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A new optimization method based on COOT bird natural life model

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ABSTRACT

Recently, many intelligent algorithms have been proposed to find the best solution for complex engineering problems. These algorithms can search volatile and multi-dimensional solution spaces and find optimal answers timely. In this paper, a new *meta*-heuristic method is proposed that inspires the behavior of the swarm of birds called Coot. The Coot algorithm imitates two different modes of movement of birds on the water surface: in the first phase, the movement of birds is irregular, and in the second phase, the movements are regular. The swarm moves towards a group of leading leaders to reach a food supply; the movement of the end of the swarm is in the form of a chain of coots, each of coot which moves behind its front coots. The algorithm then runs on a number of test functions, and the results are compared with well-known optimization algorithms. In addition, to solve several real problems, such as Tension/Compression spring, Pressure vessel design, Welded Beam Design, Multi-plate disc clutch brake, Step-cone pulley problem, Cantilever beam design, reducer design problem, and Rolling element bearing problem this algorithm is used to confirm the applicability of this algorithm. The results show that this algorithm is capable to outperform most of the other optimization methods. The source code is currently available for public from: https://www.mathworks.com/matlabcentral/fileexchange/89102-coot-optimization-algorithm.

1. Introduction

Optimization is the process of finding the best answer or the global optimal point for a problem. In optimizing the problems, the optimal global point is the minimum or maximum value of a function. Optimization issues can be found in all fields of study, which makes optimization techniques an essential and important direction of study for researchers. The *meta*-heuristic algorithms are a kind of random algorithms which used to find an optimal response. Optimization methods and algorithms are categorized into two groups of exact algorithms and approximate algorithms. Exact algorithms are capable of finding the optimal answer in a precise manner, but they are not efficient enough for strict optimization problems and their execution time expands exponentially with the dimensions of the problems. Approximate algorithms are capable of finding good (near optimal) solutions at a short time for strict optimization problems. Approximate algorithms are divided into three categories: heuristic algorithms, *meta*-heuristic and hyper heuristic algorithms. The two main problems are the innovative algorithms, catching them at the local optimum points and early convergence into these points. Meta-heuristic algorithms to solve the deficit heuristic algorithms have been proposed (Spall, 2005). In fact, *meta*-heuristic algorithms are one of a kind of approximate optimization algorithms that have solutions for Escape from Local Optimum points and can be applied to a wide range of problems. Various categories of these algorithms have been developed in recent decades (Mahdavi et al., 2015). The capabilities of the *meta*-heuristic processes can be a simple, flexible, non-inference mechanism and avoid local optimums. Meta-heuristic processes are inspired by physical phenomena, animal behavior, evolutionary concepts, and human phenomena. A category of known algorithms is presented in Fig. 1. Many articles have tried to classify optimization algorithms based on their inspiration (Ertenlice & Kalayci, 2018; Hussain et al., 2018; Sotoudeh-Anvari & Hafezalkotob, 2018).

An important question arises here, when there are famous algorithms such as those mentioned above, what is needed to offer and present new algorithms. According to the NFL theorem (Blum & Roli, 2003; Wolpert & Macready, 1997), there is no optimization algorithm that can solve all optimization problems. In fact, the average performance of the optimizers is almost the same. So there are a lot of problems that are not still well solved notwithstanding the popular optimization algorithms, and offering new algorithms can solve such problems. This is the motivation

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of the creation of a new bird collective-behavioral based optimization algorithm called Coot to compete with current algorithms in solving problems. In this paper, a new *meta*-heuristic algorithm is introduced that imitates the movements of the COOT on the water.

The rest of the paper is organized as follows: Section 2 presents a literature review of previous *meta*-heuristic algorithms. Section 3 describes the proposed COOT algorithm. The results and discussion of the proposed algorithm, benchmark functions, and real problems are presented in Sections 4–5. Finally, Section 6 concludes this article and offers suggestions for future work.

2. Related works

In recent years, optimization has become a popular research field and an economical way to find an optimal solution to complex problems. According to Fig. 1, we have divided the optimization algorithms into four categories based on the type of inspiration: Swarm Based Algorithms, which include Particle Swarm Optimization (PSO) (Eberhart & Kennedy, 2002), Firefly Algorithm (FFA) (X.-S. Yang, 2010), Ant Colony Optimization (ACO) (Colorni et al., 1991), Artificial Bee Colony (ABC) (Basturk & Karaboga, 2006), Krill Herd (KH) (Gandomi & Alavi, 2012), multi-leader whale optimization algorithm (MLWOA) (Abd Elaziz et al., 2021), and Enhanced Crow Search Algorithm (ECSA) (Ouadfel & Abd Elaziz, 2020). The second group is evolutionary algorithms such as Genetic Algorithm (GA) (Holland, 1967), Evolution Strategy (ES) (Rechenberg, 1973), Genetic Programming (GP) (Angeline, 1994), Biogeography Based Optimizer (BBO) (Simon, 2008), Evolutionary Programming (EP) (Yao et al., 1999), Differential Evolution (DE) (Storn & Price, 1995), Virulence Optimization Algorithm (VOA) (Jaderyan & Khotanlou, 2016). The third category is physics-based algorithms such as Simulated Annealing (SA) (Kirkpatrick, 2014), Black Hole Algorithm (BH) (Hatamlou, 2013), Curved Space Optimization Algorithm (CSO) (Moghaddam et al., 2012), and Ray Optimization (RO) (Kaveh & Khayatazad, 2012). The fourth category is human-based algorithms such as Harmony Search (HS) (Manjarres et al., 2013), Teaching Learning Based Optimization (TLBO) (Rao et al., 2011), Imperialist Competitive Algorithm (ICA) (Atashpaz-Gargari & Lucas, 2007), Exchanged Market Algorithm (EMA) (Ghorbani & Babaei, 2014), Thermal Exchange Optimization (TEO) (Kaveh & Dadras, 2017) and Tabu Search (TS) (Glover, 1989). There are other algorithms that are a combination of algorithms in these four categories, such as Cooperative Meta-heuristic Algorithms (Elaziz et al., 2021), volleyball premier league algorithm based on sine cosine Algorithm (VPLSCA) (Moghdani et al., 2020), and competitive chain-based Harris Hawks Optimizer (Elaziz et al., 2020). Some of the most famous are described below. Genetic Algorithm (GA) is the most popular evolutionary inspirational technique that mimics the principles of Charles Darwin's Compatibility Survival Theory. This method involves the selection process, the crossover, and the mutation process to replace the worst solution in each generation. In this algorithm, solutions are improved according to the best solutions obtained by each particle so far and the best found solution of the overall swarm. The ACO algorithm imitates the collective behavior of ants in finding the shortest path from the nest to the food source. One of the most important and most interesting behavior of ants is their behavior in finding food, and in particular how to find the shortest path between food and nest. This kind of behavior of the ants has a kind of swarm intelligence that has recently attracted the attention of scientists. In the natural world, ants of some species (initially) wander randomly, and upon finding food return to their colony while laying down pheromone trails. If other ants find such a path, they are likely not to keep travelling at random, but instead to follow the trail, returning and reinforcing it if they eventually find food. The algorithm (DE) is presented to overcome the main defect of the

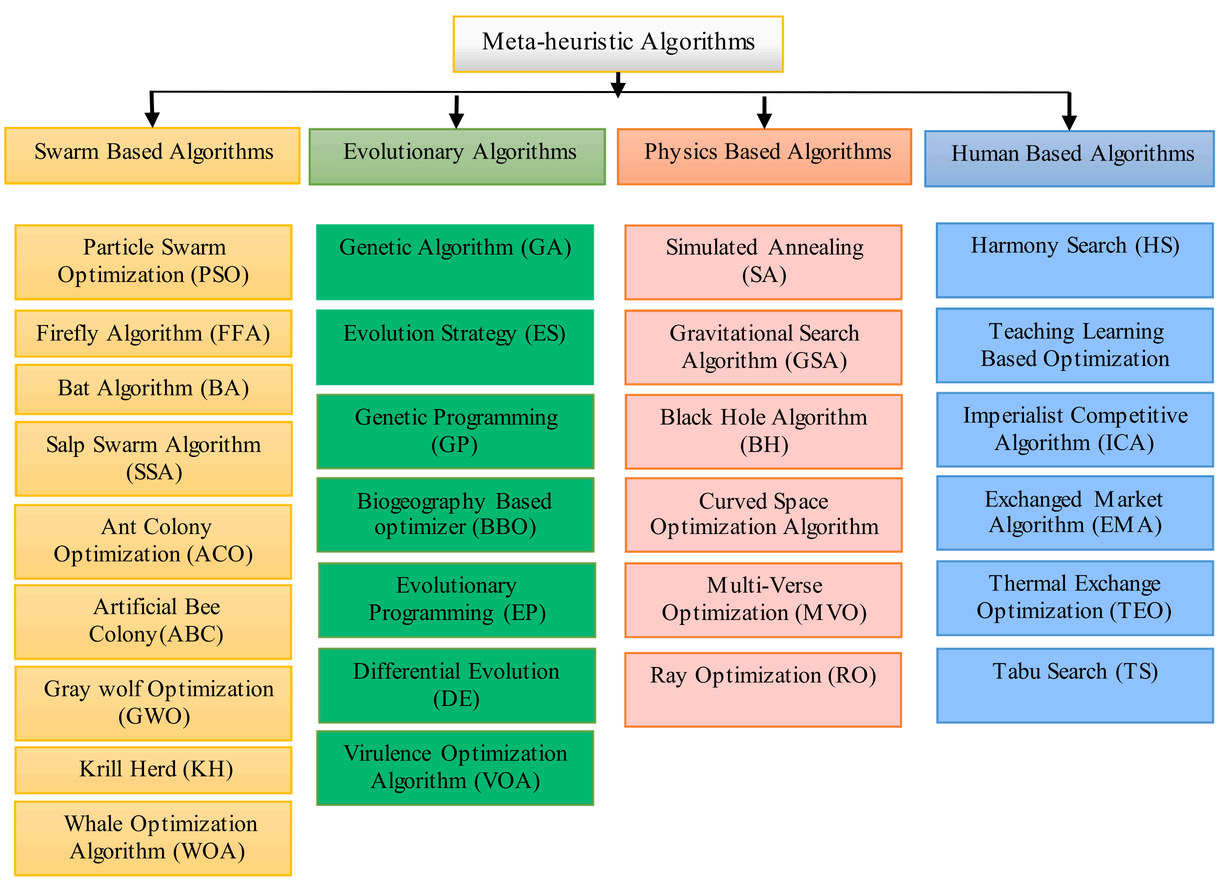

Fig. 1. Categorization of *meta*-heuristic algorithms.

genetic algorithm, namely the lack of local search in this algorithm. The main difference between genetic algorithms and algorithms (DE) is in the selection operators. In the ABC model, the colony consists of three groups of bees: employed bees, onlookers and scouts. It is assumed that there is only one artificial employed bee for each food source. In other words, the number of employed bees in the colony is equal to the number of food sources around the hive. Employed bees go to their food source and come back to hive and dance on this area. The employed bee whose food source has been abandoned becomes a scout and starts to search for finding a new food source. Onlookers watch the dances of employed bees and choose food sources depending on dances. The firefly algorithm (FA) is a *meta*-heuristic algorithm, inspired by the flashing behavior of fireflies. The primary purpose of a firefly's flash is to act as a signal system to attract other fireflies. The Firefly Algorithm is based on the life of the firefly worms. In the firefly algorithm, worms tend to be attracted to high-attraction light. The worm with less light can be absorbed into the worm with more light. In this situation, the swarm moves like a particle swarm algorithm (PSO). Of course, the degree of motion is also taken randomly to prevent the early convergence of the firefly algorithm. Bat algorithm is a heuristic algorithm proposed by Yang in 2010 and has been inspired by a property, named as echolocation, which guides the bats' movements during their flight and hunting even in complete darkness. The Salp swarm algorithm is inspired by the movement of a chain of marine species called Salp that has a clear body and its movement is like jellyfish, and water is pumped through the body and causes the salp to move. Salps form a chain to achieve fast and harmonious movement to find food. GSA is a nature-inspired algorithm based on Newton's famous law of gravity and the law of motion. In this algorithm, search agents are a set of objects that can be thought of as planets of a system. The optimum region, like a black hole, sucks the planets toward you. Information about the fitness of each object is stored in the form of gravitational masses and inertia. The exchange of information and the impact of objects on each other are under the force of gravitation.

Optimization algorithms have advantages and disadvantages.For example, the pso algorithm solves problems with small and simple dimensions well, but has poor performance in high-dimensional, complex, and hybrid problems, and suffers from premature convergence. Success-History based Adaptive Differential Evolution (SHADE) algorithm, an improved version of the DE algorithm, solves hybrid and complex problems well and does not suffer from premature convergence, but it does not solve high-dimensional problems well. Genetic algorithms and gravitational search have the same disadvantages. In this article, we tried to solve these disadvantages.

3. Coot optimization algorithms

3.1. Description

the Coots are small water birds that are members of the rail family, Rallidae. They constitute the genus Fulica, the name being the Latin for "coot" (Paillisson & Marion, 2001). Coots have prominent frontal shields or other decoration on the forehead, with red to dark red eyes and colored bills. Many, but not all, have white on the under the tail. The literature on American coot behavior includes extensive work on breeding, habitat and migratory behavior (Randler, 2005; Varo & Amat, 2008; Zhang et al., 2011). There are many behaviors and movements of coots on the water. In this article, we used the behavior of this bird on the water surface to provide a new optimization method. Coots travel at angles to their direction of motion and appear to come well within what is, for surf scoters, a zone of repulsion. The behavior of coot's swarm on water consists of Three movements. A disordered movement of activity, as shown in Fig. 2a, and a synchronized movement, shown in Fig. 2b. The coots also make a chain move on the surface of the water, and each coot moves behind its front coot, as shown in Fig. 3 (Trenchard, 2012).

Coots have different collective behaviors, our goal in this paper is to simulate collective movements (regular and irregular movements on the surface of the water. The whole group is directed toward the target (food)by a few coots in front of the group, which we consider group leaders.

Therefore, we consider the four different moves of coots on the water surface, which are as follows.

1- Random movement to this side and that side

2- Chain movement

3- Adjusting the position based on the group leaders

4- Leading the group by the leaders towards the optimal area (leader movement)

3.2. Mathematical model and algorithm

the basic framework of all optimization algorithms is the same. The algorithm starts with $(\vec{x}) = \{\vec{x}_1, \vec{x}_2, \ldots \vec{x}_n\}$ an initial random population. This random population is repeatedly evaluated by the target

Fig. 3. Moving the chain of coots on the water (Trenchard, 2012).

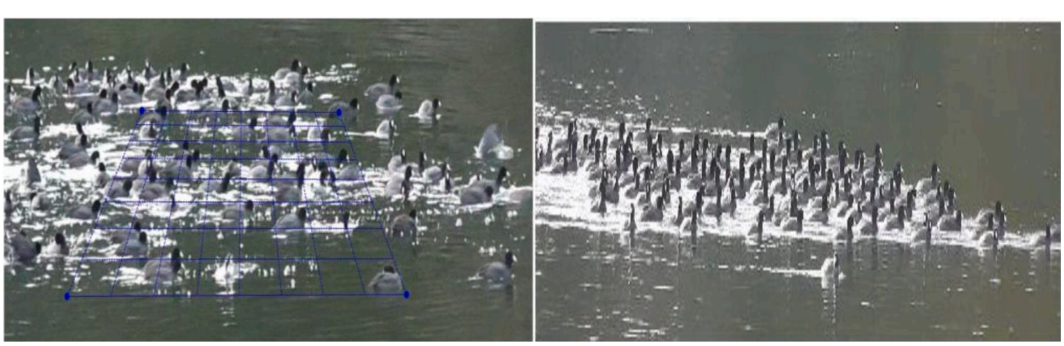

Fig. 2. (a) Disordered phase, (b) Synchronized phase, (Trenchard, 2012).

function and a target value is determined $(\overrightarrow{O}) = \{O_1, O_2, \ldots O_n\}$. It is also improved by a set of rules that are the core of an optimization technique. Because population-based optimization techniques look for the optimal amount of optimization problems, there is no guarantee of finding a solution in one run. However, with sufficient numbers of random solutions and optimization steps (iteration), the probability of finding the global optimal increases. The population is randomly generated in the sightly space using the formula (1):

$$CootPos(i) = rand(1,d).*(ub - lb) + lb \tag{1}$$

Where CootPos(i) is the coot position, d the number of variables or problem dimensions, lb is the lower bound of the search space and ub is the upper bound of the search space that is defined as the formula (2). Each variable may have a different lower bound and upper bound problem.

$$lb = [lb_1,\ lb_2, \ldots,\ lb_d]\ , ub = [ub_1,\ ub_2, \ldots,\ ub_d] \tag{2}$$

After generating the initial population and determining the position of each agent, the fitness of each solution must be calculated using $O_i = f(\overrightarrow{x})$ the objective function. We select the NL number of coots as group leaders. The choice of leaders is random.

Now, the four movements of coots on the surface of the water, which were mentioned in the previous section, are implemented.

3.2.1. Random movement to this side and that side

To implement this movement, we consider a random position according to formula (3) in the search space and move the coot towards this random position.

$$Q = rand(1,d).*(ub - lb) + lb \tag{3}$$

This coot movement explores different parts of the search space. If the algorithm gets stuck in the local optimal, this movement will cause the algorithm to escape from the local optimal. coot's new position is calculated according to formula (4).

$$CootPos(i) = CootPos(i) + A \times R2 \times (Q - CootPos(i)) \tag{4}$$

Where R2 is a random number in the interval [0, 1], A is calculated according to formula (5). Fig. 4 depicts the random movement of the coot in different directions.

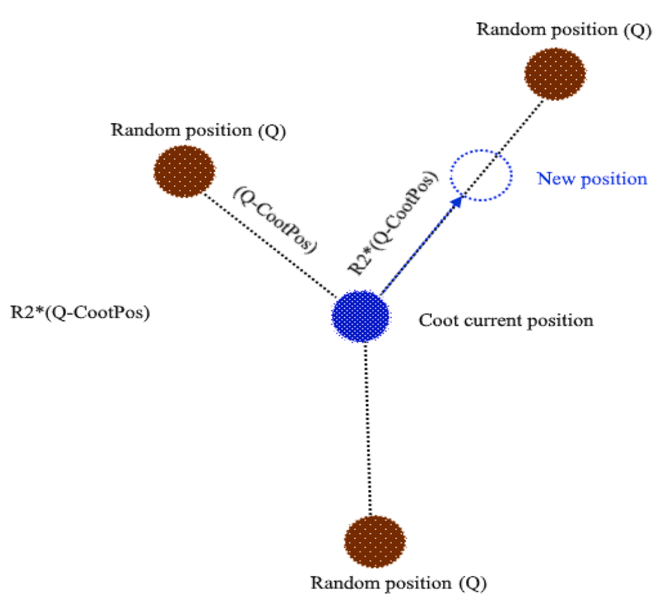

Fig. 4. Random movement to this side and that side.

$$A = 1 - L \times \left(\frac{1}{Iter}\right) \tag{5}$$

Where L is current iteration, Iter is maximum iteration.

3.2.2. Chain movement

The average position of two coots can be used to implement chain movement (Mirjalili et al., 2017). Another way to implement a chain movement is that we first calculate the distance vector between the two coots and then move the coot toward the other coot about half the distance vector. We used the first method and the new position of the coot is calculated according to formula (6).

$$CootPos(i) = 0.5 \times (CootPos(i-1) + CootPos(i))(6)$$

Where CootPos(i-1) is second coot. Fig. 5 shows the chain movement of the coots.

3.2.3. Adjusting the position based on the group leaders

Usually the group is led by a few coots in front of the group and the rest of the coots have to adjust their position based on the group's leaders and move toward them. A question that might be asked is that each coot will adjust its position based on which leader. The average position of the leaders can be considered, and the coots can update their position based on this average position. Considering the average position causes premature convergence. To implement this movement, we use a mechanism according to formula (7) to select the leader.

$$K = 1 + (i\,MOD\,NL) \tag{7}$$

Where i is the index number of the current coot, NL is the number of leaders, and K is the leader's index number. Fig. 6 shows the mechanism for choosing a leader by coot.

the coot(i) must update its position based on the leader's k. Formula (8) calculates the next position of the coot based on the selected leader.

$$\begin{aligned} CootPos(i) = {} & LeaderPos(k) + 2 \times R1 \times cos(2R\pi) \\ & \times (LeaderPos(k) - CootPos(i)) \end{aligned} \tag{8}$$

Where CootPos(i) is the current position of coot, LeaderPos(k) is selected leader position, R1 is a random number in the interval [0, 1], π is the same pi value as 3.14, and R is a random number in the interval [−1, 1].

3.2.4. Leading the group by the leaders towards the optimal area (leader movement)

The group must be directed towards a goal (optimal area), so leaders

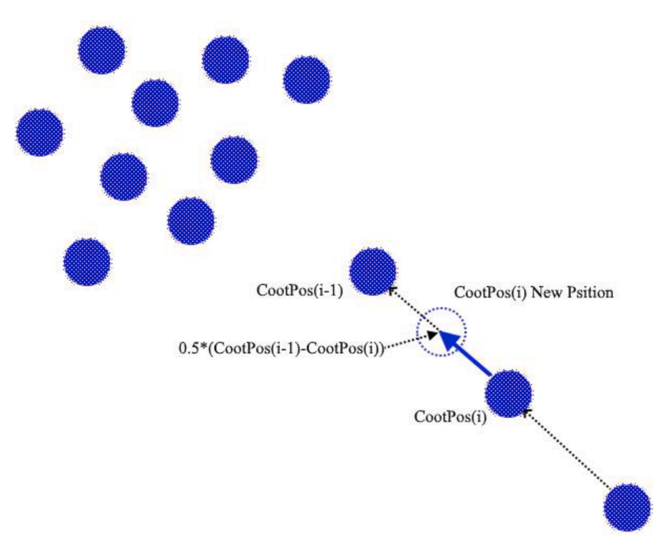

Fig. 5. Chain movement of the coots.

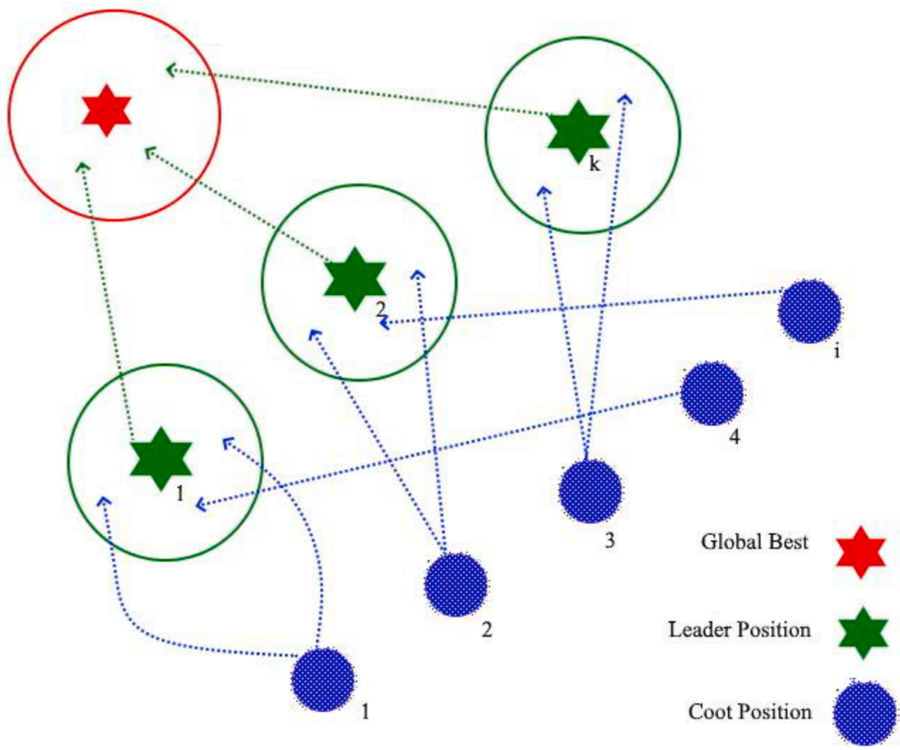

Fig. 6. mechanism for choosing a leader by coot.

need to update their position toward the goal. Formula 9 is recommended to update the position of leaders. This formula looks for better positions around this current optimal point. Sometimes leaders have to move away from the current optimal position to find better positions. This formula provides a good way of getting closer to optimal location and getting away from it.

$$LeaderPos(i) = \left\{ \begin{array}{ll} B \times R3 \times cos(2R\pi)\times & R4 < 0.5 \\ (gBest - LeaderPos(i)) + gBest & \\ B \times R3 \times cos(2R\pi)\times & R4 >= 0.5 \\ (gBest - LeaderPos(i)) - gBest & \end{array} \right\} \tag{9}$$

Where gBest is the best position ever found, R3 and R4 are random number in the interval [0, 1], π is the same pi value as 3.14, R is a random number in the interval [−1, 1], and B is calculated according to formula (10).

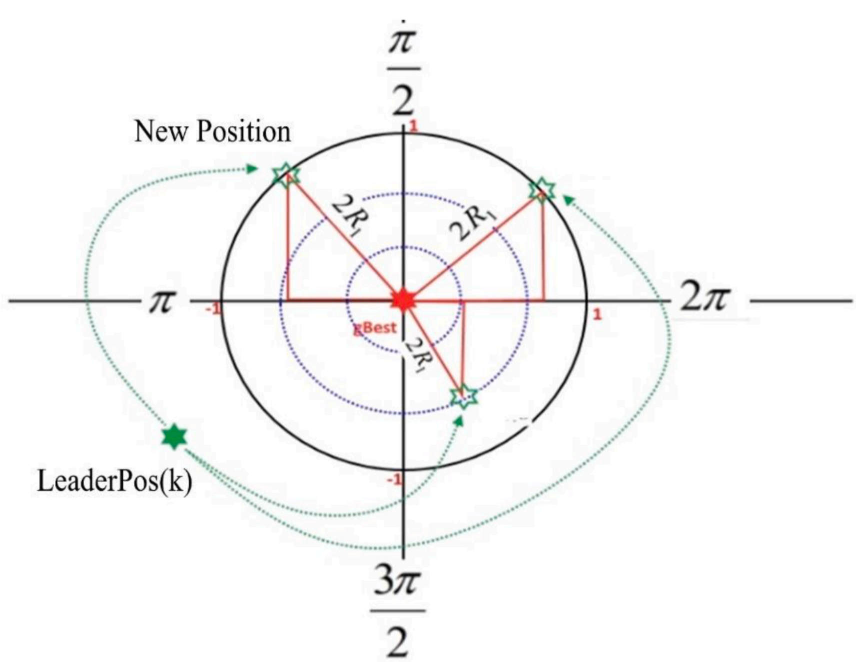

Fig. 7. update of the leaders position relative to the best position.

$$B = 2 - L \times \left(\frac{1}{Iter}\right) \tag{10}$$

Where L is current iteration, Iter is maximum iteration.

$2 \times R3$ Makes larger random movements so that the algorithm does not get trapped in the local optimum. This means that we are performing also exploration during the exploitation phase. Cos(2Rπ) Searches around the best search agent with different radius to find a better position around this search agent. Fig. 7 shows the update of the leaders position relative to the best position.

The question that arises here is how and when these different movements are performed. In order to preserve the random nature of the optimization algorithms, we consider all these movements randomly. It means that during the execution of the algorithm, the coot may move randomly, in a chain, or toward group leaders.

The pseudo-code of COOT is presented below.

```
Initialize the first population of coots randomly by Eq 1 and
Eq 2
Initialize the  parameters of P=0.5, NL (number of leaders),
Ncoot(number of coots).
Ncoot=Npop-Nl;
Random selection of leaders from the coots
Calculate the fitness of coots and leaders
Find the best coot or leader as the Globa optimum (gBest)
while the end criterion is not satisfied
Calculate A, B parameters by Eq 5 and Eq 10
 If rand< P
  R, R1, and R3 are random vectors along the dimensions of
the problem
 Else
  R, R1, and R3 are  random number
 End
  For i=1 to the number of the coots
     Calculate the  parameter of K by Eq 7
     If  rand>0.5
        Update the position of the coot by Eq 8
     Else
        If rand<0.5 i~=1
           Update the position of the coot by Eq 6
        Else
           Update the position of the coot by Eq 4
        End
     End
     Calculate the fitness of coot
     If  the fitness of coot < the fitness of leader(k)
     Temp=leader(k); leader(k)=coot; coot=Temp;
     end
  End
  For number of Leaders
     If  rand <0.5
       Update the position of the Leader by Eq 9.1
    Else
       Update the position of the Leader by Eq 9.2
    End
    If  the fitness of leader < gBest
     Temp= gBest; gBest =leader; leader=Temp;  (update
Global optimum)
     end
  End
Iter=iter+1;
end
```

4. Results and discussion

4.1. COOT algorithm performance on the classical functions

In this section, the COOT algorithm is evaluated in 13 criteria functions (with 30, 100, and 500 dimention). These are classical functions that have been used by many researchers (Mirjalili, 2016; Saremi et al., 2017). We selected these test functions to compare our results with the results of the current *meta*-heuristic algorithms. These standard functions are shown in Tables 1-2, where Dim represents the function dimensions, Range is the boundary of the search space of the function, and fmin is the optimal value.

The statistical results (mean and standard deviation) are reported in Tables 4-6. To validate the results, the COOT algorithm is compared with the PSO algorithm and Multi Verse Optimizer (MVO) (Mirjalili et al., 2016), Gray Wolf Optimization (GWO) (Mirjalili et al., 2014), and the Salp Swarm Algorithm (SSA) (Mirjalili et al., 2017), as SI-based methods and Gravitational Search Algorithm (GSA) (Rashedi et al., 2009), as a physics-based algorithm. In addition, the COOT algorithm is compared with Artificial Electric Field Algorithm (AEFA)(Anita et al., 2020), fitness dependent optimizer (FDO) (Abdullah & Ahmed, 2019), Success-History based Adaptive Differential Evolu-tion (SHADE) (Tanabe & Fukunaga, 2013), and genetic algorithm (GA) The initial controlling parameters of algorithms are shown in Table 3.

We first implemented the algorithm on 13 test functions with 30 dimensions. from these 13 functions, the first 7 are unimodal and the second 6 are multimodal. Unimodal functions are suitable for measuring the exploitation of algorithms. As Table 4 (F1-F7) shows, the proposed algorithm provides the best results in 5 test functions of 7. This shows that the coot algorithm has successfully exploited the search space. Multi-modal functions are used to measure the exploration of algorithms. These functions have many local optimum and the algorithm should avoid this local optimum. The statistical results of the algorithms on these functions are shown in Table 4 (F8-F13) . The proposed algorithm performs better than the other algorithms in the F9, F10, and F11 test functions and performs better than half the algorithms in other test functions. The results show that the proposed algorithm is capable of exploring the search space.

In the next step we implemented the proposed algorithm on the same 13 previous test functions with 100 dimensions. The results of this implementation are presented in Table 5. the results show that the proposed algorithm does not reduce performance by increasing the problem size and is stable in most test functions. The proposed algorithm at this stage performed better than all the compared algorithms in 6 unimodal test functions (F1,F2,F3,F4,F5 and F7) and 5 multimodal test functions (F9,F10,F11,F12 and F13).

In the final step, the proposed algorithm was tested on 13 test functions with 500 dimensions. The results of this experiment are shown in Table 6. The coot algorithm has performed better than all other algorithms in all test functions except function 8. The results show that the proposed algorithm is still stable with increasing dimensions, but other algorithms have shown very poor performance. The results in Tables 4 to

Table 1
Unimodal benchmark functions.

Function	Dim	Range	f_{MIN}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	30, 100, 500	[−100, 100]	0
$f_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30, 100, 500	[−10, 10]	0
$f_3(x) = \sum_{i=1}^{n} \left(\sum_{j-1}^{i} x_j\right)^2$	30, 100, 500	[−100, 100]	0
$f_4(x) = max\{\lvert x_i \rvert, 1 \leqslant i \leqslant n\}$	30, 100, 500	[−100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1} [100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2]$	30, 100, 500	[−30, 30]	0
$f_6(x) = \sum_{i=1}^{n} ([x_i + 0.5])^2$	30, 100, 500	[−100, 100]	0
$f_7(x) = max\{\lvert x_i \rvert, 1 \leqslant i \leqslant n\}$	30, 100, 500	[−1.28, 1.28]	0

Table 2
Multimodal benchmark functions.

Function	Dim	range	F_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i sin(\sqrt{\lvert x_i \rvert})$	30, 100, 500	[−500, 500]	0
$F_9(x) = \sum_{i=1}^{n}[x_i^2 - 10cos(2\pi x_i) + 10]$	30, 100, 500	[−5.12, 5.12]	0
$F_{10}(x) = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$	30, 100, 500	[−32, 32]	0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} cos(\frac{x_i}{\sqrt{i}}) + 1$	30, 100, 500	[−600, 600]	0
$F_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})] + (y_n - 1)^2\right\}$ $+\sum_{i=1}^{n} u(x_i, 10, 100, 4) + \sum_{i=1}^{n} u(x_i, 10, 100, 4) \quad y_i = 1 + \frac{x_i + 1}{4}$ $u(x_i, a, k, m) = \left\{\begin{array}{ll} k(x_i - a)^m & x_i > a \\ 0 & -a < x_i < a \\ k(-x_i - a)^m & x_i < -a \end{array}\right\}$	30, 100, 500	[−50, 50]	0
$F_{13}(x) = 0.1\{sin^2(3\pi x_1) + \sum_{i=1}^{n}(x_i - 1)^2[1 + sin^2(3\pi x_i + 1)]$ $+(x_n - 1)^2[1 + sin^2(2\pi x_n)]\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30, 100, 500	[−50, 50]	0

Table 3
Initial values for the controlling parameters of algorithms.

algorithm	parameter	value
GA	Type	Real coded
	Selection	Roulette wheel
	Crossover	Single point 0.8
	Mutation	0.3
PSO	Topology	Fully connected
	Cognitive and social constants	C1 = 2 , c2 = 2
	Inertial weight	Linearly decreases from 0.9 to 0.4
SSA	Leader position update probability	0.5
AEFA	FCheck	1
	Rpower	1
	Tag	1
	Rnorm	2
BA	Loudness (A), pulse rate (r)	0.5, 0.5
GSA	Rnorm, Rpower, alpha, and G0	2, 1, 20, 100
GWO	Convergence parameter (a)	Linear reduction from 2 to 0
SHADE	Pbest, Arc rate	0.1, 2

6 show the scalability of the proposed algorithm.

We show the Friedman mean rank of the compared algorithms for all classical functions in Fig. 8. Based on the results of Fig. 8, COOT has demonstrated a reliable and confident behavior in all classical functions compared to the other methods.

4.2. COOT algorithm performance on the CEC-2017 test functions

We used the 2017 test functions to show the performance of the proposed algorithm. This set includes 30 unimodal (F1-F3), Multimodal (F4-F10), Hybrid (F11-F20), Composition (F21-F30) test functions. These functions are very complex and challenging (Awad, Ali, Suganthan, Liang, & Qu, 2017). For more details on these functions, see the relevant article. The dimension is considered as 10 for all functions of this set. It is noted that all algorithms are run 30 times with 1000 iterations limited to 60,000 maximum number of function evaluations. The proposed algorithm was tested on these 30 test functions and compared with PSO, GA, GSA, Whale Optimization Algorithm (WOA) (Mirjalili & Lewis, 2016), SSA, AEFA, SHADE, Harris Hawks Optimization (HHO) (Heidari et al., 2019), and Gray-Wolf Optimizer (GWO) (Mirjalili et al., 2014) algorithms.

The results of Table 7 show that the proposed algorithm generally performed better on unimodal and multimodal functions than all the compared algorithms except SHADE, and the COOT algorithm ranked 2nd.

In the next step, we implemented the coot algorithm on the hybrid functions of the cec2017 collection and compared it with other algorithms. The results are shown in Table 8. The results show that the coot algorithm performed better on hybrid functions than other compared algorithms except SHADE. It is noteworthy that the results of the proposed algorithm on these functions are slightly different from the SHADE algorithm.

In the final stage, the proposed algorithm was implemented on Composition functions and compared with other algorithms. The results are shown in Table 9. The results show that the coot algorithm performed better on Composition functions than any other compared algorithm (even SHADE). the proposed algorithm is ranked number one in solving Composition functions. this indicates that the proposed algorithm has the ability to solve such functions. We show the Friedman mean rank of the compared algorithms for all three groups of CEC2017 functions in Fig. 9. Based on the results of Fig. 9, COOT has demonstrated a reliable and confident behavior in all three groups of CEC2017 functions.

The motivations of the search factors in the early stages of the optimization algorithm are sudden so that the exploration of the entire search area is performed, and the movements gradually become slower than to be exploited (He & Wang, 2007). The convergence behavior of the COOT algorithm is shown in Fig. 10, for this purpose, the search history and the path of the first search factor in the first dimension are plotted. The second column of Fig. 10 shows the search history of the search factors. The third column shows convergence. At the beginning of the algorithm, the movements are sudden and gradually converge to a point. In the PSO algorithm, the new position is updated based on the position of the best search factor and the best experienced individual position and the ratio of the previous speed. Difference between the COOT algorithm and the PSO algorithm is that we do not have the previous speed parameter in the proposed algorithm, and the position of each search agent is updated based on the current position of the search agent and the position of a number of search agents that are considered as group leaders. In addition, the proposed algorithm updates a new position based on chain movement and Random movement in different directions. Another important difference is that in the COOT algorithm, Each search factor adjusts its position on the basis of its current position and leaders, and they are not directly related to gBest (global best), and only the leaders are associated with the gBest. In PSO, all search agents

Table 4
Results for the Unimodal and Multimodal benchmark functions with 30 Dimention and 15,000 NFE.

Function		PSO	GA	GSA	MVO	SSA	FDO	AEFA	SHADE	COOT
F1	min	9.3905E-08	1.5245E+00	9.6687E-17	5.8450E-01	2.0460E-08	4.0338E+03	1.2042E-23	3.6610E-11	2.8240E-47
	max	5.0237E-05	7.4966E+00	9.1000E-03	2.0037E+00	8.2645E-07	1.3399E+04	9.9218E+01	3.6610E-11	4.3670E-23
	avg	2.8706E-06	3.6872E+00	3.0385E-04	1.2702E+00	1.7064E-07	7.8924E+03	6.0848E+00	5.3780E-10	1.4570E-24
	std	9.0693E-06	1.3063E+00	1.7000E-03	3.5250E-01	1.9364E-07	2.2788E+03	1.7971E+01	6.2524E-10	7.9728E-24
F2	min	1.1259E-04	2.2110E-01	5.1650E-08	5.0970E-01	1.3170E-01	3.2510E+01	5.4704E+00	2.0643E-06	4.0984E-24
	max	1.0800E-02	6.5960E-01	5.0147E+00	1.1893E+02	3.9579E+00	6.1353E+01	3.6682E+01	2.1998E-04	9.0834E-13
	avg	1.8000E-03	4.7510E-01	3.6620E-01	8.2030E+00	1.6660E+00	4.2446E+01	1.8795E+01	4.7532E-05	5.6782E-14
	std	2.4000E-03	1.0590E-01	1.1040E+00	2.4081E+01	1.1661E+00	7.1491E+00	9.5159E+00	5.5129E-05	1.8801E-13
F3	min	3.1124E+01	2.8924E+03	4.7366E+02	7.1790E+01	2.4401E+02	8.9605E+03	1.1117E+03	1.1341E+00	9.1935E-48
	max	4.5637E+02	1.1739E+04	1.8799E+03	4.8576E+02	3.5921E+03	3.7133E+04	6.1801E+03	7.6335E+01	8.5873E-22
	avg	1.5501E+02	5.1669E+03	9.7795E+02	2.1949E+02	1.6151E+03	2.3980E+04	2.7861E+03	1.2696E+01	2.8626E-23
	std	1.0194E+02	2.1462E+03	3.2710E+02	1.0143E+02	9.0216E+02	7.5680E+03	1.1680E+03	1.5411E+01	1.5678E-22
F4	min	1.0955E+00	4.9927E+00	3.8275E+00	8.3760E-01	4.3926E+00	3.0022E+01	3.3830E-01	3.2890E-01	9.1191E-25
	max	6.1639E+00	2.1162E+01	1.5348E+01	4.5430E+00	2.1461E+01	4.9077E+01	9.2169E+00	2.3737E+00	3.0541E-12
	avg	2.5264E+00	9.1426E+00	7.5426E+00	2.1585E+00	1.1956E+01	4.0549E+01	5.1815E+00	9.3560E-01	1.4861E-13
	std	1.0886E+00	3.0510E+00	2.5359E+00	8.2130E-01	3.9178E+00	4.7330E+00	2.0748E+00	5.6400E-01	6.0000E-13
F5	min	1.6161E+01	1.7212E+02	1.8860E+01	3.2136E+01	2.5052E+01	2.6094E+06	1.6070E+02	9.1791E+00	2.8344E+01
	max	1.0518E+02	9.0158E+02	3.7077E+02	1.5503E+03	1.5081E+03	1.1854E+07	6.3204E+04	8.9323E+01	6.6751E+01
	avg	3.7766E+01	4.1400E+02	5.5176E+01	2.2873E+02	2.5210E+02	6.1675E+06	6.4095E+03	2.7702E+01	3.1804E+01
	std	2.4842E+01	1.7821E+02	6.5717E+01	3.0975E+02	3.4243E+02	2.3139E+06	1.3272E+04	1.8017E+01	7.8738E+00
F6	min	3.8856E-08	1.5385E+00	1.0763E-16	4.7280E-01	2.8657E-08	3.6309E+03	1.0198E-04	2.6025E-11	4.7900E-02
	max	9.8496E-06	7.5916E+00	7.5898E-16	2.0189E+00	8.1567E-07	1.3997E+04	2.4047E+01	4.8932E-09	5.4550E-01
	avg	1.5514E-06	3.6049E+00	2.1296E-16	1.2045E+00	2.0255E-07	7.5626E+03	3.6599E+00	6.8013E-10	1.9360E-01
	std	2.4394E-06	1.5701E+00	1.1951E-16	3.6410E-01	1.9760E-07	2.1602E+03	5.7088E+00	1.2644E-09	1.2700E-01
F7	min	1.0200E-02	5.5600E-02	3.0500E-02	1.4000E-02	6.1500E-02	1.2952E+00	4.1100E-02	9.3000E-03	3.0816E-04
	max	5.6100E-02	2.0180E-01	1.3260E-01	5.5000E-02	4.8950E-01	8.1979E+00	3.4235E+00	4.0200E-02	2.2100E-02
	avg	2.5300E-02	1.2240E-01	7.7000E-02	3.1800E-02	1.7090E-01	3.4074E+00	5.7790E-01	2.0100E-02	4.4000E-03
	std	1.0300E-02	3.9000E-02	3.1000E-02	1.0100E-02	9.1000E-02	1.5396E+00	7.8060E-01	7.5000E-03	4.8000E-03
F8	min	−7.8895E+03	−1.1491E+04	−4.3168E+03	−9.0785E+03	−8.9386E+03	−4.0397E+03	−3.6057E+03	−1.2459E+04	−9.2190E+03
	max	−4.8503E+03	−1.0316E+04	−1.8340E+03	−5.5108E+03	−6.0596E+03	−2.1741E+03	−1.9205E+03	−1.1588E+04	−6.3040E+03
	avg	−6.5312E+03	−1.0872E+04	−2.6329E+03	−7.6056E+03	−7.4333E+03	−2.7823E+03	−2.5448E+03	−1.2156E+04	−7.5838E+03
	std	8.0228E+02	3.0684E+02	5.4839E+02	7.8459E+02	7.7441E+02	4.0470E+02	4.7170E+02	2.0127E+02	7.3974E+02
F9	min	9.5516E+01	2.8791E+00	1.4924E+01	6.4368E+01	2.8854E+01	1.6057E+02	1.9899E+01	4.4270E+00	0.0000E+00
	max	9.5516E+01	1.4370E+01	4.6763E+01	1.9867E+02	9.9496E+01	2.3677E+02	6.2682E+01	1.8577E+01	5.0591E-12
	avg	4.8255E+01	7.7912E+00	3.0844E+01	1.1275E+02	5.9345E+01	2.0104E+02	3.8149E+01	9.8402E+00	1.8948E-13
	std	1.6229E+01	2.8269E+00	6.9524E+00	3.6004E+01	1.8602E+01	1.9047E+01	9.4756E+00	3.4910E+00	9.2219E-13
F10	min	4.6059E-05	3.2410E-01	6.9696E-09	4.6500E-01	1.6462E+00	1.0229E+01	2.7100E-02	1.7372E-06	8.8818E-16
	max	2.8136E+00	1.6306E+00	3.2000E-02	4.0952E+00	4.3830E+00	1.4143E+01	3.2839E+00	1.6462E+00	1.9263E-08
	avg	1.2718E+00	7.3590E-01	1.1000E-03	1.9650E+00	2.6130E+00	1.2901E+01	1.6247E+00	4.0880E-01	6.4475E-10
	std	8.2150E-01	3.1770E-01	5.8000E-03	7.5410E-01	6.3640E-01	9.6770E-01	7.8610E-01	5.6220E-01	3.5164E-09
F11	min	3.8588E-08	9.1080E-01	1.7551E+01	7.0540E-01	8.9354E-04	3.3934E+01	2.7579E+00	2.2671E-11	0.0000E+00
	max	7.0900E-02	1.0681E+00	4.1632E+01	9.7800E-01	4.5500E-02	1.0768E+02	1.8863E+01	3.1900E-02	6.1950E-14
	avg	1.4800E-02	1.0221E+00	2.7649E+01	8.5800E-01	1.6900E-02	6.9498E+01	9.2975E+00	5.3000E-03	2.1057E-15
	std	1.6700E-02	3.0900E-02	5.5112E+00	7.9800E-02	1.1700E-02	1.9762E+01	3.9670E+00	8.6000E-03	1.1305E-14
F12	min	3.0857E-09	2.9000E-03	2.8580E-01	2.0630E-01	2.7789E+00	6.7701E+04	5.7020E-01	3.6491E-12	1.2000E-03
	max	9.3380E-01	2.4180E-01	5.9505E+00	5.3690E+00	1.1171E+01	1.0329E+07	1.4932E+01	1.0370E-01	1.3627E+00
	avg	1.7650E-01	3.1500E-02	1.9528E+00	2.0997E+00	6.1111E+00	3.5338E+06	5.0410E+00	1.3800E-02	9.7700E-02
	std	2.4850E-01	4.5400E-02	1.0741E+00	1.3981E+00	2.3871E+00	2.4166E+06	2.7787E+00	3.5800E-02	2.8540E-01
F13	min	3.4979E-07	7.6300E-02	2.0530E-01	5.4500E-02	9.0500E-02	4.0788E+06	1.3446E+01	1.1206E-11	7.0200E-02
	max	6.2250E-01	6.1460E-01	3.4698E+01	7.9380E-01	4.3504E+01	2.7790E+07	5.4673E+01	1.1000E-02	2.9777E+00
	avg	3.4600E-02	3.0580E-01	8.7784E+00	1.9020E-01	1.7499E+01	1.2712E+07	3.2278E+01	2.2000E-03	5.4750E-01
	std	1.1420E-01	1.3670E-01	6.9787E+00	1.2920E-01	1.4035E+01	6.7033E+06	1.0520E+01	4.5000E-03	5.7420E-01

