

A model of the $1/f$ noise in a forward-biased p–n diode

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Abstract

A model of $1/f$ noise in forward-biased p–n junction is suggested. The theory links current fluctuations to fluctuations of level occupancy in the density-of-states tail in the space charge region. The model is valid at forward biases for which the recombination current predominates. The $1/f$ (and $1/f$ -like) noise appears if the capture cross section of levels in the density-of-states tail decreases exponentially as the distance between a band edge and a level in the energy gap increases (multiphonon capture model). Analytical estimates for the frequency and current dependences of the $1/f$ noise are obtained.

1. Introduction

The low-frequency noise level of high-frequency devices used as oscillators or mixers in microwave communication systems is one of the major factors determining the phase noise characteristics. In addition, the low-frequency noise level is one of the most important characteristics of LEDs and semiconductor laser diodes, including quantum-dot LEDs and laser structures (see, e.g., [1–4]).

In most cases, the low frequency noise in semiconductor structures is a superposition of the generation–recombination (GR) noise and the $1/f$ noise. However, only the $1/f$ noise most frequently predominates in a wide range of currents and frequencies of analysis.

The first theory of the $1/f$ noise in forward-biased p–n junctions was put forward in [5]. This theory is based on the early model of the $1/f$ noise [6, 7] (Hooge's model). This model assumes that the $1/f$ noise in any homogeneous semiconductor ('semiconductor resistor') is due to mobility fluctuations and can be described by the empirical relation

$$\alpha = \frac{S_n}{n_0^2} f N, \quad (1)$$

where S_n/n_0^2 is the relative spectral density of the $1/f$ noise, α is the so-called empirical Hooge constant of the order of 10^{-3} for any semiconductor (and metal) and N is the total number of conduction electrons (holes) in the sample.

The theory of the $1/f$ noise caused by fluctuations of the number of carriers was developed for homogeneous

semiconductors (semiconductor resistors) in [8]. The adequacy of this theory was demonstrated for low-doped ($n_0 \sim 10^{15} \text{ cm}^{-3}$) and moderately doped ($n_0 \sim 10^{17} \text{ cm}^{-3}$) GaAs, for Si at low temperatures and for silicon carbide (see reviews in [9, 10] and references herein). However, to the best of our knowledge, there is no theory of the $1/f$ noise in the forward-biased p–n junction that is based on the model of carrier number fluctuations.

In this paper, we present a model of the $1/f$ noise in the forward-biased p–n junction. The model relates current fluctuations to fluctuations of the level occupancy in the density-of-states tail in the space charge region (SCR). The model is valid at forward biases for which the recombination current predominates and current–voltage characteristics are described by the expression $I_F \sim \exp(qV/nkT)$, with $n \sim 2$.

2. Statement of the problem

Recently, a new model of the GR noise in a forward-biased p–n junction has been suggested [11]. The model assumes that a trap level located relatively close to the conduction band edge is responsible for the observed GR noise. The model well describes both the current and frequency dependences of the GR noise observed in a forward-biased p–n SiC diode. In this paper, we use the approach developed in [11] to estimate the current fluctuations caused by fluctuations of the level occupancy in the density-of-states tail in the space charge region.

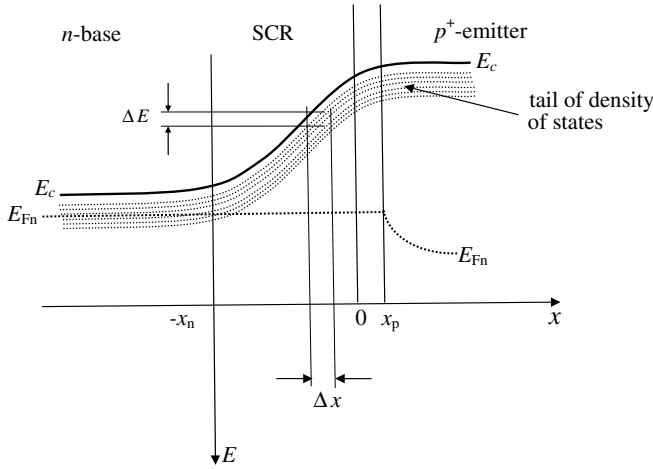


Figure 1. Qualitative distribution of the potential (band diagram) in the forward biased p^+-n junction at energies close to the conduction band edge. The level E_c is the conduction band bottom, dotted lines represent the tail levels and E_{Fn} curve shows the behavior of the electron quasi-Fermi level in the space charge region (SCR).

