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Acoustic transistor: Amplification and switch of sound by sound

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We designed an acoustic transistor to manipulate sound in a manner similar to the manipulation of electric current by its electrical counterpart. The acoustic transistor is a three-terminal device with the essential ability to use a small monochromatic acoustic signal to control a much larger output signal within a broad frequency range. The output and controlling signals have the same frequency, suggesting the possibility of cascading the structure to amplify an acoustic signal. Capable of amplifying and switching sound by sound, acoustic transistors have various potential applications and may open the way to the design of conceptual devices such as acoustic logic gates. © *2014 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4894293]

The invention of the electrical diode was important for the emergence of modern electronics. Yet the world of digital electronics only truly emerged after the realization of electrical transistors having key features of amplifying and switching electrical signals. This eventually led to technical revolutions globally. In the history of the development of the optical/thermal counterparts, the emergence of optical/thermal diodes was likewise followed by the emergence of transistors having the ability to amplify and switch light/heat and having great potential applications in their respective areas.[1–6] As an important form of classical waves, the acoustic wave has had applications ranging from non-destructive testing to medical therapy, but has long been thought to propagate as easily in either direction along a given path. Recently, the acoustic diode capable of controlling an acoustic wave in a manner similar to how an electric diode controls current has been designed and implemented.[7–9] Following this inspiration, various acoustic one-way devices have been developed with enhanced performance characteristics.[10–15] In this context, it is natural to consider the possibility of designing an acoustic transistor (AT) that would have far ranging implications for acoustic devices, acoustic applications, and the field of acoustics in general.

In this letter, we present the feasibility of constructing an AT to control acoustic waves in analogy to what the electrical transistor does for electric current. The AT is a three-terminal device consisting of a nonlinear medium and acoustic filter and having the ability to amplify and switch acoustic waves. As the acoustic analogy to an electrical transistor, the AT has the essential ability to use a small acoustic signal input through one terminal to control a signal output through the second terminal that has been amplified by transferring energy to it from a pump wave incident from the third terminal. In particular, the output acoustic signal has the same frequency as the controlling signal, which suggests the possibility of cascading the structure to amplify an acoustic signal. We anticipate an AT capable of amplifying and switching sound by sound that will have various applications and pave the way for the design of conceptual devices such as acoustic logic gates.

Figure 1(a) is a schematic of the proposed AT model with a three-terminal waveguide structure. The input signal at terminal B has small amplitude and serves as the controlling signal for manipulating the output signal at terminal C, which can have much larger amplitude as a result of the transferring of energy from the high-amplitude pump wave at terminal E. Here, the background medium filling the waveguide is chosen as water ($c_0 = 1498\,\mathrm{m/s}$ and $\rho_0 = 998\,\mathrm{kg/m^3}$). The two nonlinear media (NLM), denoted as $\mathrm{NLM}_{1,2}$, are assumed to have the same sound speed and mass density as water and a particularly strong quadratic nonlinearity characterized by effective nonlinearity parameters $\Gamma_{1,2}$. Two low-pass (LP) acoustic filters with different transmission spectra are denoted as $\mathrm{LP}_{1,2}$.

The nonlinear interaction of the signal and pump waves in NLM_1 generates waves with sum/difference frequencies $\omega_{+/-} = \omega_p \pm \omega_s$, along with the pump wave and its second harmonic wave $2\omega_p$. (The second harmonic of the signal wave is comparatively trivial.) It is possible to yield combined frequency waves with acoustic energy greater than that of the signal wave via such parametric amplification, on the

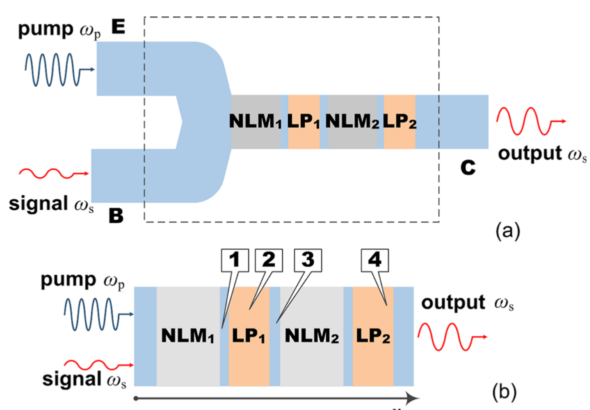

FIG. 1. (a) Schematic of an AT. (b) A simplified 1D model.

