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Differential evolution with individual-dependent topology adaptation

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ABSTRACT

Differential evolution (DE) is an efficient and robust evolutionary algorithm (EA), that has been widely and successfully applied to solve global optimization problems in diverse real-world applications. As the population structure has a major influence on the behavior of an EA, effectively incorporating population topology into DE has recently attracted increasing attention. Previous works have shown the effectiveness of different topologies in improving the performance of DE and revealed that different topologies can have different effects on the population's ability to solve optimization problems. However, the synergy of different topologies for the problems being solved has not been systematically investigated in most DE variants. Moreover, individuals with different fitness values play different roles in guiding the search during the evolutionary process. Nevertheless, the individual-dependent roles are not considered in most DE variants that consider the population topology. To overcome these drawbacks and utilize the information that is derived from the differences between the fitness values of individuals for topology adaption, we propose a multi-topology-based DE (MTDE) algorithm that includes an ensemble of multiple population topologies (MPT), an individual-dependent adaptive topology selection (ITS) scheme, and a topology-dependent mutation (TDM) strategy. In the ensemble of MPT, multiple population topologies with different degrees of connectivity are employed. In the ITS scheme, each individual adaptively selects the topology that is most compatible its role in guiding the search based on its fitness value. In the TDM strategy, the parents for mutation are chosen from the neighborhood of the current individual based on the corresponding topology to generate offspring. The effectiveness of the proposed algorithm is extensively evaluated on a suite of benchmark functions. Experimental results demonstrate the competitive performance of MTDE when compared with other state-of-the-art DE variants and EAs.

1. Introduction

Differential evolution (DE), which was proposed by Storn and Price [37], is a simple yet powerful evolutionary algorithm (EA) for global numerical optimization. Over the years, DE has been the subject of much attention and has been extended to handle multi-objective, constrained, large-scale, dynamic, and uncertain optimization problems [8]. Furthermore, in various scientific and engineering fields, different DE variants have been proposed for solving the complicated optimization problems

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in chemical engineering, economics, pattern recognition, engineering design, and so on [22,31,32,45]. The mutation strategy, as the salient feature of DE, can greatly influence the ability of DE to optimize different types of problems [4]. In most mutation operators of DE, both base and difference vectors are selected randomly from the current population. Thus, all individuals of the current population are equally likely to be selected as parents, without any selective pressure [4,8]. This high degree of randomness will cause DE to be slow in exploiting solutions. Therefore, to overcome these shortcomings and further enhance the performance of DE, many variants have been developed, mainly through the following approaches [44]: setting control parameters adaptively, designing new mutation operators, adopting an ensemble mechanism, and combining DE with other optimization methods.

Although DE has been extensively studied, neighborhood information of the population has not been systematically exploited in the algorithm design. In the literature on EA and DE, the works on using neighborhood information can roughly classified into two categories: the first is the index-based neighborhood strategy, in which the neighbors of each individual are defined based on the order of their indices in the population topology, and the second is the distance-based neighborhood strategy, in which the neighbors of each individual are identified according to the Euclidean distance in the search space. In recent years, many researchers have introduced neighborhood information into DE through the index-based neighborhood strategy [3,7,10]. The related works will be reviewed in Section 2.2.

As the earlier studies showed, the neighborhood information with index-based neighborhood can significantly improve the performance of DE for different kinds of problems. In addition, different neighborhood topologies have different influences on the behavior of DE; thus, they have different effects on the population's ability to solve optimization problems. However, the synergy of different topologies has not yet systematically considered in most DE variants. Therefore, how to utilize the neighborhood information from different population topologies to design a more efficient DE variant has long been a subject of research.

During the evolutionary process of DE, individuals with different fitness values can play different roles. Generally, the superior individuals perform local search to find better solutions in their neighborhoods, while the inferior individuals aim to explore new search areas to maintain the population diversity. Unfortunately, the individual-independent roles, which are brought about the differences in the fitness values among individuals, have not been considered when using the population topologies to solve different kinds of problems.

In this study, we propose a multi-topology-based DE algorithm (MTDE) with an individual-dependent adaptive topology mechanism. To effectively utilize the differences in the fitness values among individuals for topology adaption, MTDE includes an ensemble of multiple population topologies (MPT), an individual-dependent adaptive topology selection (ITS) scheme, and a topology-dependent mutation (TDM) strategy. In the ensemble of MPT, several population topologies with different degrees of connectivity are used, which can provide different neighborhood sizes to be chosen. The ITS scheme adaptively selects the topologies for the individuals according to their ranking values, which depend on their fitness values. Specifically, superior individuals are more likely to select a topology with a low degree of connectivity, which contributes to the search for an optimal solution within the region, while inferior individuals tend to select a larger neighborhood, which is beneficial for exploring new regions. In addition, the TDM strategy introduces the direction information based on the employed topology of the current individual into the mutation operator to generate the mutant vector.

Extensive experiments are carried out to evaluate the effectiveness of MTDE by comparing it with other state-of-the-art DE variants and well-known EAs on a suite of benchmark functions from the congress on Evolutionary Computation (CEC) 2013 and 2014 special sessions on real-parameter optimization [23,24]. The simulation results demonstrate the competitive performance of MTDE. Note that this study is an extension of our previous work [38], with more detailed descriptions and discussions of MTDE. Specifically, more extensive experiments, including comparisons with advanced DE variants, state-of-the-art EAs, and single- and multi-topology-based DE variants, and studies on the influences of different combinations of topologies, selection strategies for topology adaptation and topology parameters, are carried out to evaluate the effectiveness of MTDE comprehensively.

The rest of this paper is organized as follows. Section 2 reviews the DE algorithm and the related works on the DE variants that are enhanced with population topology. In Section 3, the proposed MTDE algorithm is presented in detail. In Section 4, the experimental results are reported and discussed. The last section presents the conclusions and discussed the possible future work.

2. DE and related works

2.1. Original DE algorithm

DE is a population-based stochastic algorithm for solving numerical optimization problems [37]. In this study, the following optimization problem is considered: *Minimize*$f(X)$, where $X \in S$, $S \subseteq R^D$ and D is the dimension of the decision variables. Similar to other EAs, DE evolves a population of vectors $X_{i,G} = [x^1_{i,G}, x^2_{i,G}, \ldots, x^D_{i,G}]$, where $i = 1, 2, \ldots, NP$, NP is the size of the population and G is the current generation number. When $G = 1$, the jth parameter of the ith vector $X_{i,G}$ is initialized as $x^j_{i,G} = L_j + rndreal(0, 1) \cdot (U_j - L_j)$, where $rndreal(0, 1)$ represents a uniformly distributed random number in the range [0, 1], and L_j and U_j are the lower and upper bounds of the jth variable, respectively.

After initialization, DE enters a simple loop that carries out the evolutionary process, which includes mutation, crossover and selection operator.

Mutation: At each generation, DE employs a mutation strategy to generate a mutant vector $V_{i,G}$ for each individual $X_{i,G}$ (called the target vector) in the current population. Six frequently used mutation strategies are listed as follows:

- DE/rand/1

$$V_{i,G} = X_{r1,G} + F \times (X_{r2,G} - X_{r3,G}) \tag{1}$$

- DE/rand/2

$$V_{i,G} = X_{r1,G} + F \times (X_{r2,G} - X_{r3,G}) + F \times (X_{r4,G} - X_{r5,G}) \tag{2}$$

- DE/best/1

$$V_{i,G} = X_{best,G} + F \times (X_{r1,G} - X_{r2,G}) \tag{3}$$

- DE/best/2

$$V_{i,G} = X_{best,G} + F \times (X_{r1,G} - X_{r2,G}) + F \times (X_{r3,G} - X_{r4,G}) \tag{4}$$

- DE/current-to-best/1

$$V_{i,G} = X_{i,G} + F \times (X_{best,G} - X_{i,G}) + F \times (X_{r1,G} - X_{r2,G}) \tag{5}$$

- DE/rand-to-best/1

$$V_{i,G} = X_{r1,G} + F \times (X_{best,G} - X_{r1,G}) + F \times (X_{r2,G} - X_{r3,G}) \tag{6}$$

In the above equations, the indices $r1$, $r2$, $r3$, $r4$ and $r5$ are random unique integers that are selected from the range [1, NP] and are different from the index i. $X_{best,\,G}$ is the best individual vector in the Gth generation. The mutation factor F is a positive control parameter for scaling the difference vector. More details can be found in [37] and [8].

Crossover: To enhance the diversity of the population, the crossover operator is applied to each pair of $X_{i,G}$ and $V_{i,G}$ to generate a trial vector $U_{i,G}$. There are two kinds of crossover scheme: binomial and exponential [37]. Binomial crossover is generally more robust and efficient than exponential crossover and is employed in this study. The process of binomial crossover is defined as follows:

$$u_{i,G}^{j} = \begin{cases} v_{i,G}^{j} & \text{if } rndreal(0,1) \le Cr \text{ or } j = j_{rand} \\ x_{i,G}^{j} & \text{otherwise} \end{cases} \tag{7}$$

where $Cr \in [0, 1]$ is the crossover rate and j_{rand} is an integer that is randomly selected from the range [1, D]. If $u_{i,G}^{j}$ is outside the boundary, it will be reinitialized within the range $[L_j, U_j]$.

Selection: To decide whether $X_{i,G}$ or $U_{i,G}$ survive in the next generation, DE uses a one-to-one greedy selection operator, which is defined as follows:

$$X_{i,G+1} = \begin{cases} U_{i,G} & \text{if } f(U_{i,G}) \le f(X_{i,G}) \\ X_{i,G} & \text{otherwise} \end{cases} \tag{8}$$

2.2. Enhanced DE algorithms with population topology

As shown in the literature, the population model has a marked influence on the performance of EAs and DE [3,10,18,19]. Over the past decades, researchers have developed many DE variants by using different population topologies. In the original DE algorithms, each individual interacts with every other individual in the whole population, which is known as the panmictic (or centralized) model. Differing from the classical population model, the structured (or decentralized) model imposes a certain structure on the population to introduce some partial isolation among individuals. In the structured population model, the flow of information is influenced by several aspects of the population topology: the degree of connectivity, the amount of clustering, and the average shortest distance from one node to another [21]. Based on these factors, the structured model has two main kinds of population topologies: coarse-grained and fine-grained. The coarse-grained population is composed of large sub-populations with loose connections, while the fine-grained population is made up of tiny sub-populations with tight connections. In this section, we will review the work related to the DE variants that are enhanced with the structured population.

2.2.1. DE with coarse-grained topology

In a coarse-grained (also called distributed) topology-based DE algorithm, the population is partitioned into several smaller subpopulation (islands). The DE algorithms independently evolve all the islands, and each island exchanges some information with a given frequency. In each island, the running DE algorithms can have different configurations.

In [49], a parallel distributed self-adaptive DE algorithm is utilized by connecting several islands in a random structure. In [42], a unidirectional ring of island is introduced into a parallel distributed DE, which attracted the interests of many researchers who are studying different configurations for the unidirectional ring topology. In [48], adaptive mechanisms for the scale factor are studied and employed for distributed DE to show its effectiveness in improving the performance of

the distributed algorithm. Differing from the distributed DEs with the same configurations of islands, the heterogeneous distributed DE algorithms are also investigated. In [47], a heterogeneous distributed DE is proposed that uses two families of subpopulations, namely, explorative and exploitative subpopulations, to play different roles for searching. In [10], a new heterogeneous distributed DE with two islands is presented. In each island, a different mechanism is employed to generate the mutant vector. In [14], an asynchronous adaptive multi-population model is proposed for distributed DE, where an asynchronous migration mechanism and adaptive procedure are used to reduce the number of control parameters in the distributed model. Recently, a distributed DE with adaptive merging and splitting of subpopulations is proposed for solving large-scale optimization problems [15].

2.2.2. DE with fine-grained topology

In a fine-grained topology-based DE algorithm, the population is made up of many small subpopulations, which are arranged in a toroidal mesh. During evolution, each individual is only allowed to interact with its neighbors, which are determined based on the topology. Over the past decades, researchers have developed many DE variants with different fine-grained population topologies (e.g., small-world, cellular and ring).

In [29], the effects of the ring and Von Neumann topologies on the behavior of DE are investigated, and the results show that DE with the ring topology (DE/Ibest/1) outperforms original DE. In [30], a bare-bones DE with both the ring and Von Neumann topologies is proposed, which is a hybrid algorithm of the bare-bones particle swarm optimizer and DE. The ring topology is also used in [7], where DE with global and local neighborhoods (DEGL) is proposed, which utilizes the concept of the neighborhood of each population member. In [5], a new DE framework, which is named neighborhood-guided DE (NGDE), is proposed. In NGDE, a neighborhood guided selection strategy is designed by combining the neighborhood information from the ring topology with the fitness information of the population.

Differing from the ring topology, the small-world topology is inspired by Milgram's "six degrees of separation" theory that two arbitrary persons can be connected via six hops. Due to the good information transfer mode of the small-world topology, it has been introduced into DE to construct neighborhoods to enhance its performance [17]. In [10], a new DE algorithm with a small-world topology population is proposed, in which the relationships among the individuals are defined based on the created small-world topology.

Cellular topology, as a widely used population scheme, was invented by Von Neumann and Ulam in 1948 and has become a powerful tool in scientific research. In [28], cellular topology is introduced into DE, and a novel evolutionary model (called CellularDE) is proposed by combining DE with cellular automata to address dynamic optimization problems. In [27], a cellular DE (cDE) algorithm is presented to study the influence of slow diffusion of information throughout the population in DE. In [10], a hierarchical cellular DE (HcDE) algorithm is proposed by arranging the solutions in cellular DE according to their fitness values. In [25], DE with cellular direction information (DE-CDI) is proposed, which uses the neighborhood information from the cellular topology to select individuals for mutation.

2.2.3. DE with multiple topologies

Differing from the single topology used in the DE variants, multiple topologies are also introduced into DE to improve its performance. Here, two representative DE variants with multiple topologies are briefly described, since they will be compared with the proposed MTDE algorithm in Section 4.7 as closely related works.

1. APT-DE [39]: In [39], a novel DE variant with adaptive population topology (APT-DE) is proposed. In APT-DE, five population topologies (cellular, distributed, regular lattice, small-world, and random graph) are randomly selected and applied to the current population. Each topology has the same probability to be selected. During the evolutionary process, if the best individual among the last three generations is not improved, another candidate topology will be selected for the next generation.
2. Na-DE [3]: To study the effects of different topologies on different functions and the synergy among multiple neighborhood topologies, a novel DE framework named neighborhood-adaptive DE (Na-DE), is proposed in [3]. In Na-DE, multiple index-based neighborhood topologies are used to define multiple neighborhood relationships for each individual. Then, the neighborhood relationship is adaptively selected for each individual on the specific functions. After that, a neighborhood-dependent directional mutation operator is used to generate a mutant vector with the selected topology for the current individual.

3. Multi-topology-based DE (MTDE)

As discussed in Section 2.2, the population topology has been successfully introduced into DE, and the promising results have been achieved. From [7,10,25], it is clear that most population topologies can provide an improvement for DE. Moreover, different population topologies exhibit obviously different influences on the behavior of DE. These observations clearly indicate that using a single topology in DE is not appropriate for optimizing different kinds of problems, which has not been well-studied in the DE algorithm design. On the other hand, different population topologies have different characteristics that affect the flow of information in the population through the social neighborhood [21]. It may be beneficial for further improving DE by using multiple topologies simultaneously during the evolutionary process. However, the synergy of different topologies for the problems being solved has not yet systematically investigated in most DE variants.

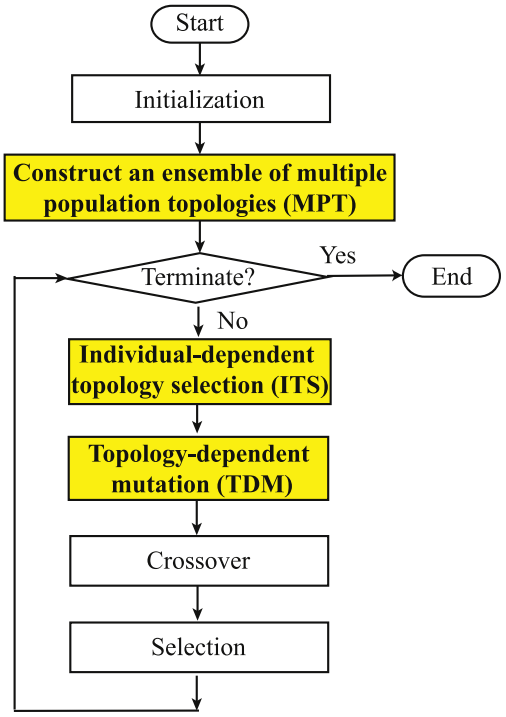

Fig. 1. The general framework of MTDE.

Furthermore, the fitness information of the population, as the feedback from the evolutionary search, is very valuable for guiding the search process of DE [5,34]. During the evolutionary process, the individuals with different fitness values play different roles. Generally, the superior individuals are expected to find the optimal solutions in their neighborhoods; thus, it is beneficial for them to have a smaller neighborhood for exploitation. In contrast, the inferior individuals are expected to explore new regions in the search space, so it is better for them to have a larger neighborhood for exploration. Unfortunately, the information that is derived from the differences between the fitness values of individuals has not yet been considered in most DE variants with the structured population.

Based on the above considerations and inspired by the individual-dependent roles during the evolutionary process, we propose a novel multi-topology-based DE (MTDE) algorithm, in which an ensemble of multiple population topologies (MPT) is formed first, and then, an individual-dependent adaptive topology selection (ITS) scheme is employed to select a suitable topology for each individual that matches its role in the search process. Finally, a topology-dependent mutation (TDM) strategy is used to guide the generation of offspring based on the selected topology. Fig. 1 illustrates the general framework of MTDE, and more details are presented in the following subsections.

3.1. Ensemble of MPT

Various types of population topologies have been successfully incorporated into DE, and their influences on the performance of DE have been examined [10]. In this study, one of the objectives is to investigate the synergy of different topologies in DE. In addition, to realize the individual-dependent roles through different neighborhood sizes, multiple population topologies with different degrees of connectivity are included to form the ensemble of MPT. Specifically, four widely used population topologies are selected, namely, small-world, ring, cellular and panmictic, which are shown in Fig. 2. The effectiveness of these topologies for DE has been validated on different kinds of problems. More details on them can be found in [10,11,21]. They are briefly described as follows:

- *Small world*: The small-world topology consists of a ring of NP vertices, each connected to its N_{sw} nearest neighbors. Then, each edge is reconnected to a vertex that is selected uniformly at random over the entire ring with probability p, where $0 < p < 1$. The detail shape is illustrated in Fig. 2(a), in which all neighbors of current individual X_i are represented by solid circles that are connected to X_i by lines.
- *Cellular*: In the cellular topology, the individuals are arranged in a 2-D toroidal mesh, and each individual only interacts with its surrounding individuals, as its neighbors, in the breeding loop. The cellular topology is illustrated in Fig. 2(b), where the compact neighborhood shape is used and the N_c neighbors of current individual X_i are shown as solid circles.
- *Ring*: In the ring topology, the individuals of the population are organized in a toroidal ring. The neighbors of an individual are the next and previous individuals that interact with it within the neighborhood radius (R_r) in the ring. The ring

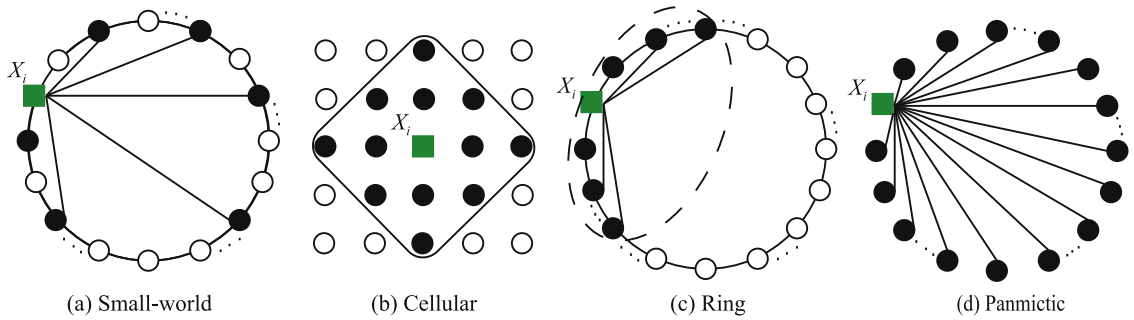

Fig. 2. Four population topologies with different degrees of connectivity. The solid circles (•) denote the neighbors of the current individual *Xi* which is represented by the solid square (■).

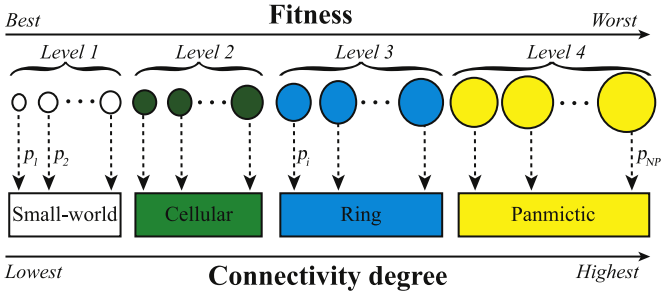

Fig. 3. Illustration of ITS scheme.

topology is illustrated in Fig. 2(c), where the neighbors of the current individual X_i are shown as the $2 \times R_r$ nearest solid circles.

- *Panmictic*: In the panmictic topology, all the individuals are connected to each other. The neighbors of the current individual X_i are all the other individuals. This topology is illustrated in Fig. 2(d), which is used in most of the DE algorithms.

Furthermore, to take advantage of the useful experience in the research of these topologies, their control parameters are set as those in the representative works: $N_{sw} = 0.1 \times NP$ and $p = 0.05$ for the small-world topology [11,21], $N_c = 13$ for the cellular topology [25,27], and $R_r = 0.1 \times NP$ for the ring topology [7]. According to the parameter settings, these four topologies have different degrees of connectivity. The small-world topology has the lowest connectivity degree, followed by the cellular and ring topologies. The panmictic topology has the highest connectivity degree.

3.2. ITS scheme

After constructing the ensemble of MPT, we need to select the population topologies for different individuals to match their roles during the evolution process. For this, we propose an individual-dependent adaptive topology selection (ITS) scheme. Fig. 3 illustrates the basic idea of the proposed ITS scheme, where a solid circle represents an individual and the size of the solid circle indicates the fitness value of the corresponding individual. The smaller the solid circle is, the better fitness the individual has. Concretely, the ITS scheme is carried out as follows.

First, all the individuals are sorted according to their fitness values. Based on their ranking values, the whole population is divided into L levels, and each level has an equal number of individuals.[1] As Fig. 3 shows, L is set as four in this paper, which is identical to the number of population topologies that are included in the ensemble of MPT.

Second, a candidate population topology is assigned to each level based on the ranking values of the individuals that belong to it. Specifically, a topology with a lower degree of connectivity is assigned to a level with superior individuals, while a topology with a higher degree of connectivity is assigned to a level with inferior individuals. As shown in Fig. 3, the small-world topology, which has the lowest degree of connectivity in the ensemble of MPT, is assigned to Level 1 that has the best individuals. Analogously, the cellular, ring and panmictic topologies are assigned to Level 2, Level 3 and Level 4, respectively.

[1] Note that the whole population may not be equally partitioned by L. In this situation, the $NP\%L$ individuals are just added into the last level.

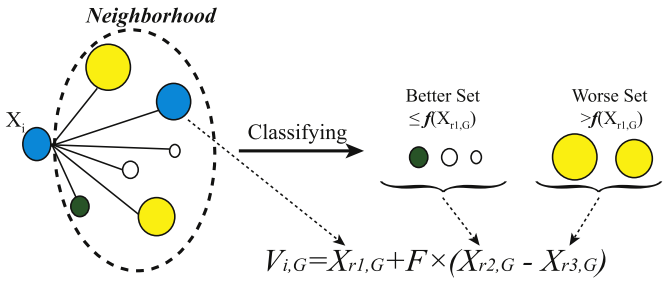

Fig. 4. Illustration of parent selection in TDM for the "DE/rand/1" mutation strategy.

Third, to increase the diversity of the population, each individual has a probability value that determines whether or not the topology that is assigned to the level to which it belongs is employed. The probability value for X_i can be calculated as

$$p_i = rank_i/NP \tag{9}$$

where $rank_i$ is the ranking value of X_i in terms of its fitness value in the whole population.

Finally, based on the probability value p_i, X_i selects a population topology T_i as follows:

$$T_i = \begin{cases} 1 & \text{if } 0 < p_i \le 0.25 \text{ and } (1.0 - p_i) > rndreal(0,1) \\ 2 & \text{if } 0.25 < p_i \le 0.50 \text{ and } (1.25 - p_i) > rndreal(0,1) \\ 3 & \text{if } 0.50 < p_i \le 0.75 \text{ and } (1.50 - p_i) > rndreal(0,1) \\ 4 & \text{if } 0.75 < p_i \le 1.0 \text{ and } (1.75 - p_i) > rndreal(0,1) \\ rand(1,4) & \text{otherwise} \end{cases} \tag{10}$$

where $rand(1, 4)$ denotes a natural number that is randomly selected from the range [1,4], and $T_i = 1, 2, 3,$ and 4 represent the small-world, cellular, ring and panmictic topologies, respectively.

3.3. TDM strategy

After selecting the population topology for each individual, the TDM strategy is employed to generate the mutant vectors based on the individual-dependent neighborhood information. In the TDM strategy, partition-based selection [3,25] is used to select parents for the mutation operator. The basic idea of parent selection in TDM for the "DE/rand/1" mutation strategy is shown in Fig. 4.

As shown in Fig. 4, the base vector in the mutation strategy (i.e., $X_{r1,G}$) is randomly selected from the neighborhood of the current individual X_i. Then, all the neighbors of X_i are divided into two sets: the superior set that contains individuals with better fitness values than $X_{r1,G}$, and the inferior set that includes individuals with worse values than $X_{r1,G}$. After that, two neighbors are randomly selected from these two sets, respectively, to construct a difference vector. Finally, a new mutant vector $V_{i,G}$ is generated with the base and difference vectors via the corresponding mutation strategy.

In the implementation of TDM, if the mutation strategy incorporates the best individual (e.g., DE/best/1, DE/best/2, DE/current-to-best/1, and DE/rand-to-best/1), the best neighbor of the current individual will be treated as the best individual. In addition, when the best or worst neighbor is selected as the base vector, two neighbors are randomly selected to construct the difference vector by directing at the better one from the worse one.

3.4. Complete procedure of MTDE

Combining the above-mentioned operators, the complete procedure of MTDE with the "DE/rand/1" mutation strategy (MTDE/rand/1, for short) is described in Algorithm 1, where the differences with respect to the original DE/rand/1 algorithm are highlighted with "*". At the beginning of MTDE, the ensemble of MPT is constructed in line 3. In this step, the four population topologies, namely, small-world, cellular, ring, and panmictic, are adopted and initialized. Then, for each individual in line 6, the ITS scheme is applied to adaptively select a specific population topology. In line 7, depending on the selected topology, a mutant vector is generated with the TDM strategy, followed by the crossover operator in line 8. In line 9, the offspring is evaluated and then compared with its parent in line 10. As shown in Algorithm 1, since the individuals independently select topologies with different connectivity degrees by the ITS scheme in line 6, each individual can effectively play its role in guiding the search during the evolutionary process of DE.

Algorithm 1 MTDE/rand/1.

Randomly generate NP individuals as the initial population and set $G = 1$;
Evaluate the fitness of each individual in population;
$*$ Construct an ensemble of multiple population topologies (MPT);
while the termination criterion is not satisfied **do**
  for each individual $X_{i,G}$ **do**
    $*$ Select a population topology T_i by using the individual-dependent adaptive topology selection (ITS) scheme;
    $*$ Generate a mutant vector $V_{i,G}$ by using the topology-dependent mutation (TDM) strategy;
    Generate a trial vector $U_{i,G}$ by the crossover operator in Eq. (7);
    Evaluate the fitness of $U_{i,G}$;
    Determine the surviving vector between $X_{i,G}$ and $U_{i,G}$ by the selection operator in Eq. (8);
  end for
  Set $G = G + 1$;
end while

3.5. Discussion

3.5.1. Complexity analysis

Compared with the original DE algorithm, the additional computations of MTDE are due to the three novel operators: constructing the ensemble of MPT, the ITS scheme, and the TDM strategy. For constructing the ensemble of MPT, the cost is $O(NP)$. For the ITS scheme, the additional cost is incurred by sorting the whole population and selecting the topologies for all the individuals using Eq. (10), which take $O(NP \times logNP)$ and $O(NP)$, respectively. For the TDM strategy, the random selection of the base vector from neighborhood takes $O(1)$, and the construction of the difference vector based on the neighborhood classification requires k times of comparisons, where k is the connectivity degree of the employed topology. Since the complexity of the original DE algorithm is $O(Gmax \times NP \times D)$, where G_{max} is the maximum number of generations, the total complexity of MTDE is $O(Gmax \times (NP \times logNP + NP + NP + NP \times k + NP \times D))$, that is $O(Gmax \times NP \times max(logNP, k, D))$. In conclusion, MTDE remains computationally efficacious in time when compared with the original DE algorithm.

3.5.2. Advantages of synthesizing multiple topologies

In MTDE, multiple population topologies are synergized for guiding the evolutionary process of DE. Compared with single-topology-based DE algorithms, MTDE has the following advantages:

- An ensemble of MPT is employed to construct multiple neighborhood relationships for each individual. Since these topologies have different degrees of connectivity and different neighborhood sizes, each individual has a chance to utilize different neighborhood information from different topologies during the evolutionary process.
- Each individual in the population can adaptively select a suitable topology for its individual-dependent role in the search process. In MTDE, the superior individuals are likely to select smaller neighborhoods so that they can search the region of an optimal point, while the inferior individuals are inclined to select larger neighborhoods so that they can explore new regions.
- By exhibiting different search behaviors through different topologies, MTDE has the potential to control the exploration versus exploitation tendencies of the population under the multi-topology framework.

Furthermore, by combining the ensemble of MPT, the ITS scheme and the TDM strategy, MTDE provides a simple and efficient way to utilize the population information from different models of interaction among individuals. The advantages of MTDE will be demonstrated through the experimental comparisons with single-topology-based DE algorithms in Section 4.6.

3.5.3. Comparison with APT-DE and Na-DE

Note that APT-DE [39] and Na-DE [3], which are methods from previous works, also use multiple population topologies to improve the performance of DE. However, there are significant differences between these two works and MTDE:

- The topologies are combined in different ways. In APT-DE, five common topologies are used: cellular, distributed, regular lattice, small-world, and random graph topologies [39]. In Na-DE, panmictic, ring, cellular and distributed topologies are selected to construct the topology pool [3]. Although they employ the similar topologies, MTDE elaborately chooses four topologies with different degrees of connectivity. Specifically, the connectivity degrees of the four topologies are set from lowest (small-world) to highest (panmictic). In this way, the individual-dependent roles can be effectively played with different types of topologies and different sizes of neighbors.
- The topologies are selected in different ways. In APT-DE, the topology is randomly selected from the candidate topologies, and the employed topology is reselected if the best individual in the population has not improved in last three generations [39]. In Na-DE [3], the topologies are selected based on the rewards of different topologies in the search until now. In contrast, the selection of topologies in MTDE is based on the differences between the fitness values of individuals. Specifically, MTDE uses an individual-dependent adaptive topology selection (ITS) scheme to select the topology for each individual.

Table 1
Mean and standard deviation of the best error values obtained by the original DE and the corresponding MTDE on all the CEC2013 functions at 30*D*.

Fun.	DE/rand/1		MTDE/rand/1	DE/rand/2		MTDE/rand/2	DE/best/1		MTDE/best/1
F1	**1.68e-030 9.22e-030**	–	7.76e-029 1.18e-028	7.08e-001 2.07e-001	+	**7.62e-009 9.68e-009**	3.04e+003 1.46e+003	+	**5.98e-028 5.22e-028**
F2	3.57e+005 2.71e+005	+	**1.39e+005 8.18e+004**	5.66e+007 1.37e+007	+	**4.10e+006 2.27e+006**	1.62e+007 1.03e+007	+	**3.27e+004 1.86e+004**
F3	**2.16e+000 1.13e+001**	–	5.27e+002 1.71e+003	5.53e+009 1.29e+009	+	**1.95e+007 1.66e+007**	1.14e+009 2.86e+009	+	**1.12e+006 1.66e+006**
F4	1.45e+003 5.83e+002	+	**1.27e+002 9.56e+001**	5.38e+004 5.58e+003	+	**4.15e+004 6.19e+003**	**1.56e-003 1.80e-003**	–	1.69e-001 2.05e-001
F5	**3.70e-029 5.52e-029**	–	4.74e-016 2.59e-015	8.21e-001 2.02e-001	+	**5.58e-006 4.03e-006**	5.80e+002 3.37e+002	+	**1.16e-014 6.12e-015**
F6	8.86e+000 1.57e+000	+	**2.39e+000 4.65e+000**	5.34e+001 2.36e+001	+	**1.34e+001 1.07e+001**	2.85e+002 1.26e+002	+	**3.48e+001 2.77e+001**
F7	**4.78e-002 9.03e-002**	–	2.44e-001 3.23e-001	9.01e+001 1.07e+001	+	**3.64e+001 1.10e+001**	1.01e+002 3.28e+001	+	**2.48e+001 1.24e+001**
F8	2.09e+001 5.71e-002	=	2.09e+001 4.75e-002	2.10e+001 4.04e-002	=	2.09e+001 5.74e-002	2.09e+001 4.45e-002	=	2.09e+001 5.41e-002
F9	3.91e+001 1.27e+000	+	**2.82e+001 1.18e+001**	3.94e+001 1.21e+000	=	3.94e+001 1.13e+000	2.38e+001 3.26e+000	+	**1.91e+001 4.03e+000**
F10	**7.89e-003 7.13e-003**	–	2.69e-002 1.20e-002	5.44e+001 1.68e+001	+	**2.10e-001 2.53e-001**	5.15e+002 2.48e+002	+	**1.70e-001 1.08e-001**
F11	1.36e+002 2.42e+001	+	**2.20e+001 7.76e+000**	2.21e+002 1.17e+001	+	**2.05e+002 1.15e+001**	1.57e+002 4.06e+001	+	**5.01e+001 1.57e+001**
F12	1.80e+002 9.23e+000	+	**1.53e+002 4.88e+001**	2.42e+002 1.20e+001	+	**2.15e+002 1.37e+001**	1.84e+002 5.29e+001	+	**5.42e+001 1.77e+001**
F13	1.79e+002 1.27e+001	+	**1.63e+002 4.05e+001**	2.45e+002 8.77e+000	+	**2.16e+002 1.43e+001**	2.63e+002 4.88e+001	+	**1.17e+002 2.94e+001**
F14	6.54e+003 4.88e+002	+	**1.13e+003 6.65e+002**	6.88e+003 2.48e+002	=	6.99e+003 1.91e+002	2.58e+003 4.31e+002	+	**1.63e+003 4.53e+002**
F15	7.18e+003 2.48e+002	=	7.28e+003 2.92e+002	7.35e+003 2.93e+002	=	7.34e+003 2.65e+002	**3.69e+003 6.98e+002**	–	6.07e+003 1.74e+003
F16	2.45e+000 2.94e-001	=	2.52e+000 3.74e-001	2.54e+000 2.48e-001	=	2.59e+000 2.32e-001	2.44e+000 2.49e-001	=	2.44e+000 3.14e-001
F17	1.84e+002 1.57e+001	+	**6.21e+001 1.44e+001**	2.68e+002 1.47e+001	+	**2.38e+002 1.17e+001**	2.50e+002 5.60e+001	+	**7.98e+001 1.49e+001**
F18	2.12e+002 8.43e+000	+	**2.02e+002 8.46e+000**	2.80e+002 1.28e+001	+	**2.50e+002 1.28e+001**	2.76e+002 5.84e+001	+	**1.64e+002 4.84e+001**
F19	1.46e+001 1.22e+000	+	**3.34e+000 1.63e+000**	2.13e+001 1.46e+000	+	**1.84e+001 1.26e+000**	1.94e+003 2.29e+003	+	**5.08e+000 1.91e+000**
F20	1.23e+001 2.14e-001	+	**1.19e+001 3.04e-001**	1.34e+001 2.34e-001	+	**1.32e+001 2.11e-001**	1.41e+001 1.20e+000	+	**1.06e+001 7.89e-001**
F21	3.02e+002 6.75e+001	=	3.18e+002 8.61e+001	3.88e+002 8.44e+001	+	**3.04e+002 5.61e+001**	1.45e+003 3.04e+002	+	**3.15e+002 7.35e+001**
F22	6.45e+003 5.49e+002	+	**7.69e+002 3.88e+002**	7.32e+003 2.60e+002	=	7.21e+003 3.40e+002	3.12e+003 6.43e+002	+	**1.66e+003 5.08e+002**
F23	7.48e+003 3.60e+002	=	7.37e+003 2.43e+002	7.79e+003 1.97e+002	+	**7.65e+003 2.02e+002**	**4.16e+003 7.59e+002**	–	5.40e+003 2.08e+003
F24	**2.00e+002 2.85e-002**	–	2.02e+002 2.35e+000	2.84e+002 5.38e+000	+	**2.22e+002 8.52e+000**	2.77e+002 1.29e+001	+	**2.37e+002 1.25e+001**
F25	**2.33e+002 1.87e+001**	–	2.43e+002 1.76e+001	3.33e+002 3.41e+000	+	**3.30e+002 3.14e+000**	3.04e+002 1.14e+001	+	**2.68e+002 7.45e+000**
F26	2.00e+002 1.12e-002	+	**2.00e+002 3.29e-003**	2.04e+002 1.04e+000	+	**2.00e+002 8.49e-002**	3.10e+002 7.88e+001	+	**2.45e+002 6.48e+001**
F27	**3.01e+002 8.73e-001**	–	3.50e+002 6.70e+001	1.32e+003 3.75e+001	+	**1.26e+003 1.22e+002**	9.70e+002 7.24e+001	+	**6.54e+002 9.04e+001**
F28	3.00e+002 8.01e-006	=	3.00e+002 0.00e+000	3.56e+002 1.03e+001	+	**3.00e+002 7.09e-003**	1.86e+003 5.23e+002	+	**2.93e+002 3.65e+001**
+/=/-	–		14/6/8	–		22/6/0	–		23/2/3

Fun.	DE/best/2		MTDE/best/2	DE/current-to-best/1		MTDE/current-to-best/1	DE/rand-to-best/1		MTDE/rand-to-best/1
F1	3.60e-028 1.76e-028	+	**2.40e-028 1.40e-028**	1.72e+003 9.48e+002	+	**4.66e-001 1.07e+000**	8.61e+002 4.49e+002	+	**7.99e-002 3.39e-001**
F2	**1.13e+005 6.52e+004**	–	2.11e+005 1.54e+005	9.29e+006 5.50e+006	+	**1.70e+004 1.05e+004**	1.13e+007 6.36e+006	+	**2.13e+004 1.77e+004**
F3	2.87e+005 8.18e+005	+	**8.15e+003 2.36e+004**	5.00e+009 2.74e+009	+	**4.50e+005 1.64e+006**	6.44e+009 2.53e+009	+	**8.63e+005 4.31e+006**
F4	**1.40e+001 1.57e+001**	–	1.51e+003 1.97e+003	5.02e+000 4.92e+000	+	**3.14e-003 5.35e-003**	9.26e+000 2.08e+001	+	**4.42e-003 6.11e-003**
F5	7.39e-015 6.28e-015	+	**2.25e-015 4.59e-015**	5.43e+002 2.84e+002	+	**1.06e+001 1.72e+001**	3.07e+002 1.42e+002	+	**6.12e+000 9.60e+000**
F6	7.18e+000 1.18e+001	=	3.60e+000 9.10e+000	2.10e+002 5.94e+001	+	**6.71e+001 2.18e+001**	1.65e+002 4.81e+001	+	**5.66e+001 2.36e+001**
F7	1.42e+001 9.84e+000	+	**7.86e-001 6.18e-001**	5.69e+001 1.86e+001	+	**1.15e+001 9.35e+000**	5.84e+001 1.86e+001	+	**1.60e+001 1.02e+001**
F8	2.09e+001 5.31e-002	=	2.09e+001 5.91e-002	2.09e+001 4.50e-002	=	2.10e+001 4.54e-002	2.09e+001 5.78e-002	=	2.09e+001 4.57e-002
F9	3.45e+001 1.07e+001	=	3.91e+001 1.13e+000	1.69e+001 2.79e+000	+	**1.44e+001 4.33e+000**	1.64e+001 2.17e+000	+	**1.15e+001 2.74e+000**
F10	2.41e-002 1.71e-002	+	**1.11e-002 1.07e-002**	2.66e+002 9.71e+001	+	**4.62e+000 3.69e+000**	2.19e+002 1.15e+002	+	**6.39e+000 6.02e+000**
F11	1.81e+002 1.95e+001	=	1.74e+002 1.31e+001	9.50e+001 3.17e+001	+	**3.35e+001 1.04e+001**	7.83e+001 1.94e+001	+	**2.99e+001 9.66e+000**
F12	1.99e+002 1.42e+001	+	**1.85e+002 1.24e+001**	1.04e+002 2.05e+001	+	**3.77e+001 2.60e+001**	8.05e+001 2.19e+001	+	**3.26e+001 8.80e+000**
F13	1.97e+002 1.87e+001	+	**1.86e+002 1.13e+001**	1.78e+002 3.70e+001	+	**1.11e+002 4.39e+001**	1.66e+002 2.81e+001	+	**7.50e+001 2.42e+001**
F14	**6.71e+003 2.93e+002**	–	6.88e+003 2.94e+002	**6.41e+003 3.60e+002**	–	6.67e+003 2.96e+002	3.94e+003 2.50e+003	+	**1.59e+003 9.67e+002**
F15	7.31e+003 3.01e+002	=	7.31e+003 2.18e+002	6.90e+003 3.42e+002	=	6.81e+003 3.16e+002	6.46e+003 9.91e+002	=	6.64e+003 4.41e+002
F16	2.49e+000 2.31e-001	=	2.41e+000 3.34e-001	2.43e+000 3.20e-001	=	2.44e+000 2.88e-001	2.53e+000 2.45e-001	=	2.43e+000 2.38e-001
F17	2.23e+002 1.91e+001	+	**2.05e+002 1.14e+001**	2.22e+002 2.44e+001	+	**1.72e+002 1.31e+001**	1.27e+002 4.67e+001	+	**6.24e+001 2.90e+001**
F18	2.31e+002 1.69e+001	+	**2.17e+002 1.03e+001**	2.45e+002 3.06e+001	+	**1.87e+002 1.11e+001**	2.11e+002 1.56e+001	+	**1.84e+002 1.04e+001**
F19	1.55e+001 1.38e+000	=	1.51e+001 1.13e+000	2.61e+002 4.79e+002	+	**7.13e+000 3.68e+000**	9.36e+001 1.08e+002	+	**3.33e+000 8.95e-001**
F20	1.23e+001 3.63e-001	=	1.22e+001 2.81e-001	1.31e+001 1.66e+000	+	**1.11e+001 4.85e-001**	1.31e+001 1.65e+000	+	**1.10e+001 3.97e-001**
F21	**2.71e+002 7.58e+001**	–	3.12e+002 5.81e+001	1.08e+003 3.19e+002	+	**3.16e+002 7.41e+001**	9.65e+002 3.18e+002	+	**3.08e+002 7.89e+001**
F22	**6.95e+003 3.36e+002**	-	7.16e+003 2.38e+002	**5.44e+003 1.09e+003**	–	6.09e+003 3.71e+002	1.30e+003 4.17e+002	=	1.43e+003 5.35e+002
F23	**7.48e+003 3.34e+002**	–	7.70e+003 2.10e+002	6.44e+003 8.60e+002	=	6.73e+003 2.66e+002	**4.49e+003 2.36e+003**	–	6.08e+003 1.12e+003
F24	2.14e+002 8.99e+000	+	**2.03e+002 5.45e+000**	2.49e+002 1.23e+001	+	**2.16e+002 6.43e+000**	2.46e+002 9.95e+000	+	**2.16e+002 6.79e+000**
F25	2.51e+002 2.24e+001	=	2.50e+002 2.74e+001	2.84e+002 1.57e+001	+	**2.46e+002 1.88e+001**	2.78e+002 1.40e+001	+	**2.55e+002 1.48e+001**
F26	2.08e+002 2.94e+001	=	2.00e+002 6.65e-003	2.52e+002 6.88e+001	=	2.51e+002 5.95e+001	2.73e+002 6.95e+001	=	**2.68e+002 6.06e+001**
F27	4.64e+002 9.06e+001	+	**3.68e+002 1.98e+002**	7.34e+002 8.36e+001	+	**4.60e+002 7.47e+001**	7.37e+002 8.91e+001	+	**4.70e+002 8.01e+001**
F28	3.00e+002 0.00e+000	=	3.00e+002 5.67e-006	1.44e+003 3.58e+002	+	**3.09e+002 2.22e+001**	1.32e+003 3.63e+002	+	**3.36e+002 2.10e+002**
+/=/-	–		11/11/6	–		21/5/2	–		22/5/1

- The neighborhood information is used in different ways. In APT-DE, the selected topology is applied to all the individuals in the current generation; thus, the neighborhood of each individual is defined in the same way [39]. In contrast, each individual in MTDE can independently select its topology based on its fitness values by Eq. (10). In this way, different individuals are equipped with different types of topologies; thus, they have different neighborhood sizes. Moreover, in APT-DE, the parents for mutation are randomly selected from the neighborhood of the current individual. In contrast, MTDE employs the TDM strategy to construct the difference vectors to direct the search in a promising direction.

Therefore, it is expected that a better balance between exploration and exploitation can be achieved by MTDE, compared with APT-DE and Na-DE. It is shown empirically that the performance of MTDE is highly competitive with the performance of these two multi-topology-based DE algorithms in Section 4.7.

Table 2
Mean and standard deviation of the best error values obtained by the original DE and the corresponding MTDE on all the CEC2013 functions at 50*D*.

Fun.	DE/rand/1		MTDE/rand/1	DE/rand/2		MTDE/rand/2	DE/best/1		MTDE/best/1
F1	**5.44e-029 8.45e-029**	-	3.94e-028 2.64e-028	2.14e+002 6.28e+001	+	**1.97e-008 3.40e-008**	1.61e+004 4.36e+003	+	**1.42e-025 7.33e-025**
F2	2.60e+006 9.62e+005	+	**4.57e+005 1.89e+005**	4.01e+008 7.12e+007	+	**3.33e+007 2.38e+007**	1.01e+008 4.07e+007	+	**2.03e+005 9.19e+004**
F3	3.40e+005 6.36e+005	=	5.17e+005 6.82e+005	6.90e+010 7.88e+009	+	**4.97e+009 3.89e+009**	7.55e+009 1.52e+010	+	**2.85e+007 5.11e+007**
F4	3.26e+004 4.68e+003	+	**6.64e+003 1.77e+003**	9.83e+004 7.70e+003	+	**8.67e+004 5.97e+003**	**2.00e-001 2.32e-001**	-	2.66e+001 2.49e+001
F5	**9.47e-016 3.08e-015**	-	4.10e-015 5.76e-015	3.20e+001 6.15e+000	+	**2.64e-005 2.44e-005**	1.98e+003 7.68e+002	+	**2.61e-014 1.34e-014**
F6	**4.35e+001 1.56e-001**	-	4.36e+001 1.04e+000	1.40e+002 3.18e+001	+	**4.35e+001 1.46e-002**	8.68e+002 2.39e+002	+	**4.79e+001 1.18e+001**
F7	**9.88e-001 1.35e+000**	-	4.56e+000 3.24e+000	1.78e+002 1.16e+001	+	**8.24e+001 1.62e+001**	1.20e+002 2.25e+001	+	**5.32e+001 1.06e+001**
F8	2.11e+001 3.52e-002	=	2.11e+001 3.43e-002	2.11e+001 3.10e-002	=	2.11e+001 3.53e-002	2.11e+001 3.94e-002	=	2.11e+001 4.05e-002
F9	7.21e+001 1.44e+000	=	7.17e+001 3.17e+000	7.25e+001 1.46e+000	=	7.23e+001 1.53e+000	5.04e+001 3.72e+000	+	**4.07e+001 8.91e+000**
F10	**3.64e-002 1.54e-002**	-	5.63e-002 2.88e-002	9.94e+002 2.51e+002	+	**8.60e-001 3.32e-001**	1.78e+003 3.91e+002	+	**1.61e-001 9.30e-002**
F11	1.81e+002 6.14e+001	+	**5.02e+001 1.43e+001**	4.77e+002 1.89e+001	+	**3.95e+002 2.08e+001**	4.41e+002 7.18e+001	+	**1.43e+002 3.32e+001**
F12	3.54e+002 9.82e+000	+	**3.22e+002 7.77e+001**	5.11e+002 2.78e+001	+	**4.15e+002 1.79e+001**	4.78e+002 7.41e+001	+	**1.47e+002 3.72e+001**
F13	3.56e+002 1.18e+001	+	**3.44e+002 1.79e+001**	5.29e+002 2.35e+001	+	**4.15e+002 2.04e+001**	5.76e+002 7.53e+001	+	**2.81e+002 5.72e+001**
F14	1.17e+004 8.78e+002	+	**1.98e+003 6.77e+002**	1.33e+004 2.55e+002	=	1.32e+004 3.90e+002	6.59e+003 6.63e+002	+	**3.75e+003 1.05e+003**
F15	1.40e+004 3.71e+002	=	1.39e+004 3.53e+002	1.43e+004 2.36e+002	=	1.42e+004 3.21e+002	**9.61e+003 3.29e+003**	-	1.36e+004 1.39e+003
F16	3.39e+000 2.47e-001	=	3.36e+000 2.45e-001	3.35e+000 2.34e-001	=	3.33e+000 3.06e-001	3.33e+000 2.09e-001	=	3.40e+000 2.64e-001
F17	3.34e+002 2.75e+001	+	**1.06e+002 1.38e+001**	5.52e+002 2.04e+001	+	**4.55e+002 1.23e+001**	7.94e+002 1.42e+002	+	**1.94e+002 3.57e+001**
F18	4.03e+002 1.23e+001	=	3.98e+002 1.25e+001	5.63e+002 2.39e+001	+	**4.63e+002 1.91e+001**	8.41e+002 1.10e+002	+	**4.22e+002 6.59e+001**
F19	2.94e+001 1.77e+000	+	**5.51e+000 2.10e+000**	1.41e+002 6.21e+001	+	**3.59e+001 1.78e+000**	2.10e+004 1.93e+004	+	**1.92e+001 6.95e+000**
F20	2.21e+001 2.58e-001	+	**2.18e+001 2.48e-001**	2.36e+001 1.83e-001	+	**2.32e+001 2.37e-001**	2.28e+001 1.52e+000	+	**2.08e+001 4.85e-001**
F21	7.08e+002 4.09e+002	=	8.38e+002 4.03e+002	6.14e+002 2.31e+002	=	6.73e+002 4.55e+002	3.48e+003 4.57e+002	+	**8.43e+002 3.51e+002**
F22	1.18e+004 1.14e+003	+	**2.08e+003 6.87e+002**	1.40e+004 2.63e+002	=	1.38e+004 4.70e+002	7.88e+003 1.21e+003	+	**4.04e+003 1.01e+003**
F23	1.44e+004 3.80e+002	=	1.44e+004 3.91e+002	1.48e+004 4.05e+002	=	1.47e+004 3.45e+002	**8.45e+003 1.70e+003**	-	1.31e+004 2.42e+003
F24	**2.02e+002 3.25e+000**	-	2.27e+002 1.02e+001	3.95e+002 7.61e+000	+	**2.71e+002 3.25e+001**	3.56e+002 1.56e+001	+	**2.90e+002 1.40e+001**
F25	**2.97e+002 3.72e+001**	-	3.07e+002 1.47e+001	4.60e+002 6.62e+000	+	**4.56e+002 5.64e+000**	4.19e+002 2.06e+001	+	**3.48e+002 1.44e+001**
F26	**2.35e+002 5.42e+001**	-	2.99e+002 6.70e+001	2.46e+002 8.91e+000	+	**2.07e+002 1.33e+001**	4.30e+002 1.20e+001	+	**3.78e+002 4.97e+001**
F27	**4.43e+002 1.38e+002**	-	7.09e+002 1.37e+002	2.27e+003 4.37e+001	+	**2.23e+003 4.50e+001**	1.71e+003 1.14e+002	+	**1.22e+003 1.21e+002**
F28	4.00e+002 0.00e+000	=	4.00e+002 5.67e-006	5.72e+002 2.72e+001	+	**4.00e+002 1.04e-003**	3.64e+003 1.03e+003	+	**5.05e+002 5.76e+002**
+/=/-	-		10/9/9	-		20/8/0	-		23/2/3

Fun.	**DE/best/2**		**MTDE/best/2**	**DE/current-to-best/1**		**MTDE/current-to-best/1**	**DE/rand-to-best/1**		**MTDE/rand-to-best/1**
F1	9.74e-028 3.98e-028	+	**7.65e-028 2.69e-028**	1.21e+004 3.65e+003	+	**6.68e+001 8.97e+001**	7.28e+003 2.00e+003	+	**2.34e+001 2.78e+001**
F2	**7.79e+005 2.85e+005**	-	2.49e+006 9.37e+005	4.42e+007 1.71e+007	+	**4.26e+005 9.50e+005**	3.59e+007 1.57e+007	+	**1.12e+006 1.60e+006**
F3	**1.60e+007 2.42e+007**	-	1.41e+008 6.04e+008	2.76e+010 1.01e+010	+	**1.41e+006 3.07e+006**	2.76e+010 7.59e+009	+	**3.97e+006 8.23e+006**
F4	**8.32e+003 3.76e+003**	-	5.23e+004 1.18e+004	7.04e+001 7.66e+001	+	**3.27e-001 2.25e-001**	7.62e+001 9.34e+001	+	**3.10e-001 1.69e-001**
F5	1.26e-014 5.22e-015	=	1.25e-014 5.03e-015	1.46e+003 4.94e+002	+	**9.14e+001 6.77e+001**	1.26e+003 3.76e+002	+	**6.04e+001 5.21e+001**
F6	4.47e+001 8.92e+000	=	4.40e+001 1.73e+000	5.16e+002 1.18e+002	+	**8.57e+001 3.87e+001**	3.94e+002 9.18e+001	+	**8.04e+001 3.01e+001**
F7	4.45e+001 1.81e+001	+	**2.12e+001 1.07e+001**	7.63e+001 1.03e+001	+	**3.11e+001 7.65e+000**	7.31e+001 1.31e+001	+	**3.41e+001 8.63e+000**
F8	2.11e+001 3.02e-002	=	2.11e+001 3.00e-002	2.11e+001 3.75e-002	=	2.11e+001 3.61e-002	2.11e+001 4.35e-002	=	2.11e+001 3.61e-002
F9	7.16e+001 1.71e+000	=	7.25e+001 1.58e+000	3.64e+001 2.95e+000	+	**2.83e+001 5.32e+000**	3.59e+001 2.66e+000	+	**2.72e+001 4.14e+000**
F10	6.11e-002 4.91e-002	+	**2.62e-002 2.13e-002**	1.24e+003 3.36e+002	+	**3.64e+001 2.00e+001**	9.68e+002 2.60e+002	+	**7.35e+001 2.78e+001**
F11	3.89e+002 2.88e+001	+	**3.64e+002 1.94e+001**	2.68e+002 5.15e+001	+	**9.78e+001 2.17e+001**	2.14e+002 3.37e+001	+	**8.32e+001 2.22e+001**
F12	4.11e+002 3.10e+001	+	**3.88e+002 1.62e+001**	2.61e+002 5.49e+001	+	**1.06e+002 6.55e+001**	2.29e+002 2.93e+001	+	**9.04e+001 2.35e+001**
F13	4.19e+002 3.21e+001	+	**3.90e+002 1.77e+001**	4.14e+002 6.76e+001	+	**3.13e+002 7.00e+001**	3.68e+002 4.82e+001	+	**1.87e+002 4.88e+001**
F14	1.30e+004 3.65e+002	=	1.31e+004 4.00e+002	**1.25e+004 5.20e+002**	-	1.29e+004 3.87e+002	6.92e+003 4.25e+003	+	**3.66e+003 9.44e+002**
F15	1.41e+004 3.67e+002	=	1.41e+004 3.21e+002	1.36e+004 5.07e+002	=	1.36e+004 3.88e+002	1.30e+004 1.74e+003	=	1.35e+004 3.28e+002
F16	3.26e+000 2.98e-001	=	3.31e+000 2.56e-001	3.32e+000 3.36e-001	=	3.35e+000 2.77e-001	3.32e+000 2.83e-001	=	3.35e+000 2.61e-001
F17	4.56e+002 3.57e+001	+	**4.21e+002 2.01e+001**	3.81e+002 1.14e+002	=	3.78e+002 1.57e+001	2.67e+002 4.48e+001	+	**1.28e+002 1.67e+001**
F18	4.88e+002 2.96e+001	+	**4.35e+002 1.80e+001**	5.94e+002 6.45e+001	+	**3.90e+002 1.90e+001**	5.17e+002 5.08e+001	+	**3.83e+002 1.73e+001**
F19	3.59e+001 4.39e+000	+	**3.16e+001 1.92e+000**	6.09e+003 6.79e+003	+	**1.89e+001 6.79e+000**	2.62e+003 2.18e+003	+	**1.80e+001 8.57e+000**
F20	**2.20e+001 4.33e-001**	-	2.25e+001 3.67e-001	2.14e+001 1.04e+000	+	**2.05e+001 4.92e-001**	2.12e+001 7.50e-001	+	**2.03e+001 5.23e-001**
F21	7.60e+002 3.92e+002	=	8.07e+002 4.16e+002	3.22e+003 3.33e+002	+	**8.93e+002 3.73e+002**	2.92e+003 3.61e+002	+	**8.10e+002 3.98e+002**
F22	**1.35e+004 5.10e+002**	-	1.38e+004 4.23e+002	**8.96e+003 4.03e+003**	-	1.25e+004 5.68e+002	3.93e+003 8.04e+002	=	3.68e+003 1.09e+003
F23	1.46e+004 4.70e+002	=	1.46e+004 3.67e+002	1.36e+004 5.30e+002	=	1.35e+004 4.91e+002	**8.97e+003 4.07e+003**	-	1.23e+004 2.82e+003
F24	2.53e+002 1.19e+001	+	**2.26e+002 1.22e+001**	3.27e+002 1.20e+001	+	**2.56e+002 1.15e+001**	3.12e+002 1.23e+001	+	**2.62e+002 1.18e+001**
F25	**3.42e+002 5.87e+001**	-	4.36e+002 2.53e+001	3.99e+002 1.19e+001	+	**3.30e+002 1.14e+001**	3.88e+002 1.19e+001	+	**3.34e+002 9.43e+000**
F26	3.12e+002 1.02e+002	+	**2.50e+002 8.89e+001**	3.86e+002 5.09e+001	+	**3.58e+002 1.13e+001**	3.84e+002 4.99e+001	+	**3.57e+002 3.14e+001**
F27	9.53e+002 2.65e+002	=	1.00e+003 4.77e+002	1.43e+003 1.03e+002	+	**9.26e+002 1.13e+002**	1.38e+003 8.25e+001	+	**1.01e+003 1.16e+002**
F28	4.00e+002 1.39e-005	=	5.01e+002 5.53e+002	3.47e+003 9.51e+002	+	**4.14e+002 1.94e+001**	3.12e+003 8.90e+002	+	**5.36e+002 5.63e+002**
+/=/-	-		11/11/6	-		21/5/2	-		23/4/1

4. Experimental results and analysis

In this section, the performance of the proposed MTDE algorithm is evaluated on a suite of benchmark functions from the CEC2013 and CEC2014 special sessions on real-parameter optimization [23,24]. First, the benchmark functions are presented. Then, the experimental setup is described. Finally, the comparisons between MTDE and the corresponding DE variants are presented.

4.1. Benchmark functions

To evaluate the performance of MTDE, 30 benchmark functions from the CEC2014 special session on real-parameter optimization [24] are used in this study. These functions span a diverse set of problem characteristics, such as multi-modal,

Table 3
Mean and standard deviation of the best error values obtained by the original DE and the corresponding MTDE on all the CEC2014 functions at 30*D*.

Fun.	DE/rand/1		MTDE/rand/1	DE/rand/2		MTDE/rand/2	DE/best/1		MTDE/best/1
F1	7.06e+004 5.19e+004	+	**4.07e+004 2.58e+004**	2.43e+007 5.41e+006	+	**9.69e+005 4.01e+005**	2.77e+007 1.66e+007	+	**7.36e+003 6.41e+003**
F2	1.06e-019 8.72e-020	+	**2.02e-023 5.07e-023**	4.03e+006 1.15e+006	+	**2.64e+000 2.47e+000**	4.85e+009 2.28e+009	+	**3.75e-021 1.69e-020**
F3	3.76e-022 3.54e-022	+	**8.67e-027 8.37e-027**	3.11e+001 6.91e+000	+	**4.10e-005 6.15e-005**	3.20e+003 2.77e+003	+	**1.46e-006 6.40e-006**
F4	1.97e-001 9.47e-002	+	**1.80e-005 4.72e-005**	1.78e+002 1.45e+001	+	**1.24e+001 2.32e+001**	3.99e+002 1.34e+002	+	**2.77e+001 4.15e+001**
F5	2.09e+001 3.96e-002	=	2.09e+001 6.08e-002	2.09e+001 4.61e-002	=	2.09e+001 6.42e-002	**2.06e+001 2.65e-001**	-	2.08e+001 2.56e-001
F6	**1.80e-004 2.92e-004**	-	8.29e-001 1.58e+000	3.50e+001 1.03e+000	+	**3.35e+001 4.43e+000**	1.82e+001 2.19e+000	+	**7.49e+000 2.61e+000**
F7	0.00e+000 0.00e+000	=	1.31e-003 3.02e-003	9.56e-001 4.12e-002	+	**2.12e-003 4.42e-003**	4.24e+001 2.17e+001	+	**1.19e-002 1.50e-002**
F8	1.35e+002 2.16e+001	+	**2.13e+001 7.09e+000**	2.14e+002 1.32e+001	+	**2.06e+002 1.14e+001**	9.17e+001 2.39e+001	+	**4.74e+001 9.40e+000**
F9	1.78e+002 1.04e+001	+	**7.52e+001 5.75e+001**	2.37e+002 8.57e+000	+	**2.15e+002 8.64e+000**	1.12e+002 2.76e+001	+	**4.37e+001 1.35e+001**
F10	5.64e+003 6.73e+002	+	**3.67e+002 2.44e+002**	6.36e+003 2.58e+002	=	6.35e+003 2.17e+002	2.26e+003 4.82e+002	+	**1.10e+003 4.30e+002**
F11	6.87e+003 2.39e+002	=	6.64e+003 9.73e+002	7.00e+003 2.51e+002	=	7.00e+003 3.03e+002	3.07e+003 5.72e+002	+	**2.74e+003 5.50e+002**
F12	2.36e+000 3.14e-001	=	2.33e+000 4.36e-001	2.45e+000 2.75e-001	=	2.41e+000 2.48e-001	3.43e-001 4.46e-001	=	5.59e-001 7.47e-001
F13	3.66e-001 3.58e-002	+	**2.94e-001 4.03e-002**	5.82e-001 5.57e-002	+	**4.92e-001 6.18e-002**	6.54e-001 4.75e-001	+	**2.98e-001 5.44e-002**
F14	2.62e-001 3.51e-002	=	2.56e-001 2.93e-002	3.28e-001 4.28e-002	+	**2.93e-001 3.23e-002**	1.12e+001 7.89e+000	+	**2.65e-001 5.25e-002**
F15	1.53e+001 1.03e+000	+	**1.36e+001 2.96e+000**	2.26e+001 1.41e+000	+	**1.83e+001 1.22e+000**	3.68e+002 6.25e+002	+	**4.33e+000 1.57e+000**
F16	1.26e+001 1.76e-001	+	**1.22e+001 7.26e-001**	1.28e+001 1.89e-001	=	1.28e+001 2.14e-001	1.07e+001 7.78e-001	+	**1.00e+001 9.42e-001**
F17	1.47e+003 1.52e+002	+	**1.02e+003 6.40e+002**	2.14e+003 1.75e+002	=	2.03e+003 1.72e+002	6.64e+004 6.40e+004	+	**5.77e+002 4.90e+002**
F18	5.40e+001 5.50e+000	+	**1.74e+001 1.59e+001**	1.05e+002 9.15e+000	+	**8.74e+001 7.14e+000**	1.17e+003 1.86e+003	+	**3.13e+001 1.75e+001**
F19	4.51e+000 2.92e-001	+	**2.35e+000 8.17e-001**	1.14e+001 1.09e+000	+	**5.91e+000 9.34e-001**	3.74e+001 2.97e+001	+	**3.74e+000 1.06e+000**
F20	3.17e+001 7.72e+000	+	**1.10e+001 4.52e+000**	7.59e+001 7.57e+000	+	**5.74e+001 7.11e+000**	3.65e+002 6.30e+002	+	**1.71e+001 7.43e+000**
F21	7.50e+002 1.68e+002	+	**2.26e+002 2.20e+002**	1.40e+003 1.32e+002	+	**1.27e+003 1.39e+002**	8.65e+003 1.74e+004	+	**2.79e+002 1.65e+002**
F22	8.07e+001 7.40e+001	=	1.20e+002 1.09e+002	5.15e+002 8.95e+001	=	5.05e+002 8.69e+001	4.22e+002 1.46e+002	+	**2.31e+002 1.31e+002**
F23	3.15e+002 0.00e+000	=	3.15e+002 1.13e-005	3.15e+002 3.35e-002	+	**3.15e+002 0.00e+000**	3.33e+002 6.52e+000	+	**3.15e+002 4.01e-006**
F24	**2.18e+002 8.94e+000**	-	2.24e+002 1.22e+000	2.21e+002 2.58e+000	+	**2.03e+002 5.50e+000**	2.61e+002 6.09e+000	+	**2.34e+002 7.20e+000**
F25	**2.03e+002 2.15e-001**	-	2.03e+002 2.84e-001	2.15e+002 1.26e+000	+	**2.06e+002 9.68e-001**	2.16e+002 5.15e+000	+	**2.04e+002 1.31e+000**
F26	**1.00e+002 4.36e-002**	-	1.00e+002 4.61e-002	1.01e+002 6.02e-002	+	**1.01e+002 4.16e-002**	1.21e+002 4.09e+001	+	**1.00e+002 6.18e-002**
F27	3.60e+002 5.01e+001	=	3.38e+002 4.88e+001	4.74e+002 9.05e+001	+	**3.65e+002 4.32e+001**	6.60e+002 1.98e+002	+	**4.62e+002 8.65e+001**
F28	**7.94e+002 2.19e+001**	-	8.52e+002 4.22e+001	1.36e+003 3.18e+001	+	**1.21e+003 3.77e+001**	1.64e+003 3.86e+002	+	**9.36e+002 7.74e+001**
F29	**2.54e+002 1.63e+002**	-	6.51e+002 2.15e+002	1.12e+003 1.07e+002	+	**3.58e+002 4.53e+001**	5.76e+004 2.93e+005	+	**5.83e+002 1.87e+002**
F30	5.77e+002 1.40e+002	+	**4.89e+002 1.14e+002**	2.50e+003 2.55e+002	+	**1.74e+003 2.04e+002**	1.06e+004 9.58e+003	+	**1.37e+003 5.40e+002**
+/=/-	-		16/8/6	-		23/7/0	-		28/1/1
Fun.	**DE/best/2**		**MTDE/best/2**	**DE/current-to-best/1**		**MTDE/current-to-best/1**	**DE/rand-to-best/1**		**MTDE/rand-to-best/1**
F1	**1.69e+004 1.34e+004**	-	1.05e+005 1.00e+005	1.23e+007 9.42e+006	+	**2.42e+003 2.70e+003**	1.44e+007 7.94e+006	+	**2.18e+003 2.77e+003**
F2	2.20e-022 2.47e-022	=	1.46e-022 1.07e-022	2.53e+009 1.29e+009	+	**5.27e+005 1.23e+006**	1.41e+009 1.09e+009	+	**2.45e+005 8.85e+005**
F3	**7.24e-027 1.42e-026**	-	1.74e-025 6.50e-026	2.81e+003 2.09e+003	+	**1.01e-002 2.68e-002**	3.00e+003 2.91e+003	+	**6.24e-003 2.72e-002**
F4	**3.55e-010 9.20e-010**	-	2.30e+000 1.19e+001	2.99e+002 9.58e+001	+	**8.83e+001 3.80e+001**	2.71e+002 7.21e+001	+	**9.88e+001 3.44e+001**
F5	2.09e+001 5.91e-002	=	2.09e+001 5.27e-002	2.10e+001 4.67e-002	=	2.10e+001 3.72e-002	2.09e+001 5.14e-002	=	2.09e+001 6.97e-002
F6	1.17e+000 1.12e+000	+	**4.09e-001 1.42e+000**	1.03e+001 2.29e+000	+	**1.96e+000 1.13e+000**	9.63e+000 1.71e+000	+	**2.16e+000 1.32e+000**
F7	9.35e-003 1.18e-002	=	4.60e-003 7.67e-003	3.09e+001 1.53e+001	+	**2.81e-001 3.06e-001**	1.48e+001 9.94e+000	+	**3.70e-002 5.43e-002**
F8	1.83e+002 1.20e+001	=	1.83e+002 1.65e+001	5.92e+001 1.16e+001	+	**3.49e+001 2.25e+001**	4.82e+001 1.01e+001	+	**2.85e+001 8.89e+000**
F9	1.98e+002 1.50e+001	=	1.95e+002 9.90e+000	6.52e+001 1.96e+001	+	**4.42e+001 3.89e+001**	5.04e+001 1.03e+001	+	**2.82e+001 9.26e+000**
F10	6.17e+003 1.91e+002	=	6.23e+003 3.36e+002	**1.44e+003 1.19e+003**	-	4.52e+003 1.37e+003	1.00e+003 2.88e+002	+	**7.34e+002 3.96e+002**
F11	6.91e+003 2.69e+002	=	7.01e+003 2.33e+002	**6.00e+003 9.20e+002**	-	6.42e+003 3.18e+002	**2.80e+003 2.06e+003**	-	4.59e+003 2.02e+003
F12	2.43e+000 2.91e-001	=	2.45e+000 2.16e-001	2.38e+000 2.69e-001	=	2.47e+000 2.60e-001	2.48e+000 2.07e-001	=	2.34e+000 4.80e-001
F13	3.57e-001 7.41e-002	=	3.58e-001 6.11e-002	5.61e-001 5.75e-001	+	**2.18e-001 4.41e-002**	3.62e-001 5.48e-002	+	**2.04e-001 3.45e-002**
F14	**2.48e-001 4.27e-002**	-	2.99e-001 1.03e-001	4.61e+000 5.59e+000	+	**2.42e-001 3.85e-002**	5.41e-001 1.47e+000	=	2.61e-001 2.67e-002
F15	1.59e+001 1.33e+000	=	1.59e+001 1.21e+000	2.99e+001 3.18e+001	+	**1.01e+001 3.00e+000**	1.24e+001 9.70e+000	+	**3.95e+000 2.53e+000**
F16	**1.25e+001 2.66e-001**	-	1.27e+001 1.96e-001	1.10e+001 4.67e-001	=	1.12e+001 4.36e-001	1.06e+001 3.96e-001	+	**1.04e+001 4.54e-001**
F17	2.00e+003 5.66e+002	+	**1.70e+003 3.23e+002**	1.44e+004 1.77e+004	+	**4.83e+002 2.90e+002**	5.06e+004 1.02e+005	+	**6.96e+002 5.26e+002**
F18	6.83e+001 1.19e+001	=	6.41e+001 7.91e+000	1.33e+002 3.63e+001	+	**4.88e+001 2.35e+001**	1.61e+002 4.79e+001	+	**5.33e+001 2.30e+001**
F19	7.22e+000 1.08e+001	=	5.06e+000 8.89e-001	2.53e+001 1.85e+001	+	**4.14e+000 1.11e+000**	2.13e+001 2.08e+001	+	**5.16e+000 1.29e+000**
F20	4.16e+001 1.05e+001	=	3.98e+001 8.60e+000	2.51e+002 4.88e+002	+	**2.16e+001 1.35e+001**	1.95e+002 2.48e+002	+	**2.31e+001 1.31e+001**
F21	8.74e+002 3.40e+002	=	9.12e+002 1.40e+002	3.29e+003 4.42e+003	+	**2.32e+002 1.25e+002**	5.87e+003 6.21e+003	+	**2.29e+002 1.46e+002**
F22	**2.26e+002 1.92e+002**	-	3.66e+002 1.75e+002	2.60e+002 7.32e+001	+	**1.55e+002 5.38e+001**	2.50e+002 1.02e+002	+	**1.72e+002 8.35e+001**
F23	3.15e+002 0.00e+000	=	3.15e+002 1.13e-005	3.28e+002 5.16e+000	+	**3.15e+002 1.48e-001**	3.27e+002 6.28e+000	+	**3.15e+002 3.65e-001**
F24	2.30e+002 6.68e+000	+	**2.21e+002 7.67e+000**	2.42e+002 6.64e+000	+	**2.28e+002 5.12e+000**	2.39e+002 6.06e+000	+	**2.29e+002 5.98e+000**
F25	2.03e+002 4.55e-001	+	**2.03e+002 2.16e-001**	2.11e+002 1.89e+000	+	**2.04e+002 9.01e-001**	2.11e+002 1.68e+000	+	**2.05e+002 1.33e+000**
F26	1.00e+002 9.06e-002	=	1.00e+002 5.61e-002	1.31e+002 4.64e+001	+	**1.04e+002 1.82e+001**	1.24e+002 4.30e+001	+	**1.07e+002 2.53e+001**
F27	3.63e+002 3.76e+001	+	**3.45e+002 4.69e+001**	5.28e+002 1.23e+002	+	**3.76e+002 4.14e+001**	5.05e+002 9.37e+001	+	**3.94e+002 5.04e+001**
F28	8.07e+002 8.22e+001	=	8.18e+002 5.46e+001	1.11e+003 1.26e+002	+	**8.68e+002 5.58e+001**	1.17e+003 2.12e+002	+	**8.88e+002 4.50e+001**
F29	**6.60e+002 2.48e+002**	-	7.98e+002 2.18e+002	4.39e+003 1.73e+004	+	**5.95e+002 2.03e+002**	1.79e+003 2.44e+003	+	**6.77e+002 3.15e+002**
F30	5.71e+002 1.48e+002	=	6.22e+002 2.86e+002	1.38e+004 2.58e+004	+	**9.22e+002 3.55e+002**	1.09e+004 1.14e+004	+	**1.59e+003 8.18e+002**
+/=/-	-		5/18/7	-		25/3/2	-		26/3/1

ill-conditioned, asymmetrical, and rotated. They can be categorized into four groups: unimodal functions (F1–F3), simple multimodal functions (F4–F16), hybrid multimodal functions (F17–F22) and composition multimodal functions (F23–F30). In addition, a suite of benchmark functions from the CEC2013 special session on real-parameter optimization [23] is used to evaluate the performance of MTDE, which includes 5 unimodal functions (F1–F5), 15 basic multimodal functions (F6–F20) and 8 composition functions (F21–F28). Detailed descriptions of the CEC2014 and CEC2013 benchmark functions can be found in [23,24], and their source codes are available from http://www3.ntu.edu.sg/home/EPNSugan/.

Table 4
Mean and standard deviation of the best error values obtained by the original DE and the corresponding CMT-DE on all the CEC2014 functions at 50*D*.

Fun.	DE/rand/1		MTDE/rand/1	DE/rand/2		MTDE/rand/2	DE/best/1		MTDE/best/1
F1	1.18e+006 4.02e+005	+	**4.25e+005 1.80e+005**	3.22e+008 5.11e+007	+	**1.13e+007 5.05e+006**	1.35e+008 5.67e+007	+	**6.92e+004 3.51e+004**
F2	3.19e+002 7.66e+002	+	**3.70e-005 1.66e-004**	7.22e+008 1.99e+008	+	**4.48e+003 5.31e+003**	2.94e+010 8.34e+009	+	**5.57e+003 6.99e+003**
F3	1.39e-001 2.01e-001	=	3.70e-001 6.56e-001	9.50e+003 1.95e+003	+	**3.77e+001 3.27e+001**	5.72e+003 8.30e+003	+	**8.47e-002 2.30e-001**
F4	7.06e+001 2.67e+001	+	**3.48e+001 3.56e+001**	6.81e+002 1.09e+002	+	**8.70e+001 1.62e+001**	3.08e+003 1.05e+003	+	**8.00e+001 4.22e+001**
F5	2.11e+001 3.88e-002	=	2.11e+001 2.21e-002	2.11e+001 3.17e-002	=	2.11e+001 3.54e-002	2.10e+001 2.24e-001	=	2.10e+001 2.25e-001
F6	**2.28e-001 5.08e-001**	-	4.60e+000 2.45e+000	6.70e+001 1.07e+000	+	**6.39e+001 6.54e+000**	4.26e+001 3.44e+000	+	**2.10e+001 3.12e+000**
F7	**2.47e-004 1.35e-003**	-	2.47e-003 3.88e-003	6.49e+000 1.60e+000	+	**7.48e-004 2.26e-003**	2.94e+002 7.67e+001	+	**2.97e-002 3.83e-002**
F8	1.95e+002 6.00e+001	+	**4.98e+001 1.30e+001**	4.56e+002 1.85e+001	+	**4.21e+002 1.61e+001**	2.60e+002 3.78e+001	+	**1.00e+002 1.84e+001**
F9	3.50e+002 1.37e+001	+	**1.05e+002 9.41e+001**	4.86e+002 1.91e+001	+	**4.22e+002 2.08e+001**	3.04e+002 4.05e+001	+	**1.08e+002 1.51e+001**
F10	1.06e+004 1.32e+003	+	**1.38e+003 4.74e+002**	1.25e+004 4.24e+002	=	1.25e+004 3.49e+002	5.81e+003 8.04e+002	+	**3.16e+003 9.14e+002**
F11	1.32e+004 3.18e+002	=	1.31e+004 3.65e+002	1.33e+004 4.21e+002	=	1.34e+004 3.32e+002	7.48e+003 8.27e+002	+	**5.33e+003 1.02e+003**
F12	3.37e+000 2.58e-001	=	3.34e+000 3.15e-001	3.39e+000 3.41e-001	=	3.27e+000 2.64e-001	**6.11e-001 7.09e-001**	-	2.02e+000 1.53e+000
F13	4.65e-001 4.48e-002	+	**3.93e-001 4.26e-002**	7.83e-001 8.39e-002	+	**6.36e-001 6.42e-002**	3.48e+000 6.20e-001	+	**4.53e-001 8.52e-002**
F14	3.15e-001 3.24e-002	=	3.23e-001 1.04e-001	4.55e-001 1.11e-001	+	**4.18e-001 2.25e-001**	7.80e+001 2.06e+001	+	**3.30e-001 9.96e-002**
F15	3.11e+001 1.22e+000	+	**2.70e+001 8.12e+000**	5.74e+002 3.56e+002	+	**3.55e+001 2.06e+000**	3.09e+004 3.13e+004	+	**1.96e+001 6.03e+000**
F16	2.23e+001 2.11e-001	+	**2.22e+001 2.40e-001**	2.25e+001 2.20e-001	=	2.25e+001 1.82e-001	1.97e+001 8.61e-001	=	1.99e+001 1.18e+000
F17	**1.36e+004 9.72e+003**	-	2.79e+004 1.80e+004	8.72e+004 3.34e+004	+	**5.32e+003 6.06e+002**	9.59e+005 1.27e+006	+	**4.38e+003 2.28e+003**
F18	**1.34e+002 9.88e+000**	-	3.16e+002 2.96e+002	3.40e+002 2.15e+001	+	**2.01e+002 1.47e+001**	1.92e+006 3.23e+006	+	**1.37e+002 3.53e+001**
F19	1.11e+001 1.23e+000	+	**8.46e+000 3.14e+000**	2.65e+001 8.47e-001	+	**1.21e+001 1.90e+000**	1.23e+002 2.75e+001	+	**1.02e+001 2.02e+000**
F20	9.95e+001 1.12e+001	+	**7.81e+001 4.70e+001**	3.10e+002 2.89e+001	+	**1.40e+002 1.02e+001**	2.01e+003 2.57e+003	+	**5.96e+001 4.58e+001**
F21	**2.70e+003 7.17e+002**	-	7.98e+003 1.04e+004	4.34e+003 3.75e+002	+	**3.21e+003 2.77e+002**	2.60e+005 2.69e+005	+	**1.73e+003 1.30e+003**
F22	8.93e+002 3.37e+002	=	8.07e+002 3.64e+002	1.60e+003 1.39e+002	+	**1.51e+003 1.47e+002**	1.03e+003 2.98e+002	+	**7.71e+002 3.13e+002**
F23	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	3.48e+002 6.95e-001	+	**3.44e+002 5.67e-006**	4.60e+002 4.57e+001	+	**3.44e+002 1.13e-005**
F24	**2.70e+002 2.28e+000**	-	2.75e+002 2.71e+000	3.07e+002 3.07e+000	+	**2.71e+002 3.01e+000**	3.49e+002 1.56e+001	+	**2.86e+002 4.97e+000**
F25	**2.06e+002 4.84e-001**	-	2.07e+002 9.26e-001	2.60e+002 6.56e+000	+	**2.16e+002 4.31e+000**	2.45e+002 8.26e+000	+	**2.12e+002 4.86e+000**
F26	1.00e+002 4.92e-002	=	1.07e+002 2.53e+001	1.01e+002 9.97e-002	+	**1.01e+002 7.13e-002**	1.95e+002 3.08e+001	+	**1.01e+002 2.03e-001**
F27	**3.28e+002 3.30e+001**	-	4.76e+002 6.46e+001	1.93e+003 3.46e+001	+	**5.79e+002 2.41e+002**	1.48e+003 1.04e+002	+	**9.01e+002 8.71e+001**
F28	**1.14e+003 4.18e+001**	-	1.27e+003 7.60e+001	4.86e+003 3.54e+002	+	**2.75e+003 3.66e+002**	4.58e+003 1.03e+003	+	**1.84e+003 3.88e+002**
F29	**7.50e+002 2.17e+002**	-	9.73e+002 2.60e+002	4.42e+005 1.69e+005	+	**1.65e+003 5.69e+002**	5.31e+006 1.07e+007	+	**8.91e+002 1.02e+002**
F30	9.46e+003 2.75e+002	=	9.39e+003 3.92e+002	3.83e+004 3.22e+003	+	**2.06e+004 9.21e+002**	1.19e+005 7.47e+004	+	**1.03e+004 1.10e+003**
+/=/-	-		11/9/10	-		25/5/0	-		27/2/1

Fun.	DE/best/2		MTDE/best/2	DE/current-to-best/1		MTDE/current-to-best/1	DE/rand-to-best/1		MTDE/rand-to-best/1
F1	**4.75e+005 2.51e+005**	-	1.23e+006 5.13e+005	7.32e+007 3.65e+007	+	**1.04e+006 1.16e+006**	6.84e+007 3.25e+007	+	**1.77e+006 1.44e+006**
F2	**6.58e-008 1.62e-007**	-	3.56e-004 7.89e-004	1.98e+010 3.97e+009	+	**8.28e+007 7.77e+007**	1.40e+010 4.16e+009	+	**5.57e+007 4.91e+007**
F3	**1.47e-001 4.10e-001**	-	1.65e+000 2.66e+000	5.43e+003 4.62e+003	+	**1.88e-009 6.06e-009**	4.24e+003 2.21e+003	+	**2.23e-009 7.35e-009**
F4	**1.89e+001 2.63e+001**	-	3.92e+001 3.75e+001	2.55e+003 8.09e+002	+	**1.84e+002 4.27e+001**	1.67e+003 4.66e+002	+	**2.09e+002 4.42e+001**
F5	2.11e+001 3.31e-002	=	2.11e+001 4.16e-002	2.11e+001 3.12e-002	=	2.11e+001 3.01e-002	2.11e+001 3.32e-002	=	2.11e+001 3.05e-002
F6	7.04e+000 2.52e+000	+	**3.25e+000 1.47e+000**	2.94e+001 2.61e+000	+	**1.38e+001 2.28e+000**	2.75e+001 2.83e+000	+	**1.24e+001 1.99e+000**
F7	4.93e-003 6.69e-003	=	2.71e-003 5.69e-003	2.33e+002 5.10e+001	+	**2.14e+000 1.28e+000**	1.38e+002 2.99e+001	+	**2.00e+000 1.29e+000**
F8	3.75e+002 2.29e+001	=	3.75e+002 1.81e+001	1.48e+002 2.73e+001	+	**7.59e+001 1.08e+001**	1.34e+002 2.81e+001	+	**7.00e+001 1.58e+001**
F9	3.94e+002 2.44e+001	+	**3.78e+002 2.06e+001**	1.55e+002 2.27e+001	+	**7.12e+001 1.16e+001**	1.58e+002 2.17e+001	+	**7.06e+001 1.44e+001**
F10	**1.23e+004 4.52e+002**	-	1.26e+004 3.27e+002	**3.76e+003 1.55e+003**	-	8.68e+003 3.43e+003	3.05e+003 6.66e+002	=	2.96e+003 8.91e+002
F11	1.33e+004 3.25e+002	=	1.32e+004 4.16e+002	1.20e+004 1.29e+003	=	1.24e+004 5.25e+002	7.44e+003 3.98e+003	=	9.54e+003 3.66e+003
F12	3.31e+000 2.99e-001	=	3.32e+000 2.14e-001	3.37e+000 2.54e-001	=	3.32e+000 2.47e-001	3.25e+000 3.08e-001	=	3.18e+000 6.19e-001
F13	4.84e-001 8.18e-002	=	5.13e-001 6.74e-002	2.83e+000 9.72e-001	+	**4.05e-001 5.12e-002**	1.31e+000 8.09e-001	+	**3.80e-001 6.84e-002**
F14	3.50e-001 1.54e-001	=	3.31e-001 9.72e-002	5.16e+001 1.46e+001	+	**3.07e-001 3.74e-002**	3.25e+001 1.23e+001	+	**3.19e-001 3.73e-002**
F15	3.41e+001 2.02e+000	+	**3.18e+001 1.93e+000**	6.72e+003 8.13e+003	+	**1.67e+001 9.27e+000**	1.73e+003 1.59e+003	+	**1.55e+001 7.82e+000**
F16	**2.22e+001 2.76e-001**	-	2.25e+001 1.60e-001	2.06e+001 4.53e-001	=	2.06e+001 3.94e-001	2.00e+001 4.74e-001	=	2.01e+001 5.72e-001
F17	3.37e+004 1.37e+004	+	**1.84e+004 1.09e+004**	3.63e+005 4.75e+005	+	**5.00e+003 1.31e+004**	4.29e+005 4.47e+005	+	**4.77e+004 3.53e+004**
F18	4.45e+002 6.18e+002	+	**2.90e+002 6.20e+002**	2.53e+005 7.22e+005	+	**1.67e+002 1.37e+002**	3.14e+005 1.32e+006	+	**6.83e+002 7.15e+002**
F19	2.24e+001 1.35e+001	+	**1.28e+001 1.02e+000**	9.87e+001 2.49e+001	+	**3.88e+001 2.21e+001**	9.12e+001 2.10e+001	+	**4.87e+001 2.10e+001**
F20	1.49e+002 3.03e+001	+	**1.21e+002 1.75e+001**	4.41e+002 2.16e+002	+	**8.10e+001 2.45e+001**	6.56e+002 6.25e+002	+	**6.09e+001 2.77e+001**
F21	9.89e+003 1.25e+004	+	**4.59e+003 2.21e+003**	1.11e+005 9.26e+004	+	**7.29e+002 3.37e+002**	1.69e+005 1.27e+005	+	**2.44e+003 8.21e+003**
F22	**1.32e+003 2.22e+002**	-	1.46e+003 1.69e+002	5.12e+002 2.01e+002	+	**2.63e+002 2.65e+002**	5.80e+002 2.22e+002	+	**3.73e+002 1.98e+002**
F23	3.44e+002 1.20e-005	=	3.44e+002 1.27e-005	4.76e+002 3.58e+001	+	**3.45e+002 1.28e+000**	4.43e+002 2.91e+001	+	**3.46e+002 3.13e+000**
F24	2.79e+002 5.03e+000	+	**2.75e+002 4.19e+000**	3.08e+002 9.17e+000	+	**2.80e+002 3.62e+000**	2.99e+002 5.40e+000	+	**2.79e+002 3.47e+000**
F25	2.07e+002 1.03e+000	+	**2.07e+002 7.28e-001**	2.35e+002 5.08e+000	+	**2.16e+002 4.25e+000**	2.34e+002 4.42e+000	+	**2.18e+002 3.71e+000**
F26	1.14e+002 3.45e+001	=	1.10e+002 3.04e+001	1.95e+002 2.53e+001	+	**1.44e+002 5.01e+001**	1.98e+002 1.84e+001	+	**1.41e+002 4.92e+001**
F27	5.58e+002 6.04e+001	+	**4.30e+002 6.16e+001**	1.18e+003 8.13e+001	+	**6.72e+002 7.10e+001**	1.09e+003 1.01e+002	+	**6.69e+002 7.98e+001**
F28	**1.27e+003 1.44e+002**	-	1.47e+003 3.30e+002	3.04e+003 4.26e+002	+	**1.47e+003 1.51e+002**	2.95e+003 3.35e+002	+	**1.51e+003 1.83e+002**
F29	1.14e+003 4.45e+002	=	1.31e+003 5.23e+002	1.01e+007 2.33e+007	+	**1.01e+003 1.93e+002**	5.43e+005 1.61e+006	+	**1.23e+003 3.09e+002**
F30	**9.48e+003 7.76e+002**	-	1.45e+004 1.80e+003	1.56e+005 1.63e+005	+	**1.22e+004 2.10e+003**	9.50e+004 4.92e+004	+	**1.18e+004 1.60e+003**
+/=/-	-		11/10/9	-		25/4/1	-		25/5/0

4.2. Experimental setup

For fair comparison, the same initial random population is used to evaluate the different algorithms, and the parameters for all the DE algorithms are set as follows, unless a change is mentioned:

- *Population size (NP)*: 100
- *Scaling factor (F)*: 0.5

Table 5
Results of the multiple-problem analysis by the Wilcoxon test between the original DE and MTDE for all the CEC2013 functions at 30*D* and 50*D*.

Algorithm at 30*D*	+/=/-	R+	R-	p-value	Á = 0.05	Á = 0.1
MTDE/rand/1 vs DE/rand/1	14/6/8	280	126	0.075916	No	Yes
MTDE/rand/2 vs DE/rand/2	22/6/0	354.5	23.5	0.00006	Yes	Yes
MTDE/best/1 vs DE/best/1	23/2/3	355.5	50.5	0.000494	Yes	Yes
MTDE/best/2 vs DE/best/2	11/11/6	225.5	152.5	0.371163	No	No
MTDE/current-to-best/1 vs DE/current-to-best/1	21/5/2	341	65	0.001611	Yes	Yes
MTDE/rand-to-best/1 vs DE/rand-to-best/1	22/5/1	321	57	0.001456	Yes	Yes
Algorithm at 50*D*	+/=/-	R+	R-	p-value	Á = 0.05	Á = 0.1
MTDE/rand/1 vs DE/rand/1	10/9/9	236.5	141.5	0.247406	No	No
MTDE/rand/2 vs DE/rand/2	20/8/0	370	8	0.000007	Yes	Yes
MTDE/best/1 vs DE/best/1	23/2/3	328	50	0.000804	Yes	Yes
MTDE/best/2 vs DE/best/2	11/11/6	151.5	226.5	1	No	No
MTDE/current-to-best/1 vs DE/current-to-best/1	21/5/2	360.5	45.5	0.000321	Yes	Yes
MTDE/rand-to-best/1 vs DE/rand-to-best/1	23/4/1	336	42	0.000395	Yes	Yes

Table 6
Results of the multiple-problem analysis by the Wilcoxon test between the original DE and MTDE for all the CEC2014 functions at 30*D* and 50*D*.

Algorithm at 30*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTDE/rand/1 vs DE/rand/1	16/8/6	357	108	0.010139	Yes	Yes
MTDE/rand/2 vs DE/rand/2	23/7/0	430	5	0.000004	Yes	Yes
MTDE/best/1 vs DE/best/1	28/1/1	462	3	0.000002	Yes	Yes
MTDE/best/2 vs DE/best/2	5/18/7	181.5	283.5	1	No	No
MTDE/current-to-best/1 vs DE/current-to-best/1	25/3/2	387	48	0.000237	Yes	Yes
MTDE/rand-to-best/1 vs DE/rand-to-best/1	26/3/1	412	23	0.000025	Yes	Yes
Algorithm at 50*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTDE/rand/1 vs DE/rand/1	11/9/10	255.5	209.5	0.628843	No	No
MTDE/rand/2 vs DE/rand/2	25/5/0	443	22	0.000014	Yes	Yes
MTDE/best/1 vs DE/best/1	27/2/1	432	3	0.000003	Yes	Yes
MTDE/best/2 vs DE/best/2	11/10/9	246	219	0.772812	No	No
MTDE/current-to-best/1 vs DE/current-to-best/1	25/4/1	425.5	39.5	0.000069	Yes	Yes
MTDE/rand-to-best/1 vs DE/rand-to-best/1	25/5/0	412	23	0.000025	Yes	Yes

- *Crossover factor (Cr)*: 0.9
- *Dimension of each function (D)*: 30 and 50
- *Maximum number of function evaluations (MaxNFEs)*: $10^4 \times D$
- *Independent number of runs (NumR)*: 30
- *Neighborhood size of small-world topology (N_{sw})*: $0.1 \times NP$
- *Random probability of each edge being reconnected for small-world topology (p)*: 0.05
- *Neighborhood size of cellular topology (N_c)*: 13
- *Neighborhood radius of ring topology (R_r)*: $0.1 \times NP$

In the experiments, six original DE algorithms (DE/rand/1, DE/rand/2, DE/best/1, DE/best/2, DE/current-to-best/1 and DE/rand-to-best/1) and six advanced DE variants (the composite DE (CoDE) [46], the adaptive DE with extern archive algorithm (JADE) [50], the modified DE with *p*-best crossover (MDEpBX) [20], the opposition DE (ODE) [35], DE with strategy adaptation (SaDE) [33] and success-history based adaptive DE (SHADE) [40]) are selected for comparison. All the parameters of these DE variants are set as in their original papers.

To evaluate the significance of the differences among the competitors, several non-parametric statistical tests are carried out by the KEEL software [1]. First, the results of single-problem analysis by the Wilcoxon test [9] at $\alpha = 0.05$ are listed in the tables as " + ", " = ", and " - ", which denote that MTDE wins, ties and loses, respectively, on the corresponding functions, compared with its competitor. Then, the multiple-problem analysis by the Wilcoxon test [9] is used to identify differences between a pair of algorithms on all functions. In addition, the Friedman test [9] is employed to obtain the ranking values of multiple algorithms on all functions.

4.3. Effect on original DE algorithms

MTDE is incorporated into the original DE algorithms to evaluate its performance on the functions in CEC2013 [23] and CEC2014 [24]. Six original DE mutation strategies (see Eqs. (1)–(6)) are used for study. The means and standard deviations of the best error values for all the functions are shown in Tables 1–4, respectively. In these tables, the better values of mean solution error, compared to the competitors, are highlighted in bold. The statistical comparison results are also summarized

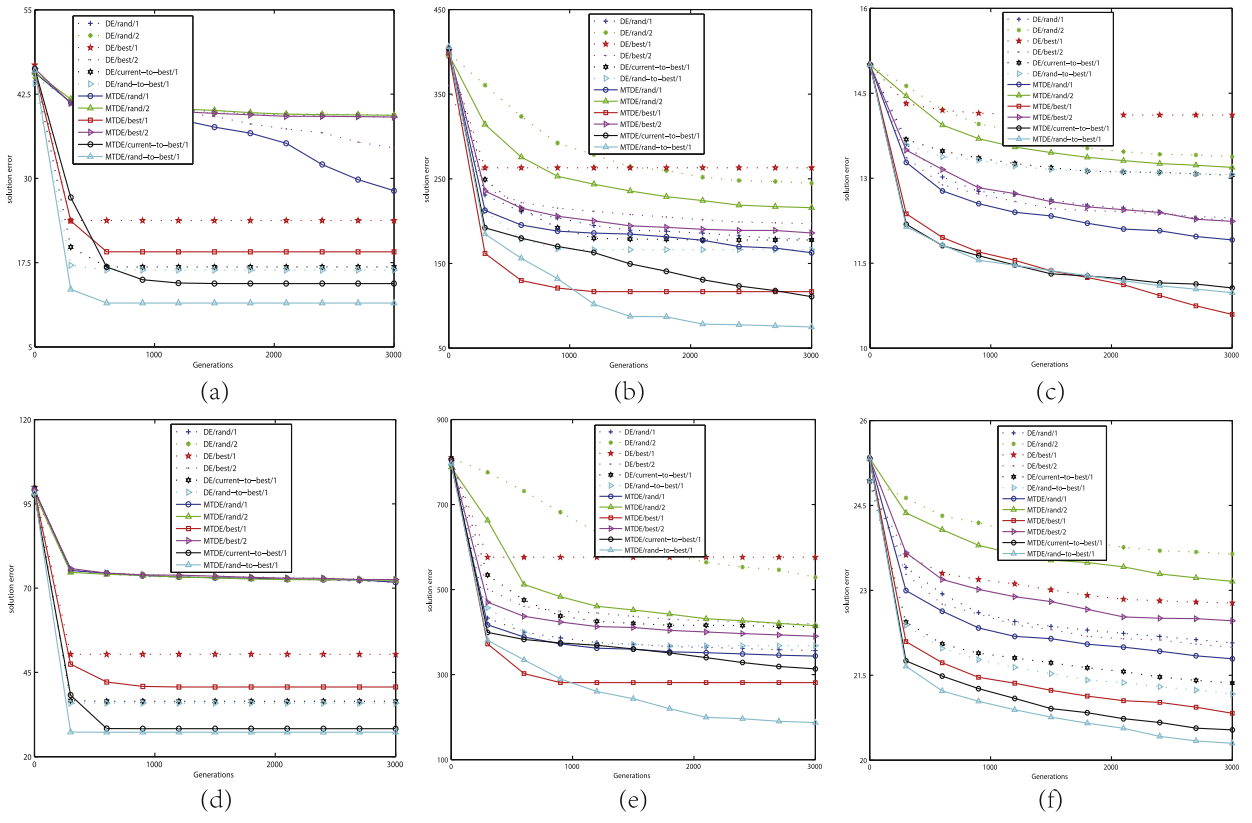

Fig. 5. Convergence graphs of MTDE and the corresponding original DE algorithms for the selected CEC2013 functions. (a) F9, 30*D*. (b) F13, 30*D*. (c) F20, 30*D*. (d) F9, 50*D*. (e) F13, 50*D*. (f) F20, 50*D*.

in Tables 5 and 6, where $R+$ and $R-$ values denote the sums of the ranks in which MTDE performs better than and worse than its competitor, respectively, based on the multiple-problem analysis. In addition, the convergence graphs for some typical functions are plotted in Figs. 5 and 6 to illustrate the mean error performance of the best solution over the total run in the respective experiments.

According to results of Table 1, MTDE is able to enhance the performances of the original DE algorithms on the CEC2013 functions at 30*D*. Specially, for DE with the exploration strategies (DE/rand/1 and DE/rand/2), MTDE performs significantly better on 14 and 22 functions, respectively. For DE with the exploitative strategies (DE/best/1, DE/best/2, DE/current-to-best/1 and DE/rand-to-best/1), MTDE obtains significant improvements on 23, 11, 21 and 22 functions, respectively. For the CEC2013 functions at higher dimension in Table 2, MTDE also performs significantly better on 10, 20, 23, 11, 21, and 23 functions than the corresponding original DE algorithms.

For the CEC2014 functions at 30*D* and 50*D*, Tables 3 and 4 show that MTDE achieves high performance. Specifically, compared with DE/rand/2, DE/best/1, DE/current-to-best/1 and DE/rand-to-best/1, MTDE obtains significantly better results on 23, 28, 25 and 26 functions at 30*D*, respectively. For the functions at 50*D* in Table 4, MTDE also performs significantly better on 25, 27, 25 and 25, respectively, than the corresponding DE algorithms.

Based on the multiple-problem analysis results shown in Table 5 for the CEC2013 functions, MTDE obtains higher $R+$ values than $R-$ values in all cases except for MTDE/best/2 at 50*D*. The p values of eight cases are less than 0.05, and those of nine cases are less than 0.1. For the CEC2014 functions, the statistical comparison results in Table 6 also show that MTDE outperforms the corresponding DE algorithms. Specifically, MTDE obtains higher $R+$ than $R-$ values in all cases except for MTDE/best/2 at 30*D*, and the p values in most cases are less than 0.05. Overall, according to the statistical comparison results in Tables 1–6, MTDE significantly outperforms most of the corresponding original DE algorithms on the test functions.

In addition, Figs. 5 and 6 show that the convergence speed of MTDE is faster than that of most of the compared algorithms.

In summary, we conclude that our proposed MTDE framework is capable of improving the performance of the original DE algorithms.

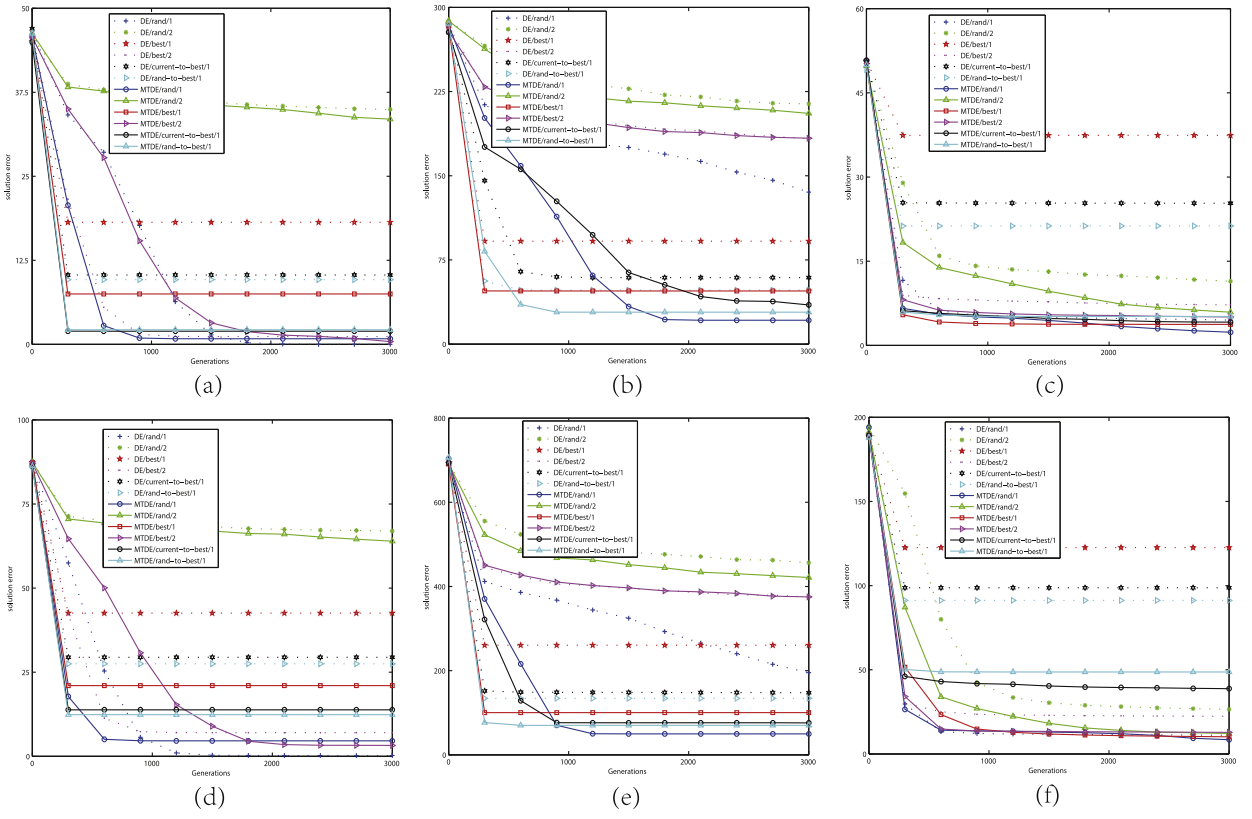

Fig. 6. Convergence graphs of MTDE and the corresponding original DE algorithms for the selected CEC2014 functions. (a) F6, 30*D*. (b) F8, 30*D*. (c) F19, 30*D*. (d) F6, 50*D*. (e) F8, 50*D*. (f) F19, 50*D*.

4.4. Effect on advanced DE variants

To further evaluate the effectiveness of the proposed algorithm, MTDE is applied to six state-of-the-art advanced DE variants: CoDE [46], JADE [50], MDEpBX [20], ODE [35], SaDE [33] and SHADE [40]. These DE variants have been proven to obtain very promising results. Here, all their parameters are set to the same values as in their original papers, except *NP* in CoDE and SaDE. To construct the same neighborhood as the other algorithms, *NP* in CoDE and SaDE is also set to 100. The results on the CEC2013 and CEC2014 functions are shown in Tables 7–10. In addition, Tables 11 and 12 provide statistics that summarize the performance comparisons.

According to the results in Tables 7 and 8, MTDE significantly improves the performance of most advanced DE variants on the CEC2013 functions. As shown in Table 7, MTDE significantly outperforms CoDE, JADE, MDEpBX, ODE, SaDE and SHADE on 7, 12, 16, 9, 13 and 12 functions at 30*D*, respectively. For the functions at 50*D* in Table 8, MTDE is significantly better on 7, 10, 16, 11, 14 and 14 functions, respectively. Based on the results of the multi-problem analysis in Table 11, it is clear that MTDE obtains higher $R+$ values than $R-$ values in all cases. In addition, according to the p values and the Wilcoxon tests at $\alpha = 0.05$ and $\alpha = 0.1$, significant statistical differences can be observed in two cases (MDEpBX and SaDE) for functions at 30*D* and in four cases (CoDE, MDEpBX, SaDE and SHADE) for functions at 50*D*.

From Tables 9 and 10, MTDE consistently outperforms the corresponding DE variants on the CEC2014 functions at both 30*D* and 50*D*. Specifically, MTDE significantly outperforms CoDE, JADE, MDEpBX, ODE, SaDE and SHADE on 9, 4, 16, 13, 10 and 9 functions at 30*D* in Table 9, respectively. For the functions at 50*D* in Table 10, MTDE significantly outperforms the corresponding DE variants on 9, 10, 21, 10, 12 and 15 functions, respectively. Moreover, as the statistical comparison results show in Table 12, MTDE obtains higher $R+$ values than $R-$ values in all cases on the CEC2014 functions. The p values in four cases (CoDE, MDEpBX, ODE, and SaDE) for functions at 30*D* and five cases (CoDE, JADE, MDEpBX, SaDE, and SHADE) for functions at 50*D* are less than 0.05, which indicates that MTDE performs significantly better than the corresponding DE variants on the CEC2014 functions overall.

The convergence curves in Figs. 7 and 8 also show that MTDE outperforms most of the corresponding DE variants in terms of convergence speed.

In general, the above results clearly demonstrate that MTDE enhances the performances of most advanced DE variants on majority of the CEC2013 and CEC2014 test functions.

Table 7
Mean and standard deviation of the best error values obtained by the advanced DE and the corresponding MTDE on all the CEC2013 functions at 30*D*.

Fun.	CoDE		MTCoDE	JADE		MTJADE	MDEpBX		MTMDEpBX
F1	1.06e-002 2.38e-003	+	**6.43e-003 1.56e-003**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.17e-001 3.12e-001	+	**2.10e-018 1.15e-017**
F2	9.84e+007 2.07e+007	=	9.15e+007 1.75e+007	1.29e+005 1.64e+005	+	**1.56e+004 9.89e+003**	1.16e+006 4.90e+005	+	**3.62e+004 3.29e+004**
F3	**7.94e+010 8.13e+010**	-	1.69e+011 1.15e+011	5.72e+005 1.44e+006	+	**3.62e+004 1.04e+005**	3.29e+007 6.86e+007	+	**8.18e+006 1.09e+007**
F4	7.09e+004 9.63e+003	=	6.95e+004 8.31e+003	3.46e+003 3.27e+003	+	**5.24e-004 1.78e-003**	3.25e+002 1.78e+003	+	**1.12e+002 6.11e+002**
F5	7.98e-002 1.61e-002	+	**5.71e-002 9.29e-003**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	7.19e+000 6.48e+000	+	**3.48e-002 8.19e-002**
F6	1.51e+002 1.20e+001	=	1.53e+002 1.34e+001	8.53e+000 3.01e+000	+	**1.33e-001 7.28e-001**	4.47e+001 2.55e+001	+	**2.29e+001 2.07e+001**
F7	**2.43e+002 7.63e+001**	-	3.21e+002 1.06e+002	5.14e+000 4.48e+000	+	**1.53e+000 2.26e+000**	2.13e+001 6.26e+000	+	**1.08e+001 7.06e+000**
F8	2.09e+001 5.52e-002	=	2.09e+001 6.21e-002	2.09e+001 6.82e-002	=	2.10e+001 4.25e-002	2.10e+001 3.91e-002	=	2.09e+001 3.79e-002
F9	3.71e+001 1.49e+000	=	3.65e+001 1.66e+000	**1.44e+001 3.08e+000**	-	2.67e+001 1.70e+000	1.40e+001 4.28e+000	+	**1.20e+001 2.32e+000**
F10	3.91e+002 5.20e+001	+	**3.08e+002 6.29e+001**	1.18e-001 6.25e-002	+	**4.30e-002 2.22e-002**	2.54e+000 3.19e+000	+	**4.48e-001 2.51e-001**
F11	3.50e+001 2.77e+000	+	**3.33e+001 2.97e+000**	1.26e+000 1.17e+000	+	**0.00e+000 0.00e+000**	1.42e+001 4.67e+000	+	**1.18e+001 4.05e+000**
F12	2.44e+002 1.32e+001	=	2.43e+002 1.19e+001	2.59e+001 5.89e+000	=	2.68e+001 4.74e+000	2.69e+001 1.26e+001	=	3.40e+001 2.18e+001
F13	2.54e+002 1.07e+001	=	2.49e+002 1.51e+001	4.99e+001 2.00e+001	=	4.58e+001 8.87e+000	6.17e+001 1.90e+001	=	5.13e+001 2.05e+001
F14	2.43e+003 2.15e+002	=	2.48e+003 2.03e+002	3.70e+001 4.06e+001	=	2.30e+001 6.26e+000	7.76e+002 2.22e+002	+	**6.35e+002 3.10e+002**
F15	7.47e+003 2.68e+002	=	7.45e+003 3.14e+002	4.86e+003 2.01e+003	+	**3.67e+003 2.69e+002**	5.02e+003 1.31e+003	=	5.07e+003 1.28e+003
F16	2.39e+000 3.26e-001	=	2.36e+000 2.51e-001	2.43e+000 3.15e-001	+	**1.97e+000 5.41e-001**	2.45e+000 3.26e-001	=	2.45e+000 2.12e-001
F17	6.97e+001 2.96e+000	=	6.98e+001 3.95e+000	3.16e+001 1.01e+000	+	**3.04e+001 2.85e-005**	4.71e+001 4.40e+000	=	4.54e+001 4.59e+000
F18	2.86e+002 1.27e+001	=	2.88e+002 1.02e+001	1.30e+002 5.99e+001	+	**8.19e+001 8.15e+000**	1.53e+002 2.16e+001	=	1.54e+002 2.65e+001
F19	1.25e+001 9.82e-001	+	**1.18e+001 9.38e-001**	**1.89e+000 4.21e-001**	-	2.18e+000 1.67e-001	4.44e+000 2.54e+000	+	**2.51e+000 5.12e-001**
F20	1.41e+001 3.13e-001	=	1.40e+001 2.90e-001	**9.79e+000 7.34e-001**	-	1.03e+001 5.38e-001	1.02e+001 6.75e-001	=	9.92e+000 6.24e-001
F21	4.88e+002 4.07e+001	+	**3.91e+002 7.63e+001**	3.01e+002 5.92e+001	=	3.19e+002 7.03e+001	3.79e+002 1.18e+002	+	**3.02e+002 6.75e+001**
F22	3.35e+003 2.41e+002	=	3.43e+003 3.14e+002	**1.43e+002 2.80e+001**	-	2.42e+002 1.14e+002	5.20e+002 2.15e+002	=	4.34e+002 1.57e+002
F23	7.90e+003 2.42e+002	=	7.86e+003 3.02e+002	3.69e+003 1.28e+003	=	3.80e+003 3.35e+002	3.60e+003 1.15e+003	=	3.69e+003 1.18e+003
F24	3.01e+002 2.46e+000	=	3.00e+002 4.48e+000	2.03e+002 2.48e+000	=	2.02e+002 2.34e+000	2.14e+002 6.37e+000	+	**2.10e+002 5.13e+000**
F25	2.98e+002 2.75e+000	=	2.98e+002 2.31e+000	**2.45e+002 2.09e+001**	-	2.69e+002 2.30e+001	2.56e+002 1.37e+001	=	2.53e+002 1.81e+001
F26	2.12e+002 2.62e+000	=	2.11e+002 2.64e+000	2.17e+002 3.94e+001	+	**2.10e+002 3.20e+001**	2.41e+002 5.50e+001	=	2.47e+002 5.48e+001
F27	1.30e+003 3.68e+001	=	1.29e+003 4.50e+001	**3.38e+002 4.49e+001**	-	4.40e+002 1.54e+002	4.47e+002 5.41e+001	+	**4.04e+002 5.98e+001**
F28	3.52e+002 1.24e+001	+	**3.33e+002 7.21e+000**	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	3.09e+002 7.51e+000	+	**3.00e+002 3.77e-003**
+/=/-	-		7/19/2	-		12/10/6	-		16/12/0
Fun.	**ODE**		**MTODE**	**SaDE**		**MTSaDE**	**SHADE**		**MTSHADE**
F1	1.22e-028 1.71e-028	=	1.73e-028 2.65e-028	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	6.01e+005 3.32e+005	+	**2.59e+005 1.65e+005**	1.31e+007 3.13e+006	+	**9.45e+006 2.62e+006**	5.99e+004 1.71e+005	+	**1.83e+004 1.02e+004**
F3	**4.68e+004 2.44e+005**	-	6.66e+004 3.63e+005	8.44e+005 1.62e+006	+	**5.04e+005 2.12e+006**	2.30e+005 8.14e+005	=	5.63e+004 2.66e+005
F4	2.43e+003 1.03e+003	+	**6.14e+002 2.91e+002**	3.27e+004 4.77e+003	+	**2.48e+004 4.61e+003**	4.67e-004 6.55e-004	+	**1.26e-005 1.28e-005**
F5	6.98e-003 3.79e-002	=	4.56e-015 6.01e-015	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	1.64e+001 1.04e+001	=	1.45e+001 1.39e+001	1.51e+001 2.76e+000	+	**8.04e+000 3.52e+000**	1.48e+001 1.52e+001	+	**1.76e+000 6.70e+000**
F7	2.90e-001 4.97e-001	=	2.88e-001 3.94e-001	2.49e+001 1.00e+001	+	**4.96e+000 2.70e+000**	4.01e+000 3.72e+000	+	**1.27e+000 1.64e+000**
F8	2.09e+001 4.87e-002	=	2.10e+001 5.41e-002	2.09e+001 4.63e-002	=	2.09e+001 6.15e-002	2.09e+001 5.88e-002	=	2.09e+001 7.09e-002
F9	1.10e+001 3.97e+000	=	1.31e+001 5.80e+000	2.80e+001 1.95e+000	=	2.82e+001 1.94e+000	**1.21e+001 3.14e+000**	-	2.81e+001 1.41e+000
F10	**2.30e-002 1.43e-002**	-	3.07e-002 1.67e-002	1.12e-001 7.03e-002	+	**3.66e-002 2.62e-002**	1.73e-001 7.64e-002	+	**5.89e-002 4.17e-002**
F11	4.63e+001 2.33e+001	+	**2.13e+001 8.73e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.13e+000 1.04e+000	+	**0.00e+000 0.00e+000**
F12	**3.48e+001 1.78e+001**	-	4.64e+001 2.25e+001	7.14e+001 7.10e+000	+	**6.45e+001 8.89e+000**	2.48e+001 6.78e+000	=	2.73e+001 5.75e+000
F13	7.96e+001 4.71e+001	=	7.14e+001 3.31e+001	1.01e+002 1.25e+001	+	**9.24e+001 1.42e+001**	5.85e+001 2.23e+001	+	**4.90e+001 1.01e+001**
F14	4.28e+003 5.77e+002	+	**1.19e+003 5.33e+002**	1.87e+000 7.49e-001	+	**1.03e+000 6.62e-001**	4.00e+001 4.49e+001	=	3.20e+001 8.43e+000
F15	4.93e+003 1.53e+003	=	4.71e+003 1.03e+003	4.88e+003 3.01e+002	=	4.96e+003 2.79e+002	**3.13e+003 4.98e+002**	-	3.53e+003 3.36e+002
F16	2.54e+000 2.67e-001	=	2.57e+000 2.33e-001	1.97e+000 2.47e-001	=	1.98e+000 1.96e-001	2.27e+000 4.79e-001	+	**1.22e+000 2.10e-001**
F17	9.57e+001 1.48e+001	+	**6.06e+001 1.19e+001**	3.04e+001 1.40e-002	+	**3.04e+001 8.50e-003**	3.13e+001 4.83e-001	+	**3.04e+001 8.42e-004**
F18	1.87e+002 2.45e+001	=	1.75e+002 2.83e+001	1.41e+002 1.08e+001	=	1.42e+002 9.39e+000	**5.23e+001 6.65e+000**	-	8.71e+001 6.80e+000
F19	7.52e+000 2.38e+000	+	**3.48e+000 1.10e+000**	2.24e+000 2.10e-001	=	2.31e+000 2.16e-001	2.04e+000 4.04e-001	=	2.10e+000 1.92e-001
F20	1.23e+001 3.02e-001	+	**1.22e+001 3.25e-001**	1.13e+001 4.10e-001	=	1.13e+001 3.38e-001	**9.25e+000 6.77e-001**	-	1.02e+001 4.13e-001
F21	3.21e+002 9.70e+001	=	3.15e+002 7.35e+001	3.03e+002 6.61e+001	=	2.90e+002 5.81e+001	3.07e+002 7.23e+001	=	3.06e+002 6.47e+001
F22	4.49e+003 7.52e+002	+	**9.49e+002 7.05e+002**	2.03e+002 3.75e+001	+	**1.56e+002 5.85e+001**	**1.35e+002 2.54e+001**	-	1.76e+002 7.47e+001
F23	4.82e+003 1.53e+003	=	5.22e+003 1.29e+003	5.45e+003 4.25e+002	=	5.41e+003 3.52e+002	**3.08e+003 6.33e+002**	-	3.84e+003 3.79e+002
F24	**2.01e+002 4.54e-001**	-	2.02e+002 2.99e+000	2.14e+002 6.87e+000	+	**2.03e+002 2.24e+000**	2.06e+002 4.08e+000	+	**2.01e+002 8.67e-001**
F25	2.40e+002 1.94e+001	=	2.37e+002 1.75e+001	2.91e+002 8.90e+000	=	2.92e+002 5.74e+000	2.46e+002 1.63e+001	=	2.36e+002 3.21e+001
F26	2.00e+002 1.06e-002	+	**2.00e+002 8.04e-003**	2.00e+002 1.27e-001	=	2.01e+002 4.47e+000	2.28e+002 4.70e+001	+	**2.07e+002 2.63e+001**
F27	**3.09e+002 8.84e+000**	-	3.19e+002 2.21e+001	8.03e+002 1.06e+002	+	**7.03e+002 1.66e+002**	3.46e+002 3.95e+001	+	**3.24e+002 3.39e+001**
F28	3.00e+002 5.67e-006	=	3.00e+002 9.82e-006	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000	3.00e+002 0.00e+000	=	3.00e+002 0.00e+000
+/=/-	-		9/14/5	-		13/15/0	-		12/10/6

4.5. Comparison with state-of-the-art EAs

According to the previous results, the MTDE framework with the individual-dependent topology adaptation effectively improves the performances of most original DE algorithms and advanced DE variants. However, a question naturally arises: how does the performance of MTDE compare to those of other state-of-the-art EAs? To address this issue, we first compare different MTDE versions. Then, the MTDE versions that perform the best overall are further compared with several state-of-the-art EAs, which are selected from the CEC2013 and CEC2014 competitions on real-parameter optimization.

The Friedman test is used to obtain the rankings of different MTDE versions on all the CEC2013 and CEC2014 functions. The final rankings of all the versions are shown in Tables 13 and 14, where “ARV” and “Rank” denote the average ranking value and the final ranking value, respectively, in the comparison. As the results show, MTSHADE obtains the best rank-

Table 8
Mean and standard deviation of the best error values obtained by the advanced DE and the corresponding MTDE on all the CEC2013 functions at 50*D*.

Fun.	CoDE		MTCoDE	JADE		MTJADE	MDEpBX		MTMDEpBX
F1	3.58e+000 5.14e-001	+	**2.71e+000 4.53e-001**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	2.78e+001 3.64e+001	+	**5.83e-002 2.22e-001**
F2	3.08e+008 4.00e+007	=	3.25e+008 3.70e+007	2.67e+005 2.96e+005	+	**4.70e+004 2.18e+004**	1.64e+006 4.85e+005	+	**5.58e+005 1.73e+005**
F3	**7.78e+010 1.03e+010**	-	8.69e+010 1.57e+010	1.59e+007 1.98e+007	+	**6.10e+006 9.71e+006**	1.05e+008 9.11e+007	+	**5.25e+007 7.83e+007**
F4	1.28e+005 1.18e+004	+	**1.21e+005 1.08e+004**	1.12e+003 2.22e+003	+	**8.55e-003 3.18e-002**	2.78e+002 1.39e+003	+	**1.84e+002 1.01e+003**
F5	4.07e+000 4.74e-001	+	**3.30e+000 3.18e-001**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.06e+002 1.79e+002	+	**5.44e+000 1.51e+001**
F6	1.99e+002 2.18e+001	+	**1.53e+002 2.49e+001**	4.34e+001 8.68e-007	=	4.24e+001 8.12e+000	5.26e+001 1.39e+001	+	**4.51e+001 2.26e+000**
F7	1.91e+002 1.40e+001	=	1.96e+002 1.53e+001	2.02e+001 6.85e+000	+	**1.25e+001 6.22e+000**	3.44e+001 7.98e+000	+	**2.61e+001 6.78e+000**
F8	2.11e+001 3.80e-002	=	2.11e+001 2.86e-002	2.11e+001 4.32e-002	=	2.11e+001 3.96e-002	2.11e+001 3.75e-002	=	2.11e+001 3.63e-002
F9	7.06e+001 1.92e+000	=	7.07e+001 2.04e+000	**3.12e+001 5.06e+000**	-	5.49e+001 2.43e+000	2.77e+001 2.50e+000	+	**2.50e+001 2.83e+000**
F10	1.66e+003 1.71e+002	=	1.60e+003 2.25e+002	1.61e-001 7.92e-002	+	**6.59e-002 3.78e-002**	9.36e+000 1.10e+001	+	**2.94e-001 1.67e-001**
F11	1.53e+002 7.89e+000	=	1.53e+002 7.04e+000	8.85e+000 3.85e+000	+	**0.00e+000 0.00e+000**	4.54e+001 9.70e+000	+	**3.43e+001 8.40e+000**
F12	5.50e+002 1.90e+001	=	5.48e+002 2.07e+001	**5.35e+001 1.67e+001**	-	6.75e+001 8.37e+000	6.55e+001 1.96e+001	+	**4.81e+001 2.07e+001**
F13	5.58e+002 2.39e+001	+	**5.40e+002 1.74e+001**	1.38e+002 3.41e+001	=	1.45e+002 2.34e+001	1.84e+002 4.06e+001	=	1.73e+002 5.40e+001
F14	6.33e+003 3.38e+002	=	6.17e+003 4.10e+002	**1.43e+002 9.05e+001**	-	3.51e+002 5.84e+001	1.95e+003 5.42e+002	=	1.90e+003 4.66e+002
F15	1.43e+004 4.53e+002	=	1.43e+004 3.73e+002	8.74e+003 3.22e+003	=	7.78e+003 4.03e+002	1.14e+004 1.55e+003	=	1.13e+004 2.48e+003
F16	3.29e+000 3.02e-001	=	3.33e+000 2.49e-001	3.33e+000 3.09e-001	+	**2.39e+000 5.55e-001**	**3.22e+000 3.11e-001**	-	3.39e+000 3.07e-001
F17	2.27e+002 1.08e+001	=	2.26e+002 1.01e+001	5.71e+001 2.20e+000	+	**5.16e+001 1.11e-001**	9.60e+001 1.05e+001	=	9.32e+001 8.02e+000
F18	5.96e+002 2.45e+001	=	6.00e+002 1.96e+001	2.08e+002 1.28e+002	+	**1.67e+002 1.05e+001**	3.17e+002 4.23e+001	=	3.17e+002 5.08e+001
F19	4.35e+001 3.37e+000	=	4.21e+001 2.97e+000	**4.32e+000 8.31e-001**	-	4.85e+000 3.74e-001	2.46e+001 1.37e+001	+	**7.77e+000 3.45e+000**
F20	2.37e+001 4.09e-001	=	2.38e+001 2.01e-001	**1.88e+001 9.16e-001**	-	1.94e+001 6.41e-001	1.95e+001 8.39e-001	=	1.95e+001 9.70e-001
F21	7.91e+002 3.19e+002	+	**3.99e+002 2.34e+002**	**7.27e+002 4.22e+002**	-	9.23e+002 3.14e+002	9.22e+002 2.78e+002	+	**6.55e+002 4.40e+002**
F22	8.20e+003 6.00e+002	=	8.14e+003 5.90e+002	**1.57e+002 1.06e+002**	-	8.84e+002 3.63e+002	1.55e+003 5.52e+002	=	1.65e+003 5.05e+002
F23	1.49e+004 4.65e+002	=	1.49e+004 2.97e+002	**6.99e+003 7.23e+002**	-	8.35e+003 8.08e+002	8.08e+003 1.88e+003	=	7.20e+003 2.47e+003
F24	3.87e+002 2.83e+000	=	3.85e+002 5.21e+000	2.28e+002 1.01e+001	=	2.27e+002 9.32e+000	2.52e+002 1.11e+001	+	**2.41e+002 1.05e+001**
F25	3.83e+002 3.14e+000	=	3.82e+002 2.55e+000	**3.14e+002 1.08e+001**	-	3.71e+002 1.63e+001	3.29e+002 9.48e+000	+	**3.18e+002 8.77e+000**
F26	2.91e+002 8.01e+001	=	2.81e+002 4.45e+001	3.19e+002 3.40e+001	+	**2.77e+002 8.60e+001**	3.51e+002 1.06e+001	+	**3.43e+002 1.07e+001**
F27	2.16e+003 3.33e+001	=	2.15e+003 3.88e+001	7.19e+002 1.26e+002	=	8.96e+002 3.56e+002	9.37e+002 1.12e+002	=	8.64e+002 1.24e+002
F28	4.55e+002 6.65e+000	+	**4.40e+002 5.01e+000**	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000	**4.35e+002 4.43e+001**	-	5.04e+002 5.48e+002
+/=/-	-		7/20/1	-		10/9/9	-		16/10/2
Fun.	**ODE**		**MTODE**	**SaDE**		**MTSaDE**	**SHADE**		**MTSHADE**
F1	1.30e-027 9.47e-028	+	**1.10e-027 2.47e-027**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F2	4.30e+006 1.21e+006	+	**7.76e+005 2.14e+005**	2.27e+007 4.28e+006	+	**1.91e+007 5.92e+006**	1.00e+005 3.45e+004	+	**2.35e+004 1.41e+004**
F3	2.87e+006 4.81e+006	=	2.97e+006 4.87e+006	2.65e+007 3.72e+007	+	**5.69e+005 1.19e+006**	4.94e+006 1.29e+007	+	**1.13e+006 2.79e+006**
F4	5.12e+004 8.38e+003	+	**1.86e+004 6.49e+003**	3.79e+004 4.43e+003	+	**2.94e+004 3.85e+003**	1.89e-002 2.32e-002	+	**1.77e-004 2.25e-004**
F5	2.27e+000 9.17e+000	+	**1.55e-014 5.22e-015**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F6	4.64e+001 9.28e+000	+	**4.38e+001 1.45e+000**	4.46e+001 7.97e-001	+	**4.34e+001 1.80e-003**	4.38e+001 1.45e+000	=	4.36e+001 1.04e+000
F7	5.02e+000 2.52e+000	=	5.07e+000 2.63e+000	5.62e+001 1.01e+001	+	**2.75e+001 1.08e+001**	1.40e+001 4.68e+000	+	**8.12e+000 3.32e+000**
F8	2.11e+001 2.96e-002	=	2.11e+001 4.40e-002	2.11e+001 3.19e-002	=	2.11e+001 3.58e-002	2.11e+001 6.65e-002	=	2.11e+001 6.87e-002
F9	**2.27e+001 7.30e+000**	-	2.77e+001 6.44e+000	5.56e+001 2.38e+000	=	5.62e+001 2.18e+000	**2.84e+001 4.73e+000**	-	5.68e+001 2.65e+000
F10	1.02e-001 6.19e-002	+	**7.24e-002 4.64e-002**	2.29e-001 1.05e-001	+	**6.22e-002 4.00e-002**	1.77e-001 6.06e-002	+	**6.51e-002 4.38e-002**
F11	8.70e+001 4.74e+001	+	**4.77e+001 1.00e+001**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	7.71e+000 4.48e+000	+	**1.78e-016 7.15e-016**
F12	1.05e+002 9.24e+001	=	8.48e+001 2.38e+001	1.81e+002 1.85e+001	+	**1.56e+002 1.25e+001**	5.78e+001 1.30e+001	+	**4.52e+001 6.71e+000**
F13	2.49e+002 9.38e+001	=	2.16e+002 8.49e+001	2.50e+002 2.07e+001	+	**2.33e+002 2.05e+001**	1.43e+002 3.39e+001	+	**1.04e+002 2.18e+001**
F14	8.60e+003 9.34e+002	+	**2.13e+003 7.21e+002**	8.61e+001 1.13e+001	+	**7.36e+001 1.27e+001**	1.10e+002 9.53e+001	+	**5.30e-002 2.16e-002**
F15	1.14e+004 2.58e+003	=	1.17e+004 2.44e+003	1.02e+004 5.13e+002	=	1.04e+004 4.26e+002	6.42e+003 6.95e+002	=	6.68e+003 4.15e+002
F16	3.45e+000 2.23e-001	=	3.32e+000 3.60e-001	2.62e+000 3.18e-001	=	2.65e+000 2.39e-001	2.54e+000 1.03e+000	+	**1.24e+000 1.81e-001**
F17	2.36e+002 2.86e+001	+	**1.17e+002 1.27e+001**	5.33e+001 4.13e-001	=	5.32e+001 3.67e-001	5.58e+001 1.80e+000	+	**5.08e+001 0.00e+000**
F18	3.98e+002 1.59e+001	=	3.94e+002 4.15e+001	2.87e+002 1.57e+001	+	**2.77e+002 1.12e+001**	**8.43e+001 1.08e+001**	-	1.21e+002 7.69e+000
F19	1.63e+001 5.66e+000	+	**6.03e+000 1.87e+000**	**4.55e+000 5.15e-001**	-	4.93e+000 5.13e-001	4.68e+000 1.66e+000	+	**2.22e+000 2.33e-001**
F20	2.22e+001 3.25e-001	=	2.21e+001 3.02e-001	2.09e+001 3.45e-001	=	2.07e+001 3.40e-001	**1.79e+001 8.48e-001**	-	1.88e+001 7.26e-001
F21	7.91e+002 3.82e+002	=	8.94e+002 3.08e+002	7.91e+002 3.82e+002	=	8.71e+002 3.61e+002	8.69e+002 3.88e+002	=	7.77e+002 4.25e+002
F22	8.87e+003 1.19e+003	+	**2.03e+003 1.01e+003**	4.79e+002 1.85e+002	+	**2.11e+002 1.29e+002**	1.84e+002 8.77e+001	+	**1.22e+001 4.50e+000**
F23	1.20e+004 2.70e+003	=	1.12e+004 2.42e+003	1.13e+004 5.71e+002	=	1.13e+004 5.09e+002	**6.30e+003 9.06e+002**	-	7.60e+003 7.73e+002
F24	**2.14e+002 6.10e+000**	-	2.22e+002 1.00e+001	2.34e+002 1.39e+001	+	**2.21e+002 9.66e+000**	2.27e+002 7.28e+000	+	**2.20e+002 1.02e+001**
F25	3.39e+002 5.88e+001	=	3.09e+002 3.72e+001	3.87e+002 6.27e+000	+	**3.84e+002 4.57e+000**	**3.03e+002 1.04e+001**	-	3.14e+002 2.22e+001
F26	2.20e+002 4.52e+001	=	2.43e+002 6.16e+001	**2.26e+002 6.96e+001**	-	3.10e+002 1.14e+002	3.07e+002 4.34e+001	=	2.62e+002 6.73e+001
F27	**5.24e+002 8.18e+001**	-	6.61e+002 1.34e+002	1.57e+003 1.22e+002	+	**1.39e+003 3.64e+002**	7.48e+002 1.22e+002	=	7.19e+002 1.44e+002
F28	4.00e+002 0.00e+000	=	4.00e+002 5.67e-006	4.00e+002 0.00e+000	=	4.00e+002 0.00e+000	5.00e+002 5.50e+002	=	4.00e+002 8.01e-006
+/=/-	-		11/14/3	-		14/12/2	-		14/9/5

ings in all the cases, followed by SHADE and MTJADE. Moreover, all the MTDE versions obtain better rankings than the corresponding DE algorithms on all the functions from CEC2013 and CEC2014.

To further evaluate the effectiveness the proposed algorithm, comparisons between MTDE and several EAs from the CEC2013 and CEC2014 competitions are performed. Based on the results in Tables 13 and 14, the MTDE versions that perform the best overall, namely, MTSHADE and MTJADE, are selected for comparison.

For the CEC2013 test functions, five EAs are selected from the CEC2013 competition for comparison due to their different features and promising results on these functions. They are hybrid variants of mean-variance mapping optimization (MVMO-SH) [36], covariance adaptation matrix evolution strategy with re-sampled inheritance search (CMA-ES-RIS) [6], frequency-based heterogeneous particle swarm optimizer (f_k-HPSO) [26], genetic algorithm with three-parent crossover (GA-TPC) [12] and DE with a success-history-based parameter adaption strategy (SHADE) [40]. The results of these algorithms

Table 9
Mean and standard deviation of the best error values obtained by the advanced DE and the corresponding MTDE on all the CEC2014 functions at 30*D*.

Fun.	CoDE		MTCoDE	JADE		MTJADE	MDEpBX		MTMDEpBX
F1	9.06e+007 2.18e+007	=	8.47e+007 2.39e+007	5.40e+003 4.45e+003	=	4.21e+003 3.90e+003	1.80e+006 1.14e+006	+	**2.09e+004 1.06e+005**
F2	1.21e+006 2.48e+005	+	**9.29e+005 1.89e+005**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	3.51e+005 5.47e+005	+	**8.35e+003 4.44e+003**
F3	1.52e+002 2.96e+001	+	**1.25e+002 2.50e+001**	**2.04e-013 3.48e-013**	-	1.58e-010 3.70e-010	2.68e-027 3.60e-027	+	**6.01e-028 1.83e-027**
F4	2.73e+002 2.91e+001	=	2.63e+002 2.48e+001	6.86e-013 3.73e-012	+	**6.80e-029 5.77e-029**	9.17e+001 3.18e+001	+	**3.67e+001 3.26e+001**
F5	2.07e+001 4.69e-002	=	2.07e+001 4.45e-002	2.04e+001 3.74e-002	=	2.04e+001 3.38e-002	2.05e+001 7.96e-002	=	2.05e+001 9.52e-002
F6	2.78e+001 1.01e+000	=	2.80e+001 1.19e+000	8.55e+000 2.12e+000	=	7.39e+000 4.24e+000	2.52e+000 1.31e+000	+	**1.23e+000 1.12e+000**
F7	5.42e-001 4.84e-002	+	**4.92e-001 5.00e-002**	0.00e+000 0.00e+000	=	2.47e-004 1.35e-003	1.71e-001 2.23e-001	+	**8.28e-004 4.48e-003**
F8	2.76e+001 2.70e+000	=	2.71e+001 3.02e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.22e+001 3.72e+000	=	1.18e+001 3.00e+000
F9	1.70e+002 1.12e+001	=	1.69e+002 9.86e+000	2.86e+001 3.88e+000	=	2.81e+001 4.33e+000	2.43e+001 5.25e+000	+	**2.00e+001 6.44e+000**
F10	1.78e+003 1.67e+002	=	1.82e+003 2.23e+002	**1.19e+000 3.08e-001**	-	4.04e+000 1.39e+000	2.19e+002 1.65e+002	=	2.16e+002 1.60e+002
F11	5.34e+003 2.82e+002	=	5.25e+003 3.01e+002	2.05e+003 2.04e+002	=	2.15e+003 2.33e+002	2.33e+003 6.22e+002	=	2.13e+003 4.83e+002
F12	1.20e+000 1.53e-001	=	1.22e+000 1.25e-001	4.51e-001 7.68e-002	=	4.74e-001 5.58e-002	4.31e-001 2.01e-001	=	3.85e-001 1.06e-001
F13	5.06e-001 5.01e-002	=	5.28e-001 4.76e-002	2.07e-001 3.25e-002	=	1.93e-001 3.65e-002	1.69e-001 4.28e-002	+	**1.50e-001 2.00e-002**
F14	2.85e-001 4.43e-002	+	**2.68e-001 3.02e-002**	2.10e-001 3.31e-002	=	2.18e-001 3.76e-002	**2.30e-001 3.50e-002**	-	2.66e-001 4.35e-002
F15	1.97e+001 1.47e+000	=	2.00e+001 1.47e+000	3.52e+000 4.18e-001	=	3.66e+000 3.28e-001	3.59e+000 8.64e-001	=	3.71e+000 1.43e+000
F16	1.20e+001 2.27e-001	=	1.19e+001 3.50e-001	1.02e+001 2.86e-001	=	1.01e+001 3.21e-001	9.26e+000 8.20e-001	=	9.14e+000 8.29e-001
F17	3.28e+006 1.19e+006	=	2.78e+006 1.06e+006	6.83e+002 3.84e+002	=	7.18e+002 3.41e+002	5.39e+003 5.56e+003	+	**9.66e+002 4.47e+002**
F18	1.64e+005 6.94e+004	+	**9.21e+004 3.84e+004**	3.17e+001 1.50e+001	=	3.25e+001 3.94e+001	7.27e+002 1.19e+003	=	1.09e+003 1.37e+003
F19	1.23e+001 1.25e+000	+	**1.08e+001 6.99e-001**	4.15e+000 7.12e-001	=	4.14e+000 8.04e-001	8.14e+000 1.08e+001	+	**4.46e+000 9.06e-001**
F20	7.17e+002 2.76e+002	=	6.53e+002 2.04e+002	3.45e+001 1.39e+001	=	3.43e+001 1.47e+001	5.41e+001 2.87e+001	+	**3.85e+001 2.34e+001**
F21	2.80e+005 1.27e+005	+	**1.93e+005 9.38e+004**	3.39e+002 4.07e+002	=	2.34e+002 1.12e+002	4.84e+002 2.25e+002	=	4.89e+002 3.01e+002
F22	2.93e+002 8.71e+001	=	3.22e+002 7.60e+001	1.33e+002 6.24e+001	=	1.37e+002 5.83e+001	2.06e+002 9.43e+001	=	2.44e+002 1.13e+002
F23	3.17e+002 4.03e-001	=	3.17e+002 3.08e-001	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	3.16e+002 4.15e-001	+	**3.15e+002 1.46e-003**
F24	2.30e+002 1.09e+000	+	**2.28e+002 1.53e+000**	2.24e+002 1.20e+000	=	2.24e+002 1.19e+000	2.28e+002 4.06e+000	=	2.28e+002 4.66e+000
F25	2.20e+002 2.54e+000	=	2.19e+002 2.80e+000	2.10e+002 2.12e+000	+	**2.04e+002 9.96e-001**	2.09e+002 1.30e+000	+	**2.07e+002 1.56e+000**
F26	1.01e+002 6.38e-002	=	1.01e+002 5.89e-002	1.04e+002 1.82e+001	=	1.04e+002 1.82e+001	1.23e+002 4.30e+001	+	**1.04e+002 1.82e+001**
F27	5.17e+002 2.54e+001	=	5.14e+002 2.72e+001	3.71e+002 5.04e+001	+	**3.22e+002 4.16e+001**	3.50e+002 3.77e+001	=	3.34e+002 3.24e+001
F28	8.37e+002 1.28e+002	+	**7.23e+002 7.44e+001**	8.61e+002 2.53e+001	+	**8.21e+002 4.70e+001**	8.89e+002 3.59e+001	=	8.86e+002 3.63e+001
F29	4.26e+002 7.79e+001	=	4.15e+002 8.66e+001	6.39e+002 2.02e+002	=	6.91e+002 1.71e+002	1.24e+003 4.47e+002	+	**9.82e+002 3.56e+002**
F30	1.62e+003 2.91e+002	=	1.60e+003 3.49e+002	1.11e+003 3.68e+002	=	1.34e+003 4.22e+002	3.54e+003 2.24e+003	+	**2.06e+003 6.58e+002**
+/=/-	-		9/21/0	-		4/24/2	-		16/13/1
Fun.	**ODE**		**MTODE**	**SaDE**		**MTSaDE**	**SHADE**		**MTSHADE**
F1	1.64e+005 1.63e+005	+	**7.24e+004 6.01e+004**	5.80e+006 1.66e+006	+	**4.35e+006 1.58e+006**	8.16e+003 4.21e+003	+	**5.63e+003 4.22e+003**
F2	1.02e+003 2.34e+003	=	1.53e-022 2.75e-022	6.67e-007 8.48e-007	+	**9.78e-008 1.75e-007**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F3	2.73e-016 3.59e-016	+	**8.53e-025 1.19e-024**	2.58e+001 1.91e+001	=	2.01e+001 1.90e+001	**8.19e-019 4.03e-018**	-	5.34e-014 2.19e-013
F4	8.86e-001 6.36e-001	+	**2.78e-001 1.79e-001**	4.39e+001 3.78e+001	+	**3.48e-002 4.96e-002**	7.86e-003 2.51e-002	+	**4.55e-018 1.63e-017**
F5	2.08e+001 1.10e-001	+	**2.07e+001 1.93e-001**	2.04e+001 4.69e-002	=	2.04e+001 3.31e-002	2.04e+001 4.48e-002	=	2.04e+001 3.49e-002
F6	8.06e-002 2.92e-001	=	3.56e-001 7.67e-001	1.30e+001 1.05e+000	=	1.27e+001 8.32e-001	**1.43e+000 2.72e+000**	-	4.73e+000 3.68e+000
F7	1.89e-003 5.12e-003	=	2.14e-003 4.47e-003	6.09e-009 3.34e-008	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F8	5.14e+001 1.92e+001	+	**2.04e+001 8.08e+000**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	3.41e+001 2.70e+001	=	3.53e+001 1.56e+001	4.85e+001 5.73e+000	=	4.85e+001 6.29e+000	**2.49e+001 3.37e+000**	-	3.04e+001 3.84e+000
F10	3.21e+003 6.30e+002	+	**5.38e+002 4.75e+002**	6.94e-004 3.80e-003	=	1.39e-003 7.60e-003	**1.80e+000 5.75e-001**	-	7.79e+000 2.74e+000
F11	3.79e+003 1.67e+003	+	**2.53e+003 1.03e+003**	2.62e+003 3.21e+002	=	2.66e+003 2.68e+002	**2.16e+003 2.15e+002**	-	2.31e+003 2.35e+002
F12	1.18e+000 5.20e-001	+	**6.32e-001 5.83e-001**	5.14e-001 6.81e-002	=	4.99e-001 6.58e-002	3.50e-001 4.74e-002	=	3.74e-001 5.60e-002
F13	3.18e-001 5.07e-002	=	3.14e-001 6.34e-002	2.64e-001 4.12e-002	=	2.77e-001 2.93e-002	2.11e-001 3.60e-002	+	**1.89e-001 2.52e-002**
F14	2.55e-001 3.47e-002	=	2.36e-001 4.25e-002	**2.14e-001 2.44e-002**	-	2.27e-001 2.88e-002	2.01e-001 2.55e-002	=	2.13e-001 3.07e-002
F15	5.52e+000 3.50e+000	=	4.78e+000 1.79e+000	6.31e+000 7.46e-001	=	6.44e+000 5.43e-001	**3.51e+000 4.31e-001**	-	4.00e+000 4.80e-001
F16	1.17e+001 7.79e-001	=	1.16e+001 1.08e+000	1.02e+001 3.52e-001	=	1.02e+001 3.01e-001	1.01e+001 3.54e-001	=	1.02e+001 3.20e-001
F17	1.58e+003 1.74e+002	+	**1.29e+003 3.44e+002**	6.65e+005 2.31e+005	+	**3.98e+005 1.46e+005**	1.28e+003 9.85e+002	+	**9.31e+002 3.32e+002**
F18	1.23e+001 1.14e+001	=	1.16e+001 5.28e+000	5.34e+003 2.58e+003	=	5.35e+003 3.40e+003	2.82e+001 1.04e+001	+	**1.85e+001 5.52e+000**
F19	2.97e+000 6.94e-001	+	**2.49e+000 5.35e-001**	5.98e+000 6.67e-001	+	**5.43e+000 5.82e-001**	4.13e+000 6.03e-001	=	3.97e+000 5.99e-001
F20	3.77e+001 4.94e+000	+	**1.28e+001 6.48e+000**	3.53e+003 1.80e+003	+	**1.98e+003 9.34e+002**	**1.48e+001 4.55e+000**	-	1.82e+001 7.31e+000
F21	8.26e+002 1.32e+002	+	**4.24e+002 2.56e+002**	9.71e+004 3.32e+004	=	8.83e+004 3.95e+004	2.76e+002 1.18e+002	=	2.89e+002 1.83e+002
F22	2.75e+002 1.22e+002	=	3.26e+002 1.82e+002	1.68e+002 4.66e+001	=	1.46e+002 5.51e+001	1.47e+002 4.53e+001	+	**1.37e+002 4.91e+001**
F23	3.15e+002 0.00e+000	=	3.15e+002 1.13e-005	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005
F24	2.17e+002 1.01e+001	=	2.18e+002 9.32e+000	2.24e+002 6.25e-001	=	2.24e+002 6.67e-001	**2.24e+002 8.11e-001**	-	2.24e+002 1.06e+000
F25	**2.03e+002 1.50e-001**	-	2.03e+002 5.81e-001	2.07e+002 7.73e-001	+	**2.06e+002 5.80e-001**	2.07e+002 2.09e+000	+	**2.04e+002 9.14e-001**
F26	**1.00e+002 4.77e-002**	-	1.00e+002 7.60e-002	1.00e+002 4.22e-002	=	1.00e+002 3.59e-002	1.04e+002 1.82e+001	=	1.00e+002 2.60e-002
F27	**4.01e+002 2.12e-001**	-	4.01e+002 2.26e-001	3.50e+002 5.44e+001	+	**3.19e+002 4.04e+001**	3.60e+002 4.90e+001	+	**3.24e+002 4.18e+001**
F28	7.85e+002 5.33e+001	=	8.13e+002 5.93e+001	9.05e+002 3.50e+001	+	**8.83e+002 1.54e+001**	8.30e+002 4.33e+001	=	8.27e+002 2.22e+001
F29	**3.89e+002 2.36e+002**	-	6.51e+002 1.95e+002	1.34e+003 8.87e+001	=	1.31e+003 1.12e+002	6.15e+002 1.72e+002	=	6.68e+002 2.14e+002
F30	7.74e+002 1.72e+002	+	**4.59e+002 7.22e+001**	1.98e+003 2.39e+002	+	**1.71e+003 3.12e+002**	1.13e+003 3.82e+002	+	**9.38e+002 4.10e+002**
+/=/-	-		13/13/4	-		10/19/1	-		9/13/8

are obtained directly from their original papers and are shown in Tables 15 and 16. From Table 15, MTSHADE and MTJADE achieve the first and fourth ranks, respectively, for the functions at 30*D*. That is, MTSHADE outperforms all five EAs overall. Similar results are obtained by MTSHADE and MTJADE compared with other EAs on the functions at 50*D* in Table 16.

For the CEC2014 test functions, three powerful EAs are selected from the CEC2014 competition for comparison: SHADE with linear population size reduction (L-SHADE) [41], which was the winner of the CEC2014 competition, mean-variance mapping optimization (MVMO) [13], and memetic DE based on fitness Euclidean-distance ratio (FERDE) [34]. The results of these algorithms are obtained directly from their original papers. Since SHADE obtains good performance on the CEC2013

Table 10
Mean and standard deviation of the best error values obtained by the advanced DE and the corresponding MTDE on all the CEC2014 functions at 50*D*.

Fun.	CoDE		MTCoDE	JADE		MTJADE	MDEpBX		MTMDEpBX
F1	2.91e+008 3.50e+007	+	**2.68e+008 4.13e+007**	4.23e+004 2.23e+004	+	**2.23e+004 1.43e+004**	2.36e+006 1.48e+006	+	**4.85e+004 3.43e+004**
F2	2.36e+008 3.56e+007	+	**1.91e+008 2.76e+007**	1.01e-020 3.03e-020	+	**2.39e-022 3.84e-022**	7.59e+007 1.19e+008	+	**1.44e+005 5.48e+005**
F3	1.81e+004 2.09e+003	=	1.74e+004 2.31e+003	**4.61e-003 2.95e-003**	-	1.36e-002 9.84e-003	3.05e-006 1.63e-005	+	**3.84e-025 4.75e-025**
F4	7.11e+002 8.49e+001	=	6.81e+002 7.23e+001	1.64e+001 2.51e+001	+	**3.27e+000 1.79e+001**	1.19e+002 3.60e+001	+	**8.09e+001 2.56e+001**
F5	2.09e+001 4.37e-002	=	2.09e+001 4.37e-002	2.06e+001 3.38e-002	=	2.06e+001 4.08e-002	2.07e+001 1.45e-001	=	2.07e+001 1.43e-001
F6	5.63e+001 1.80e+000	=	5.56e+001 1.76e+000	1.04e+001 8.30e+000	+	**5.54e+000 7.48e+000**	1.09e+001 2.40e+000	+	**6.79e+000 1.97e+000**
F7	1.32e+000 5.24e-002	+	**1.23e+000 3.74e-002**	6.80e-003 1.19e-002	+	**1.56e-003 3.57e-003**	1.64e+000 1.37e+000	+	**6.20e-002 8.48e-002**
F8	1.22e+002 7.14e+000	=	1.22e+002 7.62e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	4.07e+001 1.01e+001	+	**3.29e+001 6.41e+000**
F9	3.89e+002 1.74e+001	=	3.87e+002 1.68e+001	6.45e+001 9.89e+000	=	6.17e+001 7.94e+000	5.01e+001 9.33e+000	+	**3.99e+001 7.58e+000**
F10	5.45e+003 3.26e+002	=	5.41e+003 3.26e+002	**3.19e+001 5.08e+000**	-	8.30e+001 1.63e+001	1.14e+003 4.62e+002	=	9.79e+002 4.57e+002
F11	1.12e+004 3.19e+002	=	1.11e+004 4.64e+002	4.96e+003 4.33e+002	=	4.86e+003 3.80e+002	6.31e+003 7.77e+002	+	**5.49e+003 8.64e+002**
F12	1.87e+000 2.40e-001	=	1.92e+000 1.66e-001	5.59e-001 7.91e-002	=	5.83e-001 7.53e-002	5.97e-001 2.42e-001	=	5.80e-001 1.71e-001
F13	6.64e-001 6.35e-002	=	6.78e-001 7.70e-002	3.51e-001 5.18e-002	+	**3.03e-001 3.53e-002**	3.11e-001 4.66e-002	+	**2.52e-001 5.06e-002**
F14	3.58e-001 3.76e-002	+	**3.24e-001 3.27e-002**	2.82e-001 4.00e-002	=	2.92e-001 2.89e-002	**3.27e-001 2.91e-002**	-	3.54e-001 5.67e-002
F15	1.69e+002 3.77e+001	+	**1.27e+002 2.56e+001**	1.04e+001 1.72e+000	+	**8.92e+000 1.29e+000**	1.64e+001 1.06e+001	+	**7.23e+000 3.86e+000**
F16	2.16e+001 2.78e-001	=	2.15e+001 3.00e-001	1.89e+001 3.32e-001	=	1.89e+001 3.06e-001	1.84e+001 1.23e+000	=	1.82e+001 1.00e+000
F17	1.84e+007 5.70e+006	=	1.58e+007 4.50e+006	1.48e+004 1.44e+004	+	**9.78e+003 1.33e+004**	1.39e+005 8.42e+004	+	**2.27e+004 2.25e+004**
F18	2.49e+005 6.70e+004	+	**1.59e+005 5.42e+004**	1.28e+002 3.33e+001	=	1.33e+002 3.13e+001	5.26e+002 3.90e+002	=	4.02e+002 2.81e+002
F19	4.43e+001 7.09e+000	+	**3.68e+001 6.28e+000**	2.23e+001 1.78e+001	=	1.44e+001 6.67e+000	5.92e+001 1.67e+001	+	**1.72e+001 8.68e+000**
F20	1.37e+004 3.58e+003	+	**1.10e+004 3.33e+003**	8.72e+001 1.80e+001	=	1.00e+002 3.97e+001	1.93e+002 6.23e+001	=	2.11e+002 6.13e+001
F21	4.04e+006 1.43e+006	=	3.31e+006 1.28e+006	5.53e+003 6.26e+003	=	4.07e+003 4.91e+003	4.84e+004 3.07e+004	+	**2.19e+003 3.07e+003**
F22	1.07e+003 1.73e+002	=	1.11e+003 1.79e+002	4.91e+002 1.28e+002	=	4.77e+002 1.36e+002	5.78e+002 2.79e+002	=	4.75e+002 2.55e+002
F23	3.54e+002 1.47e+000	=	3.54e+002 1.36e+000	3.44e+002 1.27e-005	=	3.44e+002 1.13e-005	3.50e+002 4.47e+000	+	**3.44e+002 1.32e-001**
F24	**3.17e+002 3.86e+000**	-	3.22e+002 5.41e+000	**2.72e+002 2.17e+000**	-	2.74e+002 1.98e+000	2.75e+002 2.36e+000	=	2.76e+002 2.09e+000
F25	2.58e+002 6.60e+000	+	**2.56e+002 6.62e+000**	2.21e+002 4.46e+000	+	**2.15e+002 4.15e+000**	2.24e+002 1.77e+000	+	**2.22e+002 2.90e+000**
F26	1.01e+002 5.79e-002	=	1.01e+002 5.68e-002	1.07e+002 2.53e+001	=	1.00e+002 1.38e-001	1.97e+002 1.82e+001	+	**1.60e+002 4.97e+001**
F27	2.05e+003 5.29e+001	=	2.03e+003 5.84e+001	4.16e+002 5.63e+001	=	3.88e+002 5.48e+001	6.05e+002 6.01e+001	+	**5.06e+002 5.15e+001**
F28	2.25e+003 5.45e+002	=	2.02e+003 5.45e+002	1.31e+003 5.81e+001	+	**1.25e+003 4.89e+001**	1.52e+003 1.02e+002	+	**1.42e+003 8.46e+001**
F29	2.53e+003 8.39e+002	=	2.50e+003 9.29e+002	8.46e+002 9.42e+001	=	8.33e+002 1.10e+002	3.83e+003 1.86e+003	+	**1.71e+003 3.15e+002**
F30	4.39e+003 1.15e+003	=	5.19e+003 2.23e+003	1.09e+004 6.63e+002	=	1.07e+004 8.66e+002	1.80e+004 4.68e+003	+	**1.23e+004 1.61e+003**
+/=/-	-		9/20/1	-		10/17/3	-		21/8/1
Fun.	**ODE**		**MTODE**	**SaDE**		**MTSaDE**	**SHADE**		**MTSHADE**
F1	1.65e+006 4.93e+005	+	**4.51e+005 1.69e+005**	8.06e+006 2.03e+006	=	6.98e+006 2.06e+006	7.01e+004 3.58e+004	+	**3.94e+004 2.34e+004**
F2	3.66e+003 3.04e+003	=	4.47e+003 3.77e+003	3.01e+003 1.79e+003	+	**2.03e+003 1.25e+003**	3.85e-020 1.61e-019	+	**1.88e-021 6.56e-021**
F3	1.70e+000 3.72e+000	=	1.12e+000 2.24e+000	3.85e+003 1.48e+003	=	3.46e+003 1.36e+003	**5.96e-005 9.95e-005**	-	3.58e-003 3.10e-003
F4	9.34e+001 1.20e+001	+	**6.16e+001 3.41e+001**	1.02e+002 3.38e+001	+	**1.99e+001 2.80e+001**	3.40e+001 3.62e+001	+	**1.31e+001 2.88e+001**
F5	2.10e+001 8.63e-002	=	2.09e+001 2.42e-001	2.06e+001 3.47e-002	=	2.05e+001 4.37e-002	2.05e+001 3.44e-002	=	2.05e+001 3.07e-002
F6	**8.75e-001 6.94e-001**	-	2.75e+000 1.81e+000	2.84e+001 1.86e+000	+	**2.70e+001 2.44e+000**	3.28e+000 1.10e+000	+	**2.90e+000 4.05e+000**
F7	5.34e-003 6.75e-003	=	3.94e-003 5.36e-003	8.38e-003 7.23e-003	+	**3.78e-003 5.00e-003**	6.81e-003 8.05e-003	+	**2.47e-004 1.35e-003**
F8	9.34e+001 4.44e+001	+	**4.61e+001 1.06e+001**	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	7.13e+001 6.05e+001	=	6.68e+001 1.92e+001	1.22e+002 9.91e+000	=	1.19e+002 1.14e+001	**5.52e+001 7.67e+000**	-	7.05e+001 7.31e+000
F10	6.87e+003 1.33e+003	+	**1.30e+003 4.57e+002**	3.56e+000 1.06e+000	+	**2.29e+000 9.09e-001**	**3.35e+001 4.76e+000**	-	1.05e+002 2.32e+001
F11	8.14e+003 3.40e+003	+	**5.65e+003 2.17e+003**	6.36e+003 2.83e+002	=	6.22e+003 3.61e+002	**5.00e+003 3.98e+002**	-	5.62e+003 3.23e+002
F12	1.99e+000 6.48e-001	+	**6.45e-001 6.23e-001**	7.10e-001 4.52e-002	+	**6.59e-001 5.87e-002**	4.33e-001 4.79e-002	=	4.46e-001 5.32e-002
F13	4.56e-001 5.01e-002	=	4.51e-001 5.41e-002	3.87e-001 3.88e-002	=	3.66e-001 3.42e-002	3.30e-001 4.48e-002	+	**3.07e-001 2.55e-002**
F14	3.21e-001 2.53e-002	=	3.18e-001 3.86e-002	2.95e-001 2.58e-002	=	2.90e-001 2.46e-002	2.69e-001 2.30e-002	=	2.70e-001 3.18e-002
F15	1.96e+001 9.26e+000	+	**1.30e+001 6.71e+000**	1.64e+001 1.63e+000	=	1.55e+001 1.52e+000	9.48e+000 1.09e+000	=	1.01e+001 9.18e-001
F16	2.17e+001 4.30e-001	+	**2.04e+001 9.73e-001**	1.91e+001 3.65e-001	=	1.92e+001 3.32e-001	1.91e+001 3.32e-001	=	1.92e+001 3.95e-001
F17	**1.39e+004 1.32e+004**	-	2.29e+004 2.01e+004	1.62e+006 4.58e+005	=	1.41e+006 4.81e+005	1.61e+004 1.42e+004	+	**6.56e+003 1.17e+004**
F18	7.89e+001 6.62e+001	=	1.09e+002 2.97e+002	8.31e+002 4.14e+002	=	7.96e+002 5.30e+002	1.26e+002 3.61e+001	+	**1.02e+002 3.63e+001**
F19	9.08e+000 2.19e+000	+	**6.15e+000 1.40e+000**	2.88e+001 9.85e+000	+	**1.32e+001 6.59e+000**	2.34e+001 2.02e+001	=	1.48e+001 8.62e+000
F20	1.03e+002 9.11e+000	=	1.11e+002 8.08e+001	7.63e+003 2.36e+003	+	**5.60e+003 1.61e+003**	1.08e+002 5.07e+001	=	1.05e+002 4.01e+001
F21	**2.82e+003 7.10e+002**	-	5.84e+003 3.05e+003	1.20e+006 5.31e+005	=	1.05e+006 3.07e+005	5.95e+003 7.52e+003	+	**2.59e+003 2.56e+003**
F22	**7.84e+002 2.56e+002**	-	9.61e+002 2.68e+002	5.28e+002 1.54e+002	=	5.00e+002 1.14e+002	4.33e+002 1.39e+002	+	**3.57e+002 1.29e+002**
F23	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	3.44e+002 1.27e-005	=	3.44e+002 1.13e-005	3.44e+002 1.13e-005	=	3.44e+002 9.82e-006
F24	**2.69e+002 2.23e+000**	-	2.71e+002 4.11e+000	2.65e+002 5.21e+000	=	2.67e+002 4.51e+000	2.72e+002 2.45e+000	=	2.70e+002 2.49e+000
F25	2.06e+002 2.06e+000	=	2.06e+002 4.02e+000	2.17e+002 4.68e+000	+	**2.13e+002 1.80e+000**	2.21e+002 2.13e+000	+	**2.11e+002 3.34e+000**
F26	1.54e+002 5.06e+001	=	1.54e+002 5.05e+001	1.14e+002 3.46e+001	=	1.04e+002 1.83e+001	1.40e+002 4.96e+001	+	**1.04e+002 1.82e+001**
F27	**3.43e+002 3.67e+001**	-	3.99e+002 5.88e+001	7.71e+002 1.46e+002	+	**4.61e+002 1.97e+002**	3.87e+002 4.17e+001	+	**3.61e+002 4.85e+001**
F28	**1.16e+003 1.24e+002**	-	1.22e+003 7.96e+001	1.52e+003 9.01e+001	+	**1.40e+003 5.52e+001**	1.23e+003 8.84e+001	=	**1.24e+003 5.89e+001**
F29	**8.08e+002 2.73e+002**	-	1.11e+003 2.70e+002	1.32e+003 1.12e+002	=	1.31e+003 1.37e+002	9.19e+002 1.57e+002	+	**8.20e+002 1.23e+002**
F30	9.83e+003 5.47e+002	+	**9.41e+003 3.72e+002**	9.81e+003 5.68e+002	+	**9.50e+003 3.93e+002**	1.05e+004 5.96e+002	+	**1.01e+004 5.81e+002**
+/=/-	-		10/12/8	-		12/18/0	-		15/11/4

functions, it is also employed in this comparison. The results on functions at 30*D* and 50*D* are summarized in Tables 17 and 18, respectively. From Table 17, L-SHADE and MTSHADE achieve the first and second ranks, respectively, for functions at 30*D*. MTJADE obtains the fourth rank. From Table 18, L-SHADE and MVMO achieve the first and second ranks, respectively, followed by MTSHADE and MTJADE.

As the winner of the CEC2014 competition, L-SHADE uses a new version of SHADE (named SHADE 1.1) and a linear population size reduction technique to further improve the performance of SHADE [41]. Thus, it is not surprising that it obtains superior results. As a general framework, MTDE can cooperate with other advanced DE variants. It is interesting to study

Table 11
Results of the multiple-problem analysis by the Wilcoxon test between the advanced DE and MTDE for all the CEC2013 functions at 30*D* and 50*D*.

Algorithm at 30*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTCoDE vs CoDE	7/19/2	275.5	130.5	0.091735	No	Yes
MTJADE vs JADE	12/10/6	233.5	144.5	0.279642	No	No
MTMDEpBX vs MDEpBX	16/12/0	306	72	0.004759	Yes	Yes
MTODE vs ODE	9/14/5	273	133	0.107405	No	No
MTSaDE vs SaDE	13/15/0	280.5	97.5	0.025275	Yes	Yes
MTSHADE vs SHADE	12/10/6	255	151	0.231892	No	No
Algorithm at 50*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTCoDE vs CoDE	7/20/1	297	109	0.029504	Yes	Yes
MTJADE vs JADE	10/9/9	214	192	0.792844	No	No
MTMDEpBX vs MDEpBX	16/10/2	334	44	0.000387	Yes	Yes
MTODE vs ODE	11/14/3	272.5	133.5	0.110936	No	No
MTSaDE vs SaDE	14/12/2	298.5	107.5	0.028811	Yes	Yes
MTSHADE vs SHADE	14/9/5	276.5	101.5	0.034498	Yes	Yes

Table 12
Results of the multiple-problem analysis by the Wilcoxon test between the advanced DE and MTDE for all the CEC2014 functions at 30*D* and 50*D*.

Algorithm at 30*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTCoDE vs CoDE	9/21/0	366.5	68.5	0.001189	Yes	Yes
MTJADE vs JADE	4/24/2	235.5	229.5	0.942611	No	No
MTMDEpBX vs MDEpBX	16/13/1	384.5	80.5	0.001662	Yes	Yes
MTODE vs ODE	13/13/4	343	122	0.022425	Yes	Yes
MTSaDE vs SaDE	10/19/1	340.5	94.5	0.007253	Yes	Yes
MTSHADE vs SHADE	9/13/8	248.5	216.5	0.733666	No	No
Algorithm at 50*D*	+/=/-	R+	R-	p-value	$\alpha = 0.05$	$\alpha = 0.1$
MTCoDE vs CoDE	9/20/1	392.5	72.5	0.000878	Yes	Yes
MTJADE vs JADE	10/17/3	366	99	0.005848	Yes	Yes
MTMDEpBX vs MDEpBX	21/8/1	412	23	0.000025	Yes	Yes
MTODE vs ODE	10/12/8	221.5	213.5	0.922485	No	No
MTSaDE vs SaDE	12/18/0	444.5	20.5	0.000011	Yes	Yes
MTSHADE vs SHADE	15/11/4	317	118	0.030145	Yes	Yes

Table 13
Average ranking of different MTDE versions by Friedman test for the CEC2013 functions at 30*D* and 50*D*.

Algorithm at 30*D*	ARV	Rank	Algorithm at 50*D*	ARV	Rank
MTSHADE	5.0536	1	MTSHADE	4.3571	1
SHADE	6.0357	2	SHADE	6.1786	2
MTJADE	6.4643	3	MTJADE	6.5714	3
JADE	6.9821	4	JADE	6.625	4
MTSaDE	8.9821	5	MTODE	8.7143	5
MTODE	9.125	6	MTSaDE	8.75	6
MTMDEpBX	9.25	7	MTMDEpBX	9.7143	7
MTDE/rand/1	9.875	8	SaDE	9.8393	8
SaDE	9.9464	9	MTDE/rand/1	10.1071	9
ODE	10.2857	10	ODE	10.7143	10
DE/rand/1	11.4464	11	DE/rand/1	11.0893	11
MTDE/best/1	11.6964	12	MDEpBX	11.2321	12
MTDE/rand-to-best/1	12.125	13	MTDE/rand-to-best/1	12.375	13
MDEpBX	12.1607	14	MTDE/best/1	13.1786	14
MTDE/best/2	12.6607	15	MTDE/current-to-best/1	13.5179	15
DE/best/2	13.5893	16	DE/best/2	14.25	16
MTDE/current-to-best/1	13.9107	17	MTDE/best/2	14.8393	17
DE/rand-to-best/1	17.1607	18	MTDE/rand/2	16.6071	18
MTDE/rand/2	17.2143	19	DE/rand-to-best/1	16.75	19
DE/current-to-best/1	17.9107	20	MTCoDE	18.1786	20
DE/best/1	18.7679	21	DE/current-to-best/1	18.3571	21
MTCoDE	18.8393	22	CoDE	18.625	22
CoDE	19.3571	23	DE/best/1	19.1071	23
DE/rand/2	21.1607	24	DE/rand/2	20.3214	24

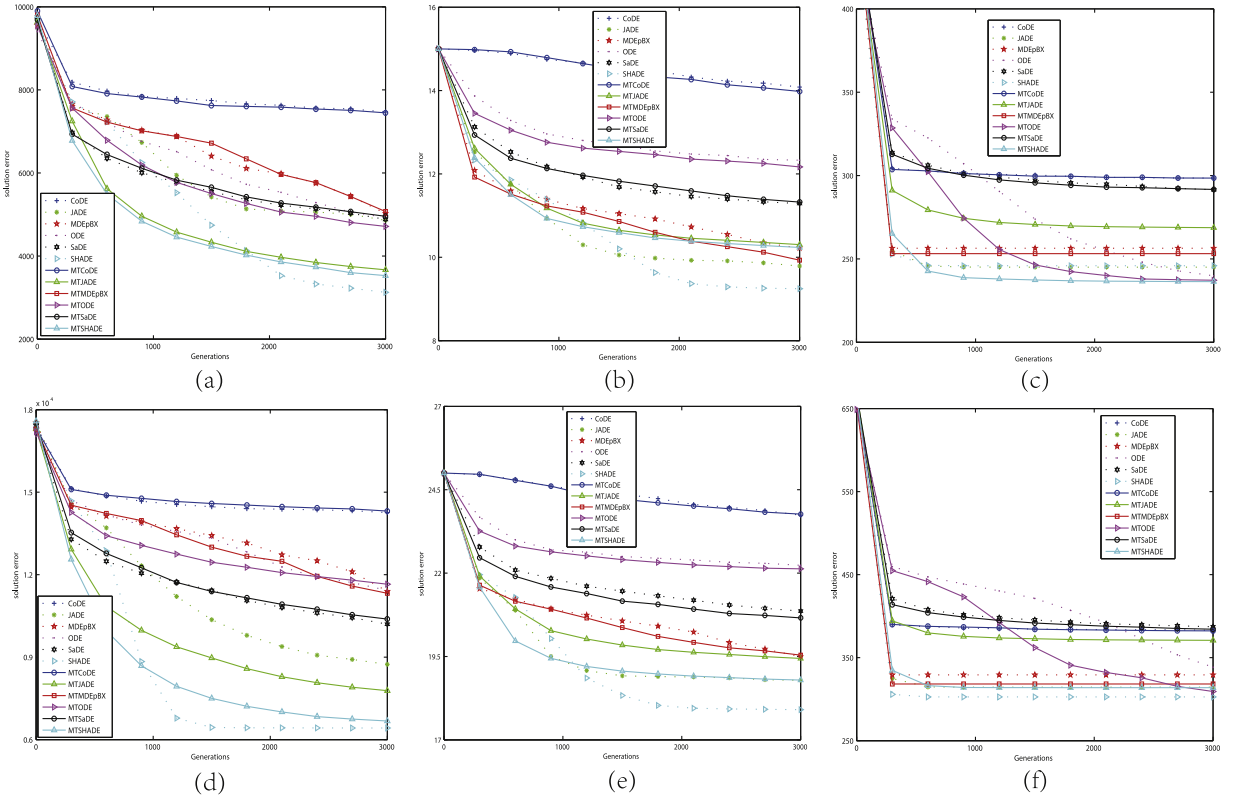

Fig. 7. Convergence graphs of MTDE and the corresponding advanced DE variants for the selected CEC2013 functions. (a) F15, 30*D*. (b) F20, 30*D*. (c) F25, 30*D*. (d) F15, 50*D*. (e) F20, 50*D*. (f) F25, 50*D*.

Table 14
Average ranking of different MTDE versions by Friedman test for the CEC2014 functions at 30*D* and 50*D*.

Algorithm at 30*D*	ARV	Rank	Algorithm at 50*D*	ARV	Rank
MTSHADE	6.0333	1	MTSHADE	4.75	1
SHADE	6.45	2	MTJADE	5.35	2
MTJADE	6.65	3	SHADE	6.6	3
JADE	7.1833	4	JADE	6.7833	4
MTODE	8.3167	5	MTODE	8.5667	5
MTDE/rand/1	8.6333	6	MTMDEpBX	9.4833	6
ODE	9.8333	7	ODE	9.5667	7
MTMDEpBX	9.8667	8	DE/rand/1	9.6667	8
MTDE/best/1	10.3667	9	MTSaDE	9.8833	9
DE/rand/1	10.4667	10	MTDE/best/1	10.2833	10
MTSaDE	11.2667	11	MTDE/rand/1	10.4667	11
MTDE/rand-to-best/1	11.6667	12	SaDE	12.25	12
MTDE/current-to-best/1	11.9167	13	MTDE/current-to-best/1	12.3833	13
MDEpBX	12.0667	14	MTDE/rand-to-best/1	12.5833	14
SaDE	12.4333	15	MDEpBX	12.9833	15
DE/best/2	12.7167	16	MTDE/best/2	13.6667	16
MTDE/best/2	13.35	17	DE/best/2	13.7167	17
MTDE/rand/2	16.1667	18	MTDE/rand/2	15.2333	18
MTCoDE	17.45	19	MTCoDE	17.7833	19
CoDE	18.3833	20	CoDE	18.5333	20
DE/rand-to-best/1	18.8667	21	DE/rand-to-best/1	19.05	21
DE/rand/2	19.55	22	DE/current-to-best/1	20.0667	22
DE/current-to-best/1	20.1	23	DE/rand/2	20.1333	23
DE/best/1	20.2667	24	DE/best/1	20.2167	24

Table 15
Mean and standard deviation of the best error values obtained by MTDE and other state-of-the-art EAs on the CEC2013 functions at 30*D*.

Fun.	MTSHADE	MVMO-SH	SHADE	MTJADE	CMAES-RIS	fk-PSO	TPC-GA
F1	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**
F2	1.83e+004 1.02e+004	1.35e-005 1.07e-005	5.99e+004 1.71e+005	1.56e+004 9.89e+003	**0.00e+000 0.00e+000**	1.54e+006 7.99e+005	2.08e+005 1.22e+005
F3	5.63e+004 2.66e+005	**2.19e-003 7.59e-003**	2.30e+005 8.14e+005	3.62e+004 1.04e+005	4.16e+003 1.56e+004	2.23e+008 2.74e+008	2.68e+007 3.99e+007
F4	1.26e-005 1.28e-005	3.44e-006 1.52e-006	4.67e-004 6.55e-004	5.24e-004 1.78e-003	**0.00e+000 0.00e+000**	4.40e+002 1.89e+002	1.74e+001 2.90e+001
F5	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**
F6	1.76e+000 6.70e+000	**0.00e+000 0.00e+000**	1.48e+001 1.52e+001	1.33e-001 7.28e-001	7.60e-004 1.92e-003	2.99e+001 1.83e+001	2.37e+001 1.12e+001
F7	1.27e+000 1.64e+000	2.34e+001 8.32e+000	4.01e+000 3.72e+000	1.53e+000 2.26e+000	3.39e+001 2.55e+001	6.29e+001 3.47e+001	2.89e+001 2.29e+001
F8	**2.09e+001 7.09e-002**	**2.09e+001 1.16e-001**	**2.09e+001 5.88e-002**	2.10e+001 4.25e-002	**2.09e+001 9.55e-002**	**2.09e+001 6.93e-002**	2.10e+001 5.66e-002
F9	2.81e+001 1.41e+000	1.42e+001 2.68e+000	**1.21e+001 3.14e+000**	2.67e+001 1.70e+000	2.37e+001 2.11e+000	1.79e+001 2.54e+000	3.78e+001 7.09e+000
F10	5.89e-002 4.17e-002	**1.18e-003 2.77e-003**	1.73e-001 7.64e-002	4.30e-002 2.22e-002	7.69e-003 5.61e-003	2.41e-001 1.50e-001	8.18e-002 3.97e-002
F11	**0.00e+000 0.00e+000**	7.02e+000 2.36e+000	1.13e+000 1.04e+000	**0.00e+000 0.00e+000**	2.59e+001 5.40e+000	2.28e+001 9.10e+000	2.40e+001 8.16e+000
F12	2.73e+001 5.75e+000	3.64e+001 1.41e+001	**2.48e+001 6.78e+000**	2.68e+001 4.74e+000	8.26e+001 4.52e+001	5.83e+001 1.67e+001	4.19e+001 1.03e+001
F13	4.90e+001 1.01e+001	7.17e+001 1.98e+001	5.85e+001 2.23e+001	**4.58e+001 8.87e+000**	1.55e+002 5.39e+001	1.24e+002 2.05e+001	8.38e+001 1.68e+001
F14	3.20e+001 8.43e+000	9.13e+002 3.92e+002	4.00e+001 4.49e+001	**2.30e+001 6.26e+000**	7.95e+002 2.08e+002	7.31e+002 2.62e+002	9.23e+002 4.39e+002
F15	3.53e+003 3.36e+002	3.33e+003 3.14e+002	3.13e+003 4.98e+002	3.67e+003 2.69e+002	**3.08e+003 3.83e+002**	3.50e+003 5.36e+002	3.98e+003 5.89e+002
F16	1.22e+000 2.10e-001	3.60e-001 1.23e-001	2.27e+000 4.79e-001	1.97e+000 5.41e-001	**1.06e-001 6.51e-002**	8.25e-001 2.18e-001	2.52e+000 6.81e-001
F17	**3.04e+001 8.42e-004**	4.84e+001 6.39e+000	3.13e+001 4.83e-001	**3.04e+001 2.85e-005**	5.51e+001 5.15e+000	5.29e+001 8.24e+000	5.30e+001 9.25e+000
F18	8.71e+001 6.80e+000	5.79e+001 9.34e+000	**5.23e+001 6.65e+000**	8.19e+001 8.15e+000	1.87e+002 3.27e+001	6.54e+001 8.64e+000	6.89e+001 1.49e+001
F19	2.10e+000 1.92e-001	**1.97e+000 5.32e-001**	2.04e+000 4.04e-001	2.18e+000 1.67e-001	2.93e+000 6.94e-001	3.37e+000 1.13e+000	3.31e+000 1.17e+000
F20	1.02e+001 4.13e-001	1.05e+001 6.23e-001	**9.25e+000 6.77e-001**	1.03e+001 5.38e-001	1.44e+001 4.09e-001	1.19e+001 8.07e-001	1.36e+001 5.08e-001
F21	3.06e+002 6.47e+001	**1.96e+002 4.55e+001**	3.07e+002 7.23e+001	3.19e+002 7.03e+001	1.84e+002 4.73e+001	3.14e+002 1.00e+002	2.95e+002 8.70e+001
F22	1.76e+002 7.47e+001	9.41e+002 4.87e+002	**1.35e+002 2.54e+001**	2.42e+002 1.14e+002	1.23e+003 2.67e+002	8.79e+002 2.84e+002	1.11e+003 3.86e+002
F23	3.84e+003 3.79e+002	3.40e+003 5.21e+002	**3.08e+003 6.33e+002**	3.80e+003 3.35e+002	3.95e+003 6.00e+002	3.47e+003 5.30e+002	4.14e+003 5.52e+002
F24	**2.01e+002 8.67e-001**	2.16e+002 5.85e+000	2.06e+002 4.08e+000	2.02e+002 2.34e+000	2.61e+002 1.04e+001	2.49e+002 8.40e+000	2.77e+002 1.76e+001
F25	**2.36e+002 3.21e+001**	2.60e+002 7.40e+000	2.46e+002 1.63e+001	2.69e+002 2.30e+001	2.80e+002 9.08e+000	2.50e+002 8.10e+000	2.98e+002 9.49e+000
F26	2.07e+002 2.63e+001	**2.00e+002 2.51e-004**	2.28e+002 4.70e+001	2.10e+002 3.20e+001	**2.00e+002 1.16e-002**	2.82e+002 7.39e+001	3.27e+002 5.75e+001
F27	**3.24e+002 3.39e+001**	5.37e+002 1.03e+002	3.46e+002 3.95e+001	4.40e+002 1.54e+002	7.22e+002 1.88e+002	7.91e+002 7.77e+001	1.04e+003 2.03e+002
F28	**3.00e+002 0.00e+000**	3.43e+002 2.15e+002	**3.00e+002 0.00e+000**	**3.00e+002 0.00e+000**	7.25e+002 1.87e+003	4.28e+002 3.82e+002	**3.00e+002 0.00e+000**
ARV	3.0357	3.0536	3.0893	3.5357	4.375	5.1071	5.75
Rank	1	2	3	4	5	6	7

Table 16
Mean and standard deviation of the best error values obtained by MTDE and other state-of-the-art EAs on the CEC2013 functions at 50*D*.

Fun.	MTSHADE	MVMO-SH	SHADE	MTJADE	CMAES-RIS	fk-PSO	TPC-GA
F1	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**
F2	2.35e+004 1.41e+004	9.26e-004 5.65e-004	1.00e+005 3.45e+004	4.70e+004 2.18e+004	**0.00e+000 0.00e+000**	2.78e+006 8.68e+005	4.76e+005 2.23e+005
F3	1.13e+006 2.79e+006	**3.89e-004 1.79e-003**	4.94e+006 1.29e+007	6.10e+006 9.71e+006	1.28e+005 4.23e+005	1.13e+009 1.13e+009	1.18e+008 1.25e+008
F4	1.77e-004 2.25e-004	2.18e-006 1.11e-006	1.89e-002 2.32e-002	8.55e-003 3.18e-002	**0.00e+000 0.00e+000**	5.08e+002 1.89e+002	2.44e+000 1.84e+000
F5	**0.00e+000 0.00e+000**	1.83e-008 8.87e-009	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	3.83e-008 1.57e-008	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**
F6	4.36e+001 1.04e+000	4.00e+001 1.20e+001	4.38e+001 1.45e+000	4.24e+001 8.12e+000	**9.75e+000 1.58e+001**	5.83e+001 2.59e+001	4.33e+001 2.23e+000
F7	8.12e+000 3.32e+000	4.41e+001 7.62e+000	1.40e+001 4.68e+000	1.25e+001 6.22e+000	4.65e+001 2.47e+001	7.94e+001 2.39e+001	4.31e+001 2.27e+001
F8	2.11e+001 6.87e-002	**2.10e+001 1.72e-001**	2.11e+001 6.65e-002	2.11e+001 3.96e-002	**2.10e+001 6.65e-002**	2.11e+001 4.91e-002	2.12e+001 4.50e-002
F9	5.68e+001 2.65e+000	3.25e+001 4.61e+000	**2.84e+001 4.73e+000**	5.49e+001 2.43e+000	4.69e+001 2.90e+000	3.83e+001 5.41e+000	7.37e+001 1.49e+000
F10	6.51e-002 4.38e-002	**5.92e-004 2.05e-003**	1.77e-001 6.06e-002	6.59e-002 3.78e-002	8.77e-003 4.93e-003	2.38e-001 1.57e-001	1.08e-001 8.96e-002
F11	1.78e-016 7.15e-016	3.55e+001 1.01e+001	7.71e+000 4.48e+000	**0.00e+000 0.00e+000**	5.28e+001 1.40e+001	8.80e+001 1.92e+001	5.71e+001 2.10e+001
F12	**4.52e+001 6.71e+000**	8.96e+001 2.42e+001	5.78e+001 1.30e+001	6.75e+001 8.37e+000	2.58e+002 9.78e+001	1.44e+002 3.01e+001	9.37e+001 2.11e+001
F13	**1.04e+002 2.18e+001**	1.84e+002 3.64e+001	1.43e+002 3.39e+001	1.45e+002 2.34e+001	4.48e+002 1.01e+002	2.75e+002 4.66e+001	1.85e+002 4.47e+001
F14	5.30e-002 2.16e-002	2.35e+003 7.63e+002	**1.10e+002 9.53e+001**	3.51e+002 5.84e+001	1.52e+003 3.68e+002	1.96e+003 4.56e+002	2.47e+003 7.85e+002
F15	6.68e+003 4.15e+002	**6.20e+003 5.82e+002**	6.42e+003 6.95e+002	7.78e+003 4.03e+002	6.34e+003 7.57e+002	6.67e+003 7.37e+002	8.99e+003 2.68e+003
F16	1.24e+000 1.81e-001	**1.10e+000 2.16e-001**	2.54e+000 1.03e+000	2.39e+000 5.55e-001	9.14e-002 5.54e-002	1.20e+000 3.03e-001	3.34e+000 4.28e-001
F17	**5.08e+001 0.00e+000**	1.02e+002 9.41e+000	5.58e+001 1.80e+000	5.16e+001 1.11e-001	1.01e+002 1.09e+001	1.17e+002 1.59e+001	1.20e+002 2.27e+001
F18	1.21e+002 7.69e+000	1.06e+002 1.47e+001	**8.43e+001 1.08e+001**	1.67e+002 1.05e+001	4.24e+002 5.11e+001	1.31e+002 1.76e+001	1.75e+002 1.06e+002
F19	**2.22e+000 2.33e-001**	3.94e+000 8.09e-001	4.68e+000 1.66e+000	4.85e+000 3.74e-001	4.97e+000 8.39e-001	8.15e+000 2.74e+000	8.89e+000 3.12e+000
F20	1.88e+001 7.26e-001	1.95e+001 6.62e-001	**1.79e+001 8.48e-001**	1.94e+001 6.41e-001	2.43e+001 6.92e-001	2.09e+001 1.07e+000	2.34e+001 8.62e-001
F21	7.77e+002 4.25e+002	**2.20e+002 1.22e+002**	8.69e+002 3.88e+002	9.23e+002 3.14e+002	2.76e+002 2.11e+002	8.72e+002 3.27e+002	8.87e+002 3.39e+002
F22	**1.22e+001 4.50e+000**	2.61e+003 8.16e+002	1.84e+002 8.77e+001	8.84e+002 3.63e+002	2.45e+003 3.66e+002	2.16e+003 6.46e+002	3.29e+003 1.74e+003
F23	7.60e+003 7.73e+002	6.85e+003 5.75e+002	**6.30e+003 9.06e+002**	8.35e+003 8.08e+002	8.35e+003 1.03e+003	7.44e+003 9.16e+002	9.84e+003 3.28e+003
F24	**2.20e+002 1.02e+001**	2.44e+002 8.68e+000	2.27e+002 7.28e+000	2.27e+002 9.32e+000	3.22e+002 2.15e+001	3.00e+002 1.74e+001	3.71e+002 2.19e+001
F25	3.14e+002 2.22e+001	3.25e+002 1.25e+001	3.03e+002 1.04e+001	3.71e+002 1.63e+001	3.63e+002 1.16e+001	**2.97e+002 1.25e+001**	3.86e+002 4.25e+000
F26	2.62e+002 6.73e+001	2.07e+002 3.56e+001	3.07e+002 4.34e+001	2.77e+002 8.60e+001	**2.00e+002 3.79e-002**	3.89e+002 4.12e+001	4.27e+002 3.46e+001
F27	**7.19e+002 1.44e+002**	1.01e+003 1.19e+002	7.48e+002 1.22e+002	8.96e+002 3.56e+002	1.27e+003 1.61e+002	1.33e+003 1.33e+002	2.01e+003 2.16e+002
F28	**4.00e+002 8.01e-006**	**4.00e+002 0.00e+000**	5.00e+002 5.50e+002	**4.00e+002 0.00e+000**	1.12e+003 1.31e+003	2.03e+003 1.60e+003	**4.00e+002 0.00e+000**
ARV	2.6964	3	3.2857	3.875	3.9643	5.2321	5.9464
Rank	1	2	3	4	5	6	7

Table 17
Mean and standard deviation of the best error values obtained by MTDE and other state-of-the-art EAs on the CEC2014 functions at 30*D*.

Fun.	L-SHADE	MTSHADE	SHADE	MTJADE	MVMO	FERDE
F1	**0.00E+000 0.00E+000**	5.63e+003 4.22e+003	8.16e+003 4.21e+003	4.21e+003 3.90e+003	1.07E-003 1.50E-004	5.41E+002 6.40E+002
F2	**0.00E+000 0.00E+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	2.38E-005 3.62E-005	2.39E-003 3.24E-003
F3	**0.00E+000 0.00E+000**	5.34e-014 2.19e-013	8.19e-019 4.03e-018	1.58e-010 3.70e-010	1.11E-003 6.89E-004	1.13E-003 7.36E-004
F4	**0.00E+000 0.00E+000**	4.55e-018 1.63e-017	7.86e-003 2.51e-002	6.80e-029 5.77e-029	4.38E-013 1.40E-013	6.25E-001 1.46E+000
F5	**2.00E+001 3.70E-002**	2.04e+001 3.49e-002	2.04e+001 4.48e-002	2.04e+001 3.38e-002	**2.00E+001 6.94E-004**	**2.00E+001 7.17E-005**
F6	**1.40E-007 9.90E-007**	4.73e+000 3.68e+000	1.43e+000 2.72e+000	7.39e+000 4.24e+000	3.62E+000 2.12E+000	1.82E+001 1.72E+000
F7	**0.00E+000 0.00E+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	2.47e-004 1.35e-003	2.99E-003 9.67E-003	**0.00E+000 0.00E+000**
F8	**0.00E+000 0.00E+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	8.58E-001 7.64E-001	**0.00E+000 0.00E+000**
F9	**6.80E+000 1.50E+000**	3.04e+001 3.84e+000	2.49e+001 3.37e+000	2.81e+001 4.33e+000	2.51E+001 7.59E+000	5.50E+001 1.04E+001
F10	**1.60E-002 1.60E-002**	7.79e+000 2.74e+000	1.80e+000 5.75e-001	4.04e+000 1.39e+000	1.79E+001 3.19E+001	1.29E+000 1.61E+000
F11	**1.20E+003 1.80E+002**	2.31e+003 2.35e+002	2.16e+003 2.15e+002	2.15e+003 2.33e+002	1.54E+003 3.36E+002	2.72E+003 4.69E+002
F12	1.60E-001 2.30E-002	3.74e-001 5.60e-002	3.50e-001 4.74e-002	4.74e-001 5.58e-002	**7.21E-002 4.08E-002**	5.64E-001 1.74E-001
F13	**1.20E-001 1.70E-002**	1.89e-001 2.52e-002	2.11e-001 3.60e-002	1.93e-001 3.65e-002	1.57E-001 3.12E-002	2.84E-001 4.41E-002
F14	2.40E-001 3.00E-002	2.13e-001 3.07e-002	2.01e-001 2.55e-002	2.18e-001 3.76e-002	**1.99E-001 2.23E-002**	2.14E-001 2.69E-002
F15	**2.10E+000 2.50E-001**	4.00e+000 4.80e-001	3.51e+000 4.31e-001	3.66e+000 3.28e-001	2.86E+000 6.44E-001	4.14E+000 7.77E-001
F16	**8.50E+000 4.60E-001**	1.02e+001 3.20e-001	1.01e+001 3.54e-001	1.01e+001 3.21e-001	1.02E+001 1.25E+000	1.12E+001 4.58E-001
F17	**1.90E+002 7.50E+001**	9.31e+002 3.32e+002	1.28e+003 9.85e+002	7.18e+002 3.41e+002	9.01E+002 3.83E+002	1.16E+003 3.73E+002
F18	**5.90E+000 2.90E+000**	1.85e+001 5.52e+000	2.82e+001 1.04e+001	3.25e+001 3.94e+001	2.89E+001 2.04E+001	2.24E+001 6.47E+000
F19	3.70E+000 6.80E-001	3.97e+000 5.99e-001	4.13e+000 6.03e-001	4.14e+000 8.04e-001	**3.08E+000 7.40E-001**	7.74E+000 7.32E-001
F20	**3.10E+000 1.50E+000**	1.82e+001 7.31e+000	1.48e+001 4.55e+000	3.43e+001 1.47e+001	1.09E+002 1.08E+002	2.80E+001 1.06E+001
F21	**8.70E+001 9.00E+001**	2.89e+002 1.83e+002	2.76e+002 1.18e+002	2.34e+002 1.12e+002	4.67E+002 2.32E+002	5.99E+002 2.15E+002
F22	**2.80E+001 1.80E+001**	1.37e+002 4.91e+001	1.47e+002 4.53e+001	1.37e+002 5.83e+001	1.45E+002 8.06E+001	1.15E+002 7.29E+001
F23	3.20E+002 0.00E+000	3.15e+002 1.13e-005	3.15e+002 1.13e-005	3.15e+002 1.13e-005	3.15E+002 5.68E-014	**3.14E+002 3.74E-009**
F24	**2.20E+002 1.10E+000**	2.24e+002 1.06e+000	2.24e+002 8.11e-001	2.24e+002 1.19e+000	2.25E+002 1.05E+000	2.25E+002 5.57E-001
F25	**2.00E+002 5.00E-002**	2.04e+002 9.14e-001	2.07e+002 2.09e+000	2.04e+002 9.96e-001	2.03E+002 5.43E-001	**2.00E+002 2.83E-002**
F26	**1.00E+002 1.60E-002**	**1.00e+002 2.60e-002**	1.04e+002 1.82e+001	1.04e+002 1.82e+001	**1.00E+002 3.31E-002**	**1.00E+002 7.61E-002**
F27	**3.00E+002 0.00E+000**	3.24e+002 4.18e+001	3.60e+002 4.90e+001	3.22e+002 4.16e+001	4.01E+002 3.64E-001	3.84E+002 2.32E+001
F28	8.40E+002 1.40E+001	8.27e+002 2.22e+001	8.30e+002 4.33e+001	8.21e+002 4.70e+001	8.77E+002 3.71E+001	**8.05E+002 2.49E+001**
F29	7.20E+002 5.10E+000	6.68e+002 2.14e+002	**6.15e+002 1.72e+002**	6.91e+002 1.71e+002	7.36E+002 4.61E+001	1.19E+003 1.06E+002
F30	1.20E+003 6.20E+002	9.38e+002 4.10e+002	1.13e+003 3.82e+002	1.34e+003 4.22e+002	2.00E+003 6.27E+002	1.07E+003 2.90E+002
ARV	2.0833	3.4333	3.5833	3.8167	3.85	4.0333
Rank	1	2	3	4	5	6

Table 18
Mean and standard deviation of the best error values obtained by MTDE and other state-of-the-art EAs on the CEC2014 functions at 50*D*.

Fun.	L-SHADE	MVMO	MTSHADE	MTJADE	FERDE	SHADE
F1	1.20E+003 1.50E+003	**7.54E-003 1.60E-003**	3.94e+004 2.34e+004	2.23e+004 1.43e+004	3.39E+002 8.57E+002	7.01e+004 3.58e+004
F2	**0.00E+000 0.00E+000**	1.52E-005 5.88E-006	1.88e-021 6.56e-021	2.39e-022 3.84e-022	8.39E-004 8.53E-004	3.85e-020 1.61e-019
F3	**0.00E+000 0.00E+000**	4.54E-003 6.42E-004	3.58e-003 3.10e-003	1.36e-002 9.84e-003	1.19E-002 8.48E-003	5.96e-005 9.95e-005
F4	5.90E+001 4.60E+001	5.77E+000 2.31E+001	1.31e+001 2.88e+001	3.27e+000 1.79e+001	**7.98E-002 5.58E-001**	3.40e+001 3.62e+001
F5	**2.00E+001 4.60E-002**	**2.00E+001 5.16E-005**	2.05e+001 3.07e-002	2.06e+001 4.08e-002	2.00E+001 2.21E-005	2.05e+001 3.44e-002
F6	**2.60E-001 5.20E-001**	1.05E+001 3.77E+000	2.90e+000 4.05e+000	5.54e+000 7.48e+000	4.62E+001 2.67E+000	3.28e+000 1.10e+000
F7	**0.00E+000 0.00E+000**	6.67E-003 5.39E-003	2.47e-004 1.35e-003	1.56e-003 3.57e-003	**0.00E+000 0.00E+000**	6.81e-003 8.05e-003
F8	2.60E-009 7.50E-009	2.28E+000 1.57E+000	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	2.82E+001 5.77E+000	**0.00e+000 0.00e+000**
F9	**1.10E+001 2.10E+000**	7.69E+001 1.83E+001	7.05e+001 7.31e+000	6.17e+001 7.94e+000	1.42E+002 2.48E+001	5.52e+001 7.67e+000
F10	**1.20E-001 4.10E-002**	8.08E+001 8.85E+001	1.05e+002 2.32e+001	8.30e+001 1.63e+001	3.25E+002 2.48E+002	3.35e+001 4.76e+000
F11	**3.20E+003 3.30E+002**	4.57E+003 5.46E+002	5.62e+003 3.23e+002	4.86e+003 3.80e+002	5.80E+003 7.11E+002	5.00e+003 3.98e+002
F12	**2.20E-001 2.80E-002**	8.93E-001 4.09E-002	4.46e-001 5.32e-002	5.83e-001 7.53e-002	8.96E-001 2.56E-001	4.33e-001 4.79e-002
F13	**1.60E-001 1.80E-002**	2.84E-001 3.84E-002	3.07e-001 2.55e-002	3.03e-001 3.53e-002	3.80E-001 6.22E-002	3.30e-001 4.48e-002
F14	3.00E-001 2.50E-002	2.38E-001 2.85E-002	2.70e-001 3.18e-002	2.92e-001 2.89e-002	**2.26E-001 2.14E-002**	2.69e-001 2.30e-002
F15	5.20E+000 5.10E-001	**5.00E+000 1.04E+000**	1.01e+001 9.18e-001	8.92e+000 1.29e+000	1.06E+001 2.07E+000	9.48e+000 1.09e+000
F16	**1.70E+001 4.80E-001**	1.89E+001 4.75E-001	1.92e+001 3.95e-001	1.89e+001 3.06e-001	2.09E+001 5.94E-001	1.91e+001 3.32e-001
F17	1.40E+003 5.10E+002	**9.65E+002 3.19E+002**	6.56e+003 1.17e+004	9.78e+003 1.33e+004	3.59E+003 7.99E+002	1.61e+004 1.42e+004
F18	9.70E+001 1.40E+001	**3.40E+001 2.94E+001**	1.02e+002 3.63e+001	1.33e+002 3.13e+001	7.92E+001 1.94E+001	1.26e+002 3.61e+001
F19	**8.30E+000 1.80E+000**	8.76E+000 1.15E+000	1.48e+001 8.62e+000	1.44e+001 6.67e+000	3.40E+001 4.56E+000	2.34e+001 2.02e+001
F20	**1.40E+001 4.60E+000**	2.29E+002 7.03E+000	1.05e+002 4.01e+001	1.00e+002 3.97e+001	1.13E+002 3.51E+001	1.08e+002 5.07e+001
F21	**5.20E+002 1.50E+002**	4.39E+003 1.99E+002	2.59e+003 2.56e+003	4.07e+003 4.91e+003	3.60E+003 1.69E+003	5.95e+003 7.52e+003
F22	**1.10E+002 7.50E+001**	4.58E+002 1.42E+002	3.57e+002 1.29e+002	4.77e+002 1.36e+002	6.58E+002 1.64E+002	4.33e+002 1.39e+002
F23	3.40E+002 4.40E-013	3.44E+002 4.55E-013	3.44e+002 9.82e-006	3.44e+002 1.13e-005	**3.37E+002 1.07E-007**	3.44e+002 1.13e-005
F24	2.80E+002 6.60E-001	**2.58E+002 3.62E+000**	2.70e+002 2.49e+000	2.74e+002 1.98e+000	2.62E+002 3.37E+000	2.72e+002 2.45e+000
F25	2.10E+002 3.60E-001	2.06E+002 1.59E+000	2.11e+002 3.34e+000	2.15e+002 4.15e+000	**2.00E+002 4.49E-002**	2.21e+002 2.13e+000
F26	**1.00E+002 1.40E+001**	1.04E+002 1.94E+001	1.04e+002 1.82e+001	**1.00e+002 1.38e-001**	**1.00E+002 1.04E-001**	1.40e+002 4.96e+001
F27	**3.30E+002 3.00E+001**	5.11E+002 7.70E+001	3.61e+002 4.85e+001	3.88e+002 5.48e+001	1.35E+003 7.16E+001	3.87e+002 4.17e+001
F28	**1.10E+003 2.90E+001**	1.33E+003 6.24E+001	1.24e+003 5.89e+001	1.25e+003 4.89e+001	1.12E+003 4.67E+001	1.23e+003 8.84e+001
F29	**7.90E+002 2.40E+001**	1.11E+003 2.31E+002	8.20e+002 1.23e+002	8.33e+002 1.10e+002	2.00E+003 2.35E+002	9.19e+002 1.57e+002
F30	8.70E+003 4.10E+002	9.82E+003 3.51E+002	1.01e+004 5.81e+002	1.07e+004 8.66e+002	**3.16E+003 5.24E+002**	1.05e+004 5.96e+002
ARV	2.0667	3.4167	3.6	3.9167	3.95	4.05
Rank	1	2	3	4	5	6

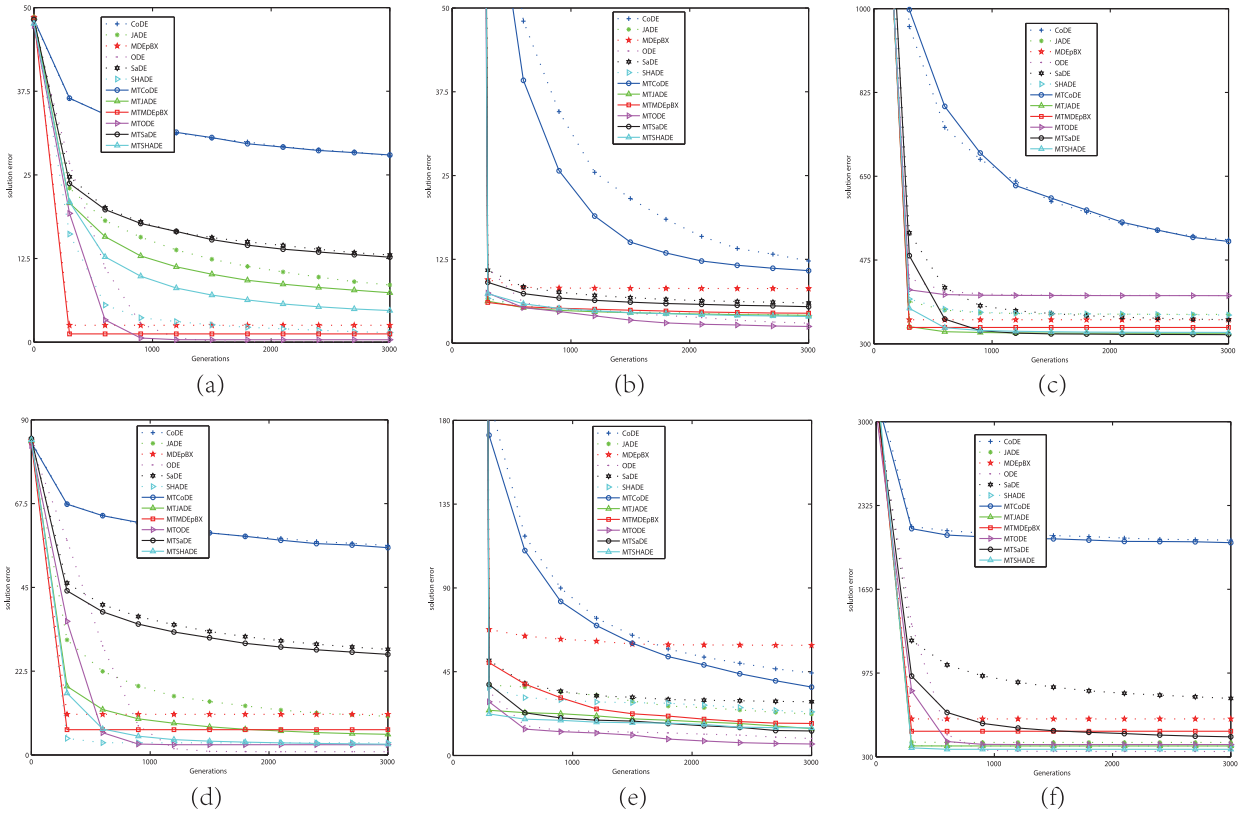

Fig. 8. Convergence graphs of MTDE and the corresponding advanced DE variants for the selected CEC2014 functions. (a) F6, 30*D*. (b) F19, 30*D*. (c) F27, 30*D*. (d) F6, 50*D*. (e) F19, 50*D*. (f) F27, 50*D*.

the benefits that can be obtained by combining MTDE with L-SHADE. Since the linear population size reduction technique is inappropriate for defining different population topologies during the evolutionary process, we only combine MTDE with the SHADE 1.1 strategy in this experiment. The resultant algorithm is named MTSHADE1.1. Then, a comparison between MTSHADE1.1 and L-SHADE is performed. The results on the CEC2014 functions are shown in Tables 19 and 20, which are obtained through independent runs with source code that was provided by the authors of L-SHADE [41]. From Table 19, MTSHADE1.1 significantly outperforms L-SHADE on four functions and is outperformed by L-SHADE on three functions at 30*D*. For the functions at 50*D*, MTSHADE1.1 significantly outperforms L-SHADE on eight functions and is outperformed by L-SHADE on five functions. Furthermore, according to the multiple-problem analysis by the Wilcoxon test in Table 20, MTSHADE1.1 obtains higher $R+$ values than $R-$ values on the functions at both 30*D* and 50*D*. These results demonstrate that the performance of MTDE can be further improved by introducing the sophisticated operators into other advanced EAs.

4.6. Comparison with single-topology-based DE variants

To evaluate the advantages of using multiple topologies over single topology, comparisons between MTDE and the DE variants with single topology are performed. Since the ensemble of MPT in MTDE includes the small-world, ring and cellular topologies, DE with the small-world topology (sw-DE), DE with the cellular topology (cell-DE), and DE with the ring topology (ring-DE) are considered in this experiment. Note that the only difference between MTDE and these single-topology-based DE variants is the population topology that is used. Two DE algorithms (JADE and SaDE) are studied here. The results on the CEC2014 functions at 30*D* and 50*D* are shown in Tables 21 and 22, respectively. The results of the multiple-problem analysis by the Wilcoxon test are summarized in Table 23. In addition, box plots for some selected functions are shown in Figs. 9 and 10.

From the results of Table 21, MTDE outperforms sw-DE, cell-DE and ring-DE overall on the functions at 30*D*. Specifically, MTDE obtains the first rank in the cases of SaDE, and obtains the second rank in the cases of JADE. In addition, as the statistical comparison results show in Table 23, MTDE obtains higher $R+$ values than $R-$ values in most cases. Based on the p values, significant differences at $\alpha = 0.05$ are observed in three cases for SaDE. According to the results in Table 22, MTDE outperforms other competitors on the functions at 50*D*, with the first rank in both cases. The results of multiple-problem analysis in Table 23 show that MTDE obtains higher $R+$ values than $R-$ values in all cases and significant differences are observed in four cases for JADE and three cases for SaDE.

Table 19
Mean and standard deviation of the best error values obtained by the MTSHADE1.1 (MTDE with SHADE1.1) and L-SHADE on all the CEC2014 functions at 30*D* and 50*D*.

	30*D*			50*D*		
Fun.	L-SHADE		MTSHADE1.1	L-SHADE		MTSHADE1.1
F1	6.85e+002 8.32e+002	=	1.63e+003 2.69e+003	**6.52e+003 4.73e+003**	-	1.22e+004 7.73e+003
F2	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.26e-017 4.18e-017	+	**1.73e-021 3.87e-021**
F3	2.29e-027 3.11e-027	=	7.89e-028 2.05e-027	1.48e-006 4.03e-006	=	4.98e-007 2.09e-006
F4	1.00e-021 5.10e-021	+	**5.51e-029 9.77e-029**	5.96e-002 3.25e-001	=	3.58e+000 1.79e+001
F5	**2.00e+001 5.98e-003**	-	2.01e+001 1.86e-002	**2.00e+001 5.25e-003**	-	2.01e+001 1.57e-002
F6	7.62e-001 1.04e+000	+	**1.97e-001 4.56e-001**	3.90e+000 1.99e+000	+	**3.02e+000 1.85e+000**
F7	3.70e-018 2.03e-017	=	2.47e-004 1.35e-003	4.76e-003 5.78e-003	+	**9.04e-004 2.78e-003**
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	1.54e-015 1.73e-015	+	**0.00e+000 0.00e+000**
F9	1.75e+001 3.29e+000	=	1.69e+001 3.17e+000	3.78e+001 4.69e+000	=	3.58e+001 4.74e+000
F10	1.80e-002 1.79e-002	=	9.28e-003 1.48e-002	2.12e-002 1.44e-002	+	**3.33e-003 5.62e-003**
F11	1.45e+003 1.75e+002	=	1.51e+003 2.36e+002	3.48e+003 3.12e+002	=	3.38e+003 3.00e+002
F12	**9.81e-002 1.84e-002**	-	1.64e-001 2.23e-002	**9.82e-002 1.43e-002**	-	1.65e-001 1.39e-002
F13	1.80e-001 2.57e-002	=	1.68e-001 3.02e-002	2.93e-001 3.46e-002	=	2.92e-001 3.74e-002
F14	2.14e-001 2.59e-002	=	2.09e-001 3.05e-002	2.82e-001 3.15e-002	=	2.67e-001 2.84e-002
F15	**2.05e+000 2.71e-001**	-	2.73e+000 2.67e-001	**4.51e+000 8.04e-001**	-	6.13e+000 6.65e-001
F16	8.95e+000 3.15e-001	=	8.95e+000 3.64e-001	**1.70e+001 4.47e-001**	-	1.74e+001 3.52e-001
F17	1.00e+003 4.01e+002	=	8.78e+002 3.31e+002	2.18e+003 8.12e+002	=	2.61e+003 2.15e+003
F18	3.59e+001 1.81e+001	=	3.46e+001 1.58e+001	1.25e+002 3.13e+001	=	1.39e+002 4.12e+001
F19	3.68e+000 4.92e-001	=	3.68e+000 7.69e-001	9.75e+000 2.04e+000	=	1.05e+001 1.45e+000
F20	1.21e+001 4.65e+000	=	9.66e+000 6.19e+000	1.56e+002 4.75e+001	=	1.50e+002 5.68e+001
F21	1.95e+002 1.14e+002	=	1.98e+002 1.01e+002	1.11e+003 4.27e+002	=	1.85e+003 2.47e+003
F22	1.44e+002 3.58e+001	=	1.20e+002 4.87e+001	3.97e+002 1.04e+002	=	3.45e+002 1.46e+002
F23	3.15e+002 0.00e+000	=	3.15e+002 0.00e+000	3.44e+002 1.27e-005	=	3.44e+002 1.33e-005
F24	2.24e+002 1.58e+000	=	2.24e+002 2.52e+000	2.73e+002 2.30e+000	=	2.73e+002 2.45e+000
F25	2.05e+002 2.02e+000	+	**2.04e+002 1.02e+000**	2.19e+002 4.11e+000	+	**2.12e+002 2.69e+000**
F26	1.00e+002 2.73e-002	=	1.00e+002 4.26e-002	1.30e+002 4.64e+001	+	**1.00e+002 1.36e-001**
F27	3.27e+002 4.52e+001	+	**3.05e+002 2.14e+001**	3.96e+002 5.72e+001	=	3.83e+002 4.88e+001
F28	8.55e+002 3.60e+001	=	8.51e+002 4.33e+001	1.30e+003 7.71e+001	+	**1.25e+003 7.55e+001**
F29	6.65e+002 1.66e+002	=	5.81e+002 2.21e+002	8.81e+002 1.60e+002	=	8.35e+002 9.39e+001
F30	8.87e+002 2.54e+002	=	1.02e+003 3.37e+002	1.03e+004 7.67e+002	=	1.03e+004 7.01e+002
+/=/-	-		4/23/3	-		8/17/5

Table 20
Results of the multiple-problem analysis by the Wilcoxon test between the MTSHADE1.1 and L-SHADE for the CEC2014 functions at 30*D* and 50*D*.

MTSHADE1.1 *vs* L-SHADE	+/=/-	$R+$	$R-$	p-value	$\alpha = 0.05$	$\alpha = 0.1$
30*D*	4/23/3	268.5	166.5	0.265453	No	No
50*D*	8/17/5	250.5	184.5	0.468834	No	No

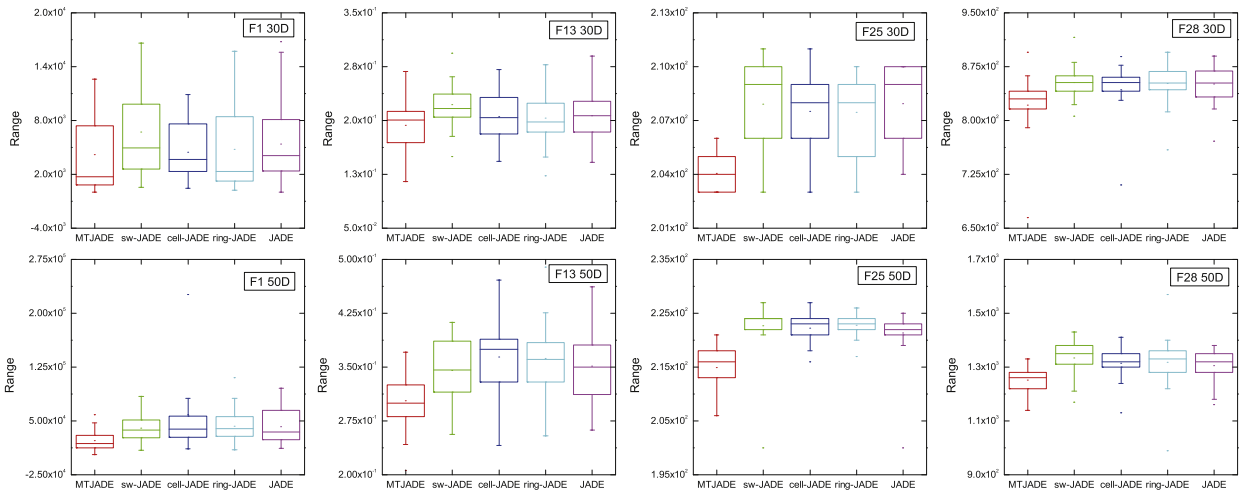

Fig. 9. Box plots of MTJADE and three JADE variants with single topology on the selected functions.

Table 21
Mean and standard deviation of the best error values obtained by MTDE and single-topology based DE on the CEC2014 functions at 30*D*.

Fun.	JADE		sw-JADE		cell-JADE		ring-JADE		MTJADE
F1	5.40e+003 4.45e+003	=	6.69e+003 4.63e+003	=	4.46e+003 3.07e+003	=	4.77e+003 4.62e+003	=	4.21e+003 3.90e+003
F2	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F3	2.04e-013 3.48e-013	=	5.80e-014 1.14e-013	=	1.66e-013 2.08e-013	=	7.19e-014 1.30e-013	-	1.58e-010 3.70e-010
F4	6.86e-013 3.73e-012	-	9.02e-011 4.38e-010	-	5.28e-012 1.15e-011	-	1.81e-011 7.33e-011	+	**6.80e-029 5.77e-029**
F5	2.04e+001 3.74e-002	=	2.04e+001 3.77e-002	=	2.04e+001 3.81e-002	=	2.04e+001 3.59e-002	=	2.04e+001 3.38e-002
F6	8.55e+000 2.12e+000	=	7.69e+000 3.80e+000	=	7.62e+000 3.26e+000	=	8.75e+000 2.39e+000	=	7.39e+000 4.24e+000
F7	0.00e+000 0.00e+000	=	7.38e-004 4.04e-003	=	0.00e+000 0.00e+000	=	2.47e-004 1.35e-003	=	2.47e-004 1.35e-003
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	2.86e+001 3.88e+000	=	2.94e+001 3.81e+000	=	2.80e+001 4.10e+000	=	2.83e+001 5.10e+000	=	2.81e+001 4.33e+000
F10	1.19e+000 3.08e-001	+	**7.71e-002 5.30e-002**	+	**3.37e-001 2.19e-001**	+	4.02e-001 1.33e-001	-	4.04e+000 1.39e+000
F11	2.05e+003 2.04e+002	=	2.02e+003 2.27e+002	=	2.03e+003 3.17e+002	=	2.05e+003 2.56e+002	=	2.15e+003 2.33e+002
F12	4.51e-001 7.68e-002	-	5.09e-001 8.24e-002	=	4.75e-001 7.40e-002	-	4.95e-001 7.53e-002	=	4.74e-001 5.58e-002
F13	2.07e-001 3.25e-002	=	2.22e-001 3.14e-002	=	2.05e-001 3.20e-002	=	2.03e-001 3.46e-002	=	1.93e-001 3.65e-002
F14	2.10e-001 3.31e-002	+	**2.04e-001 3.16e-002**	=	2.09e-001 3.14e-002	=	2.18e-001 3.99e-002	=	2.18e-001 3.76e-002
F15	3.52e+000 4.18e-001	=	3.66e+000 5.04e-001	=	3.38e+000 4.54e-001	=	3.45e+000 3.99e-001	=	3.66e+000 3.28e-001
F16	1.02e+001 2.86e-001	=	1.01e+001 3.17e-001	=	1.01e+001 3.14e-001	=	1.01e+001 3.95e-001	=	1.01e+001 3.21e-001
F17	6.23e+002 3.84e+002	-	1.19e+003 1.00e+003	=	7.08e+002 7.14e+002	=	7.66e+002 4.85e+002	=	7.18e+002 3.41e+002
F18	3.17e+001 1.50e+001	=	2.82e+001 1.33e+001	=	2.95e+001 1.80e+001	=	3.03e+001 1.21e+001	=	3.25e+001 3.94e+001
F19	4.15e+000 7.12e-001	=	4.01e+000 6.49e-001	=	4.25e+000 5.55e-001	=	4.00e+000 6.22e-001	=	4.14e+000 8.04e-001
F20	3.15e+001 1.39e+001	+	**2.12e+001 1.19e+001**	+	**2.37e+001 1.20e+001**	=	2.57e+001 1.37e+001	=	3.43e+001 1.47e+001
F21	3.39e+002 4.07e+002	=	2.87e+002 1.42e+002	=	2.72e+002 1.47e+002	=	3.05e+002 3.86e+002	=	2.34e+002 1.12e+002
F22	1.23e+002 6.24e+001	=	1.36e+002 4.91e+001	=	1.49e+002 5.68e+001	=	1.31e+002 5.65e+001	=	1.37e+002 5.83e+001
F23	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005
F24	2.24e+002 1.20e+000	=	2.24e+002 8.46e-001	=	2.24e+002 9.33e-001	=	2.24e+002 1.13e+000	=	2.24e+002 1.19e+000
F25	2.08e+002 2.12e+000	=	2.08e+002 2.50e+000	=	2.08e+002 2.11e+000	=	2.07e+002 2.28e+000	+	**2.04e+002 9.96e-001**
F26	1.04e+002 1.82e+001	+	**1.00e+002 3.22e-002**	=	1.04e+002 1.82e+001	=	1.04e+002 1.82e+001	=	1.04e+002 1.82e+001
F27	3.61e+002 5.04e+001	=	3.43e+002 4.98e+001	=	3.58e+002 5.12e+001	=	3.58e+002 5.14e+001	+	**3.22e+002 4.16e+001**
F28	8.51e+002 2.53e+001	=	8.53e+002 2.09e+001	=	8.43e+002 3.54e+001	=	8.53e+002 2.69e+001	+	**8.21e+002 4.70e+001**
F29	6.39e+002 2.02e+002	=	6.16e+002 2.17e+002	=	6.61e+002 2.03e+002	=	6.69e+002 2.11e+002	=	6.91e+002 1.71e+002
F30	1.11e+003 3.68e+002	=	1.24e+003 4.73e+002	=	1.15e+003 4.75e+002	=	1.11e+003 3.65e+002	=	1.34e+003 4.22e+002
+/=/-	-		4/25/3		2/27/1		1/27/2		4/24/2
ARV	3.15		3.0667		2.65		3.1167		3.0167
Rank	5		3		1		4		2

Fun.	SaDE		sw-SaDE		cell-SaDE		ring-SaDE		MTSaDE
F1	5.80e+006 1.66e+006	+	**4.60e+006 1.75e+006**	+	**3.66e+006 2.09e+006**	+	**4.97e+006 1.79e+006**	+	**4.35e+006 1.58e+006**
F2	6.67e-007 8.48e-007	-	2.54e-006 4.50e-006	=	6.30e-007 1.18e-006	=	7.38e-007 1.04e-006	+	**9.78e-008 1.75e-007**
F3	2.58e+001 1.91e+001	=	2.43e+001 1.67e+001	+	**2.34e+000 1.68e+000**	=	2.42e+001 2.01e+001	=	2.01e+001 1.90e+001
F4	4.39e+001 3.78e+001	=	2.71e+001 3.49e+001	-	7.74e+001 3.61e+001	+	**2.38e+001 3.28e+001**	+	**3.48e-002 4.96e-002**
F5	2.04e+001 4.69e-002	=	2.04e+001 3.52e-002	=	2.04e+001 3.74e-002	=	2.04e+001 4.60e-002	=	2.04e+001 3.31e-002
F6	1.30e+001 1.05e+000	=	1.28e+001 1.19e+000	=	1.31e+001 2.09e+000	=	1.26e+001 1.33e+000	=	1.27e+001 8.32e-001
F7	6.09e-009 3.34e-008	=	7.20e-009 3.94e-008	-	1.98e-003 4.39e-003	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	-	2.32e-001 6.23e-001	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	4.85e+001 5.73e+000	=	5.08e+001 5.67e+000	-	6.42e+001 9.12e+000	=	4.75e+001 5.06e+000	=	4.85e+001 6.29e+000
F10	6.94e-004 3.80e-003	=	6.94e-004 3.80e-003	-	1.04e+001 2.16e+001	=	6.94e-004 3.80e-003	=	1.39e-003 7.60e-003
F11	2.62e+003 3.21e+002	=	2.65e+003 2.36e+002	=	2.61e+003 2.91e+002	=	2.69e+003 2.11e+002	=	2.66e+003 2.68e+002
F12	5.14e-001 6.81e-002	=	5.13e-001 5.36e-002	=	5.02e-001 5.61e-002	=	5.06e-001 7.10e-002	=	4.99e-001 6.58e-002
F13	2.64e-001 4.12e-002	=	2.63e-001 2.97e-002	-	3.53e-001 5.27e-002	=	2.60e-001 3.36e-002	=	2.77e-001 2.93e-002
F14	2.14e-001 2.44e-002	=	2.07e-001 2.30e-002	-	2.38e-001 5.26e-002	=	2.02e-001 2.29e-002	-	2.27e-001 2.88e-002
F15	6.31e+000 7.46e-001	=	6.50e+000 8.24e-001	-	8.77e+000 1.19e+000	=	6.39e+000 7.89e-001	=	6.44e+000 5.43e-001
F16	1.02e+001 3.52e-001	=	1.03e+001 3.26e-001	=	1.03e+001 3.59e-001	=	1.03e+001 3.29e-001	=	1.02e+001 3.01e-001
F17	6.65e+005 2.31e+005	+	**5.34e+005 1.94e+005**	+	**3.45e+005 3.18e+005**	=	5.33e+005 2.32e+005	+	**3.98e+005 1.46e+005**
F18	5.34e+003 2.58e+003	+	**1.80e+003 1.13e+003**	+	**3.18e+003 2.59e+003**	+	**2.56e+003 1.43e+003**	=	5.35e+003 3.40e+003
F19	5.98e+000 6.67e-001	=	5.92e+000 5.50e-001	=	6.33e+000 7.40e-001	=	5.80e+000 4.78e-001	+	**5.43e+000 5.82e-001**
F20	3.53e+003 1.80e+003	=	3.23e+003 1.52e+003	+	**8.95e+002 1.04e+003**	=	3.32e+003 1.52e+003	+	**1.98e+003 9.34e+002**
F21	9.71e+004 3.32e+004	+	**7.47e+004 3.39e+004**	+	**5.80e+004 5.84e+004**	=	1.01e+005 3.74e+004	=	8.83e+004 3.95e+004
F22	1.68e+002 4.66e+001	=	1.73e+002 5.03e+001	=	1.83e+002 5.13e+001	=	1.85e+002 6.04e+001	=	1.46e+002 5.51e+001
F23	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 0.00e+000	=	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005
F24	2.24e+002 6.25e-001	=	2.24e+002 4.77e-001	-	2.27e+002 3.60e+000	=	2.24e+002 4.93e-001	=	2.24e+002 6.67e-001
F25	2.07e+002 7.73e-001	=	2.08e+002 9.16e-001	-	2.07e+002 1.55e+000	=	2.07e+002 7.76e-001	+	**2.06e+002 5.80e-001**
F26	1.00e+002 4.22e-002	=	1.00e+002 2.92e-002	-	1.00e+002 5.20e-002	=	1.00e+002 4.73e-002	=	1.00e+002 3.59e-002
F27	3.50e+002 5.44e+001	=	3.41e+002 4.63e+001	-	4.42e+002 7.23e+001	=	3.41e+002 5.07e+001	+	**3.19e+002 4.04e+001**
F28	9.05e+002 3.50e+001	=	9.08e+002 2.11e+001	-	9.35e+002 6.02e+001	=	9.03e+002 3.66e+001	+	**8.83e+002 1.54e+001**
F29	1.34e+003 8.87e+001	+	**1.25e+003 1.15e+002**	=	1.34e+003 2.79e+002	+	**1.28e+003 1.07e+002**	=	1.31e+003 1.12e+002
F30	1.98e+003 2.39e+002	=	1.91e+003 2.88e+002	-	2.30e+003 6.79e+002	=	2.01e+003 3.06e+002	+	**1.71e+003 3.12e+002**
+/=/-	-		5/24/1		6/10/14		4/26/0		10/19/1
ARV	3.3167		3.05		3.5833		2.7667		2.2833
Rank	4		3		5		2		1

Table 22
Mean and standard deviation of the best error values obtained by MTDE and single-topology based DE on the CEC2014 functions at 50*D*.

Fun.	JADE		sw-JADE		cell-JADE		ring-JADE		MTJADE
F1	4.23e+004 2.23e+004	=	4.02e+004 1.85e+004	=	5.89e+004 5.22e+004	=	4.25e+004 2.16e+004	+	**2.23e+004 1.43e+004**
F2	1.01e-020 3.03e-020	-	6.60e-019 3.03e-018	=	9.31e-021 1.87e-020	=	1.93e-020 4.90e-020	+	**2.39e-022 3.84e-022**
F3	4.61e-003 2.95e-003	+	**7.65e-004 7.10e-004**	+	**1.38e-003 1.03e-003**	+	**1.56e-003 1.52e-003**	-	1.36e-002 9.84e-003
F4	1.64e+001 2.51e+001	=	1.39e+001 2.44e+001	=	1.25e+001 2.50e+001	=	1.46e+001 2.33e+001	+	3.27e+000 1.79e+001
F5	2.06e+001 3.38e-002	=	2.06e+001 3.69e-002	=	2.06e+001 3.78e-002	=	2.06e+001 3.35e-002	=	2.06e+001 4.08e-002
F6	1.04e+001 8.30e+000	=	8.01e+000 7.85e+000	=	1.02e+001 7.85e+000	=	1.09e+001 8.13e+000	+	**5.54e+000 7.48e+000**
F7	6.80e-003 1.19e-002	=	4.60e-003 5.63e-003	=	5.09e-003 5.71e-003	=	6.07e-003 7.07e-003	+	**1.56e-003 3.57e-003**
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	6.45e+001 9.89e+000	=	6.63e+001 9.00e+000	=	6.29e+001 8.00e+000	=	6.59e+001 9.52e+000	=	6.17e+001 7.94e+000
F10	3.19e+001 5.08e+000	+	**3.21e-001 1.74e-001**	+	**1.61e+000 1.25e+000**	+	**6.33e-001 3.40e-001**	-	8.30e+001 1.63e+001
F11	4.96e+003 4.33e+002	=	4.91e+003 4.31e+002	=	4.87e+003 3.81e+002	=	4.88e+003 4.16e+002	=	4.86e+003 3.80e+002
F12	5.59e-001 7.91e-002	=	5.89e-001 8.19e-002	=	5.85e-001 6.45e-002	=	5.75e-001 4.67e-002	=	5.83e-001 7.53e-002
F13	3.51e-001 5.18e-002	=	3.45e-001 4.07e-002	=	3.64e-001 5.04e-002	=	3.62e-001 4.51e-002	+	**3.03e-001 3.53e-002**
F14	2.82e-001 4.00e-002	=	2.79e-001 3.43e-002	=	2.87e-001 3.09e-002	=	2.89e-001 3.64e-002	=	2.92e-001 2.89e-002
F15	1.04e+001 1.72e+000	=	9.82e+000 1.43e+000	=	9.69e+000 1.29e+000	+	**9.55e+000 1.50e+000**	+	**8.92e+000 1.29e+000**
F16	1.89e+001 3.32e-001	=	1.90e+001 3.83e-001	=	1.90e+001 3.68e-001	=	1.90e+001 3.18e-001	=	1.89e+001 3.06e-001
F17	1.48e+004 1.44e+004	=	1.84e+004 1.78e+004	-	2.49e+004 2.28e+004	=	1.77e+004 1.91e+004	+	**9.78e+003 1.33e+004**
F18	1.28e+002 3.33e+001	-	4.48e+002 3.90e+002	-	2.17e+002 1.92e+002	-	2.61e+002 3.38e+002	=	1.33e+002 3.13e+001
F19	2.23e+001 1.78e+001	-	2.66e+001 1.54e+001	=	2.51e+001 1.42e+001	=	2.67e+001 1.51e+001	=	1.44e+001 6.67e+000
F20	8.72e+001 1.80e+001	=	9.87e+001 3.89e+001	=	8.44e+001 2.81e+001	=	7.83e+001 2.03e+001	=	1.00e+002 3.97e+001
F21	5.53e+003 6.26e+003	-	9.97e+003 1.25e+004	=	9.34e+003 1.15e+004	=	8.18e+003 1.01e+004	=	4.07e+003 4.91e+003
F22	4.91e+002 1.28e+002	=	4.47e+002 9.72e+001	=	4.70e+002 1.35e+002	=	4.25e+002 1.24e+002	=	4.77e+002 1.36e+002
F23	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	=	3.44e+002 1.13e-005
F24	2.72e+002 2.17e+000	-	2.73e+002 2.14e+000	=	2.73e+002 2.12e+000	=	2.73e+002 2.23e+000	-	2.74e+002 1.98e+000
F25	2.21e+002 4.46e+000	-	2.23e+002 4.62e+000	=	2.22e+002 2.25e+000	=	2.23e+002 1.90e+000	+	**2.15e+002 4.15e+000**
F26	1.07e+002 2.53e+001	=	1.14e+002 3.44e+001	=	1.20e+002 4.05e+001	=	1.17e+002 3.77e+001	=	1.00e+002 1.38e-001
F27	4.16e+002 5.63e+001	-	4.60e+002 5.64e+001	-	4.57e+002 7.15e+001	=	4.29e+002 4.24e+001	=	3.88e+002 5.48e+001
F28	1.31e+003 5.81e+001	=	1.33e+003 6.32e+001	=	1.31e+003 5.82e+001	=	1.32e+003 8.82e+001	+	**1.25e+003 4.89e+001**
F29	8.46e+002 9.42e+001	=	9.29e+002 1.51e+002	=	8.81e+002 1.70e+002	=	8.95e+002 1.14e+002	=	8.33e+002 1.10e+002
F30	1.09e+004 6.63e+002	=	1.07e+004 6.82e+002	=	1.06e+004 5.16e+002	=	1.07e+004 7.85e+002	=	1.07e+004 8.66e+002
+/=/-	-		2/21/7		2/25/3		3/26/1		10/17/3
ARV	2.9667		3.35		3.15		3.3167		2.2167
Rank	2		5		3		4		1
Fun.	SaDE		sw-SaDE		cell-SaDE		ring-SaDE		MTSaDE
F1	8.06e+006 2.03e+006	=	7.19e+006 1.86e+006	+	**6.38e+006 2.61e+006**	+	**7.06e+006 1.26e+006**	=	6.98e+006 2.06e+006
F2	3.01e+003 1.79e+003	=	2.27e+003 1.33e+003	=	4.71e+003 4.90e+003	=	2.31e+003 1.44e+003	+	**2.03e+003 1.25e+003**
F3	3.85e+003 1.48e+003	=	3.67e+003 1.45e+003	+	**1.70e+003 1.07e+003**	=	3.71e+003 1.59e+003	=	3.46e+003 1.36e+003
F4	1.02e+002 3.38e+001	=	1.02e+002 3.78e+001	=	1.15e+002 3.12e+001	=	9.05e+001 3.27e+001	+	**1.99e+001 2.80e+001**
F5	2.06e+001 3.47e-002	=	2.06e+001 3.86e-002	=	2.06e+001 2.78e-002	=	2.06e+001 3.81e-002	=	2.05e+001 4.37e-002
F6	2.84e+001 1.86e+000	=	2.77e+001 2.13e+000	-	2.99e+001 3.04e+000	=	2.86e+001 2.11e+000	+	**2.70e+001 2.44e+000**
F7	8.38e-003 7.23e-003	=	6.75e-003 3.77e-003	=	5.75e-003 6.70e-003	+	**4.29e-003 5.16e-003**	+	**3.78e-003 5.00e-003**
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	-	1.86e+000 1.33e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	1.22e+002 9.91e+000	-	1.30e+002 1.27e+001	-	1.72e+002 1.68e+001	-	1.28e+002 9.23e+000	=	1.19e+002 1.14e+001
F10	3.56e+000 1.06e+000	+	**2.43e-001 1.34e-001**	-	1.76e+001 2.93e+001	+	**5.94e-001 2.21e-001**	+	**2.29e+000 9.09e-001**
F11	6.36e+003 2.83e+002	=	6.28e+003 3.38e+002	+	**6.10e+003 2.57e+002**	=	6.27e+003 3.39e+002	=	6.22e+003 3.61e+002
F12	7.10e-001 4.52e-002	+	**6.69e-001 6.83e-002**	+	**6.29e-001 8.29e-002**	+	6.63e-001 6.12e-002	+	**6.59e-001 5.87e-002**
F13	3.87e-001 3.88e-002	=	4.03e-001 4.49e-002	-	4.58e-001 5.79e-002	-	4.18e-001 4.11e-002	=	3.66e-001 3.42e-002
F14	2.95e-001 2.58e-002	+	**2.80e-001 2.01e-002**	=	3.12e-001 3.20e-002	=	2.90e-001 1.80e-002	=	2.90e-001 2.46e-002
F15	1.64e+001 1.63e+000	-	1.94e+001 2.09e+000	-	2.78e+001 5.02e+000	-	1.89e+001 1.59e+000	=	1.55e+001 1.52e+000
F16	1.91e+001 3.65e-001	=	1.90e+001 3.39e-001	=	1.92e+001 3.57e-001	=	1.91e+001 3.79e-001	=	1.92e+001 3.32e-001
F17	1.62e+006 4.58e+005	=	1.39e+006 3.97e+005	=	1.36e+006 9.93e+005	=	1.49e+006 4.48e+005	=	1.41e+006 4.81e+005
F18	8.31e+002 4.14e+002	+	**5.17e+002 3.06e+002**	-	1.41e+003 9.87e+002	+	**4.79e+002 1.66e+002**	=	7.96e+002 5.30e+002
F19	2.88e+001 9.85e+000	+	**1.71e+001 1.06e+001**	=	3.75e+001 2.78e+001	=	2.21e+001 1.48e+001	+	**1.32e+001 6.59e+000**
F20	7.63e+003 2.36e+003	=	7.83e+003 2.10e+003	+	**2.20e+003 1.46e+003**	=	8.12e+003 1.93e+003	+	**5.60e+003 1.61e+003**
F21	1.20e+006 5.31e+005	=	1.02e+006 2.86e+005	+	**6.48e+005 4.37e+005**	=	1.20e+006 3.84e+005	=	1.05e+006 3.07e+005
F22	5.28e+002 1.54e+002	=	5.48e+002 1.12e+002	=	6.01e+002 1.70e+002	=	5.52e+002 1.25e+002	=	5.00e+002 1.14e+002
F23	3.44e+002 1.27e-005	=	3.44e+002 1.27e-005	=	3.44e+002 9.82e-006	=	3.44e+002 1.27e-005	=	3.44e+002 1.13e-005
F24	2.65e+002 5.21e+000	+	**2.60e+002 5.42e+000**	-	2.68e+002 3.92e+000	+	**2.63e+002 5.21e+000**	=	2.67e+002 4.51e+000
F25	2.17e+002 4.68e+000	+	**2.09e+002 8.32e+000**	=	2.18e+002 2.53e+000	=	2.15e+002 5.80e+000	+	**2.13e+002 1.80e+000**
F26	1.14e+002 3.46e+001	=	1.14e+002 3.45e+001	-	1.47e+002 5.08e+001	=	1.17e+002 3.80e+001	=	1.04e+002 1.83e+001
F27	7.71e+002 1.46e+002	=	8.37e+002 1.09e+002	-	1.03e+003 1.47e+002	=	8.13e+002 1.51e+002	+	**4.61e+002 1.97e+002**
F28	1.52e+003 9.01e+001	=	1.51e+003 9.75e+001	-	1.84e+003 2.29e+002	=	1.49e+003 5.95e+001	+	**1.40e+003 5.52e+001**
F29	1.32e+003 1.12e+002	+	**1.24e+003 1.24e+002**	=	1.40e+003 2.80e+002	+	**1.24e+003 1.14e+002**	=	1.31e+003 1.37e+002
F30	9.81e+003 5.68e+002	=	9.67e+003 7.24e+002	=	1.02e+004 1.05e+003	=	9.81e+003 7.54e+002	+	**9.50e+003 3.93e+002**
+/=/-	-		8/20/2		6/13/11		7/20/3		12/18/0
ARV	3.55		2.65		3.8667		3.0167		1.9167
Rank	4		2		5		3		1

Table 23
Results of the multiple-problem analysis by the Wilcoxon test between MTDE and the corresponding single-topology based DE for the CEC2014 functions at 30*D* and 50*D*.

Algorithm at 30*D*	with JADE	*R*+	*R*−	*p*-value	with SaDE	*R*+	*R*−	*p*-value
original DE		250.5	214.5	0.703564		340.5	94.5	0.007253
sw-DE		224	211	0.879366		310.5	124.5	0.042077
cell-DE		212.5	222.5	1		261.5	173.5	0.335933
ring-DE		225	210	0.862663		334.5	130.5	0.034078
Algorithm at 50*D*	with JADE	*R*+	*R*−	*p*-value	with SaDE	*R*+	*R*−	*p*-value
original DE		366	99	0.005848		444.5	20.5	0.000011
sw-DE		370	95	0.004534		313.5	151.5	0.093676
cell-DE		326	109	0.018112		299.5	165.5	0.165027
ring-DE		354	111	0.012096		364.5	70.5	0.001384

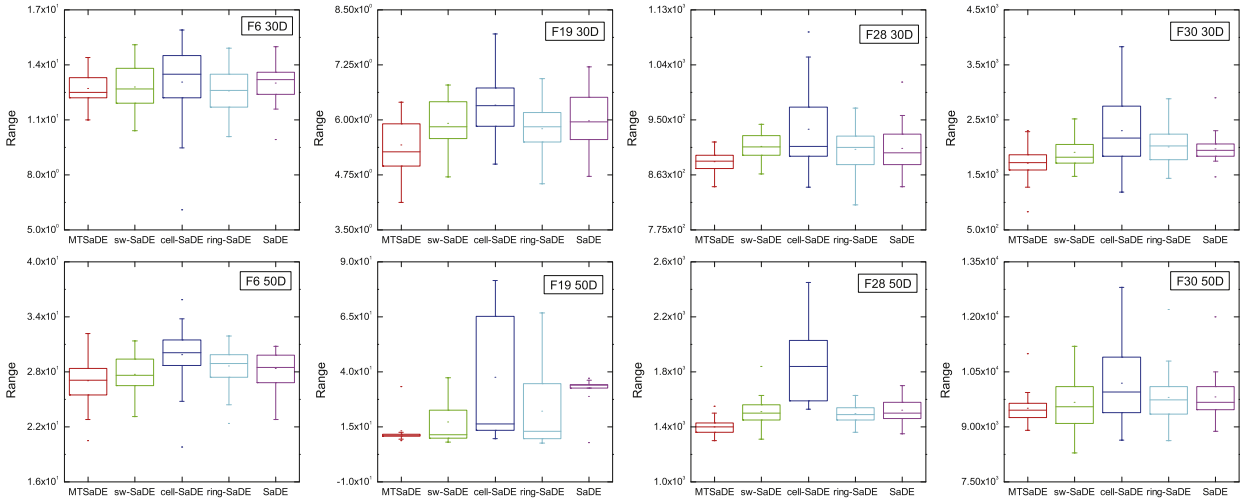

Fig. 10. Box plots of MTSaDE and three SaDE variants with single topology on the selected functions.

Furthermore, Figs. 9 and 10 show that MTDE performs the best among all the five algorithms on most selected functions.

In summary, we conclude that MTDE performs more effective than the single-topology-based DE variants by using multiple topologies simultaneously during the evolutionary process.

4.7. Comparison with multi-topology-based DE variants

To evaluate the effectiveness of MTDE with individual-dependent topology adaptation, comparisons among MTDE, APT-DE and Na-DE are performed. Among these three DE algorithmic frameworks, the main differences are the ways in which the topology is selected and the neighborhood information is utilized. Thus, the topologies in MTDE are also used in APTDE and NaDE for fair comparison. Here, MTDE, APT-DE and Na-DE are applied to four DE algorithms (DE/current-to-best/1, DE/rand-to-best/1, JADE, and MDEpBX) for comparison. The results on the CEC2014 functions at 30*D* and 50*D* are shown inTables 24–26.

According to Table 24, MTDE significantly outperforms APT-DE and Na-DE in most cases for functions at 30*D*. Specifically, MTDE performs significantly better than Na-DE on 16, 19, 5 and 10 functions, respectively, and is significantly better than APT-DE on 21, 25, 8 and 16 functions, respectively. For the higher-dimensional functions in Table 25, MTDE also obtains the results that are consistent with those in Table 24. In addition, according to the results of multiple-problem analysis by the Wilcoxon test in Table 26, MTDE obtains higher $R+$ than $R-$ values on all cases at both 30*D* and 50*D*. Based on the p values, significant differences at $\alpha = 0.05$ are observed in five cases for functions at 30*D* and in six cases for functions at 50*D*.

In addition, the box plots for MTDE, Na-DE and APT-DE with DE/current-to-best/1 and MDEpBX are shown in Figs. 11 and 12, respectively. As the figures show, MTDE achieves best performance among the three multi-topology-based DE algorithms on most selected functions.

On the whole, we conclude that MTDE outperforms the compared multi-topology-based DE algorithms due to the utilization of the differences in the fitness values among individuals for topology adaption.

Table 24
Mean and standard deviation of the best error values obtained by MTDE, Na-DE and APT-DE with four DE algorithms on the CEC2014 functions at 30*D*.

Fun.	MTDE/current-to-best/1		Na-DE/current-to-best/1		APT-DE/current-to-best/1	Fun.	MTDE/rand-to-best/1		Na-DE/rand-to-best/1		APT-DE/rand-to-best/1
F1	**2.42e+003 2.70e+003**	-	8.24e+003 9.66e+003	-	6.77e+006 6.22e+006	F1	**2.18e+003 2.77e+003**	-	1.05e+006 7.54e+005	-	1.60e+007 1.07e+007
F2	5.27e+005 1.23e+006	+	**6.41e+002 1.84e+003**	-	2.14e+009 1.14e+009	F2	**2.45e+005 8.85e+005**	-	1.13e+007 1.58e+007	-	3.33e+009 4.01e+009
F3	1.01e-002 2.68e-002	=	5.99e-003 3.27e-002	-	1.77e+003 1.40e+003	F3	**6.24e-003 2.72e-002**	-	1.86e+002 2.65e+002	-	2.76e+003 2.06e+003
F4	8.83e+001 3.80e+001	=	7.05e+001 3.91e+001	-	2.74e+002 9.72e+001	F4	9.88e+001 3.44e+001	=	1.08e+002 3.73e+001	-	3.70e+002 3.21e+002
F5	2.10e+001 3.72e-002	+	**2.04e+001 1.30e-001**	=	2.09e+001 5.92e-002	F5	2.09e+001 6.97e-002	=	2.08e+001 1.93e-001	=	2.09e+001 1.10e-001
F6	**1.96e+000 1.13e+000**	-	8.00e+000 2.30e+000	-	7.83e+000 1.84e+000	F6	**2.16e+000 1.32e+000**	-	7.74e+000 2.54e+000	-	9.20e+000 2.05e+000
F7	2.81e-001 3.06e-001	+	**4.87e-002 6.18e-002**	-	2.47e+001 1.43e+001	F7	**3.70e-002 5.43e-002**	-	7.67e-001 1.01e+000	-	3.58e+001 4.31e+001
F8	**3.49e+001 2.25e+001**	-	4.09e+001 9.86e+000	-	4.41e+001 8.76e+000	F8	**2.85e+001 8.89e+000**	-	3.50e+001 9.51e+000	-	4.15e+001 1.05e+001
F9	4.42e+001 3.89e+001	=	4.70e+001 1.02e+001	=	4.70e+001 1.15e+001	F9	**2.82e+001 9.26e+000**	-	3.69e+001 7.57e+000	-	4.75e+001 1.62e+001
F10	4.52e+003 1.37e+003	+	**1.46e+003 4.77e+002**	+	**1.69e+003 1.21e+003**	F10	**7.34e+002 3.96e+002**	-	1.17e+003 5.24e+002	-	1.06e+003 5.17e+002
F11	6.42e+003 3.18e+002	+	**2.90e+003 6.42e+002**	+	**5.37e+003 1.68e+003**	F11	4.59e+003 2.02e+003	+	**2.43e+003 7.20e+002**	+	**2.33e+003 1.12e+003**
F12	2.47e+000 2.60e-001	-	1.20e+003 1.11e-001	+	**2.35e+000 2.76e-001**	F12	2.34e+000 4.80e-001	-	1.20e+003 8.75e-001	+	**1.94e+000 7.71e-001**
F13	**2.18e-001 4.41e-002**	-	2.45e-001 5.27e-002	-	4.07e-001 3.86e-001	F13	2.04e-001 3.45e-002	=	2.29e-001 4.89e-002	-	3.80e-001 3.60e-001
F14	2.42e-001 3.85e-002	=	2.23e-001 4.66e-002	-	5.07e+000 6.90e+000	F14	2.61e-001 2.67e-002	+	**2.35e-001 3.99e-002**	=	7.70e+000 1.58e+001
F15	1.01e+001 3.00e+000	+	**5.28e+000 1.69e+000**	=	1.24e+001 6.93e+000	F15	3.95e+000 2.53e+000	=	3.87e+000 9.29e-001	-	2.93e+002 7.51e+002
F16	1.12e+001 4.36e-001	+	**1.04e+001 8.45e-001**	+	**1.08e+001 3.56e-001**	F16	1.04e+001 4.54e-001	=	1.02e+001 7.46e-001	=	1.04e+001 4.19e-001
F17	**4.83e+002 2.90e+002**	-	8.76e+002 4.65e+002	-	6.09e+003 5.62e+003	F17	**6.96e+002 5.26e+002**	-	7.26e+003 1.23e+004	-	5.92e+004 1.43e+005
F18	**4.88e+001 2.35e+001**	-	7.28e+001 2.47e+001	-	9.44e+001 4.44e+001	F18	**5.33e+001 2.30e+001**	-	7.72e+001 2.74e+001	-	2.99e+002 5.77e+002
F19	**4.14e+000 1.11e+000**	-	6.42e+000 1.36e+000	-	9.29e+000 2.93e+000	F19	**5.16e+000 1.29e+000**	-	6.66e+000 1.43e+000	-	1.71e+001 1.48e+001
F20	**2.16e+001 1.35e+001**	-	3.11e+001 2.29e+001	-	9.95e+001 4.25e+001	F20	2.31e+001 1.31e+001	=	3.10e+001 1.88e+001	-	1.50e+002 1.40e+002
F21	**2.32e+002 1.25e+002**	-	3.21e+002 1.71e+002	-	9.65e+002 1.31e+003	F21	**2.29e+002 1.46e+002**	-	3.82e+002 2.12e+002	-	6.93e+003 7.99e+003
F22	1.55e+002 5.38e+001	-	2.30e+002 1.09e+002	=	1.80e+002 7.59e+001	F22	1.72e+002 8.35e+001	=	2.04e+002 6.82e+001	-	2.17e+002 7.37e+001
F23	3.15e+002 1.48e-001	+	**3.15e+002 0.00e+000**	-	3.29e+002 7.80e+000	F23	**3.15e+002 3.65e-001**	-	3.16e+002 4.91e-001	-	3.31e+002 1.87e+001
F24	2.28e+002 5.12e+000	=	2.29e+002 5.50e+000	=	2.31e+002 9.18e+000	F24	2.29e+002 5.98e+000	=	2.30e+002 5.58e+000	-	2.36e+002 9.66e+000
F25	**2.04e+002 9.01e-001**	-	2.07e+002 2.53e+000	-	2.08e+002 2.09e+000	F25	**2.05e+002 1.33e+000**	-	2.07e+002 2.07e+000	-	2.11e+002 3.38e+000
F26	**1.04e+002 1.82e+001**	-	1.20e+002 4.06e+001	-	1.47e+002 5.06e+001	F26	1.07e+002 2.53e+001	=	1.10e+002 3.05e+001	-	1.37e+002 4.90e+001
F27	**3.76e+002 4.14e+001**	-	4.48e+002 6.82e+001	-	4.64e+002 7.13e+001	F27	**3.94e+002 5.04e+001**	-	4.49e+002 5.72e+001	-	5.02e+002 1.07e+002
F28	**8.68e+002 5.58e+001**	-	9.35e+002 1.02e+002	-	9.69e+002 8.58e+001	F28	**8.88e+002 4.50e+001**	-	9.28e+002 7.80e+001	-	1.01e+003 1.07e+002
F29	5.95e+002 2.03e+002	=	6.58e+002 1.53e+002	-	9.87e+002 1.61e+002	F29	**6.77e+002 3.15e+002**	-	9.03e+002 2.03e+002	-	1.45e+003 4.54e+002
F30	**9.22e+002 3.55e+002**	-	1.43e+003 4.70e+002	-	3.70e+003 1.19e+003	F30	**1.59e+003 8.18e+002**	-	2.07e+003 7.83e+002	-	7.56e+003 6.46e+003
+/=/-	-		8/6/16		4/5/21	w/t/l			2/9/19		2/3/25
ARV	1.5167		1.7667		2.7167	ARV	1.2667		1.9333		2.8
Rank	1		2		3	Rank	1		2		3
Fun.	MTJADE		Na-JADE		APT-JADE	Fun.	MTMDEpBX		Na-MDEpBX		APT-MDEpBX
F1	**4.21e+003 3.90e+003**	-	4.46e+003 3.75e+003	-	4.99e+003 3.94e+003	F1	2.09e+004 1.06e+005	+	**1.62e+004 2.79e+004**	-	2.14e+006 1.48e+006
F2	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	F2	8.35e+003 4.44e+003	=	1.11e+004 9.20e+003	-	6.04e+006 1.96e+007
F3	1.58e-010 3.70e-010	=	5.64e-011 6.99e-011	+	**1.06e-013 1.77e-013**	F3	**6.01e-028 1.83e-027**	-	3.38e-027 3.40e-027	-	3.27e-027 3.87e-027
F4	**6.80e-029 5.77e-029**	-	2.17e+000 1.19e+001	-	5.18e-012 2.64e-011	F4	3.67e+001 3.26e+001	=	5.14e+001 3.51e+001	-	8.35e+001 3.50e+001
F5	2.04e+001 3.38e-002	=	2.04e+001 5.04e-002	=	2.04e+001 3.64e-002	F5	2.05e+001 9.52e-002	=	2.05e+001 9.17e-002	=	2.05e+001 8.41e-002
F6	7.39e+000 4.24e+000	+	**2.76e+000 3.60e+000**	=	7.95e+000 3.85e+000	F6	**1.23e+000 1.12e+000**	-	2.52e+000 1.44e+000	-	2.85e+000 1.52e+000
F7	2.47e-004 1.35e-003	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	F7	**8.28e-004 4.48e-003**	-	7.13e-003 1.29e-002	-	2.27e-001 2.72e-001
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	F8	1.18e+001 3.00e+000	=	1.32e+001 3.84e+000	=	1.19e+001 3.86e+000
F9	2.81e+001 4.33e+000	=	2.70e+001 3.98e+000	=	2.83e+001 3.89e+000	F9	2.00e+001 6.44e+000	=	2.14e+001 6.06e+000	=	2.32e+001 7.75e+000
F10	4.04e+000 1.39e+000	+	**2.36e+000 9.91e-001**	+	**7.24e-001 2.62e-001**	F10	2.16e+002 1.60e+002	=	1.94e+002 1.33e+002	=	2.10e+002 1.19e+002
F11	2.15e+003 2.33e+002	=	2.19e+003 2.52e+002	=	2.15e+003 2.47e+002	F11	2.13e+003 4.83e+002	=	2.17e+003 5.14e+002	-	2.56e+003 5.90e+002
F12	4.74e-001 5.58e-002	-	1.20e+003 9.40e-002	=	4.82e-001 7.68e-002	F12	**3.85e-001 1.06e-001**	-	1.20e+003 1.29e-001	-	4.78e-001 1.90e-001
F13	1.93e-001 3.65e-002	=	1.82e-001 3.54e-002	=	2.04e-001 3.24e-002	F13	1.50e-001 2.00e-002	=	1.44e-001 2.40e-002	=	1.60e-001 3.57e-002
F14	2.18e-001 3.76e-002	-	2.36e-001 3.23e-002	=	2.09e-001 2.76e-002	F14	2.66e-001 4.35e-002	=	2.50e-001 5.97e-002	+	**2.31e-001 3.86e-002**
F15	3.66e+000 3.28e-001	=	3.71e+000 5.48e-001	=	3.59e+000 3.72e-001	F15	3.71e+000 1.43e+000	+	**3.09e+000 9.32e-001**	=	3.70e+000 9.35e-001
F16	1.01e+001 3.21e-001	=	1.00e+001 2.94e-001	=	1.02e+001 3.48e-001	F16	9.14e+000 8.29e-001	=	8.82e+000 8.69e-001	=	9.20e+000 8.13e-001
F17	**7.18e+002 3.41e+002**	-	1.68e+003 4.76e+003	-	9.78e+002 4.75e+002	F17	9.66e+002 4.47e+002	=	9.16e+002 3.14e+002	-	5.32e+003 5.59e+003
F18	3.25e+001 3.94e+001	=	3.27e+001 4.13e+001	=	3.63e+001 4.71e+001	F18	1.09e+003 1.37e+003	+	**4.46e+002 7.39e+002**	=	6.11e+002 7.60e+002
F19	4.14e+000 8.04e-001	=	4.06e+000 7.09e-001	=	4.12e+000 6.16e-001	F19	4.46e+000 9.06e-001	=	4.83e+000 9.37e-001	-	6.42e+000 1.15e+000
F20	3.43e+001 1.47e+001	=	3.80e+001 1.45e+001	+	**2.62e+001 1.26e+001**	F20	**3.85e+001 2.34e+001**	-	5.17e+001 2.62e+001	-	5.67e+001 2.75e+001
F21	2.34e+002 1.12e+002	=	5.11e+002 8.79e+002	-	3.27e+002 1.63e+002	F21	4.89e+002 3.01e+002	=	4.88e+002 2.12e+002	=	4.97e+002 2.85e+002
F22	1.37e+002 5.83e+001	=	1.36e+002 6.57e+001	=	1.28e+002 6.12e+001	F22	2.44e+002 1.13e+002	=	2.05e+002 1.12e+002	+	**2.03e+002 8.76e+001**
F23	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 9.82e-006	F23	**3.15e+002 1.46e-003**	-	3.15e+002 1.66e-002	-	3.16e+002 4.54e-001
F24	2.24e+002 1.19e+000	=	2.24e+002 1.06e+000	=	2.24e+002 1.17e+000	F24	2.28e+002 4.66e+000	=	2.29e+002 4.81e+000	=	2.28e+002 4.89e+000
F25	2.04e+002 9.96e-001	=	2.04e+002 9.49e-001	-	2.07e+002 2.09e+000	F25	**2.07e+002 1.56e+000**	-	2.09e+002 2.13e+000	-	2.10e+002 9.02e-001
F26	1.04e+002 1.82e+001	+	**1.00e+002 3.07e-002**	=	1.00e+002 3.00e-002	F26	1.04e+002 1.82e+001	=	1.10e+002 3.05e+001	=	1.13e+002 3.45e+001
F27	3.22e+002 4.16e+001	=	3.16e+002 3.60e+001	-	3.58e+002 5.16e+001	F27	**3.34e+002 3.24e+001**	-	3.57e+002 3.85e+001	-	3.79e+002 4.43e+001
F28	8.21e+002 4.70e+001	=	8.08e+002 4.11e+001	-	8.47e+002 2.48e+001	F28	8.86e+002 3.63e+001	=	8.92e+002 5.23e+001	=	8.94e+002 3.41e+001
F29	6.91e+002 1.71e+002	+	**5.41e+002 2.04e+002**	-	7.49e+002 2.35e+002	F29	**9.82e+002 3.56e+002**	-	1.16e+003 3.19e+002	-	1.23e+003 3.15e+002
F30	1.34e+003 4.22e+002	=	1.26e+003 6.33e+002	+	**1.26e+003 3.80e+002**	F30	**2.06e+003 6.58e+002**	-	2.57e+003 5.78e+002	-	3.33e+003 1.26e+003
+/=/-	-		4/21/5		4/18/8	+/=/-	-		3/17/10		2/12/16
ARV	1.9333		2		2.0667	ARV	1.6		1.8167		2.5833
Rank	1		2		3	Rank	1		2		3

4.8. Influence of the combination of topologies

To study the influence of topology combinations on the performance of the proposed algorithm, MTDE is compared with its variants with different combinations of topologies. For this purpose, the following five MTDE variants are investigated

Table 25
Mean and standard deviation of the best error values obtained by MTDE, Na-DE and APT-DE with four DE algorithms on the CEC2014 functions at 50*D*.

Fun.	MTDE/current-to-best/1		Na-DE/current-to-best/1		APT-DE/current-to-best/1	Fun.	MTDE/rand-to-best/1		Na-DE/rand-to-best/1		APT-DE/rand-to-best/1
F1	1.04e+006 1.16e+006	+	**3.18e+005 2.86e+005**	-	5.30e+007 2.45e+007	F1	**1.77e+006 1.44e+006**	-	5.44e+006 3.52e+006	-	8.00e+007 5.71e+007
F2	8.28e+007 7.77e+007	+	**5.18e+003 5.66e+003**	-	1.79e+010 4.25e+009	F2	**5.57e+007 4.91e+007**	-	2.00e+008 2.91e+008	-	2.21e+010 1.85e+010
F3	**1.88e-009 6.06e-009**	-	1.61e-005 7.16e-005	-	1.85e+003 3.05e+003	F3	**2.23e-009 7.35e-009**	-	9.13e-005 2.62e-004	-	5.41e+003 3.95e+003
F4	1.84e+002 4.27e+001	+	**1.30e+002 4.10e+001**	-	2.04e+003 7.05e+002	F4	2.09e+002 4.42e+001	+	**1.78e+002 4.91e+001**	-	2.50e+003 2.06e+003
F5	2.11e+001 3.01e-002	+	**2.05e+001 1.29e-001**	-	2.11e+001 4.76e-002	F5	2.11e+001 3.05e-002	=	2.11e+001 1.34e-001	-	2.11e+001 3.72e-002
F6	**1.38e+001 2.28e+000**	-	2.28e+001 5.16e+000	-	2.42e+001 2.16e+000	F6	**1.24e+001 1.99e+000**	-	2.10e+001 3.31e+000	-	2.62e+001 2.78e+000
F7	2.14e+000 1.28e+000	+	**4.39e-002 7.90e-002**	-	1.72e+002 6.15e+001	F7	2.00e+000 1.29e+000	=	2.92e+000 3.48e+000	-	2.09e+002 1.28e+002
F8	**7.59e+001 1.08e+001**	-	1.15e+002 2.14e+001	-	1.17e+002 1.81e+001	F8	**7.00e+001 1.58e+001**	-	9.05e+001 1.61e+001	-	1.24e+002 2.72e+001
F9	**7.12e+001 1.16e+001**	-	1.15e+002 2.45e+001	-	1.23e+002 1.83e+001	F9	**7.06e+001 1.44e+001**	-	8.71e+001 2.05e+001	-	1.66e+002 4.37e+001
F10	8.68e+003 3.43e+003	+	**4.21e+003 7.78e+002**	+	**4.00e+003 1.75e+003**	F10	**2.96e+003 8.91e+002**	-	3.45e+003 8.76e+002	-	3.56e+003 8.11e+002
F11	1.24e+004 5.25e+002	+	**5.81e+003 7.72e+002**	+	**1.16e+004 1.95e+003**	F11	9.54e+003 3.66e+003	+	**5.42e+003 1.72e+003**	+	**5.72e+003 2.80e+003**
F12	3.32e+000 2.47e-001	-	1.20e+003 1.95e-001	=	3.12e+000 3.54e-001	F12	3.18e+000 6.19e-001	-	1.20e+003 1.16e+000	=	3.14e+000 5.98e-001
F13	**4.05e-001 5.12e-002**	-	4.69e-001 6.52e-002	-	2.52e+000 1.05e+000	F13	**3.80e-001 6.84e-002**	-	4.36e-001 5.55e-002	-	1.71e+000 1.34e+000
F14	3.07e-001 3.74e-002	=	3.01e-001 3.22e-002	-	4.12e+001 1.34e+001	F14	3.19e-001 3.73e-002	=	3.10e-001 2.64e-002	-	3.47e+001 2.49e+001
F15	**1.67e+001 9.27e+000**	-	3.92e+001 1.55e+001	-	3.58e+003 3.73e+003	F15	**1.55e+001 7.82e+000**	-	4.20e+001 1.93e+001	-	3.24e+004 9.18e+004
F16	2.06e+001 3.94e-001	+	**1.91e+001 8.17e-001**	=	2.04e+001 5.36e-001	F16	2.01e+001 5.72e-001	+	**1.96e+001 7.94e-001**	=	1.99e+001 3.82e-001
F17	**5.00e+003 1.31e+004**	-	8.91e+003 9.80e+003	-	1.34e+005 8.23e+004	F17	**4.77e+004 3.53e+004**	-	7.71e+004 4.62e+004	-	7.55e+005 9.27e+005
F18	1.67e+002 1.37e+002	+	**1.31e+002 3.72e+001**	-	1.09e+006 4.71e+006	F18	6.83e+002 7.15e+002	=	7.25e+002 6.13e+002	-	6.10e+007 2.17e+008
F19	3.88e+001 2.21e+001	+	**2.62e+001 2.03e+001**	-	8.35e+001 2.55e+001	F19	4.87e+001 2.10e+001	=	5.81e+001 2.62e+001	-	9.91e+001 4.38e+001
F20	8.10e+001 2.45e+001	=	9.85e+001 4.41e+001	-	2.95e+002 1.73e+002	F20	6.09e+001 2.77e+001	=	6.92e+001 2.75e+001	-	1.26e+003 1.11e+003
F21	**7.29e+002 3.37e+002**	-	2.09e+003 3.03e+003	-	5.27e+004 4.38e+004	F21	**2.44e+003 8.21e+003**	-	4.42e+004 5.52e+004	-	2.38e+005 2.58e+005
F22	**2.63e+002 2.65e+002**	-	6.79e+002 2.77e+002	-	3.84e+002 1.64e+002	F22	3.73e+002 1.98e+002	=	4.51e+002 1.99e+002	=	4.39e+002 2.23e+002
F23	3.45e+002 1.28e+000	+	**3.44e+002 0.00e+000**	-	4.49e+002 4.53e+001	F23	**3.46e+002 3.13e+000**	-	3.52e+002 7.36e+000	-	4.88e+002 1.31e+002
F24	2.80e+002 3.62e+000	=	2.80e+002 4.94e+000	-	2.90e+002 8.95e+000	F24	2.79e+002 3.47e+000	=	2.80e+002 3.67e+000	-	3.04e+002 2.51e+001
F25	**2.16e+002 4.25e+000**	-	2.23e+002 5.35e+000	-	2.26e+002 6.57e+000	F25	**2.18e+002 3.71e+000**	-	2.24e+002 5.10e+000	-	2.32e+002 9.73e+000
F26	**1.44e+002 5.01e+001**	-	1.87e+002 3.40e+001	-	1.87e+002 3.44e+001	F26	**1.41e+002 4.92e+001**	-	1.80e+002 4.06e+001	-	1.94e+002 2.56e+001
F27	**6.72e+002 7.10e+001**	-	9.14e+002 1.15e+002	-	1.00e+003 7.47e+001	F27	**6.69e+002 7.98e+001**	-	8.69e+002 7.86e+001	-	1.05e+003 1.29e+002
F28	**1.47e+003 1.51e+002**	-	2.10e+003 5.46e+002	-	2.04e+003 1.91e+002	F28	**1.51e+003 1.83e+002**	-	1.88e+003 2.70e+002	-	2.28e+003 2.65e+002
F29	**1.01e+003 1.93e+002**	-	1.25e+003 3.29e+002	-	3.30e+005 1.30e+006	F29	**1.23e+003 3.09e+002**	-	1.56e+003 6.63e+002	-	5.11e+006 1.86e+007
F30	1.22e+004 2.10e+003	=	1.28e+004 2.12e+003	-	9.78e+004 8.21e+004	F30	**1.18e+004 1.60e+003**	-	1.58e+004 2.15e+003	-	1.36e+005 1.07e+005
+/=/-	-		11/4/15		2/2/26	w/t/l	-		3/8/19		1/3/26
ARV	1.5667		1.7333		2.7	ARV	1.2667		1.9333		2.8
Rank	1		2		3	Rank	1		2		3
Fun.	MTJADE		Na-JADE		APT-JADE	Fun.	MTMDEpBX		Na-MDEpBX		APT-MDEpBX
F1	**2.23e+004 1.43e+004**	-	2.85e+004 1.88e+004	-	4.39e+004 1.67e+004	F1	**4.85e+004 3.43e+004**	-	2.52e+005 3.28e+005	-	2.78e+006 1.89e+006
F2	2.39e-022 3.84e-022	=	1.54e-021 4.02e-021	-	1.24e-019 6.35e-019	F2	1.44e+005 5.48e+005	=	2.05e+006 1.04e+007	-	7.02e+007 9.95e+007
F3	1.36e-002 9.84e-003	+	**5.31e-003 4.18e-003**	+	**1.62e-003 1.85e-003**	F3	**3.84e-025 4.75e-025**	-	2.64e-022 9.58e-022	-	3.76e-007 1.52e-006
F4	**3.27e+000 1.79e+001**	-	1.31e+001 3.39e+001	-	1.30e+001 2.05e+001	F4	8.09e+001 2.56e+001	=	9.44e+001 3.85e+001	-	1.20e+002 3.18e+001
F5	2.06e+001 4.08e-002	=	2.06e+001 3.91e-002	=	2.06e+001 2.51e-002	F5	2.07e+001 1.43e-001	=	2.07e+001 9.08e-002	=	2.07e+001 8.44e-002
F6	5.54e+000 7.48e+000	=	2.42e+000 4.39e+000	=	9.20e+000 7.94e+000	F6	**6.79e+000 1.97e+000**	-	1.07e+001 2.36e+000	-	1.06e+001 2.51e+000
F7	1.56e-003 3.57e-003	=	1.15e-003 3.76e-003	=	4.27e-003 5.25e-003	F7	6.20e-002 8.48e-002	=	1.17e-001 3.74e-001	-	1.22e+000 1.42e+000
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	F8	**3.29e+001 6.41e+000**	-	3.92e+001 8.12e+000	-	4.11e+001 7.12e+000
F9	6.17e+001 7.94e+000	+	**5.73e+001 8.57e+000**	=	6.33e+001 1.00e+001	F9	**3.99e+001 7.58e+000**	-	4.46e+001 8.16e+000	-	5.57e+001 1.06e+001
F10	8.30e+001 1.63e+001	+	**7.03e+001 1.26e+001**	+	**8.28e+000 4.70e+000**	F10	9.79e+002 4.57e+002	=	1.02e+003 4.90e+002	=	1.04e+003 2.81e+002
F11	4.86e+003 3.80e+002	=	5.01e+003 4.81e+002	-	5.05e+003 2.96e+002	F11	5.49e+003 8.64e+002	=	5.44e+003 1.15e+003	-	6.13e+003 1.14e+003
F12	**5.83e-001 7.53e-002**	-	1.20e+003 1.21e-001	=	5.56e-001 6.39e-002	F12	5.80e-001 1.71e-001	-	1.20e+003 1.74e-001	=	6.18e-001 2.71e-001
F13	3.03e-001 3.53e-002	=	3.07e-001 4.23e-002	-	3.54e-001 4.85e-002	F13	**2.52e-001 5.06e-002**	-	2.79e-001 3.42e-002	-	2.99e-001 5.09e-002
F14	2.92e-001 2.89e-002	=	2.85e-001 3.45e-002	=	2.84e-001 3.11e-002	F14	3.54e-001 5.67e-002	+	**3.26e-001 3.29e-002**	=	3.30e-001 4.44e-002
F15	8.92e+000 1.29e+000	=	8.67e+000 6.86e-001	-	9.57e+000 1.15e+000	F15	**7.23e+000 3.86e+000**	-	9.58e+000 4.18e+000	-	1.72e+001 8.14e+000
F16	1.89e+001 3.06e-001	=	1.89e+001 4.02e-001	=	1.89e+001 3.48e-001	F16	1.82e+001 1.00e+000	+	**1.71e+001 1.29e+000**	=	1.84e+001 9.60e-001
F17	9.78e+003 1.33e+004	=	6.31e+003 8.21e+003	=	1.51e+004 1.60e+004	F17	**2.27e+004 2.25e+004**	-	4.81e+004 3.00e+004	-	1.63e+005 9.80e+004
F18	**1.33e+002 3.13e+001**	-	2.42e+002 3.42e+002	=	1.83e+002 2.09e+002	F18	4.02e+002 2.81e+002	=	5.20e+002 6.51e+002	=	5.53e+002 3.98e+002
F19	**1.44e+001 6.67e+000**	-	1.88e+001 1.04e+001	=	2.17e+001 1.55e+001	F19	1.72e+001 8.68e+000	=	1.43e+001 2.93e+000	-	5.66e+001 1.80e+001
F20	1.00e+002 3.97e+001	=	8.74e+001 2.60e+001	=	9.81e+001 3.49e+001	F20	2.11e+002 6.13e+001	=	2.06e+002 7.13e+001	=	2.10e+002 5.20e+001
F21	**4.07e+003 4.91e+003**	-	5.07e+003 6.20e+003	-	7.13e+003 9.15e+003	F21	**2.19e+003 3.07e+003**	-	2.56e+004 3.83e+004	-	6.40e+004 5.48e+004
F22	4.77e+002 1.36e+002	=	4.87e+002 1.22e+002	=	4.58e+002 1.40e+002	F22	4.75e+002 2.55e+002	=	4.72e+002 2.94e+002	=	5.33e+002 2.76e+002
F23	3.44e+002 1.13e-005	=	3.44e+002 1.06e-005	=	3.44e+002 1.27e-005	F23	3.44e+002 1.32e-001	=	3.44e+002 1.09e-001	-	3.50e+002 4.34e+000
F24	2.74e+002 1.98e+000	=	2.73e+002 2.19e+000	=	2.73e+002 2.58e+000	F24	2.76e+002 2.09e+000	=	2.76e+002 2.30e+000	=	2.75e+002 2.98e+000
F25	2.15e+002 4.15e+000	=	2.14e+002 4.98e+000	-	2.23e+002 1.88e+000	F25	**2.22e+002 2.90e+000**	-	2.24e+002 2.32e+000	-	2.25e+002 1.91e+000
F26	1.00e+002 1.38e-001	=	1.00e+002 1.39e-001	=	1.07e+002 2.53e+001	F26	1.60e+002 4.97e+001	=	1.80e+002 4.06e+001	-	1.90e+002 3.05e+001
F27	**3.88e+002 5.48e+001**	-	4.20e+002 6.95e+001	-	4.38e+002 5.43e+001	F27	**5.06e+002 5.15e+001**	-	5.90e+002 5.34e+001	-	5.90e+002 7.00e+001
F28	1.25e+003 4.89e+001	+	**1.21e+003 7.08e+001**	-	1.32e+003 6.43e+001	F28	**1.42e+003 8.46e+001**	-	1.50e+003 1.90e+002	-	1.50e+003 1.22e+002
F29	8.33e+002 1.10e+002	=	8.54e+002 1.34e+002	=	8.57e+002 8.72e+001	F29	1.71e+003 3.15e+002	+	**1.40e+003 2.19e+002**	-	3.64e+003 2.20e+003
F30	1.07e+004 8.66e+002	=	1.06e+004 7.50e+002	=	1.08e+004 8.63e+002	F30	1.23e+004 1.61e+003	=	1.28e+004 2.05e+003	-	1.84e+004 4.57e+003
+/=/-	-		4/20/6		2/18/10	w/t/l	-		3/14/13		0/9/21
ARV	1.85		1.8		2.35	ARV	1.4167		1.85		2.7333
Rank	2		1		3	Rank	1		2		3

empirically. Note that the proposed algorithm with all the four topologies retains the name, MTDE, as in the experimental study above, for brevity.

(1) MTDEscp: MTDE with small-world, cellular and panmictic topologies.
(2) MTDEsrp: MTDE with small-world, ring and panmictic topologies.
(3) MTDEsc: MTDE with small-world and cellular topologies.

Table 26
Results of the multiple-problem analysis by the Wilcoxon test between MTDE and the multi-topology based DE (Na-DE and APT-DE) for the CEC2014 functions at 30*D* and 50*D*.Here, DE/current-to-best/1 is abbreviated to DE/c-t-b/1. DE/rand-to-best/1 is abbreviated to DE/r-t-b/1 .

	at 30*D*	+/ = /−	*R*+	*R*−	*p*-value	at 50*D*	+/ = /−	*R*+	*R*−	*p*-value
MTDE/c-t-b/1 *vs* NaDE/c-t-b/1		16/6/8	308	127	0.049101		15/4/11	261	174	0.341391
MTDE/r-t-b/1 *vs* NaDE/r-t-b/1		19/9/2	424	41	0.000075		19/8/3	389	46	0.000192
MTJADE *vs* NaJADE		5/21/4	244.5	220.5	0.797098		6/20/4	236	199	0.680373
MTMDEpBX *vs* NaMDEpBX		10/17/3	304	161	0.135586		13/14/3	344.5	90.5	0.005834
MTDE/c-t-b/1 *vs* APT-DE/c-t-b/1		21/5/4	407	58	0.000319		26/2/2	392	43	0.000148
MTDE/r-t-b/1 *vs* APT-DE/r-t-b/1		25/3/2	435.5	29.5	0.000028		26/3/1	412	23	0.000025
MTJADE *vs* APT-JADE		8/18/4	306.5	158.5	0.125438		10/18/2	365	100	0.006095
MTMDEpBX *vs* APT-MDEpBX		16/12/2	390.5	74.5	0.001081		21/9/0	420	15	0.000011

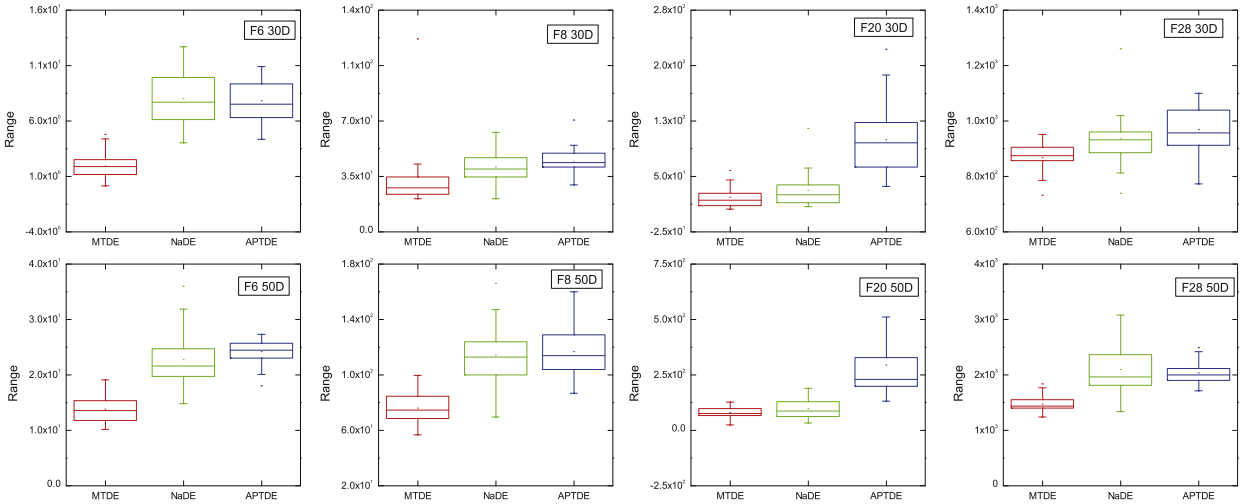

Fig. 11. Box plots of MTDE, Na-DE and APT-DE with DE/current-to-best/1 on the selected functions.

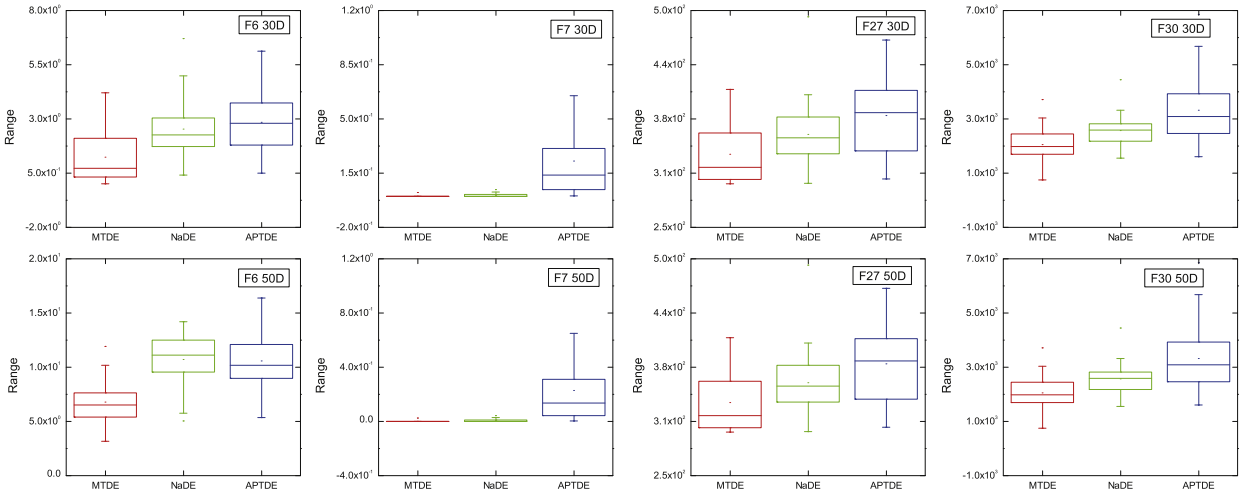

Fig. 12. Box plots of MTDE, Na-DE and APT-DE with MDE_pBX on the selected functions.

(4) MTDEsr: MTDE with small-world and ring topologies.
(5) MTDEsp: MTDE with small-world and panmictic topologies.

In this subsection, MTDE and its variants with different combinations of topologies are compared. Here, these variants are applied to two DE algorithms (DE/current-to-best/1 and SaDE) for comparison. The results on the CEC2014 functions are shown in Tables 27 and 28.

According to the results in Table 27, all the MTDE variants outperform the corresponding DE algorithms overall. Based on the Friedman test, MTDE achieves best results overall among the competitors in both cases. Specifically, in the case of

Table 27
Mean and standard deviation of the best error values obtained by MTDE and its variants with different combinations of topologies on the CEC2014 functions at 50*D*. Here, DE/current-to-best/1 is abbreviated to DE/c-t-b/1.

