UNIT 5

INTRODUCTION TO TRIGONOMETRY

Unit Outcomes

By the end of this unit, you will be able to:

- **Identify the hypotenuse, opposite and adjacent of a right-angled triangle.**
- **Describe the basic properties of a right-angled triangle.**
- **Define sine, cosine, and tangent ratio.**
- **Find trigonometric values of angles from trigonometric table.**

Unit Contents

- trigonometric table
- trigonometric ratio (sine, cosine and tangent)
- right-angled triangle
- Pythagoras theorem

INTRODUCTION

You have already studied about triangles, in particular, right-angled triangles, in your earlier classes. In this unit, you will learn the relationship between angles and sides of a right-angled triangle.

5.1 Revision on Right-angled Triangles

Definition 5.1 Right-angled triangle

A right-angled triangle is a type of triangle that has one of its angles equal to 90 degrees.

1. The opposite side to the right-angle is called hypotenuse.
2. The relationship among the three sides is: $b^2 + p^2 = h^2$.

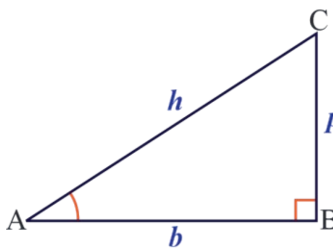


Figure 5.1

Example 1

a. What is the sum of the maximum degree measure of the two interior angles other than the right angle?

b. If one of the angles of a triangle is 90°and the other two are equal to 45°each, what do you call such a triangle?

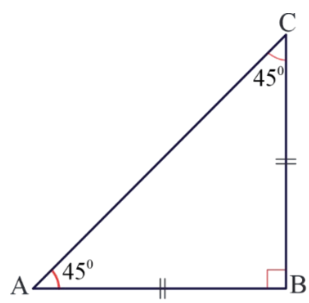


Figure 5.2

Solution:

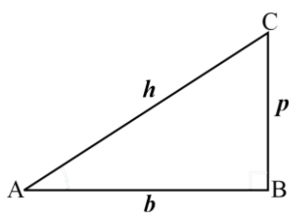
a. As you observed in figure 5.1, one of the angles of a right-angled triangle is 90°. The sum of the measure of the other two angles of this triangle is 90°. So the sum of the maximum degree measure of the two other interior angles of a right-angled triangle is 90°.

b. If one of the angles of a triangle is 90° and the other two angles measure 45° each, then the triangle is called an isosceles right-angled triangle, where the adjacent sides to 90° are equal in length (figure 5.2).

Note

For right-angled triangle, we also have the following basic properties:

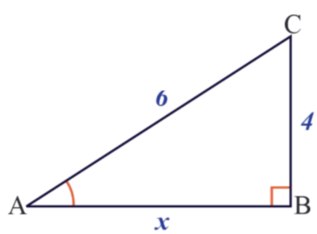
- One angle is always 90° or right angle.
- The opposite side to angle 90° is hypotenuse.
- The hypotenuse is always the longest side.
- The sum of the other two interior angles is equal to 90° (each angle is acute).



- The other two sides adjacent to the right angle are called base and perpendicular side. (b is the base and p is the perpendicular side).
- If one of the angles is 90° and the other two angles are equal to 45° each, then the triangle is called an isosceles right-angled triangle, where the adjacent sides to 90° are equal in length.
- If b, p and h are sides of a right-angled triangle as shown in the above figure, we can write a relation using the **Pythagoras theorem,** that is $b^2 + p^2 = h^2$.

Example 2

Find the length of the missing side.



Solution:

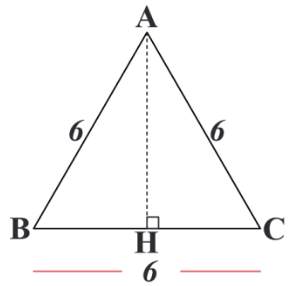
Using the Pythagoras theorem, $x^2 + 4^2 = 6^2$.

It results in $x^2 = 20$. Since $x > 0$, $x = 2\sqrt{5}$.

Example 3

Find the height of an equilateral triangle ABC with length of 6units.

Solution:



Consider equilateral triangle as shown at the right. When drawing a line segment from A and perpendicular to side BC, H is midpoint of BC. BH = 3unit. Using Pythagoras theorem, let AH $= x$ $(x > 0)$, then $BH^2 + AH^2 = AB^2$

