The geometric resistivity correction factor for several geometrical samples

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Abstract: This paper reviews the geometric resistivity correction factor of the 4-point probe DC electrical conductivity measurement method using several geometrical samples. During the review of the literature, only the articles that include the effect of geometry on resistivity calculation were considered. Combinations of equations used for various geometries were also given. Mathematical equations were given in the text without details. Expressions for the most commonly used geometries were presented in a table for easy reference.

Key words: semiconductor; four point probe; conductivity measurement; resistivity correction factor
DOI: 10.1088/1674-4926/36/8/082001 **EEACC:** 2520

1. Introduction

There is a functional relationship between the tested resistivity (ρ) of a material and its geometric shape. Therefore, a geometric resistivity correction factor (G) is used for calculating the resistivity with $R = V/I$ equation. This correction factor changes with the thickness of the sample (t), geometric dimensions, the area of the surface (A), the position and the array of the probes on the sample[1]. The resistivity is given by Equation (1) as a general Equation[2].

$$\rho = \frac{V}{I}G, \tag{1}$$

where ρ is the resistivity in Ω·cm. The total resistance (R_{total}) value can be measured with the standard 2-probe measurement technique and this parameter is equal to the summation of the resistances of the sample (R_{sample}), the wires (R_{wire}), the probes (R_{probe}) and the conductive paste (R_{paste}). The paste is located between probe and sample. Therefore, calculated resistivity with this method is higher than the real resistivity of the sample. The difference between the real and calculated resistivity of a sample becomes more important when the resistivity of the probes are higher than the sample resistivity. The resistivity of the sample (R_{sample}) can be measured using the 4-probe technique. In this measurement technique, probes numbered 1 and 4 are used for measuring the current on the surface (I), probes numbered 2 and 3 are used for measuring the potential difference between any two points on the surface (Figure 1). Due to the lack of a current between probes 2 and 3, no voltages drop in this line and probes. In this measurement technique, the resistivity is given by Equation (2).

$$\rho = \frac{V_{23}}{I_{14}}G. \tag{2}$$

The measurement set-up that has in-line probe arrangement is a practical application (Figure 2). In this measurement set-up, s is the distances between probes in cm and $s_{12} = s_{23} = s_{34} = s$ in general. In this statement the determining and calculating of the G factor is easier.

The conductivity measurement is affected by several factors such as polarization, impurity, sample geometry, cable resistances, temperature and humidity[3]. According to the sample geometry and measurement technique one or more factors are dominant. Thus the resistivity measurement technique is decided with the condition.

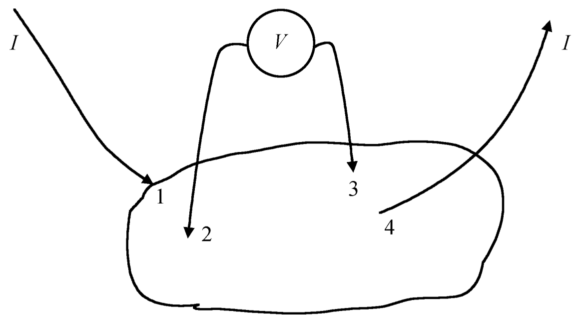

Figure 1. The measurement with random four probes on surface.

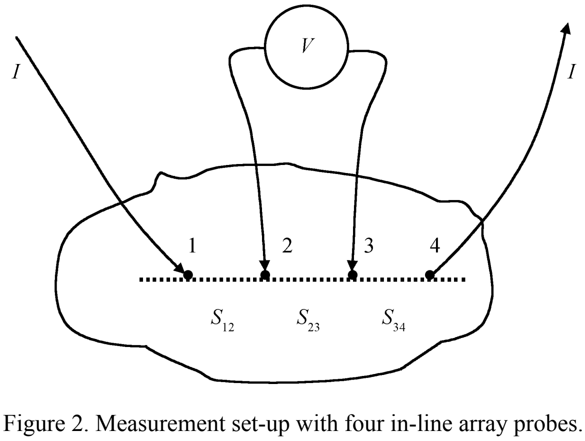

Figure 2. Measurement set-up with four in-line array probes.

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Received 7 January 2015, revised manuscript received 11 March 2015

Table 1. t and s relations commonly used for an infinite extent.

