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PHYSICS

for foreign students

Mechanics. Molecular physics and thermodynamics

Textbook

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УДК
БКК

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Посібник сприяє вивченню основ курсу фізики, поглиблення знань явищ та законів природи, які необхідні студентам для поглибленого вивчення фізики у закладах вищої освіти медико-біологічного профілю. Структура і зміст посібника відповідають програмі з фізики на підготовчих відділеннях для іноземних слухачів. Посібник містить необхідний об'єм лексики, конструкцій наукового стилю мови для адаптованого сприйняття студентами-іноземцями практичних занять та лекцій з курсу фізики англійською мовою.

CONTENTS

Preface

According to the structure and content, this textbook corresponds to the course of physics at the preparatory faculties for foreign citizens of higher educational institutions of Ukraine. Specification of goals and tasks of teaching physics is presented taking into account the specific features of the system of teaching foreign students.

The textbook provides the necessary amount of information that ensures mastery of basics of mechanics, molecular physics and thermodynamics by foreign students (part 1).

For better learning of foreign students, the textbook is adapted according to the program of studying English language for preparatory departments. Chapter 1 "Kinematics of the material point" is maximum adapted and is intended for the beginning of physics study after the completion of grammatical and phonetic courses of English language. All of topics in the textbook are constructed as follows. Text - basic concepts, lexical and grammar material - new words and terms; examples of constructs of scientific style of speech; exercises and tasks with a solution, questions and tasks for independent work. Illustrative material - drawings, graphs and general tables of basic physical quantities will also help in educational process.

In the textbook at the end of each topic a terminology table is presented.

SECTION I

MECHANICS

Chapter 1. Kinematics of the material point

§1. What is studying physics?

1. Physics is the science of nature

Science seeks to discover and describe the underlying order and simplicity in nature.

Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.

Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

The Sun, Earth, stars, water, air, chalk, molecule, atom, electron, photon are **objects of nature**.

Large size natural objects are macro objects. The Sun, Earth, Moon are **macroobjects**.

Natural objects of small size (10^{-10}-10^{-15} m) are **microobjects**. The molecule, atom, electron, atomic nucleus, proton, neutron, photon are microobjects.

Water, air, oxygen, chalk, aluminium are **substances**. Substances are **matter**.

Light is not a substance. But light is also matter. Light is a physical field. Substances and **field** are two kinds of matter.

2. Motion of matter

Matter changes all the time: Earth rotates, the satellite flies, the tree grows, sugar and salt dissolves in water. These are examples of changing matter. **Any changes in matter are called motion.**

There are such forms of motion of matter: physical, chemical, biological, social.

Mechanical motion, thermal motion, electromagnetic processes, atomic processes are physical forms of motion of matter.

Physics studies the physical properties of matter and the forms of its movement.

Words and phrases

science	macro object	shape
nature	micro object	mechanical motion
object	substance	thermo motion
molecule	physical field	process
atom	matter	light
electron	proton	sound
size	photon	atomic processes

Questions

1. Is the Sun an object of nature?
2. Is an atom an object of nature?
3. What are macro objects?
4. What are microobjects?
5. Is the Earth a macro-object?
6. The Moon, a satellite are macroobjects?
7. Is an electron a microobject?
8. Give examples of substances.
9. Give examples of physical fields.
10. What is called a motion?
11. What do you know about the form of matter?
12. What do you know about the physical forms of the motion of matter?
13. What does physics study?

Remember!
1. A molecule is a microobject
2. Any change of matter is called motion

§2. Mechanical motion.
Relativity of mechanical motion

1. Reference body

In physics, macro objects are called physical bodies or simply bodies. The Earth, a bus, a man, a table, a book there are the bodies.

The bodies are shaped and sized.

A Table, a book, The Earth have different shapes and sizes

Each body is among other bodies. For example, the book is on the desk or to the left of the notebook, to the right of the pen.

Physical bodies are in shape, size and are among other bodies. This means: physical bodies exist in space.

Example. The bus is to the left of the tree. We determined the position of the bus relative to the tree.

The position of this body in space is determined relative to other bodies.

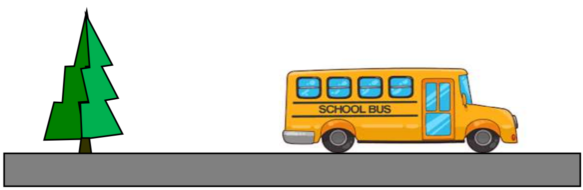
The book is on the table. In this example, the table is the body of reference.

The position of this body in space is determined relative to the reference body.

Example. The bus rides along the road (see Fig.).

The position of the bus in space changes relative to the tree. In this example, the reference body is the tree.

The position of this body in space varies with respect to the reference body, that is, this body moves relative to the reference body.



An object relative to which the position of the body in space is determined is called reference body.

2. Mechanical motion

The plane flies from Kiev to Berlin for 2 hours. The student goes from the hostel to the faculty for 5 minutes. These processes have one common property: they change over time.

Changing the position of the body in space over time is called mechanical motion.

How occurs mechanical motion?

Mechanical motion occurs in space over time.

3. Relativity of mechanical motion

Example. There is a bus. The bus is occupied by a passenger. If the reference body is a bus, then the passenger's position relative to the bus remains unchanged. The passenger does not move in relation to the bus, ie the passenger is in a state of rest on the bus.

The passenger is at rest in relation to the bus, but he moves along with the bus relative to the Earth.

Mechanical motion and state of rest (stationary state) are relative.

Words and phrases

physical body	position
size	change of position
space	relativity
counting out	state of rest
reference body	mechanical motion

Questions

1. Is the Earth a physical body?
2. Are a Satellite, a rocket physical bodies?
3. What does it mean: the bodies in space?

4. How to determine the position of a body in space?
5. What is called mechanical motion?
6. How occures mechanical motion?

Remember!
1. **The position of the body in space is determined relative to other bodies.**
2. **The man is moving towards the bus.**

§3. Material point. Trajectory

1. Material point

Physical bodies have the shape and size. Often when studying the mechanical motion of the body do not take into account its shape and size.

Example. Consider the motion of the Earth around the Sun. The distance between the Earth and the Sun is almost 25,000 times greater than the radius of the Earth. Therefore, the Earth and the Sun in this task can be considered as **material points**.

The material point is the modelofphysical body, whose dimensions can be neglected in the conditions of this problem.

Example. The train moves relative to the tree (the tree is located near the train). In this task you need to take into account the size of the train. In this problem, the train is not a material point.

In one task, the body can be a material point, in another task it is not a material point.

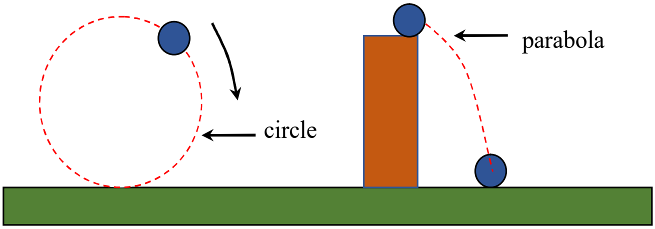
2. Trajectory

The conditional line along which the material point moves is called a trajectory.

The motion of the material point along the straight line is called rectilinear motion.

The motion of the material point along the curved line is called the curvilinear motion.

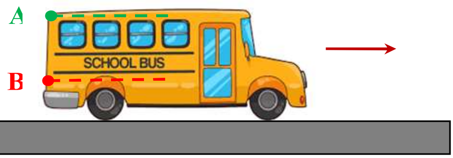
Circle movement, motion along a parabola are examples of curvilinear motion (see Fig.).

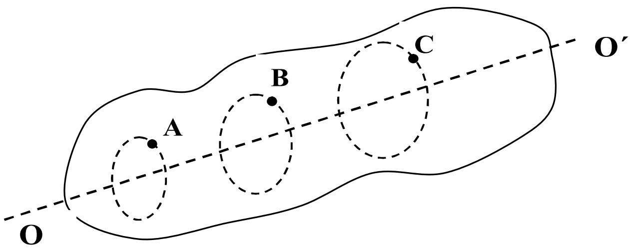


3. Types of mechanical motion

If body sizes in the task are to be taken into account, then this body is considered as a set of points.

Consider the movement of a set of two points of the bus: points A and points B (see Fig.). Trajectories of points A and B are the same. If all the points of the body describe the same trajectory, then the motion is called **translational**.





Example. In the drawing, points A, B, C move in circles.

Centers of all circles are on one straight line OO’. This straight line is called the **axis of rotation**. Movement of the body in this case is called **rotational motion**.

The motion in which the trajectory of all points of the body is circles with a center on one line is called a rotational motion.

Words and phrases

material point	rectilinear motion
to take into account	curvilinear motion
not to take into account	circle
line	parabola
trajectory	to describe
straight line	axis of rotation
curve	rotational motion

Questions

1. What is called a material point?
2. Can the Moon be considered a material point:
a) relative to a rocket that starts from Earth;
b) relative to the missile that landed on the Moon?
3. What is called a trajectory?
4. What motion is called rectilinear; curvilinear?
5. Give examples of curvilinear motion.
6. What is called a path?
7. What kind of body motion is called translational, rotational?
8. Name the bodies that move translational.
9. Give examples of the rotational motion of the bodies.

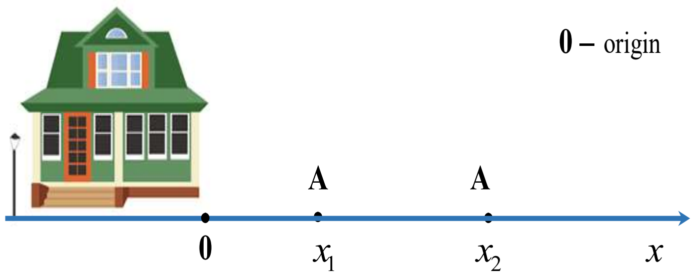
Remember!
The body is considered as a material point

§4. The main task of kinematics. Reference frame

1. The main task of kinematics

The main task of kinematics is to determine the position of the body in space at any given time.

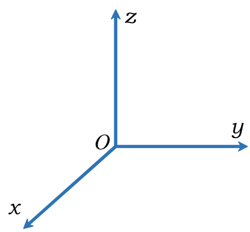
Let's consider the rectilinear motion of the point A relative to the house.



The position of a point on a straight line is determined by ***the coordinate x***. To find the point coordinate is meaninng to measure the distance from the origin to point A. This distance is called ***the coordinate***.

The OX line is called the coordinate axis. The position of each point on the line is determined by one coordinate, on the plane - by two, and in space - by three coordinates.

Axis coordinates ***OX, OY, OZ*** form ***a coordinate system***. The positions of an arbitrary point in space are determined using a rectangular coordinate system (Cartesian coordinate system) (see Fig.).



2. Reference frame

In order to determine the position of the body relative to the body of reference, it is necessary to associate with it the coordinate system. The coordinates of any point of the moving body in space will change over time. To characterize the movement, one must know which moment of time corresponds to one or another coordinate.

To determine the coordinates of a point at a given time - this means solving ***the main task of kinematics.*** To do this, you need to measure the time. Time is measured by periodic processes.

Periodic processes are processes that are repeated at regular intervals (periods of time).

The motion of the Earth relative to the Sun, the motion of the Earth relative to its axis - examples of periodic processes. A year, a day, an hour, minute (min), second (s) - these are units of time.

The main unit of time in the SI is a ***second (s)***. Different time units are related to each other as follows:

1 year=365 (366) days;

1 day =24 hours=1440 minutes=86400 seconds

1 hour =60 minutes=3600 seconds.

1s = 1 day/86400

The reference body, the coordinate system associated with it and the time measuring device are called the reference frame.

Words and phrases

distance	interval of time
coordinate	period
axis	days
coordinate system	minute
reference frame	plane

Questions

1. What is the main task of kinematics?
2. What determines the position of the point on the straight line?
3. What forms the coordinate system?
4. What does it mean to solve the main task of mechanics?
5. How is the time measured?
6. What are the units of time?
7. What unit of time is the main one?
8. What is called the reference frame?

Remember!
1. The position of the point is determined by the coordinate
2. The coordinate axes form a coordinate system

§ 5. Physical quantities

1. Physical phenomena

Subjects and events that study physics are called physical bodies and physical phenomena, respectively. **Physical phenomenon** is a set of events (changes) in nature, which occur for the same reasons or have common (common to all) signs. For example, flying an airplane, running a person, driving a bus, moving planets, and other similar events have a common feature. These bodies change their position in space. All of these examples are **mechanical phenomena**. Boiling water, melting of ice, evaporation of water, heat release during combustion of fuel are examples of **thermal phenomena**. There are other types of physical phenomena: **electromagnetic**, **optical** and **nuclear**, etc.

2. Physical quantities

Physical objects or phenomena have certain physical properties: size, density, color, aggregate state, etc.

Physical property is what other body or phenomenon is different from another body or phenomenon. For example, the physical property of any body is its linear dimensions.

Physical phenomena and physical properties of bodies are characterized by physical quantities. **The physical quantity** is a quantitative characteristic (measure) of a certain physical property of a body or phenomenon. Length, area, volume, time, temperature, velocity, force are the characteristics of physical phenomena and body properties or **physical quantities**.

For example, **length** (L) is a physical quantity that is a measure of linear body sizes; **square** (S) is a physical quantity that is a measure of the surface of the body; **volume** (V) is the measure of the space that this body takes.

A characteristic of a phenomenon or body property that can be measured is called a physical quantity.

What does it mean to find or measure a physical quantity? Measuring the physical quantity means comparing it with the unit of measurement of this quantity.

The unit of measurement is a fixed (unit) value of physical quantity. For example, for a unit of length taken 1 meter (m), for a unit of mass - 1 kilogram (kg), time - 1 second (c). The average distance from Earth to the Sun is 150 million km. This length is called the astronomical unit.

Words and phrases

phenomenon of nature	heat
test	density
physical quantity	color
property	measure
boiling	module
melting	vector quantity
combustion	scalar quantity
evaporation	aggregate state

Questions

1. What is called a physical phenomenon?
2. Give examples of physical phenomena.
3. What is called a physical property?
4. Give examples of physical properties of bodies.
5. What is a physical quantity?
6. What does it mean to measure a physical quantity?

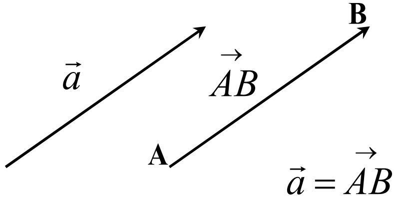
Remember!
1. Physical phenomena and properties of material objects are studied by using of observations and experiments
2. Physical quantities are divided into vector and scalar

§6. Vector and scalar quantities. Actions on vectors

1. Scalars and vectors

Physical quantities are divided into **vector and scalar quantities**. The quantities for which the direction does not matter is called **scalar quantities**, or **scalars**. Scalar quantities include length, width, area, volume, mass, density, temperature, and so on. Scalar quantities have only a numerical value.

Quantities for which not only numerical values are characteristic, but also direction, are called vector quantities, or vectors. The length of the vector is expressed by its module. Vectors are depicted in the form of directed cuts (arrows). The lengths of these segments are proportional to its numerical value - the modules. The module of the vector, or the numerical value, is the scalar value. The module of the vector $\vec{a}$ is denoted $|\vec{a}|$ either a. Vectors are equal if they have equal modules and the same direction in space (see Fig.).

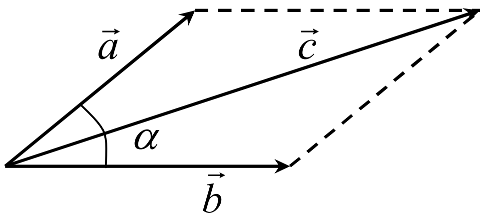


The vector values include: **speed** ($\vec{V}$), **acceleration** ($\vec{a}$), **force** ($\vec{F}$), **momentum** ($\vec{p} = m\vec{V}$), etc.

Actions on vectors: adding, subtracting, decomposing a vector into components, etc.

2. Adding of vectors

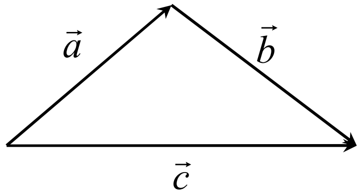
Scalar quantities are added algebraically. Vector values are added geometrically. The addition operation is written as follows: $\vec{c} = \vec{a} + \vec{\mathrm{b}}$, where $\vec{a}$ and $\vec{b}$ – initial vectors; $\vec{c}$ is resulting vector. Two vectors is adding according to the rule of **parallelogram**. The resulting vector coincides with the diagonal of the parallelogram (see Fig.).



The module of the resulting vector is determined by the formula: $c = \sqrt{a^2 + b^2 + 2ab\cos\alpha}$. The adding operation of n vectors is written as follows:

$$\vec{a}_1 + \vec{a}_2 + \ldots + \vec{a}_n = \sum_{i=1}^{n} \vec{a}_i \, .$$

Vectors $\vec{a}$ and $\vec{b}$ can be added also according to the rule of the triangle (see Fig.).

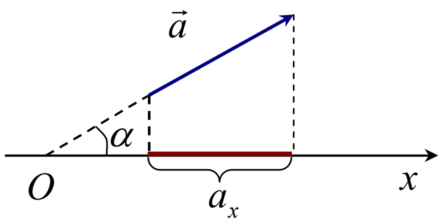


3. Scalar product of vectors

The scalar product of two vectors $\vec{a}$ and $\vec{b}$ is called an expression $(\vec{a}\cdot\vec{b}) = |\vec{a}|\cdot|\vec{b}|\cdot cos\alpha$, where α – the angle between the vectors $\vec{a}$ and $\vec{b}$.

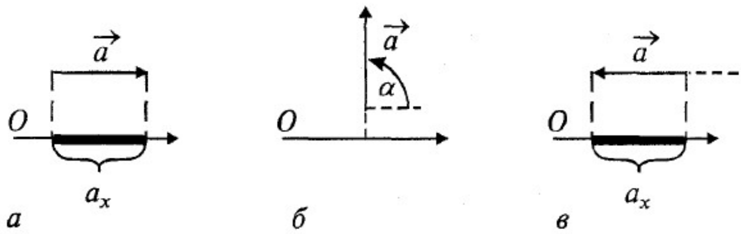
4. Projection of the vector on the axis of coordinates

The projection of the vector $\vec{a}$ on the axis Ox is called the difference in the coordinates of the end and beginning of this vector: $a_x = x - x_0$. **The projection of the vector** $\vec{a}$ on the axis Ox is determined by the formula: $a_x = |\vec{a}|\cdot cos\alpha$, where $|\vec{a}|$ is vector module $\vec{a}$, α is the angle between the direction of the axis Ox and the vector $\vec{a}$.



The projection of the vector $\vec{a}$ on the axis can be positive or negative.

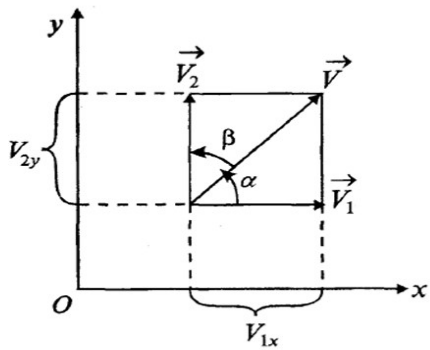
Examples



a) $\alpha = 0^\circ$; $a_x = a$. b) $\alpha = 90^\circ$; $a_x = 0$. c) $\alpha = 180^\circ$; $a_x = -a$.

5. Decomposition of a vector into components

The vector can be decomposed into components along the perpendicular directions:



$\vec{V}_1$, $\vec{V}_2$ – components of vector $\vec{V}$; $\vec{V} = \vec{V}_1 + \vec{V}_2$

$\vec{V}_{1x} = |\vec{V}| \cdot cos\alpha$; $\vec{V}_{2y} = |\vec{V}| \cdot cos\beta = |\vec{V}| \cdot \sin\alpha$

$V = \sqrt{V_{1x}^2 + V_{2y}^2}$

Words and phrases

addition	parallelogram
subtruction	diagonal
direction	decomposition
segment	sum
projection of vector	component of vector
arrow	angle

Questions

1. What physical quantities are called scalar, vector?
2. Give examples of scalar and vector quantities.
3. How do vectors add?
4. How can I decompose a vector?
5. What is a vector projection?
6. What is the scalar product of the vectors?

Remember!
1. The length of the vector is expressed by its module $V_x = V_{0x} + a_x t$ **2. The resulting vector coincides with the diagonal of the parallelogram**

§7. Uniform rectilinear motion

1. Uniform motion

Uniform motion distinguish among various mechanical movements.

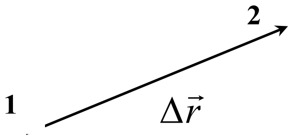
A motion in which the body at the same time intervals passes the same distance is called a **uniform motion.**

A motion with a constant in modulus and direction of speed is called a uniform rectilinear motion.

A motion in which the body at the same time intervals passes unequal distances is called a **non-uniform motion.**

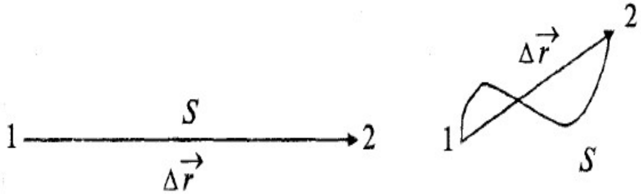
2. Displacement. Velosity and speed of uniform motion

A directed line segment that connects the initial and final position of the moving body is called a **displacement**.



$\Delta\vec{r}$ – displacement.

If the trajectory of the point's motion is a straight line and the direction of motion does not change, the module of the displacement vector is equal to the path. If the motion of the point is curvilinear, then the module of the displacement vector is less than the path.



$|\Delta\vec{r}| = S$ $\qquad\qquad$ $|\Delta\vec{r}| \neq S$

With a uniform rectilinear motion at any time intervals $\Delta t_1 = \Delta t_2 = ... = \Delta t_n$, the point makes the same displacement $\Delta\vec{r}_1 = \Delta\vec{r}_2 = ... = \Delta\vec{r}_n$.

For uniform rectilinear motion this means: $\dfrac{\Delta\vec{r}_1}{\Delta t_1} = \dfrac{\Delta\vec{r}_2}{\Delta t_2} = ... = \dfrac{\Delta\vec{r}_n}{\Delta t_n} = const$

This relation is called the velosity of uniform motion (it denotes by letter V):

$$\vec{V} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r} - \vec{r}_0}{\Delta t} \qquad Velocity = \frac{displacement}{time}$$

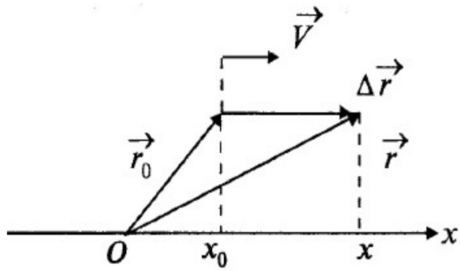
If $t_0 = 0; \Delta t = t - t_0 = t$, then $\vec{V} = \dfrac{\Delta \vec{r}}{t}$.

The physical vector quantity, which is the ratio of the displacement to the time for which this motion occurred is called the velocity of a uniform rectilinear motion.

The scalar absolute value (magnitude) of velocity is called "speed", being a coherent derived unit whose quantity is measured in the SI as metres per second (*m/s*).

Let's consider the rectilinear motion of the body along the *Ox* axis with velocity $\vec{V}$.

$\vec{r}_0$, $\vec{r}$ – radius vectors.



The vector $\Delta\vec{r} = \vec{r} - \vec{r}_0$ is called the displacement for the period of time Δt.

The projection of the displasement on *Ox* axis: $r_{ox} = x - x_0$, the projection of the velocity on *Ox* axis is equal to: $V_x = \dfrac{r_x - r_{x0}}{t} = \dfrac{x - x_0}{t}$, where

$$x = x_0 + V_x \cdot t$$

This formula is called **the equation of the coordinate of a uniform rectilinear motion**. It expresses the dependence of the coordinates on time. For a time interval $\Delta t = t$ ($t_0 = 0$) the point is moved at $S = |\Delta\vec{r}| = x - x_0 = V_x t$. Since $V_x = V$ then $S = V \cdot t$. This is the path equation. If $S_0 \neq 0$, then

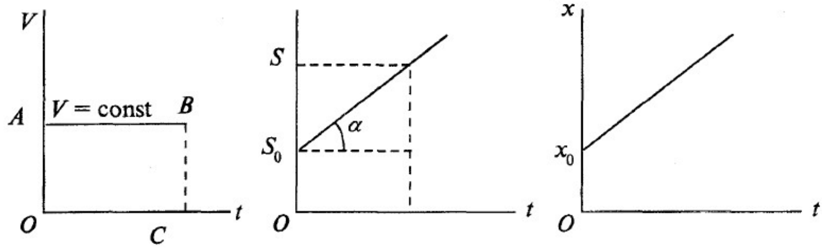
$$S = S_0 + Vt,$$

where S_0 – the initial path.

3. Graphic representation of motion

The coordinate of the material point, which moves uniformly and rectilinearly, is a linear function of time.

With a uniform motion, the velocity is constant, so the speed graph represents a direct line, parallel axis of time.



Speed graph   Path graph   Coordinate graph

The figure shows graphs of the coordinate, the path and speed of rectilinear motion. Using the speed graph, you can determine the path of the body as a product of speed and time. The speed is equal to the tangent of the angle α: $V = \dfrac{S - S_0}{t} = tg\alpha$.

On the graph of time dependence, the path S is calculated as the area of the quadrilateral *OABC*.

Problem. From two points *A* and *B* at the same time the movement of two bodies in one direction began. The distance between the points *A* and *B* is 90 *m*. The body moving from point *A* has a speed of 5 *m/s*, and the body moving from point *B* has a speed of 2 *m/s*. After what time the first body will overtake the second?

Given:

$x_{01} = 0$

$x_{02} = 90\ m$

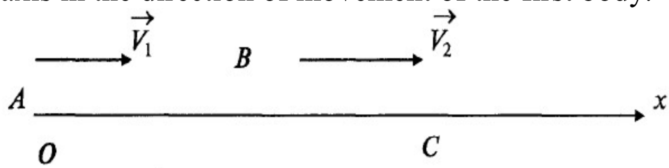
V_1=5*m/s*

V_2=2*m/s*

Find t–?

Solution

1. Let's place the beginning of the axis *Ox* in point *A* and direct the axis in the direction of movement of the first body.



2. Let's record the equation of motion of bodies: $x_1 = V_1 t, x_{01} = 0$; $x_2 = x_{02} + V_2 t, x_{02} = 90$, where x_1 and x_2 coordinates of the first and second bodies, respectively.

3. At point *C*, the first body overtakes the second one. This means that $x_1 = x_2$, that is to say $V_1 t = x_{02} + V_2 t$.

4. Let's find the time before the meeting of the bodies:

$$t = \frac{x_{02}}{V_1 - V_2} = \frac{90}{5-2} = 30\ s.$$

Words and phrases

uniform motion	equation
nonununiform motion	dependence
displacement	linear function
path	graph
velocity	area

Questions

1. What is called a uniform motion?
2. What is called a nonuniform motion?
3. What is called a displacement?
4. What is called a velocity of a uniform motion?
5. In which units is speed measured in the SI?
6. Write the equation of uniform rectylinear motion in vector and scalar form.
7. What kind of graphs have coordinates, paths and speeds of a point moving uniformly and rectilinear?

