Rotary Positional Embeddings for Length Generalization in Decision Transformers

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Abstract

In this report, I analyze the impact of positional encoding strategies on the length generalization capabilities of Decision Transformers. I compare three distinct approaches: learned absolute embeddings, fixed sinusoidal embeddings, and Rotary Positional Embeddings (RoPE). Through a Key-Door maze navigation task, trained exclusively on 8×8 grids, I evaluate performance on grids up to 20×20. It is evident from my results that RoPE outperforms the baselines, achieving an 86% success rate on 10×10 grids, whereas learned embeddings and sinusoidal encodings yield 64% and 40% respectively. On 12×12 grids, the difference is clearly visible, with RoPE maintaining 43.3% success compared to 18.3% and 1.7% for the alternatives. Further analysis of context length reveals stable performance across models, leading me to the conclusion that RoPE's advantage stems from position extrapolation rather than context capacity. Statistical tests confirm these observations with large effect sizes ($d > 0.8$), suggesting that RoPE is the optimal choice for architectures requiring robust length generalization.

1 Introduction

Decision Transformers (DT) [3] and similar architectures like Trajectory Transformers [4] frame reinforcement learning as a sequence modeling problem. While these models perform well within their training distribution, it is evident that they struggle with *length generalization*—the capacity to function on trajectories longer than those observed during training [5]. This limitation is highly relevant for offline reinforcement learning, where models must extrapolate from short, suboptimal demonstrations to longer, optimal paths.

A major suspect for this deficiency is the positional encoding scheme. The original Transformer utilizes absolute positional encodings, which link specific behaviors to specific indices. This rigid association does not make sense when the inference sequence length exceeds the training horizon. Recently, Rotary Positional Embeddings (RoPE) [2] have shown promise in language modeling by encoding position via rotation, theoretically enabling better extrapolation.

This report investigates whether RoPE improves length generalization in Decision Transformers. I train models on 8×8 Key-Door maze grids and evaluate extrapolation up to 20×20, comparing learned absolute embeddings, fixed sinusoidal embeddings, and RoPE.

2 Background

2.1 Transformer Architecture

The Transformer [1] architecture replaces recurrent processing with self-attention. This architecture serves as the foundation for my implementation.

2.1.1 Attention Mechanism

The attention mechanism enables the network to weigh token relevance and understand tokens in weeighted dependence of each other. For inputs projected into queries ($\mathbf{Q}$), keys ($\mathbf{K}$), and values ($\mathbf{V}$), the model computes a weighted sum. The scores are derived from the similarity between queries and keys, scaled by $\sqrt{d_k}$ for numerical stability:

$$\text{Attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}} + \mathbf{M}\right)\mathbf{V} \tag{1}$$

Here, $\mathbf{M}$ is a causal mask, ensuring predictions depend only on past context.

2.1.2 Multi-Head Attention

To capture diverse relationships, I employ Multi-Head Attention (MHA). The input is split into h heads, where each head i computes independent attention:

$$\text{head}_i = \text{Attention}(\mathbf{Q}\mathbf{W}_i^Q, \mathbf{K}\mathbf{W}_i^K, \mathbf{V}\mathbf{W}_i^V) \tag{2}$$

These are concatenated and projected to form the final output:

$$\text{MHA} = \text{Concat}(\text{head}_1, \ldots, \text{head}_h)\mathbf{W}^O \tag{3}$$

2.1.3 Causal Masking

To preserve the autoregressive property, I implement a causal mask $\mathbf{M}$. For a sequence of length $T = 4$, the mask is defined as:

$$\mathbf{M} = \begin{pmatrix} 0 & -\infty & -\infty & -\infty \\ 0 & 0 & -\infty & -\infty \\ 0 & 0 & 0 & -\infty \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{4}$$

The $-\infty$ entries result in zero probability after the softmax ($\exp(-\infty) = 0$). Formally:

$$\mathbf{M}_{ij} = \begin{cases} 0 & \text{if } i \geq j \\ -\infty & \text{otherwise} \end{cases} \tag{5}$$

This mask guarantees that the model does not consider future tokens in the attention computation.

2.1.4 Layer Components

Each block consists of attention and a Position-wise Feed-Forward Network (FFN):

$$\text{FFN}(x) = \text{GELU}(xW_1 + b_1)W_2 + b_2 \tag{6}$$

I employ a Pre-Layer Normalization architecture, which improves training stability [7].

