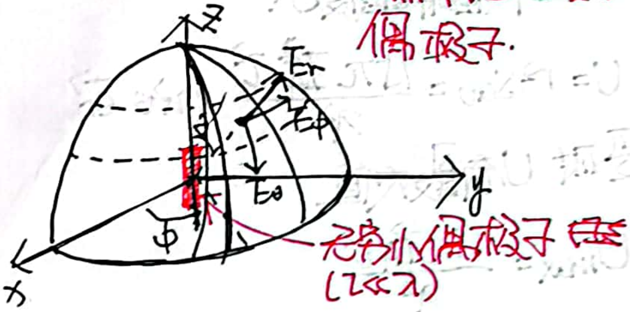
Chapter Ⅱ ~~天线基础~~ 偶极子天线

三、偶极子天线 ← 重点讨论无穷小偶极子



~~一、假设~~

1、无穷小振子 (infinitesimal dipole)

假设电流为：$i(t)=I_0\cos\omega t$

沿z轴的电流为：$I_e=\vec{e}_z I_0$

(1) 矢量位 (vector potential)

$$A=\frac{\mu_0}{4\pi}\int_c I_e\frac{e^{-jkr}}{r}dl'$$

$$=\frac{\mu_0 I l}{4\pi r}e^{-jkr}\vec{e}_z$$

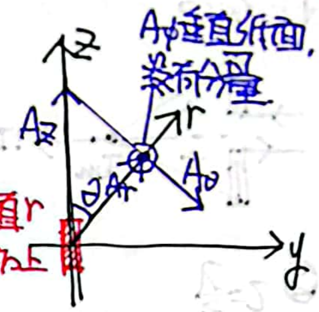
$$=A_z\vec{e}_z$$

$$\vec{A}=\vec{e}_r A_r+\vec{e}_\theta A_\theta+\vec{e}_\phi A_\phi$$

转化为球坐标

$$\begin{cases}A_r=A_z\cos\theta\\ A_\theta=-A_z\sin\theta\\ A_\phi=0\end{cases}$$

由于A_θ垂直r向下，故加上负号。



(2) 磁场 (magnetic field).

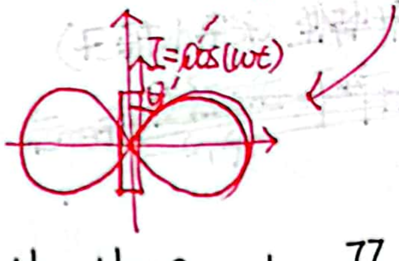
$$\vec{H}=\frac{1}{\mu_0}\nabla\times\vec{A}$$

$$=\frac{1}{\mu_0 r^2\sin\theta}\begin{vmatrix}\vec{e}_r & r\vec{e}_\theta & r\sin\theta\vec{e}_\phi\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi}\\ A_r & rA_\theta & 0\end{vmatrix}$$

$$=\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(0)-\frac{\partial}{\partial\phi}rA_\theta\right]\vec{e}_r+\frac{1}{r}\left[\frac{\partial}{\partial\phi}A_r-\frac{\partial}{\partial r}(0)\right]\vec{e}_\theta+\frac{1}{r}\left[\frac{\partial}{\partial r}rA_\theta-\frac{\partial}{\partial\theta}A_r\right]\vec{e}_\phi$$

$$=\frac{Il}{4\pi}\sin\theta\left(j\frac{k}{r}+\frac{1}{r^2}\right)e^{-jkr}\vec{e}_\phi$$

θ: 0→π过程中，H先变大后变小



• $H_r=H_\theta=0$ $H_\phi=\frac{Il}{4\pi}\sin\theta\left(j\frac{k}{r}+\frac{1}{r^2}\right)e^{-jkr}$

(3) 电场 (electric field)

$$E=\frac{1}{j\omega\varepsilon}\nabla\times H$$

$$\vec{E}=E_r\vec{e}_r+E_\theta\vec{e}_\theta$$

E和H正交，在φ上有H就没有E

$$\begin{cases}E_r=\frac{Il}{4\pi}\cdot\frac{2}{\omega\varepsilon_0}\cos\theta\left(\frac{k}{r^2}-j\frac{1}{r^3}\right)e^{-jkr}\\ E_\theta=\frac{Il}{4\pi}\frac{1}{\omega\varepsilon_0}\sin\theta\left(j\frac{k^2}{r}+\frac{k}{r^2}-j\frac{1}{r^3}\right)e^{-jkr}\end{cases}$$

2、~~电场~~ 无穷小偶极子 在近场、远场的电场。

只保留最大项

(1) 近场：$\frac{1}{kr}\ll\frac{1}{(kr)^2}\ll\frac{1}{(kr)^3}$ ……

$$\begin{cases}H_\phi=\frac{Il}{4\pi r^2}\sin\theta\\ E_r=-j\frac{Il}{4\pi r^3}\frac{2}{\omega\varepsilon_0}\cos\theta\\ E_\theta=-j\frac{Il}{4\pi r^3}\frac{1}{\omega\varepsilon_0}\sin\theta\\ E_\phi=H_r=H_\theta=0\end{cases}$$

(2) 远场

$\frac{1}{kr} \gg \frac{1}{(kr)^2} \gg \frac{1}{(kr)^3}$ $k=\frac{2\pi}{\lambda}$

$$\begin{cases} H_\varphi = j\frac{Il}{2\lambda r}\sin\theta\, e^{-jkr} = \frac{kIl}{4\pi r}\sin\theta\, e^{-jkr} \\ E_\theta = j\frac{60\pi Il}{\lambda r}\sin\theta \cdot e^{-jkr} = j\eta\frac{kI_0 l}{4\pi r}e^{-jkr}\sin\theta \\ E_r = 0 \\ H_r = H_\theta = E_\varphi = 0 \end{cases}$$

$E_\theta = \eta H_\varphi$

3. 远场下的辐射特性（元/电小振子）

~~$\vec{S}_{av} = \frac{1}{2}Re[\vec{E}\times\vec{H}^*] = \frac{15\pi I_A^2 l^2}{\lambda^2 r^2}\sin^2\theta\,\vec{e_r}$~~ 全

(1) 坡印廷矢量

$$\vec{S}_{av} = \frac{1}{2}Re[\vec{E}\times\vec{H}^*] = \frac{15\pi I_A^2 l^2}{\lambda^2 r^2}\sin^2\theta\,\vec{e_r}$$

(2) 辐射功率

$$P_r = \oiint_S \vec{S}_{av}\cdot d\vec{S} = \oiint_S \frac{1}{2}Re[\vec{E}\times\vec{H}^*]\cdot d\vec{S}$$
$$= \int_0^{2\pi} d\varphi \int_0^{\pi} \frac{15\pi I^2 l^2}{\lambda^2}\sin^3\theta\, d\theta$$
$$= 40\pi^2 I^2 \left(\frac{l}{\lambda}\right)^2$$

$P_{rad} = \oiint_S W_{rad}\cdot r^2\sin\theta\, d\theta\, d\varphi$

(3) 辐射阻抗

由于辐射的能量不再返回波源，视为天线的辐射功率完全被等效电阻吸收，称为辐射阻抗 (radiation impedance) R_r

$$P_r = \frac{1}{2}I^2 R_r$$

故 $R_r = \frac{2P_r}{I^2} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$

(4) 方向性

由 S_{av} 到辐射强度 U：

$$U = r^2 S_{av} = \frac{15\pi I_A^2 l^2}{\lambda^2}\sin^2\theta\,\vec{e_r}$$

$\theta=\frac{\pi}{2}$ 时 U 有最大值

$$U_{max} = \frac{15\pi I_A^2 l^2}{\lambda^2}$$

又 $P_r = 40\pi^2 I^2\left(\frac{l}{\lambda}\right)^2$

$$D_0 = 4\pi\frac{U_{max}}{P_r} = \frac{3}{2}$$

4. 有限长振子

(1) 指定电流 J 和磁电流密度 M

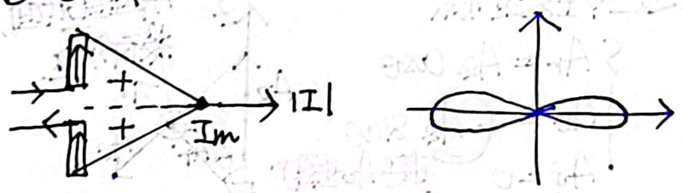
对于很细的振子（直径为零），电流为：

$$\vec{I_e} = \begin{cases} \vec{e_z} I_0 \sin[k(\frac{l}{2}-z')] & 0\leqslant z'\leqslant\frac{l}{2} \\ \vec{e_z} I_0 \sin[k(\frac{l}{2}+z')] & -\frac{l}{2}\leqslant z'\leqslant 0 \end{cases}$$