$t \gg s$		$t \ll s$	
Infinite thick	Thick	Thin (sheet)	Infinite thin (thin film)
$t \geqslant 5s$	$t \geqslant \frac{s}{2}$	$t < \frac{s}{2}$	$t < \frac{s}{10}$

2. Resistivity measurement of the semiconductors

The current density (J) is directly proportional to the electrical field magnitude (E) for the conductive material (Equation (3))[4].

$$\rho = \frac{E}{J} = \frac{E}{I/A}. \tag{3}$$

The work (W) applied by E to the charges which have flow through L way and between any randomly taken a and b points is given simply by Equation (4)[5, 6]. According to this expression, for determination of the sample's resistivity both electrical resistance and geometric extents should be known (Equation (4)).

$$W = -\Delta U = -(U_{\mathrm{b}} - U_{\mathrm{a}}) = -q\int_a^b E\mathrm{d}L, \tag{4}$$

$$\Delta V_{\mathrm{ab}} = V_{\mathrm{b}} - V_{\mathrm{a}} = -\frac{U_{\mathrm{b}} - U_{\mathrm{a}}}{q} = -\int_a^b E\mathrm{d}L = EL, \tag{5}$$

$$\rho = \frac{E}{J} = \frac{V_{\mathrm{ab}}/L}{I/A} = \frac{V_{\mathrm{ab}}}{I}\frac{A}{L} = \frac{V_{\mathrm{ab}}}{I}G. \tag{6}$$

Many semiconductor materials have a high resistivity. Because the resistance of semiconductors can rapidly decrease with temperature, the resistances of probes have become very important. The specially designed probes which have fine contact between the probe and the sample are used to measure resistivity. These probes have, in general, sharp points and a small body. When probes have a small body, they have weaknesses in regards to mechanical forces and high resistance. Thus in practice very hard materials are used to make probes such as osmium or tungsten alloys. Therefore, the resistance of probes increases when the resistivity is measured at high temperature. For this reason, the primary method used for resistivity measurements is the 4-point probe technique. However, in applications, the researcher should note that in order for the metal probes to not cause damage to the thin-film sample, the pressure of probes on the sample should be small. Nonetheless, adverse effects of the charge transfer diffusion are minimized[7].

3. Geometrical resistivity correction factor

The starting point of the geometrical factor calculation is most commonly the formulation of G for samples with infinite extent and thickness. The expression of the sample as thin or thick depends on the ratio between the thickness of the sample (t) and the spacing of the probes (s). Practically using the ratio of s and t for a slice of infinite extent is given in Table 1. In this table t and s is the sample thickness and the probe spacing in cm, respectively.

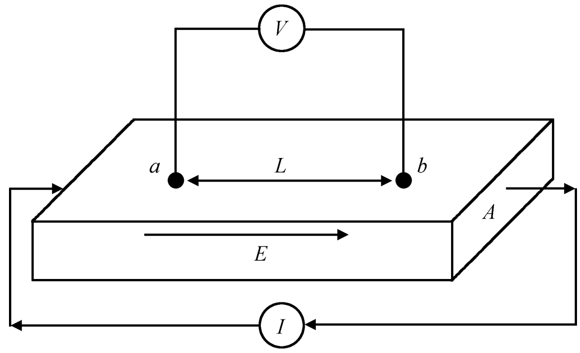

Figure 3. Electrical current on the conductive layer.

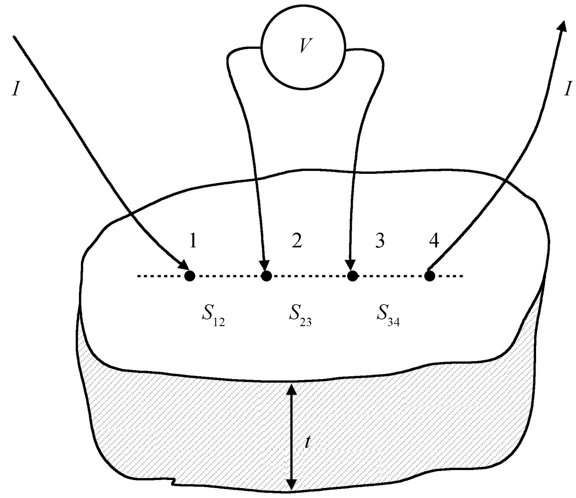

Figure 4. The sample with t thickness.

For a semi-infinite sample, the bulk resistivity is given by Equation (7)[2, 8, 9]. The semi-infinite sample is a plane that has an infinitesimal thickness and surface area. The probes are located on this plane.

$$\rho_{\mathrm{bulk}} = \frac{V}{I}G = \frac{V}{I}2\pi s. \tag{7}$$

Although infinite extensions, if the sample is a thin layer, the resistivity is computed as Equation (8)[1, 2, 10–15].

$$\rho = \frac{V}{I}G = \frac{V}{I}\frac{\pi}{\ln 2}t. \tag{8}$$

If the sample is an infinite sheet, t is negligible. Thus, the sheet resistivity is calculated from Equation (9)[2, 11].

$$\rho_0 = \rho_{\mathrm{sheet}} = \frac{V}{I}G = \frac{V}{I}\frac{\pi}{\ln 2}. \tag{9}$$

In Equation (9), ρ_{sheet} is resistivity of a sheet or an infinite thin sample surface (ρ_{surface}). In spite of the calculated unit of resistivity in Equation (9) being "Ω", t is negligible but its unit is in resistivity equation[11]. If the sample has a finite thickness and finite extent, new geometric corrections are added to Equation (7) as a factor.