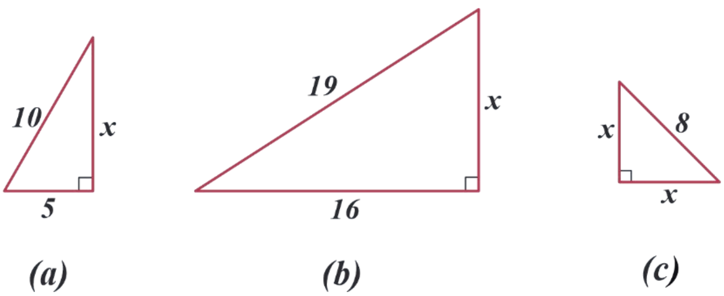
$$3^2 + x^2 = 6^2$$

$$x^2 = 27$$

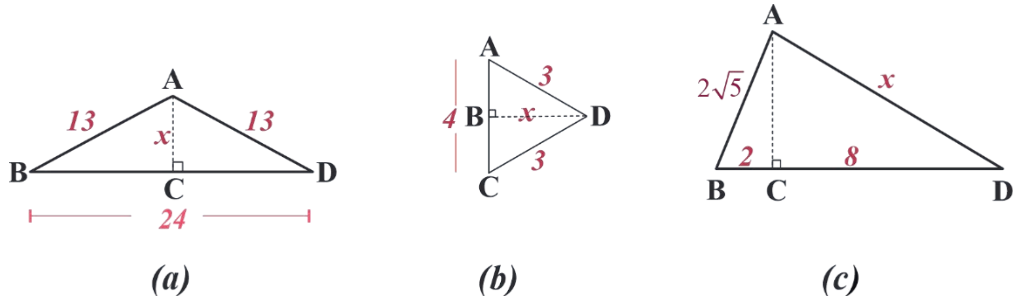
This results in $AH = x = 3\sqrt{3}$ units.

Exercise 5.1

1. Find the value of x. Express your answer in simplest radical form.



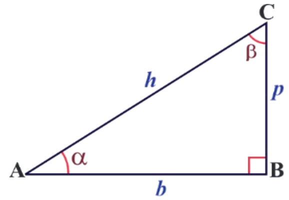
2. Find the value of x. If your answer is not an integer give it in the simplest radical form.



Conversion of the Pythagoras theorem

Conversion of the Pythagoras theorem

If $b^2 + p^2 = h^2$, then $\angle B = 90°$.



Example 1

Which one of the following can be sides of a right-angled triangle (the sides are with the same unit of length)?

a. $5, 6, 2$ **b.**$3, 4, 5$ **c.**$\sqrt{5}, 2, 3$

Solution:

a. First identify the longest side (hypotenuse). From the property of right-angled triangle, the longest side is the hypotenuse and the other would be the base and perpendicular side.

So, we can take 2 and 5 as length of the base and perpendicular side, respectively, and 6 as the hypotenuse, as shown in the figure. Now, check if Pythagoras theorem is satisfied or not, that is $b^2 + p^2 = h^2$ or not. The left hand side (LHS) of the equation is $5^2 + 2^2 = 29$ and the right hand side (RHS) of the equation is $6^2 = 36$. Here, LHS $\neq$ RHS. Hence, the triangle is not a right-angled triangle.

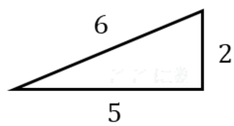


Figure 5.3

b. Consider $3, 4$ and 5 as a, b and c respectively. Check whether these numbers satisfy the theorem $b^2 + p^2 = h^2$.

LHS $= 3^2 + 4^2 = 25$ and RHS $=5^2 = 25$. Thus, LHS $=$ RHS . Therefore, it is a right-angled triangle.

c. As $2^2 < \left(\sqrt{5}\right)^2 < 3^2$, then $2 < \sqrt{5} < 3$. Hence, $h = 3$. Using equation

$b^2 + p^2 = h^2$,LHS $= (\sqrt{5})^2 + 2^2 = 9$ and RHS $= 3^2 = 9$. Thus LHS = RHS.

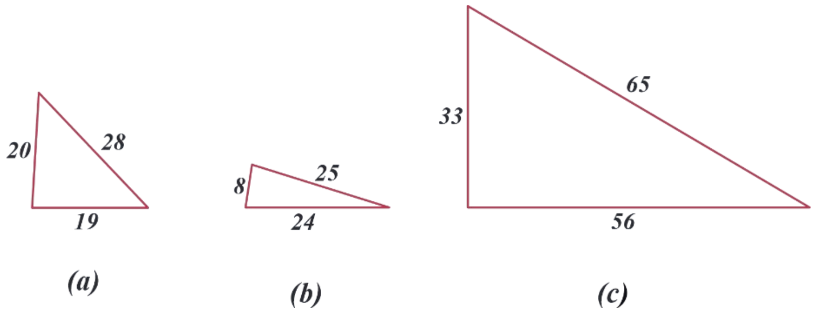
Therefore, it is a right-angled triangle.

Exercise 5.2

1. Identify whether the given sides can form a right-angled triangle or not provided that the units for each length is identical.

   a. $4, 6, 8$ b. $\sqrt{2}, \sqrt{3}, \sqrt{5}$ c. $1, 2, \sqrt{3}$ d. $\sqrt{6}, 3, \sqrt{3}$

2. Is each triangle a right-angled triangle? Explain.



5.2 Trigonometric Ratios

In this subsection, you will learn some ratios of the sides of a right-angled triangle with respect to its angles, called trigonometric ratios of the angle. We will restrict our discussion to acute angles only. However, these ratios can also be extended to other angles.

Could you think some examples from our surroundings where right-angled triangles can be imagined?

- Suppose 9th grade students of your school are visiting Addis Ababa. Now, if a student is looking at the top of the building (head office of the Commercial Bank of Ethiopia)

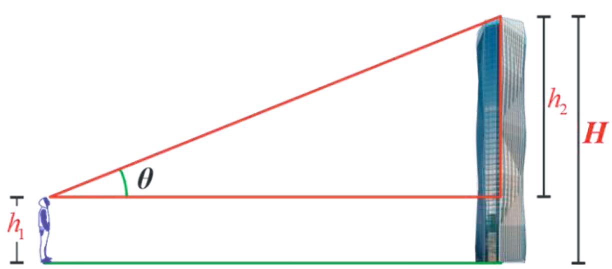


Figure 5.4

a right-angled triangle can be imagined, as shown in fig 5.4. Can you find out the height of the building without actually measuring it?

In the above situation, the height can be found by using some mathematical techniques, which come under a branch of mathematics called 'trigonometry'. Trigonometry is derived from two Greek words 'trigonon' which means 'triangle' and 'metron' meaning 'measure'. Put together, the words mean "triangle measuring". In fact, trigonometry is the study of relationships between the sides and angles of a triangle. The earliest known work on trigonometry was recorded in Egypt and Babylon. Early astronomers used it to find out the distances of the stars and planets from the Earth. Even today, most of the technologically advanced methods used in Engineering and Physical Sciences are based on trigonometric concepts.

Now let us define the trigonometric ratio.

Definition 5.2 Trigonometric ratios

The trigonometric ratios of a given angle are defined by the ratios of two sides of a right-angled triangle. These trigonometric ratios remain unchanged as long as the angle remains the same, that is, they are independent of the size of the triangle provided the angle remains the same.

There are six trigonometric ratios. But here we will define only the first three trigonometric ratios as follows.

Consider the following right-angled triangle as shown in the figure 5.5 below.

For angle A, the trigonometric ratios sine, cosine and tangent are defined as

$$\text{sine of } \angle A = \frac{\text{side opposite to angle } A}{\text{hypotenuse}} = \frac{BC}{AC}$$

$$\text{cosine of } \angle A = \frac{\text{side adjacent to angle } A}{\text{hypotenuse}} = \frac{AB}{AC}$$

$$\text{tangent of } \angle A = \frac{\text{side opposite to angle } A}{\text{side adjacent to angle } A} = \frac{BC}{AB}$$

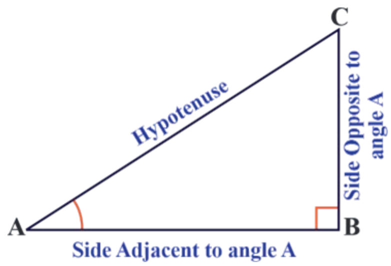


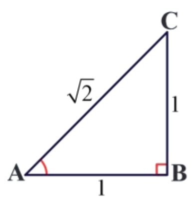
Figure 5.5

The above trigonometric ratios ***sineA***, ***cosine A*** and ***tangent A*** are abbreviated as ***sin A***, ***cos A*** and ***tan A,*** respectively.

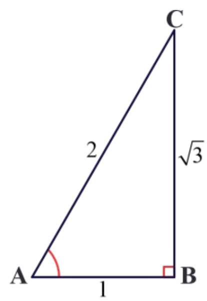
So, the trigonometric ratios of an acute angle in a right-angled triangle express the relationship between the angle and the length of its sides.

Example

Find the trigonometric ratios (sine, cosine and tangent) for angle A in the following figures.



(a)



(b)

Solution:

a. Using definition 5.2, $\sin A = \frac{Opposite}{Hypotenuse} = \frac{1}{\sqrt{2}}$, $\cos A = \frac{adjacent}{hypotenuse} = \frac{1}{\sqrt{2}}$ and

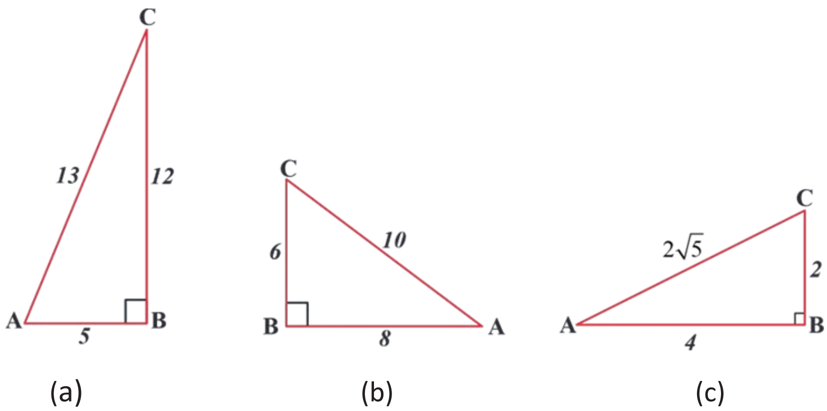
$\tan A = \frac{opposite}{adjacent} = \frac{1}{1} = 1$.

