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Social learning differential evolution

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ABSTRACT

Differential evolution (DE) has attracted much attention in the field of evolutionary computation and has proved to be one of the most successful evolutionary algorithms (EAs) for global optimization. Mutation, as the core operator of DE, is essential for guiding the search of DE. In this study, inspired by the phenomenon of social learning in animal societies, we propose an adaptive social learning (ASL) strategy for DE to extract the neighborhood relationship information of individuals in the current population. The new DE framework is named social learning DE (SL-DE). Unlike the classical DE algorithms where the parents in mutation are randomly selected from the current population, SL-DE uses the ASL strategy to intelligently guide the selection of parents. With ASL, each individual is only allowed to interact with its neighbors and the parents in mutation will be selected from its neighboring solutions. To evaluate the effectiveness of the proposed framework, SL-DE is applied to several classical and advanced DE algorithms. The simulation results on forty-three real-parameter functions and seventeen real-world application problems have demonstrated the advantages of SL-DE over several representative DE variants and the state-of-the-art EAs.

1. Introduction

Evolutionary algorithms (EAs) are stochastic optimization techniques that mimic the evolutionary process of nature. The common conceptual base of EAs is to evolve a population of candidate solutions with the help of information exchange procedures. In the last few decades, numerous EAs have been proposed based on different inspirations taken from the evolutionary process of nature. These include genetic algorithm (GA), evolution strategy (ES) , evolutionary programming (EP), particle swarm optimization (PSO), and ant colony optimization (ACO). The major differences among these EAs lie in the way new trial solutions are generated. Meanwhile, the question of how to utilize the population information to further enhance the reproduction operator's search ability is still one of the most salient and active topics in EAs.

Differential evolution (DE), proposed by Storn and Price [39], is a simple yet efficient EA for global numerical optimization. Due to its attractive characteristics, such as ease of use, compact structure, robustness and speediness, DE has been extended to handle large-scale, multi-objective, constrained, dynamic, and uncertain optimization problems [11]. Furthermore, DE has been successfully applied to many scientific and engineering fields [11], such as pattern recognition, signal processing, satellite communications, wireless sensor networks, and so on.

In DE, three main operators, i.e., mutation, crossover and selection, are used to evolve the population. Among them, mutation is the core operator that distinguishes DE from other EAs. However, we have observed, in most DE algorithms,

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the parents for mutation are selected randomly from the current population, and thus, all vectors are likely to be selected equally as parents without any selective pressure at all. Although this mutation strategy is easy to use and may be good at exploring the search space, it is slow to exploit solutions. In addition, the need for parent selection in DE has been advocated in [4,13,20,40,49]. In these work, the selection of parents for mutation has been proven to be very important to the performance of DE when solving complex problems.

Social learning, which is widely observed in animal societies, refers to the learning that is affected by the interaction with, or observation of another animal or its products [22]. As opposed to individual learning, where only a single person's learning is considered, the goal of social learning is to learn and imitate the behaviors of better people within a social group [22]. In social learning, the majority of studies focus on how the individuals within the group learn and, hence, how the entire group learns. Many mechanisms of social learning have been proposed in the literature, and they can be roughly classified into the following categories: local enhancement, stimulus enhancement, observational conditioning, matched-dependent behaviors and imitation [22]. In several EAs, these social learning mechanisms have been successfully introduced to improve their performance. In [30], an incremental social learning framework is proposed for the PSO variants with a growing population of learning agents. In [36], a social learning PSO is proposed by introducing the learning strategy each particle can learn from any better particles in the current swarm. In [27], a social learning optimization algorithm is presented that consists of three co-evolution spaces: micro, learning and belief space.

Inspired by the imitation phenomenon of social learning where people usually learn and imitate the behavior of a better person (or elite) within a social group, this study proposes an adaptive social learning (ASL) strategy for DE to develop a new DE framework, named social learning DE (SL-DE). Unlike the classical DE algorithms, where the parents in mutation are randomly selected from the current population, SL-DE uses the ASL strategy to extract neighborhood relationship information of the population to guide the selection of parents in mutation. ASL consists of four operators: in the *social ranking* operator, individuals in the current population are sorted according to their fitness values; in the *evaluating social influence* operator, the social influence of each individual is evaluated based on its ranking value; in the *building social network* operator, a new social network is built by establishing the relationships between pairs of individuals according to their social influences; in the *constructing neighborhood* operator, the neighborhood of each individual is constructed from the built social network. With ASL, each individual is only allowed to interact with its neighbors and the parents in mutation will be selected within its neighborhood. In this way, the neighborhood relationship information can be utilized effectively to guide the search of DE.

To evaluate the effectiveness of the proposed approach, we apply SL-DE to several classical DE algorithms, as well as advanced DE variants. Extensive experiments have been carried out on a set of benchmark functions from the 2013 IEEE congress on evolutionary computation (CEC) (including real-parameter optimization [25] and large-scale global optimization [24]) and the CEC 2011 on real-world application problems [10]. Simulation results have shown the advantages of SL-DE when compared with other algorithms on the test functions.

In summary, the major characteristics of SL-DE include the following:

- ASL is proposed to extract neighborhood relationship information of individuals during the evolutionary process, which shows some insights into utilizing population information with the social learning mechanism.
- In SL-DE, each individual is only allowed to interact with its neighbors and the parents in mutation will be selected from the neighborhood, which provides an alternative for selecting parents in the mutation operator of DE.
- Because the simple structure of the classical DE algorithm has been maintained, SL-DE is still very simple and can be easily applied to most advanced DE variants to further improve their performance.

The rest of this paper is organized as follows. Section 2 briefly reviews some related work. The proposed SL-DE is presented in detail in Section 3. Section 4 reports the extensive experimental results. Finally, the conclusions are drawn in Section 5.

2. Related work

2.1. DE

In this study, DE is used for solving the numerical optimization problem [39]. Without loss of generality, we consider the optimization problem to be minimized as $f(X)$, $X \in R^D$, where D is the dimension of the decision variables. DE evolves a population of NP vectors representing the candidate solutions. Each vector is denoted as $X_{i,G} = (x^1_{i,G}, x^2_{i,G}, \ldots, x^D_{i,G})$, where $i = 1, 2, \ldots, NP$, NP is the population size and G is the current generation. In the classical DE algorithms, the algorithmic schemes can be classified by using the notation "DE/x/y/z", where x means the method of selecting the parent that constitutes the base vector, y means the number of difference vectors that are used to perturb x, and z stands for the crossover type employed.

2.1.1. Initialization

In DE, the initial population should cover the entire search space as much as possible by uniformly randomizing individuals within the search space constrained by the prescribed minimum and maximum bounds. Thus, the jth parameter of the

ith individual is initialized by

$$x_{i,G}^{j} = L^{j} + rand(0,1) \times (U^{j} - L^{j}) \tag{1}$$

where $rand(0, 1)$ represents a uniformly distributed random number within the range [0, 1], and L^j and U^j represent the lower and upper bounds of the jth variable, respectively.

2.1.2. Mutation

Following initialization, DE employs the mutation strategy to generate a mutant vector $V_{i,G}$ with respect to each individual $X_{i,G}$ (called the target vector) in the current population. For example, several widely used mutation strategies are listed as follows:

- DE/rand/1

$$V_{i,G} = X_{r1,G} + F \times (X_{r2,G} - X_{r3,G}) \tag{2}$$

- DE/rand/2

$$V_{i,G} = X_{r1,G} + F \times (X_{r2,G} - X_{r3,G}) + F \times (X_{r4,G} - X_{r5,G}) \tag{3}$$

- DE/best/1

$$V_{i,G} = X_{best,G} + F \times (X_{r1,G} - X_{r2,G}) \tag{4}$$

- DE/best/2

$$V_{i,G} = X_{best,G} + F \times (X_{r1,G} - X_{r2,G}) + F \times (X_{r3,G} - X_{r4,G}) \tag{5}$$

- DE/current-to-best/1

$$V_{i,G} = X_{i,G} + F \times (X_{best,G} - X_{i,G}) + F \times (X_{r1,G} - X_{r2,G}) \tag{6}$$

- DE/rand-to-best/1

$$V_{i,G} = X_{r1,G} + F \times (X_{best,G} - X_{r1,G}) + F \times (X_{r2,G} - X_{r3,G}) \tag{7}$$

The indices $r1$, $r2$, $r3$, $r4$ and $r5$ are mutually exclusive integers randomly generated within the range [1, NP] and are also different from the index i. $X_{best,G}$ is the best individual vector at generation G, and the mutation factor F is a positive control parameter for scaling the difference vector. More details can be found in [11,39].

2.1.3. Crossover

The crossover operator is applied to each pair of $X_{i,G}$ and $V_{i,G}$ to generate a trial vector $U_{i,G}$. There are two kinds of crossover scheme: binomial and exponential [39]. The binomial crossover is widely used and can be defined as follows:

$$u_{i,G}^{j} = \begin{cases} v_{i,G}^{j} & \text{if}\, rand(0,1) \le Cr \text{ or } j = j_{rand} \\ x_{i,G}^{j} & \text{otherwise} \end{cases} \tag{8}$$

where $Cr \in [0, 1]$ is called the crossover rate. j_{rand} is a randomly chosen integer in the range [1, D]. If $u_{i,G}^{j}$ is out of the boundary, it will be reinitialized within the range [L^j, U^j].

2.1.4. Selection

The selection operator selects the better one from each pair of $X_{i,G}$ and $U_{i,G}$ for the next generation. The selection operator is given by

$$X_{i,G} = \begin{cases} U_{i,G} & \text{if } f(U_{i,G}) \le f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \tag{9}$$

2.2. Parent selection in mutation

Over the last few decades, numerous DE variants have been proposed to deal with the problems of stagnation and premature convergence [9,11]. In these DE variants, the modifications mostly focus on the following aspects: devising the new mutation operators [8,49], employing the self-adaptive strategies for control parameters [2,33], proposing the ensemble strategies [33,44], and developing the hybrid DE with other techniques [13].

In this study, we focus on the related work of parent selection in the mutation of DE. Usually, parents in DE are selected randomly from the current population. However, since the relationships between individuals in the decision space or objective space have valuable information on the problem to be optimized, it has been advocated that they be used to guide the search of the algorithm [4,20,40]. Many DE variants have been proposed along this line. In this paper, they are classified into roughly four categories, i.e., topology-based selection (TBS), distance-based selection (DBS), rank-based selection (RBS) and archive-based selection (ABS). Note that the algorithms with hybrid strategies for selecting parents will be classified into multiple categories simultaneously in this section.

TBS: Population topology is used to define the neighborhood for each individual, and the parents in mutation are only selected from the neighboring solutions of each target individual. Two main canonical kinds of population topologies are employed in DE, i.e., cellular or fine-grained topology [13], and distributed or coarse-grained topology [45]. In [31], by employing the concept of index neighborhoods, the bare bones DE was proposed. By using the ring topology, a neighborhood-based mutation operator was proposed in [8]. In [13], several population topologies, e.g., distributed, cellular, ring, and small-world, were introduced in DE to improve its performance. Recently, based on the cellular topology, a new DE framework – DE with cellular direction information – was proposed in [26].

DBS: The probabilities of individuals being selected as parents in mutation are calculated based on their distances from the target individual or from each other. In [5], a learning-enhanced DE was proposed where the neighborhood of each individual was defined based on the identified clusters. In [19], a proximity-based DE framework was proposed to select the individuals for mutation by using an affinity matrix based on the Euclidean distance. In [4], by introducing the neighborhood and direction information into DE, a neighborhood and direction information-based DE was proposed, where the Euclidean distance between a pair of individuals was used to select the base and difference vectors for mutation. In [43], a new DE framework with a multi-objective sorting based mutation operator was proposed by utilizing a Euclidean distance-based diversity metric for each individual. In [34], a Euclidean neighborhood-based mutation was integrated with various niching DE algorithms to solve multi-modal optimization problems. In [3], a fitness-and-position-based selection was proposed by combining the fitness and position information of the population simultaneously.

RBS: The fitness information of the population is used for sorting all the individuals, and the parents in mutation are proportionally selected according their ranking values. In [40], to induce and control a selective pressure on the parent and donor vectors for mutation, a rank-based parent selection operator was proposed by sorting the whole population with their fitness values. In [20], a ranking-based mutation was proposed, where some of the parents in the mutation operators are proportionally selected according to their rankings. In [43], a new DE framework with multi-objective sorting-based mutation operation was proposed, where the fitness and diversity information were utilized by using non-dominated sorting.

ABS: Archive is introduced into DE to store useful information about previously found solutions, and some parents in mutation are selected from the archive. In JADE [49], a set of recently explored inferior solutions was archived, and the solutions in the archive were selected as some parents in mutation to improve the population diversity. In [21], the successful-parent-selecting DE (SPS-DE) framework stored the successful solutions in an archive and selected all parents from the archive when stagnation had occurred. In [50], a DE framework with guiding archive (GAR-DE) was proposed to help DE escape from the situation of stagnation, where the base vector was selected from the archive if stagnation was detected.

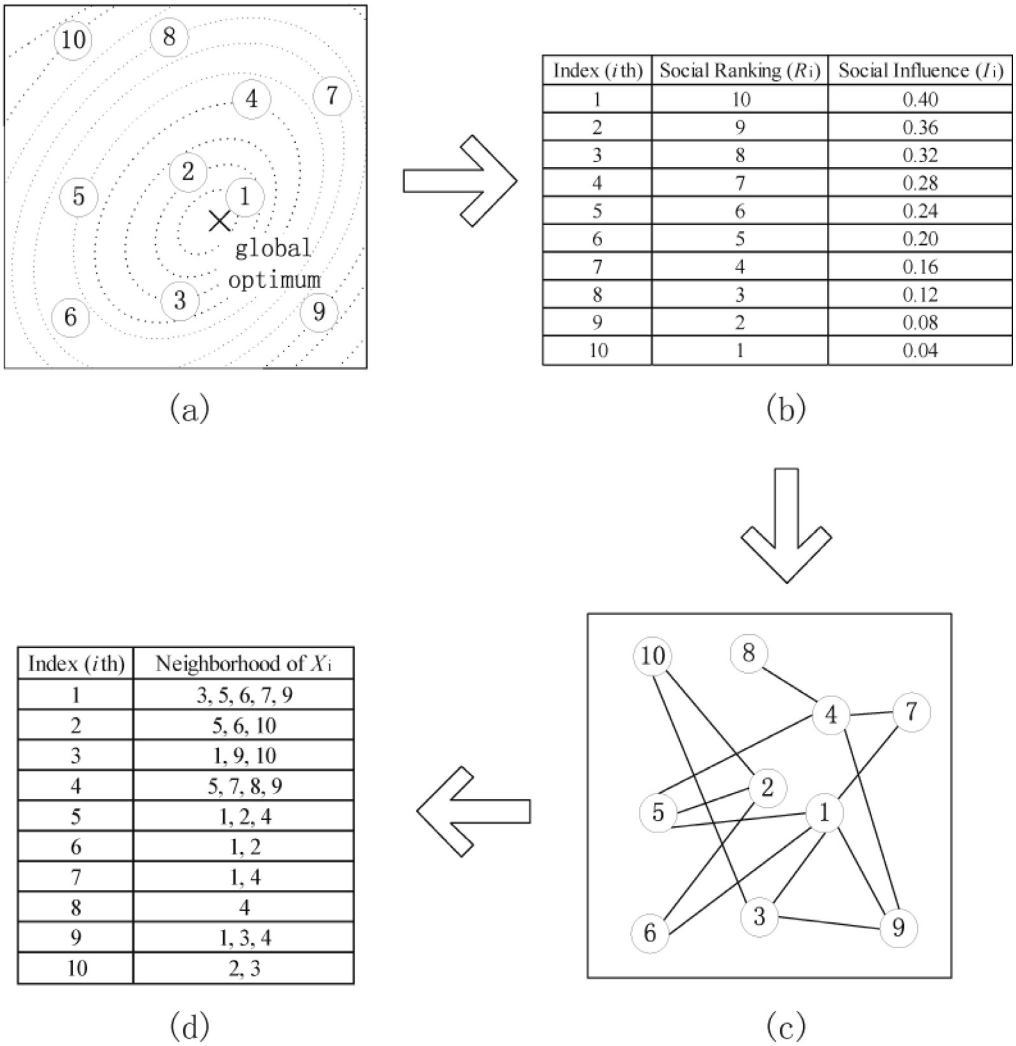
3. Proposed method

3.1. Motivations

In most DE algorithms, vectors for mutation are equally selected as parents without any selective pressure. Due to a high degree of randomness, such a mutation strategy will cause DE to be slow to exploit solutions and to be inefficient when searching in complex problem spaces. As reviewed in Section 2.2, many approaches have been developed to deal with this problem by utilizing the population information. It is clear that these attempts work well for improving the performance of DE. However, although the fixed population structure (e.g., TBS) preserves some kind of diversity, it cannot reflect the population information in time during the process of evolution. In addition, increasing the selective pressure on fitter or nearer solutions alone (e.g., DBS or RBS) will promote efficient exploitation, but diminish the exploration abilities of the mutation operators, which will trap the algorithm in a local optimum more frequently, especially when solving multi-modal problems.

In social learning, on the one hand, imitation, as a mechanism distinct from others, can lead to population-level similarities of behavior through observation [46,48]. That is, imitation can be described as the act of copying the behavior through observation of a demonstrator [46]. On the other hand, to imitate means to economize on explicit or implicit costs of evaluating each possible choice [38]. It does not require the costs of individuals' trial-and-error process when the individuals learn the behaviors from others. Due to these appealing properties of imitation in social learning, it has been successfully applied to several EAs [36]. However, to the best of our knowledge, there are few previous studies in DE that introduces the social learning mechanism to enhance the performance of DE.

Based on these considerations and inspired by the imitation phenomenon of social learning, where people usually learn and imitate the behavior of a better person (or elite) within a social group, we propose an adaptive social learning (ASL) strategy for DE (SL-DE) to provide an alternative for selecting parents in mutation.



Index (ith)	Social Ranking (R_i)	Social Influence (I_i)
1	10	0.40
2	9	0.36
3	8	0.32
4	7	0.28
5	6	0.24
6	5	0.20
7	4	0.16
8	3	0.12
9	2	0.08
10	1	0.04

Index (ith)	Neighborhood of X_i
1	3, 5, 6, 7, 9
2	5, 6, 10
3	1, 9, 10
4	5, 7, 8, 9
5	1, 2, 4
6	1, 2
7	1, 4
8	4
9	1, 3, 4
10	2, 3

Fig. 1. An example of applying ASL to a population with ten vectors. Here, *SIF* is set to 0.4. (a) The index of a vector indicates its ascending order in the whole population according to its fitness value. (b) Calculating the social ranking and social influence for each vector by Eqs. (10) and (11), respectively. (c) Building the social network based on the social influences of pairs of individuals by Eq. (12). (d) Constructing the neighborhood of each vector based on the built social network by Eq. (13).

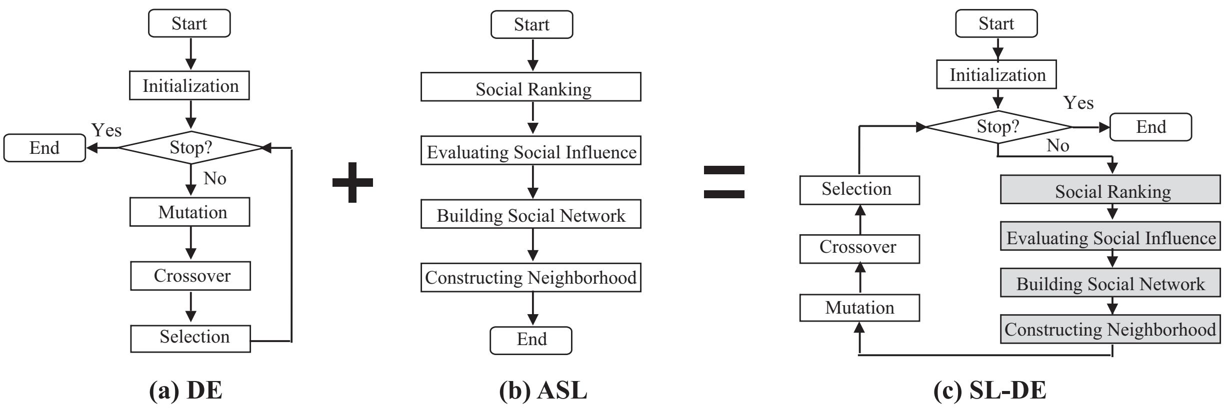


Fig. 2. Fusion of ASL and DE.

Algorithm 1 ASL.

Input: The current population;
Output: The neighbors of each individual;
/*Social ranking*/
Sorting the population according to the their fitness values;
Assign a social ranking to each individual according to Eq.(10);
/*Evaluating social influence*/
Calculate the social influence (I_i) for each individual according to Eq.(11);
/*building a social network*/
Establish a connection between each pair of individuals according to Eq. (12);
/*constructing the neighborhood*/
Identify the neighbors of each individual according to Eq. (13).

Algorithm 2 SL-DE/rand/1.

Generate the initial population P^G and set $G = 1$;
Evaluate the fitness for each individual in P^G;
While the terminated condition is not satisfied **Do**
  $*$ Apply **ASL** (Algorithm 1) to construct the neighborhood for each individual;
  For each individual $X_{i,G}$ **Do**
    $*$ Randomly select $r1, r2, r3$ from the neighborhood of $X_{i,G}$;
    Use Eq. (2) to generate a mutant vector;
    Use Eq. (8) to generate a trial vector;
    Use Eq. (9) to determine the survived vector;
  End For
  Set $G = G + 1$;
End While

Table 1
Parameter settings for the DE variants.

Parameters	Setting
Population size (*NP*)	100
Mutation factor (*F*)	0.5
Crossover factor (*Cr*)	0.9
Social factor (*p*)	0.4
Dimension of each function (*D*)	30 and 50
Maximum number of function evaluations (*MNFEs*)	$10^4 \times D$
Number of runs (*NumR*)	30

3.2. ASL

ASL consists of four main operators: social ranking, evaluating social influence, building social networks and constructing neighborhoods. Algorithm 1 shows the pseudo-code of ASL, and more details are elucidated in the following subsections. In addition, an example of applying ASL to a population with ten vectors is illustrated in Fig. 1.

3.2.1. Social ranking

Each individual is assigned a social ranking according to its fitness value, which is used to determine its status in the current population. First, the whole population is sorted in ascending order (i.e., from best to worst) based on the fitness value of each individual. Then, the social ranking (R_i) of the ith individual is assigned as follows:

$$R_i = NP + 1 - i, i = 1, 2, \ldots, NP \tag{10}$$

where NP is the population size and i is the index of the ith individual in the sorted population. According to Eq. (10), the better individuals in the population will obtain higher social ranking values.

3.2.2. Evaluating social influence

In the real world, people with different social rankings are likely to play different roles in their social networks. Based on this consideration, the social influence for each individual in the population is evaluated to show its impact on other individuals. In addition, a new parameter, named the social influence factor (*SIF*), is introduced to control the level of social influence of better individuals in guiding the population search.

Table 2
Mean and standard deviation of the best error values obtained by SL-DE and the classical DE algorithms on CEC 2013 functions at 30*D*.

Func.	DE/rand/1		SL-DE/rand/1	DE/rand/2		SL-DE/rand/2	DE/best/1		SL-DE/best/1
F1	**1.68e−030 9.22e−030**	−	7.56e−029 9.99e−029	7.08e−001 2.07e−001	+	**1.67e−016 2.14e−016**	3.04e+003 1.46e+003	+	**8.59e+001 1.21e+002**
F2	3.57e+005 2.71e+005	+	**4.77e+004 2.35e+004**	5.66e+007 1.37e+007	+	**4.93e+005 3.99e+005**	1.62e+007 1.03e+007	+	**1.54e+006 1.40e+006**
F3	**2.16e+000 1.13e+001**	−	5.82e+005 1.78e+006	5.53e+009 1.29e+009	+	**4.60e+004 4.46e+004**	1.14e+009 2.86e+009	+	**3.13e+006 4.12e+006**
F4	1.45e+003 5.83e+002	+	**7.28e+000 6.24e+000**	5.38e+004 5.58e+003	+	**1.01e+004 6.31e+003**	**1.56e−003 1.80e−003**	−	1.36e−002 2.35e−002
F5	3.70e−029 5.52e−029	=	2.01e−015 4.31e−015	8.21e−001 2.02e−001	+	**6.15e−011 5.60e−011**	5.80e+002 3.37e+002	+	**5.59e+001 4.84e+001**
F6	**8.86e+000 1.57e+000**	−	2.29e+001 2.24e+001	5.34e+001 2.36e+001	+	**4.06e+000 4.31e+000**	2.85e+002 1.26e+002	+	**7.65e+001 2.69e+001**
F7	**4.78e−002 9.03e−002**	−	7.96e−001 6.54e−001	9.01e+001 1.07e+001	+	**1.28e+001 4.67e+000**	1.01e+002 3.28e+001	+	**5.46e+001 1.65e+001**
F8	2.09e+001 5.71e−002	=	2.09e+001 5.31e−002	2.10e+001 4.04e−002	=	2.09e+001 5.14e−002	2.09e+001 4.45e−002	=	2.10e+001 5.04e−002
F9	3.91e+001 1.27e+000	+	**1.45e+001 1.08e+001**	3.94e+001 1.21e+000	=	3.97e+001 1.06e+000	2.38e+001 3.26e+000	+	**1.93e+001 3.46e+000**
F10	**7.89e−003 7.13e−003**	−	5.99e−002 3.68e−002	5.44e+001 1.68e+001	+	**2.22e−003 4.31e−003**	5.15e+002 2.48e+002	+	**4.22e+001 4.63e+001**
F11	1.36e+002 2.42e+001	+	**1.65e+001 5.88e+000**	2.21e+002 1.17e+001	+	**1.96e+002 9.03e+000**	1.57e+002 4.06e+001	+	**7.43e+001 1.37e+001**
F12	1.80e+002 9.23e+000	+	**7.68e+001 6.67e+001**	2.42e+002 1.20e+001	+	**2.11e+002 9.86e+000**	1.84e+002 5.29e+001	+	**8.76e+001 2.36e+001**
F13	1.79e+002 1.27e+001	+	**1.52e+002 4.47e+001**	2.45e+002 8.77e+000	+	**2.08e+002 1.21e+001**	2.63e+002 4.88e+001	+	**1.63e+002 4.64e+001**
F14	6.54e+003 4.88e+002	+	**7.15e+002 4.55e+002**	6.88e+003 2.48e+002	=	6.96e+003 2.23e+002	2.58e+003 4.31e+002	+	**2.04e+003 6.34e+002**
F15	7.18e+003 2.48e+002	=	7.14e+003 3.13e+002	7.35e+003 2.93e+002	=	7.31e+003 1.90e+002	**3.69e+003 6.98e+002**	−	5.37e+003 1.58e+003
F16	2.45e+000 2.94e−001	=	2.52e+000 2.34e−001	2.54e+000 2.48e−001	=	2.43e+000 2.59e−001	2.44e+000 2.49e−001	=	2.45e+000 3.37e−001
F17	1.84e+002 1.57e+001	+	**4.93e+001 5.96e+000**	2.68e+002 1.47e+001	+	**2.27e+002 1.18e+001**	2.50e+002 5.60e+001	+	**1.01e+002 2.60e+001**
F18	2.12e+002 8.43e+000	+	**1.94e+002 1.76e+001**	2.80e+002 1.28e+001	+	**2.40e+002 1.19e+001**	2.76e+002 5.84e+001	+	**1.54e+002 4.73e+001**
F19	1.46e+001 1.22e+000	+	**3.52e+000 2.89e+000**	2.13e+001 1.46e+000	+	**1.77e+001 1.01e+000**	1.94e+003 2.29e+003	+	**3.26e+001 4.22e+001**
F20	1.23e+001 2.14e−001	+	**1.15e+001 3.20e−001**	1.34e+001 2.34e−001	+	**1.29e+001 1.96e−001**	1.41e+001 1.20e+000	+	**1.07e+001 9.26e−001**
F21	3.02e+002 6.75e+001	=	2.96e+002 7.24e+001	3.88e+002 8.44e+001	+	**2.94e+002 6.46e+001**	1.45e+003 3.04e+002	+	**4.69e+002 1.16e+002**
F22	6.45e+003 5.49e+002	+	**7.00e+002 1.18e+003**	7.32e+003 2.60e+002	=	7.30e+003 2.21e+002	3.12e+003 6.43e+002	+	**1.98e+003 6.58e+002**
F23	7.48e+003 3.60e+002	=	7.29e+003 3.42e+002	7.79e+003 1.97e+002	=	7.71e+003 2.10e+002	4.16e+003 7.59e+002	+	**3.61e+003 6.35e+002**
F24	**2.00e+002 2.85e−002**	−	2.05e+002 3.83e+000	2.84e+002 5.38e+000	+	**2.08e+002 2.97e+000**	2.77e+002 1.29e+001	+	**2.51e+002 1.15e+001**
F25	**2.33e+002 1.87e+001**	−	2.42e+002 1.74e+001	3.33e+002 3.41e+000	+	**3.27e+002 4.31e+000**	3.04e+002 1.14e+001	+	**2.83e+002 6.65e+000**
F26	**2.00e+002 1.12e−002**	−	2.07e+002 2.75e+001	2.04e+002 1.04e+000	+	**2.00e+002 1.13e−002**	3.10e+002 7.88e+001	+	**2.62e+002 7.27e+001**
F27	**3.01e+002 8.73e−001**	−	3.63e+002 3.68e+001	1.32e+003 3.75e+001	+	**1.06e+003 2.42e+002**	9.70e+002 7.24e+001	+	**7.87e+002 7.83e+001**
F28	3.00e+002 8.01e−006	=	3.00e+002 8.96e−006	3.56e+002 1.03e+001	+	**3.00e+002 4.01e−006**	1.86e+003 5.23e+002	+	**3.75e+002 2.00e+002**
+/=/−	–		12/7/9	–		21/7/0	–		24/2/2

Func.	**DE/best/2**		**SL-DE/best/2**	**DE/current-to-best/1**		**SL-DE/current-to-best/1**	**DE/rand-to-best/1**		**SL-DE/rand-to-best/1**
F1	3.60e−028 1.76e−028	+	**2.98e−028 2.31e−028**	1.72e+003 9.48e+002	+	**6.32e+002 4.37e+002**	8.61e+002 4.49e+002	+	**1.04e+002 8.63e+001**
F2	1.13e+005 6.52e+004	+	**4.78e+004 2.24e+004**	9.29e+006 5.50e+006	+	**5.66e+006 3.45e+006**	1.13e+007 6.36e+006	=	8.56e+006 5.06e+006
F3	2.87e+005 8.18e+005	+	**3.09e+004 8.07e+004**	5.00e+009 2.74e+009	=	3.85e+009 2.49e+009	6.44e+009 2.53e+009	+	**2.86e+009 1.82e+009**
F4	1.40e+001 1.57e+001	+	**5.46e−001 7.75e−001**	5.02e+000 4.92e+000	=	9.26e+000 3.44e+001	9.26e+000 2.08e+001	+	**3.74e−001 6.02e−001**
F5	7.39e−015 6.28e−015	+	**3.19e−015 5.28e−015**	5.43e+002 2.84e+002	+	**2.19e+002 1.18e+002**	3.07e+002 1.42e+002	+	**1.34e+002 7.55e+001**
F6	7.18e+000 1.18e+001	+	**1.01e+000 4.85e+000**	2.10e+002 5.94e+001	+	**1.36e+002 3.10e+001**	1.65e+002 4.81e+001	+	**1.25e+002 2.53e+001**
F7	1.42e+001 9.84e+000	+	**2.61e+000 1.80e+000**	5.69e+001 1.86e+001	+	**4.29e+001 1.59e+001**	5.84e+001 1.86e+001	+	**3.86e+001 1.47e+001**
F8	2.09e+001 5.31e−002	=	2.09e+001 4.65e−002	2.09e+001 4.50e−002	=	2.09e+001 4.43e−002	2.09e+001 5.78e−002	=	2.09e+001 5.05e−002
F9	3.45e+001 1.07e+001	=	3.80e+001 5.64e+000	1.69e+001 2.79e+000	+	**1.47e+001 2.43e+000**	1.64e+001 2.17e+000	+	**1.48e+001 2.30e+000**
F10	2.41e−002 1.71e−002	=	2.41e−002 1.67e−002	2.66e+002 9.71e+001	+	**1.12e+002 5.86e+001**	2.19e+002 1.15e+002	+	**9.02e+001 4.41e+001**
F11	1.81e+002 1.95e+001	+	**1.58e+002 1.33e+001**	9.50e+001 3.17e+001	+	**6.08e+001 1.45e+001**	7.83e+001 1.94e+001	+	**4.78e+001 1.25e+001**
F12	1.99e+002 1.42e+001	+	**1.80e+002 1.02e+001**	1.04e+002 2.05e+001	+	**5.82e+001 2.05e+001**	8.05e+001 2.19e+001	+	**5.79e+001 1.59e+001**
F13	1.97e+002 1.87e+001	+	**1.78e+002 1.31e+001**	1.78e+002 3.70e+001	+	**1.34e+002 3.19e+001**	1.66e+002 2.81e+001	+	**1.12e+002 2.44e+001**
F14	6.71e+003 2.93e+002	=	6.87e+003 2.68e+002	6.41e+003 3.60e+002	=	6.38e+003 4.44e+002	3.94e+003 2.50e+003	+	**1.38e+003 3.70e+002**
F15	7.31e+003 3.01e+002	=	7.22e+003 3.81e+002	6.90e+003 3.42e+002	+	**6.61e+003 3.98e+002**	6.46e+003 9.91e+002	+	**4.50e+003 2.37e+003**
F16	2.49e+000 2.31e−001	=	2.60e+000 2.21e−001	2.43e+000 3.20e−001	=	2.42e+000 2.51e−001	2.53e+000 2.45e−001	=	**2.39e+000 3.49e−001**
F17	2.23e+002 1.91e+001	+	**1.93e+002 1.23e+001**	2.22e+002 2.44e+001	+	**1.77e+002 1.53e+001**	1.27e+002 4.67e+001	+	**6.38e+001 9.46e+000**
F18	2.31e+002 1.69e+001	+	**2.09e+002 1.23e+001**	2.45e+002 3.06e+001	+	**2.02e+002 1.30e+001**	2.11e+002 1.56e+001	+	**1.91e+002 1.45e+001**
F19	1.55e+001 1.38e+000	+	**1.39e+001 1.36e+000**	2.61e+002 4.79e+002	+	**2.04e+001 2.89e+001**	9.36e+001 1.08e+002	+	**2.19e+001 1.93e+001**
F20	1.23e+001 3.63e−001	+	**1.19e+001 4.01e−001**	1.31e+001 1.66e+000	+	**1.19e+001 1.59e+000**	1.31e+001 1.65e+000	+	**1.17e+001 1.34e+000**
F21	2.71e+002 7.58e+001	=	2.97e+002 7.94e+001	1.08e+003 3.19e+002	+	**8.07e+002 2.28e+002**	9.65e+002 3.18e+002	+	**6.57e+002 2.19e+002**
F22	6.95e+003 3.36e+002	=	7.00e+003 2.98e+002	5.44e+003 1.09e+003	+	**2.51e+003 2.05e+003**	1.30e+003 4.17e+002	+	**1.10e+003 3.68e+002**
F23	7.48e+003 3.34e+002	=	7.53e+003 3.73e+002	6.44e+003 8.60e+002	+	**4.88e+003 1.98e+003**	4.49e+003 2.36e+003	+	**2.60e+003 1.66e+003**
F24	2.14e+002 8.99e+000	+	**2.07e+002 6.09e+000**	2.49e+002 1.23e+001	+	**2.35e+002 9.23e+000**	2.46e+002 9.95e+000	+	**2.33e+002 8.36e+000**
F25	2.51e+002 2.24e+001	=	2.36e+002 2.39e+001	2.84e+002 1.57e+001	+	**2.69e+002 1.65e+001**	2.78e+002 1.40e+001	=	2.73e+002 1.28e+001
F26	2.08e+002 2.94e+001	+	**2.03e+002 1.91e+001**	2.52e+002 6.88e+001	=	2.48e+002 6.36e+001	2.73e+002 6.95e+001	=	2.70e+002 6.60e+001
F27	4.64e+002 9.06e+001	+	**3.96e+002 7.39e+001**	7.34e+002 8.36e+001	+	**5.94e+002 6.65e+001**	7.37e+002 8.91e+001	+	**6.33e+002 1.09e+002**
F28	3.00e+002 0.00e+000	=	3.00e+002 4.01e−006	1.44e+003 3.58e+002	+	**1.07e+003 3.16e+002**	1.32e+003 3.63e+002	+	**9.36e+002 3.38e+002**
+/=/−	–		17/11/0	–		22/6/0	–		23/5/0

Thus, the social influence (I_i) of the ith individual is calculated by

$$I_i = \frac{R_i \times SIF}{NP}, i = 1, 2, \ldots, NP \tag{11}$$

From Eq. (11), the individuals with better ranking values will have greater social influence on other individuals. Note that the linear model presented in this study is the simplest one, and other sophisticated models, e.g., the quadratic model or the sinusoidal model [20], will be studied to evaluate the influence on the effectiveness of ASL in our future work.

3.2.3. Building a social network

After evaluating the social influence of each individual, a social network will be built to reflect the neighborhood relationships for pairs of individuals. In this study, the neighborhood relationships of individuals in the current population will

Table 3
Mean and standard deviation of the best error values obtained by SL-DE and the classical DE algorithms on CEC 2013 functions at 50*D*.

