UNIT 7

CONGRUENCY AND SIMILARITY

Unit Outcomes

By the end of this unit, you will be able to:

- **Distinguish between congruent and similar plane figures.**
- **Apply postulates and theorems in order to prove congruency and similarity of triangles.**
- **Solve problems on perimeters and areas of similar triangles and similar polygons.**
- **State and use the criteria for similarity of triangles viz. AAA, SSS and SAS.**
- **Prove the Pythagoras Theorem.**
- **Apply these results in verifying experimentally (or proving logically) problems based on similar triangles.**

Unit Contents

KEY Words

- Ratio of area of similar plan figure
- Ratio of perimeter of similar plan figure
- Congruency
- Construction of similar plane figures
- SAS similarity
- Similarity
- Plane figures
- SSS similarity
- Proportionality
- triangles
- AA similarity
- Application of similarity

Introduction

There are maps of different countries in the world which are smaller or larger in their size or same shape or not. If you ask yourself whether a country's different maps are like or unlike, equal in size or not, you may answer it by looking at the maps. In geometry, the concept of equality in size and shape is described as congruency of triangles and likeness is called "similarity of figures". In this unit, you will learn both congruency and similarity of triangles.

7.1 Revision on Congruency of Triangles

Activity 7.1

a. What can you say about the shape and size of congruency of two triangles?

b. Can you mention some conditions of congruency of triangles?

c. For ΔABC & ΔDEF, $\overline{AB} \equiv \overline{DE}$ and $\angle B$ has the same measure as $\angle E$. .
Which of the following statements, if true, is sufficient to show that the two triangles are congruent?

i. $\overline{AB} \equiv \overline{DE}$

ii. $\angle A \equiv \angle D$

iii. $\angle C \equiv \angle F$

What is "congruency"? It means that one shape can become another using turns, flips and/or sliders. The equal sides and angles may not be on the same position (if there is a turn or a flip), but they are there. Congruent means to be identical in size and shape.

Definition 7.1

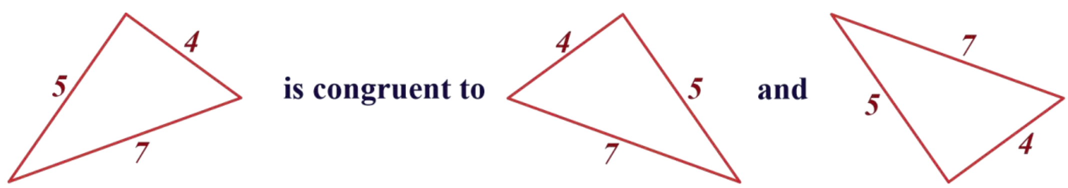
When two triangles have exactly the same three sides and the same three angles they are called congruent triangles.

Note

Same three sides means the corresponding sides have equal length and same three angles means the corresponding angles have equal measure.

Note that $\triangle ABC \equiv \triangle DEF$ means $\overline{AB} \equiv \overline{DE}$, $\overline{BC} \equiv \overline{EF}$, $\overline{AB} \equiv \overline{DE}$, $\overline{AC} \equiv \overline{DF}$ and $\angle A \equiv \angle D$, $\angle B \equiv \angle E$, $\angle C \equiv \angle F$

For example:



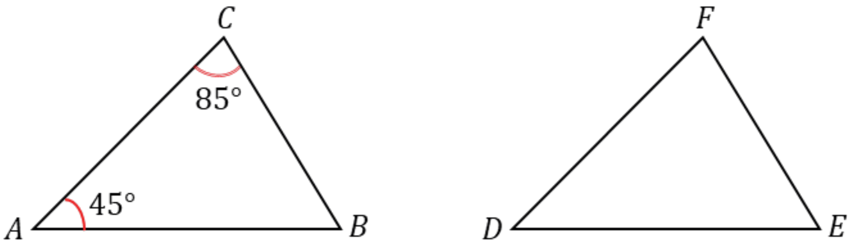
The above three triangles are congruent.

Exercise 7.1

a) If two triangles are congruent, then they have exactly same shape and size (True or False).

b) Any two equilateral triangles are congruent (True or False).

c) In the figure below, $\triangle ABC \equiv \triangle DEF$, $\angle A = 45°$, $\angle C = 85°$. Find the measures of $\angle D$ and $\angle E$.



1. SSS congruency

Side-Side-Side congruency: When two triangles have equal corresponding sides, then the triangles are congruent.

2. SAS congruency

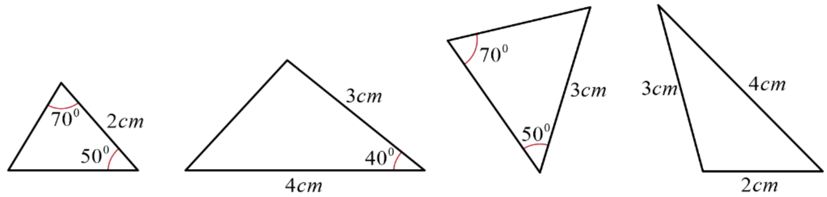
Side-Angle-Side congruency: When two triangles have two pair of congruent sides and one pair of congruent angles between the sides, then the triangles are congruent.

3. ASA congruency

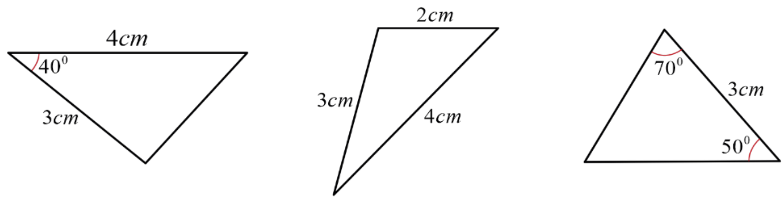
Angle-Side-Angle congruency: When two triangles have two pair of congruent angles and one pair of congruent sides between the angles, then the triangles are congruent.

Exercise 7.2

1. Identify which triangles are congruent? Why?

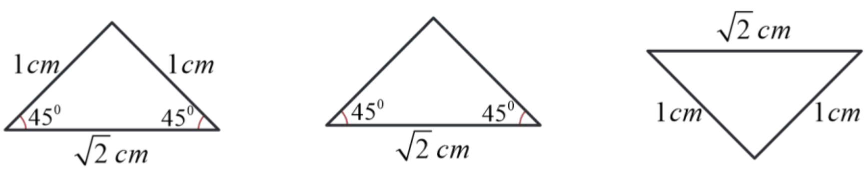


a b c d



e f g

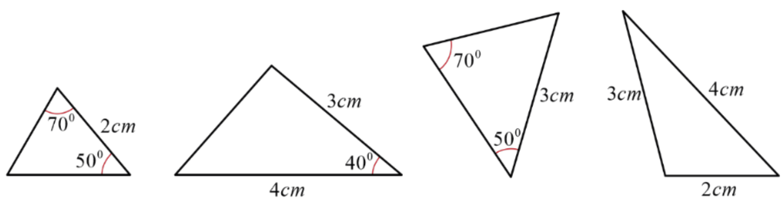
2. Which pair of triangles are congruent? Why?



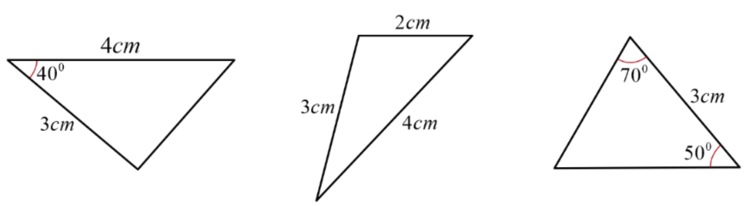
A B C

Exercise 7.3

Which of the following triangles are congruent by ASA condition?



a b c d



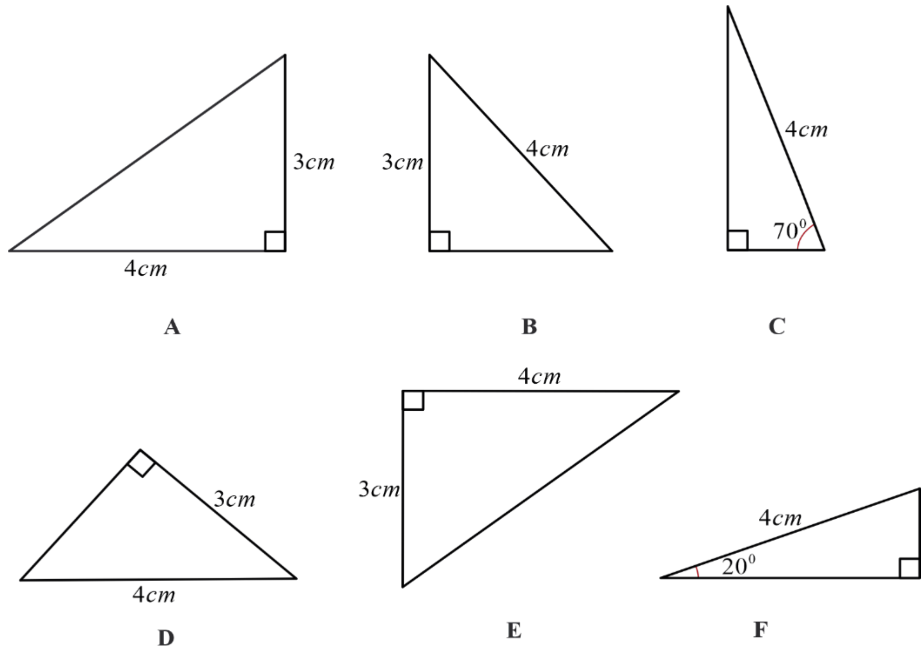
e f g

4. RHS Congruency

Right-angle hypotenuse side congruency: When two right triangles have equal hypotenuse and equal one side (leg), then the triangles are congruent.

