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A survey on recent progress in control of swarm systems

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Abstract It has been witnessed that swarm systems are superior to individual agents in performing complicated tasks. In recent years, new results in some branches of control for swarm systems have developed and investigated with respect to various objectives and scenarios. This survey is to take a glimpse into some newly developed control techniques for swarm systems, especially those presented after 2013. The covered topics include some up-to-date progress in the areas of consensus, formation, flocking, containment, optimal coverage/mission planning, and sensor networks. Contributions and connections of the mentioned references are discussed briefly. Based on the new results in control of swarm systems, some possible new future research topics are suggested.

Keywords formation, swarm systems, consensus, containment, flocking, sensor networks

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1 Introduction

Swarm system (sometimes known as multi-agent system) means a group of self-organizing agents trying to carry out tasks together. The conception of swarm system originated from many observed biological phenomena. It has been observed that, time-efficiency and energy efficiency can be improved considerably, if biological individuals construct a group to perform their functions. For example, birds are capable of forming formations during migration for energy-efficient flying; fish in flocks are more effective in defence from predators than individual ones; and ants always cooperate in groups to prey and migrate.

During the past few decades, researchers were greatly enlightened by biological swarm systems. From a perspective of system dynamics and control, researches in swarm systems can be traced back to [1], where the authors proposed a model to investigate the self-ordered motions of the system composed of particles with interactions. A simulation case study is presented to describe the motion of the particles achieving a consensus phenomenon. Theoretical analysis on the consensus phenomena in [1] is provided in [2], where some other models are also investigated. Early results on stability of swarm systems are summarized and presented in [3], where both attraction and repelling are considered in interactions among individuals.

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Up till now, most engineering swarm systems are investigated within the frameworks of control theory and graph theory [4]. In swarm systems, the state of consensus is defined as a common state (coordinated positions, velocities or attitudes) maintained by all agents [5]. For homogeneous integrator swarm systems with fixed communication topology, consensus can be accomplished by linear feedback of relative states [4]. Theoretical tools such as convexity and Lyapunov theory can be used for stability analysis of swarm systems [6]. Controllability of state-dependent and time-varying graphs is systematically studied in [7]. Impact of network topology on controllability of general complex networks is analyzed in [8].

There are some more advanced topics of consensus for swarm systems, e.g., switching communication topology, time-delay, stochastic systems, finite-time consensus, to name a few. For swarm system with switching communication topology, a sufficient condition to achieve consensus is that agents are required to be connected [5]. Further, it is proved in [9] that the condition of connectivity can be relaxed to a necessary and sufficient condition of the existence of a spanning tree. Time-delay in swarm systems can be categorized into communication delay and input delay [4], and it has been proved that only input delay influences the consensus properties of the swarm system [10]. Consensus criteria for swarm system with stochastic communication topology are presented in [11].

Consensus can be regarded as fundamentals of solving other control problems for swarm systems; for instances, formation control [12, 13] where some relative positions should be maintained by agents, flocking/cohesion [14, 15] where agents approaches each other from initial positions and maintain specific distances, consensus tracking [13, 16] where a group of consensus agents track reference trajectories. Some other more advanced problems arise in coordinated obstacle avoidance [14, 17], distributed optimization [18, 19], containment control [20], and optimal area coverage [21], to name a few. Some more higher-level topics include task allocation and mission planning [22].

Besides the above representative theoretical results, various experiments on swarm systems have been presented (mostly by using multiple autonomous ground vehicles). In [23], a new broadcast control framework is designed for coordination of a swarm system; experiments have been provided by using a group of autonomous ground vehicles to test the proposed method. In [24], the concept of passivity is used in attitude synchronization for swarm system in SE(3), and the proposed control has been tested on 2-D ground robots. A large swarm system of ground vehicles can be approximated by fluid dynamics, and techniques of hydro-dynamics can be applied to design coordinated experiments [25]. Ground vehicles are also used to test containment control approaches proposed in [26] and formation control approaches proposed in [27]. Representative examples of experiments using unmanned aerial vehicles (UAVs) include multi-UAV system with uncertain parameters [28], time-varying UAV formation [29], and multi-UAV system with switching interaction topologies [30]. Experimental results of underwater swarm systems can be found in [31, 32].

In this survey paper, the focus is on some recent developments (specifically those observed during 2013–2015) concerning swarm systems, including both theoretical results and some applications to multi-UAV systems. A preliminary version of this paper was partially presented in [33]. Topics to be covered include consensus, formation, flocking, containment control, optimization of swarm systems, and sensor networks. Please see the comprehensive survey [34] (and literature therein) for results of swarm systems and networked control systems before 2013.

2 Preliminaries

Notations: n-dimensional vectors with all its elements being 1 are denoted by $\mathbf{1}_n$; the n-dimensional identity matrix is denoted by I_n; the Kronecker product is denoted by $\otimes$; and the transpose of matrix is denoted by a superscript T. The real and imaginary parts of a complex number are denoted by $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$, respectively.

2.1 Preliminaries on graph theory

Contents of this section are cited from [35]. For more detailed contents of graph theory, please see [4].

Communications of information among agents can be described by directed/undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ [36] where $\mathcal{V} = \{\pi_1, \pi_2, \ldots, \pi_n\}$ denotes the set of all agents in the networked swarm system, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represents the set of all edges that describe information exchanges among various agents. The edge $e_{ij} = (\pi_i, \pi_j)$ in $\mathcal{G}$ indicates the information of π_i is available to π_j, and π_i can be defined as a neighbor of π_j. The set of all neighbors of node π_j is represented by $\mathcal{N}_j = \{i : (\pi_i, \pi_j) \in \mathcal{E}\}$. In an undirected graph, $(\pi_i, \pi_j) \in \mathcal{E} \Leftrightarrow (\pi_j, \pi_i) \in \mathcal{E}$. The adjacent matrix $\mathcal{A} \triangleq [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} = 1$ if $(\pi_j, \pi_i) \in \mathcal{E}$, and $a_{ij} = 0$ if $(\pi_j, \pi_i) \notin \mathcal{E}$. It is assumed that $a_{ii} = 0$. The Laplacian matrix can be defined by $\mathcal{L} \triangleq [l_{ij}] \in \mathbb{R}^{n \times n}$, where $l_{ii} = \sum_{j=1, j \neq i}^{n} a_{ij}$, and $l_{ij} = -a_{ij}$ for $i \neq j$.

In the edge $e_{ij} = (\pi_i, \pi_j)$, π_i and π_j are named as the parent node and the child node, respectively. The directed path from π_j to π_i is a sequence given by $(\pi_j, \pi_{i_1}), (\pi_{i_2}, \pi_{i_3}), \ldots, (\pi_{i_l}, \pi_i)$ in $\mathcal{G}$ with distinct nodes $\pi_{i_k}, k = 1, 2, \ldots, l$. An undirected graph is connected, if there exists a path connecting each pair of nodes. A directed tree [9] is defined as a directed subgraph, where there exists exactly one parent for each node except the root (the node has no parent), and there exists a directed path for the root to connect every other node. A spanning tree is defined as a directed tree formed by edges connecting all nodes in the graph. A graph contains a spanning tree, if some of its edges are capable of forming a spanning tree. The following lemma is of great importance in consensus control of swarm system.

Lemma 1 ([5]). For the Laplacian matrix $\mathcal{L}(\mathcal{A})$, the following properties hold: (1) $\mathcal{L}(\mathcal{A})$ always has an eigenvalue 0, and the corresponding eigenvector is $\mathbf{1}_n$; (2) Provided that $\mathcal{G}$ is fully connected, the second smallest eigenvalue of $\mathcal{L}(\mathcal{A})$ is always positive ($\lambda_2(L) > 0$).

