Computers and Structures 139 (2014) 18–27

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Colliding bodies optimization: A novel meta-heuristic method

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ARTICLE INFO

Article history:
Received 4 June 2013
Accepted 16 April 2014
Available online 9 May 2014

Keywords:
Colliding bodies optimization
Meta-heuristic algorithm
Optimal design

ABSTRACT

This paper presents a novel efficient meta-heuristic optimization algorithm called Colliding Bodies Optimization (CBO). This algorithm is based on one-dimensional collisions between bodies, with each agent solution being considered as an object or body with mass. After a collision of two moving bodies having specified masses and velocities, these bodies are separated with new velocities. This collision causes the agents to move toward better positions in the search space. CBO utilizes simple formulation to find minimum or maximum of functions and does not depend on any internal parameter. Numerical results show that CBO is competitive with other meta-heuristics.

1. Introduction

Methods of optimization can be divided into two general categories: 1. Mathematical methods such as quasi-Newton (QN) and dynamic programming (DP) [1]; 2. Meta-heuristic algorithms such as Genetic algorithms (GA) [2], Particle swarm optimization (PSO) [3], Ant colony optimization (ACO) [4], Big bang-big crunch (BB-BC) [5], Charged system search (CSS) [6], Ray optimization (RO) [7], Democratic PSO [8], Dolphin echolocation (DE) [9], Mine blast (MB) [10].

Mathematical algorithms are hard to apply and time-consuming in some optimization problems. Furthermore, they require a good starting point to successfully converge to the optimum and may be trapped in local optima [11].

Meta-heuristic algorithms try to solve optimization problems. The implementation of these algorithms can computationally be performed in a variety of ways. They often have many different variable representations and other settings that must be defined. These include the definition or representation of the solution, mechanisms for changing, developing, or producing new solutions to the problem under study, and methods for evaluating a solution's fitness or efficiency. Once a meta-heuristic algorithm is developed, a tuning process is often required to evaluate different experimental options and settings that can be manipulated by the user in order to optimize convergence behavior in terms of the algorithm's ability to find near optimal solution. A meta-heuristic algorithm is usually tuned for a specific set of problems. However, one of the nice features of efficient meta-heuristic algorithms is their applicability to a wide range of problems [6].

The main goal of this paper is introduce a new and simple optimization algorithm based on the collision between objects, which is called Colliding Bodies Optimization (CBO). The present paper is organized as follows: In Section 2, we describe the laws of collision between two bodies. In Section 3, the new method is presented. Three well-studied engineering design problems and two structural design examples are studied in Section 4. Conclusions are derived in Section 5.

2. The collision between two bodies

Collisions between bodies are governed by the laws of momentum and energy. When a collision occurs in an isolated system (Fig. 1), the total momentum of the system of objects is conserved. Provided that there are no net external forces acting upon the objects, the momentum of all objects before the collision equals the momentum of all objects after the collision.

The conservation of the total momentum demands that the total momentum before the collision is the same as the total momentum after the collision, and can be expressed by the following equation:

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \tag{1}$$

Likewise, the conservation of the total kinetic energy is expressed as:

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 + Q \tag{2}$$

where v_1 is the initial velocity of the first object before impact, v_2 is the initial velocity of the second object before impact, v'_1 is the final

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http://dx.doi.org/10.1016/j.compstruc.2014.04.005
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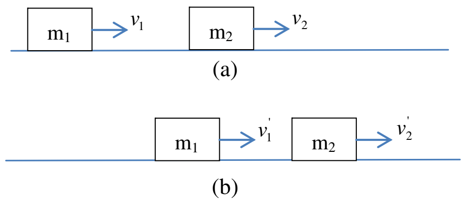

Fig. 1. The collision between two bodies. (a) Before the collision and (b) after the collision.

velocity of the first object after impact, v_2' is the final velocity of the second object after impact, m_1 is the mass of the first object, m_2 is the mass of the second object and Q is the loss of kinetic energy due to the impact [12].

The formulas for the velocities after a one-dimensional collision are:

$$v_1' = \frac{(m_1 - \varepsilon m_2)v_1 + (m_2 + \varepsilon m_2)v_2}{m_1 + m_2} \tag{3}$$

$$v_2' = \frac{(m_2 - \varepsilon m_1)v_2 + (m_1 + \varepsilon m_1)v_1}{m_1 + m_2} \tag{4}$$

where ε is the Coefficient Of Restitution (COR) of the two colliding bodies, defined as the ratio of relative velocity of separation to relative velocity of approach:

$$\varepsilon = \frac{|v_2' - v_1'|}{|v_2 - v_1|} = \frac{v'}{v} \tag{5}$$

According to the coefficient of restitution, there are two special cases of any collision as follows:

(1) A perfectly elastic collision is defined as the one in which there is no loss of kinetic energy in the collision ($Q = 0$ and $\varepsilon = 1$). In reality, any macroscopic collision between objects will convert some kinetic energy to internal energy and other forms of energy. In this case, after collision, the velocity of separation is high.
(2) An inelastic collision is the one in which part of the kinetic energy is changed to some other form of energy in the collision. Momentum is conserved in inelastic collisions (as it is for elastic collisions), but one cannot track the kinetic energy through the collision since some of it will be converted to other forms of energy. In this case, coefficient of restitution does not equal to one ($Q \neq 0$ & $\varepsilon \leqslant 1$). In this case, after collision the velocity of separation is low.

For the most real objects, the value of ε is between 0 and 1.

3. The CBO algorithm

3.1. Theory

The main objective of the present study is to formulate a new simple and efficient meta-heuristic algorithm which is called Colliding Bodies Optimization (CBO). In CBO, each solution candidate X_i containing a number of variables (i.e. $X_i = \{X_{i,j}\}$) is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e. stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects and (ii) to push stationary objects towards better positions. After the collision, new positions of colliding bodies are updated based on new velocity by using the collision laws as discussed in Section 2.

The CBO procedure can briefly be outlined as follows:

1. The initial positions of CBs are determined with random initialization of a population of individuals in the search space:

$$x_i^0 = x_{\min} + rand(x_{\max} - x_{\min}), \quad i = 1, 2, \ldots, n, \tag{6}$$

where x_i^0 determines the initial value vector of the ith CB. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable values vectors of variables; *rand* is a random number in the interval [0, 1]; and n is the number of CBs.

