

Gaussian Splats as Geometry: From Covariance Inverses to Metrics, Curvature, and Gauge Interpretations

Bjørn Remseth
Email: la3lma@gmail.com
With AI assistance

Abstract—This document explores the interpretation of Gaussian splats as defining a geometric structure on space. The inverse covariance matrices attached to splats are interpreted as metric tensors, from which we derive connections, parallel transport, and curvature. We examine this framework through three lenses: classical Riemannian geometry, gauge theory with vielbeins and spin connections, and geometric algebra with rotor fields and bivector-valued connections. Three concrete examples illustrate the theory: a 1D Gaussian metric with nontrivial connection but zero curvature, a 2D conformal metric from a Gaussian bump with explicit curvature calculation and integrated holonomy, and a rotor swirl field demonstrating topological defects. This geometric interpretation connects scene representations from computer graphics to concepts from differential geometry and gauge theory.

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Index Terms—Gaussian splatting, Riemannian geometry, curvature, parallel transport, gauge theory, geometric algebra, differential geometry

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I. INTRODUCTION

The starting question for this exploration was: “If we consider that Gaussian splats are curvature (via covariance inverses), what are the parallel transports we can use to understand this as curvature in space, and what are the corresponding gauge theories that can be seen as an interpretation?”

The answer requires first clarifying that inverse covariances define *metrics*, not curvature directly. Curvature arises from spatial variation of these metrics. This document traces the complete geometric chain:

covariance inverse \Rightarrow metric \Rightarrow connection \Rightarrow parallel transport \Rightarrow curvature

We then re-interpret this structure in gauge-theoretic language (vielbeins and spin connections) and geometric algebra (rotor fields and bivector connections). The approach

is illustrated through three concrete examples with explicit calculations.

A. Overview of Results

The main conceptual results are:

- Inverse covariance matrices Σ^{-1} naturally define a metric field, not curvature directly
- Curvature appears when examining how the metric varies in space via the Levi-Civita connection
- Parallel transport is governed by this connection; curvature measures the obstruction to path-independent parallel transport
- In gauge theory language, this is a local $SO(n)$ gauge theory of orthonormal frames with spin connection and curvature 2-form
- In geometric algebra, a rotor field $R(x)$ with connection $\mathcal{A}_\mu = (\partial_\mu R)\tilde{R}$ encodes the same structure as a bivector-valued gauge field

II. FROM GAUSSIAN SPLATS TO METRIC TENSORS

Consider a collection of Gaussian splats in \mathbb{R}^3 , each characterized by:

- Center $\mu_i \in \mathbb{R}^3$
- Covariance matrix $\Sigma_i \in \mathbb{R}^{3 \times 3}$, symmetric positive-definite
- Visual attributes (color, opacity, etc.)

The associated Gaussian density is

$$G_i(\mathbf{x}) \propto \exp\left(-\frac{1}{2}(\mathbf{x} - \mu_i)^\top \Sigma_i^{-1}(\mathbf{x} - \mu_i)\right). \quad (1)$$

The quadratic form

$$(\mathbf{x} - \mu_i)^\top \Sigma_i^{-1}(\mathbf{x} - \mu_i) \quad (2)$$

represents the squared Mahalanobis distance from μ_i in the metric defined by

$$g_i := \Sigma_i^{-1}. \quad (3)$$

This suggests interpreting Σ_i^{-1} as a local metric tensor on the tangent space at μ_i .

A. Constructing a Metric Field

For a field of splats densely covering space, we construct a smooth metric field $g_{ij}(\mathbf{x})$ by interpolating inverse covariances of nearby splats:

$$g_{ij}(\mathbf{x}) = \sum_k w_k(\mathbf{x}) (\Sigma_k^{-1})_{ij}, \quad (4)$$

where $w_k(\mathbf{x})$ are weights derived from the Gaussian densities. Provided the weighted sum remains symmetric positive-definite, this defines a legitimate Riemannian metric:

$$ds^2 = g_{ij}(\mathbf{x}) dx^i dx^j. \quad (5)$$

The slogan “*Gaussian splats are curvature*” becomes more precisely: *Gaussian splats define a metric field via their covariance inverses; curvature is then the Riemannian curvature of that metric.*

III. RIEMANNIAN GEOMETRY: CONNECTION AND CURVATURE

Given a Riemannian metric $g_{ij}(\mathbf{x})$, there exists a unique torsion-free metric-compatible connection: the Levi-Civita connection. Its coefficients (Christoffel symbols) are

$$\Gamma_{ij}^k(\mathbf{x}) = \frac{1}{2} g^{kl}(\mathbf{x}) (\partial_i g_{lj} + \partial_j g_{il} - \partial_l g_{ij}), \quad (6)$$

where g^{kl} is the matrix inverse of g_{kl} .

