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Dissertation

An empirical assessment of advanced models in option pricing

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Abstract

We investigate the performance of some relevant advanced models in pricing European option derivatives. In particular, we estimate one discrete and one continuous time volatility model (GARCH and Stochastic Volatility), and one Jump process model (Merton, 1976); then, we test the models with a specific set of market data (for options written on the S&P500, the DAX and the NIKKEI 225 indices), and compare their relative performance in terms of the errors produced by the estimating procedures. We conclude that the Jump process model outperforms the other models in almost all options written on all three indices, minimising the errors at almost every maturity and strike price. On the other hand, the GARCH model improves the performance of the standard Black-Scholes model for In-The-Money (ITM) options at medium and long term maturities. Finally, we found no conclusive evidence for the Stochastic Volatility model.

EXECUTIVE SUMMARY

The modelling of option prices has a key component in the stochastic process followed by the underlying asset. The uncertainty about the future value of the random variables involved in an option contract makes the distributional properties of those random variables an essential part in the methodology of option pricing. The purpose of this investigation is to assess empirically the performance of some relevant models that have been proposed in the literature of option pricing, which attempt to approximate the prices observed in the markets by modelling directly the random behaviour of the underlying stochastic variables. We will call them advanced models, as opposed to the standard Black and Scholes model of option pricing.

The Black and Scholes model works under the assumption that the volatility of the underlying asset price is constant through the life of the option, and that the process followed by this price is continuous. As these are considered weak assumptions, since what has been observed in the markets are price processes with changing volatility through time and discontinuous paths (jumps), models have been developed to account for these weak assumptions of the standard Black and Scholes. For our experiment we have selected to test against empirical data, particular models which incorporate changing volatility and jumps in their underlying processes. More specifically, we will test a standard discrete time volatility model (GARCH(1,1)), a standard Stochastic Volatility model (Hull and White (1987)), and a standard jump model of discontinuous paths (Merton (1976)).

To test the models we have obtained daily data for European Call options written on the S&P500, the DAX and the NIKKEI 225 equity indices. The procedure followed starts with the computation of the log-price increments of the indices and the analysis of their distributional properties. It was found that the data presents common features usually encountered in asset return series; this is, they show high kurtosis with asymmetry around the mean (skewness), and changing volatility (standard deviation) through time. Also we found that the return series presented non-linear correlation in time, and that they proved to be stationary. The models selected attempt to capture these features of the data.

To assess the performance of the models we compared the market data with the prices produced by the models. For this, we selected two measures of "goodness of fit", the Average Relative Percentage Error (ARPE) and the Root-Mean Square Error (RMSE). It was found that the Black-Scholes model

produces good results for short and medium term maturities at every strike price, however, for long maturities and in particular with for In-The-Money (ITM) options, it is outperformed by the GARCH and the Jump process models. This can be seen specially in the case of the S&P500 and the DAX indices. The Jump process model, for instance, is by far the best performer in the assessment. Once its parameters are calibrated to market data, it achieves the minimum errors for almost all the options considered in the experiment, at all maturities and all strike prices. Finally, the Stochastic Volatility model presented ambiguous results, showing improvements in some cases for In-The-Money (ITM) options, but with no discerning pattern, overall, in any of the three indices considered.

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Financial models of option prices have been developed as extensions around the seminal Black and Scholes model for pricing European Call and Put options [3]. In this fashion, Merton relaxed the assumption of non-dividend paying stocks and derived formula for pricing the exotic "down-and-out" barrier option [18], Merton introduced discontinuous paths (jumps) in the underlying asset [19], Hull and White incorporated stochastic volatility in the Black-Scholes framework [16], and Heston provided closed-form solution for a new type of stochastic volatility models [14], Dupire [9], Derman and Kani [7] and Rubinstein [24], attempted to model directly the observed "volatility smile" through the construction of an implied volatility tree for the pricing of exotic options. Discrete models for the changing volatility have also been proposed within the type of GARCH models (for example see [8], [10] and [17]), and more recently, Levy process models have been investigated to account for the jumps that characterise financial time series; for recent surveys see [6] and [25].

The motivation for the current research in pricing and hedging financial derivatives with more advanced models, is mainly concerned with the emergence of the so-called exotic options as highly traded securities in the over-the-counter markets [1]. This is summarized in the following quotation from a recent workshop on exotic derivatives and Levy models:

"In recent years more and more attention have been given to stochastic models of financial markets which depart from the traditional Black-Scholes model; that is to say both in academia and financial institutions alike. In particular focus has been placed on modelling risky assets with semi-martingales. For example Lévy process based models are able to take into account different important stylised features of financial time series. The consequence of working with more advanced stochastic models forces a number of new mathematical challenges with respect to exotic derivatives. Exotic derivatives are gaining increasing importance as financial instruments and are traded nowadays in large quantities in over the counter markets. Examples of these exotic options are lookback, barrier, Asian, Parisian, Bermudian, Russian, Israeli, Passport, Cliquet, digital, swing, corridor, Variance Swap options etc. Moreover these instruments are finding their way into other businesses like the (re-) insurance; for example catastrophe options, weather derivatives and energy derivatives are useful in handling different kinds of risk."[1]

[1]Workshop on "Exotic option pricing under advanced Lévy models". EURANDOM Institute, Eindhoven, The Netherlands May 3 and 4, 2004.

Theoretical advancements have been made in developing more realistic option pricing models that fits the properties of the financial data, however, the empirical performance of all these models needs to be contrasted to assess their validity as reliable and efficient tools for pricing and hedging derivatives in the financial markets. Previous research in this subject has been produced by Bakshi, Cao and Chen [2].

In the present investigation we will conduct an experiment in option pricing with some of these advanced models, to empirically assess their performance against a set of real market data. These include apart from the Black-Scholes model, a GARCH type volatility model, a Stochastic Volatility model and a Jump process model. We will focus on European type Call options written on equity indices (S&P500, DAX and NIKKEI 225). This is relevant since the procedures to price most exotic options are based on the estimation of parameters obtained through the modelling of the more liquid plain-vanilla options. Moreover, since we will assume that the Put-Call parity[2] holds in these very competitive markets, we do not expect to lose generality for focusing our experiment exclusively in Call option prices.

The models will be evaluated and estimated using market data. The results obtained will be contrasted in terms of the errors produced by the estimating procedures. More specifically, two measures of "goodness of fit" will be computed, the Average Relative Percentage Error (ARPE) and the Root-Mean Square Error (RMSE)[3].

Section I will investigate the distributional properties of the selected financial time series, and stylised facts for the asset returns will be presented. Section II will describe the models and their appropriateness to capture the characteristics of financial asset returns. Section III will describe the estimation procedure of the models with historical market data, and the calibration procedure for the estimation of parameter values in the Jump process model. The final assessment of the models performance, measured in term of the errors, will be offered in Section IV. Finally, Appendix A presents technical definitions and equations used throughout the investigation, and Appendix B presents tables showing the results obtained from the experiment.

[2] See Appendix A for technical notes.
[3] See Appendix A for technical definitions.

I. STYLISED FACTS OF ASSET RETURNS

Before we begin to model any financial data it is convenient to explore first the characteristics that this data possesses. Investors are interested in the returns produced by the assets they hold, then it is appropriate to focus in asset return series when we are analysing the properties of the data[4]. It have been well documented that financial asset returns exhibit certain characteristics in their distributional properties [12], [21], which are also known as stylised facts [5]. Since our experiment in option pricing will depend heavily in the process followed by the underlying assets, we shall explore here the characteristics of our data at hand and see if they conform to the assumptions that we will make later in the models, in order to consider possible misspecifications.

We have chosen to price European Call options written on three different equity indices, which are widely traded in the markets. Our data consists of daily market prices for the S&P500 index from the 22nd of May 1989 to the 17th of September 2004, daily market prices for the DAX index from the 16th of July 1999 to the 23rd of September 2004, and daily market closing prices for the NIKKEI 225 index from the 5th of October 1998 to the 1st of October 2004; as available in DATASTREAM ADVANCED 3.5 and BLOOMBERG LP financial information service, at the The Cyril Kleinwort Learning Resource Centre, Cass Business School, City University. We hope that this sample data will reflect long term and medium term characteristics of equity indices with different calculation procedures, and representing different highly developed financial markets.

The Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks in the US. The index is designed to measure performance of the broad domestic economy, through changes in the aggregate market value of 500 stocks representing all major industries. On the 23rd of September 2004, the traded volume in the index was 1.11 billion stocks with a market capitalization of 10.23 trillion US$. On the other hand, the DAX Index is a total rate of return index of 30 selected German blue chip stocks traded on the Frankfurt Stock Exchange. On the 23rd of September 2004, the traded volume in the DAX index was 92.39 million stocks with a total market capitalization of 525.4 billion EUROS. Finally, the NIKKEI 225 index is a price-weighted average of 225 top-rated Japanese companies listed in the

[4] This is also true because of the mathematical and statistical properties of log time series and their increments [5], [25], which defines a return process. See Appendix A.

first section of the Tokyo Stock Exchange. On the 1st of October 2004, the traded volume in the NIKKEI 225 index was 844.00 million stocks with a market capitalization of 213.1 trillion YENS[5].

We will use EVIEWS 4.1 and S-PLUS 6.0 to present results from the analysis of our data. To start the investigation, we show the price and return series of the three indices in figures 1, 2 and 3 respectively. We see that in all cases the returns exhibit a strong variability with different periods of apparent high and low variance (volatility clustering). This can be seen particularly for the S&P500 index, where we see that after the first few years of the 1990's, the return series started a period of low and stable volatility during the middle of the decade, which then increased sharply around 1997. The DAX index presents similar patterns as the S&P500; we can see how in 2002 the behaviour of the return changes showed higher variability than previously seen in the data (with the exception of particular points in 2001), just to go back to more "normal" levels by the end of 2003 and in 2004. Finally, the NIKKEI 225 index seems to be less "volatile" than the other indices, nevertheless, we can see how it presents the same patterns of changing volatility in the return series through the period considered.

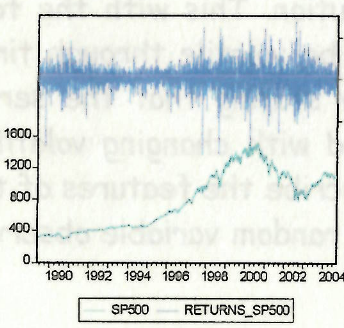


Figure 1

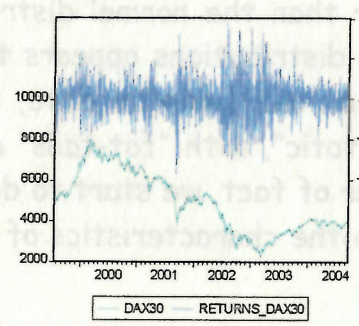


Figure 2

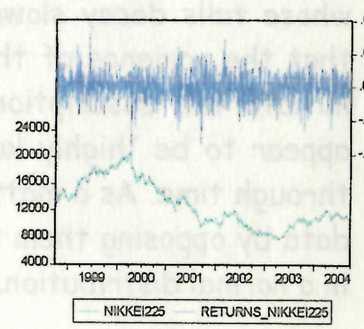


Figure 3

In financial theory the assumption of normality in the distribution of the random variables play an important role in the construction of efficient models, and in option pricing theory this is made clear when the Brownian Motion is chosen to model the process followed by the state variables; this is so since the increments of a Brownian Motion are normally distributed. It then becomes very appropriate to investigate on the distribution of the variables in our sample data, to extract information which could be used to

[5] Bloomberg L.P.

build and assess the models that we will consider in our option pricing experiment.

Figures 4, 5 and 6 present a Gaussian Kernel density estimation of the empirical distributions for the returns of the three indices considered in our sample data.

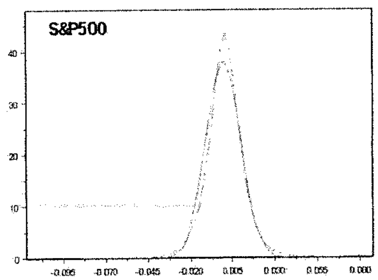
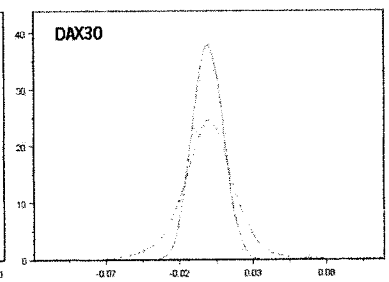


Figure 4



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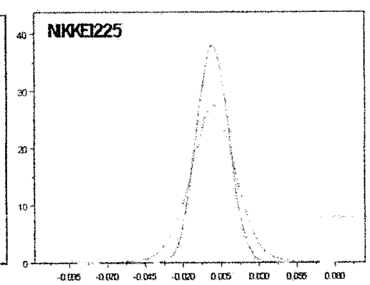

Figure 6

In the figures the green line represents the estimation of the empirical distribution, and the red line represents a standard normal distribution with zero mean and unit variance. We observe for example, that the S&P500 distribution presents a high peak which is almost symmetric around zero and whose tails decay slower than the normal distribution. This with the fact that the variance of the distributions appears to be volatile through time, initiate the description of our financial data, by stating that the series appear to be "highly kurtotic", with "fat-tails" and with changing volatility through time. As a matter of fact, we start to describe the features of the data by opposing them to the characteristics of a random variable observed in a normal distribution.

Table 1 provide us with information about the moments of the empirical distributions. For the S&P500 index, we see a slightly negative skewness of -0.157 and a high kurtosis of 7.147, showing the departure from the normal distribution whose values for these statistics are known to be zero "0" for skewness and three "3" for Kurtosis[6]. For the DAX index the moments show the same characteristics as for the S&P500 although the departure from normality is less obvious in this case, this index shows an almost zero skewness of 0.004 and kurtosis above three of 4.707. In figure 5 we see how the high peakedness of the empirical distribution disappears, however

[6] Skewness measures the asymmetry of the distribution around its mean, while Kurtosis measures the peakedness or flatness of the distribution of the series [22]. See Appendix A.

the sample data, which are partly removed whenever we take shorter sample periods for the time series [20].

We formally state now that our series depart substantially from the normal distribution as measured by the third and fourth moments, this is, they present skewness and high kurtosis, which conform with the findings of the standard literature on return series as shown in [5], [12], [21]. The modelling assumption of normality is then not consistent with the data, and extensions to the standard models that take account of this departure are justified. The validity of these "extensions" must, however, be contrasted empirically to assess if they truly outperform the standard model (Black-Scholes).

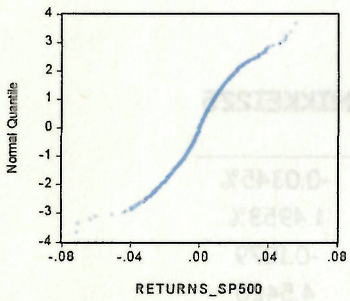

Figure 7

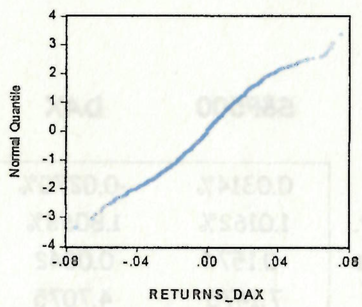

Figure 8

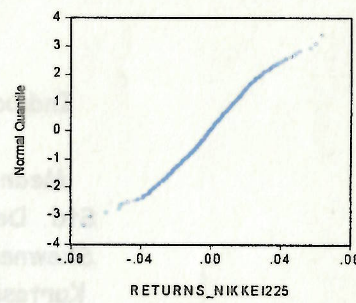

Figure 9

Other important statistical properties in financial modelling are the assumptions of independence and stationarity of the data. They are assumed whenever we assume a random process which is identically and independent distributed (i.i.d.). A series is said to be stationary if its distributional properties remain constant over time. This is to say that certain statistics of the data do not depend on time [5], [13]. Independence, on the other hand, is measured by the lack of dependence between actual and previous values in a series.

