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Meta-heuristic framework: Quantum inspired binary grey wolf optimizer for unit commitment problem☆

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ABSTRACT

This paper proposes a quantum inspired binary grey wolf optimizer (QI-BGWO) to solve unit commitment (UC) problem. The QI-BGWO integrates quantum computing concepts with BGWO to improve the hunting process of the wolf pack. The inherent properties of Q-bit and Q-gate concepts in quantum computing help in achieving better balance between exploration and exploitation properties of the search process. The position update processes of wolves at different hierarchy levels are replaced by Q-bit's individual probabilistic representation along with dynamic rotation angle and coordinate rotation gate. Therefore, solution approaches exploit the search properties of GWO and quantum computing using quantum bits, gates, superposition principle etc., to solve the unit commitment schedule efficiently. The results and statistical analysis demonstrates the effectiveness of proposed approaches in solving the UC problem and establishes the significance of proposed approaches among existing binary and quantum computing based heuristic approaches.

1. Introduction

Unit commitment (UC) is considered as one of the important tasks in efficient, reliable and optimal short term operation planning of power system. The UC problem is traditionally modelled as a cost optimization problem. The UC problem involves two steps namely commitment-decommitment (ON/OFF) schedules followed by economic dispatch of committed units. In past decades, many optimization approaches have been proposed to solve UC problem. The earliest methods in this paradigm are deterministic approaches like Legrangian relaxation (LR), priority list (PL), dynamic programming (DP), mixed integer linear programming (MILP), branch and bound techniques (BB). Each of these techniques are reported with their own advantages and disadvantages. While, the advantages being faster (LR), simple (PL), integer type solution (MILP) etc. Some of the disadvantages include high production cost (PL method), high dimensionality problems (dynamic programming

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Nomenclature

N	Number of units
H	Total number of scheduling hours
i	Thermal unit index ($i = 1, 2, 3\ldots.N$)
h	Scheduling hour index ($h = 1, 2, 3\ldots.H$)
F_c^i	Fuel cost function of ith unit
χ_i^h	Status bit (0 or 1) of ith unit for hth hour
SU_i^h	Start-up cost of ith unit for hth hour
P_i^h	Scheduled power of ith unit for hth hour
SU_i^{hot}	Hot start-up cost of ith unit
SU_i^{cold}	Cold start-up cost of ith unit
T_i^{MD}	Minimum down time of ith unit
T_i^{MU}	Minimum up time of ith unit
$T_i^{h,off}$	Consecutive hours of de-committed state of ith unit going into hth hour
$T_i^{h,on}$	Consecutive hours of committed state of ith unit going into hth hour
P_i^{min}	Minimum generation limit of ith unit
P_i^{max}	Maximum generation limit of ith unit
P_d^h	System load for hth hour
R_{sp}^h	Spinning reserve requirement for hth hour
R_i^{DR}	Ramp down rate of ith unit
R_i^{UR}	Ramp down rate of ith unit

and mixed integer linear programming), exponential rise in computational time with UC problem dimension (branch and bound), problems with numerical convergence (Legrangian relaxation method) etc.

The traditional deterministic approaches are followed by the heuristic approaches with most of them inspired from nature. Some of the heuristic and stochastic approaches for solving UC problem include genetic algorithm (GA) [1], evolutionary programming (EP) [2], simulated annealing (SA) [3], particle swarm optimization (PSO) [4], improved particle swarm optimization (IPSO) [5], firefly algorithm [6] etc. Many of the algorithms are based on natural phenomena and include a number of flukes searching for optimal solution with different mechanisms. For example, GA uses the biological evolution mechanisms like natural selection, crossover and mutation in solving the complex problems. Modified and hybrid variants of evolutionary approaches such as hybrid differential evolution with random search, ring crossover genetic algorithm (RCGA) [7] etc., have been developed in recent years to improve the solution quality and convergence of UC problem. However, complexity in basic evolutionary population based approaches limited the extension of these approaches to larger test systems. Whereas, PSO is based on coordination and social behaviour of particles amongst the group is used to optimize problems like UCP with high complexity. The other recent approaches developed to solve UC problem include hybrid variants such as grey wolf optimizer (GWO) with PSO (GWO-PSO) [8] using the hierarchical and social behaviour of grey wolf and swarm group, hybrid harmony search-random search (HHSRS) [9], fireworks algorithm (FWA), binary fireworks algorithm (BFWA) [10] that exploits the explosion principles of fireworks in sky. However, the exponential rise of fireworks/sparks in FWA approach with increasing system dimension may prove costly with respect to computational time. Other problem with the heuristic approaches is a parameter tuning for optimal performance. In this regard, GWO presents an effective mechanism in which all the parameters are evaluated in the run time in an adaptive manner. Therefore, the parameter tuning experiments can be avoided using GWO [11]. The motivation for BGWO development to solve UC problem is drawn observing the superior performance of GWO over other approaches like PSO, ACO, GA etc., on various problems.

In recent approaches, certain principles of quantum mechanics are used to model evolutionary computational techniques. These approaches employ principles like superposition, uncertainty and interference of quantum mechanics [12,13,25]. Han and Kim [14] proposed a popular evolutionary algorithm that deploys the use of concepts of quantum computing viz. quantum bits (Q-bits), quantum gates (Q-gates) and their superposition. The quantum inspired evolutionary algorithm attracted much importance due to its superior properties like balanced exploration and exploitation to converge to better solution compared to other evolutionary algorithms. The initial research involved the direct application of original QEA using Q-gates to direct best population towards the optimal solution of UC problem (QEA-UC) [15]. The conventional QEA is followed by advanced QEA approaches incorporating special initialization using Q-bit representation along with priority list to maintain diversity in the population [16,17]. The Q-bit and Q-gate concepts are also used to develop heuristic approaches like quantum inspired binary particle swarm optimization (QBPSO) [18]. In which, the velocity update operation of conventional PSO is replaced by Q-bit individual update based on probabilistic approach. Apart from these, many other approaches inspired by natural/biological phenomena are investigated [19].

Recently, Mirjalili [11] proposed a meta-heuristic approach named grey wolf optimizer (GWO) mimicking the specific behaviour of grey wolves (Canis lupus). The approach employs specific behaviour of grey wolves in hunting the prey based on leadership hierarchy. In this approach, the position of wolves in hunting process is updated using Q-bit in the proposed QI-BGWO. The constraint satisfaction is accompanied by heuristic adjustment approach to satisfy the up/down time and reserve constraints [16]. The proposed BGWO and QIBGWO approaches are applied to solve UC problem of different dimensions of 10, 20, 40, 60, 80 and 100 units with associated hourly load and reserve constraints.

The rest of the paper is organized as follows. The proposed approach of QI-BGWO is explained in Section 2 and Section 3 develops the application procedure of proposed QIBGWO approach to solve UC problem. Section 4 presents the test system, parametric analysis and computational results for different dimensions of the test system. In the same section, statistical analysis of proposed approaches is carried out to compare performance and establish statistical significance. Finally, Section 5 concludes the paper and outlines the future scope.

2. Quantum inspired BGWO (QI-BGWO)

The grey wolf optimizer (GWO) presents a near global solution search approach based on the hunting principles and hierarchy of grey wolf pack [11]. However, the search space and variable nature of the unit commitment (UC) problem requires a binary search algorithm development [18]. Therefore, the binary mapping of real valued search space to binary search space using transformation functions is imperative. The binary nature of quantum computing principles allows the application of superior exploration capabilities of quantum computing concepts to solve UC problem. In quantum computing terminology, Q-bit represents the smallest possible information unit that is saved in a two state quantum computer.

