Energy Conversion. Vol. 9, pp. 1–6. Pergamon Press, 1969. Printed in Great Britain

Determination of the Solar Cell Equation Parameters, Including Series Resistance, from Empirical Data

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(*Received* 10 *June* 1968)

Four equations are derived to determine approximate values of the parameters in the solar cell equation. The required empirical data consists of values for V_{0c}, I_{sc}, V_m, I_m, and the slope of the I–V characteristic at I_{sc}, taken from a single I–V curve at a single intensity. Trial cases have produced fits with an r.m.s. deviation of 0·4–3·6 per cent, with a worst-case single point error of +11·5 per cent. The approximate parameters may be subsequently improved by an iterative trial-and-error process, to derive the best possible parameter values to the solar-cell equation under consideration. The process readily produces reasonably accurate values of the cell series resistance, R_s, and the junction recombination factor, n. For convenience in solar array power calculations, an approximate explicit equation for $I(V)$ is derived.

1. Results of Study

The assumed solar cell equation (neglecting the contribution from shunt resistance) is

$$V = A + B \ln (I_L - I) - RI \tag{1}$$

where

I = cell current at voltage V when the cell is illuminated and under resistive load

$$\left.\begin{aligned} A &= \frac{1}{q} E_g - \frac{n}{q} kT \ln (aT^3) \\ B &= \frac{n}{q} kT \end{aligned}\right\} \text{in terms of conventional notation: } \exp\left(\frac{-A}{B}\right) = I_0$$

I_L = light generated current

R = series resistance

n = dimensionless constant compensating for non-ideal junction behavior ($n = 1$ for ideal diode; $n = 2$ when recombination in the space charge region controls the junction current).

The equations derived in the following section are summarized as follows:

$$I_L = I_{sc} - \frac{mB}{1 + mR} \simeq I_{sc} \tag{2}$$

$$B = \frac{2V_m - V_{0c}}{\dfrac{I_m}{I_L - I_m} + \ln \left[\dfrac{I_L - I_m}{I_L}\right]} \tag{3}$$

$$R = \frac{V_m}{I_m} - \frac{B}{I_L - I_m} \tag{4}$$

$$A = V_{0c} - B \ln I_L \tag{5}$$

where

I_{sc} = short circuit current, $I(V = 0)$

m = slope $\dfrac{dI}{dV}$ of I–V curve at $V = 0$

V_m = voltage at maximum power point

I_m = current at maximum power point

V_{0c} = open circuit voltage, $V(I = 0)$

At normal intensities and temperatures, m is very close to zero, whence $I_L \simeq I_{sc}$. If $m \neq 0$, it is suggested that an iteration sequence be carried out between Equations (2–4), to determine the best value of I_L.

When empirical I–V plots are used as the source of data, it must be remembered that uncertainties inherently exist in reading V_{0c} and I_{sc} and in locating and reading the maximum power point. An error of $\pm 0{\cdot}0005$ in each of these quantities will be shown to propagate to an uncertainty of $\pm 0{\cdot}09\ \Omega$ in the calculated value of the cell series resistance, as an example. (These same uncertainties applied to the method of Wolf and Rauschenback [1] would propagate to errors as large as $\pm 0{\cdot}26\ \Omega$, depending on the selected intensity differences.) The advantage of the present method is that iteration sequences offer an opportunity to reduce the parameter uncertainties to as small a value as desired, consistent with the solar cell equation under consideration—i.e., not necessarily Equation (1). Computer techniques could also be applied, with a criterion of minimizing the r.m.s. deviation to the experimental curve.

The value of n, the factor indicating the degree of recombination in the space charge region, may be of interest in comparing the relative merits of different production solar cells. Knowing the absolute temperature of the cell when the data was recorded enables the calculation to be carried out,

$$n = \frac{Bq}{kT} = 1{\cdot}16 \times 10^4 \frac{B}{T}. \tag{6}$$

At 23°C, $n = 38{\cdot}7\ B$; B should lie somewhere between 0·025 and 0·052 V.