b. Similar to (a) above, $\sin A = \frac{Opposite}{Hypotenuse} = \frac{\sqrt{3}}{2}$, $\cos A = \frac{adjacent}{hypotenuse} = \frac{1}{2}$ and

$\tan A = \frac{opposite}{adjacent} = \frac{\sqrt{3}}{1} = \sqrt{3}$.

Exercise 5.3

1. Find the trigonometric ratios sine, cosine, and tangent of the angle A for each of the following right-angled triangles:



If one of the three trigonometric ratios is given, it is possible to determine the other two trigonometric ratios.

Example

Determine the remaining two trigonometric ratios of an acute angle A of a right-angled triangle if

a. $\sin A = \frac{3}{5}$ **b.** $\tan A = \frac{1}{\sqrt{2}}$

Solution:

a. First draw a right-angled triangle ΔABC as it is given $\sin A = \frac{3}{5}$ which is equal to $\frac{BC}{AC}$. Hence, we have $\sin A = \frac{BC}{AC} = \frac{3}{5}$ and from this we immediately observe the length of side $BC = 3$ units (opposite side) and $AC = 5$ units (hypotenuse). Now, using the Pythagoras Theorem, we have $AB^2 + BC^2 = AC^2$, we determine the side length AB as follows.

$$\begin{aligned} AB &= \sqrt{AC^2 - BC^2} \\ &= \sqrt{25 - 9} \\ &= \sqrt{16} \\ &= 4 \end{aligned}$$

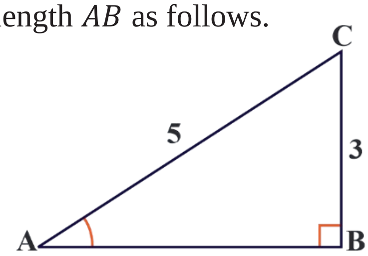


Figure 5.6

Hence, the remaining trigonometric ratios are $\tan A = \frac{BC}{AB} = \frac{3}{4}$ and $\cos A = \frac{AB}{AC} = \frac{4}{5}$.

b. Following similar procedure as (a), first we need to determine $\cos A$ and $\sin A$ from the given ratio of $\tan A = \frac{1}{\sqrt{2}}$. This is also equal to the ratio $\frac{BC}{AB}$. Hence, $\tan A = \frac{1}{\sqrt{2}} = \frac{BC}{AB}$ and if $BC = 1$, then $AB = \sqrt{2}$. Now, using the Pythagoras Theorem, we determine the length AC as

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{2+1} = \sqrt{3}.$$

Now, the remaining trigonometric ratios are $\cos A = \frac{AB}{AC} = \frac{\sqrt{2}}{\sqrt{3}}$ and $\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{3}}$.

Exercise 5.4

Determine the remaining two trigonometric ratios of an acute angle A of a right-angled triangle if

a. $\sin A = \frac{1}{2}$ b. $\cos A = \frac{2}{3}$ c. $\tan A = \frac{1}{2}$ d. $\sin A = \frac{1}{\sqrt{5}}$

Trigonometric values of basic angles

From lower grade mathematics lessons, you are already familiar with the construction of angles of $0°, 30°, 45°, 60°$ and $90°$. Now you will learn how to find the values of the trigonometric ratios for these angles.

Example 1

Find $\sin 45°, \cos 45°$ and $\tan 45°$.

Solution:

Consider a right-angled triangle ABC right-angled at B.

If one of the acute angles is $45°$, then the other acute angle is also $45°$,

that is $\angle A = \angle C = 45°$(see figure 5.7).

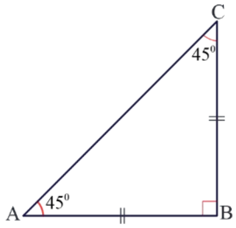


Figure 5.7

Hence, $BC = AB$(because the triangle is isosceles right-angled triangle).

Suppose $BC = AB = a$. Then by Pythagoras Theorem,

$$AC = \sqrt{AB^2 + BC^2} \text{ which implies}$$

$$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a.$$

Using the definition of trigonometric ratios we have:

$$\sin 45° = \frac{\text{side opposite to angle } 45°}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}.$$

$$\cos 45° = \frac{\text{side adjacent to angle } 45°}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}.$$

$$\tan 45° = \frac{\text{side opposite to angle } 45°}{\text{side adjacent to angle } 45°} = \frac{BC}{AB} = \frac{a}{a} = 1.$$

Example 2

Find **i)** $\sin 30°, \cos 30°$, and $\tan 30°$.

ii) $\sin 60°, \cos 60°$, and $\tan 60°$.

