# A systematic approach to the measurement of ideality factor, series resistance, and barrier height for Schottky diodes

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A nongraphical approach is proposed for measuring and evaluating the ideality $n$ factor and the series resistance of a Schottky diode. The approach involves the use of an auxiliary function and a computer-fitting routine. This technique has been found to be both accurate and reliable. The validity of this has also been confirmed by way of $I$-$V$ measurements using both commercially available and laboratory-prepared Schottky diodes.

## I. INTRODUCTION

Traditionally, the method used to extract the value of the ideality factor ($n$ factor) and the saturation current $I_s$ from the $I$-$V$ characteristic of a Schottky diode is based on the Richardson plots, $\ln I$ vs $V$ or $\ln\{I/[\exp(qV/kT)-1]\}$ vs $V$,[1-2] for which the values of $n$ and $I_s$ can be obtained from the slope and the intercept on the $\ln I$ and $\ln\{I/[\exp(qV/kT)-1]\}$ axis, respectively. However, this traditional approach does not usually account for the possible series resistance that may be present in the real diode.[3] As a result, a certain degree of uncertainty may be associated with this method which depends on the value of the series resistance $R_s$ in the system.

In order to cater for the effect of a nonzero $R_s$, Norde, in 1979,[4] proposed a method based on an auxiliary function

$$F(V)=V/2-\{\ln[I/(A_eA^*T^2)]\}/\beta$$

in order extract the value of $R_s$ and $I_s$ from the $I$-$V$ characteristics. Unfortunately, this method was still subject to a few drawbacks. First, Norde's method only works for ideal Schottky diodes, namely with $n=1$, but not for $n>1$. Second, uncertainties may arise from the determination of the minimum of $F(V_0,I_0)$ from the graph of $F(V)$ vs $V$, especially when the value of $R_s$ is small and the minimum cannot be determined precisely.

In 1985, Cibils and Buitrago[5] proposed another approach based on another auxiliary function $F(V)=V-V_a\ln I$ to obtain the value of $n$, $I_s$, and $R_s$. The method involved the determination of the minimum of the function $F(V_0)$ for different values of the independent auxiliary voltage $V_a$. The value of $n$ and $R_s$ can be obtained from the plot of $I_0$ vs corresponding $V_a$ values. Although the auxiliary function $F(V)$ here occurred in a simpler form, this method was still subject to the problem of determining the minimum from the graph of $F(V)$ vs $V$. This graphical method is particularly time consuming especially when we need to obtain a set of $I_0$ from a range of $V_a$. Moreover, the method used by Cibils and Buitrago works much better in the high-resistance case, since at low resistance the plot of $F(V)$ vs $V$ may not show a very sharp minimum turning point, so the uncertainty may be somewhat too large to be acceptable.

One year after the publication of Cibils and Buitrago, Bohlin,[6] defined another function

$$F(V)=V/\gamma-\{\ln[I/(A_eA_*T^2)]\}/\beta,$$

where $\gamma$ is a arbitrary parameter that has a value greater than $n$, in order to find the value of $R_s$, $n$, and $\phi$. Although the method adopted by Bohlin can apply to the low-resistance situation, it needs to solve two rather complicated simultaneous equations for two different $\gamma$ values before the values of $n$, $R_s$, and $\phi$ can be obtained. Moreover, the drawback of determining the minimum value of $F(V_0,\gamma)$ from a graphical method was still present in this case.

Recently, a similar approach, but working with a simpler function $F(I)=V-R_0I$, was proposed by Manifacier *et al.*, in 1988.[7] Instead of finding the minimum of $F(V)$, the method of these authors involved the determination of the maximum current $F(I_m)$. The values for $R_s$ and $n$ are obtained by solving two simultaneous equations with two different $R_0$ values. However, this method still encountered similar disadvantages in determining the graphical turning points and in solving simultaneous equations.

