

Part III - Foraging theory

Sex, Ageing and Foraging Theory

resources

energy

offspring

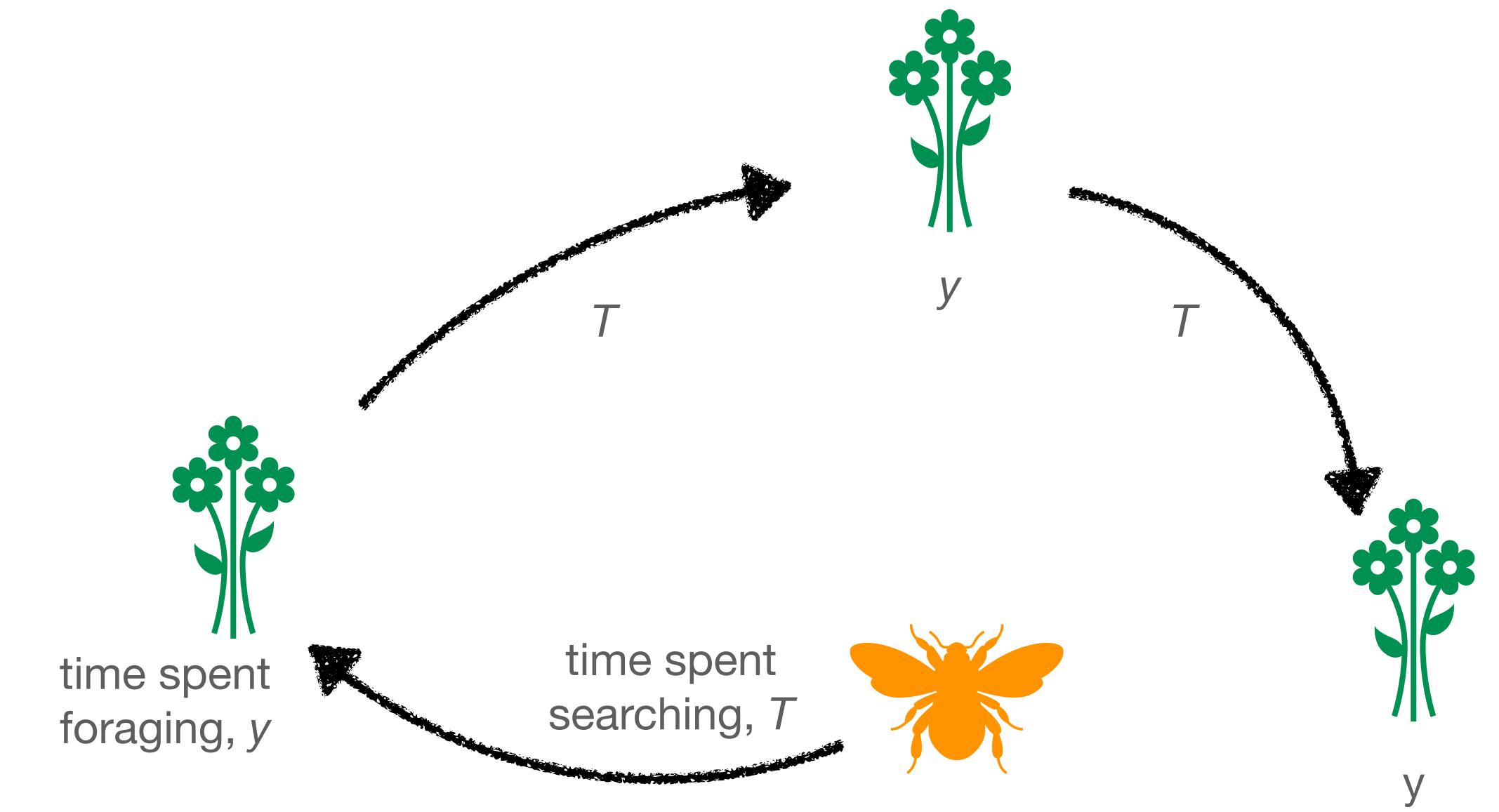
fitness



Foraging in patches

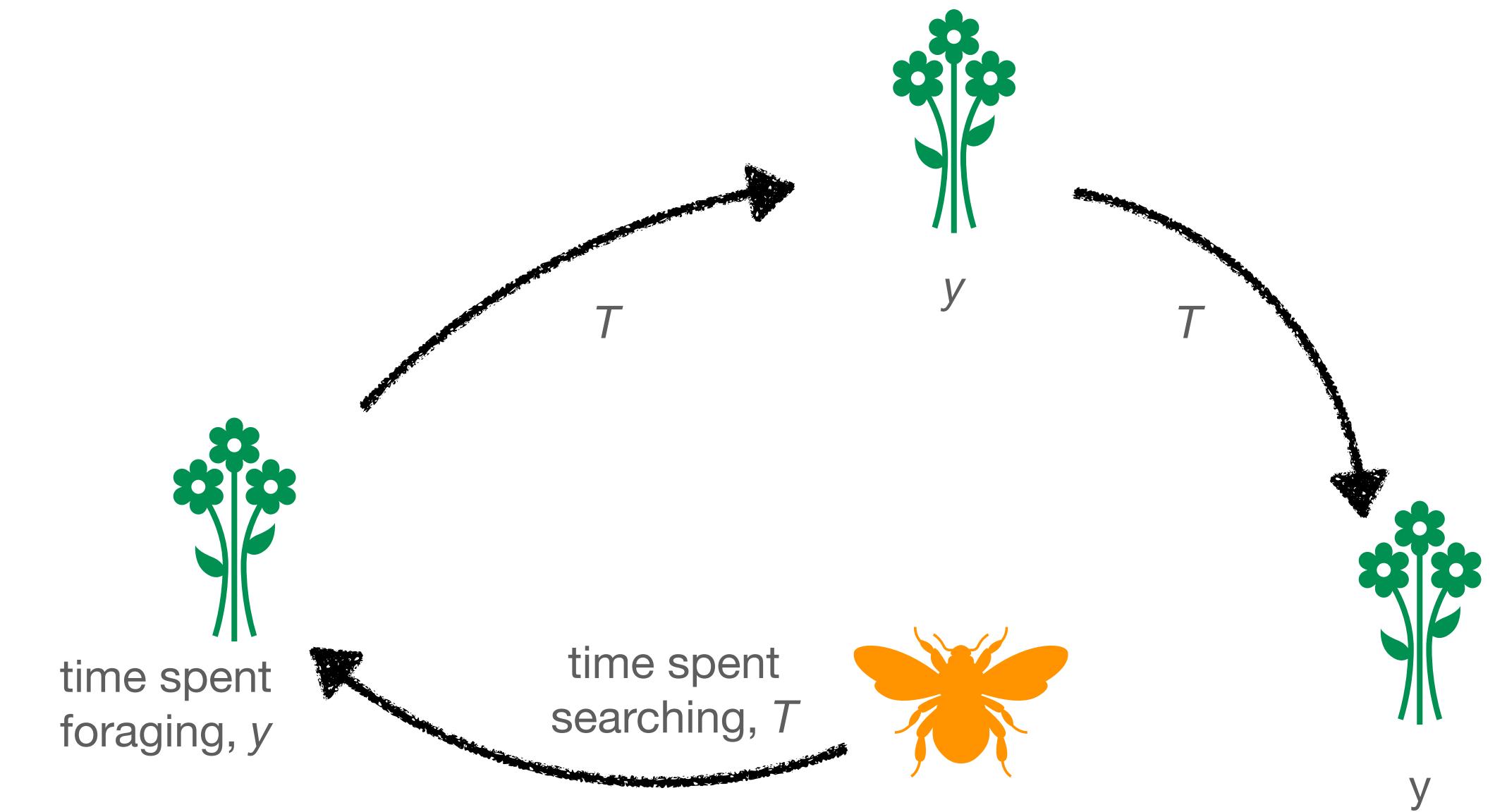
Foraging in patches

- Animal forages on multiple equivalent patches with finite amount of resources.



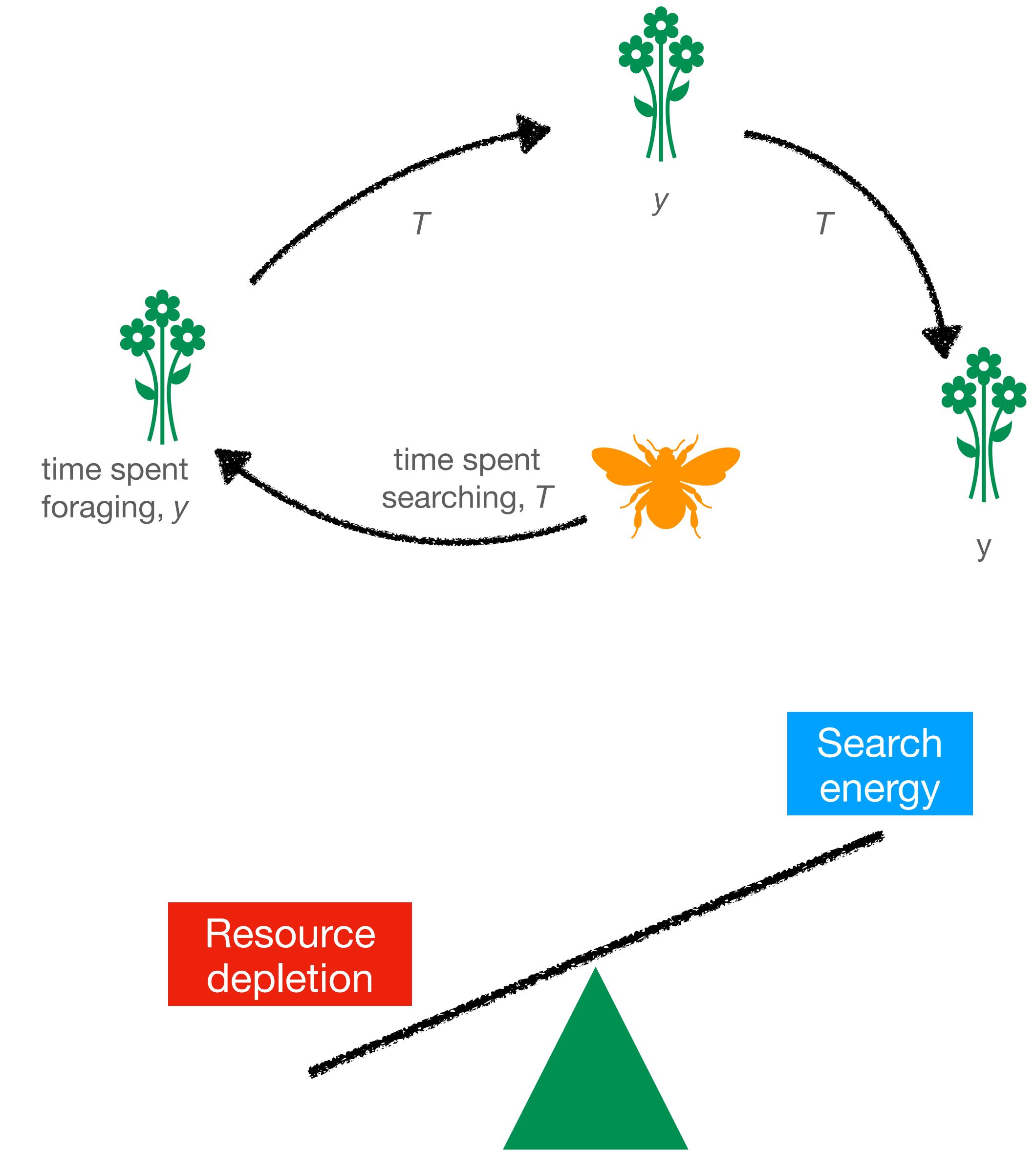
Foraging in patches

- Animal forages on multiple equivalent patches with finite amount of resources.
- How much time y should it spent foraging on a single patch when searching is costly?



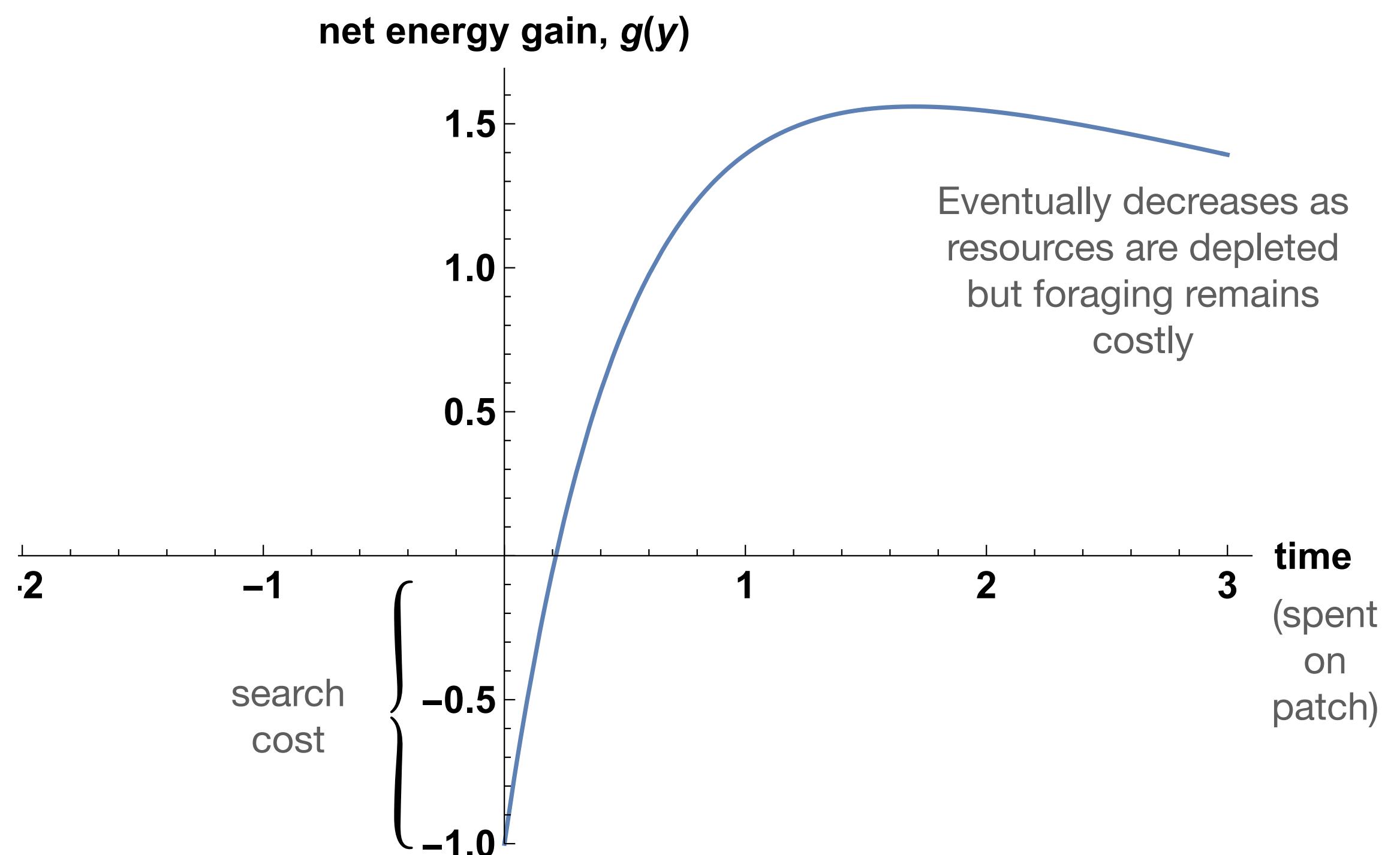
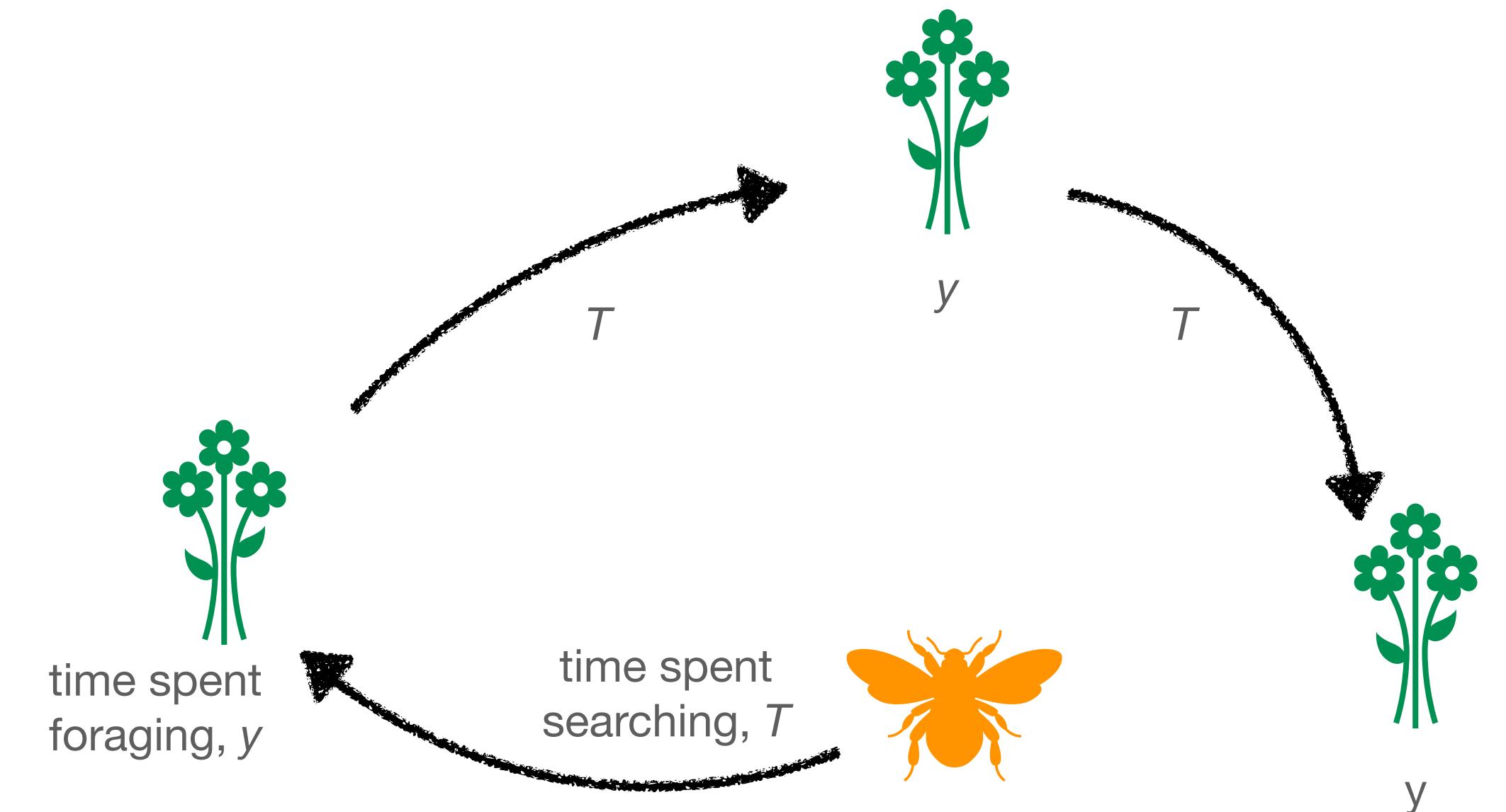
Foraging in patches

- Animal forages on multiple equivalent patches with finite amount of resources.
- How much time y should it spent foraging on a single patch when searching is costly?
- If it stays too long, resources get depleted; too short and it does not regain energy lost from search.



