# MATHEMATICS

## STUDENT TEXTBOOK GRADE 9


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**FEDERAL DEMOCRATIC REPUBLIC OF ETHIOPIA**
**MINISTRY OF EDUCATION**

**HAWASSA UNIVERSITY**

First Published August 2023 by the Federal Democratic Republic of Ethiopia, Ministry of Education, under the General Education Quality Improvement Program for Equity (GEQIP-E) supported by the World Bank, UK's Department for International Development/DFID-now merged with the Foreign, Common wealth and Development Office/FCDO, Finland Ministry for Foreign Affairs, the Royal Norwegian Embassy, United Nations Children's Fund/UNICEF), the Global Partnership for Education (GPE), and Danish Ministry of Foreign Affairs, through a Multi Donor Trust Fund.

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The Ministry of Education wishes to thank the many individuals, groups and other bodies involved – directly or indirectly – in publishing this Textbook. Special thanks are due to Hawassa University for their huge contribution in the development of this textbook in collaboration with Addis Ababa University, Bahir Dar University, Jimma University and JICA MUST project.

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**Printed by:**

GRAVITY GROUP IND LLC
P.O.Box 13TH Industrial Area, Sharjah
UNITED ARAB EMIRATES
Under Ministry of Education
Contract no. MOE/GEQIP-E/LICB/G-01/23

**ISBN: 978-99990-0-024-6**

## Welcoming Message to Students.

Dear grade 9 students, you are welcome to the first grade of secondary level education. This is a golden stage in your academic career. Joining secondary school is a new experience and transition from primary school Mathematics education. In this stage, you are going to get new knowledge and experiences which can help you learn and advance your academic, personal, and social career in the field of Mathematics.

Enjoy it!

## Introduction on Students' Textbook.

Dear students, this textbook has 9 units namely: Further on sets, the number system, Solving Equations, Solving Inequalities, Introduction to Trigonometry, Regular Polygons, Congruency and Similarity, Vectors in two Dimensions and Statistics and Probability respectively. Each of the units is composed of introduction, objectives, lessons, key terms, summary, and review exercises. Each unit is basically unitized, and lesson based. Structurally, each lesson has four components: Activity, Definition, Examples, and Exercises (ADEE).

The most important part in this process is to practice problems by yourself based on what your teacher shows and explains. Your teacher will also give you feedback, assistance and facilitate further learning. In such a way you will be able to not only acquire new knowledge and skills but also advance them further. Basically, the four steps of each of the lessons are: Activity, Definition/Theorem/Note, Example and Exercises.

### Activity

This part of the lesson demands you to revise what you have learnt or activate your background knowledge on the topic. The activity also introduces you what you are going to learn in new lesson topic.

### Definition/Theorem/Note

This part presents and explains new concepts to you. However, every lesson may not begin with definition, especially when the lesson is a continuation of the previous one.

## Example and Solution

Here, your teacher will give you specific examples to improve your understanding of the new content. In this part, you need to listen to your teacher's explanation carefully and participate actively. Note that your teacher may not discuss all the examples in the class. In this case, you need to attempt and internalize the examples by yourself.

## Exercise

Under this part of the material, you will solve the exercise and questions individually, in pairs or groups to practice what you learnt in the examples. When you are doing the exercise in the classroom either in pairs or groups, you are expected to share your opinions with your friends, listen to others' ideas carefully and compare yours with others. Note that you will have the opportunity of cross checking your answers to the questions given in the class with the answers of your teacher. However, for the exercises not covered in the class, you will be given as a homework, assignment, or project. In this case, you are expected to communicate your teacher for the solutions.

This symbol indicates that you need some time to remember what you have learnt before or used to enclose steps that you may be encouraged to perform mentally. This can help you connect your previous lessons with what it will come in the next discussions.

# Contents

# UNIT 1

# FURTHER ON SETS

## Unit Outcomes

By the end of this unit, you will be able to

- **Explain facts about sets.**
- **Describe sets in different ways.**
- **Define operations on sets.**
- **Demonstrate set operations using Venn diagram.**
- **Apply rules and principles of set theory for practical situations.**

## Unit Contents

**KEY Words**

- empty set
- union
- subset
- set description
- intersection
- proper subset
- symmetric difference
- Venn diagram
- absolute complement
- complement set

## Introduction

In Grade 7 you have learnt basic definition and operations involving sets. The concept of a set serves as a fundamental part of the present day mathematics. Today this concept is being used in almost every branch of mathematics. We use sets to define the concepts of relations and functions.

In this unit, you will discuss some further definitions, operations and applications involving sets.

### Activity 1.1

1. Define a set in your own words.
2. Which of the following are well defined sets and which are not? Justify your answer.
   a. Collection of students in your class.
   b. Collection of beautiful girls in your class.
   c. Collection of consonants of the English alphabet.
   d. Collection of hardworking teachers in a school.

## 1.1 Sets and Elements

A set is a collection of well-defined objects or elements. When we say a set is well-defined, we mean that if an object is given, we are able to determine whether the object is in the set or not.

**Note**

**i)** Sets are usually denoted by capital letters like $A, B, C, X, Y, Z$, etc.

**ii)** The elements of a set are represented by small letters like $a, b, c, x, y, z$, etc.

If $a$ is an element of set $A$, we say "$a$ belongs to $A$". The Greek symbol $\in$ (epsilon) is used to denote the phrase "belongs to". Thus, we write $\boldsymbol{a} \in \boldsymbol{A}$ if $a$ is a member of set $A$. If $b$ is not an element of set $A$, we write $\boldsymbol{b} \notin \boldsymbol{A}$ and read as "$b$ does not belong to set $A$" or "$b$ is not a member of set A".