2. The magnitude of the body mass for each CB is defined as:

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \quad k = 1, 2, \ldots, n \tag{7}$$

where $fit(i)$ represents the objective function value of the agent i; n is the population size. It seems that a CB with good values exerts a larger mass than the bad ones. Also, for maximization, the objective function $fit(i)$ will be replaced by $\frac{1}{fit(i)}$.

3. The arrangement of the CBs objective function values is performed in ascending order (Fig. 2a). The sorted CBs are equally divided into two groups:
   - The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus:

$$v_i = 0, \quad i = 1, \ldots, \frac{n}{2} \tag{8}$$

   - The upper half of CBs (moving CBs): These CBs move toward the lower half. Then, according to Fig. 2b, the better and worse CBs, i.e. agents with upper fitness value, of each group will collide together. The change of the body position represents the velocity of these bodies before collision as:

$$v_i = x_i - x_{i-\frac{n}{2}}, \quad i = \tfrac{n}{2} + 1, \ldots, n \tag{9}$$

Where, v_i and x_i are the velocity and position vector of the ith CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.

4. After the collision, the velocities of the colliding bodies in each group are evaluated utilizing Eqs. (3) and (4), and the velocity before collision. The velocity of each moving CBs after the collision is obtained by:

$$v_i' = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}}, \quad i = \frac{n}{2} + 1, \ldots, n \tag{10}$$

where v_i and v_i' are the velocity of the ith moving CB before and after the collision, respectively; m_i is mass of the ith CB; $m_{i-\frac{n}{2}}$ is mass

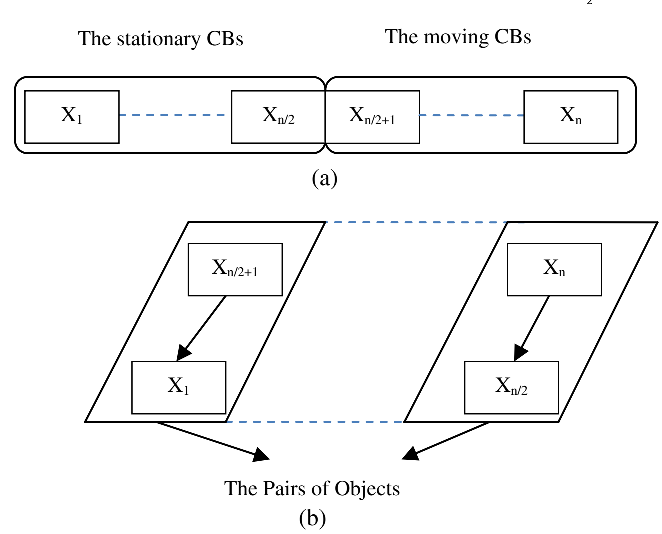

Fig. 2. (a) CBs sorted in increasing order and (b) colliding object pairs.

of the ith CB pair. Also, the velocity of each stationary CB after the collision is:

$$v'_i = \frac{\left(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}}\right) v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}}, \quad i = 1, \ldots, \frac{n}{2} \tag{11}$$

where $v_{i+\frac{n}{2}}$ and v'_i are the velocity of the ith moving CB pair before and the ith stationary CB after the collision, respectively; m_i is mass of the ith CB; $m_{i+\frac{n}{2}}$ is mass of the ith moving CB pair; ε is the value of the COR parameter whose law of variation will be discussed in the next section.

5. New positions of CBs are evaluated using the generated velocities after the collision in position of stationary CBs.
   The new positions of each moving CB is:

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v'_i, \quad i = \frac{n}{2} + 1, \ldots, n \tag{12}$$

where x_i^{new} and v'_i are the new position and the velocity after the collision of the ith moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of ith stationary CB pair. Also, the new positions of stationary CBs are obtained by:

$$x_i^{new} = x_i + rand \circ v'_i, \quad i = 1, \ldots, \frac{n}{2} \tag{13}$$

where x_i^{new}, x_i and v'_i are the new position, old position and the velocity after the collision of the ith stationary CB, respectively. *rand* is a random vector uniformly distributed in the range (−1, 1) and the sign "∘" denotes an element-by-element multiplication.

6. The optimization is repeated from Step 2 until a termination criterion, such as maximum iteration number, is satisfied. It should be noted that, a body's status (stationary or moving body) and its numbering are changed in two subsequent iterations.

Apart from the efficiency of the CBO algorithm, which is illustrated in the next section through numerical examples, parameter independency is an important feature that makes CBO superior over other meta-heuristic algorithms. Also, the formulation of CBO algorithm does not use the memory which saves the best-so-far solution (i.e. the best position of agents from the previous iterations).

The penalty function approach was used for constraint handling. The *fit*(i) function corresponds to the effective cost. If optimization constraints are satisfied, there is no penalty; otherwise the value of penalty is calculated as the ratio between the violation and the allowable limit.

3.2. The coefficient of restitution (COR)

The meta-heuristic algorithms have two phases: exploration of the search space and exploitation of the best solutions found. In the meta-heuristic algorithm it is very important to have a suitable balance between the exploration and exploitation [6]. In the optimization process, the exploration should be decreased gradually while simultaneously exploitation should be increased.

In this paper, an index is introduced in terms of the coefficient of restitution (COR) to control exploration and exploitation rate. In fact, this index is defined as the ratio of the separation velocity of two agents after collision to approach velocity of two agents before collision. Efficiency of this index will be shown using one numerical example.

In this section, in order to have a general idea about the performance of COR in controlling local and global searches, a benchmark function (Aluffi-Pentiny) chosen from Ref. [13] is optimized using the CBO algorithm. Three variants of COR values are considered. Fig. 3 is prepared to show the positions of the current CBs in 1st, 50th and 100th iteration for these cases. These three typical cases result in the following:

1. The perfectly elastic collision: In this case, COR is set equal to unity. It can be seen that in the final iterations, the CBs investigate the entire search space to discover a favorite space (global search).
2. The hypothetical collision: In this case, COR is set equal to zero. It can be seen that in the 50th iterations, the movements of the CBs are limited to very small space in order to provide exploitation (local search). Consequently, the CBs are gathered in a small region of the search space.
3. The inelastic collision: In this case, COR decreases linearly to zero and ε is defined as:

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{14}$$

where *iter* is the actual iteration number and $iter_{max}$ is the maximum number of iterations. It can be seen that the CBs get closer by increasing iteration. In this way a good balance between the global and local search is achieved. Therefore, in the optimization process COR is considered such as the above equation.