Func.	DE/rand/1		SL-DE/rand/1	DE/rand/2		SL-DE/rand/2	DE/best/1		SL-DE/best/1
F1	**5.44e−029 8.45e−029**	−	7.54e−028 5.56e−028	2.14e+002 6.28e+001	+	**2.21e−013 2.53e−013**	1.61e+004 4.36e+003	+	**7.11e+002 6.07e+002**
F2	2.60e+006 9.62e+005	+	**2.04e+005 9.73e+004**	4.01e+008 7.12e+007	+	**1.48e+007 4.78e+006**	1.01e+008 4.07e+007	+	**5.13e+006 1.94e+006**
F3	**3.40e+005 6.36e+005**	−	5.88e+006 9.50e+006	6.90e+010 7.88e+009	+	**1.07e+008 1.49e+008**	7.55e+009 1.52e+010	+	**2.82e+007 3.28e+007**
F4	3.26e+004 4.68e+003	+	**4.21e+002 2.18e+002**	9.83e+004 7.70e+003	+	**7.77e+004 9.51e+003**	2.00e−001 2.32e−001	=	2.32e−001 3.62e−001
F5	**9.47e−016 3.08e−015**	−	8.42e−015 5.63e−015	3.20e+001 6.15e+000	+	**1.00e−008 5.78e−009**	1.98e+003 7.68e+002	+	**3.35e+002 2.78e+002**
F6	4.35e+001 1.56e−001	=	4.53e+001 8.69e+000	1.40e+002 3.18e+001	+	**4.34e+001 6.57e−006**	8.68e+002 2.39e+002	+	**1.21e+002 3.68e+001**
F7	**9.88e−001 1.35e+000**	−	9.02e+000 5.01e+000	1.78e+002 1.16e+001	+	**5.39e+001 1.43e+001**	1.20e+002 2.25e+001	+	**7.58e+001 1.25e+001**
F8	2.11e+001 3.52e−002	=	2.11e+001 4.99e−002	2.11e+001 3.10e−002	=	2.11e+001 3.82e−002	2.11e+001 3.94e−002	+	**2.11e+001 3.67e−002**
F9	7.21e+001 1.44e+000	+	**5.58e+001 2.54e+001**	7.25e+001 1.46e+000	=	7.28e+001 1.28e+000	5.04e+001 3.72e+000	+	**4.20e+001 4.27e+000**
F10	**3.64e−002 1.54e−002**	−	1.00e−001 4.61e−002	9.94e+002 2.51e+002	+	**1.18e−002 7.98e−003**	1.78e+003 3.91e+002	+	**1.49e+002 7.45e+001**
F11	1.81e+002 6.14e+001	+	**4.31e+001 8.41e+000**	4.77e+002 1.89e+001	+	**3.96e+002 1.72e+001**	4.41e+002 7.18e+001	+	**2.22e+002 4.50e+001**
F12	3.54e+002 9.82e+000	+	**2.16e+002 1.44e+002**	5.11e+002 2.78e+001	+	**4.11e+002 1.76e+001**	4.78e+002 7.41e+001	+	**2.30e+002 3.85e+001**
F13	3.56e+002 1.18e+001	+	**3.47e+002 1.51e+001**	5.29e+002 2.35e+001	+	**4.13e+002 2.30e+001**	5.76e+002 7.53e+001	+	**3.83e+002 4.17e+001**
F14	1.17e+004 8.78e+002	+	**1.30e+003 5.21e+002**	1.33e+004 2.55e+002	=	1.32e+004 3.94e+002	6.59e+003 6.63e+002	+	**4.56e+003 8.43e+002**
F15	1.40e+004 3.71e+002	=	1.40e+004 4.20e+002	1.43e+004 2.36e+002	+	**1.41e+004 3.14e+002**	**9.61e+003 3.29e+003**	−	1.22e+004 3.13e+003
F16	3.39e+000 2.47e−001	=	3.36e+000 2.51e−001	3.35e+000 2.34e−001	=	3.37e+000 2.68e−001	3.33e+000 2.09e−001	=	3.36e+000 2.99e−001
F17	3.34e+002 2.75e+001	+	**9.85e+001 1.04e+001**	5.52e+002 2.04e+001	+	**4.49e+002 1.44e+001**	7.94e+002 1.42e+002	+	**2.79e+002 5.76e+001**
F18	4.03e+002 1.23e+001	+	**3.92e+002 1.53e+001**	5.63e+002 2.39e+001	+	**4.59e+002 1.98e+001**	8.41e+002 1.10e+002	+	**4.02e+002 1.19e+002**
F19	2.94e+001 1.77e+000	+	**5.19e+000 1.46e+000**	1.41e+002 6.21e+001	+	**3.49e+001 1.71e+000**	2.10e+004 1.93e+004	+	**5.57e+002 1.38e+003**
F20	2.21e+001 2.58e−001	+	**2.14e+001 4.11e−001**	2.36e+001 1.83e−001	+	**2.30e+001 3.84e−001**	2.28e+001 1.52e+000	+	**2.10e+001 7.11e−001**
F21	**7.08e+002 4.09e+002**	−	9.40e+002 3.47e+002	6.14e+002 2.31e+002	=	5.10e+002 4.21e+002	3.48e+003 4.57e+002	+	**1.36e+003 2.81e+002**
F22	1.18e+004 1.14e+003	+	**1.20e+003 4.94e+002**	1.40e+004 2.63e+002	=	1.39e+004 4.19e+002	7.88e+003 1.21e+003	+	**5.03e+003 8.18e+002**
F23	1.44e+004 3.80e+002	=	1.42e+004 3.41e+002	1.48e+004 4.05e+002	+	**1.46e+004 3.04e+002**	8.45e+003 1.70e+003	=	7.82e+003 2.67e+003
F24	**2.02e+002 3.25e+000**	−	2.33e+002 1.05e+001	3.95e+002 7.61e+000	+	**2.30e+002 1.39e+001**	3.56e+002 1.56e+001	+	**3.10e+002 1.34e+001**
F25	2.97e+002 3.72e+001	=	3.00e+002 8.13e+000	4.60e+002 6.62e+000	+	**4.51e+002 5.93e+000**	4.19e+002 2.06e+001	+	**3.74e+002 1.57e+001**
F26	**2.35e+002 5.42e+001**	−	3.11e+002 5.79e+001	2.46e+002 8.91e+000	+	**2.02e+002 1.12e+000**	4.30e+002 1.20e+001	+	**3.93e+002 5.33e+001**
F27	**4.43e+002 1.38e+002**	−	7.10e+002 1.11e+002	2.27e+003 4.37e+001	+	**2.20e+003 4.79e+001**	1.71e+003 1.14e+002	+	**1.47e+003 1.10e+002**
F28	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000	5.72e+002 2.72e+001	+	**4.00e+002 0.00e+000**	3.64e+003 1.03e+003	+	**9.01e+002 1.25e+003**
+/=/−	−		12/7/9	−		22/6/0	−		24/3/1

Func.	**DE/best/2**		**SL-DE/best/2**	**DE/current-to-best/1**		**SL-DE/current-to-best/1**	**DE/rand-to-best/1**		**SL-DE/rand-to-best/1**
F1	9.74e−028 3.98e−028	=	8.50e−028 3.17e−028	1.21e+004 3.65e+003	+	**4.22e+003 1.24e+003**	7.28e+003 2.00e+003	+	**1.23e+003 6.09e+002**
F2	7.79e+005 2.85e+005	+	**2.33e+005 8.60e+004**	4.42e+007 1.71e+007	+	**2.47e+007 1.12e+007**	3.59e+007 1.57e+007	=	3.23e+007 1.01e+007
F3	1.60e+007 2.42e+007	+	**2.50e+006 3.14e+006**	2.76e+010 1.01e+010	+	**1.80e+010 7.84e+009**	2.76e+010 7.59e+009	+	**1.73e+010 8.48e+009**
F4	8.32e+003 3.76e+003	+	**6.02e+002 3.14e+002**	7.04e+001 7.66e+001	+	**4.39e+001 7.69e+001**	7.62e+001 9.34e+001	+	**1.29e+000 1.41e+000**
F5	1.26e−014 5.22e−015	=	1.31e−014 4.36e−015	1.46e+003 4.94e+002	+	**8.91e+002 2.29e+002**	1.26e+003 3.76e+002	+	**5.64e+002 1.96e+002**
F6	4.47e+001 8.92e+000	=	4.40e+001 1.73e+000	5.16e+002 1.18e+002	+	**3.19e+002 5.67e+001**	3.94e+002 9.18e+001	+	**2.25e+002 3.99e+001**
F7	4.45e+001 1.81e+001	+	**1.73e+001 8.48e+000**	7.63e+001 1.03e+001	+	**5.38e+001 1.13e+001**	7.31e+001 1.31e+001	+	**6.29e+001 1.12e+001**
F8	2.11e+001 3.02e−002	=	2.11e+001 4.11e−002	2.11e+001 3.75e−002	=	2.11e+001 4.06e−002	2.11e+001 4.35e−002	=	2.11e+001 3.20e−002
F9	**7.16e+001 1.71e+000**	−	7.30e+001 1.20e+000	3.64e+001 2.95e+000	+	**3.31e+001 2.62e+000**	3.59e+001 2.66e+000	+	**3.36e+001 2.49e+000**
F10	6.11e−002 4.91e−002	=	4.48e−002 2.73e−002	1.24e+003 3.36e+002	+	**6.62e+002 1.69e+002**	9.68e+002 2.60e+002	+	**4.19e+002 8.96e+001**
F11	3.89e+002 2.88e+001	+	**2.06e+002 1.28e+002**	2.68e+002 5.15e+001	+	**1.65e+002 2.99e+001**	2.14e+002 3.37e+001	+	**1.26e+002 2.38e+001**
F12	4.11e+002 3.10e+001	+	**3.66e+002 1.56e+001**	2.61e+002 5.49e+001	+	**1.71e+002 3.13e+001**	2.29e+002 2.93e+001	+	**1.57e+002 2.61e+001**
F13	4.19e+002 3.21e+001	+	**3.69e+002 2.32e+001**	4.14e+002 6.76e+001	+	**3.10e+002 5.09e+001**	3.68e+002 4.82e+001	+	**2.83e+002 4.43e+001**
F14	1.30e+004 3.65e+002	=	1.31e+004 3.82e+002	1.25e+004 5.20e+002	=	1.13e+004 2.87e+003	6.92e+003 4.25e+003	+	**3.51e+003 6.96e+002**
F15	1.41e+004 3.67e+002	=	1.41e+004 4.76e+002	1.36e+004 5.07e+002	=	1.33e+004 4.54e+002	1.30e+004 1.74e+003	=	1.28e+004 2.11e+003
F16	3.26e+000 2.98e−001	=	3.29e+000 3.00e−001	3.32e+000 3.36e−001	=	3.40e+000 2.39e−001	3.32e+000 2.83e−001	=	3.38e+000 2.56e−001
F17	4.56e+002 3.57e+001	+	**4.01e+002 1.62e+001**	3.81e+002 1.14e+002	+	**2.37e+002 9.50e+001**	2.67e+002 4.48e+001	+	**1.66e+002 2.44e+001**
F18	4.88e+002 2.96e+001	+	**4.15e+002 2.01e+001**	5.94e+002 6.45e+001	+	**4.52e+002 3.45e+001**	5.17e+002 5.08e+001	+	**4.22e+002 1.93e+001**
F19	3.59e+001 4.39e+000	+	**2.56e+001 5.47e+000**	6.09e+003 6.79e+003	+	**8.89e+002 7.05e+002**	2.62e+003 2.18e+003	+	**4.47e+002 5.36e+002**
F20	2.20e+001 4.33e−001	+	**2.16e+001 4.15e−001**	2.14e+001 1.04e+000	+	**2.05e+001 4.30e−001**	2.12e+001 7.50e−001	+	**2.05e+001 6.42e−001**
F21	7.60e+002 3.92e+002	=	8.71e+002 3.61e+002	3.22e+003 3.33e+002	+	**2.61e+003 4.89e+002**	2.92e+003 3.61e+002	+	**2.35e+003 5.46e+002**
F22	1.35e+004 5.10e+002	=	1.34e+004 3.83e+002	8.96e+003 4.03e+003	+	**4.17e+003 2.45e+003**	3.93e+003 8.04e+002	=	3.41e+003 8.13e+002
F23	1.46e+004 4.70e+002	=	1.44e+004 4.94e+002	1.36e+004 5.30e+002	+	**9.99e+003 3.98e+003**	8.97e+003 4.07e+003	+	**5.94e+003 2.49e+003**
F24	2.53e+002 1.19e+001	+	**2.43e+002 9.03e+000**	3.27e+002 1.20e+001	+	**2.97e+002 1.26e+001**	3.12e+002 1.23e+001	+	**2.91e+002 1.13e+001**
F25	3.42e+002 5.87e+001	+	**3.06e+002 8.40e+000**	3.99e+002 1.19e+001	+	**3.71e+002 1.11e+001**	3.88e+002 1.19e+001	+	**3.63e+002 1.18e+001**
F26	3.12e+002 1.02e+002	=	3.26e+002 6.26e+001	3.86e+002 5.09e+001	+	**3.73e+002 4.74e+001**	3.84e+002 4.99e+001	+	**3.57e+002 6.26e+001**
F27	9.53e+002 2.65e+002	+	**8.32e+002 1.14e+002**	1.43e+003 1.03e+002	+	**1.22e+003 1.19e+002**	1.38e+003 8.25e+001	+	**1.22e+003 1.05e+002**
F28	4.00e+002 1.39e−005	=	5.00e+002 5.50e+002	3.47e+003 9.51e+002	+	**2.19e+003 7.42e+002**	3.12e+003 8.90e+002	+	**1.77e+003 8.58e+002**
+/=/−	−		14/13/1	−		24/4/0	−		23/5/0

be established in a guiding manner based on their social influences. Specifically, the relationship between X_i and X_j will be established as follows:

$$Relation(X_i, X_j) = \begin{cases} 1 & \text{if } rand(0,1) \le max(I_i, I_j) \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

where $max(I_i, I_j)$ represents the larger value between I_i and I_j. If $Relation(X_i, X_j) = 1$, there will be a connection between X_i and X_j in the social network. Otherwise, no connections will be established between them. Once the relationships for all the pairs of individuals are evaluated, a new social network will be built for the current population.

In this way, the individuals with greater social influence will have more connections with other individuals. Thus, more individuals in the population will have a chance to imitate the behavior of the better individuals.

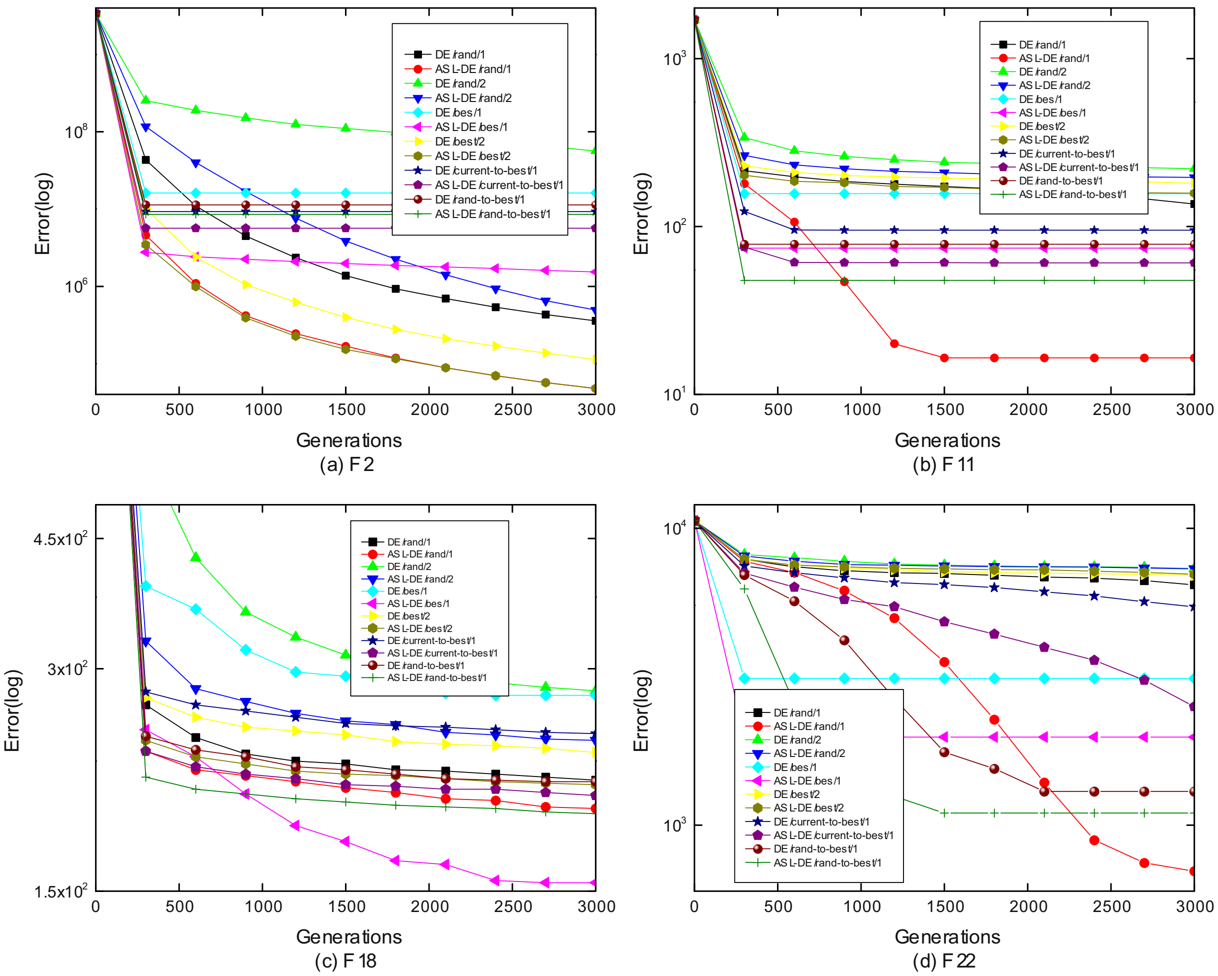


Fig. 3. Convergence graphs of SL-DE and the corresponding classical DE algorithms for the selected functions at 30*D*.

Table 4
Results of the multi-problem Wilcoxon's test for SL-DE versus the classical DE algorithms for CEC 2013 functions at 30*D* and 50*D*.

Algorithm $D = 30$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs DE/rand/1	12/7/9	286.5	119.5	5.58E−02	No	Yes
SL-DE/rand/2 vs DE/rand/2	21/7/0	380.5	25.5	4.60E−05	Yes	Yes
SL-DE/best/1 vs DE/best/1	24/2/2	376	30	7.80E−05	Yes	Yes
SL-DE/best/2 vs DE/best/2	17/11/0	276.5	101.5	3.34E−02	Yes	Yes
SL-DE/current-to-best/1 vs DE/current-to-best/1	22/6/0	373	5	9.00E−06	Yes	Yes
SL-DE/rand-to-best/1 vs DE/rand-to-best/1	23/5/0	378	0	5.00E−06	Yes	Yes
Algorithm $D = 50$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs DE/rand/1	12/7/9	236.5	141.5	2.49E−01	No	No
SL-DE/rand/2 vs DE/rand/2	22/6/0	375	5	5.00E−06	Yes	Yes
SL-DE/best/1 vs DE/best/1	24/3/1	354	24	7.00E−05	Yes	Yes
SL-DE/best/2 vs DE/best/2	14/13/1	311.5	94.5	1.21E−02	Yes	Yes
SL-DE/current-to-best/1 vs DE/current-to-best/1	24/4/0	377	1	6.00E−06	Yes	Yes
SL-DE/rand-to-best/1 vs DE/rand-to-best/1	23/5/0	377	1	6.00E−06	Yes	Yes

3.2.4. Constructing the neighborhood

With the built social network, if there is a connection between a pair of individuals, they are considered neighbors. Hence, the neighborhood of each individual can be constructed by checking the connections between it and other individuals, which is carried out as follows:

$$\text{Neighbors of } X_i = \{X_j \mid j \in [1, NP] \wedge j \neq i \wedge Relation(X_i, X_j) = 1\} \tag{13}$$

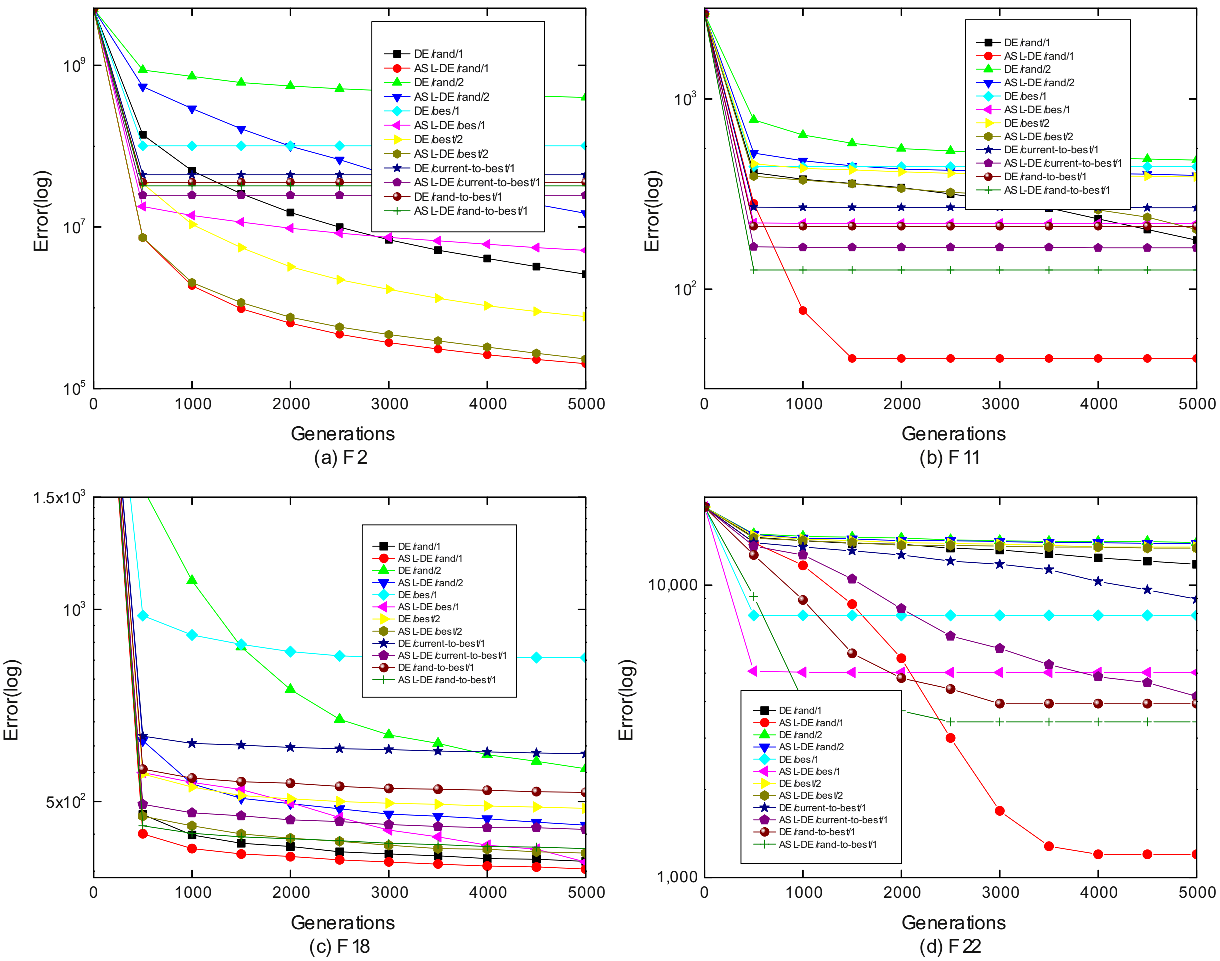


Fig. 4. Convergence graphs of SL-DE and the corresponding classical DE algorithms for the selected functions at 50*D*.

3.3. SL-DE framework

Combing ASL with DE, the framework of SL-DE is presented. As shown in Fig. 2, SL-DE alternately conducts ASL and the DE operators. At each generation of SL-DE, the neighborhood information of the current population is extracted firstly by ASL. Then, based on the obtained neighborhood information, the mutant vector for each individual will be generated by selecting the parents from its neighboring individuals.

In Algorithm 2, the SL-DE framework with the "DE/rand/1" strategy (denoted as SL-DE/rand/1) is described, where the differences with respect to DE/rand/1 are highlighted with "*". It is clear that the proposed SL-DE only affects the selection of parents in the mutation operator. Thus, it could be directly and easily applied to most DE algorithms. For the mutation operator with the best vector (e.g., DE/best/1, DE/best/2, DE/current-to-best/1 or DE/rand-to-best/1), the best neighbor of the target individual will be treated as the best vector for mutation when applying SL-DE to it.

3.4. Remarks

(1) *Complexity analysis:* Compared with the classical DE algorithm, the additional computation of SL-DE depends on constructing the neighborhood for each individual of the current population with ASL, i.e., *Step 4* in Algorithm 2. During one generation, the computational costs of social ranking, evaluating social influence, building the social network and constructing the neighborhood are $O(NP \times \log NP)$, $O(NP)$, $O(NP^2)$, and $O(NP)$, respectively. Since the complexity of the classical DE algorithm is $O(G_{\max} \times NP \times D)$, where $G_{\max}$ is the maximal number of generations, the total complexity of SL-DE is $O(G_{\max} \times (NP \times D + NP \times \log NP + NP + NP^2 + NP))$, that is, $O(G_{\max} \times NP \times \max\{NP, D\})$. In summary, the overall complexity of SL-DE is the same as those of the classical DE algorithms and many DE variants.

(2) *Compared with DE with TBS (TBS-DE):* As shown above, population topology is used in DE to select parents for mutation. SL-DE mainly differs from TBS-DE in defining the neighborhood relationship of individuals. In TBS-DE, the neighborhood

Table 5
Mean and standard deviation of the best error values obtained by SL-DE and the advanced DE variants on CEC 2013 functions at 30*D*.

Func.	CoDE		SL-CoDE	JADE		SL-JADE	jDE		SL-jDE
F1	1.06e−002 2.38e−003	+	**6.90e−005 1.75e−005**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	1.68e−030 9.22e−030
F2	9.84e+007 2.07e+007	=	9.49e+007 2.26e+007	1.29e+005 1.64e+005	+	**1.50e+004 1.12e+004**	1.50e+005 6.41e+004	+	**6.03e+004 4.08e+004**
F3	**7.94e+010 8.13e+010**	−	1.67e+011 1.06e+011	5.72e+005 1.44e+006	=	3.50e+005 7.84e+005	1.01e+006 1.52e+006	+	**4.17e+005 1.16e+006**
F4	7.09e+004 9.63e+003	+	**6.69e+004 9.35e+003**	3.46e+003 3.27e+003	+	**1.32e−002 6.56e−002**	9.76e+001 1.19e+002	+	**3.13e−001 5.16e−001**
F5	7.98e−002 1.61e−002	+	**4.12e−003 8.45e−004**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	1.51e+002 1.20e+001	+	**1.07e+002 1.80e+001**	8.53e+000 3.01e+000	+	**1.13e+000 4.78e+000**	1.16e+001 2.05e+000	+	**7.22e+000 4.34e+000**
F7	**2.43e+002 7.63e+001**	−	3.24e+002 8.79e+001	5.14e+000 4.48e+000	+	**2.21e+000 1.90e+000**	**2.14e+000 2.46e+000**	−	6.65e+000 4.19e+000
F8	2.09e+001 5.52e−002	=	2.09e+001 6.94e−002	2.09e+001 6.82e−002	=	2.09e+001 6.66e−002	2.09e+001 6.01e−002	=	2.09e+001 5.36e−002
F9	3.71e+001 1.49e+000	=	3.70e+001 1.43e+000	**1.44e+001 3.08e+000**	−	2.69e+001 1.90e+000	2.81e+001 1.82e+000	+	**2.26e+001 7.16e+000**
F10	3.91e+002 5.20e+001	+	**2.22e+002 3.80e+001**	1.18e−001 6.25e−002	+	**7.72e−002 4.34e−002**	4.07e−002 1.75e−002	=	5.48e−002 2.92e−002
F11	3.50e+001 2.77e+000	+	**2.42e+001 1.66e+000**	1.26e+000 1.17e+000	+	**0.00e+000 0.00e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F12	2.44e+002 1.32e+001	+	**2.33e+002 1.37e+001**	2.59e+001 5.89e+000	=	2.68e+001 4.97e+000	5.68e+001 9.29e+000	+	**2.96e+001 8.49e+000**
F13	2.54e+002 1.07e+001	+	**2.39e+002 1.39e+001**	4.99e+001 2.00e+001	=	5.11e+001 1.29e+001	9.40e+001 1.44e+001	+	**7.01e+001 1.91e+001**
F14	2.43e+003 2.15e+002	+	**2.21e+003 2.45e+002**	3.70e+001 4.06e+001	+	**4.92e−001 1.53e−001**	6.94e−004 3.80e−003	=	5.55e−003 1.08e−002
F15	7.47e+003 2.68e+002	=	7.38e+003 2.90e+002	4.86e+003 2.01e+003	+	**3.47e+003 2.65e+002**	5.14e+003 5.85e+002	=	5.29e+003 3.98e+002
F16	2.39e+000 3.26e−001	=	2.39e+000 3.29e−001	2.43e+000 3.15e−001	+	**2.01e+000 6.45e−001**	2.30e+000 3.78e−001	=	2.34e+000 2.60e−001
F17	6.97e+001 2.96e+000	+	**5.86e+001 2.40e+000**	3.16e+001 1.01e+000	+	**3.04e+001 0.00e+000**	3.04e+001 0.00e+000	=	3.04e+001 1.77e−006
F18	2.86e+002 1.27e+001	+	**2.70e+002 9.70e+000**	1.30e+002 5.99e+001	+	**8.11e+001 7.29e+000**	1.60e+002 1.35e+001	+	**1.41e+002 1.51e+001**
F19	1.25e+001 9.82e−001	+	**1.09e+001 1.07e+000**	**1.89e+000 4.21e−001**	−	2.13e+000 2.44e−001	1.61e+000 1.63e−001	+	**1.52e+000 1.89e−001**
F20	1.41e+001 3.13e−001	=	1.40e+001 2.39e−001	9.79e+000 7.34e−001	=	1.01e+001 6.03e−001	1.16e+001 3.72e−001	+	**1.10e+001 5.10e−001**
F21	4.88e+002 4.07e+001	+	**3.62e+002 1.00e+002**	3.01e+002 5.92e+001	=	3.02e+002 6.75e+001	2.81e+002 5.26e+001	=	2.96e+002 7.24e+001
F22	3.35e+003 2.41e+002	=	3.27e+003 3.00e+002	1.43e+002 2.80e+001	+	**1.12e+002 2.01e+001**	1.25e+002 1.52e+001	+	**1.10e+002 3.27e+000**
F23	7.90e+003 2.42e+002	=	7.88e+003 2.88e+002	3.69e+003 1.28e+003	=	3.68e+003 3.53e+002	5.64e+003 4.14e+002	=	5.52e+003 3.70e+002
F24	3.01e+002 2.46e+000	=	3.01e+002 2.47e+000	2.03e+002 2.48e+000	=	2.04e+002 3.98e+000	**2.04e+002 3.45e+000**	−	2.16e+002 8.38e+000
F25	2.98e+002 2.75e+000	=	2.99e+002 3.81e+000	2.45e+002 2.09e+001	=	2.51e+002 3.04e+001	2.82e+002 9.29e+000	+	**2.58e+002 6.60e+000**
F26	2.12e+002 2.62e+000	+	**2.09e+002 2.02e+000**	2.17e+002 3.94e+001	+	**2.00e+002 1.61e−002**	2.00e+002 3.25e−003	+	**2.00e+002 1.99e−003**
F27	1.30e+003 3.68e+001	=	1.31e+003 2.74e+001	**3.38e+002 4.49e+001**	−	3.63e+002 6.41e+001	4.94e+002 2.17e+002	=	5.26e+002 1.21e+002
F28	3.52e+002 1.24e+001	+	**3.03e+002 4.46e−001**	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000
+/=/−	−		15/11/2	−		13/12/3	−		13/13/2

Func.	**ODE**		**SL-ODE**	**SaDE**		**SL-SaDE**
F1	1.22e−028 1.71e−028	=	1.51e−028 1.26e−028	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	6.01e+005 3.32e+005	+	**7.54e+004 3.60e+004**	1.31e+007 3.13e+006	+	**7.10e+006 2.26e+006**
F3	**4.68e+004 2.44e+005**	−	1.85e+005 3.80e+005	8.44e+005 1.62e+006	=	1.44e+006 2.13e+006
F4	2.43e+003 1.03e+003	+	**7.36e+001 1.54e+002**	3.27e+004 4.77e+003	+	**1.94e+004 3.44e+003**
F5	6.98e−003 3.79e−002	+	**5.75e−004 2.60e−003**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	1.64e+001 1.04e+001	=	2.40e+001 2.14e+001	1.51e+001 2.76e+000	=	2.06e+001 2.02e+001
F7	**2.90e−001 4.97e−001**	−	9.29e−001 8.66e−001	2.49e+001 1.00e+001	+	**7.06e+000 7.11e+000**
F8	2.09e+001 4.87e−002	=	2.09e+001 1.07e−001	2.09e+001 4.63e−002	=	2.09e+001 6.17e−002
F9	**1.10e+001 3.97e+000**	−	1.41e+001 3.60e+000	2.80e+001 1.95e+000	=	2.83e+001 2.23e+000
F10	**2.30e−002 1.43e−002**	−	5.95e−002 2.53e−002	1.12e−001 7.03e−002	=	1.51e−001 9.86e−002
F11	4.63e+001 2.33e+001	+	**1.63e+001 4.01e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F12	3.48e+001 1.78e+001	=	3.57e+001 1.18e+001	7.14e+001 7.10e+000	+	**5.70e+001 8.52e+000**
F13	7.96e+001 4.71e+001	=	8.35e+001 3.01e+001	1.01e+002 1.25e+001	+	**8.52e+001 1.05e+001**
F14	4.28e+003 5.77e+002	+	**6.00e+002 2.71e+002**	1.87e+000 7.49e−001	+	**3.24e−002 5.71e−002**
F15	4.93e+003 1.53e+003	=	4.49e+003 8.02e+002	4.88e+003 3.01e+002	=	4.90e+003 2.21e+002
F16	2.54e+000 2.67e−001	+	**2.30e+000 5.37e−001**	1.97e+000 2.47e−001	=	1.95e+000 2.55e−001
F17	9.57e+001 1.48e+001	+	**5.07e+001 6.67e+000**	3.04e+001 1.40e−002	+	**3.04e+001 1.37e−006**
F18	1.87e+002 2.45e+001	+	**1.29e+002 2.41e+001**	1.41e+002 1.08e+001	+	**1.31e+002 1.11e+001**
F19	7.52e+000 2.38e+000	+	**2.74e+000 5.90e−001**	2.24e+000 2.10e−001	=	2.15e+000 2.11e−001
F20	1.23e+001 3.02e−001	+	**1.07e+001 8.49e−001**	1.13e+001 4.10e−001	+	**1.10e+001 3.33e−001**
F21	3.21e+002 9.70e+001	=	3.28e+002 7.68e+001	3.03e+002 6.61e+001	=	3.24e+002 5.44e+001
F22	4.49e+003 7.52e+002	+	**5.14e+002 2.31e+002**	2.03e+002 3.75e+001	+	**1.15e+002 1.87e+001**
F23	4.82e+003 1.53e+003	=	4.45e+003 9.56e+002	5.45e+003 4.25e+002	=	5.25e+003 3.91e+002
F24	**2.01e+002 4.54e−001**	−	2.04e+002 3.35e+000	2.14e+002 6.87e+000	+	**2.07e+002 3.97e+000**
F25	2.40e+002 1.94e+001	=	2.45e+002 1.51e+001	2.91e+002 8.90e+000	+	**2.69e+002 3.69e+001**
F26	**2.00e+002 1.06e−002**	−	2.04e+002 1.93e+001	**2.00e+002 1.27e−001**	−	2.07e+002 2.73e+001
F27	**3.09e+002 8.84e+000**	−	3.65e+002 3.75e+001	8.03e+002 1.06e+002	+	**3.78e+002 1.18e+002**
F28	3.00e+002 5.67e−006	=	2.93e+002 3.65e+001	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000
+/=/−	−		11/10/7	−		13/14/1

of each individual is constructed based on the employed population topology, where each individual only interacts with its fixed neighbors throughout the evolutionary process. In this way, the dynamics of neighborhood information in the current population cannot be utilized in time. Conversely, SL-DE uses the ASL strategy to dynamically extract the neighborhood information of individuals during the evolutionary process. Furthermore, by using the fitness information of the population, the neighborhood relationship is built in a self-organizing manner. Due to these differences, SL-DE may be more effective than TBS-DE in improving the performance of DE, which will be discussed in Section 4.6.

(3) *Compared with rank-DE:* Since both SL-DE and rank-DE [20] are proposed to modify the selection of parents in mutation with the fitness information of the population, the differences between them are discussed as follows. Although both SL-DE and rank-DE are the general frameworks for selecting the suitable parents in the mutation of DE, the ways of selecting the parents during mutation are totally different. First, rank-DE uses the fitness information of the population to select fitter

Table 6
Mean and standard deviation of the best error values obtained by SL-DE and the advanced DE variants on CEC 2013 functions at 50*D*.

Func.	CoDE		SL-CoDE	JADE		SL-JADE	jDE		SL-jDE
F1	3.58e+000 5.14e−001	+	**2.00e−001 3.38e−002**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	**0.00e+000 0.00e+000**	−	5.03e−029 7.45e−029
F2	3.08e+008 4.00e+007	=	3.16e+008 4.63e+007	2.67e+005 2.96e+005	+	**5.27e+004 2.68e+004**	5.93e+005 2.53e+005	+	**3.14e+005 1.47e+005**
F3	7.78e+010 1.03e+010	=	7.36e+010 1.81e+010	1.59e+007 1.98e+007	+	**3.14e+006 4.83e+006**	**4.03e+006 5.85e+006**	−	1.58e+007 3.22e+007
F4	1.28e+005 1.18e+004	+	**1.14e+005 1.08e+004**	1.12e+003 2.22e+003	+	**7.82e−003 1.22e−002**	2.31e+002 1.77e+002	+	**7.47e−001 5.53e−001**
F5	4.07e+000 4.74e−001	+	**7.48e−001 6.72e−002**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	2.37e−016 1.30e−015
F6	1.99e+002 2.18e+001	+	**5.70e+001 4.54e+000**	4.34e+001 8.68e−007	=	4.34e+001 1.00e−006	4.36e+001 2.16e−001	+	**4.34e+001 3.69e−003**
F7	1.91e+002 1.40e+001	=	1.88e+002 2.19e+001	2.02e+001 6.85e+000	=	2.07e+001 7.74e+000	**1.14e+001 5.35e+000**	−	2.79e+001 7.75e+000
F8	2.11e+001 3.80e−002	=	2.11e+001 4.36e−002	2.11e+001 4.32e−002	=	2.11e+001 4.42e−002	2.11e+001 3.31e−002	=	2.11e+001 4.44e−002
F9	7.06e+001 1.92e+000	=	7.05e+001 1.61e+000	**3.12e+001 5.06e+000**	−	5.41e+001 2.53e+000	5.50e+001 2.78e+000	=	4.79e+001 1.13e+001
F10	1.66e+003 1.71e+002	+	**1.40e+003 1.41e+002**	1.61e−001 7.92e−002	+	**9.54e−002 5.30e−002**	**5.14e−002 3.38e−002**	−	7.99e−002 4.02e−002
F11	1.53e+002 7.89e+000	+	**1.28e+002 7.04e+000**	8.85e+000 3.85e+000	+	**0.00e+000 0.00e+000**	0.00e+000 0.00e+000	=	3.32e−002 1.82e−001
F12	5.50e+002 1.90e+001	+	**5.19e+002 2.16e+001**	**5.35e+001 1.67e+001**	−	7.04e+001 1.04e+001	1.12e+002 1.67e+001	+	**7.08e+001 1.23e+001**
F13	5.58e+002 2.39e+001	+	**5.25e+002 1.80e+001**	1.38e+002 3.41e+001	=	1.52e+002 2.63e+001	1.79e+002 2.52e+001	+	**1.50e+002 4.03e+001**
F14	6.33e+003 3.38e+002	+	**6.07e+003 3.13e+002**	1.43e+002 9.05e+001	+	**1.18e+001 3.62e+000**	**3.39e−003 6.48e−003**	−	3.36e−002 1.25e−001
F15	1.43e+004 4.53e+002	=	1.43e+004 3.45e+002	8.74e+003 3.22e+003	=	7.57e+003 4.45e+002	1.03e+004 5.66e+002	=	1.01e+004 4.84e+002
F16	3.29e+000 3.02e−001	=	3.28e+000 3.21e−001	3.33e+000 3.09e−001	+	**2.20e+000 6.24e−001**	3.10e+000 2.62e−001	=	3.24e+000 2.93e−001
F17	2.27e+002 1.08e+001	+	**1.93e+002 7.73e+000**	5.71e+001 2.20e+000	+	**5.08e+001 1.00e−006**	5.08e+001 0.00e+000	=	5.08e+001 7.08e−007
F18	5.96e+002 2.45e+001	+	**5.74e+002 1.70e+001**	2.08e+002 1.28e+002	+	**1.57e+002 1.21e+001**	2.78e+002 2.12e+001	+	**2.45e+002 2.07e+001**
F19	4.35e+001 3.37e+000	+	**3.42e+001 2.24e+000**	4.32e+000 8.31e−001	=	4.72e+000 5.81e−001	2.72e+000 2.52e−001	=	2.64e+000 4.85e−001
F20	2.37e+001 4.09e−001	+	**2.35e+001 2.81e−001**	1.88e+001 9.16e−001	=	1.91e+001 7.46e−001	2.13e+001 3.20e−001	+	**2.05e+001 7.31e−001**
F21	7.91e+002 3.19e+002	+	**2.54e+002 1.75e+002**	7.27e+002 4.22e+002	=	7.91e+002 3.82e+002	5.81e+002 4.48e+002	=	5.40e+002 4.31e+002
F22	8.20e+003 6.00e+002	=	7.88e+003 6.94e+002	1.57e+002 1.06e+002	+	**2.87e+001 1.43e+001**	5.63e+001 3.51e+001	+	**1.54e+001 1.28e+001**
F23	1.49e+004 4.65e+002	=	1.47e+004 4.57e+002	**6.99e+003 7.23e+002**	−	8.67e+003 5.29e+002	1.09e+004 5.89e+002	+	**1.04e+004 7.94e+002**
F24	3.87e+002 2.83e+000	=	3.86e+002 4.29e+000	2.28e+002 1.01e+001	=	2.32e+002 1.02e+001	**2.30e+002 1.25e+001**	−	2.51e+002 1.20e+001
F25	3.83e+002 3.14e+000	=	3.82e+002 4.58e+000	**3.14e+002 1.08e+001**	−	3.66e+002 2.18e+001	3.62e+002 1.39e+001	+	**3.18e+002 1.00e+001**
F26	2.91e+002 8.01e+001	=	2.92e+002 6.05e+001	3.19e+002 3.40e+001	=	2.97e+002 8.51e+001	2.00e+002 2.53e−002	+	**2.00e+002 2.10e−002**
F27	2.16e+003 3.33e+001	+	**2.13e+003 5.13e+001**	**7.19e+002 1.26e+002**	−	8.26e+002 2.36e+002	1.26e+003 3.54e+002	+	**1.05e+003 1.80e+002**
F28	4.55e+002 6.65e+000	+	**4.09e+002 1.09e+000**	4.00e+002 0.00e+000	=	4.99e+002 5.44e+002	4.00e+002 5.67e−006	=	4.00e+002 0.00e+000
+/=/−	–		16/12/0	–		10/13/5	–		12/10/6

Func.	**ODE**		**SL-ODE**	**SaDE**		**SL-SaDE**
F1	1.30e−027 9.47e−028	+	**7.53e−028 5.19e−028**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	4.30e+006 1.21e+006	+	**3.22e+005 1.13e+005**	2.27e+007 4.28e+006	+	**1.04e+007 3.27e+006**
F3	**2.87e+006 4.81e+006**	−	1.04e+007 1.40e+007	2.65e+007 3.72e+007	+	**4.43e+006 6.57e+006**
F4	5.12e+004 8.38e+003	+	**1.54e+003 6.43e+002**	3.79e+004 4.43e+003	+	1.75e+004 2.95e+003
F5	2.27e+000 9.17e+000	=	1.08e+000 4.51e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	4.64e+001 9.28e+000	=	4.57e+001 8.90e+000	4.46e+001 7.97e−001	=	4.74e+001 1.24e+001
F7	**5.02e+000 2.52e+000**	−	9.54e+000 4.31e+000	5.62e+001 1.01e+001	+	**3.63e+001 1.41e+001**
F8	2.11e+001 2.96e−002	=	2.11e+001 3.78e−002	2.11e+001 3.19e−002	=	2.11e+001 3.56e−002
F9	**2.27e+001 7.30e+000**	−	2.98e+001 7.24e+000	5.56e+001 2.38e+000	=	5.51e+001 2.23e+000
F10	1.02e−001 6.19e−002	=	1.32e−001 7.97e−002	2.29e−001 1.05e−001	=	2.17e−001 1.24e−001
F11	8.70e+001 4.74e+001	+	**5.14e+001 9.79e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F12	1.05e+002 9.24e+001	=	7.67e+001 2.47e+001	1.81e+002 1.85e+001	+	**1.44e+002 1.37e+001**
F13	2.49e+002 9.38e+001	+	**1.64e+002 3.13e+001**	2.50e+002 2.07e+001	+	**2.20e+002 1.78e+001**
F14	8.60e+003 9.34e+002	+	**1.37e+003 4.94e+002**	8.61e+001 1.13e+001	+	**4.28e+001 1.17e+001**
F15	1.14e+004 2.58e+003	=	1.04e+004 2.57e+003	1.02e+004 5.13e+002	=	1.02e+004 4.31e+002
F16	3.45e+000 2.23e−001	=	3.34e+000 2.98e−001	2.62e+000 3.18e−001	=	2.63e+000 2.87e−001
F17	2.36e+002 2.86e+001	+	**1.07e+002 1.51e+001**	5.33e+001 4.13e−001	+	**5.14e+001 2.68e−001**
F18	3.98e+002 1.59e+001	=	3.64e+002 7.03e+001	2.87e+002 1.57e+001	+	**2.69e+002 1.77e+001**
F19	1.63e+001 5.66e+000	+	**6.10e+000 1.11e+000**	4.55e+000 5.15e−001	=	4.79e+000 6.46e−001
F20	2.22e+001 3.25e−001	+	**2.14e+001 7.12e−001**	2.09e+001 3.45e−001	+	**2.04e+001 5.85e−001**
F21	7.91e+002 3.82e+002	=	9.02e+002 3.41e+002	7.91e+002 3.82e+002	=	8.62e+002 3.58e+002
F22	8.87e+003 1.19e+003	+	**1.34e+003 4.85e+002**	4.79e+002 1.85e+002	+	**9.80e+001 1.64e+002**
F23	1.20e+004 2.70e+003	=	1.07e+004 2.69e+003	1.13e+004 5.71e+002	=	1.12e+004 5.40e+002
F24	**2.14e+002 6.10e+000**	−	2.29e+002 7.00e+000	2.34e+002 1.39e+001	=	2.32e+002 8.11e+000
F25	3.39e+002 5.88e+001	+	**3.00e+002 6.90e+000**	3.87e+002 6.27e+000	+	**3.81e+002 7.09e+000**
F26	**2.20e+002 4.52e+001**	−	2.77e+002 6.87e+001	2.26e+002 6.96e+001	=	2.83e+002 8.42e+001
F27	**5.24e+002 8.18e+001**	−	7.07e+002 1.22e+002	1.57e+003 1.22e+002	+	**8.52e+002 3.15e+002**
F28	4.00e+002 0.00e+000	=	4.00e+002 2.12e+000	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000
+/=/−	–		11/11/6	–		13/15/0

individuals for mutation. That is, rank-DE is a fitness-biased parent selection that increases the selective pressure on fitter solutions. In this way, rank-DE prefers to promote efficient exploitation, but it may diminish the exploration ability of DE [20]. Conversely, SL-DE utilizes the fitness information of the population to evaluate the social influence of each individual. It then constructs the neighborhood for each individual with their social influences. Second, different from rank-DE, SL-DE selects parents for mutation from the neighboring individuals instead of directly using the fitness information to guide the selection. Third, by taking advantage of the imitation mechanism, SL-DE can prompt the individuals to learn from the promising neighbors within their social groups. In this way, the transmission of good individuals through the population will be slowed down, but each individual can effectively exploit the search region of its neighborhood. Therefore, it is expected that a better balance of exploration and exploitation can be achieved by SL-DE. The comparisons between them will be given in Section 4.7.

