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Multirobot Simultaneous Path Planning and Task Assignment on Graphs with Stochastic Costs

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I. Introduction

Multi-robot task allocation problems where robots have to move to target destinations arises in a number of applications including search and rescue, goods or parts transfer in warehouses. The problem of task allocation, i.e., allocation of robots to target destinations is coupled with the problem of computing a path for the robots to the destination. We consider a situation, where there are static obstacles as well as dynamic obstacles in the environment. We assume that each robot is equipped with a local reactive collision detector and planner to avoid collision with dynamic obstacles. There could be multiple paths that may be available between a robot-destination pair [7]. The cost of travel (like energy consumed, time taken, etc.) is non-deterministic because the robot may have to slow down or stop to avoid moving obstacles.

In this paper, we present a novel algorithm for multiple robot task allocation problems where the robots have to simultaneously plan paths and select target destinations under uncertainty about the travel costs. We assume that the robots move on a graph, e.g., an actual road network or a roadmap, which captures the collision-free configuration space of the robots [3], [4], [2] with respect to static obstacles. We assume that the cost of each edge of the graph is an independent random variable with known mean and variance. Thus, the cost of any path between a robot-destination pair will be a random variable. Our goal is to *simultaneously compute the assignment of tasks (targets) to robots as well as paths to reach the tasks and a minimum cost value (say y) such that we have a guarantee that the cost of every robot's path will be less than y with high probability (say 0.95) under any realization of the random costs*. Such a solution provides a quality guarantee (albeit probabilistic) on the solution of the simultaneous task assignment and planning (STAP) problem in the presence of uncertainty about the task execution costs.

II. Chance-Constrained STAP

We model the stochastic STAP problem as a chance-constrained simultaneous task assignment and planning (CC-STAP) problem. Let $G = (V, E)$ be the graph representing the roadmap, with $|V| = N$, and let there be n robots and destinations (or tasks). Let (u, v), $u, v \in V$ denote an edge. Let ${}^{i}c_{uv}$ be the random cost for robot r_i going through edge (u, v). We assume that the mean and variance for random cost are known. Let ${}^{i}x_{uv}$ be a binary decision variable which is equal to 1 when edge (u, v) is included in the path for r_i and 0 otherwise. Each robot, r_i, is initially at a given position $s_i \in V$ and each task $t_j \in V$, is located at given position. Let $z_{ij} = 1$ if robot r_i is assigned to task t_j and 0 otherwise. Define $V' = V \setminus \{s_i, t_j\}$. The integer program formulation for the CC-STAP problem is

$$
\begin{aligned}
&\min \quad y \\
&\text{s.t.} \quad \mathbb{P}\left(\sum_{u=1}^{N}\sum_{v=1}^{N} {}^{i}c_{uv}\,{}^{i}x_{uv} \le y\right) \ge p, \quad i = 1, \dots, n. \\
&\sum_{v \in \mathcal{N}(u)} {}^{i}x_{uv} - \sum_{v \in \mathcal{N}(u)} {}^{i}x_{vu} = \begin{cases} 1, & \text{if } u = s_i, \\ -z_{ij}, & \text{if } u = t_j, \\ 0, & \forall u \in V', \end{cases} \\
&\qquad\qquad i = 1, \dots, n. \\
&\sum_{i=1}^{n} z_{ij} = 1,\ \forall j; \quad \sum_{j=1}^{n} z_{ij} = 1,\ \forall i. \\
&{}^{i}x_{uv} \in \{0, 1\}, \quad z_{ij} \in \{0, 1\}.
\end{aligned}
\tag{1}
$$

where $\mathcal{N}(u) \subset V$ denote the neighbors of node u. The third set of constraints represent that each robot is assigned to one task and each task is assigned to one robot. The second constraint ensures that each robot selects a simple path to its assigned task. The total cost of the path for each robot in any realization is less than a value y with at least a pre-specified probability p (chance constraints in Equation (1)). The objective is to minimize the value y on the cost for all paths. Note that the technical challenge in solving the problem above directly is that it is an integer nonlinear optimization problem where the nonlinearity arises due to the chance constraints. The *key contribution* of this work is to prove that the optimal solution to CC-STAP can be obtained by solving two related sub-problems, namely, (a) chance-constrained shortest path (CC-SP) problem between all robot-task pairs and (b) linear bottleneck assignment problem (LBAP) formed from the outputs of the CC-SP problems. This observation provides the recipe for a two-step algorithm described in Section III.

A. Chance-constrained shortest path problem

The CC-SP for a robot-task pair, i.e., (s_i, t_j) is: *given a graph where the means and variances of random edge cost (${}^{i}\mu_{uv}$ and ${}^{i}\sigma^2_{uv}$) are known, compute a path with minimum value ${}^{i}y_j$ such that the actual path cost in any realization is no greater than ${}^{i}y_j$ with a pre-specified probability p.* Let the feasible set corresponding to the second constraint in

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Equation (1), with $z_{ij} = 1$ be denoted by $\mathcal{X}_1$. The CC-SP for r_i going to t_j is

$$\begin{aligned} &\min\ {}^iy_j \\ &\text{s.t.}\ \sum_{u,v} {}^i\mu_{uv}\ {}^ix_{uv} + C\sqrt{\sum_{u,v} {}^i\sigma^2_{uv}\ {}^ix_{uv}} \leq {}^iy_j \\ &\qquad {}^ix_{uv} \in \mathcal{X}_1 \end{aligned} \tag{2}$$

The first constraint is equivalent to the chance constraint in (1) for r_i. It ensures that the path cost for r_i going to t_j in any realization is no greater than a value ${}^iy_j$ with at least a pre-specified probability p. If the distribution of the random costs are Gaussian, $C = \Phi^{-1}(p)$, where $\Phi^{-1}(p)$ is the inverse cumulative distribution function of a standard Gaussian. If we know only the mean and the variance of the distribution, $C = \sqrt{\frac{p}{1-p}}$. The objective is to find such path with the smallest ${}^iy_j$.

B. Linear bottleneck assignment problem

Let ℓ_{ij} be the cost of assigning r_i to t_j, which is the optimal objective value of the problem (2) for r_i to t_j, i.e., $\ell_{ij} = \min\ {}^iy_j$. The LBAP is: *given cost of the assignment ℓ_{ij}, assign each robot to a unique task such that the maximum cost of robot-task pair among the assignment is minimized.*

$$\begin{aligned} &\min\ \max_i \sum_j \ell_{ij} z_{ij} \\ &\text{s.t.}\ \sum_i z_{ij} = 1,\ \sum_j z_{ij} = 1,\ z_{ij} \in \{0,1\}\ \forall i,j. \end{aligned} \tag{3}$$

where z_{ij} is as defined in (1). The constraints guarantee that each robot-task assignment is unique. As ℓ_{ij} is obtained from (2), a feasible solution in problem (3) also satisfies the chance constraint and $\mathcal{X}_1$. Therefore the feasible solution of problem (3) is feasible to problem (1) and vice versa. We further proved that the objective in problem (3) is equivalent to objective in (1). Therefore CC-STAP can solved by two sub-problems: CC-SP and LBAP.

III. Solution Algorithm and Results

We use a two-step approach to solve the CC-STAP problem *optimally*. First, we formulate and solve CC-SP problems for each robot-task pair. Second, we use the optimal solutions for each CC-SP to formulate and solve the LBAP. LBAP can be solved by existing algorithms like threshold algorithm [1]. The key challenge is in solving the CC-SP problem for which we present a novel approach.

The solution space of the CC-SP problem can be geometrically analyzed on a two-dimensional space called variance-mean plane [8], [9]. To see this, note that for each path, we can associate a point with coordinates (σ^2, μ) where $\mu = \sum_{u,v} {}^i\mu_{uv}\ {}^ix_{uv}$ and $\sigma^2 = \sum_{u,v} {}^i\sigma^2_{uv}\ {}^ix_{uv}$. Thus all possible paths between a source destination pair can be represented as points in the mean-variance plane. The optimal solution to CC-SP is a vertex of the convex hull of this point set. However, enumerating all the points in the point set is computationally expensive. Nevertheless, using the geometric insight, we prove that the solution to problem (2) can be solved by solving a sequence of parameterized optimization problems given below:

$$\min \sum_{u,v} ({}^i\mu_{uv} + \lambda\,{}^i\sigma^2_{uv})\,{}^ix_{uv}\ \text{ s.t. }\ {}^ix_{uv} \in \mathcal{X}_1,\ \forall i \tag{4}$$

where λ is called the risk-aversion parameter. For any value of λ, the solution to problem (4), gives an extreme point (i.e., a vertex on the convex hull) on the mean-variance plane. The problem (2), thus, becomes a one-dimensional search on risk-averse parameter λ. Instead of enumerating all extreme points [6], [5], we first compute an upper bound of λ and then compute the extreme points by solving problem (4) with different value of λ within the obtained interval. The algorithmic steps and proof of correctness are analogous to our previous work on stochastic linear assignment [8].

The computational cost of our algorithm is $n^2KT_1 + T_2$ where n is the number of robots (tasks), K is the number of the deterministic shortest path problems solved, T_1 and T_2 are the computation cost for solving a deterministic shortest path problem, and a linear bottleneck assignment problem respectively. The main parameter that we do not know theoretically is the dependence of K on the number of nodes in the graph, N or the number of robots, n. Therefore, we perform simulations based on randomly generated data to study the dependence of K on n and N. Figure 1 shows the number of deterministic problems, i.e., n^2K solved as a function of the number of robots n. The quadratic dependence suggests that K is independent of n.

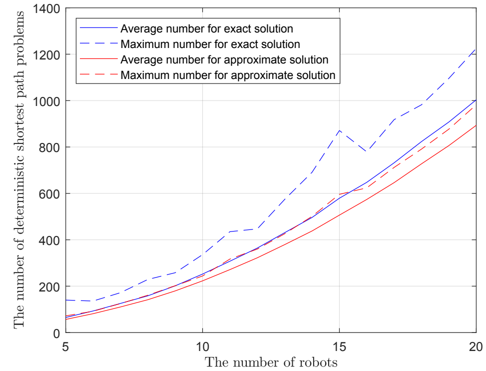

Fig. 1. Number of deterministic problems solved as the number of robots increase. The average and maximum is computed over 100 scenarios.

IV. Conclusion

We presented a novel algorithm to solve CC-STAP optimally. To the best of our knowledge, there are no available algorithms with theoretical guarantees on solution quality on stochastic STAP. Our approach is able to compute an exact solution (assuming the random edge costs are independent and Gaussian) and is more efficient than other related algorithm mentioned in [5]. The simulation results demonstrate that our algorithm is scalable with the number of robots as well as the size (i.e., number of nodes) of the road network.

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