

# **Assignment 2**

## **Image Segmentation**

### Pattern Recognition

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# 1. Load Datasets :

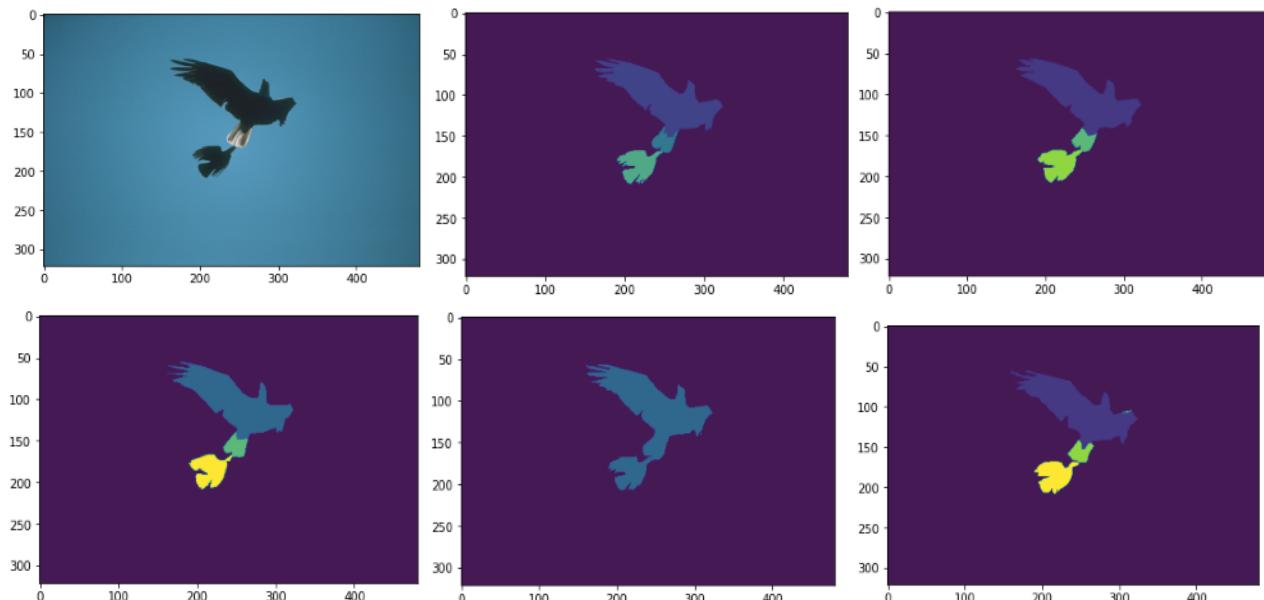
Drive authorisation

```
[ ] from google.colab import drive  
drive.mount('/content/drive')  
  
[ ] TrainPath = "/content/drive/MyDrive/data/images/train/"  
GroundTruthPath = "/content/drive/MyDrive/data/ground_truth/train/"  
NumOfPixels = 481*321  
Dimensions = 3  
NumOfImages=200  
Train, GroundTruth = DataLoading(TrainPath,GroundTruthPath,(NumOfPixels*NumOfImages,Dimensions))
```

# 2. Visualize image and groundtruth segmentation:

Function that reads an image and display an image with its associated ground truth segmentation.

```
[ ] def DataLoading(ImagesPath,GroundTruthPath,MatrixDimensions):  
    GroundTruthMatrix = []  
    ImagesList = listdir(ImagesPath)  
    ImagesMatrix = np.arange(MatrixDimensions[0] * MatrixDimensions[1]).reshape(MatrixDimensions)  
    NumOfPixels = MatrixDimensions[0] // len(ImagesList)  
    Dimensions = MatrixDimensions[1]  
    Count = 0  
    for Image in ImagesList:  
        GroundTruth = scipy.io.loadmat(GroundTruthPath+Image[:-4])  
        Img = PImage.open(ImagesPath+Image)  
        ImagesMatrix[NumOfPixels*Count:NumOfPixels*(NumOfPixels*Count),:] = np.array(Img).reshape(NumOfPixels,Dimensions)  
        for j in range(0,GroundTruth['groundTruth'].shape[1]):  
            GroundTruthMatrix.append(GroundTruth['groundTruth'][0][j][0][0])  
        Count += 1  
    return ImagesMatrix,GroundTruthMatrix  
  
[ ] plt.imshow(Train[0:NumOfPixels].reshape(321,481,3));  
plt.figure();  
plt.imshow(GroundTruth[0]);  
plt.figure();  
plt.imshow(GroundTruth[1]);  
plt.figure();  
plt.imshow(GroundTruth[2]);  
plt.figure();  
plt.imshow(GroundTruth[3]);  
plt.figure();  
plt.imshow(GroundTruth[4]);
```



### 3. Segmentation using K-means :

k-means implementation

```
[ ] def OwnKMeans(Data, NumOfClusters):
    NumberOfData = Data.shape[0]
    NumberOfFeatures = Data.shape[1]
    Mean = np.mean(Data, axis = 0)
    STD = np.std(Data, axis = 0)
    Centroids = np.random.randn(NumOfClusters,NumberOfFeatures)*STD + Mean
    OldCentroids = np.zeros(Centroids.shape)
    Clusters = np.zeros(NumberOfData)
    Distances = np.zeros((NumberOfData,NumOfClusters))
    for i in range(NumOfClusters):
        Distances[:,i] = np.linalg.norm(Data - Centroids[i], axis=1)
        Clusters = np.argmin(Distances, axis = 1)
        OldCentroids = deepcopy(Centroids)
        for i in range(NumOfClusters):
            Centroids[i] = np.mean(Data[Clusters == i], axis=0)

    return Clusters
```

a. Changing the K of the K-means algorithm between {3,5,7,9,11} clusters. Producing different segmentations and saving them as colored images {R,G,B} in ColorizedImage. Every color represents a certain group (cluster) of pixels.

```
[ ] ColorizedImages = []
Clusters = []
for K in range(3,12,2):
    for ImageNumber in range(5):
        Image = Train[NumOfPixels*ImageNumber:NumOfPixels+(NumOfPixels*ImageNumber)]
        #Model = OwnKMeans(Image,K)
        Model = ImplementedKMeans(n_clusters=K, random_state=0).fit(Image)
        Clusters.append(Model.labels_)
        ColorizedImages.append(Colorize(Model.labels_,Image,K))
```

```
[ ] def Colorize(Clusters,Image,NumOfClusters):
    ColorizedImage= np.zeros(Image.shape,dtype=np.uint8)
    Colors = []
    for i in range(NumOfClusters):
        Colors.append(np.array([randint(0, 255),randint(0, 255),randint(0, 255)]))
    for i in range(Image.shape[0]):
        ColorizedImage[i] = Colors[Clusters[i]]
    return ColorizedImage
```

- $prec_i = purity_i = \frac{1}{n_i} \max_j^k n_{ij}$
- b)i) F-measure
- $rec_i = \frac{n_{ij}}{|Tj_i|}$
- $F_i = \frac{2 * prec_i * rec_i}{prec_i + rec_i}$
- $F = \frac{1}{r} \sum_{i=1}^r F_i$

Precision: dividing the maximum element in each row over all elements in the same row(represents cluster)

