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Limitations in the application of the ideal-diode model to the analysis of luminescence from silicon solar cells

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ABSTRACT

On the basis of the ideal-diode model it is straightforward to exploit photo- and electroluminescence results from silicon solar cells. The value of the quasi-Fermi level splitting deduced from experimental luminescence data can be related to the applied voltage under the assumption of homogeneous carrier distributions. We show from numerical simulation and Shockley-diode analysis that in real solar cells discrepancies may exist between the quasi-Fermi level splitting determined from the photoluminescence radiation, which reflects the carrier distributions in the volume of the device, and the applied voltage. Implications are detailed for the construction of current–voltage characteristics from luminescence analysis.

1. Introduction

We study the application of Kirchhoff's generalised law [1,2] for the analysis of the luminescence of silicon-based diodes with regards to the transfer of ideal diode concepts to real diode applications. For an ideal diode [1] with ideal transport properties of the charge carriers, the splitting of the quasi-Fermi levels for free electrons and holes (E_{Fn}, E_{Fp}) is related to the applied voltage V_a by $E_{Fn} - E_{Fp} = eV_a$. This relation has been applied in solar cell efficiency calculations [3] and is taken to hold in real high-quality solar cells [4]. It is also useful to estimate the open-circuit voltage from the evaluation of the photoluminescence yield, taking into account smaller modifications through terms that describe the out-coupling of photons from the device. The reader is referred to the literature for the experimental application of electro- and photoluminescence for solar cell characterisation, e.g. [1–7].

The ideal transport properties of the ideal-diode model do not hold for silicon-based devices in which the values of the carrier mobilities are far from infinity. A recent study on heterojunction silicon solar cells has shown that the quasi-Fermi level splitting becomes voltage-independent and thus does not match with the applied voltage at lower voltages [8]. Here we study implications of the non-finite mobilities on the spatial variation of $E_{Fn} - E_{Fp}$ in real silicon-based solar cells by Shockley-diode analysis and numerical modelling. The latter is helpful in gaining further insight into the device physics of solar cells as it provides the numerical solution of the semiconductor device equations and assists in the discussion of the relation between the measured luminescence yield, the derived quasi-Fermi level splitting and the applied voltage of a solar cell.

2. Device modelling

In devices like *pn* diodes, the excess carrier profiles depend on the transport equations, on the boundary conditions imposed by extraction at the junction or by the loss by back-contact recombination of minority carriers and on the photogeneration rate. In addition to the Shockley-diode theory [9,10] for the analysis of the *pn*-diode properties, we apply the simulation program SC-Simul [8,11,12] that solves Poisson's equation, the continuity equations for electrons and holes and the current transport equations.

3. Results and discussion

3.1. Shockley-diode

Fig. 1 depicts the quasi-Fermi level splittings of a *pn* diode under illumination with monochromatic light at different V_a and at $T = 300$ K. The minority carrier distributions were calculated with the Shockley diode theory [9,10] and typical material-parameter values for crystalline silicon, assuming a diffusion length of 400 μm and monochromatic illumination with a photogeneration rate profile, determined by an absorption coefficient α of 1000 cm^{-1}.

It is clear that the indicated applied voltages do not correlate with the quasi-Fermi level splittings in the volume of the diode. Closer inspection shows that as a consequence of the Shockley the-

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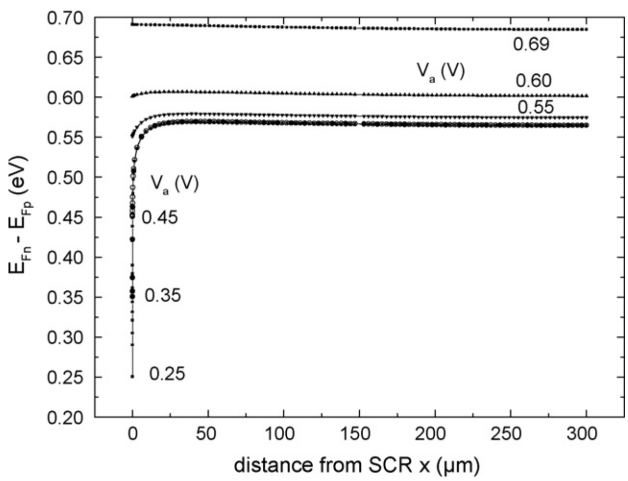

Fig. 1. Quasi-Fermi level splittings under illumination according to the Shockley-diode theory for different applied voltages. The junction and the edge of the space-charge region (SCR) are on the left.

ory only at $x = 0$ does the relation $E_{Fn} - E_{Fp} = eV_a$ hold. In the volume of the solar cell, $E_{Fn} - E_{Fp} \geq eV_a$. The equality $E_{Fn} - E_{Fp} = eV_a$ only holds at voltages around open-circuit.

3.2. Full numerical treatment of a silicon pn diode

The numerical results in Fig. 2 show the current–voltage characteristic of a 205 μm thick crystalline silicon *pn* solar cell with input material parameters for crystalline silicon with long lifetimes so that the diffusion lengths exceed the device thickness. The minority-carrier recombination velocities were set to 1 cm/s. The good photovoltaic properties are reflected by the high open-circuit voltage $V_{oc} = 788$ mV, a short-circuit current density j_{sc} of 40 mA cm^{-2} (determined largely by the photon flux and the assumed front side reflection $R_{fs} = 0$ and back side reflection $R_{bs} = 1$) and a power efficiency of 27.1%.

In Fig. 3, the band diagrams are shown for different applied voltages. The quasi-Fermi levels and the edges of the conduction and valence band, E_c and E_v, are almost constant, indicating almost constant free charge carrier profiles. It is noted that a small gradient in E_{Fn} and E_{Fp} exists for current flow. At $V_{oc} = 788$ mV, the quasi-Fermi

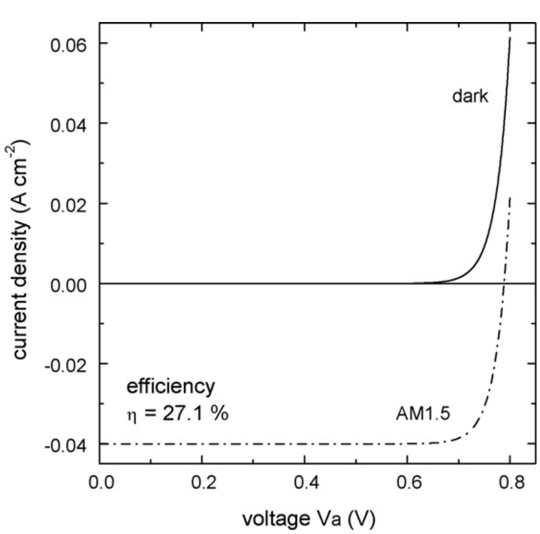

Fig. 2. Dark and illuminated current–voltage characteristics of a c-Si *pn* solar cell. With the front side reflection $R_{fs} = 0$ and back side reflection $R_{bs} = 1$ an efficiency of 27.1% is achieved.