3.1. Semi-infinite volume

If the sample is a semi-infinite volume, the resistivity expression does not include the parameters of t and surface area

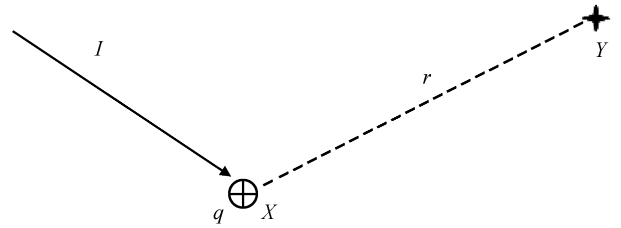

Figure 5. Y point away from single polar X point.

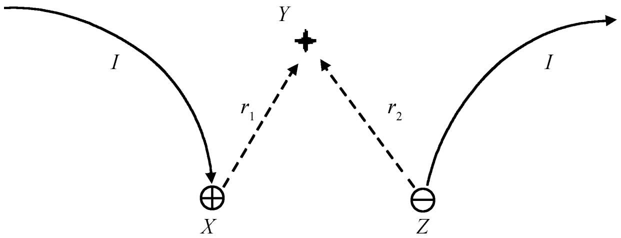

Figure 6. The Y point that r_1 and r_2 distance from electrical dipole.

for any shape. Distance between probes is s_{12}, s_{23}, s_{34}, respectively. Firstly, the current flows from X point to sample as a single polar (Figure 5). The charges are carried to Y point from this point. r is the distance between X and Y. The potential of Y is obtained as Equation (10) via calculating from Equation (5).

$$V_Y = V = -Er = -\frac{q}{r}. \tag{10}$$

Uhlir (1955) had given Equation (10) for infinite surface extent as below.

$$I = \frac{1}{\rho}AE = \frac{1}{\rho}2\pi r^2\frac{q}{r^2} = \frac{2\pi q}{\rho}. \tag{11}$$

In this expression the A is the surface area of a semi-sphere with infinite small r radius. The center of a semi-sphere is the contact point between the sample and the current probe. The V voltage for r distance from X point (Equation (12)) is obtained with combining Equations (10) and (11)[2, 8, 11, 16].

$$V = -\frac{I\rho}{2\pi}\frac{1}{r}. \tag{12}$$

In this situation, the voltage is expressed as Equation (13) for the dipole current source as seen in Figure 6. If the measurement is performed on four different points and sample as seen in Figure 7, voltage drop between probes 2 and 3 is described as Equation (14).

$$V_Y = \left(-\frac{I\rho}{2\pi}\cdot\frac{1}{r_1}\right) - \left(-\frac{I\rho}{2\pi}\cdot\frac{1}{r_2}\right) = -\frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right), \tag{13}$$

$$\Delta V_{23} = V_3 - V_2 = -\frac{I\rho}{2\pi}\left(\frac{1}{r_{31}} - \frac{1}{r_{34}} - \frac{1}{r_{21}} + \frac{1}{r_{24}}\right). \tag{14}$$

If the arrangement of electrical contacts is in-line array on the sample (Figure 2), potential difference is given as Equation (15) and the resistivity is obtained as Equation (16)[8, 16]. In these equations, only the s parameter is finite and measurable. The thickness or the surface geometry is approximately infinite. For this reason, the G only depends on the s. If distances of the probes are given as equal ($s_{12} = s_{23} = s_{34} = s$),

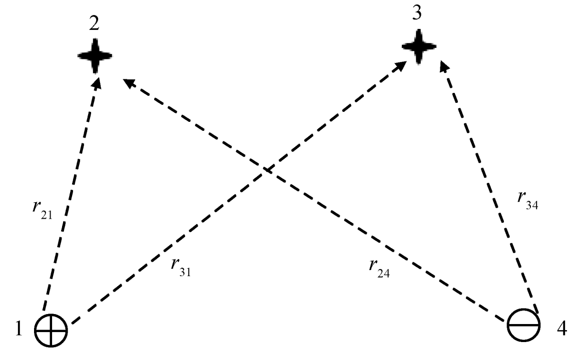

Figure 7. The two different points away from current dipole with r distance.

according to Equation (15) the correction factor is computed as Equation (17) for the semi-infinite volume[8, 16]. If the measurement set-up is organized for the $s = 1/2\pi = 0.159$ m, resistivity value is calculated as $\rho = V/I$.

$$\Delta V_{23} = V = -\frac{I\rho}{2\pi}\left(-\frac{1}{s_{12}} + \frac{1}{s_{23}+s_{34}} + \frac{1}{s_{12}+s_{23}} - \frac{1}{s_{34}}\right), \tag{15}$$

$$\rho = \frac{V}{I}\frac{2\pi}{\dfrac{1}{s_{12}} + \dfrac{1}{s_{34}} - \dfrac{1}{s_{12}+s_{23}} - \dfrac{1}{s_{23}+s_{34}}} = \frac{V}{I}G, \tag{16}$$

$$G = F_0(s) = 2\pi s. \tag{17}$$

3.2. Infinite surface area and t thickness

Figure 4 belongs to a slice of infinite extent and measurable t thickness. The G factor in resistivity expression should include parameter of the thickness for this sample. It supposes to $t \gg s$ ($t \geqslant s/2$ in practice) the sample is thick else ($t < s/2$ in practice) the sample is thin (Table 1).

