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Interaction of Isotropic Light with a Compact Plant Leaf*

WILLIAM A. ALLEN, HAROLD W. GAUSMAN, ARTHUR J. RICHARDSON, AND JAMES R. THOMAS
Southern Plains Branch, Soil and Water Conservation Research Division, Agricultural Research Service, U. S. Department of Agriculture, Weslaco, Texas 78596

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A transparent plate with rough plane-parallel surfaces is used as a theoretical model to explain the interaction of diffuse light with a compact plant leaf. Effective optical constants of a corn leaf have been determined from leaf reflectance and transmittance measured over the spectral range 0.5–2.5 μ with a recording spectrophotometer. The effective index of refraction at 0.5 μ for the corn leaf is not inconsistent with the refractive index of epicuticular wax. The effective absorption spectra of the corn leaf appears to be a superposition of the absorption coefficients of chlorophyll and pure liquid water. Residual spectral data from other leaf constituents are at the resolution limit of the spectrophotometer. The plate model of a leaf is also used to determine moisture content of the corn leaf from reflectance and transmittance measurements.

INDEX HEADINGS: Reflectance; Transmittance; Spectrophotometry; Refractive index; Spectra; Infrared; Absorption.

Allen and Richardson[1] have described the near-infrared reflectance and transmittance of plant leaves stacked in a spectrophotometer by means of the Kubelka–Munk (K–M) theory[2] for propagation of light through a diffusing medium. Basic entities in application of the K–M theory to leaves are the reflectance and transmittance of a single leaf. The purpose of this paper is to derive the reflectance and transmittance of a single typical compact plant leaf from fundamental considerations.

Willstätter and Stoll,[3] as reported by Gates,[4] explained reflectance and transmittance of a plant leaf on the basis of critical reflection of light at the cell wall–air interface of spongy mesophyll tissue. According to Myers and Allen,[5] the K–M scattering coefficient for a typical leaf can be explained by Fresnel reflections at normal incidence from 35 air interfaces along the mean optical path through the leaf. Gausman, Cardenas, and Allen[6] noted that if oblique reflections are considered, fewer interfaces account for the results. These previous concepts emerge naturally, without additional assumptions, from the leaf model investigated in this paper.

The requirement of isotropy is equivalent to the assumption that the plate surfaces are rough with respect to the wavelength of light; that is, the surfaces are assumed to be lambertian.[7] Walsh, as reported by Duntley,[8] and Stern[9] calculated the reflectivity of isotropic light of all polarizations for an interface between two dielectrics. Theory exists[10,11] for evaluating reflectance and transmittance of a plate provided that the reflectivity of each surface and the transmissivity of the plate medium are known. Conversely, if reflectance

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1 W. A. Allen and A. J. Richardson, J. Opt. Soc. Am. **58**, 1023 (1968).

2 P. Kubelka and F. Munk, Z. Techn. Physik **11**, 593 (1931).

3 R. Willstätter and A. Stoll, *Untersuchungen über die Assimilation der Kohlensäure* (Springer, Berlin, 1913).

4 D. M. Gates, H. J. Keegan, J. C. Schleter and V. R. Weidner, Appl. Opt. **4**, 11 (1965).

5 V. I. Myers and W. A. Allen, Appl. Opt. **7**, 1819 (1968).

6 H. W. Gausman, W. A. Allen, and R. Cardenas, Remote Sensing of Environment **1**, 19 (1969).

7 A uniformly diffusing or lambertian surface is one for which the irradiance in any direction varies as the cosine of the angle between that direction and the normal to the surface; such a surface appears equally bright in every direction from which it is viewed.

8 S. Q. Duntley, J. Opt. Soc. Am. **32**, 61 (1942).

9 F. Stern, Appl. Opt. **3**, 111 (1964).

10 G. G. Stokes, Proc. Roy. Soc. (London) **11**, 545 (1862).

11 W. W. Wendlandt and H. G. Hecht, *Reflectance Spectroscopy* (Wiley–Interscience, Inc., New York, 1966), Ch. 3, p. 81.

and transmittance for diffuse light are known, the optical constants of the plate can be determined.

