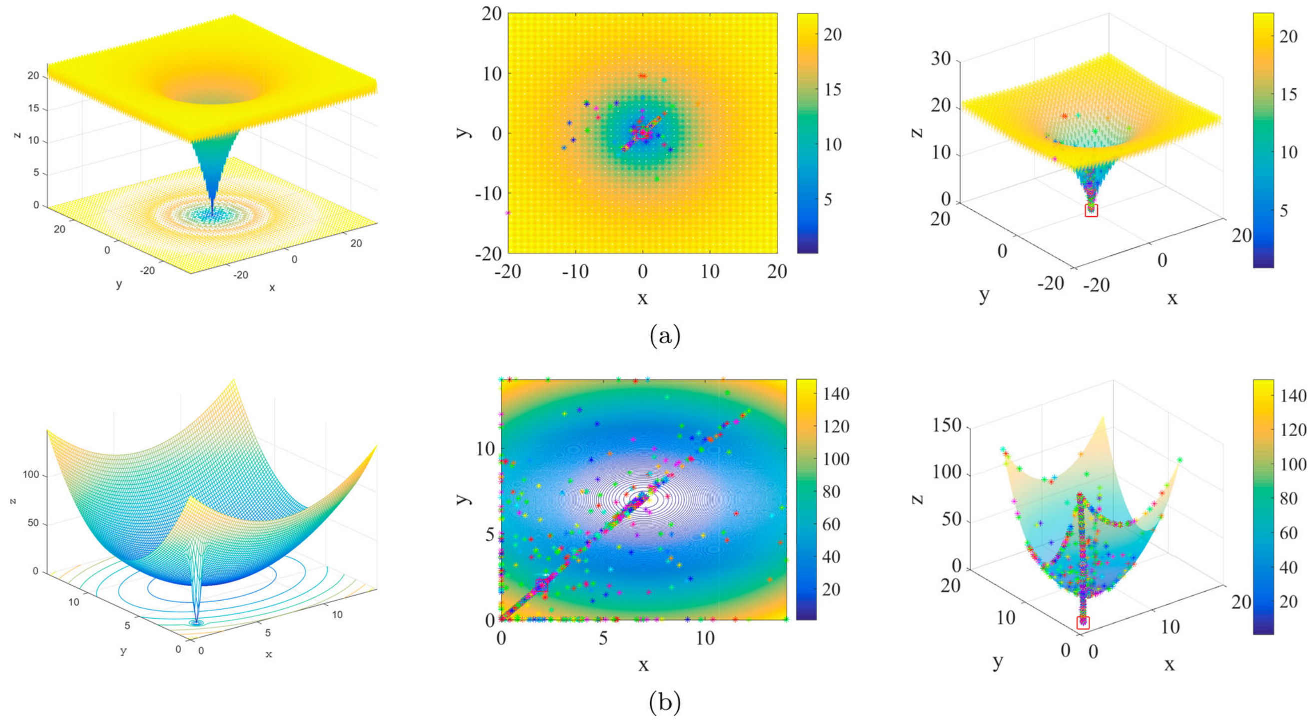

Figure 2. The paths of the SSA on the 2-D version of the test functions: (a) Ackley function, (b) Damavandi function.

Therefore, it can be employed to test the local search and global search abilities of the algorithm.

3.2.1. Analysis of the convergence accuracy

From Table 4, it can be seen that there are five test functions, namely F_8–F_{12} in which the SSA has better outperforms than the other algorithms. Then, for the F_8 test function, the SSA can achieve the solution which is closest to the optimal value compared to the other algorithms. For the F_9 test function, the SSA can successfully find an excellent solution and it always converges to the global minimum in each experiment. It is clear that the SSA has a good global search capability. The search abilities of SSA and GWO on the F_{10} test function are basically the same, followed by GSA, and the worst is PSO. On the F_{11} test function, the four algorithms can quickly converge to the optimal value of the function. But by obtaining the mean, we can notice that although GWO, GSA and PSO can find the global optimal value, it is easy to fall into the local optimum value during the iterative process. For the F_{12} test function, the SSA shows good performance in all aspect. On the whole, the SSA algorithm has strong exploration capability.

3.2.2. Analysis of the stability

On the F_8 test function, we can see that the proposed algorithm is better than the other three algorithms in terms of solution accuracy, but it is relatively poor in stability. For the remaining functions, i.e. F_9–F_{12} in which the SSA has better stability in comparison to GWO, GSA and PSO. Furthermore, from these results, we clearly know that the SSA has better performance and strong adaptability when dealing with multimodal test functions.

3.2.3. Analysis of the convergence speed

Firstly, we test the convergence speed of the algorithm on the multimodal test functions, the results of all the algorithms are shown in Figure 4. Then, for the F_8 test function, it can be reasonably concluded that the SSA converges to a value close to the optimal solution after about 200 iterations. This also makes it clearer that the SSA highlights its superiority. It can be proved from the F_9 test function that the SSA converges to the optimal value after about 20 iterations and the GWO converges to the optimal value after about 180 iterations. As a result, the proposed SSA has a much faster speed than others. For the remaining functions, the proposed algorithm also obtains very competitive results. Overall, the figure shows that the SSA has higher search efficiency and convergence rate than the other three algorithms for the multimodal functions, and the processing process is very stable.

After the above analysis, we further conclude that the SSA has global search ability and strong adaptability. Note that it is controlled by the mechanism of the SSA itself. In the SSA, the different behavioural strategies of the sparrows have made great contributions to the global search.

Table 4. Results of test functions.

F	SSA Best	SSA Ave	SSA Std	GWO Best	GWO Ave	GWO Std	PSO Best	PSO Ave	PSO Std	GSA Best	GSA Ave	GSA Std
F1	0.0	0.0	0.0	4.1687E−88	2.9804E−85	8.0928E−85	1.3375E−26	2.3453E−24	7.7381E−24	1.6686E−18	3.6298E−18	9.7994E−19
F2	0.0	2.1196E−259	0.0	4.9914E−50	4.0755E−49	4.0798E−49	1.2849E−13	9.6805E−11	3.1274E−10	7.6671E−09	9.9675E−09	1.3844E−09
F3	0.0	0.0	0.0	6.1877E−32	2.1458E−26	7.3559E−26	0.0099	0.0857	0.0712	5.7431E+01	1.1588E+02	3.6476E+01
F4	0.0	5.7257E−278	0.0	4.6452E−23	7.5793E−22	9.1734E−22	8.8377E−04	0.0100	0.0097	7.4751E−10	1.0016E−09	1.3933E−10
F5	9.9463E−10	9.2495E−07	1.9881E−06	25.1280	26.0690	0.7228	8.1702	46.5122	35.9017	2.5746E+01	2.5982E+01	1.2923E−01
F6	8.1322E−14	6.8266E−11	1.5154E−10	3.6952E−06	0.2325	0.1948	2.5356E−26	1.2547E−24	1.8180E−24	2.6556E−18	3.8571E−18	8.4977E−19
F7	9.3993E−06	1.3458E−04	1.7597E−04	3.1934E−05	2.3525E−04	1.3641E−04	3.2000E−03	7.0000E−03	2.7000E−03	2.2159E−03	5.5783E−03	1.6588E−03
F8	−9013.0	−7726.67	698.7294	−7570.5	−6347.9	615.6736	−8700.4	−6960.8	838.1568	−3996.8	−3059.1	404.0800
F9	0.0	0.0	0.0	0.0	0.2734	1.4976	29.8495	44.4083	9.8581	2.9549	7.6944	2.9088
F10	8.8818E−16	8.8818E−16	0.0	7.9936E−15	1.0125E−14	2.5721E−15	7.1942E−14	0.3429	0.6375	1.2460E−09	1.5116E−09	1.5989E−10
F11	0.0	0.0	0.0	0.0	1.4380E−03	3.7560E−03	0.0	0.0160	0.0220	0.0	0.1687	0.3537
F12	8.6262E−17	7.1744E−12	1.5378E−11	6.2991E−03	3.3584E−02	1.4395E−02	4.0845E−27	0.0484	0.1209	1.5659E−20	1.0367E−02	3.1101E−02
F13	−1.0316	−1.0316	6.5843E−16	−1.03162845	−1.0316	2.0977E−09	−1.0316	−1.0316	6.7752E−16	−1.0316	−1.0316	6.5368E−16
F14	1.7097E−13	3.9574E−10	1.3466E−09	3.7946E−06	1.7593	0.5920	−8.5487E−14	1.9333	0.3651	0.3394	1.8186	3.9740
F15	−1.0	−1.0	0.0	−1.0	−0.0336	0.1825	−1.0	−0.5732	0.3936	−0.0114	−0.0085	0.0012
F16	−1.0	−1.0	0.0	−1.0	−0.3667	0.4901	−1.0	−0.6667	0.4795	−1.0	−9.3836E−01	7.3655E−02
F17	3.0749E−04	3.1035E−04	1.5695E−05	3.0749E−04	3.3700E−04	3.7200E−04	3.0749E−04	5.9429E−04	2.6776E−04	7.8978E−04	1.9607E−03	4.9189E−04
F18	−3.8628	−3.8628	2.7101E−15	−3.8628	−3.8622	0.0020	−3.8628	−3.8628	2.7101E−15	−3.8628	−3.8628	2.6097E−15
F19	−3.3220	−3.2625	0.0605	−3.3220	−3.2347	0.0659	−3.3220	−3.2744	0.0592	−3.3220	−3.3220	1.3323E−15

3.3. Fixed-dimension test function

In order to more fully test the performance of the SSA, we selected seven fixed-dimensional functions to verify the convergence speed, stability and convergence accuracy of the algorithm.

3.3.1. Analysis of the convergence accuracy

According to Table 4, for the F_{13} test function, the simulating results indicate that the four algorithms can search the optimal value quickly and efficiently. More concretely, the four algorithms show a good balance between the exploration and the exploitation on the F_{13} test function. For the F_{14} test function, the accuracy of the SSA is much advantageous all of the comparison algorithms. Moreover, from the optimal value obtained, both GWO and PSO find a better value but the GSA gets the worst value. On the two test functions, F_{15} and F_{16}, the SSA performances almost same, but the other three algorithms are easy to fall into local optimum. For the F_{17} test function, the four algorithms may trap into local optima in each independent experiment. On the F_{18} test function, all four algorithms can find the optimal solution, but by analysing the average value we can find that GWO is slightly worse. The performance of the GSA for solving F_{19} is better than the other three algorithms.

3.3.2. Analysis of the stability

For the F_{13} test function in Table 4, the stabilities of SSA, GSA and PSO are excellent than GWO. On the F_{14} test function, it can be found that the standard deviations of GWO, PSO and GSA are relatively large, which indicates that the stability is comparatively poor. And the standard deviation of the SSA is relatively small, which means that the stability is better. The standard deviation of the SSA is zero on the F_{15} and F_{16} test functions, which shows that the sparrows can stably gather around the global best point. On the F_{17} test function, the stability of the SSA is slightly better compared with other three algorithms. For the F_{18} test function, the GSA, SSA and PSO are better stability, and GWO is the worst stability. On the F_{19} test function, the stability of the GSA is the best.

3.3.3. Analysis of the convergence speed

The convergence curve is illustrated in Figure 5. The four algorithms have high convergence speed on the F_{13} test function. From the convergence curve of F_{14}, it is obvious that the convergence trend of GWO, GSA and PSO is the same during the optimization process. For the F_{15} test function, it can be found that the SSA quickly converges to the optimal value after about 200 iterations. Thus, the convergence speed of the SSA is very fast. In addition, there are four test functions, namely F_{16}–F_{19} in which the

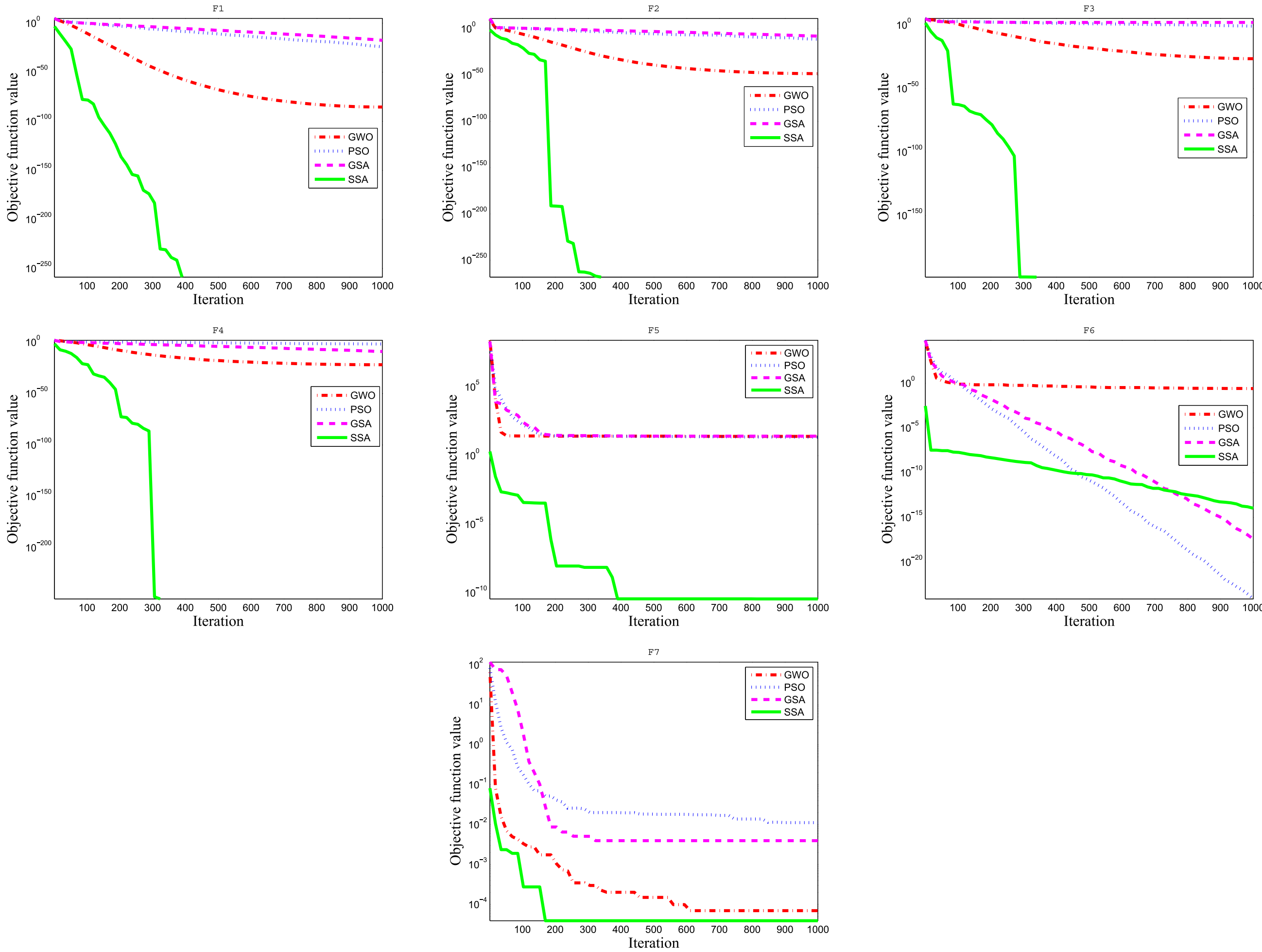

Figure 3. The convergence characteristics of the four algorithms on the unimodal test functions.

