Assignment part2

$$\begin{pmatrix} -4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix}$$

$$\det(A-\lambda I)=0$$

$$\begin{pmatrix} 4 & 8 & -1 & -2 \\ -2 & -9 & -2 & -4 \\ 0 & 10 & 5 & -10 \\ -1 & -13 & -14 & -13 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 4-\lambda & 8 & -1 & -2 \\ -2 & -9-\lambda & -2 & -4 \\ 0 & 10 & 5-\lambda & -10 \\ -1 & -13 & -14 & -13-\lambda \end{vmatrix} = 0$$

$$4-\lambda \begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix}$$

$$-8 \begin{pmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{pmatrix}$$

$$-1\begin{pmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{pmatrix}$$

$$+2\begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}$$

<u>Let start with the first matrix</u>

$$4-\lambda\begin{pmatrix} -9-\lambda & -2 & -4 \\ 10 & 5-\lambda & -10 \\ -13 & -14 & -13-\lambda \end{pmatrix}$$

$[(-9-\lambda)(5-\lambda)(-13-\lambda) + (-2)(-10)(-3) + (10)(-14)(-4)$
$-[(-4)(5-\lambda)(-13) + (-2)(10)(-13-\lambda) + (-10)(-14)(-9-\lambda)]$

$(5-\lambda)(-9-\lambda) = \lambda^2 + 4\lambda - 45$

$(\lambda^2+4\lambda-45)(-13-\lambda) = -\lambda^3 - 17\lambda^2 - 7\lambda + 585$

$-\lambda^3 - 17\lambda^2 - 7\lambda + 585 - 260 + 560 - (52(5-\lambda) - 20$
$(-13-\lambda) + 140(-9-\lambda))$

$-\lambda^3 - 17\lambda^2 - 7\lambda + 885 + 172\lambda + 740$

$-\lambda^3 - 17\lambda^2 + 165\lambda + 1625$

$4-\lambda(-\lambda^3 - 17\lambda^2 + 165\lambda + 1625)$

$-4\lambda^3 - 68\lambda^2 + 660\lambda + 6500 + \lambda^4 + 17\lambda^3 - 165\lambda^2 -$
1625λ

$$\underline{\lambda^4+13\lambda^3-233\lambda^2-965\lambda+6500}$$

$$-8\begin{pmatrix} -2 & -2 & -4 \\ 0 & 5-\lambda & -10 \\ -1 & -14 & -13-\lambda \end{pmatrix}$$

$$-8\big((-2)(5-\lambda)(-13-\lambda)+0+(-2)(-10)(-1)-$$
$$((-4)(5-\lambda)(-1)+0+(-2)(-14)(-10)\big)$$

$$-8(-2\lambda^2-12\lambda+370)$$

$$=\underline{\underline{16\lambda^2+96\lambda-2960}}$$

$$-1\begin{pmatrix} -2 & -9-\lambda & -4 \\ 0 & 10 & -10 \\ -1 & -13 & -13-\lambda \end{pmatrix}$$

$$(-2)(10)(-13-\lambda)+(-1)(-10)(-9-\lambda)-((40)+(-10)$$
$$(-13)(-2))$$

$$260+20\lambda+(-90-10\lambda)$$
$$=10\lambda+390$$
$$-1(10\lambda+390)$$
$$=\underline{-10\lambda-390}$$

$$+2\begin{pmatrix} -2 & -9-\lambda & -2 \\ 0 & 10 & 5-\lambda \\ -1 & -13 & -14 \end{pmatrix}$$

$$(-2)(10)(-14)+(-1)(-9-\lambda)(5-\lambda)-20+26(5-\lambda)$$

$$280-\lambda^2-4\lambda+45+26\lambda-150-\lambda^2+22\lambda+175$$
$$\underline{=-2\lambda^2+44\lambda+350}$$

Adding all 4 equations

$\lambda^4 + 13\lambda^3 - 233\lambda^2 - 965\lambda + 6500 + 16\lambda^2 + 96\lambda$
$- 2960 - 10\lambda - 390 - 2\lambda^2 + 44\lambda + 350$

$\lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500 = 0$

Applying newton - Raphson Law

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

$f'(\lambda) = 4\lambda^3 + 39\lambda^2 - 438\lambda - 835$

finding roots of the Equation

$f(11) = (11)^4 + 13(11)^3 - 219(11)^2 - 835(11) + 3500$
$14641 + 17303 - 26499 - 9185 + 3500$
$= -240$

$f(12) = (12)^4 + 13(12)^3 - 219(12)^2 - 835(12) + 3$
$= 5144$

There is a root between 11 and 12

$\lambda_0 = \frac{x_1 + x_2}{2}$ $\quad \lambda_0 = \frac{11+12}{2}$ $\quad \lambda_0 = 11.5$

$\lambda_1 = 11.5 - \frac{2196 - 1875}{5396.25}$

$\lambda_1 = 11.093$

$\lambda_2 = 11.93 - \frac{184.946}{4565.129}$ $\quad \lambda_2 = 11.052$

Let's find the second root

$f(2) = 16 + 104 - 876 - 1670 + 3500$

$f(2) = 1074$

$f(3) = 81 + 351 - 1971 - 2505 + 3500$

$= -544$

$x_0 = \frac{2+3}{2}$ $\quad x_0 = 2.5$

$\lambda_1 = 2.5 - \left(\frac{-285.9375}{1623.75}\right)$

$\lambda_1 = \underline{2.674}$

<u>Third root</u>

$f(\lambda) = \lambda^4 + 13\lambda^3 - 219\lambda^2 - 835\lambda + 3500$

$f(-5) = 625 - 1625 - 5475 + 4175 + 3500$

$= 1200$

$f(-6) = 1296 - 2808 - 7884 + 5010 + 3500$

$= -866$

There is a root between -5 and -6

$x_0 = \frac{-5-6}{2}$ $\quad x_0 = -5.5$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = -5.5 - \frac{219.9375}{2088.25}$

$x_1 = -5.60532$

$x_2 = -5.605321 - \frac{(-2.625)}{2141.0623}$

$\underline{x_2 = -5.6040}$

Fourth root

$f(-21) = (-21)^4 + 13(-21)^3 - 219(-21)^2 - 835(-21) + 3500$

$= 194{,}481 - 120{,}393 - 96{,}579 + 17{,}535 + 3500$

$= -1456$

$f(-22) = 234{,}256 - 138{,}425 - 105{,}996 + 18{,}370 + 3500$

$= 11{,}705$

There is a root between -21 and -22

$x_0 = \dfrac{-21 + (-22)}{2} = -21.5$

$x_1 = -21.5 + \dfrac{4{,}695.9375}{13{,}143.75}$

$x_1 = -21.1427246$

$x_2 = -21.1427 + \dfrac{249.0}{11{,}953.454}$

$x_2 = -21.124$

Let's apply formula

$(A - \lambda I)(\vec{v}) = 0$