Earth Observation Modeling Based on Layer Scattering Matrices

W. VERHOEF

National Aerospace Laboratory NLR, Anthony Fokkerweg 2, 1059 CM Amsterdam, The Netherlands

The differential equations describing radiative transfer in vegetative canopies as given by Suits are generalized and solved to derive a layer scattering matrix. Layer scattering matrices can be applied to the calculation of optical parameters for multilayer ensembles according to the Adding method. The application to atmospheric scattering is demonstrated by explaining path radiance, sky radiance, and other quantities in terms of elements from a layer scattering matrix and a surface reflectance matrix. By combining scattering matrices originating from atmospheric layers with those from earth objects, earth observation models can be constructed. These may become valuable tools in the study of various remote sensing problems.

Introduction

Remote sensing of earth objects in the visible to middle infrared wavelength region is a powerful technique for providing information on earth resources rapidly and over large areas. In the extraction of information from multispectral digital image data acquired by remote sensing missions, several levels of sophistication can be distinguished. Some of these techniques, such as visual interpretation of enhanced color composite imagery or routines for automatic classification, already offer much valuable information for relatively little effort. In other cases, however, especially when quantitative data are to be extracted, these techniques fall short because the extraction of this type of information requires more insight into the physical processes involved in the interaction of radiation with objects on earth and with the atmosphere. These interactions can be investigated by the application of radiative transfer theory to problems such as atmospheric scattering and non-Lambertian reflection characteristics of objects like vegetation canopies and water bodies. This results in mathematical models that predict the intensity of scattered or reflected radiation for given directions of sunlight and view as a function of atmospheric and object parameters. In this way the quantitative relationships between object parameters and remotely sensed data can be established for given conditions of observation. An excellent review of such models, with special emphasis on the applicability to remote sensing problems, is given in Slater (1980). Of particular interest are the vegetation canopy reflectance model of Suits (1972) and the atmospheric model of Turner (1973), since these models are not too complex, require little computer processing power, and yet are reasonably realistic.

The analytical model of Suits is based on two extensions of the Kubelka–Munk (1931) theory, which describes the scattering and extinction of isotropic diffuse fluxes in upward and downward direction. The first extension is the addition of specular sunlight with its associate extinction and scattering coefficients, which leads to the Duntley (1942) equations,

52 Vanderbilt Ave., New York, NY 10017

0034-4257/85/$3.30

also applied in the AGR-model of Allen et al. (1970); the second extension is the calculation of the radiance in the observer's direction, from which the directional reflectance can be determined. Apart from this, the coefficients of scattering and extinction are expressed in object parameters and in the angles of the sun and of observation. In Verhoef (1984) it is demonstrated that the estimation of the coefficients can be improved significantly, resulting in the more realistic SAIL model.

The Suits and SAIL models both rely on the assumption that the first iteration in the method of the self-consistent field gives a good approximation of the real flux field. This first approximation predicts the radiance at the top of the canopy under the assumption that the diffuse upward and downward fluxes are still isotropic, which, of course, is not entirely true. However, it is reasonable to assume that if absorption is strong, the diffuse fluxes are small, so that the result is dominated by single scattering, which is described accurately. Also one may expect that if absorption is weak, for instance, in the near infrared for vegetation canopies, the multiply scattered flux becomes reasonably close to isotropic, so that the first iteration still yields a fair approximation. Anyhow there does not seem to be much point in requiring an accuracy level of model output much higher than that of the model input parameters, which is usually not very high.

This paper deals with the derivation of general solutions to Suits' differential equations for homogeneous scattering layers. These solutions appear in the form of layer scattering matrices and some mathematical techniques to manipulate such matrices are discussed. Application of these techniques is not restricted to canopy reflectance, since there is no fundamental reason why Suits' approach could not be applied to atmospheric scattering, for instance. The framework presented is particularly useful for the construction of combined models of the earth–atmosphere system, leading to so-called earth observation models, for which many applications can be foreseen.

Generalized Form of Suits' Differential Equations

The Suits model for a uniform canopy layer, as given in Suits (1972), in essence consists of the definition and solution of a system of four simultaneous linear differential equations, here given in generalized form by

$$dE_s/dx = kE_s, \tag{1a}$$

$$dE_-/dx = -sE_s + aE_- - \sigma E_+, \tag{1b}$$

$$dE_+/dx = s'E_s + \sigma E_- - aE_+, \tag{1c}$$

$$dE_o/dx = wE_s + vE_- + uE_+ - KE_o, \tag{1d}$$

in which x represents the vertical dimension, defined relative to the layer's thickness and set equal to zero for the top of the layer and to -1 for the bottom of the layer. The coefficients are given in the notation of Bunnik (1978) and are dimensionless quantities. The variables are defined as follows:

E_s = direct solar irradiance,
E_- = diffuse downward irradiance (assumed isotropic),
E_+ = diffuse upward irradiance (assumed isotropic),
E_o = radiance in the observer's direction, multiplied by π, or

$$E_o = \pi L_o, \tag{2}$$

where L_o is the radiance in the observer's direction.

Equations (1a)–(1c) are the Duntley equations, also applied in the AGR model of Allen et al. (1970). Equation (1d) is introduced here as a condensed form of the two differential equations applied by Suits to describe the contributions to the radiance at the top of the canopy, which are given by

$$dF_o/dx = KF_o, \tag{3a}$$

$$\pi dL'_o/dx = (wE_s + vE_- + uE_+)F_o, \tag{3b}$$

in which F_o represents the fraction of layer area observable from outside the canopy by direct line of sight and L'_o is the radiance at the top of the canopy. For the calculation of L'_o both ways of description are equivalent, but Eq. (1d) is more compact, applies to upward radiance at any level within the layer, and can be interpreted as a general radiative transfer equation, with $J = wE_s + vE_- + uE_+$ as the source function and K as the extinction coefficient.