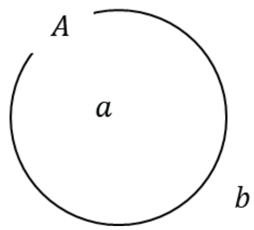



Figure 1.1

## Example 1

**a.** The set of students in your class is a well-defined set since the elements of the set are clearly known.

**b.** The collection of kind students in your school. This is not a well-defined set because it is difficult to list members of the set.

**c.** Consider $G$ as a set of vowel letters in English alphabet. Then $a \in G, o \in G, i \in G$, but $b \notin G$.

## Example 2

Suppose that $A$ is the set of positive even numbers. Write the symbol $\in$ or $\notin$ in the blank spaces.

**a.** 4 _____ $A$ **b.** 5_____ $A$ **c.** $-2$_____ $A$ **d.** 0____ $A$

**Solution:**

The positive even numbers include 2, 4, 6, 8, ... . Therefore,

**a.** $4 \in A$ **b.** $5 \notin A$, **c.** $-2 \notin A$, **d.** $0 \notin A$.

## Exercise 1.1

1. Which of the following is a well-defined set? Justify your answer.
   a. A collection of all boys in your class.
   b. A collection of efficient doctors in Black Lion Hospital.
   c. A collection of all natural numbers less than 100.
   d. The collection of songs by Artist Tilahun Gessese.
2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces:
   a. $5$___$A$  b. $8$ ___ $A$  c. $0$___$A$  d. $4$___ $A$  e. $7$___$A$

## 1.2 Set Description

Sets can be described in the following ways.

### i) Verbal method (Statement form)

In this method, the well-defined description of the elements of the set is written in an ordinary English language statement form (in words).

#### Example 1

**a.** The set of whole numbers greater than 1 and less than 20.

**b.** The set of students in this mathematics class.

### ii) Listing Methods

#### a) Complete listing method (Roster Method)

In this method, all elements of the set are completely listed. The elements are separated by commas and are enclosed within set braces, { }.

#### Example 2

**a.** The set of all even positive integers less than 7 is described in complete listing method as $\{2, 4, 6\}$.

**b.** The set of all vowel letters in the English alphabet is described in complete listing method as $\{a, e, i, o, u\}$.

## b) Partial listing method

We use this method, if listing of all elements of a set is difficult or impossible but the elements can be indicated clearly by listing a few of them that fully describe the set.

**Example 3**

Use partial listing method to describe the following sets.

**a.** The set of natural numbers less than 100.

**b.** The set of whole numbers.

**Solution:**

**a.** The set of natural numbers less than 100 are $1, 2, 3, \ldots, 99$. So, naming the set as $A$, we can express $A$ by partial listing method as $A = \{1, 2, 3, \ldots, 99\}$. The three dots after element 3 and the comma above indicate that the elements in the set continue in that manner up to 99.

**b.** Naming the set of whole numbers by $\mathbb{W}$, we can describe it as

$\mathbb{W} = \{0, 1, 2, 3, \ldots\}$.

So far, you have learnt three methods of describing a set. However, there are sets which cannot be described by these three methods. Here, below is another method of describing a set.

## iii) Set builder method (Method of defining property)

The set-builder method is described by a property that its member must satisfy the common property. This is the method of writing the condition to be satisfied by a set or property of a set.

In set brace, write the representative of the elements of a set, for example $x$, and then write the condition that $x$ should satisfy after the vertical line (|) or colon (:)

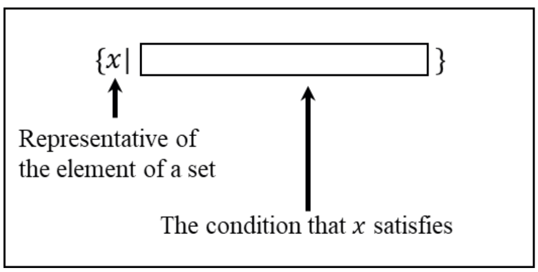



Figure 1.2

**Note**

The set of natural numbers, whole numbers, and integers are denoted by $\mathbb{N}$, $\mathbb{W}$, and $\mathbb{Z}$, respectively. They are defined as

$$\mathbb{N} = \{1, 2, 3, \ldots\}$$
$$\mathbb{W} = \{0, 1, 2, 3, \ldots\},$$
$$\mathbb{Z} = \{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

### Example 4

Describe the following sets using set builder method.

**i)** Set $A = \{1, 2, 3 \ldots 10\}$ can be described in set builder method as:

$A = \{x \mid x \in \mathbb{N} \text{ and } x < 11\}$. We read this as "$A$ is the set of all elements of natural numbers less than 11."

**ii)** Let set $B = \{0, 2, 4, \ldots.\}$. This can be described in set builder method as:

$B = \{x \mid x \in \mathbb{Z}$ and $x$ is a non-negative even integer$\}$ or

$B = \{2x \mid x = 0, 1, 2, 3, \ldots\}$ *or* $B = \{2x \mid x \in \mathbb{W}\}$.

## Exercise 1.2

1. Describe each of the following sets using a verbal method.
   a. $A = \{5, 6, 7, 8, 9\}$
   b. $M = \{2, 3, 5, 7, 11, 13\}$
   c. $G = \{8, 9, 10, \ldots.\}$
   d. $E = \{1, 3, 5, \ldots, 99\}$
2. Describe each of the following sets using complete and partial listing method (if possible):
   a. The set of positive even natural numbers below or equal to 10.
   b. The set of positive even natural numbers below or equal to 30.
   c. The set of non-negative integers.
   d. The set of even natural numbers.
   e. The set of natural numbers less than 100 and divisible by 5.
   f. The set of integers divisible by 3.
3. List the elements of the following sets:

a. $A = \{3x \mid x \in \mathbb{W}\}$  b. $B = \{x \mid x \in \mathbb{N}$ and $5 < x < 10\}$

4. Write the following sets using set builder method.

a. $A = \{1, 3, 5 \ \ldots.\}$  b. $B = \{2, 4, 6, 8\}$

c. $C = \{1, 4, 9, 16, 25\}$  d. $D = \{4, 6, 8, 10, \ldots, 52\}$

e. $E = \{-10, \ldots, -3, -2, -1, 0, 1, 2, \ldots, 5\}$  f. $F = \{1, 4, 9, \ldots.\}$

## 1.3 The Notion of Sets

### Empty set, Finite set and Infinite set

#### Empty Set

A set which does not contain any element is called an **empty set**, **void set** or **null set**. The empty set is denoted mathematically by the symbol { } or $\emptyset$.

**Example 1**

Let set $A = \{x \mid 1 < x < 2, x \in \mathbb{N}\}$. Then, $A$ is an empty set, because there is no natural number between numbers 1 and 2.

#### Finite set and Infinite set

**Definition 1.1**

A set which consists of a definite number of elements is called a finite set. A set which is not finite is called an infinite set.

**Example 2**

Identify the following sets as finite set or infinite set.

**a.** The set of natural numbers up to 10  **b.** The set of African countries

**c.** The set of whole numbers

**Solution:**

**a.** Let $A$ be a set and $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Thus, it is a finite set because it has definite (limited) number of elements.

**b.** The set of African countries is a finite set.

**c.** The set of whole numbers is an infinite set.

**Note**

The number of elements of set $A$ is denoted by $n(A)$. For instance, in the above example 2a, $n(A) = 10$. Read $n(A)$ as number of elements of set $A$

## Exercise 1.3

1. Identify empty set from the list below.
   a. $A = \{x \mid x \in \mathbb{N} \text{ and } 5 < x < 6\}$
   b. $B = \{0\}$
   c. $C$ is the set of odd natural numbers divisible by 2.
   d. $D = \{\,\}$
2. Sort the following sets as finite or infinite sets.
   a. The set of all integers
   b. The set of days in a week
   c. $A = \{x : x \text{ is a multiple of } 5\}$
   d. $B = \{x : x \in Z, x < -1\}$
   e. $D = \{x : x \text{ is a prime number}\}$

## Equal Sets, Equivalent Sets, Universal Set, Subset and Proper Subset

### Equal Sets

**Definition 1.2**

Two sets $A$ and $B$ are said to be equal if and only if they have exactly the same or identical elements. Mathematically, it is denoted as $A = B$.

Example 1

Let $A = \{1, 2, 3, 4\}$ and $B = \{4, 3, 2, 1\}$. Then, $A = B$. Set $A$ and set $B$ are equal.

## Equivalent Sets

**Definition 1.3**

Two sets $A$ and $B$ are said to be equivalent if there is a one-to-one correspondence between the two sets. This is written mathematically as $A \leftrightarrow B$ (or $A \sim B$).

**Note**

Observe that two finite sets $A$ and $B$ are equivalent, if and only if they have equal number of elements and we write mathematically this as $n(A) = n(B)$.

Example 2

Consider two sets $A = \{1, 2, 3, 4\}$ and $B = \{\text{Red}, \text{Blue}, \text{Green}, \text{Black}\}$.

In set $A$ there are four elements and in set $B$ also there are four elements. Therefore, set $A$ and set $B$ are equivalent.

## Universal Set (U)

**Definition 1.4**

A **universal set** (usually denoted by **U**) is a set which has elements of all the related sets, without any repetition of elements.

Example 3

Let set $A = \{2, 4, 6, \dots\}$ and $B = \{1, 3, 5, \dots\}$. The universal set U consists of all natural numbers, such that $U = \{1, 2, 3, 4, \dots\}$. Therefore, as we know all even and odd numbers are part of natural numbers. Hence, set U has all the elements of set $A$ and set $B$.

## Subset (⊆)

### Definition 1.5

Set $A$ is said to be a subset of set $B$ if every element of $A$ is also an element of $B$. Figure 1.3 shows this relationship. Mathematically, we write this as $\boldsymbol{A} \subseteq \boldsymbol{B}$. If set $A$ is not a subset of set $B$, then it is written as $A \nsubseteq B$.

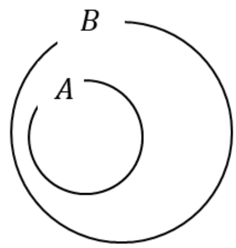


Figure 1.3

### Example 4

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$ be sets. Here, set $A$ is a subset of set $B$, or $A \subseteq B$, since all members of set $A$ are found in set $B$.

In the above set $A$, find all subsets of the set. How many subsets does set $A$ have?

**Solution:**

The subsets of set $A$ are $\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \{\ \}$. The number of subsets of set $A$ is 8.

**Note**

**i)** Any set is a subset of itself.

**ii)** Empty set is a subset of every set.

**iii)** If set $A$ is finite with $n$ elements, then the number of subsets of set $A$ is $2^n$.

In the above Example 3.b, $n(A) = 3$. Then, the number of subsets is $2^3 = 8$.

## Proper Subset (⊂)

### Definition 1.6

If $A \subseteq B$ and $A \neq B$, then $A$ is called the proper subset of set $B$ and it can be written as $A \subset B$.