Energy gains

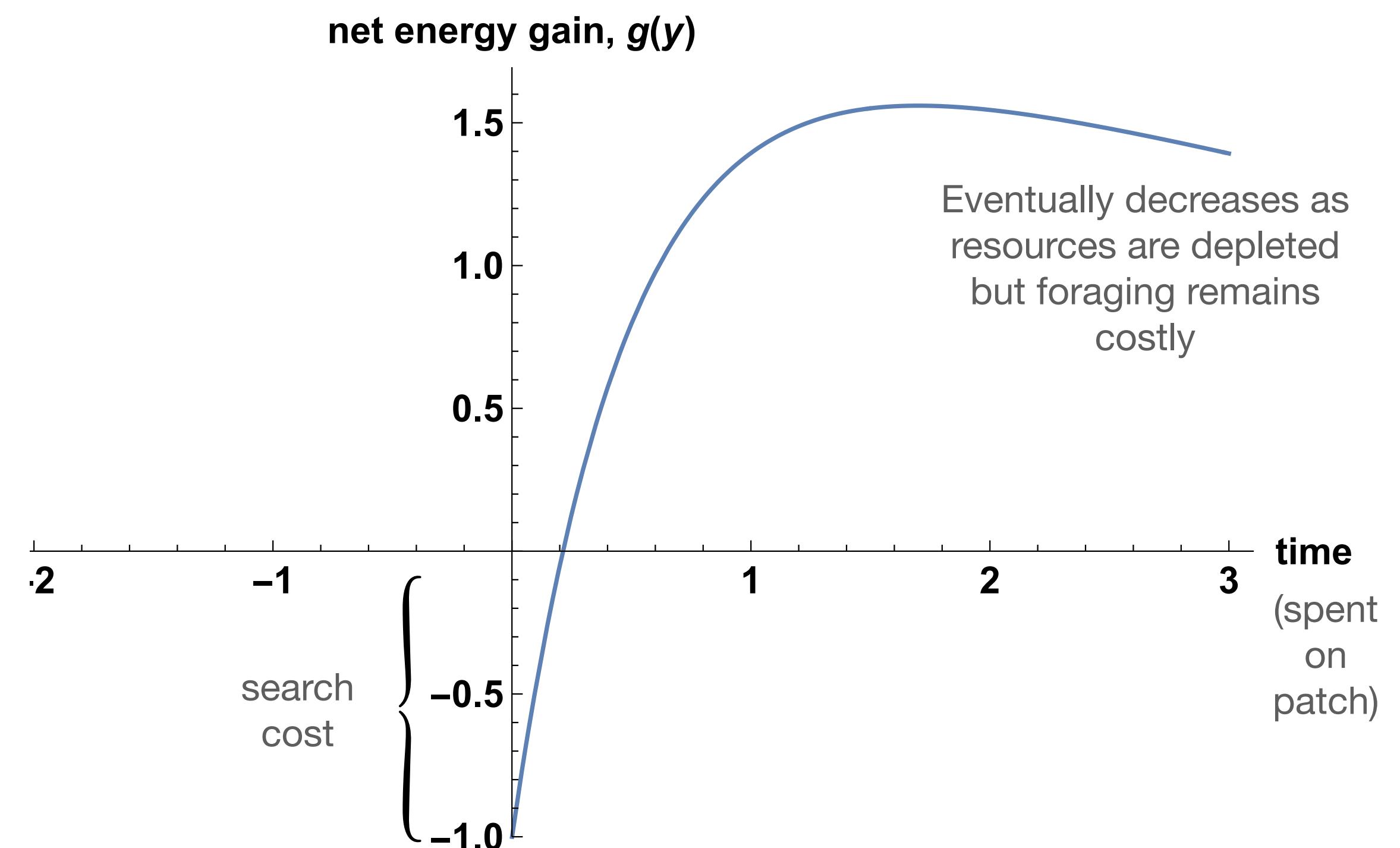
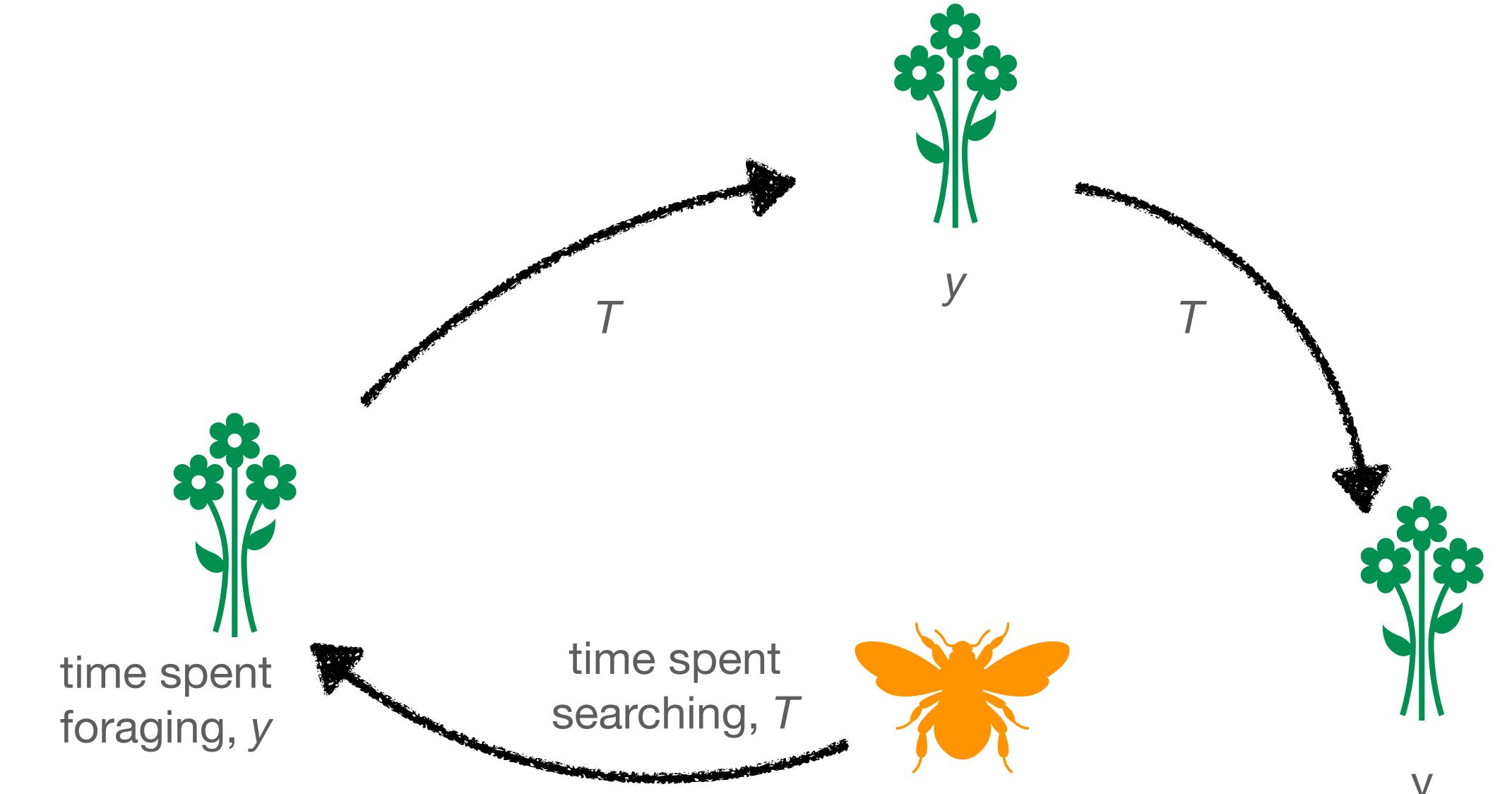
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Energy gains

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- Rate of energy gain from search + foraging :

$$R(y) = \frac{g(y)}{y + T}$$

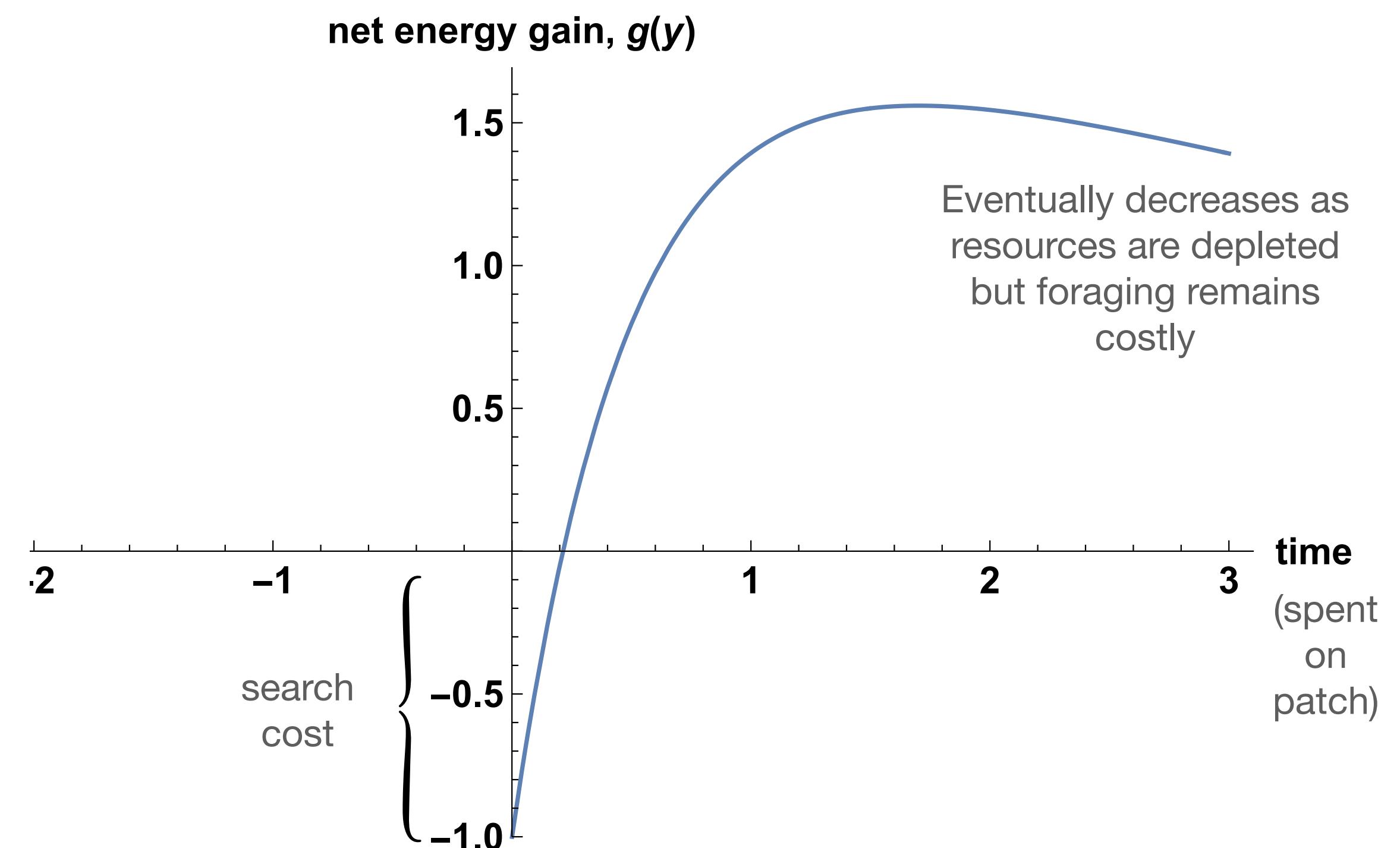
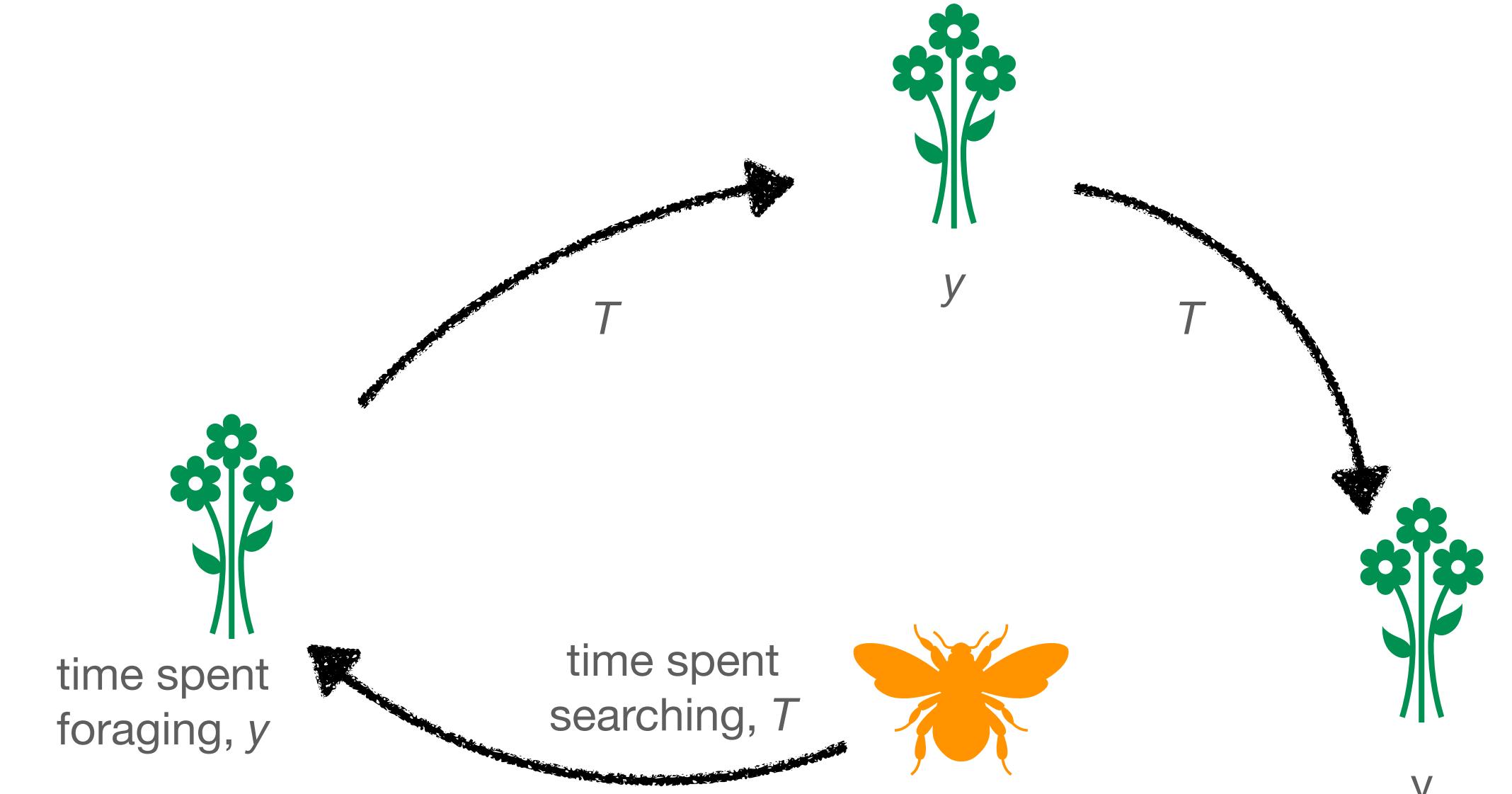