Solution:

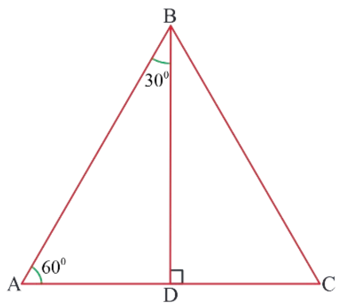


Figure 5.8

Consider half of an equilateral triangle ABC shown as below (Figure 5.8). The triangle ABD is a right-angled triangle with angles $30°$ and $60°$ (Fig.5.9). Let the length of side AB be 2. The length of the side AD is half of the length of side AC. Thus $AD = 1$.

By applying the Pythagoras Theorem,

$AD^2 + BD^2 = AB^2$.

Then, $1^2 + BD^2 = 2^2$

$$BD^2 = 4 - 1$$

$$BD^2 = 3$$

$$BD = \sqrt{3} \quad \text{because } BD > 0.$$

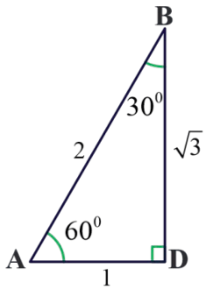


Figure 5.9

Therefore,

$$\sin 30° = \frac{\text{side opposite to angle B}}{\text{hypotenuse}} = \frac{1}{2}. \quad \sin 60° = \frac{\text{side opposite to angle A}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}.$$

$$\cos 30° = \frac{\text{side adjacent to angle B}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}. \quad \cos 60° = \frac{\text{side adjacent to angle A}}{\text{hypotenuse}} = \frac{1}{2}.$$

$\tan 30° = \frac{\text{side opposite to angle B}}{\text{side adjacent to angle B}} = \frac{1}{\sqrt{3}}$. $\tan 60° = \frac{\text{side opposite to angle A}}{\text{side adjacent to angle A}} = \frac{\sqrt{3}}{1} = \sqrt{3}$.

Example 3

Find the unknown length of the sides of the following in the right-angled triangle.

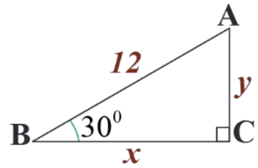


Figure 5.10

Solution:

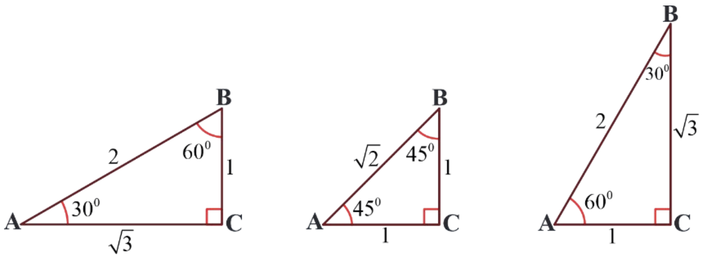
Applying the trigonometric ratios here,

$\cos 30^\circ = \frac{x}{12}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$. Then, $\frac{x}{12} = \frac{\sqrt{3}}{2}$, $x = \frac{\sqrt{3}}{2} \times 12 = 6\sqrt{3}$.

Similarly; $\sin 30^\circ = \frac{y}{12}$, $\sin 30^\circ = \frac{1}{2}$. Then, $\frac{y}{12} = \frac{1}{2}$, $y = 6$.

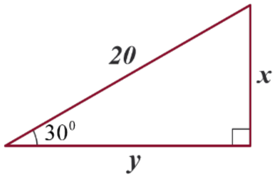
Exercise 5.5

1. Complete the following table column wise using the given triangles (a), (b) and (c) respectively.

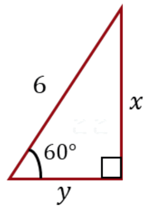


	$\angle A = 30°$	$\angle A = 45°$	$\angle A = 60°$
$\sin A$			
$\cos A$			
$\tan A$			

2. Find the unknown values of the following figures. If the value is not an integer, express it in simplest radical form.



