Numerical Analysis (089180)
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Lab 4 – Homework Solutions

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A Homework

Homework 4.1 (LU decomposition). Consider the system $\mathbb{A}\boldsymbol{x} = \boldsymbol{b}$ with

$$\mathbb{A} = \begin{bmatrix} 12 & 5 & -8 & -5 \\ -4 & -4 & 8 & -6 \\ 4 & 2 & -3 & 0 \\ 0 & -1 & 2 & -4 \end{bmatrix} \quad \text{and} \quad \boldsymbol{b} = \begin{bmatrix} 4 \\ -6 \\ 3 \\ -3 \end{bmatrix}.$$

a. Use the $\mathbb{LU}$ decomposition of matrix $\mathbb{A}$ to solve the system. Use both the functions implemented and the `lu` MATLAB/Octave command.

b. Compute the determinant of $\mathbb{A}$ using the $\mathbb{LU}$ decomposition.

c. Consider now a different right-hand side vector $\boldsymbol{c} = [-22, -12, -1, -11]^T$. Use an approach which allow us to contain the computational cost.

Solution Homework 4.1.

hw_4_1.m

```
clc
clear all
close all

A = [12 5 -8 -5;
   -4 -4 8 -6;
   4 2 -3 0;
   0 -1 2 -4;];

b = [4 -6 3 -3]';

[L, U] = lu(A)

y1 = forward_substitution(L, b);
x1 = backward_substitution(U, y1)

A\b

detA = prod(diag(U))
det(A)

c = [-22 -12 -1 -11]';

y2 = forward_substitution(L, c);
x2 = backward_substitution(U, y2)

A\c

% The solution of the system A*x2 = c is obtained without a new
% factorization, since we stored the matrices L and U the first time we
% solved the system with b. This factorization is indeed useful everytime
% the matrix A does not change and we have to solve several linear systems
% with different right-hand sides.
```