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Walrus optimizer: A novel nature-inspired metaheuristic algorithm

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ABSTRACT

Metaheuristic algorithms are intelligent optimization approaches that lead the searching procedure through utilizing exploitation and exploration. The increasing complexity of real-world optimization problem has prompted the development of more metaheuristic algorithms. Hence, this work proposes a novel swarm intelligence algorithm, Walrus optimizer (WO). It is inspired by the behaviors of walruses that choose to migrate, breed, roost, feed, gather and escape by receiving key signals (danger signals and safety signals). To test the capability of the proposed algorithm, 23 standard functions and the benchmark suite from the IEEE (Institute of Electrical and Electronics Engineers) Congress on Evolutionary Computation (CEC) 2021 are used. In addition, to evaluate the practicability of the proposed algorithm to solve various real-world optimization problems, 6 standard classical engineering optimization problems are examined and compared. For statistical purposes, 100 independent optimization runs are conducted to determine the statistical measurements, including the mean, standard deviation, and the computation time of the program, by considering a predefined stopping criterion. Some well-known statistical analyses are also used for comparative purposes, including the Friedman and Wilcoxon analysis. The results demonstrate that the proposed algorithm can provide special stability features and very competitive performance in dealing with high-dimensional benchmarks and real-world problems. The proposal of WO promotes the continuous development and application expansion of artificial intelligence, improves the efficiency of optimization calculation, and provides powerful tools for solving complex problems in the real world. The source code of WO is publicly availabe at https://ww2.mathworks.cn/matlabcentral/fileexchange/154702-walrus-optimizer-wo.

1. Introduction

The general formulation of an optimization problem is to select a set of parameters (variables) and make the design specifications (objectives) reach the optimal maximum or minimum value under a series of relevant constraints (limitations). Therefore, optimization problems can usually be expressed in the form of mathematical programming. Solving optimization problems is the norm in almost all disciplines of science and engineering, and the need for more robust solutions is ever increasing. That means, we need reasonable optimization methods that can fit the intricate nature of such up-to-date scientific and engineering challenges.

Mathematical optimization techniques used to be the only tools for optimizing problems before the proposal of heuristic optimization techniques. Mathematical optimization methods are mostly deterministic and have clear requirements on mathematical models and constraints. The typical traditional optimization methods include Linear Programming (LP), Non-Linear Programming (NLP), Mixed-Integer Programming (MIP), and Dynamic Programming (DP). Although researchers have proposed many different methods, the following problems still exist:

(1) Most of the traditional optimization methods have a high dependence on the mathematical model. If the objective function and constraint conditions cannot meet the conditions of first-order and second-order differentiability, it can fail to converge.
(2) The selection of initial points will affect the entire solution process of the problem. If the error of the selection of initial points is large, it will lead to low accuracy of the calculation results.

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(3) Actual optimization problems are usually characterized by complex structure and large scale. If traditional optimization methods are used for optimization, problems such as complex computation, difficult to find a solution and long solution time will arise.

Due to the inability of accurate optimization methods to solve complex and multidimensional problems, approximation algorithms have been proposed to solve such problems. Approximation algorithms are a new approach for efficiently solving optimization problems and are divided into two categories: heuristic and metaheuristic algorithms (Abdollahzadeh et al., 2021; Ong et al., 2021; X. S. Yang, 2010).

In mathematical programming, a heuristic algorithm is a procedure that determines near-optimal solutions to an optimization problem. However, this is achieved by trading optimality, completeness, accuracy, or precision for speed. Heuristic algorithms construct feasible solutions in a certain number of steps for a specific problem feasible set, i.e., reflected as problem dependent (Kuo & Prasad, 2000; Shin et al., 2008). A feasible solution for each instance of the optimization problem to be solved is given at an acceptable cost (in terms of computational time, computational space, etc.), and the degree of deviation of the feasible solution from the optimal solution is generally not predictable in advance. However, since they tend to be too greedy, they usually fall into a local optimum and thus usually fail to obtain a globally optimal solution.

Metaheuristic algorithms are strategies that guide the search process. The goal is to efficiently explore the search space to find near–optimal solutions. Techniques which constitute metaheuristic algorithms range from simple local search procedures to complex learning processes. They do not exploit any specificity of the problem and thus can be used as a black box, i.e., reflected as problem independent. In general, they are not greedy. In fact, they may even accept a temporary deterioration of the solution in a specific problem, which allows them to explore the solution space more thoroughly, leading to a hopefully better solution (sometimes coinciding with the global optimum).

Another difference between heuristic and metaheuristic algorithms is the presence of a “random factor”. For the same problem, given an input, the steps performed by the heuristic algorithm are fixed, and so is the output. This is not the case with metaheuristic algorithms, which include a “random factor”. In the case of a Genetic Algorithm (GA) (Holland, 1992), if the same initial population (with the same inputs) is run twice (100 generations each time), the results will probably be different. This is because the objects of the crossover and mutation operations are chosen randomly. The success of GA depends largely on the random component of the algorithm. The overall fitness gets improved over the process of generations as the best individuals are more likely to select and reproduce than the worst individuals.

Complexity theory (Brown et al., 2016; Ladyman et al., 2013) and No Free lunch (NFL) theorem (Ho & Pepyne, 2002) are two important concepts used to elucidate the limitations of metaheuristic algorithms. Complexity theory focuses on the relationship between problem difficulty and computational resources. The theory categorizes problems into P problems (polynomial time solvable) and NP problems (non-deterministic polynomial time solvable). The theory suggests that metaheuristic algorithms are usually not guaranteed to find a globally optimal solution, which is particularly evident in the case of NP-hard problems (Sörensen, 2015). They usually stop at locally optimal or suboptimal solutions. This reflects the limited nature of *meta*-heuristic algorithms, as they cannot break the fundamental limitations of complexity theory. The NFL theorem states that there is no algorithm for solving all optimization problems, which means that the performance of different algorithms is not equal when given the prior knowledge (algorithm parameters, convergence criteria) of a *meta*-heuristic algorithm for solving a particular problem (Xu et al., 2023). Concisely, there is no universal optimization procedure that works perfectly for all optimization problems and thus, the continuous flourish of the diversity of optimization algorithm is encouraged. In practical applications, the selection, design, and adjustment of metaheuristic algorithms need to consider the specific requirements and characteristics of the problem to achieve the best performance.

Combining the above two theories, it is necessary to develop a novel metaheuristic algorithm to solve combinatorial optimization problems. The specific motivations are listed below.

(1) Improving the efficiency of solving complex problems: metaheuristic algorithms are usually used to solve complex optimization problems, such as travel salesman problems, scheduling problems, path planning or even multi-objective optimization problems. The metaheuristic algorithms introduce a series of intelligent heuristic strategies and techniques to improve the efficiency of solving complex problems and reduce the computational cost. Novel metaheuristic algorithms use a combination of local and global search, which balances the trade-off between fast convergence and avoiding falling into local optimal solutions.
(2) Dealing with large-scale data: With the advent of the big data era, many problems require dealing with large-scale datasets. Metaheuristic algorithms can be used to accelerate the analysis and processing of these data. It can help decision makers get useful information faster and improve the feasibility of problem solving.
(3) Adapting to problem uncertainty: Some problems have an element of uncertainty, such as randomness or incomplete information. Through diverse strategies such as fuzzy logic, robust optimization, stochastic probabilistic modeling, and reinforcement learning, metaheuristic algorithms can be designed to be more flexible, allowing them to perform well in problems with uncertainty factors.
(4) Theoretical research: theoretical research on metaheuristic algorithms is an important motivation. By studying new metaheuristic algorithms, the theory of computation can be advanced to gain insight into the complexity and solvability of problems.
(5) Innovation and competitive advantage: In scientific research and industrial applications, proposing new metaheuristic algorithms can lead to innovation and competitive advantage. This can motivate researchers to invest more resources in developing new algorithms.

Therefore, Walrus Optimizer (WO) is proposed in this paper with a simple structure, special stability features and very competitive performance to realize the solutions of both constrained and unconstrained problems more effectively. The innovation is that the “vigilantes” in a walrus herd are the determining factor influencing the direction of the herd. The “Danger_signal” and “Safety_signal” issued by the “vigilantes” play a key role in the execution process of WO. In WO, the “Danger_signal” is used to determine whether the WO performs the exploration phase or the exploitation phase. When the “Danger_signal” meets certain conditions, the walrus herd migrates to a new domain within the solution space, which is the exploration phase in the early stage of the algorithm. On the contrary, the walrus herd reproduces, which is the exploitation phase in the late stage of the algorithm. The “Safety_signal” plays a key role in the exploitation phase, which influences whether a walrus chooses the roosting behavior or foraging. Among them, male walruses, female walruses, and juvenile walruses interact with each other in their roosting behaviors to move the population in a direction conducive to survival; foraging behaviors include the typical phenomena of gathering and fleeing, which are controlled by “Danger_signal”. The walrus herd can avoid capture or death by predators (falling into the local optimum) and achieve population growth (searching for the global optimum) in an orderly policing environment.

In the remainder of the paper, Section 2 deals with the related works. Section 3 shows the model of the proposed WO. In Section 4, the proposed WO is evaluated by 23 benchmark functions, CEC 2021 benchmark suit, 0-1 knapsack problems, and six engineering design problems. Finally, the conclusions are summarized in Section 5.

Table 1
Summary of the latest metaheuristic algorithms.

Category	Algorithm	Year	Ref.	Inspiration
SI	African Vultures Optimization Algorithm (AVOA)	2021	(Abdollahzadeh et al., 2021)	The navigation behaviors of African vultures
	Golden Eagle Optimizer (GEO)	2021	(Mohammadi-Balani et al., 2021)	The spiral flight trajectory of golden eagles
	Red Fox Optimization (RFO)	2021	(Połap & Woźniak, 2021)	The behaviors of red foxes using various tricks to distract prey
	Aquila Optimizer (AO)	2021	(Abualigah et al., 2021)	The behaviors of Aquila during prey capture
	Tuna Swarm Optimization (TSO)	2021	(Xie et al., 2021)	The cooperative foraging behavior of tuna swarm
	Wild Horse Optimizer (WHO)	2021	(Naruei & Keynia, 2021b)	The social life behaviors of wild horses
	Dingo Optimization Algorithm (DOA)	2021	(Peraza-Vázquez et al., 2021)	The hunting strategies of dingoes which are attacking by persecution, grouping tactics, and scavenging behavior
	Coot Optimization Algorithm (COOT)	2021	(Naruei & Keynia, 2021a)	The behavior of Coots on the water surface
	Wild Horse Optimizer (WHO)	2021	(Naruei & Keynia, 2021b)	The social life behaviors of wild horses
	Chameleon Swarm Algorithm (CSA)	2021	(M. S. Braik, 2021)	The navigating and hunting behaviors of chameleons
	Capuchin Search Algorithm (CapSA)	2021	(M. Braik et al., 2021)	The dynamic behavior of capuchin monkeys jumping, swinging, and climbing
	Northern Goshawk Optimization (NGO)	2021	(Dehghani et al., 2021)	The behavior of northern goshawk during prey hunting
	Hunter-Prey Optimization (HPO)	2022	(Naruei et al., 2022)	The behavior of predator and prey
	Archerfish Hunting Optimizer (AHO)	2022	(Zitouni et al., 2022)	The shooting and jumping behaviors of the archerfish for hunting aerial insects
	Honey Badger Algorithm (HBA)	2022	(Hashim et al., 2022)	The intelligent foraging behavior of honey badger
	Artificial Rabbits Optimization (ARO)	2022	(L. Wang et al., 2022)	The survival strategies of rabbits
	Beluga Whale Optimization (BWO)	2022	(Zhong et al., 2022)	The behaviors of pair swim, prey, and whale fall
	Dwarf Mongoose Optimization Algorithm (DMO)	2022	(Agushaka et al., 2022)	The foraging behavior of the dwarf mongoose
	Golden Jackal Optimization (GJO)	2022	(Chopra & Mohsin Ansari, 2022)	The collaborative hunting behavior of the golden jackals
	Sand Cat Swarm Optimization (SCSO)	2022	(Seyyedabbasi & Kiani, 2022)	The sand cat incredible ability to dig for prey
	Snake Optimizer (SO)	2022	(Hashim & Hussien, 2022)	The special mating behavior of snakes
	White Shark Optimizer (WSO)	2022	(M. Braik et al., 2022)	The behaviors of white sharks, including exceptional senses of hearing and smell
	Reptile Search Algorithm (RSA)	2022	(Abualigah et al., 2022)	The hunting behaviors of Crocodiles
	Coati Optimization Algorithm (COA)	2022	(Dehghani et al., 2023)	The behavior of coati when attacking and hunting iguanas
	Nutcracker Optimizer (NO)	2023	(Abdel-Basset et al., 2023)	The search, cache, and recovery behaviors of the nutcracker
	Bedbug Meta-Heuristic Algorithm (BMHA)	2023	(Rezvani et al., 2023)	The static and dynamic swarming behaviors of bedbugs
	Willow Catkin Optimization (WCO)	2023	(J.-S. Pan et al., 2023)	Willow trees' process of seed dispersal
	Osprey Optimization Algorithm (OOA)	2023	(Dehghani & Trojovský, 2023)	The hunting strategy of ospreys
EA	Coronavirus Herd Immunity Optimization (CHIO)	2021	(Al-Betar et al., 2021)	The herd immunity concept to tackle coronavirus pandemic (COVID-19)
	Ebola Optimization Search Algorithm (EOSA)	2022	(Olaide et al., 2022)	The propagation method of the Ebola virus disease
PhA	Solar System Algorithm (SSA)	2021	(Zitouni et al., 2021)	The orbiting behavior of some objects found in the solar system
	Crystal Structure Algorithm (CryStAl)	2021	(Talatahari et al., 2021)	The principles underlying the formation of crystal structures
	Arithmetic Optimization Algorithm (AOA)	2021	(Abualigah et al., 2021)	The arithmetic operators in mathematics
	Atomic Orbital Search (AOS)	2021	(Azizi, 2021)	The principles of quantum mechanics and the atomic model
	Volcano Eruption Algorithm (VEA)	2021	(Hosseini et al., 2021)	The nature of volcano eruption
	RUNge Kutta Optimizer (RUN)	2021	(Ahmadianfar et al., 2021)	The Runge Kutta (RK) method
	Lichtenberg Algorithm (LA)	2021	(Pereira et al., 2021)	The Lichtenberg figures patterns
	Chernobyl Disaster Optimizer (CDO)	2023	(Shehadeh, 2023)	The nuclear reactor core explosion of Chernobyl
	Energy Valley Optimizer (EVO)	2023	(Azizi et al., 2023)	The physics principles regarding stability and different modes of particle decay
Human based	Cooperation Search Algorithm (CSA)	2021	(Feng et al., 2021)	The team cooperation behaviors in modern enterprise
	Stock Exchange Trading Optimization (SETO)	2021	(Emami, 2022b)	The behavior of traders in the stock market
	Social Network Search (SNS)	2021	(Talatahari et al., 2021)	The behavior of users expressing their opinions
	Giza Pyramids Construction (GPC)	2021	(Harifi et al., 2021)	The ancient past
	Hunger Games Search (HGS)	2021	(Y. Yang et al., 2021)	Animal hunger drives activity and behavior design
	Human Felicity Algorithm (HFA)	2022	(Verij kazemi & Fazeli Veysari, 2022)	The efforts of human society to become felicity
	Ali Baba and the Forty Thieves (AFT)	2022	(M. Braik et al., 2022)	The strategies pursued by the forty thieves in the search for Ali Baba
	Alpine Skiing Optimization (ASO)	2022	(Yuan et al., 2022)	The behavior of skiers competing for the championship
	Anti-Coronavirus Optimization (ACVO)	2022	(Emami, 2022a)	The behavior of people to slow the spread of COVID-19
	Musical Chairs Algorithm (MCA)	2023	(Eltamaly & Rabie, 2023)	The musical chairs game
	Group Learning Algorithm (GLA)	2023	(Rahman, 2023)	The way individuals inside a group affect each other
	Mountaineering Team-Based Optimization (MTBO)	2023	(Faridmehr et al., 2023)	A cooperative human phenomenon

2. Related works

Metaheuristics can be classified according to various characteristics (Chica et al., 2017): nature-inspired vs. not nature-inspired; deterministic vs. stochastic; population-based vs. single-solution-based search; and iterative vs. greedy. Moreover, metaheuristic algorithms can be categorized into the following four groups based on the source of inspiration.

(1) Swarm intelligence (SI)

SI is a general term for a class of intelligent groups with self-organized behavior. SI can be seen as an intelligent behavior formed by interactions between individuals and individuals, individuals, and environment. Individuals in a group all follow a simple code of conduct with no unified central control between groups. The interaction between individuals is ultimately manifested as the intelligence of the whole group.

Hashim et al. (Hashim et al., 2022) proposed the Honey Badger Algorithm (HBA) inspired by the intelligent foraging behavior of honey badger. HBA has been applied in sliding mode controller (T. Wang et al., 2023) and optimal power flow (Akdağ, 2022). Fathollahi-Fard et al. (Fathollahi-Fard et al., 2020) proposed the Red Deer Algorithm (RDA) based on the unusual mating behavior of Scottish red deer in a breading season. RDA has been applied to solve optimal location problem of distributed generation (Lakshmi et al., 2023), Sugarcane Supply Chain Network (SSCN) (Chouhan et al., 2021a) and data clustering (Moghadam & Ahmadi, 2023). The Keshtel Algorithm (KA) (Hajiaghaei-Keshteli & Aminnayeri, 2013) derives its inspiration from the foraging behavior of the Keshtel, a dabbling duck. With its emergence, KA has been widely used in various fields, including integrated scheduling (Hajiaghaei-Keshteli & Aminnayeri, 2014), supply chain networks design (Chouhan et al., 2022; Mosallanezhad et al., 2021, 2023; Zahedi et al., 2021), and allocation-routing optimization models (Hashemi-Amiri et al., 2023).

(2) Evolutionary algorithms (EA)

EA is inspired by the biological evolution of nature and have self-organizing, self-adaptive and self-learning properties. Biological evolution is achieved through reproduction, variation, competition, and selection. The EA can solve the optimization problem mainly through selection, recombination, and mutation.

Al-Betar et al. (Al-Betar et al., 2021) proposed the Coronavirus Herd Immunity Optimizer (CHIO), which is originated from the herd immunity concept as a way to tackle coronavirus pandemic (COVID-19). Kumar et al. (Kumar et al., 2022) proposed the Multi-Objective Coronavirus Herd Immunity Optimizer (MOCHIO) and applied it to the optimization of brushless direct current (BLDC) motor design. Abu Doush et al. (Abu Doush et al., 2023) applied the CHIO to the training process of multilayer perceptron neural network (MLP) to find its optimal control parameters, thus empowering their classification accuracy.

(3) Physics-based algorithms (PhA)

PhA is stimulated from the fundamental physical laws of the universe and usually reflects the main rules of physical processes in the interconnections between search agents in the algorithm execution.

The mathematical development of the Atomic Orbital Search (AOS) is based on principles of quantum mechanics focusing on the act of electrons around the nucleus of an atom (Azizi, 2021). Talatahari et al. (Talatahari et al., 2021) proposed the Crystal Structure algorithm (CryStAl). This method is chiefly inspired by the principles underlying the formation of crystal structures from the addition of the basis to the lattice points. Shehadeh (Shehadeh, 2023) was inspired by the Chernobyl nuclear reactor core explosion to propose the Chernobyl Disaster Optimizer (CDO).

(4) Human-based algorithms

The last category of metaheuristic algorithms presented is human-based algorithms which contain algorithms inspired by human beings, including physical and non-physical activities such as thinking and social behavior.

Fathollahi-Fard et al. (Fathollahi-Fard et al., 2018) proposed the Social Engineering Optimizer (SEO). The basis of SEO depends on how an attacker attacks to a defender by four different social engineering techniques, and it precisely completes the reinforcement and the diversification phases. SEO has been widely used in supply chain network (Mousavi et al., 2021; Salehi-Amiri et al., 2021; Zahedi et al., 2021) and mixed-integer linear programming modelling (Chouhan et al., 2021b). Alkayem et al. (Alkayem et al., 2022) proposed a new algorithm merging the SEO and the particle swarm algorithm (PSO) and applied it to Structural Health Monitoring (SHM) of frame structures.

In this section, new metaheuristic algorithms from 2021 to 2023 are summarized, but is not limited to those presented in Table 1. In addition to the four major categories of metaheuristic algorithms mentioned above, many scholars have designed efficient optimization algorithms for special problems. It may contain the basic ideas of one or more algorithms, and the mixed algorithms play an irreplaceable role in specific problem solving. For example, Fathollahi-Fard et al. (Fathollahi-Fard et al., 2023) proposed an adaptive large neighborhood search algorithm based on heuristics and reconstruction for the generalized quadratic assignment problem.

The study of novel metaheuristic algorithms has been one of the important research directions in the field of computer science and artificial intelligence, but there are still many research gaps that need to be further explored and solved:

(1) Data imbalance: In many practical problems, data sets are often imbalanced. Some of these categories have a much larger sample size than others. Novel algorithms need to deal with this data imbalance to improve the performance of the model.
(2) Small-sample learning: in many domains, only a very limited amount of labeled data is available for training models. Therefore, studying algorithms for small sample learning is a challenging gap.
(3) Interdisciplinary research: the study of intelligent algorithms involves many fields such as computer science, mathematics, statistics, neuroscience and so on. Exploring the intersection between these fields for deeper understanding and innovation is a promising direction.
(4) Practical applications: despite the many theoretical potentials of intelligent algorithms, there are still challenges in applying them to solve practical problems. Researchers can focus on how to effectively apply novel intelligent algorithms to domains such as healthcare, finance, and energy management.

These research gaps represent some of the current challenges and opportunities in the field of intelligent algorithms. Addressing these challenges will help to further advance the development of Artificial Intelligence (AI) technology to better serve the needs of society and science.

3. Walrus Optimizer (WO)

The following is about the inspiration, mathematical model, procedure and complexity for WO.

3.1. Biological fundamentals

The walrus is the largest mammal in the ocean besides whales. Walruses live mainly in temperate waters in or near the Arctic. Walruses are gregarious animals, leading an amphibious life. Groups of walruses range from dozens to hundreds to thousands of individuals. Its body is cylindrical, stout, and obese, with a flattened head and blunt muzzle end. Around the upper lip there are about 400 long and hard whiskers with blood vessels and nerves, leading to a sharp sense of touch. The most unique feature of the walrus is that pair of white, well-developed upper canine teeth that keep growing throughout its life, forming tusks. The tusks can be used for self-defense, digging in the mud and sand for food such as clams, shrimp, and crabs, or to support the body when climbing on the ice (Gotfredsen et al., 2018).

Walruses look bulky with their massive bodies, but flexible in the water. Among that large number of marine animals, walruses are the best divers. They can dive in the water for 20 min and to a depth of 500 m. Walruses can stay underwater for up to 2 h after diving to the bottom, and once need fresh air, they can surface within 3 min.

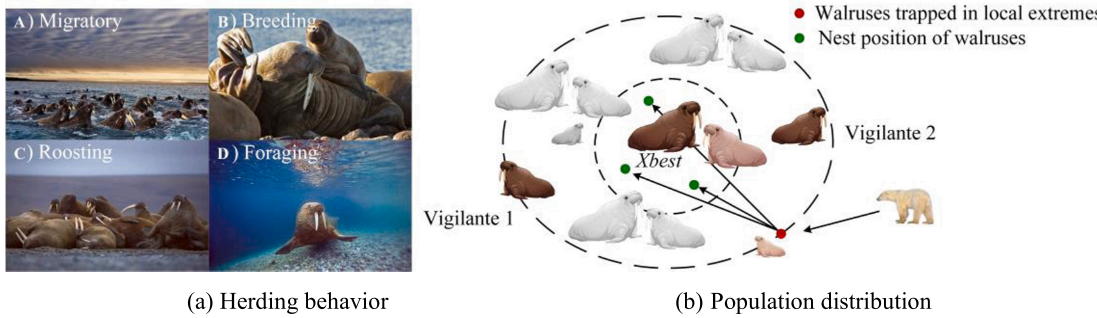


Fig. 1. Herding behavior and population distribution of walruses. (a) Herding behavior (b) Population distribution.

Walruses have a strong sense of community as shown below:

- When the breeding season begins, walruses establish their territories on the beach (Ray et al., 2016). The best positions are occupied by the strongest males. The area of the territory varies according to the number of females occupied by the male.
- Walruses are accustomed to living in deep water areas of the ocean where sunlight cannot reach. Like bats and dolphins, the walruses are lack of idiosyncratic vision; They rely on sound localization for foraging and share food information by communicating with peers.
- Walruses have strong social habits when they encounter killer whales in the water, they will adopt a collective defense strategy to defend themselves and aid their injured counterparts. The experience of the long struggle for survival prevents the walrus from letting down its guard (Jay et al., 1998). At that point, two walruses act as guards. They will go to help once their kind is injured.

Inspired from the behaviors of walrus in migrating, breeding, roosting and foraging (as shown in Fig. 1), we propose a new meta-heuristic algorithm, WO, for the first time. Two assumptions need to be clarified here:

(1) Walrus populations judge population behavior by danger and safety signals.
(2) Behavioral and role divisions in walrus populations are modeled in the walrus algorithm. Specifically, the walrus algorithm assumes social structures and interactions between male, female, and juvenile walruses.

In the WO mathematical model, the search space is the range over which the algorithm tries to find the best solution. It is usually a multidimensional space consisting of the decision variables of the problem. The solution space contains all potential solutions; A solution is the location of a walrus in the search space. It represents a potential solution to the problem. The solution can be a vector, where each component corresponds to the value of a decision variable; The decision variable is determined by the problem. WO tries to find the best solution in the search space, i.e., the solution that optimizes the objective function of the problem.

