C4-exam

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ity of are (i,j) must hold; otherwise, we could label node j (via a forward arc) and node j would not be in V'.Now consider an are(i,j)such that i is in V' and j is in V.Then $x_{ij} = 0$ must hold; otherwise, we could label noe i (via a backward arc) and node i would not be in V'.Now (3) shows that the current flow must satisfy

$$\text{Capacity of CUT} = \text{current flow from source to sink}$$

which is the desired result.

From the remarks following Lemma 1,when the sink cannot be labeled, the maximum flow from source to sink has been obtained.

Summary and Illustration of the Ford-Fulkerson Method

Step 1 Find a feasible flow (setting each arc's flow to zero will do).

Step 2 Using the labeling procedure, try to label the sink. If the ink cannot be labeled, then the current feasible flow is a maximum flow; if the sink is labeled, then go on to step 3.

Step3 Using the method previously described, adjust the feasible flow and increase the flow from the source to the sink. Return to step 2.

To illustrate the Ford-Fulkerson method, we find the maximum flow from source to sink for Sunco Oil, Example 3(see Figure 6).We begin by letting the flow in each are equal zero.We then try to label the sink-label the source, and then arc(so,1)and node 1;then label arc (1,2) and node 2; finally, label arc (2,si) and node si.Thus, C = (so,1)-(1,2)-(2,si). Each are in C is a forward arc, so we can increase the flow through each are in C by min(2,3,2) = 2 units. The resulting flow is pictured in Figure 15.

As we saw previously (Figure 12), we can label the sink by using the chain C = (so, 2)-(1,2)-(1,3)-(3,si).We cna increase the flow through the forward arcs (so,2),(1,3),and(3,si)by 1 unit and decrease the flow through the backward arc(1,2) by 1 unit. The resulting flow is pictured in Figure 16. It is now impossible ot label the sink. Any attempt to label the sink must begin by labeling are (so,2)and node 2; then we could label are(1,2) and are (1,3).But there is no way to label the sink.

We can verify that the curent flow is maximal by finding the capacity of the cut corresponding to the set of unlabeled vertices(in this cas,si) The cut corresponding to si i the set of arcs(2,si)and(3,si),with capacity 2+1=3. Thus,Lemma I implies that any feasible flow can transport at most 3 units from source to sink. Our current flow trans-ports 3 units from source to sink, so it must be an optimal flow.

Another example of the Ford-Fulkerson metho is given in Figure 17. Note that without the concept of a backward arc, we could not have obtained the maximum flow of 7