RESULTS AND DISCUSSION

In order to obtain the Boron profile in Silicon substrate the signal $S_I(S)$ given by the probe and related to the Boron impurity in the sample is compared to the signal $S_I(C)$ related to the Boron in a Silicon check-sample with a known impurity profile. The density $C_I(S)$ of Boron in the sample is given by [4]:

$$C_I(S) = \frac{C_I(C)}{S_I(C)} \cdot \frac{S_{si}(C)}{S_{si}(S)} \cdot S_I(S) \tag{1}$$

where $C_I(C)$, $S_{si}(C)$ and $S_{si}(S)$ are respectively the impurity density in the check-sample and the signal related to the Silicon atoms in both the check-sample and the studied device.

The Boron profile in the Silicon as given by relation (1) is shown in Fig. 2, which shows good agreement between experiment and the two types of theoretical results in the upper part of the profile.

The poor agreement observed at a slow concentration between experimental results and the results of SUPREM can be attributed to the exponential term empirically introduced in the SUPREM program to account for the channelling effect[3]. Indeed when the Silicon substrate is covered by a SiO_2 layer the impinging Boron ions have randomly distributed directions when they reach the SiO_2–Si interface. Consequently the probability of channeling for these ions is very low. On the other hand agreement with Furukawa's theory is not surprising since it does not consider the channeling effect.

As far as the impurity profile in the Silicon dioxide is concerned relation (1) will not apply because the Si atoms concentration $C_{si}(C)$ in the check-sample is different from $C_{si}(S)$ in the SiO_2 layer. In this case it can be easily shown that the impurity concentration is given by:

$$C_I(S) = \frac{C_I(C)}{S_I(C)} \cdot \frac{S_{si}(C)}{S_{si}(S)} \cdot \frac{C_{si}(S)}{C_{si}(C)} S_I(S) \tag{2}$$

where the $C_{si}(S)/C_{si}(C)$ ratio can be calculated from the molecular weight and the density of the two species and is equal to 0.45.

Under these conditions the implanted Boron profile as given by relation (2) is also represented in Fig. 2 which shows good agreement between both theoretical and experimental results. The high concentration observed near the SiO_2 surface in disagreement with the two theories is fictive and caused by an exaltation phenomenon of the ionizing probe signal at the start of ionic analysis[4].

As a summary both Furukawa's and SUPREM theories give satisfactory agreement with experimental results in the SiO_2 layer and in the underlaying Silicon. However it can be noticed that SUPREM program does not account for the deeper region of the profile and this can be caused by a lack of channeling effect in presence of the SiO_2 layer.

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SOLAR CELL FILL FACTORS: GENERAL GRAPH AND EMPIRICAL EXPRESSIONS†

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1. INTRODUCTION

Two contributions are made to techniques for evaluating solar cell fill factors. A graph is given which allows fill factors to be determined for any combination of the parameters used to characterize cell performance. Also given are simple empirical expressions for these factors. These contributions are of use in predicting characteristics of cells and solar cells systems as well as deducing cell parameters from experimental data.

The performance of individual solar cells and solar cell systems is commonly characterized in terms of three parameters. These are the open circuit voltage (V_{OC}), the short circuit current (I_{SC}), and the fill factor (*FF*). The maximum power output of the device or system is the product of these three quantities. Even though the fill factor is important in solar cell calculations, there is no explicit solution for it in terms of other cell parameters and iterative methods have been necessary to calculate it. In the present correspondence, a general solar cell fill factor graph is given which allows the fill factor of a cell to be found under a wide range of conditions. Additionally, empirical expressions are given for the fill factor under similarly general conditions and their accuracy discussed.

Over its normal operating region, the following variables are normally adequate to describe solar cell performance: V_{OC} and I_{SC}, the light generated current (I_L), the cell temperature (T), the "ideality factor" (n), the diode saturation current (I_0), and the parasitic series and shunt resistances, R_s and R_{sh}[1]. The output current-voltage relationship under illumination is then given by[1].:

$$I = I_L - I_0\left\{\exp\left[\frac{q(V+IR_s)}{nkT}\right] - 1\right\} - \frac{V+IR_s}{R_{sh}} \tag{1}$$

where k is Boltzmann's constant with kT/q having the value of 0.02586 V at 300°K.

For a cell in which the shunt resistance, R_{sh}, is large and does not have to be taken into consideration, it has been shown possible to derive an expression clarifying the interrelationship of the remaining parameters. Defining the "characteristic resistance", R_{ch}, of a cell as the ratio V_{OC}/I_{SC}, and the thermal voltage, V_{th}, as nkT/q, allows normalized resistances and voltages to be defined. A normalized series resistance, r_s, is defined as R_s/R_{ch} and a normalized open circuit voltage, v_{oc}, as V_{OC}/V_{th}.

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Additionally, a normalized current, f_i, is defined as I/I_{SC}. This gives the following expression for the fill factor in this case[2]:

$$FF_s = \left| f_i \left\{ (1 - f_i r_s) + \frac{1}{v_{oc}} \ln [1 - f_i (1 - \exp(v_{oc} r_s - 1))] \right\} \right|_{\text{maximum}}.$$

The annotation "maximum" indicates that the fill factor is given by the maximum value of the preceeding expression as f_i varies for fixed r_s and v_{oc}. The above expression shows that in the case considered, the fill factor depends only on the normalized parameters r_s and v_{oc}. This results in the general family of curves shown as solid lines in Fig. 1 which completely specifies the effect of series resistance on solar cell fill factors[2].

Similarly in the case where series resistance is small and negligible, a normalized shunt resistance, r_{sh}, is defined as R_{sh}/R_{ch} and a normalized voltage, f_v, is defined as V/V_{OC}. This gives an expression for the fill factor in this case as:

$$FF_{sh} = \left| f_v \left\{ 1 - \frac{[\exp(f_v v_{oc}) - 1](1 - 1/r_{sh})}{[\exp(v_{oc}) - 1]} - f_v/r_{sh} \right\} \right|_{\text{maximum}}. \quad (3)$$

Again this shows that the fill factor in this case depends only on the normalized parameters, r_{sh} and v_{oc}. This gives rise to a second family of curves shown dashed in Fig. 1 which completely specifies the effect of shunt resistance on solar cell fill factors.

The case where both series and shunt resistances are important is more complex. However an *approximate* technique based on Fig. 1 has been found to give reasonably accurate results. The technique is best demonstrated by an example. A cell at 28°C gives a V_{OC} of 583 mV, and an I_{SC} of 2.02 A. It has an ideality factor, n, of 1.3, a series resistance of 0.0578 Ω and a shunt resistance of 1.444 Ω. This gives normalized values of 17.3, 0.2, and 5 for v_{oc}, r_s, and r_{sh}. To find the fill factor, the point on Fig. 1 is found corresponding to the case where r_s and $1/r_{sh}$ equal to zero as indicated. One then moves down vertically to the solid line corresponding to the correct value of r_s (0.2 in this case). Then one extrapolates horizontally to the left until the line corresponding to r_s and $1/r_{sh}$ equal to zero is again intercepted. The final step is to again move down vertically to the dashed line corresponding this time to the correct value of $1/r_{sh}$ (0.2 in this case). For the present cell this gives an estimate of the fill factor of 0.54. The exact value is calculated as 0.556. In less demanding examples where r_s and $1/r_{sh}$ are smaller or one is small compared to the other, the accuracy can be very much better.