In financial theory the assumption of highly liquid competitive markets means that prices follow what is known as a *white noise*, that is, there is no dependence between the actual value in the series and its past history, and the price processes are unpredictable, this also known as the *Markov property* of the series. The absence of significant linear correlation is often cited in support for the "efficient market hypothesis" [10]. The measure of linear dependence in time series is known as the Autocorrelation Function (AFC)[9], Independence, however implies that any nonlinear function of

[9] See Appendix A

its heavy tails becomes more obvious. Finally, for the NIKKEI 225, we see as well a slightly negative skewness of -0.107 and a kurtosis above three of 4.541. Again, figure 6 shows no high peakedness for this series, but the fat tails are very obvious. We can say here that the heavy tails observed in the empirical data are indicative that "extreme events" are more likely to occur in reality than can be predicted from a normal distribution.

We can test normality in the distributions, using the Jarque-Bera statistic[7], which measures the difference of the skewness and kurtosis of the series with those from the normal distribution. We see in Table 1 that the Jarque-Bera statistic presents high values for all distributions with probability zero, meaning that we can reject at any relevant confidence level the null hypothesis that the series is normally distributed.

Index	S&P500	DAX	NIKKEI225
Mean	0.0314%	-0.0270%	-0.0345%
Std. Dev.	1.0162%	1.8063%	1.4953%
Skewness	-0.1574	0.0042	-0.1079
Kurtosis	7.1475	4.7075	4.5420
Jarque-Bera	2882.9816	164.3703	142.0183
Probability	0.0000	0.0000	0.0000

Table 1

Another way to see if the series are normally distributed is by a Quantile-Quantile (Q-Q) plot against the normal distribution[8]. Figures 7, 8 and 9 show the shape of the Q-Q plot for each index, respectively. If the series were normally distributed the graphs would show a straight line of 45degrees, meaning that the quantiles of the empirical distributions equal those of a theoretical normal distribution; while an S-shaped curve signifies that the empirical distribution has fatter tails than the normal distribution. As we can see, each index plot shows an S-shaped curve, revealing the high kurtosis in their distributions, however it is less pronounced in the case of the DAX and NIKKEI 225 indices. This last observation, can be explained by the fact that the non-gaussianity observed in financial series corresponds mainly to the presence of large jumps experienced by the series through the range of

[7] See Appendix A.
[8] See Appendix A.

returns (such as the squared returns) will also have no autocorrelation. When these non-linear series exhibit positive autocorrelation, this is a signal of the phenomenon known as "volatility clustering", that is, large returns variations are more likely to be followed by another large return variation.

Figures 10, 11 and 12 show the Autocorrelation Function (ACF) for the returns and squared returns series for each of the three indices, with up to 90 lagged values. The unpredictability of returns and the clustering of volatility can be appreciated by looking at these figures. Predictability appears whenever there are significant autocorrelation in returns, and volatility clustering when there are significant autocorrelation in squared returns. Under conventional criteria [10], autocorrelation bigger than 0.033 in absolute value, would be significant at a 5% level. We observe, that returns autocorrelations are almost all non-significant for the three series, while the square returns have (almost) all significant positive autocorrelations, except for the NIKKEI 225 index, where we see that it decreases quickly and becomes insignificant at relatively few lagged values.

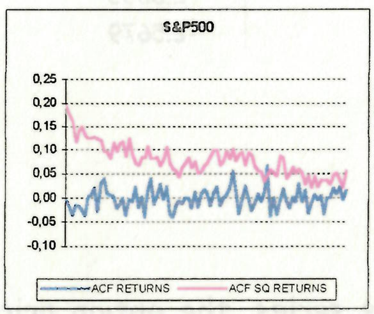

Figure10

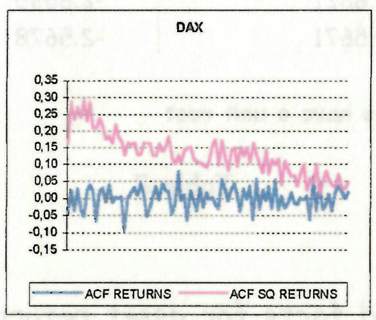

Figure11

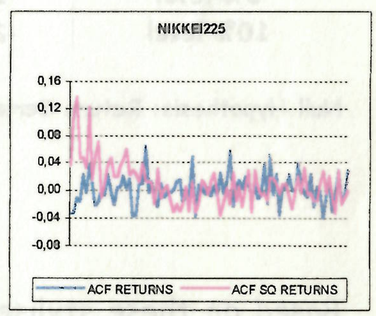

Figure12

The figures give consistent evidence for both the unpredictability of returns (linear independence) and the volatility clustering (non-linear dependence). However, in the case of the NIKKEI 225 index the clustering of volatility is not so clear. Nevertheless, we can state here, for the purpose of assessing the models described later in this investigation, that the return series are (linearly) unpredictable, but present in general non-linear correlation in the volatility.

Finally, the stationarity of the series is addressed by performing unit root statistical tests[10] on the series and evaluating their significance at various confidence levels. The stationarity of the data is a basic requirement of any

[10] See Appendix A.

statistical analysis, since we need some stable statistical properties upon which to base our estimation procedures [22], [23]. Table 2 shows the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test statistics of unit roots for the three indices considered. The presence of unit roots in the estimating equations account for the non-stationarity of the data. We observe that for all three indices, both test statistics present lesser values than the critical values at all significant levels with almost zero probabilities; this lead us to reject the null hypothesis of unit root in the estimating equations, and to accept the stationary hypothesis in the series [13], [22].

	S&P500		DAX		NIKKEI 225	
	t-Statistic	Prob.*	t-Statistic	Prob.*	t-Statistic	Prob.*
Test statistics:						
Augmented Dickey-Fuller	-63.5958	0.0001	-38.4719	0.0000	-37.5987	0.0000
Phillips-Perron	-64.0082	0.0001	-38.4994	0.0000	-37.6231	0.0000
Test critical values:						
1% level	-3.4318		-3.4350		-3.4350	
5% level	-2.8621		-2.8635		-2.8635	
10% level	-2.5671		-2.5678		-2.5679	

Null Hypothesis: Return Series have a unit root

Table 2

Based on these stylised facts for asset returns series, the option pricing models to be considered here are attempts to approximate the observed behaviour in the underlying assets of option contracts, with the purpose of correcting the "mispricing" of the standard model (Black-Scholes) whose assumptions, as we will see, differ from the characteristics that we can observe in the real data. However, the features of the data mentioned above are not easily reproduced by a single model all at once, and should be viewed as a set of constraints that a stochastic process must obey if it wants to replicate approximately the characteristics of asset return series [5].

In the next section we will start by describing the Black-Scholes option pricing model, stating its assumptions and its explicit formulae to compute European Call options. Then we will introduce the discrete-time Volatility Models and the continuous-time Stochastic Volatility Model. Finally, a model incorporating Jumps in the stochastic processes will be presented to complete the selection of models to be contrasted in the experiment.

II. OPTION PRICING MODELS

II.1 Black-Scholes

The Black-Scholes model starts with the assumption that the underlying asset (S) follows a Geometric Brownian Motion (GBM) [3] of the form,

$$dS_t = S_0(\mu dt + \sigma dW_t), \qquad 0 < t < T, \qquad S_0 > 0 \qquad (1)$$

where the drift rate μ and the volatility σ of the asset price are constants, and W is a Standard Brownian Motion. Under the risk-neutral probability measure the process (1) has solution [3], [25],

$$S_t = S_0 \exp((r - q - 1/2\sigma^2)\, t + \sigma W_t) \qquad (2)$$

Where r is the risk-free interest rate in the economy, and q is the dividend yield earned by the asset.