2. Derivation of Equations

Equation (1) is assumed and two values of voltage are subtracted to eliminate A:

$$V_m - V_{0c} = B \ln \frac{I_L - I_m}{I_L} - RI_m. \tag{7}$$

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To evaluate V_m and I_m the derivative of power ($= IV$) is taken and set equal to zero

$$\frac{\mathrm{d}(IV)}{\mathrm{d}I} = A + B\ln(I_L - I) - \frac{IB}{I_L - I} - 2RI = 0,$$

and substituting Equation (1) and solving:

$$V_m = \frac{BI_m}{I_L - I_m} + RI_m \tag{8}$$

Equations (7) and (8) are now added to eliminate RI_m, and solved for B:

$$B = \frac{2V_m - V_{0c}}{\dfrac{I_m}{I_L - I_m} + \ln\dfrac{I_L - I_m}{I_L}} \tag{3}$$

Knowing B, we can explicitly determine R from Equation (8). The initial calculation of B should assume $I_L \simeq I_{sc}$.

To determine A, we evaluate Equation (1) at $I = 0$,

$$V_{0c} = A + B\ln I_L,$$

which produces Equation (5).

Finally, in the event of data taken at extremes of intensity or temperature, I_L may not equal I_{sc}. We examine the derivative of Equation (1) at I_{sc}:

$$\left.\frac{\mathrm{d}V}{\mathrm{d}I}\right|_{V=0} = \frac{-B}{I_L - I_{sc}} - R \tag{9}$$

Since V is usually treated as the independent variable in most laboratory plots, we calculate the slope m

$$m = \frac{\mathrm{d}I}{\mathrm{d}V} = \left(\frac{\mathrm{d}V}{\mathrm{d}I}\right)^{-1}$$

and solve Equation (9) for I_L to produce Equation (2).

3. An Explicit Approximation for $I(V)$

Equation (1) cannot be solved explicitly for $I(V)$ when the series resistance term is included. A somewhat convenient approximation may be derived which is valid for small $R(R \gtrsim 0{\cdot}2)$ by expansion of the exponential.

$$\begin{aligned} I_L - I &= \exp\left(\frac{V - A + RI}{B}\right) \\ &= \exp\left(\frac{V - A}{B}\right)\exp\left(\frac{RI}{B}\right) \\ &= \exp\left(\frac{V - A}{B}\right)\left(1 + \frac{RI}{B} + \frac{R^2I^2}{2B^2} + \dots\right). \end{aligned} \tag{10}$$

Since RI/B is, in practice, close to unity between I_m and I_{sc}, total neglect of this term will introduce an error in the value of $(I_L - I)$ of a factor of $2{\cdot}7$. The first order approximation reduces this factor to $1{\cdot}3$, and the second order to $1{\cdot}08$. The first order solution of Equation (1) is

$$I = (I_L - E)\frac{1}{1 + \dfrac{R}{B}E} \tag{11}$$

where

$$E = \exp\left(\frac{V - A}{B}\right)$$

4. Iteration to Higher Accuracy

Figures 1, 2, and 3 show the effect of variation of A, B, and R on the I–V characteristic. Figure 4 shows the effect of the variation of R under the first order approximation for I as a function of V. It is observed that variations in A (and also, obviously, I_L) have negligible effect on the shape of the I–V curve and serve

