

Elemental charge and the fine structure constant

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Definition of the medium

We assume all space to be filled with a superfluid like medium, which behavior can be defined by four fundamental constants: the speed of light, the viscosity of the medium, Planck's constant and elementary charge e .

Hereby, we define the value of viscosity by the inverse of vacuum permeability μ_0 , which used to be defined as: $4\pi \times 10^{-7}$ H/m.

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Vacuum_permeability\)](https://en.wikipedia.org/wiki/Vacuum_permeability))

Further, we define the value of elementary charge exactly as defined in the SI system, but assign a unit of measurement in kilogram per second [kg/s]:

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Elementary_charge\)](https://en.wikipedia.org/wiki/Elementary_charge))

For more background information about the characteristics of the medium see my work in progress, where in the current version the symbol ν instead of k is used for the quantum circulation constant:

(Ref: [Researchgate \(https://www.researchgate.net/publication/341931087\)](https://www.researchgate.net/publication/341931087))

```
In [1]: from math import *

c      = 299792458.0      # speed of light      [m/s]
eta    = 1/(4*pi*1e-7)   # viscosity          [kg/m-s], [Pa-s]
h      = 6.62607015e-34  # Planck's constant [kg/m^2-s], [J-s]
e      = 1.602176634e-19 # elementary charge [kg/s]
```

From here, we can define further constants, starting with the diffusivity, kinematic viscosity or quantum circulation constant k , which has a unit of measurement in $[m^2/s]$ and a value equal to light speed squared.

Then we have elementary mass, which is quite unusual. This originates at the quantized vortices observed in rotating superfluids, with a circulation equal to $\kappa_o = h/m$, so if the circulation is given by the quantum circulation constant k , we can compute elementary mass by: $m = h/k$:

(Ref: [Vortices in rotating superfluid He \(https://www.pnas.org/doi/10.1073/pnas.96.14.7760\)](https://www.pnas.org/doi/10.1073/pnas.96.14.7760))

"Around one vortex line κ equals the circulation quantum, $\kappa_o = h/m$."

```
In [2]: k      = c*c      # quantum circulation constant           [m^2/s]
          # 8.987551787368176e+16

m      = h/k      # elementary mass 7.372497323812708e-51 [kg]

rho    = eta/k    # mass density 8.85418781762039e-12 [kg/m^3]
```

Vector Laplace operator

Next, we consider the vector Laplace operator, defined by:

$$\Delta \vec{F} = \text{grad div} \vec{F} - \text{curl curl} \vec{F}. \quad (\text{eq.1})$$

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Laplace_operator#Vector_Laplacian\)](https://en.wikipedia.org/wiki/Laplace_operator#Vector_Laplacian))

The vector Laplace operator describes the second spatial derivative in three dimensions, which would be the three dimensional generalization of the one dimensional d^2/dx^2 , and has a unit of measurement in per meter squared $[/m^2]$. What is very interesting is that together with the quantum circulation constant, we can actually define the time derivative of any given vector field in three dimensional space-time as follows:

$$\frac{d\vec{F}}{dt} = -k\Delta\vec{F}. \quad (\text{eq.2})$$

This is very significant, because it enables us to define higher order Laplace and Poisson equations in three dimensions and it also gives us the opportunity to obtain a much deeper understanding of the Nature of space and time in our Universe.

What the Laplace operator does is establish a Helmholtz decomposition of the vector field \vec{F} , whereby the term $\text{grad div} \vec{F}$ results in a **linear** component, the curl free or irrotational field usually denoted \vec{E} , and the term $\text{curl curl} \vec{F}$ results in an **angular** component, the divergence free or incompressible field usually denoted \vec{B} . And these two components may be evaluated independently of one another.

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Helmholtz_decomposition\)](https://en.wikipedia.org/wiki/Helmholtz_decomposition))

So when we consider the ratio of charge to mass density q/ρ within this model, we obtain a unit of measurement in $[kg/s][m^3/kg] = [m^3/s]$, whereby the m^3 describes a volume V , so what this ratio describes is a rate of change of the volume V of some charged object per second.

And since the grad div operator has a unit in $[/m^2]$ we obtain a unit of measurement in $[m/s]$ or velocity when we compute $\text{grad div } e/\rho$.

The grad div operator in the vector Laplacian can be thought of as a surface integral over a spherical volume $(4/3)\pi R^3$, divided by radius R , whereby the limit of R is taken to zero. However, we can compute the same operation without taking the limit of R to zero by computing the radius to volume ratio R/V for such an object, such as a ring vortex with big

toroidal radius R and small poloidal radius r , and then use multiplication, provided the fields involved are symmetrical with respect to the surface area of the object. Fortunately, this is the case in this exercise.

