bounds is in order. While the lower bound would be particularly important, the improved upper bound would also be useful.

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An Approximation to the Fermi Integral $F_{1/2}(x)$

By H. Werner and G. Raymann

The Fermi Integral as defined, for instance, in the *Handbuch der Physik*, Bd. XX, S. 58 [1], is given by

$$F_p(x) = \int_0^\infty \frac{t^p}{e^{t-x} + 1}\, dt. \tag{1}$$

The function $F_{1/2}(x)$ has for negative values of x an expansion of the form

$$F_{1/2}(x) = \frac{\sqrt{\pi}}{2} \sum_{\nu=1}^{\infty} (-1)^{\nu-1} \cdot \frac{e^{\nu x}}{\nu^{3/2}}, \tag{2}$$

and for large positive x the asymptotic expansion

$$\begin{aligned} F_{1/2}(x) \sim x^{3/2} \Bigg[\frac{2}{3} + \frac{\pi^2}{12 \cdot x^2} + \binom{\frac{1}{2}}{3} \cdot \frac{7}{60} \cdot \frac{\pi^4}{x^4} + \cdots \\ + \binom{\frac{1}{2}}{2n-1} \frac{2^{2n-1} - 1}{n} \mid B_{2n} \mid \cdot \frac{\pi^{2n}}{x^{2n}} + \cdots \Bigg]; \end{aligned} \tag{3}$$

compare [2], formulas (10) and (12);

B_{2n} are the Bernoulli numbers, given for example in [3], page 298. We obtained Chebyshev approximations to $F_{1/2}(x)$, based upon the table by McDougall and Stoner [4]. This table was subtabulated by interpolation with a fifth-degree polynomial. The approximations are

$$\begin{aligned} F_{1/2}^*(x) &= e^x \sum_{\nu=0}^{5} a_\nu e^{\nu x} && \text{for} \quad -\infty < x \leqq +1, \\ F_{1/2}^*(x) &= x^{3/2} \left[\frac{2}{3} + \sum_{\nu=0}^{5} \frac{b_\nu}{x^{2\nu+2}} \right] && \text{for} \quad +1 < x < +\infty, \end{aligned} \tag{4}$$

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the coefficients

ν	a_ν	b_ν
0	+0.8860 7596	+0.8435 00
1	−0.3087 1705	+0.7108 09
2	+0.1463 8520	−3.7124 56
3	−0.0584 3877	+6.7056 28
4	+0.0143 1771	−5.5948 77
5	−0.0015 0176	+1.7777 87

With these approximations, the relative error $|F_{1/2}(x) - F^*_{1/2}(x)|/F_{1/2}(x)$ is less than $2 \cdot 10^{-4}$ and $5 \cdot 10^{-4}$, respectively.

Another intensive table of $F_p(x)$ has been given by G. A. Chisnall [5] who also discusses in [6] a method for the interpolation of the existing tables of $F_{1/2}(x)$. It is not difficult to obtain analogous Chebyshev approximations to $F_p(x)$ for any fixed values of p to a prescribed degree of accuracy if one is able to generate the function with this (or slighty more) accuracy.

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On the Congruences $(p-1)! \equiv -1$ and $2^{p-1} \equiv 1 \pmod{p^2}$

By Erna H. Pearson

The results of computations to determine primes p such that one of the relations

$$(p-1)! \equiv -1 \pmod{p^2}, \tag{1}$$

$$2^{p-1} \equiv 1 \pmod{p^2} \tag{2}$$

holds have been published previously [1–5]. The known Wilson primes (those satisfying (1)) are 5, 13, and 563, the last having been determined by Goldberg [3] in testing $p < 10^4$. Froberg [4] tested $10^4 < p < 30{,}000$ without finding additional Wilson primes.

Froberg [4] determined $p = 1093$ and $p = 3511$ to be the only primes less than

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