MaximumPosition: finds the maximum element in which column

SummationOverPosition: sums the elements in the column determined by MaxPosition

Recall: divide maximum element in row over sum of the column

Applying the FMeasure rule then dividing it by ContingencyTable.shape which is the total number of elements (r)

```
[ ] def FMeasure(Clusters,Variables):
    FMeasure = 0
    ContingencyTable = contingency_matrix(Clusters,Variables)
    for i in range(ContingencyTable.shape[0]):
        SummationOverColumns = 0
        Precision = ContingencyTable.max(axis=1)[i] / np.sum(ContingencyTable[i])
        MaximumPosition = ContingencyTable.argmax(axis=1)[i]
        for j in range(ContingencyTable.shape[0]):
            SummationOverColumns += ContingencyTable[j][MaximumPosition]
        Recall = ContingencyTable.max(axis=1)[i] / SummationOverColumns
        FMeasure += 2*Precision*Recall / (Precision+Recall)
    return FMeasure/ContingencyTable.shape[0]
```

```
[ ] K = [3,5,7,9,11]
FMeasures = []
Count = 0
for j in range(0,25,5):
    for i in range(0,5):
        GroundTruthImage = GroundTruth[i].reshape(481*321)
        FMeasureValue = FMeasure(Clusters[j],GroundTruthImage)
        print("For K = ",K[Count]," and Ground Truth Image = ",i+1," F-Measure is ",FMeasureValue)
    Count +=1
    FMeasures.append(FMeasureValue)
print("Average of the M trials = ",sum(FMeasures)/len(FMeasures))
plt.scatter(K,FMeasures)
plt.title('F-Measure versus K for Ground Truth Image 5');
plt.gcf().set_size_inches(15,5);
plt.xlabel('K');
plt.ylabel('F-measure');
```

```
For K =  3  and Ground Truth Image =  1  F-Measure is  0.733609265856848
For K =  3  and Ground Truth Image =  2  F-Measure is  0.7295116629968853
For K =  3  and Ground Truth Image =  3  F-Measure is  0.7302320535106962
For K =  3  and Ground Truth Image =  4  F-Measure is  0.7497413292857241
For K =  3  and Ground Truth Image =  5  F-Measure is  0.7275946767018934
For K =  5  and Ground Truth Image =  1  F-Measure is  0.4912085358951834
For K =  5  and Ground Truth Image =  2  F-Measure is  0.4887564511051098
For K =  5  and Ground Truth Image =  3  F-Measure is  0.4891618523419341
For K =  5  and Ground Truth Image =  4  F-Measure is  0.5001493558707896
For K =  5  and Ground Truth Image =  5  F-Measure is  0.48757425890930106
For K =  7  and Ground Truth Image =  1  F-Measure is  0.4437953671489502
For K =  7  and Ground Truth Image =  2  F-Measure is  0.4339851856086162
For K =  7  and Ground Truth Image =  3  F-Measure is  0.43569961244645333
For K =  7  and Ground Truth Image =  4  F-Measure is  0.37503069922219556
For K =  7  and Ground Truth Image =  5  F-Measure is  0.4309020247250897
For K =  9  and Ground Truth Image =  1  F-Measure is  0.35863754662925157
For K =  9  and Ground Truth Image =  2  F-Measure is  0.34976384523644577
For K =  9  and Ground Truth Image =  3  F-Measure is  0.34491786390119633
For K =  9  and Ground Truth Image =  4  F-Measure is  0.30899422568959384
For K =  9  and Ground Truth Image =  5  F-Measure is  0.3477837148026481
For K =  11 and Ground Truth Image =  1  F-Measure is  0.34973526200935334
For K =  11 and Ground Truth Image =  2  F-Measure is  0.34142771854398285
For K =  11 and Ground Truth Image =  3  F-Measure is  0.3412101629572837
For K =  11 and Ground Truth Image =  4  F-Measure is  0.25993233981646496
For K =  11 and Ground Truth Image =  5  F-Measure is  0.34028302502015095
Average of the M trials =  0.46682754003181665
```

b)ii) Conditional Entropy

$$H(\mathcal{T}|C_i) = - \sum_{j=1}^k \left( \frac{n_{ij}}{n_i} \right) \log \left( \frac{n_{ij}}{n_i} \right)$$

$$H(\mathcal{T}|\mathcal{C}) = \sum_{i=1}^r \frac{n_i}{n} H(\mathcal{T}|C_i)$$

Created if condition for ContingencyTable so it skip if  $n_{ij} = 0$

```
def ConditionalEntropy(Clusters, Variables):
    ContingencyTable = contingency_matrix(Clusters, Variables)
    ConditionalEntropies = []
    for i in range(ContingencyTable.shape[0]):
        ConditionalEntropy = 0
        NumberOfElementsInCluster = np.sum(ContingencyTable[i])
        for j in range(ContingencyTable.shape[1]):
            if(ContingencyTable[i][j] != 0): ConditionalEntropy -= (ContingencyTable[i][j]/NumberOfElementsInCluster) * (np.log2(ContingencyTable[i][j]/NumberOfElementsInCluster))
        ConditionalEntropies.append(ConditionalEntropy/NumberOfElementsInCluster)
    return sum(ConditionalEntropies)/np.sum(ContingencyTable)
```

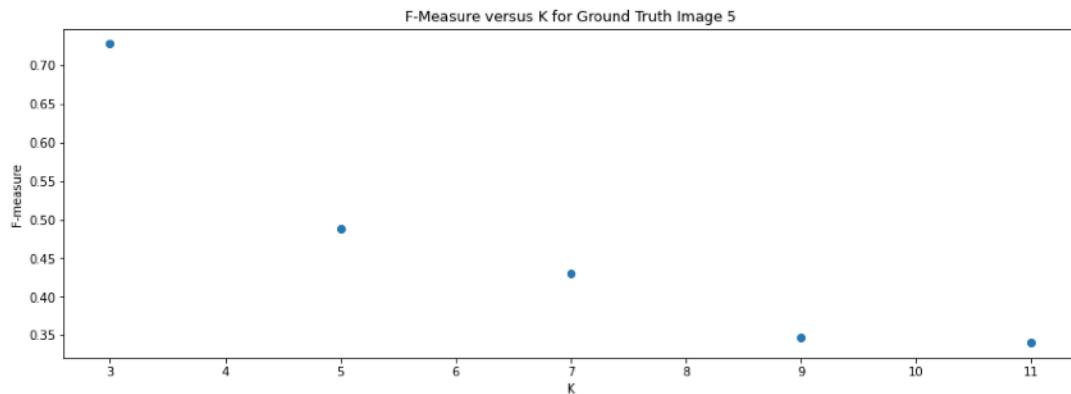
```
[ ] K = [3,5,7,9,11]
ConditionalEntropies = []
Count=0
for j in range(0,25,5):
    for i in range(0,5):
        GroundTruthImage = GroundTruth[i].reshape(481*321)
        ConditionalEntropyValue = ConditionalEntropy(Clusters[j],GroundTruthImage)
        print("For K =",K[Count]," and Ground Truth Image =",i+1," Conditional Entropy is ",ConditionalEntropyValue)
    Count +=1
    ConditionalEntropies.append(ConditionalEntropyValue)
print("Average of the M trials = ",sum(ConditionalEntropies)/len(ConditionalEntropies))
plt.scatter(K,ConditionalEntropies)
plt.title('Conditional Entropy versus K for Ground Truth Image 5');
plt.gcf().set_size_inches(15,5);
plt.xlabel('K');
plt.ylabel('Conditional Entropy');
```

```
For K = 3 and Ground Truth Image = 1 Conditional Entropy is 0.10152969371736463
For K = 3 and Ground Truth Image = 2 Conditional Entropy is 0.10779588196287074
For K = 3 and Ground Truth Image = 3 Conditional Entropy is 0.10324334469251822
For K = 3 and Ground Truth Image = 4 Conditional Entropy is 0.06480372543451053
For K = 3 and Ground Truth Image = 5 Conditional Entropy is 0.11565521755228422
For K = 5 and Ground Truth Image = 1 Conditional Entropy is 0.09849018683546802
For K = 5 and Ground Truth Image = 2 Conditional Entropy is 0.10470293857867158
For K = 5 and Ground Truth Image = 3 Conditional Entropy is 0.10017939207788484
For K = 5 and Ground Truth Image = 4 Conditional Entropy is 0.062107291192656726
For K = 5 and Ground Truth Image = 5 Conditional Entropy is 0.11198678655047652
For K = 7 and Ground Truth Image = 1 Conditional Entropy is 0.08658652772909427
For K = 7 and Ground Truth Image = 2 Conditional Entropy is 0.0943329422294311
For K = 7 and Ground Truth Image = 3 Conditional Entropy is 0.08936518644072566
For K = 7 and Ground Truth Image = 4 Conditional Entropy is 0.051666739996496944
For K = 7 and Ground Truth Image = 5 Conditional Entropy is 0.10219320314991216
For K = 9 and Ground Truth Image = 1 Conditional Entropy is 0.07394001624664394
For K = 9 and Ground Truth Image = 2 Conditional Entropy is 0.0827527282300966
For K = 9 and Ground Truth Image = 3 Conditional Entropy is 0.07861393835166973
For K = 9 and Ground Truth Image = 4 Conditional Entropy is 0.04079864010489978
For K = 9 and Ground Truth Image = 5 Conditional Entropy is 0.0905134569039672
For K = 11 and Ground Truth Image = 1 Conditional Entropy is 0.06558941674308778
For K = 11 and Ground Truth Image = 2 Conditional Entropy is 0.07405629508892825
For K = 11 and Ground Truth Image = 3 Conditional Entropy is 0.06931670151042064
For K = 11 and Ground Truth Image = 4 Conditional Entropy is 0.03476522903902387
For K = 11 and Ground Truth Image = 5 Conditional Entropy is 0.08203258984609278
Average of the M trials = 0.10047625080054656
```