2.2 Consensus control for fixed communication topologies

Consider the 1st-order integrator swarm system composed by n agents:

$$\dot{x}_i = u_i, \quad i \in \mathcal{I}_n = \{1, 2, \ldots, n\}, \tag{1}$$

where x_i is the state of agent i, and u_i represents the protocol (the control effort to be designed). Assume that the information exchange is described by the graph defined in Subsection 2.1, and each agent only accesses the information of its neighbors. The aim is to design some protocol u_i, such that all agents in (1) can reach an agreement: $\lim_{t \to +\infty} \|x_i(t) - x_j(t)\| = 0,\ i \in \mathcal{I}_n,\ j \in \mathcal{I}_n,\ i \neq j$, which can be defined as asymptotic consensus [5]. If agents satisfy $\lim_{t \to +\infty} x_i(t) = \frac{1}{n} \sum_{j=1}^{n} x_j(0)$, it then follows that the swarm system reaches an average consensus [5]. For the 1st-order integrator swarm system, the protocol can be designed by [5]

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i \right), \tag{2}$$

and it can be proved by using Lemma 1 that the swarm system reaches asymptotic consensus and average consensus. It is displayed in [9] that connected graph is a sufficient condition for consensus, and a more relaxed condition (which is necessary and sufficient) is that there exists a spanning tree in the graph.

The swarm system (1) is capable of consensus tracking with the protocol

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i \right) + a_{ir} \left(x_r - x_i \right), \tag{3}$$

where x_r is the time-varying reference trajectory to be tracked, and $a_{ir} = 1$ if x_r is available to the ith agent ($a_{ir} = 0$ otherwise).

However, very limited numbers of real cases are described by 1st-order integrator swarm system (1); consequently, the consensus problem of 2nd-order integrator swarm system is proposed, such that the protocol can be implemented for more applications. The 2nd-order integrator swarm system is given by

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i \in \mathcal{I}_n = \{1, 2, \ldots, n\}, \tag{4}$$

where x_i, v_i and u_i can be seen as positions, velocities and accelerations of Newton systems, respectively. The consensus protocol for the 2nd-order integrator swarm system can be designed by [37]

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} \left((x_j - x_i) + \gamma(v_j - v_i) \right), \ i \in \mathcal{I}, \tag{5}$$

where $\gamma > 0$ is a scaling factor.

It has been proved in [37] that spanning tree is necessary (but insufficient) for asymptotic consensus. A sufficient condition is that, there exists a spanning tree in $\mathcal{G}$, and the scaling factor γ satisfies

$$\gamma > \max_{\mu_i \neq 0} \sqrt{\frac{2}{|\mu_i| \cos(\frac{\pi}{2} - \tan^{-1} \frac{-\mathrm{Re}(\mu_i)}{\mathrm{Im}(\mu_i)})}}. \tag{6}$$

The detailed proof can be found in [37].

2.3 Some other classical results

Consensus protocol with time-switching graphs can be referred to [9, 38]. For 1st-order integrator swarm systems with switching communication graphs, the asymptotic consensus is guaranteed, if the union of graphs of each time-switching intervals always contains a spanning tree; for 2nd-order integrator swarm systems with switching communication graph, the sufficient condition is more strict, and it requires that there always be a spanning tree in each switching interval.

Some early results in flocking/cohesion are presented in [14], where topics such as time-delay, obstacle avoidance and consensus tracking are considered. Another flocking algorithm is given in [39], where each individual, with the proposed protocol, is capable of repelling its over-close neighbors and align to a consensus velocity with appropriate distances. Analysis on controllability of swarm systems with state-dependent graphs are provided in [7]. Model predictive control (MPC) can be applied to swarm systems to address the constraints on system states and control inputs [40]. Event-triggered control were used for consensus such that the computational burden could be significantly reduced [41].

Consensus control can be designed for higher-order or complex swarm systems. In [42], a consensus control is proposed for Euler-Lagrangian system with actuator saturations. Synthesis of a flock and segregation controller for a heterogeneous multi-agent system can be found in [43], where the concept of differential artificial potential is used to prove the results. A necessary and sufficient condition is analyzed in [44] for synchronization of a multi-agent system constructed by heterogeneous linear models.

Based on results of consensus control, classic results of formation of swarm systems can be referred to [45], where a unified framework of formation is proposed. Formation protocol of multi-agent systems with constraints on actuator inputs can be addressed by using receding horizon control (or MPC) [46]. Evolutionary game theoretic approaches can be applied to formation protocols of swarm systems, and MPC can be used in finding Nash equilibria [47]. Backstepping is another approach that has been applied to formation protocols. In [48], backstepping control is proposed for triangle formation with three coleader agents. In [49], a leader-follower formation control is proposed by using adaptive backstepping integrated with sliding mode control. Guidance laws for formation of multi-UAVs can be referred to [50].

Some other classic results include formation and consensus tracking [13], distributed optimization [19], and mission planning [51], to name a few.

3 Recent developments in consensus control

Consensus control of swarm systems is still among the most popular research topics, since it is the fundamental of some other more complicated topics. Some recent results in consensus protocols for swarm systems are obtained in the following research areas such as consensus with time delay, cooperative output regulation, finite/fixed time consensus, and consensus for higher-order systems.

3.1 Consensus protocol for swarm systems with time-delay

Quite often there may exist delays in communications among agents. For swarm systems (1) with time-delays in acquiring neighboring information, the control for consensus can be designed by [5]

$$u_i = \sum_{\pi_j \in \mathcal{N}_i} a_{ij} \left(x_j(t - \tau_{ij}) - x_i(t - \tau_{ij}) \right), \tag{7}$$

where $\tau_{ij} > 0$ denotes the time-delay occurred with edge e_{ij}.

It is proved in [5] that, the multi-agent system (1) with delay τ_{ij} and protocol (7) is capable of average consensus, if its communication graph is fixed, undirected and fully connected, and the delay satisfies $\tau_{ij} = \tau \in (0,\ \tau^*)$ with $\tau^* = \frac{\pi}{2\lambda_{\max}(L)}$, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue.

In [52], consensus protocol with time-delays in neighboring information is proposed for the swarm system described by Lagrangian equation with parameter uncertainties; the adaptive consensus control is designed with a sliding vector. Another adaptive consensus protocol for Lagrangian systems with time-delays in accessing neighboring information can be found in [53], where the theory of small gain is applied to prove average consensus. The consensus protocol with communication time-delay and intermittent information interaction has been proposed in [54] for 2nd-order multi-agent systems (including Lagrangian systems). Time-delay has been investigated in event-triggered consensus control for multi-agent systems [55]. Consensus with time-delay for discrete-time multi-agent systems can be found in [56].

3.2 Cooperative output regulation

Output regulation is proposed to design a static or dynamic controller, such that the controlled plant could achieve asymptotic tracking of the reference signal in existence of external disturbances. For detailed results on output regulation, please see the textbook [57].

The theory of output regulation has been applied to multi-agent systems for cooperative consensus, e.g., [58, 59]. Cooperative output regulation for linear multi-agent systems can be found in [60], which is further extended in [61]. In most existing researches, the exosystem is regarded as the leader to be followed, and the multi-agent plants are considered to be followers. Distributed internal models are designed such that the output regulation of the original multi-agent system can be transformed into stabilization of an augmented system.