4. Test problems and optimization results

Three well-studied engineering design problems and two structural design problems taken from the optimization literature are used to investigate the efficiency of the proposed approach. These examples have been previously studied using a variety of other techniques, which are useful to show the validity and effectiveness of the proposed algorithm. In order to assess the effect of the initial population on the final result, these examples are independently optimized with different initial populations.

For engineering design examples, 30 independent runs were performed for CBO, considering 20 individuals and 200 iterations; the corresponding number of function evaluations is 4000. The number of function evaluations set for the GA-based algorithm developed by Coello [16], the PSO-based method developed by He and Wang [18], the evolution strategies developed by Montes and Coello [19] is 900,000, 200,000 and 25,000, respectively. Similar to CBO, the number of function evaluations for the charged system search algorithm developed by Kaveh and Talatahari [6] is 4000.

In the truss design problems, 20 independent runs were carried out, considering 40 individuals and 400 iterations: hence, the maximum number of structural analyses was 16,000. The CBO algorithm was coded in MATLAB. Structural analysis was performed with the direct stiffness method.

4.1. Test problem 1: design of welded beam

As the first example, design optimization of the welded beam shown in Fig. 4 is carried out. The welded beam design problem was often utilized to evaluate performance of different optimization methods [6,14–19]. The objective is to find the best set of design variables to minimize the total fabrication cost of the structure subject to shear stress (τ), bending stress (σ), buckling load (Pc), and end deflection (δ) constraints. Assuming $x_1 = h$, $x_2 = l$, $x_3 = t$, and $x_4 = b$ as the design variables, the mathematical formulation of the problem can be expressed as:

Find

$$\{x_1, x_2, x_3, x_4\} \tag{15}$$

To minimize

$$\cos t(x) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14 + x_2) \tag{16}$$

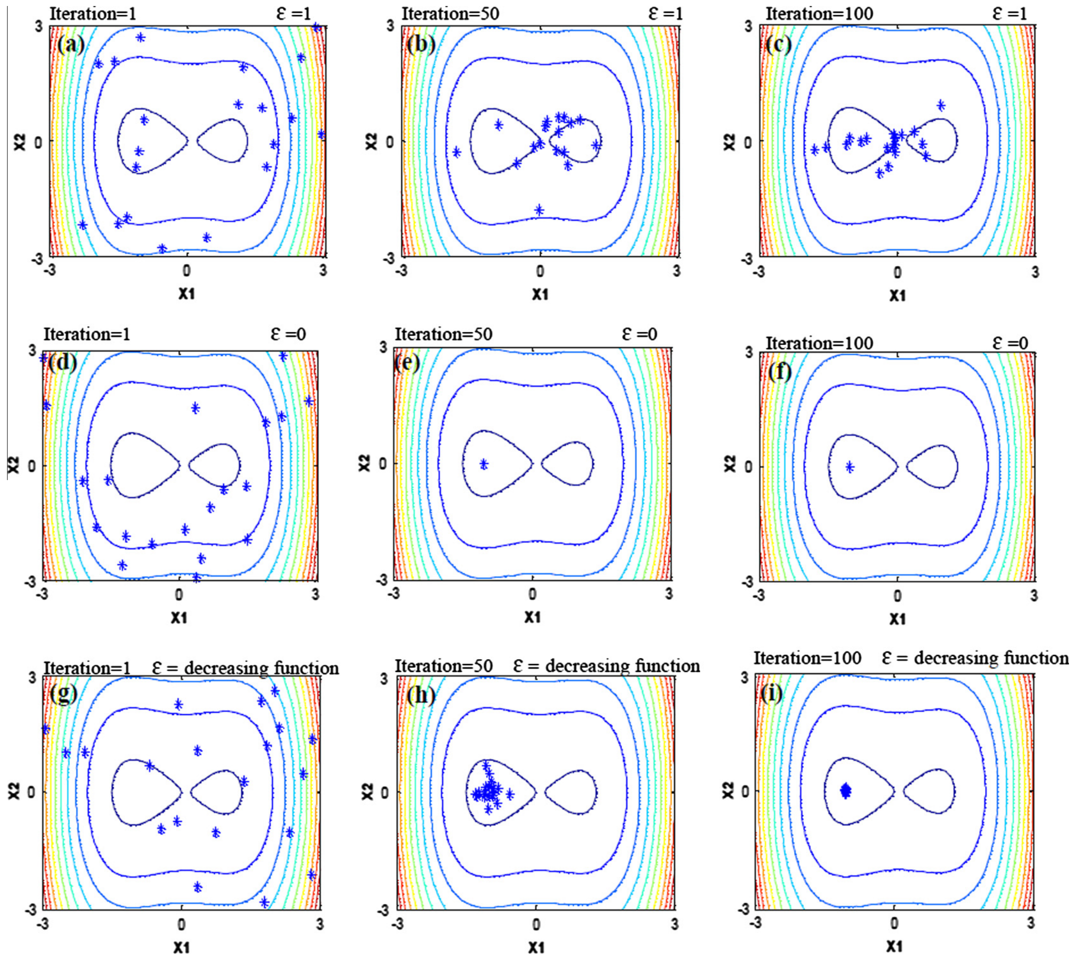

Fig. 3. Evolution of the positions of CBs during optimization history for different definitions of the coefficient of restitution (Aluffi-Pentiny benchmark function).

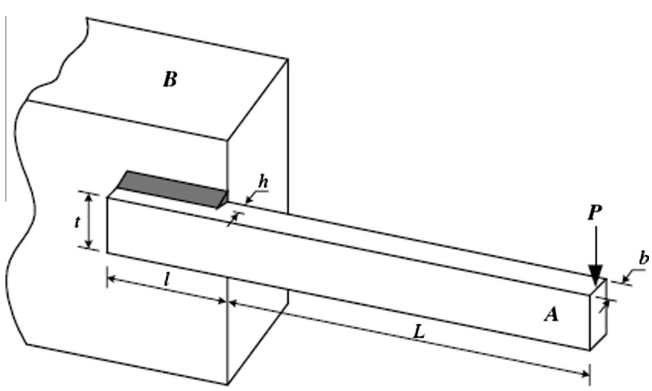

Fig. 4. Schematic of the welded beam structure with indication of design variables.

Subjected to

$$\begin{aligned}
&g_1(x) = \tau(x) - \tau_{\max} \leqslant 0 \\
&g_2(x) = \sigma(x) - \sigma_{\max} \leqslant 0 \\
&g_3(x) = x_1 - x_4 \leqslant 0 \\
&g_4(x) = 0.10471x_1^2 + 0.04811x_3x_4(14 + x_2) - 5 \leqslant 0 \\
&g_5(x) = 0.125 - x_1 \leqslant 0 \\
&g_6(x) = \delta(x) - \delta_{\max} \leqslant 0 \\
&g_7(x) = p - p_c(x) \leqslant 0
\end{aligned} \tag{17}$$

The bounds on the design variables are:

$$0.1 \leqslant x_1 \leqslant 2, \quad 0.1 \leqslant x_2 \leqslant 10, \quad 0.1 \leqslant x_3 \leqslant 10, \quad 0.1 \leqslant x_4 \leqslant 2 \tag{18}$$

where

$$\begin{aligned}
&\tau(x) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \\
&\tau' = \frac{P}{\sqrt{2}x_1x_2} \quad \tau'' = \frac{MR}{J} \quad M = P\left(L + \frac{x_2}{2}\right) \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \\
&J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\} \quad \sigma(x) = \frac{6PL}{x_4x_3^2} \quad \delta(x) = \frac{4PL^3}{Ex_3^3x_4} \\
&P_c(x) = \frac{4.013\sqrt{E(x_3^2x_4^6/36)}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)
\end{aligned} \tag{19}$$

The constants in Eqs. (17) and (19) are chosen as follows: $P = 6000$ lb, $L = 14$ in., $E = 30 \times 10^6$ psi, $G = 12 \times 10^6$ psi, $\tau_{\max} = 13{,}600$ psi, $\sigma_{\max} = 30{,}000$ psi, and $\delta_{\max} = 0.25$ in.

Radgsdell and Phillips [14] compared optimal results of different optimization methods which were mainly based on mathematical optimization algorithms. Deb [15], Coello [16] and Coello and Montes [17] solved this problem using GA-based methods. Also, He and Wang [18] used effective co-evolutionary particle swarm optimization, Montes and Coello [19] solved this problem utilizing evolution strategies, and Kaveh and Talatahari [6] employed charged system search.

Table 1 compares the optimized design and the corresponding cost obtained by CBO with those obtained by other meta-heuristic algorithms documented in literature. It can be seen that the best solution obtained by CBO is better than those quoted for the other algorithms. The statistical data on 30 independent runs reported in Table 2 also demonstrate the better search ability of CBO with respect to the other algorithms: in fact the best, worst and average cost, and the standard deviation (S.D.) of the obtained solutions are better than literature. The lowest standard deviation achieved by CBO proves that the present algorithm is more robust than other meta-heuristic methods.

4.2. Test problem 2: design of a pressure vessel

Design optimization of the cylindrical pressure vessel capped at both ends by hemispherical heads (Fig. 5) is considered as the second example. The objective of optimization is to minimize the total manufacturing cost of the vessel based on the combination of welding, material and forming costs. The vessel is designed for a working pressure of 3000 psi and a minimum volume of 750 ft^3 regarding the provisions of *ASME* boiler and pressure vessel code. Here, the shell and head thicknesses should be multiples of 0.0625 in. The thickness of the shell and head is restricted to 2 in. The shell and head thicknesses must not be less than 1.1 in. and 0.6 in, respectively. The design variables of the problem are x_1 as the shell thickness (T_s), x_2 as the spherical head thickness (T_h), x_3 as the radius of cylindrical shell (R), and x_4 as the shell length (L). The problem formulation is as follows:

Find

$$\{x_1, x_2, x_3, x_4\} \tag{20}$$

Table 3
Comparison of CBO optimized designs with literature for the pressure vessel problem.

Methods	X_1 (T_s)	X_2 (T_h)	X_3 (R)	X_4 (L)
Sandgren [20]	1.125000	0.625000	47.70000	117.7010
Kannan and Kramer [21]	1.125000	0.625000	58.29100	43.6900
Deb and Gene [22]	0.937500	0.500000	48.32900	112.6790
Coello [16]	0.812500	0.437500	40.32390	200.0000
Coello and Montes [17]	0.812500	0.437500	42.09739	176.6540
He and Wang [18]	0.812500	0.437500	42.09126	176.7465
Montes and Coello [19]	0.812500	0.437500	42.09808	176.6405
Kaveh and Talatahari [6]	0.812500	0.812500	0.812500	176.572656
Present work	0.779946	0.385560	40.409065	198.76232

Table 1
Comparison of CBO optimized designs with literature for the welded beam problem.

Design variables	Best solution found							
	Ragsdell and Phillips [14]	Deb [15]	Coello [16]	Coello and Montes [17]	He and Wang [18]	Montes and Coello [19]	Kaveh and Talatahari [6]	Present work
$x_1(h)$	0.245500	0.248900	0.208800	0.205986	0.202369	0.202369	0.20582	0.205722
$x_2(l)$	6.1960000	6.173000	3.420500	3.471328	3.544214	3.544214	3.468109	3.47041
$x_3(t)$	8.273000	8.178900	8.997500	9.020224	9.04821	9.04821	9.038024	9.037276
$x_4(b)$	0.245500	0.253300	0.210000	0.20648	0.205723	0.205723	0.205723	0.205735
$f(x)$	1.728024	2.433116	−1.748310	1.728226	1.728024	1.728024	1.724866	1.724663

Table 2
Statistical results from different optimization methods for the welded beam design problem.