To price plain-vanilla European Call (and Put) options in the Black-Scholes model, we discount the expectation (under the risk-neutral probability measure) of the terminal payoff of the option, at the risk-free interest rate [25], i.e.;

$$V_t = \exp(-r\tau)\, E_Q[G(S_T) \mid F_t], \qquad 0 < t < T \qquad (3)$$

Where V_t is the current value of the option, τ is the time to expiration, $E_Q[.]$ is the expectation operator under the risk-neutral probability measure Q conditional on the set of information F available at time t, and $G(S_T)$ is the payoff function of the option at the expiration date T. For a Call option with a strike price K, the payoff function is given by $G(S_T) = (S_T - K)^+$.

Now, equation (3) can be re-expressed using equation (2) and the definition of conditional expectation (with t = 0) as [25],

$$V_0 = \exp(-r\tau) \int^{\infty} [G(S_T) f_{xnormal}(0,t)]dx$$

$$V_0 = \exp(-r\tau) \int_{-\infty}^{\infty} [G(S_0 \exp((r-q-1/2\sigma^2)t + \sigma x)) f_{xnormal}(0,t)]dx \qquad (4)$$

where x is a standard normally distributed random variable with mean zero and variance t, and f_x is density function of x under the risk-neutral probability measure Q. Additionally, from equation (2) we know that S_t is log-normally with mean and variance as,

$$\ln S_t \sim N(\ln S_0 + (r - q - 1/2\sigma^2)t, \sigma\sqrt{t}) \qquad \text{for } 0 < t \leq T$$

Then, the Black-Scholes model explicit formula for a European Call option can be found by solving the integral in (4) considering the lognormal property of S_t [15]. The value of a European Call option is given by,

$$C = C(K,T) = \exp(-qt)\, S_0\, N(d_1) - K \exp(-rT)\, N(d_2) \qquad (5)$$

where

$$d_1 = \frac{\log(S_0/K) + (r - q + 1/2\sigma^2)T}{\sigma\sqrt{T}},$$

$$d_2 = \frac{\log(S_0/K) + (r - q - 1/2\sigma^2)T}{\sigma\sqrt{T}},$$

and N(x) is the cumulative distribution function of a standard normally distributed random variable x.

The assumptions of the GBM in the Black and Scholes, implies that returns under this model are Gaussian and that it does not comply exactly with the stylised facts of asset returns. Nevertheless, the BS model produces easy to compute option prices for European Calls and Puts, although observations from the market show that it is misspecified and its price outputs are not accurate. One way to show this is by calculating the *implied volatility* of the asset price; this is, given some market prices for Call (or Put) options, the implied volatility is the required volatility that the BS needs to replicate those prices.

The implied volatility can be computed by taking the option market prices as inputs into the BS, and then solving for the unknown volatility from the BS Call (or Put) formula. This can be done for a set of options with different strike prices (K) and times to maturity (τ), giving rise to what is known as the volatility surface, this is, the whole set of volatilities implicit in option prices with different combinations of strikes and maturity times; which can be expressed as $\sigma(K, \tau)$. Figures 13 through 18 show the implied volatility for different maturities as calculated from a set of option values on each of the three equity indices.

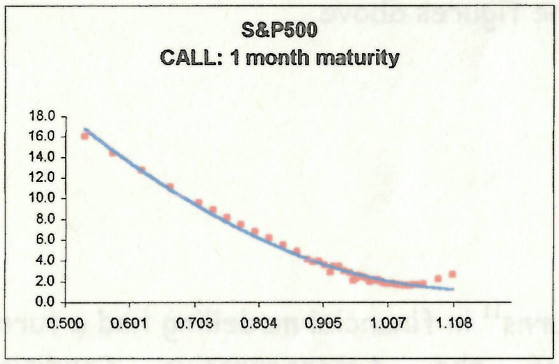

Figure 13

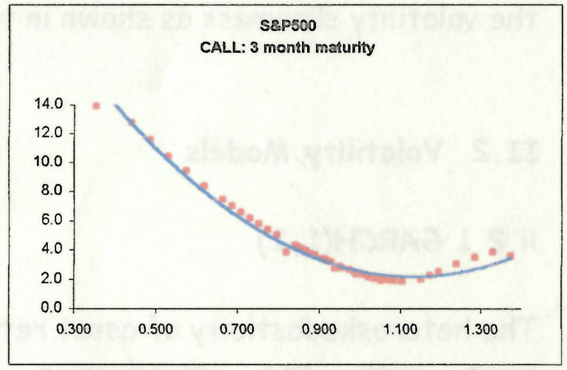

Figure 14

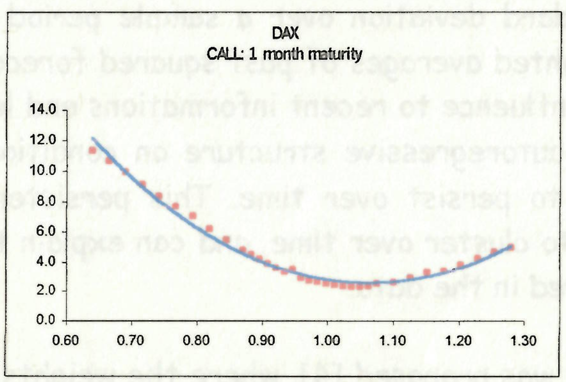

Figure 15

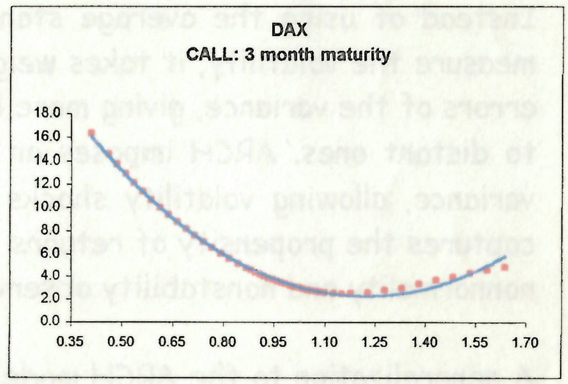

Figure 16

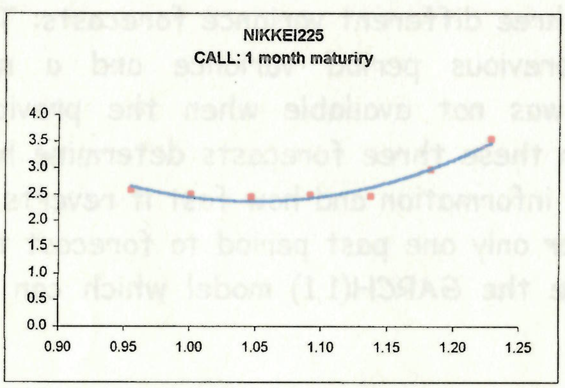

Figure 17

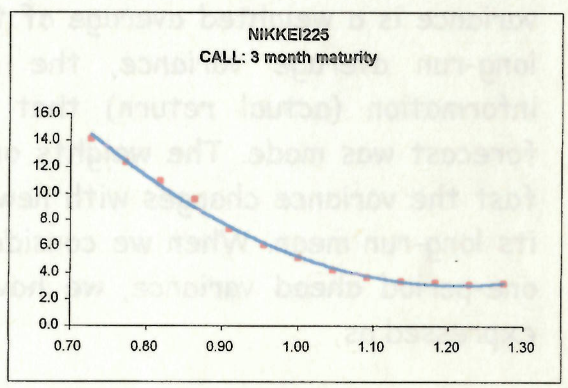

Figure 18

From the figures, we observe that the Black-Scholes model seems not to be consistent with the data. This is, since under the BS the volatility is a constant value, the implied volatility from the option prices should look like a flat line, however we see in the figures that they present in all cases a kind of U-shaped curve. This is also known as the "volatility smile" or "volatility skew". From this, the natural extension to the BS model has been to account for the non-constant volatility observed in the markets, which is reflected in the volatility skewness as shown in the figures above.