[a)]Author to whom correspondence should be addressed. Electronic mail: jccheng@nju.edu.cn condition that $\omega_1 < \omega_s < \omega_2 \ll \omega_p$ and $|p_s| \ll |p_p|$.[16] Here, ω_q and $|p_q|$ refer to the angular frequency and acoustic pressure amplitude with subscripts $q = p, s$ representing the pump and signal waves, respectively, and $\omega_{1,2}$ refer to the lower and upper limits of the operating band. The AT should work for any monochromatic wave within this band, but the nonlinear nature of the AT will make the response to a multi-frequency signal very complicated. Owing to the huge discrepancy between the frequencies and the amplitudes of the pump and signal waves, further interaction between the second harmonic of the pump wave and the signal wave will consume the energy of the signal wave, but only result in undesired frequency components. Therefore, LP_1 is used to eliminate $2\omega_p$, and thereby enhance the efficiency of amplification. In NLM_2, the pump wave continues to interact with the generated wave with combined frequencies, and LP_2 filters out all the waves with frequencies outside the operating band. As a result, the output signal has a frequency identical to that of the original signal, while the amplitude has undergone two-stage amplification, which is the key conceptual difference from previous works on acoustic amplification.[16] This means the acoustic energy flux passing through terminal C (an analogue to the collector of the electrical transistor) can be switched by the signal at terminal B (analogue to the base) with the aid of the power supply from terminal E (analogue to the emitter).

Note that the acoustic wave will propagate within channels with subwavelength cross-sections in the absence of a cut-off frequency.[17] As long as the waveguide is sufficiently thin, the model shown in Fig. 1(a) can be approximately simplified to the one-dimensional (1D) system shown in Fig. 1(b). Figures 2(a)–2(d) schematically illustrate the variations of the frequency components during the amplification process. However, we must stress that there necessarily exist particular limitations on the parameters of the system that are crucial for the occurrence of the key feature of acoustic amplification. As the underlying physical mechanism, the amplification stems from the fact that the energy transferred to the signal wave via such three-wave mixing exceeds the loss of energy due to nonlinear interaction, which can be regarded as the converse of the common use of three-wave mixing to suppress sound by sound. This requires that the generated waves with sum/difference frequencies must combine in such a manner that the additional energy provided to the original signal frequency be in phase with the initial signal. To achieve this goal, two conditions need to be met. (1) The pump and signal waves need to have the same propagating phase when entering the nonlinear medium, which means that the original phases of these two waves at the respective terminals need to be adjusted appropriately. (2) The nonlinear medium needs to be non-dispersive, which can be guaranteed by employing a bubbly medium that works under a quasi-static condition, so that the various spectral components generated at sum and difference frequencies continue to propagate in phase; otherwise, the in-phase condition will be destroyed, and complicate phase-matching techniques must be used, as is generally done in the optical wave-mixing process for which the optical media are usually dispersive. Only when such specific conditions are satisfied can the whole system behave as described by the simplified 1D system having the potential to amplify sound by sound.

For the 1D nonlinear layered structure shown in Fig. 1(b), the wave propagation can be solved numerically employing the extended transfer-matrix method.[18] The simplicity of the model also allows an analytical estimation of the propagation properties, which may provide deeper insight into the underlying physics and facilitate the design of the resulting devices.

For the pressures of the signal and pump waves, we have

$$p_s(x,t) = |p_s| \exp\left[i(k_s x - \omega_s t)\right], \tag{1}$$

$$p_p(x,t) = |p_p| \exp\left[i(k_p x - \omega_p t)\right], \tag{2}$$

where $k_p = \omega_p/c_0$ and $k_s = \omega_s/c_0$ are the respective wave numbers.

By solving the Burgers equation governing the nonlinear wave interaction between the signal and pump waves, one has[16]

$$p_+(x) \approx \frac{\Gamma_1 + 2}{4} \frac{p_p p_s \omega_+}{\rho_0 c_0^3} \alpha_p D_1 \exp(ik_+ D_1 - \alpha_p D_1), \tag{3}$$

$$p_-(x) \approx \frac{\Gamma_1 + 2}{4} \frac{p_p p_s^* \omega_-}{\rho_0 c_0^3} \alpha_p D_1 \exp(ik_- D_1 - \alpha_p D_1), \tag{4}$$

where $p_\pm(x)$ denotes the space-dependent parts of pressures of the sum and difference waves, Γ_1 and D_1 are the effective nonlinearity parameter and thickness of NLM_1, respectively, and α_p is the attenuation coefficient for water only at ω_p (i.e., the attenuation at ω_s is negligible owing to the assumption $\omega_s \ll \omega_p$).[19]

For an ideal low-pass filter, the transmission should be unity in the pass band and zero in the stop band. Following the same procedure, within the operating band, the pressure of the output acoustic wave through terminal C can be approximately expressed as[16]

$$|p_o| = \frac{(\Gamma_1 + 2)(\Gamma_2 + 2)}{4} \frac{p_p^2 \omega_s^2}{\rho_0 c_0^4 \omega_p^2} \alpha_p^2 D_1 D_2 e^{-\alpha_p (D_1 + D_2)} |p_s|, \tag{5}$$

where Γ_2 and D_2 are the effective nonlinearity parameter and thickness of NLM_2.

In practice, the two low-pass filters have to be implemented with realistic structures, chosen as superlattices (SLs) composed of alternate layers of water and glass with appropriately tuned parameters, which have been proven effective in the design of acoustic diodes and have a simple structure suitable for the extended transfer-matrix method.[17] In this study, the frequency of the pump wave is chosen as 1 MHz. Accordingly, the thicknesses of layers are chosen as $d_{w,1} = 0.125$ mm and $d_{G,1} = 0.475$ mm for SL_1 and $d_{w,2} = 0.375$ mm and $d_{G,2} = 1.44$mm for SL_2. The material parameters of glass are $\rho_G = 2511\,\mathrm{kg/m^3}$ and $c_G = 5754\,\mathrm{m/s}$. The numbers of periods are chosen as 15 for $SL_{1,2}$. The two SLs thus yield the desirable transmission properties: SL_1 and SL_2 have stop bands centered at 2 and 1 MHz, respectively, as shown in Fig. 2(e). It can be readily inferred that the upper limit of the operating band is approximately $\omega_2 \approx 2\pi \times 160$ kHz, which will be verified later in numerical simulations. Note also that the finite sizes of the

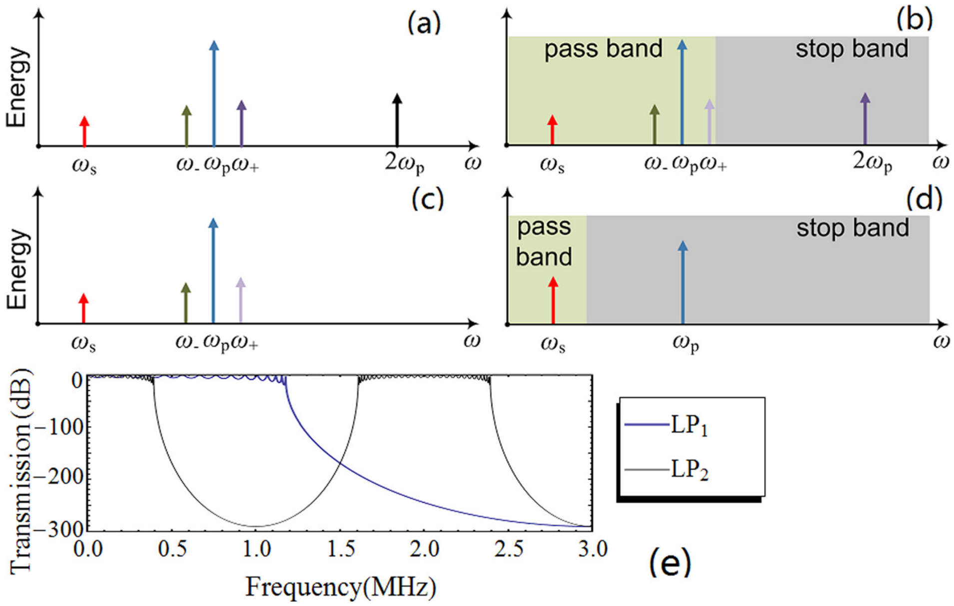

FIG. 2. Schematic illustration of frequency components in four regions numbered 1 (a), 2 (b), 3 (c), and 4 (d) in Fig. 1(b). Here, the amplitudes and locations of the frequency components have been scaled for better visual effect. (e) Transmission spectra of the two SLs acting as LP_1 and LP_2, respectively.

two SLs will necessarily result in imperfect AT devices, which will be discussed later.

In the theoretical model, the nonlinear media are required to have particularly strong nonlinearity yet negligible viscosity. In practice, this can be realized using a bubbly soft medium for which the quasi-static condition is satisfied.[20] In such cases, the effective nonlinear parameter Γ can be adjusted by tuning the bubble concentration β and reaches a maximum value $11\lambda/64\mu$ at an optimal bubble concentration $4\mu/3\lambda$ with λ and μ being the Lame coefficient and β, respectively. In soft media that always satisfy $\mu \ll \lambda$, the acoustic nonlinearity can be particularly high with a very low bubble concentration, which results in negligible bubble-induced impedance mismatch. Such theoretical prediction has been verified experimentally, which suggests the possibility of yielding a theoretical maximum of the nonlinearity parameter as high as 10^5 using plastizole, as a specific example.[20] It has also been demonstrated, both theoretically and experimentally, that an effective nonlinearity parameter with magnitude of 10^4 to 10^5 can be realized in water containing cylindrical bubbles.[21] One then has $\Gamma_{1,2} = \Gamma$ for $NLM_{1,2}$. The value of Γ is adjusted manually in the numerical simulations, and the thicknesses of $NLM_{1,2}$ are chosen as $D_1 = D_2 = 5\,\text{cm}$.

Figure 3(a) illustrates the frequency dependence of the gain of the AT for two different values of Γ when $|p_p| = 0.2\,\text{MPa}$. The gain is defined as $|I_o||I_s|^{-1}$ with I_o and I_s being the acoustic intensity at terminals C and B, respectively, in analogy to the gain of electrical transistors defined as the ratio between the collector and base currents. Under specified conditions, the proposed model can yield an output signal with much larger amplitude than the original input signal. It is therefore reasonable to identify this system as an effective AT capable of amplifying the input acoustic wave with the aid of an external acoustic source. In particular, the output signal has a frequency identical to that of the original input wave, suggesting that this structure can be cascaded to yield higher gain. This is a significant feature of a transistor model.[22] The dependence of the gain on the nonlinearity of the nonlinear medium can be observed from the results shown in Fig. 3(a). The upper limit of the bandwidth is determined by the frequency response of $SL_{1,2}$, agreeing well with the results in Fig. 3. However, the lower limit of the working band of the AT, ω_1, is related to the relationship for $|p_o|$, where the output signal is proportional to $(\omega_s/\omega_p)^2$ rather than the corner frequencies of the filters. The gain fluctuates within the effective operating bandwidth, which stems from Gibbs oscillation due to the finite size of the SLs. We have also investigated the dependence of the output acoustic intensity at terminal C on the acoustic pressure exerted on terminal E for different cases in which the acoustic intensity at terminal B differs, while $\omega_s = 100\,\text{kHz}$. Typical results are shown in Fig. 3(b), which is a direct analogue to the characteristic curve of an electrical transistor, in which the gate voltage controls the relationship between the collector current and collector–emitter voltage. This implies the potential of the proposed model to switch sound by sound, even suggesting the possibility of building acoustic logic gates.

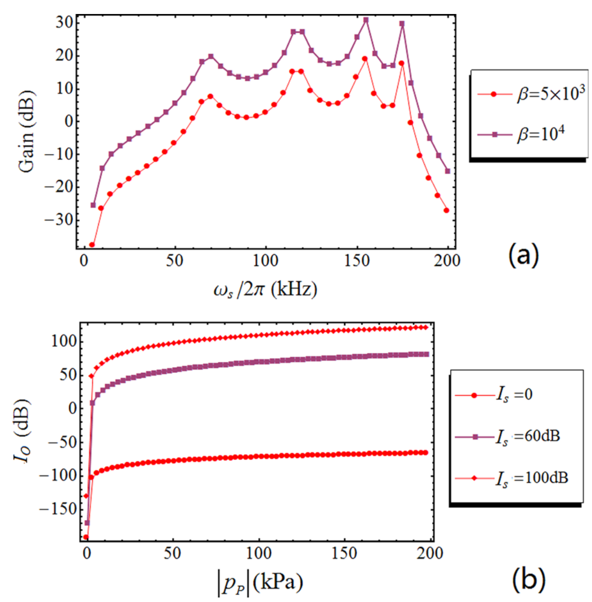

FIG. 3. (a) Frequency dependence of the gain of the AT for two different values of Γ. (b) Dependence of the output acoustic intensity I_o on the value of $|p_p|$, for different values of I_s.

In summary, we have presented an AT model that is a three-terminal device consisting of nonlinear media and acoustic filters. Numerical analysis of the wave propagation showed that the AT, as the acoustic analogy to an electrical transistor, has the essential ability to use a small acoustic signal passing through the first terminal to control a signal that is output through the second terminal and is amplified by a transferring of energy from a pump wave incident at the third terminal. Moreover, the output acoustic signal has the same frequency as the controlling signal, showing the possibility of cascading ATs to yield higher gain. Our design may open possibilities in the acoustic field, and has various potential applications ranging from acoustic device design to medical ultrasound diagnosis and therapy.

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