## Example 5

Given that sets $A = \{2,5,7\}$ and $B = \{2,5,7,8\}$. Set $A$ is a proper subset of set $B$, that is, $A \subset B$ since $A \subseteq B$ and $A \neq B$. Observe also that $B \not\subset$ A.

In the above set $A$, find all the proper subsets. How many proper subsets does set $A$ have?

**Solution:**

The proper subsets of set $A$ are $\{2\}, \{5\}, \{7\}, \{2,5\}, \{5,7\}, \{2,7\}, \{\,\}$. There are seven subsets.

**Note**

**i)** For any set $A$, $A$ is not a proper subset of itself.

**ii)** The number of proper subsets of set $A$ is $2^n - 1$.

**iii)** Empty set is the proper subset of any other sets.

**iv)** If set $A$ is subset of set $B$ $(A \subseteq B)$, conversely $B$ is super set of $A$ written as $B \supset A$.

## Exercise 1.4

1. Identify equal sets, equivalent sets or which are neither equal nor equivalent.
   a. $A = \{1,2,3\}$ and $B = \{4,5\}$
   b. $C = \{q,s,m\}$ and $D = \{6,9,12\}$
   c. $E = \{3,7,9,11\}$ and $F = \{3,9,7,11\}$
   d. $G = \{I,J,K,L\}$ and $H = \{J,K,I,L\}$
   e. $I = \{x \mid x \in \mathbb{W},\ x < 5\}$ and $J = \{x \mid x \in \mathbb{N}, x \leq 5\}$
   f. $K = \{x \mid x \text{ is a multiple of } 30\}$ and $L = \{x \mid x \text{ is a factor of } 10\}$
2. List all the subsets of set $H = \{1,3,5\}$. How many subsets and how many proper subsets does it have?
3. Determine whether the following statements are true or false.
   a. $\{a,\ b\} \not\subset \{b,c,a\}$
   b. $\{a,\ e\} \subseteq \{x \mid x \text{ is a vowel in the English alphabet}\}$ c. $\{a\} \subset \{a,b,c\}$

4. Express the relationship of the following sets, using the symbols $\subset$, $\supset$, or $=$

   a. $A = \{1, 2, 5, 10\}$ and $B = \{1, 2, 4, 5, 10, 20\}$

   b. $C = \{x \mid x$ is natural number less than $10\}$ and $D = \{1, 2, 4, 8\}$

   c. $E = \{1, 2\}$ and $F = \{x \mid 0 < x < 3, x \in \mathbb{Z}\}$

5. Consider sets $A = \{2, 4, 6\}$, $B = \{1, 3\ 7, 9, 11\}$ and $C = \{4, 8, 11\}$, then

   a. Find the universal set

   b. Relate sets $A, B, C$ and $U$ using subset.

## 1.4 Operations on Sets

There are several ways to create new sets from sets that have already been defined. Such process of forming new set is called set operation. The three most important set operations namely Union$(\cup)$, Intersection$(\cap)$, Complement$(')$ and Difference $(-)$ are discussed below.

### Union and Intersection

**Activity 1.2**

Let the universal set is the set of natural numbers $\mathbb{N}$ which is less than 12, and sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 3, 5, 7, 9\}$.

a. Can you write a set consisting of all natural numbers that are in $A$ or in $B$?

b. Can you write a set consisting of all natural numbers that are in $A$ and in $B$?

c. Can you write a set consisting of all natural numbers that are in $A$ and not in $B$?

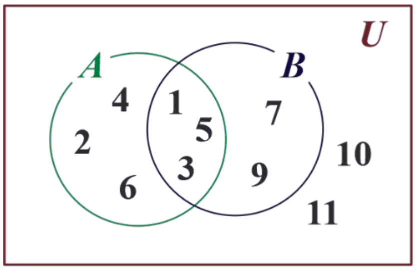



Figure 1.4

### Venn diagrams

A Venn diagram is a schematic or pictorial representation of the sets involved in the discussion. Usually sets are represented as interlocking circles, each of which is enclosed in a rectangle, which represents the universal set. Figure 1.4 above is an example of Venn diagram.

## Definition 1.7

The union of two sets $A$ and $B$, which is denoted by $\boldsymbol{A} \cup \boldsymbol{B}$, is the set of all elements that are either in set $A$ or in set $B$ (or in both sets). We write this mathematically as

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$.

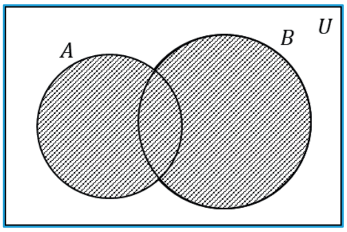



Figure 1.5

## Definition 1.8

The intersection of two sets $A$ and $B$, denoted by $A \cap B$, is the set of all elements that are both in set $A$ and in set $B$. We write this mathematically as

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

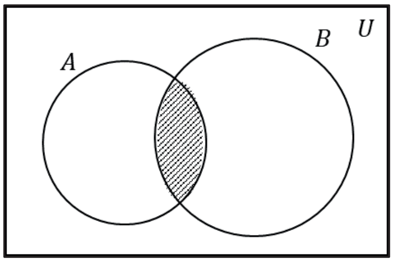



Figure 1.6

### Note

Two sets $A$ and $B$ are disjoint if $A \cap B = \emptyset$

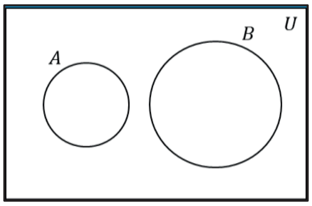



Figure 1.7

### Example 1

Let $A = \{0, 1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 6, 7\}$ be sets. Draw the Venn diagram and find $A \cup B$ and $A \cap B$.