(a)



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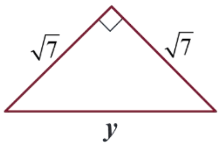

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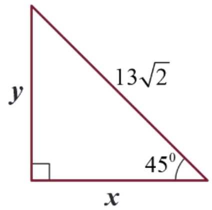

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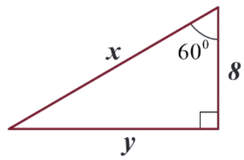

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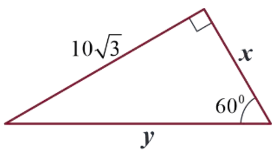

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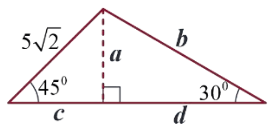

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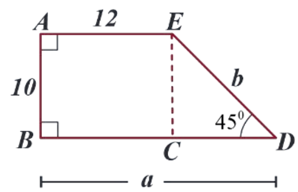

(h)

Trigonometric Ratios of 0° to 90°

We discussed how to determine trigonometric ratio of $30°, 45°$ and $60°$ using examples. To derive the trigonometric ratios of $0°$ and $90°$ needs advanced concept of mathematics and will not be treated at this level. The trigonometric ratios of these two angles together with $30°, 45°$ and $60°$ are indicated in the following table 5.1.

Table 5.1

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

Remark: From the above table, you can observe that as $\angle A$ increases from $0°$ to $90°$, $\sin A$ increases from 0 to 1 while $\cos A$ decreases from 1 to 0. On the other hand, the value of $\tan A$ increases from 0, become 1 at 45° and continues to increase until 90°.

Trigonometrical values of angles from a table

$(\sin\theta, \cos\theta \text{ and } \tan\theta \ \ for\ 0° \leq \theta \leq 90°)$

In the previous discussion we have seen how to determine the trigonometric ratios of $\sin\theta, \cos\theta$ and $\tan\theta$ when θ is a special angle $0°, 30°, 45°, 60°$ and $90°$. Following the same procedure of getting the trigonometric ratio of the given special angles, it is possible to determine the trigonometric ratio of any angle. There are trigonometric tables of approximate values of trigonometric ratios of acute angles that have already been constructed by advanced arithmetical processes. One of such tables is found at the back of this textbook.

Follow the next steps to read the trigonometric values from a trigonometric table:-

- ✓ Identify the trigonometric ratio from the first row if the degree you need is between $0°$ and $45°$ and the last row if the degree is between $45°$and $90°$.
- ✓ Using the column of the trigonometric ratio you need to find and move down or up till you get the required degree.
- ✓ The intersection of the row and column of the above two steps is the trigonometric value of the required degree.

Example

Using a trigonometric table, find the value of each of the following.

a. $\sin 18°$ **b.** $\cos 37°$ **c.** $\tan 70°$

Solution:

a. We can read the trigonometric ratio from a table. Since the given degree is 18, use the first row and take the column of sin then move down till we get

18°. The number we obtain is a value for sin 18°.

That is $\sin 18° \approx 0.309017$. (You can observe the steps from table 5.2)

Table 5.2

Move Down

Move to the right

	sin	cos	tan
0	0	1	0
1	0.017452	0.999848	0.017455
2	0.03490	0.999391	0.034921
3	0.052336	0.99863	0.052408
4	0.069756	0.997564	0.069927
5	0.087156	0.996195	0.087489
6	0.104528	0.994522	0.10510
7	0.121869	0.992546	0.122784
8	0.139173	0.990268	0.140541
9	0.156434	0.987688	0.158384
10	0.173648	0.984808	0.176327
11	0.190809	0.981627	0.19438
12	0.207912	0.978148	0.212556
13	0.224951	0.97437	0.230868
14	0.241922	0.97030	0.249328
15	0.258819	0.965926	0.267949
16	0.275637	0.961262	0.286745
17	0.292371	0.95630	0.30573
18	0.309017	0.951057	0.324919
19	0.325568	0.945519	0.344327

b. In a similar way, $\cos 37° \approx 0.798636$.

c. $\tan 70° \approx 2.747480$

Exercise 5.6

Find the value of each of the following using trigonometric table.

a. $\sin 23°$ b. $\cos 9°$ c. $\tan 40°$

d. $\sin 52^{\circ}$ e. $\cos 68^{\circ}$ f. $\tan 82^{\circ}$

Example 1

Given $\cos A = 0.819152$. Using the trigonometric table, find the angle A.

Solution:

On the column corresponding to 'cos', find the given number 0.819152. Corresponding to this number is 35, that is $A = 35°$. (see table 5.3 below).

