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Analytical and quasi-explicit four arbitrary point method for extraction of solar cell single-diode model parameters

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ABSTRACT

In this paper, the five parameters of the solar cell single-diode model are analytical and quasi-explicitly extracted for the first time, just using the coordinates of four arbitrary points of the characteristic *I*–*V* curve and the slopes of the curve in these points. The new method presented, called Analytical and Quasi-Explicit (AQE) method, is exact because no simplifications of the model nor *a priori* approximations of the parameters are used and, it is quasi-explicit in the sense that all the parameters except one are explicitly given. The unique parameter not explicitly computed is easily obtained by solving a five-degree polynomial equation. Accurate and practical conditions are provided to select which solution of the previous equation is the desired parameter.

It is also introduced a very easy method to obtain, directly from real data measurements, the needed four points of the *I*–*V* curve as well as the slopes in these points, without using any kind of sophisticated techniques.

Finally, some experimental results are presented to demonstrate the high accuracy and simplicity of the new method. The results are compared with the well-known analytical five-point method and the recent oblique asymptote method.

1. Introduction

The single diode model or one exponential diode model is one of the most used for characterizing the electrical behavior of a solar cell or a photovoltaic (PV) module. This is because the theoretical curve generated by this model fits the real *I*–*V* characteristic of most of the PV modules with very good accuracy (see, for instance, [1,2]) under a minimum of illumination (about half AM1 according to [1]). If the five parameters of the model are extracted for certain conditions of irradiance and temperature, then the electrical behavior of the PV module can be precisely predicted for any other conditions of irradiance and temperature.

Since the sixties, a lot of methodologies have been suggested for the extraction of the parameters. Some of the previous methodologies consist on solving a system of five non-linear equations obtained after substituting real data, measured on the *I*–*V* curve or provided by the manufacturer in the datasheet, on both, the model equation and the equation obtained differentiating (implicitly) in the previous one. To solve the previous system, a lot of procedures have been proposed. The so-called *exact methods* try to solve the system numerically using, for instance, the *Newton*–*Raphson* technique for simultaneous equations [3]. In general, the exact methods require extensive computation and also good initial guesses for the iterations to converge. The so-called *analytical methods* try to extract the parameters explicitly, for instance, considering some approximations on some terms of the system equations [4–6]. These methods are reliable and accurate and they moreover have the advantage of simplicity. Nevertheless, these methods also have some drawbacks. First, they just provide approximate solutions of the model but, even more important, they usually need to know some information (for instance, the slopes) of the *I*–*V* curve in the short circuit point, the open circuit point and the maximum power point, which can entail some difficulties to be accurately measured in practice.

Another kind of methodologies consists on the best (in some sense) theoretical curve which fits the real one. This is accomplished by solving an optimization problem (typically a least square problem) that consists on minimizing, between all the possible parameters (in other words, between all the possible theoretical

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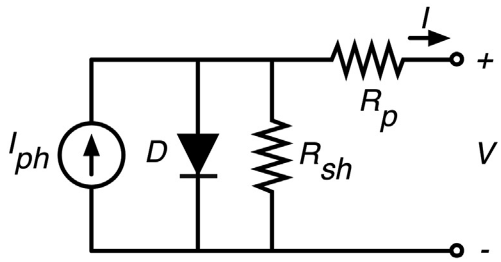

Fig. 1. Single-diode model equivalent electrical circuit.

curves), certain objective function that depends on both, the real and the theoretical data. These methods are called *curve fitting-methods*. For instance, the so-called vertical optimization problem consists essentially on minimizing the differences between the real and the theoretical current intensities [7,8]. If intensities are changed by voltages in the previous problem, the lateral optimization method [9] is obtained. A combination of the vertical and the lateral optimization methods has been also suggested in Ref. [10]. Another possibility consists on minimizing the difference between the conductances [11] or the area between the real and the theoretical curve [1]. There are a lot of possibilities that have been studied but many others could also be explored. The curve fitting methods are very good methods for fitting the entire curve in some sense, although in general the fitting techniques require a lot of computation and time consumption besides powerful mathematical tools. In addition, the main problem of the curve fitting method is that they strongly need initial data near the solution so they can converge.

There are also methods which use the non-elementary *Lambert W* function to extract the parameters [12,13], taking the advantage that current can be written as an explicit function of the voltage and vice versa. Nevertheless, the *Lambert W* function is itself defined implicitly and, therefore, at least the numerical methods needed to obtain the solutions that provide the *Lambert W* function are also needed to obtain the parameters of the model.

Some papers dealing with simplified models of three and four parameters [6,14,15] have been able to obtain exactly and explicitly the corresponding parameters just by using information of the I–V curve near the operating point of the PV module, that is, the point where the PV module obtains its maximum power. Nevertheless, these models are only valid for some specific conditions in which, for instance, the shunt resistance is so high that can be neglected.

The new method presented in this paper, called Analytical Quasi-Explicit (AQE) method, does not need any kind of assumptions about the value of the parameters, neither needs to use special points, in fact, the required four points can be arbitrarily located along the I–V curve. It also does not need too much computation and neither initial guesses. It just needs acceptable accuracy of the data provided and to solve a five-degree polynomial equation. This equation, which only depends on the data, is very easy and quick to solve with common numerical methods, most of them already implemented in math programs. Since the polynomial has odd degree, the equation always has a real solution, however, the equation may have more than one solution, which occurs in practice. Any case, an easy and tight bound on the parameter to be obtained by the equation is provided allowing to select which solution is the correct parameter. The presented method requires to have four points of the I–V curve and the slope of the curve at these points. An easy method to obtain the four points and the corresponding slopes is also provided in this work, this method is based on a new function called **α-power function**. This function allows obtaining the slope at a point in a similar way as the slope at the maximum power point is obtained in many papers.

The structure of the paper is the following: In section 2 the theory of the AQE method is presented. In section 3 it is introduced the α-power function for obtaining the points of the I–V curve with their slopes. Section 4 presents the experimental results and their comparison with some well-known parameters extraction methods. Finally, section 5 summarizes and concludes the study.

2. Theory of the analytical quasi-explicit method

The single diode model equivalent electrical circuit of a solar cell is depicted in Fig. 1.

This model can be extrapolated to a photovoltaic (PV) model with n_p cells in parallel and n_s cells in series. At a given illumination, the relation between the current and the voltage is given by model equation (1):

$$I = n_p I_{ph} - n_p I_{sat}\left(e^{\frac{\frac{V}{n_s}+\frac{IR_s}{n_p}}{nV_T}} - 1\right) - n_p\frac{\frac{V}{n_s}+\frac{IR_s}{n_p}}{R_{sh}} \tag{1}$$

where I_{ph} is the photocurrent, I_{sat} is the diode saturation current, n is the diode ideality factor, $V_T = \frac{k}{q}T$, being T the temperature of the cell, k the Boltzmann's constant and, q the electronic charge, R_s is the series resistance and, R_{sh} is the shunt resistance.