It is well known that the density of states in real semiconductor single crystals does not vanish at the conduction and valence band edges. Any crystal lattice imperfections give rise to tails of the density of states, $\rho(E)$, which exponentially decay into the energy gap (density-of-states tails): $\rho(E) = \rho_0 \exp(-E/E_0)$, where E_0 is a characteristic constant. Here and hereinafter, we suppose for definiteness that the $1/f$ noise is caused by fluctuations of the occupancy of tail states lying near the conduction band edge. The energy E is measured down from the bottom of the conduction band. It is noteworthy that the total concentration of the tail levels is given by

$$N_0 = \int_0^\infty \rho_0 \exp(-E/E_0) dE = E_0 \rho_0. \quad (2)$$

If capture into the tail states follows the multiphonon scenario [12], the capture cross section of the levels in the tails, σ , exponentially decreases into the energy gap: $\sigma(E) = \sigma_0 \exp(-E/E_1)$, where E_1 is a constant equal to the rate at which the capture cross section decreases as the energy E becomes higher. (Accordingly, the time constant $\tau(E) \sim 1/\sigma(E)$ exponentially increases with E growth.) As demonstrated earlier [8–10], this gives rise to the $1/f$ noise in homogeneous semiconductors (semiconductor resistors).

Figure 1 shows qualitatively the distribution of the potential (band diagram) in a forward-biased p^+-n junction at energies close to the conduction band edge. The level E_c is the bottom of the conduction band, the dotted lines represent the tail levels and the E_{Fn} curve shows the trend of the electron quasi-Fermi level in the space charge region.

Let us single out a narrow energy band ΔE in the density-of-states tail in the SCR (figure 1). The contribution to the total noise from this band is fully equivalent to the contribution to the noise from a local level with the same energy position E and the concentration $N_t(E) = \int_E^{E+\Delta E} \rho(E') dE'$. It is clear that the noise spectrum associated with this single ‘level’ is a Lorentzian and summing of such Lorentzians over the density-of-states tail should give the $1/f$ (or $1/f$ -like) noise due to very

wide dispersion of time constants τ , caused by the exponential dependence of the capture cross section on energy E (equation (2)).

The results obtained in [11] enable direct calculation of the noise associated with the density-of-states tail.

3. Results and discussion

Just like in [11], we assume that the recombination current in the forward biased $p-n$ junction can be written as [13]

$$I \approx \frac{\pi}{2} n_i A \exp(eV/2kT) \frac{kT}{F_0 \tau_R}, \quad (3)$$

where n_i is the intrinsic concentration; A is the diode area; F_0 is the characteristic field at a coordinate x_0 , where the recombination rate of electrons and holes is at a maximum; τ_R is the recombination time constant in the SCR; $F_0 \approx [eN_D(2V_{bin} - V)/\epsilon\epsilon_0]^{1/2}$ [13], where $V_{bin} = (kT/e) \ln(N_D/n_i)$ and N_D is the donor concentration in the base (low-doped region) of the diode. The dominant contribution to the recombination current comes from a region of length kT/eF_0 at x_0 .

When a constant dc voltage V is applied to the diode, the current fluctuates due to fluctuations of the field F_0 [11]. In turn, the fluctuations of the electric field F_0 are caused by tail state occupancy fluctuations arising from the exchange of electrons between the tail states and the conduction band. As mentioned in [11], the carrier concentration in the SCR of a forward-biased $p-n$ junction is high enough to screen the excess charge. The characteristic screening length depends on the current density and fluctuation coordinate. Therefore, an accurate solution of the problem is only possible in the framework of a 3D simulation. We use in this paper, just like in [11], a simplified approach assuming that the excess charge is uniformly spread over the whole SCR.

Analysis of the results obtained in [11] shows that such a simplified approach gives no way of estimating the noise amplitude. However, it enables a description of both the frequency and current dependences of the noise. It is noteworthy that the single currently existing theory of the $1/f$ noise in a forward-biased $p-n$ junction cannot estimate the noise amplitude, because all the results of this theory are expressed in terms of the Hooge ‘constant’ α . Meantime, the values of α may vary from seven to nine orders of magnitude even for the same semiconductor material [9, 10].

