A Novel Search Algorithm based on Fish School Behavior

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Abstract— Search problems are sometimes hard to compute. This is mainly due to the high dimensionality of some search spaces. Unless suitable approaches are used, search processes can be time-consuming and ineffective. Nature has evolved many complex systems able to deal with such difficulties. Fish schools, for instance, benefit greatly from the large number of constituent individuals in order to increase mutual survivability. In this paper we introduce a novel approach for searching in high-dimensional spaces taking into account behaviors drawn from fish schools. The derived algorithm – Fish-School Search (FSS) – is mainly composed of three operators: feeding, swimming and breeding. Together these operators afford the evoked computation: (i) wide-ranging search abilities, (ii) automatic capability to switch between exploration and exploitation, and (iii) self-adaptable global guidance for the search process. This paper includes a detailed description of the novel algorithm. Finally, we present simulations where the FSS algorithm is compared with, and in some cases outperforms, well-known intelligent algorithms such as Particle Swarm Optimization in high-dimensional searches.

Keywords—swarm intelligence, fish school, social behaviour, search algorithms.

I. Introduction

Many pelagic fish species, as with other animals, present gregarious behavior, mainly to increase their survivability. We shall understand this phenomenon in at least two different ways, that is, for mutual protection and synergy of collective tasks. By protection we mean reducing the chances of being chased and caught by predators; and by synergy, we refer to an active means of achieving collective goals such as finding food.

On the other hand, the 'choice' or habit of some fish species of living their lives in schools reduces individual freedom for swimming movements and increases competition in sometimes not-so-rich food regions. Concerning fish aggregation, evidence across oceans and rivers shows that the benefits largely outweigh the drawbacks. This paper aims at presenting this novel computational intelligent search technique inspired largely in the above-mentioned interesting behavior.

Although we have taken great care not to depart too much from the original inspiration, as with all other nature-inspired (computer) models, FSS contains a few abstractions and simplifications that have been introduced to afford efficiency and usability to the algorithm.

However, the main characteristics derived from real fish schools and incorporated into the core of our approach are sound. They were grouped in observable behaviors such as:

- Feeding: inspired by the natural instinct of individuals (fishes) to find food in order to grow strong and be able to breed. Notice that food here is a metaphor for the evaluation of candidate solutions in the search process. As opposed to the concept of food intake, we have also considered that an individual can lose weight by swimming;

- Swimming: the most elaborate observable behavior utilized in our approach. It aims at mimicking the coordinated and only apparent collective contained movement produced by all the fishes in the school. Swimming is driven by feeding and, ultimately, it will guide the search process;

- Breeding: inspired by the natural selection mechanism that endows successful beings with the capacity to procreate, whereas others, *i.e.* the weak individuals, are more likely to perish. Notice that offspring here is a metaphor to enable exploitation abilities for better adapted candidate solutions of the search process.

II. Background

A. Search problems and algorithms

Although there are several approaches for searching, there is, unfortunately, no general optimal search strategy [1]. Thus, solving search problems is sometimes more of an art form than an engineering practice. Although custom-made algorithms are valuable options for specific problems, a more general automatic search engine would be a great boon for tackling problems of high dimensionality.

Search problems can be of various kinds. For example, they can be classified into two groups with regard to the structure of their search-space, which can be either structured or unstructured. For the former, there are many traditional

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techniques that are, on average, quite efficient. The same observation does not apply to the latter, that is, there is no overall good approach for search spaces on which there is no prior information.

We think that FSS might be a valuable option for searching in high dimensional and unstructured spaces.

B. Population-based algorithm (PBA)

Many nature-inspired algorithms such as genetic algorithms (GA) [2,3], artificial immune systems [4], ant colony optimization [5] and particle swam optimization (PSO) [6,7,8] are based on the concept of populations. In all these approaches, the computing discrimination power and memorization ability of past experiences are distributed in various degrees among the individuals of the population.

Distributed representation and computation are interesting features to incorporate into search algorithms because of the parallelization they imply. The obvious trade-off is the cost of control (*i.e.* communication among individuals), as opposed to the lower costs associated with centralized control.