**Solution:**

Figure 1.8 shows Venn diagram of set $A$ and $B$.

Thus, $A \cup B = \{0, 1, 2, 3, 4, 5, 6, 7\}$ and

$A \cap B = \{1, 3, 7\}$.

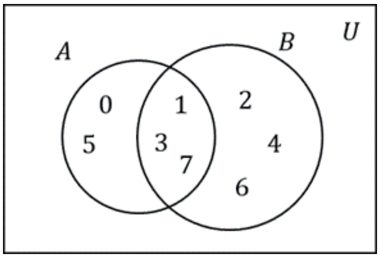



Figure 1.8

## Example 2

Let $A = \{2, 4, 6, 8, 10, \ldots\}$ and $B = \{3, 6, 9, 12, 15, \ldots\}$ be sets. Then, find $A \cup B$ and $A \cap B$.

**Solution:**

$A \cup B = \{x \mid x \text{ is a positive integer that is either even or a multiple of } 3\}$

$= \{2, 3, 6, 9, 12, 15, \ldots.\}$

$A \cap B = \{x \mid x \text{ is a positive integer that is both even and a multiple of } 3\}$

$= \{6, 12, 18, 24, \ldots.\}$

**Note**

**i)** Law of $\emptyset$ and $U$: $\emptyset \cap A = \emptyset$, $U \cap A = A$.

**ii)** Commutative law: $A \cap B = B \cap A$.

**iii)** Associative Law: $(A \cap B) \cap C = A \cap (B \cap C)$.

## Exercise 1.5

1. Let $A = \{0, 2, 4, 6, 8\}$ and $B = \{0, 1, 2, 3, 5, 7, 9\}$. Draw the Venn Diagram and find $A \cup B$ and $A \cap B$.
2. Let $A$ be the set of positive odd integers less than 10 and $B$ is the set of positive multiples of 5 less than or equal to 20. Find a) $A \cup B$, b) $A \cap B$.
3. Let $C = \{x \mid x \text{ is a factors of } 20\}$, $D = \{y \mid y \text{ is a factor of } 12\}$.

   Find a) $C \cup D$, b) $C \cap D$.

## Complement of sets

### Definition 1.9

Let $A$ be a subset of a universal set $U$. The absolute complement (or simply complement) of $A$, which is denoted by $\boldsymbol{A'}$, is defined as the set of all elements of $U$ that are not in $A$. We write this mathematically as

$$A' = \{x : x \in U \text{ and } x \notin A\}.$$

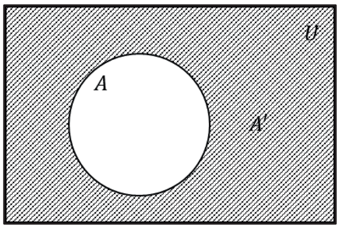



Figure 1.9

**Example 1**

**a.** Let $U = \{0, 1, 2, 3, 4\}$ and $A = \{3, 4\}$. Then, $A' = \{0, 1, 2\}$.

**Example 2**

Let $U = \{1, 2, 3, \ldots, 10\}$ be a universal set,

$A = \{x \mid x$ is a positive factor of 10 in $U\}$ and

$B = \{x \mid x$ is an odd integer in $U\}$ be sets.

**a.** Find $A'$ and $B'$.

**b.** Find $(A \cup B)'$ and $A' \cap B'$. What do you observe from the answers?

**Solution:**

**a.** $A = \{1, 2, 5, 10\}$, $B = \{1, 3, 5, 7, 9\}$, Thus,

$A' = \{3, 4, 6, 7, 8, 9\}$, $B' = \{2, 4, 6, 8, 10\}$.

**b.** First, we find $A \cup B$. Hence, $A \cup B = \{1, 2, 3, 5, 7, 9, 10\}$ and

$$(A \cup B)' = \{4, 6, 8\}.$$

On the other hand, from $A'$ and $B'$, we obtain $A' \cap B' = \{4, 6, 8\}$. Hence, we immediately observe $(A \cup B)' = A' \cap B'$.

In general, for any two sets $A$ and $B$, $(A \cup B)' = A' \cap B'$. It is called the first statement of De Morgan's law.

## De Morgan's Law

For the complement set of $A \cup B$ and $A \cap B$,

1st statement: $(A \cup B)' = A' \cap B'$,

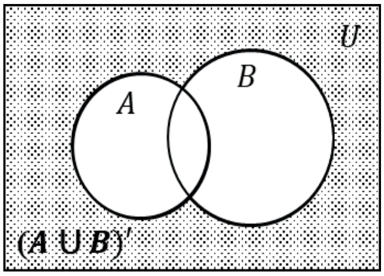



Figure 1.10

2nd statement: $(A \cap B)' = A' \cup B'$.

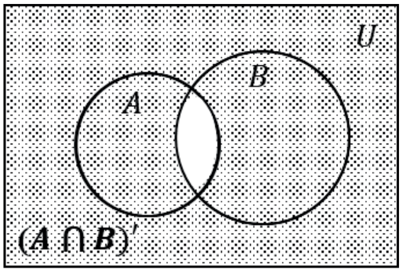



Figure 1.11

### Exercise 1.6

1. If the universal set $U = \{0, 1, 2, 3, 4, 5\}$, and $A = \{4, 5\}$, then find $A'$.
2. Let the universal set $U = \{1, 2, 3, \ldots, 20\}$, $A = \{x \mid x \text{ is a positive factor of } 20\}$ and $B = \{x \mid x \text{ is an odd integer in } U\}$. Find $A'$, $B'$, $(A \cup B)'$ and $A' \cap B'$.
3. Let the universal set be $U = \{x \mid x \in \mathbb{N}, x \leq 10\}$. When $A = \{2, 5, 9\}$, and $B = \{1, 5, 6, 8\}$, find a) $A' \cap B'$ and b) $A' \cup B'$.

## Difference of sets

### Definition 1.10

The difference between two sets $A$ and $B$, which is denoted by $A - B$, is the of all elements in $A$ and not in $B$; this set is also called the relative complement of $A$ with respect to $B$. We write this mathematically as $A - B = \{x \mid x \in A \text{ and } x \notin B\}$.

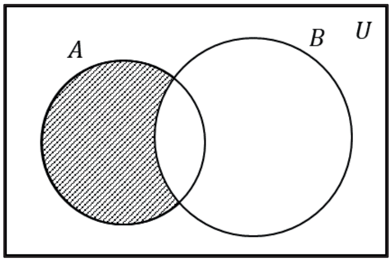



Figure 1.12

**Note**

The notation $A - B$ can be also written as $A \backslash B$.

### Example 1

If sets $A = \{0, 1, 2, 3, 4\}$ and $B = \{3, 4\}$, then $A - B$ or $A \backslash B = \{0, 1, 2\}$.

### Example 2

Let $U$ be a universal set of the set of one-digit numbers, $A$ be the set of even numbers, $B$ be the set of prime numbers less than 10. Find the following:

**a.** $A - B$ or $A \backslash B$ **b.** $B - A$ or $B \backslash A$

**c.** $A \cup B$ **d.** $U - (A \cup B)$ or $U \backslash (A \cup B)$

**Solution:**

Here, $A = \{0, 2, 4, 6, 8\}, B = \{2, 3, 5, 7\}$. Then, we illustrate the sets using a Venn diagram as follows. From the Venn diagram we observe:

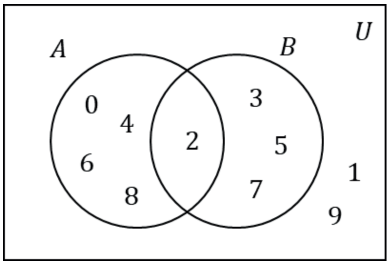



Figure 1.13

**a.** $A - B = \{0, 4, 6, 8\}$

**b.** $B - A = \{3, 5, 7\}$

**c.** $A \cup B = \{0, 2, 3, 4, 5, 6, 7, 8\}$

**d.** $U - (A \cup B) = \{1, 9\}$

### Example 3

For the same sets in Example 2, find the following. What can you say from Example 2 a. and b.? What about d. and Example 2, a.?

**a.** $A'$ **b.** $U - A$

**c.** $B'$ **d.** $A \cap B'$

**Solution:**

**a.** $A' = \{1, 3, 5, 7, 9\}$ **b.** $U - A = \{1, 3, 5, 7, 9\}$

**c.** $B' = \{0, 1, 4, 6, 8, 9\}$ **d.** $A \cap B' = \{0, 4, 6, 8\}$

From a. and b., we can say, $A' = U - A$. From d. and Example 2, a., we can say, $A - B = A \cap B'$.

**Theorem 1.1**

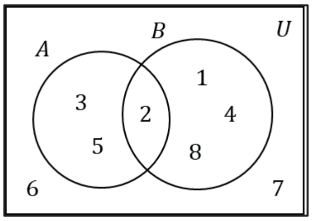
For any two sets $A$ and $B$, each of the following holds true.

$(A')' = A \qquad A' = U - A$

$A - B = A \cap B' \qquad A \subseteq B \Leftrightarrow B' \subseteq A'$

## Exercise 1.7

From the given Venn diagram, find each of the following:




**a.** $A - B$ or $A \backslash B$ **b.** $B - A$ or $B \backslash A$

**c.** $A \cup B$ d. $U - (A \cup B)$ or $U \backslash (A \cup B)$

## Symmetric Difference of Two Sets

**Definition 1.11 Symmetric Difference**

For two sets A and B, the symmetric difference between these two sets is denoted by $A\Delta B$ and is defined as:

$A\Delta B = (A\backslash B) \cup (B\backslash A)$, which is $(A - B) \cup (B - A)$

or

$= (A \cup B)\backslash(A \cap B)$

In the Venn diagram, the shaded part represents $A\Delta B$

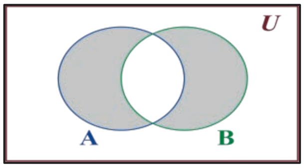



Figure 1.14

### Example 1

Consider sets $A = \{1, 2, 4, 5, 8\}$ and $B = \{2, 3, 5, 7\}$.

Then, find $A\Delta B$.

**Solution:**

First, let us find $A\backslash B = \{1, 4, 8\}$ and $B\backslash A = \{3, 7\}$.

Hence, $A\Delta B = (A\backslash B) \cup (B\backslash A)$

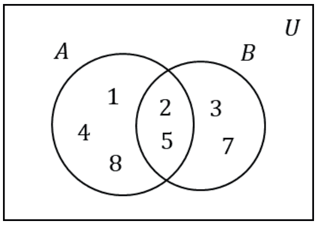



Figure 1.15

$= \{1, 4, 8\} \cup \{3, 7\} = \{1, 3, 4, 7, 8\}.$

Or $A \Delta B = (A \cup B) \backslash (A \cap B) = \{1, 3, 4, 7, 8\}.$

### Example 2

Given sets $A = \{d, e, f\}$ and $B = \{4, 5, 6\}$. Then, find $A \Delta B$.