have the same constant movement and velocity at the same time, while in coot it is not, and a coot may have a chain movement or randomly movement.

The COOT algorithm differs from the Multi-Leader PSO (MLPSO) algorithm (Liu & Liu, 2017).The proposed algorithm is inspired by coot movements on the water, and like multi-leader pso, has not a speed parameter. Another important difference is in the mechanism of leader selection. We use a simple mechanism, but in multi-leader pso the game theory mechanism is used. The multi-leader pso algorithm requires a lot of memory to store the best experienced positions, whereas the proposed algorithm does not use memory. All individuals of the multi-leader pso algorithm use one mechanism to update , whereas the proposed

Table 5
Results for the Unimodal and Multimodal benchmark functions with 100 Dimention and 15,000 NFE.

Function		PSO	GA	GSA	MVO	SSA	FDO	AEFA	SHADE	COOT
F1	min	6.9653E+01	5.3853E+02	2.4878E+03	1.2229E+02	9.4142E+02	3.7611E+04	1.6677E+03	7.5320E-01	3.0297E-53
	max	4.1826E+02	1.1458E+03	5.7871E+03	2.2844E+02	2.1602E+03	6.7058E+04	5.3680E+03	1.8241E+01	6.8693E-16
	avg	1.7389E+02	8.1352E+02	4.1617E+03	1.6478E+02	1.4219E+03	4.8258E+04	2.8607E+03	4.3805E+00	2.2900E-17
	std	7.7260E+01	1.5153E+02	9.1449E+02	2.5006E+01	3.2757E+02	6.9383E+03	7.9407E++02	3.9703E+00	1.2542E-16
F2	min	6.4414E+00	1.3601E+01	1.0218E+01	3.4536E+02	3.6213E+01	2.0918E+02	1.5568E+02	1.2592E+00	2.8292E-24
	max	2.1111E+01	1.9811E+01	3.6337E+01	2.5402E+29	6.4264E+01	1.0060E+11	2.4606E+02	1.1634E+01	1.8238E-09
	avg	1.0261E+01	1.6708E+01	1.8472E+01	8.4672E+27	4.7148E+01	5.6671E+09	2.0603E+02	3.8326E+00	6.1960E-11
	std	3.2575E+00	1.6276E+00	5.8549E+00	4.6377E++28	6.6816E+00	2.0563E+10	2.3275E+01	1.9173E++00	3.3281E-10
F3	min	1.5460E+04	2.3725E++04	8.1910E+03	5.4104E+04	1.5188E+04	1.1725E+05	1.2915E+04	1.1516E+04	5.4708E-50
	max	4.5354E+04	6.6340E+04	2.5060E+04	7.8728E+04	9.4962E+04	4.3629E+05	4.0775E+04	2.1990E+04	1.4417E-17
	avg	2.9170E+04	4.4276E+04	1.4853E+04	6.6850E+04	4.8369E+04	3.1000E+05	2.2100E+04	1.6642E+04	4.8057E-19
	std	7.8458E+03	1.1133E+04	3.9454E+03	7.4447E+03	2.0308E+04	7.9865E+04	7.0625E+03	2.6321E+03	2.6321E-18
F4	min	1.9029E+01	2.2930E+01	1.5925E+01	4.1073E+01	1.8872E+01	5.8822E+01	1.3870E+01	2.2190E+01	6.0397E-25
	max	3.3467E+01	4.4164E+01	2.2626E+01	7.0415E+01	3.4763E+01	7.2361E+01	2.2128E+01	3.5270E+01	3.9719E-08
	avg	2.3898E+01	3.1569E+01	1.8677E+01	6.0921E+01	2.7256E+01	6.6501E+01	1.7209E+01	2.7347E+01	1.3252E-09
	std	3.0609E+00	4.9567E+00	1.5567E+00	6.0115E+00	3.7300E+00	3.3843E+00	2.1228E+00	2.9998E+00	7.2514E-09
F5	min	4.2886E+03	2.4757E+04	3.4972E+04	2.9795E+03	5.7971E+04	2.1160E+07	7.7741E+05	4.3150E+02	9.8321E+01
	max	4.5207E+04	9.2137E+04	2.1749E+05	4.0191E+04	7.4674E+05	1.2226E+08	3.8182E+06	2.9802E+03	1.6813E+03
	avg	1.2503E+04	5.1870E+04	8.3437E+04	1.1509E+04	1.9647E+05	5.3902E+07	1.9727E+06	8.4938E+02	1.6269E++02
	std	1.0208E+04	1.6860E+04	3.9322E+04	9.4857E+03	1.4563E+05	2.4793E+07	7.4679E+05	4.5672E+02	2.9090E+02
F6	min	5.4796E+01	5.5967E+02	2.5107E+03	1.2456E+02	9.3516E+02	3.1718E+04	8.6381E+02	5.9980E-01	6.9538E+00
	max	6.7712E+02	1.0809E+03	5.9838E+03	2.1477E+02	2.4838E+03	6.6621E+04	4.2775E+03	1.1422E+01	1.0185E+02
	avg	2.0617E+02	7.8926E+02	4.1108E+03	1.5895E+02	1.4946E+03	4.7909E+04	2.7105E+03	3.1604E+00	1.6013E+01
	std	1.3756E+02	1.3942E+02	8.8509E+02	2.0218E+01	4.0356E+02	9.2522E+03	8.1223E+02	2.4939E+00	2.0123E+01
F7	min	3.1330E-01	1.0732E+00	1.5553E+00	3.9310E-01	1.2984E+00	3.8034E+01	1.9142E+01	2.5900E-01	4.4694E-04
	max	1.2000E+00	2.0674E+00	6.4619E+00	1.0035E+00	5.0470E+00	1.9305E+02	1.1556E+02	7.9920E-01	3.9800E-02
	avg	5.0590E-01	1.5439E+00	3.6457E+00	6.6220E-01	2.8223E+00	7.8950E+01	4.8133E+01	4.6310E-01	7.6000E-03
	std	1.9450E-01	1.5439E+00	1.3259E+00	1.5210E-01	8.5180E-01	3.4165E+01	2.3702E+01	1.1280E-01	7.4000E-03
F8	min	−2.2982E+04	−2.9413E+04	−5.9794E+03	−2.5986E+04	−2.6220E+04	−7.4285E+03	−7.4198E+03	−2.5338E+04	−2.5166E+04
	max	−1.5181E+04	−2.3490E+04	−3.2591E+03	−1.9777E+04	−1.8242E+04	−4.3012E+03	−3.2762E+03	−2.1154E+04	−1.4408E+04
	avg	−1.9454E+04	−2.6667E+04	−4.6545E+03	−2.3136E+04	−2.1859E+04	−5.5331E+03	−4.9507E+03	−2.3174E+04	−1.8551E+04
	std	1.9668E+03	1.4553E+03	6.8949E+02	1.5920E+03	1.7589E+03	7.5107E+02	1.1329E+03	1.2689E+03	3.1119E+03
F9	min	1.4270E+02	1.1965E+02	1.2272E+02	5.6876E+02	1.7487E+02	7.3325E+02	3.5786E+02	3.7147E+02	0.0000E+00
	max	2.7619E+02	2.0523E+02	2.2285E+02	8.5875E+02	3.6922E+02	9.4409E+02	5.4809E+02	5.8470E+02	1.0800E-11
	avg	1.8986E+02	1.6581E+02	1.7463E+02	7.2435E+02	2.4724E+02	8.5991E+02	4.5794E+02	4.7953E+02	7.6928E-13
	std	3.3360E+01	2.0264E+01	2.5843E+01	7.9788E+01	4.1850E+01	4.5033E+01	5.4736E+01	4.6983E+01	2.2776E-12
F10	min	4.1052E+00	4.3875E+00	3.6487E+00	4.3924E+00	7.7675E+00	1.4740E+01	7.0457E+00	2.3368E+00	8.8818E-16
	max	7.0695E+00	5.5574E+00	5.3455E+00	2.0271E+01	1.1901E+01	1.8344E+01	1.0238E+01	5.3882E+00	1.4984E-07
	avg	5.4362E+00	4.9694E+00	4.5101E+00	7.2739E+00	1.0173E+01	1.6212E+01	8.7752E+00	3.7818E+00	4.9956E-09
	std	7.6760E-01	3.0500E-01	5.1920E-01	5.1781E+00	9.9020E-01	7.3760E-01	7.4500E-01	7.5190E-01	2.7357E-08
F11	min	1.5479E+00	6.1492E+00	6.0568E+02	2.1535E+00	7.7537E+00	3.0337E+02	5.6742E+01	3.0350E-01	0.0000E+00
	max	9.9589E+00	1.0079E+01	7.6007E+02	2.9745E+00	2.2365E+01	5.8944E+02	1.0335E+02	1.0966E+00	6.2061E-14
	avg	2.9901E+00	8.3353E+00	6.8062E+02	2.5185E+00	1.4746E+01	4.4997E+02	8.4289E+01	7.7190E-01	2.1427E-15
	std	1.5860E+00	9.5600E-01	4.5807E+01	2.3850E-01	4.1558E+00	6.8384E+01	1.2865E+01	2.2090E-01	1.1320E-14
F12	min	3.9423E+00	3.0565E+00	5.7002E+00	1.0048E+01	1.6992E+01	8.8284E+06	2.3268E+01	8.4128E+00	7.9800E-02
	max	2.2947E+01	8.6318E+00	2.6535E+01	4.0049E+01	5.3981E+01	7.7700E+07	1.6884E+04	2.1658E+01	8.1560E-01
	avg	8.7237E+00	5.4697E+00	1.0926E+01	2.0978E+01	3.7215E+01	3.3948E+07	1.4319E+03	1.3624E+01	2.1890E-01
	std	3.4332E+00	1.4374E+00	4.2637E+00	7.3682E+00	1.0021E+01	1.4375E+07	3.1733E+03	3.2159E+00	1.3430E-01
F13	min	7.0222E+01	7.7039E+01	1.6243E+02	1.1854E+02	2.0083E+02	5.2761E+07	1.9174E+04	5.1764E+01	8.8830E+00
	max	2.0390E+03	2.1652E+03	5.3155E+04	2.1097E+02	1.7924E+04	2.2474E+08	1.5009E+06	1.5118E+02	3.2611E+01
	avg	1.9408E+02	2.3802E+02	2.9855E+03	1.6390E+02	4.0649E+03	1.2553E+08	3.4665E+05	1.0330E+02	1.3338E+01
	std	3.5310E+02	4.2503E+02	9.6396E+03	2.3688E+01	5.4699E+03	4.8783E+07	3.1617E+05	3.1436E+01	6.2886E+00

algorithm has different movements such as chain, random, and positioning with leaders. Leaders in the multi-leader pso have no movement and are considered to be several global best, whereas in the proposed algorithm, leaders update their position based on a search mechanism.

5. The function of the COOT algorithm on engineering problems

There are eight limited problems in engineering design used by many researchers: Tension/compression spring, welded beam, pressure vessel designs, Multi-plate disc clutch brake, Step-cone pulley problem, Cantilever beam design, reducer design problem, Rolling element bearing problem. These problems have several constraints of equality and inequality, so the COOT algorithm must be equipped with a constraint control method so that it can also optimize constraint problems. The results of COOT is compared to various conventional and modified optimizers proposed in previous studies. The swarm size and maximum iterations of all problems are set to 30 and 500, respectively. Details of these problems given in Table 10.

Table 6
Results for the Unimodal and Multimodal benchmark functions with 500 Dimention and 15,000 NFE.

Function		PSO	GA	GSA	MVO	SSA	FDO	AEFA	SHADE	COOT
F1	min	3.6808E+04	6.5848E+04	4.6211E+04	1.0512E+05	7.9364E+04	3.0852E+05	3.9413E+04	4.9384E+04	2.6511E-48
	max	6.9384E+04	1.0037E+05	6.0399E+04	1.3781E+05	1.0684E+05	4.7681E+05	5.4746E+04	8.2090E+04	6.6401E-14
	avg	5.0027E+04	8.1438E+04	6.0399E+04	1.1933E+05	9.3964E+04	3.8279E+05	4.5439E+04	6.5632E+04	2.2134E-15
	std	6.7210E+03	7.5700E+03	3.5116E+03	7.1706E+03	6.4288E+03	4.1505E+04	3.5913E+03	9.3896E+03	1.2123E-14
F2	min	3.2376E+02	4.3959E+02	2.6372E+02	3.4268E+163	4.8017E+02	1.4127E+43	8.0209E+02	4.0100E+02	3.7287E-24
	max	9.8879E+02	5.2330E+02	1.6959E+259	1.4004E+212	5.7101E+02	1.7450E+82	9.2151E+02	5.1187E+02	1.6401E-08
	avg	3.9069E+02	4.7627E+02	1.0599E+258	8.7528E+210	5.3654E+02	1.0932E+81	8.6421E+02	4.4930E+02	5.5242E-10
	std	1.6151E+02	2.5733E+01	Inf	Inf	2.1185E+01	4.3619E+81	3.8702E+01	3.3219E+01	2.9934E-09
F3	min	5.5263E+05	7.6767E+05	2.7650E+05	1.6668E+06	4.9857E+05	3.8826E+06	2.4146E+05	6.1753E+05	8.3862E-47
	max	1.0601E+06	1.9840E+06	1.0094E+07	2.3972E+06	3.4396E+06	1.1265E+07	8.7262E+05	1.0379E+06	4.5877E-10
	avg	8.3123E+05	1.1108E+06	1.5087E+06	2.0102E+06	1.2558E+06	6.9854E+06	4.9216E+05	8.2917E+05	1.5292E-11
	std	1.3207E+05	2.6948E+05	1.8017E+06	1.8025E+05	7.0119E+05	1.9268E+06	1.9127E+05	1.0465E+05	8.3760E-11
F4	min	4.1343E+01	4.3802E+01	2.4621E+01	9.1515E+01	3.4224E+01	8.4876E+01	2.3103E+01	4.5755E+01	2.0232E-20
	max	5.8294E+01	6.2875E+01	3.1326E+01	9.6844E+01	4.9069E+01	9.2259E+01	2.9201E+01	5.5919E+01	5.1787E-06
	avg	4.7236E+01	5.2207E+01	2.7836E+01	9.4505E+01	4.9069E+01	8.9575E+01	2.5918E+01	5.1845E+01	1.8379E-07
	std	3.6291E+00	4.7943E+00	1.4761E+00	1.3172E+00	3.3986E+00	2.0731E+00	1.5457E+00	2.6468E+00	9.4531E-07
F5	min	1.2760E+07	2.2638E+07	4.1780E+06	1.2258E+08	2.8498E+07	4.2613E+08	1.4721E+07	1.9523E+07	4.9828E+02
	max	3.3610E+07	4.4365E+07	1.0161E+07	2.3352E+08	4.6954E+07	1.2913E+09	2.6641E+07	4.3955E+07	1.9703E+03
	avg	1.8984E+07	3.3082E+07	7.9065E+06	1.7020E+08	3.7868E+07	7.9560E+08	2.0463E+07	3.2895E+07	5.6329E+02
	std	5.4629E+06	6.4320E+06	1.3849E+06	3.1479E+07	4.7897E+06	2.2299E+08	3.3034E+06	5.3305E+06	2.7088E+02
F6	min	3.8423E+04	6.8924E+04	4.9078E+04	1.0480E+05	8.2207E+04	2.9911E+05	3.9328E+04	5.2145E+04	9.2880E+01
	max	7.7666E+04	9.4504E+04	5.9150E+04	1.3683E+05	1.0853E+05	4.7573E+05	5.3357E+04	7.7902E+04	8.4136E+02
	avg	5.1448E+04	7.9750E+04	5.3027E+04	1.2070E+05	9.4904E+04	3.7512E+05	4.4402E+04	6.5248E+04	1.3305E+02
	std	8.6422E+03	7.2140E+03	2.6357E+03	9.1593E+03	5.9897E+03	4.5469E+04	2.9581E+03	6.7303E+03	1.3415E+02
F7	min	9.2101E+01	1.5171E+02	5.6037E+02	9.1386E+02	2.0484E+02	2.5102E+03	6.2370E+03	1.4543E+02	1.3000E-03
	max	1.9293E+02	3.8234E+02	1.1141E+03	1.3747E+03	3.5613E+02	1.0356E+04	1.1507E+04	4.0811E+02	3.2200E-02
	avg	1.3554E+02	2.4666E+02	8.2546E+02	1.1319E+03	2.7803E+02	5.8119E+03	8.2084E+03	2.4212E+02	6.5000E-03
	std	2.4390E+01	5.6000E+01	1.2642E+02	1.2296E+02	3.5019E+01	2.1375E+03	1.1084E+03	5.9078E+01	6.7000E-03
F8	min	−8.8583E+04	−8.5571E+04	−1.4681E+04	−8.3260E+04	−7.0119E+04	−1.5196E+04	−1.5666E+04	−1.4771E+03	−6.2898E+04
	max	−5.5065E+04	−6.8678E+04	−7.4462E+03	−6.9011E+04	−4.5715E+04	−8.7892E+03	−7.8757E+03	−2.8987E+04	−3.6691E+04
	avg	−7.1488E+04	−6.8678E+04	−1.0578E+04	−7.4162E+04	−6.0211E+04	−1.1925E+04	−1.0485E+04	−3.6666E+04	−5.1558E+04
	std	7.0120E+03	5.0063E+03	1.8797E+03	3.5437E+03	5.1730E+03	1.3665E+03	1.7106E+03	1.1601E+04	8.1843E+03
F9	min	2.3523E+03	2.4012E+03	2.4620E+03	6.0437E+03	2.9262E+03	4.9024E+03	4.1803E+03	3.6510E+03	0.0000E+00
	max	3.1291E+03	2.8384E+03	3.0258E+03	6.7914E+03	3.3706E+03	5.3181E+03	4.8004E+03	5.0782E+03	1.1823E-11
	avg	2.6147E+03	2.6084E+03	2.7332E+03	6.3620E+03	3.1562E+03	5.1090E+03	4.4925E+03	4.7186E+03	6.6696E-13
	std	1.7167E+02	1.0663E+02	1.3266E+02	1.5323E+02	1.1179E+02	1.1195E+02	1.7302E+02	2.9646E+02	2.4929E-12
F10	min	1.1586E+01	1.2011E+01	9.8522E+00	2.0750E+01	1.3772E+01	1.7884E+01	1.0765E+01	1.3252E+01	8.8818E-16
	max	1.4579E+01	1.3542E+01	1.0599E+01	2.0925E+01	1.4701E+01	1.8851E+01	1.1964E+01	1.5380E+01	2.9115E-10
	avg	1.2611E+01	1.2804E+01	1.0309E+01	2.0848E+01	1.4233E+01	1.8448E+01	1.1246E+01	1.4174E+01	9.8985E-12
	std	7.1990E-01	3.8490E-01	2.3190E-01	4.1800E-02	2.2560E-01	2.2720E-01	3.0710E-01	4.6880E-01	5.3122E-11
F11	min	3.4119E+02	6.0434E+02	8.3054E+03	9.2894E+02	7.1616E+02	2.9611E+03	1.4612E+03	4.5174E+02	0.0000E+00
	max	4.9681E+02	8.8882E+02	8.9899E+03	1.2469E+03	1.0289E+03	4.0857E+03	1.6730E+03	8.2975E+02	5.5511E-16
	avg	4.3280E+02	7.2506E+02	8.5724E+03	1.0621E+03	8.5258E+02	3.4910E+03	1.5562E+03	5.9419E+02	3.3307E-17
	std	4.5487E+01	7.1661E+01	1.6570E+02	8.9525E+01	6.4001E+01	2.9346E+02	4.9317E+01	8.8492E+01	1.0967E-16
F12	min	2.6379E+05	6.0886E+05	3.4457E+02	1.1626E+08	4.3341E+05	1.8557E+08	8.3501E+03	1.7592E+06	4.7600E-01
	max	5.9586E+06	5.6498E+06	8.3174E+04	3.6330E+08	3.2591E+06	2.3435E+09	8.1940E+05	9.6591E+06	2.2340E+00
	avg	2.0894E+06	2.2370E+06	1.1235E+04	1.7237E+08	1.3326E+06	1.3394E+09	1.4178E+05	4.7423E+06	7.3480E-01
	std	1.4684E+06	1.5781E+06	1.6197E+04	5.8659E+07	6.2813E+05	6.2396E+08	1.4986E+05	2.0955E+06	3.7320E-01
F13	min	1.0736E+07	1.6556E+07	1.3007E+06	3.8990E+08	2.1665E+07	1.2736E+09	3.4324E+06	3.1543E+07	4.9964E+01
	max	5.4981E+07	6.3362E+07	4.7603E+06	7.1762E+08	5.0646E+07	4.9895E+09	1.4697E+07	7.5620E+07	1.4627E+02
	avg	2.7536E+07	3.4847E+07	2.7957E+06	5.2545E+08	3.3483E+07	2.5448E+09	7.7103E+06	4.6371E+07	6.6950E+01
	std	9.9799E+06	1.1122E+07	9.3455E+05	8.8629E+07	6.8832E+06	9.6179E+08	2.6282E+06	1.0160E+07	1.9292E+01

Table 7
Results for the CEC-2017 (unimodal-multimodal) test functions with 10 Dimention and 60,000 NFE.

Function		PSO	GA	GSA	WOA	SSA	GWO	HHO	AEFA	SHADE	COOT
F1	min	1.0101E+02	3.2788E+03	1.0091E+02	4.4479E+04	1.6052E+02	6.1039E+03	1.4732E+05	1.0095E+02	1.0000E+02	1.0021E+02
	max	7.6872E+03	6.8707E+04	1.5331E+03	2.8587E+07	1.0526E+04	3.8108E+08	1.0891E+06	1.1758E+03	1.0000E+02	5.2961E+03
	avg	1.3090E+03	1.5662E+04	2.8798E+02	2.4365E+06	3.9400E+03	2.7370E+07	3.5793E+05	3.3198E+02	1.0000E+02	1.0417E+03
	std	1.9790E+03	1.5527E+04	2.9812E+02	5.9762E+06	2.8021E+03	9.2021E+07	1.9299E+05	2.7238E+02	0.0000E+00	1.2215E+03
	rank	5	7	2	9	6	10	8	3	1	4
F2	min	2.0000E+02	2.0000E+02	5.5800E+03	4.4100E+02	2.0000E+02	4.2740E+03	2.0000E+02	1.9749E+04	2.0000E+02	2.0000E+02
	max	2.0000E+02	1.9660E+03	1.4039E+05	5.0468E+05	4.8540E+03	1.6530E+09	2.9486E+04	3.7782E+07	2.0000E+02	2.0000E+02
	avg	2.0000E+02	3.8743E+02	3.7135E+04	8.0734E+04	6.7360E+02	6.2489E+07	1.5394E+03	3.6525E+06	2.0000E+02	2.0000E+02
	std	0.0000E+00	4.0091E+02	3.4949E+04	1.2337E+05	1.1175E+03	3.0075E+08	5.3107E+03	7.5837E+06	0.0000E+00	0.0000E+00
	rank	1	2	5	6	3	8	4	7	1	1
F3	min	3.0000E+02	4.7019E+02	7.6273E+03	3.7519E+02	3.0000E+02	3.1074E+02	3.0032E+02	5.0277E+03	3.0000E+02	3.0000E+02
	max	3.0000E+02	5.7982E+03	1.2572E+04	7.2820E+03	3.0000E+02	4.7251E+03	3.0245E+02	1.6631E+04	3.0000E+02	3.0000E+02
	avg	3.0000E+02	2.2333E+03	1.0098E+04	1.3320E+03	3.0000E+02	1.2847E+03	3.0121E+02	1.1786E+04	3.0000E+02	3.0000E+02
	std	4.4783E-14	1.2425E+03	1.2314E+03	1.4511E+03	6.5485E-10	1.3874E+03	6.3660E-01	2.6733E+03	0.0000E+00	6.2685E-04
	rank	2	8	9	7	3	6	5	10	1	4
F4	min	4.0182E+02	4.0002E+02	4.0409E+02	4.0083E+02	4.0060E+02	4.0281E+02	4.0033E+02	4.0642E+02	4.0000E+02	4.0000E+02
	max	4.0555E+02	4.0773E+02	4.0782E+02	5.3715E+02	4.7195E+02	4.6335E+02	4.9599E+02	4.0739E+02	4.0000E+02	4.0600E+02
	avg	4.0366E+02	4.0451E+02	4.0662E+02	4.2774E+02	4.0688E+02	4.0980E+02	4.1186E+02	4.0701E+02	4.0000E+02	4.0363E+02
	std	8.2670E-01	2.6136E+00	8.3070E-01	3.5478E+01	1.2364E+01	1.0667E+01	2.1624E+01	2.1600E-01	0.0000E+00	1.5650E+00
	rank	3	4	5	10	6	8	9	7	1	2
F5	min	5.1592E+02	5.0498E+02	5.3383E+02	5.1102E+02	5.0597E+02	5.0413E+02	5.2020E+02	5.0100E+02	5.0201E+02	5.0712E+02
	max	5.6467E+02	5.3284E+02	5.7762E+02	5.9735E+02	5.4079E+02	5.3456E+02	5.8564E+02	5.0895E+02	5.0510E+02	5.3483E+02
	avg	5.3837E+02	5.1702E+02	5.5852E+02	5.4938E+02	5.2047E+02	5.1355E+02	5.4931E+02	5.0434E+02	5.0318E+02	5.1756E+02
	std	1.1543E+01	5.8266E+00	9.3088E+00	2.0066E+01	1.0058E+01	7.4670E+00	1.6039E+01	2.1464E+00	7.3300E-01	6.0319E+00
	rank	7	4	10	9	6	3	8	2	1	5
F6	min	6.0000E+02	6.0000E+02	6.0634E+02	6.1207E+02	6.0021E+02	6.0003E+02	6.1252E+02	6.0000E+02	6.0000E+02	6.0004E+02
	max	6.2525E+02	6.0019E+02	6.5065E+02	6.5205E+02	6.3713E+02	6.0270E+02	6.6538E+02	6.0010E+02	6.0000E+02	6.0367E+02
	avg	6.0598E+02	6.0007E+02	6.2539E+02	6.2966E+02	6.1064E+02	6.0044E+02	6.3058E+02	6.0001E+02	8.8557E-06	6.0103E+02
	std	5.5578E+00	5.0500E-02	9.2849E+00	1.0904E+01	8.2406E+00	6.5170E-01	1.2740E+01	2.0700E-02	8.8557E-06	1.1766E+00
	rank	6	3	8	9	7	4	10	2	1	5
F7	min	7.1567E+02	7.1187E+02	7.1093E+02	7.3385E+02	7.1355E+02	7.1448E+02	7.4159E+02	7.1060E+02	7.1127E+02	7.1463E+02
	max	7.3648E+02	7.3548E+02	7.1967E+02	8.6444E+02	7.5709E+02	7.5592E+02	8.1903E+02	7.1588E+02	7.1579E+02	7.4256E+02
	avg	7.2265E+02	7.2011E+02	7.1364E+02	7.7900E+02	7.3368E+02	7.2925E+02	7.8134E+02	7.1221E+02	7.1347E+02	7.2849E+02
	std	4.7989E+00	5.0502E+00	1.7193E+00	2.8339E+01	1.1772E+01	1.0742E+01	2.0192E+01	1.2611E+00	9.4370E-01	7.1546E+00
	rank	5	4	3	9	8	7	10	1	2	6
F8	min	8.0497E+02	8.0398E+02	8.1293E+02	8.1300E+02	8.0497E+02	8.0398E+02	8.0919E+02	8.0100E+02	8.0125E+02	8.0597E+02
	max	8.2786E+02	8.2090E+02	8.2985E+02	8.6570E+02	8.5373E+02	8.2189E+02	8.5189E+02	8.0597E+02	8.0703E+02	8.2487E+02
	avg	8.1625E+02	8.1052E+02	8.2046E+02	8.3792E+02	8.2205E+02	8.1083E+02	8.3031E+02	8.0342E+02	8.0360E+02	8.1328E+02
	std	5.9545E+00	4.2911E+00	4.4701E+00	1.2600E+01	1.2005E+01	4.1088E+00	9.3409E+00	1.3748E+00	1.2856E+00	5.0667E+00
	rank	6	3	7	10	8	4	9	1	2	5
F9	min	9.0000E+02	9.0005E+02	9.0000E+02	9.1751E+02	9.0000E+02	9.0005E+02	9.9346E+02	9.0000E+02	9.0000E+02	9.0000E+02
	max	9.0000E+02	9.1805E+02	9.0000E+02	2.2762E+03	1.1725E+03	9.2576E+02	1.7072E+03	9.0000E+02	9.0000E+02	9.0254E+02
	avg	9.0000E+02	9.0347E+02	9.0000E+02	1.5068E+03	9.2310E+02	9.0390E+02	1.3350E+03	9.0000E+02	9.0000E+02	9.0037E+02
	std	2.9856E-14	4.7377E+00	0.0000E+00	4.2929E+02	6.0725E+01	7.1371E+00	1.9276E+02	0.0000E+00	0.0000E+00	6.6450E-01
	rank	2	4	1	8	6	5	7	1	1	3
F10	min	1.1301E+03	1.1891E+03	2.0301E+03	1.3929E+03	1.2486E+03	1.0339E+03	1.4329E+03	1.5644E+03	1.0074E+03	1.2699E+03
	max	2.1534E+03	2.1369E+03	3.2416E+03	2.7076E+03	2.3394E+03	1.8613E+03	2.4216E+03	2.8143E+03	1.2704E+03	2.1845E+03
	avg	1.8274E+03	1.6484E+03	2.8415E+03	2.0599E+03	1.7744E+03	1.5255E+03	2.0282E+03	2.1872E+03	1.0953E+03	1.7231E+03
	std	2.3720E+02	2.0451E+02	2.9475E+02	3.5685E+02	2.5290E+02	2.0879E+02	2.2503E+02	3.5907E+02	6.4229E+01	2.2356E+02
	rank	6	3	10	8	5	2	7	9	1	4
Friedman mean rank		4.3	4.2	6.0	8.5	5.8	5.7	7.6	4.3	1.2	3.9
RANK		5	3	8	10	7	6	9	4	1	2

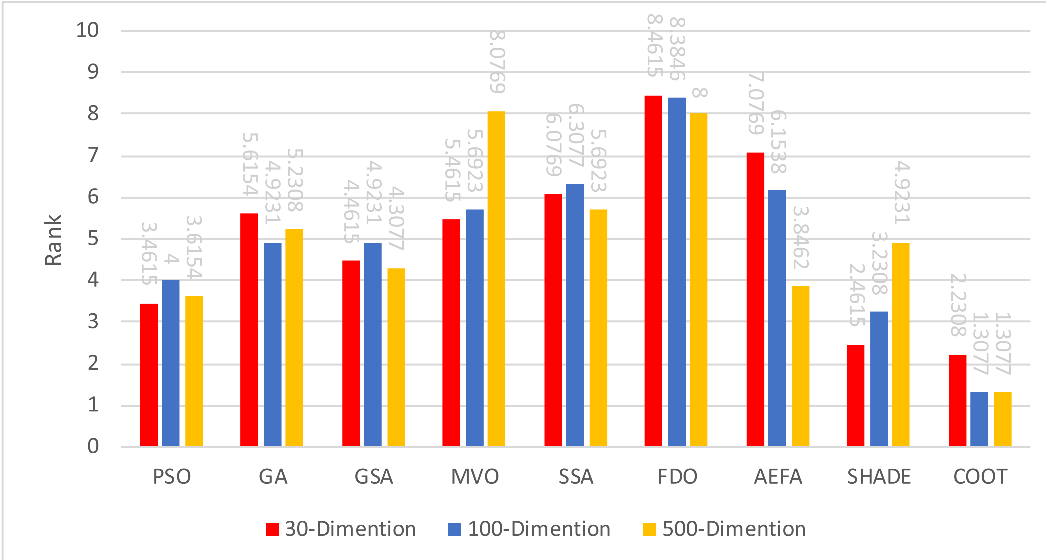

Fig. 8. Friedman mean rank (Classical functions).

Remarkably, since many algorithms have implemented these problems with different conditions, we have mentioned only the algorithms' best solution.

5.1. Tension/Compression spring design problem

As shown in Fig. 11. The main goal of this engineering design problem is to minimize the weight of the spring involving three decision variables which are wire diameter (d), mean coil diameter (D) and a number of active coils (N) (Khalilpourazari & Khalilpourazary, 2019). This problem is subjected to three inequality constraints and an objective function given in Eq. (11).

$$\vec{x} = [x_1\, x_2\, x_3] = [dDN],$$

$$f(\vec{x}) = (x_3 + 2)x_2x_1^2,$$

$$g_1(\vec{x}) = 1 - \frac{x_2^3 x_3}{71785x_1^4} \leqslant 0,$$

$$g_2(\vec{x}) = \frac{4x_2^2 - x_1x_2}{12566(x_2x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} \leqslant 0,$$

$$g_3(\vec{x}) = 1 - \frac{140.45x_1}{x_2^2 x_3} \leqslant 0,$$

$$g_4(\vec{x}) = \frac{x_1 + x_2}{1.5} - 1 \leqslant 0,$$

$$0.05 \leqslant x_1 \leqslant 2.00,$$

$$0.25 \leqslant x_2 \leqslant 1.30,$$

$$2.00 \leqslant x_3 \leqslant 15.0$$

This problem has been solved with many mathematical and exploratory approaches (Han et al., 2018; Xia et al., 2018). The results of comparing of the COOT algorithm with well - known optimization algorithms such as the Opposition-Based Sine Cosine Algorithm (Abd Elaziz et al., 2017), Grasshopper Optimization Algorithm using Opposition-based Learning (Ewees et al., 2018), Modified Spider Monkey Optimization Based on Nelder-Mead (Singh et al., 2018), Harris hawks optimizer (Heidari et al., 2019), salp swarm algorithm (Mirjalili et al., 2017), thermal exchange optimization (Kaveh & Dadras, 2017), Moth-flame optimization (Mirjalili, 2015b), Stochastic Fractal Search (Salimi, 2015), Grey Wolf Optimizer (Mirjalili et al., 2014), Whale Optimization Algorithm (Mirjalili & Lewis, 2016), Water Evaporation Optimization (Kaveh & Bakhshpoori, 2016), Bat algorithm (X. Yang & Hossein Gandomi, 2012), chaotic water cycle algorithm (Heidari et al., 2017), Sine cosine grey wolf optimizer (Gupta et al., 2020), Spotted hyena optimizer (Dhiman & Kumar, 2017), Grey Prediction Evolution Algorithm Based on Accelerated Even (Gao et al., 2020), Artificial ecosystem-based optimization (Zhao et al., 2020), chaotic multi-verse optimization (Sayed et al., 2018), I-ABC greedy (Sharma & Abraham, 2020), and Arithmetic Optimization Algorithm (AOA) (Abualigah et al., 2021), are presented in Table 11. The results showed that the AOA algorithm performs better than all of the compared optimization methods. However, the COOT algorithm was able to take third rank.

5.2. Pressure vessel design

The objective of this problem is to minimize the total cost consisting of material, forming, and welding of a cylindrical vessel as in Fig. 12. Both ends of the vessel are capped, and the head has a hemispherical shape. There are four variables in this problem:

The thickness of the shell (Ts).

The thickness of the head (Th).

Inner radius (R).

Length of the cylindrical section without considering the head (L).

This problem is subject to four constraints. These constraints and the problem are formulated as follows:

$$\vec{x} = [x_1\, x_2\, x_3 x_4] = [T_s T_h R L],$$

$$f(\vec{x}) = 0.6224x_1x_3x_4 + 1.7781x_2x_3^2 + 3.1661x_1^2x_4 + 19.84x_1^2x_3,$$

$$g_1(\vec{x}) = -x_1 + 0.0193x_3 \leqslant 0,$$

$$g_2(\vec{x}) = -x_3 + 0.00954x_3 \leqslant 0,$$

$$g_3(\vec{x}) = -\pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 + 1296000 \leqslant 0,$$

$$g_4(\vec{x}) = x_4 - 240 \leqslant 0,$$

$$0 \leqslant x_1 \leqslant 99,$$

$$0 \leqslant x_2 \leqslant 99,$$

$$10 \leqslant x_3 \leqslant 200,$$

$$10 \leqslant x_4 \leqslant 200$$

The results of comparing of the COOT algorithm with well - known

Table 8
Results for the CEC-2017 (Hybrid) test functions with 10 Dimention and 60,000 NFE.

Function		PSO	GA	GSA	WOA	SSA	GWO	HHO	AEFA	SHADE	COOT
F11	min	1.1050E+03	1.1072E+03	1.1157E+03	1.1244E+03	1.1035E+03	1.1097E+03	1.1164E+03	1.1524E+03	1.1000E+03	1.1031E+03
	max	1.1777E+03	1.1582E+03	1.2017E+03	1.4367E+03	1.3270E+03	1.1560E+03	1.3936E+03	2.2352E+03	1.1037E+03	1.1623E+03
	avg	1.1334E+03	1.1283E+03	1.1411E+03	1.1904E+03	1.1561E+03	1.1280E+03	1.1810E+03	1.3937E+03	1.1009E+03	1.1187E+03
	std	1.9642E+01	1.3444E+01	2.2300E+01	6.7302E+01	4.7268E+01	1.3313E+01	6.4190E+01	2.1862E+02	1.2291E+00	1.3122E+01
	rank	5	4	6	9	7	3	8	10	1	2
F12	min	2.0205E+03	8.8482E+03	3.3082E+04	8.3933E+03	6.0567E+03	2.4931E+04	1.1031E+04	1.6169E+04	1.2002E+03	3.0122E+03
	max	5.2710E+04	5.5615E+06	2.2181E+06	1.6536E+07	6.3502E+06	3.1953E+06	1.0386E+07	3.1185E+06	1.5567E+03	5.6675E+04
	avg	1.3683E+04	1.9830E+06	7.9328E+05	5.0955E+06	1.6929E+06	9.2055E+05	2.2004E+06	1.3799E+06	1.3311E+03	1.2721E+04
	std	1.0944E+04	1.6473E+06	5.9468E+05	5.9709E+06	1.8402E+06	8.8092E+05	2.4818E+06	9.7197E+05	1.0528E+02	1.1177E+04
	rank	3	8	4	10	7	5	9	6	1	2
F13	min	1.6661E+03	1.3227E+03	6.5051E+03	1.8785E+03	1.4797E+03	1.9781E+03	1.9571E+03	4.5702E+03	1.3000E+03	1.4203E+03
	max	1.6708E+04	2.2735E+04	1.4100E+04	4.4093E+04	3.2077E+04	2.9204E+04	4.8159E+04	1.5647E+04	1.3085E+03	1.7229E+04
	avg	8.9503E+03	8.5837E+03	1.0880E+04	1.5420E+04	1.0597E+04	1.2243E+04	1.5712E+04	1.0557E+04	1.3040E+03	2.6986E+03
	std	4.3964E+03	5.5962E+03	2.1540E+03	1.1291E+04	8.1626E+03	7.4492E+03	1.2789E+04	3.3467E+03	2.6210E+00	2.9599E+03
		4	3	7	9	6	8	10	5	1	2
F14	min	1.4398E+03	1.4265E+03	4.1537E+03	1.4793E+03	1.4404E+03	1.4561E+03	1.4661E+03	1.5660E+03	1.4000E+03	1.4270E+03
	max	3.3246E+03	1.2481E+04	9.7948E+03	5.1903E+03	1.9378E+03	5.4546E+03	1.7986E+03	1.2958E+04	1.4050E+03	1.5829E+03
	avg	1.8370E+03	2.9988E+03	6.8379E+03	1.8633E+03	1.5168E+03	2.6755E+03	1.5452E+03	5.2641E+03	1.4008E+03	1.4711E+03
	std	5.5978E+02	2.3360E+03	1.4141E+03	9.2987E+02	8.7588E+01	1.7167E+03	6.6511E+01	2.6841E+03	1.0223E+00	3.6752E+01
	rank	5	8	10	6	3	7	4	9	1	2
F15	min	1.5055E+03	1.5158E+03	9.9033E+03	1.6626E+03	1.6812E+03	1.5267E+03	1.6138E+03	3.5834E+03	1.5000E+03	1.5046E+03
	max	4.0262E+03	1.4147E+04	3.2097E+04	2.3249E+04	6.6676E+03	8.2476E+03	5.9188E+03	2.4865E+04	1.5005E+03	1.7616E+03
	avg	1.8305E+03	4.3278E+03	1.8085E+04	6.0438E+03	2.4824E+03	3.1302E+03	3.1737E+03	1.2953E+04	1.5003E+03	1.5925E+03
	std	5.4564E+02	3.5510E+03	4.8772E+03	4.2689E+03	1.2865E+03	1.8963E+03	1.4318E+03	5.2319E+03	2.0020E-01	5.4971E+01
	rank	3	7	10	8	4	5	6	9	1	2
F16	min	1.6005E+03	1.6003E+03	2.0114E+03	1.6111E+03	1.6032E+03	1.6127E+03	1.7228E+03	1.8580E+03	1.6001E+03	1.6019E+03
	max	2.0558E+03	2.0573E+03	2.5156E+03	2.0990E+03	2.0725E+03	1.9909E+03	2.1215E+03	2.1939E+03	1.6018E+03	1.7346E+03
	avg	1.8521E+03	1.7627E+03	2.1481E+03	1.8582E+03	1.7109E+03	1.7012E+03	1.9254E+03	2.0278E+03	1.6007E+03	1.6441E+03
	std	1.3169E+02	1.3086E+02	1.0549E+02	1.5659E+02	1.1269E+02	9.6556E+01	1.1565E+02	8.8870E+01	3.8920E-01	5.2051E+01
	rank	6	5	10	7	4	3	8	9	1	2
F17	min	1.7236E+03	1.7027E+03	1.7480E+03	1.7287E+03	1.7272E+03	1.7258E+03	1.7271E+03	1.7461E+03	1.7001E+03	1.7125E+03
	max	1.8642E+03	1.7644E+03	2.1526E+03	1.9164E+03	1.9994E+03	1.8637E+03	1.9184E+03	1.9055E+03	1.7014E+03	1.7952E+03
	avg	1.7541E+03	1.7348E+03	1.8768E+03	1.8050E+03	1.7719E+03	1.7495E+03	1.7790E+03	1.7972E+03	1.7005E+03	1.7465E+03
	std	3.1020E+01	1.8185E+01	1.1287E+02	5.3134E+01	5.1275E+01	2.6872E+01	4.1064E+01	5.3334E+01	3.7910E-01	1.8145E+01
	rank	5	2	10	9	6	4	7	8	1	3
F18	min	1.9528E+03	2.2642E+03	4.2162E+03	3.2654E+03	2.1302E+03	2.4486E+03	2.5697E+03	2.0504E+03	1.8000E+03	1.8835E+03
	max	4.3593E+04	2.7081E+04	2.1639E+04	4.2602E+04	5.4077E+04	4.2668E+04	3.8970E+04	2.1200E+04	1.8210E+03	8.3704E+03
	avg	1.0734E+04	7.1771E+03	9.9634E+03	1.5614E+04	1.8821E+04	2.9075E+04	1.8639E+04	9.2131E+03	1.8044E+03	3.1618E+03
	std	1.1036E+04	6.4815E+03	3.6033E+03	1.0615E+04	1.1823E+04	1.2376E+04	1.2090E+04	4.4185E+03	8.0845E+00	1.3668E+03
	rank	6	3	5	7	9	10	8	4	1	2
F19	min	1.9241E+03	1.9035E+03	8.1905E+03	2.0864E+03	1.9175E+03	1.9113E+03	2.0512E+03	4.3743E+03	1.9000E+03	1.9064E+03
	max	8.5734E+03	1.0040E+04	9.2625E+04	4.1341E+05	6.6029E+03	1.4826E+04	3.0920E+04	1.1276E+05	1.9015E+03	2.0660E+03
	avg	4.0595E+03	4.4735E+03	5.3071E+04	5.3308E+04	2.5661E+03	4.9518E+03	8.8128E+03	2.7654E+04	1.9001E+03	1.9408E+03
	std	2.0294E+03	2.3365E+03	1.9849E+04	9.9186E+04	1.2089E+03	4.9356E+03	8.7504E+03	2.8409E+04	2.7170E-01	3.6707E+01
	rank	4	5	9	10	3	6	7	8	1	2
F20	min	2.0053E+03	2.0000E+03	2.1824E+03	2.0613E+03	2.0227E+03	2.0206E+03	2.0393E+03	2.0382E+03	2.0000E+03	2.0168E+03
	max	2.3442E+03	2.1516E+03	2.5374E+03	2.2652E+03	2.2206E+03	2.2261E+03	2.3086E+03	2.3974E+03	2.0003E+03	2.1458E+03
	avg	2.0838E+03	2.0282E+03	2.2601E+03	2.1602E+03	2.0828E+03	2.0561E+03	2.1298E+03	2.1723E+03	2.0000E+03	2.0458E+03
	std	7.9273E+01	3.2827E+01	8.5042E+01	5.8918E+01	4.8942E+01	4.5769E+01	6.9280E+01	7.0197E+01	7.9200E-02	2.4581E+01
	rank	6	2	10	8	5	4	7	9	1	3
Friedman mean rank		4.7	4.7	8.1	8.3	5.4	5.5	7.4	7.7	1.0	2.2
RANK		4	3	9	10	5	6	7	8	1	2

optimization algorithms such as the Opposition-Based Sine Cosine Algorithm (Abd Elaziz et al., 2017), Grasshopper Optimization Algorithm using Opposition-based Learning (Ewees et al., 2018), Modified Spider Monkey Optimization Based on Nelder-Mead (Singh et al., 2018), Harris hawks optimizer (Heidari et al., 2019), Moth-flame optimization (Mirjalili, 2015b), Grey Wolf Optimizer (Mirjalili et al., 2014), genetic algorithm (Deb, 1991), Whale Optimization Algorithm (Mirjalili & Lewis, 2016), Water Evaporation Optimization (Kaveh & Bakhshpoori, 2016), Bat algorithm (Yang & Hossein Gandomi, 2012), Sine cosine grey wolf optimizer (Gupta et al., 2020), Grey Prediction Evolution Algorithm Based on Accelerated Even (Gao et al., 2020), charged system search (Kaveh & Talatahari, 2010), Artificial ecosystem-based optimization (Zhao et al., 2020), chaotic multi-verse optimization (Sayed et al., 2018), I-ABC greedy (Sharma & Abraham, 2020), and Arithmetic Optimization Algorithm (Abualigah et al., 2021), are presented in Table 12. The results showed that the OBSCA algorithm performs better

Table 9
Results for the CEC-2017 (Composition) test functions with 10 Dimention and 60,000 NFE.

Function		PSO	GA	GSA	WOA	SSA	GWO	HHO	AEFA	SHADE	COOT
F21	min	2.2000E+03	2.2054E+03	2.3035E+03	2.2072E+03	2.2000E+03	2.2001E+03	2.2012E+03	2.2648E+03	2.2000E+03	2.1000E+03
	max	2.3587E+03	2.3301E+03	2.4127E+03	2.3992E+03	2.3380E+03	2.3330E+03	2.3813E+03	2.3159E+03	2.3067E+03	2.3146E+03
	avg	2.2991E+03	2.3021E+03	2.3625E+03	2.2960E+03	2.2567E+03	2.3077E+03	2.3064E+03	2.3045E+03	2.2778E+03	2.2015E+03
	std	6.1421E+01	3.5656E+01	1.9131E+01	7.0453E+01	5.9857E+01	2.7431E+01	6.3670E+01	8.9703E+00	4.5641E+01	2.8257E+01
	rank	5	6	10	4	2	9	8	7	3	1
F22	min	2.2000E+03	2.3016E+03	2.3000E+03	2.2613E+03	2.2221E+03	2.2121E+03	2.2569E+03	2.3000E+03	2.2318E+03	2.2305E+03
	max	3.1794E+03	2.3153E+03	2.3003E+03	2.3400E+03	2.3085E+03	2.5890E+03	2.3236E+03	2.3003E+03	2.3006E+03	2.3068E+03
	avg	2.3222E+03	2.3062E+03	2.3000E+03	2.3104E+03	2.2984E+03	2.3109E+03	2.3100E+03	2.3000E+03	2.2979E+03	2.2969E+03
	std	1.6429E+02	3.3745E+00	7.3000E-02	1.8348E+01	1.9979E+01	5.7727E+01	1.5190E+01	5.2800E-02	1.2499E+01	1.7386E+01
	rank	10	6	5	8	3	9	7	4	2	1
F23	min	2.6366E+03	2.6130E+03	2.6964E+03	2.6181E+03	2.6070E+03	2.6026E+03	2.6113E+03	2.6029E+03	2.6004E+03	2.6060E+03
	max	2.7525E+03	2.6492E+03	2.8845E+03	2.6879E+03	2.6408E+03	2.6357E+03	2.7238E+03	2.6197E+03	2.6083E+03	2.6308E+03
	avg	2.6724E+03	2.6291E+03	2.7572E+03	2.6477E+03	2.6219E+03	2.6178E+03	2.6663E+03	2.6123E+03	2.6055E+03	2.6166E+03
	std	2.8527E+01	8.9443E+00	5.4501E+01	1.7093E+01	9.7182E+00	8.5743E+00	3.0081E+01	4.3441E+00	1.4945E+00	7.5665E+00
	rank	9	6	10	7	5	4	8	2	1	3
F24	min	2.5000E+03	2.5001E+03	2.5000E+03	2.5233E+03	2.7363E+03	2.7001E+03	2.5007E+03	2.7279E+03	2.5000E+03	2.5000E+03
	max	2.8792E+03	2.7807E+03	2.8519E+03	2.8170E+03	2.7603E+03	2.7714E+03	2.9115E+03	2.7370E+03	2.7365E+03	2.7594E+03
	avg	2.7636E+03	2.7215E+03	2.5905E+03	2.7644E+03	2.7470E+03	2.7461E+03	2.7519E+03	2.7319E+03	2.7181E+03	2.6180E+03
	std	1.0998E+02	8.9360E+01	1.3976E+02	4.9661E+01	6.6626E+00	1.3317E+01	1.2159E+02	2.4709E+00	5.9313E+01	1.2082E+02
	rank	9	4	1	10	7	6	8	5	3	2
F25	min	2.8977E+03	2.8981E+03	2.8980E+03	2.8995E+03	2.8978E+03	2.8983E+03	2.8981E+03	2.9068E+03	2.8977E+03	2.8977E+03
	max	2.9459E+03	2.9533E+03	2.9438E+03	3.0255E+03	2.9523E+03	2.9512E+03	2.9713E+03	2.9520E+03	2.9458E+03	2.9500E+03
	avg	2.9180E+03	2.9385E+03	2.9405E+03	2.9536E+03	2.9252E+03	2.9330E+03	2.9323E+03	2.9476E+03	2.9270E+03	2.9193E+03
	std	2.3824E+01	1.9914E+01	1.1349E+01	2.2288E+01	2.3771E+01	2.0086E+01	2.3977E+01	7.9839E+00	2.2500E+01	2.3452E+01
	rank	1	7	8	10	3	6	5	9	4	2
F26	min	2.6000E+03	2.6009E+03	2.8000E+03	2.6075E+03	2.9000E+03	2.8055E+03	2.8103E+03	2.8000E+03	2.9000E+03	2.6000E+03
	max	3.9118E+03	3.3896E+03	4.4414E+03	4.4517E+03	3.9202E+03	3.9599E+03	4.5943E+03	3.3638E+03	2.9471E+03	3.0358E+03
	avg	3.1141E+03	3.0645E+03	3.6125E+03	3.3825E+03	2.9512E+03	3.0165E+03	3.2923E+03	2.8788E+03	2.9016E+03	2.8993E+03
	std	2.9980E+02	1.5631E+02	7.3867E+02	5.3713E+02	1.8745E+02	2.9726E+02	4.5733E+02	1.0366E+02	8.6083E+00	6.8279E+01
	rank	8	7	10	9	5	6	4	1	3	2
F27	min	3.1027E+03	3.0932E+03	3.1477E+03	3.0960E+03	3.0879E+03	3.0894E+03	3.0948E+03	3.0946E+03	3.0890E+03	3.0876E+03
	max	3.2958E+03	3.1393E+03	3.3352E+03	3.1936E+03	3.1565E+03	3.1033E+03	3.2411E+03	3.1656E+03	3.0900E+03	3.0997E+03
	avg	3.1615E+03	3.1114E+03	3.2598E+03	3.1237E+03	3.0942E+03	3.0937E+03	3.1393E+03	3.1133E+03	3.0895E+03	3.0926E+03
	std	4.7787E+01	1.2029E+01	4.4027E+01	2.8218E+01	1.2099E+01	3.2018E+00	4.2710E+01	1.7947E+01	1.8960E-01	3.0994E+00
	rank	9	5	10	7	4	3	8	6	1	2
F28	min	3.1000E+03	3.1002E+03	3.4208E+03	3.1192E+03	3.1000E+03	3.1678E+03	3.1012E+03	3.3837E+03	3.1000E+03	3.1000E+03
	max	3.4465E+03	3.4465E+03	3.4947E+03	3.7494E+03	3.7362E+03	3.4323E+03	3.7318E+03	3.4494E+03	3.4118E+03	3.4118E+03
	avg	3.3640E+03	3.2888E+03	3.4547E+03	3.3957E+03	3.2400E+03	3.3789E+03	3.4018E+03	3.3935E+03	3.2964E+03	3.1991E+03
	std	9.5907E+01	1.5356E+02	1.9988E+01	1.6582E+02	1.4245E+02	6.9775E+01	1.2939E+02	1.9166E+01	1.4077E+02	1.2217E+02
	rank	5	3	10	8	2	6	9	7	4	1
F29	min	3.1469E+03	3.1641E+03	3.2821E+03	3.2161E+03	3.1382E+03	3.1456E+03	3.1829E+03	3.1797E+03	3.1307E+03	3.1526E+03
	max	3.3397E+03	3.2811E+03	3.7052E+03	3.6245E+03	3.2800E+03	3.3432E+03	3.4817E+03	3.5410E+03	3.1572E+03	3.2597E+03
	avg	3.2250E+03	3.2208E+03	3.4357E+03	3.3384E+03	3.1994E+03	3.1775E+03	3.2925E+03	3.3097E+03	3.1391E+03	3.1925E+03
	std	4.8234E+01	3.4343E+01	1.3226E+02	9.2970E+01	3.7332E+01	3.5933E+01	7.5201E+01	1.0211E+02	5.3253E+00	2.6912E+01
	rank	6	5	10	9	4	2	7	8	1	3
F30	min	4.0214E+03	4.4796E+04	5.3047E+05	2.8159E+04	4.3280E+03	8.3716E+03	1.2165E+04	3.3338E+05	3.3945E+03	3.5493E+03
	max	1.2533E+06	2.7991E+06	1.5276E+06	8.0262E+06	1.7460E+06	3.0366E+06	5.4770E+06	8.0925E+06	8.2058E+05	8.2058E+05
	avg	2.7642E+05	3.0087E+05	1.0915E+06	7.2469E+05	2.3287E+05	5.9625E+05	9.6804E+05	1.4848E+06	1.9410E+05	1.0146E+05
	std	4.3354E+05	5.3384E+05	3.0205E+05	1.5155E+06	4.8909E+05	8.7162E+05	1.5536E+06	1.4083E+06	3.5152E+05	2.4928E+05
	rank	4	5	9	7	3	6	8	10	2	1
Friedman mean rank		6.6	5.4	8.3	7.9	3.8	5.7	7.2	5.9	2.4	1.8
RANK		7	4	10	9	3	5	8	6	2	1

than all of the compared optimization methods. However, the COOT algorithm was able to take second rank.

5.3. Welded beam design

The welded beam is shown in Fig. 13 is designed for minimum cost subject to constraints on shear stress in weld (τ), bending stress in the beam (σ), buckling load on the bar (Pc), end deflection of the beam (δ), and side constraints.

This problem has four variables such as the thickness of weld (h), length of the attached part of the bar (l), the height of the bar (t), and thickness of the bar (b). The mathematical formulation is as follows:

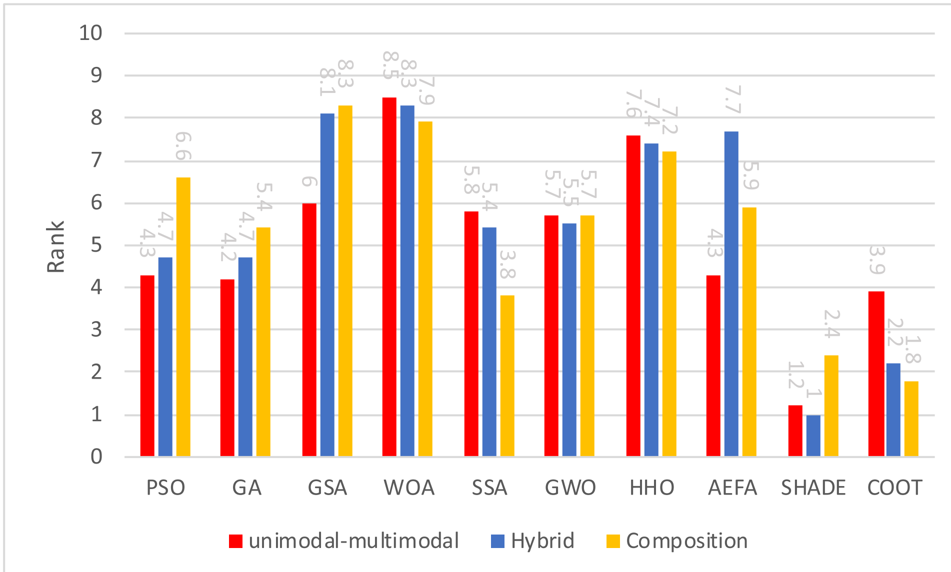

Fig. 9. Friedman mean rank (CEC2017).

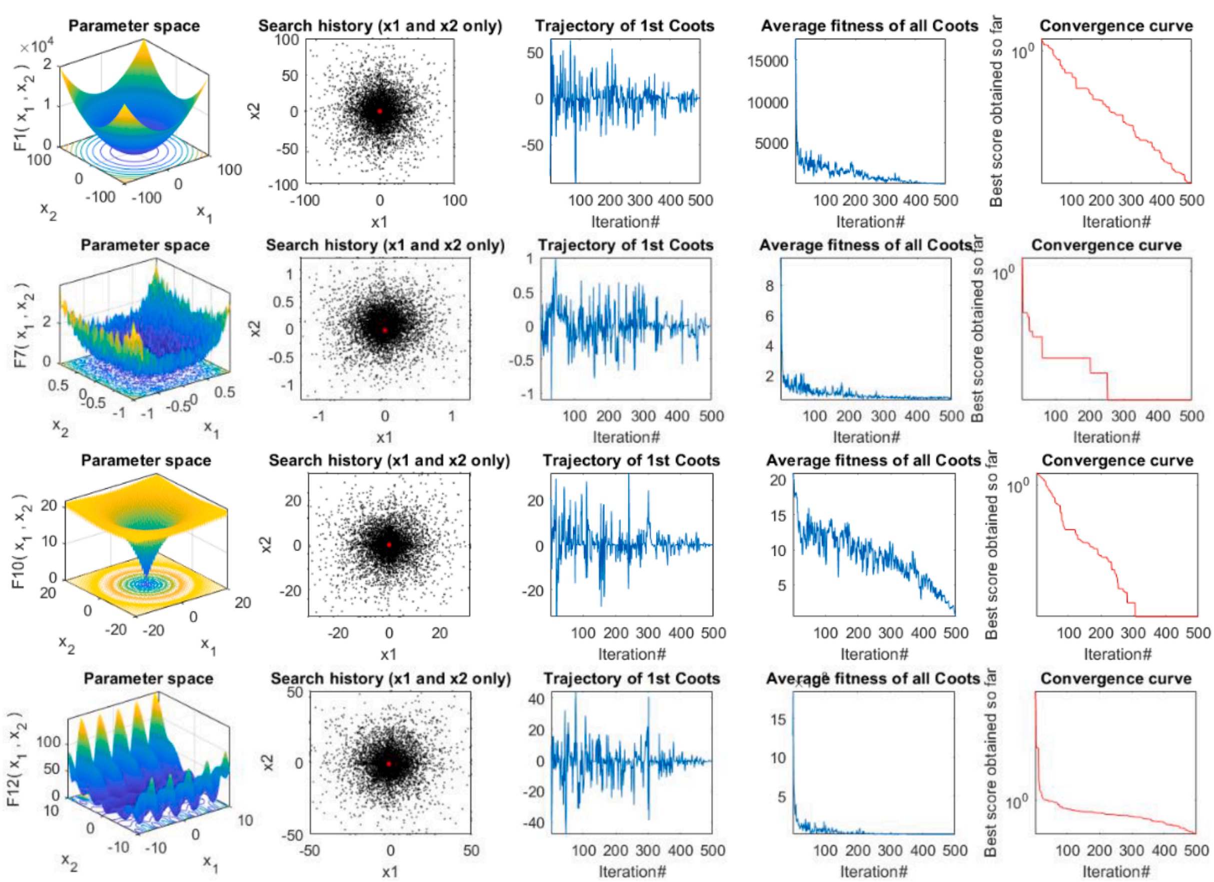

Fig. 10. Search history and trajectory of the first coot in the first dimension.