Standard Solution

The general solution of system (1) is found by a standard method, similar to one applied by Chance and Cantu (1975) to the Duntley equations (1a)–(1c). In this case system (1) is written in matrix-vector notation by

$$\frac{d}{dx}(\mathbf{E}) = M\mathbf{E}, \tag{4}$$

where $\mathbf{E}$ is the flux vector and M is the matrix of coefficients of system (1). By a linear transformation Y of vector $\mathbf{E}$, a vector of independent variables $\mathbf{F}$ is formed, for which the following differential equation applies:

$$\frac{d}{dx}(\mathbf{F}) = \Lambda\mathbf{F}, \tag{5a}$$

$$\text{with } \Lambda = YMY^{-1}, \tag{5b}$$

and where Λ is a diagonal matrix of coefficients, which are the eigenvalues of matrix M, equal to $k, m = (a^2 - \sigma^2)^{1/2}$, $-m$, and $-K$. If the components of $\mathbf{F}$ are represented by F_1, F_2, F_3, F_4, then the general solution is given by

$$F_1 = \delta_1 e^{kx}, \tag{6a}$$

$$F_2 = \delta_2 e^{mx}, \tag{6b}$$

$$F_3 = \delta_3 e^{-mx}, \tag{6c}$$

$$F_4 = \delta_4 e^{-Kx}, \tag{6d}$$

in which $\delta_1 - \delta_4$ are constants to be determined from boundary equations. In terms of the original fluxes, the general solution is derived by applying the inverse transformation Y^{-1} to $\mathbf{F}$, which yields

$$E_s = \delta_1 e^{kx}, \tag{7a}$$

$$E_- = D_s\delta_1 e^{kx} + h_1\delta_2 e^{mx} + h_2\delta_3 e^{-mx}, \tag{7b}$$

$$E_+ = C_s\delta_1 e^{kx} + \delta_2 e^{mx} + \delta_3 e^{-mx}, \tag{7c}$$

$$E_o = H_s\delta_1 e^{kx} + (h_1C_o - D_o)\delta_2 e^{mx} + (h_2C_o - D_o)\delta_3 e^{-mx} + \delta_4 e^{-Kx}. \tag{7d}$$

The relation between $\mathbf{F}$ and $\mathbf{E}$ can also be expressed by applying the forward transformation Y to $\mathbf{E}$, which yields

$$\delta_1 e^{kx} = E_s, \tag{8a}$$

$$(h_1 - h_2)\delta_2 e^{mx} = (h_2 C_s - D_s)E_s + E_- - h_2 E_+, \tag{8b}$$

$$(h_1 - h_2)\delta_3 e^{-mx} = (-h_1 C_s + D_s)E_s - E_- + h_1 E_+, \tag{8c}$$

$$\delta_4 e^{-Kx} = -H_o E_s - C_o E_- + D_o E_+ + E_o. \tag{8d}$$

The constants introduced in Eqs. (7) and (8) are given by

$$h_1 = (a + m)/\sigma, \tag{9a}$$

$$h_2 = (a - m)/\sigma = 1/h_1, \tag{9b}$$

$$C_s = [s'(k - a) - s\sigma]/(k^2 - m^2), \tag{10a}$$

$$C_o = [v(K - a) - u\sigma]/(K^2 - m^2), \tag{10b}$$

$$D_s = [-s(k + a) - s'\sigma]/(k^2 - m^2), \tag{11a}$$

$$D_o = [-u(K + a) - v\sigma]/(K^2 - m^2), \tag{11b}$$

$$H_s = (uC_s + vD_s + w)/(K + k), \tag{12a}$$

$$H_o = (sC_o + s'D_o + w)/(K + k). \tag{12b}$$

An important relation for the proof of reciprocity relations is given by

$$H_o + C_o D_s = H_s + C_s D_o, \tag{13}$$

which follows from applying $YY^{-1} = I$.

The general solutions presented by Eqs. (7) and (8) can be applied to calculate the directional reflectance if the boundary equations for a particular case are substituted. Since such a restriction to a particular case would limit the applicability of the solution obtained, this paper will focus on the more general case of known incident fluxes at the top and the bottom of the layer.

Derivation of a Layer Scattering Matrix

Equations (7) and (8) can be applied to calculate the outward fluxes from an isolated layer if the incident fluxes are given. The incident and outward fluxes of such a layer are illustrated in Fig. 1. The incident fluxes are the downward fluxes at the top, $E_s(0)$ and $E_-(0)$, and the upward fluxes at the bottom of the layer, $E_+(-1)$ and $E_o(-1)$.
In this case the outward fluxes are found as follows:

A. Substitute $E_s(0)$, $E_-(0)$, and $E_+(-1)$ in (7a), (7b), and (7c) to solve the constants δ_1, δ_2, and δ_3.
B. Apply (7a), (7b), and (7c) again to calculate the outward fluxes $E_s(-1)$, $E_-(-1)$, and $E_+(0)$.
C. Substitute the solved outward fluxes, $E_s(-1)$ and $E_-(-1)$, and the given incident fluxes, $E_+(-1)$ and $E_o(-1)$, in (8d) to solve the constant δ_4.
D. Apply (8d) again for the top of the layer to calculate the outward flux $E_o(0)$.