§8. Relativity of motion. The law of addition of velocities

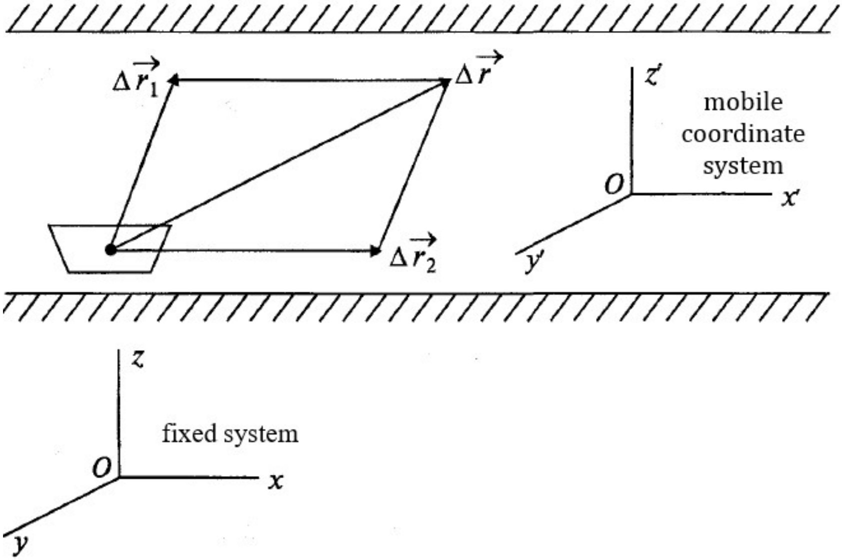
1. The law of addition of velocities

Consider the motion of a body (boat) in different reference systems. Denote: xyz – fixed coordinate system; $x'y'z'$ – mobile coordinate system; $\vec{r}_1$ – displacement of the body relative to water; $\vec{r}_2$ – displacement of the water relative to the shore; Δt – time interval (see fig.).

The displacement $\Delta\vec{r}$ of the body relative to the shore is a vector sum of two displacements, namely: $\Delta\vec{r}_1 + \Delta\vec{r}_2$.

Let's this equality divide into Δt:

$\dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\Delta\vec{r}_1}{\Delta t} + \dfrac{\Delta\vec{r}_2}{\Delta t}$, where $\vec{V} = \dfrac{\Delta\vec{r}}{\Delta t}$ – body velocity relative to a fixed coordinate system, $\vec{V}' = \dfrac{\Delta\vec{r}_1}{\Delta t}$ – body velocity relative to the mobile coordinate system, $U = \dfrac{\Delta\vec{r}_2}{\Delta t}$ – water velocity relative to the shore (the velocity of the mobile coordinate system relative to the fixed one)



We will obtain:

$\vec{V} = \vec{V}' + \vec{U}$

*This equation expresses the classical **law of addition of velocities**:*

The velocity $\vec{V}$ of a body in a fixed frame of reference is equal to the vector sum of the velocity $\vec{V}'$ of a body in a mobile reference frame and the velocity $\vec{U}$ of a mobile reference frame relative to fixed one.

Mechanical motion is relative. Body coordinates, the displacement and the velocity relative to different reference systems are different: the trajectory of motion, the path, the displacement and the velocity depend on the choice of the reference frame.

Problem

A steamboat with a speed of 18 *km/h* in relation to water came out from point *A* against stream. Water speed is 2 *m/s*. What distance between point *A* and the steamboat will be after 2 hours?

Given:
V =18 *km/h* = 5 *m/s*
U = 2 *m/s*
$t = 2\ h = 7{,}2\cdot10^3\ s$
Find: x –?

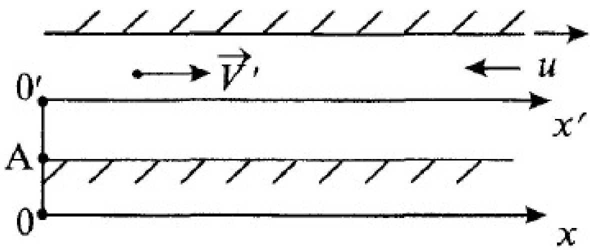
Solution

1. Let Ox is a fixed axis, Ox' – mobile one. At time moment t_0=0, $x = x'_0 = 0$.

2. In a mobile reference frame: $x'_0 = V'_x t$.

3. In fixed one: $x = x'_0 + U_x t = V'_x t + U_x t$ or $x = (V' - U)t$;

x=(5-2)·7,2·10^3≈ 22 *km*.



Words and phrases

mobile	flow
fixed	down stream
shore	against stream
steamboat	law

Questions

1. Which expression the law of addition of velocities is called?

2. Formulate the law of addition of velocities.

3. Is mechanical motion relative?

4. From what the trajectory of the body, the path, the displacement, the speed depend upon?

Remember!
1. Water velocity relative to the shore
2. Displacement the boat relative to the shore

Problems

1. The boat moves against stream of river at a speed of 12 *km/h* relative to shore, and downstream – at a speed of 16 *km/h*. How big is the river stream velocity?

2. The ship passes the distance between two points *A* and *B* along the river for 3 hours, and the raft passes the same distance in 12 hours. How many time will the ship spent to return?

3. The boat moves at an angle of 120º to the direction of the river. The speed of the boat relative to the water is 1 *m/s*, the flow speed has the same value. Find the boat speed module relative to the shore.

4. The boat crosses the river in the width of 0.5 *km*. The velocity of the boat relative to the water is directed perpendicular to the shore, its module is 4 *m/s*. At what distance the river stream will carry the boat for 1 hour of the crossing, if the stream speed is 1 *m/s*? What distance will the boat passed?

5. The athlete's speed is 9 *km/h*, and the wind speed is 6 *km/h*. Determine the athlete's speed relative to the air if: a) the wind is opposite; b) the wind perpendicular to the direction of the run.

§9. Nonuniform rectilinear motion

1. Average speed

Motion, in which the body has different displacement for the equal time intervals, is called nonuniform or variable motion.

The variable motion is characterized by average and instantaneous velocity.

The average velocity (vector $\vec{V}_{av}$) and the average speed (scalar V_{av}) are distinguished

$$V_{av} = \frac{\Delta S}{\Delta t}$$

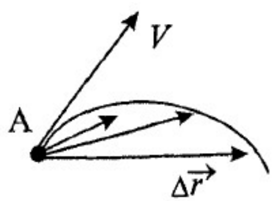
Let the body in successive intervals of time Δt_n passes respectively the segments of the path ΔS_n. For all time $t = \sum_n \Delta t_n$ the body will go all the path $S = \sum_n \Delta S_n$.

Then the average speed $V_{av} = \dfrac{\sum_n \Delta S_n}{\sum_n \Delta t_n}$.

2. Instantaneous speed

During nonuniform motion, body speed changes over time. If, during time Δt, the displacement of point A is equal to $\Delta \vec{r}$, then the ratio $\dfrac{\Delta \vec{r}}{\Delta t}$ determines the average speed during this time.

If we reduce the time interval Δt, the direction of the displacement vector $\Delta \vec{r}$ approaches the tangent at the point A of the trajectory of motion, through which the body passes at the instant of time.



Therefore, the velocity vector lies in the tangent to the trajectory of the motion of the body at the point A.

The speed of the body at a given moment in a given point of the trajectory is called instantaneous velocity.

The instantaneous velocity is determined using the formula:

$$\vec{V} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t}$$

Instantaneous velocity equals the boundary of the displacement ratio to the time interval, if this time period is infinitely small.

3. Uniformly accelerated motion. Acceleration

A motion, during which, the speed of the body for any the same time interval varies by the same magnitude is called uniformly accelerated rectilinear motion.

A vector quantity equal to the ratio of the velocity vector change $\Delta \vec{V}$ ***to the interval of time*** Δt ***during which this change occurred is called the acceleration*** $\vec{a}$ ***of a body:***

$\vec{a} = \dfrac{\vec{V} - \vec{V}_0}{t - t_0} = \dfrac{\Delta \vec{V}}{\Delta t}$, where $\vec{V}_0$ – initial velocity, $\vec{V}$ – final velocity.

Acceleration characterizes the change in speed per time unit. A unit of acceleration is meter per second in a square (m/s^2).

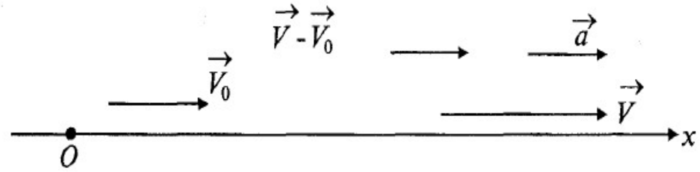
If you count the time from the moment $t_0 = 0$, then $\vec{a} = \dfrac{\Delta \vec{V}}{t}$.

In accordance with this formula, the value of the velocity vector at the time: $\vec{V} = \vec{V}_0 + \vec{a}t$

In projections on the axis Ox: $a_x = \dfrac{V_x - V_{0x}}{t}$.

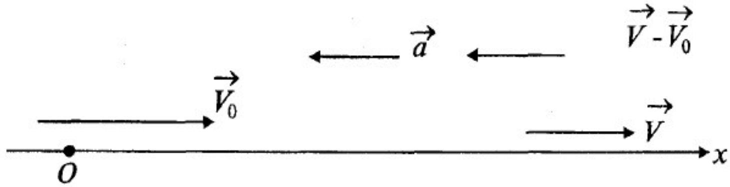
In Figure a) examples of uniformly accelerated motion in the positive direction of the axis Ox are given.

а)



In Figure b) examples of uniformly decelerated motion in the positive direction of the axis *Ox* are given.

б)



For the following examples of nonuniform motion, the following formulas can be written:

а) $\vec{a} = \dfrac{\Delta\vec{V}}{t}$; $a_x = \dfrac{V_x - V_{0x}}{t}$, $V_x > V_{0x}, \Delta V > 0$, $a_x > 0$ – uniformly accelerated motion;

б) $\vec{a} = \dfrac{\Delta\vec{V}}{t}$; $a_x = \dfrac{V_x - V_{0x}}{t}$, $V_x < V_{0x}$, $\Delta V < 0$, $a_x < 0$ – uniformly decelerated motion;

For accelerated motion, the direction of velocity coincides with the acceleration vector, and for decelerated motion, the velocity and acceleration vectors have opposite directions.

Problem

The first half of the way the body was moving at a speed of 60 *km/h*, and the second half of the path - at a speed of 40 *km/h*. Find the average speed of the body.

Given:

$V_1 = 60$ *km/h*

$V_2 = 40$ *km/h*

Find V_{av} – ?

Solution:

1. Average speed along the way is equal: $V_{av} = \frac{S}{t}$,

where $S = S_1 + S_2$, a $t = t_1 + t_2$. Under condition $S_1 = S_2 = \frac{S}{2}$.

Then $t_1 = \frac{S}{2V_1}$, and $t_2 = \frac{S}{2V_2}$.

2. Let's find average speed:

$$V_{av} = \frac{S}{\frac{S}{2V_1} + \frac{S}{2V_2}} = \frac{2V_1V_2}{V_1 + V_2}, \quad V_{av} = \frac{2 \cdot 60 \cdot 40}{60 + 40} = 48 \ \kappa m/h$$

Answer: V_{av} = 48 *κm/h*.

Words and phrases

ununiform motion	instantaneous velocity
average velosity	tangent line
instantaneous motion	positive direction
acceleration	negative direction
relation	increasing module
projection of velocity	decreasing module

Questions

1. What is called nonuniform motion?
2. What is called the average speed?
3. What is called instantaneous speed?
4. What motion is called uniformly accelerated motion?
5. What is called acceleration?
6. What is the acceleration unit in the *SI*?
7. Write a set of formulas for the speed, the path and the coordinates of the uniformly accelerated motion.
8. Draw a graph of the projection of velocity on the *Ox* axis for the body uniformly decelerated (the speed module decreases).

9. Draw a graph of the change of coordinates with time (the speed increases).

Remember!
Average speed is entered to characterize the nonuniform motion

Problems

1. The car drove a distance from *A* to *B* at a speed of 60 *km/h*, and returned with a speed of 20 *km/h*. What is the average speed of a car?

2. The cyclist rode from one city to another. Half way he traveled at a speed of 12 *km/h*. Then half of the rest time of motion drove at a speed of 6 *km/h*, and then walked at the end of the path at a speed of 4 *km/h*. Determine the average speed of the cyclist's journey.

3. For the first second of uniformly accelerated motion without initial velocity the body passed 5 *m*. What distance the body passed for the first 3 *s* of motion? For the first 10 *s*?

4. The train begins to move from resting and moves uniformly accelerated. On the first kilometer of the way, its speed increased to 10 *m/s*. How much does it grow on the second kilometer?

5. The body moves uniformly accelerated. During the fourth second the body passed 35 *m*. What was acceleration of the body?

6. A car is traveling at 67 *mph* (30 *m/s*) when the driver steps on the brake slowing the car down at a rate of 3 m/s^2. How long and what distance will be required for the car to stop on dry pavement?

§10. The equation of velocity and path of uniformly accelerated rectilinear motion

1. The equation of velocity

For uniformly accelerated motion with initial velocity $\vec{V}_0$ acceleration $\vec{a} = \dfrac{\vec{V} - \vec{V}_0}{t}$. Hence the velocity $\vec{V} = \vec{V}_0 + \vec{a}t$.

In projection on the axis Ox: $V_x = V_{0x} + a_x t$.

1. The velocity of the point increases ($\vec{V} > \vec{V}_0$) and the direction of acceleration is coincides with the direction of velocity. In this case $V_{0x} = V_0$, $V_x = V$, $a_x = a$ and then

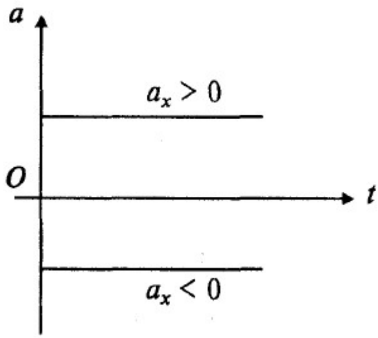
$$V = V_0 + at.$$

2. The speed of the point decreases ($\vec{V} < \vec{V}_0$) and the direction of acceleration is opposite to velocity direction. In this case $V_{0x} = V_0$, $V_x = V$, $a_x = -a$ and then

$$V = V_0 - at.$$

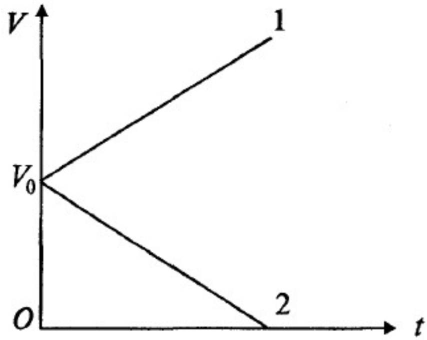
3. Graphs of acceleration projection ($a_x = a$).

The dependence of the projection of acceleration on time is a straight line parallel to the axis of time.



4. Graphs of velocity projection ($V_x = V$).

Chart 1 shows the dependence of speed on time $V = V_0 + at$ (speed increases), chart 2 shows the dependence of speed on time $V = V_0 - at$ (speed decreases).



2. The equation of path of uniformly accelerated rectilinear motion

For a uniformly accelerated motion, the dependence of the velocity projection on time is linear. If $V_x = V$, then $V = V_0 + at$.

It is known that the average value of a linear function is equal to the arithmetic mean, that is $V_{av} = \dfrac{V_0 + V}{2}$.

The path of the body for uniformly accelerated motion can be found by the formula $S = V_{av} \cdot t$. Let's substitute in this formula the expression for average speed. We will get:

$S = \dfrac{V_0 + V}{2} t$. We replace V by $V = V_0 + at$:

$$S = \frac{V_0 + V_0 + at}{2} t = V_0 t + \frac{at^2}{2}$$

$$S = V_0 t + \frac{at^2}{2}.$$

This is the equation of the path of uniformly accelerated motion.

If $\vec{V} < \vec{V}_0$, the path equation has the form $S = V_0 t - \frac{at^2}{2}$.

In the case $V_0 = 0$ we have $S = \frac{at^2}{2}$.

The equation of the coordinate of a point with an uniformly accelerated motion has the form

$$x = x_0 + V_{0x} t + \frac{a_x t^2}{2},$$

Where x_0 – initial coordinate; V_{0x} – projection of initial velocity; a_x – projection of acceleration.

Let`s find an expression that binds the path and velocity of the uniformly accelerated motion. Average speed is $V_{av} = \frac{V_0 + V}{2}$.

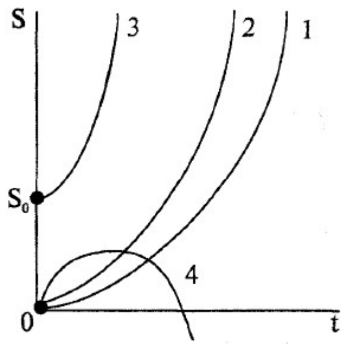
From $V = V_0 + at$ we will find *t*. We will substitute V_{av} and t in formula for the path of uniformly accelerated motion:

$$t = \frac{V - V_0}{a},\ S = V_{av} t = \frac{V_0 + V}{2} \cdot \frac{V - V_0}{a} = \frac{V^2 - V_0^2}{2a}.$$

From here $V^2 = V_0^2 + 2aS$.

3. Graphs of the path of uniformly accelerated motion

The graph of the dependence of the path on time for uniformly accelerated motion is a parabola (see Fig.):



$$1 - S = \frac{at^2}{2};\ 2 - S = V_0 t + \frac{at^2}{2};\ 3 - S = S_0 + V_{0x} t + \frac{a_x t^2}{2};$$

$$4 - S = V_0 t - \frac{at^2}{2}.$$

Problem 1.

The body motion equation has the form $x = 15t + 0{,}4t^2$. Find the acceleration of the body. Determine initial speed and speed after 5 seconds of motion.

Given:

$\underline{t = 5\ s}$

Find: $a - ?\ V_0 - ? V - ?$

Solution

1. Compare this equation of motion of a body with the equation of motion in the general form: $x = x_0 + V_{0x} t + \frac{a_x t^2}{2}$ and $x = 15t + 0{,}4t^2$.

Obviously, $x_0 = 0$, $V = 15\,m/s$, $\frac{a}{2} = 0{,}4\ m/s^2$, hence $a = 0{,}8\,m/s^2$.

2. The speed of the body in 5 *s* can be found using the formula $V = V_0 + at$. $V = 15 + 0{,}8 \cdot 5 = 19\,m/s$.

Answer: $V = 19\,m/s$.

Problem 2.

The body moves uniformly accelerate. For a fifth second it passes the path of 45 *m*. Find the acceleration of the body and the path for a tenth second.

Given:

$\Delta S_5 = 45\ m$

Find: $a -\ ?\ \Delta S_{10} - ?$

Solution

1. The path for 5-th second equal to $\Delta S_5 = S_5 - S_4$,

where $S_5 = \dfrac{at_5^{\,2}}{2}$, $S_4 = \dfrac{at_4^{\,2}}{2}$.

$$45 = \frac{a}{2}(t_5^2 - t_4^2) = \frac{a}{2}(25-16)$$

$$a = \frac{2\cdot 45}{9} = 10\, m/s^2.$$

$$x = x_0 + V_{0x}t + \frac{a_x t^2}{2} \text{ and } x = 15t + 0{,}4t^2 .$$

2. In the same way

$$\Delta S_{10} = S_{10} - S_9 = \frac{at_{10}^2}{2} - \frac{at_9^2}{2} = \frac{a}{2}(10^2 - 9^2) = \frac{10}{2}19 = 95\, m.$$

$$\Delta S_{10} = 95\, m.$$

Answer: $a = 10\, m/s^2$, $\Delta S_{10} = 95\, m$.

Words and phrases

linear dependence	to get a formula
Arithmetical average	projection of acceleration

Questions

1. What motion is called uniformly accelerated?
2. What is called acceleration?
3. What is acceleration unit?
4. Write the set of formulas for speed, the path and the coordinates of the accelerated motion.
5. Draw graphs of the projection of speed for a body moving uniformly accelerated (the speed decreases).
6. Draw a graph of the coordinates change with the time (the speed increases).

Problems

1. The train travels at a speed of 72 *km/h*. During braking to a full stop, it crossed the distance of 200 *m*. Find the time of braking?

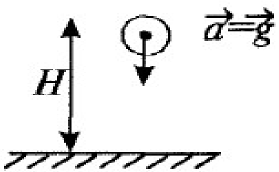
2. The car moves smoothly ($V_0 = 0$) with an acceleration of 0,5 m/s^2. What is the car's path for the third second?

3. The train moves with constant acceleration ($V_0 = 0$). For a fifth second, it passed by 1 *m* more than for the third second. What is equal the acceleration of a train?

§11. Examples of uniformly accelerated motion

1. Free fall of bodies

Free fall is referred to as the uniformly accelerated motion of bodies on Earth in a vacuum under the action of gravity of the Earth.



When free fall, all bodies, regardless of their mass, move equally – with constant acceleration.

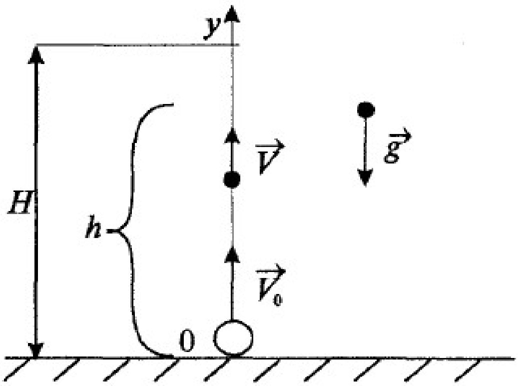
The acceleration $\vec{a} = \vec{g}$ is called acceleration of free fall.

On the surface of the Earth, the acceleration module is approximately equal $g = 9{,}8\, m/s^2$.

Numerical value of g depends on altitude and geographic latitude. So at the equator $g = 9{,}780\, m/s^2$, but at the poles $g = 9{,}832\, m/s^2$.

In formulas describing the smooth accelerated motion of the path S it's height H or h, acceleration a – acceleration of free fall g. The body speed at any time: $\vec{V} = \vec{g}t$. The path passed by the body (height of fall): $h = \frac{gt^2}{2}$. The free fall duration: $t = \sqrt{\frac{2h}{g}}$. The speed body that falls from height *h*: $V = \sqrt{2gh}$.

2. Motion of a body thrown upright



The body moves upwards at an initial speed V_0 and acceleration $a_y = -g$. Acceleration during lifting the body upwards is oppositely directed velocity of the body. At the highest point, the body stops. Then the free fall of the body begins to surface of the Earth. The speed of motion: $V = V_0 - gt$, where V_0 – initial speed.

The time of movement of the body up (*V*=0) is found from the expression $t = \frac{V_0}{g}$.

Height of body lift $H = V_0 t - \frac{gt^2}{2}$. From other side

$$H = V_0 \frac{V_0}{g} - \frac{g\left(V_0/g\right)^2}{2} = \frac{V_0^2}{2g}.\ H = \frac{V_0^2}{2g}.$$

The lifting height of the body upwards is equal to the path of the body when it falls freely $h = \frac{V_к^2}{2g}$. Therefore, we get equality

$$\frac{V_0^2}{2g} = \frac{V_к^2}{2g}, \; V_0 = V_к.$$

The initial rate of throwing the body is equal to the final speed of its fall.

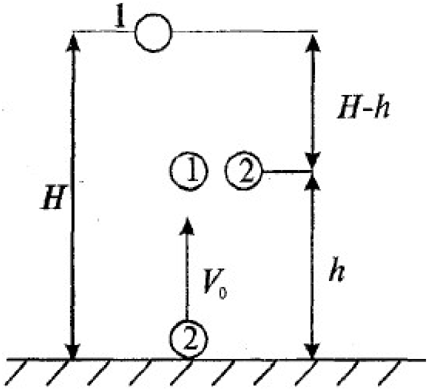
The lifting time of the body thrown up to its maximum height is equal to the time of its free fall.

Problem

The body freely falls from a height of 98 *m*. At the same time, another body was thrown upright up at a speed of 39,2 *m/s*. After what time and at what height from the Earth these bodies will meet?

Given:
H =98 *m*
V_0 = 39,2 *m/s*
g = 9,8 *m/s*^{2}

Find t–?, h–?



Solution

1. The path that passes through the first body during free fall to the meeting with the second body is equal to $H - h = \frac{gt^2}{2}$.

2. Height to which the second body will rise at the same time t: $h = V_0 t - \frac{gt^2}{2}$.

3. Let's add these two equalities. We'll get it $H - h + h = \frac{gt^2}{2} + V_0 t - \frac{gt^2}{2}$, $V_0 t = H$.

$t = \frac{H}{V_0} = \frac{98}{39,2} = 2,5\, s,\ h = H - \frac{qt^2}{2} = 98 - 30,6 = 67,4\, m.$

Answer: $h = 67,4\, m,\ t = 2,5\, s.$

Words and phrases

free falling	equator
acceleration of free falling	pole
geographical breadth	throw
height	simultaneously

Questions

1.What is called a free fall?
2.What kind of motion is a free fall?
3.How directed the acceleration of free fall?
4.The value of the acceleration of free fall depends on what?

Problems

1. The body falls from a height of 490 *m*. Determine how far the body will pass in the last second of the fall?

2. The body falls from a height of 2 *km*. What time will it pass the last 100 *m* of its way?

3. The ball, tucked up vertically, fell to the ground after 3 seconds. How fast was the ball thrown and how high did it rise?

4. The body is thrown vertically upwards at a speed of 4.9 *m/s*. Simultaneously with the limiting height that this body can achieve, a second body begins to fall down with the same initial velocity. Determine the time that a body will meet?

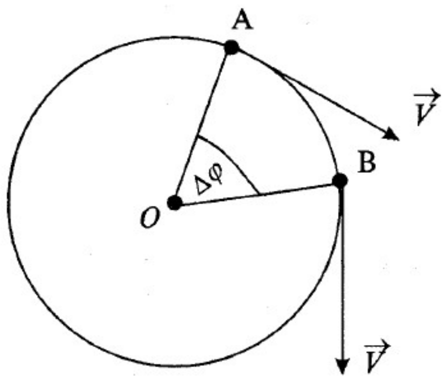
4. Two bodies are thrown vertically at intervals of time *t* seconds one relative to enother with the same initial velocity. With what speed will the second body move relative to the first? Determine the law of the distance change between bodies?

§12. Uniform circular motion

1. Linear and angular velocity

Let`s consider the motion of a point traversing a circular path at constant speed. The position of the point moving along a circle is determined by the radius vector drawn from the center of the circle to a given point. At each point of the circle, the instantaneous velocity is directed along a tangent to a circle; it is perpendicular to the velocity vector. Uniform circular motion is characterized by **linear and angular velocities.**

Uniform circular motion is called a motion along circular trajectory with constant speed.



For time period Δt it is occur displacement of the point long the arc *AB*, that is $\Delta S = \overset{\cup}{AB}$.

Linear speed is: $V = \dfrac{\Delta S}{\Delta t}$, where ΔS the length of the circle arc passed in a time interval Δt.The linear speed is numerically equal to the instantaneous velocity module.

The ***period of rotation*** T is the time of the complete revolution of the material point in a circle. The circumference of the circle is $S = 2\pi R$ (where R – radius of the circle). Then from the expression for the linear velocity we obtain:

$$V = \frac{2\pi R}{T}.$$

The number of revolutions of a point in 1 second is called ***the frequency of the rotation***. The frequency of the rotation is indicated by the letter n. The period is a quantity inversely proportional to the frequency: $T = \dfrac{1}{n}$. Then $V = 2\pi Rn$.

With uniform circular motion at any equal time interval the point do the same angular displacement. This conclusion can be written as follows: $\dfrac{\Delta\varphi}{\Delta t} = const$.

The ratio $\dfrac{\Delta\varphi}{\Delta t}$ is called the ***angular speed*** of a point that moves uniformly in a circle and is denoted by a letter ω.

An angular speed of a body is called a physical quantity equal to the ratio an angle $\Delta\varphi$, to which the radius vector R is rotated, to time interval Δt:

$$\omega = \frac{\Delta\varphi}{\Delta t}$$

The unit of measurement of angular velocity is the radian per second (*rad/s*). Radian is called an angle whose arc length is equal to the radius of a circle ($2\pi = 360^{\circ}$). One radian is equal to $57^{\circ}17'44''$.