2.2 Decision Transformer Formulation

I utilize the Decision Transformer, reframing RL as autoregressive modeling. Trajectories τ consist of return-to-go ($\hat{R}$), state (s), and action (a):

$$\tau = (\hat{R}_1, s_1, a_1, \ldots, \hat{R}_T, s_T, a_T) \tag{7}$$

Standard inputs combine modality and timestep:

$$\mathbf{x}_t = \text{Embed}(\text{modality}_t) + \text{Embed}_{\text{time}}(t) \tag{8}$$

However, explicit absolute embeddings hinder generalization when t exceeds training ranges.

2.3 Positional Encoding Strategies

I compare two methods: Sinusoidal encoding and RoPE.

2.3.1 Sinusoidal Positional Embeddings

The original Transformer [1] adds absolute encodings. For position m and dimension i:

$$PE(m, 2i) = \sin\left(\frac{m}{10000^{2i/d}}\right) \tag{9}$$

$$PE(m, 2i+1) = \cos\left(\frac{m}{10000^{2i/d}}\right) \tag{10}$$

Limitations in RL include reliance on absolute positions and potential interference with semantic information.

2.3.2 Rotary Positional Embeddings (RoPE)

RoPE rotates query and key vectors. For embedding pairs, rotating by $\theta_i \cdot m$ encodes position m:

$$R_{\theta,m} = \begin{pmatrix} \cos(m\theta) & -\sin(m\theta) \\ \sin(m\theta) & \cos(m\theta) \end{pmatrix} \tag{11}$$

RoPE naturally encodes relative distance. The inner product of query q (pos m) and key k (pos n) becomes:

$$q_m'^T k_n' = q_m^T R_{\theta,m}^T R_{\theta,n} k_n = q_m^T R_{\theta,(n-m)} k_n \tag{12}$$

Eq. 12 shows attention depends only on relative distance $(n - m)$, enabling translation invariance and generalization to unseen trajectory lengths.

Efficient Implementation I apply rotation component wise to avoid matrix multiplication:

$$x'_{rot} = x \odot \cos(\Theta) + \text{rotate\_half}(x) \odot \sin(\Theta) \tag{13}$$

where $\text{rotate\_half}([x_1, x_2, \ldots]) = [-x_2, x_1, \ldots]$ and $\theta_i = \text{base}^{-2i/d}$.

Property	Sinusoidal	RoPE
Method	Addition	Rotation
Type	Absolute	Relative (implicit)
Application	Token emb.	Query/Key vectors
Translation Inv.	No	Yes

Table 1: Comparison of positional encoding approaches.

3 Experimental Setup

3.1 Environment and Data

I utilize a Key-Door Maze environment (Figure 1) requiring sequential logic: collect key $\rightarrow$ open door $\rightarrow$ reach goal.

- **Data Generation:** 5,000 trajectories on fixed 8×8 grids (70% optimal, 30% sub-optimal).
- **Generalization Task:** Models trained *only* on $8{\times}8$ grids, evaluated on sizes up to $20{\times}20$.

3.2 Model Architecture

I adopt the Decision Transformer with modifications for grid-world generalization (Table 2).

Embeddings: To support variable grid sizes, I employ a grid agnostic State Encoder. Integer inputs are first processed through item embeddings, followed by a two-layer Convolutional Neural Network (CNN) utilizing 64 filters, 3×3 kernels, and ReLU activations. To resolve spatial variability, I apply an

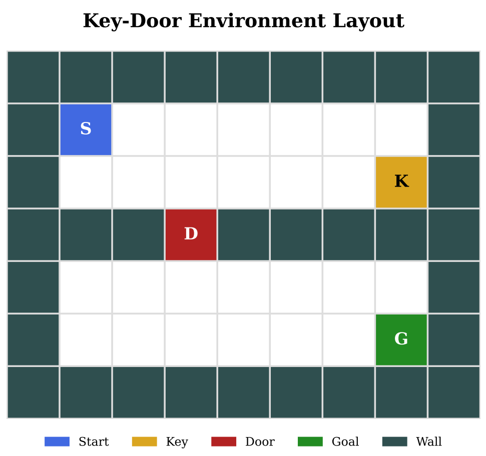

Figure 1: Key-Door Maze environment. The agent (2) must navigate walls (1) to find the key (4), unlock the door (5), and reach the goal (3).