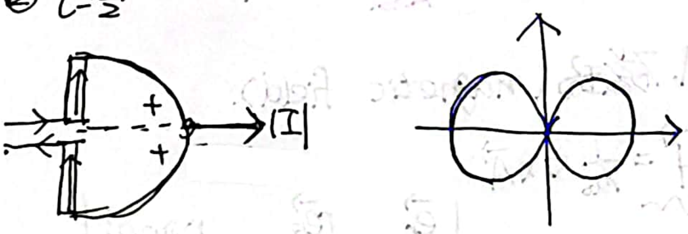
~~$l \ll \lambda$~~

• 电流在天线上的分布和辐射方向图

① $l \ll \lambda$

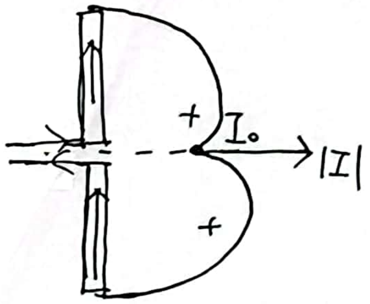


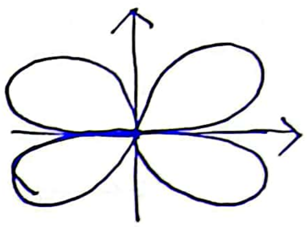
② $l=\frac{\lambda}{2}$



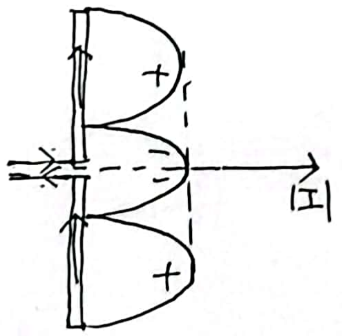
Chapter II 偶极子天线

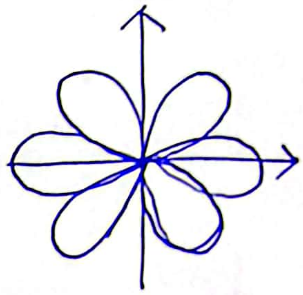
③ $\frac{\lambda}{2} < l < \lambda$.





④ $\lambda < l < \frac{3}{2}\lambda$.





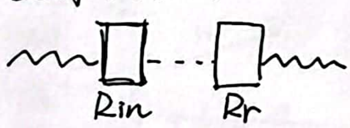
(2) 确定辐射阻抗R_r和输入阻抗R_{in}.

$$R_{in} = \left[\frac{I_0}{I_{in}}\right]^2 R_r = \frac{R_r}{\sin^2(\frac{kl}{2})}$$

$\sin^2(\frac{\pi}{2}) = 1$. 即 要 $kl = \frac{2\pi}{\lambda} \cdot l = \pi$

得 $l = \frac{\lambda}{2}$

此时 $R_{in} = R_r$.



$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}.$$

故 $l = \frac{\lambda}{2}$时. $R_{in} = R_r$. 天线效率最高

但 $l = 1.5\lambda$. $l = 2.5\lambda$ 时 依然有$R_{in} = R_r$

(3) ~~确定有效~~

(3) 最大有效面积.

① 极小偶极子天线的有效面积

$$A_{rm} = 0.119\lambda^2$$

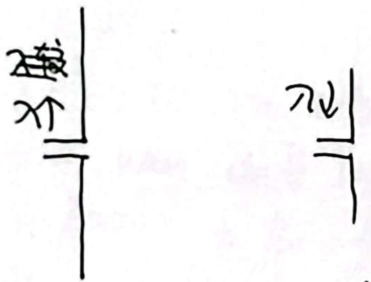
② 各向同性天线有效面积

$$A_{tm} = \frac{A_{rm}}{D_{r0}} = \frac{0.119\lambda^2}{1.5} = \frac{\lambda^2}{4\pi}$$

③ 通常, 最大有效面积:

$$A_{em} = \frac{\lambda^2}{4\pi} D_0.$$

• 比较: 两根天线的 $l = \frac{\lambda}{2}$, 但λ不同



f↓	f↑
A_{rm}↑	A_{rm}↓
P_r↑	P_r↓

波长的频率越高, 接收到的功率越小