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- Fitness : $w(y, x) \propto R(y)$



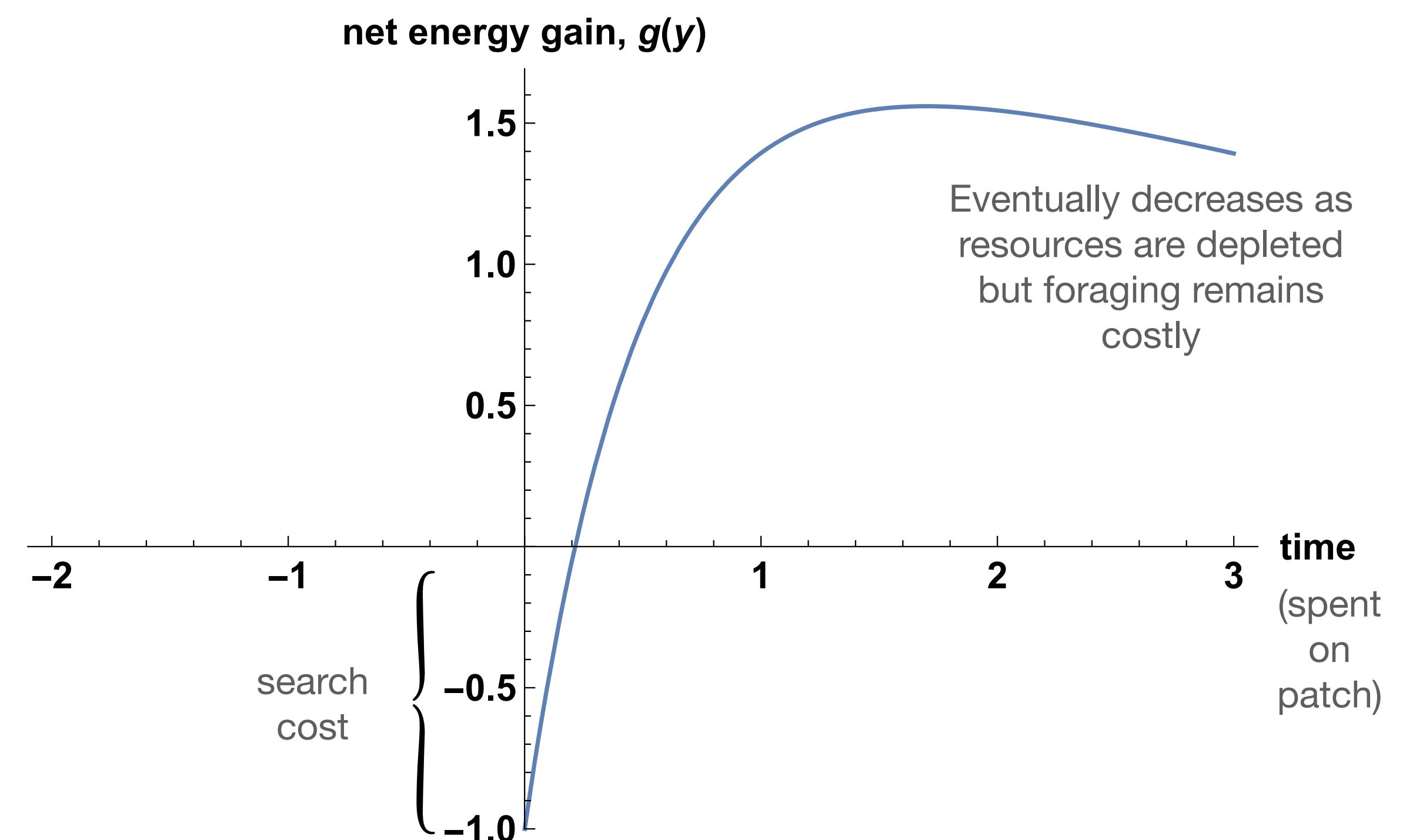
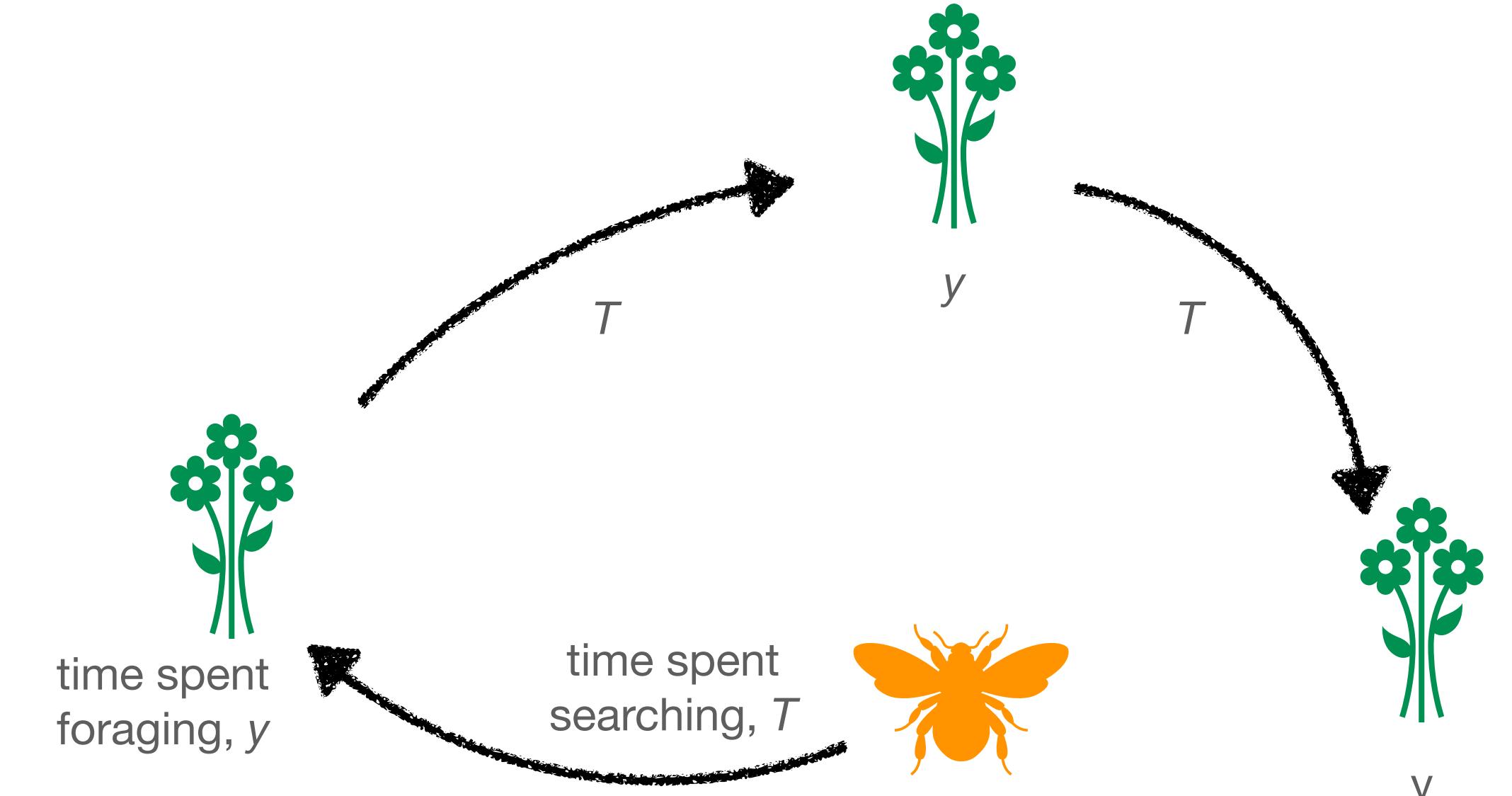
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- Selection gradient :

$$s(x) \propto \frac{g'(x)}{x + T} - \frac{g(x)}{(x + T)^2}$$

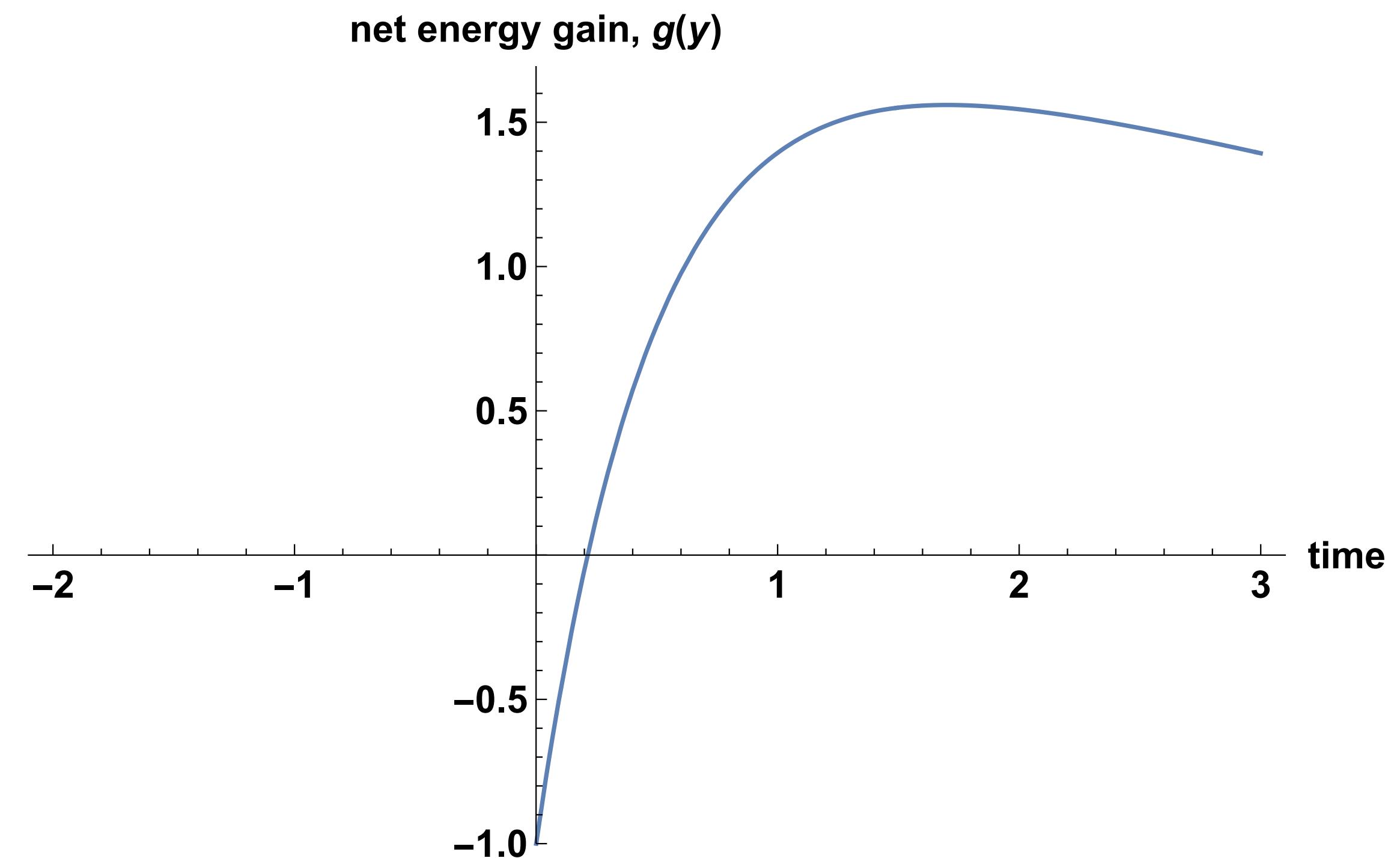
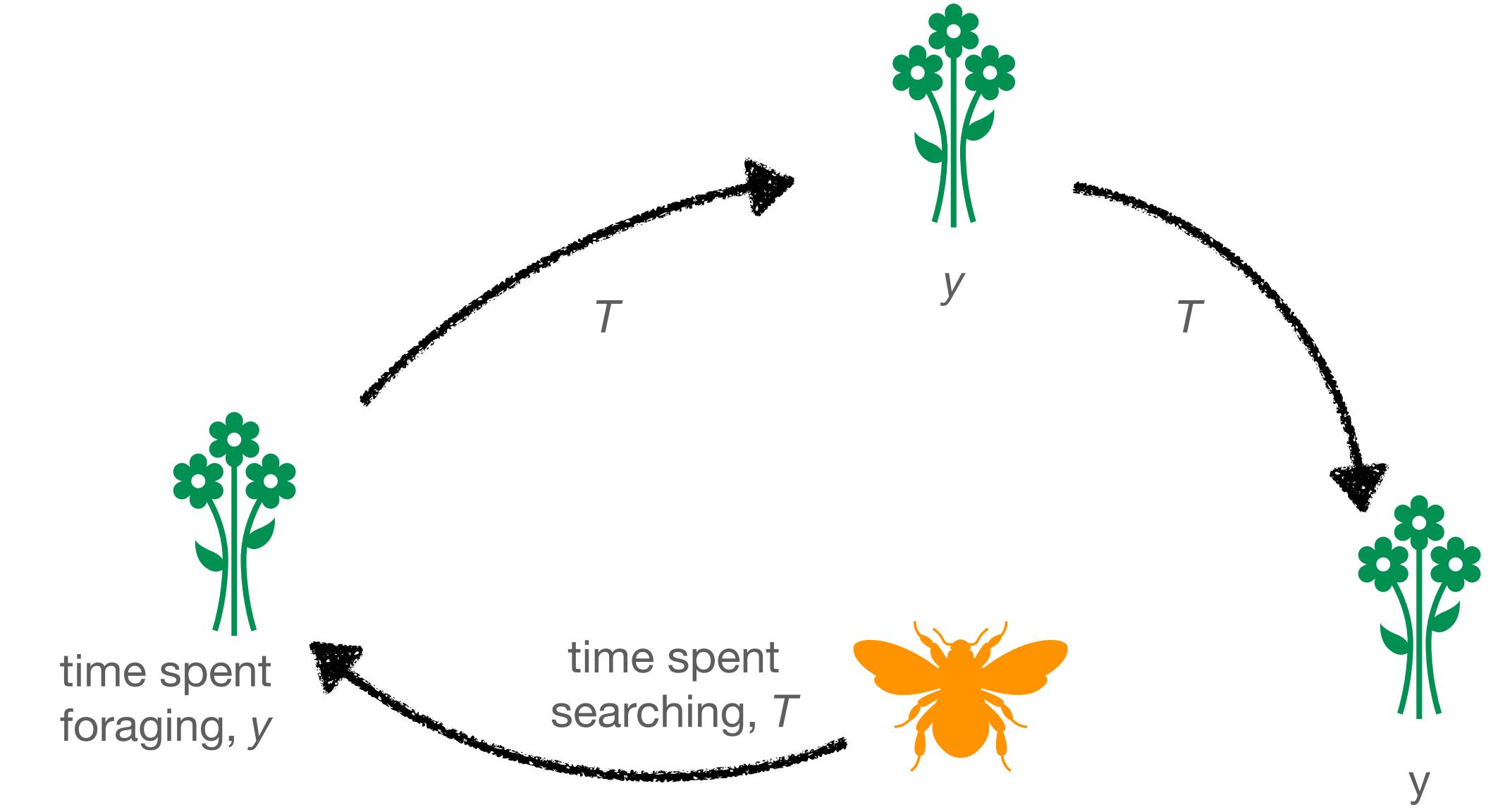


