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Small-World Optimization Algorithm for Function Optimization

Haifeng Du, Xiaodong Wu, and Jian Zhuang

Key Laboratory of Education Ministry for Modern Design and Rotor-Bearing System
Xi'an Jiaotong University, Xi'an 710049, China
Haifengdu@mail.xjtu.edu.cn

Abstract. Inspired by the mechanism of small-world phenomenon, some small-world optimization operators, mainly including the local short-range searching operator and random long-range searching operator, are constructed in this paper. And a new optimization algorithm, Small-World Optimization Algorithm (SWOA) is explored. Compared with the corresponding Genetic Algorithms (GAs), the simulation experiment results of some complex functions optimization indicate that SWOA can enhance the diversity of the population, avoid the prematurity and GA deceptive problem to some extent, and have the high convergence speed. SWOA is shown to be an effective strategy to solve complex tasks.

1 Introduction

The small-world phenomenon has its roots in experiments performed by the social psychologist Stanley Milgram in the 1960s to trace out short paths through the social networks of the United States [1]. The results indicate that any two individuals in the social network are all linked by short chains of acquaintances (popularly known as "six degrees of separation"). After Duncan Watts and Steve Strogatzs' paper - "Collective Dynamics of Small-World Networks" published in Nature in 1998 [2], the small-world phenomenon has been introduced to network theory by some computer scientists. And it's with great expectations to become an important theory related to computer science after systematology, information theory and cybernetics. The small-world phenomenon is an interdisciplinary problem involving with sociology, mathematics and computation science, which has become hotspots in the theoretical discussion of Artificial Intelligence (AI) and Complexity Theory. It has been successfully used in internet control [3], AIDS diffusion forecast [4] and dynamical research of biological protein networks [5], yet little discussion about small-world phenomenon has been found in the field of optimization algorithm.

If the precision is set, optimization space can be considered as a grid space (network space) constructed by the variables. We can take the optimization as a process that information transmits from candidate solution to optimal solution in search space (networks). In this paper, therefore, a new optimal search algorithm is constructed which makes use of the effective information transmission mechanism inspired by the small-world phenomenon. We proceed as follows: Section 2 provides

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a description of the general theories about small-world phenomenon and introduces the optimal search process from the point view of the small-world principle. Based on the local shortcuts search operator and random long-range search operator of small-world, section 3 discusses a kind of Small World Optimization Algorithm (SWOA). The experiments and the corresponding analyses bout SWOA are described in Sections 4. Finally, Section 5 states some conclusions.

2 Small-World Networks and Optimal Process

Roughly speaking, the small-world phenomenon reveals a most effective mode of information transmission in the movement of a lot of complex networks (such as social network), which are high clustering subnest including “local contacts” nodes and some random long-range shortcuts that conduce to produce shortcuts to increase efficiency of information transmission. The small-world phenomenon still has no accurate definition. Ordinarily, if the mean distance L between two nodes of the networks increases logarithmically with the number of network nodes N (namely $L \propto \ln N$), it can be considered that the network has the small-world phenomenon.

The small-world network of Watts and Strogatz is an intermediate between regular and random networks [2], namely, it combined with a subnet that has local clustering shortcuts and some random long-range shortcuts. Kleinberg points out that the small-world phenomenon is an efficiency question about routing algorithm, where local knowledge suffices to find effective paths to get to destination [1].

Without loss of generality, we consider the following global optimization problem P:

$$\text{maximize} \quad f(\boldsymbol{x}) \quad \text{subject to} \quad \boldsymbol{d} \le \boldsymbol{x} \le \boldsymbol{u} \tag{1}$$

Where, $\boldsymbol{x} = \{x_1, x_2, \cdots, x_m\} \in \Re^m$ is a variable vector, $\boldsymbol{d} = \{d_1, d_2, \cdots, d_m\}$ and $\boldsymbol{u} = \{u_1, u_2, \cdots, u_m\}$ define the feasible solution space, namely, $x_i \in [d_i, u_i]$ $i = 1,2,\cdots,m$,$f(\boldsymbol{x})$ is the objective function.

When m=2, x_1-x_2 constructs the search space as figure 1 shown, in which x_1,x_2 get its value only at the circle. Let T be the optimal solution and B be the initial candidate solution, thus the resolving procedure of optimal question P is the search from B to T, which can be considered as the information transmission process from B to T, as the broken line shown in figure 1. The optimal search networks that we were talking about above is very similar to “the planar lattice small-world networks with many random contact”[2] constructed by Watts and Strogatz. If we consider the small-world phenomenon in search process, which is to say that we introduce global random search (random long-range contacts) into local search (local shortcuts) and emphasize the global result induced by local action, an optimal search algorithm with small-world phenomenon will be constructed. We call this kind of algorithms as Small world Optimization Algorithm (SWOA).

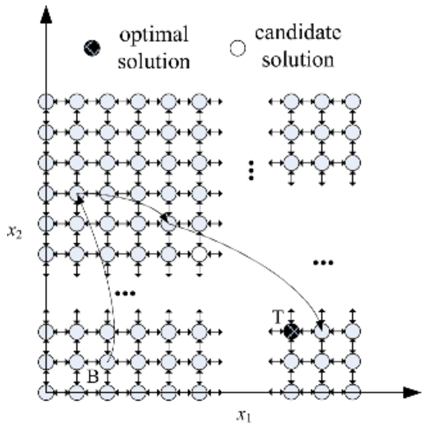

Fig. 1. Node Lattice of Two-dimensional Space Search Question

3 Small-World Operator and the Algorithm

3.1 Basic Definition

Considering the efficiency of algorithm and Stanley Milgram's design about sociological examination [1], the point which participates in search or information transmission should not be a simple one but a set of candidate solutions. We denote these transmission node populations as set $\boldsymbol{S}$. Furthermore, $\boldsymbol{s} = s_1 s_2 \cdots s_l$, $\boldsymbol{s} \in \boldsymbol{S}$, is the node coding of the variable $\boldsymbol{x}$, described by $\boldsymbol{s} = \hbar(\boldsymbol{x})$, and $\boldsymbol{x}$ is called the decoding of node $\boldsymbol{s}$, described as $\boldsymbol{x} = \hbar^{-1}(\boldsymbol{s})$. Let τ be the number of the possible values of a_i. For binary coding and decimal coding, there are $\tau = 2$ and $\tau = 10$ accordingly. Generally, antibody bit string is divided into m parts, and the length of each part is l_i, where $l = \sum_{i=1}^{m} l_i$, and each part is denoted as $x_i \in [d_i, u_i]$ $\quad i = 1,2,\cdots,m$. Especially, for binary coding, we use the decoding method as follow:

$$x_i = d_i + \frac{u_i - d_i}{2^{l_i} - 1}\left(\sum_{j=1}^{l_i} s_j 2^{j-1}\right) \tag{2}$$

In this paper, we adopt 0-1 binary encoding.

And small-world search space ($\boldsymbol{s} \in \boldsymbol{I}$) is:

$$\boldsymbol{I}^n = \{\boldsymbol{S} : \boldsymbol{S} = (\boldsymbol{s}_1, \boldsymbol{s}_2 \cdots \boldsymbol{s}_n),\ \ \boldsymbol{s}_k \in \boldsymbol{I},\ \ 1 \le k \le n\} \tag{3}$$

Where the positive integer n is the size of node population. The node population $\boldsymbol{S}=\{\boldsymbol{s}_1,\boldsymbol{s}_2\cdots\boldsymbol{s}_n\}$, which is an n-dimension group of node $\boldsymbol{s}$, is a point in the node group space $\boldsymbol{I}^n$

The global optimal solutions for set of problem P is defined as the following

$$\boldsymbol{T}\equiv\left\{\boldsymbol{s}\in\boldsymbol{I}:f(e^{-1}(\boldsymbol{s}))=f^{*}\equiv\min(f(\boldsymbol{x}):\boldsymbol{d}\le\boldsymbol{x}\le\boldsymbol{u})\right\} \tag{4}$$

Where f^{*} is the optimal value of objective function f. For the node population $\boldsymbol{S}$, $\vartheta(\boldsymbol{S})\equiv|\boldsymbol{S}\cap\boldsymbol{T}|$ denotes the number of optimal solutions in antibody population $\boldsymbol{S}$.

Define the distance between $\boldsymbol{s}_i,\boldsymbol{s}_j\in\boldsymbol{S}$ is $d\left(\boldsymbol{s}_i,\boldsymbol{s}_j\right)=\left\|\boldsymbol{s}_i-\boldsymbol{s}_j\right\|$, where $\|\bullet\|$ denotes an arbitrary norm. For binary coding, we use Hamming distance; while for decimal coding we generally use Euclidean distance.

The ℓ neighborhood set of the node $\boldsymbol{s}_i$ can be defined as

$$\varsigma^{\ell}(\boldsymbol{s}_i)=\left\{\boldsymbol{s}_j\,\middle|\,0<\left\|\boldsymbol{s}_i-\boldsymbol{s}_j\right\|\le\ell,\quad \boldsymbol{s}_j\in\boldsymbol{S}\right\} \tag{5}$$

And $\left|\varsigma^{\ell}(\boldsymbol{s}_i)\right|$ is the number of elements in $\varsigma^{\ell}(\boldsymbol{s}_i)$, $\overline{\varsigma^{\ell}(\boldsymbol{s}_i)}$ is the non- ℓ neighborhood set of $\boldsymbol{s}_i$. Obviously, there is

$$\overline{\varsigma^{\ell}(\boldsymbol{s}_i)}=\left\{\boldsymbol{s}_j\,\middle|\,\ell<\left\|\boldsymbol{s}_i-\boldsymbol{s}_j\right\|,\quad \boldsymbol{s}_j\in\boldsymbol{S}\right\} \tag{6}$$

Furthermore, if $\boldsymbol{s}_i\notin\boldsymbol{T}$ and $\vartheta\left(\varsigma^{\ell}(\boldsymbol{s}_i)\right)\equiv\left|\varsigma^{\ell}(\boldsymbol{s}_i)\cap\boldsymbol{T}\right|=0$, there is $\vartheta\left(\overline{\varsigma^{\ell}(\boldsymbol{s}_i)}\right)\equiv\left|\overline{\varsigma^{\ell}(\boldsymbol{s}_i)}\cap\boldsymbol{T}\right|\ge 1$.

3.2 Local Shortcuts Search Operator

The main action of local shortcuts search operator $\boldsymbol{\Psi}$ is transmitting the information from node $s_i(k)$ to the corresponding node $s_i(k+1)$ which is the nearest node to the goal set T in $\varsigma^{\ell}\left(\boldsymbol{s}_i(k)\right)$ with minor ℓ. We denote this process with $\boldsymbol{s}_i(k+1)\leftarrow\Psi\left(\boldsymbol{s}_i(k)\right)$. In practice, $\left|\varsigma^{\ell}\left(\boldsymbol{s}_i(k)\right)\right|$ is very big. We often randomly select $n_l(k)<\left|\varsigma^{\ell}\left(\boldsymbol{s}_i(k)\right)\right|$ nodes from $\varsigma^{\ell}\left(\boldsymbol{s}_i(k)\right)$ to construct a temporary local transmission network to realize local search. For example: 20 bits binary encoding node $\boldsymbol{s}_i(k)$, $\left|\varsigma^{1}\left(\boldsymbol{s}_i(k)\right)\right|=19$ but $\left|\varsigma^{2}\left(\boldsymbol{s}_i(k)\right)\right|=\left|\varsigma^{1}\left(\boldsymbol{s}_i(k)\right)\right|+\mathrm{C}_{20}^{2}$. In detail, we construct the following local shortcuts search operator $\boldsymbol{\Psi}$ for binary encoding.

$\boldsymbol{s}_i'(k)\leftarrow\boldsymbol{s}_i(k)\times 0$ get a temporary zero bit string which has the same construction as $\boldsymbol{s}_i(k)$ .And $\oplus$, $\neg$, $\otimes$ corresponding to bit-oriented adding、minus and multiply.

Local Shortcuts Search Operator $\boldsymbol{\Psi}$

given $n_l(k) < \left|\varsigma^{\ell}(\boldsymbol{s}_i)\right|$; $\boldsymbol{s}_i(k+1) \leftarrow \boldsymbol{s}_i(k)$;

begin

$j \leftarrow 0$;

repeat

$\boldsymbol{s}_i'(k) \leftarrow \boldsymbol{s}_i(k) \times 0$;

Stochastic selecting r($0 < r \le \ell$) bits from $\boldsymbol{s}_i'(k)$ and change them into 1;

$\boldsymbol{s}_i'(k) \leftarrow \boldsymbol{s}_i(k) \neg 2 \times \left(\boldsymbol{s}_i(k) \otimes \boldsymbol{s}_i'(k)\right) \oplus \boldsymbol{s}_i'(k)$;

if $f\left(e^{-1}\left(\boldsymbol{s}_i'(k)\right)\right) < f\left(e^{-1}\left(\boldsymbol{s}_i(k+1)\right)\right)$

$\boldsymbol{s}_i(k+1) \leftarrow \boldsymbol{s}_i'(k)$;

end if

$j \leftarrow j+1$;

until $j = n_l(k)$

end.

Fig. 2. Flowchart of Local Link Search Operator

3.3 Random Long-Range Search Operator

Essentially, a point in $\overline{\varsigma^{\ell}(\boldsymbol{s}_i)}$ is stochastically selected as the information transmission object point, $\boldsymbol{s}_i'(k)$ of $\boldsymbol{s}_i(k)$, by random long-range search operator $\boldsymbol{\Gamma}$ according to the preset probability distribution, which can be described as $\boldsymbol{S}'(k) \leftarrow \boldsymbol{\Gamma}\left(\boldsymbol{S}(k)\right)$ (ℓ gets a bigger value, namely $\left\|\boldsymbol{s}_i - \boldsymbol{s}_i'(k)\right\|$ is bigger). Considering the convenience and availability, the Inverse Operation in evolutionary computation is used to construct the operator $\boldsymbol{\Gamma}$. The random long-range search operator $\boldsymbol{\Gamma}$ in this paper is described as the figure 3.

Where rand(0-1) generates uniform distributed random number whose values fall between 0 and 1. $\boldsymbol{s}_i'(k)\big|_{\mu}^{\nu}$ means overturning positive sequence of the characters' from bit μ to bit ν ,for example:

$$\boldsymbol{s}_i'(k) \quad \underset{\mu}{1}\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ \underset{\nu}{1}\ 1\ 0\ 0 \xrightarrow{\boldsymbol{s}_i'(k)\big|_{\mu}^{\nu}} 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 0$$

p_l in the operator $\boldsymbol{\Gamma}$ is corresponding to the inversion probability of inversion operation in evolutionary algorithm except that a limit for the distance between μ and ν is set.

Random Long-range Search Operator $\boldsymbol{\Gamma}$

Given global long-range probability p_l and ℓ ;

begin

$i \leftarrow 0$;

Repeat

$\boldsymbol{s}_i'(k) \leftarrow \boldsymbol{s}_i(k)$;

$p \leftarrow \text{rand}(0-1)$;

if $p_l < p$ and $\boldsymbol{s}_i(k)$ is not the optimal solution in current set of node,

Stochastic generate two integer μ 、 ν ,where $1 \le \mu < \nu \le l$ and $|\mu - \nu| > \ell$;

$\boldsymbol{s}_i(k) \leftarrow \boldsymbol{s}_i'(k)\big|_{\mu}^{\nu}$;

end if

$i \leftarrow i+1$;

until $i = n$

end.

Fig. 3. Flowchart of Random Long-range Search Operator

3.4 Small World Optimal Algorithm

Based on the local shortcuts operator $\boldsymbol{\Psi}$ and random long-range operator above, the step of realizing SWOA is shown in figure 4.

The stop conditions are defined as either the restricted iterative number or the time when the solutions are not improved at successively iterations, or both of the two. Combining the enactment iterative times with hunting condition, here the algorithm is halted as the following criterion:

$$\left|f^* - f^{best}\right| < \varepsilon \tag{7}$$

Where f^* is the global optimum of f, f^{best} is the current best function value. If $0 < \left|f^*\right| < 1$, the following equation holds:

$$\left|f^* - f^{best}\right| < \varepsilon\left|f^*\right| \tag{8}$$

Compared with the evolutionary algorithm, the SWOA does not adopt crossover operator or global selection operation; instead, local selecting is applied so that local information transmission in small world phenomenon is emphasized. Instead of simple point search strategy, the algorithm adopts population as in the evolutionary algorithm, which is consistent with socialist investigative experiment [1].

According to the analysis of the two-dimensional grid network just like figure 2, Jon Kleinberg proved that the expected delivery time of the decentralized algorithm is at most $\alpha(\log N)^2$ [6]. Inspired by Jon Kleinberg's analyzing and thinking of the

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Small World Optimal Algorithm (SWOA)

Initialization: Enactment the size of node population n, inversion probability p_l , ℓ and other parameters. initialize the node population $\boldsymbol{S}(0)=\{\boldsymbol{s}_1(0),\boldsymbol{s}_2(0),\cdots,\boldsymbol{s}_n(0)\}$;

$k \leftarrow 0$;

begin

while terminative condition doesn't satisfy;

$\boldsymbol{S}'(k) \leftarrow \boldsymbol{S}(k)$;

$\boldsymbol{S}'(k) \leftarrow \boldsymbol{\Gamma}(\boldsymbol{S}'(k))$;

$i \leftarrow 0$

Repeat

$\boldsymbol{s}_i'(k+1) \leftarrow \boldsymbol{\Psi}(\boldsymbol{s}_i'(k))$, $\boldsymbol{s}_i'(k) \in \boldsymbol{S}'(k)$;

if $f\left(e^{-1}(\boldsymbol{s}_i'(k+1))\right) < f\left(e^{-1}(\boldsymbol{s}_i(k))\right)$

$\boldsymbol{s}_i(k+1) \leftarrow \boldsymbol{s}_i'(k+1)$;

else

$\boldsymbol{s}_i(k+1) \leftarrow \boldsymbol{s}'(k)$;

end if

$i \leftarrow i+1$;

until $I = n$

$k \leftarrow k+1$;

end while

end.

Fig. 4. Flowchart of SWOA

characteristic of SWOA, we can roughly estimate SWOA's computation complexity as $\alpha(l \log \tau)^2$.

