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Symbolic Algebra for the Calculation of the Series and Parallel Resistances in PV module model

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***Abstract*—In this paper a symbolic analysis of the equations that allow to calculate the values of the series and parallel resistances appearing in the single diode model of a photovoltaic module is presented. Such a model requires the identification of five values, that are the photo-induced current, the diode ideality factor and saturation current, the series and parallel resistances. This calculation can be done on the basis of the few data taken from the module data sheet or by using a fitting process starting from experimental measurements. In this paper the first case is considered: it allows to identify the parameters' values in the standard test conditions and requires a solution of a non linear system of equations. In this paper the problem is afforded by using symbolic calculations only: the main result is the explicit expressions giving the values of all the five parameters. The explicit formulas allow to avoid the convergence problems that are typical of the methods based on the solution of a non linear system of equations. Nonetheless, the values obtained by using the proposed procedure can be used as guess solution for a more accurate calculation.**

I. Introduction

The simulation of PhotoVoltaic (PV) arrays is of fundamental importance in many contexts. It is required for the design of the Maximum Power Point Tracking (MPPT) algorithm and of the switching converter performing this action. It is also mandatory for the prediction of the power produced on different time scales, from seconds to, especially in terms of energy, years. The accurate simulation of the PV module is also useful for model based diagnostics, in order to understand the differences between the actual module operation and the predicted one.

Some PV models are presented and used in literature, but the one more extensively recalled, because of the good compromise it offers between accuracy and complexity, is the single diode model shown in Fig.1.

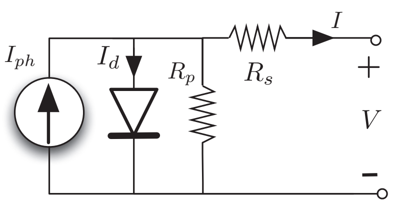

Figure 1. Single diode model of a PV unit.

Under the assumption that the PV unit works in uniform conditions, e.g. in terms of temperature and irradiance, the model shown in Fig.1 is scalable. This means that it can be used to describe a cell or any series/parallel combination of cells, provided that a proper scaling up or down of the parameters is operated.

The single diode model ensures a good reconstruction of the current vs voltage (I-V) and power vs voltage (P-V) curves. It requires a proper setting of the parameters appearing in the model that are, according to (1), $I_{ph}, I_{sat}, \eta, R_s, R_p$:

$$I = I_{ph} - I_{sat} \cdot \left(e^{\frac{V+I\cdot R_s}{n_s \eta V_t}} - 1 \right) - \frac{V + I \cdot R_s}{R_p} \quad (1)$$

Equation (1) gives, at the module level, the relationship between the current and the voltage at its terminals. In (1), n_s is the number of series connected cells forming the PV module and I_{ph} is the photo-induced current, whose value depends on the material the cells are made of, on the cell temperature and area, as well as on the irradiance level they receive. I_{sat} is the saturation current of the diode, representing the semiconductor junction, in the model. V_t is the thermal voltage (2):

$$V_t = \frac{k \cdot T}{q} \quad (2)$$

in which $k = 1.3806503 \cdot 10^{-23} J/K$ is the Boltzmann constant, $q = 1.60217646 \cdot 10^{-19} C$ is the electron charge and T is the junction temperature. η is the ideality factor, while R_s and R_p model the loss mechanisms taking place in the PV module: the former is influenced by the quality of the contact between the ribbon and the cells and by any current-related mechanisms. The parallel resistance is related to recombination mechanisms [1].

By referring to a commercial PV module, the values of the parameters appearing in (1), in particular of the set $I_{ph}, I_{sat}, \eta, R_s, R_p$, can be calculated by using a fitting strategy with a set of experimental measurements performed on the module itself. This approach allows an accurate parametric identification in the conditions at which the measurements where acquired, but it requires an experimental effort. Regardless of the operating conditions at which the values of the five parameters involved in the single diode model are identified, the *conversion* of the results of experimental measurements in the $\{I_{ph}, I_{sat}, \eta, R_s, R_p\}$ values requires the solution of a non linear system of equations.

It is a much more common practice to identify the parameters values on the basis of the data available in the module data sheet and referring to the performances of the PV module in Standard Test Conditions (STC). This approach does not require any experiment, but is forced to use a very small amount of data, thus it is a challenging task. Some approaches have been presented in literature, e.g. in [1], [2], [3], [4] and [5]. The largest part of them require, again, the solution of a non linear system of equations, at least of the second order. It is well known that this requires a guess solution, which is not always easy to be chosen, and, due to the great non linearity of the problem, the convergence might not be reached at all. The problem especially arises when the values of the series and parallel resistances must be determined. In fact, the calculation of the first three parameters $\{I_{ph}, I_{sat}, \eta\}$ is almost direct. If the parallel resistance R_p is initially neglected, the model is simplified significantly and also the series resistance R_s can be determined by means of an explicit equation, without solving any system of equation [1] [2]. The role of R_p is marginal in some working condition and for some kinds of analyses, but it is important to keep it into account if partial shadowing needs to be simulated. In fact, if the PV cell receives a high irradiation but it is put in series with shadowed cells producing a lower current, the current difference in the model shown in Fig.1 flows through the R_p, which has to be correctly identified if an accurate simulation is needed.

In this paper a novel approach to the calculation of the series and parallel resistances of the single diode model representing a commercial PV module is presented. The procedure taken into consideration for the evaluation of $\{I_{ph}, I_{sat}, \eta\}$ is the one proposed in [1], thus based on the assumption that R_p can be neglected for the identification of these first three parameters. Afterwards, instead of manipulating some equations in order to achieve a couple of mutually dependent non linear equations to be solved for calculating R_s and R_p, as done in [1], two independent formulas are obtained. A reasonable approximation, consisting in neglecting the quantity $R_s \cdot I_{MPP,STC}$ with respect to $2 \cdot V_{MPP,STC}$, ensures values for the two resistances that are very close to those ones which could be obtained at the price of solving a non linear system of equations. A wide class of PV modules actually on the market have, in STC, the voltage in the Maximum Power Point (MPP) which is two or three times larger than the current in the same point. Moreover, they have few hundreds of milli-ohm of series resistance [3].

The model parameters identified at STC usually make the PV model (1) far from the reality at low irradiation levels. This error can be compensated by using suitable translational formulas available in literature. In the final part of the paper it is demonstrated that the parameters values obtained by means of the proposed procedure guarantee, also far from the STC, an accuracy that is close to that one ensured by the parameters determined through a precise procedure.

II. IDENTIFICATION OF $\{I_{ph}, I_{sat}, \eta\}$ PARAMETERS.

The following information is given usually in PV modules data sheets:

- Voltage in open circuit conditions: $V_{OC,STC}$
- Voltage temperature coefficient: $\alpha_v = \left.\frac{dV_{OC}}{dT}\right|_{STC}$
- Current in short circuit conditions: $I_{SC,STC}$
- Current temperature coefficient: $\alpha_i = \left.\frac{dI_{SC}}{dT}\right|_{STC}$
- Operation at the MPP: $I_{MPP,STC}$ and $V_{MPP,STC}$

By neglecting, at a first approximation level, as also done in [1], the PV module parallel resistance R_p in (1), the equation whose parameters need to be identified is the (3).

$$I = I_{ph} - I_{sat} \cdot \left[e^{\frac{(V+I \cdot R_s)}{n_s \eta V_t}} - 1 \right] \tag{3}$$

The photo induced current is calculated straightforwardly, because it is significantly higher than the diode current, so that it is assumed to be equal to the short circuit current in STC:

$$I_{ph,STC} = I_{SC,STC} \tag{4}$$

The second step starts by applying (3) in the open circuit condition:

$$\begin{aligned} 0 &= I_{ph,STC} - I_{sat,STC} \cdot \left(e^{\frac{V_{OC,STC}/n_s}{\eta V_{t,STC}}} - 1 \right) \\ &\approx I_{ph,STC} - I_{sat,STC} \cdot e^{\frac{V_{OC,STC}/n_s}{\eta V_{t,STC}}} \end{aligned} \tag{5}$$

where the second equivalence has been obtained by neglecting the unitary term with respect to the exponential one. Consequently, it is:

$$V_{OC,STC} \approx -\eta \cdot n_s \cdot V_{t,STC} \cdot \ln\left(\frac{I_{sat,STC}}{I_{ph,STC}} \right) \tag{6}$$

The open circuit voltage temperature coefficient can be calculated as follows:

$$\begin{aligned} \alpha_v &= \frac{dV_{OC,STC}}{dT} \\ &= n_s \cdot \eta \cdot V_{t,STC} \cdot \frac{d}{dT} \left(\ln I_{ph,STC} - \ln I_{sat,STC} \right) \end{aligned} \tag{7}$$

By accounting for the temperature dependence of the thermal voltage (2), it is possible to write that:

$$\begin{aligned} \alpha_v = & \frac{n_s \cdot \eta \cdot V_{t,STC}}{T_{STC}} \cdot \ln\left(\frac{I_{ph,STC}}{I_{sat,STC}} \right) + \\ & + n_s \cdot \eta \cdot V_{t,STC} \cdot \frac{1}{I_{ph,STC}} \cdot \frac{dI_{ph,STC}}{dT} + \\ & - n_s \cdot \eta \cdot V_{t,STC} \cdot \frac{1}{I_{sat,STC}} \cdot \frac{dI_{sat,STC}}{dT} \end{aligned} \tag{8}$$

The second term in the parentheses can be calculated by keeping into account that the saturation current is expressed as follows:

$$I_{sat} = C \cdot T^3 \cdot e^{\left(-\frac{E_{gap}}{kT} \right)} \tag{9}$$

where E_{gap} is the band gap of the semiconductor material, e.g. $E_{gap} = 1.124eV = 1.8 \cdot 10^{-19}$J for crystalline silicon, and C is the temperature coefficient [1].

Thus, by means of (9), it results that:

$$\frac{1}{I_{sat,STC}} \cdot \frac{dI_{sat,STC}}{T_{STC}} = \frac{3}{T_{STC}} + \frac{E_{gap}}{k \cdot {T_{STC}}^2} \tag{10}$$

while the first term is related to the current temperature coefficient α_i, so that:

$$\alpha_v = \frac{n_s \cdot \eta \cdot V_{t,STC}}{T_{STC}} \cdot \ln\left(\frac{I_{ph,STC}}{I_{sat,STC}}\right) + \\ + n_s \cdot \eta \cdot V_{t,STC} \cdot \left(\frac{\alpha_i}{I_{ph,STC}} - \frac{3}{T_{STC}} - \frac{E_{gap}}{k \cdot {T_{STC}}^2}\right) \tag{11}$$

From (6) it is:

$$\alpha_v = \frac{V_{OC,STC}}{T_{STC}} + n_s \cdot \eta \cdot V_{t,STC} \cdot \frac{\alpha_i}{I_{ph,STC}} + \\ - n_s \cdot \eta \cdot V_{t,STC} \cdot \frac{3}{T_{STC}} - \frac{E_{gap}}{k \cdot {T_{STC}}^2} \tag{12}$$

thus, the diode ideality factor is calculated by means of (13):

$$\eta = \frac{\alpha_v - \frac{V_{OC,STC}}{T_{STC}}}{n_s \cdot V_{t,STC} \cdot \left(\frac{\alpha_i}{I_{ph,STC}} - \frac{3}{T_{STC}} - \frac{E_{gap}}{k \cdot {T_{STC}}^2}\right)} \tag{13}$$

In order to calculate I_{sat}, the open circuit conditions can be used as well, so that from (5) it is:

$$I_{sat,STC} \approx I_{ph,STC} \cdot e^{-\frac{V_{OC,STC}}{\eta \cdot n_s \cdot V_{t,STC}}} \tag{14}$$

so that, the temperature coefficient C in (9) can be evaluated and I_{sat} can be calculated at the desired value of the temperature T.

$$C = \frac{I_{sat,STC}}{{T_{STC}}^3 \cdot e^{-\frac{E_{gap}}{k \cdot T_{STC}}}} \tag{15}$$

In conclusion, the two resistances remain undetermined. The following section shows the details of the novel approach proposed in this paper.

III. R_s AND R_p IDENTIFICATION BY EXPLICIT FORMULAS

In the sequel, just for simplifying the notation, to the values calculated in STC, that are $V_{MPP,STC}, I_{MPP,STC}, V_{t,STC}, I_{sat,STC}$, the subscript STC has been removed.

The following change of variable is firstly fixed:

$$x = \frac{V_{MPP} + R_s \cdot I_{MPP}}{n_s \eta V_t} \tag{16}$$

This allows to express the series resistance as:

$$R_s = \frac{x n_s \eta V_t - V_{MPP}}{I_{MPP}} \tag{17}$$

The parallel resistance, from (1), can be expressed as:

$$R_p = \frac{V_{MPP} + I_{MPP} \cdot R_s}{I_{ph} - I_{MPP} - I_{sat} \cdot \left(e^{\frac{V_{MPP} + I_{MPP} \cdot R_s}{n_s \eta V_t}} - 1\right)} \tag{18}$$

so that:

$$R_p = \frac{x n_s \eta V_t}{I_{ph} - I_{MPP} - I_{sat} \cdot (e^x - 1)} \tag{19}$$

At the MPP the following condition holds:

$$\frac{\partial (V \cdot I)}{\partial V}\bigg|_{MPP} = 0 \rightarrow I_{MPP} + V_{MPP} \cdot \frac{\partial I}{\partial V}\bigg|_{MPP} = 0 \tag{20}$$

where all the quantities are referred to STC. By using (1) it results that:

$$\frac{\partial I}{\partial V}\bigg|_{MPP} = -\frac{\frac{1}{R_p} + \frac{I_{sat}}{n_s \eta \cdot V_t} \cdot e^{\frac{V_{MPP} + I_{MPP} \cdot R_s}{n_s \eta V_t}}}{1 + \frac{R_s}{R_p} + \frac{Rs \cdot I_{sat}}{n_s \eta \cdot V_t} \cdot e^{\frac{V_{MPP} + I_{MPP} \cdot R_s}{n_s \eta V_t}}} \tag{21}$$

so that (20) gives:

$$I_{MPP} - V_{MPP} \cdot \frac{\frac{1}{R_p} + \frac{I_{sat}}{n_s \eta \cdot V_t} \cdot e^{\frac{V_{MPP} + I_{MPP} \cdot R_s}{n_s \eta V_t}}}{1 + \frac{R_s}{R_p} + \frac{Rs \cdot I_{sat}}{n_s \eta \cdot V_t} \cdot e^{\frac{V_{MPP} + I_{MPP} \cdot R_s}{n_s \eta V_t}}} = 0 \tag{22}$$

At this point, as shown in [2], the non linear system of equations obtained by collecting (18) and (22) must be solved. Its convergence is not guaranteed and the problem of choosing a good guess solution is open.

Instead, by substituting (17) and (19) in (22) and by neglecting the term small quantity R_s^2, a few algebra steps lead to:

$$\eta V_t x^2 \approx -\frac{V_{MPP}^2}{n_s \eta V_t} + 2 \cdot V_{MPP} \cdot x \tag{23}$$

Consequently, it results that solving (22) means solving (24) with respect to x:

$$2V_{MPP} \cdot (I_{MPP} - I_{ph} - I_{sat}) + (I_{ph} + I_{sat})\, n_s \eta V_t \cdot x + \\ + I_{sat} \cdot e^x \left[-n_s \eta V_t x + V_{MPP} \cdot \left(2 - \frac{V_{MPP}}{n_s \eta V_t}\right)\right] = 0 \tag{24}$$

The first two terms in (24) can be simplified, by accounting for (16), with $I^* = I_{ph} + I_{sat}$ as follows:

$$\begin{aligned} & 2V_{MPP} \cdot (I_{MPP} - I^*) + I^* n_s \eta V_t x = \\ & = V_{MPP} \cdot (2 \cdot I_{MPP} + I^*) + \\ & + I^* \cdot (R_s \cdot I_{MPP} - 2 \cdot V_{MPP,STC}) \approx \\ & \approx V_{MPP} \cdot (2I_{MPP} + I^*) - 2 \cdot I^* \cdot V_{MPP} = \\ & = V_{MPP} \cdot (2 \cdot I_{MPP} - I^*) \end{aligned} \tag{25}$$

The approximation adopted in (25) is justified by the fact that, for a large class of PV modules on the market, the quantity $R_s \cdot I_{MPP,STC}$ is almost two orders of magnitude smaller than $2 \cdot V_{MPP,STC}$. In [3], the series resistance value as well as the current and voltage values in the MPP have been reported for a large number of commercial modules. Looking at those numbers it is easy to verify that the assumption made above is reasonable.

In this way, the non linear equation (24) can be written as follows:

$$V_{MPP} \cdot (2 \cdot I_{MPPC} - I_{ph}) + \\ + I_{sat} e^x \left[-n_s \eta V_t x + V_{MPP} \cdot \left(2 - \frac{V_{MPP}}{n_s \eta V_t} \right) \right] = 0 \quad (26)$$

This equation admits an analytical solution based on the use of the Lambert W function [6], which is the solution of the equation:

$$f(x) = x \cdot e^x \quad (27)$$

Finally, it is:

$$x = W \left[\frac{V_{MPP} (2 I_{MPP} - I_{ph}) e^{\frac{V_{MPP}(V_{MPP} - 2 n_s \eta V_t)}{n_s \eta^2 V_t^2}}}{n_s \eta I_{sat} V_t} \right] + \\ + 2 \frac{V_{MPP}}{n_s \eta V_t} - \frac{V_{MPP}^2}{n_s \eta^2 V_t^2} \quad (28)$$

In conclusion, if the approach proposed in [1] and [2] is used, the five parameters of the single diode model are determined in STC by using the equations (4), (13), (14) and by solving the system consisting of (18) and (22).

The approach proposed in this paper, instead, does not require the solution of any non linear system, thus needing the use of (4), (13), (14) and the substitution of the value obtained by (28) in (17) and (19), so that the values of the parallel and series resistances of the PV module are calculated.

IV. Simulation results

A first application example concerns the Suntech STP260-24/Vb-1 PV module, whose parameters are listed in table I.

Table I
SUNTECH STP245S-20/WD: DATA SHEET VALUES

PV array	Values @ STC
Short-circuit current I_{SC}	8.09 A
Open-circuit voltage V_{OC}	44.0 V
MPP current I_{mpp}	7.47 A
MPP voltage V_{mpp}	34.8 V
Temperature coefficient of I_{SC} (α_I)	0.055 %/^{o}C
Temperature coefficient of V_{OC} (α_V)	-0.34 %/^{o}C
Number of cells in series	72

The model parameters obtained applying the procedure described in this paper are listed in Table II and in Table III. A comparison with data already published in literature reveals a good agreement with the values of the reciprocal of the slopes of the I-V curve in open circuit and short circuit conditions.

Table II
FIRST THREE PARAMETERS OF THE PV CIRCUITAL MODEL

Parameter	Value
$I_{ph}[A]$	8.09
η	1.0283
$I_{sat}[A]$	$7.1957 \cdot 10^{-10}$

Table III
RESISTANCE VALUES COMPARISON

	Explicit	N-L system	Method [3]	Meas. [3]
$R_s[\Omega]$	0.4622	0.4952	0.61	0.40
$R_p[\Omega]$	168.96	224.05	188	227

A further validation is proposed by comparing the results obtained by means of the numerical procedure proposed in this paper and those ones presented in [4]. Such a validation considers a Kyocera KC200GT PV module, which parameters in STC are given in Table IV, obtaining the parameters values reported in Table IV.

Table IV
KYOCERA KC200GT: DATA SHEET VALUES

PV array	Values @ STC
Short-circuit current I_{SC}	8.21 A
Open-circuit voltage V_{OC}	32.9 V
MPP current I_{mpp}	7.61 A
MPP voltage V_{mpp}	26.3 V
Temperature coefficient of I_{SC} (α_I)	0.0032 V/K
Temperature coefficient of V_{OC} (α_V)	-0.1230 A/K
Number of cells in series	54

Table V
PARAMETERS VALUES COMPARISON

	Explicit	N-L system	Method [4]
$I_{ph}[A]$	8.21	8.21	8.214
η	1.0755	1.0755	1.3
$I_{sat}[A]$	$2.1546 \cdot 10^{-9}$	$2.1546 \cdot 10^{-9}$	$9.825 \cdot 10^{-8}$
$R_s[\Omega]$	0.2844	0.3052	0.221
$R_p[\Omega]$	157.536	214.171	415.405

With such values, both the explicit model discussed in this paper as well as the calculation of the two resistances by means of the non linear system of equations allows to obtain a current vs. voltage curve that is almost coincident with that one obtained by using the parameters reported in [4]. This is evident by looking at Fig. 2.

The curves in Fig. 2 have been reported in the P-V plane and the percentage errors with respect to the curve obtained through the set of parameters given in [4] has been calculated. These errors in terms of power are plotted in Fig.3: they are below the threshold of 1% in the neighborhood of the maximum power point.

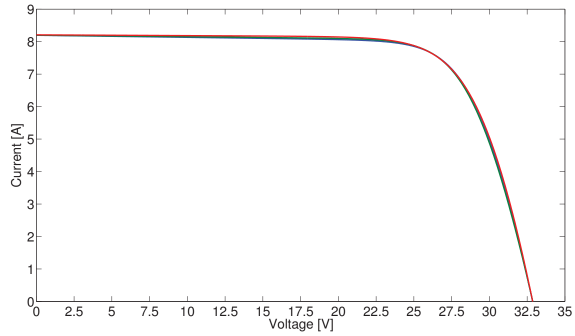

Figure 2. Current vs. voltage curves of the Kyocera PV module: the curves obtained by means of the three sets of parameters given in Tab.V are indistinguishable.

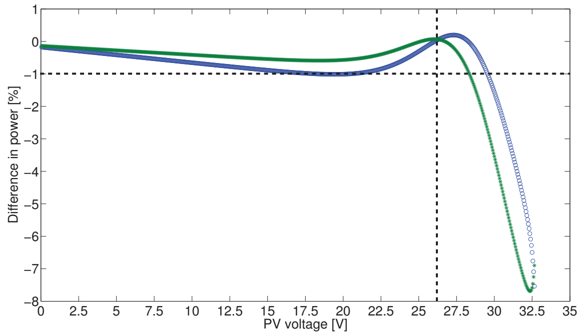

Figure 3. Percentage errors in term of power between the curves obtained by the parameters in the first two columns of Tab.V and the curve corresponding to the set of parameters in the third column of Tab.V. The horizontal dashed line gives the -1% threshold. The vertical dashed line puts into evidence the voltage at which the MPP occurs.

V. Conclusions

In this paper a novel symbolic approach for the calculation of the values of the series and parallel resistances in single diode model of photovoltaic modules is proposed. This calculation usually requires the solution of a non linear system of equations, but the approach proposed in this manuscript allows a direct calculation of these two values. The symbolic method uses the Lambert W function that allows to manipulate exponential expression and is the key tool for obtaining the explicit expression of the two resistances.

VI. Acknowledgments

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