Exercise 7.4

a) Identify the triangles into pairs of congruent figures.



b) Among the following statements, which the following conditions from (i) to (viii) will represent for ΔABC and ΔPQR to be congruent?

i. $AB \equiv PQ, BC \equiv QR, AC \equiv PR$

ii. $BC \equiv QR, \angle B \equiv \angle Q, \angle C \equiv \angle R$

iii. $AB \equiv PQ, BC \equiv QR, \angle C \equiv \angle P$

iv. $AB \equiv PQ, BC \equiv QR, \angle B \equiv \angle Q$

v. $AB \equiv PQ, \angle A \equiv \angle P, \angle C \equiv \angle R$

vi. $AB \equiv PQ, BC \equiv QR$

vii. $\angle B \equiv \angle Q, \angle C \equiv \angle R$

7.2 Definition of Similar Figures

Activity 7.2

1. If two plane figures have the same number of sides, what can you say about the plane figures?
2. If two plane figures have the same area, what can you say about similarity and congruency the plane figures?

Definition 7.2

Two plane figures are similar if their corresponding angles are congruent and the ratios of their corresponding sides are proportional. This common ratio is called the scale factor.

Two plane figures are similar means that

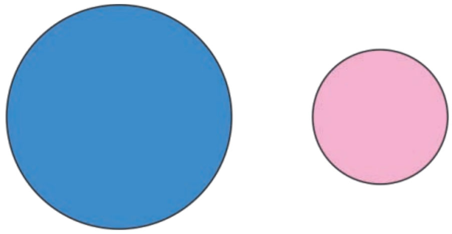
- All corresponding angles are congruent
- All corresponding sides are proportional

When we magnify or diminish similar figures, one exactly places on the other and vice versa.

1. If the two triangles, the two rectangles and the two pentagons below satisfy the above properties then they are similar to each other.

   Note that if ΔABC and ΔDEF are similar, then we write mathematically as $\Delta ABC \sim \Delta DEF$.

2. Any two circles (of any radii) have the same shape and hence they are always similar.



Similarly, two squares (of any side lengths) are similar.

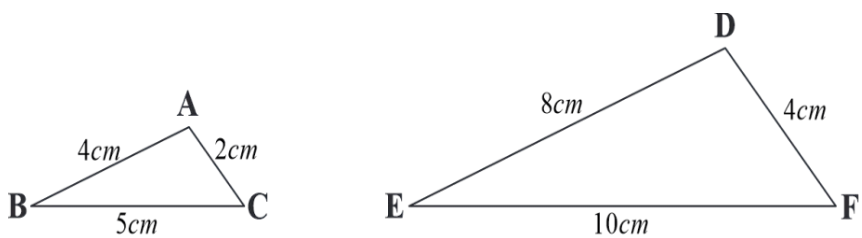
Exercise 7.5

Any two regular polygons are similar (True or False). Why?

Example 1

Similar triangles and quadrilaterals:

Given $\Delta ABC \sim \Delta\, DEF$, find the common ratio of their corresponding sides.



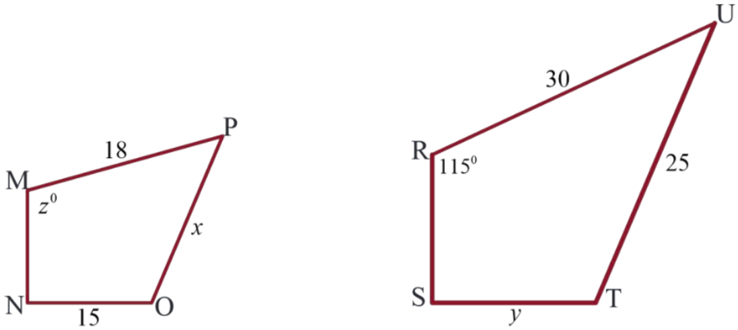
Solution:

$$\frac{DE}{AB} = \frac{8}{4} = 2\ , \quad \frac{EF}{BC} = \frac{10}{5} = 2, \quad \frac{FD}{CA} = \frac{4}{2} = 2,$$

Thus, the common ratio is 2.

Example 2

In the following quadrilaterals, assume $PMNO \sim URST$. Find the unknowns x, y, z



In the similarity statement, $\angle \mathrm{M} = \angle \mathrm{R}$, So, $z^0 = 115^0$.

To find the unknown lengths x and y set up proportions of the corresponding sides as

$$\frac{\mathrm{PM}}{\mathrm{UR}} = \frac{\mathrm{PO}}{\mathrm{UT}}$$

$$\frac{18}{30} = \frac{x}{25}$$

$$x = 15$$

Similarly, to find y,

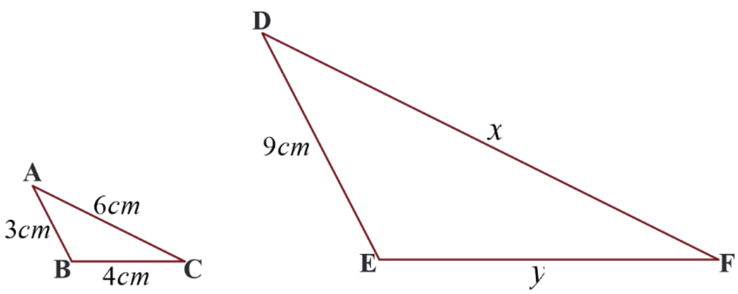
$$\frac{\mathrm{PO}}{\mathrm{UT}} = \frac{\mathrm{NO}}{\mathrm{ST}}$$

$$\frac{15}{25} = \frac{15}{y}$$

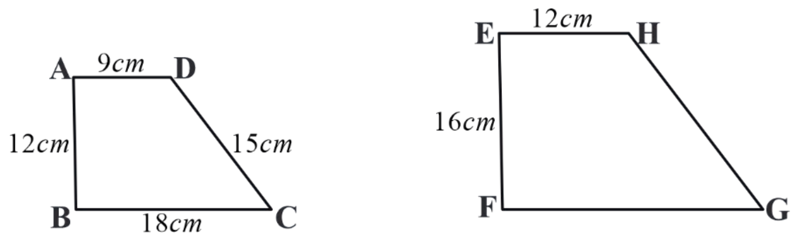
$$y = 25$$

Exercise 7.6

a) Given $\Delta ABC \sim \Delta DEF$. Find x and y.

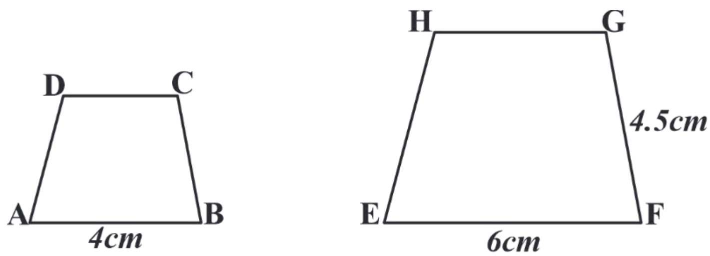


b) Figure $ABCD$ is similar to figure $EFGH$. Find the perimeter of $EFGH$.



c) Given $ABCD \sim EFGH$, find the common ratio of their corresponding sides.

Find BC



Exercise 7.7

For the following questions, determine whether each of the statement is true or false.

a. All equilateral triangles are similar.

b. All isosceles triangles are similar.

c. All isosceles right triangles are similar.

d. All rectangles are similar.

e. All rhombuses are similar.

f. All squares are similar.

g. All congruent polygons are similar.

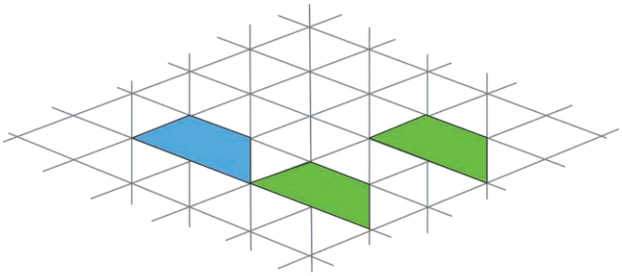
h. All similar polygons are congruent.

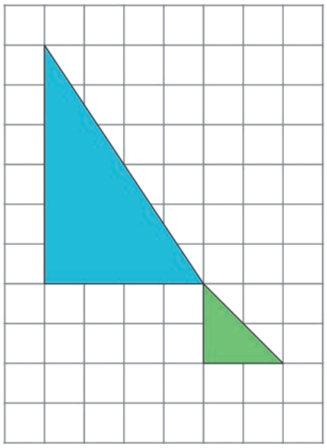
i. All regular pentagons are similar.

7.3 Theorems on similar plane figures

Activity 7.3

Determine whether the following figures are similar or not.





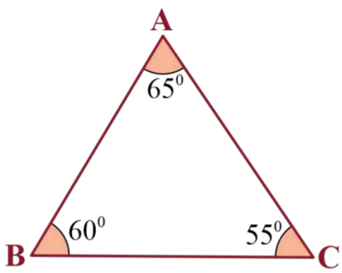
Simply as two people can look at a painting and notice or sense differently the piece of the art work, there is always more than one way to create a proper proportion given similar triangles. These theorems are very useful in such a way that we can conclude the similarity of a pair of triangles by showing only three pair of sides or /and angles among the six pairs of congruent angles and sides.

For example, we can say: The three theorems: Angle - Angle (AA), Side - Side - Side (SSS) and Side - Angle - Side (SAS) are important for determining similarity in triangles.

7.3.1 AA similarity Theorem

The AA similarity theorem for triangles states that if the two angles of one triangle are respectively congruent to the two angles of the other, then the triangles are similar. In short, equiangular triangles are similar. Ideally, the name of this criterion should then be the AAA (Angle-Angle-Angle) similarity theorem, but we call it as AA similarity theorem because we need only two pairs of angles to be equal - the third pair will then automatically be equal by the angle sum property of triangles. The notation ~ is used to denote similarity.

Consider the following figures, in which ΔABC and ΔDEF are equiangular.

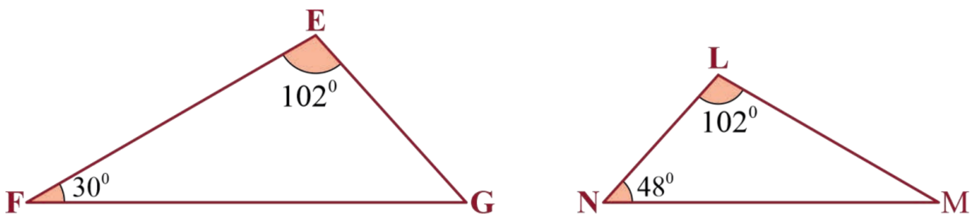


Using the *AA* similarity theorem, we can say that these triangles are similar.

Mathematically, it is written as $\Delta ABC \sim \Delta DEF$.

Example

Determine whether or not the following two triangles are similar.

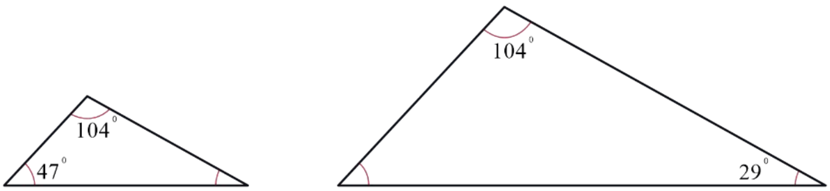


Solution:

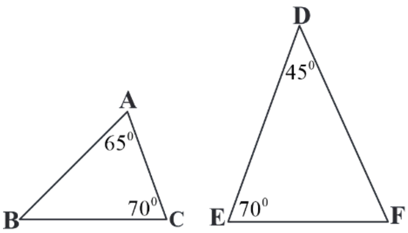
Compare the angles to see if we can use the AA similarity theorem. Using triangle angle sum theorem, $m(\angle G) = 48^o$ & $m(\angle M) = 30^o$. Hence, $\Delta EFG \sim \Delta LMN$ by AA similarity theorem.

Exercise 7.8

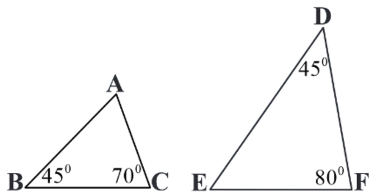
Are the following pairs of triangles similar or not by AA similarity theorem or not? Why?



a



b



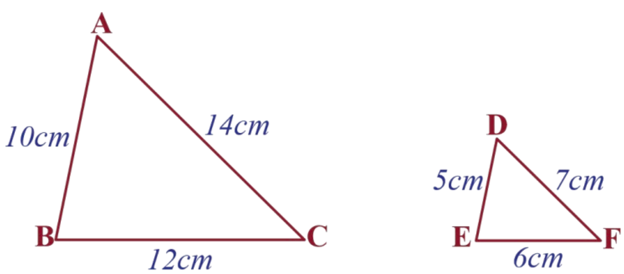
c

7.3.2 SSS similarity Theorem

If the corresponding three sides of two triangles are proportional to each other, then the triangles are similar. This essentially means that any such pair of triangles are equiangular (all corresponding angle pairs are congruent).

Example 1

Determine whether the following pairs of triangles are similar by SSS similarity theorem.



Solution:

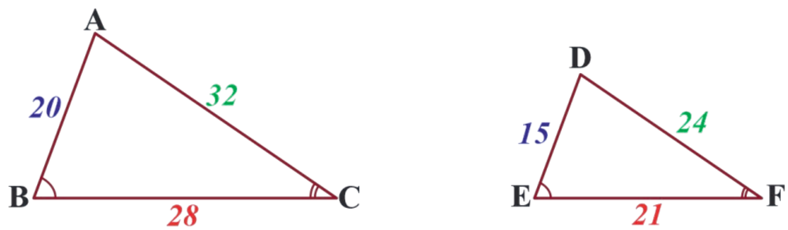
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 2.$$

Hence, $\Delta ABC \sim \Delta DEF$ by SSS similarity theorem.

Example 2

When the corresponding sides of two triangles are proportional, the triangles are similar. What are the corresponding sides? Using the triangles below, we see how the sides are line up in the diagram.

Solution:

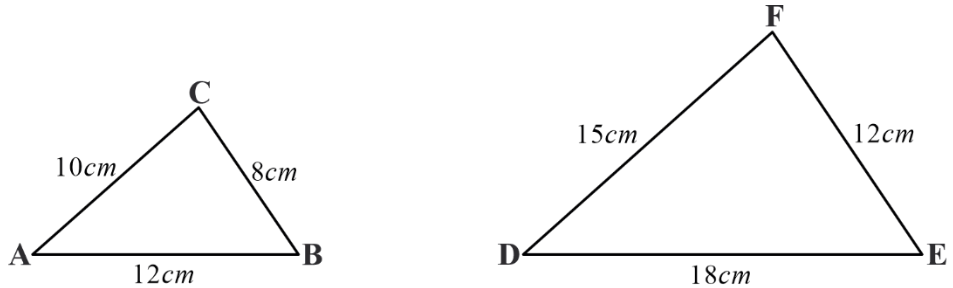


Since $\frac{28}{21} = \frac{32}{24} = \frac{20}{15} = \frac{4}{3}$ the same color are the corresponding sides.

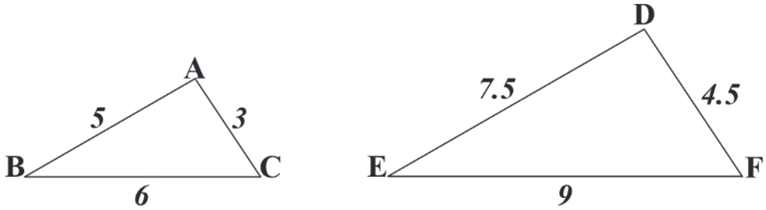
Therefore, the triangles are similar by SSS similarity theorem.

Exercise 7.9

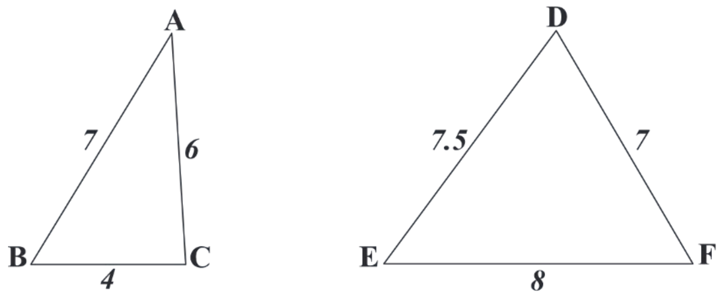
a) Determine whether the following pairs of triangles are similar by SSS similarity theorem or not.



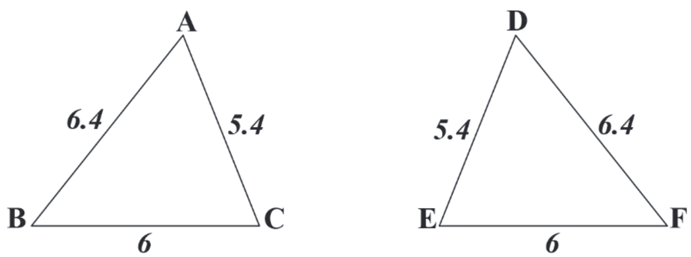
b) Determine whether the following pairs of triangles are similar.



(1)



(2)

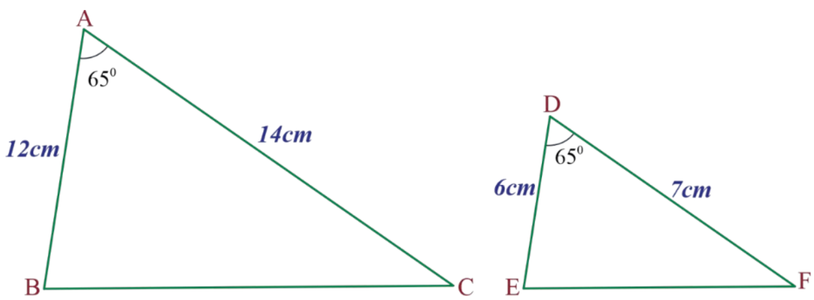


(3)

7.3.3 SAS similarity theorem

If the corresponding two sides of two triangles are proportional and the included angles are congruent, then the two triangles are similar.

Note the emphasis on the word included. If the congruent angles are non-included angles, then the two triangles may not be similar. Consider the following figures.



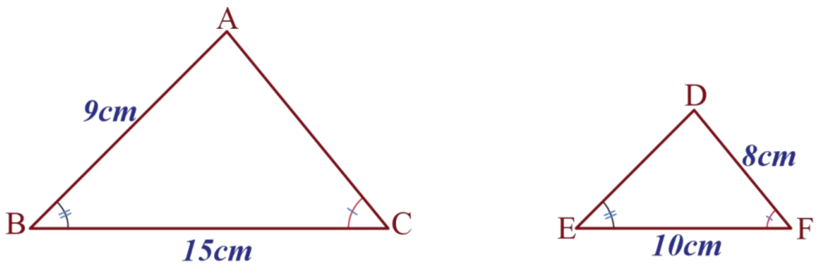
$\frac{AB}{DE} = \frac{12}{6} = 2$, $\frac{AC}{DF} = \frac{14}{7} = 2$, then the two corresponding sides are in the same ratio and $\angle A \equiv \angle D$

Hence, $\Delta ABC \sim \Delta DEF$ by SAS similarity theorem.

