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ORIGINAL ARTICLE

A novel chaotic Henry gas solubility optimization algorithm for solving real-world engineering problems

Betül Sultan Yıldız[1] · Nantiwat Pholdee[2] · Natee Panagant[2] · Sujin Bureerat[2] · Ali Riza Yildiz[3] · Sadiq M. Sait[4]

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Abstract

The paper proposes a novel metaheuristic based on integrating chaotic maps into a Henry gas solubility optimization algorithm (HGSO). The new algorithm is named chaotic Henry gas solubility optimization (CHGSO). The hybridization is aimed at enhancement of the convergence rate of the original Henry gas solubility optimizer for solving real-life engineering optimization problems. This hybridization provides a problem-independent optimization algorithm. The CHGSO performance is evaluated using various conventional constrained optimization problems, e.g., a welded beam problem and a cantilever beam problem. The performance of the CHGSO is investigated using both the manufacturing and diaphragm spring design problems taken from the automotive industry. The results obtained from using CHGSO for solving the various constrained test problems are compared with a number of established and newly invented metaheuristics, including an artificial bee colony algorithm, an ant colony algorithm, a cuckoo search algorithm, a salp swarm optimization algorithm, a grasshopper optimization algorithm, a mine blast algorithm, an ant lion optimizer, a gravitational search algorithm, a multi-verse optimizer, a Harris hawks optimization algorithm, and the original Henry gas solubility optimization algorithm. The results indicate that with selecting an appropriate chaotic map, the CHGSO is a robust optimization approach for obtaining the optimal variables in mechanical design and manufacturing optimization problems.

Keywords Hybrid metaheuristics · Henry gas solubility optimization · Chaotic maps · Mechanical and manufacturing design · Diaphragm spring

1 Introduction

The emergence of numerical control (NC) technology has greatly influenced both the cost and machining quality of the part with any geometry to be machined. As a result of the discovery of NC-based machining methods, intensive research has been focused on automated process planning methods to best determine machines, manufacturing methods, and machining parameters. Therefore, both innovations and technological advances can save money and time for any workpiece. Basically, matching the minimization of processing time to the maximization of profit, as for every product to be processed, has gained great importance when it is necessary to benefit from economies of scale, particularly in economies of scale. However, the machining economy essentially involves the optimal determination of cutting parameters that affect the cost, quality, and productivity at the highest level. Often, in today's competitive markets, to make them more attractive and efficient, new features are added to products. To achieve such goals, complicated optimization problems are inevitable. Product cost minimization is a typical design objective function, where production time plays a vital part in reducing the total cost. For the last few decades, especially in the last decade, the use of metaheuristics (MHs) to successfully tackle such optimization problems has been reported [1–13]. It can be dated

✉ Ali Riza Yildiz
aliriza@uludag.edu.tr

1 Department of Electric and Energy, Bursa Uludag University, Görükle, 16059 Bursa, Turkey

2 Department of Mechanical Engineering, Faculty of Engineering, Sustainable Infrastructure Research and Development Center, Khon Kaen University, Khon Kaen 40002, Thailand

3 Department of Automotive Engineering, Faculty of Engineering, Bursa Uludag University, Görükle, 16059 Bursa, Turkey

4 Department of Computer Engineering, King Fahd University of Petroleum and Minerals, Dhahran, Saudi Arabia

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back to the early 1900s when optimal cutting parameters were recognized to be important [14]. Nevertheless, as the problems were difficult to solve back then, little enhancement had been made [15]. Traditional methods related to integer, geometric, and dynamic programming as well as some deterministic heuristics [3, 16–33] were applied for solving processing parameters optimization; however, the results are merely acceptable with the problem of being trapped at local optima. As a result, the use of alternative optimizers that are metaheuristics has been investigated. The metaheuristic optimizers can be categorized into several types, including swarm intelligence, nature-inspired optimization, evolutionary algorithms, and physics-based optimization. Over the past few decades, such algorithms have been implemented on a wide variety of real-world applications [34–72]. Practical manufacturing optimization problems, such as turning, grinding and milling, in our previous studies, have been solved by using numerous MHs e.g. a multi-verse optimization algorithm, a Harris hawks optimization algorithm, genetic algorithms, a grasshopper optimization algorithm, particle swarm optimization, differential evolution, an immune algorithm, cuckoo search algorithms, a moth- flame algorithm, etc. [5–11, 58]. The advantages of MHs are that they are derivative-free, thus, making them able to solve almost any kind of optimization. They are capable of solving global optimization due to the use of population-based concepts and randomization and can handle complicated problems that gradient-based optimizers struggle to solve. However, their weak points are the low convergence rate due to randomization in their procedures. As a result, it is always a challenging task to improve or upgrade existing MHs.

The concept of MH search is to have an efficient balance between its search diversification and intensification. There have been several ways to achieve such a goal, and one of them is too good hybrid features of several algorithms [74], as this can achieve a good balance between diversification (global exploration) and intensification (local search) [75–87].

The HGSO is a recently invented metaheuristic that has been proven one of the powerful MHs. In our recent paper [7], the HGSO was used for the optimization of a vehicle brake pedal. It has been found that there is still room for further improvement of the optimizer. From the previous study, it is shown that integrating the Nelder-Mead algorithm into HGSO [86] is more efficient and robust than the original HGSO. In this work, chaotic maps have been added to the main procedure of HGSO, leading to the hybrid chaotic Henry gas solubility optimization (CHGSO). The method is then implemented on the optimization problems for manufacturing and mechanical design. The results obtained from using the proposed method are superior to those from using an ant colony algorithm, a genetic algorithm, scatter search, particle swarm optimization, differential evolution, harmony search, an artificial bee colony algorithm, simulated annealing, a hybrid particle swarm algorithm, an improved differential evolution algorithm, teaching learning-based algorithm, a grasshopper optimization algorithm, a cuckoo search algorithm, a multi-verse optimizer, Harris hawks optimization, and the original Henry gas solubility optimization. Statistically comparative results related to the engineering conventional test problems, the manufacturing, and the diaphragm spring design problems show the superiority of the CHGSO.

2 Henry gas solubility optimisation algorithm

In this section, the main idea of the Henry gas solubility optimization algorithm (HGSO). The HGSO is based on Henry's law [7, 43]. The mathematical steps of the HGSO can be explained as follows.

2.1 Initialization step

Decide the number of gases (N) and the initial positions of the gases. The partial pressure on gases should be initialized. The position of the *i*th gas is denoted by $X_{i,\mathrm{j}}$ and can be obtained using the following formula:

$$X_i(t+1) = X_{\min} + rx\left(X_{\max} - X_{\min}\right), \tag{1}$$

where r is a randomly chosen number considered between 0 and 1, and $X_{\max}$ and $X_{\min}$ are the upper and lower limits.

2.2 Clustering

One should decide on the number of clusters. The gases of the same type should belong to the same cluster.

2.3 Evaluation

This has two steps. First, we should find the best gas in each cluster, and then, we determine the best gas among all clusters. The objective function can be used to rank the gases. The location of the best gas in each cluster (j) is denoted by $X_{j,\,\mathrm{best},}$ and the position of the best gas among clusters is denoted by X_{best}.

2.4 Calculation of Henry's coefficient

Henry's coefficient of each cluster is calculated using the equation below:

$$H_j(t+1) = H_j(t)x\exp\left(-C_j\left(\frac{1}{T(t)} - \frac{1}{T^\theta}\right)\right), T(t) = \exp\left(-\frac{t}{\text{iter}}\right). \tag{2}$$

In the above equation, *T (t)* is the temperature in the *t*th iteration, iter defines the maximum number of iteration, and T^Θ shows a constant value.

2.5 Calculation of solubility

The solubility of each gas is calculated as follows:

$$S_{i,j}(t) = KxH_j(t+1)xH_{i,j}(t), \tag{3}$$

where $P_{ij}(t)$ denotes the partial pressure on the gas i that belongs to cluster j, and K defines a constant.

2.6 Update position step

The location is updated as given below.

The positions of particles are calculated by the following equation:

$$\begin{aligned} X_{i,j}(t+1) = & X_{i,j}(t) + F \times r \times \gamma \times \left(X_{j,\text{best}}(t) - X_{i,j}(t)\right) \\ & + F \times r \times \alpha \times \left(S_{i,j}(t) \times X_{\text{best}}(t) - X_{i,j}(t)\right), \end{aligned}$$

$$\gamma = \beta \times \exp\left(\frac{\mathrm{F_{best}(t)} + \varepsilon}{\mathrm{F_{i,j}(t)} + \varepsilon}\right), \varepsilon, \tag{4}$$

In Eq. (4), r defines a random number, α, β, and γ are constants. $F_{i,j}(t)$ is the fitness of the gas i, which belongs to cluster j. $F_{\text{best}}(t)$ is the best value of fitness among all clusters. The fitness is usually taken equal to the objective function value.

2.7 Escape from local optimal

The aim of the current step to escape from the optimal local value. Selection of the number of worst agents (N_w) is calculated as follows:

$$N_w = N \times \left(\text{rand}\left(c_2 - c_1\right) + c_1\right), c_1 = 0.1\text{and}c_2 = 0.2, \tag{5}$$

where N defines a number of search agents.

2.8 Calculation of the position of the worst agents

For the N_w worst gases, we apply a different formula to obtain their new positions. The formula is given below:

$$G_{(i,j)} = G_{\min(i,j)} + r \times \left(G_{\max(i,j)} - G_{\min(i,j)}\right). \tag{6}$$

In the above equation, r is a random number m, and $G_{\min(i,J)}$ and $G_{\max(i,j)}$ denote the lower and upper bounds.

3 Chaotic maps

In the next subsections, we discuss the most common chaotic maps applied in this paper.

3.1 Chebyshev map

The following equation expresses the family of Chebyshev map [78]:

$$x_{k+1} = \cos\left(k\cos^{-1}\left(x_k\right)\right). \tag{7}$$

3.2 Circle map

The Circle map can be written as [79]

$$x_{k+1} = x_k + b - (a - 2\pi)\sin\left(2\pi x_k\right)\text{mod}(1). \tag{8}$$

With $a = 0.5$ and $b = 0.2$, the chaotic sequence in (0, 1) is created.

3.3 Gauss/mouse map

The Gaussian map is defined as follows [80]:

$$x_{k+1} = \begin{cases} \varepsilon + x_k + cC_k^n, 0 < x_k \le P \\ \frac{X_k - P}{1+P}, P < x_k < 1 \end{cases}. \tag{9}$$

$$1 \Big/ \frac{1}{x_k \text{mod}(1)} = \frac{1}{x_k} - \left[\frac{1}{x_k}\right]. \tag{10}$$

3.4 Iterative map

The following expression is the iterative chaotic map [82]:

$$X_{k+1} = \sin\left(\frac{a\pi}{X_k}\right), \tag{11}$$

where $\alpha \in (0, 1)$ is a predefined parameter.

3.5 Logistic map

The following equation represents the logistic map [83]:

$$X_{k+1} = \alpha X_k\left(1 - X_k\right). \tag{12}$$

3.6 Piecewise map

Equation (13) details the piecewise map [84]:

$$f(x) = \begin{cases} \frac{x_k}{P} 0 \le x_k < P \\ \frac{x_p - P}{0.5 - P} P \le x_k < \frac{1}{2} \\ \frac{1 - P - X_k}{0.5 - P} \frac{1}{2} \le x_k < 1 - P \\ \frac{1 - X_k}{P} 1 - P \le x_k < 1 \end{cases} . \tag{13}$$

3.7 Sine map

The sine map can be formulated as follows [85]:

$$x_{k+1} = \frac{\alpha}{4} \sin\left(\pi x_k\right). \tag{14}$$

3.8 Singer map

The Singer map is defined as follows [86]:

$$x_{k+1} = \mu\left(7.86x_k - 23.31x_k^2 + 28.75x_k^3 - 13.3028.75x_k^4\right) \tag{15}$$

where μ is a parameter set in the range of 0.9 and 1.08.

3.9 Sinusoidal map

The Sinusoidal map is expressed as follows:

$$x_{k+1} = ax_k^2 \sin\left(\pi X_k\right). \tag{16}$$

For $a = 2.3$ and $x_0 = 0.7$ it has the following simplified form [83]:

$$x_{k+1} = \sin\left(\pi x_k\right). \tag{17}$$

3.10 Tent map

The tent map is calculated by the equation [87]:

$$X_{k+1} = \begin{cases} \frac{X_k}{0.7}, X_k < 0.7 \\ \frac{10}{3}(1 - X_k), X_k \ge 0.7 \end{cases} . \tag{18}$$

4 Engineering cases studies

Three engineering design problems, including welded-beam and cantilever beam problems, are used to prove the superiority of the CHGSO. Further details of the design problems are given in Appendix A. Moreover, after evaluating and confirming that the present method is efficient, a manufacturing problem related to the grinding process is considered. The numbers of function evaluations (NFE) used with CHGSO are reported in Table 1.

Table 1 The NFE of the CHGSO algorithm for each engineering benchmark

Problem	FENs
Cantilever beam	8.000
Welded beam	8.000

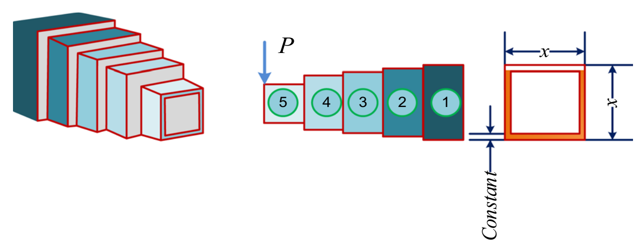


Fig. 1 Cantilever beam structure [44]

4.1 Cantilever beam design problem

The stepped cantilever beam is displayed in Fig. 1. The objective function is the minimization of the structural mass of the beam. Design variables of the stepped beams are those cross-sectional parameters, i.e., the widths and heights of the beam elements [44].

The optimum result obtained from using CHGSO is compared with those found in the literature [44, 45, 48, 50, 52, 88, 89, 102] and reported in Table 2. It is shown that the result of CHGSO is better than those presented in the literature. The minimum weight obtained is 1.33995. CHGSO is terminated after 8.000 function evaluations. The search history of CHGSO for this design problem is illustrated in Fig. 2.

4.2 Welded beam design optimization problem

The optimization problem is posed to minimize the manufacturing cost of the welded beam (see Fig. 3) [44]. The beam is subject to a point load, as shown. The design variables include the parameters h, l, t, and b, while design constraints are bending and shear stresses on the beam, buckling load, and deflection.

The result obtained from CHGSO is compared to those reported in the literature [43, 46, 94], as shown in Table 3. It is displayed that CHGSO is better than the others based on the obtained feasible cost. The termination criterion is fulfilled at 8.000 function evaluations. CHGSO outperforms its original version HGSO as the latter gives the

Table 2 Comparison of the best results for the cantilever beam design optimization problem

Methods	Optimal values of design variables					Objective	NFE
	x_1	x_2	x_3	x_4	x_5		
CHGSO	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995	8.000
HWOANM[37]	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995	8.000
GOA [44]	6.011674	5.31297	4.48307	3.50279	2.16333	1.33996	13.000
MFO[48]	5.98487177	5.31672692	4.497332585	3.51361646	2.16162	2.16162029	15.000
ALO [50]	6.01812	5.31142	4.48836	3.49751	2.158329	1.33995	14.000 3.49751 2.158329 1.33995 14,0 0 0
MVO [52]	6.023940221548	5.30601123355	4.4950113234	3.4960223242	2.152726	1.3399595	14.000 3.49751 2.158329 1.33995 14,0 0 0
CS [89]	6.0089	5.3049	20.000	3.5077	2.1504	1.33999	2500
SOS [102]	6.01878	5.30344	32.000	3.49896	2.15564	1.33996	15000

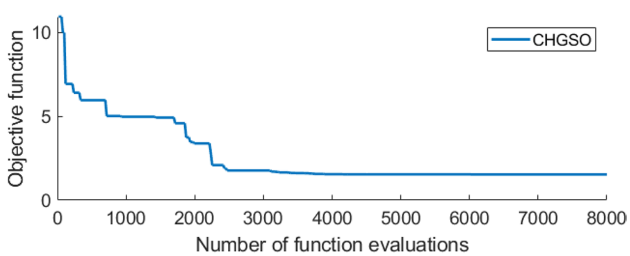


Fig. 2 Convergence diagram for the cantilever beam problem

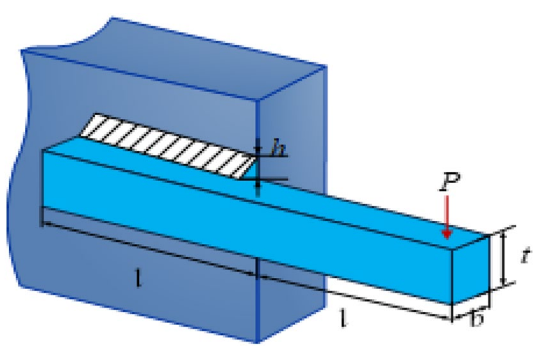


Fig. 3 Welded beam structure [44]

best result of 1.7260 with 10.000 function evaluations, while the former gives the best cost of 1.72491 with fewer function evaluations. The search history of CHGSO for the welded beam design is plotted in Fig. 4.

5 Optimization of the grinding problem

Further performance evaluation of CHGSO is made. The real-world grinding optimization problem presented in Wen et al. [59] is used to test the proposed method. The optimization problem is posed with the design variables as workpiece speed, lead of dressing, wheel speed, and depth of dressing to minimize production cost and maximize the final surface quality and production rate. Figure 5 displays the schematic illustration of the grinding machine.

Grinding composes two types as rough grinding, where the production rate is increased, and production costs are decreased. The second type is finish grinding, where the production cost is minimized to obtain the optimum finish surface quality. The optimization problem is formulated in Wen et al. [59] and expressed below.

The optimum design problem has four design variables, including wheel speed (Vs), the depth of dressing (doc), lead of dressing (L), and workpiece speed (V_w). Cost minimization is assigned to the design problem, while design constraints are as follows:

5.1 Minimize

$$\mathrm{COF}\left(V_\mathrm{s}, V_\mathrm{w}, \mathrm{Doc}, L\right) = \frac{C_T}{C_T^*} - \frac{\mathrm{WRP}}{\mathrm{WRP}^*} + \frac{R_a}{R_a^*}, \tag{19}$$

$$0 \le W_i \le 1,$$

$$1000 \le V_\mathrm{s} \le 2023(13),$$

Table 3 Comparison of the best results between CHGSO and previous algorithms for welded beam

Methods	Optimal values of design variables				Objective
	h	l	t	b	
CHGSO	0.2057	3.4714	9.0366	0.2057	1.72491
HGSO [43]	0 .2054	3 .4476	9 .0269	0 .2060	1 .7260
HWOANM [37]	0.2057	3.4714	9.0366	0.2057	1.72491
NAMDE [35]	0.2057296397	3.4704886656	9.0366239103	0.2057296397	1.72485230
GSA [49]	0.182129	3.856979	10.0000	0.202376	1.87995
CPSO [102]	0.202369	3.544214	9.048210	0.205723	1.73148
GA [94]	0.1829	4.0483	9.3666	0.2059	1.82420

