parts, but also offers a degree of standardization combined with flexibility and a ready adaptation to civil or military purposes not possible using earlier equipment practice.

Although this article has dealt specifically with the application of UCP to line-transmission equipment, it is thought that the design principles of the new practice could be applied with advantage in other fields of electronic engineering. Essentially the design approach is to analyze the particular application envisaged, by means of functional block schematics, to make sure that the various circuit functions occur sufficiently frequently to justify the method of attack, and then to design a minimum number of self-contained functional units which can be used over and over again as the "bricks" to build different equipments.

ACKNOWLEDGMENTS

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Switching Time in Junction Diodes and Junction Transistors*

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Summary—**The time in which a junction diode may be switched from forward to reverse conduction is of great importance in computing networks. By considering the behavior of the minority carriers in a diode in a representative switching circuit an approximate solution for the switching transient may be derived. The transient is separated into two phases: first, one of constant current, where the flow is limited by the external resistance, and second, a "collection" phase, where the current decays at a rate determined by the minority carrier lifetime and the dimensions of the diode. A critical parameter in the solution is the ratio of the short-circuit reverse current to the forward current before switching. The mathematical treatment is a boundary value solution of the minority carrier diffusion equations which is accomplished by the use of Laplace transformations. The duration of the two phases of current flow is determined for a planar junction, a hemispherical junction, and for a planar junction with junction-to-contact distance small compared to a diffusion length. The last treatment is extended to the junction transistor and the behavior of the collector current is calculated. The general results indicate that for a given minority carrier lifetime the last two of the three diode structures will give the smallest switching times. In addition it is found generally that the time is minimized by decreasing lifetime and increasing the ratio of reverse to forward current.**

INTRODUCTION

IN THE USE of junction diodes in computer-switching circuits the switching time of the diode from forward to reverse voltage is of great importance. The problem may be represented as follows: In Fig. 1, a forward current, I_f, is flowing in the diode; at time, $t=0$, the switch, S, is thrown to the right. The switching time may now be defined as that time in which the voltage, V_D, reaches 90 per cent of the battery voltage. (The final static resistance of the diode is assumed to be much greater than R_0.)

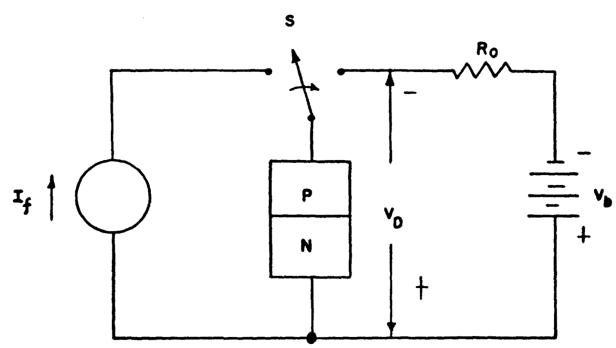

Fig. 1—Representative switching circuit.

PHYSICAL MODEL[1]

For simplicity, the treatment to follow is applied to a p-n junction where the conductivity of the p-type material is much greater than that of the n-type material. Practical embodiments of such a model are the familiar bonded n-type diodes and indium alloy-process devices. This limitation gives the qualitative picture of the carrier and potential distribution shown in Fig. 2. Here, the energy-band representation shows a forward current, I_f, proportional to the negative gradient of the hole concentration at the right of the barrier. The electron density in the p-type material is negligible since $\sigma_p \gg \sigma_n$ and is therefore neglected.

Given the above initial picture of the junction, it is now necessary to consider the behavior of the system after $t=0$, when the diode is switched into the reverse-biasing circuit. To anticipate the results it may be stated immediately that the initial transient in the circuit will be a constant current, given by V_b/R_0. That is, for a reasonable length of time after switching, the voltage

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[1] The physical description follows the methods and notation of W. Shockley, "Electrons and Holes in Semiconductors," D. Van Nostrand Co., Inc., New York, N. Y.; 1950, or "The theory of p-n junctions in semiconductors and p-n junction transistors," *Bell Sys. Tech. Jour.*, vol. 28, p. 435; 1949.

across the junction will be small compared to the battery voltage; therefore, the current will be completely determined by the series resistance. (A practical circuit might have a battery voltage of 20 v and a series resistance of 20,000 ohms.) It should be emphasized that the voltage across the junction (the "space charge" voltage) does not change abruptly at $t=0$; actually, it is a dependent function of the hole density at the barrier which in turn is determined by the flow of diffusion current across the barrier. If the term p_{n0} in Fig. 2, b is neglected, the behavior of the hole density and space-charge potential may be represented as in Fig. 3.