Parameter	Value
Embedding dim. (d_{model})	320
Attention heads	10
Head dimension	32
Transformer layers	8
Context length	30 timesteps
FFN hidden dim.	1280 (4×)
Dropout	0.1

Table 2: Decision Transformer architecture configuration.

`AdaptiveAvgPool2d` layer, which reduces spatial dimensions to a fixed $(1, 1)$ output before it is projected to d_{model}.

Actions and returns-to-go are handled via learned lookup tables and linear projections, respectively. This modular approach ensures that the input dimensionality remains constant, even when the underlying environment scale shifts during evaluation.

3.3 Positional Encoding Variants

I distinguish two types of positional information:

1. **Global Episode Timestep (t)**: Encoded using fixed sinusoidal embeddings in all variants.

2. **Local Context Index (k)**: Position within the context window.

Three strategies for encoding k are compared: Baseline (learned), Sinus (fixed), and RoPE. While t is encoded with Sinusoidal encodings.

3.4 Training Protocol

- **Dataset:** 5,000 episodes (70/15/15 split).
- **Optimization:** AdamW [8], lr $= 10^{-4}$, weight decay 0.01.
- **Training:** 100 epochs, batch size 64.
- **Seeds:** 3 random seeds.
- **Evaluation:** 100 episodes per grid size per seed.

4 Results and Analysis

4.1 Training Performance

All three variants converge successfully (Table 3, Figure 2). RoPE achieves marginally lower training loss (0.356) compared to the others, while maintaining comparable validation performance. This suggests efficient optimization without overfitting.

4.2 The Loss Generalization Disconnect

Training metrics (Table 3) mask a fundamental disconnect. Because loss only measures single-timestep replication, it fails to distinguish between memorizing absolute coordinates and learning causal navigation.

On 8×8 training grids, both strategies yield low loss. The deficiency becomes evident only during extrapolation: the Baseline's coordinate-dependence triggers a collapse on 12×12 grids, whereas RoPE enables generalization. Thus, validation loss is an insufficient proxy for OOD performance.

Model	Final Loss	Val Loss	Success (%)
Baseline	0.413	0.698	97.3
Sinus	0.411	0.697	98.7
RoPE	0.356	0.805	99.3

Table 3: Training metrics on 8×8 grids.

4.3 Length Generalization

I now analyze the generalization capabilities of the models. Table 4 summarizes the results, which are visualized in Figure 3.

4.3.1 Key Findings

RoPE Advantage. RoPE yields superior results, achieving 86% success on 10×10 grids and 43.3% on 12×12. At extreme extrapolation (20×20), RoPE maintains 5.3% success while alternatives reach 0–2.3%.

Learned vs. Fixed Encodings. Learned absolute embeddings (64% on 10×10) significantly outperform fixed sinusoidal versions (40%). This gap alludes to optimization dynamics: learned weights start near-zero for task-specific specialization, whereas sinusoidal structures may impose rigid constraints. However, on larger grids, both perform poorly.

Variance Analysis. The standard deviations reveal instability in absolute encodings. Baseline shows ±25.2% on 10×10, Sinus shows ±40.6%, while RoPE

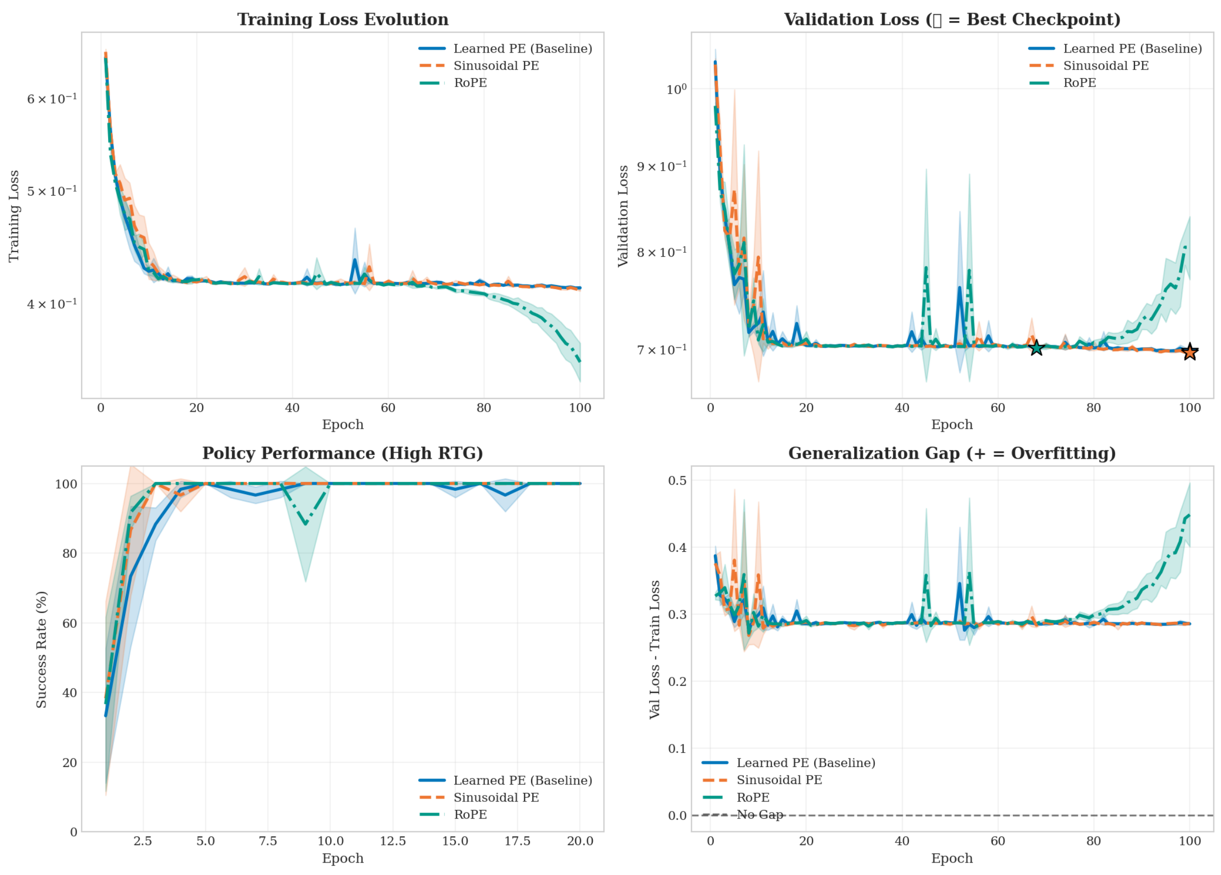

Figure 2: Training dynamics comparison. Top: training and validation loss converge similarly across models. Bottom: validation success rate and overfitting indicator show comparable in-distribution performance.

Grid	Baseline	Sinus	RoPE	Δ vs Base	Δ vs Sinus
8×8	100.0 ± 0.0	100.0 ± 0.0	100.0 ± 0.0	—	—
10×10	64.0 ± 25.2	40.0 ± 40.6	86.0 ± 12.1	+22.0	+46.0
12×12	18.3 ± 19.7	1.7 ± 1.2	43.3 ± 18.1	+25.0	+41.6
15×15	1.3 ± 0.9	0.0 ± 0.0	13.7 ± 12.9	+12.4	+13.7
20×20	2.3 ± 3.3	0.0 ± 0.0	5.3 ± 5.6	+3.0	+5.3

Table 4: Length generalization success rates (%, mean ± std across 3 seeds).

maintains ±12.1%. This high variance indicates seed-dependent generalization when extrapolating beyond trained indices.

4.3.2 Path Efficiency

Table 5 compares the average steps to reach the goal.

Grid	Baseline	Sinus	RoPE	Eff. Ratio
8×8	12.6	12.6	13.1	1.04
10×10	30.5	38.5	**22.3**	1.4
12×12	56.0	59.7	**49.1**	2.2
15×15	74.6	75.0	**70.0**	2.4
20×20	98.5	100.0	**98.7**	2.4

Table 5: Average steps to goal (lower is better).

The efficiency analysis highlights a crucial distinction: while RoPE degrades in success rate on larger grids, the successful episodes remain reasonably efficient (1.4 optimal on 10×10). In contrast, baseline models often succeed via inefficient random walks.

4.4 Context Length Scaling

I tested whether RoPE's advantage stems from better context window utilization. Models were evaluated with context lengths from 30 to 90 tokens (Table 6).

Context	Sinus	RoPE	RoPE Δ
30 (1.0×)	28.7	50.1	+21.3
45 (1.5×)	30.0	50.2	+21.2
60 (2.0×)	29.6	50.1	+20.5
90 (3.0×)	29.7	50.1	+20.4

Table 6: Average success rate (%) at varying context lengths across all grid sizes. RoPE's advantage remains constant.

4.4.1 Key Findings

Performance remains stable across context lengths. RoPE's ∼21% advantage persists regardless of window size. This leads me to the conclusion that RoPE's advantage is **not** about handling more context tokens, but about **extrapolating to unseen sequence positions**. The distinction between context capacity and position extrapolation is worth emphasizing, as it is central to interpreting the performance gap:

- **Context capacity**: The total number of tokens the model can attend to simultaneously within its fixed window.
- **Position extrapolation**: The model's ability to transfer patterns learned at indices 1–30 to novel positions such as 50–80.

Crucially, since I did not explicitly train the networks to leverage extended sequences, the observed stability across window sizes is expected. This finding effectively isolates the variable of interest: the performance advantage of RoPE stems from a superior ability to apply learned attention patterns to novel position indices. It is evident that while absolute encodings become "lost" at unseen indices, RoPE's relative inductive bias allows the causal mechanism of the task to remain intact regardless of the specific index value.

4.5 Attention Pattern Analysis

Figure 4 visualizes attention weights. RoPE exhibits consistent banded patterns where attention concentrates at fixed relative offsets. In contrast, learned embeddings show position-specific patterns that fail beyond training indices.

4.6 Translation Invariance Verification

I empirically validated RoPE's translation invariance by comparing attention patterns for shifted sequences:

- Mean error: 0.014 ± 0.004
- Maximum error: 0.024

Errors below 3% confirm that RoPE maintains attention patterns regardless of absolute position, validating the theoretical property from Equation 12.

4.7 Statistical Analysis

Table 7 summarizes statistical comparisons, with effect sizes visualized in Figure 5.

Grid	Cohen's d	p-val	Signif.
10×10	1.23	0.208	No
12×12	2.56	0.035	Yes
15×15	1.21	0.213	No
20×20	1.18	0.223	No

Table 7: Statistical comparison (RoPE vs Sinus).

Effect sizes consistently exceed $d = 0.8$. The 12×12 comparison achieves significance. Non-significant results elsewhere reflect limited statistical power rather than absent effects.

4.8 Failure Mode Analysis

Examining unsuccessful 20×20 episodes reveals systematic failures at positions > 50 steps. While RoPE's frequency progression (base $10000^{-2i/d}$) produces wavelengths up to ∼35,000 positions, which makes its theoretical capacity sufficient, the empirical generalization degrades earlier. This suggests that the model struggles to process increasingly complex spatial states, a hypothesis I explore in the Discussion.

5 Discussion

5.1 Mechanism Analysis

Why Learned Embeddings Partially Succeed. Learned embeddings can adjust representations to form an implicit relative encoding. However, this is

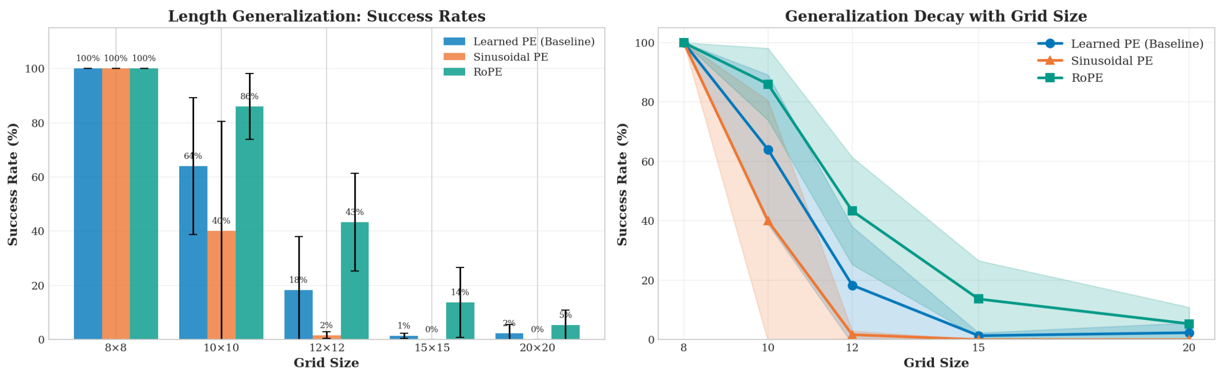

Figure 3: Length generalization comparison across grid sizes. RoPE maintains substantially higher success rates beyond training distribution. Error bars indicate standard deviation across 3 seeds.