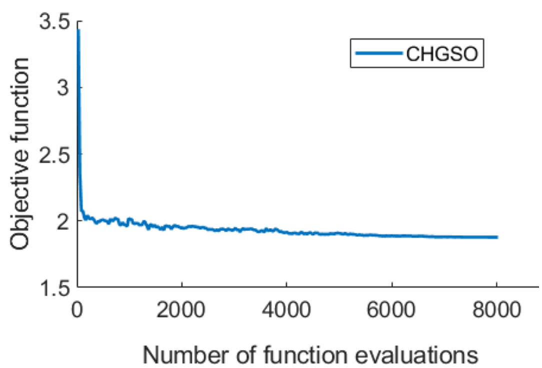


Fig. 4 Convergence diagram for the welded beam problem

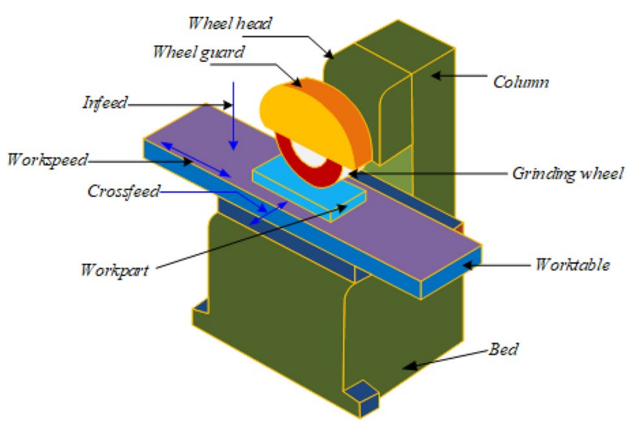


Fig. 5 Schematic illustration of a grinding machine

$$10 \le V_w \le 22.7, \tag{20}$$

$$0.01 \le \mathrm{Doc} \le 0.137, \tag{21}$$

$$0.01 \le L \le 0.137, \tag{22}$$

5.2 Subject to

$$g_1 = U - U^* \le 0, \tag{23}$$

$$g_2 = G - \frac{\mathrm{WRP}}{\mathrm{WWP}} \le 0, \tag{24}$$

$$g_3 = \frac{|R_{em}|}{K_m} - \mathrm{MSC} \le 0, \tag{25}$$

$$g_4 = R_a - 1.8 \le 0, \tag{26}$$

where

$$\begin{aligned} \mathrm{CT}\left(\frac{\$}{pc}\right) = & \frac{M_c}{60p}\left(\frac{L_w+L_e}{1000V_w}\right)\left(\frac{b_w+b_e}{f_b}\right) \\ & \left(\frac{a_w}{a_p}S_p + \frac{a_w b_w L_w}{\pi D_e b_s a_p G}\right) + \frac{M_c}{60p}\left(\frac{S_d}{V_r} + t_1\right) \\ & + \frac{M_c}{60}\frac{t_{ch}}{N_t} + \frac{M_c}{60p}\frac{1}{N_d}\frac{\pi D_e b_s}{1000LV_s} + C_s\left(\frac{a_w b_w L_w}{pG}\right) + \frac{C_d}{pN_{td}}, \end{aligned} \tag{27}$$

$$T_{\mathrm{ave}} = 12.5 * 10^3 \frac{d_g^{\frac{16}{27}} a_p^{\frac{19}{27}}}{D_e^{\frac{8}{27}}}\left(1 + \frac{doc}{L}\right) L^{\frac{16}{27}} \left(\frac{V_w}{V_s}\right)^{\frac{16}{27}}, \tag{28}$$

$$R_a = \begin{cases} 0.4587 T_{ave}^{0.3} 0 < T_{ave} \le 0.254 \\ 0.78667 T_{ave}^{0.72} 0.254 < T_{ave} \le 2.54 \end{cases}, \tag{29}$$

$$\mathrm{WRP} = 95.4 \frac{\left(1 + \left(\frac{2doc}{3L}\right)L^{\frac{11}{19}} (V_w/W_s)^{3/19} V_s\right.}{D_e^{43/304} VOL^{0.47} d_g^{5/38} R_c^{27/19}}, \tag{30}$$

$$U = 13.8 + \left(\frac{9.64 * 10^{-4} V_s}{a_p V_w}\right) + \left(6.9 * 10^{-3} \frac{2102.4 V_w}{D_e V_s}\right) * \left(A_0 + \frac{K_u V_s L_w a_w}{V_w D_e^{\frac{1}{2}} a_p^{\frac{1}{2}}}\right) \frac{V_s D_e^{\frac{1}{2}}}{V_w a_p^{\frac{1}{2}}}, \tag{31}$$

$$U^*\left(\frac{J}{\text{mm}}^3\right) = 6.2 + 1.76\left(\frac{D_e^{\frac{1}{4}}}{a_p^{\frac{3}{4}} V_w^{\frac{1}{2}}}\right), \tag{32}$$

$$\text{WWP} = \left(\frac{k_p a_p d_g^{5/38} R_c^{27/29}}{D_c^{\frac{1.2}{VOL}-43/304} VOL^{0.38}}\right) * \frac{(1 + (doc/L)L^{\frac{27}{19}}(V_s/V_w)V_w}{\left(1 + \left(\frac{2doc}{3L}\right)\right)}, \tag{33}$$

$$K_s\left(\frac{N}{\text{mm}}\right) = \frac{1000V_s f_b}{\text{WWP}} = \frac{1000D_e^{\frac{1.2}{VOL}-\frac{43}{304}} VOL^{0.38} f_b}{K_a a_p d_g^{\frac{5}{38}} R_c^{\frac{27}{19}}} * \frac{\left(1 + \frac{2Doc}{3L}\right)\left(\frac{v_s}{v_w}\right)^{\frac{16}{19}}}{\left(1 + \frac{Doc}{L}\right) L^{\frac{27}{19}}}, \tag{34}$$

$$K_c\left(\frac{N}{\text{mm}}\right) = \frac{1000V_w f_b}{\text{WRP}} = \frac{1000D_e^{43/304} VOL^{0.47} d_g^{5/38} R_c^{27/19} f_b \left(\frac{v_w}{v_s}\right)^{16/19}}{95.4(1 + \frac{2Doc}{3L})R^{11/19}}, \tag{35}$$

$$MSC = \frac{1}{2K_c}\left(1 + \frac{V_w}{V_s G}\right) + \frac{1}{K_s} \tag{36}$$

Tables 4 and 5 give the comparative results of that found using CHGSO and those reported in the previous papers. From Table 4, it is illustrated that the production cost (CT) obtained from CHGSO is 17.24% bigger than that from quadratic programming (QP). The *WRP* found using CHGSO is 6.57% higher than the genetic algorithm (GA).

Table 4 The best results for the rough grinding problem

Method	V_s	V_w	Doc	L	CT	WRP	COF	NFE
Wen et al. [59]	2000	19.96	0.055	0.044	6.204233	17.50635	– 0.12745	–
Saravanan et al. [60]	1998	11.30	0.101	0.065	7.863146	22.25564	– 0.16323	2000
Baskar et al. [61]	2010	10.19	0.118	0.081	8.301986	24.20161	– 0.18994	–
Krishna and Rao [66]	2023	10.00	0.129	0.068	8.4201	25.18881	– 0.20872	–
Lee et al. [63]	2023	13.17	0.074	0.137	7.266622	23.7012	– 0.2292	3000
Zhang et al. [64]	2023	13.2882	0.0729	0.137	7.236179	23.64125	– 0.22922	8000
Krishna [65]	2023	10.00	0.130	0.1093	8.422366	26.24735	– 0.23507	–
Lin and Li [66]	2023	10.492	0.1164	0.0815	8.187158	24.29603	– 0.19804	–
Slowik and Slowik [68]	2023	10.88	0.101	0.137	7.991014	25.21858	– 0.23091	–
Pawar et al. [70]	2023	10	0.11	0.137	8.338127	25.61568	– 0.22349	1600
SA [70]	2023	11.48	0.089	0.137	7.755	24.45	– 0.223	1600
HS [70]	2019.35	12.455	0.079	0.136 7	7.455	23.89	– 0.225	1600
ABC [70]	2023	10.973	0.097	0.137	7.942	25.00	– 0.226	1600
TLBO [71]	2023	11.537	0.0899	0.137	7.742	24.551	– 0.226	600
CS [72]	2023	13.002	0.0756	0.137	7.311	23.790	– 0.229	4.000
TLCS [72]	2023	12.779	0.0778	0.137	7.372	23.910	– 0.229	4.000
MPEDE [73]	1990.709	15.11179	0.058408	0.12790	6.828333	22.02423	– 0.20919	350.000
HCLPSO [73]	2023	13.85756	0.067691	0.137	7.096093	23.3531	– 0.22902	350.000
RSFS [73]	2023	13.14565	0.07426	0.137	7.272989	23.71612	– 0.22924	32.000
GOA [5]	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	30.000
MVO [5]	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	30.000
HHO [5]	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	27.000
HGSO	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	30.000
WOA [37]	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	32.000
HWOANM [37]	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	20.000
CHGSO	2023	13.143	0.074284	0.137	7.2739	23.718	– 0.22925	19.000

Table 5 Comparison of the best results for the finish grinding problem

Method	*Vs*	*Vw*	Doc	L	*CT*	WRP	COF	NFE
Wen et al. [59]	2000	19.99	0.052	0.091	7.719465	20.08839	0 .554714	–
Saravanan et al. [60]	1986	21.40	0.024	0.136	7.371263	20.61583	0.542687	2000
Baskar et al. [61]	2023	19.36	0.019	0.134	7.693813	20.01146	0.528092	–
Krishna and Rao [62]	1921	18.34	0.016	0.124	7.882011	18.02011	0.521453	–
Lee et al. [63]	2023	22.66	0.01	0.137	7.135612	20.0013	0.52247	3000
Zhang et al. [64]	2023	22.7	0.0109	0.137	7.129877	20	0.5224	8000
Krishna [65]	2170	17.49	0.008	0.137	8.032829	20.086	0.50506	–
Lin and Li [66]	2023	20.216	0.015	0.137	7.524477	20.00704	0.525173	–
Slowik and Slowik [68]	2022	21.95	0.013	0.136	7.243562	20.00027	0.523749	–
Pawar et al. [69]	2023	22.66	0.011	0.137	7.135612	20.0013	0.52247	1600
MPEDE [73]	2023	22.14358	0.011992	0.137	7.2115	20.0199	0.5232	350.000
HCLPSO [73]	1964.832	18.74019	0.033769	0.127065	7.8749	20.2596	0.5488	350.000
RSFS [73]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	32.000
GOA [5]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	30.000
MVO [5]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	33.000
HHO [5]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	27.000
WOA [37]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	32.000
HWOANM [37]	2023	22.7	0.010926	0.137	7.129877	20	0.5224	20.000
HGSO	2023	22.7	0.010926	0.137	7.129877	20	0.5224	30.000
CHGSO	2023	22.7	0.010926	0.137	7.129877	20	0.5224	19.000

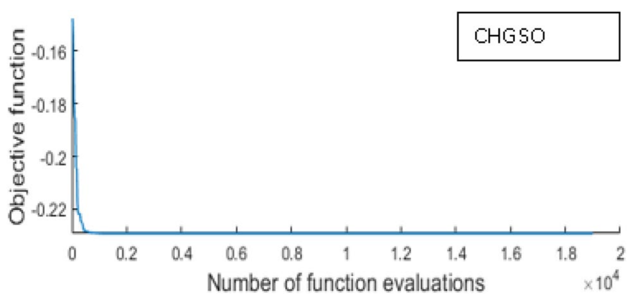


Fig. 6 Convergence diagram for the rough grinding problem

The *WRP* found using CHGSO is 35.48% higher than QP. The CHGSO has accomplished considerable development of 40.56% over the GA for *COF*. The search history for the rough grinding problem by CHGSO is given in Fig. 6.

It is seen in Table 5 that the *WRP* value obtained from using CHGSO is and 2.99% lower than GA. For the finish-grinding process, it is 10.99% higher than the PSO. The CT obtained from using CHGSO is 3.27% lower than the GA, 7.64% lower than the QP, 5.24% lower than the PSO, and 7.33% lower than the ACO. Using CHGSO leads to the improvement of 3.74% over the GA, 5.83% over the QP, 0.53% over the PSO, 1.08% over the ACO, and 0.153% over the hybrid PSO based on COF. Figure 7 shows the convergence history of the finish grinding process performed by CHGSO.