This model fits the I–V characteristic of most of the PV cells and modules with very good accuracy. If the five parameters I_{ph}, I_{sat}, n, R_s, and R_{sh} are extracted for the standard conditions of irradiance and temperature, then the electrical behavior can be precisely predicted for any other conditions of irradiance and temperature. The photovoltaic module model equation (1) can be rewritten as

$$I = A - B\left(C^V D^I - 1\right) - EV \tag{2}$$

where

$$A = n_p I_{ph}\frac{R_{sh}}{R_{sh}+R_s}, B = n_p I_{sat}\frac{R_{sh}}{R_{sh}+R_s}, C = e^{\frac{1}{n_s nV_T}}, D = e^{\frac{R_s}{n_p nV_T}}, E = \frac{n_p}{n_s}\frac{1}{R_{sh}+R_s}$$

If the new parameters A, B, C, D, and E are obtained, then the original parameters are also determined by

$$I_{ph} = A\frac{1}{n_p}\frac{\ln(C)}{\ln(C)-E\ln(D)}, I_{sat} = B\frac{1}{n_p}\frac{\ln(C)}{\ln(C)-E\ln(D)}, nV_T = \frac{1}{n_s}\frac{1}{\ln(C)}, R_s = \frac{n_p}{n_s}\frac{\ln(D)}{\ln(C)}, R_{sh} = \frac{n_p}{n_s}\left(\frac{1}{E} - \frac{\ln(D)}{\ln(C)}\right)$$

Note that, if at the moment of obtaining the data it is known the temperature T of the cells, the diode ideality factor n can be obtained as $n = \frac{1}{V_T}\frac{1}{n_s}\frac{1}{\ln(C)}$.

2.1. Resolution of the model by means of the AQE method

In this subsection it is explained the mathematical development of the new method (AQE) to extract the parameters of the solar cell single-diode model.

Rearranging equation (2) and denoting $K = A + B$, it is obtained

$$I = \frac{1}{\ln(D)}\left(\ln(K - I - EV) - V\ln(C) - \ln(B)\right) \tag{3}$$

Now, taking the derivative with respect to V in (3)

$$I' = \frac{-1}{\ln(D)}\left(\frac{I'+E}{K-I-EV} + \ln(C)\right) \tag{4}$$

Assume now that we have four points P_1, P_2, P_3 and P_4 of the I–V curve with coordinates (V_j, I_j) and slopes I'_j, for $j = 1,2,3,4$. Forcing that the model satisfies these data then

$$I(V_j) = I_j \text{ and } I'(V_j) = I'_j \text{ for } j = 1,2,3,4 \tag{5}$$

Considering the quotients

$$\frac{I'(V_1) - I'(V_2)}{I'(V_2) - I'(V_3)} \text{ and } \frac{I'(V_2) - I'(V_3)}{I'(V_3) - I'(V_4)},$$

the following system with two equations and two unknowns, E and K, is obtained from (4) and (5)

$$\begin{cases} \dfrac{I'_1 - I'_2}{I'_2 - I'_3} = \dfrac{(K-I_3-EV_3)\big((K-I_2-EV_2)(I'_1+E) - (K-I_1-EV_1)(i'_2+E)\big)}{(K-I_1-EV_1)\big((K-I_3-EV_3)(I'_2+E) - (K-I_2-EV_2)(I'_3+E)\big)} \\ \dfrac{I'_2 - I'_3}{I'_3 - I'_4} = \dfrac{(K-I_4-EV_4)\big((K-I_3-EV_3)(I'_2+E) - (K-I_2-EV_2)(I'_3+E)\big)}{(K-I_2-EV_2)\big((K-I_4-EV_4)(I'_3+E) - (K-I_3-EV_3)(I'_4+E)\big)} \end{cases} \tag{6}$$

Using elementary algebraic calculus, the previous system becomes the following equivalent system

$$\begin{cases} K(p_1(E) - p_3(E)) = p_2(E) - p_4(E) \\ K(q_1(E) - q_3(E)) = q_2(E) - q_4(E) \end{cases} \tag{7}$$

where $p_1(E),\ p_3(E),\ q_1(E)$, and $q_3(E)$ are polynomials in E of degree 2, and $p_2(E),\ p_4(E),\ q_2(E)$, and $q_4(E)$ are polynomials in E of degree 3 given by

$$p_1(E) = (I'_1 - I'_2)\big((I_2+EV_2)(I'_3+E) - (I_3+EV_3)(I'_2+E) - (I_1 + EV_1)(I'_2 - I'_3)\big)$$

$$p_2(E) = (I'_1 - I'_2)(I_1+EV_1)\big((I_2+EV_2)(I'_3+E) - (I_3+EV_3)(I'_2+E)\big)\big)$$

$$p_3(E) = (I'_2 - I'_3)\big((I_1+EV_1)(I'_2+E) - (I_2+EV_2)(I'_1+E) - (I_3 + EV_3)(I'_1 - I'_2)\big)$$

$$p_4(E) = (I'_2 - I'_3)(I_3+EV_3)\big((I_1+EV_1)(I'_2+E) - (I_2+EV_2)(I'_1+E)\big)$$

$$q_1(E) = \left(I'_2 - I'_3\right)\big((I_3+EV_3)(I'_4+E) - (I_4+EV_4)(I'_3+E) - (I_2 + EV_2)(I'_3 - I'_4)\big)$$

$$q_2(E) = (I'_2 - I'_3)(I_2+EV_2)\big((I_3+EV_3)(I'_4+E) - (I_4+EV_4)(I'_3+E)\big)$$

$$q_3(E) = \left(I'_3 - I'_4\right)\big((I_2+EV_2)(I'_3+E) - (I_3+EV_3)(I'_2+E) - (I_4 + EV_4)(I'_2 - I'_3)\big)$$

$$q_4(E) = (I'_3 - I'_4)(I_4+EV_4)\big((I_2+EV_2)(I'_3+E) - (I_3+EV_3)(I'_2+E)\big)$$

From system (7) the following polynomial equation which only depends on E is obtained.

$$(p_2(E) - p_4(E))(q_1(E) - q_3(E)) - (p_1(E) - p_3(E))(q_2(E) - q_4(E)) = 0 \tag{8}$$

Let us call

$$p(x) = (p_2(x) - p_4(x))(q_1(x) - q_3(x)) - (p_1(x) - p_3(x))(q_2(x) - q_4(x)) \tag{9}$$

Observe that p is a polynomial of degree 5. Computing the roots of p, the solutions of (8) are obtained and, one of these solutions is precisely the parameter E.

