Basis Functions and Feature Engineering

Linear in Which Sense?

We are training linear models in the form:

$$f(x;\theta) = \theta_0 + \sum_{j=1}^{n} \theta_j x_j$$

- Where x_j is the j-th input attribute
- ...θ_j is the corresponding weight, θ_0 is the intercept

However, we originally introduced linear models in this form:

$$f(x;\theta) = \sum_{i=1}^{n} \theta_j \phi_j(x)$$

Where $\phi_j(x)$ is any "basis function"

Feature Engineering

The original formulation:

$$f(x;\theta) = \sum_{i=1}^{n} \theta_j \phi_j(x)$$

- Implies that we can pre-compute any function of the input attributes
- ...And the consider a linear combination of those

The approach is still technically Linear Regression

We can thing of this process as building new attributes

- Since attributes are also referred to as "features"
- The technique is known as feature engineering

It works with any kind of ML model

Why Engineering Features?

By using basis functions we can exploit non-linear dependencies

Let's load again the data for our Taiwan real estate problem

In [26]:
```
import pandas as pd
import numpy as np
from sklearn.model_selection import train_test_split

data = pd.read_csv('data/real_estate.csv', sep=',')
cols = data.columns
X = data[cols[:-1]]
y = np.log(data[cols[-1]])
X_tr, X_ts, y_tr, y_ts = train_test_split(X, y, test_size=0.34, random_state=42)
```

Let's recall what the input attributes are:

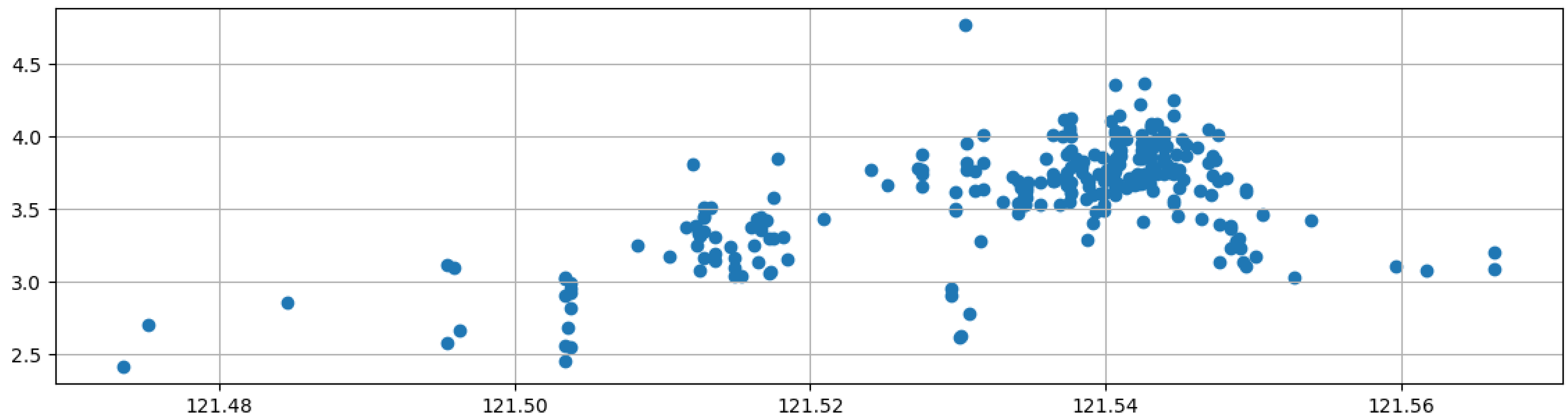
In [28]:
```
X_tr.columns
```

Out[28]:
```
Index(['house age', 'dist to MRT', '#stores', 'latitude', 'longitude'], dtype='object')
```

Why Engineering Features?

Let's build a scatter plot between "longitude" and the target

In [36]:
```
plt.figure(figsize=(14, 3.5))
plt.scatter(X_tr['longitude'], y_tr)
plt.grid(':')
```

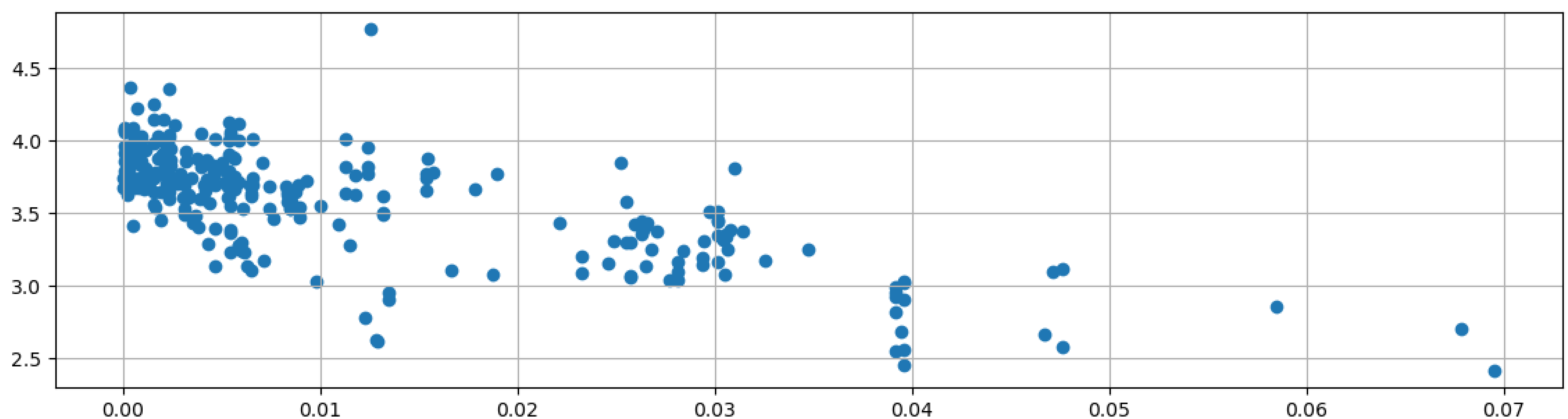


There is clearly a dependency, but it is non-linear

Why Engineering Features?

...But we can linearize the dependency by applying a custom function

In [41]:
```
plt.figure(figsize=(14, 3.5))
plt.scatter(np.abs(X_tr['longitude']-121.543), y_tr)
plt.grid(':')
```



- In theory, we could do the same with any other input attributes
- ...But for this dataset there are no other clear-cut cases

Empirical Evaluation

In our example, the technique enables a modest accuracy gain

In [54]:
```
from sklearn.linear_model import LinearRegression
from sklearn.metrics import r2_score

m = LinearRegression()
m.fit(X_tr, y_tr)
y_pred_tr, y_pred_ts = m.predict(X_tr), m.predict(X_ts)
print(f'R2: {r2_score(y_tr, y_pred_tr):.3} (training), {r2_score(y_ts, y_pred_ts):.3} (test)')

X_tr2, X_ts2 = X_tr.copy(), X_ts.copy()
X_tr2['longitude'] = np.abs(X_tr2['longitude']-121.543)
X_ts2['longitude'] = np.abs(X_ts2['longitude']-121.543)

m2 = LinearRegression()
m2.fit(X_tr2, y_tr)
y_pred_tr2, y_pred_ts2 = m2.predict(X_tr2), m2.predict(X_ts2)
print(f'R2: {r2_score(y_tr, y_pred_tr2):.3} (training), {r2_score(y_ts, y_pred_ts2):.3} (test)')
```

```
R2: 0.691 (training), 0.645 (test)
R2: 0.694 (training), 0.647 (test)
```