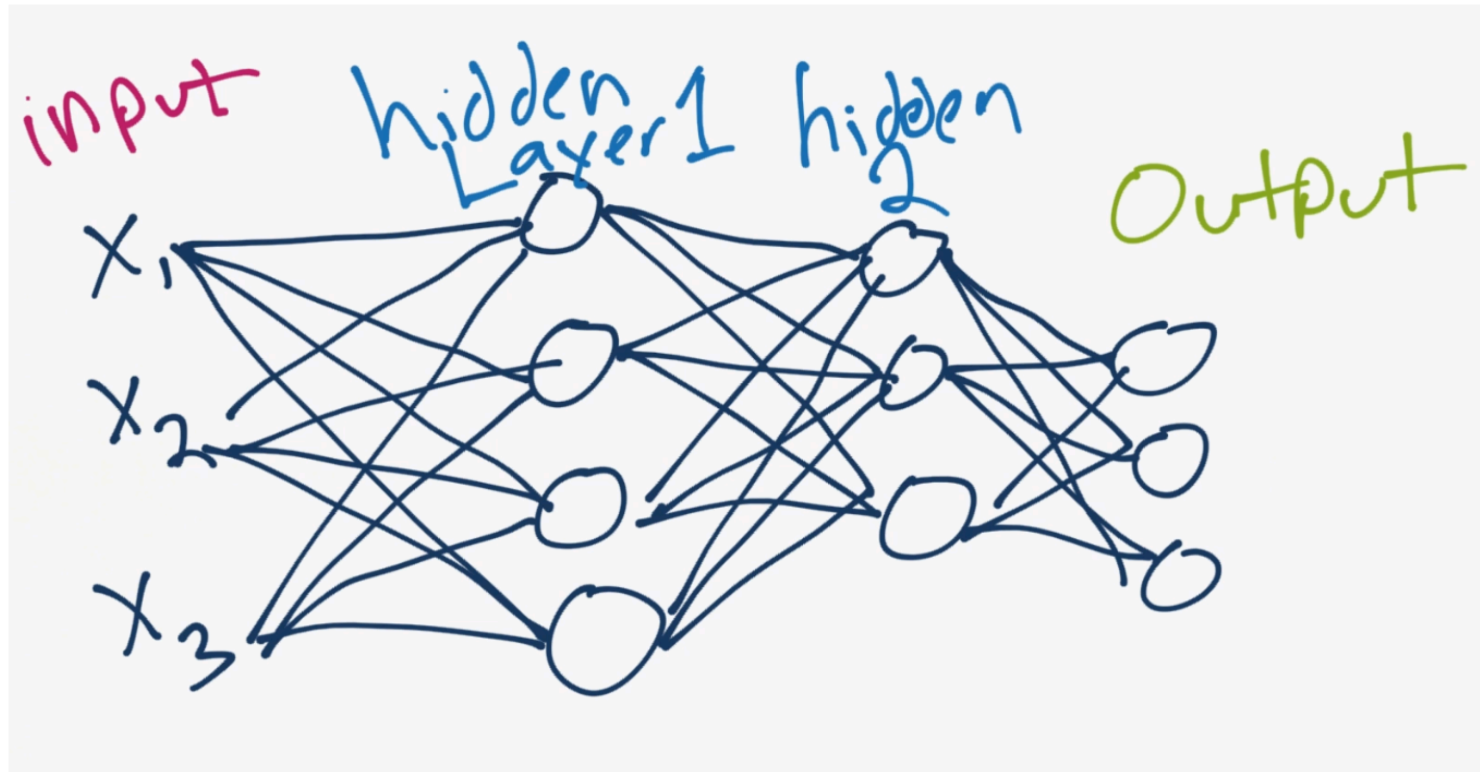
Convolutional Neural Networks

DL as Representation Learning

A nice perspective on DNs is to view them as "space tranformation machines"



- The input is encoded in its natural space
- The first layer morphs the input into a transformed space and so on
- The last layer applies linear/logistic regression to a learned representation

DL as Representation Learning

From this perspective, DNs can act as automatic feature extractor

This is the reason why they work so well on certain domains

- E.g. images, audio, natural language processing
- ...Meaning domains with complex or perceptual representations

Consider a digit recognition application

With classical ML, one would need to:

- Design high quality features (which requires a lot of domain expertise)
- ...And then train a model

With DL, the first step can be (partially skipped)

This is a huge advantage!

Image Data

We will try to use Deep Learning on image data

In particular we will use the MNIST Digit Recognition Dataset

- Code to download MNIST is available directy in Keras

```
In [2]: from keras.datasets import mnist
        # load the data, shuffled and split between train and test sets
        (x_train, y_train), (x_test, y_test) = mnist.load_data()

Downloading data from https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.np
z
11490434/11490434 ━━━━━━━━━━━━━━━━━━━━ 1s 0us/step
```

The MNIST data is now stored in pairs of numpy arrays.

- The `x_train` and `x_test` arrays contain the greyscale value of each pixel
- The `y_train` and `y_test` arrays contain the class (digit) as an integer

Image Data

Let's inspect the output

```
In [3]: print(f'Shape of y_train: {y_train.shape}')
        print(f'Shape of y_test: {y_test.shape}')
        n_tr = y_train.shape[0]
        n_ts = y_test.shape[0]

Shape of y_train: (60000,)
Shape of y_test: (10000,)
```

- There are 60,000 training examples
- ...And 10,000 test examples

The target arrays are one-dimensional

Let's check a sample:

```
In [4]: y_train

Out[4]: array([5, 0, 4, ..., 5, 6, 8], dtype=uint8)
```

Image Data

Let's inspect the input

In [5]:
```
print(f'Shape of x_train: {x_train.shape}')
print(f'Shape of x_test: {x_test.shape}')
x_h = x_train.shape[1]
x_w = x_train.shape[2]
```

```
Shape of x_train: (60000, 28, 28)
Shape of x_test: (10000, 28, 28)
```

- The dataset input consists of 28x28 matrices

In [6]:
```
print(f'Minimum: {x_train.min()} (train), {x_train.min()} (test)')
print(f'Maximum: {x_train.max()} (train), {x_train.max()} (test)')
```

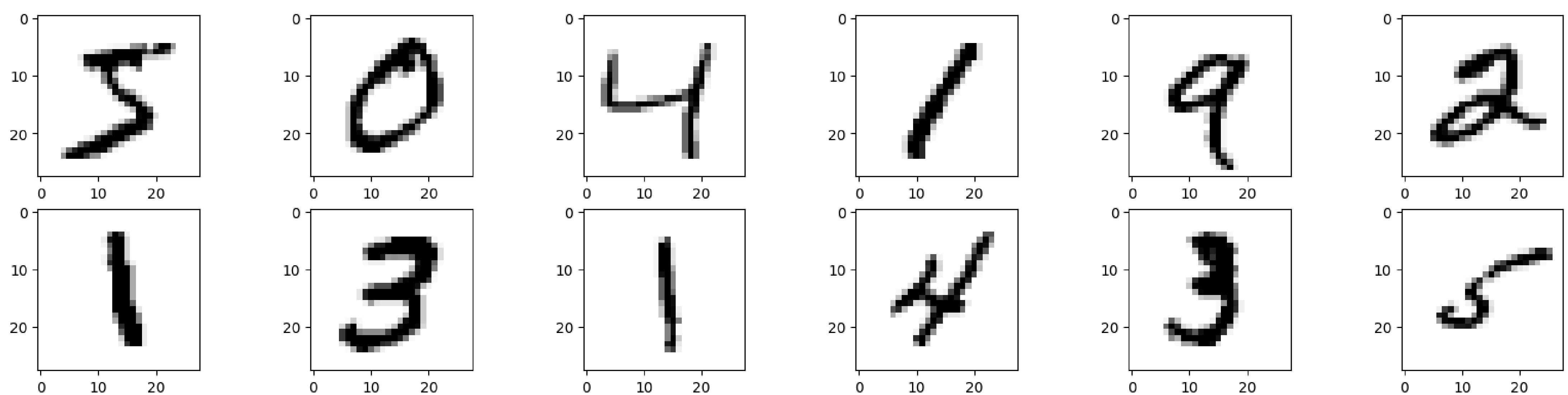
```
Minimum: 0 (train), 0 (test)
Maximum: 255 (train), 255 (test)
```

- The content of the matrix cells ranges from 0 to 255
- ...And it represents 8 bit brightness values

Image Data

Let's see some sample images

In [7]:
```
m, n = 2, 6
plt.figure(figsize=figsize)
for i in range(m):
    for j in range(n):
        plt.subplot(m, n, i*n + j + 1)
        plt.imshow(x_train[i*n + j], cmap='Greys')
plt.show()
```