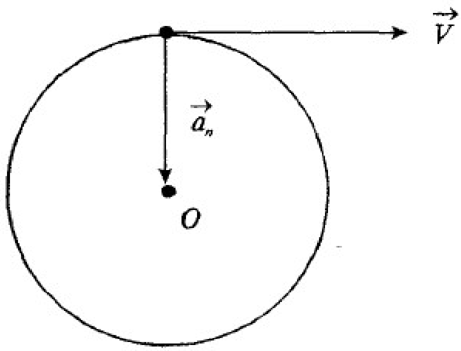
If the point carries a complete spin in a circle, then the angle $\Delta\varphi = 2\pi$, and the time of full revolutions is equal to the period of rotation, that is $\Delta t = T$.

Then $\omega = \frac{2\pi}{T}$. Consider that $T = \frac{1}{n}$. Then $\omega = 2\pi n$. From expression for $V = \frac{2\pi R}{T}$ we can establish the relationship between linear and angular velocity: $V = \omega R$

The linear velocity is equal to the product of the angular velocity and circle radius.

2. Centripetal acceleration

During a uniform circular motion, the velocity module (speed) has a steady value $\left|\vec{V}\right| = const$. The direction of the velocity vector changes over time $\vec{V} \neq const$. The velocity change vector is always directed to the center of the circle. Therefore, uniform circular motion is a motion with acceleration:



Acceleration, which characterizes the rate of change of the vector of instantaneous velocity of a point during its circular motion, is called centripetal acceleration:

$$a_n = \frac{V^2}{R}$$

The centripetal acceleration vector is directed to the center of the circle, that is, perpendicular to the instantaneous velocity vector.

Words and phrases

linear velocity	frequency of rotation
angularvelocity	centripetal (normal) acceleration
radian	period of rotation
Circle	arc

Questions

1. What determines the position of the point moving along a circle?
2. What is called an angular velocity?
3. How to determine a linear velocity?
4. In which units does measure an angular velocity?
5. What is called a period?
6. What is called a rotational freaquency?
7. What is called a centripetal acceleration? What can we say about the direction of the centripetal acceleration vector?

Problems

1. Determine the linear and angular velocity of an artificial Earth satellite, if its period of rotation in orbit is equal to 105 minutes, and the flight altitude is of 1200 *km*? The average radius of the Earth is of 6400 *km*.
2. Find the speed and the acceleration of the points of the Earth's equator, due to the daily rotation of the Earth. The radius of the Earth is of 6400 *km*.
3. Find the speed caused by the Earth's daily rotation and the acceleration of the point on the Earth's surface at the 60th parallel.
4. Astronauts are trained to overload in special centrifuges. With what frequency should the centrifuge be rotated, so that the

astronaut, which is at a distance of 3 m from the axis of rotation, was moving at an acceleration $a = 5g$?

5. A ball on a thread describes a circle in a horizontal plane, making one spin in time t. The length of the thread L, the thread line forms with a vertical line constant angle α. What is the speed and acceleration of the ball?

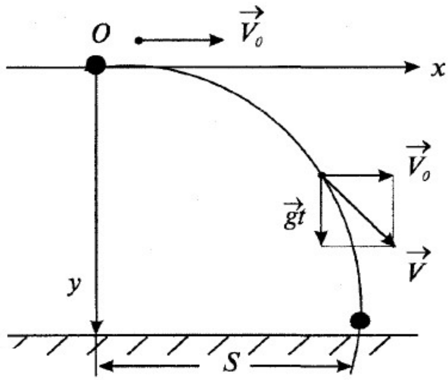
Remember!
Linear speed $V = \dfrac{\Delta S}{\Delta t}$; $V = \dfrac{2\pi R}{T}$
Angular speed $\omega = \dfrac{\Delta \varphi}{\Delta t}$; $\omega = \dfrac{2\pi}{T}$
Period of rotation $T = \dfrac{1}{n}$; $T = \dfrac{2\pi R}{V}$
Freaquency of rotation $n = \dfrac{1}{T}$
Connection between linear and angular speed $V = \omega R$
Centripetal acceleration $a_n = \dfrac{V^2}{R} = \omega^2 R$

§13. Examples of curvilinear motion

Now we discuss some example of curved motion or two dimensional motion of constant acceleration such as the motion of constant acceleration such as the motion of a particle projected at certain angle with the horizontal in vertical x-y plane (this type of motion is called projectile motion). Air resistance to the motion of the body is to be assumed absent in this type of motion.

1. Motion of the body that thrown horizontally

This motion can be decomposed into two independent motions: a rectilinear uniform motion along the axis Ox and a free fall motion from the height at which the body was at the moment of throwing along the axis Oy.



The total velocity vector $\vec{V} = \vec{V}_0 + \vec{g}t$ is decomposed into two components. The projection of this vector on the axis Ox: $x = V_0 t$. Similarly, on the axis Oy we have: $y = \frac{gt^2}{2}$. These equations allow you to find the position of a body in a given reference frame at any moment of time. From the equation $y = \frac{gt^2}{2}$ we find t and substitute it into the equation for the coordinate x. As a result, we obtain:

$$x = V_0 \sqrt{\frac{2y}{g}} \text{ or } S = V_0 \sqrt{\frac{2y}{g}}.$$

The maximum distance that the body overcomes in a horizontal direction to the point of incidence is called the ***range of flight*** or ***horizontal range***.

From the equation $x = V_0 t$ we find t and substitute it into the equation $y = \frac{gt^2}{2}$. We obtain $y = \frac{g}{2V_0^2} x^2$. As you can see, the equation of the body trajectory is a ***parabola***.

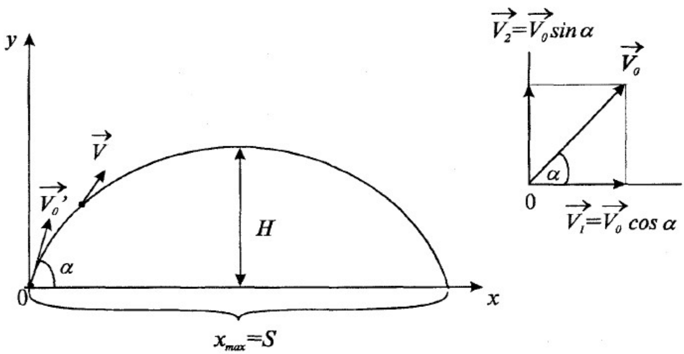
2. Motion of the body that thrown at some angle with horizontal

The motion of a body that thrown at some angle to the horizon can be decomposed into two independent motions: a uniform rectilinear motion in the direction of *Ox* at a speed $V_x = V_0 \cos\alpha$ and a uniform accelerated motion of a body thrown upright along the axis *Oy* at an initial velocity $V_{0y} = V_0 \sin\alpha$.

At time t, the projections of velocity on the axis of this reference system are:

$$V_x = V_0 \cos\alpha$$

$$V_y = V_0 \sin\alpha - gt$$



Equations of a body coordinates:

$$x = V_0 \cos\alpha \cdot t, \quad y = V_0 \sin\alpha \cdot t - \frac{gt^2}{2}$$

Next, we solve the system of these two equations and obtain the equation of the trajectory:

$$y = tg\alpha \cdot x - \frac{g}{2V_0^2 \cos^2\alpha} \cdot x^2 .$$

This is also a parabola equation.

Consequently, the trajectory of the motion of a body thrown at some angle to the horizon is a parabola whose branches are directed downwards. It`s mean that a lifting time to the maximum height is equal to the falling time from the maximum height.

At a maximum height $V_y = V_0\, sin\,\alpha - gt = 0$, from here $t = \dfrac{V_0 \sin\alpha}{g}$, where t – a lifting time of the body to the height H:

$$H = V_0 \sin\alpha \cdot \frac{V_0 \sin\alpha}{g} - \frac{g}{2}\left(\frac{V_0 \sin\alpha}{2g}\right)^2 = \frac{V_0^{\,2} \sin^2\alpha}{2g}$$

$$H = \frac{V_0^{\,2} \sin^2\alpha}{2g}$$

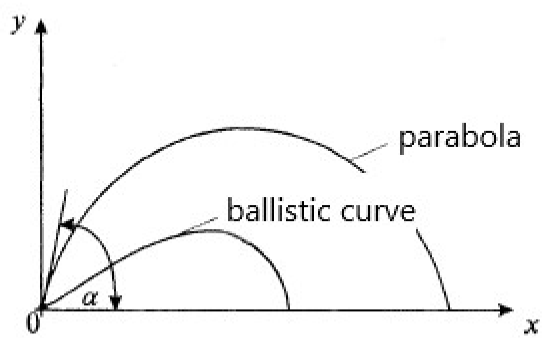
The body will fall to the ground through time $t' = 2t$:

$$t = \frac{2V_0 \sin\alpha}{g}$$

Range of flight:

$$S = x_{max} = V_0 \cos\alpha \cdot \frac{2V_0 \sin\alpha}{g} = \frac{V_0^{\,2} \sin 2\alpha}{g}\,.$$

Analysis of this expression shows that with an angle $\alpha = 45^\circ$ is the maximum flight range of the body. In terrestrial conditions, due to air resistance, the trajectory of the body motion does not correspond to a mathematical parabola. In real conditions, the trajectory of the body is a ballistic curve (see Fig.).



Problem

The body was thrown at the angle $\alpha = 30^{\circ}$ to the horizon from a position with a coordinate $y_0 = 5\,m$ above the ground. The initial body speed was of 10 *m/s*. Determine the coordinate of the highest point of the trajectory of the body y_{max}, the coordinate of the point of incidence x_n and the velocity V_n at that point.

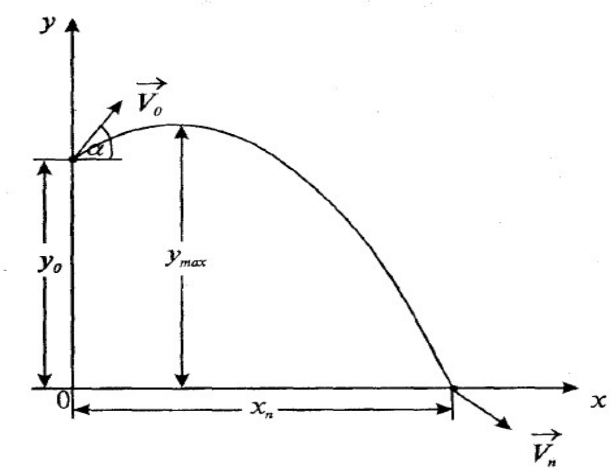
Given:

$\alpha = 30^{\circ}$

$y_0 = 5\,m$

$V_0 = 10\ m/s$

Find: y_{max}–?, x_n– ?, V_n–?.



Solution

1. The coordinate y_{max} is determined by the formula $y_{max} = y_0 + \dfrac{V_0^2 \sin^2 \alpha}{g}$, $y_{max} = \left(5 + \dfrac{10^2 \cdot 0{,}5^2}{2 \cdot 9{,}8}\right) m \approx 6{,}3\,m.$

2. To determine x_n under condition $x_n = 0$ we will find the flight time of the body until the collision with the ground:

$$0 = y_0 + V_0 t_n \sin\alpha - \frac{g t_n^2}{2}, \text{ from here}$$

$$t_n = \frac{V_0 \sin\alpha + \sqrt{V_0^2 \sin^2\alpha + 2 g y_0}}{g} \approx 1{,}6\, s,$$

$$x_n = V_0 t_n \cos\alpha = 10 \cdot 1{,}6 \cdot 0{,}87\ m \approx 14\ m$$

3. Final speed V_n is:

$$V_n = \sqrt{(V_0 \cos\alpha)^2 + (V_0 \sin\alpha - g t_n)^2} = 14\, m/s.$$

Answer: $y_{max} \approx 6{,}3\, m,\ t_n \approx 1{,}6\, s,\ V_n \approx 14\ m/s.$

Words and phrases

maxheight	final speed
distance of flight	equation of parabola
time of falling	ballistic curve

Questions

1. On which components you can decompose the motion of the body thrown horizontally?
2. What does it mean to find the trajectory of the flight?
3. What is the range of flight?
4. What kind of line is the trajectory of the motion of the body thrown horizontally?
5. Write parabola equations.
6. What value of the angle does provide the maximum flight range of the body thrown at some angle to the horizon?

Problems

1. The plane flies horizontally at a speed of 360 *km/h* at an altitude of 490 *m*. When it flies above point *A*, a subject is thrown out of it. At what distance from point *A* the object will fall to the Earth?

2. How the time and the range of flight of the body thrown horizontally from a certain height will change with the increase of the throwing speed twice?

3. The ball moved on the table at a speed of 2 *m/s*. When it reaches the edge it fell at a distance of 0.8 *m* from the table. Determine the height of the table?

4. The range of flight of a body thrown horizontally at a speed of 10 *m/s* is equal to the height of throwing. From what height the body was thrown?

Chapter 2. The dynamics of the material point

§ 14. Force. Newton's first law. Galileo's principle of relativity

1. Newton's first law. Inertia

The main task of the dynamics is to determine the position of the moving body at any given time according to the known position of the body, the initial speed and the forces acting on the body.

At the heart of the dynamics are the three ***laws of Newton***, which are the basic laws of classical mechanics. Newton's laws are valid for macroscopic bodies, the velocity of which is much smaller than the speed of light in a vacuum.

The first Newton`s law refers to bodies on which other bodies (forces) do not act or the action of these bodies (forces) is compensated, while the body retains a state of rest or uniform rectilinear motion.

The body on which other bodies or fields do not act or their actions are compensated is called **isolated body**.

Reference systems, in which the body is in a state of rest or moves uniformly and rectilinearly, are called **inertial systems**.

Any reference system moving relative to the selected inertial system uniormly and rectilinearly will also be inertial.

The phenomenon of conservation by the body of rest state or uniform rectilinear motion in the absence or compensation of external interactions is called **inertia**.

The rectilinear and uniform motion of the body in the initial reference frame is called the motion by inertia, and **Newton's first law is the law of inertia.**

The first Newton`s law affirms the existence of inertial reference frames.

Newton's first law is worded as follows:

There are such reference frames in which the body retains a state of rest or uniform rectilinear motion.

The **non-inertial reference frame** is the reference frame moving with acceleration relative to the inertial reference frame.

2. Galileo's principle of relativity

In inertial reference frame, all mechanical processes occur in the same way. During the transition from one inertial reference system to another, the speed of the body, the displacement, the trajectory, but not the acceleration, hence, and not the laws of motion, can change.

Thus, the **Galileo's principle of relativity** argues:

In any inertial reference systems, the laws of mechanical motion are the same.

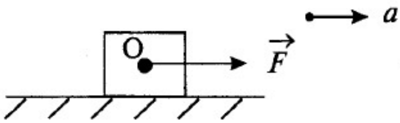
The Galileo's principle of relativity or the mechanical principle of relativity testifies to the physical equality of all inertial reference frames.

The principle of relativity was extended by **Albert Einstein** to all phenomena of nature: **All processes of nature occur identically in all inertial reference frames**.

3. Force. Net force

The physical vector quantity, which is a measure of action on the body from other bodies or fields is called the ***force***.

The force is completely determined if its module, direction and point of application are given. The direction of the force vector coincides with the acceleration vector of the body on which the force acts**.**



The unit of force in SI is **Newton (*N*)**: $1N = 1\frac{kg \cdot m}{s^2}$. **One Newton is a force that gives the body a mass of 1 *kg* of acceleration of 1 *m/s*^{2}.**

The force, the action of which can be replaced by the action of several forces applied to the body at one point is called the **net force**.

Net force is equal to the vector sum of all forces acting on the body:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \ldots + \vec{F}_n = \sum_{i=1}^{n} \vec{F}_i$$

If two forces act on the body at an angle to each other, the direction of net force coincides with the diagonal of the parallelogram the sides of which these forces are. If the forces and are directed equally, the net force is directed in the same direction, and the module is equal to the sum of their modules. The net foce of two oppositely directed forces is directed as greater force.

The forces acting between parts of a closed system of bodies are called **internal forces**.

The forces acting on the systems of bodies from the outside are called **external forces**.

A closed system of bodies that is not under action of external forces is called **isolated system**.

Words and phrases

dynamics	inertial system
compensation	uninertial system
isolated system	net force
force	principle of relativity

Questions

1. What is the main task of the dynamics?
2. Formulate the first Newton`s law.
3. What systems are called inertial reference frame?
4. What is called a force?
5. What force is called net force?
6. Formulate the Galileo’s principle of relativity.
7. What system is called isolated?

Remember!
1. The action of a body on another body 2. The interaction of bodies is the cause of acceleration

§15. Newton's laws

1. Inertia. Body mass

The property of the body, by which its acceleration depends on interaction with other bodies, is called inertia.

The phenomenon of **inertia** is manifested in the fact that different bodies under the action of the same forces get different accelerations.

Inertia has a quantitative measure. A physical quantity, which is a measure of inertia of the body, is called an **inert mass** or simply a body mass. Inert mass – the main dynamic characteristics of the body.

The mass characterizes one more property of bodies – the ability to attract with other bodies. In these cases, the mass acts as a measure of gravity, and it is called a **gravity mass**.

In modern physics, the identity of the values of the inert and gravity mass of a given body is established.

To measure mass in SI there is a standard – **one kilogram** ($m_{st} = 1\ kg$).

There are two ways to measure the mass. The first way is to bring the body of an unknown mass to interact with the standard. The mass ratio of the interacting bodies is inversely proportional to the modulus of their accelerations:

$$\frac{m_1}{m_2} = \frac{|\Delta\vec{V}_2|}{|\Delta\vec{V}_1|} = \frac{a_2}{a_1}$$

Therefore, the mass of the body m_b can be determined using the reference mass m_{st} by formula:

$$\frac{m_b}{m_{st}} = \frac{\left|\Delta\vec{V}_{st}\right|}{\left|\Delta\vec{V}_b\right|}; \; m_b = m_{st}\frac{\left|\Delta\vec{V}_{st}\right|}{\left|\Delta\vec{V}_b\right|} = m_{st}\frac{a_{st}}{a_b},$$

where m_{st} – reference mass, a_b, a_{st} – acceleration of the body and standard, respectively.

In practice, another method of mass measurement is used – a way of comparing masses of bodies on lever scales.

In Newton's mechanics it is believed that:

1) the mass of the body does not depend on its speed;

2) the mass of the body is equal to the aggregate mass of particles from which it is composed;

3) for a given set of bodies the law of mass conservation is fulfilled: **for any processes occurring in the system of bodies, its mass remains constant.**

Density of a substance ρ is a physical quantity equal to the ratio of body weight to its volume:

$$\rho = \frac{m}{V}$$

The physical meaning of density is that it shows what mass of this substance is contained in a volume unit. A unit of density in SI is kilograms per cubic meter (*kg/m*^{3}).

Numerical values of the densities of different substances is differ significantly one to each other. For example, for water ρ=1000 *kg/m*^{3}, for ice ρ=900 *kg/m*^{3}, for iron ρ=7800 *kg/m*^{3}.

2. Newton's second law

The second law states that the acceleration of an object is dependent upon two variables – the net force acting upon the object and the mass of the object. ***The acceleration of an object depends directly upon the net force acting on the object, and inversely on the mass of the object.*** As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased:

$$\vec{a} = \frac{\vec{F}}{m}, \text{ or } \vec{F} = m\vec{a} .$$

Consequently, the product of the mass for acceleration is equal to the force.

In the case of action on the body at the same time several forces, a net force must be considered as force $\vec{F}$. In this case, the second law takes the form of:

$$\vec{a} = \frac{\sum_{i=1}^{n} \vec{F}_i}{m} \text{ or } \sum_{i=1}^{n} \vec{F}_i = m\vec{a}$$

This equation is called the ***basic equation of dynamics*** or ***the law of motion.***

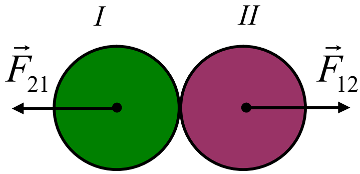
If the effect of other bodies on the body is offset, that means $\sum_{i=1}^{n} \vec{F}_i = 0$. Forces, for which the net force is equal to zero are called balanced.

$\sum_{i=1}^{n} \vec{F}_i = 0$, then $\vec{a} = 0$ and we obtain the following definition of **Newton's first law**:

If on the body acts balanced forces, then the body retains a state of rest or uniform rectilinear motion.

3. Newton's third law

Newton's third law: the bodies act one to each other with forces equal in modulus and directed opposite



$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right|$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

The forces $\vec{F}_{12}$ and $\vec{F}_{21}$ that arise during the interaction are applied to different bodies and therefore can not balance each other. Equality of forces during the interaction takes place always and does not depend on whether the bodies move or are in relative peace.

The forces of interaction between two bodies always have the same nature. So, for example, the Earth attracts the Moon by the force of gravity. Then the force from the Moon (the same in modulus and opposite in direction) is also a force of gravity.

Problem

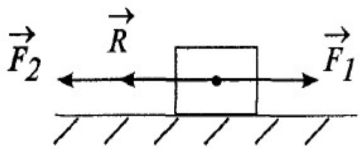
Two forces F_1= 10 N and F_2= 25 N are applied at one point and directed along one line in opposite directions. Define the equalized and net forces, determine the directions of the actions of these forces.

Given:

F_1=10 N

F_2=25 N

Find: $\vec{R}$ i $\vec{F}$



Solution

1. The net force is equal to the difference of two forces and directed along the force F_2: $R = F_2 - F_1$=15 N.

2. The equalized force F in modulus is equal to the net force R, but directed opposite. This force F =15 N and directed along $\vec{F}_2$.

Words and phrases

inertia	balanced force
mass	etalon of mass

Questions

1. What is called inertia?
2. What is called a body mass?
3. What does Newton's first law say?
4. Formulate the second law of Newton.
5. Formulate the third law of Newton.
6. For what kind of reference frame the laws of Newton is valid?

Remember!
The body mass is a quantitative characteristic of inertia

Problems

1. The body, weight of 8 *kg*, starts to move under the action of the constant force, it passed 10 *m* in the first second of its motion. Determine the value of the force.

2. The balloon of mass *m* began to drop with constant acceleration *w*. Determine the mass of the ballast, which should be dropped overboard, so that the balloon received the same acceleration, but directed upward. The air resistance can be neglected.

3.A net force of 7.5 *kN*, west acts on a 1208 *kg* race car. At what rate will the car accelerate? (a = 6.2 m/s^2, west)

4. A car, mass 1485 *kg*, travelling south at 116 *km/h*, slows to a stop in 10.25 seconds. What is the magnitude and direction of the net force that acted on the truck? (F = 4.67 *kN*, north)

5. A horizontal force of 26.3 *N* is necessary to push a 99 *N* wooden box across a tile floor at constant velocity. Find the coefficient of sliding friction between the two surfaces. (μ = 0.27)

6. The stone that slid along the horizontal surface of the ice stopped after a distance of S=48 *m*. Find the initial speed of the

stone if the frictional force of the stone sliding on the ice is 0.06 of the normal stone pressure on the ice.

7. The body of mass m without initial velocity slides from an inclined plane with an angle α from height *h*. The coefficient of friction of a body on a surface is equal to μ. How many time will elapse before the body reach the foot?

8. A force of 25 *N*, directed at an angle of 30^0 to the horizon upwards, acts on a body weighing 2 *kg*, which is on a horizontal straight line. Determine the frictional force if the coefficient of friction is of 0.2.

9. Determine the height of the inclined plane if the body, moving without the initial speed from the top, reached the ground in 4 seconds. The length of the plane is 10 *m*.

§16. Momentum of the body. The law of conservation of momentum

1. Momentum of the body

The second Newton`s law can be written in a different form, if we enter a new physical quantity – the momentum of the body.

The momentum of a body is a vector quantity equal to the product of the body mass and its velocity:

$$\vec{p} = m\vec{V}$$

The unit of momentum in SI is kilogram-meter per second (*kg·m/s*).

Given that the acceleration $\vec{a} = \frac{\Delta\vec{V}}{\Delta t}$, we substitute this expression in the equation of the Newton`s second law. Finaly, we obtain:

$$\vec{F} = m\frac{\Delta\vec{V}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

Consequently, the force is equal to the momentum change per unit time. The last formula can be written as follows:

$$\vec{F}\Delta t = \Delta\vec{p}$$

The product of force and the time of its action is called ***the impulse***.

Now the second Newton`s law is formulated using a momentum:

As a result of force action, the momentum of the body changes. The change in body momentum is equal to the impulse.

2. The law of conservation of momentum

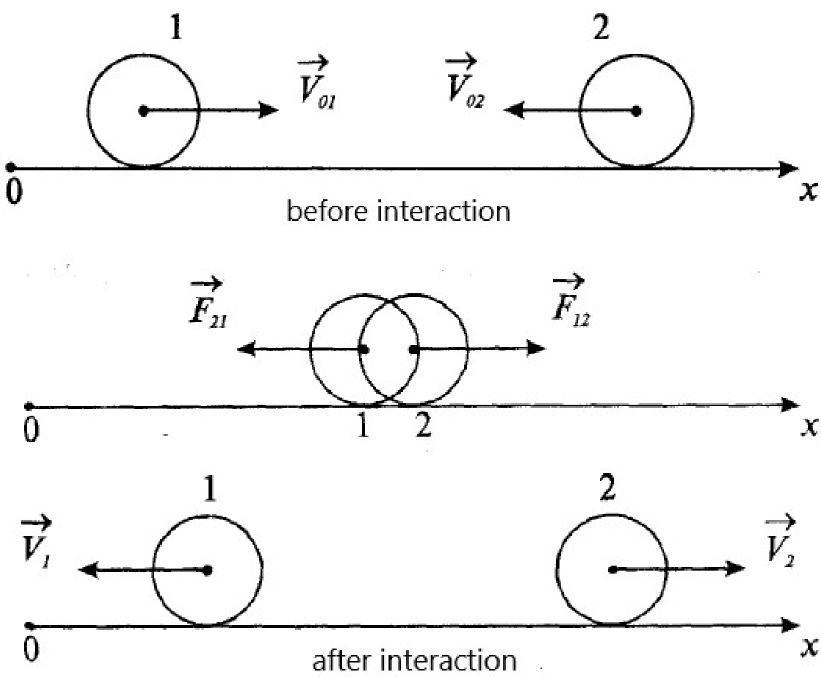
Let`s consider the interaction of two bodies in an isolated system.

Denote:

m_1, m_2 – masses of bodies;

$\vec{V}_{01}, \vec{V}_{02}$ – initial velocities of bodies;

$\vec{V}_1, \vec{V}_2$ – the final velocities of the bodies after the interaction.



According to the third Newton`s law, the forces acting on the bodies when they interact, are equal in modulus and have opposite directions:

$$\vec{F}_{12} = -\vec{F}_{21}$$

For each body we write a momentum change:

$$\vec{F}_{12}t = m_1\vec{V}_1 - m_1\vec{V}_{01},$$

$$-\vec{F}_{21}t = m_2\vec{V}_2 - m_1\vec{V}_{02},$$

where t is the time of interaction of bodies.

Then we obtain:

$$m_1\vec{V}_{01} + m_2\vec{V}_{02} = m_1\vec{V}_1 + m_2\vec{V}_2.$$

This expression can be written as follows:

$$\vec{p}_{01} + \vec{p}_{02} = \vec{p}_1 + \vec{p}_2$$

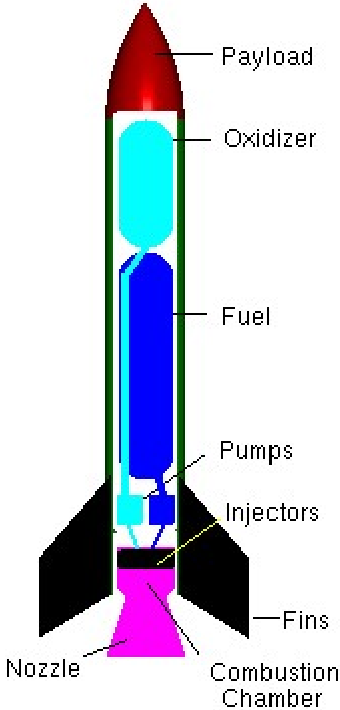
The last equality expresses the law of conservation of momentum for two interacting bodies.