The result can be expressed by the following set of equations:

$$E_s(-1) = \tau_{ss} E_s(0), \tag{14a}$$

$$E_-(-1) = \tau_{sd} E_s(0) + \tau_{dd} E_-(0) + \rho_{dd} E_+(-1), \tag{14b}$$

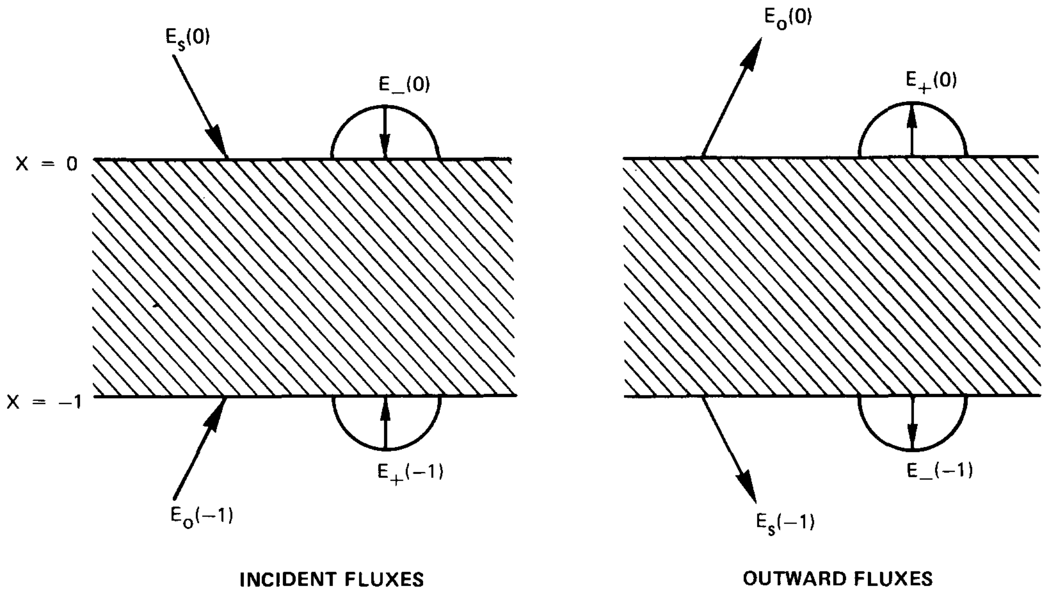

FIGURE 1. Illustration of the incident and outward fluxes for an isolated layer.

$$E_+(0) = \rho_{sd}E_s(0) + \rho_{dd}E_-(0) + \tau_{dd}E_+(-1), \tag{14c}$$

$$E_o(0) = \rho_{so}E_s(0) + \rho_{do}E_-(0) + \tau_{do}E_+(-1) + \tau_{oo}E_o(-1), \tag{14d}$$

in which the coefficients are given by

$$\begin{aligned}
\tau_{ss} &= e^{-k},\\
\tau_{oo} &= e^{-K},\\
\rho_{dd} &= (e^m - e^{-m})/(h_1e^m - h_2e^{-m}),\\
\tau_{dd} &= (h_1 - h_2)/(h_1e^m - h_2e^{-m}),\\
\rho_{sd} &= C_s(1 - \tau_{ss}\tau_{dd}) - D_s\rho_{dd},\\
\tau_{sd} &= D_s(\tau_{ss} - \tau_{dd}) - C_s\tau_{ss}\rho_{dd},\\
\rho_{do} &= C_o(1 - \tau_{oo}\tau_{dd}) - D_o\rho_{dd},\\
\tau_{do} &= D_o(\tau_{oo} - \tau_{dd}) - C_o\tau_{oo}\rho_{dd},\\
\rho_{so} &= H_o(1 - \tau_{ss}\tau_{oo})\\
&\quad - C_o\tau_{sd}\tau_{oo} - D_o\rho_{sd}.
\end{aligned} \tag{15}$$

The coefficients are divided in reflectances ρ and transmittances τ. Double indices are used to indicate the types of incident and outward flux involved, i.e., s refers to the direct solar flux E_s, d refers to the diffuse fluxes E_- and E_+, and o refers to the flux in the observer's direction, symbolized by $E_o = \pi L_o$. In matrix notation the result is expressed by

$$\begin{bmatrix} E_s(-1)\\ E_-(-1)\\ E_+(0)\\ E_o(0) \end{bmatrix} = \begin{bmatrix} \tau_{ss} & 0 & 0 & 0\\ \tau_{sd} & \tau_{dd} & \rho_{dd} & 0\\ \rho_{sd} & \rho_{dd} & \tau_{dd} & 0\\ \rho_{so} & \rho_{do} & \tau_{do} & \tau_{oo} \end{bmatrix} \begin{bmatrix} E_s(0)\\ E_-(0)\\ E_+(-1)\\ E_o(-1) \end{bmatrix}, \tag{16}$$

or $\mathbf{E}_{\text{out}} = \mathbf{Z}\mathbf{E}_{\text{in}}$, where $\mathbf{Z}$ is the layer scattering matrix.

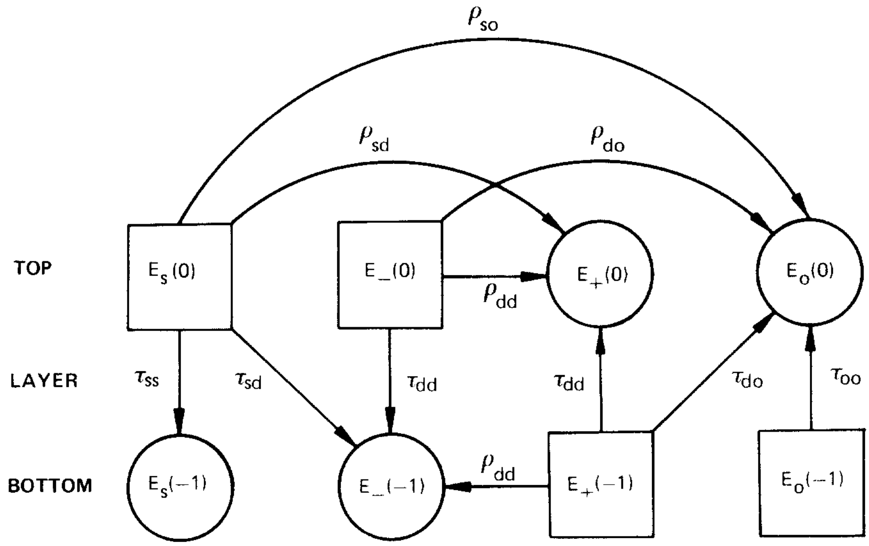

FIGURE 2. Flux interactions for an isolated homogeneous scattering layer.