Up to now, CHGSO provides the best result in the literature for rough grinding using 19.000 number of function evaluations (NFE). Meanwhile, 350,000 NFE are required by MPEDE and HCLSPO to obtain an optimum. The HGSO, WOA [37], HWOANM [37], HHO [5], MVO [5], GOA [5], and RSFS [87], respectively, require 30.000, 32.000, 20.000, 27.000, 30.000, 30.000, and 32.000 NFEs to reach an optimum.

It can obviously be concluded that the proposed CHGSO is superior to those metaheuristics presented in the literature by Wen et al. [59], Baskar et al. [61], Saravanan et al. [60], Krishna and Rao [62], Lin and Li [66], Krishna [65], Rao and Pawar [69], Zhang et al. [64], Lee et al. [63], Pawar and Rao [71], Khalilpourazari and Khalilpourazary [73], Huang et al. [72], Yildiz [37], and Yildiz et al.[5].

6 Optimization of diaphragm spring using the CHGSO optimization algorithm

In this section, the shape optimization of a vehicle diaphragm spring taken from the automotive industry is completed using the CHGSO. The optimization problem can be defined to search for structural shape as design variables, x, to minimize structural mass $f(x)$ while the stress constraint $g(x)$ is imposed. It can be mathematically written as

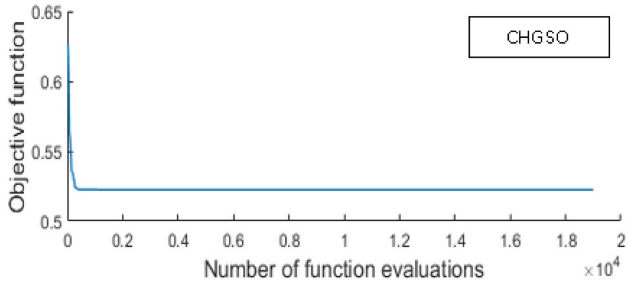


Fig. 7 Convergence diagram for the finish grinding problem

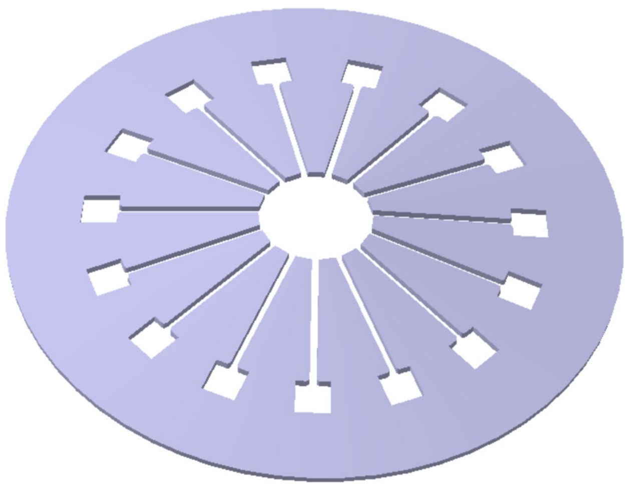


Fig. 8 Initial design domain of the spring

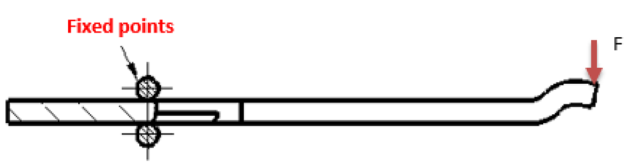


Fig. 9 Boundary conditions of the spring

$$MinF(x) = \text{mass}(x), \tag{37}$$

$$\text{Constraint}: g(x) \leq 0, \tag{38}$$

$$x_i^l \leq x_i \leq x_i^u, i = 1, \text{NDV}, \tag{39}$$

where NDV is the number of shape design variables, whose lower and upper bounds are x^l and x^u. In the shape optimization of the spring, mass minimization is considered as the objective function. The constraint function is the maximum stress of the component under boundary conditions (Fig. 9).

Initial design space and boundary conditions of the diaphragm spring are given in Figs. 8, 9, respectively.

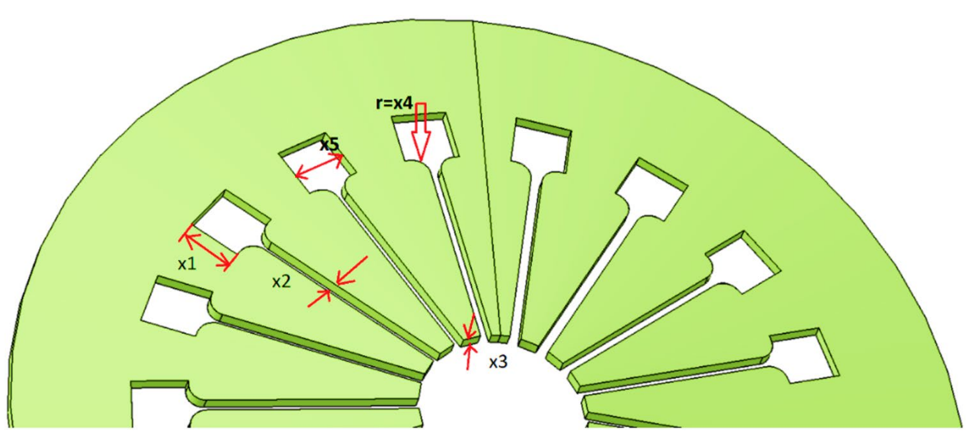
In this research, shape optimization of the automobile diaphragm spring is carried out using the CHGSO. The design variables given in Fig. 10 are $x1, x2, x3, x4$, and $x5$. The upper and lower limits of the design variables in shape optimization are considered as $30 < X1 < 40$, $3 < X2 < 5$, $2 < X3 < 4$, $5 < X4 < 10$ and, $35 < X5 < 45$ where the unit is in millimeters. The optimum design of the diaphragm spring after shape optimization using the CHGSO is illustrated in Fig. 11.

