MSP - xdokou14

December 11, 2022

0.1 1. Úkol

[1]:
```
# zadání 4
import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import scipy.stats as stats
import statsmodels.api as sm
from matplotlib import cm
```

Import dat, kde poslední sloupec tvoří mnou zjištěné hodnoty.

[2]:
```
# data 1
city = np.array(['big', 'big', 'medium', 'medium', 'medium', 'small', 'small',␣
 ↪'small'])
cat = np.array(
    ['Praha', 'Brno', 'Znojmo', 'Tišnov', 'Rokytnice nad Jizerou', 'Jablunkov',␣
 ↪'Dolní Věstonice', 'student'])
count = np.array([1327, 915, 681, 587, 284, 176, 215, 21])
winter = np.array([510, 324, 302, 257, 147, 66, 87, 11])
summer = np.array([352, 284, 185, 178, 87, 58, 65, 4])
change = np.array([257, 178, 124, 78, 44, 33, 31, 4])
indifferent = np.array([208, 129, 70, 74, 6, 19, 32, 2])
```

Pro zjištění rovnosti procentuálního zastoupení obyvatel měst použijeme test dobré shody. $H_0 : p_1 = p_2 ... = p_8$

[3]:
```
def chi_test(str, cat, count, dof=1):
    #print("got: ", cat.astype(int), "\nexp: ", (([cat.sum() / count.sum()] *␣
 ↪len(count)) * count).astype(int), '\n')
    print(str, stats.chisquare(cat, f_exp=([cat.sum() / count.sum()] *␣
 ↪len(count)) * count, ddof=dof))


chi_test("winter", winter, count)
chi_test("summer", summer, count)
chi_test("change", change, count)
print("critical value: ", stats.chi2.ppf(0.95, 6))
```

```
winter Power_divergenceResult(statistic=21.310706163566906,
pvalue=0.001613039676565202)
summer Power_divergenceResult(statistic=7.299595273038772,
pvalue=0.2940271928800054)
change Power_divergenceResult(statistic=12.346058276016716,
pvalue=0.05467923612892293)
critical value:  12.591587243743977
```

Pro zimní čas zamítamé H_0 ($p < 0.05$), tudíž se percentuální zastoupení mezi městy liší. Pro změnu času a letní čas H_0 nezamítamé.

Pro zjištění rovnosti procentuálního zastoupení mezi různě velkými městy (velké, střední, malé) použijeme obdobný test nad seskupenými daty.

[4]:
```
# group by city
city_group = pd.DataFrame({'city': city, 'count': count, 'winter': winter,␣
↪'indifferent': indifferent})
city_group = city_group.groupby('city').sum()

chi_test("group_winter", city_group['winter'], city_group['count'])
chi_test("group_indifferent", city_group['indifferent'], city_group['count'])
```

```
group_winter Power_divergenceResult(statistic=15.616786433175259,
pvalue=7.756296276494824e-05)
group_indifferent Power_divergenceResult(statistic=20.57102536251306,
pvalue=5.745922029341851e-06)
```

H_0 zamítáme jak pro zimní čas, tak pro nerozhodnuté obyvatele.

Pro odhadnutí z které skupiny student pocházi (velké, střední, malé) použijeme opět χ^2 test. Očekávané hodnoty pro příslušnost studenta tvoři bodový odhad z daných skupin.

[5]:
```
city_group = pd.DataFrame(
    {'city': city, 'cat': cat, 'count': count, 'winter': winter, 'summer':␣
↪summer, 'change': change,
     'indifferent': indifferent})
city_group = city_group[city_group['cat'] != 'student']
city_group = city_group.drop('cat', axis=1)

city_group = city_group.groupby(['city']).sum()
city_group.loc['student'] = [count[-1], winter[-1], summer[-1], change[-1],␣
↪indifferent[-1]]
# separate count column
counts = city_group['count']
city_group = city_group.drop('count', axis=1)

# chi_test student to X cities
student = city_group.loc['student'].values

chi_test("student=big city", student, city_group.loc['big'].values, dof=0)
```

```
chi_test("student=medium city", student, city_group.loc['medium'].values, dof=0)
chi_test("student=small city", student, city_group.loc['small'].values, dof=0)
```

```
student=big city Power_divergenceResult(statistic=2.369349379132745,
pvalue=0.49936572713839256)
student=medium city Power_divergenceResult(statistic=1.0717291713049328,
pvalue=0.7839027804135497)
student=small city Power_divergenceResult(statistic=2.261937024132146,
pvalue=0.5198519535559099)
```

Ze získaných dat nelze vyvodit příslušnost studenta do kterékoliv skupiny ($p > 0.05$ pro každou skupinu). Závěr nejde vyvodit nejspíše kvůli podobnosti kategorii a malému vzorku mých dat, avšak největší podobnost se ukazuje ke středně velkým městům.

