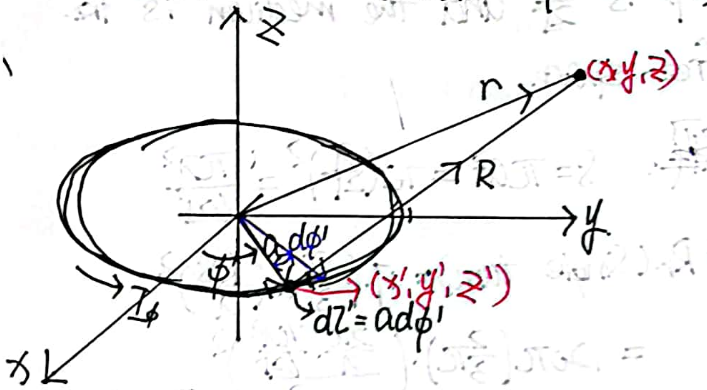
Chapter IV 环形天线 (Assignment)

Loop wire ~~antenan~~ Antennas

一、小型环状天线 (Small Loop Antennas)



圆环置于x,y平面上

1.小型环状天线的辐射场.

$$A(x,y,z)=\frac{\mu}{4\pi}\int_C I_e(x',y',z')\frac{e^{-jkR}}{R}dl'$$

$$I_e=\hat{a}_x I_x+\hat{a}_y I_y+\hat{a}_z I_z$$

由直角坐标 → 圆柱坐标 → 球坐标

$$I_e=\hat{a}_r I_\phi \sin\theta\sin(\phi-\phi')+\hat{a}_\theta I_\phi\cos\theta\sin(\phi-\phi')+\hat{a}_\phi I_\phi\cos(\phi-\phi')$$

从环的任意一点到观察点距离:

$$R=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$ 球坐标

$$=\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}$$

$$dl'=ad\phi'$$

$$A_\phi=\frac{a\mu}{4\pi}\int_0^{2\pi}I_\phi\cos(\phi-\phi')\cdot\frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}\cdot d\phi'$$

$$=\frac{a\mu I_0}{4\pi}\int_0^{2\pi}\cos\phi'\frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos(\phi-\phi')}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}d\phi'$$

令 $$f=\frac{e^{-jk\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}}{\sqrt{r^2+a^2-2ar\sin\theta\cos\phi'}}$$

麦克劳林级数:

$$f=f(0)+f'(0)a+\frac{1}{2!}f''(0)a^2+\cdots+\cdots$$

只保留前两项

$$f\approx f(0)+f'(0)a$$

$$=\left[\frac{1}{r}+a\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta\cos\phi'\right]e^{-jkr}$$

f代入A_ϕ得

$$A_\phi=\frac{a^2\mu I_0}{4}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta$$

$$A_r=A_\theta=\phi$$

故 $$A=\hat{a}_\phi A_\phi=\hat{a}_\phi\cdot\frac{a^2\mu I_0}{4}e^{-jkr}\left(\frac{jk}{r}+\frac{1}{r^2}\right)\sin\theta$$

$$=\hat{a}_\phi j\frac{k\mu a^2 I_0\sin\theta}{4r}\left(1+\frac{1}{jkr}\right)e^{-jkr}$$

磁通量 $\vec{B_A}=\mu\vec{H_A}=\nabla\times\vec{A}$

• 故磁场强度各分量:

$$\begin{cases}H_r=j\dfrac{ka^2I_0\cos\theta}{2r^2}\left(1+\dfrac{1}{jkr}\right)e^{-jkr}\\ H_\theta=-\dfrac{(ka)^2I_0\sin\theta}{4r}\left[1+\dfrac{1}{jkr}-\dfrac{1}{(kr)^2}\right]e^{-jkr}\\ H_\phi=0\end{cases}$$

• J=0时电场分量

$$\begin{cases}E_r=E_\theta=0\\ E_\phi=\eta\dfrac{(ka)^2I_0\sin\theta}{4r}\left[1+\dfrac{1}{jkr}\right]e^{-jkr}\end{cases}$$

2、功率密度和辐射电阻

(1) 复数功率密度：

$$\vec{W}_{rad}=\frac{1}{2}(\vec{E}\times\vec{H}^*)$$
$$=\frac{1}{2}[(\hat{a}_\phi E_\phi)\times(\hat{a}_r H_r^*+\hat{a}_\theta H_\theta^*)]$$
$$=\frac{1}{2}[-\hat{a}_r E_\phi H_\theta^*+\hat{a}_\theta E_\phi H_r^*]$$

(2) 复数功率

$$P_r=\oiint_S W\,ds=\eta\cdot\frac{(ka)^4}{32}|I_0|^2\cdot\int_0^{2\pi}\int_0^{\pi}\left[1+j\frac{1}{(kr)^3}\right]\sin^3\theta\,d\theta\,d\phi$$

$$=\eta\left(\frac{\pi}{12}\right)(ka)^4|I_0|^2\left[1+j\frac{1}{(kr)^3}\right]$$

$kr\gg1$ 远场中

$$=\eta\left(\frac{\pi}{12}\right)(ka)^4|I_0|^2$$

(3) 辐射电阻

由 $P_{rad}=\frac{1}{2}|I_0|^2R_r$

$$R_r=\frac{2P_r}{|I_0|^2}=\eta\left(\frac{\pi}{6}\right)(k^2a^2)^2$$
$$=\eta\cdot\left(\frac{2}{3}\pi\right)\cdot\left(\frac{kS}{\lambda}\right)^2=20\pi^2\left(\frac{C}{\lambda}\right)^4$$
$$\approx31171\left(\frac{S^2}{\lambda^4}\right)$$

圆环周长 $C=2\pi a$.

