

for $\lambda = -21.124$

$$\begin{pmatrix} 4-21.124 & 8 & -1 & -2 \\ -2 & -9-21.124 & -2 & -4 \\ 0 & 10 & 5-21.124 & -10 \\ -1 & -13 & -14 & -13-21.124 \end{pmatrix}$$

$$\begin{pmatrix} -17.124 & 8 & -1 & -2 \\ -2 & -30.124 & -2 & -4 \\ 0 & 10 & -16.124 & -10 \\ -1 & -13 & -14 & -34.124 \end{pmatrix} \begin{pmatrix} \vec{V}_1 \\ \vec{V}_2 \\ \vec{V}_3 \\ \vec{V}_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -17.124 \vec{V}_1 + 8 \vec{V}_2 - \vec{V}_3 - 2 \vec{V}_4 = 0 & (eq1) \\ -2 \vec{V}_2 - 30.124 \vec{V}_2 - 2 \vec{V}_3 - 4 \vec{V}_4 = 0 & (eq2) \\ 10 \vec{V}_2 - 16.124 \vec{V}_3 - 10 \vec{V}_4 = 0 & (eq3) \\ -\vec{V}_2 - 13 \vec{V}_2 - 14 \vec{V}_3 - 34.124 \vec{V}_4 = 0 & (eq4) \end{cases}$$

$$\vec{V}_3 = -17.124 \vec{V}_1 + 8 \vec{V}_2 - 2 \vec{V}_4$$

Let replace \vec{V}_3 in Equation 2 (eq2)

$$\begin{aligned} -2 \vec{V}_2 - 30.124 \vec{V}_2 - 2(-17.124 \vec{V}_1 + 8 \vec{V}_2 - 2 \vec{V}_4) - 4 \vec{V}_4 &= 0 \\ -2 \vec{V}_2 - 30.124 \vec{V}_2 + 34.248 \vec{V}_1 - 16 \vec{V}_2 + 4 \vec{V}_4 - 4 \vec{V}_4 &= 0 \end{aligned}$$

$$32.248 \vec{V}_1 - 46.124 \vec{V}_2 = 0$$

$$46.124 \vec{V}_2 = 32.248 \vec{V}_1$$

$$\vec{V}_2 = \frac{32.248}{46.124} \vec{V}_1$$

$$\vec{V}_2 = 10 \quad \underline{\underline{\vec{V}_2 = 0.7 \vec{V}_1}}$$

$$\begin{cases} -\vec{V}_4 - 13(0.7\vec{V}_1) - 14(-17.124\vec{V}_2 + 8\vec{V}_3 - 2\vec{V}_4) - 39.124\vec{V}_4 = 0 \\ -10\vec{V}_2 + 239.736\vec{V}_4 - 112\vec{V}_2 + 22\vec{V}_4 - 38.124\vec{V}_4 = 0 \end{cases}$$

$$229.636\vec{V}_4 - 112\vec{V}_2 - 6.124\vec{V}_4 = 0$$

$$229.636\vec{V}_4 - 112(0.7\vec{V}_1) - 6.124\vec{V}_4 = 0$$

$$229.636\vec{V}_4 - 78.4\vec{V}_1 - 6.124\vec{V}_4 = 0$$

$$151.236\vec{V}_4 - 6.124\vec{V}_1 = 0$$

$$\vec{V}_4 = \frac{151.236}{6.124} \vec{V}_1$$

$$\vec{V}_4 = 24.696\vec{V}_1$$

$$\begin{cases} 10\vec{V}_2 - 16.124\vec{V}_3 - 10\vec{V}_4 = 0 \end{cases}$$

$$10(0.7\vec{V}_1) - 16.124\vec{V}_3 - 10(24.696\vec{V}_1) = 0$$

$$7\vec{V}_1 - 16.124\vec{V}_3 - 246.96\vec{V}_1 = 0$$

$$-16.124\vec{V}_3 = 239.96\vec{V}_1$$

$$\vec{V}_3 = \frac{-239.96}{16.124} \vec{V}_1$$

$$\vec{V}_3 = -14.882\vec{V}_1$$

$$\text{Eigenvektor} = \begin{pmatrix} \vec{V}_1 \\ 0.7\vec{V}_1 \\ -14.882\vec{V}_1 \\ 24.696\vec{V}_1 \end{pmatrix} = \vec{V}_1 \begin{pmatrix} 1 \\ 0.7 \\ -14.882 \\ 24.696 \end{pmatrix}$$

$$\text{Eigenvektor is } \begin{pmatrix} 1 \\ 0.7 \\ -14.882 \\ 24.696 \end{pmatrix}$$