Very Sensitive Measurement Method of Electron Device Current Noise

M. Macucci and B. Pellegrini

***Abstract*—The problem of measuring very low levels of current noise in bipoles (linear or not) is dealt with, and a new measurement technique is proposed, which allows us to measure noise power spectra 6–10 dB lower than the equivalent input power spectrum of the amplifier necessary to perform the measurement. Thus we obtain an improvement of 16–20 dB in the sensitivity with respect to the one of conventional methods, which, for an acceptable accuracy, require the noise of the bipole under test to be 10 dB larger than the equivalent input one of the amplifier. Our method is based upon the accurate measurement of the amplifier transimpedance with respect to the input current noise sources, and on the precise evaluation and subtraction of the contribution from all the spurious sources to the total noise. The whole procedure is implemented by means of a dual channel signal analyzer, and almost completely automatized. The experimental results obtained with the application of our method agree very well with theoretical previsions.**

I. Introduction

As it is well known, a limit in measuring the power spectral density S_{I_B} of current noise in electron devices is represented by the equivalent input current noise S_{I_D} of the amplifier used to perform the measurement itself.

In parallel with the effort in designing and in realizing amplifiers with increasingly better noise performances , it is also important to develop measurement methods capable of enhancing the sensitivity reachable with a given amplifier [3].

To this end we have developed a new noise measurement technique based on the accurate determination of the transimpedance between the current noise generator to be measured and the amplifier output, and on the precise evaluation and subtraction of the noise due to the amplifier and to other parasitic sources.

This way we solve the problems deriving both from the spurious noise sources and from the unknown impedances of the bipole under test (BUT), of its biasing network, and of the amplifier input.

The transimpedance is obtained by injecting noise from a generator into the input circuit, and measuring the effect at the amplifier output, while the contribution of the spurious noise sources is evaluated by substituting the BUT with a bipole having the same impedance but a known noise behavior. Finally, the results for the transimpedance and for the additional noise contributions are used to elaborate and correct the total noise measurements, and, thus to evaluate levels of the BUT noise current which may be 6–10 dB below those of the equivalent noise generator of the amplifier.

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Except for the impedance measurements (needed for the BUT and its substitution impedance), all the other ones and the mathematical operations required for the application of our method are implemented and automatized by means of a dual channel signal analyzer.

This new technique has been tested by applying it to measure the power spectra of the noise given by known generators, of the Nyquist noise produced by bipoles made up of resistors and capacitors, and of shot noise in p-n junctions.

In all cases we have been able to measure, within 10% accuracy, power spectral densities S_{I_B} of current noise which were at least four times smaller than the power spectrum S_{I_D} of the equivalent input generator of the amplifier. We have therefore obtained a 16–20 dB increase in sensitivity with respect to conventional methods, which, for the same 10% accuracy, require S_{I_B} of the BUT to be 10 dB larger than S_{I_D}.

II. Measurement System and Method

Two of the most general circuit configurations for the measurement of current noise in a bipole are shown in Fig. 1(a) and in Fig. 1(b). In both cases we have a biasing network, a Bipole Under Test (which can be linear or not), an amplifier and a measuring instrument. The BUT, in its turn, may be any one-port network.

In Fig. 1(a) the biasing network, having an output impedance Z_P, and the BUT are connected in parallel with the amplifier input. In Fig. 1(b), instead, the BUT is connected in series to the amplifier input and to the biasing network, whose output impedance Z_S can be made to be negligible in most practical cases.

For the parallel case of Fig. 1(a) we can use an equivalent circuit like the one shown in Fig. 2(a), where Z_{in} and Z_O are the input and output impedances, respectively, of the amplifier, and A is its gain. Z_B is the BUT impedance, I_P the noise current generator representing the noise sources inside the biasing network according to the Norton theorem, I_B the BUT noise generator we want to measure, and E_n and I_n are the equivalent input noise generators of the amplifier [5], [6]. E_n and I_n do not depend on the impedance connected between the input terminals of the amplifier. The circuit of Fig. 2(a), for $Z_P = \infty$ and $I_P = 0$, includes also the series case of Fig. 1(b) if $Z_S \approx 0$.