圆环面积 $S=\pi a^2$

若有N匝线圈

$$R_r=\eta\cdot\left(\frac{\pi}{6}\right)(k^2a^2)^2N^2$$

[例]: Find the radiation resistance of a single-turn and an eight-turn small circular loop. The radius of the loop is $\frac{\lambda}{25}$ and the medium is free-space.

解：$S=\pi a^2=\pi\left(\frac{\lambda}{25}\right)^2=\frac{\pi\lambda^2}{625}$

$$R_r(\text{Single turn})=\eta\cdot\left(\frac{2\pi}{3}\right)\cdot\left(\frac{kS}{\lambda}\right)^2$$
$$=120\pi\left(\frac{2}{3}\pi\right)\cdot\left(\frac{\frac{2\pi}{\lambda}\cdot\frac{\pi\lambda^2}{625}}{\lambda}\right)^2$$
$$=0.788\,\Omega$$

$$R_r(8\text{ turns})=0.788\times(8)^2=50.43\,\Omega$$

3、辐射强度和方向性

(1) 强度

$$U=r^2W_{rad}=\frac{\eta}{2}\cdot\left(\frac{ka^2}{4}\right)^2|I_0|^2\sin^2\theta$$

$$U_{max}=U|_{\theta=\frac{\pi}{2}}=\frac{\eta}{2}\left(\frac{ka^2}{4}\right)^2|I_0|^2$$

(2) 方向性：

$$D_0=4\pi\cdot\frac{U_{max}}{P_{rad}}=\frac{3}{2}$$

(3). 最大有效面积

$$A_{em}=\left(\frac{\lambda^2}{4\pi}\right)D_0=\frac{3\lambda^2}{8\pi}$$

各向同性天线的有效面积

~~目录~~ Chapter IV 环形天线

[例] The radius of a small loop of constant current is $\frac{\lambda}{25}$. ~~Find~~ Find the physical area of the loop and compare it with its maximum effective aperture.

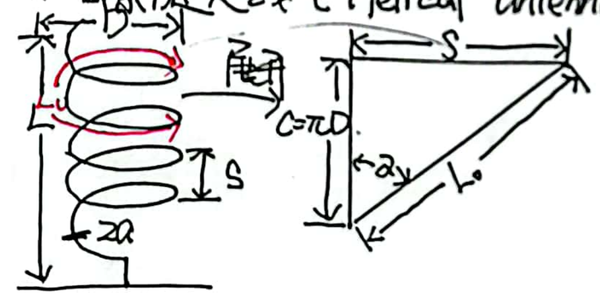
解：$S(physical) = \pi a^2 = \pi\left(\frac{\lambda}{25}\right)^2 = 5.03\times10^{-3}\lambda^2$

$$A_{em} = \frac{3\lambda^2}{8\pi} = 0.119\lambda^2$$

$$\frac{A_{em}}{S} = \frac{0.119\lambda^2}{5.03\times10^{-3}\lambda^2} = 23.66.$$

• 从电学尺度看，环的有效面积相当于它的物理尺寸的24倍。这就是为什么使用环形作为天线.

二、螺旋天线 (Helical antennas)



• 俯仰角 α (pitch angle)：控制螺旋天线在z方向上的增长.

• 两种模式：①普通模式 ②端射模式

1、普通模式：

$$E_\theta = j\eta\frac{kI_0Se^{-jkr}}{4\pi r}\sin\theta$$

$$E_\phi = \eta\frac{k^2\left(\frac{D}{2}\right)^2I_0e^{-jkr}}{4r}\sin\theta$$

• 轴比 (axial ratio)

$$AR = \frac{|E_\theta|}{|E_\phi|} = \frac{4S}{\pi kD^2} = \frac{2\lambda S}{(\pi D)^2}$$

当 $E_\theta=0$ 时 → 线性水平极化

$E_\phi=0$ $AR=\infty$ 时 → 垂直水平极化

当 $AR=1$ 时. $\frac{2\lambda S}{(\pi D)^2}=1$

$C=\pi D=\sqrt{2\lambda_0 S}$

$\tan\alpha = \frac{S}{\pi D} = \frac{\pi D}{2\lambda_0}$

2、端射模式 (end-fire).

螺旋天线的辐射方向沿着螺旋的轴线，这种模式下，电磁波在轴向之外的方向上辐射出去.

$C\approx\lambda$, $S\approx\frac{\lambda}{4}$
周长　间距

• $R\approx140\left(\frac{C}{\lambda_0}\right)$

~~HPBW = 52λ₀~~

• $HPBW = \frac{52\lambda_0^{\frac{3}{2}}}{C\sqrt{NS}}$

• $FPBW = \frac{115\lambda_0^{\frac{3}{2}}}{C\sqrt{NS}}$

• $D_0 = 15N\frac{C^2S}{\lambda_0^3}$

• $AR = \frac{2N+1}{2N}$

• 在远场下，

$$E = \sin\left(\frac{\pi}{2N}\right)\cdot\cos\theta\,\frac{\sin\left[\left(\frac{N}{2}\right)\psi\right]}{\sin\left(\frac{\psi}{2}\right)}.$$

其中 $\psi = k_0\left(S\cos\theta - \frac{L_0}{p}\right)$.

波速

$$P=\frac{L_0/\lambda_0}{S/\lambda_0+1}$$ (ordinary end-fire radiation)

$$P=\frac{L_0/\lambda_0}{S/\lambda_0+\left(\frac{2N+1}{2N}\right)}$$ (Hansen-Woodyard End-fire radiation)