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Extraction of non-ideal junction model parameters from the explicit analytic solutions of its I–V characteristics

Adelmo Ortiz-Conde *, Francisco J. García Sánchez

Laboratorio de Electrónica del Estado Sólido (LEES), Universidad Simón Bolívar, Sartenejas, Caracas 1080A, Venezuela

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Abstract

In modeling semiconductor junctions the extraction of the model's parameters is often hindered by the presence of parasitic series resistance and shunt conductance. We propose a method to extract the intrinsic and extrinsic model parameters using the exact explicit analytic solutions for current and voltage of the junction's I–V characteristics, which are expressed in terms of Lambert W functions. However, direct numerical fitting of these solutions to extract the model's parameters would be an unwieldy and computationally inefficient task. To circumvent this difficulty, the proposed method is based on first calculating the Integral Difference Function, D, from the explicit analytic solutions for I and V. This produces a purely algebraic equation in I and V whose coefficients contain the model's parameters. The coefficients of this auxiliary equation can be quickly determined by direct numerical fitting. From them, all the intrinsic and extrinsic model parameters are then readily obtained at once. The method is tested on representative synthetic I–V characteristics to illustrate the computation process.

1. Introduction

Modeling semiconductor junctions, bipolar and Schottky frequently requires considering the presence of parasitic series resistance and shunt conductances [1]. When such a model is to be used, the necessary procedure to extract the values of the intrinsic junction parameters, ideality factor and reverse current, and the extrinsic, or parasitic, parameters from the experimental current–voltage characteristics might not be a straightforward matter, since the intrinsic parameters are often obscured by the presence of the parasitic elements. The issue has received considerable attention and several extraction methods have been proposed to the effect. Most of them are step-by-step procedures that essentially rely on extracting each parameter from restricted regions of the I–V characteristics where the effect of other parameters is assumed to be negligible. Such approaches work as long as these distinct regions are actually present in the characteristics, which is clearly not the case when the junction exhibits considerable parasitic series resistance and shunt conductance, as happens in some devices, for instance in some organic semiconductor junctions.

2. Explicit analytic solutions

We will consider a generic diode which may be modelled by a single exponential-type ideal junction, series parasitic resistance (R_s), and for completeness sake we

* Corresponding author. Tel.: +58 212 906 4010; fax: +58 212 906 4025.
E-mail addresses: ortizc@usb.ve, ortizc@ieee.org (A. Ortiz-Conde).

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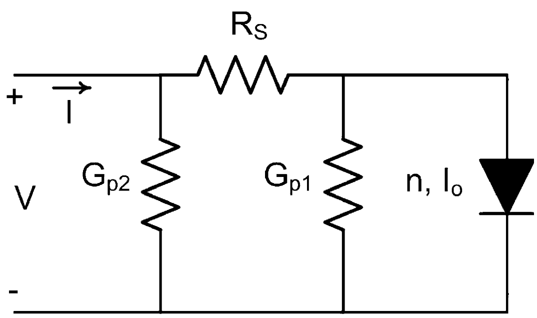


Fig. 1. Generic single-exponential diode equivalent circuit including parasitic series and parallel conductances.

will include the possibility of two shunt loss mechanisms: parallel parasitic conductances, G_{p1} and G_{p2}, at the junction and at the periphery, respectively. Fig. 1 presents the equivalent circuit for such model.

The I–V characteristics mathematical model of this circuit is given by the equation:

$$I = I_0\left\{\exp\left[\frac{V(1+R_sG_{p2}) - IR_s}{nV_{th}}\right] - 1\right\} + (V - IR_s)G_{p1} + VG_{p2}(1 + R_sG_{p1}) \quad (1)$$

where I is the terminal current, V is the terminal voltage, I_0 is the reverse saturation current, n is the ideality factor, and $V_{th} = kT/q$ is the thermal voltage. Parallel conductances, G_{p1} and G_{p2}, instead of parallel resistances, $R_{p1} = 1/G_{p1}$ and $R_{p2,} = 1/G_{p2}$, are used for mathematical convenience. For example, in all the equations the effects of G_{p1} disappear when $G_{p1} = 0$, which is easier to handle than the evaluation of the limit when R_{p1} goes to infinity. This notation allows to easily go from the general model to a particular model.