As shown in [11], the spectral density of the noise generated by a single level in a forward-biased p^+-n junction is given by the following integral over the SCR:

$$S_I = \frac{4N_t^* I^2 \tau_e}{Ax_n^2 N_D^2} \int_0^{x_n} \frac{\eta(z) dz}{[\eta(z) + 1][(\eta(z) + 1)^2 + \omega^2 \tau_e^2]}. \quad (4)$$

Here N_t^* is the ‘effective’ concentration of the noisy level (the introduction of this parameter emphasizes the fact that the simplified approach employed here yields no information about the noise amplitude), I is the forward current, $x_n = [2\epsilon\epsilon_0(V_{bi} - V)/eN_D]^{1/2}$ is the width of the SCR in the low-doped base and $V_{bi} = (kT/e) \ln(N_D N_A/n_i^2)$ is the built-in potential. The parameter τ_e in equation (4) is the characteristic

emission time, $\omega = 2\pi f$ is the circular frequency, $z = x + x_n$, and $\eta(z) = n(z)/n_t$, where n is the free electron concentration in the SCR, $n_t = N_c \exp(-\frac{E_c - E_t}{kT})$, N_c is the density of states in the conduction band and $(E_c - E_t)$ is the energy spacing between the bottom of the conduction band and the noisy level.

The spectral noise density generated by all levels in the density-of-states tail is given by the integral over energy from the conduction band edge to infinity. Repeating word for word the arguments analyzed in detail in [11] and taking (2) into account, we can show that the expression for the spectral noise density generated by levels in the density-of-states tail has the form

$$S_I = \frac{4\rho_0^* I^2}{A x_n^2 N_D^2} \int_0^\infty dE \int_0^{x_n} dz \times \frac{\tau_c(E) \eta(z, E) e^{-\frac{E}{E_0}}}{[(\eta(z, E) + 1)[(\eta(z, E) + 1)^2 + \omega^2 \tau_c^2]} \quad (5)$$

Here $\tau_c(E) = \tau_0 e^{\frac{E}{E_1} + \frac{E}{kT}}$, $\eta(z, E) = \frac{N_D}{n_t(E)} \exp(-\frac{z^2}{2l_D^2})$, $\tau_0 = \frac{1}{\sigma_0 v_T N_c}$, where v_T is the thermal electron velocity and $l_D = (\epsilon \epsilon_0 kT / e^2 N_D)^{1/2}$ is the Debye length in the neutral part of the base.

Strictly speaking, integration over energy in integral (5) should be made from zero to $E_g/2$ (E_g is the energy gap). The integration from zero to infinity in (5) is due to the fact that E_1 is considerably smaller than E_g . Indeed, the scale for E_1 is equal (in the order of magnitude) to the energy of a local optical phonon [12]. For Si and GaAs, E_1 is ~ 0.01 eV [8, 9]. We use this same value of E_1 for SiC as well.

Denoting

$$\eta_0(z) = \frac{N_D}{N_c} \exp\left(-\frac{z^2}{2l_D^2}\right) \quad \text{and} \quad s = \frac{z}{l_D}, \quad (6)$$

we can rewrite expression (5) in the form more suitable for an analytical treatment:

$$S_I = \frac{4\rho_0^*}{A N_D^2 l_D} \tilde{I}, \quad (7)$$

where

$$\tilde{I} = I^2 \tau_0 \left(\frac{l_D^2}{x_n^2} \right) \int_0^\infty dE \int_0^{x_n/l_D} ds \times \frac{\eta_0(z) e^{\frac{E}{E_1} - \frac{E}{kT} - \frac{E}{E_0}}}{(\eta_0(z) + e^{-\frac{E}{kT}})[(\eta_0(z) + e^{-\frac{E}{kT}})^2 + \omega^2 \tau_0^2 e^{\frac{2E}{E_1}}]} \quad (8)$$

Below, we use for estimates the same values of the parameters as those used in [11] where a SiC p⁺-n diode with a doping level in the n-base equal to $N_D = 2 \times 10^{14} \text{ cm}^{-3}$ was considered.

