which trivially rearranges to give the recurrence relation (18.115). To obtain the recurrence relation (18.116), we begin by differentiating the generating function (18.114) with respect to x, which yields and thus we have

Equating coefficients of h^n on each side then gives which immediately simplifies to give (18.116).

18.8 Associated Laguerre functrions

The associated Laguerre equation has the form

$$xy + (m + 1 - x)y + ny = 0$$

; it has a regular singularity at x = 0 and an essential singularity at $x = \infty$. We restrict our attention to the situation in which the parameters n and m are both non-negative integers, as is the case in nearly all physical problems. The associated Laguerre equation occurs most frequently in quantum-mechanical applications.Any solution of (18.118) is called an *associated Laguerre function.* Solution of (18.118) for non-negative integers $n$and m are given by the *associated Laguerre polynomials*

$$L(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

, where $L_n(x)$ are the ordinary Laguerre polynomials.

Show that the functions L(x) defined in (18.119) are solutions of (18.118).

Since the Laguerre polynomials $L_n(x)$ are solutions of Laguerre's equation (18.107), we have

$$xL_{n+m} + (1 - x)L_{n+m} + (n + m)L_n{+}_m = 0$$

Differentiating this equation m times using Leibnitz' theorem and rearranging, we find

$$xL^(m + 2)_n +_m +(m + 1 - x)L^(m + 1)_{n+m} + nL^(m)_{n+m} = 0.$$

Note that some authors define the associated Laguerre polynomials as , which is thus related to our expression

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1 Introduction