It is well known that the above transcendental equation may not be solved explicitly in general for I or V using common elementary functions. Therefore, it has been customary to use explicit approximate solutions for modeling purposes. Several of these approximate solutions have been proposed [2,3] which use only elementary functions. However, nowadays exact explicit analytical solutions for I and V already exist [4] that make use of the special function known as Lambert W function [5], a special function which is not expressible in terms of elementary analytical functions. The Lambert W function is defined as the solution to the equation $W(x)\exp[W(x)] = x$. Although this function has not yet been widely used in electronics problems, it has already proved useful in other Physics applications [6,7]. It has also been used for solving some previously analytically unsolved but basic diode [4] and bipolar transistor circuit analysis problems [8], as well as in device modeling formulations [9]. This type of solutions can also be used directly to study illuminated solar cells, as was recently done [10], by adding the photogenerated current to the ideal junction current.

The solution for each variable I and V as an explicit function of the other and of the device model parameters are presented in Table 1. In these equations (2)–(11) W represents the usual short-hand notation for the principal branch of the Lambert W function, $R_{12} \equiv 1/(G_{p1} + G_{p2} + G_{p1}G_{p2}R_s)$ has units of resistance, and $d_1 \equiv 1/(1 + R_sG_{p1})$ and $d_2 \equiv 1/(1 + R_sG_{p2})$ are dimensionless quantities.

These explicit representations are convenient and efficient computational alternatives to iteratively solving the original transcendental Eq. (1) to be used in device models for circuit simulation. However they still result unsuitable for the purpose of extracting the model's parameters directly by numerical fitting.

3. Integral difference auxiliary function

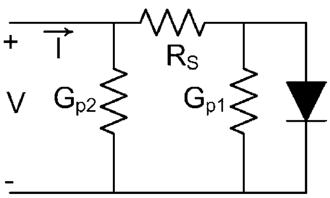
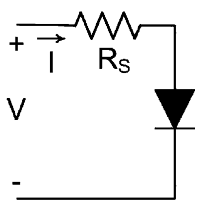
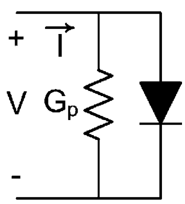
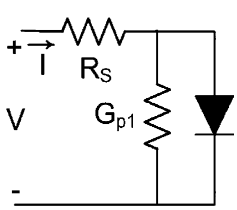
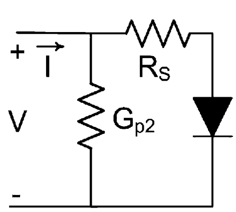
The problem of extracting the model parameters could be attempted by direct vertical optimization of the parameters using the measured I–V data by minimizing the quadratic error on the vertical axis (i.e., the current). However this method would be quite computationally intensive because of the implicit nature of the equation. Recently direct lateral optimization was proposed [11] for a diode exhibiting only significant series resistance, based on the approach of minimizing the error on the horizontal axis (i.e., the voltage). The motivation for doing so, in the case of a diode with only series resistance ($G_{p1} \approx G_{p2} \approx 0$), is that the voltage can be explicitly solved from (1) as a function of the current [Eq. (5)], significantly reducing the computation time.

Other extraction procedures make use of auxiliary functions or operators [12–15]. A useful method involves the integration of the current with respect to voltage and has been successfully used to extract the parameters of a diode [16,17]. The use of integration, instead of the more commonly employed differentiation, makes this method more immune to measurement errors because of the low-pass filter nature of integration. Furthermore, in the case of negligible parallel resistance ($G_{p1} \approx G_{p2} \approx 0$) and for values of $I \gg I_0$, it was proved that the following auxiliary function eliminates the effects of the series resistance [18]:

$$G(I,V) \equiv \frac{IV - 2\int_0^V I\,dV}{I} \approx nV_{th}\left[\ln\left(\frac{I}{I_0}\right) - 2\right]. \quad (12)$$