A. Parallel Transport

A vector field $v^i(t)$ along a curve $\gamma(t)$ is parallel transported if

$$\frac{Dv^k}{dt} := \frac{dv^k}{dt} + \Gamma_{ij}^k(\gamma(t)) \dot{\gamma}^i(t) v^j(t) = 0. \quad (7)$$

This preserves the metric length:

$$\frac{d}{dt} (g_{ij} v^i v^j) = 0. \quad (8)$$

In the splat interpretation, parallel transport means adjusting vectors as one moves through the field so that lengths and inner products remain invariant according to the local covariance-inverse metric.

B. Curvature: Failure of Path Independence

The Riemann curvature tensor measures the failure of parallel transport to be path-independent:

$$R_{lij}^k = \partial_i \Gamma_{lj}^k - \partial_j \Gamma_{li}^k + \Gamma_{mi}^k \Gamma_{lj}^m - \Gamma_{mj}^k \Gamma_{li}^m. \quad (9)$$

For tangent vectors X, Y and a vector v , the curvature operator is

$$R(X, Y)v = \nabla_X \nabla_Y v - \nabla_Y \nabla_X v - \nabla_{[X, Y]} v. \quad (10)$$

Parallel transporting v around a small parallelogram spanned by X and Y yields $v + R(X, Y)v + O(\text{area}^2)$. If $R = 0$ everywhere, the space is flat.

In the Gaussian-splat setting, curvature quantifies how metric differences between nearby splats produce net rotation when transporting vectors around loops.

IV. DISCRETE GRAPH PICTURE

Alternatively, treating each splat as a graph node provides a discrete analogue of metric-compatible parallel transport.

A. Edge Transport Operators

Let each splat i be a node with metric $g_i = \Sigma_i^{-1}$. Connect nearby nodes with edges. A discrete parallel transport along edge $i \rightarrow j$ is a linear map

$$P_{ij} : T_i \rightarrow T_j \quad (11)$$

preserving metric lengths:

$$v^\top g_i w = (P_{ij} v)^\top g_j (P_{ij} w), \quad \forall v, w, \quad (12)$$

equivalently,

$$P_{ij}^\top g_j P_{ij} = g_i. \quad (13)$$

One construction: factorize $g_i = A_i^\top A_i$ (e.g., via Cholesky), then define

$$P_{ij} = A_j^{-1} R_{ij} A_i, \quad (14)$$

where $R_{ij} \in \text{SO}(3)$ is a rotation representing gauge freedom.

B. Discrete Curvature via Holonomy

For a loop of nodes $i_1 \rightarrow i_2 \rightarrow \dots \rightarrow i_n \rightarrow i_1$, compose edge transports:

$$P_{\text{loop}} = P_{i_1 i_2} P_{i_2 i_3} \dots P_{i_n i_1}. \quad (15)$$

If $P_{\text{loop}} = I$, parallel transport is trivial around that loop (locally flat). Otherwise, P_{loop} is a rotational holonomy encoding discrete curvature.

V. GAUGE THEORY INTERPRETATION

We now express the metric structure using local frames (vielbeins) and gauge connections.

A. Vielbeins: From Metric to Frame

Given $g_{ij}(x)$, introduce an orthonormal frame field $e_i^a(x)$ (vielbein) such that

$$g_{ij}(x) = \delta_{ab} e_i^a(x) e_j^b(x), \quad (16)$$

with a, b running over frame indices. The vielbein is a “square root” of the metric.

For a Gaussian splat, one natural choice: diagonalize $\Sigma^{-1}(x) = R(x) \Lambda(x) R(x)^\top$ with $R(x) \in \text{SO}(n)$ and $\Lambda(x)$ diagonal with positive entries $\lambda_a(x)$. Define $S(x) = \text{diag}(\sqrt{\lambda_1}, \dots, \sqrt{\lambda_n})$. Then

$$e_i^a(x) = (R(x) S(x))_i^a \quad (17)$$

gives $g = e^\top e$.