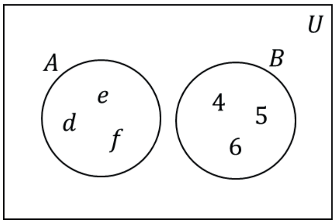



Figure 1.16

**Solution:**

First, we find $A \backslash B = \{d, e, f\}$ and $B \backslash A = \{4,5,6\}$.

Hence, $A \Delta B = (A \backslash B) \cup (B \backslash A)$

$= \{d, e, f\} \cup \{4, 5, 6\} = \{d, e, f, 4, 5, 6\}.$

## Exercise 1.8

1. Given $A = \{0, 2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3, 5, 7, 9\}$. Then, find $A \Delta B$.
2. If $A \Delta B = \emptyset$, then what can be said about the two sets?
3. For any two sets $A$ and $B$, can we generalize $A \Delta B = B \Delta A$ ? Justify your answer.

## Cartesian Product of Two Sets

### Definition 1.12 Cartesian Product of Two Sets

The Cartesian product of two sets $A$ and $B$, denoted by $A \times B$, is the set of all ordered pairs $(a, b)$ where $a \in A$ and $b \in B$. This also can be expressed as

$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}.$

### Example 1

Let $A = \{1, 2\}$ and $B = \{a, b\}$. Then, find **a)** $A \times B$ **b)** $B \times A$

**Solution:**

**a.** $A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

**b.** $B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

### Example 2

If $A \times B = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$, then find sets $A$ and $B$.

**Solution:**

$A$ is the set of all first components of $A \times B$, that is, $A = \{1, 2, 3\}$, and

$B$ is the set of all second components of $A \times B$, that is, $B = \{a, b\}$.

### Exercise 1.9

1. Let $A = \{1, 2, 3\}$ and $B = \{e, f\}$. Then, find a) $A \times B$ b) $B \times A$.
2. If $A \times B = \{(7,6), (7,4), (5,4), (5,6), (1,4), (1,6)\}$, then find sets $A$ and $B$.
3. If $A = \{a, b, c\}$, $B = \{1, 2, 3\}$ and $C = \{3, 4\}$, then find $A \times (B \cup C)$.
4. If $A = \{6, 9, 11\}$, then find $A \times A$.
5. If the number of elements of set $A$ is 6 and the number of elements of set $B$ is 4, then the number of elements of $A \times B$ is ________________.

## 1.5 Application

### Number of Elements of union of two sets

For the two subsets $A$ and $B$ of a universal set $U$, the following formula on the number of elements holds. That is

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

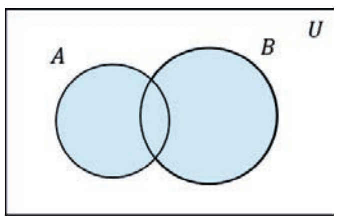



Figure 1.17

Especially in the case $A \cap B = \emptyset$, thus,

$n(A \cap B) = 0$, and the following holds:

$n(A \cup B) = n(A) + n(B)$.

If $A \cap B \neq \emptyset$, then

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

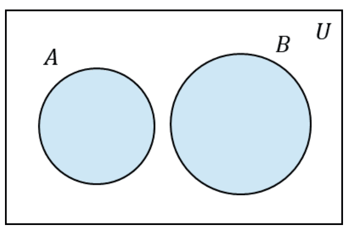



Figure 1.18

### Example

Let $A$ and $B$ be two finite sets such that $n(A) = 20$, $n(B) = 28$, and $n(A \cup B) = 36$, then find $n(A \cap B)$.

**Solution:**

Using the formula $n(A \cup B) = n(A) + n(B) - n(A \cap B)$ we have

$$36 = 20 + 28 - n(A \cap B).$$

This gives $n(A \cap B) = (20 + 28) - 36 = 48 - 36 = 12$.

## Exercise 1.10

1. Let $A$ and $B$ be two finite sets such that $n(A) = 34$, $n(B) = 46$ and $n(A \cup B) = 70$. Then, find $n(A \cap B)$.
2. There are 60 people attending a meeting. 42 of them drink tea and 27 drink coffee. If every person in the meeting drinks at least one of the two drinks, find the number of people who drink both tea and coffee. (Hint: Use a Venn diagram).

## Summary

1. A set is a collection of well-defined objects or elements. When we say a set is well-defined, we mean that given an object we are able to determine whether the object is in the set or not.
2. Sets can be described in the following ways:

   **Verbal method (Statement form)**

   In this method, the well-defined description of the elements of the set is written in an ordinary English language statement form (in words).

   **Complete listing method (Roster Method)**

   In this method all the elements of the sets are completely listed. The elements are separated by commas and are enclosed within set brace, { }.

   **Partial listing method**

   We use this method, if listing of all elements of a set is difficult or impossible but the elements can be indicated clearly by listing a few of them that fully describe the set.

   **Set builder method (Method of defining property)**

   The set-builder method is described by a property that its member must satisfy. This is the method of writing the condition to be satisfied by a set or property of a set.
3. A set which does not contain any element is called an empty set, void set or null set. The empty set is denoted mathematically by the symbol { } or Ø.
4. Two sets A and B are said to be equal if and only if they have exactly same or identical elements. Mathematically, we write this as $A = B$.
5. Set $A$ is said to be a subset of set $B$ if every element of $A$ is also an element of $B$. Mathematically, we write this as $A \subseteq B$.
   - Any set is a subset to itself.
   - Empty set is a sub set of every set.