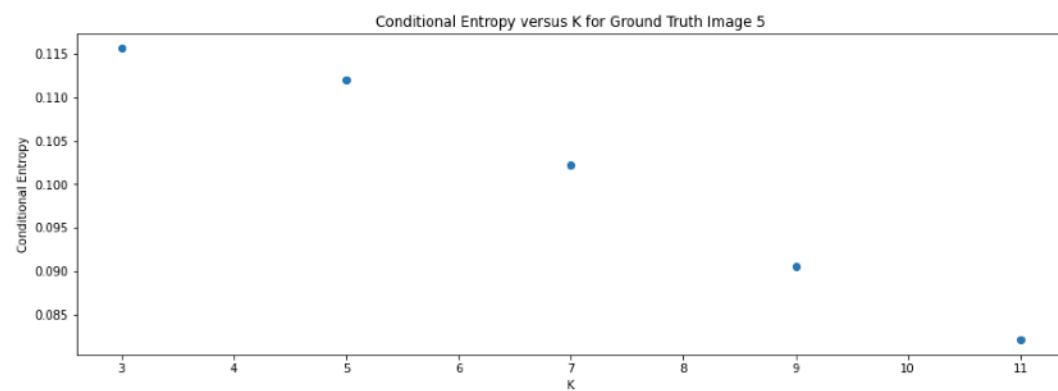
### Comment on F-measure & Entropy:

Both, F-measure and Conditional Entropy, decreases as the K for GroundTruth increases.

In F-measure it started from 0.7 and decreased till 0.3 due to oversampling



Conditional Entropy, measure of uncertainty, started 0.115 and decreased till 0.083



## 4. Big Picture

- a. Select the first five images and display their corresponding groundtruth against segmentation results using K-means at K=5.

Comment on the results

Disparity between results, when k increases over-clustering occurs

Image 1:

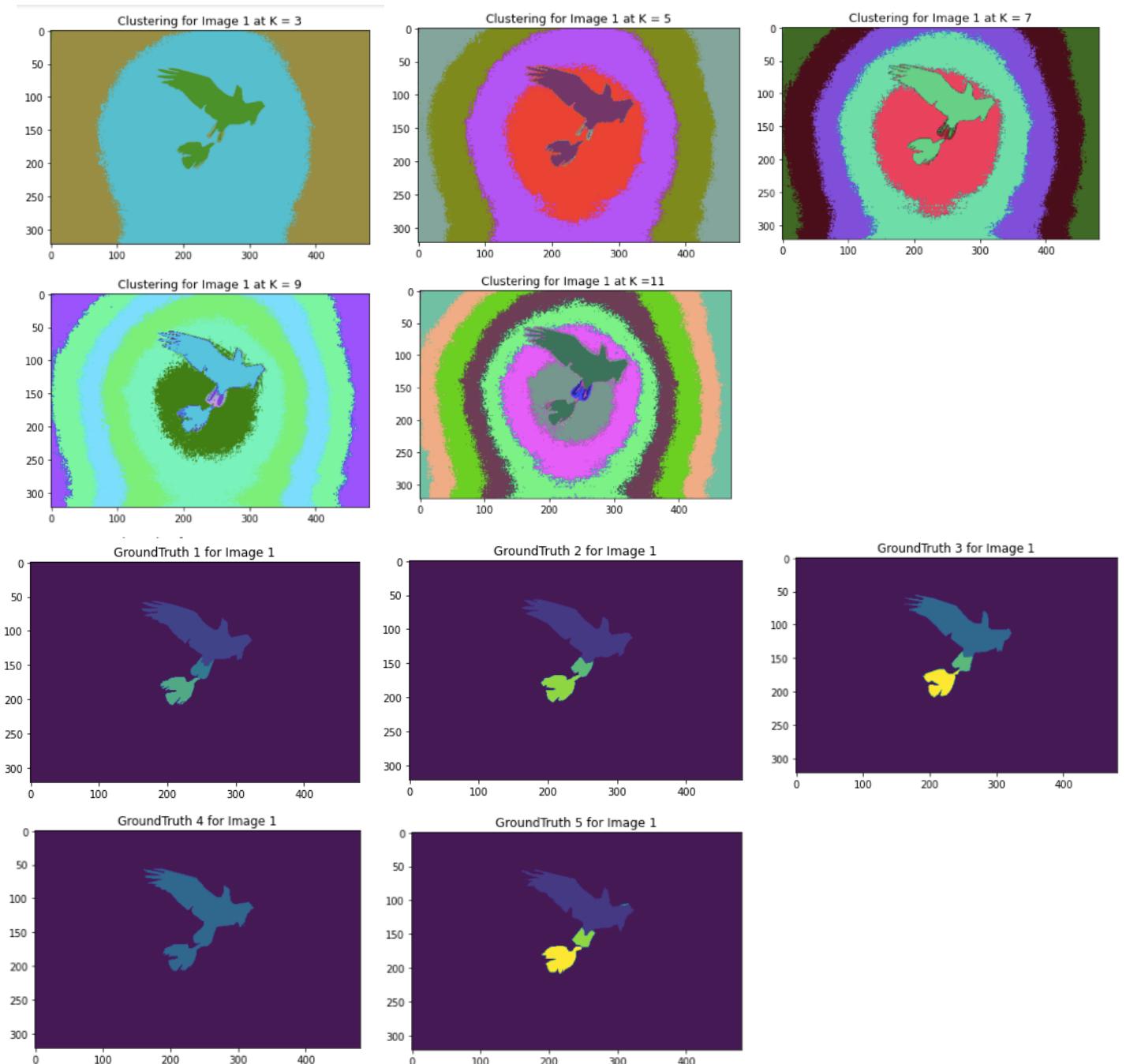


Image 2:

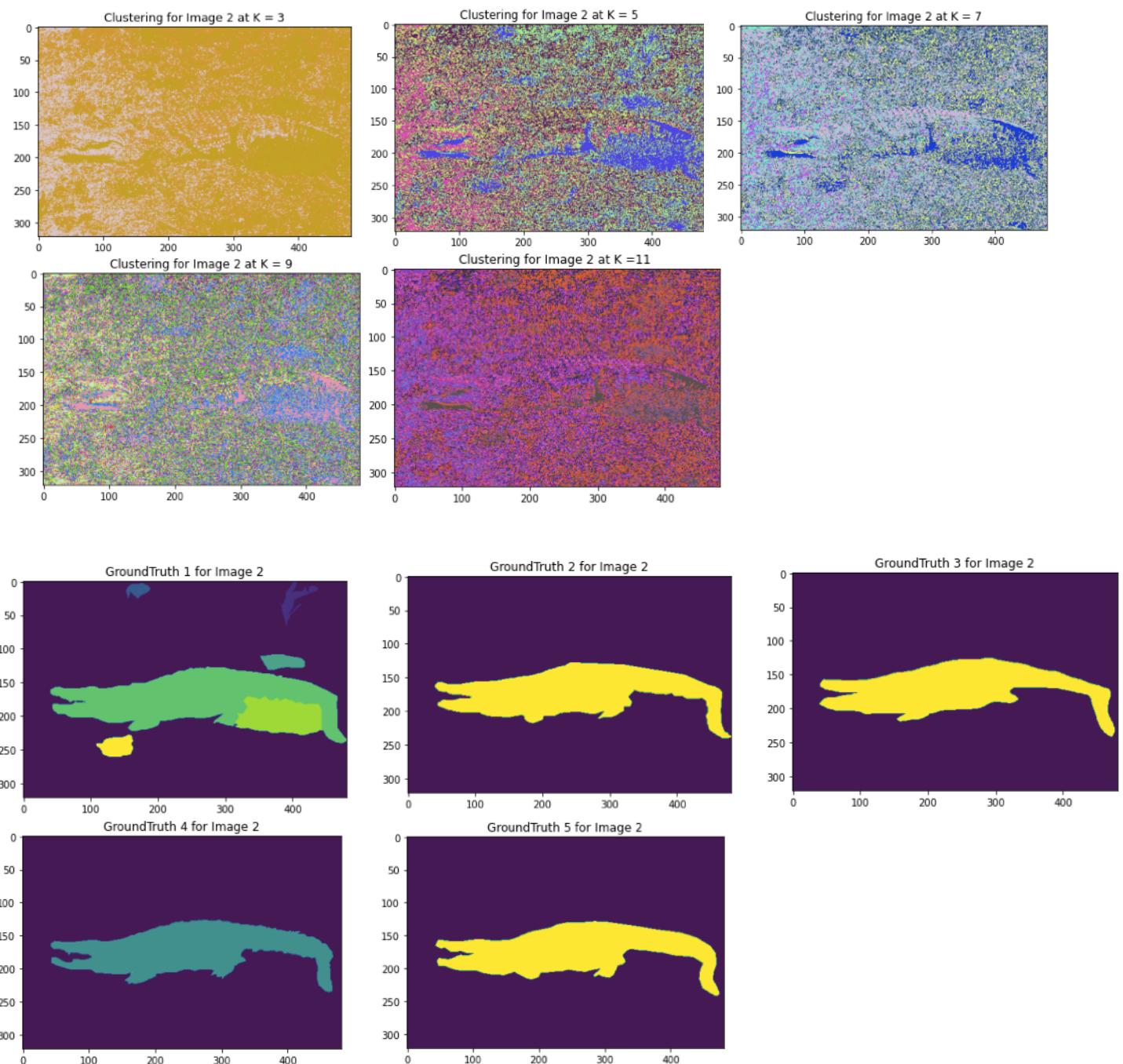


Image 3:

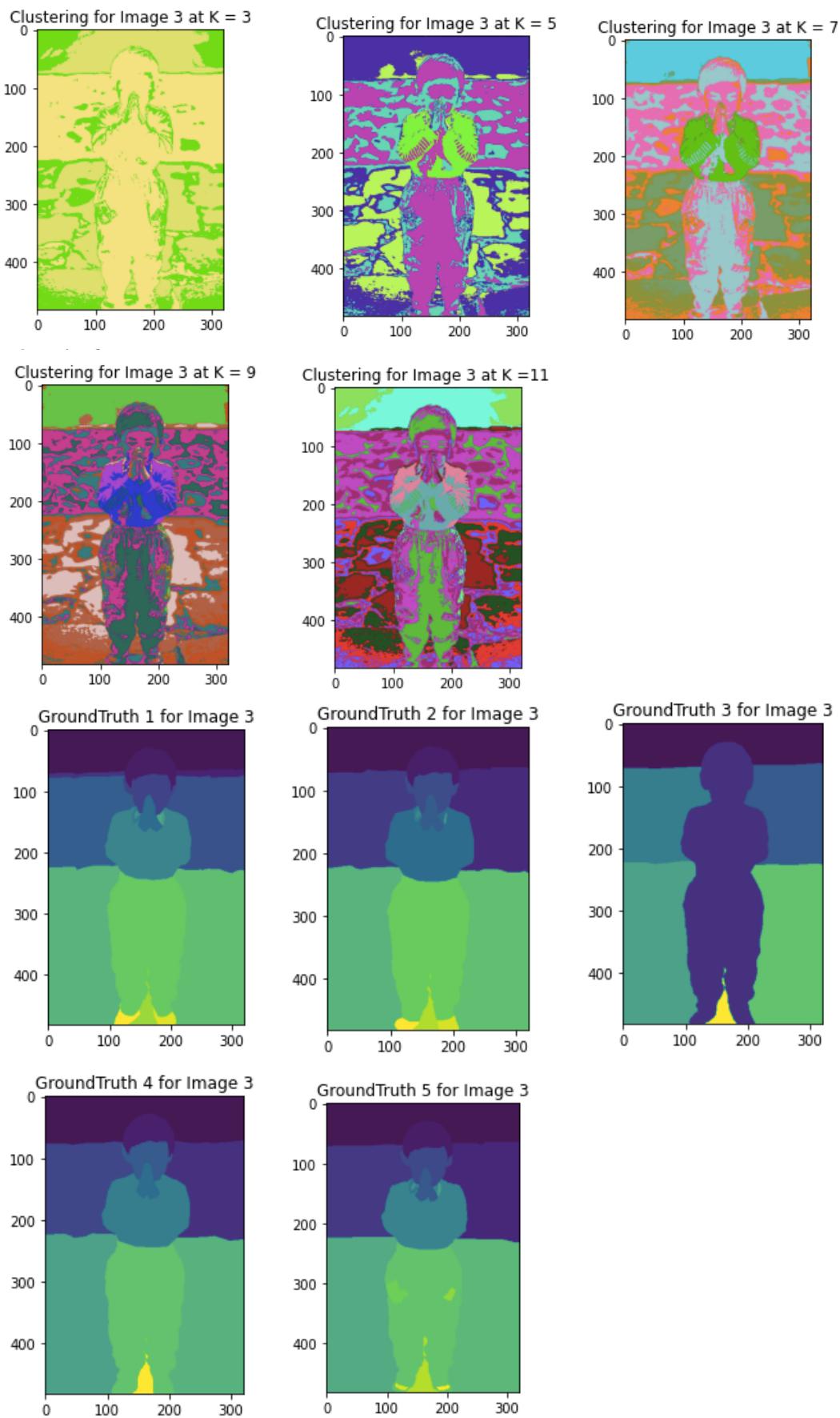


Image 4:

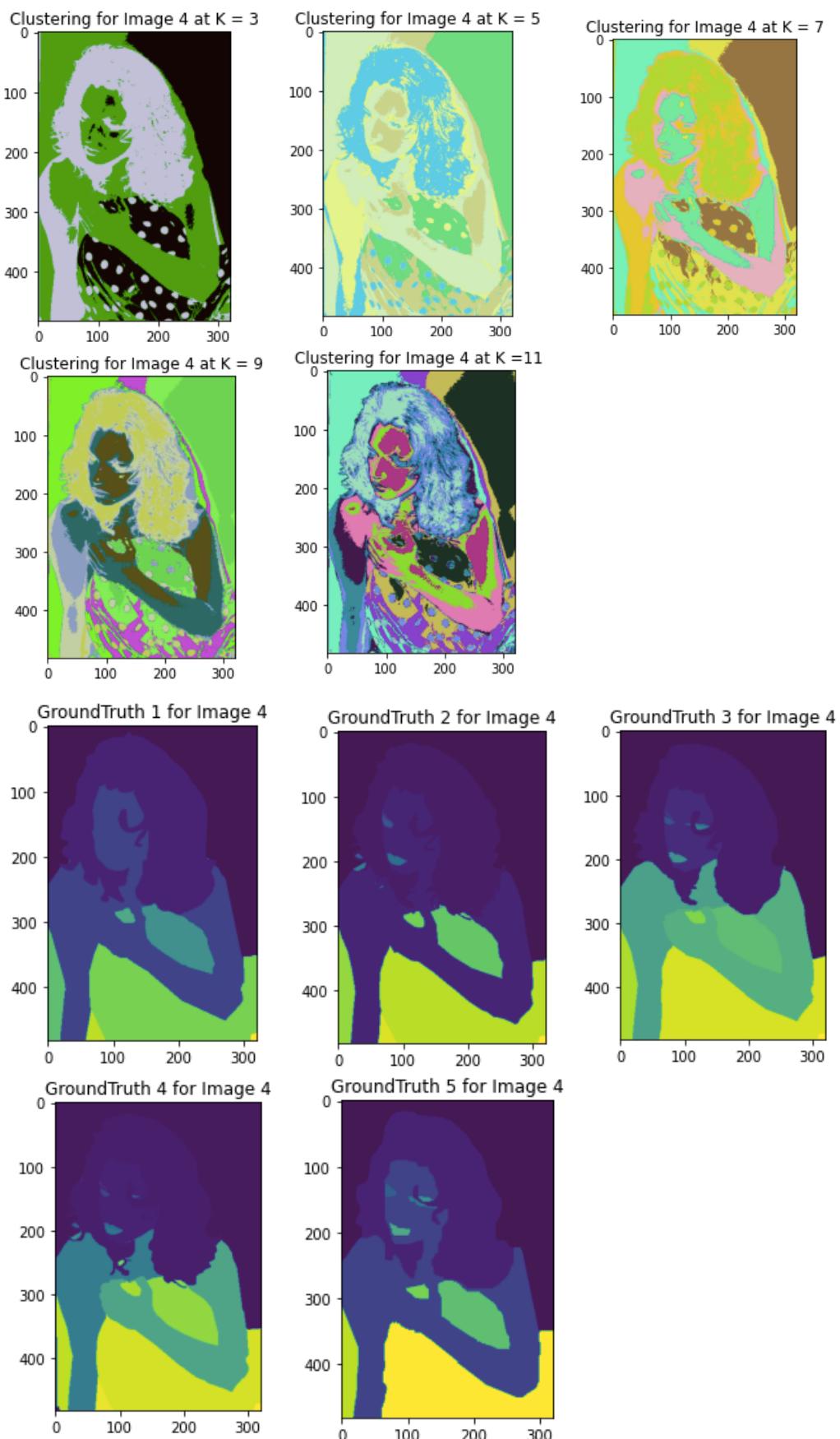
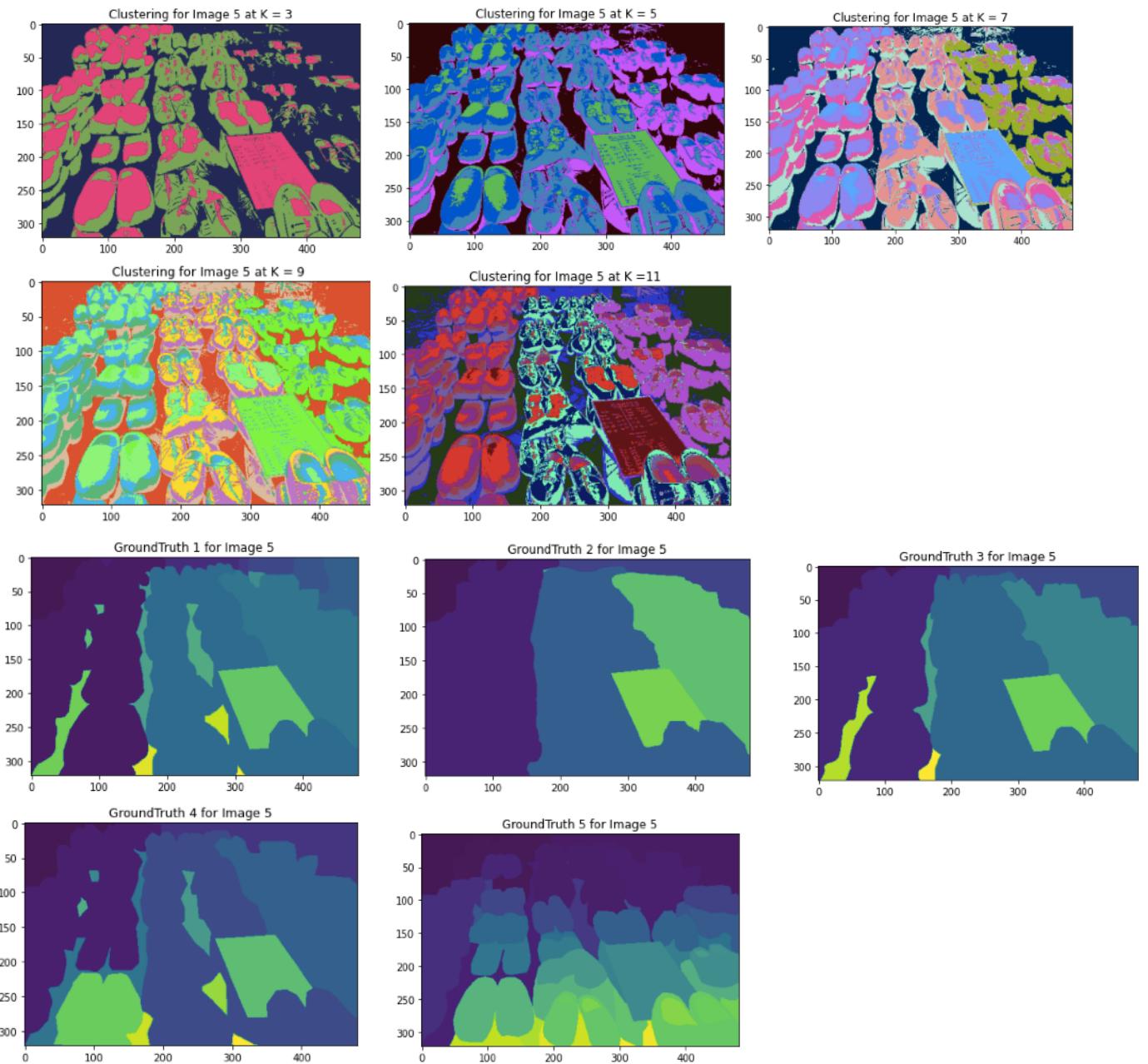


Image 5:



b. Select same five images and display their corresponding ground truth against segmentation results using Normalized-cut for the 5- NN graph, at K=5.

When running the code the first time, it didn't work hence reduced the images size by 25%.

```
[ ] TrainPath = "/content/drive/MyDrive/dataResized/images/"
GroundTruthPath = "/content/drive/MyDrive/dataResized/ground_truth/train/"
NumOfPixels = 120*80
Dimensions = 3
NumOfImages=200
Train, GroundTruth = DataLoading(TrainPath,GroundTruthPath,(NumOfPixels*NumOfImages,Dimensions))
```

NormalisedCut algorithm

---

### ALGORITHM 16.1. Spectral Clustering Algorithm

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#### SPECTRAL CLUSTERING ( $\mathbf{D}, k$ ):