Methods	Best result	Average optimized cost	Worst result	Std dev.
Ragsdell and Phillips [14]	2.385937	N/A	N/A	N/A
Deb [15]	2.433116	N/A	N/A	N/A
Coello [16]	1.748309	1.771973	1.785835	0.011220
Coello and Montes [17]	1.728226	1.792654	1.993408	0.074713
He and Wang [18]	1.728024	1.748831	1.782143	0.012926
Montes and Coello [19]	1.737300	1.813290	1.994651	0.070500
Kaveh and Talatahari [6]	1.724866	1.739654	1.759479	0.008064
Present work	1.724662	1.725707	1.725059	0.0002437

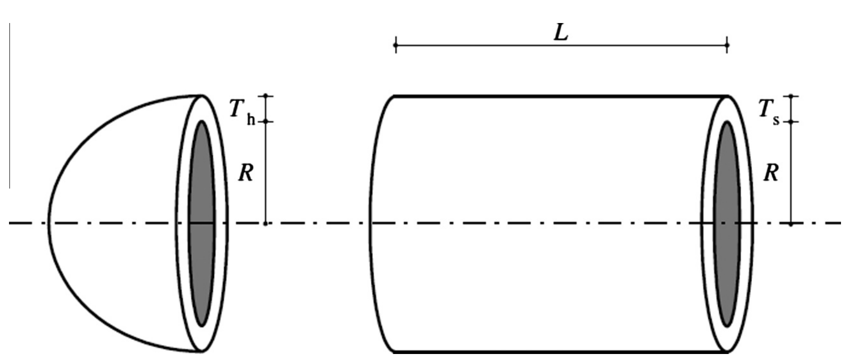

Fig. 5. Schematic of the spherical head and cylindrical wall of the pressure vessel with indication of design variables.

Table 4
Statistical results from different optimization methods for the pressure vessel problem.

Methods	Best result	Average optimized cost	Worst result	Std dev.
Sandgren [20]	8129.103	N/A	N/A	N/A
Kannan and Kramer [21]	7198.042	N/A	N/A	N/A
Deb and Gene [22]	6410.381	N/A	N/A	N/A
Coello [16]	6288.744	6293.843	6308.149	7.4133
Coello and Montes [17]	6,059.946	6177.253	6469.322	130.9297
He and Wang [18]	6061.077	6147.133	6363.804	86.4545
Montes and Coello [19]	6059.745	6850.004	7,332.879	426.0000
Kaveh and Talatahari [6]	6059.088	6067.906	6085.476	10.256
Present work	5889.911	5934.201	6213.006	63.5417

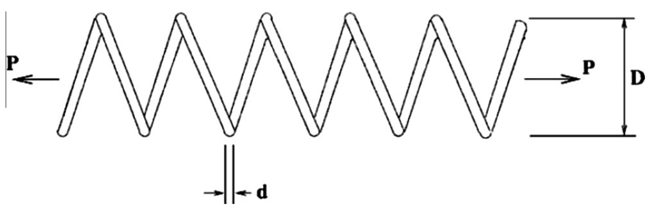

Fig. 6. Schematic of the tension/compression spring with indication of design variables.

To minimize

$$\cos t(x) = 0.6224x_3x_1x_4 + 1.7781x_3^2x_2 + 3.1611x_1^2x_4 + 19.8621x_3x_1^2 \quad (21)$$

Subject to

$$\begin{aligned} g_1(x) &= 0.0193x_3 - x_1 \leqslant 0 \\ g_2(x) &= 0.00954x_3 - x_2 \leqslant 0 \\ g_3(x) &= 750 \times 1728 - \pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 \leqslant 0 \\ g_4(x) &= x_4 - 240 \leqslant 0 \end{aligned} \quad (22)$$

The bounds on the design variables are:

$$1.125 \leqslant x_1 \leqslant 2, \quad 0.625 \leqslant x_2 \leqslant 2, \quad 10 \leqslant x_3 \leqslant 240, \quad 10 \leqslant x_4 \leqslant 240 \quad (23)$$

It can be seen from Table 3 that the present algorithm found the best design overall which is about 3% lighter than the best known design quoted in literature (5889.911 vs. 6059.088 of Ref. [6]). The statistical data reported in Table 4 indicate that the standard deviation of CBO optimized solutions is the third lowest among those quoted for the different algorithms compared in this test case. Statistical results given in Table 4 indicate that CBO is in general more robust than the other meta-heuristic algorithms. However, the worst optimized design and standard deviation found by CBO are higher than for CSS.

4.3. Test problem 3: design of a tension/compression spring

This problem was first described by Belegundu [23] and Arora [24]. It consists of minimizing the weight of a tension/compression spring subject to constraints on shear stress, surge frequency, and minimum deflection as shown in Fig. 6. The design variables are the wire diameter d (=x_1);the mean coil diameter D (=x_2), and the number of active coils N (=x_3). The problem can be stated as follows:

Find

$$\{x_1, x_2, x_3\} \quad (24)$$

To minimize

$$\cos t(x) = (x_3 + 2)x_2x_1^2 \quad (25)$$

Table 5
Comparison of CBO optimized designs with literature for the tension/compression spring problem.

Methods	Optimal design variables			Constraints				$f(x)$
	$x_1(d)$	$x_2(D)$	$x_3(N)$	$g_1(x)$	$g_2(x)$	$g_3(x)$	$g_4(x)$	
Belegundu [23]	0.050000	0.315900	14.250000	−0.000014	−0.003782	−3.938302	−0.756067	0.0128334
Arora [24]	0.053396	0.399180	9.185400	−0.053396	−0.000018	−4.123832	−0.698283	0.0127303
Coello [16]	0.051480	0.351661	11.632201	−0.002080	−0.000110	−4.026318		0.0127048
Coello and Montes [17]	0.051989	0.363965	10.890522	−0.000013	−0.000021	−4.061338	−0.722698	0.0126810
He and Wang [18]	0.051728	0.357644	11.244543	−0.000845	−1.2600e−05	−4.051300	−0.727090	0.0126747
Montes and Coello [19]	0.051643	0.355360	11.397926	−0.001732	−0.0000567	−4.039301	−0.728664	0.012698
Kaveh and Talatahari [6]	0.051744	0.358532	11.165704	8.78603e−6	0.0011043	−4.063371	−0.726483	0.0126384
Present work	0.051894	0.3616740	11.007846	−3.1073e−4	−1.4189e−5	−4.061846	−0.724287	0.0126697

Table 6
Statistical results from different optimization methods for tension/compression string problem.