Generally speaking, the above-mentioned methods can help to extract the real $R_s$ and $n$ values from the $I$-$V$ characteristics of a Schottky diode under certain circumstances. Although each of these methods has its own advantage over the others, they have common disadvantages in the graphical determination of turning points and in their inconvenient working procedures. The inconvenience and uncertainties associated with the graphical method still remain the main limitation to the applicability of this method in this field.

In this paper, we would like to propose another method, which goes with the trend of using an auxiliary function $F(V,I)$ to determine the value of $n$ and $R_s$, but with a more convenient and computer-compatible approach.

## II. METHOD

Among the four above-mentioned auxiliary functions that have been suggested for finding the value of $n$ and $R_s$, we adopt the simple function $F(V)=V-V_a\ln I$ suggested by Cibils and Buitrago. However, instead of plotting the function $F(V)$ vs $V$ and obtaining the minimum by a graphical method, we carry out our calculation from another point of view.

  

We start with the simple function

$$F(V)=V-V_a \ln I. \tag{1}$$

For $V>3kT/q$, the $I$-$V$ characteristics of the Schottky diodes with the inclusion of a series resistance $R_s$ may be approximated by

$$I \simeq I_s \exp\left(\frac{q(V-IR_s)}{nkT}\right), \tag{2}$$

where $I_s=A_e A^{**}T^2 \exp(-q\Phi/kT)$ and $n$ is called the ideality factor.

If we use $I$ instead of $V$ as the independent variable in Eq. (1) and substitute Eq. (2) into Eq. (1) we can obtain $F(I)$ as

$$F(I)=IR_s+[(nkT/q)-V_a]\ln I-(nkT/q)\ln I_s, \tag{3}$$

where $V_a$ is an arbitrary voltage independent of $V$. Equation (3) may be written in a simpler form,

$$F(I)=aI+b \ln I+c, \tag{4}$$

where

$$a=R_s, \quad b=\frac{nkT}{q}-V_a, \quad \text{and} \quad c=\frac{nkT}{q}\ln I_s.$$

Thus, for a diode at a constant temperature, for a given value of $V_a$, $a$, $b$, and $c$ can be assumed constant.

With the view of proving that minimum obtained from the plot of $F(I)$ vs $I$ is the same as the one obtained by Cibils and Buitrago using $F(V)$. We put $dF/dI$ equal to zero. Since

$$\frac{dF}{dI}=a+\frac{b}{I},$$

putting $dF/dI=0$ and gives

$$I_0=-b/a=(V_a/R_s)-(nkT/qR_s), \tag{5}$$

which is identical to the result obtained by Cibils and Buitrago from the $F(V)$ vs $V$ plot.

Here the form of Eq. (4) is a more convenient form which can be fitted by a simple least-squares-fitting technique using a standard computer algorithm.[8] This computer-fitting procedure can help us determine the value of $a$, $b$, and $c$ from a set of $I$-$V$ data. As a result, the minimum $I_0=-b/a$ can easily be obtained. By varying the value of $V_a$ in the computer loop, we can thus generate a set of $V_a$ and $I_0$ values within a short time.

Finally, we can extract the value of $n$ and $R_s$ from the plot of $I_0$ vs $V_a$ as described by Cibils and Buitrago. Moreover, this last step can also be included in the fitting program and thus we can directly obtain the value of $n$ and $R_s$ after the computation.

The advantage of this approach is that we can minimize the uncertainties in determining the minimum $F(I)$ vs $I$ plot and we can use the computer to speed up the calculation procedure. Furthermore, our approach can be improved in accuracy by adding an external resistance $R_v$ in series to the diode and the consistency of the $n$ factor and $R_s$ value can be checked for different $R_v$ values.

