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Reply to "Comment on a new approximation of Fermi-Dirac integrals of order 1/2 for degenerate semiconductor devices"

There are no solution mistakes or procedural mistakes in our paper [1].

The Corrigendum [2] provides the needed corrections for the inadvertent typographical errors, which do not affect the results and conclusions of our paper [1]. We thank Huang and Xiao [3] for pointing out to us.

Although the values of the constants we have reported in the paper [1] are correct to the decimal numbers as in Ref. [1], full decimal accuracy of the constants up to 14 digits must be used, as we have used in our work. This is also reported in the Corrigendum. Truncating the decimal accuracy impacts the *accuracy* of our approximation, especially for a certain range like $-2 \leq \eta \leq 0$. Fig. 1 shows the impact of the number of decimals used for the constants on the error profiles.

Huang and Xiao [3] is right in pointing to the linear transformation (k-field transformation), and it is the standard Prony's method, however if μ_i are known or selected *a priori*, one can solve for C's [5]. As we have seen from our analysis and can also be seen from equation (21) from Huang and Xiao's Comment [3], there is a fixed relationship or *mapping* between a_i (and hence μ_i) as well as C_i with and without linear transformation. In Fig. 2, we show the results of our approximation and the relative errors profiles with and without the linear ("k-field") transformation. The results are, as expected, almost identical. The difference between relative error profiles with and without linear transformations is of the order of 10^{-7}. We have provided the sets of constants C_i and a_i which we have obtained in these two cases (See Tables 1 and 2). All our constants are real numbers.

In the following we make certain observations and criticisms of Huang and Xiao Comment.

They had made a few mistakes and had made incorrect claims.

(1) Under Section 3.1 they claim to make a shift to "k-field" and state "$\eta(k) = 6/(N-1)*k - 2$ with $\eta(0) = 0$ and $\eta(N-1) = 4$". One can clearly see that when $k = 0$, $\eta(0) \neq 0$. It is $\eta(0) = -2$ as it should be. We have done linear transformations correctly and our results are displayed in Fig. 2. The differences are insignificant (of the order of 10^{-7}).

(2) Huang and Xiao [3] give in equation (15), what they characterize as 'correct' values of α_i and report μ_i values as in equation (16). The roots μ_i found for the polynomial with coefficients α_i as given by Huang and Xiao are easily seen to be incorrect. We show in Fig. 3, Wolfram Alpha screenshot. Thus their solutions, arrived at in equation (19), and their conclusions about our results are incorrect. We show in Fig. 4, a comparison of error profiles resulting from equation (19) of [3] and our results (equation (22) of [1] with the full decimal accuracy). We have a maximum relative error of 5.1% whereas equation (19) gives an error of 37.505%. Our results yield real coefficients and not complex numbers.

Other remarks:

(a) Accuracy, in principle, can be improved, by adopting suitable weighting functions in the Least Squares minimization stage, which we have not done.

(b) Further, based on our Source data of Blakemore [4], we had only one minimum step size of 0.1 and hence we did not vary the step size. Results can vary based on the step size.

(c) The values of $\alpha_i$used by Huang and Xiao from the Thesis [6] also should have the full decimal complement. They are $\alpha_1 = -3.494039322147419$, $\alpha_2 = 4.576627801340779$, $\alpha_3 = -2.663418909991258$, $\alpha_4 = 0.581063087121608$

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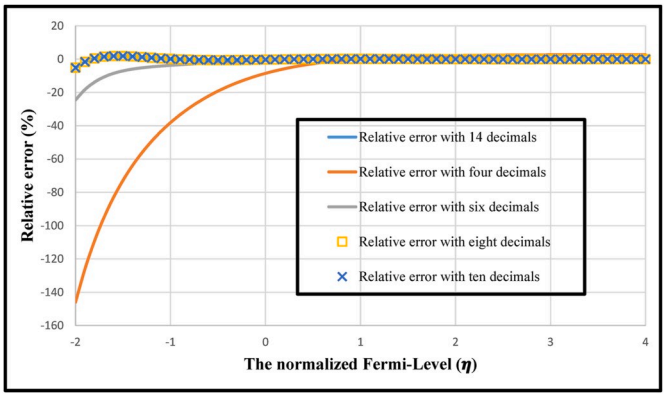

Fig. 1. Relative error profiles of approximated values obtained in Ref. [2,6] to actual values of $\mathscr{F}$ with different numbers of decimals.

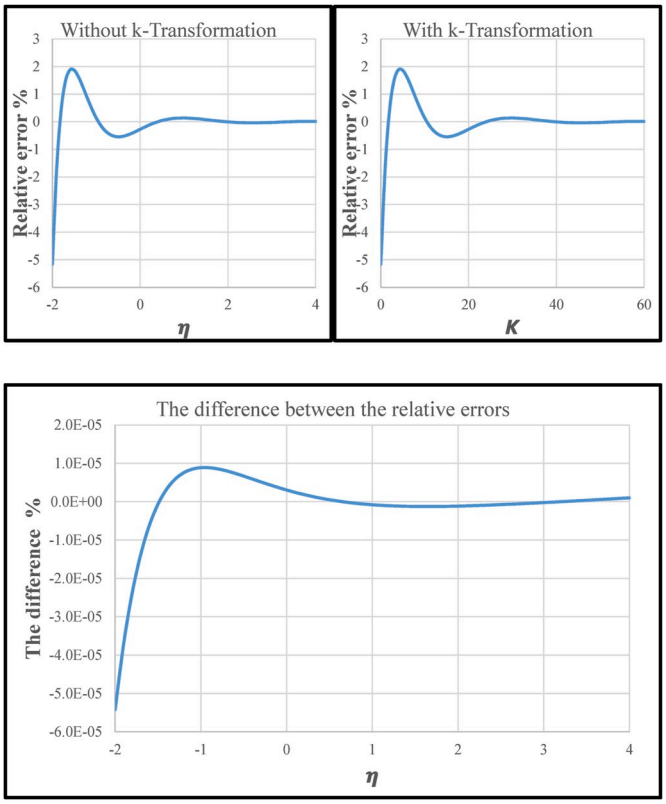

Fig. 2. The relative errors for the calculations with and without k-transformation and the difference between two error profiles.

Table 1
The obtained C's and a's in our paper (i.e. without k-transformation).

C_1	C_2	C_3	C_4
5795.540857	−8358.440387	7038.341888	−4474.679296
a_1	a_2	a_3	a_4
−0.099204871	−0.110982279	−0.15989276	−0.172816032

Table 2
The obtained C's and a's with k-transformation.

C_1	C_2	C_3	C_4
7067.44261461371	−10435.741105219	9690.61906405507	−6322.19794617126
a_1	a_2	a_3	a_4
−0.00992048714594	−0.011098227853958	−0.015989276244864	−0.017281603189133

Fig. 3. The screenshot of obtained values of μ_i using the values of α_i from Eq.15 in Ref. [3]. (Note. The calculations have been done using Alpha Wolfram online tool).

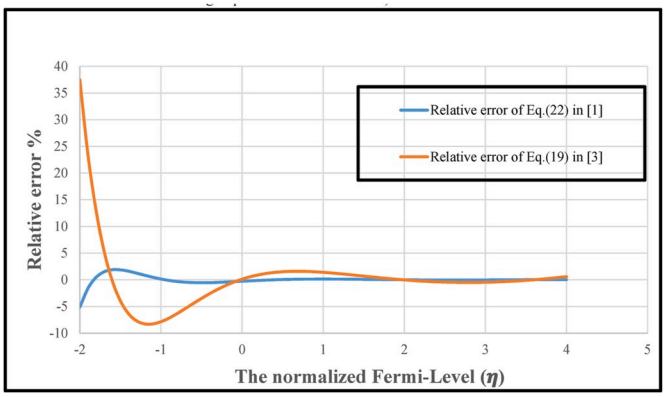

Fig. 4. Relative error profiles of approximated values obtained using Eq.(22) in Ref. [2] and using Eq. (19) in Ref. [3] to actual values of $\mathscr{F}$.

1. Conclusion

Huang and Xiao's claim that they "proved that the developed approximation in Ref. [1] is not true." is completely false. Our approximation is not only valid, but it is, we think, significant in that, for a non-asymptotic numerical approximation, it is holding good with good accuracy even for *twice differentiated functions* matching other FDIs.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.spmi.2020.106568.

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