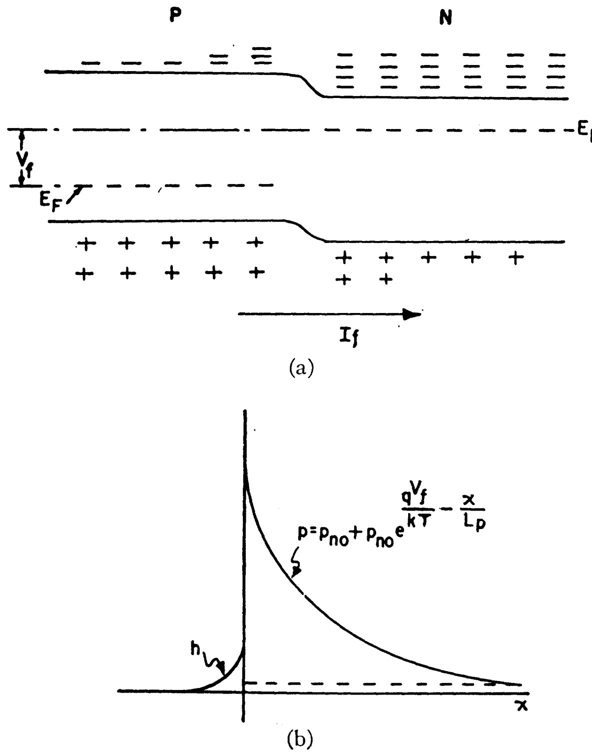

Fig. 2—(a) Energy bands; and (b) Minority-carrier densities for forward biased p-n junction.

Between $t=0$ and T_I, a constant current, $I_r=V_b/R_0$, flows to the left across the barrier; the junction voltage is given by $kT/q \ln p_0/p_{n0}$ plus a small ohmic contribution which is neglected. The hole density, p_0, at the boundary may be determined as a function of time from a solution of the diffusion equation. (The dashed lines represent the functions used in the solutions below.) At or near T_I, the hole density approaches zero, V tends toward minus infinity and a new set of conditions now holds. These are that the hole density at the barrier is practically zero, therefore V is now much larger than kT/q and the diffusion current is no longer constant. The behavior of the hole density is now calculated according to this condition, $(p(0)=0)$, and $J=-Ddp/dx$ gives the current as a function of time. It might be noted that the electron current at the barrier is neglected throughout the treatment. This assumption is found to be valid since any electron flow must come either from the space charge region or the p-type material. The former is unlikely since the space-charge current, $I_n=CdV/dt$, is found to be negligible compared to the diffusion current, and the latter is ruled out by the high conductivity of the p-type material. The remainder of the treatment is concerned with the mathematical solution of the diffusion equations subject to the boundary conditions.

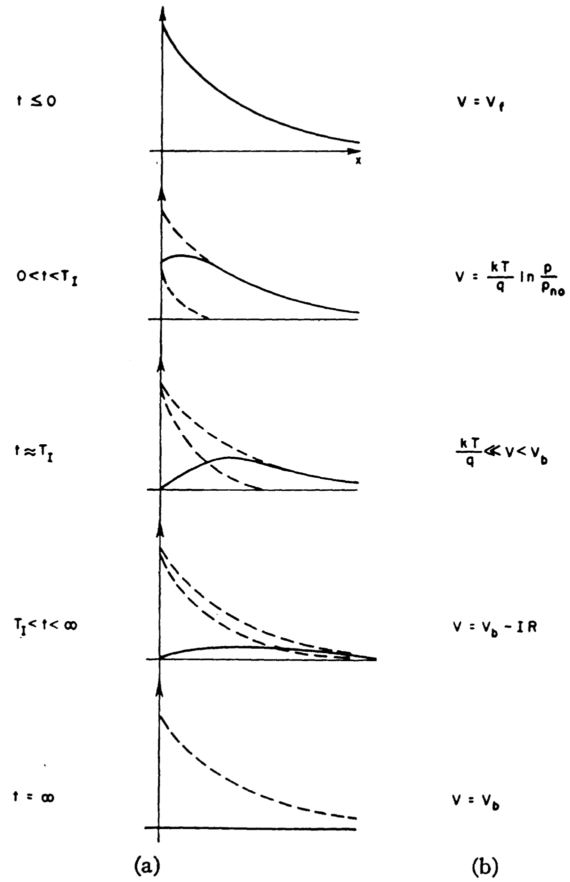

Fig. 3—(a) Hole density, and (b) junction potential, during switching process.