Fun.	DE/c-t-b/1	MTDE/c-t-b/1	MTDEscp/c-t-b/1	MTDEsrp/c-t-b/1	MTDEsc/c-t-b/1	MTDEsr/c-t-b/1	MTDEsp/c-t-b/1
F1	7.32e+007 3.65e+007	1.04e+006 1.16e+006	1.60e+006 1.56e+006	2.71e+006 2.22e+006	**3.05e+004 2.20e+004**	5.75e+004 3.83e+004	3.35e+006 2.31e+006
F2	1.98e+010 3.97e+009	8.28e+007 7.77e+007	1.90e+008 2.07e+008	5.49e+008 5.27e+008	**9.34e+006 2.98e+007**	1.39e+007 5.10e+007	1.13e+009 6.22e+008
F3	5.43e+003 4.62e+003	1.88e-009 6.06e-009	6.60e-009 2.57e-008	**1.28e-009 4.14e-009**	1.42e-005 4.07e-005	4.70e-006 9.61e-006	1.63e-009 4.93e-009
F4	2.55e+003 8.09e+002	1.84e+002 4.27e+001	2.19e+002 6.43e+001	3.02e+002 7.96e+001	**1.01e+002 3.98e+001**	1.29e+002 4.09e+001	3.57e+002 8.35e+001
F5	**2.11e+001 3.12e-002**	**2.11e+001 3.01e-002**	**2.11e+001 4.64e-002**	**2.11e+001 4.20e-002**	**2.11e+001 3.24e-002**	2.12e+001 2.93e-002	**2.11e+001 4.66e-002**
F6	2.94e+001 2.61e+000	1.38e+001 2.28e+000	**1.24e+001 2.56e+000**	1.54e+001 2.75e+000	1.46e+001 4.05e+000	2.18e+001 4.20e+000	1.61e+001 2.29e+000
F7	2.33e+002 5.10e+001	2.14e+000 1.28e+000	3.30e+000 2.37e+000	5.31e+000 3.11e+000	**1.47e-001 3.19e-001**	2.26e-001 4.93e-001	1.54e+001 4.95e+000
F8	1.48e+002 2.73e+001	7.59e+001 1.08e+001	**7.33e+001 1.67e+001**	8.44e+001 1.53e+001	7.86e+001 1.54e+001	1.01e+002 1.39e+001	8.28e+001 1.93e+001
F9	1.55e+002 2.27e+001	**7.12e+001 1.16e+001**	7.40e+001 1.50e+001	8.09e+001 1.33e+001	7.40e+001 1.46e+001	9.60e+001 2.25e+001	8.04e+001 1.33e+001
F10	**3.76e+003 1.55e+003**	8.68e+003 3.43e+003	7.59e+003 3.67e+003	6.12e+003 3.87e+003	8.87e+003 3.80e+003	4.36e+003 1.57e+003	5.67e+003 3.65e+003
F11	**1.20e+004 1.29e+003**	1.24e+004 5.25e+002	1.24e+004 4.07e+002	1.24e+004 3.20e+002	1.27e+004 3.95e+002	1.24e+004 1.36e+003	1.24e+004 4.37e+002
F12	3.37e+000 2.54e-001	3.32e+000 2.47e-001	3.31e+000 2.99e-001	3.31e+000 2.91e-001	3.32e+000 2.93e-001	**3.30e+000 3.18e-001**	3.38e+000 2.79e-001
F13	2.83e+000 9.72e-001	**4.05e-001 5.12e-002**	4.26e-001 6.59e-002	4.60e-001 7.32e-002	4.73e-001 7.07e-002	4.67e-001 8.08e-002	4.61e-001 7.18e-002
F14	5.16e+001 1.46e+001	3.07e-001 3.74e-002	3.04e-001 3.24e-002	3.05e-001 3.92e-002	3.04e-001 2.65e-002	**3.03e-001 3.31e-002**	3.39e-001 8.57e-002
F15	6.72e+003 8.13e+003	1.67e+001 9.27e+000	1.97e+001 9.79e+000	2.15e+001 9.11e+000	**1.43e+001 7.97e+000**	2.00e+001 5.13e+000	2.22e+001 8.28e+000
F16	2.06e+001 4.53e-001	2.06e+001 3.94e-001	2.06e+001 4.43e-001	2.05e+001 5.13e-001	2.10e+001 4.04e-001	2.06e+001 4.27e-001	**2.04e+001 5.20e-001**
F17	3.63e+005 4.75e+005	5.00e+003 1.31e+004	3.56e+003 5.07e+003	4.59e+003 1.21e+004	**3.35e+003 3.03e+003**	6.13e+003 1.37e+004	4.43e+003 9.54e+003
F18	2.53e+005 7.22e+005	1.67e+002 1.37e+002	2.11e+002 2.97e+002	1.75e+002 7.81e+001	1.58e+002 7.03e+001	2.15e+002 4.02e+002	**1.51e+002 4.17e+001**
F19	9.87e+001 2.49e+001	3.88e+001 2.21e+001	3.31e+001 2.51e+001	4.44e+001 2.50e+001	**1.39e+001 6.21e+000**	1.88e+001 1.37e+001	4.86e+001 2.37e+001
F20	4.41e+002 2.16e+002	8.10e+001 2.45e+001	8.92e+001 3.37e+001	1.09e+002 3.63e+001	**6.02e+001 1.73e+001**	8.63e+001 3.37e+001	1.15e+002 5.28e+001
F21	1.11e+005 9.26e+004	**7.29e+002 3.37e+002**	9.84e+002 4.54e+002	9.16e+002 3.68e+002	1.07e+003 6.74e+002	1.06e+003 5.09e+002	1.20e+003 8.90e+002
F22	5.12e+002 2.01e+002	2.63e+002 2.65e+002	**2.46e+002 1.78e+002**	2.80e+002 1.93e+002	4.41e+002 3.62e+002	3.71e+002 1.76e+002	3.12e+002 1.88e+002
F23	4.76e+002 3.58e+001	3.45e+002 1.28e+000	3.47e+002 2.77e+000	3.49e+002 5.08e+000	**3.44e+002 1.58e-001**	**3.44e+002 3.55e-002**	3.58e+002 6.79e+000
F24	3.08e+002 9.17e+000	2.80e+002 3.62e+000	2.81e+002 3.74e+000	2.81e+002 3.71e+000	**2.78e+002 4.52e+000**	2.80e+002 2.97e+000	2.86e+002 4.04e+000
F25	2.35e+002 5.08e+000	**2.16e+002 4.25e+000**	2.18e+002 4.20e+000	2.20e+002 4.04e+000	2.18e+002 6.04e+000	2.21e+002 4.29e+000	2.21e+002 4.33e+000
F26	1.95e+002 2.53e+001	1.44e+002 5.01e+001	1.34e+002 4.73e+001	1.45e+002 4.94e+001	**1.10e+002 3.04e+001**	1.50e+002 5.06e+001	1.54e+002 4.80e+001
F27	1.18e+003 8.13e+001	**6.72e+002 7.10e+001**	7.11e+002 6.54e+001	7.41e+002 4.67e+001	6.86e+002 7.91e+001	8.63e+002 1.21e+002	7.55e+002 6.79e+001
F28	3.04e+003 4.26e+002	**1.47e+003 1.51e+002**	1.50e+003 1.60e+002	1.57e+003 1.69e+002	1.57e+003 1.58e+002	1.73e+003 2.10e+002	1.59e+003 1.65e+002
F29	1.01e+007 2.33e+007	1.01e+003 1.93e+002	1.06e+003 2.41e+002	1.16e+003 3.81e+002	**9.34e+002 2.00e+002**	1.04e+003 2.32e+002	1.13e+003 3.47e+002
F30	1.56e+005 1.63e+005	1.22e+004 2.10e+003	1.32e+004 2.20e+003	1.53e+004 3.50e+003	**1.06e+004 7.90e+002**	1.16e+004 1.36e+003	1.82e+004 3.40e+003
ARV	6.3667	2.7833	3.1333	4.1167	2.8833	3.9	4.8167
Rank	7	1	3	5	2	4	6
Fun.	SaDE	MTSaDE	MTSaDEscp	MTSaDEsrp	MTSaDEsc	MTSaDEsr	MTSaDEsp
F1	8.06e+006 2.03e+006	6.98e+006 2.06e+006	6.88e+006 1.82e+006	**6.10e+006 1.30e+006**	6.71e+006 1.63e+006	6.79e+006 1.55e+006	6.95e+006 1.82e+006
F2	3.01e+003 1.79e+003	**2.03e+003 1.25e+003**	2.45e+003 1.86e+003	2.95e+003 2.06e+003	3.74e+003 2.64e+003	2.73e+003 2.30e+003	2.80e+003 1.42e+003
F3	3.85e+003 1.48e+003	3.46e+003 1.36e+003	**2.98e+003 9.72e+002**	3.15e+003 1.12e+003	3.47e+003 1.02e+003	3.25e+003 1.34e+003	3.36e+003 1.23e+003
F4	1.02e+002 3.38e+001	**1.99e+001 2.80e+001**	2.13e+001 3.06e+001	2.40e+001 3.33e+001	3.61e+001 3.67e+001	5.05e+001 3.57e+001	2.84e+001 3.36e+001
F5	2.06e+001 3.47e-002	**2.05e+001 4.37e-002**	2.06e+001 3.69e-002	2.06e+001 2.91e-002	2.06e+001 3.25e-002	2.06e+001 2.86e-002	2.06e+001 3.99e-002
F6	2.84e+001 1.86e+000	2.70e+001 2.44e+000	2.76e+001 2.55e+000	2.77e+001 2.27e+000	2.82e+001 2.01e+000	**2.66e+001 2.37e+000**	2.73e+001 2.24e+000
F7	8.38e-003 7.23e-003	3.78e-003 5.00e-003	1.97e-003 3.68e-003	**1.05e-003 3.01e-003**	4.74e-003 5.81e-003	2.89e-003 4.19e-003	4.60e-003 5.48e-003
F8	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**	**0.00e+000 0.00e+000**
F9	1.22e+002 9.91e+000	**1.19e+002 1.14e+001**	1.22e+002 9.24e+000	1.22e+002 1.17e+001	1.21e+002 1.13e+001	1.24e+002 1.02e+001	**1.19e+002 1.17e+001**
F10	3.56e+000 1.06e+000	2.29e+000 9.09e-001	2.89e+000 1.06e+000	1.86e+000 8.19e-001	1.46e+000 7.67e-001	**3.36e-001 2.02e-001**	2.54e+000 9.31e-001
F11	6.36e+003 2.83e+002	**6.22e+003 3.61e+002**	6.39e+003 3.87e+002	6.24e+003 3.11e+002	6.31e+003 3.34e+002	6.31e+003 2.91e+002	6.29e+003 4.03e+002
F12	7.10e-001 4.52e-002	**6.59e-001 5.87e-002**	6.74e-001 7.88e-002	6.75e-001 6.76e-002	6.66e-001 6.33e-002	6.79e-001 7.33e-002	6.86e-001 4.65e-002
F13	3.87e-001 3.88e-002	**3.66e-001 3.42e-002**	3.76e-001 3.63e-002	3.79e-001 4.57e-002	3.77e-001 4.26e-002	3.89e-001 3.10e-002	3.70e-001 3.55e-002
F14	2.95e-001 2.58e-002	2.90e-001 2.46e-002	2.92e-001 2.27e-002	**2.78e-001 2.74e-002**	2.88e-001 2.12e-002	2.89e-001 2.07e-002	2.79e-001 2.33e-002
F15	1.64e+001 1.63e+000	1.55e+001 1.52e+000	**1.53e+001 1.45e+000**	1.57e+001 1.44e+000	1.60e+001 1.14e+000	1.63e+001 1.50e+000	**1.53e+001 1.11e+000**
F16	1.91e+001 3.65e-001	1.92e+001 3.32e-001	1.92e+001 2.66e-001	**1.90e+001 4.39e-001**	1.91e+001 3.55e-001	1.91e+001 3.13e-001	1.92e+001 2.35e-001
F17	1.62e+006 4.58e+005	1.41e+006 4.81e+005	1.46e+006 5.37e+005	1.48e+006 5.19e+005	1.36e+006 4.02e+005	**1.10e+006 3.66e+005**	1.49e+006 3.82e+005
F18	8.31e+002 4.14e+002	7.96e+002 5.30e+002	7.41e+002 3.98e+002	6.03e+002 3.78e+002	7.52e+002 5.32e+002	**4.45e+002 2.61e+002**	5.98e+002 3.42e+002
F19	2.88e+001 9.85e+000	1.32e+001 6.59e+000	1.73e+001 9.91e+000	**1.15e+001 1.17e+000**	1.61e+001 9.18e+000	1.60e+001 9.12e+000	1.68e+001 9.99e+000
F20	7.63e+003 2.36e+003	5.60e+003 1.61e+003	5.47e+003 1.67e+003	**5.31e+003 1.61e+003**	5.53e+003 1.92e+003	5.33e+003 1.70e+003	5.44e+003 1.44e+003
F21	1.20e+006 5.31e+005	1.05e+006 3.07e+005	1.09e+006 3.08e+005	1.02e+006 4.31e+005	**9.39e+005 2.88e+005**	9.76e+005 3.25e+005	1.06e+006 3.59e+005
F22	5.28e+002 1.54e+002	5.00e+002 1.14e+002	**4.95e+002 1.64e+002**	5.69e+002 1.15e+002	5.15e+002 9.53e+001	5.05e+002 1.51e+002	5.26e+002 1.37e+002
F23	**3.44e+002 1.27e-005**	**3.44e+002 1.13e-005**	**3.44e+002 1.27e-005**	**3.44e+002 1.20e-005**	**3.44e+002 1.27e-005**	**3.44e+002 1.06e-005**	**3.44e+002 1.27e-005**
F24	**2.65e+002 5.21e+000**	2.67e+002 4.51e+000	2.67e+002 4.38e+000	2.68e+002 2.85e+000	2.66e+002 4.47e+000	2.67e+002 3.25e+000	2.68e+002 3.24e+000
F25	2.17e+002 4.68e+000	**2.13e+002 1.80e+000**	2.14e+002 1.92e+000	**2.13e+002 1.84e+000**	**2.13e+002 2.21e+000**	**2.13e+002 1.50e+000**	**2.13e+002 1.83e+000**
F26	1.14e+002 3.46e+001	**1.04e+002 1.83e+001**	**1.04e+002 1.83e+001**	1.14e+002 3.47e+001	1.10e+002 3.06e+001	1.17e+002 3.80e+001	1.20e+002 4.08e+001
F27	7.71e+002 1.46e+002	**4.61e+002 1.97e+002**	4.94e+002 2.07e+002	5.83e+002 2.21e+002	6.28e+002 2.19e+002	5.87e+002 2.09e+002	4.69e+002 2.01e+002
F28	1.52e+003 9.01e+001	**1.40e+003 5.52e+001**	1.43e+003 1.03e+002	**1.40e+003 7.04e+001**	1.45e+003 9.43e+001	1.41e+003 4.25e+001	**1.40e+003 7.20e+001**
F29	1.32e+003 1.12e+002	1.31e+003 1.37e+002	1.29e+003 1.17e+002	1.30e+003 1.84e+002	**1.23e+003 1.48e+002**	1.24e+003 1.63e+002	1.33e+003 1.56e+002
F30	9.81e+003 5.68e+002	9.50e+003 3.93e+002	**9.49e+003 5.42e+002**	9.54e+003 5.18e+002	**9.49e+003 5.00e+002**	9.56e+003 4.55e+002	9.60e+003 4.93e+002
ARV	6.0667	3.1	3.8	3.4167	3.8167	3.6667	4.1333
Rank	7	1	4	2	5	3	6

DE/current-to-best/1, MTDE obtains the first rank, followed by MTDEsc, MTDEscp, MTDEsr, MTDEsrp, and MTDEsp. In the case of SaDE, MTDE also obtains the first rank, followed by MTDEsrp, MTDEsr, MTDEscp, MTDEsc, and MTDEsp.

Furthermore, the results of multiple-problem analysis by the Wilcoxon test in Table 28 show that MTDE obtains higher $R+$ than $R-$ in most of the cases. According to the p values, significant differences at $\alpha = 0.05$ are observed in three cases for DE/current-to-best/1 and in one case for SaDE.

Therefore, the above results indicate that the MTDE variants with different combinations of topologies are able to improve the performance of DE. In addition, the benefit of combining the four topologies in MTDE is revealed, compared with its variants with different combinations of topologies.

Table 28
Results of the multiple-problem analysis by the Wilcoxon test between MTDE and its variants with different combinations of topologies on the CEC2014 functions 50*D*. Here, DE/current-to-best/1 is abbreviated to DE/c-t-b/1.

	with DE/c-t-b/1	$R+$	$R-$	p-value	with SaDE	$R+$	$R-$	p-value
MTDEscp		300.5	134.5	0.070229		261	174	0.341391
MTDEsrp		391.5	73.5	0.000975		260	205	0.562663
MTDEsc		196	269	1		236	199	0.680373
MTDEsr		263.5	171.5	0.314666		233	232	0.983548
MTDEsp		382.5	82.5	0.001912		330	135	0.043291

4.9. Influence of the selection strategy for topology adaptation

In MTDE, an individual-dependent adaptive topology selection (ITS) scheme is proposed for selecting the topology for each individual that best matches its role during the searching process. To study the effect of the topology selection strategy, three adaptive operator selection (AOS) techniques are considered in this subsection: the adaptive pursuit method (AP) [43], the upper-confidence-bound-based multi-armed bandit method (MAB) [2], and the probability matching method (PM) [16]. These AOS techniques are the distinguished selection schemes in EAs. More details on them can be found in [2,16,43]. These MTDE variants with different selection techniques are denoted as MTDE_{AP}, MTDE_{MAB}, and MTDE_{PM}, respectively. All the parameters of these AOS techniques are set as in their original papers. In addition, the MTDE variant with the random selection mechanism (MTDE_{rand}) is considered as the baseline. Two DE algorithms (DE/current-to-best/1 and SaDE) are used for comparison. The results on the CEC2014 functions are shown in Tables 29 and 30.

From Table 29, MTDE obtains significantly better results than its variants with different AOS techniques in all the cases. Specifically, in the case of DE/current-to-best/1, MTDE performs significantly better on 20 functions, compared with each competitor. In the case of SaDE, MTDE significantly outperforms MTDE_{AP}, MTDE_{MAB}, MTDE_{PM} and MTDE_{rand} on 10, 14, 11 and 11 functions, respectively. Based on the Friedman test, MTDE obtains the first rank in the case of DE/current-to-best/1, followed by MTDE_{PM} and MTDE_{rand}. In the case of SaDE, MTDE also obtains the first rank, followed by MTDE_{AP} and MTDE_{PM}.

In addition, the results of multiple-problem analysis in Table 30 show that MTDE obtains higher $R+$ values than $R-$ values in all cases. With the multiple-problem Wilcoxon test, significant differences at $\alpha = 0.05$ are observed in all the cases for both DE/current-to-best/1 and SaDE, which suggests that MTDE significantly outperforms all its variants overall under this condition.

Generally, we conclude that the ITS scheme is more effective than other AOS techniques for MTDE on the test functions.

4.10. Influence of the topology parameters

In MTDE, each population topology has a control parameter (N_{sw} in Small-world topology, N_c in cellular topology and R_r in ring topology) that decides its degree of connectivity. To study the influences of these parameters, MTDE is compared with its variants under different parameter combinations. In this subsection, based on previous studies [11,21,27], N_{sw} is set to $0.06NP$ and $0.1NP$, N_c is set to 9 and 13, and R_r is set to $0.1NP$ and $0.15NP$. Since the connectivity degree of each topology should match the corresponding level of the population in the ITS scheme, six parameter combinations are considered: (N_{sw}, N_c, R_r) = ($0.06NP$, 9, $0.1NP$), ($0.06NP$, 9, $0.15NP$), ($0.06NP$, 13, $0.1NP$), ($0.06NP$, 13, $0.15NP$), ($0.1NP$, 13, $0.15NP$), and ($0.1NP$, 13, $0.1NP$). The MTDE variant as a function of its parameter combination is denoted as MTDE(N_{sw}, N_c, R_r). Here, the MTDE variants are applied to two DE algorithms (DE/rand/1 and SaDE) for comparison. The results on the CEC2014 functions at 30*D* are shown in Figs. 13 and 14. In Figs. 13 and 14, the pie charts describe the proportion of the functions on which each algorithm performs the best in terms of the mean error.

As the figures show, in the case of DE/rand/1, MTDE($0.1NP$, 13, $0.1NP$) performs the best on 22% of the functions, and MTDE($0.06NP$, 13, $0.1NP$) achieves second place on 18%, followed by MTDE($0.06NP$, 9, $0.1NP$) and MTDE($0.1NP$, 13, $0.15NP$). Moreover, MTDE($0.06NP$, 9, $0.15NP$), MTDE($0.1NP$, 13, $0.1NP$), MTDE($0.06NP$, 9, $0.1NP$) and MTDE($0.06NP$, 13, $0.15NP$) obtain the largest pie section on unimodal functions, simple multimodal functions, hybrid functions and composition functions, respectively. In the case of SaDE, MTDE($0.1NP$, 13, $0.1NP$) and MTDE($0.06NP$, 13, $0.1NP$) achieve first and second places with percentages of 24% and 20%, respectively, followed by MTDE($0.1NP$, 13, $0.15NP$) and MTDE($0.06NP$, 6, $0.1NP$). MTDE($0.1NP$, 13, $0.1NP$) obtains the largest pie section on simple multimodal functions (22%), hybrid functions (33%) and composition functions (23%).

By close inspection of Figs. 13 and 14, some interesting observations are made. First, the parameter combination that is used in the above experiments, i.e., (N_{sw}, N_c, R_r) = ($0.1NP$, 13, $0.1NP$), obtains the best results compared with its competitors. Second, no MTDE variant performs significantly better than the other variants. Third, different parameter combinations have different influences on the performance of MTDE for different types of functions. From the above analysis, it seems that the optimal setting of these parameters is ad hoc to the specific type of function. Therefore, adaptive or self-adaptive parameter control techniques for setting these parameters will be studied in the future.

Table 29
Mean and standard deviation of the best error values obtained by the MTDE variants with different selection strategies for topology adaptation on the CEC2014 functions at 30*D*. Here, DE/current-to-best/1 is abbreviated to DE/c-t-b/1.

Fun.	MTDE/c-t-b/1		MTDE$_{AP}$/c-t-b/1		MTDE$_{MAB}$/c-t-b/1		MTDE$_{PM}$/c-t-b/1		MTDE$_{rand}$/c-t-b/1
F1	**2.42e+003 2.70e+003**	-	4.63e+006 2.50e+006	-	5.58e+006 6.65e+006	-	6.37e+006 3.05e+006	-	4.70e+006 3.41e+006
F2	**5.27e+005 1.23e+006**	-	9.38e+008 7.56e+008	-	1.51e+009 1.87e+009	-	5.54e+008 4.35e+008	-	6.76e+008 4.42e+008
F3	**1.01e-002 2.68e-002**	-	1.24e+003 1.11e+003	-	9.71e+002 1.59e+003	-	6.76e+002 8.77e+002	-	7.50e+002 7.68e+002
F4	**8.83e+001 3.80e+001**	-	2.17e+002 7.08e+001	-	1.96e+002 1.01e+002	-	1.94e+002 4.87e+001	-	1.89e+002 5.15e+001
F5	2.10e+001 3.72e-002	=	2.09e+001 5.04e-002	=	2.09e+001 5.11e-002	+	**2.09e+001 5.96e-002**	=	2.09e+001 4.32e-002
F6	**1.96e+000 1.13e+000**	-	6.30e+000 1.86e+000	-	9.76e+000 2.58e+000	-	6.64e+000 1.57e+000	-	5.96e+000 1.58e+000
F7	**2.81e-001 3.06e-001**	-	1.23e+001 7.43e+000	-	2.19e+001 2.17e+001	-	8.61e+000 6.43e+000	-	8.82e+000 5.39e+000
F8	**3.49e+001 2.25e+001**	-	3.89e+001 8.86e+000	-	4.23e+001 1.06e+001	-	4.02e+001 8.53e+000	-	4.28e+001 1.14e+001
F9	4.42e+001 3.89e+001	=	3.57e+001 1.08e+001	=	4.98e+001 1.23e+001	=	3.97e+001 8.52e+000	=	4.34e+001 9.12e+000
F10	4.52e+003 1.37e+003	+	**1.49e+003 1.16e+003**	+	**1.69e+003 1.24e+003**	+	**1.67e+003 1.17e+003**	+	**1.80e+003 1.39e+003**
F11	6.42e+003 3.18e+002	+	**5.89e+003 8.97e+002**	+	**5.61e+003 1.26e+003**	+	**5.40e+003 1.56e+003**	+	**5.69e+003 1.16e+003**
F12	2.47e+000 2.60e-001	=	2.38e+000 3.21e-001	=	2.38e+000 2.64e-001	=	2.41e+000 2.59e-001	=	2.37e+000 2.60e-001
F13	**2.18e-001 4.41e-002**	-	3.40e-001 5.72e-002	-	3.84e-001 2.41e-001	-	3.43e-001 7.83e-002	-	3.16e-001 6.69e-002
F14	2.42e-001 3.85e-002	=	4.82e-001 9.41e-001	=	3.08e+000 5.62e+000	=	2.21e-001 4.17e-002	=	2.28e-001 4.28e-002
F15	1.01e+001 3.00e+000	+	**7.48e+000 3.87e+000**	=	1.16e+001 9.64e+000	+	**7.04e+000 3.79e+000**	+	**7.17e+000 2.90e+000**
F16	1.12e+001 4.36e-001	+	**1.08e+001 5.14e-001**	=	1.09e+001 4.48e-001	+	**1.08e+001 3.74e-001**	+	**1.08e+001 3.72e-001**
F17	**4.83e+002 2.90e+002**	-	1.13e+004 2.04e+004	-	1.14e+004 3.14e+004	-	8.30e+003 1.07e+004	-	4.67e+003 4.64e+003
F18	**4.88e+001 2.35e+001**	-	9.12e+001 2.60e+001	-	9.53e+001 2.92e+001	-	9.19e+001 2.61e+001	-	9.98e+001 3.42e+001
F19	**4.14e+000 1.11e+000**	-	1.28e+001 1.49e+001	-	1.36e+001 1.50e+001	-	8.89e+000 2.99e+000	-	1.30e+001 1.53e+001
F20	**2.16e+001 1.35e+001**	-	7.99e+001 3.77e+001	-	9.21e+001 5.63e+001	-	6.00e+001 2.45e+001	-	8.53e+001 3.90e+001
F21	**2.32e+002 1.25e+002**	-	7.79e+002 9.30e+002	-	1.70e+003 1.97e+003	-	1.13e+003 2.29e+003	-	7.52e+002 5.04e+002
F22	1.55e+002 5.38e+001	=	1.82e+002 6.90e+001	=	1.91e+002 7.91e+001	=	1.56e+002 6.95e+001	=	1.98e+002 7.58e+001
F23	**3.15e+002 1.48e-001**	-	3.21e+002 2.47e+000	-	3.19e+002 5.25e+000	-	3.19e+002 2.21e+000	-	3.19e+002 1.88e+000
F24	2.28e+002 5.12e+000	=	2.30e+002 5.98e+000	=	2.31e+002 6.50e+000	=	2.29e+002 5.82e+000	=	2.29e+002 5.25e+000
F25	**2.04e+002 9.01e-001**	-	2.09e+002 1.60e+000	-	2.08e+002 2.38e+000	-	2.08e+002 1.86e+000	-	2.08e+002 2.03e+000
F26	**1.04e+002 1.82e+001**	-	1.24e+002 4.29e+001	-	1.24e+002 4.29e+001	-	1.24e+002 4.29e+001	-	1.30e+002 4.65e+001
F27	**3.76e+002 4.14e+001**	-	4.46e+002 5.68e+001	-	4.72e+002 8.69e+001	-	4.40e+002 4.38e+001	-	4.49e+002 5.68e+001
F28	**8.68e+002 5.58e+001**	-	9.36e+002 7.57e+001	-	9.34e+002 8.94e+001	-	9.13e+002 8.06e+001	-	9.52e+002 1.13e+002
F29	**5.95e+002 2.03e+002**	-	1.08e+003 1.93e+002	-	9.19e+002 1.36e+002	-	1.06e+003 2.17e+002	-	9.89e+002 1.62e+002
F30	**9.22e+002 3.55e+002**	-	3.49e+003 1.11e+003	-	3.88e+003 3.70e+003	-	3.28e+003 1.70e+003	-	3.77e+003 2.37e+003
w/t/l	-		4/6/20		2/8/20		5/5/20		4/6/20
ARV	1.9333		3.2667		4.0667		2.6		3.1333
Rank	1		4		5		2		3
Fun.	MTSaDE		MTSaDE$_{AP}$		MTSaDE$_{MAB}$		MTSaDE$_{PM}$		MTSaDE$_{rand}$
F1	4.35e+006 1.58e+006	-	5.78e+006 1.58e+006	-	5.41e+006 2.07e+006	=	5.24e+006 1.85e+006	-	5.86e+006 1.78e+006
F2	**9.78e-008 1.75e-007**	-	1.18e-006 1.51e-006	-	1.71e-006 3.56e-006	-	1.67e-006 2.49e-006	-	3.76e-006 8.30e-006
F3	2.01e+001 1.90e+001	=	2.16e+001 1.39e+001	=	2.11e+001 1.93e+001	-	3.63e+001 3.59e+001	=	2.70e+001 2.55e+001
F4	**3.48e-002 4.96e-002**	-	6.39e+001 2.87e+001	-	4.90e+001 4.51e+001	-	4.96e+001 3.29e+001	-	5.82e+001 3.44e+001
F5	2.04e+001 3.31e-002	=	2.04e+001 3.74e-002	-	2.04e+001 3.79e-002	=	2.04e+001 3.39e-002	-	2.04e+001 3.50e-002
F6	1.27e+001 8.32e-001	=	1.31e+001 1.20e+000	=	1.29e+001 1.18e+000	=	1.32e+001 1.32e+000	=	1.28e+001 1.25e+000
F7	0.00e+000 0.00e+000	=	1.35e-006 7.37e-006	-	7.07e-005 2.27e-004	=	4.37e-008 2.39e-007	=	3.90e-012 2.12e-011
F8	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000	=	6.01e-009 3.28e-008	=	0.00e+000 0.00e+000	=	0.00e+000 0.00e+000
F9	4.85e+001 6.29e+000	=	5.02e+001 5.37e+000	=	5.14e+001 6.92e+000	=	4.81e+001 5.16e+000	=	4.88e+001 6.37e+000
F10	1.39e-003 7.60e-003	=	4.86e-003 1.05e-002	-	1.36e+000 2.04e+000	-	4.18e-003 8.46e-003	=	2.08e-003 6.35e-003
F11	2.66e+003 2.68e+002	=	2.63e+003 2.60e+002	=	2.64e+003 2.49e+002	=	2.63e+003 2.18e+002	=	2.66e+003 2.22e+002
F12	4.99e-001 6.58e-002	=	5.13e-001 5.60e-002	=	5.15e-001 6.05e-002	=	5.27e-001 6.31e-002	=	5.03e-001 7.63e-002
F13	2.77e-001 2.93e-002	=	2.70e-001 2.69e-002	=	2.66e-001 3.49e-002	+	**2.52e-001 2.80e-002**	+	**2.65e-001 3.97e-002**
F14	2.27e-001 2.88e-002	=	2.13e-001 2.72e-002	=	2.20e-001 2.85e-002	=	2.17e-001 2.70e-002	=	2.17e-001 2.65e-002
F15	6.44e+000 5.43e-001	=	6.24e+000 6.03e-001	=	6.66e+000 8.22e-001	=	6.54e+000 5.90e-001	=	6.46e+000 9.64e-001
F16	1.02e+001 3.01e-001	=	1.03e+001 3.76e-001	=	1.03e+001 2.75e-001	=	1.02e+001 3.79e-001	=	1.04e+001 2.94e-001
F17	**3.98e+005 1.46e+005**	-	5.87e+005 2.37e+005	-	5.18e+005 2.59e+005	-	5.84e+005 2.72e+005	-	6.26e+005 2.43e+005
F18	5.35e+003 3.40e+003	=	4.12e+003 2.30e+003	+	**2.61e+003 1.67e+003**	=	3.78e+003 2.13e+003	=	4.51e+003 2.36e+003
F19	**5.43e+000 5.82e-001**	-	5.89e+000 6.61e-001	-	5.86e+000 8.34e-001	-	5.99e+000 5.57e-001	-	6.10e+000 5.22e-001
F20	1.98e+003 9.34e+002	-	3.00e+003 1.31e+003	=	2.38e+003 1.74e+003	-	3.18e+003 1.70e+003	-	3.04e+003 1.42e+003
F21	8.83e+004 3.95e+004	=	8.87e+004 4.47e+004	=	8.88e+004 5.11e+004	=	9.30e+004 5.52e+004	=	7.70e+004 3.65e+004
F22	1.46e+002 5.51e+001	=	1.62e+002 5.64e+001	=	1.72e+002 6.90e+001	=	1.66e+002 5.44e+001	=	1.70e+002 4.47e+001
F23	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005	=	3.15e+002 8.01e-006	=	3.15e+002 1.13e-005	=	3.15e+002 1.13e-005
F24	2.24e+002 6.67e-001	=	2.24e+002 5.89e-001	-	2.25e+002 1.00e+000	+	**2.24e+002 6.36e-001**	=	2.24e+002 7.17e-001
F25	**2.06e+002 5.80e-001**	-	2.07e+002 8.17e-001	-	2.07e+002 8.12e-001	-	2.08e+002 9.29e-001	-	2.08e+002 6.50e-001
F26	1.00e+002 3.59e-002	+	**1.00e+002 2.98e-002**	-	1.00e+002 4.23e-002	=	1.00e+002 3.36e-002	=	1.00e+002 3.30e-002
F27	**3.19e+002 4.04e+001**	-	3.42e+002 5.41e+001	-	3.74e+002 5.48e+001	-	3.46e+002 5.23e+001	-	3.53e+002 5.60e+001
F28	**8.83e+002 1.54e+001**	-	9.04e+002 2.27e+001	-	9.23e+002 5.50e+001	-	9.00e+002 2.96e+001	-	9.04e+002 3.23e+001
F29	1.31e+003 1.12e+002	=	1.34e+003 8.68e+001	=	1.32e+003 1.55e+002	=	1.30e+003 1.10e+002	=	1.29e+003 8.25e+001
F30	**1.71e+003 3.12e+002**	-	2.08e+003 2.69e+002	-	2.10e+003 4.14e+002	-	2.10e+003 2.82e+002	-	2.02e+003 4.12e+002
w/t/l			1/19/10	-	1/15/14		2/17/11	-	1/18/11
ARV	2.0167		3.0667		3.5833		3.0833		3.25
Rank	1		2		5		3		4

Table 30
Results of the multiple-problem analysis by the Wilcoxon test between MTDE and its variants with different selection strategies for topology adaptation on the CEC2014 functions at 30*D*. Here, DE/current-to-best/1 is abbreviated to DE/c-t-b/1.

	with DE/c-t-b/1	*R*+	*R*−	*p*-value	with SaDE	*R*+	*R*−	*p*-value
$MTDE_{AP}$		388	77	0.001334		352	83	0.003335
$MTDE_{MAB}$		409	56	0.000262		374.5	60.5	0.000576
$MTDE_{PM}$		383	82	0.001797		351.5	113.5	0.013975
$MTDE_{rand}$		388	77	0.001187		357.5	107.5	0.009842

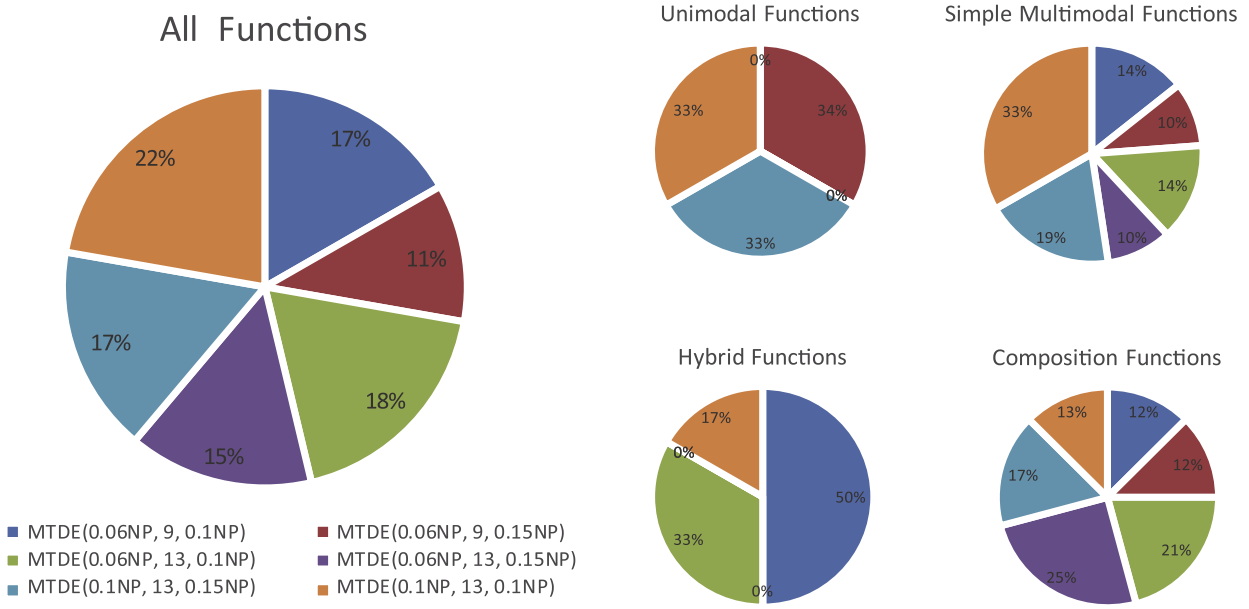

Fig. 13. Pie charts for the MTDE/rand/1 variants with different parameter combinations. Each slice in pie chart illustrates the proportion of functions for which each variant obtains the best results in terms of the mean error.

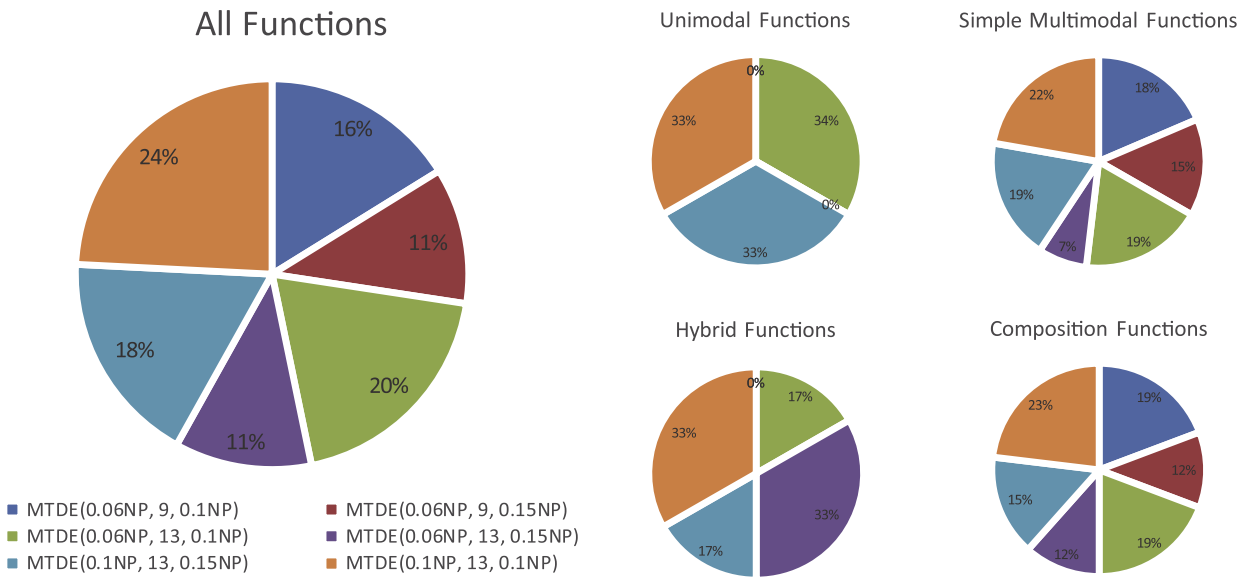

Fig. 14. Pie charts for the MTSaDE variants with different parameter combinations. Each slice in pie chart illustrates the proportion of functions for which each variant obtains the best results in terms of the mean error.

5. Conclusion and future research

In the literature, many researchers have introduced population topologies into DE to improve its performance. As previous results show, different topologies have different influences on the behavior of DE when facing different kinds of problems and at different evolutionary stages. Moreover, during the evolutionary process, individuals with different fitness values exhibit different search behaviors and play different roles in guiding the search. Generally, superior individuals benefit from having smaller neighborhoods for exploitation, while inferior individuals prefer larger neighborhoods for exploration. However, in most previous works, the individual-dependent roles are always neglected when incorporating with the population topology.

In this study, we propose a multi-topology-based DE (MTDE) algorithm, with the purpose of using the information that is extracted from the differences in the fitness values among individuals to guide the selection of the population topology. To allow individuals with different roles to have neighborhoods of different sizes in which to exhibit their search behaviors, multiple topologies with different degrees of connectivity are employed in MTDE. Specifically, the proposed MTDE contains three main components: an ensemble of multiple population topologies (MPT), an individual-dependent adaptive topology selection (ITS) scheme and a topology-dependent mutation (TDM) strategy. In MTDE, the ensemble of MPT is formed by combining multiple topologies with different connectivity degrees. Then, with the ITS scheme, the individuals in the current population are sorted according to their fitness values, and each individual selects a topology from the ensemble of MPT that matches its role based on the ranking value. After that, partition-based parent selection for mutation is employed in the TDM strategy to guide the generation of the mutant vectors according to the individual-dependent topology.

To evaluate the effectiveness of the proposed algorithm, extensive experiments on a suite of benchmark functions from the special session of the 2013 and 2014 Congresses on Evolutionary Computation are carried out. Through comparing the performance of MTDE with those of the original DE algorithms, advanced DE variants, single- and multi-topology-based DE algorithms, and other powerful EAs, it has been confirmed that MTDE achieves competitive results.

As a continuation of this study, first, we will further investigate other sophisticated individual-dependent mechanisms for topology adaption in the MTDE framework. Second, more population topologies and their combinations will be incorporated into MTDE to study its effectiveness comprehensively. Third, applying the proposed algorithm to solve real-world problems would be worthwhile work in the future.

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