Application to a thin superfluid vortex ring

(Also see: [Toroidal and poloidal coordinates](https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates) (https://en.wikipedia.org/wiki/Toroidal_and_poloidal_coordinates))

The volume of a ring vortex with big toroidal radius R and small poloidal radius r is given by:

$$V = 2\pi^2 R r^2. \quad (\text{eq. 3})$$

and thus the ratio R/V becomes:

$$R/V = R/2\pi^2 R r^2 = 1/2\pi^2 r^2. \quad (\text{eq. 4})$$

With this, we can define linear momentum $P = mv$ for a ring vortex with mass m and charge q by:

$$P = m \text{ grad div } (q/\rho) = \frac{m}{(2\pi^2 r^2)} (q/\rho). \quad (\text{eq. 5})$$

By substituting $m = \rho 2\pi^2 R r^2$, we obtain

$$P = \rho 2\pi^2 R r^2 1/(2\pi^2 r^2) (q/\rho) = Rq. \quad (\text{eq. 6})$$

or:

$$P = Rq = mv. \quad (\text{eq. 7})$$

The angular momentum of a quantized ring vortex in a superfluid is given by:

$$P = \rho \Gamma \pi R^2 (1 + 3/4 \varepsilon^2), \quad (\text{eq. 8})$$

with ε the ratio of the ring radius R to the radius of the hollow core a , or $\varepsilon = a/R$.

(Ref. eq. 1.8 in: [Dynamics of thin vortex rings](https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings) (https://www.researchgate.net/publication/232025984_Dynamics_of_thin_vortex_rings))

Under the assumption that $a \ll R$, this reduces to:

$$P = \rho \Gamma \pi R^2. \quad (\text{eq. 9})$$

Because of momentum conservation, the angular and linear momentum of such an object must be equal in magnitude, so we can equate them and simplify:

$$P = \rho \Gamma \pi R^2 (1 + 3/4 \epsilon^2) = Rq$$

$$\rho \Gamma \pi R^2 (1 + 3/4 \epsilon^2) = Rq$$

$$\rho \Gamma \pi R (1 + 3/4 \epsilon^2) = q$$

(eq. 10)

And solve for R:

$$R = \frac{q}{\rho \Gamma \pi (1 + 3/4 \epsilon^2)}$$

(eq. 11)

Under the assumption that $a \ll R$, this reduces to:

$$R = \frac{q}{\rho \Gamma \pi}$$

(eq. 12)

This way, we can approximate R and confirm both the angular and linear momentum are equal for the elemental ring vortex with charge e and circulation $\Gamma=k$:

```
In [3]: R = e/(rho*k*pi)      # Approximate radius of elemental vortex ring
                                     # 6.408706536e-26 [m]
print(R)
print(rho*k*pi*R**2)
print(R*e)

6.408706536e-26
1.0267879866142279e-44
1.0267879866142279e-44
```

And by dividing linear momentum by mass, we can compute a velocity. For the elementary ring vortex with mass m, this computes to about 3.7 times the speed of the sun relative to the cosmic microwave background:

```
In [4]: print((R*e)/(m*1000), "km/s.")

1392.7275135085724 km/s.
```

Definition of elemental charge from the fine structure constant definition

The fine structure constant α is defined by:

$$\alpha = \frac{e^2}{4\pi\epsilon_0 \frac{h}{2\pi} c}$$

(eq. 13)

(Ref: [Wikipedia \(https://en.wikipedia.org/wiki/Fine-structure_constant\)](https://en.wikipedia.org/wiki/Fine-structure_constant))

Within our model, we substitute $\epsilon_0 = \rho$. And then we can simplify to:

$$\alpha = \frac{e^2}{2\rho h c}.$$

(eq. 14)

And then we can rewrite this to define elementary charge:

$$e^2 = e * e = 2\alpha\rho h c.$$

(eq. 15)

Next, we can approximate α by:

$$\alpha = \frac{1}{8\pi^2 \sqrt{3}}.$$

(eq. 16)

(Ref: eq. 16 in: [The Atomic Vortex Hypothesis, a Forgotten Path to Unification \(https://vixra.org/pdf/1310.0237v1.pdf\)](https://vixra.org/pdf/1310.0237v1.pdf))

And we substitute α by this approximation and simplify to obtain:

$$e * e = \frac{\rho h c}{4\pi^2 \sqrt{3}}.$$

(eq. 17)

Next step is to multiply by m/m , R/R and π/π and re-arrange:

$$e * e = \rho \frac{h}{m} \pi R * \frac{m c}{4\pi^3 \sqrt{3} R}.$$

(eq. 18)

```
In [5]: angular = rho*(h/m)*pi*R
linear  = (m*c)/(4*pi**3*sqrt(3)*R)

print("angular or magnetic: ", angular)
print("linear or dielectric:", linear)
```

```
angular or magnetic:  1.602176634e-19
linear or dielectric: 1.605442311612586e-19
```

Obviously, if we were to multiply these values and compute the square root, we obtain a

value that is very close to the defined value of elemental charge, e .

What is interesting, however, is that the angular value is already exactly equal to elemental charge e . And its unit of measurement evaluates to $[kg/m^3] * [m^2/s] * [m] = [kg/s]$, the unit we assigned to electrical charge. And as can be expected, the unit of measurement for the linear term also works out to $[kg/s]$.