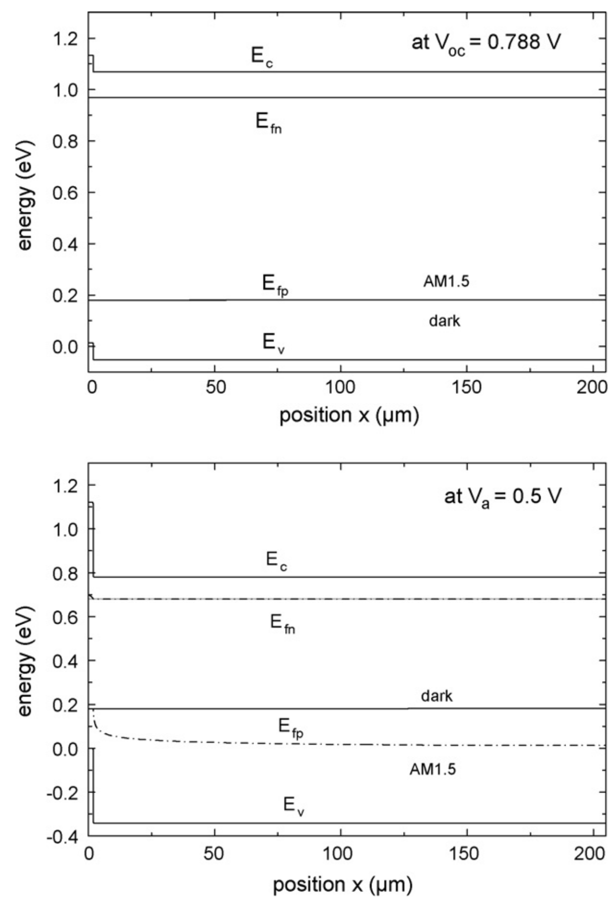

Fig. 3. Band diagrams at open circuit and 0.5 V together with the spatial distributions of the quasi-Fermi levels. The full (dashed) lines indicate E_{fn} and E_{fp} in the dark (under illumination). Illumination is from the left.

level splitting that determines the luminescence yield is almost the same in the dark and under illumination, i.e., in an electroluminescence or photoluminescence experiment the same yield would be detected at open circuit. In fact, $E_{Fn} - E_{Fp} = 0.788$ eV, independent of the position x. At $V_a = 0.5$ V, the quasi-Fermi level splitting under illumination is much larger than in the dark. In agreement with the exponential behaviour of the radiative recombination rate with applied voltage [2] we have $E_{F\,n} - E_{Fp} = 0.5$ eV at $V_a = 0.5$ V in the dark. However, under illumination $E_{Fn} - E_{Fp} > 0.5$ eV in the volume of the cell except at the junction where $E_{Fn} - E_{Fp} = 0.5$ eV.

According to the ideal diode model a quasi-Fermi level splitting of 0.5 eV is expected at a voltage of 0.5 V. Here, in the c-Si diode the excess carrier density in the volume is higher because the mobility is too low compared with the mobility of loss-free transport in the ideal diode. The increased minority-carrier density in the real diode results in a higher photoluminescence yield that becomes voltage independent at lower voltages. Additional simulations show that by increasing the carrier mobility $E_{Fn} - E_{Fp}$ approaches 0.5 eV at 0.5 V under illumination.

A current–voltage characteristic can be constructed from voltage-dependent photoluminescence yields [6]. Under the assumption that the radiative recombination rate R_{BB} is proportional to $\exp[(E_{Fn} - E_{Fp})/(kT)]$ and that $E_{Fn} - E_{Fp}$ is about spatially constant, we construct a voltage scale V by

$$V = V_{oc} - kT \ln\left(\frac{R_{BB,oc}}{R_{BB}}\right) \tag{1}$$

where $R_{BB,oc}$ is R_{BB} at open circuit. The integrated radiative recombination rate R_{BB} is defined by the integral $\int Cnp\,\mathrm{d}x$ where C is the

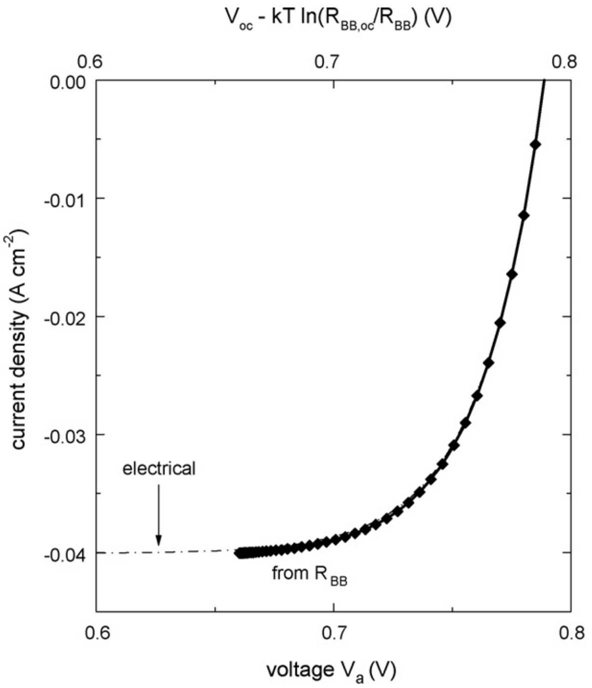

Fig. 4. Current–voltage characteristics with the voltage scale calibrated from photoluminescence for each current-density value (symbols, top abscissa) and of the current density vs. applied voltage (dashed line).

bimolecular recombination coefficient. Fig. 4 shows the electrically measured (simulated) current–voltage characteristic. In addition, the data points represented by symbols show the characteristic constructed from R_{BB} with the top abscissa. No data points exist for the latter for voltages smaller than 0.66 V. The current–voltage construction from the PL data will thus never reach short-circuit because a quasi-Fermi level splitting of 0.66 eV is maintained at $V < 0.66$ V.

The apparently very good agreement between the two characteristics does not hold when the voltage scales are directly compared in Fig. 5. Here it is evident that the voltages determined from R_{BB} and thus the quasi-Fermi level splittings have nothing to do with V_a for voltages smaller than 0.66 V. The constructed voltage agrees with V_a only between MPP at 0.7 V and open circuit. However, for the electroluminescence evaluation there is perfect agreement between $E_{Fn} - E_{Fp}$ and eV_a. The simulated radiative band-to-band recombination rate is confirmed to be exponentially dependent on the applied voltage as suggested by Würfel [2].

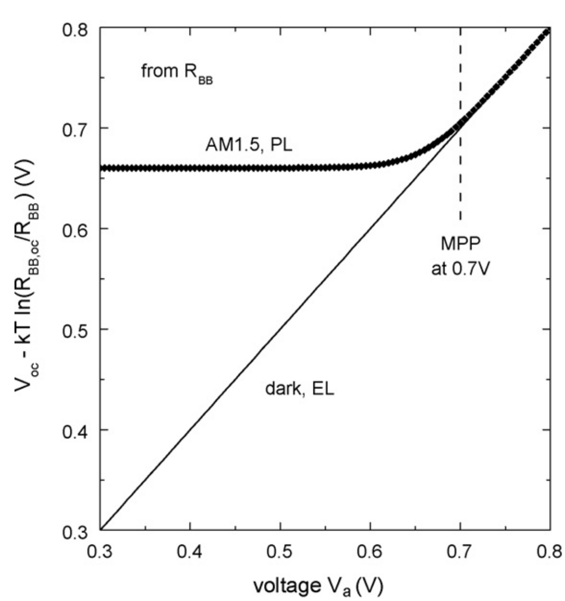

Fig. 5. Comparison of the voltage determined from the radiative recombination rates in the dark and under illumination. The diagonal indicates that in the dark, i.e., for electroluminescence, $E_{Fn} - E_{Fp} = eV_a$. Under illumination, this relation is only valid between MPP and open circuit.

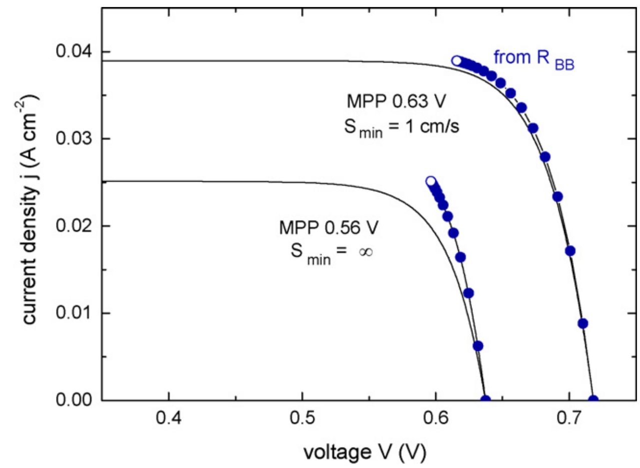

Fig. 6. Current–voltage characteristics of two solar cells with different minority-carrier surface recombination velocities. The voltage scale is calibrated from photoluminescence for each current-density value (symbols). The electrical characteristics are given by full lines. The open symbols indicate the data sets that correspond to short circuit.

3.3. Additional diodes

Fig. 6 illustrates the current–voltage characteristic of a 200 μm thick a-Si:H/c-Si heterojunction solar cell for monochromatic illumination (980 nm, $2.5 \times 10^{17}\,cm^{-2}\,s^{-1}$ photon flux). Here, the results for two different back-contact properties show a reduction in short-circuit current, open-circuit voltage and fill factor for the Ohmic back contact in comparison with the case of low surface recombination velocity. The reconstructed current voltage characteristics with the abscissa values based on Eq. (1) deviate from the electrical characteristic at lower voltages. For the diode with Ohmic back contact the maximum-power point is in a range that cannot be accessed by the reconstructed curve as the quasi-Fermi level splitting even at short circuit (open symbol) is too high.

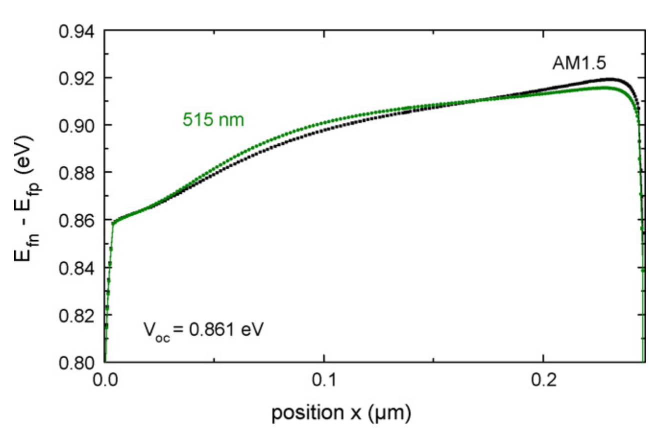

Fig. 7. Quasi-Fermi level splittings in a-Si:H *pin* diode at open circuit for AM1.5 and monochromatic illumination. Illumination is from the left.

Ferraioli et al. [6] studied the influence of contact-related series resistance on the constructed current–voltage characteristic from photoluminescence data of a high efficiency silicon solar cell. The results in Fig. 6 indicate that discrepancies between constructed and electrical current–voltage characteristics are not necessarily related to series resistance but charge carrier loss and lower than ideal fill factor and discrepancies between applied voltage and quasi-Fermi level splitting. Despite these possible large deviations between $E_{Fn} - E_{Fp}$ and eV_a under illumination, it is nevertheless useful to evaluate the photoluminescence yield as an indicator of the open circuit voltage.

Fig. 7 illustrates that in low-mobility thin-film solar cells sufficient gradients in the quasi-Fermi levels may exist. The figure shows the simulated quasi-Fermi level splittings of an amorphous silicon *pin* solar cell at open-circuit for monochromatic illumination (photon flux $5.6 \times 10^{16}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$). Because of the spatial variation of $E_{Fn} - E_{Fp}$, a simple analysis of the relation between photoluminescence yield and applied voltage cannot be applied.

4. Conclusions

The simulated quasi-Fermi level splittings for a high efficiency crystalline silicon solar cell indicate that the ideal diode model can only be applied in the range between maximum power point and open circuit. At lower voltages a higher quasi-Fermi level splitting than predicted by the ideal-diode model is maintained under illumination, the value of which becomes voltage-independent as shown previously for simulated heterojunction silicon solar cells [8]. In contrast, the electroluminescence radiation rate depends exponentially on the applied voltage as the quasi-Fermi level splitting tracks the applied voltage. For a solar cell with lower efficiency and lower fill factor the reconstruction of the current–voltage characteristics from PL data did not follow the electrical curve. For low-mobility solar cells we demonstrated that a precise link between PL yield and open-circuit voltage yet needs to be established.

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