All the generators other than I_B can be represented, as it is shown in Fig. 2(b), with a single current generator I_D whose power spectrum S_{I_D} is given by (see Appendix):

$$S_{I_D} = S_{I_P} + S_{I_n} + \frac{S_{E_n}}{|Z_L|^2} + 2\,\mathrm{Re}\left\{S_{E_nI_n}\right\}\mathrm{Re}\left\{Z_L^{-1}\right\} - 2\,\mathrm{Im}\left\{S_{E_nI_n}\right\}\mathrm{Im}\left\{Z_L^{-1}\right\} \quad (1)$$

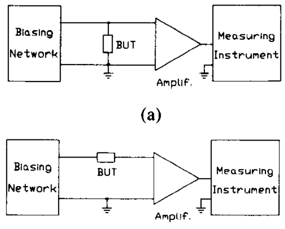


Fig. 1. (a) Circuit configuration for the measurement of the noise current in a bipole having a biasing network connected in parallel or, (b), in series.

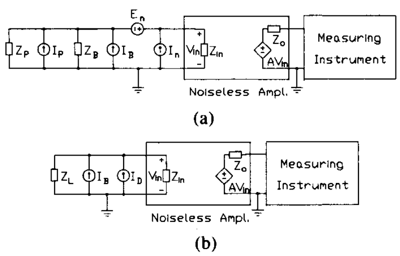


Fig. 2. (a) Circuit diagram of the measuring system, showing all the generators contributing to the total noise. (b) Its equivalent network, in which $Z_L = Z_P//Z_B$, and the effect of all the noise generators other than I_B is represented with a single generator I_D.

where S_{I_P}, S_{I_n}, S_{E_n} are the power spectra of I_P, I_n, E_n, respectively, and $S_{E_nI_n}$ is the cross-spectrum of E_n and I_n, which can in principle be correlated. I_P and I_B, on the contrary, are certainly uncorrelated with each other and with I_n and E_n, being the result of independent physical phenomena. Z_L is the parallel of Z_B and Z_P and it is, more in general, the total equivalent impedance seen from the amplifier input.

Our measurement system is shown in Fig. 3. The impedance Z_T can either be connected in parallel with the amplifier input or between the amplifier input and a noise generator V_S, according to the position of the switch SW_1. For Z_T we use a capacitor, since it is the circuit element which yields the most ideal behavior on a wide frequency range.

As a measuring instrument we use a two-channel HP3562A signal analyzer which performs all the measurements and the mathematical operations needed for the application of our method. The analyzer provides also the noise source V_S and controls the switch SW_1, automatically executing the sequence of measurements and calculations described in the present and in the following sections.

Initially SW_1 is in the upper position and on channel B we can measure the power spectrum S_{V_0} at the amplifier output, which can be written as

$$S_{V_0} = S_I|A|^2|Z_P//Z_B//Z_{in}//Z_T|^2 = S_I|A|^2Z_L//Z_{in}|^2 \tag{2}$$

where $S_I = S_{I_B} + S_{I_D}$, with S_{I_D} given by (1) for $Z_L = Z_P//Z_B//Z_T$.

Then, with SW_1 in the lower position, and $S_{V_S}//|Z_T|^2 >> S_I$ (S_{V_S} is the power spectrum of V_S) we can measure, using both channels A and B of the analyzer, the frequency response F between V_S and the amplifier output. Since

$$|F|^2 = |A|^2\left|\frac{Z_P//Z_{in}//Z_B}{Z_P//Z_{in}//Z_B + Z_T}\right|^2 \tag{3}$$

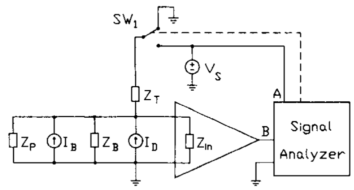


Fig. 3. Measurement system for the proposed method.

from (2) we get

$$S_I = \frac{S_{V_O}}{|F|^2|Z_T|^2} = \frac{\omega^2 C_T^2}{|F|^2} S_{V_O} \tag{4}$$

where C_T is the value of the capacitor used for Z_T.

It is worth noticing that, according to the circuit of Fig. 3 and to (3) or (4), FZ_T represents the actual transimpedance of the amplifier with respect to the current generators I_B and I_P, taking into account the effect of the impedances Z_P, Z_B, and Z_T itself.

With the above described procedure we do not need to perform separate measurements for the amplifier gain and for the impedances appearing in (2), thus avoiding the related significative sources of error. Up to this point, in particular, we wouldn't need to measure the impedance of the bipole, but its value will be required later for the evaluation of S_{I_D}.

In the following section we shall examine how the contribution S_{I_D} of I_D can be separately evaluated in order to subtract it from the result S_I we have already obtained.

III. Correction Procedure and Errors

A. Measurement of S_{E_n}, S_{I_n}, S_{I_P} and $S_{E_nI_n}$

According to (1), once we have measured the spectra S_{I_n}, S_{E_n}, Re $\{S_{E_nI_n}\}$, Im $\{S_{E_nI_n}\}$ relative to the amplifier and the one S_{I_P} of the noise due to the biasing network, we can evaluate S_{I_D} for every value of the BUT impedance. S_{I_n}, S_{E_n} and the real and imaginary parts of $S_{E_nI_n}$ could be measured using techniques with input impedances kept at different temperatures, like in [7], [8] or by means of the procedure described below.

From (1) and (A4), where S_{I_P} is now replaced by S_{I_R}, the Nyquist noise of R_L (for the meaning of R_L see below), we have

$$S_{E_n} = \frac{S_{V_{O0}}}{|A|^2} \tag{5}$$

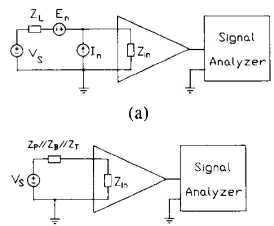
$$S_{I_n} = \frac{S_{V_{O\infty}}}{|Z_{in}|^2|A|^2} \tag{6}$$

$$S_{I_n} + \omega^2 C_L^2 S_{E_n} - 2\omega C_L \operatorname{Im}\{S_{E_nI_n}\} = S_{I_{DC}} \tag{7}$$

$$S_{I_n} + \frac{S_{E_n}}{R_L^2} + \left(\frac{2}{R_L}\right) \operatorname{Re}\{S_{E_nI_n}\} = S_{I_{DR}} - S_{I_R} \tag{8}$$

where $S_{V_{O0}}$ and $S_{V_{O\infty}}$ are the power spectra at the amplifier output when the input is short-circuited or open, respectively. $S_{I_{DC}}$ and $S_{I_{DR}}$ are the total equivalent input current noise spectra obtained with the procedure presented in the previous section when, in the circuit of Fig. 3, $I_B = 0$, and Z_P and Z_B are replaced with a capacitor C_M or with a resistor R_M, respectively. In the former case Z_T is a capacitor (C_T) and $C_L = C_M + C_T$, while in the latter Z_T is a resistor (R_T) and $R_L = R_M//R_T$.

$|A|$ and Z_{in} can be determined using the setup shown in Fig. 4(a): for $Z_L = 0$ we have $A = V_O'/V_S$, and, for a known or



(a)

(b)

Fig. 4. (a) Setup for the measurement of $|A|$ and Z_{in} of the amplifier. (b) Setup for the measurement of the total impedance connected in parallel with the amplifier input.

measured $Z_L = \overline{Z_L}$, we have

$$Z_{\text{in}} = \frac{\overline{Z_L}}{A/F_L - 1} \tag{9}$$

where $F_L = V_O''/V_S$ is the frequency response between V_S and the amplifier output, V_O' and V_O'' being the output voltages in the two cases.

Therefore from (5)–(9) we evaluate S_{E_n}, S_{I_n}, and $S_{E_nI_n}$.

In order to obtain the part of S_{I_D} due to E_n and I_n we should also know the value of the parallel $Z_P//Z_B//Z_T$. We could measure it by means of the circuit arrangement of Fig. 4(b), from which we obtain

$$Z_P//Z_B//Z_T = \frac{Z_{\text{in}}}{A/F_L' - 1} \tag{10}$$

where F_L' is the frequency response between V_S and V_O.

Moreover, in order to complete the measurement of I_D according to (1), we could evaluate the contribution S_{I_P} from the biasing network by performing a measurement of the equivalent input current noise S_I with a noiseless BUT or just without any BUT. In such a case $S_I = S_{I_D}$, and S_{I_P} can be finally obtained from (1) and (5)–(10).

Finally, by means of all these quantities and of (1), we get the total spectrum S_{I_D}.

B. Direct Measurement of S_{I_D}

It is possible to evaluate S_{I_D} also with a different and more straightforward technique.

As it is apparent from Eq. (1), the value of S_{I_D} depends on the impedance Z_B of the BUT. We can proceed, replacing the BUT itself with an equivalent impedance Z_N having a known noise behavior. Since in general we cannot have available an impedance equal to Z_B on the whole frequency range on which we perform our investigation, this substitution and the related measurements must be repeated for every single value ω_m of the angular frequency in which we are interested, in such a way that $Z_N(\omega_m) = Z_B(\omega_m)$. The impedance Z_N may, in the simplest case, be made up of a resistor in parallel with a proper reactance, both kept at a known constant temperature. Also the temperature of the reactance is important, because at high frequencies it gives a significant contribution, due to nonideality effects, to the total noise.

The procedure we now need to apply in order to determine S_{I_D} is exactly the same as the one examined in Section II for the measurement of S_I. With SW_1 in the upper position we measure the power spectrum of the amplifier output, and, by means of Eq. (4), we obtain, this time, $S_I' = S_{I_D} + S_{I_Z}$, where S_{I_Z} is the power spectrum of the Nyquist noise of the substitution impedance Z_N. We do not need to repeat the measurement for F since it remains the same as with the BUT, being $Z_N(\omega_m) = Z_B(\omega_m)$. According to the Nyquist theorem S_{I_Z} is simply given by

$$S_{I_Z} = 4kT\,\text{Re}\,\{Z_N^{-1}\} \tag{11}$$

where k is the Boltzmann constant and T the temperature at which Z_N is kept. We now subtract S_{I_Z} from S_I' and we get S_{I_D}.

C. Errors

The separate measurement of S_{E_n}, S_{I_n}, $S_{E_nI_n}$ and S_{I_P} would have the advantage of yielding a complete noise characterization of the measuring system. This characterization would be complete in the sense that, once obtained, it could be used to correct the measurement on every BUT, regardless of its impedance.

Unfortunately, such a procedure cannot in general be applied with satisfactory accuracy. S_{I_D}, as obtained with the technique described in Section III-A, is a function of many experimentally evaluated quantities, in fact we have:

$$S_{I_D} = S_{I_D}(S_{V_{O0}}, S_{V_{O\infty}}, S_{I_{DC}}, S_{I_{DR}}, S_{I_P}, \omega, C_L, R_L, \overline{Z_L}, A, F_L, F_L'). \tag{12}$$

The number of these parameters does not relevantly change also using the procedure presented in [5], [6] for the determination of S_{I_n}, S_{E_n}, and $S_{E_nI_n}$.

The relative error $\Delta S_{I_D}/S_{I_D}$ can be expressed as

$$\frac{\Delta S_{I_D}}{S_{I_D}} = \sum_{i=1}^{N} W_{S_{I_D}}^{x_i} \frac{\Delta x_i}{x_i} \tag{13}$$

where x_i is the ith of the $N = 12$ quantities on which S_{I_D} depends, Δx_i is the error on x_i and

$$W_{S_{I_D}}^{x_i} = \frac{x_i}{S_{I_D}} \frac{\partial S_{I_D}}{\partial x_i} \tag{14}$$

is the sensitivity of S_{I_D} with respect to x_i.

Since N is large and the relative error on some of the quantities is significant, good accuracy is not attainable. Sometimes there are also problems in performing some of the needed measurements. For example it is not always possible to measure $S_{V_{O\infty}}$, because of the amplifiers. For instance, some commercial transresistive amplifiers such as the Brookdeal 5002 do not work properly with an open-circuited input.

The direct measurement of S_{I_D}, instead, yields a value of S_{I_D} itself which depends on a smaller set of measured quantities:

$$S_{I_D} = S_{I_D}(S_{V_O}, \omega, C, Z_N, F, T). \tag{15}$$

All of them can be measured with good accuracy and for every amplifier.

This is the ultimate reason why we have selected the direct measurement of S_{I_D} in the application of all our measurements.

IV. Experimental Results

We have tested our method by performing three main kinds of measurements:

a) Measurement of the power spectrum due to a known adjustable noise generator;
b) Measurement of the Nyquist noise of bipoles made up of resistors and capacitors kept at a known temperature;
c) Measurement of shot noise in almost-ideal n^+-p junctions.

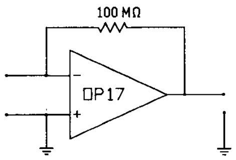


Fig. 5. Circuit diagram of the transresistive amplifier used in the measurements.