Preprocessing

Before we can start training we need to do some preprocessing

We will apply a min-max encoding to the input

- ...Since minima and maxima are clearly define

```
In [8]: x_train_norm = x_train / 255.0
        x_test_norm = x_test / 255.0
```

We will adopt a one-hot encoding for the output

- ...Since we will need to build a network with one neuron per class
- Keras provides a utility function for the conversion

```
In [9]: y_train_cat = keras.utils.to_categorical(y_train)
        y_test_cat = keras.utils.to_categorical(y_test)
        y_train_cat[:2, :]

Out[9]: array([[0., 0., 0., 0., 0., 1., 0., 0., 0., 0.],
               [1., 0., 0., 0., 0., 0., 0., 0., 0., 0.]])
```

Adding Channel Information

When working with image data, one extra step is needed

...Since images are not necessarily greyscale!

- Color images are often represented using the RGB color space
- ...I.e. each pixel has an associated value for the red, green, and blue hue.

For this reason, an image is best described by a tensor not a matrix

- An i, j pair identifies a pixel
- ...And a third dimension specifies the channel (RGB)

Even if we have a single channel, is best to make Keras aware of that

In [10]:
```
x_train_c = x_train_norm.reshape(-1, x_h, x_w, 1)
x_test_c = x_test_norm.reshape(-1, x_h, x_w, 1)
input_shape = (x_h, x_w, 1)
output_shape = (10,)
print(f'New shape of the training set: {x_train_c.shape}')
```

```
New shape of the training set: (60000, 28, 28, 1)
```

Training a Baseline Model

As a baseline, we will build an MLP model

```
In [11]: def build_mlp(input_shape, output_shape, hidden, rate=0.05):
             mdl = keras.Sequential()
             mdl.add(keras.Input(shape=input_shape))
             mdl.add(keras.layers.Flatten())
             for k, h in enumerate(hidden):
                 mdl.add(Dense(h, activation='relu'))
                 mdl.add(keras.layers.Dropout(rate))
             mdl.add(Dense(output_shape[0], activation='softmax'))
             return mdl
```

A classical MLP is not designed to handle images

- ...For this reason with start with a special `Flatten` layer
- ...Which discards all tensor dimensions (except the number of samples)

We also need to build one output neuron per class

- ...And we need to use a softmax activation function

Training a Baseline Model

We can now train a 2-layer network as a baseline

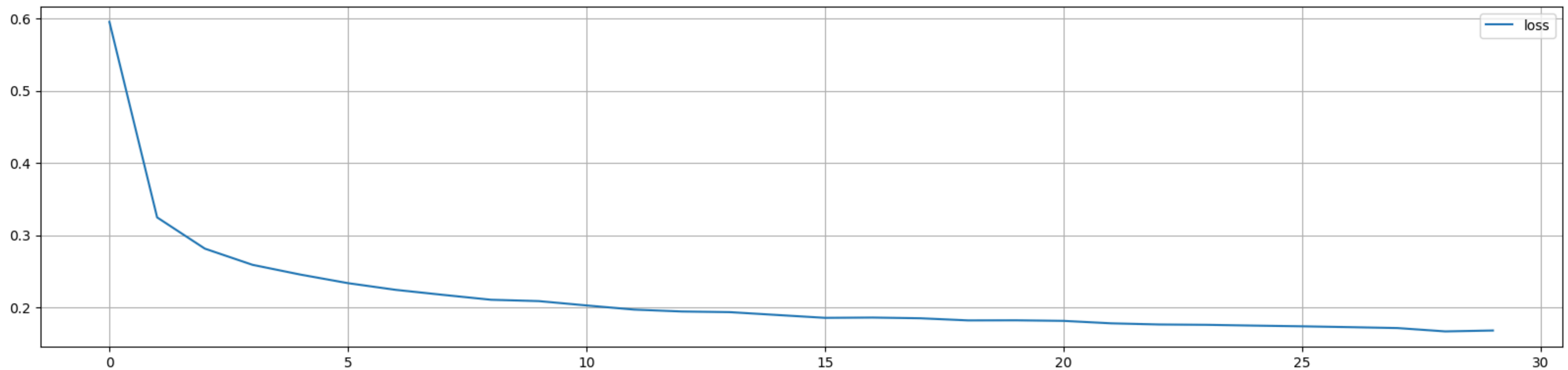
In [12]:
```
def train_nn(nn, X_tr, y_tr, batch_size, epochs, verbose=1):
    nn.compile(loss='categorical_crossentropy', optimizer='adam')
    history = nn.fit(X_tr, y_tr, batch_size=batch_size, epochs=epochs, verbose=verbose)
    cols = [k for k in history.history.keys()]
    vals = np.array([history.history[c] for c in cols]).T
    return pd.DataFrame(data=vals, columns=cols)

nn1 = build_mlp(input_shape, output_shape, hidden=[16, 16])
history = train_nn(nn1, x_train_c, y_train_cat, batch_size=32, epochs=30, verbose=1)
```

```
Epoch 1/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 1s 253us/step - loss: 0.9242
Epoch 2/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 248us/step - loss: 0.3415
Epoch 3/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 243us/step - loss: 0.2781
Epoch 4/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 245us/step - loss: 0.2542
Epoch 5/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 247us/step - loss: 0.2425
Epoch 6/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 246us/step - loss: 0.2309
Epoch 7/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 243us/step - loss: 0.2263
Epoch 8/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 0s 255us/step - loss: 0.2215
```