Marginal value theorem

Optimum x^* such that $s(x^*) = 0$,

i.e., such that

$$g'(x^*) = \frac{g(x^*)}{x^* + T} = R(x^*)$$

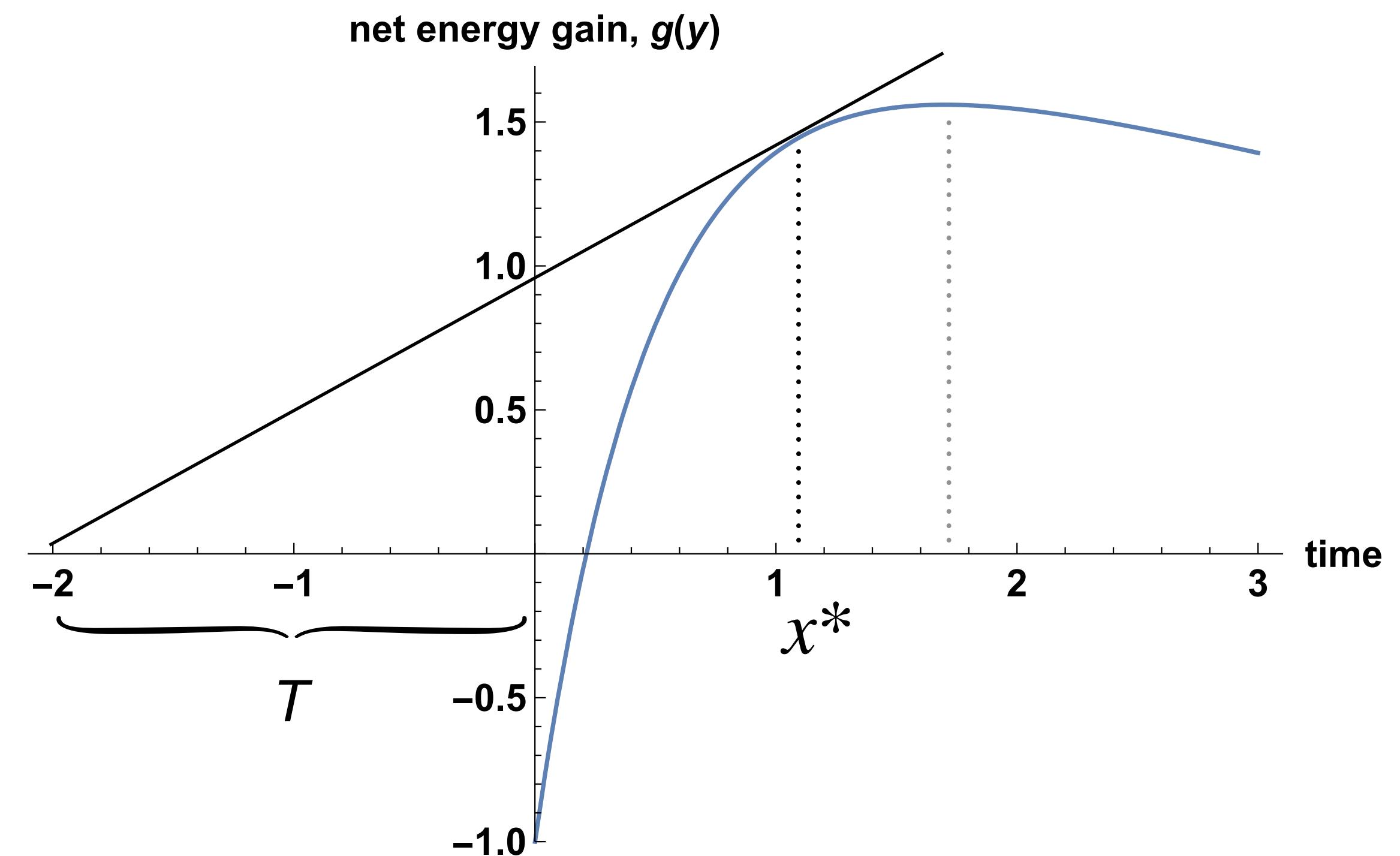
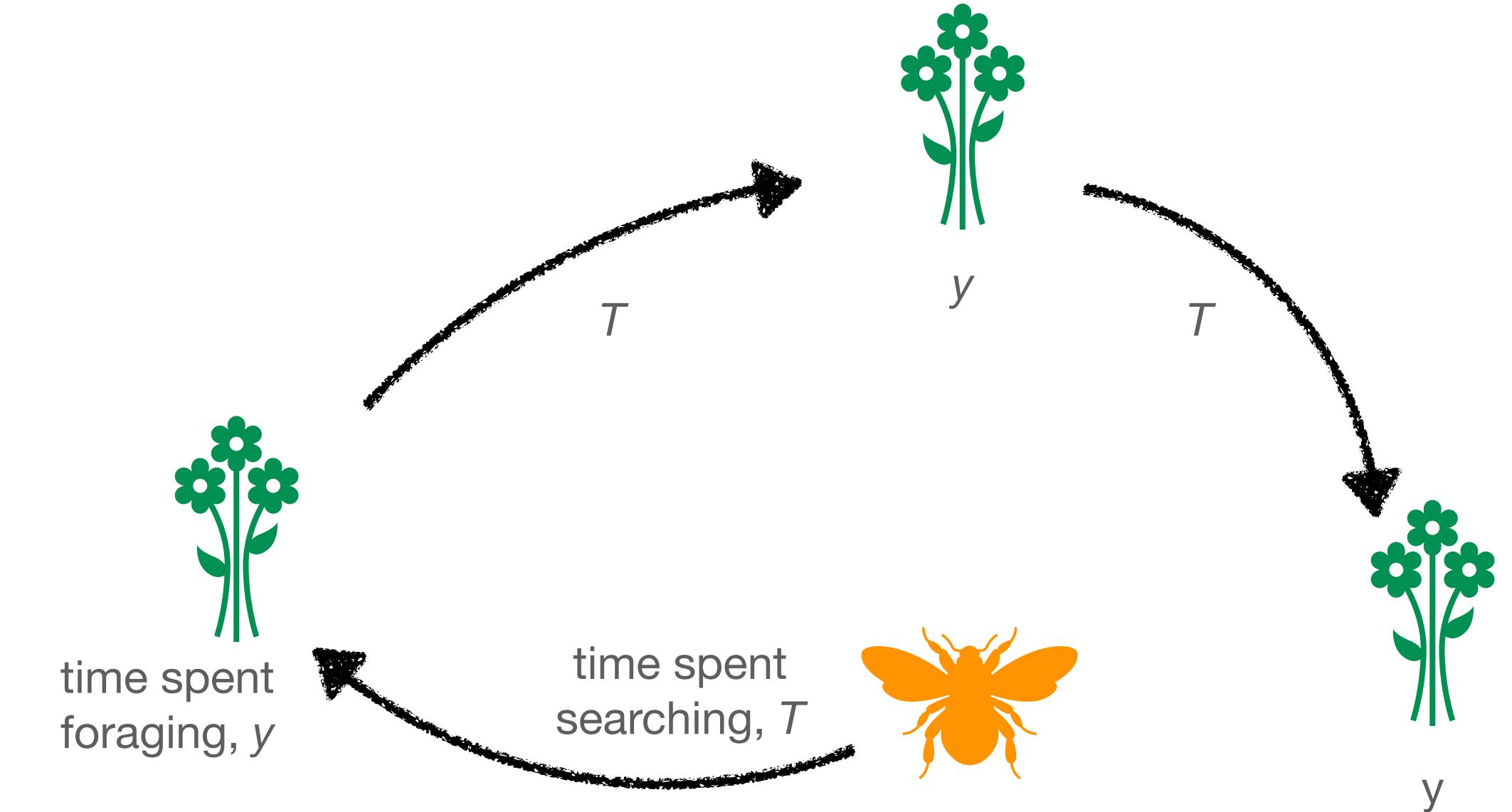


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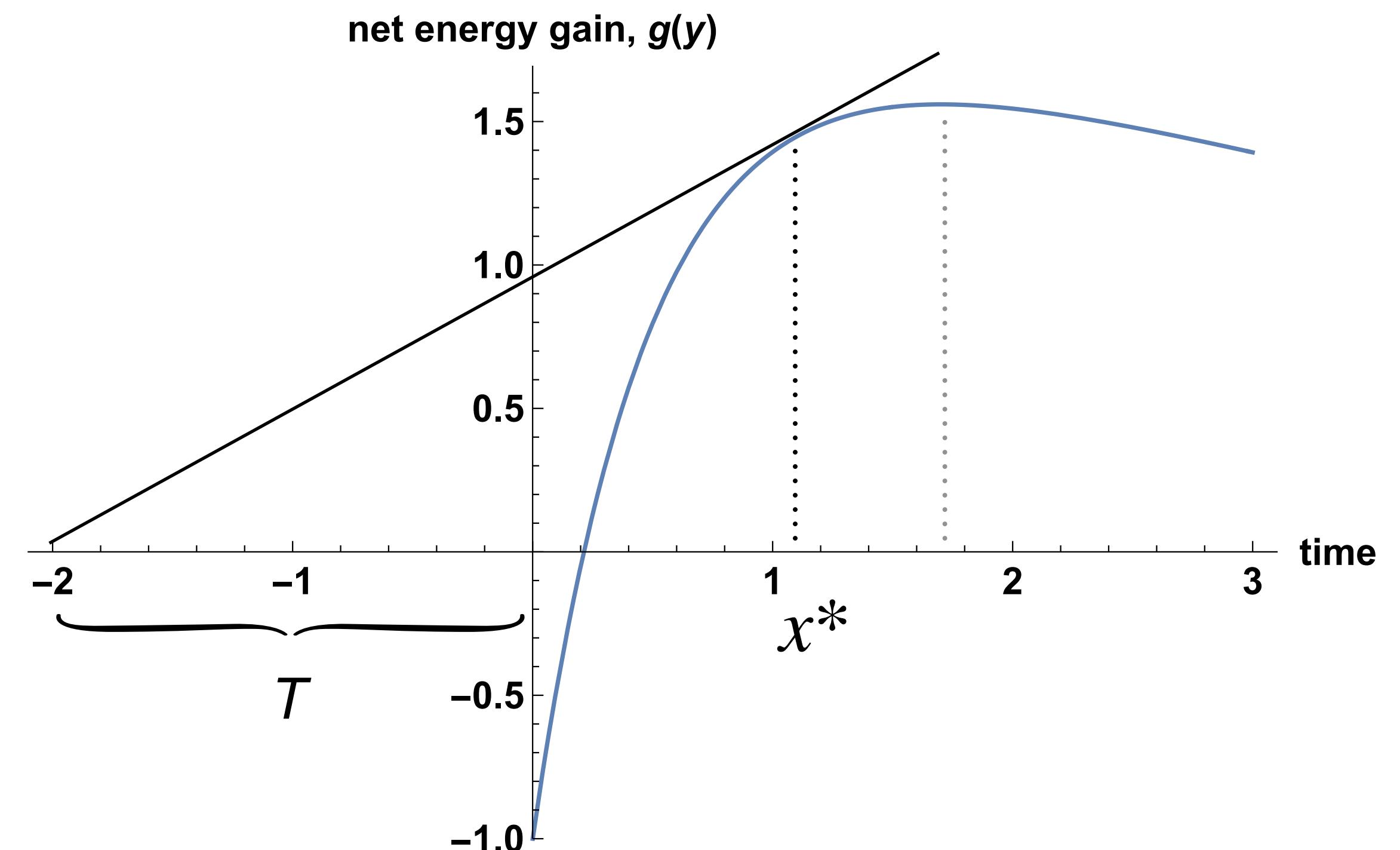
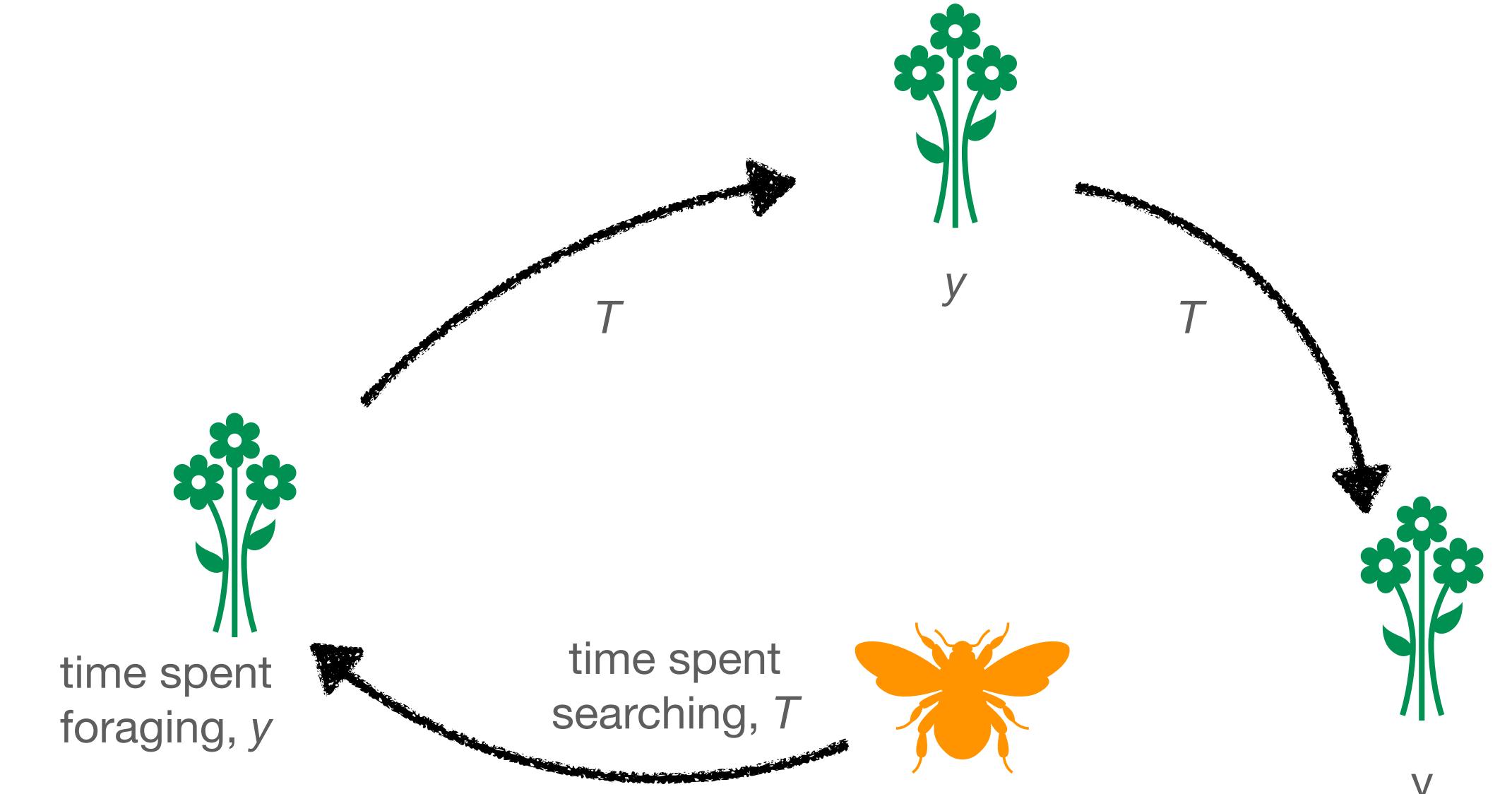
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An animal should leave when the marginal (or instantaneous) rate of energy gain $g'(x^*)$ has fallen to the rate of energy gain $R(x^*)$



When selection favours risky foraging?

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Variation in relationship with uncertainty



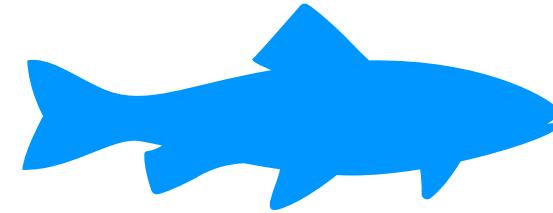
When selection favours risky foraging?

State-dependent payoffs and uncertainty

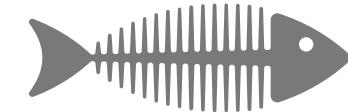
When selection favours risky foraging?

State-dependent payoffs and uncertainty

High condition
e.g., well-fed



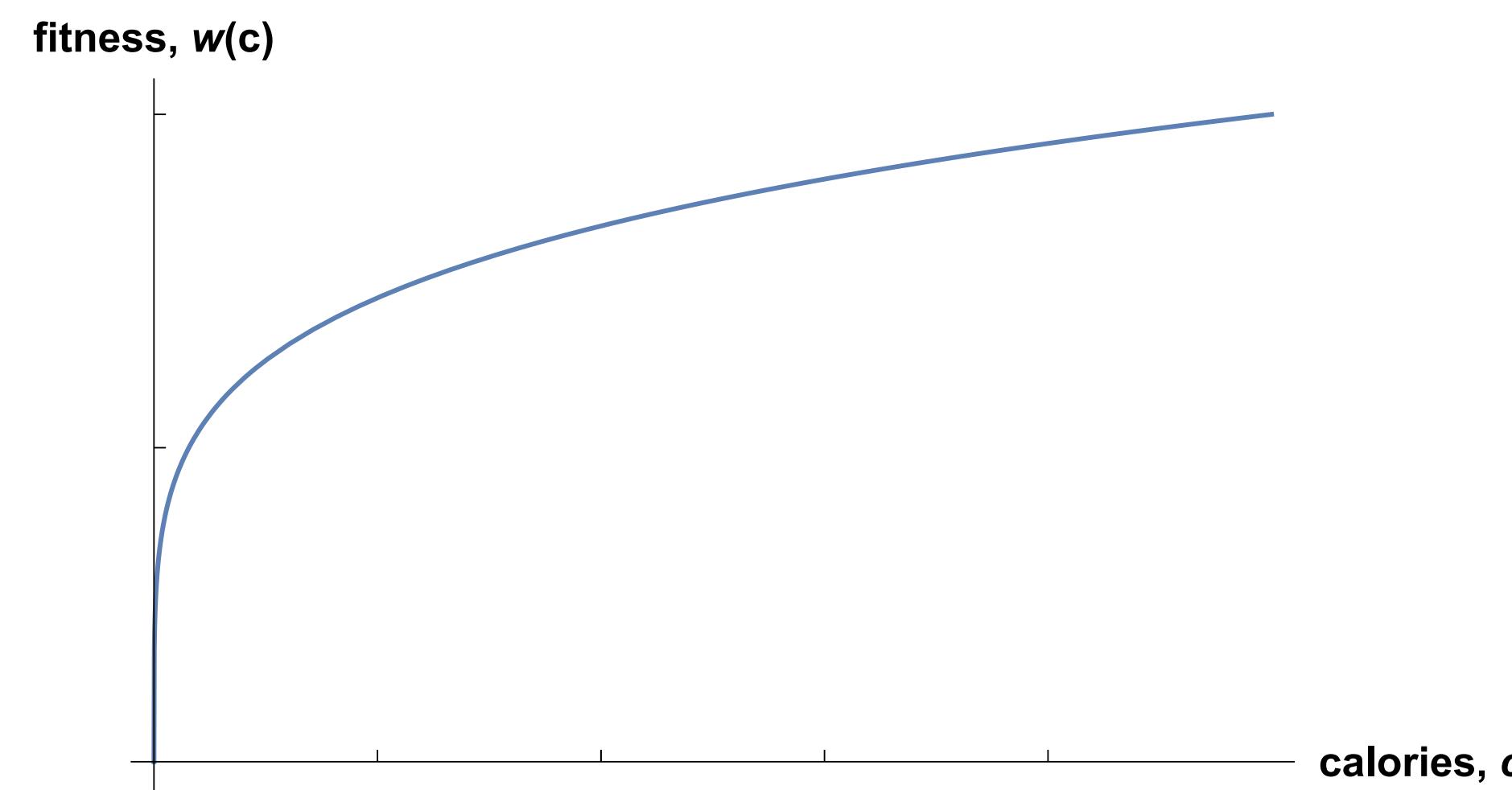
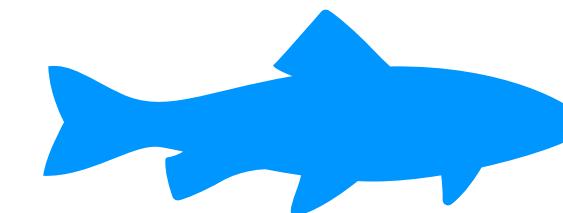
Low condition
e.g., poorly-fed



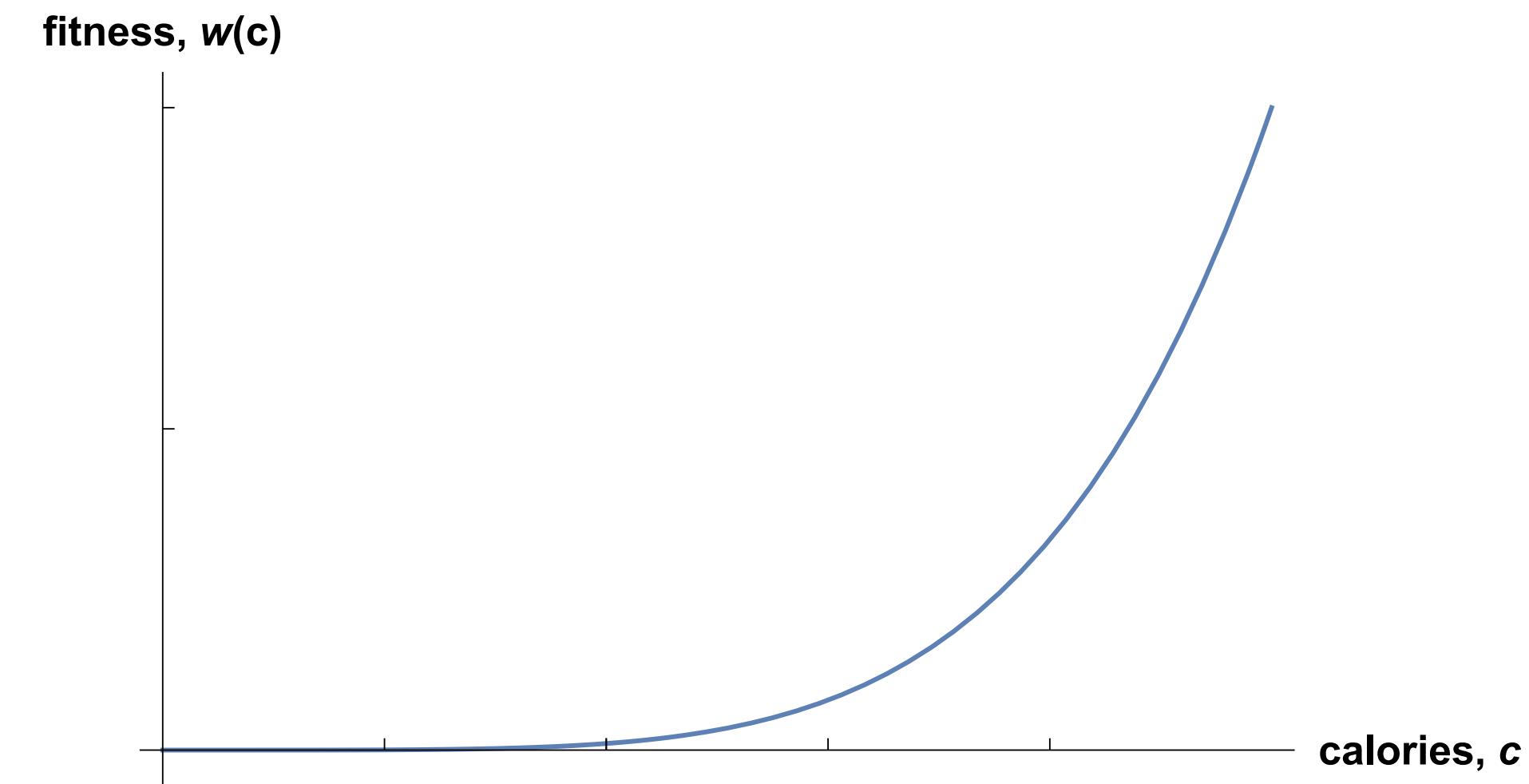
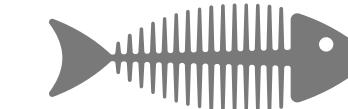
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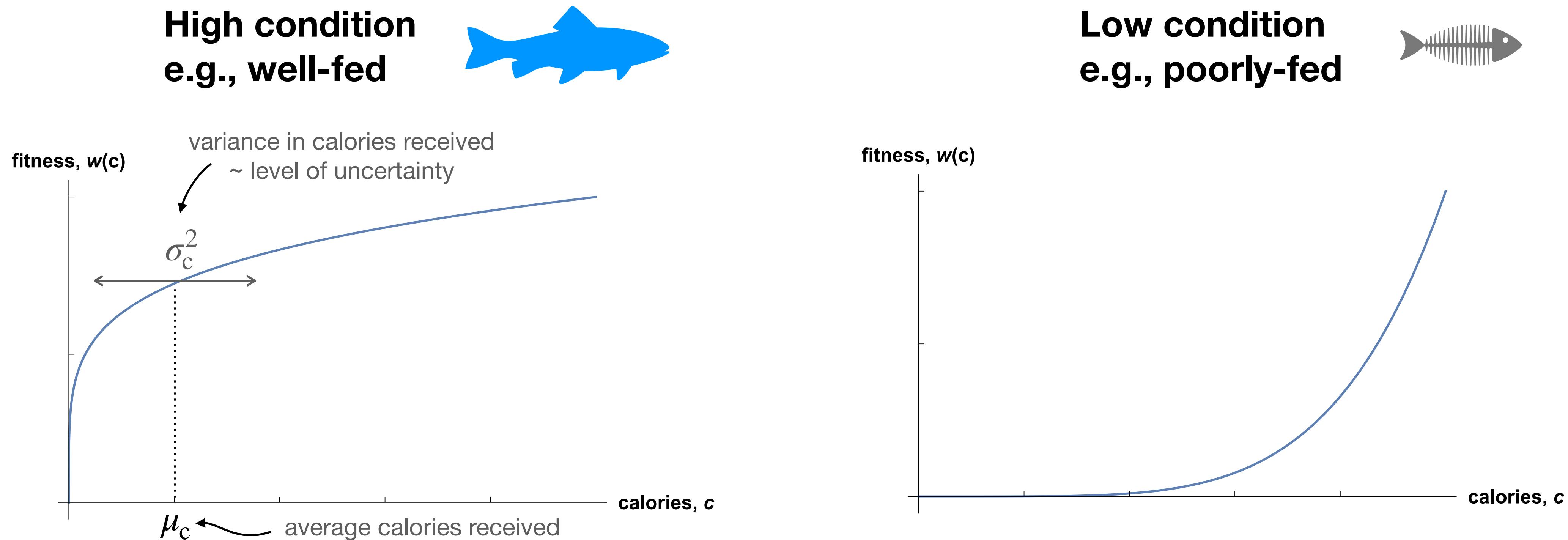


Low condition
e.g., poorly-fed



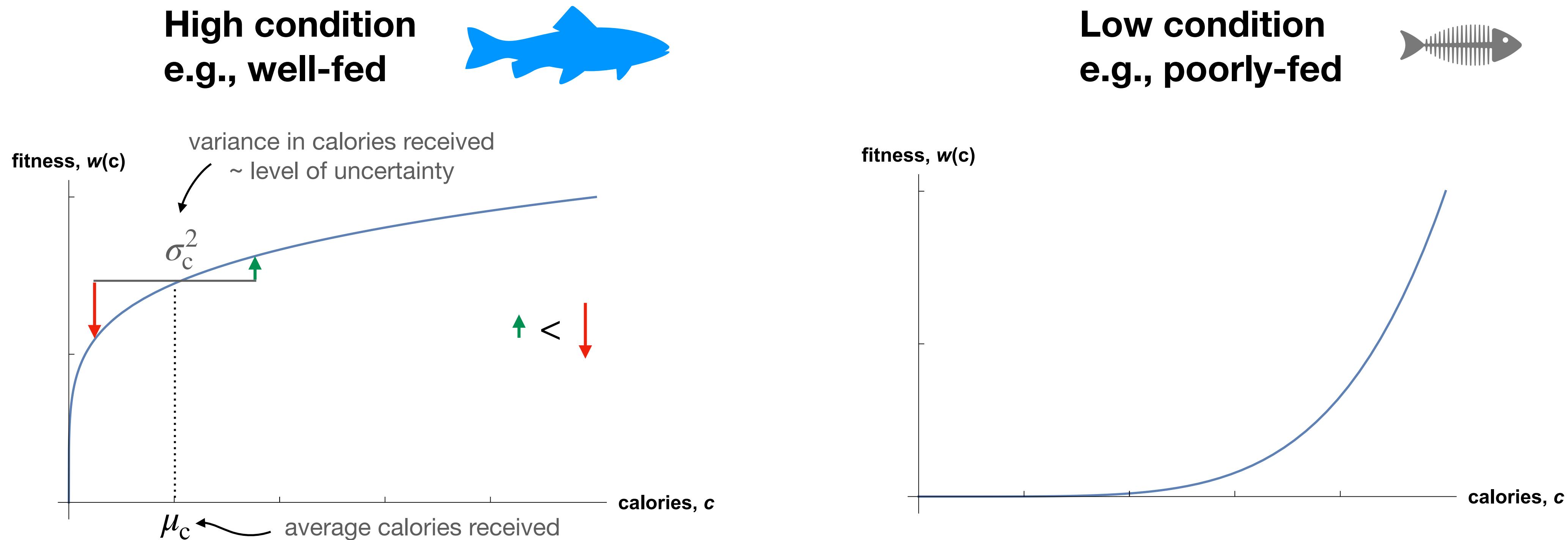
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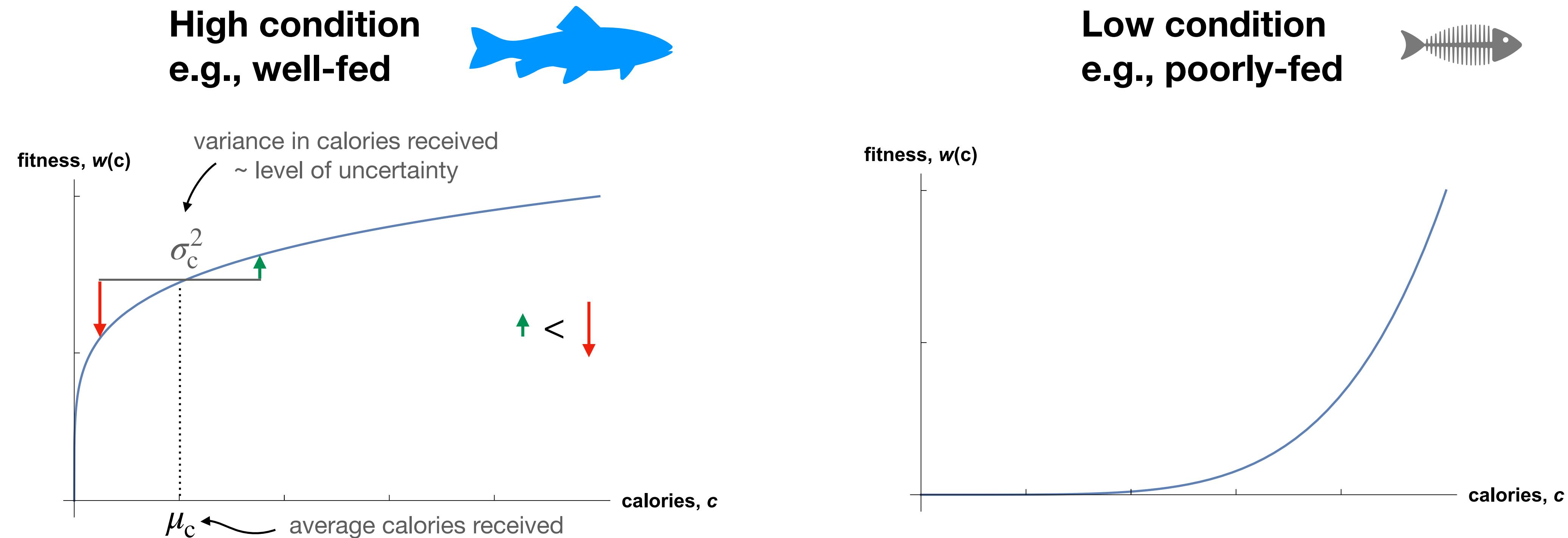
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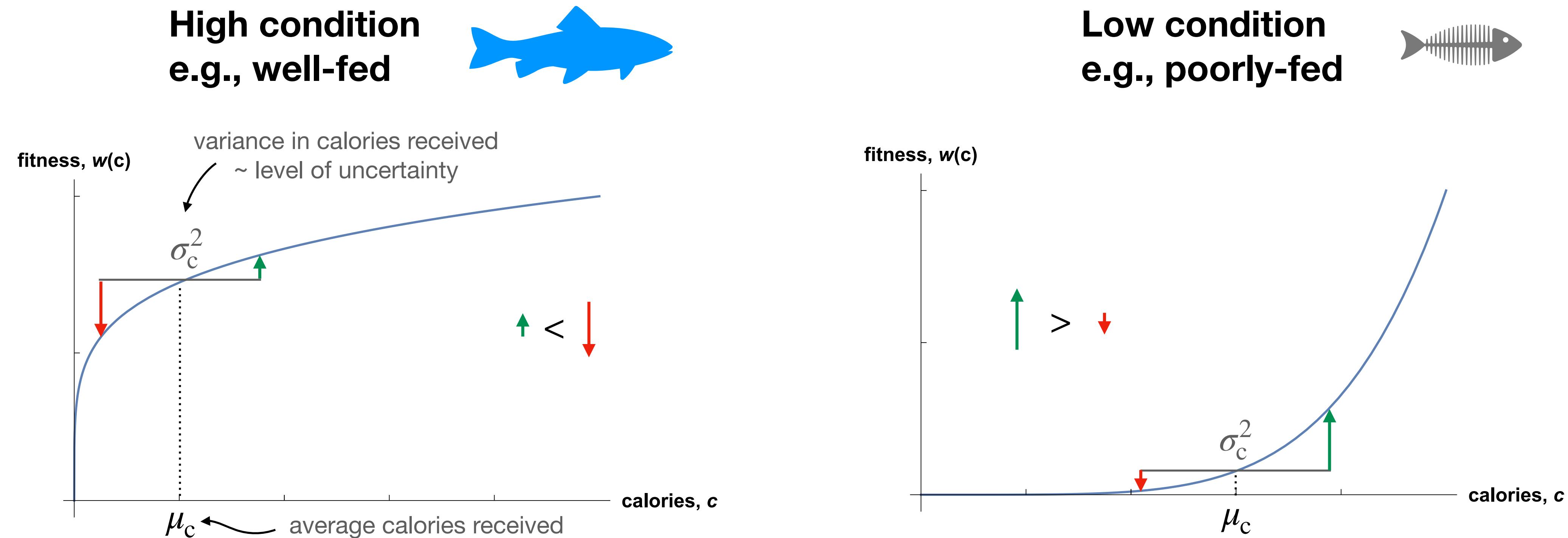
State-dependent payoffs and uncertainty



Risk not worth taking: fitness cost of bad times
outweighs fitness benefits of good times

When selection favours risky foraging?

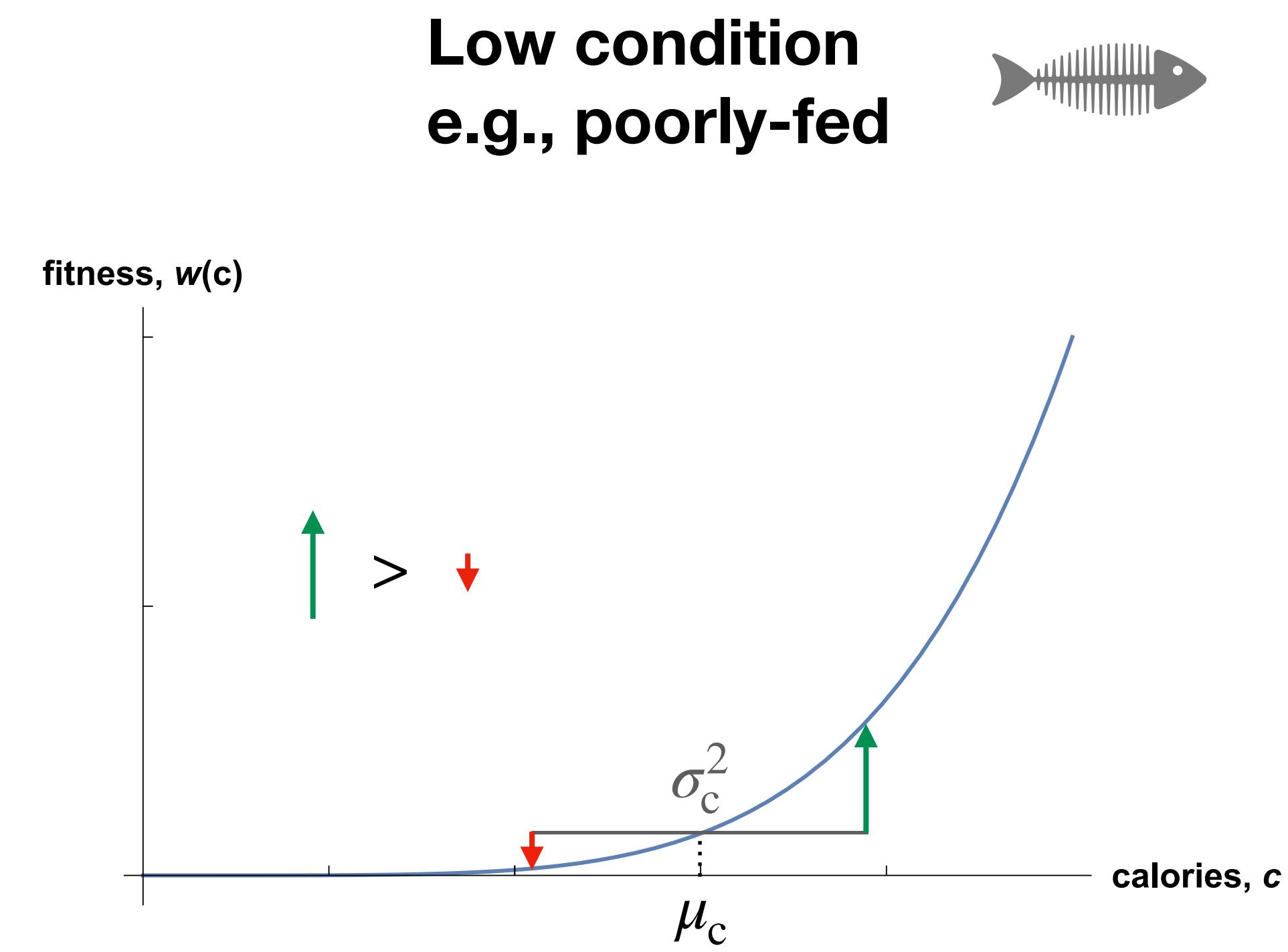
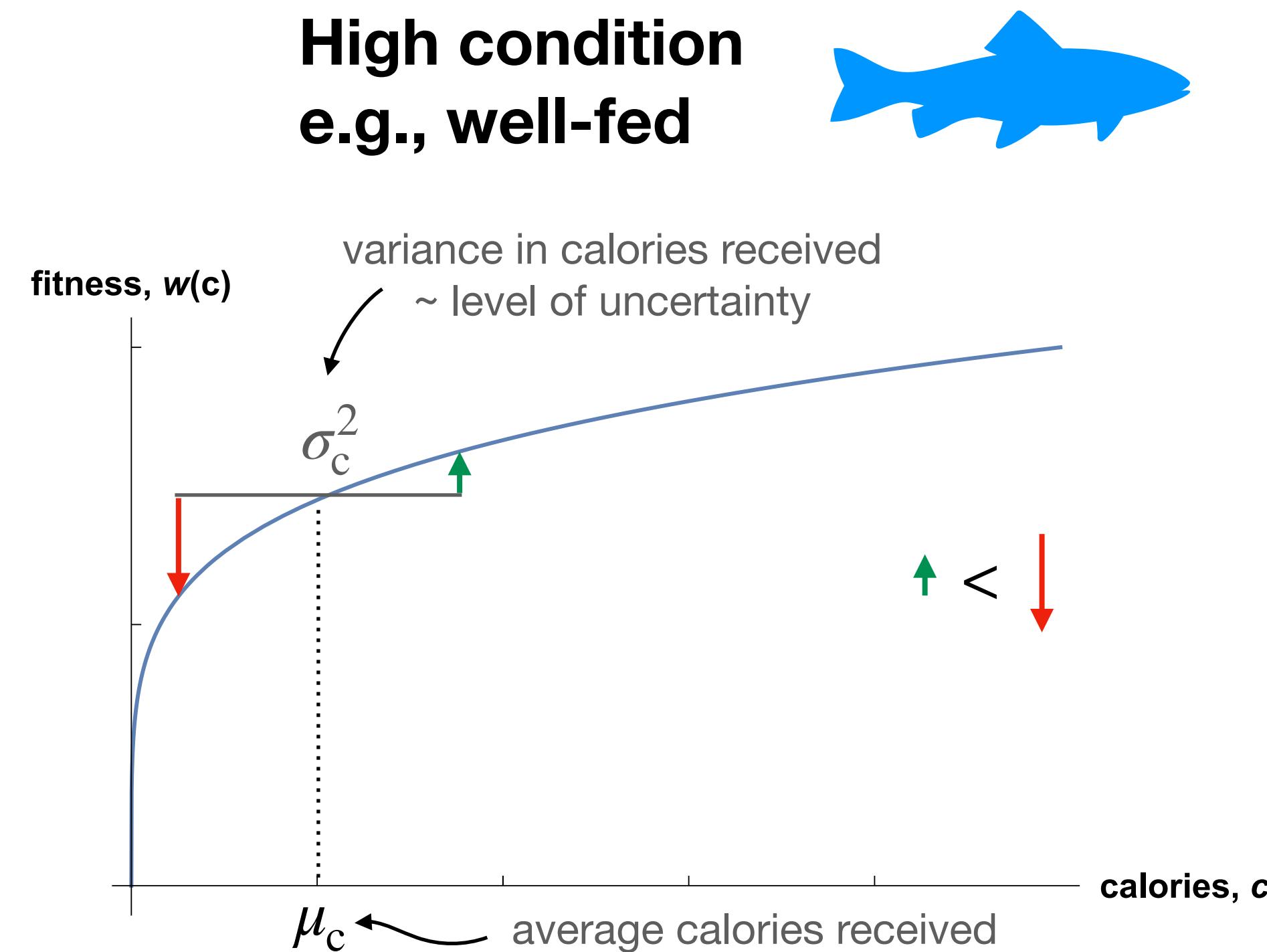
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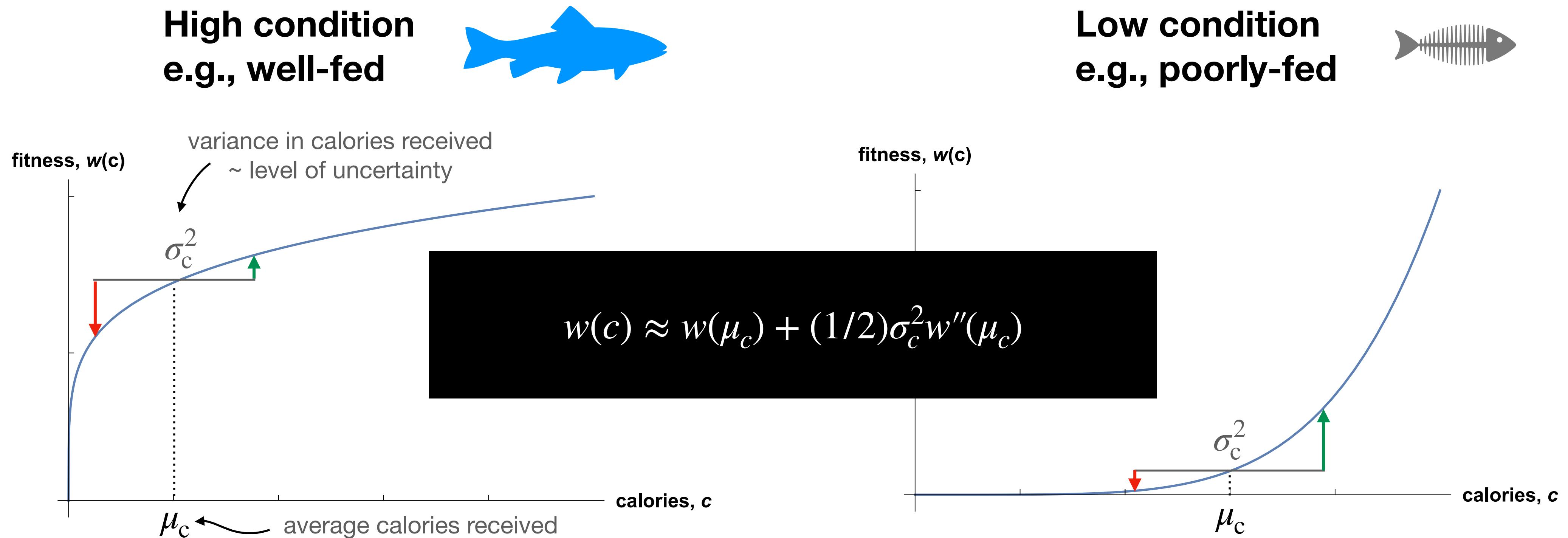


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The exploitation of renewable resources

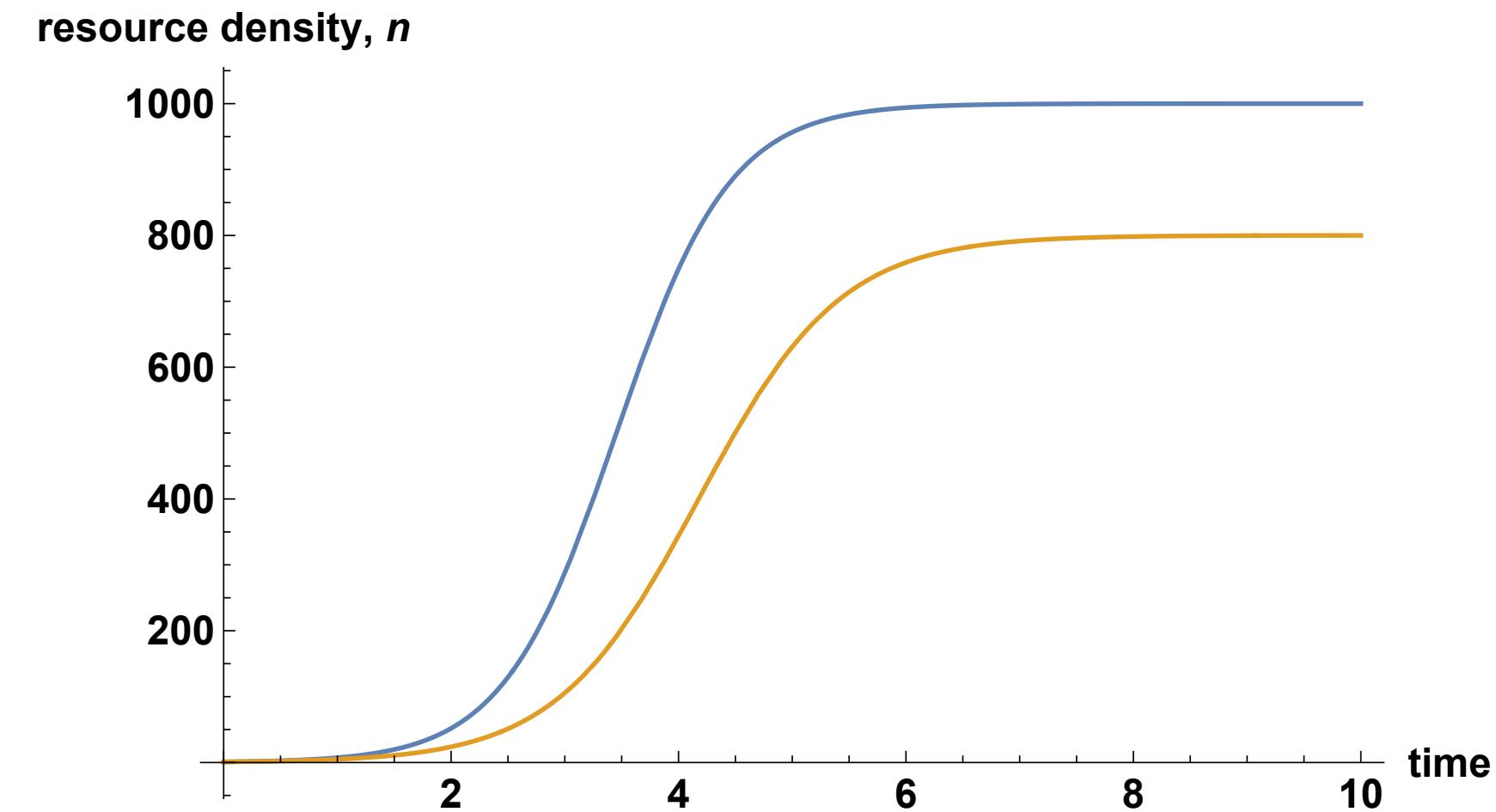
The exploitation of renewable resources

Shaefer's model

- Biotic resource with density n ,

$$\frac{dn}{dt} = \underbrace{r \left(1 - \frac{n}{K} \right)}_{\text{logistic growth}} n - \underbrace{n_c h(x) n}_{\text{harvesting by population of } n_c \text{ consumers with foraging effort } x}$$

foraging function



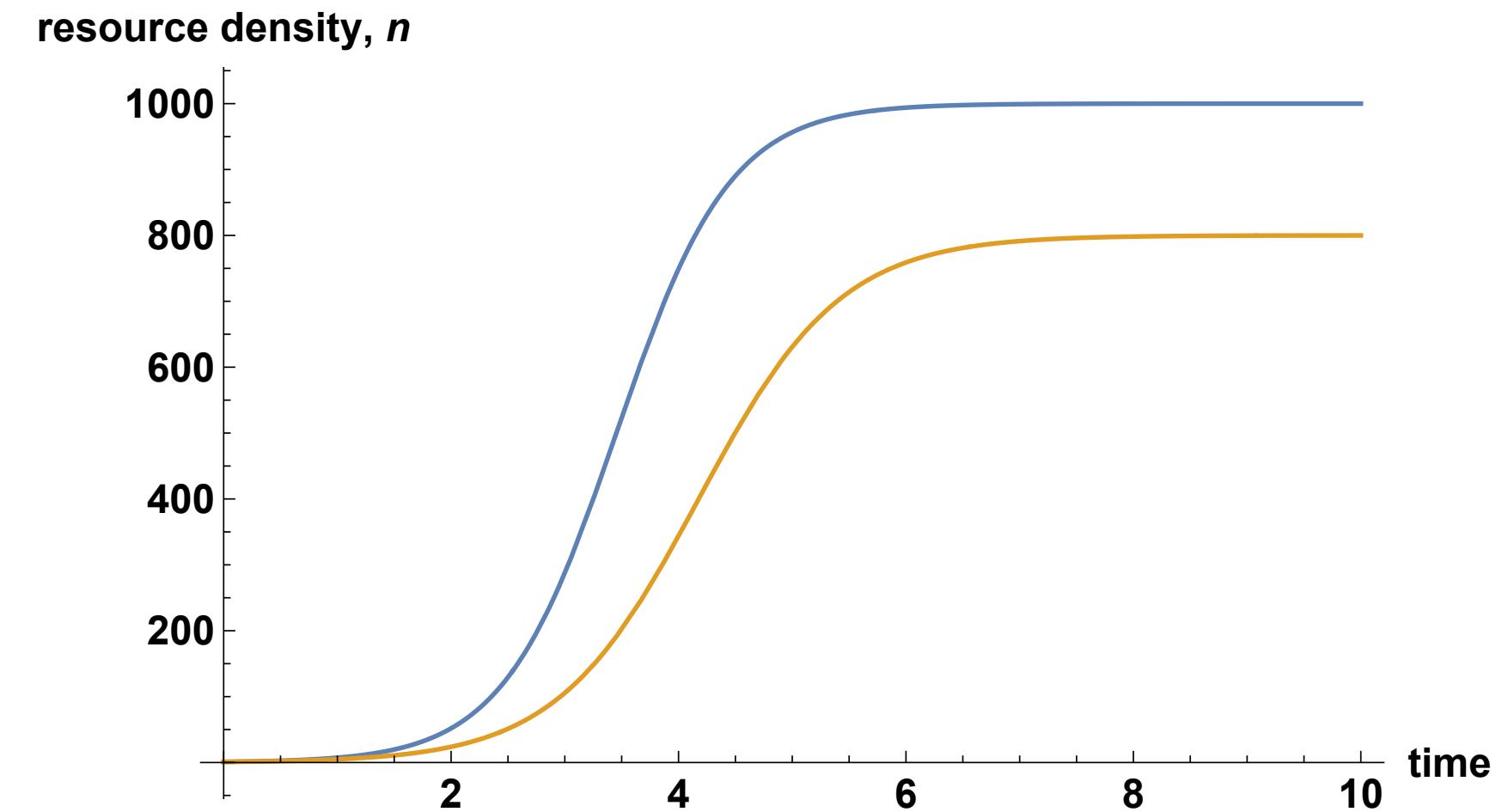
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- Equilibrium resource density $\hat{n}(x)$ such that