For all the measurements we have used the simple transresistive amplifier shown in Fig. 5. The operational amplifier is a PMI OP17, which has a very low equivalent input current noise. For the equivalent generator I_D defined by (1) (see Fig. 2(b)) we have measured, for impedances Z_L connected at the input having a resistive part with a value higher than a few MΩ and a reactive part of a few hundred pF, a power spectrum of about 200×10^{-30} A^2/Hz at frequencies below 1 kHz. This is mainly due to the 100 MΩ feedback resistor, which from the Nyquist theorem, gives a thermal noise of 165.7×10^{-30} A^2/Hz at room temperature. At higher frequencies we observe an increase of 20 dB per frequency decade in the equivalent input current noise. This can be explained by the effect of the shunt capacitances at the amplifier input, which make the contribution of the equivalent input noise voltage generator of the OP17 itself become prevalent with respect to the other noise sources.

We have used for the capacitor C_T appearing in (4) a value of 46.45 pF in the measurements described in Section IV-A and 34.60 pF in all the other ones.

A. Noise Produced by a Known Generator

A noise generator with a known power spectrum has been synthesized according to the circuit diagram shown in the inset of Fig. 6, where V_G is the same internal noise source of the signal analyzer which is also used for the frequency response measurement. The Norton equivalent of the part included in the dashed box, which represents our BUT, has a power spectrum shaped in such a way as to be approximately proportional to the one of S_{I_D} on the whole frequency range of interest.

In this particular situation we do not need to replace the BUT with an equivalent impedance; we simply turn off the noise source V_G and perform the correction measurement. If the temperature of the components is kept constant, the contribution due to the thermal noise of the resistive elements is the same both in the main and in the correction measurement. Therefore we do not have to subtract the term given by (11) from the power spectrum obtained from the correction measurement, moreover we do not add any error due to differences between the value of the BUT impedance Z_B and that of the substitution one Z_N.

These favorable conditions have allowed us to measure, within 15% accuracy, noise levels which are about one-tenth of the amplifier noise, as it is shown in Fig. 6, where the dots represent the experimental points of S_{I_B}, the dashed line indicates its theoretical value, and the solid one shows the power spectrum S_{I_D} of the amplifier noise.

With the same meaning of the symbols we have reported in Fig. 7 the results for a measurement on the same equivalent generator, but with V_G adjusted in such a way that the noise power spectrum of the equivalent BUT is about ten times larger than the one of the amplifier. The dot-dashed line indicates, for purposes of comparison, the results which would be obtained with a conventional procedure, without any correction, and which contain a relative error of about 10%.

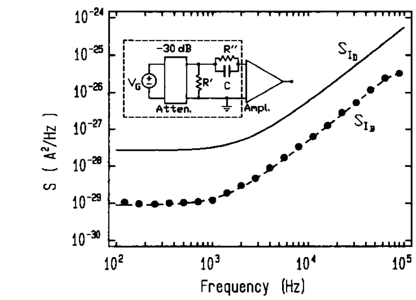


Fig. 6. Spectral density S_{I_B} of the noise current produced by the bipole represented in the inset, for V_G adjusted in such a way that the noise power spectrum of the bipole is about one-tenth of the one due to the amplifier ($R' = 50$ Ω, $R'' = 200$ MΩ, $C = 0.57$ pF; Atten.: attenuator). The plot represents the measured value of S_{I_B} (dots), its theoretical one (dashed line), and the spectrum S_{I_D} of the amplifier noise (solid line).

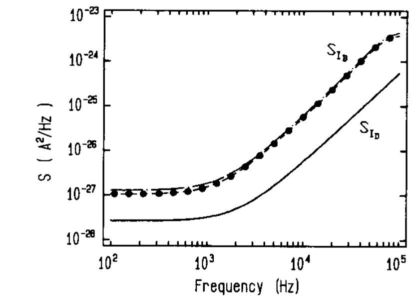


Fig. 7. Spectral density S_{I_B} of the noise current produced by the bipole represented in the inset of Fig. 6, for V_G adjusted in such a way that the noise power spectrum of the bipole is about ten times larger than the one due to the amplifier. The plot represents the value of S_{I_B} obtained with a conventional method (dot-dashed line), the results obtained with our technique (dots), the theoretical value (dashed line), and the spectrum S_{I_D} of the amplifier noise (solid line).