The flux interactions implied by the coefficients of the layer scattering matrix are illustrated in Fig. 2. Here the incident fluxes are placed in a box, whereas the outward fluxes resulting from the interactions are encircled.

For the calculation of the directional reflectance it appears convenient to introduce subvectors of **E** and submatrices of Z as follows:

$$\mathbf{E}^d = \begin{bmatrix} E_s \\ E_- \end{bmatrix}, \tag{17a}$$

$$\mathbf{E}^u = \begin{bmatrix} E_+ \\ E_o \end{bmatrix}, \tag{17b}$$

$$T_d = \begin{bmatrix} \tau_{ss} & 0 \\ \tau_{sd} & \tau_{dd} \end{bmatrix}, \tag{18a}$$

$$R_b = \begin{bmatrix} 0 & 0 \\ \rho_{dd} & 0 \end{bmatrix}, \tag{18b}$$

$$R_t = \begin{bmatrix} \rho_{sd} & \rho_{dd} \\ \rho_{so} & \rho_{do} \end{bmatrix}, \tag{19a}$$

$$T_u = \begin{bmatrix} \tau_{dd} & 0 \\ \tau_{do} & \tau_{oo} \end{bmatrix}. \tag{19b}$$

By these definitions Eq. (16) can be rewritten as

$$\begin{bmatrix} \mathbf{E}^d(b) \\ \mathbf{E}^u(t) \end{bmatrix} = \begin{bmatrix} T_d & R_b \\ R_t & T_u \end{bmatrix} \begin{bmatrix} \mathbf{E}^d(t) \\ \mathbf{E}^u(b) \end{bmatrix}, \tag{20}$$

in which the indices refer to downward (d), upward (u), top of the layer (t), and bottom of the layer (b).

Applications

Calculation of the directional reflectance of a single homogeneous layer on a non-Lambertian reflecting surface

The non-Lambertian reflectance of a surface can be expressed by a surface reflectance matrix R_s, defined by

$$R_s = \begin{bmatrix} r_{sd} & r_{dd} \\ r_{so} & r_{do} \end{bmatrix}. \tag{21}$$

For the flux interactions at the surface, which is also the bottom of the layer, this yields

$$\mathbf{E}^u(b) = R_s \mathbf{E}^d(b). \tag{22}$$

From Eq. (20) it follows that

$$\mathbf{E}^d(b) = T_d \mathbf{E}^d(t) + R_b \mathbf{E}^u(b), \tag{23a}$$

$$\mathbf{E}^u(t) = R_t \mathbf{E}^d(t) + T_u \mathbf{E}^u(b). \tag{23b}$$

By expressing the reflected flux vector $\mathbf{E}^u(t)$ in the incident flux vector $\mathbf{E}^d(t)$ one obtains

$$\mathbf{E}^u(t) = \left[R_t + T_u (I - R_s R_b)^{-1} R_s T_d \right] \times \mathbf{E}^d(t) = R_t^* \mathbf{E}^d(t), \tag{24}$$

where R_t^* is the resulting surface reflectance matrix of the ensemble and which is given by

$$R_t^* = \begin{bmatrix} r_{sd}^* & r_{dd}^* \\ r_{so}^* & r_{do}^* \end{bmatrix} = R_t + T_u (I - R_s R_b)^{-1} R_s T_d. \tag{25}$$

The elements of R_t^* are described as follows:

r_{dd}^* = hemispherical reflectance for hemispherical incidence,

r_{sd}^* = hemispherical reflectance for direct incidence,

r_{do}^* = directional reflectance for hemispherical incidence,

r_{so}^* = bidirectional reflectance.

Applying matrix equation (25) yields for these reflectances

$$r_{dd}^* = \rho_{dd} + \tau_{dd} r_{dd} \tau_{dd} / (1 - r_{dd} \rho_{dd}), \tag{26a}$$

$$r_{sd}^* = \rho_{sd} + (\tau_{ss} r_{sd} + \tau_{sd} r_{dd}) \tau_{dd} / (1 - r_{dd} \rho_{dd}), \tag{26b}$$

$$r_{do}^* = \rho_{do} + \tau_{dd} (r_{dd} \tau_{do} + r_{do} \tau_{oo}) / (1 - r_{dd} \rho_{dd}), \tag{26c}$$

$$r_{so}^* = \rho_{so} + \tau_{ss} r_{so} \tau_{oo} + \frac{(\tau_{ss} r_{sd} + \tau_{sd} r_{dd}) \tau_{do} + (\tau_{sd} + \tau_{ss} r_{sd} \rho_{dd}) r_{do} \tau_{oo}}{1 - r_{dd} \rho_{dd}}. \tag{26d}$$

The flux interaction diagram associated with these equations is presented in Fig. 3. The directional reflectance factor of the ensemble, r, is given by

$$r = \left[r_{so}^* E_s(t) + r_{do}^* E_-(t) \right] / \left[E_s(t) + E_-(t) \right], \tag{27}$$

in which $E_s(t)$ and $E_-(t)$ are the direct solar irradiance and the diffuse sky irradiance at the top of the layer.

