

Eigenvalue Calculations

$$\begin{bmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{bmatrix}$$

Forming a new matrix by  $-\lambda$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

$$\det \begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix}$$

$$= -10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(14\lambda+10) + (5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(\lambda-9)-52)-210) - 1950$$

$$-10\lambda - 10(4-\lambda)(2\lambda-30) + 10(4-\lambda)(4\lambda+10) + (5-\lambda)(-14\lambda + (4-\lambda)((-\lambda-13)(\lambda-9)-52)-210) - 1950 = 0$$

$$\lambda = 1, \\ \lambda = -1$$

$$A - I = \begin{bmatrix} 4-\lambda & -2 & 10 & -1 \\ 8 & 9-\lambda & -13 & -1 \\ -1 & -2 & 5-\lambda & -14 \\ -2 & -4 & -10 & -13-\lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 0 & -1 \\ 8 & 8 & 10 & -13 \\ -1 & -2 & 4 & -14 \\ -2 & -4 & -10 & -14 \end{bmatrix}$$

$$-10\lambda + (5-\lambda)(-14\lambda + (4-\lambda)(-\lambda-13)(-\lambda-9) - 210) - (40-10\lambda) \\ (2\lambda-30) + (40-10\lambda)(14\lambda+10) - 1910 = 0 \\ (5-\lambda)(-14\lambda + (4-\lambda)(-\lambda-13)(-\lambda-9) - 210) - (40-10\lambda) \\ (2\lambda-30) + (40-10\lambda)(14\lambda+10) - 1910 = 0$$

$$\lambda = -5.60402060796358$$

$$\begin{bmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -\lambda-9 & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -\lambda-13 \end{bmatrix} =$$

$$\begin{bmatrix} 9.604020 & 8 & -1 \\ -2 & -3.5959793 & -2 \\ 0 & 10 & 10.6040206 \\ -1 & -13 & -14 \end{bmatrix}$$

$$\begin{bmatrix} -2 \\ -4 \\ -10 \\ -7.395979392 \end{bmatrix}$$

$$\lambda = 2.67459$$

For  $X = 2.67459$

$$M = A \cdot XI$$

$$M = \begin{pmatrix} 4 - 2.67459 & 8 & -1 & -2 \\ -2 & 9 - 2.67459 & -2 & -4 \\ 0 & 10 & 5 - 2.67459 & -16 \\ -1 & -15 & -14 & -13 - 2.67459 \end{pmatrix}$$

$$= \begin{pmatrix} -1.32541 & 8.00000 & -1.00000 & -2.00000 \\ -2.00000 & 6.32541 & -2.00000 & -4.00000 \\ 0.00000 & 10.00000 & 2.32541 & -10.00000 \\ -1.00000 & -15.00000 & -14.00000 & -15.67459 \end{pmatrix}$$

eliminating

$$\begin{pmatrix} 1.32541 & 8.00000 & -1.00000 & -2.00000 \\ 0 & 15.3934 & -3.00000 & -7.0120 \\ 0 & 10.0000 & 2.32541 & -10.0000 \\ 0 & -6.9676 & -14.753 & -17.1532 \end{pmatrix}$$

$$\begin{pmatrix} 1.32541 & 8.00000 & -1.00000 & -2.00000 \\ 0 & 15.3934 & -3.00000 & -7.0120 \\ 0 & 0 & 4.2344 & -6.1539 \\ 0 & 0 & 0 & -43.3352 \end{pmatrix}$$

$$y_1 = \begin{bmatrix} -8.494361505869146 \\ 1.427101712256019 \\ -1.83841494602921 \\ 1 \end{bmatrix}$$