II.2 Volatility Models

ii.2.1 GARCH(1,1)

The heteroskedasticity of asset returns[11] in financial modelling had a turning point with the introduction of the Autoregressive Conditional Heteroskedasticity (ARCH) model [11]. The ARCH model is a mechanism that forecasts the variance of a time series in terms of current observables. Instead of using the average standard deviation over a sample period to measure the volatility, it takes weighted averages of past squared forecast errors of the variance, giving more influence to recent informations and less to distant ones. ARCH imposes an autoregressive structure on conditional variance, allowing volatility shocks to persist over time. This persistence captures the propensity of returns to cluster over time, and can explain the nonnormality and nonstability observed in the data.

A generalization to the ARCH model was proposed [4], where the weights on past square residuals were assumed to decline geometrically at a rate to be estimated from the data. This is called the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model. The GARCH forecast variance is a weighted average of three different variance forecasts: The long-run average variance, the previous period variance and a new information (actual return) that was not available when the previous forecast was made. The weights on these three forecasts determine how fast the variance changes with new information and how fast it reverts to its long-run mean. When we consider only one past period to forecast the one-period ahead variance, we have the GARCH(1,1) model which can be expressed as,

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

where the conditional variance (σ_t^2) is a function of its most recent observation (σ_{t-1}^2), the new information (ε_{t-1}^2, the actual asset return) which

[11] This is, when the series do not have constant variance. See [13].

is assumed to be conditionally normally distributed with variance changing through time [17], constant parameters α and β, and the long-run average variance which determines ω.

The GARCH option pricing model is developed from a framework of risk-neutral valuation that includes the heteroskedasticity of the asset return process. In this way, it accounts for the changing volatility observed in the data. Under a risk-neutral pricing measure Q, the conditional variance process follows,

$$\sigma_t^2 = \omega + \alpha(\xi_{t-1} - \lambda\sqrt{\sigma_{t-1}})^2 + \beta\sigma_{t-1}^2 \tag{6}$$

where the normalised random variable $\xi_{t-1}/\sqrt{\sigma_{t-1}}$ is standard normal distributed and λ is the unit risk premium under Q. The process, is then, not a GARCH process [8].

The valuation framework of this pricing model computes the terminal asset price at a future point in time through the temporal aggregation of one-period asset returns. Under the risk-neutral pricing measure Q, the asset price follows a martingale process, and at time T it is,

$$X_T = X_t \exp[(T-t)r - 1/2\sum\sigma_s^2 + \sum\xi_s] \tag{7}$$

where r is the risk-free rate of interest, and the summations ($\sum$) are from time t to time T, for $t \leq s \leq T$.

The price of a European Call option in the GARCH pricing model is the discounted value, at the risk-free rate of interest, of the expected positive terminal payoff (when the option finishes In-The-Money (ITM)), or zero otherwise, this is,

$$C_t^{GH} = \exp(-(T-t)r)\, E^Q[\max(X_T - K, 0) |\ F_t] \tag{8}$$

Where $E^Q[.]$ is the expectation operator under the risk neutral probability measure Q, conditional on the set of information at time t (F_t). The terminal payoff of a European call option is, as we know, $(X_T - K, 0)^+$.

The GARCH option pricing model extends the Black and Scholes to a framework where the variance is heteroskedastic, it reproduces the leptokurtotic behaviour of the asset return series, with a stationary variance process that is correlated with the lagged asset returns [8]. Some correction in the biased produced by the Black and Scholes model is expected from this model, which incorporates features closer to the observed characteristics of the market data. The model is implemented through the simulation of the asset price process, discounting the expected payoff, at the risk-free rate of interest.

ii.2.4 Stochastic Volatility (Hull and White)

So far we have considered discrete type volatility models where the conditional variance is correlated with its past history. Here, we present another method to model the time changing volatility, with the introduction of a different approach, where the volatility of the asset price is directly modelled via a stochastic process. These are continuous type volatility models that are able to replicate accurately the behaviour observed in asset returns.

One of the standard stochastic volatility models is the model of Hull and White [16]. In this model the asset price S and the instantaneous variance σ_t^2 obey, under a risk-neutral probability measure, the following stochastic processes,

$$dS = rSdt + \sigma Sdz$$

$$d\sigma_t^2 = \alpha\sigma_t^2 dt + \xi\sigma_t^2 dw \tag{9}$$

where r, the risk-free rate is assumed constant, α and ξ are independent of S, and dz and dw are independent Wiener processes (Brownian Motions). The mean variance over some interval [0,T] in this setting is expressed as,

$$V_{mean} = 1/T \int^T [\sigma_t^2] dt \tag{10}$$

The distribution of the return series in this model, conditional upon the mean variance (V_m), is normal with mean $rT-V_mT/2$ and variance V_mT. This is, the distribution depends only on the risk-free rate, the time to maturity, the initial asset price, and the mean variance over the period [0,T]. Thus, any path followed by σ_t^2 will have the same terminal distribution if they have the same mean variance, which is true for an infinite number of paths when σ_t^2 is stochastic. From this Hull and White concludes that for a stochastic variance, the terminal distribution of the asset price, conditional on the mean variance, is lognormal [16].

The model states that the price of a European option with zero correlation between the asset price and the stochastic volatility, is the Black-Scholes price integrated over the probability distribution of the average variance rate during the life of the option [15]. This is,

$$\int^{\infty} [c(V_m)f_{Vm}]dV_m \qquad (11)$$

where $c(V_m)$is the Black-Scholes Call option price with variance V_m, and f_{Vm} is the density function of the mean variance under a risk-neutral probability measure.

The correlation between the underlying asset and the volatility has an impact on the terminal distribution of the asset price in this Stochastic Volatility model, and is a very important modelling assumption for financial time series. When the correlation is positive, high asset prices produces high volatilities, and it is more likely that high asset prices will result; on the other hand, low asset prices produces low volatilities, and low terminal asset prices will be more probable. The net effect is that the terminal asset price distribution will be more positively skewed than the lognormal distribution. Finally, for the case when the correlation is negative, the reverse holds, high asset prices reduces volatility, so that it is unlikely that very high asset prices will result, while low asset prices increases volatility, so that very low prices for the asset become less likely. The net effect in this case is that the terminal asset price distribution will be more peaked than the lognormal distribution [8].

II.3 Jump Model

ii.3.1 Merton (1976)

The Merton model of discontinuous paths is a Jump process model superimposed upon a Geometric Brownian Motion [15]. It follows the stochastic process,

$$dS = (\mu - \lambda k)dt + \sigma dz + dJ$$

where μ represents the expected return from the asset net of the dividend yield, λ represents the frequency of jumps per year, k the mean jump size as percentage of the asset price, dz is a Wiener process and dJ is the process generating the Jumps[12]. The process has volatility σ.

The option prices of European options in the Merton model applying risk-neutral valuation[13] can be obtained as a series where each term involves a Black-Scholes formula. This can be written as,

$$\sum_{n=0}^{\infty} (\exp(-\lambda' T)(\lambda' T)^n / n!) C_{BS} \qquad (12)$$

The Black-Scholes Call in this setting has variance,

$$\sigma_t^2 + (n\delta^2 / T) \qquad (13)$$

where δ is the standard deviation of a normal distribution. Additionally, the risk-free rate is,

$$r - \lambda k + (n\gamma/T) \qquad (14)$$

where the variable $\gamma = \ln(1 + k)$ is normally distributed. Finally, the Merton model reproduces appropriately the kurtosis observed in financial return series, showing heavier tails than the Gaussian distribution, with finite exponential moments [6].

III. Estimation procedures

The contracts for the S&P500 are Call options traded in US$ at the Chicago Board Options Exchange (CBOE), with a size of 100 options per contract. A total of 241 Call option (closing) prices were collected on the 17th of September 2004, for eight different maturities and a range of up to 60 different strike prices. On the other hand, the contracts for the DAX are those traded in EUROS in the EUREX exchange market, with a contract size of 5 Call options. A total of 247 Call option (closing) prices were selected on the 23rd of September 2004, for seven different maturities and a range of up to 49 different strike prices. Finally, the contracts selected for the NIKKEI, traded in YENS, have a size of 1000 options per contract, and a total of 44 Call option (closing) prices were collected on the 1st of October

[12] This is a Poisson process with intensity parameter λ.
[13] This assumes that the jump component of the asset returns represents nonsystematic risk [15].