$$\hat{n}(x) = K \left(1 - n_c \frac{h(x)}{r}\right)$$

The exploitation of renewable resources

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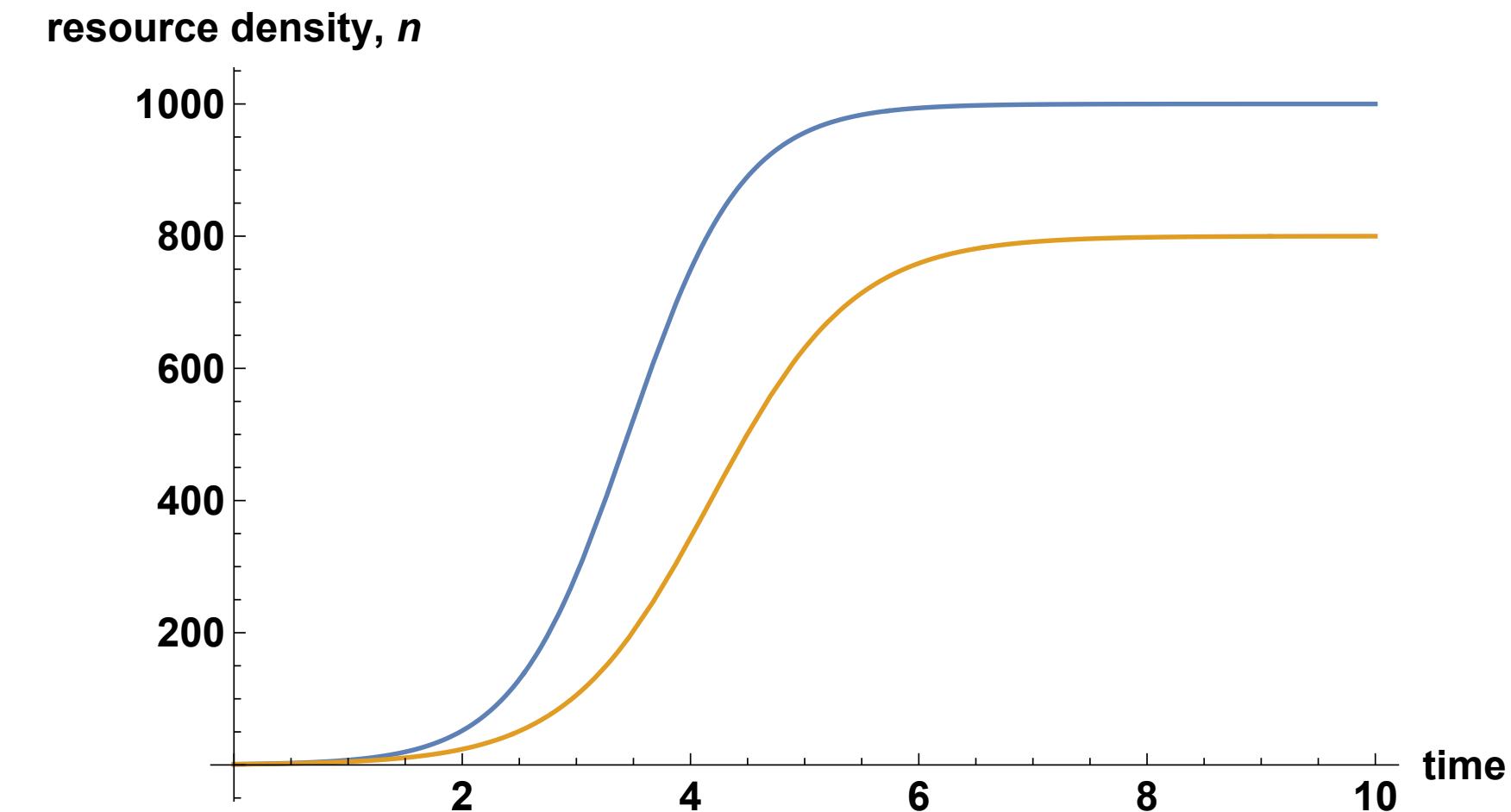
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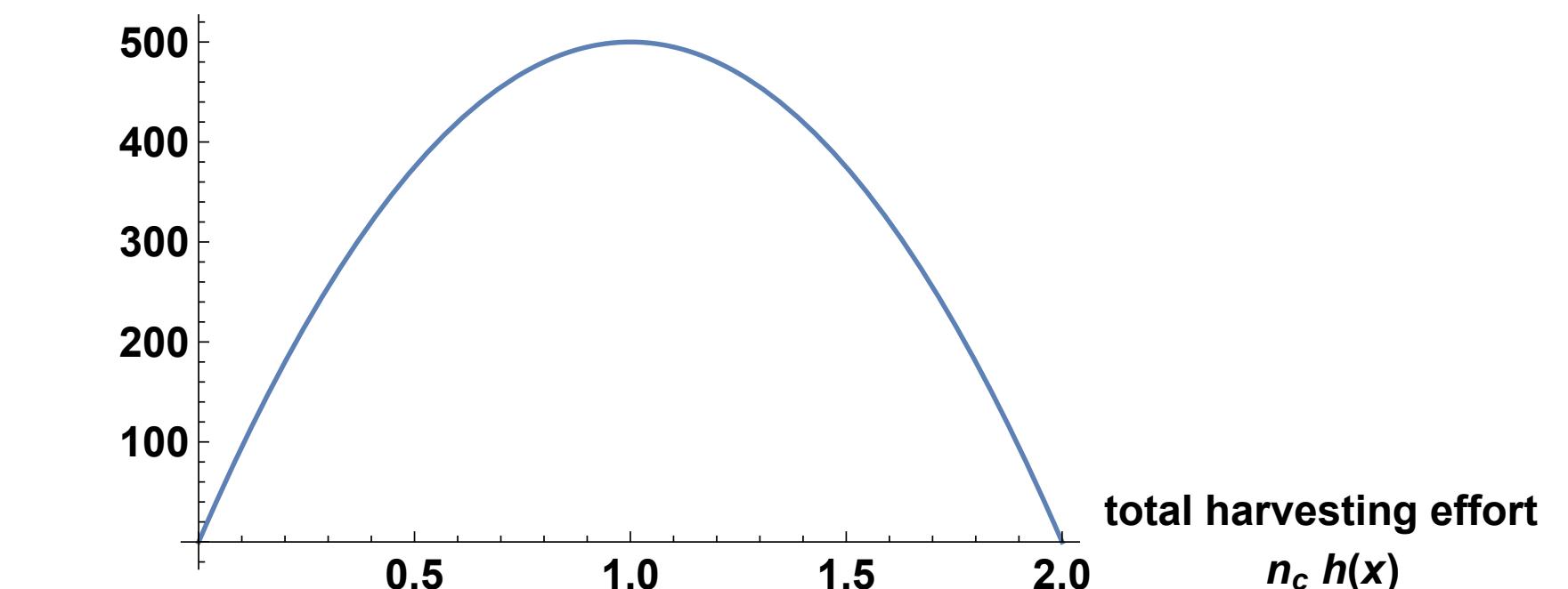
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total yield
 $Y = n_c h(x) \hat{n}(x)$



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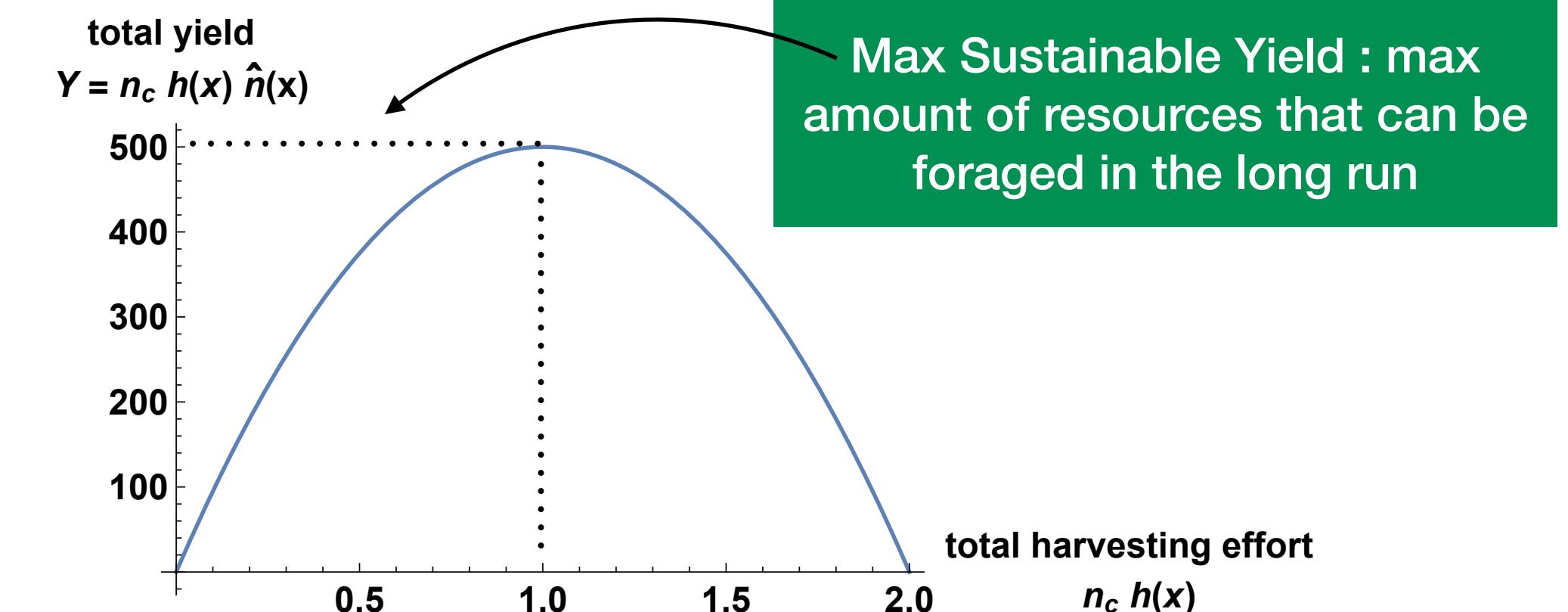
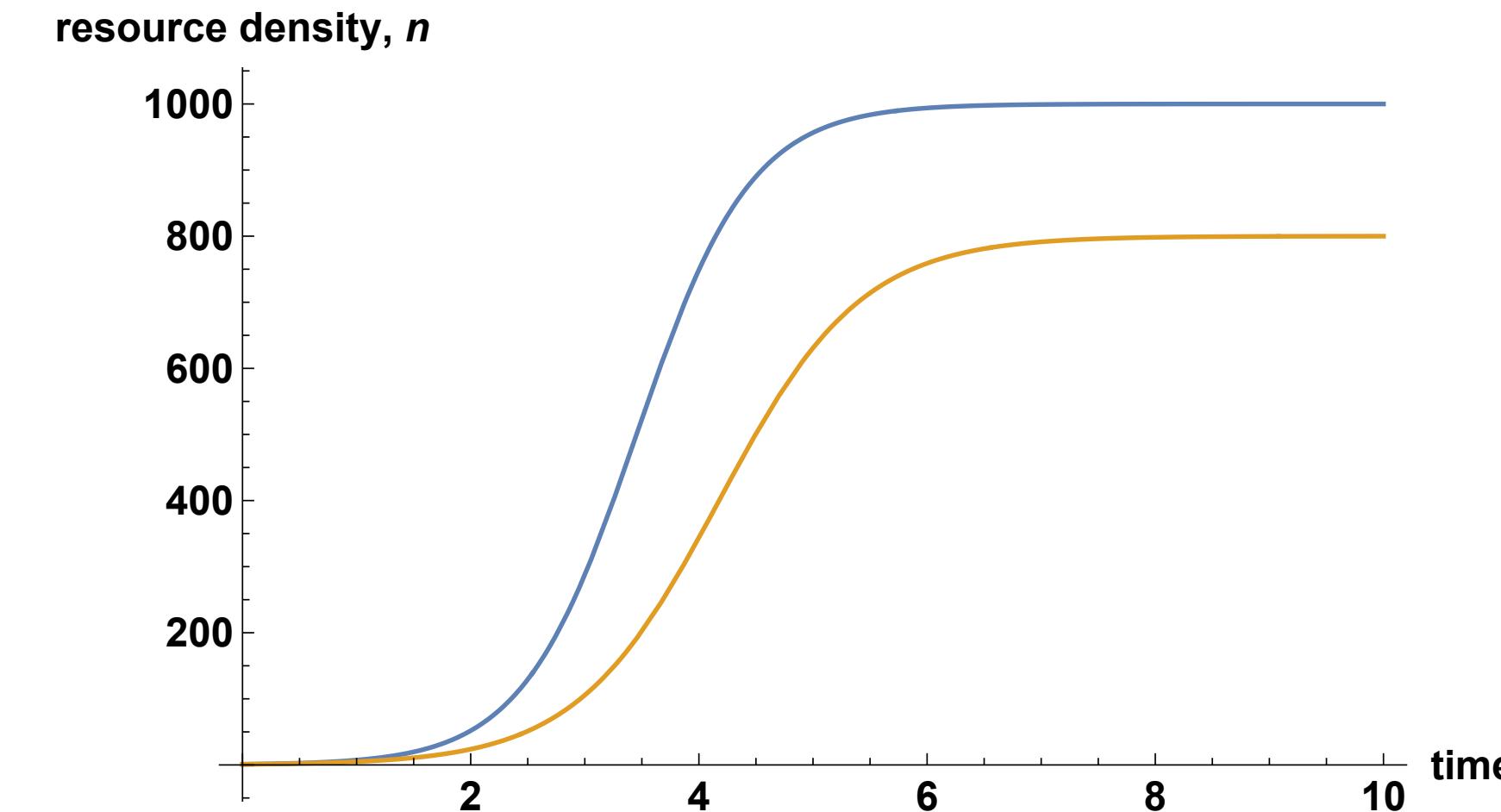
logistic growth

foraging function

harvesting by population
of n_c consumers with
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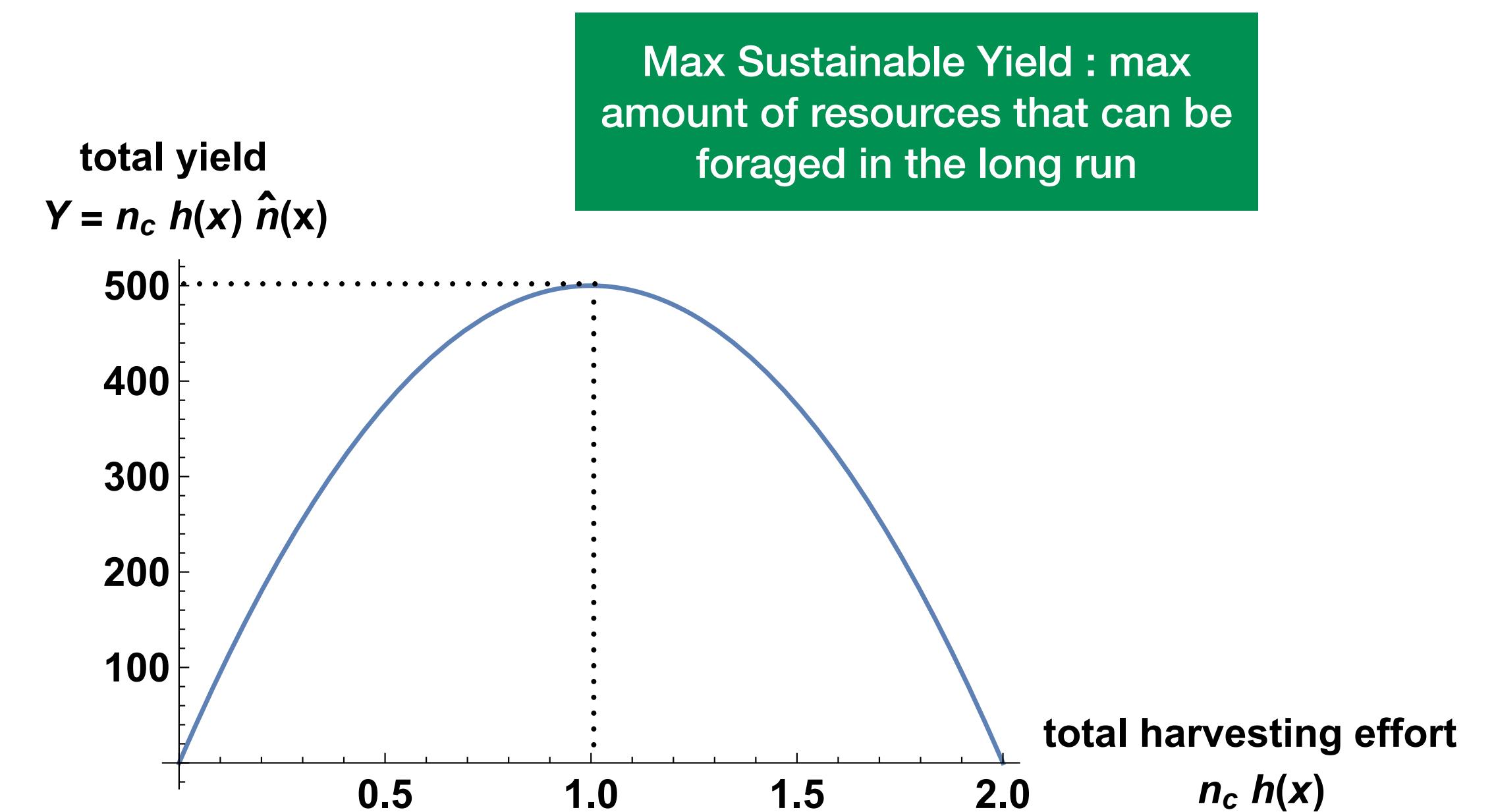
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The exploitation of renewable resources

