which trivially rearranges to give the recurrence relation (18.115). To obtain the recurrence relation (18.116), we begin by differentiating the generating function (18.114) with respect to x, which yields

$$\frac{\delta G}{\delta x} = -\frac{he^{-xh/(1-h)}}{(1-h)^2} = \sum L\,'_n h^n,$$

and thus we have

$$-h \sum L_n h^n = (1-h) \sum L\,'_n h^n,$$

Equating coefficients of h^n on each side then gives

$$-L_{n-1} = L\,'_n - L\,'_{n-1}$$

which immediately simplifies to give (18.116).

18.8 Associated Laguerre functrions

The associated Laguerre equation has the form

$$xy\,'' + (m+1-x)y;' + ny = 0;$$

; it has a regular singularity at x = 0 and an essential singularity at $x = \infty$. We restrict our attention to the situation in which the parameters n and m are both non-negative integers, as is the case in nearly all physical problems. The associated Laguerre equation occurs most frequently in quantum-mechanical applications.Any solution of (18.118) is called an *associated Laguerre function.* Solution of (18.118) for non-negative integers $n$and m are given by the *associated Laguerre polynomials*

$$L(x) = (-1)^m \frac{d^m}{dx^m} L_{n+m}(x)$$

, where $L_n(x)$ are the ordinary Laguerre polynomials.

→Show that the functions L(x) defined in (18.119) are solutions of (18.118).

Since the Laguerre polynomials $L_n(x)$ are solutions of Laguerre's equation (18.107), we have

$$xL_{n+m} + (1-x)L_{n+m} + (n+m)Ln + m = 0$$

Differentiating this equation m times using Leibnitz' theorem and rearranging, we find

$$xL^(m+2)_{n+m} + (m+1-x)L^(m+1)_{n+m} + nL^(m)_{n+m} = 0.$$

Note that some authors define the associated Laguerre polynomials as , which is thus related to our expression