In recent years particle swarm optimization has presented good results for search problems with high dimensionality. In PSO, candidate solutions emerge by flocking behavior around more successful individuals. Particles in PSO utilize the notion of adjustable speed according to the degree of success achieved. Bratton *et al.* [8] proposed a standard for performance comparison of PSO implementations.

III. FISH-SCHOOL SEARCH (FSS)

A. FSS Computational principles

The search process in FSS is carried out by a population of limited-memory individuals – the fishes. Each fish represents a possible solution to the problem. Similarly to PSO or GA, search guidance in FSS is driven by the success of some individual members of the population.

The main feature of the FSS paradigm is that fishes contain an innate memory of their successes – their weights. In comparison to PSO, this information is highly relevant because it can obviate the need to keep a log of best positions visited by all individuals, their velocities and other competitive global variables.

Another major feature of FSS is the idea of evolution through a combination of information of parents (after breeding) and some collective swimming (*i.e.* "reasoning") that selects among different modes of operation during the search process, on the basis of instantaneous results. As a matter of fact, only phenotypical and environmental information of parents is required during reproduction (*i.e.* actual evolution of offspring).

As for dealing with the high dimensionality and lack of structure of the search space, the authors think that FSS should at least incorporate principles such as the following:

(i) simple computations in all individuals;

(ii) various means of storing distributed memory of past computations;

(iii) local computations (preferably within small radiuses);

(iv) low communications between neighboring individuals;

(v) minimum centralized control (preferably none); and

(vi) some diversity among individuals.

A brief rationale for the above-mentioned principles is given, respectively: (i) this reduces the overall computation cost of the search; (ii) this allows for adaptive learning; (iii), (iv) and (v) these keep computation costs low as well as allowing some local knowledge to be shared, thereby speeding up convergence; and finally, (vi) this might also speed up the search due to the differentiation/specialization of individuals.

B. Overview of the new approach

The inspiration mentioned in Section I, together with the principles just stated above, are instantiated in our approach in the form of three operators that comprise the main routines of the FSS algorithm. To understand the operators, a number of concepts need to be defined.

The concept of food is related to the function to be optimized in the process. For example, in a minimization problem the amount of food in a region is inversely proportional to the function evaluation in this region. The "aquarium" is defined by the delimited region in the search space where the fishes can be positioned.

The operators are grouped in the same manner in which they were observed when drawn from the fish school. They are as follows:

- Feeding: food is a metaphor for indicating to the fishes the regions of the aquarium that are likely to be good spots for the search process;

- Swimming: actually a collection of operators that are responsible for guiding the search effort globally towards subspaces of the aquarium that are collectively sensed by all individual fishes as more promising with regard to the search process;

- Breeding: the last operator, it is responsible for refining the search performed. It was conceived to allow automatic transitioning from exploration to exploitation abilities.

C. FSS operators

1) The Feeding operator

As in real situations, the fishes of FSS are attracted by food that is scattered in the aquarium at various concentrations. In order to find greater amounts of food, the fishes in the school can perform independent movements (see individual movements in the next section). As a result, each fish can grow or diminish in weight, depending on its success or failure in the search for food. We propose that fish weight variation is proportional to the normalized difference between the evaluation of fitness function of previous and current fish position with regard to food concentration of these spots. The assessment of 'food' concentration considers all problem dimensions, as shown in (1),

$$W_i(t+1) = W_i(t) + \frac{f[x_i(t+1)] - f[x_i(t)]}{\max\{|f[x_i(t+1)] - f[x_i(t)]|\}}, \quad (1)$$

where $W_i(t)$ is the weight of the fish i, $\vec{x}_i(t)$ is the position of the fish i and $f[\vec{x}_i(t)]$ evaluates the fitness function (*i.e.* amount of food) in $\vec{x}_i(t)$.

A few additional measures were included to ensure that convergence toward rich areas of the aquarium occurs rapidly, namely:

- Fish weight variation is evaluated once at every FSS cycle;
- An additional parameter, named weight scale (W_{scale}) was created to limit the weight of a fish. The fish weight can vary between "1" and W_{scale}.
- All the fishes are born with weight equal to $\frac{W_{scale}}{2}$.

2) The Swimming operators

A basic animal instinct is to react after stimulation (or sometimes, lack of it). In our approach swimming is considered to be an elaborate form of reaction regarding survivability. In FSS, the swimming patterns of the fish school are the result of a combination of three different causes (*i.e.* movements).

For fishes, swimming is directly related to all important individual and collective behaviors such as feeding, breeding, escaping from predators, moving to more livable regions of the aquarium or, simply being gregarious.

This panoply of motivations to swim away inspired us to group causes of swimming into three classes: (i) individual, (ii) collective-instinct and (iii) collective-volition. Below we provide further explanations on how computations are performed on each of them.

a) Individual movement

Individual movements occur for each fish in the aquarium at every cycle of the FSS algorithm. The swim direction is randomly chosen. Provided the candidate destination point lies within the aquarium boundaries, the fish assesses whether the food density there seems to be better than at its current location. If this is not the case or if the step-size is not possible (i.e. it lies outside the aquarium or is blocked by, say, reefs), the individual movement of the fish does not occur. Soon after each individual movement, feeding occurs, as detailed above.

For this movement, we define a parameter to determine the fish displacement in the aquarium called individual step ($step_{ind}$). Each fish moves $step_{ind}$ if the new position has more food than the previous position. Actually, to include more randomness in the search process we multiply the individual step by a random number generated by a uniform distribution in the interval [0,1]. In our simulation we decrease the individual step linearly in order to provide exploitation abilities in later iterations.

b) Collective-instinctive movement

After all fishes have moved individually, a weighted average of individual movements based on the instantaneous success of all fishes of the school is computed. This means that fishes that had successful individual movements influence the resulting direction of movement more than other ones. When the overall direction is computed, each fish is repositioned. This movement is based on the fitness evaluation enhancement achieved, as shown in (2).

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \frac{\sum_{i=1}^{N} \Delta\vec{x}_{ind\,i}\{f[x_i(t+1)] - f[x_i(t)]\}}{\sum_{i=1}^{N}\{f[x_i(t+1)] - f[x_i(t)]\}}, \quad (2)$$

Where $\Delta\vec{x}_{ind\,i}$ is the displacement of the fish i due to the individual movement in the FSS cycle.

Fig. 2 shows how a fish school evokes harmonious collective instinctive movements. The size of the circle of each fish in the figure corresponds to its weight.
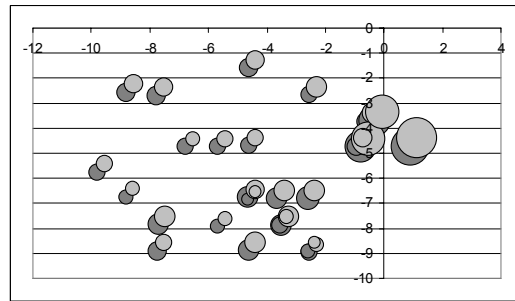

Figure 2. Collective instinctive movements illustrated here before (dark gray circles) and after (light gray circles) their occurrence.

c) Collective-volitive movement

After individual and collective-instinctive movements are performed, one additional positional adjustment was still necessary for all fishes in the school: the collective-volitive movement. This movement was devised as an overall success/failure evaluation based on the incremental weight variation of the fish school as a whole. In other words, this last movement will be based on the overall performance of the fish school.

The rationale is as follows: if the fish school is putting on weight (meaning the search has been successful), the radius of the school should contract; if not, it should dilate. This operator is deemed to help greatly in enhancing the exploration abilities in FSS. This phenomenon might also occur in real swarms, but the reasons are as yet unknown.

The fish-school dilation or contraction is applied as a small step drift to every fish position with regard to the school barycenter. The fish-school barycenter is obtained by considering all fish positions and their weights, as shown in (3).

$$Bari\,(t) = \frac{\sum_{i=1}^{N} \vec{x}_i(t)W_i(t)}{\sum_{i=1}^{N} \vec{x}_i(t)}. \qquad (3)$$

Collective-volitive movement will be inwards or outwards (in relation to the fish-school barycenter), according to whether the previously recorded overall weight of the school has increased or decreased, respectively, in relation to the new overall weight observed at the end of the current FSS cycle.

For this movement, we also defined a parameter called volitive step ($step_{vol}$). We evaluate the new position as in (4) if the overall weight of the school increases in the FSS cycle; if the overall weight decreases, we use (5).

$$\vec{x}_i(t+1) = \vec{x}_i(t) - step_{vol}\,.rand\,.[\vec{x}_i(t) - Bari\,(t)], \qquad (4)$$

$$\vec{x}_i(t+1) = \vec{x}_i(t) + step_{vol}\,.rand\,.[\vec{x}_i(t) - Bari\,(t)], \qquad (5)$$

where *rand* is a random number uniformly generated in the interval [0,1]. We also decreased the linear $step_{vol}$ along the iterations.

Fig. 3 shows how a fish school evokes focused collective-volitive movement towards the search objectives. Contractions and expansions of the school will occur according to success or failure.
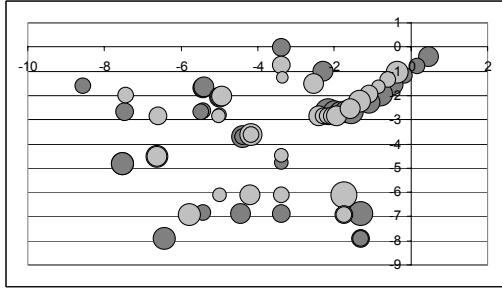

Figure 3. Collective-volitive movements illustrated here before (dark gray circles) and after (light gray circles) their occurrence.

3) The Breeding operator

Breeding in FSS, as in nature, can be viewed as a strong indicator that 'things are going well'. We have considered that for procreation to occur a pair of fishes should have individually met a minimum threshold (*i.e.* an indication of success or maturity in the search process). A heuristic value for this could be the top of the scale.

The selection of candidates for breeding considers all fishes that have reached a predefined threshold. The winner is the fish that presents the maximum ratio of weight over distance in relation to the breeding candidate.

We have decided that only one child is generated by a couple at one given time. The size of the new fish *k* will be the average of sizes of its parents *i* and *j*, as shown in (6). The initial position of the new fish is the midpoint between its parents, as shown in (7). Pragmatically, this will also represent the desirable refinement of the search process in the event of success, *i.e.* the concept of exploitation [8].

$$\vec{W}_k(t+1) = \frac{\vec{W}_i(t) + \vec{W}_j(t)}{2}, \qquad (6)$$

$$\vec{x}_k(t+1) = \frac{\vec{x}_i(t) + \vec{x}_j(t)}{2}, \qquad (7)$$

In order to keep the number of fishes in the school constant, whenever a new fish is created, the smallest fish is removed. Although this may seem unrealistic, it emulates the decease of the weak – which is indeed a natural phenomenon.

D. FSS cycle and Stop conditions

The FSS algorithm starts by randomly generating a fish school according to parameters that control fish sizes and their initial positions.

Regarding dynamics, the central idea of FSS is that all bio-inspired operators perform independently of each other across the three conceived classes.

The search process (*i.e.* FSS at work) is enclosed in a loop, where invocations of the previously presented operators will occur until at least one stop condition is met.

As of now, stop conditions conceived for FSS are as follows: limit of cycles (the stopping condition of all experiment of this paper), time limit, maximum school radius, minimum school weight, maximum fish number and maximum breeding number.

IV. FISH SCHOOL AT WORK

A. Experimental set-up

Five benchmark functions were used to carry out simulations and are described in (8), (9), (10), (11), and (12). Table I shows the search space, the initialization range, and the optimum for each function. All searches were carried out in 30 dimensions. All five functions are used for minimization problems. Two of these functions namely, Rosenbrock and Schwefel 1.2, represent simple unimodal problems; the other three, Rastrigin, Griewank, and Ackley, are highly complex multimodal functions that contain many local optima.

$$F_{Rosenbrock}\,(x) = \sum_{i=1}^{n-1}\left[100\left(x_{i+1} - x_i^2\right)^2 + \left(1 - x_i\right)^2\right], \qquad (8)$$

$$F_{Rastrigin}\,(x) = 10\,n + \sum_{i=1}^{n}\left[x_i^2 - 10\cos\left(2\pi x_i\right)\right], \qquad (9)$$

$$F_{Griewank}\,(x) = 1 + \sum_{i=1}^{n}\frac{x_i^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right), \qquad (10)$$

$$F_{Ackley}(x) = -20 \exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n}x_i^2}\right), \quad (11)$$

$$-\exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$$

$$F_{Schwefel\ 1.2}(x) = \sum_{i=1}^{n}\left(\sum_{j=1}^{i}x_j\right)^2. \quad (12)$$

TABLE I.   FUNCTIONS USED: SEARCH SPACE, INITIALIZATION RANGE, AND OTIMA.

Function	Parameters		
	Search space	Initialization	Optima
Rosenbrock	$-30 \le x_i \le 30$	$15 \le x_i \le 30$	1.0^D
Rastrigin	$-5.12 \le x_i \le 5.12$	$2.56 \le x_i \le 5.12$	0.0^D
Griewank	$-600 \le x_i \le 600$	$300 \le x_i \le 600$	0.0^D
Ackley	$-32 \le x_i \le 32$	$16 \le x_i \le 32$	0.0^D
Schwefel 1.2	$-100 \le x_i \le 100$	$50 \le x_i \le 100$	0.0^D

A factorial planning of experiments was performed to find suitable parametric combination of individual and volitive steps at both initial and final limits (*i.e. $step_{ind,initial}$, $step_{ind,final}$, $step_{vol,initial}$, $step_{vol,final}$*). We have associated the individual and volitive steps as percentages of the actual search space. Percentage values considered for initial and final limits were, respectively, as follows: 10; 1; 0.1 and 0.1; 0.01; 0.001; 0.0001. W_{scale} was set as 5000; this is half of the number of considered iterations.

All the simulations were performed using 30 fishes and 10,000 iterations. After 30 simulations the mean and the standard deviation were recorded. All the fishes were randomly initialized in areas of the aquarium that are far from the optimal solution regarding every dimension.

We compared our results with PSO simulation results presented in an earlier landmark paper [8]. Three PSO approaches were considered for comparisons: original PSO with the g_{best} topology, constriction PSO with the g_{best} topology and constriction PSO with the L_{best} topology. All the PSO simulations included 30 trials, each of which performed 300,000 evaluations, and simulations that considered 30 dimensions and used 30 particles. Therefore, we considered that a fair convergence analysis between the FSS and PSO approaches could be made.

B.   Simulation Results

Tables II, III, IV, V and VI show the best simulation results for function Rosenbrock, Rastrigin, Griewank, Ackley and Schwefel 1.2, respectively. Only the best results obtained are presented for each function (in all tables). Result in bold face are the best for each function.

Table VII subsumes the comparison between the FSS and PSO approaches. It contains the best results achieved for the five benchmark functions used to evaluate the performance of the four algorithms.

TABLE II.   SIMULATION RESULTS FOR ROSENBROCK FUNCTION – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

W_{scale}	$step_{ind,initial}$	$step_{ind,final}$	$Step_{vol,initial}$	$Step_{vol,initial}$	Fitness
5000	**0.1%**	**0.001%**	**1%**	**0.01%**	**16.118 (0.729)**
5000	0.1%	0.001%	1%	0.001%	16.462 (0.797)
5000	0.1%	0.0001%	1%	0.01%	16.403 (0.853)
5000	0.1%	0.0001%	1%	0.001%	16.447 (0.770)

TABLE III.   SIMULATION RESULTS FOR RASTRINGIN FUNCTION – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

W_{scale}	$step_{ind,initial}$	$step_{ind,final}$	$Step_{vol,initial}$	$Step_{vol,initial}$	Fitness
5000	10%	0.1%	10%	0.1%	14.128 (3.680)
5000	10%	0.1%	10%	0.01%	13.737 (2.889)
5000	**10%**	**0.01%**	**10%**	**0.1%**	**13.386 (4.005)**
4000	10%	0.01%	10%	0.1%	13.686 (2.827)

TABLE IV.   SIMULATION RESULTS FOR GRIEWANK FUNCTION – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

W_{scale}	$step_{ind,initial}$	$step_{ind,final}$	$Step_{vol,initial}$	$Step_{vol,initial}$	Fitness
5000	1%	0.001%	10%	0.01%	0.0037 (0.002)
5000	**1%**	**0.001%**	**1%**	**0.01%**	**0.0027 (0.002)**
5000	0.1%	0.0001%	1%	0.01%	0.0049 (0.004)
5000	0.1%	0.0001%	1%	0.001%	0.0037 (0.004)

TABLE V.   SIMULATION RESULTS FOR ACKLEY FUNCTION – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

W_{scale}	$step_{ind,initial}$	$step_{ind,final}$	$Step_{vol,initial}$	$Step_{vol,initial}$	Fitness
5000	**10%**	**0.01%**	**10%**	**0.1%**	**0.0400 (0.020)**
5000	10%	0.01%	10%	0.01%	0.0839 (0.041)
5000	10%	0.01%	1%	0.01%	0.1583 (0.032)
5000	10%	0.01%	1%	0.001%	0.1633 (0.031)

TABLE VI.   SIMULATION RESULTS FOR SCHWEFEL 1.2 FUNCTION – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

W_{scale}	$step_{ind,initial}$	$step_{ind,final}$	$Step_{vol,initial}$	$Step_{vol,initial}$	Fitness
5000	1%	0.01%	1%	0.01%	0.0972 (0.026)
5000	1%	0.01%	1%	0.001%	0.0947 (0.032)
5000	**1%**	**0.001%**	**1%**	**0.01%**	**0.0808 (0.022)**
5000	1%	0.001%	1%	0.01%	0.0915 (0.032)

Notice that our novel search algorithm outperforms the original PSO in all the cases. Moreover, FSS achieved

excellent results for notoriously hard multimodal functions such as the Rastrigin, Griewank, and Ackley.

TABLE VII. SIMULATION RESULTS – MEAN FITNESS AND (STANDARD DEVIATION) FOR 30 TRIALS.

Function	Mean Fitness and Standard Deviation			
	Orig. PSO	Constricted PSO (G_{best})	Constricted PSO (L_{best})	FSS
Rosenbrock	54.6867 (2.8570)	8.1579 (2.7835)	12.6648 (1.2304)	**16.118 (0.729)**
Rastrigin	400.7194 (4.2981)	140.4876 (4.8538)	144.8155 (4.4066)	**13.386 (4.005)**
Griewank	1.0111 (0.0031)	0.0308 (0.0063)	0.0009 (0.0005)	**0.0027 (0.002)**
Ackley	20.2769 (0.0082)	17.6628 (1.0232)	17.5891 (1.0264)	**0.0400 (0.020)**
Schefel 1.2	5.4572 (0.1429)	0.0 (0.0)	0.1259 (0.0178)	**0.0808 (0.022)**

V. DISCUSSION AND CONCLUSIONS

In this paper we have introduced the general ideas and principles embedded in FSS. This novel search algorithm is very promising as a search tool for dealing with unstructured high dimensional spaces, as may be concluded from the results of the previous section.

The performance of FSS on some multimodal functions was surprisingly good when compared to the unimodal, which was also good.

Although previous works in the literature [10] [11] [12] have similar titles or even motivation as this present one, they targeted different computational problems and used approaches that are dramatically distinct from the ones adopted in this paper.

The bio-inspired operators, based on the fish school devised here, produce a very interesting balance between exploration and exploitation abilities, both of which are highly desirable in a search tool. As demonstrated, the new approach presents an interesting capacity of self-directing the search effort, as well as self-regulating the search granularity.

We foresee that FSS will most likely receive a great number of extensions in the near future, namely, sea currents, springs, predators, reefs, corals and other barriers to the school progression, all of them, situations to be avoided or taken advantage of. Taken altogether, these extensions may allow FSS to deal with noise, attractors, repulsors and no-go regions.

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