2004, for five different maturities and a range of up to 12 different strike prices.

The Black and Scholes model is computed using equation (5) for European Call options for all maturities and strike prices. A historical volatility of 90 days was used for the price calculations, since it produced better results than other alternative windows (30 and 180 days). The risk-free rate was approximated using rates from maturities of two weeks (DAX) to one month (S&P500 and NIKKEI), while the dividend yield used was obtained from Bloomberg LP. The Risk-free interest rate selected for the S&P500 was the yield of the 4-weeks US-Treasury bills on the 17^{th}, September 2004. For the DAX it was the two-weeks EURO repo rate, and for the NIKKEI, it was the 1-month (Japan) Certificate of Deposit (CD) rate. Table 3 shows the parameters and the initial values used in the computation of the Black-Scholes option prices for each index.

	S&P500	DAX	NIKKEI
Risk-Free interest rate	1.58%	2.00%	0.51%
Dividend Yield	1.74%	2.08%	0.90%
Historical Volatility	10.35%	14.6%	16.66%
Index Initial Value	1128.55	3905.66	10985.17

Table 3

The GARCH(1,1) option pricing model is computed simulating the price process in equation (7) 102 times, and then taking the expectation given in equation (8) to discount it at the risk-free rate of interest. For the simulation, 102 standard normal random variables produced by S-PLUS 6.0 were used as the variable $\xi_{t-1}/\sqrt{\sigma_{t-1}}$ in equation (6), then, the random variable is multiplied by $\sqrt{\sigma_{t-1}}$ to have an estimate of the variable ξ_{t-1} itself. Here the variable σ_{t-1} starts with the long-run variance, which along with the other parameters of the GARCH model are computed using EVIEWS 4.1. Table 4 shows the estimated parameters (with standard deviations) of the GARCH model using the Maximum Likelihood method[14].

[14] See Appendix A.

	S&P500		DAX		NIKKEI	
	Parameter	Std. Dev.	Parameter	Std. Dev.	Parameter	Std. Dev.
ω	4.96E-7	9.58E-8	3.23E-6	1.35E-6	1.52E-5	4.98E-6
α	0,0456	0.0034	0,0847	0.0138	0,0628	0.0168
β	0,9501	0.0037	0,9050	0.0145	0,8659	0.0365
λ	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007

Table 4

The Stochastic Volatility model is computed calculating the mean-variance in equation (10), and then using this mean-variance to integrate the Black-Scholes (BS) price as in equation (11). To compute the mean-variance, we divide the maturity time T into η equal subintervals to generate the variance at time $t + i(T-t)/\eta$. The formula for the variance is,

$$V_i = V_{i-1} \exp((\mu - \xi^2/2)\Delta t + \nu_i \xi \sqrt{\Delta t})$$

where $\Delta t = (T-t)/\eta$, ξ is a constant, ν_i are independent standard normal variates and $1 \leq i \leq \eta$. The variable μ is made dependent on σ and t, with the process $\mu = a(\sigma^* - \sigma)$, and a and σ^* constants. The BS price is calculated using the arithmetic mean of V_i after 1000 simulations. To compute the integral in equation (11), twelve simulations for the mean-variance were performed to compute twelve different BS prices. The Call option price in the model is calculated as the arithmetic average of the twelve computed BS prices. Table 5 shows the parameters used in the computation of the Stochastic Volatility Call option prices written on each of the three indices considered, where the values of a and ξ are chosen as conservative estimates [16].

	S&P500	DAX	NIKKEI
Long-Run Variance	0,00012	0,00031	0,00021
A	10	10	10
ξ	1	1	1

Table 5

Finally, **the Jump-diffusion Merton model** is computed using equation (12), combined with equations (13) and (14) for the calculation of the BS price.

In this model the parameters were estimated using a calibration procedure with the set of option prices available. The calibration procedure is an iteration method that assigns values to the parameters of the model in order to minimise the errors produced by them. It is a method that improves the performance of historical-based parameter estimators for these class of models, where different historical price processes may lead to different risk-neutral pricing measures [6].

We selected the Average Relative Percentage Error (ARPE) as the measure of errors to be minimised in the calibration. Table 6 presents parameter values obtained from the calibration of the model to market data, for option prices with maturities of 45, 42 and 50 days, for the S&P500, DAX and NIKKEI respectively.

	S&P500	DAX	NIKKEI
δ	2,84%	1,37%	0,06%
λ	122,91	122,90	20,60
K	0,4%	1,4%	2,9%

Table 6

IV. Final Assessment

The Black and Scholes (BS) model implies completeness in the markets, this means that there exist a unique equivalent probability measure under which the underlying process is a Martingale [6]. The introduction of new random variable (Stochastic Volatility and Jumps) makes the models incomplete, signifying that there is no a unique Equivalent Martingale Measure (EMM) to price the options, and we have to choose an appropriate EMM according to particular criteria such as utility maximisation [25].

The models we have investigated are imperfect approximations to the real stochastic behaviour of the random variables, however, they are designed to produce a better "fit" to the market data than the standard Black and Scholes model. What follows present the results obtained from our experiment concerning the performance of the different models in terms of pricing errors, when we use the Average Relative Percentage Error (ARPE) as the goodness of fit measure[15].

[15] Tables showing the numerical results of the experiment for the ARPE% and RMSE indicators are included in Appendix B.

Figures 19, 20 and 21 present the Average Relative Percentage Error (ARPE) of the Black-Scholes model[16], for all the options on the S&P500, the DAX and the NIKKEI225 indices respectively, at different maturity times.

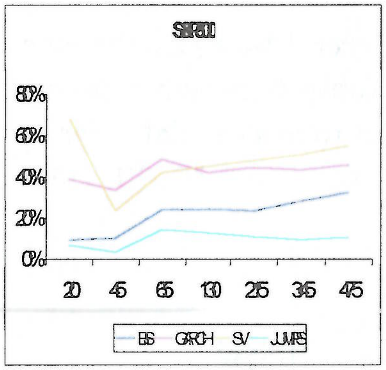

Figure 19

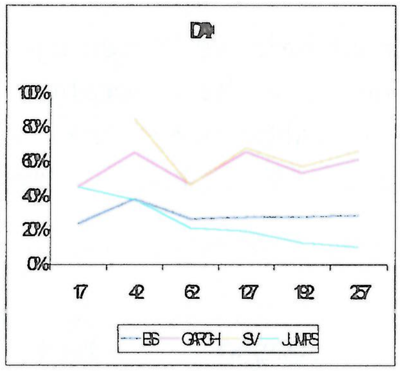

Figure 20

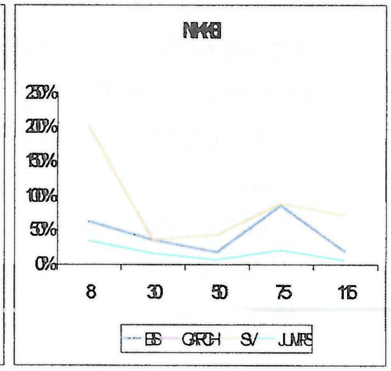

Figure 21

We observe how the Jump model minimises the ARPE at almost every maturity time, however, the Black and Scholes model still outperforms the GARCH and the Stochastic Volatility models when the whole range of maturity times is considered. Similar patterns can be seen for the RMSE.

When only In-The-Money (ITM) options are considered we observe in figures 22, 23 and 24 a better performance of the GARCH model relative to the standard BS, this is especially true in the case of the S&P500 index.

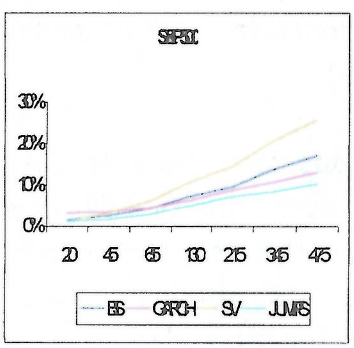

Figure 22

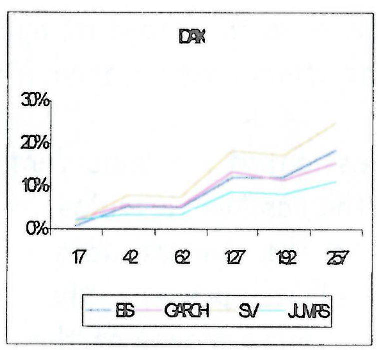

Figure 23

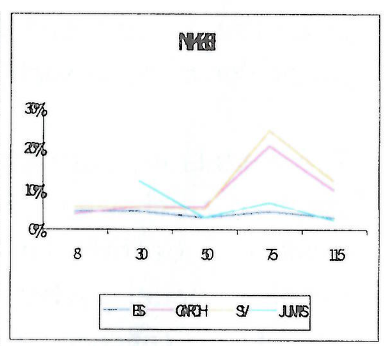

Figure 24

[16] Our criteria is to considered options with a *Moneyness* (Asset Price/Strike Price) in the range (0.97,0.97) to be At-The-Money (ATM), options less or equal than 0.97, to be In-The-Money (ITM) , and options greater or equal than 1.03, to be Out-of-The-Money; see [16].

For At-The-Money (ATM) and Out-of-The-Money (OTM) options, the patterns are not discernible, since the Black and Scholes in many cases still outperforms its competitors[17]. All the models perform better at short and medium term maturity times, and degenerate at long term maturity times.

The Jump-diffusion model of Merton [19] appears as the best performer in our experiment. These class of models incorporating Jumps have been investigated recently using Levy processes, and they seem to produce in general better results than the diffusion type pricing models [25]. It have been argued that the Jump type pricing models possesses various empirical, computational and statistical features that improve considerably the methodology of pricing and hedging financial derivatives, and that they deliver qualitative different results about key issues in option pricing and risk management, in general [6], [25].

Finally, we show in table 7, some key results taken from the book of Cont and Tankov [6], about the properties of the Jump and diffusion models, concerning the empirical facts observed in the markets. This encourages the investigation of Jump type pricing models as a better alternative to the understanding of today´s highly complex derivative markets. The results of our experiment point out in this direction.

Empirical Facts	Diffusion models	Models with Jumps
Heavy Tails	Possible by choosing nonlinear volatility strutures	Generic property
Concentration of losses in a few large downward moves	Price movements are conditionally Gaussian; large sudden movements do not occur	Jumps/discontinuities in prices can give rise to large losses
Markets are incomplete; some risks cannot be hedged away	Markets are complete	Markets are incomplete

Table 7

[17] In the case of the GARCH pricing model, this is due mainly to the fact that it assigns a value of zero to most of the OTM options; recall the component $E^Q[\max(X_T-K,0)| F_t]$ in the valuation formula in equation (8).

APPENDIX A

1. **The Put-Call parity,** is a relationship that holds by no-arbitrage principles, and can be defined as,

$$P_t + S_t \exp(-q(T-t)) = C_t + K \exp(-r(T-t))$$

where P_t , C_t and S_t represent the Put, Call and asset values at time t respectively, K is the strike price, q the dividend yield, and r the risk-free rate.

2. **Error measures**

The Average Relative Percentage error is,

$$ARPE = \frac{1}{\#\text{ options}} \sum_{\text{options}} \frac{|\text{market price} - \text{model price}|}{\text{market price}}$$

And the Root Mean Square Error is,

$$RMSE = \sqrt{\sum_{\text{options}} \frac{(\text{market price} - \text{model price})^2}{\#\text{options}}}$$

3. **The log return**, r_t, of an asset is the difference of the natural logarithm of its price (value), and can be defined for an Index as,

$$r_t = \log(I_t/I_{t-1}) = \log(I_t) - \log(I_{t-1})$$

where I_t represents the level of the Index at time t.

4. The **Standard Deviation** is a measure of dispersion or spread in a series,

$$\alpha_2 = \sqrt{\frac{\sum (y_i - Y)^2}{(N-1)}}$$

where N is the number of observations in the current sample, y_i are sample values, Y is the mean of the series, and the summation of squared residuals ranges from i=1 to N.

5. **Skewness** measures the asymmetry of the distribution of the series around its mean,

$$\alpha_3 = \frac{\sum (y_i - Y / \sigma)^3}{N} = \mu_3$$

where σ is an estimator for the standard deviation that is based on the biased estimator for the variance, N is the number of observations in the current sample, y_i are sample values, Y is the mean of the series, and the summation ranges from i=1 to N.

6. **Kurtosis** measures the peakedness or flatness of the distribution of the series,

$$\alpha_4 = \frac{\sum (y_i - Y / \sigma)^4}{N} = \mu_4$$

where again, σ is an estimator for the standard deviation that is based on the biased estimator for the variance, N is the number of observations in the current sample, y_i are sample values, Y is the mean of the series, and the summation ranges from i=1 to N.

7. The **Jarque-Bera** statistics compares the third and fourth moments of the residuals to those from the normal distribution,

$$JB = N \left[\frac{S^2}{6} + \frac{(K-3)^2}{24}\right]$$

where N is the sample size, S represents the Skewness and K represents the Kurtosis of the distribution.

8. The **Quantile-Quantile (Q-Q) plot** is a tool for comparing two data series. This plots the quantiles of a chosen (empirical) distribution against the quantiles of another empirical or theoretical distribution.

9. The ***Autocorrelation Function (ACF)*** of a series at lag k is,

$$\rho_k = \frac{\text{Covariance with lag k}}{\text{Variance}} = \frac{\sum(y_i - Y)(y_{i+k} - Y)}{\sum (y_i - Y)^2}$$

where Y is the sample mean of the series, and y_i are sample values. This is the correlation coefficient for values of the series k periods apart.

10. The **Unit Root** test is a test of the null hypothesis H_0: $\rho = 1$ against the one-sided alternative H_1: $\rho < 1$, where ρ is the parameter for the lagged value y_{t-1} in equation $y_t = \rho y_{t-1} + \varepsilon$, where the error term ε has no autocorrelation, with mean zero and constant variance. The Augmented Dickey-Fuller and Phillip-Perron tests are unit root that take account of series with more than one lagged values and the asymptotic distribution of the test statistic, respectively.

11. The **Maximum Likelihood Method** is a parameter estimation procedure that seeks to maximise the likelihood that the data at hand occurs. It maximises the function,

$$F = \exp[(-\tfrac{1}{2}\sum\varepsilon^2/\sigma^2)] / \sigma^n(\sqrt{2\pi})^n$$

For an error as $\varepsilon_t = Y_t - \beta_1 - \beta_2 X_t$. See [22].

APPENDIX B

1. ARPE% and RMSE for options on the S&P500, at different maturities.

All Options:							
ARPE %	20	45	65	130	215	345	475
BS	9,40%	10,19%	23,82%	24,26%	23,12%	28,21%	32,56%
GARCH	38,95%	33,71%	48,71%	42,17%	44,46%	43,46%	45,85%
SV	68,50%	23,6%	42,0%	45,13%	48,12%	50,83%	55,01%
JUMPS	6,98%	3,29%	14,28%	12,78%	10,99%	9,38%	10,61%
RMSE	20	45	65	130	215	345	475
BS	1,47	2,95	4,75	9,60	14,09	23,85	32,33
GARCH	5,20	6,50	10,67	14,36	20,29	29,59	37,44
SV	1,81	4,80	10,07	17,15	24,89	39,90	52,84
JUMPS	0,94	2,03	2,95	6,49	10,44	14,34	23,79
In-The-Money:							
ARPE %	20	45	65	130	215	345	475
BS	1,51%	2,57%	4,21%	7,31%	9,54%	13,84%	17,07%
GARCH	3,25%	3,48%	4,43%	6,47%	8,78%	10,71%	13,09%
SV	1,07%	3,2%	6,2%	10,8%	14,38%	20,77%	25,56%
JUMPS	1,22%	1,67%	2,99%	5,1%	7,18%	8,44%	10,23%
RMSE	20	45	65	130	215	345	475
BS	1,46	3,22	5,24	10,95	16,44	27,06	36,79
GARCH	2,66	4,09	6,24	11,49	16,30	24,81	32,22
SV	0,91	3,71	7,61	15,90	24,02	39,56	53,85
JUMPS	1,07	2,39	3,84	7,94	12,93	17,45	30,14
At-The-Money:							
ARPE %	20	45	65	130	215	345	475
BS	8,38%	20,79%	25,00%	32,36%	32,67%	38,76%	41,81%
GARCH	76,64%	91,83%	85,69%	85,35%	87,85%	79,30%	75,02%
SV	17,92%	63,0%	74,4%	90,13%	92,59%	94,69%	95,87%
JUMPS	4,19%	7,37%	8,66%	15,18%	17,37%	18,21%	14,51%
RMSE	20	45	65	130	215	345	475
BS	1,76	4,32	7,21	13,86	18,30	30,42	40,55
GARCH	9,84	18,51	24,09	35,43	48,50	61,74	72,26
SV	2,95	12,59	21,12	38,24	51,76	74,37	92,98
JUMPS	0,95	1,78	2,74	6,73	9,81	14,19	14,18
Out-The-Money:							
ARPE %	20	45	65	130	215	345	475
BS	32,52%	28,25%	52,33%	54,49%	45,64%	54,31%	59,30%
GARCH	na	na	na	na	na	na	na
SV	332,65%	68,1%	86,5%	99,93%	99,96%	99,99%	100,00%
JUMPS	27,64%	6,51%	33,09%	26,75%	16,57%	9,05%	10,34%
RMSE	20	45	65	130	215	345	475
BS	0,20	0,83	2,02	4,17	6,46	12,36	17,82
GARCH	1,36	3,81	6,85	10,66	16,90	25,72	33,59
SV	2,00	1,99	5,85	10,62	16,87	25,72	33,58
JUMPS	0,15	0,07	0,45	1,34	2,56	3,02	4,01

2. ARPE% and RMSE for options on the DAX, at different maturities.

All Options:						
ARPE %	**17**	**42**	**62**	**127**	**192**	**257**
BS	23,69%	38,24%	26,43%	27,54%	27,5%	28,40%
GARCH	45,41%	65,16%	46,63%	65,50%	53,48%	61,09%
SV	na	85,11%	46,41%	67,81%	57,03%	65,98%
JUMPS	45,24%	37,69%	20,98%	19,35%	12,41%	10,02%
RMSE	**17**	**42**	**62**	**127**	**192**	**257**
BS	2,19	12,85	22,73	50,80	76,68	106,57
GARCH	14,87	29,67	42,00	87,49	103,87	130,80
SV	7,63	28,22	45,27	98,74	127,09	165,52
JUMPS	13,80	7,96	14,12	36,14	50,58	64,19
In-The-Money:						
ARPE %	**17**	**42**	**62**	**127**	**192**	**257**
BS	0,80%	5,36%	5,06%	12,14%	12,28%	18,49%
GARCH	2,18%	5,79%	5,42%	13,51%	11,50%	15,46%
SV	1,56%	8,10%	7,47%	18,36%	17,25%	24,93%
JUMPS	2,17%	3,43%	3,25%	8,9%	8,30%	11,16%
RMSE	**17**	**42**	**62**	**127**	**192**	**257**
BS	2,74	19,38	27,47	70,27	95,97	143,76
GARCH	10,60	21,84	29,93	76,83	87,72	117,96
SV	6,17	28,61	39,23	101,05	127,64	183,61
JUMPS	7,70	12,74	17,99	52,06	65,08	91,86
At-The-Money:						
ARPE %	**17**	**42**	**62**	**127**	**192**	**257**
BS	3,32%	16,93%	23,46%	31,02%	35,92%	37,79%
GARCH	76,17%	81,31%	84,63%	87,99%	89,16%	88,65%
SV	10,30%	68,13%	81,8%	92,25%	97,59%	196,29%
JUMPS	74,78%	6,88%	10,91%	20,00%	21,92%	16,34%
RMSE	**17**	**42**	**62**	**127**	**192**	**257**
BS	2,57	17,74	31,95	67,70	97,93	123,74
GARCH	36,36	78,45	110,95	188,90	240,32	287,77
SV	4,20	67,46	108,82	199,89	266,53	455,39
JUMPS	36,05	8,02	15,55	43,97	60,01	53,77
Out-The-Money						
ARPE %	**17**	**42**	**62**	**127**	**192**	**257**
BS	64,80%	63,04%	59,14%	38,39%	45,07%	36,40%
GARCH	na	na	na	na	na	na
SV	na	na	na	na	na	na
JUMPS	100,00%	65,10%	49,82%	27,09%	15,97%	8,12%
RMSE	**17**	**42**	**62**	**127**	**192**	**257**
BS	0,41	1,99	6,93	20,27	33,16	46,27
GARCH	4,20	13,35	26,87	57,99	82,64	106,45
SV	10,10	9,99	25,26	57,99	82,64	106,45
JUMPS	4,20	0,58	1,88	11,27	16,99	15,94

3. ARPE% and RMSE for options on the NIKKEI, at different maturities.

All Options					
ARPE %	8	30	50	75	115
BS	62,16%	34,86%	17,02%	84,91%	18,8%
GARCH	na	na	na	na	na
SV	201,90%	34,80%	42,47%	88,01%	69,97%
JUMPS	33,76%	15,80%	6,09%	20,47%	5,52%
RMSE	8	30	50	75	115
BS	51,83	44,95	39,17	22,57	22,18
GARCH	na	na	na	na	na
SV	55,12	71,97	95,90	152,57	189,33
JUMPS	633,25	375,16	34,58	27,26	25,64
In-The-Money					
ARPE %	8	30	50	75	115
BS	4,43%	4,23%	2,58%	4,54%	2,63%
GARCH	3,72%	5,69%	5,41%	20,81%	9,95%
SV	5,53%	5,23%	4,74%	24,30%	12,03%
JUMPS	na	12,05%	2,72%	6,6%	2,05%
RMSE	8	30	50	75	115
BS	56,62	58,92	50,02	29,03	28,63
GARCH	52,59	66,37	82,74	133,20	114,59
SV	61,90	63,33	68,72	155,55	132,84
JUMPS	749,26	502,83	42,27	42,23	31,35
At-The-Money					
ARPE %	8	30	50	75	115
BS	74,57%	9,99%	6,23%	8,16%	4,33%
GARCH	127,87%	105,94%	97,42%	98,06%	95,52%
SV	53,37%	71,09%	85,77%	92,18%	93,90%
JUMPS	14,33%	21,07%	13,21%	13,84%	9,62%
RMSE	8	30	50	75	115
BS	52,20	21,99	18,37	28,96	19,72
GARCH	89,51	233,06	287,39	348,12	434,62
SV	37,36	156,39	253,04	327,24	427,24
JUMPS	10,03	46,35	38,97	49,14	43,77
Out-The-Money					
ARPE %	8	30	50	75	115
BS	338,42%	94,18%	44,99%	116,33%	31,34%
GARCH	na	na	na	na	na
SV	na	71,10%	97,68%	99,91%	99,94%
JUMPS	0,00%	20,27%	10,20%	24,57%	6,78%
RMSE	8	30	50	75	115
BS	6,77	10,71	11,78	19,42	26,98
GARCH	na	na	na	na	na
SV	26,64	26,48	57,62	79,57	132,01
JUMPS	0,00	11,23	8,94	14,17	14,51

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