For a system of interacting bodies, the law of conservation of a momentum of an isolated system is formulated as follows:

In an isolated system, the vector sum of the bodies momentumes befor the interaction is equal to the vector sum of the bodies momentumes after the interaction. That is, the total momentum of an isolated system remains constant.

3. Jet motion

The motion that occurs as a result of separation and removal from a body its parts is called ***the jet motion***. The other part of the body thus moves in the opposite direction. The basis of the jet motion is the law of momentum conservation.



An example of a jet motion is the movement of a rocket. Let's consider how a rocket engine works. Through the jet nozzle, the gas produced as a result of combustion of fuel is thrown at high speed. Gas is the separated part of the rocket. The change in the speed of the rocket is greater, the higher the speed of emitted gas, and the greater the ratio of the mass of the gas to the mass of the rocket.

According to the law of momentum conservation the vector sum of momentums remains constant. It`s mean, after turning on the rocket engine, the vector sum of the rocket momentum and the momentum of the gases flowing out of the engine is zero:

$$M\vec{V}_r + m\vec{V}_g = 0 \ ,$$

where $M, \vec{V}_r$ – mass and velocity of the rocket, $m, \vec{V}_g$ – mass and velocity of the gas.

$$\vec{V}_r = -\frac{m\vec{V}_g}{M}.$$

According to the law of conservation of the momentum the rocket moves in the opposite direction relative to the direction of gas movement. Modern rockets can move in outer space.

Problem

A platform with a mass of 500 *kg* travels on a horizontal road at a speed of 4 *m/s*. Toward it another platform weighing 300 *kg* at a speed of 8 *m/s* is moving. Find the speed of the platforms, if after they collide they moves together?

Given:

m_1=500 *kg*

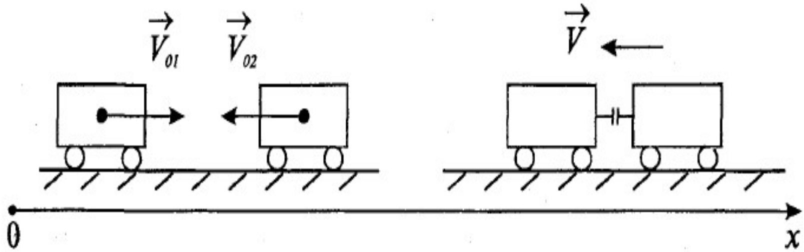
V_{01}=4 *m/s*

m_2=300 *kg*

V_{02}=8 *m/s*

Find: V -?

Solution



1. Let's write the law of conservation of momentum in a vector form:

$$m_1\vec{V}_{01} + m_2\vec{V}_{02} = (m_1 + m_2)\vec{V}$$

2. Next, we write the law of conservation of momentum in the projection on the axis *Ox*:

$$m_1 V_{01} - m_2 V_{02} = (m_1 + m_2)V$$

3. Then we calculate the speed V :

$$V = \frac{m_1 V_{01} - m_2 V_{02}}{m_1 + m_2} = \frac{500 \cdot 4 - 300 \cdot 8}{800} m/s = -0,5\, m/s$$

$Answer: V = -0,5\, m/s$.

Words and phrases

momentum	separation
law of conservation	impulse
jet motion	rocket

Questions

1. What is called the momentum of the body?
2. Formulate the law of conservation of momentum for an isolated system.
3. Formulate the second Newton`s law using momentum.
4. What is called the jet motion?

Remember!
The law of conservation of momentum is at the core of the jet motion

Problems

1. How much momentum does a stationary 5500 *kg* mass have?
2. A 20 *kg* projectile flies horizontally at a speed of 500 *m/s*, hits a platform with sand of 10 tons and stuck in it. How fast did the platform start moving?
3. From a two-stage rocket with a total mass of 1 *t* at the time of reaching a speed of 170 *m/s*, its second stage of mass of 400 *kg* was separated, the speed of which thus increased to 185 *m/s*. Determine how fast the first stage of rocket began to move? (speedes are given relative to the Earth).
4. A man jumped up on a moving trolley. What will be after that the speed of the trolley, if the mass of a man is 70 *kg*, the weight of the trolley is 30 *kg* and the initial speed of the trolley is 1,5 *m/s*?

5. The meteorite and the rocket move towards each other at an angle of 90^0. The rocket hits the meteorite and gets stuck in it. Meteorite mass is M , rocket mass – m , meteorite velocity – V_1 , rocket speed – V_2 . Determine the momentum of the meteorite with the rocket in it after the accident.

6. An object has a momentum of 55 *kg·m/s* and hits a stationary object making the second object starts to move. If the first object ends with a momentum of 13 *kg·m/s*, what is the momentum of the second object?

7. An elastic collision occurs in one dimension, in which a 10 *kg* block traveling at 5 *m/s* collides with a 5 *kg* block traveling at 3 *m/s* in the same direction. What are the velocities of the two blocks immediately after the collision? (Answer: 3.67 *m/s*, 5.67 *m/s*).

8. A frog of mass *m* sits at the end of a board of mass *M* and length *L*. The board rests on the surface of the pond. The frog jumps at an angle α to the horizon along the board. What should be the initial speed V_0 of the frog so that after the jump the frog is at the other end of the board? Water resistance can be neglected.

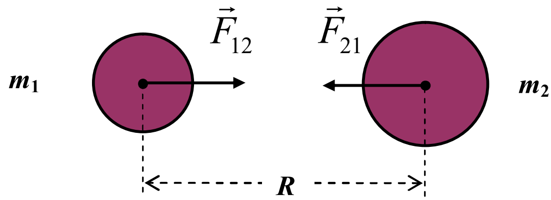
§17. Types of forces in mechanics

1. Forces of universal gravitation

Forces of universal gravitation are forces of mutual attraction between all material bodies in nature. In accordance with the third Newton`s law, the two bodies interacts with equal in modulus and opposite in direction forces of gravitational attraction.

The force of mutual attraction that acts between the celestial bodies - the Sun, the Moon, planets, comets, and stars - Newtton called as ***a force of universal gravitation*** and discovered the ***law of universal gravitation***.

Newton has shown that the forces of mutual attraction of two bodies depend on the mass of the bodies and the distance between them.



Consider two bodies masses m_1 and m_2 at a distance R. Between two masses gravitational forces $\vec{F}_{12}$ and $\vec{F}_{21}$ act. The gravitational forces are determined by the law of universal gravitation, formulated by Newton:

The law of universal gravitation: all bodies are attracted to each other by forces of universal gravitation, which are directly proportional to the masses of these bodies, and inversely proportional to the square of the distance between them:

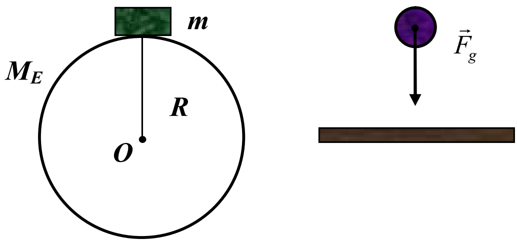
$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = G\frac{m_1 m_2}{R^2},$$

where G=6,67·10^{-11} $N{\cdot}m^2/kg^2$ – the **gravitational constant**, also known as the **universal gravitational constant**, or as **Newton's constant**, is an empirical physical constant involved in the calculationof gravitational effects in **Sir Isaac Newton's law of universal gravitation** and **in Albert Einstein's general theory of relativity**.

The gravitational constant is numerically equal to the attraction force of two bodies weighing 1 *kg* each, if the distance between them is 1 *m*. This is its physical meaning. The form of the writing of the law of universal gravitation is valid for two material points and for bodies having the form of a sphere.

2. Force of gravity

The force with which the Earth acts on every body and gives it an acceleration is called the force of gravity.



This force is directed to the center of the Earth. In accordance with the second Newton's law, the force of gravity for a body mass m is equal to :

$$\vec{F}_g = m\vec{g} \, ,$$

where $\vec{g}$ – free fall acceleration. The value of $\vec{g}$ depends on the geographical latitude of the place and its height above sea level.

If we ignore the rotation of the Earth and assume the reference frame associated with the Earth is inertial, then force of gravity will be equal to the force of universal gravitation:

$$mg = G\frac{mM_E}{R_E^2} \, ,$$

where $M_E = 5{,}98 \cdot 10^{24}\,kg$ – mass of the Earth; R_E – radius of the Earth.

From where we have a formula for calculating the acceleration of free fall due to mass and radius of the Earth:

$$g = G\frac{M_E}{R_E^2}$$

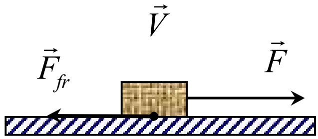
Thus, the acceleration of free fall is determined by the mass and radius of the Earth and does not depend on the mass of the body. Let's calculate this expression and get the numerical value of the acceleration of free fall – ***9.81 m/s²***. The acceleration of free fall has different values at different points of the Earth. Usually in tasks use the value of $\vec{g}$ at latitude 45º – 9.81 m/s^2

If the body is at an altitude of H, then

$$g_H = G\frac{M_E}{(R_E + H)^2}$$

3. The forces of friction

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.



Friction forces are tangent to the surface. These forces impede the motion of one body relative to another.

Friction forces are due to the interaction of molecules and have electromagnetic nature.

Static sliding friction – friction that exists between tuched surfaces when there is no their mutual movement. The static friction force at each instant is equal to the modulus of the applied external force and is directed opposite to the force that tries to move the body along the surface.

$$\vec{F}_{fr} = -\vec{F}$$

The **static sliding friction** increases when increasing external force until slipping begins.

When sliding, there is also a force that prevents the movement – the force of sliding friction. It is approximately equal to the maximum static sliding friction.

The law of sliding friction is experimentally established: **the force of sliding friction is always directed against motion and**

by value is directly proportional to the force of the normal body pressure on the support:

$$\left|\vec{F}_{fr}\right| = \mu\left|\vec{N}\right|,$$

where μ – a coefficient of friction, $\vec{N}$ – the force of normal pressure (or equal to it by the modulus of the force of the reaction of the support). The force of normal pressure is directed perpendicular to the surface on which the body is located.

The coefficient of friction depends on the material and the quality of surface treatment.

If one body rolls over the surface of another, there is a **rolling friction**.

Rolling friction is the resistive force that slows down the motion of a rolling ball or wheel. It is also called **rolling resistance**.When a force or torque is applied to a stationary wheel, there is a small static rolling friction force holding back the rolling motion. However, resistance from **static sliding friction** is what really causes the wheel to start rolling.

Once it is rolling, the resistance to the motion is typically a combination of several friction forces at the point of contact between the wheel and the ground or other surface.

Provided the same quality of rolling surface friction is less than tens of times. Therefore, in mechanisms seek to replace sliding friction by rolling friction.

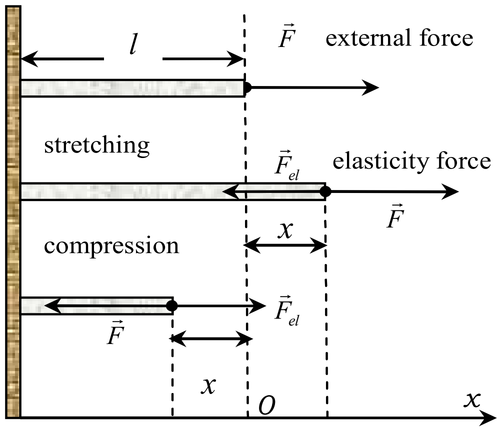
4. Elasticity forces

Deformation is called a change in the shape, size or volume of the body under the influence of external forces.

If the deformations completely disappear after the cessation of action on the body of external forces, they are called elastic. During elastic deformations in the body there are forces of elasticity. Under the action of external force, body particles (atoms, molecules) are displaced from their equilibrium positions, while forces in the body arise that attempt to turn particles into equilibrium positions.

The forces of elasticity have electromagnetic nature. For elastic deformations, **Hooke's law** is experimentally established. For one-sided tension or compression, under the action of external force, directed along the rod Hooke's law is worded as follows:

The force of the elasticity that occurs during deformation of the body is directly proportional to the elongation of the body and directed toward the opposite external force.

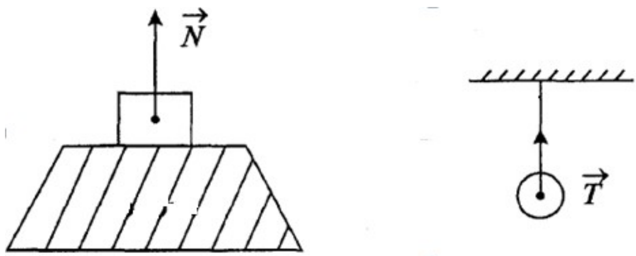


Projection of force of elasticity on the axis *Ox*:

$$(F_{el})_x = -kx$$

The coefficient k is called **stiffness**. Its value depends on the size and material of the body. Unit of stiffness – Newton per meter (*N/m*).

For example, a book lying on the table deforms it and causes the force of elasticity $\vec{N}$, which is directed vertically up. The force of elasticity acting on the body from the support side, is called the **force of the reaction of the support**. The force of elasticity $\vec{T}$ acting on the body from the side of the suspension is called the **tension force of the suspension**.



The deformation is called **elastic**, if after the termination of force the deformation completely disappears.

Deformation is called **plastic**, if after deactivation of force the deformation is preserved.

Problem 1. At what point on the straight line connecting the Earth and the Moon, the body will be attracted by the Earth and the Moon with the same force? The distance between the Earth and the Moon is equal to 60 terrestrial radii, the Earth's mass is 81 times the mass of the Moon.

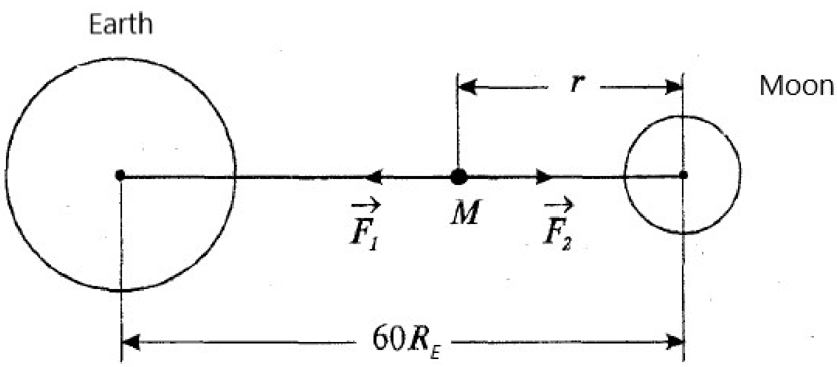
Given:

$S = 60R_E$

$R_E = 6370km$

$M_E = 60M_M$

Find: r -?



Solution

1. The force of gravitational interaction between the Earth and body mass m is: $F_1 = G\dfrac{mM_E}{(60R_E - r)^2}$,

but between the body and the Moon: $F_2 = G\dfrac{mM_M}{r^2}$.

2.Under the condition of the task $F_1 = F_2$,

$$G\frac{mM_E}{(60R_E - r)^2} = G\frac{mM_M}{r^2},$$

from here $\dfrac{81}{r^2} = \dfrac{1}{(60R_E - r)^2}$, $r = 54R_E$.

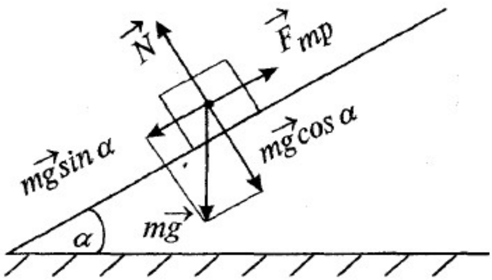
Answer: $r = 54R_E$.

Problem 2. The body slides down uniformly on 40º inclined plane. Determine the coefficient of friction.

Given:

$\alpha = 40^\circ$

Find: μ -?



1. Let's write the equation of motion in vector and scalar forms:

$m\vec{g} + \vec{N} + \vec{F}_{fr} = m\vec{a} = 0$, $mg\sin\alpha - F_{fr} = 0$, $N - mg\cos\alpha = 0$.

2. Friction force is $F_{fr} = \mu N = \mu mg\cos\alpha$. From here:

$$\mu = \frac{\sin\alpha}{\cos\alpha} = tg\alpha \approx 0{,}84.$$

Answer: $\mu \approx 0{,}84$.

Words and phrases

law of universal gravitation	roling friction
attraction	deformation
gravity	elastic deformation
attractive force	forces of resiliency
force of friction	lenear expensivity
static friction	reaction of support
sliding	compression
sliding friction	inflexibility
coefficient of friction	stiffnes

Questions

1. Formulate the law of universal gravitation.
2. What is the physical content of the gravitational constant?
3. What is gravity?
4. Write the formula for calculating the acceleration of free fall.
5. What is gravity dependent on?
6. What are the forces of friction?
7. What are the types of friction forces?
8. What is the coefficient of friction?
9. How to reduce slip friction?
10. What is deformation of the body?
11. Formulate Hooke's law.
12. What is the stiffnes coefitient dependent on?

Remember!
1. The law of universal gravitation is valid for material points **2. The force of friction is opposite to the direction of motion**

Problems

1. How many times will the force of attraction to the Earth of an artificial satellite diminish when it be at a distance equal to the five radiuses of the Earth relative to being on the ground?

2. What is the acceleration of the free fall on a planet, whose radius is 4 times greater than the radius of the Earth, and the mass is 51 times greater than the mass of the Earth? The planet does not rotate.

3. A spacecraft emerged in a circle orbit with a radius of 10 million *km* around the star opened by it. What is the mass of this star, if the period of rotation of the ship is of 628000 *s*?

4. A bar, weight of 1.6 *kg*, uniformly drag along the table with a spring, stiffnes 40 *N/m*. What is the elongation of the spring, if the coefficient of friction between the bar and the table is of 0.3?

5. The body thrown on an ice rink at a speed of 36 *km/h*, goes to a stop of 40 *m*. What is the coefficient of friction between the body and ice?

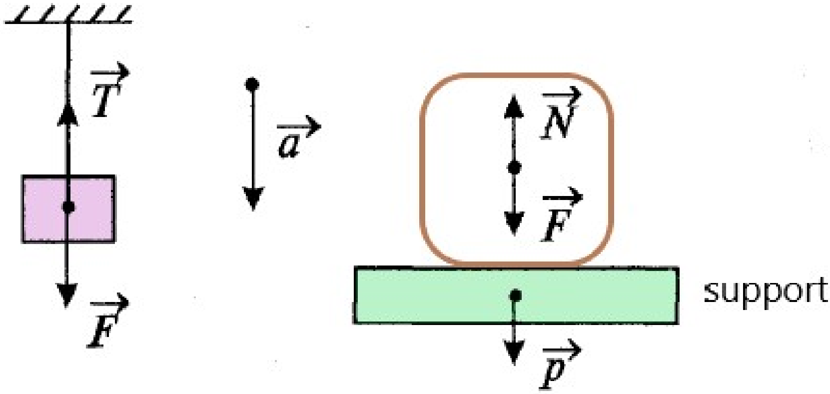
6. At which acceleration the cable will break when lifting a load of 500 *kg*, if the maximum tensile force that the cable holds without breaking, is 15 *kN*?

7. Find an extension of a cable with a stiffness of 100 *kN*/*m* when towing a car weighing 2 *t* with an acceleration of 0.5 *m/s*.

§18. Weight of a body. Weightlessness. Escape velocities

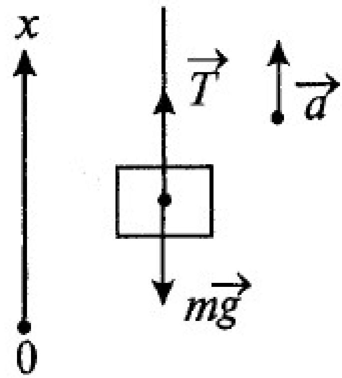
1. Weight of a body

The force with which the body acts on the support or hanger due to the attraction to the Earth is called the weight of a body.



If the support or hanger is stationary or moves at a constant speed ($\vec{a} = 0$), the weight by module is equal to the gravity force: $|\vec{P}| = |\vec{N}|$ or $|\vec{P}| = |\vec{T}|$. According to second Newton's Law $\vec{F} + \vec{T} = m\vec{a}$. $\vec{F} + \vec{T} = 0$, or projected onto the vertical axis $F - T = 0$, $F = T = mg$

Let's look at some examples. Let the surface (support, hanger) move so that the acceleration is directed upwards.



Then the main law of the dynamics for the body of mass m, which is contained on the support:

$$\vec{F} + \vec{T} = m\vec{a}\text{, where } \vec{F} = m\vec{g}$$

Let's write this equation in projections on the axis Ox:

$$T - F = ma$$

The force of the reaction of the support is:

$$T = F + ma$$

According to Newton's third law, the body weight P equals:

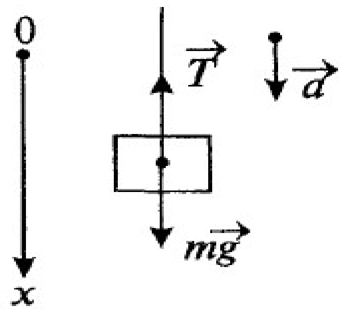
$$\left|\vec{P}\right| = \left|\vec{T}\right|$$

$$P = F + ma, P = mg + ma, P = m(g + a),$$

where a – acceleration module.

Thus, if the body is on a support that moves with acceleration, directed upward, the body weight increases. There is an overload – an increase in body weight caused by an accelerated motion of the support.

Overload occurs in two cases: when the support with the body moves upward uniformly accelerated or moves down uniformly decelerated.



If the body along with the support moves with acceleration, directed vertically downward, the weight of the body decreases:

$ma = mg - T$, $T = m(g - a)$, $P = m(g - a)$

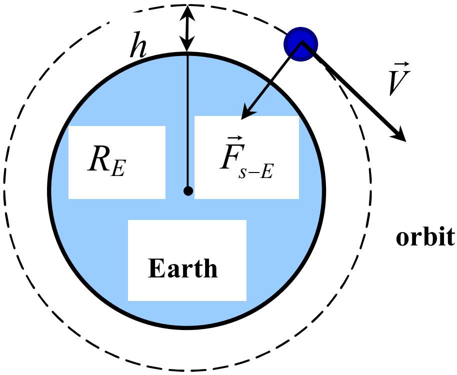
If the body along with the support freely falls, then $a = g$. From the last formula it follows that $P = 0$.

The loss of weight during the body motion with the acceleration of free fall is called **weightlessness**. It is obvious that the state of free fall of any body is a state of its weightlessness.

In the state of weightlessness on all bodies of the mobile system, only the forces of gravity, which give all bodies of the system of the same acceleration, act only. Therefore, these bodies do not push one to another and on supports and do not cause deformation of bodies.

2. Motion of artificial satellites of the Earth

Let's consider the motion of a body in a circular orbit around the Earth.



The satellite moves under the action of gravitational force $F_{s\text{-}E}$:

$$F_{s\text{-}E} = G\frac{mM_E}{\left(R_E + h\right)^2},$$

where $R_E + h$ – orbit radius, R_3 – Earth radius, h – body height above the Earth.

Since the body moves in a circle, its acceleration is centripetal and in modulus it is equal to:

$$a = \frac{V^2}{R_E + h}$$

According to the second Newton's law $F_{s\text{-}E} = ma$ or

$$G\frac{mM_E}{(R_E+h)^2}=\frac{mV^2}{R_E+h},$$

where m, M_E – the mass of the satellite and the Earth.

From this expression we obtain an equation for speed:

$$V=\sqrt{\frac{GM_E}{R_E+h}}.$$

If the body has the velosity in a horizontal direction at a height above the Earth determined by the last formula, then it will move in a circle around the Earth. Such a body is called an ***artificial satellite*** of the Earth.

The speed with which a body (satellite) moves around the Earth in a circular orbit only under the action of gravity is called the ***first cosmic velocity***. When moving, all the bodies in the satellite are in a state of weightlessness.

For an artificial Earth satellite, rotating near the Earth's surface, one can calculate the value of the first cosmic velocity, taking values h=0 and R_E=6370 *km*, g=9,8 *m/s*^{2}. From expression for $g=G\frac{M_E}{R_E^2}$ we obtain $GM_E=gR_E^2$. Then the expression for the satellite speed becomes:

$$V=\sqrt{\frac{GM_E}{R_E+h}}=\sqrt{\frac{gR_E{}^2}{R_E+h}},\ V=V_0=\sqrt{gR_E}\approx 7{,}9\ km/s.$$

At an initial velocity greater than 7.9 *km/s*, but less than 11.2 *km/s*, an artificial Earth satellite moves around the Earth along the ellipse. The higher the initial velocity, the more elongated elipse. At an initial velocity equal to 11.2 *km/s*, the ellipse turns into a parabola and the satellite stops moving around the Earth and becomes a satellite of the Sun. This speed is called the ***second cosmic velocity***.

Thus, the second cosmic velocity is the velocity that the body must recieved to overcome the gravity of the Earth and become a satellite of the Sun.

The ***third space velocity*** is called the velocity, which is required to give the spacecraft near the Earth's surface so that it overcomes the gravity of the Solar system and becomes the satellite of the Galaxy.

The second and third cosmic velocities are the ***escape velocities.***

Words and phrases

weight	escape velocity
support	ellipse
hanger	hyperbola
weightlessness	galaxy
artificial satellite	planetary system

Questions

1. What is the body weight?
2. In which case, body weight is equal to gravity?
3. What is called weightlessness, overload?
4. What is called the first cosmic velocity?
5. In what orbital is the body moving at a second cosmic velocity?

Problems

1. Determine the first cosmic velocity on the surface of the moon, if its radius is 1760 *km*, and the acceleration of free fall on the moon is 6 times less than on the Earth.

2. The rotation time of Jupiter is 12 times greater than the Earth's rotation around the Sun. What is the distance from Jupiter to the Sun, if the distance from Earth to the Sun is $150 \cdot 10^9$ *m*? Orbits are considered circles.

3. Considering the orbits of the Earth's moon is approximately circular, calculate the ratio of masses of the Earth and the Sun. It is known that the Moon makes 13 turns throughout

the year and that the distance from the Sun to the Earth is 390 times greater than the distance from the Moon to Earth.

4. Otis L. Evaderz is conducting his famous elevator experiments. Otis stands on a bathroom scale and reads the scale while ascending and descending the John Hancock building. Otis' mass is 80 *kg*. He notices that the scale readings depend on what the elevator is doing. What is the scale reading when Otis accelerates upward at 0.40 m/s^2.

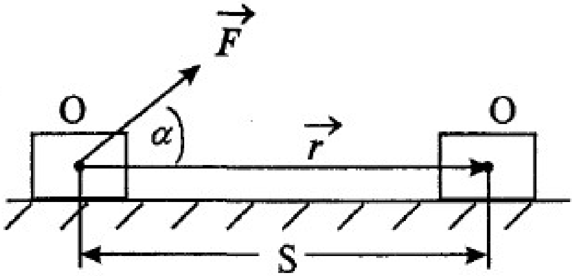
5. Find the height of the geostationary orbit of the Earth.

Chapter 3. Work and energy. Statics

§19. Mechanical work. Power. Efficiency

1. Mechanical work

The physical quantity, which is equal to the scalar product of force and the displacement of the point of its application is called mechanical work.



$\vec{r}$ – the displacement, $S = |\vec{r}|$ – the pass.