Example 1

Assume $\Delta ABC \sim \Delta DEF$

Find the length of the remaining sides of the triangles.



Solution:

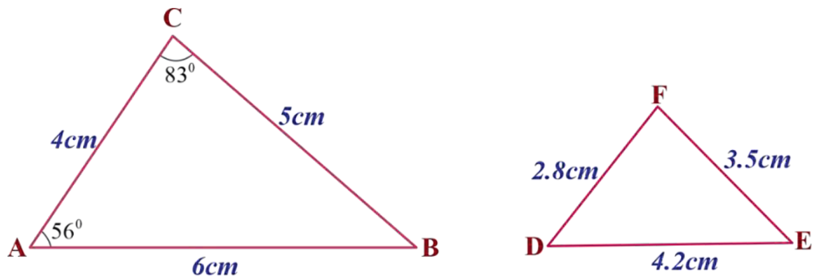
Since the triangles are similar, we have relation

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}.$$

Here, $\frac{9}{DE} = \frac{15}{10}$. This implies $DE = 6cm$. Also, $\frac{9}{6} = \frac{AC}{8cm}$, gives $AC = 12cm$.

Example 2

Consider the following figures and find the value of $\angle E$.



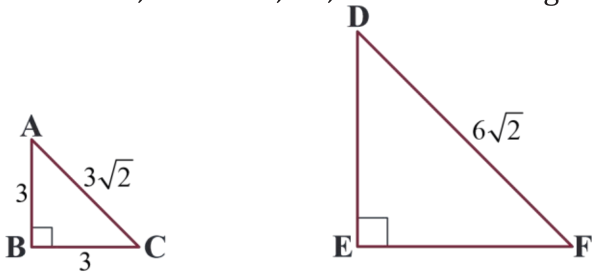
Solution:

Match the longest side with the longest side and the shortest side with the shortest side and check all the three ratios. We note that the three sides of the two triangles are respectively proportional, that is $\frac{6}{4.2} = \frac{5}{3.5} = \frac{4}{2.8} = \frac{10}{7}$. Therefore, $\Delta ABC \sim \Delta DEF$

by SSS similarity theorem. Hence, the corresponding angles are congruent and hence $\angle E = \angle B = 180^0 - (56^0 + 83^0) = 41^0$.

Example 3

Given $\Delta ABC \sim \Delta DEF$, Find $\angle D, \angle E, \angle F$ and the lengths of DE .



Solution:

As $\Delta ABC \sim \Delta DEF$, $\angle D = \angle A = 45°$ since the triangles are isosceles triangles.

$\angle E = \angle B = 90°$,

$\angle F = \angle C = 45°$,

$$\frac{DE}{AB} = \frac{DF}{AC}$$

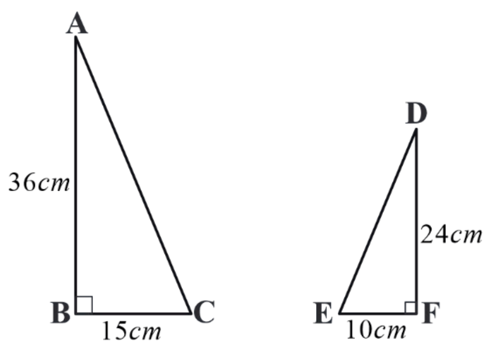
$$\frac{DE}{3} = \frac{6\sqrt{2}}{3\sqrt{2}}$$

$$\frac{DE}{3} = 2$$

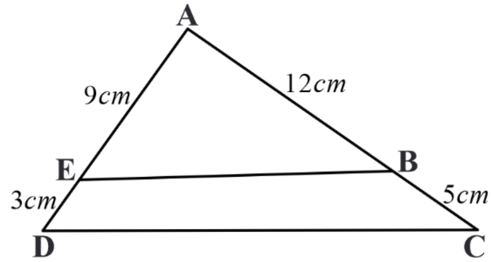
$$\text{DE} = 6$$

Exercise 7.10 a)

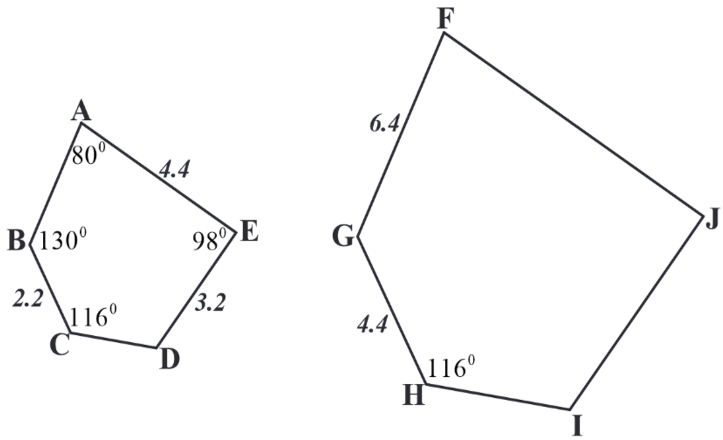
a. Are these two triangles similar? Why?



b. Are there similar triangles in the figure below? Why?



c. Given ABCDE ~FGHIJ, Find ∠D, ∠F, ∠J and ∠I and the lengths of IJ and JF.



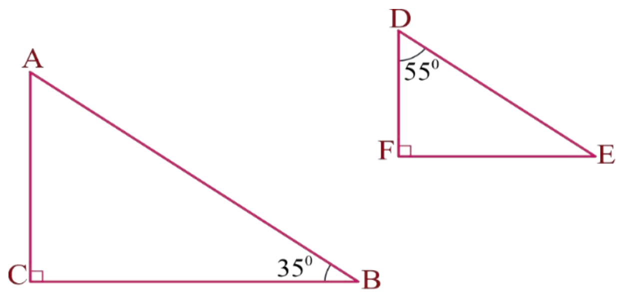
Exercise 7.10 b)

1. Answer the following questions referring the triangles below:

   Given $AB = 20$ and $DE = 15$.

   i. Are these triangles similar?

   ii. Find BC if $FE = 12$.



2. What do we mean when we say two triangles are congruent? Two triangles are similar? What is the difference between congruency and similarity?
3. If $\Delta BIG \sim \Delta HAT$, then

a. list the congruent angles and proportions of the sides.

b. If $BI = 9$ and $HA = 15$, find the scale factor.

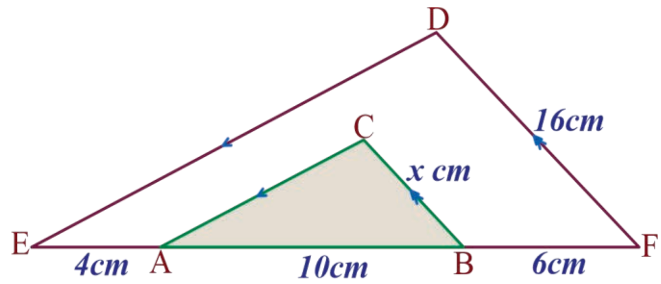
c. If $BG = 21$, find HT. (Use the scale factor above)

d. If $AT = 45$, find IG.

4. As shown in the figure below, DE is parallel to CA and DF is parallel to CB. Answer the following questions:

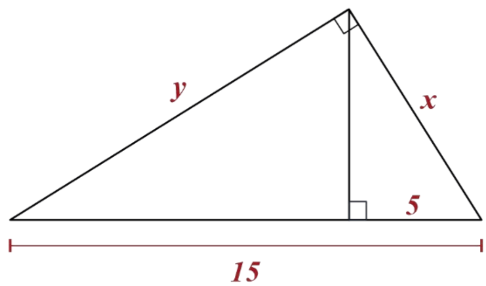
a. Show, $\Delta ABC \sim \Delta EFD$

b. Find the value of x.

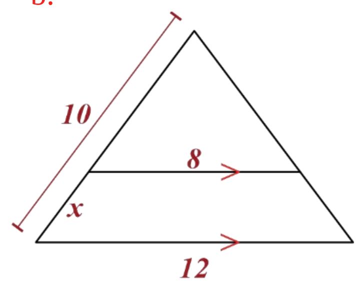


5. Find the unknown numbers *x*, *y* and *z* of the following figures.

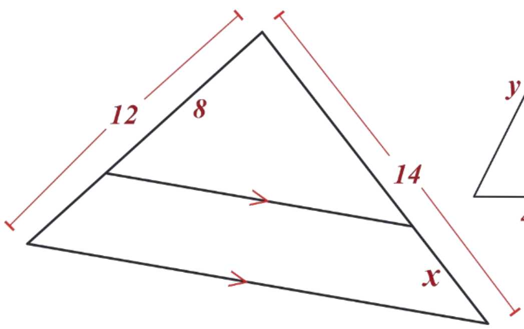
a.



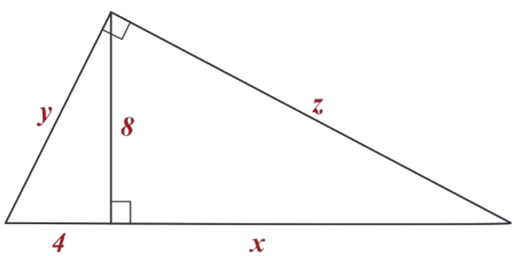
b.



c.



d.

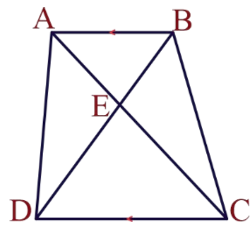


6. Suppose $\overline{AB}$ and $\overline{DC}$ are parallele in the figure below. Use congruency of alternate interior angles.