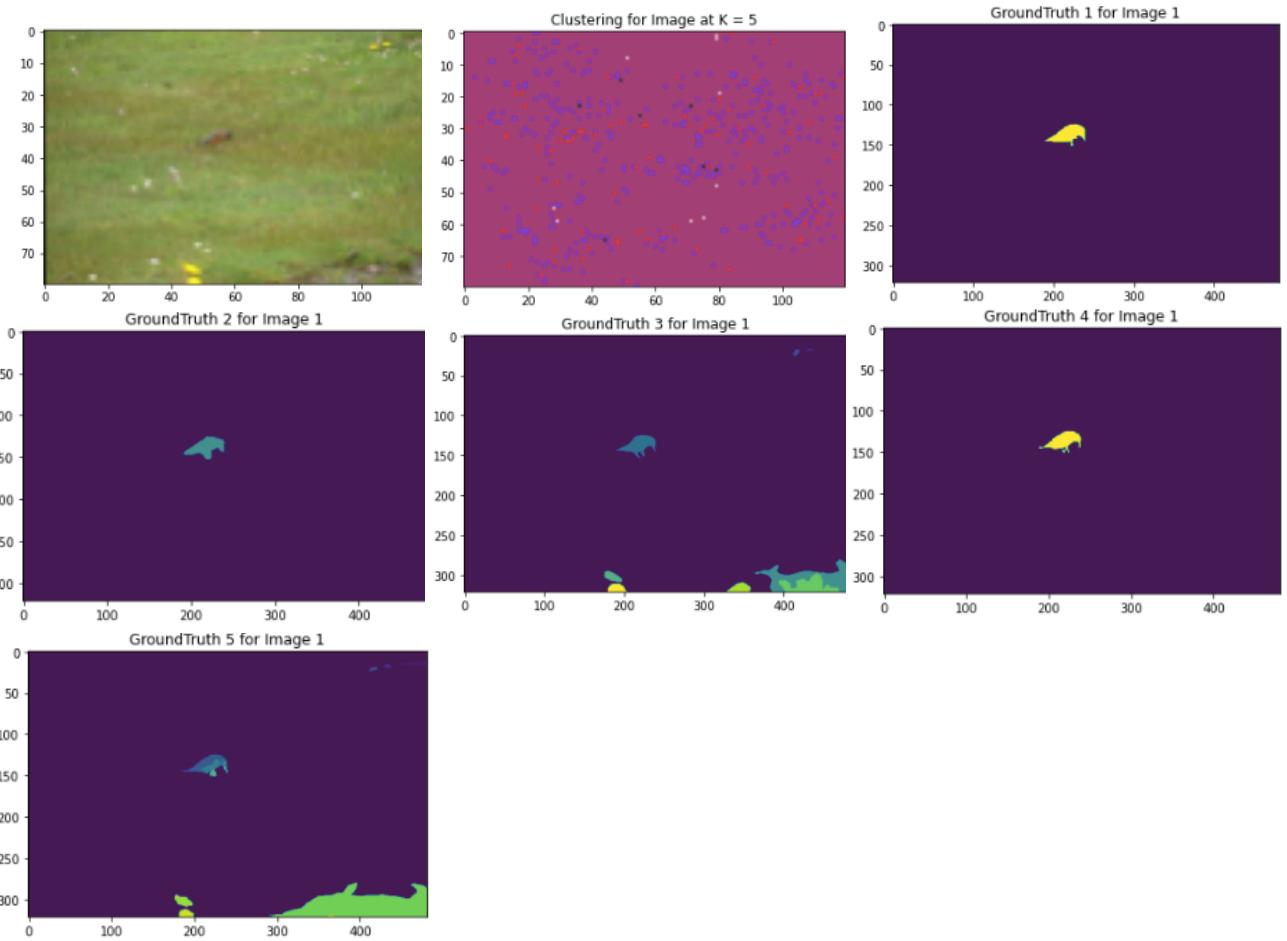
- 1 Compute the similarity matrix  $\mathbf{A} \in \mathbb{R}^{n \times n}$
  - 2 **if** ratio cut **then**  $\mathbf{B} \leftarrow \mathbf{L}$
  - 3 **else if** normalized cut **then**  $\mathbf{B} \leftarrow \mathbf{L}^s$  or  $\mathbf{L}^a$
  - 4 Solve  $\mathbf{B}\mathbf{u}_i = \lambda_i \mathbf{u}_i$  for  $i = n, \dots, n - k + 1$ , where  $\lambda_n \leq \lambda_{n-1} \leq \dots \leq \lambda_{n-k+1}$
  - 5  $\mathbf{U} \leftarrow (\mathbf{u}_n \quad \mathbf{u}_{n-1} \quad \dots \quad \mathbf{u}_{n-k+1})$
  - 6  $\mathbf{Y} \leftarrow$  normalize rows of  $\mathbf{U}$  using Eq. (16.19)
  - 7  $\mathcal{C} \leftarrow \{C_1, \dots, C_k\}$  via K-means on  $\mathbf{Y}$
- 

```
[ ] def NormalizedCut(Data, NumOfClusters, Mode, Gamma=1.0, NumOfNeighbors=1):
    if(Mode == 'rbf'):
        similarityMatrix = rbf(Data,Data, Gamma)
    elif(Mode == 'knn'):
        nearest_neighbor = nn(n_neighbors=NumOfNeighbors)
        nearest_neighbor.fit(Data)
        similarityMatrix = nearest_neighbor.kneighbors_graph(Data, mode='connectivity').toarray()

    degreeMmatrix = np.diag(np.sum(similarityMatrix, axis=1))
    laplacianMatrix = degreeMmatrix - similarityMatrix
    nomalizedAsymmertircLaplacianMatrix = np.dot(np.linalg.pinv(degreeMmatrix), laplacianMatrix)
    values,vectors = np.linalg.eig(nomalizedAsymmertircLaplacianMatrix)
    idx = np.real(values).argsort()[:NumOfClusters]
    vectors = np.real(vectors[:,idx])
    Normalization = np.linalg.norm(vectors, axis=1)
    NormalizedVectors = (vectors.T / Normalization).T
    Clusters = ImplementedKMeans(n_clusters=NumOfClusters,n_jobs=-1).fit_predict(NormalizedVectors)
    return Clusters
```

Cutting image before clustering

```
[ ] ColorizedImages = []
Count = 2
#Clusters=NormalizedCut(Train[NumOfPixels*Count:NumOfPixels+(NumOfPixels*Count)],5,'knn',NumOfNeighbors=5)
Clusters=SpectralClustering(n_clusters=5,affinity='nearest_neighbors',n_neighbors=5,n_jobs=-1).fit_predict(Train[NumOfPixels*Count:NumOfPixels+(NumOfPixels*Count)])
ColorizedImages.append(Colorize(Clusters,Train[NumOfPixels*Count:NumOfPixels+(NumOfPixels*Count)],5))
```



## 5. Extra

- i) Suggest a way to modify the feature vector to include spatial layout.

By adding 2 more dimensions, the position of each pixel, {R,G,B,X,Y}

- ii) 4a

For loop to determine j = x-axis , k = y-axis for the first 5 images

```
[ ] SpatialTrain=np.arange(NumOfPixels*NumOfImages*5).reshape(NumOfPixels*NumOfImages,5)
    SpatialPosition = []
    for i in range(5):
        for j in range(321):
            for k in range(481):
                SpatialPosition.append([j,k])
    TrainData = Train[0:NumOfPixels*5]
    SpatialTrain =np.hstack([TrainData,SpatialPosition])
```

WithoutPosition removes last 2 column, as it won't matter in the colorizing process  
In spatial layout, k-means take into consideration each pixel's position not only the colours {R,G,B}

```
[ ] ColorizedImages = []
    Clusters = []
    for K in range(3,12,2):
        for Count in range(5):
            Image = SpatialTrain[NumOfPixels*Count:NumOfPixels+(NumOfPixels*Count)]
            WithoutPosition = SpatialTrain[NumOfPixels*Count:NumOfPixels+(NumOfPixels*Count),0:3]
            #Model = OwnKMeans(Image,K)
            Model = ImplementedKMeans(n_clusters=K, random_state=0).fit(Image)
            Clusters.append(Model.labels_)
            ColorizedImages.append(ColorizeSpatial(Model.labels_,WithoutPosition,K))
```

```
[ ] def ColorizeSpatial(Clusters,Image,NumOfClusters):
    ColorizedImages= np.zeros(Image.shape,dtype=np.uint8)
    Colors = []
    for i in range(NumOfClusters):
        Colors.append(np.array([randint(0, 255),randint(0, 255),randint(0, 255)]))
    for i in range(Image.shape[0]):
        ColorizedImages[i] = Colors[Clusters[i]]
    return ColorizedImages
```

Image 1:

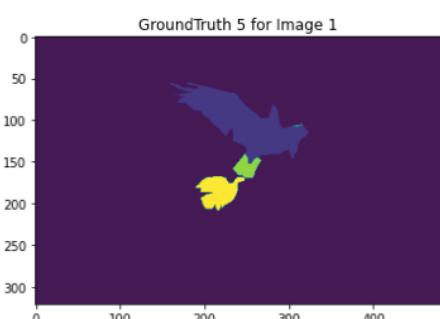
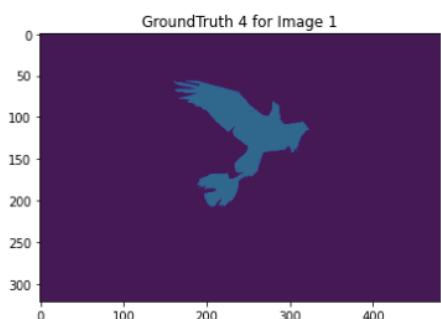
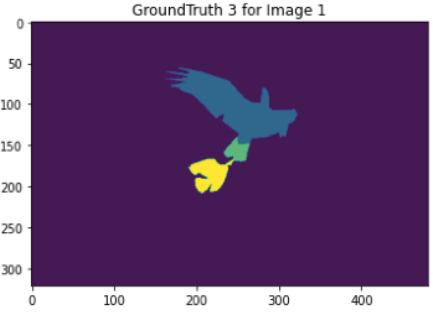
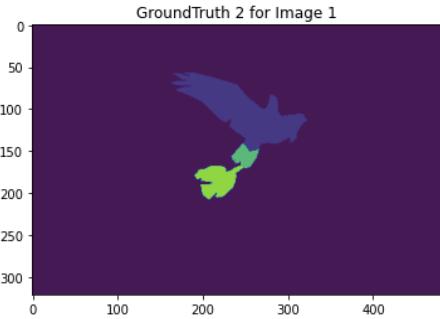
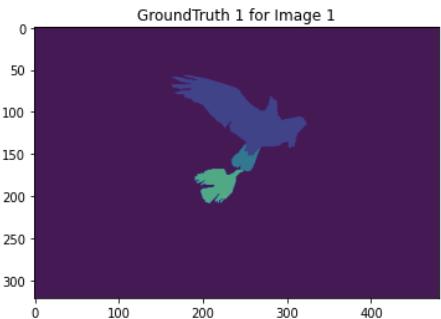
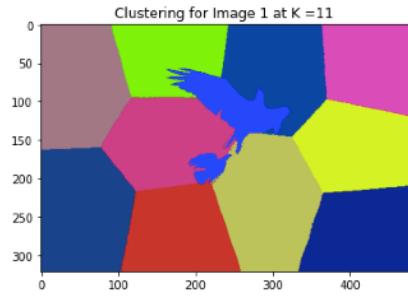
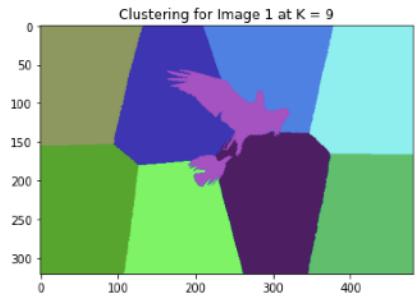
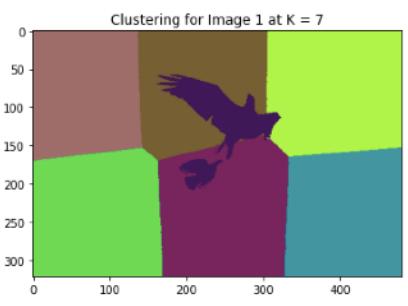
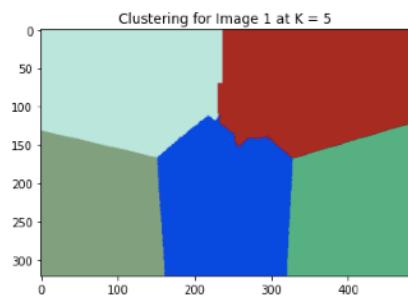
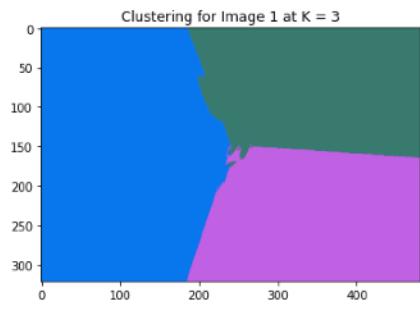


Image 2:

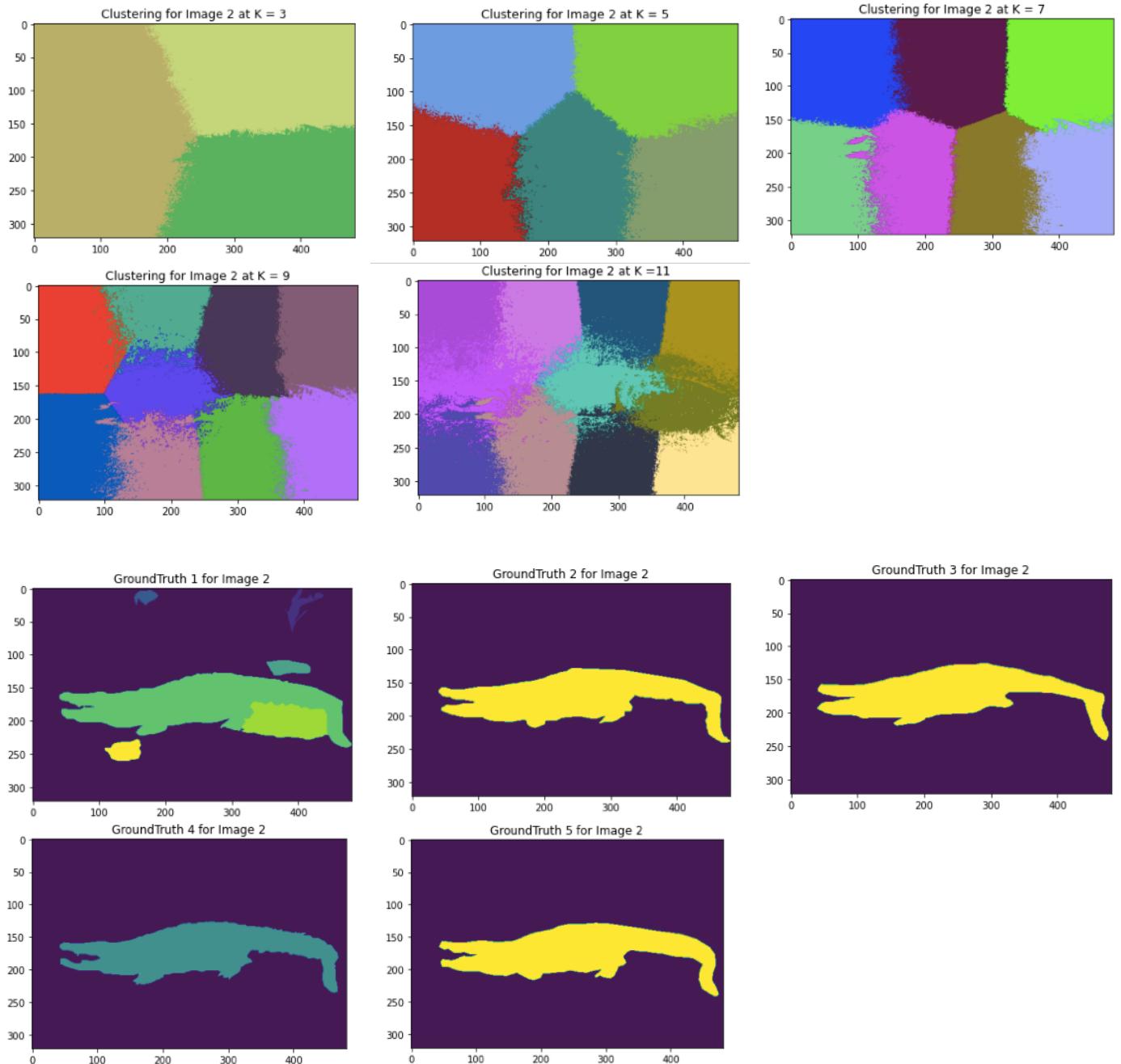
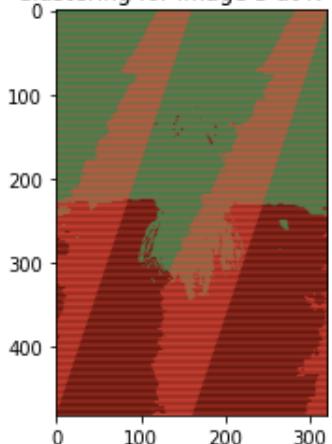
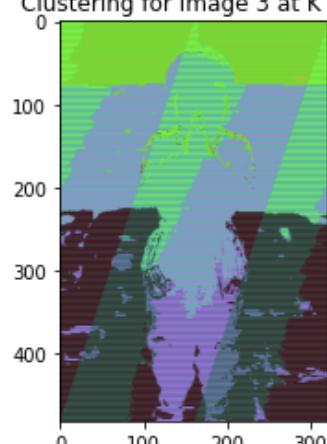


Image 3:

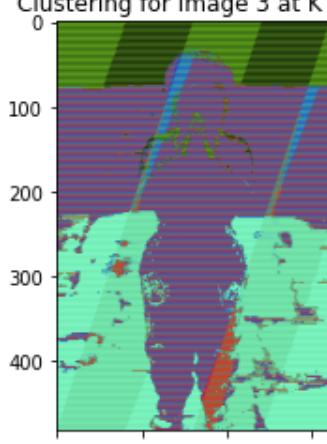
Clustering for Image 3 at K = 3



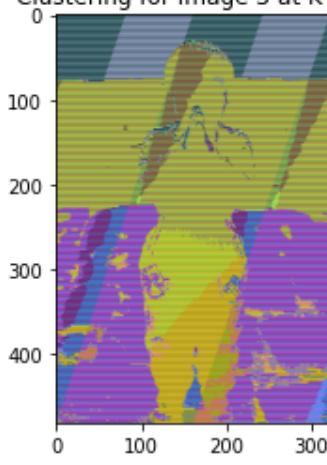
Clustering for Image 3 at K = 5



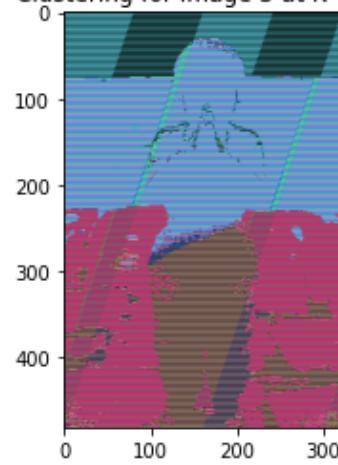
Clustering for Image 3 at K = 7



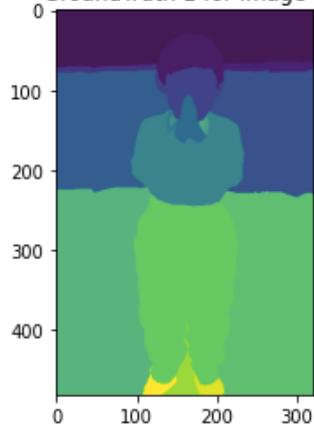
Clustering for Image 3 at K = 9



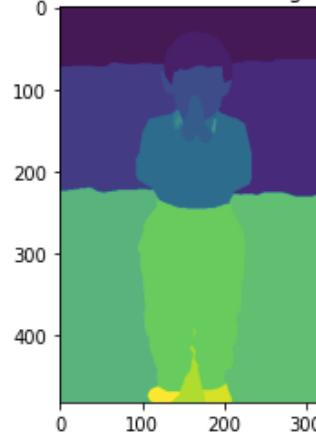
Clustering for Image 3 at K = 11



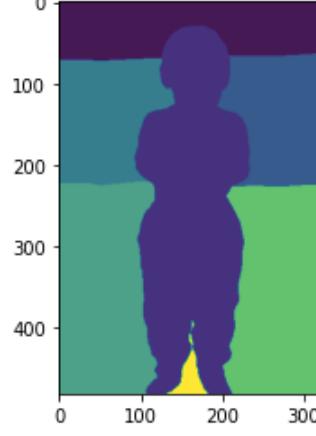
GroundTruth 1 for Image 3



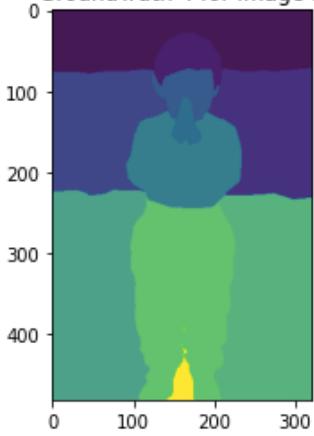
GroundTruth 2 for Image 3



GroundTruth 3 for Image 3



GroundTruth 4 for Image 3



GroundTruth 5 for Image 3

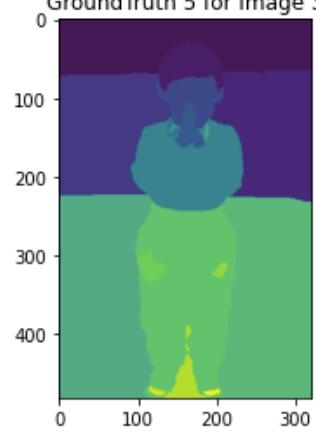


Image 4:

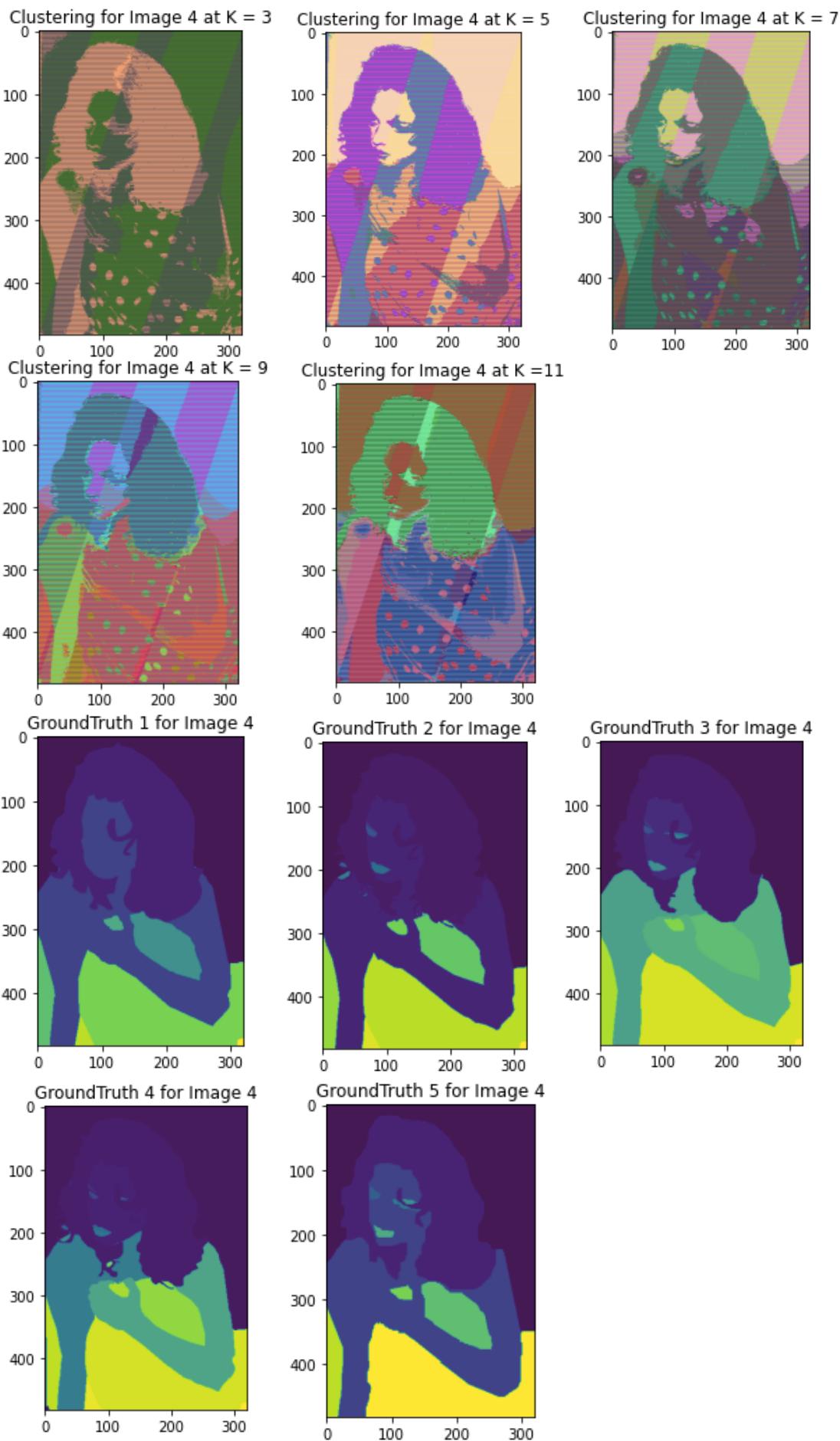


Image 5:

