Elemental charge and the fine structure constant

By Arend Lammertink, MScEE. E-mail: lamare at gmail dot com.

Definition of the medium

We assume all space to be filled with a superfluid like medium, which behavior can be defined by four fundamental constants: the speed of light, the viscosity of the medium, Planck's constant and elementary charge e.

Hereby, we define the value of viscosity by the inverse of vacuum permeability μ_0, which used to be defined as: $4\pi \times 10^{-7}$ H/m.

(Ref: [Wikipedia (https://en.wikipedia.org/wiki/Vacuum_permeability)](https://en.wikipedia.org/wiki/Vacuum_permeability))

Further, we define the value of elementary charge exactly as defined in the SI system, but assign a unit of measurement in kilogram per second [kg/s]:

(Ref: [Wikipedia (https://en.wikipedia.org/wiki/Elementary_charge)](https://en.wikipedia.org/wiki/Elementary_charge))

For more background information about the characteristics of the medium see my work in progress, where in the current version the symbol ν instead of k is used for the quantum circulation constant:

(Ref: [Researchgate (https://www.researchgate.net/publication/341931087)](https://www.researchgate.net/publication/341931087))

In [1]:
```
from math import *

c     = 299792458.0     # speed of light     [m/s]
eta   = 1/(4*pi*1e-7)   # viscosity          [kg/m-s],    [Pa-s]
h     = 6.62607015e-34  # Planck's constant  [kg/m^2-s], [J-s]

e     = 1.602176634e-19 # elementary charge  [kg/s]
```

From here, we can define further constants, starting with the diffusivity, kinematic viscosity or quantum circulation constant k, which has a unit of measurement in $[m^2/s]$ and a value equal to light speed squared.

Then we have elementary mass, which is quite unusual. This originates at the quantized vortices observed in rotating superfluids, with a circulation equal to $\kappa_o = h/m$, so if the circulation is given by the quantum circulation constant k, we can compute elementary mass by: $m = h/k$:

(Ref. [Vortices in rotating superfluid He (https://www.pnas.org/doi/10.1073/pnas.96.14.7760)](https://www.pnas.org/doi/10.1073/pnas.96.14.7760))

"Around one vortex line κ equals the circulation quantum, $\kappa_o = h/m$."

In [2]:
```
k      = c*c     # quantum circulation constant
                 # 8.987551787368176e+16                [m^2/s]

m      = h/k     # elementary mass 7.372497323812708e-51 [kg]

rho    = eta/k   # mass density   8.85418781762039e-12   [kg/m^3]
```

Vector Laplace operator

Next, we consider the vector Laplace operator, defined by:

$$\Delta \vec{F} = \text{grad div} \vec{F} - \text{curl curl} \vec{F}.$$

(eq.1)

(Ref. [Wikipedia (https://en.wikipedia.org/wiki/Laplace_operator#Vector_Laplacian)](https://en.wikipedia.org/wiki/Laplace_operator#Vector_Laplacian))

The vector Laplace operator describes the second spatial derivative in three dimensions, which would be the three dimensional generalization of the one dimensional d^2/dx^2, and has a unit of measurement in per meter squared [$/m^2$]. What is very interesting is that together with the quantum circulation constant, we can actually define the time derivative of any given vector field in three dimensional space-time as follows:

$$\frac{d\vec{F}}{dt} = -k \Delta \vec{F}.$$

(eq.2)

This is very significant, because it enables us to define higher order Laplace and Poisson equations in three dimensions and it also gives us the opportunity to obtain a much deeper understanding of the Nature of space and time in our Universe.

What the Laplace operator does is establish a Helmholtz decomposition of the vector field $\vec{F}$, whereby the term grad div $\vec{F}$ results in a **linear** component, the curl free or irrotational field usually denoted $\vec{E}$, and the term curl curl $\vec{F}$ results in an **angular** component, the divergence free or incompressible field usually denoted $\vec{B}$. And these two components may be evaluated independently of one another.

(Ref. [Wikipedia (https://en.wikipedia.org/wiki/Helmholtz_decomposition)](https://en.wikipedia.org/wiki/Helmholtz_decomposition))

So when we consider the ratio of charge to mass density q/ρ within this model, we obtain a unit of measurement in $[kg/s][m^3/kg] = [m^3/s]$, whereby the m^3 describes a volume V, so what this ratio describes is a rate of change of the volume V of some charged object per second.

And since the grad div operator has a unit in [$/m^2$] we obtain a unit of measurement in [m/s] or velocity when we compute grad div e/ρ.

The grad div operator in the vector Laplacian can be thought of as a surface integral over a spherical volume $(4/3)\pi R^3$, divided by radius R, whereby the limit of R is taken to zero. However, we can compute the same operation without taking the limit of R to zero by computing the radius to volume ratio R/V for such an object, such as a ring vortex with big toroidal radius R and small poloidal radius r, and then use multiplication, provided the fields involved are symmetrical with respect to the surface area of the object. Fortunately, this is the case in this exercise.

Application to a thin superfluid vortex ring

(Also see: [Toroidal and poloidal coordinates (https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates)](https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates))

The volume of a ring vortex with big toroidal radius R and small polloidal radius r is given by:

$$V = 2\pi^2 Rr^2.$$

(eq. 3)

and thus the ratio R/V becomes:

$$R/V = R/2\pi^2 Rr^2 = 1/2\pi^2 r^2.$$

(eq. 4)

With this, we can define linear momentum $P = mv$ for a ring vortex with mass m and charge q by:

$$P = m \text{ grad div } (q/\rho) = \frac{m}{(2\pi^2 r^2)}(q/\rho).$$

(eq. 5)

By substituting $m = \rho 2\pi^2 Rr^2$, we obtain

$$P = \rho 2\pi^2 Rr^2 1/(2\pi^2 r^2)(q/\rho) = Rq.$$

(eq. 6)

or:

$$P = Rq = mv.$$

(eq. 7)

The angular momentum of a quantized ring vortex in a superfluid is given by:

$$P = \rho\Gamma\pi R^2(1 + 3/4\varepsilon^2),$$

(eq. 8)

with ε the ratio of the ring radius R to the radius of the hollow core a, or $\varepsilon = a/R$.

(Ref. eq. 1.8 in: [Dynamics of thin vortex rings (https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings)](https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings))

Under the assumption that $a << R$, this reduces to:

$$P = \rho\Gamma\pi R^2.$$

(eq. 9)

Because of momentum conservation, the angular and linear momentum of such an object must be equal in magnitude, so we can equate them and simplify:

$$P = \rho \Gamma \pi R^2 (1 + 3/4\varepsilon^2) = Rq$$

$$\rho \Gamma \pi R^2 (1 + 3/4\varepsilon^2) = Rq$$

$$\rho \Gamma \pi R (1 + 3/4\varepsilon^2) = q$$

(eq. 10)

And solve for R:

$$R = \frac{q}{\rho \Gamma \pi (1 + 3/4\varepsilon^2)}$$

(eq. 11)

Under the assumption that $a << R$, this reduces to:

$$R = \frac{q}{\rho \Gamma \pi}$$

(eq. 12)

This way, we can approximate R and confirm both the angular and linear momentum are equal for the elemental ring vortex with charge e and circulation Γ=k:

In [3]:
```
R = e/(rho*k*pi)      # Approximate radius of elemental vortex ring
                      # 6.408706536e-26 [m]
print(R)
print(rho*k*pi*R**2)
print(R*e)
```

```
6.408706536e-26
1.0267879866142279e-44
1.0267879866142279e-44
```

And by dividing linear momentum by mass, we can compute a velocity. For the elementary ring vortex with mass m, this computes to about 3.7 times the speed of the sun relative to the cosmic microwave background:

In [4]:
```
print((R*e)/(m*1000), "km/s.")
```

```
1392.7275135085724 km/s.
```

Definition of elemental charge from the fine structure constant definition

The fine structure constant α is defined by:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \frac{h}{2\pi} c}$$

(eq. 13)

(Ref. Wikipedia (https://en.wikipedia.org/wiki/Fine-structure_constant))

Within our model, we substitute $\epsilon_0 = \rho$. And then we can simplify to:

$$\alpha = \frac{e^2}{2\rho h c}.$$

(eq. 14)

And then we can rewrite this to define elementary charge:

$$e^2 = e * e = 2\alpha\rho h c.$$

(eq. 15)

Next, we can approximate α by:

$$\alpha = \frac{1}{8\pi^2\sqrt{3}}.$$

(eq. 16)

(Ref: eq. 16 in: The Atomic Vortex Hypothesis, a Forgotten Path to Unification (https://vixra.org/pdf/1310.0237v1.pdf))

And we substite α by this approximation and simplify to obtain:

$$e * e = \frac{\rho h c}{4\pi^2\sqrt{3}}.$$

(eq. 17)

Next step is to multiply by m/m, $\boldsymbol{R}/\boldsymbol{R}$ and π/π and re-arrange:

$$e * e = \rho\frac{h}{m}\pi\boldsymbol{R} * \frac{mc}{4\pi^3\sqrt{3}\boldsymbol{R}}.$$

(eq. 18)

In [5]:
```
angular = rho*(h/m)*pi*R
linear  = (m*c)/(4*pi**3*sqrt(3)*R)

print("angular or magnetic: ", angular)
print("linear or dielectric:", linear)
```

```
angular or magnetic:  1.602176634e-19
linear or dielectric: 1.605442311612586e-19
```

Obviously, if we were to multiply these values and compute the square root, we obtain a value that is very close to the defined value of elemental charge, e.

What is interesting, however, is that the angular value is already exactly equal to elemental charge e. And its unit of measurement evaluates to $[kg/m^3] * [m^2/s] * [m] = [kg/s]$, the unit we assigned to electrical charge. And as can be expected, the unit of measurement for the linear term also works out to $[kg/s]$.

However, there is a slight mismatch between the two values, which should be equal. In order to solve that problem, we compute P^2 using the relation we already found, namely $P = Rq = mv$:

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3R}}.$$

(eq. 19)

And simplify to:

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \frac{\rho hc}{4\pi^2 \sqrt{3}}.$$

(eq. 20)

Next, we also compute the angular momentum squared, P^2, for a thin vortex ring using eq. 1.8 from the already referenced paper "Dynamics of thin vortex rings":

$$P = \rho \Gamma \pi R^2 (1 + 3/4\varepsilon^2).$$

(eq. 21)

This way we obtain:

$$P^2 = (\rho \Gamma \pi R^2 (1 + 3/4\varepsilon^2))^2.$$

$$P^2 = \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\varepsilon^2)^2.$$

(eq. 22)

And then we equate both equations:

$$P^2 = \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\varepsilon^2)^2 = R^2 * \frac{\rho hc}{4\pi^2 \sqrt{3}}.$$

(eq. 23)

And rewrite to:

$$(1 + 3/4\varepsilon^2)^2 = \frac{hc}{4\pi^2 \sqrt{3} \rho \Gamma^2 \pi^2 R^2}$$

(eq. 24)

In order to simplify further calculations, we take the right term together to define a variable X:

$$X = \frac{hc}{4\pi^2\sqrt{3\rho\Gamma^2\pi^2 R^2}},$$

(eq. 25)

thus:

$$(1 + 3/4\varepsilon^2)^2 = X.$$

(eq. 26)

And solve for ε:

$$\varepsilon = \sqrt{\frac{4}{3}(\sqrt{X} - 1)}.$$

(eq. 27)

And since ε is defined as the ratio a/R we can compute a along:

$$a = \varepsilon R.$$

(eq. 28)

In [6]:
```
X   = (h*c)/(4*pi*pi*sqrt(3)*rho*k*k*pi*pi*R*R)
eps = sqrt( (4/3)* (sqrt(X)- 1) )
a   = eps*R    # radius of hollow core 2.361813747378764e-27 [m]

print("eps:", eps)
print("a:  ", a)
```

```
eps: 0.03685320484112684
a:   2.361813747378764e-27
```

Corrected definition of elemental charge

Next step is to correct our definition for linear elementary charge by including the term $(1 + 3/4\varepsilon^2)^2$ to come to our new definition of elemental charge squared:

$$e * e = \rho\frac{h}{m}\pi R * \frac{mc}{4\pi^3\sqrt{3}R(1 + 3/4\varepsilon^2)^2}.$$

or:

$$e * e = \rho\frac{h}{m}\pi R * mc\alpha.$$

(eq. 29)

This illustrates that α actually represents a coupling factor between the linear (electric) and angular (magnetic) fields describing the dynamics of the elemental superfluid vortex ring in the medium.

And then we evaluate both terms again:

In [7]:
```
a_e = 1 + (3/4) * eps *eps

linear  = (m*c)/(4*pi**3*sqrt(3)*R*a_e*a_e)
angular = rho*(h/m)*pi*R
q       = sqrt(linear*angular)

print("angular or magnetic charge: ", angular)
print("linear or dielectric charge:", linear)
print("elemental charge:           ", q)
```

```
angular or magnetic charge:  1.602176634e-19
linear or dielectric charge: 1.6021766340000001e-19
elemental charge:            1.602176634e-19
```

New definition of fine structure constant α

Next step is to return to the equation we started with:

$$e^2 = e * e = 2\alpha\rho hc.$$

(eq. 30)

And equate this to our corrected equation:

$$e * e = 2\alpha\rho hc = \rho\frac{h}{m}\pi R * \frac{mc}{4\pi^3\sqrt{3}R(1+3/4\varepsilon^2)^2}.$$

(eq. 31)

And solve for α to obtain a new definition for α:

$$\alpha = \frac{1}{8\pi^2\sqrt{3}(1+3/4\varepsilon^2)^2}.$$

(eq. 32)

We can compute this and compare to the the 2018 CODATA recommended value of α:

$$\alpha = 0.007297352569311$$

(eq. 33)

(Ref. Wikipedia (https://en.wikipedia.org/wiki/Fine-structure_constant))

In [8]:
```
alpha        = 1/(8*pi*pi*sqrt(3)*(1+(3/4)*eps*eps)*(1+(3/4)*eps*eps)
alpha_codata = 0.007297352569311

print("alpha:     ",alpha)
print("codata:    ",alpha_codata)
print("relative: ",alpha_codata/alpha)
```

```
alpha:     0.007297352565305213
codata:    0.007297352569311
relative:  1.0000000005489371
```

Obviously, the result is very close to the official value, so it seems we have solved an important problem in theoretical physics.

Also, since this definition of α has been directly derived from our new definition for elementary charge, this result goes a lot further than just offering an explanation for the physical origin of the fine structure constant α, it also offers a much deeper understanding of what charge is and, by extension, the electromagnetic fields.

The source of this document is available at: https://bit.ly/ElementalCharge (https://bit.ly/ElementalCharge)

A pdf version is available at: https://bit.ly/ElementalChargePDF (https://bit.ly/ElementalChargePDF)

Join the discussion about this work at: https://bit.ly/ElementalChargeDiscussion (https://bit.ly/ElementalChargeDiscussion)

In [ ]: