PSET 06 - Hw 03 (20171049)

(a) $P(\omega_1) = \frac{7}{14} = 0.5$, $P(\omega_2) = \frac{7}{14} = 0.5$

(b) $\mu_{w_1} = \frac{(0,0)+(0,1)+(2,0)+(3,2)+(3,3)+(2,2)+(2,0)}{7}$

$\mu_{w_1} = \left(\frac{12}{7}, \frac{8}{7}\right) = (1.714, 1.143)$

Also, $\mu_{w_2} = \frac{(7,7)+(8,6)+(9,7)+(8,10)+(7,10)+(8,9)+(7,11)}{7}$

$\mu_{w_2} = \left(\frac{54}{7}, \frac{60}{7}\right) = (7.714, 8.571)$

Now, $\Sigma_{w_1} = \frac{1}{7\times 6}\begin{bmatrix} 66 & 37 \\ 37 & 62 \end{bmatrix} = \begin{bmatrix} 1.571 & 0.881 \\ 0.881 & 1.476 \end{bmatrix}$

Also, $\Sigma_{w_2} = \frac{1}{7\times 6}\begin{bmatrix} 24 & -27 \\ -27 & 152 \end{bmatrix} = \begin{bmatrix} 0.571 & -0.643 \\ -0.643 & 3.619 \end{bmatrix}$

(c) $(\Sigma_{w_1})^{-1} = \begin{bmatrix} 0.957 & -0.571 \\ -0.571 & 1.018 \end{bmatrix}$, $(\Sigma_{w_2})^{-1} = \begin{bmatrix} 2.189 & 0.389 \\ 0.389 & 0.345 \end{bmatrix}$

$|\Sigma_{w_1}| = 1.543$, $|\Sigma_{w_2}| = 1.656$

Now, to derive equation for decision boundary, $P(w_1|x) = P(w_2|x)$

$\Rightarrow P(x|w_1).P(w_1) = P(x|w_2)P(w_2)$

$\Rightarrow \frac{1}{2}\times\frac{1}{(\sqrt{2\pi})^2|\Sigma_1|^{1/2}} e^{-\frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1)} = \frac{1}{2}\times\frac{1}{(\sqrt{2\pi})^2|\Sigma_2|^{1/2}} e^{-\frac{1}{2}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2)}$

$\Rightarrow \frac{1}{2}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2) - \frac{1}{2}(x-\mu_1)^T\Sigma_1^{-1}(x-\mu_1) = \ln\left|\frac{\Sigma_1}{\Sigma_2}\right|^{1/2}$

$\Rightarrow \cancel{\tfrac{1}{2}}(x-\mu_2)^T\Sigma_2^{-1}(x-\mu_2) - \cancel{\tfrac{1}{2}}(x-\mu_1)\Sigma_1^{-1}(x-\mu_1) = \cancel{\tfrac{1}{2}}\ln\left|\frac{\Sigma_1}{\Sigma_2}\right|$

Putting values of $x=[x\ y]^T$, μ_1, μ_2, Σ_1^{-1}, Σ_2^{-1}, $|\Sigma_1|$, $|\Sigma_2|$, we get,

$\Rightarrow 1.231x^2 - 0.67y^2 + 1.92xy - 38.42x - 11.54y + 205 = 0.$

Decision boundary

(As $h = \frac{1.92}{2}$, $a = 1.231$, $b = -0.67$ $\Rightarrow h^2 - ab > 0$)