3.1. Analytical analysis of the solution

Let us first of all note that integral (8) becomes divergent at $\omega \rightarrow 0$. This result follows from the conditions $E_1 < E_0$ and $E_1 < kT = 0.026$ eV. Indeed, the numerator of the integrand in integral (8) increases monotonically with energy, whereas the denominator monotonically decreases (at $\omega \rightarrow 0$). (It is noteworthy that the conditions $E_1 < E_0$ and $E_1 < kT$ should also be necessarily satisfied for the $1/f$ noise to be caused by tail

state occupancy fluctuations in homogenous semiconductors [8, 9].)

This kind of divergence is highly characteristic of theories in which no physical restrictions on the low-frequency limit of noise are considered. In the case under consideration, we can note at least two evident physical restrictions on such a low-frequency limit of noise. First, as mentioned above, the integration over energy in integral (5) should be made, strictly speaking, from zero to $E_g/2$ (rather than to infinity). The second physical restriction results from the fact that at any reasonable value of ρ_0^* and high enough value of the energy E_{lim} , $\rho(E) = \rho_0^* \exp(-E_{\text{lim}}/E_0)$ becomes so small that the statistical approach to fluctuation processes makes no sense. Taking as a rough estimate the condition $N(E_{\text{lim}}) \sim 1$, we have $E_{\text{lim}} \sim 1.1$ eV at $N_0^* = \rho_0^* E_0 \sim 10^{16} \text{ cm}^{-3}$. As can be seen for SiC ($E_g > 3$ eV), this restriction is an even stronger constraint on the upper limit of integration than the condition associated with the value of $E_g/2$.

At a small (but non-zero) value of $\omega \tau_0$, the integrand in integral (8) increases exponentially with energy E until the term $\omega^2 \tau_0^2 \exp(2E_1/E)$ in the denominator is smaller than $[\frac{N_D}{N_c} \exp(-s^2/2) + \exp(-E/kT)]^2$. With a further increase in energy, the integrand in (8) exponentially decreases: $\tilde{I} \sim \exp(-E_1/E)$.

To simplify further analysis, let us show that the condition

$$\frac{N_D}{N_c} \exp(-s^2/2) < \exp(-E/kT) < \omega \tau_0 \exp(E/E_1) \quad (9)$$

is valid for the actual frequencies $\omega = 2\pi f$ and given E_1 and E_0 values.

As a first step, let us find when the left part of inequality (9), $(N_D/N_c) \exp(-s^2/2) < \exp(-E/kT)$, is valid. Obviously, this inequality holds if $(N_D/N_c) < \exp(-E/kT)$ because s is positive within the integration limits. Hence, the left part of (9) is valid if $E < kT \ln(N_c/N_D)$.

Now, we find when the right part of inequality (9), $\exp(-E/kT) < \omega \tau_0 \exp(E/E_1)$, is valid. Denoting $\beta = E_1/kT$, we find that this is so if

$$E/E_1 > \frac{1}{1 + \beta} \ln\left(\frac{1}{\omega \tau_0}\right). \quad (10)$$

Hence, we obtain that inequality (9) is valid at

$$\omega \tau_0 > (N_D/N_c)^{\frac{1+\beta}{\beta}}. \quad (11)$$

With $N_D = 2 \times 10^{14} \text{ cm}^{-3}$, $N_c = 1.7 \times 10^{19} \text{ cm}^{-3}$ [11], $\beta = E_1/kT \approx 0.38$ and characteristic τ_0 value $\tau_0 \sim 10^{-15} \text{ s}$ [8, 9], condition (11) is satisfied at $\omega \tau_0 \geq 10^{-18}$, or at characteristic frequencies of analysis $f = \omega/2\pi \geq 10^{-4} \text{ Hz}$.

Taking (9) into account, we can rewrite the double integral in expression (8) as

$$\begin{aligned} \tilde{I}_1 &= \int_0^\infty dE \int_0^{x_n/l_D} ds \\ &\times \frac{\frac{N_D}{N_c} e^{-s^2/2} e^{\frac{E}{E_1} - \frac{E}{kT} - \frac{E}{E_0}}}{\left(\frac{N_D}{N_c} e^{-s^2/2} + e^{-\frac{E}{kT}}\right) \left[\left(\frac{N_D}{N_c} e^{-s^2/2} + e^{-\frac{E}{kT}}\right)^2 + \omega^2 \tau_0^2 e^{\frac{2E}{E_1}}\right]} \\ &\approx E_1 \frac{N_D}{N_c} \int_0^{x_n/l_D} ds e^{-s^2/2} \int_0^\infty d\varepsilon \frac{e^{\varepsilon - \nu \varepsilon}}{e^{-2\beta \varepsilon} + \omega^2 \tau_0^2 e^{2\varepsilon}}, \quad (12) \end{aligned}$$

where $\varepsilon = E/E_1$, $\nu = E_1/E_0$.

