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Evaluation of solar cell parameters by nonlinear algorithms

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Abstract. A numerical procedure is described to calculate the parameters of the equivalent circuit of a solar cell, based on the application of algorithms to optimise nonlinear functions defining the difference between the experimental characteristics and the theoretical model. We have obtained empirical data from dark $I(V)$ characteristics. A study of the method to obtain the initial solution is also included. Results obtained for space solar cells and commercial cells (terrestrial use) are given. Modifications of the commonly used equivalent-circuit model of solar cells are suggested.

1. Introduction

The solar cell can be modelled by an equivalent circuit in which the parameters involved have a physical meaning and are hence related to the physical phenomena occurring in the device.

In a silicon homojunction solar cell, the current is basically due to recombination in the space charge region (SCR) and to the diffusion of the minority carriers in quasineutral emitter and base regions. These phenomena can be modelled by two exponential standard terms.

The analytical model for the $I(V)$ characteristics of the solar cell can be written as follows (Wolf *et al* 1977)

$$I = I_{01}\left[\exp\left(\frac{q(V-IR_s)}{nkT}\right)-1\right] + I_{02}\left[\exp\left(\frac{q(V-IR_s)}{kT}\right)-1\right] + \frac{V-IR_s}{R_{sh}} \qquad (1)$$

where I_{01} and I_{02} are the saturation currents corresponding to the SCR recombination and diffusion terms. The parameter n is the ideality factor that usually has a value between 1 and 2, and depends on the recombination process in the SCR. The parameters k, T and q have their usual meaning.

There are two other phenomena electrically important in solar cell behaviour. They have been included in our model (equation (1)) by two lumped parameters:

(i) The 'shunt resistance' parameter, R_{sh}, used by Stirn (1972), in which all possible origins of shunt current are included, such as short circuit of the junction, for instance due to the technological manipulation. These phenomena are considered as a linear term in the model.

(ii) The 'series resistance' parameter, R_s, that accounts for the $I(V)$ characteristic behaviour for moderately high bias level. The series resistance effects are well known due to their important influence on the operating electrical parameters such as fill factor or the maximum power point. This resistance arises from conduction losses in the emitter and base bulk regions and from grid geometry and resistance, and several methods have been developed to measure its value (Wolf and Rauschenbach 1963, Rajkanan and Shewchun 1979).

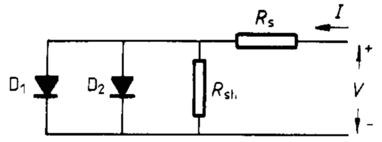

Figure 1. Circuit model for a solar cell, corresponding to equation (1).

Equation (1) can be represented by the equivalent circuit, shown in figure 1. The model considered here is useful because of the presence of five parameters having different origins. The analysis of their values can give us very important information about the influence of some external conditions on the mechanisms giving rise to the terminal current in the cells. Technological problems in the manufacturing process of a general purpose solar cell or irradiation effects on space solar cells, are well known examples.

It has been recently pointed out that the superposition principle is not applicable in most cases (Rothwarf 1978), and consequently, the use of illuminated $I(V)$ characteristics would produce different results. We consider only dark $I(V)$ characteristics as experimental data in the method to evaluate the five parameters involved in the model. This is the main difference between this paper and previous works in this field (Mottet 1980).

2. Method

The problem that we must solve is to evaluate the five parameters: I_{01}, n, R_{sh}, I_{02} and R_s in order to fit a given $I(V)$ experimental characteristic with the analytical model of equation (1).

The basis of the method is to define an object function for the difference between both characteristics, experimental and theoretical. An optimisation process with this function will give values of the parameters. In order to define this object function we will first consider an error function

$$e(\varphi) = [F(\varphi, V) - S(V)]W(V) \tag{2}$$

where $\varphi = [I_{01}, n, R_{sh}, I_{02}, R_s]^T$ is a vector containing the parameters, F is the analytical model of the cell, in this case given by equation (1), S is the experimental characteristic, W is a particular weight function and V is the voltage across the cell and will be our independent variable.

We will consider the definition of $e(\varphi)$ at N points of the $I(V)$ characteristic; equation (2) takes the following form:

$$e_i(\varphi) = W_i \left\{ I_{01}\left[\exp\left(\frac{qA_i}{nkT}\right) - 1\right] + I_{02}\left[\exp\left(\frac{qA_i}{kT}\right) - 1\right] + \frac{A_i}{R_{sh}} - I_i \right\} \tag{3}$$

where $A_i = V_i - I_i R_s$. After some practical considerations we have modified equation (3) as follows

$$e_i(\varphi) = \frac{1}{I_i}\left\{ I_{01}^2\left[\exp\left(\frac{qA_i}{n^2kT}\right) - 1\right] + I_{02}^2\left[\exp\left(\frac{qA_i}{kT}\right) - 1\right] + \frac{A_i}{R_{sh}^2} - I_i \right\} \tag{4}$$

with $A_i = V_i - I_i R_s^2$.

The $I(V)$ characteristics of solar cells are usually measured over more than five decades on the current axis. This wide range of current values leads to very different values of $e_i(\varphi)$ depending on the $I(V)$ point where it is calculated. If no weight function is used, the overall error can be reduced but high values of $e_i(\varphi)$ can be found at some points of the characteristics.

In order to equalise the values of the $e_i(\varphi)$ function we have used a weight function W defined as the inverse of the current at the point.

$$W_i(V_i) = \frac{1}{I_i}. \tag{5}$$

The vector φ has been replaced by the vector φ^2 in order to avoid negative values of the parameters. The change of variable φ to φ^2 does not lead to dimensional inconsistencies because we also replace all the parameters by their square roots.

Now we can define our object function $U(\varphi)$ from the error function $e_i(\varphi)$ given by equation (4). We have used two criteria: the first is the least mean square criterion

$$U(\varphi) = \sum_{i=1}^{N} e_i^2(\varphi) \tag{6}$$

and the second is the Tchebyshev error or minimax criterion

$$U(\varphi) = \max_i |e_i(\varphi)| \qquad i = 1, 2, \ldots, N. \tag{7}$$

It must be pointed out that the previous considerations made in the error function, to define equation (4), are independent of the selected criterion (least mean squares or minimax) in the object function formulation. A suitable algorithm has to be applied to object functions given by equations (6) or (7), to obtain the vector φ for a minimum value of $U(\varphi)$.

Nonlinear algorithms are necessary because the error function is a nonlinear function of the parameters involved and so the object function is also nonlinear.

2.1. *Least mean squares*

If the criterion chosen to define the object function is the least mean squares, the Gauss–Newton method may be used to optimise $U(\varphi)$ (Wilde and Beighther 1967). The basis of this iterative method is the evaluation of the Jacobian matrix J, which elements are defined by

$$J_{ij} = \frac{\partial e_i}{\partial \varphi_j}. \tag{8}$$

The iterative function is given by

$$\Delta\varphi = -(J^TJ)^{-1}J^Te(\varphi) \tag{9}$$

where

$$e(\varphi) = [e_1(\varphi), e_2(\varphi), \ldots, e_N(\varphi)]^T.$$

The increment $\Delta\varphi$ given by equation (9) has to be added to the vector φ to obtain the next value of the vector φ of the parameters.

In some cases it is convenient to use a completely equivalent iteration function

$$\Delta\varphi = -\alpha^TA^Te(\varphi) \tag{10}$$

where α and A are matrices obtained from the Jacobian matrix J by an orthonormalisation process. It is necessary to point out that the columns of matrix A are orthonormals. The application of equation (10) avoids the inversion of the matrix (J^TJ) and reduces calculation time.

In some cases $\Delta\varphi$ is too large and it is convenient to limit its maximum value by the use of the Levenberg parameter λ, giving

$$\Delta\varphi_0 = \lambda\Delta\varphi. \tag{11}$$

The parameter λ is determined by the unidimensional minimisation of the function $U(\lambda)$, that results from the substitution of equation (11) in equation (6). An efficient method to evaluate λ is the 'golden section' method (Wilde and Beighther 1967).

2.2. *Minimax*

If the object function is defined according to equation (7) the most convenient algorithm is that of Madsen (1975).

The optimisation method basically consists of the generation of an equivalent problem that can be described in terms of linear programming after a Taylor series expansion. The introduction of a change of variable allows a simplex solution of the problem.

In defining a dual problem the increment $\Delta\varphi$ can be found. The Madsen algorithm introduces an improvement in the calculation of $\Delta\varphi$ based on the restriction of the modulus of $\Delta\varphi$

$$\max_K |\Delta\varphi_K| < \eta \qquad K = 1, \ldots, m \qquad \text{(number of parameters)} \tag{12}$$

where η adjusts itself in each iteration in such a way that

$$U(\varphi + \Delta\varphi) < U(\varphi). \tag{13}$$

3. Initial conditions

In the application of Gauss–Newton or Madsen algorithms an initial solution must be chosen close to the optimum. There are a number of methods to calculate values of the solar cell parameters based on experimental $I(V)$ characteristics. In this work we have developed a numerical routine that calculates the initial solution before the iterative optimisation procedure is applied to the function $U(\varphi)$.

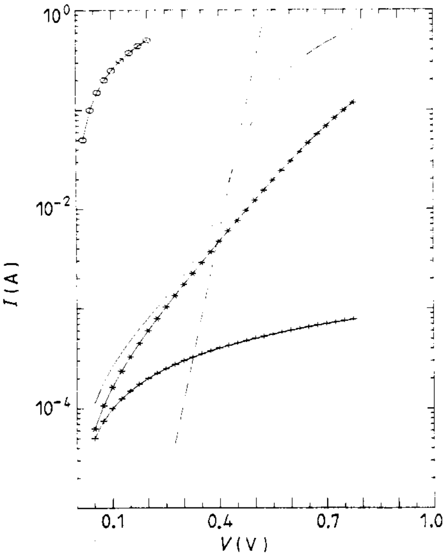

Figure 2. Dark $I(V)$ characteristic of a solar cell, showing the contribution of the different phenomena involved. (*) Space charge recombination; (+) R_{sh} contribution; (---) diffusion term; (○) R_s contribution; (——) data.

Figure 2 shows a typical $I(V)$ characteristic and the different terms involved according to equation (1). The strategy we have followed to obtain the initial conditions is summarised as follows:

(i) The series resistance R_s is first calculated by a trial and error method to detect the value that will explain the higher region of the $I(V)$ characteristic, leading to a diffusion term with an ideality factor equal to one after subtraction point-to-point of the term I_iR_s. In this way the initial solution for the parameter I_{02} of the diffusion term is obtained simultaneously.

(ii) The shunt resistance R_{sh} can be evaluated from the slope of the characteristic at the origin, which is a method widely used in the literature.

(iii) The diffusion term and the shunt resistance term of the characteristics are then subtracted from the value of the current from the experimental data and the term corresponding to the recombination in the SCR is obtained.

(iv) Finally a numerical adjustment of the points to an analytical exponential expression is made in a range of applied voltage where unity is negligible with respect to the exponential term in equation (1). The initial values of I_{01} and n are calculated.

This strategy leads in general to good results because the most appropriate region of the $I(V)$ curve is selected to calculate the initial value of the parameters related to the phenomena occurring in that region. Another benefit of the method is the use of only a dark $I(V)$ characteristic, avoiding the use of several dark and illuminated $I(V)$ curves.

The validity of a solution can be tested calculating the standard deviation (SD) between the data $I(V)$ curve and the calculated curve obtained with equation (1) using the numerical values of the parameters.

