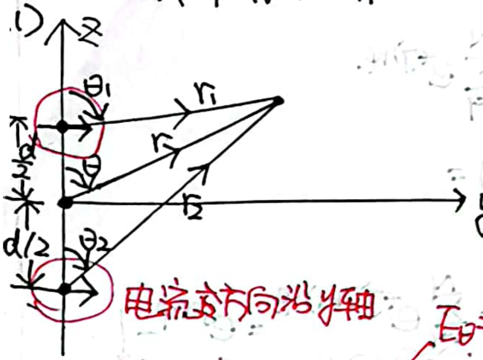
Chapter III 天线阵列 (Antenna arrays).

一、二元阵列. (two element arrays)

1、二元阵列的辐射

(1)



电流方向沿y轴

$E_\theta = j\eta \frac{kI_0 l}{4\pi r} e^{-jkr}\sin\theta$

两个极小偶极子沿z轴水平放置

合场强：

$$E_t = E_1 + E_2 = \hat{a}_\theta j\eta \frac{kI_0 l}{4\pi}\left\{ \frac{e^{-j[kr_1 - (\beta/2)]}}{r_1}\cos\theta_1 + \frac{e^{-j(kr_2+\frac{\beta}{2})}}{r_2}\cos\theta_2 \right\}$$

沿z轴偶极子场为(sinθ),现在为沿y轴,为(cosθ).

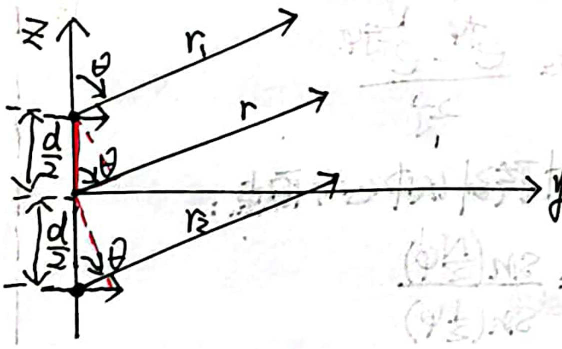
两个偶极子之间相位差

(2) 远场:

对于相位 $\begin{cases} \theta_1 \approx \theta_2 \approx \theta \\ r_1 \approx r - \frac{d}{2}\cos\theta \\ r_2 \approx r + \frac{d}{2}\cos\theta \end{cases}$

对于幅度: $r \approx r_1 \approx r_2$

天线的极林函数中有体现



合场强,

$$E_t = \hat{a}_\theta j\eta \frac{kI_0 l}{4\pi}\cdot\frac{e^{-jkr}}{r}\cos\theta \,\cancel{\times}\left\{2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]\right\}$$

• 阵列因子 (array factor):

$$AF = 2\cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]$$

归一化

$$AF_n = \cos\left[\frac{1}{2}(kd\cos\theta+\beta)\right]$$

3 ~~E(total)~~

• 阵列乘积规律:

E(total) = E(单个极小偶极子) × (阵列因子)

2、零值点 (null)

[例题]. Find the nulls of the total field when $d=\frac{\lambda}{4}$ and:

a. $\beta=0$ b. $\beta=+\frac{\pi}{2}$ c. $\beta=-\frac{\pi}{2}$

解:

a. $\beta=0$

归一化电场:

$E_{tn} = \cos\theta\cos\left[\frac{\pi}{4}\cos\theta\right]\Big|_{\theta=\theta_n} = 0$ 分别讨论

$\cos\theta = 0 \Rightarrow \theta_{null} = \frac{\pi}{2}$

$\cos(\frac{\pi}{4}\cos\theta) = 0 \Rightarrow \frac{\pi}{4}\cos\theta = \frac{\pi}{2} \Rightarrow \cos\theta = 2$

θ_{null} = does not exist.

故 $\theta_{null} = \frac{\pi}{2}$

b. $\beta = +\frac{\pi}{2}$

The nulls are obtained by setting the total normalized field equal to zero.

$E_{tn} = \cos\theta\cos\left[\frac{\pi}{4}(\cos\theta+1)\right]\Big|_{\theta=\theta_n} = 0$

$\cos\theta = 0 \Rightarrow \theta_{null} = \frac{\pi}{2}$

$\cos\left[\frac{\pi}{4}(\cos\theta+1)\right] = 0 \Rightarrow \theta_{null} = 0.$

$c.\ \beta = -\frac{\pi}{2}$

$$E_{tn} = \cos\theta \cdot \cos\left[\frac{\pi}{4}(\cos\theta - 1)\right]\Big|_{\theta=\theta_n} = 0.$$

$\cos\theta = 0 \Rightarrow \theta_{null} = \frac{\pi}{2}$

$\cos\left[\frac{\pi}{4}(\cos\theta_{null} - 1)\right] = 0 \Rightarrow \frac{\pi}{4}(\cos\theta_{null} - 1) = -\frac{\pi}{2}$

$\Rightarrow \theta_{null} = \pi$

• 求解零值点：

令归一化电场为0.

$$E_{tn} = \cos\theta \cdot \cos\left[\frac{1}{2}(kd\cos\theta + \beta)\right]\Big|_{\theta=\theta_n} = 0$$