Once obtained the parameter E, the remaining parameters are extracted consecutively as follows:

Parameter K is obtained, for instance, from the first equation of (7) as

$$K = \frac{p_2(E) - p_4(E)}{p_1(E) - p_3(E)} \tag{10}$$

Now, D is obtained from (4) and (5) considering the difference $I'(V_3) - I'(V_4)$:

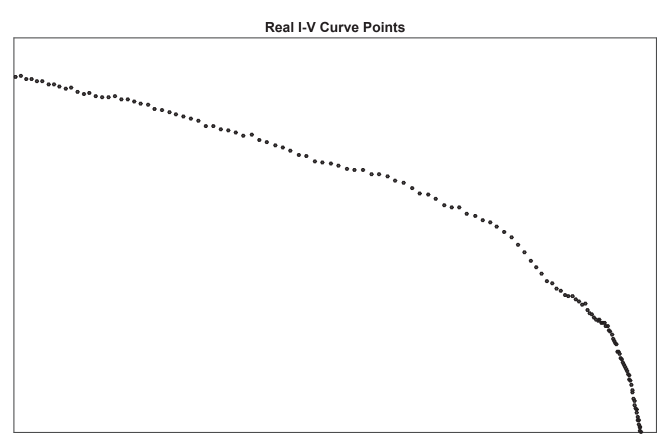

Fig. 2. Aspect of part of a real I–V curve.

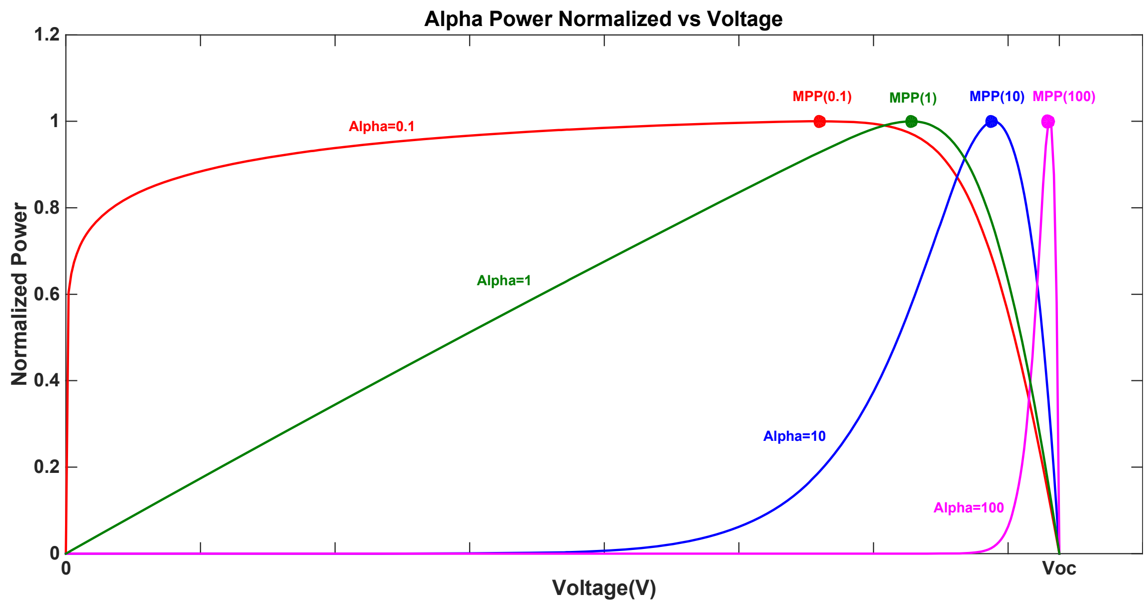

Fig. 3. Example of normalized α-power functions.

Table 1
Points used in Test 1.

Point	Voltage [V]	Current [A]	Derivative [A/V]
I_{sc}	0	3.9840637	−0.0079681
M_{PP}	31.3719970	3.6091172	−0.1150426
P_3	35.2719834	1.8	−0.9318895
V_{oc}	36.9004017	0	−1.2559520

Table 2
Theoretical and extracted parameters.

	I_{ph} [A]	I_{sat} [A]	nV_T [J C]	R_s [Ω]	R_{sh} [Ω]
Theoretical	4	1e-14	1.1	0.5	125
AQE Method	4	1e-14	1.1	0.5	125
5p method	3.9999	1.0858e-14	1.1028	0.49734	125.5
OA method	4	1e-14	1.1	0.5	125

$$D = e^{-\frac{1}{I'_3 - I'_4}\left(\frac{I'_3+E}{K-I_3-EV_3} - \frac{I'_4+E}{K-I_4-EV_4}\right)} \tag{11}$$

Again from (4) and (5), C is obtained as:

$$C = e^{-I'_4 \ln(D) - \frac{I'_4+E}{K-I_4-EV_4}} \tag{12}$$

Parameter B is obtained from (2) and (5) as:

$$B = \frac{K - I_4 - EV_4}{C^{V_4} D^{I_4}} \tag{13}$$

Finally,

$$A = K - B \tag{14}$$

Theoretically, the parameters can be obtained with another data points but, in our tests, we have used the previous ones with very good results.

2.2. About the parameter *E*

The fact that polynomial p has odd degree, ensures that it has at least a real root. In principle, the number of solutions is not known (of course less than or equal to five) and neither which of these solutions corresponds to the parameter E. Nevertheless, from Ref. [16] we have that

$$E < -I' \tag{15}$$

for any derivative I' and, in particular,

$$E < -I'(0) \tag{16}$$

Since I' is decreasing (see again [16]), the best upper bound obtained for E is precisely the absolute value of the slope of the I–V curve at short circuit point, that is, $|I'(0)|$. In practice, this value is not always known so the lowest value between the absolute values of the slopes at the selected points will be taken as upper bound of E. This value normally will correspond to the absolute value of the slope of the I–V curve at the selected point with minimum voltage.

On the other hand, in some cases, in real conditions, due to measurement and calculus errors, none of the polynomial solutions is lower than the upper bound selected. If this is the case, E is forced to be the absolute value of the slope of the I–V curve at the known point with minimum voltage ($E = |I'_1|$).

3. The α-power function: obtaining slopes on the *I*–*V* curve

To solve the model, it is needed a set of four points of the I–V curve as well as the slopes of the curve at these points. In a theoretical curve, it is very easy to obtain the desired data and the method works perfectly for four arbitrary points on the I–V curve. In practice, there is a big difficulty for obtaining the slopes of points of the I–V curve because the data are a finite collection of points which moreover have errors of measurements, noise of the electrical instruments, rounding errors, etc (the real I–V curve usually has the aspect shown in Fig. 2).

With points in such a way, it is very difficult to obtain the right slopes using standard mathematical techniques, such as

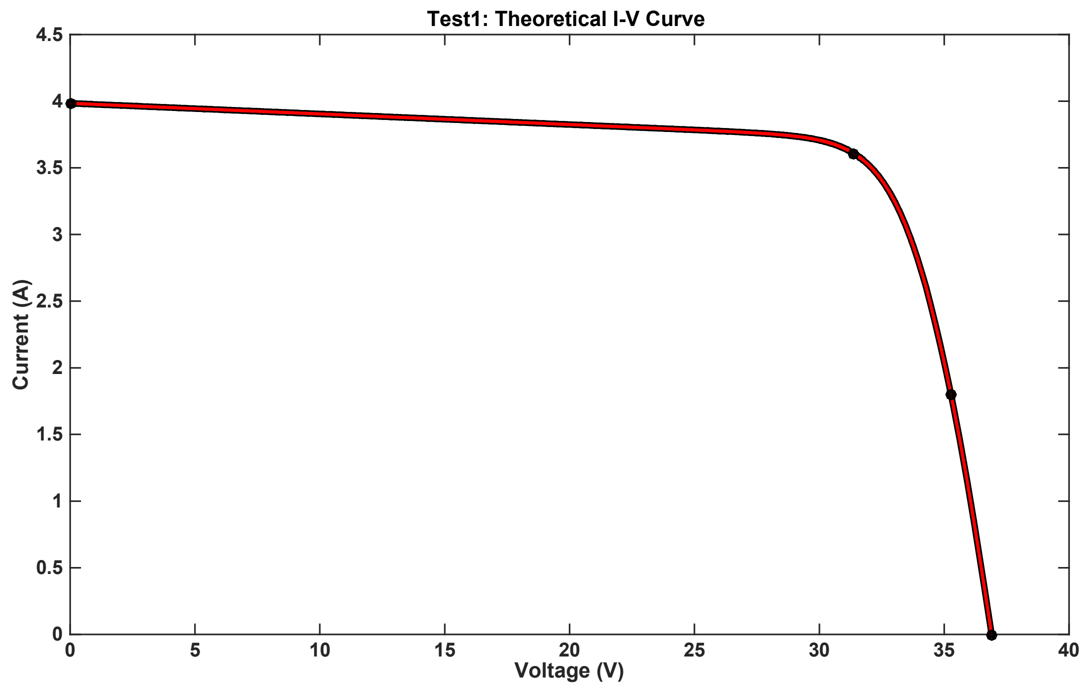

Fig. 4. Test 1 results. Black: Theoretical curve – red: AQE extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 3
Points around MPP used in Test 1.

Point	Voltage [V]	Current [A]	Derivative [A/V]
P_1	30.8719970	3.6570030	−0.0787646
P_2	31.37199703	3.6091172	−0.1150426
P_3	31.871997	3.5393627	−0.1668987
P_4	32.371997	3.4390144	−0.2379233

approximations of the derivative by means of the corresponding incremental quotients. The method proposed in this paper works fine in these bad conditions and it is based on a new function we are going to introduce now. Given a positive real number α, the **α-power function** associated to a PV cell/module is defined as:

$$P_\alpha = V^\alpha I \tag{17}$$

Let us assume that I is a continuous, differentiable and decreasing function with derivative function also decreasing. It was shown at [16] that the function I given by the single-diode model satisfies these hypotheses. Then, function P_α is also continuous and differentiable and attains a global maximum at the compact interval $[0, V_{oc}]$. Since this function vanishes at $V = 0$ and $V = V_{oc}$ and, it is positive at $]0, V_{oc}[$, the maximum will belong to the open interval $]0, V_{oc}[$, specifically, the maximum will be attained at a critical point of the function, that is, a point where the derivative with respect to V is zero. So, if we derivate with respect to V in (17) we obtain that

$$P'_\alpha = \alpha V^{\alpha-1} I + V^\alpha I' = V^{\alpha-1}(\alpha I + V I')$$

and, using that $V > 0$, one has $P'_\alpha = 0$ if and only if

$$I' = -\alpha \frac{I}{V} \tag{18}$$

Taking into account the previous hypotheses over I, it is not difficult to see that there exists a unique voltage V_α satisfying condition (18). Let us denote $I_\alpha = I(V_\alpha)$ and $I'_\alpha = I'(V_\alpha)$, that is, I'_α is the slope of the I–V curve at the point (V_α, I_α).Therefore, the maximum will be attained at the unique point V_α satisfying that

$$I'_\alpha = -\alpha \frac{I_\alpha}{V_\alpha} \tag{19}$$

The point (V_α, I_α) will be called α-power point. Observe that the 1-power point is just the maximum power point.

Now, similarly as one obtains in practice the maximum power point of the PV module and its slope, we can obtain for a given α the maximum α-power point and its slope, providing then a new point of the I–V curve and its slope. Obviously it will depend on the quality of the data but the results are amazingly very good using directly the measured data in real conditions. In Fig. 3 it is shown an example of P_α for a given I–V curve and for different α (0.1, 1, 10 and 100). The curves are normalized dividing each one of them by its corresponding maximum MPP(α) in order to represent all of them in the same figure. As can be seen, each curve has a different MPP(α) which provides a point (V_α, I_α) of the I–V curve with slope $I'_\alpha = -\alpha \frac{I_\alpha}{V_\alpha}$.

4. Experimental results

With the aim to show the effectiveness of the proposed method, several parameters extraction tests are presented in this section. When it is possible the results are compared with the ones obtained using the well-known analytical five-point (5p) method [1] and the recent oblique asymptote (OA) method [16].

4.1. Test 1. Theoretical I–V curve

From a theoretical set of given parameters (*I_{ph} = 4A, I_{sat} = 1e-14A, nV_T = 1.1 JC, R_s = 0.5 Ω and R_{sh} = 125 Ω*), simulating a PV module, a theoretical curve has been generated using equation (1).

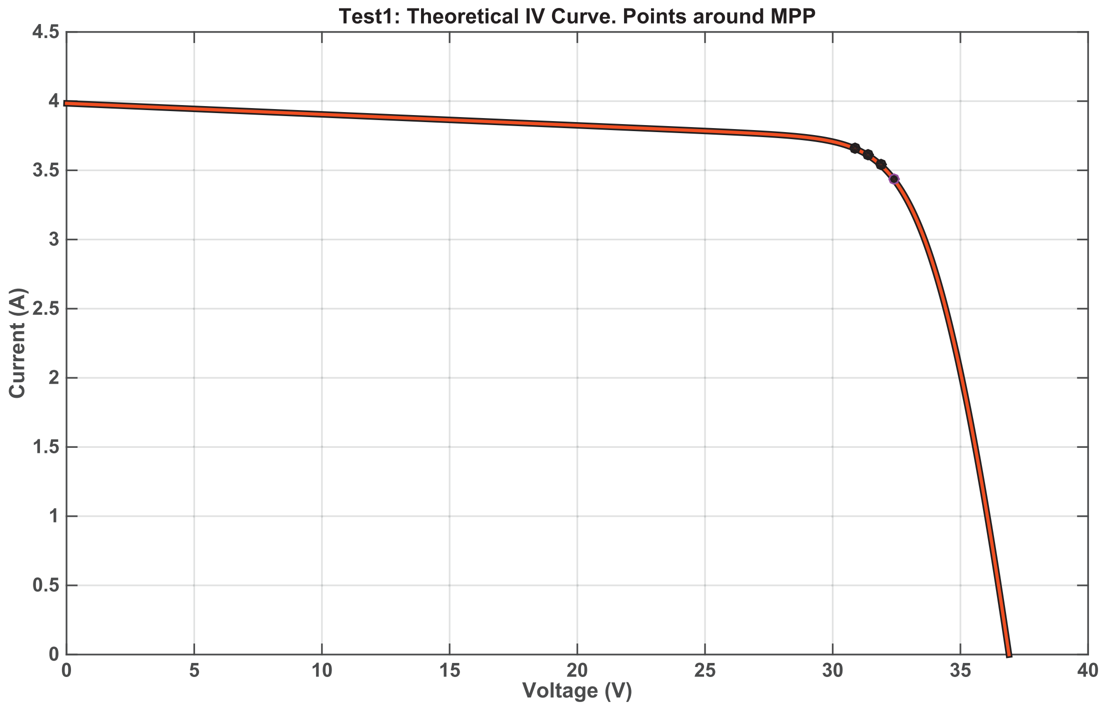

Fig. 5. Test 1 results, points around MPP. Black: Theoretical curve – Red: AQE method extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 4
Points used in Test 2.

Point	Voltage [V]	Current [A]	Derivative [A/V]
I_{sc}	0	3.9857266	−0.0084803
M_{PP}	31.5	3.6010521	−0.1143191
P_3 ($\alpha = 10$)	35.2719834	1.8	−0.9318895
V_{oc}	36.8937502	0	−1.2455982

Table 5
Theoretical and extracted parameters (noisy curve).

	I_{ph} [A]	I_{sat} [A]	nV_T [J C]	R_s [Ω]	R_{sh} [Ω]
Theoretical	4	1e-14	1.1	0.5	125
AQE Method	4.002	5.9457e-14	1.1615	0.48993	123.32
5p method	4.0035	0.064e-14	1.0168	0.526	117.92
OA method	4.0035	0.067e-14	1.0181	0.525	117.39

In order to perform the parameter extraction, four points of the curve have been calculated (with seven decimals accuracy). The coordinates of the points have been obtained from equation (1) and, the corresponding slopes have been computed by means of the following formula (see [7, equation (7)])

$$i' = -\frac{E + BC^v D^i \ln C}{1 + BC^v D^i \ln D}$$

To compare this method with the 5p and the OA methods, the points chosen are: the short circuit (SC) point, the maximum power point (MPP), the open circuit (OC) point and one point located between the MPP and OC points (only in the OA and the AQE methods). In Table 1 are detailed the points used.

In this case, the solutions obtained for E from equation (9) are (0.0079681, 0.1150426, 0.4638778, 0.9318895, 1.1672003). The upper-bound used for obtaining the 'correct' E parameter is $-I'_1$ (0.0079681). In fact, the first solution is lower than this value, although due to the rounding errors they seem equals, so this is the one selected to continue with the calculus.

As can be seen in the results presented in Table 2 and Fig. 4, the AQE method achieves an exact extraction of the parameters, as well as the OA method. The 5p method, due to its intrinsic approximations, does not achieve the exact resolution but a very closed one.

As expected, the results of this test demonstrate that, with accurate data points, the proposed method is exact and the model parameters of any solar cell or PV module can be easily extracted. The great advantage of the AQE method with respect the other ones is that it is also able to do an exact extraction of the parameters with any set of four points and their derivatives (it is not necessary to use the Open Circuit and the Short Circuit points). An example is presented using a set of points near the MPP, given in Table 3, and with these points the exact parameters are also extracted.

In this test, the solutions obtained for E from equation (9) are (0.0079681, 0.1139509, 0.1150426, 0.16689874, 0.4602883). The upper-bound used for obtaining the 'correct' E parameter is $-I'_1$ (0.0787646). Only the first solution is lower than this value, so this is the one selected to continue with the calculus.

In Fig. 5 it is drawn the points used, the theoretical curve (black) and the curve generated from the extracted parameters (red).

4.2. Test 2. Theoretical I–V curve with addition of white Gaussian noise

In this test, with the aim to prove the robustness of the extraction methods, a white Gaussian noise (using the matlab *awgn* function with signal to noise ratio = 50 dB) has been added to the Test 1 theoretical curve. The resulting curve is quite similar to a real measured one, and then the three methods have been tested again. In this case, the I_{sc} point and its slope have been calculated using a linear regression of the first 30% points of the curve, and the same technique has been applied to calculate the V_{oc} point but with the last 1.5% points of the curve. The MPP has been located searching

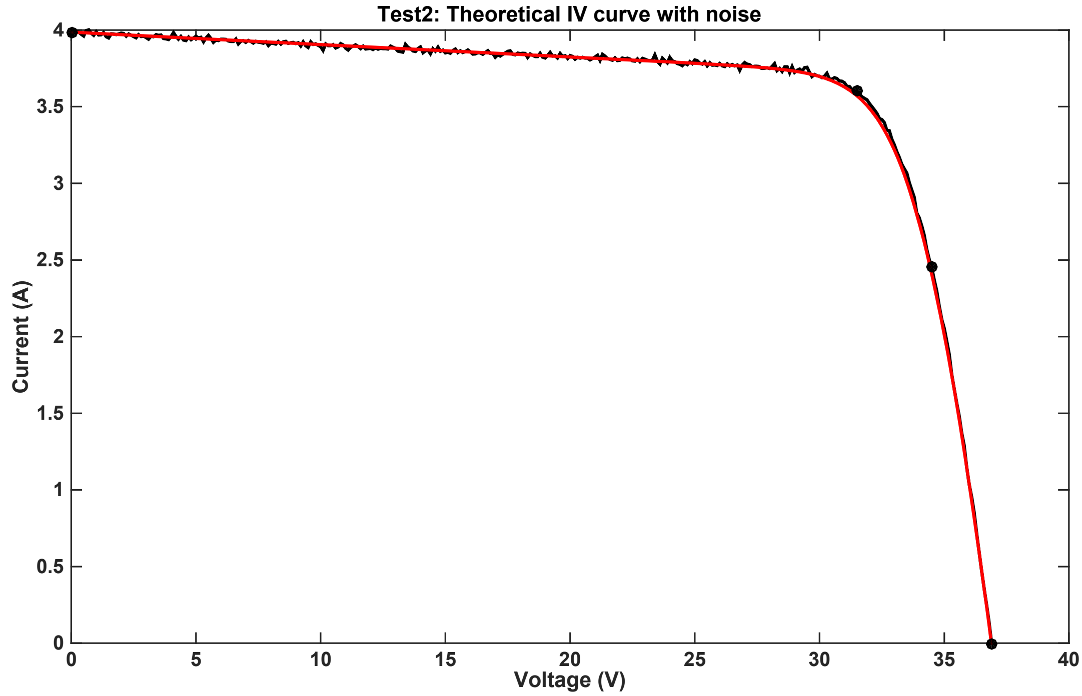

Fig. 6. Test 2 results. Black: Theoretical curve – Red: AQE method extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

for the point with the maximum power, and its slope is given by $-I_{mpp}/V_{mpp}$. The third point and its slope needed for the AQE method has been calculated with the **α-power function** using $\alpha = 10$. In Table 4 are detailed the points used.

The solutions obtained for E from equation (9) are (0.0080767, 0.1143191, 0.3823892, 0.7112709, 1.0985174). The upper-bound used for obtaining the 'correct' E parameter is $-I'_1$ (0.0084803). Only the first solution is lower than this value, so this is the one selected to continue with the calculus.

The theoretical parameters and the parameters extracted with the three methods are listed in Table 5. Due to the added noise, neither method is able to extract the exact parameters but all of

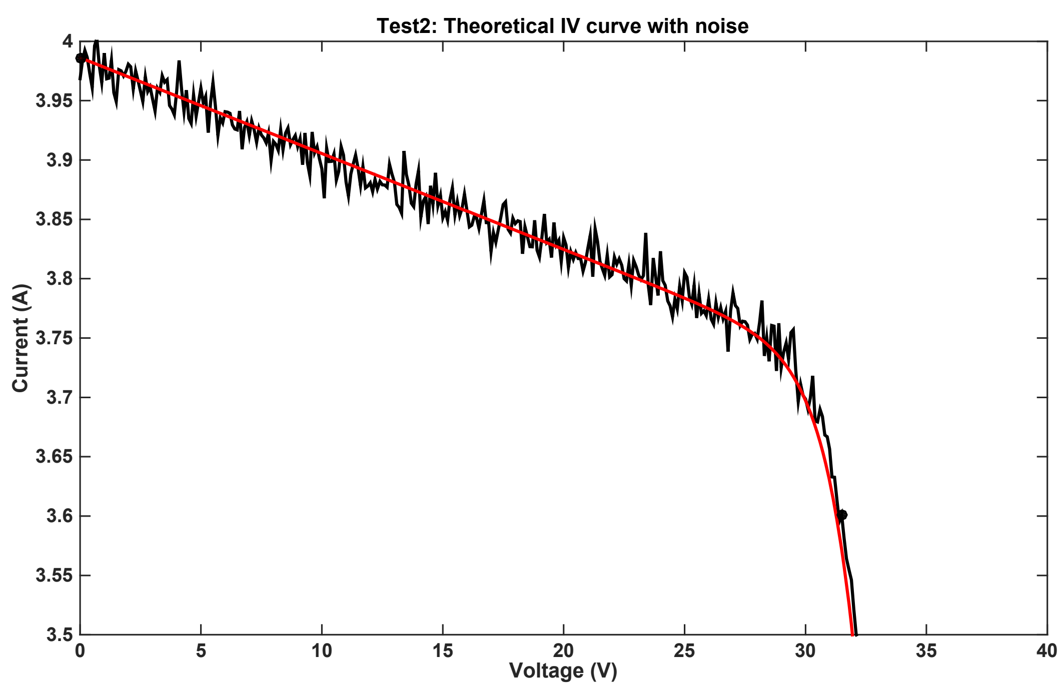

Fig. 7. Test 2 results zoom. Black: Theoretical curve – Red: AQE method extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 6
Points used in Test 2 with arbitrary points.

Point	Voltage [V]	Current [A]	Derivative [A/V]
P_1 ($\alpha = 0.06$)	26.2	3.7793351	−0.0086550
P_2 ($\alpha = 1$)	31.4	3.6028524	−0.1147405
P_3 ($\alpha = 4$)	33.2	3.1956019	−0.3850123
P_4 ($\alpha = 8$)	34.2	2.6895793	−0.6291413

Table 7
Theoretical and extracted parameters.

	I_{ph} [A]	I_{sat} [A]	nV_T [J C]	R_s [Ω]	R_{sh} [Ω]
Theoretical	4	1e-14	1.1	0.5	125
AQE Method	4.016	0.656e-14	1.09	0.527	115.01

them achieve very good approximations. In Fig. 6 it is shown the whole noisy curve (black) and the curve generated from the AQE method extracted parameters (red). A detailed zoom of Fig. 6, where the noise is clearly visible, is presented in Fig. 7.

The advantage of AQE method of extracting the parameters with any set of four points and their slopes has also been tested with the noisy curve. The four points and their slopes have been calculated with the **α-power function** using $\alpha = 0.06$, $\alpha = 1$, $\alpha = 4$ and $\alpha = 8$. In Table 6 are detailed the points used.

From these points the solutions obtained for E from equation (9) are (0.0105392, 0.1147405, 0.2262503, 0.3850123, 0.6620485). The upper-bound used for obtaining the 'correct' E parameter is $-I'_1$ (0.0086550). None of the polynomial solutions is lower than the upper bound selected so E is forced to be the upper bound (E = 0.0086550). From this solution, the AQE method has extracted the parameters listed in Table 7. As it was expected the results are not as good as using the extreme points (I_{sc} and V_{oc}) with the linear regression to calculate their derivatives, this is because the linear regression uses a set of points and it is a kind of low pass-filter that reduces the noise effect. On the other hand, it is worth noting that using four arbitrary points the AQE method works and its results are quite good as can be seen in Fig. 8.

4.3. Test 3. Real I–V curve

Finally, a real *I–V* curve has been used to compare the parameter extraction methods. A 100 points *I–V* curve has been measured from an Aerospace High Efficiency Silicon Cell (due to confidentiality reasons it is not possible to specify the model) at irradiance 1000 W/m^2, temperature 25 °C and air mass 0 (AM0 space conditions). As in Test 2, the I_{sc} and V_{oc} points with their corresponding slopes have been calculated using linear regressions. Also, the MPP has been located searching for the point with the maximum power and the fourth point and its slope needed for the AQE method has been calculated again with the **α-power function** using $\alpha = 10$. In Table 8 are detailed the points used.