2.1. Quantum computing

The smallest information that can be saved in the Q-bit is represented by,

$$\begin{bmatrix}\alpha\\ \beta\end{bmatrix};\ \forall\ \alpha,\beta\ \in[0,\ 1] \tag{1}$$

Where, α and β are related to the existence of Q-bits in 0 or 1 state through the following relation.

$$|\alpha|^2+|\beta|^2=1 \tag{2}$$

In the above equation, $|\alpha|^2$ and $|\beta|^2$ denote the probabilities of driving the Q-bit to state 0 and 1 respectively i.e., higher the value of $|\alpha|^2$, higher is the probability that, the Q-bit finds itself in state $|0\rangle$ and vice versa. Thus, the Q-bit can be said to obey the linear superposition principle of quantum computing as given by,

$$|\varphi\rangle=\alpha|0\rangle+\beta|1\rangle \tag{3}$$

The string of Q-bits represented as a Q-bit individual is given by,

$$\left[\begin{array}{c|c|c|c}\alpha_1 & \alpha_2 & \alpha_3 & \ldots\ldots\ \alpha_n\\ \beta_1 & \beta_2 & \beta_3 & \ldots\ldots\ \beta_n\end{array}\right] \tag{4}$$

Where, n denotes the total number of Q-bits in a Q-bit individual and each bit satisfying the following relation.

$$|\alpha_i|^2+|\beta_i|^2=1;\ \ \forall\ i\in\ \{1,2,3,\ldots..n\} \tag{5}$$

The probability that gives equal chances for a Q-bit to attain 0 or 1 states is achieved when α and β converge to a value $1/\sqrt{2}$. In this case, for all the possible states with equal probability $(1/\sqrt{2})$, the linear superposition principle results in,

$$|\varphi\rangle=\sum_{k=1}^{2^n}\frac{1}{\sqrt{2^n}}|X_k\rangle \tag{6}$$

Where X_k denotes the state k represented by a binary string $(x_1,x_2,x_3\ldots..x_n)$ with each and every bit is a binary value "0" or "1". Each Q-bit is updated in each iteration (t) using the Q-gate variation operator $U(\Delta\theta_{ji}^t)$ as given by,

$$\begin{bmatrix}\alpha_{ji}^t\\ \beta_{ji}^t\end{bmatrix}=U\left(\Delta\theta_{ji}^t\right)\begin{bmatrix}\alpha_{ji}^{t-1}\\ \beta_{ji}^{t-1}\end{bmatrix} \tag{7}$$

Where,

$$U\left(\Delta\theta_{ji}^t\right)=\begin{bmatrix}cos\left(\Delta\theta_{ji}^t\right) & -sin\left(\Delta\theta_{ji}^t\right)\\ sin\left(\Delta\theta_{ji}^t\right) & cos\left(\Delta\theta_{ji}^t\right)\end{bmatrix}\ \ \forall\ j\in n, i\in m \tag{8}$$

In the above equations, $n\,and\,m$ are the number of Q-bits and number of Q-bit individuals respectively, $\Delta\theta_{ji}^t$ denotes the rotation angle of jth Q-bit of ith Q-individual for tth iteration. The direction and magnitude of variation of Q-bit for

Table 1
Lookup table of sample rotation angles (F = *False*, T = True).

x_{ji}^t	b_j^t	$F(X_j^t) \geq F(B^t)$	$\Delta\theta_{ji}^t$
0	0	F	θ_1
0	1		θ_2
1	0		θ_3
1	1		θ_4
0	0	T	θ_5
0	1		θ_6
1	0		θ_7
1	1		θ_8

every iteration is decided by $\Delta\theta_{ji}^t$. For the best solution (B), the $\Delta\theta_{ji}^t$ can be predefined [13] as given in look up Table 1. In the lookup table, x_{ji}^t represents the *j*th bit of *i*th individual at *t*th iteration/generation for the binary solution *X* and b_j^t is the *j*th bit of best solution *B* for iteration *t*. The angle vector $\Theta = [\theta_1, \theta_2, \theta_3, \ldots\ldots\theta_8]^T$ is set with values $\theta_1 = 0$, $\theta_2 = 0.01\pi$, $\theta_3 = -0.01\pi$, $\theta_4 = 0$, $\theta_5 = 0$, $\theta_6 = 0$, $\theta_7 = 0$, $\theta_8 = 0$ based on the experimental tests performed on the knapsack problems [13].

2.2. GWO with quantum computing

In the proposed approach, the position of each wolf occupies a value of either "0" or "1" depending upon the probabilities $|\alpha|^2$ and $|\beta|^2$ respectively. The Q-bit update replaces the transformation based update process of traditional BGWO. Therefore, in the proposed QI-BGWO approach, the update coefficients $\vec{A}$ and $\vec{C}$ are replaced by a single Q-gate variation operator $U(\Delta\theta_{ji}^t)$. The position of *i*th wolf ($X_i = \{x_{i1}, x_{i2}, x_{i3}, \ldots x_{im}\}$) is updated using $|\beta_i|^2$ which is the probability that drives the state towards 1. The *j*th element of *i*th wolf is updated as follows:

$$x_{ij} = \begin{cases} 1, & n_{ij}^{rand} < \left|\beta_{ij}\right|^2 \\ 0, & otherwise \end{cases} \quad \forall i \in \{1, 2, 3 \ldots NP\};\ j \in \{1, 2, 3 \ldots\ldots n\} \tag{9}$$

Where *NP* denotes the population size, *n* is the number of elements in each population and n_{ij}^{rand} is the random number distributed uniformly over the range [0, 1]. The traditional update of Q-bit (α, β) from previous iteration ($t-1$) to current iteration (t) requires the rotation angle to be predefined. Whereas in the proposed approach the rotation gate angle $\Delta\theta_{ij}$ is updated using the position of α, β and δ wolves as given by,

$$\Delta\theta_{ij}^t = \theta \times \left\{\left(\gamma_{1i}x_j^\alpha - x_{ij}\right) + \left(\gamma_{2i}x_j^\beta - x_{ij}\right) + \left(\gamma_{3i}x_j^\delta - x_{ij}\right)\right\} \quad \forall i \in \{1, 2, 3 \ldots NP\};\ j \in \{1, 2, 3 \ldots\ldots n\} \tag{10}$$

Where θ is the rotation angle magnitude and γ_{1i},γ_{2i} *and* γ_{3i} can be evaluated using the fitness of current wolf with α, β and δ wolves as given by,

$$\gamma_{1i} = \begin{cases} 1, & F(x_i) < F(x^\alpha) \\ 0, & otherwise \end{cases} \tag{11}$$

$$\gamma_{2i} = \begin{cases} 1, & F(x_i) > F(x^\alpha) \&\& F(x_i) < F\left(x^\beta\right) \\ 0, & otherwise \end{cases} \tag{12}$$

$$\gamma_{3i} = \begin{cases} 1, & F(x_i) > F(x^\alpha) \&\& F(x_i) > F\left(x^\beta\right) \&\& F(x_i) < F\left(x^\delta\right) \\ 0, & otherwise \end{cases} \tag{13}$$

Where $F(X_i)$, $F(x^\alpha)$, $F(x^\beta)$ and $F(x^\delta)$ are the fitness values of objective $F(X)$ for current, α, β and δ wolves respectively. The speed of convergence and quality of the solution is greatly affected by magnitude of rotation angle (θ) as given by [18],

$$\theta = \theta_{max} - (\theta_{max} - \theta_{min}) \times \frac{t}{t_{max}} \tag{14}$$

3. Solution methodology for UC using QI-BGWO

3.1. Problem formulation

3.1.1. Objective function

The objective function of UC problem is modelled as a minimization problem of total cost which constitutes of fuel cost, start-up and shut down costs.

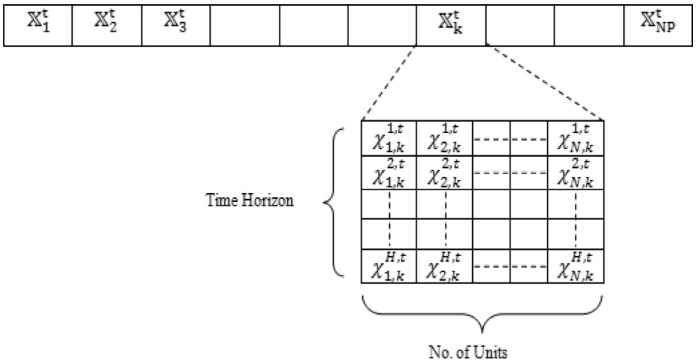

Fig. 1. Representation of population structure of UC problem with QI-BGWO.

Fuel cost: The fuel cost is expressed as a quadratic equation given by,

$$F_c^i\left(P_i^h\right) = a_i + b_i\left(P_i^h\right) + c_i\left(P_i^h\right)^2 \tag{15}$$

Where a_i,b_i *and* c_i are the fuel cost coefficients of *i*th unit.

Start-up cost: The start-up cost applicable for *i*th unit during *h*th hour is given by,

$$SU_i^h = \begin{cases} SU_i^{hot} & if\ T_i^{MD} \le T_i^{off} \le T_i^{MD} + T_i^{cold} \\ SU_i^{cold} & if\ T_i^{off} \ge T_i^{MD} + T_i^{cold} \end{cases} \tag{16}$$

Therefore, the total objective function is given by,

$$\begin{aligned} TC = \sum_h \sum_i F_c^i\left(P_i^h\right)\chi_i^h + SU_i^h\left(1 - \chi_i^h\right)\chi_i^h \\ \forall\, h \in H;\, i \in N;\, \chi_i^h \in \{0, 1\} \end{aligned} \tag{17}$$

3.1.2. Constraints

Generation limits: The actual generation of the units in committed state should comply with the generation limits as given by,

$$\chi_i^h P_i^{min} \le P_i^h \chi_i^h \le \chi_i^h P_i^{max} \tag{18}$$

Load balance constraints: The system demand-supply balance constraint given by,

$$\sum_i P_i^h \chi_i^h = P_d^h;\ \forall\, h \in H;\, i \in N \tag{19}$$

Spinning reserve: The hourly additional online capacity can be summed up as follows.

$$\sum_i \chi_i^h P_i^{max} = P_d^h + R_{sp}^h;\ \forall\, h \in H;\, i \in N \tag{20}$$

Minimum up/down time constraints: The time that should elapse between commitment and de-commitment events of units is predefined based on the reliability and satisfactory performance of particular unit.

$$\chi_i^h = \begin{cases} 1, & if\ 1 \le T_{i,h-1}^{on} < T_i^{MU} \\ 0, & if\ 1 \le T_{i,h-1}^{off} < T_i^{MD} \\ 0\ or\ 1, & otherwise \end{cases} \tag{21}$$

Ramp up/down rates: The ramp up/down rate constraints are given by,

$$\chi_i^h P_i^{h,min} < P_i^h < \chi_i^h P_i^{h,max} \tag{22}$$

Where,

$$\begin{aligned} P_i^{h,min} = max\left(P_i^{min}, P_i^{h-1} - R_i^{DR}\right) \\ P_i^{h,max} = min\ \left(P_i^{max}, P_i^{h-1} + R_i^{UR}\right) \end{aligned} \tag{23}$$

3.2. QI-BGWO implementation to UC problem

The generalized representation of UC problem is shown in Fig. 1. The commitment status of *i*th generator of *k*th wolf during *h*th hour of *t*th iteration is represented by$\chi_{i,k}^{h,t}$. Thus, the commitment of *i*th generator for *h*th is confirmed by assigning "1" and vice versa for de-commitment. Hence, the status bits of *k*th wolf in a population of *NP* range from $\chi_{1,k}^{1,t}$ to $\chi_{N,k}^{H,t}$ for *t*th iteration. The commitment matrix of all units of *k*th wolf over 24 scheduling horizon during *t*th iteration is denoted by $\mathbb{X}_k^t$.

3.2.1. Initialization of Q-bits and wolf positions

In the process of initialization, every Q-bit has equal chances of storing either 0 or 1. Therefore, the Q-bit of ith unit of kth wolf for hth hour is initialized (iteration, t = 0) with equal probabilities i.e., both $\alpha_{i,k}^{h,0} = 1/\sqrt{2})$ and $\beta_{i,k}^{h,0} = 1/\sqrt{2}$. The initial position of wolves is estimated using Eq. (15) i.e., random number $nr_i^{h,0}$ is generated which is normally distributed over [0, 1] and compared with $(\beta_{i,k}^{h,0})^2$ which is 1/2 as initialized. Therefore, if the random number $nr_{i,k}^{h,t}$ is less than 1/2, the commitment bit of corresponding unit in the wolf for that particular hour is initialized as 1 ($\chi_{i,k}^{h,0} = 0$) and vice versa. This can be summarised as follows.

$$\chi_{i,k}^{h,0} = \begin{cases} 1, & nr_i^{h,0} < 1/2 \\ 0, & otherwise \end{cases} \tag{24}$$

The initial positions of α, β *and* δ wolves are determined as the wolf's positions with first, second and third lowest cost respectively.

3.2.2. Updating Q-bits using Q-gate

The Q-gate update is given by,

$$\Delta\theta_{i,k}^{h,t+1} = \theta \times \left\{ \left(\gamma_{1i}^t \chi_i^{\alpha,h,t} - \chi_{i,k}^{h,t}\right) + \left(\gamma_{2i}^t \chi_i^{\beta,h,t} - \chi_{i,k}^{h,t}\right) + \left(\gamma_{3i}^t \chi_i^{\delta,h,t} - \chi_{i,k}^{h,t}\right) \right\} \tag{25}$$

Where, $\gamma_{1i}^t, \gamma_{1i}^t$ *and* γ_{1i}^t for α, β *and* δ wolves for t^{th} iteration are given by,

$$\gamma_{1i}^t = \begin{cases} 1, & F\left(\mathbb{X}_k^t\right) < F\left(\mathbb{X}^{\alpha,t}\right) \\ 0, & otherwise \end{cases} \tag{26}$$

$$\gamma_{2i}^t = \begin{cases} 1, & F\left(\mathbb{X}_k^t\right) > F\left(\mathbb{X}^{\alpha,t}\right) \&\& F\left(\mathbb{X}_k^t\right) < F\left(\mathbb{X}^{\beta,t}\right) \\ 0, & otherwise \end{cases} \tag{27}$$

$$\gamma_{3i}^t = \begin{cases} 1, & F\left(\mathbb{X}_k^t\right) > F\left(\mathbb{X}^{\alpha,t}\right) \&\& F\left(\mathbb{X}_k^t\right) > F\left(\mathbb{X}^{\beta,t}\right) \&\& F\left(\mathbb{X}_k^t\right) < F\left(\mathbb{X}^{\delta,t}\right) \\ 0, & otherwise \end{cases} \tag{28}$$

Where, $F(\mathbb{X}_k^t)$ is the objective fitness for search vector/wolf position vector $\mathbb{X}_k^t$ in kth iteration which can be estimated using economic load dispatch (ELD) of committed thermal units. This paper adopts the Lambda iteration approach for solving ELD [18]. Therefore, the updated Q-bits of a Q-bit individual are given by,

$$\begin{bmatrix} \alpha_{i,k}^{h,t+1} \\ \beta_{i,k}^{h,t+1} \end{bmatrix} = \begin{bmatrix} cos\left(\Delta\theta_{i,k}^{h,t+1}\right) & -sin\left(\Delta\theta_{i,k}^{h,t+1}\right) \\ sin\left(\Delta\theta_{i,k}^{h,t+1}\right) & cos\left(\Delta\theta_{i,k}^{h,t+1}\right) \end{bmatrix} \begin{bmatrix} \alpha_{i,k}^{h,t} \\ \beta_{i,k}^{h,t} \end{bmatrix} \tag{29}$$

The Q-bit update process is constrained with respect to the following relation.

$$\left|\alpha_{i,k}^{h,t+1}\right|^2 + \left|\beta_{i,k}^{h,t+1}\right|^2 = 1 \tag{30}$$

3.2.3. Updating wolf position

The position vector ($\mathbb{X}_k^t$) of kth wolf during ith iteration is carried out using the updated Q-bit information as given by,

$$\chi_{i,k}^{h,t} = \begin{cases} 1, & if\ nr_i^{h,t} < \left|\beta_{i,k}^{h,t}\right|^2 \\ 0, & otherwise \end{cases} \tag{31}$$

3.2.4. Updating position of α, β and *δ wolves*

The position update of each of α, β *and* δ depends on possibility of finding better position vector ($\mathbb{X}_k^t$) as given by,

$$\mathbb{X}^{\alpha,t+1} = \begin{cases} \mathbb{X}_k^{t+1}, & if\ F\left(\mathbb{X}_k^{t+1}\right) \le F\left(\mathbb{X}^{\alpha,t}\right) \\ \mathbb{X}^{\alpha,t}, & otherwise \end{cases} \tag{32}$$

$$\mathbb{X}^{\beta,t+1} = \begin{cases} \mathbb{X}_k^{t+1}, & if\ F\left(\mathbb{X}^{\alpha,t}\right) \le F\left(\mathbb{X}_k^{t+1}\right) \le F\left(\mathbb{X}^{\beta,t}\right) \\ \mathbb{X}^{\beta,t}, & otherwise \end{cases} \tag{33}$$

$$\mathbb{X}^{\delta,t+1} = \begin{cases} \mathbb{X}_k^{t+1}, & if\left\{F\left(\mathbb{X}^{\alpha,t}\right) \&\& F\left(\mathbb{X}^{\beta,t}\right)\right\} \le F\left(\mathbb{X}_k^{t+1}\right) \le F\left(\mathbb{X}^{\delta,t}\right) \\ \mathbb{X}^{\delta,t}, & otherwise \end{cases} \tag{34}$$

3.2.5. Termination criteria

The proposed approach considers number of iterations as termination criteria. Therefore, the QI-BGWO stops as soon as the iteration reaches pre-defined maximum number of iterations. Fig. 2 shows the flow chart of proposed QI-BGWO approach for solving UC problem.

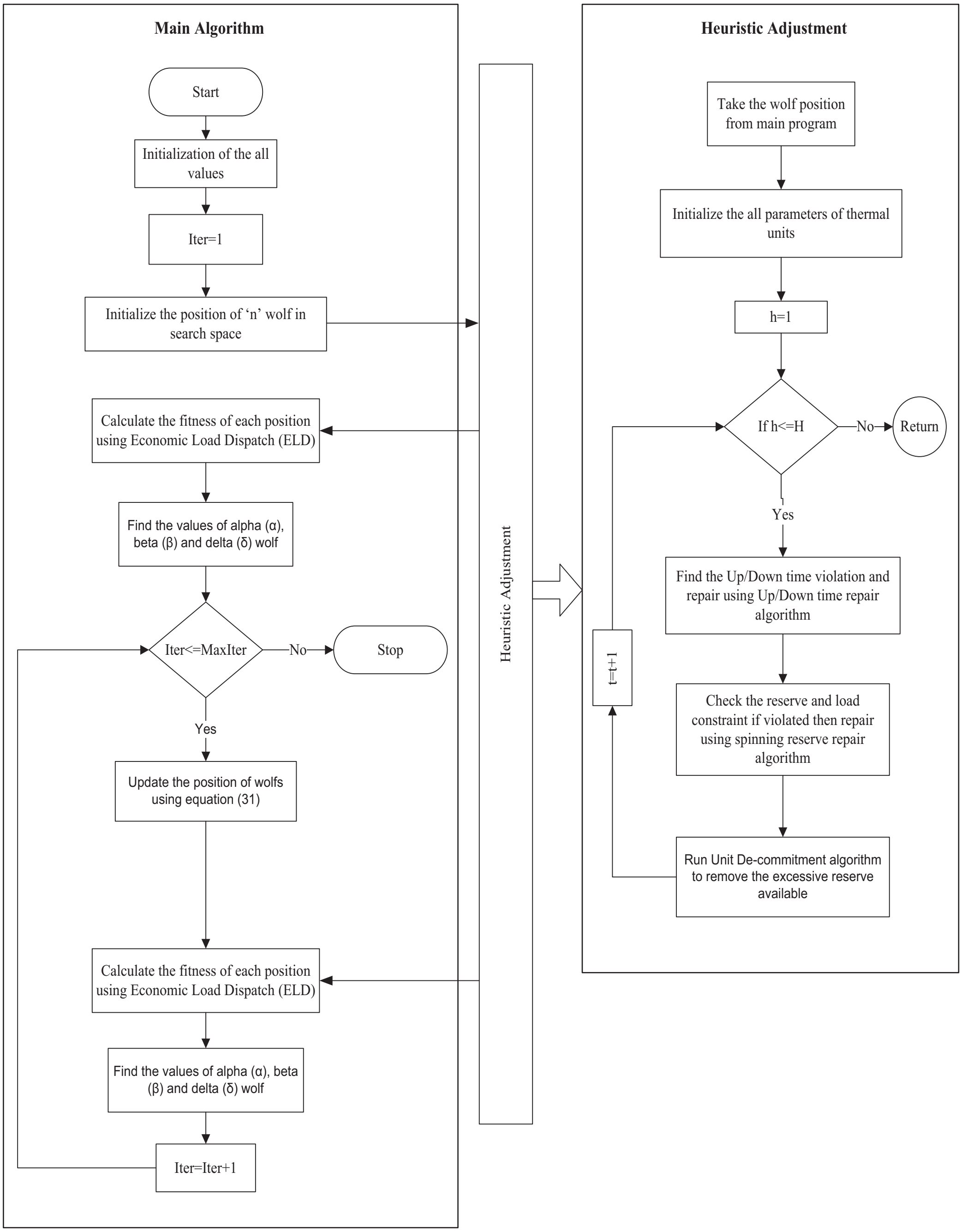

Fig. 2. Flowchart of proposed QI-BGWO for UC problem.

3.3. Constraint repair techniques

The heuristic approach of handling constraints based on rule based mechanism is adopted in this work [16]. In the process of random initialization and update process of evolutionary algorithms like QI-BGWO, often the state transition of units may violate minimum-up down time constraints.

3.3.1. Minimum up/down constraints

The commitment and De-commitment event of unit must abide the up/down constraints of the same. This paper uses rule based heuristic adjustment process to tackle the constraint repair as explained in Algorithm 1.

Table 2
Effect of rotation angle on solution quality (generation cost/fitness value).

$\theta_{max}-\theta_{min}$ (rad.)	Best ($)	Average ($)	Worst ($)	Std. Deviation ($)
0.05π–$0.01\ \pi$	2,244,343	2,245,184	2,245,861	463.3794
$0.05\ \pi$–$0.001\ \pi$	2,244,429	2,244,912	2,245,673	441.9851
$0.04\ \pi$–$0.01\ \pi$	**2,242,947**	**2,244,071**	**2,244,279**	**190.8368**
$0.04\ \pi$–$0.001\ \pi$	2,243,028	2,244,102	2,244,247	165.7347
$0.03\ \pi$–$0.01\ \pi$	2,243,322	2,244,188	2,244,511	258.7272
$0.03\ \pi$–$0.001\ \pi$	2,243,918	2,244,124	2,244,462	207.1592
$0.02\ \pi$–$0.01\ \pi$	2,244,088	2,244,190	2,244,388	85.04488
$0.02\ \pi$–$0.001\ \pi$	2,243,878	2,244,323	2,244,505	187.1011

Table 3
Numerical results of proposed approaches for different test systems.

Units	Approach	Cost ($)			
		Best ($)	Average ($)	Worst ($)	Std. Deviation
10	QEP	563,936.3	563,980.04	564,017.6	18.97852
	BGWO (tanh)	563,937.3	563,945.17	563,976.64	25.44
	QI-BGWO	563,936.3	563,936.3	563,936.3	0
20	QEP	1,123,404	1,123,455.9	1,123,505	36.9198
	BGWO (tanh)	1,124,552.9	1,124,805.41	1,124,891	222.0301
	QI-BGWO	1,123,294	1,123,458.6	1,123,526	41.65551
40	QEP	2,245,520	2,245,675.4	2,245,933	164.8325
	BGWO (tanh)	2,248,138	2,248,304.7	2,248,428	414.7316
	QI-BGWO	2,242,947	2,244,071.4	2,244,279	190.8368
60	QEP	3,364,972	3,366,000.2	3,367,247	927.0687
	BGWO (tanh)	3,367,473	3,367,765	3,367,861	193.1321
	QI-BGWO	3,361,766	3,364,280.2	3,364,873	452.8217
80	QEP	4,489,856	4,492,184.7	4,493,418	956.229
	BGWO (tanh)	4,492,477	4,492,527.5	4,492,648	124.9673
	QI-BGWO	4,481,925	4,486,760.6	4,487,935	452.8217
100	QEP	5,608,527	5,609,920.5	5,611,208	1042.705
	BGWO (tanh)	5,610,159	5,612,106	5,612,368	449.772
	QI-BGWO	5,602,365	5,605,773.4	5,606,974	712.6927

3.3.2. Spinning reserve and load satisfaction repair

The load and spinning reserve satisfaction constraints are enforced to guarantee the reliability of the system. In case of these constraint violation, the de-committed units are committed until the demand and spinning reserve constraints are satisfied. The detailed procedure of load and reserve constraint repair strategy is explained in Algorithm 2 as follows.

3.3.3. De-commitment algorithm under excessive spinning reserve

Algorithm 3 presents the procedure of De-commitment to avoid excess online capacity.

4. Numerical results and discussion

The proposed QI-BGWO is modelled to solve UC problem with test systems of different dimensions starting from 10 to 100 units for 24 h scheduling horizon [18]. The test systems of 20 units, 40 units, 60 units, 80 units and 100 units are generated by duplicating the 10 unit test system [18]. Similarly, the hourly load for the test systems other than 10 unit system is estimated by duplicating/multiplying with appropriate scalar value.

4.1. Parameter settings for QI-BGWO

4.1.1. Rotation angle determination

The solution of QI-BGWO is affected by θ which in turn depends on the variation range decided by θ_{min}andθ_{max}. In this approach, the rotation angle θ is dynamically updated every iteration within the rangeθ_{min}*to* θ_{max}. Therefore choosing the optimal range of maximum and minimum rotation angle is very important to attain solution of good quality. For the same, the ranges of rotation angles are tested with a population size of 30 and solution quality is observed Table 2). It can be observed that, range $0.04\ \pi$–$0.01\ \pi$ resulted in best possible lowest generation cost. Thus this range is selected as optimum rotation angle for dynamic rotation angle update ((20).

4.1.2. Population size determination

The population size influences the effective exploration of search space. However, oversizing the same may lead to increased computational time with considerably less improvement in solution quality. Therefore, selection of optimal popula-

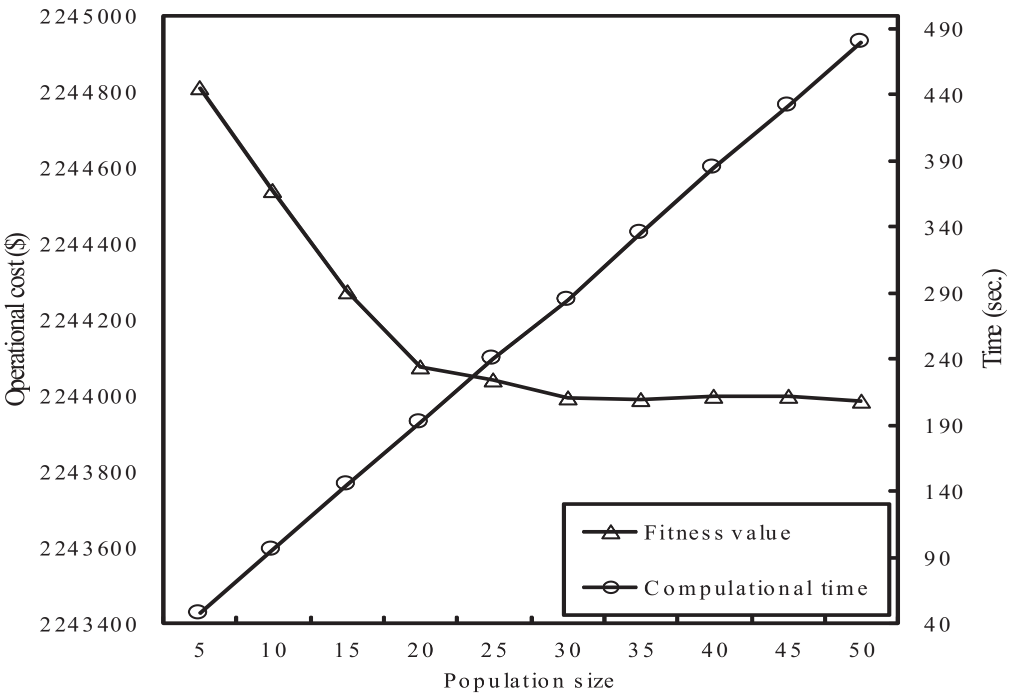

Fig. 3. Variation of solution quality and computational time as a function of population size for 40 unit test system.

tion size is a key aspect of any optimization problem. In this paper, simulation experiments are performed to observe the solution quality and computational time for different population sizes.

The effect of population size on solution quality and computational time is observed for population sizes varying from 5 to 50 with $\theta_{max} = 0.04\,\pi$ and$\theta_{min} = 0.01\,\pi$. It can be observed (Fig. 3) that, the solution quality saturated at higher values of population sizes but the computational time increased with population size. Therefore, population size of 30 has been selected for UC problem considering higher execution times. Similarly, the range of rotation angle for which lowest mean fitness is observed for range 0.04π–0.001π.

4.2. Case studies: BGWO and QI-BGWO performance and convergence

The effectiveness of proposed BGWO and QI-BGWO is verified though simulations on test systems with 10–100 units. The experimental results are tabulated in Table 3. The hyperbolic tanh/V-shaped function is considered for BGWO comparison because of its superior performance over s-shaped function.

The population size and rotation angle used for the numerical experiments are decided by the parametric analysis. The optimal range of rotation angle used $0.04\,\pi$–$0.01\,\pi$ (Table 2) and optimal population size of 30 is selected to achieve a compromise between solution quality and computational efficiency. The cost convergence characteristics of BGWO and QI-BGWO for 10–100 unit systems with maximum iteration set to 500 are shown in Fig. 4. It can be observed that, BGWO has rapid convergence characteristic compared to QI-BGWO. However, the solution quality of QI-BGWO has surpassed that of BGWO. The comparison for convergence characteristics of proposed BGWO and QI-BGWO with traditional evolutionary algorithm implemented with quantum principles is shown in Fig. 5.

4.3. Comparison of proposed approaches with various other approaches

The comparison with other algorithms for various test systems from 10–100 units is presented in Table 4. The comparison of both the proposed approaches with other approaches reveals the effectiveness of BGWO with respect to GA [1], EP [2], SA [3] and IPSO [5] for all test systems from 10–100 units. The proposed heuristic approach QI-BGWO showed superior performance compared to other quantum based approaches viz. IQEA [16], QEA [14], QBPSO [18] etc. However, no particular inferences can be drawn from comparison for computational times owing to the differences in computational facilities of different approaches.

The commitment and economic dispatch schedule for 10 unit and 24 h horizon using proposed QI-BGWO approach is shown in Table 5. Similarly, the commitment schedule of larger system with 100 thermal units is presented in Table 6. The generation scheduling corresponding to commitment schedule presented in Table 8 can be obtained by performing ELD of committed units.

Table 4
Comparison of proposed approaches with various other approaches.

	10 Unit system					20 Unit Cost ($)				
Approach	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. time (S)	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. time (S)
GA [1]	565,825	–	570,032	0.74	221	1,126,243	–	1,132,059	0.52	733
MINLP (GAMS)	567,022	–	–	–	0.083	1,138,513	–	–	–	2.137
EP [2]	564,551	565,352	566,231	0.3	100	1,125,494	1,127,257	1,129,793	0.38	340
SA [3]	565,828	565,988	566,260	0.08	3	1,126,251	1,127,955	1,129,112	0.25	17
IPSO [5]	563,954	564,162	564,579	0.11	–	1,125,279	–	1,127,643	0.21	–
BFWA [10]	563,977	564,018	564,855	–	65.42	1,124,858	1,124,941	1,125,087	–	106.03
IBPSO [20]	563,977	564,155	565,312	0.025	27	1125,216	1125,448	1125,730	0.015	55
FuzzySA DP [21]	563,978	–	–	–	–	1123,390	–	–	–	–
DACGA [22]	563,987	–	–	–	–	–	–	–	–	–
hGADE/r1 [23]	563,938	564,044	–	–	–	1123,383	1124,436	–	–	–
MILP [24]	563,937.7	–	–	–	0.58	1123,297.4	–	–	–	16.4
HHSRS [9]	563,937.6	563,965.31	563,995.33	–	16.83	1,124,889	1,124,912.8	1,124,951.5	–	35.01
GWO-PSO [8]	565,210.25	–	–	–	–	–	–	–	–	–
HDE-RS [9]	558,681.	560,154.4	561,809.1	–	227.36	1121,013	1121,096	1122,104	–	696.8
RCGA [7]	563,937	564,019	564,219	3.001E-05	–	1123,297	1123,851	1124,537	4.26842E-05	–
IQEA [16]	563,977	563,977	563,977	0	15	1,123,890	1,124,320	1,124,504	0.05	42
QEA [14]	563,938	563,969	564,672	0.13	19	1,123,607	1,124,689	1,125,715	0.19	28
QBPSO [18]	563,977	563,977	563,977	0	18	1,123,297	1,123,981	1,124,294	0.09	50
BGWO	**563,936**	563,936	563,936	0	71.2	1,124,614	1,124,857	1,125,156	0.01973851	148.1
QI-BGWO	**563,936**	563,936	563,936	0	62.3	**1,123,294**	1,123,459	1,123,526	0.01976309	112.12
	40 Unit Cost ($)					**60 Unit Cost ($)**				
Approach	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. Time (Sec)	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. Time (Sec)
GA [1]	2,251,911	–	2,259,706	0.35	2697	3,376,625	–	3,384,252	0.23	5840
MINLP (GAMS)	2,257,248	–	–	–	11.522	3,392,140	–	–	–	69.721
EP [2]	2,249,093	2,252,612	2,256,085	0.31	1176	3,371,611	3,376,255	3,381,012	0.28	2267
SA [3]	2,250,063	2,252,125	2,254,539	0.2	88	–	–	–	–	–
IPSO [5]	2,248,163	–	2,252,117	0.18	–	3,370,979	–	3,379,125	0.24	–
BFWA [10]	2,248,228	2,248,572	2,248,645	–	238.02	3,367,445	3,367,828	3,367,974	–	422.29
IBPSO [20]	2248,581	2248,875	2249,302	0.011	110	3367,865	3368,278	3368,779	0.009708225	172
FuzzySA DP [21]	2244,334	–	–	–	–	3366,975	–	–	–	–
hGADE/r1 [23]	2243,724	2245,321	–	–	–	3363,470	3365,587	–	–	–
HHSRS [9]	2248,508	2,248,652.7	2248,757	–	179.6	–	–	–	–	–
HDE-RS [9]	2241,564	2248,379	2255,194	–	2394.9	–	–	–	–	–
RCGA [7]	2242,887	2243,569	2244,117	3.34207E-05	–	3365,337	3366,052	3366,873	2.49551E-05	–
MILP [24]	2242,575	–	–	–	271	3359,954.8	–	–	–	960
IQEA [16]	2,245,151	2,246,026	2,246,701	0.07	132	3,365,003	3,365,667	3,366,223	0.04	273
QEA [14]	2,245,557	2,246,728	2,248,296	0.12	43	3,366,676	3,368,220	3,372,007	0.16	54
QBPSO [18]	2,242,957	2,244,657	2,245,941	0.13	158	3,361,980	3,363,763	3,365,707	0.11	328
BGWO	2,248,024	2,248,350	2,249,076	0.0184	284.5	3,367,276	3,367,550	3,367,755	0.0057	477.1
QI-BGWO	**2,242,947**	2,244,071	2,244,279	0.018	196.46	**3,361,766**	3,364,280	3,364,873	0.0057	355.54