More recently, attentions are paid to cooperative output regulation for nonlinear multi-agent systems, e.g., [62, 63]. Solvability of output regulation for a typical class of nonlinear multi-agent system is discussed in [64], where it has been proved that the distributed output regulation can be solved if (1) the leader information can be reached by each follower through some undirected path, and (2) the global robust output regulation can be solved for each subsystem. Backstepping technique is applied to solve the distributed output regulation for nonlinear multi-agent systems in block lower triangular form [65], where some parametric uncertainties are also considered. The issue of switching communication network in cooperative output regulation for nonlinear multi-agent systems is addressed in [66], where the technique of average dwell time is applied to prove the solvability of the problem. If there exists bounded uncertain input in leader information, the non-smooth function of tracking errors can be applied such that the cooperative output regulation is still solvable globally [67]. Sometimes there might exist large parametric uncertainties and unknown control directions, and an adaptive cooperative output regulation with Nussbaum function [68] can be designed to treat this situation.

Other recent representative results on this topic include adaptive distributed observer [69], discrete-time adaptive linear cooperative output regulation [70], and adaptive cooperative output regulation over directed graphs [71], and some references therein.

3.3 Finite-time/fixed-time consensus protocols

The consensus is named finite-time consensus, if it can be achieved within some finite-time $t < t^*$ [72,73]. The idea of finite-time consensus originates from finite-time stabilization in the general control theory.

Finite-time stabilization is challenging since it incorporates non-Lipschitz dynamics. For detailed theory on finite-time stabilization, please see some early examples including sliding model control [74] and Hölder continuous feedback [75, 76]. A detailed survey on finite-time control can be found in [77].

Early work on finite-time consensus can be found in [78], where the nonsmooth gradient flows are analyzed and applied for the multi-agent system. One of the pioneering results on continuous nonsmooth control for finite-time consensus is provided in [79]. More recent result on finite-time consensus control for the 1st-order integrator swarm system (1) is given in [72] by

$$u_i = \sum_{j \in \mathcal{N}_i} a_{ij} \mathrm{sign}\left(x_j - x_i\right) \left\| x_j - x_i \right\|^{\alpha_{ij}}, \tag{8}$$

where $0 < \alpha_{ij} < 1$ are control parameters. It can be noted that, if $\alpha_{ij} = 1$, then the protocol (8) becomes (2). The protocol guarantees the finite-time consensus, provided there always exists a spanning tree.

An extension of the finite-time consensus control (8) is proposed in [73], where issues of control constraints and communication failures can be addressed. For 2nd-order integrator swarming system (4), a finite-time consensus control is proposed in [80], where finite-time tracking is also discussed. Another example of finite-time tracking can be found in [81]. Based on the result of finite-time consensus, finite-time optimal formation can be designed [82].

The transient process of finite-time consensus may be related to the initial system states. If asymptotic consensus is reached within finite-time, and the transient process does not necessarily depend on initial system states, then it is called a fixed-time consensus. Pioneering results on fixed-time consensus are provided in [35], where two types of fixed-time consensus protocols have been proposed for the 1st-order integrator swarm system (1) by

$$u_i = \alpha \left(\sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i\right) \right)^{2-\frac{p}{q}} + \beta \left(\sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i\right) \right)^{\frac{p}{q}} \tag{9}$$

which achieves global consensus, and

$$u_i = \alpha \sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i\right)^{2-\frac{p}{q}} + \beta \sum_{j \in \mathcal{N}_i} a_{ij} \left(x_j - x_i\right)^{\frac{p}{q}} \tag{10}$$

which achieves global average consensus. In (9) and (10), α, β, p and q are positive control parameters satisfying $p < q$. An extension of the results obtained in [35] to the 2nd-order integrator swarm system (4) is given in [83], where consensus tracking problem is also addressed.

Other recent results on finite-time and fixed-time consensus of swarm systems contain: adaptive control for finite-time consensus of undetermined 2nd-order integrator swarm systems [84] and mechanical systems [85], finite-time consensus for swarm systems with uncertain inherent nonlinear dynamics [86], finite-time consensus with stochastic matrices [87], finite-time consensus of 2nd-order integrator swarm system with input saturation [88], fixed-time consensus with linear and nonlinear state measurements [89], and robust fixed-time consensus [90].

3.4 Consensus protocols for higher-order swarm systems

General swarm systems are far more complicated than the simplest integrator systems. Progress has been witnessed in consensus for more realistic higher-order systems, i.e., general linear systems described by $\dot{x}_i = A_i x_i + B_i u_i$; higher-order integrator systems given by

$$\begin{cases} \dot{x}_{i,j} = x_{i,j+1}, \\ \dot{x}_{i,n} = u_i, \end{cases} \quad j = 1, 2, \ldots, n; \tag{11}$$

Euler-Lagrangian systems described by

$$M(q, \dot{q})\ddot{q}_i + N(q, \dot{q})\dot{q}_i + G(q, \dot{q}) = u_i; \tag{12}$$

and heterogeneous swarm systems where each of the agents has different dynamics.

In [91], the general linear system with uncertain parameters is investigated

$$\dot{x}_i = (A_i + \Delta A_i)x_i + B_i u_i, \tag{13}$$

and the problem of consensus can be proved to be equivalent to the problem of H_∞ control for decoupled subsystems, which is then addressed with linear matrix inequity (LMI). Results of continuous-time system are further extended to discrete-time cases. Some recent results on quantized consensus protocols for general linear swarm system can be found in [92].

Consensus for higher-order systems (11) has been extensively studied. For this type of swarm system, a consensus protocol can be proposed by integrating closed-loop control and local communications [93, 94]:

$$u_i = b\sum_{k=1}^{l-1} x_i^{(k)} + \sum_{j=1}^{n} c_{ij}\left(\sum_{k=0}^{l-2}\gamma_k(x_j^{(k)} - x_i^{(k)})\right), \tag{14}$$

where b and γ_k are control gains; c_{ij} are elements of adjacent matrix; and $x_i^{(k)}$ denotes the kth derivative of x_i. Other researches on consensus control for high-order swarm systems can be referred to [95, 96].

Early researches on consensus protocols for swarm Euler-Lagrangian system (12) can be referred to [42]. A typical consensus control for Euler-Lagrangian swarm system has been proposed in [42] by

$$u_i = -\sum_{j=1}^{n} a_{ij}(q_i - q_j) - \sum_{j=1}^{n} b_{ij}(\dot{q}_i - \dot{q}_j) - K_i\dot{q}_i, \tag{15}$$

where a_{ij} is the element of adjacent matrix associated with graph G_A for q_i, and b_{ij} is the element of adjacent matrix associated with graph G_B for $\dot{q}_i$; and K_i is a control gain. It is proved that, if G_A is undirected and connected, and G_B is undirected, then the closed-loop system reaches a consensus. The fundamental result is extended to address problems such as input saturation and unmeasurable generalized coordinate derivatives. Other results on swarm Euler-Lagrangian systems can be found in [52, 53].

3.5 Other results and open topics on consensus

Some other new progress in consensus control of swarm systems contain: consensus path-following [97], consensus tracking control [80,98,99], consensus control subject to spatial constraints [100,101], consensus protocol for stochastic swarm systems [102–105], consensus control for nonlinear systems [106–108], event-triggered consensus [109], and bipartite consensus [110]. It has to be acknowledged that there are surely other results that are not listed here due to limitation of our knowledge.

In the current stage, there exist some open topics in consensus including consensus for heterogeneous nonlinear multi-agent systems, quantized consensus for general multi-agent systems, and finite-time consensus with general directed graph.

4 Recent developments in formation control for swarm systems

Consensus control can be regarded as a fundamental for other forms of swarm system control. A direct extension of consensus for swarm systems is the distributed control for formation.