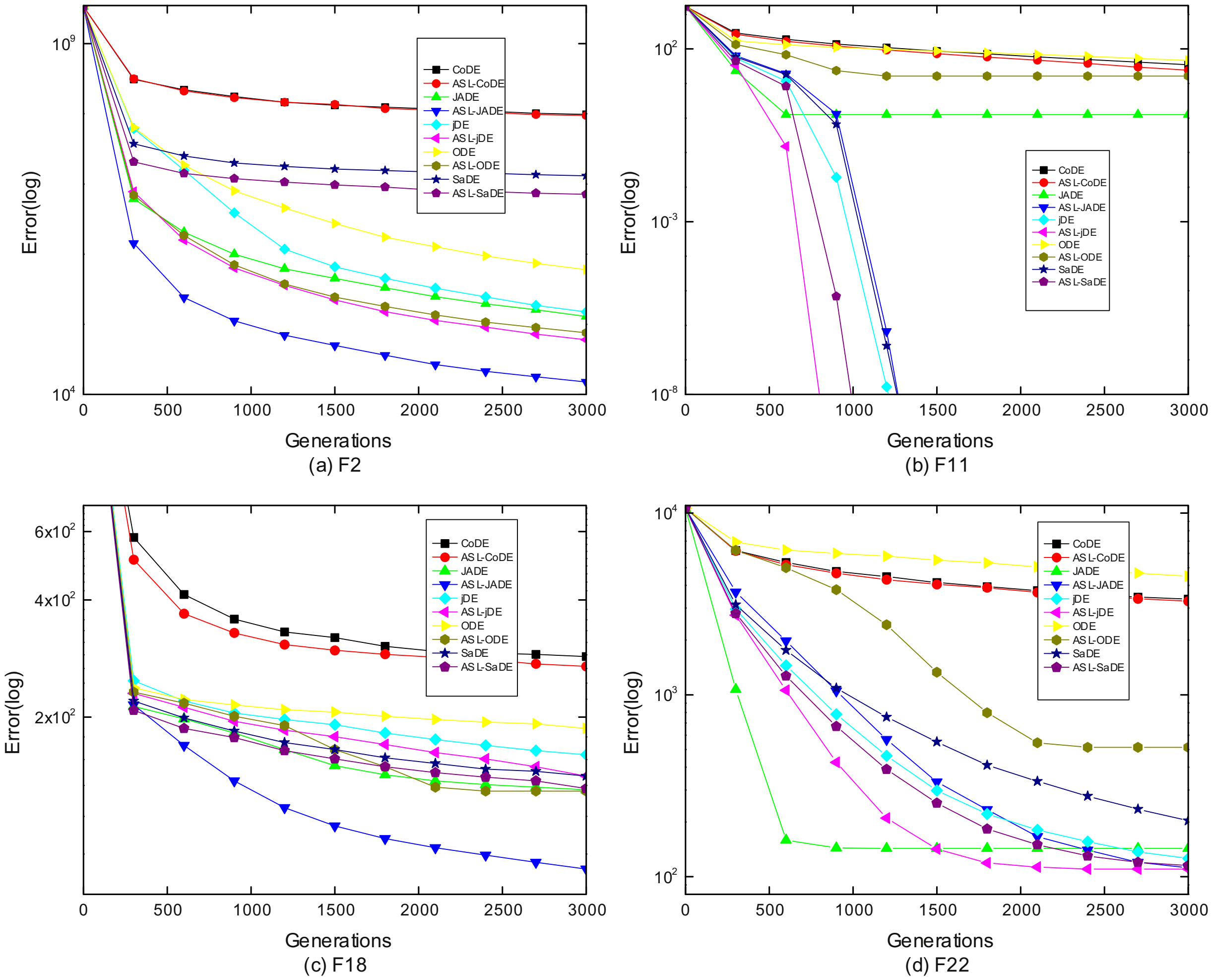


Fig. 5. Convergence graphs of SL-DE and the corresponding advanced DE variants for the selected functions at 30*D*.

4. Experimental result

In this section, extensive experiments are carried out to evaluate the performance of SL-DE. A test suite of the benchmark functions is used, including the CEC2013 special sessions on real-parameter optimization [25] and large-scale global optimization [24], and the CEC2011 on real-world application problems [10]. Detail definitions can be found in [24,25] and [10], respectively.

The experiments can be divided into eight parts:

(1) Sections 4.2 and 4.3 investigate what benefits can be obtained by the use of ASL in the classical DE algorithms and advanced DE variants, respectively.

(2) Sections 4.4 and 4.5 investigate how good the performance of SL-DE is compared with several up-to-date DE variants and non-DE EAs, respectively.

(3) Section 4.6 studies the performance of SL-DE when compared with the DE variants with topology-based parent selection (TBS-DE).

(4) Sections 4.7 and 4.8 analyze what the advantages SL-DE has over three related DE frameworks, i.e., ProDE [19], rank-DE [20] and MS-DE [43].

(5) Section 4.9 compares the performance of SL-DE with a recently DE variant with ABS, i.e., SPS-DE [21].

(6) SL-DE introduces a new parameter, the social influence factor (*SIF*). Section 4.10 conducts a sensitivity analysis on it.

(7) Section 4.11 studies the scalability of SL-DE on the benchmark functions at 1000*D* from CEC2013.

(8) To show the performance on real-world problems, Section 4.12 applies SL-DE to solve the real-world application problems from CEC2011.

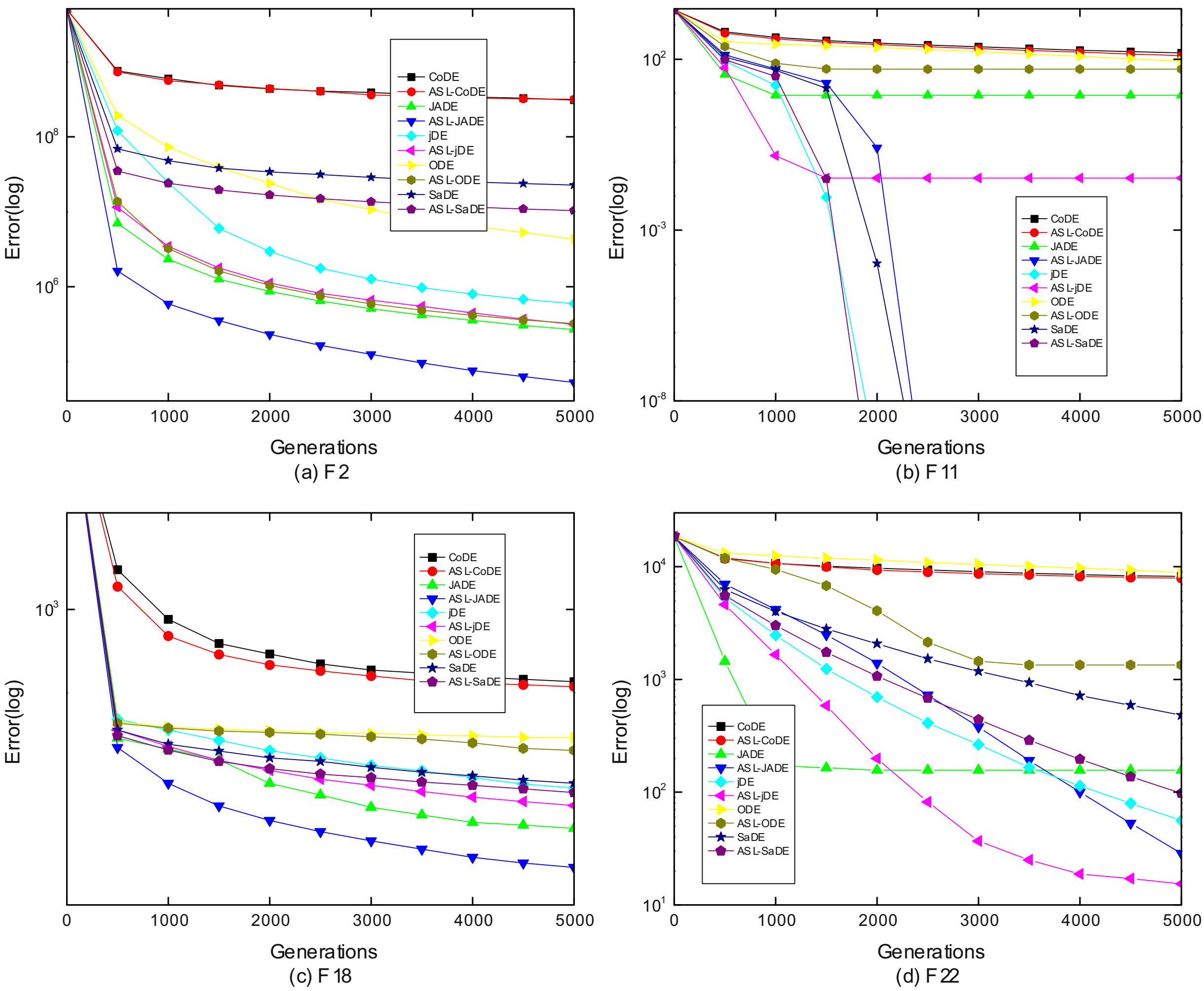


Fig. 6. Convergence graphs of SL-DE and the corresponding advanced DE variants for the selected functions at 50*D*.

Table 7
Results of the multi-problem Wilcoxon's test for SL-DE versus the advanced DE variants for CEC 2013 functions at 30*D* and 50*D*.

Algorithm $D = 30$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-CoDE vs CoDE	15/11/2	312.5	65.5	2.89E−03	Yes	Yes
SL-JADE vs JADE	13/12/3	289.5	116.5	4.63E−02	Yes	Yes
SL-jDE vs jDE	13/13/2	265	141	1.54E−01	No	No
SL-ODE vs ODE	11/10/7	236.5	141.5	2.47E−01	No	No
SL-SaDE vs SaDE	13/14/1	281.5	124.5	7.04E−02	No	No
Algorithm $D = 50$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-CoDE vs CoDE	16/12/0	370.5	35.5	1.04E−04	Yes	Yes
SL-JADE vs JADE	10/13/5	223	183	6.41E−01	No	No
SL-jDE vs jDE	12/10/6	296	110	3.32E−02	Yes	Yes
SL-ODE vs ODE	11/11/6	284.5	121.5	6.19E−02	No	Yes
SL-SaDE vs SaDE	13/15/0	322.5	83.5	6.14E−03	Yes	Yes

4.1. Experimental settings

To maintain a fair and reliable comparison between SL-DE and its corresponding competitors, the same random initial population is employed in this study. The parameter settings are shown in Table 1 unless a change is mentioned. All the experiments are carried out on an Intel Core i3 duo PC with 3.30-GHz CPU and 4-GB RAM.

Table 8
Average ranking values (ARV) of different SL-DE variants by Friedman test for CEC 2013 functions at 30*D* and 50*D*.

$D = 30$			$D = 50$		
Algorithm	ARV	Final Rank	Algorithm	ARV	Final Rank
SL-JADE	2.59	1	**SL-JADE**	2.77	1
SL-jDE	3.68	2	**SL-jDE**	3.23	2
SL-ODE	3.88	3	**SL-SaDE**	4.39	3
SL-SaDE	4.70	4	**SL-ODE**	4.50	4
SL-DE/rand/1	4.95	5	**SL-DE/rand/1**	5.09	5
SL-DE/best/2	6.32	6	**SL-DE/best/2**	6.59	6
SL-DE/rand-to-best/1	7.25	7	**SL-DE/rand-to-best/1**	7.20	7
SL-DE/rand/2	7.57	8	**SL-DE/best/1**	7.63	8
SL-DE/best/1	7.70	9	**SL-DE/rand/2**	7.77	9
SL-DE/current-to-best/1	8.20	10	**SL-DE/current-to-best/1**	8.13	10
SL-CoDE	9.18	11	**SL-CoDE**	8.71	11

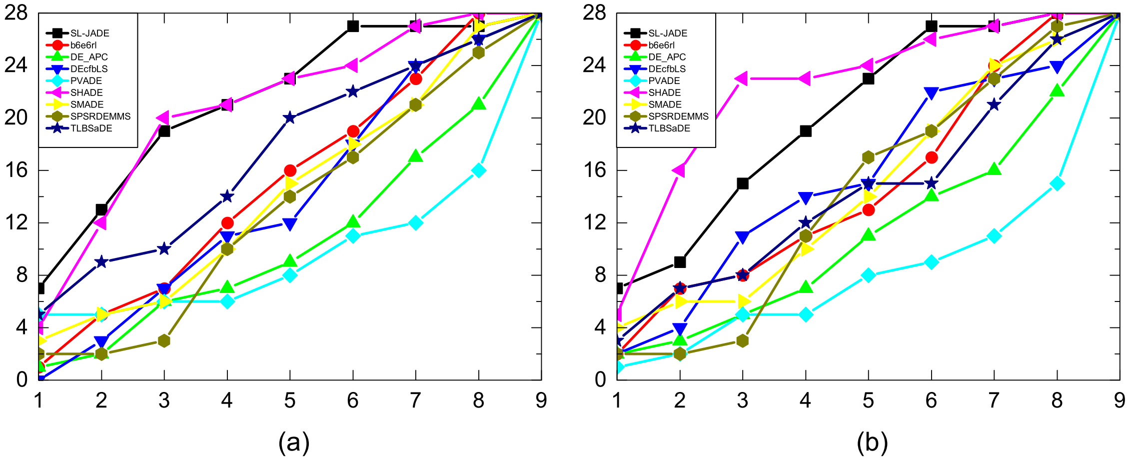


Fig. 7. The Top-K, $K = 1, 2, \ldots, 9$, curves of the up-to-date DE variants and SL-DE for the functions at (a) 30*D* and (b) 50*D*.

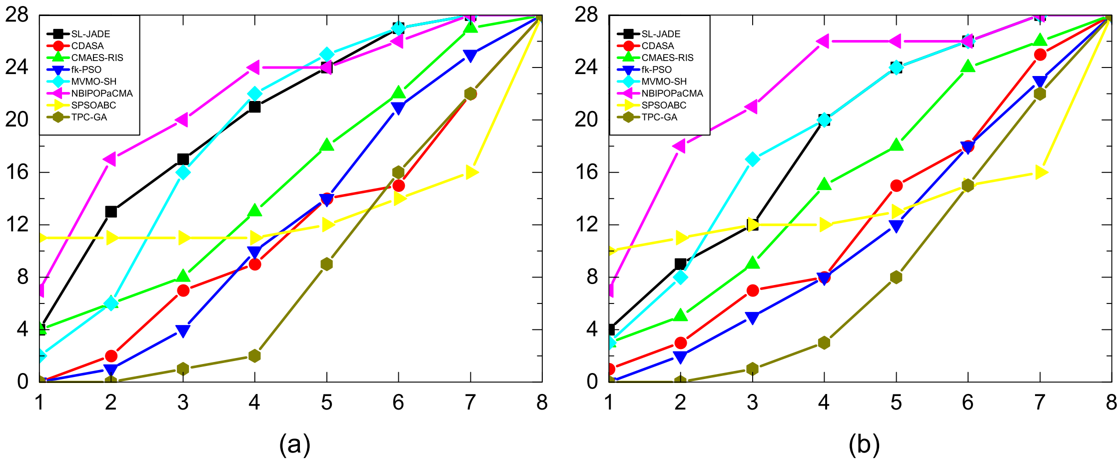


Fig. 8. The Top-K, $K = 1, 2, \ldots, 8$, curves of non-DE EAs and SL-DE for the functions at (a) 30*D* and (b) 50*D*.

To show significant differences among the algorithms, nonparametric statistical tests [12] are carried out by the KEEL software[1]. The results of the single-problem Wilcoxon signed-rank test at $\alpha = 0.05$ are firstly shown in the tables as " + ", " = ", or " − ", which means that SL-DE is significantly better than, not significantly different from or significantly worse

[1] KEEL: a software tool to assess EAs to data mining problems, which can be available from http://www.keel.es/.

Table 9
Mean and standard deviation of the best error values obtained by SL-JADE and the up-to-date DE variants on CEC 2013 functions at 30*D*.

	SL-JADE		b6e6rl		DE-APC		DEcfbLS		PVADE		SHADE		SMADE		SPSRDEMMS		TLBSaDE
F1	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	1.50e+004 1.12e+004	−	6.83e+004 4.44e+004	−	2.11e+005 1.56e+005	−	2.00e+005 1.15e+005	−	2.22e+006 1.39e+006	+	**9.81e+003 8.59e+003**	+	**0.00e+000 0.00e+000**	−	9.17e+004 4.38e+004	+	**5.00e+003 1.81e+003**
F3	3.50e+005 7.84e+005	=	5.26e+003 1.42e+004	=	1.34e+006 3.36e+006	−	2.65e+006 5.62e+006	=	1.81e+003 2.78e+003	+	**6.82e+001 2.76e+002**	=	1.65e+004 6.47e+004	−	1.00e+007 1.26e+007	+	**2.60e+000 2.75e+000**
F4	1.32e−002 6.56e−002	−	1.95e−002 3.57e−002	−	2.55e−001 7.66e−001	−	3.94e+002 5.27e+002	−	1.65e+004 3.24e+003	=	1.54e−004 2.64e−004	+	**0.00e+000 0.00e+000**	−	2.41e+000 3.32e+000	−	1.09e+000 5.26e−001
F5	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	−	1.70e−007 2.12e−007	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	−	2.27e−013 1.19e−013	=	0.00e+000 0.00e+000
F6	1.13e+000 4.78e+000	−	5.89e+000 1.04e+001	−	9.04e+000 1.77e+000	−	6.34e+000 1.39e+000	−	9.07e+000 6.24e+000	+	**1.01e+000 4.85e+000**	−	3.65e+000 9.11e+000	−	1.59e+001 6.60e+000	+	**8.22e−007 1.64e−006**
F7	2.21e+000 1.90e+000	−	2.29e+001 7.74e+000	−	2.53e+001 2.13e+001	−	5.90e+001 1.75e+001	+	**1.32e+000 1.36e+000**	−	4.82e+000 5.37e+000	−	3.17e+001 1.66e+001	−	1.07e+001 6.10e+000	−	1.32e+001 2.71e+000
F8	2.09e+001 6.66e−002	=	2.09e+001 4.68e−002	=	2.09e+001 4.53e−002	=	2.09e+001 9.90e−002	=	2.09e+001 5.35e−002	+	**2.07e+001 1.84e−001**	=	2.09e+001 5.31e−002	−	2.10e+001 4.68e−002	+	**2.08e+001 5.65e−002**
F9	2.69e+001 1.90e+000	−	2.89e+001 9.51e−001	=	3.02e+001 9.57e+000	+	**2.39e+001 2.16e+000**	+	**6.78e+000 3.99e+000**	=	2.73e+001 1.68e+000	+	**2.29e+001 3.06e+000**	+	**2.47e+001 3.63e+000**	=	2.51e+001 4.29e+000
F10	7.72e−002 4.34e−002	+	2.04e−002 1.34e−002	=	6.14e−002 4.46e−002	+	**1.90e−002 1.52e−002**	+	**2.10e−002 1.28e−002**	=	7.65e−002 4.14e−002	+	**1.69e−002 9.87e−003**	+	**5.10e−002 3.34e−002**	+	**1.13e−002 4.17e−003**
F11	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	−	3.08e+000 4.90e+000	=	3.32e−002 1.82e−001	−	5.65e+001 1.07e+001	=	0.00e+000 0.00e+000	−	1.07e+001 4.59e+000	−	2.65e−014 2.88e−014	=	0.00e+000 0.00e+000
F12	2.68e+001 4.97e+000	−	8.16e+001 1.34e+001	−	3.11e+001 9.40e+000	−	5.42e+001 8.47e+000	−	1.17e+002 1.14e+001	+	**2.29e+001 4.11e+000**	−	5.57e+001 1.41e+001	−	4.43e+001 1.42e+001	−	4.77e+001 2.94e+000
F13	5.11e+001 1.29e+001	−	1.14e+002 1.35e+001	−	7.45e+001 2.25e+001	−	1.03e+002 1.84e+001	−	1.29e+002 1.36e+001	=	5.11e+001 1.46e+001	−	1.27e+002 3.81e+001	−	8.14e+001 2.12e+001	−	8.04e+001 7.23e+000
F14	4.92e−001 1.53e−001	+	**2.01e−002 2.29e−002**	−	3.90e+003 4.24e+002	−	3.45e+001 1.35e+001	−	3.19e+003 4.19e+002	+	**3.12e−002 2.37e−002**	−	1.31e+002 1.40e+002	−	3.52e+000 7.56e+000	−	2.34e+001 6.69e+000
F15	3.47e+003 2.65e+002	−	4.60e+003 3.62e+002	−	4.07e+003 8.87e+002	=	3.34e+003 9.86e+002	−	5.59e+003 3.40e+002	+	**3.23e+003 2.32e+002**	−	4.03e+003 7.20e+002	−	4.45e+003 7.21e+002	=	3.49e+003 2.01e+002
F16	2.01e+000 6.45e−001	=	1.92e+000 4.64e−001	−	2.41e+000 5.47e−001	+	8.85e−001 1.01e+000	−	2.44e+000 2.48e−001	+	**9.35e−001 1.60e−001**	+	1.23e−001 7.08e−002	−	2.31e+000 4.07e−001	+	1.36e+000 1.77e−001
F17	3.04e+001 0.00e+000	=	3.04e+001 4.50e−005	−	5.93e+001 5.72e+000	−	3.53e+001 1.12e+000	−	1.01e+002 1.12e+001	=	3.04e+001 9.37e−007	−	3.51e+001 1.31e+000	−	3.04e+001 7.48e−003	−	3.20e+001 3.88e−001
F18	8.11e+001 7.29e+000	−	1.70e+002 1.33e+001	+	**6.03e+001 8.86e+000**	=	7.77e+001 9.61e+000	−	1.82e+002 1.36e+001	+	**7.23e+001 5.46e+000**	=	8.04e+001 1.40e+001	−	9.60e+001 3.11e+001	+	**7.24e+001 6.31e+000**
F19	2.13e+000 2.44e−001	+	**1.85e+000 1.50e−001**	=	2.26e+000 5.19e−001	+	**1.54e+000 1.89e−001**	−	5.41e+000 9.21e−001	+	**1.36e+000 1.33e−001**	−	2.61e+000 5.45e−001	+	**1.22e+000 1.46e−001**	−	2.53e+000 1.87e−001
F20	1.01e+001 6.03e−001	−	1.18e+001 3.14e−001	−	1.26e+001 8.13e−001	−	1.17e+001 7.12e−001	−	1.14e+001 3.57e−001	−	1.05e+001 4.52e−001	=	1.05e+001 8.49e−001	−	1.13e+001 5.30e−001	−	1.04e+001 3.39e−001
F21	3.02e+002 6.75e+001	=	2.79e+002 8.08e+001	+	**2.61e+002 6.04e+001**	−	3.33e+002 9.37e+001	−	3.03e+002 4.58e+001	−	3.21e+002 5.88e+001	=	3.23e+002 8.90e+001	=	2.94e+002 6.46e+001	+	**2.43e+002 5.04e+001**
F22	1.12e+002 2.01e+001	−	1.21e+002 1.44e+001	−	4.68e+003 4.01e+002	−	2.40e+002 7.63e+001	−	2.46e+003 3.61e+002	+	**9.75e+001 2.75e+001**	−	1.68e+002 3.25e+001	+	**7.54e+001 5.02e+001**	−	2.06e+002 4.15e+001
F23	3.68e+003 3.53e+002	−	4.96e+003 4.25e+002	−	4.26e+003 1.01e+003	=	3.58e+003 5.29e+002	−	5.78e+003 4.78e+002	=	3.52e+003 3.83e+002	−	4.12e+003 8.75e+002	−	4.99e+003 7.33e+002	−	4.14e+003 3.44e+002
F24	2.04e+002 3.98e+000	−	2.50e+002 1.27e+001	−	2.91e+002 2.05e+001	−	2.63e+002 1.04e+001	+	**2.02e+002 1.34e+000**	=	2.05e+002 4.79e+000	−	2.30e+002 2.80e+001	−	2.52e+002 8.96e+000	−	3.02e+002 2.69e+000
F25	2.51e+002 3.04e+001	−	2.77e+002 1.67e+001	−	2.99e+002 8.81e+000	−	2.83e+002 6.91e+000	+	**2.24e+002 2.22e+001**	=	2.57e+002 2.19e+001	−	2.76e+002 1.18e+001	=	2.66e+002 8.06e+000	−	2.95e+002 1.79e+000
F26	2.00e+002 1.61e−002	−	2.17e+002 5.06e+001	−	3.28e+002 5.46e+001	−	2.00e+002 7.35e−003	−	2.24e+002 4.42e+001	−	2.04e+002 1.94e+001	=	2.13e+002 5.29e+001	−	2.00e+002 5.21e−003	+	**2.00e+002 0.00e+000**
F27	3.63e+002 6.41e+001	−	9.97e+002 8.74e+001	−	1.23e+003 1.29e+002	−	9.39e+002 5.40e+001	+	**3.22e+002 7.69e+000**	=	3.90e+002 1.16e+002	−	6.35e+002 1.30e+002	−	8.66e+002 9.31e+001	−	1.15e+003 1.46e+002
F28	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.00e+002 2.50e−005	=	3.00e+002 0.00e+000	=	4.28e+002 3.75e+002	=	3.00e+002 0.00e+000	=	2.93e+002 3.65e+001
+/=/−	–		3/9/16		2/8/18		4/9/15		6/5/17		11/14/3		5/9/14		4/5/19		9/6/13

Table 10
Mean and standard deviation of the best error values obtained by SL-JADE and the up-to-date DE variants on CEC 2013 functions at 50*D*.

	SL-JADE		b6e6rl		DE-APC		DEcfbLS		PVADE		SHADE		SMADE		SPSRDEMMS		TLBSaDE
F1	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	–	2.12e−013 5.77e−014	=	0.00e+000 0.00e+000
F2	5.27e+004 2.68e+004	–	3.21e+005 1.52e+005	–	3.90e+005 1.73e+005	–	6.73e+005 4.23e+005	–	2.16e+005 8.15e+004	+	**2.50e+004 1.07e+004**	+	**0.00e+000 0.00e+000**	–	4.72e+005 1.84e+005	–	1.50e+005 2.28e+004
F3	3.14e+006 4.83e+006	=	7.93e+006 2.48e+007	=	6.55e+006 1.04e+007	–	2.42e+008 2.56e+008	–	9.05e+006 8.76e+006	+	**8.82e+005 1.93e+006**	+	**4.50e+005 1.69e+006**	–	4.22e+007 3.47e+007	+	**4.59e+005 2.40e+005**
F4	7.82e−003 1.22e−002	–	1.87e−001 2.27e−001	–	1.49e+000 1.45e+000	–	1.02e+003 1.17e+003	–	2.05e+002 1.00e+002	+	**1.72e−003 1.58e−003**	+	**0.00e+000 0.00e+000**	–	5.14e+000 5.69e+000	–	5.50e+002 1.55e+002
F5	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	–	9.60e−004 6.50e−004	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	–	6.21e−013 3.17e−013	=	0.00e+000 0.00e+000
F6	4.34e+001 1.00e−006	–	4.34e+001 0.00e+000	+	**3.90e+001 1.63e+000**	=	4.34e+001 0.00e+000	–	6.83e+001 2.81e+001	=	4.22e+001 7.10e+000	=	4.28e+001 8.31e+000	=	4.38e+001 1.44e+000	+	**3.89e+001 8.67e+000**
F7	2.07e+001 7.74e+000	–	8.15e+001 1.61e+001	–	3.66e+001 1.55e+001	–	1.04e+002 8.93e+000	=	2.24e+001 1.03e+001	=	2.40e+001 7.40e+000	–	3.90e+001 1.62e+001	–	3.14e+001 8.74e+000	–	4.68e+001 5.33e+000
F8	2.11e+001 4.42e−002	=	2.11e+001 4.55e−002	=	2.11e+001 3.35e−002	=	2.11e+001 1.02e−001	–	2.11e+001 3.51e−002	+	**2.09e+001 1.62e−001**	=	2.11e+001 4.03e−002	=	2.11e+001 4.87e−002	+	**2.10e+001 2.56e−002**
F9	5.41e+001 2.53e+000	–	5.64e+001 2.81e+000	–	6.18e+001 1.59e+001	+	**4.72e+001 3.15e+000**	+	**2.62e+001 3.20e+000**	=	5.54e+001 2.11e+000	+	**4.32e+001 3.23e+000**	+	**5.03e+001 5.58e+000**	–	6.00e+001 1.60e+000
F10	9.54e−002 5.30e−002	+	3.61e−002 1.86e−002	+	**6.67e−002 4.58e−002**	+	**3.41e−002 1.73e−002**	–	5.91e−001 3.60e−001	=	7.66e−002 3.82e−002	+	**2.25e−002 1.35e−002**	+	**6.51e−002 4.47e−002**	+	**1.27e−002 4.24e−003**
F11	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	–	3.66e+001 1.43e+001	–	4.15e+000 4.09e+000	–	1.70e+002 4.18e+001	=	0.00e+000 0.00e+000	–	5.00e+001 1.73e+001	–	7.20e−014 3.32e−014	=	0.00e+000 0.00e+000
F12	7.04e+001 1.04e+001	–	1.92e+002 2.65e+001	+	**5.69e+001 1.39e+001**	–	1.38e+002 3.04e+001	–	2.59e+002 2.27e+001	+	**5.95e+001 1.29e+001**	–	1.54e+002 4.68e+001	–	8.90e+001 2.36e+001	–	1.15e+002 7.18e+000
F13	1.52e+002 2.63e+001	–	2.53e+002 2.89e+001	=	1.63e+002 2.96e+001	–	2.43e+002 5.19e+001	–	3.06e+002 2.85e+001	=	1.43e+002 2.12e+001	–	3.36e+002 5.77e+001	=	1.60e+002 2.76e+001	–	2.10e+002 1.74e+001
F14	1.18e+001 3.62e+000	+	**3.21e−002 2.04e−002**	–	9.93e+003 4.63e+002	–	2.29e+002 9.31e+001	–	7.22e+003 6.21e+002	+	**3.91e−002 2.09e−002**	–	3.07e+002 2.00e+002	–	2.40e+001 1.43e+001	–	7.56e+002 8.46e+001
F15	7.57e+003 4.45e+002	–	9.24e+003 4.79e+002	–	9.72e+003 3.52e+003	+	**6.09e+003 7.82e+002**	–	1.25e+004 5.61e+002	+	**6.84e+003 3.76e+002**	–	8.66e+003 8.90e+002	–	8.98e+003 9.62e+002	=	7.53e+003 2.80e+002
F16	2.20e+000 6.24e−001	=	2.33e+000 6.41e−001	–	3.24e+000 4.43e−001	=	1.56e+000 1.40e+000	–	3.34e+000 3.37e−001	+	**1.29e+000 1.98e−001**	+	**8.00e−002 3.09e−002**	–	2.93e+000 6.82e−001	+	**1.68e+000 1.77e−001**
F17	5.08e+001 1.00e−006	+	**5.08e+001 0.00e+000**	–	1.75e+002 1.27e+001	–	6.56e+001 2.07e+000	–	2.37e+002 2.47e+001	+	**5.08e+001 0.00e+000**	–	6.51e+001 4.57e+000	–	5.08e+001 2.51e−002	–	7.83e+001 1.30e+000
F18	1.57e+002 1.21e+001	–	3.28e+002 4.58e+001	+	**1.07e+002 1.39e+001**	=	1.54e+002 2.02e+001	–	3.86e+002 1.69e+001	+	**1.35e+002 1.39e+001**	–	1.88e+002 3.80e+001	–	1.91e+002 4.81e+001	–	1.76e+002 6.28e+000
F19	4.72e+000 5.81e−001	+	**3.39e+000 2.12e−001**	=	5.10e+000 2.06e+000	+	**2.92e+000 2.86e−001**	–	2.14e+001 4.41e+000	+	**2.58e+000 2.72e−001**	–	5.23e+000 9.86e−001	+	**2.05e+000 3.23e−001**	–	7.28e+000 4.29e−001
F20	1.91e+001 7.46e−001	–	2.15e+001 3.57e−001	–	2.23e+001 8.56e−001	–	2.18e+001 7.29e−001	–	2.08e+001 3.81e−001	=	1.93e+001 8.38e−001	=	1.93e+001 8.49e−001	–	2.07e+001 8.48e−001	=	1.91e+001 3.22e−001
F21	7.91e+002 3.82e+002	+	**4.67e+002 4.18e+002**	=	6.40e+002 4.34e+002	+	**5.21e+002 4.10e+002**	–	9.70e+002 1.45e+002	=	8.83e+002 3.36e+002	=	8.52e+002 3.55e+002	=	6.02e+002 4.44e+002	+	**2.00e+002 0.00e+000**
F22	2.87e+001 1.43e+001	=	3.86e+001 2.44e+001	–	1.06e+004 4.85e+002	–	6.65e+002 1.57e+002	–	7.67e+003 8.64e+002	+	**1.41e+001 8.75e+000**	–	3.28e+002 2.10e+002	–	3.92e+001 3.25e+001	–	2.35e+003 3.25e+002
F23	8.67e+003 5.29e+002	–	9.80e+003 5.36e+002	=	9.11e+003 3.29e+003	+	**7.83e+003 1.95e+003**	–	1.15e+004 1.37e+003	+	**7.58e+003 6.99e+002**	–	9.45e+003 1.95e+003	–	9.28e+003 9.39e+002	–	9.47e+003 3.91e+002
F24	2.32e+002 1.02e+001	–	3.33e+002 1.74e+001	–	3.84e+002 9.85e+000	–	3.32e+002 6.66e+000	–	2.79e+002 1.47e+001	=	2.34e+002 9.97e+000	–	2.99e+002 1.16e+001	–	3.11e+002 1.39e+001	–	3.97e+002 2.11e+000
F25	3.66e+002 2.18e+001	=	3.64e+002 2.28e+001	–	3.84e+002 3.94e+000	=	3.61e+002 7.18e+000	+	**3.48e+002 1.45e+001**	+	**3.46e+002 2.77e+001**	=	3.69e+002 1.48e+001	+	**3.34e+002 1.42e+001**	–	3.78e+002 2.56e+000
F26	2.97e+002 8.51e+001	=	3.38e+002 1.21e+002	–	4.02e+002 5.54e+001	+	**2.00e+002 3.43e−002**	–	3.52e+002 5.26e+001	=	2.60e+002 8.22e+001	=	2.91e+002 9.99e+001	=	2.89e+002 1.11e+002	+	**2.00e+002 5.09e−002**
F27	8.26e+002 2.36e+002	–	1.72e+003 1.33e+002	–	2.14e+003 4.84e+001	–	1.55e+003 9.15e+001	–	1.13e+003 1.17e+002	–	9.91e+002 3.23e+002	–	1.20e+003 1.66e+002	–	1.49e+003 1.46e+002	–	2.15e+003 3.14e+001
F28	4.99e+002 5.44e+002	=	4.00e+002 0.00e+000	=	7.72e+002 9.70e+002	=	4.00e+002 0.00e+000	+	4.00e+002 1.60e+000	=	4.00e+002 5.67e−006	=	1.23e+003 1.41e+003	=	7.01e+002 9.17e+002	=	4.00e+002 0.00e+000
+/=/–	–		5/10/13		4/9/15		7/8/13		3/2/23		14/13/1		6/9/13		4/6/18		7/6/15

Table 11
Results of the multi-problem Wilcoxon's test for SL-JADE versus the up-to-date DE variants for CEC 2013 functions at 30D and 50D.

Algorithm at 30D	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
b6e6rl	311.5	94.5	1.31E−02	Yes	Yes
DE_APC	364.0	42.0	2.35E−04	Yes	Yes
DEcfbLS	287.0	91.0	1.80E−02	Yes	Yes
PVADE	290.5	87.5	1.43E−02	Yes	Yes
SHADE	151.5	254.5	1.00E+00	Yes[a]	Yes[a]
SMADE	291.5	86.5	1.33E−02	Yes	Yes
SPSRDEMMS	335.0	71.0	2.55E−03	Yes	Yes
TLBSaDE	251.0	155.0	2.69E−01	No	No
Algorithm at 50D	**$R+$**	**$R-$**	p-value	$\alpha = 0.05$	$\alpha = 0.1$
b6e6rl	311.5	94.5	1.28E−02	Yes	Yes
DE_APC	321.5	56.5	1.40E−03	Yes	Yes
DEcfbLS	245.0	161.0	3.33E−01	No	No
PVADE	370.5	35.5	1.25E−04	Yes	Yes
SHADE	95.0	311.0	1.00E+00	Yes[a]	Yes[a]
SMADE	280.5	97.5	2.71E−02	Yes	Yes
SPSRDEMMS	323.0	83.0	5.93E−03	Yes	Yes
TLBSaDE	272.0	134.0	1.14E−01	No	No

[a] Means the competitor is significantly better than SL-DE based on the statistical analysis.

Table 12
Average ranking values (ARV) by Friedman test and final rank values of SL-JADE and the up-to-date DE variants for CEC 2013 functions at 30D and 50D.

$D = 30$			$D = 50$		
Algorithm	ARV	Final rank	Algorithm	ARV	Final rank
SHADE	3.16	1	SHADE	2.73	1
SL-JADE	**3.57**	**2**	**SL-JADE**	**3.75**	**2**
TLBSaDE	4.04	3	TLBSaDE	4.84	3
SMADE	5.11	4	DEcfbLS	4.93	4
b6e6rl	5.30	5	SMADE	4.95	5
DEcfbLS	5.43	7	b6e6rl	5.29	6
SPSRDEMMS	5.43	7	SPSRDEMMS	5.30	7
DE-APC	6.46	8	DE-APC	6.23	8
PVADE	6.50	9	PVADE	6.98	9

than the compared algorithm on the corresponding function, respectively. The summarized results of the statistical tests are also shown in the last row of the tables as "$+/=/-$".

4.2. Comparison with classical DE algorithms

In this section, SL-DE is compared with six DE algorithms (see Eqs. (2)–(7)) to show the benefits of ASL for the classical DE mutation strategies. The results for the functions from CEC2013 at 30D and 50D are shown in Tables 2 and 3, respectively. The better values in terms of mean solution error compared between SL-DE and its corresponding DE algorithm are highlighted in boldface. Furthermore, convergence graphs for some test functions are plotted in Figs. 3 and 4.

For the results of functions at 30D, Table 2 shows that SL-DE provides significantly better results when compared with the corresponding DE algorithms in most functions. Specifically, for DE/rand/1 and DE/rand/2, SL-DE is significantly better on 12 and 21 functions, respectively. For DE/best/1, DE/best/2, DE/current-to-best/1, and DE/rand-to-best/1, SL-DE obtains significant improvements on 24, 17, 22, and 23 functions, respectively. Additionally, the results of multi-problem Wilcoxon signed-rank tests are shown in Table 4, where $R+$ and $R-$ mean the sum of ranks for the functions where SL-DE performs better than or worse than its competitor, respectively. From Table 4, it is clear that SL-DE can obtain higher $R+$ values than $R-$ values in all cases. In most cases, the p values are less than 0.05. These results indicate that SL-DE is able to significantly improve the performance of most classical DE algorithms for the functions at 30D.

For the results of functions at 50D, Table 3 also shows that SL-DE is consistently superior to most corresponding DE algorithms. Based on the statistical results for the functions at 50D in Table 4, it is clear that SL-DE can significantly enhance the performance of the corresponding DE algorithms in most cases except for DE/rand/1. In the case of DE/rand/1, SL-DE also obtains higher $R+$ than $R-$ values.

Figs. 3 and 4 also show that SL-DE is better than the corresponding DE algorithms in terms of the convergence rate for most selected functions.

Table 13
Mean and standard deviation of the best error values obtained by SL-SHADE and SHADE on CEC 2013 functions at 30D and 50D.

30D				50D		
	SHADE		SL-SHADE	SHADE		SL-SHADE
F1	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F2	1.51e+04 1.05e+04	=	**1.15e+04 1.09e+04**	3.39e+04 1.33e+04	=	**2.73e+04 1.12e+04**
F3	**1.08e+05 5.58e+05**	=	1.12e+05 4.16e+05	**1.70e+06 2.71e+06**	=	4.92e+06 8.55e+06
F4	2.31e-03 4.06e-03	+	**1.18e-03 2.88e-03**	6.29e-03 9.18e-03	+	**3.12e-03 2.72e-03**
F5	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F6	1.01e+00 4.77e+00	+	**0.00e+00 0.00e+00**	4.34e+01 2.49e-13	=	**4.28e+01 5.54e+00**
F7	2.81e+00 2.76e+00	=	**1.62e+00 1.20e+00**	1.98e+01 8.62e+00	=	**1.64e+01 8.28e+00**
F8	2.08e+01 1.90e-01	=	**2.07e+01 1.63e-01**	2.09e+01 1.37e-01	=	2.09e+01 1.37e-01
F9	2.75e+01 1.72e+00	=	**2.73e+01 2.05e+00**	5.57e+01 1.98e+00	=	**5.51e+01 1.87e+00**
F10	1.36e-01 8.09e-02	+	**8.10e-02 5.02e-02**	1.35e-01 7.09e-02	=	**1.29e-01 5.81e-02**
F11	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F12	2.38e+01 3.80e+00	+	**2.12e+01 3.52e+00**	5.96e+01 1.01e+01	=	**5.51e+01 8.89e+00**
F13	4.71e+01 1.39e+01	=	**4.63e+01 1.32e+01**	1.48e+02 2.07e+01	+	**1.36e+02 2.03e+01**
F14	**2.43e-02 2.64e-02**	=	3.68e-02 2.44e-02	**2.46e-02 1.75e-02**	−	6.25e-02 2.30e-02
F15	3.22e+03 2.73e+02	=	**3.09e+03 2.94e+02**	6.77e+03 5.02e+02	=	**6.74e+03 3.28e+02**
F16	9.45e-01 1.41e-01	=	**9.22e-01 2.41e-01**	1.34e+00 1.45e-01	+	**1.20e+00 2.26e-01**
F17	3.04e+01 2.57e-14	=	**3.04e+01 2.49e-14**	**5.08e+01 4.00e-14**	=	5.08e+01 4.80e-14
F18	**7.07e+01 6.19e+00**	=	7.13e+01 5.86e+00	1.37e+02 7.12e+00	=	**1.34e+02 1.26e+01**
F19	1.38e+00 1.26e-01	=	**1.36e+00 1.23e-01**	**2.70e+00 3.81e-01**	=	2.81e+00 4.19e-01
F20	1.05e+01 6.53e-01	=	**1.03e+01 6.45e-01**	**1.93e+01 6.35e-01**	=	1.95e+01 5.77e-01
F21	3.21e+02 5.78e+01	=	**2.92e+02 7.32e+01**	9.21e+02 3.39e+02	=	**8.00e+02 3.81e+02**
F22	**1.02e+02 2.21e+01**	=	1.07e+02 1.98e+00	**1.13e+01 1.04e+00**	=	1.31e+01 7.48e+00
F23	**3.37e+03 4.58e+02**	=	3.40e+03 3.76e+02	7.63e+03 5.57e+02	+	**7.16e+03 6.12e+02**
F24	**2.04e+02 3.13e+00**	=	2.05e+02 4.07e+00	**2.32e+02 9.65e+00**	=	2.35e+02 8.31e+00
F25	2.62e+02 2.05e+01	+	**2.51e+02 1.33e+01**	3.43e+02 2.95e+01	+	**3.07e+02 2.66e+01**
F26	**2.07e+02 2.61e+01**	=	2.14e+02 3.54e+01	2.82e+02 7.96e+01	=	**2.77e+02 6.98e+01**
F27	3.93e+02 1.43e+02	=	**3.47e+02 4.03e+01**	8.09e+02 2.70e+02	=	**7.50e+02 1.74e+02**
F28	3.00e+02 0.00e+00	=	3.00e+02 0.00e+00	4.98e+02 5.26e+02	=	**4.97e+02 5.23e+02**
+/=/−	-		5/23/0	-		5/22/1

With respect to the features of benchmark functions, a close inspection of the results in Tables 2–4 and Figs. 3–4 indicates that SL-DE can obtain better results in most cases when solving different kinds of functions. For the unimodal functions ($F1-F5$), SL-DE significantly improves the performance of the classical DE algorithms in 46 out of 60 cases and is outperformed in 6 cases. For the multi-modal functions and the composition function ($F6-F28$), SL-DE can obtain significantly better results on $F11$, $F12$, $F13$, $F17$, $F18$, $F19$ and $F20$.

As these results show, ASL is able to bring benefits to the classical DE algorithms for most test functions. The reason may be in that ASL can dynamically extract the neighborhood relationship information of individuals and allow individuals to interact with the promising neighbors during the evolutionary process. In this way, the population information can be utilized effectively to guide the search of DE.

4.3. Comparison with advanced DE variants

To evaluate the effectiveness of ASL for the advanced DE mutation strategies, ASL is applied to five DE variants: CoDE [44], JADE [49], jDE [2], ODE [35] and SaDE [33]. These advanced DE variants can be classified into three different kinds of modifications: ensemble strategies (CoDE and SaDE), self-adaptive control parameters (JADE and jDE) and new mutation operators (ODE and JADE). The results on the 28 benchmark functions from CEC2013 at 30D and 50D are presented in Tables 5 and 6, respectively. The convergence graphs for some test functions are also plotted in Figs. 5 and 6.

From Table 5, it is clear that SL-DE exhibits potential improvements for most advanced DE variants. Specifically, for the DE variants with ensemble strategies, SL-CoDE is significantly better than CoDE on 15 functions, while SL-SaDE significantly outperforms SaDE on 13 functions. For the DE variants with self-adaptive control parameters, SL-JADE and SL-jDE significantly outperform JADE and jDE on 13 and 13 functions, respectively. For ODE, SL-ODE is significantly better on 11 functions.

In Table 6, SL-DE can also significantly improve the performance of most of the functions at 50D. For CoDE and SaDE, SL-DE is significantly better on 16 and 13 functions, respectively. For JADE and jDE, SL-DE can obtain significantly better results on 10 and 12 functions, respectively. For ODE, SL-DE is significantly better on 11 functions.

Figs. 5 and 6 show that SL-DE is better than the advanced DE variants in terms of the convergence speed for most selected functions.

Furthermore, the multi-problem Wilcoxon signed rank tests at $\alpha = 0.05$ and $\alpha = 0.1$ are carried out, and the results are shown in Table 7. It is clear that SL-DE can obtain the higher $R+$ values than $R-$ values in all cases. According to the statistics of $p = 0.05$ and 0.1, significant differences can be observed in two and three cases, respectively, for the functions

Table 14
Mean and standard deviation of the best error values obtained by SL-JADE and the other non-DE EAs on CEC 2013 functions at 30*D*.

	SL-JADE		CDASA		CMAES-RIS		f_k-PSO		MVMO-SH		NBIPOPaCMA		SPSOABC		TPC-GA
F1	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	+	−1.40e+003 0.00e+000	=	0.00e+000 0.00e+000
F2	1.50e+004 1.12e+004	−	8.57e+005 3.98e+005	+	0.00e+000 0.00e+000	−	1.43e+006 8.01e+005	+	1.38e−005 1.06e−005	+	0.00e+000 0.00e+000	−	7.77e+005 1.51e+006	−	2.20e+005 1.53e+005
F3	3.50e+005 7.84e+005	−	3.45e+007 4.70e+007	=	3.54e+003 1.43e+004	−	2.17e+008 2.55e+008	+	1.94e−003 6.95e−003	+	9.10e−005 4.94e−004	−	5.61e+007 8.84e+007	−	2.62e+007 3.81e+007
F4	1.32e−002 6.56e−002	−	2.41e−001 7.02e−001	+	0.00e+000 0.00e+000	−	4.68e+002 2.20e+002	+	3.54e−006 1.41e−006	+	0.00e+000 0.00e+000	−	5.07e+003 2.21e+003	−	1.74e+001 2.70e+001
F5	0.00e+000 0.00e+000	−	7.88e−006 3.34e−006	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	+	−1.00e+003 0.00e+000	=	0.00e+000 0.00e+000
F6	1.13e+000 4.78e+000	−	3.61e+001 2.72e+001	+	6.33e−004 1.77e−003	−	2.81e+001 1.75e+001	+	0.00e+000 0.00e+000	+	0.00e+000 0.00e+000	+	−8.90e+002 7.59e+000	−	2.35e+001 1.02e+001
F7	2.21e+000 1.90e+000	−	7.41e+001 3.24e+001	−	3.67e+001 2.61e+001	−	6.31e+001 3.65e+001	−	2.39e+001 8.05e+000	−	2.40e+000 6.78e+000	+	−7.48e+002 1.84e+001	−	2.99e+001 2.26e+001
F8	2.09e+001 6.66e−002	=	2.09e+001 9.06e−002	+	2.09e+001 8.88e−002	=	2.09e+001 6.60e−002	=	2.09e+001 1.07e−001	=	2.09e+001 5.11e−002	+	−6.79e+002 5.10e−002	−	2.10e+001 5.41e−002
F9	2.69e+001 1.90e+000	+	2.25e+001 3.62e+000	+	2.37e+001 2.03e+000	+	1.82e+001 2.82e+000	+	1.44e+001 2.67e+000	+	3.34e+000 1.13e+000	+	−5.70e+002 2.37e+000	−	3.73e+001 7.62e+000
F10	7.72e−002 4.34e−002	+	3.51e−002 1.99e−002	+	7.48e−003 5.32e−003	−	2.28e−001 1.40e−001	+	9.86e−004 2.56e−003	+	0.00e+000 0.00e+000	+	−5.00e+002 5.69e−002	=	8.15e−002 3.93e−002
F11	0.00e+000 0.00e+000	−	1.09e+000 1.26e+000	−	2.53e+001 5.45e+000	−	2.35e+001 9.02e+000	−	6.94e+000 2.40e+000	−	3.08e+000 1.12e+000	+	−4.00e+002 0.00e+000	−	2.39e+001 7.55e+000
F12	2.68e+001 4.97e+000	−	1.14e+002 3.17e+001	−	7.67e+001 4.36e+001	−	5.78e+001 1.61e+001	−	3.76e+001 1.37e+001	+	3.08e+000 1.49e+000	+	−2.36e+002 1.64e+001	−	4.21e+001 9.75e+000
F13	5.11e+001 1.29e+001	−	1.81e+002 3.93e+001	−	1.52e+002 5.55e+001	−	1.25e+002 2.10e+001	−	7.13e+001 1.87e+001	+	3.05e+000 1.48e+000	+	−8.51e+001 2.34e+001	−	8.41e+001 1.65e+001
F14	4.92e−001 1.53e−001	−	6.27e+002 2.96e+002	−	7.43e+002 2.29e+002	−	7.09e+002 2.47e+002	−	9.00e+002 4.01e+002	−	7.93e+002 3.95e+002	+	−8.42e+001 6.00e+000	−	9.43e+002 4.13e+002
F15	3.47e+003 2.65e+002	−	3.84e+003 7.04e+002	+	3.05e+003 4.14e+002	=	3.42e+003 5.46e+002	=	3.34e+003 3.04e+002	+	7.80e+002 3.26e+002	−	3.67e+003 3.38e+002	−	3.99e+003 6.15e+002
F16	2.01e+000 6.45e−001	+	3.08e−001 1.43e−001	+	1.06e−001 6.15e−002	+	8.39e−001 2.20e−001	+	3.69e−001 1.29e−001	+	3.87e−001 8.81e−001	−	2.01e+002 2.32e−001	−	2.52e+000 6.23e−001
F17	3.04e+001 0.00e+000	−	3.47e+001 1.55e+000	−	5.47e+001 5.68e+000	−	5.26e+001 7.82e+000	−	4.84e+001 6.35e+000	−	3.46e+001 1.85e+000	−	3.31e+002 1.24e−001	−	5.34e+001 9.64e+000
F18	8.11e+001 7.29e+000	−	1.97e+002 5.66e+001	−	1.87e+002 3.08e+001	+	6.70e+001 8.94e+000	+	5.68e+001 9.17e+000	=	6.57e+001 5.21e+001	−	4.91e+002 7.85e+000	+	6.88e+001 1.37e+001
F19	2.13e+000 2.44e−001	=	2.17e+000 5.75e−001	−	2.87e+000 6.75e−001	−	3.21e+000 1.10e+000	=	2.00e+000 5.19e−001	=	2.23e+000 3.83e−001	−	5.02e+002 5.42e−001	−	3.34e+000 1.11e+000
F20	1.01e+001 6.03e−001	−	1.48e+001 4.40e−001	−	1.42e+001 7.11e−001	−	1.19e+001 7.81e−001	=	1.04e+001 6.38e−001	−	1.29e+001 6.73e−001	−	6.11e+002 8.24e−001	−	1.37e+001 4.97e−001
F21	3.02e+002 6.75e+001	+	2.70e+002 6.78e+001	+	1.87e+002 4.34e+001	=	3.16e+002 9.42e+001	+	2.07e+002 5.21e+001	+	1.93e+002 2.54e+001	−	1.03e+003 8.03e+001	=	2.99e+002 9.16e+001
F22	1.12e+002 2.01e+001	−	4.41e+002 1.80e+002	−	1.22e+003 2.58e+002	−	8.51e+002 3.10e+002	−	9.42e+002 4.53e+002	−	8.32e+002 5.27e+002	−	8.90e+002 3.91e+001	−	1.22e+003 5.79e+002
F23	3.68e+003 3.53e+002	−	5.48e+003 8.46e+002	=	3.96e+003 5.67e+002	=	3.59e+003 6.02e+002	+	3.38e+003 5.16e+002	+	6.97e+002 2.39e+002	−	5.08e+003 5.41e+002	−	4.23e+003 6.69e+002
F24	2.04e+002 3.98e+000	−	2.93e+002 4.05e+001	−	2.61e+002 1.06e+001	−	2.48e+002 8.51e+000	−	2.16e+002 6.77e+000	+	1.68e+002 3.13e+001	−	1.25e+003 1.34e+001	−	2.75e+002 1.89e+001
F25	2.51e+002 3.04e+001	−	3.14e+002 7.16e+000	−	2.80e+002 9.60e+000	=	2.50e+002 8.33e+000	−	2.59e+002 7.34e+000	+	2.20e+002 1.33e+001	−	1.38e+003 9.74e+000	−	2.99e+002 8.80e+000
F26	2.00e+002 1.61e−002	−	2.74e+002 8.07e+001	−	2.00e+002 1.23e−002	−	2.93e+002 7.24e+001	−	2.00e+002 2.26e−004	+	1.61e+002 3.30e+001	−	1.46e+003 7.60e+001	−	3.31e+002 5.33e+001
F27	3.63e+002 6.41e+001	−	1.09e+003 3.19e+002	−	7.41e+002 1.81e+002	−	7.89e+002 7.58e+001	−	5.37e+002 1.05e+002	−	4.57e+002 7.33e+001	−	2.19e+003 1.62e+002	−	1.05e+003 1.95e+002
F28	3.00e+002 0.00e+000	=	3.69e+002 2.94e+002	=	6.47e+002 1.71e+003	=	4.07e+002 3.50e+002	=	3.29e+002 2.01e+002	=	2.60e+002 8.14e+001	−	1.72e+003 2.06e+002	=	3.00e+002 0.00e+000
+/=/−	–		4/4/20		9/5/14		3/8/17		10/8/10		15/6/7		11/0/17		1/5/22

Table 15
Mean and standard deviation of the best error values obtained by SL-JADE and the other non-DE EAs on CEC 2013 functions at 50*D*.

	SL-JADE		CDASA		CMAES-RIS		f_k-PSO		MVMO-SH		NBIPOPaCMA		SPSOABC		TPC-GA
F1	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	+	−1.40e+003 0.00e+000	=	0.00e+000 0.00e+000
F2	5.27e+004 2.68e+004	–	1.77e+006 5.68e+005	+	0.00e+000 0.00e+000	–	2.83e+006 9.21e+005	+	1.04e−003 7.86e−004	+	0.00e+000 0.00e+000	–	5.13e+005 1.53e+005	–	4.75e+005 2.04e+005
F3	3.14e+006 4.83e+006	–	2.34e+008 2.18e+008	+	1.25e+005 3.89e+005	–	1.05e+009 1.06e+009	+	5.76e−004 2.10e−003	+	2.93e+001 1.58e+002	–	8.84e+007 7.86e+007	–	1.09e+008 1.17e+008
F4	7.82e−003 1.22e−002	=	1.36e−002 3.81e−002	+	0.00e+000 0.00e+000	–	5.25e+002 2.13e+002	+	2.20e−006 1.09e−006	+	0.00e+000 0.00e+000	–	3.74e+003 1.03e+003	–	3.59e+000 6.11e+000
F5	0.00e+000 0.00e+000	–	8.99e−006 2.16e−006	–	3.96e−008 1.53e−008	=	0.00e+000 0.00e+000	–	1.85e−008 1.15e−008	=	0.00e+000 0.00e+000	+	−1.00e+003 0.00e+000	=	0.00e+000 0.00e+000
F6	4.34e+001 1.00e−006	–	4.96e+001 1.67e+001	+	9.14e+000 1.51e+001	–	5.59e+001 2.42e+001	=	3.77e+001 1.50e+001	+	0.00e+000 0.00e+000	+	−8.62e+002 2.05e+001	=	4.51e+001 1.00e+001
F7	2.07e+001 7.74e+000	–	1.06e+002 1.85e+001	–	4.49e+001 2.34e+001	–	7.68e+001 2.33e+001	–	4.34e+001 7.53e+000	+	5.21e+000 5.61e+000	+	−7.25e+002 1.14e+001	–	4.12e+001 2.15e+001
F8	2.11e+001 4.42e−002	=	2.11e+001 5.91e−002	+	2.10e+001 6.69e−002	=	2.11e+001 4.96e−002	+	2.11e+001 1.64e−001	=	2.11e+001 4.81e−002	+	−6.79e+002 3.07e−002	–	2.12e+001 4.40e−002
F9	5.41e+001 2.53e+000	+	4.65e+001 4.88e+000	+	4.72e+001 2.92e+000	+	3.85e+001 5.48e+000	+	3.24e+001 4.28e+000	+	7.26e+000 1.93e+000	+	−5.42e+002 4.42e+000	–	7.37e+001 1.53e+000
F10	9.54e−002 5.30e−002	+	4.29e−002 2.52e−002	+	9.03e−003 5.06e−003	–	2.27e−001 1.45e−001	+	4.93e−004 1.88e−003	+	0.00e+000 0.00e+000	+	−5.00e+002 8.75e−002	=	1.03e−001 8.38e−002
F11	0.00e+000 0.00e+000	–	2.12e+000 1.56e+000	–	5.48e+001 1.57e+001	–	8.82e+001 1.85e+001	–	3.64e+001 1.11e+001	–	5.54e+000 2.82e+000	+	−4.00e+002 6.08e−002	–	5.50e+001 2.03e+001
F12	7.04e+001 1.04e+001	–	2.51e+002 4.60e+001	–	2.73e+002 9.86e+001	–	1.43e+002 3.02e+001	–	9.13e+001 2.30e+001	+	5.82e+000 2.63e+000	+	−1.24e+002 3.60e+001	–	9.55e+001 2.15e+001
F13	1.52e+002 2.63e+001	–	4.24e+002 4.51e+001	–	4.53e+002 9.55e+001	–	2.78e+002 4.77e+001	–	1.81e+002 3.64e+001	+	8.59e+000 6.36e+000	+	8.17e+001 4.18e+001	–	1.79e+002 4.69e+001
F14	1.18e+001 3.62e+000	–	1.08e+003 4.20e+002	–	1.51e+003 3.47e+002	–	1.96e+003 4.81e+002	–	2.35e+003 7.68e+002	–	1.38e+003 5.71e+002	+	−7.44e+001 7.87e+000	–	2.39e+003 7.50e+002
F15	7.57e+003 4.45e+002	=	7.29e+003 7.79e+002	+	6.27e+003 7.22e+002	+	6.73e+003 7.79e+002	+	6.22e+003 5.54e+002	+	1.50e+003 4.68e+002	=	7.53e+003 5.15e+002	–	9.31e+003 2.69e+003
F16	2.20e+000 6.24e−001	+	4.92e−001 1.51e−001	+	8.94e−002 5.08e−002	+	1.21e+000 2.83e−001	+	1.12e+000 2.11e−001	+	6.37e−001 1.34e+000	–	2.01e+002 1.97e−001	–	3.38e+000 4.37e−001
F17	5.08e+001 1.00e−006	–	5.91e+001 1.53e+000	–	1.01e+002 1.11e+001	–	1.18e+002 1.77e+001	–	1.01e+002 9.33e+000	–	5.79e+001 3.11e+000	–	3.52e+002 2.31e−001	–	1.16e+002 2.22e+001
F18	1.57e+002 1.21e+001	–	4.43e+002 1.15e+002	–	4.26e+002 5.32e+001	+	1.32e+002 1.71e+001	+	1.06e+002 1.46e+001	=	1.42e+002 1.14e+002	–	6.17e+002 2.31e+001	=	1.77e+002 1.08e+002
F19	4.72e+000 5.81e−001	+	3.75e+000 6.95e−001	=	5.03e+000 8.71e−001	–	8.13e+000 2.65e+000	+	4.01e+000 7.76e−001	=	4.53e+000 5.51e−001	–	5.05e+002 1.66e+000	–	8.86e+000 3.21e+000
F20	1.91e+001 7.46e−001	–	2.41e+001 1.06e+000	–	2.43e+001 6.35e−001	–	2.08e+001 1.08e+000	=	1.95e+001 6.45e−001	–	2.27e+001 9.57e−001	–	6.20e+002 6.09e−001	–	2.34e+001 8.01e−001
F21	7.91e+002 3.82e+002	=	6.79e+002 3.74e+002	+	2.85e+002 2.20e+002	=	8.94e+002 3.08e+002	+	2.33e+002 1.55e+002	+	1.97e+002 1.83e+001	–	1.55e+003 3.26e+002	=	8.34e+002 3.81e+002
F22	2.87e+001 1.43e+001	–	7.01e+002 2.63e+002	–	2.45e+003 3.62e+002	–	2.12e+003 6.39e+002	–	2.70e+003 8.65e+002	–	1.42e+003 6.07e+002	–	8.51e+002 1.28e+001	–	3.28e+003 1.64e+003
F23	8.67e+003 5.29e+002	–	1.04e+004 1.28e+003	=	8.34e+003 9.41e+002	+	7.35e+003 9.46e+002	+	6.82e+003 6.07e+002	+	1.59e+003 7.05e+002	–	9.95e+003 7.24e+002	=	9.59e+003 3.18e+003
F24	2.32e+002 1.02e+001	–	3.70e+002 7.21e+001	–	3.21e+002 2.05e+001	–	3.00e+002 1.66e+001	–	2.44e+002 8.22e+000	=	2.40e+002 2.19e+001	–	1.31e+003 2.33e+001	–	3.74e+002 2.13e+001
F25	3.66e+002 2.18e+001	–	4.04e+002 6.14e+000	=	3.63e+002 1.13e+001	+	2.99e+002 1.50e+001	+	3.23e+002 1.24e+001	+	2.47e+002 6.06e+000	–	1.46e+003 1.84e+001	–	3.85e+002 4.30e+000
F26	2.97e+002 8.51e+001	=	3.30e+002 1.19e+002	+	2.00e+002 3.63e−002	–	3.90e+002 3.77e+001	+	2.06e+002 3.25e+001	+	1.93e+002 1.83e+001	–	1.59e+003 9.96e+001	–	4.25e+002 3.42e+001
F27	8.26e+002 2.36e+002	–	1.53e+003 2.49e+002	–	1.28e+003 1.73e+002	–	1.33e+003 1.31e+002	–	1.03e+003 1.33e+002	=	7.55e+002 1.16e+002	–	2.93e+003 1.97e+002	–	2.02e+003 2.12e+002
F28	4.99e+002 5.44e+002	=	1.05e+003 1.32e+003	=	1.10e+003 1.29e+003	–	2.07e+003 1.59e+003	=	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000	–	2.25e+003 1.16e+003	=	4.00e+002 0.00e+000
+/=/−	-		4/7/17		11/5/12		6/4/18		14/4/10		15/8/5		11/1/16		0/8/20

Table 16
Results of the multi-problem Wilcoxon's test for SL-JADE versus the non-DE EAs for CEC 2013 functions at 30*D* and 50*D*.

Algorithm at 30*D*	*R+*	*R-*	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
CDASA	369.5	36.5	1.43E-04	Yes	Yes
CMAES-RIS	264	142	1.61E-01	No	No
f_k-PSO	313.5	64.5	2.67E-03	Yes	Yes
MVMO-SH	208	198	9.00E-01	No	No
NBIPOPaCMA	107.5	270.5	1.00E+00	Yes[a]	Yes[a]
SPSOABC	284.5	121.5	6.11E-02	Yes	Yes
TPC-GA	359.5	18.5	4.00E-05	Yes	Yes
Algorithm at 50*D*	***R+***	***R-***	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
CDASA	339.5	66.5	1.81E-03	Yes	Yes
CMAES-RIS	205.0	173.0	6.92E-01	No	No
f_k-PSO	301.5	76.5	6.63E-03	Yes	Yes
MVMO-SH	162.5	243.5	1.00E+00	Yes[a]	Yes[a]
NBIPOPaCMA	80.5	297.5	1.00E+00	Yes[a]	Yes[a]
SPSOABC	284.0	122.0	6.35E-02	No	Yes
TPC-GA	385.5	20.5	3.10E-05	Yes	Yes

[a] means the competitor is significantly better than SL-DE based on the statistical analysis.

Table 17
Average ranking values (ARV) by Friedman test and final rank values of SL-JADE and the other non-DE EAs for CEC 2013 functions at 30*D* and 50*D*.

$D = 30$			$D = 50$		
Algorithm	ARV	Final Rank	Algorithm	ARV	Final Rank
NBIPOPaCMA	2.61	1	NBIPOPaCMA	2.45	1
SL-JADE	**3.43**	**2**	MVMO-SH	3.50	2
MVMO-SH	3.46	3	**SL-JADE**	**3.77**	**3**
CMAES-RIS	4.63	4	CMAES-RIS	4.50	4
SPSOABC	4.93	5	SPSOABC	4.82	5
f_k-PSO	5.34	6	CDASA	5.32	6
CDASA	5.61	7	f_k-PSO	5.59	7
TPC-GA	6.00	8	TPC-GA	6.05	8

at 30*D*, and three and four cases, respectively, for the functions at 50*D*. These results indicate that SL-DE is better than its corresponding advanced DE variant for all functions overall, according to the statistical analysis.

With respect to the features of benchmark functions, a close inspection of the results in Tables 5 and 6 indicates that SL-DE is able to obtain better results than the corresponding advanced DE variants for solving most unimodal and multimodal functions. For unimodal functions ($F1 - F5$), SL-DE is significantly better than the advanced DE variants in 27 out of 50 cases and is outperformed in 5 cases. For multi-modal functions and the composition function ($F6 - F28$), SL-DE also obtains better results in most cases, especially in the cases of CoDE and SaDE.

In general, the overall results of Tables 5–7 and Figs. 5 and 6 clearly show that SL-DE can improve the performance of the advanced DE variants. Furthermore, SL-DE is proven to be an efficient framework to further improve the performance of newly proposed DE variants that combine with different kinds of modifications.

4.4. Comparison with up-to-date DE variants

As the results show above, it is clear that SL-DE can improve the performance of most classical DE algorithms and advanced DE variants. According to the Friedman test, we can obtain the rankings of different SL-DE variants for all the test functions. Table 8 shows the final rankings of all SL-DE variants. Overall, SL-JADE and SL-jDE obtain the first two ranks in both cases. SL-ODE obtains the third rank for the functions at 30*D*, while SL-SaDE obtains the third rank for the functions at 50*D*.

To provide more comparisons, the best overall performing SL-DE variant, SL-JADE, is selected for further comparison with eight up-to-date DE variants from the CEC 2013 competition [25]. The eight DE variants are as follows:

(1) b6e6rl: DE with twelve combinations of competing mutation strategies and control parameter settings [42];
(2) DE-APC: DE with an automatic parameter configuration [18];
(3) DEcfbLS: DE with a concurrent fitness-based local search [32];
(4) PVADE: DE with a population's variance-based adaptive strategy [15];
(5) SHADE: DE with a success-history-based parameter adaption strategy [41];
(6) SMADE: DE with a super-fit multi-criteria strategy [7];

Table 18
Mean and standard deviation of the best error values obtained by SL-DE and the DE variants with different population topologies on CEC 2013 functions at 30*D*. In addition, average ranking values (ARV) by Friedman test and final rank values of different DE variants are also shown in the table.

Func.	ring-CoDE	cell-CoDE	sw-CoDE	SL-CoDE	ring-JADE	cell-JADE	sw-JADE	SL-JADE
F1	6.31e−03 1.60e−03	7.21e−03 1.39e−03	1.23e−02 3.05e−03	**6.90e−05 1.75e−05**	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F2	9.27e+07 1.94e+07	9.43e+07 1.81e+07	1.01e+08 1.50e+07	9.49e+07 2.26e+07	1.65e+05 2.08e+05	1.00e+05 1.23e+05	1.33e+05 1.74e+05	**1.50e+04 1.12e+04**
F3	**3.86e+10 1.49e+10**	**4.25e+10 1.36e+10**	**6.07e+10 4.37e+10**	1.67e+11 1.06e+11	2.33e+06 3.44e+06	3.06e+06 4.51e+06	1.91e+06 4.69e+06	**3.50e+05 7.84e+05**
F4	7.00e+04 7.87e+03	7.27e+04 7.95e+03	7.50e+04 1.09e+04	6.69e+04 9.35e+03	3.38e+03 3.11e+03	1.80e+03 2.73e+03	5.04e+03 3.07e+03	**1.32e−02 6.56e−02**
F5	5.81e−02 9.68e−03	6.59e−02 7.95e−03	8.96e−02 1.20e−02	**4.12e−03 8.45e−04**	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F6	1.48e+02 1.34e+01	1.45e+02 1.73e+01	1.56e+02 1.45e+01	**1.07e+02 1.80e+01**	4.79e+00 4.70e+00	1.02e+01 4.58e+00	1.13e+01 1.20e+01	**1.13e+00 4.78e+00**
F7	**1.92e+02 3.34e+01**	**1.95e+02 3.88e+01**	**2.38e+02 6.88e+01**	3.24e+02 8.79e+01	5.26e+00 4.90e+00	4.73e+00 3.55e+00	3.96e+00 3.07e+00	**2.21e+00 1.90e+00**
F8	2.10e+01 5.01e−02	2.09e+01 5.66e−02	2.10e+01 4.92e−02	2.09e+01 6.94e−02	2.09e+01 5.29e−02	2.09e+01 5.53e−02	2.09e+01 4.85e−02	2.09e+01 6.66e−02
F9	3.66e+01 1.49e+00	3.70e+01 1.50e+00	3.73e+01 1.47e+00	3.70e+01 1.43e+00	**1.71e+01 4.07e+00**	**1.57e+01 3.92e+00**	**1.34e+01 2.61e+00**	2.69e+01 1.90e+00
F10	2.93e+02 5.81e+01	2.87e+02 5.18e+01	3.89e+02 6.78e+01	**2.22e+02 3.80e+01**	1.30e−01 5.72e−02	1.22e−01 4.23e−02	1.17e−01 5.41e−02	**7.72e−02 4.34e−02**
F11	3.27e+01 3.15e+00	3.38e+01 3.07e+00	3.53e+01 3.19e+00	**2.42e+01 1.66e+00**	3.02e+00 1.98e+00	2.55e+00 2.38e+00	1.69e+00 1.66e+00	**0.00e+00 0.00e+00**
F12	2.43e+02 1.56e+01	2.46e+02 1.21e+01	2.46e+02 1.74e+01	**2.33e+02 1.37e+01**	2.42e+01 7.85e+00	2.45e+01 7.92e+00	2.63e+01 7.55e+00	2.68e+01 4.97e+00
F13	2.53e+02 1.00e+01	2.46e+02 1.83e+01	2.54e+02 1.43e+01	**2.39e+02 1.39e+01**	5.91e+01 1.84e+01	5.26e+01 2.25e+01	5.28e+01 2.24e+01	5.11e+01 1.29e+01
F14	2.45e+03 2.01e+02	2.40e+03 2.76e+02	2.53e+03 2.15e+02	**2.21e+03 2.45e+02**	1.05e+02 9.06e+01	8.46e+01 9.86e+01	4.99e+01 5.54e+01	**4.92e−01 1.53e−01**
F15	7.39e+03 2.44e+02	7.42e+03 3.37e+02	7.48e+03 2.76e+02	7.38e+03 2.90e+02	4.35e+03 1.85e+03	4.85e+03 2.04e+03	4.79e+03 1.98e+03	3.47e+03 2.65e+02
F16	2.43e+00 2.63e−01	2.35e+00 3.52e−01	2.46e+00 2.66e−01	2.39e+00 3.29e−01	2.37e+00 4.84e−01	2.28e+00 5.41e−01	2.39e+00 4.60e−01	**2.01e+00 6.45e−01**
F17	6.85e+01 3.54e+00	7.05e+01 3.38e+00	7.18e+01 2.86e+00	**5.86e+01 2.40e+00**	3.32e+01 1.61e+00	3.24e+01 8.45e−01	3.16e+01 9.25e−01	**3.04e+01 0.00e+00**
F18	2.81e+02 1.27e+01	2.75e+02 1.54e+01	2.87e+02 1.35e+01	**2.70e+02 9.70e+00**	1.24e+02 6.29e+01	9.88e+01 5.87e+01	1.09e+02 5.98e+01	**8.11e+01 7.29e+00**
F19	1.23e+01 9.33e−01	1.25e+01 1.10e+00	1.22e+01 1.23e+00	**1.09e+01 1.07e+00**	**1.80e+00 3.96e−01**	**1.82e+00 3.50e−01**	**1.96e+00 2.76e−01**	2.13e+00 2.44e−01
F20	1.41e+01 2.51e−01	1.40e+01 3.45e−01	1.40e+01 2.96e−01	**1.40e+01 2.39e−01**	9.88e+00 6.74e−01	**9.70e+00 4.62e−01**	**9.52e+00 6.70e−01**	1.01e+01 6.03e−01
F21	4.27e+02 5.12e+01	4.52e+02 7.98e+00	4.99e+02 4.33e+01	**3.62e+02 1.00e+02**	3.14e+02 6.62e+01	3.11e+02 6.94e+01	3.02e+02 8.38e+01	3.02e+02 6.75e+01
F22	3.35e+03 2.91e+02	3.25e+03 2.79e+02	3.33e+03 3.26e+02	3.27e+03 3.00e+02	1.62e+02 4.44e+01	1.73e+02 5.83e+01	1.45e+02 2.60e+01	**1.12e+02 2.01e+01**
F23	7.80e+03 2.55e+02	7.89e+03 3.02e+02	7.82e+03 3.75e+02	7.88e+03 2.88e+02	3.75e+03 1.36e+03	3.83e+03 1.42e+03	4.08e+03 1.64e+03	3.68e+03 3.53e+02
F24	3.01e+02 2.44e+00	3.01e+02 2.47e+00	3.01e+02 3.32e+00	3.01e+02 2.47e+00	2.04e+02 5.24e+00	2.05e+02 3.60e+00	**2.02e+02 2.35e+00**	2.04e+02 3.98e+00
F25	3.00e+02 2.91e+00	2.98e+02 2.55e+00	2.98e+02 2.48e+00	2.99e+02 3.81e+00	2.53e+02 1.42e+01	2.50e+02 1.46e+01	2.54e+02 1.19e+01	2.51e+02 3.04e+01
F26	2.11e+02 2.05e+00	2.12e+02 3.05e+00	2.12e+02 2.83e+00	**2.09e+02 2.02e+00**	2.28e+02 4.78e+01	2.11e+02 3.39e+01	2.31e+02 4.77e+01	**2.00e+02 1.61e−02**
F27	1.30e+03 2.94e+01	1.30e+03 2.63e+01	1.30e+03 3.99e+01	1.31e+03 2.74e+01	3.61e+02 6.57e+01	3.50e+02 6.02e+01	**3.33e+02 3.42e+01**	3.63e+02 6.41e+01
F28	3.41e+02 8.25e+00	3.47e+02 8.73e+00	3.60e+02 1.06e+01	**3.03e+02 4.46e−01**	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00
ARV	2.36	2.46	3.41	1.77	2.98	2.61	2.55	1.86
Final Rank	2	3	4	1	4	3	2	1

(*continued on next page*)

Table 18 (*continued*)

Func.	ring-jDE	cell-jDE	sw-jDE	SL-jDE	ring-SaDE	cell-SaDE	sw-SaDE	SL-SaDE
F1	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	1.68e−30 9.22e−30	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F2	1.59e+05 8.31e+04	1.63e+05 9.85e+04	1.72e+05 1.52e+05	**6.03e+04 4.08e+04**	1.13e+07 2.52e+06	1.10e+07 2.61e+06	1.17e+07 3.02e+06	**7.10e+06 2.26e+06**
F3	2.16e+06 2.59e+06	2.18e+06 4.18e+06	9.12e+05 1.47e+06	**4.17e+05 1.16e+06**	5.54e+05 1.04e+06	1.20e+06 2.19e+06	1.70e+06 3.87e+06	1.44e+06 2.13e+06
F4	8.38e+01 7.56e+01	9.27e+01 7.54e+01	9.30e+01 7.28e+01	**3.13e−01 5.16e−01**	3.38e+04 5.00e+03	3.39e+04 4.69e+03	3.39e+04 4.43e+03	**1.94e+04 3.44e+03**
F5	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F6	1.51e+01 1.08e+01	1.33e+01 2.67e+00	1.19e+01 3.50e+00	**7.22e+00 4.34e+00**	1.80e+01 1.40e+01	1.32e+01 5.61e+00	2.39e+01 2.13e+01	2.06e+01 2.02e+01
F7	5.05e+00 4.16e+00	**4.05e+00 2.55e+00**	**2.48e+00 2.47e+00**	6.65e+00 4.19e+00	4.03e+01 1.42e+01	4.39e+01 9.85e+00	3.78e+01 1.47e+01	**7.06e+00 7.11e+00**
F8	2.09e+01 6.08e−02	2.09e+01 4.86e−02	2.09e+01 6.68e−02	2.09e+01 5.36e−02	2.09e+01 4.15e−02	2.10e+01 4.52e−02	2.09e+01 4.53e−02	2.09e+01 6.17e−02
F9	2.70e+01 3.48e+00	2.75e+01 2.26e+00	2.81e+01 1.59e+00	**2.26e+01 7.16e+00**	2.80e+01 1.55e+00	2.83e+01 1.32e+00	2.78e+01 1.69e+00	2.83e+01 2.23e+00
F10	5.01e−02 2.73e−02	5.74e−02 3.87e−02	**3.37e−02 2.56e−02**	5.48e−02 2.92e−02	**6.88e−02 4.89e−02**	**1.02e−01 7.62e−02**	**7.97e−02 5.35e−02**	1.51e−01 9.86e−02
F11	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F12	4.91e+01 1.26e+01	4.83e+01 1.05e+01	5.60e+01 8.98e+00	**2.96e+01 8.49e+00**	7.68e+01 8.42e+00	7.55e+01 8.67e+00	7.52e+01 9.36e+00	**5.70e+01 8.52e+00**
F13	7.88e+01 1.95e+01	8.14e+01 1.52e+01	9.26e+01 1.52e+01	**7.01e+01 1.91e+01**	1.10e+02 1.53e+01	1.13e+02 1.06e+01	1.10e+02 1.48e+01	**8.52e+01 1.05e+01**
F14	2.08e−03 6.35e−03	5.85e−10 2.22e−09	1.39e−03 5.28e−03	5.55e−03 1.08e−02	1.73e−01 8.54e−02	1.03e+00 6.15e−01	2.24e+00 1.11e+00	**3.24e−02 5.71e−02**
F15	5.22e+03 3.19e+02	5.19e+03 4.49e+02	5.26e+03 4.79e+02	5.29e+03 3.98e+02	4.88e+03 2.82e+02	4.89e+03 2.70e+02	4.87e+03 3.84e+02	4.90e+03 2.21e+02
F16	2.30e+00 2.74e−01	2.32e+00 2.30e−01	2.39e+00 3.12e−01	2.34e+00 2.60e−01	1.99e+00 2.67e−01	1.93e+00 2.33e−01	1.93e+00 2.04e−01	1.95e+00 2.55e−01
F17	3.04e+01 2.32e−06	3.04e+01 1.17e−06	3.04e+01 6.13e−07	3.04e+01 1.77e−06	3.04e+01 2.94e−03	3.05e+01 1.81e−02	3.05e+01 1.80e−02	**3.04e+01 1.37e−06**
F18	1.51e+02 1.37e+01	1.51e+02 1.56e+01	1.62e+02 1.53e+01	**1.41e+02 1.51e+01**	1.45e+02 9.64e+00	1.46e+02 9.63e+00	1.43e+02 8.05e+00	**1.31e+02 1.11e+01**
F19	1.52e+00 2.09e−01	1.56e+00 1.94e−01	1.62e+00 1.56e−01	1.52e+00 1.89e−01	2.05e+00 3.67e−01	2.15e+00 2.37e−01	2.12e+00 2.52e−01	2.15e+00 2.11e−01
F20	1.16e+01 3.84e−01	1.16e+01 4.05e−01	1.17e+01 3.74e−01	**1.10e+01 5.10e−01**	1.14e+01 4.32e−01	1.14e+01 3.20e−01	1.14e+01 3.00e−01	**1.10e+01 3.33e−01**
F21	**2.55e+02 6.14e+01**	2.82e+02 7.95e+01	3.01e+02 7.72e+01	2.96e+02 7.24e+01	**2.79e+02 5.47e+01**	**2.71e+02 6.38e+01**	**2.80e+02 6.40e+01**	3.24e+02 5.44e+01
F22	**9.91e+01 2.63e+01**	1.06e+02 2.47e+01	1.28e+02 1.71e+01	1.10e+02 3.27e+00	1.49e+02 3.12e+01	1.59e+02 3.52e+01	1.70e+02 3.72e+01	**1.15e+02 1.87e+01**
F23	5.68e+03 5.32e+02	5.48e+03 4.99e+02	5.47e+03 5.49e+02	5.52e+03 3.70e+02	5.37e+03 2.75e+02	5.45e+03 3.37e+02	5.40e+03 4.23e+02	5.25e+03 3.91e+02
F24	2.14e+02 6.47e+00	**2.12e+02 7.11e+00**	**2.07e+02 5.28e+00**	2.16e+02 8.38e+00	2.18e+02 6.08e+00	2.20e+02 6.53e+00	2.18e+02 7.96e+00	**2.07e+02 3.97e+00**
F25	2.78e+02 7.76e+00	2.79e+02 1.16e+01	2.82e+02 9.97e+00	**2.58e+02 6.60e+00**	2.92e+02 5.33e+00	2.91e+02 8.47e+00	2.92e+02 5.94e+00	**2.69e+02 3.69e+01**
F26	2.00e+02 5.29e−03	2.00e+02 3.81e−03	2.00e+02 3.08e−03	**2.00e+02 1.99e−03**	**2.00e+02 9.94e−02**	**2.00e+02 9.60e−02**	**2.00e+02 1.08e−01**	2.07e+02 2.73e+01
F27	5.82e+02 1.66e+02	6.14e+02 1.65e+02	5.87e+02 2.39e+02	**5.26e+02 1.21e+02**	8.15e+02 9.58e+01	8.03e+02 8.45e+01	8.21e+02 8.49e+01	**3.78e+02 1.18e+02**
F28	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00	3.00e+02 0.00e+00
ARV	2.34	2.54	2.89	2.23	2.36	2.84	2.71	2.09
Final Rank	2	3	4	1	2	4	3	1

Table 19
Mean and standard deviation of the best error values obtained by SL-DE and the DE variants with different population topologies on CEC 2013 functions at 50*D*. In addition, ARV by Friedman test and final rank values of different DE variants are also shown in the table.

Func.	ring-CoDE	cell-CoDE	sw-CoDE	SL-CoDE	ring-JADE	cell-JADE	sw-JADE	SL-JADE
F1	2.38e+00 3.58e−01	2.96e+00 4.01e−01	4.33e+00 7.10e−01	**2.00e−01 3.38e−02**	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F2	3.16e+08 4.47e+07	2.97e+08 5.22e+07	3.19e+08 3.44e+07	3.16e+08 4.63e+07	2.48e+05 2.99e+05	2.04e+05 2.41e+05	2.21e+05 2.78e+05	**5.27e+04 2.68e+04**
F3	7.92e+10 8.64e+09	7.60e+10 1.05e+10	8.21e+10 1.18e+10	**7.36e+10 1.81e+10**	1.86e+07 2.62e+07	1.51e+07 1.94e+07	1.37e+07 1.47e+07	**3.14e+06 4.83e+06**
F4	1.20e+05 1.14e+04	1.23e+05 1.05e+04	1.22e+05 1.11e+04	**1.14e+05 1.08e+04**	2.08e+03 2.92e+03	1.92e+03 2.45e+03	1.60e+03 2.31e+03	**7.82e−03 1.22e−02**
F5	3.25e+00 3.67e−01	3.47e+00 3.88e−01	4.05e+00 4.65e−01	**7.48e−01 6.72e−02**	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F6	1.64e+02 2.13e+01	1.87e+02 2.30e+01	2.11e+02 3.09e+01	**5.70e+01 4.54e+00**	4.38e+01 1.44e+00	4.26e+01 4.49e+00	4.34e+01 2.01e−03	4.34e+01 1.00e−06
F7	1.84e+02 1.16e+01	1.82e+02 1.06e+01	1.89e+02 1.40e+01	1.88e+02 2.19e+01	2.30e+01 7.27e+00	1.92e+01 5.77e+00	1.80e+01 7.12e+00	2.07e+01 7.74e+00
F8	2.11e+01 3.95e−02	2.11e+01 4.22e−02	2.11e+01 3.18e−02	2.11e+01 4.36e−02	2.11e+01 4.48e−02	2.11e+01 6.55e−02	2.11e+01 3.38e−02	2.11e+01 4.42e−02
F9	7.03e+01 1.89e+00	7.03e+01 1.82e+00	7.01e+01 1.99e+00	7.05e+01 1.61e+00	**3.96e+01 6.18e+00**	**3.39e+01 6.41e+00**	**2.93e+01 5.34e+00**	5.41e+01 2.53e+00
F10	1.56e+03 1.80e+02	1.55e+03 1.68e+02	1.78e+03 1.47e+02	**1.40e+03 1.41e+02**	1.92e−01 8.68e−02	1.55e−01 8.46e−02	1.66e−01 9.03e−02	**9.54e−02 5.30e−02**
F11	1.48e+02 8.00e+00	1.49e+02 1.03e+01	1.56e+02 1.03e+01	**1.28e+02 7.04e+00**	1.42e+01 3.70e+00	1.31e+01 4.05e+00	9.13e+00 2.71e+00	**0.00e+00 0.00e+00**
F12	5.44e+02 1.67e+01	5.55e+02 2.05e+01	5.43e+02 2.29e+01	**5.19e+02 2.16e+01**	**5.62e+01 1.55e+01**	**5.48e+01 1.30e+01**	**5.60e+01 1.21e+01**	7.04e+01 1.04e+01
F13	5.58e+02 2.80e+01	5.49e+02 2.36e+01	5.58e+02 2.45e+01	**5.25e+02 1.80e+01**	1.46e+02 3.03e+01	1.54e+02 3.25e+01	1.35e+02 3.33e+01	1.52e+02 2.63e+01
F14	6.06e+03 3.88e+02	6.04e+03 4.19e+02	6.17e+03 3.31e+02	6.07e+03 3.13e+02	3.52e+02 1.82e+02	3.23e+02 1.51e+02	2.07e+02 1.30e+02	**1.18e+01 3.62e+00**
F15	1.43e+04 4.18e+02	1.43e+04 4.05e+02	1.43e+04 3.41e+02	1.43e+04 3.45e+02	8.86e+03 3.26e+03	8.97e+03 2.99e+03	8.62e+03 3.24e+03	7.57e+03 4.45e+02
F16	3.28e+00 2.78e−01	3.28e+00 2.12e−01	3.31e+00 3.34e−01	3.28e+00 3.21e−01	2.96e+00 8.11e−01	3.25e+00 4.47e−01	3.11e+00 4.74e−01	**2.20e+00 6.24e−01**
F17	2.21e+02 7.47e+00	2.27e+02 8.31e+00	2.29e+02 8.81e+00	**1.93e+02 7.73e+00**	6.24e+01 3.13e+00	5.95e+01 2.46e+00	5.71e+01 1.87e+00	**5.08e+01 1.00e−06**
F18	5.94e+02 2.52e+01	5.97e+02 1.98e+01	6.02e+02 1.62e+01	**5.74e+02 1.70e+01**	2.14e+02 1.24e+02	1.91e+02 1.32e+02	2.10e+02 1.27e+02	**1.57e+02 1.21e+01**
F19	4.22e+01 3.27e+00	4.23e+01 3.52e+00	4.54e+01 3.73e+00	**3.42e+01 2.24e+00**	5.19e+00 1.43e+00	4.84e+00 1.31e+00	4.54e+00 1.28e+00	4.72e+00 5.81e−01
F20	2.37e+01 3.60e−01	2.37e+01 2.65e−01	2.39e+01 3.09e−01	**2.35e+01 2.81e−01**	1.88e+01 9.32e−01	**1.86e+01 1.26e+00**	**1.87e+01 8.38e−01**	1.91e+01 7.46e−01
F21	4.18e+02 8.84e+01	5.44e+02 2.30e+02	7.30e+02 3.44e+02	**2.54e+02 1.75e+02**	8.59e+02 3.85e+02	8.19e+02 3.96e+02	7.48e+02 4.10e+02	7.91e+02 3.82e+02
F22	8.09e+03 5.02e+02	8.12e+03 5.29e+02	8.21e+03 4.07e+02	7.88e+03 6.94e+02	3.83e+02 2.11e+02	3.20e+02 1.34e+02	1.69e+02 9.90e+01	**2.87e+01 1.43e+01**
F23	1.49e+04 2.95e+02	1.49e+04 3.33e+02	1.50e+04 2.19e+02	**1.47e+04 4.57e+02**	**7.02e+03 1.30e+03**	**6.91e+03 9.35e+02**	**7.17e+03 1.53e+03**	8.67e+03 5.29e+02
F24	3.85e+02 3.91e+00	3.86e+02 4.11e+00	3.85e+02 4.28e+00	3.86e+02 4.29e+00	2.27e+02 8.83e+00	2.30e+02 9.27e+00	2.28e+02 1.12e+01	2.32e+02 1.02e+01
F25	3.82e+02 4.50e+00	3.81e+02 3.97e+00	3.83e+02 3.90e+00	3.82e+02 4.58e+00	**3.21e+02 1.45e+01**	**3.18e+02 1.03e+01**	**3.14e+02 1.21e+01**	3.66e+02 2.18e+01
F26	**2.66e+02 2.21e+01**	2.77e+02 4.73e+01	2.76e+02 4.56e+01	2.92e+02 6.05e+01	3.05e+02 5.98e+01	3.24e+02 3.61e+01	3.18e+02 4.24e+01	2.97e+02 8.51e+01
F27	2.15e+03 3.34e+01	2.16e+03 4.39e+01	2.16e+03 3.68e+01	2.13e+03 5.13e+01	8.01e+02 1.39e+02	8.06e+02 1.37e+02	7.27e+02 1.71e+02	8.26e+02 2.36e+02
F28	4.45e+02 5.25e+00	4.52e+02 5.80e+00	4.62e+02 8.30e+00	**4.09e+02 1.09e+00**	4.00e+02 5.67e−06	4.00e+02 0.00e+00	4.99e+02 5.42e+02	4.99e+02 5.44e+02
ARV	2.23	2.59	3.48	1.70	3.11	2.54	2.09	2.27
Final Rank	2	3	4	1	4	3	1	2

(*continued on next page*)

Table 19 (*continued*)

Func.	ring-jDE	cell-jDE	sw-jDE	SL-jDE	ring-SaDE	cell-SaDE	sw-SaDE	SL-SaDE
F1	**0.00e+00 0.00e+00**	**0.00e+00 0.00e+00**	**0.00e+00 0.00e+00**	5.03e−29 7.45e−29	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F2	6.12e+05 2.34e+05	5.65e+05 2.30e+05	6.04e+05 2.09e+05	**3.14e+05 1.47e+05**	2.11e+07 4.67e+06	1.98e+07 4.01e+06	2.25e+07 3.72e+06	**1.04e+07 3.27e+06**
F3	1.29e+07 2.01e+07	8.41e+06 1.54e+07	**4.86e+06 7.17e+06**	1.58e+07 3.22e+07	1.64e+07 3.33e+07	3.16e+07 5.57e+07	2.76e+07 5.28e+07	**4.43e+06 6.57e+06**
F4	2.77e+02 1.73e+02	3.15e+02 3.62e+02	4.43e+02 4.93e+02	**7.47e−01 5.53e−01**	3.82e+04 4.61e+03	3.86e+04 4.67e+03	4.04e+04 4.86e+03	**1.75e+04 2.95e+03**
F5	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	2.37e−16 1.30e−15	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F6	4.39e+01 5.03e−01	4.43e+01 8.15e−01	4.38e+01 5.14e−01	**4.34e+01 3.69e−03**	**4.52e+01 1.04e+00**	4.94e+01 1.32e+01	**4.52e+01 9.15e−01**	4.74e+01 1.24e+01
F7	**2.12e+01 6.11e+00**	**2.09e+01 5.84e+00**	**1.35e+01 5.61e+00**	2.79e+01 7.75e+00	6.98e+01 9.17e+00	7.11e+01 1.04e+01	6.15e+01 8.34e+00	**3.63e+01 1.41e+01**
F8	2.11e+01 4.39e−02	2.11e+01 3.70e−02	2.11e+01 3.49e−02	2.11e+01 4.44e−02	2.11e+01 2.81e−02	2.11e+01 3.22e−02	2.11e+01 2.98e−02	2.11e+01 3.56e−02
F9	5.41e+01 2.87e+00	5.49e+01 2.54e+00	5.56e+01 2.20e+00	**4.79e+01 1.13e+01**	5.65e+01 1.73e+00	5.59e+01 2.99e+00	5.52e+01 2.39e+00	**5.51e+01 2.23e+00**
F10	1.46e−01 9.79e−02	1.23e−01 8.37e−02	**5.66e−02 3.50e−02**	**7.99e−02 4.02e−02**	2.46e−01 1.21e−01	2.47e−01 1.82e−01	2.12e−01 1.35e−01	2.17e−01 1.24e−01
F11	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	3.32e−02 1.82e−01	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00	0.00e+00 0.00e+00
F12	8.87e+01 2.14e+01	9.55e+01 1.68e+01	1.16e+02 1.53e+01	**7.08e+01 1.23e+01**	1.89e+02 1.79e+01	1.94e+02 1.80e+01	1.85e+02 1.76e+01	**1.44e+02 1.37e+01**
F13	1.71e+02 2.21e+01	1.82e+02 3.58e+01	1.78e+02 2.40e+01	**1.50e+02 4.03e+01**	2.83e+02 1.85e+01	2.71e+02 2.46e+01	2.70e+02 2.15e+01	**2.20e+02 1.78e+01**
F14	**3.75e−03 6.68e−03**	**2.51e−03 5.08e−03**	1.52e−02 3.53e−02	3.36e−02 1.25e−01	3.88e+01 8.33e+00	5.92e+01 6.57e+00	9.70e+01 1.12e+01	**4.28e+01 1.17e+01**
F15	1.02e+04 4.73e+02	9.84e+03 5.39e+02	9.93e+03 5.84e+02	1.01e+04 4.84e+02	**1.00e+04 3.20e+02**	1.03e+04 3.92e+02	1.03e+04 4.22e+02	1.02e+04 4.31e+02
F16	**2.97e+00 3.39e−01**	3.07e+00 4.14e−01	**3.04e+00 3.83e−01**	3.24e+00 2.93e−01	2.63e+00 2.45e−01	2.58e+00 2.61e−01	2.70e+00 2.45e−01	2.63e+00 2.87e−01
F17	5.08e+01 0.00e+00	5.08e+01 0.00e+00	5.08e+01 0.00e+00	5.08e+01 7.08e−07	5.26e+01 2.78e−01	5.33e+01 4.36e−01	5.40e+01 4.72e−01	**5.14e+01 2.68e−01**
F18	2.60e+02 2.13e+01	2.57e+02 2.70e+01	2.73e+02 2.05e+01	**2.45e+02 2.07e+01**	3.07e+02 1.10e+01	3.01e+02 2.00e+01	2.93e+02 1.67e+01	**2.69e+02 1.77e+01**
F19	2.70e+00 2.69e−01	2.73e+00 2.60e−01	2.74e+00 2.65e−01	2.64e+00 4.85e−01	4.60e+00 4.88e−01	4.57e+00 4.48e−01	**4.40e+00 3.83e−01**	4.79e+00 6.46e−01
F20	2.11e+01 5.56e−01	2.13e+01 4.87e−01	2.14e+01 5.26e−01	**2.05e+01 7.31e−01**	2.08e+01 3.89e−01	2.09e+01 4.08e−01	2.08e+01 3.92e−01	**2.04e+01 5.85e−01**
F21	5.50e+02 4.41e+02	5.19e+02 4.32e+02	6.11e+02 4.53e+02	5.40e+02 4.31e+02	7.03e+02 3.52e+02	6.91e+02 3.71e+02	7.79e+02 4.02e+02	8.62e+02 3.58e+02
F22	2.48e+01 2.09e+01	2.53e+01 2.09e+01	5.04e+01 3.34e+01	**1.54e+01 1.28e+01**	2.79e+02 1.92e+02	2.20e+02 1.07e+02	4.54e+02 1.15e+02	**9.80e+01 1.64e+02**
F23	1.09e+04 6.82e+02	1.05e+04 7.13e+02	1.09e+04 4.51e+02	**1.04e+04 7.94e+02**	1.12e+04 5.73e+02	1.13e+04 4.64e+02	1.15e+04 5.23e+02	1.12e+04 5.40e+02
F24	2.52e+02 1.07e+01	2.47e+02 1.12e+01	**2.35e+02 1.52e+01**	2.51e+02 1.20e+01	2.49e+02 1.31e+01	2.51e+02 1.19e+01	2.40e+02 1.27e+01	**2.32e+02 8.11e+00**
F25	3.52e+02 1.80e+01	3.63e+02 9.83e+00	3.65e+02 1.20e+01	**3.18e+02 1.00e+01**	3.88e+02 6.81e+00	3.89e+02 5.66e+00	3.90e+02 6.04e+00	**3.81e+02 7.09e+00**
F26	2.00e+02 3.78e−02	2.00e+02 2.71e−02	2.00e+02 2.69e−02	**2.00e+02 2.10e−02**	**2.02e+02 1.22e+00**	**2.03e+02 2.06e+00**	**2.03e+02 1.60e+00**	2.83e+02 8.42e+01
F27	1.28e+03 1.94e+02	1.31e+03 2.23e+02	1.30e+03 3.28e+02	**1.05e+03 1.80e+02**	1.55e+03 1.36e+02	1.60e+03 8.13e+01	1.57e+03 1.36e+02	**8.52e+02 3.15e+02**
F28	4.00e+02 0.00e+00	4.00e+02 5.67e−06	4.00e+02 0.00e+00	4.00e+02 0.00e+00	4.00e+02 0.00e+00	4.00e+02 0.00e+00	4.00e+02 0.00e+00	4.00e+02 0.00e+00
ARV	2.55	2.50	2.77	2.18	2.45	2.95	2.77	1.84
Final Rank	3	2	4	1	2	4	3	1

Table 20
Results of the multi-problem Wilcoxon's test for SL-DE versus the DE variant with different population topologies for CEC 2013 functions at 30*D* and 50*D*.

Algorithm at 30D		+/ = /−	*R+*	*R-*	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
CoDE	**ring-DE**	14/12/2	261.5	116.5	7.50E-02	=	+
	cell-DE	13/13/2	275.5	130.5	9.55E-02	=	+
	sw-DE	15/11/2	317	89	8.93E-03	+	+
JADE	**ring-DE**	13/13/2	322.5	55.5	1.24E-03	+	+
	cell-DE	12/13/3	324.5	81.5	5.34E-03	+	+
	sw-DE	14/9/5	300	78	7.39E-03	+	+
jDE	**ring-DE**	10/16/2	246.5	131.5	1.63E-01	=	=
	cell-DE	12/14/2	258.5	147.5	2.01E-01	=	=
	sw-DE	12/13/3	294.5	111.5	3.62E-02	+	+
SaDE	**ring-DE**	13/12/3	272.5	133.5	1.11E-01	=	=
	cell-DE	14/11/3	288.5	117.5	4.95E-02	+	+
	sw-DE	13/12/3	328	50	8.04E-04	+	+
Algorithm at 50D		+/ = /−	***R+***	***R-***	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
CoDE	**ring-DE**	16/11/1	321	57	1.27E-03	+	+
	cell-DE	15/13/0	322.5	83.5	6.14E-03	+	+
	sw-DE	18/13/0	380.5	25.5	3.90E-05	+	+
JADE	**ring-DE**	10/14/4	247.5	130.5	1.56E-01	=	=
	cell-DE	9/14/5	243	135	1.89E-01	=	=
	sw-DE	9/14/5	219	159	4.64E-01	=	=
jDE	**ring-DE**	13/11/4	320	86	7.46E-03	+	+
	cell-DE	12/13/3	263	143	1.66E-01	=	=
	sw-DE	13/9/6	273	133	1.07E-01	=	=
SaDE	**ring-DE**	13/12/3	270.5	107.5	4.88E-02	+	+
	cell-DE	15/12/1	318	60	1.81E-03	+	+
	sw-DE	14/11/3	311	67	3.25E-03	+	+

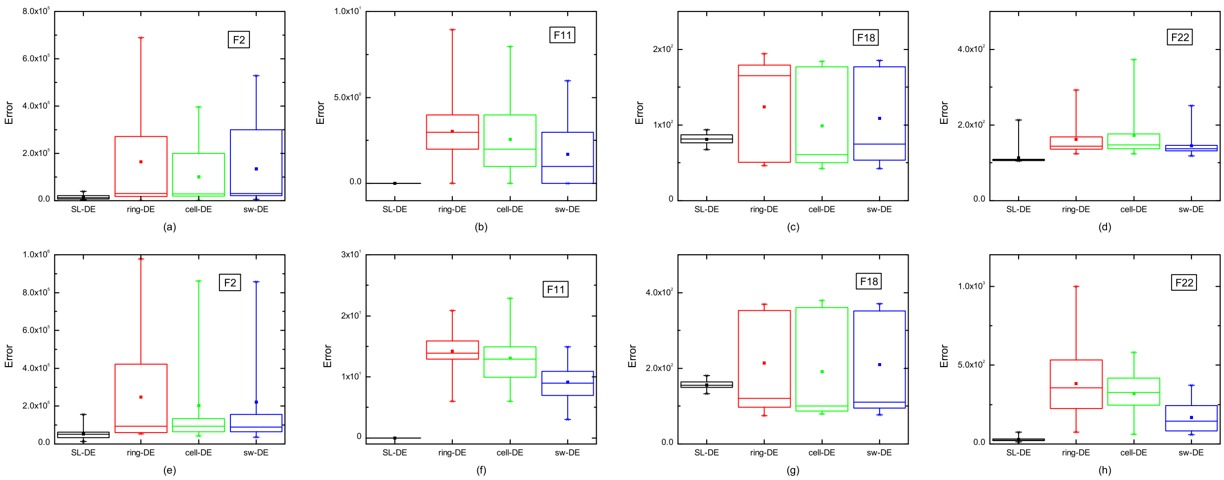


Fig. 9. Box plots of ring-DE, cell-DE, sw-DE and SL-DE with JADE on F2, F11, F18 and F22 at 30*D* ((a)–(d)) and 50*D* ((e)–(h)).

(7) SPSRDEMMS: DE with a structured population size and multiple mutation strategies [47];
(8) TLBSaDE: DE with a teaching and learning best strategy [1].

These DE variants are able to obtain promising results for the test functions. Thus, they are chosen as representative of the comparisons. The results of these variants are directly obtained from their original papers. Tables 9 and 10 show the results for the functions at 30*D* and 50*D*, respectively. The multi-problem Wilcoxon signed rank tests and the Friedman test are also conducted and the results are shown inTables 11 and 12, respectively.

FromTables 9 and 10, it is clear that SL-JADE can obtain better results than most DE variants for functions at 30*D* and 50*D*. In addition, as Table 11 shows, SL-JADE obtains higher $R+$ values than $R-$ values in most cases. Based on the results of the Friedman test in Table 12, SL-JADE obtains the second rank for the functions at 30*D* and 50*D*. Overall, SL-JADE is better than seven DE variants and is only worse than SHADE.

Furthermore, based on the pairwise Wilcoxon rank-sum test, the number of times an algorithm obtains the *K*th best result (i.e., Top-K, $K = 1, 2, \ldots, 9$) for all the test functions is counted. The curves are shown in Fig. 7, where if the curve of an algorithm is above those of other algorithms and reaches 28 (the total number of test functions) with the smallest

Table 21
Mean and standard deviation of the best error values obtained by ProDE, rank-DE and SL-DE on CEC 2013 functions at 30*D*.

Func.	ProDE/rand/1	rank-DE/rand/1	SL-DE/rand/1	ProDE/rand/2	rank-DE/rand/2	SL-DE/rand/2	ProDE/best/1	rank-DE/best/1	SL-DE/best/1
F1	**8.41e−030 3.77e−029**	**2.10e−029 5.21e−029**	7.56e−029 9.99e−029	7.18e−001 1.88e−001	9.71e−006 5.38e−006	**1.67e−016 2.14e−016**	3.35e+003 1.41e+003	3.43e+003 1.47e+003	**8.59e+001 1.21e+002**
F2	4.11e+005 2.62e+005	1.08e+005 5.98e+004	**4.77e+004 2.35e+004**	6.03e+007 1.41e+007	9.80e+006 3.58e+006	**4.93e+005 3.99e+005**	2.06e+007 1.17e+007	2.79e+007 1.46e+007	**1.54e+006 1.40e+006**
F3	**1.38e+000 4.38e+000**	**2.20e+005 8.35e+005**	5.82e+005 1.78e+006	5.75e+009 1.24e+009	3.50e+008 1.82e+008	**4.60e+004 4.46e+004**	1.91e+009 6.16e+009	1.70e+009 3.17e+009	**3.13e+006 4.12e+006**
F4	1.44e+003 5.38e+002	6.77e+001 6.22e+001	**7.28e+000 6.24e+000**	5.31e+004 6.82e+003	3.92e+004 5.85e+003	**1.01e+004 6.31e+003**	**2.05e−003 4.20e−003**	**2.04e−003 3.83e−003**	1.36e−002 2.35e−002
F5	1.61e−028 2.21e−028	**0.00e+000 0.00e+000**	2.01e−015 4.31e−015	7.72e−001 1.96e−001	6.68e−004 3.36e−004	**6.15e−011 5.60e−011**	5.35e+002 3.01e+002	6.69e+002 3.33e+002	**5.59e+001 4.84e+001**
F6	**8.49e+000 1.85e+000**	**8.09e+000 6.58e+000**	2.29e+001 2.24e+001	5.41e+001 1.62e+001	1.34e+001 7.35e−001	**4.06e+000 4.31e+000**	2.54e+002 9.33e+001	2.44e+002 6.97e+001	**7.65e+001 2.69e+001**
F7	**4.58e−002 6.76e−002**	**8.97e−002 1.33e−001**	7.96e−001 6.54e−001	8.85e+001 7.09e+000	5.41e+001 6.91e+000	**1.28e+001 4.67e+000**	1.02e+002 2.33e+001	1.03e+002 3.07e+001	**5.46e+001 1.65e+001**
F8	2.09e+001 5.15e−002	2.09e+001 6.26e−002	2.09e+001 5.31e−002	2.09e+001 6.00e−002	2.09e+001 5.96e−002	2.09e+001 5.14e−002	2.09e+001 4.23e−002	2.09e+001 3.86e−002	2.10e+001 5.04e−002
F9	3.85e+001 1.48e+000	3.51e+001 9.90e+000	**1.45e+001 1.08e+001**	3.91e+001 1.48e+000	3.94e+001 1.10e+000	3.97e+001 1.06e+000	2.40e+001 2.71e+000	2.54e+001 3.23e+000	**1.93e+001 3.46e+000**
F10	**8.13e−003 6.07e−003**	**1.91e−002 1.22e−002**	5.99e−002 3.68e−002	5.85e+001 1.50e+001	1.43e+000 2.86e−001	**2.22e−003 4.31e−003**	5.78e+002 2.10e+002	6.73e+002 2.77e+002	**4.22e+001 4.63e+001**
F11	1.34e+002 2.52e+001	2.09e+001 1.49e+001	**1.65e+001 5.88e+000**	2.26e+002 8.39e+000	2.04e+002 1.42e+001	**1.96e+002 9.03e+000**	1.80e+002 5.09e+001	1.51e+002 4.86e+001	**7.43e+001 1.37e+001**
F12	1.76e+002 1.30e+001	1.75e+002 7.79e+000	**7.68e+001 6.67e+001**	2.39e+002 1.80e+001	2.23e+002 1.65e+001	**2.11e+002 9.86e+000**	1.61e+002 4.04e+001	1.96e+002 3.80e+001	**8.76e+001 2.36e+001**
F13	1.79e+002 1.12e+001	1.72e+002 1.17e+001	**1.52e+002 4.47e+001**	2.48e+002 1.04e+001	2.26e+002 1.12e+001	**2.08e+002 1.21e+001**	2.62e+002 6.41e+001	2.45e+002 4.55e+001	**1.63e+002 4.64e+001**
F14	6.53e+003 4.76e+002	4.60e+003 1.56e+003	**7.15e+002 4.55e+002**	6.92e+003 2.59e+002	6.89e+003 2.73e+002	6.96e+003 2.23e+002	2.64e+003 4.76e+002	3.05e+003 5.82e+002	**2.04e+003 6.34e+002**
F15	7.21e+003 2.74e+002	7.22e+003 3.23e+002	7.14e+003 3.13e+002	7.25e+003 3.09e+002	7.40e+003 3.00e+002	7.31e+003 1.90e+002	**4.26e+003 1.38e+003**	**4.04e+003 8.29e+002**	5.37e+003 1.58e+003
F16	2.59e+000 2.56e−001	2.50e+000 2.19e−001	2.52e+000 2.34e−001	2.49e+000 2.93e−001	2.45e+000 3.06e−001	2.43e+000 2.59e−001	2.46e+000 2.59e−001	2.35e+000 4.37e−001	2.45e+000 3.37e−001
F17	1.81e+002 1.67e+001	1.14e+002 1.98e+001	**4.93e+001 5.96e+000**	2.65e+002 1.77e+001	2.52e+002 1.09e+001	**2.27e+002 1.18e+001**	2.18e+002 4.61e+001	2.52e+002 5.18e+001	**1.01e+002 2.60e+001**
F18	2.11e+002 8.71e+000	2.04e+002 9.09e+000	**1.94e+002 1.76e+001**	2.78e+002 1.62e+001	2.56e+002 1.18e+001	**2.40e+002 1.19e+001**	2.53e+002 5.98e+001	2.88e+002 5.39e+001	**1.54e+002 4.73e+001**
F19	1.49e+001 1.13e+000	1.25e+001 3.14e+000	**3.52e+000 2.89e+000**	2.14e+001 1.10e+000	1.90e+001 1.14e+000	**1.77e+001 1.01e+000**	1.62e+003 1.93e+003	1.60e+003 2.10e+003	**3.26e+001 4.22e+001**
F20	1.22e+001 2.59e−001	1.18e+001 3.02e−001	**1.15e+001 3.20e−001**	1.34e+001 2.12e−001	1.31e+001 2.54e−001	**1.29e+001 1.96e−001**	1.42e+001 1.18e+000	1.43e+001 1.06e+000	**1.07e+001 9.26e−001**
F21	3.11e+002 6.94e+001	3.04e+002 7.49e+001	2.96e+002 7.24e+001	3.51e+002 5.99e+001	3.14e+002 6.62e+001	**2.94e+002 6.46e+001**	1.24e+003 3.07e+002	1.46e+003 3.02e+002	**4.69e+002 1.16e+002**
F22	6.37e+003 5.22e+002	4.14e+003 1.44e+003	**7.00e+002 1.18e+003**	7.33e+003 1.89e+002	7.16e+003 2.92e+002	7.30e+003 2.21e+002	3.09e+003 4.67e+002	3.48e+003 6.01e+002	**1.98e+003 6.58e+002**
F23	7.44e+003 3.64e+002	7.36e+003 2.38e+002	**7.29e+003 3.42e+002**	7.78e+003 2.43e+002	7.59e+003 2.98e+002	7.71e+003 2.10e+002	4.23e+003 9.19e+002	4.78e+003 6.00e+002	**3.61e+003 6.35e+002**
F24	**2.00e+002 1.67e−002**	**2.00e+002 6.69e−001**	2.05e+002 3.83e+000	2.84e+002 5.48e+000	2.56e+002 7.96e+000	**2.08e+002 2.97e+000**	2.77e+002 1.15e+001	2.74e+002 1.11e+001	**2.51e+002 1.15e+001**
F25	**2.33e+002 1.88e+001**	**2.37e+002 1.55e+001**	2.42e+002 1.74e+001	3.32e+002 3.05e+000	3.29e+002 3.60e+000	**3.27e+002 4.31e+000**	3.01e+002 8.03e+000	3.07e+002 8.85e+000	**2.83e+002 6.65e+000**
F26	**2.00e+002 7.63e−003**	**2.00e+002 3.34e−003**	2.07e+002 2.75e+001	2.04e+002 8.08e−001	2.01e+002 2.07e−001	**2.00e+002 1.13e−002**	3.03e+002 7.87e+001	2.95e+002 8.31e+001	**2.62e+002 7.27e+001**
F27	**3.01e+002 1.44e+000**	**3.08e+002 1.06e+001**	3.63e+002 3.68e+001	1.32e+003 3.28e+001	1.29e+003 4.60e+001	**1.06e+003 2.42e+002**	9.77e+002 8.35e+001	9.87e+002 7.86e+001	**7.87e+002 7.83e+001**
F28	3.00e+002 9.82e−006	3.00e+002 6.94e−006	3.00e+002 8.96e−006	3.59e+002 1.06e+001	3.00e+002 8.95e−002	**3.00e+002 4.01e−006**	1.78e+003 5.07e+002	1.95e+003 4.20e+002	**3.75e+002 2.00e+002**

Func.	**ProDE/best/2**	**rank-DE/best/2**	**SL-DE/best/2**	**ProDE/current-to-best/1**	**rank-DE/current-to-best/1**	**SL-DE/current-to-best/1**	**ProDE/rand-to-best/1**	**rank-DE/rand-to-best/1**	**SL-DE/rand-to-best/1**
F1	3.31e−028 1.44e−028	4.67e−028 1.98e−028	**2.98e−028 2.31e−028**	1.76e+003 1.09e+003	1.90e+003 1.31e+003	**6.32e+002 4.37e+002**	1.08e+003 7.32e+002	4.72e+002 2.60e+002	**1.04e+002 8.63e+001**
F2	1.53e+005 7.05e+004	8.48e+004 4.67e+004	**4.78e+004 2.24e+004**	9.69e+006 7.05e+006	9.34e+006 7.23e+006	**5.66e+006 3.45e+006**	1.16e+007 6.28e+006	9.34e+006 6.74e+006	8.56e+006 5.06e+006
F3	1.41e+005 6.19e+005	2.04e+005 5.44e+005	**3.09e+004 8.07e+004**	6.97e+009 4.72e+009	6.05e+009 3.63e+009	**3.85e+009 2.49e+009**	6.93e+009 4.41e+009	6.25e+009 3.62e+009	**2.86e+009 1.82e+009**
F4	1.22e+001 1.12e+001	1.75e+000 1.39e+000	**5.46e−001 7.75e−001**	6.41e+000 1.31e+001	**2.39e−001 3.56e−001**	9.26e+000 3.44e+001	1.23e+001 1.55e+001	3.26e−001 8.39e−001	**3.74e−001 6.02e−001**
F5	5.80e−015 5.92e−015	6.28e−015 6.31e−015	**3.19e−015 5.28e−015**	4.70e+002 1.94e+002	4.26e+002 2.47e+002	**2.19e+002 1.18e+002**	3.18e+002 1.52e+002	3.06e+002 1.38e+002	**1.34e+002 7.55e+001**
F6	4.40e+000 1.00e+001	3.17e+000 8.00e+000	**1.01e+000 4.85e+000**	2.11e+002 7.57e+001	1.86e+002 4.79e+001	**1.36e+002 3.10e+001**	1.55e+002 3.09e+001	1.45e+002 4.46e+001	**1.25e+002 2.53e+001**
F7	2.17e+001 1.08e+001	1.58e+001 1.01e+001	**2.61e+000 1.80e+000**	5.52e+001 1.66e+001	5.83e+001 1.87e+001	**4.29e+001 1.59e+001**	5.84e+001 1.74e+001	5.76e+001 1.79e+001	**3.86e+001 1.47e+001**
F8	2.10e+001 3.73e−002	2.09e+001 4.69e−002	2.09e+001 4.65e−002	2.10e+001 3.54e−002	2.09e+001 4.66e−002	2.09e+001 4.43e−002	2.10e+001 4.61e−002	2.10e+001 5.32e−002	2.09e+001 5.05e−002
F9	3.80e+001 5.34e+000	3.70e+001 6.38e+000	3.80e+001 5.64e+000	1.70e+001 2.33e+000	1.68e+001 2.35e+000	**1.47e+001 2.43e+000**	1.65e+001 2.02e+000	1.67e+001 2.57e+000	**1.48e+001 2.30e+000**
F10	2.43e−002 1.32e−002	2.98e−002 1.76e−002	2.41e−002 1.67e−002	2.84e+002 1.16e+002	2.27e+002 1.28e+002	**1.12e+002 5.86e+001**	2.34e+002 1.48e+002	1.70e+002 8.81e+001	**9.02e+001 4.41e+001**
F11	1.77e+002 1.39e+001	1.75e+002 1.21e+001	**1.58e+002 1.33e+001**	9.10e+001 2.24e+001	8.61e+001 2.73e+001	**6.08e+001 1.45e+001**	8.49e+001 2.32e+001	7.27e+001 1.78e+001	**4.78e+001 1.25e+001**
F12	1.97e+002 1.77e+001	1.90e+002 1.65e+001	**1.80e+002 1.02e+001**	1.02e+002 2.08e+001	8.63e+001 2.51e+001	**5.82e+001 2.05e+001**	8.08e+001 2.55e+001	8.71e+001 2.45e+001	**5.79e+001 1.59e+001**
F13	1.98e+002 1.41e+001	1.94e+002 1.44e+001	**1.78e+002 1.31e+001**	1.74e+002 3.51e+001	1.59e+002 4.09e+001	**1.34e+002 3.19e+001**	1.67e+002 3.62e+001	1.53e+002 3.05e+001	**1.12e+002 2.44e+001**
F14	6.86e+003 2.29e+002	6.84e+003 3.03e+002	6.87e+003 2.68e+002	6.32e+003 3.84e+002	6.10e+003 9.18e+002	6.38e+003 4.44e+002	4.03e+003 2.55e+003	2.08e+003 1.74e+003	**1.38e+003 3.70e+002**
F15	7.30e+003 3.18e+002	7.28e+003 2.92e+002	7.22e+003 3.81e+002	6.76e+003 4.01e+002	6.74e+003 3.75e+002	6.61e+003 3.98e+002	6.11e+003 1.69e+003	4.79e+003 2.33e+003	**4.50e+003 2.37e+003**
F16	2.45e+000 2.66e−001	**2.35e+000 3.35e−001**	2.60e+000 2.21e−001	2.47e+000 2.98e−001	2.47e+000 2.70e−001	2.42e+000 2.51e−001	2.38e+000 2.42e−001	2.52e+000 2.72e−001	2.39e+000 3.49e−001
F17	2.17e+002 2.34e+001	2.10e+002 1.33e+001	**1.93e+002 1.23e+001**	2.15e+002 3.23e+001	1.99e+002 3.64e+001	**1.77e+002 1.53e+001**	1.15e+002 4.92e+001	8.58e+001 2.63e+001	**6.38e+001 9.46e+000**
F18	2.31e+002 1.46e+001	2.31e+002 1.84e+001	**2.09e+002 1.23e+001**	2.48e+002 2.98e+001	2.42e+002 3.34e+001	**2.02e+002 1.30e+001**	2.14e+002 2.01e+001	2.19e+002 1.47e+001	**1.91e+002 1.45e+001**
F19	1.54e+001 1.77e+000	1.53e+001 1.65e+000	**1.39e+001 1.36e+000**	2.90e+002 5.75e+002	2.32e+002 4.66e+002	**2.04e+001 2.89e+001**	9.28e+001 1.12e+002	1.00e+002 2.07e+002	**2.19e+001 1.93e+001**
F20	1.22e+001 3.18e−001	1.20e+001 4.44e−001	**1.19e+001 4.01e−001**	1.31e+001 1.70e+000	1.31e+001 1.80e+000	**1.19e+001 1.59e+000**	1.36e+001 1.61e+000	1.30e+001 1.60e+000	**1.17e+001 1.34e+000**
F21	3.17e+002 8.00e+001	3.31e+002 8.85e+001	2.97e+002 7.94e+001	1.07e+003 2.78e+002	1.06e+003 3.30e+002	**8.07e+002 2.28e+002**	9.26e+002 3.11e+002	8.44e+002 3.54e+002	**6.57e+002 2.19e+002**
F22	7.02e+003 3.51e+002	6.90e+003 3.30e+002	7.00e+003 2.98e+002	4.43e+003 2.05e+003	3.39e+003 2.25e+003	**2.51e+003 2.05e+003**	1.32e+003 4.85e+002	1.48e+003 3.76e+002	1.10e+003 3.68e+002
F23	7.58e+003 3.77e+002	7.61e+003 3.31e+002	7.53e+003 3.73e+002	6.71e+003 6.72e+002	5.84e+003 1.77e+003	**4.88e+003 1.98e+003**	4.83e+003 2.25e+003	3.02e+003 1.37e+003	**2.60e+003 1.66e+003**
F24	2.15e+002 1.17e+001	2.15e+002 1.06e+001	**2.07e+002 6.09e+000**	2.50e+002 1.06e+001	2.53e+002 1.17e+001	**2.35e+002 9.23e+000**	2.45e+002 1.03e+001	2.47e+002 1.14e+001	**2.33e+002 8.36e+000**
F25	2.52e+002 1.92e+001	2.57e+002 1.29e+001	**2.36e+002 2.39e+001**	2.83e+002 1.39e+001	2.82e+002 1.04e+001	**2.69e+002 1.65e+001**	2.85e+002 1.14e+001	2.83e+002 8.02e+000	**2.73e+002 1.28e+001**
F26	2.15e+002 3.95e+001	2.13e+002 3.87e+001	**2.03e+002 1.91e+001**	2.60e+002 6.99e+001	2.57e+002 7.04e+001	2.48e+002 6.36e+001	2.50e+002 6.68e+001	2.84e+002 6.97e+001	2.70e+002 6.60e+001
F27	4.94e+002 9.93e+001	5.08e+002 9.88e+001	**3.96e+002 7.39e+001**	7.52e+002 7.50e+001	7.31e+002 8.69e+001	**5.94e+002 6.65e+001**	7.35e+002 9.33e+001	7.12e+002 7.09e+001	**6.33e+002 1.09e+002**
F28	2.93e+002 3.65e+001	3.75e+002 2.85e+002	3.00e+002 4.01e−006	1.45e+003 4.11e+002	1.34e+003 4.20e+002	**1.07e+003 3.16e+002**	1.36e+003 3.54e+002	1.33e+003 3.42e+002	**9.36e+002 3.38e+002**

Table 22

Mean and standard deviation of the best error values obtained by ProDE, rank-DE and SL-DE on CEC 2013 functions at 50*D*.