By definition, the mechanical work of constant force is equal to:

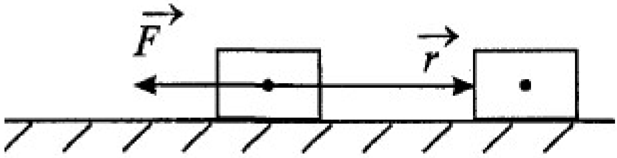
$$A = (\vec{F} \cdot \vec{r}) = |\vec{F}| \cdot |\vec{r}| \cdot \cos\alpha\,,\ A = F \cdot S \cdot \cos\alpha$$

***Unit of work – Joule*:** 1 J = 1 $N·m$. One joule is equal to the work performed by force in one newton while moving one meter if its direction coincides with the direction of motion. The value of the work can be positive or negative, depending on the sign $\cos\alpha$.

Forces which direction coincides with the direction of displacement are **called traction forces**.

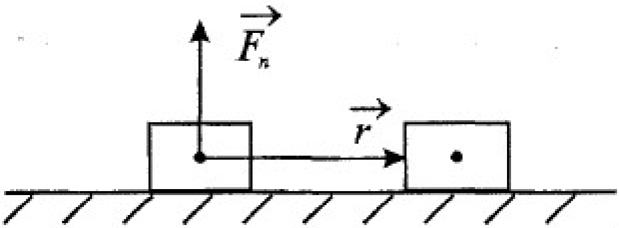
$A = (\vec{F} \cdot \vec{r}) = |\vec{F}| \cdot |\vec{r}| \cdot \cos 0^\circ = FS > 0$ – the work of traction forces is always positive.

If the force of resistance $\vec{F}$ acts on the body, then the work of this force is negative.



$$A = (\vec{F} \cdot \vec{r}) = |\vec{F}| \cdot |\vec{r}| \cdot \cos 180^\circ = FS \cdot (-1) < 0$$

If the normal force $\vec{N}$ acts on the body, then the work of this force is equal to zero.

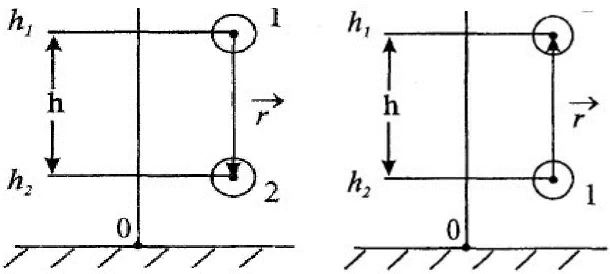


$$A = (\vec{F} \cdot \vec{r}) = |\vec{F}| \cdot |\vec{r}| \cdot \cos 90^\circ = FS \cdot 0 = 0$$

Examples. **The work of gravity** is calculated by the formula:

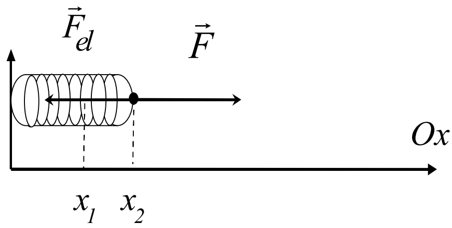
$A = mg(h_1 - h_2)$, where $h_1 - h_2$ – difference in heights in the initial and final position

The body moves down *The body moves upwards*



The body moves down – work is positive. The body moves upward – the work is negative.

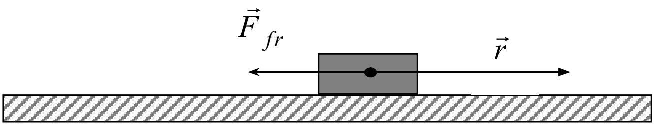
The work of the elastic force $\vec{F}_{el}$ is calculated by the formula:



$$A = \frac{k(x_1^{\,2} - x_2^{\,2})}{2},$$

where k – stiffness, x_1, x_2 – the initial and final coordinates of the free end of the deformed spring.

The work of friction force is always negative, since during friction the force of friction is directed oppositely to the displacement:



$$\mathrm{A} = \left|\vec{F}_{fr}\right| \cdot \left|\vec{r}\right| \cdot \cos 180^{\circ} = FS \cdot (-1) < 0,$$

where F – module of the friction force, S – module of the displacement.

2. Conservative forces

Let the body, under the action of gravity or elasticity, move along a closed trajectory and return to its original position. Then in the previous formulas $h_1 = h_2$, $x_1 = x_2$. Thus, $A = 0$.

The work of gravity or elasticity forces on a closed trajectory is zero.

Forces that have this property are called **conservative.** The forces of gravity and elasticity are conservative forces.

Friction forces are non-conservative forces, their work on a closed trajectory is never zero.

3. Power

The scalar quantity, which is equal to the ratio of the work performed to the interval of time during which it was performed is called power:

$$N = \frac{A}{t}$$

Unit of power measurement in SI is **Watts (*W*)**: 1W=1J/1s. 1W is the power at which 1 J work is performed during 1 s. In science and technology, units of measurement of kilowatt (1 kW = 10^3 W) and megawatts (1 MW = 10^6 W) are often used.

If under the action of constant force of draft F, the body moves uniformly at a speed V, then the power equals:

$$N = \frac{F \cdot S}{t} = FV.$$

With a uniform motion, the power is equal to the product of the force and the speed of motion. With uniform motion, average power is calculated:

$$N_{av} = FV_{av}$$

4. The efficiency (Eff)

In practice, the full work done by the mechanism is always a little bigger than useful work. Part of the work is carried out against the frictional force in the mechanism and the movement of its individual parts.

The ratio of useful work to full work is called the efficiency of the mechanism.

Abbreviated designation of the efficiency is *Eff*. It is denoted by the Greek letter η (read "eta"):

$$\eta = \frac{A_u}{A} = \frac{N_u}{N},$$

where A_u (N_u) – useful work (power), A (N) – full work (power).

Efficiency is expressed as a percentage.

Problem

Engines of the electric locomotive at a speed of 54 *km/h* develop power of 900 *kW*. Determine the traction force, if the efficiency of the engines is 80%.

Given:

V =54 *km/h*=15 *m/s*

$N = 900$ *kW*=$9\cdot10^5$ *W*

$\eta = \underline{80\ \%=0{,}8}$

Find: F –?

Solution

1. Usefull power $N_u = FV$.

2. Efficiancy $\eta = \frac{N_u}{N} = \frac{FV}{N}$.

3. Traction force $\mathrm{F} = \frac{N\eta}{V} = \frac{9\cdot10^5\cdot0{,}8}{15}N = 48\,kN$

Answer: $F = 48\,kN$.

Words and phrases

work	power
joule	watt
conservative forces	efficiency
useful work	full work

Questions

1. What is called work?
2. What is work equal?

3. What is the unit of measurement of work?

4. By what formula calculates the work of gravity force, elasticity force, friction force?

5. What forces are called conservative forces?

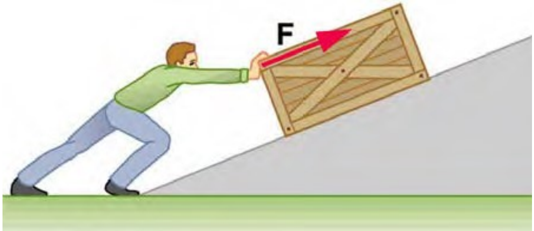
6. What is called power?

7. What is the unit of power measurement?

Remember!
Efficiency is expressed as a percentage

Problems

1. A 75.0-*kg* person climbs stairs, gaining 2.50 meters in height. Find the work done to accomplish this task.
2. A lift of 6 tons rises upwards with a constant acceleration of 1.4 *m/s*. What is the work done if the elevator rises to 100 *m*?
3. A train with a weight of $6{\cdot}10^5$ *kg* uniformly rises to the mountain. Mountain climb 5 *m* per kilometer of the way. Friction force is of 10 *N*. Train speed is of 72 *km/h*. Determine the power of the electric locomotive.
4. A car, mass 3.5 tons, traveled horizontally 10 *km*. What work did the resistance forces, if the resistance coefficient is of 0.06?
5. A car, mass 1.5 tons, goes uphill. What work do the force of gravity and force of the reaction of the support on the segment of the path of 1 *km*, if the slope is of 5^0?
6. If a person who normally requires an average of 12,000 *kJ* (3000 *kcal*) of food energy per day consumes 13,000 *kJ* per day, he will steadily gain weight. It is nown, that 400 *W* are used when cycling at a moderate speed. How much bicycling per day is required to work off this extra 1000 *kJ*?
7. Calculate the work done by an 85.0-kg man who pushes a crate 4.00 *m* up along a ramp that makes an angle of 20.0º with the horizontal (See Fig.). He exerts a force of 500 *N* on the crate parallel to the ramp and moves at a constant speed. Be certain to include the work he does on the crate *and* on his body to get up the ramp.



8. A shopper pushes a grocery cart 20.0 *m* at constant speed on levelground, against a 35.0 *N* frictional force. He pushes in a direction 25.0º below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper?

§20. Mechanical energy

1. Mechanical energy

The general quantitative measure of the motion of matter and various interactions is called ***energy***. Energy characterizes the state of the system and its ability to perform work in the transition from one state to another.

In physics, different forms of energy are considered: mechanical, thermal, electrical, magnetic, gravitational, nuclear, etc.

Changing energy is always associated with performance of work.

The mechanical energy of the body (system) is the scalar physical quantity that characterizes the mechanical state of the body and whose change equals the mechanical work that the system can perform:

$$\Delta E = A\,,\ E_2 - E_1 = A\,,$$

Where E_1– initial energy of the body, E_2 – final energy of the body, A – mechanical work.

The unit of energy measurement in SI is Joule (*J*).

2. Kinetic energy

Let`s find a work that is performed by the force F to move a body, mass m, on a distance S on a horizontal surface. If the direction of force coincides with the direction of displacement, and the initial and final velocities of the body are equal to V_1 and V_2, respectively, then from the formulas for the uniform accelerated motion $V_2 = V_1 + at$ and $S = V_1 t + \frac{at^2}{2}$ we will obtain an expression for displacement

$$S = \frac{V_2^2 - V_1^2}{2a}.$$

The work of a force:

$$A = F \cdot S = ma\frac{V_2^2 - V_1^2}{2a} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2}$$

The scalar quantity $E_K = \frac{mV^2}{2}$, which has the dimension of work, is called **kinetic energy**.

The energy, which is determined by the speed of the body, is called kinetic energy of the body:

$$E_K = \frac{mV^2}{2}$$

Kinetic energy – the relative quantity, it depends on the choice of reference frame. If a force or several forces acts at the same time on the body during its motion, the kinetic energy of the body changes – the body accelerates or stops.

The work of net force applied to the body is equal to the change in the kinetic energy of the body:

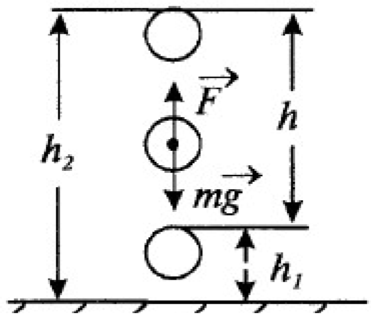
$$A = E_{K_2} - E_{K_1} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2}$$

The last statement and formula expressing it is called the **kinetic energy theorem.**

3. Potential energy

Potential energy is the energy of the interaction of bodies, which depends on the relative positions of bodies or parts of the same body. For example, a body raised above the Earth, a stretched or compressed spring, etc.

Let the body of mass m rises to the height h relative to the ground under the action of force F with constant velocity ($V = const$).



Let`s find the work of gravity force when lifting a body from height h_1 to height h_2:

$$A = mgh \cdot \cos\pi = -mg(h_2 - h_1)$$
$$A = -(mgh_2 - mgh_1)$$

The last formula shows that the work of gravity force is equal to the change of a physical quantity equal to mgh.

The physical quantity, equal to the product of the mass of the body, acceleration of free fall and height, is called gravitational potential energy

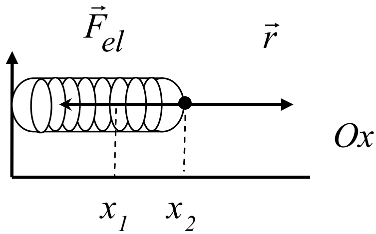
$$E_P = mgh.$$

The last formula is valid if the surface of the Earth is considered as zero energy level and calculate energy at altitudes much smaller than the radius of the Earth. The work of gravity force and the change in the potential energy of the body always has the opposite sign:

$$A = -(E_{P_2} - E_{P_1}).$$

The formula for the work of gravity force can be valid for the work of the elasticity force. The work carried out by the force of elasticity in the deformation of the spring from the initial coordinate x_1 to the final coordinate x_2:

$$A = \frac{k(x_1^2 - x_2^2)}{2}$$



Physical quantity, equal to half of the product of stiffness of the body and magnitude of its deformation, is called the potential elastic potential energy:

$$E_P = \frac{kx^2}{2}$$

Consequently, the work of the forces of elasticity is equal to the change in the potential energy of an elastically deformed body taken with the opposite sign:

$$A = -(E_{P_2} - E_{P_1})$$

Elastic potential energy is energy stored as a result of applying a force to deform an elastic object. The energy is stored until the force is removed and the object springs back to its original shape, doing work in the process. The deformation could involve compressing, stretching or twisting the object. Many objects are designed specifically to store elastic potential energy, for example:

- The coil spring of a wind-up clock
- An archer's stretched bow
- A bent diving board, just before a divers jump
- The twisted rubber band which powers a toy airplane
- A bouncy ball, compressed at the moment it bounces off a brick wall.

An object designed to store elastic potential energy will typically have a high ***elastic limit***, however all elastic objects have a limit to the load they can sustain. When deformed beyond the elastic limit, the object will no longer return to its original shape. In earlier generations, wind-up mechanical watches powered by coil springs were popular accessories. Nowadays, we don't tend to use wind-up smartphones because no materials exist with high enough ***elastic limit*** to store elastic potential energy with high enough energy density.

Potential energy is the energy of interaction between bodies. For example, when it comes to the potential energy of a body raised to a height, it means that the body interacts with the Earth, and the body-Earth system has energy.

The value of potential energy depends on the choice of zero reference level.

The potential elastic energy is the energy of interaction of individual parts of the body among themselves.

Thus, potential energy is the energy of the interaction of bodies, which is determined by the mutual positions of bodies or parts of the body.

4. The Law of Energy Conservation

Consider the interaction of two bodies that form a closed system, that is, they interact only with each other. When the bodies interact with each other through forces of elasticity or gravity, the work done by forces is equal to the change of the potential energy of the systemthe taken with the opposite sign:

$$A = -(E_{P_2} - E_{P_1})$$

This work is equal to the change in kinetic energy, by the theorem of kinetic energy:

$$A = E_{K_2} - E_{K_1} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2}$$

Comparing these expressions, you can write:

$$E_{K_2} - E_{K_1} = -(E_{P_2} - E_{P_1})$$

We see that how much the potential energy decreases, the kinetic increases so much.

The last formula can be rewritten as follows:

$$E_{K_2} + E_{P_2} = E_{K_1} + E_{P_1}$$

Consequently, the sum of the kinetic and potential energy of the bodies that make up the closed system remains unchanged at any given time.

The sum of the kinetic and potential energy of the bodies is called total mechanical energy.

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The law of conservation of energy can be stated as follows:

The total amount of mechanical energy, in a closed system in the absence of dissipative forces (e.g. friction, air resistance), remains constant:

$$E_K + E_P = \text{const} \;\text{ or }\; \Delta E = 0.$$

The law of conservation of energy reveals the physical meaning of the concept of work: the work is equal to the energy that has converted from one form into another.

If the system, together with the forces of gravity and elasticity, have frictional forces or resistance, the total mechanical energy decreases. At the same time, the internal energy of the interacting bodies increases. The reduction of mechanical energy equals the increase of the internal one.

In general, the law of energy conservation and convertation is formulated as follows:

Energy cannot be created or destroyed, but is merely changed from one form into another.

Problem

The body at height of 2.2 *m* above the ground has a speed of 10 *m/s*. At what speed will the body move near the ground? Air resistance can be neglected.

Given:

h_1=2,2 *m*

h_2=0 *m*

V_1=10 *m/s*

g=9,8 m/s^2

Find: V_2-?

Solution

1. We denote the mass of the body m. The change in the kinetic energy of a body equals the change in the potential energy with the opposite sign:

$$A = E_{K_2} - E_{K_1} = \frac{mV_2^2}{2} - \frac{mV_1^2}{2} = -(mgh_2 - mgh_1)$$

$$V_2^2 - V_1^2 = 2g(h_1 - h_2).$$

From here

$$V_2 = \sqrt{2g(h_1 - h_2) + V_1^2}\ ;\ V_2 = \sqrt{2 \cdot 9{,}8 \cdot 2{,}2 + 100} = \sqrt{144} \approx 12\ m/s.$$

Answer: $V_2 \approx 12\ m/s$.

Words and phrases

energy	law of energy conservation
kinetic energy	internal energy
potencial energy	convertation of energy
zero level	equivalent

Questions

1. What is called energy?
2. What is the energy change associated with?
3. What is called kinetic energy?
4. Formulate the kinetic energy theorem.
5. What is called potential energy?
6. Formulate the law of energy conservation for a closed system.
7. Formulate the law of conservation of energy in the general case.

Problems

1. The stone was thrown at angle α to the horizon, giving it an initial velocity of 15 *m/s*. What will be the speed of a stone at height of 10 *m*?

2. Express the kinetic energy of the body through its mass *m* and momentum *p*.

3. The dynamometer, whose spring is stretched by 10 *cm*, shows the force of 100 *N*. What is the potential energy of the stretched spring?

4. The stone was thrown upright at a speed of 15 *m/s*. At what height its kinetic energy will be 2 times higher than the potential energy?

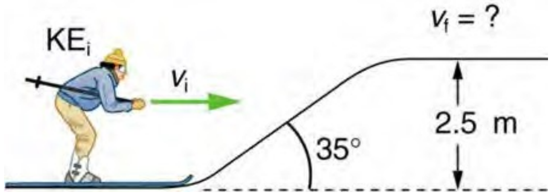
5. The body was thrown horizontally at an initial speed of 25 *m/s*. How many time will the kinetic energy of the body be doubled?

6. A body, mass 2 *kg*, was thrown at an angle to the horizon. At the top of the trajectory at a height of 20 *m*, its kinetic energy was equal to 100 *J*. What was the initial body velocity? At what angle to the horizon it was thrown?

7. Boxing gloves are padded to lessen the force of a blow. Calculate the force exerted by a boxing glove on an opponent's face, if the glove and face compress 7.50 *cm* during a blow in which the 7.00-*kg* arm and glove are brought to rest from an initial speed of 10.0 *m/s*.

8. How fast must a 3000-*kg* elephant move to have the same kinetic energy as a 65.0-*kg* sprinter running at 10.0 *m/s*?

9. A 60.0-*kg* skier with an initial speed of 12.0 *m/s* coasts up a 2.50-*m* high rise as shown in figure below. Find her final speed at the top, given that the coefficient of friction between her skis and the snow is 0.08. (Hint: Find the distance traveled up the incline assuming a straight-line path as shown in the figure.)



10. A 5×10^5-*kg* subway train is brought to a stop from a speed of 0.5 *m/s* in 0.4 *m* by a large spring bumper at the end of its track. What is the force constant (stiffness) *k* of the spring?

§21. Basic concepts of statics

1. Conditions of equilibrium of a body

Static studies **conditions of equilibrium** of a material point or a body under the action of applied forces.

The state of the body, in which it is motionless or moves uniformly rectilinearly, or uniformly rotates around the axis passing through its **center of mass**, is called **equilibrium.**

A point whose radius vector is determined by the expression below is called **center of mass of a system of material points**:

$$\vec{r}_c = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i},$$

where m_i – the mass of the *i*-th material point of the system, $\vec{r}_i$ – *i*-th radius vector, n – the number of material points.

The center of mass is the point in which the entire mass of the system is concentrated, and all external forces acting on this system act on it.

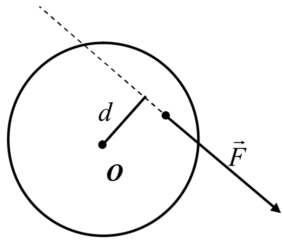
For equilibrium of a material point it is necessary that the geometric sum of forces applied to the point be equal to zero:

$$\sum_{i=1}^{n} F_i = 0 .$$

If we decompose all the forces acting on the material point into the components in the direction of the X and Y axes, then the equilibrium condition will have the form:

$$\sum_{i=1}^{n} F_{ix} = 0 ; \sum_{i=1}^{n} F_{iy} = 0$$

The equilibrium of a solid depends not only on the size and direction of the forces acting, but also on the point at which point they are applied.



The magnitude, direction, and point of application of the force are incorporated into the definition of the physical quantity called torque. **Torque is the rotational equivalent of a force. It is a measure of the effectiveness of a force in changing or accelerating a rotation (changing the angular velocity over a period of time).**

Torque is a vector quantity. The direction of the torque vector depends on the direction of the force on the axis. We will consider just the module of torque (projection of torque on the chosen direction – axis of rotation).

In equation form, the magnitude of torque is defined to be

$$\vec{M} = \vec{F} \times \vec{d} \ ,$$

where M is the symbol for torque, d is the **perpendicular lever arm**. The perpendicular lever arm is the shortest distance from the pivot point to the line along which force F acts.

The torque that cause to turn the body relative to the axis of rotation counter clockwise is considered positive, and in the clockwise direction it is negative.

The **SI unit of torque** is newtons times meters, usually written as $N{\cdot}m$. For example, if you push perpendicular to the door with a force of 40 N at adistance of 0.800 m from the hinges, you exert a torque of 32 $N{\cdot}m$ (0.800 m×40 N×sin 90º) relative to the hinges. If you reduce the force to 20 N, the torque is reduced to 16 $N{\cdot}m$, and so on.

For the body equilibrium, it is necessary to fulfill two conditions:

1. The algebraic sum of torques relative to any point must be equal to zero:

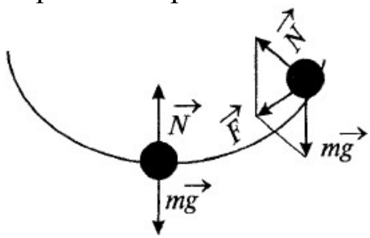
$$\sum_{i=1}^{n} M_i = 0\text{, where } n \text{ – number of torques.}$$

2. The vector sum of all forces applied to the body must be zero:

$$\sum_{i=1}^{n} \vec{F_i} = 0\text{, where } n \text{ – number of forces.}$$

2. Types of equilibrium

Stable equilibrium is a state where, for a slight deviation from the equilibrium position, there is a net force or torque that returns the body to the equilibrium position.

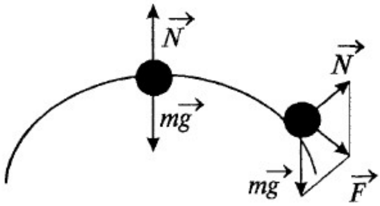


A system is said to be in stable equilibrium if, when displaced from equilibrium, it experiences a net force or torque in a direction opposite to the direction of the displacement. For example, a body at the bottom of a bowl will experience a *restoring* force when displaced from its equilibrium position. This force moves it back toward the equilibrium position. Most systems are in stable equilibrium, especially for small displacements.

Stable equilibriumis a state with minimal potential energy.

Unstable equilibrium is a state, when, for a body deviates from equilibrium position, a net force or torque in the same direction as the displacement from equilibrium arise.

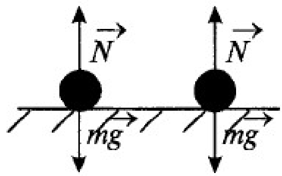
A system is in **unstable equilibrium** if, when displaced, it experiences a net force or torque in the *same* direction as the displacement from equilibrium. A system in unstable equilibrium accelerates away from its equilibrium position if displaced even slightly. An obvious example is a ballresting on top of a hill. Once displaced, it accelerates away from the crest.



Unstable equilibrium is a state with maximum potential energy.

Neutral equilibrium is the state when, for any displacement of the body from the equilibrium position, the net force and torque remains equal to zero.

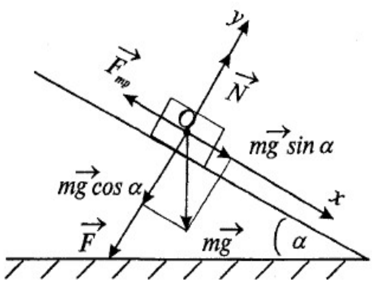
A system is in **neutral equilibrium** if its equilibrium is independent of displacements from its original position. A body on a flat horizontal surface is an example.



Problem 1

The body lies on an inclined plane. What force F should this body be pressed to the inclined plane so that it remains in equilibrium on it? The body mass is of 2 kg, length of inclined plane – 1 m, height – 60 cm.The coefficient of friction betwen the body and inclined plane is of 0.4.

Given:
m=2 kg
l=1 m
h=0.6 m
μ=0.4
Find: F-?



1. Let`s concider the forces acting on the body: $m\vec{g}$ – body weight, $\vec{N}$ – the force of the normal reaction of the inclined plane, $\vec{F}_{fr}$ – friction force, $\vec{F}$ – the force that pressure the body to the inclined plane.

2. Let`s write the equation of equilibrium for the body:

$$\sum_{i=1}^{n} \vec{F}_i = 0 , \ m\vec{g} + \vec{N} + \vec{F} + \vec{F}_{fr} = 0 ,$$

3. Let`s write the equation of equilibrium in projections on the axis *Ox* and *Oy*:

$$mg\sin\alpha - F_{fr} = 0 , \ N - mg\cos\alpha - F = 0 .$$

Consider that $\mathrm{F}_{fr} = \mu N$, and $\sin\alpha = h/l$, $\cos\alpha = \dfrac{\sqrt{l^2 - h^2}}{l}$.

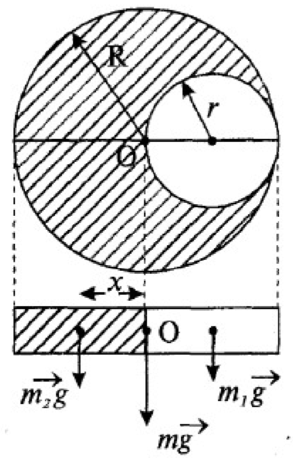
We solve the obtained equation system and find the force *F*:

$$F = \frac{mg}{l}\left(\frac{h}{\mu} - \sqrt{l^2 - h^2}\right) = \frac{2 \cdot 9{,}8}{1}\left(\frac{0{,}6}{0{,}4} - \sqrt{1-(0{,}6)^2}\right) \approx 14H.$$

Answer: $F \approx 14\,\mathrm{N}$

Problem 2

A homogeneous flat plate has the form of a circle of radius R, from which a circle of twice the smaller radius, touches the center of the plate, is carved out. Determine the position of the center of the mass of the plate.



Given:
R
r=R/2
Find: *x -?*

1. If you insert the carved part into the previous place, then the weight of the entire body can be given as sum of two

forces: $m_1\vec{g}$ and $m_2\vec{g}$ – the weights of the carved part and the remaining part, respectively. The plate will be in equilibrium relative to the axis passing through the point O.

2. The condition of equilibrium of the system relative this axis has the form:

$-M_1 + M_2 = 0$, here $M_1 = m_1 gr$ and $M_2 = m_2 gx$.

$$-m_1 gr + m_2 gx = 0, \quad x = \frac{m_1}{m_2} r .$$

3. The masses of homogeneous plates of the same thickness are:

$$m = \rho sh = \rho\pi R^2 h, \quad m_1 = \rho s_1 h = \rho\pi r^2 h,$$

$$m_2 = m - m_1 = \rho\pi h(R^2 - r^2),$$

where ρ – density, s_1 – cutout area, s – area of the entire plate, h – thickness of the plate.