4.1 Results on formation control

In formation of swarm systems, individual agents are required to maintain a fixed (or predefined time-varying) relative position with its neighbors. Classic methods [37] of formation protocols include (1) leader-follower scheme, (2) behavioral scheme, and (3) virtual leader scheme. For early developments of these approaches, please see [111–113].

Similar expressions can be given for consensus problem and formation control problem, as proposed by [29]. In the task of formation control for 2nd-order integrator systems, the objective is to design distributive control, such that $\lim_{t\to+\infty}(\theta_i - h_i - c(t)) = 0$, where θ_i represents the state of agent i; $h_i(t) = [h_{ix}, h_{iv}]^{\mathrm{T}}$ is a function describing the formation to be accomplished; and $c(t)$ denotes a formation center function. The formulation of formation problem can be reduced to that of consensus problem by $\lim_{t\to+\infty}(\theta_i - s(t)) = 0$, where $s(t)$ denotes a consensus function. In [29], a time-varying reference formation is accomplished by

$$u_i = K_1(\theta_i - h_i) + K_2 \sum_{j\in\mathcal{N}_i} w_{ij}\left((\theta_j - h_j) - (\theta_i - h_i)\right) + \dot{h}_{iv}, \tag{16}$$

where w_{ij} denotes the corresponding element of the adjacent matrix; and K_1 and K_2 are control gains that can be obtained by solving a Reccati equation. Results proposed in [29] have been expanded to address time-varying reference formation with time-varying topology [30]. Experimental results with a swarm UAV system are provided in [29] to illustrate theoretical results.

Formation control can be designed by using position-based strategy, displacement-based strategy, or distance-based strategy [114]. Position-based strategy is more prevailingly considered, where full information of relative positions can be fed-back to achieve reference formation patterns. In displacement-based formation control [115], relative distances and relative orientations are measured for feedback, but relative positions cannot be directly measured. The principle is that, by using consensus control, orientations of agents can be aligned; displacement-based formation control can then be designed by using relative distances in the way of position-based strategy. In distance-based formation, only reference relative distances are required to be maintained. Properties of rigidity are often required in distance-based formation, such that the reference formation patterns can be guaranteed "rigid" (thus not subject to change if the relative distances are specified). Recently, an affine formation control is proposed, such that agents can reshape between position-based formation and distance-based formation [116].

A two-leader formulation of formation strategy is proposed in [117], where the reference formation is given by a certain pattern $F_\xi = c_1\mathbf{1} + c_2\xi$, and ξ is defined as formation basis. The objective is to find some distributed protocols, such that $\lim_{t\to+\infty} z(t) = F_\xi$. In [117], the protocols are designed by

$$v_i = \begin{cases} v_0(t), & i = 1, 2, \\ \sum_{j\in\mathcal{N}_i} w_{ij}(z_j - z_i) + v_0(t), & i = 3, \ldots, n, \end{cases} \tag{17}$$

and

$$a_i = \begin{cases} -\gamma v_i + a_0(t), & i = 1, 2, \\ \sum_{j\in\mathcal{N}_i} w_{ij}(z_j - z_i) - \gamma v_i + a_0(t), & i = 3, \ldots, n, \end{cases} \tag{18}$$

for first-order integrator systems and second-order integrator systems, respectively, where w_{ij} denotes the corresponding element of a complex adjacent matrix. Experimental results can be found in [118].

Distributed control for formation of a multi-robot system is designed in [119], where MPC is applied to stabilize the reference formation. In MPC, the optimization is processed with a Nash-equilibrium strategy. Terminal constraints and a novel repairing algorithm are designed to guarantee the stability of the swarm system. Experiments are given to support the theoretical results. Another example of application of MPC to formation control can be found in [120], where a new brain storm optimization is used to improve performances.

4.2 Collision avoidance

One of the key concern of formation control is collision avoidance during transient process.

Potential function can be regarded as the most widely applied strategy for avoiding collisions. A general form of potential function ϕ should satisfy

$$\phi(\lambda) = 0, \text{ if } \|\lambda\| \geqslant \Delta, \tag{19}$$

$$0 < \phi(\lambda) < \infty, \text{ if } \delta < \|\lambda\| < \Delta, \tag{20}$$

$$\lim_{\|\lambda\| \to \delta^+} \phi(\lambda) = +\infty, \tag{21}$$

where $0 < \delta < \Delta \leqslant 2$, and $\|\lambda\| \to \delta^+$ indicates that $\|\lambda\|$ approaches δ from a larger number. By using the potential function, repulsive forces would be generated if agents are close to each other, such that inter-agent collisions can be circumvented. A typical potential function is proposed in [121] for avoiding inter-agent collisions. In the formation control in [122], potential functions are applied to generate repulsive forces, so that collisions with obstacles can be avoided. The strategy is based on behavioral structure, and it can also be applied to leader-follower and virtual leader structures [122]. Other examples of using potential functions to avoid collisions can be found in [123, 124].

A recent formation protocol with inter-agent collision avoidance and obstacle avoidance for multi-robot systems is presented in [25], where terms of artificial viscosity are included to circumvent possible collisions. The basic principle is that, by using the viscosity terms, the swarm system would behave like "viscus" fluid, such that particle penetration can be avoided, and no agents would occupy the same position. The viscosity term could be regarded as a special type of potential function. The proposed formation protocol is applicable to both holonomic robots and non-holonomic robots.

Possible collisions may be considered as constraints, and distributed receding horizon control [71] is proposed, such that constrains in formation control can be addressed explicitly. In distributed receding horizon control, transient performances can be improved by using corresponding cost functions.

4.3 Extensions and applications

Circumnavigation [125] for swarm non-holonomic system is one interesting formulation of formation control. The swarm non-holonomic system can be given by $\dot{x}_i = v_i \cos\psi_i$, $\dot{y}_i = v_i \sin\psi_i$, $\dot{\psi}_i = \omega_i$, where (x_i, y_i) denotes the position of agent i in 2-dimensional space; ψ_i represents the direction of agent i; and v_i and ω_i are considered as the control inputs, denoting the speed and rotational rate of the agent, respectively. The agents are required to navigate along concentric orbits with the target being the center: $\lim_{t\to+\infty} r_i(t) = r_d$, where $r_i \triangleq \sqrt{(x_i - x_d)^2 + (y_i - y_d)^2}$; (x_d, y_d) denotes the target position; and r_d is the pre-defined radius of circumnavigation. The reference formation can be formulated on either the same orbit [126] or multiple orbits [125] according to specific requirements. In [125], control laws of circumnavigation for different agents are heterogeneous, such that different behaviors with respect to their neighbors and the target can be different. A systematic methodology is proposed to calculate parameters for heterogeneous protocols.

Another similar formulation is set-surrounding formation control [127], where agents are required to surround a given convex set X (instead of the target in circumnavigation):

$$\lim_{t\to+\infty} \left(w_{ij}\left(x_j(t) - P_X(x_j(t))\right) - \left(x_i(t) - P_X(x_i(t))\right)\right) = 0, \tag{22}$$

where x_i denotes the state of the i-th agent; $P_X(x_i)$ denotes the projector vector of the i-th agent onto the convex set X; and w_{ij} denote the weight coefficients of the edges. There are consistent cases, where agents should surround the convex set with same nonzero distances to X and desired projection angles between each pair of agents; and inconsistent cases, where agents should converge to the convex set.

Leader-escort control [128] can be formulated based on set-surrounding and bipartite control [110]. The bipartite control indicates that agents can be divided into two groups, where agents in the same group are cooperative, and agents from different groups are antagonistic. The antagonistic scheme of agents can be implemented by assigning non-positive elements to the adjacent matrix. Leader-escort is achieved by enabling the two antagonistic groups of agents to surround the same target or convex set with desired distances.

Practical applications of formation control may include synchronizing attitudes of multiple satellites. A typical result is proposed in [129], where the satellites are modeled in quaternion. Nonlinear distributed observer is designed for each follower to estimate the states of the leader, and distributed formation control can be designed for individual satellite.

4.4 Other results and open topics on formation control

Other recent results in the research of formation include target-point formation [130], where the followers are controlled to a virtual target point; iterative learning formation control [131], where stochastic factors can be considered; adaptive formation control [132], where uncertain Euler-Lagrangian swarm systems are considered; adaptive configuration control [28], where failures of some agents are considered; switching formation control with time-delays [133]; merging control [134], where a sub-formation of agents are controlled to merge into another sub-formation. Due to limitation of our knowledge, there exist other achieved results that are not listed.

Some open topic in formation control include experimental implementation with complex nonlinear dynamics and collision avoidance strategy, formation control with real-time localization, and pure distance-based formation.

5 Recent developments in flocking for swarm systems

The flocking problem for swarm systems distinguishes from distributed formation. In flocking control, it requires agents maintain some certain distances, but not necessarily in any rigid formations. For instance, in flocking control, agents are required to form a triangle or rectangle (or other shapes of) formation with fixed length of edges, so that each agent maintains its distances with others; possible transitions among various shapes are allowed if the distances between agents are maintained. Comparatively, in formation control, the formations should usually be in specified time-invariant or time-varying shapes.

There exist three basic rules proposed in [135] as fundamental requisites of flocking, namely flock centering, collision avoidance, and velocity matching. Early results can be traced back to [14], where several flocking strategies are designed for second-order multi-agent systems in different scenarios. Fundamentally, flocking protocols in [14] are constructed by three terms including a gradient-based term, a consensus term and a navigational feedback term given as follows:

$$u_i = \sum_{j\in\mathcal{N}_i} \phi\left(\|x_j - x_i\|_\sigma\right) \boldsymbol{n}_{ij} + \sum_{j\in\mathcal{N}_i} a_{ij}(x)\left(v_j - v_i\right) - c_1(x_i - x_r) - c_2(v_i - v_r), \tag{23}$$

where $\phi(\cdot)$ is a potential function; $\boldsymbol{n}_{ij}$ is a vector connecting agent i and j; a_{ij} denotes the corresponding element of the adjacent matrix; x_r and v_r are reference position and velocity, respectively.

Flocking control with dynamic external disturbances can be referred to a recent work [136]. The dynamic flocking control law is designed in the similar form of (23) by

$$u_i = -\sum_{j\in\mathcal{N}_i(t)} \nabla_{q_i}\psi\left(\|q_i - q_j\|\right) - \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t)(\xi_{2i} - \xi_{2j}) - D_i\hat{\omega}_i, \tag{24}$$

where ψ is the specific potential function [137] to avoid collisions; and ξ_{2i} and $\hat{\omega}_i$ are outputs of state observer and disturbance observer, respectively. Some adaptive terms with updating laws can be added to cancel the parametric uncertainties in the flocking control of swarm systems [138].

The main drawback of flocking control is that it is incapable of covering relatively large area; and anti-flocking control is proposed to overcome this drawback. Three main rules should be satisfied in anti-flocking control, namely de-centering, collision avoidance, and selfishness [139]. Semi-flocking is proposed to combine the advantages of both flocking and anti-flocking [139]. In semi-flocking control, small groups of agents are flocked around specific targets, while some agents are left for searching new targets. From the flocking control law (23) (or (24)), it can be seen that flocking control is composed by three parts, i.e., a gradient-based component, a velocity consensus component, and navigational feedback. Semi-flocking is different from flocking in that, the navigation feedback is modified, such that each agent is capable of deciding whether to track a target or to search for new target.

With the strategy of MPC, flocking with constraints can be guaranteed for 2nd-order swarm systems [140]. In the proposed flocking control, linear MPC is applied to explicitly address input constraints. In MPC for flocking, the aim of optimization is to minimize total distances of agents from α-Lattice; control efforts are also considered in the cost function. A challenge in distributed MPC for swarm systems is that, prediction of neighboring accelerations is often unavailable, thus cannot be processed in its classic way. It is supposed in [140] that predictive neighboring accelerations are always zeros; through this assumption, the predictions can be processed without real neighboring accelerations. Similar assumptions on zero neighboring accelerations have been witnessed in some other projects on distributed MPC for multi-agent systems [141]. Geometric properties of the optimal path are used to prove the feasibility and stability of the closed-loop system with MPC.

Flocking protocols for a 2nd-order swarm system in a constrained space are proposed in [142], where the conception of mirror velocity is proposed to describe bouncing phenomenon if the agent hits the boundary. The flocking control is designed by

$$u_i = -\sum_{j\in\mathcal{N}_i} a\left(\|p_{ij}\|\right)\left(v_i - L_i L_j v_j\right), \tag{25}$$

where $a(\cdot)$ is a non-increasing Lipschitz function; p_{ij} denotes the relative position with respect to the i-th agent and the j-th agent; v_i denotes the velocity of the i-th agent; and L_i is a switching matrix describing the bouncing phenomenon of agents.

Deep analysis and simulations indicate that, behaviors of individuals in flocking swarm systems are more influenced by number of its neighbors rather than the size of the swarm system [143]. It is also discovered that, in specific communication topologies, nodes are ranked by considering their influences on the overall system, and ranked nodes and influences can be utilized to steer the swarm system among various states [144]. In a swarm system, there might be some non-aligners, and their influences on flocking against aligners are thoroughly investigated in [145], where a critical fraction of non-aligners is proposed to indicate that the swarming would be abolished in case of considerable number of non-aligners.

Application cases of flocking control can be found in [146], where robotic swarm systems subjected to non-holonomic constraints are considered. A unified configuration of formation, flocking, and path-following is established. The obtained theoretical results are demonstrated by simulation and experiments.

Experiments of integrated flocking, formation and localization implemented with large swarm of robots can be found in [147], where 1024 robots named Kilobot are programmed to perform swarming and self-assembling by using only limited local information. Additional programming has been conducted to rectify collective movements if there are possible outliers.

Some other recent progresses in flocking protocols of swarm system include flocking by using position-only measurement [148], styled-velocity flocking [149], adaptive flocking control for Euler-Lagrangian systems with parametric uncertainties and time-delays [150], and flocking in random communication radius [151]. An interesting experimental implementation of a swarm robotic fish system in flocking has been displayed in [31].

There are still some open issues in flocking of swarm systems, e.g., flocking and tracking with connectivity preservation for uncertain exosystems, and adaptive flocking for nonlinear multi-agent systems with uncertain parameters and time-delays.

6 Recent developments in containment control

The containment control is proposed for swarm systems to perform more advanced functions than consensus or formation. In containment, all followers are required to converge into the convex hull generated by leaders of the the swarm system. Suppose that, in the swarm system, there exist M leaders and $N-M$ followers. The aim of containment could be stated mathematically by

$$\lim_{t\to+\infty}\left(x_i(t) - \sum_{j=M+1}^{N} \alpha_j x_j(t)\right) = 0, \tag{26}$$

where $\alpha_j \geqslant 0$ and $\sum_{j=M+1}^{N}\alpha_j = 1$. Physically, containment control distinguishes from formation or flocking; it does not necessarily request a constant distance between agents. Early results of containment control for swarm systems are reported in [20], where a hybrid Stop-Go policy is introduced.