At comparatively small currents, the x_n/l_D ratio in the upper limit of integral (12) is $x_n/l_D \approx 6.4 \gg 1$. Hence, this ratio can be substituted for infinity when making the analytical estimate. Then $\int_0^\infty ds e^{-s^2/2} = \sqrt{\pi/2}$.

Using the substitution $x = e^\varepsilon$, we can bring the integral over $d\varepsilon$ to the form

$$\int_0^\infty d\varepsilon \frac{e^{\varepsilon-\nu\varepsilon}}{e^{-2\beta\varepsilon} + \omega^2\tau_0^2 e^{2\varepsilon}} = \int_1^\infty \frac{x^{2\beta-\nu} dx}{1 + \omega^2\tau_0^2 x^{2\beta+2}}. \quad (13)$$

Integral (13) can be expressed in terms of a hypergeometric function. However, the approximate value of (13) can be expressed in terms of elementary functions by further transformations. Using successively the substitutions $y = x^{1+\beta}$ and $z = \lambda y$ (where $\lambda = \omega\tau_0$) and taking into account that $[(\beta - \nu)/1 + \beta] \ll 1$, we have from (13)

$$\begin{aligned} \int_1^\infty \frac{x^{2\beta-\nu} dx}{1 + \omega^2\tau_0^2 x^{2\beta+2}} &\approx \frac{1}{1+\beta} \frac{1}{\lambda^{1+\frac{\beta-\nu}{1+\beta}}} \int_0^\infty \frac{z^{\frac{\beta-\nu}{1+\beta}} dz}{1+z^2} \\ &\approx \frac{1}{1+\beta} \frac{1}{\lambda} \int_0^\infty \frac{dz}{1+z^2} \approx \frac{\pi}{2} \frac{1}{1+\beta} \frac{1}{\lambda}. \end{aligned} \quad (14)$$

Thus, the analytical estimate of integral \tilde{I} has the form

$$\tilde{I} \approx I^2\tau_0 \left(\frac{l_D^2}{x_n^2} \right) E_1 \frac{N_D}{N_c} \left(\frac{\pi}{2} \right)^{3/2} \frac{1}{1 + \frac{E_1}{kT}} \frac{1}{(\omega\tau_0)^{1 + \frac{E_1/kT - E_1/E_0}{1 + E_1/kT}}}. \quad (15)$$

Let us note, first of all, that expression (15) predicts that the spectral noise density depends on frequency as $S \sim 1/f^\gamma$, where $\gamma = 1 + \frac{E_1/kT - E_1/E_0}{1 + E_1/kT} \approx 1.04$ is very close to unity ($1/f$ noise).

Then, expression (13) predicts, at comparatively small current densities, when inequality $x_n/l_D \gg 1$ is valid, the current dependence of the noise in the form $S_I \sim I^2$. As the current increases and, accordingly, x_n becomes smaller, the noise intensity grows with current somewhat more steeply. It is noteworthy that such current and frequency dependences of noise have been observed experimentally, for example, in [14, 15] on forward-biased 4H-SiC p-n junctions.

It is also noteworthy that, because γ is very close to unity, the results obtained with the model are virtually insensitive to the value of τ_0 , which is known rather poorly for semiconductors.

3.2. Numerical calculations

The results of numerical calculations agree very well with the analytical estimate obtained above. Figure 2 shows the results of numerical calculations of integral (8), which can be rewritten as

$$\begin{aligned} \tilde{I} &= I^2\tau_0 \left(\frac{l_D^2}{x_n^2} \right) \int_0^1 dE \int_0^{x_n/l_D} ds \\ &\times \frac{\eta_0(z) e^{\frac{E}{E_1} - \frac{E}{kT} - \frac{E}{E_0}}}{(\eta_0(z) + e^{-\frac{E}{kT}}) [(\eta_0(z) + e^{-\frac{E}{kT}})^2 + \omega^2\tau_0^2 e^{\frac{2E}{E_1}}]}. \end{aligned} \quad (16)$$

As mentioned above, we used in our numerical calculations the same values of the parameters as those in [11] for a p⁺-n SiC diode (4H-SiC, donor doping concentration $N_D = N_D - N_A = 2 \times 10^{14} \text{ cm}^{-3}$, working area $A \approx 6.4 \times 10^{-3} \text{ cm}^2$).