3.2.1. Thick samples

The resistivity for the thick sample in $t \geqslant s/2$ is expressed as Equation (18)[2, 11, 12]. In this expression $F_1(t/s)$ is the extra geometrical correction factor for the finite t thickness sample and is expressed as Equation (20)[17]. This equation is more useful and simple than the one described by Uhlir (1955) (Equation (23)).

$$\rho = \frac{V}{I}\cdot F_0(s)\cdot F_1(t/s), \tag{18}$$

$$G = 2\pi s\cdot F_1(t/s), \tag{19}$$

$$F_1(t/s) = \frac{t/s}{2\ln\dfrac{\sinh(t/s)}{\sinh(t/2s)}}. \tag{20}$$

$F_1(t/s) \to 1$ while $t \to \infty$ (Table 2). With reference to this approach, extra geometrical correction factor can be given as $F_1(t/s) = 1$ for $t/s > 5$[5]. In this statement, the sample thickness can be accepted as semi-infinite for the $t > 5s$ thickness and geometrical factor is transform to Equation (17).

Table 2. $F_1(t/s)$ factor values versus t/s ratio.

t/s	$F_1(t/s)$
1	0.615
3	0.969
5	0.997
10	0.999

Table 3. The effect of the thickness.

t/s	$\frac{\sinh(t/s)}{\sinh(t/2s)}$
0.5	2.063
0.3	2.023
0.1	2.003

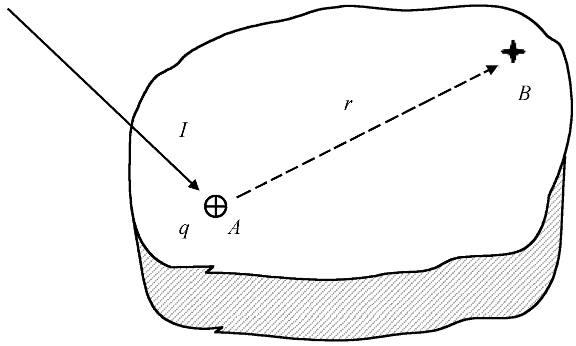

Figure 8. B reference point on the sample with semi-infinite extensions and t thickness.

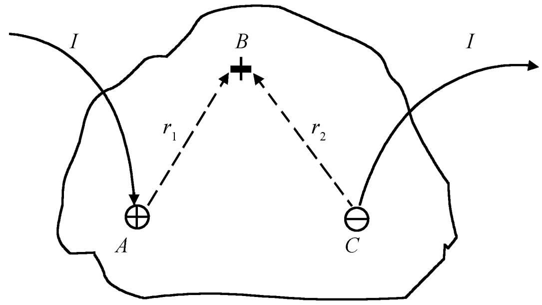

Figure 9. Randomly selected the B point that is r distance from current dipole A and C point.

3.2.2. Thin samples

$\frac{\sinh(t/s)}{\sinh(t/2s)} \cong 2$ can be acceptable for the finite and thin measurable slice and $t < s/2$ thickness (Table 3) and the function of $F_1(t/s)$ transform to Equation (21)[16].

$$F_1(t/s) \cong \frac{t/s}{2\ln 2}. \tag{21}$$

As can be seen in Table 3 and Equation (21), if $t < s/2$, s has great value compared with the thickness of the sample. In this case the resistivity expression for the thin sample which has three-dimensional geometry, infinitive extension and finite but small t thickness is as in Equation (22)[1, 2, 5, 10–16].

$$G = F_0(s) \cdot F_1(t/s) = \frac{\pi}{\ln 2} t. \tag{22}$$

Uhlir (1955) obtained the same result using a different method of calculation[16]. This study is the first in the literature that contains the geometric factor equation taking into account thickness. Because the current flows through a surface t units thick in an infinitely large volume, the volume of the sample is not considered to be infinite. In this case the resistivity expression should contain the thickness term and Uhlir expressed such as Equation (23)[16]. In this equation CD (correction divisor) is an additional correction factor which is derived for the finite thickness and $1/\text{CD} = F_{\text{Uhlir}}(s/t)$.

$$\rho = 2\pi s \frac{\Delta V}{I} \frac{1}{\text{CD}}, \tag{23}$$

$$G = 2\pi s \frac{1}{\text{CD}} = F_0(s) \cdot F_{\text{Uhlir}}(s/t). \tag{24}$$

Uhlir, solution of the $F_{\text{Uhlir}}(s/t)$ function for the $t < s/2$ given as $\cong 2\ln 2\frac{s}{t}$[16]. If this expression is used in Equation (24), Equation (22) is obtained again.