3.2. Mathematical model and algorithm

3.2.1. Initialization

In WO, the optimization process starts with a set of randomly generated candidate solutions (X).

$$X = LB + rand(UB - LB) \tag{1}$$

where, LB and UB are the lower and upper boundary of the problem variables, $rand$ is a uniform random vector in the range 0 to 1.

Walruses are agents that perform the optimization process. Their positions are continuously updated with iterations.

$$\mathrm{X} = \begin{bmatrix} X_{1,1}X_{1,2}\cdots X_{1,d} \\ X_{2,1}X_{2,2}\cdots X_{2,d} \\ \vdots\vdots\vdots\vdots \\ \vdots\vdots\vdots\vdots \\ X_{n,1}X_{n,2}\cdots X_{n,d} \end{bmatrix}_{n\times d} \tag{2}$$

where, n is the population size, d is the dimension of design variables.

The fitness values corresponding to all search agents are stored as:

$$\mathrm{F} = \begin{bmatrix} (f_{1,1}f_{1,2}\cdots f_{1,d}) \\ (f_{2,1}f_{2,2}\cdots f_{2,d}) \\ \vdots\vdots\vdots\vdots \\ \vdots\vdots\vdots\vdots \\ (f_{n,1}f_{n,2}\cdots f_{n,d}) \end{bmatrix}_{n\times d} \tag{3}$$

Walrus populations are divided into adults and juveniles, which account for 90 % and 10 % of the population, respectively. Among adult walruses, the male to female ratio is 1:1.

3.2.2. Danger signals and safety signals

Walruses are very vigilant, both in foraging and roosting. There will be 1 to 2 walruses as guards patrolling around, and danger signals will be sent out immediately once unexpected situations are found. The danger signal and safety signal in WO are defined as follows:

$$\mathrm{Danger\_signal} = A^*R \tag{4}$$

$$\alpha = 1 - t/T \tag{5}$$

$$A = 2 \times \alpha \tag{6}$$

$$R = 2 \times r_1 - 1 \tag{7}$$

where, A and R are danger factors, α decreases from 1 first to 0 with the number of iterations t, and T is the maximum iteration.

The safety signal corresponding to the danger signal in WO is defined as follows:

$$\mathrm{Safety\_signal} = r_2 \tag{8}$$

where, r_1 and r_2 are random numbers lie in the range of (0, 1).

3.2.3. Migration (exploration)

When risk factors are too high, walrus herds will migrate to areas more suitable for population survival. In this phase, the walrus position is updated as follows:

$$X_{i,j}^{t+1} = X_{i,j}^{t} + \text{Migration\_step} \tag{9}$$

$$\text{Migration\_step} = (X_m^t - X_n^t) \bullet \beta \bullet {r_3}^2 \tag{10}$$

$$\beta = 1 - \frac{1}{1 + \exp(-\frac{t - \frac{T}{2}}{T} \times 10)} \tag{11}$$

where, $X_{i,j}^{t+1}$ is the new position for the i th walrus on the j th dimension, $X_{i,j}^{t}$ is the current position of the i th walrus on the j th dimension, Migration_step is the step size of walrus movement, two vigilantes are randomly selected from the population, the positions of the vigilantes correspond to X_m^t and X_n^t, β is the migration step control factor, which varies with iteration as a smooth curve, and r_3 is a random number lies in the range of (0, 1).

3.2.4. Reproduction (exploitation)

In contrast to migration, walrus herds tend to breed in currents when risk factors are low. During reproduction, there are mainly two behaviors, onshore roosting, and underwater foraging. The mathematical model is as follows.

(1) Roosting behavior

The male, female and juvenile walruses are our classification of population members. They have different ways of renewing their position.

Step 1: Redistribution of male walruses

The population diversity is crucial to the later iterative search for superiority. In the quasi-Monte Carlo method, the Halton sequence is a widely used method to generate randomly distributed sequences. Adopting Halton sequence distribution for male walrus position update can allow a broader distribution of the population with search space. The principle is to divide the search area evenly into several parts and select a random point in each part. This ensures both randomness and uniformity.

Step 2: Position update of female walruses

The female walrus is influenced by male walrus ($Male_{i,j}^t$) and the lead walrus (X_{best}^t). As the process of iteration, the female walrus is gradually influenced less by the mate and more by the leader.

$$Female_{i,j}^{t+1} = Female_{i,j}^{t} + \alpha \bullet \left(Male_{i,j}^{t} - Female_{i,j}^{t}\right) + (1 - \alpha) \bullet \left(X_{best}^{t} - Female_{i,j}^{t}\right) \tag{12}$$

where, $Female_{i,j}^{t+1}$ is the new position for the i th female walrus on the j th dimension, $Male_{i,j}^t$ and $Female_{i,j}^t$ are the positions of the i th male and female walruses on the j th dimension.

Step 3: Position update of juvenile walruses

Juvenile walruses at the edge of the population are often targeted by killer whales and polar bears. Therefore, juvenile walruses need to update their current position to avoid predation.

$$Juvenile_{i,j}^{t+1} = \left(O - Juvenile_{i,j}^{t}\right) \bullet P \tag{13}$$

$$O = X_{best}^{t} + Juvenile_{i,j}^{t} \bullet LF \tag{14}$$

where, $Juvenile_{i,j}^{t+1}$ is the new position for the i th juvenile walrus on the j th dimension, $Juvenile_{i,j}^{t}$ is the position of the i th juvenile walrus on the j th dimension, P is the distress coefficient of juvenile walrus and is a random number of $(0,1)$, O is the reference safety position, LF is a vector of random numbers based on Lévy distribution representing Lévy movement.

$$\text{Levy}(\alpha) = 0.05 \times \frac{x}{|y|^{\frac{1}{\alpha}}} \tag{15}$$

where x and y are two normally distributed variables, x $\mathrm{N}(0, \sigma_x^2)$, y $\mathrm{N}(0, \sigma_y^2)$.

$$\sigma_x = \left[\frac{\Gamma(1+\alpha)\sin\left(\frac{\pi\alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right)\alpha 2^{\frac{(\alpha-1)}{2}}}\right]^{\frac{1}{\alpha}}, \sigma_y = 1, \alpha = 1.5 \tag{16}$$

where, σ_x and σ_y are the standard deviations, $\Gamma(x) = (x+1)!$.

(2) Foraging behavior

Underwater foraging includes fleeing and gathering behaviors.

a) Fleeing behavior

Walruses are also attacked by natural predators during underwater foraging, and they will flee from their current activity area based on danger signals from their peers. This behavior occurs in the late iteration of the WO, and a certain degree of perturbation to the population helps walruses to conduct global exploration.

$$\sigma_x = \left[\frac{\Gamma(1+\alpha)\sin\left(\frac{\pi\alpha}{2}\right)}{\Gamma\left(\frac{1+\alpha}{2}\right)\alpha 2^{\frac{(\alpha-1)}{2}}}\right]^{\frac{1}{\alpha}}, \sigma_y = 1, \alpha = 1.5 \tag{17}$$

where, $\left|X_{best}^t - X_{i,j}^t\right|$ denotes the distance between the current walrus and the best walrus, r_4 is a random number lies in the range of (0, 1).

b) Gathering behavior

Walruses can cooperate to forage and move according to the location of other walruses in the population and sharing location information can help the whole walrus herd to find the sea area with higher food abundance.

$$X_{i,j}^{t+1} = (X_1 + X_2)/2 \tag{18}$$

$$\begin{cases} X_1 = X_{best}^t - a_1 \times b_1 \times \left|X_{best}^t - X_{i,j}^t\right| \\ X_2 = X_{second}^t - a_2 \times b_2 \times \left|X_{second}^t - X_{i,j}^t\right| \end{cases} \tag{19}$$

$$a = \beta \times r_5 - \beta \tag{20}$$

$$b = \tan(\theta) \tag{21}$$

where, X_1 and X_2 are two weights affecting the gathering behavior of walrus, X_{second}^t is the position of the second walrus in the current iteration, $\left|X_{second}^t - X_{i,j}^t\right|$ denotes the distance between the current walrus and the second walrus, a and b are the gathering coefficients, r_5 is a random number lies in the range of (0, 1), and θ takes values ranging from 0 to π.

3.3. The procedure of WO

In WO, the danger signal is used to determine whether the WO performs the exploration phase or the exploitation phase. When the absolute value of the danger signal is not less than 1, the walrus herd migrates to a new domain within the solution space, which is the exploration phase in the early stage of the algorithm; On the contrary, the walrus herd reproduces, which is the exploitation phase in the late stage of the algorithm. The security signal plays a key role in the exploitation phase, which influences whether individual walrus choose roosting behavior or foraging behavior. Among them, foraging behavior includes two typical phenomena, gathering and fleeing, which are controlled by danger signals. The pseudo code and flowchart of WO are described in detail in Algorithm 1 and Fig. 2, respectively.

Algorithm 1: The pseudo code of WO

Input: Algorithm parameters (population size N, maximum iteration T)
Initialize the population and define the related parameters
Evaluate the fitness values and obtain the best solution
While $t \leq T$
  If $|\text{Danger\_signal}| \geq 1$ {Exploration phase}
    Update new position of each walrus using Eq. (9)
  Else {Exploitation phase}
    If $\text{Safety\_signal} \geq 0.5$ // Breeding behavior //
    For each male walrus
      Update new position based on Halton sequence
    End For
    For each female walrus
      Update new position using Eq. (12)
    End For
    For each juvenile walrus
      Update new position using Eq. (13)
    End For
   Else // Foraging behavior //
    If $|\text{Danger\_signal}| \geq 0.5$ // Gathering behavior //
      Update new position of each walrus using Eq. (17)
    Else // Fleeing behavior //
      Update new position of each walrus using Eq. (18)
    End If
   End If
  End If
  Update the walrus position
  Calculate the fitness value and update the current best solution
  $t = t + 1$
End While
Output: the best solution

3.4. Computational complexity

When solving optimization problems, the computational complexity is very useful to evaluate the efficiency of an algorithm and depends on three main processes: initialization, fitness evaluation and updating of the solution. The computational complexity of the initialization process and the updating mechanism are $O(N)$ and $O(N \times T) + O(N \times T \times D)$, where N is the population size, T is the maximum iterations, and D is the dimension of given problem. Therefore, computational complexity of WO is $O(N \times (T + T \times D + 1))$. The amount of memory space temporarily occupied by an algorithm during operation can be measured in terms of space complexity. The space complexity of WO algorithm is the maximum amount of space used at any one time which is considered during its initialization process. Thus, the space complexity of WO algorithm is $O(N \times Dim)$.

3.5. Walrus herd behaviors

To visualize the behavior of the proposed algorithm, the simulations of walrus herd behavior are shown in this section. Fig. 3 depict the group behavior of a walrus herd of size 30 searching for optimal solutions of unimodal and multimodal functions in 3D space. The black dots indicate the 30 walruses, and the red dots indicate the global optimal positions. When $t = 1$, the walruses are randomly generated in the solution space with a wide scattering range. As the iteration continues, the search range of all walruses gradually narrows. Finally, all walruses converge to the red dots. The search results of the unimodal function can show that WO has good exploitation ability, and the search results of the multimodal function can reflect the strong exploration ability of WO.

4. Results and discussions

All the algorithms are coded in MATLAB programming software and simulations are run on PC with Intel i7 CPU with 16 GB RAM. The codes of all comparison algorithms are published by their original authors. Note that, all algorithms have been simulated under the same conditions (population size and maximum number of iterations) in the same class of tests.

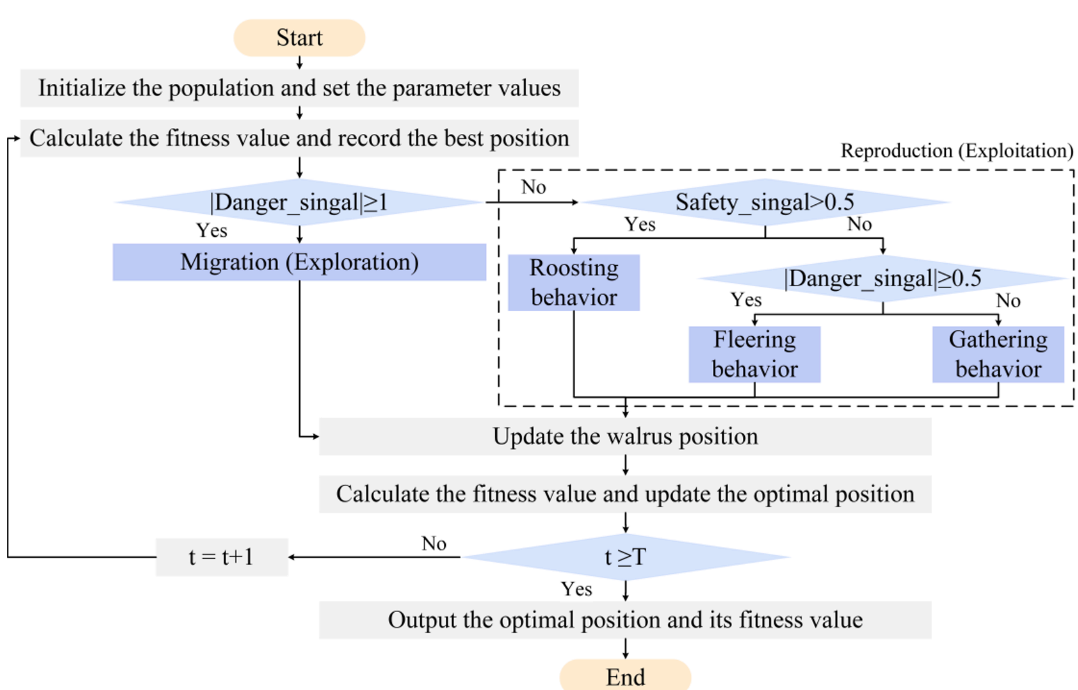


Fig. 2. Flow chart of WO.

4.1. Standard benchmark functions analysis

4.1.1. Experimental setup

A set of benchmark functions is used to verify the optimization capability of the WO. This set of functions consists of three different groups of unimodal, multimodal, and fixed-dimension multimodal. Unimodal benchmark functions (F1-F7), which only have one best global solution, are used for assessing the exploitation ability of optimization methods. Multimodal benchmark functions (F8-F23) have many local optima. Thus, they are used for assessing the exploration and exploitation balance in metaheuristic algorithms. The features of all the functions are shown in Table 2.

Some well-known optimizers, including Artificial Bee Colony Algorithm (ABC) (Karaboga & Basturk, 2007), Atomic Orbital Search (AOS) (Azizi, 2021), Butterfly Optimization Algorithm (BOA) (S. Arora & Singh, 2019), Black Widow Optimization Algorithm (BWOA) (Hayyolalam & Pourhaji Kazem, 2020), Chimp Optimization Algorithm (ChOA) (Khishe & Mosavi, 2020), Fruit fly Optimization Algorithm (FOA) (W. T. Pan, 2012), Genetic Algorithm (GA) (Deb, 1991), Grey Wolf Optimizer (GWO) (Mirjalili et al., 2014), Moth-flame Optimization Algorithm (MFO) (Mirjalili, 2015a), Particle Swarm Optimization (PSO) (Kennedy & Eberhart, 1995), Sand Cat Swarm Optimization (SCSO) (Seyyedabbasi & Kiani, 2022), Spotted Hyena Optimizer (SHO) (Dhiman & Kumar, 2017), Seagull Optimization Algorithm (SOA) (Dhiman & Kumar, 2019), Sparrow Search Algorithm (SSA) (Xue & Shen, 2020), and Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016) are used for comparisons with the WO. The dimensions of F1-F13 are set to 30. The maximum iterations are set to 2000 with a population size (search agents) of 100. All the algorithms are independently run for 100 times. The parameters of compared algorithms are shown in Table 3.

4.1.2. Qualitative analysis

The qualitative results of WO are evaluated using several unimodal and multimodal standard criterion functions, and the results are evaluated using four different criteria of WO, including search history, trajectory of the first agent, average fitness, and convergence curve. The search history graph reveals all locations visited by walrus individuals during the iteration; The trajectory graph monitors how the first walrus has changed during the iteration; The average fitness graph shows the overall change in walrus population; The convergence behavior plot records the best solution after each iteration of the population.

Ten typical functions are selected from 23 benchmark tests for analysis. It can be observed from the search history in Fig. 4 that WO tends to show a similar pattern when dealing with different situations, including promoting diversification, exploring favorable areas of solution space, and developing the optimal location neighborhoods.

According to the trajectory in Fig. 4, we can see the sudden movement of the walrus at the beginning of the search process. The amplitude of these fluctuations decreases with iterations. This point ensures the shift of the iterative process from the exploratory trend to the developmental step. Finally, the movement pattern of the walrus becomes very stable. By monitoring the average fitness of all agents, we can note the process of decreasing fitness values. There is a rich population diversity in the initial phase of the iteration. Therefore, WO can dynamically focus on more promising domains during the iterative process. Based on the convergence curves, we can observe the pattern of accelerated decline in all curves.

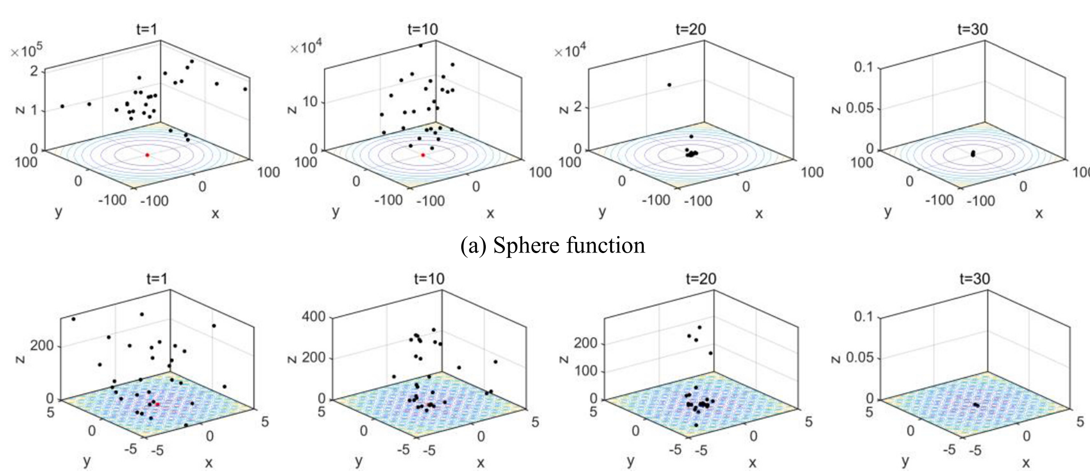
4.1.3. Quantitative analysis

In this subsection, the performance of WO is quantified. 100 independent runs of each algorithm in each test are implemented, and the statistical results are summarized in Table 4 and Table 5. The last three lines of tables are reported to display statistical analysis. The first line shows three symbols (W|T|L) that denote the number of the functions in which the performance of the algorithm is the best (win) | indistinguishable (tie) | inferior (loss) to the others. The second line illustrates the Friedman mean rank. The third line refers to the final rank values of all algorithms (Abualigah et al., 2021).

The unimodal functions (F1-F7) have only one global optimal solution and are used to examine the ability of the algorithm to exploit the global solution. The statistical results in Table 4 confirm the superiority of the WO in the exploitation of the optimal solutions, as it obtains the minimum of the best and worst solutions for the six functions tested in this set, as well as being smaller than the other comparison algorithms in terms of average and standard deviation.

The multiple local optimal solutions of the multidimensional benchmark functions (F8-F13) and the fixed-dimensional benchmark functions (F14-F23) can test the diversification capability of the algorithm. We can see from the results in Table 4 and Table 5 that the WO obtains first rank both on average and standard deviation in F8-F13, achieves first rank on average value in F14-F23, achieves second rank on standard deviation value in F17, and achieves third rank on standard deviation in F18. BWOA achieves the best solution in F1-F4. However, the ABC, AOS, GWO, MFO, PSO, SCSO, SHO, SSA, WOA, and BWOA in the comparison algorithms show a higher level in the best values in some of the function tests. Compared with the WO, they fail to perform as well in terms of the worst, average, or standard deviation values.

The time rows in Table 4 and Table 5 record the average time obtained by running 100 tests independently. The statistics show that a few algorithms take less computation time than WO, while most of the



(a) Sphere function

(b) Rastrigin function

Fig. 3. Swarm behaviors of 30 individuals.

Table 2
Summary of the unimodal benchmark functions.

Functions	Description	Dimensions	Range	f_{min}
Sphere	$F1 = \sum_{i=1}^{d} x_i^2$	d	[-100,100]	0
Schwefel 2.22	$F2 = \sum_{i=1}^{d} \lvert x_i \rvert + \prod_{i=1}^{d} \lvert x_i \rvert$	d	[-10,10]	0
Schwefel 1.2	$F3 = \sum_{i=1}^{d} \left(\sum_{j=1}^{i} x_j \right)^2$	d	[-100,100]	0
Schwefel 2.21	$F4 = \max\{\lvert x_i \rvert, 1 \le i \le d\}$	d	[-100,100]	0
Rosenbrock	$F5 = \sum_{i=1}^{d-1} \left[100 \left(x_{i+1} - x_i^2 \right)^2 + (x_i - 1)^2 \right]$	d	[-30,30]	0
Step	$F6 = \sum_{i=1}^{d} ([x_i + 0.5])^2$	d	[-100,100]	0
Quartic	$F7 = \sum_{i=1}^{d} i x_i^4 + rand[0,1)$	d	[-128,128]	0
Schwefel 2.26	$F8 = \sum_{i=1}^{d} (x_i \sin(\sqrt{\lvert x_i \rvert}))$	d	[-500,500]	-418.9829 d
Rastrigin	$F9 = \sum_{i=1}^{d} [x_i^2 - 10\cos 2\pi x_i + 10n]$	d	[-5.12,5.12]	0
Ackley	$F10 = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2} \right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d} \cos 2\pi x_i \right) + 20 + \exp(1)$	d	[-32,32]	0
Griewank	$F11 = \frac{1}{4000}\sum_{i=1}^{d} x_i^2 - \prod_{i=1}^{d} \cos \frac{x_i}{\sqrt{i}} + 1$	d	[-600,600]	0
Penalized [a)]	$F12 = \frac{\pi}{d}[10\sin(\pi y_1)] + \sum_{i=1}^{d-1} (y_i - 1)^2 \left[1 + 10\sin^2(\pi y_{i+1}) + \sum_{i=1}^{d} u(x_i, 10, 100, 4) \right]$	d	[-50,50]	0
	$F13 = 0.1\left(\sin^2(3\pi x_1) + \sum_{i=1}^{d} (x_i - 1)^2 [1 + \sin^2(3\pi x_1 + 1)] + (x_d - 1)^2 + \sin^2(2\pi x_d) \right) + \sum_{i=1}^{d} u(x_i, 5, 100, 4)$	d	[-50,50]	0
Foxholes	$F14 = [\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=1}^{2} (x_i - a_{ij})^6}]^{-1}$	2	[-65.536,65.536]	0.998
Kowalik	$F15 = \sum_{i=1}^{11} [a_i - \frac{x_1 (b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}]^2$	4	[-5,5]	0.0003075
Camel-Back	$F16 = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[-5,5]	-1.0316
Branin	$F17 = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6)^2 + 10(1 - \frac{1}{8\pi})\cos x_1 + 10$	2	[-5,5]	0.398
Goldstein-Price	$F18 = \left[1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2) \right] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_i + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2)]$	2	[-2,2]	3
Hartman's Family	$F19 = -\sum_{i=1}^{4} c_i \exp(-\sum_{i=1}^{3} a_{ij}(x_j - p_{ij})^2)$	3	[-1,2]	-3.86
Shekel's Family	$F20 = -\sum_{i=1}^{4} c_i \exp(-\sum_{i=1}^{6} a_{ij}(x_j - p_{ij})^2)$	6	[0,1]	-3.32
	$F21 = -\sum_{i=1}^{5} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	[0,1]	-10.1532
	$F22 = -\sum_{i=1}^{7} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	[0,1]	-10.4028
	$F23 = -\sum_{i=1}^{10} [(X - a_i)(X - a_i)^T + c_i]^{-1}$	4	[0,1]	-10.5363

[a)] $where y_i = 1 + \frac{x_i + 1}{4}, u(x_i, a, k, m) \begin{cases} K(x_i - a)^m if x_i > a \\ 0 - a \le x_i \ge a \\ K(-x_i - a)^m - a x_i \end{cases}$

remaining algorithms (e.g., GWO, SOA, WOA, etc.) take more computation time than WO. The result obtained using the Wilcoxon rank sum test is a parameter called p-value, which measures whether the advantage of the algorithm being measured is significant compared to the competing algorithms. In this test, an algorithm is statistically substantial if it has a p-value smaller than 0.05. Combined with the solution results of 100 experimental records, the "+", "-", and "=" symbols in Table 6 indicate that the optimization performance of WO is better than, worse than, and equal to that of the comparison algorithm, respectively. In most cases, WO performs significantly better than the other comparison algorithms.

4.1.4. Convergence analysis

The convergence curves of all algorithms in unimodal benchmark functions for the best score obtained so far are shown in Fig. 5(a-g). All algorithms completed convergence in F1, F2 and F5, but WO shows the fastest convergence rate. Although FOA appears to produce the fastest iteration on F1, F3, and F4-F6, its accuracy is too low compared with other algorithms, which can be verified by combining the data in Table 6. In the F3-F4 and F6-F7, ABC, MFO and FOA stagnate at local extremes, which confirms that the exploitation phase of WO is efficient and has reliable exploration value. The convergence curves of multimodal benchmark function and fixed-dimension benchmark function tests are shown in Fig. 5(h-m) and Fig. 5(n-w). From the curves of F8-F9, F11-F13, F17 and F19-F20, the proposed WO can obtain the highest accuracy of fitness value and the fastest convergence speed.

