

# Elemental charge and the fine structure constant

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## Definition of the medium

We assume all space to be filled with a superfluid like medium, which behavior can be defined by four fundamental constants: the speed of light, the viscosity of the medium, Planck's constant and elementary charge  $e$ .

Hereby, we define the value of viscosity by the inverse of vacuum permeability  $\mu_0$ , which used to be defined as:  $4\pi \times 10^{-7}$  H/m.

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Vacuum\\_permeability\)](https://en.wikipedia.org/wiki/Vacuum_permeability) )

Further, we define the value of elementary charge exactly as defined in the SI system, but assign a unit of measurement in kilogram per second [kg/s]:

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Elementary\\_charge\)](https://en.wikipedia.org/wiki/Elementary_charge) )

For more background information about the characteristics of the medium see my work in progress, where in the current version the symbol  $\nu$  instead of  $k$  is used for the quantum circulation constant:

(Ref: [Researchgate \(https://www.researchgate.net/publication/341931087\)](https://www.researchgate.net/publication/341931087) )

```
In [1]: from math import *

c      = 299792458.0      # speed of light      [m/s]
eta    = 1/(4*pi*1e-7)   # viscosity          [kg/m-s], [Pa-s]
h      = 6.62607015e-34  # Planck's constant [kg/m^2-s], [J-s]
e      = 1.602176634e-19 # elementary charge [kg/s]
```

From here, we can define further constants, starting with the diffusivity, kinematic viscosity or quantum circulation constant  $k$ , which has a unit of measurement in  $[m^2/s]$  and a value equal to light speed squared.

Then we have elementary mass, which is quite unusual. This originates at the quantized vortices observed in rotating superfluids, with a circulation equal to  $\kappa_o = h/m$ , so if the circulation is given by the quantum circulation constant  $k$ , we can compute elementary mass by:  $m = h/k$ :

(Ref: [Vortices in rotating superfluid He \(https://www.pnas.org/doi/10.1073/pnas.96.14.7760\)](https://www.pnas.org/doi/10.1073/pnas.96.14.7760) )

"Around one vortex line  $\kappa$  equals the circulation quantum,  $\kappa_o = h/m$ ."

```
In [2]: k      = c*c      # quantum circulation constant          [m^2/s]
          # 8.987551787368176e+16

m      = h/k      # elementary mass 7.372497323812708e-51 [kg]

rho    = eta/k     # mass density 8.85418781762039e-12 [kg/m^3]
```

## Vector Laplace operator

Next, we consider the vector Laplace operator, defined by:

$$\Delta \vec{F} = \text{grad div} \vec{F} - \text{curl curl} \vec{F}. \quad (\text{eq.1})$$

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Laplace\\_operator#Vector\\_Laplacian\)](https://en.wikipedia.org/wiki/Laplace_operator#Vector_Laplacian) )

The vector Laplace operator describes the second spatial derivative in three dimensions, which would be the three dimensional generalization of the one dimensional  $d^2/dx^2$ , and has a unit of measurement in per meter squared  $[/m^2]$ . What is very interesting is that together with the quantum circulation constant, we can actually define the time derivative of any given vector field in three dimensional space-time as follows:

$$\frac{d\vec{F}}{dt} = -k\Delta \vec{F}. \quad (\text{eq.2})$$

This is very significant, because it enables us to define higher order Laplace and Poisson equations in three dimensions and it also gives us the opportunity to obtain a much deeper understanding of the Nature of space and time in our Universe.

What the Laplace operator does is establish a Helmholtz decomposition of the vector field  $\vec{F}$ , whereby the term  $\text{grad div} \vec{F}$  results in a **linear** component, the curl free or irrotational field usually denoted  $\vec{E}$ , and the term  $\text{curl curl} \vec{F}$  results in an **angular** component, the divergence free or incompressible field usually denoted  $\vec{B}$ . And these two components may be evaluated independently of one another.