- If set $A$ is finite with $n$ elements, then the number of subsets of set $A$ is $2^n$.

**6.** If $A \subseteq B$ and $A \neq B$, then $A$ is called the proper subset of $B$ and it can be written as $A \subset B$.

- For any set $A$, $A$ is not a proper subset to itself.
- If set $A$ is finite with $n$ elements the number of proper subsets of set $A$ is $2^n - 1$.
- Empty set is a proper subset of any other sets.

**7.** A universal set (usually denoted by $U$) is a set which has elements of all the related sets, without any repetition of elements.

**8.** The union of two sets $A$ and $B$, which is denoted by $A \cup B$, is the set of all elements that are either in set $A$ or in set B (or in both sets). We write this mathematically as

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

**9.** The intersection of two sets $A$ and $B$, which is denoted by $A \cap B$, is the set of all elements that are in set $A$ and in set $B$. We write this mathematically as

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

**10.** The difference between two sets $A$ and $B$, which is denoted by $A - B$, is the set of all elements in set $A$ and not in set $B$; this set is also called the relative complement of set $A$ with respect to set $B$. We write this mathematically as

$$A - B = A\backslash B = \{x \mid x \in A \text{ and } x \notin B\}.$$

**11.** Let $A$ be subset of a universal set $U$. The absolute complement (or simply complement) of set $A$, which is denoted by $A'$, is defined as the set of all elements of $U$ that are not in $A$. We write this mathematically as

$$A' = \{x \mid x \in U \text{ and } x \notin A\}.$$

**12.** A Venn diagram is a schematic or pictorial representation of the sets involved in the discussion. Usually sets are represented as interlocking circles, each of which is enclosed in a rectangle, which represents the universal set.

**13.** For two sets $A$ and $B$ the symmetric difference between these two sets is denoted by $A\Delta B$ and is defined as $A\Delta B = (A\backslash B) \cup (B\backslash A) = A \cup B)\backslash(A \cap B)$.

**14.** For any two finite sets $A$ and $B$, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.

## Review Exercise

**1.** Express following sets using the listing method.

**a.** $A$ is the set of positive factors of 18

**b.** $B$ is the set of positive even numbers below or equal to 30

**c.** $C = \{2n \mid n = 0, 1, 2, 3, \dots\}$

**d.** $D = \{x \mid x^2 = 9\}$

**2.** Express following sets using the set-builder method.

**a.** $\{2, 4, 6, \dots..\}$

**b.** $\{1, 3, 5, \dots, 99\}$

**c.** $\{1, 4, 9, \dots, 81\}$

**3.** Find all the subsets of the following sets.

**a.** $\{3, 4, 5\}$

**b.** $\{a, b\}$

**4.** Find $A \cup B$ and $A \cap B$ of the following.

**a.** $A = \{2, 3, 5, 7, 11\}$ and $B = \{1, 3, 5, 8, 11\}$

**b.** $A = \{x \mid x \text{ is the factor of } 12\}$ and $B = \{x \mid x \text{ is the factor of } 18\}$

**c.** $A = \{3x \mid x \in \mathbb{N}, x \leq 20\}$ and $B = \{4x \mid x \in \mathbb{N}, x \leq 15\}$

**5.** If $B \subseteq A, A \cap B' = \{1, 4, 5\}$, and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, then find set $B$.

**6.** Let $A = \{2, 4, 6, 7, 8, 9\}$, $B = \{1, 3, 5, 6, 10\}$ and

$C = \{x \mid \in \mathbb{Z}, 3x + 6 = 0 \text{ or } 2x + 6 = 0\}$.

Find **a)** $A \cup B$

**b)** Is $(A \cup B) \cup C = A \cup (B \cup C)$?

7. Suppose the universal set $U$ be the set of one-digit numbers, and set $A = \{x \mid x \text{ is an even natural number less than or equal to } 9\}$.

   Describe each set by complete listing method:

   a. $A'$

   b. $A \cap A'$

   c. $A \cup A'$

   d. $(A')'$

   e. $\phi \setminus U$

   f. $\phi'$

   g. $U'$

8. Let $U = \{1, 2, 3, 4, \ldots \ldots, 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{1, 2, 3, 4\}$. Evaluate $U \setminus (A \Delta B)$.

9. Consider a universal set $U = \{1, 2, 3, . . . , 14\}$, $A = \{2, 3, 5, 7, 11\}$, $B = \{2, 4, 8, 9, 10, 11\}$. Then, which one of the following is true?

   A. $(A \cup B)' = \{1, 4, 6, 12, 13, 14\}$

   B. $A \cap B = A' \cup B'$

   C. $A \Delta B = (A \cap B)'$

   D. $A \setminus B = \{3, 5, 7\}$

   E. None

10. Let $A = \{3, 7, a^2\}$ and $B = \{2, 4, a + 1, a + b\}$ be two sets and all the elements of the two sets are integers. If $A \cap B = \{4, 7\}$, then find $a$ and $b$. In addition, find $A \cup B$.

11. In a survey of 200 students in Motta Secondary School, 90 students are members of Nature club, 31 students are members of Mini-media club, 21 students are members of both clubs. Answer the following questions.

    a. How many students are members of either of the clubs?

    b. How many students are not members of either of the clubs?

c. How many students are only in Nature club?

12. A survey was conducted in a class of 100 children and it was found out that 45 of them like Mathematics whereas only 35 like Science and 10 students like both subjects. How many like neither of the subjects?

A) 70 B) 30 C) 100 D) 40