4.1.5. Stability analysis

As can be seen from Fig. 6(a), most of the tested algorithms show relatively stable performance in the unimodal benchmark functions. Individual algorithms such as GA, MFO, ABC, SHO, WOA, and FOA exhibit discrete characteristics, with more singular values for GA and WOA, indicating a large variation in the 100-test results for these algorithms.

The results of multimodal benchmark functions (see Fig. 6(b)) show that the stability of most algorithms, such as ABS, AOS, and BOA, is obviously weakened, and MFO in F10 is the most obvious ones. As shown in the test results of F14-F23 (see Fig. 6(c)), many singular values (red "+") appear in BWOA, FOA, GA and SHO, indicating that their 100 times results have the characteristics of high discreteness. In contrast, the proposed WO still maintains high stability. The optimizer can still search for the global optimal value stably when a lot of local optimal value interferes. This phenomenon is consistent with the statistical results of standard deviation from Table 4 to Table 5.

4.1.6. Sensitivity analysis

The process of finding the best parameter settings for a metaheuristic algorithm in solving a given problem is a major issue. Different values of the control parameters of an algorithm might yield different results as reported in the literature (Braik, 2021). Hence, there is a need to tune the control parameters of each metaheuristic that could provide more adaptability and robustness in solving a broad range of problems in various fields of science. The optimal parameter set of the proposed algorithm was selected based on the use of Design of Experiment (DoE)

Table 3
Parameter values of algorithms.

Algorithms	Parameters	Values
ABC	The percentage of onlooker bees	50%
	The percentage of employed bees	50%
	The number of scout bees	1
AOS	Maximum number of layers around nucleus (n)	5
	Photon rate for position determination of electrons (PR)	0.1
BOA	Modular modality (c)	0.01
	Power exponent (a)	0.1
	Switch probability (p)	0.8
BWOA	Procreating rate (PP)	0.6
	Cannibalism rate (CR)	0.44
	Mutation rate (Pm)	0.4
ChOA	m	chaotic vector
GA	Crossover percentage (pc)	0.8
	Mutation rate (pm)	0.1
MFO	A constant of the logarithmic spiral (b)	1
PSO	The maximum and minimum of inertia weight ($\omega_{\max}, \omega_{\min}$)	0.9, 0.6
	c_1, c_2	2
SCSO	Simulated hearing characteristics of the sand cat (S_M)	2
SOA	Control parameters (f_c)	2
	Constants to define the spiral shape (u, v)	1
SSA	Initial speed (v_0)	0
WOA	A constant of the logarithmic spiral (b)	1
AFT	Initial estimate of the tracking distance at the first iteration (α_0)	1
	Final rough estimation of the probability at the end iteration (β_0)	0.1
	Constant values that control exploration and exploitation ability (α_1, β_1)	2
ALO	Constant defined based on the current iteration (w)	2, 3, 4, 5
CSA	Probability of a chameleon finding its prey (P_p)	0.1
	Three constant values that control exploration and exploitation ability (γ, α, β)	1, 3.5, 3
	Two positive constants that control the velocity of the chameleon tongue (c_1, c_2)	1.75
	Two positive numbers that control exploration ability (p_1, p_2)	0.25, 1.75
	A positive number that controls exploitation ability (ρ)	1
DMOA	The alpha female's vocalization that keeps the family within a path ($peep$)	2
	Number of babysitters	3
GOA	The intensity and length scale of attraction (f, l)	0.5, 1.5
	The upper and lower limits of the decreasing coefficient ($c_{\max}, c_{\min}$)	1, 0.00004
MVO	Wormhole existence probability ($\mathrm{WEP}_{\max}, \mathrm{WEP}_{\min}$)	1, 0.2
	The exploitation accuracy over the iterations (p)	6
NGO	A random number I	1 or 2
SCA	A constant for movement direction (a)	2
	Three constants for food quantity, exploration, and exploitation (c_1, c_2, c_3)	0.5, 0.05, 2
	Threshold for food quantity	0.25, 0.6
SS	Initial speed (v_0)	0
WSO	The acceleration coefficient (τ)	4.125
	The upper and lower frequencies of the undulating motion ($f_{\max}, f_{\min}$)	0.75, 0.07
	Three positive constants employed to manage exploration and exploitation behaviors (a_0, a_1, a_2)	6.25, 100, 0.0005

framework through empirical testing of the proposed algorithm on a selected set of test functions.

Here, the proposed parameter setting method examines the sensitivity of the maximum number of iterations (T), number of search agents (N) and the proportion of male walruses to the walrus herd (p). F5, F6, F12, F13, F14 and F15 from the unimodal and multimodal benchmark functions were selected for testing. The values of each parameter for the sensitivity analysis design were defined as follows: $T = \{200, 500, 1000, 2000\}$, $N = \{30, 50, 80, 100\}$, $p = \{0.3, 0.35, 0.4, 0.45\}$. Each function was solved with 30 independent runs. The results of the sensitivity analysis, in terms of the mean fitness and convergence curves, for the above four functions with the three control parameters are illustrated below.

Table 10 shows the average fitness of WO when used to simulate six different benchmark functions. Fig. 7(a) and (b) show the WO convergence curves of F1 under different maximum iterations and number of search agents, respectively. Fig. 7(c) displays the convergence curve of WO on F5 with the different proportion of male walruses to the walrus herd. The computational results and the visualization demonstrate that WO converges to the optimum solution when the maximum number of iterations increases. Moreover, it can be seen from Fig. 7(b) that the number of iterations decreases when the number of search agents increases. Fig. 7(c) displays relatively stable behavior of WO when p ranges from 0.3 to 0.4. Based on this analysis, the best value for p, with WO having the best results, can be suggested as its largest value (i.e., $p = 0.45$). In F14 and F15 tests, WO has relatively slight sensitivity to these parameters, where it presented small differences between the results.

4.1.7. Scalability analysis

The scalability of the proposed WO is tested by varying the dimensions of F1-F13 to determine the effect of increasing the dimensions on the computational results of different test functions. The dimensions of the test functions are 50, 100, and 500. The 15 algorithms in Section 4.1.1 (function dimension is 30) are selected for comparative analysis. This reveals the effect of dimensionality change on the convergence ability of WO, which is used to diagnose the effectiveness of WO for low and high dimensional problems.

The scalability analysis included 39 tests (13 standard functions). From Tables 7 to 9, the WO outperforms the comparison algorithms in most cases, with 49 out of 52 cases (94.230 %) achieving the best mean and standard deviation values, which is higher than BWOA (53.846 %), SCSO (38.462 %), WOA (17.308 %), SHO (13.462 %), SOA (5.769 %), AOS (3.846 %), BOA (1.923 %), and other algorithms (0 %). In addition, Friedman test is performed for all algorithms. The results in Fig. 8 show that the WO is ranked first, while SCSO, SHO and WOA are ranked second, third and fourth, respectively.

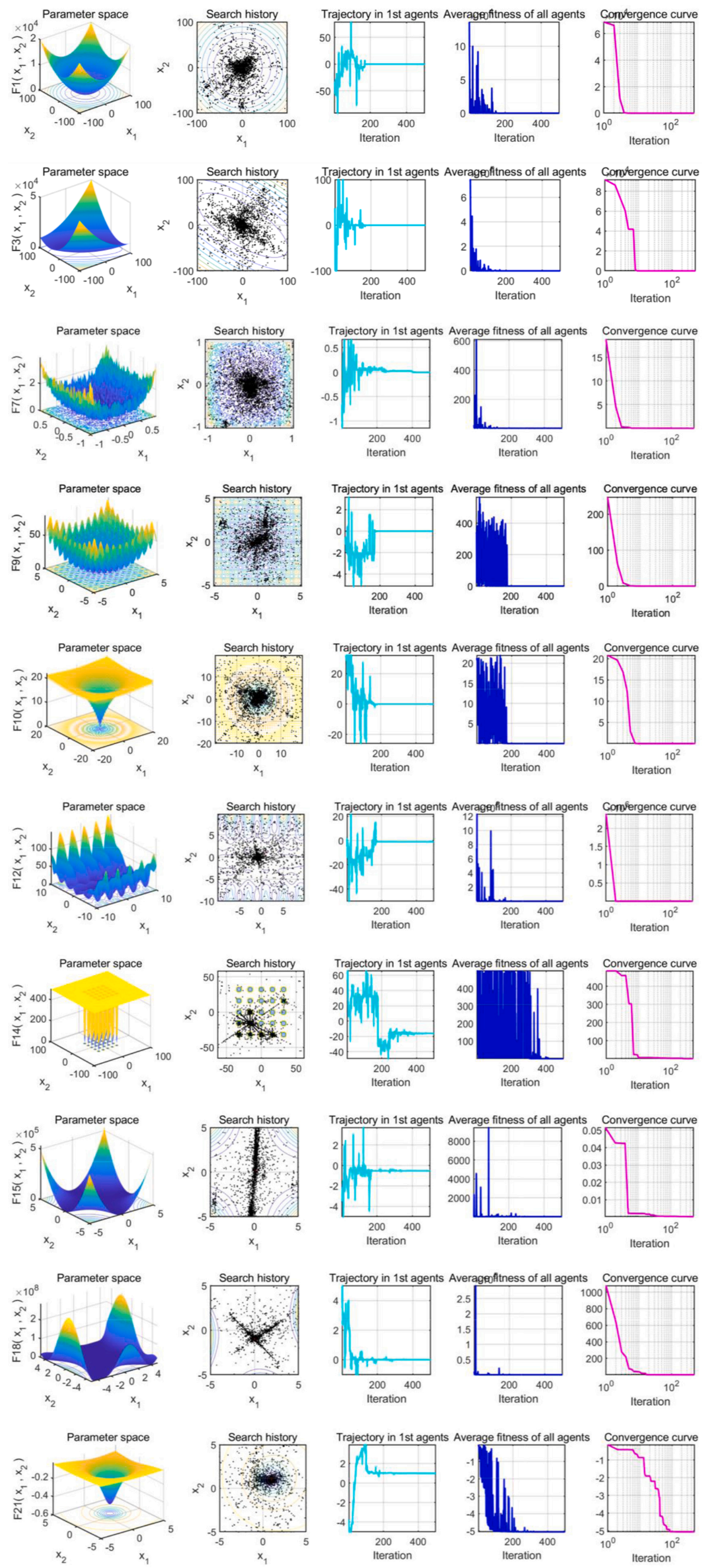


Fig. 4. Qualitative results of WO.

Table 4
Results of unimodal and multimodal benchmark functions (F1-F13) with $d = 30$.

Function		Comparative algorithms															
		ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
F1	Best	2.42E-10	1.74E-266	1.64E-17	**0.00E+00**	2.00E-109	6.80E-04	8.35E-03	1.33E-178	6.73E-17	5.69E-05	**0.00E+00**	**0.00E+00**	1.13E-57	3.26E-09	**0.00E+00**	**0.00E+00**
	Worst	1.33E-08	1.12E-248	2.13E-17	**0.00E+00**	4.23E-57	1.19E-03	3.31E-02	8.36E-173	1.00E+04	4.36E-03	**0.00E+00**	1.93E-278	3.56E-48	7.06E-09	**0.00E+00**	**0.00E+00**
	Avg	2.12E-09	1.12E-250	1.93E-17	**0.00E+00**	5.87E-59	9.18E-04	1.61E-02	3.64E-174	1.80E+03	7.55E-04	**0.00E+00**	1.93E-280	5.00E-50	5.09E-09	**0.00E+00**	**0.00E+00**
	Std	1.80E-09	0.00E+00	9.38E-19	**0.00E+00**	4.47E-58	1.17E-04	4.65E-03	0.00E+00	3.86E+03	7.06E-04	**0.00E+00**	**0.00E+00**	3.58E-49	8.64E-10	**0.00E+00**	**0.00E+00**
	Time (s)	4.06E+00	1.91E+00	5.62E-01	2.32E+00	2.52E+01	4.59E-01	4.81E-01	1.20E+00	7.03E-01	4.29E-01	1.13E+01	2.06E+00	2.08E+00	1.08E+00	1.78E+01	8.33E-01
F2	Best	1.76E-08	6.75E-138	3.58E-15	**0.00E+00**	9.16E-65	1.36E+00	1.80E-01	1.22E-101	3.35E-11	2.84E-03	3.81E-274	**0.00E+00**	5.77E-42	3.30E-05	6.75E-243	**0.00E+00**
	Worst	2.79E-06	5.98E-129	1.67E-14	**0.00E+00**	4.25E-37	1.90E+00	4.47E-01	2.44E-98	1.10E+02	5.47E-02	5.07E-263	6.31E-146	2.38E-37	2.69E+00	6.71E-222	**0.00E+00**
	Avg	2.63E-07	1.31E-130	1.51E-14	**0.00E+00**	5.81E-39	1.59E+00	3.19E-01	1.22E-99	2.96E+01	1.43E-02	7.15E-265	6.31E-148	1.11E-38	2.08E-01	7.61E-224	**0.00E+00**
	Std	4.37E-07	7.43E-130	1.74E-15	**0.00E+00**	4.31E-38	1.15E-01	4.70E-02	2.84E-99	2.19E+01	7.87E-03	**0.00E+00**	6.31E-147	3.13E-38	4.54E-01	**0.00E+00**	**0.00E+00**
	Time (s)	4.61E+00	2.14E+00	6.11E-01	2.97E+00	2.39E+01	5.65E-01	5.03E-01	1.20E+00	7.29E-01	4.32E-01	1.13E+01	1.95E+00	2.18E+00	1.11E+00	1.70E+01	8.31E-01
F3	Best	2.39E+04	3.74E-216	1.72E-17	**0.00E+00**	4.64E-31	1.86E-01	4.19E-01	2.65E-66	2.20E-01	9.26E+00	**0.00E+00**	2.80E-02	6.50E-14	8.23E-08	1.90E+00	**0.00E+00**
	Worst	6.12E+04	1.87E-193	2.24E-17	**0.00E+00**	1.89E-16	3.80E-01	3.26E+03	1.15E-53	4.50E+04	3.22E+01	**0.00E+00**	2.11E+04	1.75E-05	7.33E-06	5.89E+03	**0.00E+00**
	Avg	4.34E+04	1.89E-195	2.02E-17	**0.00E+00**	2.19E-18	2.57E-01	2.44E+02	1.97E-55	1.36E+04	1.80E+01	**0.00E+00**	4.10E+03	7.77E-07	4.39E-07	6.25E+02	**0.00E+00**
	Std	7.75E+03	**0.00E+00**	9.95E-19	**0.00E+00**	1.89E-17	3.85E-02	4.89E+02	1.35E-54	1.20E+04	4.89E+00	**0.00E+00**	4.94E+03	2.70E-06	8.17E-07	8.04E+02	**0.00E+00**
	Time (s)	6.98E+00	3.53E+00	3.17E+00	9.05E+00	2.52E+01	1.92E+00	1.75E+00	2.62E+00	1.97E+00	1.66E+00	1.26E+01	4.58E+00	3.71E+00	2.43E+00	1.67E+01	2.36E+00
F4	Best	3.21E+01	3.06E-129	1.43E-14	**0.00E+00**	2.64E-23	6.20E-03	1.01E-01	9.47E-47	2.05E+00	4.88E-01	8.15E-226	**0.00E+00**	1.57E-12	3.50E-05	5.08E-13	**0.00E+00**
	Worst	4.80E+01	7.24E-121	1.83E-14	**0.00E+00**	1.80E-12	1.01E-02	2.07E-01	3.94E-42	4.08E+01	1.26E+00	6.14E-212	8.52E-73	1.87E-07	3.70E+00	8.13E+01	**0.00E+00**
	Avg	4.11E+01	1.94E-122	1.64E-14	**0.00E+00**	2.77E-14	8.20E-03	1.57E-01	8.73E-44	1.42E+01	8.77E-01	7.88E-214	8.52E-75	4.00E-09	1.25E-01	1.33E+01	**0.00E+00**
	Std	3.28E+00	7.87E-122	7.52E-16	**0.00E+00**	1.82E-13	7.85E-04	1.89E-02	4.17E-43	8.62E+00	1.48E-01	**0.00E+00**	8.52E-74	1.97E-08	4.22E-01	2.05E+01	**0.00E+00**
	Time (s)	4.12E+00	2.09E+00	5.60E-01	2.66E+00	2.55E+01	4.93E-01	4.73E-01	1.28E+00	6.92E-01	4.19E-01	1.13E+01	2.08E+00	2.61E+00	1.22E+00	1.53E+01	8.75E-01
F5	Best	4.98E+01	2.43E+01	2.88E+01	2.88E+01	2.65E+01	2.82E+01	9.50E+00	2.41E+01	1.80E+00	2.23E+01	2.50E+01	2.69E-04	2.64E+01	9.30E+00	3.05E-02	**2.66E-08**
	Worst	1.18E+03	2.53E+01	2.90E+01	2.90E+01	2.90E+01	2.91E+01	8.20E+01	2.85E+01	9.01E+04	5.29E+02	2.88E+01	2.80E+01	2.87E+01	2.77E+02	2.60E+01	**2.78E-03**
	Avg	2.56E+02	2.48E+01	2.89E+01	2.89E+01	2.87E+01	2.88E+01	3.38E+01	2.59E+01	2.11E+04	9.60E+01	2.68E+01	1.30E+01	2.73E+01	4.30E+01	2.48E+01	**2.91E-04**
	Std	1.70E+02	1.94E-01	2.60E-02	3.96E-02	4.13E-01	1.85E-01	2.11E+01	8.57E-01	3.79E+04	7.34E+01	1.11E+00	1.37E+01	5.32E-01	3.69E+01	2.51E+00	**5.07E-04**
	Time (s)	4.62E+00	2.20E+00	8.40E-01	2.92E+00	2.59E+01	6.89E-01	6.12E-01	1.44E+00	8.84E-01	5.41E-01	1.14E+01	2.22E+00	2.53E+00	1.42E+00	1.60E+01	9.46E-01
F6	Best	4.16E-10	1.22E-04	3.56E+00	2.26E+00	1.22E+00	7.64E+00	7.37E+00	8.70E-07	**5.00E-17**	4.72E-05	1.60E-07	4.53E-05	1.04E-02	2.46E-09	8.05E-06	4.39E-11
	Worst	2.93E-08	4.61E-04	5.94E+00	6.83E+00	2.60E+00	7.69E+00	8.05E+00	1.01E+00	1.98E+04	3.93E-03	1.01E+00	1.01E+00	7.56E-01	7.90E-09	5.76E-05	**1.58E-07**
	Avg	2.29E-09	2.71E-04	4.86E+00	4.71E+00	1.88E+00	7.66E+00	7.85E+00	1.29E-01	1.10E+03	6.80E-04	2.62E-01	1.12E-01	2.28E-01	5.24E-09	2.52E-05	**3.59E-08**
	Std	3.18E-09	6.46E-05	5.93E-01	9.62E-01	2.87E-01	1.07E-02	1.01E-01	1.91E-01	3.43E+03	7.36E-04	2.18E-01	1.87E-01	1.72E-01	9.31E-10	1.02E-05	**3.49E-08**
	Time (s)	3.90E+00	1.99E+00	5.22E-01	2.51E+00	2.48E+01	4.80E-01	4.52E-01	1.22E+00	7.14E-01	4.12E-01	1.10E+01	2.04E+00	2.46E+00	1.34E+00	1.72E+01	8.08E-01
F7	Best	1.92E-02	3.05E-05	7.18E-05	9.84E-07	8.27E-06	7.85E+00	3.81E-01	1.68E-05	4.91E-03	6.72E-02	5.83E-08	7.90E-07	1.85E-03	5.90E-03	1.43E-06	**2.86E-07**
	Worst	7.78E-02	3.90E-04	4.07E-04	5.54E-05	8.61E-04	4.95E+01	1.51E+00	4.13E-04	1.88E+01	4.94E-01	1.19E-04	3.16E-04	2.05E-02	4.84E-02	1.02E-03	**7.42E-05**
	Avg	4.65E-02	1.42E-04	2.26E-04	1.19E-05	1.47E-04	2.14E+01	8.56E-01	1.23E-04	1.68E+00	1.93E-01	1.18E-05	2.22E-05	8.44E-03	1.59E-02	2.03E-04	**1.80E-05**
	Std	1.13E-02	7.50E-05	8.01E-05	9.71E-06	1.63E-04	7.50E+00	2.29E-01	7.87E-05	4.30E+00	8.26E-02	1.54E-05	3.43E-05	3.71E-03	6.71E-03	2.28E-04	**1.83E-05**
	Time (s)	5.48E+00	2.87E+00	2.05E+00	3.16E+00	2.75E+01	1.30E+00	1.20E+00	1.96E+00	1.49E+00	1.15E+00	1.18E+01	3.60E+00	3.13E+00	2.10E+00	1.94E+01	1.65E+00
F8	Best	-3.49E+243	-9.15E+03	-4.10E+03	-7.80E+03	-6.00E+03	-3.95E+00	-3.92E+03	-8.51E+03	-1.09E+04	-9.87E+03	-9.14E+03	**-1.26E+04**	-6.07E+03	-9.31E+03	**-1.26E+04**	**-1.26E+04**
	Worst	-1.58E+233	-6.24E+03	-6.33E+03	-3.95E+03	-5.75E+03	-7.74E-01	-1.59E+03	-4.10E+03	-7.22E+03	-5.09E+03	-5.05E+03	**-1.26E+04**	-4.81E+03	-6.08E+03	-8.56E+03	**-1.26E+04**
	Avg	-5.07E+241	-7.78E+03	-4.78E+03	-5.75E+03	-5.84E+03	-2.48E+00	-2.66E+03	-6.45E+03	-9.00E+03	-6.81E+03	-7.27E+03	**-1.26E+04**	-5.31E+03	-7.73E+03	-1.22E+04	**-1.26E+04**
	Std	6.55E+04	6.58E+02	3.65E+02	7.90E+02	5.93E+01	9.57E-01	5.04E+02	6.60E+02	8.50E+02	8.56E+02	7.83E+02	3.24E-01	2.74E+02	7.30E+02	7.26E+02	**5.44E-04**
	Time (s)	6.12E+00	2.25E+00	1.73E+00	2.74E+00	2.63E+01	6.26E-01	1.08E-02	1.37E+00	8.72E-01	6.12E-01	1.19E+01	2.43E+00	2.59E+00	1.40E+00	1.79E+01	1.02E+00
F9	Best	1.56E+02	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.59E+01	7.66E-01	**0.00E+00**	3.58E+01	2.31E+01	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.09E+01	**0.00E+00**	**0.00E+00**
	Worst	2.17E+02	6.62E+01	1.89E+02	**0.00E+00**	3.45E+00	8.71E+01	3.15E+00	**0.00E+00**	2.38E+02	9.93E+01	**0.00E+00**	**0.00E+00**	**0.00E+00**	8.86E+01	**0.00E+00**	**0.00E+00**
	Avg	1.95E+02	1.19E+01	1.89E+00	**0.00E+00**	4.40E-02	6.52E+01	1.82E+00	**0.00E+00**	1.18E+02	4.86E+01	**0.00E+00**	**0.00E+00**	**0.00E+00**	4.26E+01	**0.00E+00**	**0.00E+00**
	Std	1.12E+01	1.71E+01	1.89E+01	**0.00E+00**	3.57E-01	8.41E+00	5.46E-01	**0.00E+00**	3.82E+01	1.35E+01	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.57E+01	**0.00E+00**	**0.00E+00**

(*continued on next page*)

Table 4 (*continued*)

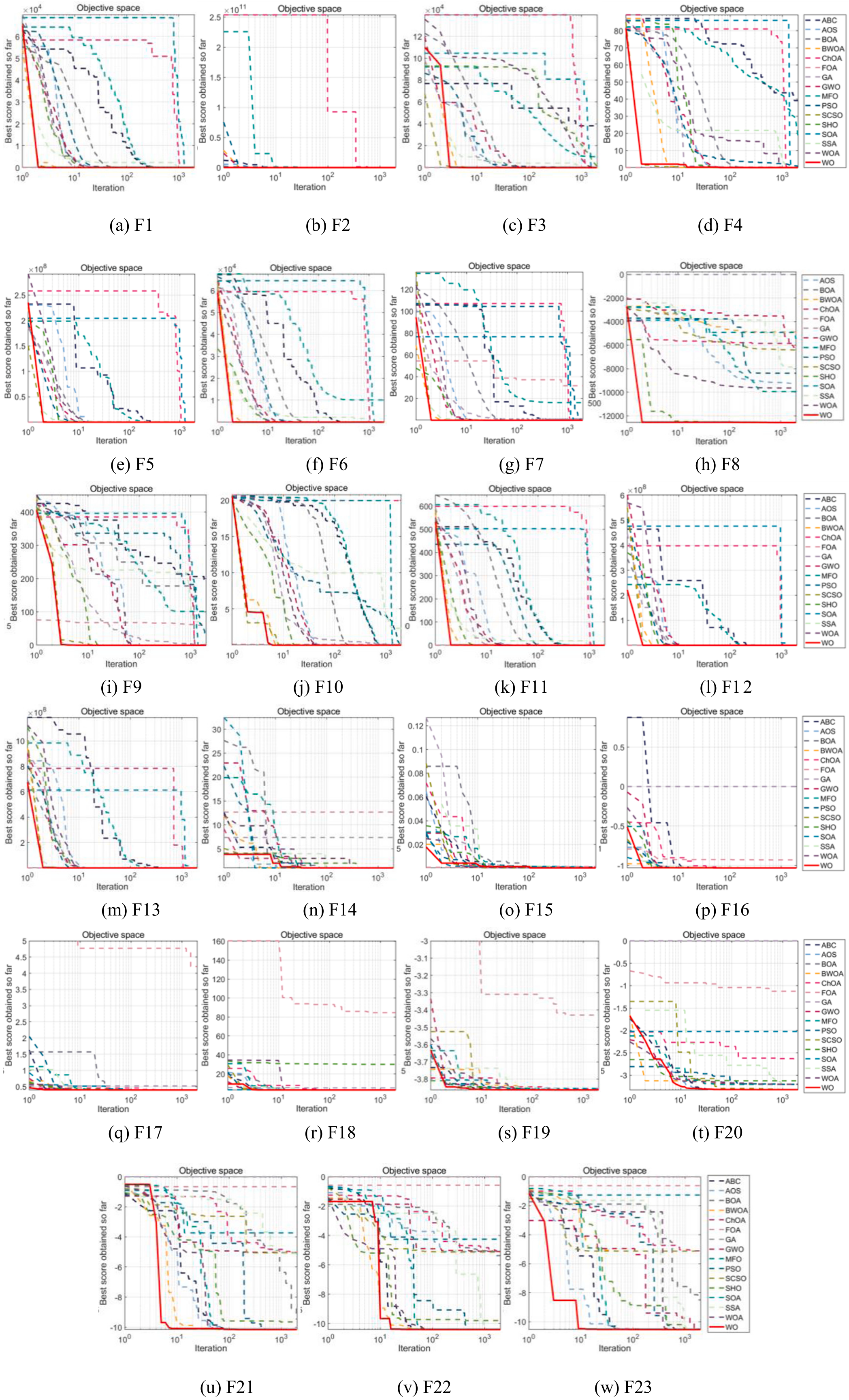
Function		Comparative algorithms															
		ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
	Time (s)	4.16E+00	2.21E+00	8.30E-01	2.46E+00	2.46E+01	5.84E-01	5.28E-01	1.21E+00	8.14E-01	5.50E-01	1.13E+01	1.99E+00	2.49E+00	1.35E+00	1.77E+01	9.16E-01
F10	Best	1.17E-04	8.88E-16	4.44E-15	**8.88E-16**	2.00E+01	6.64E-02	5.19E-02	4.44E-15	2.66E-09	4.42E-03	**8.88E-16**	**8.88E-16**	4.44E-15	1.22E-05	**8.88E-16**	**8.88E-16**
	Worst	9.98E-03	4.44E-15	1.51E-14	**8.88E-16**	2.00E+01	1.06E-01	1.21E-01	1.51E-14	2.00E+01	8.66E-02	**8.88E-16**	7.99E-15	7.99E-15	2.89E+00	7.99E-15	**8.88E-16**
	Avg	1.61E-03	4.12E-15	1.04E-14	**8.88E-16**	2.00E+01	8.55E-02	8.41E-02	8.03E-15	9.65E+00	1.81E-02	**8.88E-16**	1.21E-15	4.48E-15	1.30E+00	3.62E-15	**8.88E-16**
	Std	1.42E-03	1.02E-15	3.60E-15	**0.00E+00**	1.54E-03	7.56E-03	1.60E-02	9.44E-16	9.52E+00	1.10E-02	**0.00E+00**	1.14E-15	3.55E-16	9.00E-01	2.47E-15	**0.00E+00**
	Time (s)	4.46E+00	2.22E+00	6.59E-01	2.47E+00	2.41E+01	5.70E-01	5.54E-01	1.19E+00	8.18E-01	5.47E-01	1.10E+01	2.18E+00	2.45E+00	1.35E+00	1.85E+01	9.42E-01
F11	Best	8.53E-08	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	9.66E-07	2.71E-04	**0.00E+00**	1.11E-16	5.07E-06	**0.00E+00**	**0.00E+00**	**0.00E+00**	8.98E-09	**0.00E+00**	**0.00E+00**
	Worst	1.61E-02	9.92E-03	**0.00E+00**	**0.00E+00**	1.05E-02	2.39E-06	1.24E-03	1.52E-02	9.07E+01	5.67E-02	**0.00E+00**	7.82E-01	**0.00E+00**	3.44E-02	3.17E-02	**0.00E+00**
	Avg	3.32E-04	5.21E-04	**0.00E+00**	**0.00E+00**	2.84E-04	1.48E-06	5.91E-04	4.52E-04	4.53E+00	1.41E-02	**0.00E+00**	7.82E-03	**0.00E+00**	8.89E-03	9.95E-04	**0.00E+00**
	Std	1.68E-03	2.09E-03	**0.00E+00**	**0.00E+00**	1.64E-03	2.90E-07	1.75E-04	2.30E-03	1.98E+01	1.21E-02	**0.00E+00**	7.82E-02	**0.00E+00**	9.46E-03	4.57E-03	**0.00E+00**
	Time (s)	4.65E+00	2.34E+00	9.84E-01	2.76E+00	2.36E+01	7.31E-01	6.58E-01	1.26E+00	9.36E-01	6.09E-01	1.14E+01	2.44E+00	2.54E+00	1.46E+00	1.74E+01	1.05E+00
F12	Best	1.68E+00	1.71E-05	1.10E-01	1.30E-01	7.90E-02	1.71E+00	1.45E+00	7.40E-08	**5.15E-17**	2.38E-07	2.70E-08	2.88E-07	9.41E-04	1.05E-11	1.43E-06	1.27E-12
	Worst	1.47E+01	5.68E-05	4.86E-01	9.00E-01	6.72E-01	1.73E+00	1.73E+00	3.95E-02	1.77E+00	2.22E-05	5.31E-02	4.75E-02	5.44E-02	7.90E+00	1.20E-05	**2.58E-09**
	Avg	6.87E+00	3.53E-05	2.86E-01	4.35E-01	1.73E-01	1.72E+00	1.69E+00	1.28E-02	1.82E-01	4.08E-06	2.12E-02	1.03E-02	1.21E-02	1.89E+00	4.22E-06	**3.42E-10**
	Std	2.50E+00	9.14E-06	8.29E-02	1.45E-01	1.15E-01	3.66E-03	4.39E-02	9.31E-03	3.48E-01	4.43E-06	1.14E-02	1.05E-02	8.00E-03	1.75E+00	1.96E-06	**4.72E-10**
	Time (s)	8.50E+00	4.16E+00	5.07E+00	6.17E+00	2.54E+01	2.70E+00	2.65E+00	3.24E+00	2.84E+00	2.56E+00	1.30E+01	6.18E+00	4.76E+00	3.51E+00	2.21E+01	3.10E+00
F13	Best	1.25E+00	2.57E-04	7.90E-01	1.47E+00	2.27E+00	2.79E+00	8.36E-04	1.38E-06	**1.98E-16**	5.87E-06	3.79E-07	3.04E-07	2.09E+00	1.32E-10	1.73E-05	2.37E-12
	Worst	1.87E+01	2.16E-02	2.91E+00	3.00E+00	3.00E+00	2.87E+00	4.40E-03	5.00E-01	3.60E+00	1.13E-02	2.60E+00	8.04E-01	2.48E+00	2.10E-02	1.11E-02	**1.90E-07**
	Avg	6.72E+00	4.67E-03	1.73E+00	2.81E+00	2.85E+00	2.83E+00	1.93E-03	1.55E-01	1.56E-01	9.67E-04	1.54E+00	1.80E-01	2.33E+00	4.87E-03	1.20E-03	**1.52E-08**
	Std	4.18E+00	6.33E-03	4.44E-01	3.63E-01	1.45E-01	1.89E-02	7.13E-04	1.24E-01	6.39E-01	2.62E-03	6.75E-01	2.25E-01	7.50E-02	6.87E-03	3.30E-03	**2.56E-08**
	Time (s)	8.42E+00	4.28E+00	4.92E+00	6.04E+00	2.55E+01	2.65E+00	2.55E+00	3.17E+00	2.85E+00	2.58E+00	1.27E+01	6.24E+00	4.09E+00	3.47E+00	2.25E+01	3.07E+00
(W|T|L)	(0|9|4)	(3|10|0)	(2|11|0)	(3|10|0)	(2|10|1)	(0|7|6)	(0|11|2)	(2|11|0)	(3|10|0)	(0|13|0)	(4|9|0)	(3|10|0)	(2|11|0)	(0|13|0)	(3|10|0)	(9|4|0)	
Friedman mean rank	12.15	5.81	9.35	5.69	9.08	12.38	11.92	6.62	13.69	10.92	4.42	5.81	8.69	11.46	6.00	2.00	
Final rank	14	4	10	3	9	15	13	7	16	11	2	4	8	12	6	1	

Table 5
Results of fixed-dimension multimodal benchmark functions (F14-F23).

		Comparative algorithms															
	Function	ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
F14	Best	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	1.27E+01	9.99E-01	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**
	Worst	**9.98E-01**	1.99E+00	9.98E-01	4.95E+00	2.98E+00	1.27E+01	1.27E+01	1.08E+01	**9.98E-01**	1.99E+00	2.98E+00	2.98E+00	1.01E+00	**9.98E-01**	2.98E+00	**9.98E-01**
	Avg	**9.98E-01**	1.01E+00	**9.98E-01**	1.45E+00	1.02E+00	1.27E+01	8.89E+00	2.61E+00	**9.98E-01**	1.17E+00	1.51E+00	1.20E+00	9.98E-01	**9.98E-01**	1.02E+00	**9.98E-01**
	Std	3.52E-11	9.94E-02	1.26E-05	1.02E+00	1.98E-01	2.83E-11	3.98E+00	3.01E+00	2.57E-15	3.75E-01	8.75E-01	5.98E-01	6.63E-04	2.03E-15	1.98E-01	**1.80E-15**
	Time (s)	1.34E+01	6.03E+00	8.16E+00	1.07E+01	5.51E+00	4.56E+00	4.33E+00	4.21E+00	4.37E+00	4.07E+00	4.80E+00	8.47E+00	3.98E+00	4.52E+00	8.54E+00	4.56E+00
F15	Best	7.77E-04	**3.08E-04**	3.08E-04	**3.08E-04**	1.22E-03	3.08E-04	**3.08E-04**	**3.08E-04**	**3.08E-04**	**3.08E-04**	**3.08E-04**	3.10E-04	**3.08E-04**	**3.08E-04**	**3.08E-04**	**3.08E-04**
	Worst	1.17E-03	1.22E-03	3.42E-04	2.04E-02	1.29E-03	1.21E-03	9.02E-03	2.04E-02	1.49E-03	1.06E-03	1.22E-03	2.59E-03	1.22E-03	1.33E-03	1.23E-03	**3.08E-04**
	Avg	1.04E-03	4.74E-04	3.14E-04	2.40E-03	1.24E-03	3.38E-04	6.79E-04	1.76E-03	7.38E-04	5.80E-04	3.55E-04	9.60E-04	9.28E-04	7.31E-04	4.83E-04	**3.08E-04**
	Std	5.07E-05	3.53E-04	5.09E-06	6.03E-03	1.28E-05	1.00E-04	1.11E-03	5.13E-03	3.63E-04	2.93E-04	2.01E-04	7.60E-04	4.07E-04	3.65E-04	3.05E-04	**4.57E-08**
	Time (s)	4.43E+00	1.72E+00	5.28E-01	2.63E+00	3.86E+00	3.95E-01	3.99E-01	3.11E-01	2.78E-01	1.27E-01	1.57E+00	4.30E-01	4.18E-01	4.36E-01	4.20E+00	3.94E-01
F16	Best	-9.53E-01	**-1.03E+00**	-9.54E-01	**-1.03E+00**	**-1.03E+00**	-9.56E-01	-1.03E+00	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**
	Worst	-6.49E-01	**-1.03E+00**	-6.91E-01	**-1.03E+00**	**-1.03E+00**	-6.42E-01	-3.51E-02	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**
	Avg	-8.92E-01	**-1.03E+00**	-8.94E-01	**-1.03E+00**	**-1.03E+00**	-8.99E-01	-6.62E-01	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**	**-1.03E+00**
	Std	5.68E-02	3.35E-09	6.02E-02	3.71E-05	1.02E-06	5.91E-02	2.93E-01	4.44E-10	1.56E-15	1.56E-15	3.00E-12	8.93E-10	2.45E-09	2.43E-15	3.01E-14	**1.46E-15**
	Time (s)	4.64E+00	1.73E+00	7.74E-01	2.37E+00	1.76E+00	3.87E-01	1.06E-02	2.64E-01	2.59E-01	1.24E-01	9.05E-01	3.36E-01	3.22E-01	4.15E-01	3.11E+00	3.70E-01
F17	Best	2.83E+00	3.98E-01	3.90E+00	3.98E-01	3.98E-01	4.24E+00	4.03E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	**3.98E-01**
	Worst	4.24E+01	3.98E-01	4.41E+01	3.98E-01	3.99E-01	4.15E+01	2.68E+00	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	**3.98E-01**
	Avg	1.64E+01	3.98E-01	1.59E+01	3.98E-01	3.98E-01	1.51E+01	8.39E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	3.98E-01	**3.98E-01**
	Std	1.25E+01	5.95E-06	1.26E+01	**1.06E-15**	1.45E-04	1.20E+01	4.20E-01	1.07E-05	**1.06E-15**	**1.06E-15**	2.64E-10	1.66E-05	1.33E-07	1.60E-15	1.34E-09	1.14E-15
	Time (s)	4.04E+00	1.83E+00	4.43E-01	2.27E+00	1.66E+00	3.61E-01	3.30E-01	2.00E-01	2.01E-01	6.41E-02	8.37E-01	2.26E-01	2.58E-01	4.18E-01	3.25E+00	3.25E-01
F18	Best	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	8.40E+01	3.01E+00	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**
	Worst	**3.00E+00**	**3.00E+00**	3.04E+00	3.00E+01	**3.00E+00**	8.04E+02	8.21E+01	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	3.49E+01	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**
	Avg	**3.00E+00**	**3.00E+00**	3.00E+00	3.27E+00	**3.00E+00**	1.10E+02	1.70E+01	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**	6.12E+00	**3.00E+00**	**3.00E+00**	**3.00E+00**	**3.00E+00**
	Std	9.48E-16	5.33E-08	4.47E-03	2.70E+00	1.28E-06	1.18E+02	1.58E+01	2.80E-07	4.25E-15	**8.43E-16**	4.98E-08	8.94E+00	5.95E-08	2.46E-14	2.36E-08	1.41E-15
	Time (s)	3.97E+00	1.69E+00	4.17E-01	2.37E+00	1.65E+00	3.31E-01	3.23E-01	1.92E-01	1.94E-01	6.34E-02	8.16E-01	2.10E-01	2.57E-01	3.40E-01	3.36E+00	3.01E-01
F19	Best	**-3.86E+00**	**-3.86E+00**	-3.82E+00	**-3.86E+00**	**-3.86E+00**	-3.82E+00	-3.85E+00	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**	-3.86E+00	**-3.86E+00**	**-3.86E+00**	**-3.86E+00**
	Worst	**-3.86E+00**	-3.86E+00	-2.48E+00	-3.86E+00	-3.85E+00	-2.61E+00	-2.85E+00	-3.86E+00	**-3.86E+00**	**-3.86E+00**	-3.86E+00	-3.09E+00	-3.86E+00	**-3.86E+00**	-3.86E+00	**-3.86E+00**
	Avg	**-3.86E+00**	**-3.86E+00**	-3.71E+00	**-3.86E+00**	-3.86E+00	-3.70E+00	-3.59E+00	-3.86E+00	**-3.86E+00**	**-3.86E+00**	-3.86E+00	-3.84E+00	-3.86E+00	**-3.86E+00**	-3.86E+00	**-3.86E+00**
	Std	6.25E-15	7.88E-04	2.08E-01	7.88E-04	1.71E-03	1.93E-01	2.11E-01	2.11E-03	6.25E-15	6.25E-15	2.66E-03	1.11E-01	2.38E-04	6.71E-15	1.44E-03	**3.28E-15**
	Time (s)	4.28E+00	1.79E+00	1.69E+00	3.39E+00	2.50E+00	4.32E-01	1.08E-02	3.32E-01	3.12E-01	1.68E-01	1.27E+00	4.47E-01	4.06E-01	4.76E-01	4.38E+00	4.35E-01
F20	Best	-3.32E+00	-3.32E+00	-1.63E+00	-3.32E+00	-3.31E+00	-1.84E+00	-3.05E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.32E+00	-3.31E+00	-3.09E+00	-3.32E+00	-3.32E+00	**-3.32E+00**
	Worst	-3.20E+00	-3.13E+00	-3.66E-01	-3.04E+00	-2.27E+00	-4.95E-01	-1.07E+00	-3.09E+00	-3.20E+00	-3.20E+00	-2.27E+00	-2.37E+00	-1.43E+00	-3.20E+00	-3.09E+00	**-3.32E+00**
	Avg	-3.27E+00	-3.26E+00	-9.51E-01	-3.26E+00	-2.87E+00	-9.58E-01	-2.05E+00	-3.26E+00	-3.23E+00	-3.26E+00	-3.24E+00	-3.13E+00	-2.78E+00	-3.22E+00	-3.26E+00	**-3.32E+00**
	Std	5.96E-02	6.98E-02	2.97E-01	6.98E-02	2.73E-01	2.69E-01	4.36E-01	6.64E-02	4.87E-02	5.97E-02	1.21E-01	1.36E-01	3.95E-01	4.02E-02	6.65E-02	**2.21E-15**
	Time (s)	4.33E+00	1.83E+00	1.73E+00	3.34E+00	4.71E+00	4.51E-01	1.07E-02	4.44E-01	3.89E-01	2.08E-01	2.34E+00	6.28E-01	5.85E-01	5.50E-01	5.86E+00	4.85E-01
F21	Best	**-1.02E+01**	**-1.02E+01**	-9.47E+00	**-1.02E+01**	-5.05E+00	-8.56E-01	-3.01E+00	**-1.02E+01**	**-1.02E+01**	**-1.02E+01**	**-1.02E+01**	**-1.02E+01**	-5.06E+00	**-1.02E+01**	**-1.02E+01**	**-1.02E+01**
	Worst	**-1.02E+01**	-5.06E+00	-4.87E+00	-2.63E+00	-8.81E-01	-4.58E-01	-3.39E-01	-5.06E+00	-2.63E+00	-5.06E+00	-5.06E+00	-8.80E-01	-6.19E-01	-5.06E+00	**-1.02E+01**	**-1.02E+01**
	Avg	**-1.02E+01**	-9.80E+00	-6.54E+00	-9.19E+00	-4.57E+00	-5.44E-01	-8.44E-01	-9.70E+00	-8.18E+00	-8.73E+00	-6.84E+00	-8.56E+00	-2.39E+00	-9.09E+00	**-1.02E+01**	**-1.02E+01**
	Std	2.07E-14	1.29E+00	1.49E+00	2.10E+00	1.30E+00	7.75E-02	4.83E-01	1.46E+00	2.76E+00	2.29E+00	2.44E+00	2.69E+00	1.54E+00	2.07E+00	5.51E-05	**1.79E-14**
	Time (s)	4.39E+00	1.89E+00	1.45E+00	3.35E+00	3.34E+00	5.08E-01	1.08E-02	4.37E-01	4.11E-01	2.30E-01	1.69E+00	6.10E-01	5.22E-01	6.01E-01	4.69E+00	5.46E-01
F22	Best	-1.04E+01	-1.04E+01	-9.68E+00	-1.04E+01	-8.90E+00	-8.08E-01	-4.27E+00	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-1.04E+01	-5.46E+00	-1.04E+01	-1.04E+01	**-1.04E+01**
	Worst	-1.04E+01	-5.09E+00	-4.79E+00	-1.84E+00	-9.08E-01	-4.66E-01	-5.14E-01	-5.09E+00	-2.75E+00	-5.09E+00	-5.09E+00	-2.28E+00	-7.27E-01	-2.77E+00	-2.77E+00	**-1.04E+01**
	Avg	-1.04E+01	-1.02E+01	-6.64E+00	-8.45E+00	-5.09E+00	-5.57E-01	-9.37E-01	-1.02E+01	-9.20E+00	-9.56E+00	-7.00E+00	-8.83E+00	-2.35E+00	-1.00E+01	-1.03E+01	**-1.04E+01**
	Std	1.60E-14	1.05E+00	1.46E+00	2.78E+00	6.66E-01	5.04E-02	4.51E-01	9.11E-01	2.54E+00	1.95E+00	2.56E+00	2.44E+00	1.53E+00	1.46E+00	9.26E-01	**1.30E-14**
	Time (s)	4.82E+00	1.91E+00	1.88E+00	3.67E+00	3.34E+00	5.76E-01	1.08E-02	5.04E-01	4.71E-01	2.88E-01	1.74E+00	7.55E-01	5.68E-01	6.38E-01	4.73E+00	5.98E-01
F23	Best	**-1.05E+01**	**-1.05E+01**	-1.01E+01	**-1.05E+01**	-8.31E+00	-1.15E+00	-3.36E+00	**-1.05E+01**	**-1.05E+01**	**-1.05E+01**	**-1.05E+01**	**-1.05E+01**	-5.13E+00	**-1.05E+01**	**-1.05E+01**	**-1.05E+01**
	Worst	**-1.05E+01**	-3.84E+00	-4.83E+00	-2.43E+00	-9.46E-01	-5.07E-01	-6.15E-01	-1.05E+01	-2.81E+00	-5.13E+00	-5.13E+00	-1.91E+00	-9.45E-01	-5.13E+00	**-1.05E+01**	**-1.05E+01**
	Avg	**-1.05E+01**	-1.00E+01	-7.24E+00	-9.06E+00	-5.09E+00	-6.05E-01	-1.06E+00	-1.05E+01	-1.02E+01	-1.04E+01	-7.19E+00	-9.73E+00	-2.28E+00	-1.02E+01	**-1.05E+01**	**-1.05E+01**
	Std	1.61E-14	1.75E+00	1.54E+00	2.61E+00	5.28E-01	8.55E-02	3.59E-01	2.89E-05	1.47E+00	9.27E-01	2.64E+00	1.80E+00	1.41E+00	1.28E+00	4.25E-05	**1.42E-14**
	Time (s)	5.41E+00	2.00E+00	2.44E+00	4.27E+00	3.42E+00	6.66E-01	1.10E-02	6.22E-01	5.63E-01	3.94E-01	1.85E+00	9.24E-01	6.67E-01	7.19E-01	5.03E+00	
(W|T|L)	(0|10|0)	(0|10|0)	(0|9|1)	(0|8|2)	(0|10|0)	(0|9|1)	(0|6|4)	(0|10|0)	(1|8|1)	(2|8|0)	(0|9|1)	(0|10|0)	(0|10|0)	(1|9|0)	(0|10|0)	**(7|3|0)**	7.01E-01
Friedman mean rank	4.70	8.60	11.05	10.50	8.85	9.10	12.10	9.00	6.75	6.45	10.40	12.40	9.60	5.85	6.45	**4.20**	
Final rank	2	7	14	13	8	10	15	9	6	4	12	16	11	3	5	**1**	

Table 6
Results of the Wilcoxon rank-sum test on benchmark functions (F1-F23).

Function	ABC-WO		AOS-WO		BOA-WO		BWOA-WO		ChOA-WO		FOA-WO		GA-WO		GWO-WO		MFO-WO		PSO-WO		SCSO-WO		SHO-WO		SOA-WO		SSA-WO		WOA-WO	
	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H
F1	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.19E-39	+	5.64E-39	+	5.64E-39	+	8.28E-02	-	5.64E-39	+	5.64E-39	+	N/A	=
F2	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	4.18E-39	+	5.64E-39	+	5.64E-39	+	8.28E-02	-	5.64E-39	+	5.64E-39	+	5.64E-39	+
F3	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
F4	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	1.23E-03	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
F5	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	3.07E-34	+	1.25E-35	+	1.25E-35	+	1.25E-35	+
F6	4.18E-24	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	9.27E-21	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	1.25E-35	+	2.97E-14	+	1.25E-35	+
F7	2.56E-34	+	4.12E-32	+	2.72E-34	+	1.33E-05	+	5.15E-22	+	2.56E-34	+	2.56E-34	+	2.31E-28	+	2.56E-34	+	2.56E-34	+	2.40E-07	+	8.71E-04	+	2.56E-34	+	2.56E-34	+	3.33E-20	+
F8	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.52E-34	+	2.32E-25	+	2.52E-34	+	2.52E-34	+	5.65E-33	+
F9	5.64E-39	+	1.20E-11	+	1.28E-32	+	N/A	=	1.33E-02	+	5.64E-39	+	5.64E-39	+	N/A	=	5.64E-39	+	5.64E-39	+	5.64E-39	=	N/A	=	N/A	=	5.63E-39	+	N/A	=
F10	5.64E-39	+	5.26E-38	+	2.16E-39	+	N/A	=	5.64E-39	+	5.64E-39	+	5.64E-39	+	2.32E-44	+	5.64E-39	+	5.64E-39	+	5.64E-39	=	4.03E-03	+	5.76E-45	+	5.48E-39	+	1.01E-20	+
F11	5.64E-39	+	1.33E-02	+	N/A	=	N/A	=	1.28E-32	+	5.64E-39	+	5.64E-39	+	4.44E-02	+	5.59E-39	+	5.64E-39	+	5.64E-39	=	N/A	=	N/A	=	5.64E-39	+	2.42E-02	+
F12	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	1.43E-01	-	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.21E-23	+	2.56E-34	+
F13	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	4.62E-01	-	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	1.98E-02	+	2.56E-34	+
F14	N/A	=	3.22E-02	+	2.52E-28	+	1.43E-06	+	6.60E-32	+	3.52E-45	+	3.77E-39	+	1.05E-11	+	N/A	=	1.73E-05	+	5.01E-08	+	2.14E-15	+	2.16E-38	+	N/A	=	3.22E-02	+
F15	2.52E-34	+	1.80E-02	+	2.52E-34	+	9.92E-19	+	2.52E-34	+	2.52E-34	+	6.00E-34	+	1.49E-20	+	3.71E-34	+	2.41E-26	+	7.50E-24	+	2.52E-34	+	1.13E-29	+	3.88E-19	+	7.18E-34	+
F16	1.38E-37	+	4.45E-03	+	1.38E-37	+	1.23E-03	+	7.21E-26	+	1.38E-37	+	1.38E-37	+	3.51E-02	+	1.23E-03	+	1.23E-03	+	1.23E-03	+	3.59E-03	+	2.16E-06	+	1.23E-03	+	1.23E-03	+
F17	1.15E-38	+	6.99E-36	+	1.15E-38	+	1.58E-02	+	3.24E-37	+	1.15E-38	+	1.15E-38	+	3.74E-34	+	1.58E-03	+	1.58E-03	+	2.67E-01	-	4.72E-36	+	9.07E-36	+	1.58E-01	+	1.22E-02	+
F18	N/A	=	1.10E-36	+	5.64E-39	+	3.22E-02	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	2.15E-38	+	N/A	=	N/A	=	1.81E-34	+	4.07E-36	+	8.41E-31	+	N/A	=	1.61E-25	+
F19	N/A	=	5.64E-39	+	5.64E-39	+	3.22E-02	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	N/A	=	N/A	=	1.74E-34	+	5.64E-39	+	5.62E-39	+	N/A	=	5.64E-39	+
F20	5.41E-15	+	9.86E-10	+	2.21E-35	+	2.08E-02	+	2.74E-32	+	2.21E-35	+	2.21E-35	+	4.71E-10	+	2.33E-06	+	4.82E-01	-	9.62E-12	+	7.14E-23	+	2.21E-35	+	4.98E-22	+	1.04E-10	+
F21	5.64E-39	+	5.64E-39	+	5.64E-39	+	9.34E-06	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	1.05E-10	+	7.00E-09	+	5.15E-39	+	5.64E-39	+	5.64E-39	+	1.39E-06	+	5.64E-39	+
F22	5.64E-39	+	5.64E-39	+	5.64E-39	+	1.97E-10	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.08E-06	+	3.27E-05	+	5.27E-39	+	5.64E-39	+	5.64E-39	+	7.31E-03	+	5.64E-39	+
F23	5.64E-39	+	5.64E-39	+	5.64E-39	+	1.03E-07	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	4.44E-02	+	8.27E-02	-	5.22E-39	+	5.64E-39	+	5.64E-39	+	1.33E-02	+	5.63E-39	+



(a) F1 (b) F2 (c) F3 (d) F4

(e) F5 (f) F6 (g) F7 (h) F8

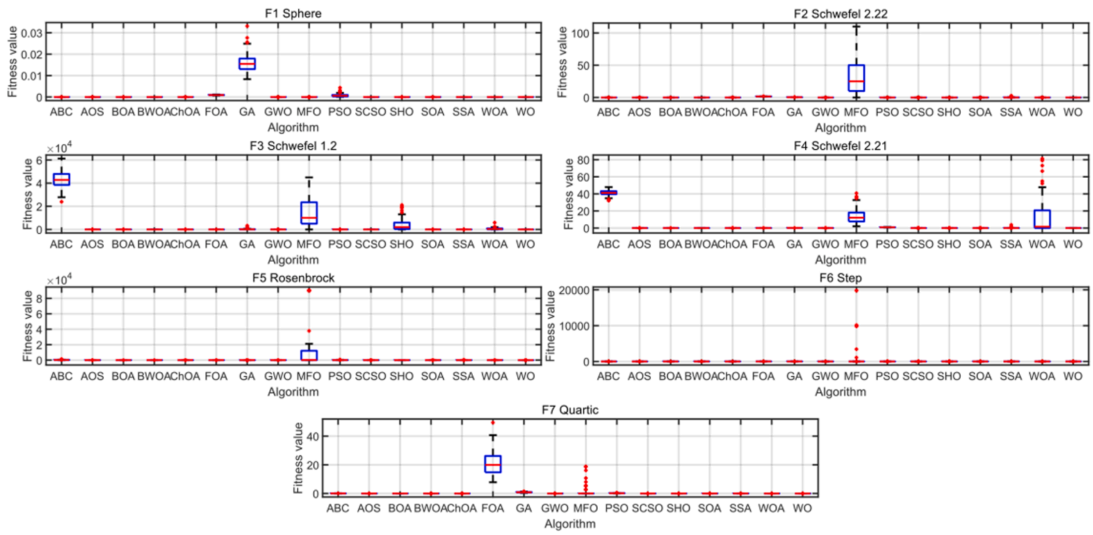
(i) F9 (j) F10 (k) F11 (l) F12

(m) F13 (n) F14 (o) F15 (p) F16

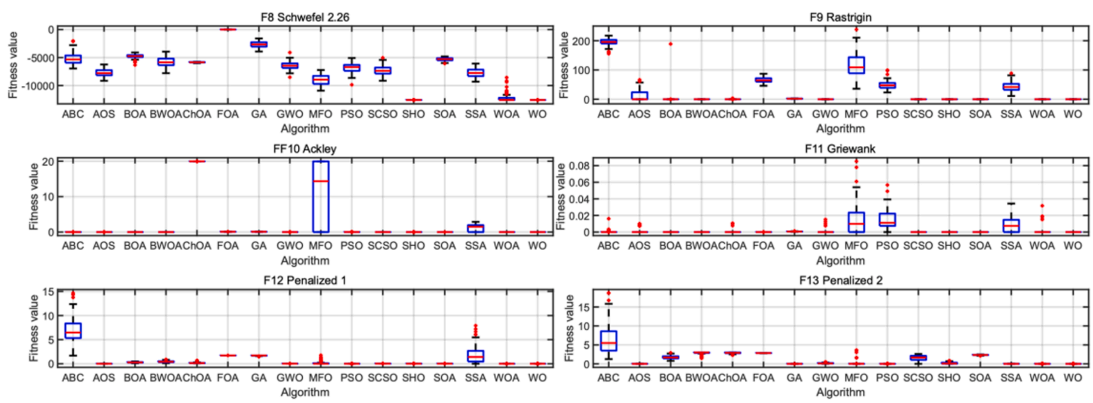
(q) F17 (r) F18 (s) F19 (t) F20

(u) F21 (v) F22 (w) F23

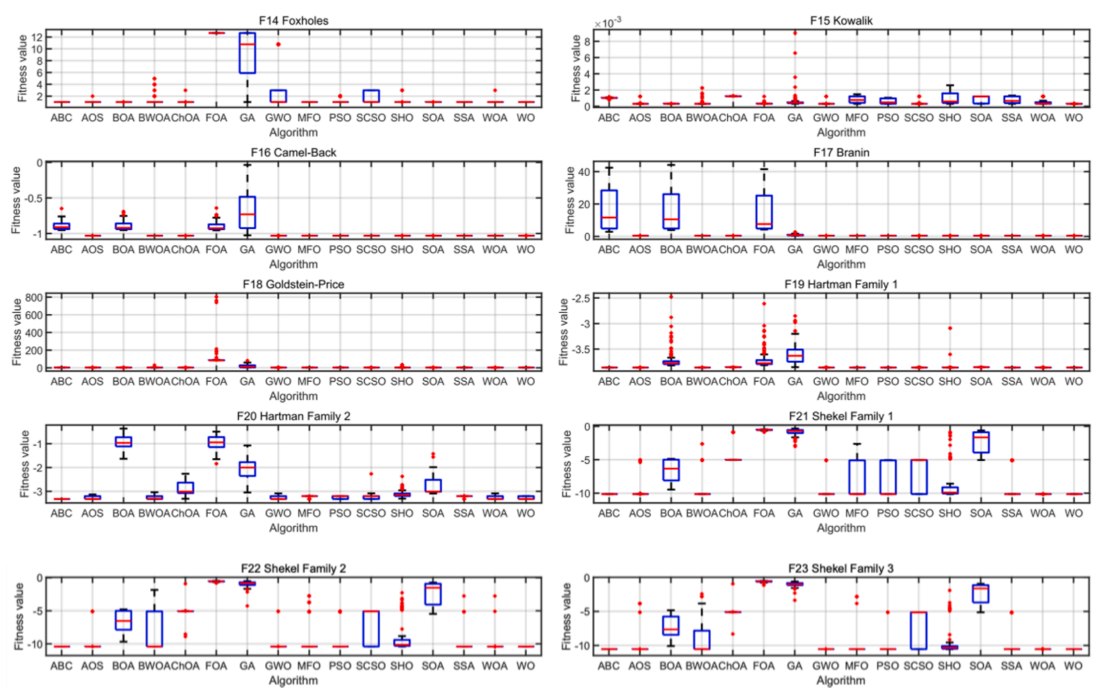
Fig. 5. Iteration diagram of benchmark functions.



(a) Results of unimodal benchmark functions

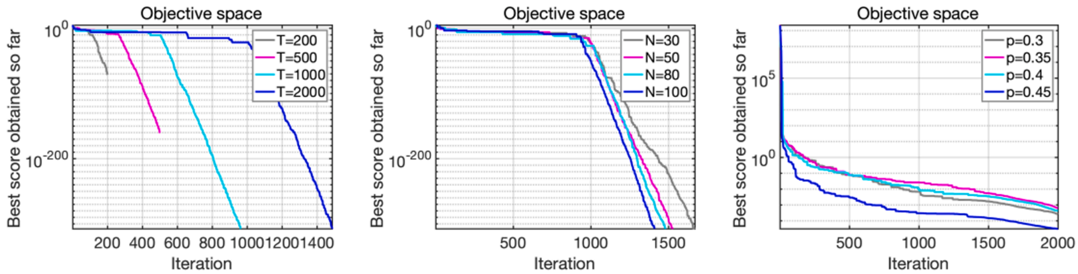


(b) Results of multimodal benchmark functions



(c) Results of fixed-dimension multimodal benchmark functions

Fig. 6. Boxplot of benchmark functions.



(a) T ($N = 100, p = 0.45$) (b) N ($T = 2000, p = 0.45$) (c) p ($T = 2000, N = 100$)

Fig. 7. Sensitivity analysis of WO.

4.2. CEC 2021 benchmark suite analysis

4.2.1. Experimental setup

A set of single objective real-parameter numerical optimizations from the latest Competitions on Evolutionary Computation 2021 (CEC 2021) was selected for testing, including 10 benchmark test functions and eight transformations of each function (A. W. Mohamed et al., 2023). CF1-CF10 is the Basic transformation functions of CEC 2021, and CF11-CF20 is the Bias transformation functions of CEC 2021. The details of the CEC 2021 are reported in Table 11. The maximum iterations and population size (search agents) are set as 500 and 100 respectively. Each test was run 100 times independently.

This section considers combining the proposed WO capabilities with fifteen algorithms in the optimization field include Ali Baba and the forty thieves (AFT) (M. Braik et al., 2022), Ant Lion Optimizer (ALO) (Mirjalili, 2015b), Chameleon Swarm Algorithm (CSA) (M. S. Braik, 2021), Dragonfly Algorithm (DA) (Mirjalili, 2016a), Dwarf Mongoose Optimization Algorithm (DMOA) (Agushaka et al., 2022), Grasshopper Optimization Algorithm (GOA) (Abualigah & Diabat, 2020), Moth-Flame Optimization Algorithm (MFO) (Mirjalili, 2015a), Multi-Verse Optimizer (MVO) (Mirjalili et al., 2015), Northern Goshawk Optimization (NGO) (Dehghani et al., 2021), Simulated Annealing (SA) (Kirkpatrick et al., 1983), Sine Cosine Algorithm (SCA) (Mirjalili, 2016b), Snake Optimizer (SO), Salp Swarm Algorithm (SS) (Mirjalili et al., 2017), Sparrow Search algorithm (SSA) (Xue & Shen, 2020), White Shark Optimizer (WSO) (M. Braik et al., 2022) were compared. The parameters of other compared algorithms are based on the recommendations in the literature, and the parameter settings are shown in Table 3.

4.2.2. Quantitative analysis

Table 12 reports the optimal fitness values achieved by the selected algorithms on the studied benchmarks suite. The P-values at the significance level $\alpha = 0.05$ by Wilcoxon rank-sum test are shown in Table 13. From the results, WO outperformed the other metaheuristic algorithms on 10 and 10 out of 10 Basic transformation functions and Bias transformation functions in CEC 2021 benchmarks suite, respectively. For comparison algorithms, NGO steadily found the global optimal in CF3, CF8, CF11, CF13, CF15 and CF18 tests, and tied for the first place with WO in these tests. DA can find the global optimal in tests other than CF20, but not every time, as reflected in the other three statistical indicators. Friedman test and Wilcoxon rank-sum test results show that the WO can take the first rank in solving CEC 2021.

4.2.3. Convergence analysis

The convergence curves of all algorithms are shown in Fig. 9. By examining these curves, the WO reflects the fastest convergence. NGO can complete convergence in CF2-CF4 and CF11-CF13 tests. However, most of the algorithms fall into local optima in the face of mixed and composite function optimization problems. Compared to the statistical results of WO most of the compared algorithms exhibit extreme instability. In general, WO can be considered as a reliable algorithm in terms of its convergence speed and accuracy.

4.2.4. Stability analysis

Stability is particularly important for the algorithms. As with the 23 benchmark function tests, test results were recorded and statistically analyzed for 100 independent runs of the CEC 2021 benchmark suite. On CF1 and CF11 (Unimodal function), most of the tested algorithms showed relatively stable performance, except for the DA algorithm, which showed a high number of singular values. For the remaining 18 complex functions, the test results of the comparison algorithms have a high degree of dispersion. In contrast, the proposed WO still maintains a high stability (see Fig. 10). When the local optimum is disturbed, the optimizer is still able to search for the global optimum in a stable manner. This phenomenon is consistent with the standard deviation statistics in Table 12.

4.3. Experimental results on 0–1 knapsack problems

The 0–1 knapsack problem (0–1 KP) is one of the most studied NP-hard combinatorial problems in the last few decades (Lim et al., 2016). It has practical applications in numerous areas including budget control, telecommunication, and resource allocation, to name but a few.

The classical 0–1 KP can be defined as: Given a set of M items with p_j and w_j representing the profit and weight of each item j, respectively; the goal is to choose a subset of the items such that its total profit is maximized without exceeding the knapsack capacity, C. The problem can be formulated as:

$$\text{Maximize} \sum_{j=1}^{M} p_j x_j \tag{22}$$

$$\text{Subject to} \sum_{j=1}^{M} w_j x_j \leq C,\ x_j \in \{0,1\},\ j = 1,2,\cdots,M \tag{23}$$

where, x_j is a binary decision variable with $x_j = 1$ if item j is included in the knapsack, 0 otherwise. Without loss of generality, it may be assumed that all weights and profits are positive, all weights are smaller than C, and the overall weight of items exceeds C.

The original data of the simulation example is shown in Table 14. The maximum iterations and population size (search agents) are set as 500 and 100 respectively.

The test results are shown in Table 15. Compared with GA and SA, both WO and BPSO find the optimal solution. WO takes an average of 0.224849 s for 10 runs, indicating that the algorithm finds the optimal solution from 2^{50} feasible solutions in less than 1 s. It shows the effectiveness of WO in dealing with NP-hard problems.

4.4. Experimental results on real-world engineering optimization problems

The proposed WO was also employed to solve 6 different engineering

Table 7
Results of benchmark functions (F1-F13) with$d = 50$.

Function		Comparative algorithms															
		ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
F1	Avg	1.83E-03	1.58E-242	1.84E-03	**0.00E+00**	3.00E-40	1.89E-03	1.07E+00	5.22E-128	6.50E+03	3.84E-01	**0.00E+00**	3.94E-281	4.89E-18	1.78E-08	**0.00E+00**	**0.00E+00**
	Std	2.22E-04	0.00E+00	2.27E-04	**0.00E+00**	1.68E-39	2.43E-04	1.60E-01	2.46E-127	8.33E+03	2.04E-01	**0.00E+00**	0.00E+00	3.28E-17	2.37E-09	**0.00E+00**	**0.00E+00**
F2	Avg	2.83E+00	9.92E-127	2.85E+00	**0.00E+00**	7.14E-02	2.83E+00	4.74E+00	2.32E-74	6.61E+01	1.09E+00	3.61E-250	7.26E-167	2.26E-18	7.81E-01	3.00E-222	**0.00E+00**
	Std	1.45E-01	4.06E-126	1.40E-01	**0.00E+00**	7.14E-01	1.36E-01	4.38E-01	3.89E-74	3.53E+01	4.04E-01	0.00E+00	0.00E+00	5.52E-18	8.14E-01	0.00E+00	**0.00E+00**
F3	Avg	1.33E+00	2.00E-182	1.34E+00	**0.00E+00**	6.66E-04	1.34E+00	6.21E+02	6.68E-29	3.74E+04	2.99E+02	**0.00E+00**	5.42E+04	9.27E+02	3.34E+01	1.33E+04	**0.00E+00**
	Std	1.43E-01	0.00E+00	1.46E-01	**0.00E+00**	6.64E-03	1.55E-01	7.49E+02	6.07E-28	2.43E+04	6.25E+01	**0.00E+00**	3.13E+04	1.64E+03	2.67E+01	9.38E+03	**0.00E+00**
F4	Avg	1.01E-02	8.45E-118	1.01E-02	**0.00E+00**	1.18E-08	1.02E-02	4.96E-01	5.01E-29	6.59E+01	2.25E+00	3.25E-206	2.25E-29	5.14E-01	5.65E+00	3.02E+01	**0.00E+00**
	Std	7.71E-04	4.54E-117	8.18E-04	**0.00E+00**	5.46E-08	7.59E-04	3.00E-02	8.76E-29	6.76E+00	2.23E-01	0.00E+00	2.25E-28	2.03E+00	1.94E+00	3.16E+01	**0.00E+00**
F5	Avg	4.90E+01	4.53E+01	4.90E+01	4.89E+01	4.88E+01	4.90E+01	1.63E+02	4.62E+01	3.22E+06	5.65E+02	4.70E+01	1.16E+01	4.82E+01	8.96E+01	4.44E+01	**1.30E-03**
	Std	1.97E-01	4.57E-01	1.71E-01	4.14E-02	2.00E-01	1.64E-01	2.15E+01	8.60E-01	1.58E+07	4.29E+02	1.07E+00	2.03E+01	5.91E-01	9.69E+01	6.38E+00	**3.20E-03**
F6	Avg	1.28E+01	2.94E-03	1.28E+01	9.90E+00	5.58E+00	1.28E+01	1.86E+01	1.03E+00	6.40E+03	3.84E-01	1.39E+00	4.18E-01	2.77E+00	9.41E-07	4.01E-04	**1.73E-08**
	Std	1.45E-02	9.28E-04	1.45E-02	1.12E+00	3.97E-01	1.42E-02	6.70E-01	4.35E-01	8.70E+03	2.11E-01	5.07E-01	6.10E-01	5.76E-01	1.32E-06	1.30E-04	**2.32E-09**
F7	Avg	8.61E+01	1.69E-04	8.99E+01	2.42E-05	1.96E-04	9.02E+01	3.81E+00	1.86E-04	1.38E+01	5.10E+01	1.21E-05	2.45E-05	4.88E-02	4.35E-02	2.81E-04	**1.16E-05**
	Std	2.41E+01	8.63E-05	2.34E+01	2.44E-05	1.50E-04	2.87E+01	7.90E-01	1.19E-04	2.11E+01	5.55E+01	1.44E-05	4.44E-05	1.90E-02	1.28E-02	3.28E-04	**9.45E-06**
F8	Avg	-3.58E+00	-1.20E+04	-2.81E+00	-7.41E+03	-9.41E+03	-3.44E+00	-3.37E+03	-9.77E+03	-1.37E+04	-1.11E+04	-1.16E+04	-2.10E+04	-8.21E+03	-9.55E+04	-2.03E+04	**-2.10E+04**
	Std	5.94E+00	8.64E+02	8.40E-01	9.04E+02	6.99E+01	6.03E+00	6.62E+02	9.75E+02	1.60E+03	1.23E+03	1.01E+03	1.19E+01	4.29E+02	1.70E+04	1.32E+03	**1.60E-02**
F9	Avg	1.24E+02	5.80E+00	1.25E+02	**0.00E+00**	2.19E-01	1.25E+02	6.44E+01	5.68E-16	2.50E+02	1.55E+02	**0.00E+00**	**0.00E+00**	**0.00E+00**	6.50E+01	**0.00E+00**	**0.00E+00**
	Std	1.28E+01	1.83E+01	1.19E+01	**0.00E+00**	1.33E+00	1.30E+01	5.96E+00	5.68E-15	5.59E+01	3.26E+01	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.18E+01	**0.00E+00**	**0.00E+00**
F10	Avg	9.45E-02	4.41E-15	9.50E-02	**8.88E-16**	2.00E+01	9.40E-02	1.09E+00	1.42E-14	1.78E+01	1.06E+00	**8.88E-16**	1.03E-15	8.10E-11	2.05E+00	3.27E-15	**8.88E-16**
	Std	6.27E-03	3.55E-16	6.38E-03	**0.00E+00**	9.73E-04	6.14E-03	1.25E-01	2.04E-15	5.40E+00	5.62E-01	**0.00E+00**	7.00E-16	2.95E-10	6.59E-01	2.43E-15	**0.00E+00**
F11	Avg	1.93E-06	7.68E-04	1.92E-06	**0.00E+00**	1.44E-03	1.94E-06	2.62E-02	2.27E-04	4.97E+01	1.42E-02	**0.00E+00**	**0.00E+00**	1.50E-16	4.24E-03	4.71E-04	**0.00E+00**
	Std	3.34E-07	3.28E-03	2.87E-07	**0.00E+00**	6.06E-03	3.56E-07	3.82E-03	1.61E-03	6.47E+01	7.59E-03	**0.00E+00**	**0.00E+00**	9.16E-16	6.17E-03	2.69E-03	**0.00E+00**
F12	Avg	1.52E+00	2.74E-03	1.52E+00	6.23E-01	3.81E-01	1.52E+00	1.90E+00	4.23E-02	1.28E+07	9.07E-03	4.24E-02	1.39E-02	1.53E-01	4.11E+00	2.19E-05	**4.67E-09**
	Std	3.05E-03	1.25E-02	2.75E-03	1.60E-01	1.21E-01	3.01E-03	5.35E-02	3.36E-02	5.61E+07	2.03E-02	1.79E-02	1.59E-02	5.42E-02	2.05E+00	7.68E-06	**5.04E-09**
F13	Avg	4.81E+00	5.29E-02	4.81E+00	4.98E+00	4.86E+00	4.81E+00	1.58E-01	7.89E-01	1.64E+07	1.35E-01	4.23E+00	3.43E-01	4.68E+00	7.42E-01	3.84E-03	**2.27E-07**
	Std	2.00E-02	1.53E-01	1.72E-02	1.27E-01	1.26E-01	1.69E-02	3.10E-02	2.64E-01	8.08E+07	6.29E-02	3.83E-01	4.22E-01	8.46E-02	5.66E+00	5.36E-03	**2.93E-07**
(W|T|L)	(0|13|0)	(0|13|0)	(0|12|1)	(7|6|0)	(0|12|1)	(0|12|1)	(0|13|0)	(0|13|0)	(0|4|9)	(0|13|0)	(5|8|0)	(2|10|1)	(0|13|0)	(0|13|0)	(2|11|0)	(13|0|0)	
Friedman mean rank	11.15	5.46	11.69	5.65	9.54	11.77	12.54	6.85	14.69	10.69	4.42	5.12	8.73	9.85	5.85	2.00	
Final rank	12	4	13	5	9	14	15	7	16	11	2	3	8	10	6	1	

Table 8
Results of benchmark functions (F1-F13) with$d$ $= 100$.

.Function		Comparative algorithms															
		ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
F1	Avg	4.53E-03	7.97E-232	4.49E-03	**0.00E+00**	4.04E-24	4.47E-03	2.09E+01	2.43E-86	1.75E+04	2.50E+01	**0.00E+00**	1.76E-247	3.28E+02	1.07E-07	**0.00E+00**	**0.00E+00**
	Std	3.48E-04	0.00E+00	3.83E-04	**0.00E+00**	2.68E-23	4.01E-04	6.44E-01	3.16E-86	1.24E+04	5.72E+00	**0.00E+00**	0.00E+00	7.06E+02	1.35E-08	**0.00E+00**	**0.00E+00**
F2	Avg	6.20E+00	1.54E-122	6.15E+00	**0.00E+00**	3.08E-17	6.17E+00	3.75E+01	6.58E-51	1.42E+02	3.17E+01	2.39E-241	4.86E-152	6.11E-04	3.94E+00	5.82E-224	**0.00E+00**
	Std	2.22E-01	6.22E-122	2.63E-01	**0.00E+00**	5.07E-17	2.61E-01	7.20E-01	5.15E-51	4.99E+01	7.58E+00	0.00E+00	4.86E-151	7.26E-04	1.80E+00	0.00E+00	**0.00E+00**
F3	Avg	1.22E+01	2.44E-173	1.25E+01	**0.00E+00**	9.82E+01	1.25E+01	8.78E+03	2.07E-10	1.33E+05	5.90E+03	**0.00E+00**	5.55E+05	1.10E+05	4.39E+03	2.71E+05	**0.00E+00**
	Std	9.97E-01	0.00E+00	1.06E+00	**0.00E+00**	3.62E+02	9.92E-01	2.26E+03	1.15E-09	5.28E+04	1.14E+03	**0.00E+00**	1.28E+05	3.01E+04	3.17E+03	7.34E+04	**0.00E+00**
F4	Avg	1.35E-02	1.08E-113	1.35E-02	**0.00E+00**	1.50E+01	1.34E-02	8.86E-01	2.13E-13	8.91E+01	5.50E+00	3.30E-206	9.94E-06	2.83E+01	1.35E+01	5.92E+01	**0.00E+00**
	Std	9.58E-04	5.56E-113	1.05E-03	**0.00E+00**	2.79E+01	8.60E-04	1.06E-02	1.39E-12	2.74E+00	5.69E-01	0.00E+00	9.94E-05	8.95E+00	1.76E+00	3.34E+01	**0.00E+00**
F5	Avg	9.96E+01	9.59E+01	9.96E+01	9.89E+01	9.88E+01	9.97E+01	1.20E+03	9.65E+01	4.40E+07	1.77E+04	9.76E+01	1.02E+01	2.39E+07	1.79E+02	9.45E+01	**4.69E-03**
	Std	2.21E-01	7.90E-01	2.51E-01	4.31E-02	1.42E-01	2.72E-01	3.30E+01	9.49E-01	6.04E+07	5.34E+03	9.34E-01	2.94E+01	1.61E+07	1.96E+02	9.38E+00	**1.07E-02**
F6	Avg	2.56E+01	1.64E-01	2.56E+01	2.22E+01	1.62E+01	2.56E+01	8.37E+01	6.17E+00	1.73E+04	2.47E+01	6.18E+00	6.89E-01	3.10E+02	5.16E-05	1.18E-02	**1.05E-07**
	Std	2.50E-02	6.34E-02	2.51E-02	1.25E+00	6.51E-01	2.31E-02	1.10E+00	8.06E-01	1.22E+04	7.02E+00	9.87E-01	1.17E+00	4.67E+02	7.35E-05	2.68E-03	**1.28E-08**
F7	Avg	6.08E+02	2.34E-04	6.07E+02	2.48E-05	3.51E-04	6.29E+02	4.30E+02	3.25E-04	9.66E+01	1.22E+03	1.17E-05	1.91E-05	2.08E+01	1.94E-01	2.44E-04	**1.43E-05**
	Std	1.33E+02	1.37E-04	1.45E+02	2.44E-05	2.36E-04	1.37E+02	2.20E+01	1.79E-04	8.94E+01	2.74E+02	1.43E-05	2.23E-05	1.69E+01	4.44E-02	2.43E-04	**1.13E-05**
F8	Avg	-3.59E+00	-1.96E+04	-3.45E+00	-1.09E+04	-1.83E+04	-4.31E+00	-4.82E+03	-1.70E+04	-2.49E+04	-2.13E+04	-2.17E+04	-4.19E+04	-1.46E+04	-1.75E+05	-4.09E+04	**-4.19E+04**
	Std	2.05E+00	1.58E+03	5.52E-01	1.58E+03	8.63E+01	6.61E+00	8.55E+02	2.19E+03	2.90E+03	2.22E+03	1.68E+03	1.78E+02	7.54E+02	2.77E+04	1.81E+03	**3.13E-01**
F9	Avg	2.97E+02	1.25E+00	2.92E+02	**0.00E+00**	7.43E-02	2.94E+02	7.28E+02	2.27E-15	5.80E+02	6.87E+02	**0.00E+00**	**0.00E+00**	1.30E+02	1.17E+02	2.27E-15	**0.00E+00**
	Std	1.86E+01	1.25E+01	2.07E+01	**0.00E+00**	7.43E-01	1.86E+01	1.78E+01	1.60E-14	8.40E+01	1.10E+02	**0.00E+00**	**0.00E+00**	5.19E+01	3.58E+01	1.60E-14	**0.00E+00**
F10	Avg	1.07E-01	4.55E-15	1.07E-01	**8.88E-16**	2.00E+01	1.06E-01	3.38E+00	2.43E-14	1.98E+01	3.71E+00	**8.88E-16**	1.17E-15	1.30E+00	3.20E+00	4.12E-15	**8.88E-16**
	Std	6.11E-03	7.91E-16	5.38E-03	**0.00E+00**	3.50E-04	5.65E-03	3.57E-02	3.39E-15	3.27E-01	3.10E-01	**0.00E+00**	1.09E-15	1.54E+00	6.03E-01	2.43E-15	**0.00E+00**
F11	Avg	2.62E-06	9.87E-05	2.63E-06	**0.00E+00**	2.13E-03	2.63E-06	2.95E-01	2.06E-04	2.12E+02	2.97E-01	**0.00E+00**	**0.00E+00**	3.35E+00	3.45E-03	**0.00E+00**	**0.00E+00**
	Std	3.36E-07	9.87E-04	3.22E-07	**0.00E+00**	7.81E-03	3.65E-07	1.23E-02	1.45E-03	1.30E+02	6.71E-02	**0.00E+00**	**0.00E+00**	4.26E+00	5.39E-03	**0.00E+00**	**0.00E+00**
F12	Avg	1.38E+00	1.23E-02	1.38E+00	8.62E-01	6.43E-01	1.38E+00	3.46E+00	1.23E-01	4.61E+07	6.01E-01	9.18E-02	1.60E-02	6.91E+07	6.57E+00	1.56E-04	**6.04E-08**
	Std	1.98E-03	1.18E-02	2.27E-03	1.13E-01	7.83E-02	2.10E-03	5.73E-02	3.43E-02	1.05E+08	3.65E-01	2.22E-02	3.14E-02	5.01E+07	1.54E+00	4.11E-05	**7.43E-08**
F13	Avg	9.77E+00	6.16E+00	9.77E+00	9.99E+00	9.66E+00	9.77E+00	3.12E+00	4.52E+00	1.60E+08	1.56E+01	9.45E+00	7.00E-01	1.25E+08	1.15E+02	3.10E-02	**4.32E-06**
	Std	1.91E-02	2.78E+00	2.07E-02	4.92E-03	1.46E-01	2.33E-02	9.10E-02	4.47E-01	2.60E+08	6.30E+00	1.54E-01	9.82E-01	8.38E+07	2.64E+01	2.56E-02	**6.47E-06**
(W|T|L)	(0|13|0)	(0|13|0)	(0|12|1)	(7|6|0)	(0|12|1)	(0|13|0)	(0|12|1)	(0|13|0)	(0|6|7)	(0|12|1)	(5|8|0)	(2|10|1)	(0|12|1)	(0|13|0)	(2|11|0)	**(13|0|0)**	
Friedman mean rank	10.73	5.62	10.58	5.31	9.23	10.54	12.39	6.58	14.31	12.31	3.85	4.69	13.00	9.69	5.39	**1.81**	
Final rank	12	6	11	4	8	10	14	7	16	13	2	3	15	9	5	**1**	

Table 9
Results of benchmark functions (F1-F13) with $d = 500$.

Function		Comparative algorithms															
		ABC	AOS	BOA	BWOA	ChOA	FOA	GA	GWO	MFO	PSO	SCSO	SHO	SOA	SSA	WOA	WO
F1	Avg	3.43E-02	3.16E-225	3.42E-02	**0.00E+00**	8.50E-09	3.41E-02	1.36E+02	4.44E-38	6.24E+05	3.68E+03	**0.00E+00**	4.78E-262	4.90E+05	1.50E+03	**0.00E+00**	**0.00E+00**
	Std	1.89E-03	0.00E+00	1.51E-03	**0.00E+00**	1.12E-08	1.86E-03	1.46E+00	3.30E-38	3.26E+04	2.37E+02	**0.00E+00**	0.00E+00	3.75E+04	2.08E+02	**0.00E+00**	**0.00E+00**
F2	Avg	3.55E+01	4.95E-118	3.55E+01	**0.00E+00**	1.21E-06	3.54E+01	2.20E+02	5.14E-23	1.63E+03	4.63E+107	1.07E-228	1.13E-148	4.82E+02	1.43E+02	9.00E-221	**0.00E+00**
	Std	7.61E-01	1.33E-117	7.20E-01	**0.00E+00**	6.58E-07	6.64E-01	1.12E+00	1.66E-23	7.43E+01	4.59E+108	0.00E+00	1.13E-147	5.24E+01	9.70E+00	0.00E+00	**0.00E+00**
F3	Avg	2.06E+03	1.83E-158	2.06E+03	**0.00E+00**	9.39E+05	2.06E+03	2.66E+05	5.53E+03	2.47E+06	3.10E+05	**0.00E+00**	2.24E+07	2.82E+07	2.61E+05	2.13E+07	**0.00E+00**
	Std	9.25E+01	1.41E-157	9.47E+01	**0.00E+00**	5.43E+05	8.40E+01	5.08E+04	6.08E+03	5.18E+05	6.45E+04	**0.00E+00**	6.08E+06	6.64E+06	1.51E+05	4.35E+06	**0.00E+00**
F4	Avg	2.83E-02	1.16E-108	2.82E-02	**0.00E+00**	9.74E+01	2.85E-02	9.72E-01	2.89E+01	9.85E+01	2.38E+01	1.40E-201	8.30E-34	7.80E+01	2.37E+01	7.43E+01	**0.00E+00**
	Std	1.72E-03	6.22E-108	1.48E-03	**0.00E+00**	1.15E+00	1.51E-03	2.91E-03	8.08E+00	3.44E-01	1.08E+00	0.00E+00	8.30E-33	1.74E+01	1.55E+00	2.78E+01	**0.00E+00**
F5	Avg	5.12E+02	4.97E+02	5.12E+02	4.99E+02	4.99E+02	5.12E+02	7.98E+03	4.97E+02	2.32E+09	1.64E+07	4.98E+02	1.37E+01	3.56E+09	8.21E+04	4.94E+02	**5.12E-02**
	Std	1.36E+00	4.28E-01	1.26E+00	4.18E-02	1.87E-01	1.38E+00	1.06E+02	3.84E-01	2.10E+08	2.14E+06	3.66E-01	6.95E+01	2.59E+08	1.87E+04	3.81E-01	**1.64E-01**
F6	Avg	1.29E+02	3.74E+01	1.29E+02	1.22E+02	1.12E+02	1.29E+02	4.81E+02	8.43E+01	6.27E+05	3.71E+03	7.64E+01	5.74E+00	4.89E+05	1.55E+03	1.49E+00	**2.03E-03**
	Std	7.84E-02	2.11E+00	6.33E-02	1.35E+00	1.03E+00	7.01E-02	2.63E+00	2.21E+00	3.47E+04	2.74E+02	4.02E+00	9.41E+00	4.29E+04	2.15E+02	2.33E-01	**4.14E-03**
F7	Avg	6.44E+04	2.46E-04	6.36E+04	1.21E-05	2.05E-03	6.14E+04	1.73E+04	1.18E-03	1.74E+04	5.46E+04	2.64E-05	1.46E-05	2.51E+04	7.57E+00	3.58E-04	**2.19E-05**
	Std	1.00E+04	1.60E-04	8.71E+03	1.14E-05	1.67E-03	9.28E+03	2.76E+02	3.96E-04	1.91E+03	2.09E+03	3.45E-05	1.39E-05	1.84E+03	7.75E-01	4.22E-04	**2.87E-05**
F8	Avg	-4.98E+00	-5.03E+04	-4.61E+00	-2.58E+04	-8.66E+04	-4.71E+00	-1.33E+04	-6.63E+04	-8.88E+04	-1.05E+05	-9.02E+04	-2.09E+05	-5.49E+04	-8.00E+04	-2.06E+05	**-2.10E+05**
	Std	7.41E+00	6.97E+03	4.40E+00	3.73E+03	5.27E+02	4.81E+00	2.22E+03	1.14E+04	7.96E+03	1.05E+04	5.27E+03	2.79E+02	2.38E+03	1.05E+04	6.68E+03	**2.65E+01**
F9	Avg	1.83E+03	**0.00E+00**	1.84E+03	**0.00E+00**	2.16E-02	1.85E+03	4.42E+03	1.22E-12	5.37E+03	7.18E+03	**0.00E+00**	**0.00E+00**	2.23E+03	6.57E+02	**0.00E+00**	**0.00E+00**
	Std	4.74E+01	**0.00E+00**	6.27E+01	**0.00E+00**	2.15E-01	5.54E+01	3.62E+01	5.52E-13	1.52E+02	4.09E+02	**0.00E+00**	**0.00E+00**	2.57E+02	9.29E+01	**0.00E+00**	**0.00E+00**
F10	Avg	1.37E-01	4.55E-15	1.38E-01	**8.88E-16**	2.00E+01	1.38E-01	3.67E+00	9.16E-14	2.00E+01	1.06E+01	**8.88E-16**	1.10E-15	1.82E+01	9.75E+00	3.48E-15	**8.88E-16**
	Std	4.58E-03	6.09E-16	3.83E-03	**0.00E+00**	8.25E-03	4.13E-03	1.16E-02	8.02E-15	2.44E-02	2.32E-01	**0.00E+00**	9.87E-16	3.43E-01	4.11E-01	2.57E-15	**0.00E+00**
F11	Avg	4.99E-06	**0.00E+00**	4.93E-06	**0.00E+00**	2.27E-03	4.96E-06	4.78E-01	4.78E-04	5.65E+03	1.95E+00	**0.00E+00**	**0.00E+00**	4.43E+03	1.49E+01	**0.00E+00**	**0.00E+00**
	Std	4.33E-07	**0.00E+00**	4.02E-07	**0.00E+00**	1.27E-02	4.08E-07	9.45E-03	2.76E-03	3.07E+02	6.12E-02	**0.00E+00**	**0.00E+00**	3.65E+02	1.81E+00	**0.00E+00**	**0.00E+00**
F12	Avg	1.26E+00	1.37E-01	1.26E+00	1.12E+00	1.04E+00	1.26E+00	3.78E+00	6.23E-01	5.13E+09	5.67E+04	3.63E-01	1.69E-02	1.07E+10	1.52E+01	2.13E-03	**3.26E-07**
	Std	1.21E-03	1.23E-02	1.19E-03	3.71E-02	2.42E-02	1.15E-03	2.50E-02	2.83E-02	6.28E+08	2.53E+04	3.46E-02	2.93E-02	6.81E+08	3.21E+00	3.22E-04	**6.29E-07**
F13	Avg	4.96E+01	4.98E+01	4.96E+01	5.00E+01	4.89E+01	4.96E+01	2.02E+01	4.38E+01	9.87E+09	1.12E+06	4.96E+01	3.15E+00	1.73E+10	9.15E+02	8.63E-01	**1.32E-04**
	Std	3.46E-02	1.94E-02	3.91E-02	4.96E-03	2.15E-01	4.13E-02	2.23E-01	6.15E-01	1.12E+09	3.10E+05	8.38E-02	5.02E+00	1.31E+09	4.32E+01	2.13E-01	**3.02E-04**
(W|T|L)	(0|12|1)	(0|11|2)	(0|12|1)	(7|6|0)	(0|12|1)	(0|13|0)	(0|13|0)	(0|13|0)	(0|9|4)	(0|11|2)	(3|10|0)	(2|11|0)	(0|9|4	(0|13|0)	(3|10|0)	(13|0|0)	
Friedman mean rank	10.12	5.54	10.04	5.04	9.31	9.65	11.08	7.39	14.23	12.92	4.15	4.39	14.31	10.85	4.96	2.04	
Final rank	11	6	10	5	8	9	13	7	15	14	2	3	16	12	4	1	

Table 10
Average fitness values of WO using different parameters.

Parameter	Value	F5	F6	F12	F13	F14	F15
T	200	2.25E-04	3.32E-08	2.98E-10	1.23E-08	9.98E-01	3.08E-04
	500	8.82E-04	5.91E-08	5.97E-10	2.01E-08	9.98E-01	3.08E-04
	1000	1.62E-04	6.56E-08	1.27E-09	6.82E-09	9.98E-01	3.08E-04
	2000	1.48E-05	2.14E-08	2.05E-10	1.97E-10	9.98E-01	3.07E-04
N	30	8.42E-03	1.45E-05	1.12E-07	1.35E-06	9.98E-01	3.14E-04
	50	6.64E-04	5.24E-07	1.73E-09	1.03E-07	9.98E-01	3.09E-04
	80	4.89E-04	1.58E-07	4.37E-10	1.53E-08	9.98E-01	3.08E-04
	100	2.17E-05	2.83E-08	2.78E-10	1.87E-10	9.98E-01	3.07E-04
p	0.3	1.40E-02	6.10E-04	3.51E-06	1.66E-05	9.98E-01	3.82E-04
	0.35	9.17E-04	3.19E-05	1.12E-07	1.35E-07	9.98E-01	3.12E-04
	0.4	2.86E-04	5.58E-07	5.00E-09	1.56E-08	9.98E-01	3.08E-04
	0.45	1.73E-05	2.83E-08	3.14E-10	4.11E-10	9.98E-01	3.07E-04

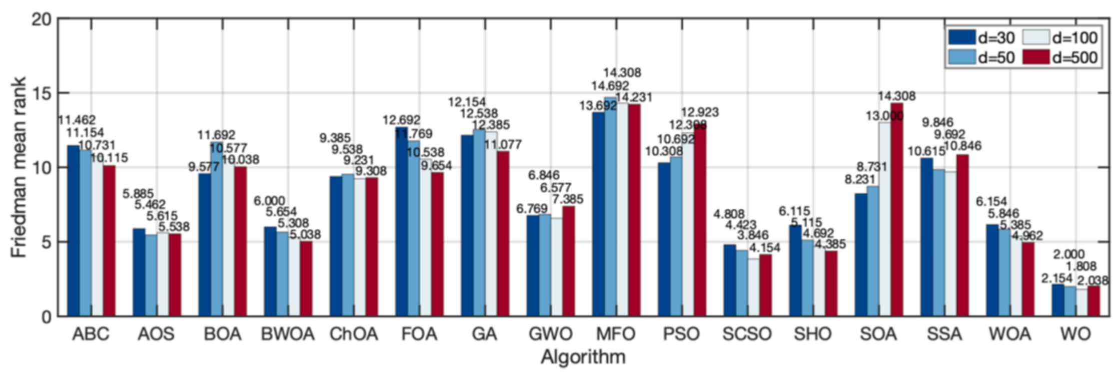


Fig. 8. Friedman mean rank results for different dimensions.

Table 11
Summary of the CEC 2021 benchmark suite.

Type	No.	Functions	f_{min} Basic	Bias
Unimodal function	CF1	CEC 2017 F1 (Awad et al., 2017)	0	100
Basic functions	CF2	CEC 2014 F11 (Liang et al., 2014)	0	1100
	CF3	CEC 2017 F7	0	700
	CF4	CEC 2017 F19	0	1900
Hybrid functions	CF5	CEC 2014 F17	0	1700
	CF6	CEC 2017 F16	0	1600
	CF7	CEC 2014 F21	0	2100
Composition functions	CF8	CEC 2017 F22	0	2200
	CF9	CEC 2017 F24	0	2400
	CF10	CEC 2017 F25	0	2500
# For all functions: search range is $[100, 100]^d$				

design problems and the results are presented in this section (Gandomi, 2014). The engineering optimization problems used in this study find the optimal solutions under special conditions, such as design principles, resource limitations, and safety requirements (Han et al., 2022). Since the problems of this section have different constraints, we need to employ a constraint handling method. There are different types of penalty functions in the literature (Coello Coello, 2002): static, dynamic, annealing, adaptive, co-evolutionary, and death penalty. The last penalty function, death penalty, is the simplest method that assigns a large objective value to the fitness function (in the case of minimization). This approach causes the algorithm to discard solutions that are much larger than the normal range during the optimization process. For the sake of simplicity, this section equips the WO with a death penalty function to handle constraints. The maximum iterations and population size (search agents) are set as 500 and 100 respectively.

4.4.1. Case 1: Gear train design

The gear train design problem refers to the cost optimization of the gear ratio of a compound gear train (see Fig. 11(a)). The current gear design problem has four gears and the error between a required gear ratio (1/6.931) and an obtained gear ratio are shown to be minimized. Therefore, the objective function is considered a discrete problem with error reduction. The problem variables are the teeth for each gearwheel. The design engineering constraint is defined as the number of teeth on any gear that should only be in the range of $[12, 60]$. To handle discrete variables, each search agent was rounded to the nearest integer number before the fitness evaluation (Mirjalili, 2015b; Peraza-Vázquez et al., 2021). The mathematical model of pressure vessel design is as follows:

Suppose $x = [x_1x_2x_3x_4] = [G_AG_BG_CG_D]$

$$\text{Minimize} f(x) = \left(\frac{1}{6.931} - \frac{x_3x_2}{x_1x_4}\right)^2 \quad (24)$$

where $x_{1,2,3,4} \in [12, 60]$ are integer design variables.

Table 16 shows the values of the four design variables and the optimal cost. From the table, it can be observed that the WO provides better weight than other methods and is suitable to solve discrete constrained problems, followed by Cuckoo Search (CS).

4.4.2. Case 2: Cantilever beam design

The design of cantilever beam is a type of concrete engineering optimization problems (see Fig. 11(b)). The weight of the cantilever beam is minimized by optimizing five hollow square section parameters. The mathematical model of cantilever beam design is as follows:

Suppose $x = [x_1x_2x_3x_4x_5]$

$$\text{Minimize } f(x) = 0.06224(x_1 + x_2 + x_3 + x_4 + x_5) \quad (25)$$

$$\text{Subject to } g(x) = \frac{60}{x_1^3} + \frac{27}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \leq 0 \quad (26)$$

Table 12
Results of the comparative methods on CEC 2021 benchmark suite.