The standard deviation is defined as follows:

$$\mathrm{SD} = \left[\frac{1}{N}\sum_{i=1}^{N}\left(\frac{I_{\mathrm{calc}}(V_i)}{I_{\mathrm{data}}(V_i)}-1\right)^2\right]^{1/2}. \tag{14}$$

In general it is considered a good adjustment between calculated and data $I(V)$ characteristics if SD $< 10\%$.

In some cases we have obtained initial solutions that verify this condition, so they could have been considered as final results.

4. Results

We present here the results that were obtained for a number of solar cells using the above method. In all cases the experimental $I(V)$ curve has been measured in dark conditions with an experimental arrangement in which the temperature can be controlled.

The $I(V)$ characteristic is registered on an X–Y plotter up to 0.8 V. The minimum number of points we have used is 30 which is more than five times greater than the number of unknown parameters.

We have considered:

(i) Space-use solar cells: n^+–p structure, 8 Ω cm base resistivity, area 3 cm^2, junction depth 1 μm. We have evaluated three samples that have been irradiated with 10 MeV protons with fluences of 10^{11}, 3×10^{11} and 10^{12} cm^{-2}.

(ii) Commercial, terrestrial-use solar cells: n^+–p structure on 3 in silicon wafers, Ag–Ti contacts with an adequate geometry for a low R_s value.

Figure 3 shows two experimental $I(V)$ curves corresponding to the cells studied. It was possible to control the temperature of measurement to different values. This is an important aspect because it gives us information about the evolution of the values of the parameters as a function of temperature. Figure 4 shows an $I(V)$ curve obtained at three different temperatures.

We have systematically applied the two criteria in the definition of the object function

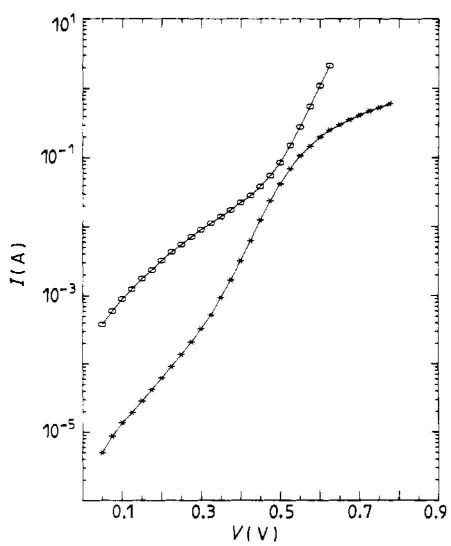

Figure 3. Experimental $I(V)$ curves used as data in the numerical routines. (*) Solar cell for space use; (○) commercial terrestrial solar cell.

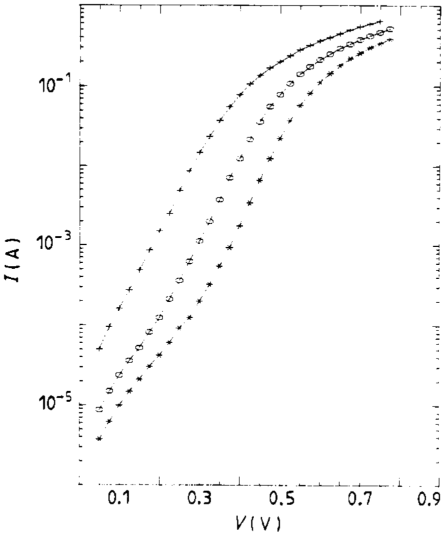

Figure 4. Experimental $I(V)$ characteristics obtained at different temperatures (space-use solar cell). (∗) 293 K; (○) 330 K; (+) 373 K.

to our $I(V)$ characteristics. The results obtained are adequate, as can be seen in table 1, showing the parameter values obtained from the application of both criteria. It is difficult to say which criterion is best to apply in order to obtain the object function.

Table 2 and table 3 give the best results obtained, paying attention to the standard deviation value instead of the selected criterion.

4.1. Space solar cells

Table 2 shows a summary of the parameter values obtained for three samples at different measurement temperatures. Figure 5 shows a comparison between the experimental $I(V)$ curve and the characteristic obtained with equation (1) using the values of the parameters calculated by the method described in this paper and taken from table 2 (samples 2 and 3).

Table 1. Parameter values obtained from $I(V)$ characteristics for two different samples of space-use solar cells at 56 °C.

	Sample 1		Sample 2	
	Minimax	Least mean squares	Minimax	Least mean squares
I_{01} (μA)	1.6	2.1	0.61	0.75
n	1.75	1.81	1.41	1.46
R_{sh} (kΩ)	6.05	7.43	5.94	6.23
I_{02} (nA)	3.9	3.6	1.3	2.4
R_s (Ω)	0.47	0.468	0.47	0.48
SD (%)	3.3	2.5	2.8	2.1

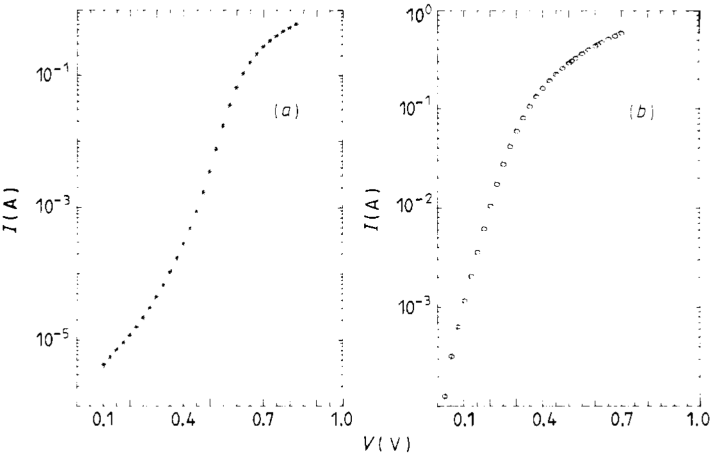

Figure 5. Comparison between data and simulated $I(V)$ curves at two temperatures (space-use solar cell). (*a*) 260 K; (*b*) 410 K. Points (∗), (○), simulation; full curves from data.

If we consider the results in table 2 for the values of the saturation current of the diffusion term I_{02}, it can be seen that they conform quite closely to the Shockley theory. Furthermore, the Shockley theory predicts a temperature dependence of I_{02} as follows

$$I_{02} = kT^3 \exp(-E_g/kT). \tag{15}$$

Considering the temperature dependence of E_g (Neville 1978) we obtain

$$\frac{I_{02}}{kT^3 \exp[2.8 \times 10^{-4}(T-300)/kT]} = \exp\left(\frac{-E_{g(300\ \mathrm{K})}}{kT}\right). \tag{16}$$

A semilog plot of equation (16) would show a linear behaviour, the slope being the band gap of silicon at 300 K.

Table 2. Results obtained for space-use solar cells at different temperatures.

	Temperature (°C)	I_{01} (μA)	n	R_{sh} (kΩ)	I_{02} (nA)	R_s (Ω)	DS (%)
Sample 1	22	0.47	2.06	19.5	0.037	0.42	4.4
	56	2.12	1.81	7.4	3.65	0.46	2.5
	80	3.73	1.56	3.2	26.8	0.46	3
	92	8.24	1.47	2.1	128.4	0.47	2.5
	112	14.3	1.22	0.77	1080	0.47	5
Sample 2	22	0.46	1.96	13.8	0.06	0.53	5.7
	56	0.74	1.46	6.2	2.45	0.49	2.1
	80	2.94	1.31	3.6	5.3	0.46	2
	100	4.66	1.16	1.2	140	0.47	10
	137	108.7	1.3	0.7	2370	0.56	6.2
Sample 3	−13	0.08	2.19	26.6	0.0004	0.31	5.2
	22	0.79	1.99	11.2	0.16	0.36	3.2
	56	5.08	1.83	3.9	17.05	0.44	2.1
	80	13.9	1.73	2.99	221.2	0.46	1.6