Training a Baseline Model

Let's inspect the training curve

In [13]:
```
history.plot(figsize=figsize, grid=':');
```



There's still something to go before convergence, but we'll stop here

Evaluation

Now we can compute the model accuracy

In [14]:
```
nn1_p_tr = nn1.predict(x_train_c, verbose=0).argmax(axis=1)
nn1_p_ts = nn1.predict(x_test_c, verbose=0).argmax(axis=1)

nn1_acc_tr = accuracy_score(y_train, nn1_p_tr)
nn1_acc_ts = accuracy_score(y_test, nn1_p_ts)

print(f'2-layer network accuracy: {nn1_acc_tr:.3f} (train), {nn1_acc_ts:.3f} (test)')
```

```
2-layer network accuracy: 0.969 (train), 0.951 (test)
```

We are doing already pretty well!

- What can we do to improve the results?
- Beyond "stacking more layers" the answer is not clear

Exploiting Structural Information

DNs are very flexible learning models

- ...Since we can choose both how many layer to use
- ...And how big they should be

However, it's difficult to develop an intuition of which options work

- This is due to the poor interpretability of DNs
- ...To the point that a fully fledged research field focuses on automatic tuning

There is one type of choice that is intuitive and has a big impact

...This concerns the idea of exploiting structural information

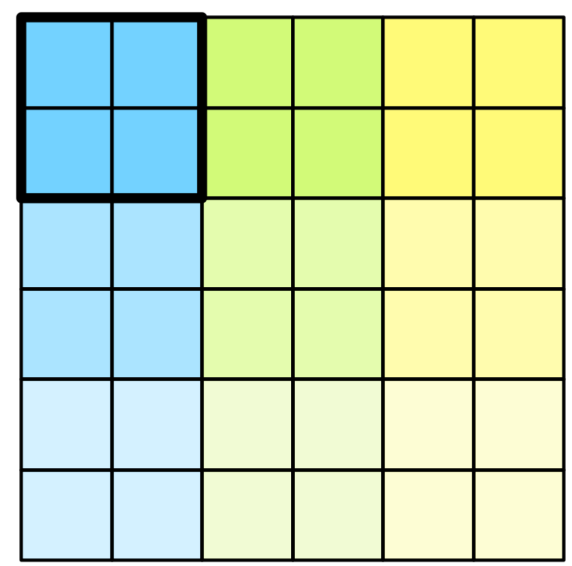
- For example, nearby pixels in an image may be semantically linked
- ...And the same goes for nearby points in time
- ...Or nearby words in a sentence

Convolutional Layers

This idea is at the basis of convolutional layers

A 2D convolution layer...

- Starts from an input tensor with shape (m, n, c)
- ...And slides a linear n_f, m_f filter (or kernel) on top of the image
- ...With a certain step size (stride)