B. Gauge Symmetry and Connection

Rotating the orthonormal frame at each point is a local $\text{SO}(n)$ gauge symmetry:

$$e_i^a(x) \mapsto \Lambda_b^a(x) e_i^b(x), \quad \Lambda(x) \in \text{SO}(n). \quad (18)$$

A connection 1-form $\omega_{b\mu}^a(x)$ with values in $\mathfrak{so}(n)$ describes how frames change under parallel transport:

$$\nabla_\mu e_i^a = \partial_\mu e_i^a + \omega_{b\mu}^a e_i^b - \Gamma_{i\mu}^k e_k^a = 0 \quad (19)$$

expresses metric-compatibility in the orthonormal basis.

C. Curvature as Field Strength

The curvature of this connection is the $\mathfrak{so}(n)$ -valued 2-form

$$R_{b\mu\nu}^a = \partial_\mu \omega_{b\nu}^a - \partial_\nu \omega_{b\mu}^a + \omega_{c\mu}^a \omega_{b\nu}^c - \omega_{c\nu}^a \omega_{b\mu}^c. \quad (20)$$

In gauge theory language:

- Gauge group: $\text{SO}(n)$ (or $\text{Spin}(n)$)
- Gauge field: spin connection ω
- Field strength: curvature R

In the splat framework: the covariance-inverse field gives g , orthonormal frames give e , and the spin connection ω encodes how principal axes of Gaussian ellipsoids twist through space.

VI. GEOMETRIC ALGEBRA INTERPRETATION

Geometric algebra packages orthonormal frames and rotations into multivector objects called rotors [1], [2].

A. Rotor Field as Gauge Field

In Euclidean 3-space, a rotor $R(x)$ is an even multivector satisfying $R\tilde{R} = 1$, acting by sandwiching:

$$v' = Rv\tilde{R}, \quad (21)$$

which rotates vector v .

Let $e_a^{(0)}$ be a fixed reference frame. A rotor field $R(x)$ defines a local orthonormal frame:

$$e_a(x) = R(x) e_a^{(0)} \tilde{R}(x). \quad (22)$$

Combining $R(x)$ with diagonal scaling $S(x)$ gives a frame transformation

$$A(x) = R(x)S(x), \quad (23)$$

and the metric can be written $g(x) = A(x)^\top A(x)$.

B. Bivector Connection and Curvature

Define a bivector-valued gauge potential:

$$\mathcal{A}_\mu(x) = (\partial_\mu R(x))\tilde{R}(x), \quad (24)$$

living in the Lie algebra of $\text{Spin}(n)$ (the bivector space). This measures how the rotor field changes.

The curvature 2-form is the bivector-valued

$$\mathcal{F}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu]. \quad (25)$$

Parallel transport along curves can be expressed via path-ordered exponentials of \mathcal{A}_μ (the holonomy rotor).

In geometric algebra, the “metric from splats, curvature from metric” story becomes: inverse covariances define a metric; evolving eigenframes are encoded by rotor field $R(x)$; the bivector connection $\mathcal{A}_\mu = (\partial_\mu R)\tilde{R}$ is the gauge field; its curvature measures eigenframe twist around loops.

VII. EXAMPLE 1: 1D GAUSSIAN-MODIFIED METRIC

Consider a 1D line with coordinate x and metric

$$ds^2 = g(x) dx^2, \quad g(x) > 0. \quad (26)$$

Interpret $g(x) = \sigma^{-2}(x)$ from a local Gaussian width $\sigma(x)$.

A. Conformal Parameterization

Write $g(x) = e^{2\phi(x)}$ for a scalar field $\phi(x)$. A Gaussian bump:

$$\phi(x) = \alpha \exp\left(-\frac{x^2}{2a^2}\right). \quad (27)$$

B. Connection and Parallel Transport

In 1D, the single Christoffel symbol is

$$\Gamma_{xx}^x = \frac{1}{2} g^{-1}(x) \frac{dg}{dx} = \phi'(x). \quad (28)$$

Parallel transport of $v^x(t)$ along $x(t)$ satisfies

$$\frac{dv^x}{dt} + \phi'(x(t)) \dot{x}(t) v^x = 0, \quad (29)$$

with solution

$$v^x(t) = C e^{-\phi(x(t))}. \quad (30)$$

The coordinate component rescales with the conformal factor to keep metric length invariant.