Denote the leader set and the follower set by $\mathcal{R}$ and $\mathcal{F}$, respectively. A fundamental containment protocol [152] is proposed for first-order swarm system with stationary leaders by

$$u_i = 0, \quad i \in \mathcal{R}, \tag{27}$$

$$u_i = -\sum_{j\in\mathcal{F}\cup\mathcal{R}} a_{ij}\left(x_i - x_j\right), \quad i \in \mathcal{F}, \tag{28}$$

and containment protocol with dynamic leaders can be proposed based on results of stationary leaders. Distributed containment protocol for heterogeneous swarm systems is investigated in [153], where both first-order and second-order followers are included.

Performances of the closed-loop swarm system with containment control can be improved by including neighboring control efforts; however, the problem of time-delay should be well addressed if neighboring input information is included in the protocol. In [154], a containment protocol with neighboring controls and time-delay is proposed by

$$u_i = \frac{1}{d}\left(-\gamma \sum_{j\in\mathcal{F}\cup\mathcal{R}} a_{ij}\left(x_i - x_j\right) + \sum_{j\in\mathcal{F}\cup\mathcal{R}} a_{ij} u_j(t-\tau)\right), \ i \in \mathcal{F}, \tag{29}$$

where τ denotes the time-delay in acquiring neighboring input information. It can be proved that, provided there exists a united spanning tree in its communication graph, then the distributed containment could be accomplished asymptotically with the protocol given by (29).

A containment protocol for high-order singular swarm systems with time-delays has been designed in [155], where the problem of containment for high-order system is decoupled into stabilization problems of several lower-dimensional systems with time-delays, and the problem is solved by using the LMI approach. Containment control with output feedback for high-order swarm system can be found in [156].

Apart from time-delay, sampled-data problem is addressed in some recent researches [157]. The concern is that it is almost unrealistic to obtain information continuously from large numbers of neighbors. In [157], aperiodic sampling is considered, indicating that neighboring information is sampled at time-varying sampling intervals. It is proved that, containment control design for the aperiodic sampled-data swarm system can be transformed into control design for linear discrete-time system, and the small-gain approach can be applied to ensure the stability of the swarm system.

In [158], an adaptive protocol is designed for containment of nonlinear swarm systems with uncertainties. The swarm system in [158] is composed by heterogeneous agents represented by paramaterizable uncertain nonlinear functions. The adaptive control for containment is formulated and designed by using the new M-Matrix approach. The containment protocol for heterogeneous multi-agent systems could be designed within the theoretical framework of output regulation [159], where the leaders are described by exogenous systems. Dynamic state-feedback control law is applied to solving the containment control problem. Researches on finite-time containment control can be found in [88].

Some interesting topics for further investigation include containment control with switching communication graph, sampled-data containment control for heterogeneous swarm systems, and containment for time-varying nonlinear swarm systems.

7 Recent developments in optimization for swarm systems

In this section, the topic of optimization is divided into optimal coverage control and optimal mission planning of swarm systems.

7.1 Optimal coverage control

In optimal coverage control, it requires that the agents in the swarm systems be assigned to cover some spatial areas to optimize some indexes of time, energy or communication efficiency. In coverage control problem, the individual agent is usually a sensor A_i with its position $p_i \in P$ and sensing radius r_i connected to a sensing network. The entire area Q to be covered can be given by a convex area or a non-convex area.

There are static coverage problem and dynamic coverage problem; and most nowadays research efforts are given to dynamic coverage problem, where locations of events in the area are subject to variation, and agents should respond to cover them. Theoretically, distribution density $\phi : Q \to \mathbb{R}^+$ can be defined as the function to reflect the probability of events that occur over the area Q [21]. A function $f(\|q - p_i\|)$, which is non-decreasing, can be proposed to evaluate the measurement performance of agent i located at p_i with respect to the event occured at q. The area Q can be described and covered by a partition $\mathcal{W} = \{W_1, W_2, \ldots, W_n\}$, and the performance index to be optimized can be designed by

$$\mathcal{H}(P, \mathcal{W}) = \sum_{i=1}^{n} \int_{W_i} f(\|q - p_i\|)\phi(q)\mathrm{d}q, \tag{30}$$

indicating that the optimization is solved to obtain the optimal locations of agents and partitions of the area to be covered. It should be noted that the partition is continuously updated. Partition can be optimized by using Voronoi Partition [160], where the partition can be transformed to a function with respect to location of agents, and the index of performance can be described by $\mathcal{H}(P, \mathcal{W}(P))$. It follows that Lloyd algorithm can be used to solve the coverage problem [21].

The optimal coverage with respect to persistent awareness for a convex area with communication loss is proposed in [161], where a closed path is used to design the coverage controller. The optimization in the coverage control design can be solved through mixed-integer nonlinear programming (MINLP).

The optimal coverage for non-convex area with swarm systems can be found in [162,163]. Difficulties in optimally covering non-convex area include that, even if the distance between two agents is short, information would still be lost due to spatial obstacles. In [162], a new technique of vision-based power diagram is applied for partition of non-convex areas, such that heterogeneous sensing capability is addressed. A gradient-based coverage control is proposed, such that local optimization of index is achieved.

In [164], the optimal coverage can be accomplished with searching agents and servicing agents. The searching agents are applied as mobile sensors detecting potential events within neighborhood; and the servicing agents conduct optimal coverage by using the information acquired from searching agents. Typical coverage algorithm is applied in this research, and experimental results are obtained.

Other new recent results in optimal coverage by using swarm systems contain: optimal coverage with stochastic intermittent communications [165], clustering optimal coverage [166], robust adaptive coverage in case of external disturbances, noises and uncertainties [167], and optimal coverage with time-varying density functions [168].

7.2 Optimal mission planning

The objective of optimal mission planning for swarm systems is to optimally allocate resources or/and assign tasks to agents. Mission planning can be solved by optimization techniques, sometimes with mixed integer programing. The objective function (or index) to be optimized can be formulated in higher levels for optimal energy-efficiency and/or time-efficiency. Constraints can be constructed by modeling physical actuator saturations and spatial limitations. The MPC framework can be used in optimal mission planning [32] of swarm systems because of its potential of handling explicit constraints and robust responses with respect to external disturbances.

For the theoretic statement of the problem of the high-level mission planning, a language named linear temporal logic ($\mathrm{LTL}_{-\mathrm{X}}$) is designed in [51], where the missions for the swarm system are routing problems for vehicles. The language of $\mathrm{LTL}_{-\mathrm{X}}$ can be applied in other types of optimal mission planning such as the case in [169]. Based on $\mathrm{LTL}_{-\mathrm{X}}$, other types of languages for more advanced mission planning can be proposed, temporal action logic (TAL) in [22] for example.

Integrated search, task allocation and tracking was proposed in [98] for swarm fixed wing aerial vehicles. The high-level control logic is constructed based on a finite state automaton including several flight modes.

In [170], genetic algorithm (GA) has been used for optimal mission planning in cooperative surveillance, such that robust and flexibility can be guaranteed for the swarm system. A new heuristic method is proposed for optimal mission planning and routing of multi-UAV system in [171] in case of communication limits. In [172], the optimal mission planning for the multi-UAV system is accomplished by two steps: optimal path planning between tasks, and optimal routing subject to requirements on time-efficiency. A specific mission planning for protecting a harbor area is reported in [173].

7.3 Some open topics in optimization of swarm systems

There currently exist some open topics in optimization of swarm systems. At the current stage, it is of significant interest to design optimal coverage control with time-varying coverage metrics. Coverage for mobile targets is also considerably challenging. Noisy stochastic communications are still expected to be incorporated in optimal coverage control and optimal mission planning. Moreover, it is fairly desirable to consider nonlinear heterogeneous agents in optimal mission planning.

