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# OPTIMIZING COMPLEX FUNCTIONS BY CHAOS SEARCH

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During past decades, the role of optimization has steadily increased in many fields. It is a hot problem in research on control theory. In practice, optimization problems become more and more complex. Traditional algorithms cannot solve them satisfactorily. Either they are trapped to local minima or they need much more search time. Chaos often exists in nonlinear systems. It has many good properties such as ergodicity, stochastic properties, and "regularity." A chaotic motion can go nonrepeatedly through every state in a certain domain. By use of these properties of chaos, an effective optimization method is proposed: the chaos optimization algorithm (COA). With chaos search, some complex optimization problems are solved very well. The test results illustrate that the efficiency of COA is much higher than that of some stochastic algorithms such as the simulated annealing algorithm (SAA) and chemotaxis algorithm (CA), which are often used to optimize complex problems. The chaos optimization method provides a new and efficient way to optimize kinds of complex problems with continuous variables.

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With the development of society and the progress of science, optimization problems have become more and more important and exist almost everywhere, even in our daily life. Today, many optimization problems are very large in scale and complex in nature; more and more new and efficient optimization algorithms are required to meet their needs. Instead of traditional methods, some stochastic optimization algorithms such as SAA, GA, and CA have attracted much more attention. They are widely used in some areas. But there are still some problems. For example, in SAA, there is a trade-off between the quality of the final solution obtained and the execution time required. In GA, the problems of premature convergence and evolving too slowly often exist. CA is more easily trapped to local minima than SAA and GA, and it is not suitable for optimizing discrete problems.

Chaos often exists in nonlinear systems. It is the highly unstable motion of deterministic systems in finite phase space. High instability means that the neighbor traces depart exponentially with time. Because of the instability, the long-time motion of systems shows some confusion and typical stochastic properties. But chaos is not really chaotic, it has exquisite structure.

The theory and applications of modern nonlinear dynamics have achieved many results in areas of mechanics, physics, mathematics, chemistry, biology, medicine, and so on. There are also some research works on the complex movement in nonlinear control systems from eighty decades. Most of the results that have been obtained are about the possibility of some complex movements such as bifurcation and chaos generated in a control system (Chao et al., 1985; Mees, 1986; Ushio et al., 1983). Now there are also some research works on chaos control (Ott et al., 1990; Kapitaniak, 1993; Wu et al., 1995). In fact, chaos has some good properties such as ergodicity, stochastic properties, and "regularity." Figure 1 shows an example of chaos of a nonlinear vibration problem (Zhu, 1995). A chaotic movement can go through every state in a certain area according to its own regularity, and every state is obtained only once. Taking advantage of chaos, a new optimization algorithm is presented that is called the chaos optimization algorithm (COA). COA is not like some stochastic optimization algorithms that escape from local minima by accepting some bad solutions according to a certain probability. It searches on the regularity of chaotic motions. COA can more easily escape from local minima than stochas-

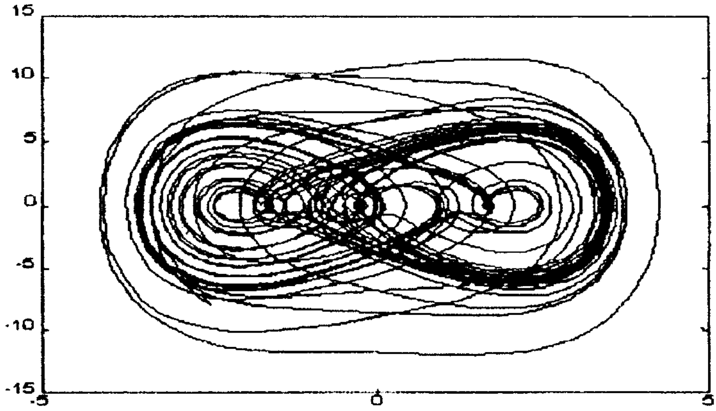


**Figure 1.** The phase plot of a nonlinear vibration problem.

tic optimization algorithms. Another optimization algorithm concerned with chaos, which is different from COA, has also been presented (Chen & Aihara, 1995). It uses a transient chaotic neural network and bifurcation theory for optimization of combinatorial problems.

# THE CHAOS OPTIMIZATION ALGORITHM

## The Chaos System Selected for Optimization

As explained, the long-time behavior of a chaos system shows a kind of "stochastic" property. These systems are very sensitive to their initial conditions. Very small differences in the initial conditions will cause large differences in the long-time behavior of systems.

In the well-known logistic mapping (Lu et al., 1990):

$$x_{n+1} = f(\mu, x_n) = \mu x_n(1 - x_n) \tag{1}$$

where $\mu$ is a control parameter and $n = 0, 1, 2, \ldots$ . Suppose $0 \leqslant x_0 \leqslant 1$, $0 \leqslant \mu \leqslant 4$. It is easy to find that Eq. (1) is a deterministic dynamic system without any stochastic disturbance. It seems that its long-time behavior can be predicted. But that is not true. The behavior of system (1) is greatly changed with the variation of $\mu$. Test results are shown in Figures 2 to 5.

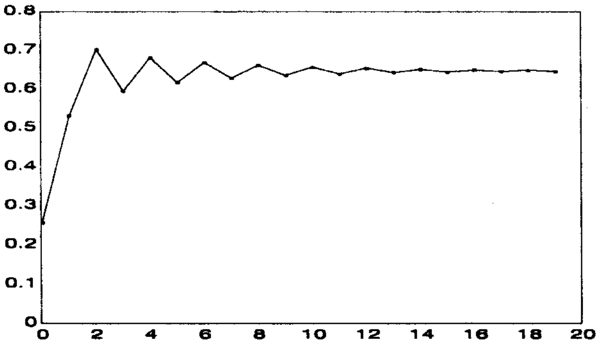


**Figure 2.** The output of system (1) when $\mu = 2.8$.

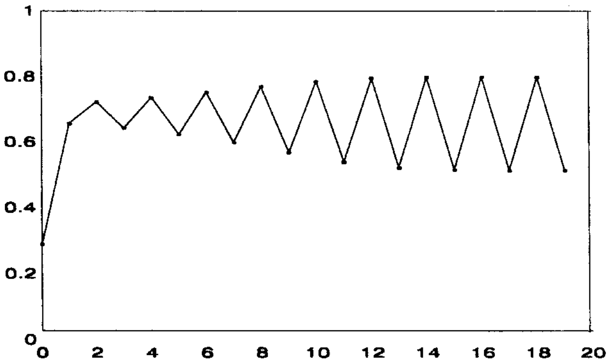


**Figure 3.** The output of system (1) when $\mu = 3.2$.

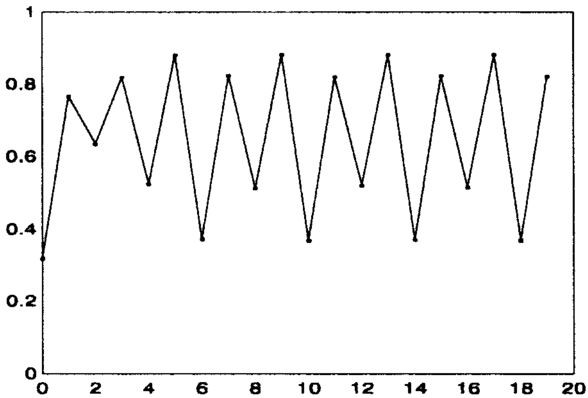


**Figure 4.** The output of system (1) when $\mu = 3.53$.

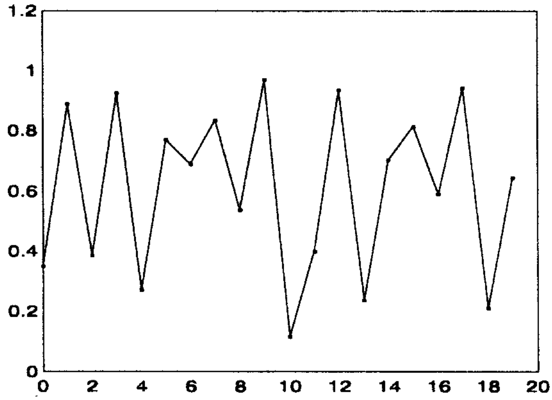


**Figure 5.** The output of system (1) when $\mu = 3.9$.

When $\mu = 4$, system (1) becomes chaotic, and the equation is changed to

$$x_{n+1} = 4x_n(1 - x_n) \tag{2}$$

For $x_0 = 0.1$ and 0.1001, the simulation results are shown in Table 1. When $n > 10$, the initial information of the system is completely lost. Its output is like a stochastic output. Very small difference in the initial value causes large difference in its long-time behavior, which is the basic characteristic of chaos. System (2) is selected for optimization.

## Chaos Optimization Algorithm

If the optimization problems are continuous problems rather than discrete problems (e.g., combinatorial problems) and the constraints of the variables are known, the optimization problems can be described as

$$\min f(x_i) \qquad i = 1, \ldots, n$$

$$\text{s.t. } a_i \leqslant x_i \leqslant b_i \tag{3}$$

Then COA can be used to optimize these problems. First, using the "carrier wave" method, make optimization variables vary to chaos variables. Second, "amplify" the ergodic area of chaotic motion to the variation ranges of every variable, because the chaos system we selected

**Table 1.** Simulation results for system (2)

| Iterative number | Values of $x$ when $x_0$ is 0.1 | Values of $x$ when $x_0$ is 0.1001 |
|---|---|---|
| 1 | 0.36 | 0.36032 |
| 2 | 0.9215999 | 0.921958 |
| 3 | 0.289014 | 0.2878059 |
| 4 | 0.8219397 | 0.8198946 |
| 5 | 0.5854193 | 0.5906698 |
| 6 | 0.9708142 | 0.967116 |
| 7 | 0.1133361 | 0.1272106 |
| 8 | 0.401964 | 0.4441122 |
| 9 | 0.9615558 | 0.9875062 |
| 10 | 0.1478651 | 0.04935078 |
| 11 | 0.5040039 | 0.1876611 |
| 12 | 0.9999359 | 0.6097777 |
| 13 | 2.565219e − 4 | 0.9517955 |
| 14 | 1.025825e − 3 | 0.1835234 |
| 15 | 4.99089e − 3 | 0.5993704 |
| 16 | 1.632915e − 2 | 0.9605021 |
| 17 | 6.425002e − 2 | 0.1577511 |
| 18 | 0.2404878 | 0.5148908 |
| 19 | 0.7306138 | 0.9991131 |
| 20 | 0.7872692 | 3.544522e − 3 |

has a certain ergodic area of 0–1. Finally, use the chaos search method to optimize problems. The main procedures of the algorithm are shown as follows:

Step 1. Generate $i$ chaos variables: Give $i$ initial values which have very small differences to $x(n)$ of Eq. (2); then $i$ chaotic states can be obtained by Eq. (2).

Step. 2. First carrier wave: By the carrier wave method, change $i$ optimization variables to chaos variables. Furthermore, "amplify" the ergodic areas of the $i$ chaotic variables to the variance ranges of optimization variables by Eq. (4):

$$x_i'(n+1) = c_i + d_i x_i(n+1) \tag{4}$$

where $x_i(n+1)$ are $i$ chaotic states generated by Eq. (2), $c_i$ and $d_i$ are constants such as amplification gain. Equation (4) is an algebraic sum, $x_i'(n+1)$ are $i$ chaotic search variables of the optimization problem.

Step 3. "Rough" search: Let $x_i^* = x_{i0}$, and calculate the value of the objective function $f$, $f^* = f$.

Do

$$n = n + 1,$$

$$x_i = x_i'(n + 1)$$

calculate the value of objective function $f$.

If $f \leqslant f^*$ then $f^* = f$, $x_i^* = x_i$.

Else if $f \geqslant f^*$ then give up the solution.

Loop until $f^*$ does not improve after $k$ searches where $k$ is a integer.

Step 4. Second carrier wave: Do the second carrier wave by Eq. (5).

$$x_i''(n + 1) = x_i^* + \alpha x_i(n + 1) \tag{5}$$

where $x_i^*$ is the best solution up to now, $\alpha$ is a constant, and $\alpha x_i(n + 1)$ generates $i$ chaotic states with small ergodic ranges around $x_i^*$.

Step 5. "Fine" search:

Do

$$n = n + 1$$

Let $x_i = x_i''(n + 1)$, and calculating the objective function $f$.

If $f \leqslant f^*$ then $f^* = f$, $x_i^* = x_i''$.

Else if $f \geqslant f^*$ then give up the solution.

Loop until the stop criterion is satisfied.

Step 6. Stop the search process and put out $x^*$, $f^*$ as the best solution.

## Notes:

1. About the initial values of $x_i(n)$: The initial values of $x_i(n)$ used for the carrier wave have very small differences, for example, 0.1, 0.1001, and 0.1002. The very small differences of the initial values make the chaotic search variables vary in different traces, and the search process goes on normally.
2. About the second carrier wave: Although chaotic states have ergodicity in a certain area, it may take a long time to get some states. If the global optimum just appears at those states, it will take a long search

time to get the optimum. We use a second carrier wave to solve this problem. After the first carrier wave, an approximate optimum can be obtained rapidly. It is often in the neighbor domain of the optimum. Then, by the second carrier wave, the search process searches in the small neighbor domain of the approximate optimum, and the global minimum can be obtained.

3. Other chaos systems can also be used to generate chaotic states for the carrier wave besides Eq. (2).

## APPLICATION OF THE CHAOS OPTIMIZATION ALGORITHM

Five complex functions shown in Eqs. (6)–(10) (Goldberg et al., 1989; Potts et al., 1994) have been selected to test the COA. These functions have the following characteristics:

Continuous/ discontinuous
Convex/ nonconvex
Unimodal/ multimodal
Quadratic/ nonquadratic
Low dimensionality/ high dimensionality

Graphs of these functions are shown in Figures 6 to 9. The results obtained with COA are summarized in Table 2. SAGACIA in Table 2 is

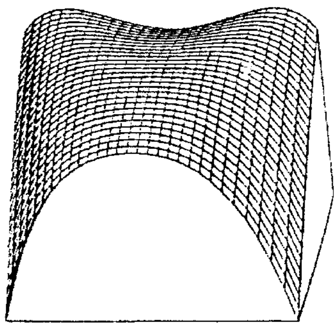

**Figure 6.** The graph of function 1.

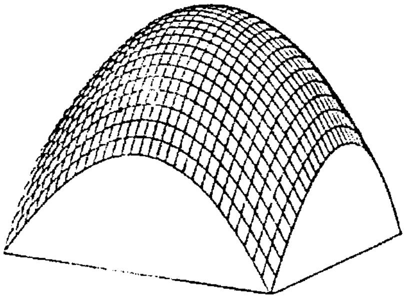

**Figure 7.** The graph of function 2.

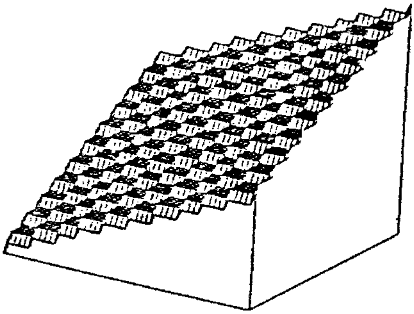

**Figure 8.** The graph of $F_3$.

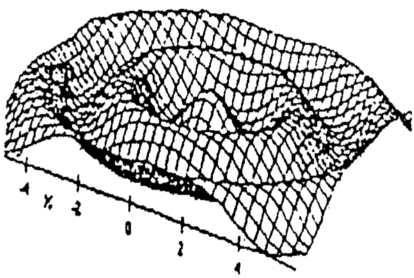

**Figure 9.** The graph of $F_5$.

**Table 2.** Optimization results

| Functions | Optimization algorithms | Ratios of getting the global optimum (%) | Numbers of searched states when the global maximum is obtained |
|---|---|---|---|
| $F_1$ | COA | 100 | 2905 |
| | SAA | 100 | 4530 |
| | CA | 55 | 3154 |
| | SAGACIA | 100 | 1440 |
| $F_2$ | COA | 100 | 758 |
| | SAA | 100 | 7160 |
| | CA | 100 | 5833 |
| | SAGACIA | 100 | 4000 |
| $F_3$ | COA | 100 | 1786 |
| | SAA | 100 | 6310 |
| | CA | 72 | 5122 |
| | SAGACIA | 100 | 2380 |
| $F_4$ | COA | 100 | 1104 |
| | SAA | 100 | 3170 |
| | CA | 87 | 3040 |
| | SAGACIA | 100 | 1130 |
| $F_5$ | COA | 100 | 1092 |
| | SAA | 48 | 6610 |
| | CA | 14 | 500 |
| | SAGACIA | 97.5 | 1140 |

a new optimization algorithm that is based on SAA, GA, and CA (Li, 1996).

$$F_1 = 100(x_1 - x_2) + (1 - x_1)^2 \qquad -2.048 \leqslant x_i \leqslant 2.048 \tag{6}$$

$$F_2 = \sum_1^3 x_i^2 \qquad -5.12 \leqslant x_i \leqslant 5.12 \tag{7}$$

$$F_3 = \sum_1^5 \text{integer}(x_i) \qquad -5.12 \leqslant x_i \leqslant 5.12 \tag{8}$$

$$\begin{aligned} F_4 = {} & \left[1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)\right] \\ & \times \left[30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 \right. \\ & \left. - 36x_1x_2 + 27x_2^2)\right] \qquad -2 \leqslant x_i \leqslant 2 \end{aligned} \tag{9}$$

$$F_5 = 0.5 - \frac{\sin^2\sqrt{x_1^2 + x_2^2} - 0.5}{\left[1 + 0.001(x_1^2 + x_2^2)\right]^2} \qquad -4 \leqslant x_i \leqslant 4 \tag{10}$$

## CONCLUSION

By use of the ergodicity, stochastic properties, and regularity of chaos states, COA is much more powerful than some stochastic optimization algorithms. It has much higher search speed than SAA, CA, and SAGACIA. COA supplies a new and efficient approach for a kind of complex optimization problems.

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