$\Downarrow$

Hyperbola

Plot $\longrightarrow$

Rough shape $\Rightarrow$

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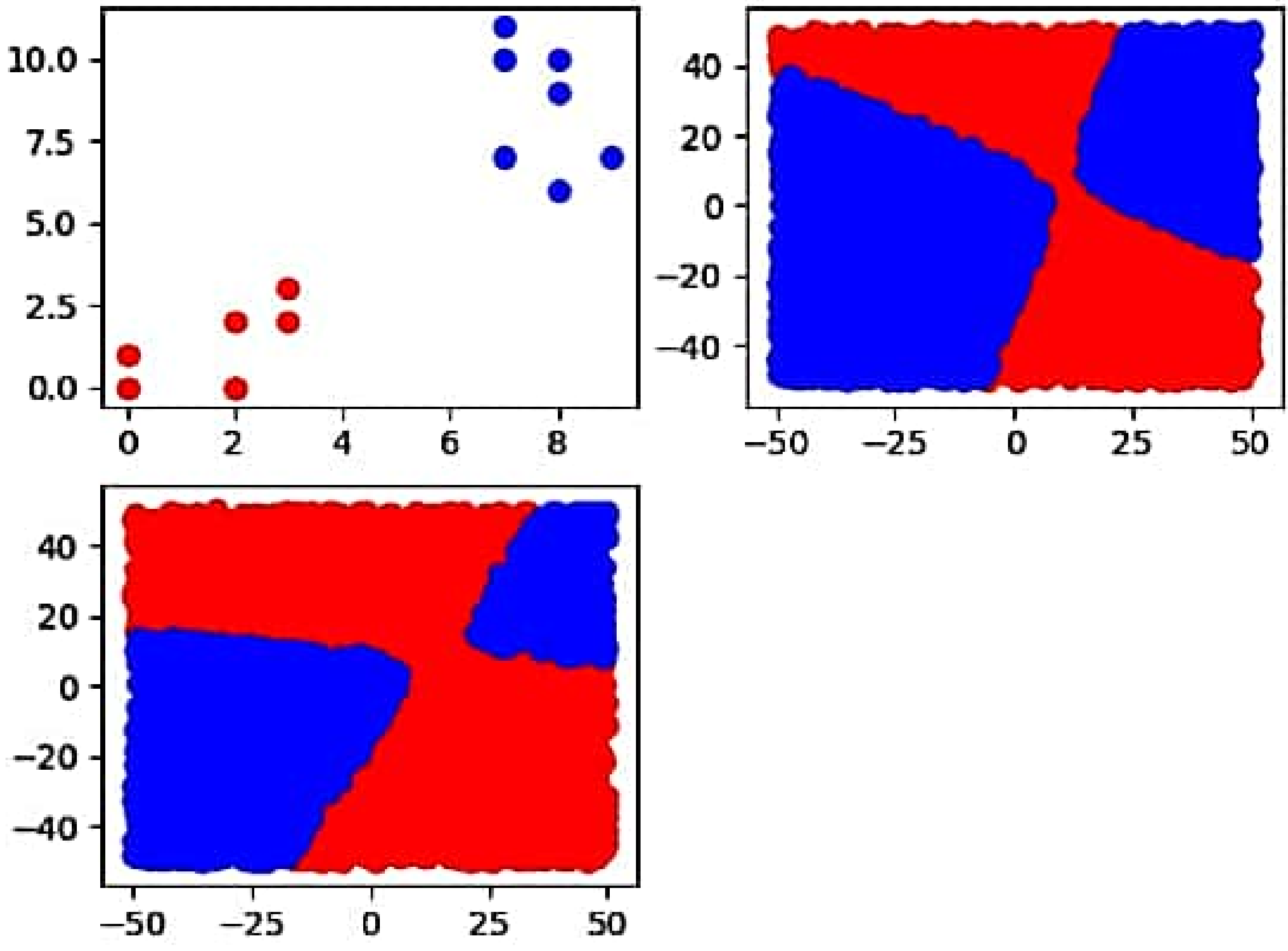
10.0
7.5
5.0
2.5
0.0
0
2
4
6
8
40
20
0
−20
−40
−50
−25
0
25
50

{ (d) is plotted above }

(e.) Let the penalties for each misclassification are different for the 2 classes, ω_1 and ω_2, & let $\dfrac{Penalty(\omega_1)}{Penalty(\omega_2)} = k$. (2017104 9)

Now, $P(\omega_1|x) = \dfrac{P(x|\omega_1)P(\omega_1)}{P(x)} \times \left(\dfrac{k}{k+1}\right)$.

Also, $P(\omega_2|x) = \dfrac{P(x|\omega_2)\cdot P(\omega_2)}{P(x)} \times \left(\dfrac{1}{k+1}\right)$.

Now, $P(\omega_1|x) = P(\omega_2|x)$ and $P(\omega_1) = P(\omega_2) = 1/2$.

$\Rightarrow \quad P(x|\omega_1) \times k = P(x|\omega_2)$

$\Rightarrow \quad (x-\mu_2)^T \Sigma_2^{-1} (x-\mu_2) - (x-\mu_1)\Sigma_1^{-1}(x-\mu_1) = \ln \left| \dfrac{|\Sigma_1|}{|\Sigma_2| \times k} \right|$

Hence, the new decision boundary will change according to k, when k increases, the decision boundary moves towards the region of less penalty, and vice versa.