C. Curvature in 1D

Any 1D metric is locally flat (can be reparameterized to constant g). The Riemann curvature vanishes identically. This example has nontrivial connection and parallel transport but no genuine curvature. To see curvature, we need at least 2D.

VIII. EXAMPLE 2: 2D CONFORMAL METRIC FROM GAUSSIAN BUMP

Now consider a 2D plane (x, y) with a conformal metric built from a Gaussian bump, where curvature appears nontrivially.

A. Metric from Scalar Gaussian Field

Take a radially symmetric scalar field

$$\phi(r) = \alpha \exp\left(-\frac{r^2}{2a^2}\right), \quad r = \sqrt{x^2 + y^2}, \quad (31)$$

and define

$$ds^2 = e^{2\phi(r)}(dx^2 + dy^2). \quad (32)$$

Interpret ϕ as derived from overlapping Gaussian splats collectively warping distance.

B. Curvature Calculation

For a 2D conformal metric $ds^2 = e^{2\phi(x,y)}(dx^2 + dy^2)$, the Gaussian curvature is

$$K(x, y) = -e^{-2\phi(x,y)} \Delta \phi(x, y), \quad (33)$$

where Δ is the flat Laplacian. For radial $\phi(r)$,

$$\Delta \phi(r) = \phi_{rr}(r) + \frac{1}{r} \phi_r(r). \quad (34)$$

Computing derivatives:

$$\phi_r(r) = -\alpha \frac{r}{a^2} e^{-\frac{r^2}{2a^2}}, \quad (35)$$

$$\phi_{rr}(r) = -\alpha \frac{1}{a^2} e^{-\frac{r^2}{2a^2}} + \alpha \frac{r^2}{a^4} e^{-\frac{r^2}{2a^2}}, \quad (36)$$

thus

$$\Delta\phi(r) = \alpha \frac{r^2 - 2a^2}{a^4} e^{-\frac{r^2}{2a^2}}. \quad (37)$$

The curvature is

$$K(r) = -e^{-2\phi(r)} \Delta\phi(r) = -\alpha \frac{r^2 - 2a^2}{a^4} e^{-\frac{r^2}{2a^2}} e^{-2\alpha e^{-\frac{r^2}{2a^2}}}. \quad (38)$$

At the origin:

$$K(0) = \frac{2\alpha}{a^2} e^{-2\alpha} \approx \frac{2\alpha}{a^2} \quad (\alpha \ll 1). \quad (39)$$

Qualitatively:

- Near $r = 0$: $K(0) > 0$ (positive curvature)
- As r grows: ring of negative curvature
- For large r : $K(r) \rightarrow 0$

A single Gaussian bump creates a curvature hill at the center with a compensating negative-curvature ring.

C. Integrated Curvature and Holonomy

The total curvature inside a disk of radius R :

$$\mathcal{K}(R) := \int_{B_R} K dA_g. \quad (40)$$

For our conformal metric, $dA_g = e^{2\phi} dx dy$, so $K dA_g = -\Delta\phi dx dy$. In polar coordinates:

$$\mathcal{K}(R) = -2\pi \int_0^R \Delta\phi(r) r dr = -2\pi [r\phi_r(r)]_0^R = 2\pi\alpha \frac{R^2}{a^2} e^{-\frac{R^2}{2a^2}}. \quad (41)$$

As $R \rightarrow \infty$, $\mathcal{K}(R) \rightarrow 0$: positive curvature near the bump is balanced by negative curvature in the outer ring.

The angle by which a vector rotates when parallel transported around a small loop is approximately the integrated curvature:

$$\delta\theta \approx \mathcal{K}(R) \approx 2\pi\alpha \frac{R^2}{a^2} \quad (42)$$

for small R and α . This is the “splats \Rightarrow metric \Rightarrow curvature \Rightarrow holonomy” story explicitly computed.

IX. EXAMPLE 3: ROTOR SWIRL FIELD AS TOPOLOGICAL DEFECT

The previous example had smooth curvature. Now consider a case where the anisotropy field’s orientation winds around the origin, producing a topological defect with nontrivial holonomy even with constant eigenvalues.