MSY and over-consumption

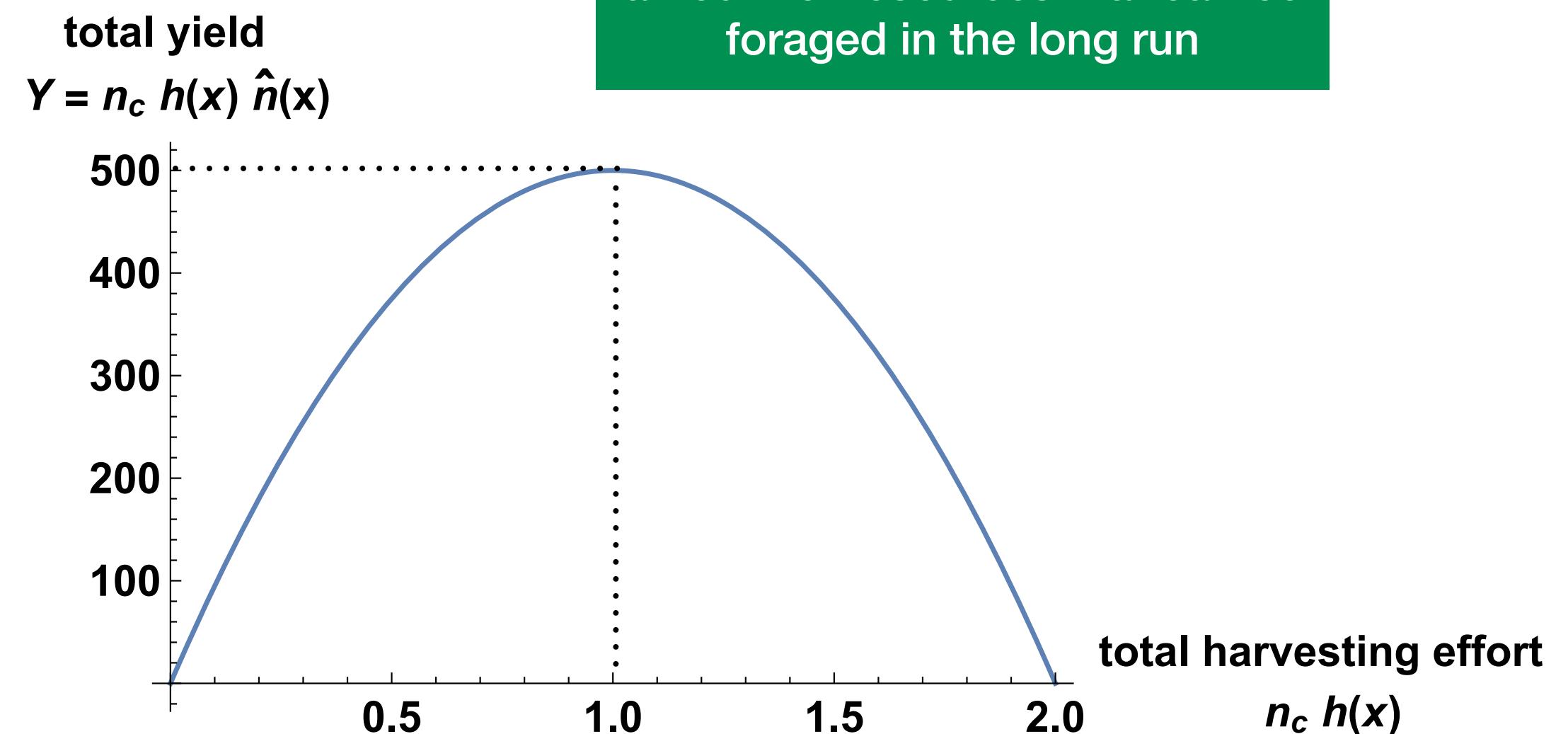


The exploitation of renewable resources

MSY and over-consumption

$$\text{Total yield} = n_c h(x) \times \hat{n}(x) = n_c x \times K \left(1 - n_c \frac{x}{r} \right)$$

$h(x) = x$

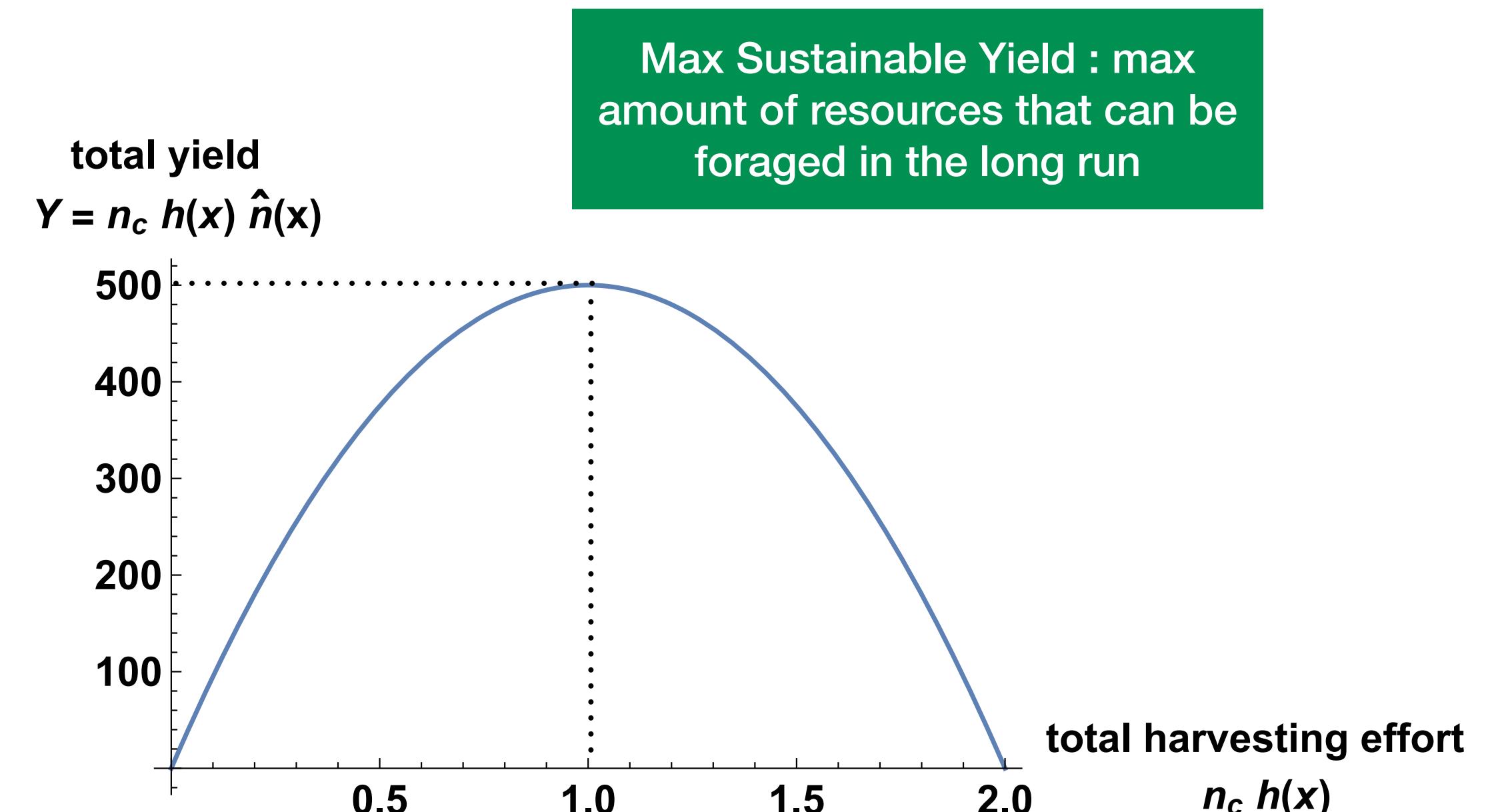


The exploitation of renewable resources

MSY and over-consumption

- Total yield = $n_c h(x) \times \hat{n}(x) = n_c x \times K \left(1 - n_c \frac{x}{r} \right)$
- x_{MSY} : Foraging effort that maximises total yield =

$$x_{\text{MSY}} = \frac{1}{n_c} \frac{r}{2}$$
- MSY = $n_c h(x_{\text{MSY}}) \times \hat{n}(x_{\text{MSY}}) = \frac{Kr}{4}$
- Resource density = $\hat{n}(x_{\text{MSY}}) = \frac{K}{2}$



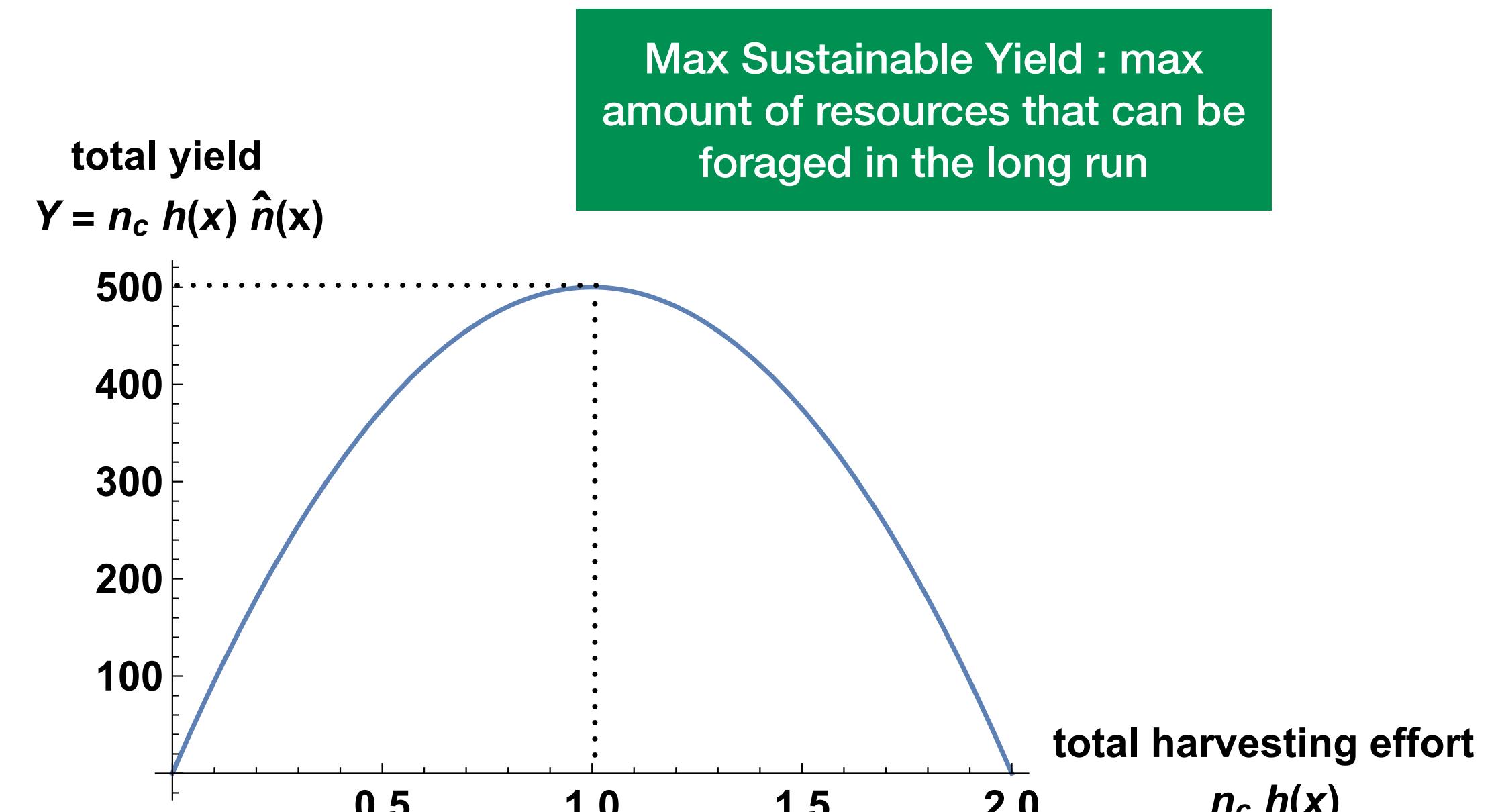
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- Any effort above x_{MSY} amounts to over-exploitation.



The exploitation of renewable resources

How evolution shapes foraging of biotic resources

The exploitation of renewable resources

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- Well-mixed population where individuals all exploit the same resource and compete with one another.

The exploitation of renewable resources

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- Fitness of a mutant with foraging effort y in a resident population x ,

individual yield - individual cost of effort

$$w(y, x) \propto y\hat{n}(x) - c(y)$$

The exploitation of renewable resources

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$$\bullet \quad x^* = x_{\text{MSY}} \frac{2Kn_c}{Kn_c + c_0 r}$$

$$h(x) = x$$
$$c(x) = \frac{c_0}{2}x^2$$

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When cost is large, $c_0 \geq \frac{Kn_c}{r}$ then $x^* \leq x_{\text{MSY}}$.
Otherwise, $x^* > x_{\text{MSY}}$.
When $c_0 = 0$, evolution leads to resource extinction.

The exploitation of renewable resources

How evolution shapes foraging of biotic resources

- Well-mixed population where individuals all exploit the same resource and compete with one another.
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individual yield - individual cost of effort

Due to competition, evolution typically leads to over-exploitation and lower yield than if individuals were coordinated.

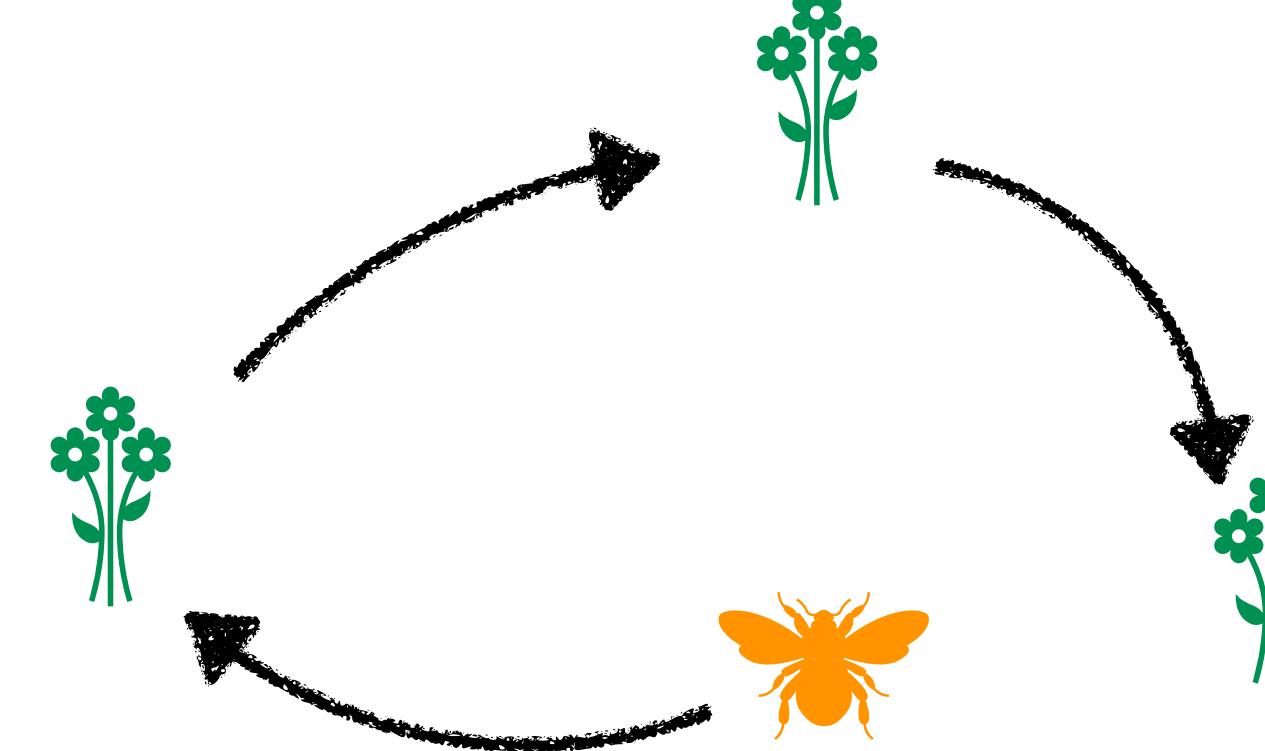
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Summary



- Marginal value theorem allows to understand when an organism should leave for new pastures: leave when the *marginal* rate of energy gain has fallen to the total rate of gain.
- Risky foraging behaviours can be explained from state dependent payoffs where the fitness of low condition individuals accelerates with energy.
- For biotic resources, there may exist a foraging effort such that yield is maximised and resources are maintained. Due to competition, however, natural selection tends to favour over-consumption.