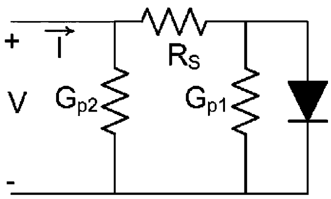
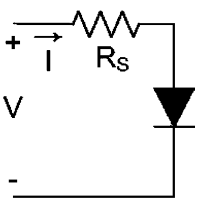
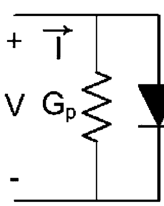
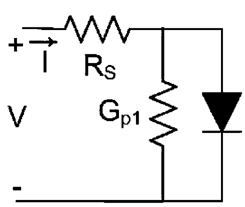
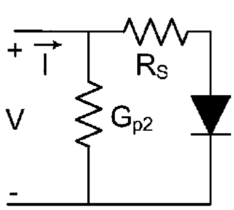
The right hand side of (12) indicates that a plot of auxiliary function G versus $\ln(I)$, for $I \gg I_0$ should produce a straight line. Therefore, the slope and the intercept on the voltage axis allow the determination of the intrinsic n and I_0 parameters, respectively, without any

Table 1
Exact explicit solutions for the several cases

Case	Explicit solutions
	$$I = \frac{nV_{\rm th}}{R_{\rm s}} W\left\{\frac{I_0R_{\rm s}d_1}{nV_{\rm th}}\exp\left[\frac{d_1(V+I_0R_{\rm s})}{nV_{\rm th}}\right]\right\} + d_1(VG_{\rm p1}-I_0)+VG_{\rm p2} \quad (2)$$ $$V = -nV_{\rm th}d_2W\left\{\frac{I_0R_{12}}{nV_{\rm th}d_2}\exp\left[\left(I+\frac{I_0}{d_2}\right)\frac{R_{12}}{nV_{\rm th}}\right]\right\} + Id_2[R_{\rm s}+R_{12}]+I_0R_{12} \quad (3)$$
	$$I = \frac{nV_{\rm th}}{R_{\rm s}} W\left\{\frac{I_0R_{\rm s}}{nV_{\rm th}}\exp\left[\frac{(V+I_0R_{\rm s})}{nV_{\rm th}}\right]\right\} - I_0 \quad (4)$$ $$V = IR_{\rm s} + nV_{\rm th}\ln\left(\frac{I+I_0}{I_0}\right) \quad (5)$$
	$$I = I_0\left[\exp\left(\frac{V}{nV_{\rm th}}\right)-1\right]+G_{\rm p}V \quad (6)$$ $$V = -nV_{\rm th}W\left[\frac{I_0}{nV_{\rm th}G_{\rm p}}\exp\left(\frac{I+I_0}{nV_{\rm th}G_{\rm p}}\right)\right]+\frac{I+I_0}{G_{\rm p}} \quad (7)$$
	$$I = \frac{nV_{\rm th}}{R_{\rm s}} W\left\{\frac{d_1I_0R_{\rm s}}{nV_{\rm th}}\exp\left[\frac{d_1(V+I_0R_{\rm s})}{nV_{\rm th}}\right]\right\} + d_1(VG_{\rm p1}-I_0) \quad (8)$$ $$V = -nV_{\rm th}W\left[\frac{I_0}{nV_{\rm th}G_{\rm p1}}\exp\left(\frac{I+I_0}{nV_{\rm th}G_{\rm p1}}\right)\right] + I\left(R_{\rm s}+\frac{1}{G_{\rm p1}}\right)+\frac{I_0}{G_{\rm p1}} \quad (9)$$
	$$I = \frac{nV_{\rm th}}{R_{\rm s}} W\left\{\frac{I_0R_{\rm s}}{nV_{\rm th}}\exp\left[\frac{(V+I_0R_{\rm s})}{nV_{\rm th}}\right]\right\} - I_0 + VG_{\rm p2} \quad (10)$$ $$V = -d_2nV_{\rm th}W\left[\frac{I_0}{d_2nV_{\rm th}G_{\rm p2}}\exp\left(\frac{I+\frac{I_0}{d_2}}{nV_{\rm th}G_{\rm p2}}\right)\right] + \frac{I+I_0}{G_{\rm p2}} \quad (11)$$

interference from the parasitic series resistance, $R_{\rm s}$. Once these two parameters are known, the value of $R_{\rm s}$ can be found readily. It should be mentioned that the lower limit of integration in (12) does not have to be zero, that is, function G can be applied to any particular region of the forward I–V characteristics. This could be useful whenever it is expected that the intrinsic parameters do not remain constant throughout the whole I–V characteristics and thus a multiple exponential junction model is called for.

Table 2
Coefficients of the equation: $D(I,V) \equiv D_{V1}V + D_{I1}I + D_{I1V1}VI + D_{V2}V^2 + D_{I2}I^2$

Case	D_{I1}	D_{V1}	D_{I1V1}	D_{I2}	D_{V2}
	$-2R_sI_0$ $-2nV_{th}(1+G_{p1}R_s)$	$2R_snV_{th}G_{p1}G_{p2}$ $+2I_0(R_sG_{p2}+1)$ $+2nV_{th}(G_{p1}+G_{p2})$	$1+2R_s(G_{p1}+G_{p2})$ $+2R_s^2G_{p1}G_{p2}$	$-R_s(1+G_{p1}R_s)$	$-G_{p2}-G_{p1}$ $-2R_sG_{p1}G_{p2}$ $-R_sG_{p2}^2-R_s^2G_{p1}G_{p2}^2$
	$-2R^sI_0-2nV^{th}$	$2I_0$	1	$-R_s$	0
	$-2nV_{th}$	$2I_0+2nV_{th}G_p$	1	0	$-G_p$
	$-2R_sI_0$ $-2nV_{th}(1+G_{p1}R_s)$	$2I_0+2nV_{th}G_{p1}$	$1+2R_sG_{p1}$	$-R_s(1+G_{p1}R_s)$	$-G_{p1}$
	$-2R_sI_0-2nV_{th}$	$2I_0(R_sG_{p2}+1)$ $+2nV_{th}G_{p2}$	$1+2R_sG_{p2}$	$-R_s$	$-G_{p2}(1+R_sG_{p2})$

The numerator of Eq. (12) represents what is known as the Integral Difference Function $D(I,V)$ defined as [18]:

$$D(I,V) \equiv \int_0^I V\,\mathrm{d}I - \int_0^V I\,\mathrm{d}V = IV - 2\int_0^V I\,\mathrm{d}V$$
$$= 2\int_0^I V\,\mathrm{d}I - IV. \tag{13}$$

As before, it should be stressed that, although the integrals' lower limits shown in (13) are defined here at the origin for simplicity, this does not need be so. That is, function D can be applied to any particular region between any two points of the forward I–V characteristics. Function D has also found useful applications in other areas, such as for quickly calculating device harmonic distortion [19].

Substitution of (3) into (13) and integrating with respect to I results in a long expression that contains Lambert W functions, V, and I (See Appendix A). Replacing the terms that contain Lambert W functions of V, using Eq. (3), and after some algebraic manipulation, function $D(I,V)$ may be conveniently expressed by a purely algebraic equation of the form:

$$D(I,V) \equiv D_{V1}V + D_{I1}I + D_{I1V1}VI + D_{V2}V^2 + D_{I2}I^2, \tag{14}$$

where the five coefficients are given in terms of the model parameters. They are shown in Table 2 for the several cases considered here. It should be emphasized that algebraic expression (14) is arrived at without using any approximations and that as such it is valid for any region of operation, including $I < I_0$. Unfortunately the five coefficients are not all independent. For example, we easily see that:

$$D^2_{I1V1} = 1 + 4D_{I2}D_{V2}. \tag{15}$$

There are actually four independent coefficients and therefore only four unknowns may be extracted uniquely. In the general case of significant parasitic series resistance and both shunt conductances, the five diode model parameters, n, I_0, R_s, G_{p1}, and G_{p2}, may not be extracted uniquely. The general solution of n, I_0, G_{p1}, and G_{p2}, in terms of R_s, and the five coefficients is presented in Appendix B. Nevertheless, parasitics in practical diodes may be frequently modeled by a single series resistance in combination with a single shunt conductance, representing the dominant shunt loss either at the junction or at the periphery. In such cases all four model parameters may be extracted uniquely. Let us next analyze these cases.

4. Parameter extraction

The parameter extraction procedure consists of fitting (14) to the experimental data to produce the values of the coefficients given by in Table 2; and from them, calculate the diode model parameters. Let us analyze the cases when the coefficients may be uniquely determined.

4.1. Only series resistance

In the not infrequent event that all shunt loss mechanisms could be neglected, case already considered by Banwell and Jayakumar [20], the current can be solved explicitly as (4) (same as Eq. (4) in Ref. [20]), and the solution for the voltage (5) is in terms of common elementary functions. Notice that two of the diode model parameters I_0 and R_s, are given directly by two of the coefficients, D_{V1}, and D_{I2}, respectively. Then, the third parameter, n, is determined from D_{I1}.

4.2. Only shunt loss

Conversely, if the series resistance is negligible, the combination of any shunt loses present yields directly the explicit solution (6) for the current in terms of elementary functions. However solving for the voltage involves a transcendental equation whose explicit solution again may be obtained in terms of Lambert W functions as indicated by (7). Once again, two of the diode model parameters n, and G_p, are given directly by two of the coefficients, D_{I1} and D_{V2}, respectively. The third parameter, I_0, is finally determined from D_{V1}.

4.3. Series resistance and shunt loss only at the junction

When the significant shunt loss occurs only at the junction, and a series parasitic resistance is also present, explicit solutions for I and V using the Lambert W functions are given by (8) and (9). The coefficients obtained from the fitting are used to uniquely calculate the four diode model parameters, n, I_0, R_s, and G_{p1}.

4.4. Series resistance and only peripheral shunt loss

When only shunt losses at the periphery are relevant, the explicit solutions for the current and voltage are given by (10) and (11). As before, fitting (14) to the experimental data produces the corresponding coefficients, and from them the four diode model parameters, n, I_0, R_s, and G_{p2}, can be uniquely calculated.

5. Discussions

Fig. 2 presents synthetic I–V characteristics for the case of series resistance and only peripheral shunt loss, obtained from (10), using parameters values of $I_0 = 10^{-12}$ A, $n = 1.5$, $G_{p1} = 0$ and various combinations of R_s and G_{p2}. The calculations were done with a 5 mV increment using Maple [21] with 20 digits precision. Symbols used in this and in the following figures are

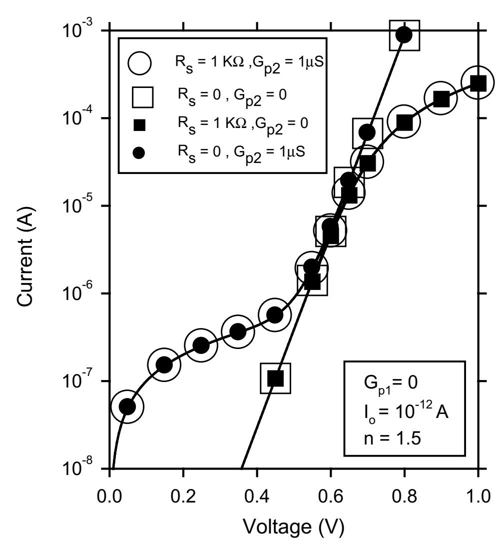


Fig. 2. Test synthetic I–V characteristics for the case of series resistance and only peripheral shunt loss. Symbols are used to identify the several cases and do not represent data points.

not data points but are used to identify the several cases. The ideal case of $R_s = 0$ and $G_{p2} = 0$, identified by large hollow squares, is a straight line. The case when $R_s = 1\ k\Omega$ is significant and $G_{p2} = 0$, identified by small solid squares, produces a straight line for low voltage that bends down for high voltage (i.e., the effects R_s become important at high voltage). The case when only G_{p2} is significant ($G_{p2} = 1\ \mu S$ and $R_s = 0$), is identified by small solid circles. It is a straight line at high voltage and bends up at low voltage (i.e., the effects G_{p2} are important at low voltage). When R_s and G_{p2} are both simultaneously significant ($R_s = 1\ k\Omega$ and $G_{p2} = 1\ \mu S$) is identified by large hollow circles. What is important to notice is that the plot in this case does not present any segment from which the intrinsic parameters could be obtained, because the overlapping effects of R_s and G_{p2} conceal the intrinsic characteristics everywhere. Therefore the intrinsic parameters may not be obtained by any traditional method directly from any portion of these *I*–*V* characteristics.

Fig. 3 presents the calculated *D* as a function of terminal voltage for the *I*–*V* characteristics illustrated in the previous figure. We observe that the ideal case ($R_s = 0$ and $G_{p2} = 0$) and the case of only significant G_{p2} ($G_{p2} = 1\ \mu S$ and $R_s = 0$) both display the same $D(V)$ everywhere. This is because the operator *D* when represented as a function of the terminal voltage *V* eliminates the effect of G_{p2}. We also observe that in the case of only significant R_s ($R_s = 1\ k\Omega$ and $G_{p2} = 0$), *D* differs from the ideal case only at high voltage, where R_s becomes important. The case of $R_s = 1\ k\Omega$ and $G_{p2} = 1\ \mu S$ produces the same *D* for all *V* as the case $R_s = 1\ k\Omega$ and $G_{p2} = 0$.

Fig. 4 shows the same calculated *D* but now as a function of terminal current for the same *I*–*V* characteristics illustrated in Fig. 2. The plots indicate that the ideal case ($R_s = 0$ and $G_{p2} = 0$) and the case of significant R_s ($R_s = 1\ k\Omega$ and $G_{p2} = 0$) both display the same $D(I)$ everywhere. This is because the operator *D* when represented as a function of *I* eliminates the effect of R_s. We also observe that for the case of only significant G_{p2} ($R_s = 0$ and $G_{p2} = 1\ \mu S$), *D* differs from the ideal case only at low voltage where the effect of G_{p2} is significant. The case when $R_s = 1\ k\Omega$ and $G_{p2} = 1\ \mu S$ gives the same *D* for all *I* as the case $R_s = 0$ and $G_{p2} = 1\ \mu S$.

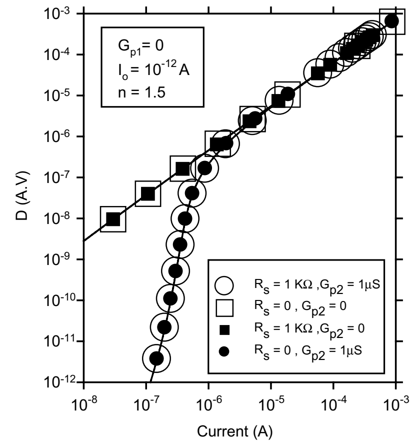


Fig. 4. Calculated *D* as a function of current. Symbols are used to identify the several cases and do not represent data points.

The previous combinations, as well as several other additional cases, were simulated and the quadratic equation of *D* as a function of current and voltage, defined in (14), was then used to extract the simulated parameters. In all cases the extraction procedure succeeded in producing the exact original parameters, within computational accuracy. This means that the errors between the original and the extracted parameters depend only on the computational precision and accuracy.

If we wish to model separately diffusion and generation–recombination current components, a two exponential diode model should be considered. A procedure to convert from single to double exponential diode model was presented previously [22].

Although this method is presented as a parameter extraction technique in diodes, it is applicable to other physical problems such as, for example, the post breakdown behaviour of MOS capacitors [23] or organic solar cells [24].

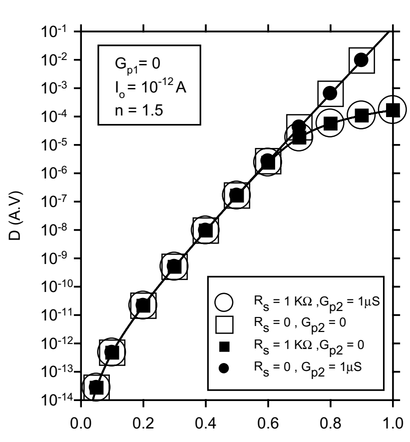


Fig. 3. Calculated *D* as a function of voltage, for the case of series resistance and only peripheral shunt loss. Symbols are used to identify the several cases and do not represent data points.

6. Conclusions

A new method to extract the intrinsic and extrinsic model parameters of semiconductor junctions containing parasitic resistance and shunt conductance has been presented. The Integral Difference Function *D* was applied to the exact explicit analytical solutions of the junction's *I*–*V* characteristics. Since the resulting equation of D is expressed in terms of Lambert W functions, parameter extraction by direct numerical fitting of the equations was not attempted as it would result computationally impractical. Instead, the resulting D is reduced to a purely algebraic equation with terms in I, V, IV, I^2, and V^2. The intrinsic and extrinsic model parameters can then be readily determined from the coefficients of these terms. The applicability of the procedure was exemplified for the cases of only series resistance, only shunt conductance, and two series resistance and shunt conductance combinations. The quadratic two-dimensional fitting process represents a fast and accurate parameter extraction procedure. The application of the method to non-ideal synthetic I–V characteristics demonstrates that it is theoretically exact, its correctness depending only on numerical computation inaccuracies.

Appendix A

The integration of V, defined in (3), with respect to I in terms of Lambert W functions is:

$$\int_0^I V\,\mathrm{d}I = \mathrm{p}_1 - \mathrm{p}_2 W[x\exp(y)] - \frac{\mathrm{p}_2}{2}W[x\exp(y)]^2, \quad \text{(A.1)}$$

where

$$\mathrm{p}_1 \equiv nV_{\mathrm{th}}I_0 + \frac{(1+R_{\mathrm{s}}G_{\mathrm{p2}})I_0^2 + I(I + IR_{\mathrm{s}}G_{\mathrm{p1}} + 2I_0)}{2(G_{\mathrm{p1}} + G_{\mathrm{p2}} + G_{\mathrm{p1}}G_{\mathrm{p2}}R_{\mathrm{s}})}, \quad \text{(A.2)}$$

$$\mathrm{p}_2 \equiv \frac{G_{\mathrm{p1}} + G_{\mathrm{p2}} + G_{\mathrm{p1}}G_{\mathrm{p2}}R_{\mathrm{s}}}{(1+R_{\mathrm{s}}G_{\mathrm{p2}})}(nV_{\mathrm{th}})^2, \quad \text{(A.3)}$$

$$y \equiv \frac{I + I_0(1+G_{\mathrm{p2}}R_{\mathrm{s}})}{nV_{\mathrm{th}}(G_{\mathrm{p1}} + G_{\mathrm{p2}} + G_{\mathrm{p1}}G_{\mathrm{p2}}R_{\mathrm{s}})}, \quad \text{(A.4)}$$

and

$$x \equiv \frac{I_0(1+G_{\mathrm{p2}}R_{\mathrm{s}})}{nV_{\mathrm{th}}(G_{\mathrm{p1}} + G_{\mathrm{p2}} + G_{\mathrm{p1}}G_{\mathrm{p2}}R_{\mathrm{s}})}. \quad \text{(A.5)}$$

The variables p_1 and p_2 have units of power and x and y are dimensionless.

Appendix B

The general solution of n, I_0, G_{p1}, and G_{p2}, in terms of R_{s}, D_{V1}, D_{I1}, D_{I1V1}, D_{V2} and D_{I2} is as follows. Solving forG_{p1} from D_{I2} (fourth column Table 2),

$$G_{\mathrm{p1}} = -\frac{D_{I2} + R_{\mathrm{s}}}{R_{\mathrm{s}}^2}, \quad \text{(B.1)}$$

and solving for G_{p2} from D_{V2} (fifth column Table 2),

$$G_{\mathrm{p2}} = \frac{-1 - 2R_{\mathrm{s}}G_{\mathrm{p1}} + \left(1 - 4R_{\mathrm{s}}D_{\mathrm{V2}} - 4R_{\mathrm{s}}^2G_{\mathrm{p1}}D_{V2}\right)^{1/2}}{2R_{\mathrm{s}}(1+R_{\mathrm{s}}G_{\mathrm{p1}})}. \quad \text{(B.2)}$$

Next, I_0 and n are obtained by solving the system of two linear equations described by D_{I1} and D_{V1} (first and second columns Table 2),

$$I_0 = \frac{1}{2}\left(D_{\mathrm{V1}} + G_{\mathrm{p1}}D_{\mathrm{I1}} + G_{\mathrm{p2}}D_{\mathrm{I1}} + G_{\mathrm{p1}}R_{\mathrm{s}}D_{\mathrm{V1}} + G_{\mathrm{p1}}R_{\mathrm{s}}G_{\mathrm{p2}}D_{\mathrm{I1}}\right), \quad \text{(B.3)}$$

and

$$n = -\frac{R_{\mathrm{s}}D_{\mathrm{V1}} + R_{\mathrm{s}}G_{\mathrm{p2}}D_{\mathrm{I1}} + D_{\mathrm{I1}}}{2V_{\mathrm{th}}}. \quad \text{(B.4)}$$

It is important to point out that given a set of values of D_{V1}, D_{I1}, D_{V2}, and D_{I2} [D_{I1V1} is obtained from (15)] defines a unique current voltage characteristic which can be generated with numerous combinations of R_{s}, n, I_0, G_{p1}, and G_{p2}. For example, given a set of values for D_{V1}, D_{I1}, D_{V2}, D_{I2}, the values of n, I_0, G_{p1}, and G_{p2} can be obtained from (B.1)–(B4) for any arbitrary value of R_{s}.

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