REFLECTANCE AND TRANSMITTANCE OF A PLATE IN ISOTROPIC LIGHT

Figure 1 illustrates unit isotropic radiant flux I_0 emanating from medium 1, interacting with the interface between media 1 and 2, passing through medium 2 of thickness D, interacting with the interface between media 2 and 3, and emerging eventually into both media 1 and 3. Light emergence into medium 1 is designated reflectance R and light emergence into medium 3 is termed transmittance T. Media 1 and 3 will be regarded as air and medium 2 will be specified by the relative index of refraction n between air and medium 2 and by the absorption coefficient k of medium 2. Transmissivity at an interface between media i and j will be designated T_{ij}; the corresponding reflectivity is given by $R_{ij}=1-T_{ij}$. The irradiances associated with multiple reflections within the plate are referenced in Fig. 1 by regions 2, 6, 8, 12 $\cdots$, and 3, 5, 9, 11 $\cdots$. These regions are assumed to be infinitesimally close to the surface. Analytical values associated with the various regions in Fig. 1 are listed in Table I. Reflectance R is determined by contributions from regions 1, 7, 13, $\cdots$; transmittance is determined by contributions from regions 4, 10, $\cdots$. Reflectance R and transmittance T, respectively, consist of the sums of the infinite series

$$R=R_{12}+T_{12}T_{21}\tau^2R_{23}(1+\tau^2R_{23}R_{21}+\cdots), \tag{1}$$

$$T=T_{12}\tau T_{23}(1+\tau^2R_{23}R_{21}+\cdots). \tag{2}$$

The subscripts in Eqs. (1)–(2) refer to media 1, 2, and 3; the quantity τ is the transmissivity of the plate. We sum Eqs. (1) and (2) to obtain the relations

$$R=R_{12}+\frac{T_{12}\tau^2R_{23}T_{21}}{1-\tau^2R_{23}R_{21}}, \tag{3}$$

$$T=\frac{T_{12}\tau T_{23}}{1-\tau^2R_{23}R_{21}}. \tag{4}$$

Equations (3) and (4) describe a plane-parallel rough transparent plate in isotropic light. Diffuse light is trapped by medium 2 for those rays that exceed the critical angle, thus $T_{12}\neq T_{23}$. The transmissivities $T_{21}=T_{23}$ can be calculated either by the relation $T_{21}=n^{-2}T_{12}$[9] or measured by procedures suggested by Duntley.[8] If the calculated value of T_{21} is used, Eqs. (3) and (4) can be written in the forms

$$R=(1-T_{12})+\frac{\tau^2T_{12}^2(n^2-T_{12})}{n^4-\tau^2(n^2-T_{12})^2}, \tag{5}$$

$$T=\frac{\tau n^2T_{12}^2}{n^4-\tau^2(n^2-T_{12})^2}. \tag{6}$$

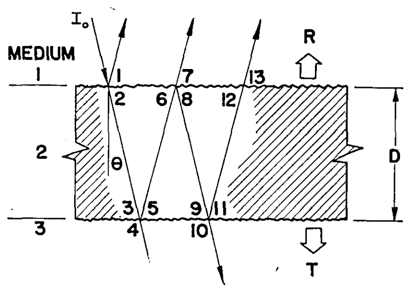

FIG. 1. Multiple reflections produced by a transparent plate with rough surfaces.

The absorptance of the plate can be expressed by the relation

$$1-R-T=\frac{T_{12}(1-\tau)n^2}{n^2-\tau(n^2-T_{12})}. \tag{7}$$

INDEX OF REFRACTION OF A PLATE

Equations (5) and (6) can be used to evaluate the index of refraction n of a plate if R and T are known. Eliminate τ from Eqs. (5) and (6) to obtain the relation

$$F(n)\equiv[T^2-(R-R_{12})^2](n^2-T_{12})-T_{12}^2(R-R_{12})=0. \tag{8}$$

The transmissivity $T_{12}=1-R_{12}$ for diffuse light at an interface between two dielectrics of relative index of refraction n can be written[9] in the forms

$$T_1=\frac{4}{3}\frac{2n+1}{(n+1)^2}, \tag{9}$$

$$T_2=\frac{4n^3(n^2+2n-1)}{(n^2+1)^2(n^2-1)}-\frac{2n^2(n+1)}{(n^2-1)^2}\log n+\frac{2n^2(n^2-1)^2}{(n^2+1)^3}\log\frac{n(n+1)}{n-1}. \tag{10}$$

The subscript 1 in Eqs. (9) and (10) designates the case in which the electric vector is perpendicular to the plane of incidence, and the subscript 2 pertains to the case in

TABLE I. Irradiance for regions shown in Fig. 1.

Region	Irradiance
1	R_{12}
2	T_{12}
3	$T_{12}\tau$
4	$T_{12}\tau T_{23}$
5	$T_{12}\tau R_{23}$
6	$T_{12}\tau^2R_{23}$
7	$T_{12}\tau^2R_{23}T_{21}$
8	$T_{12}\tau^2R_{23}R_{21}$
9	$T_{12}\tau^3R_{23}R_{21}$
10	$T_{12}\tau^3R_{23}R_{21}T_{23}$
11	$T_{12}\tau^3R_{23}R_{21}R_{23}$
12	$T_{12}\tau^4R_{23}^2R_{21}$
13	$T_{12}\tau^4R_{23}^2R_{21}T_{21}$
..	...

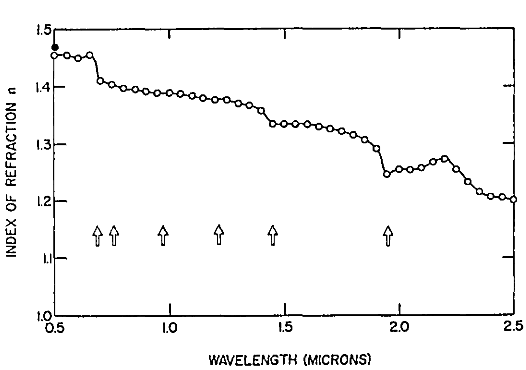

FIG. 2. Effective refractive index of a typical corn leaf. Arrows mark absorption bands due to chlorophyll and liquid water. The additional • point at 0.5 μ is the published index of refraction for carnauba wax.

which the electric vector lies in the plane of incidence. Light transmissivity of intermediate polarization can be written in the form

$$T_{12}=\mu T_1+(1-\mu)T_2, \tag{11}$$

where $1>\mu>0$. We set $\mu=\frac{1}{2}$ for the case of random polarization.

The appropriate root of Eq. (8) can be evaluated by means of the Newton–Raphson[12] method in which the relation

$$F'(n)=2n[T^2-(R-R_{12})^2] \\ -[T^2+T_{12}^2+2n(R-R_{12})-(R-R_{12})^2]dT_{12}/dn \tag{12}$$

must be used. The quantity $dT_{12}/dn=\mu dT_1/dn+(1-\mu)\,dT_2/dn$ can be calculated exactly, but with some difficulty, from Eqs. (9)–(11). Over the restricted range $1.2<n<1.5$, however, the quantity dT_{12}/dn can be calculated with requisite accuracy from the approximation cubics $T_i=A_i+B_in+C_in^2+D_in^3$ where the polynomial coefficients of T_i are listed in Table II.

TRANSMISSIVITY OF A PLATE IN ISOTROPIC LIGHT

The transmissivity τ of a plate in isotropic light can be obtained from Eq. (5) in the form

$$\tau^2=\frac{n^4(R-R_{12})}{T_{12}^2(n^2-T_{12})+(n^2-T_{12})^2(R-R_{12})}. \tag{13}$$

The transmissivity τ of the plate can be related to the absorption coefficient k of the plate medium. The transmissivity of the plate is the irradiance at the lower surface in Fig. 1 relative to the irradiance at the upper surface. Let k be the absorption coefficient of the plate medium. The length of a typical slant ray through the plate of Fig. 1 can be designated $D\sec\theta$, where θ is measured from the plate normal. The transmissivity of the plate along this slant ray is given by $\exp(-kD\sec\theta)$.

[12] H. Margenau and G. M. Murphy, *The Mathematics of Physics and Chemistry* (D. Van Nostrand Company, Inc., New York, 1947), p. 475.

TABLE II. Coefficients of approximation cubics $T_i=A_i+B_in+C_in^2+D_in^3$ for transmissivity of polarized diffuse light across a dielectric interface.[a] The coefficients T_i are used in calculation of dT_{12}/dn of Eq. (12).

Coefficients	$i=1$	$i=2$
A_i	+1.435 228	+1.516 853
B_i	−0.549 374	−1.117 852
C_i	+0.127 332	+0.779 349
D_i	−0.013 136	−0.186 815

[a] Valid for the range $1.2<n<1.5$.

Flux emerging from unit area of a rough or lambertian surface varies as $\cos\theta$. The total flux emanating from unit area of the upper plate surface that reaches the lower surface is obtained by straightforward integration of all flux over the hemisphere $0<\theta<\pi/2$. The transmissivity τ for isotropic light passing through a rough plate of thickness D is given by the relation

$$G(k)\equiv\tau-(1-z)e^{-z}-z^2\int_z^\infty x^{-1}e^{-x}dx=0, \tag{14}$$

where $z=kD$. The exponential integral in Eq. (14) is a tabulated function.[13] The root of Eq. (14) can be evaluated readily by means of the Newton–Raphson method where the relation

$$G'(k)=2D\left[e^{-z}-z\int_z^\infty x^{-1}e^{-x}dx\right] \tag{15}$$

must be used.

EXPERIMENTAL VERIFICATION

The elementary theory advanced in this paper applies to a compact leaf, an immature leaf, or a leaf in which the intercellular space has been infiltrated with water. A corn leaf, for example, is a compact leaf; that is, the leaf structure is characterized by relative absence of intercellular space. Figure 2 is the effective dispersion curve of a typical corn leaf, determined from reflectance and transmittance measurements. The effective dispersion curve of this leaf is not inconsistent with values

TABLE III. Values of Ω in the Gauss criterion[a] for the best fit between the absorption coefficients of the corn leaf and those of pure liquid water over the spectral range 1.4–2.5 μ. The equivalent water thickness D is held constant over the range.

Genera	K–M theory	Plate theory
Corn[b]	0.0161	0.0035
Cotton[c]	0.0159	0.0027
Citrus[d]	0.0227	0.0077

[a] Equation (16).
[b] Compact leaf.
[c] Immature leaf.
[d] Infiltrated with water.

[13] Walter Grutichi and William F. Cahill in *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, M. Abramowitz and I. A. Stegun, Eds. (U. S. Gov't Printing Office, Washington, D. C., 1964; Dover Publications, Inc., New York, 1965), Ch. 5, p. 221.

of the epicuticular wax[14] found on the surfaces of many leaves. The additional data point at 0.5 μ is the average published[15] index of refraction, $n=1.4687$ at 40°C, for carnauba wax, a substance obtained from the leaf surface of the carnauba palm, *Capernicia cerifera.*

The solid line in Fig. 3 is the effective absorption spectrum of the corn leaf, calculated by use of the plate model presented in this paper. The data points in Fig. 3 are the published absorption coefficients of pure liquid water.[16] The equivalent water thickness D of the corn leaf was determined by fitting the plate model to the Curcio[16] data over the spectral range 1.4–2.5 μ. The equivalent water thickness was determined to be 146 μ while the measured leaf thickness was 188 μ. Justification of the above procedure for moisture determination is based upon the premise that absorption of the corn leaf in the spectral range 1.4–2.5 μ is caused principally by pure liquid water. The dashed lines in Fig. 3 are the measured effective absorption curves resolved into components corresponding to pure liquid water and chlorophyll. Residuals between the leaf absorption curve and data for pure liquid water over the range 1.4–2.5 μ may be regarded as differential absorption spectra of leaf substances other than water and/or uncertainties of the fundamental constants for pure liquid water.

The plate-model theory of a leaf advanced in this paper contains two substantial improvements upon the K–M formulation. First, the K–M scattering coefficient s has been interpreted in terms of Fresnel reflections from the leaf or cell surfaces. Polarization phenomena have been included in the plate theory in a natural manner. Second, the absorption coefficient of a leaf determined by the plate model is in closer agreement with that of pure liquid water than calculations made from the K–M formulation.

Table III indicates the agreement between absorption coefficients k calculated from Eq. (14) and those values k_0 observed for pure liquid water[16] for the 23 spectral values $\mu=1.40,\ 1.45,\ \cdots,\ 2.50\,\mu$. Table III also illustrates the agreement between the K–M absorption coefficient k and those for pure liquid water for the same spectral values. In both the plate and K–M comparisons, an optimum constant water thickness D over the range 1.4–2.5 μ was assumed in order to obtain the best fit on a semilog plot such as Fig. 3. The Gauss criterion for goodness of fit[17]

$$\Omega\equiv[\Sigma(\log k_0-\log k)^2/n-m]=\text{a minimum} \qquad (16)$$

[14] G. Eglinton and R. J. Hamilton, Science **156**, 1322 (1967).

[15] *International Critical Tables* (McGraw-Hill Book Co., New York, 1927), Vol. II, p. 213.

[16] J. A. Curcio and C. C. Petty, J. Opt. Soc. Am. 41, 302 (1951).

[17] Archie G. Worthing and Joseph Geffner, *Treatment of Experimental Data* (John Wiley & Sons, Inc., New York, 1943), Ch. 11, p. 261.

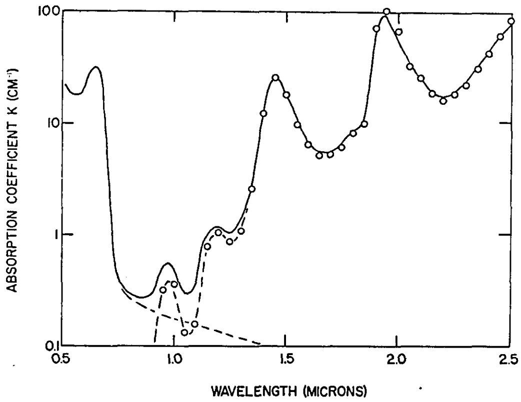

FIG. 3. The solid line indicates the equivalent absorption coefficient of a typical corn leaf with 146 μ equivalent water thickness (EWT). The dashed lines are components due to chlorophyll and pure liquid water. The data points are the Curcio values for pure liquid water.

was used. The values $n=23$ and $m=1$ were assumed in Eq. (16). Preliminary calculations listed in Table III suggest that the plate-model values for Ω are substantially better than corresponding K–M values over the same spectral region. The genera listed in Table III correspond, respectively, to a compact leaf, an immature leaf, and a leaf infiltrated with water.

The theory advanced in this paper can be extended easily to a noncompact leaf. Such a leaf can be regarded as a pile of N-compact cell layers separated by $N-1$ air spaces, where N need not be an integer. With this interpretation, the results of Ref. 1 apply. A consequence of the modified theory is that a measure can be obtained for the intercellular air space of a leaf.

CONCLUSIONS

The reflectance and transmittance spectra of a typical compact plant leaf can be synthesized with considerable accuracy by interaction of isotropic light with a transparent plate. The plate model of a leaf has been justified by successful predictions. The effective index of refraction of a corn leaf is not inconsistent with the published value of epicuticular wax. The effective absorption coefficient of a corn leaf is largely a superposition of two partial coefficients associated, respectively, with chlorophyll and pure liquid water. Differential spectra due to other leaf substances have not been established.

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