Accurate Calculation of the Absorptance Enhances Efficiency Limit of Crystalline Silicon Solar Cells With Lambertian Light Trapping

Sören Schäfer and Rolf Brendel

***Abstract*—The widely accepted limiting efficiency for crystalline silicon solar cells with Lambertian light trapping under 1 sun was previously calculated to be 29.43% for a 110-μm-thick device by using the commonly applied weak absorption approximation for light trapping. However, the short-circuit current density increases by 0.17 mA/cm^2 when modeling the optical absorptance of an ideal Lambertian light trapping scheme exactly. The resulting new 1-sun efficiency limit is 29.56% and holds for a cell that is 98.1 μm in thickness.**

***Index Terms*—Efficiency limit, photon recycling, silicon, solar cell.**

I. Introduction

Lambertian light trapping is a widely accepted benchmark for excellent light trapping. The upper limit for 1-sun energy conversion efficiency of a crystalline Si solar cell with Lambertian light trapping was calculated to be 29.43% by Richter *et al.* [1] for a nondoped cell of optimum thickness of 110 μm at 298.15 K. The first row of Table I lists the parameters of the illuminated current–voltage (*J*–*V*) curve of this cell.

The current experimental record efficiency is 26.7% [2], [3] and, thus, leaves a gap to the aforementioned limit of only $2.7\%_{\rm abs}$. It is, therefore, of increasing interest to calculate the limiting benchmark efficiency accurately.

Large progress toward a precise determination of the limiting efficiency was made by Richer *et al.* [1], who recalculated the previous efficiency limit [4] by using improved data and/or improved modeling techniques for:

1) the solar spectrum;
2) the optical constants of Si;
3) the parametrization of the free carrier absorption (FCA);
4) the parametrization of intrinsic recombination;
5) bandgap narrowing.

Each of these five improvements changed the efficiency limit by $-0.14\%_{\rm abs}$ to $+0.27\%_{\rm abs}$. The combined effect was a $+0.38\%_{\rm abs}$ increase of the limiting efficiency from previously 29.05% [4] to the now widely accepted value of 29.43% [1].

TABLE I
Limiting Efficiency η of Si Solar Cells of Thickness W as Calculated Using (1) or (2)

Eq.	W [µm]	η [%]	$V_{\rm oc}$ [mV]	$J_{\rm sc}$ [mA/cm^2]	FF [%]	$V_{\rm mpp}$ [mV]	Ref.
(1)	110	29.43	761.3	43.31	89.26	697.3	[1]
(1)	110	29.43	761.4	43.30	89.28	697.4	t.p.
(2)	110	29.55	761.4	43.47	89.28	697.4	t.p.
(2)	98.1	29.56	763.3	43.36	89.31	699.3	t.p.

The illumination is 1 sun of AM1.5G and the cell temperature is $T = 298.15$ K. Open-circuit voltage $V_{\rm oc}$, short-circuit current density $J_{\rm sc} = J_L\,(0\,{\rm V})$, fill factor FF, and maximum power point voltage $V_{\rm mpp}$ are also given. The simulations in this paper use (6) and (8) to calculate the *J*–*V* curve. The last digit of all figures from this paper (t.p.) is rounded.

Ideal Lambertian light trapping is by definition characterized by zero front surface reflectance, unity rear side reflectance, and a complete randomization of the direction of light propagation when passing through the front surface [5]–[8]. Previous studies on efficiency limits of Si solar cells [1], [4], [9] calculated the optical absorptance for Lambertian light trapping using the weak absorption approximation. This approximation is, however, known to underestimate the Lambertian absorption as first pointed out by Tiedje *et al.* [9].

This paper builds on the approach of [1] unless stated otherwise. However, we use the exact rather than an approximate description of Lambertian light trapping to increase the limiting efficiency. In order to do so and in contrast with [1], we apply the generalized Planck's law [10] for treating photon recycling.

II. Lambertian Absorption

When including the FCA, the weak absorption limit of the optical absorption by electron–hole (e–h) pair generation of a cell with Lambertian light trapping is [9]

$$A_{\rm bb}(\lambda, V) = \frac{\alpha_{\rm bb}}{\alpha_{\rm bb} + \alpha_{\rm fca} + \frac{1}{4n^2W}} \tag{1}$$

and depends on the absorption coefficient $\alpha_{\rm bb}$ due to e–h pair generation by band-to-band transitions, the absorption coefficient $\alpha_{\rm fca}$ due to FCA, and the loss $1/4n^2W$ per internal path length of light escaping through the front of a Si wafer with a refractive index n and a thickness W. The absorptance (1) depends on the wavelength of light λ via $\alpha_{\rm bb}$, $\alpha_{\rm fca}$ and n, and on voltage V via the carrier concentrations that enter $\alpha_{\rm fca}$.