Let's substitute magnitudes of m_1 and m_2 into equation for x, also consider that $r = R/2$:

The center of mass (gravity) is located at a distance x from the center of the plate.

$$x = \frac{\pi\rho hr^3}{\pi\rho h(R^2 - r^2)} = \frac{r^3}{R^2 - r^2} = \frac{R^3}{8\left(R^2 - \frac{R^2}{4}\right)} = \frac{R}{6}.$$

Answer: $x = R/6$.

Words and phrases

equilibrium	stable equilibrium
center of mass(gravity)	unstableequilibrium
condition of equilibrium	neutral equilibrium
torque	Statics

Questions

1. What does statics study?
2. What is called body balance?
3. What is called torque?

4. What is called the center of mass (center of gravity) of the body?

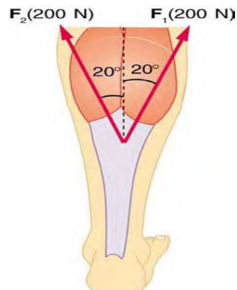
5. What are the conditions for the equilibrium of the material point and the body under the action of the applied forces?

6. What types of balance do you know? Give examples.

Problems

1. A painting of mass 3 *kg* is hung off a wall with two strings attached to the top two corners of the painting. Each string makes an angle of 45 degrees with the ceiling. What is the tension in the strings?

2. Two muscles in the back of the leg pull on the Achilles tendon asshown in the figure below. What total force do they exert?

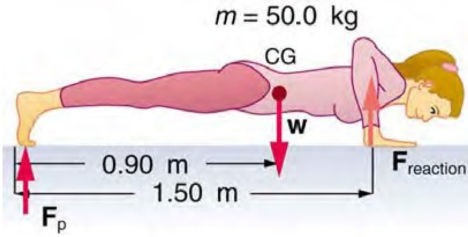


3. When opening a door, you push on it perpendicularly with a force of 55.0 *N* at a distance of 0.850 m from the hinges. What torque are youexerting relative to the hinges? Does it matter if you push at the same height as the hinges?

4. Two children of mass 20 *kg* and 30 *kg* sit balanced on a seesaw with the pivot point located at the center of the seesaw. If the children are separated by a distance of 3 *m*, at what distance from the pivot point isthe small child sitting in order to maintain the balance?

5. A person carries a plank of wood 2 *m* long with one hand pushing down on it at one end with a force F_1 and the other hand holding it up at 50 *cm* from the end of the plank with force F_2. If the plank has a mass of 20 *kg* and its center of gravity is at the middle of the plank, what are the forces F_1 and F_2?

6. What force should the woman in the figure belowexert on the floorwith each hand to do a push-up? Assume that she moves up at a constant speed. The triceps muscle at the back of her upper armhas an effective lever arm of 1.75 *cm*, and she exerts force on the floorat a horizontal distance of 20.0 *cm* from the elbow joint. Calculate themagnitude of the force in each triceps muscle, and compare it to herweight.



7. To get up on the roof, a person (mass 70.0 *kg*) places a 6.00-*m* aluminum ladder (mass 10.0 *kg*) against the house on a concrete pad with the base of the ladder 2.00 *m* from the house. The ladder rests against a plastic rain gutter, which we can assume to be frictionless. The center of mass of the ladder is 2 *m* from the bottom. The person is standing 3 *m* from the bottom. What are the magnitudes of the forces on the ladder at the top and bottom?

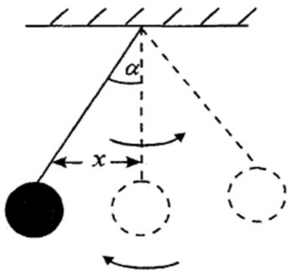
Chapter 4. Mechanical oscilations and waves

§22. Mechanical oscilations

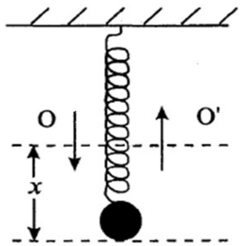
1. Characteristics of mechanical oscilations

Oscillations are called the following types of motion or changes in the state of the system, which are periodically repeated in time. Due to their physical nature, oscillations are divided into **mechanical** and **electromagnetic ones.** By the nature of oscillations it can be **free, forced** and **self-oscillating.**

Mechanical oscilations are periodic movements with a repetition of the equilibrium position of the system.



simple pendulum



spring pendulum

system are called **free oscillations.**

Oscillations that arise in the system under the action of periodic external forces are called **forced oscillations.**

Oscillations caused by a constant external influence on a system that regulates itself are **auto-oscillation**. An example of a self-oscillating system is a clock with a pendulum or a mechanical wristwatch.

Oscillations are characterized by the following parameters: **amplitude, phase, period, frequency**.

Frequency ν is the number of occurrences of a repeating event per second.

Cyclic frequency is the number of oscillations per 2π seconds.

The SI derived unit of frequency is the herz (Hz), named after the German physicist Heinrich Hertz; one hertz means that an event repeats once per second. The cyclic frequency is measured in seconds minus first degree.

The amplitude of oscillation is the maximum displacement of the system from the equilibrium position.

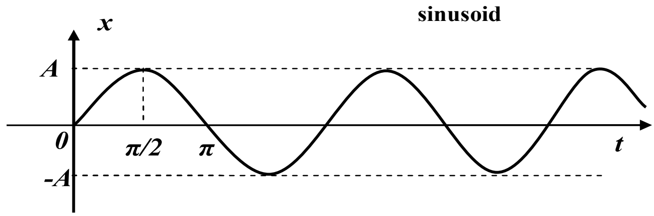
The period of oscillations is the time interval through which the value of the parameter characterizing the oscillatory system is repeated.

Phase of oscilations $\varphi = \omega t + \varphi_0$ is a quantity that specifies the instantaneous values of the variables of the oscillation system parameters. At the beginning of the time frame ($t = 0$) the phase is equal to the initial phase φ_0.

The oscillations of the body relative to the equilibrium position is given by a function $f(t)$ that describes the dependence of the displacement x on time t: $x = f(t)$.

2. Harmonic oscillations

Periodic oscillations of the system, during which its parameters change according to the law of the sinus or cosine, are called **harmonic oscillations**.



The equation of harmonic oscillations is the equation of the form:

$$x = A sin(\omega t + \varphi_0).$$

Where – x displacement of the vibrational point from the equilibrium position, $A = x_0$ amplitude, ω – cyclic frequency, t –

time, $(\omega t+\varphi_0)$ – phase of oscillations, φ_0 – initial phase. **The phase** determines the degree of deviation from the position of equilibrium at the moment t.

The speed during harmonic oscillations is determined by the formula:

$$V=\omega A\cos(\omega t+\varphi_0)=V_0\sin\left(\omega t+\frac{\pi}{2}+\varphi_0\right),$$

where $V_0=\omega A$ – amplitude of speed. The phase of the speed ahead of the phase of displacement at $\frac{\pi}{2}$.

The acceleration during harmonic oscillations:

$$a=-\omega^2 A\sin(\omega t+\varphi_0)=-a_0\sin(\omega t+\varphi_0)=-\omega^2 x,$$

where $a_0=\omega^2 A$ – amplitude of acceleration. The phase of the acceleration ahead of the phase of displacement at π.

The acceleration phase is ahead of the phase of coordinates at π radians (in antiphase). **The phase of oscillation is measured in radians (rad).**

During harmonic oscillations the acceleration is directly proportional to the displacement:

$$a=-\omega^2 x$$

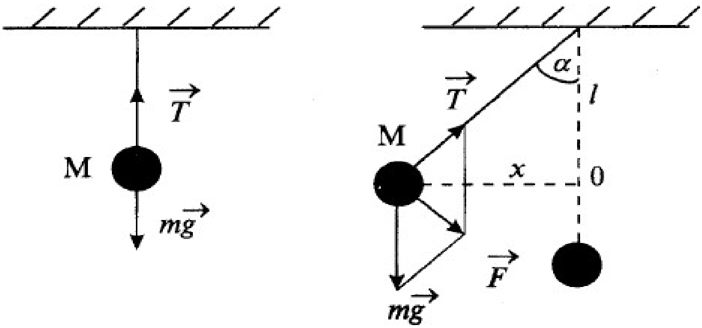
Consequently, the projection of force on the direction of oscillation equals:

$$F_x=ma=-m\omega^2 x$$

Remember another definition of harmonic oscillations. Oscillations that occur under the action of force, directed to the equilibrium position of the system and directly proportional to the displacement from this position, are called harmonic oscillations.

3. Simple pendulum

A simple pendulum is called a material point suspended on a weightless and rigid thread, which moves in a vertical plane under the action of gravity.



The pendulum oscillates by the force of gravity $m\vec{g}$ and the force of the tension $\vec{T}$ of the thread. The module of the net force is equal $F = mg\sin\alpha$.

At small angles of deviation $\sin\alpha = \frac{x}{l}$, where x is displacement of the pendulum from the equilibrium position, l – thread length.

Taking into account that the direction of displacement and net force are opposite, we obtain:

$$F = -mg\frac{x}{l}.$$

The acceleration of pendulum is: $a = -g\frac{x}{l} = -\omega_0{}^2 x$,

where $\omega_0 = \frac{g}{l}$, ω_0 – natural cyclic frequency.

The period of oscillations of the simple pendulum is determined by the formula:

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$$

Similarly, one can find the oscillation period for a ***spring pendulum***. The spring pendulum is called a system consisting of a spring of small mass with stiffness k, to which the body mass m is attached.

Period of oscillations of the spring pendulum is determined by the formula:

$$T=\frac{2\pi}{\omega_0}=2\pi\sqrt{\frac{m}{k}}$$

Let`s find the kinetic and potential energy of a spring pendulum:

$$E_K=\frac{mV^2}{2}=\frac{m\omega^2A^2}{2}\sin^2\omega t,$$

$$E_P=\frac{kx^2}{2}=\frac{m\omega^2A^2}{2}\cos^2\omega t.$$

In the process of oscillation, the transformation of potential energy into a kinetic and vice versa occurs. The value of total energy at each instant of time is equal to the maximum kinetic or maximum potential energy:

$$E_K+E_P=\frac{m\omega^2A^2}{2}\sin^2\omega t+\frac{m\omega^2A^2}{2}\cos^2\omega t=\frac{m\omega^2A^2}{2}.$$

During oscillations the ***law of conservation of mechanical energy*** is carried out.

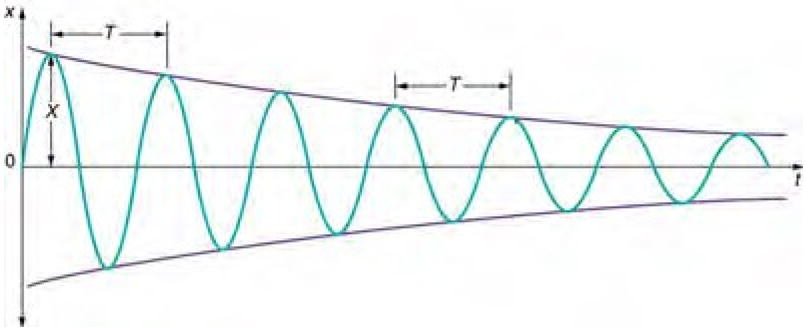
The total mechanical energy of the vibrational system remains unchanged in the process of harmonic oscillations:

$$E=E_K+E_P=\frac{m\omega^2A^2}{2}=\frac{kA^2}{2}=const.$$

4. Damped oscillation

Oscillations, whose energy decreases over time are called damped oscillations. Damping occurs due to the effects of friction and resistance forces.

A mass-spring system or a pendulum will not go on swinging for ever. Energy is gradually lost to the surroundings due to air resistance or some other resistive force and the oscillations die away. This effect is called **damping**.



In road vehicles, dampers (wrongly called 'shock absorbers') are fitted to the suspension springs so that unwanted oscillations die away quickly. Some systems (for example moving-coil ammeters and voltmeters) have so much damping that no real oscillations occur. The minimum damping needed for this is called **critical damping**.

The rate at which the amplitude falls depends on the fraction of the existing energy that is lost during each oscillation.

In a **lightly damped system** only a small fraction is lost so that the amplitude of one oscillation is only slightly lower than the one before.

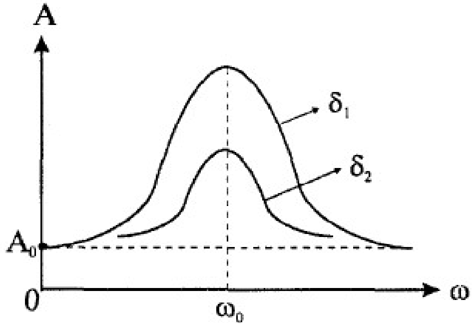
5. Forced oscillations. Resonance

Forced oscillations can be caused by external force, which periodically changes. The frequency of forced oscillations is equal to the frequency of external force. If the external force $F(t)$ varies according to the harmonic law $F(t) = F_0 \cos \omega t$ (F_0 – amplitude), then in the system also establishes harmonic oscillations.

If the frequency of external force approaches natural frequency of the pendulum, the amplitude of the system fluctuations will increase sharply.

The phenomenon of a sharp increase in the amplitude of forced oscillations of the system when the frequency of

external force approaches natural frequency of oscillations of the system is called resonance: $\omega = \omega_0$ **.**



The less friction, the greater the amplitude during resonance. The figure shows two resonant curves – the least friction is characteristic of the curve δ_1 ($\delta_1 < \delta_2$).

Problem 1

The simple pendulum with a period of oscillations of 2 *s* has a length of 0.9973 *m*. Determine the acceleration of gravity.

Given:
$T = 2\ s$
$l = 0.9973\ m$
Find: g-?

Solution

Period of oscillations of simple pendulum is

$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{l}{g}}$. From here $g = \frac{4\pi^2 l}{T^2} \approx 9{,}882\ m/s^2$.

Answer: $g \approx 9{,}882\ m/s^2$.

Problem 2

The point carries out harmonic oscillations with a period of 2 seconds. The amplitude of oscillation is 10 *cm*. Find displacement, speed and acceleration of the point in 2 *s* after passing the equilibrium position. The beginning of oscillations coincides with the equilibrium position.

Given:
$T = 2\ s$
$A = 10\ cm = 0.1\ m$
$t = 2\ s$
$\varphi_0 = 0$
Find: x, V, a -?

Solution

1. Displacement of the point is:

$$x = A sin(\omega t + \varphi_0),$$

$$x = A\sin\frac{2\pi}{T}t,\ x = 0.1\sin\frac{2\cdot 180}{2}\cdot 0.2 = 0.1\cdot\sin 36^\circ \approx 0{,}06\,m.$$

2. Speed of the point is:

$$V = \omega A\cos(\omega t + \varphi_0);\ V = \frac{2\pi}{T}A\cos\frac{2\pi}{T}t,$$

$$V = \frac{2\cdot 3.14}{2}\cdot 0.1\cdot\cos 36^\circ \approx 0{,}25\,m/s.$$

3. Acceleration of the point is:

$$a = -\omega^2 A\sin(\omega t + \varphi_0),\ a = -\omega^2 A\sin(\omega t + \varphi_0),\ a = -\frac{4\pi^2}{T^2}x,$$

$$|a| = \frac{4\cdot 3.14^2}{4}\cdot 0.06 \approx 0{,}57\ m/s^2.$$

Answer: $x \approx 0{,}06\ m,\ V \approx 0{,}25\ m/s,\ |a| \approx 0{,}57\ m/s^2.$

Words and phrases

oscillations	period
periodic motion	frequency
oscillating system	cyclic frequency
mechanical oscillations	amplitude
electromagnetic oscillations	harmonic oscillations
free oscillations	simple pendulum
forcing oscillations	spring pendulum
auto-oscillations	resonance

Questions

1. What are called oscillations?
2. What are free and forced oscillations?
3. What are auto-oscillations?
4. What do you know about the characteristics of oscillation processes?
5. What oscillations are called harmonic oscillations?
6. What is the phase of oscillation?
7. Draw a graph of harmonic oscillation.
8. How the period of oscillations of the simple pendulum depends on its length?
9. How does the energy conversion occur during the oscillation of the spring pendulum?
10. What is called the resonance phenomenon?

Remember!
The frequency of natural oscillations coincides with the frequency of external forces

Problems

1. If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?
2. A tire has a tread pattern with a crevice every 2.00 *cm*. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 *m/s*?
3. How does the period of spring pendulum oscillation change if the amplitude of the oscillations is doubled?
4. A 0.500-*kg* mass suspended from a spring oscillates with a period of 1.50 *s*. How much mass must be added to the object to change the period to 2.00 *s*?
5. Find the stiffness of the spring, if a 700-*g* load suspended from it performs 18 oscillations in 21 seconds?
6. Determine the acceleration of free fall at a point of the Earth, if the simple pendulum with period of oscillation of 1 *s* placed there will be 0.995 *m* length.

7. A pendulum that has a period of 3 *s* and that is located where the acceleration due to gravity is 9.79 m/s^2 is moved to a location where it the acceleration due to gravity is 9.82 m/s^2. What is its new period?

8. A diver on a diving board is undergoing simple harmonic motion. Her mass is 55.0 *kg* and the period of her motion is 0.8 *s*. The next diver is a male whose period of simple harmonic oscillation is 1.05 *s*. What is his mass if the mass of the board is negligible?

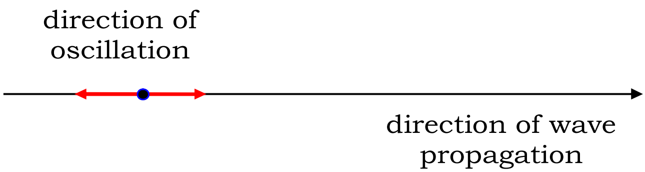
9. Calculate the wave velocity of the ocean wave if the distance between wave crests is 10 *m* and the time for a seagull to bob up and down is 5 *s*.

§23. Mechanical waves

1. Longitudinal and transverse waves

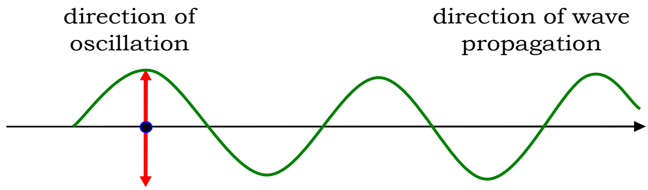
The process of propagation of oscillations in an elastic medium is called ***mechanical waves***. If the direction of oscillation coincides with the direction of propagation of the wave, then such a wave is called a ***longitudinal wave***.

Model of longitudinal wave.



If the direction of oscillation is perpendicular to the direction of propagation of the wave, then the wave is ***transverse***.

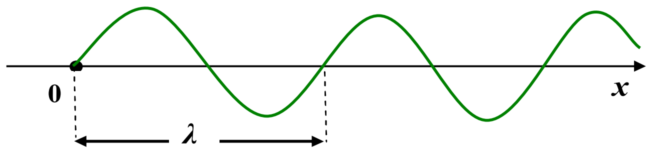
Model of transverse wave.



The distance that the wave passed through during its period is called the **wavelength**:

$$\lambda = VT \ ,$$

where λ is wavelength, T – period of oscillations, V – speed of wave propagation (see fig.).



The **wavelength** is the spatial period of a periodic wave – the distance over which the wave's shape repeats.

The wavelength can be written in a different form:

$$\lambda = VT = \frac{V}{\nu} = \frac{2\pi V}{\omega},$$

where $\nu = \frac{1}{T}$ – frequency of oscillation, ω – cyclic frequency.

If the medium is gas or liquid, then it can only be longitudinal waves, which represent the alternation of stretching and compression. If the medium is solid, then the elastic wave can be both transverse and longitudinal.

2. Sound

Sound is an elastic wave in a physical medium that, by its frequency and intensity, can be perceived by the organs of hearing. Sound oscillations, which have a frequency of less than 18 *Hz*, are called infrasound, from 20 *kHz* to 10^9 *Hz* – ultrasound, and higher than 10^9 *Hz* – hypersounds.

The ***speed of sound*** in the air (at 0^0 *C*) is 332 *m/s*, in water – 1500 *m/s*, in steel – 5500 *m/s*. In vacuum, the sound does not spread.

The speed of sound depends on the elastic properties of the medium, temperature, pressure. In most liquids, the speed of sound decreases with a decrease in temperature. Exception is water. In water, the speed of sound when heated by 1 *K* increases by 2.5 *m/s*.

Sound is a complex phenomenon: on the one hand, it is elastic vibrations characterized by a certain frequency, intensity, on the other hand – psychophysiological sensation of a certain height, loudness and timbre in human perception. Human hearing organs are capable of receiving sound waves at frequencies from **18** to **20,000** *Hz*. Human is best suited for sound oscillation with frequencies of 1000 - 3000 *Hz*.

In the perception of sound, a person distinguishes three characteristics of the sound – ***height***, ***timbre***, ***volume*** or ***loudness***. These characteristics are subjective – the same sound can be evaluated differently by different people. The musical sound has a linear spectrum of frequencies. This means that sound is a complex oscillation, which can be decomposed into several simple or harmonic oscillations with certain frequencies and amplitudes.

Oscillations with the lowest frequency gives the height of the musical tone and is called the ***main tone***, or the ***first harmonic***. The higher the frequency of the main tone, the higher the perceived sound. Oscillations with higher frequencies are called ***overtones*** or ***harmonics***. A set of overtones determines the sound of the tone.

Objective volume estimation is its intensity or sound power.

The intensity of the sound is measured by the average value of the sound energy, which passes through the unit time through a unit of area perpendicular to the direction of sound propagation.

The ***sound power*** is proportional to the square of the amplitude.

Words and phrases

mechanical waves	infrasound
transverse waves	ultrasound
longitudinal waves	sound power
wavelength	volume
speed of sound	timbre
sound	overtone

Questions

1. What is called a wave motion or a wave?
2. What types of mechanical waves do you know?
3. What wave is called longitudinal?
4. What wave is called transverse?
5. In which media does longitudinal and transverse waves are propagated?
6. What is called wavelength?
7. What oscillations are called sound?
8. What is the timbre, volume and intensity of the sound wave?

Remember!
1. Mechanical waves propagates in an elastic medium   **2. The tone of sound is determined by the main frequency**

Problems

1. What is the speed of sound in a material in which sound waves with a frequency of 900 *Hz* have a wavelength of 5 *m*?

2. How many times does the length of the sound waves change when the wave moves from air to water? The speed of sound propagation in the air and water is 340 *m/s* and 1450 m/s, respectively.

3. Storms in the South Pacific can create waves that travel all the wayto the California coast, which are 12,000 *km* away. How long does it take them if they travel at 15 *m/s*?

4. How many times a minute does a boat bob up and down on oceanwaves that have a wavelength of 40 *m* and a propagation speed of 5 *m/s*?

5. Scouts at a camp shake the rope bridge they have just crossed andobserve the wave crests to be 8 *m* apart. If they shake it the bridgetwice per second, what is the propagation speed of the waves?

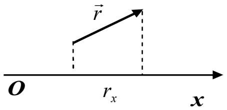
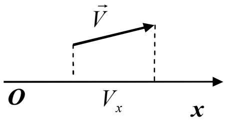
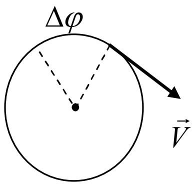
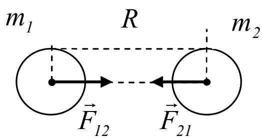
6. What is the wavelength of an earthquake that shakes you with a frequency of 10 *Hz* and gets to another city 84 *km* away in 12 *s*?

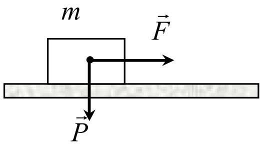
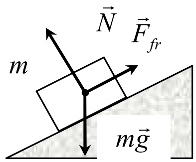
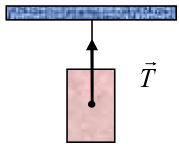
7. Your ear is capable of differentiating sounds that arrive at the ear just 1 *ms* apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different time son a day when the speed of sound is 340 *m/s*?

8. What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2 *Hz* and the waves propagate at 0.8 *m/s*?

9. Seismographs measure the arrival times of earthquakes with a precision of 0.1 s. To get the distance to the epicenter of the quake, they compare the arrival times of *S*- and *P*-waves, which travel at different speeds. If *S*- and *P*-waves travel at 4 and 7.2 *km/s*, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined?

§24. Basic physical quantities of mechanics

Figure	Denotation	Unit of measurement
 	$\vec{r}$ – radius vectors of displacement $\lvert\vec{r}\rvert$ – module of displacement r_x – projection of displacement S – path x – coordinate x_0 – initial coordinate	1 m (meter)
	t – time	1 s (second)
	$\vec{V}$ – velocity $\lvert\vec{V}\rvert$ – module of velocity V_x – projection of velocity	1 m/s (meter per second)
 	$\Delta\varphi$ – turning angle $\vec{V}$ – linear velocity ω – angular velocity T – period ν – frequency of rotation m – mass $\vec{F}_{12}, \vec{F}_{21}$ – gravity forces R – distance G – universal gravitational constant	1 rad (radian) 1 m/s (meter per second) 1 rad/s (radian per second) 1 s (second) 1 Hz (herz) 1 kg (kilogram) 1 N (newton) 1 m (meter) $6{,}67\cdot10^{-11}m^3/(kg\cdot s^2)$

  	μ – coefficient of friction k – coefficient stiffness $\vec{P}$ – body weight $m\vec{g}$ – gravity force $\vec{N}$ – force of normal pressure $\vec{F}_{fr}$ – friction force $\vec{T}$ – tension force	dimensionless value 1 N/m (Newton per meter) 1 N (newton)
	$\vec{p} = m\vec{V}$ – momentum $\vec{F}t$ – impulse A – mechanical work N – power E – energy η – efficiency	1 kg·m/s (kilogram· meter per second) 1 N·s (newton·second) 1 J (Joule) 1 W (watt) 1 J (Joule) % (percent)

SECTION II

MOLECULAR PHYSICS AND THERMODYNAMICS

Chapter 5. Molecular physics

§25. Kinetic theory

1. Basic principles of kinetic theory

Molecular physics studies thermal phenomena caused by the motion and interaction of molecules of substances.

Kinetic theory is a scientific theory that explains thermal phenomena, physical properties of bodies and substances in various aggregate states by their molecular structure, interaction and motion of molecules.

1. Physical bodies have a discrete structure. They consist of particles (molecules, atoms, ions).
2. The particles are in a continuous chaotic motion.
3. Particles interact with each other. This interaction is characteristic of the forces of attraction and repulsion.

The discrete structure of matter, the existence of atoms and molecules is proved by a large number of observations and experiments: spread of odor, sound, the diffusion phenomenon, expansion and compression of bodies, evaporation, microscopic photos of molecules, etc.

2. Size and mass of molecules

Modern research methods allow you to accurately determine the size of molecules and atoms. These dimensions are approximately the same for molecules of any substance.

The linear dimensions of atoms and molecules are of the order of 10^{-10} *m*. The size of the complex molecules is much larger. So protein molecules have linear dimensions of $43 \cdot 10^{-10}$ *m*.

The size of an atom is determined by the distance from the center of the nucleus to the edge of the orbit, which contains the external valence electrons.

Mass of a molecule is very small. For example, mass of hydrogen molecule $m_{H_2} = 3.34 \cdot 10^{-27}$ *kg*, mass of water molecule $m_{H_2O} = 30.0 \cdot 10^{-27}$ *kg*.

In practice, using such numbers is not convenient. Therefore, for the unit of measurement of the mass of molecules adopted **атоmic mass unit (amu)**.

The atomic mass unit is 1/12 of the mass of the most common isotope of carbon. It has been experimentally established that **amu** equals to $1.66 \cdot 10^{-27}$ *kg*.

The more molecules in the body, the greater the amount of substance contained in this body. The relative number of atoms or molecules in the body is called **amount of substance**.

The ratio of the number of molecules *N* to the number of atoms N_A in 0.012 *kg* of carbon C_6^{12} is called the amount of substance:

$$n = \frac{N}{N_A}$$

The amount of substance is measured in moles.