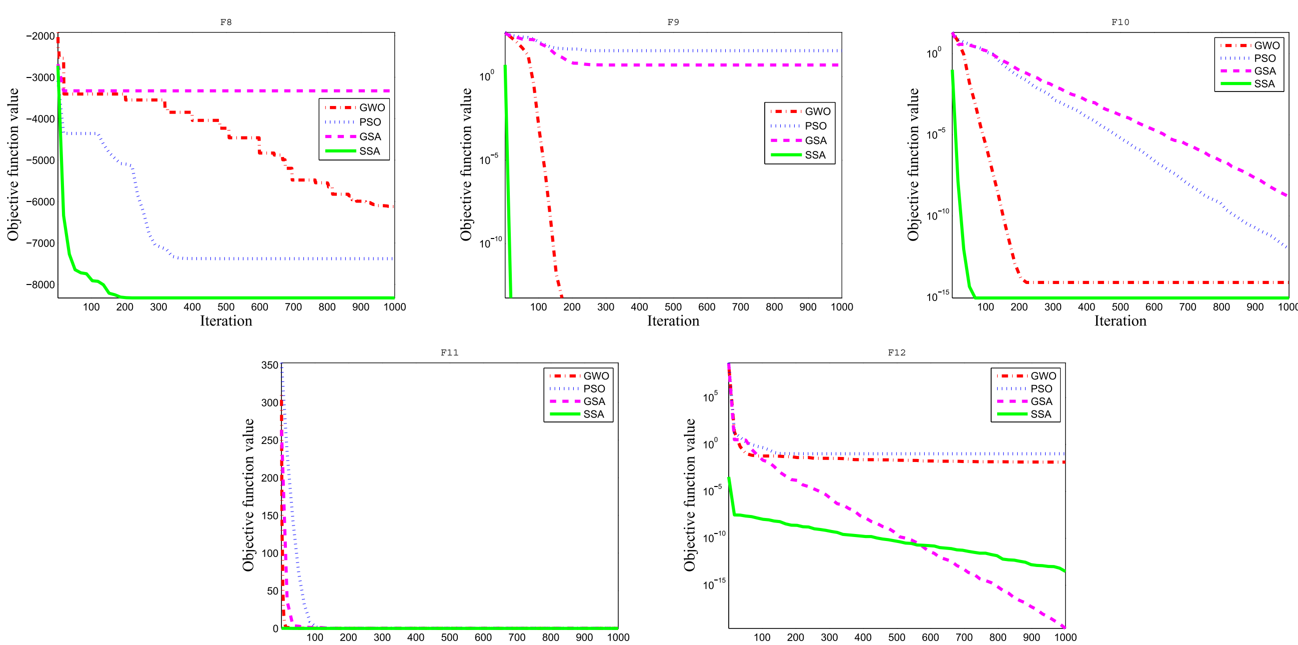

Figure 4. The convergence characteristics of the four algorithms on the multimodal test functions.

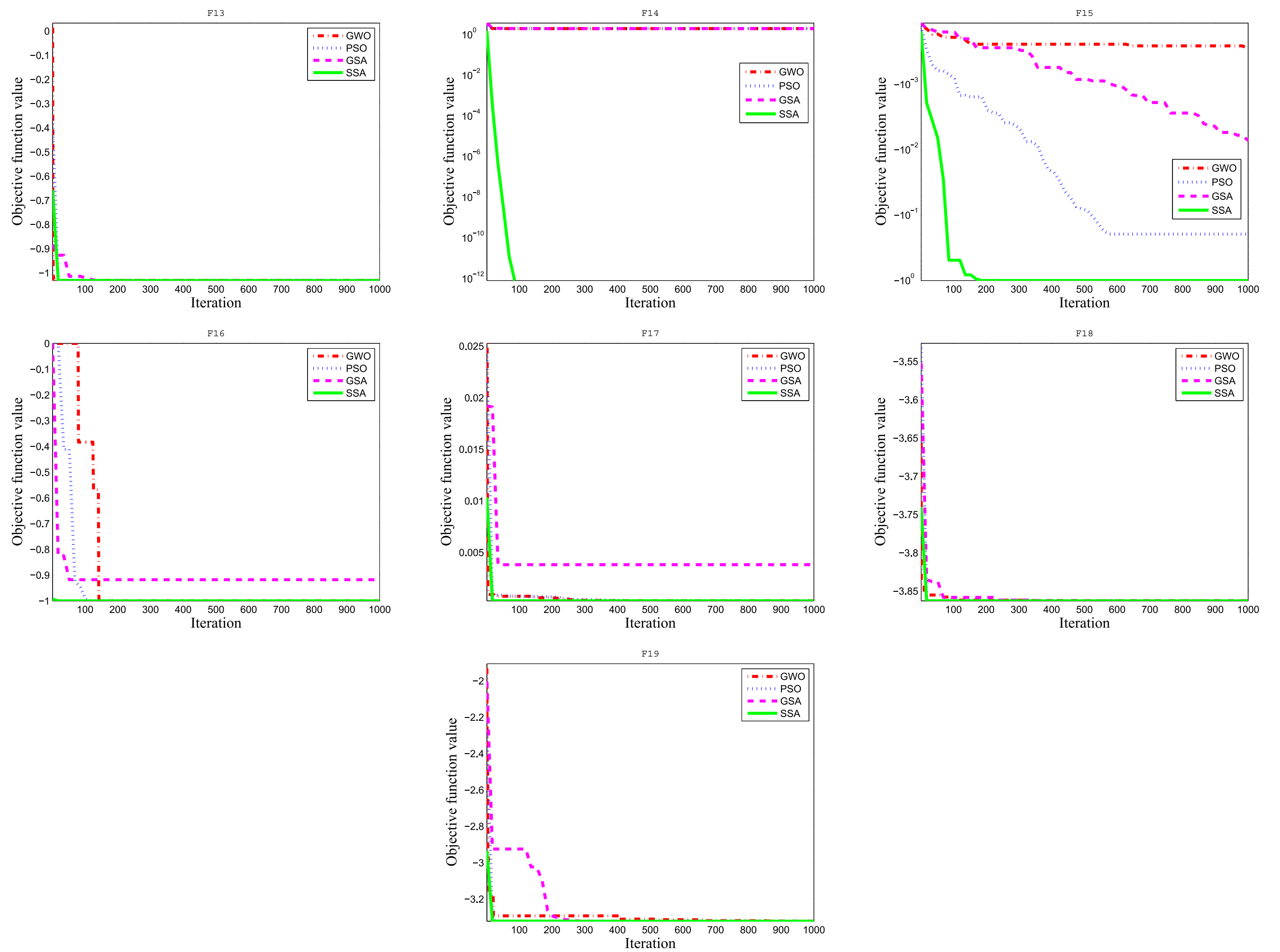

Figure 5. The convergence characteristics of the four algorithms on the fixed-dimension test functions.

convergence speed of the SSA is faster than that of GWO, PSO and GSA. This is because the SSA rapidly converges to a stable value at the beginning of iteration.

The simulation results show that the SSA has strong optimization ability for the optimization of the unimodal test functions, multimodal test functions and fixed-dimension test functions. Moreover, it is seen that the SSA has a certain competitiveness with other state-of-the-art algorithms. Therefore, it is believed that the SSA is able to achieve a certain balance between global exploration and local exploitation.