3.2.3. Infinitesimal thin samples

The resistivity correction factor for an infinitesimally thin ($t \cong 0$) and infinite extension was first reported by Smits (1958) and Vaughan (1961)[2, 11, 14]. The beginning expression was achieved for the resistivity equation. Figure 8 is a sample which has t thickness and semi-infinite extensions. A and B points are arbitrary locations on this sample. The A point is single polar current source and the B is any under the test point. Also r is the distance between points A and B. In this statement the potential difference between points A and B is described as Equation (25)[5, 6].

$$\Delta V = -\int E\,\mathrm{d}r = -\int \rho J\,\mathrm{d}r = -\frac{I\rho}{2\pi t}\ln r, \tag{25}$$

$$\Delta V = V - V_0 = -\frac{I\rho}{2\pi}\ln r. \tag{26}$$

If thickness is infinitesimal ($t \cong 0$), the potential difference is given as Equation (26) for a point on the slice[11, 14]. In this equation V is the measured potential and V_0 is the reference potential. The potential difference on the randomly selected single B point location for the dipole current source which is on the $t \cong 0$ slice is given as Equation (27) (Figure 9). On the other hand the potential difference on the randomly selected two point location on the $t \cong 0$ slice is expressed as Equation (28) (Figure 10).

$$V_{\text{B}} = -\frac{I\rho}{2\pi}\ln\frac{r_1}{r_2}, \tag{27}$$

$$\Delta V = V_3 - V_2 = -\frac{I\rho}{2\pi}\left(\ln\frac{r_{31}}{r_{34}} - \ln\frac{r_{21}}{r_{24}}\right) = \frac{I\rho}{2\pi}\ln\frac{r_{31}r_{24}}{r_{34}r_{21}}. \tag{28}$$

If the probe distances are equal ($s_{12} = s_{23} = s_{34} = s$) and four-probe points are in-line array (Figure 2), the voltage difference between probes 2 and 3 and the geometric contribution are obtained as Equations (29) and (30), respectively.

$$\Delta V = V_3 - V_2 = V = \frac{I\rho}{2\pi}\ln\frac{2s \cdot 2s}{s \cdot s} = I\rho\frac{\ln 2}{\pi}, \tag{29}$$

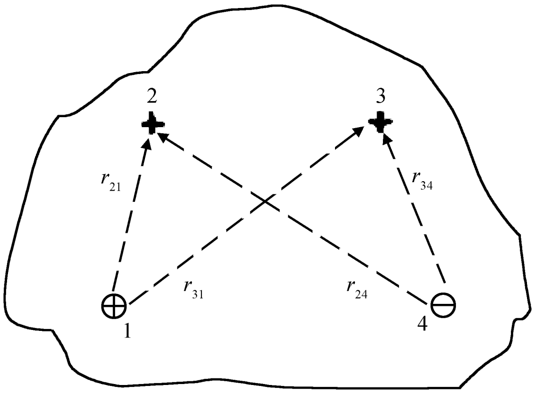

Figure 10. Randomly selected two points (1 and 2) that are r distance from current dipole 1 and 4 point.

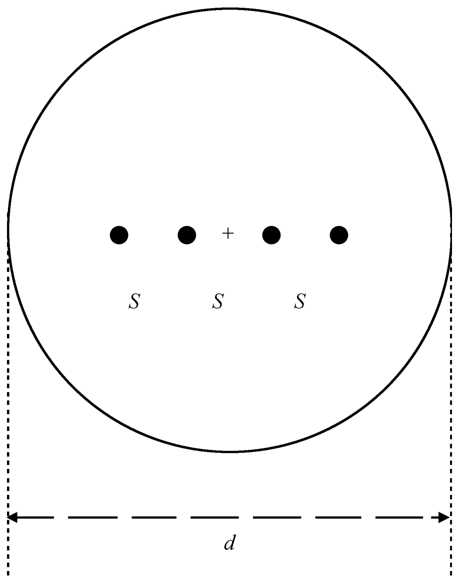

Figure 11. Infinitesimal thin and circular surface.

$$G = \frac{\pi}{\ln 2}. \tag{30}$$

3.3. Circular sample

In this part, a circular sample is observed for a slice of finite extensions, d radius and finitely or very thin thickness. The probes are in-line arrangements, centered on the circular surface and equal distances (s) on the sample. Again, the G changes depending on the t thickness of sample for the circular sample with finitely surface area. These different conditions on thickness should be investigated respectively.

3.3.1. Infinitesimal thin circular sample

Figure 11 indicates the circular sample with finite surface area and infinitesimal t thickness. So thickness is ignored. In this system, distance between probes (s) and statements of the probes on the sample are very important. A ratio function of circle radius to probe spacing contributes to the potential (Equation (31))[2, 11].

Table 4. Computed F_2 values for different d/s ratios.

d/s	$F_2(d/s)$
10	0.9203
50	0.9966
100	0.9991
1000	0.9999

$$V = \frac{I\rho}{\pi}\left[\ln 2 + \ln\frac{(d/s)^2+3}{(d/s)^2-3}\right]. \tag{31}$$

In this case the resistivity for the circular, $t < s/2$ and with d radius sample is expressed as Equation (32)[2, 11, 13].

$$\begin{aligned}\rho &= \frac{V}{I}\left\{\pi\left[\ln 2 + \ln\frac{(d/s)^2+3}{(d/s)^2-3}\right]^{-1}\right\}\\ &= \frac{V}{I}\frac{\pi}{\ln 2}\left[1+\frac{1}{\ln 2}\ln\frac{(d/s)^2+3}{(d/s)^2-3}\right]^{-1}.\end{aligned} \tag{32}$$

Then the resistivity equation and additional correction factor is given by Equations (33) and (34) for the circular sample, respectively[2, 10, 11, 13]. The solutions of $F_2(d/s)$ function are reported by Smits (1958) and Swartzendruber (1964) and results are given in Table 4[11, 12]. As seen in Table 4, if $(d/s) \to \infty$, $F_2(d/s) \to 1$. In this case the G expression transforms to Equation (30) for the $(d/s) \geqslant 40$ geometry. Equation (30) belongs to finitely s and semi-infinitive slice $(d \to \infty)$.

$$\rho = \frac{V}{I}G = \frac{V}{I}\left[\frac{\pi}{\ln 2}F_2(d/s)\right], \tag{33}$$

$$F_2(d/s) = \left[1+\frac{1}{\ln 2}\ln\frac{(d/s)^2+3}{(d/s)^2-3}\right]^{-1}. \tag{34}$$

Vaughan (1961) obtained the same equations for the circular samples using different mathematical methods. The resistivity is expressed below for an in-line arrangement of four-probes, assuming an infinitesimally thin (sheet) and a circular geometry[14].

$$\rho = \frac{V}{I}G = \frac{V}{I}\frac{2\pi}{\ln 4L}. \tag{35}$$

Here the term L is additional correction factor that is obtained by Vaughan (1961). This correction factor includes the t thickness, r radius and a function that includes coordinates of distance between the center of in-line arrangement probes and current dipole. According to the calculation in this publication, the result of L equation is equal to $L = 1$ for $t/r = 0$. In this instance $\lim_{t\to 0} L = 1$ namely the sample is infinitely thin. So the surface resistivity is obtained again as Equation (30) for the two dimension geometry and infinitely thin (thin film) samples.

$$\rho = \frac{V}{I}\frac{2\pi}{\ln 4} = \frac{V}{I}\frac{\pi}{\ln 2}. \tag{36}$$

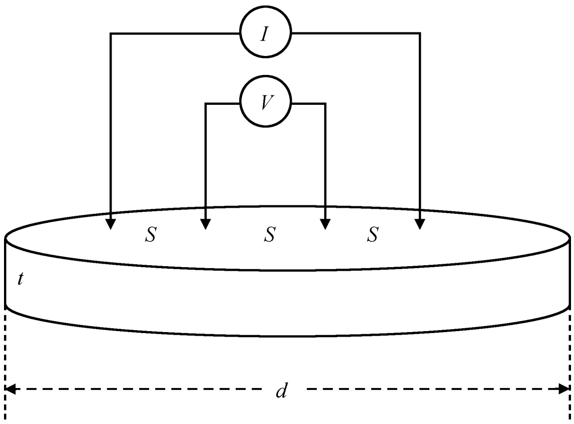

Figure 12. Thin circular sample with t thickness.

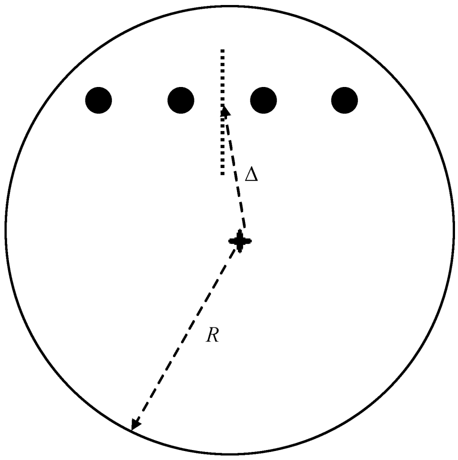

Figure 13. Placements of probes on circular sample.

3.3.2. Thin circular sample

The G function of a circular sample which has finite extensions and thin t thickness is given as below (Equation (37))[2, 10–13, 18].

$$\rho = \frac{V}{I}G = \frac{V}{I}\left[F_0(s)\cdot F_1(t/s)\cdot F_2(d/s)\right] = \frac{V}{I}\frac{\pi}{\ln 2}t\cdot F_2(d/s). \tag{37}$$

Equation (37) is the geometric factor that includes the effect of finitely t thickness (Equation (22)), s distances of probes and surface dimensions limited with d diameter ($F_2(d/s)$). In here, mean of the thin thickness is $t/s \ll 1$ and in practice $t/s < 0.5$. Swartzendruber (1964) reported the $F_2(d/s)$ function in general form[13]. Swartzendruber (1964) and Yamashita (1989) explained the resistivity expression for a circular sample as Equation (38).

$$\rho = \frac{V}{I}\frac{\pi}{\ln 2}t\frac{1}{1+\eta} = \rho_0\left(1+\eta\right)^{-1} = \rho_0\cdot F_2\left(d/s\right). \tag{38}$$

In Equation (38), the dimensionless η magnitude is reported by Swartzendruber (1964) and Yamashita (1989) performed by different calculations but with the same results as Equation (39)[13, 18]. In Equation (39), R and $\varDelta$ are radius ($d/2$) and distance between center of the in-line array probes (Figure 13) and circle, respectively. In Equation (39), placements of the probes on the samples are taken into account during the calculation of geometric factor. If the probes are placed on the circle center ($\varDelta = 0$), the equation of η is computed as Equation (40). In this case, if the η equation is used in resistivity expression (Equation (38)), Equation (34) is re-obtained.

$$\eta = \frac{1}{2\ln 2}\times \ln\frac{\left[1-\left(\frac{\varDelta}{R}+\frac{s}{2R}\right)\left(\frac{\varDelta}{R}-\frac{3s}{2R}\right)\right]\left[1-\left(\frac{\varDelta}{R}-\frac{s}{2R}\right)\left(\frac{\varDelta}{R}+\frac{3s}{2R}\right)\right]}{\left[1-\left(\frac{\varDelta}{R}-\frac{s}{2R}\right)\left(\frac{\varDelta}{R}-\frac{3s}{2R}\right)\right]\left[1-\left(\frac{\varDelta}{R}+\frac{s}{2R}\right)\left(\frac{\varDelta}{R}+\frac{3s}{2R}\right)\right]}, \tag{39}$$

$$\eta = \frac{1}{2\ln 2}\ln\frac{\left[1-\left(\frac{s}{d}\right)\left(-\frac{3s}{d}\right)\right]\left[1-\left(-\frac{s}{d}\right)\left(\frac{3s}{d}\right)\right]}{\left[1-\left(-\frac{s}{d}\right)\left(-\frac{3s}{d}\right)\right]\left[1-\left(\frac{s}{d}\right)\left(\frac{3s}{d}\right)\right]} = \frac{1}{2\ln 2}\ln\left[\frac{1+3\left(s/d\right)^2}{1-3\left(s/d\right)^2}\right]^2. \tag{40}$$

The most useful side of the η equation, it includes the effect of the probes array on the sample for all sample material shapes. As seen in Table 4, because $\lim\limits_{d\to\infty} F_2(d/s) = 1$, the geometric factor is given as $F_2(d/s) \cong 1$ for $d/s \geqslant 40$ geometry. In this instance the G factor is re-obtained as Equation (22) for finite extensions and thickness.

3.3.3. Thick circular sample

If t thickness is not small ($t/s \geqslant 0.5$), the volume resistivity expression is given as Equation (41)[47]. This expression includes Equations (17), (20) and (34) functions.

$$\rho = \frac{V}{I}\cdot F_0(s)\cdot F_1(t/s)\cdot F_2(d/s) = \frac{V}{I}G. \tag{41}$$

3.4. Rectangular sample

Yamashita (1987) investigated the effect of thickness of the probes and geometry of the rectangular prism sample on the geometric factor. The rectangular prism has small t thickness (Figure 14). The contact points between current probes numbered 1 and 4 (s_1 and s_4) and the sample are the equivalent centers of the current distribution in the current probes. The displacement δ is the distance between the center of the probes and the current distribution center. The probes numbered 2 and 3 (s_2 and s_3) are voltage probes. The V_{23} is given by Equation (42)[1].

$$V_{23} = \frac{\rho I_{14}}{2\pi t}\ln\frac{(s_1+s_2-\delta)(s_2+s_3-\delta)}{(s_1-\delta)(s_3-\delta)}. \tag{42}$$

Equation (42) resistivity equation includes t and s parameters. In this equation the displacement δ (proximity effect) is given by $\delta = \frac{1}{2}\left[L-(L^2-4R^2)^{1/2}\right]$. L is the spacing between the 1 and 4 probes where $L = s_1 + s_2 + s_3$ and R is radius of the probes' tips. If the equivalent current distribution

Table 5. The Geometric Factor equations for the commonly used geometries.

Thickness	Surface area	Geometric factor equations (G)	
∞ ($t \geqslant 5s$)	∞	$G = F_0(s) = \dfrac{2\pi}{\dfrac{1}{s_1} + \dfrac{1}{s_3} - \dfrac{1}{s_1 + s_2} - \dfrac{1}{s_2 + s_3}}$ $G = F_0(s) = 2\pi s$ ($s_{12} = s_{23} = s_{34} = s$)	
t	∞	Thick ($t \geqslant s/2$)	$G = F_0(s) \cdot F_1(t/s)$ ($t \geqslant 5s$) $\Rightarrow$ $F_1(t/s) = 1$
		Thin ($t < s/2$)	$G = \frac{\pi}{\ln 2} t$
		Inf. thin ($t < s/10$)	$G = \frac{\pi}{\ln 2}$
	Circular	Thick ($t \geqslant s/2$)	$G = F_0(s) \cdot F_1(t/s) \cdot F_2(d/s)$ ($t \geqslant 5s$) $\Rightarrow$ $F_1(t/s) = 1$
		Thin ($t < s/2$)	$G = \frac{\pi}{\ln 2} t \cdot F_2(d/s)$
		Inf. thin ($t < s/10$)	$G = \frac{\pi}{\ln 2} \cdot F_2(d/s)$

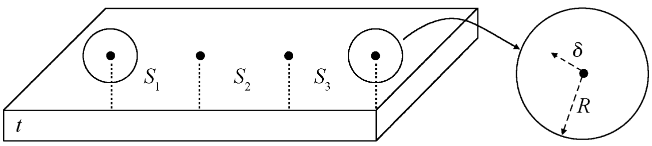

Figure 14. Rectangular prism sample with t thickness and probes with R radius.

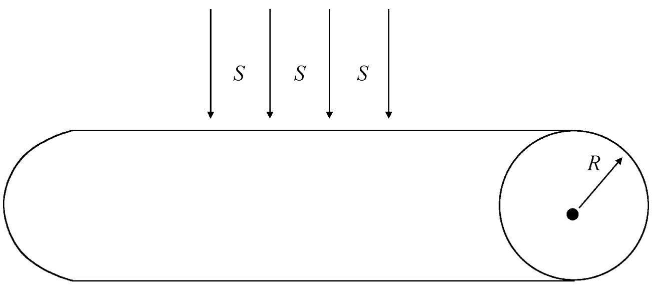

Figure 15. Cylindrical sample with R radius.

center is centered on the probes or the radius of the probes are zero (pinpoint probes) or $s_1 = s_2 = s_3 = s$ (so $L = 3s$), proximity effect becomes $\delta = 0$. In this statement Equation (42) becomes Equation (43) and the G factor is obtained as Equation (22).

$$V_{23} = \frac{\rho I_{14} \ln 2}{\pi t}. \tag{43}$$

In this expression, the G increase rapidly due to the experimental results for the $t \geqslant 0.5$ mm and higher values. The contribution of t has very small value for $t < 0.5$ mm. Therefore generally the contribution of t should be ignored for the very thin samples and the G factor is obtained again as Equation (30)[1].

3.5. Cylindrical geometry

Gegenwarth (1968) reported the correction factor for a cylindrical semiconductor that has an infinitely long radius R[19]. The resistivity function includes the current density, the area of contact of the electrodes, the probe spacing, and a modified Bessel function.

The resistivity equation for the infinitely long bulk semiconductor (Figure 15) is described as Equation (44)[19]. In this equation K is an expression in the potential equation which depends on the s value and the position of the probes.

$$\rho = \frac{\Delta V}{I} \frac{\pi^2 R}{2} \frac{1}{K} = \frac{\Delta V}{I} G. \tag{44}$$

If Equation (44) is used for the $s \ll R$ measurement setup, the G equation is computed again as Equation (17)[19]. In this computing, R has to go to infinity. This result is the same with the semi-infinitive volume statement and independent of the thickness.

$$\lim_{\frac{s}{R} \to 0} K = \frac{\pi R}{4s}. \tag{45}$$

Equation (46) is applicable for very high s distances according to radius ($s \gg R$) and resistivity is obtained as Equation (47)[19]. Again here, in order to obtain Equation (47), R should go to zero so the sample should be very thin. This expression belongs to a cylinder that has finitely thin thickness.

$$\lim_{\frac{s}{R} \to \infty} K = \frac{\pi R}{s}, \tag{46}$$

$$\rho = \frac{\Delta V}{I} \frac{\pi R^2}{s} = \frac{\Delta V}{I} G. \tag{47}$$

Yamashita calculated the correction factor from Poisson equations for the hollow conducting cylinders and depends on the extensions of the sample and arrangement of probes[20]. However the computing of the correction factor which is obtained by Yamashita (1987, 1988) calculating from the Poisson equation for the circular disk pellet is converged to $2\pi s$ and $\pi/\ln 2$[9, 15].

4. Conclusion

Fundamental geometric factors of $2\pi s$ and $\pi/\ln 2$ are always valid when the extensions are negligible. Special cases of different correction factors have always used these two coefficients. As shown above, there are many equations and mathematic methods that have been obtained by different researchers and all of them give the same results. Assuming a correct calculation of resistivity and a sample geometry as given above (or using an equation closer to our described samples), the correction factor is correct with a small error. The expressions for the commonly used geometries are presented in a table for handy reference (Table 5).

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