Mathematical Treatment[2]

Three cases are to be treated mathematically subject to the above criteria. They are the planar diode, the hemispherical diode, and the narrow-base diode. These are represented in Fig. 4 and discussed as treated.

A. Planar Diode

Planar junction with length of n-type material much greater than a diffusion length. (Fig. 4a. $W \gg L_p$.) The diffusion equation

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \frac{p}{\tau_p} \tag{1}$$

may be rewritten to read

$$\frac{\partial p}{\partial T} = \frac{\partial^2 p}{\partial X^2} - p \tag{2}$$

[2] B. Lax and S. F. Neustadter (to be published) have treated the planar junction more rigorously than the treatment to follow, and their results are in good agreement with the approximations used in this paper.

where

$$T = \frac{t}{\tau_p} \quad \text{and} \quad X = \frac{x}{L_p}\cdot$$

Since the initial distribution of holes at $t=0$ is a steady-state solution to the diffusion equation; a complete solution may be found by adding to this a new transient solution. The over-all solution for the constant-current phase should satisfy the boundary condition that $J(X=0)$ is constant and equal to $(-J_r)=(-V_b/AR_0)$,

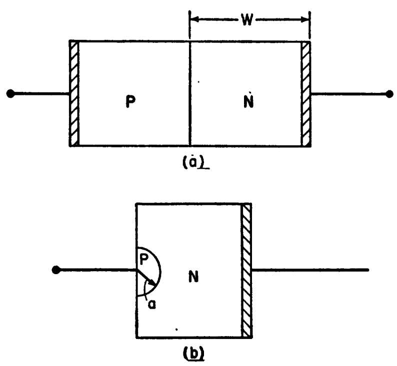

Fig. 4—Junction models used for calculation.

where A is the junction area. Now the current due to the steady state solution is J_f; therefore, the transient solution should satisfy the diffusion equation and the boundary condition that $J(X=0)=(J_f+J_r)$. When this solution, which we call $p_I(X, T)$ is subtracted from the initial steady-state solution, sketched in Fig. 3, required answer is obtained, since now $J(X=0)=J_f-(J_f+J_r)$ $=-J_r$. By Laplace transformation method[3]

$$P_I = \frac{(J_f+J_r)L_p}{D_p} I(X, \sqrt{T})$$

where

$$I(x, z) \overset{\Delta}{=} \frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}e^{-x^2/4u^2}du, \tag{3}$$

$$I(0, z) = \operatorname{erf} z$$

and

$$I(x, \infty) = e^{-x}.$$

The diode voltage for $0<T<T_I$ is thus given by

$$V = \frac{kT}{q}\ln\frac{p_0 - p_I(0, T)}{p_{n0}} \tag{4}$$

which is plotted in Fig. 5 for several values of J_r/J_f. As may be seen from the graph the voltage across the space charge region remains of the order of kT/q (0.026 v at room temperature) until a time very near T_I when it decreases very rapidly to minus infinity. It is this low value of voltage up to T_I which validates the assumption of constant current during the first phase of the diode transient. The time, T_I, is found by equating the transient solution at $X=0$, to the steady-state value of

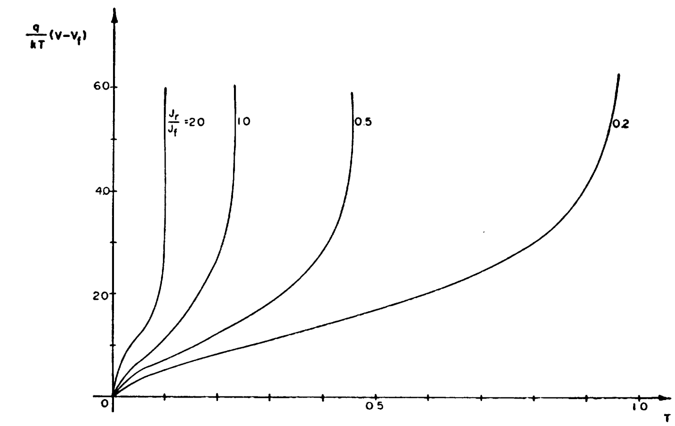

Fig. 5—Junction voltage during constant-current phase.