4. Case studies

In this section, the two practical engineering problems are chosen to illustrate the competitiveness of the proposed algorithm in solving constrained optimization problems with mix variables. For the processing of the inequality constraints in the problem we use penalty functions, which embeds constraints into the objective function. The formula described as follows:

$$\check{f}(\mathbf{x}) = f(\mathbf{x}) + \phi \sum_{j=1}^{p} g_j^{\kappa}(\mathbf{x})\delta(g_j(\mathbf{x})) \tag{6}$$

where $\kappa \in \{1, 2\}$, and $\phi \gg 1$ is the penalty parameter. $\check{f}(\mathbf{x})$ is the penalized objective function, and $f(\mathbf{x})$ is the original fitness function. p denotes the number of the inequality constraints. $g_j(\mathbf{x})$(j $=$ 1,2, . . . ,p) are the inequality constraints. In addition, $\delta(g_j(\mathbf{x}))$ is defined as

$$\delta(g_j(\mathbf{x})) = \begin{cases} 1, & \text{if } g_j(\mathbf{x}) > 0 \\ 0, & \text{if } g_j(\mathbf{x}) \leq 0 \end{cases}$$

4.1. *Case I. Himmelblau's nonlinear optimization problem*

The Himmelblau's nonlinear optimization is a well-known benchmark problem, which has been applied to many fields. The problem is outlined as

$$\min: \ f(x) = 5.3578547x_3^2 + 0.8356891x_1x_5$$

$$
\begin{aligned}
&\quad + 37.293239x_1 - 40,792.141 \\
\text{s.t. } g_1(x) &= 85.334407 + 0.0056858x_2x_5 \\
&\quad + 0.0006262x_1x_4 - 0.0022053x_3x_5 \\
g_2(x) &= 80.51249 + 0.0071317x_2x_5 \\
&\quad + 0.0029955x_1x_2 + 0.0021813x_3^2 \\
g_3(x) &= 9.300961 + 0.0047026x_3x_5 \\
&\quad + 0.0012547x_1x_3 + 0.0019085x_3x_4 \\
&0 \le g_1(x) \le 92 \\
&90 \le g_2(x) \le 110 \\
&20 \le g_3(x) \le 25 \\
&78 \le x_1 \le 102 \\
&33 \le x_2 \le 45 \\
&27 \le x_3 \le 45 \\
&27 \le x_4 \le 45 \\
&27 \le x_5 \le 45 \qquad (7)
\end{aligned}
$$

We independently run 30 times to generate the statistical results. Table 5 illustrates the optimized results obtained by the SSA for the Himmelblau's nonlinear optimization problem. Moreover, the feasible best solution is $x = (78, 33, 29.9953, 45, 36.7758)$ with $f(x) = -30,665.5387$. The constraint values are $g = (92, 98.8405, 20)$. It can be clearly seen that the proposed algorithm is feasible on this issue.

4.2. Case II. Speed reducer design

The Figure 6 presents the designing of the speed reducer, which is to minimize the total weight and subject to constraints on the bending stress of the gear teeth, surfaces stress, transverse deflections of the shafts and stresses in the shafts. The variables $b(x_1), m(x_2), z(x_3), l_1(x_4), l_2(x_5), d_1(x_6), d_2(x_7)$ are the face width, module of teeth, number of teeth on pinion, length of shaft one between bearings, length of shaft two between bearings and the diameter of the first and second shafts, repectively. These constraints and the problem are designed as below:

Table 5. Results obtained by the SSA for the Himmelblau's nonlinear optimization problem.

Best	Worst	Mean	Std	No. sparrow
−30,665.5387	−30,662.8505	−30,665.3808	0.5713	40

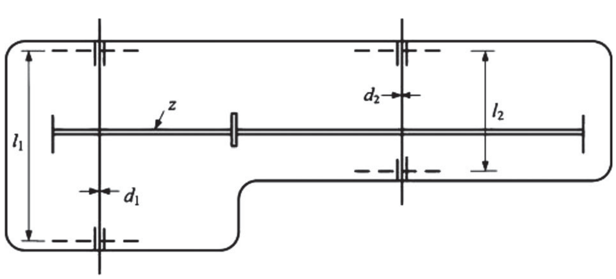

Figure 6. Schematic of the speed reducer design (Gandomi, Yang, & Alavi, 2013).