4 Simulation Experiment

4.1 Test Function and Conditions

Seven functions adopted in our simulation experiment are shown as formulas from (9) to (15). These functions' space shapes are so complex that their optimal question can not be better solved by using general algorithm.

$$\max f_1(x,y) = 1 + x\times\sin(4\pi x) - y\times\sin(4\pi y+\pi) + \frac{\sin(6\sqrt{x^2+y^2})}{6\sqrt{x^2+y^2}+10^{-15}} \quad x,y\in[-1,1],\ f^*=2.118 \tag{9}$$

$$\max f_2(x,y) = 1 + x\times\sin(4\pi x) - y\times\sin(4\pi y+\pi) \quad x,y\in[-1,1],\ f^*=2.055 \tag{10}$$

$$\max f_3(x,y) = \left(\frac{b}{a+(x^2+y^2)}\right)^2 + (x^2+y^2)^2 \quad x,y\in[-5.12,5.12],\ f^*=3600 \text{ where } a=0.05\ b=3 \tag{11}$$

$$\min f_4(x,y)=\left\{\sum_{i=1}^{5} i\cos[(i+1)x+i]\right\}\times\left\{\sum_{i=1}^{5} i\cos[(i+1)y+i]\right\} \qquad x,y\in[-10,10],\ f^*=-186.73 \tag{12}$$

$$\min f_5(x,y)=\left(4-2.1x^2+x^{\frac{4}{3}}\right)x^2+xy+\left(-4+4y^2\right)y^2 \qquad x,y\in[-5.12,5.12]\quad f^*=--1.031628 \tag{13}$$

$$\max f_6(x,y)=-\left(x^2+y^2\right)^{0.25}\left(\sin^2 50\left(x^2+y^2\right)^{0.1}+1.0\right) \qquad x,y\in[-5.12,5.12]\ f^*=0 \tag{14}$$

$$\min f_7(x,y)=20+x^2-10\cos(2\pi x)+y^2-10\cos(2\pi y) \qquad x,y\in[-5.12,5.12]\quad f^*=0 \tag{15}$$

All numerical experiments in this paper are implemented on PC of PIV2.4G、512MRAM by using Matlab language.

4.2 Experiment Result and Analysis

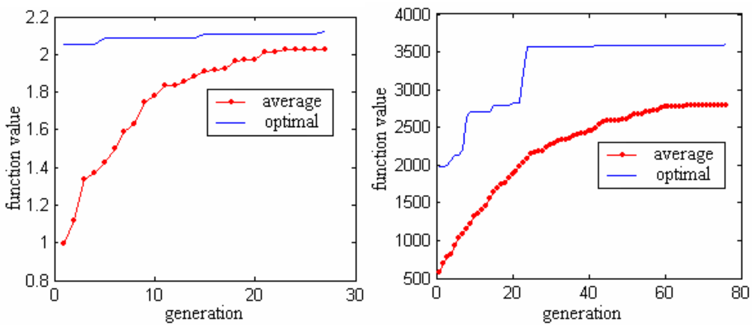
The performance comparison of SWOA and a GA (MGA) is shown in table 1. MGA is a modified genetic algorithm which adopts new selection operator and population update strategy [7]. The population size of MGA is 30 and the encoding length is 80, maximal generation is 1500, crossover probability is 0.7, mutation probability is 0.0017, Generation gap (GGAP, namely how many new individuals are created) is 0.9. In order to keep the approximate computational complexity with MGA, the population size $\boldsymbol{S}$ of SWOA is30 and $n_l(k)$ is 4, p_l is 0.3, ℓ is 1, node's encoding length is also 80.The statistical results in table 1 are obtained from 50 simulations.

Table 1. Performance comparison of MGA and SWOA

Contrast item		f_1	f_2	f_3	f_4	f_5	f_6	f_7
N_{best}	MGA	21	50	45	19	49	8	49
	SWOA	50	50	50	50	50	50	50
N_{max}	MGA	74	26	1382	706	200	1031	1005
	SWOA	113	29	1136	166	49	396	88
N_{min}	MGA	2	1	45	20	1	161	19
	SWOA	5	2	32	7	7	43	23
N_{mean}	MGA	14.48	6.24	387.9	78.95	22.43	593.	363.0
	SWOA	31.92	8.56	115.7	59.84	25.9	116.3	45.92
N_{std}	MGA	15.57	5.24	308.6	153.68	27.43	339.0	266.3
	SWOA	22.02	5.07	166.1	34.84	11.75	54.50	14.84
ε		1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-1}	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}	1.0×10^{-2}