8 Recent developments in sensor networks

In spite of the promising benefits such as abundant information free of spatial limit that wireless sensor networks are capable of providing, the theory of estimation still confronted new challenges in recent years. Some negative influences brought by wireless sensor networks on performances of estimation have been investigated in [174]. The unreliability of interaction/communication channels (for example, data packet quantization, and data dropout) has been researched in [175]. The aforementioned work mainly focus on the estimator stability analysis under the imperfect communication scenarios. The new conception of controlling communications among remote estimators and sensors originates based on the limited resources of communication among individual sensors in the wireless network. Some previous researches address the constrained resource problem by using a certain amount of bits to construct an analog measurement, such that the constrained bandwidth could be sufficient for distributive wireless sensor networks [176]. In [177], the identification problem is investigated for an ARMA model with quantized and intermittent measurements, and the MLE estimator are implemented by using the EM-based algorithm. The estimation problem subject to energy constraint is analyzed in [178], where an online energy saving schedule is proposed for the battery-powered sensors with multiple power options for packet transmission. For the efficiency of the communication resources, at each time, the sensors purposely abandon some data packets that are not so important. Adaptive packet transmission is applied to estimate states of a dynamic system [179, 180]. The problem of maximum likelihood estimation is formulated in [180] for a linear time-invariant system, where a deterministic event-based sensor schedule is designed, and it is illustrated that the complexity of computation accumulates with the length of the time horizon. A one-step event-based MLE is proposed to compromise the increasing complexity. Liu et al. [181] focused on the distributed event-based estimation, where individual sensors only send the important information to their neighbors. The upper bound has been calculated for the covariance of estimation obtained by the distributed filter.

9 Case studies and future topics

9.1 Guidance laws for swarm systems

Early researches on distributed guidance and cooperative guidance can be dated back to [182], where multiple missiles are required to attack one stationary targets simultaneously. Results in [182] can be extended to incorporate constraints on accelerations [183]. Another approach to synchronize terminal times of missiles is to adaptively adjust navigation gains [184, 185].

Sometimes there possibly exist multiple intruders, such that the interception cannot be simply completed by only one rotary UAV. An intuitive strategy is to employ multiple UAVs to intercept the

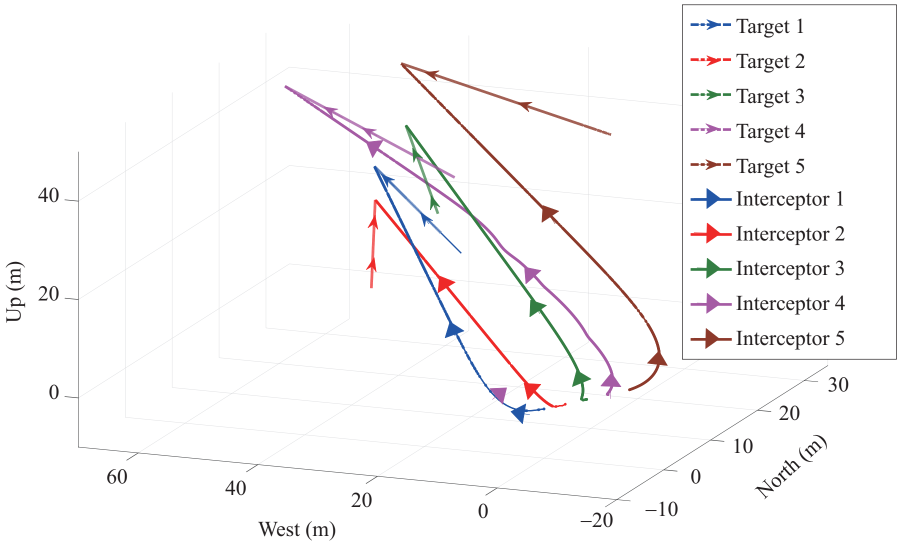

Figure 1 (Color online) Interception by using multi-UAVs system.

intruders, implying that it is necessary to design guidance laws for the multi-UAVs system. In the aforementioned works on distributed or cooperative guidance, however, the scenarios under consideration are often multiple interceptors versus one target; there are very limited results on guidance laws for UAVs or missiles to intercept multiple targets.

In our recent research, distributed guidance and control are designed for the multi-UAVs system to intercept multiple intruders. The proposed distributed guidance and control are based on classic parallel navigation (PN) integrated with velocity feedback. Potential functions are applied such that neither inter-agent collisions nor mis-interceptions would ever happen. The main novelties of the proposed distributed guidance and control contain: (1) it does not necessarily depend on previous results of consensus; and (2) the scheme of collision avoidance is orientation-based (or line-of-sight based, instead of position-based or distance-based). It is proven that, with the proposed strategy of distributed guidance and control, the UAVs are capable of intercepting the intruding targets with finite times and within non-zero hitting velocities; there would be neither inter-agent collisions nor mis-interceptions before interceptions. A simulation result is shown in Figure 1, where the multi-UAV system is capable of intercepting all intruding targets; neither interagent collisions nor mis-interceptions have ever been observed.

Some open problems following this research include mission planning based on time/energy-efficiency, path planning and event-triggered control.

9.2 Event-triggered control for swarm systems

A swarm system is usually deployed in hostile environments, where agents are powered by batteries with limited amount of energy. Existing protocols for consensus, flocking/cohesion, formation, are mostly based on the assumption that the communication bandwidth can be assigned arbituarily large such that the interaction among agents are compatible with continuous communications. When energy efficiency is a major concern, it is crucial to decide what and to whom to communicate and when the communication should occur. The concept of event-triggered distributed control and optimization has been applied in swarm systems regarding to energy efficiency and distributed scenarios [186, 187].

Consider the simplest case where the agent i is modeled by single integrator $\dot{x}_i(t) = u_i(t)$. Our purpose is to (1) design a communication protocol, also called "event-triggering condition", to mediate the communication from one agent to another; and (2) design a control algorithm based on shared data through intermittent communication so that all agents reach consensus.

The challenge of using event-triggered control for swarm systems is to reduce the control updates and communication cost simultaneously. This difficulty task was overcome in [188], where a sampled-data approach is proposed to prove that the inter-event times for any agents are lower bounded by a

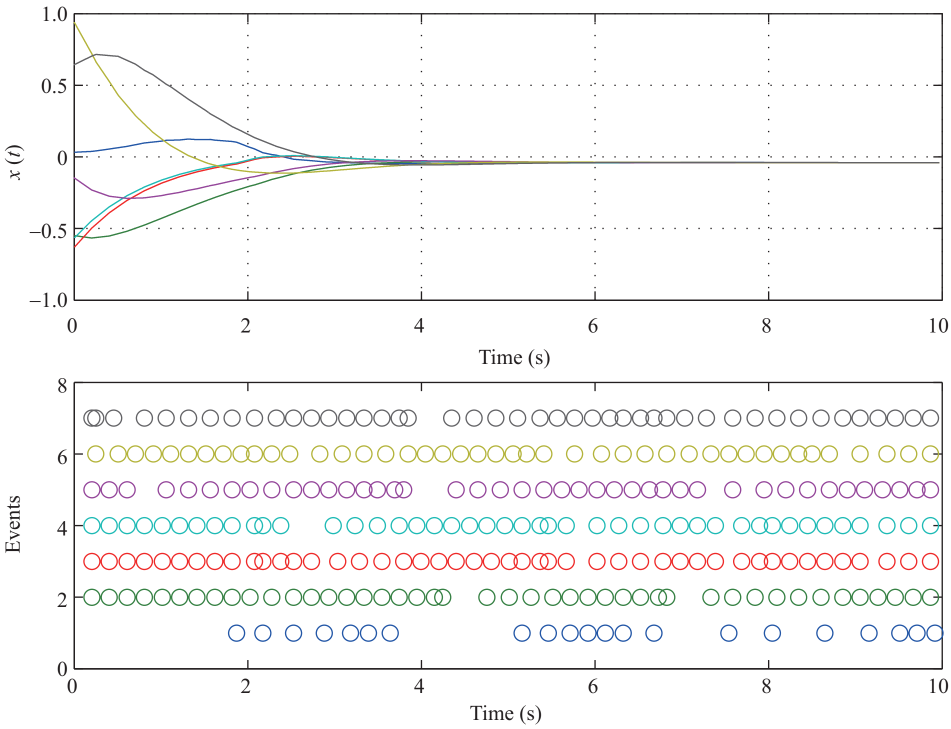

Figure 2 (Color online) State trajectories and event instants.