Methods	Best result	Average optimized cost	Worst result	Std dev.
Belegundu [23]	0.0128334	N/A	N/A	N/A
Arora [24]	0.0127303	N/A	N/A	N/A
Coello [16]	0.0127048	0.012769	0.012822	3.9390e−5
Coello and Montes [17]	0.0126810	0.0127420	0.012973	5.9000e−5
He and Wang [18]	0.0126747	0.012730	0.012924	5.1985e−5
Montes and Coello [19]	0.012698	0.013461	0.16485	9.6600e−4
Kaveh and Talatahari [6]	0.0126384	0.012852	0.013626	8.3564e−5
Present work	0.126697	0.1272964	0.128808	5.00376e−5

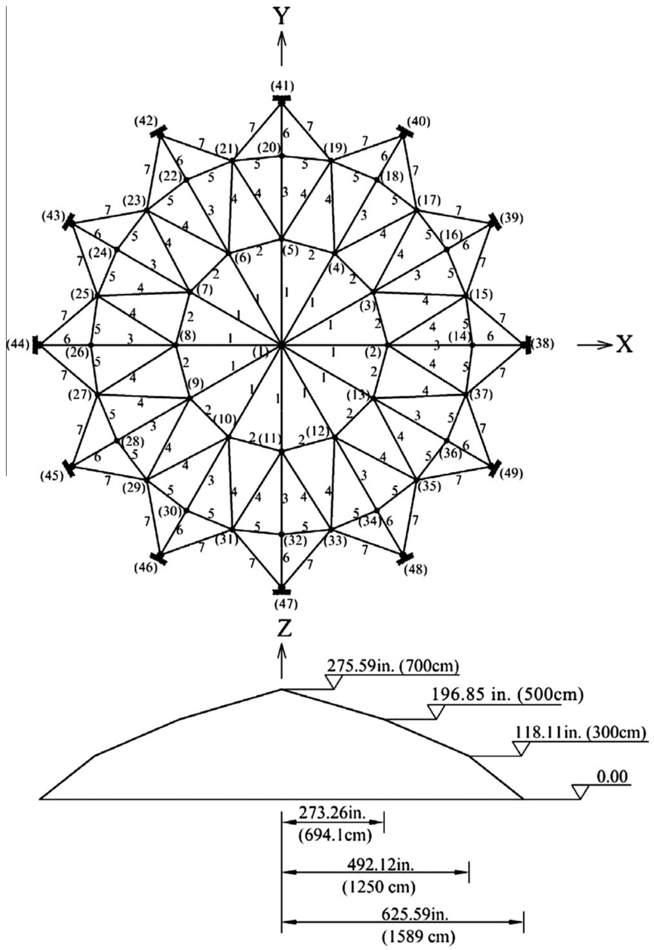

Fig. 7. Schematic of the spatial 120-bar dome truss with indication of design variables and main geometric dimensions.

Subject to

$$
\begin{aligned}
g_1(x) &= 1 - \frac{x_2^3 x_3}{71785 x_1^4} \leqslant 0 \\
g_2(x) &= \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \leqslant 0 \\
g_3(x) &= 1 - \frac{140.45 x_1}{x_2^2 x_3} \leqslant 0 \\
g_4(x) &= \frac{x_1 + x_2}{1.5} - 1 \leqslant 0
\end{aligned} \tag{26}
$$

The bounds on the design variables are:

$$0.05 \leqslant x_1 \leqslant 2, \quad 0.25 \leqslant x_2 \leqslant 1.3, \quad 2 \leqslant x_3 \leqslant 15, \tag{27}$$

This problem has been solved by Belegundu [23] using eight different mathematical optimization techniques. Arora [24] also solved this problem using a numerical optimization technique called a constraint correction at the constant cost. Coello [16] as well as Coello and Montes [17] solved this problem using GA-based method. Additionally, He and Wang [18] utilized a co-evolutionary particle swarm optimization (CPSO). Recently, Montes and Coello [19], Kaveh and Talatahari [6] used evolution strategies and the CSS to solve this problem, respectively.

Tables 5 and 6 compare the best results obtained in this paper and those of the other researches. Once again, CBO found the best design overall. In fact, the lighter design found by Kaveh and Talatahari in [6] actually violates the first two optimization constraints. The statistical data reported in Table 6 show that the standard deviation on optimized cost seen for CBO is fully consistent with literature.

4.4. Test problem 4: weight minimization of the 120-bar truss dome

The fourth test case solved in this study is the weight minimization problem of the 120-bar truss dome shown in Fig. 7. This test case was investigated by Soh and Yang [25] as a configuration optimization problem. It has been solved later as a sizing optimization

Table 7
Comparison of CBO optimized designs with literature in the 120-bar dome problem.

Element group	Optimal cross-sectional areas (in^2)							
	Case 1				Case 2			
	Ref.[26] HS	Ref. [27] HPSACO	Ref. [7] RO	Present work	Ref.[26] HS	Ref. [27] HPSACO	Ref. [7] RO	Present work
1	3.295	3.311	3.128	3.1229	3.296	3.779	3.084	3.0832
2	3.396	3.438	3.357	3.3538	2.789	3.377	3.360	3.3526
3	3.874	4.147	4.114	4.1120	3.872	4.125	4.093	4.0928
4	2.571	2.831	2.783	2.7822	2.570	2.734	2.762	2.7613
5	1.150	0.775	0.775	0.7750	1.149	1.609	1.593	1.5918
6	3.331	3.474	3.302	3.3005	3.331	3.533	3.294	3.2927
7	2.784	2.551	2.453	2.4458	2.781	2.539	2.434	2.4336
Best weight (Ib)	19707.77	19491.3	19476.193	19454.7	19893.34	20078.0	20071.9	20064.5
Average weight (Ib)	–	–	–	19466.0		–	–	20098.3
Std (Ib)	–	–	33.966	7.02		–	112.135	26.17

	Case 3			Case 4			
	Ref. [27] HPSACO	Ref. [7] RO	Present work	Ref. [27] HPSACO	Ref. [7] RO	Ref.[28] IRO	Present work
1	2.034	2.044	2.0660	3.095	3.030	3.0252	3.0273
2	15.151	15.665	15.9200	14.405	14.806	14.8354	15.1724
3	5.901	5.848	5.6785	5.020	5.440	5.1139	5.2342
4	2.254	2.290	2.2987	3.352	3.124	3.1305	3.119
5	9.369	9.001	9.0581	8.631	8.021	8.4037	8.1038
6	3.744	3.673	3.6365	3.432	3.614	3.3315	3.4166
7	2.104	1.971	1.9320	2.499	2.487	2.4968	2.4918
Best weight (Ib)	31670.0	31733.2	31724.1	33248.9	33317.8	33256.48	33286.3
Average weight (Ib)	–	–	32162.4	–	–	–	33398.5
Std (Ib)	–	274.991	240.22	–	354.333	–	67.09

problem by Lee and Geem [26], Kaveh and Talatahari [27], Kaveh and Ilchi [28], and Kaveh and Khayatazad [7].

The allowable tensile and compressive stresses are set according to the AISC ASD (1989) [29] code, as follows:

$$\begin{cases} \sigma_i^+ = 0.6F_y & \text{for } \sigma_i \geqslant 0 \\ \sigma_i^- & \text{for } \sigma_i \leqslant 0 \end{cases} \tag{28}$$

where σ_i^- is calculated according to the slenderness ratio

$$\sigma_i^- = \begin{cases} \left[\left(1 - \frac{\lambda_i^2}{2C_c^2}\right)F_y\right] \Big/ \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3}\right) & \text{for } \lambda_i < C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geqslant C_c \end{cases} \tag{29}$$

where E is the modulus of elasticity, F_y is the yield stress of steel, C_c is the slenderness ratio (λ_i) dividing the elastic and inelastic buckling regions ($C_c = \sqrt{2\pi^2 E/F_y}$), λ_i is the slenderness ratio ($\lambda_i = \frac{KL_i}{r_i}$), K is the effective length factor, L_i is the member length and r_i is the radius of gyration.

The modulus of elasticity is 30,450 ksi (210,000 MPa) and the material density is 0.288 lb/in^3 (7971.810 kg/m^3). The yield stress of steel is taken as 58.0 ksi (400 MPa). On the other hand, the radius of gyration (r_i) is expressed in terms of cross-sectional areas as $r_i = aA_i^b$ [30]. Here, a and b are constants depending on the types of sections adopted for the members such as pipes, angles, and tees. In this example, pipe sections (a = 0.4993 and b = 0.6777) are adopted for bars [31]. All members of the dome are divided into seven groups, as shown in Fig. 7. The dome is considered to be subjected to vertical loads at all the unsupported joints. These are taken as −13.49 kips (60 kN) at node 1, −6.744 kips (30 kN) at nodes 2 through 14, and −2.248 kips (10 kN) at the remaining of the nodes. The minimum cross-sectional area of elements is 0.775 in^2 (5 cm^2). In this example, four cases of constraints are considered: with stress constraints and no displacement constraints (Case 1), with stress constraints and displacement limitations of ±0.1969 in (5 mm) imposed on all nodes in x- and y-directions (Case 2), no stress constraints but displacement limitations of ±0.1969 in (5 mm) imposed on all nodes in z-directions (Case 3), and all constraints explained above (Case 4). For Case 1 and Case 2, the maximum cross-sectional area is 5.0 in^2 (32.26 cm^2) while for Case 3 and Case 4 is 20.0 in^2 (129.03 cm^2).

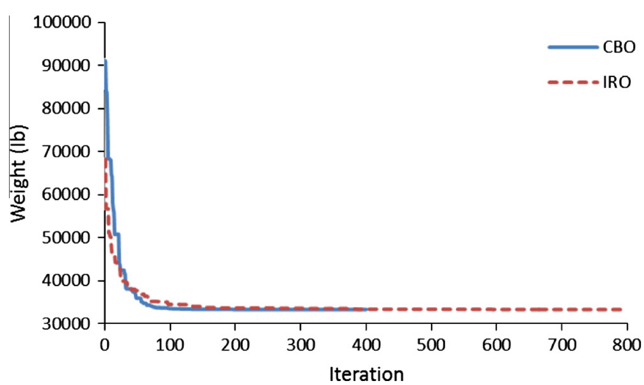

Fig. 8. Convergence curves obtained for Case 4 of the 120-bar dome problem.

Table 7 compares the optimization results obtained in this study with previous research presented in literature. It can be seen that CBO always designed the lightest structure except for Cases 3 and 4 where HPSACO and IRO converged to a slightly lower weight. CBO always completed the optimization process within 14,960 structural analyses (40 agents × 374 optimization iterations) while HPSACO required on average 10,000 analyses (400 optimization iterations) and HS required 35,000 analyses. The average number of analyses required by the RO and IRO algorithms were instead 19,900 and 18,300, respectively. Fig. 8 shows the convergence diagram of the CBO and IRO for the case 4. CBO always converged to feasible designs that are critical with respect to the optimization constraints set for each problem variant.

4.5. Test problem 5: design of Forth truss bridge

The last test case was the layout optimization of the forth bridge shown in Fig. 9a which is a 16 m long and 1 m high truss of infinite span. Because of infinite span, the cross section of the

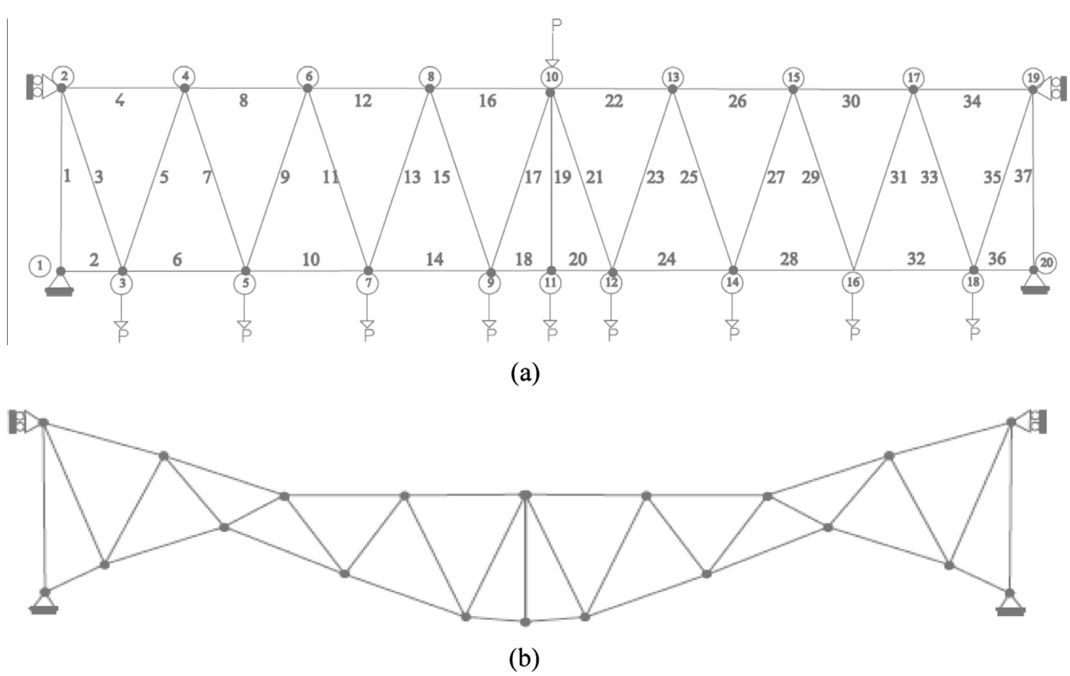

Fig. 9. (a) Schematic of the Forth truss bridge and (b) optimized layout of the forth bridge.

Table 8
Comparison of CBO optimization results with literature for the forth bridge problem.

No	Design variable	Kaveh and Khayatazad [7]			Present work
		BB-BC	PSO	RO	
1	A_1	56.41	25.20	20.54	23.314
2	A_2	58.20	97.60	44.62	36.867
3	A_3, A_5	53.89	35.00	6.37	9.847
4	A_4	60.21	64.30	50.10	49.679
5	A_6	56.27	14.51	30.39	26.563
6	A_7	57.08	37.91	17.61	12.737
7	A_8	49.19	69.85	41.04	37.120
8	A_{10}	48.67	8.76	8.55	1.545
9	A_9, A_{11}	45.43	47.54	33.93	28.35
10	A_{12}	15.14	6.36	0.63	0.891
11	A_{14}	45.31	27.13	26.92	24.110
12	A_{13}	62.91	3.82	23.42	9.112
13	A_{18}	56.77	50.82	42.06	29.071
14	A_{15}, A_{17}	46.66	2.70	2.01	8.222
15	A_{16}	57.95	5.46	8.51	8.715
16	A_{19}	54.99	17.62	1.27	2.107
17	Δy_2, Δy_{19}	6.89	140	70.88	11.093
18	Δy_3, Δy_{18}	17.74	139.65	64.88	50.352
19	Δy_4, Δy_{17}	1.81	117.59	−6.99	−50.529
20	Δy_5,Δy_{16}	23.57	139.70	128.31	119.315
21	Δy_6, Δy_{15}	3.22	−16.51	−64.24	−124.378
22	Δy_7, Δy_{14}	5.85	139.06	139.29	34.219
23	Δy_8, Δy_{13}	4.01	−127.74	−109.62	−120.867
24	Δy_9, Δy_{12}	10.52	−81.03	21.82	−41.323
25	Δy_{10}	−25.99	60.16	−55.09	−115.609
26	Δy_{11}	2.74	−139.97	2.29	−54.590
Best weight (kg)		37132.3	20591.9	11215.7	10250.9
Average weight (kg)		40154.1	25269.3	11969.2	11112.63
Std (kg)		1235.4	2323.7	545.5	522.54

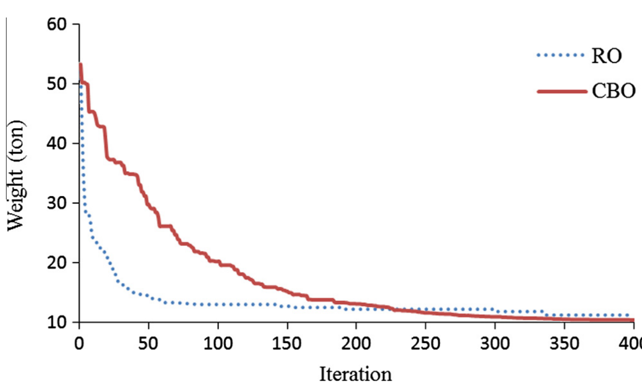

Fig. 10. Convergence curves obtained in the forth bridge problem.

bridge can be modeled as symmetric about the axis joining nodes 10 and 11. Structural symmetry allowed the 37 elements of which the bridge is comprised to be grouped into 16 groups (see Table 8): hence, there are 16 independent sizing variables. Nodal coordinates were included as layout variables: *X*-coordinates of nodes could not vary while *Y*-coordinates (except those of nodes 1 and 20) were allowed to change between −140 and 140 cm with respect to the initial configuration of Fig. 9a. Thus, the optimization problem included also 10 layout variables. The cross-sectional areas (sizing variables) could vary between 0.5 and 100 cm^2.

Material properties were set as follows: modulus of elasticity of 210 GPa, allowable stress of 250 MPa, specific weight of 7.8 ton/m^3. The structure is subject to self-weight and concentrated loads shown in Fig. 9a.

Table 8 compares CBO optimization results with literature. It appears that CBO found the best design overall saving about 1000 kg with respect to the optimum currently reported in literature. Furthermore, the standard deviation on optimized weight observed for CBO in 20 independent optimization runs was lower than for the other meta-heuristic optimization algorithms taken as basis of comparison.

The optimized layout of the bridge is shown in Fig. 9b. Fig. 10 compares the convergence behavior of CBO and RO. Although RO was considerably faster in the early optimization iterations, CBO converged to a significantly better design without being trapped in local optima.

5. Concluding remarks

This paper presents a novel efficient meta-heuristic optimization algorithm called Colliding Bodies Optimization (CBO). The governing laws from the physics provide a theoretical foundation to the CBO algorithm and determine the movement process of the objects in the optimization search. In CBO, each agent (candidate solutions) is considered as a colliding body. After the collision of two moving bodies with specified masses and velocities, these agents are separated from each other with new velocities to explore the design space.

Apart from the efficiency and robustness of the CBO algorithm, which are proven by the optimization results gathered by solving five classical test problems, the ease in its implementation and being parameter independent are definite strength points of the CBO. The proposed approach performed very well both in terms of the quality of optimized design and the number of function evaluations to find the optimum.

Acknowledgement

The first author is grateful to Iran National Science Foundation for the support.

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