The result from the CHGSO is listed in Table 6, along with those obtained from using a genetic algorithm and the CHGSO. It can be concluded that the minimum mass of 255 gr with 245 MPa is obtained with CHGSO, which is superior to the other two algorithms.

7 Conclusions

This paper introduces a novel hybrid of chaotic Henry gas solubility algorithm. As presented in Tables 2 and 3, the CHGSO has provided wonderful results when used to optimize well-known design problems. Finally, a grinding problem and real-world engineering diaphragm spring design problem taken from the automotive industry are

Fig. 10 Design variables



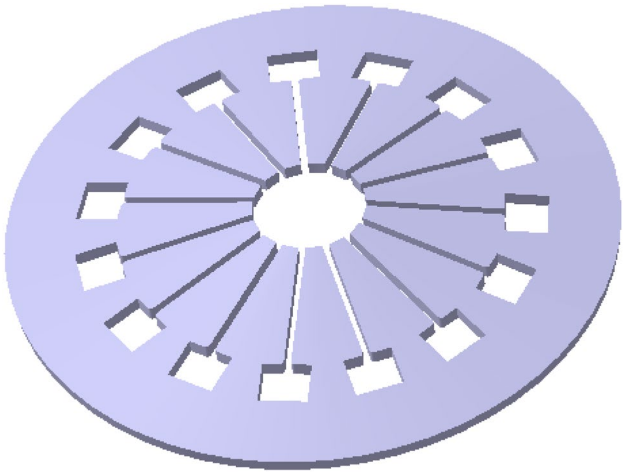


Fig. 11 Optimum diaphragm spring design using the CHGSO algorithm

Table 6 Comparison of the best results for the vehicle diaphragm spring

Method	Mass (gram)	Stress (MPa)	Number of function evaluations
Initial design	312.6	268	
Genetic algorithm	290	248	500
Sine–cosine optimizer	281	247	500
Dragonfly optimizer	280	245	500
Salp swarm optimizer	275	240	500
HGSO	273	246	500
CHGSO	255	245	500

solved using the CHGSO. From Tables 4, 5, 6, it can be proved that when compared to other famous optimization algorithms such as genetic algorithms, an ant colony algorithm, scatter search algorithm, particle swarm optimization, differential evolution, simulated annealing, an improved differential evolution algorithm, an artificial bee colony algorithm, harmony search, hybrid particle swarm algorithm, a cuckoo search algorithm, teaching learning-based optimization, grasshopper optimization, a dragonfly optimizer, salp swarm optimization, sine–cosine optimizer, an ant lion optimizer, gravitational search, a multi-verse optimizer, a mine blast algorithm, a Harris hawks algorithm, and the original Henry gas solubility optimization algorithm, the proposed method is effective when being employed to search for the optimal machining parameters to obtain better solutions. Better performance is due to the circular map gives a perfect balance between diversification and intensification, which makes the algorithm more powerful and effective.

Appendix A

Cantilever problem

The problem can be formulated as follows:

Minimize:

$$f(X) = 0.0624\left(x_1 + x_2 + x_3 + x_4 + x_5\right).$$

Subject to:

$$\mathrm{g}(x) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \le 0.$$

The design variables are the heights (or widths) of the different beam elements, and the thickness is held fixed (here $t = 2/3$). The bound constraints are set as $0.01 \le x_j \le 100$.

Welded beam design problem

The problem can be formulated as follows:

Minimize: $f(x) = 1.10471h^2l + 0.04811tb(14.0 + l)$.

Subject to:

$$g_1(x) = \tau(x) - \tau_{max} \le 0, g_2(x) = \sigma(x) - \sigma_{max} \le 0,$$

$$g_3(x) = h - b \le 0, g_4(x) = 0.1047h^2 + 0.04811tb(14 + l) - 5 \le 0,$$

$$g_5(x) = 0.125 - h \le 0, g_6(x) = \delta(x) - \delta_{max} \le 0,$$

$$g_7(x) = P - P_c(x) \le 0,$$

where $\tau = \sqrt{(\tau')^2 + (\tau'')^2 + 2\tau'\tau''\frac{1}{2R}}$, $\tau' = \frac{P}{\sqrt{2}hl}, \tau'' = \frac{MR}{J}$, $M = P\left(L + \frac{1}{2}\right), R = \sqrt{\frac{l^2}{4} + \frac{(h+t)^2}{4}}$,

$$J = 2\left\{\sqrt{2}hl\left[\frac{l^2}{12} + \frac{(h+t)^2}{4}\right]\right\}, \sigma = \frac{6PL}{bt^2},$$
$$P_c = \frac{4.013E\sqrt{t^2b^6/36}}{bt^2}\left(1 - \frac{t}{2L}\sqrt{\frac{E}{4G}}\right),$$

$$P = 6000lp, l = 14in, E = 30 \times 10^6 psi, G = 12 \times 10^6 psi,$$

$$\tau_{max} = 13600psi, \delta_{max} = 0.25in, 0.1 \le h, b \le 2,$$

and $1 \le l, t \le 10$.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interests.

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