Multilayer extensions

The resultant surface reflectance matrix of an ensemble of many (say N) homogeneous layers on a non-Lambertian surface can be calculated easily if matrix equation (25) is applied recursively. This results in the following algorithm:

Set $R_t^* = R_s$.
For layer $J = 1 - N$ (lowest to highest):
  set $R_s = R_t^*$;
  compute T_d, R_b, R_t, and T_u for layer J;
  $R_t^* = R_t + T_u (I - R_s R_b)^{-1} R_s T_d$.
Next J.

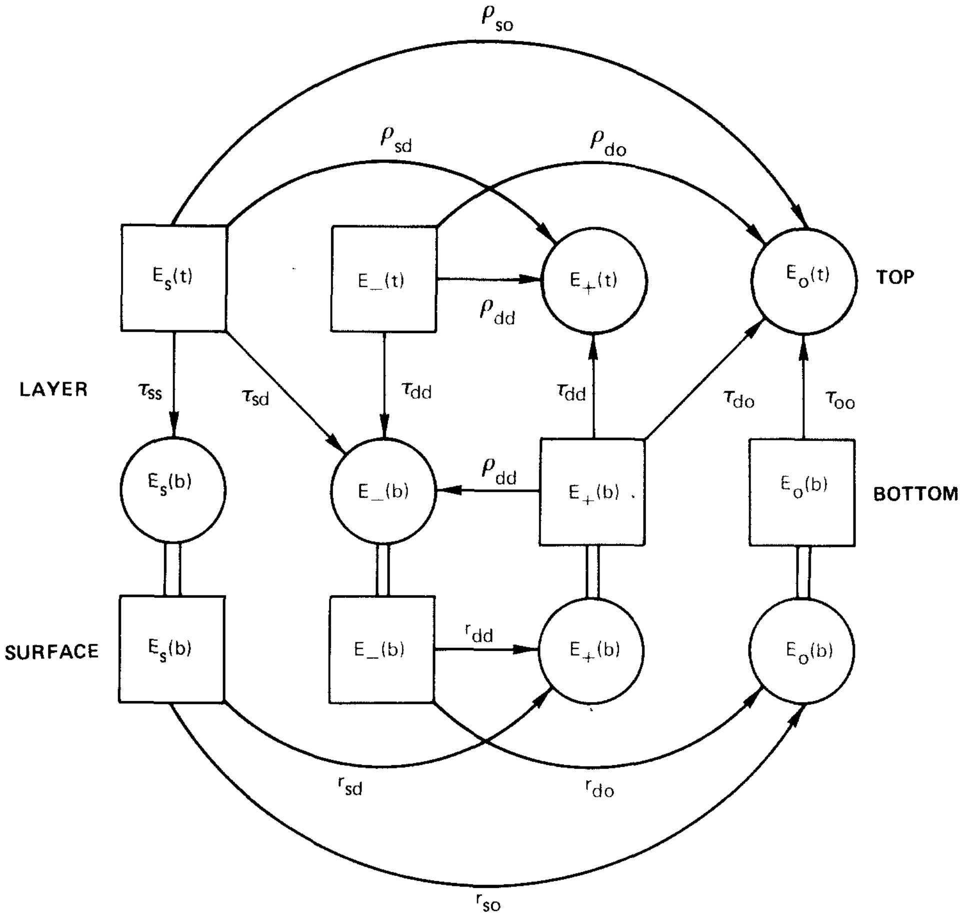

FIGURE 3. Flux interaction diagram for the combination of a scattering layer on top of a non-Lambertian reflecting surface.

A similar procedure, known as the Adding method of van de Hulst (1980), has been used by Cooper et al. (1982). They applied it to the calculation of canopy reflectance according to a finite element method based on finite thickness leaf layers and finite angular intervals of leaf slope and zenith angles of incidence and scattering.

Sometimes, for instance, when flux intensities at intermediate levels in a multilayer ensemble have to be calculated, it will be necessary to determine the complete scattering matrix of an N-layer combination. This can be achieved by first deriving the resultant scattering matrix of a two-layer combination as follows:

If the layers are indexed 1 and 2 for upper and lower, and the levels are indicated by 0, 1, and 2, then the following equations apply:

$$\mathbf{E}^d(1) = T_{d1}\mathbf{E}^d(0) + R_{b1}\mathbf{E}^u(1), \quad (28a)$$

$$\mathbf{E}^u(0) = R_{t1}\mathbf{E}^d(0) + T_{u1}\mathbf{E}^u(1), \quad (28b)$$

$$\mathbf{E}^d(2) = T_{d2}\mathbf{E}^d(1) + R_{b2}\mathbf{E}^u(2), \quad (29a)$$

$$\mathbf{E}^u(1) = R_{t2}\mathbf{E}^d(1) + T_{u2}\mathbf{E}^u(2). \quad (29b)$$

The scattering matrix of the two-layer

ensemble is defined by

$$\begin{bmatrix} \mathbf{E}^d(2) \\ \mathbf{E}^u(0) \end{bmatrix} = \begin{bmatrix} T_d^* & R_b^* \\ R_t^* & T_u^* \end{bmatrix} \begin{bmatrix} \mathbf{E}^d(0) \\ \mathbf{E}^u(2) \end{bmatrix}. \quad (30)$$

Application of Eqs. (28) and (29) yields

$$\begin{bmatrix} T_d^* & R_b^* \\ R_t^* & T_u^* \end{bmatrix} = \begin{bmatrix} T_{d2}(I - R_{b1}R_{t2})^{-1}T_{d1} \\ R_{b2} + T_{d2}(I - R_{b1}R_{t2})^{-1}R_{b1}T_{u2} \\ R_{t1} + T_{u1}(I - R_{t2}R_{b1})^{-1}R_{t2}T_{d1} \\ T_{u1}(I - R_{t2}R_{b1})^{-1}T_{u2} \end{bmatrix}. \quad (31)$$

Recursive application of (31) for calculation of the scattering matrix of an N-layer ensemble leads to an algorithm more complex, but otherwise similar to that for the surface reflectance matrix of an N-layer ensemble.