Function		AFT	ALO	CSA	DA	DMOA	GOA	MFO	MVO	NGO	SA	SCA	SO	SS	SSA	WSO	WO
CF1	Best	1.57E-31	1.60E-02	2.81E-02	**0.00E+00**	4.89E-19	3.41E-01	1.03E-11	1.23E+03	8.09E-99	6.06E-18	2.08E-20	5.80E-125	1.40E-02	2.17E-01	4.89E-11	**3.39E-279**
	Worst	4.36E-27	5.94E+03	3.02E+03	1.36E+08	7.79E-17	9.84E+03	1.00E+04	1.62E+04	2.48E-95	1.40E-06	1.31E-10	1.01E-116	9.92E+03	4.71E+03	3.87E-02	**8.68E-212**
	Avg	2.21E-28	9.26E+02	3.93E+02	4.89E+06	1.33E-17	2.01E+03	3.50E+03	6.84E+03	1.65E-96	1.41E-08	2.85E-12	3.73E-118	6.84E+02	6.97E+02	7.27E-04	**8.69E-214**
	Std	5.08E-28	1.10E+03	5.54E+02	1.70E+07	1.25E-17	2.58E+03	4.79E+03	3.65E+03	3.00E-96	1.40E-07	1.37E-11	1.55E-117	1.26E+03	9.64E+02	4.14E-03	**0.00E+00**
CF2	Best	3.75E-01	3.67E+00	1.25E-01	**0.00E+00**	7.61E+02	3.79E+00	6.83E+00	3.68E+00	**0.00E+00**	6.25E-02	**0.00E+00**	**0.00E+00**	5.40E-09	1.25E-01	6.29E-02	**0.00E+00**
	Worst	8.37E+02	1.37E+03	7.41E+02	1.56E+03	1.38E+03	1.68E+03	1.52E+03	1.30E+03	9.40E-10	1.37E+01	9.40E+02	2.64E+01	1.28E+03	9.50E+02	2.79E+02	**0.00E+00**
	Avg	2.76E+02	7.46E+02	1.40E+02	5.34E+02	1.10E+03	9.10E+02	4.70E+02	5.05E+02	1.19E-11	5.24E+00	1.90E+01	4.87E+00	4.23E+02	4.08E+02	3.90E+01	**0.00E+00**
	Std	1.93E+02	3.00E+02	1.77E+02	4.34E+02	1.45E+02	3.44E+02	3.53E+02	2.79E+02	9.70E-11	3.65E+00	1.01E+02	5.49E+00	2.72E+02	2.81E+02	7.14E+01	**0.00E+00**
CF3	Best	4.98E+00	8.96E+00	3.98E+00	**0.00E+00**	1.63E+01	9.95E+00	4.98E+00	7.99E+00	**0.00E+00**	3.65E-17	6.66E-21	7.89E-31	3.98E+00	4.98E+00	3.84E-01	**0.00E+00**
	Worst	4.80E+01	8.10E+01	2.93E+01	1.98E+02	3.82E+01	6.03E+01	4.15E+01	4.22E+01	**0.00E+00**	1.30E+01	9.68E+01	1.61E+01	5.77E+01	5.67E+01	4.06E+01	**0.00E+00**
	Avg	2.17E+01	3.12E+01	1.56E+01	3.05E+01	3.08E+01	2.61E+01	2.25E+01	2.51E+01	**0.00E+00**	1.07E+01	1.12E+01	7.79E+00	2.42E+01	2.33E+01	2.16E+01	**0.00E+00**
	Std	8.02E+00	1.16E+01	4.65E+00	2.48E+01	3.74E+00	1.13E+01	6.84E+00	6.95E+00	**0.00E+00**	2.18E+00	2.81E+01	5.15E+00	9.98E+00	1.10E+01	7.57E+00	**0.00E+00**
CF4	Best	2.51E-01	3.03E-01	3.30E-01	**0.00E+00**	1.12E+00	3.59E-01	2.17E-01	2.62E-01	**0.00E+00**	6.52E-02	**0.00E+00**	**0.00E+00**	3.20E-01	3.06E-01	8.22E-01	**0.00E+00**
	Worst	2.49E+00	3.80E+00	2.38E+00	1.02E+01	2.93E+00	3.63E+00	3.03E+00	3.48E+00	8.54E-02	1.75E+00	5.50E+00	1.28E-01	5.03E+00	3.86E+00	2.45E+00	**0.00E+00**
	Avg	9.57E-01	1.29E+00	9.60E-01	1.99E+00	2.12E+00	1.53E+00	1.10E+00	1.32E+00	1.93E-03	6.61E-01	5.18E-01	3.50E-03	1.47E+00	1.43E+00	1.83E+00	**0.00E+00**
	Std	3.97E-01	6.31E-01	4.13E-01	1.68E+00	3.56E-01	6.79E-01	5.64E-01	5.57E-01	1.12E-02	3.05E-01	1.20E+00	1.96E-02	8.15E-01	7.24E-01	3.19E-01	**0.00E+00**
CF5	Best	4.16E-01	3.36E+02	7.43E+00	**0.00E+00**	5.43E+00	6.41E+02	1.20E+00	1.63E+02	9.19E-22	8.49E-20	4.81E-19	2.50E-117	2.72E+02	2.92E+02	2.72E-12	**3.10E-295**
	Worst	4.93E+02	1.13E+04	5.00E+02	1.33E+05	4.52E+01	1.35E+04	1.71E+03	1.38E+03	1.34E-19	2.26E+01	2.61E+01	2.85E+02	6.57E+03	1.09E+04	6.76E+00	**4.78E-243**
	Avg	1.14E+02	3.96E+03	1.22E+02	5.29E+03	1.65E+01	5.37E+03	1.32E+02	6.58E+02	1.65E-20	2.49E+00	4.78E-01	4.03E+01	2.12E+03	2.07E+03	1.29E+00	**4.79E-245**
	Std	1.07E+02	2.83E+03	1.27E+02	1.43E+04	7.54E+00	3.54E+03	2.41E+02	2.78E+02	1.66E-20	4.62E+00	3.11E+00	6.68E+01	1.46E+03	1.73E+03	1.12E+00	**0.00E+00**
CF6	Best	2.38E-01	2.29E+00	8.38E-01	**0.00E+00**	1.25E+00	2.27E+00	3.63E-01	1.53E+00	1.46E-03	2.29E-02	2.17E-04	1.13E-14	6.53E-01	4.68E-01	1.78E-01	**0.00E+00**
	Worst	1.41E+02	2.53E+02	1.51E+01	3.61E+02	5.86E+00	2.77E+02	1.72E+02	2.43E+02	8.34E-02	1.24E+01	1.84E+01	1.19E+01	1.20E+02	1.61E+02	1.16E+01	**0.00E+00**
	Avg	1.08E+01	3.11E+01	2.86E+00	5.44E+01	2.45E+00	8.85E+01	2.53E+01	3.92E+01	1.84E-02	2.46E+00	4.49E-01	9.29E-01	7.92E+00	1.23E+01	3.29E+00	**0.00E+00**
	Std	1.91E+01	4.08E+01	1.88E+00	7.88E+01	6.41E-01	7.41E+01	3.36E+01	5.35E+01	1.10E-02	4.11E+00	1.90E+00	1.35E+00	1.33E+01	2.21E+01	2.59E+00	**0.00E+00**
CF7	Best	4.57E-02	5.69E+01	8.05E-01	**0.00E+00**	9.58E-01	4.25E+02	1.42E-02	1.46E+01	6.42E-05	1.19E-02	7.35E-06	6.66E-08	1.52E+01	1.02E+02	2.91E-02	**5.06E-302**
	Worst	2.50E+02	1.29E+04	3.37E+02	1.13E+04	6.34E+00	1.23E+04	4.43E+02	6.23E+02	1.80E-03	3.40E+01	3.92E-01	1.36E+02	9.10E+03	6.02E+03	2.97E+00	**1.14E-241**
	Avg	4.42E+01	3.56E+03	3.59E+01	1.58E+03	1.80E+00	4.81E+03	3.55E+01	1.53E+02	5.09E-04	3.12E+00	4.82E-02	5.90E+00	1.57E+03	1.21E+03	4.97E-01	**1.39E-243**
	Std	6.10E+01	3.18E+03	6.37E+01	2.77E+03	7.50E-01	3.19E+03	6.92E+01	1.23E+02	2.82E-04	6.43E+00	7.40E-02	1.96E+01	1.88E+03	1.12E+03	4.62E-01	**0.00E+00**
CF8	Best	**0.00E+00**	1.04E+01	1.11E-15	**0.00E+00**	1.14E-02	4.91E-05	1.53E-13	9.97E+00	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	1.11E+01	1.14E+01	8.22E-14	**0.00E+00**
	Worst	6.98E+02	9.62E+02	1.87E+02	1.43E+03	5.74E+02	1.18E+03	1.16E+03	1.15E+03	**0.00E+00**	4.14E+02	1.42E+02	4.19E+01	4.64E+02	6.10E+02	2.21E+01	**0.00E+00**
	Avg	9.99E+01	2.52E+02	1.88E+01	2.80E+02	9.53E+01	2.41E+02	1.60E+02	2.41E+02	**0.00E+00**	5.97E+01	1.42E+00	3.16E+00	1.56E+02	1.77E+02	3.33E+00	**0.00E+00**
	Std	1.54E+02	1.91E+02	2.63E+01	2.86E+02	1.16E+02	2.39E+02	2.44E+02	3.06E+02	**0.00E+00**	1.12E+02	1.42E+01	8.74E+00	1.07E+02	1.28E+02	5.87E+00	**0.00E+00**
CF9	Best	8.88E-15	5.91E-05	8.88E-15	**0.00E+00**	3.25E-05	1.41E-03	2.40E-08	1.50E-01	8.34E-33	2.01E-11	6.16E-12	1.45E-127	4.94E-05	4.65E-05	2.73E-08	**2.41E-296**
	Worst	4.84E+00	6.70E+01	1.05E+01	7.88E+01	7.41E+01	8.58E+00	4.73E+00	8.17E+00	8.88E-15	9.91E+02	9.71E-08	3.51E-114	5.12E+01	6.76E+01	7.76E-06	**5.80E-230**
	Avg	1.42E-01	5.56E+00	3.26E+00	1.51E+01	3.54E+00	1.36E+00	1.37E-01	1.14E+00	8.79E-15	1.52E+02	5.58E-09	4.85E-116	2.02E+00	2.30E+00	3.65E-07	**7.31E-232**
	Std	8.12E-01	1.54E+01	3.44E+00	1.67E+01	1.02E+01	2.42E+00	7.86E-01	1.88E+00	8.88E-16	3.54E+02	1.27E-08	3.61E-115	5.52E+00	8.54E+00	1.01E-06	**0.00E+00**
F10	Best	4.81E+01	4.83E+01	4.83E+01	**0.00E+00**	1.15E+01	4.83E+01	4.82E+01	4.82E+01	9.18E-04	6.15E-02	1.80E-03	9.81E-05	2.17E-02	7.35E-03	4.85E+01	**2.77E-277**
	Worst	8.21E+01	8.88E+01	5.11E+01	1.81E+02	4.98E+01	8.10E+01	8.24E+01	8.07E+01	5.09E+01	8.64E+01	8.65E+01	5.57E+01	1.07E+02	9.15E+01	4.95E+01	**3.64E-212**
	Avg	5.07E+01	5.03E+01	4.92E+01	6.62E+01	4.81E+01	5.04E+01	4.98E+01	4.93E+01	1.40E+01	5.50E+01	4.94E+01	9.43E+00	5.25E+01	5.27E+01	4.91E+01	**3.97E-214**
	Std	7.47E+00	6.20E+00	5.90E-01	3.19E+01	4.51E+00	5.55E+00	5.41E+00	3.20E+00	2.07E+01	1.52E+01	2.59E+01	1.62E+01	1.18E+01	1.21E+01	2.08E-01	**0.00E+00**
CF11	Best	**100.00**	100.01	100.02	**100.00**	**100.00**	100.70	**100.00**	1136.75	**100.00**	**100.00**	**100.00**	**100.00**	100.13	100.03	**100.00**	**100.00**
	Worst	**100.00**	6831.36	3960.34	72786433.70	**100.00**	10086.80	10100.00	16911.14	**100.00**	**100.00**	**100.00**	**100.00**	4673.64	5264.36	100.15	**100.00**
	Avg	**100.00**	1256.91	562.85	3957605.24	**100.00**	2047.71	4000.02	7357.93	**100.00**	**100.00**	**100.00**	**100.00**	735.18	683.43	100.00	**100.00**
	Std	1.03E-14	1.31E+03	6.70E+02	1.07E+07	0.00E+00	2.81E+03	4.90E+03	3.92E+03	**0.00E+00**	3.12E-07	3.57E-11	0.00E+00	9.22E+02	7.90E+02	1.51E-02	**0.00E+00**
CF12	Best	1100.25	1106.89	1100.13	**1100.00**	1733.88	1100.25	1107.02	1103.92	**1100.00**	1100.06	**1100.00**	**1100.00**	1100.06	1100.06	1100.06	**1100.00**
	Worst	2141.91	2505.36	1688.04	2866.98	2629.67	2977.58	2407.48	2273.42	1100.53	1116.95	2620.05	1146.43	2304.67	2146.46	1543.34	**1100.00**
	Avg	1390.90	1820.15	1245.02	1595.42	2226.17	1949.77	1561.16	1608.21	1100.01	1105.32	1126.42	1104.79	1546.63	1520.08	1147.44	**1100.00**
	Std	2.16E+02	3.33E+02	1.68E+02	4.07E+02	1.81E+02	3.58E+02	3.09E+02	2.53E+02	5.64E-02	3.73E+00	1.59E+02	6.28E+00	2.99E+02	2.92E+02	7.36E+01	**0.00E+00**
CF13	Best	703.98	710.95	705.97	**700.00**	723.58	704.98	708.96	708.03	**700.00**	**700.00**	**700.00**	**700.00**	704.98	704.98	700.88	**700.00**
	Worst	744.38	771.09	730.74	865.31	738.50	764.67	739.59	752.49	**700.00**	712.31	797.42	717.71	760.69	757.71	739.64	**700.00**
	Avg	720.98	731.02	716.38	729.28	730.60	723.99	721.80	724.07	**700.00**	710.23	712.12	708.54	724.22	725.06	723.15	**700.00**
	Std	7.98E+00	1.02E+01	4.71E+00	2.43E+01	3.64E+00	1.01E+01	6.58E+00	7.58E+00	**0.00E+00**	3.11E+00	2.91E+01	5.50E+00	1.23E+01	1.19E+01	7.31E+00	**0.00E+00**
CF14	Best	1900.40	1900.34	1900.40	**1900.00**	1900.89	1900.32	1900.47	1900.45	**1900.00**	1900.09	**1900.00**	**1900.00**	1900.17	1900.22	1900.98	**1900.00**
	Worst	1902.32	1902.69	1902.44	1913.00	1902.93	1903.20	1903.12	1902.55	1900.00	1901.77	1904.80	1900.29	1903.57	1904.36	1902.42	**1900.00**

(*continued on next page*)

Table 12 (*continued*)

Function		AFT	ALO	CSA	DA	DMOA	GOA	MFO	MVO	NGO	SA	SCA	SO	SS	SSA	WSO	WO
	Avg	1901.03	1901.29	1900.99	1902.17	1902.16	1901.47	1901.18	1901.28	1900.00	1900.68	1900.39	1900.02	1901.28	1901.44	1901.90	**1900.00**
	Std	3.58E-01	5.49E-01	4.08E-01	1.95E+00	4.11E-01	6.09E-01	5.64E-01	4.66E-01	2.28E-04	2.78E-01	1.03E+00	5.39E-02	6.07E-01	7.55E-01	3.26E-01	**0.00E+00**
CF15	Best	1701.41	2339.99	1702.78	**1700.00**	1705.02	2380.69	**1700.00**	1807.64	**1700.00**	**1700.00**	**1700.00**	**1700.00**	2091.75	1847.09	**1700.00**	**1700.00**
	Worst	2259.24	15745.85	2134.63	83615.26	1744.88	14710.54	2540.16	3141.40	**1700.00**	1818.44	**1700.00**	1976.90	11155.53	11871.52	1711.38	**1700.00**
	Avg	1816.93	4886.03	1801.11	5418.03	1716.47	7135.67	1782.95	2311.84	**1700.00**	1704.79	**1700.00**	1743.21	3786.89	3685.05	1701.60	**1700.00**
	Std	1.26E+02	2.62E+03	1.12E+02	1.01E+04	8.13E+00	3.58E+03	1.34E+02	2.87E+02	**0.00E+00**	1.36E+01	6.09E-06	5.41E+01	1.72E+03	1.57E+03	1.78E+00	**0.00E+00**
CF16	Best	1600.33	1601.23	1600.16	**1600.00**	1601.29	1601.74	1600.39	1601.75	1600.00	1600.10	**1600.00**	1600.00	1601.01	1600.63	1600.37	**1600.00**
	Worst	1730.71	1779.35	1616.49	1839.05	1605.00	1883.52	1756.02	1858.60	1600.31	1623.36	1615.82	1604.89	1738.26	1737.26	1611.85	**1600.00**
	Avg	1611.80	1633.50	1602.97	1635.96	1602.69	1687.73	1623.18	1651.10	1600.02	1601.59	1600.39	1601.03	1610.85	1611.70	1602.73	**1600.00**
	Std	2.17E+01	3.97E+01	2.53E+00	5.10E+01	6.97E-01	7.72E+01	3.05E+01	6.05E+01	3.53E-02	3.26E+00	1.66E+00	9.26E-01	1.66E+01	2.05E+01	2.48E+00	**0.00E+00**
CF17	Best	2100.08	2254.54	2100.62	**2100.00**	2100.97	2275.27	2100.12	2117.03	**2100.00**	2100.02	**2100.00**	2100.01	2119.46	2137.64	2100.03	**2100.00**
	Worst	2366.15	13997.10	2407.73	15759.04	2103.31	15423.09	2538.88	2672.93	2100.00	2134.11	2100.26	2219.25	10750.27	10692.16	2102.17	**2100.00**
	Avg	2155.18	4986.52	2128.32	3614.09	2101.71	6754.74	2124.27	2236.98	**2100.00**	2102.58	2100.03	2108.54	3254.18	3218.11	2100.42	**2100.00**
	Std	6.93E+01	2.57E+03	4.91E+01	2.82E+03	5.03E-01	3.36E+03	4.93E+01	1.07E+02	2.32E-04	5.94E+00	5.32E-02	2.09E+01	1.29E+03	1.29E+03	3.33E-01	**0.00E+00**
CF18	Best	**2200.00**	2220.33	**2200.00**	**2200.00**	2200.02	2220.76	**2200.00**	2209.93	**2200.00**	**2200.00**	**2200.00**	**2200.00**	2219.85	2209.90	**2200.00**	**2200.00**
	Worst	2926.85	3127.98	2310.10	3272.34	2936.43	2980.61	3370.39	3247.49	**2200.00**	2579.28	3777.11	2263.13	2901.68	2750.20	2223.58	**2200.00**
	Avg	2308.99	2455.02	2221.41	2468.89	2326.90	2437.48	2387.30	2433.89	**2200.00**	2277.94	2215.77	2208.95	2362.73	2341.61	2202.17	**2200.00**
	Std	1.60E+02	1.88E+02	2.26E+01	2.68E+02	1.43E+02	1.74E+02	2.51E+02	2.84E+02	**0.00E+00**	1.24E+02	1.58E+02	1.21E+01	1.16E+02	1.21E+02	4.72E+00	**0.00E+00**
CF19	Best	**2400.00**	**2400.00**	**2400.00**	**2400.00**	**2400.00**	2400.00	**2400.00**	2400.17	**2400.00**	**2400.00**	**2400.00**	**2400.00**	**2400.00**	**2400.00**	**2400.00**	**2400.00**
	Worst	2405.62	2473.68	2409.03	2468.38	2473.01	2408.76	2405.61	2406.85	**2400.00**	3391.15	**2400.00**	**2400.00**	2411.74	2408.37	**2400.00**	**2400.00**
	Avg	2400.11	2406.97	2403.33	2413.86	2406.21	2401.83	2400.06	2401.39	**2400.00**	2591.33	**2400.00**	**2400.00**	2401.51	2401.43	**2400.00**	**2400.00**
	Std	7.71E-01	1.69E+01	3.20E+00	1.67E+01	1.20E+01	2.81E+00	5.61E-01	2.01E+00	3.60E-13	3.89E+02	1.33E-08	6.46E-14	2.66E+00	2.45E+00	2.15E-06	**0.00E+00**
CF20	Best	2548.06	2548.28	2548.27	**2500.00**	2509.55	2548.32	2548.17	2548.15	2500.00	2500.04	2500.00	**2500.00**	2535.02	2500.02	2548.26	**2500.00**
	Worst	2581.35	2578.91	2557.91	2666.06	2549.77	2583.07	2579.28	2581.81	2550.89	2592.21	2586.88	2556.11	2608.87	2602.77	2549.59	**2500.00**
	Avg	2550.01	2549.76	2549.27	2562.78	2547.55	2550.46	2549.75	2549.86	2513.14	2552.69	2551.83	2511.02	2552.65	2554.60	2549.07	**2500.00**
	Std	6.17E+00	3.06E+00	1.11E+00	3.11E+01	5.98E+00	5.93E+00	5.10E+00	5.15E+00	2.09E+01	1.18E+01	2.57E+01	1.77E+01	1.10E+01	1.40E+01	2.25E-01	**0.00E+00**
(W|T|L)	(0|20|0)	(0|20|0)	(0|20|0)	(0|9|11)	(0|20|0)	(0|17|3)	(0|20|0)	(0|18|2)	(6|14|0)	(0|18|2)	(0|18|2)	(0|20|0)	(0|20|0)	(0|20|0)	(0|20|0)	**(20|0|0)**	
Friedman mean rank	8.50	12.30	7.15	15.40	6.43	12.80	10.35	10.75	3.25	7.20	8.15	5.23	10.90	11.35	5.05	**1.20**	
Final rank	9	14	6	16	5	15	10	11	2	7	8	4	12	13	3	**1**	

Table 13
Results of the Wilcoxon rank-sum test on CEC 2021 benchmark suite (CF1-CF20).

Function	AFT-WO		ALO-WO		CSA-WO		DA-WO		DMOA-WO		GOA-WO		MFO-WO		MVO-WO		NGO-WO		SA-WO		SCA-WO		SO-WO		SS-WO		SSA-WO		WSO-WO	
	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H	P-value	H
CF1	5.64E-39	+	5.64E-39	+	5.64E-39	+	6.99E-33	+	5.64E-39	+	5.64E-39	+	3.48E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF2	5.62E-39	+	5.64E-39	+	5.59E-39	+	4.07E-36	+	5.64E-39	+	5.64E-39	+	5.63E-39	+	5.64E-39	+	1.58E-03	+	5.64E-39	+	8.28E-31	+	1.46E-35	+	5.62E-39	+	5.62E-39	+	5.64E-39	+
CF3	5.64E-39	+	5.64E-39	+	5.63E-39	+	3.04E-37	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	N/A	=	5.62E-39	+	5.64E-39	+	5.64E-39	+	5.61E-39	+	5.62E-39	+	5.64E-39	+
CF4	5.64E-39	+	5.64E-39	+	5.64E-39	+	3.04E-37	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	7.32E-03	+	5.64E-39	+	2.68E-12	+	2.04E-04	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF5	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.64E-29	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.53E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+
CF6	2.56E-34	+	2.56E-34	+	2.56E-34	+	1.61E-30	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+
CF7	5.64E-39	+	5.64E-39	+	5.64E-39	+	3.04E-37	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF8	2.16E-38	+	5.64E-39	+	5.62E-39	+	2.16E-38	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	N/A	=	1.82E-34	+	7.49E-23	+	2.72E-06	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF9	3.48E-41	+	5.64E-39	+	5.28E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.76E-45	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF10	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.64E-29	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+	2.56E-34	+
CF11	N/A	=	5.64E-39	+	5.64E-39	+	4.80E-35	+	N/A	=	5.64E-39	+	1.80E-13	+	5.64E-39	+	N/A	=	4.70E-39	+	N/A	=	N/A	=	5.64E-39	+	5.64E-39	+	5.74E-26	+
CF12	5.63E-39	+	5.64E-39	+	5.58E-39	+	3.03E-37	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	8.28E-03	+	5.64E-39	+	9.14E-10	+	8.08E-32	+	5.62E-39	+	5.60E-39	+	5.64E-39	+
CF13	5.64E-39	+	5.64E-39	+	5.62E-39	+	1.12E-36	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	N/A	=	5.64E-39	+	3.68E-09	+	4.07E-36	+	5.61E-39	+	5.60E-39	+	5.64E-39	+
CF14	5.64E-39	+	5.64E-39	+	5.64E-39	+	4.07E-36	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	1.58E-03	+	5.64E-39	+	5.53E-08	+	5.10E-06	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF15	5.63E-39	+	5.64E-39	+	5.64E-39	+	3.04E-37	+	5.64E-39	+	5.64E-39	+	2.15E-38	+	5.64E-39	+	N/A	=	5.63E-39	+	5.53E-08	+	2.16E-38	+	5.64E-39	+	5.64E-39	+	8.14E-38	+
CF16	5.64E-39	+	5.64E-39	+	5.64E-39	+	4.07E-36	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF17	5.64E-39	+	5.64E-39	+	5.64E-39	+	3.04E-37	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.63E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+
CF18	1.12E-36	+	5.64E-39	+	8.30E-29	+	4.07E-36	+	5.64E-39	+	5.64E-39	+	2.16E-38	+	5.64E-39	+	5.63E-39	+	5.62E-39	+	5.64E-39	+	7.54E-23	+	5.64E-39	+	5.64E-39	+	5.82E-13	+
CF19	1.12E-36	+	5.62E-39	+	5.60E-18	+	2.16E-33	+	5.64E-39	+	5.64E-39	+	5.07E-08	+	5.64E-39	+	5.63E-39	+	5.18E-39	+	5.64E-39	+	7.54E-23	+	5.59E-39	+	5.60E-39	+	1.23E-03	+
CF20	5.64E-39	+	5.64E-39	+	5.64E-39	+	4.07E-36	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+	5.64E-39	+

where, $x_{1,2,3,4,5} \in [0.01, 100]$ are design variables.

The results obtained by WO and other metaheuristic algorithms collected from the literatures are given in Table 17. From this table, it can be seen that the optimal cost obtained by WO (i.e., 13.013640) and SMA has the similar performance (i.e., 13.339957). The remaining ranking order is ALO, MVO, Symbiotic Organisms Search (SOS), MFO, CS, Method of Moving Asymptotes (MMA), Generalized Convex Approximation (GCA) in its version I and II (GCA I and GCA II, respectively).

4.4.3. *Case 3: tension/compression spring design*

This work was done by optimizing the wire diameter (*D*), average coil diameter (*d*) and effective coil number (*N*) to minimize the weight of the spring (see Fig. 11(c)). The constraints include minimum deviation (g_1), shear stress (g_2), impact frequency (g_3), and outer diameter limit (g_4). The mathematical model of spring design is described as follows:

Suppose $x = [x_1 x_2 x_3] = [dDN]$

$$\text{Minimize} f(x) = x_1^2 x_2 (2 + x_3) \tag{27}$$

$$\text{Subject to} \begin{cases} g_1 = 1 - \dfrac{x_2^3 x_3}{71785 x_1^4} \leq 0 \\ g_2 = \dfrac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \dfrac{1}{5108 x_1^2} - 1 \leq 0 \\ g_3 = 1 - \dfrac{140.45 x_1}{x_2^2 x_3} \leq 0 \\ g_4 = \dfrac{x_1 + x_2}{1.5} - 1 \leq 0 \end{cases} \tag{28}$$

where,$x_1 \in [0.05, 2]$, $x_2 \in [0.25, 1.3]$, $x_3 \in [2, 15]$ are design variables.

Examining the optimal costs in Table 18, we can clearly observe that WO and HHO produced the best solution and achieved the best design with a cost of 0.012665, which no other can achieve.

4.4.4. *Case 4: Pressure vessel design*

The purpose of this design is to find the minimum cost of the cylindrical vessel design (see Fig. 11(d)). There are four parameters to be optimized: shell thickness (*Ts*), head thickness (*Th*), inner radius (*R*) and cylinder length (*L*). The objective function is constrained by four constraint functions. The mathematical model of welding beam design is as follows:

Suppose $x = [x_1 x_2 x_3 x_4] = [TsThRL]$

$$\text{Minimize} f(x) = 0.6224 x_1 x_3 x_4 + 1.7781 x_2 x_3^2 + 3.1661 x_1^2 x_4 + 19.84 x_1^2 x_3 \tag{29}$$

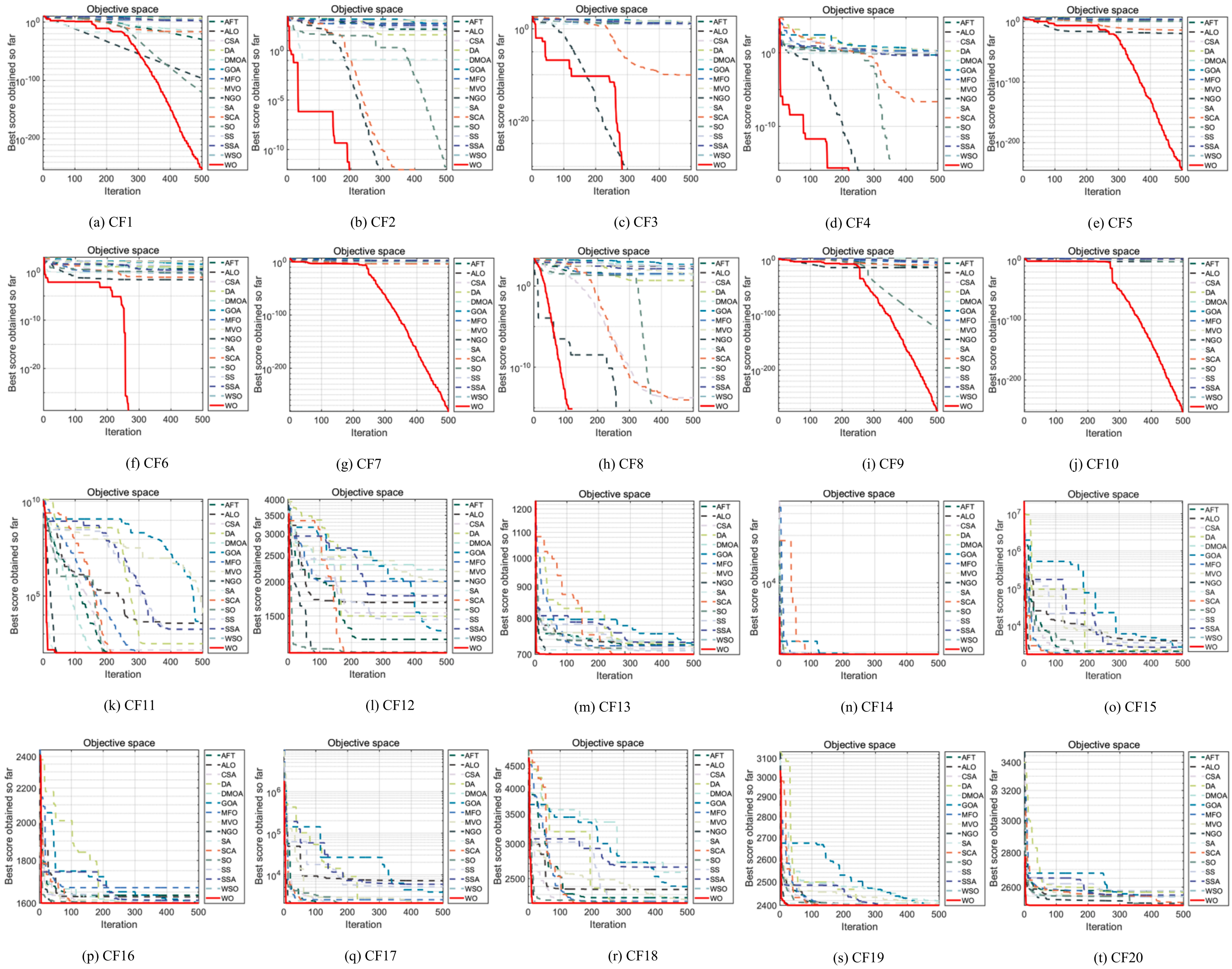
$$\text{Subject to} \begin{cases} g_1 = 0.0193 x_3 - x_1 \leq 0 \\ g_2 = 0.00954 x_3 - x_3 \leq 0 \\ g_3 = -\pi x_3^2 x_4 - \dfrac{4}{3}\pi x_3^3 + 1296000 \leq 0 \\ g_4 = x_4 - 240 \leq 0 \end{cases} \tag{30}$$

where,$x_{1,2} \in [0.1, 99]$ and $x_{3,4} \in [10, 200]$ are design variables.

Some of the algorithms that are chosen for comparison are SMA, Harris Hawks Optimization (HHO), ACO, WOA, GA, MVO, Co-evolutionary Particle Swarm Optimization (CPSO), Harmony Search (HS) and Gravitational Search Algorithm (GSA). The results obtained by WO and their comparison with the aforementioned metaheuristics are reported in Table 19. In this table, the comparison results show that the WO is ranked as the first best solution obtained.

4.4.5. *Case 5: Welded beam design*

The minimum cost is the objective function of the welded beam design problem (see Fig. 11(e)), and the objective function is constrained by seven inequalities. The decision variables are weld thickness (*h*), steel bar connection length (*l*), steel bar height (*t*) and steel bar



(a) CF1 (b) CF2 (c) CF3 (d) CF4 (e) CF5

(f) CF6 (g) CF7 (h) CF8 (i) CF9 (j) CF10

(k) CF11 (l) CF12 (m) CF13 (n) CF14 (o) CF15

(p) CF16 (q) CF17 (r) CF18 (s) CF19 (t) CF20

Fig. 9. Iteration diagram of CEC 2021 benchmark suite results.

thickness (b) respectively. The mathematical model of welding beam design is as follows:

Suppose $x = [x_1x_2x_3x_4] = [hltb]$

$$Minimize f(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2)\# \tag{31}$$

$$\text{Subject to}\begin{cases} g_1 = \tau(X) - 13600 \le 0 \\ g_2 = \sigma(X) - 30000 \le 0 \\ g_3 = x_1 - x_4 \le 0 \\ g_4 = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \le 0 \\ g_5 = 0.125 - x_1 \le 0 \\ g_6 = \delta(X) - 0.25 \le 0 \\ g_7 = 6000 - P_c(x) \le 0 \end{cases} \tag{32}$$

where,$x_{1,4} \in [0.1, 2]$ and $x_{2,3} \in [0.1, 10]$ are design variables, $\tau(X) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$ is shear stress, $\tau' = \frac{6000}{\sqrt{2}x_1x_2}$, $\tau'' = \frac{MR}{J}$, $M = 6000\left(14 + \frac{x_2}{2}\right)$, $R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1+x_3}{2}\right)^2}$, $J = 2\{\sqrt{2}x_1x_2[\frac{x_2^2}{12} + (\frac{x_1+x_3}{2})^2]\}$, $\sigma(X) = \frac{504000}{x_3^2x_4}$ is beam bending stress, $\delta(X) = \frac{65856000}{(30\times10^6)x_3^3x_4}$ is beam deflection, $P_c(x) = \left\{\left[4.013(30\times10^6)\sqrt{\frac{x_3^2x_4^6}{36}}/196\right] \times \left[1 - [x_3\sqrt{\frac{30\times10^6}{4(12\times10^6)}}]/28\right]\right.$ is critical buckling load.

The WO was tested on this problem and the results were compared with AO, HPO, MPA, HHO, an effective Co-evolutionary Differential Evolution (CDE), GSA, SIMPLEX, APPROX and DAVID. The results are shown in Table 20. The WO has discovered better optimal values than other algorithms and has been able to rank first.

4.4.6. Case 6: Speed reducer design

The problem of minimizing the weight of a speed reducer involves seven design variables and four constraints (see Fig. 11(f)). The design variables include the face width (x_1), the module of teeth (x_2), a discrete design variable on behalf of the teeth in the pinion (x_3), length and diameter of the first shaft between bearings (x_4 and x_6), length and diameter of the second shaft between bearings (x_5 and x_7). Four constraints covering stress (g_1), bending stress of the gear teeth (g_2), stresses in the shafts (g_3), and transverse deflections of the shafts (g_4). The mathematical model of welding beam design is as follows:

Suppose $x = [x_1x_2x_3x_4x_5x_6x_7]$

$$\begin{aligned}\text{Minimize} f(x) =& 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) \\ & - 1.508x_1\left(x_6^2 + x_7^2\right) + 7.4777\left(x_6^3 + x_7^3\right)\end{aligned} \tag{33}$$

where $x_1 \in [2.6, 3.6]$, $x_2 \in [0.7, 0.8]$, $x_3 \in [17, 28]$, $x_4 \in [7.3, 8.3]$, $x_5 \in [7.8, 8.3]$, $x_6 \in [2.9, 3.9]$, and $x_7 \in [5.0, 5.5]$ are design variables.

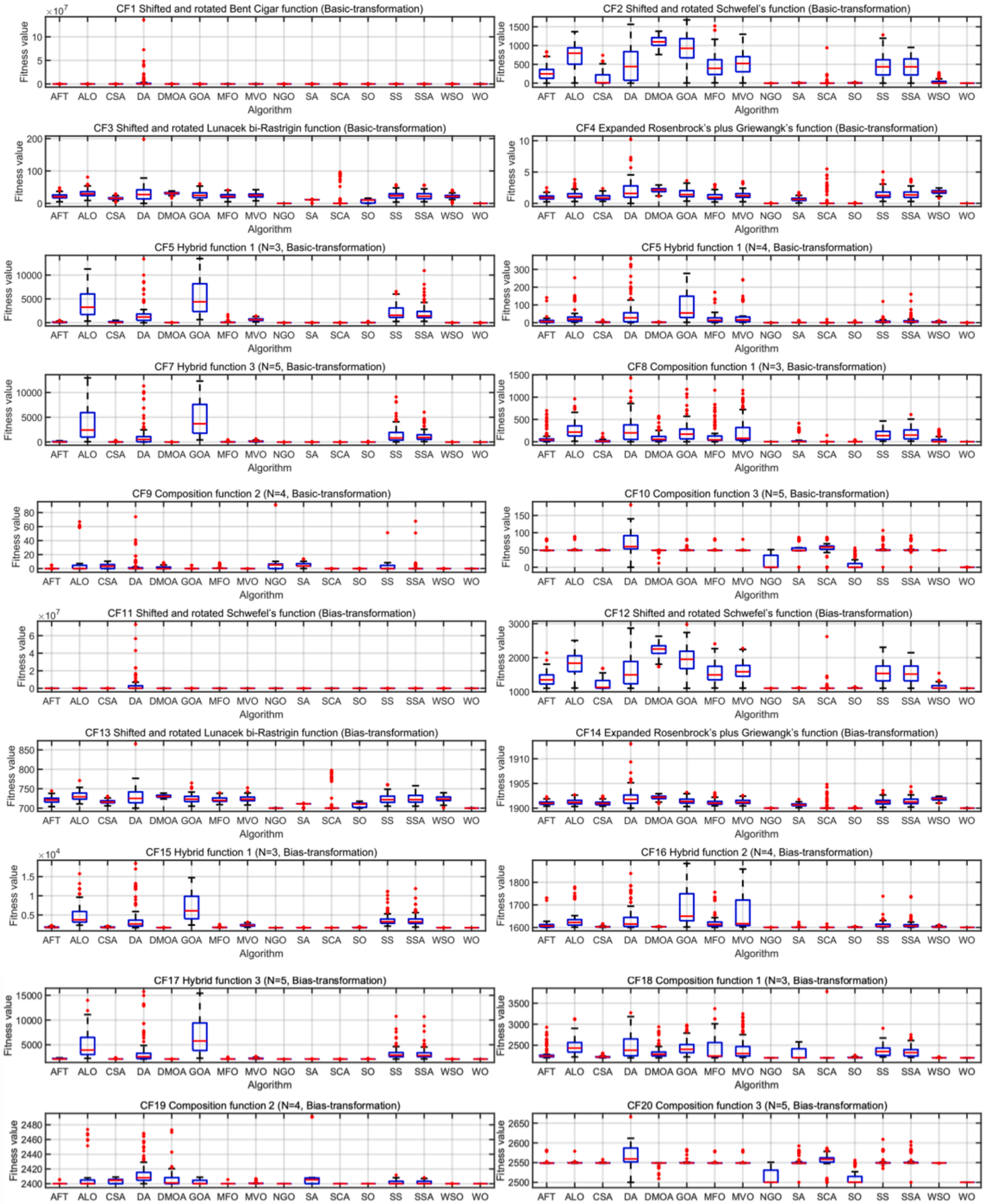


Fig. 10. Boxplot of test results of CEC 2021 benchmark suite.

Table 14
The original data of the simulation example.

Parameters	Value
$P = \{p_1, p_2, \cdots, p_M\}$	{72,490,651,833,833,489,359,337,267,441,70,934,467,661,220,329,440,774,595,98,424,37,807,320,501,309,834,851,34,459,111,253,159,858,793,145, 651,856,400,285,405,95,391,19,96,273,152,473,448,231}
$W = \{w_1, w_2, \cdots, w_M\}$	{438,754,699,587,789,912,819,347,511,287,541,784,676,198,572,914,988,4,355,569,144,272,531,556,741,489,321,84,194,483,205,607,399,747,118, 651,806,9,607,121,370,999,494,743,967,718,397,589,193,369}
V	11258

This problem has been solved by WO and compared with the literature in Table 21. The comparison is made between Method of GWO, AO, multidisciplinary design optimization (MDO), HS, hybrid Harris Hawks-Sine Cosine Algorithm (hHHO-SCA), SCA, GSA, GA and PSO. Note that WO outperforms other techniques when obtaining the lowest weight and shows the high performance of the WO is demonstrated by its ability to approximate the global optimum for this problem.

In addition, the best, mean, worst and standard deviation of WO after

Table 15
The original data of the simulation example.

Algorithms	Solution	Function value
GA (Jin & Ma, 2004)	{10111000000111100110101010111110111011010000000111}	14865
SA (Jin & Ma, 2004)	{01111101010101000110101010110110011011011000000110}	15844
BPSO (Ma et al., 2006)	{01111001010111000110101010110100011011011010000110}	16052
WO	{01111001010111000110101010110100011011011010000110}	16052

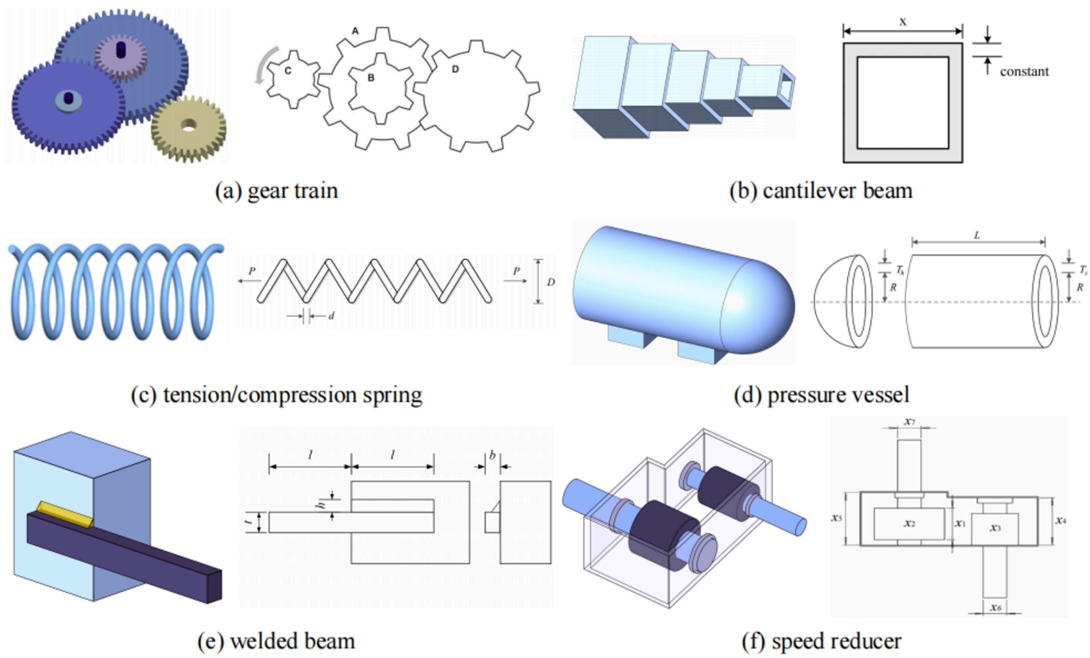


Fig. 11. Description of engineering design problems.

Table 16
Comparison of optimization results of gear train design

Algorithms	Optimal values for variables				$f(x)$
	G_A	G_B	G_C	G_D	
WO	**43**	**16**	**19**	**43**	**2.700857E-12**
CS (Gandomi et al., 2013)	43	16	19	49	2.700900E-12
CSA (Gandomi et al., 2013)	19	16	43	49	2.701000E-12
WOA (Mirjalili & Lewis, 2016)	47	12	13	23	9.921570E−10
GeneAS (Deb & Goyal, 1996)	33	14	17	50	1.362000E-09
Simulated annealing (Zhang & Wang, 1993)	52	15	30	60	2.360000E-09
Mixed integer discrete continuous optimization (Kannan & Kramer, 1994)	33	15	13	41	2.146000E-08
Augmented Lagrange Multiplier (Kannan & Kramer, 1994)	13	15	33	41	2.146000E-08
Mixed integer discrete continuous programming (Fu et al., 1991)	47	29	14	59	4.500000E-06
Nonlinear integer and discrete programming (Sandgren, 1990)	18	22	45	60	5.712000E-06

10 independent runs were obtained. The statistical results of WO are shown in Table 22.

5. Conclusions

In this article, we present a nature-inspired migration and reproduction of the famous animal - walrus. This mammal has adapted to land and sea environments with social intelligence. We propose a model-independent novel metaheuristic algorithm models behavior of walrus into optimization technique and called Walrus Optimizer. In the proposed algorithm, we have modeled global search simulating migration behavior of walruses and local search simulating reproduction behavior of walruses. Moreover, we have also modeled landing process and feeding process during the reproduction.

A series of mathematical problems (23 benchmark functions, CEC 2021 benchmark suite, 0–1 knapsack problem and real-world engineering optimization problems) are used to evaluate the proposed WO and compared with other metaheuristics. The statistics from the benchmark tests show rapid convergence. WO locates most individuals around the best value with a small number of iterations, which gives WO

Table 17
Comparison of optimization results of cantilever beam design

Algorithms	Optimal values for variables					$f(x)$
	x_1	x_2	x_3	x_4	x_5	
WO	**5.947300**	**4.866000**	**4.434143**	**3.474900**	**2.186440**	**13.013640**
SMA (Li et al., 2020)	6.017757	5.310892	4.493758	3.501106	2.150159	13.339957
ALO (Naruei et al., 2022)	6.018120	5.311420	4.488360	3.497510	2.158329	13.399500
MVO (Naruei et al., 2022)	6.023940	5.306011	4.495011	3.496022	2.152726	13.399595
SOS (Naruei et al., 2022)	6.018780	5.303440	4.495870	3.498960	2.155640	13.399600
MFO (Mirjalili, 2015a)	5.983000	5.316700	4.497300	3.513600	2.161600	13.399880
CS (Naruei et al., 2022)	6.008900	5.304900	4.502300	3.507700	2.150400	13.399900
MMA (Peraza-Vázquez et al., 2021)	6.010000	5.300000	4.490000	3.490000	2.150000	13.400000
GCA I (Peraza-Vázquez et al., 2021)	6.010000	5.300000	4.490000	3.490000	2.150000	13.400000
GCA II (Peraza-Vázquez et al., 2021)	6.010000	5.304000	4.490000	3.498000	2.150000	13.400000

Table 18
Comparison of optimization results of tension/compression spring design.

Algorithms	Optimal values for variables			$f(x)$
	d	D	N	
WO	**0.050000**	**0.311500**	**14.892300**	**0.012665**
HHO (Heidari et al., 2019)	0.051796	0.359305	11.138859	0.012665
SHO (Naruei et al., 2022)	0.051144	0.343751	12.09550	0.012674
WOA (Naruei et al., 2022)	0.051207	0.345215	12.004032	0.012676
RO (Abualigah et al., 2022)	0.051370	0.349096	11.762790	0.012679
ES (Abualigah et al., 2022)	0.051643	0.355360	11.397926	0.012698
GSA (Naruei et al., 2022)	0.050276	0.323680	13.525410	0.012702
GA (Coello, 2000)	0.051480	0.351661	11.632201	0.012705
MVO (Abualigah et al., 2022)	0.052510	0.376020	10.335130	0.012790
CC (J. S. Arora, 2004)	0.050000	0.315900	14.250000	0.012833

Table 19
Comparison of optimization results of pressure vessel design.

Algorithms	Optimal values for variables				$f(x)$
	Ts	Th	R	L	
WO	**0.778190**	**0.384659**	**40.320119**	**199.993131**	**5885.349088**
SMA (Li et al., 2020)	0.793100	0.393200	40.671100	196.217800	5994.185700
HHO (Heidari et al., 2019)	0.817584	0.407293	42.091746	176.719635	6000.462590
ACO (Kaveh & Talatahari, 2010)	0.812500	0.437500	42.098353	176.637751	6059.725800
WOA (Naruei et al., 2022)	0.812500	0.437500	42.098270	176.638998	6059.741000
GA (Abualigah, et al., 2022)	0.812500	0.437500	42.097398	176.654050	6059.946340
MVO (Abualigah, et al., 2022)	0.812500	0.437500	42.090738	176.738690	6060.806600
CPSO (Abualigah, et al., 2022)	0.812500	0.437500	42.091266	176.746500	6061.077700
HS (Abualigah, et al., 2022)	1.125000	0.625000	58.290150	43.692680	7197.730000
GSA (Rashedi et al., 2009)	1.125000	0.625000	55.988660	84.454203	8538.835900

Table 20
Comparison of optimization results of welded beam design.

Algorithms	Optimal values for variables				$f(x)$
	h	l	t	b	
WO	**0.168066**	**4.065890**	**9.997888**	**0.168071**	**1.587354**
AO (Abualigah et al., 2021)	0.163100	3.365200	9.020200	0.206700	1.656600
HPO (Naruei et al., 2022)	0.198812	3.337754	9.192016	0.198833	1.670240
MPA(Faramarzi et al., 2020)	0.205728	3.470509	9.036624	0.205730	1.724853
HHO (Heidari et al., 2019)	0.204039	3.531061	9.027463	0.206147	1.731991
CDE (Naruei et al., 2022)	0.203137	3.542998	9.033498	0.206179	1.733462
GSA (Naruei et al., 2022)	0.182129	3.856979	10.00000	0.202376	1.879950
SIMPLEX (Abualigah et al., 2022)	0.279200	5.625600	7.751200	0.279600	2.530700
APPROX (Abualigah et al., 2022)	0.244400	6.218900	8.291500	0.244400	2.381500
DAVID (Abualigah et al., 2022)	0.243400	6.255200	8.291500	0.244400	2.384100

a good advantage in improving the accuracy of the result. In addition, the Friedman test and Wilcoxon rank sum test quantitatively verified the advantages of WO over the fifteen comparison algorithms. WO optimization performance is superior to other algorithms in different engineering examples.

WO maintains a leading position in terms of optimality (i.e., the capacity to reach optimal values), scalability (i.e., ability to deal with large-scale problems) and computation time (especially for large-scale problems). However, its limitations are shown in competitiveness compared to approximation algorithms in the performances of modeling

Table 21
Comparison of optimization results of speed reducer design

Algorithms	Optimal values for variables							$f(x)$
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
WO	**3.500**	**0.700**	**17.000**	**7.300**	**7.715**	**3.350**	**5.287**	**2994.471066**
GWO (M. Braik et al., 2021)	3.507	0.700	17.000	7.381	7.816	3.358	5.287	3001.288000
AO (Abualigah et al., 2021)	3.502	0.700	17.000	7.310	7.748	3.364	5.299	3007.732800
MDO (Lu & Kim, 2010)	3.500	0.700	17.000	7.300	7.670	3.542	5.246	3019.583365
HS (Geem et al., 2001)	3.520	0.700	17.000	8.370	7.800	3.367	5.289	3029.002000
hHHO-SCA (Abualigah et al., 2022)	3.506	0.700	17.000	7.300	7.991	3.453	5.287	3029.873076
CA (M. Braik et al., 2021)	3.509	0.700	17.000	7.300	7.800	3.461	5.289	3030.563000
GSA (M. Braik et al., 2021)	3.600	0.700	17.000	8.300	7.800	3.370	5.289	3051.120000
GA (M. Braik et al., 2021)	3.510	0.700	17.000	8.350	7.800	3.362	5.288	3067.561000
PSO (Stephen et al., 2018)	3.500	0.700	17.000	7.518	7.783	3.351	5.287	3145.922000

Table 22
Statistical results of WO in real-world engineering optimization problems.

Problems	Best	Mean	Worst	Std
Case 1	2.700857E-12	5.943261E-10	2.357641E-09	7.953806E-10
Case 2	13.012130	13.012978	13.013670	6.078283E-04
Case 3	0.012665	0.013363	0.014771	7.069479E-04
Case 4	5885.334775	6179.253361	6486.857909	2.268564E+02
Case 5	1.587154	1.588547	1.591209	1.243997E-03
Case 6	2994.471071	2994.471272	2994.472253	3.705633E-04

$$Subjectto\begin{cases} g_1 = \frac{27}{x_1x_2^2x_3} - 1 \le 0, g_2 = \frac{397.5}{x_1x_2^2x_3^2} - 1 \le 0 \\ g_3 = \frac{1.93x_4^3}{x_2x_3x_6^4} - 1 \le 0, g_4 = \frac{1.93x_5^3}{x_2x_3x_7^4} - 1 \le 0 \\ g_5 = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9\times 10^6}}{110.0x_6^3} - 1 \le 0, g_6 = \frac{\sqrt{\left(\frac{745x_4}{x_2x_3}\right)^2 + 16.9\times 10^6}}{110.0x_6^3} - 1 \le 0 \\ g_7 = \frac{x_2x_3}{40} - 1 \le 0, g_8 = \frac{5x_2}{x_1} - 1 \le 0, g_9 = \frac{x_1}{12x_2} - 1 \le 0 \\ g_{10} = \frac{1.5x_6 + 1.9}{x_4} - 1 \le 0, g_{11} = \frac{1.1x_7 + 1.9}{x_5} - 1 \le 0 \end{cases} \# \tag{34}$$

(i.e., capacity to develop models that accurately represent the real-life system) and uncertainty (i.e., ability to cope with non-deterministic scenarios). Therefore, it also opens up some promising research directions and potentials. If WO is chosen to optimize a specific case in a subsequent study, WO can be further improved according to the specific situation and characteristics of relevant cases, so as to ensure that the algorithm has better optimization performance when solving specific problems. WO can be combined with different fields, applied to real problems, and provide more possibilities for solving complex problems.

The future research is directed towards solving NP-hard problems and optimizing the design of complex engineering cases using WO. Considering the specific requirements and characteristics of the problem, the algorithm strategy and parameters are adjusted to achieve the best performance. In addition, a multi-objective optimization version of WO can be developed and applied to different problems. To explore the practicality and innovation of the new intelligent optimization algorithm in designing engineering cases.

CRediT authorship contribution statement

Muxuan Han: Conceptualization, Software, Visualization, Writing – original draft. **Zunfeng Du:** Methodology, Validation, Formal analysis, Supervision. **Kum Fai Yuen:** Supervision, Writing – review & editing. **Haitao Zhu:** Validation, Formal analysis. **Yancang Li:** Methodology, Formal analysis. **Qiuyu Yuan:** Methodology, Software, Visualization, Formal analysis.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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