The mole is defined as the amount of substance that contains an equal number of elementary entities as there are atoms in 12 *g* of carbon C_6^{12}.

This number is called Avogadro's number and has the value $6.022140857(74) \cdot 10^{23}$.

The ratio of the number of substance molecules to the amount of substance is called **Avogadro's number**:

$$N_A = \frac{N}{n}.$$

Avogadro's number and has the value $N_A = 6.02 \cdot 10^{23}\ mol^{-1}$. It shows how many atoms or molecules are contained in one mole of substance.

The molar mass *M* is a quantity equal to the ratio of the mass of the substancem to the amount of substance ***n***:

$$M = \frac{m}{n}.$$

The unit of measurement of molar mass is kilogram per mole (***kg/mol***).

To determine the mass of the molecule m_m, you have to divide the molar mass by Avogadro's number:

$$m_m = \frac{M}{N_A}.$$

3. Forces of interaction between molecules

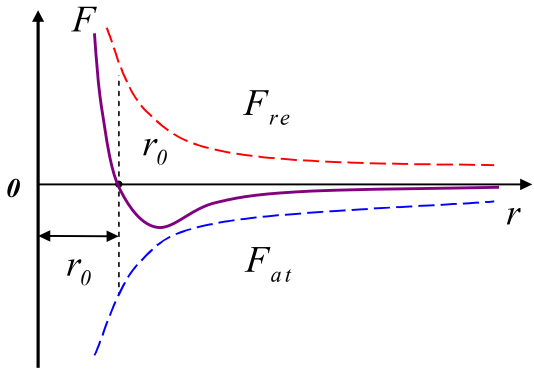
The forces of intermolecular interaction are a combination of forces of attraction and repulsion. According to experimental and theoretical studies, the forces of interaction between molecules are inversely proportional to the distances between the particles: the forces of attraction $F_{at} = -\frac{b}{r^n}$, repulsive forces $F_{re} = \frac{a}{r^m}$, where $r-$ distance between particles, a,b,m,n – constants for a given substance.

The dependence of the interaction forces $F(r)$ of molecules on the distance is depicted on the graph. Let one molecule be at the origin of the coordinate *O* and the other one at a distance from it.

From the graph it is clear that the interaction forces are very much dependent on the distance between the molecules, especially the repulsive force.

For $r > r_0$, the attraction of molecules exceeds repulsion, and when $r < r_0$, the repulsion molecules exceed the attraction.

For $r = r_0$, the force of attraction is equal to the forces of repulsion. This state corresponds to the most stable position of the molecules. The distance $r = r_0$ is called the ***equilibrium distance***.



The simultaneous existence of the forces of attraction and repulsion means that on the particles actes net force $F(r)$. The analysis of the function $F(r)$ gives an explanation of the molecular mechanism of the appearance of forces of elasticity in solids. The force of stretching prevents the stretching of the body and promotes the return of particles to their original position. Conversely, the forces of repulsion counteract the external forces of compression.

4. Thermal motion of molecules

The continuous chaotic motion of particles is called thermal motion.

The thermal motion is characterized by an root mean square velocity V_{sq}:

$$V_{sq} = \sqrt{\overline{V}^2} = \sqrt{\frac{V_1^2 + V_2^2 + \ldots + V_N^2}{N}}.$$

Where $\overline{V}^2$ – mean square of speed. It should not be confused with the square of the average speed $\overline{V}^2 \neq (\overline{V})^2$.

The nature of thermal motion of molecules, atoms, ions depends on the physical state of the substance.

In gases, the force of attraction is not able to keep particles together, they flight and occupy the entire volume.

In gases, particles move straight and chaotic to collide with other particles. When particles meeting, they changes their directionsof motion. The trajectory of each particle is a broken line. The kinetic energy of the molecule is much greater than the potential energy of the interaction of molecules $E_K >> E_P$.

In solids, the particles carry out chaotic oscillatory motions near the equilibrium position. In solids $E_K << E_P$.

In liquids the particles carry out chaotic oscillatory motion near the equilibrium position and rectilinear displacement into new equilibrium position. In liquids $E_K \approx E_P$.

Words and phrases

theory	diffusion
discrete	brownian motion
continuous	mole
chaotic	avogadro's number
thermal motion	molar mass
attracting forces	oscillating motion
repulsive forces	root mean square velocity (speed)

Questions

1. What does molecular physics study?
2. Formulate the basic principles of kinetic theory.
3. What are the linear sizes of molecules?
4. What is the amount of substance?
5. What is Avogadro's number?
6. How do the forces of interaction between molecules depend on the distance between them?

7. What is called thermal motion?
8. How gas particles moves?

Problems

1. Calculate the mass of the nitrogen molecule ($M_{N_2} = 28 \cdot 10^{-3}\ kg/mol$, $N_A = 6.02 \cdot 10^{23}\ mol^{-1}$).

2. How many molecules is contained in 5 cubic centimeter of carbon dioxide gas under normal conditions (T_0=273 K, P_0=1.01 · 10^5 Pa)?

§26. Basic equation of kinetic theory

1. Ideal gas

In gases, the forces of interaction of molecules are so small that they can be neglected. Dimensions of gas molecules are also small compared with the distances between them. Therefore, to simplify the calculations, one can neglect the volume of molecules and the forces acting between them. This simplification makes it possible to replace the study of real gases by studying their approximate model - ***ideal gas***.

Ideal gas is a model of real gas, which ignore the size of the molecules whose only interactions are perfectly elastic collisions.

The concept of ideal gas helps to abstract from the features of real gas and formulate laws that are common to all gases.

2. Temperature and molecular velocity

The molecule of the ideal gas moving with speed V_i has kinetic energy $E_{Ki} = \dfrac{mV_i^2}{2}$.

The sum of the kinetic energies of all gas molecules is the total kinetic energy. If all the molecules are the same, then:

$E_K = \frac{m}{2}\sum_{i=1}^{N} V_i^2$, where N is a number of molecules.

Let`s Divide the total kinetic energy of molecules E_K into the number of molecules N: $\frac{E_K}{N} = \frac{m}{2N}\sum_{i=1}^{N} V_i^2$.

The ratio $\frac{E_K}{N}$ is called the ***average kinetic energy*** of the translational motion of a molecule. The last formula can be written as follows:

$$\overline{E}_K = \frac{m}{2}\overline{V^2}.$$

The quantity of $\overline{V^2}$ is called average of the molecular speed squared.

Experiments and calculations show that the average kinetic energy of the translational motion of a molecule is directly proportional to the absolute temperature:

$$\overline{E}_K = \frac{m}{2}\overline{V^2} = \frac{3}{2}kT,$$

where $\frac{3}{2}k$ is coefficient of proportionality.

Consequently, **absolute temperature *T* is the measure of the average kinetic energy of the molecules thermal motion**. This is the physical content of absolute temperature.

Coefficient k is called Boltzmann's constant ($k = 1.38 \cdot 10^{-23}\ J/K$). It shows how much the kinetic energy of one molecule changes with temperature change per Kelvin.

From equality $\overline{E}_K = \frac{m}{2}\overline{V^2} = \frac{3}{2}kT$, it follows that $\overline{V^2} = \frac{3kT}{m}$.

The square root of this value is called root mean square speed (*rms*) of the molecules thermal motion:

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}},$$

where $R = k \cdot N_A$ is molar gas constant; M – molar mass, m – mass of a molecule.

Molar gas constant is:

$$R = 8.31 \frac{J}{K \cdot mol}.$$

Let`s calculate the root mean square speed of the molecules. For example, at $T = 273\ K$ for nitrogen atoms ($M = 14 \cdot 10^{-3}\ kg/mol$) $V_{rms} \approx 450\,m/s$. Hydrogen molecules at this temperature have a root mean square speed $V_{rms} \approx 1700\,m/s$.

The speed of gas molecules under normal conditions is very high. The molecules move faster than sound in the air.

3. Gas pressure

Gas pressure is a result of the impact of a large number of molecules on the wall of the vessel. Since the number of molecules are very large and they interact with the wall very often, it is possible to replace their total action on the surface of the wall with one continuous force. The value of this force per unit of a surface of the wall determines the gas pressure acting on the wall of a vessel:

$$P = \frac{F}{S},$$

where P is pressure, F – force, S – surface area.

Pressure is a scalar quantity equal to the force acting on a unit of surface area perpendicular to this surface.

Unit of pressure in SI – ***Pascal (Pa)***: 1 Pa=1N/m^2.

The force of molecules impact, at a constant mass of gas, depends on the value of kinetic energy of thermal motion of the molecules. Calculations show that the pressure of the ideal gas is:

$$P = \frac{2}{3} n_0 \overline{E_K},$$

where $\overline{E_K}$ – the average value of the kinetic energy of an ideal gas, $n_0 = \frac{N}{V}$ – the concentration of molecules, that is, the number of molecules per unit volume: N – number of molecules, V – gas volume.

This equation is called the **basic equation of kinetic theory of ideal gas.**

The pressure of ideal gas is proportional to the product of molecules concentration and average kinetic energy of translational motion of molecules.

The basic equation of the kinetic theory establishes the relationship between the microscopic quantities (molecular mass, the number of particles per unit volume, the average kinetic energy of a molecule) and the macroscopic quantity -pressure that characterizes the gas as a whole and can be measured in the experiment.

Words and phrases

Boltzmann'sconstant	concentration of molecules
ideal gas	pressure
absolute temperature	molecular-kinetic theory

Questions

1. What is called ideal gas?
2. What is the physical content of absolute temperature?
3. What is a root mean square speed?
4. How can you determine gas pressure?
5. **5.** Formulate the basic equation of the kinetic theory of gases and write down its formula.

6. Between what physical quantities the basic equation of kinetic theory sets the relationship?

Remember!
Gas pressure is the result of molecules impacts

Problems

1. Determine the root mean square speed and the average kinetic energy of the thermal motion of oxygen molecules at a temperature of $27^{\circ}\ C$.

2. What is the average kinetic energy of hydrogen molecule if the gas mass of 2.5 kg takes a volume of 3 m^3 with pressure of 150 kPa?

§27. Properties of gases

1. Gas laws

The state of gas is determined by its volume, pressure and temperature. The equations connecting these quantities are called ***gas laws***. P – gas pressure, V – volume and T – temperature are the ***parameters of the gas state***.

Isoprocesses are processes in which one of the parameters of the gas state remains constant.

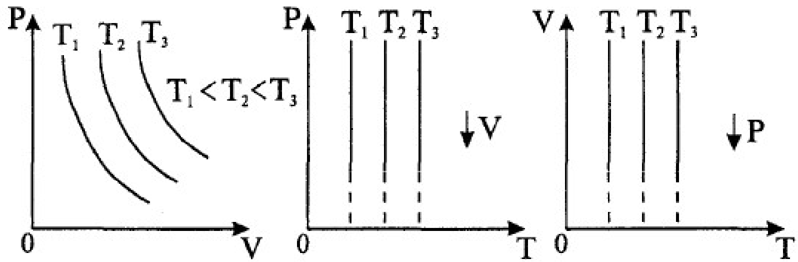
The **isothermal process** is a process of changing the state of gas at a constant temperature.

Boyle-Mariotte law: for a given mass of gas at constant temperature, the product of gas volume on the gas pressure is a constant value.

$$PV = const\ ,\ (m = const; T = const\ ).$$

For any two states: $P_1V_1 = P_2V_2$.

isotherms



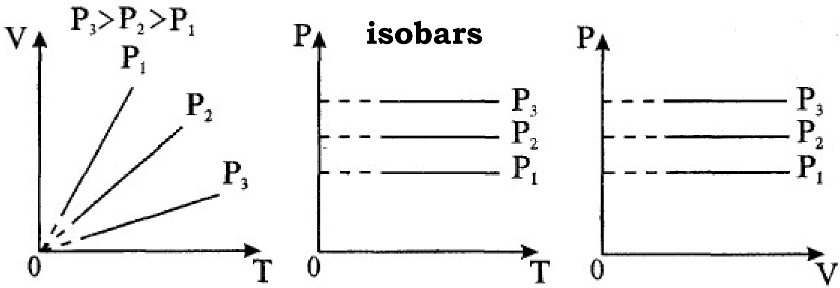
The isobar process is the process of changing the state of gas at a constant pressure.

Gay-Lussac law: heating a gas by 1K at constant pressure leads to an increase in gas volume by 1/273 parts of volume at $T_0 = 273K$:

$$V_t = V_0(1+\beta t) \text{ or } V_T = V_0 \beta T,$$

where $V_T = V_t - V_0$, β is volumetric thermal expansion coefficient. For all of gases $\beta = \frac{1}{273} K^{-1}$.

For two states of gas: $\frac{V_1}{V_0} = \frac{T_1}{T_0}$ or $\frac{V_1}{T_1} = \frac{V_0}{T_0}$



The isochloric process is a process of changing the state of gas at a constant volume.

Charles law: heating a gas by 1 K at a constant volume leads to an increase in gas pressure by 1/273 parts of pressure at $T_0 = 273K$:

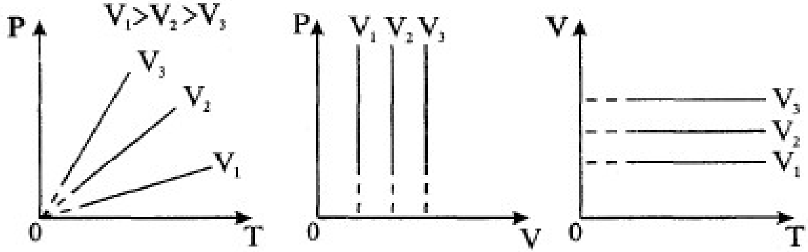
$$P_t = P_0(1+\gamma t)\,, P_T = P_0\gamma T$$

where $P_T = P_t - P_0$, γ is thermal pressure coefficient. For all of gases $\gamma = \frac{1}{273}K^{-1}$.

For two states of gas:

$$\frac{P_1}{P_0} = \frac{T_1}{T_0} \text{ or } \frac{P_1}{T_1} = \frac{P_0}{T_0}\,.$$

isochorou



2. Clapeyron's equation

Clapeyron's equation generalizes gas laws.
At $m = const$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}\,.$$

For a given mass of gas, the product of pressure on the volume, divided by the absolute temperature, is a constant value:

$$\frac{PV}{T} = const\,.$$

Often, the state of gas is compared with the state under normal conditions in the SI: $P_0 = 1.01\cdot 10^5\ N/m^2$; $T_0 = 273K$.

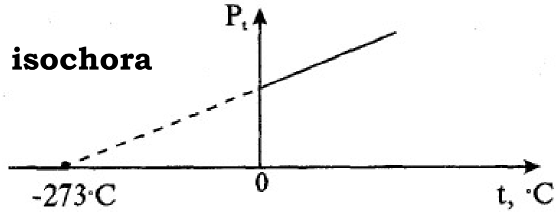
3. Absolute zero temperature

The dependence of gas pressure on temperature on the scale of Celsius has the form:

$$P_t = P_0(1+\gamma t),$$

where P_0 is gas pressure at $T_0 = 273K$.

Let's illustrate this dependence on the graph:



The continuation of the isochora crosses the axis of temperature at a point $t = -273^\circ C$ at which the gas pressure is zero:

$P = 0$, $0 = P_0(1+\gamma t)$. But $P_0 \neq 0$, it means $1+\gamma t = 0$.

From here $t = -\dfrac{1}{\gamma} = -273{,}15^\circ C \approx -273^\circ C$.

The point $t = -273^\circ C$, taken as the origin (zero) of the **Kelvin scale**:

$$0K \approx -273^\circ C.$$

The physical content of the absolute zero is that at this temperature gas pressure disappears. In practice, it is impossible to reach a temperature of 0 *K*, and this temperature can be approaching infinitely close. In modern physics, the temperature reaches 10^{-7} *K*.

4. Dalton's Law

If the pressure creates a mixture of n gases, then the next expression is valid:

$$P = \sum_{i=1}^{n} P_i \ ,$$

where P_i is partial pressure.

Partial pressure is the pressure that would create each gas individually.

Dalton's Law: The pressure of a gas mixture equals the sum of their partial pressures.

Problem 1. The gas at a temperature of 455 K has a volume of V_0. At what temperature is it necessary to cool the gas at constant pressure to make it volume $(2/5)V_0$?

Given:

$T_1 = 477K$

$V_1 = V_0$

$V_2 = \dfrac{2}{5}V_0$

$P_1 = P_2$

Find: $T_2 - ?$

Solution

From the law of Gay-Lussac

$$\frac{V_1}{V_2} = \frac{T_1}{T_2} \ .$$

From here $T_2 = \dfrac{V_2 T_1}{T_1}$, $T_2 = \dfrac{\frac{2}{5}V_0 \cdot 455K}{V_0} = 182K$.

Answer: $T_2 = 182K$.

Problem 2. The outdoor temperature is -13° *C*, in the room 22° *C*. How will the pressure in the gas cylinder change if the balloon is brought into the room? In the room, the pressure gauge on the cylinder showed 1.5 *MPa* ($1.5 \cdot 10^6$ *Pa*).

Given:

$T_1 = 273K - 13K = 260K$

$T_2 = 273K + 22K = 295K$

$P_2 = 1{,}5 \cdot 10^6\ \Pi a$

Find: ΔP - ?

Solution

1. The process of heating the gas in a closed cylinder is isochoric ($V = const$).

By the law of Charles $\frac{P}{T} = const$. Pressure change when heating gas $\Delta P = P_2 - P_1$, $P_1 = P_2 - \Delta P$.

2. Charles's equation for two states $\frac{P_2}{T_2} = \frac{P_2 - \Delta P}{T_1}$, from here

$$\Delta P = \frac{P_2(T_2 - T_1)}{T_2}, \ \Delta P = \frac{1.5 \cdot 10^6 \cdot 35}{295} Pa = 0.18 \cdot 10^6 Pa = 0.18 MPa.$$

Answer: $\Delta P = 0.18 MPa$.

Words and phrases

state parameters	thermal pressure coefficient
isothermal process	Kelvin
isobar process	Dalton`slaw
isochoric process	parcial pressure
volumetric thermal expansion coefficient	manometer

Questions

1. What are called isoprocesses?
2. What do you know isoprocesses?
3. Formulate the gas laws.

4. What is the physical content of absolute zero?
5. Formulate the Dalton`slaw.

Remember!
The temperature is approaching infinitely close to absolute zero

Problems

1. At a certain temperature, the gas pressure is $2\cdot10^5$ *Pa*. On how many degrees the gas temperature will rise at a constant volume, if the gas pressure has become $2.5\cdot10^5$ *Pa*.

2. At 0° *C*, the gas has a pressure of $2\cdot10^6$ *Pa*. What is the gas pressure at 819° *C* if the gas volume does not change?

3. Determine the air mass in a room of 60 m^3 at a temperature of 20° *C* and a pressure of 10^5 *Pa*, if the air density under normal conditions is 1.29 kg/m^3.

§28. Equation of state

1. The ideal gas law

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volumeoccupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product *PV* is aconstant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of *V*. The ideal gas law describes thebehavior of real gases under most conditions.

In the state of thermal equilibrium for a given mass of gas, the relation is valid:

$$\frac{PV}{T} = const = B\ .$$

To calculate the constant B we use Avogadro's law at $P_0 = 1.013 \cdot 10^5\,Pa$; $T_0 = 273K$, $V_M = 0.0224\,m^3/mol$:

$$\frac{P_0 V_0}{T_0} = \frac{1.013 \cdot 10^5\,Pa \cdot 0.0224\,m^3/mol}{273K} = 8.31\frac{J}{K \cdot mol}\ .$$

This constant is denoted by the letter R and is called **the universal gas constant**.

Therelationship between the pressure, volume, and temperature is given by **the equation of state** called **the ideal gas law**. For a one molethis equation is as follows:

$$PV_M = RT,$$

where V_M is **molar volume** (the volume of one mole of gas).

For an arbitrary mass of gas $m = nM$, where n – number of moles, M – molar mass. For n moles the volume $V = nV_M$.

Let`s multiply the equation $PV_M = RT$ by n: $PnV_M = nRT$, frome here

$$PV = \frac{m}{M}RT\ .$$

This equation is called **the equation of state of ideal gas or the ideal gas law.**

Here P is gas pressure, V – volume, M – molar mass, T – temperature, R – universal gas constant.

Problem 1. How many molecules of air is in a room of 240 m^3 at a temperature of 288 K and a pressure of 10^5 Pa?

Given:

$V_1 = 240 м^3$

$T_1 = 288K$

$P_1 = 10^5\,Па$

Find: N -?

Solution

1. The number of molecules in the room is determined by the formula: $N = nN_A$, where n is number of moles, N_A – Avogadro's number.

2. The number of moles we find from the ideal gas law: $P_1V_1 = nRT$. From here $n = \dfrac{P_1V_1}{RT_1}$. Then $N = \dfrac{P_1V_1}{RT_1}N_A$.

$$N = \frac{6.02\cdot 10^{23}mol^{-1}\cdot 10^5\dfrac{N}{m^2}\cdot 240m^3}{8,31\dfrac{J}{K\cdot mol}\cdot 288K} \approx 6\cdot 10^{27}.$$

Answer: $N \approx 6\cdot 10^{27}$.

Problem 2. In a 5 liter cylinder it is 5 *kg* of oxygen at 300 *K*. What mass of gas should be released from the cylinder so that at a temperature of 350 *K* the pressure decreased by $2\cdot 10^4$ *Pa*?

Solution

1. Let`s write the ideal gas law for two states of gas:

$P_1V = \dfrac{m_1}{M}RT_1$ and $P_2V = \dfrac{m_1 - \Delta m}{M}RT_2$. Let's subtract second equation from the first:

$$(P_1 - P_2)V = \Delta PV = \frac{R}{M}(m_1T_1 - m_2T_2).$$

From here $\Delta m\dfrac{R}{M}T_2 = \Delta PV - \dfrac{R}{M}(m_1T_1 - m_2T_2)$,

$$\Delta m = M\frac{\Delta PV + \dfrac{Rm_1}{M}(T_2 - T_1)}{RT_2} = \frac{M\Delta PV}{RT_2} + \frac{m_1(T_2 - T_1)}{T_2},$$

$$\Delta m = \frac{32kg\cdot 5\cdot 10^{-3}m^3\cdot 2\cdot 10^4 kg\cdot m\cdot s^2\cdot mol\cdot K}{10^3 mol\cdot s^2\cdot m^2\cdot 8,3kg\cdot m^2\cdot 350K} + \frac{5kg\cdot 50K}{350K} = 0,71kg.$$

Answer: $\Delta m = 0,71kg$.

Words and phrases

equation of state	ideal gas law
molar volume	universal gas constant

Questions

1. What is the equation of state?
2. How can we determine the universal gas constant?
3. Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?
4. A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

Remember!
The ideal gas law relates the pressure and volume of the gas to the number of gas molecules and the temperature of the gas

Problems

1. Determine the mass of air required to fill the car's tire if its volume is 12 liters. The tire is pumped at a temperature of $27^0\,C$ to a pressure of $2.2{\cdot}10^5\,Pa$.
2. A 100 liter cylinder contains 5.76 kg of oxygen. At what temperature there is a danger of explosion, if the cylinder withstands pressure up to $5{\cdot}10^6\,Pa$?
3. How many air molecules are contained in a room of 240 m^3 at temperature of $15^0\,C$ and a pressure of 750 $mmHg$?
4. The vessel is completely filled with hydrogen at T_1=291 K and external pressure P_1=$1.01{\cdot}10^5\,Pa$. As a result of an increase in the temperature of the outside air to T_2=310 K and a decrease in pressure to P_2=$0.985{\cdot}10^5\,Pa$, the excess gas leaked through the

opening, thereby reducing the weight of the vessel by a value of 59 *N*. Determine the initial mass of hydrogen.

5. The gauge pressure in your car tires is $2.5{\cdot}10^5$ N/m^2 at a temperature of 35º *C* when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to – 40º *C*?

6. What is the gauge pressure in a 25.0º*C* car tire containing 3.60 *mol* of gas in a 30.0 *l* volume? What will its gauge pressure be if you add 1.00 *l* of gas originally at atmospheric pressure and 25.0º*C*? Assume the temperature returns to 25.0º*C* and the volume remains constant.

7. Calculate the number of moles in the 2-*l* volume of air in the lungs of the average person. Note that the air is at 37º*C* (body temperature).

8. An airplane passenger has 100 cm^3 of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.5{\cdot}10^4$ N/m^2?

9. The number density of gas atoms at a certain location in the space above our planet is about 1.00×10^{11} m^{-3}, and the pressure is $2.75{\cdot}10^{-10}$ N/m^2 in this space. What is the temperature there?

10. In the deep space between galaxies, the density of atoms is as low as 106 atoms/m^3, and the temperature is a frigid 2.7 *K*. What is the pressure? What volume (in m^3) is occupied by 1 mol of gas? If this volume is a cube, what is the length of its sides in kilometers?

Chapter 6. Thermodynamics

§29. Basic concepts of thermodynamics

1. The internal energy of ideal gas

Thermodynamics is a section of physics that studies heat phenomena in gases, liquids and solids, based on the transformation of the thermal form of matter motion into other types of energy.

Atoms and molecules not only are in a chaotic motion, but all the time interact with each other. In gases it is a short-term collision. In liquids and solids it is a constant interaction of atoms, molecules, or ions, in which they fluctuate around relatively stable equilibrium positions.

The internal energy of the body consists of the kinetic energy of the thermal motion of atoms and molecules and the potential energy of their interaction:

$$U = \sum_N E_{K_N} + \sum_N E_{P_N} \ ,$$

where N is number of particles.

A body or a set of bodies in which processes associated with changes in internal energy occurs is called a ***thermodynamic system.***

Ideal gas is a thermodynamic system.

The internal energy of an ideal gas is equal to the sum of the kinetic energies of the motion of molecules:

$U = E_{K_1} + E_{K_2} + ... + E_{K_N}$, where N is number of molecules.

The average kinetic energy of the molecules of an ideal gas is determined by the expression:

$$\overline{E_K} = \frac{3}{2} kT \ .$$

If the gas contains N molecules, then their total energy is:

$$U = \frac{3}{2} kTN$$

Hence, it is fair to assert that the change in the internal energy of an ideal gas in a thermodynamic process is determined only by a change in its temperature:

$$\Delta U = U_2 - U_1 = \frac{3}{2} Nk(T_2 - T_1).$$

There are two ways to change the internal energy of gas:
1) when performing work (compression or expansion of gas);
2) at heat exchange (heating or cooling of gas).

The change in internal energy of a body as a result of work equal to the value of this work:

$$A = \Delta U = U_2 - U_1,$$

where ΔU is change in internal energy of a body.

The internal energy of the body can change with heat exchange between the bodies without work. A quantitative change in the internal energy when heat exchanged is called **amount of heat** and is denoted by the letter ***Q***. The amount of heat is measured in units of energy *J* (Joule).

The change in internal energy due to heat exchange is equal to the amount of heat that a body received or lost as a result of heat exchange:

$$Q = \Delta U = U_2 - U_1,$$

where U_1 and U_2 are initial and final value of internal energy, respectively.

The given amount of heat causes an increase in body temperature or a change in its aggregate state.

There are three types of heat exchange: **thermal conduction, convection, radiation.**

Thermal conductionis the process of transferring heat from the heated parts of the body to the less heated, which leads to temperature equalization without transfer of matter.

Metals have the highest thermal conductivity, among which the best conductors of heat are copper (Cu) and silver (Ag).

The worst conductors of heat are gases.

During the transfer of heat in liquids and gases, heat-exchange processes, which are accompanied by the transfer of matter, take place.

Heat transfer due to the transfer of matter in gases and liquids is called **convection**.

There is a kind of heat exchange that does not require an environment. This particular type of heat exchange is called **radiation**.

Thermal energy is emitted by all without exception of the body, because they all have internal energy. The energy emitted by the body depends on its temperature: the higher the body temperature, the more energy it emits.

