Reinforcement Learning HW 3

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1 Recap

a)

b)

c)

2 TD(λ)

2.1 a)

$$\begin{aligned}
G_t^{(n)} &= R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{n-1} R_{t+n} + \gamma^n V(S_{t+n}) \\
G_{t+1}^{(n-1)} &= R_{t+2} + \gamma R_{t+3} + \cdots + \gamma^{n-2} R_{t+n} + \gamma^{n-1} V(S_{t+n}) \\
G_t^{(n)} &= R_{t+1} + \gamma\big(R_{t+2} + \gamma R_{t+3} + \cdots + \gamma^{n-2} R_{t+n} + \gamma^{n-1} V(S_{t+n})\big) \\
&= R_{t+1} + \gamma G_{t+1}^{(n-1)}
\end{aligned}$$

b)

$$G_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_t^{(n)}$$

Using part (a), $$G_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1}\big(R_{t+1} + \gamma G_{t+1}^{(n-1)}\big)$$

$$= R_{t+1}(1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} + \gamma(1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_{t+1}^{(n-1)}$$

By the geometric series,

$$\sum_{n=1}^{\infty} \lambda^{n-1} = \frac{1}{1-\lambda}$$

$$
\begin{aligned}
G_t^\lambda &= R_{t+1} + \gamma(1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} G_{t+1}^{(n-1)} \\
&= R_{t+1} + \gamma(1-\lambda)\sum_{n=0}^{\infty} \lambda^{n} G_{t+1}^{(n)} \\
&= R_{t+1} + \gamma\Big[(1-\lambda)V(S_{t+1}) + \lambda G_{t+1}^\lambda\Big]
\end{aligned}
$$

$$
\boxed{G_t^\lambda = R_{t+1} + \gamma\Big[(1-\lambda)V(S_{t+1}) + \lambda G_{t+1}^\lambda\Big]}
$$