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Helmholtz\\_decomposition\)](https://en.wikipedia.org/wiki/Helmholtz_decomposition) )

So when we consider the ratio of charge to mass density  $q/\rho$  within this model, we obtain a unit of measurement in  $[kg/s][m^3/kg] = [m^3/s]$ , whereby the  $m^3$  describes a volume  $V$ , so what this ratio describes is a rate of change of the volume  $V$  of some charged object per second.

And since the grad div operator has a unit in  $[/m^2]$  we obtain a unit of measurement in  $[m/s]$  or velocity when we compute  $\text{grad div} e/\rho$ .

The grad div operator in the vector Laplacian can be thought of as a surface integral over a spherical volume  $(4/3)\pi R^3$ , divided by radius  $R$ , whereby the limit of  $R$  is taken to zero. However, we can compute the same operation without taking the limit of  $R$  to zero by computing the radius to volume ratio  $R/V$  for such an object, such as a ring vortex with big

toroidal radius  $R$  and small poloidal radius  $r$ , and then use multiplication, provided the fields involved are symmetrical with respect to the surface area of the object. Fortunately, this is the case in this exercise.

## Application to a thin superfluid vortex ring

(Also see: [Toroidal and poloidal coordinates](https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates) ([https://en.wikipedia.org/wiki/Toroidal\\_and\\_poloidal\\_coordinates](https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates)) )

The volume of a ring vortex with big toroidal radius  $R$  and small poloidal radius  $r$  is given by:

$$V = 2\pi^2 R r^2. \quad (\text{eq. 3})$$

and thus the ratio  $R/V$  becomes:

$$R/V = R/2\pi^2 R r^2 = 1/2\pi^2 r^2. \quad (\text{eq. 4})$$

With this, we can define linear momentum  $P = mv$  for a ring vortex with mass  $m$  and charge  $q$  by:

$$P = m \text{ grad div } (q/\rho) = \frac{m}{(2\pi^2 r^2)} (q/\rho). \quad (\text{eq. 5})$$

By substituting  $m = \rho 2\pi^2 R r^2$ , we obtain

$$P = \rho 2\pi^2 R r^2 1/(2\pi^2 r^2) (q/\rho) = Rq. \quad (\text{eq. 6})$$

or:

$$P = Rq = mv. \quad (\text{eq. 7})$$

The angular momentum of a quantized ring vortex in a superfluid is given by:

$$P = \rho \Gamma \pi R^2 (1 + 3/4 \varepsilon^2), \quad (\text{eq. 8})$$

with  $\varepsilon$  the ratio of the ring radius  $R$  to the radius of the hollow core  $a$ , or  $\varepsilon = a/R$ .

(Ref. eq. 1.8 in: [Dynamics of thin vortex rings](https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings) ([https://www.researchgate.net/publication/232025984\\_Dynamics\\_of\\_thin\\_vortex\\_rings](https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings)) )

Under the assumption that  $a \ll R$ , this reduces to:

$$P = \rho \Gamma \pi R^2. \quad (\text{eq. 9})$$

Because of momentum conservation, the angular and linear momentum of such an object must be equal in magnitude, so we can equate them and simplify:

$$P = \rho \Gamma \pi R^2 (1 + 3/4 \epsilon^2) = Rq$$

$$\rho \Gamma \pi R^2 (1 + 3/4 \epsilon^2) = Rq$$

$$\rho \Gamma \pi R (1 + 3/4 \epsilon^2) = q$$

(eq. 10)

And solve for R:

$$R = \frac{q}{\rho \Gamma \pi (1 + 3/4 \epsilon^2)}$$

(eq. 11)

Under the assumption that  $a \ll R$ , this reduces to:

$$R = \frac{q}{\rho \Gamma \pi}$$

(eq. 12)

This way, we can approximate R and confirm both the angular and linear momentum are equal for the elemental ring vortex with charge e and circulation  $\Gamma=k$ :

```
In [3]: R = e/(rho*k*pi)      # Approximate radius of elemental vortex ring
                                     # 6.408706536e-26 [m]
print(R)
print(rho*k*pi*R**2)
print(R*e)

6.408706536e-26
1.0267879866142279e-44
1.0267879866142279e-44
```

And by dividing linear momentum by mass, we can compute a velocity. For the elementary ring vortex with mass m, this computes to about 3.7 times the speed of the sun relative to the cosmic microwave background:

```
In [4]: print((R*e)/(m*1000), "km/s.")

1392.7275135085724 km/s.
```

## Definition of elemental charge from the fine structure constant definition

The fine structure constant  $\alpha$  is defined by:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \frac{h}{2\pi} c}$$

(eq. 13)

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Fine-structure\\_constant\)](https://en.wikipedia.org/wiki/Fine-structure_constant) )

Within our model, we substitute  $\epsilon_0 = \rho$ . And then we can simplify to:

$$\alpha = \frac{e^2}{2\rho h c}.$$

(eq. 14)

And then we can rewrite this to define elementary charge:

$$e^2 = e * e = 2\alpha\rho h c.$$

(eq. 15)

Next, we can approximate  $\alpha$  by:

$$\alpha = \frac{1}{8\pi^2 \sqrt{3}}.$$

(eq. 16)

(Ref: eq. 16 in: [The Atomic Vortex Hypothesis, a Forgotten Path to Unification \(https://vixra.org/pdf/1310.0237v1.pdf\)](https://vixra.org/pdf/1310.0237v1.pdf) )

And we substitute  $\alpha$  by this approximation and simplify to obtain:

$$e * e = \frac{\rho h c}{4\pi^2 \sqrt{3}}.$$

(eq. 17)

Next step is to multiply by  $m/m$ ,  $R/R$  and  $\pi/\pi$  and re-arrange:

$$e * e = \rho \frac{h}{m} \pi R * \frac{m c}{4\pi^3 \sqrt{3} R}.$$

(eq. 18)

```
In [5]: angular = rho*(h/m)*pi*R
linear  = (m*c)/(4*pi**3*sqrt(3)*R)

print("angular or magnetic: ", angular)
print("linear or dielectric:", linear)
```

```
angular or magnetic:  1.602176634e-19
linear or dielectric: 1.605442311612586e-19
```

Obviously, if we were to multiply these values and compute the square root, we obtain a

value that is very close to the defined value of elemental charge,  $e$ .

What is interesting, however, is that the angular value is already exactly equal to elemental charge  $e$ . And its unit of measurement evaluates to  $[kg/m^3] * [m^2/s] * [m] = [kg/s]$ , the unit we assigned to electrical charge. And as can be expected, the unit of measurement for the linear term also works out to  $[kg/s]$ .

However, there is a slight mismatch between the two values, which should be equal. In order to solve that problem, we compute  $P^2$  using the relation we already found, namely  $P = Rq = mv$ :

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R}. \quad (\text{eq. 19})$$

And simplify to:

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \frac{\rho h c}{4\pi^2 \sqrt{3}}. \quad (\text{eq. 20})$$

Next, we also compute the angular momentum squared,  $P^2$ , for a thin vortex ring using eq. 1.8 from the already referenced paper "Dynamics of thin vortex rings":

$$P = \rho \Gamma \pi R^2 (1 + 3/4\epsilon^2). \quad (\text{eq. 21})$$

This way we obtain:

$$\begin{aligned} P^2 &= (\rho \Gamma \pi R^2 (1 + 3/4\epsilon^2))^2. \\ P^2 &= \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\epsilon^2)^2. \end{aligned} \quad (\text{eq. 22})$$

And then we equate both equations:

$$P^2 = \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\epsilon^2)^2 = R^2 * \frac{\rho h c}{4\pi^2 \sqrt{3}}. \quad (\text{eq. 23})$$

And rewrite to:

$$(1 + 3/4\epsilon^2)^2 = \frac{hc}{4\pi^2 \sqrt{3} \rho \Gamma^2 \pi^2 R^2} \quad (\text{eq. 24})$$

In order to simplify further calculations, we take the right term together to define a variable  $X$ :

$$X = \frac{hc}{4\pi^2 \sqrt{3} \rho \Gamma^2 \pi^2 R^2},$$

(eq. 25)

thus:

$$(1 + 3/4\epsilon^2)^2 = X.$$

(eq. 26)

And solve for  $\epsilon$ :

$$\epsilon = \sqrt{\frac{4}{3}(\sqrt{X} - 1)}.$$

(eq. 27)

And since  $\epsilon$  is defined as the ratio  $a/R$  we can compute  $a$  along:

$$a = \epsilon R.$$

```
In [6]: X = (h*c)/(4*pi*pi*sqrt(3)*rho*k*k*pi*pi*R*R)
eps = sqrt( (4/3)* (sqrt(X) - 1) )
a = eps*R      # radius of hollow core 2.361813747378764e-27 [m]

print("eps:", eps)
print("a:  ", a)
```

```
eps: 0.03685320484112684
a: 2.361813747378764e-27
```

## Corrected definition of elemental charge

Next step is to correct our definition for linear elementary charge by including the term  $(1 + 3/4\epsilon^2)^2$  to come to our new definition of elemental charge squared:

$$e * e = \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R (1 + 3/4\epsilon^2)^2}.$$

or, with a sneak preview of what's to come:

$$e * e = \rho \frac{h}{m} \pi R * 2mc\alpha.$$

(eq. 29)

This illustrates that  $\alpha$  actually represents a coupling factor between the linear (electric) and angular (magnetic) fields describing the dynamics of the elemental superfluid vortex ring in the medium.

And then we evaluate both terms again:

```
In [7]: a_e = 1 + (3/4) * eps * eps

linear = (m*c)/(4*pi**3*sqrt(3)*R*a_e*a_e)
angular = rho*(h/m)*pi*R
q        = sqrt(linear*angular)

print("angular or magnetic charge: ", angular)
print("linear or dielectric charge:", linear)
print("elemental charge:           ", q)
```

```
angular or magnetic charge: 1.602176634e-19
linear or dielectric charge: 1.6021766340000001e-19
elemental charge:          1.602176634e-19
```

## New definition of fine structure constant $\alpha$

Next step is to return to the equation we started with:

$$e^2 = e * e = 2\alpha phc.$$

(eq. 30)

And equate this to our corrected equation:

$$e * e = 2\alpha phc = \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R (1 + 3/4\epsilon^2)^2}.$$

(eq. 31)

And solve for  $\alpha$  to obtain a new definition for  $\alpha$ :

$$\alpha = \frac{1}{8\pi^2 \sqrt{3} (1 + 3/4\epsilon^2)^2}.$$

(eq. 32)

We can compute this and compare to the the 2018 CODATA recommended value of  $\alpha$ :

$$\alpha = 0.007297352569311$$

(eq. 33)

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Fine-structure\\_constant\)](https://en.wikipedia.org/wiki/Fine-structure_constant) )



```
In [8]: alpha          = 1/(8*pi*pi*sqrt(3)*(1+(3/4)*eps*eps)*(1+(3/4)*eps*eps))
alpha_codata = 0.007297352569311

print("alpha:      ",alpha)
print("codata:     ",alpha_codata)
print("relative:   ",alpha_codata/alpha)

alpha:      0.007297352565305213
codata:     0.007297352569311
relative:   1.0000000005489371
```

Obviously, the result is very close to the official value, so it seems we have solved an important problem in theoretical physics.

Also, since this definition of  $\alpha$  has been directly derived from our new definition for elementary charge, this result goes a lot further than just offering an explanation for the physical origin of the fine structure constant  $\alpha$ , it also offers a much deeper understanding of what charge is and, by extension, the electromagnetic fields.

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