the hole density, p_0, giving

$$\operatorname{erf}\sqrt{T_I} = \frac{1}{1+J_r/J_f} \tag{5}$$

since

$$J_f = \frac{D_pP_0}{L_p}\cdot \tag{6}$$

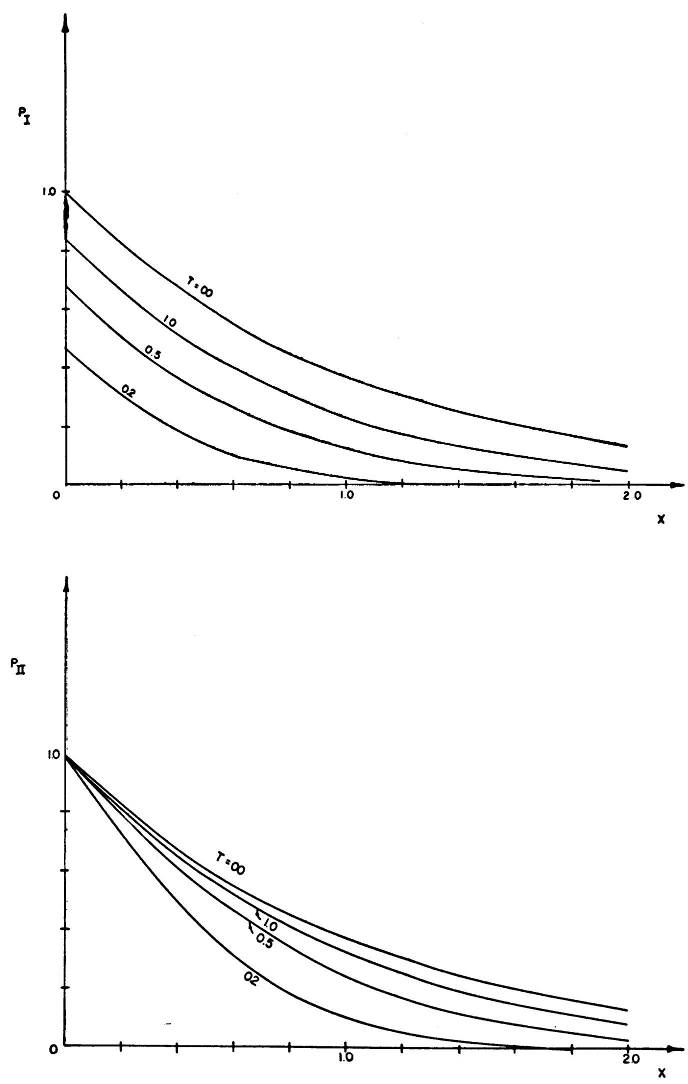

Fig. 6—The functions, p_I and p_{II}, for the planar junction.

[3] See Appendix for a discussion of the mathematical treatment.

For the remainder of the solution, the diffusion equation is solved subject to the boundary condition that $p(X=0)=0$ for all T, or specifically, $p_{II}(0, T)$ is constant and equal to p_0. This gives

$$p_{II} = \frac{J_f L_p}{D_p}\left[e^{-X} - I\left(X, \frac{X}{2\sqrt{T}}\right)\right] \tag{7}$$

and

$$J_{II} = -D_p\left(\frac{\partial p_{II}}{\partial X}\right)_{X=0} = J_f\left[\operatorname{erf}\sqrt{T} + \frac{e^{-T}}{\sqrt{\pi T}}\right]. \tag{8}$$

For the complete solution of the problem, p_{II} when $J_{II}=(J_f+J_r)$ is assumed to equal p_I at $T=T_I$. Actually, the magnitude and slope of p_I and p_{II} at $(X=0)$ are set equal at the end of the constant-current phase. Fig. 6 shows the normalized functions from which it may be