$$
\begin{aligned}
\min: \quad & f(b;m;z;l_1;l_2;d_1;d_2) = 0.7854bm^2(3.3333z^2 \\
&\quad + 14.9334z - 43.0934) - 1.508b(d_1^2 + d_2^2) \\
&\quad + 7.4777(d_1^3 + d_2^3) + 0.7854(l_1d_1^2 + l_2d_2^2) \\
\text{s.t.} \quad & g_1(x) = \frac{27}{bm^2z} - 1 \le 0, \\
& g_2(x) = \frac{397.5}{bm^2z^2} - 1 \le 0 \\
& g_3(x) = \frac{1.93l_1^3}{mzd_1^4} - 1 \le 0 \\
& g_4(x) = \frac{1.93l_2^3}{mzd_2^4} - 1 \le 0 \\
& g_5(x) = \frac{\sqrt{M^2 + 16.9 \times 10^6}}{110d_1^3} - 1 \le 0 \\
& g_6(x) = \frac{\sqrt{H^2 + 157.5 \times 10^6}}{85d_2^3} - 1 \le 0 \\
& g_7(x) = \frac{mz}{40} - 1 \le 0 \\
& g_8(x) = \frac{5m}{b} - 1 \le 0 \\
& g_9(x) = \frac{b}{12m} - 1 \le 0 \\
& g_{10}(x) = \frac{1.5d_1 + 1.9}{l_1} - 1 \le 0 \\
& g_{11}(x) = \frac{1.1d_2 + 1.9}{l_2} - 1 \le 0 \\
& 2.6 \le b \le 3.6 \\
& 0.7 \le m \le 0.8 \\
& 17 \le z \le 28 \\
& 7.3 \le l_1 \le 8.3 \\
& 7.8 \le l_2 \le 8.3 \\
& 2.9 \le d_1 \le 3.9 \qquad (8)
\end{aligned}
$$

where

$$M = \frac{745l_1}{mz}, \quad H = \frac{745l_2}{mz}$$

In the SSA, the maximum number of the iterations is 3000 and the sparrow population size is set to 300. We independently run 30 times. Table 6 shows the optimization results for the speed reducer problem. In this

Table 6. Results obtained by the SSA for the speed reducer problem.

Best	Worst	Mean	Std
2996.7077	3008.1638	2997.7101	2.8209

case, the optimal solution obtained by our algorithm is $x = (3.500059, 0.7, 17, 7.3, 7.8, 3.351209, 5.286813)$ with a function value of 2996.7077. Here, the constraint values are $g = (-0.073931, -0.19801, -0.49977, -0.90148, -0.00089021, -7.374e-05, -0.7025, -1.6971e-05, -0.58333, -0.051121, -0.010834)$. Hence, our result is feasible and verifies the effectiveness of the proposed algorithm.

5. Conclusion

In this paper, we present an effective optimization technique, the sparrow search algorithm, which simulates the foraging and anti-predation behaviours of sparrows. Then, it introduces the mathematical model and the framework of the proposed algorithm. Finally, the performance of the SSA is compared with that of the GWO, PSO and GSA on 19 test functions. The results demonstrate that the proposed SSA can provide highly competitive results compared with the other state-of-the-art algorithms in terms of searching precision, convergence speed, and stability. Moreover, the results of the two practical engineering problems also exhibit that the SSA has high performance in diverse search spaces. As analysis above, it can be seen that the SSA has a good ability to explore the potential region of the global optimum, and hence the local optimum issue is avoided effectively.

In our further research, we would continue to do more in-depth analysis and research on the SSA. Also, we would try to apply this algorithm in more complex practical engineering problems, such as travelling salesman problem (TSP), robot path planning problem, etc. Moreover, we would extend the current SSA to deal with the multi-objective optimization problem.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported in part by the National Natural Science Foundation of China [grant numbers 61873059 and 61922024], the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning of China, and the Natural Science Foundation of Shanghai [grant number 18ZR1401500].

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