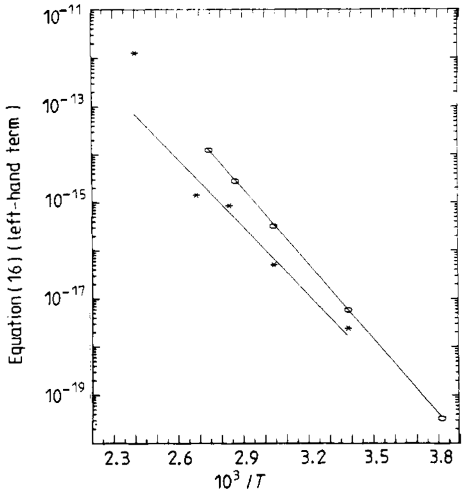

Figure 6. Plot of equation (16) on a semilog axis. The slope is $E_{g(300K)}$. (*) Sample 2; (○) sample 3.

From our results we have observed such behaviour in the samples we have studied. Figure 6 shows the results for samples 2 and 3. The slopes are between 0.938 eV and 1.1 eV. A similar study can be made of the experimental values of the saturation current I_{01}. In this case our results for the slope do not give close values to the Shockley–Read–Hall theory (SRH theory) predictions. We found values between 0.52 eV and 0.72 eV.

The explanation of these results may be that the SRH theory calculates the recombination current if a recombination centre in the middle of the energy gap is considered. As far as solar cells for space use are considered, several recombination centres, induced by irradiation by protons, are present and active in the band gap, and certainly, the most important of them are not located in the middle of the gap.

Another point of interest concerns the values of the parameter n, which is the ideality factor of the recombination term of equation (1). Previous work in this field considered the factor n equal to 2 (Mottet 1980), whereas our work takes n as the fifth parameter in the evaluation procedure. Values of n other than 2 can be explained in terms of the recombination centre density N_t and the activation energy of the levels (Ashburn *et al* 1975).

In the samples we have studied, the ideality factor n depends also on the temperature. The values we have found vary between 2 for $T = -13$ °C and 1.22 for $T = 112$ °C. The consideration of n as the fifth parameter can lead to further insight in the interpretation of the results which is not possible if n is taken as a constant, equal to 2.

More precisely, according to previous theory (Wolf *et al* 1977), values of $n = 1$ can be justified if only shallow levels in the band gap are considered, and values of $n = 1.5$ (Anderson and Buckingham 1977) can be explained if a uniform distribution of levels with two different energies is considered. The method described in this paper can be an interesting tool to further work in this field.

4.2. *Results on commercial solar cells* (*terrestrial use*)

Some of the values obtained are presented in table 3. It is important to note that we have

Table 3. Results obtained for commercial solar cells, designed for terrestrial use.

Parameter	Sample T1	Sample T2	Sample T3
I_{01} (mA cm^{-2})	0.201	0.153	0.406
n	4.3	4.22	4.5
R_{sh} (Ω cm^2)	4200	>10,000	>10,000
I_{02} (nA cm^{-2})	0.027	0.033	0.029
R_s (Ω cm^2)	0.015	0.04	0.03
SD (%)	3.2	14.8	8.9

reached values of n up to 4, that can not be justified theoretically. Our interpretation is that the SCR term loses its physical meaning in order to explain low bias behaviour of the cell. From this point of view, the double exponential $I(V)$ characteristic is not sufficient to describe the $I(V)$ behaviour in the sense that we have obtained a poor adjustment and/or unexpected values for the parameters related to the phenomena for the low-bias, low-current region of the cell.

In some cases a constant value of the shunt resistance R_{sh} does not give good adjustment. We have tried to explain this phenomenon in two ways. First by deducing a function $R_{sh}(V)$ which allows a good adjustment in the lower region of the curve and secondly by trying a three-exponential model for the $I(V)$ characteristic modelling.

The first solution has been used in figure 7, corresponding to sample T1, where the function $R_{sh}(V)$ is formed by decomposition of the curve in a similar way to the one described in § 3. The standard deviation obtained is 8.6% and the corresponding value of the parameters is $I_{01} = 4.2\ \mu$A cm^{-2}, $n = 2.2$ and the same for I_{02} and R_s (see table 3).

Conversely the three-exponential function has been used as follows

$$I = \frac{V - IR_s}{R_{sh}} + \sum_{i=1}^{3} I_{01}\left(\exp\frac{q(V - IR_s)}{n_i kT} - 1\right). \tag{17}$$

The results obtained in this case with equation (15) have been given elsewhere (Cabestany 1982).

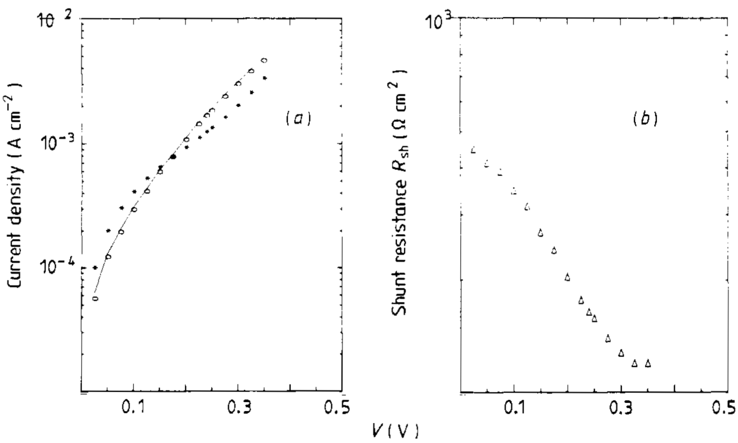

Figure 7. (*a*) Comparison of commercial (terrestrial) cell $I(V)$ data with simulated curves obtained from models of equation (1) and equation (15). (*) R_{sh}; (○) simulated with $R_{sh}(V)$. (*b*) Shape of the $R_{sh}(V)$ function used in the latter case.

Our interpretation of the third exponential term in equation (17) is the modelling of junction short-circuit phenomena due to the influence of the metal contact formation. It is not the purpose of this paper to analyse in detail the physical interpretation that can be given to the third exponential term, but this mechanism may be proposed for development in further research.

There are several works showing the influence that the metal contact formation and associated heat treatments can have on the $I(V)$ characteristics in the low-bias region. For instance Salama (1978) showed degradation of electrical behaviour in solar cells due to the junction short circuit as a result of the titanium migration in silicon (mainly in the presence of oxygen) giving electrically active TiO_2. More recent research (Ross 1982, Weizer *et al* 1982) associates with the same phenomena the so-called 'flat spot' effect in the $I(V)$ characteristic.

We feel that the third exponential term can account for the development underneath the contacts, with the localised metal–semiconductor barriers randomly distributed, and characteristics which will depend on the surface state of the silicon material, contact metal and resistivity of the base of the cell. Quantitative theoretical and experimental work in this field can be done with the help of the routines presented in this paper and will be the object of future research.

5. Conclusions

A methodology to calculate the parameters of the equivalent circuit of a solar cell is described in this paper. The method is based on the definition of an object function for the difference between the experimental $I(V)$ curve and the model. The object function $U(\varphi)$ is found to be nonlinear and hence the appropriate algorithms have to be used to find a solution that optimises the $U(\varphi)$ function.

We have used two criteria, least mean square and minimax, both of them with good results. A method to find an initial solution is also described which is critical in the time calculations of the routine.

Several space-use solar cells have been considered and their parameters have been evaluated. We have also worked with some commercial, terrestrial, cells and a model modification for the $I(V)$ characteristics has been considered in order to explain all the possible phenomena involved.

It must be pointed out that there are two additional questions:

(i) The $I(V)$ characteristics considered as data for the numerical method, are obtained in dark conditions.

(ii) This methodology for parameter evaluation is completely general, in the sense that it is independent of the shape of the $I(V)$ curves of the cells (see figures 3 and 4).

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