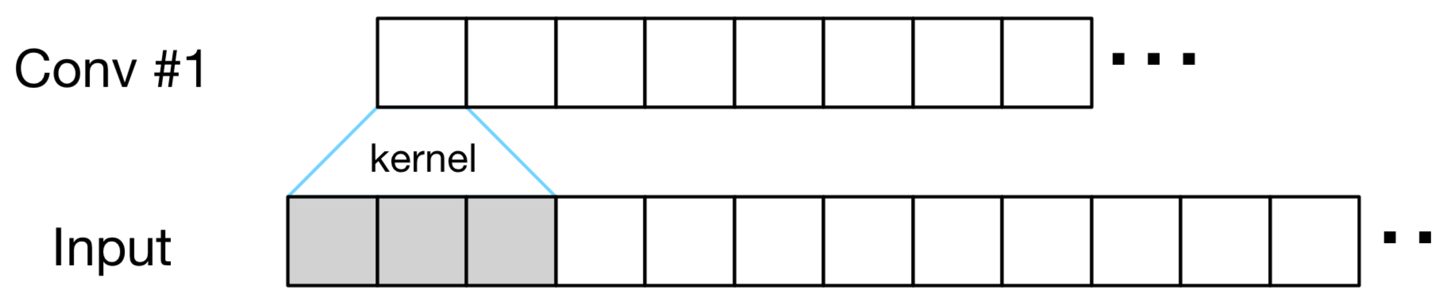
- You can think of that as moving an n_f, m_f mask across an image
- The figure shows a 2x2 convolution with stride 2

Convolutional Layers

Each application of the kernel...

- Compute a dot product (involving all channels) to obtain a scalar
- ...The optionally applies an activation function

Here we see the effect along 1 dimension:



Therefore, by applying a (m_f, n_f) 2D convolution to an (m, n, c) tensor

...We get a $(m - m_f + 1, n - n_f + 1)$ output tensor

- Intuitively, starting from a multi-channel image
- ...We get a slightly smaller single-channel image

Convolutional Layers

If we don't want to reduce the image size

- ...We can include some padding

If we want to undersample the image

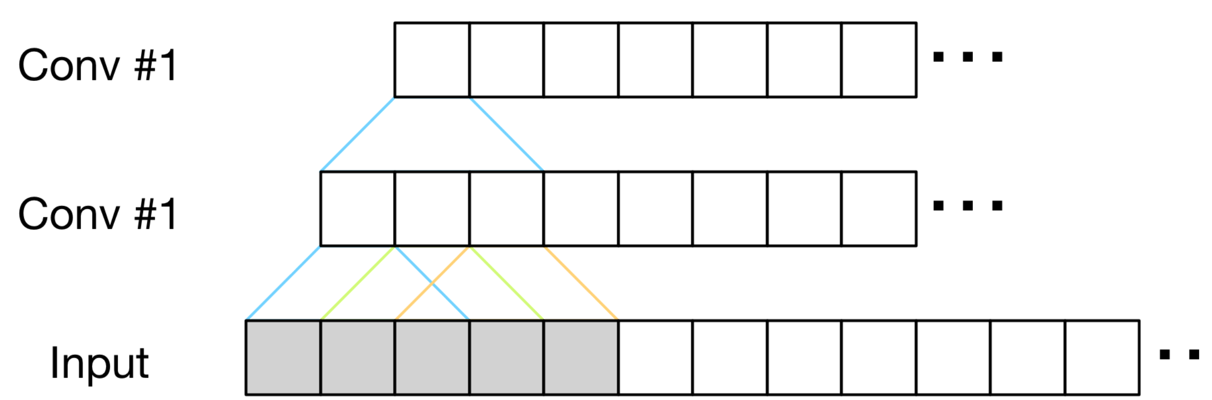
- ...We can use a non unary stride

If we use multiple filters for the same level

- ...We obtain multiple output "images"

If we stack multiple convolutional layers

- ...Later layers will be able to access information from a larger receptive field



Convolutional Layers

Convolutional layers have some interesting properties

Their weights are associated only to the filter

- So, if we have c channels and a (m_f, n_f) filter
- ...We have $m_f n_f c$ weights regardless of the input size

This allows a huge reduction in terms of number of weights

The price to pay is a higher bias

...But the trick is that it's a bias that makes sense!