Where N_{bes} denote the time of reaching the optimal value, N_{max}, N_{min}, N_{mean}, and N_{std} denote the maximum, minimum , average and standard deviation of the evaluation number reaching the optimal value respectively. N_{max}, N_{min}, N_{mean}, and N_{std} in table 1 are only the statistic results without immersing into the local optimal value, because MGA can break away from the local optimal triumphantly only for function

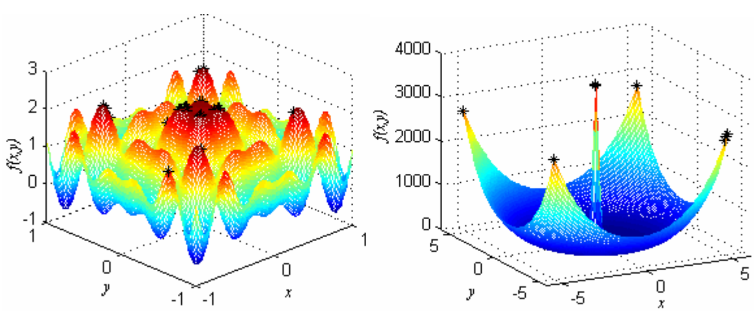
f_2 in 50 simulations. For f_2, the performance of SWOA is equivalent with MGA. For other functions, SWOA has more strong ability to break away from the local optimum than MGA; and the convergent speed of ADCSA is generally faster than MGA. N_{std} of SWOA is less than that of MGA universally, which indicate that SWOA has better robustness and stability than MGA. f_3 is a function which is used in Evolutionary Computation to test algorithm's ability to overcome deceptive problem. The optimal result shows that SWOA overcomes the deceptive problem to a certain extent. The relatively changing of the objective function denotes the algorithm's diversity essentially. For functions f_1 and $f_{3,}$ the change of objective function in one experiment is shown in fig 5. It can be seen that SWOA keeps the population diversities better.



(a) f_1 (b) f_3

Fig. 5. The changing of the function value for SWOA

Figure 6 gives one of the optimization solution distributed in function space for f_1 and $f_{3,}$ where "*" denotes the final solution.



(a) f_1 (b) f_3

Fig. 6. The optimized result of SWOA

Obviously, SWOA has searched a lot of local optimal solution at the same time. The corresponding evolutionary computation, such as MGA, can also search the optimal solution but the distribution of the solution is centralized, furthermore the individual of population may become completely the same. Accordingly, SWOA has better diversity than MGA.

5 Conclusions and Prospecting

Based on the view of small-world phenomenon, the text gives a new description of optimal search process. It considers that the search process is essentially an information transmission process in search space networks. Different from six degrees of separation phenomenon experiment, we know less or even no knowledge about the behavior of object point so that the question will be more complex. Making use of theories of small-world phenomenon, the paper constructs local shortcuts search operator and random long-range search operator; furthermore it constructs a Small World Optimization Algorithm used for optimization. The experiment of some typical function optimal questions shows that the SWOA has following advantages: avoiding the problem of getting into local minimum; increasing the diversity of population; improving convergence rate and overcoming deceptive problem to a certain extent.

Main work of the paper is concentrating on constructing a new optimal algorithm based on the Small-World Phenomenon mechanism, at the same time it exploits a new field in application of small-world phenomenon. Kleinberg found that a subtle variant of the Watts–Strogatz network will in fact support efficient search: Rather than adding the long-range shortcuts uniformly at random, add links between nodes of this network with a probability that decays like the d^{th} power of their distance (in d dimensions)[1]. Therefore how to improve the searching efficiency of small world algorithm and the interrelated theoretical analysis are foci of our further research.

Acknowledgements

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