However, there is a slight mismatch between the two values, which should be equal. In order to solve that problem, we compute P^2 using the relation we already found, namely $P = Rq = mv$:

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R}. \quad (\text{eq. 19})$$

And simplify to:

$$P^2 = R^2 q^2 = R^2 e * e = R^2 * \frac{\rho h c}{4\pi^2 \sqrt{3}}. \quad (\text{eq. 20})$$

Next, we also compute the angular momentum squared, P^2 , for a thin vortex ring using eq. 1.8 from the already referenced paper "Dynamics of thin vortex rings":

$$P = \rho \Gamma \pi R^2 (1 + 3/4\epsilon^2). \quad (\text{eq. 21})$$

This way we obtain:

$$\begin{aligned} P^2 &= (\rho \Gamma \pi R^2 (1 + 3/4\epsilon^2))^2. \\ P^2 &= \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\epsilon^2)^2. \end{aligned} \quad (\text{eq. 22})$$

And then we equate both equations:

$$P^2 = \rho^2 \Gamma^2 \pi^2 R^4 (1 + 3/4\epsilon^2)^2 = R^2 * \frac{\rho h c}{4\pi^2 \sqrt{3}}. \quad (\text{eq. 23})$$

And rewrite to:

$$(1 + 3/4\epsilon^2)^2 = \frac{hc}{4\pi^2 \sqrt{3} \rho \Gamma^2 \pi^2 R^2} \quad (\text{eq. 24})$$

In order to simplify further calculations, we take the right term together to define a variable X :

$$X = \frac{hc}{4\pi^2 \sqrt{3} \rho \Gamma^2 \pi^2 R^2},$$

(eq. 25)

thus:

$$(1 + 3/4\epsilon^2)^2 = X.$$

(eq. 26)

And solve for ϵ :

$$\epsilon = \sqrt{\frac{4}{3}(\sqrt{X} - 1)}.$$

(eq. 27)

And since ϵ is defined as the ratio a/R we can compute a along:

$$a = \epsilon R.$$

```
In [6]: X = (h*c)/(4*pi*pi*sqrt(3)*rho*k*k*pi*pi*R*R)
eps = sqrt( (4/3)* (sqrt(X) - 1) )
a = eps*R      # radius of hollow core 2.361813747378764e-27 [m]

print("eps:", eps)
print("a: ", a)
```

```
eps: 0.03685320484112684
a: 2.361813747378764e-27
```

Corrected definition of elemental charge

Next step is to correct our definition for linear elementary charge by including the term $(1 + 3/4\epsilon^2)^2$ to come to our new definition of elemental charge squared:

$$e * e = \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R (1 + 3/4\epsilon^2)^2}.$$

(eq. 29)

And then we evaluate both terms again:

```
In [7]: a_e = 1 + (3/4) * eps * eps

linear = (m*c)/(4*pi**3*sqrt(3)*R*a_e*a_e)
angular = rho*(h/m)*pi*R
q        = sqrt(linear*angular)

print("angular or magnetic charge: ", angular)
print("linear or dielectric charge:", linear)
print("elemental charge:           ", q)
```

```
angular or magnetic charge: 1.602176634e-19
linear or dielectric charge: 1.6021766340000001e-19
elemental charge:          1.602176634e-19
```

New definition of fine structure constant α

Next step is to return to the equation we started with:

$$e^2 = e * e = 2\alpha phc.$$

(eq. 30)

And equate this to our corrected equation:

$$e * e = 2\alpha phc = \rho \frac{h}{m} \pi R * \frac{mc}{4\pi^3 \sqrt{3} R (1 + 3/4\epsilon^2)^2}.$$

(eq. 31)

And solve for α to obtain a new definition for α :

$$\alpha = \frac{1}{8\pi^2 \sqrt{3} (1 + 3/4\epsilon^2)^2}.$$

(eq. 32)

We can compute this and compare to the the 2018 CODATA recommended value of α :

$$\alpha = 0.007297352569311$$

(eq. 33)

(Ref. [Wikipedia \(https://en.wikipedia.org/wiki/Fine-structure_constant\)](https://en.wikipedia.org/wiki/Fine-structure_constant))


```
In [8]: alpha          = 1/(8*pi*pi*sqrt(3)*(1+(3/4)*eps*eps)*(1+(3/4)*eps*eps))
alpha_codata = 0.007297352569311

print("alpha:      ",alpha)
print("codata:     ",alpha_codata)
print("relative:   ",alpha_codata/alpha)

alpha:      0.007297352565305213
codata:     0.007297352569311
relative:   1.0000000005489371
```

Obviously, the result is very close to the official value, so it seems we have solved an important problem in theoretical physics.

Also, since this definition of α has been directly derived from our new definition for elementary charge, this result goes a lot further than just offering an explanation for the physical origin of the fine structure constant α , it also offers a much deeper understanding of what charge is and, by extension, the electromagnetic fields.

The source of this document is available at: <https://bit.ly/ElementalCharge> (<https://bit.ly/ElementalCharge>)

A pdf version is available at: <https://bit.ly/ElementalChargePDF> (<https://bit.ly/ElementalChargePDF>)

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In []: