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Critical analysis of weighting functions for the deep level transient spectroscopy of semiconductors

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Abstract. More than 20 weighting functions proposed for DLTS are critically compared. It is shown that the variety of DLTS peak widths of different weighting functions was primarily due to the variety of shapes of the high-temperature side of the peak. A refinement of the classification scheme of the weighting functions is proposed. It is shown that the high-temperature side of the DLTS peak can be made independent of the weighting function, by considering the delay time t_d between the end of the filling pulse and the beginning of the weighting function as an integral part of each weighting function. An optimum delay time (specific for each weighting function) is obtained by maximizing the figure of merit defined as the signal-to-noise ratio of the correlator divided by the DLTS peak width. Using the concept of optimum delay time, the functions can be classified into several groups according to their selectivity. The functions from one group differ only by their sensitivity, making it easy to select the functions with the best signal-to-noise ratios. In the second part of the paper, several misunderstandings concerning exponential correlators and double boxcars that frequently appear in the literature are revealed.

1. Introduction

Deep level transient spectroscopy (DLTS) [1] has become an important experimental tool for the detection and characterization of deep level defects in semiconductors. The sensitivity and selectivity of a DLTS system are strongly affected by the choice of the weighting function used to correlate the capacitance transients [2]. According to the theory of signal processing, the best sensitivity is provided by the weighting function which has the form of the noise-free signal itself; that is, for a DLTS system it should be a decaying exponential [2]. Because the exponential correlator has the poorest selectivity, compared with those of other weighting functions, more than 20 different correlation functions have been proposed during the past 20 years [2–24] to improve the resolution of DLTS. Each weighting function was claimed to have advantages over the already known ones. However, because there were neither self-consistent methods of classification of the weighting function nor commonly accepted parameters which would help to quantify the differences between correlators, there was a good deal of misunderstanding of the importance of the correct choice of the correlation function for the performance of a DLTS system.

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In the first part of the paper the refinement of the classification of weighting functions proposed by Crowell and Alipanahi [3] is suggested. This refinement makes the classification self-consistent and convenient for the comparative analysis. On the basis of this classification, the most frequently used weighting functions are critically compared in the second part of the paper. Parameters of the discussed weighting functions are determined and presented in the form of tables.

2. The method of comparison of weighting functions

For the analysis and comparison of weighting functions $W(t)$, the output signal of each correlator was calculated, using a formula [2] with modifications [14] necessary to take into account the delay time t_d between the end of the filling pulse and the beginning of the correlation:

$$S(T, t_d) = t_c^{-1} \int_{t_d}^{t_d+t_c} \exp(-t/\tau(T)) W(t - t_d)\, \mathrm{d}t \qquad (1)$$

where t_c is the duration of the weighting function. The role of the delay time t_d will be discussed in section 4. Initially, one can assume that $t_d = 0$. The following parameters were calculated as functions of the delay time. The width

w of the DLTS peak was calculated as the ratio of the time constants of exponential relaxations τ_{min} and τ_{max} at which half the maximum amplitude $S(T_{max})$ of the DLTS peak is reached:

$$w = \tau_{max}/\tau_{min}. \tag{2}$$

It is easy to show that the time constants of two exponential relaxations of the same amplitude should differ at least by a factor of w in order for them to be resolved in the DLTS spectrum. In contrast to the actual DLTS peak width ΔT and the ratio $\Delta T/T$, the relative width w is independent of the parameters of the deep centre (E_a and σ) and the temperature position T of the peak and characterizes only the correlation function. $\Delta T/T$ can be obtained from w by using the following approximate relation [4]:

$$\frac{\Delta T}{T} = \frac{\ln(w)}{2 + E_a/(kT)}. \tag{3}$$

In terms of the activation energy, the resolution can be evaluated as

$$\Delta E_a = kT \ln(w). \tag{4}$$

The sensitivity of a correlator was determined as the signal-to-noise ratio (SNR) of the correlator, defined as SNR $= S/N$, where S was the amplitude of the DLTS peak, given by equation (1), and the noise N was found from the expression [3, 5]

$$N = \left(\int_{t_d}^{t_d+t_c} [W(t - t_d)]^2 \, \mathrm{d}t \right)^{1/2}. \tag{5}$$

The inverse value SNR^{-1} gives an estimate of the signal-to-noise ratio which the input transient should have in order for one to obtain a signal-to-noise ratio exceeding unity in the output spectrum. It should be noted that the SNR value given by equation (5) is calculated for a single transient. Averaging the sequence of transients noticeably improves the signal-to-noise ratio of the DLTS spectrum.

To characterize the weighting functions in terms of a single parameter, the figure of merit M was used. In [2] the figure of merit was the ratio of the DLTS signal amplitude S to the peak width w. In [3] it was actually the signal-to-noise ratio SNR. In this work, these two formulae were combined and the figure of merit M was defined as the ratio of the signal-to-noise ratio SNR and the DLTS peak width w:

$$M = 1000 \times \mathrm{SNR}/w = 1000 \times S/(N \times w). \tag{6}$$

This definition reflects the goal of finding a weighting function with the highest signal-to-noise ratio and the lowest peak width.

3. Classification of the weighting functions, based on the asymptotic analysis of the low-temperature side of the DLTS peak

The classification of weighting functions originally proposed by Crowell and Alipanahi [3] was based on a simple approximation for the low-temperature side of the DLTS peak. For slow transients with the time constant τ_s, corresponding to the low-temperature side of the DLTS peak ($\tau_s \gg \tau_0$, $\tau_0 \simeq (0.1$–$0.6)t_c$) the ratio t/τ_s will be much less than unity and the exponent under the integral in equation (1) can be expanded in a Taylor series:

$$S(\tau_s) = t_c^{-1} \int_{t_d}^{t_d+t_c} \left(1 - \frac{t}{\tau_s} + \frac{1}{2}\frac{t^2}{\tau_s^2} - \cdots \right) W(t - t_d) \, \mathrm{d}t. \tag{7}$$

If the weighting function satisfies the condition

$$\int_{t_d}^{t_d+t_c} t^k W(t - t_d) \, \mathrm{d}t = 0 \qquad 0 \le k < k_0 \tag{8}$$

then the first k_0 terms in equation (7) can be neglected and equation (1) can be approximated by

$$S(\tau_s) \approx t_c^{-1} \int_{t_d}^{t_d+t_c} \text{constant} \times \frac{t^{k0}}{\tau_s^{k0}} W(t - t_d) \, \mathrm{d}t = \text{constant} \times \tau_s^{-k0}. \tag{9}$$

The output signal of the correlator will be proportional to τ_s^{-k0} for slow transients. A filter with the characteristic $S(\tau_s) \propto \tau_s^{-k0}$ is called in electronics 'the k_0-order filter' and, following Crowell and Alipanahi [3], we use the same term to denote correlators with this kind of characteristic. Filters from the first to the fifth and higher orders are known and were proposed as weighting functions. Crowell and Alipanahi [3], Thurzo *et al* [6] and Hodgart [7] developed techniques to construct step-like weighting functions of a given order. The mathematical basis and properties of filters with binary coefficients were discussed in detail by Dmowski [8]. Atanasov [9, 10] developed filters of the sixth and higher orders, which were the sum of sine waves with different amplitudes and periods.

Filters of the first order reject the DC component of the input transients. Filters of the second order additionally reject the linear drifts over time, filters of the third order also reject quadratic drifts, and so on. Filters of orders higher than the first can be effectively used for measurement of samples with a strong dependence of the capacitance on temperature.

An approximation of the slow-transient (low-temperature) side of the DLTS signal given by equation (9) provided a basis for the classification of weighting functions into several large groups according to the order of the filter for slow transients [3]. A large collection of different correlation functions is presented in figure 1. Their parameters, obtained from equations (2)–(5) for zero delay time $t_d = 0$, are listed in table 1. Aside from the DLTS peak width w, signal-to-noise ratio SNR and figure of merit M, the following parameters were calculated: the amplitude of the DLTS signal S, corresponding to unity amplitude of the transient, and the ratio of the time constant of the transient to the period of correlation t_c at the DLTS maximum.

The first two functions in table 1 are the exponential correlator and the linear ramp, which, if used 'as they are', do not reject the DC component and are zeroth order filters. To obtain a maximum in the DLTS spectrum, Miller *et al* [2] proposed to restore the DC component of the relaxation during the period t_c. In practice, a sample-and-hold circuit is used to subtract the capacitance at the end of each transient from the next transient: $\Delta C(t) = C(t) - C(t_c + t_d)$. The output signal of the restored exponential correlator is given by

$$S(\tau_s) = t_c^{-1} \int_{t_d}^{t_d+t_c} \{\exp(-t/\tau_s) - \exp[-(t_c + t_d)/\tau_s]\} \times W(t - t_d)\,\mathrm{d}t. \qquad (10)$$

By expanding both exponents, we obtain

$$\begin{aligned} S(\tau_s) &= t_c^{-1} \int_{t_d}^{t_d+t_c} \left(1 - \frac{t}{\tau_s} + \cdots - 1 + \frac{t_c + t_d}{\tau_s} - \cdots\right) \times W(t - t_d)\,\mathrm{d}t \\ &= t_c^{-1} \int_{t_d}^{t_d+t_c} \left(\frac{t_c + t_d - t}{\tau_c} + \cdots\right) W(t - t_d)\,\mathrm{d}t \\ &\cong \text{constant} \times \tau_c^{-1}. \end{aligned} \qquad (11)$$

In other words, after the restoration of the DC component the exponential correlator becomes the first-order filter and can be used for DLTS.

Though the classification scheme of table 1 is based on fundamental filtering properties of weighting functions, there is a large scattering in the values of the parameters of the functions. DLTS peak widths and signal-to-noise ratios of functions of the same order may differ by an order of magnitude. Hence, this type of classification permits one to draw only a limited number of conclusions. Indeed, the exponential correlator has the highest signal-to-noise ratio. The linear ramp, which can be considered as an approximation for the exponential one, has a value of the SNR close to that of the exponential correlator. As was expected, all other functions have poorer sensitivity, though they generally have better selectivity. Though generally functions of higher order have smaller line widths and worse signal-to-noise ratios, it is not clear how to compare functions within the same order. Unfortunately, the classification according to the order of the filter alone only slightly helps one to understand the properties of weighting functions and to compare them. To improve the classification, we will analyse the high-temperature side of DLTS peaks and introduce the notion of the delay time.

4. Analysis of the high-temperature side of the DLTS peak and the influence of the delay time on the parameters of the correlators

The dependence $S(\tau)$ for $\tau \ll \tau_0$ (the high-temperature side of the DLTS peak) is determined by fast transients, decaying quickly after the filling pulse. Since the area under the plot of a decaying exponent $\exp(-t/\tau)$ is proportional to τ, an arbitrary weighting function (even $W(t) \equiv$ constant) will provide at least a first-order filter ($S(\tau_s) \propto \tau_s$ as $\tau_s \to 0$) for the fast transients:

$$S(\tau_s) = \int_0^{t_c} \exp(-t/\tau_s)\,\mathrm{d}t = \tau_s \times \text{constant}. \qquad (12)$$

The waveform of the weighting function W can in principle increase the order of the filter for fast transients. In fact, our simulations revealed that the waveform of most of the correlation functions only slightly affected the dependence $S(\tau_s)$ for $\tau_s \to 0$, shifting it along the τ/τ_{max} axis or changing its slope within about 5–10%. Only two exceptions were found, namely the sine wave and the split sine wave, which increased the slope of the dependence $S(\tau_s)$ to $S(\tau_s) \propto \tau_s^2$ for fast decays ($\tau_s \to 0$).

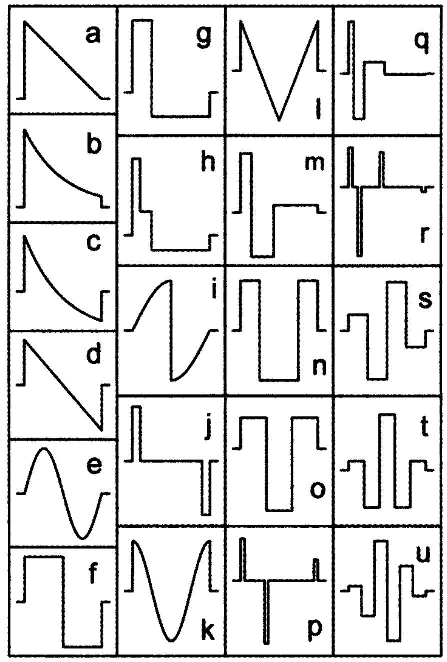

Figure 1. Waveforms of various weighting functions: (a) linear ramp, $w = 1 - t^*$, t^* being the normalized time $t^* = (t - t_d)/t_c$, $t^* \subset [0, 1]$; (b) exponential correlator, $w = \exp(-2t^*)$; (c) shifted exponential, $w = \exp(-2t^*) - [\exp(-2) - 1]/2$; (d) shifted linear ramp, $w = \frac{1}{2} - t^*$; (e) sine wave, $w = \sin(2\pi t^*)$; (f) rectangular lock-in, $w = \mathrm{sign}(\frac{1}{2} - t^*)$, $\mathrm{sign}(x)$ is defined as -1, if $x < 0$, and 1, if $x \geq 0$; (g) rectangle function of Crowell, step heights 1 and $-\frac{1}{3}$, step durations $0.25t^*$ and $0.75t^*$; (h) rectangle function of Hodgart, step heights 1, 0.309 and -0.196, step durations $0.1t^*$, $0.15t^*$ and $0.75t^*$; (i) split sine wave, $w = \sin(\pi t^*) \times \mathrm{sign}(\frac{1}{2} - t^*)$; (j) double boxcar, $w = 1$, $0 \leq t^* < 0.1$, $w = 0$, $0.1 \leq t^* \leq 0.9$, $w = -1$, $0.9 < t^* \leq 1$; (k) cosine, $w = \cos(2\pi t^*)$; (l) triangular, $w = 1 - 4t^*$, $0 \leq t^* < 0.5$, $4t^* - 3$, $0.5 \leq t^* < 1$; (m) three-step functions of Crowell, step heights 1, $-\frac{3}{4}$, $\frac{1}{8}$, step durations $\frac{1}{7}t^*$, $\frac{2}{7}t^*$ and $\frac{4}{7}t^*$; (n) square wave, step height 1, -1 and 1, step durations $\frac{1}{4}t^*$, $\frac{1}{2}t^*$ and $\frac{1}{4}t^*$; (o) HiRes-3, heights of equal-duration steps 1, -2 and 1; (p) three-point function of Dmowski, $w = 1$, $-\frac{3}{2}$ and $\frac{1}{2}$ at $t^* = 0$, $\frac{1}{3}$ and 1, strobe width 0.05; (q) four-step function of Crowell, step heights 1, $-\frac{7}{8}$, 7/32 and $-1/64$, step durations $(1/15)t^*$, $(2/15)t^*$, $(4/15)t^*$ and $(8/15)t^*$; (r) four-point function of Dmowski, strobe heights 1, $-\frac{7}{4}$, $\frac{7}{8}$ and $-\frac{1}{8}$, strobe positions 0, $t^*/7$, $3t^*/7$ and t^*, width of strobes $\Delta t^* = 0.05$; (s) HiRes-4, heights of equal-width step 1, -3, 3 and -1; (t) HiRes-5, heights of equal-width steps 1, -4, 6, -4 and 1; and (u) HiRes-6, heights of equal-width steps 1, -5, 10, -10, 5 and -1.

The dependence $S(\tau_s)$ for fast transients can be made steeper, introducing a delay time t_d between the end of the filling pulse and the beginning of the weighting function. The idea of increasing the resolution of the spectrum by delaying the weighting function was briefly mentioned in the early works of Miller *et al* [2], Crowell and Alipanahi [3] and Tokuda *et al* [11, 12]. Later Dmowski [8] obtained

Table 1. Parameters of the weighting functions from figure 1, calculated without a delay time. The letters used to denote weighting functions in figure 1, are given in the second column.

Order of the filter	Weighting function	Figure of merit M	DLTS peak width w	Signal-to-noise ratio SNR	Output signal amplitude S	τ_{max}/t_c
0(1)	(a) Linear ramp	19.4	19.3	0.37	0.216	0.500
	(b) Exponential	18.5	20.4	0.38	0.187	0.488
1	(c) Shifted exponential	10.6	24.2	0.26	0.062	0.331
	(d) Shifted linear ramp	11.0	21.9	0.24	0.139	0.372
	(e) Sine	11.8	15.2	0.18	0.126	0.424
	(f) Lock-in	11.1	18.3	0.20	0.204	0.398
	(g) Rectangular (Crowell)	10.3	21.8	0.23	0.130	0.289
	(h) Rectangular (Hodgart)	8.6	27.2	0.24	0.089	0.248
	(i) Split sine	9.0	14.2	0.13	0.090	0.440
	(j) Double boxcar	4.9	36.8	0.18	0.081	0.300
2	(k) Cosine	9.6	11.8	0.11	0.079	0.157
	(l) Triangular	9.1	12.6	0.12	0.066	0.154
	(m) Three-step (Crowell)	9.0	11.8	0.11	0.060	0.108
	(n) Square wave	9.1	10.6	0.096	0.096	0.164
	(o) HiRes-3	7.8	9.8	0.076	0.108	0.175
	(p) Three-point (Dmowski)	4.9	18.2	0.089	0.038	0.103
3	(q) Four-step (Crowell)	6.2	10.0	0.062	0.026	0.046
	(r) Four-point (Dmowski)	5.4	10.6	0.057	0.029	0.0542
	(s) HiRes-4	4.2	7.6	0.032	0.071	0.107
4	(t) HiRes-5	2.1	6.6	0.014	0.052	0.0752
5	(u) HiRes-6	1.1	6.0	0.0063	0.041	0.0571

analytical expressions for the determination of rate windows of a lock-in with different delay times. To obtain a straight line on the Arrhenius plot after the introduction of a delay time, it was proposed [11–15] to keep the t_d/t_c ratio fixed. This keeps the proportionality between the time constant τ_0 of the capacitance relaxation at the point of the maximum of the DLTS peak and the duration of the weighting function t_c. Nolte and Haller [16] demonstrated that the correlation of the exponential transient is equivalent to the Laplace transform of the weighting function and showed that the functional form of the high-temperature (relative to the position of the DLTS peak) response is simply the Laplace transform of the leading term in the Taylor expansion of the weighting function. To decrease the width of the DLTS peak, the leading term in the Taylor expansion of the weighting function should be of as high an order as possible, which means nothing other than weighting to later times [16].

The introduction of the delay time t_d changes the dependence equation (12) to the form

$$S(\tau_s) = \int_{t_d}^{t_d+t_c} \exp(-t/\tau_s)\,\mathrm{d}t = \text{constant} \times \exp(-t_d/\tau_s)\tau_s. \tag{13}$$

The exponential $\exp(-t_d/\tau_s)$ is a rapidly decaying function for $\tau_s \to 0$ and it suppresses, for small values of τ_s, any linear or quadratic dependence of $S(\tau_s)$. The fast-transient side of the dependence $S(\tau_s)$ becomes non-linear on a double-logarithmic scale (see for example figure 2(b), discussed in section 5). Its slope (which was unity before the introduction of the delay time) is in the range 2–3 in the part of the dependence that is most important for practical purposes, close to the DLTS maximum ($1 > S(\tau_s)/S(\tau_{max}) > 0.1$).

The influence of the delay time on the parameters of several correlation functions was discussed previously [14]. The calculations showed that a relatively small delay time was required, in order to make the slope of the fast-transient side of the dependence dominated by the exponential $\exp(-t_d/\tau_s)$. The optimum value of the delay time which provided a noticeable increase in the resolution (up to 20%) with almost no losses in the signal-to-noise ratio was specific for each weighting function and was in the $t_d \simeq (0.02\text{–}0.07)t_c$ range. The optimum values of the delay time for various weighting functions and the parameters of these functions with optimum delay times are given in table 2.

Table 2. Parameters of the weighting functions from figure 1, calculated with the optimum delay time. The letters used to denote weighting functions in figure 1, are given in the second column.

Order of the filter	Weighting function	Optimum delay time t_d/t_c	Figure of merit M	DLTS peak width w	Signal-to-noise ratio SNR	Output amplitude S	τ_{max}/t_c
0(1)	(a) Linear ramp	0.076	21.0	15.5	0.33	0.188	0.596
	(b) Exponential	0.086	20.7	15.6	0.32	0.160	0.598
1	(c) Shifted exponential	0.082	12.8	16.2	0.21	0.050	0.444
	(d) Shifted linear ramp	0.077	12.6	15.9	0.20	0.116	0.475
	(e) Sine	0	11.8	15.2	0.18	0.126	0.424
	(f) Lock-in	0.048	11.6	15.7	0.18	0.182	0.459
	(g) Rectangular (Crowell)	0.057	11.4	16.5	0.19	0.109	0.368
	(h) rectangular (Hodgart)	0.075	10.9	16.8	0.18	0.069	0.360
	(i) Split sine	0	9.0	14.2	0.13	0.090	0.440
	(j) Double boxcar	0.131	7.9	16.5	0.13	0.058	0.501
2	(k) Cosine	0.032	10.6	8.8	0.093	0.066	0.185
	(l) Triangular	0.037	10.4	8.8	0.092	0.053	0.186
	(m) Three-step (Crowell)	0.018	9.7	9.4	0.091	0.051	0.125
	(n) Square wave	0.023	9.5	8.8	0.084	0.084	0.183
	(o) HiRes-3	0.019	8.1	8.5	0.069	0.097	0.190
	(p) Three-point (Dmowski)	0.040	6.7	9.7	0.065	0.028	0.146
3	(q) Four-step (Crowell)	0.008	6.6	8.0	0.053	0.022	0.0525
	(r) Four-point (Dmowski)	0.011	6.0	7.9	0.047	0.024	0.0631
	(s) HiRes-4	0.011	4.3	6.7	0.029	0.064	0.1137
4	(t) HiRes-5	0.007	2.2	5.9	0.013	0.048	0.0788
5	(u) HiRes-6	0.005	1.1	5.4	0.0058	0.038	0.0594

Apart from a certain improvement in resolution, the adjustment of the fast-transient response to the $\tau_s \exp(-t_d/\tau_s)$ form has very important consequences for the classification of weighting functions. Let us consider three weighting functions representing the first-order filters, for example the double boxcar, rectangular lock-in and sine wavefunctions. If no delay time is introduced, the peak widths of these three functions differ by a factor of more than two (see table 1). The DLTS peak's form and the dependence of the correlator output signal on the time constant of the transient are shown in figure 2(a). Since all three functions are the first-order filters, the low-temperature part of the signal response, corresponding to slow transients ($\tau/\tau_0 \gg 1$), is proportional to τ^{-1} and practically coincides for all three functions. The difference in peak widths of the correlation functions is due to the fast-transient (high-temperature) side of the DLTS peaks (left side of the peak in figure 2). The introduction of the optimum delay time $t_{d\,opt}$ increases the slope of the high-temperature side of the DLTS peak (figure 2(b)), making it essentially independent of the weighting function. The values of the peak widths of the three considered weighting functions become very close after the introduction of the optimum delay time (see table 2). As expected, the low-temperature sides of the curves (figure 2(b)), which correspond to slow transients, are not affected by the delay time. Finally, if we compare weighting functions of different orders (figure 2(c)), we see that the response to fast transients (the high-temperature side of the DLTS peak) is determined by the optimally chosen delay time and does not depend on the order of the weighting function. At the same time, the slope of the low-temperature ($\tau/\tau_0 \gg 1$) side of the peak is equal to the order of the filter.

The most important consequence of the introduction of the optimum delay time is that the peak widths of functions from the same group became nearly identical. The only difference between functions of the same order is in their signal-to-noise ratio. The classification according to the order of the filter now becomes self-consistent and allows us to draw conclusions in the following sections about the optimal functions for practical work.

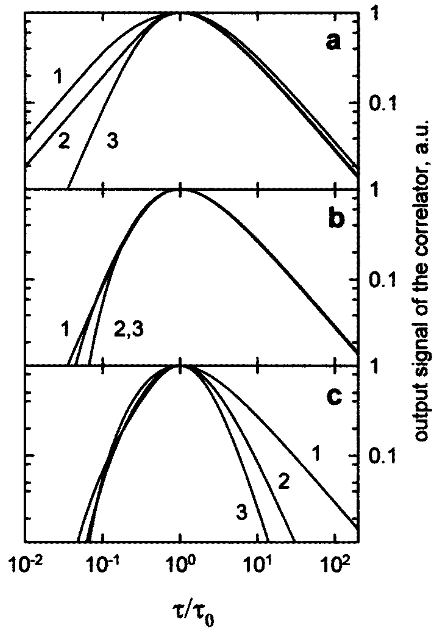

Figure 2. Influences of the introduction of the optimum delay time and of the order of the weighting function on the form of peaks in a DLTS spectrum. (a) Three weighting functions of the first order without a delay time: curve 1, double boxcar; curve 2, rectangular lock-in; and 3, sine wave. (b) Three weighting functions of the first order with the optimum delay time $t_d = t_{d\,opt}$: curve 1, double boxcar; curve 2, rectangular lock-in; and curve 3, sine wave. (c) Weighting functions of different orders with the optimum delay time $t_d = t_{d\,opt}$; curve 1, double boxcar (order 1); curve 2, cosine wave (order 2); and curve 3, HiRes-4 (order 3).

5. Comparative analysis of the previously reported weighting functions

As has already been discussed above, the exponential correlator has the highest output signal-to-noise ratio among other correlators. However, due to technical difficulties with the DC restoration of capacitance transients, the exponential correlator is seldom employed in the form of equation (10). To simplify the experimental set-up and to avoid baseline restoration, the reference waveform is frequently level-shifted to satisfy equation (8) for $k_0 = 1$:

$$W(t) = \exp(-t/\tau_0) - [1 - \exp(-t_c/\tau_0)]. \qquad (14)$$

It is generally assumed [3, 13, 17, 18], that the shifted exponential correlator has the same parameters as the original one. However, our simulations show that this is not the case. Its signal-to-noise ratio is more than 30% worse and the DLTS line is 30% wider than that of the non-shifted correlator. The overall performance determined by the figure of merit is no better than the performance of the rectangle and sine lock-in (compare the exponential and the shifted exponential in tables 1 and 2). Substitution of the capacitance decay $f(t, T)$ by $f(t, T) - C$ (C being a constant which satisfies the condition $\int[f(T, t) - C]\,\mathrm{d}t = 0$, which also may be considered as a way to avoid the baseline restoration) has the same negative effect on the parameters of the exponential correlator.

It follows from our simulations that the optimum waveform of the exponential correlator is $\exp(-2t^*)$, where t^* is the normalized time $t^* = (t - t_d)/t_c$, so that $t^* \subset [0, 1]$ for $t \subset [t_d, t_d + t_c]$. Small deviations from $\exp(-2t^*)$ are not critical, though we would not recommend working with reference waveforms faster than $\exp(-4t^*)$ or slower than $\exp(-0.5t^*)$, because it leads to decreases both in signal-to-noise ratio and in the figure of merit of the correlator.

The most widespread first-order correlation function is the double boxcar, which belongs to the first-order correlation functions. In order to improve its signal-to-noise ratio, measurements are typically performed using relatively wide gates rather than the narrow sampling periods originally proposed. The effect of the widths of the sampling gates Δt on the properties of the double boxcar was discussed in detail by Day *et al* [19] and Dmowski and Jakubowski [20].

The positioning of the sampling gates t_1 and t_2 (in our notation $t_1 = t_d$ and $t_2 = t_d + t_c$) affects the sensitivity and the resolution of a DLTS system [21, 22]. To generalize our approach, the parameters of the double boxcar in table 1 were calculated without a delay time; that is, for $t_1 = 0$. In practice, the double boxcar is always used with a delay time $t_d = t_1 > 0$. This case is considered in table 2. It was suggested [22] that, for very narrow strobes Δt, the optimum value of $\alpha = t_2/t_1$ falls within the 50–150 range. For wider strobes ($\Delta t \simeq 0.1t_2$) the optimum value shifts to $\alpha \simeq 9$ [14] (these values of Δt and α were used in table 2) and even to $\alpha \simeq 4$ for $\Delta t \simeq 0.25t_2$ [23]. Lower values of α may be chosen if one is interested in better resolution. However, the improvement of resolution for $\alpha < \alpha_{opt}$ ($t_d > t_{d\,opt}$) is insignificant compared with the losses in signal-to-noise ratio.

Another widely used correlation function is the rectangular lock-in. It is actually a double boxcar with extremely wide gates, equal to half of the period, which results in a better signal-to-noise ratio than that of the standard double boxcar (tables 1 and 2). Some commercially available lock-in amplifiers utilize a sinusoidal weighting function. The other functions of the first order, to the best of our knowledge, have never been employed in practice after they were proposed. Among them are rectangular functions proposed by Crowell and Alipanahi [3] and Hodgart [24], which can be considered approximations of the shifted exponential and split sine wave suggested by Miller *et al* [2].

The next group of correlation functions are filters of the second order. They are cosine functions together with its approximations (a triangular function and a square wave), proposed by Crowell and Alipanahi [3] and a three-step function HiRes-3, developed by Hodgart [7]. The second-order filters have noticeably better peak width, but the twofold decrease in the signal-to-noise ratio makes the figure of merit about 20% worse than that of the first-order filters. Three third-order filters, presented in tables 1 and 2, were constructed using the algorithms of Crowell and Alipanahi [3], Hodgart [7] and Dmowski [13]. The gain in selectivity of the third-order filters compared with that of the second-order filters is less than the loss in the SNR. Finally, the penalty for the rather insignificant decrease of the peak width of the fourth- and fifth-order filters (functions HiRes-5 and HiRes-6 (figures 1(t) and (u)) of Hodgart [7]) is a drastic decrease in SNR and the figure of merit.

The most efficient peak form, from the point of view of resolution, is the symmetrical form, which is the case for second- and third-order filters (see figure 2(c)). The improvement in resolution of the weighting functions of higher order is not significant compared with the losses in signal-to-noise ratio and their usage is therefore not justified. The resolution which can be obtained with the third-order weighting function is about $w \approx 6$ (see table 2).

6. Discussion and conclusions

With the development of inexpensive fast desk-top computers, it has become easier and cheaper to digitize capacitance transients and to correlate them numerically than to use expensive analogue correlators. In an automated set-up, one can easily process the same transients with different correlation functions (corresponding to different rate windows or different resolution capacities). Therefore, a proper choice of the correlation function becomes the least expensive means to achieve a better resolution in DLTS. It is possible to obtain a threefold gain in the energy resolution compared with that of the standard double boxcar without considerable programming effort.

Most of the known weighting functions are effective filters only for slow transients, so it is natural to classify them according to their response to slow decays (the order of the filter). However, this classification becomes consistent and logical only if the optimum delay time is considered an integral part of every weighting function. If an optimum delay time is introduced, the form of the DLTS peaks becomes function-independent for all correlation functions within the same filter order. The only substantial difference between functions belonging to the same order becomes their signal-to-noise ratio, thus providing a criterion for choosing the best function from each order.

The weighting functions of zeroth order (the exponential correlator and non-shifted linear ramp) have the best sensitivity. However, none of them reject the DC component of the capacitance transients and they do not form a maximum in the DLTS spectrum unless the baseline of the transient is restored according to equation (13), as was suggested by Miller *et al* [2]. If the weighting function is shifted to satisfy the condition $\int w(t)\,\mathrm{d}t = 0$ or the capacitance transient is zero-balanced in a similar way, the parameters of both functions become significantly (about 30%) worse and they lose their advantages over the other first-order filters.

The rectangular approximations of the exponent proposed by Crowell and Alipanahi [3], the rectangular lock-in and the sine lock-in, have rather similar properties and each of them is a good choice for routine work. The double boxcar has the worst signal-to-noise ratio among the first-order filters studied. Though it is often believed to have a better resolution, in fact all first-order functions have the same resolution after the introduction of the optimum delay time. The apparently better resolution of the double boxcar was only due to the fact that, unlike the other weighting functions, it was always used with a significant delay time, which is the position of the first strobe: $t_1 = t_d$.

The best second-order functions are the cosine wave, its triangular approximation and the three-step function of Crowell and Alipanahi [3]. Among the third-order functions analysed in this paper the best is the four-step function of Crowell and Alipanahi [3]. These filters decrease the peak width from the value of $w \simeq 16$–18, which is typical for such commonly used filters as rectangular lock-in and sine filters, to $w \simeq 7$–8. This gain of selectivity is accompanied by a fourfold decrease in the signal-to-noise ratio, which can be tolerated in most practical cases. The improvement in resolution due to the usage of weighting functions of order higher than three is insignificant compared with the losses in the signal-to-noise ratio and their application is not justified. The resolution limit of traditional correlation DLTS has recently been studied in [26]. It was shown that the limitations in resolution of the weighting functions in figure 1 are a drawback of the previously reported weighting functions rather than of the correlation procedure itself. Generally, there is no principal difference between correlation DLTS and inverse Laplace transformation of the capacitance transients. Recently, the narrow-band weighting functions based on Gaver-Stehfest algorithm of the inverse Laplace transform were suggested [25]. These functions enable one to obtain resolution as high as $w \approx 3.4$, for a high, but still realistic signal-to-noise ratio of the input transients of about 900.

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References

[1] Lang D V 1974 *J. Appl. Phys.* **45** 3014
Lang D V 1974 *J. Appl. Phys.* **45** 3023
[2] Miller G L, Ramirez J V and Robinson D A H 1975 *J. Appl. Phys.* **46** 2638
[3] Crowell C R and Alipanahi S 1981 *Solid-State Electron.* **24** 25
[4] Gordeev D M and Shmagin V B 1992 *Sov. Phys.–Semicond.* **26** 538
[5] Dmowski K and Pioro Z 1987 *Rev. Sci. Instrum.* **58** 75
[6] Thurzo I, Pogany D and Gmucova K 1992 *Solid-State Electron.* **35** 1737
[7] Hodgart M S 1979 *Electron. Lett.* **15** 724
[8] Dmowski K 1992 *J. Appl. Phys.* **71** 2259
[9] Atanassov R D 1982 *Bulg. J. Phys.* **9** 661
[10] Atanasov R D 1983 *Rev. Sci. Instrum.* **54** 1362
[11] Tokuda Y, Shimizu N and Usami A 1979 *Japan. J. Appl. Phys.* **18** 309
[12] Tokuda Y, Hayashi M and Usami A 1981 *J. Phys. D: Appl. Phys.* **14** 895

[13] Dmowski K 1990 *Rev. Sci. Instrum.* **61** 1319
[14] Vyvenko O F and Istratov A A 1992 *Sov. Phys.–Semicond.* **26** 947
[15] Ferenczi G and Kiss J 1981 *Acta Phys. Acad. Sci. Hung.* **50** 285
[16] Nolte D D and Haller E E 1987 *J. Appl. Phys.* **62** 900 Erratum 1988 *J. Appl. Phys.* **63** 592
[17] Borsuk J A and Swanson R M 1980 *IEEE Trans. Electron. Dev.* **27** 2217
[18] Wagner E E, Hiller D and Mars D E 1980 *Rev. Sci. Instrum.* **51** 1205
[19] Day D S, Tsai M Y, Streetman B G and Lang D V 1979 *J. Appl. Phys.* **50** 5093
[20] Dmowski K and Jakubowski A 1989 *Rev. Sci. Instrum.* **60** 106
[21] Balasubramanyam N and Kumar V 1988 *J. Appl. Phys.* **64** 6311
[22] Thomas H 1985 *J. Appl. Phys.* **57** 4619
[23] Shmatov A A 1992 *Sov. Phys.–Semicond.* **26** 267
[24] Hodgart M S 1978 *Electron. Lett.* **14** 388
[25] Istratov A A 1997 *J. Appl. Phys.* **82** 2965
[26] Istratov A A 1997 *Rev. Sci. Instrum.* **68** 3861