Infinite reflectances

The surface reflectance matrix of a layer of infinite optical thickness follows immediately from Eqs. (15) by setting all transmittances equal to zero and by taking the limit $m \to \infty$ for ρ_{dd}, which equals h_2. The result can be expressed by

$$R_t^\infty = \begin{bmatrix} r_{sd}^\infty & r_{dd}^\infty \\ r_{so}^\infty & r_{do}^\infty \end{bmatrix} = \begin{bmatrix} C_s - D_s h_2 & h_2 \\ H_o - D_o(C_s - D_s h_2) & C_o - D_o h_2 \end{bmatrix}. \quad (32)$$

Proof of reciprocity relations

The coefficients of Suits' differential equations (1), as determined according to Suits (1972) or as given for the SAIL model in Verhoef (1984), show the following reciprocity relations: If the directions of solar incidence and of observation are interchanged, the couples (k, K), (s, u) and (s', v) commute and the coefficients a, σ, and w remain the same. As a consequence, for the constants of Eqs. (9)–(12) it is found that the couples (C_s, C_o), (D_s, D_o), and (H_s, H_o) also commute, whereas h_1 and h_2 are invariant. For the coefficients of the layer scattering matrix of Eqs. (15) this means that the couples (τ_{ss}, τ_{oo}), (ρ_{sd}, ρ_{do}), and (τ_{sd}, τ_{do}) commute and ρ_{dd}, τ_{dd}, and ρ_{so} are invariant. The reciprocity relation for ρ_{so} of Eqs. (15) is not obvious, since the expression mentioned is not symmetric. A more symmetric expression is found if τ_{sd} and ρ_{sd} are substituted. This yields

$$\begin{aligned} \rho_{so} = {} & (H_o + C_o D_s)(1 - \tau_{ss}\tau_{oo}) \\ & - [C_o D_s(1 - \tau_{dd}\tau_{oo}) \\ & + C_s D_o(1 - \tau_{dd}\tau_{ss})] \\ & + [C_o C_s \tau_{oo}\tau_{ss} + D_o D_s]\rho_{dd}, \end{aligned} \quad (33)$$

which is symmetric except for the first term. However, from Eq. (13) it follows that $H_o + C_o D_s$ equals $H_s + C_s D_o$, so that ρ_{so} is also invariant under interchanging the positions of sun and observer.

The reflectance matrix of a single homogeneous layer on top of a reflecting surface shows reciprocity relations if the reflectance matrix of the surface also has this property, i.e., the couple (r_{sd}, r_{do}) should commute and r_{dd} and r_{so} should be invariant. In this case it can be shown, by employing Eqs. (26), that the couple (r_{sd}^*, r_{do}^*) commutes and r_{dd}^* and r_{so}^* are invariant. By applying Eqs. (26) recursively, the proof of the reciprocity relations can be extended to any number of homogeneous layers.

Atmospheric scattering

The procedures outlined in the previous sections can also be applied to the modeling of radiative transfer in the atmosphere. In this case the scattering and extinction coefficients of system (1) are to be estimated on the basis of optical depths, scattering phase functions, and absorption characteristics of different atmospheric constituents, such as air, aerosols, water vapor, etc. If this has been done, then the reflectance matrix of the combination atmosphere–earth can be calculated by Eqs. (26), of which especially Eq. (26d) is interesting because of its applicability to earth observation problems in the visible to middle infrared region of the spectrum. Assuming that the extraterrestrial spectral solar irradiance on a plane perpendicular to the sunrays is given by E_s^0, and that there is no diffuse downward irradiance at the top of the atmosphere, the upward radiance at the top of the atmosphere $L_o(t)$ follows from

$$\pi L_o(t) = (E_s^0 \cos\theta_s) r_{so}^*, \tag{34}$$

where θ_s is the solar zenith angle. Substitution of (26d) in (34) leads to an expression in which several terms can be identified as quantities relevant to satellite remote sensing in the optical spectral range. The result can be expressed by

$$\begin{aligned}\pi L_o(t) &= (E_s^0 \cos\theta_s)\rho_{so} \\ &\quad + (E_s^0 \cos\theta_s)(\tau_{ss} r_{sd} + \tau_{sd} r_{dd})\tau_{do}/ \\ &\qquad (1 - r_{dd}\rho_{dd}) \\ &\quad + (E_s^0 \cos\theta_s)\tau_{ss} r_{so}\tau_{oo} \\ &\quad + (E_s^0 \cos\theta_s)(\tau_{sd} + \tau_{ss} r_{sd}\rho_{dd}) r_{do}\tau_{oo}/ \\ &\qquad (1 - r_{dd}\rho_{dd}),\end{aligned} \tag{35}$$

which can be equated to

$$\begin{aligned}\pi L_o(t) &= \pi L_{pa} + \pi L_{pb} + E_{\text{sun}} r_{so} T \\ &\quad + E_{\text{sky}} r_{do} T\end{aligned} \tag{36}$$

or

$$\pi L_o(t) = \pi L_p + E_{\text{tot}} r T, \tag{37}$$

and in which the following quantities have been identified:

- L_{pa} = atmospheric path radiance,
- L_{pb} = background albedo contribution to path radiance,
- L_p = total path radiance,
- E_{sun} = solar irradiance at ground level,
- E_{sky} = sky irradiance at ground level,
- E_{tot} = total downward irradiance at ground level,
- r_{so} = object's bidirectional reflectance factor,
- r_{do} = object's directional reflectance for hemispherical incidence,
- r = object's directional reflectance factor,
- T = object-sensor transmittance.

The separate terms of Eqs. (36) and (37) are given by

$$\pi L_{pa} = \left(E_s^0 \cos\theta_s\right)\rho_{so}, \tag{38a}$$

$$\pi L_{pb} = E_{\text{upw}}\tau_{do}, \tag{38b}$$

with

$$E_{\text{upw}} = \left(E_s^0 \cos\theta_s\right)\left(\tau_{ss}r_{sd} + \tau_{sd}r_{dd}\right)/\left(1 - r_{dd}\rho_{dd}\right), \tag{39}$$

$$E_{\text{sun}} = \left(E_s^0 \cos\theta_s\right)\tau_{ss}, \tag{40a}$$

$$T = \tau_{oo}, \tag{40b}$$

$$E_{\text{sky}} = \left(E_s^0 \cos\theta_s\right)\left(\tau_{sd} + \tau_{ss}r_{sd}\rho_{dd}\right)/\left(1 - r_{dd}\rho_{dd}\right), \tag{41}$$

$$L_p = L_{pa} + L_{pb}, \tag{42}$$

$$E_{\text{tot}} = E_{\text{sun}} + E_{\text{sky}}, \tag{43}$$

$$r = \left(r_{so}E_{\text{sun}} + r_{do}E_{\text{sky}}\right)/E_{\text{tot}}. \tag{44}$$

It should be emphasized that the ρ and τ parameters refer to the atmosphere and the r parameters to reflectance of the earth's surface. Since the surface of the earth is not homogeneous, the parameters r_{sd} and r_{dd}, which determine the diffuse upwelling irradiance at the bottom of the atmosphere, should be interpreted as average reflectances, describing the earth's albedo over a large area. In order to extend this simple atmospheric model so that it may include also calculation of the sky radiance distribution over the hemisphere, an expression for the bidirectional transmittance of a single homogeneous layer is derived. In this case the flux E_o is directed downwards from top to bottom of the layer and the differential equation for E_o becomes

$$dE_o/dx = -wE_s - uE_- - vE_+ + KE_o, \tag{45}$$

which leads to the general solution

$$\delta_4 e^{Kx} = -H_o'E_s - C_o'E_- + D_o'E_+ + E_o, \tag{46}$$

where the constants are given by

$$H_o' = \left(sD_o + s'C_o + w\right)/(K - k), \tag{47a}$$

$$C_o' = -D_o, \tag{47b}$$

$$D_o' = -C_o, \tag{47c}$$

and in which C_o and D_o are defined as in Eqs. (10b) and (11b). The flux E_o at the bottom of the layer, $E_o(-1)$, is solved as follows:

A. Assume that the incident fluxes $E_s(0)$, $E_-(0)$, $E_+(-1)$, and $E_o(0)$ are given and calculate the outward fluxes $E_s(-1)$, $E_-(-1)$, and $E_+(0)$ according to equations (7a)–(7c).
B. Substitute $E_s(0)$, $E_-(0)$, $E_+(0)$, and $E_o(0)$ in Eq. (46) to solve δ_4.
C. Apply (46) again for the bottom of the layer ($x = -1$) to solve $E_o(-1)$.

The result can be expressed by

$$E_o(-1) = \tau_{so}E_s(0) + \tau_{do}E_-(0) + \rho_{do}E_+(-1) + \tau_{oo}E_o(0), \tag{48}$$

in which τ_{do}, ρ_{do}, and τ_{oo} are as given by Eqs. (15), and the bidirectional transmittance τ_{so} of the layer is given by

$$\tau_{so} = H_o'(\tau_{ss} - \tau_{oo}) - C_o\rho_{sd}\tau_{oo} - D_o\tau_{sd}. \tag{49}$$

The term $H_0'(\tau_{ss} - \tau_{oo})$ becomes indeterminate for $K = k$, which occurs if the zenith angles of the sun and of observation are equal. In that case this term must be replaced by the limit for $K \to k$, which equals $(sD_o + s'C_o + w)\tau_{ss}$.

Since $E_-(0)$ and $E_o(0)$ can be assumed equal to zero, and $E_+(-1)$ for the atmosphere–earth system equals E_{upw} as given by Eq. (39), Eq. (48) can be rewritten as

$$\pi L_{\text{sky}} = \left[\tau_{so} + \rho_{do}(\tau_{ss}r_{sd} + \tau_{sd}r_{dd})/ (1 - r_{dd}\rho_{dd})\right] E_s^0 \cos\theta_s. \tag{50}$$

This expression can be used to estimate the sky radiance at any position in the sky (except the solar disc) for any solar zenith angle.

In some regions of the spectrum the atmospheric absorption coefficient α may be assumed equal to zero. This case requires a special solution of the diffuse reflectance ρ_{dd} and the diffuse transmittance τ_{dd}, since these become indeterminate for $\alpha = 0$, if calculated according to Eqs. (15). This is explained as follows: The extinction coefficient for diffuse fluxes, κ, can, because of energy conservation, be expressed as

$$\kappa = \alpha + \sigma + \sigma', \tag{51}$$

in which σ and σ' are the backscatter coefficient and the forward scattering coefficient for diffuse fluxes, respectively. The attenuation coefficient a, which is used in Suits' differential equations, is given by

$$a = \kappa - \sigma' = \alpha + \sigma, \tag{52}$$

which means that eigenvalue $m = (a^2 - \sigma^2)^{1/2}$ approximates $(2\alpha\sigma)^{1/2}$ if α is small, and which becomes equal to zero for $\alpha = 0$. Taking the limit $m \to 0$ in the expressions for ρ_{dd} and τ_{dd}, it is found that these become

$$\rho_{dd} = \sigma/(a+1) = \sigma/(\sigma+1), \tag{53a}$$

$$\tau_{dd} = 1/(a+1) = 1/(\sigma+1). \tag{53b}$$

The sum of both equals 1, which could be expected since it was assumed that there was no absorption.

Earth observation simulation models

The techniques discussed in the previous sections are suitable as "modules" for the construction of more complex models including, for instance, atmospheric scattering and directional reflectance properties of several types of objects like vegetation, bare soil, and water surfaces. Such earth observation models may become valuable tools for the study of various problems related to remote sensing data collection and interpretation, of which some are mentioned below:

- —sensor design and mission planning;
- —correction for atmospheric effects;
- —correction for object-dependent directional reflectance effects;
- —extraction of object-related information.

A very powerful combination results if an earth observation simulation model is interfaced to an image processing system, since this will provide the possibility to test image processing techniques on model-generated images and to evaluate models on the basis of comparison with real imagery.

Discussion

The mathematical framework presented, with the theoretical examples of applications shown in the preceding section, seems attractive as a simple but powerful tool for the construction of combined radiative transfer models of the atmosphere–earth system. The method is quite comprehensible, easy to visualize by means of flux interaction diagrams, flexible in the number of layers and different media, and requires little computer processing power. However, the relative simplicity of the model also forces a drastic abstraction of reality, which in some cases may lead to unacceptable errors. In addition to this, it should be noted that the method relies on a first iteration in the calculation of the radiance field. This approximation is composed of a contribution due to single scattering and one due to multiple scattering by interaction with the initially assumed semi-isotropic diffuse fluxes. It may be expected, however, that this first approximation of the radiance field is already reasonably close to the one obtained after convergence to the final self-consistent flux field (which satisfies the complete radiative transfer equation in any direction and at any depth in the layer), at least with regard to average intensity. If the diffuse flux is low with respect to the direct solar flux, the result is dominated by single scattering, which is modeled as accurately as possible. On the other hand, if the diffuse flux is relatively high, then multiple scattering dominates, but in that case the diffuse flux becomes closer to isotropic, so that the assumption of semi-isotropic fluxes pertaining to Suits' equations still holds in first approximation.

Considering the complexity of radiative transfer in the atmosphere–earth system with its massive amounts of data necessary to describe the boundary conditions, the variability of scattering and reflection processes, and the vertical and horizontal heterogeneities found in reality, it is not feasible or even desirable to construct a model which takes all this into account. For complex systems of this kind, abstractions of reality are inevitable and with the rather drastic simplifications in the present model the accuracy of its simulation results is not expected to be better than 10% or 20% on average.

Finally, it should be emphasized that the proposed theory is not meant to suggest an improvement of any existing special purpose atmospheric or vegetation scattering model, but rather first priority was given to putting both types of model into a common framework. Results of simulation experiments in the future will have to establish the necessity of making refinements.

Conclusion

The five differential equations applied by Suits to the calculation of the directional canopy reflectance have been reduced to four generalized differential equations, implying a four-flux radiative transfer theory with nine coefficients. If the nine coefficients of scattering and extinction are expressed as dimensionless quantities, then the thickness of a layer becomes a redundant parameter. The generalized differential equations have been applied to a single homogeneous scattering layer leading to a solution in terms of various reflectance and transmittance factors, which form nine elements of a layer scattering matrix. Layer scattering matrices can be combined easily by Adding methods to calculate optical parameters of multilayer ensembles and they greatly facilitate the proof of reciprocity relations. It has been demonstrated that in principle it is possible to apply the method to atmospheric scattering as well, since quantities like the path radiance, sky radiance, and solar and sky irradiance can be explained in terms of elements of layer scattering matrices. Application of the method to the combined atmosphere–earth system leads to the development of earth observation models, which can be employed for the study of many problems related to remote sensing in the visible to middle infrared region of the spectrum.

References

Allen, W. A., Gayle, and Richardson (1970), Plant canopy irradiance specified by the Duntley equations, *J. Opt. Soc. Am.* 60(3):372–376.

Bunnik, N. J. J. (1978), The multispectral reflectance of shortwave radiation by agricultural crops in relation with their morphological and optical properties, thesis, H. Veenman en Zonen, Wageningen, The Netherlands.

Chance, J. E., and Cantu, J. M. (1975), A study of plant canopy reflectance models, Final report on faculty research grant, Pan American University, Edinburg, TX 78539.

Cooper, K., et al. (1982), Reflectance of a vegetation canopy using the Adding method, *Appl. Opt.* 21(22):4112–4118.

Duntley, S. Q. (1942), The optical properties of diffusing materials, *J. Opt. Soc. Am.* 32(2):61–70.

Kubelka, P., and Munk, F. (1931), Ein Beitrag zur Optik der Farbanstriche, *Ann. Tech. Phys.*, 11:593–601.

Slater, P. N. (1980), *Remote Sensing. Optics and Optical Systems*, Addison-Wesley, Advanced Book Program, Reading, MA.

Suits, G. H. (1972), The calculation of the directional reflectance of a vegetative canopy, *Remote Sens. Environ.* 2:117–125.

Turner, R. E. (1973), Atmospheric effects in remote sensing, in *Remote Sensing of Earth Resources*, II (F. Shahrockhi, Ed.), University of Tennessee, pp. 549–583.

Van de Hulst, R. C. (1980), *Multiple Light Scattering*, Academic, London.

Verhoef, W. (1984), Light scattering by leaf layers with application to canopy reflectance modelling: the SAIL model, *Remote Sens. Environ.*, 16:125–141.

Received 26 March 1984; revised 4 September 1984.