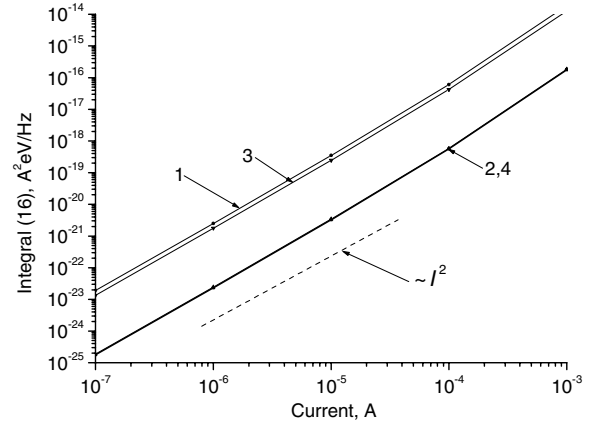


Figure 2. Calculated dependences of integral \tilde{I} (16) on current at two values of τ_0 and two frequencies of analysis, $f = \omega/2\pi$. Curves 1 and 2: $\tau_0 = 10^{-15} \text{ s}$, curves 3 and 4: $\tau_0 = 10^{-18} \text{ s}$. Curves 1, 3: $f = 1 \text{ Hz}$, curves 2, 4: $f = 100 \text{ Hz}$.

As can be seen from figure 2, the value of integral (16) and, consequently, the noise intensity depend very weakly on τ_0 , in full agreement with the analytical solution. A change in τ_0 by three orders of magnitude has virtually no effect on the noise (curves 2 and 4 coincide to within less than the line thickness). Curves 1 and 3 ($f = 1 \text{ Hz}$) are also very close. It is noteworthy that the calculations at $\tau_0 = 10^{-12} \text{ s}$ give the results that virtually coincide with the data presented in figure 2. At small currents, the current dependence of noise, given by integral (16), is close to $S_I \sim I^2$. As the current increases, the slope of the $S_I(I)$ dependence becomes somewhat steeper.

Comparing curves 1 and 2 and 3 and 4, we can conclude that the frequency dependence of noise is very close to $S \sim 1/f$, as it follows from the analytical expression (15): an increase in the frequency f by two orders of magnitude is accompanied by a decrease in the value of integral (16) (and, accordingly, by a decrease in the noise intensity calculated in terms of the model) by approximately a factor of 100.

Finally, at $I = 10^{-7} \text{ A}$, $\tau_0 = 10^{-15} \text{ s}$, $x_n/l_D \approx 6.4$, $N_D/N_c = 10^{-5}$, $E_1/kT \approx 0.38$, $E_1/E_0 \approx 0.33$ and $f = 100 \text{ Hz}$, the value of the integral \tilde{I} , calculated using (15), is $1.6 \times 10^{-25} \text{ A}^2 \text{ eV Hz}^{-1}$. As can be seen in figure 2, the analytically and numerically calculated values virtually coincide.

It is important to emphasize that the frequency dependence of noise, $S \sim 1/f$, is not postulated in advance (as was done in [5]), but emerges as natural consequence of the superposition of characteristic times τ distributed in a very wide frequency range [16, 17]. The main defect of the theory is, as mentioned above, that the theory cannot estimate the noise amplitude. However, the theory describes correctly the frequency and current dependences of the noise.

4. Conclusion

The model suggested for describing the $1/f$ noise in a forward-biased p⁺-n junction relates the current fluctuations to fluctuations of level occupancies in the density-of-states tail

in the space charge region. The model correctly describes the frequency and current dependences of the noise. In particular, the model yields, in agreement with numerous experiments, a $1/f$ spectrum with a noise level inversely proportional to the diode area A . At comparatively small current densities, which correspond to the situation when the recombination current predominates, the noise amplitude S is proportional to a squared current ($S \sim I^2$).

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