Materials Science and Engineering B 165 (2009) 57–59

Contents lists available at ScienceDirect

Materials Science and Engineering B

journal homepage: www.elsevier.com/locate/mseb

Schottky diode parameters extraction using Lambert W function

W. Jung [a,*], M. Guziewicz [b]

[a] Department of Analysis of Semiconductor Nanostructures, Institute of Electron Technology, Al. Lotników 32/46, 02-668 Warsaw, Poland
[b] Department of Semiconductor Processing for Photonics, Institute of Electron Technology, Al. Lotników 32/46, 02-668 Warsaw, Poland

ARTICLE INFO

Article history:
Received 18 June 2008
Received in revised form 11 February 2009
Accepted 18 February 2009

Keywords:
Schottky diode
Lambert function

ABSTRACT

Exact analytical solution is presented for the current flow in Schottky barrier diode with inclusion of series resistance and shunt conductance using the Lambert W function. The presented expression enables determination of more realistic values for the parameters of various quality Schottky diodes.

1. Introduction

Extraction of Schottky diode parameters ideality factor n, barrier height ϕ, series resistance R_s and shunt conductance G_p is important from a point of view of carriers transport mechanisms and technology optimization. Various methods have been developed to extract the parameters of Schottky diodes. Among the most widely used methods one can mention:

- Standard method which requires the presence of linear region in the ln(I) versus V plot [1]. Only two parameters n and ϕ can be obtained from the slope and intercept of linear region and this relatively simple analysis fails when large series resistance R_s is present.
- Methods using some auxiliary functions that allow the separation of the effect of R_s [2,3].
- Method using the derivative of the current with respect to voltage [4]. This method is very sensitive to measurements errors.
- Method using the integration of the current with respect to voltage [5]. This method fails in the presence of leakage current.
- Recently published methods [6,7] the so-called lateral and vertical optimization. These methods are based on minimizing the sum of squares of the relative differences between the measured and the fitting values of current. This optimization problem needs solution of sets differential equations usually by Newton's method.

The method presented in this paper is based on exact analytical solution for current flow through Schottky diode with series resistance and parallel conductance. Similar problem was presented in [8], but only for three parameters and in this case using Lambert function is not necessary because there exists simple relation between voltage and current as shown in [6].

2. Parameter extraction method

General equation describing current flow through Schottky diode is given in the form:

$$I = I_d + I_p = I_s\left\{\exp\left[\frac{\beta}{n}(V - R_sI)\right] - 1\right\} + G_p(V - R_sI) \quad (1)$$

$$I_s = AA^{**}T^2\exp(-\beta\phi) \quad (2)$$

where I_d is the diode current at bias V, $\beta = (q/kT)$ is the usual inverse thermal voltage, n is the ideality factor, I_s is the saturation current, A is the diode area, A^{**} is the modified Richardson constant, ϕ is the Schottky barrier height, R_s is series resistance, I_p is the shunt current through the shunt resistance $R_p = 1/G_p$.

An explicit expression for $I(V)$ or $V(I)$ given by Eq. (1) cannot be constructed from just common elementary functions, but solution for this equation can be found in terms of the Lambert W function. Eq. (1) can be transformed to the form $We^w = x$ as shown below.

$$\frac{\beta R_s}{n}\left(I + \frac{I_s}{1+G_pR_s} - \frac{G_pV}{1+G_pR}\right)e^{(\beta R_s/n)(I+(I_s/1+G_pR_s)-(G_pV/1+G_pR))}$$
$$= \frac{\beta R_sI_s}{n(1+G_pR_s)}e^{(\beta V/n)+(\beta R_sI_s/n(1+G_pR_s))-(\beta VG_pR_s/n(1+G_pR_s))} \quad (3)$$

The solution of this transcendental equation is a multi-valued function $w = W_k(x)$ referred to as the Lambert W function. Only the basic branch of Lambert function is important for general Eq. (1), because it satisfies the condition $W_o(x) = 0$ for $x = 0$.

* Corresponding author. Tel.: +48 22 5487 784; fax: +48 22 8470 631.
E-mail address: jung@ite.waw.pl (W. Jung).

0921-5107/$ – see front matter
doi:10.1016/j.mseb.2009.02.013

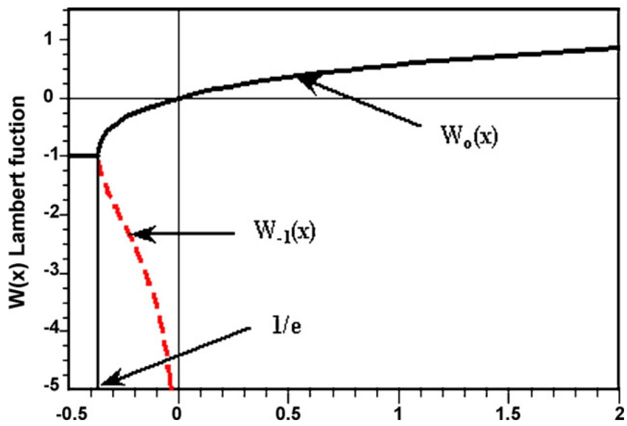

Fig. 1. Plots of real valued branches of Lambert function $W_o(x)$ and $W_{-1}(x)$.

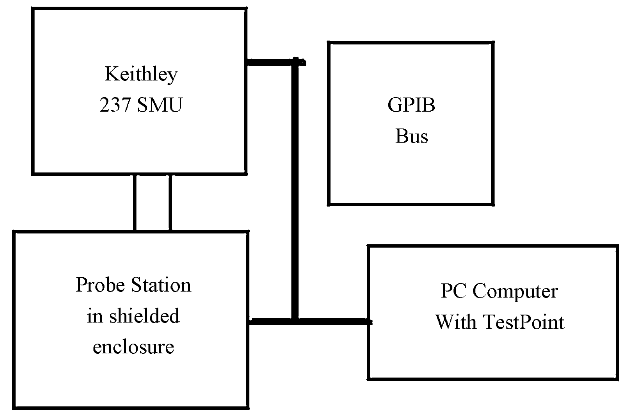

Fig. 2. Block diagram of the measurement system.

An explicit solution of Eq. (1) is given by:

$$I = \frac{nW_o[\beta R_s I_s e^{(\beta V/n)-(\beta V R_s G_p/n(1+R_s G_p))+(\beta R_s I_s/n(1+R_s G_p))}/n(1+R_s G_p)]}{\beta R_s} + \frac{VG_p}{1+R_s G_p} - \frac{I_s}{1+R_s G_p} \quad (4)$$

and after inserting expression on I_s analytical dependence current on voltage is given by:

$$I = \frac{nW_o[\beta R_s AA^{**}T^2 e^{(\beta V/n)-\beta\phi-(\beta V R_s G_p/n(1+R_s G_p))+(\beta R_s AA^{**}T^2 e^{-\beta\phi}/n(1+R_s G_p))}/n(1+R_s G_p)]}{\beta R_s} + \frac{VG_p}{1+R_s G_p} - \frac{AA^{**}T^2 e^{-\beta\phi}}{1+R_s G_p} \quad (5)$$

Using similar procedure one can obtain voltage–current dependence in which case:

$$V = -\frac{nW_o[\beta AA^{**}T^2 e^{-\beta\phi+(\beta I/nG_p)+(\beta AA^{**}T^2 e^{-\beta\phi}/nG_p)}/nG_p]}{\beta} + \frac{I}{G_p} + IR_s + \frac{AA^{**}T^2 e^{-\beta\phi}}{G_p} \quad (6)$$

The Lambert function is implemented in many mathematical systems like Mathematica by Wolfram Research under the name ProductLog or Matlab by MathWorks under the name Lambert. A plot of real valued Lambert function is shown in Fig. 1.

Some basic features of Lambert function deserve mention. For $x > 0$ W function is single-valued. For $-1/e < x < 0$ Lambert function is bivalued (two real values) indicated as $W_{-1}(x)$ and $W_o(x)$. It corresponds to stable and unstable operating points of a circuit having local positive feedback. For $x < -1/e$, W is the complex-valued function. Limiting behaviour of Lambert function determines asymptotic expansion which gives $W_o(x) \sim \ln(x)$ for $x \to \infty$ and $W_{-1}(x) \sim \ln(-x)$ for $x \to 0^-$.

Parameters extraction of Schottky barrier diode was done using the FindFit function in the Mathematica software package. Function FindFit gives the best fit parameters for a set of data and a given model function. The model function used in this work for parameter extraction was given by Eq. (5). Function FindFit can be used for linear and nonlinear problems using last square curve fitting. In the case of nonlinear problems the function FindFit can use internally several methods like: Newton, quasi Newton, Levenberg–Marquardt, conjugate gradient and principal axis method of Brent [9], which can be chosen depending on solved problem. The function FindFit works most efficiently that means finds globally optimal fit if the given input parameters values are not much different from the optimal and if symbolic derivatives can be computed. Both of these conditions are performed due to analytical form of Eq. (5).

The presented method assumes as the most published methods, that all parameters are bias independent. While this is of course limitation, it turns out not to be significant for most practical devices, although some publications, for example [10,11] address bias dependencies.

Next limit of the method is connected with semiconductor property. For doping levels below approximately 10^{18} per cm^3, the carrier transport occurs via thermionic emission over the barrier given by Eq. (1), while for larger doping levels tunnel or field emission through the then narrower depletion layer is predominant.

The proposed procedure was experimentally verified using Gold–Gallium Arsenide and Iridium–Silicon Carbide Schottky barrier diodes. Ir film as Schottky contact was deposited on (0 0 0 1) SiC 4H epilayer, and ohmic contact was previously formed on the bottom side by annealing of (150 nm) Ni film at 1000 °C for 2 min. The current–voltage characteristics were measured using the measurement setup shown in Fig. 2.

Results of measurement and simulation in the case of Iridium–Silicon Carbide Schottky barrier diode are presented in Fig. 3.

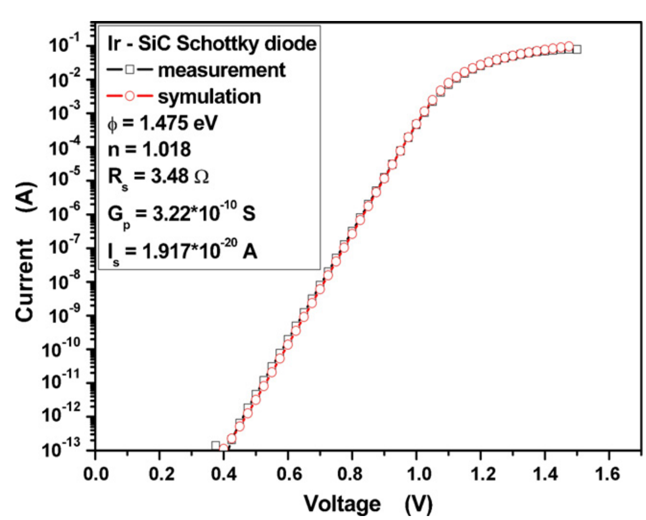

Fig. 3. Current–voltage characteristics of Ir–4H SiC Schottky diodes (measured—open squares; simulated—open circles).

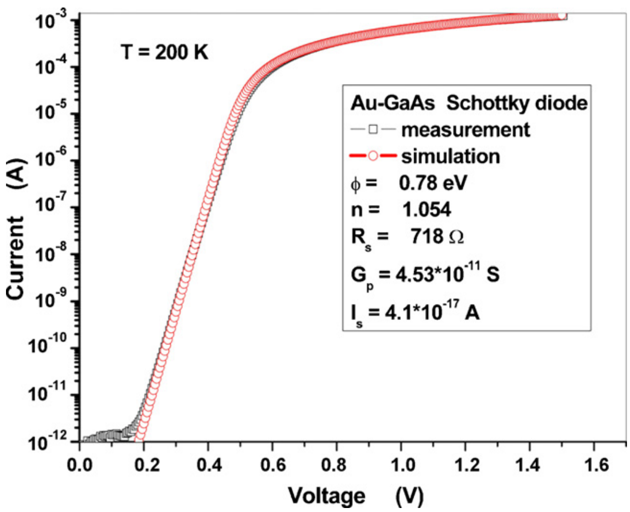

Fig. 4. Current–voltage characteristics of Au–GaAs Schottky diodes (measured at 200 K—open squares; simulated—open circles.

The presented method has also been successfully applied in parameter extraction from current–voltage characteristics measured as a function of temperature. Current–voltage characteristics of Gold–Gallium Arsenide Schottky barrier diode measured at temperature 200 K and calculated for extracted parameters are presented in Fig. 4.

3. Conclusions

We have presented the exact analytical solutions for the current flow in Schottky barrier diode with inclusion of series resistance and shunt conductance. The closed-form solutions are derived using the Lambert W function. Due to closed-form solution the process of Schottky barrier parameter extraction is simple more controllable and quite accurate even when electrical noise or random errors occur during measurement of *I–V* characteristics. The proposed method is fast, enables to use the whole bias range and allows automatization of the measurement process.

Acknowledgement

This work was supported by the Polish Min. of Science and Higher Education project no. N N515 415634.

References

[1] S.M. Sze, Physics of Semiconductor Devices, Wiley, New York, 1981.
[2] H.J. Norde, J. Appl. Phys. 50 (1979) 5052.
[3] S.K. Cheung, N.W. Cheung, Appl. Phys. Lett. 49 (1986) 85.
[4] V. Mikhelashvili, et al., J. Appl. Phys. 85 (1999) 6873.
[5] A. Ortiz-Conde, et al., Solid-State Electron. 38 (1995) 265.
[6] A. Ortiz-Conde, et al., Proceedings of the 22nd International Conference on Microelectronics (MIEL 2000), vol. 1, Niz, Serbia, 14–17 May, 2000.
[7] A. Ferhat-Hamida, et al., Solid-State Electron. 46 (2002) 615.
[8] T.C. Banwall, A. Jaykumar, Electron. Lett. 36 (2000) 291.
[9] R.P. Brent, Algorithms for Minimization without Derivatives, Dover, New York, 2002.
[10] M. Lyakas, R. Zaharia, M. Eizenberg, J. Appl. Phys. 78 (1995) 5481.
[11] C. Popescu, J.C. Manifacier, R. Ardebili, Phys. Status Solidi A 158 (1996) 611.