Func.	ProDE/rand/1	rank-DE/rand/1	SL-DE/rand/1	ProDE/rand/2	rank-DE/rand/2	SL-DE/rand/2	ProDE/best/1	rank-DE/best/1	SL-DE/best/1
F1	**7.57e**−029 9.16e−**029**	**2.15e**−028 1.82e−**028**	7.54e−028 5.56e−028	2.31e+002 6.05e+001	2.84e−002 1.54e−002	**2.21e**−013 2.53e−**013**	1.72e+004 4.18e+003	1.82e+004 4.76e+003	**7.11e+002 6.07e+002**
F2	2.66e+006 8.37e+005	4.71e+005 1.34e+005	**2.04e+005 9.73e+004**	4.35e+008 6.01e+007	1.89e+008 6.98e+007	**1.48e+007 4.78e+006**	1.02e+008 4.60e+007	1.14e+008 6.13e+007	**5.13e+006 1.94e+006**
F3	**2.46e+005 4.89e+005**	**9.24e+005 1.20e+006**	5.88e+006 9.50e+006	7.64e+010 1.01e+010	2.93e+010 8.86e+009	**1.07e+008 1.49e+008**	1.04e+010 1.76e+010	1.55e+010 2.24e+010	**2.82e+007 3.28e+007**
F4	3.43e+004 9.34e+003	4.47e+003 1.39e+003	**4.21e+002 2.18e+002**	1.02e+005 8.62e+003	8.83e+004 7.71e+003	**7.77e+004 9.51e+003**	2.73e−001 3.02e−001	**7.98e**−002 7.59e−**002**	2.32e−001 3.62e−001
F5	**4.74e**−016 2.59e−**015**	**3.77e**−015 5.39e−**015**	8.42e−015 5.63e−015	3.24e+001 5.09e+000	1.43e−001 6.00e−002	**1.00e**−008 5.78e−**009**	2.23e+003 7.43e+002	2.59e+003 9.45e+002	**3.35e+002 2.78e+002**
F6	4.34e+001 9.26e−003	4.34e+001 0.00e+000	4.53e+001 8.69e+000	1.46e+002 3.17e+001	4.47e+001 6.00e−001	**4.34e+001 6.57e**−**006**	9.01e+002 2.65e+002	9.99e+002 3.37e+002	**1.21e+002 3.68e+001**
F7	**1.38e+000 1.18e+000**	**2.57e+000 2.57e+000**	9.02e+000 5.01e+000	1.74e+002 1.29e+001	1.34e+002 1.44e+001	**5.39e+001 1.43e+001**	1.14e+002 2.70e+001	1.32e+002 2.16e+001	**7.58e+001 1.25e+001**
F8	2.11e+001 2.81e−002	2.11e+001 3.23e−002	2.11e+001 4.99e−002	2.11e+001 4.84e−002	2.11e+001 3.78e−002	2.11e+001 3.82e−002	2.11e+001 3.64e−002	2.11e+001 3.33e−002	2.11e+001 3.67e−002
F9	7.21e+001 1.53e+000	7.02e+001 1.01e+001	**5.58e+001 2.54e+001**	7.24e+001 1.85e+000	7.24e+001 1.47e+000	7.28e+001 1.28e+000	4.98e+001 4.05e+000	5.09e+001 3.65e+000	**4.20e+001 4.27e+000**
F10	**3.22e**−002 2.26e−**002**	**5.56e**−002 2.93e−**002**	1.00e−001 4.61e−002	1.02e+003 1.65e+002	3.38e+001 1.37e+001	**1.18e**−002 7.98e−**003**	1.82e+003 4.14e+002	1.87e+003 5.52e+002	**1.49e+002 7.45e+001**
F11	2.01e+002 4.39e+001	**2.80e+001 5.43e+000**	4.31e+001 8.41e+000	4.77e+002 1.89e+001	4.29e+002 1.60e+001	**3.96e+002 1.72e+001**	4.45e+002 7.52e+001	4.26e+002 8.57e+001	**2.22e+002 4.50e+001**
F12	3.56e+002 9.60e+000	3.48e+002 9.94e+000	**2.16e+002 1.44e+002**	5.16e+002 2.23e+001	4.55e+002 2.06e+001	**4.11e+002 1.76e+001**	4.35e+002 6.25e+001	4.77e+002 9.46e+001	**2.30e+002 3.85e+001**
F13	3.57e+002 1.36e+001	3.52e+002 1.54e+001	**3.47e+002 1.51e+001**	5.23e+002 2.32e+001	4.55e+002 2.01e+001	**4.13e+002 2.30e+001**	5.61e+002 1.06e+002	5.72e+002 1.03e+002	**3.83e+002 4.17e+001**
F14	1.13e+004 8.97e+002	6.08e+003 2.61e+003	**1.30e+003 5.21e+002**	1.32e+004 3.54e+002	1.33e+004 2.99e+002	1.32e+004 3.94e+002	6.44e+003 9.63e+002	6.59e+003 7.89e+002	**4.56e+003 8.43e+002**
F15	1.41e+004 2.43e+002	1.38e+004 4.68e+002	1.40e+004 4.20e+002	1.41e+004 3.63e+002	1.42e+004 3.60e+002	1.41e+004 3.14e+002	**9.70e+003 3.22e+003**	**9.43e+003 3.24e+003**	1.22e+004 3.13e+003
F16	3.36e+000 3.11e−001	3.48e+000 1.84e−001	**3.36e+000 2.51e**−**001**	3.29e+000 3.14e−001	3.32e+000 2.64e−001	3.37e+000 2.68e−001	3.34e+000 2.25e−001	3.38e+000 2.72e−001	3.36e+000 2.99e−001
F17	3.30e+002 1.98e+001	1.94e+002 3.60e+001	**9.85e+001 1.04e+001**	5.59e+002 1.85e+001	4.82e+002 2.14e+001	**4.49e+002 1.44e+001**	7.28e+002 1.58e+002	7.78e+002 1.16e+002	**2.79e+002 5.76e+001**
F18	4.06e+002 1.33e+001	3.96e+002 1.46e+001	**3.92e+002 1.53e+001**	5.64e+002 2.24e+001	5.04e+002 1.53e+001	**4.59e+002 1.98e+001**	7.54e+002 1.02e+002	7.98e+002 9.42e+001	**4.02e+002 1.19e+002**
F19	2.92e+001 1.49e+000	1.95e+001 7.29e+000	**5.19e+000 1.46e+000**	1.62e+002 6.55e+001	3.90e+001 1.72e+000	**3.49e+001 1.71e+000**	2.47e+004 1.80e+004	3.57e+004 3.97e+004	**5.57e+002 1.38e+003**
F20	2.21e+001 2.64e−001	2.18e+001 2.98e−001	**2.14e+001 4.11e**−**001**	2.36e+001 2.61e−001	2.33e+001 3.30e−001	**2.30e+001 3.84e**−**001**	2.27e+001 1.51e+000	2.26e+001 1.70e+000	**2.10e+001 7.11e**−**001**
F21	**5.52e+002 4.20e+002**	**7.08e+002 4.09e+002**	9.40e+002 3.47e+002	5.95e+002 1.78e+002	5.10e+002 4.40e+002	5.10e+002 4.21e+002	3.51e+003 5.19e+002	3.62e+003 4.30e+002	**1.36e+003 2.81e+002**
F22	1.21e+004 1.08e+003	5.95e+003 2.90e+003	**1.20e+003 4.94e+002**	1.40e+004 3.50e+002	1.40e+004 3.01e+002	1.39e+004 4.19e+002	7.89e+003 1.44e+003	8.05e+003 1.21e+003	**5.03e+003 8.18e+002**
F23	1.44e+004 4.46e+002	1.42e+004 2.69e+002	1.42e+004 3.41e+002	1.48e+004 3.16e+002	1.47e+004 3.74e+002	1.46e+004 3.04e+002	9.24e+003 9.38e+002	9.64e+003 1.46e+003	**7.82e+003 2.67e+003**
F24	**2.03e+002 4.23e+000**	**2.14e+002 7.51e+000**	2.33e+002 1.05e+001	3.94e+002 6.85e+000	3.65e+002 1.56e+001	**2.30e+002 1.39e+001**	3.65e+002 1.52e+001	3.63e+002 1.41e+001	**3.10e+002 1.34e+001**
F25	3.01e+002 4.31e+001	**2.86e+002 6.85e+000**	3.00e+002 8.13e+000	4.61e+002 6.10e+000	4.54e+002 4.91e+000	**4.51e+002 5.93e+000**	4.20e+002 1.42e+001	4.19e+002 1.65e+001	**3.74e+002 1.57e+001**
F26	**2.51e+002 5.51e+001**	**2.71e+002 5.92e+001**	3.11e+002 5.79e+001	2.52e+002 1.13e+001	2.23e+002 6.99e+000	**2.02e+002 1.12e+000**	4.23e+002 4.21e+001	4.09e+002 6.91e+001	**3.93e+002 5.33e+001**
F27	**3.76e+002 5.54e+001**	**5.53e+002 1.08e+002**	7.10e+002 1.11e+002	2.26e+003 4.94e+001	2.24e+003 4.34e+001	**2.20e+003 4.79e+001**	1.74e+003 1.36e+002	1.77e+003 1.28e+002	**1.47e+003 1.10e+002**
F28	4.00e+002 0.00e+000	4.00e+002 5.67e−006	4.00e+002 0.00e+000	5.74e+002 2.99e+001	4.02e+002 4.30e−001	**4.00e+002 0.00e+000**	3.64e+003 1.06e+003	3.98e+003 9.74e+002	**9.01e+002 1.25e+003**

Func.	**ProDE/best/2**	**rank-DE/best/2**	**SL-DE/best/2**	**ProDE/current-to-best/1**	**rank-DE/current-to-best/1**	**SL-DE/current-to-best/1**	**ProDE/rand-to-best/1**	**rank-DE/rand-to-best/1**	**SL-DE/rand-to-best/1**
F1	9.43e−028 3.38e−028	1.22e−027 8.18e−028	**8.50e**−**028 3.17e**−**028**	1.13e+004 3.52e+003	9.96e+003 3.03e+003	**4.22e+003 1.24e+003**	7.86e+003 2.11e+003	5.54e+003 1.37e+003	**1.23e+003 6.09e+002**
F2	7.90e+005 2.70e+005	4.64e+005 1.66e+005	**2.33e+005 8.60e+004**	4.95e+007 1.96e+007	4.43e+007 1.41e+007	**2.47e+007 1.12e+007**	4.06e+007 1.79e+007	4.54e+007 1.58e+007	**3.23e+007 1.01e+007**
F3	7.00e+006 9.43e+006	1.62e+007 2.10e+007	**2.50e+006 3.14e+006**	2.98e+010 9.81e+009	2.85e+010 1.19e+010	**1.80e+010 7.84e+009**	2.53e+010 8.86e+009	2.46e+010 8.25e+009	**1.73e+010 8.48e+009**
F4	8.93e+003 4.58e+003	2.55e+003 1.44e+003	**6.02e+002 3.14e+002**	1.25e+002 1.94e+002	**9.01e+000 1.10e+001**	4.39e+001 7.69e+001	5.95e+001 6.42e+001	8.91e+000 3.68e+001	**1.29e+000 1.41e+000**
F5	1.62e−014 9.49e−015	1.45e−014 5.51e−015	1.31e−014 4.36e−015	1.31e+003 3.98e+002	1.29e+003 3.65e+002	**8.91e+002 2.29e+002**	1.07e+003 2.78e+002	1.10e+003 4.40e+002	**5.64e+002 1.96e+002**
F6	4.32e+001 4.73e+000	4.53e+001 9.40e+000	4.40e+001 1.73e+000	5.36e+002 1.40e+002	4.85e+002 1.03e+002	**3.19e+002 5.67e+001**	3.97e+002 8.37e+001	3.69e+002 7.35e+001	**2.25e+002 3.99e+001**
F7	5.19e+001 1.83e+001	5.67e+001 2.35e+001	**1.73e+001 8.48e+000**	7.34e+001 1.43e+001	7.24e+001 1.03e+001	**5.38e+001 1.13e+001**	7.03e+001 1.25e+001	7.98e+001 1.63e+001	**6.29e+001 1.12e+001**
F8	2.11e+001 2.76e−002	2.11e+001 4.85e−002	2.11e+001 4.11e−002	2.11e+001 3.26e−002	2.11e+001 3.51e−002	2.11e+001 4.06e−002	2.11e+001 3.36e−002	2.11e+001 4.56e−002	2.11e+001 3.20e−002
F9	7.29e+001 1.39e+000	**7.18e+001 2.02e+000**	7.30e+001 1.20e+000	3.80e+001 3.12e+000	3.69e+001 2.76e+000	**3.31e+001 2.62e+000**	3.58e+001 2.70e+000	3.64e+001 3.41e+000	**3.36e+001 2.49e+000**
F10	5.44e−002 3.00e−002	4.94e−002 3.07e−002	4.48e−002 2.73e−002	1.20e+003 3.63e+002	1.14e+003 3.11e+002	**6.62e+002 1.69e+002**	8.19e+002 2.15e+002	7.75e+002 1.76e+002	**4.19e+002 8.96e+001**
F11	3.84e+002 2.80e+001	3.46e+002 7.75e+001	**2.06e+002 1.28e+002**	2.73e+002 5.28e+001	2.58e+002 5.97e+001	**1.65e+002 2.99e+001**	2.12e+002 2.99e+001	1.97e+002 2.67e+001	**1.26e+002 2.38e+001**
F12	4.11e+002 2.77e+001	4.23e+002 3.39e+001	**3.66e+002 1.56e+001**	2.37e+002 4.31e+001	2.30e+002 4.22e+001	**1.71e+002 3.13e+001**	2.40e+002 5.24e+001	2.35e+002 4.07e+001	**1.57e+002 2.61e+001**
F13	4.23e+002 3.52e+001	4.15e+002 2.87e+001	**3.69e+002 2.32e+001**	4.21e+002 7.07e+001	4.02e+002 4.20e+001	**3.10e+002 5.09e+001**	3.65e+002 4.26e+001	3.55e+002 5.97e+001	**2.83e+002 4.43e+001**
F14	1.30e+004 3.67e+002	1.31e+004 3.25e+002	1.31e+004 3.82e+002	1.24e+004 5.94e+002	1.17e+004 2.18e+003	1.13e+004 2.87e+003	5.46e+003 3.89e+003	3.82e+003 6.78e+002	3.51e+003 6.96e+002
F15	1.42e+004 4.08e+002	1.42e+004 4.04e+002	1.41e+004 4.76e+002	1.37e+004 3.85e+002	1.35e+004 4.81e+002	**1.33e+004 4.54e+002**	1.31e+004 1.60e+003	1.28e+004 1.99e+003	1.28e+004 2.11e+003
F16	3.37e+000 2.71e−001	3.35e+000 2.85e−001	3.29e+000 3.00e−001	3.38e+000 2.63e−001	3.40e+000 3.20e−001	3.40e+000 2.39e−001	3.38e+000 3.42e−001	3.36e+000 1.76e−001	3.38e+000 2.56e−001
F17	4.52e+002 3.16e+001	4.54e+002 2.42e+001	**4.01e+002 1.62e+001**	4.03e+002 1.00e+002	3.13e+002 8.85e+001	**2.37e+002 9.50e+001**	2.81e+002 6.81e+001	2.50e+002 4.62e+001	**1.66e+002 2.44e+001**
F18	4.80e+002 3.21e+001	4.81e+002 3.84e+001	**4.15e+002 2.01e+001**	5.95e+002 6.01e+001	5.80e+002 5.58e+001	**4.52e+002 3.45e+001**	5.08e+002 4.80e+001	4.95e+002 4.56e+001	**4.22e+002 1.93e+001**
F19	3.39e+001 3.74e+000	3.36e+001 4.36e+000	**2.56e+001 5.47e+000**	4.85e+003 4.14e+003	4.43e+003 4.18e+003	**8.89e+002 7.05e+002**	2.77e+003 2.38e+003	1.58e+003 1.29e+003	**4.47e+002 5.36e+002**
F20	2.20e+001 3.82e−001	2.19e+001 3.31e−001	**2.16e+001 4.15e**−**001**	2.12e+001 9.37e−001	2.08e+001 6.46e−001	**2.05e+001 4.30e**−**001**	2.12e+001 1.26e+000	2.14e+001 9.85e−001	**2.05e+001 6.42e**−**001**
F21	7.62e+002 3.66e+002	8.23e+002 3.43e+002	8.71e+002 3.61e+002	3.15e+003 3.84e+002	3.06e+003 4.32e+002	**2.61e+003 4.89e+002**	2.84e+003 4.95e+002	2.81e+003 4.80e+002	**2.35e+003 5.46e+002**
F22	1.36e+004 4.06e+002	1.34e+004 5.05e+002	**1.34e+004 3.83e+002**	8.03e+003 4.13e+003	5.96e+003 3.38e+003	**4.17e+003 2.45e+003**	4.21e+003 1.69e+003	4.64e+003 5.89e+002	**3.41e+003 8.13e+002**
F23	1.45e+004 4.58e+002	1.45e+004 3.41e+002	1.44e+004 4.94e+002	1.37e+004 7.28e+002	1.28e+004 2.38e+003	**9.99e+003 3.98e+003**	1.03e+004 3.63e+003	7.78e+003 3.17e+003	**5.94e+003 2.49e+003**
F24	2.51e+002 1.36e+001	2.58e+002 1.47e+001	**2.43e+002 9.03e+000**	3.33e+002 1.30e+001	3.26e+002 1.38e+001	**2.97e+002 1.26e+001**	3.14e+002 1.36e+001	3.16e+002 1.51e+001	**2.91e+002 1.13e+001**
F25	3.33e+002 4.88e+001	3.28e+002 4.36e+001	**3.06e+002 8.40e+000**	3.92e+002 1.16e+001	3.92e+002 1.18e+001	**3.71e+002 1.11e+001**	3.88e+002 1.24e+001	3.78e+002 1.14e+001	**3.63e+002 1.18e+001**
F26	3.13e+002 1.02e+002	3.00e+002 7.80e+001	3.26e+002 6.26e+001	3.74e+002 6.85e+001	4.00e+002 7.48e+000	**3.73e+002 4.74e+001**	3.78e+002 6.04e+001	3.81e+002 4.90e+001	**3.57e+002 6.26e+001**
F27	1.09e+003 4.64e+002	9.59e+002 1.23e+002	**8.32e+002 1.14e+002**	1.44e+003 1.14e+002	1.47e+003 1.15e+002	**1.22e+003 1.19e+002**	1.40e+003 9.14e+001	1.39e+003 1.07e+002	**1.22e+003 1.05e+002**
F28	6.04e+002 7.77e+002	1.12e+003 1.34e+003	5.00e+002 5.50e+002	3.39e+003 7.96e+002	3.35e+003 1.00e+003	**2.19e+003 7.42e+002**	3.11e+003 8.80e+002	2.91e+003 1.01e+003	**1.77e+003 8.58e+002**

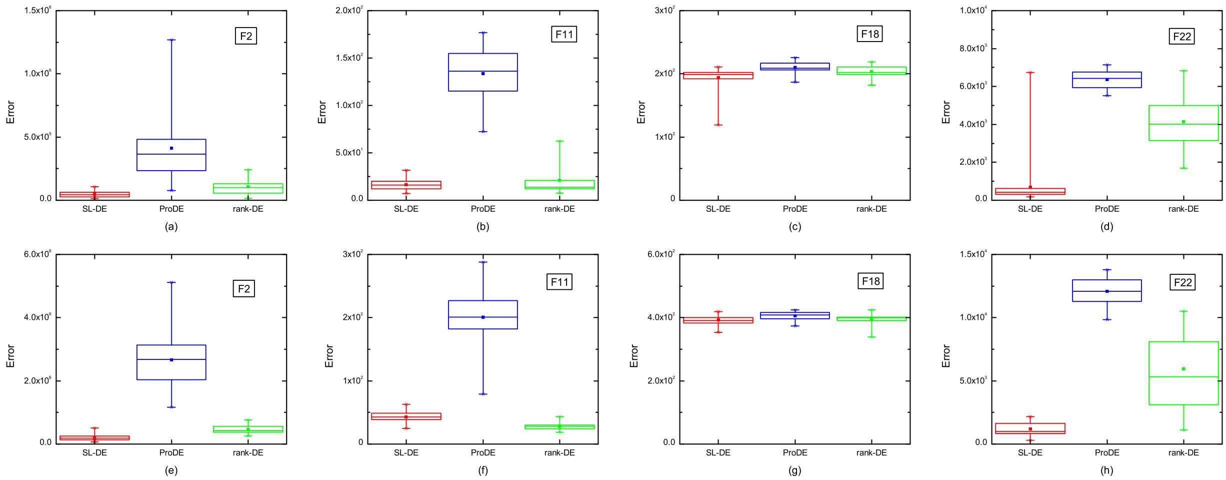


Fig. 10. Box plots of Pro-DE, rank-DE, and SL-DE with the "DE/rand/1" strategy on F2, F11, F18 and F22 at 30*D* ((a) and (d)) and 50*D* ((e) - (h)).

Table 23
Results of the multi-problem Wilcoxon's test for SL-DE versus ProDE and rank-DE, respectively, for CEC 2013 functions at 30*D* and 50*D*.

Algorithm at 30D		+/ = /−	R+	R-	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
DE/rand/1	**ProDE**	13/6/9	295.5	110.5	3.42E−02	Yes	Yes
	rank-DE	10/8/10	282.5	123.5	6.77E−02	No	Yes
DE/rand/2	**ProDE**	21/7/0	340.5	37.5	2.29E−04	Yes	Yes
	rank-DE	20/8/0	329.5	76.5	3.83E−03	Yes	Yes
DE/best/1	**ProDE**	24/2/2	378.5	27.5	5.60E−05	Yes	Yes
	rank-DE	24/2/2	378.0	28.0	6.40E−05	Yes	Yes
DE/best/2	**ProDE**	16/12/0	353.0	25.0	6.30E−05	Yes	Yes
	rank-DE	16/11/1	324.0	54.0	1.06E−03	Yes	Yes
DE/current-to-best/1	**ProDE**	22/6/0	386.0	20.0	2.90E−05	Yes	Yes
	rank-DE	21/6/1	351.0	27.0	9.50E−05	Yes	Yes
DE/rand-to-best/1	**ProDE**	23/5/0	396.0	10.0	1.00E−05	Yes	Yes
	rank-DE	20/8/0	405.0	1.0	4.00E−06	Yes	Yes
Algorithm at 50D		+/ = /−	***R+***	***R-***	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
DE/rand/1	**ProDE**	12/7/9	248.5	129.5	1.49E−01	No	No
	rank-DE	10/7/11	195.5	182.5	8.66E−01	No	No
DE/rand/2	**ProDE**	20/8/0	369.5	8.5	1.20E−05	Yes	Yes
	rank-DE	19/9/0	393.5	12.5	7.00E−06	Yes	Yes
DE/best/1	**ProDE**	24/3/1	356.0	22.0	5.70E−05	Yes	Yes
	rank-DE	24/2/2	355.0	23.0	6.30E−05	Yes	Yes
DE/best/2	**ProDE**	15/13/0	317.0	61.0	1.78E−03	Yes	Yes
	rank-DE	15/12/1	339.5	38.5	2.74E−04	Yes	Yes
DE/current-to-best/1	**ProDE**	25/3/0	377.0	1.0	6.00E−06	Yes	Yes
	rank-DE	23/4/1	395.5	10.5	1.10E−05	Yes	Yes
DE/rand-to-best/1	**ProDE**	24/4/0	404.5	1.5	4.00E−06	Yes	Yes
	rank-DE	23/5/0	401.5	4.5	6.00E−06	Yes	Yes

K value, the corresponding algorithm is regarded as the best among these competitors. It is clear that SHADE and SL-JADE first reach 28 at $K = 8$ for the functions at 50*D*. For the functions at 30*D*, SL-JADE is slower than SHADE and b6e6rl to reach 28. However, the curve of SL-JADE is never beneath those of other competitors, except for SHADE. Therefore, SL-JADE is the second best-performing algorithm in this aspect.

As the results show, SHADE can perform best for the test functions in this comparison. Thus, it is interesting to study what benefits can be obtained by combining SL-DE with SHADE. Along this line, ASL is applied to SHADE and the new algorithm is named SL-SHADE. A comparison between SHADE and SL-SHADE is also made. These two algorithms are independently run with the source code provided by the authors of SHADE [41]. The results are shown in Table 13. We can observe that SL-SHADE can obtain better results than SHADE overall for the functions both at 30D and 50D. Based on the Wilcoxon signed rank tests between SL-SHADE and SHADE, $R+ = 250$, $R- = 128$, and $p = 0.138$ for the functions at 30D, and $R+ = 291.5$, $R- = 86.5$, and $p = 0.0128$ for the functions at 50D. Therefore, we can draw the conclusion that ASL can also enhance the performance of SHADE.

Table 24
Mean and standard deviation of the best error values obtained by SL-DE and MS-DE on CEC 2013 functions at 30*D*.

Func.	MS-DE/rand/1		SL-DE/rand/1	MS-DE/rand/2		SL-DE/rand/2	MS-DE/best/2		SL-DE/best/2
F1	**0.00e+000 0.00e+000**	−	7.56e−029 9.99e−029	2.83e−002 1.21e−002	+	**1.67e−016 2.14e−016**	2.72e−014 2.92e−014	+	**2.98e−028 2.31e−028**
F2	3.77e+005 1.98e+005	+	**4.77e+004 2.35e+004**	3.04e+007 7.22e+006	+	**4.93e+005 3.99e+005**	5.17e+005 3.60e+005	+	**4.78e+004 2.24e+004**
F3	**2.77e−002 9.32e−002**	−	5.82e+005 1.78e+006	6.11e+009 1.47e+009	+	**4.60e+004 4.46e+004**	1.26e+006 5.55e+006	+	**3.09e+004 8.07e+004**
F4	5.53e+002 3.67e+002	+	**7.28e+000 6.24e+000**	4.88e+004 6.37e+003	+	**1.01e+004 6.31e+003**	1.31e+004 4.71e+003	+	**5.46e−001 7.75e−001**
F5	**5.65e−023 3.09e−022**	−	2.01e−015 4.31e−015	9.44e−003 5.09e−003	+	**6.15e−011 5.60e−011**	**1.66e−015 4.04e−015**	−	3.19e−015 5.28e−015
F6	**2.13e+000 6.61e+000**	−	2.29e+001 2.24e+001	1.84e+001 2.41e+000	+	**4.06e+000 4.31e+000**	5.08e+000 9.71e+000	+	**1.01e+000 4.85e+000**
F7	**7.35e−002 1.38e−001**	−	7.96e−001 6.54e−001	9.64e+001 9.78e+000	+	**1.28e+001 4.67e+000**	2.58e+001 1.17e+001	+	**2.61e+000 1.80e+000**
F8	2.09e+001 5.62e−002	=	2.09e+001 5.31e−002	2.09e+001 4.94e−002	=	2.09e+001 5.14e−002	2.10e+001 6.00e−002	=	2.09e+001 4.65e−002
F9	3.79e+001 4.67e+000	+	**1.45e+001 1.08e+001**	3.92e+001 1.16e+000	=	3.97e+001 1.06e+000	3.91e+001 1.79e+000	=	3.80e+001 5.64e+000
F10	**1.20e−002 1.03e−002**	−	5.99e−002 3.68e−002	3.63e+000 1.25e+000	+	**2.22e−003 4.31e−003**	**7.31e−003 6.35e−003**	−	2.41e−002 1.67e−002
F11	5.69e+001 3.91e+001	+	**1.65e+001 5.88e+000**	2.23e+002 1.30e+001	+	**1.96e+002 9.03e+000**	1.86e+002 9.87e+000	+	**1.58e+002 1.33e+001**
F12	1.87e+002 9.67e+000	+	**7.68e+001 6.67e+001**	2.49e+002 1.38e+001	+	**2.11e+002 9.86e+000**	1.96e+002 1.80e+001	+	**1.80e+002 1.02e+001**
F13	1.85e+002 8.71e+000	+	**1.52e+002 4.47e+001**	2.46e+002 1.40e+001	+	**2.08e+002 1.21e+001**	2.01e+002 1.12e+001	+	**1.78e+002 1.31e+001**
F14	5.19e+003 1.41e+003	+	**7.15e+002 4.55e+002**	6.94e+003 2.45e+002	=	6.96e+003 2.23e+002	6.88e+003 3.13e+002	=	6.87e+003 2.68e+002
F15	7.25e+003 2.75e+002	=	7.14e+003 3.13e+002	7.33e+003 1.97e+002	=	7.31e+003 1.90e+002	7.29e+003 3.47e+002	=	7.22e+003 3.81e+002
F16	2.43e+000 2.21e−001	=	2.52e+000 2.34e−001	2.49e+000 2.47e−001	=	2.43e+000 2.59e−001	**2.44e+000 2.56e−001**	−	2.60e+000 2.21e−001
F17	1.30e+002 2.22e+001	+	**4.93e+001 5.96e+000**	2.69e+002 1.52e+001	+	**2.27e+002 1.18e+001**	2.17e+002 1.25e+001	+	**1.93e+002 1.23e+001**
F18	2.17e+002 1.15e+001	+	**1.94e+002 1.76e+001**	2.81e+002 1.11e+001	+	**2.40e+002 1.19e+001**	2.33e+002 1.40e+001	+	**2.09e+002 1.23e+001**
F19	1.45e+001 1.62e+000	+	**3.52e+000 2.89e+000**	2.13e+001 1.17e+000	+	**1.77e+001 1.01e+000**	1.69e+001 9.44e−001	+	**1.39e+001 1.36e+000**
F20	1.23e+001 2.63e−001	+	**1.15e+001 3.20e−001**	1.34e+001 1.78e−001	+	**1.29e+001 1.96e−001**	1.30e+001 2.26e−001	+	**1.19e+001 4.01e−001**
F21	3.60e+002 9.69e+001	+	**2.96e+002 7.24e+001**	3.97e+002 8.80e+001	+	**2.94e+002 6.46e+001**	3.93e+002 9.51e+001	+	**2.97e+002 7.94e+001**
F22	4.88e+003 1.43e+003	+	**7.00e+002 1.18e+003**	7.29e+003 2.54e+002	=	7.30e+003 2.21e+002	7.55e+003 3.13e+002	+	**7.00e+003 2.98e+002**
F23	7.63e+003 2.38e+002	+	**7.29e+003 3.42e+002**	7.90e+003 2.37e+002	+	**7.71e+003 2.10e+002**	7.87e+003 2.90e+002	+	**7.53e+003 3.73e+002**
F24	**2.00e+002 2.76e−001**	−	2.05e+002 3.83e+000	2.90e+002 7.69e+000	+	**2.08e+002 2.97e+000**	2.08e+002 6.74e+000	=	2.07e+002 6.09e+000
F25	2.38e+002 2.38e+001	=	2.42e+002 1.74e+001	3.32e+002 3.87e+000	+	**3.27e+002 4.31e+000**	3.31e+002 5.32e+000	+	**2.36e+002 2.39e+001**
F26	**2.00e+002 9.54e−003**	−	2.07e+002 2.75e+001	2.02e+002 7.52e−001	+	**2.00e+002 1.13e−002**	**2.00e+002 1.35e−002**	−	2.03e+002 1.91e+001
F27	**3.03e+002 3.24e+000**	−	3.63e+002 3.68e+001	1.33e+003 4.08e+001	+	**1.06e+003 2.42e+002**	1.21e+003 2.40e+002	+	**3.96e+002 7.39e+001**
F28	3.00e+002 0.00e+000	=	3.00e+002 8.96e−006	3.12e+002 3.50e+000	+	**3.00e+002 4.01e−006**	3.00e+002 5.67e−006	=	3.00e+002 4.01e−006
+/ = /−	–		14/5/9	–		22/6/0	–		18/6/4

(*continued on next page*)

Table 24 (*continued*)

Func.	MS-CoDE		SL-CoDE	MS-JADE		SL-JADE	MS-SaDE		**SL-SaDE**
F1	7.94e−003 1.90e−003	+	**6.90e−005 1.75e−005**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	1.06e+008 2.19e+007	+	**9.49e+007 2.26e+007**	9.93e+004 1.03e+005	+	**1.50e+004 1.12e+004**	1.24e+007 3.07e+006	+	**7.10e+006 2.26e+006**
F3	1.40e+011 1.65e+011	=	1.67e+011 1.06e+011	1.46e+006 3.25e+006	=	3.50e+005 7.84e+005	1.15e+006 1.72e+006	=	1.44e+006 2.13e+006
F4	7.22e+004 8.61e+003	+	**6.69e+004 9.35e+003**	4.50e+003 3.36e+003	+	**1.32e−002 6.56e−002**	3.18e+004 3.61e+003	+	**1.94e+004 3.44e+003**
F5	4.11e−002 6.59e−003	+	**4.12e−003 8.45e−004**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	1.53e+002 1.84e+001	+	**1.07e+002 1.80e+001**	9.00e+000 1.24e+001	+	**1.13e+000 4.78e+000**	**1.27e+001 1.24e+000**	−	2.06e+001 2.02e+001
F7	3.69e+002 1.80e+002	=	3.24e+002 8.79e+001	3.48e+000 3.32e+000	=	2.21e+000 1.90e+000	1.89e+001 7.15e+000	+	**7.06e+000 7.11e+000**
F8	2.10e+001 3.32e−002	=	2.09e+001 6.94e−002	2.09e+001 7.18e−002	=	2.09e+001 6.66e−002	2.09e+001 6.15e−002	=	2.09e+001 6.17e−002
F9	3.70e+001 1.18e+000	=	3.70e+001 1.43e+000	**1.44e+001 2.65e+000**	−	2.69e+001 1.90e+000	2.85e+001 1.28e+000	=	2.83e+001 2.23e+000
F10	4.44e+002 6.73e+001	+	**2.22e+002 3.80e+001**	1.09e−001 4.36e−002	+	**7.72e−002 4.34e−002**	**4.93e−002 4.28e−002**	−	1.51e−001 9.86e−002
F11	3.12e+001 2.47e+000	+	**2.42e+001 1.66e+000**	2.06e+000 1.38e+000	+	**0.00e+000 0.00e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F12	2.48e+002 1.64e+001	+	**2.33e+002 1.37e+001**	**2.38e+001 7.24e+000**	−	2.68e+001 4.97e+000	7.09e+001 6.84e+000	+	**5.70e+001 8.52e+000**
F13	2.52e+002 1.59e+001	+	**2.39e+002 1.39e+001**	5.50e+001 2.23e+001	=	5.11e+001 1.29e+001	9.50e+001 1.26e+001	+	**8.52e+001 1.05e+001**
F14	2.34e+003 2.16e+002	+	**2.21e+003 2.45e+002**	6.33e+001 7.71e+001	+	**4.92e−001 1.53e−001**	1.31e+000 6.58e−001	+	**3.24e−002 5.71e−002**
F15	7.38e+003 3.85e+002	=	7.38e+003 2.90e+002	4.50e+003 1.60e+003	+	**3.47e+003 2.65e+002**	4.96e+003 3.10e+002	=	4.90e+003 2.21e+002
F16	2.45e+000 2.55e−001	=	2.39e+000 3.29e−001	2.21e+000 6.90e−001	=	2.01e+000 6.45e−001	1.90e+000 2.59e−001	=	1.95e+000 2.55e−001
F17	6.82e+001 3.49e+000	+	**5.86e+001 2.40e+000**	3.17e+001 7.49e−001	+	**3.04e+001 0.00e+000**	3.05e+001 2.05e−002	+	**3.04e+001 1.37e−006**
F18	2.89e+002 1.44e+001	+	**2.70e+002 9.70e+000**	1.37e+002 5.66e+001	+	**8.11e+001 7.29e+000**	1.44e+002 1.13e+001	+	**1.31e+002 1.11e+001**
F19	1.30e+001 1.08e+000	+	**1.09e+001 1.07e+000**	**1.80e+000 3.17e−001**	−	2.13e+000 2.44e−001	2.28e+000 2.06e−001	+	**2.15e+000 2.11e−001**
F20	1.40e+001 2.45e−001	=	1.40e+001 2.39e−001	9.85e+000 7.34e−001	=	1.01e+001 6.03e−001	1.14e+001 3.48e−001	+	**1.10e+001 3.33e−001**
F21	4.72e+002 2.90e+001	+	**3.62e+002 1.00e+002**	2.80e+002 6.40e+001	=	3.02e+002 6.75e+001	**2.65e+002 5.97e+001**	−	3.24e+002 5.44e+001
F22	3.44e+003 3.13e+002	=	3.27e+003 3.00e+002	1.39e+002 2.37e+001	+	**1.12e+002 2.01e+001**	2.42e+002 4.03e+001	+	**1.15e+002 1.87e+001**
F23	7.88e+003 2.91e+002	=	7.88e+003 2.88e+002	4.04e+003 1.13e+003	=	3.68e+003 3.53e+002	5.46e+003 3.68e+002	+	**5.25e+003 3.91e+002**
F24	3.02e+002 2.93e+000	=	3.01e+002 2.47e+000	**2.03e+002 3.72e+000**	−	2.04e+002 3.98e+000	2.15e+002 8.47e+000	+	**2.07e+002 3.97e+000**
F25	3.00e+002 3.07e+000	=	2.99e+002 3.81e+000	2.51e+002 1.86e+001	=	2.51e+002 3.04e+001	2.93e+002 5.29e+000	+	**2.69e+002 3.69e+001**
F26	2.12e+002 2.77e+000	+	**2.09e+002 2.02e+000**	2.14e+002 3.59e+001	+	**2.00e+002 1.61e−002**	**2.01e+002 1.43e−001**	−	2.07e+002 2.73e+001
F27	1.31e+003 3.12e+001	=	1.31e+003 2.74e+001	**3.28e+002 3.33e+001**	−	3.63e+002 6.41e+001	8.64e+002 9.09e+001	+	**3.78e+002 1.18e+002**
F28	3.61e+002 1.44e+001	+	**3.03e+002 4.46e−001**	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000
+/ = /−	–		16/12/0	–		11/12/5	-		15/9/4

Table 25
Mean and standard deviation of the best error values obtained by SL-DE and MS-DE on CEC 2013 functions at 50*D*.

Func.	MS-DE/rand/1		SL-DE/rand/1	MS-DE/rand/2		SL-DE/rand/2	MS-DE/best/2		SL-DE/best/2
F1	**4.08e−029 1.50e−028**	−	6.58e−028 4.97e−028	6.82e+000 2.70e+000	+	**3.18e−013 4.86e−013**	2.49e−017 1.51e−017	+	**5.41e−028 2.07e−028**
F2	1.58e+006 5.95e+005	+	**2.08e+005 6.00e+004**	3.24e+008 5.99e+007	+	**1.62e+007 5.33e+006**	4.28e+006 1.52e+006	+	**3.23e+005 1.10e+005**
F3	**1.45e+005 1.97e+005**	−	2.93e+006 4.22e+006	6.90e+010 1.25e+010	+	**2.52e+008 4.26e+008**	5.58e+006 1.34e+007	=	1.61e+006 1.98e+006
F4	1.90e+004 5.26e+003	+	**4.39e+002 2.10e+002**	9.36e+004 9.23e+003	+	**7.88e+004 8.20e+003**	5.70e+004 8.83e+003	+	**1.42e+003 5.31e+002**
F5	**2.23e−015 4.80e−015**	−	9.58e−015 8.85e−015	6.58e−001 2.66e−001	+	**1.21e−008 7.55e−009**	5.95e−015 5.79e−015	=	8.75e−015 7.80e−015
F6	4.36e+001 1.04e+000	=	4.55e+001 8.49e+000	4.88e+001 1.89e+000	+	**4.34e+001 5.23e−006**	4.34e+001 0.00e+000	=	4.31e+001 1.20e+001
F7	**8.16e−001 7.79e−001**	−	8.59e+000 3.75e+000	1.76e+002 1.33e+001	+	**5.30e+001 1.05e+001**	2.67e+001 1.13e+001	+	**9.46e+000 5.76e+000**
F8	2.11e+001 3.39e−002	=	2.11e+001 2.83e−002	2.11e+001 4.68e−002	=	2.11e+001 4.85e−002	2.11e+001 3.26e−002	=	2.11e+001 3.50e−002
F9	7.24e+001 1.69e+000	+	**4.34e+001 2.56e+001**	7.28e+001 1.27e+000	=	7.28e+001 1.19e+000	7.29e+001 1.18e+000	=	7.28e+001 1.44e+000
F10	**2.75e−002 1.68e−002**	−	1.09e−001 6.14e−002	1.16e+002 4.80e+001	+	**1.11e−002 8.51e−003**	**1.72e−002 1.06e−002**	−	3.67e−002 1.84e−002
F11	5.52e+001 4.16e+001	=	3.70e+001 7.51e+000	4.59e+002 2.04e+001	+	**3.92e+002 1.84e+001**	3.64e+002 1.56e+001	+	**3.22e+002 2.33e+001**
F12	3.61e+002 1.21e+001	+	**2.09e+002 1.46e+002**	4.94e+002 1.97e+001	+	**4.12e+002 1.70e+001**	3.88e+002 1.71e+001	+	**3.60e+002 1.70e+001**
F13	3.61e+002 1.36e+001	+	**3.46e+002 1.19e+001**	4.87e+002 2.36e+001	+	**4.13e+002 1.60e+001**	3.76e+002 1.83e+001	=	3.69e+002 1.35e+001
F14	7.61e+003 1.77e+003	+	**1.30e+003 4.26e+002**	1.33e+004 3.11e+002	=	1.32e+004 3.31e+002	1.32e+004 3.86e+002	=	1.31e+004 3.47e+002
F15	1.41e+004 3.07e+002	=	1.40e+004 3.16e+002	1.41e+004 4.29e+002	=	1.41e+004 3.14e+002	1.42e+004 2.74e+002	+	**1.40e+004 3.98e+002**
F16	3.27e+000 2.35e−001	=	3.21e+000 2.95e−001	3.35e+000 2.14e−001	=	3.36e+000 3.30e−001	3.24e+000 3.51e−001	=	3.32e+000 3.28e−001
F17	2.24e+002 4.57e+001	+	**9.60e+001 1.24e+001**	5.33e+002 2.01e+001	+	**4.50e+002 1.87e+001**	4.10e+002 1.97e+001	+	**3.94e+002 1.47e+001**
F18	4.10e+002 1.44e+001	+	**3.94e+002 1.43e+001**	5.31e+002 3.00e+001	+	**4.55e+002 2.17e+001**	4.30e+002 1.30e+001	+	**4.13e+002 1.58e+001**
F19	2.48e+001 6.14e+000	+	**4.95e+000 1.43e+000**	5.23e+001 8.97e+000	+	**3.47e+001 1.74e+000**	3.22e+001 1.63e+000	+	**2.67e+001 4.39e+000**
F20	2.23e+001 3.03e−001	+	**2.14e+001 3.46e−001**	2.36e+001 3.39e−001	+	**2.31e+001 1.88e−001**	2.28e+001 1.99e−001	+	**2.18e+001 2.87e−001**
F21	**5.00e+002 4.10e+002**	−	7.06e+002 4.32e+002	**3.09e+002 1.47e+002**	−	6.42e+002 4.55e+002	**3.35e+002 3.12e+002**	−	8.29e+002 3.99e+002
F22	7.86e+003 2.23e+003	+	**1.15e+003 4.28e+002**	1.41e+004 2.54e+002	=	1.39e+004 3.43e+002	1.43e+004 3.47e+002	+	**1.36e+004 3.18e+002**
F23	1.46e+004 3.39e+002	+	**1.43e+004 4.01e+002**	1.49e+004 3.53e+002	=	1.47e+004 3.53e+002	1.50e+004 3.05e+002	+	**1.46e+004 3.59e+002**
F24	**2.04e+002 4.99e+000**	−	2.31e+002 9.90e+000	4.00e+002 6.40e+000	+	**2.34e+002 2.10e+001**	**2.08e+002 7.00e+000**	−	2.32e+002 1.02e+001
F25	3.14e+002 5.80e+001	=	2.99e+002 7.78e+000	4.62e+002 5.43e+000	+	**4.52e+002 6.03e+000**	4.55e+002 6.92e+000	+	**3.04e+002 3.63e+001**
F26	**2.11e+002 3.43e+001**	−	3.02e+002 5.78e+001	2.40e+002 8.96e+000	+	**2.02e+002 1.26e+000**	**2.01e+002 2.86e−001**	−	2.67e+002 6.86e+001
F27	**4.34e+002 1.29e+002**	−	7.44e+002 1.26e+002	2.27e+003 4.18e+001	+	**2.22e+003 5.76e+001**	2.05e+003 3.50e+002	+	**7.15e+002 1.32e+002**
F28	4.00e+002 0.00e+000	=	5.99e+002 7.56e+002	4.27e+002 8.31e+000	+	**4.00e+002 5.67e−006**	4.00e+002 0.00e+000	=	4.00e+002 8.01e−006
+/ = /−	–		12/7/9	–		20/7/1	–		15/9/4

(*continued on next page*)

Table 25 (*continued*)

Func.	MS-CoDE		SL-CoDE	MS-JADE		SL-JADE	MS-SaDE		SL-SaDE
F1	3.06e+000 4.11e−001	+	**2.07e−001 3.57e−002**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	3.30e+008 6.71e+007	=	3.14e+008 5.42e+007	2.19e+005 2.38e+005	+	**5.24e+004 2.07e+004**	2.57e+007 4.85e+006	+	**1.18e+007 4.34e+006**
F3	8.04e+010 1.14e+010	=	7.91e+010 1.49e+010	1.03e+007 2.44e+007	=	6.20e+006 1.25e+007	2.54e+007 3.40e+007	+	**5.64e+006 7.27e+006**
F4	1.29e+005 1.12e+004	+	**1.16e+005 1.00e+004**	2.48e+003 3.02e+003	+	**3.48e−002 1.51e−001**	3.29e+004 2.96e+003	+	**1.78e+004 2.78e+003**
F5	2.43e+000 2.44e−001	+	**7.74e−001 1.06e−001**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	2.22e+002 2.88e+001	+	5.86e+001 4.28e+000	4.50e+001 7.72e+000	=	4.36e+001 1.04e+000	4.47e+001 8.12e−001	=	4.49e+001 1.68e+000
F7	1.97e+002 1.61e+001	=	**1.92e+002 1.82e+001**	1.85e+001 7.54e+000	=	1.68e+001 6.81e+000	5.69e+001 1.15e+001	+	**3.45e+001 1.36e+001**
F8	2.11e+001 3.82e−002	=	2.11e+001 2.04e−002	2.11e+001 5.25e−002	=	2.11e+001 3.32e−002	2.11e+001 2.58e−002	=	2.11e+001 4.01e−002
F9	7.11e+001 2.03e+000	+	**7.03e+001 1.63e+000**	**3.26e+001 4.39e+000**	−	5.49e+001 2.62e+000	5.65e+001 1.90e+000	=	5.59e+001 2.13e+000
F10	1.86e+003 1.38e+002	+	**1.41e+003 1.36e+002**	1.63e−001 1.06e−001	+	**1.14e−001 7.15e−002**	1.68e−001 1.08e−001	=	1.66e−001 7.16e−002
F11	1.55e+002 9.95e+000	+	**1.27e+002 7.62e+000**	9.43e+000 3.36e+000	+	**0.00e+000 0.00e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F12	5.52e+002 2.40e+001	+	**5.25e+002 1.66e+001**	**5.12e+001 1.23e+001**	−	7.07e+001 1.01e+001	1.74e+002 1.60e+001	+	**1.47e+002 1.64e+001**
F13	5.55e+002 2.37e+001	+	**5.17e+002 2.17e+001**	1.47e+002 3.78e+001	=	1.40e+002 2.09e+001	2.42e+002 2.05e+001	+	**2.17e+002 2.22e+001**
F14	6.17e+003 3.02e+002	=	6.07e+003 3.18e+002	2.04e+002 1.28e+002	+	**1.38e+001 4.14e+000**	9.00e+001 1.58e+001	+	**4.08e+001 7.38e+000**
F15	1.44e+004 2.88e+002	=	1.44e+004 2.52e+002	8.34e+003 3.32e+003	=	7.75e+003 4.75e+002	1.05e+004 3.19e+002	+	**1.01e+004 3.98e+002**
F16	3.38e+000 2.28e−001	=	3.27e+000 3.51e−001	2.95e+000 8.55e−001	+	**2.50e+000 6.07e−001**	2.72e+000 3.02e−001	=	2.70e+000 2.08e−001
F17	2.26e+002 9.89e+000	+	**1.95e+002 8.39e+000**	5.79e+001 2.23e+000	+	**5.08e+001 8.99e−006**	5.37e+001 3.45e−001	+	**5.14e+001 2.19e−001**
F18	6.02e+002 2.28e+001	+	**5.74e+002 1.82e+001**	1.86e+002 1.26e+002	=	1.54e+002 1.48e+001	2.86e+002 9.08e+000	+	**2.73e+002 1.43e+001**
F19	4.71e+001 3.77e+000	+	**3.42e+001 1.98e+000**	5.18e+000 1.31e+000	=	4.84e+000 6.48e−001	4.84e+000 4.55e−001	=	4.96e+000 6.79e−001
F20	2.37e+001 2.74e−001	=	2.36e+001 2.21e−001	1.89e+001 1.11e+000	=	1.93e+001 5.94e−001	2.08e+001 3.08e−001	+	**2.03e+001 4.83e−001**
F21	5.14e+002 2.09e+002	+	**2.24e+002 1.95e+001**	8.00e+002 3.87e+002	=	8.00e+002 3.87e+002	7.04e+002 4.53e+002	=	9.14e+002 3.12e+002
F22	8.29e+003 4.52e+002	=	8.34e+003 4.80e+002	2.06e+002 1.30e+002	+	**2.62e+001 8.65e+000**	5.82e+002 1.56e+002	+	**5.51e+001 5.18e+001**
F23	1.49e+004 3.46e+002	+	**1.47e+004 4.00e+002**	**7.21e+003 1.69e+003**	−	8.33e+003 7.59e+002	1.14e+004 5.09e+002	=	1.14e+004 4.67e+002
F24	3.87e+002 3.41e+000	=	3.87e+002 3.01e+000	2.25e+002 9.84e+000	=	2.30e+002 9.72e+000	**2.30e+002 1.64e+001**	−	2.38e+002 9.97e+000
F25	**3.81e+002 4.72e+000**	−	3.84e+002 3.31e+000	**3.13e+002 1.02e+001**	−	3.65e+002 2.44e+001	3.90e+002 4.80e+000	+	**3.81e+002 9.46e+000**
F26	2.88e+002 5.85e+001	=	3.06e+002 7.53e+001	3.11e+002 5.16e+001	+	**2.65e+002 7.18e+001**	**2.03e+002 4.56e−001**	−	2.80e+002 7.96e+001
F27	2.16e+003 4.61e+001	=	2.15e+003 4.65e+001	7.14e+002 1.52e+002	=	8.18e+002 2.96e+002	1.62e+003 1.24e+002	+	**7.82e+002 2.25e+002**
F28	4.58e+002 6.92e+000	+	**4.09e+002 1.10e+000**	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000	4.00e+002 0.00e+000	=	5.02e+002 5.59e+002
+/ = /−	–		15/12/1	–		9/15/4	–		14/12/2

Table 26
Results of the multi-problem Wilcoxon's test for SL-DE versus MS-DE for CEC 2013 functions at 30*D* and 50*D*.

Algorithm $D = 30$	$+/=/-$	*R+*	*R-*	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs MS-DE/rand/1	14/5/9	290.5	115.5	4.51E-02	Yes	Yes
SL-DE/rand/2 vs MS-DE/rand/2	22/6/0	350	28	9.00E-05	Yes	Yes
SL-DE/best/2 vs MS-DE/best/2	18/6/4	359	19	4.00E-05	Yes	Yes
SL-CoDE vs MS-CoDE	16/12/0	346	32	1.47E-04	Yes	Yes
SL-JADE vs MS-JADE	11/12/5	282	96	2.47E-02	Yes	Yes
SL-SaDE vs MS-SaDE	15/9/4	290	88	1.47E-02	Yes	Yes
Algorithm $D = 50$	$+/=/-$	***R+***	***R-***	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs MS-DE/rand/1	12/7/9	242	136	1.97E-01	No	No
SL-DE/rand/2 vs MS-DE/rand/2	20/7/1	349.5	28.5	1.05E-04	Yes	Yes
SL-DE/best/2 vs MS-DE/best/2	15/9/4	333.5	72.5	2.85E-03	Yes	Yes
SL-CoDE vs MS-CoDE	15/12/1	337.5	40.5	3.30E-04	Yes	Yes
SL-JADE vs MS-JADE	9/15/4	261	117	8.15E-02	No	Yes
SL-SaDE vs MS-SaDE	14/12/2	286	92	1.92E-02	Yes	Yes

Table 27
Average computational time (in seconds) used by all the DE algorithms and their corresponding MS-DE and SL-DE variants.

	30*D*					50*D*				
Func.	DE	MS-DE	SL-DE	*Ratio*1[a]	*Ratio*2[b]	DE	MS-DE	SL-DE	*Ratio*1[a]	*Ratio*2[b]
F1	2.877	32.180	6.724	2.337	4.786	6.080	75.063	13.116	2.157	5.723
F2	4.552	34.222	8.560	1.881	3.998	13.100	80.678	21.021	1.605	3.838
F3	5.948	35.423	9.965	1.675	3.555	19.283	85.858	27.361	1.419	3.138
F4	4.113	33.465	8.057	1.959	4.153	11.885	80.169	19.657	1.654	4.078
F5	3.675	33.053	7.408	2.016	4.462	8.300	77.461	15.622	1.882	4.958
F6	4.084	33.390	8.076	1.978	4.135	11.885	92.345	19.951	1.679	4.629
F7	8.050	37.253	11.741	1.458	3.173	25.213	111.747	32.573	1.292	3.431
F8	6.752	35.930	10.498	1.555	3.423	21.296	108.362	29.039	1.364	3.732
F9	81.151	104.993	82.800	1.020	1.268	227.358	313.728	234.690	1.032	1.337
F10	4.854	34.018	8.278	1.705	4.110	13.993	102.486	21.239	1.518	4.825
F11	4.260	33.492	7.710	1.810	4.344	9.864	100.004	17.106	1.734	5.846
F12	7.777	38.046	11.074	1.424	3.436	26.324	115.022	34.079	1.295	3.375
F13	8.057	41.585	11.388	1.413	3.652	27.122	115.641	34.868	1.286	3.317
F14	5.187	36.366	8.874	1.711	4.098	12.614	101.962	20.345	1.613	5.012
F15	6.398	39.999	9.839	1.538	4.065	18.149	108.889	25.912	1.428	4.202
F16	39.065	73.673	41.951	1.074	1.756	110.938	203.040	118.777	1.071	1.709
F17	3.810	37.031	7.461	1.958	4.963	8.915	98.074	16.518	1.853	5.937
F18	6.160	39.399	9.774	1.587	4.031	20.145	109.149	27.378	1.359	3.987
F19	4.348	37.753	7.963	1.831	4.741	12.698	102.354	20.311	1.600	5.039
F20	6.166	39.128	9.434	1.530	4.147	19.510	108.607	26.649	1.366	4.075
F21	14.212	47.581	17.054	1.200	2.790	48.668	134.994	54.650	1.123	2.470
F22	12.034	44.456	15.281	1.270	2.909	31.239	112.748	38.789	1.242	2.907
F23	15.531	47.936	18.854	1.214	2.543	48.031	115.608	55.576	1.157	2.080
F24	91.639	122.892	93.083	1.016	1.320	265.061	281.968	274.431	1.035	1.027
F25	91.785	117.002	95.104	1.036	1.230	265.777	385.597	274.690	1.034	1.404
F26	96.726	120.514	104.347	1.079	1.155	283.974	415.545	292.597	1.030	1.420
F27	95.105	118.316	102.399	1.077	1.155	276.882	408.145	285.374	1.031	1.430
F28	19.804	48.366	24.122	1.218	2.005	67.409	171.423	73.209	1.086	2.342
Avg.	23.361	53.481	27.065	**1.159**	**1.976**	68.275	154.167	75.912	**1.112**	**2.031**

[a] *Ratio*1: the cost of SL-DE is divided by that of the classical DE algorithm.
[b] *Ratio*2: the cost of MS-DE is divided by that of the SL-DE.

4.5. Comparison with non-DE EAs

In this section, SL-JADE is further compared with seven non-DE EAs that are also selected from the CEC 2013 competition [25]. The selected algorithms are presented as follows:

(1) CDASA: an adaptive ACO algorithm for continuous optimization [23];
(2) CMAES-RIS: a CMA-ES super-fit scheme for the re-sampled inheritance search [6];
(3) f_k-PSO: a new self-adaptive heterogeneous PSO [29];
(4) MVMO-SH: the MVMO with embedded local search and multi-parent crossover strategies [37];
(5) NBIPOPaCMA: one version of the covariance matrix adaptation evolution strategy (CMAES) with restarts [28];
(6) SPSOABC: a hybrid PSO and artificial bee colony algorithm [16];
(7) TPC-GA: a GA with a new three-parent crossover [17].

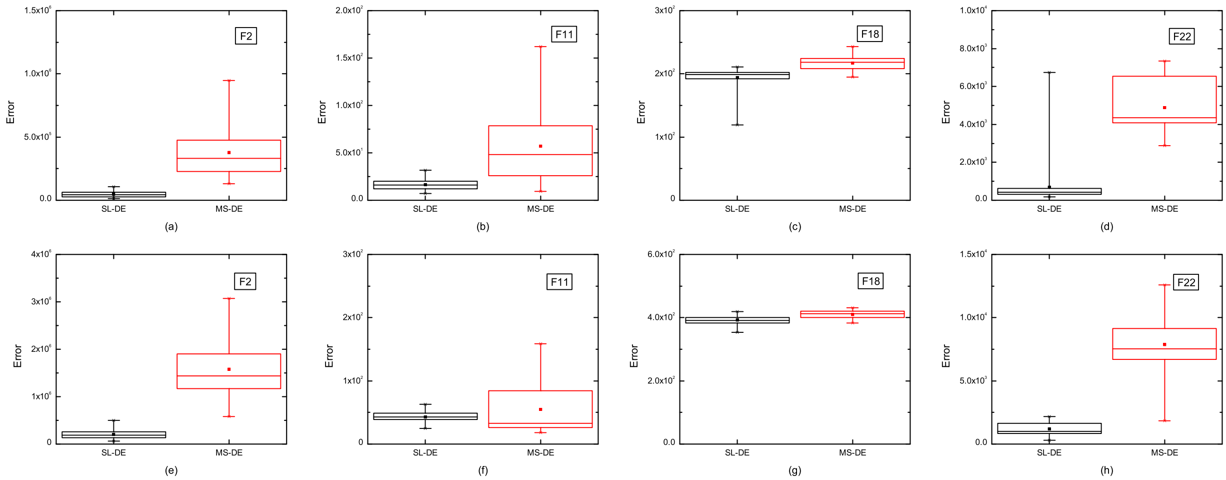


Fig. 11. Box plots of MS-DE and SL-DE with the “DE/rand/1” strategy on F2, F11, F18 and F22 at 30*D* ((a)–(d)) and 50*D* ((e)–(h)).

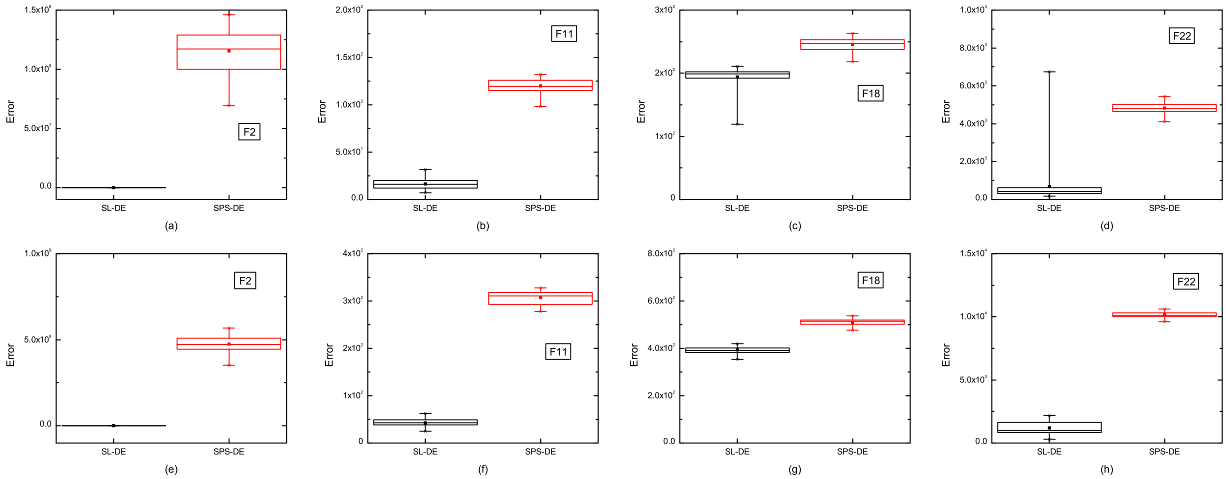


Fig. 12. Box plots of SPS-DE and SL-DE with the “DE/rand/1” strategy on F2, F11, F18 and F22 at 30*D* ((a) - (d)) and 50*D* ((e) - (h)).

These EAs are very powerful algorithms with different features and are thus chosen for the comparisons. Their results are obtained from their original papers and are summarized in Tables 14 and 15. The results of the multi-problem Wilcoxon signed rank tests and the Friedman test are shown in Tables 16 and 17, respectively. Overall, SL-DE can obtain the second rank for functions at 30*D*. It outperforms most competitors and is only outperformed by NBIPOPaCMA. For functions at 50*D*, SL-DE obtains the third rank. It is outperformed by NBIPOPaCMA and MVMO-SH, and it outperforms the other competitors.

The Top-*K* ($K = 1, 2, \ldots, 8$) curves of the eight competitors for these test functions are shown in Fig. 8. It can be observed that SL-JADE, NBIPOPaCMA and MVMO-SH first reach 28 at $K = 7$, and the curve of SL-JADE is only beneath those of NBIPOPaCMA and MVMO-SH.

Note that the main contribution of this study is that it proposes an SL-DE framework with an ASL-based parent selection scheme, not a “Best” algorithm to defeat other state-of-the-art algorithms. In this sense, the results of Tables 14–17 and Fig. 8 clearly demonstrate the effectiveness of SL-DE at solving the test functions when compared with other powerful EAs. In the future work, SL-DE will be further enhanced by introducing a sophisticated local search, e.g., the local search in MVMO-SH.

4.6. Comparison with TBS-DE

Population topology has been widely used in EAs and DE to select parents for mutation. To verify the effectiveness of SL-DE when compared with DE with topology-based selection (TBS-DE), three representative population topologies, i.e., ring, cellular and small-world, are used in this section. These topologies have been introduced into many EAs and have obtained very promising results [13]. Their details can be found in [13]. Here, the DE variants with ring, cellular and small-world topology are named as ring-DE, cell-DE and sw-DE, respectively. In addition, the neighborhood radius is set to 10%*NP* for

Table 28
Mean and standard deviation of the best error values obtained by SL-DE and SPS-DE on CEC 2013 functions at 30*D*.

Func.	SPS-DE/rand/1		SL-DE/rand/1	SPS-DE/rand/2		SL-DE/rand/2
F1	2.88e−10 1.28e−10	+	**7.56e−29 9.99e−29**	8.24e+01 1.12e+01	+	**1.67e−16 2.14e−16**
F2	1.15e+08 1.81e+07	+	**4.77e+04 2.35e+04**	1.72e+08 2.88e+07	+	**4.93e+05 3.99e+05**
F3	3.61e+08 1.04e+08	+	**5.82e+05 1.78e+06**	1.43e+10 2.03e+09	+	**4.60e+04 4.46e+04**
F4	4.96e+04 7.44e+03	+	**7.28e+00 6.24e+00**	5.86e+04 6.86e+03	+	**1.01e+04 6.31e+03**
F5	9.55e−06 2.74e−06	+	**2.01e−15 4.31e−15**	1.89e+01 2.54e+00	+	**6.15e−11 5.60e−11**
F6	**2.21e+01 3.23e+00**	=	2.29e+01 2.24e+01	2.43e+02 2.49e+01	+	**4.06e+00 4.31e+00**
F7	4.92e+01 4.16e+00	+	**7.96e−01 6.54e−01**	1.22e+02 1.22e+01	+	**1.28e+01 4.67e+00**
F8	2.09e+01 5.55e−02	=	2.09e+01 5.31e−02	2.09e+01 5.58e−02	=	2.09e+01 5.14e−02
F9	3.90e+01 1.19e+00	+	**1.45e+01 1.08e+01**	**3.96e+01 1.06e+00**	=	3.97e+01 1.06e+00
F10	1.06e+02 2.03e+01	+	**5.99e−02 3.68e−02**	7.87e+02 9.48e+01	+	**2.22e−03 4.31e−03**
F11	1.20e+02 7.74e+00	+	**1.65e+01 5.88e+00**	**1.82e+02 1.28e+01**	−	1.96e+02 9.03e+00
F12	2.14e+02 1.13e+01	+	**7.68e+01 6.67e+01**	2.66e+02 1.64e+01	+	**2.11e+02 9.86e+00**
F13	2.12e+02 1.28e+01	+	**1.52e+02 4.47e+01**	2.69e+02 1.41e+01	+	**2.08e+02 1.21e+01**
F14	4.19e+03 3.69e+02	+	**7.15e+02 4.55e+02**	**5.02e+03 2.56e+02**	−	6.96e+03 2.23e+02
F15	7.28e+03 2.80e+02	=	**7.14e+03 3.13e+02**	7.34e+03 2.47e+02	=	**7.31e+03 1.90e+02**
F16	**2.48e+00 2.92e−01**	=	2.52e+00 2.34e−01	2.46e+00 2.86e−01	=	**2.43e+00 2.59e−01**
F17	1.57e+02 1.07e+01	+	**4.93e+01 5.96e+00**	2.54e+02 1.22e+01	+	**2.27e+02 1.18e+01**
F18	2.45e+02 1.05e+01	+	**1.94e+02 1.76e+01**	3.21e+02 1.48e+01	+	**2.40e+02 1.19e+01**
F19	1.55e+01 1.06e+00	+	**3.52e+00 2.89e+00**	4.41e+01 6.04e+00	+	**1.77e+01 1.01e+00**
F20	1.29e+01 2.24e−01	+	**1.15e+01 3.20e−01**	1.34e+01 2.30e−01	+	**1.29e+01 1.96e−01**
F21	**2.91e+02 5.25e+01**	=	2.96e+02 7.24e+01	1.16e+03 1.96e+02	+	**2.94e+02 6.46e+01**
F22	4.82e+03 3.06e+02	+	**7.00e+02 1.18e+03**	**5.58e+03 3.02e+02**	−	7.30e+03 2.21e+02
F23	7.63e+03 2.67e+02	+	**7.29e+03 3.42e+02**	7.82e+03 3.46e+02	=	7.71e+03 2.10e+02
F24	2.49e+02 4.82e+00	+	**2.05e+02 3.83e+00**	2.98e+02 5.81e+00	+	**2.08e+02 2.97e+00**
F25	3.19e+02 3.04e+00	+	**2.42e+02 1.74e+01**	**3.26e+02 4.04e+00**	=	3.27e+02 4.31e+00
F26	2.10e+02 1.64e+00	+	**2.07e+02 2.75e+01**	2.16e+02 3.03e+00	+	**2.00e+02 1.13e−02**
F27	1.19e+03 4.98e+01	+	**3.63e+02 3.68e+01**	1.32e+03 2.56e+01	+	**1.06e+03 2.42e+02**
F28	3.00e+02 5.88e−04	+	**3.00e+02 8.96e−06**	1.02e+03 4.60e+01	+	**3.00e+02 4.01e−06**
+/=/−	–		23/5/0	–		19/6/3
Func.	**SPS-ODE**		**SL-ODE**	**SPS-SaDE**		**SL-SaDE**
F1	**0.00e+00 0.00e+00**	−	1.51e−28 1.26e−28	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F2	1.32e+08 1.77e+07	+	**7.54e+04 3.60e+04**	1.37e+07 2.83e+06	+	**7.10e+06 2.26e+06**
F3	5.29e+06 9.03e+06	+	**1.85e+05 3.80e+05**	**5.22e+05 5.82e+05**	=	1.44e+06 2.13e+06
F4	5.28e+04 7.29e+03	+	**7.36e+01 1.54e+02**	3.32e+04 5.91e+03	+	**1.94e+04 3.44e+03**
F5	**0.00e+00 0.00e+00**	−	5.75e−04 2.60e−03	1.15e−30 1.70e−30	+	**0.00e+00 0.00e+00**
F6	**1.64e+01 8.34e+00**	=	2.40e+01 2.14e+01	**1.06e+01 6.64e−01**	−	2.06e+01 2.02e+01
F7	2.74e+01 1.87e+01	+	**9.29e−01 8.66e−01**	**1.53e+00 9.94e−01**	−	7.06e+00 7.11e+00
F8	2.10e+01 4.83e−02	=	**2.09e+01 1.07e−01**	2.09e+01 5.04e−02	=	2.09e+01 6.17e−02
F9	2.80e+01 2.89e+00	+	**1.41e+01 3.60e+00**	**2.81e+01 3.69e+00**	=	2.83e+01 2.23e+00
F10	**4.71e−02 2.90e−02**	=	5.95e−02 2.53e−02	**1.48e−02 1.08e−02**	−	1.51e−01 9.86e−02
F11	9.94e+01 1.55e+01	+	**1.63e+01 4.01e+00**	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F12	1.54e+02 3.46e+01	+	**3.57e+01 1.18e+01**	9.42e+01 9.25e+00	+	**5.70e+01 8.52e+00**
F13	1.59e+02 2.70e+01	+	**8.35e+01 3.01e+01**	1.06e+02 1.75e+01	+	**8.52e+01 1.05e+01**
F14	4.46e+03 3.19e+02	+	**6.00e+02 2.71e+02**	1.98e+01 4.24e+00	+	**3.24e−02 5.71e−02**
F15	6.47e+03 1.03e+03	+	**4.49e+03 8.02e+02**	5.59e+03 2.29e+02	+	**4.90e+03 2.21e+02**
F16	2.50e+00 3.53e−01	=	**2.30e+00 5.37e−01**	2.17e+00 2.63e−01	+	**1.95e+00 2.55e−01**
F17	1.09e+02 1.87e+01	+	**5.07e+01 6.67e+00**	3.10e+01 1.32e−01	+	**3.04e+01 1.37e−06**
F18	2.10e+02 1.08e+01	+	**1.29e+02 2.41e+01**	1.58e+02 1.18e+01	+	**1.31e+02 1.11e+01**
F19	1.19e+01 1.38e+00	+	**2.74e+00 5.90e−01**	2.79e+00 2.67e−01	+	**2.15e+00 2.11e−01**
F20	1.30e+01 2.49e−01	+	**1.07e+01 8.49e−01**	1.14e+01 3.39e−01	+	**1.10e+01 3.33e−01**
F21	3.30e+02 8.28e+01	=	**3.28e+02 7.68e+01**	3.36e+02 9.06e+01	=	**3.24e+02 5.44e+01**
F22	4.69e+03 3.59e+02	+	**5.14e+02 2.31e+02**	2.83e+02 6.93e+01	+	**1.15e+02 1.87e+01**
F23	6.56e+03 1.08e+03	+	**4.45e+03 9.56e+02**	6.01e+03 3.48e+02	+	**5.25e+03 3.91e+02**
F24	2.52e+02 1.45e+01	+	**2.04e+02 3.35e+00**	**2.03e+02 1.16e+00**	−	2.07e+02 3.97e+00
F25	3.21e+02 2.51e+00	+	**2.45e+02 1.51e+01**	2.83e+02 1.99e+01	=	**2.69e+02 3.69e+01**
F26	2.09e+02 3.06e+00	+	**2.04e+02 1.93e+01**	**2.01e+02 1.46e−01**	−	2.07e+02 2.73e+01
F27	1.25e+03 3.25e+01	+	**3.65e+02 3.75e+01**	4.68e+02 1.91e+02	+	**3.78e+02 1.18e+02**
F28	3.00e+02 8.01e−06	=	**2.93e+02 3.65e+01**	3.00e+02 0.00e+00	=	3.00e+02 0.00e+00
+/=/−	–		20/6/2	–		15/8/5

ring-DE; the neighborhood shape is set to $C13$ for cell-DE; and k is set to 2 and β is set to 0.2 for sw-DE with the adding edges policy [14]. Note that the parameters of these topologies, which are recommended in [8,26] and [14], respectively, may be not optimum for the test functions; their optimal setting is beyond the scope of this study. The results of all the functions at 30*D* and 50*D* are summarized in Tables 18 and 19. The results of the multi-problem Wilcoxon signed-rank tests are also shown in Table 20. In addition, the box plots for some tested functions at 30*D* and 50*D* are shown in Fig. 9.

From the results of the functions at 30*D* in Table 18, it is clear that SL-DE can obtain the best results overall in all cases. According to the results of the statistical analysis in Table 20, SL-DE obtains higher $R+$ values than $R-$ values in all cases,

Table 29
Mean and standard deviation of the best error values obtained by SL-DE and SPS-DE on CEC 2013 functions at 50*D*.

Func.	SPS-DE/rand/1		SL-DE/rand/1	SPS-DE/rand/2		SL-DE/rand/2
F1	1.30e−02 2.06e−03	+	**7.54e−28 5.56e−28**	8.90e+03 7.95e+02	+	**2.21e−13 2.53e−13**
F2	4.74e+08 5.22e+07	+	**2.04e+05 9.73e+04**	6.49e+08 7.79e+07	+	**1.48e+07 4.78e+06**
F3	2.44e+10 2.82e+09	+	**5.88e+06 9.50e+06**	9.62e+10 8.60e+09	+	**1.07e+08 1.49e+08**
F4	8.61e+04 7.43e+03	+	**4.21e+02 2.18e+02**	1.06e+05 7.56e+03	+	**7.77e+04 9.51e+03**
F5	4.26e−01 6.11e−02	+	**8.42e−15 5.63e−15**	2.92e+02 1.85e+01	+	**1.00e−08 5.78e−09**
F6	4.68e+01 2.20e−01	+	**4.53e+01 8.69e+00**	9.60e+02 7.77e+01	+	**4.34e+01 6.57e−06**
F7	1.24e+02 5.54e+00	+	**9.02e+00 5.01e+00**	1.94e+02 1.11e+01	+	**5.39e+01 1.43e+01**
F8	2.11e+01 4.72e−02	=	2.11e+01 4.99e−02	2.11e+01 3.85e−02	=	2.11e+01 3.82e−02
F9	7.24e+01 1.74e+00	+	**5.58e+01 2.54e+01**	**7.19e+01 1.31e+00**	−	7.28e+01 1.28e+00
F10	1.15e+03 1.34e+02	+	**1.00e−01 4.61e−02**	3.79e+03 3.34e+02	+	**1.18e−02 7.98e−03**
F11	3.07e+02 1.48e+01	+	**4.31e+01 8.41e+00**	4.93e+02 2.39e+01	+	**3.96e+02 1.72e+01**
F12	4.63e+02 1.39e+01	+	**2.16e+02 1.44e+02**	6.45e+02 3.14e+01	+	**4.11e+02 1.76e+01**
F13	4.63e+02 1.83e+01	+	**3.47e+02 1.51e+01**	6.59e+02 1.99e+01	+	**4.13e+02 2.30e+01**
F14	9.42e+03 3.19e+02	+	**1.30e+03 5.21e+02**	**1.07e+04 2.49e+02**	−	1.32e+04 3.94e+02
F15	1.42e+04 2.72e+02	+	**1.40e+04 4.20e+02**	1.42e+04 4.44e+02	=	**1.41e+04 3.14e+02**
F16	**3.28e+00 2.61e−01**	=	3.36e+00 2.51e−01	**3.36e+00 2.71e−01**	=	3.37e+00 2.68e−01
F17	3.76e+02 1.48e+01	+	**9.85e+01 1.04e+01**	8.87e+02 4.72e+01	+	**4.49e+02 1.44e+01**
F18	5.10e+02 1.66e+01	+	**3.92e+02 1.53e+01**	9.31e+02 4.59e+01	+	**4.59e+02 1.98e+01**
F19	4.11e+01 1.89e+00	+	**5.19e+00 1.46e+00**	8.58e+03 2.54e+03	+	**3.49e+01 1.71e+00**
F20	2.28e+01 2.08e−01	+	**2.14e+01 4.11e−01**	2.36e+01 2.73e−01	+	**2.30e+01 3.84e−01**
F21	**7.63e+02 4.27e+02**	=	9.40e+02 3.47e+02	3.23e+03 2.85e+02	+	**5.10e+02 4.21e+02**
F22	1.02e+04 2.29e+02	+	**1.20e+03 4.94e+02**	**1.17e+04 3.09e+02**	−	1.39e+04 4.19e+02
F23	1.46e+04 2.69e+02	+	**1.42e+04 3.41e+02**	1.48e+04 4.65e+02	=	**1.46e+04 3.04e+02**
F24	3.54e+02 6.31e+00	+	**2.33e+02 1.05e+01**	3.98e+02 4.45e+00	+	**2.30e+02 1.39e+01**
F25	4.30e+02 3.38e+00	+	**3.00e+02 8.13e+00**	**4.42e+02 6.03e+00**	−	4.51e+02 5.93e+00
F26	**3.07e+02 3.51e+01**	=	3.11e+02 5.79e+01	3.15e+02 2.79e+01	+	**2.02e+02 1.12e+00**
F27	2.15e+03 4.84e+01	+	**7.10e+02 1.11e+02**	2.24e+03 3.98e+01	+	**2.20e+03 4.79e+01**
F28	4.01e+02 1.73e−01	+	**4.00e+02 0.00e+00**	1.82e+03 7.67e+01	+	**4.00e+02 0.00e+00**
+/ = /−	–		24/4/0	–		20/4/4
Func.	**SPS-ODE**		**SL-ODE**	**SPS-SaDE**		**SL-SaDE**
F1	**1.35e−29 5.12e−29**	−	7.53e−28 5.19e−28	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F2	4.88e+08 5.97e+07	+	**3.22e+05 1.13e+05**	2.73e+07 9.06e+06	+	**1.04e+07 3.27e+06**
F3	1.37e+10 1.68e+10	+	**1.04e+07 1.40e+07**	8.14e+06 8.48e+06	+	**4.43e+06 6.57e+06**
F4	9.53e+04 7.41e+03	+	**1.54e+03 6.43e+02**	3.92e+04 5.86e+03	+	**1.75e+04 2.95e+03**
F5	**9.47e−16 3.61e−15**	−	1.08e+00 4.51e+00	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F6	**4.34e+01 7.52e−05**	−	4.57e+01 8.90e+00	**4.34e+01 4.98e−06**	−	4.74e+01 1.24e+01
F7	1.36e+02 7.96e+00	+	**9.54e+00 4.31e+00**	**1.00e+01 4.68e+00**	−	3.63e+01 1.41e+01
F8	2.11e+01 4.21e−02	=	2.11e+01 3.78e−02	2.11e+01 3.75e−02	=	2.11e+01 3.56e−02
F9	5.84e+01 3.81e+00	+	**2.98e+01 7.24e+00**	**4.19e+01 1.15e+01**	−	5.51e+01 2.23e+00
F10	7.19e+01 3.78e+02	+	**1.32e−01 7.97e−02**	**5.39e−02 2.89e−02**	−	2.17e−01 1.24e−01
F11	2.66e+02 1.75e+01	+	**5.14e+01 9.79e+00**	0.00e+00 0.00e+00	=	0.00e+00 0.00e+00
F12	3.17e+02 6.44e+01	+	**7.67e+01 2.47e+01**	2.09e+02 1.91e+01	+	**1.44e+02 1.37e+01**
F13	3.54e+02 2.21e+01	+	**1.64e+02 3.13e+01**	2.42e+02 2.43e+01	+	**2.20e+02 1.78e+01**
F14	9.68e+03 3.99e+02	+	**1.37e+03 4.94e+02**	2.00e+02 2.09e+01	+	**4.28e+01 1.17e+01**
F15	1.21e+04 1.70e+03	+	**1.04e+04 2.57e+03**	1.14e+04 4.70e+02	+	**1.02e+04 4.31e+02**
F16	3.53e+00 2.55e−01	+	**3.34e+00 2.98e−01**	2.95e+00 2.59e−01	+	**2.63e+00 2.87e−01**
F17	2.87e+02 2.77e+01	+	**1.07e+02 1.51e+01**	5.78e+01 8.60e−01	+	**5.14e+01 2.68e−01**
F18	4.03e+02 1.67e+01	+	**3.64e+02 7.03e+01**	3.12e+02 1.33e+01	+	**2.69e+02 1.77e+01**
F19	2.50e+01 1.94e+00	+	**6.10e+00 1.11e+00**	6.41e+00 3.88e−01	+	**4.79e+00 6.46e−01**
F20	2.30e+01 2.45e−01	+	**2.14e+01 7.12e−01**	2.10e+01 3.84e−01	+	**2.04e+01 5.85e−01**
F21	**8.33e+02 3.47e+02**	=	9.02e+02 3.41e+02	**5.71e+02 4.39e+02**	−	8.62e+02 3.58e+02
F22	1.03e+04 5.87e+02	+	**1.34e+03 4.85e+02**	7.41e+02 1.61e+02	+	**9.80e+01 1.64e+02**
F23	1.33e+04 1.21e+03	+	**1.07e+04 2.69e+03**	1.22e+04 8.00e+02	+	**1.12e+04 5.40e+02**
F24	3.67e+02 6.37e+00	+	**2.29e+02 7.00e+00**	**2.13e+02 6.11e+00**	−	2.32e+02 8.11e+00
F25	4.33e+02 5.13e+00	+	**3.00e+02 6.90e+00**	**3.58e+02 3.37e+01**	−	3.81e+02 7.09e+00
F26	2.90e+02 6.75e+01	=	**2.77e+02 6.87e+01**	**2.14e+02 4.63e+01**	−	2.83e+02 8.42e+01
F27	2.17e+03 3.87e+01	+	7.07e+02 1.22e+02	**7.39e+02 1.83e+02**	=	8.52e+02 3.15e+02
F28	4.00e+02 5.67e−06	=	4.00e+02 2.12e+00	4.00e+02 0.00e+00	=	4.00e+02 0.00e+00
+/ = /−	–		21/4/3	–		14/6/8

and the p values are less than 0.05 and 0.1 in seven and nine cases, respectively. These results clearly mean SL-DE is better than its corresponding DE variant for all functions at 30*D*.

For the functions at 50*D* in Table 19, SL-DE is consistently superior to most of the corresponding DE variants. SL-DE can obtain the best results overall in three out of four cases. In the cases of JADE, sw-DE obtains the best results, while SL-DE follows the close behind. The multi-problem Wilcoxon signed-rank test also shows that SL-DE can obtain higher $R+$ values than $R-$ values in all cases, and the p values are less than 0.05 and 0.1 in seven cases for both scenarios.

Moreover, Fig. 9 shows that SL-DE is the best among all the DE variants with different topologies in all the cases.

Table 30
Results of the multi-problem Wilcoxon's test for SL-DE versus SPS-DE for CEC 2013 functions at 30D and 50D.

Algorithm $D = 30$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs SPS-DE/rand/1	23/5/0	384.5	21.5	4.10E−05	+	+
SL-DE/rand/2 vs SPS-DE/rand/2	19/6/3	320.0	58.0	1.65E−03	+	+
SL-ODE vs SPS-ODE	20/6/2	390.0	16.0	2.06E−05	+	+
SL-SaDE vs SPS-SaDE	15/8/5	307.0	99.0	1.64E−02	+	+
Algorithm $D = 50$	$+/=/-$	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs SPS-DE/rand/1	24/4/0	354.0	24.0	7.37E−05	+	+
SL-DE/rand/2 vs SPS-DE/rand/2	20/4/4	331.0	47.0	6.46E−04	+	+
SL-ODE vs SPS-ODE	21/4/3	377.5	28.5	7.04E−05	+	+
SL-SaDE vs SPS-SaDE	14/6/8	249.0	129.0	1.28E−01	=	=

In summary, the overall results of Tables 18–20 and Fig. 9 clearly indicate that SL-DE can improve the performance of DE more effectively than the TBS-DE variants. The superior performance of SL-DE is attributed to the better balance between exploration and exploitation achieved by utilizing the fitness information of the population to guide the selection of parents while preserving the diversity of the population with the built social network.

4.7. Comparison with ProDE and rank-DE

As mentioned above, ProDE [19] and rank-DE [20] are also proposed to improve the performance of DE by selecting promising individuals for mutation. To show the effectiveness of SL-DE as an alternative DE framework, ProDE and rank-DE are considered for comparison. Six classical DE mutation strategies are used. The results of the functions at 30D and 50D are shown in Tables 21 and 22, respectively. The box plots of the DE variants with the "DE/rand/1" strategy for some tested functions at 30D and 50D are shown in Fig. 10.

From the results of functions at 30D in Tables 21 and 23, SL-DE is significantly better than ProDE on 13, 20, 24, 16, 22 and 23 functions in the cases of DE/rand/1, DE/rand/2, DE/best/1, DE/best/2, DE/current-to-best/1, and DE/rand-to-best/1, respectively. SL-DE is significantly superior to rank-DE on 10, 20, 24, 16, 23 and 22 functions in the corresponding case, respectively. From the results of the functions at 50D in Tables 22 and 23, SL-DE is consistently superior to ProDE and rank-DE in most cases, such as those of the functions at 30.

The results of the multi-problem Wilcoxon signed-rank tests in Table 23 show that SL-DE obtains higher $R+$ values than $R-$ values in all cases. For the functions at 30D, there are significant differences in six cases between SL-DE and ProDE and five cases between SL-DE and rank-DE, based on the Wilcoxon test at $\alpha = 0.05$. According to the Wilcoxon test at $\alpha = 0.1$, significant differences are observed in six cases for both scenarios. For the functions at 50D, according to the Wilcoxon tests at $\alpha = 0.05$ and 0.1, there are significant differences in five cases for both scenarios. Moreover, Fig. 10 shows that SL-DE can obtain better results than the competitor in most cases.

In general, these results show that SL-DE is the best among these three DE frameworks overall for the test functions. The superior performance of SL-DE may be due to the fact that the proposed framework can effectively extract the neighborhood relationship information of individuals to guide the mutation operator to generate new solutions.

4.8. Comparison with MS-DE

Recently, a new DE framework, MS-DE, was proposed to enhance the performance of DE [43]. In MS-DE, the fitness and diversity information are simultaneously used to select parents for mutation. Here, a comparison with MS-DE is conducted to further study the performance of SL-DE. Three classical DE algorithms (i.e., DE/rand/1, DE/rand/2 and DE/best/2) and three advanced DE variants (i.e., CoDE, JADE and SaDE) are used for comparison. The results for functions at 30D and 50D are shown in Tables 24 and 25, respectively. In addition, the results of multi-problem Wilcoxon signed-rank tests are summarized in Table 26. The box plots for MS-DE and SL-DE with the "DE/rand/1" strategy are also displayed in Fig. 11.

From Table 24, it is clear that SL-DE is significantly better than MS-DE in most cases. In the cases of DE/rand/1, DE/rand/2 and DE/best/2 at 30D, SL-DE significantly outperforms MS-DE on 14, 22 and 18 functions, respectively. In the cases of CoDE, SaDE and JADE, SL-DE significantly outperforms MS-DE on 16, 15 and 11 functions, respectively. For the functions at 50D shown in Table 25, similar results are obtained as those in Table 24. The results of the statistical analysis in Table 26 also shows that SL-DE obtains higher $R+$ values than $R-$ values in all cases, and the p values are less than 0.05 and 0.1 in most cases. In addition, Fig. 11 clearly demonstrates that SL-DE is better than MS-DE in all the cases.

As discussed in Section 3.4, the complexity of SL-DE is $O(G_{\max} \times NP \times \max\{NP, D\})$, which is lower than that of MS-DE (i.e., $O(G_{\max} \times NP \times NP \times D)$ [43]). To show the efficiency of SL-DE, the average run-times obtained by DE, MS-DE and SL-DE on the test functions are shown in Table 27. In Table 27, *Ratio*1 is the cost of SL-DE divided by the cost of the classical DE algorithm, and *Ratio*2 is the cost of MS-DE divided by the cost of SL-DE. From Table 27, we can see that most *Ratio*1

Table 31
Mean and standard deviation of the best error values obtained by JADE and SL-JADE with different social influence factor (*SIF*) on CEC 2013 functions at 30*D*. The average ranking values (ARV) and final rank by Friedman test are also shown in the last two rows of the table.

	JADE	SL-JADE											
Func.	–		*SIF* = 0.3		*SIF* = 0.4		*SIF* = 0.5		*SIF* = 0.6		*SIF* = 0.7		*SIF* = 0.8
F1	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	1.29e+005 1.64e+005	+	**1.37e+004 7.99e+003**	+	**1.50e+004 1.12e+004**	+	**1.49e+004 9.75e+003**	+	**1.24e+004 9.49e+003**	+	**1.45e+004 1.14e+004**	+	**1.75e+004 1.04e+004**
F3	5.72e+005 1.44e+006	=	4.07e+005 1.10e+006	=	3.50e+005 7.84e+005	=	5.39e+005 1.82e+006	=	3.40e+005 9.97e+005	+	**1.84e+005 6.91e+005**	+	**7.93e+004 4.23e+005**
F4	3.46e+003 3.27e+003	+	**4.17e−002 1.58e−001**	+	**1.32e−002 6.56e−002**	+	**3.74e−003 8.59e−003**	+	**4.22e−001 2.31e+000**	+	**1.10e−002 3.82e−002**	+	**4.59e−002 2.39e−001**
F5	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	8.53e+000 3.01e+000	+	**1.21e+000 4.77e+000**	+	**1.13e+000 4.78e+000**	+	**1.94e−001 2.23e−001**	+	**1.03e+000 4.80e+000**	+	**9.39e−001 4.81e+000**	+	**1.85e−001 7.26e−001**
F7	5.14e+000 4.48e+000	+	**2.93e+000 3.63e+000**	+	**2.21e+000 1.90e+000**	+	**2.27e+000 2.24e+000**	+	**2.46e+000 2.34e+000**	+	**2.29e+000 2.42e+000**	+	**3.72e+000 7.02e+000**
F8	2.09e+001 6.82e−002	=	2.09e+001 5.85e−002	=	2.09e+001 6.66e−002	=	2.10e+001 6.61e−002	=	2.09e+001 4.87e−002	=	2.09e+001 6.51e−002	=	2.09e+001 9.81e−002
F9	**1.44e+001 3.08e+000**	–	2.61e+001 1.91e+000	–	2.69e+001 1.90e+000	–	2.65e+001 1.50e+000	–	2.61e+001 1.92e+000	–	2.66e+001 1.53e+000	–	2.66e+001 2.16e+000
F10	1.18e−001 6.25e−002	+	**8.04e−002 4.87e−002**	+	**7.72e−002 4.34e−002**	+	**7.30e−002 4.73e−002**	+	**7.35e−002 4.06e−002**	+	**6.59e−002 3.29e−002**	+	**7.53e−002 3.96e−002**
F11	1.26e+000 1.17e+000	+	**0.00e+000 0.00e+000**	+	**0.00e+000 0.00e+000**	+	**0.00e+000 0.00e+000**	+	**0.00e+000 0.00e+000**	+	**0.00e+000 0.00e+000**	+	**0.00e+000 0.00e+000**
F12	2.59e+001 5.89e+000	=	2.79e+001 4.11e+000	=	2.68e+001 4.97e+000	=	2.65e+001 5.09e+000	=	2.82e+001 5.43e+000	=	2.73e+001 4.68e+000	=	2.53e+001 5.24e+000
F13	4.99e+001 2.00e+001	=	5.06e+001 1.45e+001	=	5.11e+001 1.29e+001	=	4.87e+001 1.32e+001	=	5.01e+001 1.21e+001	=	4.97e+001 7.73e+000	=	5.18e+001 1.15e+001
F14	3.70e+001 4.06e+001	+	**5.58e−001 1.75e−001**	+	**4.92e−001 1.53e−001**	+	**4.97e−001 1.83e−001**	+	**5.21e−001 2.14e−001**	+	**5.26e−001 2.06e−001**	+	**4.73e−001 1.77e−001**
F15	4.86e+003 2.01e+003	+	**3.42e+003 2.97e+002**	+	**3.47e+003 2.65e+002**	+	**3.55e+003 2.99e+002**	+	**3.42e+003 3.18e+002**	+	**3.55e+003 3.13e+002**	+	**3.47e+003 2.62e+002**
F16	2.43e+000 3.15e−001	+	**2.04e+000 5.83e−001**	+	**2.01e+000 6.45e−001**	+	**2.08e+000 5.47e−001**	+	**2.11e+000 5.65e−001**	+	**1.94e+000 4.94e−001**	+	**2.04e+000 5.45e−001**
F17	3.16e+001 1.01e+000	+	**3.04e+001 3.54e−007**	+	**3.04e+001 0.00e+000**	+	**3.04e+001 0.00e+000**	+	**3.04e+001 3.54e−007**	+	**3.04e+001 6.13e−007**	+	**3.04e+001 3.54e−007**
F18	1.30e+002 5.99e+001	+	**8.05e+001 7.46e+000**	+	**8.11e+001 7.29e+000**	+	**8.10e+001 6.39e+000**	+	**8.01e+001 7.50e+000**	+	**7.93e+001 6.50e+000**	+	**8.04e+001 7.78e+000**
F19	**1.89e+000 4.21e−001**	–	2.14e+000 1.74e−001	–	2.13e+000 2.44e−001	–	2.15e+000 2.14e−001	–	2.11e+000 2.26e−001	–	2.14e+000 2.38e−001	–	2.19e+000 1.77e−001
F20	**9.79e+000 7.34e−001**	–	1.03e+001 4.40e−001	=	1.01e+001 6.03e−001	=	1.00e+001 4.35e−001	–	1.02e+001 4.40e−001	–	1.02e+001 4.98e−001	–	1.03e+001 5.15e−001
F21	3.01e+002 5.92e+001	=	3.23e+002 7.37e+001	=	3.02e+002 6.75e+001	=	3.33e+002 6.18e+001	=	2.99e+002 7.01e+001	=	3.12e+002 7.64e+001	=	3.02e+002 6.75e+001
F22	1.43e+002 2.80e+001	+	**1.08e+002 3.76e+000**	+	**1.12e+002 2.01e+001**	+	**1.08e+002 3.83e+000**	+	**1.08e+002 2.92e+000**	+	**1.13e+002 2.09e+001**	+	**1.04e+002 1.71e+001**
F23	3.69e+003 1.28e+003	=	3.88e+003 4.63e+002	=	3.68e+003 3.53e+002	=	3.81e+003 3.74e+002	=	3.85e+003 4.33e+002	=	3.72e+003 3.35e+002	=	3.75e+003 4.10e+002
F24	2.03e+002 2.48e+000	=	2.03e+002 2.89e+000	=	2.04e+002 3.98e+000	=	2.03e+002 2.96e+000	=	2.04e+002 3.44e+000	=	2.03e+002 2.29e+000	=	2.03e+002 2.38e+000
F25	2.45e+002 2.09e+001	=	2.58e+002 3.40e+001	=	2.51e+002 3.04e+001	=	2.58e+002 3.26e+001	=	2.57e+002 3.19e+001	=	2.57e+002 3.06e+001	–	2.64e+002 2.89e+001
F26	2.17e+002 3.94e+001	+	**2.07e+002 2.81e+001**	+	**2.00e+002 1.61e−002**	+	**2.07e+002 2.68e+001**	+	**2.03e+002 1.86e+001**	+	**2.03e+002 1.84e+001**	+	**2.04e+002 1.93e+001**
F27	3.38e+002 4.49e+001	=	3.41e+002 2.95e+001	–	3.63e+002 6.41e+001	=	3.50e+002 4.24e+001	=	3.60e+002 7.86e+001	=	3.63e+002 7.93e+001	=	3.59e+002 6.37e+001
F28	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	=	3.33e+002 1.81e+002	=	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000
+/=/−	–		13/12/3		13/12/3		13/13/2		13/12/3		14/11/3		14/10/4
ARV	4.64		4.34		3.82		3.89		3.84		3.55		3.91
Final Rank	7		6		2		4		3		1		5

Table 32
Summary of the multi-problem Wilcoxon's test for JADE versus SL-JADE with different social influence factor (*SIF*) on CEC 2013 functions at 30*D*.

Algorithm	(1)	(2)	(3)	(4)	(5)	(6)	(7)
JADE (1)	–	131.0	116.5#	134.0	128.5	119.5#	119.0#
SL-JADE with $SIF = \mathbf{0.3}$ **(2)**	247.0	–	170.5	210.0	130.5#	131.0#	177.0
SL-JADE with $SIF = \mathbf{0.4}$ **(3)**	289.5[a]	235.5	–	192.0	179.5	202.0	206.5
SL-JADE with $SIF = \mathbf{0.5}$ **(4)**	272.0	168.0	186.0	–	191.5	163.5	178.5
SL-JADE with $SIF = \mathbf{0.6}$ **(5)**	249.5	275.5	226.5	186.5	–	173.5	191.0
SL-JADE with $SIF = \mathbf{0.7}$ **(6)**	258.5	275.0	176.0	214.5	232.5	–	220.0
SL-JADE with $SIF = \mathbf{0.8}$ **(7)**	259.0	201.0	199.5	227.5	187.0	186.0	–

[a] means the method in the row significantly improves the method of the corresponding column. # means the method in the column significantly improves the method of the corresponding row. Upper diagonal of level significance is $\alpha = 0.9$ and lower diagonal level of significance is $\alpha = 0.95$.

values are very close to 1, which means that the additional computational cost of SL-DE is trivial in these the cases when compared with the classical DE algorithms. On the other hand, most *Ratio*2 values are higher than 3 for $F1 - F20$. These results clearly demonstrate the efficiency of SL-DE when compared with MS-DE.

Overall, the results from Tables 24 to 27 show that SL-DE can obtain better results than MS-DE on the test functions with lower complexity.

4.9. Comparison with SPS-DE

In the DE variants, the archive-based technology has been introduced to guide the selection of parents for mutation. To show the effectiveness of SL-DE when compared with DE with archive-based selection (ABS), a recent DE variant, SPS-DE [21], is considered here. In SPS-DE, $F = 0, 7$ and $Cr = 0.5$, which are the same as [21]. Four DE algorithms, i.e, DE/rand/1, DE/rand/2, ODE, and SaDE, are used for comparison. The results for functions at 30*D* and 50*D* are shown in Tables 28 and 29, respectively. The results of the multi-problem Wilcoxon signed-rank tests are shown in Table 30. The box plots for SPS-DE and SL-DE with the "DE/rand/1" strategy are shown in Fig. 12.

For the functions at 30*D* in Table 28, it is clear that SL-DE obtains significantly better results than SPS-DE in all cases. Specifically, in the cases of DE/rand/, DE/rand/2, ODE and SaDE, SL-DE is significantly better than SPS-DE on 23, 19, 20 and 15 functions, respectively. From the multi-problem Wilcoxon signed-rank test in Table 30, SL-DE can obtain higher $R+$ values than $R-$ values in all cases, and the *p* values of these cases are all less than 0.05 and 0.1. These results indicate that SL-DE is superior to the corresponding SPS-DE overall for the functions at 30*D*. For the functions at 50*D*, SL-DE is consistently superior to the corresponding SPS-DE in all cases. The results of the Wilcoxon signed-rank test in Table 30 also show the better performance of SL-DE when compared with the corresponding SPS-DE for the functions at 50*D*. Moreover, Fig. 12 shows that SL-DE is better than SPS-DE overall for the selected functions.

4.10. Parameter study

In SL-DE, there is a control parameter, termed the social influence factor (*SIF*), that decides the social influence of better individuals in a social network. To study the sensitivity of SL-DE to the *SIF* value, comparisons of SL-DE with different *SIF* vales are made. In this section, JADE is used for comparison, and six different values of *SIF*, i.e., 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8, are considered. The results for the functions at 30*D* are shown in Table 31. Multi-problem Wilcoxon signed-rank tests results are summarized in Table 32, where the value in the row means the sum of ranks for the functions that the method in the current row outperforms the method in the corresponding column.

From Table 31, it is clear that the SL-JADE variants with different *SIF* values are better than JADE overall. Specifically, when *SIF* is set to 0.3, 0.4, 0.5, 0.6, 0.7, and 0.8, respectively, SL-JADE is significantly better than JADE on 13, 13, 13, 13, 14 and 14 functions, respectively. In addition, based on the Friedman test, the average ranking values (ARVs) of the different SL-JADE variants are very close. The results from Table 32 show that the $R+$ and $R-$ values obtained by the different SL-JADE variants are very close in most cases, which means there is no significant difference in the performance of SL-JADE with different *SIF* values.

In general, these results indicate that SL-JADE with different *SIF* values can obtain significant improvements for JADE. Furthermore, we can conclude that the performance of SL-DE is insensitive to the value of *SIF*, and $SIF \in [0.3, 0.8]$ is a good choice for the test functions in this study.

4.11. Scalability study

To study the scalability of the proposed algorithm, SL-DE is tested on 15 benchmark functions at 1000*D* from the CEC 2013 special session on large-scale global optimization [24]. These functions, denoted as $LF1 - LF15$, are classified into

Table 33
Mean and standard deviation of the best error values obtained by the selected DE algorithms and advanced DE variants and their corresponding SL-DE variants on CEC 2013 functions at 1000*D*.

Func.	DE/rand/2		SL-DE/rand/2	DE/best/1		SL-DE/best/1	DE/current-to-best/1		SL-DE/current-to-best/1
LF1	8.59e+009 8.82e+008	+	**8.85e+007 4.22e+007**	1.31e+011 1.18e+010	+	**2.97e+010 3.92e+009**	1.13e+011 5.84e+009	+	**8.88e+010 4.98e+009**
LF2	2.14e+004 6.40e+002	+	**1.12e+004 4.46e+002**	5.24e+004 2.36e+003	+	**3.46e+004 1.43e+003**	4.28e+004 1.07e+003	+	**3.65e+004 7.04e+002**
LF3	2.16e+001 5.15e−003	=	2.16e+001 7.26e−003	2.13e+001 3.82e−002	+	**2.07e+001 7.16e−002**	2.16e+001 4.65e−003	=	2.16e+001 5.62e−003
LF4	6.64e+011 4.64e+010	+	**2.94e+010 9.32e+009**	1.14e+012 5.39e+011	+	**4.59e+011 2.34e+011**	7.60e+011 1.62e+011	+	**5.81e+011 1.34e+011**
LF5	1.13e+007 6.38e+005	+	**9.39e+006 3.65e+005**	1.81e+007 2.25e+006	+	**1.08e+007 1.37e+006**	1.32e+007 1.76e+006	+	**9.76e+006 1.19e+006**
LF6	1.06e+006 1.25e+003	=	1.06e+006 8.43e+002	1.06e+006 1.16e+003	=	1.06e+006 1.22e+003	**1.06e+006 1.07e+003**	−	1.06e+006 1.15e+003
LF7	4.44e+012 1.59e+012	+	**1.48e+009 4.89e+008**	1.14e+012 8.15e+011	+	**6.82e+009 4.38e+009**	7.56e+011 4.03e+011	+	**2.14e+011 8.04e+010**
LF8	2.08e+013 2.34e+012	−	**8.03e+013 4.76e+013**	1.21e+016 1.35e+016	+	**1.21e+015 8.16e+014**	4.21e+015 3.68e+015	+	**1.62e+015 1.37e+015**
LF9	9.70e+008 2.90e+007	+	**7.61e+008 3.02e+007**	1.32e+009 1.29e+008	+	**8.54e+008 1.24e+008**	1.05e+009 1.19e+008	+	**8.03e+008 6.25e+007**
LF10	9.40e+007 2.31e+005	=	9.40e+007 2.25e+005	9.41e+007 3.39e+005	=	9.41e+007 3.54e+005	9.40e+007 2.69e+005	=	9.41e+007 2.60e+005
LF11	1.19e+015 3.59e+014	+	**1.31e+011 4.83e+010**	1.39e+014 7.12e+013	+	**1.09e+012 6.42e+011**	1.03e+014 3.78e+013	+	**3.34e+013 1.02e+013**
LF12	4.20e+011 4.01e+010	+	**1.83e+010 5.15e+009**	2.90e+012 1.68e+011	+	**4.67e+011 4.89e+010**	1.92e+012 8.33e+010	+	**1.43e+012 4.81e+010**
LF13	6.52e+014 2.00e+014	+	**1.30e+010 3.22e+009**	8.31e+013 3.99e+013	+	**3.28e+010 1.54e+010**	6.01e+013 2.32e+013	+	**1.79e+013 7.07e+012**
LF14	1.39e+015 4.06e+014	+	**2.13e+011 8.30e+010**	1.07e+014 6.70e+013	+	**6.29e+011 2.59e+011**	8.05e+013 3.25e+013	+	**2.36e+013 8.75e+012**
LF15	1.85e+012 6.26e+011	+	**1.74e+009 2.94e+009**	1.56e+015 5.50e+014	+	**1.29e+013 1.08e+013**	4.79e+014 1.82e+014	+	**2.24e+014 5.19e+013**
+/ = /−	–		11/3/1	–		13/2/0	-		12/2/1
Func.	**CoDE**		**SL-CoDE**	**JADE**		**SL-JADE**	**SaDE**		**SL-SaDE**
LF1	3.34e+010 7.02e+008	+	**3.03e+010 7.54e+008**	8.83e+003 4.24e+004	=	5.45e+004 2.72e+005	**5.74e+001 2.78e+002**	−	4.36e+005 1.26e+006
LF2	3.99e+004 5.77e+002	+	**3.92e+004 3.81e+002**	**1.54e+004 6.40e+002**	−	1.68e+004 9.43e+002	**1.83e+003 1.40e+002**	−	8.65e+003 3.42e+003
LF3	2.14e+001 1.58e−002	=	2.14e+001 1.34e−002	2.05e+001 1.98e−002	+	**2.02e+001 5.23e−002**	2.08e+001 9.97e−003	=	2.08e+001 9.68e−003
LF4	1.07e+012 1.10e+011	+	**8.52e+011 1.02e+011**	1.22e+010 5.15e+009	+	**7.58e+009 2.49e+009**	1.16e+011 2.89e+010	+	**5.53e+010 2.02e+010**
LF5	1.24e+007 3.42e+005	+	**1.16e+007 4.29e+005**	**2.42e+006 3.84e+005**	−	2.83e+006 4.87e+005	5.96e+006 4.29e+005	+	**4.93e+006 4.70e+005**
LF6	1.06e+006 1.85e+003	=	1.06e+006 1.25e+003	**1.04e+006 1.34e+004**	−	1.05e+006 3.74e+003	1.05e+006 1.34e+003	=	1.05e+006 1.16e+003
LF7	1.26e+011 3.12e+010	+	**8.97e+010 1.90e+010**	2.94e+007 1.11e+007	+	**1.61e+007 4.76e+006**	3.30e+008 2.88e+008	+	**2.53e+007 8.66e+006**
LF8	1.63e+016 3.88e+015	+	**1.50e+015 1.08e+015**	1.58e+014 5.33e+013	+	**6.57e+013 3.88e+013**	3.02e+015 1.16e+015	+	**1.65e+015 7.51e+014**
LF9	1.03e+009 2.61e+007	+	**9.82e+008 3.26e+007**	**1.88e+008 2.72e+007**	−	2.24e+008 3.07e+007	4.44e+008 4.03e+007	+	**3.80e+008 3.34e+007**
LF10	9.39e+007 2.58e+005	=	9.40e+007 1.66e+005	9.37e+007 1.07e+006	+	**9.30e+007 8.59e+005**	9.31e+007 2.27e+005	=	9.33e+007 3.16e+005
LF11	9.82e+013 1.64e+013	+	**7.98e+013 1.11e+013**	2.02e+009 2.33e+009	+	**5.95e+008 3.02e+008**	7.74e+008 1.42e+008	=	7.30e+008 2.45e+008
LF12	2.38e+012 5.02e+010	+	**2.31e+012 3.44e+010**	1.98e+006 9.85e+006	+	**5.44e+003 4.98e+003**	**1.12e+004 1.43e+004**	−	1.09e+008 5.07e+008
LF13	6.20e+013 1.19e+013	+	**4.83e+013 9.64e+012**	1.57e+009 4.64e+008	+	**9.56e+008 3.32e+008**	1.47e+009 3.02e+008	+	**1.19e+009 3.18e+008**
LF14	1.44e+014 1.57e+013	+	**1.11e+014 2.29e+013**	1.76e+010 8.64e+009	+	**3.88e+009 1.92e+009**	2.03e+010 7.71e+009	+	**8.51e+009 4.85e+009**
LF15	4.32e+013 4.18e+012	+	**3.40e+013 3.14e+012**	1.12e+007 6.93e+006	=	8.62e+006 4.28e+006	3.19e+007 3.89e+006	+	**7.52e+006 5.62e+006**
+/ = /−	–		12/3/0	–		9/2/4	-		8/4/3

Table 34
Results of the multi-problem Wilcoxon's test for SL-DE versus the selected DE algorithms on CEC 2013 functions at 1000*D*.

Algorithm at 1000*D*	+/=/−	*R*+	*R*−	*p*-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/2 vs DE/rand/2	11/3/1	92.5	12.5	1.10E−02	Yes	Yes
SL-DE/best/1 vs DE/best/1	13/2/0	118.5	1.5	8.05E−04	Yes	Yes
SL-DE/current-to-best/1 vs DE/current-to-best/1	12/2/1	114.5	5.5	1.79E−03	Yes	Yes
SL-CoDE vs CoDE	12/3/0	114.5	5.5	1.79E−03	Yes	Yes
SL-JADE vs JADE	9/2/4	96	24	3.82E−02	Yes	Yes
SL-SaDE vs SaDE	8/4/3	96.5	23.5	3.56E−02	Yes	Yes

five types of functions [24]: fully-separable functions ($LF1 - LF3$), functions with a separable subcomponent ($LF4 - LF7$), functions with no separable subcomponents ($LF8 - LF11$), overlapping functions ($LF12 - LF14$) and non-separable functions ($LF15$). In this study, 25 independent runs are carried out for each problem, and *MNFEs* is set to 3×10^6. Three DE algorithms (DE/rand/2, DE/best/1, and DE/current-to-best/1) and three advanced DE variants (CoDE, JADE, and SaDE) are used here. The results are summarized in Tables 33 and 34.

As the results show, SL-DE significantly outperforms the corresponding DE algorithm on most functions. For example, in the cases of DE/best/1 and CoDE, SL-DE is significantly better on 13 and 12 functions, respectively. The results of the multi-problem Wilcoxon signed-rank test in Table 34 show that SL-DE obtains higher $R+$ values than $R-$ values in all cases, and the p values of all cases are less than 0.05. These results clearly demonstrate that SL-DE can benefit the DE algorithms studied in solving large-scale optimization problems.

4.12. Application to real-world problems

To evaluate the performance of SL-DE on real-world problems, 17 functions from the CEC 2011 competition are selected [10]: parameter estimation of frequency-modulated sound waves ($P1$), Lennard-Jones potential ($P2$), optimal control of a nonlinear stirred tank reactor ($P4$), spread spectrum radar polly phase code design ($P7$), transmission network expansion planning ($P8$), large scale transmission pricing ($P9$), circular antenna array design ($P10$), dynamic economic dispatch instance 1 ($P11.1$), dynamic economic dispatch instance 2 ($P11.2$), static economic load dispatch instance 1 ($P11.2$), static economic load dispatch instance 2 ($P11.3$), static economic load dispatch instance 3 ($P11.4$), static economic load dispatch instance 4 ($P11.4$), static economic load dispatch instance 5 ($P11.5$), hydrothermal scheduling instance 1 ($P11.8$), hydrothermal scheduling instance 2 ($P11.9$), and hydrothermal scheduling instance 3 ($P11.10$). More details can be found in [10]. Here, *MNFEs* is set to 150,000, and 30 independent runs are carried out for each problem. The results are shown in Tables 35 and 36.

From the results in Table 35, we can find that SL-DE obtains significantly better results than the corresponding DE algorithms in most cases. For example, in the cases of DE/best/1, DE/best/2, DE/current-to-best/1, CoDE and SaDE, SL-DE is significantly better on 8, 10, 5, 5 and 8 functions, respectively, without losing any functions. Moreover, as the statistical analysis results shown in Table 36, SL-DE obtains higher $R+$ values than $R-$ values in all cases, and the p value is less than 0.05 in five cases and is less than 0.1 in eight cases. These results indicate that SL-DE is an effective alternative for solving real-world applications.

5. Conclusion and future research

Inspired by the imitation phenomenon of social learning in animal societies, an adaptive social learning (ASL) strategy is proposed, and a new DE framework named social learning DE (SL-DE) is developed by introducing ASL into DE. Unlike the classical DE algorithms, SL-DE extracts neighborhood relationship information of individuals in the current population to guide the selection of parents.

Extensive experiments have been carried out to evaluate the effectiveness of SL-DE by comparing it with classical DE algorithms, advanced DE variants, up-to-date EAs and DE variants with other parent selection schemes. The simulation results on a suite of benchmark functions show that SL-DE can achieve a better performance than most algorithms studied. Furthermore, sensitivity to the control parameter and the scalability of SL-DE have been experimentally investigated.

In the future, the present work could be extended in the following directions. First, different calculation models for evaluating social influence will be studied. Second, adaptive or self-adaptive techniques for setting the social influence factor will be investigated. Finally, an extension of SL-DE to the multi-objective optimization will be studied.

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Table 35
Mean and standard deviation of the best error values obtained by SL-DE and the corresponding DE algorithms on real-world problems.

Prob.	DE/rand/1		SL-DE/rand/1	DE/rand/2		SL-DE/rand/2	DE/best/1		SL-DE/best/1
P1	4.10e−001 2.25e+000	=	3.90e−001 2.14e+000	1.32e+001 2.48e+000	+	**3.49e−001 1.73e+000**	1.35e+001 5.86e+000	+	**1.03e+001 5.15e+000**
P2	−9.53e+000 6.55e−001	+	**−1.54e+001 5.87e+000**	−6.93e+000 7.20e−001	+	**−7.61e+000 7.70e−001**	−1.70e+001 3.28e+000	+	**−2.42e+001 2.09e+000**
P4	**1.68e+001 3.14e+000**	−	1.87e+001 3.20e+000	1.68e+001 3.23e+000	=	1.78e+001 3.46e+000	1.71e+001 3.41e+000	=	1.85e+001 3.27e+000
P7	1.72e+000 1.23e−001	=	1.58e+000 3.86e−001	1.78e+000 1.14e−001	=	1.75e+000 8.01e−002	8.89e−001 1.29e−001	+	**7.89e−001 1.20e−001**
P8	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000
P9	1.96e+009 1.28e+008	+	**2.82e+008 5.09e+007**	4.30e+009 1.04e+008	+	**3.95e+009 1.03e+008**	2.66e+009 1.71e+008	+	**4.57e+008 8.86e+007**
P10	**−2.17e+001 1.01e−001**	−	−2.16e+001 1.39e−001	−2.13e+001 1.77e−001	+	**−2.18e+001 9.71e−002**	−1.90e+001 2.75e+000	+	**−2.13e+001 8.32e−001**
P11.1	1.29e+005 2.44e+004	+	**5.26e+004 5.72e+002**	7.06e+007 9.48e+006	+	**2.11e+006 2.65e+005**	2.51e+006 6.85e+005	+	**1.88e+005 7.75e+004**
P11.2	7.27e+007 3.82e+006	=	7.24e+007 3.69e+006	**7.12e+007 2.98e+006**	−	7.25e+007 2.61e+006	7.17e+007 2.59e+006	=	7.21e+007 3.25e+006
P11.3	**1.54e+004 1.81e−004**	−	1.54e+004 1.10e+001	1.54e+004 0.00e+000	=	1.54e+004 2.56e−004	1.55e+004 1.44e+001	=	1.55e+004 1.44e+001
P11.4	1.83e+004 1.55e+002	=	1.82e+004 7.17e+001	1.87e+004 4.32e+001	+	**1.86e+004 5.64e+001**	1.85e+004 1.40e+002	+	**1.84e+004 1.32e+002**
P11.5	3.28e+004 2.09e+001	=	3.28e+004 3.11e+001	3.30e+004 3.09e+001	+	**3.28e+004 3.96e+001**	3.29e+004 8.44e+001	=	3.29e+004 6.76e+001
P11.6	1.38e+005 2.80e+003	+	**1.34e+005 2.54e+003**	1.44e+005 4.07e+003	=	1.43e+005 4.68e+003	1.35e+005 3.93e+003	=	1.35e+005 2.33e+003
P11.7	2.97e+006 6.05e+005	+	**2.31e+006 2.18e+005**	7.29e+008 2.20e+008	+	**7.12e+007 5.18e+007**	2.04e+006 2.40e+005	=	1.99e+006 1.45e+005
P11.8	2.70e+006 7.15e+005	+	**1.27e+006 6.45e+004**	4.95e+007 3.62e+006	+	**2.88e+007 3.97e+006**	1.35e+006 1.29e+006	=	9.78e+005 1.03e+005
P11.9	3.39e+006 6.44e+005	+	**1.78e+006 1.40e+005**	4.96e+007 3.15e+006	+	**3.15e+007 3.73e+006**	2.51e+006 2.27e+006	+	**1.27e+006 1.79e+005**
P11.10	2.70e+006 7.15e+005	+	**1.27e+006 6.45e+004**	4.95e+007 3.62e+006	+	**2.88e+007 3.97e+006**	1.35e+006 1.29e+006	=	9.78e+005 1.03e+005
+/ = /−	–		8/6/3	–		11/5/1	–		8/9/0

Prob.	DE/best/2		SL-DE/best/2	DE/current-to-best/1		SL-DE/current-to-best/1	DE/rand-to-best/1		SL-DE/rand-to-best/1
P1	5.24e+000 5.74e+000	+	**2.35e+000 4.37e+000**	1.02e+001 5.46e+000	=	9.54e+000 6.93e+000	1.43e+001 4.00e+000	+	**1.09e+001 5.31e+000**
P2	−1.02e+001 7.91e−001	+	**−1.12e+001 8.20e−001**	−2.28e+001 4.36e+000	=	−2.21e+001 3.96e+000	−2.22e+001 2.42e+000	+	**−2.40e+001 1.85e+000**
P4	1.60e+001 3.04e+000	=	1.69e+001 3.34e+000	1.63e+001 3.09e+000	=	1.69e+001 3.26e+000	**1.63e+001 3.13e+000**	−	1.84e+001 3.26e+000
P7	1.75e+000 9.52e−002	=	1.74e+000 1.03e−001	1.45e+000 3.79e−001	=	1.61e+000 2.90e−001	9.34e−001 3.84e−001	=	1.01e+000 3.73e−001
P8	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000
P9	2.64e+009 1.33e+008	+	**1.02e+009 8.19e+007**	3.64e+009 1.04e+008	+	**2.89e+009 1.11e+008**	3.47e+009 1.22e+008	+	**2.19e+009 1.24e+008**
P10	−2.17e+001 1.49e−001	=	−2.17e+001 1.46e−001	−2.06e+001 1.11e+000	=	−2.10e+001 1.32e+000	−2.01e+001 1.91e+000	=	−2.00e+001 1.82e+000
P11.1	1.93e+005 1.39e+005	+	**5.26e+004 4.80e+002**	1.74e+006 2.32e+005	+	**1.14e+006 1.63e+005**	1.63e+006 2.65e+005	+	**8.75e+005 1.59e+005**
P11.2	7.25e+007 3.31e+006	=	7.15e+007 3.11e+006	7.25e+007 3.09e+006	=	7.16e+007 2.86e+006	7.23e+007 3.07e+006	=	7.25e+007 2.95e+006
P11.3	1.54e+004 8.69e+000	=	1.55e+004 1.31e+001	1.54e+004 8.34e+000	=	1.55e+004 1.30e+001	1.55e+004 1.03e+001	=	1.55e+004 1.49e+001
P11.4	1.81e+004 4.57e+001	=	1.82e+004 5.65e+001	1.83e+004 8.07e+001	=	1.83e+004 8.35e+001	1.83e+004 9.83e+001	=	1.83e+004 8.31e+001
P11.5	3.28e+004 2.68e+001	+	**3.28e+004 2.55e+001**	3.29e+004 5.97e+001	=	3.29e+004 6.24e+001	3.29e+004 7.73e+001	+	**3.29e+004 5.45e+001**
P11.6	1.38e+005 3.52e+003	+	**1.36e+005 3.25e+003**	1.33e+005 2.16e+003	=	1.32e+005 1.92e+003	1.33e+005 2.57e+003	=	1.33e+005 2.51e+003
P11.7	3.21e+006 8.82e+005	+	**2.52e+006 4.08e+005**	1.96e+006 5.98e+004	=	2.00e+006 1.46e+005	2.02e+006 1.77e+005	=	1.98e+006 1.36e+005
P11.8	2.57e+006 8.56e+005	+	**1.21e+006 1.04e+005**	1.63e+007 5.48e+006	+	**9.53e+005 1.42e+004**	8.37e+006 6.18e+006	+	**9.85e+005 1.31e+005**
P11.9	3.82e+006 9.24e+005	+	**1.71e+006 1.90e+005**	1.65e+007 5.45e+006	+	**1.31e+006 5.07e+005**	8.18e+006 5.44e+006	+	**1.17e+006 1.00e+005**
P11.10	2.57e+006 8.56e+005	+	**1.21e+006 1.04e+005**	1.63e+007 5.48e+006	+	**9.53e+005 1.42e+004**	8.37e+006 6.18e+006	+	**9.85e+005 1.31e+005**
+/ = /−	–		10/7/0	–		5/12/0	–		8/8/1

Prob.	CoDE		SL-CoDE	JADE		SL-JADE	jDE		SL-jDE
P1	1.02e+001 3.36e+000	=	9.54e+000 3.92e+000	3.74e−001 2.05e+000	+	**3.01e−002 6.31e−002**	3.12e−001 4.99e−001	+	**0.00e+000 0.00e+000**
P2	−1.12e+001 9.08e−001	=	−1.10e+001 6.84e−001	**−2.71e+001 5.70e−001**	−	−2.38e+001 8.37e−001	−2.15e+001 9.56e−001	+	**−2.23e+001 6.98e−001**
P4	1.66e+001 3.25e+000	=	1.59e+001 3.05e+000	1.47e+001 1.71e+000	=	1.55e+001 2.77e+000	**1.62e+001 3.05e+000**	−	1.87e+001 3.05e+000
P7	1.59e+000 1.03e−001	=	1.59e+000 8.87e−002	**9.24e−001 2.82e−001**	−	1.17e+000 6.93e−002	1.33e+000 8.12e−002	=	1.33e+000 1.12e−001
P8	2.55e+002 2.61e+001	=	2.55e+002 2.39e+001	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000
P9	1.19e+009 1.02e+008	=	1.17e+009 9.40e+007	1.03e+007 6.57e+006	+	**7.33e+006 7.35e+006**	8.73e+006 1.22e+006	=	1.22e+007 1.24e+007
P10	−1.48e+001 1.03e+000	+	**−1.67e+001 1.58e+000**	−2.15e+001 1.25e−001	=	−2.15e+001 1.12e−001	−2.16e+001 1.32e−001	=	−2.16e+001 1.63e−001
P11.1	5.43e+007 8.00e+006	+	**5.00e+007 8.39e+006**	5.23e+004 5.35e+002	=	5.24e+004 5.46e+002	6.40e+004 4.80e+003	+	**5.23e+004 6.34e+002**
P11.2	7.26e+007 3.46e+006	=	7.13e+007 3.12e+006	7.24e+007 3.77e+006	=	7.27e+007 3.29e+006	7.26e+007 3.29e+006	=	7.20e+007 2.54e+006
P11.3	1.55e+004 2.92e+000	+	**1.54e+004 1.05e+000**	1.54e+004 0.00e+000	=	1.54e+004 8.03e−001	1.54e+004 2.23e−001	=	1.54e+004 3.73e−001
P11.4	1.96e+004 3.22e+002	=	1.95e+004 2.63e+002	1.83e+004 4.09e+002	=	1.82e+004 2.71e+002	**1.83e+004 8.73e+001**	−	1.83e+004 8.94e+001
P11.5	3.30e+004 3.49e+001	=	3.30e+004 4.34e+001	3.29e+004 6.11e+001	=	3.29e+004 1.17e+002	**3.28e+004 3.50e+001**	−	3.28e+004 3.89e+001
P11.6	6.06e+005 2.00e+005	+	**3.98e+005 1.98e+005**	1.36e+005 3.41e+003	+	**1.33e+005 4.37e+003**	1.34e+005 1.60e+003	+	**1.30e+005 1.11e+003**
P11.7	7.52e+007 6.11e+007	+	**3.94e+007 2.69e+007**	2.04e+006 9.89e+004	+	**1.93e+006 2.26e+004**	2.13e+006 1.52e+005	+	**2.01e+006 8.07e+004**
P11.8	1.48e+007 3.27e+006	=	1.41e+007 2.61e+006	9.47e+005 1.13e+004	+	**9.42e+005 5.12e+003**	1.04e+006 4.90e+004	+	**9.48e+005 1.15e+004**
P11.9	1.68e+007 2.75e+006	=	1.65e+007 2.26e+006	1.26e+006 9.26e+004	+	**1.01e+006 1.07e+005**	1.56e+006 1.21e+005	+	**1.17e+006 1.68e+005**
P11.10	1.48e+007 3.27e+006	=	1.41e+007 2.61e+006	9.47e+005 1.13e+004	+	**9.42e+005 5.12e+003**	1.04e+006 4.90e+004	+	**9.48e+005 1.15e+004**
+/ = /−	–		5/12/0	–		7/8/2	–		8/6/3

Prob.	ODE		SL-ODE	SaDE		SL-SaDE
P1	**2.68e+000 4.61e+000**	−	5.63e+000 6.42e+000	2.63e+000 2.54e+000	+	**7.15e−001 8.46e−001**
P2	−2.07e+001 5.62e+000	+	**−2.70e+001 6.89e−001**	−2.16e+001 8.12e−001	+	**−2.22e+001 5.67e−001**
P4	**1.51e+001 2.34e+000**	−	1.85e+001 3.31e+000	1.47e+001 2.13e+000	=	1.53e+001 2.44e+000
P7	6.65e−001 1.10e−001	=	6.73e−001 1.46e−001	1.23e+000 8.33e−002	=	1.27e+000 9.05e−002
P8	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000	2.20e+002 0.00e+000	=	2.20e+002 0.00e+000
P9	2.33e+009 1.31e+008	+	**4.64e+008 5.00e+007**	7.80e+007 5.67e+006	+	**1.06e+007 1.20e+006**
P10	−2.17e+001 9.24e−002	=	−2.17e+001 1.51e−001	−2.12e+001 1.03e−001	+	**−2.15e+001 1.15e−001**
P11.1	7.63e+004 6.09e+003	+	**5.27e+004 6.39e+002**	6.14e+004 1.42e+003	+	**5.40e+004 6.74e+002**
P11.2	6.34e+007 9.32e+005	−	6.40e+007 1.36e+006	7.24e+007 3.04e+006	=	7.27e+007 3.59e+006
P11.3	**1.54e+004 0.00e+000**	=	1.54e+004 8.25e+000	1.55e+004 2.02e+000	=	1.55e+004 3.11e+000
P11.4	1.84e+004 1.31e+002	+	**1.82e+004 5.26e+001**	1.92e+004 1.42e+002	=	1.92e+004 1.32e+002
P11.5	3.28e+004 2.02e+001	=	3.28e+004 3.18e+001	3.30e+004 2.50e+001	=	3.30e+004 2.85e+001
P11.6	1.41e+005 2.19e+003	+	**1.36e+005 2.08e+003**	1.37e+005 1.69e+003	=	1.37e+005 1.34e+003
P11.7	3.29e+006 7.71e+005	+	**2.49e+006 4.54e+005**	2.01e+006 5.15e+004	=	2.02e+006 8.09e+004
P11.8	3.10e+006 6.25e+005	+	**1.32e+006 1.08e+005**	1.03e+006 2.71e+004	+	**1.01e+006 2.60e+004**
P11.9	4.29e+006 9.35e+005	+	**1.90e+006 2.07e+005**	1.61e+006 1.23e+005	+	**1.54e+006 1.08e+005**
P11.10	3.10e+006 6.25e+005	+	**1.32e+006 1.08e+005**	1.03e+006 2.71e+004	+	**1.01e+006 2.60e+004**
+/ = /−	–		9/5/3	–		8/9/0

Table 36
Results of the multi-problem Wilcoxon's test for SL-DE versus the corresponding DE algorithms on real-world problems.

Algorithm	+/ = /−	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
SL-DE/rand/1 vs DE/rand/1	8/6/3	124.5	11.5	3.01E−03	Yes	Yes
SL-DE/rand/2 vs DE/rand/2	11/5/1	134.5	18.5	5.34E−03	Yes	Yes
SL-DE/best/1 vs DE/best/1	8/9/0	128.0	25.0	1.33E−02	Yes	Yes
SL-DE/best/2 vs DE/best/2	10/7/0	115.5	20.5	1.19E−02	Yes	Yes
SL-DE/current-to-best/1 vs DE/current-to-best/1	5/12/0	101.5	34.5	7.68E−02	No	Yes
SL-DE/rand-to-best/1 vs DE/rand-to-best/1	8/8/1	101.0	35.0	8.12E−02	No	Yes
SL-CoDE vs CoDE	5/12/0	131.5	4.5	8.00E−04	Yes	Yes
SL-JADE vs JADE	7/8/2	102.5	50.5	2.04E−01	No	No
SL-jDE vs jDE	8/6/3	116.5	36.5	5.38E−02	No	Yes
SL-ODE vs ODE	9/5/3	118.0	35.0	4.55E−02	Yes	Yes
SL-SaDE vs SaDE	8/9/0	92.0	44.0	2.02E−01	No	No

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