For the AQE method, the solutions obtained for E are (0.0051379, 1.5682446, 4.274466, 9.1984787, 18.176239). The upper-bound used for obtaining the 'correct' E parameter is $-I'_1$ (0.0037647). None of the polynomial solutions is lower than the upper bound selected so E is forced to be the upper bound (E = 0.0037647). From this parameter the other ones are explicitly calculated.

The parameters extracted with the three methods are listed in Table 9.

In this case the real parameters are not known, so in order to compare the real *I–V* curve with the ones generated from the estimated parameters, the area error between the real and the theoretical curves has been calculated using the technique based on the trapezoidal rule [16]. Table 10 represents the values of the areas between the real curve and the estimated ones, normalized dividing by the total area under the real curve.

It is evident from the results that all the methods fit very well the real *I–V* curve in terms of the area between the curves. In this case OA method is a little bit better than 5p and both of them are

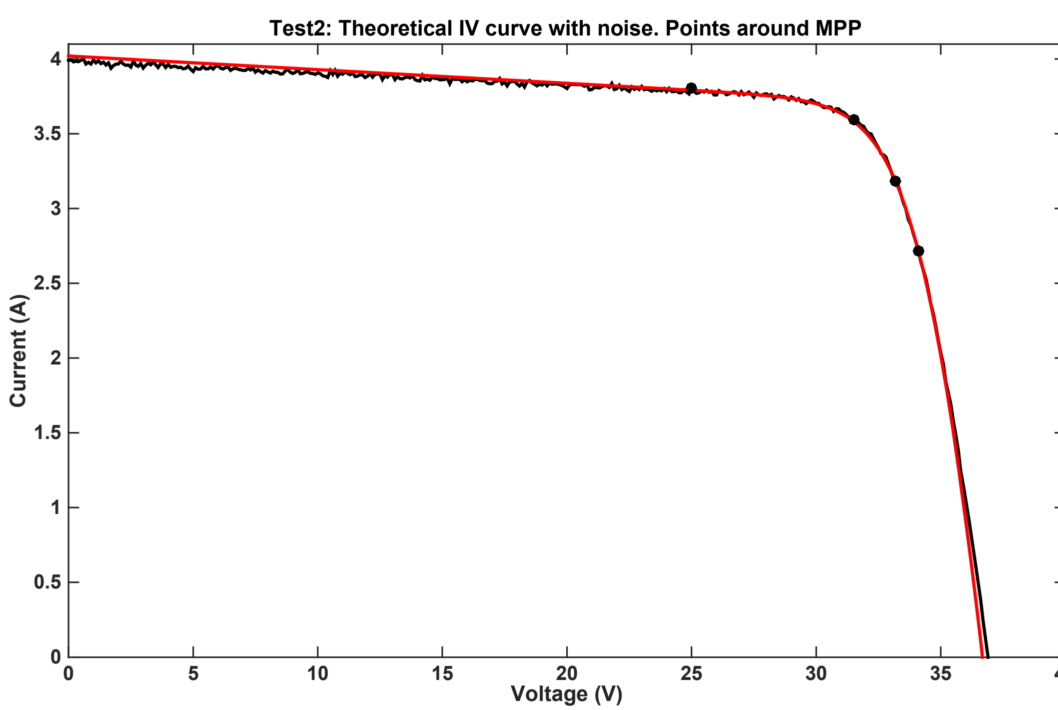

Fig. 8. Test 2 results, points around MPP. Black: Theoretical curve – Red: AQE method extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 8
Points used in Test 3.

Point	Voltage [V]	Current [A]	Derivative [A/V]
I_{sc}	0	0.8777560	−0.0037647
M_{PP}	0.522342	0.81916	−0.1568245
P_3 ($\alpha = 10$)	0.58745	0.540452	−9.1984785
V_{oc}	0.624143	0	−0.6241430

Table 9
Test 3 extracted parameters.

	I_{ph} [A]	I_{sat} [A]	nV_T [J C]	R_s [Ω]	R_{sh} [Ω]
AQE Method	0.87772	2.1416e-08	3.5614e-02	2.2187e-03	194.63
5p method	0.87776	4.0714e-08	3.6967e-02	7.0633e-04	265.62
OA method	0.87777	2.7398e-08	3.6120e-02	3.5391e-03	265.62

Table 10
Test 3 extracted parameters.

	Normalized area error
AQE Method	4.1898e-3
5p method	3.3038e-3
OA method	2.6617e-3

better than the AQE method. Anyway, in Fig. 9 it is shown real curve (black) the curve generated from the AQE method extracted parameters (red), and the points used. As can be seen, the precision of the method is not the best but it is very high.

As commented before, the great advantage of AQE method is that it is able to extract the parameters with any set of four points. This characteristic has also been tested with the real I–V curve. The four points and their slopes have been calculated with the α-power function using $\alpha = 0.02$, $\alpha = 1$, $\alpha = 4$ and $\alpha = 12$. In Table 11 are detailed the points used.

Table 11
Points used in Test 3 with arbitrary points arround the MPP.

Point	Voltage [V]	Current [A]	Derivative [A/V]
P_1 ($\alpha = 0.02$)	0.385159	0.878113	−0.0455974
P_2 ($\alpha = 1$)	0.522342	0.81916	−1.568244
P_3 ($\alpha = 4$)	0.56165	0.711404	−5.0662898
P_4 ($\alpha = 12$)	0.590769	0.508271	−10.3242587

Table 12
Test 3, points around MPP, extracted parameters.

	I_{ph} [A]	I_{sat} [A]	nV_T [J C]	R_s [Ω]	R_{sh} [Ω]	Normalized area error
AQE Method	0.90132	7.0677e-14	3.3569e-2	8.31158e-3	13.7464	9.20135e-3

The solutions obtained for E are (0.14710034, 1.5682446, 2.7413249, 5.0665288, 9.6075411). The upper-bound used for obtaining the ‘correct’ E parameter is I'_1 (0.0455974). None of the polynomial solutions is lower than the upper bound selected so E is forced to be the upper bound (E = 0.0455974). From this parameter the other ones are explicitly calculated. From these points the AQE method has extracted the parameters listed in Table 12.

Although the results are not as good as using the extreme points (I_{sc} and V_{oc}), it is worth noting that using only four points the AQE method works and its results are quite good as can be seen in Fig. 10.