① $\theta_{null} = \frac{\pi}{2}$

② $\theta_{null} = \arccos\left(\frac{\lambda}{2\pi d}[-\beta \pm (2n+1)\pi]\right)$

$\theta = 90^\circ$ 的零点是由阵列中的单元方向图引起的，而其余的是由阵列形成的。当单元间无相位差（$\beta = 0$）时，要由阵列的作用产生一个零点，$d \geq \frac{\lambda}{2}$

二、N元阵列（N-elements arrays）

$$\boxed{AF} = 1 + e^{+j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + \cdots + e^{+j(N-1)(kd\cos\theta+\beta)}$$

$$\boxed{= \sum_{n=1}^{N} e^{+j(n-1)(kd\cos\theta+\beta)}}$$

令 $\psi = (kd\cos\theta + \beta)$.

故 $AF = \sum_{n=1}^{N} e^{+j(n-1)\psi}$

1、阵列因子.

※推导：

$$AF = \sum_{n=1}^{N} e^{+j(n-1)\psi}$$

两边同乘 $e^{j\psi}$.

$$AF e^{j\psi} = \sum_{n=1}^{N} e^{+jn\psi}.$$

同时减去 AF

~~$AF e^{j}$~~

$$\underbrace{\sum_{n=1}^{N} e^{+jn\psi}}_{保留最后项} - \underbrace{\sum_{n=1}^{N} e^{+j(n-1)\psi}}_{保留最后一项} = e^{jN\psi} - 1$$

$$AF(e^{j\psi} - 1) = (e^{jN\psi} - 1)$$

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1}$$

$$= e^{j\left[\frac{N-1}{2}\right]\psi}\left[\frac{e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}}{e^{j\frac{1}{2}\psi} - e^{-j\frac{1}{2}\psi}}\right]$$

分子提出了 $e^{j\frac{N}{2}\psi}$

分母提出了 $e^{j\frac{1}{2}\psi}$

$$= e^{j\left[\frac{N-1}{2}\right]\psi}\left[\frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}\right]$$

$$\sin\psi = \frac{e^{j\psi} - e^{-j\psi}}{2j}$$

如果~~阵列~~阵列以中心为原点：

$$AF = \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)}$$

~~当~~ ψ 很小时.

$$\lim_{\psi\to 0} AF = \frac{\sin(\frac{N}{2}\psi)}{\frac{1}{2}\psi}.$$

$\sin x \approx x$.

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· $\lim_{\psi\to 0} \frac{\sin(\frac{N}{2}\psi)}{\sin(\frac{1}{2}\psi)} = N$.

故AF的最大值约为N.

归一化的阵列因子:

$$AF_n = \frac{\sin(\frac{N}{2}\psi)}{N\sin(\frac{1}{2}\psi)} \approx \frac{\sin(\frac{N}{2}\psi)}{\frac{N}{2}\psi}$$

ψ很小或N足够大时

2、阵列零值点

$\sin(\frac{N}{2}\psi)=0 \Rightarrow \frac{N}{2}\psi\Big|_{\theta=\theta_{null}} = \pm n\pi$.

其中 $\psi = kd\cos\theta+\beta$.

算出 $\theta_{null} = \arccos\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$

3、最大值.

$\sin(\frac{1}{2}\psi)\to 0$ 时有最大值.

$$\frac{\psi}{2} = \frac{1}{2}(kd\cos\theta+\beta)\Big|_{\theta=\theta_{max}} = \pm n\pi$$

$$\Rightarrow \theta_{max} = \arccos\left[\frac{\lambda}{2\pi d}(-\beta\pm 2m\pi)\right]\Big|_{m=0}$$

$$\Rightarrow \theta_{max} = \arccos\left(\frac{\beta\lambda}{2\pi d}\right)$$

4、波束宽度:

$$N\cdot\frac{\psi}{2} = \frac{N}{2}(kd\cos\theta+\beta)\Big|_{\theta=\theta_{3dB}} = \pm 1.391$$

$$\Rightarrow \theta_{3dB} = \arccos\left[\frac{\lambda}{2\pi d}\left(-\beta\pm\frac{2.782}{N}\right)\right]$$

若 $d \gg \lambda$.

$$\theta_{3dB} \approx \frac{\pi}{2} - \frac{\lambda}{2\pi d}\left(-\beta\pm\frac{2.782}{N}\right).$$

$$HPBW = 2|\theta_{max}-\theta_{3dB}|$$

5、方向性:

阵列因子的方向性:

$$D_0 = \frac{U_{max}}{U_0} \approx \frac{1}{\frac{\pi}{Nkd}} = \frac{Nkd}{\pi} = 2N\left(\frac{d}{\lambda}\right)$$

$d=\frac{\lambda}{2}$时. $D_0 = N$

[例]: Given a linear, broadside, uniform array of 10 ~~isthe~~ isotropic elements ($N=10$) with a seperation of $\lambda/4$ ($d=\frac{\lambda}{4}$) between ~~the~~ the elements, find the directivity of the array.

解: $D_0 = 2N\left(\frac{d}{\lambda}\right) = 5$ (dimemisonless) $= 10\lg(5) = 6.99\,dB$.