(*continued on next page*)

Table 4 (*continued*)

Approach	**80 Unit Cost ($)**					**100 Unit Cost ($)**				
	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. Time (Sec)	Best ($)	Average ($)	Worst ($)	Difference (%)	Avg. Time (Sec)
GA [1]	4,504,933	–	4,510,129	0.12	10,036	5,627,437	–	5,637,914	0.19	15,733
MINLP (GAMS)	4,526,022	–	–	–	204.03	5,646,219	–	–	–	524.11
EP [2]	4,498,479	4,505,536	4,512,739	0.32	3584	5,623,885	5,633,800	5,639,148	0.27	6120
SA [3]	4,498,076	4,501,156	4,503,987	0.13	405	5,617,876	5,624,301	5,628,506	0.19	696
IPSO [5]	4,495,032	–	4,508,943	0.31	–	5,619,284	–	5,628,506	0.24	–
BFWA [10]	4,491,284	4,492,550	4,493,036	–	676.53	5,610,954	5,612,422	5,612,790	–	1043.47
IBPSO [20]	4491,083	4491,681	4492,686	0.0120	235	5610,293	5611,181	5612,265	0.015	295
FuzzySA DP [21]	4490,844	–	–	–	–	5610,217	–	–	–	–
hGADE/r1 [23]	4486,180	4489,500	–	–	–	5604,787	5610,074	–	–	–
RCGA [7]	4486,991	4487,476	4487,949	2.18386E-05	–	5606,663	5607,088	5607,850	2.372E-05	–
MILP [24]	–	–	–	–	–	5597,770.1				6341
IQEA [16]	4,486,963	4,487,985	4,489,286	0.05	453	5,606,022	5,607,561	5,608,525	0.04	710
QEA [14]	4,488,470	4,490,128	4,492,839	0.1	66	5,609,550	5,611,797	5,613,220	0.07	80
QBPSO [18]	4,482,085	4,485,410	4,487,168	0.11	554	5,602,486	5,604,275	5,606,178	0.07	833
BGWO	4,492,340	4,492,480	4,492,605	0.002	751.2	5,611,557	5,612,051	5,612,665	0.0080	1036.8
QI-BGWO	**4,481,925**	4,486,761	4,487,935	0.002	639.987	**5,602,365**	5,605,773	5,606,974	0.00802	912.43

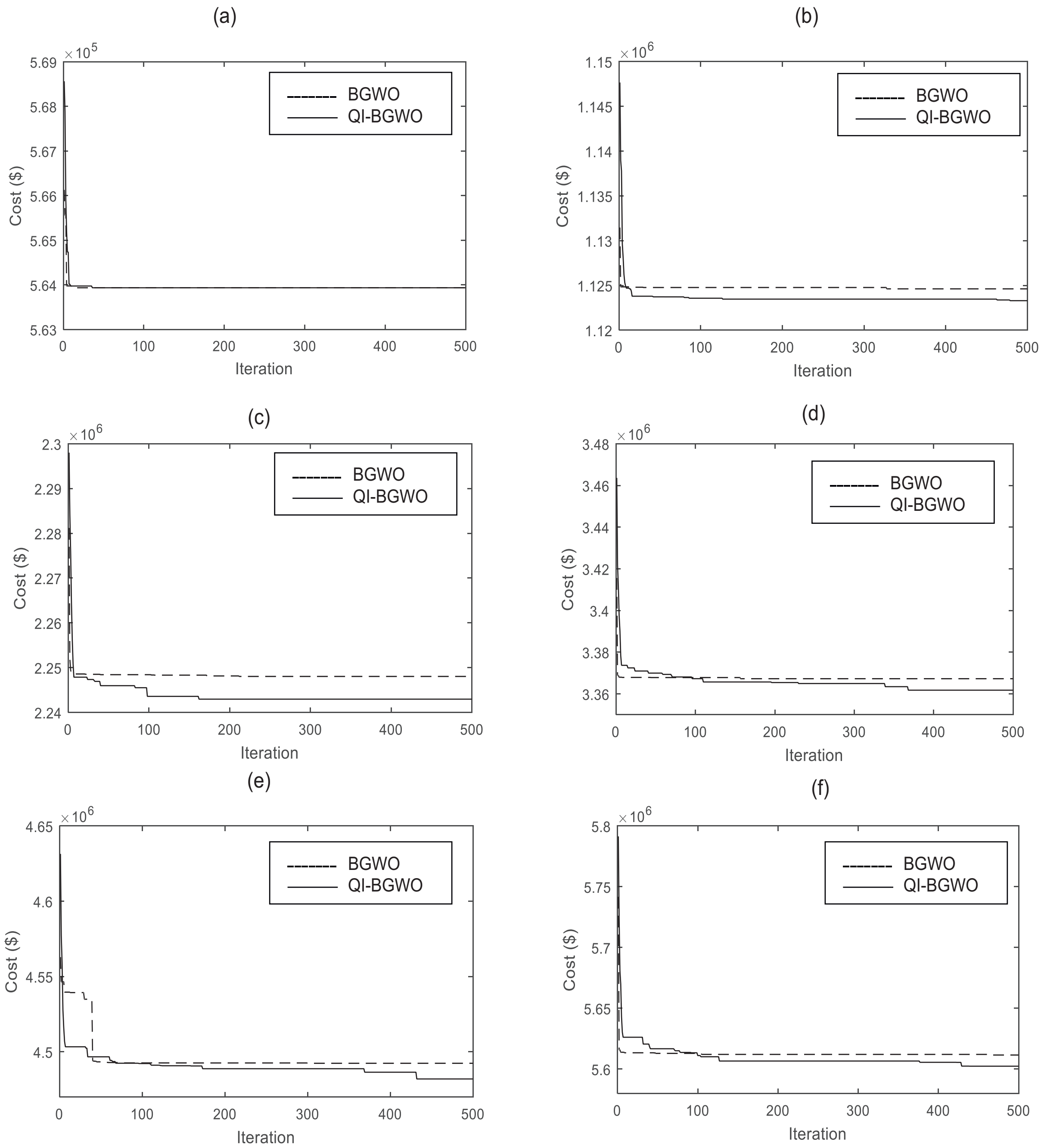

Fig. 4. Convergence characteristics of different test systems with proposed approaches. (a) 10 unit case (b) 20 unit case (c) 40 unit case (d) 60 unit case (e) 80 unit case (f) 100 unit case.

4.4. Statistical analysis

Apart from the performance comparison of optimization approaches in terms of metrics such as best fitness, mean fitness and standard deviation, statistical test provide more insights on the true performance of various approaches. Also, the statistical significance of a new algorithm with respect to existing approaches can also be demonstrated using statistical tests. The same have been used in recent studies to establish the statistical significance of computational approaches for solving UC problem [23]. The most commonly used tests include Friedman test (aligned ranks, on-aligned ranks) and Wilcoxon test. Generally, the best and mean fitness along with standard deviation are mentioned in performance and the individual trail values are seldom reported. Therefore, the most relative approaches are modelled in this work for comparing

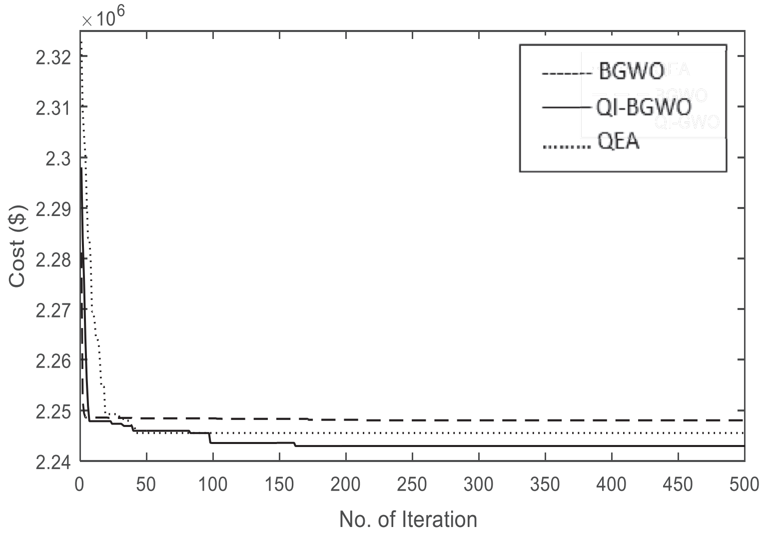

Fig. 5. Convergence characteristic comparison of proposed approaches with traditional quantum approaches.

Table 5
Unit commitment schedule and reserve available for 10 unit system.

Hour	Unit1	Unit2	Unit3	Unit4	Unit5	Unit6	Unit7	Unit8	Unit9	Unit10	Load	Reserve
1	455	245	0	0	0	0	0	0	0	0	700	210
2	455	295	0	0	0	0	0	0	0	0	750	160
3	455	370	0	0	25	0	0	0	0	0	850	222
4	455	455	0	0	40	0	0	0	0	0	950	122
5	455	390	0	130	25	0	0	0	0	0	1000	202
6	455	360	130	130	25	0	0	0	0	0	1100	232
7	455	410	130	130	25	0	0	0	0	0	1150	182
8	455	455	130	130	30	0	0	0	0	0	1200	132
9	455	455	130	130	85	20	25	0	0	0	1300	197
10	455	455	130	130	162	33	25	10	0	0	1400	152
11	455	455	130	130	162	73	25	10	10	0	1450	157
12	455	455	130	130	162	80	25	43	10	10	1500	162
13	455	455	130	130	162	33	25	10	0	0	1400	152
14	455	455	130	130	85	20	25	0	0	0	1300	197
15	455	455	130	130	30	0	0	0	0	0	1200	132
16	455	310	130	130	25	0	0	0	0	0	1050	282
17	455	260	130	130	25	0	0	0	0	0	1000	332
18	455	360	130	130	25	0	0	0	0	0	1100	232
19	455	455	130	130	30	0	0	0	0	0	1200	132
20	455	455	130	130	162	33	25	10	0	0	1400	152
21	455	455	130	130	85	20	25	0	0	0	1300	197
22	455	455	0	0	145	20	25	0	0	0	1100	137
23	455	425	0	0	0	20	0	0	0	0	900	90
24	455	345	0	0	0	0	0	0	0	0	800	110

the performance using non-parametric statistical tests. The algorithms selected for this purpose include binary approaches and quantum computing based approaches such as binary particle swarm optimization (BPSO), binary fireworks algorithm (BFWA), quantum evolutionary programming (QEP), and quantum inspired binary particle swarm optimization (QBPSO) etc. The performance of proposed BGWO and QI-BGWO approaches is evaluated for 10 unit, 40 unit and 100 unit test systems to account for small, medium and large scale systems. The distribution of independent samples/solutions for 10 unit, 40 unit and 100 unit test systems using various binary and quantum approaches along with BGWO and QI-BGWO are presented in Fig. 6. The statistical significance of proposed approaches is demonstrated at a significance level of 0.05. The *p-value 1* denotes the statistical significance (significant differences) of proposed BGWO and QI-BGWO over all the algorithms considered for comparison. On the other hand, *p-value 2* takes into account the significance differences between BGWO and QI-BGWO at the significance level considered.

The Friedman non-aligned ranks test provides performance comparison in terms of simple ranks. The performance of proposed BGWO and QI-BGWO in terms of Friedman ranks is compared to other approaches in Table 7. It can be observed that both BGWO and QI-BGWO performed to same extent for smaller test system (10 unit). However, QI-BGWO has clear superior performance when compared to BGWO for medium (40 unit) and large scale (100 unit) test systems. Also, BGWO

Table 6
Commitment schedule of 100 unit test system using QI-BGWO.

Units	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1–10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
11–18	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
19–20	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
21	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
22–25	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
26–30	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
31–32	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
33	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
34–39	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
40	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
41–45	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
48–47	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
48–50	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
51–60	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	0	0
61–63	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	1	1	0	0
64–70	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
71–78	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	0	0
79–80	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
81–88	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0	0	0	0
98	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0
90	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
91–98	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
99–100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

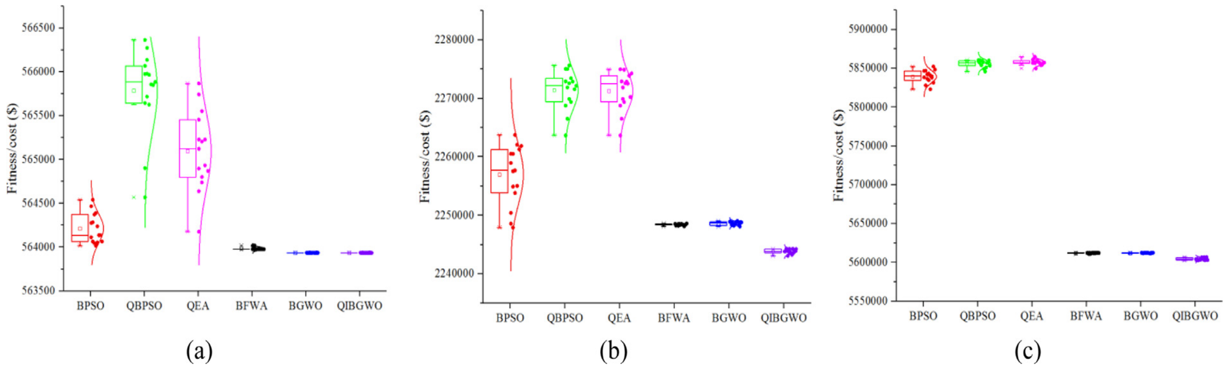

Fig. 6. Solution/independent sample distribution for (a) 10 unit system (b) 40 unit system and (c) 100 unit system.

Table 7
Friedman simple (non-aligned) ranks and associated p-values for various approaches across different test systems.

Approach	*10 unit system*			*40 unit system*			*100 unit system*			*Mean rank*
	Rank	*p-value1*	*pvalue-2*	*Rank*	*p-value1*	*pvalue-2*	*Rank*	*p-value1*	*pvalue-2*	
BPSO	4.066667			3.866667			4			3.977778
QBPSO	5.8			5.533333			5.466667			5.6
QEA	5.133333	2.52E−14	1	5.466667	6.08E−14	1.08E−04	5.533333	7.79E−14	1.08E−04	2.447634
BFWA	3			2.2			2.266667			2.488889
BGWO	**1.5**			2.933333			2.733333			2.388889
QIBGWO	**1.5**			**1**			**1**			**1.166667**

lacked superiority in performance compared to BFWA at large scale systems. The overall mean rank suggest the dominance of QI-BGWO on other approaches including BGWO. For BGWO also, the mean performance edges over other binary and quantum based approaches. The p-value 1 for all the test systems is expectedly higher compared to p-value 2 i.e., BGWO and QI-BGWO have higher differences with other approaches compared to the differences within themselves.

The performance of a particular algorithm compared to the average performance of all algorithms can be evaluated using Friedman aligned ranks method. The ranks of various algorithms with respect to the average performance are presented in Table 8. The same reveals a mere shift in the range of ranks with relative performances of a particular algorithm with respect to other algorithms remained same as that of the non-aligned Friedman ranks. This can be attributed to the fact that the order performance in case of absolute samples as well as samples w.r.t average performance is same. The p-value

Table 8
Friedman aligned ranks and associated p-values for various approaches across different test systems.

Approach	10 unit system		40 unit system		100 unit system		Mean rank
	Rank	p-value	Rank	p-value	Rank	p-value	
BPSO	49		51.6		53.1333		51.24443
QBPSO	81.1333		75.9333		74.0667		77.04443
QEA	69.4667	6.53E−14	75.0667	7.04E−16	76.8	3.96E−16	44.26668
BFWA	29.8667		29.8667		29.9333		29.8889
BGWO	21.7667		32.5333		31.0667		28.45557
QIBGWO	**21.7667**		**8**		**8**		**12.5889**

Table 9
Wilcoxon pairwise comparison for 10 unit test system.

	BPSO	QBPSO	QEA	BFWA	BGWO	QIBGWO
BPSO	1	6.10E−05	0.000122	6.10E−05	6.10E−05	6.10E−05
QBPSO	6.10E−05	1	0.003357	6.10E−05	6.10E−05	6.10E−05
QEA	0.000122	0.003357	1	6.10E−05	6.10E−05	6.10E−05
BFWA	6.10E−05	6.10E−05	6.10E−05	1	6.10E−05	6.10E−05
BGWO	6.10E−05	6.10E−05	6.10E−05	6.10E−05	1	1
QIBGWO	6.10E−05	6.10E−05	6.10E−05	6.10E−05	1	1

Table 10
Wilcoxon pairwise comparison for 10 unit test system.

	BPSO	QBPSO	QEA	BFWA	BGWO	QIBGWO
BPSO	1	6.10E−05	6.10E−05	0.000183	0.000183	6.10E−05
QBPSO	6.10E−05	1	0.910156	6.10E−05	6.10E−05	6.10E−05
QEA	6.10E−05	0.910156	1	6.10E−05	6.10E−05	6.10E−05
BFWA	0.000183	6.10E−05	6.10E−05	1	0.012451	6.10E−05
BGWO	0.000183	6.10E−05	6.10E−05	0.012451	1	6.10E−05
QIBGWO	6.10E−05	6.10E−05	6.10E−05	6.10E−05	6.10E−05	1

Table 11
Wilcoxon pairwise comparison for 10 unit test system.

	BPSO	QBPSO	QEA	BFWA	BGWO	QIBGWO
BPSO	1	6.10E−05	6.10E−05	6.10E−05	6.10E−05	6.10E−05
QBPSO	6.10E−05	1	0.719727	6.10E−05	6.10E−05	6.10E−05
QEA	6.10E−05	0.719727	1	6.10E−05	6.10E−05	6.10E−05
BFWA	6.10E−05	6.10E−05	6.10E−05	1	0.187622	6.10E−05
BGWO	6.10E−05	6.10E−05	6.10E−05	0.187622	1	6.10E−05
QIBGWO	6.10E−05	6.10E−05	6.10E−05	6.10E−05	6.10E−05	1

of algorithms/approaches for Friedman aligned ranks test is lower compared to its non-aligned counterpart. The same reveals that, the statistical significance (significance differences) for various solution vectors compared to average solution vector is lower when compared to differences in absolute solution vector comparison.

The Friedman and Friedman aligned ranks test can be used to test the hypothesis of statistical significance among a group of approaches. However, fails to compare individual algorithm significance with respect to other algorithm. The same can be verified using Wilcoxon pairwise comparison test. The Wilcoxon pairwise comparison of various approaches with respect to each other in terms of *p-values* across difference test systems is presented from Tables 9–11. It can be observed from Table 9 all the approaches differ significantly from each other for 10 unit system. However, the differences between QEP and QBPSO are not significant for medium (40 unit) size and large scale (100 unit) test systems. Also, there are no significant differences between BGWO and BFWA at large scale test systems. Nevertheless, QI-BGWO is observed to have significant differences with each of other algorithms including BGWO considered for medium and large scale test systems. Therefore, from the Friedman and Wilcoxon tests, the superior performance and statistical significance can be established for proposed QI-BGWO when compared to existing binary and quantum inspired approaches.

5. Conclusion and future scope

A quantum inspired binary grey wolf optimizer (QI-BGWO) is proposed using Q-bit to solve unit commitment problem. The proposed approach uses the hunting behaviour of grey wolf and the Q-bit, Q-gate, superposition principles of quantum computing. The probabilistic representation of Q-bit individual is used along with coordinate Q-gate rotation angle and dynamic update of rotation angle magnitude to update the position of wolves of grey wolf pack. The advanced concepts of

Algorithm 1 Pseudo code of up/down time constraint repair algorithm.

```
Begin
    For i = 1 to MaxUnit (i.e. 'N') do
        If h == 1
            If χ_i^h == 1 then
                T_{i,h}^on ← T_{i,h−1}^on + 1;
                T_{i,h}^off ← 0;
            Else If χ_i^h == 0 then
                T_{i,h}^off ← T_{i,h−1}^off + 1;
                T_{i,h}^on ← 0;
            End If
        Else
            If χ_i^h == 1 then
                If T_{i,h−1}^off < T_i^MD then χ_i^h ← 0;
                    T_{i,h}^off ← T_{i,h−1}^off + 1;
                    T_{i,h}^on ← 0;
                Else
                    T_{i,h}^on ← T_{i,h−1}^on + 1;
                    T_{i,h}^off ← 0;
                End If
            Else If χ_i^h == 0 then
                If T_{i,h−1}^on < T_i^MU then χ_i^h ← 1;
                    T_{i,h}^on ← T_{i,h−1}^on + 1;
                    T_{i,h}^off ← 0;
                Else
                    T_{i,h}^off ← T_{i,h−1}^off + 1;
                    T_{i,h}^on ← 0;
                End If
            End If
        End If
    End For
End
```

Algorithm 2 Pseudo code of reserve and load constraint repair algorithm.

```
Begin
    Sort the units in order of Max generation capacity and store position in 'g';
    For i = 1 to MaxGen (i.e. 'N') do
        k ← g(i);
        If Σ_i χ_i^h P_i^max < P_d^h + R_sp^h then
            If χ_k^h == 1 then Continue with Next 'i';
            Else
                χ_k^h ← 1;
                If T_{k,h}^off > T_k^MD then
                    T_{k,h}^on ← T_{k,h−1}^on + 1;
                    T_{k,h}^off ← 0;
                Else
                    l ← t − T_{k,h}^off + 1;
                    If l ≤ 0 then l ← 1 End If
                    While l > t do
                        χ_k^l ← 1;
                        T_{k,l}^on ← T_{k,l−1}^on + 1;
                        T_{k,l}^off ← 0;
                    End While
                End If
            End If
        Else
            Break For loop;
        End If
    End For
End
```

quantum computing improved the balance between exploration and exploitation of search space there by resulting in better solution quality at considerably lower population size. The performance of proposed heuristic quantum computing based approach QI-BGWO concludes its effectiveness in solving UC problem. Therefore, it can be concluded that, proposed approach can be applied to large scale UC problems effectively. In addition, statistical tests are performed to show demonstrate the superior performance of proposed QIBGWO. The proposed approaches can be applied to solve other binary problems in power systems problems such as profit based unit commitment, under the scenario of restructured power systems. Similarly, the

Algorithm 3 Pseudo code of Unit De-commitment algorithm.

```
Begin
    Sort the units in order of Min generation capacity and store position in 'g';
    For i = 1 to MaxGen (i.e. 'N') do
        k ← g(i);
            If χ_k^h == 0 then Continue with Next 'i';
            Else
                If Σ_i χ_i^h P_i^max − P_k^max ≥ P_d^h + R_sp^h then
                    If T_{k,h}^on > T_k^MU or T_{k,h}^on == 1 then
                        χ_k^h ← 0;
                        T_{k,h}^off ← T_{k,h−1}^off + 1;
                        T_{k,h}^on ← 0;
                    Else
                            Continue with Next 'i';
                    End If
                Else
                  Break For loop;
                End If
            End If
        End For
End
```

proposed QI-BGWO can be extended to solve combined economic and emission dispatch (CEED) of thermal units in emission constrained power systems. Also, the proposed BGWO and QI-BGWO approaches can be implemented in conjugation with other real coded approaches as hybrid algorithms to solve power system problem with binary as well as real valued variables such as demand response scheduling, electric vehicle scheduling in modern power systems.

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