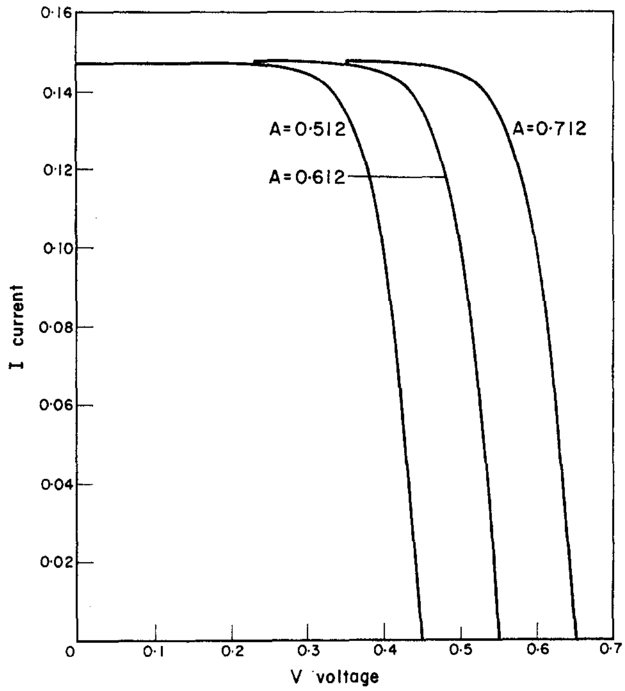

Fig. 1. Effect of A variation on I–V curve.
$V = A + B\ln(I_L - I) - RI$
Where: $B = 0{\cdot}0322$
$R = 0{\cdot}20$
$I_L = 0{\cdot}1472$

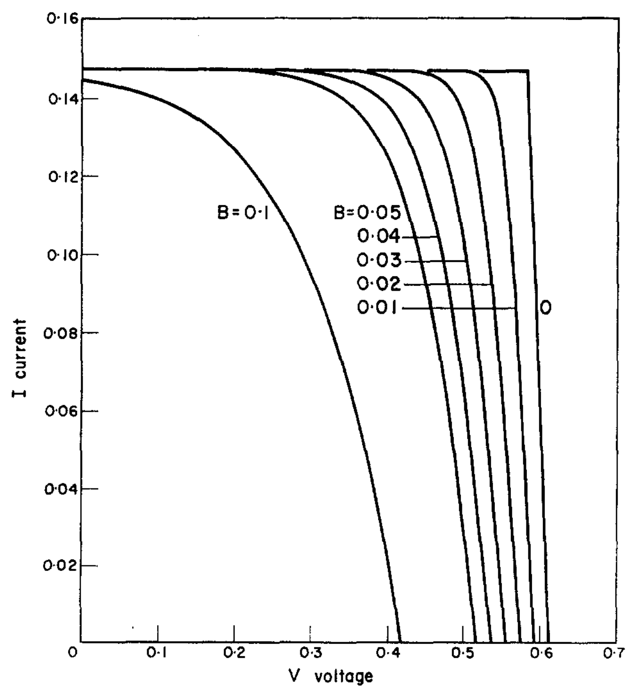

Fig. 2. Effect of B variation on I–V curve.
$V = A + B\ln(I_L - I) - RI$
Where: $A = 0{\cdot}612$
$R = 0{\cdot}20$
$I_L = 0{\cdot}1472$

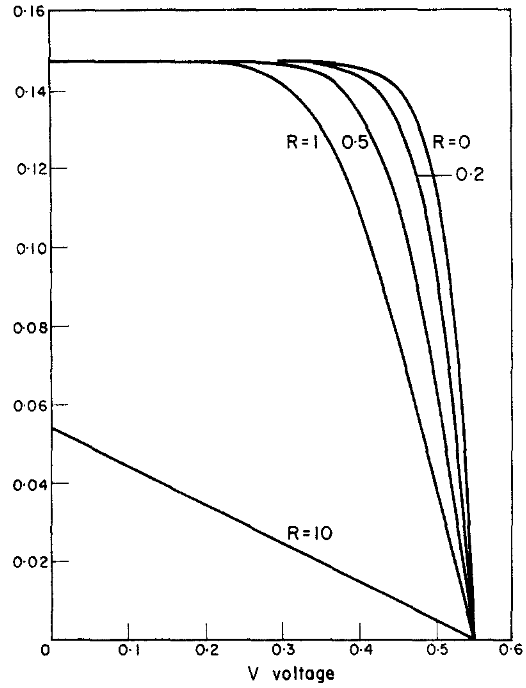

Fig. 3. Effect of R variation on I–V curve.