A. Anisotropic Gaussian with Rotating Eigenframe

In 2D, suppose the inverse covariance is

$$\Sigma^{-1}(x) = R(\theta(x)) \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} R(\theta(x))^\top, \quad (43)$$

with fixed $\lambda_1, \lambda_2 > 0$ and rotation angle $\theta(x)$.

In geometric algebra, rotations are encoded by rotors

$$R(\theta) = e^{-\frac{\theta}{2} e_{12}}, \quad (44)$$

where $e_{12} = e_1 e_2$ is the unit bivector.

Define a swirl eigenframe:

$$\theta(r, \varphi) = k\varphi, \quad (45)$$

where (r, φ) are polar coordinates and $k \in \mathbb{Z}$ is a winding number. The rotor field is

$$R(\varphi) = e^{-\frac{k\varphi}{2} e_{12}}. \quad (46)$$

As you circle the origin, the principal axes rotate k times.

B. GA Connection and Holonomy

The GA connection along the angular direction is

$$\mathcal{A}_\varphi(\varphi) = (\partial_\varphi R) \tilde{R}. \quad (47)$$

Differentiating:

$$\partial_\varphi R(\varphi) = -\frac{k}{2} e_{12} R(\varphi), \quad (48)$$

so

$$\mathcal{A}_\varphi(\varphi) = -\frac{k}{2} e_{12}, \quad (49)$$

which is constant in φ .

The holonomy rotor around the circle $\varphi : 0 \rightarrow 2\pi$ is

$$R_{\text{hol}} = \exp\left(-\frac{k}{2} e_{12} \cdot 2\pi\right) = e^{-k\pi e_{12}}. \quad (50)$$

A rotor $e^{-\frac{\alpha}{2} e_{12}}$ rotates vectors by angle α , so this holonomy corresponds to net rotation $2\pi k$.

For $k = 1$: holonomy is a 2π rotation (nontrivial spin structure). This is a vortex in the eigenframe field: walking around the origin, principal axes twist by $2\pi k$. In Riemannian terms, this corresponds to curvature concentrated at the origin (like a cone with deficit angle), whereas the Gaussian bump example had smeared curvature.

X. SYNTHESIS AND IMPLICATIONS

We have established a coherent geometric interpretation of Gaussian splats:

- Inverse covariances are metrics, not curvature directly
- Promoting Σ_i^{-1} to a smooth metric field $g(x)$ via interpolation yields the Levi-Civita connection and its curvature
- Parallel transport preserves vector lengths in the g -metric
- Curvature measures failure of parallel transport to be path-independent
- In discrete splat graphs, edge transports P_{ij} preserve local metrics, with curvature as holonomy around discrete loops
- In gauge theory, decomposing g into vielbein e_i^a and spin connection $\omega_{b\mu}^a$ gives a local $\text{SO}(n)$ gauge theory
- In geometric algebra, rotor field $R(x)$ encodes eigenframes, with $\mathcal{A}_\mu = (\partial_\mu R) \tilde{R}$ as bivector-valued gauge potential

The three examples illustrated:

- 1) 1D Gaussian metric: nontrivial connection but zero curvature
- 2) 2D conformal metric: explicit curvature hill with compensating negative ring, and integrated holonomy

- 3) Rotor swirl: topological-defect curvature with nontrivial holonomy from constant eigenvalues but winding eigenframe

This geometric view connects scene representations from computer graphics to differential geometry and gauge theory. Viewing splats as defining an anisotropic, spatially varying Riemannian metric provides a framework for understanding how directions and distances behave in splat-space. The gauge-theoretic and geometric-algebra formulations offer powerful mathematical language for expressing this structure.

While not the conventional description in graphics literature, this interpretation is mathematically coherent and potentially fruitful—especially for connecting scene representations to ideas from gauge theory, geometric algebra, and curved configuration spaces in robotics.

XI. CONCLUSION

We have developed a rigorous mathematical framework interpreting Gaussian splats as defining geometric structures on space. The key insight is that inverse covariance matrices naturally define metric tensors, from which standard differential-geometric structures follow: connections, parallel transport, and curvature.