0.2 2. Úkol

import dat

[6]:
```
# data2
x = np.array(
    [0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00, 2.22, 2.22, 2.22, 2.22, 2.22, 2.
 ↪22, 2.22, 4.44, 4.44, 4.44, 4.44, 4.44,
     4.44, 4.44, 6.67, 6.67, 6.67, 6.67, 6.67, 6.67, 6.67, 8.89, 8.89, 8.89, 8.
 ↪89, 8.89, 8.89, 8.89, 11.11, 11.11,
     11.11, 11.11, 11.11, 11.11, 11.11, 13.33, 13.33, 13.33, 13.33, 13.33, 13.
 ↪33, 13.33, 15.56, 15.56, 15.56, 15.56,
     15.56, 15.56, 15.56, 17.78, 17.78, 17.78, 17.78, 17.78, 17.78, 17.78, 20.
 ↪00, 20.00, 20.00, 20.00, 20.00, 20.00,
     20.00])
y = np.array(
    [0.00, 1.67, 3.33, 5.00, 6.67, 8.33, 10.00, 0.00, 1.67, 3.33, 5.00, 6.67, 8.
 ↪33, 10.00, 0.00, 1.67, 3.33, 5.00, 6.67,
     8.33, 10.00, 0.00, 1.67, 3.33, 5.00, 6.67, 8.33, 10.00, 0.00, 1.67, 3.33, 5.
 ↪00, 6.67, 8.33, 10.00, 0.00, 1.67,
     3.33, 5.00, 6.67, 8.33, 10.00, 0.00, 1.67, 3.33, 5.00, 6.67, 8.33, 10.00, 0.
 ↪00, 1.67, 3.33, 5.00, 6.67, 8.33,
     10.00, 0.00, 1.67, 3.33, 5.00, 6.67, 8.33, 10.00, 0.00, 1.67, 3.33, 5.00, 6.
 ↪67, 8.33, 10.00],
    dtype='float64')
z = np.array(
    [3.51678419592411, 0.1849332370251, -6.99049059346124, -20.8617205435173,␣
 ↪-42.18343555059, -64.1081000325912,
     -97.0458957178397, 9.94388450504941, 9.8986323460927, 0.00208487443735393,␣
 ↪-11.9988247882815, -34.3924062967239,
     -57.8057880895009, -88.5027847946918, 27.2265088682819, 24.4284903127453,␣
 ↪18.2145118228463, 1.94336256143575,
     -16.7727492407465, -42.0967616131069, -71.1967634077031, 56.3446765711985,␣
 ↪52.378628730395, 44.868666552744, 4.56,
```

```
    11.1141991787491, -14.9444563191628, -46.3430249668268, 92.5713346195499,␣
↪89.3713833654496, 81.3928266038351,
    65.624785460706, 44.8871370272759, 23.7631895332308, -8.64772334749607, 139.
↪152701188226, 135.142403459449,
    125.462388521822, 112.482276630433, 94.9110448098603, 67.9275602260165, 39.
↪4220662346769, 195.442542098054,
    192.351096305394, 185.89808996746, 169.086590035809, 150.799896551493, 126.
↪327402037799, 95.0588038056046,
    260.907635326248, 257.874160634941, 251.576673225094, 235.967305190209, 217.
↪589979963901, 192.992167354398,
    160.733689709696, 337.506810454201, 334.670521354979, 324.663539748041, 313.
↪044243941327, 293.633223156008,
    269.455800707765, 235.257507397646, 423.522399438186, 421.287561597445, 412.
↪428044955087, 397.834243660025,
    382.450519904685, 355.996853248479, 322.553027375103])
```

Pomocné funkce pro vizualizaci regrese.

[7]:
```
x1 = np.linspace(0, 20, 100)
y1 = np.linspace(0, 10, 100)
x1, y1 = np.meshgrid(x1, y1)



def plot_3d(x, y, z, x1, y1, z1):
    plt.figure()
    ax = plt.axes(projection='3d')
    ax.plot_surface(x1, y1, z1, cmap=cm.coolwarm, linewidth=0,␣
↪antialiased=False, alpha=0.5)
    # add data points
    ax.scatter(x, y, z, c='r', marker='x')
    ax.set_xlabel('x')
    ax.set_ylabel('y')
    ax.set_zlabel('z')
    # rotate view
    ax.view_init(15, 75)
    plt.show()
```

Vypočtená statistika pro obecnou kvadratickou regresi

[8]:
```
# alpha = .05 is default
F = sm.add_constant(np.column_stack((x, y, x ** 2, y ** 2, x * y)))
modelres = sm.OLS(z, F).fit()
print(modelres.summary(xname=['const', 'x', 'y', 'x^2', 'y^2', 'xy']))
print()
print('R^2', modelres.rsquared)

z1 = modelres.predict(sm.add_constant(
```

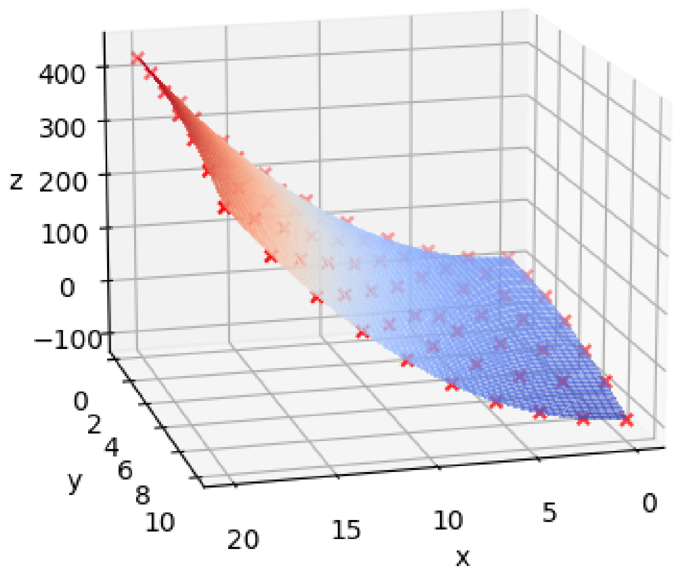
```
    np.column_stack((x1.ravel(), y1.ravel(), x1.ravel() ** 2, y1.ravel() ** 2,
↪x1.ravel() * y1.ravel())))).reshape(
    x1.shape)

plot_3d(x, y, z, x1, y1, z1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  0.999
Method:                 Least Squares   F-statistic:                 2.610e+04
Date:                Sun, 11 Dec 2022   Prob (F-statistic):          1.79e-104
Time:                        16:49:17   Log-Likelihood:                -180.16
No. Observations:                  70   AIC:                             372.3
Df Residuals:                      64   BIC:                             385.8
Df Model:                           5
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.2580      1.589      2.680      0.009       1.084       7.432
x              0.8635      0.248      3.484      0.001       0.368       1.359
y             -0.3672      0.468     -0.785      0.435      -1.302       0.568
x^2            1.0080      0.011     91.195      0.000       0.986       1.030
y^2           -0.9655      0.041    -23.415      0.000      -1.048      -0.883
xy             0.0014      0.019      0.074      0.941      -0.036       0.039
==============================================================================
Omnibus:                      132.161   Durbin-Watson:                   2.220
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             6863.247
Skew:                          -6.429   Prob(JB):                         0.00
Kurtosis:                      49.774   Cond. No.                         839.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

R^2 0.9995097318397491
```



Pro redukci modelu využijeme T-test na nulovou hodnotu koeficientu. Koeficient můžeme při analýze zanedbat při zamítnuti nulové hypotézy rovnosti parametrů ((P>|t|) > 0.05).

[9]:
```
print(modelres.t_test("x5 = 0"))   #xy

F = sm.add_constant(np.column_stack((x, y, x ** 2, y ** 2)))
modelres = sm.OLS(z, F).fit()

print(modelres.t_test("x2 = 0"))   #y
```

```
                             Test for Constraints
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
c0             0.0014      0.019      0.074      0.941      -0.036       0.039
==============================================================================
                             Test for Constraints
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
c0            -0.3535      0.426     -0.830      0.410      -1.204       0.497
==============================================================================
```

Výsledný regresní model: $Z = B_1 + B_2X + B_3X^2 + B_4Y^2$. 2b) Hodnoty koeficientů jsou vyznačeny v tabulce pod sloupcem “coef”. 95% interval spolehlivosti je vyznačen v posledních 2 sloupcích [0.025

0.975].

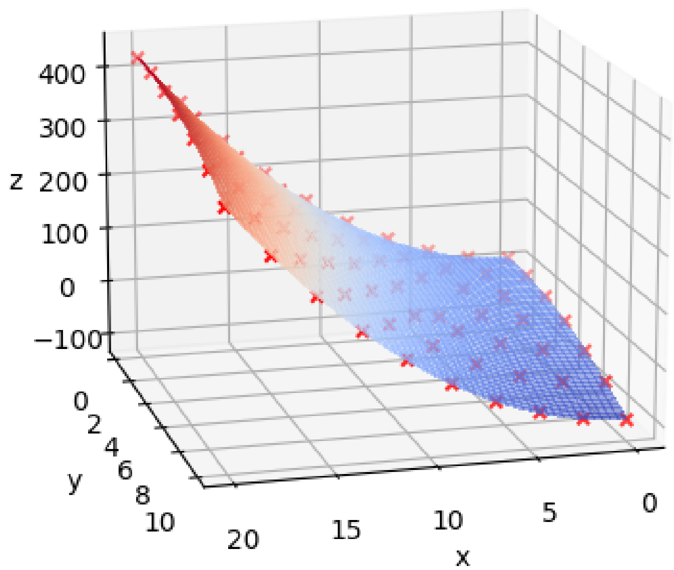
[10]:
```
F = sm.add_constant(np.column_stack((x, x ** 2, y ** 2)))
modelres = sm.OLS(z, F).fit()
print(modelres.summary(xname=['const', 'x', 'x^2', 'y^2']))
print()
print('R^2', modelres.rsquared)
print("rozptyl závislé proměnné", modelres.mse_resid)

z1 = modelres.predict(sm.add_constant(
    np.column_stack((x1.ravel(), x1.ravel() ** 2, y1.ravel() ** 2)))).reshape(
    x1.shape)
plot_3d(x, y, z, x1, y1, z1)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       1.000
Model:                            OLS   Adj. R-squared:                  0.999
Method:                 Least Squares   F-statistic:                 4.438e+04
Date:                Sun, 11 Dec 2022   Prob (F-statistic):          5.66e-109
Time:                        16:49:18   Log-Likelihood:                -180.54
No. Observations:                  70   AIC:                             369.1
Df Residuals:                      66   BIC:                             378.1
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          3.6001      1.058      3.402      0.001       1.487       5.713
x              0.8704      0.227      3.829      0.000       0.416       1.324
x^2            1.0080      0.011     92.118      0.000       0.986       1.030
y^2           -0.9982      0.011    -88.172      0.000      -1.021      -0.976
==============================================================================
Omnibus:                      134.606   Durbin-Watson:                   2.195
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7420.478
Skew:                          -6.620   Prob(JB):                         0.00
Kurtosis:                      51.671   Cond. No.                         534.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.

R^2 0.9995044933271784
rozptyl závislé proměnné 10.795343055493214
```



Pro test rovnosti na nulu dvou koeficientů zároveň jsem zvolil F-test s B_0 a B_1. H_0: $B_0 = B_1 = 0$

[11]:
```python
print(modelres.f_test("const = x1 = 0"))
```

```
<F test: F=51.68770554193782, p=3.11091337215992e-14, df_denom=66, df_num=2>
```

Na hladině významnosti $\alpha = 0.05$ ((P>|t|) < 0.05) zamítáme H_0, koeficienty se tudíž zároveň liší od nuly.

Pro test rovnosti dvou koeficientů jsem zvolil T-test mezi B_0 a B_1. H_0: $B_0 = B_1$

[12]:
```python
print(modelres.t_test("const = x1 "))
```

```
                             Test for Constraints
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
c0             2.7297      1.237      2.206      0.031       0.259       5.200
==============================================================================
```

Na hladině významnosti $\alpha = 0.05$ ((P>|t|) < 0.05) zamítáme H_0, koeficienty se tudíž liší.