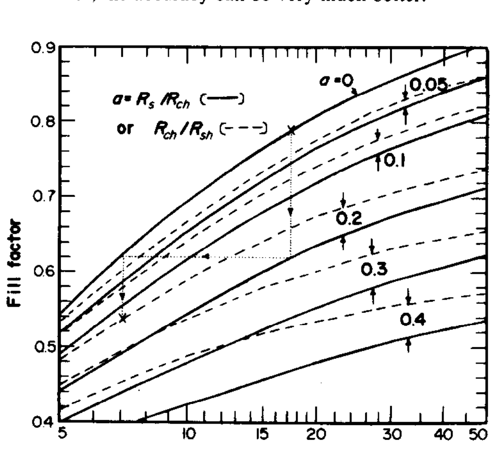

Fig. 1. The general solar cell fill factor diagram. V_{OC} is the open circuit voltage, V_{th} is nkT/q, R_s and R_{sh} are parasitic series and shunt resistances, while R_{ch} is the characteristic resistance of the cell given by V_{OC}/I_{SC}. I_{SC} is the short-circuit current. The solid lines allow the fill factor to be determined when shunt resistance is very large while the dashed lines allow this to be done when series resistance is small. The dotted path demonstrates an *approximate* technique for estimating the fill factor when both resistances are important.

(The accuracy can be estimated by repeating the above procedure except first going vertically down to intercept the appropriate $1/r_{sh}$ curve and the appropriate r_s curve on the second vertical translation. This approach can be shown rigorously to underestimate the fill factor. For the example above it gives a value of 0.52, about 0.02 lower than the preferred technique. For all combinations tested, the correct value of the fill factor has been found to lie no further from the estimate obtained from the preferred technique than the corresponding difference between estimates. Hence it is possible to estimate the fill factor for the example above as 0.54 ± 0.02.)

Graphical methods are not always convenient or capable of giving the required accuracy. The second part of this correspondence describes empirical expressions for the fill factors in terms of the parameters already discussed. Related expressions for two of the four cases considered have been given previously[3] but the present expressions are not only simpler but more accurate.

For the case where both R_s and R_{sh} both have negligible effect upon cell performance, a simple but very accurate expression for the fill factor is:

$$FF_0 = \frac{v_{oc} - \ln(v_{oc} + 0.72)}{v_{oc} + 1} \quad (4)$$

where v_{oc} is the open circuit voltage normalized to the thermal voltage, nkT/q. This expression is accurate to one digit in the fourth significant place for all normalized open circuit voltages > 10. This accuracy is more than adequate for all solar cell work. It is also about one significant digit better over this range than a similar expression given elsewhere[3].

For the case where R_{sh} is so large as to be negligible but R_s is important, it has not been possible to find a simple expression of similar accuracy. The best compromise is the very simple expression:

$$FF_s = FF_0(1 - r_s) \quad (5)$$

where r_s is the series resistance normalized to the "characteristic resistance", V_{OC}/I_{SC}. This expression is better than 2% accurate for normalized voltage > 10 and normalized resistance < 0.4. This means that it is accurate to about one digit in the second significant place, but again better over this range than a more complex expression given elsewhere[3].

For the case where series resistance is negligible but shunt resistance is important, a slightly more complicated expression gives excellent results;

$$FF_{sh} = FF_0 \left[1 - \frac{(v_{oc} + 0.7)}{v_{oc}} \frac{FF_0}{r_{sh}} \right] \quad (6)$$

where r_{sh} is the shunt resistance normalized to the cell's characteristic resistance and v_{oc} is the open circuit voltage normalized to nkT/q. This expression is valid to about one digit in the third significant place for all $v_{oc} > 10$ and $r_{sh} > 2.5$, again adequate for most solar cell work.

If both series and shunt resistances are important, the preferred approach is to calculate FF_s using eqn (5) and substitute this for FF_0 whenever it appears in eqn (6). For the rather demanding example discussed earlier, the fill factor calculated by this approach is 0.547 compared to the exact value of 0.556. This is the largest error detected in examples done to date, giving a high level of confidence in an accuracy of one digit in the second significant place.

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