B. Thermal Noise of Resistors

We have measured the noise produced by a resistor cooled down in liquid nitrogen, so that the desired noise power spectrum, about one-fourth of the one due to the amplifier, can be obtained with not too large a resistance value. A bipole at room temperature has been used for the substitution impedance.

The resistor has been found to have a value of 78.125 MΩ at a temperature of 77 K, so that, from the Nyquist theorem, its theoretical noise power spectrum is of 54.53×10^{-30} A^2/Hz. The measured power spectrum S_{I_B} is shown in Fig. 8, together with the theoretical value and the power spectrum S_{I_D} of the amplifier noise. As it can be seen, there is good agreement between experimental points and theoretical value (dashed line), even near the upper end of the frequency interval, where the amplifier noise (solid line) significantly increases, and becomes eight times larger than the measured noise.

In order to perform a check of our method on a wider frequency range, we have synthesized a bipole yielding a power spectrum of the Nyquist noise with the same frequency behavior as the one produced by the amplifier, but with an intensity about six times smaller between 1 kHz and 100 kHz. In this case the bipole was kept at room temperature, while the substitution impedance was cooled down in liquid nitrogen. The results are shown in Fig. 9, where the inset contains the circuit diagram of the bipole; the dots indicate the experimental results, while the

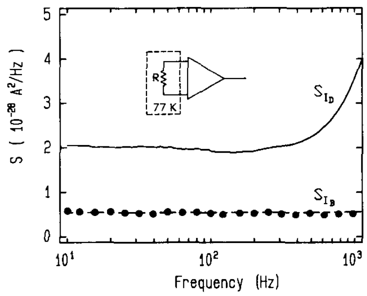


Fig. 8. Spectral density S_{I_B} of the Nyquist noise of a 78.125 MΩ resistor cooled down in liquid nitrogen. The plot represents the measured value of S_{I_B} (dots), its theoretical value (dashed line), and the spectrum S_{I_D} of the amplifier noise (solid line). The inset shows the circuit diagram of the BUT.

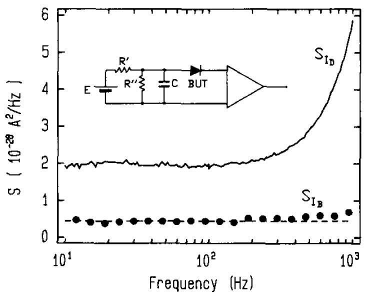


Fig. 10. Spectral density S_{I_B} of the shot noise of a n^+-p junction forward biased with a current of 136 pA. The plot represents the measured value of S_{I_B} (dots), the theoretical shot noise (dashed line), and the amplifier noise S_{I_D} (solid line). The inset shows the BUT and the relative biasing network. $E = 1.3$ V, $R' = 26$ kΩ, $R'' = 5$ kΩ, $C = 10$ μF.

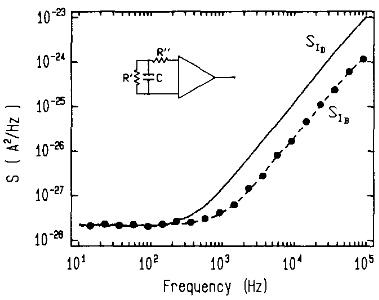


Fig. 9. Spectral density S_{I_B} of the Nyquist noise of the bipole shown in the inset ($R' = 78.125$ MΩ, $R'' = 3.27$ kΩ, $C = 280$ pF). The plot represents the measured value of S_{I_B} (dots), its theoretical value (dashed line), and the spectrum S_{I_D} of the amplifier noise (solid line).

dashed and solid lines represent the theoretical and the amplifier noise, respectively.

C. Shot Noise in n^+-p Junctions

The diodes we have tested are n^+-p junctions manufactured by SGS-Thomson Microelectronics with a particular technological process aimed at reducing the impurity concentration as much as possible, so that they show an almost ideal behavior both for I–V characteristics [9], [10] and noise.

The junction under test has been kept at a constant temperature of 0°C in a bath of melting ice [10], forward biased with currents ranging from 136 pA to 100 nA.

In Fig. 10 the dots represent the values of S_{I_B} obtained for the most critical case, the lowest value of the bias current, 136 pA. The setup used to perform the measurement is shown in the inset of the same figure. The theoretical shot noise (dashed line) has a spectral density $S_{I_B} = 2qI = 43.57 \times 10^{-30}$ A²/Hz, which corresponds, within a few percent, to the measured values. The increase which can be noticed at frequencies close to 1 kHz is not spurious, but it is due to the appearance, in the differential admittance of the diode, of a frequency-dependent dissipative part, which is revealed also by impedance measurements. In this case, at the frequency of 1 kHz, the amplifier noise S_{I_D} is about 12 times larger than the one S_{I_B} of the diode.

It is finally worth noticing that, to the best of our knowledge, the shot noise level of 6.6 fA/$\sqrt{\mathrm{Hz}}$ evaluated with our method is by far the smallest one ever directly measured in p-n junctions with comparable accuracy.

V. Conclusion

The experimental verification of our method has shown that it makes possible the measurement of current noise levels which are about one-fourth to one-tenth of the level of the amplifier noise, with a relative error smaller than 10%. With traditional techniques an accuracy of the same order would be attainable only for noise levels at least ten times larger than the one of the amplifier. Therefore, with respect to them, we obtain a 16–20-dB improvement in sensitivity.

A further appreciable increase in sensitivity might be reached by using the proposed method in conjunction with the cross-spectrum technique [3], [11], [12], which consists in amplifying the noise under test by means of two distinct amplifiers, and evaluating the cross spectrum of their outputs. This way the overall background noise of the measurement system is reduced, with respect to what would be obtained with a single amplifier, by the amount of the contributions which in the two-amplifier system are uncorrelated.

It is also possible to realize a complete automatization of the measurement procedure, by means of a computer-controlled impedance meter and an impedance synthesizer. This should allow not only a faster and more efficient application of the method, but also a refinement in the precision of the substitution procedure, so that further increases in sensitivity might be obtained.

Appendix

In this appendix we are going to show how the output noise of the amplifier can be evaluated from the knowledge of the power spectra of the input equivalent noise generators and the related cross-spectra. With reference to the circuit of Fig. 2, where we now consider $I_B = 0$, since the only pair of correlated generators is represented by E_n and I_n, the power spectrum S_V at the output is given by

$$S_V = S_{I_n^O} + S_{I_P^O} + S_{E_n^O} + 2\,\mathrm{Re}\,\{S_{E_n^O I_n^O}\} \qquad \text{(A1)}$$

where $S_{I_n^O}$, $S_{I_P^O}$, $S_{E_n^O}$ are the power spectra of the stochastic processes present at the output as a result of the amplification of I_n, I_P, E_n, respectively, and $S_{E_n^O I_n^O}$ is the cross-spectrum of the processes due to E_n and I_n.

This is an example of linear filtering of random processes; thus the power spectra at the output can be obtained multiplying those at the input by the square modulus of the related frequency response, and the expression for the cross-spectrum

$S_{E_n^o I_n^o}$ will read

$$S_{E_n^o I_n^o} = S_{E_n I_n} H_E(j\omega)\, H_I^*(j\omega) \tag{A2}$$

where $H_E(j\omega)$ and $H_I(j\omega)$ are the frequency responses for E_n and I_n, respectively. Therefore we obtain

$$\begin{aligned} S_V = {} & |A|^2 |Z_P//Z_B//Z_{\text{in}}|^2 \\ & \cdot \left\{ S_{I_n} + S_{I_P} + \frac{S_{E_n}|Z_{\text{in}}|^2}{|Z_P//Z_B//Z_{\text{in}}|^2 |Z_{\text{in}} + (Z_B//Z_P)|^2} \right\} \\ & + 2\,\text{Re}\left\{ S_{E_n I_n} \cdot \frac{Z_{\text{in}} A}{Z_{\text{in}} + (Z_P//Z_B)} \cdot (Z_P//Z_B//Z_{\text{in}})^* A^* \right\}. \end{aligned} \tag{A3}$$

With simple manipulations we get

$$\begin{aligned} S_V = {} & |A|^2 |Z_P//Z_B//Z_{\text{in}}|^2 \left\{ S_{I_n} + S_{I_P} + \frac{S_{E_n}}{|Z_P//Z_B|^2} \right. \\ & \left. + 2\,\text{Re}\left\{ S_{E_n I_n} \frac{1}{Z_P//Z_B} \right\} \right\} \end{aligned} \tag{A4}$$

which, dividing both sides by $|A|^2 |Z_P//Z_B//Z_{\text{in}}|^2$, substituting Z_L for $Z_B//Z_P$, and expanding the last term in the right hand side, yields (1).

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