This interpretation was examined through three complementary lenses:

- Classical Riemannian geometry (metrics, Christoffel symbols, Riemann curvature)
- Gauge theory (vielbeins, spin connections, $SO(n)$ gauge symmetry)
- Geometric algebra (rotor fields, bivector connections, holonomy rotors)

The three concrete examples provided explicit calculations demonstrating how Gaussian-shaped metrics produce nontrivial geometric phenomena, from simple parallel transport in 1D to curvature hills and topological defects in 2D.

Future directions could explore applications to:

- Path planning in splat-based scene representations
- Gauge-theoretic regularization of splat reconstructions
- Geometric algebra formulations of splat rendering
- Connections to physics-based scene understanding

This work demonstrates that the mathematical machinery of differential geometry, gauge theory, and geometric algebra provides natural and powerful tools for understanding Gaussian splat representations beyond their immediate graphics applications.

ACKNOWLEDGMENTS

This document was created through collaborative dialogue with AI systems (ChatGPT and Claude), exploring the geometric interpretation of Gaussian splats.

ABOUT THIS DOCUMENT

This document represents a technical exploration created through a collaborative process between human and AI. The production process followed these steps:

- 1) **Discovery:** This work emerged from curiosity about whether Gaussian splats could be interpreted geometrically, specifically whether their covariance matrices could be viewed as defining curvature in space. The question arose while considering connections between scene representations in computer graphics and concepts from differential geometry.
- 2) **Initial Exploration:** The initial dialogue with ChatGPT explored the mathematical chain from covariance inverses to metrics, connections, parallel transport, and curvature. The conversation clarified that inverse covariances define metrics (not curvature directly) and developed the gauge-theoretic and geometric-algebra interpretations.
- 3) **Synthesis:** The ChatGPT conversation output was organized into a structured IEEE conference paper format, with sections reordered for pedagogical clarity and explicit examples added to illustrate the abstract theory.
- 4) **Implementation:** Three concrete examples were developed with explicit calculations: 1D Gaussian metric (connection without curvature), 2D conformal metric (curvature from Gaussian bump with integrated holonomy), and rotor swirl field (topological defect).
- 5) **Visualization:** This document primarily uses mathematical notation. Future versions could include visualizations of curvature fields, parallel transport paths, and holonomy around loops.
- 6) **Verification:** All references were scrutinized for authenticity. URLs were tested for accessibility where available, author names were verified against real publications, and publication venues were confirmed. This verification process is documented in the following section.

This exploration demonstrates how AI-assisted dialogue can help develop novel connections between established mathematical frameworks and emerging computational techniques. The geometric interpretation presented here is mathematically rigorous and potentially useful for future work connecting computer graphics to differential geometry and gauge theory.

NOTE ON REFERENCES AND VERIFICATION

This document contains AI-generated content. All references have been subject to rigorous verification to ensure academic integrity.

Verification Process:

- All URLs were tested for accessibility using automated web search
- Author names were verified against real publications and institutional affiliations
- DOIs were confirmed where available through publisher websites
- Publication venues (journals, conferences, publishers) were validated
- Content relevance was checked against citations through web searches
- ISBN numbers were verified for books through multiple sources

Verification Status in References: Each reference includes a `note` field indicating its verification status:

- “Verified: URL accessible” – URL was tested and confirmed working
- “Verified: DOI accessible” – DOI was confirmed through publisher
- “Standard reference” – Well-known textbook or established foundational work
- “Verified: Web search confirms existence” – Publication confirmed through multiple independent sources

Reference Note: The original ChatGPT output cited “Dorst et al. 2007, *Geometric Algebra for Physicists*” but this is actually authored by Doran & Lasenby (2003). Dorst’s 2007 book is “*Geometric Algebra for Computer Science*”. The corrected citation uses Doran’s book as it better matches the physics-oriented content.

Important Notice: Due to the AI-assisted nature of this document’s creation, readers should independently verify any references used for critical applications. This level of scrutiny is essential when working with AI-generated academic content.

REFERENCES

- [1] D. Hestenes, *Space-Time Algebra*. New York, NY, USA: Gordon and Breach, 1966, standard reference: Foundational work on geometric algebra. Historical reference.
- [2] C. Doran and A. Lasenby, *Geometric Algebra for Physicists*. Cambridge, UK: Cambridge University Press, 2003, standard reference: Comprehensive treatment of geometric algebra with applications to physics. Includes chapters on rotations, spacetime algebra, and gauge theory.