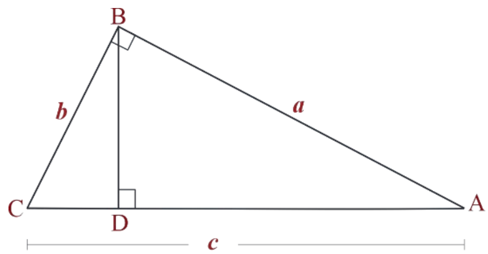
a. Mention two similar triangles. How you check they are similar?

b. Can you mention triangles which are not similar?

c. If $AB = 10, AE = 7$ and $DC = 22,$ then find AC.



7. Prove Pythagoras theorem using similarity of triangles. (Hint: Consider a right angle triangle, ΔABC and draw segment BD perpendicular to AC. The three triangles are similar.)



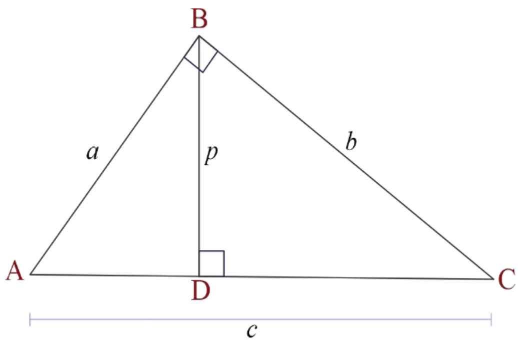
8. ABC is a right angle triangle, right angled at B. If BD, the length of the perpendicular from B on AC, is p,

$BC = b, AC = c$ and $AB = a$

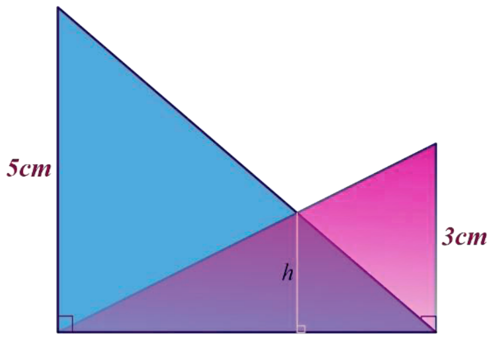
($the\ figure\ below$), show that:

i. $pc = ab$

ii. $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



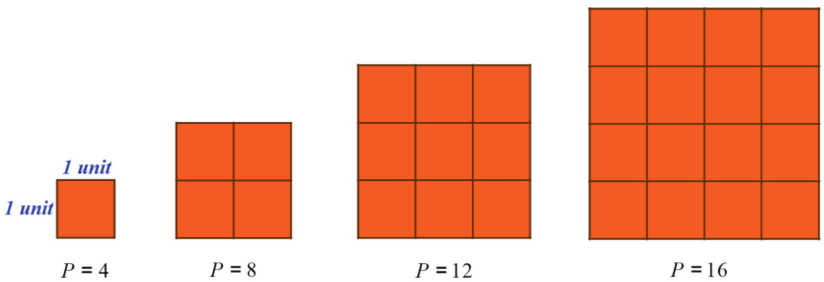
9. Find the perpendicular distance from the point of intersection E of the hypotenuses of the right angled triangles to their common base. Is there any other method to solve this problem? What can you conclude?

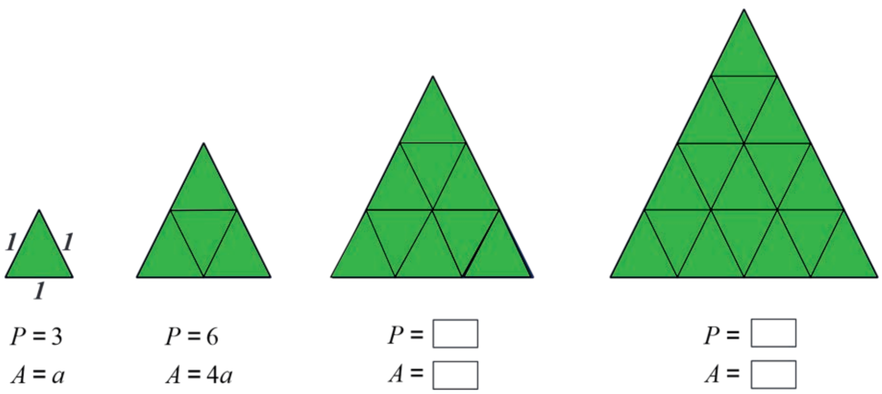


7.4 Ratio of perimeters of similar plane figures

Activity 7.4

1. What is ratio?
2. What is a scale factor?
3. Use pattern blocks to make a figure whose dimensions are 1, 2, 3 and 4 times larger than the original figure. Find the perimeter P of each larger figure.





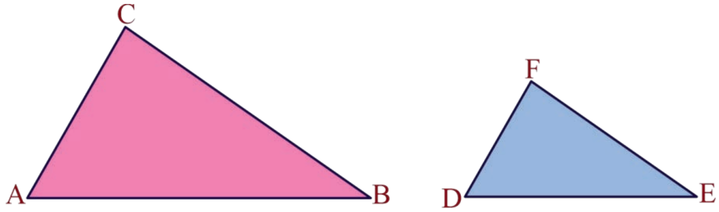
4. How do changes in dimensions of similar figures affect the perimeters of the figures? Is this always so? What can you generalize?

Definition 7.3

If two figures are similar, then the ratio of their perimeters is equal to ratio of their corresponding side lengths.

Example 1

Assume that $\Delta ABC \sim \Delta DEF$ and the ratio of the lengths of their sides is 2. Then, find the ratio of the corresponding perimeters. The perimeter of ΔABC is $6 + 8 + 10$ and the perimeter of ΔDEF is $3 + 4 + 5$.



Solution:

The perimeter of $\Delta ABC = 6 + 8 + 10 = 24$ and

The perimeter of $\Delta DEF = 3 + 4 + 5 = 12$.

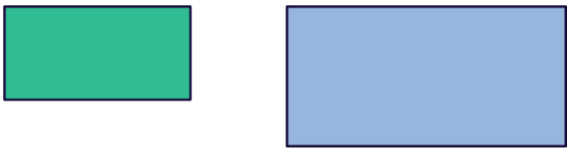
$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{24}{12} = 2$ is equal to the ratio of the corresponding sides

$\left(\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}\right)$

Note: $\frac{AB}{DE}$ is the same as AB: DE. Both refer to the ratio of AB to DE.

Example 2

Find the ratio (purple to blue) of the perimeters of the similar rectangles below if the lengths of the rectangles are 4cm and 6cm for purple and blue, respectively.



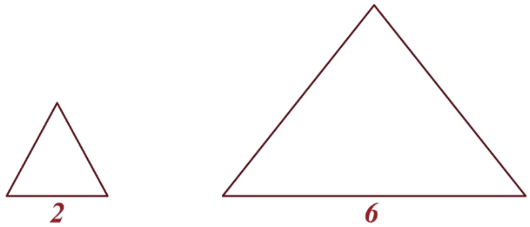
Solution:

$$\frac{\text{Perimeter of the Purple rectangle}}{\text{Perimeter of the Blue rectangle}} = \frac{4\text{cm}}{6\text{cm}} = \frac{2}{3}$$

Exercise 7.11

Use the formula $\frac{p_1}{p_2} = \frac{s_1}{s_2}$, for two similar plane figures where p_1 and p_2 are the perimeters and s_1 and s_2 are the corresponding sides of the figures

1. Find the ratio of the perimeter of the similar triangles below.



2. The ratio of the perimeters of two similar triangles is $\frac{3}{5}$.

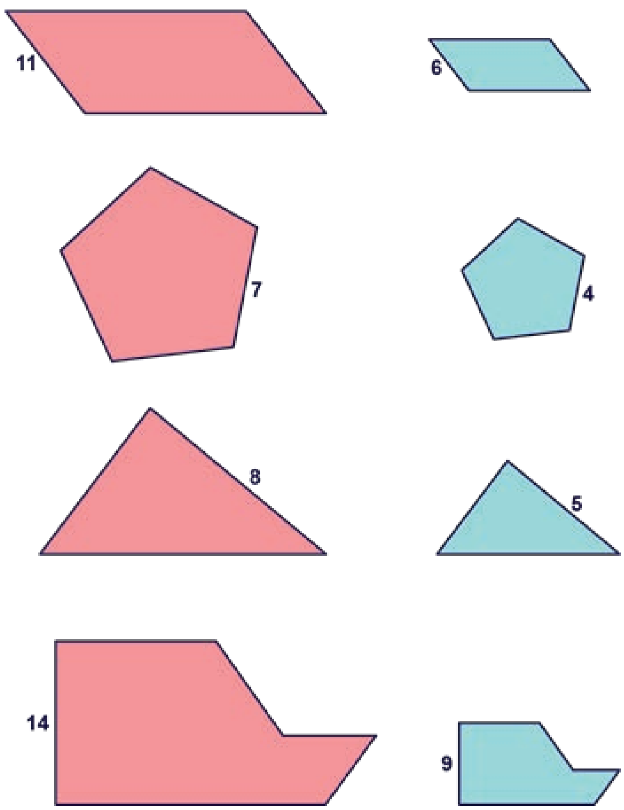
   The scale factor is _________

3. Two circles have radii 8 cm and 12 cm, respectively. The ratio of their circumferences is _______

4. How does doubling the side lengths of a rectangle affect its perimeter?

5. How does quadrupling the side lengths of a rectangle affect its perimeter?

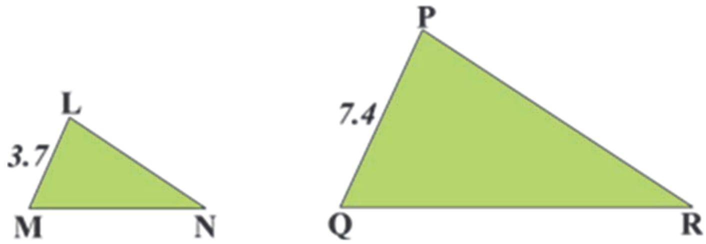
6. The pairs of figures below are similar. Find the ratio of their perimeters (purple to blue)



7.5 Ratio of areas of similar plane figures

Activity 7.5

1. Repeat activity 7.4 question number 3 and 4 for comparing areas of similar plane figures.
2. The two triangles below are similar. Drag any orange dot (vertex) at P, Q, R. The areas of the triangles are 9.2 and 36.8. Verify that the square of the ratio of the two corresponding sides is equal to the ratio of the areas.

