Homework - 5 ;- Q1) (20171049)

Given, $\mu_1 = [3, 3]$ & $\mu_2 = [7, 7]$.

Now, for decision boundaries, making Mahalanobis distance same from both the classes.

(i) $\Sigma_1 = \Sigma_2 = 3I = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$.

Now, $(X-\mu_1)^T \Sigma_1^{-1} (X-\mu_1) = (X-\mu_2)^T \Sigma_2^{-1} (X-\mu_2)$

$\Sigma_1^{-1} = \Sigma_2^{-1} = \frac{1}{3} I$.

$\Rightarrow (X-\mu_1)^T (\frac{1}{3} I)(X-\mu_1) = (X-\mu_2)^T (\frac{1}{3} I)(X-\mu_2)$

$\Rightarrow (X-\mu_1)^T (X-\mu_1) = (X-\mu_2)^T (X-\mu_2)$

$\Rightarrow \|X-\mu_1\|^2 = \|X-\mu_2\|^2$

$\Rightarrow (x-\mu_{x_1})^2 + (y-\mu_{y_1})^2 = (x-\mu_{x_2})^2 + (y-\mu_{y_2})^2$

$\Rightarrow 2x\mu_{x_1} + 2y\mu_{y_1} - \mu_{x_1}^2 - \mu_{y_1}^2 = 2x\mu_{x_2} + 2y\mu_{y_2} - \mu_{x_2}^2 - \mu_{y_2}^2$.

$\Rightarrow \underline{ax + by + c = 0}$. ← A straight line.

$\Rightarrow 6x + 6y - 18 = 14x + 14y - 98$

$\Rightarrow 8x + 8y = 80$

$\Rightarrow \boxed{x + y = 10}$.

(ii) $\Sigma_1 = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$ ← As Σ are always symmetric.

$\Rightarrow (X-\mu_1)^T \Sigma_1^{-1} (X-\mu_1) = (X-\mu_2)^T \Sigma_2^{-1} (X-\mu_2)$.

$\Rightarrow \Sigma_1^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix}$, $\Sigma_2^{-1} = \frac{1}{5}\begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$

$\Rightarrow 5\begin{bmatrix} x-3 \\ y-3 \end{bmatrix}^T \begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x-3 \\ y-3 \end{bmatrix} = 2\begin{bmatrix} x-7 \\ y-7 \end{bmatrix}^T \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x-7 \\ y-7 \end{bmatrix}$.

$\Rightarrow 5\begin{bmatrix} x-3 \\ y-3 \end{bmatrix}^T \begin{bmatrix} 3x-2y-3 \\ -2x+3y \end{bmatrix} = 2\begin{bmatrix} x-7 \\ y-7 \end{bmatrix}^T \begin{bmatrix} 3x-y-14 \\ -x+2y-7 \end{bmatrix}$.

$\Rightarrow 5(x-3)(3x-2y-3) + 5(y-3)(-2x+3y) = 2(x-7)(3x-y-14) + 2(y-7)(-x+2y-7)$

$\Rightarrow ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$.

$\underline{a = 11}$, $b = 4$, $h = -8$ $\Rightarrow \underline{h^2 - ab > 0}$ ← $\underline{\text{Hyperbola}}$.

Homework-5, Q2:)

(a) Dimensions of D must be 2000 X 200.

Practical Rank of D = 35.

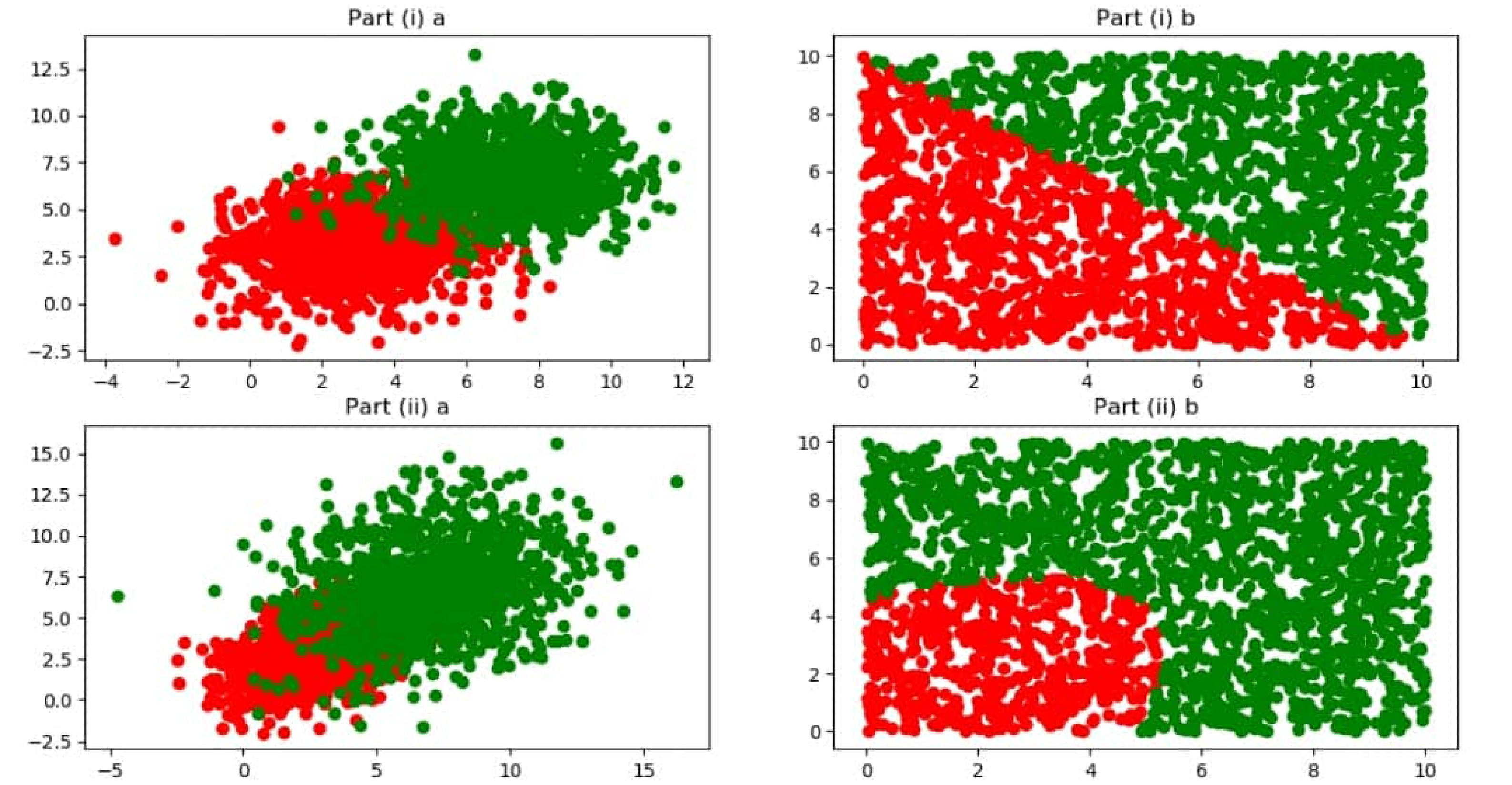
Now, D_{ij} = No of times student i orders from restaurant j.

As students who have not ordered yet from any restaurant can order now, so the value of D_{ij} increases from 0 to say 1, 2, ... So, practical rank can only increase and never decrease.

(b) (i) Using SVD, we can get the eigen values, and we can see that first 35 eigen values are greater than 0, so our practical rank is 35. Now, we take eigen vectors corresponding to these values and so, can get the canonical students this way.

(ii) For any new student, take its features on a new vector, and we can take dot product with the canonical students then the student with the highest dot product must be the closest and recommended food item.

(c) As a new restaurant is to be opened, we take the transpose of the solution in 'a' case, and we can do the experiment 'b' again, and as the rank remains same on transposing, so, we can repeat the 'b' part and can recommend accordingly.


Part (i) a
Part (i) b
Part (ii) a
Part (ii) b

Homework 5 - Q3) (20171049)

For a vector X, covariance matrix, $\Sigma = E\{(X-\bar{X})(X-\bar{X})^T\}$,

where X is a $n \times 1$ vector, $\bar{X}$ is mean vector.

Now, for a symmetric matrix 'A' of size $n \times n$, it is semi-definite only if $U^t A U \geq 0$, for every $n \times 1$ column vector U.

→ We know that covariance matrix is always symmetric.

So, consider any $n \times 1$ vector U,

$$\therefore \quad U^t \Sigma U = U^T E\{(X-\bar{X})(X-\bar{X})^T\} U.$$

$$= E\{U^T(X-\bar{X})(X-\bar{X})^T U\}.$$

$$= E\{\underbrace{(U^T(X-\bar{X}))}_{1\times 1}\underbrace{(U^T(X-\bar{X}))^T}_{1\times 1}\}.$$

$$= E(\sigma^2) = \underline{\underline{\sigma^2}} \geq 0 \text{ (always)}.$$

Hence, Σ is always a PSD.

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