5. Conclusions

As far as we know, in this paper is presented for the first time an analytical and quasi-explicit method, called AQE method, which is able to obtain the five parameters of the solar cell single-diode model just using four arbitrary points of the I–V curve and their

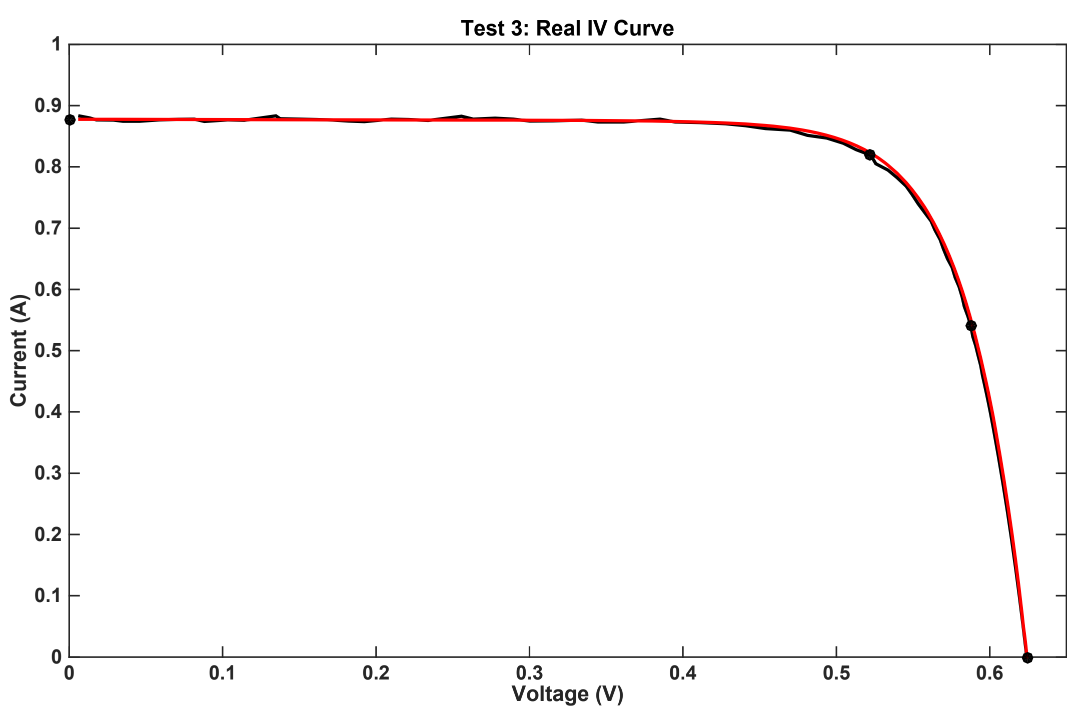

Fig. 9. Test 3 results. Black: Real curve – Red: AQE method extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

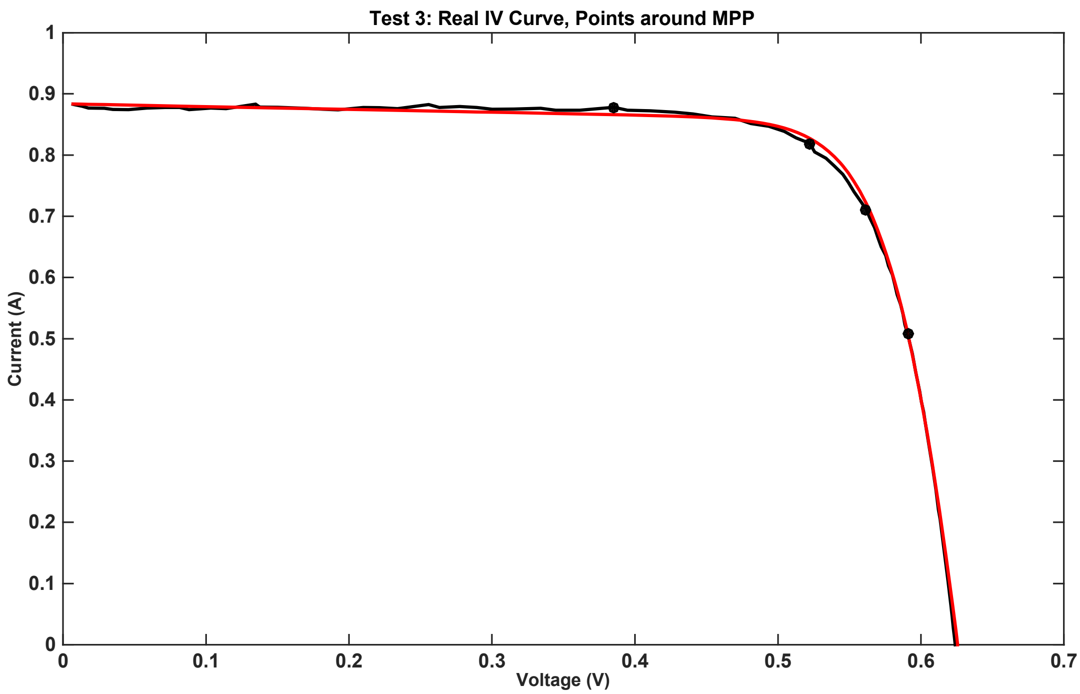

Fig. 10. Test 3 results points around MPP. Black: Real curve – Red: extracted parameters curve. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

slopes. The calculus of the parameters is reduced to compute the roots of a five-degree polynomial equation with only one unknown. Once this equation is solved, one of the roots is precisely one of the searched parameters and, the remaining four parameters, are explicitly given in terms of the extracted parameter and the data. We point out that the presented method does not uses any kind of approximations of the parameters nor any simplification of the equations of the model.

One of the main difficulties to apply this method is to know the slopes in the four points of the I–V curve. To overcome this difficulty, we introduce a new function, called α-power function, which works similarly to the power function to obtain the slope at the maximum power point. Moreover, this function could be used in many other works which need to know the slopes of a curve given by a cluster of points.

The AQE method has been tested and compared with other well-known methods, namely, the analytical five-point (5p) method and the asymptote oblique (AO) method. We have performed three tests corresponding to three kind of curves. The first curve has been theoretically produced while the second one has been obtained by introducing white noise in the theoretical one, and the third one is a real I–V curve measured from an Aerospace High Efficiency Silicon Cell.

To compare justly the three methods, 5p, AO and AQE, one should use the same data with each method to solve the single-diode model. Since it is not completely possible, we have distinguished two types of data:

Type 1) Data on the extreme points of the I–V curve and any other central point, as the MPP: all the methods are applicable
Type 2) Data around the MPP: only the AQE method is applicable

We have seen that with data of type 1, all the methods work very well, providing very good results in all kind of curves. The great difference between AQE method and the other ones is that it is the unique able to work with data of type 2. We can see in the previous tests that this method works perfect (i.e. extract the exact parameters) in a theoretical curve where the coordinates and the slope of any point can be exactly known. Also it works well with a theoretical curve with white noise, providing very good approximations of the real parameters. For a real curve, the AQE method works fine taking into account that measured real data have different type of errors and, moreover, the curve has a small quantity of points near the MPP. With curves having more points and using optimization techniques to minimize the effect of data errors, the AQE method could provide very good results even with data around the operating point, which is one of the objectives of the industry to control the behavior of the solar cells in real time and it has been an unsolved problem until this moment.

Acknowledgments

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