Table 10
Details of the eight real-world constrained optimization problem. D is the total number of decision variables of the problem, g is the number of inequality constraints, and h is the number of equality constraints.

No.	Name	D	g	h	Objective
1	Tension/Compression spring	3	4	0	Minimize
2	Pressure vessel design	4	4	0	Minimize
3	Welded Beam Design	4	7	0	Minimize
4	Multi-plate disc clutch brake	5	8	0	Minimize
5	Step-cone pulley problem	4	1	0	Minimize
6	Cantilever beam design	5	1	1	Minimize
7	reducer design problem	7	11	0	Minimize
8	Rolling element bearing problem	10	9	1	Maximize

$$
\begin{aligned}
&\vec{x} = [x_1\, x_2\, x_3 x_4] = [hltb],\\
&f(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2),\\
&g_1(\vec{x}) = \tau(\vec{x}) - \tau_{max} \leqslant 0,\\
&g_2(\vec{x}) = \sigma(\vec{x}) - \sigma_{max} \leqslant 0,\\
&g_3(\vec{x}) = \delta(\vec{x}) - \delta_{max} \leqslant 0,\\
&g_4(\vec{x}) = x_1 - x_4 \leqslant 0,\\
&g_5(\vec{x}) = P - P_c(\vec{x}) \leqslant 0,\\
&g_6(\vec{x}) = 0.125 - x_1 \leqslant 0,\\
&g_7(\vec{x}) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leqslant 0\\
&0.1 \leqslant x_1 \leqslant 2,\\
&0.1 \leqslant x_2 \leqslant 10,\\
&0.1 \leqslant x_3 \leqslant 10,\\
&0.1 \leqslant x_4 \leqslant 2\\
&\tau(\vec{x}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2},\\
&\tau' = \frac{p}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}, M = P(L + \frac{x_2}{2}),\\
&R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2},\\
&J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\},\\
&\sigma(\vec{x}) = \frac{6PL}{x_4x_3^2}, \delta(\vec{x}) = \frac{6PL^3}{Ex_3^2x_4}\\
&P_c(\vec{x}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}),\\
&P = 6000\,lb, L = 14\,in., \quad \delta_{max} = 0.25\,in.,\\
&E = 30 \times 1^6\,psi,\; G = 12 \times 10^6\,psi,\\
&\tau_{max} = 13600\,psi,\; \sigma_{max} = 30000\,psi
\end{aligned}
\tag{13}
$$

The results of comparing the COOT algorithm with different optimization methods with the same penalty function are given in Table 13. The results show the COOT algorithm has better performance in finding optimum values of the welded beam problem. So the COOT algorithm can be a good alternative to the old optimization algorithms.

5.4. Multiple disk clutch brake design problems

The main objective of this problem is to minimize the mass of a multiple disk clutch brake. In this problem, five integer decision variables are used which are inner radius (r_i), outer radius (r_0), disk thickness (t), the force of actuators (F), and several frictional surfaces (Z). Fig. 14 shows the structural parameters of this problem. This problem contains nine non-linear constraints. The problem can be defined as follows.

$$
\begin{aligned}
&Minimize:\\
&f(\bar{x}) = \pi(x_2^2 - x_1^2)x_3(x_5 + 1)_\rho\\
&subject\ to:\\
&g_1(\bar{x}) = -p_{max} + p_{rz} \leqslant 0,\\
&g_2(\bar{x}) = p_{rz}v_{sr} - v_{sr,max P max} \leqslant 0,\\
&g_3(\bar{x}) = \Delta R + x_1 - x_2 \leqslant 0,\\
&g_4(\bar{x}) = -L_{max} + (x_5 + 1)(x_3 + \delta) \leqslant 0,\\
&g_5(\bar{x}) = sM_s - M_h \leqslant 0,\\
&g_6(\bar{x}) = T \geqslant 0,\\
&g_7(\bar{x}) = -v_{sr,max} + v_{sr} \leqslant 0,\\
&g_8(\bar{x}) = T - T_{max} \leqslant 0,\\
&where\\
&M_h = \frac{2}{3}\mu x_4x_5\frac{x_2^3 - x_1^3}{x_2^2 - x_1^2}N.mm,\\
&\omega = \frac{\pi n}{30}rad/s,\\
&A = \pi(x_2^2 - x_1^2)mm^2,\\
&p_{rz} = \frac{x_4}{A}N/mm^2,\\
&v_{sr} = \frac{\pi R_{sr}n}{30}mm/s,\\
&R_{sr} = \frac{2}{3}\frac{x_2^3 - x_1^3}{x_2^2x_1^2}mm,\\
&T = \frac{I_z\omega}{M_h + M_f},\\
&\Delta R = 20mm, L_{max} = 30mm, \mu = 0.6,\\
&V_{sr,max} = 10m/s, \delta = 0.5mm, s = 1.5,\\
&T_{max} = 15s, n = 250rpm, T_z = 55Kg.m^2,\\
&M_s = 40Nm, M_f = 2Nm, and\ p_{max} = 1.\\
&with\ bounds:\\
&60 \leqslant x_1 \leqslant 80, 90 \leqslant x_2 \leqslant 110, 1 \leqslant x_3 \leqslant 3,\\
&0 \leqslant x_4 \leqslant 1000, 2 \leqslant x_5 \leqslant 9.
\end{aligned}
\tag{14}
$$

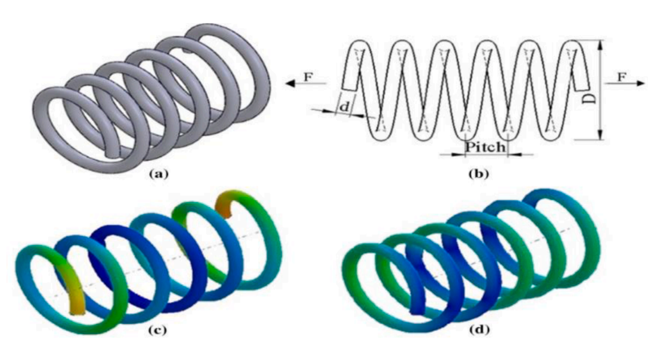

Fig. 11. (a) 3D view of the spring, (b) 2D view of the spring, (c) displacement heat map, (d) stress heat map (Khalilpourazari & Khalilpourazary, 2019).

Table 11
Results for Tension/compression spring.

Algorithms	d	D	N	Optimal cost
COOT	0.051652753877	0.355844265085	11.340383344758	0.012665293034089
OBSCA(Abd Elaziz et al., 2017)	0.05230	0.31728	12.54854	0.012625
OBLGOA(Ewees et al., 2018)	0.0530178	0.38953229	9.6001616	0.01270136
SMONM(Singh et al., 2018)	0.051918	0.362248	10.97194	0.012666
HHO (Heidari et al., 2019)	0.051796393	0.359305355	11.138859	0.012665443
SSA (Mirjalili et al., 2017)	0.051207	0.345215	12.004032	0.0126763
TEO (Kaveh & Dadras, 2017)	0.051775	0.3587919	11.16839	0.012665
MFO (Mirjalili, 2015b)	0.051994457	0.36410932	10.868422	0.0126669
SFS (Salimi, 2015)	0.051689061	0.356717736	11.288966	0.012665233
GWO (Mirjalili et al., 2014)	0.05169	0.356737	11.28885	0.012666
WOA (Mirjalili & Lewis, 2016)	0 0.051207	0 0.345215	12 0.004032	0 0.0126763
WEO (Kaveh & Bakhshpoori, 2016)	0.051685	0.356630	11.294103	0.012665
BA (X. Yang & Hossein Gandomi, 2012)	0.05169	0.35673	11.2885	0.012665
CWCA (Heidari et al., 2017)	0.051709	0.35710734	11.270826	0.012672
SC-GWO (Gupta et al., 2020)	0.051511	0.352376	11.552600	0.012672
SHO (Dhiman & Kumar, 2017)	0.051144	0.343751	12.09550	0.012674000
GPEAae (Gao et al., 2020)	0.051685	0.356631	11.294000	0.012665
AEO (Zhao et al., 2020)	0.051897	0.361751	10.879842	0.0126662
CMVO (Sayed et al., 2018)	0.051689	0.356716	11.289012	0.012665
I-ABC greedy (Sharma & Abraham, 2020)	0.051686	0.356014	11.202765	0.012665
AOA (Abualigah et al., 2021)	0.0500	0.349809	11.8637	0.012124

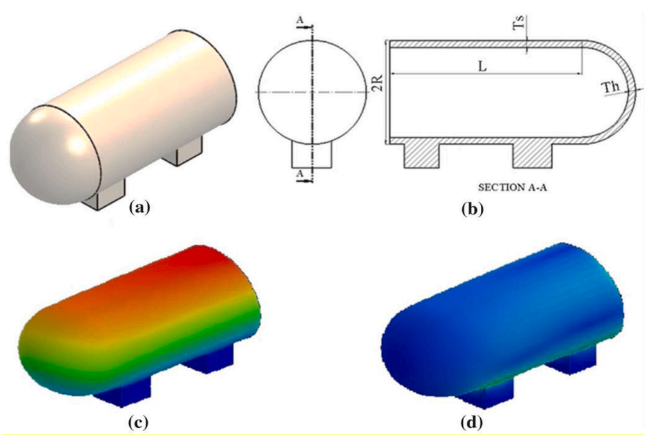

Fig. 12. (a) 3D shape of the pressure vessel, (b) 2D shape of the pressure vessel, (c) displacement heat map, (d) stress heat map (Khalilpourazari & Khalilpourazary, 2019).

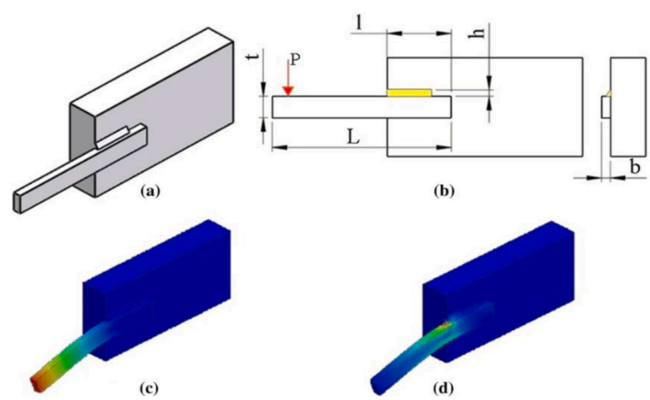

Fig. 13. (a) 3D view of the welded beam, (b) 2D view of the welded beam, (c) displacement heat map, (d) stress heat map (Khalilpourazari & Khalilpourazary, 2019).

Table 12
Results for the pressure vessel.

Algorithms	Ts	Th	R	L	Optimal cost
COOT	0.77817	0.384651	40.319618	200	5885.3487
OBSCA(Abd Elaziz et al., 2017)	1.2500	0.0625	59.1593	70.8437	5833.9892
OBLGOA(Ewees et al., 2018)	0.81622	0.40350	42.291138	174.811191	5966.67160
SMONM(Singh et al., 2018)	0.778322	0.384725	40.3275957	199.8889	5885.595
HHO (Heidari et al., 2019)	0.81758383	0.4072927	42.09174576	176.7196352	6000.46259
GWO (Mirjalili et al., 2014)	0.8125	0.4345	42.089181	176.758731	6051.5639
GA (Deb, 1991)	0.812500	0.437500	42.097398	176.654050	6059.9463
WEO (Kaveh & Bakhshpoori, 2016)	0.812500	0.437500	42.098444	176.636622	6059.71
BA (X. Yang & Hossein Gandomi, 2012)	0.812500	0.437500	42.098445	176.636595	6059.7143
MFO (Mirjalili, 2015b)	0.8125	0.4375	42.098445	176.636596	6059.7143
CSS (Kaveh & Talatahari, 2010)	0.812500	0.437500	42.103624	176.572656	6059.0888
WOA (Mirjalili & Lewis, 2016)	0 0.812500	0.437500	42 0.0982699	176 0.638998	6059 0.7410
SC-GWO (Gupta et al., 2020)	0.8125	0.4375	42.0984	176.63706	6059.7179
GPEAae (Gao et al., 2020)	0.812500	0.437500	42.098497	176.635954	6059.708025
AEO (Zhao et al., 2020)	0.8374205	0.413937	43.389597	161.268592	5994.50695
CMVO (Sayed et al., 2018)	0.8125	N/A	42.0984	176.6372	6059.7208
I-ABC greedy (Sharma & Abraham, 2020)	0.8125	0.4375	42.0984	176.6369	6059.7124
AOA (Abualigah et al., 2021)	0.8303737	0.4162057	42.75127	169.3454	6048.7844

The optimal results of proposed COOT in compared to those revealed by Harris Hawks Optimizer (Heidari et al., 2019), Teaching–learning-based optimization (Rao et al., 2011), Water cycle algorithm (Eskandar et al., 2012), Passing vehicle search algorithms (Savsani & Savsani, 2016), Flying Squirrel Optimizer (Azizyan et al., 2019), quantum-behaved simulated annealing algorithm-based moth-flame optimization (Yu et al., 2020), Artificial ecosystem-based optimization (Zhao et al., 2020), chaotic multi-verse optimization (Sayed et al., 2018), and I-ABC greedy (Sharma & Abraham, 2020). Table 14 shows the attained results of different optimizers for this test case. From Table 14, we can recognize that the COOT attains the best rank and can outperform all the compared algorithms in terms of quality of solutions.

5.5. Step-cone pulley problem

This problem's main objective is to minimize the weight of 4 step-cone pulleys using five variables in which four variables are the diameters of each step of the pulley, and the last one is the pulley's width. This problem contains 11 non-linear constraints to assure that the transmit power must be at 0.75 hp. The mathematical formulation of this problem can be defined as follows.

Minimize:

$$f(\overline{x}) = \rho\omega\left[d_1^2\left\{11+\left(\frac{N_1}{N}\right)^2\right\}+d_2^2\left\{1+\left(\frac{N_2}{N}\right)^2\right\}+d_3^2\left\{1+\left(\frac{N_3}{N}\right)^2\right\}+d_4^2\left\{1+\left(\frac{N_4}{N}\right)^2\right\}\right]$$

Subject to:

$$\begin{aligned}
h_1(\overline{x}) &= C_1 - C_2 = 0,\\
h_2(\overline{x}) &= C_1 - C_3 = 0,\\
h_3(\overline{x}) &= C_1 - C_4 = 0,\\
g_{i=1,2,3,4}(\overline{x}) &= -R_i \leqslant 2,\\
g_{i=1,2,3,4}(\overline{x}) &= (0.75\times 745.6998) - P_i \leqslant 0\\
&\textit{where},\\
C_i &= \frac{\pi d_i}{2}\left(1+\frac{N_i}{N}\right)+\frac{\left(\frac{N_i}{N}-1\right)^2}{4a}+2a,\ i=(1,2,3,4),\\
R_i &= exp\left(\mu\left\{\pi - 2sin^{-1}\left\{\left(\frac{N_i}{N}-1\right)\frac{d_i}{2a}\right\}\right\}\right),\ i=(1,2,3,4),\\
P_i &= st\omega(1-R_i)\frac{\pi d_i N_i}{60},\ i=(1,2,3,4),
\end{aligned} \tag{15}$$

$$t = 8mm,\quad s = 1.75MPa,\quad \mu = 0.35,\ \rho = 7200kg/m^3,\ a = 3mm.$$

A schematic view of this problem is illustrated in Fig. 15.

This problem has been solved by COOT and compared with the literature in Table 15. It may be seen that the comparison is made between Method of TLBO (Rao et al., 2011), ABC (Rao et al., 2011), PVC (Savsani & Savsani, 2016), and AEFA (Anita et al., 2020). Table 15 shows that the COOT algorithm outperforms other algorithms. This shows the high performance of the COOT algorithm in approximating the global optimum for this problem.

5.6. Cantilever beam design problem

A cantilever beam includes five hollow elements with square-shaped cross-section. Fig. 16 shows that each element is defined by one variable, while the thickness is constant, so there is a total of 5 structural parameters. The objective is to minimize the weight of the beam. There is also one vertical displacement constraint that should not be violated by the final optimal design. The problem formulation is as follows:

$$\begin{aligned}
&\textit{Minimize}:\\
&\overline{x} = [x_1x_2x_3x_4x_5],\\
&f(\overline{x}) = 0.6224(x_1+x_2+x_3+x_4+x_5),\\
&g(\overline{x}) = \frac{61}{x_1^3}+\frac{37}{x_2^3}+\frac{19}{x_3^3}+\frac{7}{x_4^3}+\frac{1}{x_5^3}\leqslant 1,\\
&0.01\leqslant x_1,x_2,x_3,x_4,x_5\leqslant 100.
\end{aligned}$$

This problem has been solved by COOT and compared with the literature in Table 16. It may be seen that the comparison is made between Method of MVO (Mirjalili et al., 2016), MMA (CHICKERMANE & GEA, 1996), GCA_I (CHICKERMANE & GEA, 1996), GCA_II (CHICKERMANE & GEA, 1996), CS (Gandomi et al., 2013), SOS (Cheng & Prayogo, 2014), ALO (Mirjalili, 2015a), Interactive autodidactic school (Jahangiri et al., 2020), Grey Prediction Evolution Algorithm Based on Accelerated Even (Gao et al., 2020), and Artificial ecosystem-based optimization (Zhao et al., 2020). Table 16 shows that the COOT algorithm outperforms other algorithms. This shows the high performance of the COOT algorithm in approximating the global optimum for this problem.

5.7. Speed reducer design problem

The speed reducer design problem is an engineering design problem. It has seven design variables, as shown in Fig. 17. The main objective of this design problem is to minimize the weight of speed reducer with subject to the following constraints (Mezura-Montes & Coello, 2005):

- Bending stress of the gear teeth.
- Surface stress.
- Transverse deflections of the shafts.
- Stresses in the shafts.

There are seven design variables ($z1 - z7$) such as face width (b), the module of teeth (m), number of teeth in the pinion (p), length of the first shaft between bearings (l_1), length of the second shaft between bearings (l_2), diameter of first (d_1) shafts, and diameter of second shafts (d_2). The mathematical formulation of this problem is described as follows:

Table 13
Results for the welded beam.