Table 5.3

Move down till you get cosine value 0.819152

	sin	cos	tan		
0	0	1	0		90
1	0.017452	0.999848	0.017455	57.2900	89
2	0.03490	0.999391	0.034921	28.63628	88
3	0.052336	0.99863	0.052408	19.08115	87
4	0.069756	0.997564	0.069927	14.30068	86
5	0.087156	0.996195	0.087489	11.43006	85
6	0.104528	0.994522	0.10510	9.514373	84
7	0.121869	0.992546	0.122784	8.144353	83
8	0.139173	0.990268	0.140541	7.115376	82
9	0.156434	0.987688	0.158384	6.313757	81
10	0.173648	0.984808	0.176327	5.671287	80
11	0.190809	0.981627	0.19438	5.144558	79
12	0.207912	0.978148	0.212556	4.704634	78
13	0.224951	0.97437	0.230868	4.33148	77
14	0.241922	0.97030	0.249328	4.010784	76
15	0.258819	0.965926	0.267949	3.732054	75
16	0.275637	0.961262	0.286745	3.487418	74
17	0.292371	0.95630	0.30573	3.270856	73
18	0.309017	0.951057	0.324919	3.077686	72
19	0.325568	0.945519	0.344327	2.904214	71
20	0.34202	0.939693	0.36397	2.74748	70
21	0.358368	0.933581	0.383864	2.605091	69
22	0.374606	0.927184	0.404026	2.475089	68
23	0.390731	0.920505	0.424474	2.355855	67
24	0.406736	0.913546	0.445228	2.246039	66
25	0.422618	0.906308	0.466307	2.144509	65
26	0.438371	0.898794	0.487732	2.050306	64
27	0.45399	0.891007	0.509525	1.962612	63
28	0.469471	0.882948	0.531709	1.880728	62
29	0.484809	0.87462	0.554308	1.80405	61
30	0.5000	0.866026	0.57735	1.732053	60
31	0.515038	0.857168	0.60086	1.664281	59
32	0.529919	0.848048	0.624869	1.600336	58
33	0.544639	0.838671	0.649407	1.539867	57
34	0.559192	0.829038	0.674508	1.482563	56
35	0.573576	0.819152	0.700207	1.42815	55
36	0.587785	0.809017	0.726542	1.376383	54

Move to the left till to get the required angle

Example 2

Given $\sin A° = 0.7565$ find the measure of the acute angle A, correct to the nearest degree.

Solution:

In the column of 'sin', search the given number. If we get it directly, take the degree which corresponds to the number. Otherwise, take the approximation to the nearest one. In our case, the given number is not directly available on the table. Two values closest to the given value 0.7565 are 0.75471, the smaller one and 0.766045 the larger one. These values correspond to $49°$ and $50°$, respectively.

The given number is closest to 0.75471(since $|0.7565 - 0.75471| < |0.7565 - 0.766045|$), and it corresponds to $49°$. Therefore, $A = 49°$(to the nearest degree).

Exercise 5.7

1. Find the measure of an acute angle A, correct to the nearest degree if

a. $\sin A = 0.173648$ b. $\cos A = 0.961262$ c. $\tan A = 0.60086$

d. $\sin A = 0.798636$ e. $\cos A = 0.438371$ f. $\tan A = 2.355855$

g. $\sin A = 0.2300$ h. $\cos A = 0.9960$ i. $\tan A = 1.2000$

2. Find the angle between the diagonal and the base of a rectangle where the base is 8 cm and the height is 5 cm. (give your answer to the nearest degree)

Summary

1. For a right-angled triangle with hypotenuse h and other sides b and p, the Pythagoras theorem can be written as equation $b^2 + p^2 = h^2$.

2. Let ΔABC be a right-angled triangle with hypotenuse(H) and opposite side(O) and adjacent side (A), as shown in figure 5.11. Then, the trigonometric ratios are defined as

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AC}$$
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AB}{AC}$$
$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AB}$$

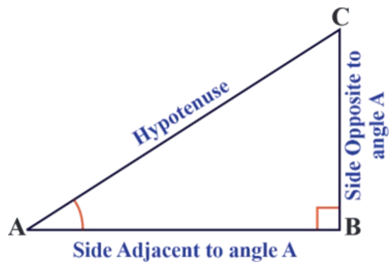

Figure 5.11

- For A and C as acute angle of the same right-angled triangle as shown in figure 5.11, $\sin A = \cos C$ and $\sin C = \cos A$.

3. The trigonometric values of basic angles $(0°, 30°, 45°, 60°$ and $90°$) are given as:

- ➢ $\sin 0° = 0, \sin 30° = \frac{1}{2}, \sin 45° = \frac{1}{\sqrt{2}}, \sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 90° = 1$
- ➢ $\cos 0° = 1, \cos 30° = \frac{\sqrt{3}}{2}, \sin 45° = \frac{1}{\sqrt{2}}, \cos 60° = \frac{1}{2}$ and $\cos 90° = 0$
- ➢ $\tan 0° = 0, \tan 30° = \frac{1}{\sqrt{3}}, \tan\ 45° = 1, \tan 60° = \sqrt{3}$ and $\tan 90°$ is undefined.

4. The following steps could be taken in order to read values of trigonometric ratio of sine, cosine and tangent from a trigonometric table.
   - Identify the trigonometric ratio from the first row if the degree you need is between 0° and 45° and the last row if the degree is between 45° and 90°.
   - Using the column of the trigonometric ratio you need to find and move down or up till you get the required degree.
   - The intersection of the row and column of the above two steps is the trigonometric value of the required degree.