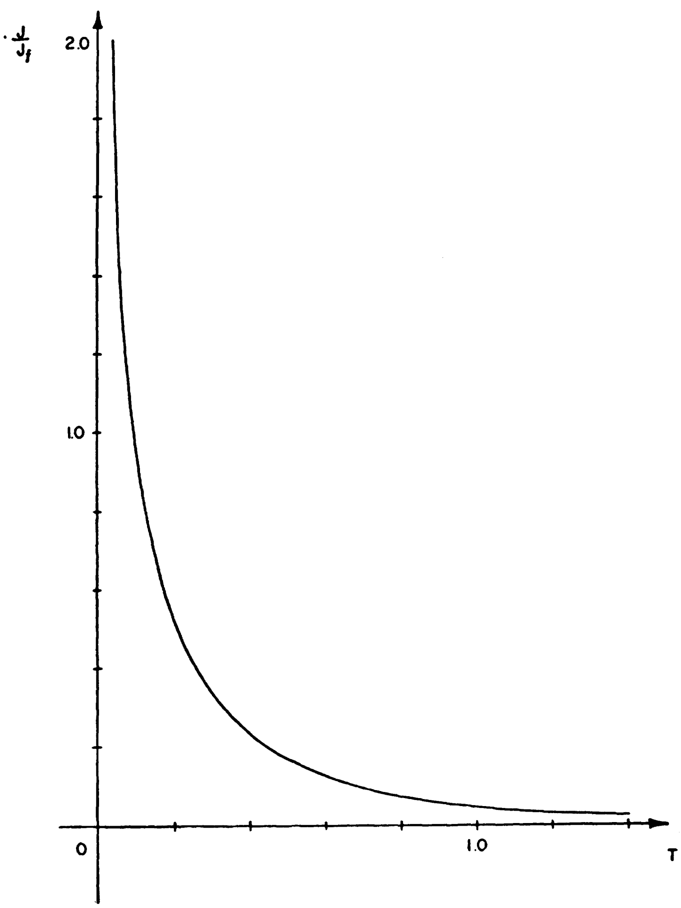

Fig. 7.—Junction reverse current after $T=T_I$.

seen that the above approximation is reasonable. If the true diffusion current $(J_{II}-J_f)$ is now plotted[4] as in Fig. 7, the reverse current after $T=T_I$ is given by this curve starting at $J=J_r$. The current previous to this time is, of course, J_r, from $T=0$ to $T=T_I$. (Note that the time scale may be shifted in the two functions p_I and p_{II}.)

[4] This particular equation has been derived previously by Lax and Neustadter, op. cit.; R. G. Shulman and M. E. McMahon, "Recovery currents in germanium p-n junction diodes," paper presented at the AIEE Winter General Meeting, New York, N. Y., January 20, 1953, also *Jour. Appl. Phys.*, vol. 24, p. 1267, 1953; and E. M. Pell, "Recombination rate in germanium by observation of pulsed reverse characteristics," *Phys. Rev.*, vol. 90, p. 228; 1953.

B. Hemispherical Diode

Semi-infinite n-type material. (Fig. 4b) The diffusion equation in spherical co-ordinates becomes

$$\frac{\partial u}{\partial T} = \frac{\partial^2 u}{\partial R^2} - u \tag{9}$$

where $u=rp$, $R=r/L_p$, and $T=t/\tau_p$. The solution for the first phase is then

$$p_I = \frac{(J_f+J_r)}{D_p}\left\{\frac{A^2}{A^2-1} I(R-A, \sqrt{T}) + \frac{A}{A^2-1}\left[e^{T(1/A^2-1)}e^{(R-A/A)} \operatorname{erfc}\left(\frac{R-A}{2\sqrt{T}} + \frac{\sqrt{T}}{A}\right) + I\left(R-A, \frac{R-A}{2\sqrt{T}}\right) - I(R-A, \infty)\right]\right\} \tag{10}$$

where $A=a/L_p$. Now the forward current is given by

$$J_f = \frac{Dp_0}{L_p}\left(\frac{L_p}{a}+1\right) \tag{11}$$

therefore, at $R=A$, the hole density becomes zero when $p_{II}=p_0$ or

$$\frac{1}{1+J_r/J_f} = \frac{A}{A-1}\operatorname{erf}\sqrt{T} + \frac{1}{1-A}\left[1 - e^{T(1-A^2/A^2)} \operatorname{erfc}\left(\frac{\sqrt{T}}{A}\right)\right]. \tag{12}$$

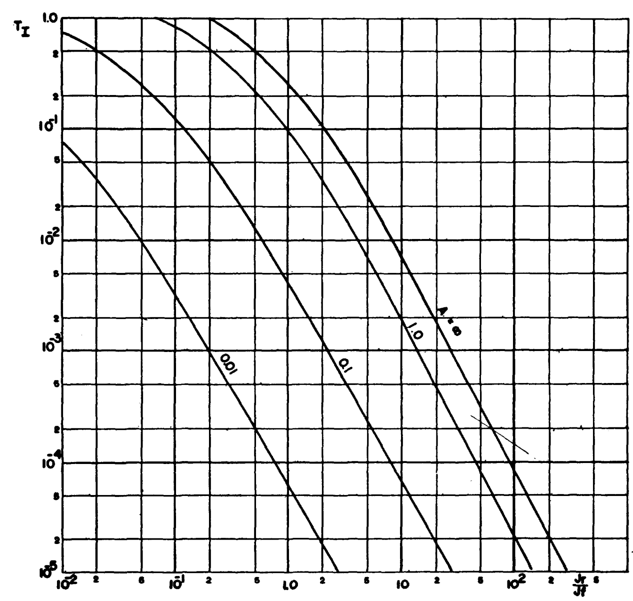

Fig. 8—Storage time, T_I, as a function of A and J_r/J_f.

This relationship is plotted in Fig. 8, which also gives the storage time for the planar case if A is set equal to infinity. The transient solution for the decaying phase is determined as in the linear case giving

$$p_{II} = p_0 \frac{a}{r}\left[I(R - A, \infty) - I\left(R - A, \frac{R - A}{2\sqrt{T}}\right)\right] \quad (13)$$

and

$$J_{II} = \frac{D_p p_0}{a} + \frac{D_p p_0}{L_p}\left[\operatorname{erf} \sqrt{T} + \frac{1}{\sqrt{\pi T}} e^{-T}\right]. \quad (14)$$

Subtracting J_f from the above equation gives the net diode reverse current

$$J_{II} - J_f = \frac{A}{A + 1}\left[\operatorname{erf} \sqrt{T} - 1 + \frac{1}{\sqrt{\pi T}} e^{-T}\right] \quad (15)$$

after use of (11). This is exactly the same equation as that plotted in Fig. 7, after multiplication by the factor $A/(A+1)$, and the method of determining the decay current is as in the previous solution. Fig. 9 shows the general curve for the decay time during the second phase as a function of the current ratio, J_r/J_f, and the radius of the contact, $A = a/L_p$. Here the decay time is defined as that in which the current falls from J_r to 10 per cent of J_r. This is the same time as that required for the diode-voltage to reach 90 per cent of the battery value.

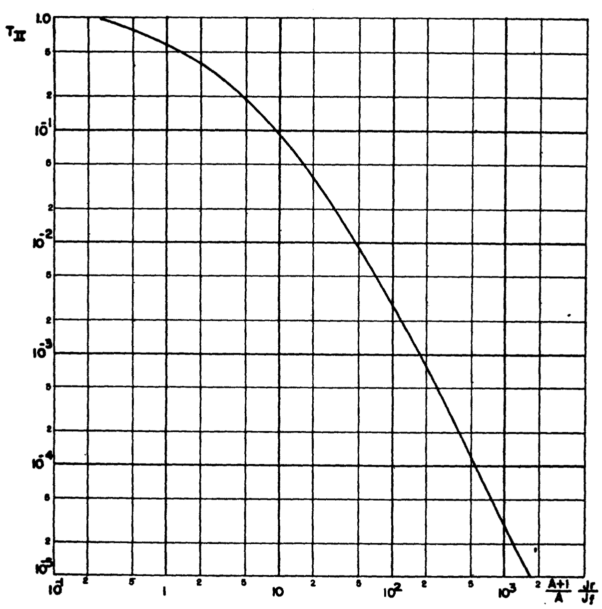

Fig. 9—Decay time, T_{II}, as a function of A and J_r/J_f.

C. Narrow-Base Diode

Width of n-type material, W; lifetime assumed infinite. (Fig. 4a. $W \ll L_p$.)

Here the boundary conditions will be slightly different than the two preceding treatments. If the ohmic contact to the diode is assumed to be a sink for holes the hole density at $x = W$ must be zero at all times. This is not generally true for all diode contacts but the calculation will be made on this basis since it is directly applicable to the junction transistor where the collector junction is, almost by definition, a sink for holes.

The diffusion equation may be modified to read:

$$\frac{\partial p}{\partial T} = \frac{\partial^2 p}{\partial X^2} \quad (16)$$

where $T = D_p t/W^2$ and $X = x/W$. By the previous methods the constant-current phase solution is found to be

$$p_I = \frac{(J_f + J_r)W}{D}\left(\frac{2}{\pi^2}\right)\sum_{n=0}^{\infty} \frac{(-1)^n(1 - e^{-\pi^2 T(n+1/2)^2})}{(n + \frac{1}{2})^2} \cdot \sin \pi(n + \tfrac{1}{2})(1 - X) \quad (17)$$

and the solution for the decaying phase with the hole-density constant at $X = 0$, is

$$P_{II} = \frac{J_f W}{D}\left[1 - X + \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n e^{-\pi^2 n^2 t}}{n} \cdot \sin \pi n(1 - X)\right]. \quad (18)$$

These solutions are similar to those plotted in Fig. 6 except that they vanish at $X = 1$. The net decay current at the barrier and at the collector or ohmic contact, $(X = W)$, is plotted in Fig. 10.

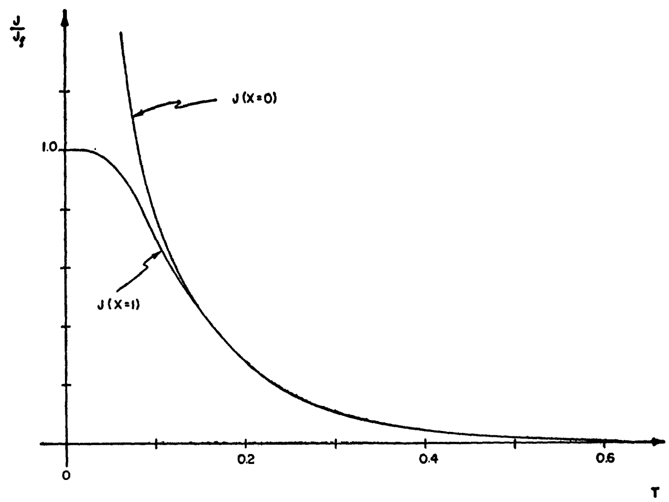

Fig. 10—Net current flow during decaying-current phase (narrow-base).

The decay time T_{II}, defined as before, is plotted in Fig. 11 with the constant-current phase time, T_I. Also shown on the same plot is T_C, which is the time required for the collector current in a junction transistor, $(J_{II}(W) - J_f)$, to fall to 10 per cent of J_f, the initial collector current, after the emitter is switched from forward to reverse. The time, T_C, includes the time, T_I, for the constant-current phase. For $J_r/J_f \ll 1$, T_C is obtained from the equation

$$J_I(W) = (J_f + J_r)\left[1 - \frac{2}{\pi}\sum_{n=0}^{\infty} (-1)^n \frac{e^{-\pi^2 T(n+1/2)^2}}{(n + \frac{1}{2})}\right] \quad (19)$$

which is the transient current at $X = W$ for the constant-current phase.

Discussion of Results

The calculations for the planar diode are seen to be a special case of the hemispherical diode with an infinite radius of curvature. Most interesting in the resultant curves is the rapid decrease of the storage times, T_I and T_{II}, with decreasing junction radius. It should be emphasized that these calculations really give an upper limit on the switching times, since, as the radius decreases, the current density, in practical cases becomes so large that the recombination near the junction is no longer linear. The net result of this conductivity modulation should be a decrease in the effective lifetime caused by the large increase in majority carrier density required to neutralize the excess minority carriers. This does not mean that the optimum over-all design results are obtained with the smallest possible junction radius. It should be remembered that forward direction spreading resistance will increase not only with decreasing radius but also with decreasing lifetime, since effective radius for spreading resistance calculation is of the order of the junction radius plus diffusion length.

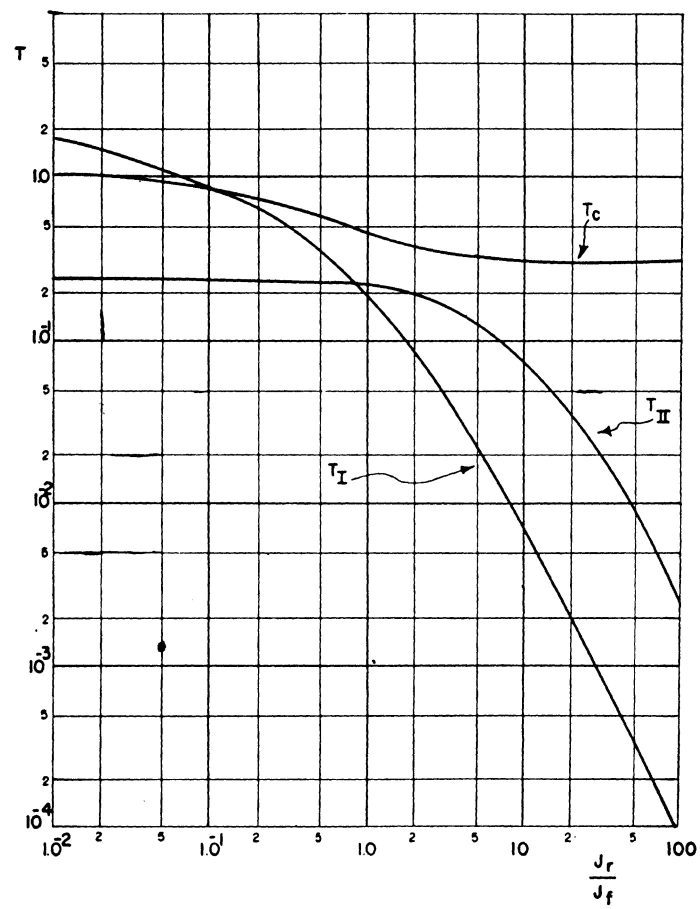

Fig. 11—Time constants, T_I, T_{II}, and T_C, for the narrow-base diode and junction transistor.

The resultant curves for the narrow-base diode are quite similar to the linear diode except for the scale. The normalized parameter, T, which was based on the lifetime in the first two cases is now based on the width of the semiconductor body. In fact, when considering the junction transistor application, the time variable may be written as

$$T = \frac{Dt}{W^2} = \left(\frac{D}{\pi W^2}\right)\pi t = \pi f_{c \cdot o} t \tag{20}$$

where f_{co} is the frequency cutoff for α as calculated from the diffusion equation.[5] In this connection, it should be remembered that the T_C curves give the behavior of the current source αI_e while the actual collector voltage will also be a function of the collector impedance. Similarly, if the time constant of reverse emitter capacitance and circuit resistance is comparable to calculated switching time of emitter, solution is no longer accurate.

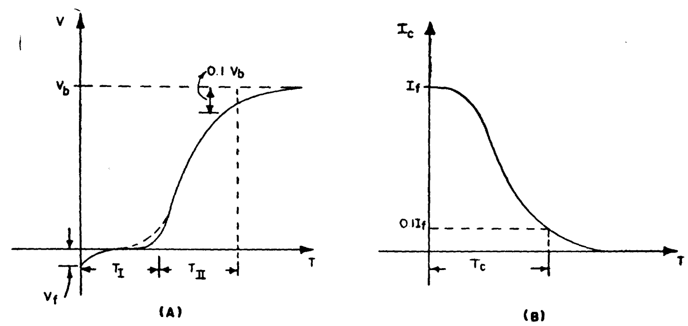

Fig. 12—Switching transients in junction diodes and transistors.

Conclusion

The voltage, V_D, across the diode in Fig. 1 will behave as shown in Fig. 12(a), where the times indicated are shown in Figs. 8, 9, and 11. If the diode is the emitter of a junction transistor, then the collector current will behave as in Fig. 11(b) where T_C is given in Fig. 10. The ratio J_r/J_f is also equal to V_B/I_fR_0.

Acknowledgment

The mathematical treatment was aided by the contributions of J. Keilson, S. F. Neustadter, G. C. Hunt, and the computing group of this laboratory. Of especial value were the suggestions and encouragement of E. Rawson, B. Lax, J. E. Thomas, and R. B. Adler.

Appendix

Mathematical Methods

The solution of the diffusion equation for the different boundary conditions was obtained by Laplace transformation in the time domain and solution of the resultant equation by standard techniques. For the linear and hemispherical diodes the appropriate inverse transforms were found in Magnus and Oberhettinger,[6] and Hameister.[7] The transforms for the narrow-base diode are again in Magnus and Oberhettinger.[6]

The integral of (3) of the text may be written

$$I(x, z) = \frac{\sqrt{\pi}}{4}\left[e^{-x}\operatorname{erfc}\left(\frac{x}{2z} - z\right) - e^{x}\operatorname{erfc}\left(\frac{x}{2z} + z\right)\right].$$

S. F. Neustadter of this laboratory is responsible for this form.

[5] W. Shockley, M. Sparks and G. K. Teal, "*P-n* junction transistors," *Phys. Rev.*, vol. 83, p. 151; 1951.
[6] W. Magnus and F. Oberhettinger, "Formulas and Theorems for the Special Functions of Mathematical Physics," Chelsea Pub. Co., New York, N. Y., p. 129 and p. 136; 1949.
[7] E. Hameister, "Laplace Transformation," Verlag von R. Oldenbourg, Munich and Berlin, Germany, p. 136; 1943.