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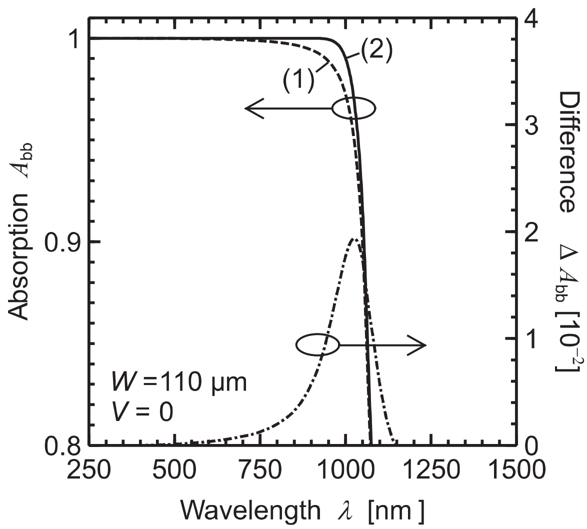

Fig. 1. Band-to-band absorptance spectra A_{bb} as calculated with (1) (broken line) and (2) (solid line) for a crystalline Si wafer of thickness of $W = 110\ \mu$m and at short-circuit condition $V = 0$. The integrated difference (dash-dotted line) corresponds to a short-circuit current gain of 0.17 mA/cm^2.

The exact expression for the e–h pair generating fraction of the Lambertian absorptance [6], [7] is

$$A_{\rm bb}(\lambda, V) = \frac{\alpha_{\rm bb}}{\alpha_{\rm bb} + \alpha_{\rm fca}} \frac{(1 - T_{\rm r})(1 + T_{\rm r})\ n^2}{n^2 - (n^2 - 1)\, T_{\rm r}^2} \tag{2}$$

with

$$T_{\rm r} = \exp(-\alpha_{\rm tot} W)(1 - \alpha_{\rm tot} W) + (\alpha_{\rm tot} W)^2 E_1(\alpha_{\rm tot} W) \tag{3}$$

and the total absorption coefficient

$$\alpha_{\rm tot} = \alpha_{\rm bb} + \alpha_{\rm fca}. \tag{4}$$

We evaluate the exponential integral function [11]

$$E_1(x) = \int_{\rm x}^{\infty} t^{-1} \exp(-t)\, dt \tag{5}$$

appearing in (3) by using the respective function implemented in MATLAB (version 2015Rb). The absorption coefficients for band-to-band transitions, the FCA, and the air mass 1.5 global solar spectrum (AM1.5G) are taken from the same source as in [1]. The formula for calculating the light-generated current density is [1, eq. (3)]. We apply trapezoidal integration and a piecewise cubic interpolation of the optical constants at the tabulated wavelength values of the solar spectrum.

The broken line in Fig. 1 illustrates the calculated absorptance of a 110-μm-thick Si layer applying the approximation (1) and the solid line applying (2). The photogenerated short-circuit current density that we calculate using (1) is $J_{\rm sc} = 43.30$ mA/cm^2, which is in agreement with [1] to within one count of the fourth digit. When using (2), we find a current density of $J_{\rm sc} = 43.47$ mA/cm^2 that is larger by 0.17 mA/cm^2.

III. Radiative Recombination and Photon Recycling for Lambertian Absorption

We have to calculate the *J–V* curve in order to determine the new limiting efficiency and the new optimum wafer thickness that result from the enhanced short-circuit current density. The current–voltage curve

$$J(V) = J_{\rm L}(V) - J_{\rm Aug}(V) - J_{\rm rad}(V) \tag{6}$$

depends on the light-generated current density $J_{\rm L}$, the Auger recombination current density $J_{\rm Aug}$, and the radiative recombination current density $J_{\rm rad}$. Note that $J_{\rm L}$ also depends weakly on the voltage V via the FCA. We calculate $J_{\rm Aug}$ as explained in [1] and determine the radiative recombination current density via the one-to-one correspondence to the photon emission rate