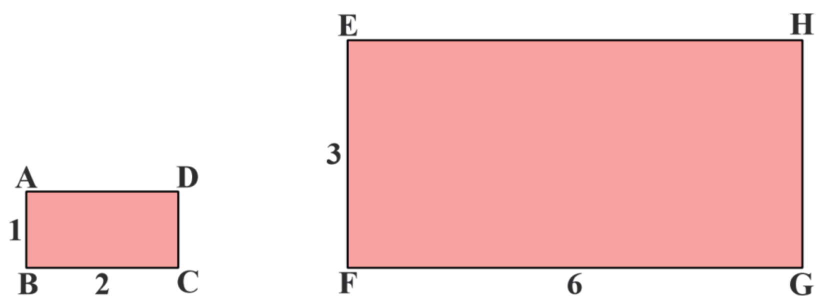


Definition 7.4

If two plane figures are similar, then the ratio of their areas is equal to the square of the ratio of their corresponding side lengths.

Example 1

Consider a pair of similar rectangles 1cm by 2cm and 3cm by 6cm below. The ratio of the corresponding sides is 3. Find the ratio of their areas.



Area of the smaller rectangle $= 1 \times 2 = 2cm^2$

Area of the larger rectangle $= 3 \times 6 = 18cm^2$

The ratio of area $= \frac{Area\ of\ the\ larger\ rectangle}{Area\ of\ the\ smaller\ rectangle}$

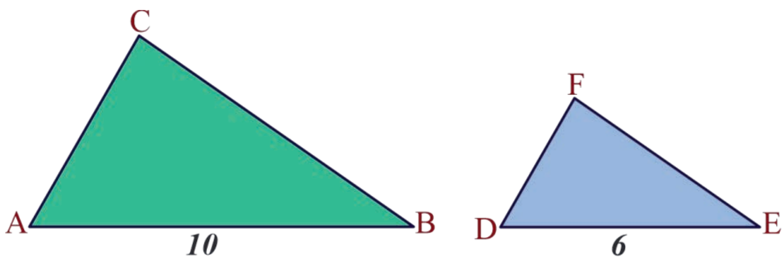
$= \frac{18}{2}$

$= 9 = 3^2 =$ (the ratio of the length of thier corresponding side)^2

In other words, since the ratio of length $= 3:1$, then ratio of area $= 3^2:1^2 = 9:1$

Example 2

If ΔABC and ΔDEF are similar as indicated below. Find the ratio of the area of the larger triangle to the smaller triangle.



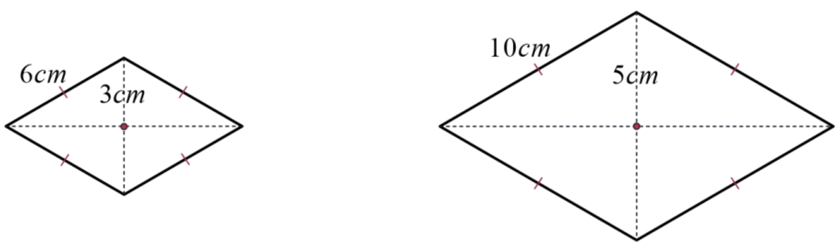
Solution:

The ratio of area $= \frac{\text{Area of the larger triangle}}{\text{Area of the smaller triangle}} = \left(\frac{10}{6}\right)^2 = \frac{25}{9}$.

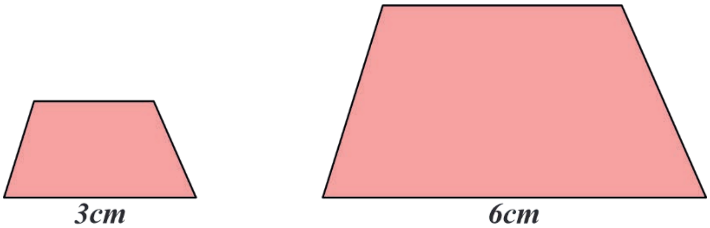
Therefore, the ratio of the areas is $\frac{25}{9}$

Exercise 7.12

a. Find the ratio of the areas of rhombi below. The rhombi are similar.



b. Find the ratio of areas of similar quadrilaterals below.



c. The ratio of the areas of two squares is $64:49$. The scale factor is ____________ and the ratio of their perimeters is_____________.

d. Two circles have radii 8 cm and 12 cm, respectively. The ratio of their circumferences is ____________ and the ratio of their areas is ____________.
(Hint: All circles are similar. The scale factor is the ratio of their radii.)

e. The ratio of the areas of two similar rectangles is $25:36$. The scale factor is ____________ and the ratio of their perimeters is ____________.

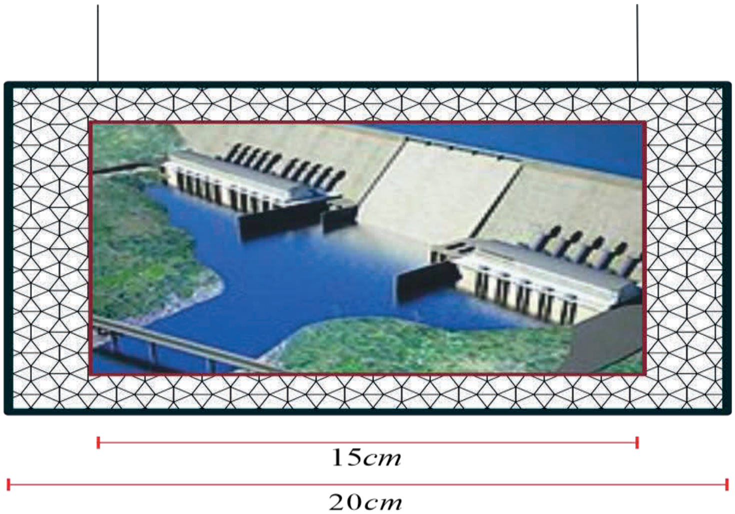
f. Two similar triangles have corresponding sides 3 and 9. If the area of the first triangle is 12, what is the area of the second triangle?

g. Two similar triangles have areas 144 cm^2 and 169 cm^2. If a side of the larger triangle is 26cm, find the length of the corresponding side of the smaller triangle.

h. How does doubling the side lengths of a rectangle affect its area?

i. How does quadrupling the side lengths of a rectangle affect its area?

j. You place a picture on a page. The page and the picture are similar rectangles.



1. How many times greater is the area of the page than the area of the picture?
2. If the area of the picture is 45 cm^2, what is the area of the page?

k. A triangle with area of $10m^2$ has a base of $4m$. A similar triangle has an area of $90m^2$. What is the height of the larger triangle?

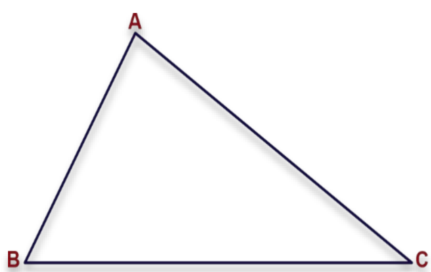
7.6 Construction of similar figures

Activity 7.6

1. What is a scale factor?
2. Given a line segment, how do you draw a line segment parallel to the given lines segment?
3. What does it mean that the scale factor is greater than 1? Less than1? Equal to 1?

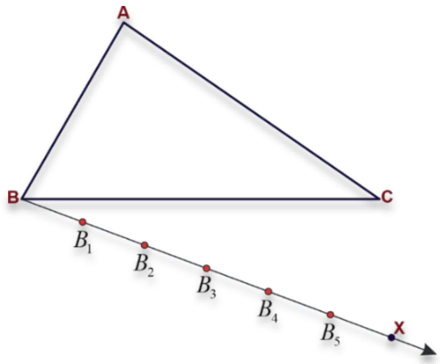
The construction of a similar triangle involves two different cases. On one hand, the triangle to be constructed could be bigger (or larger), and on the other hand, the triangle may be smaller than the given triangle. Also, the scale factor determines the ratio of the sides of the triangle to be constructed with the corresponding sides of the given triangle.

Example: construct a triangle similar to the given triangle ABC for example, with **scale factor $\frac{5}{3}$.**

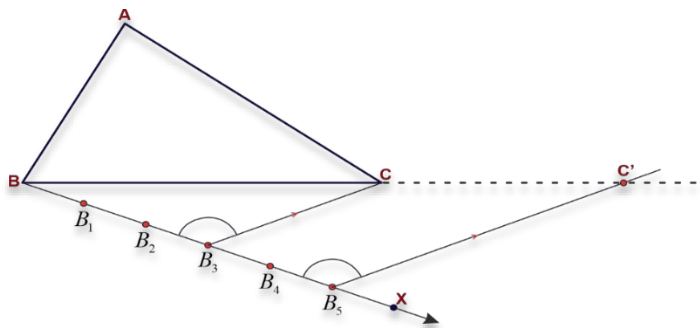


Steps of construction:

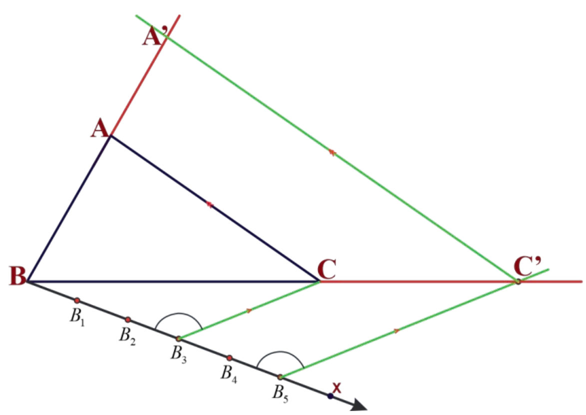
Step 1: Draw a ray BX making an acute, acute with the base BC and mark 5 points B_1, B_2, B_3, B_4, B_5 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$.



Step 2: Join B_3C and draw a line B_5C' such that B_3C is parallel to B_5C' , where C' lie on the produced BC.



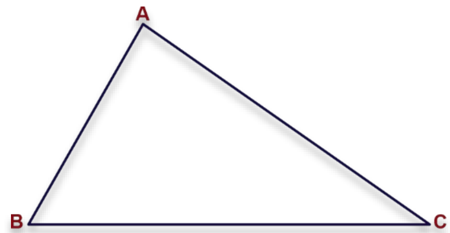
Step 3: Now draw another line parallel to AC at C' such that it meets the produced BA at A'.



Hence, $\Delta A'B'C'$ is the required triangle similar to the ΔABC.

Exercise 7.13

Construct a triangle similar to the given triangle ABC, with scale factor 1, $\frac{1}{2}$ and $\frac{2}{3}$.



7.7 Applications of similarity

Activity 7.7

Given a ladder $5m$ long lying on a wall. If the height of the wall to the top of the ladder is 3m. How far is the bottom of the ladder from the wall?

Similar triangles are used to work out the heights of tall objects such as trees, buildings and towers without climbing, which are difficult to measure for us.

Example 1

Almaz is 1.6 meters tall and is standing outside next to her younger brother. She notices that she can see both of their shadows and decides to measure each shadow. Her shadow is 2 meters long and her brother's shadow is 1.5 meters long. How tall is Almaz's brother?

Solution:

Let x be the height of the younger brother. Using proportionality of corresponding heights and length of shadows, we have

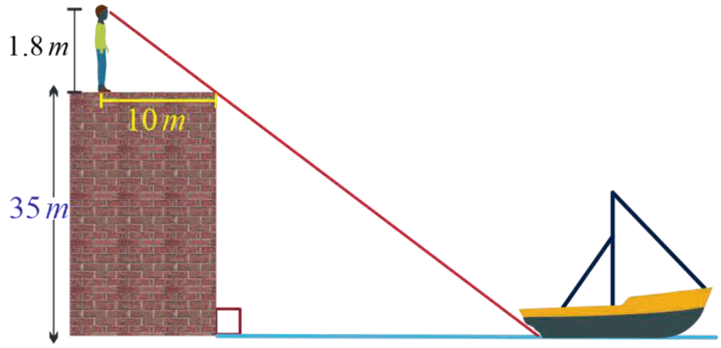
$\frac{1.5}{2} = \frac{x}{1.6}$, Hence, $x = 1.2$ meters.

Therefore, Almaz's brother is $1.2m$ long.

Example 2

Abebe's boat has come untied and floated away on Lake Tana. He is standing on the top of a building that is 35 m above the water from the lake. If he stands 10 m from the edge of the building, he can visually align the top of the building with the water at the back of his boat. His eye level is 1.8 m above the top of the building. Approximately, how far is Abebe's boat from the building?

Solution:



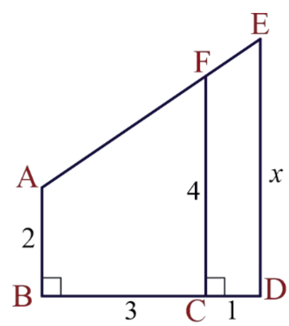
Let x be the distance from the boat to the building. Using proportionality of corresponding sides of the two triangles, we have

$\frac{35}{1.8} = \frac{x}{10}$, Hence, $x \sim 194.4$m.

Therefore, the boat is approximately 194.4m from the building.

Example 3

Suppose BA, CF and DE be the heights of three buildings. Let $DC = 1, BC = 3,$ $CF = 4$ and $AB = 2$ as indicated in the figure below show. Find the height x of the tallest building.



Solution:

Extend FA and CB so that they meet at G. Now, we have ΔDGE~ΔCGF by AA similarity theorem.

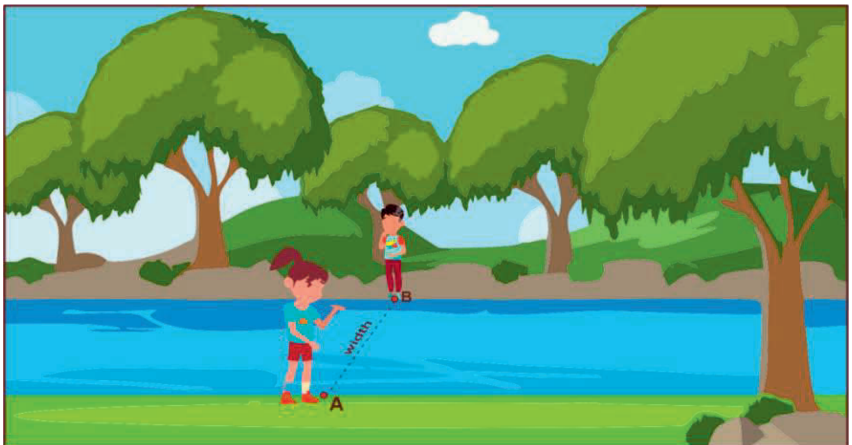
$\frac{\text{DG}}{\text{CG}} = \frac{\text{ED}}{\text{FC}}$. Hence, $\frac{\text{GB}+4}{\text{GB}+3} = \frac{x}{4}$. Now, consider similar triangles AGB and FGC. From this, we get $\frac{\text{GB}}{\text{GC}} = \frac{\text{AB}}{\text{FC}}$.

$\frac{GB}{GB+3} = \frac{2}{4}$. Therefore, $GB = 3$ and $x = \frac{14}{3}$

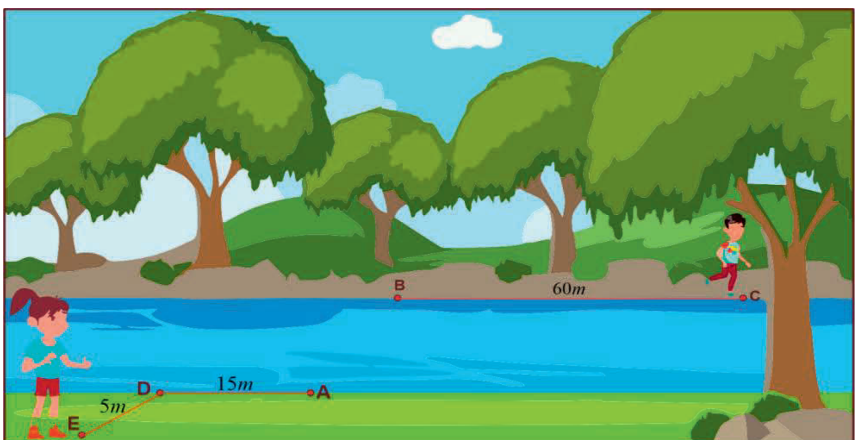
Example 4

Two persons stand at point A and B and want indirectly to measure the width of the river. How long is the width of the river? It is done as follows: Person A moves 15 m to the left and person B moves 60 m to the right. Moreover, person A moves 5m away from the river. The line connecting the places after the two people have moved passes through point A.

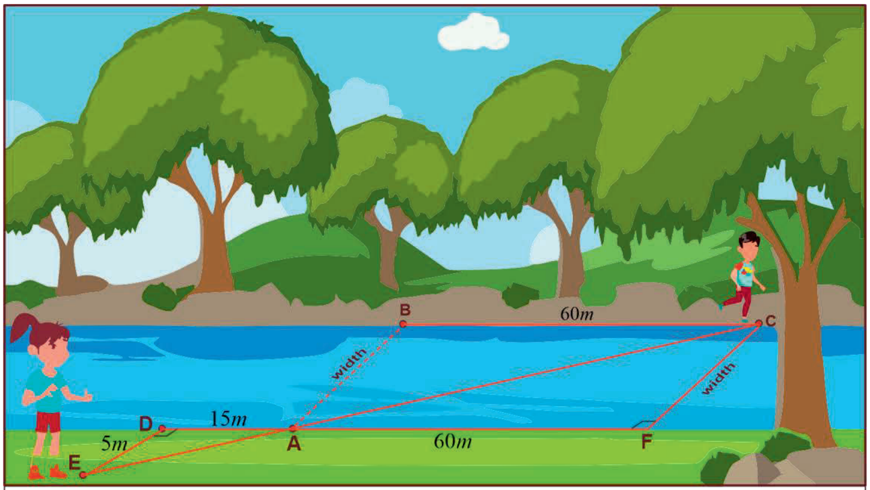
Solution:



Their Original Position



Person at B moves the right 60m at C
and, Person at A moves to left 15m at D and then 5m away from the river at E



Similar Triangles ADE and AFC are formed

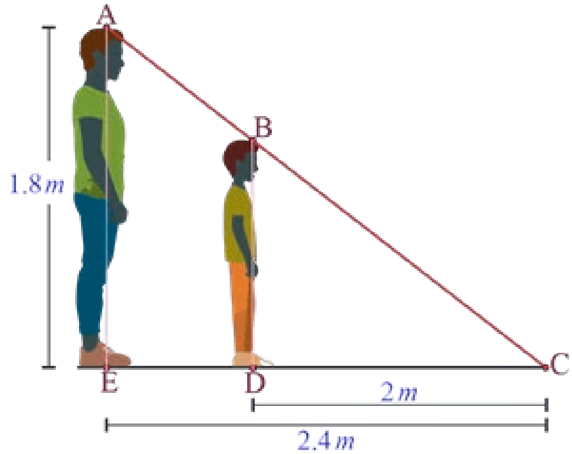
Therefore, $\frac{AD}{AF} = \frac{DE}{FC} = \frac{AE}{AC}$

$\frac{15}{60} = \frac{5}{FC} = \frac{AE}{AC}$.