Review Exercise

1. Which one of the following is not correct about a right-angled triangle?

   A. The angle opposite to the hypotenuse is a right angle

   B. .If the base, height and hypotenuse of a right-angled triangle has lengths b, p and h units,respectively, then $b^2 + p^2 = h^2$.

   C. For an isosceles right-angled triangle, two sides of a triangle are equal in length.

   D. If θ is one of the angles of a right-angled triangle, then $\cos\theta$ can be greater than 1.

2. In ΔABC, right-angled at B, $AB = 24$ cm , $BC = 7$cm. Determine

   a. $\sin A$, $\cos A$

   b. $\sin C$, $\cos C$

3. State whether the following are true or false. Justify your answer.

   a. The value of $\tan A$ is always less than 1.

   b. $\sin\theta = \frac{4}{3}$, for some acute angle θ.

   c. If $\sin\theta = \frac{1}{3}$, then $\cos\theta = \frac{2\sqrt{2}}{3}$.

   d. When $0° \leq \theta \leq 90°$ is an angle of a right-angled triangle, both $\sin\theta$ and $\cos\theta$ are between 0 and 1.

4. Given $0° < \theta < 90°$. If $\tan\theta = 1$, then which one of the following is not true?

   A. $\cos\theta = \frac{\sqrt{2}}{2}$ B. $\cos\theta = \sin\theta$

   C. $\theta = 45°$ D. $\tan\theta = \sin\theta$

5. If $\sin\theta = \frac{2}{3}$ for an acute angle θ, then which one of the following is correct?

   A. $\cos\theta = \frac{\sqrt{5}}{2}$ B. $\tan\theta = \frac{2}{\sqrt{5}}$

C. $\cos(90° - \theta) = \frac{3}{2}$ D. $\sin\theta > \cos\theta$

6. Find the value of each of the following using trigonometric table.
   a. $\sin 18°$
   b. $\cos 57°$
   c. $\tan 71°$

7. Find the measure of an acute angle A, correct to the nearest degree if $\tan A = 12.10$.

8. The angle formed by the top of the building at a distance of 50 m from its foot on horizontal plane is found to be 60 degree. Find the height of the building. (See figure 5.12)

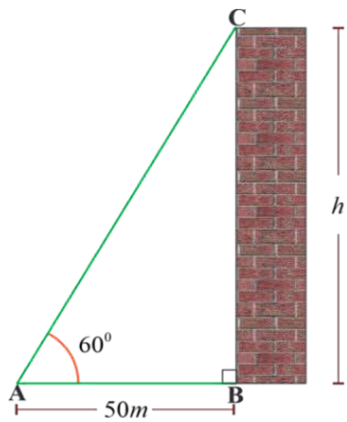

Figure 5.12

9. A building with its height of 60 m above sea level is located at Bahir Dar city near to Lake Tana. The angle between the line segment from the sailing boat to the top of the building and surface of the water is 25° as shown in figure 5.13.
   a. How far is the boat from the base of the building to the nearest meter?
   b. What is the distance between the man at the top of the building and the boat? (Use fig 5.13)

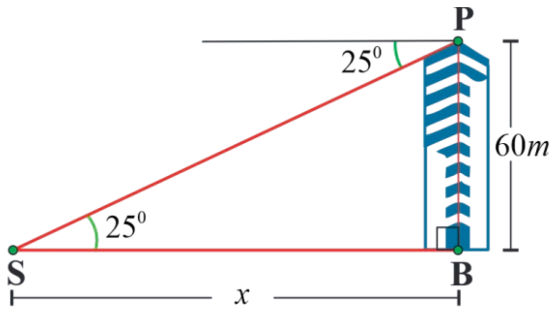

Figure 5.13

10. Two women A and B lie on the leveled ground at opposite sides of 150 m tall tower. If A observes the top of the tower at an angle of 60° and B observes the same point at an angle of 30°, then how far the two women can be away from each other?

A. $100\sqrt{3}$ m **B.** $600\sqrt{3}$ m

C. $200\sqrt{3}$ m **D.** $450\sqrt{3}$ m

11. In figure 5.14, if the height of the man is 1.74 m, $\theta = 63°$ and he is located at 100 m away from the building, find the height of the building.

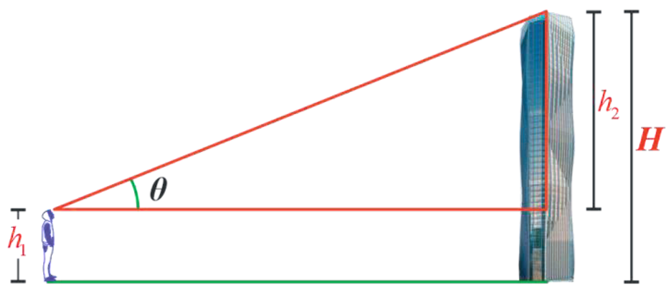

Figure 5.14