- Intuitively, filters will learn to recognize local features
- Earlier convolutions will focus on fine-grain details
- ...While later convolution will aggregate them

This property allows CNN to work very well on image data

CNNs in Keras

Convolutional layer are available in Keras as `Conv2D` objects

In [15]:
```python
from keras.layers import Conv2D

def build_cnn(input_shape, output_shape, hidden, convs, rate=0.05):
    mdl = keras.Sequential()
    mdl.add(keras.Input(shape=input_shape))
    for nf in convs:
        mdl.add(Conv2D(nf, kernel_size=(3,3), activation='relu'))
    mdl.add(keras.layers.Flatten())
    for h in hidden:
        mdl.add(Dense(h, activation='relu'))
        mdl.add(keras.layers.Dropout(rate))
    mdl.add(Dense(output_shape[0], activation='softmax'))
    return mdl
```

- The first parameter is the number of filters
- Then we have the filter (kernel) size and the activation function

Training a CNN

CNNs can be trained as usual, but the process is much slower

...Since even with few weights, we still need to do a lot of computations

- Using GPUs can considerably accelerate this step

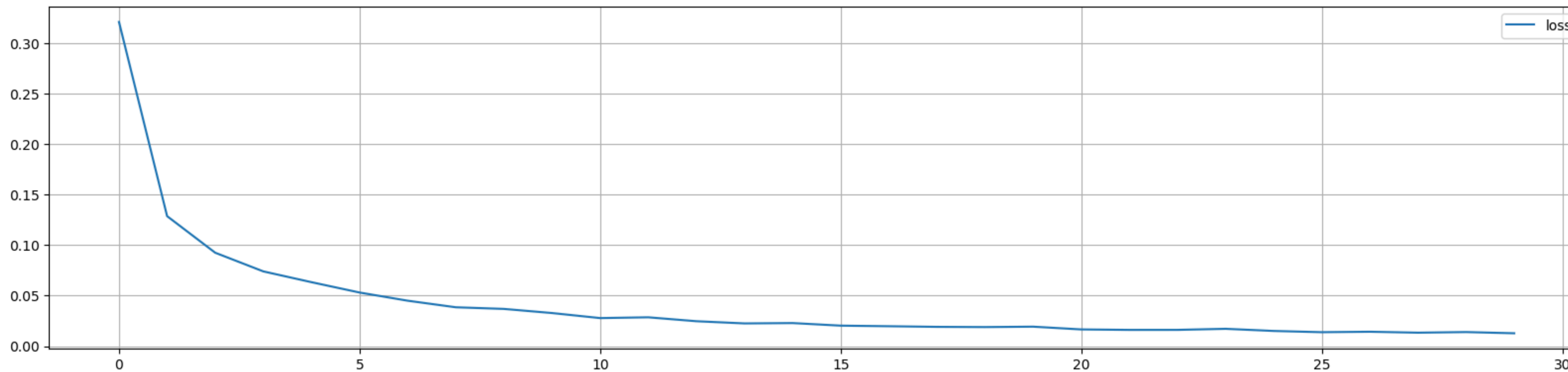
```
In [16]: cnn1 = build_cnn(input_shape, output_shape, hidden=[16], convs=[16])
         history2 = train_nn(cnn1, x_train_c, y_train_cat, batch_size=32, epochs=30, verbose=1)
```

```
Epoch 1/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 4s 2ms/step - loss: 0.5502
Epoch 2/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.1367
Epoch 3/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0918
Epoch 4/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0722
Epoch 5/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0627
Epoch 6/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0508
Epoch 7/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0416
Epoch 8/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0380
Epoch 9/30
1875/1875 ━━━━━━━━━━━━━━━━━━━━ 3s 2ms/step - loss: 0.0350
Epoch 10/30
```

Training a CNN

Let's check the training curve

In [17]:
```
history2.plot(figsize=figsize, grid=':');
```



Again, there is still some way to go, but we'll stop here for a fair comparison

Quality Evaluation

```
In [18]: cnn1_p_tr = cnn1.predict(x_train_c, verbose=0).argmax(axis=1)
         cnn1_p_ts = cnn1.predict(x_test_c, verbose=0).argmax(axis=1)

         cnn1_acc_tr = accuracy_score(y_train, cnn1_p_tr)
         cnn1_acc_ts = accuracy_score(y_test, cnn1_p_ts)

         print(f'Shallow network accuracy: {nn1_acc_tr:.3f} (train), {nn1_acc_ts:.3f} (test)')
         print(f'Convolutional network accuracy: {cnn1_acc_tr:.3f} (train), {cnn1_acc_ts:.3f} (test)

         Shallow network accuracy: 0.969 (train), 0.951 (test)
         Convolutional network accuracy: 0.999 (train), 0.981 (test)
```

The results are much better!

- Even if the CNN has much fewer weights than the fully connected one
- ...And the same number of hidden layers

Exploiting structural information is a powerful idea in DL

- Rather than focusing o low-level design choices (e.g. crafting features)
- ...We focus on building architecture that can exploit general properties