common sampling period. For clarity, two time sequences are given first: event sequence $\mathcal{E} = \{t_k^i\}_{k\in\mathbb{Z}_+}$ and detection sequence $\mathcal{T} = \{kh\}_{k\in\mathbb{Z}_+}$. Here i means agent i, and h is a common sampling period shared by all agents. Based on these two sequences, we can define $\hat{x}_i(t) = x_i(t_k^i)$, for $t \in \left[t_k^i, t_k^{i+1}\right)$, and $\bar{x}_i\,(t) = x_i(kh)$, for $t \in [kh, kh+h)$. The event instants $\mathcal{E}$ are the time instants when agent i broadcasts $x_i(t_k^i)$ to all its neighbors. Assume that the initial event instants $t_0^i = 0$ for all i. Then, the event instants $\mathcal{E}$ can be determined iteratively

$$t_{k+1}^i = \inf\left\{t \in \mathcal{T} |\, \|e_i\,(t)\|_2^2 \geqslant \sigma_i \,\|z_i\,(t)\|_2^2\right\},$$

where $e_i\,(t) = \hat{x}_i\,(t) - \bar{x}_i\,(t)$, and $z_i\,(t) = \sum_{j\in\mathcal{N}_i}(\bar{x}_i\,(t) - \bar{x}_j\,(t))$. In this way, we reduce the unnecessary transmissions between neighboring agents. The control input for agent i is given by $u_i\,(t) = -z_i\,(t)$. It is easy to see that the control signal is updated whenever events take place in agent i or its neighbors. Therefore, the control update frequency is reduced as well.

Recently, sampled-data average consensus is investigated for swarm systems within the event-triggered theoretical framework to reduce the frequency of communications among various agents [189]. In comparison with some previous results on sampled-data consensus [188], the novelties are threefold. The event-triggered sampled-data protocol is designed for average consensus of swarm systems with balanced and fully connected graphs, guaranteeing a positive-definite lower bound of inter-event intervals for all agents. Effectiveness of the proposed event-triggered protocol is analyzed in detail, and it is shown that states of agents with balanced and fully connected graph are capable of asymptotically converging to the average of initial states of all agents with appropriately selecting sampling period and control parameters. The maximum allowable sampling period is carefully characterized for the proposed protocol, and it is shown that requirements found in [188] can be relaxed for undirected graphs. It is shown that the proposed event-triggered protocol requires no more information exchanges than the periodic communication and control algorithm, as illustrated by Figure 2.

There are still lots of open problems to be tackled in event-triggered control of swarm systems. As well known, the classical consensus algorithms use only the relative information. However, in most event triggered control, absolute information is used for event detection. The challenge still exists in detecting

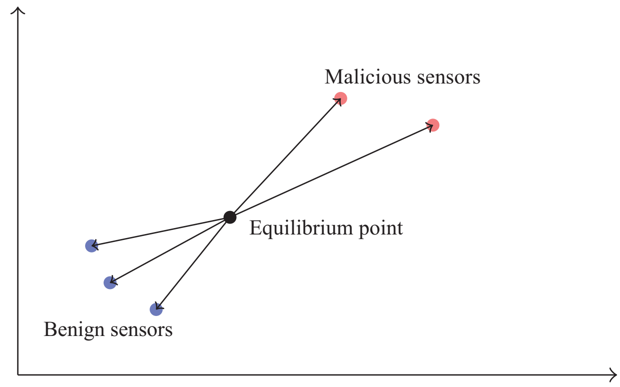

Figure 3 (Color online) Force analogy of resilient estimation. The equilibrium point where the sum force is zero should be bounded.

an event by using only relative information. Moreover, the problem of excluding Zeno behavior for dynamic synchronization is still open.

9.3 Resilient estimation for swarm systems

The security challenges in swarm systems have been discussed in [190–192]. Among all the threats, the insecure sensory information acquisition may cause the most severe consequences since many system functional units such as controllers and planners rely on the accurate system state information. For example, in a target following mission, the swarm robots monitor the position of a moving target, and exchange measurements with each other. To maintain an appropriate trajectory, each robot must generate a reliable estimate of the target state using the collective measurements. However, if the inter-agent communication, typically using radio-frequency and infra-red technologies, is hampered or even blocked, the precision of the target state estimate will be reduced. If some agents are compromised and manipulated data are injected in the information flow, the error of the state estimate may diverge.

In our recent work [193], we study a general problem of resilient estimation for swarm systems under integrity attacks. Integrity attack in our context means that a subset of agents are fully controlled by the attacker. Since in most cases we know nothing about the attackers, the malicious measurements are assumed to be arbitrary. The maximum number of them is bounded by a constant due to the limited capability of attackers.

The choice of an estimator for each agent is now debatable. A least square based estimator is not resilient to integrity attacks since single malicious measurement can drive the mean arbitrarily far. Similarly, the classical Kalman filter cannot generate a reliable estimate even if one sensor is manipulated, because Kalman filter is taking all measurements equivalently. We make our efforts to answer the questions: given an estimator for an agent, what can we talk about its resilience and performance? And are there any systematic analysis procedures we can employ? We propose an analytical approach to solve the problem by taking a convex optimization based estimator as an example: $\hat{x} = \arg\min_x \sum_{j \in \mathcal{S}} \varphi_j(y_j - x)$, where x is the state of interest and y_j is the measurement of jth sensor, and φ_j is a convex function. Our proposed approach can be applied to analyze the resilience of a class of commonly used estimators, and the estimation performance can be quantified, which is always ignored in the existing works. The intuition of our approach for resilience analysis comes from force analogy. Imagine that the function φ_j is a potential field and each measurement exerts a force on the optimal estimate $\hat{x}$. Under integrity attacks, we have to guarantee the equilibrium point (i.e., the point where sum force is zero) must be bounded, as illustrated by Figure 3. We have analyzed the sufficient and necessary conditions for resilience. One special case is that, for resilient estimation, benign sensors must be more than malicious sensors if all agents are homogeneous.

One of the open problems is how to analyze the resilience of an estimator against correlated attacks. Since it is not easy to decouple the attacks exerted on different sensors, a new methodology may be needed. Another interesting problem from the attacker's view is to design a optimal integrity attack to induce the worst possible damage. The tools in game theory may be useful to handle the design and analysis.

10 Conclusion

In this survey, we have reviewed and summarized some significantly new progresses on control and optimization approaches for swarm systems, especially those within 2013–2015, including some newly proposed results in distributed consensus, coordinated formation, flocking, optimal containment, optimal coverage and mission planning, and sensor networks. The purpose of this survey is to give a brief overview of this research area in perspective of control engineers. There inevitably exist some other significant researches on swarm systems that have not been discussed due to the limitation of our knowledge. Based on our current research works, several new topics on swarm systems are suggested.

Conflict of interest The authors declare that they have no conflict of interest.

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