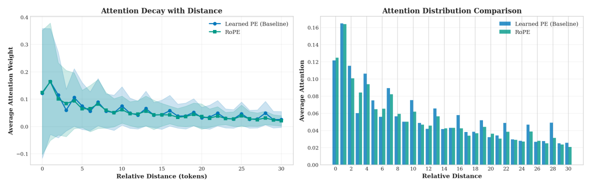

Figure 4: Attention distance decay analysis. RoPE maintains smoother decay with distance. Baseline shows irregular patterns.

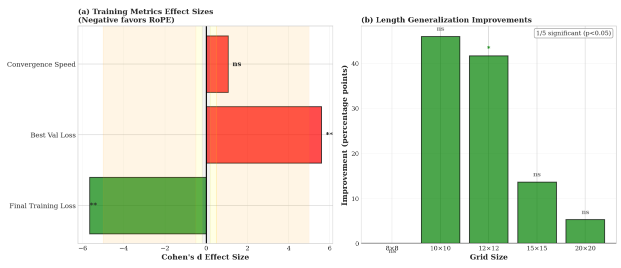

Figure 5: Effect sizes (Cohen's d) and significance levels.

fragile: the high variance and rapid degradation highlight fundamental limitations.

The Role of Global Timesteps. All models utilize a fixed sinusoidal encoding for global episode timestep t. This dual encoding explains the efficiency gap. Baselines likely rely on the global horizon to spur exploration, whereas RoPE's relative signals facilitate directed navigation.

Why RoPE Provides Robust Generalization. RoPE structurally enforces $q'_m k'^T_n = f(m-n)$. Because absolute indices are never computed, a learned rule like "key seen two steps ago" transfers identically to any position.

5.2 The Encoder Information Bottleneck

Despite RoPE's strengths, performance drops to 5.3% on 20×20 grids. The state encoder is a likely bottleneck. Moving from 8×8 to 20×20 increases the compression ratio by $\sim$6×. This loss may cause "state aliasing." This suggests that even with ideal positional encodings, navigation fails if the visual encoder cannot distinguish states.

5.3 Implications for Offline RL

The results have significant implications for Offline RL. My findings suggest that RoPE enhances the capability to combine suboptimal trajectories by allowing the attention mechanism to maintain temporal logic across longer horizons.

5.4 Limitations

Statistical Power. Three seeds provide limited confidence. Yet, the large effect sizes are notable. Therefore, further research with additional seeds is necessary to confirm these assumptions.

Task Specificity. Key-Door maze results may not generalize to all domains. It might be worthwhile to evaluate on additional task types.

Extreme Extrapolation. Performance degrades substantially beyond 2× training length.

No complete Isolation. Each version of the model uses sinusoidal encoding for the timestep encoding. As such a complete generalization for non dual positional encoding is at best limited.

RoPE Variants. I tested RoPE for sequence positions. Applying RoPE to timesteps may provide additional benefits and is an interesting direction for future work.

6 Conclusion

This report evaluated positional encoding strategies for Length Generalization.

Findings:

1. **Generalization Hierarchy.** RoPE yields better results, achieving 86% on 10×10 grids. Learned embeddings showed resilience but collapsed on larger grids.

2. **Variance Indicates Instability.** Learned embeddings exhibited high variance compared to RoPE, indicating seed-dependent generalization.

3. **Structural Guarantees Enable Robustness.** RoPE ensures attention depends only on relative distances.

4. **Position Extrapolation, Not Context Scaling.** Context length experiments showed stable performance, confirming RoPE addresses position index generalization.

Implications: For offline RL, reliance on absolute positional encodings is a bottleneck. RoPE offers a robust solution. However, I am skeptical whether positional encoding alone is sufficient; future architectures must also address spatial resolution to achieve unbounded generalization.

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Use of Ai: Throughout the course of the project LLMs have been used for coding, documentation and text refinement.