$$V = A + B \ln (I_L - I) - RI$$

Where: $A = 0{\cdot}612$
$B = 0{\cdot}0322$
$I_L = 0{\cdot}1472$

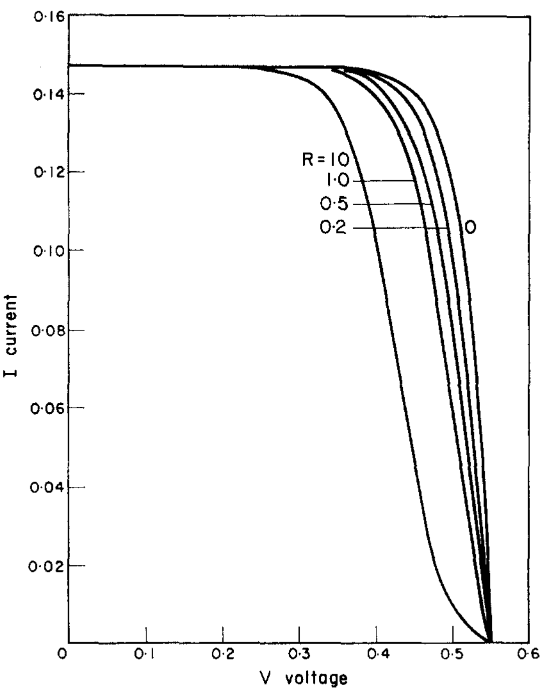

Fig. 4. Effect of R variation on approximate V–I curve.

$$I = \frac{I_L - E}{1 + \frac{R}{B} E}. \quad \text{Where } E = \exp \frac{V - A}{B}$$

(This is explicit solution for $I = I(V)$ where series resistance has been included as a first order approximation).

$A = 0{\cdot}612$
$B = 0{\cdot}0322$
$I_L = 0{\cdot}1472$

primarily to fix the "endpoints". A first set of parameters should be inserted into the equation for $V(I)$ and values of voltage compared with the experimental curve. The following guidelines to iteration are currently suggested:

(i) *Deviation small near V_{0c} and generally increasing "upwards" towards I_{sc}*

(*a*) Increase R, if calculated $V(I)$ are larger than experimental values and vice versa.
(*b*) Recalculate B from Equation (4).
(*c*) Recalculate A from Equation (5).

(ii) *Deviation mostly above V_m (maximum power point)*

(*a*) Increase B, if calculated $V(I)$ are larger, and vice versa.
(*b*) Recalculate R and A.

(iii) *$V(I)$ too large mostly below V_m*

(*a*) Cell may not have negligible shunt resistance, P. The values of $I(V)$ are lowered at higher voltages by the presence of the term $- V/P$. Equation (1) thus may not be an adequate representation of cell under test.

In general, B is limited by Equation (6) and the accepted range of n; and R, P, A, B, and I_L are restricted to be positive constants.

5. Propagation of Errors

Equations 1–5 may each be analysed by the "chain-rule" of partial differentiation to derive the effect of uncertainties in their constituant variables:

$$\Delta f(A, B, C) = \left|\frac{\partial f}{\partial A}\right| \Delta A + \left|\frac{\partial f}{\partial B}\right| \Delta B + \left|\frac{\partial f}{\partial C}\right| \Delta C. \tag{12}$$

The equation for series resistance, R, was analysed in detail because this quantity is currently under closest scrutiny at the Philco–Ford Space Power Laboratory. Applying Equation (12) we obtain:

$$\begin{aligned}\Delta R = &\left|\frac{1}{I_m} - \frac{2}{I_m + Q \ln (Q/I_L)}\right| \Delta V_m \\ &+ \left|\frac{1}{I_m + Q \ln (Q/I_L)}\right| \Delta V_{0c} \\ &+ \left|\frac{(2V_m - V_{0c})(\ln (Q/I_L) + I_m/I_L)}{[I_m + Q \ln (Q/I_L)]^2}\right| \Delta I_{sc} \\ &+ \left|\frac{-V_m}{I_m^2} + \frac{(2V_m - V_{0c}) \ln (Q/I_L)}{[I_m + Q \ln (Q/I_L)]^2}\right| \Delta I_m \end{aligned} \tag{13}$$

where $Q = (I_L - I_m)$.

Using the numerical values of Example I in the following section, we calculate

$$\begin{aligned}\Delta R &= 13{\cdot}6\, \Delta V_m + 10{\cdot}9\, \Delta V_{0c} + 49{\cdot}3\, \Delta I_{sc} + 108\, \Delta I_m \\ &= \pm 0{\cdot}09\, \Omega\end{aligned}$$

where $\Delta V_m = \Delta V_{0c} = \Delta I_{sc} = \Delta I_m = \pm 0{\cdot}0005$.

The importance of accurate values for the current at the maximum power point and at $V = 0$ is thus demonstrated.

The same uncertainties applied to a typical example of the Wolf and Rauschenback method of determining series resistance [1] produces:

$$\Delta R = \left|\frac{1}{I_2 - I_1}\right| (\Delta V_1 + \Delta V_2) + \left|\frac{V_1 - V_2}{(I_2 - I_1)^2}\right| (\Delta I_1 + \Delta I_2)$$

$$= 200\,(\Delta V_1 + V_2) + 60\,(\Delta I_1 + \Delta I_2)$$

$$= \pm 0{\cdot}26\ \Omega$$

where, we have in addition assumed,

$$I_{L1} = 0{\cdot}135$$
$$I_{L2} = 0{\cdot}130$$
$$I_1 = 0{\cdot}130, \qquad V_1 = 0{\cdot}3720$$
$$I_2 = 0{\cdot}125, \qquad V_2 = 0{\cdot}3735.$$

It is pointed out, however, that a larger separation of I_1 and I_2 will reduce ΔR.

6. Two Random Examples using Laboratory I–V Plots

Example I: $I_L = I_{sc} = 0{\cdot}135$
$I_m = 0{\cdot}122$
$V_{0c} = 0{\cdot}552$
$V_m = 0{\cdot}420$

(*i*) Calculated parameters: $B = 0{\cdot}0409$
(No iteration) $R = 0{\cdot}298$
$A = 0{\cdot}6338$

(*ii*) Comparison of curves:

I	V (calc.)	V (obs.)	Δ
0·130	0·3786	0·372	+1·8%
0·125	0·4084	0·407	+0·3
0·120	0·4264	0·427	−0·1
0·110	0·4503	0·452	−0·4
0·100	0·4670	0·468	−0·2
0·080	0·4915	0·494	−0·5
0·060	0·5101	0·512	−0·4
0·040	0·5257	0·527	−0·2
0·020	0·5395	0·540	−0·1
0	0·5520	0·552	0

r.m.s. $\Delta = 0{\cdot}4$ per cent.

Example II: $I_L = I_{sc} = 0{\cdot}132$
$I_m = 0{\cdot}119$
$V_m = 0{\cdot}422$
$V_{0c} = 0{\cdot}543$

(*i*) Calculated parameters: $B = 0{\cdot}044$
$R = 0{\cdot}159$
$A = 0{\cdot}63216$

I	V (calc.)	V (obs.)	Δ
0·130	0·3378	0·303±0·007	+11·5%
0·125	0·3938	0·383	+2·8
0·120	0·4183	0·4175	+0·2
0·115	0·4344	0·433	+0·3
0·110	0·4466	0·445	+0·4
0·100	0·4647	0·4625	+0·5
0·08	0·4892	0·4875	+0·3
0·06	0·5068	0·505	+0·4
0·04	0·5207	0·520	+0·1
0·02	0·5326	0·5325	+0·02
0	0·5430	0·543	0

r.m.s. $\Delta = 3{\cdot}6$ per cent.

Note: A subsequent iteration, suggested by the large deviation of the first entry, selected an increased B and reduced the first entry above from + 11·5 to + 3·0 per cent, and reduced the r.m.s. Δ to 2·5 per cent. This solar cell, an ion-implanted cell, may have possessed a number of irregularities not usually encountered with standard diffused cells.

7. An Alternate Method for Determining A, B, and R

The explicit solutions for the parameters of the solar cell equation [Equation (1)] have been given in terms of three points on the I–V curve: I_{sc}, V_{0c}, and the maximum power point. The resultant solutions have particularly simple forms. An alternate method will be briefly discussed which, although more complicated, does not require the precise location of the maximum power point but, instead, makes use of any other *two* points (preferably above and below P_{max} on the knee of the curve). These points will be designated (V_1, I_1) and (V_2, I_2).

The parameter A is eliminated in Equation (1) by forming the two relations ($V_1 - V_2$) and ($V_2 - V_{0c}$). Then dividing by the resultant coefficient of R, we subtract again to obtain:

$$B = \frac{(I_1 - I_2)(V_2 - V_{0c}) - I_2(V_1 - V_2)}{(I_1 - I_2)\ln\dfrac{I_L - I_2}{I_L} - I_2 \ln\dfrac{I_L - I_1}{I_L - I_2}} \tag{14}$$

To obtain R, we again form the relations ($V_1 - V_2$) and ($V_2 - V_{0c}$), transpose the entire R term, and divide:

$$R = \frac{(V_2 - V_{0c})\,L - (V_1 - V_2)}{(I_1 - I_2) - LI_2} \tag{15}$$

where

$$L = \frac{\ln[(I_L - I_1)/(I_L - I_2)]}{\ln[(I_L - I_2)/I_L]}$$

The expressions for A and I_L remain unchanged.

A test case using the curve of Example II above has been evaluated. The two new points are

$$(V_1, I_1) = (0{\cdot}400,\ 0{\cdot}1235)$$
$$(V_2, I_2) = (0{\cdot}450,\ 0{\cdot}1075)$$

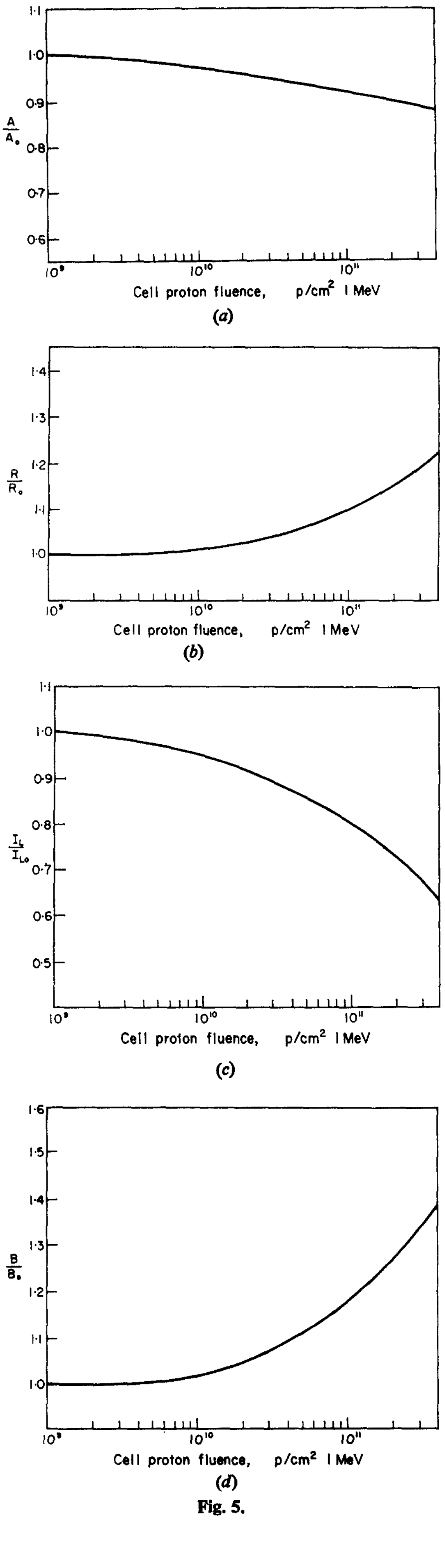

Fig. 5.

and the resultant calculated parameters are

$$B = 0\cdot04475$$
$$R = 0\cdot164$$
$$A = 0\cdot6336$$
$$I_L = I_{sc} = 0\cdot132$$

These parameters insure that the curve will, at least, pass exactly through the four selected points. The match elsewhere on the curve may not be so fortuitous.

8. Variable Parameters as a Means for Expressing Radiation Degradation†

The effect of radiation on the I–V characteristic of a solar cell is usually presented in terms of degradation to I_{sc}, V_{0c}, and $P_{\max}$ vs. radiation fluence. Such three-point presentation concisely summarizes the total effect of radiation damage to Equation (1), but requires the design engineer to tediously reconstruct the total I–V curve before he can utilize this knowledge. Not knowing the location of either the voltage or current points at the value of maximum power, he must resort to certain approximations or trial-and-error curve reconstructing. He is usually content if the results are correct to ± 2 or 3 per cent. When one examines the effect on vehicle lifetime at five years in synchronous orbit, however, it is observed that a ± 2 per cent

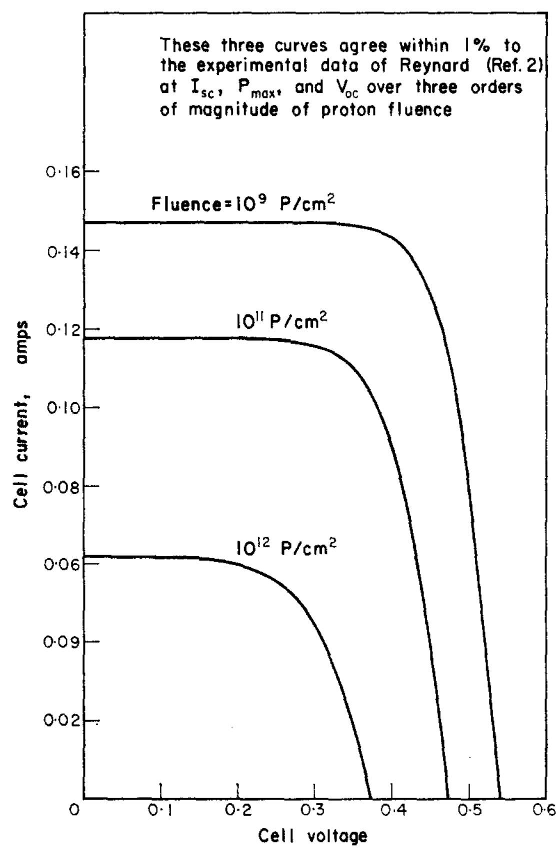

Fig. 6. A typical set of I–V curves which are easily generated using the parametric presentation of radiation degradation shown in Fig. 5.

† This section was submitted June 12, 1968 at the suggestion of Mr. Paul Rappaport, the editor for Photoelectric Processes and is intended to briefly indicate work currently in progress. Comments or criticism from readers on this approach would be deeply appreciated by the author.

variation in power translates to an uncertainty of several years.

One solution to this problem is the presentation of radiation degradation as changes to the "fixed" parameters, A, B, I_L, and R. Thus, assuming the accuracy of Equation (1) over (at least) a portion of the I–V curve of interest, the exact translations and shape changes (e.g., "knee softening") will all be accommodated.

Figure 5 presents examples of this type of approach (these curves were assembled from Reynard's [2] 1-Mev proton degradation data and are not intended to contain the accuracy that a thorough analysis would produce). Figure 6 shows a family of I–V curves produced using Equation (1) and the parameter "degradations" of these four graphs.

Presentation of degradation data in parametric form on a purely phenomenological basis provides a number of advantages to the design engineer who is interested in fast, accurate, and complete I–V curves. Such an approach, admittedly, is inherently unsatisfying to the physicist who wants a direct "cause and effect" justification for the observed behavior. The diffusion length approach, used extensively in the past, is an example of behavior explanation rooted in established semiconductor physics. Verification of the final parameter curves versus degradation should thus be carried out and the exact equivalence of the two presentations should be demonstrated. In the work currently in progress at Philco–Ford an attempt is being made to generalize on the final parameter curve shapes to include the effects of both electron and proton degradation.

References

[1] M. Wolf and H. Rauschenback, Series Resistance Effects on Solar Cell Measurements, *Advd Energy Conversion*, 3, 455 (1963). Also presented at the 1961 Pacific General Meeting of the AIEE, Salt Lake City, Utah, August 23–25 (1961).

[2] D. L. Reynard, Proton and Electron Irradiation of N/P Silicon Solar Cells, Contract AF 04(647)–787, Lockheed Report LMSC 3–56–65–4, 12 April (1965).