Due to thermal radiation, heat is transmitted in vacuum, in space. The energy of the Sun get in on Earth and supports life on it.

2. Specific heat

The amount of heat provided to a body due to heating depends on its mass, substance and temperature difference in final and initial states:

$$Q = Cm(T_2 - T_1) = Cm\Delta T,$$

If the body is cooling, then $T_1 > T_2$, so the amount of heat released during cooling the body is equal to: $Q = Cm(T_1 - T_2)$,
where C – specific heat. The unit of specific heat in SI is ***J/(kg·K***).

$$C = \frac{Q}{m\Delta T}.$$

The physical quantity, which is determined by amount of heat that one needs to give 1 kg of substance to increase its temperature by 1 K, is called specific heat of substance.

Numerical value of specific heat of substance determines the change in internal energy of a body weighing 1 *kg* with a temperature change by 1 *K*. It is established that different substances have different specific heat. Hidrogen and water has the highest specific heat: $C_{H_2} = 14.2 \cdot 10^3$ $J/(kg \cdot K)$, $C_{H_2O} = 4.18 \cdot 10^3$ $J/(kg \cdot K)$.

Problem. A metal body, whose specific heat equal to C=125 $J/(kg \cdot K)$, falls from height of h=1200 m. During the collision with ground the body heats up. How many degrees did the body temperature increase, if 50% of mechanical energy turned into internal energy of the body?

Given:

$C = 125 \dfrac{J}{kg \cdot K}$

$U = 0.5E_P$

$\underline{h = 1200\ m}$

Find ΔT –?

Solution

1. Change of internal energy of the body: $\Delta U = Q = Cm\Delta T$.
2. The body at altitude h has potential energy $E_P = mgh$.

Under condition $0.5E_P = Q$. So $0.5mgh = Cm\Delta T$.

From here $\Delta T = \dfrac{0.5gh}{C} = \dfrac{0.5 \cdot 9.8 \cdot 1200}{125} K = 47K$.

Answer: $\Delta T = 47K$.

Words and phrases

thermodynamics	thermodynamic system
internal energy	amount of heat
heat exchange	specific heat
thermal conductivity	thermal conduction
cooling	Radiation
heating	Convection

Questions

1. What does thermodynamics study?
2. What is internal energy of the body?
3. What is called thermodynamic system?

4. What are the ways of changing the internal energy of the body?

5. What is called amount of heat?

6. What is called specific heat?

7. What kinds of heat transfer exist in nature? What is the physical content of specific heat of substance?

Problems

1. What body has a greater internal energy: a piece of ice at 0^0 *C* or water produced from this piece of ice at 0^0 *C*?

2. It was mixed of 20 liters of water at 12^0 *C* and 40 liters of water at 80^0 *C*. Determine final temperature of the mixture if, during mixing, the heat loss was of 100 *J*?

3. The body flies at a speed of 200 *m/s*, its specific heat is 125 *J*/(*kg·K*). How will the body temperature change if all the kinetic energy turns into internal energy of the body?

4. A locomotive weighing 200 *t*, moving at a speed of 72 *km/h*, was stopped by the brakes. How much heat was released during braking?

§30. The laws of thermodynamics

1. The first law of thermodynamics

Internal energy of the body can be changed in the process of heat transfer or work, and both processes can take place simultaneously.

For example, gas in a cylinder under a piston can be heated by transferring to it certain amount of heat, and at the same time work is being done to expand or compress it.

The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. **The first law of thermodynamics states that the change in internal energy of a system equals the net heat transfer into the system minus the net work done by the system. In equation form, the first law of thermodynamics is**

$$\Delta U = Q - A,$$

here *ΔU* is the change in internal energy *U* of the system. *Q* is the net heat transferred into the system – that is, *Q* is the sum of all heat transfer into and out of the system. *A* is the net work done by the system – that is, *A* is the sum of all work done on or by the system. We use the following sign conventions: if *Q* is positive, then there is a net heat transfer into the system; if *A* is positive, then there is net work done by the system. So positive *Q* adds energy to the system and positive *A* takes energy from the system.

Let's consider the first law of thermodynamics in different iso-processes:

1) **isothermal process** ($T = const,\ \Delta U = 0$): all the heat goes to the gas work $Q = A$.

2) **isobaric process**: $Q = \Delta U + A$.

3) **isochoric process** ($V = const$), $A = 0$: the total amount of heat is spent on changing the internal energy of the gas $Q = A$.

Given the equivalence of the quantities Q and A, the first law of thermodynamics can be formulated as follows: **energy does not appear and does not disappear, but only transforms from one of its form to another**.

Adiabatic process occurs without heat exchange with the environment, and gas can perform work on external bodies due to its internal energy.

The establishment of the first law of thermodynamics was due to the failure of the creation of a machine that would have infinitely long operation without the flow of heat from the outside. In thermodynamics, such mashine is called the **"eternal" engine (Perpetuum Mobile) of the first kind: it is impossible to work indefinitely for a long time due to the internal energy of the system.**

2. Heat balance equation

Let's consider an isolated thermodynamic system in which the change in the internal energy of the bodies occurs only in the heat transfer process $\Delta U = Q$. All of bodies that are in heat exchange eventually reach the state of thermal equilibrium when their temperatures are aligned.

Being in the heat exchange process, some bodies give the amount of heat that others receive. This condition is called **heat balance**.

Heat balance equation is based on the fact that the sum of amount of heat received by the body is equal to the sum of amount of heat that was given to other bodies due to heat exchange:

$$\sum_{i=1}^{n} Q_i^{out} = \sum_{j=1}^{m} Q_j^{in} ,$$

where n – the number of bodies giving heat, n – the number of bodies receiving heat.

Heat balance equation: the amount of heat given by the bodies whose internal energy decreases is equal to the amount of heat received by the body, whose internal energy is increasing.

3. Specific heat of combustion of fuel

When combustion of different types of fuels a different amount of heat is allocated.

Specific heat of combustion of fuel is the amount of heat that is released when the combustion is 1 kg of fuel:

$$q = \frac{Q}{m} ,$$

where q – specific heat of combustion of fuel. The value q is measured in joules per kilogram (*J/kg*).

Always when burning fuel in the system, more heat will be allocated than is used for useful purposes.

The ratio of useful heat Q_u to the heat quantity which released with the full fuel combustion Q, is called the efficiency of the heater:

$$\eta = \frac{Q_u}{Q} = \frac{Cm_1 \Delta T}{mq},$$

where η – efficiency; C – specific heat of a body; m_1 – mass of a body; ΔT – change in body temperature when heated; q – specific heat of combustion of fuel; m – mass of fuel.

Typically, the heater's efficiency is expressed as a percentage:

$$\eta = \frac{Q_u}{Q} \cdot 100\%.$$

4. The second law of thermodynamics

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only – that is, they are **irreversible**, under a given set of conditions. Although **irreversibility** is seen in day-to-day life – a broken glass does not resume its original state, for instance – complete irreversibility is a statistical statement that cannot be seen during the lifetime of the universe. More precisely, an **irreversible process** is one that depends on path. If the process can go in only one direction, then the reverse path differs fundamentally and the process cannot be **reversible**. For example, as noted in the previous section, heat involves the transfer of energy from higher to lower temperature. A cold object in contact with a hot one never gets colder, transferring heat to the hot object and making it hotter. Furthermore, mechanical energy, such as kinetic energy, can be completely converted to thermal energy by friction, but the reverse is impossible. A hot stationary object never spontaneously cools off and starts moving. Yet another example is the expansion of a puff of gas introduced into one corner of a vacuum chamber.

The gas expands to fill the chamber, but it never regroups in the corner. The random motion of the gas molecules could take them all back to the corner, but this is never observed to happen.

The process is called **reversible** if the system returns to its original state without any changes in the environment.

If after the completion of the process in the surrounding bodies or in this system there are some changes, the process is **irreversible**.

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur–none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but which in fact are equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes. The already familiar direction of heat transfer from hot to cold is the basis of our first version of the second law of thermodynamics.

The provision on the irreversibility of natural processes is one of the general formulations of the second law of thermodynamics.

The physical content of this law is revealed in this formulation: **heat can not pass from the body of the less heated to the body more heated.**

From the wording of this law it follows that one can not build a machine that would only work at the expense of obtaining heat from the environment.

Such a hypothetical mashine was called the **"eternal" engine of the second kind**.

Therefore, the second law of thermodynamics is often worded as follows: **the eternal engine of the second kind is**

impossible (it is impossible to construct an engine that would carry out work only by cooling the thermal reservoir).

Problem 1. In a 0.13 *kg* calorimeter containing 0.3 *kg* of water, at a temperature of 8.4 ° C, a metal body weight 0.2 kg temperature 100 °*C* was placed. In calorimeter temperature 21.5 °*C* established. Determine the specific heat of the body if the specific heat of the calorimeter was $0.38\cdot10^3$ *J*/(*kg*·*K*).

Given:

$m_1 = 0{,}13kg$

$m_2 = 0{,}3kg$

$m_3 = 0{,}2kg$

$T_1 = T_2 = 273^\circ C + 8{,}4^\circ C = 281{,}4K$

$T_3 = 273^\circ C + 100^\circ C = 373K$

$\theta = 273^\circ C + 21{,}5^\circ C = 294{,}5K$

Find: C_3–?

Solution

1. Three bodies are involved in heat exchange: calorimeter, water and metal. Metal gives heat, and water and a calorimeter receives heat.

2. Let's write the heat balance equation:

$Q_1 + Q_2 = Q_3$.

$Q_1 = C_1 m_1(\theta - T_1)$; $Q_2 = C_2 m_2(\theta - T_1)$; $Q_3 = C_3 m_3(T_3 - \theta)$;

$C_3 m_3(T_3 - \theta) = C_1 m_1(\theta - T_1) + C_2 m_2(\theta - T_1)$

From here: $C_3 = \dfrac{(C_1 m_1 + C_2 m_2)(\theta - T_1)}{m_3(T_3 - \theta)}$;

$$C_3 = \frac{(0.38\times10^3\cdot0.13 + 4.19\times10^3\cdot0.3)\cdot(294.5 - 281.4)}{0.2\cdot(373 - 294.5)}\frac{J}{kg\cdot K} \approx$$

$$\approx 9.2\cdot10^2\frac{J}{kg\cdot K}.$$

Answer: $C_3 \approx 9.2\cdot10^2\dfrac{J}{kg\cdot K}$.

Problem 2. How much coal with a calorific value of $3{,}1\cdot10^7$ *J/kg* it's necessary to burn to heat up 100 *kg* of water at 303 *K* to 373 *K*, if efficiency of the heater is 60%.

Given:

$q = 1.3\cdot10^7\ J/K$

$m_1 = 100\ kg$

$T_1 = 303\ K$

$T_2 = 373\ K$

Find: *m – ?*

Solution

1. Let's find the amount of useful heat Q needed to heat water from 303 K to 373 K:

$$Q_K = Cm_1\Delta T \text{, де } C=4200\ J/(kg\cdot K).$$

2. Then we find the amount of heat that is emitted when coal is burned by mass m: $Q = qm$.

3. Let's write the formula for the efficiency of the heater:

$$\eta = \frac{Q_u}{Q}\cdot100\%,$$

So $\eta = \dfrac{Cm_1\Delta T}{qm}\cdot100\%$, mass of coal $m = \dfrac{Cm_1\Delta T}{\eta q}\cdot100\%$;

$$m = \frac{4200\cdot100\cdot70\cdot100}{60\cdot3.1\cdot10^7} \approx 1{,}5\ kg$$

Answer: $m \approx 1.5\ kg$

Words and phrases

equivalent	fuel burning
engine	heater
heat balance	refrigerator
fuel	irreversible process
calorimeter	reservoir

Questions

1. Formulate the first law of thermodynamics and write it in mathematical form.
2. Formulate heat balance equation.
3. What is the specific heat of combustion?
4. What is called a heater efficiency?
5. Formulate the second law of thermodynamics.

Problems

1. It was mixed 10 *kg* of water at $9^{\circ}C$, 20 *kg* of water at $40^{\circ}C$ and 6 *kg* of water at $100^{\circ}C$. Determine the temperature of the mixture.

2. Determine the efficiency of the heater, if you can boil 11 *kg* of water at $20^{\circ}C$, when combustion 0.2 *kg* of gasoline.

§31. Melting and crystallization

1. The concept of phase

In thermodynamics, a set of homogeneous, identical in properties parts of a system is called **phase**.

Example. There is water and steam above it in a closed vessel. Water and steam is a system of two phases. If some ice is added to the water, it forms third phase of water.

The transition of a substance from one phase state to another is called **phase transformation** or **transition**.

At phase transitions there is **heat absorption** or**release**.

2. Melting and crystallization of solids

The heat process that results in the phase transition of a substance from a solid to a liquid is called melting.

The temperature at which the melting occurs at the normal pressure is called **melting point**.

Melting occurs necessarily with heat absorption when the body reaches the melting point. If you stop heat transfer to the body, then the process of melting itself will stop.

The process of transition of liquids into a solid state is called crystallization. The heat of melting is equal to the heat of crystallization:

$$Q_{mel} = Q_{cr}$$

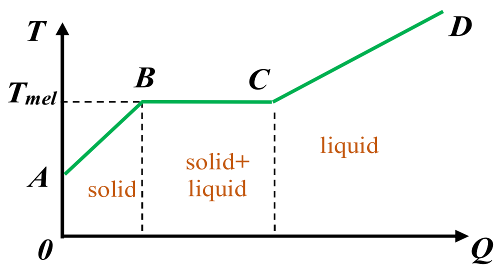
The temperature at the normal pressure at which crystallization occurs is called **crystallization temperature**.

Thermal processes of melting and crystallization occur at constant temperature, because during these processes there are structural changes in the ordering of atoms and molecules of matter in crystalline lattices or, conversely, their destruction.

There are many substances that do not have a constant melting and crystallization temperature. Such substances are called amorphous (wax, resin, glass, etc.).

The figure shows a schematic diagram of the change in aggregate state of a substance.

The crystallization temperature is equal to the melting point. When the solid is heated its temperature rises to the melting point $T_{mel} = T_{cr}$ (see fig., line *AB*).



With further heating of the body its temperature remains constant (line *BC*) until the whole body melts (point C). After full melting, the temperature rises again (line *CD*).

The process of crystallization has a reverse direction. Line *DC* - liquid cooling, *CB* - crystallization at a constant temperature

$T_{cr} = T_{mel}$, point B - complete crystallization of the body, BA - cooling of the solid.

3. Specific heat of melting

When melting, the internal energy of the body changes.

The amount of heat needed to convert 1 kg of substance from a solid state to a liquid at a melting point is called specific heat of melting:

$$\lambda = \frac{Q_{mel}}{m},$$

Specific heat of melting is indicated by the letter g and is measured in joules per kilogram (*J/kg*). For example, the specific heat of melting for ice is 332 *kJ/kg*. This means that in order to melt 1 *kg* of ice at 0° C, it is necessary to give it 332 *kJ* of heat.

To determine the amount of heat needed to melt the solid, it is necessary to multiply specific heat of melting of the substance by mass of the body:

$$Q_{mel} = \lambda m .$$

The process of transition of liquids into a solid state is called crystallization. The heat of melting equals the heat of crystallization.

Problem. A piece of ice of mass m at 0°C is placed in a calorimeter of mass m_1, which contains water with mass of m_2. Let C_1 - specific heat of calorimeter, C_2 - specific heat of water, T_1 - initial temperature of water and calorimeter, T_2 - final temperature of the mixture. Determine the specific heat of melting for ice λ.

Given:

m – ice mass

m_1 – calorimeter mass

m_2 – mass of water

T_1 – initial temperature of water and calorimeter

T_2 – final temperature of the mixture

C_1 – specific heat of calorimeter

C_2 – specific heat of water

Find: λ – ?

Solution

1. The amount of heat required for melting ice is:

$$Q = \lambda m .$$

2. The amount of heat for heating the water of mass m from $T_{mel} = 273K$ to T equal to:

$$Q_2 = C_2 m(T - 273K) .$$

3. The amount of heat given by the calorimeter and water to melt the ice and heating this water from 273 K to the final T, is equal:

$$Q_3 = C_1 m_1 (T_1 - T) ;\ Q_4 = C_2 m_2 (T_1 - T) .$$

4. Let's write heat balance equation:

$$Q_1 + Q_2 = Q_3 + Q_4 ;$$

$$\lambda \mathrm{m} + C_2 m(T - 273K) = C_1 m_1 (T_1 - T) + C_2 m_2 (T_1 - T) .$$

From here

$$\lambda = \frac{(C_1 m_1 + C_2 m_2)(T_1 - T) - C_2 m(T - 273K)}{m} .$$

Answer: $\lambda = \dfrac{(C_1 m_1 + C_2 m_2)(T_1 - T) - C_2 m(T - 273K)}{m}$.

Words and phrases

Phase	melting point
phase transition	heat of melting
Melting	specific heat of melting
Crystallization	temperature of mixture

Questions

1. What is called a phase?
2. What is called phase transformation?
3. What process is called melting?
4. What is the physical content of specific heat of melting?
5. What of the thermal process is opposite to melting?
6. Compare the temperature of melting and crystallization of a certain substance?

Problems

1. What will be the temperature of water, if you mix 100 grams of boiling water and 100 grams of water the temperature of 20º *C*?

2. How much heat energy released with the formation of 5 *kg* of ice?

3. At what speed, a lead ball should strike an obstacle to melt, if the temperature of the ball was 100º C before the impact? During impact, only 60% of the ball energy is converted into heat.

4. To what temperature can you heat 50 liters of water at a temperature of 20 °C, burning 0.1 *kg* of gasoline?

§32. Vaporization

1. Evaporation and condensation

The phenomenon of the transition of liquids into a gaseous state is called evaporation.

Evaporation is inherent not only in liquids, but also in solids. In the case of solids, this process is called **sublimation - the transition from a solid state to a gaseous one**.

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of "smoke" from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

Evaporation of liquids occurs with energy absorption. Energy expenditure on evaporation of liquids characterized by the specific heat of vaporization.

A physical quantity equal to the ratio of the amount of heat required to change liquid into steam at the temperature of vaporization to its mass is called the specific heat of vaporization:

$$r = \frac{Q_{vap}}{m}.$$

Specific heat of vaporization in SI is measured in joules per kilogram (***J/kg***).

To determine the amount of heat needed to evaporate a certain mass of liquid, it is necessary to multiply its specific heat of vaporization by mass:

$$Q_{vap} = rm.$$

The process of transition of matter from a gaseous state to a liquid is called condensation. Condensation is accompanied by the release of heat, which is quantified by the same formula as the heat of evaporation.

Specific heat of vaporization is equal to the specific heat of condensation.

Rate of evaporation depends on the following reasons:

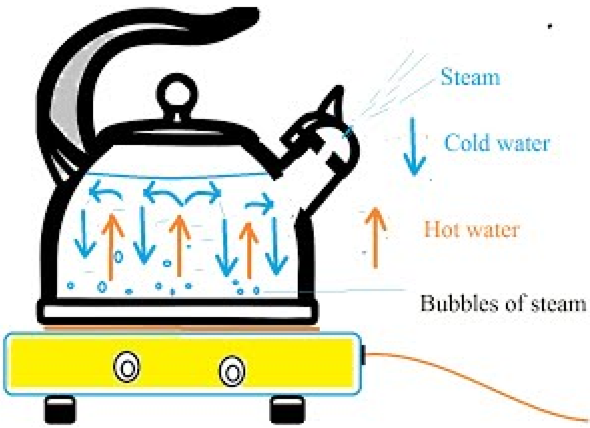
1) type of liquid - different liquids have different value of r;
2) temperature - evaporation increases with increasing temperature;
3) value of surface area - the larger the surface area, the more intense the process of evaporation;

4) air pressure above the surface - the higher the pressure, the higher the probability of returning to the liquid molecules that are above the surface.

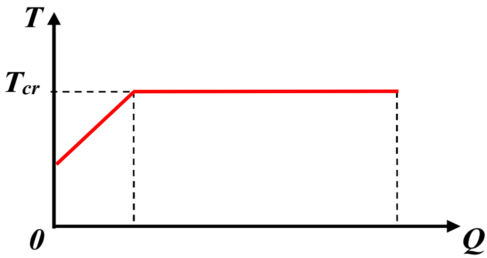
2. Boiling

Boiling is an internal evaporation of a liquid, resulting in the formation of bubbles of vapour (steem) inside liquid, which rise on the surface.

The formation of bubbles during boiling is shown in the figure. Water and other fluids inside have microscopic air bubbles. When the liquid is heated, the evaporation is carried out inside these bubbles. With increasing pressure in the bubble, the volume of the bubble increases. The boiling process occurs when the vapor pressure in the bubble exceeds the external pressure. The bubble rises to the surface and breaks. Movement of bubbles in a liquid creates a **mixing** or **convection** of a liquid.



The temperature at which the liquid boils is called the **boiling point**. Each substance is characterized by a certain boiling point. The boiling point of the liquids depends on the external pressure, because the bubbles of steam for exiting outside need to overcome its counteraction. By increasing external pressure, you can increase the boiling point of the liquid. When the external pressure decreases, the boiling temperature decreases. For example, in normal conditions, water boils at 100 °C, and in mountains, where the air pressure is lower, it can boil even at 81.5 °C. During boiling, the liquid temperature stays constant until all the liquids turn into steam.



Problem. How much coal is needed to convert 1 *t* of water at 10°*C* to steam, if the boiling point of the water at $P=8\cdot10^5$ *Pa* is 170°*C*, and the heater efficiency is 70%?

Given:

$m_1 = 1000\ kg$

$C_1 = 4{,}2\cdot10^3\ J/kg$

$T_b = 170^\circ C$

$m_2 = 1000\ kg$

$r = 22.6\cdot10^5\ J/kg$

$q = 2.93\cdot10^7\ J/kg$

$\eta = 70\%$

Find: m –?

Solution

1. Used amount of heat $Q = qm$.
2. Useful amount of heat $Q_u = Q\eta = qm\eta$.
3. The heat transferred to the water when heated to the boiling point is equal to: $Q_1 = C_1 m_1 (T_b - T_1)$.
4. Heat, which is absorbed by steam, equals: $Q_2 = rm_2$.
5. Let's write heat balance equation:

$$Q_u = Q_1 + Q_2;\ qm\eta = C_1 m_1 (T_b - T_1) + rm_2.$$

From the last equation we find mass of coal:

$$m = \frac{C_1 m_1 (T_b - T_1) + r m_2}{q\eta} \approx 143\ kg$$

Answer: $m \approx 143\ kg$.

Words and phrases

vaporization	specific heat of vaporization
evaporation	boiling
sublimation	bubble
condensation	interfusion
boiling point	convection

Questions

1. What is called the phenomenon of the transition of matter from the liquid state to the gaseous?

2. Give the determination of the specific heat of vaporization. What is its physical meaning?

3. What is called condensation?

4. Rate of evaporation depends on what?

5. What does it mean boiling?

6. How does the boiling temperature depend on external pressure?

Problems

1. What amount of heat should be given to 50 g of water, the temperature of which is 0 ° C, to bring it to a boil and to convert half of it into a steam?

2. How much heat should be spent on converting 6 kg of ice at -20 ° C into steam at a temperature of 100 ° C?

§33. Basic physical quantities in molecular physics and thermodynamics

Physical quantity	Unit
N - number of molecules m - mass of substance	1 kg (kilogram)
m_0 - mass of a molecule	
n - amount of substance	1 mole
N_A - Avogadro's number	$N_A = 6{,}023 \cdot 10^{23}$ mole^{-1}
P - pressure	1 Pa (Pascal)
n_0 - concentration of molecules	1 m^{-3}
$\overline{V^2}$ - mean square of molecule speed	m^2/s^4
$\overline{E}$ - average value of kinetic energy of molecules thermal motion	1 J (Joule)
t - temperature on the Celsius scale	1ºC (degrees Celsius)
T - absolute temperature	1 K (Kelvin)
k - Boltzmann constant	$k = 1{,}38 \cdot 10^{-23}$ J/K
V - volume	1 m^3(cubic meter)
R - universal gas constant	R=8,31 J/(mole·K)
U - internal energy ΔU - change of internal energy Q - amount of heat A - work	1 J(Joule)
C - specific heat	1 J/(kg·K)
r - specific heat of vaporization	1 J/kg
λ - specific heat of melting	1 J/kg

Appendix

1. Some physical constants

Physical constant	Symbol	Unit
Earth radius	R_3	$6{,}37\cdot10^6$ m
Earth mass	M_3	$5{,}97\cdot10^{24}$ kg
Free fall acceleration	g	9,8 m/s^2
Gravitational constant	G	$6{,}67\cdot10^{-11}$ m^3/(kg·s^2)
Avogadro's number	N_A	$6{,}02\cdot10^{23}$ mole^{-1}
Boltzmann constant	k	$1{,}38\cdot10^{-23}$ J/K
Universal gas constant	R	8,31 J/(mole·K)
The volume of one mole under normal conditions	V_0	22,4 m^3/mole
Elementary charge	e	$1{,}6\cdot10^{-19}$ C
Electron mass	m	$9{,}11\cdot10^{-31}$ kg
Faraday's constant	F	$9{,}65\cdot10^4$ C/mole
Speed of light in a vacuum	c	$3\cdot10^8$ m/s
Planck's constant	h	$6{,}62\cdot10^{-34}$ J·s
Rydberg constant	R'	$1{,}097\cdot10^7$ m^{-1}

2. Density of some substances

Solids, 10^3 kg/m^3			
Aluminum	2,7	Copper	8,9
Tree	0,8	Nickel	8,8
Iron	7,8	Lead	11,3
Brick	1,8	Silver	10,5
Ice	0,9	Steel	7,8
Liquids, 10^3 kg/m^3			
Pure water	1	Machine oil	0,9
Sea water	1,03	Mercury	13,6
Kerosene	0,8	Alcohol	0,8
Gases, kg/m^3			
Hydrogen	0,089	Helium	0,18
Air	1,29	Oxygen	1,43

3. Young's module, 10^{11} Pa

Aluminum	0,7	Copper	1,2
Iron	2,1	Steel	2,2
Brass	0,9	Lead	0,17

4. Surface tension of liquids (at room temperature), 10^{-2} N/m

Aniline	4,3	Soap solution	4
Water	7,4	Alcohol	2,2
Kerosene	3,6	Mercury	47,1

5. Specific heat, 10^3 J/(kg·K)

Nitrogen	1,05	Ice	2,1
Aluminum	0,88	Copper	0,39
Water	4,19	Tin	0,23
Hydrogen	14,2	Air	1,005
Iron	0,46	Lead	0,13
Oxygen	0,92	Alcohol	2,42
Brass	0,38	Steel	0,46

6. Melting point of solids, K

Aluminum	933	Copper	1356
Iron	1803	Tin	505
Brass	1173	Lead	600
Ice	273	Silver	1233

7. Specific heat of melting of solids, 10^5J/kg

Aluminum	3,9	Tin	0,58
Ice	3,35	Lead	0,25
Copper	1,8	Silver	1,01

8. Evaporation temperature, К

Water	373	Alcohol	351
Mercury	630	Ether	308

9. Specific heat of vaporization, 10^5 J/kg

Water	22,6	Alcohol	9,05
Mercury	2,82	Ether	3,68

10. Specific heat of combustion, 10^7 J/kg

Gasoline	4,61	Kerosene	4,61
Wood	1,26	Oil	4,61
Coal	2,93	Alcohol	2,93

11. Linear coefficient of thermal expansion of solids, 10^{-5} $К^{-1}$

Aluminum	2,4	Copper	1,7
Iron	1,2	Lead	2,9
Invar	0,15	Steel	1,1
Brass	1,9	Glass	0,9

12. Volumetric coefficient of thermal expansion, 10^{-4} $К^{-1}$

Water	1,8	Mercury	1,8
Kerosene	10,0	Sulfuric acid	5,6
Oil	10,0	Alcohol	11,0

Note. The values of physical quantities are given under normal conditions.

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