$$J_\gamma(V) = B_{\rm rel}(V)\pi \int_{250\,\rm nm}^{1450\,\rm nm} A_{\rm bb}(\lambda, V)\, \frac{2\,{\rm c}}{\lambda^4} \frac{1}{{\rm e}^{\frac{hc}{\lambda kT} - \frac{qV}{kT}} - 1} d\lambda \tag{7}$$

that follows from Würfel's generalized Planck's law [10]. We have to include the relative coefficient of radiative recombination $B_{\rm rel}$ [12] here, because $A_{\rm bb}$ uses $\alpha_{\rm bb}$ at zero voltage and, thus, does not account for the injection dependence of $\alpha_{\rm bb}$. Formula (7) uses Planck's constant h, the vacuum velocity of light c, the Boltzmann constant k, and the temperature T of the Si lattice. The net radiative recombination current density then is

$$J_{\rm rad}(V) = qB_{\rm rel}(V)\pi \int_{250\,\rm nm}^{1450\,\rm nm} A_{\rm bb}(\lambda, V)\, \frac{2{\rm c}}{\lambda^4} \times \left[\frac{1}{{\rm e}^{\frac{hc}{\lambda kT} - \frac{qV}{kT}} - 1} - \frac{1}{{\rm e}^{\frac{hc}{\lambda kT}} - 1}\right] d\lambda \tag{8}$$

with q denoting the elementary charge. The second term in the square bracket describes the emission of radiation in thermal equilibrium. Note that this generalized Planck's law fully accounts for the effect of photon recycling [10]. An extra correction factor for photon recycling that was used in previous works [1], [4], [9] is not required when using (8).

Now the question arises if (8) is identical to the treatment of photon recycling that was chosen in [1]. We verify analytically that both the treatments are mathematically equivalent if $A_{\rm bb}$ is taken in the weak absorption limit (1), if the FCA is neglected, and if the two terms -1 in (8) are neglected. We were, however, not able to verify this equivalence when using (2) for $A_{\rm bb}$ or when including FCA. This is why we use (7) rather than the approach applied in [1] for calculating the limiting efficiency with the exact formula (2) for Lambertian absorptance.

IV. New Efficiency Limit

When running our model using (6) and (8) for a cell of 110 μm in thickness and when using (1) for calculating the Lambertian absorptance, we find agreement with [1] in all parameters of the illuminated *J–V* curve to within two counts of the fourth digit. The results are listed in the second row of Table I.

We now use (2) instead of (1) and find an efficiency of 29.55%, as listed in the third row of Table I. Finally, we run a thickness optimization using (2). A cell with a thickness of $W = 98.1\ \mu$m reaches the highest efficiency. The new limiting efficiency for Lambertian light trapping is 29.56%. Note that we assume an intrinsic Si wafer.

The efficiency is 29.34% at an optimum thickness of $W = 78.7$ μm when assuming an n-type Si with a donator concentration of 1.5×10^{15} cm^{-3}, as used in [3]. However, when assuming a p-type Si with an acceptor concentration of 1.5×10^{15} cm^{-3}, the limiting efficiency is 29.55% at an optimum thickness of $W = 97.7$ μm. This latter value for the p-type Si is, thus, very close to the maximum value of 29.56% for an intrinsic Si material.

V. Conclusion

We calculated the theoretical maximum of the energy conversion efficiency of a crystalline Si solar cell exhibiting Lambertian light trapping under illumination by 1 sun. Applying the generalized Planck's law for simulating radiative recombination simplifies the treatment of photon recycling and avoids approximations concerning this effect. Using (2) for the optical absorptance by a Lambertian light trapping scheme enhances the maximum efficiency from 29.43% [1] to the new value of 29.56% while using the same material parameters and the same solar spectrum as in [1]. The optimum wafer thickness reduces from 110 to 98.1 μm for a nondoped Si wafer.

Comparison with the results of [1] shows that the correction of the Lambertian absorptance does not change the relative reduction of the efficiency due to doping and FCA.

The correction of the limiting efficiency value by $+0.13\%_{\mathrm{abs}}$ is small but not negligible when compared with the $+0.38\%_{\mathrm{abs}}$ reported in [1]. The increase of the limiting efficiency reported here enhances the scope for experimental efficiency improvements by 5% relative to at least $2.8\%_{\mathrm{abs}}$ efficiency points until meeting the Lambertian limit.

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Authors' photographs and biographies not available at the time of publication.