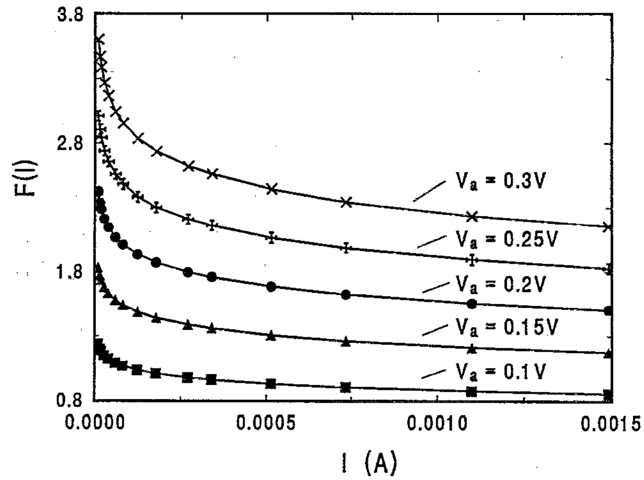


FIG. 1. Experimental plot of $F(I)$ data for SB130 Schottky diode, for $V_a$ between 0.1 and 0.3 V. The fitted curves are from Eq. (5) with $a$, $b$, and $c$ as fitting parameters.

The plot of $n$ vs $R_v$ gives use the mean value of $n$ and the plot of $R_s$ vs $R_v$ will indicate the value of $R_s$ from the interception on the $R$ axis.

## III. EXPERIMENT

With the view of demonstrating the procedure of our approach, a Motorola SB130 commercial metal-silicon Schottky diode and homemade Sb/$p$-Si diode are used. The Sb/$p$-Si diode is fabricated on a 5–10 Ω cm $p$-type silicon substrate. The substrate is degreased and etched in $H_2SO_4$:$H_2O_2$:$H_2O$ (10:1:1) then etched in aqueous HF. Antimony is evaporated at $10^{-6}$ Torr pressure to a thickness of 1000 Å, and a junction area $A=0.0314$ cm$^2$ is defined. Heat treatment is performed for 10 min in a forming gas (80% $H_2$, 20% $N_2$) atmosphere. An ohmic contact is formed on the back surface of the substrate by alloying an Al contact at 550 °C for 15 min. All $I$-$V$ measurements are carried out inside a well-regulated cryostat (Oxford Instrument, helium gas exchange type) with a programmable power supply and picoammeter system.

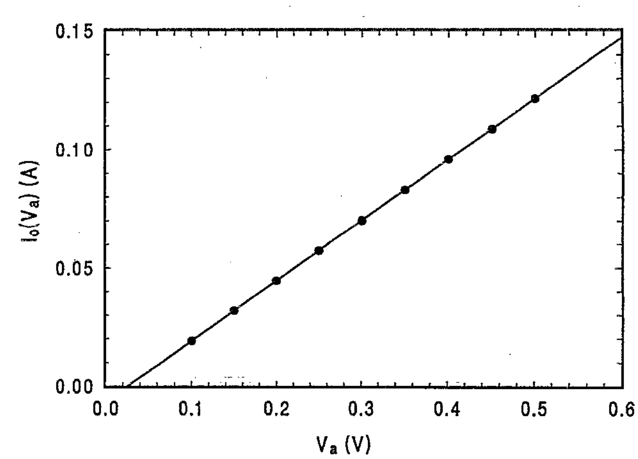


FIG. 2. Plot of $I_0(V_a)$ data for the SB130 Schottky diode.

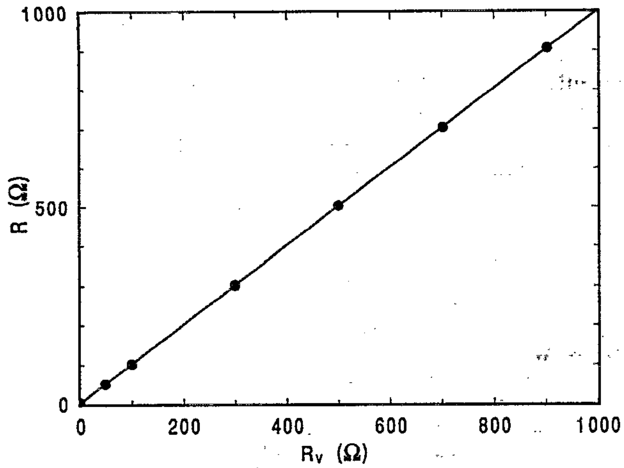


FIG. 3. The extended resistance method. $R$ is the total resistance $R_s+R_v$, determined by our method, and is shown plotted as a function of external resistance $R_v$.

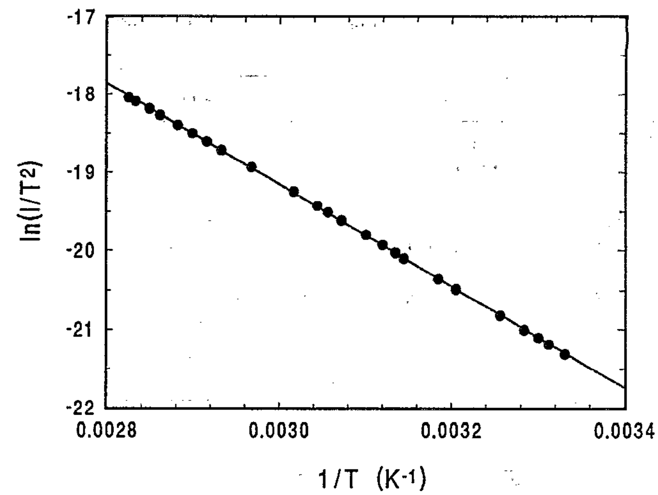


FIG. 5. Activation energy plot for the SB130 diode with forward bias of 0.1 V.

## IV. WORKING PROCEDURE

First, the $I$-$V$ trace of the SB130 diode at 295 K is taken for analysis. Based on our approach, we proceed to obtain the minimum value of $I_0$ corresponding to different values of $V_a$ by the computer-fitting method. Figure 1 shows the plot of $F(I)$ vs $I$ for $V_a$=0.1–0.3 V and their corresponding fitting curves. The plot of $I_0$ vs $V_a$ for $V_a$ =0.1–0.5 is shown in Fig. 2. The values of $n$ and $R_s$ obtained are 0.995±0.02 and 3.95±0.5 Ω, respectively. In an attempt to confirm the validity of our technique, that method making use of an external resistor has been adopted. Figures 3 and 4 show the plots of $R$ vs $R_v$ and $n$ vs $R_v$, respectively, from their measurements. The averaged value of $n$ is 0.995∓0.015 and the regression value of $R_s$ is 3.1±0.45 Ω. For the value of the barrier height, we can obtain the value of $\ln(A_eA^{**})$ from the activation energy plot as shown in Fig. 5.[1] Upon substituting this $\ln(A_eA^{**})$ together with the $n$ and $R_s$ values for SB130 into Eq. (2), we obtained the barrier height $\phi_{i-v}$=0.66±0.01 V which is in agreement with the value of $\phi_a$=0.656±0.02 V obtained from the activation energy plot.

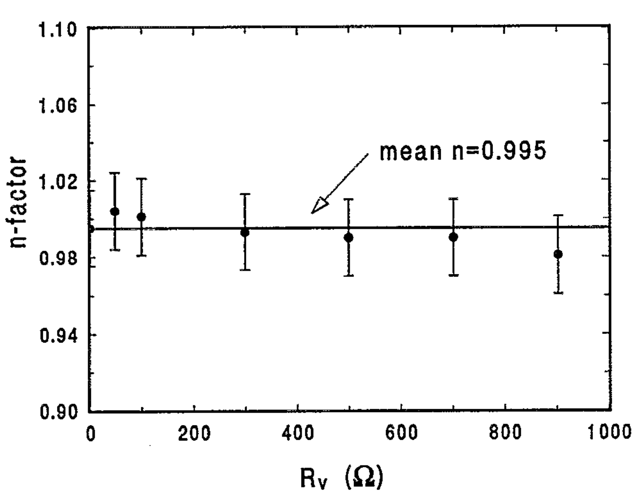


FIG. 4. The extended resistance method. $n$ is shown plotted as a function of external resistance $R_v$.

## V. DISCUSSION

In order to compare the accuracy and values obtained from our approach with other methods, a comparison is made with the other techniques in the analysis of the $I$-$V$ data from the SB130 and Sb/$p$-Si diodes at 295 K. Tables I and II display the comparison of the results obtained from various methods on the SB130 and Sb/$p$-Si diode, respectively. In Table I, we find that the results of $n$ and $R_s$ obtained from methods 4 and 5 (even compared with the traditional method), are unreliable and inaccurate. For methods 3, 4, and 6 an external resistance of 300 Ω is added to the system, since these methods only work for high-resistance diodes. As for the results shown in Table II, the values of $n$ and $R_s$ obtained for the Sb/$p$-Si diode by methods 4 and 6 are much more reasonable. However, methods 3 and 5 still cannot give us satisfactory values.

In view of the two tables, we observe that the values of $n$ and $R_s$ obtained from our approach are much more reasonable and accurate when compared with the other methods even under a rather low-resistance situation.

TABLE I. Result of $n$ and $R_s$ obtained from various methods on SB130.

| Method | $n$ | $R_s$/Ω | Remark | Φ/V |
|---|---|---|---|---|
| 1 (traditional) | 1.028 | ⋯ | ⋯ | 0.663±0.02 |
| 2 (our method) | 0.995±0.015 | 3.1±0.45 | ⋯ | 0.66±0.015 |
| 3 (Ref. 5) | 0.95±0.18 | 308±6 | $R_v$=300 Ω | 0.67±0.03 |
| 4 (Ref. 6) | 1.28±0.15 | 306.2±30 | $R_v$=300 Ω, $\gamma_1$=2.5, and $\gamma_2$=3.5. | 0.63±0.06 |
| 5 (Ref. 7) | 0.91±0.09 | 12.1±2.5 | $R_1$=100 Ω and $R_2$=50 Ω | 0.66±0.04 |
| 6 (Ref. 4) | 1 | 316.2±19 | Assumed $n$=1 | 0.663±0.03 |

TABLE II. Result of $n$ and $R_s$ obtained from various method on Sb/$p$-Si diode.

| Method | $n$ | $R_s/\Omega$ | Remark | $\Phi/V$ |
|---|---|---|---|---|
| 1 (traditional) | 1.17±0.05 | ··· | ··· | 0.6±0.02 |
| 2 (our method) | 1.001±0.02 | 21.3±0.9 | ··· | 0.62±0.02 |
| 3 (Ref. 5) | 0.96±0.4 | 321.9±13 | $R_v=300\ \Omega$ | 0.625±0.06 |
| 4 (Ref. 6) | 1.03±0.1 | 21.1±2 | $\gamma_1=2.5$ and $\gamma_2=3$. | 0.62±0.06 |
| 5 (Ref. 7) | 0.74±0.26 | 41.6±15 | $R_1=100\ \Omega$ and $R_2=200\ \Omega$. | 0.65±0.05 |
| 6 (Ref. 4) | 1 | 24.0±2 | Assumed $n=1$ | 0.618±0.05 |

## VI. CONCLUSION

In this work, a more systematic, computer-fitting approach is suggested for extracting the values of the ideality factor $n$ and the series resistance $R_s$ from the $I$-$V$ characteristics of Schottky diodes. With the help of the computer curve-fitting technique, this approach reduces the uncertainties and inconvenience that have been associated with past proposed methods. Consistent results have been obtained for both commercial and homemade Schottky diodes in the regime where the series resistance is low.

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