$FC = 20$. Hence, the width of the river is $20m$

Exercise 7.14

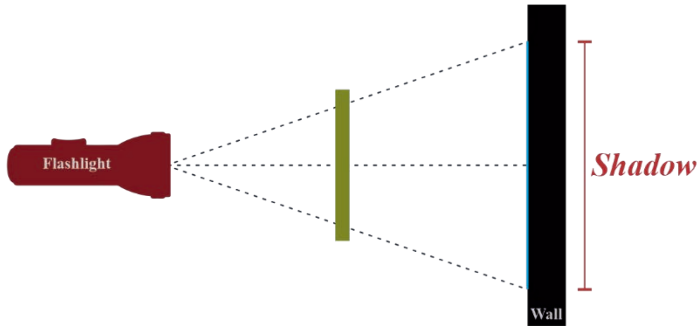
a. Abdi is 1.8 meters tall and is standing outside next to his younger brother. He notices that he can see both of their shadows and decides to measure each shadow. His shadow is 2.4 meters long and his brother's shadow is 2 meters long. How tall is Abdi's brother?



b. Abdisa's boat has come untied and floated away on Lake Hawasa. He is standing on the top of a building that is 75 m above the water from the lake. If he stands 15 m from the edge of the building, he can visually align the top of the building with the water at the back of his boat. His eye level is 1.75 m above the top of the building. Approximately, how far is Abebe's boat from the building?

Exercise 7.15

1. A giraffe is $6m$ tall and casts a shadow of 4m. Another small giraffe casts a shadow of 2m. How long is the smaller giraffe?
2. A flagpole casts a shadow $9m$ long. A person 1.8 m long, standing nearby casts a shadow $4m$ long. How tall is the flagpole?
3. A photograph measuring 4cm wide and 5cm long is enlarged to make a wall mural. If the mural is 120cm wide, how long is the mural?
4. A $9m$ ladder leans against a building 6m above the ground. At what height would a $15m$ ladder touch the building if both ladders form the same angle from the ground?
5. A ladder is placed against a wall such that it aches up to a height of 4m of the wall. If the foot of the ladder is 3m away from the wall, find the length of the ladder.
6. P and Q are points on the sides CA and CB, respectively of ΔABC, right angled at C. Prove that $AQ^2 + BP^2 = AB^2 + PQ^2$.
7. A $12 - centimeter$ rod is held between a flashlight and a wall as shown. Find the length of the shadow on the wall if the rod is 24 cm from the wall and 15 cm from the light. Assume that the dotted line segments meet at a point inside the flashlight.


Flashlight
Shadow
Wall

Summary

1. Two plane figures are similar if:

   i. their corresponding sides are proportional.

   ii. their corresponding angles are congruent.

2. Under an enlargement

   a. Lines and their images are parallel.

   b. Angles remain the same.

   c. All lengths are increased or decreased in the same ratio.

3. Scale factor- the ratio of corresponding sides usually expressed numerically so that:

$$\text{Scale factor} = \frac{\text{length of the line segment on the enlagement}}{\text{length of the line segment on the original}}$$

4. AA Similarity theorem

   If two angles of one triangle are congruent to the corresponding two angles of another triangle, then the two triangles are similar.

   SSS Similarity theorem

   If the corresponding three sides of two triangles are proportional to each other, then the triangles are similar.

   SAS Similarity theorem

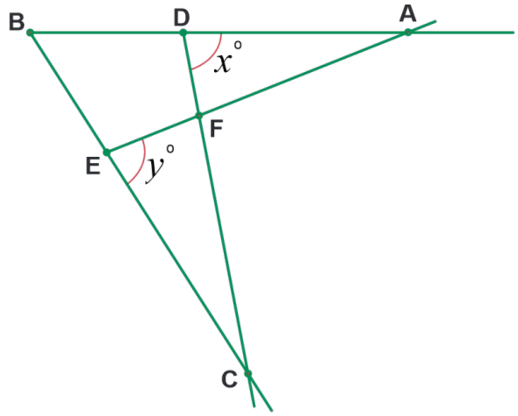
   If the corresponding two sides of two triangles is proportional and the included angles are congruent, then the two triangles are similar.

5. If two figures are similar, then the ratio of their perimeters is equal to ratio of their corresponding side lengths.

6. If two figures are similar, then the ratio of their areas is equal to the square of the ratio of their corresponding side lengths.

Review Exercise

1. In the given figure, if $x = y$ and AB = CB, then prove that AE = CD.



2. AD is an altitude of an isosceles ΔABC in which $AB \equiv AC$.

Show that

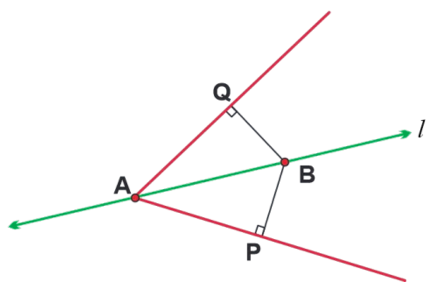
(i) AD bisects

(ii) AD bisects $\angle A$.

3. In the given figure, line l is the bisector of $\angle A$ and B is any point on l. If segment BP and BQ are perpendiculars from point B to the arms of $\angle A$, show that

(i) $\Delta APB \equiv \Delta AQB$

(ii) $BP \equiv BQ$, i.e., B is equidistant from the arms of $\angle A$.

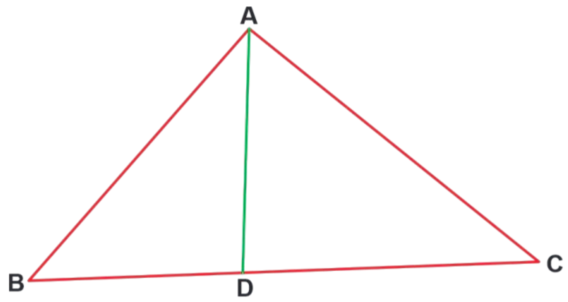


4. In a triangle, a line is drawn parallel to one side and a small triangle is cut off. Prove that the triangle cut off is similar to the original triangle.

5. Consider the four triangles made by joining all the midpoints of a triangle. Show these triangles are congruent and they are similar to the largest triangle.

6. Prove that a parallelogram is formed by joining the midpoints of any quadrilateral,

7. Assume that D is a point on side BC of triangle ABC such that $\angle ADC = \angle BAC$ as in the figure. Show that $(CA)^2 = \mathrm{CB} \cdot \mathrm{CD}$.



8. L and M are the mid-points of the sides AB and AC of $\Delta\mathrm{ABC}$, right angled at B. Show that $4(\mathrm{LC})^2 = (\mathrm{AB})^2 + 4(\mathrm{BC})^2$.

9. The perimeter of two similar triangles ABC and DEF are 12 cm and 18 cm. Find the ratio of the area of $\Delta\mathrm{ABC}$ to that of $\Delta\mathrm{DEF}$

10. The altitudes AD and PS of two similar triangles ABC and PQR are of length 2.5 cm and 3.5 cm. Find the ratio of area of ΔABC to that of $\Delta\mathrm{PQR}$.

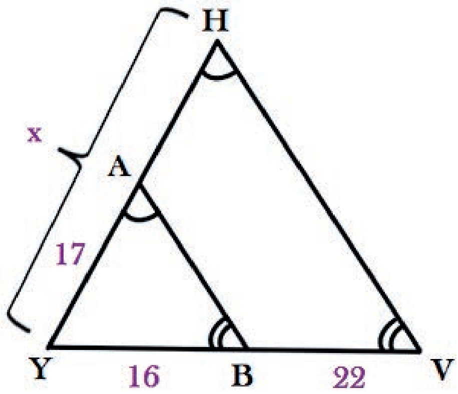
11. Two poles of heights 12 m and 17 m, stand on a plane ground and the distance between their feet is 12 m. Find the distance between their top.

12. A ladder is placed against a wall and its top reaches a point at a height of 8 m from the ground. If the distance between the wall and foot of the ladder is 6 m, find the length of the ladder.

13. In an equilateral triangle, show that three times the square of a side equals four times the square of a median.

14. A triangle with an area of 60 square meters has a base of 4 meters. A similar triangle has an area of 50 square meters. Find the height of the smaller triangle.

15. The value of x in the diagram below is:



A. 38 B. 33.375 C. 40.375 D. 39.75

16. If $\Delta ABC \sim \Delta DEF$, then which of the following is not true?

A. $\frac{AB}{DE} = \frac{BC}{EF}$ B. $\frac{AB}{DE} = \frac{EF}{BC}$ C. $\frac{AC}{DF} = \frac{BC}{EF}$ D. $\frac{DE}{AB} = \frac{EF}{BC}$

17. If the ratio of sides of triangles ABC and DEF is k, then the ratio of their perimeters is:

A. k B. k^2 C. $\frac{1}{k}$ D. $k+1$