Algorithms	τ	σ	Pc	δ	Optimal cost
COOT	0.1988271	3.337971	9.191986	0.1988345	1.670301154263369
OBSCA(Abd Elaziz et al., 2017)	0.230824	3.069152	8.988479	0.208795	1.722315
OBLGOA(Ewees et al., 2018)	0.205769	3.471135	9.032728	0.2059072	1.7257
SMONM(Singh et al., 2018)	0.20573	3.470489	9.036624	0.20573	1.724852
HHO (Heidari et al., 2019)	0.204039	3.531061	9.027463	0.206147	1.73199057
HS (Lee & Geem, 2004)	0.2442	6.2231	8.2915	0.2443	2.3807
GSA (Rashedi et al., 2009)	0.182129	3.856979	10	0.202376	1.879952
ESs (Mezura-Montes & Coello, 2005)	0.199742	3.61206	9.0375	0.206082	1.7373
CDE (Huang et al., 2007)	0.203137	3.542998	9.033498	0.206179	1.733462
LFD (Houssein et al., 2020)	0.1857	3.9070	9.1552	0.2051	1.7700
TSA (Kaur et al., 2020)	0.203290	3.471140	9.035100	0.201150	1.721020
SHO (Dhiman & Kumar, 2017)	0.205563	3.474846	9.035799	0.205811	1.725661
IAS (Jahangiri et al., 2020)	0.2057	3.4705	9.0366	0.2057	1.7249
GPEAae (Gao et al., 2020)	0.205731	3.470467	9.036624	0.205730	1.724851
AEO (Zhao et al., 2020)	0.2057296	3.4704886	9.0366239	0.2057296	1.7248520
CMVO (Sayed et al., 2018)	0.20573	3.4705	9.03662	0.20573	1.724852
I-ABC greedy (Sharma & Abraham, 2020)	0.2057294	3.47048861	9.03662389	0.20572876	1.7248210
AOA (Abualigah et al., 2021)	0.194475	2.57092	10.000	0.201827	1.7164

$$
\begin{aligned}
&Consider\,\bar{z} = [z_1\, z_2\, z_3\, z_4\, z_5\, z_6\, z_7] = [b\, m\, p\, l_1\, l_2\, d_1\, d_2],\\
&Minimize\, f(\bar{z}) = 0.7854 z_1 z_2^2 (3.3333 z_3^2 + 14.9334 z_3 - 43.0934)\\
&-1.508 z_1 (z_6^2 + z_7^2) + 7.4777 (z_6^3 + z_7^3) + 0.7854 (z_4 z_6^2 + z_5 z_7^2),\\
&Subject\ to:\\
&g_1(\bar{z}) = \frac{27}{z_1 z_2^2 z_3} - 1 \leqslant 0,\\
&g_2(\bar{z}) = \frac{397.5}{z_1 z_2^2 z_3^2} - 1 \leqslant 0,\\
&g_3(\bar{z}) = \frac{1.93 z_4^3}{z_2 z_7^4 z_3} - 1 \leqslant 0,\\
&g_4(\bar{z}) = \frac{1.93 z_4^3}{z_2 z_7^4 z_3} - 1 \leqslant 0,\\
&g_5(\bar{z}) = \frac{\left[(745(z_4/z_2 z_3))^2 + 16.9\times 10^6\right]^{1/2}}{110 z_6^3} - 1 \leqslant 0,\\
&g_6(\bar{z}) = \frac{\left[(745(z_5/z_2 z_3))^2 + 157.5\times 10^6\right]^{1/2}}{85 z_7^3} - 1 \leqslant 0,\\
&g_4(\bar{z}) = \frac{1.93 z_4^3}{z_2 z_7^4 z_3} - 1 \leqslant 0,\\
&g_7(\bar{z}) = \frac{z_2 z_3}{40} - 1 \leqslant 0,\\
&g_8(\bar{z}) = \frac{5 z_2}{z_1} - 1 \leqslant 0,\\
&g_9(\bar{z}) = \frac{z_1}{12 z_2} - 1 \leqslant 0,\\
&g_{10}(\bar{z}) = \frac{1.5 z_6 + 1.9}{z_4} - 1 \leqslant 0,\\
&g_{11}(\bar{z}) = \frac{1.1 z_7 + 1.9}{z_5} - 1 \leqslant 0,\\
&where,\\
&2.6 \leqslant z_1 \leqslant 3.6,\ 0.7 \leqslant z_2 \leqslant 0.8,\ 17 \leqslant z_3 \leqslant 28,\ 7.3 \leqslant z_4 \leqslant 8.3,\\
&7.3 \leqslant z_5 \leqslant 8.3,\ 2.9 \leqslant z_6 \leqslant 3.9,\ 5.0 \leqslant z_7 \leqslant 5.5.
\end{aligned}
\tag{17}
$$

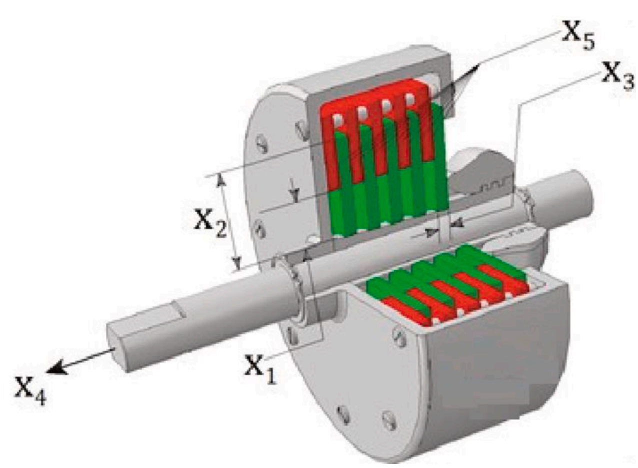

Fig.14. Multiple disk clutch brake (Azizyan et al., 2019).

Table 17 reveals the obtained optimal solutions by different algorithms on this design problem. The proposed COOT is compared with well-known metaheuristic algorithms namely (Kaur et al., 2020)(TSA), Spotted Hyena Optimizer (SHO) (Dhiman & Kumar, 2017), Grey Wolf Optimizer (GWO) (Mirjalili et al., 2014), Particle Swarm Optimization (PSO) (Eberhart & Kennedy, 2002), Multi-verse Optimizer (MVO) (Mirjalili et al., 2016), Sine Cosine Algorithm (SCA) (Mirjalili, 2016), Gravitational Search Algorithm (GSA) (Rashedi et al., 2009), Genetic Algorithm (GA) (Holland, 1967), Emperor Penguin Optimizer (EPO) (Dhiman & Kumar, 2018), Sine cosine grey wolf optimizer (Gupta et al., 2020), Grey Prediction Evolution Algorithm Based on Accelerated Even (Gao et al., 2020), Artificial ecosystem-based optimization (Zhao et al., 2020), chaotic multi-verse optimization (Sayed et al., 2018), I-ABC greedy (Sharma & Abraham, 2020), and Arithmetic Optimization Algorithm (Abualigah et al., 2021). Examining the results in Table 17, we found that COOT is the best optimizer to deal with this problem and can achieve superior results compared to other techniques.

5.8. Rolling element bearing design problem

This problem is formulated to optimize the load-carrying capacity of a rolling element bearing using five design variables and five design parameters. These design variables are pitch diameter (D_m), ball diameter (D_b), outer and inner raceway curvature coefficients (f_0 and f_i) and the total number of balls (Z). The design parameters are e, ε, ζ, K_{Dmax}, and K_{Dmin} appeared in only constraints. These all are considered as variables, i.e. five design variables and five design parameters. This problem contains nine non-linear constraints based on manufacturing and kinematic factors. The mathematical formulation of this problem is described as follows:

Maximize:

$$f(\overline{x}) = \begin{Bmatrix} f_c Z^{2/3} D_b^{1.8}, \text{ if } D_b \leqslant 25.4\, mm \\ 3.647 f_c Z^{2/3} D_b^{1.4}, \text{ otherwise} \end{Bmatrix}$$

Subject to:

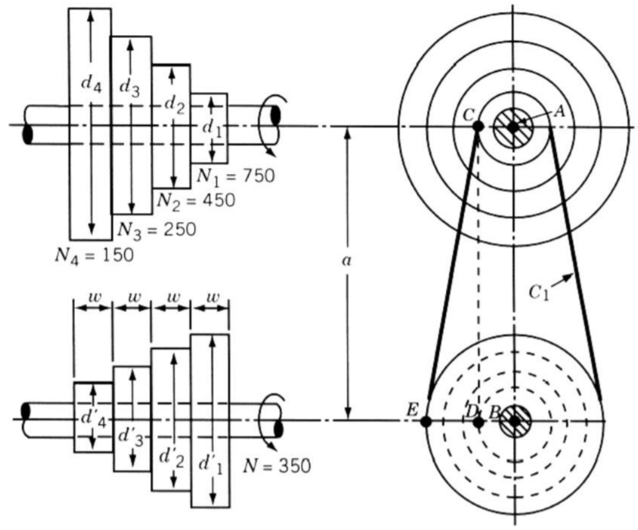

Fig.15. Step-cone pulley problem (Savsani & Savsani, 2016).

$$g_1(\overline{x}) = Z - \frac{\phi_0}{2sin^{-1}(D_b/D_m)} - 1 \leqslant 0,$$

$$g_2(\overline{x}) = 2D_b - K_{Dmin}(D-d) > 0,$$

$$g_3(\overline{x}) = K_{Dmax}(D-d) - 2D_b \geqslant 0,$$

$$g_4(\overline{x}) = \zeta B_\omega - D_b \leqslant 0,$$

$$g_5(\overline{x}) = D_m - 0.5(D+d) \geqslant 0,$$

$$g_6(\overline{x}) = (0.5+e)(D+d) - D_m < 0,$$

$$g_7(\overline{x}) = 0.5(D - D_m - D_b) - \varepsilon D_b \geqslant 0,$$

$$g_8(\overline{x}) = f_i \geqslant 0.515,$$

$$g_9(\overline{x}) = f_0 \geqslant 0.515,$$

where

$$f_c = 37.91\left[1 + \left\{1.04\left(\frac{1-\gamma}{1+\gamma}\right)^{1.72}\left(\frac{f_i(2f_0-1)}{f_0(2f_i-1)}\right)^{0.41}\right\}^{10/3}\right]^{-0.3}, \gamma = \frac{D_b}{D_m}, f_i = \frac{r_i}{D_b}, f_0 = \frac{r_0}{D_b},$$

$$\phi_0 = 2\pi - 2 \times cos^{-1}\left(\frac{\{(D-d)/2 - 3(T/4)\}^2 + \{D/2 - (T/4) - D_b\}^2 - \{d/2 + (T/4)\}^2}{2\{(D-d)/2 - 3(T/4)\}\{D/2 - (T/4) - D_b\}}\right)$$

$$T = D - d - 2D_b,\ D = 160,\ d = 90,\ B_w = 30.$$

Table 14
Comparison of results for multi-plate disc clutch brake.

Algorithm	$r_i(x_1)$	$r_0(x_2)$	$t(x_3)$	$F(x_4)$	$Z(x_5)$	Optimal cost
COOT	70	90	1	214.13	2	0.23524245790
HHO (Heidari et al., 2019)	69.9999999992493	90	1	1000	2.3128	0.259768993
TLBO (Rao et al., 2011)	70	90	1	810	3	0.313656
WCA (Eskandar et al., 2012)	70	90	1	910	3	0.313656
PVS (Savsani & Savsani, 2016)	70	90	1	980	3	0.31366
FSO (Azizyan et al., 2019)	70	90	1	870	3	0.313657
QSMFO (Yu et al., 2020)	80	101.3002	3	600	9	0.2902
AEO (Zhao et al., 2020)	70	90	1	810	3	0.3136566
CMVO (Sayed et al., 2018)	70	90	1	910	3	0.313656
I-ABC greedy (Sharma & Abraham, 2020)	70	90	1	900	3	0.313766

Table 15
Comparison of results for Step-cone pulley problem.

Algorithm	X_1	X_2	X_3	X_4	X_5	Optimal weight
COOT	38.5852399	53.0944977	70.787126	84.87239	89.82746	16.203291
TLBO (Rao et al., 2011)	40	54.7643	73.01318	88.42842	85.98624	16.63451
ABC(Rao et al., 2011)	NAN	NAN	NAN	NAN	NAN	16.634655
PVC (Savsani & Savsani, 2016)	40	54.7643021	73.013177	88.428419	85.98624	16.63450513
AEFA (Anita et al., 2020)	39.25346	54.01469	72.01386	86.34195	89.03809	16.6218705

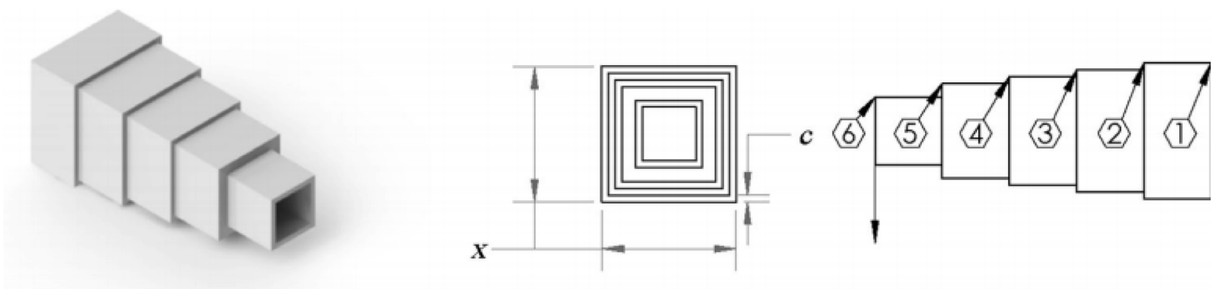

Fig. 16. Cantilever beam design problem (Mirjalili et al., 2016).

Table 16
Comparison results for the cantilever design problem.

Algorithm	Optimal values for variables					Optimal weight
	X_1	X_2	X_3	X_4	X_5	
COOT	6.02743657	5.3385748	4.4904867	3.483437	2.134591	1.3365745
MVO (Mirjalili et al., 2016)	6.0239402	5.3060112	4.4950113	3.496022	2.1527261	1.3399595
MMA (CHICKERMANE & GEA, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
GCA_I (CHICKERMANE & GEA, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
GCA_II (CHICKERMANE & GEA, 1996)	6.0100	5.3000	4.4900	3.4900	2.1500	1.3400
CS (Gandomi et al., 2013)	6.0089	5.3049	4.5023	3.5077	2.1504	1.33999
SOS (Cheng & Prayogo, 2014)	6.01878	5.30344	4.49587	3.49896	2.15564	1.33996
ALO (Mirjalili, 2015a)	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995
IAS (Jahangiri et al., 2020)	5.99140	5.30850	4.51190	3.50210	2.16010	1.34000
GPEAae (Gao et al., 2020)	6.014808	5.306724	4.493232	3.505168	2.153781	1.339982
AEO (Zhao et al., 2020)	6.028850	5.316521	4.462649	3.508455	2.157761	1.339965

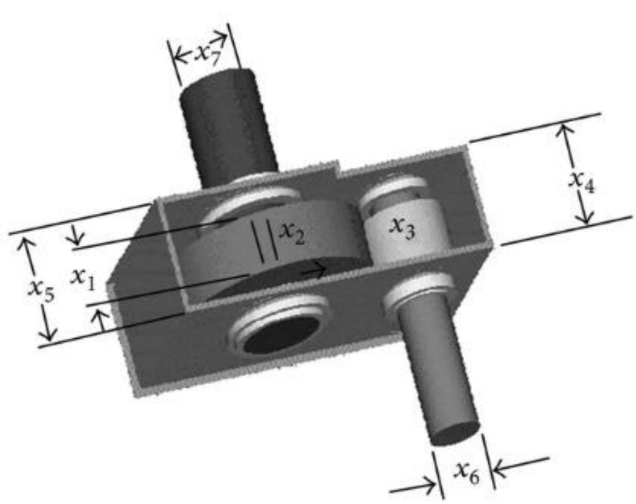

Fig. 17. Speed reducer design (Hassan et al., 2005).

Table 17
Comparison of the best solution obtained from different algorithms for speed reducer design problem.

Algorithm	Optimum variables							Optimum cost
	b	m	p	L_1	L_2	D_1	D_2	
COOT	3.24132	0.7	17	7.3	7.715336	3.350215	5.28665	2892.7461
TSA (Kaur et al., 2020)	3.50120	0.7	17	7.3	7.8	3.33410	5.26530	2990.9580
EPO (Dhiman & Kumar, 2018)	3.50123	0.7	17	7.3	7.8	3.33421	5.26536	2994.2472
SHO (Dhiman & Kumar, 2017)	3.50159	0.7	17	7.3	7.8	3.35127	5.28874	2998.5507
GWO (Dhiman & Kumar, 2017)	3.506690	0.7	17	7.380933	7.815726	3.357847	5.286768	3001.288
PSO (Dhiman & Kumar, 2017)	3.500019	0.7	17	8.3	7.8	3.352412	5.286715	3005.763
MVO (Dhiman & Kumar, 2017)	3.508502	0.7	17	7.392843	7.816034	3.358073	5.286777	3002.928
SCA (Dhiman & Kumar, 2017)	3.508755	0.7	17	7.3	7.8	3.461020	5.289213	3030.563
GSA (Dhiman & Kumar, 2017)	3.600000	0.7	17	8.3	7.8	3.369658	5.289224	3051.120
GA (Dhiman & Kumar, 2017)	3.510253	0.7	17	8.35	7.8	3.362201	5.287723	3067.561
SC-GWO (Gupta et al., 2020)	3.50064	0.7	17	7.30643	7.80617	3.35034	5.28694	2996.9859
GPEAae (Gao et al., 2020)	3.499997	0.7	17	7.300001	7.715311	3.350214	5.286653	2994.468240
AEO (Zhao et al., 2020)	3.5	0.7	17	7.3	7.7153199	3.3502146	5.2866545	2994.471066
CMVO (Sayed et al., 2018)	3.5	0.7	17	7.3	7.715319	3.350214	5.286654	2994.471
I-ABC greedy (Sharma & Abraham, 2020)	3.50021	0.7	17	7.3	7.71531189	3.3502147	5.2866554	2994.4710315
AOA (Abualigah et al., 2021)	3.50384	0.7	17	7.3	7.72933	3.35649	5.2867	2997.9157

With bounds:

$$
\begin{gathered}
0.5(D+d)\leqslant D_m\leqslant 0.6(D+d),\\
0.15(D-d)\leqslant D_b\leqslant 0.45(D-d),\\
4\leqslant Z\leqslant 50,\\
0.515\leqslant f_i\leqslant 0.6,\\
0.515\leqslant f_0\leqslant 0.6,\\
0.4\leqslant K_{Dmin}\leqslant 0.5,\\
0.6\leqslant K_{Dmax}\leqslant 0.7,\\
0.3\leqslant \varepsilon\leqslant 0.4,\\
0.02\leqslant e\leqslant 0.1,\\
0.6\leqslant \zeta\leqslant 0.85.
\end{gathered}
$$

A schematic view of this problem is illustrated in Fig. 18.

This case covers near 1.5% of the feasible area of the target space.

The results of COOT is compared to HHO (Heidari et al., 2019), SCA (Mirjalili, 2016), GA4 (Gupta et al., 2007), TLBO (Rao et al., 2011), and PVS (Savsani & Savsani, 2016) techniques. Table 18 tabulates the results of COOT versus those of other optimizers. Table 18 shows that the proposed COOT has detected the best solution with the maximum cost with substantial progress compared to HHO, SCA, GA4, TLBO, and PVS algorithms.

As can be seen, the proposed COOT algorithm performed very well in solving real-world engineering problems. It should be noted that most of the compared algorithms achieved the above results with better conditions such as more iterations, more search agents and more number function evaluations (NFE). However, the proposed algorithm found a better solution. Therefore, the proposed algorithm can replace previous

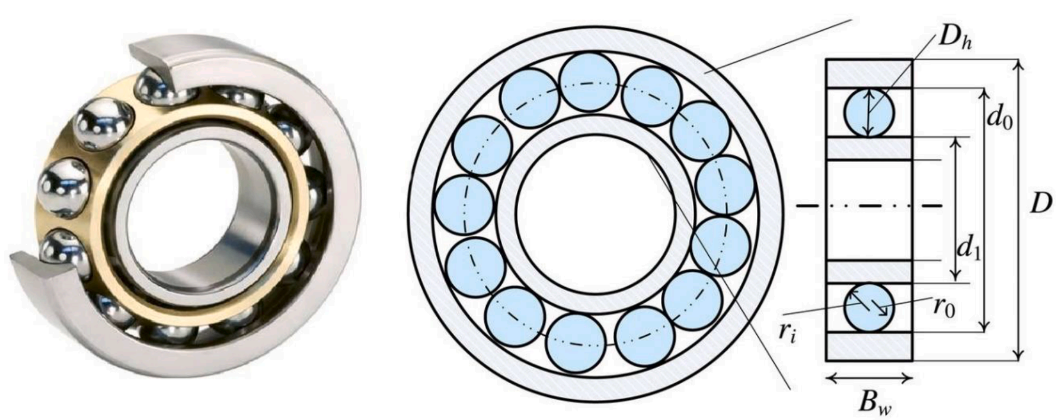

Fig. 18. Rolling element bearing problem (Heidari et al., 2019).

Table 18
Comparison of results for rolling element bearing design problem.

Algorithm	GA4	TLBO	PVS	SCA	HHO	COOT
D_m	125.717100	125.7191	125.719060	125	125.000000	125.0000
D_b	21.423000	21.42559	21.425590	21.14834	21.000000	21.8750
Z	11.000000	11.000000	11.000000	10.92928	11.092073	10.7770
F_i	0.515000	0.515000	0.515000	0.515	0.515000	0.5150
F_0	0.515000	0.515000	0.515000	0.515	0.515000	0.5150
K_{Dmin}	0.415900	0.424266	0.400430	0.5	0.400000	0.4319
K_{Dmax}	0.651000	0.633948	0.680160	0.7	0.600000	0.6529
ε	0.300043	0.300000	0.300000	0.3	0.300000	0.3000
e	0.022300	0.068858	0.079990	0.02778	0.050474	0.0200
ζ	0.751000	0.799498	0.700000	0.62912	0.600000	0.6000
Maximum cost	81843.30	81859.74	81859.741210	83431.117	83011.88329	83918.492

Table 19
Results of the proposed COOT algorithm in solving real-world engineering problems.

No.	Name	Optimal value	Rank
1	Tension/Compression spring	0.012665293034089	3
2	Pressure vessel design	5885.3487	2
3	Welded Beam Design	1.670301154263369	1
4	Multi-plate disc clutch brake	0.23524245790	1
5	Step-cone pulley problem	16.203291	1
6	Cantilever beam design	1.3365745	1
7	reducer design problem	2892.7461	1
8	Rolling element bearing problem	83918.492	1

algorithms.

In summary, Table 19 shows the results of the proposed COOT algorithm in solving real-world engineering problems.

6. Conclusion

In this paper, a new swarm-based optimization algorithm was suggested that inspired regular and irregular movements of birds called Coot on the surface of the water. Unique features such as swarm leadership by a leading group and chain movement at the end of the swarm are the main motive for creating this optimization algorithm. In order to evaluate the algorithm in terms of exploration and exploitation, 13 test functions (with 30, 100, and 500 dimention) including 7 single-modal test functions for evaluating the exploitation of the algorithm and 6 multi-modal test functions for evaluation of algorithm exploration was used. We used the 2017 test functions to show the performance of the proposed algorithm. This set includes 30 unimodal (F1-F3), Multimodal (F4-F10), Hybrid (F11-F20), Composition (F21-F30) test functions. The results showed that the coot algorithm has a good and competitive performance compared to other well-known optimization algorithms. In addition, in order to evaluate the algorithm in unknown search spaces, the proposed algorithm was applied to a number of well-known engineering design issues, which showed that the Coot algorithm is efficient in solving problems with unknown search spaces.

effective and efficient results can highlight the COOT as an alternative optimization algorithm to classical methods. Solving other optimization problems in different disciplines is recommended with COOT. Since COOT is a population-based algorithm developing a binary and multi-objective version of COOT would be a valuable contribution.

CRediT authorship contribution statement

Iraj Naruei: Investigation, Validation, Writing - original draft. **Farshid Keynia:** Conceptualization, Data curation, Methodology, Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

Abd Elaziz, M., Lu, S., & He, S. (2021). A multi-leader whale optimization algorithm for global optimization and image segmentation. *Expert Systems with Applications, 175*, 114841. https://doi.org/10.1016/j.eswa.2021.114841

Abd Elaziz, M., Oliva, D., & Xiong, S. (2017). An improved opposition-based sine cosine algorithm for global optimization. *Expert Systems with Applications, 90*, 484–500. https://doi.org/10.1016/j.eswa.2017.07.043

Abdullah, J. M., & Ahmed, T. (2019). Fitness Dependent Optimizer: Inspired by the Bee Swarming Reproductive Process. *IEEE Access, 7*, 43473–43486. https://doi.org/10.1109/Access.628763910.1109/ACCESS.2019.2907012

Abualigah, L., Diabat, A., Mirjalili, S., Abd Elaziz, M., & Gandomi, A. H. (2021). The arithmetic optimization algorithm. *Computer Methods in Applied Mechanics and Engineering, 376*, 113609. https://doi.org/10.1016/j.cma.2020.113609

Angeline, P. J. (1994). Genetic programming: On the programming of computers by means of natural selection,. In Biosystems (Vol. 33, Issue 1). MIT press. https://doi.org/10.1016/0303-2647(94)90062-0.

Anita, Yadav, A., & Kumar, N. (2020). Artificial electric field algorithm for engineering optimization problems. *Expert Systems with Applications, 149*, 113308. https://doi.org/10.1016/j.eswa.2020.113308

Atashpaz-Gargari, E., & Lucas, C. (2007). Imperialist competitive algorithm: An algorithm for optimization inspired by imperialistic competition. *IEEE Congress on Evolutionary Computation, 2007*, 4661–4667. https://doi.org/10.1109/CEC.2007.4425083

Awad, N H, Ali, M. Z., Suganthan, P. N., Liang, J. J., Qu, B. Y. (2017). Problem Definitions and Evaluation Criteria for the CEC 2017 Special Session and Competition on Single Objective Real-Parameter Numerical Optimization. 2017 IEEE Congress on Evolutionary Computation (CEC).

Azizyan, G., Miarnaeimi, F., Rashki, M., & Shabakhty, N. (2019). Flying Squirrel Optimizer (FSO): A novel SI-based optimization algorithm for engineering problems. *Iranian Journal of Optimization, 11*(2), 177–205. http://ijo.iaurasht.ac.ir/article_663726.html.

Basturk, B., Karaboga, D. (2006). An artificial bee colony (ABC) algorithm for numeric function optimization. In Proceedings of the IEEE swarm intelligence symposium, Indianapolis, IN, USA. May, 2006, 12–14. https://ci.nii.ac.jp/naid/20001441290/.

Blum, C., & Roli, A. (2003). Metaheuristics in combinatorial optimization. *ACM Computing Surveys, 35*(3), 268–308. https://doi.org/10.1145/937503.937505

Cheng, M.-Y., & Prayogo, D. (2014). Symbiotic organisms search: A new metaheuristic optimization algorithm. *Computers & Structures, 139*, 98–112. https://doi.org/10.1016/j.compstruc.2014.03.007

Chickermane, H., & Gea, H. C. (1996). Structural optimization using a new local approximation method. *International Journal for Numerical Methods in Engineering, 39*(5), 829–846. https://doi.org/10.1002/(SICI)1097-0207(19960315)39:5<829::AID-NME884>3.0.CO;2-U

Colorni, A., Dorigo, M., & Maniezzo, V. (1991). Distributed Optimization by Ant Colonies. *European Conference on Artificial Life, 142*, 134–142.

Deb, K. (1991). Optimal design of a welded beam via genetic algorithms. *AIAA Journal, 29*(11), 2013–2015. https://doi.org/10.2514/3.10834

Dhiman, G., & Kumar, V. (2017). Spotted hyena optimizer: A novel bio-inspired based metaheuristic technique for engineering applications. *Advances in Engineering Software, 114*, 48–70. https://doi.org/10.1016/j.advengsoft.2017.05.014

Dhiman, G., & Kumar, V. (2018). Emperor penguin optimizer: A bio-inspired algorithm for engineering problems. *Knowledge-Based Systems, 159*, 20–50. https://doi.org/10.1016/j.knosys.2018.06.001

Eberhart, R., & Kennedy, J. (2002). A new optimizer using particle swarm theory. *MHS'95*. In *Proceedings of the Sixth International Symposium on Micro Machine and Human Science* (pp. 39–43). https://doi.org/10.1109/MHS.1995.494215

Abd Elaziz, M., Ewees, A. A., Neggaz, N., Ibrahim, R. A., Al-qaness, M. A. A., & Lu, S. (2021). Cooperative meta-heuristic algorithms for global optimization problems. *Expert Systems with Applications, 176*, 114788. https://doi.org/10.1016/j.eswa.2021.114788

Elaziz, M. A., Heidari, A. A., Fujita, H., & Moayedi, H. (2020). A competitive chain-based Harris Hawks Optimizer for global optimization and multi-level image thresholding problems. *Applied Soft Computing, 95*, 106347. https://doi.org/10.1016/j.asoc.2020.106347

Ertenlice, O., & Kalayci, C. B. (2018). A survey of swarm intelligence for portfolio optimization: Algorithms and applications. *Swarm and Evolutionary Computation, 39*, 36–52. https://doi.org/10.1016/j.swevo.2018.01.009

Eskandar, H., Sadollah, A., Bahreininejad, A., & Hamdi, M. (2012). Water cycle algorithm – A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers & Structures, 110–111*, 151–166. https://doi.org/10.1016/j.compstruc.2012.07.010

Ewees, A. A., Abd Elaziz, M., & Houssein, E. H. (2018). Improved grasshopper optimization algorithm using opposition-based learning. *Expert Systems with Applications, 112*, 156–172. https://doi.org/10.1016/j.eswa.2018.06.023

Gandomi, A. H., & Alavi, A. H. (2012). Krill herd: A new bio-inspired optimization algorithm. *Communications in Nonlinear Science and Numerical Simulation, 17*(12), 4831–4845. https://doi.org/10.1016/j.cnsns.2012.05.010

Gandomi, A. H., Yang, X.-S., & Alavi, A. H. (2013). Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems. *Engineering with Computers, 29*(1), 17–35. https://doi.org/10.1007/s00366-011-0241-y

Gao, C., Hu, Z., Xiong, Z., & Su, Q. (2020). Grey Prediction Evolution Algorithm Based on Accelerated Even Grey Model. *IEEE Access, 8*, 107941–107957. https://doi.org/10.1109/Access.628763910.1109/ACCESS.2020.3001194

Ghorbani, N., & Babaei, E. (2014). Exchange market algorithm. *Applied Soft Computing, 19*, 177–187. https://doi.org/10.1016/j.asoc.2014.02.006

Glover, F. (1989). Tabu Search—Part I. *ORSA Journal on Computing, 1*(3), 190–206. https://doi.org/10.1287/ijoc.1.3.190

Gupta, S., Tiwari, R., & Nair, S. B. (2007). Multi-objective design optimisation of rolling bearings using genetic algorithms. *Mechanism and Machine Theory, 42*(10), 1418–1443. https://doi.org/10.1016/j.mechmachtheory.2006.10.002

Gupta, S., Deep, K., Moayedi, H., Foong, L. K., & Assad, A. (2020). Sine cosine grey wolf optimizer to solve engineering design problems. *Engineering with Computers*. https://doi.org/10.1007/s00366-020-00996-y

Han, X., Liu, Q., Wang, H., & Wang, L. (2018). Novel fruit fly optimization algorithm with trend search and co-evolution. *Knowledge-Based Systems, 141*, 1–17. https://doi.org/10.1016/j.knosys.2017.11.001

Hassan, R., Cohanim, B., de Weck, O., & Venter, G. (2005). April 18). A Comparison of Particle Swarm Optimization and the Genetic Algorithm. In *46th AIAA/ASME/ASCE/*

AHS/ASC Structures, Structural Dynamics and Materials Conference. https://doi.org/10.2514/6.2005-1897

Hatamlou, A. (2013). Black hole: A new heuristic optimization approach for data clustering. *Information Sciences, 222*, 175–184. https://doi.org/10.1016/j.ins.2012.08.023

He, Q., & Wang, L. (2007). An effective co-evolutionary particle swarm optimization for constrained engineering design problems. *Engineering Applications of Artificial Intelligence, 20*(1), 89–99. https://doi.org/10.1016/j.engappai.2006.03.003

Heidari, A. A., Ali Abbaspour, R., & Rezaee Jordehi, A. (2017). An efficient chaotic water cycle algorithm for optimization tasks. *Neural Computing and Applications, 28*(1), 57–85. https://doi.org/10.1007/s00521-015-2037-2

Heidari, A. A., Mirjalili, S., Faris, H., Aljarah, I., Mafarja, M., & Chen, H. (2019). Harris hawks optimization: Algorithm and applications. *Future Generation Computer Systems, 97*, 849–872. https://doi.org/10.1016/j.future.2019.02.028

Holland, J. H. (1967). Genetic Algorithms understand Genetic Algorithms. *Surprise 96, 1* (1), 12–15. https://doi.org/10.2307/24939139

Houssein, E. H., Saad, M. R., Hashim, F. A., Shaban, H., & Hassaballah, M. (2020). Lévy flight distribution: A new metaheuristic algorithm for solving engineering optimization problems. *Engineering Applications of Artificial Intelligence, 94*, 103731. https://doi.org/10.1016/j.engappai.2020.103731

Huang, F., Wang, L., & He, Q. (2007). An effective co-evolutionary differential evolution for constrained optimization. *Applied Mathematics and Computation, 186*(1), 340–356. https://doi.org/10.1016/j.amc.2006.07.105

Hussain, K., Mohd Salleh, M. N., Cheng, S., & Shi, Y. (2018). Metaheuristic research: A comprehensive survey. *Artificial Intelligence Review, 52*(4), 2191–2233. https://doi.org/10.1007/s10462-017-9605-z

Jaderyan, M., & Khotanlou, H. (2016). Virulence optimization algorithm. *Applied Soft Computing, 43*, 596–618. https://doi.org/10.1016/j.asoc.2016.02.038

Jahangiri, M., Hadianfard, M. A., Najafgholipour, M. A., Jahangiri, M., & Gerami, M. R. (2020). Interactive autodidactic school: A new metaheuristic optimization algorithm for solving mathematical and structural design optimization problems. *Computers & Structures, 235*, 106268. https://doi.org/10.1016/j.compstruc.2020.106268

Kaur, S., Awasthi, L. K., Sangal, A. L., & Dhiman, G. (2020). Tunicate Swarm Algorithm: A new bio-inspired based metaheuristic paradigm for global optimization. *Engineering Applications of Artificial Intelligence, 90*, 103541. https://doi.org/10.1016/j.engappai.2020.103541

Kaveh, A., & Bakhshpoori, T. (2016). Water evaporation optimization: A novel physically inspired optimization algorithm. *Computers & Structures, 167*, 69–85. https://doi.org/10.1016/j.compstruc.2016.01.008

Kaveh, A., & Dadras, A. (2017). A novel meta-heuristic optimization algorithm: Thermal exchange optimization. *Advances in Engineering Software, 110*, 69–84. https://doi.org/10.1016/j.advengsoft.2017.03.014

Kaveh, A., & Khayatazad, M. (2012). A new meta-heuristic method: Ray optimization. *Computers & Structures, 112–113*, 283–294. https://doi.org/10.1016/j.compstruc.2012.09.003

Kaveh, A., & Talatahari, S. (2010). A novel heuristic optimization method: Charged system search. *Acta Mechanica, 213*(3–4), 267–289. https://doi.org/10.1007/s00707-009-0270-4

Khalilpourazari, S., & Khalilpourazary, S. (2019). An efficient hybrid algorithm based on Water Cycle and Moth-Flame Optimization algorithms for solving numerical and constrained engineering optimization problems. *Soft Computing, 23*(5), 1699–1722. https://doi.org/10.1007/s00500-017-2894-y

Kirkpatrick, S. (2014). Optimization by Simulated Annealing Optimization by Simulated Annealing. Science, 220(January 1983), 671–680. https://doi.org/10.1142/9789812799371.

Lee, K. S., & Geem, Z. W. (2004). A new structural optimization method based on the harmony search algorithm. *Computers & Structures, 82*(9–10), 781–798. https://doi.org/10.1016/j.compstruc.2004.01.002

Liu, P., & Liu, J. (2017). Multi-leader PSO (MLPSO): A new PSO variant for solving global optimization problems. *Applied Soft Computing, 61*, 256–263. https://doi.org/10.1016/j.asoc.2017.08.022

Mahdavi, S., Shiri, M. E., & Rahnamayan, S. (2015). Metaheuristics in large-scale global continues optimization: A survey. *Information Sciences, 295*, 407–428. https://doi.org/10.1016/j.ins.2014.10.042

Manjarres, D., Landa-Torres, I., Gil-Lopez, S., Del Ser, J., Bilbao, M. N., Salcedo-Sanz, S., & Geem, Z. W. (2013). A survey on applications of the harmony search algorithm. *Engineering Applications of Artificial Intelligence, 26*(8), 1818–1831. https://doi.org/10.1016/j.engappai.2013.05.008

Mezura-Montes, E., & Coello, C. A. C. (2005). A simple multimembered evolution strategy to solve constrained optimization problems. *IEEE Transactions on Evolutionary Computation, 9*(1), 1–17. https://doi.org/10.1109/TEVC.2004.836819

Mirjalili, S. (2015a). The ant lion optimizer. Advances in Engineering Software, 83, 80–98. https://doi.org/https://doi.org/10.1016/j.advengsoft.2015.01.010.

Mirjalili, S. (2015b). Moth-flame optimization algorithm: A novel nature-inspired heuristic paradigm. *Knowledge-Based Systems, 89*, 228–249. https://doi.org/10.1016/j.knosys.2015.07.006

Mirjalili, S. (2016). SCA: A sine cosine algorithm for solving optimization problems. *Knowledge-Based Systems, 96*, 120–133. https://doi.org/10.1016/j.knosys.2015.12.022

Mirjalili, S., Gandomi, A. H., Mirjalili, S. Z., Saremi, S., Faris, H., & Mirjalili, S. M. (2017). Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems. *Advances in Engineering Software, 114*, 163–191. https://doi.org/10.1016/j.advengsoft.2017.07.002

Mirjalili, S., & Lewis, A. (2016). The whale optimization algorithm. *Advances in Engineering Software, 95*, 51–67. https://doi.org/10.1016/j.advengsoft.2016.01.008

Mirjalili, S., Mirjalili, S. M., & Hatamlou, A. (2016). Multi-Verse optimizer: A nature-inspired algorithm for global optimization. *Neural Computing and Applications, 27*(2), 495–513. https://doi.org/10.1007/s00521-015-1870-7

Mirjalili, S., Mirjalili, S. M., & Lewis, A. (2014). Grey wolf optimizer. *Advances in Engineering Software, 69*, 46–61. https://doi.org/10.1016/j.advengsoft.2013.12.007

Moghaddam, F. F., Moghaddam, R. F., & Cheriet, M. (2012). Curved Space Optimization: A Random Search based on General Relativity Theory. ArXiv Preprint ArXiv: 1208.2214. http://arxiv.org/abs/1208.2214.

Moghdani, R., Elaziz, M. A., Mohammadi, D., & Neggaz, N. (2020). An improved volleyball premier league algorithm based on sine cosine algorithm for global optimization problem. *Engineering with Computers*. https://doi.org/10.1007/s00366-020-00962-8

Ouadfel, S., & Abd Elaziz, M. (2020). Enhanced crow search algorithm for feature selection. *Expert Systems with Applications, 159*, 113572. https://doi.org/10.1016/j.eswa.2020.113572

Paillisson, J.-M., & Marion, L. (2001). Interaction between coot (Fulica atra) and waterlily (Nymphaea alba) in a lake: The indirect impact of foraging. *Aquatic Botany, 71*(3), 209–216. https://doi.org/10.1016/S0304-3770(01)00182-6

Randler, C. (2005). Coots Fulica atra reduce their vigilance under increased competition. *Behavioural Processes, 68*(2), 173–178. https://doi.org/10.1016/j.beproc.2004.12.007

Rao, R. V., Savsani, V. J., & Vakharia, D. P. (2011). Teaching–learning-based optimization: A novel method for constrained mechanical design optimization problems. *Computer-Aided Design, 43*(3), 303–315. https://doi.org/10.1016/j.cad.2010.12.015

Rashedi, E., Nezamabadi-pour, H., & Saryazdi, S. (2009). GSA: A gravitational search algorithm. *Information Sciences, 179*(13), 2232–2248. https://doi.org/10.1016/j.ins.2009.03.004

Rechenberg, I. (1973). Evolution strategy: Optimization of technical systems according to the principles of biological evolution. *Frommann-Holzboog, Stuttgart, 104*, 15–16.

Salimi, H. (2015). Stochastic Fractal Search: A powerful metaheuristic algorithm. *Knowledge-Based Systems, 75*, 1–18. https://doi.org/10.1016/j.knosys.2014.07.025

Saremi, S., Mirjalili, S., & Lewis, A. (2017). Grasshopper optimisation algorithm: Theory and application. *Advances in Engineering Software, 105*, 30–47. https://doi.org/10.1016/j.advengsoft.2017.01.004

Savsani, P., & Savsani, V. (2016). Passing vehicle search (PVS): A novel metaheuristic algorithm. *Applied Mathematical Modelling, 40*(5–6), 3951–3978. https://doi.org/10.1016/j.apm.2015.10.040

Sayed, G. I., Darwish, A., & Hassanien, A. E. (2018). A new chaotic multi-verse optimization algorithm for solving engineering optimization problems. *Journal of Experimental & Theoretical Artificial Intelligence, 30*(2), 293–317. https://doi.org/10.1080/0952813X.2018.1430858

Sharma, T. K., & Abraham, A. (2020). Artificial bee colony with enhanced food locations for solving mechanical engineering design problems. *Journal of Ambient Intelligence and Humanized Computing, 11*(1), 267–290. https://doi.org/10.1007/s12652-019-01265-7

Simon, D. (2008). Biogeography-based optimization. *IEEE Transactions on Evolutionary Computation, 12*(6), 702–713. https://doi.org/10.1109/TEVC.2008.919004

Singh, P. R., Elaziz, M. A., & Xiong, S. (2018). Modified Spider Monkey Optimization based on Nelder-Mead method for global optimization. *Expert Systems with Applications, 110*, 264–289. https://doi.org/10.1016/j.eswa.2018.05.040

Sotoudeh-Anvari, A., & Hafezalkotob, A. (2018). A bibliography of metaheuristics-review from 2009 to 2015. *International Journal Of Knowledge-Based And Intelligent Engineering Systems, 22*(1), 83–95.

Spall, J. C. (2005). *Introduction to stochastic search and optimization: estimation, simulation, and control* (Vol. 65). John Wiley & Sons.

Storn, R., & Price, K. (1995). Differential evolution- A simple and efficient adaptive scheme for global optimization over continuous spaces. Technical Report TR-95-012, 11(4), 1–12. https://doi.org/https://doi.org/10.1023/A:1008202821328.

Tanabe, R., & Fukunaga, A. (2013). Success-history based parameter adaptation for Differential Evolution. *IEEE Congress on Evolutionary Computation, 2013*, 71–78. https://doi.org/10.1109/CEC.2013.6557555

Trenchard, H. (2012). American coot collective on-water dynamics. ArXiv.Org, 5929. http://adsabs.harvard.edu/cgi-bin/nph-data_query?bibcode=2012arXiv1205.5929T&link_type=ABSTRACT.

Varo, N., & Amat, J. A. (2008). Differences in foraging behaviour of sympatric coots with different conservation status. *Wildlife Research, 35*(7), 612. https://doi.org/10.1071/WR07179

Wolpert, D. H., & Macready, W. G. (1997). No free lunch theorems for optimization. *IEEE Transactions on Evolutionary Computation, 1*(1), 67–82. https://doi.org/10.1109/4235.585893

Xia, X., Gui, L., He, G., Xie, C., Wei, B.o., Xing, Y., … Tang, Y. (2018). A hybrid optimizer based on firefly algorithm and particle swarm optimization algorithm. *Journal of Computational Science, 26*, 488–500. https://doi.org/10.1016/j.jocs.2017.07.009

Yao, X., Liu, Y., & Lin, G. (1999). Evolutionary programming made faster. *IEEE Transactions on Evolutionary Computation, 3*(2), 82–102. https://doi.org/10.1109/4235.771163

Yang, X.-S. (2010). Firefly Algorithm, Stochastic Test Functions and Design Optimisation. ArXiv Preprint ArXiv:1003.1409. http://arxiv.org/abs/1003.1409.

Yang, X., & Hossein Gandomi, A. (2012). Bat algorithm: A novel approach for global engineering optimization. *Engineering Computations, 29*(5), 464–483. https://doi.org/10.1108/02644401211235834

Yu, C., Heidari, A. A., & Chen, H. (2020). A quantum-behaved simulated annealing algorithm-based moth-flame optimization method. *Applied Mathematical Modelling, 87*, 1–19. https://doi.org/10.1016/j.apm.2020.04.019

Zhang, W., Liu, W., & Ma, J. (2011). Territory and territorial behavior of migrating Common Coot (Fulica atra). *Journal of Forestry Research, 22*(2), 289–294. https://doi.org/10.1007/s11676-011-0164-x

Zhao, W., Wang, L., & Zhang, Z. (2020). Artificial ecosystem-based optimization: A novel nature-inspired meta-heuristic algorithm. *Neural Computing and Applications, 32*(13), 9383–9425. https://doi.org/10.1007/s00521-019-04452-x

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